

The design and operation of X-Rotor type wind turbines

PhD Thesis

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Abstract

In order to successfully limit anthropogenic global heating, the rate of deployment for offshore wind energy needs to increase significantly over the coming decades. From an engineering perspective, this can be facilitated by a reduction in the cost of energy, and the avoidance of structural barriers to the installation of offshore wind energy.

The X-Rotor is a vertical axis wind turbine concept that uses secondary rotors for power take-off. Studies have shown that this turbine concept has the potential to significantly decrease the cost of energy for offshore wind. The use of secondary rotors also reduces the required mass of rare earth metals, reducing the reliance on a geographically concentrated supply chain, and the use of a vertical axis primary rotor can also facilitate an increase in wind farm density, easing the constraints on site availability.

This thesis investigates the design and operation of X-Rotor type turbines, understanding the dynamics of power take-off with secondary rotors, the design of both primary and secondary rotors, and the operation of X-Rotor type turbines.

The aerodynamic aspects of power take-off with secondary rotors are derived, treating the secondary rotor as both an actuator disk and an optimally designed rotor. It is found that conversion efficiencies between the primary and secondary rotors of between 80%-85% are achievable, relying on low induction secondary rotors and high primary rotor tip speeds.

The design of high tip speed ratio VAWTs is then investigated, and it is shown that the use of straight bladed VAWTs with blades incline to the vertical plane can increase both the optimum tip speed ratio and the maximum power available to the primary rotor. The design of an X-Rotor type turbine is then completed coupling the optimum

secondary rotor design process with a large sample of primary rotor designs. It is shown that an inclination angle is necessary to achieve a 5MW 2 bladed design with an unsupported blade length less than 100m, and that a minimum blade of 78m is required, using a blade chord length of 5m and an inclination angle of 30 degrees. It is found that the use of inclined blades reduces the maximum operational thrust, operational out of blade blade root bending moment, and the extreme blade root bending moment for rotors with equivalent power ratings. Rotor designs which minimise the combined blade mass of the upper and lower blades, whilst achieving a power rating of 5MW at 12m/s are identified to have chord lengths between 4m-6m, blade lengths of 78m-80m, and inclination angles between 30° and 40°.

Finally, the control of X-Rotor type wind turbines is investigated. The maximum power coefficient of the 5MW 2 bladed X-Rotor design, investigated as part of the Horizon 2020 X-Rotor project, including the conversion of power between the primary and secondary rotors is calculated to be 0.41. Following this, a recommended operational strategy for the 5MW 2 bladed X-Rotor design is presented which ensures high conversion efficiency at low wind speeds, limits power capture in above rated wind speeds, and conforms to strict limits on secondary rotor speed.

Chapter 1

Introduction

1.1 Motivations

Thorough documentation on the science of climate change and its global impacts has been produced by the international panel on climate change (IPPC) in [8]. The report states both that *"Human activities, principally through emissions of greenhouse gases, have unequivocally caused global warming"* and that *"This has led to widespread adverse impacts and related losses and damages to nature and people"*, underlining the need to urgently mitigate anthropogenic global heating. In this context, the limitation of global temperature increases to well below 2° above pre-industrial levels, with an ambition to limit the increase to 1.5°, was committed to in the Paris agreement, a legally binding international treaty adopted in December 2015 [9].

Key to preserving a 1.5° future is the replacement of fossil fuel use with renewable energy through the electrification of fossil fuel dependent processes and renewable generation of electricity [10]. Offshore wind energy already represents a significant portion of renewable energy generation in Europe and China, and will play an expanding roll in the energy systems of the near future [10]. Offshore wind energy is often favoured compared to onshore wind energy due to the better quality wind resource, with generally higher wind speeds and less turbulent flows, and a larger available area for development, mitigating some complex land use issues and seeing generally more positive societal acceptance.

Chapter 1. Introduction

In order to meet global climate goals, it is estimated that the global installed capacity of offshore wind energy must meet almost 500GW by 2030, and 2500GW by 2050 [10]. The global energy transition is not currently on track to meet the requirements for a 1.5° future, and global installed capacity stands at 64.3GW at the end of 2022 [11]. In order to meet the 1.5° target, the installation rate must increase from an average of 3GW/yr in the previous decade, to 54GW/yr over the next ten years [10]. Whilst a ramp up in deployment is underway, with the average deployment rate over the last 3 years (2020-2022) reaching 12GW/yr [11], a 4.5 fold increase in the deployment rate is required. From an engineering perspective, the required increase in the global rate of deployment of offshore wind capacity can be encouraged through the development of wind turbine technologies that reduce the cost of energy and circumvent potential bottlenecks in wind energy deployment.

Wind turbines have converged on a 3 bladed horizontal axis, variable speed, variable pitch design. The 3 bladed horizontal axis wind turbine (HAWT) design emerged from the 1980's as the preferred solution for sub MW onshore wind turbines. In the preceding period, a number of wind energy conversion systems had been developed in the wake of the 1979 oil shock, including a significant number of vertical axis wind turbines (VAWTs). However, the horizontal axis machines had a higher aerodynamic efficiency, more favourable torque speed characteristics, and a simpler loading regime. Whilst many techno-economic factors contribute to the success of a technology, it is generally considered that these characteristics led to more cost effective designs and eventual commercial dominance of the HAWT. From this period onwards, onshore wind turbines have significantly increased in size, facilitated by innovations in every aspect of the design and control of the machine; however, the basic topology has remained consistent.

Moving into the offshore environment has increased the importance of balance of plant and operations and maintenance (O&M) costs in terms of the levelised cost of energy (LCOE), and has removed some issues associated with the transport of large components on-shore. This has incentivised significant further upscaling of individual wind turbines and has led to the power density of individual wind turbines decreasing,

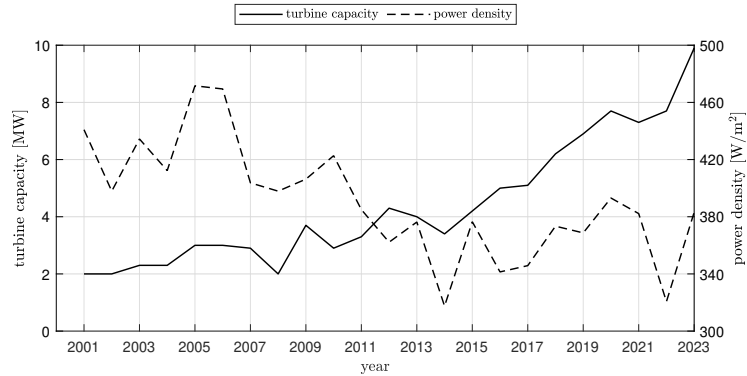


Figure 1.1: Trends in offshore wind turbine capacity and power density, data from [1].

increasing the capacity factor of offshore wind farms and decreasing the LCOE. These trends are demonstrated in figure 1.1. Other design trends encouraged from the transition offshore include an increase in the capacity of individual wind farms, and the transition towards direct drive drive-trains, employing permanent magnet synchronous generators, due to their increased efficiency and reliability [12].

Another key trend in the development of offshore wind turbines is the transition to floating substructures. Floating offshore wind turbines unlock the significant wind energy potential in waters deeper than 50m-60m [13], which are typically considered un-economical for fixed bottom offshore wind turbines. The cost of energy for floating offshore wind is currently significantly higher than for fixed bottom projects and technological development is required to lower the cost of energy and increase commercial competitiveness [14].

Over the course of the last 40 years of wind turbine development, the effectiveness of the horizontal axis design has been evidenced both by the significant reduction in the LCOE, especially of the last 15 years of offshore wind development, and by the low absolute value of the LCOE for both onshore and offshore wind compared to other forms of power generation. Despite this, a number of challenges are associated with this traditional turbine concept which have been exacerbated by the shifting industry trends outlined above:

Upscaling

Whilst O&M and balance of plant considerations have made turbine upscaling a more economic option for turbines in the context of offshore wind farms, the trends in cost associated with upscaling individual turbines have long been considered negative [15]. This can be most readily understood by the 'square-cube' law. Essentially, the power available to a wind turbine scales with the swept area of the rotor, the square of a common length scale, whilst the costs associated with the rotor scale with the mass/volume, which increases with the cube of the common length scale [16]. Whilst the additional power available higher in the boundary layer has led to an increased exponent associated with the power, and innovations in engineering technologies have led to a decreased exponent associated with the turbine cost, the fundamentals of similarity scaling still provide a downward pressure on the maximum turbine size. Under constrained rotor tip speeds, this square-cubed law also applied to the rotor drive-train, with gearbox and generator masses typically associated with the input torque, and the input torque increasing with the cube of the common length scale.

In addition to the 'square cube' law, a consideration that provides further downward pressure on the maximum size of individual wind turbines is the self weight loads associated with gravity. These loads impart a once per revolution reversing in-plane bending moment on rotor blades. Gravitational moments scale with the 4th power of the common length scale and will therefore become design drivers as turbines continue to increase in size. Once gravitational loads become design driving, the amount of material required to limit the bending stresses in the blade must exceed the, already unfavourable, cubic scaling discussed above.

Decreasing power density

Decreasing the power density of wind turbine rotors, in order to increase their capacity factors, necessitates an increase in the rotor diameter which relates to an increase in the blade cost. An additional cost increase is introduced by the increase in rotor torque incurred by increasing the rotor radius under a constrained tip speed which increases the weight and cost of the drive-train components.

Large wind farms

For large offshore wind farms, wake losses limit the capacity density for offshore wind farms and necessitate larger farms taking up larger portions of the sea bed and incurring higher costs.

Direct drive drive-trains

The transition towards direct drive PMSG based drive-trains, in conjunction with the trend towards larger power ratings and lower power density, has led to very large, very heavy generators that require a significant amount of rare earth elements. These generators represent a significant portion of the wind turbine cost [17], and issues around the supply chain for rare earth elements have been highlighted as a potential bottleneck for wind turbine supply [18].

Floating substructures

Traditional offshore HAWTs have drive-trains situated at the rotor hub height, well over 100m above the ocean surface. This leads to a high centre of mass, increasing costs for floating substructures. As well as the high centre of mass, HAWTs also have a high centre of thrust, which further increases the costs associated with floating platform development.

In light of these issues facing the deployment of traditional HAWTs offshore, a number of novel wind turbine concepts have been introduced, aimed at further decreasing the cost of energy for offshore wind.

Multi-rotors, wind turbines that incorporate more than one rotor nacelle assembly on a single support structure, have recently attracted considerable academic and industrial attention for use in offshore applications. Through using multiple single rotors the 'square-cube' law can be circumvented for the rotor nacelle assembly as an increase in area is achieved through using multiple units, leading to costs scaling directly with power capacity, i.e. with the square of the common length scale [16]. The mass scaling

associated with support structures do not undergo a similar, favourable, shift. A thorough discussion is outside of the scope of this brief introduction and interested readers are pointed to [16]. Significant academic and industrial interest in the concept has been demonstrated by the recent multi-rotor symposiums hosted at HAW Hamburg in 2023 and the University of Strathclyde in 2024, and commercial projects, such as Wind Catching Systems' 126MW concept [19]. Whilst multi-rotor turbines are a promising development, uncertainties around the O&M implications of including a large number of smaller rotor nacelle assemblies, and the design challenges around the scale of the turbine support structure and substructure have thus far limited commercial actualisation.

VAWTs, turbines that rotate about the vertical - rather than horizontal - axis, were developed for commercial application in the 1980s with over 100MW of capacity installed by Flowind Inc. [20]. However, both commercial and academic interest in the technologies waned as HAWTs became the dominant technology [21]. A thorough review of the history of VAWT development can be found in [22]. In the context of offshore wind farms, VAWTs have recently been reconsidered due to a number of possible design synergies:

Firstly, VAWTs do not experience cyclic gravitational loads, thus the potential techno-economic ceiling for single unit capacity is potentially larger than that of a HAWT. The wakes of VAWTs have also been shown to re-energise at a faster rate than HAWT wakes, and effective wake control with VAWTs has been shown to be achievable through introducing a fixed pitch offset [23]. This allows for higher density wind farms, and a reduction in the cost of energy. VAWTs also, typically, have the heavy drive-train equipment located close to sea level; this both lowers the centre of gravity decreasing the costs of floating platforms, and has potential benefits for maintenance operations. The potential of VAWTs to decrease the cost of energy for offshore wind has led to significant academic interest, with large scale projects such as DeepWind [24] and Nova [25], and ongoing commercial ventures such as SeaTwirl [19].

Whilst there is significant academic and commercial interest in the use of VAWTs offshore, there are a number of drawbacks to the technology. Firstly, VAWT blades

experience cyclic aerodynamic loads as they rotate, which can lead both to lower aerodynamic efficiencies and more costly rotors. These loads also propagate to the support structure which can further increase the mass and cost. The cyclic loads lead to a cyclical variation in the torque input to the drive-train and, when combined with the high torque output from VAWT rotors, significantly increase the drive-train costs. Experience from the NOVA project indicated that this increase in drive-train costs was a key prohibitive factor in the economically feasible deployment of multi-MW VAWTs. Whilst it has been shown through simulation work that VAWTs at the multi-megawatt scale can reach similar efficiencies of modern HAWTs [26], the increased costs due to the more complex load regime have so far limited commercial offshore VAWT deployment.

This thesis aims to investigate the design and control of a third type of novel wind turbine concept, the X-Rotor. This turbine concept aims to directly address issues around upscaling, low power density, wake losses, and drive-train cost. This thesis will only consider the turbine with reference to a fixed bottom sub-structure; however, there is no identifiable reason why the X-Rotor will not preserve the design synergies between conventional VAWTs and floating substructures. The following section will introduce the X-Rotor concept and provide an overview of the published academic work on the turbine.

1.2 X-Rotor

In order to preserve the benefits of VAWTs in the offshore environment whilst overcoming some of the drawbacks inherent in VAWT design, the X-Rotor wind turbine concept was developed at the University of Strathclyde in 2010/2011 [27], and first presented in academic literature in [20]. It consists of an X-shaped primary rotor, which is allowed to rotate freely, without any power taken off from the low speed shaft. Attached to the tips of the lower blades are secondary HAWT rotors, which provide the means of power take-off, and produce a retarding torque which controls the speed of the primary rotor. The operating principle of the X-Rotor is therefore that there is balance between the aerodynamic torque on the primary rotor, and the revolution averaged retarding torque arising from aerodynamic thrust on the secondary rotors. An illustration of the

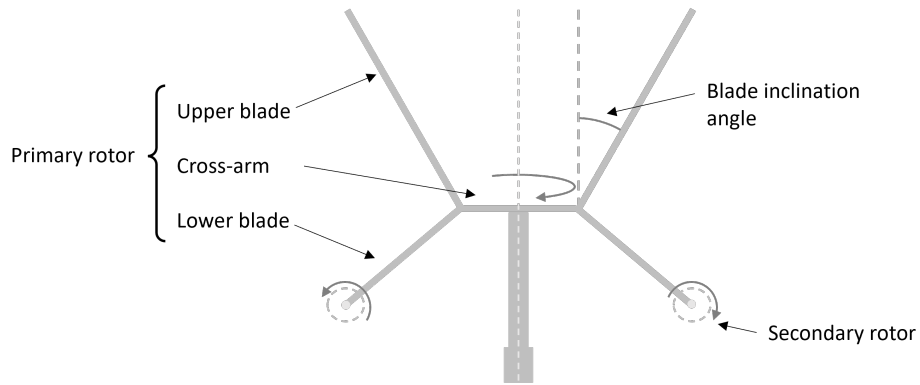


Figure 1.2: Illustration of the X-Rotor concept.

X-Rotor concept is provided in figure 1.2.

The average wind speed incident on the secondary rotors is equal to the primary rotor tip speed, which allows the secondary rotors to be shrunk significantly whilst achieving an equivalent power output to the primary rotor. Assuming a similar power coefficient between the primary and secondary rotors, if the primary rotor was operating at a tip speed ratio of λ_p , the secondary rotor area can be reduced by a factor of $1/\lambda_p^3$ and achieve a similar power output since power scales with the cube of wind speed. The reduced rotor area allows the secondary rotors to rotate significantly faster with the system essentially acting as an aerodynamic gearbox. In this manner the need for a gearbox can be avoided without requiring the use of large multi-pole generators. This aerodynamic gearing therefore allows for significant reductions in the drive-train cost and weight. In addition to the reduction in drive-train costs, secondary rotors have been shown to facilitate significant savings in O&M costs [20, 28] since, as the secondary rotors are light and located close to the ocean surface, no heavy lift vessels are required for major component replacements or repairs in the drive-train.

The geometry of the primary rotor increases the tip speed of the primary rotor relative to that of a conventional H-Rotor VAWT, a requirement for efficient power conversion using secondary rotors [20, 27], and increases the swept area of the primary rotor, increasing the maximum power available for a given blade and cross-arm length compared to a conventional H-Rotor. The lower blades also counteract the overturning moment from the upper blades compared to a conventional V-Rotor [20], where the

centre of thrust is significantly above the cross arm.

Discussing the power capture of the X-Rotor requires 4 specific definitions:

1. **Primary rotor power coefficient** (C_{Pp}) the ratio of the mechanical power captured by the primary rotor, averaged over a primary rotor revolution, normalised by the kinetic power that would pass through the swept area of the primary rotor with the rotor absent.
2. **Secondary rotor power coefficient** (C_{Ps}) the ratio of the mechanical power captured by the secondary rotor normalised by the kinetic power that would pass through the swept area of the secondary rotor with the secondary rotor absent, in the reference frame of the secondary rotor.
3. **Turbine power coefficient** (C_{Pt}) the ratio of the total mechanical power captured by all of the secondary rotors, averaged over a primary rotor revolution, normalised by the kinetic power that would pass through the swept area of the primary rotor with the rotor absent.
4. **Conversion efficiency** (η) the ratio of the total mechanical power captured by all of the secondary rotors, averaged over a primary rotor revolution, normalised by the mechanical power captured by the primary rotor, averaged over a primary rotor revolution.

The primary and secondary rotor power coefficients follow conventional definitions of the power coefficient of wind turbines, as in [16]. The turbine power coefficient is a convenient metric to compare the power coefficient of an X-Rotor turbine to a conventional VAWT or HAWT as it is a measure of the mechanical power available to the drive-train. A key goal of any coupled X-Rotor design is therefore the maximise the turbine power coefficient. Finally, the conversion efficiency facilitates the evaluation of how well designed a set of primary and secondary rotors are to each other. High conversion efficiency implies that the majority of the aerodynamic power captured by the primary rotor is available for conversion to electrical power in the drivetrain, a low conversion efficiency indicates that there are significant losses associated with poorly designed primary/secondary rotor coupling.

Chapter 1. Introduction

The conceptualisation of the X-Rotor first began in the early 2010s [27], and grant funding for a feasibility study was secured in 2017 [29], which led to the development of a 5MW 2 bladed concept, presented in [20], and [30]. This output then provided the basis for a successful Horizon 2020 bid, with the project running from January 2021 until April 2024 [31], taking the 5MW 2 bladed design from technology readiness level 1 to 3, and developing the design and simulation tools required for more detailed study of the concept. At present, work continues at the University of Strathclyde. The majority of the work presented in this thesis was completed during the Horizon 2020 project. The rest of this section will provide a thorough overview of the peer-reviewed published literature on the X-Rotor concept.

Following the initial confidential report, [27], the first academic publication on the X-Rotor concept was produced at the conclusion of a feasibility study [20]. This was further reported in more detail in [30]. These publications describe the X-Rotor concept, and investigate the design of a pair of baseline 5MW turbines with 2 and 3 blades, including a basic LCOE evaluation. Detailed primary rotor blade structural designs were presented, using the 50 year extreme wind speed with a parked rotor as the design driving load case. A quasi-static cycle counting method was used to dimension the substructure based on fatigue loads. A feasible operational strategy for the two bladed turbine was also presented. Whilst blade planforms are provided, the aerodynamic design process for the primary rotor was not discussed. The secondary rotor design is discussed in a qualitative fashion, indicating that low induction rotors with a low design tip speed ratio should be utilised. No secondary rotor blade planforms are provided.

The power curve for the primary rotor was obtained through aerodynamic simulation, using the lifting line free vortex wake model implemented in QBlade [32]. The blade polars were not representative of the aerofoil sections operating at the Reynolds numbers experienced during rotor operation, and the predicted aerodynamic characteristics do not agree with those found by more representative aerodynamic simulation work presented in Chapter 3. The power coefficient of the primary rotor was underestimated in this study, and the sensitivity of the power coefficient to a reduction in tip speed ratio was over-estimated, indicating that a low amplitude cyclic pitch strategy of

the primary rotor upper blades was viable for rotor speed control in above rated wind speeds.

The LCOE analysis found that the X-Rotor concept had the potential to cut the LCOE of offshore wind energy by up to 26% compared to traditional HAWT turbines, stemming from the capital cost savings on the drive-train, and O&M savings primarily achieved through not requiring jack up vessels for drive-train failures. The large saving in LCOE was calculated on the basis that the capital costs for all turbine components but the drive-train, replaced by the secondary rotors and generators, are equal. As stressed in the conclusion of [30], the large projected LCOE savings do not account for the increased total blade mass of the rotor, and increase in support structure mass for the two-bladed design, are therefore likely an over-estimate.

The requirements for models to investigate O&M costs for the X-Rotor, and a discussion of modifications to existing models that could be implemented to facilitate these investigations were discussed in [33]. Following the recommendations put forward in [33], the Strathclyde O&M tool [34] was modified to model the X-Rotor [28]. It was found that the X-Rotor could provide O&M savings of 19% compared to a conventional direct drive turbine, and 40% compared to conventional geared turbine. These savings were dependent on the ability of the turbine to operate at reduced capacity with a single secondary rotor in the event of a secondary rotor failure, and that secondary rotor modules were replaced, then repaired onshore. Sensitivities to the modelling assumptions were further investigated in [35], and the impact of the main bearing failure rate, was found to be a significant driver of O&M costs.

Secondary rotor generator designs were explored in [36], and it was found that outer rotor machines could provide a higher power density, a smaller diameter, and lower losses. A novel wireless rotary power transformer for transferring power between the rotating frame of the primary rotor and the stationary frame of the tower were presented in [37] and [38]. An investigation into the jacket substructure was presented in [39], which found a significantly higher jacket mass compared to the feasibility study. A detailed design of the piles for the proposed jacket structure was also provided in [40].

In terms of rotor aerodynamics, a code-comparison between a number of low, mid,

and high fidelity simulations of the X-Rotor primary rotor was reported in [41]. This study demonstrated that the aerodynamic power coefficient of the primary rotor was significantly under-predicted in [20] and [30]. Select results from this publication are re-produced in Chapter 3 to inform the choice of appropriate aerodynamic modelling methods for rotor design studies. The wake of the X-Rotor has been studied both experimentally [42] (on a 1:100 scale model), and numerically [41], using the actuator line method. In both cases, it was found that utilising a pitch offset could help the wake recover faster, potentially facilitating higher density wind farms. However, this effect was more significant for more traditional H-Rotors than the X-Rotor. The effect of secondary rotors on the primary rotor aerodynamics was experimentally investigated, on a 1:100 scale model, in [43], demonstrating that the primary rotor torque could be counteracted by the thrust on the secondary rotors.

1.3 Research questions

As demonstrated in the above review, aside from the qualitative discussion presented in [30], an investigation into the aerodynamic design of secondary rotors has not yet been presented in literature. Additionally, the aerodynamic design of X-Rotor primary rotors has not been investigated. This thesis therefore aims to address this literature gap and explore the aerodynamic design and operation of X-Rotor type wind turbines. The primary research question explored by this thesis is:

1. *Can tip mounted secondary rotors be used efficiently for wind turbine power take-off?*

This primary question is answered through a series of secondary questions, each of which are explored in the following chapters:

2. *How can secondary rotors be designed for efficient power take-off?*
3. *How can a vertical axis wind turbine be designed for efficient use with secondary rotors?*
4. *How should a vertical axis wind turbine using secondary rotors be operated?*

1.4 Research output

Peer reviewed journal publications:

1. **Morgan, L.**, Leithead, W., & Carroll, J. (2024). On the use of secondary rotors for vertical axis wind turbine power take-off. *Wind Energy*, 27(6), 569-582.
2. **Morgan, L.**, Amiri, A. K., Leithead, W., & Carroll, J. (2024). Effect of Blade Inclination Angle for Straight Bladed Vertical Axis Wind Turbines. *Wind Energy Science Discussions*, 2024, 1-26.
3. Giri Ajay, A., **Morgan, L.**, Wu, Y., Bretos, D., Cascales, A., Pires, O., & Ferreira, C. (2023). Aerodynamic model comparison for an X-shaped vertical-axis wind turbine. *Wind Energy Science Discussions*, 2023, 1-25.
4. Leithead, W. E., Amiri, A. M. K., Camciuc, A., **Morgan, L.**, Carroll, J., & Feuchtwang, J. (2025). X-Rotor, an innovative offshore wind turbine to reduce cost of energy. *Energies*, 18(10), 2549.

Peer reviewed conference publications:

1. **Morgan, L.**, & Leithead, W. (2022, April). Aerodynamic modelling of a novel vertical axis wind turbine concept. In *Journal of Physics: Conference Series* (Vol. 2257, No. 1, p. 012001). IOP Publishing.
2. Recalde-Camacho, L., Leithead, W., **Morgan, L.**, & Amiri, A. K. (2024, June). Controller design for the X-Rotor offshore wind turbine concept. In *Journal of Physics: Conference Series* (Vol. 2767, No. 3, p. 032050). IOP Publishing.
3. Jamieson, P., Simao Ferreira, C., Dalhoff, P., Störtenbecker, S., Collu, M., Salo, E., McMillan, D., McMorland, J., **Morgan, L.**, & Buck, A. (2022, April). Development of a multi rotor floating offshore system based on vertical axis wind turbines. In *Journal of Physics: Conference Series* (Vol. 2257, No. 1, p. 012002). IOP Publishing.

Book chapter:

1. Jamieson, P., & **Morgan, L.** (2022). 2.25-Trends, prospects and R&D directions in wind turbine technology. In *Comprehensive Renewable Energy* (pp. 817-853).

Horizon 2020 Technical reports:

1. **Morgan, L.**, Stock, A., & Leithead, W. E. (2022). Control simulation model for X-ROTOR concept. <https://doi.org/10.5281/zenodo.6967521>
2. **Morgan, L.**, & Leithead, W. (2023). D3.1 Definition of operational strategy. X-ROTOR H2020 Project. <https://doi.org/10.5281/zenodo.11395317>
3. Leithead, W., **Morgan, L.**, & Stock, A. (2022). Control model for X-ROTOR. <https://doi.org/10.5281/zenodo.6973714>
4. Leithead, W., & **Morgan, L.** (2024). D3.4 Design controller to realise operational strategy. X-ROTOR H2020 Project. <https://doi.org/10.5281/zenodo.11395295>
5. Campos-Gaona, D., Stock, A., Leithead, W., Anaya-Lara, O., & **Morgan, L.** (2021). Secondary rotors and powertrain design. <https://doi.org/10.5281/zenodo.6973850>

1.5 Thesis structure

This thesis is split into 4 further chapters, with 3 technical chapters followed by a detailed discussion with concluding remarks. Four appendices are included.

Chapter 2 provides a general overview on the use of secondary rotors for wind turbine power take-off, informing the design of both primary and secondary rotors. Chapter 3 investigates how a VAWT rotor can be designed to efficiently use secondary rotors, specifically looking at the effect of blade inclination angle. Chapter 4 presents and evaluates potential options for the operational strategy of the 5 MW X-Rotor turbine. Chapter 5 explores the implications of the results presented in the previous chapters and outlines their methodological limitations. Following this, a discussion of

Chapter 1. Introduction

the technological readiness of the X-Rotor turbine is presented, detailing significant potential for future work and the original research questions are addressed directly. Appendix A discusses the validity of the assumptions made in Chapter 2, Appendix B presents a DMS model for VAWT simulation developed by the author, Appendix C describes the 2DAC method extensively used for VAWT simulation in Chapter 3, and Appendix D details the aerodynamic design of the 2 bladed 5MW X-Rotor.

Chapter 2

Power take-off with secondary rotors

This chapter aims to address the primary research question posed by this thesis: *"Can tip mounted secondary rotors be used efficiently for wind turbine power take-off?"*. Additionally, the secondary question *"How can secondary rotors be designed for efficient power take-off?"* is addressed.

The secondary rotor concept is introduced in Section 2.1 and a survey of the existing literature (both academic and patent) is provided in Section 2.2. Within this survey, numerous examples of wind turbine concepts utilising secondary rotors are identified. However, it is found that a systematic study of the use of secondary rotors, including the coupling between primary and secondary rotors and the limitations inherent in secondary rotor design/operation, has not been completed. The rest of this chapter aims to provide a thorough overview of the aerodynamic aspects of wind power conversion using secondary rotors. The chapter first takes a general approach considering secondary rotors agnostic the specific configuration of the primary rotor. Following this the elements of aerodynamic conversion efficiency specific to both HAWT and VAWT rotors are derived and a brief discussion of other aspects of secondary rotor design including the loads, operability, and other design synergies is also provided in the context of a comparison between HAWT axis and VAWT primary rotors.

Section 2.3 underlines the basic principles of wind turbine design and control with

secondary rotors and the fundamental limits of secondary rotor operation. Section 2.4 then represents secondary rotors first as actuator disks, then as optimally designed rotors. Within this section, investigations into the achievable range of torque reduction between primary and secondary rotors, the power conversion efficiency, the optimal design points for primary rotor operation, and the sensitivity of power conversion efficiency to a number of design parameters are presented. A brief investigation into the use of ducted secondary rotors is then presented in Section 2.5. Sections 2.6 and 2.7 look at the specific aspects of using secondary rotors for power conversion with horizontal and vertical axis primary rotors respectively. Following this Section 2.8 compares the potential benefits and drawbacks of each configuration, widening the scope from the primary focus of aerodynamic efficiencies to a broader discussion of the additional loads on the primary and secondary rotors, turbine operability, and other design synergies. Finally, Section 2.9 will give an overview of key conclusions that can be drawn from this chapter.

2.1 The secondary rotor concept

Traditionally wind turbine power take-off is completed through an electrical machine connected to the rotor shaft either directly or through a gearbox as a sub-system referred to as the drive-train [44]. Due to the low rotational speed of the wind turbine rotor, the input torque is typically very high leading to heavy, expensive drive-trains. Additionally, as these drive-trains are located at the rotor hub height (typically in excess of 100m above sea level), the operations and maintenance costs associated with the drive-train are also large due to the cost of scheduling heavy lift vessels for large component repairs. In general, a wind turbine's nacelle accounts for approximately 50% of turbine capital costs [17] and maintenance associated with the drive-train accounts for between 49% and 59% of the wind turbine downtime (based of drive-train configuration) [45] and 69% of the material cost of repairs, as calculated combining failure rate data and material cost data from [46]. This indicates that innovation in wind turbine power take-off has the potential to significantly decrease the cost of energy, both in terms of capital and operational expenditure.

In order to increase the conversion efficiency and the drive-train reliability, large offshore wind turbines are converging on the use of permanent magnet synchronous generators [12], either in a direct drive configuration or utilising a medium speed gear-box. The production of these PMSGs require the extraction of rare earth metals including neodymium, praseodymium, dysprosium and terbium [47]. Humanitarian issues have been raised around the methods of extraction, and the geographic distribution of the mineral deposits (with the majority based in China) have led to concerns around geopolitical disruptions to the supply chain [48]. Finally the limited size of these mineral deposits and projected scale of global demand has led to predictions of a bottleneck in the offshore wind supply chain [18], although the scale of this issue is contended [49]. Whilst, for reliability reasons, PMSGs are likely to be employed in secondary rotor systems, the torque reduction facilitated by the use of secondary rotor significantly decreases the volume of material required. This is because, considering a constant shear stress over the air-gap, the generator mass must scale cubically with input torque [50].

In addition to the current costs associated with wind turbine drive-trains, the traditional drive-train concept also has issues with up-scaling. The power available to wind turbines scales with the swept area (l^2); however, as the rotor tip speed is constrained to limit aero-acoustic emissions and leading edge erosion, the rotor speed scales inversely to the radius increase (l^{-1}), thus the rotor torque, and therefore generator volume, scales with the cube of the length scale (l^3). This square-cube relationship between the revenue (power captured) and costs (proportional to the mass/volume of components) is often highlighted when considering the costs of the rotor structure [15]; however, here it is referring directly to the generator costs when upscaled subject to a tip speed limit. Despite this square-cube law, rotor up-scaling has thus far remained attractive as it can offer significant savings to the balance of plant and operations and maintenance costs. Whilst it has been shown that real up-scaling has thus far 'beaten' the square-cube law [15], likely due to technological development occurring parallel to upscaling, wind turbine designs which circumvent these scaling laws altogether should be considered to facilitate economically viable up-scaling in future. The recent increase in research work on multi rotor systems [51] indicates that the wind energy community

is taking seriously the prospect of radical innovation to facilitate the further up-scaling of wind turbine systems. Secondary rotors provide a means of increasing the power take-off capacity that scales linearly with the power captured, as additional secondary rotor systems can be added rather than up-scaling the individual drive-train and can therefore provide a solution to the up-scaling problem. Additionally the torque-related cost associated with upscaling is present when decreasing the power density of wind turbine rotors and, as demonstrated in this chapter, can be circumvented through the use of secondary rotors.

These issues associated with traditional wind turbines can be categorised into a high capital costs, high associated O&M costs, high use of magnetic material, problematic upscaling behaviour, and problematic costs associated with lowering the turbine power density (i.e. the rated wind speed of the turbine). Whilst these issues are all currently present in the case of HAWTs, excessive capital cost and material use represent a further challenge in the case of VAWTs, with the inherently lower rotational speed and higher torque rating. The secondary rotor concept aims to overcome these issues through utilising additional rotors attached to the blade tips of the primary rotor which see an increased wind speed, facilitating a small rotor with a high rated power which can be connected to a relatively conventional power-train.

Due to its small radius, the secondary rotor can spin at high rotational speeds, essentially acting as an aerodynamic gearbox. The reduced input torque significantly reduces the cost of the drive-train whilst forgoing the additional costs associated with a gearbox reducing the overall CAPEX associated with the drive-train [20]. The secondary rotors are both light and easy to access and do not, therefore, require a jack up vessel for maintenance, facilitating significant O&M savings [28]. Through reducing the input torque to the generators, the use of rare earth metals can be significantly reduced compared to conventional direct drive drive-trains. Additionally, through coupling the power take-off to the primary rotor tip speed, rather than the primary rotor shaft speed primary rotor expansion to decrease the power density of the turbine will incur no additional drive-train costs if the primary rotor tip speed is maintained, as an identical secondary rotor unit can be used. In terms of upscaling, a linear upscaling of

power take off with respect to power rating is achievable through increasing the number of secondary rotors, rather than increasing their size. Finally, as the drive-train costs are seen as one of the potential roadblocks for the development of VAWTs, secondary rotors can be seen as an enabling technology, allowing the economical deployment of VAWTs whilst preserving many of the benefits of the technology.

2.2 Secondary rotors within the literature

The use of secondary rotors attached to the tips of wind turbine blades to act as a means of reducing the torque input to a wind turbine drive-train was first proposed by Ufimtsev in 1924, as identified by in an article in the *Izvestiya Otdeleniya Tekhnicheskikh Nauk, Akademiya Nauk SSSR* (Proceedings of the Department of Technical Sciences, Academy of Sciences of the USSR) [52]. No original copies of Ufimtsev's work on secondary rotors have been obtained, and only the abstract of Krasovskiy's paper is available to access. It is not certain whether a translated version of the text exists. The abstract indicates that the article focuses on aerodynamic transmission (use of secondary rotors) for turbines with a rating between 100kW and 3MW. The abstract notes the high (40m/s-70m/s) wind speed incident on the secondary rotors facilitating rotational speeds in excess of 500rpm with efficiencies up to and above 80%. The veracity of these claims will be discussed within this chapter. This work identifies two possible transmission configurations; the first utilises shafts running down the primary rotor blades into a single generator at the primary rotor hub, whilst the second utilises generators attached to the blade tips. Mechanical transmission from the blade tips to the rotor hub can be considered to be relatively unfeasible for modern wind turbines with long blades that undergo significant deflection during operation.

Later, in a review of fluid dynamic aspects of wind energy conversion, Vries [53] comments on the use of tip rotors as a means of overcoming the high torque - low speed power-train input inherent in wind turbine operation. The concept is dismissed out of hand, as the additional blade loading due to the secondary rotors, and the gyroscopic loads on the secondary rotors are considered more problematic than the high input torque problem that they are addressing. It should be noted that this judgement was

made when the typical wind turbine had a rating of 30kW and a diameter of 10m [16]. Considering that rotors have up-scaled far beyond this point, a reconsideration of the technology is most certainly valuable.

Secondary rotors were again discussed in a broader review of novel wind turbine concepts at the 5th biennial wind energy conference and workshop (1981) by Ljungstroem [54]. Again, whilst the book of abstracts has been located, sadly the paper itself has not yet been found despite contacting the conference organisers. The abstract refers to these machines as Dual Flow Turbines (DFTs) and considers both axial and cross-flow primary rotors with axial and cross-flow secondary rotors.

To the authors knowledge, the next published academic discussion of secondary rotors for wind turbine power take-off was completed in the development of the X-rotor concept, with the feasibility study results published in [20]. This article presents the key results of a feasibility study, including the definition of an exemplar 5MW turbine, based on that in [27], and the estimated cost of energy savings achieved by the X-Rotor system. It highlights the design synergies between using a VAWT with secondary rotors, and details cost of energy calculations which demonstrate the potential benefits from secondary rotors, savings of up to 26% on the cost of energy. Due to it's broad scope, the discussions on the use of secondary rotors is limited, and neither the mechanics of power take off with secondary rotors nor the process by which secondary rotors are designed are discussed. A more detailed discussion of the outcomes from this feasibility study is provided in [30].

A survey of emerging technologies in the wind energy sector [51] included a discussion on the use of secondary rotors attached to the rotor blade tips, including a brief discussion on the potential power density of the generators. A thorough discussion of the concept was however outside of the scope of the review paper.

In [55], the design of secondary rotors was expanded on further. Specifically, the use of an actuator disk representation including tip losses allowed the author to examine the effects of the design induction factor on the efficiency of energy conversion. In this work the secondary rotor radius is taken as a free design variable and the constraints on secondary rotor design are not explicitly imposed. The use of twin secondary rotors

is also discussed, noting that the low induction factor increases the potential for energy capture.

The use of secondary rotors on a downwind horizontal axis machine is briefly discussed in [56] as the WTx2 concept. Here, 6 bladed secondary rotors are attached to the tips of a 3 bladed downwind HAWT. It is indicated that the combined mass of the secondary rotor and generators will be 6 tons compared to a conventional 250 ton drive-train for a 10MW machine. Analysis from work package 5 [57] of the X-Rotor project indicates that a 10 ton drive-train mass is achievable for a 2.5MW secondary rotor drive-train, which would correspond to a 40 ton mass for a 10MW machine, whilst lower drive-train masses are possible from shrinking the secondary rotors, this would come at a conversion efficiency penalty, and it seems that the 6 ton estimate is considerably lower than expected.

In addition to the academic work, a number of patents have been filed, which are discussed here. US4197056 [58] describes a HAWT with articulated blades that utilises secondary rotors for power take-off. A large portion of the patent is associated with the articulation of the primary rotor blades and the ease of installation. The secondary rotors are introduced as a means of reducing the wind turbine nacelle mass, with the shrunken diameter and high rotational speed facilitating either a direct connection, or a connection with a small gear ratio, to an electrical generator. Additionally, the option to use secondary rotors as propellers for start up is identified. There is no mention of how secondary rotors should be designed in order to maximise conversion efficiency, nor is there mention of how the primary or secondary rotors could be controlled in order to ensure maximum efficiency.

Later, US5151610 [59] introduces secondary rotors for both horizontal and vertical axis wind turbines. The stated aim of using secondary rotors is again the same, to increase the rotational speed input to an electrical generator without using a gearbox. The patent describes the secondary rotor design as a horizontal axis rotor, with a multi-bladed windmill type rotor, common in American farms, or a two bladed propeller given as examples. Later patents have pointed out that using an American style wind turbine would lead to inefficient energy conversion. However, it seems clear from the patent

Chapter 2. Power take-off with secondary rotors

authors wording:

" The [secondary] rotor is an axial flow rotor ... The rotor in the example shown is a multi-bladed propeller, such as a type known as a multi-blade American farm windmill... However, the number of blades can be decreased down to a two-bladed propeller as shown by the propeller type rotor ... if so desired, and in accordance with the local wind speed conditions and the design of the entire wind machine" ,

that two extremes of potential secondary rotor designs are presented, and the author indicates that the intended secondary design will fall somewhere between these examples. Additionally, the use of a shroud is encouraged. No gearbox is recommended, and a rotor speed of between 2000 and 3000 rpm is considered. Assuming that the secondary rotor tip speed is limited to 200m/s, this would imply a rotor radius between 0.94m and 0.64m. As discussed in this chapter, low induction secondary rotor designs are required for efficient energy conversion between primary and secondary rotors, as the ratio of power coefficient to thrust coefficient is the determining variable. This implies that, for utility scale rotors, larger secondary rotor diameters and lower secondary rotor speeds than those assumed in this patent are required.

This patent is the first to describe the use of secondary rotors on a VAWT; however, it does not engage in any specific details of primary or secondary rotor configurations that would increase the conversion efficiency between the primary and secondary rotor. It does not represent a considerable step forward in terms of the fundamentals of power take-off with secondary rotors from the patent of [58].

The patent WO02086312 [60] introduces a HAWT in a downwind configuration that utilises secondary rotors for power take-off, again utilising a direct drive configuration. This patent is the first to describe how secondary rotors might be designed to increase the efficiency of energy conversion between the primary and secondary rotor. The patent defines the power coefficient of the secondary rotor(s) as the power captured by the secondary rotor normalised by the power required to move the secondary rotor at a given velocity. This definition of power coefficient refers to the conversion efficiency between the primary and secondary rotor, rather than the aerodynamic power coeffi-

cient as traditionally defined for wind turbines. A brief derivation, further explained in section 2.3, yields the results that the conversion efficiency is given by $(1 - a)$ where a represents the axial induction factor of the secondary rotors, indicating that secondary rotors designed with a low axial induction factor yield the highest conversion efficiency. In this way, the patent claims that the power coefficient of secondary rotors can overcome the Betz limit. However, this is a misleading claim as the rotors are operating well below the Betz limit and it is through the re-definition of the power coefficient for secondary rotors that the Betz limit is 'broken'. The patent recommends a secondary rotor thrust coefficient of approximately 0.4, corresponding to an axial induction factor of 0.11, an aerodynamic power coefficient of 0.35 and a conversion efficiency of 0.89 in the case of an ideal actuator disk. Based on this size, the required secondary rotor radius for a range of primary rotor tip speeds is presented for a 2 bladed 3MW design with rotor radii varying from 1.3m to 2.4m. This patent proposes that the secondary rotor tip speed ratio should be limited in order to lower the maximum tip speed of the secondary rotor to avoid compressibility effects. However, the relationship governing the rotational speed of the secondary rotor, the secondary rotor radius and the maximum allowable tip speed is not discussed significantly, and no meaningful contribution is made in quantifying the achievable torque reduction.

On the operation of the primary and secondary rotors, it is stated that the secondary rotors should be operated at a low tip speed ratio to limit the secondary rotor tip speed, that the secondary rotor thrust coefficient should be held below 0.4, and that the primary rotor should be held at its maximum power coefficient. However there is no description of how primary and secondary rotors could be designed to ensure this is the case. In terms of the control of the wind turbine, pitch control of the secondary rotor, speed control of the secondary rotor, pitch control of the primary rotor, and control of the angle between the secondary rotor plane and the primary rotor plane are discussed as options.

The patent identifies that the main method of turbine control is to maintain a constant primary rotor power coefficient through blade pitch and rotor speed control of the secondary rotor. In order to slow the primary rotor, the patent proposes pitch

control of the secondary rotors or increasing the tip speed ratio of secondary rotors in order to increase the secondary rotor thrust coefficient.

In terms of turbine control, the specific method of maintaining a constant primary rotor tip speed ratio is not discussed, and the interactions between the strict limitations on secondary rotor tip speed and the control strategy are not discussed. This is most pertinent when discussing the potential to increase secondary rotor speed to decrease the primary rotor speed. This control action would take place at high wind speeds, when the tip speed of the secondary rotors are already at their upper limit, further increasing the secondary rotor tip speed is therefore likely implausible. Additionally, pitch actuators are a common point of failure within wind turbines and it is therefore advisable to avoid secondary rotor pitch control if possible.

In general, the patent WO02086312 represents a significant step forward in the design of secondary rotors, introducing the necessity of a low induction secondary rotor design and deriving a basic formulation of the conversion efficiency between primary and secondary rotors. However, the specific design of wind turbines is not discussed. In terms of rotor control, the patent does not define any specifics.

The patent GB247750 [61] introduces a wind turbine concept that utilises secondary rotors on a drag driven VAWT. Since drag driven turbines have an inherently low power coefficient, they are typically unsuitable for MW scale applications as the rotor swept area is required to be excessively large. In addition to this, these rotors typically operated at very low tip speed ratios. It is shown in this thesis that significantly higher tip speed ratios are required for efficient power conversion between primary and secondary rotors. Additionally, low primary rotor tip speed operation implies that the wind speed seen by the secondary rotors is not greatly enhanced and that gearboxes would be required, negating a significant benefit of using secondary rotors for power take-off. For both of these reasons, the patent is not considered further here.

The patent US 2021/0388812 A1 [62] describes the X-Rotor, an X-shaped VAWT primary rotor using secondary rotors attached to the tips of the lower blades for power take-off. The ability to use direct drive generators with a low number of pole pairs whilst maintaining high efficiency is highlighted as the key design driver and is achievable with

X-Rotor configuration. Secondly, the maintainability of the detachable secondary rotor systems, due to their proximity to the surface of the sea, is recognised as an important benefit of the system. This patent, again, indicates the necessity of low induction secondary rotor design, and additionally highlights that the product of the tip speed ratios of the primary and secondary rotors must be kept below 16, such that the secondary rotor does not exceed 60% of the speed of sound. Such a limit is imposed as higher tip speeds would lead to decreased aerodynamic efficiency of the secondary rotors. The proposed control methodologies for the turbine are pitching of either the upper or lower (or both) of the primary rotor blades, and control of the secondary rotor speeds. Blade planforms for a potential primary and secondary rotor configuration capable of generating 5MW are also presented.

The patent represents a significant development in the literature, firstly through detailing a practicable solution to mounting secondary rotors on a VAWT primary rotor, through using an X-shaped primary rotor, and secondly through highlighting that the tip speed limits on the secondary rotor imply that the combined tip speed ratio of the primary and secondary rotor must be limited, and that this inherently leads to a coupled design.

After a survey of both the academic literature and the available patents, it is clear that there has been an interest in the use of secondary rotors for wind turbine power take-off for a long time. The general operational concept of increasing the power density of the flow incident on a secondary rotor through attachment to the tips of a primary rotor, facilitating a low rotor radius and high rotational speeds is consistent across the literature. The work of Jamieson [55] Madsen [60], and Leithead [62] represent the most developed work on secondary rotors. The work of Madsen is the first that points to the key conclusion that the secondary rotors must be designed to be low induction. The work of Jamieson [55] moves further looking at a specific design and quantifying the torque reduction and investigating some pertinent aspects of rotor design, finally the work of Leithead [62] discusses the coupled nature of primary and secondary rotor design through the maximum allowable secondary rotor tip speed. Whilst the coupling between the primary and secondary rotors are discussed in [62] and have clearly been

accounted for in the presented rotor designs, the equations describing the fundamental relationships between design variables are not presented or derived.

The maximum size of the secondary rotors is predominantly linked to the maximum allowable tip speeds of the secondary rotor(s) and the design rotational speed of the secondary rotor(s), whilst the required axial induction is a function of the primary rotor power rating, the primary rotor tip speed ratio, and the secondary rotor area. This coupling of the primary and secondary rotors is key to generating sensible designs for wind turbines utilising secondary rotors, and to looking at the efficacy of secondary rotors as a means of reducing the drive-train torque.

2.3 Basic principles

In order for the turbine to be in equilibrium, the aerodynamic torque on the primary rotor must be matched by the retarding torque from the thrust on the secondary rotors. Whilst transient imbalances may exist, this condition must be enforced over a primary rotor revolution. Thus, assuming the primary rotor rotational speed is constant, the energy captured by the primary rotor over a single revolution must be matched by the work done by the thrust on the secondary rotors.

$$\frac{1}{2}\rho A_p C_{Pp} U_0^3 \left[\frac{2\pi}{\Omega_p} \right] = N_s \int_0^{2\pi} R_p \frac{1}{2} \rho A_s C_{Ts} U_I^2(\theta_p) d\theta_p. \quad (2.1)$$

Here, ρ refers to the density of air, A to the rotor area, C_P to the power coefficient, U_0 to the free wind speed, Ω to the rotor speed, R to the rotor radius, N to the number of rotors, C_T to the thrust coefficient, U_I to the wind speed incident on the secondary rotor plane without the presence of the secondary rotors - equivalent to the free-stream wind-speed in a traditional HAWT, and θ the angle of azimuth. The subscripts $[_{p/s}]$ indicate variables pertaining to the primary or secondary rotor(s) respectively. Assuming that the secondary rotors are operating at a fixed value of C_{Ts} , this equilibrium condition can be used to find the required operating point (C_{Ts}) for secondary rotors to hold a given turbine configuration in equilibrium:

$$C_{Ts} = \frac{A_p}{N_s A_s} \frac{C_{Pp}}{\lambda_p} \frac{U_0^2}{\overline{U_I^2}}, \quad (2.2)$$

where λ_p represents the tip speed ratio of the primary rotor, given by

$$\lambda_p = \frac{\Omega_p R_p}{U_0}, \quad (2.3)$$

and $\overline{U_I^2}$ represents the mean of the square of the wind speed incident on the secondary rotors.

Defining η as the ratio of the aerodynamic power captured by the secondary rotor(s) to the aerodynamic power captured by the primary rotor, equation 2.1 can be used to represent the power captured by the primary rotor in terms of the secondary rotor. Assuming that the secondary rotors are held at a constant tip speed ratio, i.e. C_{Ps} and C_{Ts} remain constant this gives

$$\eta = \frac{C_{Ps}}{C_{Ts}} \frac{\int_0^{2\pi} U_I^3(\theta_p) d\theta_p}{\Omega_p R_p \int_0^{2\pi} U_I^2(\theta_p) d\theta_p}. \quad (2.4)$$

The efficiency of power conversion is dictated by two factors; the ratio of the secondary rotor power and thrust coefficient, and a term relating to the incident wind speed on the secondary rotors. This term represents the power that can be extracted from the ambient wind speed incident on the turbine and so on the secondary rotors, in addition to the power extracted by the secondary rotors due to their translation by the primary rotor.

Considering only the apparent wind speed due to primary rotor rotation and assuming that the secondary rotors are attached at the tips of the primary rotor, they see a wind speed of $\Omega_p R_p$. At a constant primary rotor speed, equations 2.1 and 2.4 then reduce to

$$C_{Ts} = \frac{1}{N_s A_s} \frac{C_{Pp} A_p U_0^3}{(\Omega_p R_p)^3} = \frac{A_p}{N_s A_s} \frac{C_{Pp}}{\lambda_p^3} \quad (2.5)$$

and

$$\eta = \frac{C_{Ps}}{C_{Ts}} \quad (2.6)$$

From these equations, an important result can already be deduced, indicating the

applicability of secondary rotor use for low wind speed sites.

To ensure high capacity factors and to exploit low wind speed sites, wind turbines are often designed for lower power densities. Whilst this can be beneficial for the overall cost of energy, the rotor expansion necessary to maintain the turbine power rating leads to decreased rotor speeds and increased drive-train torques. A basic approximation for traditional wind turbine drive-trains is that the costs and mass scale approximately with rotor torque. The costs of both the expanded rotor and the higher torque drive-train must be considered against the benefits of a higher capacity factor.

In the case of a wind turbine employing a secondary rotor system, the drive-train costs are not affected by the power density of the primary rotor if a constant tip speed and rated power is assumed. From equation 2.2 it is clear that, if the power rating is maintained, the product $C_{Pp}A_pU_0^3$ remains constant, and thus if the primary rotor tip speed ($\Omega_p R_p$) is also held constant, an identical secondary rotor design (N_s, A_s, C_{Ts}) can be used. This rotor would have the same conversion efficiency and torque/speed characteristics leading to unchanged drive-train costs. For considerable decreases in the power density of the wind turbine, the primary rotor tip speed may not be able to be conserved, as the design tip speed ratio of the primary rotor can not be arbitrarily increased without consequence.

This is an important result; however, to further explore the use of secondary rotors, and to understand the achievable conversion efficiencies, the actual representation of the secondary rotors must be discussed.

