Asymmetric Reciprocity, Reference Wage Formation, and the Theory of Wages and Unemployment

Marco Fongoni

A Thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

Department of Economics
University of Strathclyde
Glasgow

June 2017

Declaration

This thesis is the result of the author's original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.

Signed: Date:

Abstract

This thesis contributes to the theory of wages and unemployment through an in-depth theoretical analysis of firms' wage setting and hiring decisions and workers' perceptions of fairness and attitude in the production process.

Chapter 1 develops a microeconomic theory of wage setting behaviour based on contractual incompleteness, fairness, reciprocity and reference dependence and loss aversion in the evaluation of wage contracts by workers. The chapter makes the following contributions: it provides a theoretical explanation for wage rigidity in a dynamic environment; it offers a psychological foundation for asymmetric reciprocity, identifying loss aversion as the driver of negative reciprocity being stronger than positive reciprocity; and it analyses the implications of "asymmetric reference-dependent reciprocity" and anticipated wage rigidity for optimal wage setting and hiring behaviour.

Chapter 2 incorporates the theory developed in Chapter 1 into a canonical search and matching framework and analyses its macroeconomic implications. In so doing the chapter contributes to the literature of labour market fluctuations from a novel behavioural perspective. In contrast to existing theoretical results, in the presence of reference-dependent reciprocity the cyclicality of the hiring wage is shown to be irrelevant for the volatility of vacancies and unemployment. Moreover, the novel behavioural aspects introduced turn out to be qualitatively and quantitatively important in determining the size of the surplus from new employment relationships. Finally, by considering the role of uncertainty, it is shown that the expectation by firms of downward wage rigidity dampens hiring incentives and increases the volatility of both job creation and unemployment.

Chapter 3 explores the concept of the reference "fair" wage in depth. Building on a large body of research that has explored the concepts of fairness, reference dependence, and social norms and identity, this chapter develops a general, and portable, analytical framework to model reference wage formation. Several inherent properties of the reference wage are formalised: the intrinsic tendency of workers to adapt their reference wage over time; the role of readily available information, which can also be "manipulated" by the firm and/or third parties; and asymmetries in fairness evaluations. This framework is applied to study the implications of asymmetric partial adaptation of the reference wage for wage and reciprocity dynamics; and the effect of relative wage comparisons between newly hired and incumbent workers for the cyclical behaviour of vacancies and unemployment.

Acknowledgements

Writing this thesis has been a life journey and I would like to take this opportunity to thank all the people who have supported me in this accomplishment.

First and foremost I am infinitely grateful to my supervisors Julia Darby and Alex Dickson. It is not an overstatement if I say that this journey would have not been possible without their human and professional support. They engaged in their supervision role with enthusiasm and patience and I can only be thankful for that. It must have been challenging at the beginning, to read my lengthy, Italian-English sentences and to try to digest my shaky mathematical models. Most probably it still is. So I am truly grateful for their punctual and constructive feedback; but also for giving me the intellectual freedom to pursue all my ideas and make my own mistakes. I do not think I could have been luckier than this.

In addition, I would like to thank Rod Cross and Rodolphe Desbordes for their feedback and encouragement.

I am also thankful to all my friends, colleagues, sport-mates and flatmates here in Glasgow who have managed to put up with me in those years. In particular I would like to thank Shukhrat Kholmuminov for his unique humour and friendship; Anastasia Charalampidou and Tajana Stankovic for the evening philosophy/gossip chats; Stewart Summerhill for his great calmness when listening to my weekly complains at lunch time; Matthew Robertson for being always up for an after-work beer (and for reading the never-ending first drafts of my thesis chapters); my all-time best flatmates Lynne Myers and Julie Menassier who made the "161 High Street" feel like home; and Elena Gelibter, Tommaso Villella and Tiago Loncan for being there for me during the past year, and for supporting me in the arduous choice of how many sharing plates to order at restaurants. I would also like to thank a special person who has spent almost two of these years with me, listening to my theories and talks to take shape, and filling my days with great happiness. I am really thankful she has been there. Finally, I am very grateful to Evie, especially for the strength, energy and positivity she managed to convey during the final stage of this this journey.

A special thank also goes to my amazing friends Gianluca Tarsetti, Marco Jerome Gasparrini, Riccardo Carino, Ruben David, Filippo Damiani, Benedetta Giungi, Lucia Polidori, Agnese Caporossi, and to my sister Laura. They literally picked me up and put me back on my feet in the hardest and most delicate moments. Their warm support, even though from distance, has been essential to keep the focus and complete this thesis.

Finally, I am grateful to my dad, for his continuous encouragement and love; and to my mum, for her unique spiritual and intellectual support.

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Variables

$\underline{\omega}$	minimum legal wage in the market
θ	labour market tightness
e	worker's effort
p	aggregate productivity
q	idiosyncratic match productivity
r	worker's reference wage
s	shift determinant
u	unemployment rate
v	vacancy rate
w	wage
w^{-i}	wage of other workers
x	entitlement determinant

Parameters

α	speed of adaptation
$\bar{\beta}$	weighting function: integrated mechanism
$\bar{\gamma}$	weighting function: segregated mechanism
$\bar{m}$	efficiency of matching
β	scaling parameter: integrated mechanism

δ	discount factor
η	worker's subjective weight on gain-loss utility
γ	subjective relative prevalence of μ: segregated mechanism
κ	fixed vacancy-posting cost
λ	degree of loss aversion
ν	variance of the shock
ρ	job destruction probability
σ	elasticity of the matching function with respect to unemployment
ε	productivity shocks
ξ	worker's subjective weight on relative wage comparisons in the wage utility

Functions

$\bar{m}(\cdot)$	matching function
$\tilde{e}(\cdot)^+$	optimal effort: positive reciprocity
$\tilde{e}(\cdot)^-$	optimal effort: negative reciprocity
$\tilde{e}_n$	optimal effort: normal effort
$\tilde{v}(\cdot)$	worker's effort utility
$\Omega(\cdot)$	worker's first-order condition
$\overline{w}(\cdot)$	present discounted value of output
$\overline{w}(\cdot)$	present discounted value of the wage
$\Phi(\cdot)$	marginal effect of current wage on future profits
$\pi(\cdot)$	firm's instantaneous per-worker profit
$\Psi(\cdot)$	myopic firm's first-order condition
$\underline{w}(\cdot)$	worker's reservation wage
$\underset{\sim}{w}(\cdot)$	wage threshold below which a worker exerts zero effort

$\underline{q}(\cdot)$ firm's reservation productivity

$\Upsilon(\cdot)$ forward-looking firm's first-order condition

$\underset{\sim}{q}(\cdot)$ match productivity threshold below which a firm pays the wage $\underset{\sim}{w}$