2.4 Representing secondary rotors

2.4.1 The secondary rotor as an actuator disk

The simplest method of representing the secondary rotors is to describe them as actuator disks. In this case the power and thrust coefficients are given by

$$C_P = 4a(1 - a)^2 \tag{2.7}$$

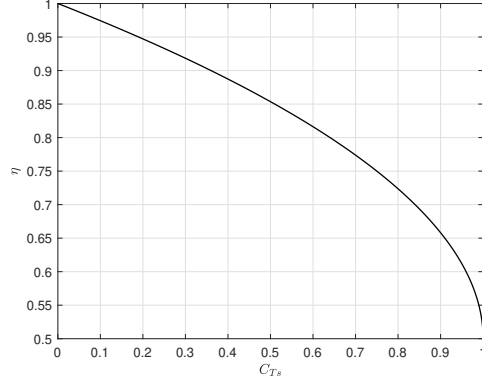


Figure 2.1: The efficiency of power conversion between a primary and secondary rotor, η , as a function of design secondary rotor thrust coefficient, C_{Ts} , as calculated with an actuator disk.

and

$$C_T = 4a(1 - a), \quad (2.8)$$

where a represents the axial induction factor. Recalling equation 2.6 it is clear that the conversion efficiency is given by

$$\eta = \frac{C_{Ps}}{C_{Ts}} = (1 - a_s) = \frac{1 + \sqrt{1 - C_{Ts}}}{2}, \quad (2.9)$$

implying that, for efficient energy conversion between the primary and secondary rotors, secondary rotors should be designed to minimise the axial induction factor through minimising the design thrust coefficient. This is quite different to conventional wind turbine design, in which the goal is to maximise the power coefficient, typically designing for $a = 1/3$, $C_T = 8/9$. Figure 2.1 shows the relationship between the secondary rotor thrust coefficient and the efficiency of power conversion between the primary and secondary rotor. From this, it is clear that designed to operate at the Betz limit with an axial induction factor of $1/3$ may only achieve a maximum conversion efficiency of $2/3$, whilst the conversion efficiency approaches 1 as the secondary rotor thrust coefficient approaches 0.

Equation 2.2 demonstrates how the design thrust coefficient for the secondary rotor can be minimised. For a given primary rotor configuration, the secondary rotor thrust

coefficient is minimised through maximising the effective swept area of the secondary rotors. This can be achieved through increasing either the number of secondary rotors, or the secondary rotor swept area. The number of secondary rotors is a design variable, whilst the swept area of each secondary rotor is limited by the maximum allowable tip speed, U_{max} , with

$$R_s \Omega_s \leq U_{max}. \quad (2.10)$$

Thus, for a given secondary rotor speed, the most efficient rotor design is obtained through taking the upper limit of equation 2.10. Re-casting equation 2.2 in terms of the rated torque of the primary rotor, Q_p , and using the expression for the maximum secondary rotor radius one obtains

$$C_{Ts} = \frac{2}{\pi \rho} \frac{Q_p}{N_s U_{max}^2 R_p^3} \left(\frac{\Omega_s}{\Omega_p} \right)^2. \quad (2.11)$$

In conjunction with equation 2.9, this facilitates the examination of the secondary rotor system as an aerodynamic gearbox, in terms of the effective speed ratio, $n_\omega = \Omega_s/\Omega_p$ and the input torque, Q_p . It is interesting to note that, under similarity scaling of the primary rotor, the rotor torque is proportional to the cube of the primary rotor radius. This implies that the required secondary rotor thrust coefficient, and therefore the efficiency of energy conversion, is invariant to the scaling of the primary rotor for a given effective speed ratio.

Figure 2.2 presents the required design thrust coefficient and conversion efficiency for a baseline rotor design with $N_s = 1$, $R_p = 75\text{m}$, $U_{max} = 184\text{m/s}$, and $\rho = 1.225\text{kg/m}^3$ for a range of input torques from 2.5MNm to 10MNm. Assuming that the primary rotor is operating with a tip speed ratio of 5 and a rated wind speed of 12m/s, these correspond to rated powers of 2MW to 8MW, with power densities ranging from 141Wm^{-2} to 566Wm^{-2} . It is clear that, for a given effective speed ratio, a higher input torque requires larger thrust coefficients leading to lower conversion efficiencies. Similarly, to facilitate higher effective speed ratios, the secondary rotor must be made smaller as in equation 2.10, increasing the required thrust coefficient and decreasing the conversion efficiency. It is clear that high conversion efficiencies are possible, especially

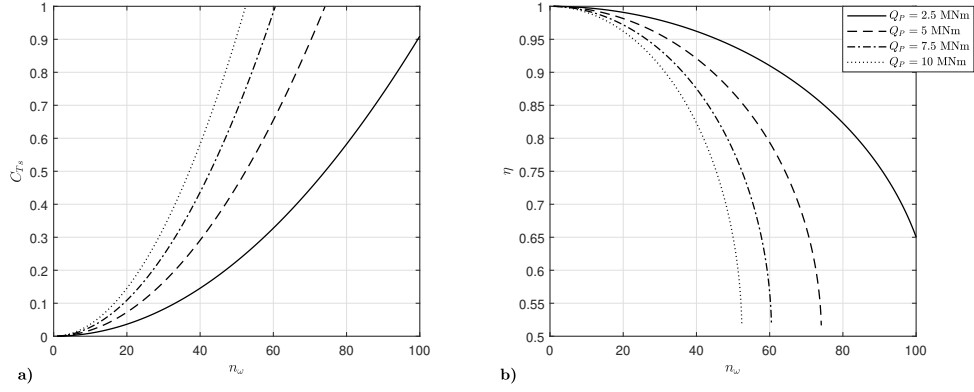


Figure 2.2: The effect of effective speed ratio n_ω and primary rotor torque, Q_p on the design secondary rotor thrust coefficient, C_{Ts} , a), and conversion efficiency, η , b), as calculated with an actuator disk.

for relatively low effective speed ratios.

It is also important to understand the effect of using multiple secondary rotors. Through introducing multiple secondary rotors, the machine is transformed into a multiple generator drive-train configuration, as explored for traditional drive-trains in [63]. Since the input power is distributed between the secondary rotors, the total torque reduction between the primary and secondary rotor is still given by the speed ratio, n_ω . The effect of using multiple secondary rotors on the conversion efficiency as a function of both effective speed ratio is shown in figure 2.3. Here, the same baseline turbine used as in figure 2.2 with a primary rotor torque of 5MNm. The most efficient means of ensuring a given effective speed ratio is the inclusion of more secondary rotors, since this increases the combined swept area of the secondary rotors and decreases the required thrust coefficient.

The effect of the primary rotor radius and the maximum secondary rotor speed on conversion efficiency are shown in figure 2.4, these are generated with the baseline rotor variables used to generate figure 2.2, with an input torque of 5MNm and an effective torque reduction of 25. Figure 2.4 demonstrates that, for a given speed ratio, the efficiency of power conversion between the primary and secondary increases as the primary rotor radius increases, and as the maximum allowable secondary rotor tip speed increases. This is because the the required secondary rotor thrust coefficient

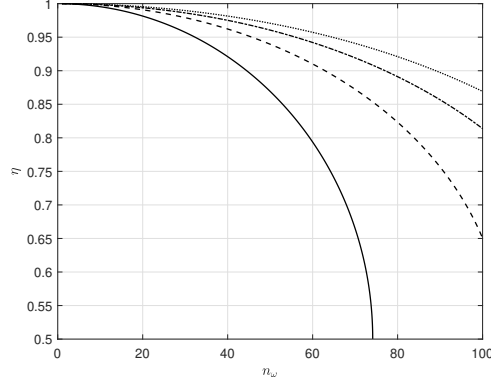


Figure 2.3: The effect of rotor number, N , effective speed ratio, n_ω on conversion efficiency between a primary and secondary rotor, η , for a 5MNm input torque as calculated treating the secondary rotor as an actuator disk.

is inversely proportional to the cube of the primary rotor radius and the square of the maximum allowable tip speed, as shown in equation 2.11, and lower thrust designs facilitate more efficient power conversion between the primary and secondary rotor, as shown in equation 2.9. In the case of the maximum allowable tip speed, it is important to understand that an accurate estimation of the maximum allowable tip speed has not yet been determined, and that a thorough investigation is required to characterise this limitation. It is clear that the maximum tip speed has a large effect on the maximum achievable conversion efficiency, and uncertainty within this area affects both the optimal primary rotor design, and the specific design of the secondary rotor and the rest of the power-train.

Considering a primary rotor with a given power density, Φ_p , and a given tip speed, $U_{tip,p}$, the rotor torque is given by

$$Q = \frac{\Phi_p k_p R_p^3}{U_{tip,p}} \quad (2.12)$$

where k_p represents the geometrical parameter defining the relationship between the square of the rotor tip radius and the swept area of the turbine, $k_p = \pi$ for a HAWT. Incorporating these into equation 2.9, the efficiency of power conversion between primary

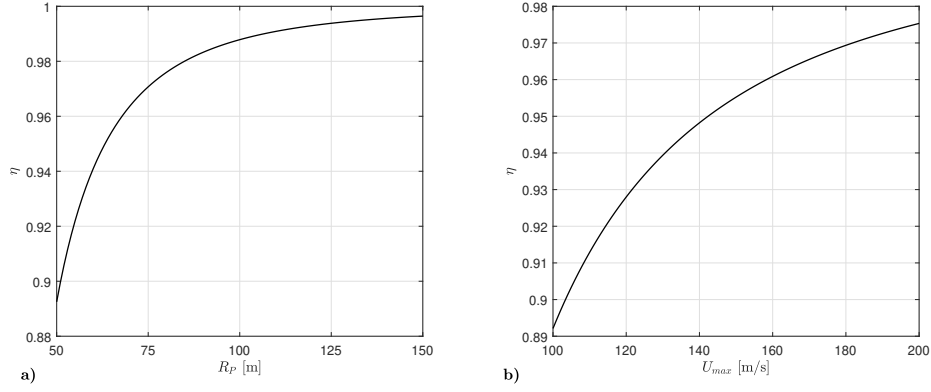


Figure 2.4: Effect of primary rotor radius, R_p , a), and maximum tip speed, U_{max} , b), on conversion efficiency between a primary and secondary rotor, η with a single secondary rotor, an input torque of 25MNm and an effective torque reduction factor of 25, as calculated with an actuator disk representation.

and secondary rotors is given by

$$\eta = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{2\Phi_p n_\omega^2 k_p}{\rho N_s U_{max}^2 U_{tip,p} \pi}}. \quad (2.13)$$

demonstrating that the efficiency of power conversion with secondary rotors is independent of rotor scale. This also indicates that rotor geometries which maximise the tip radius for a given swept area and that larger primary rotor tip speeds are favourable for efficient secondary rotor design.

Through the representation of secondary rotors as actuator disks, this section has shown the following:

1. Efficient energy conversion between the primary and secondary rotor(s) is dependent on low induction, (and therefore low thrust coefficient) design. This means that secondary rotor design will be a considerable departure from traditional wind turbine rotor design.
2. Secondary rotors can provide an efficient means of power-train torque reduction; however, their efficiency is reduced as the rotor torque increases and the required speed ratio increases.
3. For a given effective speed ratio (or torque reduction ratio), the efficiency of power

conversion between the primary and secondary rotors is invariant to upscaling the primary rotor.

4. The efficiency of power conversion between the primary and secondary rotor is highly dependent on the primary rotor configuration, with the required thrust coefficient inversely proportional to the cube of the primary rotor tip speed ratio.
5. The efficiency of power conversion between the primary and secondary rotor at a given secondary rotor rotational speed is highly dependent on the maximum allowable tip speed for the secondary rotors, with the secondary rotor thrust coefficient inversely proportional to the square of the maximum allowable tip speed. More research is required to understand this maximum tip speed limit.

Whilst the representation of secondary rotors as actuator disks has proved insightful, some of the underlying dynamics behind power conversion between the primary and secondary rotors are not well captured by this model. The maximum achievable power coefficient of the secondary rotors will be limited by the lift to drag ratio of the aerofoil sections used to make up the secondary rotor blade. Additionally, both tip losses and tangential induction will introduce a tip speed ratio dependence on both the power and thrust coefficients. Whilst the required thrust coefficient decreases as the primary rotor tip speed increases, the tip speed ratio of the secondary rotors (given by U_{max}/U_{tip} in the limiting case) will decrease as the primary rotor tip speed increases and the secondary rotor performance will drop. Finally, the hub of the secondary rotors will contribute the rotor thrust, but will not contribute to the power captured. Including this parasitic thrust from the rotor hub is important to understand the effects of hub geometry and the limitations inherent on very low induction secondary rotor designs, where most of the required thrust coefficient may be provided by the parasitic thrust of the secondary rotor hub.

2.4.2 The optimally designed secondary rotor

In order to properly capture the effects of finite aerofoil lift to drag ratios, tip and root losses, tangential induction, and parasitic thrust from the rotor hub, this section will

introduce optimal rotor design theory, modifying the presentation in [16] to account for an arbitrary design thrust coefficient. This approach uses the BEM method, but iterative schemes for the calculation of the flow angle are circumvented through the assumption of an optimal twist distribution. This allows the on-design characteristics of the rotor to be calculated without numerical simulation.

Firstly, the design thrust coefficient, tip speed ratio, and blade number must be input. Following this the design axial induction can be calculated, including the effects of tip loss. Following this, assuming an optimal twist distribution and using the design lift to drag ratio of the aerofoil, the tangential induction distribution and the rotor power coefficient can be calculated. Finally, using the design lift coefficient and design angle of attack of the aerofoil section, the optimal chord and twist distributions can be calculated. This work will be presented for a blade comprising of a single aerofoil section; however, the method can be readily adapted to include a distribution of aerofoil sections.

For a HAWT rotor with tip losses, the thrust coefficient is given by

$$C_T = \begin{cases} 8a(1-a) \int_{x_h}^1 F x dx & (a \leq 0.4) \\ 2 \int_{x_{hub}}^1 \left\{ \frac{8}{9} + \left[4F - \frac{40}{9} \right] a + \left[\frac{50}{9} - 4F \right] a^2 \right\} x dx & (a > 0.4) \end{cases} \quad (2.14)$$

where x represents the normalised spanwise coordinate, x_{hub} represents the normalised hub radius, and F represents the end loss factor. The low blockage ($a \leq 0.4$) case represents the typical result for an actuator disk with tip losses, and the high blockage case ($a > 0.4$) represents the integrated form of Buhl's tip correction for high blockage cases [64]. The end loss function is given by the product of the tip and root loss functions

$$F = F_r F_t \quad (2.15)$$

with

$$F_t = \frac{2}{\pi} \text{acos} \left(\exp \left[-\frac{B}{2} \frac{(1-x)}{x} \sqrt{1 + \left\{ \frac{\lambda x}{(1-a)} \right\}^2} \right] \right) \quad (2.16)$$

$$F_r = \frac{2}{\pi} a \cos \left(\exp \left[-\frac{B}{2} \frac{(x - x_{hub})}{x_{hub}} \sqrt{1 + \left\{ \frac{\lambda x}{(1 - a)} \right\}^2} \right] \right) \quad (2.17)$$

where B represents the number of blades. The formulation of the tiploss functions here is identical to those used in NREL's Aerodyn modelling tool [2].

For a given rotor configuration (λ, B) , the axial induction factor required to achieve a desired thrust coefficient is therefore given by a piece-wise function with

$$a = \frac{1}{2} \left(1 - \sqrt{1 - \frac{C_T / (1 - x_{hub}^2)}{\int_{x_{hub}}^1 2Fxdx}} \right) \quad (2.18)$$

for $(C_T \leq 1.92 \int_{x_h}^1 F(x)xdx)$ and

$$a = \frac{1}{72 \int_{x_{hub}}^1 Fxdx - 50(1 - x_{hub}^2)} \left(36 \int_{x_{hub}}^1 Fxdx - 20(1 - x_{hub}^2) \right. \\ \left. - 3 \sqrt{24 \int_{x_{hub}}^1 Fxdx \left[6 \int_{x_{hub}}^1 Fxdx - 4(1 - x_{hub}^2) - 72C_T \right] + 50(1 - x_{hub}^2)C_T} \right) \quad (2.19)$$

for $(C_T > 1.92 \int_{x_h}^1 F(x)xdx)$.

As the end loss coefficient is dependent on the induction factor, an iterative scheme is required to calculate the axial induction factor. Initialising the calculation with no end losses, the induction factor can be calculated with equations 2.18 & 2.19. The end loss factor can then be updated with equation 2.15, and the induction factors re-calculated. This process can be repeated until the results are suitably converged. Convergence (with a maximum residual of 10^{-5}) is typically reached within 5 iterations, as shown in the example in figure 2.5.

With the axial induction factor calculated, the tangential induction factor, a_t can be readily obtained with

$$a_t(x) = \frac{\sqrt{\lambda_S^2 \kappa^2 x^2 + 2\lambda \kappa x - 4a\kappa[\lambda x - \kappa(1 - a)] + 1 - (\lambda \kappa x + 1)}}{2\lambda \kappa x} \quad (2.20)$$

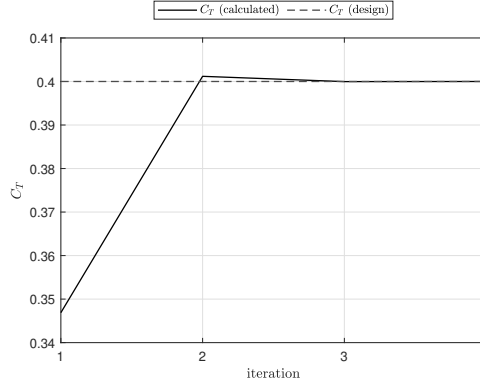


Figure 2.5: Typical convergence behaviour for calculating the required axial induction factor for a given thrust coefficient, C_T , with a maximum residual of 1×10^{-5} .

where κ represents the design lift to drag ratio of the aerofoil section. The design lift to drag ratio of the aerofoil section can be used and no iterative scheme is required due to the a-priori assumption that an optimal twist distribution is employed.

Considering the forces on a blade element,

$$dT = \frac{1}{2} \rho U_{rel}^2 B c [C_L \cos(\phi) + C_D \sin(\phi)] dr \quad (2.21)$$

where U_{rel} represents the apparent wind speed, ϕ represents the flow angle and c represents the local chord length. The velocity triangle associated with the blade element is shown in figure 2.6. The local thrust coefficient for a rotor annulus is therefore given by

$$C_T = \frac{(1-a)^2}{\sin^2(\phi)} \frac{B c C_L}{2\pi R x} \left[\cos(\phi) + \frac{1}{\kappa} \sin(\phi) \right] \quad (2.22)$$

evaluating the velocity triangle in figure 2.6, the non-dimensional lift distribution ($\Lambda = c B C_L / R$) can be obtained with

$$\Lambda = \frac{2\pi C_T}{\lambda(1+a_t) \sqrt{(1-a)^2 + [\lambda x(1+a_t)]^2} \left[1 + \frac{1}{\kappa} \frac{1-a}{\lambda x(1+a_t)} \right]}, \quad (2.23)$$

from which the chord distribution can be readily obtained if the design lift coefficient of the aerofoil is known. Similarly, with the axial and tangential induction factors calculated, the flow angle can be evaluated and the optimal twist distribution is given

by

$$\beta = \phi - \alpha_0, \quad (2.24)$$

where α_0 is the design angle of attack for the aerofoil section.

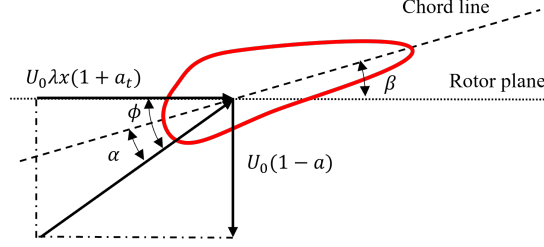


Figure 2.6: Velocity triangle for a blade element of a HAWT rotor.

From a similar analysis, the power produced by a rotor annulus, in terms of the non-dimensional lift distribution, is given by

$$dP = \Omega dQ = \frac{1}{2} \rho U_{rel}^2 B \Lambda R^2 \lambda x \left[\sin(\phi) - \frac{1}{\kappa} \cos(\phi) \right] dx. \quad (2.25)$$

Integrating this across the active rotor radius, substituting in equation 2.23, and normalising, the rotor power coefficient can be obtained:

$$C_P = \int_{x_{hub}}^1 \frac{2C_T(\kappa[1 - a] - \lambda x[1 + a_t])}{\lambda \kappa x(1 + a_t) + (1 - a)} dx. \quad (2.26)$$

Maintaining the thrust coefficient explicitly in this formulation allows this approach to be extended beyond $a = 0.4$ into the high blockage region with equations 2.19.

To evaluate the validity of this framework, rotor designs defined by equations 2.23 & 2.24 can be simulated, and the thrust and power coefficients can be compared to the design thrust coefficient and the predicted power coefficient, as calculated with 2.26.

For the validation study a 3 bladed rotor was used with a 4.7m rotor radius and a 0.8m hub radius with the blades made using a single FFA-W3-241 aerofoil section. The aerofoil polars were generated using X-foil [65] at a Reynolds number of 1 million, a mach number of 0, and a critical amplification factor of 6. A range of design thrust coefficients from 0.1 to 1.5, and a range of design tip speed ratios from 2 to 8 were

Chapter 2. Power take-off with secondary rotors

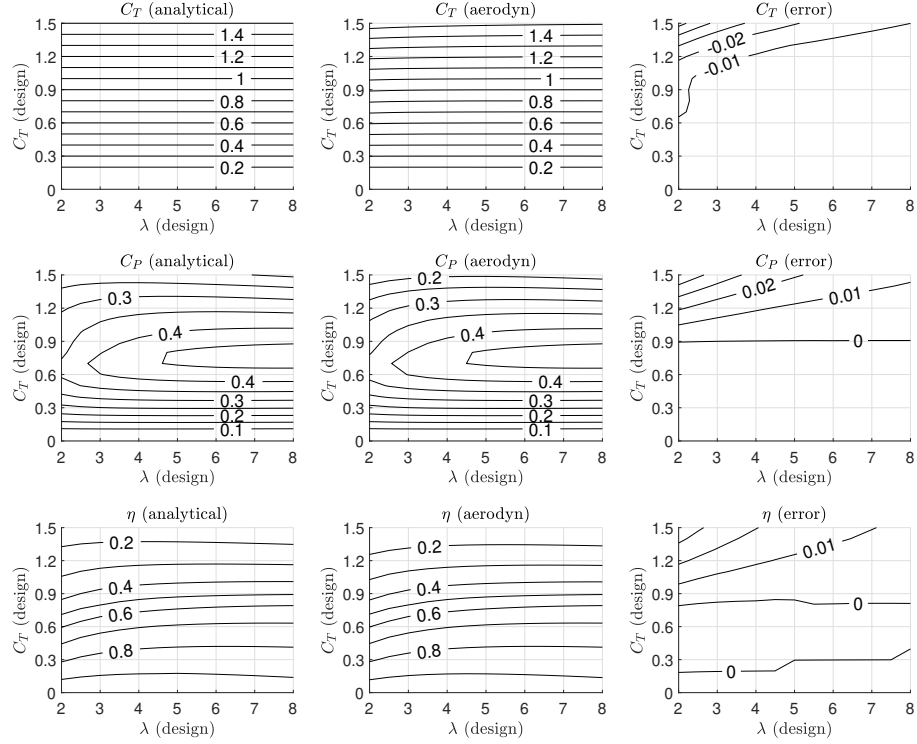


Figure 2.7: Comparison of rotor power coefficient, thrust coefficient and the ratio of power to thrust coefficient as calculated with the semi-analytical optimal design approach, and with the BEM code aerodyn [2].

considered. This comparison is shown in figure 2.7.

In the case of this comparison, the use of aerofoil polar is relatively arbitrary, so long as it remains consistent across the BEM solver and the optimal design methodology. However, it should be noted that, whilst X-Foil is commonly used in wind turbine design and evaluation, it is inherently limited in this context due the fact it ignores 3D effects, and is less accurate as the aerofoil approaches stall. Since this study is looking at the on-design behaviour of the turbine operating well below stall, the use of X-Foil is considered adequate.

For low blockage cases ($C_T \leq 0.9$), there is very strong agreement between the predicted on-design rotor coefficients and the actual on-design rotor coefficients, with no significant error between the two results. For high blockage cases ($C_T > 0.9$) errors

begin to appear in the low design tip speed ratio range. This error in this high blockage, low tip speed ratio range is likely due to the end loss corrections which are not well defined for high blockage cases. High conversion efficiency ($>80\%$) is achieved for $C_T < 0.5$, thus the analytical on-design representation of secondary rotors is considered valid for parametric design studies for wind turbine systems using secondary rotors, as this is the area in which sensible rotor designs exist.

In addition to an accurate description of the aerodynamically active portion of the rotor, the secondary rotor hub must be accounted for. For a typical 5MW wind turbine, the rotor hub represents approximately 0.06% of the rotor swept area [66] however for the X-rotor secondary rotor design, the rotor hub represents up to 4.5% of the rotor swept area. Including the parasitic thrust from the secondary rotor hub is therefore important. The thrust coefficient of the rotor including the contribution of the rotor hub is given by

$$C_T = C_{T_{rotor}} + C_{T_{hub}} \left(\frac{r_{hub}}{R} \right)^2. \quad (2.27)$$

In order to estimate the hub radius of a secondary rotor generator, similarity scaling can be applied to an initially representative design with

$$r_{hub} = k_{hub} Q^{1/3}. \quad (2.28)$$

The design provided in [57] indicates that $k_{hub} = 0.025$. The opportunity for integrated blade root/hub designs could be further explored to increase secondary rotor performance. An investigation into wind turbine hub design was completed due to the high performance of the 2MW ENERCON E70 which utilised a large 'spinner' on the rotor hub and reported a significant increase in the power coefficient [67]. This study used rotor resolved CFD simulations of a typical wind turbine hub, an augmented wind turbine hub with an extended blade, and a hub utilising a large spinner. The study does not find conclusive evidence that an extend optimal blade design or a design with a spinner would increase the ratio between the ratio of the power and thrust coefficient. However, as previously discussed, the optimal secondary rotor design differs significantly from traditional wind turbine design and a specific study is required to properly

understand this effect.

The framework for generating an optimal secondary rotor design, based on the set of equations described above is represented diagrammatically in figure 2.8.

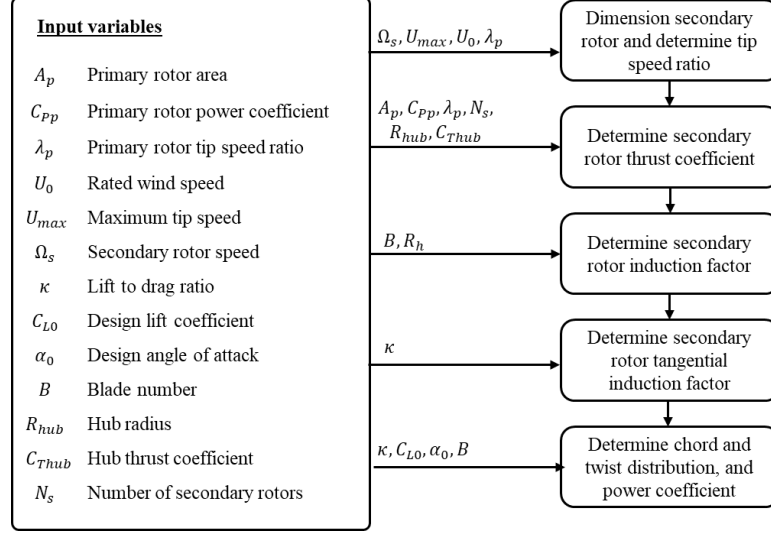


Figure 2.8: Diagrammatic representation of the framework for producing optimal secondary rotor designs for a given primary rotor configuration.

To demonstrate the efficacy of the above framework, a case study is presented below. The generic parameters for a coupled primary/secondary rotor system are given in table 2.1. The rotational speed of the secondary rotor is taken considering a 4 pole pair generator with an output frequency of 25Hz, and the hub radius is taken from [57]. The aerofoil lift to drag ratio is typical for a wind turbine aerofoil at Reynolds numbers of approximately 10^6 . The primary rotor area is scaled to ensure that the total output power of the full turbine system (i.e. the combined output power of the secondary rotors) is equal to 5MW, requiring an iterative processes. It is assumed that a primary rotor power coefficient of 0.45 is achievable over the range of tip speed ratios considered in this study. It should be noted that any change in primary rotor power coefficient would simply result in a larger required primary rotor area and from equation 2.2 it is clear that the secondary rotor operating point and consequently the conversion efficiency between the primary and secondary rotor(s) would be identical as the product $C_{Pp}A_p$ exists in the expression.

Variable	Value
ρ [kg/m ³]	1.225
U_{max} [m/s]	184
P [MW]	5
U_{rated} [m/s]	12
N_s	2
C_{Pp}	0.45
B	5
κ	100
Ω_s [rad/s]	39.27
R_{hub} [m]	1
C_{Thub}	0.4

Table 2.1: Parameters for the baseline wind turbine considered using secondary rotors for power take-off.

Initially, it is instructive to see what the optimum tip speed ratio of the primary rotor should be in this design considering optimally designed secondary rotors. The conversion efficiency between the primary and secondary rotor as a function of primary rotor tip speed ratio is shown in figure 2.9. At each primary rotor tip speed ratio, an optimised secondary rotor is produced using the procedure described above and the total output power of the system is calculated. The area of the primary rotor is then scaled such that the output is equal to the desired output power (5MW). A new optimised secondary rotor is designed around this point and the process is iterated until the output power has converged at the desired value.

To better understand the losses associated with power conversion, figure 2.10 shows the conversion efficiency between the primary and secondary rotor as a function of primary rotor tip speed ratio as calculated with different fidelity secondary rotor representations.

In the low tip speed ratio range, the required thrust coefficient is high, leading to inefficient power conversion in all cases. In the high tip speed ratio range, as the required thrust coefficient decreases, the conversion efficiency as calculated with an actuator disk approach increases. However, when the rotor hub is taken into account the proportion of power dissipated by the parasitic thrust on the rotor hub increases. At high tip speed ratios, the thrust coefficient contribution from the rotor hub remains

constant but the total required thrust coefficient from the secondary rotor decreases. Therefore, a smaller proportion of the primary rotor power is available to be converted. With the primary rotor operating at a tip speed ratio of 8.5, only 77% of the primary rotor power is available to the aerodynamically active portion of the secondary rotors, with 23% dissipated by the parasitic thrust on the secondary rotor hub. As this loss increases with primary rotor tip speed ratio, it offsets the increase in efficiency due to a lower axial induction factor and leads to the defined optimal primary tip speed ratio.

The effect of tip and root losses on conversion efficiency are slightly nuanced. In the case of an actuator disk with tip loss, the tip loss function affects the power and thrust coefficient equally. Therefore rotor designs with an equal axial induction factor will be equally efficient with or without tip loss. However, if a specific thrust coefficient is required, the axial induction required to obtain that thrust coefficient increases. The increase in axial induction factor leads to a decrease in the total conversion efficiency as in equation 2.9. Whilst the magnitude of the tip loss function increases as the primary rotor tip speed ratio increase due to the corresponding reduction in secondary rotor tip speed ratio, the rotor is very lightly loaded at these high primary rotor tip speed ratios and the effect of tip losses are relatively small, leading to conversion efficiency losses associated with tip losses decreasing from 5.5% for $\lambda_p = 4.5$ to 0.5% for $\lambda_p = 8.5$.

The effect of moving from an actuator disk representation to an optimised rotor representation including drag losses and tangential induction are shown to introduce additional losses between 7.4% at $\lambda_p = 4.5$ down to 4% at $\lambda_p = 8.5$. With all of this considered, the maximum achievable conversion efficiency achievable with the power-train described in table 2.1 is 81%, as shown in figure 2.9.

It is also interesting to consider the effect of the rated wind speed on power conversion. Figure 2.11 shows the effect of rated wind speed on conversion efficiency. For a given primary rotor tip speed ratio there exists an optimum rated wind speed for which power conversion is maximised. However the value of the maximum achievable efficiency is independent of rated wind speed if the primary rotor tip speed ratio is treated as a free variable, as shown in figure 2.11. It appears that the optimum primary rotor tip speed ratio is inversely proportional to the rated wind speed, indi-

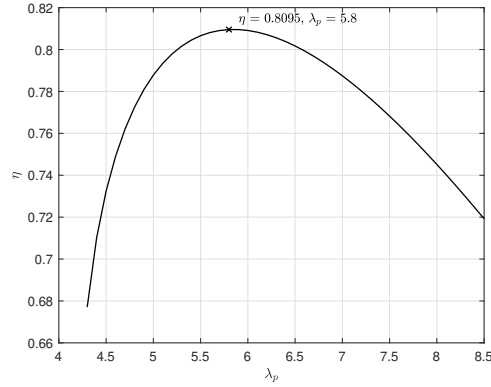


Figure 2.9: Effect of primary rotor tip speed ratio, λ_p , on conversion efficiency, η for a coupled primary/secondary rotor system with the parameters given in table 2.1.

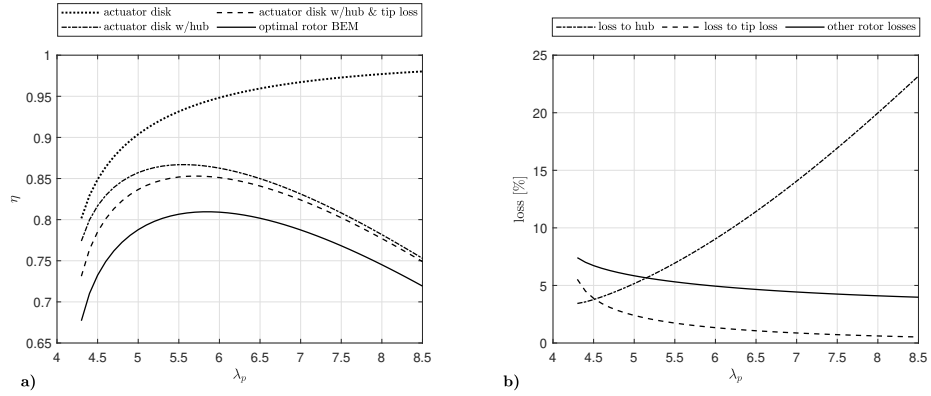


Figure 2.10: Factors contributing towards power conversion efficiency, η , a), and losses, b), as a function of primary rotor tip speed ratio, λ_p . The dotted line represents an ideal actuator disk, the dot-dash line represents an actuator disk with an thrust producing hub region, the dashed line represents an actuator disk with tip loss and a thrust producing hub region, and the solid black line represents the optimally designed BEM rotor.

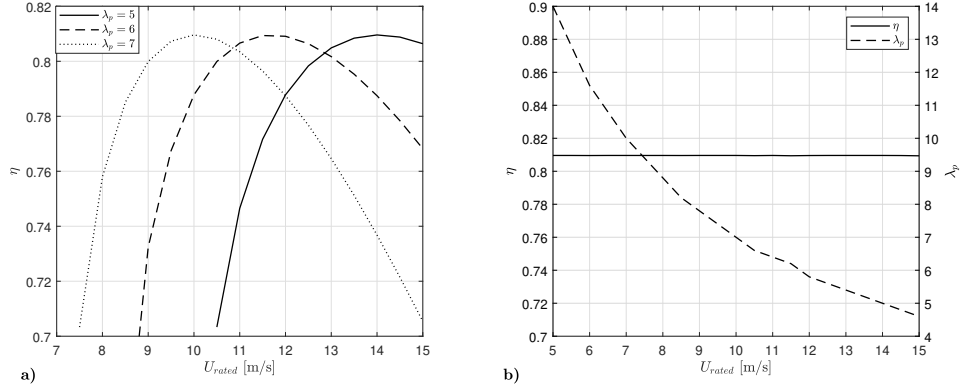


Figure 2.11: Effect of rated wind speed, U_{rated} , on conversion efficiency, η , a), and optimal tip speed ratio, λ_p , b), for a coupled primary-secondary rotor system described in table 2.1.

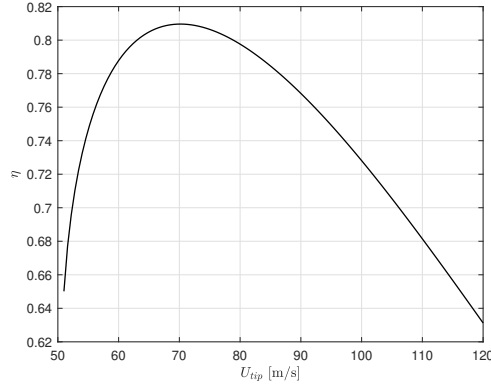


Figure 2.12: Effect of primary rotor tip speed, U_{tip} , on conversion efficiency, η , for the baseline primary-secondary rotor system described in table 2.1.

cating that the primary rotor tip speed is the key variable. Plotting the conversion efficiency against the primary rotor tip speed ($U_{tip,p} = \Omega_p R_p$), the different curves collapse onto each other, and the underlying relationship is revealed, as shown in figure 2.12. This demonstrates that under the present configuration, the optimal tip speed for the primary rotor is 70m/s. A consequence of this is that, for a given secondary rotor drive-train, the key driver of conversion efficiency is the primary rotor tip speed, and that this should be considered when designing turbines for sites with different mean wind speeds.

The effect of the rotor hub radius on the efficiency of power conversion is shown

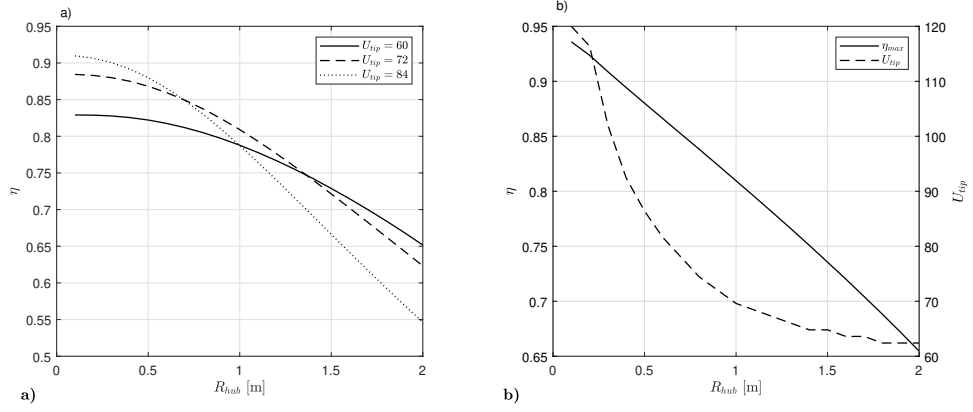


Figure 2.13: Effect of secondary rotor hub radius, R_{hub} , on conversion efficiency, η , a), and optimal tip speed, U_{tip} , b), for a coupled primary-secondary rotor system described in table 2.1.

in figure 2.13. This figure shows the effect of increase hub radius for constant primary rotor tip speed ratios of 4, 5, and 6, and the effect of hub radius on the maximum achievable conversion efficiency and the corresponding primary rotor tip speed ratio.

It is clear that, for a given primary rotor tip speed ratio, there is a negative trend between the hub radius and conversion efficiency. As the hub radius is reduced to smaller values, the effect on the conversion efficiency reduces, i.e. $|d\eta/dR_{hub}|$ decreases. This shows the diminishing returns from efforts to shrink the secondary rotor nacelle. Another clear trend is that the efficiency is more sensitive to changes in the hub radius at higher primary rotor tip speeds. It is also demonstrated that as the hub radius is reduced, operation at higher primary rotor tip speed ratios is favoured as less power is lost to parasitic thrust. This facilitates significantly more efficient power conversion.

The effect of the maximum allowable tip speed for the secondary rotor on the conversion efficiency between the primary and secondary rotors is shown in figure 2.14 a). As the rotational speed of the secondary rotors is fixed, the swept area of the secondary rotors is proportional to U_{max}^2 , a consequence of equation 2.10. It follows that, as the maximum allowable secondary rotor tip speed decreases, the required thrust coefficient for the secondary rotors increases and the efficiency of power conversion drops. Additionally, for a given primary rotor tip speed, a higher maximum secondary rotor tip speed facilitates operation at higher tip speed ratios, decreasing tip losses. The

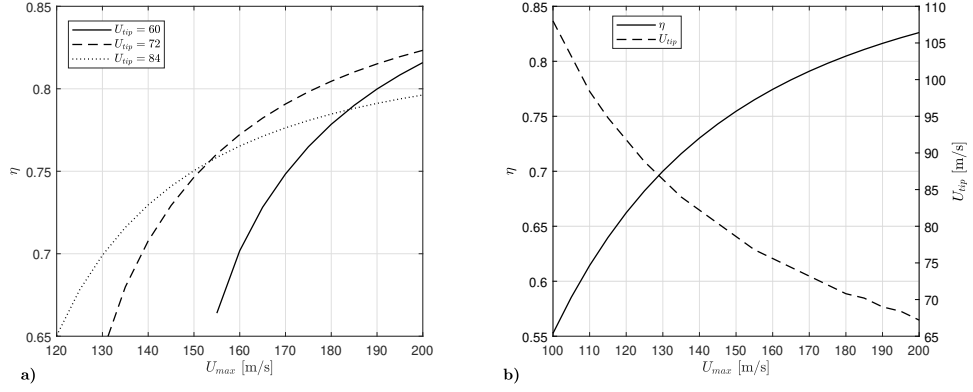


Figure 2.14: Effect of maximum allowable tip speed, U_{max} , on conversion efficiency, η , a), and optimal primary rotor tip speed, U_{tip} , b), for a coupled primary-secondary rotor system described in table 2.1.

relationship between the maximum secondary rotor tip speed, and both the optimal primary rotor tip speed and the maximum achievable power conversion efficiency is shown in figure 2.14 b). It is clear that, as the maximum secondary rotor tip speed decreases, the optimal primary rotor tip speed increases. The increase in primary rotor tip speed causes a relative decrease in the secondary rotor thrust coefficient, yielding more efficient power conversion.

More specific to the secondary rotor, the effects of blade number, and aerofoil lift to drag ratio can be investigated. This is completed considering a turbine with the baseline parameters provided in table 2.1, operating at 12m/s at the optimal primary rotor tip speed ratio, design $\lambda_p = 5.8$. The effect of blade number and aerofoil lift to drag ratio are shown in figure 2.15. Increasing the lift to drag ratio of the aerofoil section used in the blade is shown to have a limited effect on power conversion efficiency. The key effect enabled by an increased lift to drag ratio is a larger power coefficient for a given design thrust coefficient.

Increasing blade number also increases the conversion efficiency. Increasing the blade number reduces the magnitude of tip loss effect, which are equally weighted between the power and thrust coefficients. The increase in conversion efficiency is caused by a decrease in the required induction factor due to the decreased tip losses and the associated increase in power to thrust coefficient ratio.

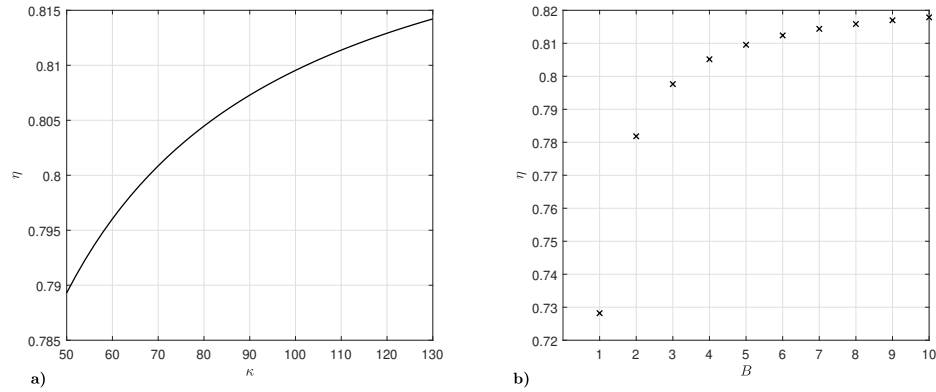


Figure 2.15: Effect of aerofoil lift to drag ratio, κ , on conversion efficiency, η , a), and secondary rotor blade number, B , on conversion efficiency, b), for a coupled primary-secondary rotor system described in table 2.1.

It should be noted that the design lift to drag ratio of an aerofoil section increases with increasing Reynolds number, and that the chord length (and therefore Reynolds number) is inversely proportional to the number of blades employed. In order to maximise conversion efficiency, a balance must be struck between the increase in efficiency obtained through increasing blade number, and the decrease due to decreasing Reynolds number. A more practical limitation on blade number is another consequence of the associated decrease in the design chord length. Assuming similarity scaling, the bending stresses within the blade increase with c^{-3} , structural limitations imposed by the blade design and material choice are therefore likely to be the key limiting factor in deciding the maximum blade number.

Changing the rated power of a wind turbine system using secondary rotors has a number of effects: in terms of the secondary rotor drive-train, if the rotor speed remains the same, the torque on the secondary rotors will increase. This will lead to an increase in the generator diameter and a corresponding increase in the rotor hub radius as in equation 2.28. Additionally, the increased power from the primary rotor will require a larger thrust coefficient from the secondary rotors (for a given primary rotor tip speed ratio), with consequences for the conversion efficiency. A method of preserving a relatively high conversion efficiency as turbines increase in power rating is through the inclusion of more secondary rotor systems. Figure 2.16 shows the efficiency of power

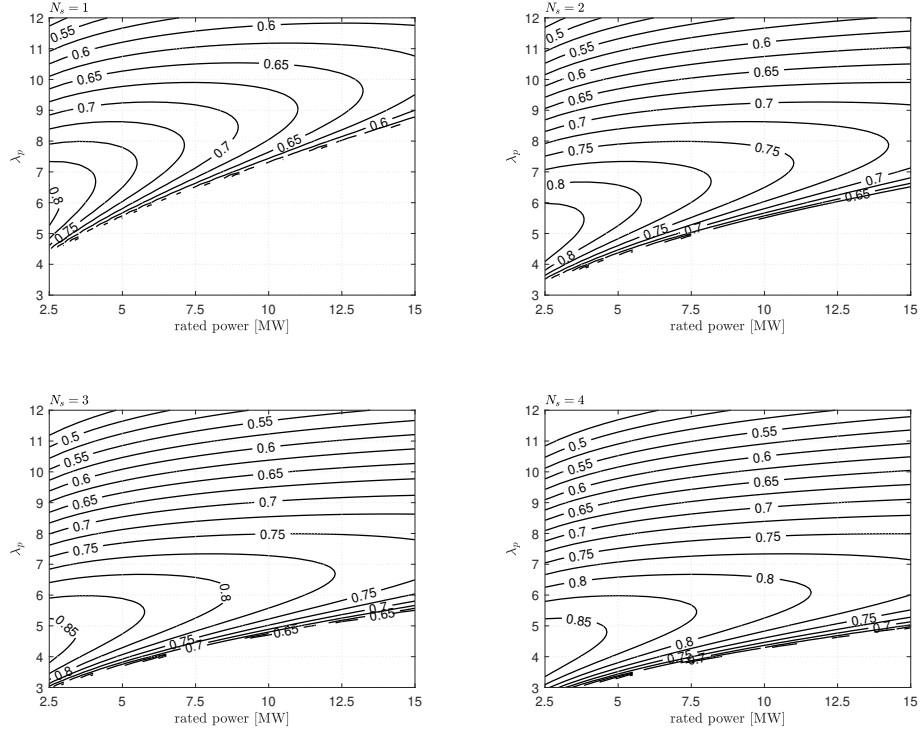


Figure 2.16: Effect of rated power and primary rotor tip speed ratio, λ_p , on conversion efficiency for a coupled primary-secondary rotor system described in table 2.1 with the number of secondary rotors increasing from 1 to 4.

conversion between the primary and secondary rotor as a function of primary rotor tip speed and rated power for turbines utilising 1, 2, 3, and 4 secondary rotors with power-trains described by table 2.1. This demonstrates that turbines with greater power ratings typically require higher primary rotor tip speed ratios to ensure low induction operation and high conversion efficiency. Additionally, the use of multiple secondary rotors increases total swept area and facilitates more efficient power conversion at lower tip speed ratios and higher power ratings.

Through representing the secondary rotors as optimally designed rotors, insight into the coupled design of primary and secondary rotors has been drawn and a procedure for the optimal aerodynamic design of secondary rotors for a given primary rotor configuration, which can be used as a baseline for more holistic secondary rotor design,

has been put forward.

In general, three factors have been identified that affect the conversion efficiency between the primary and secondary rotor

1. The design axial induction factor of the secondary rotor:

Much like the case of the actuator disk, it has been found that reducing the design axial induction factor increases the conversion efficiency between the primary and secondary rotors. The design axial induction factor is predominantly driven by the design thrust coefficient which can be decreased by increasing the relative swept area of the secondary rotors ($N_s A_s / A_p$), or increasing the primary rotor tip speed ratio. Specific to the design of the secondary rotors, increasing the number of blades mitigates the effects of tip losses and decreases required axial induction for a given thrust coefficient and increasing the secondary rotor tip speed ratio has a similar effect. The secondary rotor tip speed ratio is, however, dictated by a number of other variables such as the primary rotor tip speed and the maximum allowable tip speed.

2. The proportion of power lost to parasitic thrust:

Another key factor that affects the power conversion between the primary and secondary rotor is the fraction of power lost to parasitic thrust on the secondary rotor hub. This can be represented as the proportion of the design thrust coefficient that is provided by the secondary rotor hub. As the thrust coefficient contribution from the rotor hub remains constant, the proportion of the available power dissipated by the thrust on the secondary rotor hub increases as the design thrust coefficient of the secondary rotor decreases. This effect lowers the conversion efficiency for very low induction secondary rotors.

3. The maximum achievable power coefficient for a given axial induction factor:

The final factor identified that effects the efficiency of power conversion between the primary and secondary rotors is the maximum achievable power coefficient for a given axial induction factor. This can be increased through operation at

a higher secondary rotor tip speed ratio, increasing the lift to drag ratio of the aerofoil section, and increasing the number of blades.

An additional conclusion that has been drawn is that, for a given power rating, the efficiency of secondary rotors is dependent on the primary rotor tip speed. This implies that secondary rotors coupled with high tip speed ratio primary rotors represent an effective means of power take-off for low wind speed sites. For conventional wind turbines, when moving to low wind speed sites, the rotor radius must be increased to maintain a given power rating. As the tip speed of the rotor is limited, the rotational speed of the rotor must decrease resulting in increased drive-train torque. In the case of a coupled primary-secondary rotor system, an identical secondary rotor system may be employed if the primary rotor tip speed is maintained, as the secondary rotor speed and power rating is constant, there is no torque penalty on the drive-train.

2.5 Flow augmentation

The use of flow augmentation devices, such as ducts or diffusers, have been considered for wind turbine applications from the late 1950s, with the work of Lilley and Rainbird [68]. The general idea of the use of such devices is that the flow can be locally augmented, increasing the mass flow rate through the rotor plane and enabling enhanced power capture. A thorough review of the state of the art in ducted turbine design is given in [69], and the fluid dynamic aspects of ducted turbines are well described in [70].

The use of flow augmentation devices as part of a secondary rotor system has not previously been investigated. It has been assumed that the additional thrust force acting on a flow augmentation device would negate any increase in the rotor power coefficient that could be achieved, and because of this no analysis has been completed.

The challenge of duct design will not be undertaken here, but the simplified model of an actuator disk in augmented flow put forward by Jamieson [71] will be used to explore the design potential for ducted secondary rotors.

In this method, the power and thrust coefficient of an actuator disk in augmented

Chapter 2. Power take-off with secondary rotors

flow is given by

$$C_P = \frac{4(a - a_d)(1 - a)^2}{(1 - a_d)^2} \quad (2.29)$$

$$C_T = \frac{4(a - a_d)(1 - a)}{(1 - a_d)^2} \quad (2.30)$$

where a_d represents the induction from the flow augmentation device at the rotor plane without the presence of power extraction (defined in the same sense as the induction from the actuator disks i.e. a negative value of a_d represents flow acceleration).

In addition to the thrust on the rotor, the flow augmentation device experiences an additional thrust force, as shown in [16], this is given by

$$T_D = -a_d T_R \quad (2.31)$$

where T_R represents the rotor thrust. Normalising for the swept area of rotor, the total thrust coefficient for the secondary rotor and duct is given by

$$C_T = \frac{4(a - a_d)(1 - a)}{(1 - a_d)}. \quad (2.32)$$

The efficiency of conversion between the primary and secondary rotors is therefore given by

$$\eta = \frac{(1 - a)}{(1 - a_d)}. \quad (2.33)$$

For a given thrust coefficient and duct induction, the axial induction factor of the secondary rotor is given by

$$a = \frac{(1 + a_d) - \sqrt{(1 + a_d)^2 - C_T(1 - a_d) - 4a_d}}{2}, \quad (2.34)$$

and the efficiency of conversion between the primary and secondary rotor becomes

$$\eta = \frac{1}{2} + \frac{1}{2} \sqrt{\frac{1 - C_T - a_d}{(1 - a_d)}}. \quad (2.35)$$

Considering a efficient combined primary and secondary rotor design, with a design secondary rotor thrust coefficient of 0.36, indicating an ideal conversion efficiency of

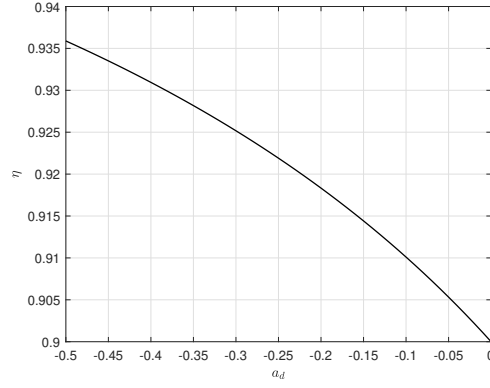


Figure 2.17: Effect of duct induction, a_d , on secondary rotor conversion efficiency, η .

0.9, the conversion efficiency as a function of duct induction factor is shown in figure 2.17. It can be seen that as the magnitude of the duct induction increases, the efficiency of power conversion increases and from equation 2.35, it is clear that the conversion efficiency tends to 1 for any value of C_T if the duct induction is significantly large. It should be noted that this approach assumes an ideal diffuser with no additional forces from skin friction, and the increase in conversion efficiency should be treated as an upper bound, rather than a performance prediction.

In this ideal case, it has been demonstrated that flow augmentation devices could be highly effective in increasing the efficiency of secondary rotor systems, and further work in this area is recommended; however, a more thorough investigation into the design of ducted secondary rotors is outside of the scope of this thesis. Duct design and the simulation of ducted rotors represents a significant challenge even in plane inflow, and the design of a duct for secondary rotors would be complicated further due to the orbital motion of the secondary rotors. It may be useful to employ an approach based on conformal mapping, where a duct is designed for plane inflow, then a conformal map similar to that put forward by Migliorie [72] could be employed to obtain a duct with similar characteristics in orbital motion.

Thus far the primary rotor has been represented in a generic manner and the effects of the specific primary rotor configuration (either horizontal or vertical axis) have not been considered. The following two sections will discuss specific issues around the

implementation of secondary rotors on HAWTs and VAWTs respectively.

2.6 Application to HAWTs

As discussed in Section 2.2, secondary rotors have previously been considered for application to HAWTs. In this case, the apparent wind speed normal to the secondary rotor can be assumed to be given by

$$U_I = \Omega_p R_p \cos(\chi) + U_0 \sin(\chi) \quad (2.36)$$

where the first term represents the contribution due to the rotation of the primary rotor blade, and the second term represents the contribution of the ambient wind speed incident on the secondary rotor. χ represents the angle of the secondary rotor axis relative to the primary rotor plane, as shown in figure 2.18.

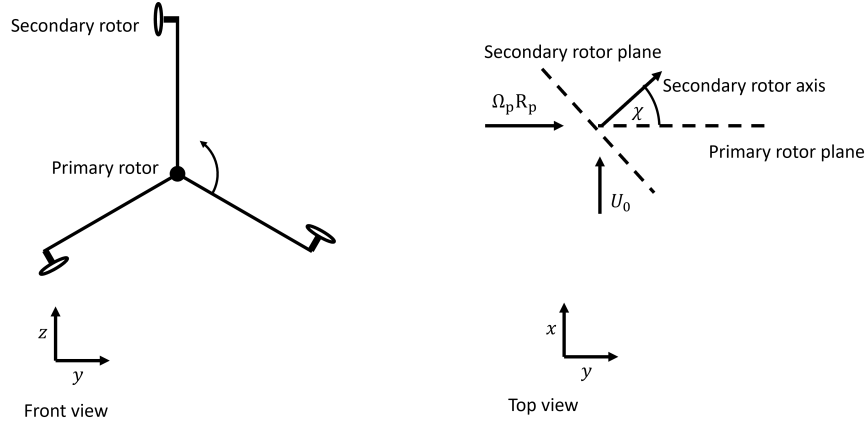


Figure 2.18: Definition of the angle of the secondary axis relative to the primary rotor plane, χ .

The inflow speed is maximised with

$$\chi = \text{atan} \left(\frac{U_0}{\Omega_p R_p} \right) = \text{atan} \left(\frac{1}{\lambda_p} \right) \quad (2.37)$$

giving

$$U_I = (\Omega_p R_p) \sqrt{1 + \frac{1}{\lambda_p^2}}. \quad (2.38)$$

Taking this maximum and using equations 2.36 and 2.1, the thrust coefficient required to hold the primary rotor at a given operating point is given by

$$C_{Ts} = \frac{C_{Pp}A_p}{N_sA_s} \frac{1}{\lambda_p^2} \frac{1}{\sqrt{1 + 1/\lambda_p^2}} \quad (2.39)$$

and the conversion efficiency at that operating point is given by

$$\eta = \frac{C_{Ps}}{C_{Ts}} \sqrt{1 + \frac{1}{\lambda_p^2}}. \quad (2.40)$$

The aerodynamic efficiency of a horizontal axis primary rotor can be evaluated using the optimally designed rotor representation discussed in Section 2.4.2. Whilst a full optimisation would include a variable induction distribution, for simplicity the primary rotor can be designed for a constant axial induction of $a = 1/3$ (the optimal value for an actuator disk). Again, for simplicity, a constant lift to drag ratio can be assumed across the primary rotor, here a value of $\kappa = 125$ is assumed. Whilst rotors will typically employ thicker, less efficient aerofoil sections towards the blade root, the majority of power production is located in the outboard sections where this assumption is valid. The power coefficient is presented as a function of design tip speed ratio for both a two bladed and a three bladed primary rotor in figure 2.19.

It can be seen that, for the secondary rotor drive-train considered here, full turbine power coefficients of up to 0.38 are achievable for a two bladed 5MW turbine, and up to 0.4 for a three bladed 5MW turbine. Increasing the number of secondary rotors increases their swept area, decreasing the optimum tip speed ratio for the primary rotor.

As discussed in Section 2.4.2 secondary rotors appear to be a useful means of ensuring that the cost of power take-off does not increase as the primary rotor reduces in specific power (i.e. decreasing the rated wind speed). Figure 2.20 shows the achievable full turbine power coefficient for a HAWT utilising a pair of secondary rotors with a 5MW rating as a function of rated wind speed and primary rotor tip speed ratio.

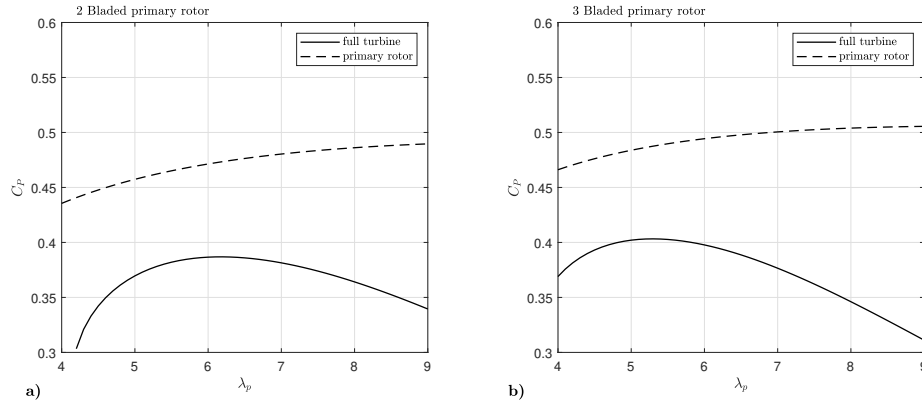


Figure 2.19: Effect of primary rotor tip speed ratio, λ_p , on primary rotor power coefficient and full turbine power coefficient, for a HAWT primary rotor with a baseline secondary rotor power-train defined in table 2.1 with two blades, a), and three blades, b).

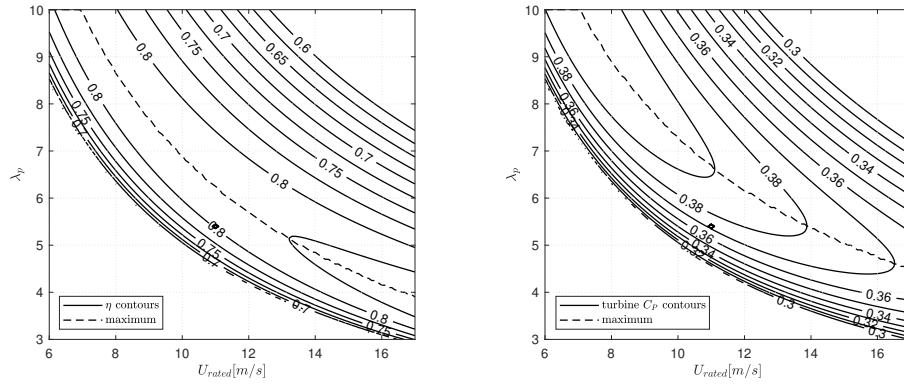


Figure 2.20: Maximum power conversion efficiency, a), and turbine power coefficient, b), for a two bladed 5MW turbine with a HAWT primary rotor as a function of primary rotor tip speed ratio, λ_p , and rated wind speed, U_{rated} .

2.7 Application to VAWTs

When secondary rotors are applied to the blade tips of VAWTs, they see an inflow velocity given by

$$U_I = \Omega_p R_p \cos(\chi) + U_0 \sin(\theta + \chi) \quad (2.41)$$

where χ represents the angle between the secondary rotor normal axis and the tangent to the primary rotor circumference. The maximum average velocity over a revolution is obtained with $\chi = 0$, the corresponding inflow speed is given by

$$U_I = \Omega_p R_p + U_0 \sin(\theta). \quad (2.42)$$

Taking this maximum and using equations 2.2 and 2.1, the thrust coefficient required to hold the primary rotor at a given operating point is given by

$$C_{Ts} = \frac{C_{Ps} A_p}{N_s A_s} \frac{1}{\lambda_p^3} \frac{1}{1 + \frac{1}{2} \lambda_p^{-2}} \quad (2.43)$$

and the conversion efficiency at that operating point is given by

$$\eta = \frac{C_{Ps}}{C_{Ts}} \frac{1 + \frac{3}{2} \lambda_p^{-2}}{1 + \frac{1}{2} \lambda_p^{-2}}. \quad (2.44)$$

The assumption that the inflow velocity is independent of the induced velocities by the primary rotor, as characterised by equation 2.41, is verified in appendix A. In terms of primary rotor representation, there does not exist a framework similar to that for HAWTs to estimate the maximum achievable conversion efficiency for a given VAWT rotor configuration and the challenge of VAWT design is discussed at length in Chapter 3. This means that the full turbine power coefficient, a useful figure of merit, cannot be readily calculated in parametric studies. Despite this, the achievable power conversion efficiency for VAWTs of various power ratings and primary rotor tip speed ratios can be calculated as any decrement in power coefficient is responded to by a proportionate increase in rotor area, and the secondary rotor operating point is maintained, as in equation 2.43. The corresponding conversion efficiency is then only a function of the

maximum achievable secondary rotor power coefficient and the primary rotor tip speed ratio.

The approach used in Section 2.4.2 can be employed, modifying equations 2.14 and 2.6 with equations 2.43 and 2.44 to obtain conversion efficiency estimates for a 5MW VAWT at different design tip speed ratios with the power-train described in table 2.1 and optimally design secondary rotors. This is shown in figure 2.21. The validity of the representation of the secondary rotors using the equations developed in section 2.4.2 is discussed in appendix A. It can be seen that conversion efficiencies in excess to 0.8 are achievable.

Additionally, the possibility to deploy VAWTs in low wind speed sites is interesting. Figure 2.22 shows the conversion efficiency of a 5MW Turbine utilising 2 secondary rotors as a function of primary rotor tip speed ratio and rated wind speed.

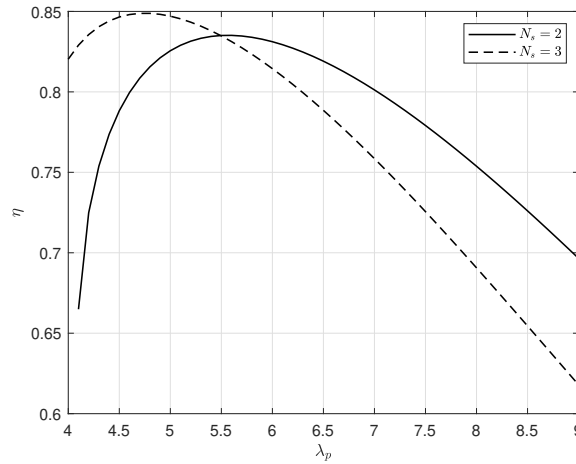


Figure 2.21: Effect of primary rotor tip speed ratio, λ_p , on conversion efficiency, η , for a VAWT primary rotor with a baseline secondary rotor power-train defined in table 2.1 for a two bladed and 3 bladed rotor.

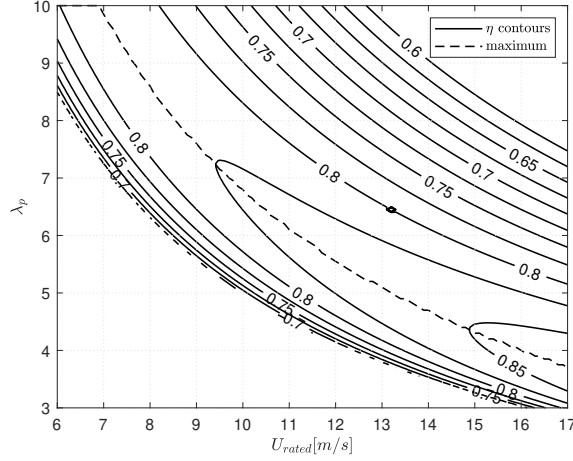


Figure 2.22: Effect of primary rotor tip speed ratio, λ_p , and rated wind speed, U_{rated} , on conversion efficiency for a VAWT primary rotor with a baseline secondary rotor power-train defined in table 2.1.

2.8 HAWT vs VAWT primary rotor

At this point, with the aerodynamic efficiency of secondary rotors for applications on both HAWT and VAWT rotors understood, it is instructive to compare each case. This section considers the application of secondary rotors to both HAWTs and VAWTs in terms of aerodynamic efficiency, additional loads imparted by the secondary rotors, potential for control, and other design synergies.

2.8.1 Aerodynamic efficiency

Figure 2.23 shows the efficiency of power conversion for a HAWT, a VAWT, and the case where the secondary rotor inflow is considered to be equal to the tip speed of the rotor, considering the rotor parameters provide in table 2.1. From this, it is clear that the maximum conversion efficiency achievable by a VAWT rotor is higher than that achievable by a HAWT rotor with an equal power rating for any given primary rotor tip speed ratio. It should also be noted that the ideal primary rotor tip speed ratio drops from 5.8 for the case without considering the free wind speed, to 5.7 for a HAWT, to 5.6 for a VAWT.

This is driven by the difference in inflow velocities as given in equations 2.36 &

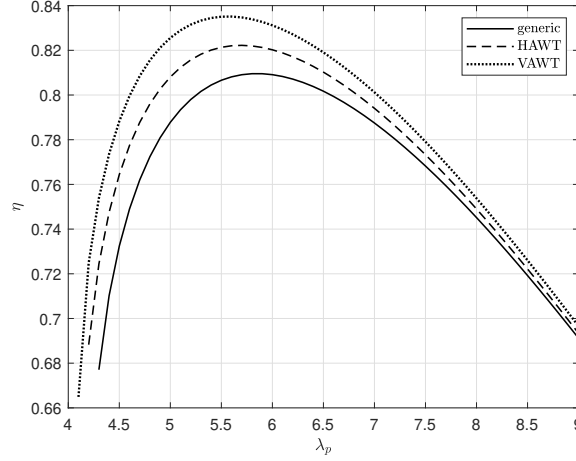


Figure 2.23: Change in conversion efficiency, η for HAWT and VAWT primary rotors as a function of primary rotor tip speed ratio, λ_p , as calculated with the baseline turbine parameters given in table 2.1.

2.41. The increase in conversion efficiency for both HAWTs and VAWTs when the full inflow velocity to the secondary rotors is considered arises from two effects. Firstly, the increase in the square of the revolution averaged inflow velocity compared to the primary rotor tip speed lowers the required thrust coefficient of the secondary rotor. This allows the secondary rotors to operate at a lower axial induction and achieve a higher conversion efficiency. Secondly, the secondary rotors capture power from the free wind speed as well as the wind speed induced by the rotation of the primary rotor. The increase in power capture associated with these effects is equal to the average of cube of the incident wind speed over the product of the primary rotor tip speed and the mean of the square of the wind speed on the secondary rotors.

The increase in thrust coefficient is shown in figure 2.24a and the increase in conversion efficiency due to the power extraction from the incident wind speed is shown in figure 2.24b. The higher average incident wind speed on the secondary rotors experienced by a HAWT facilitate operation at lower thrust coefficients compared to VAWTs; however, the cyclic nature of the inflow speed on VAWTs means that more energy can be extracted from the free wind speed.

In terms of the achievable conversion efficiency between primary and secondary rotors, VAWT rotors are marginally more effective than HAWT rotors with an maximum

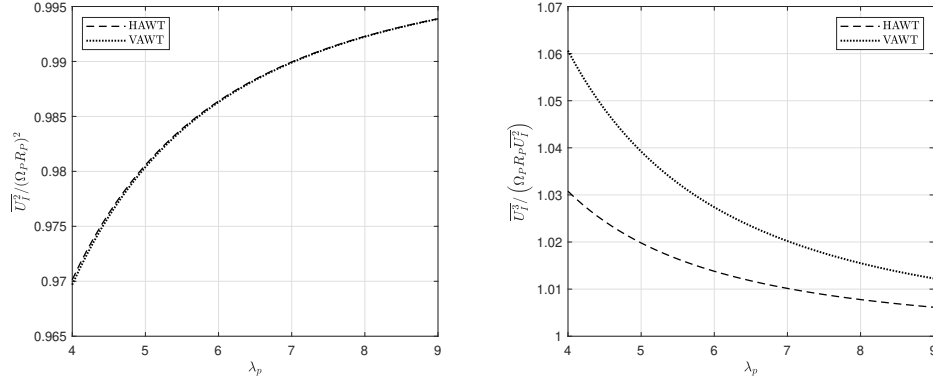


Figure 2.24: Change in effective thrust coefficient, a), and increase in power capture, b), due to the effect of the incident wind speed for HAWT and VAWT primary rotors as a function of primary rotor tip speed ratio, λ_p .

conversion efficiency approximately 1.5% higher in the optimal case. In terms of the optimal tip speed ratio, the optimum tip speed ratio is dependent on the rated wind speed of the turbine, and the optimum primary rotor tip speed (for a given secondary rotor tip speed/area) is a more fundamental quantity. The alignment of the optimal tip speed ratio in terms of primary rotor power coefficient and efficient power conversion must be considered on a case by case basis. In this case the optimal tip speed ratio of 5.5-5.7 which is larger than those typically associated with a VAWT and smaller than those typically associated with a HAWT. Whilst an optimal rotor design framework exists for HAWTs, no such framework exists for the optimal design of VAWTs. The design of high tip speed ratio VAWTs is discussed in Chapter 3 and it is found that straight bladed VAWTs can be designed to operate at sufficiently high power coefficients for this tip speed ratio range through inclining the rotor blades with respect to the vertical axis, as shown in figure 1.2.

2.8.2 Operation / control

A discussion of operational strategies for wind turbines utilising secondary rotors is provided in Chapter 4; however, a brief discussion of control options is discussed here as a means of comparing the suitability of secondary rotors for HAWT and VAWT primary rotors.

Conventional wind turbines have converged on variable speed variable pitch operation, with the rotor speed changing to track the optimal tip speed ratio in below rated conditions, and the blades pitching to feather to shed aerodynamic power and maintain a constant rotor speed in above rated conditions. Rotor speed control in below rated conditions is applied through controlling the generator torque.

For a wind turbine utilising secondary rotors, a similar strategy seems sensible whereby the turbine system is controlled to maximise power capture below rated wind speed and a constant power output is achieved once rated power is obtained. This can be readily achieved through maintaining the primary rotor at a constant rotor speed in above rated conditions.

Below rated operation

The below rated operation of both HAWTs and VAWTs with secondary rotors is relatively similar, with maximum power point tracking achieved through operating the secondary rotors at a constant tip speed ratio, corresponding to the thrust coefficient given by equation 2.2. And it is only in above rated conditions that significant differences occur.

Above rated operation

In terms of above rated control, in the case of vertical axis primary rotors, the most favoured configuration employs separate upper and lower blades, with the secondary rotors attached to the lower blade tips. This allows pitch control of the upper blades without changing the rotor normal direction for the secondary rotors. Deterministic power fluctuations are inherent in the VAWT configuration due to fluctuations in the incident wind speed, as shown in equation 2.41, and the magnitude of these fluctuations increase as the primary rotor tip speed ratio decreases. As previously discussed, these fluctuations increase the available power from the secondary rotors. The relative power fluctuations are a function of the primary rotor tip speed ratio with

$$\frac{[\max(/ \min(P_s))]}{\overline{P_s}} = \frac{\lambda_p^3 \pm 3\lambda_p^2 + 3\lambda_p \pm 1}{\lambda_p^3 + \frac{3}{2}\lambda_p}. \quad (2.45)$$

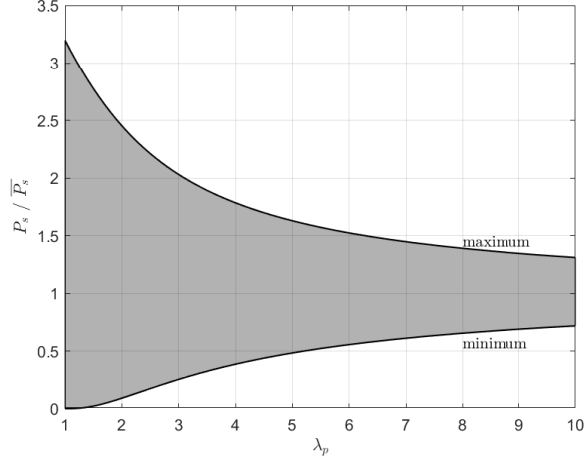


Figure 2.25: Fluctuations in secondary rotor power as a function of primary rotor tip speed ratio, λ_p .

where $+$ is taken for the maximum and $-$ is taken for the minimum. The fluctuations in available power to the secondary rotors is shown in figure 2.25. A primary rotor with a tip speed ratio of 5 and a rated wind speed of 12m/s will be operating at a tip speed ratio of 2.4 at 25m/s, it is unlikely to be economically beneficial to over-rate the secondary rotor drive-train to deal with power fluctuations up to 2.25 times the mean power, and some cyclic variation in secondary rotor tip speed ratio is likely to be required to limit power fluctuations. A further discussion of this is given in Chapter 4.

In the case of horizontal axis primary rotors, pitching the primary rotor blades changes the inclination of the secondary rotor plane. This introduces a yaw offset that may have significant effects on the power captured by the secondary rotor. A typical pitching range for HAWT (the NREL 5MW reference wind turbine is taken as an example here) may be up to 24° [66]. According to equation 2.37, the optimum angle between the primary rotor plane and the secondary rotor normal direction increases from 10.1° at rated wind speed to 21.4° at cut out wind speed for a primary rotor operating at an optimal tip speed ratio of $\lambda_p = 5.6$, with a rated wind speed of 11.4m/s (set to align with the rated wind speed of the NREL 5MW reference wind turbine). Assuming that the secondary rotors are attached at the optimal angle during maximum

power point tracking $\chi_0 = \text{atan}(1/\lambda'_p)$, the yaw misalignment increases from 0 to a maximum of approximately 10° at 25m/s, and the inflow velocity to the secondary rotors increases up to 5% at the rated wind speed of 25m/s, corresponding to an equivalent increase in power capture. Pitching the primary rotor blades to shed loads does not, therefore, seem to have a large negative impact on power capture. However, there could be load implications. The gyroscopic loads on the secondary rotor blades and shaft are dependent of the effective yaw rate. For a typical horizontal axis machine, the maximum pitch rate is approximately $3^\circ/\text{s}$, corresponding to a maximum effective secondary rotor yaw rate of 0.05 rad/s. The gyroscopic loads induced by the rotation of the primary rotor are proportional to the rotational speed of the primary rotor. For vertical axis machines this is typically around 0.8 rad/s, and is expected to be slightly higher for horizontal axis machines designed for use with secondary rotors. This implies that gyroscopic loads from blade pitching will not be a considerable load source compared to the gyroscopic effects associated with primary rotor rotation.

Deterministic secondary rotor power fluctuations will be present due to the sampling of the wind shear profile. Assuming a shear exponent of 0.12, and a primary rotor tip speed ratio of 5.6, fluctuations of -3% and +2% about the mean power are experienced. Considering constant speed operation in above rated conditions these power swings increase to +15% -19% at a wind speed of 25m/s. Whilst considerably lower than the case for a vertical axis primary rotor, some cyclic secondary rotor control may be required to limit the magnitude of power fluctuations.

Overview

In general, the control of HAWTs and VAWTs using secondary rotors in below rated conditions is very similar, with both configurations employing constant tip speed ratio control of the secondary rotors. VAWTs will have considerably larger deterministic power fluctuations from a single generator (approximately $\pm 50\%$ compared to $\pm 2.5\%$). In terms of above rated control, the complexity of control for HAWTs is increased compared to VAWTs as the primary rotor pitch angle is coupled to the secondary rotor yaw offset in the former case and can be decoupled in the latter. A thorough study of

the effect of this coupling on the increased complexity of the controller is outside of the scope of this thesis. In terms of deterministic power fluctuations, larger fluctuations are experienced by the VAWT at cut out wind speeds (125% compared to 15%) require cyclic secondary rotor control to limit power fluctuations.

The operation of both VAWTs and HAWTs utilising secondary rotors appears feasible, with VAWTs requiring cyclic secondary rotor speed control to decrease power fluctuations but with HAWTs having more complex coupled dynamics between the primary rotor pitch control and the secondary rotors.

2.8.3 Loads

In terms of the rotor loads, one can consider separately the loads on the primary rotor and the loads of the secondary rotor.

Primary rotor loads

The loads imparted onto a primary rotor from the secondary rotors are fourfold: Aerodynamic, gravitational, torque-reaction, and inertial.

The primary aerodynamic force acting on the secondary rotors is the thrust force. This provides the retarding torque that counters the aerodynamic torque on the primary rotor. In the case of horizontal axis primary rotors, each blade would have a secondary rotor at the blade tip, leading to very low edgewise blade root bending moments due to load cancellation. In the case of vertical axis primary rotors in the X-Rotor configuration, the upper blades do not have secondary rotors attached. Because of this, the resultant mean torque on the lower blades, including the effects of the secondary rotors, must be equal and opposite to the mean torque the upper blades, the edgewise cancellation of loads at the blade root is therefore not achieved in the case of vertical axis primary rotors in the X-configuration. In both configurations the secondary rotors provide significant damping to the edgewise vibration of the rotor blades to which they are attached.

The aerodynamic loads on the primary rotor differ significantly between HAWTs and VAWTs. HAWTs experience a steady aerodynamic load in a uniform inflow. In

operation, the aerodynamic load fluctuations are caused by both deterministic velocity fluctuations, tower shadow and wind shear, and stochastic velocity fluctuations, turbulence. VAWT rotors experience strong cyclic loading in uniform inflow conditions, corresponding to the change in angle of attack as the blade moves into and out-of the incoming wind direction. In operation, wind shear is not a cause of load fluctuations and the specific rotor geometry determines the salience of tower shadow on the loads. Stochastic velocity fluctuations due to turbulence are also a key source of loads for VAWT rotors. It is expected that the loading on a VAWT rotor will be more severe than on an equivalent HAWT rotor, primarily due to the deterministic load fluctuations associated with the blade tracking into, and out-of the incoming wind direction.

The gravitational loads due to the presence of secondary rotors vary in character significantly between horizontal and vertical axis primary rotors. In the case of vertical axis primary rotors, the gravitational load is constant, with a value of $F_g = mg$. A component of this forces acts as an additional, constant, flapwise bending moment on the lower blades given by $mgL_L \sin(\psi_L)$. The other component, given by $mg \cos(\psi_L)$, acts along the axis of the blade, holding it in tension and increasing the stiffness of the blade. Here L_L represents the length of the lower blade and ψ_L represents the inclination angle of the lower blade. For a VAWT rotor, the additional gravitational load on the primary rotor blades, caused by the presence of the secondary rotors, is therefore expected to be relatively benign. In the case of a horizontal axis primary rotor, the gravitational force of the secondary rotor provides a time varying edgewise bending moment with given by $mgR_p \sin(\theta)$. This leads to reversing once per revolution (1P) loads on the blade. These could significantly increase the damage to the turbine blade and potentially act as a barrier towards upscaling due to the unfavourable scalability of gravitational loads.

Since the secondary rotors will produce the same amount of aerodynamic torque at the same power rating, the generator reaction torque which must be supported by the blade will be the same for both HAWT and VAWT primary rotors. Additionally, the gyroscopic load is proportional to the product of the two rotor speeds, and this product will be similar for both configurations. The gyroscopic loads due to the change

in direction of the secondary rotor angular momentum vector will therefore be similar.

The centrifugal load due to the secondary rotors is also likely to be similar; however, in the case of a VAWT primary rotor in the X-Rotor configuration, this force has a component acting as an additional flapwise load and a component acting along the axis of the blade increasing the apparent stiffness. The flapwise load generated from the centrifugal force on the secondary rotors acts in the opposite direction to that caused by gravitational loads. In the case of a horizontal axis primary rotor the centrifugal load acts predominantly in the radial direction unless a significant cone angle is applied, and will therefore lead to increased centrifugal stiffening.

Secondary rotor loads

In terms of the loads on the secondary rotors, there are 3 primary load components: aerodynamic, gravitational, and inertial.

The aerodynamic loads on the secondary rotor blades have deterministic components acting at two frequencies, $1P_p$ and $1P_s$. In the case of a HAWT primary rotor, the inflow velocity is relatively steady, only changing with the shear profile of incoming wind speed. However, with a VAWT primary rotor, the inflow velocity to the secondary rotors changes with equation 2.41 which causes $1P_p$ oscillations in secondary rotor blade loads. In both cases a $1P_s$ component arises from the shear profile induced by the primary rotor rotation. However, this is exacerbated further by the $1P_p$ periodic fluctuations in yaw offset experienced by a secondary rotor attached to a VAWT. In above rated conditions, a secondary rotor attached to a HAWT primary rotor will also experience significant yaw loads.

Due to the light weight of the secondary rotor blades, the gravitational loads on the secondary rotors are negligible compared to the aerodynamic loads.

Since the primary and secondary rotors will be operating at similar rotational speeds in both the HAWT and the VAWT cases, the inertial loads (both centrifugal and gyroscopic) are expected to be similar. However, in the case of a HAWT primary rotor, the secondary rotor would experience additional gyroscopic loads due to primary rotor pitch action. Since the effective yaw rate is expected to be an order of magnitude slower

than the primary rotor speed, the additional gyroscopic loads due to the pitching of the primary rotor blades are not expected to be significant.

2.8.4 Other design synergies

O&M

Through employing a much lighter power take-off system which is located at an easier to access location there are expected to be O&M benefits associated with the use of secondary rotors for both HAWTs and VAWTs. Typically the power take-off is located at hub height in the wind turbine nacelle, and any major replacements required a large, expensive jack up vessel. In the case of wind turbines using secondary rotors, jack-up vessels are not required for power take-off component replacement as rotors are lighter and can be located relatively close to the sea surface.

It is expected that the benefits will be more profound in the case VAWTs, since the secondary rotors are attached to the lower blades which means that they are consistently in an easily accessible location close to the sea level. However, in the case of HAWTs, secondary rotors will not necessarily be in an easy to access location. It is feasible that some mechanism could be employed to rotate the primary rotor in order to bring the secondary rotor requiring maintenance close to sea level, however this would add cost and complexity to the hub design.

The estimated savings on O&M costs for the X-Rotor concept compared to a traditional HAWT have been up to 25% [20], predominantly arising from the reduction in the use of jack-up vessels, further more detailed work has given a range from 18% to 38% based on the HAWT drive-train configuration considered [28]. It can be imagined that most of these savings would be translated to HAWT primary rotor. However the additional complexity of rotating the primary rotor to ensure accessibility to the secondary rotors would be a non-trivial task. Additionally, failures associated with the yaw drive would still be present in the case of a HAWT.

Mooring for VAWTs

A problem unique to conventional floating VAWTs is that the reaction torque from the generator is translated through the floating substructure and into the mooring system. This can be problematic for some mooring systems. In the case of VAWTs utilising secondary rotors, there is no torque transfer from the rotating primary rotor to the stationary platform and no additional torque reaction is therefore required from the mooring system.

2.8.5 Overview

It is shown that secondary rotors can perform power take-off for both Horizontal and Vertical axis wind turbines with conversion efficiencies up to approximately 85%. Secondary rotors can, in principal, be used with both horizontal and vertical axis primary rotors, with only a small inherent difference in conversion efficiency favouring a vertical axis primary rotor (which may well be eliminated by the difference in primary rotor power coefficient). Outside of this scope, however, the use of secondary rotors on vertical axis primary rotors may be more favourable as there are no oscillating gravitational loads transferred to the primary rotor. Additionally, rotor control can be considered more simple as the pitch control of the primary rotor can be decoupled from the attachment angle of the secondary rotors. In the case of VAWTs using secondary rotors, larger power fluctuations will be present. Some over-rating of power-train components is therefore necessary, and cyclic secondary rotor control is required to limit these fluctuations. The potential O&M benefits are more pronounced as the secondary rotors are consistently at an accessible location from the sea surface.

Generally, when comparing HAWTs and VAWTs, HAWTs are considered to be more aerodynamically efficient, experience lower periodic loading, and have evidently been found to be the most appealing solution to wind energy thus far and are more technologically developed. VAWTs are now being considered for offshore wind energy as they have a lower centre of mass, a lower centre of thrust which can reduce the cost of floating offshore platforms. They do not require a yaw drive and depending on the configuration may be able to have a significantly smaller tower. Additionally, it

is shown that wind farm density can be significantly increased through using VAWTs, especially when employing a fixed pitch offset which induces a strong side-force and acts as a vortex generator at the atmospheric scale [23]. When comparing with traditional power-trains, the inherently lower speeds and the large torque ripple typically means that VAWTs require significantly larger and more costly power-trains than HAWTs of a similar power rating.

Secondary rotors can be seen an enabling technology for VAWTs allowing the preservation of the benefits whilst mitigating the torque penalty associated with VAWTs. Additionally, in the case of secondary rotor - VAWT configurations, the additional loads on the turbine imparted by the secondary rotors are less significant than in the case of HAWTs. Whilst the periodic aerodynamic loads on the turbine structure must be dealt with, VAWTs have, here, been identified as the most promising primary rotor type for use with secondary rotors.

2.9 Conclusion

Secondary rotors have been shown as a technically viable method of reducing power-train torque without using a gearbox. A key consideration is that secondary rotors couple the power take-off to the tip speed of the primary rotor, rather than the shaft speed. This can confer larger benefits for turbines that would typically see a large torque penalty such as those designed for a low rated wind speed, as a marginal change in design (increase in the primary rotor tip speed ratio) can allow for there to be no additional costs associated with power take-off.

Additionally, secondary rotors may be seen as an enabling technology for VAWTs, as a key drawback to VAWTs that is often identified are the expensive drive-trains caused by the low speed - high torque operation and the inherent torque ripple. Both of these effects are mitigated by the use of secondary rotors.

Chapter 3

Vertical axis wind turbine design for secondary rotors

The previous chapter explored the use of secondary rotors for wind turbine power take-off, concluding with a comparative discussion on the use of secondary rotors for HAWTs and VAWTs. Due to a number of design benefits inherent to VAWT rotors and a number of synergies identified between VAWT rotors and secondary rotors, VAWTs were identified as a primary rotor configuration to investigate. This chapter will investigate VAWT design, focusing specifically on VAWTs with inclined blades (V/X-VAWTs) and how these rotors facilitate high tip speed ratio operation, necessary for power take-off with secondary rotors, and higher power capture. In essence, the research question addressed in this chapter is: *"How can a vertical axis wind turbine be designed for efficient use with secondary rotors?"*

VAWTs can generally be classified based on the primary aerodynamic force exploited for torque generation, either lift or drag. Drag based rotors are often better at self starting, are more simple to construct, and inherently limit their maximum power output. However, they typically operate with a low power coefficient, and with unfavourable torque speed characteristics. Lift based devices typically operate at higher power coefficients, with better torque speed characteristics; however, self starting and power limitation are often more complex. Since aerodynamic efficiency and torque speed characteristics become dominant concerns as rotors up-scale, utility scale VAWT

concepts typically focus on lift based rotors. Additionally, in the context of rotor design for use with secondary rotors, drag based rotors operate at low tip speed ratios which are not conducive to efficient power conversion between the primary and secondary rotors. For these reasons, this chapter will be focused on the design of lift type VAWTs.

Lift type VAWTs have many potential configurations, with any 2D shape with vertical symmetry representing a possible rotor configuration. For large offshore VAWTs, practical design considerations have led to a convergence on 2 key designs: Darrieus VAWTs, with curved blades attached at both the blade root and tip to rotating torque tube, and straight bladed VAWTs, typically configured with blades orientated parallel to the axis of rotation in an H configuration attached to a central member with struts. Since rotors with straight blades facilitate secondary rotor attachment close to the ocean surface, a key design synergy identified in Chapter 2, this chapter will be focused on straight bladed lift type VAWTs.

This chapter is structured as follows: Firstly, the aerodynamics of lift-type VAWTs is discussed in Section 3.1, following this an overview of VAWT modelling techniques are presented in 3.2. A review of VAWT design parameters is provided in Section 3.3 and the effect of these parameters on power capture and tip speed ratio is discussed. In this review it is identified that blade inclination angle could provide a means of increasing power capture and increasing the optimum tip speed ratio to the values required for efficient power take-off. Following this, a thorough literature review on the development of straight bladed VAWTs with inclined blades is provided in Section 3.4. Section 3.5 describes the numerical study undertaken within this chapter in order to investigate VAWT design for secondary rotors, and the results of this study are presented in Section 3.6. These results are discussed in Section 3.7, and final conclusions are drawn in Section 3.8.

3.1 Aerodynamics of lift type VAWTs

This discussion of VAWT aerodynamics draws heavily from the work of De Tavernier in [73], to which the reader is pointed for a more complete discussion of VAWT aerodynamics. As a VAWT blade rotates, it experiences an aerodynamic force distributed across its span that can, in the traditional manner, be decomposed into a lift force, F_L - perpendicular to the relative velocity, and a drag force, F_D - parallel to the relative velocity. These forces can be projected into the force tangential to the blade path, F_t , producing torque, and a force normal force blade path F_n . These forces are dependent on the flow conditions around the blade, which are, in turn, dependent on the magnitude of these aerodynamic forces.

Considering a blade element of a VAWT rotor blade, the rotating blade element experiences a relative velocity $\overrightarrow{U_{rel}}$, made up of the sum of the velocity of the blade element due to rotor rotation Ωr , the free wind speed $\overrightarrow{U_0} = U_0 \hat{x}$, and velocity induced by the rotors interaction with the flow $\overrightarrow{U_{ind}}$. These velocities are typically expressed either a Cartesian coordinate system corresponding to the incoming wind field:

$$\begin{aligned} U_x &= U_0 + \Omega r \cos(\theta) + \overrightarrow{U_{ind}} \cdot \hat{x}, \\ U_y &= \Omega r \sin(\theta) + \overrightarrow{U_{ind}} \cdot \hat{y}, \\ U_z &= \overrightarrow{U_{ind}} \cdot \hat{z}, \end{aligned} \tag{3.1}$$

or the cylindrical (or conical) coordinate system corresponding to the rotor:

$$\begin{aligned} U_t &= U_x \cos(\theta) + U_y \sin(\theta), \\ U_n &= [U_x \sin(\theta) - U_y \cos(\theta)] \cos(\psi) + U_z \sin(\psi), \\ U_s &= -[U_x \sin(\theta) - U_y \cos(\theta)] \sin(\psi) + U_z \cos(\psi), \end{aligned} \tag{3.2}$$

The radial velocity in the plane of a single 2D rotor segment may also be of interest

$$U_r = U_n \cos(\psi) + U_s \sin(\psi), \tag{3.3}$$

and the relative velocity in the plane of the blade element is also used frequently in blade element calculations

$$\vec{U}_{rel} = U_t \hat{t} + U_n \hat{n}.$$

Here, Ω represents the rotor speed; r represents the local rotor radius; θ represents the azimuth angle; ψ represents the blade inclination angle; $\hat{x}$, $\hat{y}$, and $\hat{z}$ represent the unit vectors in the stream-wise, lateral, and vertical directions; $\hat{n}$, $\hat{t}$, and $\hat{s}$ represent the unit vectors in the normal, tangential and span-wise directions; and $\hat{r}$ represents the unit vector radial direction. The direction in which velocity is represented is denoted by the subscript. These unit vectors, the azimuth angle, and the inclination angle are shown in figure 3.1 for clarity.

The force on the blade element is dependent on both the magnitude of the relative velocity, and the angle of attack. Typically, the force intensity is evaluated in the 2D plane of the blade element, disregarding the spanwise velocity U_s . The force intensity is then given by

$$\begin{aligned} f_L &= \frac{1}{2} \rho U_{rel}^2 c C_L(\alpha), \\ f_d &= \frac{1}{2} \rho U_{rel}^2 c C_d(\alpha), \end{aligned} \tag{3.4}$$

where ρ represents the density of air, c represents the chord length, C_L the lift coefficient, C_D the drag coefficient and α the angle of attack. The angle of attack is defined as the sum of the flow angle, ϕ , and the pitch angle, β . The flow angle is defined as the angle between the relative velocity vector and the blade's circular path, and the pitch angle is defined as the angle between the blade's chord line and the tangent to blade's circular path. These angles are shown in figure 3.2.

The normal and tangential forces are obtained through rotation by the flow angle ϕ . In the case of an inclined blade element, the radial force is typically required to calculate the flow field using momentum methods, is obtained by multiplying the normal force factor of $\cos(\psi)$, under the assumption of zero span-wise force.

$$f_t = f_l \sin(\phi) - f_d \cos(\phi), \tag{3.5}$$

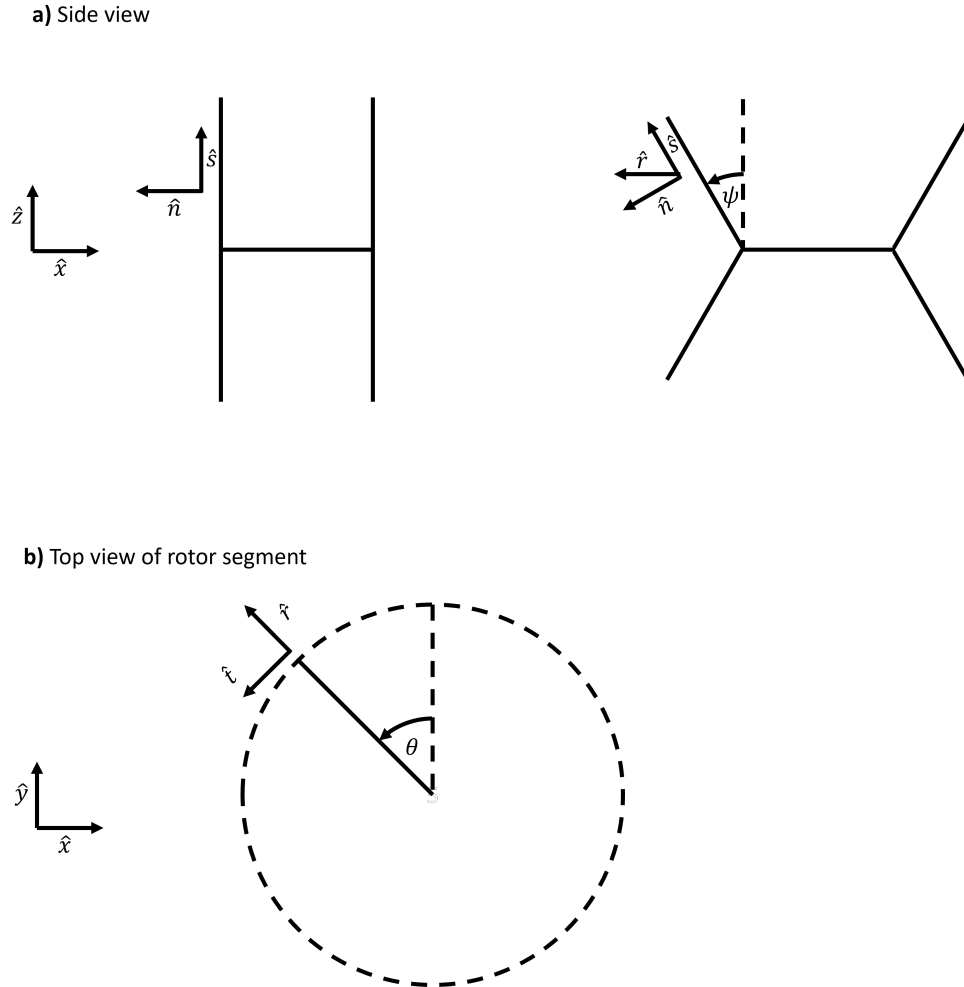


Figure 3.1: a) side view, and b) top view of a VAWT rotor. ψ represents the inclination angle; θ represents the azimuth angle; $\hat{x}$, $\hat{y}$, and $\hat{z}$ represent the unit vectors in the stream-wise, lateral, and vertical directions; $\hat{n}$, $\hat{t}$, and $\hat{s}$ represent the unit vectors in the normal, tangential and span-wise directions; and $\hat{r}$ represents the unit vector in the radial direction.

$$f_n = [f_l \cos(\phi) + f_d \sin(\phi)],$$

$$f_r = f_n \cos(\psi).$$

Figure 3.2 shows the relative velocities at a blade element, the angles associated with these velocities, and the resultant forces on the blade elements.

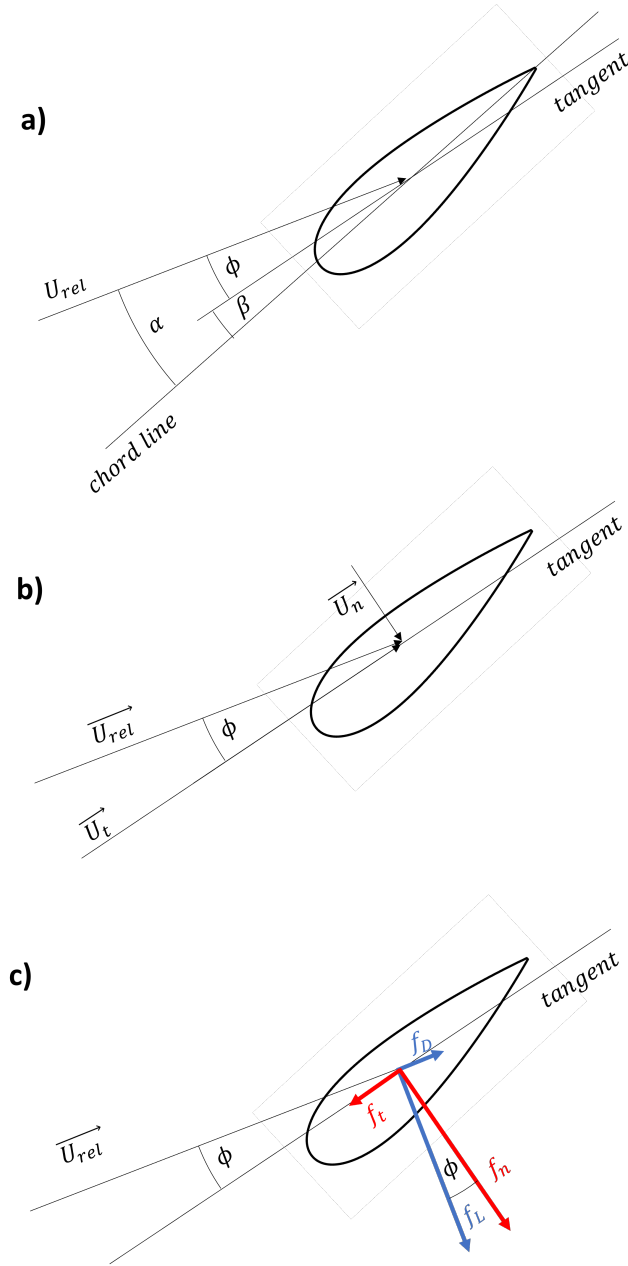


Figure 3.2: a) angles, b) velocities, and c) forces on a 2D blade element of a VAWT rotor.

The value of $\vec{U}_{ind}$ is a function of the forces exerted by the blade which are, as demonstrated above, determined by the local flow conditions. The coupled problem of determining both U_{ind} and $f_{[n/t]}$ represents the overarching challenge of VAWT aerodynamic modelling. Methods for the determination of the flow field are described

in Section 3.2.1, and methods for the determination of the forces are described in Section 3.2.2.

The torque generated by a VAWT blade can be readily calculated by integrating the corresponding moment distribution along the blade:

$$Q_b = \int_0^L r(l) f_t(l) dl \quad (3.6)$$

noting that the radius may change over the blades span in the blades that are inclined and must therefore be included within the integral. The power produced by a rotor can then readily be calculated by summing the revolution averaged torque contributions of each blade

$$P = \frac{1}{2\pi} \Omega N_b \int_0^{2\pi} Q_b d\theta. \quad (3.7)$$

The aerodynamics of a VAWT are fundamentally more complex than those of a HAWT, both at the blade scale, and at the rotor scale. In a uniform wind field, a HAWT blade element maintains a constant angle of attack whereas a VAWT blade experiences large fluctuations between positive and negative angles of attack. At the scale of the rotor, the flow passes through the rotor plane twice instead of once, leading to a more complex induced velocity field. Due to this complexity, no simple analogue to actuator disk theory can be accurately applied to derive a Betz type limit on maximum energy extraction. However, in general, similar behaviour is observed. A lightly loaded VAWT rotor will have little interaction with the flow field and will therefore not extract a significant proportion of the power available; conversely, a highly loaded VAWT rotor will induce a large negative velocity in the streamwise direction and will therefore limit the availability of power to the rotor.

In general, rotor loading is typically increased as the rotor solidity is increased or the tip speed ratio is increased. The solidity of a vertical axis wind turbine represents the total planform area of the blades, as a fraction of the rotor swept area in the plane perpendicular to the incident wind. The solidity is typically described locally to a 2D rotor segment with

$$\sigma = \frac{Nc}{r}. \quad (3.8)$$

Here, N represents the number of blades, c represents the chord length, and r represents the local rotor radius.

For a given tip speed ratio, increasing the solidity, either through increasing the blade number or the chord length, increases the force exerted by the rotor on the flow since there is additional area exposed to a similar dynamic pressure, although the change in induced wind speed also affects the angle of attack which will temper the increase in the aerodynamic force exerted by the rotor. Increasing the tip speed ratio for a given rotor solidity, increases the dynamic pressure at the blades and therefore increases the force exerted by these blades on the flow. Again, however, this increase is tempered by the decrease in the angle of attack fluctuations as the tangential component of velocity increases. Various models of the interaction between the rotor and the flow, are presented in Section 3.2.1. For a more complete overview of VAWT aerodynamics and modelling, the reader is pointed to [73].

3.2 Modelling of VAWTs

In order to successfully investigate the effect of VAWT design parameters on aerodynamic performance, one must compare the modelling methods available and select a model at an appropriate fidelity level for the task at hand. The following section describes a number of common modelling paradigms. Following this, a comparison between aerodynamic models is presented, and a modelling paradigm is selected for use in the investigations presented in this chapter. The model comparison is further explored in the publication [41]. Section 3.2.1, provides a brief description of VAWT flow models, Section 3.2.2 provides a description of the blade element representation of VAWT blades, Section 3.2.3 provides a description of strut modelling techniques, and Section 3.2.4 shows the results of a benchmark of a number of VAWT simulation codes.

3.2.1 VAWT flow models

VAWT flow models can be broadly categorised into momentum models, vortex models, and viscous computational fluid dynamics (CFD) approaches. Momentum models

are most commonly used as design codes since they are computationally fast, numerically stable, and are simple to implement. Parameters associated with the simulation code can typically be left constant over a range of simulations allowing for batched simulations to be completed. Vortex models are more computationally expensive than momentum models, and are more complex to execute numerically. However, they are a more physical representation of the problem and can thus be used to verify the results of momentum models. Since vortex models can typically still be run on personal computers they are a useful tool for the quick validation of momentum models. CFD simulations are typically more complex to execute accurately and very computationally expensive. However, they directly model the aerodynamic phenomena unique to VAWTs, and therefore can be essential for validating lower fidelity simulations. A discussion of CFD simulation is outside of the scope of this thesis, and useful discussions can be found in [74]. The following section will discuss momentum models and free vortex wake models for VAWT rotors.

Momentum models

Momentum models use results derived from Eulerian flow analysis, to calculate the velocity induced by revolution averaged forces at the rotor. The dimensionality of the model is, here, defined by the number of dimensions in which these induced velocities are calculated. This does not represent the dimensionality of the rotor considered for simulation. For example, 1D momentum models can be used to obtain transverse distributions of induced streamwise velocity through a 2D rotor segment through discretisation into multiple streamtubes, and both 1D and 2D moment models can be used to model 3D rotors through discretising the 3D rotor into a number of 2D rotor segments. Whilst, in most cases, the forces on a rotor blade are calculated using tabulated polars - often corrected for dynamic effects, generalised force distributions can be used to investigate generalised load cases [75]. The process of vertical discretisation, and solution of 2D rotor segments using various 1D and 2D methods is shown in figure 3.3. These 1D and 2D solution methods are further described in the following subsections.