ε_x elasticity of any variable x with respect to p

$\tilde{w}(\cdot)^+$ optimal wage: wage gift

$\tilde{w}(\cdot)^-$ optimal wage: unfair wage

$\tilde{w}(\cdot)^=$ optimal wage: fair wage

$\tilde{u}(\cdot)$ worker's wage utility

$b(\cdot)$ worker's benefit of effort

$c(\cdot)$ worker's cost of effort

$f(\cdot)$ job finding probability

$h(\cdot)$ probability that a vacant job is matched with a worker

$J(\cdot)$ firm's value of the employment relationship

$M(\cdot)$ worker's morale function

$m(\cdot)$ worker's utility from absolute wage level

$n(\cdot) \equiv \mu(\cdot)$ gain-loss value function

$q_{u,l}(\cdot)$ upper,lower productivity thresholds, optimal wage setting rule

$r_{U,L}(\cdot)$ upper,lower reference wage thresholds, optimal wage setting policy

$s(\cdot)$ firm's per-worker labour cost

$u(\cdot)$ worker's instantaneous utility

V firm's value of a vacancy

$y(\cdot)$ firm's per-worker output

Others

$\bar{r}$ upper bound on the state and control space

$\mathcal{E}$ steady-state equilibrium tuple

$\mathcal{H}^*(\lambda)$ high steady-state equilibrium tuple

$\mathcal{L}^*$ low steady-state equilibrium tuple

$\mathcal{R}$ state space, firm's problem

$\mathcal{W}$ control space, firm's problem

$\mathcal{C}$ set of worker's employment status, social categories

$\mathcal{M}^*$ moderate steady-state equilibrium tuple

$\mathcal{I}$ Identity: set of relevant reference wage determinants

$\mathcal{P}$ set of fair wage norms

$\mathcal{S}$ set of shift determinants

$\mathbb{A}$ indicator variable: asymmetric adaptation

$\mathbb{B}$ indicator variable: asymmetric social comparison (integrated)

$\mathbb{C}$ indicator variable: asymmetric social comparison (segregated)

$\mathbb{I}$ indicator variable: existence of productivity thresholds

$\mathcal{X}$ set of entitlement determinants

Ω universal set of reference wage determinants

$\overline{\varphi}$ distribution of newly hired worker's reference wage

τ starting period of employment relationships

E worker's social category: incumbent

N worker's social category: newly hired

S worker's social category: social comparison

U worker's social category: unemployed

Introduction

A fundamental question in economics relates to the existence, and cyclical behaviour, of unemployment. Research aimed at understanding these aspects has led to the development of several complementary explanations: search frictions, implicit contracts, efficiency wages, insider-outsider relations, and matching frictions among others. These theories have provided a range of possibilities for why the labour market does not clear, and therefore why unemployment exists. However, they have been less successful in explaining cyclical behaviour: the unemployment rate rises faster than it falls, is highly counter-cyclical and also very persistent.

Throughout the history and development of macroeconomic theory, the most popular, and fervently debated, explanation to account for the cyclical behaviour of unemployment has been based on nominal and real wage rigidities, in particular, downward wage rigidity. This focus is justified by the logical appeal of the underlying theoretical argument, as well as by the empirical evidence that has accumulated. In a situation in which firms are facing a recession and need to reduce labour costs, if they cannot adjust through "prices", the adjustment will be implemented through "quantities". The empirical evidence appears to support this hypothesis: as opposed to unemployment fluctuations, wages rise more rapidly than they fall; and virtually every distribution of wage changes has a spike at zero, with wage cuts less frequent than wage increases. Explanations based on wage rigidity are simple, intuitive and substantially improve the explanatory power of the labour market theories mentioned above. Any modern macroeconomic model of the business cycle—be it based on efficiency wages, search and matching frictions, other market imperfections, or a combination of these—requires a degree of wage rigidity to be built-in to sufficiently explain unemployment fluctuations.

However, this considerable reliance on the role of downward wage rigidity poses an additional set of fundamental questions. Why do firms not adjust wages downwards during recessions? What is the source of this constraint? Is it the product of institutional impediments, or does it have roots in the very nature of employment relationships? Moreover, does the anticipation of downward wage rigidity affect firms' wage setting and hiring decisions? If so, what are the implications of these considerations for the cyclical behaviour of unemployment? The answers to these questions have important consequences for the theory of wages and unemployment, not only from a purely positive scientific stance, but also from a normative perspective. It is only through a correct diagnosis of the causes and consequences of wage rigidity that appropriate policy interventions to cure the unemployment problem can be designed. In this respect, wage setting behaviour is at the core of macroeconomics.

This thesis aims to address such questions through an in-depth theoretical analysis of the behaviour of workers and firms engaged in employment relationships. By drawing insights from behavioural economics, psychology and sociology, this approach will place a considerable emphasis on the behavioural aspects that influence firms' wage setting behaviour and workers' attitudes in the production process, abstracting from potential legislative and institutional constraints. The purpose of this is to understand whether downward wage rigidity is an inherent feature of employment relationships, in order to subsequently identify its sources and to analyse the consequences for labour markets fluctuations.

The crucial importance of this approach lies in the identification of the behavioural mechanisms underlying the functioning of employment contracts and wage setting decisions. A labour market characterised by downward wage rigidity is essentially a dynamic economic system exhibiting asymmetric fluctuations and a certain degree of irreversibility. These features could be the result of institutions such as legally-binding firing costs and labour unions. However, if asymmetry and irreversibility are the outcome of workers and firms optimising behaviour, even absent institutional constraints, existing economic policy may have to be revised to take account of this, and new "behavioural" interventions may have to be designed. Nevertheless, before one even engages with the development of such policies, the questions set out at the beginning of this section will have to be addressed; and a rigourous theoretical framework capturing the behavioural nature, and the macroeconomic consequences, of the wage setting process must be developed. The methodological advantage of this approach is that macroeconomic models equipped with more realistic microfoundations will not only better fit stylised facts (without backward engineering), but will also provide well-grounded and novel predictions.

The thesis begins this inquiry in Chapter 1 by formalising a theory of reciprocity and wage setting behaviour within a worker-firm employment relationship. The worker evaluates wage contracts with respect to a reference "fair" wage and is assumed to be loss averse, that is, wage cuts below the reference wage that are perceived as unfair have a disproportionate impact on utility compared to equivalent size wage gifts. This assumption, inspired by the work of Kahneman and Tversky (1979), is a first-order departure from the literature and lies at the heart of many of the results derived throughout the thesis. The theory characterises the worker's optimal effort response to wage changes as being reference dependent, where positive and negative reciprocity are defined as relative deviations from an intrinsically motivated level of effort, and loss aversion is identified as the psychological foundation for the stronger intensity of negative reciprocity. This wage-effort relationship, labelled "asymmetric reference-dependent reciprocity", reconciles under a unique theory several stylised facts documented by the empirical literature on reciprocity in employment relationships (e.g. Campbell and Kamlani (1997), Mas (2006), Bewley (2007) and Fehr, Goette, and Zehnder (2009)).