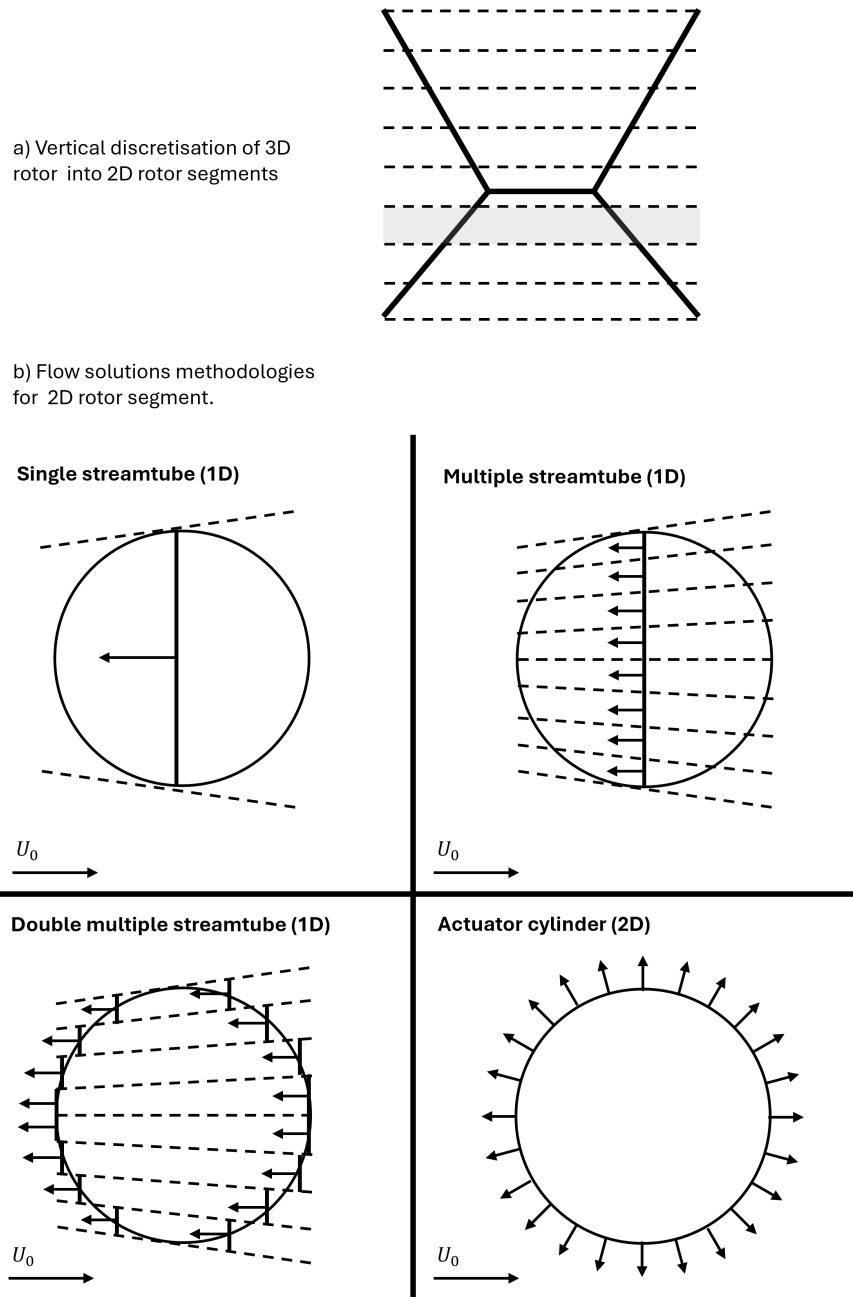


Figure 3.3: Diagrammatic representation of how various 1D and 2D momentum methods are employed to solve the flow around a 3D VAWT rotor. Sub-figure a) shows the discretisation of the 3D rotor into 2D rotor segments, and sub-figure b) shows the solution of this 2D rotor segments using various 1D and 2D momentum models.

1D momentum models

A simple set of approaches to modelling the flow around a VAWT are 1D momentum models. These model the rotor's interaction with the flow as either a single actuator disk or a number of actuator disks, parallel to the incoming flow. The thrust force exerted by the disk is given by the rotation average stream-wise force on the portion of the rotor represented by the disk. At each actuator surface, the induced velocity, $\overrightarrow{U_{ind}}$, is defined only in the stream-wise direction, and is given by the canonical actuator disk relationship:

$$\begin{aligned}\overrightarrow{U_{ind}} &= aU_0\hat{x}, \\ C_T &= 4a(1 - a).\end{aligned}\tag{3.9}$$

The single streamtube model was first proposed by Templin [76], this represented the rotor as a single actuator disk with a thrust force equal to the rotor averaged stream-wise force, as shown in figure 3.3. The model has two key limitations. Firstly, the induced velocity has a single value over the entire rotor disk, despite the fact that the stream-wise force varies significantly in the transverse direction. Secondly, the downwind half of the rotor is in the wake of the upwind half, and experiences larger induced velocities in the stream-wise direction. In order to capture the distribution of induced velocities in the transverse direction, multiple streamtube models were proposed [77, 78], shown in figure 3.3. These are effectively quasi-2D models, they discretise the rotor into a number of independent parallel streamtubes, calculating the induced velocities for each streamtube based on the rotation averaged stream-wise force over the portion of the rotor covered by each streamtube.

The multiple streamtube model was further extended by Paraschivou [79] and Sharpe [80] to the double multiple streamtube (DMS) model, shown in figure 3.3. This model includes a series of two actuator disks in each parallel streamtube to represent the flows interaction with the upwind and a downwind rotor half. There is one-way coupling of the upwind and downwind actuator surfaces, with the inflow velocity for the downwind actuator disk set equal to the wake velocity on the upwind actuator disk where atmospheric pressure has been recovered. This allows a more representative

induced velocity distribution which accounts for both the transverse distribution in induced velocity and the fundamentally different flow conditions at the upwind and downwind actuator surfaces. Since its conception, the DMS model has been used widely for VAWT aerodynamic simulation [81–83], and remains a common and useful tool for VAWT analysis. However, a number of comparative studies have highlighted that the method does not perform well in benchmark comparisons where the goal is accurately reproducing the results of higher fidelity models [26,84]. There are a number of limitations to the DMS framework, which include:

1. **No transverse induced velocity:** DMS methods do not model any induced velocities in the transverse direction. It has been argued that, due to symmetry, the rotation averaged transverse forces on a VAWT rotor are small compared to the stream-wise force. However, especially when asymmetric aerofoil profiles are used or rotor blades are pitched, large transverse forces can exist locally and transverse induced velocities should be modelled.
2. **Coupling between the upwind and downwind actuator surfaces:** DMS methods simplify the relationship between the upwind and downwind actuator surfaces such that there is no velocity induced by the downwind actuator surface at the upwind actuator surface, and that the inflow to the downwind actuator surface is given by the outflow of the upwind actuator surface. This pair of simplifications can be considered unjustified, especially in the case where the downwind actuator surface is highly loaded, or in streamtubes where the distance between actuator surface is very small.
3. **Streamtube independence:** A key assumption of multiple streamtube and DMS models is that the streamtubes remain independent. This assumption is invalidated by the existence of transverse velocities, and by the necessity of streamtube expansion to preserve the mass flow through the streamtube. Whilst corrections have been introduced to account for streamtube expansion [80,85], these models still assume a set of independent streamtubes which fundamentally do not exist.

2D momentum models

The limitations of a streamtube representation of a VAWT rotor discussed in the previous subsection can be largely overcome by representing the rotor by a circular actuator surface, coinciding with the swept area of the VAWT, as proposed by Madsen [86]. This approach is referred to as the 2D Actuator Cylinder (2DAC) model, and is shown in figure 3.3. The 2DAC model applies the rotation averaged force distribution over the actuator surface in the normal and tangential directions as body forces and the flow field is resolved in a Eulearian framework. The resulting velocity field can then be defined as the sum of a linear solution, valid for lightly loaded rotors, and a non-linear solution, required to accurately resolve the flow-field around a heavily loaded rotor. Due to the computational time required for the calculation of the non-linear solution, the linear solution is typically coupled with a non-linear correction factor [87].

The 2DAC model inherently calculates the induced velocities in both the stream-wise and the transverse directions, and the the flow field at each point on the rotor surface is calculated based on the rotation averaged force distribution along the entire rotor. This directly overcomes the limitations of the DMS method, and benchmarking studies have shown that the 2DAC model can more accurately predict VAWT performance. There are, however, a number of limitations to the 2DAC approach:

1. **No vertical induced velocity:** In calculating the performance of a 3D rotor, the 2DAC model still requires the summation of a number of independent rotor segments and assumes no vertical velocity. Especially in the case of rotors with inclined blades, such as the X-Rotor, velocities are induced in the vertical direction and should be accounted for. The presence vertical velocities also invalidates the assumption of rotor segment independence.
2. **Flowfield based on rotation averaged forces:** The 2DAC model uses the rotation averaged rotor force to calculate the flow field. This smearing of the force is not representative of reality, and further, since the wake from each blade is not modelled, it eliminates blade wake interaction, which can be significant [88].
3. **Tiploss model required:** Since the blade-flow interaction is not resolved in

the 2DAC approach, tip loss models are required to approximate the effects of trailing vorticity on the blades aerodynamic performance, and the effect of finite blade number on the rotor wake. Physically consistent tip loss models have not been derived for VAWT rotors, and approaches from aircraft, and HAWT rotors are typically employed [85].

Free vortex wake models

Free vortex wake models are time domain models which do not rely on rotor averaged force fields. The method is based on potential flow theory, representing the rotor wake as vortex elements convected at the local flow velocity. The velocity induced by each vortex element can be calculated using the Biot-Savard law, and, since potential flow is governed by the (linear) Laplace equation, the total induced velocity is given by the sum of the velocity induced by each wake element and the velocity induced by the bound vorticity on the blade. The bound vorticity at each blade section is calculated using the Kutta-Joukowski theorem, with the lift coefficient of the blade typically calculated using tabulated polars.

At each time step, vortex elements are added to the wake due to changes in the bound circulation on the blade. Trailing vorticity is added as a consequence of a span-wise distribution in bound vorticity and shed vorticity is added to the wake as a consequence of changes in the strength of bound vorticity over time. This is a consequence of Helmholtz' first theorem; *'The strength of a vortex filament is constant along its length'* and Kelvin's theorem *'the circulation around a closed curve, enclosing the same fluid elements, moving with the fluid remains constant with time'* respectively.

Free vortex wake models was first employed by Strickland for VAWT simulation [89]; however, more recently released simulation tools codes such as CACTUS [90], QBlade [32], and OLAF [91] include implementations of free vortex wake models. The computational time associated with free vortex wake simulation has been significantly reduced through exploiting the linearity of the Laplace equation to parallelise induced velocity computations, and computing them on graphics cards.

Free vortex wake models overcome a number of the limitations of the 2DAC model,

through inherently modelling vertical induction, modelling the flow field in the time domain and therefore avoiding the use of rotation averaged force fields, and through directly modelling tip losses. The drawbacks of Free vortex wake models include:

1. **Vortex core modelling** The Biot-Savard law tends towards infinite induced velocity approaching the centre of the vortex. In reality vortices have a core rotating at a constant speed, and the induced velocity reduces to zero at the core centre. In addition to this, the discretisation of a continuous vortex sheet into a number of discrete vortex filaments, caused by the discretisation of both the blade and time, can cause load spikes.
2. **Wake breakdown/Viscous effects** In reality viscous effects lead to wake recovery and a breakdown of the vortex structure. This is ignored by the free vortex wake models.

3.2.2 Blade element representation

The flow modelling methods discussed in Section 3.2.1 required tabulated blade polars relating the lift and drag force coefficients to the angle of attack. These allow the forces on a 2D blade section to be calculated for a given flow condition. Blade polars can either be generated through wind tunnel experiments or numerical simulation. For small scale rotors, wind tunnel testing allows polars to be obtained experimentally; however, for large scale rotors, the high Reynolds number means that polars are typically obtained numerically due to high costs and complexity of experimental campaigns.

Typically, aerofoil polars are generated through simulation for values up to the static stall angle, and, for angles of attack well past the static stall angle, it is assumed that aerofoils share similar characteristics and empirical rules are applied to extrapolate the polars to $\pm 180^\circ$. Numerical simulations are typically completed using either a viscous panel method, such as XFOIL [65], or through 2D CFD simulations. For post stall extrapolation, the Viterna method is commonly applied [92]. Careful interpolation between the simulated values and the extrapolated values is required.

Whether gathered experimentally or through simulations, blade polars are typically representative of static, two dimensional rectilinear flow around the aerofoil section. The flow around a vertical axis wind turbine is dynamic, non-rectilinear, and 3 dimensional. There are therefore a number of phenomena that are not well described by the blade element approach such as:

1. **Dynamic stall:** Dynamic stall occurs when the unsteady motion of an aerofoil leads to the angle of attack exceeding the static stall angle. It is typically characterised by an increase in the lift coefficient beyond the maximum static value at angles of attack exceeding the static stall angle as flow separation is delayed, followed by a sharp increase in drag coefficient and drop in the lift coefficient as the flow separates. During reattachment, lift coefficient remains below it's static value until the flow has reattached. A thorough discussion of the phenomena can be found in [93]. This effect is present during VAWT operation [94], and has been shown to considerably alter VAWT rotor performance [95,96]. Dynamic stall can be modelling using additional engineering models such as those described in [85].
2. **Flow curvature:** Flow curvature refers to the change in aerofoil behaviour between rectilinear and curvilinear flow. Typically, blade element polars are obtained from either wind tunnels or from simulation tools, both of which enforce rectilinear flow conditions. A blade section orbiting a central point sees curvilinear streamlines changing the local velocity and pressure distributions around the aerofoil and introducing a virtual camber and virtual incidence effects [97,98].
3. **Finite span effects:** Finite span effects refer to the change in rotor behaviour when moving from a 2D (quasi-infinite) rotor segment to a 3D rotor. In general there are two phenomena underway, backwash and the effect of a finite blade number. The effect of backwash is typically described using Pradtl-Lanchaster theory for an elyptical circulation distribution, and the effect of finite blade number is typically described using Willmer's modified version of the Prandtl method for screw propellers [85].

3.2.3 Strut modelling

VAWT blades are attached to the rotating hub or torque tube with struts, such as the crossarm shown in figure 1.2 and these struts typically have a significant effect on the aerodynamic efficiency of the rotor. For example, the X-Rotor uses a single strut, the crossarm, to attach the blades to the central hub, as shown in figure 1.2. Investigations on the 17m Sandia test bed rotor showed that the peak power coefficient of the rotor dropped from 0.454 to 0.376 when struts were included on the rotor [99]. Struts oriented with an angle outside of the horizontal have been used for wake re-direction [100], and have complex aerodynamics since they have changing, non-zero, angle of attack over each revolution. Since this study only considers a horizontal strut, the contribution of the parasitic drag to the rotor torque can be simplified to

$$Q_s = -N_b \rho c_s \frac{1}{4\pi} \int_0^{2\pi} \left(\int_{R_t}^{R_0} r C_{Ds} U_s^2 dr + R_0 c_s C_{Di} U_s^2 \Big|_{r=R_0} \right) d\theta \quad (3.10)$$

where R_t represents the radius of the tower, c_s represents the strut chord length, U_s represents the apparent wind speed at the strut, and C_{Ds} and C_{Di} represent the strut drag coefficient and the interference drag coefficient respectively. The strut drag coefficient can be calculated using empirical formulations for the profile drag of streamlined foil sections [101] and the interference drag due to the blade/strut attachment can be calculated using the empirical relation for a t-junction of two foils [101]. This approach was proposed in [102].

Considering only the incident wind speed due to rotation and extending the strut to the centre of the rotor, the above can be simplified to

$$Q_s = -\frac{1}{2} N_b \rho c_s \Omega^2 R_0^3 \left(\frac{R_0 C_{Ds}}{4} + c_s C_{Di} \right) \quad (3.11)$$

corresponding to a reduction in power coefficient given by

$$C_{Ps} = -N_b c_s \frac{1}{A} \left(\lambda \frac{R_0}{R_{tip}} \right)^3 \left(\frac{R_0}{4} C_{Ds} + c C_{Di} \right). \quad (3.12)$$

3.2.4 VAWT model benchmarking

As outlined in section 3.2.1, a number of aerodynamic models for VAWT rotors exist which represent rotor aerodynamics at different fidelity levels, with increased fidelity typically correlated with an increase in both accuracy and computational cost. Since the investigation presented in this chapter requires a large number of simulations, a lower fidelity model is favoured; however, the utility of the conclusions drawn from this investigation is dependent on the accuracy of the model. Therefore, selecting an appropriate aerodynamic model, which has a low enough computational cost to facilitate a broad design study but is accurate enough to provide useful conclusions, is of paramount importance.

The appropriate aerodynamic model for this study can be determined through the evaluation of the VAWT aerodynamic code benchmarking completed as part of the X-Rotor project, reported in [41]. This benchmark included: 1 DMS model, developed by the author and described in Appendix B; 1 2DAC model, presented in [103] and described in Appendix C; 2 FVW models, QBlade [32] and CACTUS [90]; and 1 blade resolved CFD simulation, detailed in [41]. The outcome of the benchmarking and further details of the numerical simulations are reported in full in [41]. For clarity, in discussing the results of the benchmarking the momentum based methods will be referred to as low fidelity, the vortex methods will be referred to as mid fidelity, and the blade resolved CFD simulation will be referred to as high fidelity.

The test case used for the benchmarking study was the 5MW X-Rotor primary rotor, defined in appendix D. The rotor is essentially a pair of 2 bladed V rotors, connected to a common crossarm at an inner radius of 25m. The upper blades have a length of 100m, and are attached with an inclination angle of 30° , the lower blades have a length of 65.6m and is attached at an inclination angle of 130° (equivalent to 50° if not inverted). Both the upper and lower blades have a tip radius of 75m. The upper blades are variable pitch whilst the lower blades are fixed with a 0° pitch angle. The full primary rotor geometry is provided in Appendix D, and a diagram with dimensions is provided in figure 3.4

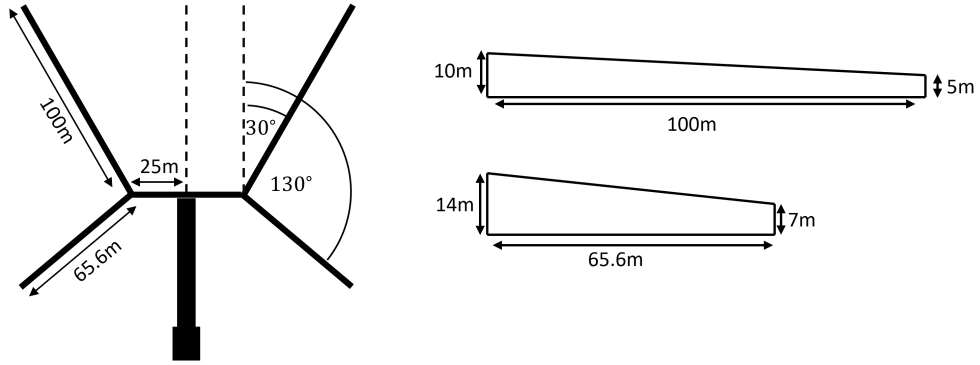


Figure 3.4: Geometry of the 5MW 2 bladed X-Rotor primary rotor.

Firstly, it is informative to consider the rotor in the case where the upper blades have a 0° pitch offset. Figure 3.5 shows the power coefficient and thrust coefficient as a function of tip speed ratio for this case. Below a tip speed ratio of 5, both of the low fidelity models and QBlade show good agreement whilst CACTUS predicts a larger power coefficient over the full range of tip speed ratios. At a tip speed ratio of 5, both of the mid-fidelity models over-predict the power coefficient compared to the low fidelity models and the high fidelity simulation. In terms of rotor thrust coefficient, the low fidelity models consistently underestimate the rotor thrust compared to the mid and high fidelity models, with the DMS model significantly under-estimating the rotor thrust coefficient for tip speed ratios above 3.5 compared to all of the other models.

Figure 3.6 shows the power coefficient as a function of pitch angle for 3 different tip speed ratios. At all tip speed ratios, the DMS model tends to over-predict the power coefficient at negative pitch values, with the effect becoming more pronounced as the tip speed ratio increases. This can be understood as a breakdown of the flow model as the rotor becomes loaded asymmetrically towards the downwind rotor half. The other 3 models show similar trends in terms of power coefficient as the pitch angle changes.

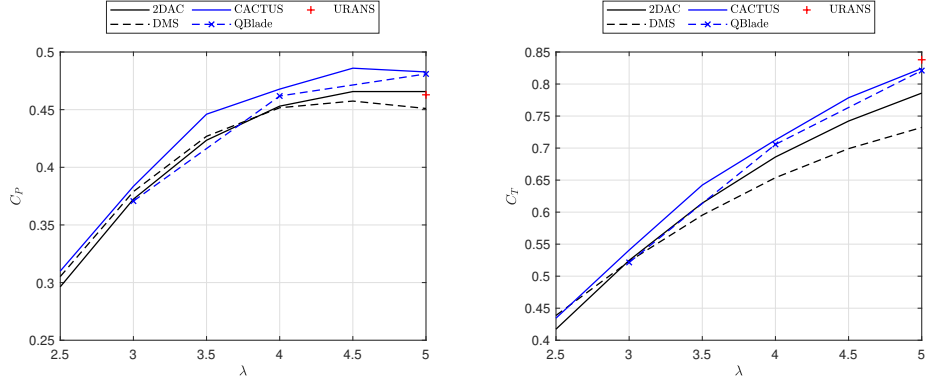


Figure 3.5: X-Rotor power and thrust coefficient, C_P and C_T , as a function of tip speed ratio, λ , for a fixed pitch offset of 0° as calculated using 5 different modelling techniques. DMS model was developed by the author, with details provided in appendix B.

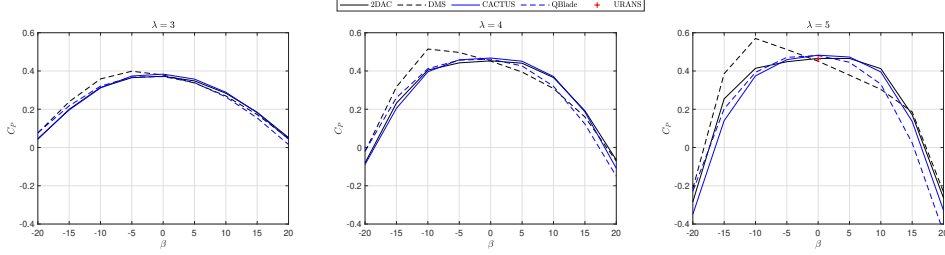


Figure 3.6: X-Rotor power coefficient, C_P , as a function of pitch angle, β , for tip speed ratios, λ , of 3, 4 and 5 as calculated using 5 different modelling techniques. DMS model was developed by the author, with details provided in appendix B.

Figure 3.7 shows the azimuthal variation in the integrated normal, F_n , and tangential, F_t , force on a single upper blade at tip speed ratios of 3, 4, and 5 operating at 12m/s with an air density of 1.225kg/m^3 . The normal force is well reproduced across the 4 low and mid fidelity models. At a tip speed ratio of 5, the high fidelity simulation predicts a wider peak in the normal force distribution in the upwind rotor half, and a shallower trough in the downwind rotor half when compared to the 4 other models.

Figure 3.8 shows the integrated normal, and tangential force as a function as azimuth angle for a single lower blade, again for tip speed ratios of 3, 4, and 5 operating at 12m/s with an air density of 1.225kg/m^3 . The normal force distribution is well reproduced across the four low and mid fidelity models, discounting the clear examples of

blade vortex interaction from the vortex models visible in the downwind rotor half at all tip speed ratios. The high fidelity simulation predicts a higher normal force magnitude in the upwind rotor half and a higher normal force magnitude in the downwind rotor half. In terms of the tangential force distribution, there is good agreement between the vortex models and the 2DAC model in the upwind rotor half; however, the DMS model consistently predicts a more peaked distribution with a larger maximum value. In the downwind rotor half, the vortex models show strong agreement and the 2DAC model predicts a peak in the tangential force earlier in the revolution than it is predicted by the vortex models. The high fidelity simulation agrees well with the vortex models and the 2DAC model in the upwind rotor half. In the downwind rotor half, the blade vortex interaction does not lead to significant changes in the integrated tangential force, and the induced peak in tangential force is at approximately 270 degrees, whilst the 2DAC model predicts the peak tangential force to occur significantly earlier in the revolution at approximately 200 degrees. Despite this, the peak tangential force in the downwind rotor half is well predicted by both the 2DAC model and the DMS model.

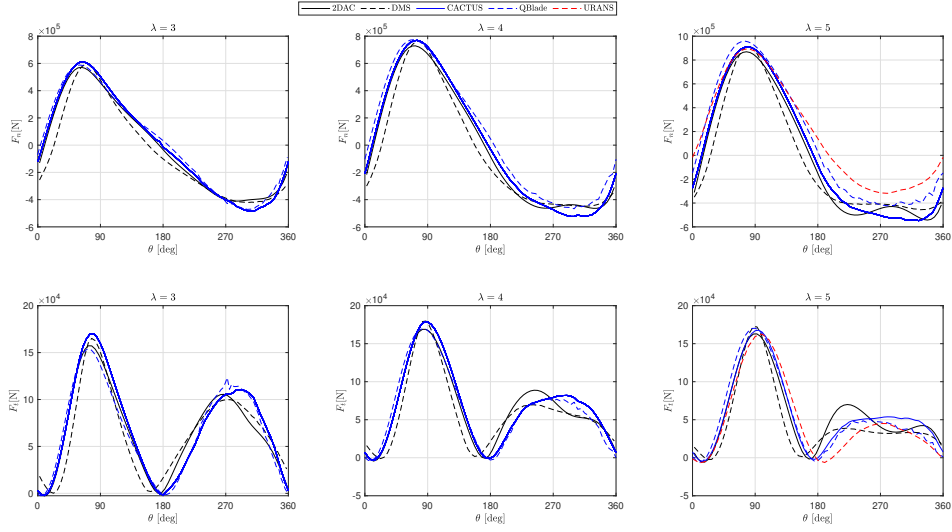


Figure 3.7: X-Rotor upper blade normal and tangential integrated blade forces, F_n and F_t , as a function of azimuth angle, θ , at a wind speed of 12m/s as calculated using 5 different modelling techniques. DMS model was developed by the author, with details provided in appendix B.

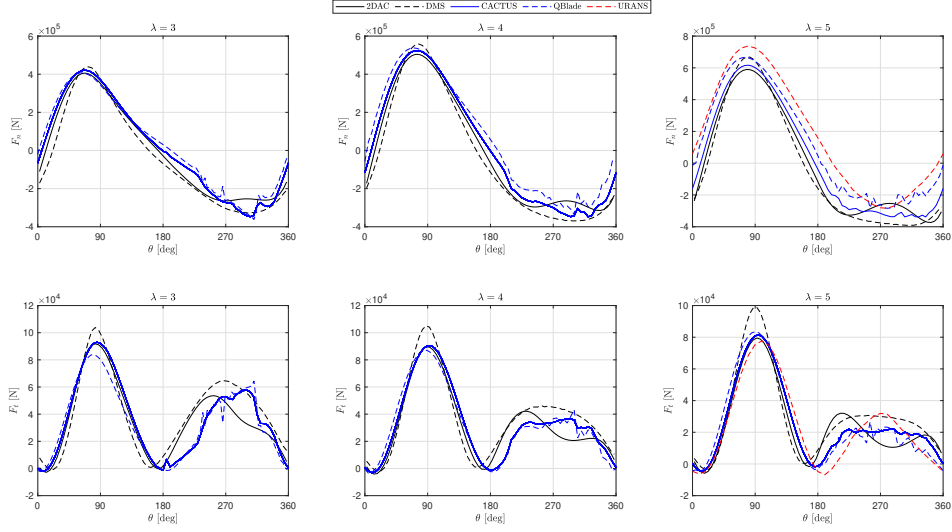


Figure 3.8: X-Rotor lower blade normal and tangential integrated blade forces, F_n and F_t , as a function of azimuth angle, θ , at a wind speed of 12m/s as calculated using 5 different modelling techniques. DMS model was developed by the author, with details provided in appendix B.

Of the two low fidelity codes, the 2DAC model more accurately reproduce the results of mid fidelity models, and accurately reproduces the power coefficient compared to the high fidelity simulation. For this reason, subsequent aerodynamic analysis will be completed using the 2DAC simulation code.

It should also be noted that the power coefficient and the normal and tangential force on the blades is well reproduced by the 2DAC model compared to the high fidelity URANS simulation despite the 2DAC not including a correction factor for streamline curvature or dynamic stall. This indicates that the aerodynamic model can be used without correction factors for early stage design investigations.

3.3 Vertical axis wind turbines design

The previous chapter has shown that a relatively high primary rotor tip speed ratio is required for efficient power take-off with secondary rotors. The following section will discuss the key design parameters of VAWTs focusing on their effect on power coefficient and optimum tip speed ratio.

3.3.1 Rotor solidity

The rotor solidity can be considered to be the most important design factor in determining the aerodynamic characteristics of VAWT rotor. Through increasing the rotor solidity, the optimum tip speed ratio of the rotor decreases since a lower apparent velocity at blade is required to achieve optimum streamwise induction. Since the rotor is operating at a lower tip speed ratio, the blade will experience larger fluctuations in angle of attack. The range of angle of attack variations has an influence on rotor efficiency, since the tangential force coefficient varies as a function of angle of attack. Straight bladed turbines with un-tapered blades and no blade inclination can be described by a single solidity value; however, more complex rotor designs, employing an inclination angle or a variable chord distribution will have variable solidity.

Figure 3.9 shows the effect of solidity of the performance of a 2 bladed 2m diameter Darrieus wind turbine at a constant Reynolds number of 150,000, reproduced from [3]. The turbine did not include struts, therefore the effects of parasitic drag were minimised. It is clear that the lower solidity rotors operate at higher tip speed ratios, with a wider $C_p - \lambda$ curve, whilst rotors with a higher solidity operate at lower tip speed ratios and have a more peaked $C_p - \lambda$ curve.

The optimum solidity for VAWT rotors, has a wide reported range in literature and is dependent on rotor configuration, and rotor scale. For Darrieus VAWTs, solidities of 0.2-0.25 have been shown [3] to maximise the power coefficient, whilst for straight bladed VAWTs, solidities of 0.25-0.5 have been reported, [104, 105]. Bianchini [106], provided a thorough optimisation of small straight bladed VAWT rotors for lifetime energy capture at a range of sites. A large range of solidities were reported, between 0.18 and 0.56. However, when considering symmetrical aerofoil shapes at high wind speed sites with a larger swept areas (the maximum swept area considered was 9m^2), optimum solidity values of 0.18 - 0.26 were found.

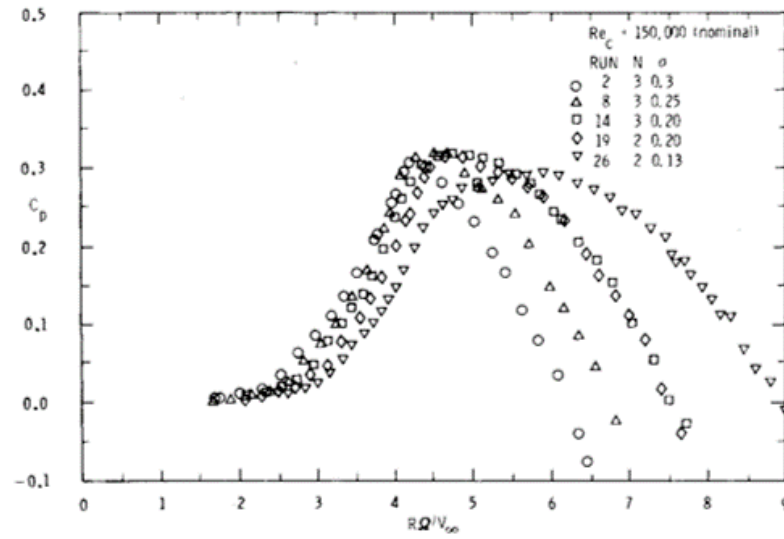


Figure 3.9: Effect of rotor solidity on the power coefficient - tip speed ratio, C_P - λ , curve for a 2m Darrieus rotor, reproduced from [3].

The rotor solidity is typically varied through changing either the chord length of the blades or the blade number. These have different effects and are discussed individually in the following subsections.

Chord length

Increasing the blade chord length increases both the rotor solidity and the blade Reynolds number, and decreases the aspect ratio of the blade. The increase in Reynolds number is often desirable for small rotors, where increasing the Reynolds number can significantly improve the efficiency of the rotor. In these cases, the Reynolds number is in the transition zone and both the peak tangential force coefficient and the range of angles of attack over which the tangential force coefficient is positive are significantly increased with increasing Reynolds number. For larger rotors, the Reynolds number is typically above the transition zone and further increase in the Reynolds number typically has less effect on rotor performance.

Increasing the chord length decreases the aspect ratio of the blade. This typically increases the finite span effects, increasing the induced drag and decreasing the lift coefficient. These effects combine to decrease the efficiency of the rotor [85].

Finally, decreasing the chord length reduces the 2nd moment of area of the 2D blade section, increasing the stress experienced within in the blade for a given load. Whilst the extreme loads experienced in parked condition decrease proportionally with the chord length, the 2nd moment of area decreases with the forth power of chord length. This implies that the chord length cannot be arbitrarily decreased since the bending stresses will become too large and the blade will fail.

Blade number

Increasing blade number increases blade wake interaction [88], increases the rotor solidity, increases the number of struts required to support the blades, and decreases the magnitude of the torque ripple.

Increasing blade wake interaction is expected to decrease the efficiency of the turbine [88]; however, this was not observed in wind tunnel tests by Blackwell [3]. Further studies have demonstrated a reduction in performance associated with an increase in blade number for a given solidity [107]; however, unlike Blackwell these experiments do not also maintain a constant Reynolds number, and it is unclear if the performance detriment was caused by a reduction in Reynolds number or the increased incidence of blade wake interaction. The effect of blade number of VAWT performance in terms of peak power coefficient and tip speed ratio, normalised with controlled solidity and Reynolds number is therefore relatively under researched however the work of Blackwell [3] indicates that the difference between a 2 bladed rotor and 3 bladed rotor is relatively small.

Increasing blade number also increases the number of struts, since at least one strut is required per blade. This increases parasitic drag and will hinder performance at higher tip speed ratios. Additionally, increasing blade number reduces the amplitude of torque and thrust fluctuations since the combination of 3 sets of blade loads with a phase different of $\pm 120^\circ$ typically has a lower amplitude than the sum of 2 sets of blade loads with a phase difference of 180° .

3.3.2 Pitch angle

Introducing a fixed pitch angle to a VAWT blade changes the angle of attack along the blade and rotates the plane in which the aerodynamic force is projected. A positive pitch angle, as defined in figure 3.2, will increase the loading in the upwind rotor half, decrease the loading in the downwind rotor half and introduce a net force in the windward direction. Similarly a negative pitch angle will decrease the loading in the upwind rotor half, increase the loading in the downwind rotor half and introduce a net force in the leeward direction.

Studies have shown that a pitch angle of -2° typically increases the peak power coefficient [108–110]. It has been observed that a reduction in the pitch angle reduces the optimum tip speed ratio of the rotor [109, 110]. The results from Bianchini [110] indicate that small changes in the pitch angle can provide a marginal increase to the power coefficient, with small changes to the optimum tip speed ratio (up to 7%), and that large changes to the pitch angle significantly reduce rotor performance.

3.3.3 Aerofoil profile

Modifying the shape of an aerofoil changes the relationship between the non-dimensional force coefficients and the angle of attack. Furthermore, the change in shape directly affects the 2nd moment of area of the section, modifying the stiffness of the blade. The majority of VAWT research uses symmetric NACA 4-series aerofoils [21]; however, the profile thickness, camber, leading edge radius and trailing edge shape are all variables that can alter aerofoil performance.

Symmetric profiles are typically favoured for blade design [21] since VAWT blades experience positive-to-negative angle of attack fluctuations during operation. Despite this, cambered aerofoils have been proposed and small power gains have been demonstrated through the introduction of slight cambers [111].

An increase in profile thickness typically decreases the aerodynamic performance of a rotor [112] due to the increase in drag coefficient. However, it significantly increases the stiffness of the blade and is therefore desirable from a structural perspective.

Two notable departures from the NACA 4 series aerofoil family are the SAND00

series and the DU 06-W-200 [111]. In the SAND00- [113] series, the leading edge radius and trailing edge shape and location of maximum thickness have been modified in order to promote natural laminar flow, delaying the transition to a turbulent boundary layer and theoretically reducing the drag coefficient and increasing VAWT performance. The DU 06-W-200 is also designed to promote natural laminar flow, and has a 0.8% camber.

In general, the selection of aerofoil profile is determined by a trade-off between aerodynamic and structural properties. Modern VAWT research primarily utilises profiles that are either symmetric or have a slight camber, with thicknesses between 18% and 21%. A larger aerofoil thickness may permit the use of a small chord length blade operating at a higher tip speed ratio. However the aerodynamic effect of increasing the aerofoil thickness is not expected to have a significant effect on the optimal tip speed ratio of the rotor.

3.3.4 Blade taper

The blade taper describes how the chord length changes from blade root to blade tip, with the blade root typically defined as the attachment location to the rotor cross-arm. Introducing a taper to a blade introduces two structurally favourable characteristics, firstly it lowers the centre of force on the blade, reducing the bending moments at the blade root. This is due to reducing the chord length toward the blade tip and increasing the chord length toward the blade root. Secondly, it increases the second moment of area towards the blade root (where the bending moment is higher), this decreases the stress at the blade root and can lead to a lower blade cost. Additionally, it has been demonstrated that a taper of up to 50% can increase maximum power coefficient of a straight bladed VAWT rotor [108], without significantly changing the optimum tip speed ratio.

3.3.5 Rotor aspect ratio

The rotor aspect ratio is defined for a straight bladed VAWT as the ratio between the height and width of the rotor swept area. For rotors with a fixed solidity and swept

area, increasing the aspect ratio will lead to a lower chord length, a longer blade length, and a higher rotational speed.

For large turbines, the increase in Reynolds number has a very small impact on power production. Large aspect ratios are therefore favoured from a purely aerodynamic perspective as the reduction in finite span effects can increase power capture [114]. The reduction in induced drag can also slightly increase the optimum tip speed ratio of the rotor [114]. From a structural perspective however, lower aspect ratios are favoured and blades will experience smaller aerodynamic bending moments, lower centrifugal loads, and have a larger second moment of area.

The parasitic drag from struts increases as the radius increases due to the increase in strut length; however, the parasitic drag also increases as the rotational speed increases due to the higher apparent wind speed. For rotors with identical swept area and tip speed ratios, and no inclination angle ($R_0 = R_{tip}$), equation 3.12 shows that the power lost to parasitic drag increases as the rotor radius increases. This shows that higher aspect ratio rotors would experience lower losses from parasitic drag if the strut dimensions can be kept constant. It should, however, be noted that high aspect ratio blades may require additional struts per blade which will further increase the parasitic drag.

In general, whilst high aspect ratio are typically favoured for large scale straight bladed VAWTs, the optimal aspect ratio is dependent on a number of design specific factors. The effect of aspect ratio on optimum tip speed ratio is small, and typically governed by the nature of the parasitic drag on the rotor struts. For a given rotor solidity, the rotor aspect ratio is not expected to have a significant impact on the optimal tip speed ratio.

3.3.6 Inclination angle

Introducing a blade inclination angle modifies the swept area of the rotor, changes the relative velocity along the blade span, changes the area swept by each blade element, rotates the plane in which the blade forces are projected, and changes the blade length per unit height of the rotor.

The increase in swept area can lead to an increase in power capture, since the power available to the rotor increases. The change in relative velocity over the blade span introduces a variation in local tip speed ratio along the blade span, whereby the blade tips are operating at a higher tip speed ratio than the blade root. The change in area swept by each blade section also means that a constant chord length blade will have variable solidity along its span.

The rotation of the plane in which the blade forces are projected indicates that a larger force is required to obtain a similar blockage value. However, this is cancelled by the fact that the blade length per unit height increases by the same factor. The tangential force component remains in the direction of rotation. The change in the orientation of the blade elements reduce the normal component of velocity in the plane of the blade element, and therefore the angle of attack at the blade section.

3.3.7 Conclusions

Chapter 2 demonstrated that high tip speed ratios are required for efficient power take-off with secondary rotors. The two key rotor variables associated with a change in optimum tip speed ratio are identified to be the rotor solidity and the blade inclination angle.

Figure 3.10 shows the power coefficient as a function of tip speed ratio for a large scale 2 bladed H-Rotor with an aspect ratio of 1.5 for 3 different chord lengths, as calculated using the a 2D actuator cylinder model discussed in Section 3.2. It is clear that lowering the solidity of the rotor increases its design tip speed ratio; however, a significant penalty in power coefficient is incurred as the solidity is decreased and the optimum tip speed ratio increases. This is predominantly due to the increase in parasitic drag from the rotor crossarm. With a chord to radius ratio of 0.05, the H-Rotor has an optimal tip speed ratio of 4.25, with a power coefficient of 0.35, a 30% decrease from the power coefficient of a rotor with a chord to radius ratio of 0.25. Additionally, the reduction in the blade chord reduces the second moment of area of the blade and increases the internal stresses under a given bending moment. A significant reduction in the blade chord is therefore impracticable from a structural standpoint,

requiring strong materials, or more blade material, to compensate and thereby leading to increased costs.

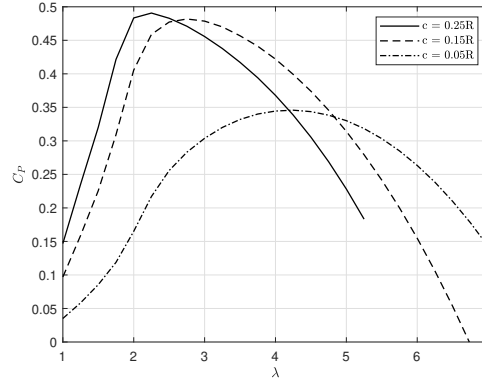


Figure 3.10: Effect of blade chord length, c , on the power coefficient - tip speed ratio, C_p - λ , curve for a 2 bladed large scale H-Rotor with radius R , and a blade length to inner radius ratio of 3, as calculated using the 2DAC model.

Figure 3.11 shows the power coefficient as a function of tip speed ratio for a large scale 2 bladed VAWT with a blade length to inner radius ratio of 3 (effective aspect ratio of 1.5) and a chord to inner radius ratio of 0.25. Inclination angles of 0° , 20° and 40° are shown. These rotors have identical blade lengths, inner radii, and chord lengths, they are only differentiated by the inclination angle of the blades.

It is clear that the introduction of an inclination angle facilitates an increase in optimal tip speed ratio without such a significant penalty in power coefficient. At an inclination angle of 40° , the optimum tip speed ratio is increased to 5 and with a power coefficient of 0.46, only 7% lower than the equivalent H-rotor value of 0.49.

Additionally, since inclining the rotor blades changes the swept area of the turbine, the changes in rotor power are not well captured by only referring to the power coefficient. Figure 3.12 shows the power captured by the rotor at a wind speed of $12m/s$ as a function of λ . Due to the large increase in swept area, the rotor with an inclination angle of 40° can achieve an additional 40% power capture whilst operating at a significantly higher tip speed ratio.

These preliminary results provide the motivation to further study the effects of blade inclination angle on VAWT rotor aerodynamics, specifically looking at VAWTs

coupled with secondary rotors. The following section provides a thorough overview of the existing literature around straight bladed VAWTs with inclined blades.

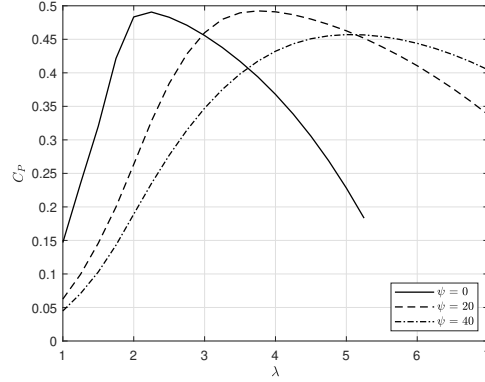


Figure 3.11: Effect of blade inclination angle, ψ , on the power coefficient - tip speed ratio, C_P - λ , curve for a large scale straight bladed VAWT with a chord to radius ratio of 0.25, a blade length to inner radius ratio of 3, and 2 blades, as calculated using the 2DAC model.

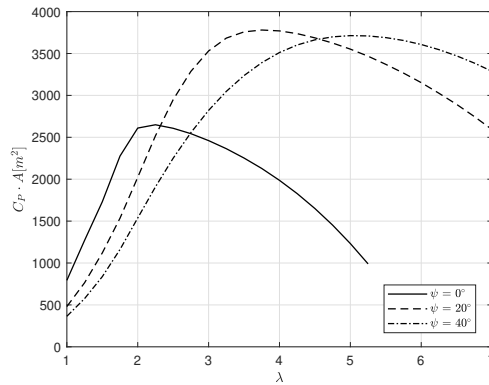


Figure 3.12: Effect of blade inclination angle, ψ , on the $(C_P A)$ - λ curve for a large scale straight bladed VAWT with a chord to radius ratio of 0.25, a blade length to inner radius ratio of 3, and 2 blades, as calculated using the 2DAC model.

3.4 Development of VAWTs with inclined blades

The use of straight bladed VAWTs with inclined blades was first introduced by Ljungström [115], with the author noting that straight bladed VAWTs facilitated cyclic pitching more easily than Darrieus type rotors. A variable geometry H-Rotor with inclinable

blades was later proposed by Musgrove [116], this used a reefing system to incline the blades as a means of reducing power capture at high wind speeds. Wind tunnel tests on a scale model conducted by Willmer [117] demonstrated that the power coefficient decreased significantly as the blade inclination angle was decreased from 0° to -60° . Figure 3.13a) shows the 'Musgrove 250' turbine with blades in the 'reefed' position. It should be noted that, since the rotor used blade inclination to limit power, the blades were inclined such that the swept area was reduced.

Further work on straight bladed VAWTs with inclined blades was completed by David Shapre on rotors with the blade connected directly to a central hub. An initial study [118] determined the power coefficient for inclination angles of 35° , 45° and 55° for a blade aspect ratio of 11.9, finding the maximum power coefficient occurring with an inclination angle of 55° . Additionally, increasing blade aspect ratio was found to increase the peak power coefficient for an inclination angle of 45° . The aerodynamic simulation work was completed using the DMS approach described in [119]. Further work was completed, validating the aerodynamic simulation work and demonstrating how the rotor could be controlled using a tip-pitch system in [120], and a 5kW system was field tested in 1987 [121]. This system is shown in figure 3.13b). Following these tests, a single bladed design utilizing a counterbalancing weight was proposed [6], see figure 3.13c).

Further academic work on VAWTs with inclined blades was undertaken as part of the NOVA project with the design of the Aerogenerator-X, as shown in figure 3.13e) [6]. This 10MW V-rotor had rotor blades that were augmented with sails, aimed and counteracting the aerodynamic overturning moments inherent in V-VAWT designs. The rotor design work is presented in [6]. A gradient based optimisation approach was coupled to a BEM based aerodynamic solver [122] and was used to find a rotor geometry that minimised the rotor volume (used as a proxy for the rotor cost) under a number of design constraints including ensuring a rated power of 10MW at 13m/s with rated rotor speed of 4rpm. The inclination of the rotor blades was treated as a free variable; however, the optimisation procedure did not significantly alter the initial inclination angle of 59° . The aerodynamic simulation tool developed by Shires used a

single induction factor over each half cycle, essential applying a double actuator disk approach. This does not accurately represent the aerodynamic behaviour of the rotor in either the upwind or downwind rotor half.

In [123], Abusannuga introduces a VAWT rotor geometry similar to the X-Rotor primary rotor, with inclined upper and lower blades attached to a single cross-arm. The study uses a multiple streamtube method to calculate the aerodynamic power coefficient of a number of rotor configurations. It is shown that inclination angles of 25° to 45° maximise the power coefficient and that blade inclination increases the optimum tip speed ratio. This study was limited in a number of ways. Firstly, it did not explicitly account for how the change in rotor area affects power capture, and therefore did not meaningfully capture the effect of blade inclination on power capture. Secondly, there was no discussion of blade loads. Finally, the use of a multiple streamtube modelling tool indicates that the inflow conditions to the downwind rotor half are not well described by the model.

From this review of the literature, it can be seen that previous investigations have predominantly investigated V-Rotors, rather than X-Rotors. A key design synergy between VAWT rotors and secondary rotors has been identified as the near ocean surface attachment location of the secondary rotors. Since the blade tips of V-rotors are the top of the rotor, there is no near-ocean-surface attachment location for secondary rotors. X-shaped VAWTs are considered in [123]; however, the investigation presents only the power coefficient, and does not consider the effects of the change in rotor swept area on power capture. Additionally, the investigation only reports power coefficient, and does not consider the aerodynamic loads on the rotor. Finally, none of the papers identified within this literature review have considered the conversion efficiency between primary and secondary rotors when designing a VAWT rotor. The proceeding sections of this chapter will aim to fill the identified literature gap through a numerical modelling campaign.



Figure 3.13: a) Musgrove 250 turbine [4], b) 5kW V-Rotor tested by Sharpe and Taylor [5], c) Single bladed V-VAWT concept [6], d) Taylor V-Turbine [5], e) The aerogenerator concept [6], and f) the X-Rotor wind turbine concept.

3.5 Methodology

The investigation presented in this chapter examines a number of parametrised X-Shaped VAWT rotor designs, using a 2DAC model to evaluate power capture and loads. The extreme loads are calculated based on a blade element approach, and significant post processing is applied to evaluate the potential designs. Section 3.5.1 describes the rotors designs considered in this study, Section 3.5.2 described how the rotor power production and associated loads are calculated, and Section 3.5.3 described how extreme loads are calculated.

3.5.1 Rotor geometry

In order to preserve a key design synergy identified between straight bladed VAWT rotors and secondary rotors, primary rotors in an X-configuration are considered. This provides attachment positions for the secondary rotors close to the ocean surface and minimises the overturning moment at the rotor hub. Figure 3.14 shows an example of the rotors considered in this study. The test geometries are developed from 4 base rotors which share a fixed crossarm length 25m, and have blade lengths of 25m, 50m, 75m, and 100m extending outwards in both directions from the crossarm. These correspond to effective aspect ratios of 1, 2, 3 and 4 respectively. Each upper and lower blade has a blade root defined as its connection point to the crossarm and a blade tip, and the end of the blade.

Each configuration is simulated with inclination angles of 0° to 50° , constant chord lengths ranging from 0.625m ($0.025r_0$) to 12.5m($0.5r_0$) with an interval of 0.625m, over a tip speed ratio range from 1.5 to 6 with an interval of 0.25. The blades have a constant NACA0018 section, with the polars calculated at Reynolds number of 15 million, and extrapolated using the Viterna method. Table 3.1 shows the range of rotor design variables considered in this study.

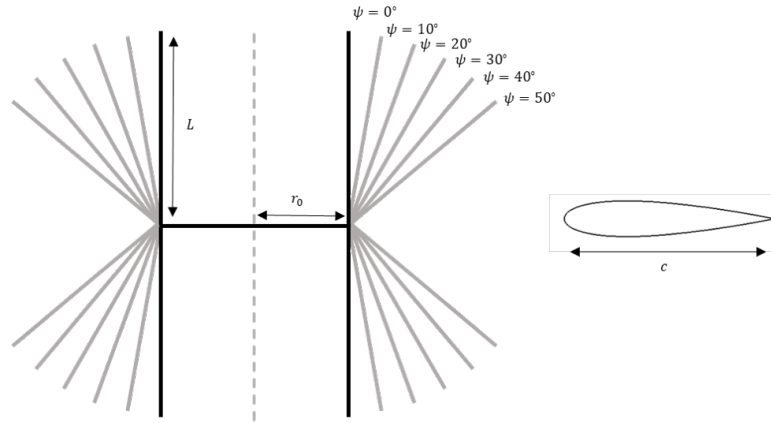


Figure 3.14: Representation of the rotor configurations considered in this study.

variable	value
r_0 [m]	25
L [m]	25,50,75,100
ψ [°]	0,10,20,...,50
c [m]	0.625,1.250,1.875,...,12.50
aerofoil	NACA 0018
N_b	2
U_0 [m/s]	12
λ	1.50,1.75,2.00,...,6
Re	15,000,000

Table 3.1: Range of parameters for primary rotors considered in this study.

3.5.2 Power production

Power production and operational loads are computed using the 2DAC model validated in Section 3.2.4. The aerodynamic model is updated to include the parasitic drag from the strut interaction, for a strut chord length equal to 1/2 of the blade chord length, with a 30% thickness, and a Reynolds number of 400,000 (representative of the average chord-wise Reynolds number of the strut over the tip speed ratio range considered in this study at a wind speed of 12m/s).