The implications of asymmetric reference-dependent reciprocity for wage setting behaviour are further explored in a dynamic setting in which the worker adapts their reference wage to the most recent wage contract. This analysis establishes that the adaptation of the reference wage and the relatively large cost of negative reciprocity that stems from loss aversion are the key fundamental drivers of downward wage rigidity; and that the reference-dependent nature of reciprocity, combined with the adaptation of the reference wage, leads to a "re-normalisation" of effort during the course of the employment relationship: reciprocity is essentially a temporary phenomenon. This latter prediction is consistent with the recent experimental study of Sliwka and Werner (2017), and has important implications for the optimal employment contract offered by a forward-looking firm anticipating these dynamics. A recurring theme in the models developed throughout the thesis is in fact the characterisation of an inter-temporal trade-off faced by a firm when choosing the wage: due to the worker's adaptation of the reference wage, the optimal wage contract will be set to balance the marginal benefits of inducing more effort in the current employment period *versus* the expected marginal cost of employing a worker with a relatively higher reference wage in the future. The analysis of this inter-temporal trade-off sheds new light on the implications of the asymmetry and dynamics of reciprocity for a firm's wage compression incentive (Elsby, 2009); and for the expected value of the employment relationship which determines hiring decisions.

The fundamental contribution made in Chapter 1 is the formal demonstration that after accounting for intrinsic cognitive aspects that characterise the evaluation of wage contracts by workers, asymmetry and irreversibility are inherent features of reciprocity and wage setting behaviour in employment relationships. Hence it follows: what are the implications of these behavioural mechanisms for unemployment fluctuations?

This question is addressed in Chapter 2, in which the macroeconomic implications of the theory developed are analysed through the lens of a canonical search and matching framework. This choice is motivated by the difficulties in reconciling wage and unemployment fluctuations that have been identified in the recent theoretical and empirical literature. Advances in the theory of labour market fluctuations have placed particular emphasis on the role of new hires' wage rigidity to explain the observed volatility of job creation (Shimer, 2005), shifting away from the view that downward wage rigidity in existing employment relationships is an important driver of large and persistent unemployment fluctuations (e.g. Pissarides (2009); Elsby, Shin, and Solon (2016)). However, recent empirical evidence has reported that wage offers to newly hired workers are substantially pro-cyclical (Martins, Solon, and Thomas, 2012; Carneiro, Guimarães, and Portugal, 2012; Haefke, Sonntag, and Rens, 2013; Stüber, 2017); and that the existing framework cannot simultaneously replicate the volatilities of both the hiring wage and the vacancy-unemployment ratio (Kudlyak, 2014). Thus it is not yet clear whether the emphasis on new hires' wage cyclicality is well placed, or what impact the wage rigidity of incumbent workers has on the cyclical behaviour of unemployment and vacancies.

Chapter 2 contributes to understanding these issues from the novel behavioural perspective that has been advanced in the theory developed in Chapter 1. The consideration by firms of the workers' asymmetric reference-dependent reciprocity affects the expected value of new employment relationships at the time of hiring. Based on this premise, this chapter establishes the existence of a range of path-dependent equilibrium outcomes, the realisation of which crucially depends on the level of new hires' wage entitlements in the labour market. When applied to the analysis of the amplitude and cyclical co-movement of vacancies and unemployment, this framework delivers the perhaps surprising result that in the presence of reference-dependent reciprocity the cyclicality of the hiring wage is irrelevant for the volatility of job creation. Essentially, the cyclical changes in positive and negative reciprocity induced by firms' optimal wage setting behaviour contribute to amplify the impact of exogenous shocks, offsetting the effect of changes in the hiring wage. The quantitative importance of this prediction is further assessed through a simple calibration exercise, which shows that the model can simultaneously accommodate plausible empirical estimates of the volatilities of both the hiring wage and the vacancy-unemployment ratio. Nevertheless a question remains: what is the role of expected downward rigidity in long-term employment relationships for firms' hiring decisions? By introducing uncertainty around the evolution of a job-match productivity, it is shown that even if downward wage rigidity does not generate endogenous, ex-post inefficient, layoffs (as suggested by Eliaz and Spiegler (2014)), the expected relatively large cost of implementing wage cuts—that is, the anticipation of stronger negative reciprocity by incumbent workers—negatively influences firms' expected surplus from new employment relationships, dampening hiring incentives and increasing the volatility of both job creation and unemployment.

The micro- and macro-economic frameworks developed and discussed thus far are based on the assumption that new hires' reference wages are exogenous and that, once employed, incumbent workers adapt their reference wage to their most recent wage contract. Adaptation of entitlements, as it is defined, is the most corroborated hypothesis in the empirical literature and it is grounded in theories of habit formation and other studies in the psychology of decision making (Kahneman and Thaler, 1991; Baucells, Weber, and Welfens, 2011). Nevertheless it is plausible to expect that a newly hired worker's reference wage is in part endogenously determined by the labour market environment and that an incumbent worker's reference wage might be influenced by more than just the previous wage contract. These aspects are crucial for a comprehensive account of reference-dependent reciprocity and wage setting behaviour. Moreover, they could potentially shed light on the nature of the asymmetry and irreversibility features of employment relationships, and whether and how they can be exogenously "corrected". As such, gaining an understanding of the process of workers' reference wage formation may be insightful, especially in the context of the theories developed throughout this thesis.

Chapter 3 explores the concept of the reference "fair" wage in depth. The reference wage is an artefact of economic models that captures a broader set of feelings, entitlements, information and norms about what is perceived to be a fair remuneration. Despite its prominent and crucial role in the analysis of labour markets, the existing literature does not yet offer a systematic approach that provides a comprehensive understanding of the workers' reference wage formation process and determinants. This chapter aims to fill this gap by providing a general, and portable, analytical framework to model reference wage formation. This approach has been inspired by a large body of research that has explored the concepts of fairness in labour relations (Fehr et al., 2009); reference dependence in behavioural decision theory (Kahneman, 1992); and social norms and identity from sociology (Akerlof and Kranton, 2000). Workers' perceptions of fairness depend on a complex interaction of experience, history, social norms and the institutional context. Moreover, shaped by these interactions, the reference wage formation process and the information set affecting its determinants evolve over time and exhibit asymmetries: workers adapt more rapidly to experienced gains (wage gifts) than to losses (unfair wages), and disadvantageous pay inequality has a substantially larger effect on fairness judgements and morale. This state-contingent, and context-dependent, nature of fairness perceptions does not easily lend itself to the development of a self-contained theory of reference wage formation. To overcome this issue, the chapter proposes a general behavioural principle based on the social identity approach in economics. This approach enhances the portability of the framework developed to a variety of economic settings, and leaves the theorist/analyst with the task to design a stylised labour market—consisting of workers' social categories and wage norms—that should appropriately characterise the purpose of the inquiry.

The chapter concludes by analysing the implications of two salient features of reference wage formation: the partial and asymmetric adaptation of wage entitlements to past contracts; and the spill-over effect that stems from relative wage comparisons between new hires and incumbent workers. By incorporating these aspects into the models developed in Chapters 1 and 2 these applications deliver insightful and transparent results in the analysis of long-term employment relationships and labour markets. Asymmetric partial adaptation generates asymmetric endogenous persistence in wage and reciprocity dynamics—also providing a coherent explanation for the observed asymmetry in the intensity(loss aversion) and persistence(partial adaptation) of negative *versus* positive reciprocity; while relative wage comparisons generate hysteresis in the cyclical behaviour of wages, vacancies and unemployment, that is, temporary cyclical shocks have permanent equilibrium effects.

The theories developed in these chapters are therefore centered around the existence of asymmetries and irreversibility in the cyclical behaviour of wages and unemployment. This thesis seeks to uncover the behavioural mechanisms underlying these inherent aspects of employment relationships, and to formally analyse their implications for the macroeconomics of labour markets.