A dynamic stall sub-model has not been included since, for inclined blades at low tip speed ratios, the local tip speed ratio drops well below 1 at the blade root, which is outside of the range for which many dynamic stall models have been validated, additionally at the high tip speed ratios required for efficient power conversion between

the primary and secondary rotors, the model has been successfully validated against CFD simulations which inherently model dynamic stall. Flow curvature corrections have also been omitted due to the more accurate performance of the model compared to blade resolved CFD results without the correction factor, as in [124]. A Prandtl type tip loss correction is applied as in [103], as is recommended in [85].

To preserve generality, no wind shear profile is considered. This allows a single V rotor to be simulated, and for a mirror image of that turbine to be added in the post processing to obtain the results for the full rotor, as shown in figure 3.15. This symmetry means that there is no overturning moment at the rotor hub, and that the overturning moment at the tower base is given by the product of the rotor thrust and the hub height.

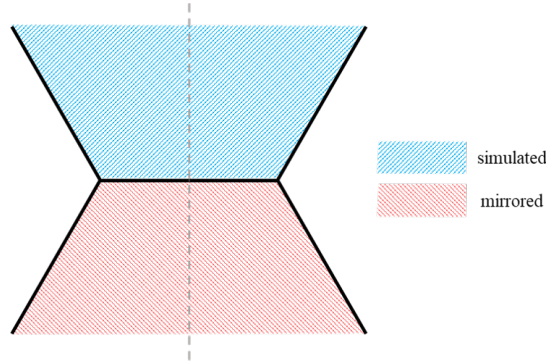


Figure 3.15: Example showing how symmetry can be exploited for X-Rotor simulation.

A useful metric for understanding the operational loads on the turbine is operational out of plane (flapwise) blade root bending moment, given by

$$M_{flap}(\theta) = \int_0^L f_n(\theta, l) l dl. \quad (3.13)$$

Whilst the operational loading regime of VAWT rotor is complex and a full fatigue analysis is outside of the scope of this investigation, comparing the maximum operational flapwise blade root bending moment can give an indication of how operational loads are expected to differ between turbine designs.

Following the aerodynamic simulation of the primary rotor, the conversion efficiency between the primary and secondary rotors can be calculated using the methodology

provided in Chapter 2, and electrical power available to the turbine can be determined. The secondary rotors are designed with the parameters given in table 2.1.

3.5.3 Extreme loads

Typically in literature, it is assumed, based on [20, 57] that VAWT blade design is dictated by extreme loads, as in [125]. The extreme wind field can be obtained using the the IEC extreme wind model without turbulence,

$$U = 1.4U_{ref} \left(\frac{z}{z_{ref}} \right)^{0.11} \quad (3.14)$$

with $U_{ref} = 50\text{m/s}$ for an IEC class 1 turbine, corresponding to a high wind speed site. Here z_{ref} is given by the hub height considering a lower blade tip clearance of 25m,

$$z_{ref} = 25 + l \cos(\psi). \quad (3.15)$$

As in [125] the maximum load is assumed to occur at an azimuth angle of 90° , where the coefficient of drag is at its maximum value. The bending moment at the spanwise location l along the blade is then given by

$$M(l) = \frac{1}{2} \rho c C_D \int_l^L U_{BE}^2 (\tilde{l} - l) d\tilde{l} \quad (3.16)$$

where $U_{BE} = U \cos(\psi)$ is the projection of the wind speed onto the plane of the blade element. The blade root bending moment is obtained by evaluating this at $l=0$. This simplified analysis allows the trends in extreme aerodynamic loads to be estimated. Although the self-weight load is not considered in this analysis, the gravitational load due to the secondary rotor is included for the lower blade with the term

$$M_{SR}(l) = m_{sr} g \sin(\psi)(L - l). \quad (3.17)$$

In this case, a secondary rotor mass of 10 tonnes is assumed. Assuming that the design driving load is the extreme load case described above, a representative estimate of the blade mass can be generated using this load case.

This estimate is calculated assuming that the load bearing structural element in the blade is a pair of uni-axial glass fibre spar caps with variable thickness t_{spar} separated centre-to-centre by the distance $t_f - t_{spar}$, where t_f represents the aerofoil thickness. Including the mass of an outer shell of tri-axial glass fibre with a fixed thickness of $t_{shell} = 0.005m$, the total mass of the blade is given by

$$m_{blade} = \int_0^L \rho_{spar} l_{spar} t_{spar}(l) + \rho_{shell} t_{shell} l_{shell} dl \quad (3.18)$$

where the surface profile length of the aerofoil, l_{shell} is given by $l_{shell} = 2.076c$ for the NACA0018 section considered here. The length of the spar is chosen to be $l_{spar} = 0.4c$ as in [126]. In order to evaluate equation 3.18, the spar thickness distribution must be obtained. This can be calculated by assuming that the spar is designed efficiently, such that the maximum stress under load is equal to the maximum allowable stress.

At any given point along the blade, the stress, σ_{bld} , in the spar cap at a distance γ from the neutral axis is given by

$$\sigma_{bld} = M \frac{\gamma}{I}. \quad (3.19)$$

Where I represents the 2nd moment of area, and the bending moment M can be calculating using equation 3.16. The location of maximum bending stress occurs at the point furthest from the neutral axis, $\gamma = t_f/2$.

Assuming the spar caps are a pair of rectangular sections with length l_{spar} , thickness t_{spar} , separated centre-to-centre by a distance $t_f - t_s$, and centred on the neutral axis, the second moment of area is given by:

$$I = \frac{l_s}{12} (t_f^3 - [t_f - 2t_{spar}]^3). \quad (3.20)$$

To ensure that the bending stress is lower than the maximum allowable value, the spar thickness must therefore satisfy:

$$4t_{spar}^3 - 6t_f t_{spar}^2 + 3t_f^2 t_{spar} - 3 \frac{M t_f}{l_s \sigma_{max}} = 0, \quad (3.21)$$

which can be solved using an appropriate value of σ_{max} . The maximum bending stress

under compression for uni-axial glass fibre can be taken from [20] as $\sigma_{max} = 625.8\text{MPa}$, and the density of both uniaxial and tri-axial can be obtained from the same source as $\rho_{spar} = 1915\text{kg/m}^3$ and $\rho_{shell} = 1845\text{kg/m}^3$.

3.6 Results

3.6.1 Power production

The power coefficient as a function of tip speed ratio for fixed chord lengths of 2.5m to 10m are shown in figure 3.16. For every configuration presented in figure 3.16 it is clear that increasing the inclination angle increases the optimum tip speed ratio and that increasing the blade chord length, and thus the rotor solidity, decreases the optimum tip speed ratio. Power coefficients between 0.49 and 0.5 are achievable in many configurations, and the peak power coefficient of 0.498 is found for the rotor with an inclination angle of 0, a blade length of 100m, and a chord length 5m (1/5th of the inner radius) at a tip speed ratio of 2.5.

The peak power coefficient for each rotor configuration and the corresponding tip speed ratio are shown in figure 3.17. This makes clear the positive trend between rotor inclination angle and optimum tip speed ratio. It also shows higher chord length blades can ensure that the power coefficient remains high when a blade is inclined. A limitation of the current study is also highlighted in this figure. At low chord lengths, the optimum tip speed ratio reaches the upper limit considered in this study at relatively low blade inclination angles, indicating the actual optimum tip speed ratio may be higher, and that higher power coefficients could be attained outside of the search range.

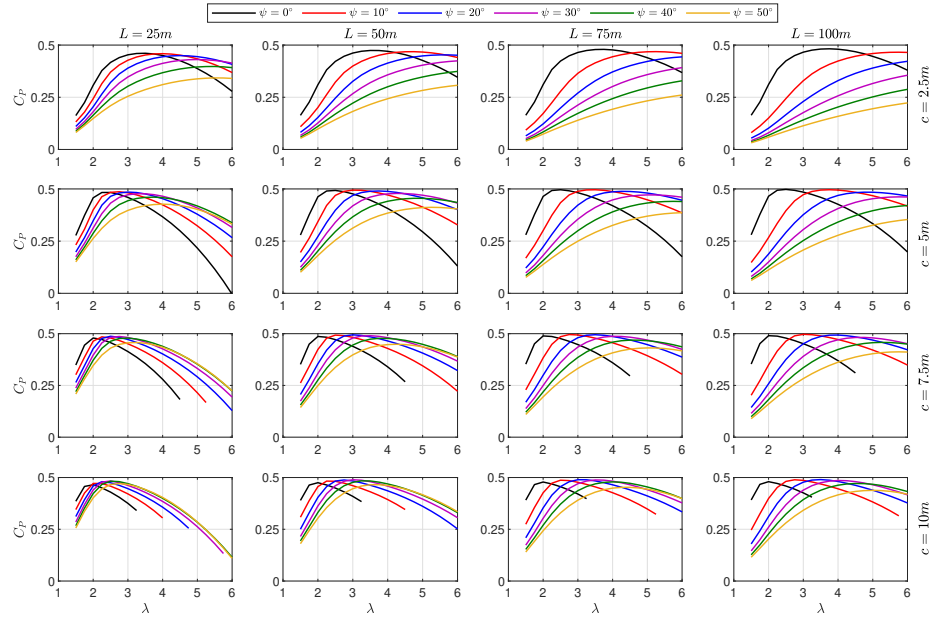


Figure 3.16: Power coefficient - tip speed ratio, $C_P - \lambda$, curves of various VAWT rotor configurations with chord length c , blade length L , and inclination angle, ψ , as calculated using the 2DAC model.

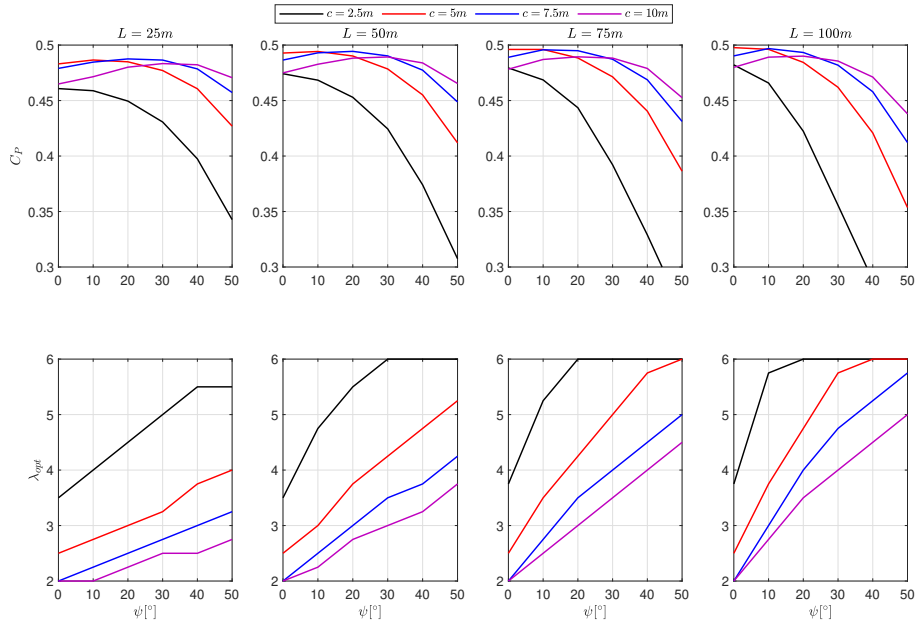


Figure 3.17: Peak power coefficient, C_p , and corresponding optimum tip speed ratio, λ_{opt} , as a function of blade inclination angle, ψ , for various VAWT rotor configurations with chord length c , and blade length L .

Since the rotor geometry is dependent on both the blade length and the inclination angle, it is more informative to look at the power captured by the rotor, rather than the power coefficient, when comparing rotor configurations. Figure 3.18 shows the power captured by each rotor configuration operating at its optimum tip speed ratio at a wind speed of 12m/s with an air density of 1.225kg/m³. This clearly demonstrates potential benefits of blade inclination, allowing for a significant increase in power production without a change in the blade planform. It is also clear that more power is available to rotors with inclined blades as the aspect ratio increases. This is due to the increase in swept area, since longer blades facilitate a larger increase in area when the blade is inclined.

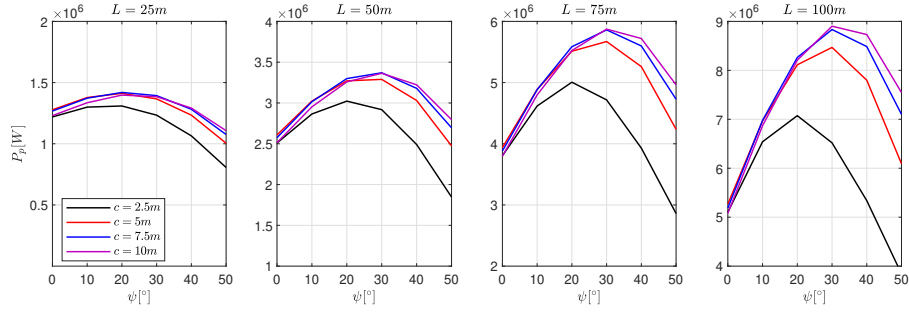


Figure 3.18: Rated power, P_p , as a function of inclination angle, ψ , for various VAWT configurations with chord length c , and blade length L , considering a wind speed of 12m/s and an air density of 1.225kg/m³.

Using the methodology described in Chapter 2, the maximum achievable conversion efficiency for a secondary rotor designed with the parameters provided in table 2.1 can be readily calculated for each of the primary rotor configurations presented in figure 3.18. This allows the calculation of the power available to the secondary rotors for each rotor configuration at a wind speed of 12m/s with an air density of 1.225kg/m³. Since the conversion efficiency is significantly dependent on the primary rotor rotor tip speed ratio, for some rotor configurations the optimum tip speed ratio of the combined primary-secondary rotor shifts compared to the optimum tip speed ratio of the primary rotor. The maximum power at the output of the secondary rotors is presented in figure 3.19. It is clear that there is a significant power increase associated with blade inclination for blades of identical planform which increases as the aspect ratio increases. Table 3.2 shows the rated power of each H-Rotor configuration, the maximum rated power of each X-Rotor configuration and the corresponding optimum inclination angle. Power gains between 4% and 152% are available through blade inclination. For most rotor configurations, the optimum inclination angle is 30°; however, rotors with lower aspect ratios and low solidities have lower optimum inclination angles, and rotors with high aspect ratios and high solidities have higher optimum inclination angles.

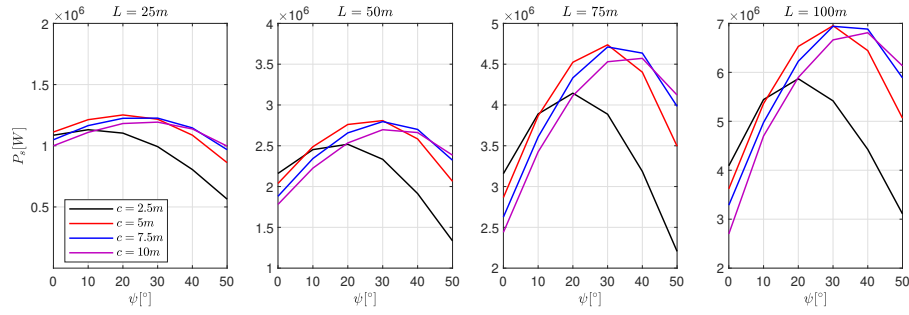


Figure 3.19: Power output of optimally designed tip mounted secondary rotors, P_s , as a function of blade inclination angle, ψ , for various VAWT configurations with chord length c , and blade length L , considering a wind speed of 12m/s and an air density of 1.225kg/m^3 .

		$c = 2.5\text{m}$	$c = 5\text{m}$	$c=7.5\text{m}$	$c=10\text{m}$
$l = 25\text{m}$	P_H (MW)	1.09	1.09	1.00	0.91
	P_X (MW)	1.13	1.24	1.20	1.15
	ΔP (%)	4.03	13.74	19.76	25.85
	ΔT (%)	0.87	-0.32	-3.51	3.80
	ψ (°)	10.00	20.00	30.00	30.00
$l = 50\text{m}$	P_H (MW)	2.17	2.01	1.80	1.62
	P_X (MW)	2.52	2.80	2.77	2.65
	ΔP (%)	16.38	39.45	53.85	63.90
	ΔT (%)	0.87	-0.32	-3.51	3.80
	ψ (°)	20.00	30.00	30.00	30.00
$l = 75\text{m}$	P_H (MW)	3.17	2.81	2.50	2.27
	P_X (MW)	4.15	4.73	4.69	4.54
	ΔP (%)	30.99	67.95	87.54	100.51
	ΔT (%)	11.10	21.95	35.13	35.50
	ψ (°)	20.00	30.00	30.00	40.00
$l = 100\text{m}$	P_H (MW)	4.10	3.56	3.14	2.70
	P_X (MW)	5.87	6.94	6.92	6.78
	ΔP (%)	43.18	94.87	120.26	151.74
	ΔT (%)	11.62	35.16	52.84	55.80
	ψ (°)	20.00	30.00	30.00	40.00

Table 3.2: Rated power of different straight bladed VAWT configurations using secondary rotors for power take-off in a H configuration and the X configuration which maximises power capture. ΔP and ΔT represent the change in power and thrust moving from an H configuration to the X configuration which maximises power capture, and ψ_{opt} represents the inclination angle that maximises power capture.

The information presented in figure 3.19 can be condensed into a series of con-

four plots, showing the maximum available power for the primary-secondary rotor configuration as a function of blade length and inclination angle for different chord lengths. These contour plots of maximum available power are presented alongside the corresponding rotation averaged thrust values, maximum blade root bending moment values, and rated rotor speed values in figure 3.20.

In the case of a HAWT, the rotor area is dependent only on the blade length, allowing similar rotors to be compared using the power coefficient. However, in the case of a X-VAWT, the rotor area is dependent on both the blade length, inclination angle, and crossarm length. This implies that non-dimensionalising by the rotor area to compare the power coefficients is not informative, since two rotors employing blades of equal length and producing the same amount of power may have significantly different power coefficients due to the changes in swept area associated with blade inclination angle, as can be seen comparing figures 3.17 and 3.18. Additionally, the conversion efficiency of secondary rotors is dependent on dimensional properties including the rotor tip speed and the rated power. For this reason, the effects of blade inclination angle are further investigated considering turbines with a 5MW power rating, and a fixed crossarm length of 25m.

The minimum blade length required for a 5MW power rating as a function of blade inclination angle is shown in figure 3.21. This demonstrates that, for blades of constant chord length, the inclination angle which facilitates the minimum blade length increases as the chord length increases. The inclination angle corresponding to a minimum blade length ranges from 20° to 40° . To obtain a power rating of 5MW with an un-inclined blade, a blade length of more than 100m from crossarm to tip, or 200m from lower blade tip to upper blade tip, is required. However, with a blade inclination angle of 30 degrees, a blade length of only 78m, crossarm to tip, is required with a chord length of 5m.

Figures 3.22 and 3.23 show the rotation averaged thrust and maximum out of plane blade root bending moment for the rotors described in figure 3.21 at rated wind speed. It is clear that for every configuration, the thrust decreases as the inclination angle increases for a given power rating. This can be understood as the rotor operating at a

lower axial induction, facilitated by the increased area, and therefore having a higher ratio of power to thrust coefficient. The blade root bending moment decreases such that it is at a minimum at the minimum blade length, this is to be expected as the blade root bending moment is proportional to the square of the blade length.

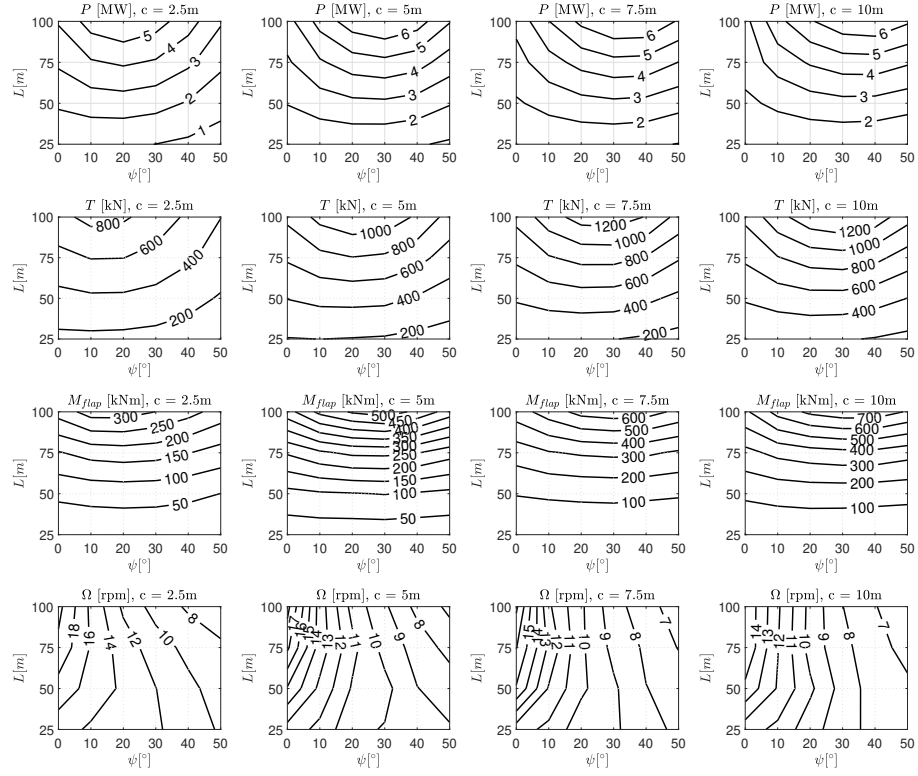


Figure 3.20: Power, P , thrust, T , maximum blade root bending moment, M_{flap} , and optimum rotor speed, Ω , contours for various rotor configurations with chord length c , blade length L , and inclination angle ψ , operating at their optimum tip speed ratio at 12m/s with an air density of 1.225kg/m³.

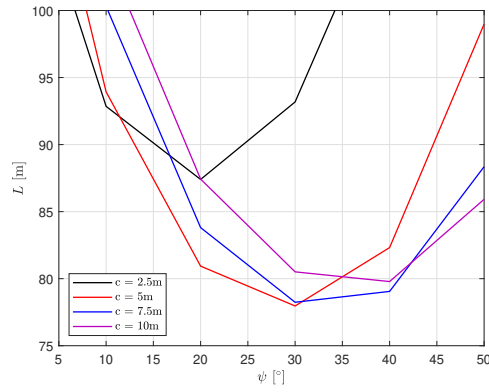


Figure 3.21: Minimum blade length, L , required for a power rating of 5MW as a function of blade inclination angle, ψ , for rotor configuration with chord length c , at a wind speed of 12m/s with an air density of 1.225kg/m³.

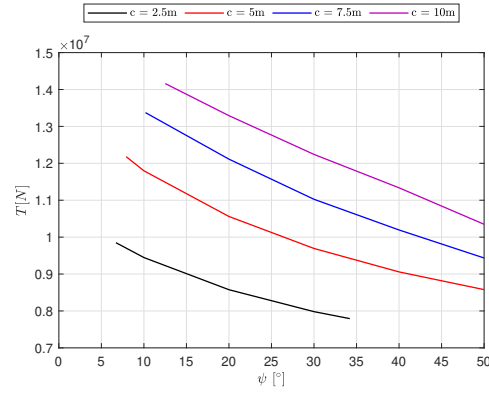


Figure 3.22: Rotation averaged thrust, T , at rated power as a function of blade inclination angle, ψ , for rotor configurations with chord length c , rated at 5MW at a wind speed of 12m/s with an air density of 1.225kg/m³.

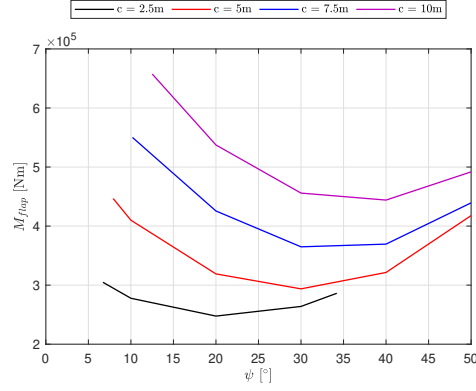


Figure 3.23: Maximum blade root bending moment, M_{flap} , at rated power as a function of blade inclination angle, ψ , for rotor configurations with chord length c , rated at 5MW at a wind speed of 12m/s with an air density of 1.225kg/m³.

Unlike a variable pitch HAWT, where the maximum operational loads typically occur around the rated wind speed, the maximum operational loads for a VAWT typically occur closer to cut out wind speed, as discussed in Chapter 4, and are therefore dependent on the above rated operational strategy. For this reason the downward trend in thrust loads as the inclination angle increases can only be seen as indicative that the operational loads would likely be reduced. A full design of the turbine including the structural design, operational strategy, and controller must be completed before the loads across the full operational envelope can be determined.

3.6.2 Extreme loads

The flapwise upper and lower blade root bending moment calculated for a parked rotor, using the IEC class 1 extreme wind field without turbulence is shown in figure 3.24 alongside the corresponding blade mass calculated using equation 3.18.

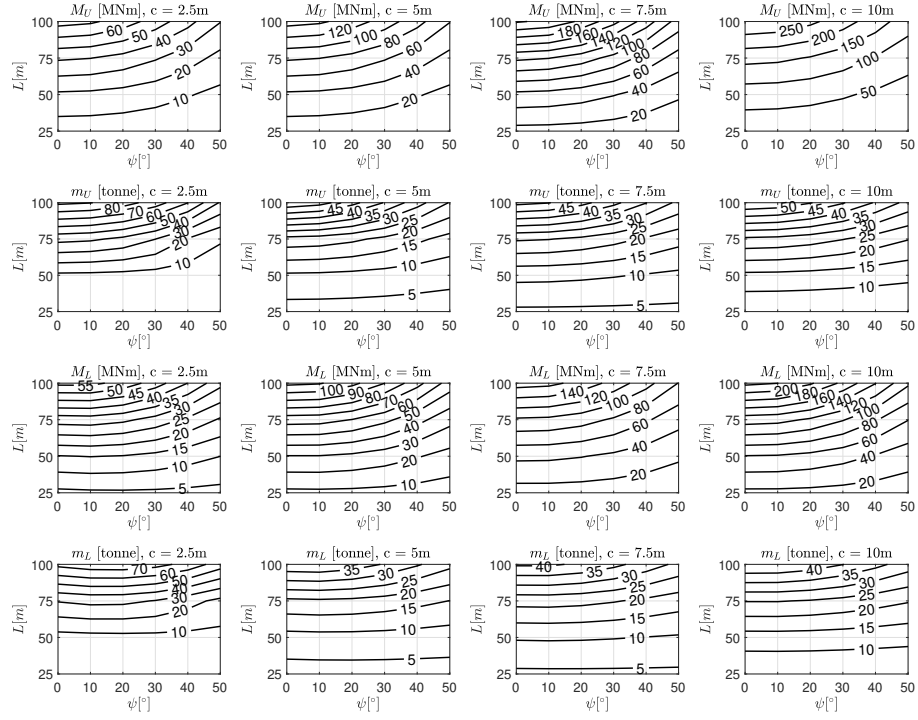


Figure 3.24: Extreme flapwise blade root bending moment for upper, M_U , and lower, M_L , blades, and estimated blade mass for the upper, m_U , and lower, m_L , blades of various rotor configurations with chord length c , blade length L , and inclination angle ψ .

Figure 3.24 shows that blades of equal blade length and chord length will experience lower extreme blade root bending moments as the blades are inclined. This is to be expected since the component of wind speed normal to the blade drops with $\cos(\psi)$, leading to the force on the blade dropping with $\cos^2(\psi)$. Due to the gravitational load from the secondary rotor increasing with $\sin(\psi)$, this effect is less apparent for the lower blades, especially at low chord lengths, since the relative size of the gravitation load from the secondary rotor increases compared to the aerodynamic load.

The increase in blade root bending moment for any rotor configuration with a fixed chord length and inclination angle is also visible in figure 3.24. This is expected since, under constant force intensity, the blade root bending moment increases with the square of blade length. The translation of these lower loads, into a lower blade mass is also

clear.

Figure 3.25 shows the extreme blade root bending moment as a function of inclination angle for the 5MW turbines presented in figure 3.21, and their associated blade masses. The reduction in wind speed projected into the plane of the blade element leads to a reduction in blade root bending moment for the 5MW blades as the blade inclination angle increases. It is also clear that the inclination angle which minimises the blade mass increases as the blade chord length increases. For the chord lengths presented here, both the upper and lower rotor blade mass is minimised with a chord length of 5m and an inclination angle of 30° .

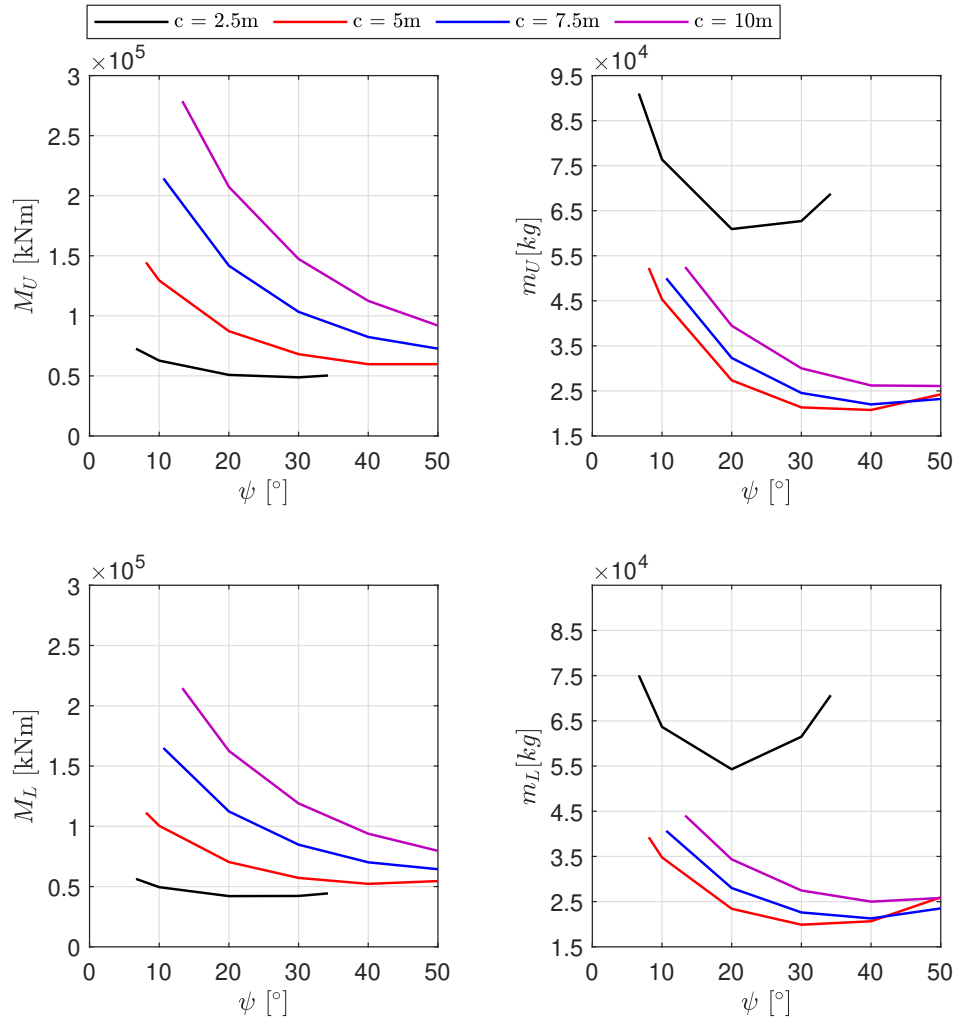


Figure 3.25: Extreme flap-wise blade root bending moment for both upper, M_U , and lower, M_L blades and blade mass for both upper m_U , and lower, m_L , blades as a function of blade inclination angle, ψ , for rotor configurations with chord length c , rated at 5MW at a wind speed of 12m/s with an air density of 1.225kg/m^3 .

Figure 3.26 shows the minimum combined blade mass for a 5MW turbine rating as a function of blade chord length, and the corresponding optimum blade length, inclination angle, and operational tip speed ratio.

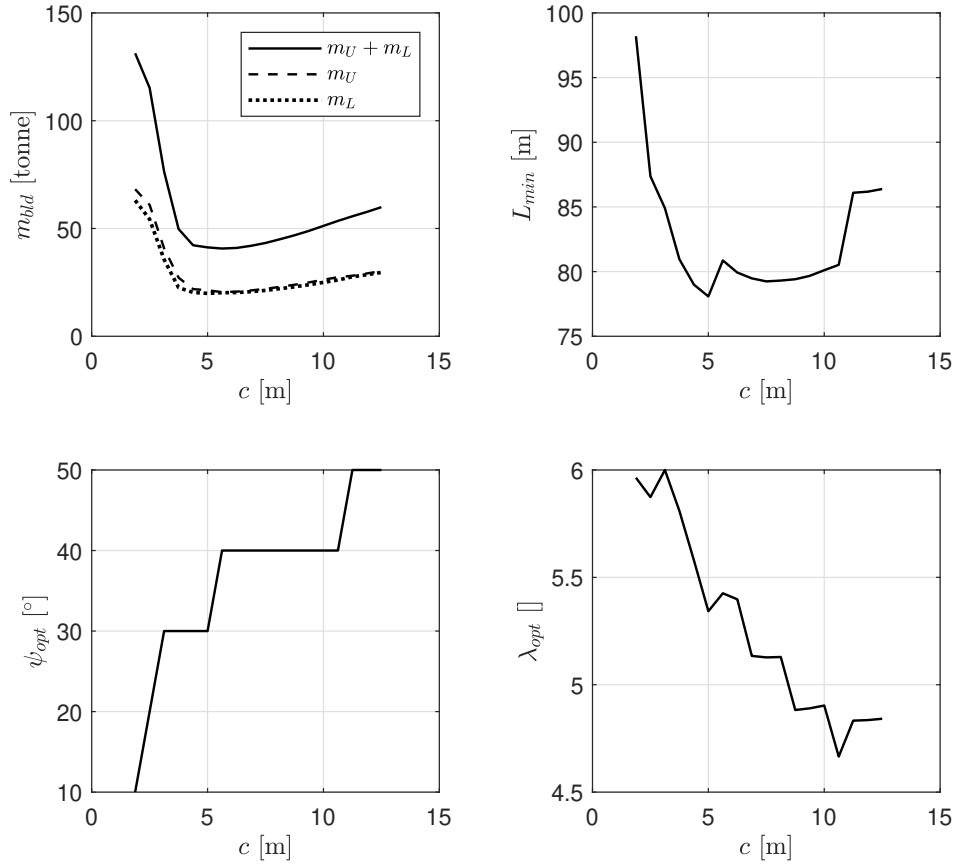


Figure 3.26: Minimum blade mass, m_{bld} , for a 5MW X-Rotor type turbine with a rated wind speed of 12m/s, as a function of blade chord length, c , and corresponding values of blade length, L_{min} , inclination angle, ψ_{opt} , and optimum tip speed ratio, λ_{opt} .

It is clear from figure 3.26 that an optimum fixed chord length between 4m-6m minimises the blade mass for a 5MW X-Rotor type turbine with a crossarm length of 25m. This corresponds to a blade length of 78-80m, an inclination angle of 30°-40°, and an optimum tip speed ratio between 5.8 and 5.4. Furthermore, it is clear that as the rotor increases in solidity, the optimum inclination angle increases, and the optimum tip speed ratio reduces.

3.7 Discussion

The C_P - λ curves shown in figure 3.16 provides the basis for the power production results. For H-Rotors, the peak power coefficient increases as the blade length increases, as the tip losses are reduced, and the proportion of power lost to parasitic drag decreases. Increasing the chord length is also shown to decrease the optimum tip speed ratio, as expected. The optimum chord length is found to be 5m, corresponding to a solidity of 0.4.

Generally, as the blade is inclined the peak power coefficient drops and the optimum tip speed ratio increases. The reduction in peak power coefficient is due to the blade operating at a range of λ values, which means that the full rotor is not operating optimally at the single optimum value of λ . Furthermore, the reduction in the range of angle of attack fluctuations can lead to a reduction in peak efficiency. The increase in λ_{opt} occurs for two key reasons. Firstly, the reduction in the range of angle of attack fluctuations reduces the magnitude of averaged normal force coefficient, which means that a larger dynamic pressure is required to reach optimal blockage. Secondly, the blade tips will be operating above their local λ_{opt} , when the rotor is operating at its λ_{opt} since the optimum value for the rotor accounts for the whole rotor, and the inboard sections of the blade operate at a lower local value of λ than the blade tip.

Figure 3.16 also shows that rotors with inclined blades have a wider peaked C_P - λ curve, with a less precipitous reduction in C_P as the tip speed ratio reduces below the peak value. This is expected, since locally the blades are operating over a range of tip speed ratios, and the blades therefore stall gradually from root to tip as the tip speed ratio decreases. The implication of this is that passive power regulation is far less feasible and that a control action such as pitch regulation is likely required in above rated conditions.

In terms of power production, figures 3.17 and 3.18, show that introducing a blade inclination angle increases the optimum tip speed ratio of a straight bladed VAWT rotor, and can significantly increase the power captured by the rotor. Whilst there is typically a decrease in power coefficient, the increase in swept area overcomes this

deficit.

Considering power take-off with optimally designed secondary rotors, figure 3.19 shows that there is a significant increase in the power available to secondary rotors for primary rotors employing inclined blades. Table 3.2 indicates that blade inclination can facilitate a power increase between 4% at low solidity low aspect rotors, and 152% at high solidity high aspect ratio rotors. This is due to the increase in primary rotor power capture and the increased performance at high tip speed ratios.

Whilst an increase in primary rotor thrust associated with blade inclination is also shown, the increase in thrust is significantly lower than the increase in power for the majority of configurations. This is because the rotor is effectively operating at a lower induction value, increasing the ratio of power to thrust coefficient. This is exacerbated by the fact that, in the high solidity cases, the optimum H-rotor tip speed ratio, accounting for the conversion efficiency of the secondary rotors, is significantly larger than the optimum tip speed ratio of the lone H-rotor. This means that the H-rotor is operating at a significantly higher induction value.

For a 5MW rated VAWT using secondary rotors as power take-off, figures 3.21, 3.22, 3.23 and 3.25 show that blade inclination is required when considering a maximum unsupported blade length of 100m, and that the inclination angle that minimises the blade length lies between 20° and 40° depending on chord length. There is also a clear downward trend in terms of primary rotor thrust at rated power with respect to blade inclination angle, indicating the potential for a reduced support structure cost associated with blade inclination. It should also be noted that the tower height required to obtain a tip clearance of 25m reduces with increased inclination angle, which can facilitate further reductions in the tower base bending moment. However, the hub height is likely decided not only by minimum tip clearance but also the desired position in the boundary layer, and this reduction may, therefore, not be realised. The maximum blade root bending moment at rated wind speed is minimised when the blade length is minimised for each configuration, which is to be expected.

In figure 3.26, it is found that, for fixed chord length X-Rotors rated at 5MW, the optimum fixed chord length to minimise blade mass lies between 4m-6m and that

this corresponds to a blade length of 78-80m, an inclination angle of 30° - 40° , and an optimum tip speed ratio between 5.8 and 5.4. Although only high level structural design was considered, this indicates that the large chord lengths associated with 5MW X-Rotor design developed in [27] could be reduced, and that the blade length could also be reduced. This has the potential to reduce the capital cost of the primary rotor. Practical designs would, of course, include a blade taper, and issues around the frequencies of the blade modes, blade buckling, and rotor performance over the full operational envelope must be considered.

A major limitation of this study is that it has only considered fixed chord length blades. Variable chord length blades, which taper towards the blade tip, are beneficial in three ways. Firstly, the larger chord length of the blade at the blade root provides a larger second moment area, reducing the volume of material required to reduce the stress to an allowable value and reducing the blade mass. Secondly, in the extreme load case, the decrease in chord length towards the blade tip reduces the force intensity at blade locations with a large moment arm, reducing the extreme load, again reducing the volume of material required to limit the bending stress in the blade. Finally, it has been shown in [124], that a span-wise optimisation of chord length to maximise power capture introduces a tapered blade, indicating that utilising a taper can increase the rotors power coefficient. Future studies should consider introducing more complex blade designs including a tapered blade. A further limitation of this study is the use of a simplified design load case, and a very simple methodology for obtaining a representative estimate of the blade mass.

3.8 Conclusions

This chapter aimed to answer the research question *"How can a vertical axis wind turbine be designed for efficient use with secondary rotors?"* After a survey of VAWT design parameters, it was identified that the blade inclination angle was a key design variable for straight bladed VAWT rotors that could facilitate an increase in optimum tip speed ratio up to values appropriate for efficient power take-off with secondary rotors. Using a 2DAC aerodynamic simulation model, verified against higher fidelity

models, and CFD simulations, the effect of blade inclination angle of straight bladed VAWT performance is then investigated. A simple methodology for the estimation of a representative blade mass was also introduced through the evaluation of extreme loads.

It is shown that, through the introduction of an inclination angle to both the upper and lower blades, a straight bladed VAWT can increase power capture and increase its operation tip speed ratio, making the rotor more suited for use with secondary rotors. For a 5MW rotor, a fixed chord length between 4m-6m was found to minimise both upper and lower blade mass, corresponding to a blade length of 78m-80m. These rotor designs had an inclination angle of 30° - 40° . Rotors without a blade inclination angle require an unsupported blade span of over 100m to obtain a 5MW power rating, outside the range of blade lengths considered in this study.

Chapter 4

Control strategies for the X-Rotor turbine

This chapter addresses the research question "*How should a vertical axis wind turbine using secondary rotors be operated?*". Using the 2 bladed 5MW X-Rotor turbine presented in [20] and [30] as a case study, this chapter introduces a range of potential operating strategies for an X-Rotor type turbine, evaluates them based on power production and loads, then provides a recommended control strategy for the turbine.

The chapter is structured as follows: Section 4.1 discusses the difference in control strategies between conventional HAWTs, VAWTs, and the X-Rotor turbine, and reviews the existing literature on X-Rotor control. Section 4.2 then provides the aerodynamic characteristics of the X-Rotor primary and secondary rotors and Section 4.3 gives an overview of X-Rotor control. Following this, Section 4.4 defines 4 potential control strategies for an X-Rotor type turbine and Section 4.5 evaluates these strategies in terms of power capture and rotor loads. These results are discussed in Section 4.6 and, following this discussion, a recommended control strategy is presented in Section 4.7; final conclusions are drawn in Section 4.8.

4.1 Introduction

The control strategy of a wind turbine defines the operational state of the turbine over the full range of wind speeds in which the turbine is 'active' i.e. generating power. Typically, the operational state can be defined using the target values of blade pitch and rotor speed in steady state at each wind speed. From this, the steady state power production and loads can be obtained.

Large offshore HAWTs utilise variable speed, variable pitch control strategies, tracking C_{Pmax} in variable speed mode below rated power, and pitching the blades in order to operate at a constant speed and torque, and therefore power, in above rated conditions. A constant rotor speed region is typically defined between the variable speed zone and the constant power zone, where the blades remain un-pitched and the torque is allowed to increase with wind speed up to the rated power. This is employed to limit tip speed and the associated aero-acoustic emissions and leading edge erosion. Collective pitch control of the blades is also utilised for active damping of the tower vibrations [127], and individual pitch control can be used to reduce the yaw moment and rotor/blade loads [128].

Variable speed operation is facilitated through the use of power converters interfacing the generator with the grid. In the case of Doubly Fed Induction Generators (DFIGs) the power converter is rated at a fraction of the generator output power and interfaces with the rotor coils, this facilitates partial speed control. Alternatively, a fully rated converter can interface between the generator, typically synchronous, and the grid, facilitating full speed control. Fully rated converters are now favoured for large offshore turbines [12]. To enact speed control, the wind turbine controller demands a torque from the generator, using feedback control to achieve a given rotor speed set point. Pitch control is typically applied collectively, and blades are pitched to feather above the rated wind speed. Figure 4.1 shows a typical torque-speed curve for a variable speed, variable pitch HAWT. The trajectory A-B1-C1-E shows the control strategy described above.

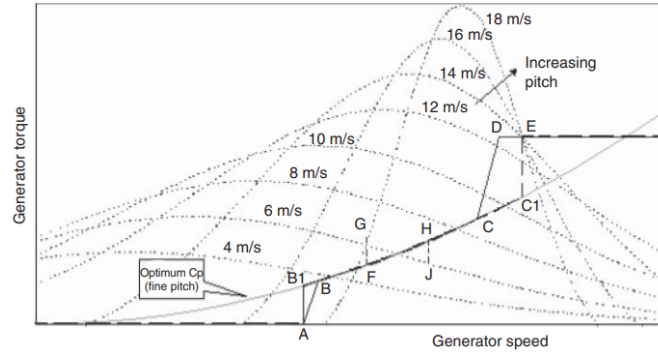


Figure 4.1: Typical torque-speed curve for variable speed, variable pitch HAWT, adapted from [7].

Due to the lack of technical maturity and the variation in rotor concepts, the control of VAWTs has not converged in the same manner as HAWTs, and a number of control options have been proposed. Turbines are typically operated in variable speed mode, tracking C_{Pmax} up to a certain speed and then relying on either passive or active stall to limit power in above rated conditions. Variable rotor geometry [116, 129] and blade pitch angle [20, 82] control have also been proposed for power regulation in VAWTs. Azimuthally varying, or cyclic, pitch actuation has also been investigated to increase power capture and to alleviate loads [82, 130].

Early work on the control of VAWT rotors, including the 34m Sandia concept, scheduled the rotor speed based on the wind speed using an anemometer reading of the wind speed [131]. However, it is now conventional to approach the control of VAWTs in a similar manner to a HAWT and schedule the generator torque and other auxiliary control actions based on internal variables such as the rotor speed, since single point wind speed measurements are not representative of the wind field over the entire rotor.

A detailed discussion of potential control strategies for the 5MW Deepwind VAWT is presented in [132]. Since the Deepwind turbine concept only employs torque control on the primary rotor shaft, with no auxiliary control actuators, the turbine is operated in a similar manner to a variable speed stall controlled HAWT [126]. At low wind speeds the turbine tracks the C_{Pmax} curve, and a number of options are available at higher wind speeds. A more conservative approach involves tracking C_{Pmax}

up to a certain rotor speed then maintaining a constant rotor speed after this point, a more aggressive approach involves extending the range over which C_{Pmax} tracking is implemented, then actively reducing the rotor speed to induce stall in above rated conditions. Both options were considered in [132]. It was found that the more conservative control strategy required modified aerofoil characteristics to successfully reach rated power whilst stalling effectively at high wind speeds. Comparing the conservative and aggressive control strategies, as expected, the conservative strategy had a lower annual energy production and the damage equivalent loads were lower compared to more aggressive strategy. Goude [133] proposed a method of variable speed VAWT control which scheduled the electrical power extraction based on rotor speed to maintain operation at C_{Pmax} together with a second order correction employed to ensure that the strategy remains stable at low wind speeds.

The variable rotor geometry approach for power regulation was pioneered by Peter Musgrove [116] and culminated in the building of VAWT-450 at Carmarthen Bay [4] in 1986. This turbine utilised a 'reefing' system to fold the blades in on themselves and reduce the swept area of the turbine to limit power capture. It was, however, found that passive blade stall was effective at power limitation, and the subsequent larger demonstration turbine design did not include variable rotor geometry control. More recently, the ARCUS VAWT concept has been proposed by Sandia National Labs [129]. This is a tower-less Darrieus VAWT design, which utilises rotor area control to reduce loads at high wind speeds.

Cyclic pitch strategies are often presented in the context of increased power capture in below rated conditions. Parachivou [130] presented an investigation into the optimisation of VAWT pitch control using a genetic algorithm, this predicted gains in annual energy production of almost 30% using momentum based models. The rotors considered for pitch optimisation were constant speed designs, which are not representative of control strategies that would likely be employed for multi-MW turbines. Additionally, the CADRAAV code used in the optimisation did not include the effects of streamtube expansion and therefore did not accurately model the flow in the down-wind rotor half. In [134], the change in rotor loads due to the use of a non-disclosed variable pitch

scheme were presented. This demonstrated an increase in power coefficient of 15% in maximum power point tracking mode, and highlighted the ability of variable pitch schemes to limit power and thrust in above rated conditions. This numerical study was completed using a 3D free vortex wake code including dynamic stall effects; the simulation work should therefore be considered to be reasonably representative. It should, however, be noted that the actual pitch scheme was not presented due to commercial sensitivity, and limits on pitch rates were not mentioned in the paper. Further to these numerical studies, an online optimisation of the pitch schedule for a lab-scale VAWT using a genetic algorithm is presented in [135]. The paper experimentally demonstrates a 3 fold increase in power coefficient and a 77% decrease in blade loads in off-design conditions. It should be noted that the 3 fold increase in power coefficient is only possible due to the very low power coefficient of the original turbine.

The control of the X-Rotor differs significantly from both HAWTs and VAWTs for a number of reasons. In terms of primary rotor control, the retarding torque applied by the secondary rotors is inherently proportional to the square of primary rotor speed whereas, in the case of a conventional turbine, the generator can theoretically provide any retarding torque below the maximum value at any speed. The constant of proportionality relating the square of the primary rotor speed with the retarding torque is controlled through modifying the secondary rotor thrust coefficient. As the maximum secondary rotor thrust coefficient is relatively low, the number of achievable equilibrium operating conditions is limited. Figure 4.2 shows representative torque-speed curves for an X-Rotor primary rotor in blue, with the corresponding retarding torques in red. For simplicity the secondary effect of the ambient wind speed on the secondary rotors has been neglected in the retarding torque calculation. In reality, the effect of the secondary rotors tracking into the ambient wind speed mean that the retarding torque from the secondary rotors is not a true quadratic curve and is dependent on the ambient wind speed, although this is a secondary effect. At a given wind speed, equilibrium operating states can be obtained through finding the intersection of the primary rotor torque-speed curve and the secondary rotor retarding torque curve.

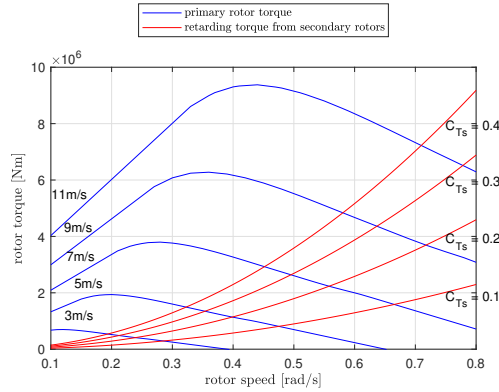


Figure 4.2: Demonstrative torque-speed curve for an X-Rotor wind turbine.

In above rated conditions, primary rotor pitch control is required to limit turbine power capture. Since the secondary rotors are attached to the lower blades of the X-Rotor, only the upper blades are variable pitch, and a significant amount of pitch actuation is therefore required to limit the primary rotor torque.

An additional feature of X-Rotor control is that power take off is completed through secondary rotors. This leads to more complex efficiency curves for power take off compared to a traditional generator. Additionally, the secondary rotors experience large periodic fluctuations in the incident wind speed, corresponding to the tracking into and out-of the ambient wind speed.

Previous works detailing potential control strategies for the X-Rotor turbine are presented in [20] and [30], the latter reference with contributions from the author. In [20], an overview of the X-Rotor concept is presented and a basic control strategy for the primary rotor is described which tracks $C_{P_{max}}$ in below rated conditions, and includes cyclic pitching of the upper blades in above rated conditions to limit power capture. The presented pitch distribution is not feasible, following a zero-mean square wave with the sign switching between the upwind and downwind rotor halves. Additionally, the aerodynamic simulations were based on blade polars that were not representative of the aerofoil sections at the operational Reynolds number, leading to lower primary rotor power coefficient estimates compared to later work [26]. The investigation reported in [20] is reported in much greater detail in [30], albeit with an updated, more feasible,

cyclic pitching strategy. The proposed control of the secondary rotors involves tracking a constant secondary rotor tip speed ratio in all conditions, and pitching the primary rotor blades to maintain a constant primary rotor speed in above rated conditions, therefore keeping a constant mean incident wind speed on the secondary rotors. The potential to control the secondary rotors in constant speed mode in above rated wind speeds, limiting the maximum tip speed, is discussed, but not investigated. Similarly, cyclic pitching of the primary rotor to increase power capture in below rated conditions, and reduce loads in above rated conditions was discussed but not investigated. The aerodynamic characteristics of the secondary rotors are also only represented using an actuator disk approach with a single induction factor, and specific details of secondary rotor design and control are therefore inherently limited.

Expanding on the previously published work on control strategies for the X-Rotor turbine, this chapter will provide a thorough overview of control options, focussing primarily on secondary rotor control. Four practicable control options are presented which satisfy the key limitations on turbine operation, namely limiting the secondary rotor tip speed and the total power output. Each strategy will then be evaluated in terms of power capture and rotor loads.

4.2 Characteristics of the X-Rotor

The definition of the X-Rotor primary and secondary rotors is provided in Appendix D. The primary rotor is consistent with the design presented in [20] and [30]. The secondary rotor is designed following the optimal design process presented in Chapter 2. The secondary rotor hub radius is taken as 1m, which is sufficient to accommodate a generator, see [57].