Chapter 1

A Theory of Reciprocity and Wage Setting Behaviour

1.1 Introduction

Virtually every macroeconomic model of the business cycle—be it based on efficiency wages, search frictions, New Keynesian imperfections, or a combination of these—requires wage rigidity to be built-in to sufficiently explain unemployment fluctuations. Whilst compelling reasons might be advanced to *assume* wage rigidity, a convincing microeconomic account of wage setting behaviour that captures the empirical regularities of employment relationships is much more desirable.

The aim of this chapter is to provide a comprehensive and tractable microeconomic model of wage setting behaviour that stands as a theoretical foundation for wage rigidity, inspired by ideas advanced in the behavioural economics literature and a synthesis of recent convergent insights from anthropological and experimental research. In doing so, the chapter makes two further key contributions: i) it offers a psychological foundation for asymmetric reciprocity, identifying loss aversion as the driver of negative reciprocity being stronger than positive reciprocity; and ii) it analyses the implications of "*asymmetric* reference-dependent reciprocity" and *anticipated* wage rigidity for a firm's wage setting behaviour and hiring decision in a two-period decision making environment.

The basic premise of the theory is that there is contractual incompleteness over effort in an employment relationship, which is at least in part discretionary (Williamson, 1985). A worker evaluates the fairness of the wage they are paid relative to a reference "fair" wage. A wage that exceeds the reference wage is perceived as a gift, whilst if it falls below the reference wage it is perceived as unfair. Central to the adopted modelling approach is the inclusion of a "morale function" in the worker's payoff, which measures their evaluation of the productive effort they undertake in light of the fairness of the wage they are paid: if the worker is paid the *fair* wage they will exert their intrinsically-motivated normal effort (independent of the absolute wage level); if the worker perceives their wage as a *gift* they will receive an increase in utility from increasing their effort (a gift to the firm); whilst if they perceive their wage to be *unfair* the worker's utility will increase by reducing their effort (reducing the firm's payoff). As such, a worker's payoff exhibits both positive and negative reciprocity—defined as relative deviations from normal effort—that stems from their reference-dependent preferences. Moreover, since the worker is loss averse, which implies that unfair wages generate a disproportionate decrease in payoff giving rise to an asymmetry in reciprocity, negative reciprocity is stronger than positive reciprocity.

The inclusion of reciprocity establishes a wage-effort relationship in which the optimal effort of the worker is increasing in the wage paid, and the asymmetry between negative and positive reciprocity implies there is a kink at the reference wage if the worker is loss averse. This wage-effort relationship is defined as the worker's "asymmetric reference-dependent reciprocity". If a firm is considering paying a loss averse worker below their reference wage there will be a relatively large negative impact on their effort and consequent reduction in the output produced, consideration of which gives rise to wage rigidity.

The implications of the theory are further explored in a two-period employment relationship in which the evolution of the match productivity is uncertain, and the worker adapts their feelings of entitlement once they become employed: whilst the worker's reference wage at the start of the employment relationship is exogenously given, in the subsequent period it is determined endogenously by the wage paid in the initial employment contract. The analysis illustrates several inherent features of the theory developed in this chapter for wage and effort dynamics. The worker's adaptation of the reference wage and the relatively large cost of negative reciprocity that stems from loss aversion are identified as the key fundamental drivers of wage rigidity in a dynamic environment. Moreover, the reference-dependent nature of the worker's reciprocity behaviour, combined with the adaptation of the reference wage, leads to a "re-normalisation" of effort during the course of the employment relationship: the same amount of reciprocity cannot be sustained by paying the same wage in two consecutive employment periods.[1] A forward-looking firm will anticipate the effects of

[1]This prediction, which is a distinctive feature of the dynamic model developed in this chapter, is consistent with the recent experimental study of Sliwka and Werner (2017). They conduct a laboratory experiment in which individuals work on a real-effort task and are paid different wage profiles which vary in the frequency and size of wage increases. They find that the positive effect on effort of a wage increase only lasts one period; and that in the following periods, absent subsequent increases in the wage, working performance converges back towards the level associated with a constant wage.

these dynamics on the value of the employment relationship and will adjust the initial employment contract accordingly.

An optimal employment contract is characterised by the initial wage paid to the worker and by the firm's reservation productivity determining hiring. Building on the insight of Elsby (2009) this framework is used to analyse a forward-looking firm's incentive to compress the initial wage in anticipation of wage rigidity. In contrast with Elsby's prediction, the analysis suggests that even in the absence of expected wage rigidity—that is, even if a worker is not loss averse and reciprocity is symmetric—the firm still faces an incentive to compress wages in order to offset the dynamic re-normalisation of effort that stems from the worker's adaptation of the reference wage.[2] Moreover, it is shown that the overall effect of wage rigidity on wage compression is ambiguous: a firm contracting with a more loss averse worker faces an inter-temporal tradeoff between stronger negative reciprocity at the start of the employment relationship if the optimal wage is perceived as unfair, and a greater cost of cutting the wage in the future if the match productivity decreases. As such, if the former incentive dominates, a firm may optimally *increase* the initial wage to attenuate the current effect of negative reciprocity. Finally, the analysis investigates how asymmetric reference-dependent reciprocity influences the firm's expected value of the employment relationship, from which a reservation productivity determining hiring is derived. Independently of whether wage rigidity reinforces or tempers the incentive to compress initial wages, the anticipation of stronger negative reciprocity and expected wage rigidity unambiguously reduce a firm's incentive to hire.

Besides being based on a large and growing body of research documenting worker and firm behaviour in employment relationships, the model developed in this chapter can be thought of as a comprehensive descriptive theory of wage setting behaviour, consistent with a number of ideas advanced in the theoretical literature that seem not to have been considered in a common analytical framework. The asymmetric effort function derived is closely related with the gift-exchange model of Akerlof (1982), which captured what is defined here as positive reciprocity; and the fair wage-effort hypothesis of Akerlof and Yellen (1990), which considered what is defined here as negative reciprocity, the relative strength of which, in the theory exposed here, depends

[2]Elsby (2009) investigates wage compression in an infinite-horizon dynamic model under uncertainty, featuring an ongoing employment relationship where downward wage rigidity binds. Elsby's model attributes the incentive to actively compress wages entirely to the firm's anticipation of downward wage rigidity in the expected continuation value of the employment relationship. As it is shown in Section 1.4, the theory developed here provides a framework that elucidates the actual behavioural forces behind active wage compression. In particular, the reduced-form effort function assumed by Elsby (2009), while insightful, does not capture the dynamic re-normalisation of effort, which is identified here as the main driver of the wage compression incentive.