The power coefficient of the X-Rotor primary rotor, as calculated using the 2DAC model presented in Chapter 3, is shown as a function of the upper blade pitch angle and the primary rotor tip speed ratio in figure 4.3. The maximum power coefficient is shown to be 0.49, which is achieved at a tip speed ratio of 4.75 and a pitch angle of 2° . In order to limit the duty cycle on the pitch actuator, the results presented in this chapter only consider applying a fixed pitch offset varying as a function of wind speed

rather than utilising cyclic pitching.

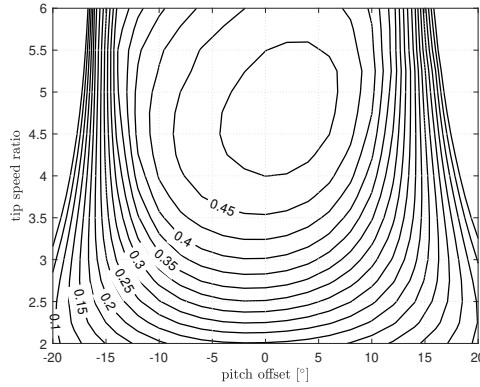


Figure 4.3: Contours representing primary rotor power coefficient as a function of tip speed ratio and pitch offset

The key aerodynamic characteristics of the secondary rotor, the thrust and power coefficients, are presented in figure 4.4 as a function of tip speed ratio. The aerodynamic simulations are completed using the BEM simulation tool QBlade [32]. The thrust is augmented by adding the effect of the rotor hub, with a radius of 1m and a thrust coefficient of 0.4 - representative of a hemisphere facing the flow. The maximum power coefficient of the secondary rotor is equal to 0.29 corresponding to a thrust coefficient of 0.36 achieved at a tip speed ratio of 3.6. Essentially, the rotor is designed to operate at an axial induction value of approximately 0.1. Key to understanding the efficiency of power take-off with secondary rotors is the ratio of the power coefficient to the thrust coefficient, this is presented as a function of tip speed ratio in figure 4.5. The C_P to C_T ratio has a broad, flat peak with a maximum value of approximately 0.8, indicating that conversion efficiencies in excess of 80% are achievable over a range of conditions. The reduction in thrust coefficient at high tip speed ratios is unusual for wind turbine rotors, and is a consequence of the low thrust coefficient design.

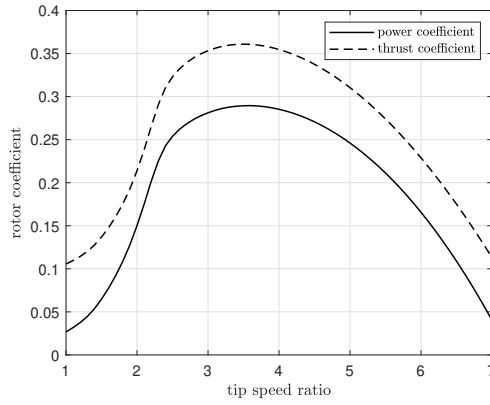


Figure 4.4: Secondary rotor power and thrust coefficient as a function of tip speed ratio.

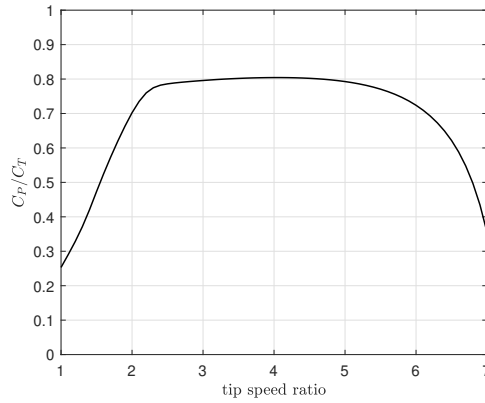


Figure 4.5: Secondary rotor power coefficient to thrust coefficient ratio as a function of tip speed ratio.

4.3 Control of the X-Rotor turbine

The basic dynamics of a vertical axis wind turbine utilising secondary rotors for power take off are discussed in Chapter 2. In essence, there must be balance between the revolution averaged aerodynamic torque on the primary rotor and the revolution averaged retarding torque provided by the secondary rotors; this results in an equilibrium rotational speed, or tip speed ratio, for the primary rotor. This can equivalently be represented as a balance in the energy captured by the primary rotor over a single revolution and the work done to translate the secondary rotors through a single revolution

of the primary rotor, or a balance between the average aerodynamic power captured by the primary rotor and the average power required to translate the secondary rotors through a single revolution. Using this last representation, and non-dimensionalising the average power required to rotate the secondary rotors over a single revolution by the power available to the primary rotor, the equilibrium condition can be defined as

$$C_{Pp}(\lambda_p, \beta_p) = \frac{N_s A_s}{A_p} \frac{\lambda_p}{2\pi} \int_0^{2\pi} C_{Ts}(\lambda_s) [\lambda_p + \cos(\theta_p)]^2 d\theta_p. \quad (4.1)$$

The primary rotor pitch angle, β_p , and the secondary rotor tip speed ratio, λ_s , are the two control variables, the primary and secondary rotor area, A_p and A_s , are defined by the turbine design, and the primary rotor tip speed ratio λ_p , is determined through the evaluation of the equilibrium condition. The primary rotor power coefficient is a function of λ_p and β_p , and is determined by the design of the primary rotor. C_{Pp} contours are shown as a function of tip speed ratio and pitch offset in figure 4.3. The secondary rotor thrust coefficient is a function of λ_s , and dependent on the geometry of the secondary rotor, the thrust coefficient is shown as a function of tip speed ratio in figure 4.4.

Once the equilibrium primary rotor tip speed ratio has been determined, the power available to the secondary rotor generators can be readily calculated. Non-dimensionalising this by the power available to the primary rotor defines a turbine power coefficient, C_{Pt} , which can be used to calculate the aerodynamic power available to the generators in a similar manner to the conventional power coefficient for a HAWT;

$$C_{Pt} = \eta C_{Pp} = \frac{N_s A_s}{A_p} \frac{1}{2\pi} \int_0^{2\pi} C_{Ps}(\lambda_s) [\lambda_p + \cos(\theta_p)]^3 d\theta_p. \quad (4.2)$$

In this chapter, only collective pitch control of the primary rotor is considered. This refers to pitching the primary rotor blades collectively to a single pitch value constant over the primary rotor rotation. This pitch value is defined in the control strategy as a single value as a function of wind speed. Following the convention in Chapter 3, a positive pitch angle refers to nose-in pitching, increasing the amplitude of the angle of attack in the upwind rotor half, and decreasing the amplitude of the angle of attack in

the downwind rotor half.

A simple method of control is to operate the secondary rotors at a constant tip speed ratio. This can readily be achieved through optimal torque control of the secondary rotors as described in [7]. In this case, the retarding torque from the generator is proportional to the square of the secondary rotor speed. Further discussion is provided in Section 4.4.1. If the secondary rotors are operated at a constant tip speed ratio, the thrust coefficient remains constant over a revolution and the thrust coefficient required to maintain a given primary rotor tip speed ratio can be readily calculated using equation 4.1, as shown in figure 4.6. The range of achievable primary rotor tip speed ratios is limited by the maximum thrust coefficient of the secondary rotors, shown in red on the contour plot, which peaks at a very low value due to the low induction rotor design.

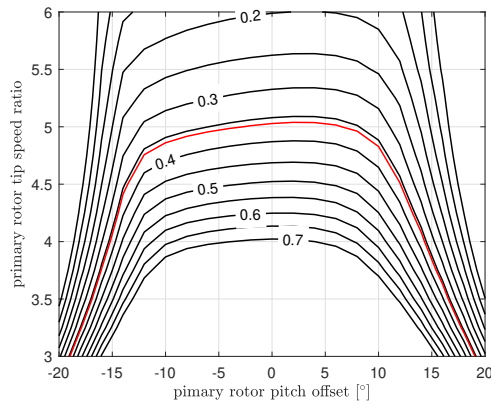


Figure 4.6: Contours representing the secondary rotor thrust coefficient required for equilibrium conditions as a function of primary rotor pitch offset and primary rotor tip speed ratio. The red contour marks the maximum thrust coefficient available from the secondary rotors.

Through combining the information presented in figure 4.6, with the secondary rotor aerodynamic characteristics presented in figure 4.4, the equilibrium primary rotor tip speed ratio, calculating using equation 4.1, can be presented as a function of secondary rotor tip speed ratio and primary rotor pitch offset. This is shown in figure 4.7. At each equilibrium operating point, the power captured by the secondary rotors can be easily evaluated, and the turbine power coefficient can be calculated using equation 4.2.

The turbine power coefficient contours are presented as a function of secondary rotor tip speed ratio and primary rotor pitch offset in figure 4.8.

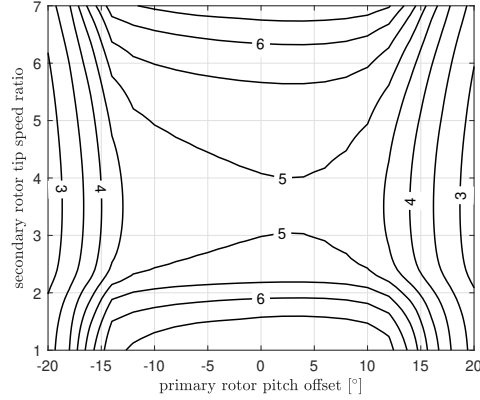


Figure 4.7: Contours representing the primary rotor tip speed ratio at equilibrium conditions as a function of primary rotor pitch offset and secondary rotor tip speed ratio.

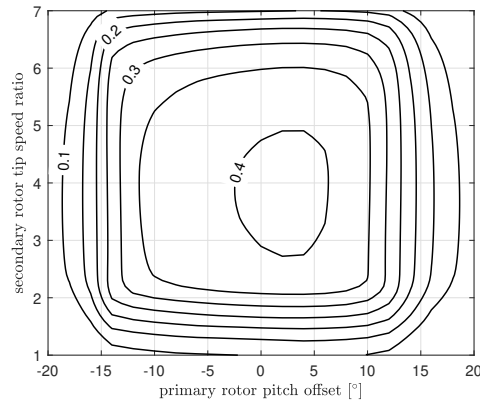


Figure 4.8: Contours representing turbine power coefficient, as described in equation 4.2, at equilibrium conditions as a function of primary rotor pitch offset and secondary rotor tip speed ratio.

The maximum turbine power coefficient is 0.41, achieved at a secondary rotor tip speed ratio of 3.90 and a primary rotor pitch offset of 2° , this corresponds to a primary rotor tip speed ratio of 4.99. Operating at this maximum power coefficient, a rated power of 5MW can be achieved at a wind speed of 11.59 m/s. It is notable that there is a relatively large area within which the turbine power coefficient equal or greater than

0.4 i.e. within 2.5% of the maximum value. This indicates that power capture should not be significantly affected by minor fluctuations around the turbine operating point.

It is recommended that the primary rotor blades are collectively pitched to feather in the upwind rotor half (a negative pitch angle as defined in this thesis). This is to avoid the blade stalling in the upwind rotor half where the inflow velocity is highest, thereby maximising blade aerodynamic damping.

4.4 Determination of control strategies

In order to determine a viable control strategy for the X-Rotor turbine, the goals and limitations of the control strategy must be clearly defined. The goals of the control strategy are to maximise the power captured whilst minimising the turbine loads under the constraints of not exceeding the rated power of the turbine and not exceeding the maximum allowable tip speed for the secondary rotors. The maximum tip speed limit for the secondary rotors, U_{max} is considered to be 184m/s, as in [20] and [30], and the turbine is rated at 5MW.

In order to determine a control strategy which fulfils these requirements, more complex control of the secondary rotors is required than that described in the previous section. The following section discusses four possible control options that conform to the above limitations. Section 4.5 then evaluates the efficacy of these options in terms of rotor load mitigation and power production. In each case, collective primary rotor pitch control is applied in above rated wind speeds.

The four control strategies considered in this section are:

1. Collective λ_s power regulation
2. Collective λ_s primary rotor speed regulation
3. Dynamically constrained secondary rotor speed
4. Collective secondary rotor speed

Collective λ_s control refers to the two secondary rotors being controlled at the same tip speed ratio, which is held constant at a given primary rotor operating point. In

the definition of the control strategy, the secondary rotor tip speed ratio is therefore scheduled as a function of wind speed. At high wind speeds, the secondary rotor tip speed ratio must be reduced significantly to conform to the limitations on the secondary rotor tip speed. In collective λ_s power regulation, the primary rotor pitch angle is scheduled to ensure a constant power output from the secondary rotors in above rated wind speeds. It is found that this mode of control leads to high primary rotor speeds at very high wind speeds, which is detrimental from a loads perspective. To avoid this, collective λ_s primary rotor speed regulation is developed. This control strategy again employs collective λ_s control of the secondary rotors, but in above rated conditions the primary rotor pitch angle is scheduled to ensure a constant primary rotor speed rather than a constant power output.

Dynamically constrained secondary rotor speed control refers to the control strategy where each secondary rotor is operating at a target tip speed ratio, λ_s^* ; however, strict upper and lower limits are imposed on the secondary rotor speed. The secondary rotor can operate at the target tip speed ratio during the portion of the primary rotor rotation where $U_I(\theta_p)\lambda_s^* \leq U_{max}$, and in cases where $U_I(\theta_p)\lambda_s^* > U_{max}$, the limit on the maximum secondary rotor enforces secondary rotor operation at a lower tip speed ratio. The secondary rotor tip speed ratio is therefore not necessarily constant at a given primary rotor operating point. In above rated conditions, the primary rotor blades are pitched such that the power output from the secondary rotors is held constant at the rated power. As the issue of increasing primary rotor speed in above rated conditions is not encountered, primary rotor speed regulation in above rated conditions is not investigated as a separate strategy.

Collective secondary rotor speed control refers to the control strategy where the secondary rotors are operated a collective speed. This allows the secondary rotors to share the same rotor side power converter, reducing the fluctuations in power input to the power converter and reducing the combined rating of power converters required for the turbine. Rather than operate at a single secondary rotor speed, this strategy allows the windward secondary rotor to dictate the speed at which both turbines are operating, with the windward turbine employing dynamically constrained secondary

rotor speed control. Again, in above rated conditions, the primary rotor blades are pitched such that the power output from the secondary rotors is held constant at the rated power. As the issue of increasing primary rotor speed in above rated conditions is, again, not encountered, primary rotor speed regulation in above rated conditions is not investigated as a separate strategy.

4.4.1 Collective λ_s power regulation

Collective λ_s control refers to secondary rotors being collectively operated at a fixed value of λ_s over a primary rotor revolution, which varies as a function of wind speed. No additional capping of the secondary rotor speed is applied. The control strategy therefore defines the value of λ_s based on the wind speed incident on the primary rotor. Power regulation control refers to pitching the primary rotor to maintain a constant secondary rotor output power in above rated conditions.

Controlling the secondary rotors to operate at a constant tip speed ratio can be readily implemented through k_c control; that is, in response to a demanded k_c , the secondary rotor speed is adjusted such that

$$Q_g = k_c \Omega_s^2, \quad (4.3)$$

where Q_g represents the generator torque. The value of k_c is calculated externally to the controller, based on the desired tip speed ratio, with

$$k_c = \frac{1}{2} \rho A_s R_s^3 \frac{C'_{Q_s}}{\lambda_s'^2} \quad (4.4)$$

where λ_s' and C'_{Q_s} represent the desired secondary rotor tip speed ratio and corresponding torque coefficient.

Since the turbine is operating in power regulation control, in above rated conditions, the primary rotor is pitched to maintain a constant value of revolution averaged power from the secondary rotors.

To determine the value of λ_s required to ensure the secondary rotor tip speed does not exceed the upper limit, one must first determine the relationship between the

maximum secondary rotor tip speed, and the other variables describing rotor operation. During collective λ_s regulation, the maximum secondary rotor tip speed occurs whilst the rotor plane is normal to the ambient wind and rotating into it i.e. when $\theta_p = 0$. When this occurs, the incident wind speed on the secondary rotor is given by $U_0(1 + \lambda_p)$, and the tip speed of the secondary rotor is given by

$$\Omega_S R_s = U_0(1 + \lambda_p)\lambda_S. \quad (4.5)$$

In order to constrain the secondary rotor tip speed, the non dimensional quantity, $(1 + \lambda_p)\lambda_S$, must be below U_{max}/U_0 . This quantity will henceforth be referred to as the maximum turbine tip speed ratio, λ_{tm} . For each combination of primary rotor pitch angle, and secondary rotor tip speed ratio, the equilibrium primary rotor tip speed ratio can be calculated by enforcing equation 4.1, these contours are presented in figure 4.7. From this, the maximum turbine tip speed ratio can be readily calculated as a function primary rotor pitch angle and secondary rotor tip speed ratio, as presented in figure 4.9. Figure 4.10 shows the upper limit of the maximum turbine tip speed ratio required to limit the secondary rotor tip speed to 184m/s as a function of wind speed.

In order to maintain rated power in above rated conditions, the turbine power coefficient must follow

$$C_{Pt} = \frac{P_{rated}}{\frac{1}{2}\rho A U_0^3}. \quad (4.6)$$

A line representing the required power coefficient as a function of wind speed is also plotted in figure 4.10.

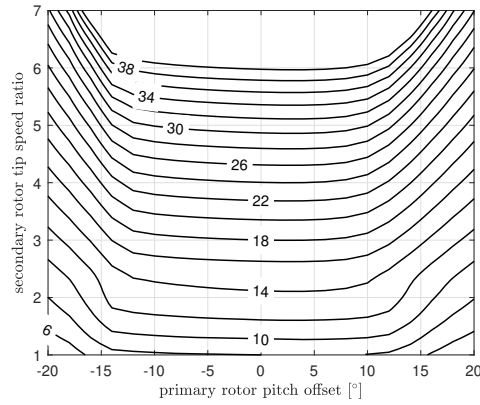


Figure 4.9: Contours representing the maximum turbine tip speed ratio ($\lambda_s[1 + \lambda_p]$) during equilibrium operation, as a function of primary rotor pitch offset and secondary rotor tip speed ratio.

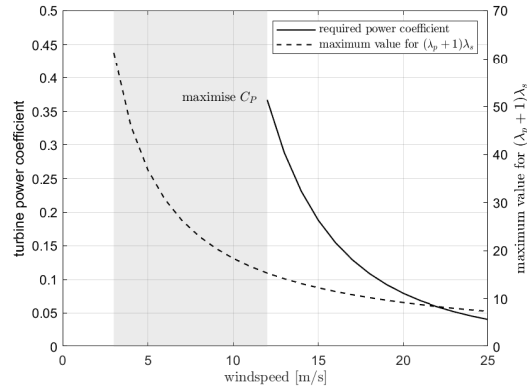


Figure 4.10: Required power coefficient and maximum turbine tip speed ratio as a function of wind speed for definition of control strategy.

To define a control strategy which limits the maximum tip speed, maximises power capture in below rated conditions, and maintains a constant power in above rated conditions, a simple algorithm can be followed using the contours plots shown in figures 4.8 and 4.9.

Firstly, the maximum turbine tip speed ratio corresponding to the maximum turbine power coefficient, λ_{tm}^* , must be evaluated. In all cases where λ_{tm}^* is below the upper limit, the turbine can operate in maximum power point tracking. As the wind speed increases, the upper limit on λ_{tm} decreases. In conditions where λ_{tm}^* exceeds the upper

limit, C_{Pt} must be evaluated along the contour corresponding to the upper limit of λ_{tm} . If the maximum value of C_{Pt} on this contour is below the upper limit on C_{Pt} , shown as a function of wind speed in figure 4.10, then the operational point corresponding to the maximum value can be selected. If the maximum value is equal to or exceeds the upper limit of C_{Pt} , then the operational point corresponding to the intersection of the $C_{Pt} = 2P_{rated}/\rho AU_0^3$ contour and the $\lambda_{tm} = U_{max}/U_0$ contour must be selected.

Figure 4.11 shows how a control strategy is determined through the intersection of C_{Pt} contours and λ_{tm} contours as functions of the primary rotor pitch angle and secondary rotor tip speed ratio. Contours corresponding to the upper limit of λ_{tm} , in red, and the C_{Pt} required, in blue, at wind speeds between 6m/s and 24m/s are shown. For 6m/s and below, λ_{tm}^* is below the upper limit. At 8m/s and 10m/s, in order to conform to the upper limit on λ_{tm} whilst maximising power coefficient, the secondary rotor tip speed ratio drops below the optimum value and the primary rotor pitch angle remains at $+2^\circ$. At 12m/s and above, both λ_{tm} and C_{Pt} must be limited to constrain the the secondary rotor tip speed and turbine power output.

The primary rotor pitch angle, and secondary rotor tip speed ratio are shown as functions of wind speed in figure 4.12. The corresponding primary and secondary rotor speeds are shown in figure 4.13, and the power curve is shown in figure 4.14. The secondary rotor speed as a function of primary rotor azimuth angle at a number of different wind speeds is shown in figure 4.15, demonstrating how the tip speed is limited through the change in secondary rotor tip speed ratio.

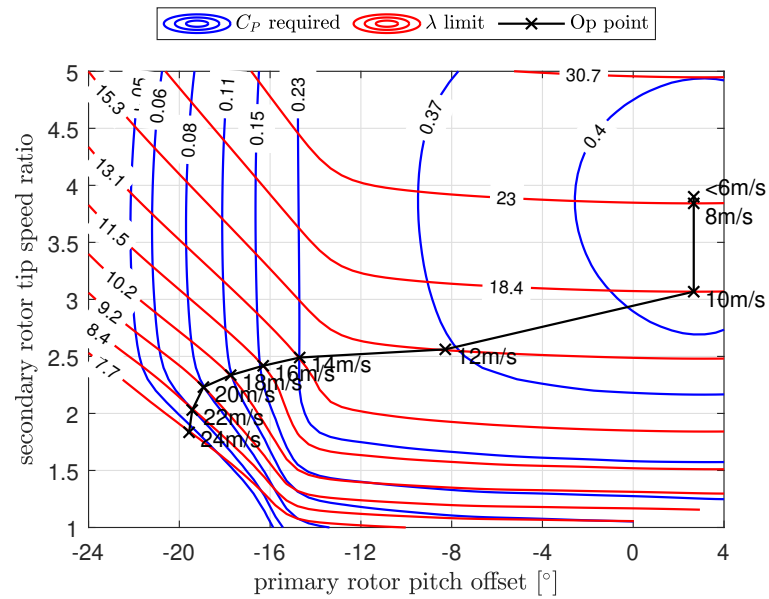


Figure 4.11: Calculation of collective λ_s strategy

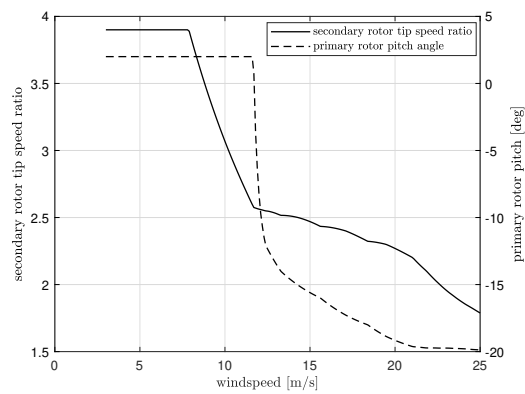


Figure 4.12: Definition of collective λ_s strategy showing the secondary rotor tip speed ratio, λ_s , and the primary rotor pitch angle, β_p , as a function of wind speed.

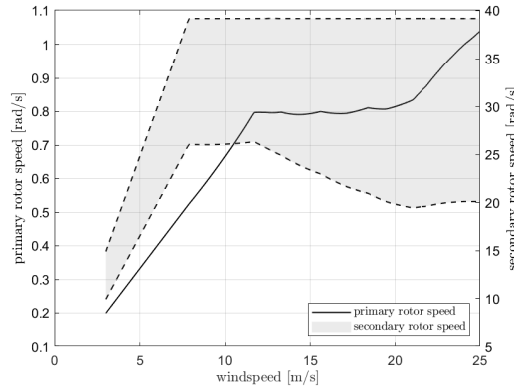


Figure 4.13: Primary and secondary rotor speed as a function of wind speed. The black line shows the primary rotor speed, and the grey shaded area, bounded by the dashed black lines, show the range of secondary rotor speeds.

Whilst the secondary rotor speed is successfully limited, as shown in figure 4.13, and an acceptable power curve is obtained, as shown in figure 4.14, a significant issue is presented by the increase in primary rotor speed in above rated conditions. Since the secondary rotor tip speed ratio must be significantly reduced to ensure that the secondary rotor tip speed does not exceed the limiting value, the efficiency of power conversion between the primary and secondary rotors is reduced, exemplified by the drop in C_{P_s}/C_{T_s} at low tip speed ratios shown in figure 4.5. Because of this the primary rotor must operate at higher power coefficients, and therefore higher rotor speeds. This is evident in the significant increase in primary rotor power capture at high wind speeds, as shown in figure 4.14. These phenomena can also be understood as the secondary rotor operating at a significantly lower power coefficient and requiring higher incident wind speeds, and therefore higher primary rotor speeds, to maintain the desired rated power. The required increase in primary rotor speed at high wind speeds is likely to have significant detrimental effects on the primary rotor loads, these are discussed in Section 4.5.3.

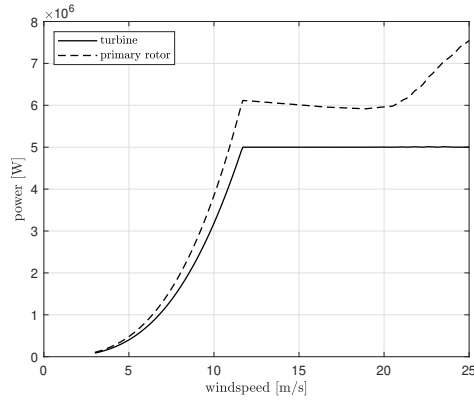


Figure 4.14: Power curve for collective λ_s strategy.

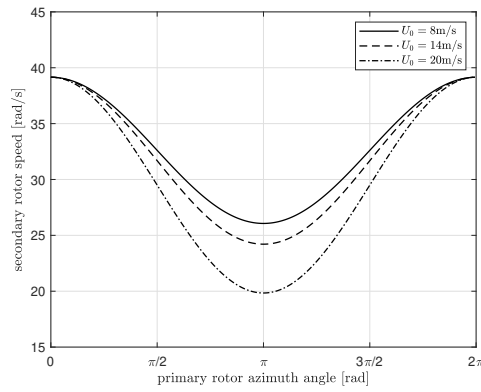


Figure 4.15: Secondary rotor speed as a function of azimuth angle for collective λ_s strategy.

4.4.2 Collective λ_s primary rotor speed regulation

The collective λ_s , limited Ω_p strategy employs collective λ_s control, as described in Section 4.4.1, but in above rated conditions the primary rotor is pitched to maintain a constant primary rotor speed, rather than a constant revolution averaged power output from the secondary rotors. This strategy is employed to overcome primary rotor over-speed at high wind speeds. Limiting the primary speed can be achieved whilst simultaneously maintaining the limit on secondary rotor tip speed; however, a constant power output is not also achievable simultaneously.

The realisation of this control strategy can be achieved in a similar manner to the

process shown in figure 4.11 but using intersecting primary rotor tip speed ratio contours with maximum turbine tip speed ratio contours in conditions where the primary rotor speed exceeds the upper limit. The primary rotor tip speed contours are calculated using the equilibrium condition stated in equation 4.1, and are presented in figure 4.7. The intersection of primary rotor tip speed ratio contours and maximum turbine tip speed ratio contours ensuring a maximum primary rotor speed 0.79 rad/s and a maximum secondary rotor tip speed of 184m/s are shown in figure 4.16. The corresponding secondary rotor tip speed ratio, and primary rotor pitch angle are shown as a function of wind speed in figure 4.17.

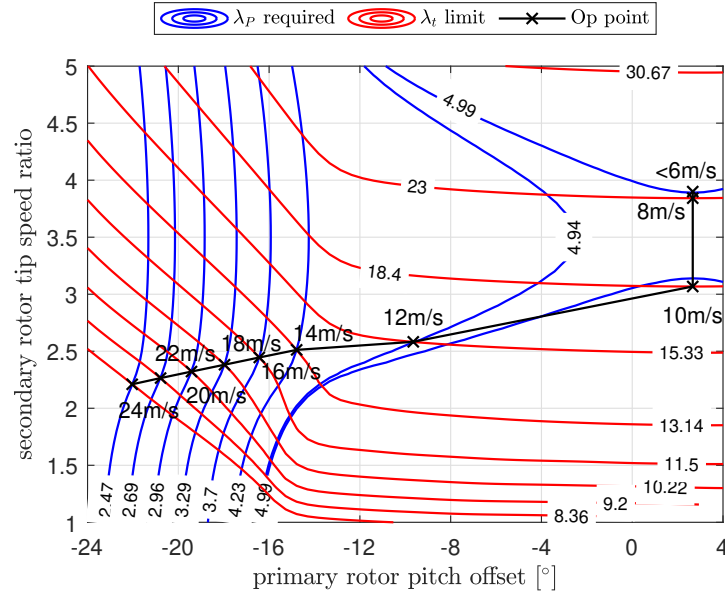


Figure 4.16: Determination of collective λ_s , limited Ω_p strategy.

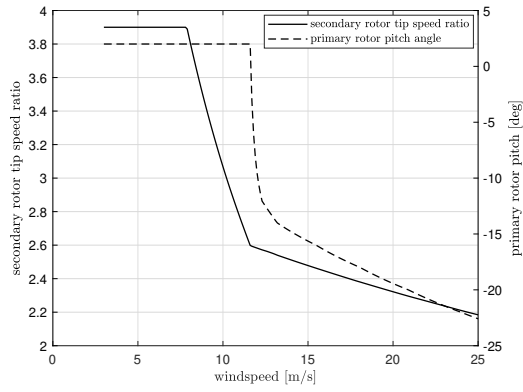


Figure 4.17: Definition of collective λ_s , limited Ω_p strategy.

The corresponding primary and secondary rotor speeds are shown in figure 4.18. It is clear that the strategy maintains the required limits on both primary and secondary rotor speed. The corresponding power curve is shown in figure 4.19.

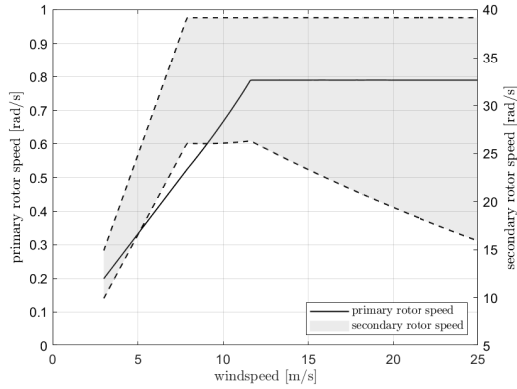


Figure 4.18: Primary and secondary rotor speeds corresponding to collective λ_s , limited Ω_p strategy. The black line shows the primary rotor speed, and the grey shaded area, bounded by the dashed black lines, show the range of secondary rotor speeds.

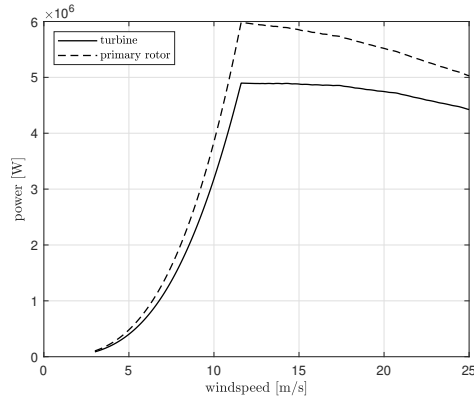
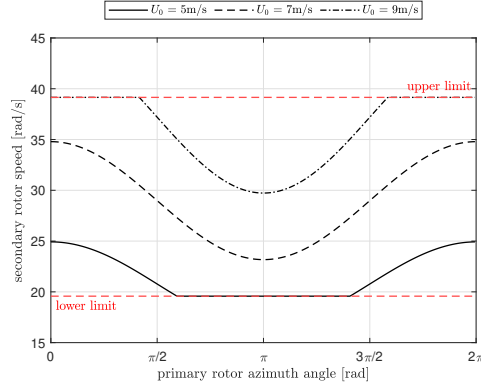


Figure 4.19: Power curve corresponding to collective λ_s , limited Ω_p strategy.

The cost, in terms of turbine power capture, of maintaining a constant primary rotor speed whilst constraining the secondary rotor tip speed ratio using a collective λ_s strategy is clear. The power output of the turbine drops from it's rated value of 5MW by approximately 10% to a value of 4.5MW by the cut-out wind speed. Since very high wind speeds are not experienced often, the effect on lifetime energy production is likely to be low; this is quantified in Section 4.5.2.

4.4.3 Dynamically controlled secondary rotor speed

As an alternative to maintaining a constant secondary rotor tip speed ratio over each primary rotor revolution, the secondary rotor tip speed can be controlled by imposing a strict upper and lower limit whilst operating at a target value of λ_s between these limits, as represented in figure 4.20. The lower limit on Ω_s is also applied to ensure more efficient power conversion in the power train. In this case the secondary rotor speed is limited to 1/2 of the maximum value.


 Figure 4.20: Secondary rotor speed under limited Ω_s control.

This method of secondary rotor speed control can be described using a target secondary rotor tip speed ratio, λ_s^* , and a maximum and minimum allowable secondary rotor speed, Ω_{s-max} and Ω_{s-min} . The secondary rotor speed varies over a revolution to maintain $\lambda_s = \lambda_s^*$; however, the rotor speed is 'cut off' and limited to the maximum and minimum values. Taking the maximum rotor speed as $\Omega_{s-max} = U_{max}/R_s$ and the minimum rotor speed to $\Omega_{s-min} = \Omega_{s-max}/2$, the secondary rotor tip speed ratio, for a given primary rotor tip speed ratio, is given by

$$\lambda_s(\theta_p, U_0) = \begin{cases} \frac{U_{max}}{2U_0[\lambda_p + \cos(\theta_p)]}, & \text{if } \lambda_s^*[\cos(\theta_p) + \lambda_p] < \frac{U_{max}}{2U_0} \\ \lambda_s^*, & \text{if } \frac{U_{max}}{2U_0} \leq \lambda_s^*(\cos(\theta_p) + \lambda_p) \leq \frac{U_{max}}{U_0} \\ \frac{U_{max}}{U_0[\lambda_p + \cos(\theta_p)]}, & \text{otherwise.} \end{cases} \quad (4.7)$$

In order to find equilibrium operating states, the secondary rotor speed can be determined for a given wind speed over a range of primary rotor tip speed ratios using equation 4.7. Following this, the corresponding thrust coefficient at each azimuthal position can be evaluated. This can then be used with equation 4.1 to determine the equilibrium primary rotor tip speed ratio for a given pitch offset, and the turbine power coefficient can be evaluated using equation 4.2. In above rated conditions, the required primary rotor pitch angle, corresponding to the desired turbine power coefficient, can be calculated through repeating the process for a range of primary rotor pitch offsets. The pitch value corresponding to the desired power coefficient can then be readily obtained

through interpolation. This process is shown in figure 4.21.

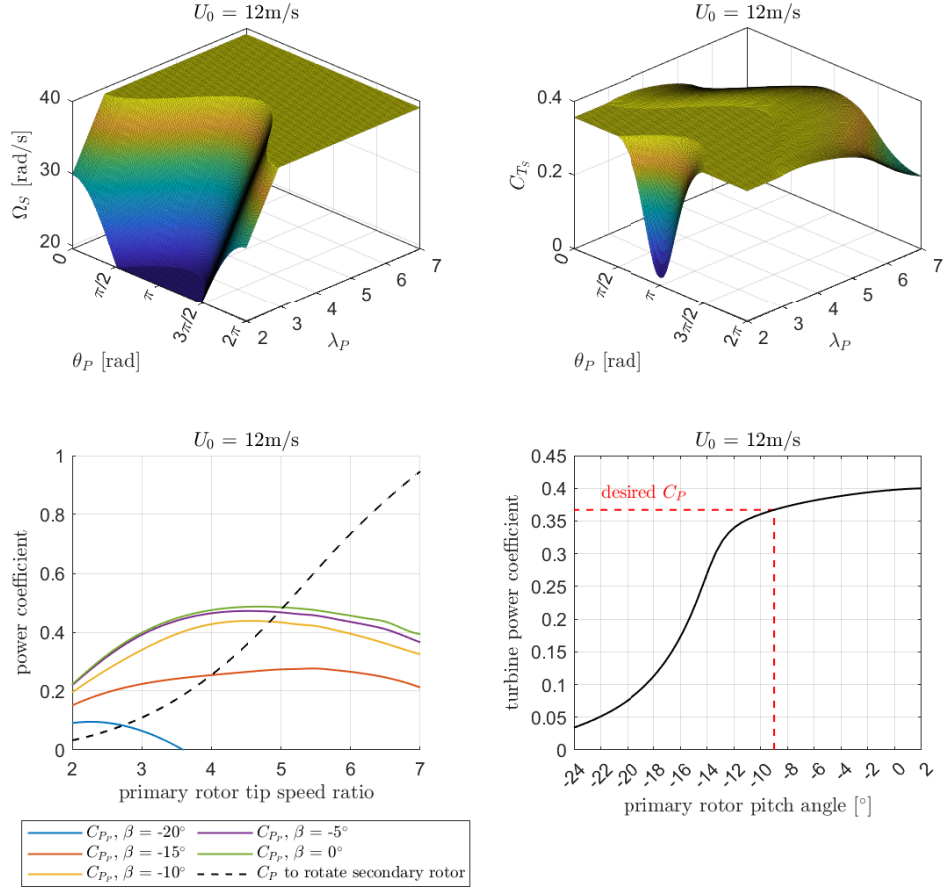


Figure 4.21: Example calculation of equilibrium operation points and required primary rotor pitch angle for limited Ω_s control strategy.

Taking the target secondary rotor tip speed ratio to be the value corresponding to the maximum turbine power coefficient, and taking the approach to maximise the power coefficient in below rated conditions and following the power coefficient curve shown in figure 4.10 to maintain a constant power of 5MW in above rated conditions, the strategy can be readily determined. Figure 4.22 shows the primary rotor pitch angle and target secondary rotor tip speed ratio as a function of wind speed. Figure 4.23 shows the corresponding primary rotor speed and secondary rotor speed range. It is clear that the secondary rotor speed is successfully constrained by this strategy

and that there is no run-off in the primary rotor speed at high wind speeds. In fact, a small reduction in primary rotor speed as the ambient wind speed increases is visible in above rated conditions.

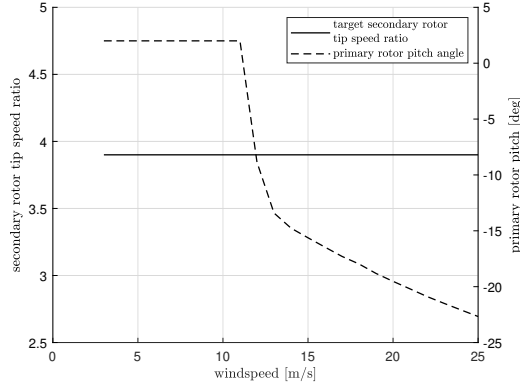


Figure 4.22: Strategy definition for limited Ω_s control strategy.

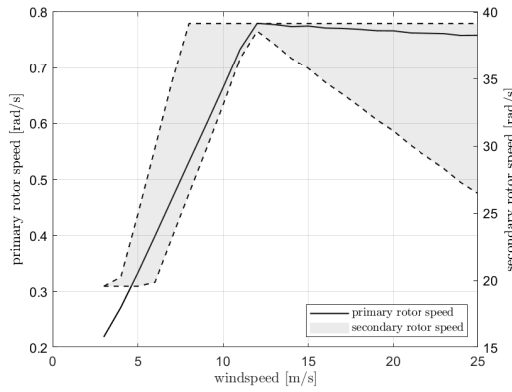


Figure 4.23: Primary and secondary rotor speed as a function of wind speed for limited Ω_s control strategy, the black line shows the primary rotor speed, and the grey shaded area, bounded by the dashed black lines, show the range of secondary rotor speeds.

The corresponding power curve is shown in figure 4.24, demonstrating that the power can be successfully limited at 5MW. The implications of this strategy on lifetime energy yield and primary rotor loads are discussed in Section 4.5.2.

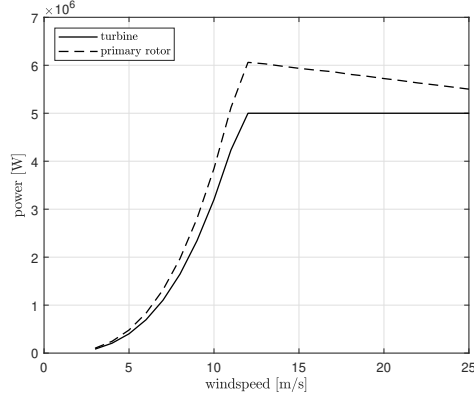


Figure 4.24: Power curve for limited Ω_s control strategy.

4.4.4 Collective secondary rotor speed control

Due to the periodic fluctuations in inflow speed, the power output from the secondary rotors fluctuates significantly over the primary rotor revolution. Assuming the secondary rotors operate at a constant tip speed ratio, the maximum power output from the secondary rotors, normalised by the mean power output, is given by

$$\frac{\max(P_s)}{\bar{P}_s} = \frac{(\lambda_p + 1)^3 (\lambda_p^2 + \frac{1}{2})}{\lambda_p^3 (\lambda_p^2 + \frac{3}{2})}. \quad (4.8)$$

This implies that each secondary power train must be capable of operating, temporarily, at above 2 times the nominal rated power of the secondary rotor at the cut-out wind speed. When using the dynamically controlled secondary rotor speed strategy, the secondary rotor tip speed ratio is reduced at the peak inflow speed and the power fluctuations are therefore slightly reduced. The power output of each secondary rotor at the cut-out wind speed, where the maximum power swings are experienced, are shown in figure 4.25.

Also shown in figure 4.25 is the combined power output from the secondary rotors, it is clear that the power swing is significantly smaller. Again, assuming that the secondary rotors operate at a constant tip speed ratio, the maximum power output of the combined secondary rotors, normalised by the mean power output, is given by

$$\frac{\max(\sum P_s)}{\sum \bar{P}_s} = \frac{(\lambda_p + 1)^3 + (\lambda_p - 1)^3 (\lambda_p^2 + \frac{1}{2})}{2\lambda_p^3 (\lambda_p^2 + \frac{3}{2})}. \quad (4.9)$$

The approximate maximum power output is therefore 1.28 times the mean power output at the cut-out wind speed. This is, again, reduced due the lower conversion efficiency of the secondary rotors as the tip speed ratio is reduced at the peak inflow speed.

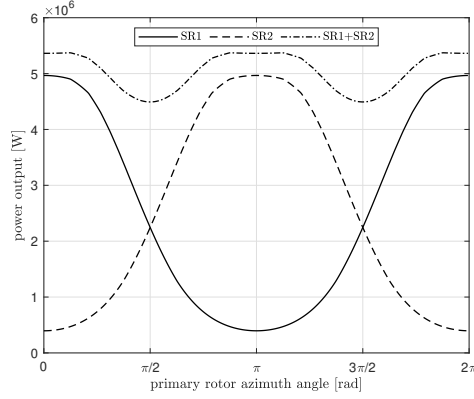


Figure 4.25: Deterministic secondary rotor power swings for limited Ω_s control strategy.

If individual speed control is applied, both secondary rotors require individual generator side power converters, as shown in the power-train configuration in figure 4.26 a). Since both the generator and the generator side power converter must be designed to operate, at least temporarily, at the maximum power output of a single secondary rotor, both generator side power converters must be rated at 5MW, despite only transferring a mean power of 2.5MW. Since the grid side converter must be rated to transfer the maximum combined power from the secondary rotors, shown in figure 4.26 b), the total capacity of installed power converters must be 15.4MW, as opposed to the 10MW required for a conventional turbine.

If collective speed control is applied, the two secondary rotors can share the same rotor side power converter, as shown in figure 4.26 b). This implies that the single generator side power converter must be rated at 5.4MW. Since the grid-side power converter must also be rated at 5.4MW, this reduced the total installed power converter capacity to 10.8MW, a reduction of approximately 30%, bringing the turbine approximately

in-line with conventional turbines employing a fully rated power converter.

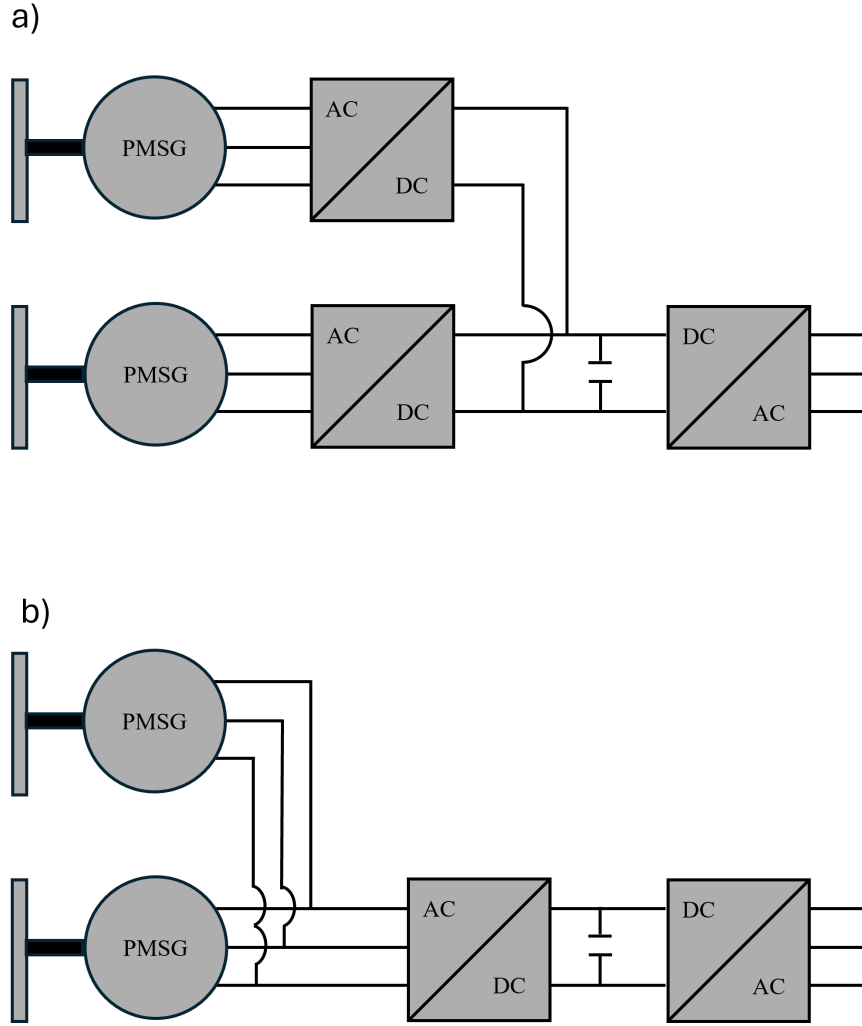


Figure 4.26: Potential power train configurations for the 2 bladed X-Rotor turbine.

To maintain a high conversion efficiency under collective speed control, the collective speed of the rotors is dictated by the windward secondary rotor, which follows the same strategy put forward in Section 4.4.3. The secondary rotor tip speed ratio is therefore given by

$$\lambda_s = \begin{cases} \text{use equation 4.7} & \text{if } \theta_p \leq \pi/2 \text{ or } \theta_p > 3\pi/2 \\ \lambda_s(\theta_p^*) \frac{U_I(\theta_p^*)}{U_I(\theta_p)} & \text{if } \pi/2 < \theta_p \leq 3\pi/2. \end{cases} \quad (4.10)$$

Here, θ_p^* represents the azimuth angle of the windward travelling secondary rotor, which

is given by

$$\theta_p^* = \text{mod}(\theta_p + \pi, 2\pi). \quad (4.11)$$

The corresponding secondary rotor speed is shown as a function of primary rotor azimuth angle in figure 4.27.

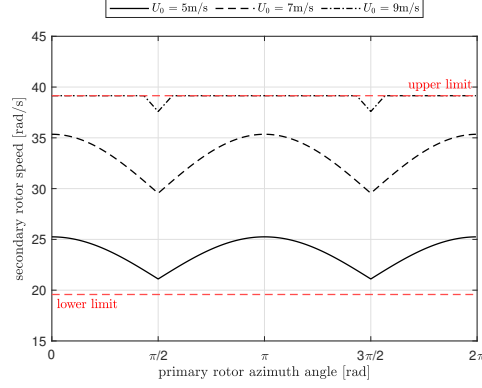


Figure 4.27: Secondary rotor speed as a function of primary rotor azimuth angle under the collective Ω_s control strategy.

In order to obtain equilibrium operating points, the same procedure described in Section 4.4.3 can be applied, with the secondary rotor tip speed ratio now given by equation 4.10 rather than equation 4.7. Using the same target secondary rotor tip speed ratio, and defining a strategy that maximises power capture below rated wind speed and limits the rated power to 5MW, the required primary rotor pitch angle is shown in figure 4.28, and the corresponding primary and secondary rotor speeds are shown in figure 4.29. Finally, the corresponding power curve is shown in figure 4.30.

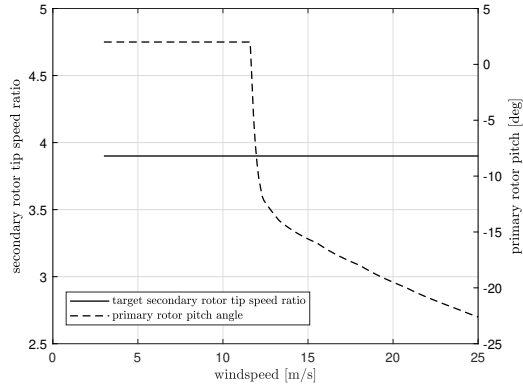


Figure 4.28: Strategy definition for collective Ω_s control strategy.

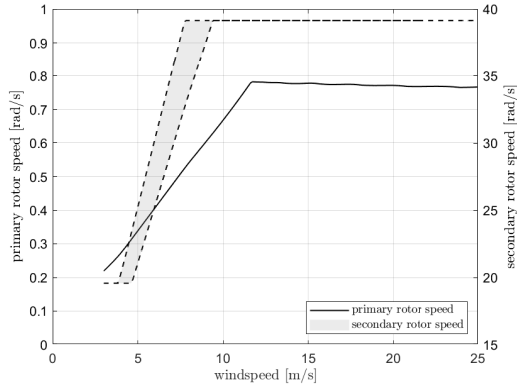


Figure 4.29: Primary and secondary rotor speed as a function of wind speed for collective Ω_s control strategy. The black line shows the primary rotor speed, and the grey shaded area, bounded by the dashed black lines, show the range of secondary rotor speeds.

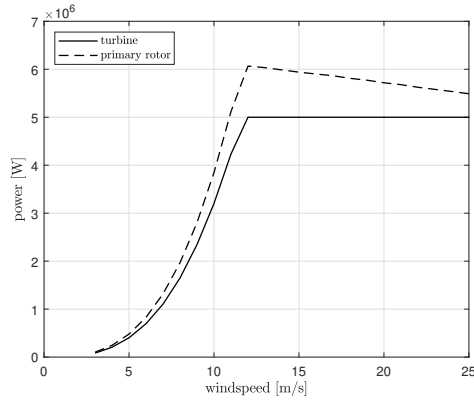


Figure 4.30: Power curve for collective Ω_s control strategy.

4.5 Evaluation of control options

4.5.1 Weibull distribution

To evaluate the control options presented in the previous section, the Weibull distribution used in [20], representative of a typical north sea site, is used. This has a shape parameter of 1.94, and a scale parameter of 10.2, and is shown in figure 4.31.

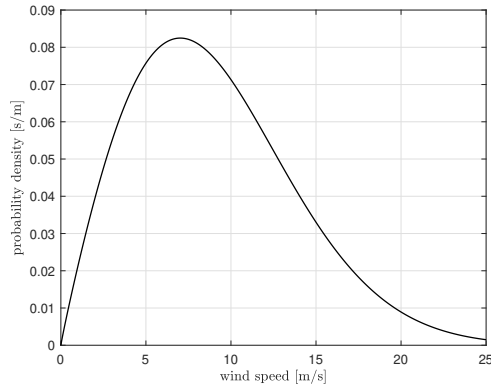


Figure 4.31: Representative Weibull distribution used to compare power production and loads.

4.5.2 Power production

The power curves for each of the four strategies discussed in the previous section are shown in figure 4.32. As the peak turbine power coefficient for each strategy is 0.41,

the 3 strategies that engage in maximum power point tracking, then constant power control, have near identical power curves. The power curve for the collective λ_s limited Ω_p strategy shows the reduction in turbine power in above rated conditions.

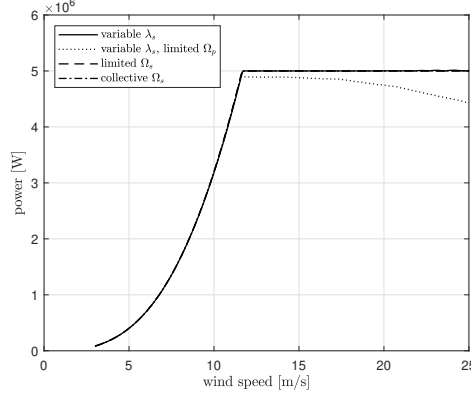


Figure 4.32: Power curves for each control strategy.

Using the Weibull distribution presented in figure 4.31, an upper limit on the lifetime energy capture of the turbine can be readily calculated (assuming 100% turbine availability). Figure 4.33 shows the lifetime energy production (LEP) in each of the considered wind speed bins. The reduction in energy capture for the collective λ_s limited Ω_p strategy is clear, and the similarity of the other strategies in terms of energy capture is also evident. The limited Ω_s strategies perform slightly better at wind speeds between 8m/s and 12m/s. Secondary rotor control to limit tip speed begins to occur at 7.8m/s, and limiting secondary rotor tip speed using speed saturation has less effect on the turbine power coefficient than shifting the mean secondary rotor tip speed ratio. This can be seen in figure 4.34. Table 4.1 shows the lifetime energy capture and the capacity factor for each strategy. It should be noted that these changes in energy capture are very small, and corresponding to at maximum a 1.2% reduction or a 0.1% gain in energy capture compared to the collective λ_s strategy.

	Collective λ_s	Collective λ_s limited Ω_p	limited Ω_s	collective Ω_s
LEP [MWhrs]	422382	417136	422773	422843
Capacity factor	0.482	0.476	0.483	0.483

Table 4.1: Lifetime energy capture and capacity factor for each control strategy.

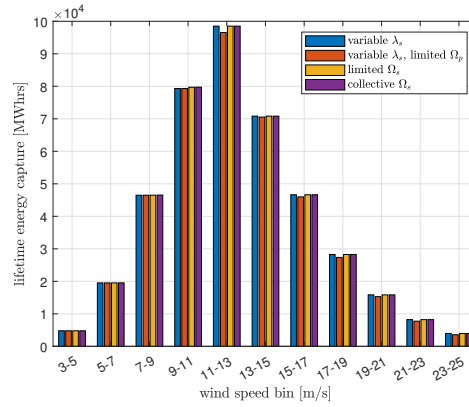


Figure 4.33: Lifetime Energy capture in each wind speed bin.

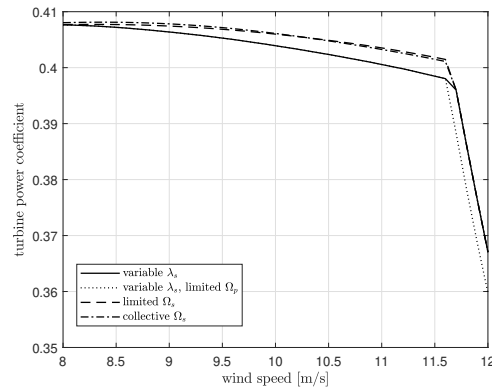


Figure 4.34: Detailed view of turbine power coefficient near rated power for each control strategy.

4.5.3 Loads

To fully assess the loads on the turbine, a full aeroelastic model must be developed, including a turbulent wind field model. For HAWTs, the aerodynamic loads caused by the stochastic, non-homogenous wind field dominate the loading spectrum. For VAWTs, the large deterministic fluctuations in loading are expected to dominate. Thus, an evaluation of the deterministic load fluctuations is useful in evaluating a proposed control strategy. The lack of aero-elastic model represents a significant limitation, and the obtained loads are used as an indicative measure for comparison between strategies, rather than an estimation of the actual loads experienced by the turbine. The primary

rotor loads presented here are the steady state aerodynamic loads as calculated using the actuator cylinder model described in Appendix C, and verified in Chapter 3. The load data corresponding the equilibrium operating point was obtained for a look-up table generated through simulating the rotor over a wide range of operating conditions. The secondary rotor loads were calculated using a look up table generated from BEM simulations of the secondary rotor.

The secondary rotor loads are calculated using a look-up based approach

The loads considered in this section are the primary rotor thrust, T_p , the out of plane bending moment on both the upper and lower blade, M_U and M_L , and the thrust on the secondary rotors, T_s . Example load traces are shown in figure 4.35.

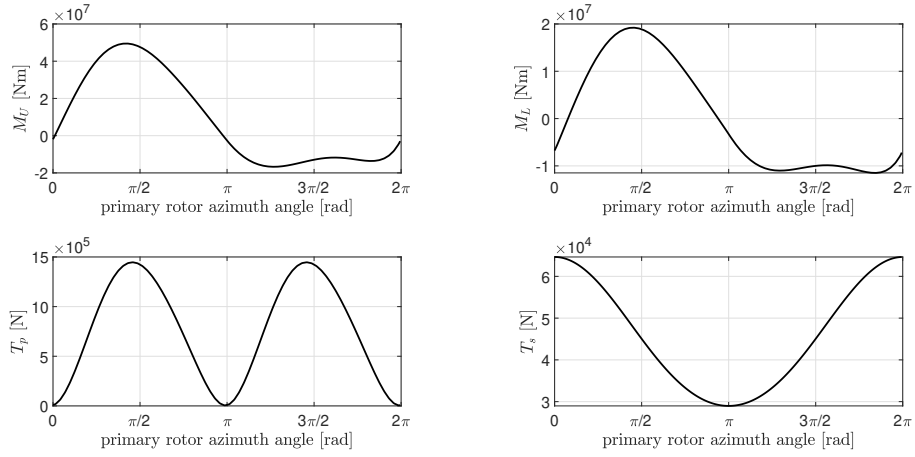


Figure 4.35: Exemplary load signals including the primary rotor thrust, T_p , the out of plane bending moment on both the upper and lower blade, M_U and M_L , and the thrust on the secondary rotors, T_s . Loads calculated for the X-Rotor primary rotor operating in collective λ_s control at 11m/s.

To calculate the damage equivalent loads (DELs), these loads are treated as sinusoids with a 1P frequency, apart from the rotor thrust which is dominated by a 2P component and represented accordingly. The absolute amplitudes from a discrete Fourier transform (DFT) of the zero-mean loads are shown in figure 4.36. The blade bending moments and secondary rotor thrust loads are normalised by the 1P component, and the primary rotor thrust load is normalised by the 2P component. This demonstrates that the assumption that the blade bending moments and secondary ro-

tor thrust loads are represented by their equivalent load range acting at 1P, and that the primary rotor thrust is represented by its load range operating at 2P, is not too egregious and is acceptable for comparative purposes.

Both the mean values and amplitudes are used in the DEL calculation, these are shown in figure 4.37.

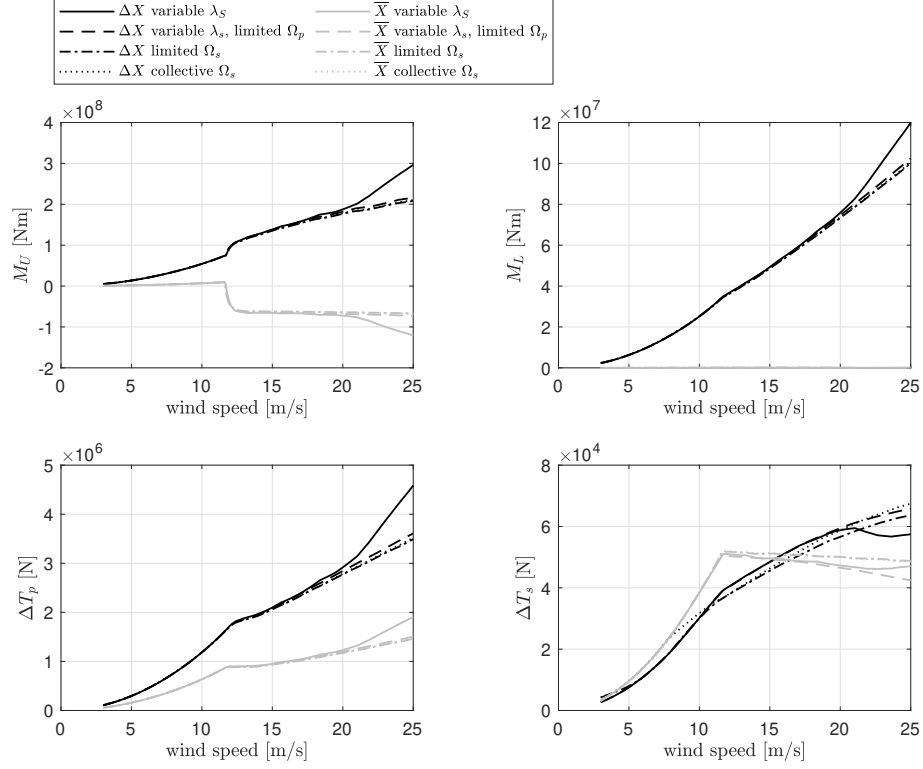


Figure 4.37: Load ranges and mean loads for each control strategy.

The 1Hz damage equivalent load for each wind speed bin is calculated using

$$L_{DEL,i} = (L_{cor,i}^w n_i)^{1/w}, \quad (4.12)$$

where w represents the Wöhler exponent, n_i represents the number of load cycles and $L_{cor,i}$ represents the Goodman-corrected load range, given by

$$L_{cor} = \frac{L_r}{1 - \frac{L_m}{1.5L_{max}}}, \quad (4.13)$$

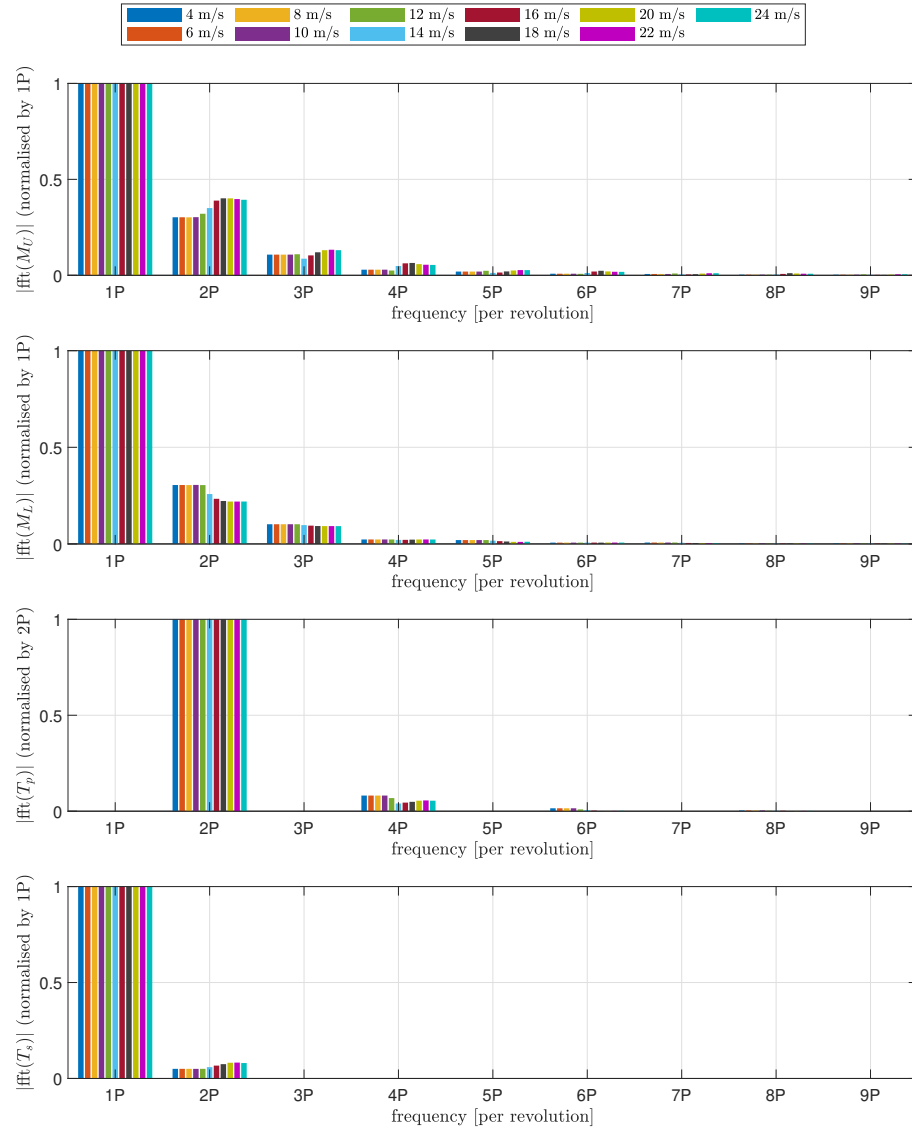


Figure 4.36: DFT amplitudes of the four load components considered in this analysis.

where L_r represents the load range, L_m represents the mean load and L_{max} represents the ultimate load for the component. In this case it is assumed that the component is designed such that the ultimate load is twice the maximum operational load. The number of load cycles is given by

$$n_i = p_i * f_i * t_l, \quad (4.14)$$

where p_i represents the probability of occurrence of the wind speed bin, obtained through integration of the Weibull distribution, t_l represents the turbine lifetime in seconds, here taken to be 25 years, and f_i represents the load frequency in Hz (either 1P or 2P as described above). The Wöhler exponent for the blade bending moments is taken as 10, representative of GRP, and the Wöhler exponent for the thrust load on the primary and secondary rotor is taken as 4, representative of steel. This is because the primary rotor thrust load is predominantly acting on the support structure, and the secondary rotors are likely to be made of metal, rather than GRP.

Using this approach, the 1Hz damage equivalent load for the 4 signals considered here at each wind speed bin for each strategy are shown in figure 4.38.

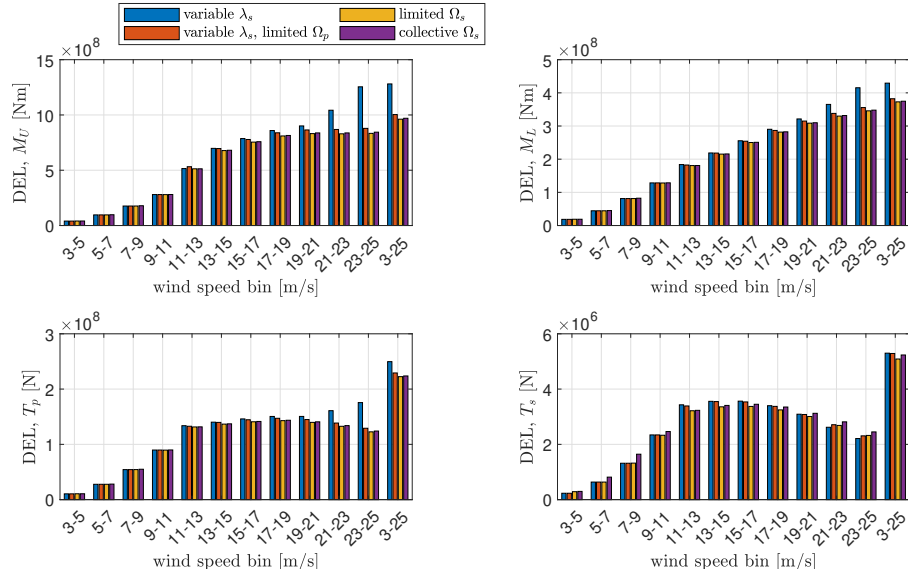


Figure 4.38: Lifetime DELs for considered control strategies at each wind speed bin.

The lifetime DEL is given by

$$L_{DEL} = \left(\sum_i L_{DEL,i}^w \right)^{1/w}, \quad (4.15)$$

and is shown for in table 4.2. It can be seen that the two strategies employing a saturation limit on secondary rotor speed facilitate primary rotor operation that reduces the lifetime damage equivalent load for both the support structure and the blades. A 2.75% increase in the secondary rotor thrust DEL is incurred when using collective secondary rotor speed control, compared to individual control.

	Collective λ_s	Collective λ_s limited Ω_p	limited Ω_s	collective Ω_s
DEL(M_{nU}) [MNm]	1,281	1,004	962	970
DEL(M_{nL}) [MNm]	429	382	373	375
DEL (T_P) [MN]	250	229	222	224
DEL (T_S) [kN]	5.30	5.29	5.09	5.23

Table 4.2: Lifetime DELs for considered control strategies.

4.6 Discussion

4.6.1 Energy capture

Section 4.5 evaluates the four potential X-Rotor control strategies proposed in 4.4. In terms of energy capture, strategies 1, 3, and 4 can achieve very similar power curves, whilst strategy 2 cannot maintain a rated power of 5MW in above rated conditions. The inability of strategy 2 to maintain a power of 5MW in above rated conditions is interesting and deserves further discussion.

Previous work has described that the power available to secondary rotors increases as the ambient wind speed increases and the primary rotor is held at a constant speed and power output [20,30]. The efficiency of power conversion between the primary and secondary rotor is given by

$$\eta = \frac{C_{Ps}}{C_{Ts}} \frac{1 + \frac{3}{2}\lambda_p^{-2}}{1 + \frac{1}{2}\lambda_p^{-2}}. \quad (4.16)$$

This indicates that, in the case of constant primary rotor speed and power operation,

the power available to the secondary rotors increases with increasing wind speed since the efficiency of power conversion increases with $(\lambda_p^2 + 3/2)/(\lambda_p^2 + 1/2)$. This effect relies on the secondary rotors maintaining operation at a constant tip speed ratio. Since constant primary rotor speed operation occurs at high wind speeds, this effect is tempered since secondary rotor tip speed must be constrained and the secondary rotor tip speed ratio cannot maintain its optimum value. This reduction in secondary rotor tip speed ratio will alter the ratio of the power to thrust coefficient, following the curve presented in 4.5, away from its maximum value. The curve has a relatively wide, flat top with a relatively sharp elbow showing a reduction in C_P/C_T for $\lambda_s < 2.25$. Thus, the ideal increase in power capture at constant primary rotor speed in increasing wind speed cannot be realised if the secondary rotor speed is controlled effectively in collective λ_s operation, since limiting the secondary rotor tip speed requires significant reductions in the secondary rotor tip speed ratio. If the secondary rotors are operating in limited Ω_s mode or collective Ω_s mode, an increase in power capture if the primary rotor is held in constant speed mode is expected. Figures 4.29 and 4.23 show that the primary rotor speed must be marginally reduced in above rated wind speeds in order to maintain a constant power output from the secondary rotors.

Disregarding the limits imposed on the secondary rotor tip speed, the desirability of the utilisation of this additional energy available to the secondary rotors may also be questioned. Whilst the generator is able to temporarily produce power above its nameplate rating, the turbine may operate at the high wind speed sections of its power curve for an extended period of time, and the generator must therefore be rated to accommodate this additional power. Furthermore, in a wind farm setting, all infrastructure must be uprated to allow for this additional capacity. Since this additional capital cost is expected to be relatively large, the most efficient utilisation of this additional cost may simply be to operate the turbine at a marginally higher rated wind speed under a traditional operating strategy, increasing the time spent at high power operation and further increasing LEP, although the effect on primary rotor loads must also be considered. For this reason, the strategies presented in this chapter have considered a conventional power curve, as it represents a more conservative estimate on

expected power capture. A detailed cost benefit analysis must be conducted in order to understand whether or not the exploitation of the additional power available to the secondary rotors, through operating the primary rotor in constant speed mode in above rated conditions, is worthwhile.

4.6.2 Loads

The deterministic lifetime damage equivalent loads are shown in table 4.2. The lowest primary and secondary rotor loads occur for the limited Ω_s strategy.

The weighted DELs for each wind speed bin are shown in figure 4.38. Despite the low probability of occurrence, operation in high wind speeds exposes the turbine to a high DEL. Figure 4.39 shows the damage equivalent load for each wind speed bin, normalised by the energy production over the operational lifetime in that wind speed bin. It is clear that operating at high wind speeds represents a significant cost, in terms of the the damage caused by operation i.e. fatigue accumulation, for a relatively low return in energy capture.

If the design driving loads for the turbine components are fatigue loads in regular operation, it could be recommended that the turbine should not be operated at the highest wind speed bins. In terms of the secondary rotor, it is expected that the rotors will be detached and replaced relatively frequently, in the range of 2-5 years, due to erosion caused by high tip speed operation [136]. It is therefore considered unlikely that operational fatigue loads are likely to be the design drivers. For the primary rotor, as discussed in 3, in literature VAWT blade designs are often completed considering the extreme load case as design drivers. However, an analysis of the design load cases has not yet been completed for the X-Rotor, and the option to reduce fatigue consumption with limited impact on lifetime energy capture may prove valuable.

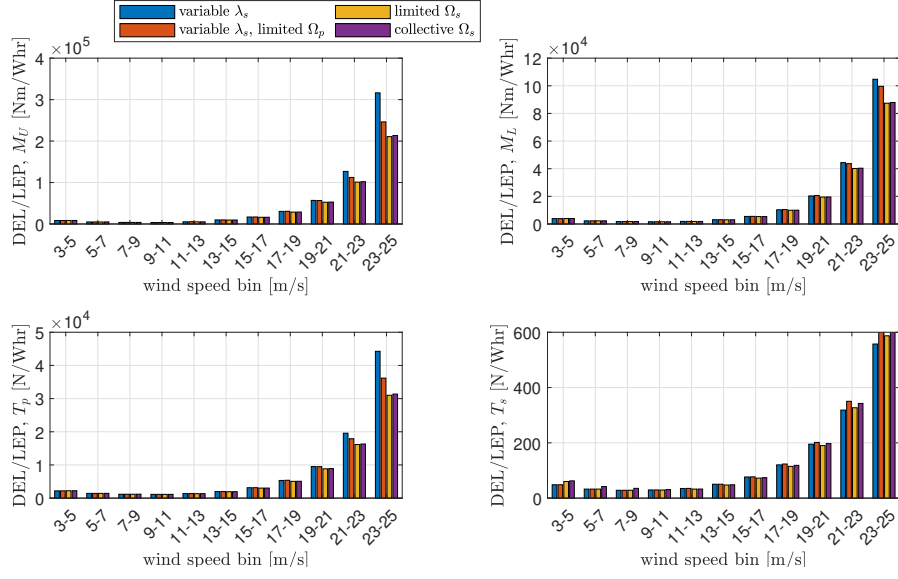


Figure 4.39: Lifetime DEL normalised by LEP for each wind speed bin.

Further primary rotor load reduction could be possible through introducing a variable/individual pitch strategy. Here, the pitch angle could follow a deterministic variation, as in [137], using the power output from the secondary rotors to estimate the wind direction. Otherwise, pitch control based on strain gauges at the blade root could be utilised in a similar manner to individual pitch control for HAWTs [128].

4.6.3 Secondary rotor operability

An additional consideration is the aerodynamic stability of the secondary rotors. When the secondary rotor stalls, the gradient of the constant wind speed torque-speed curve changes sign and the rotor exhibits negative aerodynamic damping for in-plane rotation. This means that an increase in the secondary rotor speed produces an increase in secondary rotor torque, leading to a further acceleration in the secondary rotor speed. This can lead to mode excitation, excessive loads, and controller instabilities. The rotor is in stall when the gradient of the C_Q - λ plot, shown in figure 4.40, is positive, i.e. $\lambda_s < 2.5$.

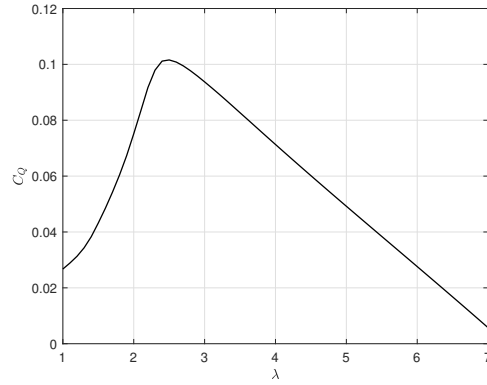


Figure 4.40: Secondary rotor torque coefficient as a function of tip speed ratio.

When employing limited Ω_s or collective Ω_s strategies, at a given ambient wind speed, the secondary rotor may only be stalled temporarily, since the tip speed ratio changes due to the change in incident wind speed. If the rotor is only stalled for a fraction of the primary rotor rotation, it is expected that the positive aerodynamic damping experienced over the other fraction will limit mode excitation and excessive loading. The fraction of a primary rotor rotation for which the secondary rotor is stalled is shown as a function of wind speed in figure 4.41. It is clear that the use of collective λ_s strategies to limit secondary rotor speed lead to the secondary rotor being operated in stall at all wind speeds above 14.3m/s, and that limited Ω_s strategies limit the fraction of time the secondary rotor experiences stall to a maximum of 27.5%. To limit the amount of time under which the secondary rotors are stalled, the limited Ω_s or collective Ω_s strategies are therefore recommended.

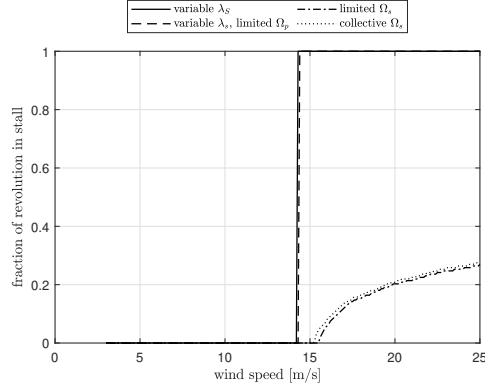


Figure 4.41: Proportion of revolution during which secondary rotor is stalled.

4.6.4 Recommendations for control strategies

Considering the primary and secondary rotor loads, the lifetime energy capture, and the operational regime of the secondary rotors, this chapter recommends using strategy limited Ω_s or collective Ω_s control of the secondary rotors.

Collective Ω_s control facilitates a reduction in the cumulative capacity of power electronics in the power train; however, the controller required for this strategy is expected to be marginally more complex, specifically due to the switching of secondary rotor speed input. It is therefore advised that limited Ω_s control is applied to facilitate a simple, robust controller for the secondary rotor, similar to the rotor speed controller for a conventional variable speed HAWT.

4.7 Final definition of limited control strategy

The recommended control strategy follows the limited Ω_s control defined in subsection 4.4.3, with the addition of pre-emptive pitching employed between 11.2m/s and 12.1m/s. Pre-emptive pitching is typical in the control of HAWTs, to limit the maximum pitch rate close to the rated wind speed. As is clear from figures 4.12, 4.17, 4.22, and 4.28, the rate of change in pitch angle is very large immediately above the rated wind speed. In order to limit excessive pitch actuation in conditions where the wind speed is fluctuating close to the rated wind speed, a smooth ramp up to the required

pitch angle is employed with pitch actuation initiating below the rated wind speed. The exact application of a pre-emptive pitch scheme should be further evaluated through simulation with an aero-servo-elastic simulation of the X-Rotor. Figure 4.42 shows the rotor performance coefficients, the primary rotor pitch angle and rotor speed, and the secondary rotor speed range as a function of wind speed.

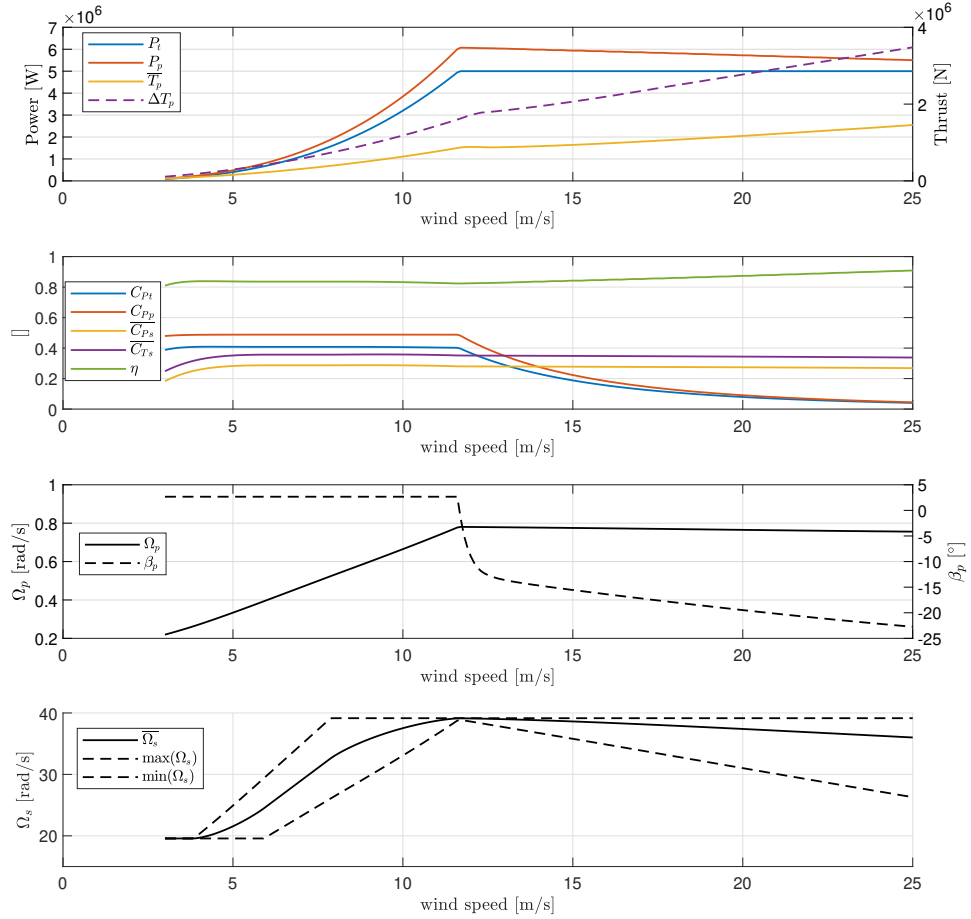


Figure 4.42: Definition of recommended control strategy.

The primary rotor torque-speed curve is shown in figure 4.43, and the secondary rotor torque-speed curve is shown in figure 4.44. The primary rotor is operated marginally above it's optimum tip speed ratio, since the primary rotor has an optimum tip speed ratio of 4.75, whilst the maximum turbine power coefficient is achieved at a primary

rotor tip speed ratio of 4.99. At low wind speeds, the primary rotor operates at even higher tip speed ratios, since the secondary rotors are operating lower thrust coefficients due to their high tip speed ratio operation. At high wind speeds, the primary rotor speed is not held constant, but reduces marginally to limit the power captured by the secondary rotors. This can be achieved through controlling the primary rotor pitch angle on the rotation averaged power captured by the secondary rotors.

The secondary rotor is operated in a similar manner to a variable speed wind turbine, with a low speed constant speed mode, a maximum power point tracking mode, and a high speed constant speed mode. The rotors are operated at a higher than optimum tip speed ratio, since this corresponds the maximum turbine efficiency. The majority of secondary rotor operation is far from the stall front, however the secondary rotors do operate in stall for a small portion of each primary rotor revolution at high wind speeds, as shown in figure 4.41.

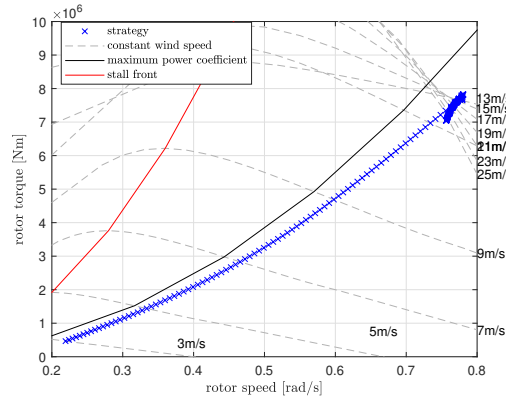


Figure 4.43: Primary rotor torque-speed curve.

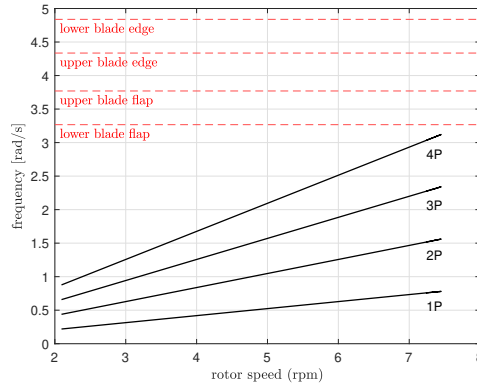


Figure 4.45: Campbell diagram for the recommended X-Rotor control strategy.

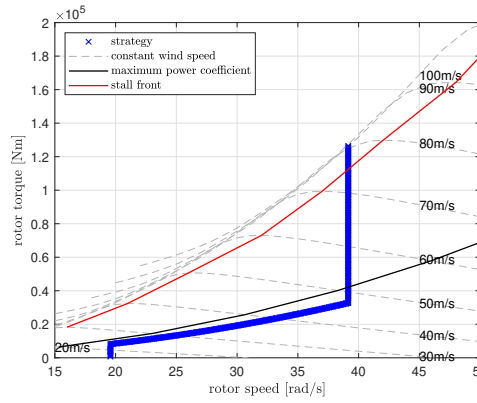


Figure 4.44: Secondary rotor torque-speed curve.

A Campbell diagram corresponding to the primary rotor is shown in figure 4.45. The natural frequencies of the blades are taken from [30]. The 4P component of the maximum rotor speed falls within 5% (4.49%) of the lower blade flapwise frequency. Since reducing the primary rotor speed close to the rated wind speed is expected to have a significant effect on power capture, it is advisable to reinforce the blade to increase the stiffness in the flap-wise direction. It should be noted that the lower blade flapwise frequency is expected to increase with rotor speed due to centrifugal stiffening, which is exacerbated by the mass of the secondary rotor at the blade tip, however this has not yet been modelled.

4.8 Conclusion

This chapter has provided a discussion of the potential control options for the 5MW, 2 bladed X-Rotor concept presented in [20] and [30]. The unique dynamics of the X-Rotor turbine are introduced and the equilibrium conditions for constant primary rotor speeds are introduced. A review of the prior literature including control strategies for an X-Rotor turbine revealed that simultaneous limitation on the secondary rotor tip speed and the turbine output power in above rated wind speeds has not been explicitly demonstrated. Following this, 4 secondary rotor control strategies were implemented which are coupled with a corresponding primary rotor pitch strategy in above rated conditions.

A comparison found that operating the secondary rotors in a similar manner to a variable speed wind turbine is successful in maximising power output whilst minimising the loads on the primary rotor. Collective speed operation of the secondary rotors, facilitating shared generator side power converters, was also investigated. It is found there was no significant penalty in terms of energy capture or primary rotor loads, and a small increase of 2.75% on the secondary rotor thrust DEL was incurred.

It is also found that operation of secondary rotors in collective λ_s mode i.e. at a constant tip speed ratio over the full primary rotor rotation with no clipping of Ω_s , incurs either significant load penalty or a reduction in LEP. Additionally, this form of operation results in the secondary rotors operating in a stalled condition for a significant portion of rotor operation, leading to concerns around negative damping and mode excitation. For these reasons, operation of the secondary rotors in collective λ_s mode is not advised.

A recommended control strategy is provided for the 5MW X-Rotor concept which achieved a 5MW rated power at 12m/s and maintained the upper and lower limits on primary rotor speed. To ensure controller simplicity, collective speed operation is not employed, and the limited Ω_s strategy is used. A pre-emptive pitching strategy is employed to limit the maximum required pitch rate. The control strategy is detailed in torque-speed curves for both the primary and secondary rotors in figures 4.43 and 4.44

respectively. A Campbell diagram reveals a potential 4P excitation of the first lower blade mode in the constant speed region. Since reducing the speed here would cause a significant change in the rated power of the machine, it is recommended that the first mode of the lower blade should be shifted to avoid resonance.

This study was limited by the simplified steady-state approach taken to study both rotor loads and power capture. A full aero-elastic model of the turbine is required to more thoroughly understand and evaluate the control strategies presented here further. Additionally, cyclic pitch control of the primary rotor blades was not considered, this could provide significant opportunities for load reduction on the upper blades of the primary rotor.

Chapter 5

Discussion and conclusions

The previous chapters of this thesis investigate the use of secondary rotors for wind turbine power take-off, primarily focussing on their integration with VAWT primary rotors. It is shown that conversion efficiencies between 80%-85% are achievable for commercial scale (5MW) machines, facilitating turbine power coefficients of approximately 0.4. This is enabled by the use of optimally designed secondary rotors, and the high tip speed ratio operation of VAWT primary rotors. Through the application of inclined blades, it is shown that VAWT primary rotors can be designed to operate at the tip speeds required for efficient power take off using secondary rotors and achieve a higher rated power than H-Rotors. In terms of turbine operation, a number of feasible operational strategies are investigated for an X-Rotor type turbine. Multiple control strategies capable of high conversion efficiency in below rated conditions and power regulation in above rated conditions, which conform to the strict limitations on secondary rotor speed, are developed, demonstrating the feasibility of the turbine concept.

This chapter discusses the results and limitations of Chapters 2-4 in Section 5.1. Following this, future work is presented in Section 5.2 and Section 5.3 synthesises these discussions into the key next steps recommended to further develop the X-Rotor concept. Finally, Section 5.4 discusses how the research questions posed in Chapter 1 are addressed.

5.1 Overview of work presented

5.1.1 Power take off with secondary rotors

In Chapter 2 of this thesis, a thorough literature review of previous work on the use of secondary rotors for wind turbine power take off is presented, including both academic and patent literature. It is found that the most complete theoretical representation of secondary rotors was provided in the patent WO02086312 [60], which applied to secondary rotors coupled to a HAWT and represented the rotors as actuator disks. Chapter 2 extends this work in a number of ways. Firstly, using this actuator disk representation, the maximum achievable efficiency of power conversion between the primary and secondary rotors is defined as a function of the turbine geometry, the environmental limitations, and the torque reduction ratio for the generic case of a turbine utilising secondary rotors in equation 2.13. This equation informs the top-down design of turbines utilising secondary rotors, primarily indicating that high conversion efficiency requires high primary rotor tip speeds, and indicates that the efficiency of power conversion reduces as the torque reduction ratio increases, indicating a trade off between power train cost reduction and energy capture.

Following this, an analytical method of determining the aerodynamic characteristics of an optimally designed secondary rotor using BEM theory is developed and validated. This facilitates the design of secondary rotors for any given primary rotor configuration. Whilst this is formally a one-way optimisation, due to very low computational cost of generating the on-design rotor performance coefficients for the optimised secondary rotor, this methodology enables rudimentary coupled primary/secondary rotor design, as demonstrated in Chapter 3. A key finding arising from this representation of the secondary rotors is that the parasitic drag from the rotor hub consumes more of the available power as the design thrust coefficient of the secondary rotor decreases, providing an upper limit on conversion efficiency. A range of generic trait studies indicate that practicable designs can have achieve conversion efficiencies in the range of 0.8-0.85.

The application of flow augmentation devices, such as ducts, is briefly investigated

using the formulation presented in [71]. The results indicate that ducted rotors could provide a means of increasing the conversion efficiency. This basic analysis does not provide enough evidence to encourage the use of ducts for wind turbines employing secondary rotors but does encourage further research into the topic.

Finally, the specifics of secondary rotor application to both VAWT and HAWT primary rotors are discussed. The differences in incident wind speed and corresponding changes to aerodynamic conversion efficiencies are derived and calculated and it is found that VAWT rotors can achieve a higher conversion efficiency between the primary and secondary rotors. Additional concerns including secondary rotor loads, turbine operability, and unique design synergies are also discussed. This chapter concludes that the use of secondary rotors for VAWT power take off is compelling due to design synergies associated with the ease of turbine maintenance, and the fact that power take off is a significant issue for VAWTs and secondary rotors can therefore be seen as an enabling technology to use VAWTs for large offshore wind farms, facilitating higher density wind farms and potentially lower cost floating sub-structures.

The methodological limitations of this chapter are numerous. Firstly, whilst a FVW model is used to validate a BEM based approach to secondary rotor design in Appendix A, both models still utilise a simple blade element approach to calculate the blade forces. The blade sections are operating at a significant fraction of the speed of sound, with the maximum tip speed taken at Mach 0.54. The validity of the polars at this high Mach number is questionable, as they were calculated at a free-stream Mach number of 0. At these intermediate Mach numbers, the flow can locally reach Mach numbers greater than one as the flow accelerates over the suction side of the aerofoil section. This can lead to shock formation, and the associated increases in drag and onset of premature flow separation and un-steady loads [138]. At a turbine level, it is expected that operating at high Mach numbers may cause an increase in the thrust coefficient of the secondary rotor, a decrease in the power coefficient of the secondary rotor as found in [139], and therefore a decrease in the conversion efficiency between the primary and secondary rotors.

Blade-resolved compressible CFD simulations or experimental campaigns will be re-

quired to understand the effect of high Mach number operation. Possible modifications to the BEM approach include the use of Prandtl–Glauert transformation, as in [139], modifying the lift and drag coefficients by a factor of $1/\sqrt{1-M^2}$ [140], or the use of polar tables generated over a range of Mach numbers using compressible Navier-Stokes CFD. The study presented in [141] found that, for a traditional wind turbine operating at a tip speed of 150m/s, CFD simulation including compressibility effects indicated a 1.4% increase in power output and a 4.2% increase in thrust output compared to the incompressible case. This would correspond to a 2.6% decrease in conversion efficiency between the primary and secondary rotors.

Furthermore, the analysis presented in this chapter did not include the mutual aerodynamic interaction between the primary and secondary rotor or the mutual interactions between multiple secondary rotors. In Appendix A, the rotor disk averaged velocity at the design location of a secondary rotor for the X-Rotor is found to be be relatively well described by the sum of the rotational and free stream velocity, indicating that the interaction between the primary rotor and the secondary rotor is not likely to be significant. In terms of the interaction between the secondary rotor and the primary rotor, Bensason [43] found that, during wind tunnel testing, the secondary rotors cause an accelerated diffusion of the tip vortex, leading to a decrease in induced in-plane velocity gradients both on the edge and within the volume of rotation of the rotor, indicating a reduction in the loads on the primary rotor blade. It should, however, be noted that this effect was likely exaggerated by the location of the secondary rotors, directly on the blade tip of the primary rotor lower blade. Further studies of the coupled primary and secondary rotor systems with the explicit aim of quantifying the magnitude of the mutual aerodynamic interactions are required.

Finally, the design work in this section assumed a maximum allowable tip speed of 184m/s, as reported in [20]. It has been shown, in figure 2.14, that the maximum achievable conversion efficiency is highly dependent on the maximum allowable tip speed. Clarification of this maximum tip speed through high fidelity simulation and consultation with both blade manufacturers and environmental experts is required to better understand the limitations on secondary rotor tip speed and determine an

acceptable design value.

5.1.2 X-Rotor primary rotor design

Chapter 3 provides a review of design parameters for VAWTs, and the use of VAWTs with inclined blades is shown to be a feasible method of increasing the design tip speed ratio of a VAWT into the range at which efficient power conversion with secondary rotors can occur. To ensure the maintainability of secondary rotor units, near-ocean-surface mounting locations are required, resulting in an X-shaped primary rotor design. Following this, a review of the academic literature on the design, modelling and testing of straight bladed VAWTs with inclined blades is presented.

To facilitate a design study of X-shaped VAWTs, a benchmarking exercise is completed, fully reported in [41], which compares a number of low-fidelity and mid-fidelity models with high fidelity blade-resolved U-RANS simulation. It is found the 2DAC model accurately reproduces the power production and rotor loading compared to higher fidelity models, and this model is therefore taken forward to complete a design study into X-shaped VAWT rotors.

A set of 24 rotor geometries are investigated, each rotor geometry is investigated with 20 possible chord lengths over a tip speed ratio range between 1.5 and 6.5. It is found that, alongside an increase in optimal tip speed ratio, due to the increase in rotor swept area, blade inclination can increase the power captured by the rotor. Following this, the optimal secondary rotor design methodology presented in Chapter 2 is employed to calculate the power output of the turbine considering power take off using an optimally designed secondary rotor. The results show that rotors employing inclined blades can generate significantly more power than traditional H-Rotors with the same blade length. High solidity, high aspect ratio rotors benefitted the most from the introduction of inclined blades.

For a set of rotors with identical chord lengths, the minimum blade length required to achieve a rated power of 5MW (when using secondary rotors) is calculated as a function blade inclination angle. For every considered solidity, blade lengths in excess of 100m (outside of the range of rotor geometries considered) are required for H-Rotors;

however, power ratings of 5MW are achievable with blade lengths of 78m when inclined blades are employed. The optimum blade inclination angle is found to increase with increasing chord length. The revolution averaged thrust for a 5MW rotor is found to reduce as a function of inclination angle and rotor solidity, and the maximum operational blade root bending moment is found to be minimised at blade inclination angles of 20° - 40° . Blade inclination is also found to reduce the primary rotors exposure to extreme loads.

Following this, the rotor designs which correspond to the in the minimum blade mass are calculated as a function of blade chord length. It is found that the combined upper and lower blade mass is minimised for a fixed chord length between 4m-6m, corresponding to a blade length of 78m-80m, and a blade inclination angle of 30° - 40° .

The analysis presented in this chapter demonstrates that X-Shaped VAWTs represent a design solution for VAWT primary rotors that out-perform H-Shaped VAWTs in terms of both power capture and load mitigation. Due to the increase in optimal tip speed ratio and the increase power output for a given blade length, driven by the increase in rotor swept area, X-Shaped VAWT facilitate the use of shorter blades that can reduce both operational and extreme loads, and are expected to reduce the primary rotor mass and cost. Optimal inclination angles for practicable blade designs lie in the range of 30° to 40° , with the optimal inclination angle increasing with increasing blade solidity.

A key methodological limitation of the study is the aerodynamic model used for the design process. Unsteady blade element effects, including dynamic stall and flow curvature, are not modelled and generally can be considered relevant to VAWT design. However, the simulation benchmark provided in [41] demonstrated that the uncorrected model performed well against a blade-resolved URANs simulation considering both blade and rotor loads.

5.1.3 Control strategies for the X-Rotor

Chapter 4 reviews typical control strategies for conventional variable speed, variable pitch HAWTs, presents proposed control strategies for large offshore VAWTs, and dis-

cusses the key differences between the control of conventional turbines and the control of the X-Rotor. Turbine power coefficient contours for the 2 bladed 5MW X-Rotor design are derived and presented as a function of the secondary rotor tip speed ratio and the primary rotor pitch angle, and limitations on secondary rotor operational speed are introduced.

Four possible control strategies for the secondary rotors are presented which achieve high conversion efficiency at below rated wind speeds and limit the power output of the turbine to 5MW in above rated wind speeds whilst conforming to the limits on the maximum secondary rotor speed. Each of these control strategies are evaluated in terms of energy capture and deterministic load fluctuations through the calculation of lifetime DELs. It is found that operating the secondary rotors in a similar manner to variable speed HAWTs provides the highest yields in LEP and the lowest DELs for each of the considered load signals. This involves operating with a low-wind-speed constant speed region, a maximum power point tracking region, and a high-wind-speed constant speed region. Collective speed operation of the secondary rotors, facilitating the use of a single generator side power converter, is investigated and it is found that this did not have a significant impact on LEP or the lifetime DELs.

Finally, a recommended control strategy is defined which implements simple control logic and maintains the limits on minimum and maximum secondary rotor speed whilst achieving efficient energy conversion at low wind speeds and limiting power output at high wind speeds. Corresponding torque-speed curves for both the primary and secondary rotor are produced and discussed.

The primary methodological limitations of the work presented in this chapter concern the evaluation of the different control strategies. The strategies are evaluated in terms of their steady state performance in a uniform ambient wind field, and only the aerodynamic loads are evaluated. The development of a full aero-elastic turbine model, and the design and implementation of controllers for each strategy is required for a more complete analysis.

5.2 Future work

5.2.1 Secondary rotor design

Addressing uncertainties in secondary rotor design and performance

A key unknown in the aerodynamic performance of the secondary rotors is the mutual interaction between the secondary rotors and the primary rotor. The implications for both the primary rotor and secondary rotor loads and power production must be evaluated using high fidelity coupled aerodynamic simulations or through experiments.

An additional uncertainty in the aerodynamic performance of the secondary rotors is driven by the abnormally large rotor hub. There have not yet been detailed studies on the effect of the large rotor hub on secondary rotor performance, apart from assuming annular independence and treating the hub and a body which produces only drag (thrust). However, it is expected that the local blockage at the rotor hub may direct flow from the centre of the rotor outward towards the blades, increasing the rotor power and thrust outputs. An investigation into the effects of large rotor hub designs was completed in [67] using blade-resolved CFD simulations of multiple wind turbine hubs. Marginal increases in both the power and thrust coefficients were reported, between 1% and 3.6%; however, there was not significant evidence for a change in the ratio of power coefficient to thrust coefficient. This implies that, whilst the rotor characteristics might change marginally, the conversion efficiency between the primary and secondary rotor is likely to remain relatively unchanged.

The maximum tip speed of the secondary rotors represents a key variable in determining the maximum achievable efficiency of power conversion for any given torque reduction ratio, and thus far there has not been significant research on clarifying the currently assumed value of 184m/s, first proposed in [20]. Blade-resolved compressible CFD simulations, such as those presented in [141], of the secondary rotors at a range of rotor speeds are required to more accurately estimate the maximum allowable tip speed for the secondary rotors.

Another contributing uncertainty originates from the secondary rotor operating at an oscillating yaw angle, corresponding to the rotation of the primary rotor tip into,

and out of, the free stream wind. The yaw correction models currently employed in BEM codes have been shown to be insufficient, and the analytical BEM representation derived in Chapter 2 does not consider the effect of yaw offset. It should, however, be noted that a comparison of a look-up based approach, using rotor coefficients derived using the optimal design methodology derived in Chapter 2, and a lifting line free vortex wake simulation, which inherently models the velocities induced during yawed operation, show good agreement, as presented in Appendix A.

A further uncertainty arises from the design of the subsystem which will attach the secondary rotors to the tip of the lower blades of the primary rotor. The ability to easily replace secondary rotor modules has been identified as a promising method of reducing O&M costs [35]; however, the connection must be designed to withstand significant loading. Furthermore, the apparent wind speed at the point of connection will be very high (equal to the wind speed incident on the secondary rotors), implying that any parasitic drag will cause a significant reduction in power capture. For example, if the drag induced by the connection was to increase the effective thrust coefficient of the secondary rotors by 10%, this would correspond to a reduction in the peak turbine power coefficient of 9%. Design work is required to estimate the front facing area of the subsystem and the its effective drag coefficient.

Finally, the design driving loads for the secondary rotors are also a source of uncertainty. The maximum operational loads are expected to be significantly larger than the parked loads during extreme wind events. This is because the maximum wind speeds incident on the secondary rotors are higher during operation, up to 72m/s, than in parked conditions using the extreme wind model from [142]. Additionally, large deterministic fluctuations in the secondary rotor loads are experienced over a primary rotor revolution due to them tracking into and out-of the ambient wind. This indicates that fatigue loads under normal operation may be the design driver and research must be undertaken to design the rotors accordingly. An evaluation of the IEC design load cases [142] for the secondary rotors is required to determine the design driving load cases.

Improving design methodology

At present the secondary rotor design is completed using the BEM based optimal aerodynamic design procedure described in Chapter 2 and does not consider the structural design including the blade internals. The design process could be improved by considering a coupled aero-structural design of the secondary rotors. The current design employs a single aerofoil section, the FFA-W3-241, it is expected that a more structurally efficient design could be achieved by using a variable aerofoil distribution, using thicker aerofoils at the blade root to increase the second moment of area and reduce bending stresses. Furthermore, adjustments to the blade chord distribution could be employed towards to blade root to reduce bending stresses.

5.2.2 Primary rotor design

Addressing uncertainties in primary rotor design

The primary rotor design work presented in Chapter 3 considers a generic cross-arm chord length to account for strut losses. The design work for the 2-Bladed 5MW X-Rotor presented in [20] and [30] does not consider the drag arising from the rotor cross-arm, nor is any design of the rotor cross-arm presented. Because of this, the control strategy presented in Chapter 4 also does not account for cross-arm losses. Understanding the design driving loads for the cross-arm and developing a simple method for scaling the cross-arm dimensions would make the results of general studies such as that presented in Chapter 3 more representative, allowing for the trade off between cross-arm mass and drag, and increased swept area to be more thoroughly understood, reducing the uncertainty in the primary rotor design.

Da Silva [143] presented a cross-arm design that used a hollow cylinder which tapered from a diameter of 6.9m at the cross-arm centre to 6.5m at the crossarm tip. This design was verified to withstand fatigue and extreme loads. Significant parasitic drag losses were identified, up to 5% of the primary rotor rated power at the rated wind speed. The potential to introduce fairings onto the cross-arm to reduce the parasitic drag losses was discussed; however, this has not yet been investigated. Additionally,

work completed at the University of Strathclyde as part of an industrial project has explored the potential of a lattice based crossarm, which could significantly reduce the cross-arm mass. Again, it may be advised to employ fairings to reduce the parasitic drag. A representative design of the rotor cross-arm is required to obtain turbine mass estimates, cost estimates, and understand the effects of parasitic drag on aerodynamic performance. The development of generic design rules would also facilitate more rigorous aero-structural primary rotor design studies considering the trade off between cross-arm mass and drag, and increased swept area using optimisation codes.

In terms of the blade mass estimation process used in Chapter 3, the design driving load is assumed to be the parked rotor in extreme winds, and no detailed fatigue analysis has yet been completed. This is also the case for the more involved blade designs presented in [20] and [30], which are designed such that they qualify for simplified strain verification and do not require fatigue analysis. This simplified strain verification is no longer part of up to date standards for wind turbine design, and the further applicability of this to the X-Rotor requires investigation.