on the degree loss aversion. Perhaps similar in spirit is the model of Danthine and Kurmann (2007), who assume that workers' preferences exhibit reciprocity à la Rabin (1993) from whence gift exchange à la Akerlof (1982) can be derived.[3] However they do not capture negative reciprocity in the form of sub-normal effort, and their model generates wage rigidity only under certain assumptions about the nature of shocks, workers' reference wages and the functional form of the workers' gift. In a more recent contribution Eliaz and Spiegler (2014) incorporate reference dependence, contractual incompleteness and negative reciprocity, subsumed under a reduced-form reference-dependent production function, in a search and matching framework à la Mortensen and Pissarides (1994). However, Eliaz and Spiegler (2014) capture an extreme form of negative reciprocity: a wage cut below the reference wage, no matter how large, will induce a worker to exert zero discretionary effort, leaving the adverse effect on output to be randomly determined by a parameter that represents the incompleteness of the labour contract (which is independent of the size of the wage cut).[4] In contrast, the theory developed in this chapter identifies the severity of the adverse effect of a wage cut on output with a worker's degree of loss aversion, since this determines the strength of negative reciprocity.[5]

This framework lends support to some aspects of these models, but has the advantage of being clear about the nature of the behavioural forces at play, their driving factors and their implications. As it is shown in the analysis of the optimal employment contract and its potential implications, this type of approach is particularly important for a better understanding of the dynamics of workers' reciprocity; and to identify the sources of wage rigidity, wage compression and hiring incentives. In models in which

[3] In Danthine and Kurmann (2007) the firms' gift is always positive in equilibrium, implying that workers always exert supra-normal effort levels. Their analysis focuses on the static macroeconomic implications of gift exchange when workers' reference wages are also influenced by firms' ability to pay ("internal reference"), the relative importance of which determines the extent of wage flexibility subject to demand and technology shocks. Danthine and Kurmann (2010) estimate the effects of this gift-exchange mechanism in a DSGE model and conclude that past wage levels and firms' ability to pay are among the most important determinants of wage setting. These empirical results are consistent with the predictions of the model developed in this chapter.

[4] As they show in an appendix, Eliaz and Spiegler (2014) can derive their reduced-form production function from an optimisation problem for a worker that yields an expression for discretionary effort: if a worker is paid a wage at least equal to their reference wage, the worker is *assumed* to exert effort normalised to unity; if they are paid an unfair wage below their reference wage, effort is zero. Since the relative importance of discretionary effort in a firm's output is given by the extent to which the labour contract is incomplete, this also determines the random fraction of output that is destroyed when a worker is paid below their reference wage. As such, Eliaz and Spiegler's (2014) model suggests that the more a contract is incomplete, the greater is the adverse effect of wage cuts on output.

[5] The key assumptions and predictions of the models of Elsby (2009), Danthine and Kurmann (2007, 2010) and Eliaz and Spiegler (2014) are presented in Section A.1.1 of Appendix A. The objective of this analysis is to elucidate the similarities as well as the most distinctive differences (in terms of theory and modelling approach) between these models and the theory developed in this chapter.

reciprocity is not explicitly derived and wage rigidity comes from *ad hoc* mechanisms such predictions would either not be possible, or their interpretation could be more difficult and potentially misleading. Moreover, the model developed in this chapter stands as benchmark organising framework for a rigorous microeconomic analysis of the behaviour of workers and firms, which can potentially deliver novel implications for the macroeconomic behaviour of wages and unemployment. Section 1.5 provides a discussion of these implications, as well as some extensions of the model that could account for other aspects of workers' behaviour and reference wage formation, some of which are explored further in Chapters 2 and 3 of this thesis.

The next section exposes a synthesis of a theory of wage setting behaviour in employment relationships. The model based on contractual incompleteness, fairness, reciprocity, and reference dependence and loss aversion is set out in Section 1.3, where the implications for the worker's optimal effort decision and the firm's wage-setting rule are also derived. Section 1.4 explores the dynamic implications of the model and the properties of the optimal employment contract. Section 1.5 discusses further implications in relation to the worker's reference wage and the literature on labour market fluctuations, and Section 1.6 offers some concluding remarks.

1.2 Morale, Fairness and Reciprocity in Employment Relationships

There is an emerging consensus in the literature that behavioural concerns such as fairness, workers' morale and reciprocity influence firms' wage setting behaviour. These intrinsic aspects of the employment relationship are also considered to be key behavioural forces that underly the observation of downward wage rigidity. In this section it is argued that these ideas have been considered in the literature at least since the turn of the twentieth century, and that it is thanks to recent convergent findings in anthropological and experimental research combined with theories from behavioural economics that a unified consensus has emerged. Hence what follows proposes a synthesis of the literature, which will provide the underlying conceptual framework for the theory and the analysis developed in the remainder of the chapter.

1.2.1 Early Insights

That workers' morale and productivity is linked with their perceptions of fairness, and that employers are concerned about these issues when deciding upon wage policies, has long been acknowledged by economists. Marshall (1890) often expressed the reasons why employers would pay workers high wages and discussed the negative impacts on 'efficiency' and work 'intensity' of otherwise lower wages. Slichter (1920) placed workers' feelings of being treated unfairly as one of the most important causes of low morale and the resulting non-cooperative behaviour of workers towards the employer. Hicks (1963), Solow (1979) and Okun (1981) advanced similar arguments when discussing the possible sources of the Keynesian wage floor (Keynes, 1936): they argued that resistance to cut nominal wages comes from employers, concerned about the effects of wage cuts on workers' morale, 'ability' and 'willingness to work' (Hicks, 1963, p. 94-95). The gift-exchange model of Akerlof (1982) and the fair wage-effort hypothesis of Akerlof and Yellen (1990) provide the first contributions that formalise some of these insights, appealing to what has become known in the behavioural economics literature as positive and negative reciprocity.

1.2.2 Anthropological Evidence

Within the last three decades, thanks to the ground-breaking work of several economists including Blinder and Choi (1990), Campbell (1997), Bewley (1999) and more recently Galuscak, Keeney, Nicolitsas, Smets, Strzelecki, and Vodopivec (2012), Druant, Fabiani, Kezdi, Lamo, Martins, and Sabbatini (2012) and Du Caju, Kosma, Lawless, Messina, and Rõõm (2015), the understanding of the employment relationship has been greatly enhanced. By interviewing firms' managers and labour leaders in several countries these studies provide insight into the validity of the behavioural assumptions advanced in the theoretical literature.[6]

A central finding is that firms' managers are concerned about treating workers fairly. Wage reductions that are perceived as unfair damage morale, inducing grievance among workers who negatively reciprocate the employer with lower effort and productivity (Bewley, 2007). On the other hand wage increases could generate improvements in effort and cooperation among workers. As such, these findings suggest the existence of a relationship between wage *changes* and workers' effort.

Such a relationship is not universally straightforward, however: for instance Campbell (1997) find that effort responds more intensely to wage cuts than to increases in wages, and that any positive effect of wage increases on effort is believed by managers to be temporary, since workers rapidly get used to the wage received. On the other hand, wage reductions without impacts on morale are also achievable by employers, though only when workers understand their necessity in avoiding the firm shutting down or to

[6]Other anthropological studies include those by Kaufman (1984), Baker, Gibbs, and Holmstrom (1994), Agell and Lundborg (1995, 2003) and Agell and Bennmarker (2007). Reviews of this literature can be found in Howitt (2002) and Bewley (2007).

prevent mass layoffs (Bewley, 2007).

1.2.3 Experimental Evidence

There is an additional stream of evidence that comes from laboratory and field experiments. Overall the most important finding is confirmation of the existence of reciprocal behaviour in the employment relationship: when people receive extra pay in excess of their standards of fairness they reciprocate with higher effort (positive reciprocity); when people perceive they have been treated unfairly they reciprocate by exerting minimum or lower effort (negative reciprocity). However, field experiments document evidence that positive reciprocity is weaker than negative reciprocity (see, for instance, Kube, Maréchal, and Puppe (2013) and Malmendier, Velde, and Weber (2014)). In a combined laboratory and field experiment Cohn, Fehr, and Goette (2014) try to address this inconsistency. They infer that positive reciprocity exists but may quickly disappear, which is consistent with the previously discussed anthropological findings.

One interpretation attributes this result to the asymmetric nature of workers' reciprocity behaviour: negative reciprocity is stronger than positive reciprocity (Fehr et al., 2009). Another interpretation suggests that the weak, or temporary, response of effort to wage rises is the outcome of a shift of the workers' standards of fairness to the higher wage received (Gneezy and List, 2006). The latter conjectuer has been recently corroborated by the laboratory experiment of Sliwka and Werner (2017).

Taken together, evidence from laboratory and field experiments offer complementary insights into understanding the impacts of wage changes on effort, by reinforcing the existence of an asymmetric wage-effort relationship.

1.2.4 The Proposed Synthesis

This section proposes a synthesis of a theory of wage setting behaviour that captures the essential features of wage setting and the employment relationship that emerge from several ideas and perspectives. The theory is built around four core concepts: workers' morale; their perceptions of fairness; reciprocity; and contractual incompleteness.

Workers' Morale. Morale represents the workers' state of mind when performing a productive activity. As concluded by Bewley (2007), good morale is not related to happiness or job satisfaction, but with the willingness of workers to cooperate and work to achieve the firm's goals; and when morale is low workers tend to hold back cooperation and cease to identify themselves with the firm. This idea is captured by assuming that workers' willingness to exert effort is directly related to their morale: when morale is good, cooperation is enhanced and performing the productive activity generates a psychological benefit; when morale is low, workers are less motivated and the psychological cost of exerting effort increases.

Perceptions of Fairness. Changes in workers' morale depend on whether workers feel they are treated fairly by their employer. Following the standard approach in the literature these perceptions are captured within a reference "fair" wage relative to which the fairness of a wage contract is evaluated: a wage below the reference wage is perceived as *unfair*, while a wage above is perceived as a *gift*.[7] Moreover, by incorporating the intrinsic psychological aspect of human decision making of loss aversion (Kahneman and Tversky, 1979), enables to capture the idea that morale is most affected when workers feel they are being treated unfairly. This implies that a wage cut below the reference wage (perceived as a loss) has a greater impact on morale than an increase in wage of the same amount (perceived as a gain).[8]

Reciprocity. The idea that is perhaps most prominent from the literature discussed is that the employment relationship is based on a mutual understanding of reciprocal behaviour. If a firm sets a wage contract that is considered unfair, workers start to feel a grievance against the firm and morale will decrease. As a consequence effort will become more psychologically costly and workers will negatively reciprocate the treatment they perceive as unfair by exerting less effort. A similar response, but in the opposite direction, would arise if the firm sets a wage that is above the workers' reference wage. Moreover, due to the assumption that unfair behaviour has a stronger impact on workers' morale (loss aversion), effort will be more responsive to wage changes that are considered unfair as opposed to wage changes considered as gifts. Thus, workers are characterised by intentions-based reciprocity (e.g. Rabin, 1993; Dufwenberg and Kirchsteiger, 2004; Falk and Fischbacher, 2006), while firms, although self-interested, are concerned about fairness because of the effect of workers' responses to wage changes on profitability.[9]

Contractual Incompleteness. When thinking about firms' wage setting behaviour, and more generally about the employment contract, it is considered a negotiation in

[7]As it will be argued in Chapter 3, the reference "fair" wage is an artifact that simplifies the broader concept of workers' perceptions of fairness. The literature has captured the same concept with different names such as "fair wage" in Marshall (1890) and Hicks (1963), "wage norm" in Lindbeck and Snower (1986), "perceptions of entitlement" in Kahneman, Knetsch, and Thaler (1986), "feelings of entitlement" in Hart and Moore (2008) and "the reference frame of fairness judgements" in Fehr et al. (2009).

[8]This assumption is consistent with the evidence reported by surveys and experiments as discussed above. While wage increases have a weak impact on morale, unfair wage cuts damage workers' morale due to an "insult effect" and a "standard of living effect" (Bewley, 2007, p. 161). Fehr et al. (2009, p. 377) argue that evidence of such behaviour suggests the existence of "reference-dependent fairness concerns".

[9]This type of reciprocity is conceptually different from the idea of inequity aversion (Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000) used in a model of wage setting by Benjamin (2015). For a comparison of the various models of reciprocity advanced in the literature see Malmendier et al. (2014).

which an employer (the buyer) offers a wage in exchange for productive activity by a worker (the seller). However, unlike in goods markets, the employer is not able to contract upon the "quality" of workers' productive activity: effort is discretionary and therefore not contractible. This peculiarity of labour markets brought Okun (1981) to the conclusion that the employment relationship is governed by an "invisible handshake" and Williamson (1985) to define the employment contract as an "incomplete agreement". According to Williamson (1985, p. 262-63), only the minimum job performance can be enforced by the contract (the "perfunctory cooperation"), while workers "enjoy discretion" about the quality of their service, in terms of cooperation, effort and efficiency (the "consummate cooperation"). This latter aspect is influenced by the worker's evaluation of the fairness of the wage they are offered.

The key intuition of the theory developed in this chapter is that there is a cost associated with reducing wages. If workers perceive wage reductions to be unfair their effort and willingness to work will drop, reducing productivity and influencing the firm's profitability. Thus the firm's managers may refrain from cutting wages in light of adverse economic conditions if, at the margin, the related cost in the form of negative reciprocity is greater than the benefit of paying lower wages. This insight confirms the predictions of early prominent hypotheses, and offers a psychological foundation for it.

1.3 The Model

To begin with, consider the wage setting behaviour of a firm for a single period of employment with an established worker. It is assumed a setting of complete and perfect information. At the start of the employment period the firm learns the match productivity q and the worker's exogenously-given reference wage r. It then decides whether to continue the employment relationship and, if so, the wage w to offer to the worker. The match productivity is a realisation of the random variable Q which is distributed on $[0, \infty)$ with cumulative distribution function F and density function f, and captures the interaction between the firm's technology, per-worker capital and the idiosyncratic productivity of the worker. For simplicity, it is assumed that the worker will accept any contract offered.[10] After considering the wage in relation to their reference wage, the worker decides on a (non-negative) level of effort e which generates output for the firm. Payoffs are then realised, the form of which is described next. Since this is a game of complete information in which choices are made sequentially and the firm is assumed to be motivated only by profit, it can be solved by backward induction.

[10] As such, the reservation utility for the worker is not modelled here, the effect of which is to add an additional threshold to the model. The implications of the worker's reservation utility are nevertheless explored in Appendix A.2, Section A.2.2 of Chapter 2 in the context of a search and matching framework.