Addressing the uncertainty in the design driving loads for the X-Rotor primary rotor blades requires the simulation of the X-Rotor turbine over the full range of design load cases listed in IEC61400 [142] with appropriate modifications pertaining the differences between a conventional HAWT and the X-Rotor. From this, the appropriate design load cases can be identified. This process has been completed for the Deepwind 5MW Rotor [144], and smaller more rigid H-rotor designs [145]. However, since the X-Rotor is a significant departure from both of these turbines, a turbine-specific study must be completed. In order to complete this task, an aero-elastic model capable of simulating the X-Rotor over the full range of DLCs is required, such a model is not available and its development is of paramount importance.

The conformation/identification of the design driving load cases for the primary rotor blades would either provide more confidence in the blade mass estimation presented in Chapter 3, or inform a new approach that must be based on a different load estimate. Additionally, the specification of design driving loads for the primary rotor blades is important for more detailed design work, such as that presented in [20] and [30].

Another current design uncertainty around the X-Rotor primary rotor is the nature of the connection between the rotor blades and the crossarm. This connection would have to be designed to endure the significant bending moments at the blade roots and be able to accommodate the pitch actuators for the upper blades. Typically, blades transition to a cylindrical section at the blade root and are attached to the low speed shaft through a large, cast iron, hub. The design work presented in Chapter 3 does not consider a transition to a cylindrical section at the blade root. The blade design for the X-Rotor primary rotor presented in [20] and [30], and used in Chapter 4, also end at a very large chord length, with no such transition. A detailed design of the blade roots of the upper and lower blades, and the hub which attaches these blades to the crossarm must be completed to understand both the mass and cost of the structure, and the potential aerodynamic losses associated with parasitic drag from the structure.

In a conventional HAWT, the blade root experiences a very low apparent wind speed due to rotation, since it is very close to the rotor centre. In the case of the X-Rotor, the blade roots of the upper and lower blades are located at the cross-arm radius, and consequently sees a significantly higher apparent wind speed, approximately 1/3rd of the tip speed. In the case of the 2 bladed 5MW X-Rotor, if each of the blades transitioned to a cylindrical section that had a diameter of 5m and a length of 3m, the drag losses would amount to 500kW at the rated rotor speed. Such losses are not inevitable, but this rough calculation highlights the importance of more detailed design work to understand the potential for aerodynamically efficient and structurally practical blade roots for the X-Rotor blades.

Prospective improvements in primary rotor blade design

The blade planform properties for the X-Rotor primary rotor are unconventional for wind turbine blades, with a very large maximum chord, a large overall planform area, and a low relative blade thickness. For reference, the NREL 5MW reference wind turbine (RWT) can be used to compare blade designs for the lower and upper blades presented in [30], and one of the potential blade configurations presented in Chapter 3. The NREL 5MW turbine is chosen for comparison because it has a similar power

rating. The chord length and relative thickness of the NREL 5MW turbine, the X-Rotor upper and lower blades from [30], and a potneital blade configuration from Chapter 3 are shown in figure 5.1. Since the X-Rotor primary rotor is a VAWT; operates at a lower tip speed ratio; and has 2 blades, rather than 3, it is not expected that the chord distribution should be similar. This comparison aims to illustrate the significant size difference between the currently designed X-Rotor blades, and blades employed for a traditional HAWT. It should also be noted that a 2 bladed X-Rotor uses 4 blades, 2 upper and 2 lower, whilst the HAWT uses 3.

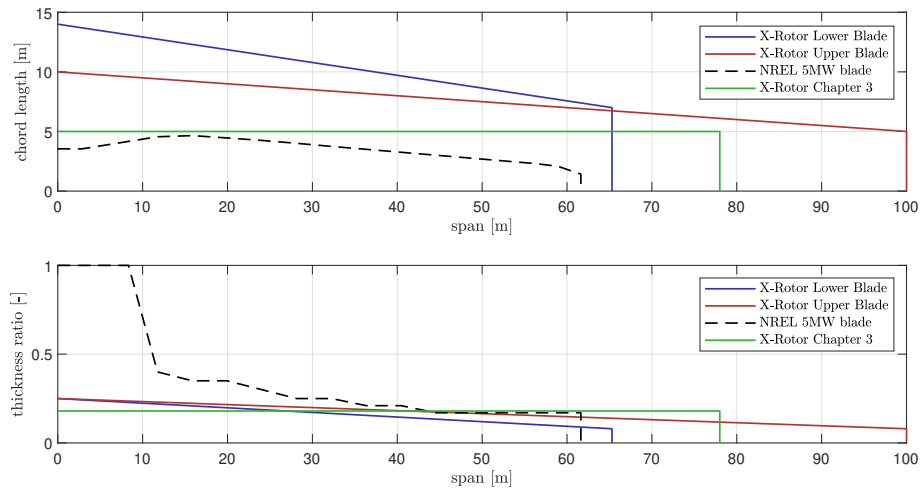


Figure 5.1: Comparison between the 5MW X-Rotor blade and typical HAWT blades of similar length.

The significant increase in blade span and chord length for the X-Rotor blades compared to HAWT blades from a turbine with a similar power rating is clear. Additionally, the low relative thickness of the aerofoil sections compared to the reference HAWT blades is evident. The thickness of the blade is important since increasing the thickness reduces the bending stresses experienced in the blade, facilitating a reduction in the material required to oppose a given bending moment. At the blade root, the blade properties are more dominated by structural considerations than aerodynamic considerations, hence the increase in the relative aerofoil thickness as the blades approach the blade root. Introducing significantly higher thickness aerofoil sections towards the

blade root of the X-Rotor is encouraged to achieve the required thickness for structural efficiency without the significant increase in chord length.

The increase in chord length and blade length represents an increase in blade material and therefore an increase in blade mass/cost. Additionally, in extreme wind speeds, the large planform area of the blades exposes them to larger loads, requiring more material to reduce stresses in the structure - further increasing mass and cost. These large chord lengths also propose a challenge in terms of buckling of the shell, due to large unsupported sections of the blade shell between the spar cap and the rear reinforcement. Finally an upper limit of blade chord length is also typically avoided due to logistical constraints for transport/installation [146].

In [20] and [30], the total combined blade mass for the 2 bladed 5MW X-Rotor design was calculated to be 127,768kg, which is 2.4 times larger than the combined blade mass for the NREL 5MW turbine, at 53,220kg. The papers do not consider the additional cost incurred by this significant increase in blade mass in the cost of energy estimations, since the rotor mass of the NREL 5MW turbine, including the cast iron hub, is 110,000kg and this is similar to the combined blade mass of the X-Rotor primary rotor. This analysis does not reflect the fact that: 1) the cast iron hub will be significantly less expensive per kilo than the composite blades, and 2) that the X-Rotor primary rotor will require a hub like structure to attach the primary rotor blades to the cross-arm and that this mass of this component should be included in any comparison of rotor mass. It can therefore be concluded that the significant cost represented by the increase in blade mass has not been accounted for in these works, although this limitation is discussed in the conclusion of [30].

The design of lighter weight blades for the primary rotor is therefore important to ensure the competitiveness of the X-Rotor concept. The work presented in Chapter 3 demonstrated that shorter, lower solidity, blades could be employed in the design of 5MW turbines coupled with secondary rotors. Designing the primary rotor to operate at a slightly higher tip speed ratio, 5.3, enables the use of a shorter, lower solidity primary rotor blade, using a constant chord length of 5m and a blade length of 78m. More practical designs would employ a variable thickness and a tapered chord length. These

could be generated through coupling the methodology presented in 3 for aerodynamic simulation and blade mass estimation with an optimisation code, and enabling a wider range of variables to be considered i.e. blade taper, aerofoil thickness, and separate upper and lower blade planforms. This would allow for the algorithmic design of blade platforms which could significantly reduce the mass of the blades.

5.2.3 Operational strategy

Cyclic pitch control of the primary rotor

The use of cyclic pitch control for the primary rotor is not investigated in Chapter 4. Numerous studies have shown the potential for cyclic pitching to increase power production and decrease loads for VAWT rotors [130, 134, 135, 137], and their application to the X-Rotor should be investigated. A unique characteristic of the X-Rotor which could help facilitate the application of cyclic pitch strategies is that the power signal from the secondary rotors can be used to estimate the incoming wind direction. The potential benefits of employing cyclic pitching must be evaluated considering appropriate limits on pitch rate, and the consequences of the increased duty cycle on the lifetime of the pitch actuators must be understood.

Operation with a single secondary rotor

A possible advantage of using multiple secondary rotors for power take off is the opportunity for redundancy. If a single power take off unit fails, the turbine can continue to operate under a modified operational strategy.

The basic methodology presented in Chapter 4 can be used to evaluate the potential for operation with a single secondary rotor. Setting the number of secondary rotors to 1, and including additional parasitic drag to represent the inactive secondary rotor, a turbine power coefficient of 0.3 is still achievable, as shown in figure 5.2. This requires secondary rotor operation at a tip speed ratio of 3.7 as opposed to 3.9, and a primary rotor pitch offset of 4° as opposed to 2° . The equilibrium primary rotor tip speed ratio in maximum power point tracking therefore increases 5.72, as opposed to 4.99. Of course, the maximum power must be limited to the capacity of a single secondary

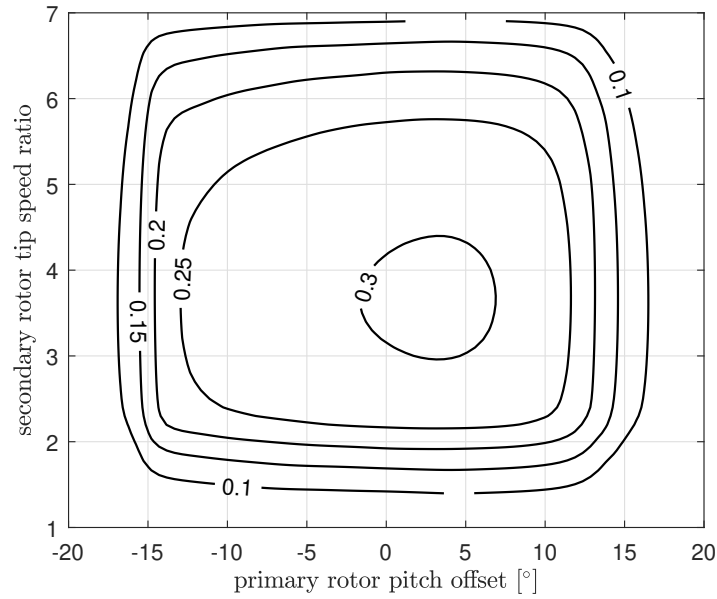


Figure 5.2: Turbine power coefficient contours as a function of secondary rotor tip speed ratio and primary rotor pitch angle for X-Rotor operation with a single secondary rotor.

rotor power train, and limits on both the primary and secondary rotor speed must be upheld; but at low wind speeds, this shows that operation using a single secondary rotor is possible, and the power captured by the turbine can be 3/4 of the power available with both secondary rotors active.

The costs associated with this approach are the additional loads due asymmetrical operation, and off-design operation for both the primary and secondary rotors. These must be evaluated and compared to the benefits, primarily a reduction in turbine downtime and consequent increase in energy capture. Further research on both the loads associated with single secondary rotor operation and the additional energy captured through reduced down-time must be completed to better understand if single secondary rotor operation should be employed. As the partial load efficiency of some generators is very low, single secondary rotor operation could also be employed at low wind speeds to increase the efficiency of power capture.

Reduction in the power production of the lower blades

An issue with the rotor design presented in [20] and [30] and used in Chapter 4, is the excessive power production of the lower blades of the primary rotor at high wind speeds. The initial aerodynamic evaluation of the rotor behaviour, presented in [20] and [30] used polars that were not representative of the aerofoil section and Reynolds number, and predicted that the rotor would stall in an aggressive manner. Using more representative aerofoil polars for the aerodynamic simulation of the primary rotor, it is found that the lower blades do not stall effectively at high wind speeds, and that the upper blades is required to produce negative torque, acting as an aerodynamic brake, to ensure that the primary rotor power is limited successfully. At wind speeds above 16.7m/s, the upper blades must produce negative power. This results in excessive loading of both the upper and lower blades of the primary rotor, and should be avoided in future designs. A compelling design option to address this challenge is the introduction of a positive (nose in) pitch offset on the lower blades, to encourage them to stall at higher tip speed ratios, i.e. at lower wind speeds.

5.2.4 Further applications

Beyond the scope of this thesis, the optimal secondary rotor design methodology developed in Chapter 2, validated over a large range of results, has the potential to be employed for a number of other uses where turbines are not designed to operate at the Betz limit. This could include low induction rotors [147], wind-cars [148], energy ships [149], or Fly-Gen airborne wind energy systems [150]. The research on VAWTs with inclined blades may also provide useful for traditional VAWT design.

5.3 Steps forward for further X-Rotor development

For the further development of the X-Rotor concept, a number of steps are recommended:

1. **Aero-elastic simulations covering a range of load cases:** At present the structural design of the primary and secondary rotors are based on load cases

which have been determined by the based on engineering judgement and experience of wind turbine turbine design. These load cases have been obtained from the combination of aerodynamic simulations and calculation of inertial and gravitational loads. The development of an aero-elastic code able to simulate the turbine over the range of DLCs stipulated in IEC 61400 [142], modifying where appropriate due to the unique nature of the turbine, would represent a significant step forward in the understanding of the turbine. This is due to the more accurate loads obtained from such a model, and the ability to accurately determine the most significant DLCs.

2. **High fidelity aerodynamic simulations to understand maximum tip speeds for secondary rotors:** The maximum secondary rotor tip speed is a key variable required for the coupled design of the primary and secondary rotors, and in determining the control strategy of the turbine. High fidelity aerodynamic simulations of the secondary rotors including compressibility effects are required to understand how secondary rotor performance is dependent on the tip speed, and determine the maximum allowable tip speed for the secondary rotors.
3. **Scale model demonstration to validate tools and investigate mutual aerodynamic interactions:** A 1:100 scaled model of the X-Rotor turbine was presented in [43]; however, the investigation predominantly focussed on flow-field measurement and not the validation of design and simulation codes which focus on rotor performance. A testing campaign on a larger scaled model could allow both the validation of numerical codes, and the characterisation of the mutual aerodynamic interaction between the primary and secondary rotors.
4. **Development of coupled primary/secondary rotor optimisation tool:** The development and validation of an algorithmic design tool that integrates primary and secondary rotors, with aim of minimising turbine cost would facilitate a more accurate estimation of the cost of energy reductions achievable with the X-Rotor. This could be achieved through the combination of the optimal secondary rotor design methodologies presented in Chapter 2, and the primary rotor

aerodynamic simulation work and blade mass estimation methodology presented in Chapter 3 into an optimisation framework. This could allow for an investigation into a wider range of primary rotor design variables such as the chord distribution, twist angle, upper and lower blade lengths, upper and lower blade inclination angles, and aerofoil section thickness.

5.4 Addressing the research questions

The research questions posed by this thesis in Chapter 1 are:

1. *Can tip mounted secondary rotors be used efficiently for wind turbine power take off?*
2. *How can secondary rotors be designed for efficient power take off?*
3. *How can a vertical axis wind turbine be designed for efficient use with secondary rotors?*
4. *How should a vertical axis wind turbine using secondary rotors be operated?*

In terms of the primary research question, it is demonstrated that tip mounted secondary rotors can be used efficiently for wind turbine power take off. Chapter 2 finds that, if a primary rotor is operated at the required tip speed, low induction secondary rotor design can ensure conversion efficiencies of between 80%-85% between the primary and secondary rotor. In Chapter 3, it is verified that VAWT primary rotors in a X-configuration using secondary rotors for power take off are able to achieve a 5MW power rating, including the efficiency of conversion between primary and secondary rotors, with blades lengths below 80m. Finally, in Chapter 4, it is demonstrated that a number of feasible operating strategies X-Rotor turbine exist which ensure high conversion efficiency in below rated conditions, limit power capture in above rated conditions, and conform to the strict limitations on secondary rotor speed.

Chapter 2 derives a procedure for the optimal aerodynamic design of secondary rotors, directly addressing research question 2. Chapter 3 addresses research question 3, indicating that an X-shaped VAWT primary with an inclination angle between 30° - 40° ,

Chapter 5. Discussion and conclusions

both facilitates the required increase in primary rotor tip speed required to efficiently operate with secondary rotors as power take off, and increases the power captured by the primary rotor compared to an H-Rotor. Finally, research question 4 is examined in Chapter 4 which provides a recommended control strategy for the 2 bladed 5MW X-Rotor turbine presented in [20] and [30] and investigated as part of the Horizon 2020 X-Rotor research project.

Appendix A

Verification of inflow and BEM modelling for secondary rotors

A.1 Introduction

The appendix reproduces the evidence, first presented by the author in [151], on the validity of the assumptions around inflow velocity and the validity of the analytical BEM equations for secondary rotors attached to a VAWT primary rotor.

A.2 Verification of inflow conditions

The inflow conditions of the secondary rotor have been assumed to be characterised by equation 2.41. Implicit in this characterisation is the assumption that velocity field induced by the primary rotor does not significantly affect the inflow of the secondary rotor. Primary rotor simulations using Lifting line free vortex wake (LLFVW) models allow the velocity field around the primary rotor to be evaluated, facilitating the validation of this assumption through the evaluation of the velocity field at the attachment position of the secondary rotors.

In this case, an exemplar primary rotor from the X-Rotor project, presented in [20] and [30] and reproduced in appendix D, is simulated in the open source LLFVW solver QBlade [32]. The rotor is simulated for 12 full rotations until the rotor thrust is fully

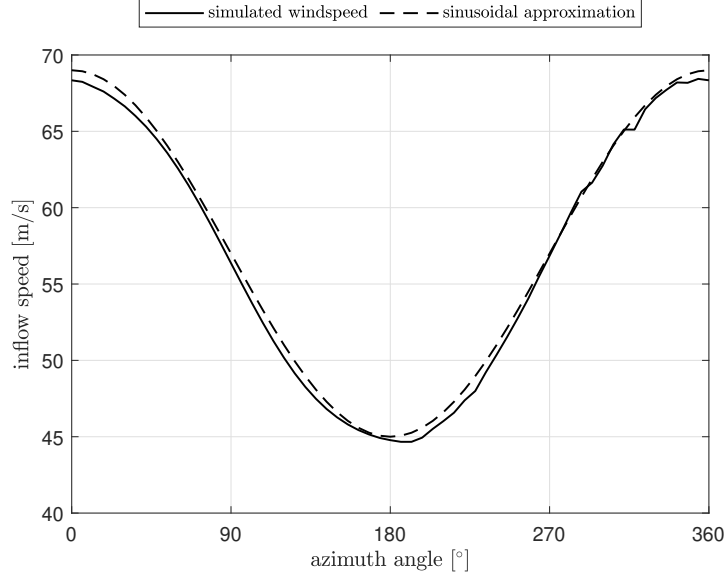


Figure A.1: Verification of the assumption of an undisturbed inflow speed to the secondary rotors through the evaluation of the wind field normal to the secondary rotor plane from LLFVW simulations of the primary rotor.

converged. The flow field in the plane of the secondary rotor is then evaluated at 5 degree intervals over the full primary rotor rotor sweep. The secondary rotor plane is defined such that the secondary rotors blade tip is 4.7m (one rotor radius) below the primary rotor blade tip. The velocity, normal to the secondary rotor plane, is then averaged over the secondary rotor area, and can be compared to the approximation in equation 2.41 as shown in figure A.1. There is good agreement between the evaluated velocity field from the LLFVW simulations.

A.3 Verification of BEM models for secondary rotors

With the simplistic representation of the inflow velocity verified, there are still a number of uncertainties associated with the unique aerodynamic environment in which the secondary tip rotors operate. The orbital motion of the secondary rotors produces a curved wake. This curved wake may have implications for the tip loss formulations for the secondary rotor, and is not consistent with the wake assumptions inherent in BEM theory. Additionally the orbital motion induces a horizontal shear on the secondary

Appendix A. Verification of inflow and BEM modelling for secondary rotors

rotor, and the combination of the rotational velocity and the mean flow also means that the secondary rotor is operating with an oscillating yaw offset. Whilst these are not considered in the optimal BEM representation of the rotor described in chapter 2, these effects are intrinsically modelled by LLFVW codes. Thus, a comparison between the simulated behaviour of the secondary rotor in orbital motion using the LLFVW method and the behaviour of the secondary rotor characterised by the inflow velocity given in equation 2.41 and the power and thrust coefficients calculated by the optimal design approach detailed in section 2 allows for the approach to be validated.

The motion of the simulated secondary rotor is prescribed such that it follows a circular orbit with a radius of 75m and a rotational speed of 0.76rad/s, the ambient wind speed is set to 12 m/s. This is equivalent to the operation of the 5MW exemplar X-Rotor design presented in [20] with a tip speed ratio of 4.75. The yaw angle of the secondary is scheduled such that the rotor normal aligns with the tangent to the circular motion, and the rotational speed of the secondary rotor is scheduled such that it operates at the optimal tip speed ratio given the inflow assumption in equation 2.41.

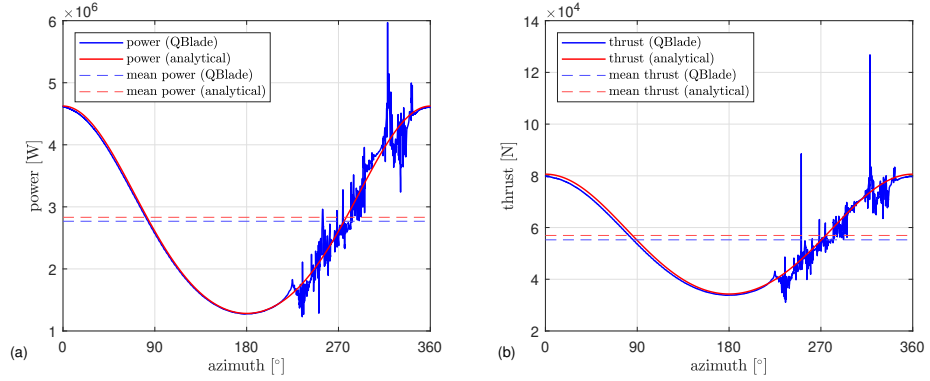


Figure A.2: Comparison of an optimised BEM representation of a secondary rotor and an LLFVW representation.

The results, in terms of rotor power production and axial thrust loading, are shown in figure A.2. In the upwind rotor half, between 0° and 180°, the optimized BEM representation provides an excellent characterisation of both power production and thrust loading on the rotor compared to the LLFVW simulations. In the downwind rotor half, however, the rotor encounters its own wake which leads to significant fluctuations in

the power capture and thrust loading. This self-wake interaction is not modelled by the optimized BEM representation. It should be noted that secondary rotor wake will be subject to strong interactions from the vortex system of the primary rotor wake, thus a fully coupled aerodynamic simulation is required to fully characterise the secondary rotor behaviour in the downwind rotor half. Additionally, the LLFVW model does not include viscous forces which drive the dissipation of the rotor wake. The large fluctuations occur due to the un-physically large induced velocities at the rotor passes through vortex filaments and higher fidelity Navier-Stokes CFD based approaches are required to fully understand the secondary rotor behaviour in the downwind rotor sweep.

Nonetheless, in terms of rotation averaged behaviour, the models show good agreement, with the optimised BEM representation providing a rotation averaged power production estimate 2.2% higher than the LLFVW model, and a rotation average thrust estimate 3.1% higher than the LLFVW model.

Appendix B

DMS model definition

B.1 Introduction

This appendix reproduces the description of the DMS model, originally presented in [152], implemented by the author and used in the code comparison described in chapter 3. The model follows the formulation of Sharpe [119], allowing for streamtube expansion and conservation of stream wise momentum. It should be noted that this model uses a reference frame which is shifted by 90° compared to the descriptions of VAWTs presented in Chapter 3. The geometry associated with this model is presented in figure B.2.

B.2 Definition of DMS model

In order to simulate a 3D turbine through DMS, the rotor must first be discretised vertically into a number of 2D segments parallel to the plane of rotation, as shown in figure B.1*a*. Each 2D segment is then discretised into a number of streamtubes with the flow conditions in each streamtube characterised by the flow in the central streamline, as shown in figure B.1*b*. Each streamtube crosses two actuator surfaces representing the interaction between the upwind and downwind rotor half, as shown in figure B.1*c*. The flow through each streamtube can then be calculated using momentum balance equations under the assumption that the far wake from the upstream actuator surface can be taken as the inflow condition at the downwind actuator surface.

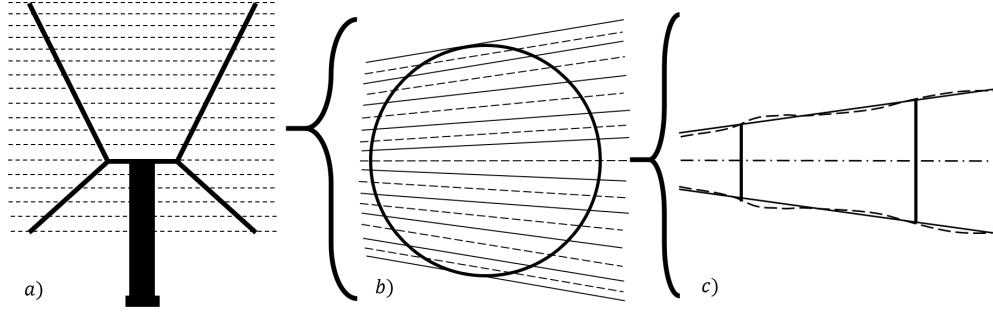


Figure B.1: The discretisation procedure employed during DMS modelling.

The DMS model thus relies on the solution of momentum balance equations to resolve the local flow conditions. These are used to obtain the local blade forces which can, in turn, be integrated to obtain the rotor and blade variables of interest. This picture is complicated by the fact that the streamtube discretisation scheme cannot be completed with respect to any factor pertaining to the physical rotor geometry. This is because the location of the streamtubes are not known a priori due to their dependence on the rotor loading distribution, and the consequent streamtube expansion due to momentum conservation. To circumvent this issue, the rotor can be discretised by the angle subtended between the rotor radius vector and the incident streamline, henceforth referred to as the blade angle θ , rather than the angle of azimuth Θ . It should be noted that in the case of an unloaded rotor, the streamlines remain parallel and the blade angle reduces to the angle of azimuth ($\theta = \Theta$). Both θ and Θ are labelled in figure B.2a for clarity.

The solution procedure therefore involves three steps, the determination of the flow conditions at each blade angle (as calculated through momentum balance equations), the determination of streamtube geometry (as calculated through momentum conservation), and the integration of local blade forces into relevant rotor variables. These are each discussed in the subsections below.

B.2.1 Local flow calculations

At each actuator surface, the flow is reduced to a one dimension problem where the retardation of the flow through an actuator surface is characterised by the induction

Appendix B. DMS model definition

factor, a , with

$$U_{I[u/d]} = (1 - a_{[u/d]})U_{N[u/d]} \quad (\text{B.1})$$

where U_I represents the wind speed at the actuator surface, U_N represents the wind-speed at the actuator surface if the surface was not present and the subscripts $[u/d]$ are introduced to specify if variables refer to the upwind or downwind actuator surfaces. Momentum theory dictates that the thrust coefficient at each actuator surface is given by

$$C_{T[u/d]} = 4a_{[u/d]}(1 - a_{[u/d]}). \quad (\text{B.2})$$

This can be equated to the thrust coefficient as calculated though the non-dimensionalised, rotation averaged forces on the blade element projected onto the streamtube's central streamline, giving

$$C_{T[u/d]} = \left[\sec(\theta) \frac{Nc}{2\pi R} \right] \left[\frac{U_{a[u/d]}}{U_{N[u/d]}} \right]^2 \left[C_n(\alpha_{[u/d]})\cos(\theta) - C_t(\alpha_{[u/d]})\sin(\theta) \frac{1}{\cos(\psi)} \right]. \quad (\text{B.3})$$

The first term represents the rotation averaging, with N representing the number of blades, c representing the local chord length, and R representing the rotor radius. The second term is an artifact of the de-dimensionalisation and represents the ratio between apparent velocity at the blade element, U_a and the incident velocity at the actuator surface in the no-load scenario, U_N . The final term represents the projection of the blade element force onto the streamline, with C_n and C_t representing the normal and tangential force coefficients, α representing the angle of attack, and ψ representing the angle between the blade normal plane and the rotor plane. The lift, drag, normal and tangential axes, as well as the associated blade angles are shown figure B.2b.

At the upwind actuator surface, the no-load velocity is simply given by the free stream velocity ($U_{N[u]} = U_0$) whilst at the downwind actuator surface atmospheric pressure is assumed to have recovered with the no-load velocity given by

$$U_{N[d]} = (1 - 2a_{[u]})U_0. \quad (\text{B.4})$$

The apparent velocity at the blade element is given by the sum of the incident wind-

Appendix B. DMS model definition

speed and rotation induced velocity

$$U_{a[u/d]} = \sqrt{[\omega R + U_{I[u/d]} \cos(\theta)]^2 + [U_{I[u/d]} \sin(\theta) \cos(\psi)]^2}. \quad (\text{B.5})$$

where ω represents the rotational speed of the blade element.

Finally, the angle of attack is given as the sum of the inflow angle, ϕ and the pitch offset, β

$$\alpha_{[u/d]} = \phi_{[u/d]} + \beta = \text{atan} \left(\frac{U_{I[u/d]} \sin(\theta) \cos(\psi)}{\omega R + U_{I[u/d]} \cos(\theta)} \right) + \beta. \quad (\text{B.6})$$

This scheme fully describes the flow in terms of two equations with two unknowns; the induction factors $[a_u, a_d]$. The solution to any streamtube can therefore be found by equating equations B.2 & B.3 at the upwind actuator surface to calculate $a_{[u]}$ then at the downwind actuator surface to calculate $a_{[d]}$.

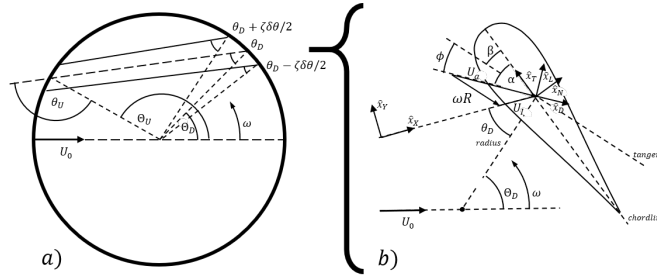


Figure B.2: The streamline and blade element geometries involved in the DMS procedure.

B.2.2 Determination of streamtube geometry

The geometry of a single streamtube is shown in figure B.2a, it has a central streamline which crosses the rotor at blade angles $(\theta_{[u]}, \theta_{[d]})$, and its boundaries are given by the streamlines that pass at $(\theta_{[u]} \pm \zeta_{[u]} \delta \theta / 2, \theta_{[d]} \mp \zeta_{[d]} \delta \theta / 2)$.

The definition of a streamtube implies that the mass flow must be conserved which dictates that

$$U_{[u]} A_{[u]} = U_{[m]} A_{[m]} = U_{[d]} A_{[d]}, \quad (\text{B.7})$$

where A represents the cross sectional area of the streamtube, and the subscript $[m]$ is introduced to represent the position at the at the mid-point between the surfaces.

Appendix B. DMS model definition

Assuming that the streamlines are straight within the region enclosed by the rotor segment, the area at the midpoint between the two actuator surfaces is equal to the mean streamtube area. The area of the upwind and downwind actuator surfaces can therefore be related to the mean streamtube area with

$$A_{[u]} = \left(\frac{2U_{[d]}}{U_{[u]} + U_{[d]}} \right) A_{[m]} = \underbrace{\left[\frac{2(1 - 2a_{[u]})(1 - a_{[d]})}{(1 - 2a_{[u]})(1 - a_{[d]}) + (1 - a_{[u]})} \right]}_{\zeta_{[u]}} A_{[m]}, \quad (\text{B.8})$$

$$A_{[d]} = \left(\frac{2U_{[u]}}{U_{[u]} + U_{[d]}} \right) A_{[m]} = \underbrace{\left[\frac{2(1 - a_{[u]})}{(1 - 2a_{[u]})(1 - a_{[d]}) + (1 - a_{[u]})} \right]}_{\zeta_{[d]}} A_{[m]}, \quad (\text{B.9})$$

where the square bracketed terms represent the upwind and downwind expansion factors, given by ζ_U and ζ_D respectively. To facilitate the solution of the problem, it is assumed that the midpoint streamtube area is given by

$$A_{[m]} = R \cos(\theta) \delta \theta, \quad (\text{B.10})$$

which is independent of streamtube loading. With this, all variables are described and the geometry of any given streamtube can be determined.

As the central streamline is not deflected, the angle of azimuth corresponding to each streamline can be calculated though integrating outward from the central streamline:

$$\Theta_{[u]}(\theta) = \pi + \int_{\pi}^{\theta_{[u]}} \zeta_{[u]}(\tilde{\theta}) d\tilde{\theta} \quad (\text{B.11})$$

$$\Theta_{[d]}(\theta) = \int_0^{\theta_{[d]}} \zeta_{[d]}(\tilde{\theta}) d\tilde{\theta}. \quad (\text{B.12})$$

As the expansion factor is unknown prior before the streamtube is solved, cases using a cyclic pitch regime must employ a forward estimate technique to estimate the angle of azimuth

$$\Theta^{[i+1]} = \Theta^{[i]} + \zeta_{[u/d]}^{[i]} \delta \theta. \quad (\text{B.13})$$

A key impact of the proper inclusion of streamtube expansion, is that the power ex-

traction from the tandem actuator surfaces becomes coupled through the scaling of the actuator surface area.

B.2.3 Resolving and integrating blade Forces

With the geometry determined, the flow field is fully resolved, and all that remains is the integration of blade element forces to obtain rotor/blade variables of interest. The blade torque can be obtained by numerically integrating the elemental torque contribution over the blade span

$$Q_B(\Theta) = \int_0^L \frac{1}{2} \rho U_I^2 c(l) D_Q(\Theta) dl \quad (\text{B.14})$$

where the local torque contribution is given by

$$D_Q = R[C_L \sin(\phi) - C_D \cos(\phi)] \pm (0.25 - \xi) c[\{C_L \cos(\phi) + C_D \sin(\phi)\} \cos(\psi)]. \quad (\text{B.15})$$

Here, the $\pm$ sign refers to the upwind and downwind case respectively and ξ represents the relative attachment position of the aerofoil (measured from the leading edge). Care must be taken in the evaluation of equation B.14 to ensure that the integration is taken at a specific azimuth angle rather than at a blade angle as streamtube expansion is non-uniform over the rotor height. The rotor torque is found by summing the torque contributions from each blade with an appropriate phase offset

$$Q_R(\theta) = \sum_{i=1}^N Q_B(\Theta + i \frac{2\pi}{N}). \quad (\text{B.16})$$

With the rotor torque obtained, the power is given by the product of the rotor speed and the integral of the torque over a single rotation

$$P = \omega \int_0^{2\pi} Q_R(\Theta) d\Theta. \quad (\text{B.17})$$

Blade bending moments and other rotor force coefficients can be obtained through the integration and normalisation of the appropriate force vectors.

B.3 Additional aerodynamic phenomena

For the purposes of the code comparison presented in chapter 3, the corrections to account for dynamic stall, and flow curvature reported in [152], were not included in the model. The tip loss model, and the high-blockage correction are described below.

B.3.1 Tip losses

The aerodynamic force on a blade can be understood to be caused by the bound vorticity on the blade, this bound vorticity leaves the blade at the blade tips in the form of tip-vortices, generating a complex structure behind the blade. This reduces the circulation at the blade tips, lowering the blade loads. This complex behaviour can be approximated using a tip-loss function, f . In the context of wind energy, the simplified tip loss model of Prandtl is typically invoked, using the assumption that the wake can be represented by a number of impermeable disks translated at the free-stream velocity. Numerous modifications of the tip-loss function exist for HAWTs and a thorough review is presented in [153].

With respect to VAWTs, these tip-loss functions are modified to maintain the same form of the tip-loss factor, but altering the disk spacing to better represent the vortex wake behind the rotor. The generalised form of the tip loss function is given by

$$f = \frac{2}{\pi} \text{acos} \left(\exp \left[-\pi \frac{s}{d} \right] \right), \quad (\text{B.18})$$

where s represents the distance from the blade tip, and d represents the distance between wake disks. Paraschivoiu [85] proposes that the distance, d is given by

$$d = U_{w[u/d]} \frac{\pi}{N\omega}, \quad (\text{B.19})$$

indicating that each of the N blades sheds a disk propagating at velocity U_W twice every rotation. Alternatively, Sharpe [119] proposed that the distance between the shed disks is given by

$$d = U_{w[u/d]} \frac{\pi R}{NU_0}. \quad (\text{B.20})$$

Appendix B. DMS model definition

These models are reconciled if the tip-speed ratio is equal to 1, however as the tip-speed ratio increases the tip loss function of Paraschivoiu estimates efficiency gains that are not realised in the case of Sharpe’s correction.

The difference in these approaches is often masked by the normalisation scheme, applied to ensure that the tip-loss function is equal to unity (thus having no effect) at the blade center when considering H-Rotors. Paraschivoiu calculates the tip-loss contribution from the nearest blade tip, and normalises with respect to the tip-loss at the center, whilst Sharpe takes the product of both tip-loss functions, and normalised by the value of this product at the blade center. The fact that both approaches require normalisation at the blade center may imply that they may not be accurate for use in the case of V-Rotors or X-Rotors whereby the normalisation cannot occur as there is no position on the rotor blade where the tip-loss function is known to be equal to 1.

A further difference in approach can be found in how the tip-loss factor is applied. Here there are 3 options. Paraschivoiu [79] uses the tip-loss function directly in the blade element calculations, retarding the flow velocity normal to the blade element by the calculated tip loss factor f , whilst Sharpe [119], indicates that the tip loss function should be applied such that the induction factor a , is replaced by the product af in the momentum theory calculations. In other works, such as that of [122], the approach used in HAWT BEM codes such as aerodyn [2] is applied, where the tip loss factor is applied such that

$$C_T = 4a(1 - a)f. \quad (\text{B.21})$$

The formulation used by Aerodyn is helpful, as it allows for Buhl’s [64] correction for the turbulent wake state to be readily applied. The model presented in this abstract utilises the tip-loss function proposed by Sharpe, with correction applied as in equation B.21.

B.3.2 Turbulent wake state

Experimentally, the parabolic relationship between induction factor and rotor thrust provided in equation B.2 has been found to break down for induction values around ($a > 0.4$) after which an approximately linear relationship emerges [2]. It is understood

Appendix B. DMS model definition

that this occurs because the wake behind the rotor disk enters a turbulent state and the relationship derived using actuator disk theory (based on Bernoulli's equation) breaks down. The classic correction applied in this state is obtained by taking a linear extrapolation of the thrust parabola, however this becomes problematic when a tip loss correction is applied to the thrust coefficient, leading to a discontinuity at the point of transition which can interfere with the iterative solution process. Buhl [64] proposed replacing the linear interpolation with a second order polynomial, which transitions smoothly from the momentum parabola to the maximum value at ($a = 1$). The value of the two endpoints, and the smoothness condition provide the 3 boundary conditions required to calculate the momentum coefficient.

A new development in actuator modelling in general was introduced by Ayati [154] who utilised an extension of Rankine-Froude momentum theory proposed by Sterios and Hultmark which is valid for a much wide range of loading, and doesn't predict wake reversal for highly loaded situations. This approach seems very promising for the application of DMS at higher tip-speed ratios where the rotor loading is generally above the range at which Rankine-Froude momentum theory is applicable.

The model presented in this abstract, and used in the code comparison presented in chapter 3, utilises the correction factor proposed by Buhl.

Appendix C

2DAC model definition

C.1 Introduction

This appendix describes the linearised 2DAC method used to simulate VAWT rotors as part of chapter 3. The original code used for the simulations in chapter 3 was developed by a number of researchers at TU-Delft [103], and external wrapper scripts were developed by the author to facilitate the parametric study presented in chapter 3. The 2DAC method was developed by Madsen [86], and the derivation presented here follows those presented in [73] and [155].

C.2 Background theory

In order to simulate a 3D turbine using the 2DAC method, the rotor must first be discretised vertically into a number of 2D segments, parallel to the plane of rotation. In each of these 2D segments, the incompressible Euler equations can then be solved based on the revolution averaged forces exerted by the rotor in order to obtain the induced velocity distribution.

Consider a flow with uniform velocity U_0 in the x direction, with a small disturbance

Appendix C. 2DAC model definition

in the velocity in the x and y directions given by $U_0 u_x$ and $U_0 u_y$ respectively;

$$\begin{aligned} U_x &= U_0(1 + u_x) \\ U_y &= U_0 u_y. \end{aligned} \tag{C.1}$$

In this case, the Euler equations in two dimensions for inviscid, incompressible flow under the application of a body force f are given by [156]

$$\frac{du_x}{dx} = -\frac{1}{\rho U_0^2} \frac{dp}{dx} + \frac{1}{\rho U_0^2} f_x - u_x \frac{du_x}{dx} - u_y \frac{du_x}{dy} \tag{C.2}$$

$$\frac{du_y}{dy} = -\frac{1}{\rho U_0^2} \frac{dp}{dy} + \frac{1}{\rho U_0^2} f_y - u_x \frac{du_y}{dx} - u_y \frac{du_y}{dy} \tag{C.3}$$

$$\frac{du_x}{dx} + \frac{du_y}{dy} = 0, \tag{C.4}$$

where p represents the pressure, and ρ represents the fluid density. These can be combined to give

$$\frac{d^2 p}{dx^2} + \frac{d^2 p}{dy^2} = \left(\frac{df_x}{dx} + \frac{df_y}{dy} \right) + \left(\frac{dg_x}{dx} + \frac{dg_y}{dy} \right). \tag{C.5}$$

where g_x and g_y represent the second order forces. Ignoring these second order forces, the pressure distribution can be obtained with

$$p(x, y) = \frac{1}{2\pi} \int_S \frac{f_x(x - \eta)}{(x - \eta)^2 + (y - \zeta)^2} + \frac{f_y(y - \zeta)}{(x - \eta)^2 + (y - \zeta)^2} d\eta d\zeta, \tag{C.6}$$

where η and ζ represent the coordinates in space where the forces, f_x and f_y are applied on the x axis and y axis respectively [73]. Once the pressure distribution is obtained, the velocity can be obtained from equations C.2 and C.3:

$$u_x = -\frac{1}{\rho U_0^2} p(f) + \frac{1}{\rho U_0^2} \int_{-\infty}^x f_x d\tilde{x}, \tag{C.7}$$

$$u_y = \frac{1}{\rho U_0^2} \int_{-\infty}^x \frac{\partial}{\partial y} p(\tilde{x}) d\tilde{x} + \frac{1}{\rho U_0^2} \int_{-\infty}^x f_y(\tilde{x}) d\tilde{x}. \tag{C.8}$$

Appendix C. 2DAC model definition

In the case of a 2D circular actuator surface (or infinite cylindrical actuator surface), centred on the zero coordinate with a radius of 1, considering only the radial force, the induced velocities are given by

$$u_x(x, y) = \frac{1}{2\pi} \int_0^{2\pi} \frac{\overline{f_r^*}(\theta)}{\rho U_0^2} \frac{[x + \sin(\theta)] \sin(\theta) - [y - \cos(\theta)] \cos(\theta)}{[x + \sin(\theta)]^2 - [y - \cos(\theta)]^2} d\theta$$

$$\underbrace{-\overline{f_r^*}(\arccos(y)) + \overline{f_r^*}(-\arccos(y))}_{\substack{\text{inside cylinder} \\ \text{in wake}}} \quad (\text{C.9})$$

$$u_y(x, y) = \frac{1}{2\pi} \int_0^{2\pi} \frac{\overline{f_r^*}(\theta)}{\rho U_0^2} \frac{[x + \sin(\theta)] \cos(\theta) + [y - \cos(\theta)] \sin(\theta)}{[x + \sin(\theta)]^2 + [y - \cos(\theta)]^2} d\theta. \quad (\text{C.10})$$

Here, $\overline{f_r^*}$ represents the time averaged radial force applied by the actuator surface, normalised to the geometry of the actuator surface. The normalised, time averaged force can be related to the force experienced by the blade element with

$$\overline{f_r^*}(\theta) = \frac{B}{2\pi R} f_r(\theta)$$

$$\overline{f_t^*}(\theta) = \frac{B}{2\pi R} f_t(\theta) \quad (\text{C.11})$$

where the factor $B/2\pi$ represents the time averaging of the force, and the factor R normalises the force with respect to the actuator surface which has a radius of 1.

The induced velocities calculated using equations C.9 and C.10 neglect the effect of 'second-order' forces, and require a correction factor to give accurate results in moderate to high loading conditions. Madsen [87], proposes that a correction factor k_a can be applied to the induced velocities such that

$$(u_x, u_y) = k_a \cdot (u_x, u_y)_{\text{linear}}, \quad (\text{C.12})$$

with

$$k_a = \frac{1}{1 - a}, \quad (\text{C.13})$$

where a represents the axial induction factor, which can be calculated back calculated

Appendix C. 2DAC model definition

from the thrust coefficient averaged over the rotor segment.

The induced velocities u_x and u_y relate the induced velocity discussed in Chapter 3 with

$$\overrightarrow{U_{ind}} = U_0 u_x \cdot \hat{x} + U_0 u_y \cdot \hat{y}. \quad (\text{C.14})$$

C.3 Solution procedure

Similar to the blade element momentum method, equations C.9 and C.10 allow the calculation of the velocity induced by the force distribution $f_r(\theta)$, and the calculation of $f_r(\theta)$ is dependent on these induced velocities. An iterative scheme, whereby the the induced velocity is first set to zero and is subsequently updated based on the force distribution of the previous iteration, can therefore be implemented to solve the flow on a 2D VAWT segment.

The 2D rotor segment is first discretised in N_{el} control points, with an azimuthal resolution of $\Delta\theta = 2\pi/N_{el}$. The forces, f_r and f_t , at the blade element are then calculated using equation 3.5, using a look up table for the blade polars and the blade geometry. The revolution averaged, normalised forces at each control point are then calculated using equation C.11.

To calculate the induced velocities, a pair of square matrixes which weight the contribution of the radial force at the i^{th} control point to the induced velocity at the j^{th} control point, based on equations C.9 and C.10, can be calculated. These are specific only to rotor geometry and are independent of loading. The influence matrices can therefore be calculated once, outside of the induced velocity calculation loop and re-used for each iteration.

The evaluation of the induced velocity is complicated by the fact that equations C.9 and C.10 cannot be evaluated on the actuator surface due to a singularity. To circumvent this, the evaluation points for the induced velocity are given as points

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slightly offset from the radius with

$$\begin{aligned}x_e &= -\xi r \sin(\theta), \\y_e &= \xi r \cos(\theta),\end{aligned}\tag{C.15}$$

where ξ is a number very close to 1. If ξ is less than one, all of the evaluation points are within the cylinder, if ξ is greater than one, the points corresponding to the upwind rotor half are outside of the cylinder and not in the wake of the cylinder, whilst the points corresponding to the downwind rotor half are outside of the cylinder in the wake of the cylinder.

The influence matrices are therefore given by

$$(R_x)_{ij} = \int_{\theta_i - \Delta\theta/2}^{\theta_i + \Delta\theta/2} \frac{-[(x_e)_j + \sin(\theta)] \sin(\theta) + [(y_e)_j - \cos(\theta)] \cos(\theta)}{[(x_e)_j + \sin(\theta)]^2 + [(y_e)_j - \cos(\theta)]^2} d\theta, \tag{C.16}$$

and

$$R_y(i, j) = \int_{\theta_i - \Delta\theta/2}^{\theta_i + \Delta\theta/2} \frac{-[(x_e)_j + \sin(\theta)] \cos(\theta) - [(y_e)_j - \cos(\theta)] \sin(\theta)}{[(x_e)_j + \sin(\theta)]^2 + [(y_e)_j - \cos(\theta)]^2} d\theta. \tag{C.17}$$

In the case where $\xi = 1 + \delta$ where δ is small and positive, the induced velocity calculation is therefore

$$(w_x)_j = k_a \sum_{i=1}^{N_{el}} (Q_n)_i (R_x)_{i,j} - \underbrace{(\overline{f_r^*})_j + (\overline{f_r^*})_{N-j}}_{\text{in downwind rotor half}}, \tag{C.18}$$

$$(w_y)_j = k_a \sum_{i=1}^{N_{el}} (Q_n)_i (R_y)_{i,j}, \tag{C.19}$$

where k_a represents the linear correction factor as calculated using equation C.13. For the calculation of the linear correction factor, the thrust coefficient is given by

$$C_T = \int_0^{2\pi} \overline{f_r^*}(\theta) \sin(\theta) + \overline{f_t^*}(\theta) \cos(\theta) d\theta, \tag{C.20}$$

and the axial induction is calculated using an empirical fit which includes a high block-

Appendix C. 2DAC model definition

age correction:

$$a = 0.0892C_T^3 + 0.0544C_T^2 + 0.2511C_T - 0.0017. \quad (\text{C.21})$$

Once the induced velocities are calculated, the induced velocity estimate is recalculated using an appropriate relaxation factor, and the process is repeated iteratively until the induced velocities have converged. The force distribution for each blade element is be stored.

This is repeated for each 2D rotor segment, after which the force distribution over each blade has been fully resolved, and the relevant rotor loads are calculated through integration.

Tip loss correction

In this model implementation, a tip loss correction is be applied such that

$$\begin{aligned} w_{x,\text{corrected}} &= \frac{w_{x,\text{original}}}{f}, \\ w_{y,\text{corrected}} &= \frac{w_{y,\text{original}}}{f}, \end{aligned} \quad (\text{C.22})$$

where the tip loss function f is given by Prandtl tip loss correction factor as calculated for a HAWT [2]. This uses an estimate of the rotor averaged induction factor.

Appendix D

Two Bladed 5MW X-Rotor definition

D.1 Primary rotor definition

The rotor geometry for the 2 bladed 5MW X-Rotor primary rotor [20] is shown table D.1.

Crossarm length [m]	25
Upper blade length [m]	100
Lower blade length [m]	65.3
Upper blade inclination angle [°]	30
Lower blade inclination angle [°]	130 (equiv 50)
Tip radius [m]	75
Rotor area [m ²]	12870

Table D.1: Geometry for the 5MW 2 bladed X-Rotor primary rotor.

D.1.1 Blade plan-form

The primary rotor blades have no twist distribution, and utilise symmetric NACA00-series aerofoils. The blade is designed with a linear taper in chord and a linear taper in relative thickness. The span-wise distribution of blade chord and relative aerofoil thickness are shown in table D.2.

Appendix D. Two Bladed 5MW X-Rotor definition

Upper blade			Lower blade		
Span[m]	Chord [m]	t/c [%]	Span [m]	Chord [m]	t/c [%]
0.00	10.00	25	0.00	14.00	25
5.88	9.71	24	3.84	13.59	24
11.76	9.41	23	7.68	13.18	23
17.65	9.12	22	11.52	12.76	22
23.53	8.82	21	15.36	12.35	21
29.41	8.53	20	19.20	11.94	20
35.29	8.24	19	23.04	11.53	19
41.18	7.94	18	26.88	11.12	18
47.06	7.65	17	30.72	10.71	17
52.94	7.35	16	34.55	10.29	16
58.82	7.06	15	38.39	9.88	15
64.71	6.76	14	42.23	9.47	14
70.59	6.47	13	46.07	9.06	13
76.47	6.18	12	49.91	8.65	12
82.35	5.88	11	53.75	8.24	11
88.24	5.59	10	57.59	7.82	10
94.12	5.29	9	61.43	7.41	9
100.00	5.00	8	65.27	7.00	8

Table D.2: X-Rotor primary rotor blade plan-form.

D.2 Secondary rotor definition

The secondary rotors are designed using the methodology described in Chapter 2 and are designed for optimal power take-off with a primary rotor operating at a power coefficient of 0.475 with a tip speed ratio of 4.75 at a rated wind speed of 12m/s. The maximum secondary rotor tip speed is given by 184m/s and the design rotor speed is 39rad/s, this corresponds to a secondary rotor radius of 4.7m and a swept area of $69.4m^2$. The secondary rotors are designed using an FFA-W3-241 aerofoil section, with a design lift coefficient of 1.3 and a peak lift to drag ratio of 103 at an angle of attack of 8° , as calculated at a Reynolds number of 1,000,000 using X-foil [65].

Using the methodology described in Chapter 2, the secondary rotor is designed with a thrust coefficient of 0.4, and an operational tip speed ratio of 3.23. The corresponding chord and twist distributions are shown in figure D.1.

Appendix D. Two Bladed 5MW X-Rotor definition

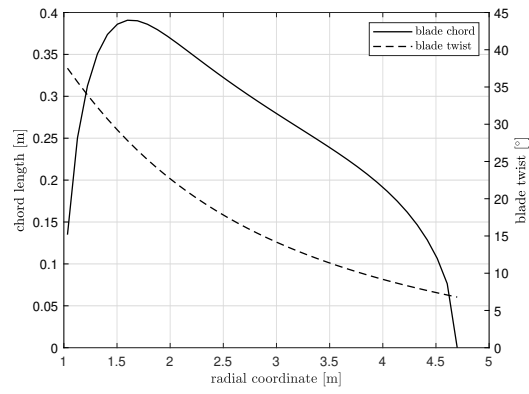


Figure D.1: Secondary rotor chord and twist distribution for the 5MW 2 bladed X-Rotor.

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