1.3.1 Payoffs

The per-worker output in an employment relationship (the price of which is normalised to one) is a function of both the match productivity and the effort chosen by the worker, and is denoted $y(q, e)$. The per-worker cost of production is $s(w)$ where w is the wage paid to the worker. The firm is materially motivated by profit:

$$\pi(w; q, e) = y(q, e) - s(w). \tag{1.1}$$

The following assumptions are made:

F1. $s'(w) > 0$ and $s''(w) \geq 0$.

F2. $\dfrac{\partial y(q,e)}{\partial e}, \dfrac{\partial y(q,e)}{\partial q} > 0, \dfrac{\partial^2 y(q,e)}{\partial e^2}, \dfrac{\partial^2 y(q,e)}{\partial q^2} \leq 0$ and $\dfrac{\partial^2 y(q,e)}{\partial q \partial e} > 0.$

Notice that assumption F2 implies the marginal product of effort is increasing in the match productivity.

To capture behaviour consistent with the synthesis exposed in Section 1.2—namely: reference dependence and loss aversion of the wage in relation to the reference wage; and reciprocity in relation to the worker's perception of the fairness of the wage—the worker's preferences are specified by an additively separable utility function composed of a *wage utility*, $\tilde{u}$, and an *effort utility*, $\tilde{v}$:

$$u(e; w, r) = \tilde{u}(w, r) + \tilde{v}(e; w, r).$$

The *wage utility* $\tilde{u}(w, r)$ represents the worker's perceived utility from wage evaluations which is supposed to be comprised of a standard utility function and a gain-loss function $n(w|r)$:

$$\tilde{u}(w, r) = m(w) + \xi n(w|r).$$

The function $m(w)$ captures the effect of absolute wage levels on the worker's utility, and $n(w|r)$ captures the worker's evaluation of the wage relative to the reference wage, the functional form of which will be defined shortly. The parameter $\xi \geq 0$ measures the worker's subjective weight of relative wage comparisons in the wage utility. It is assumed that:

W1. $m'(w) > 0$ and $m''(w) < 0$.

The *effort utility* $\tilde{v}(e; w, r)$ that the worker derives from engaging in productive activity takes the form

$$\tilde{v}(e; w, r) = b(e) - c(e) + M(e; w, r),$$

where $b(e)$ represents the worker's intrinsic psychological benefit of being productive, and $c(e)$ their intrinsic psychological and physical cost of productive activity.[11] It is assumed that:

W2. $b'(e) > 0$ and $b''(e) \leq 0$; $c'(e) > 0$ and $c''(e) > 0$; and $b'(0) > c'(0)$.

The function $M(e; w, r)$ is the "morale function", a key component of the model that lies at the heart of many of the results that follow. Consider a morale function of the form

$$M(e; w, r) \equiv g(e)n(w|r) \tag{1.2}$$

where:

W3. $g'(e) > 0$ and $g''(e) = 0$.

Hence, define $g'(e) \equiv \zeta > 0$, so that $M(e; w, r) = \zeta e n(w|r)$.

Morale depends on the worker's evaluation of the wage in relation to the reference wage, the functional form of which is defined next. It is assumed that $n(w|r) \equiv \mu(m(w) - m(r))$ where $\mu(\cdot)$ is a gain-loss value function that exhibits loss aversion in the spirit of Kahneman and Tversky (1979). As such, for a loss averse worker the evaluation of utility differences between the wage and the reference wage will be steeper for wages below the reference wage than for those above it. Consider the following assumption about the functional form of the gain-loss utility.[12]

W4. $\mu(0) = 0$ and, whilst $\mu(x)$ is non-differentiable at $x = 0$, it is continuous with $\mu'(x) > 0$ and $\mu''(x) = 0$ for all $x \neq 0$. Moreover, for any $x > 0$, $\mu'(-x)/\mu'(x) \equiv \lambda \geq 1$.

Under this assumption, it follows that the gain-loss utility is piecewise-linear:

$$n(w|r) \equiv \mu(m(w) - m(r)) = \begin{cases} \eta[m(w) - m(r)] & \text{if } w \geq r \\ \lambda\eta[m(w) - m(r)] & \text{if } w < r \end{cases} \tag{1.3}$$

where $\eta > 0$ is a scaling parameter that represents the importance of gain-loss utility for the worker, and $\lambda \geq 1$ represents the worker's degree of loss aversion.[13]

[11] The model is developed as to capture the idea that "normal effort" (i.e. effort that a worker would exert absent morale considerations) is not zero. This necessitates the inclusion of intrinsic benefits and costs of productive activity. Whilst this approach contrasts with, for example, shirking models in the efficiency wage literature (e.g. Shapiro and Stiglitz (1984)) it is consistent with the idea that workers perceive positive satisfaction from engaging with productive activity (see, for example, the discussion in Altmann, Falk, Grunewald, and Huffman (2014, Appendix)).

[12] Assumption W4 closely resembles the assumptions of Kőszegi and Rabin (2006) over the properties of their 'universal gain-loss function', except that is does not capture diminishing sensitivity.

[13] The gain-loss function enters the worker's utility twice, crucially in the morale function but also in

The morale function, being dependent on $n(w|r)$, captures an additional psychological cost/benefit of productive effort associated with the worker's perception of fairness. If the wage exceeds the reference wage (it is perceived as a 'gift') the worker gains some additional benefit of productive effort and an increase in effort (a "gift" to the firm) will increase utility. If the wage falls short of the reference wage (it is perceived as "unfair") there is an additional psychological cost of productive effort and a reduction in effort increases utility. As such, the morale function implies the worker's payoff exhibits *reciprocity*, and since morale is linked to loss aversion, negative reciprocity is stronger than positive reciprocity.

Since morale depends on effort, these same considerations apply to the margins of the worker's payoff function: for a loss averse worker the reduction in the marginal utility of effort for an unfair wage will be larger than the increase in marginal utility for a wage an equivalent amount above the reference wage. Consequently, the effect on optimal effort, which is determined by these margins, will be asymmetric. The remainder of the section will derive the relationship between effort and the wage.

1.3.2 The Worker's Choice of Effort

A worker's choice of effort, after the wage has been set, will depend on their evaluation of the wage in relation to their reference wage. Given a reference wage r and a wage offer w the worker will seek to

$$\max_{e \geq 0} u(e; w, r).$$

Recall that

$$u(e; w, r) = m(w) + \xi\mu(m(w) - m(r)) + b(e) - c(e) + \zeta e\mu(m(w) - m(r)),$$

and from (1.3) that $\mu(m(w)-m(r))$ is piecewise-linear; and denote the utility-maximising effort by $\tilde{e}(w, r, \lambda)$. The worker's optimal effort choice is therefore characterised by the following first-order condition

$$\Omega(e; w, r, \lambda) \equiv b'(e) - c'(e) + \zeta\mu(m(w) - m(r)) \leq 0, \tag{1.4}$$

in which the inequality is replaced with an equality if $e > 0$.[14]

The following theorem defines the properties of the worker's optimal effort function,

the wage utility. Whilst the latter is not important for the theory developed in this chapter (indeed, it is possible that $\xi = 0$), a realistic model of wage evaluation should allow loss aversion to influence the utility received from a wage offer. This would become important if the analysis considered the worker's reservation utility.

[14]The second-order sufficient condition $b''(e) - c''(e) < 0$ is satisfied under assumption W2.

which exhibits what it will be defined as "asymmetric reference-dependent reciprocity".

Theorem 1. *For any given wage offer w relative to their reference wage r, the worker's optimal effort function takes the form*

$$\tilde{e}(w,r,\lambda) = \begin{cases} \tilde{e}(w,r)^{+} & \text{if } w > r \\ \tilde{e}_n & \text{if } w = r \\ \tilde{e}(w,r,\lambda)^{-} & \text{if } w < r \end{cases} \tag{1.5}$$

where $\tilde{e}_n$ is "normal" effort is implicitly defined by $b'(e) = c'(e)$; $\tilde{e}(w,r)^{+} > \tilde{e}_n$ is implicitly defined by $b'(e) + \zeta\eta[m(w) - m(r)] = c'(e)$; and $\tilde{e}(w,r,\lambda)^{-} < \tilde{e}_n$ is implicitly defined by $b'(e) \leq c'(e) + |\zeta\lambda\eta[m(w) - m(r)]|$ (with equality if $e > 0$, which is true for all $w > \underset{\sim}{w}(r,\lambda)$, defined in the proof).

a) For a given r, $\tilde{e}(w,r,\lambda)$ is a continuous, increasing and concave function of w for all $w \neq r$. Moreover,

$$\lim_{\epsilon \to 0} \frac{\partial \tilde{e}(r-\epsilon,r,\lambda)^{-}}{\partial w} = \lambda \cdot \lim_{\epsilon \to 0} \frac{\partial \tilde{e}(r+\epsilon,r)^{+}}{\partial w},$$ [15]

implying that the optimal effort function has a kink at $w = r$ if $\lambda > 1$.

b) For a given w, $\tilde{e}(w,r,\lambda)$ is a continuous and decreasing function of r for all $w \neq r$.

c) Finally, for all $w < r$, $\tilde{e}(w,r,\lambda)$ is a continuous and decreasing function of λ; and $\partial\tilde{e}(w,r,\lambda)^{-}/\partial w$ is increasing in λ.

Proof. See Appendix B.1. ■

Theorem 1 establishes that if a worker is paid their reference wage then they will exert normal effort $\tilde{e}_n$, which is independent of the absolute wage level;[16] and it identifies a positive relationship between effort and changes in the wage relative to the reference wage. Moreover, if the worker is loss averse, the effect of these changes will be asymmetric due to a kink in the effort function at the reference wage. This is illustrated in Figure 1.1.

[15]Throughout the thesis, where sequences of ϵ are considered over which limits are taken, it is specified that $\{\epsilon_n\}_{n=1}^{\infty} \subset \mathbb{R}_{+}$, meaning that where the wage is specified to be $r - \epsilon$ and the limit is taken as $\epsilon \to 0$, it is considered as the wage increasing to the reference wage, and likewise when the wage is specified to be $r + \epsilon$ and the limit is taken as $\epsilon \to 0$, it is considered as the wage decreasing to the reference wage.

[16]Inspired by the findings reported in Bewley (2007), that it is not wage levels but changes in wages that influence effort, normal effort should be a non-pecuniary concept and is therefore modelled as being independent of the wage. This approach, and its dynamic implications that are analysed in Section 1.4, are consistent with the recent laboratory experiment of Sliwka and Werner (2017).

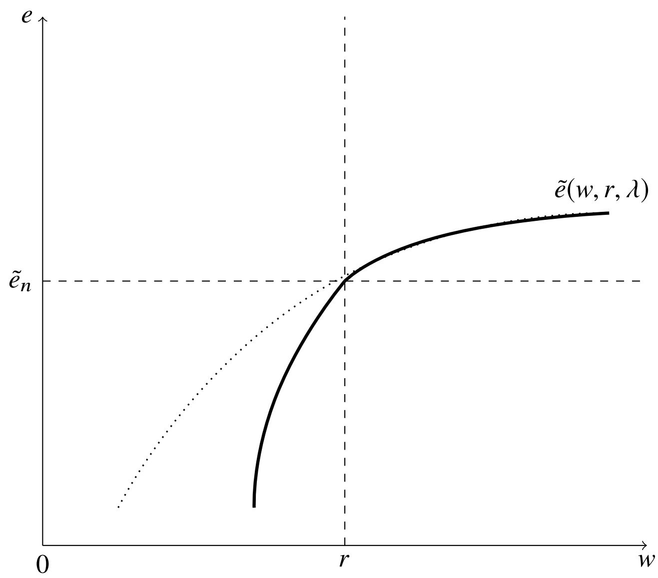

Figure 1.1:
Asymmetric Reference-dependent Reciprocity

The positive relationship between effort and the wage is driven by the morale function: an increase in the wage gives a higher marginal utility of effort which consequently results in higher optimal effort. The asymmetric nature of effort responses has the particular implication that for changes in the wage from an initial wage equal to the reference wage, the effect of negative reciprocity that results from a reduction in the wage will be greater than the effect of positive reciprocity resulting from an increase in the wage. The extent of this "asymmetric reference-dependent reciprocity" depends on the worker's degree of loss aversion: whilst when the wage exceeds the reference wage optimal effort is independent of λ, below the reference wage more loss averse workers exert less effort, which decreases faster as the wage gets further from the reference wage. Indeed, if a worker is not loss averse ($\lambda = 1$), reciprocity is symmetric.

The derived effort relationship establishes the model as providing a micro-foundation for effort functions that exhibit asymmetric reciprocity that are commonly assumed in the literature, but that are not explicitly modelled. This micro-foundation is based on perceptions of fairness coupled with loss aversion, and is consistent with the evidence suggesting that workers are subject to such forces, as it was summarised in Section 1.2.[17]

[17]The idea that loss aversion gives rise to asymmetric reciprocity in the effort relationship is consistent with the intuition presented in, for example, Campbell (1997), Mas (2006), Bewley (2007) and Fehr et al. (2009).

1.3.3 The Firm's Wage Setting Rule

Next consider the firm's problem in setting the wage given that it anticipates the behaviour of the worker in response to the wage offer. Suppose a firm is facing a worker who has an exogenously-given reference wage r and for whom the match productivity is q. The firm will seek to maximise its payoff given in (1.1) where the worker's effort is determined as in (1.5). As such, the firm's problem is to

$$\max_{w \geq 0} \pi(w; q, \tilde{e}(w, r, \lambda))$$

where $\pi(w; q, e) = y(q, e) - s(w)$. The firm will continue the employment relationship with the worker at the optimal wage only if it is profitable; otherwise the employment relationship will be terminated.

The firm's output depends on the match productivity and on the effort of the worker, and the firm seeks to balance the marginal product of labour with the per-worker marginal cost. Consider two workers with differing match productivity that are otherwise identical (and in particular have the same reference wage). Conventional thinking implies that the worker with the lower productivity should be paid a lower wage. The model outlined here captures the idea that if the firm paid the worker its preferred wage for a given match productivity, and this falls below the worker's reference wage, the worker's morale may be affected that will impact their effort and therefore the output produced from the employment relationship. This implies a cost of reducing the wage below the reference wage, borne from the effect of negative reciprocity which may be large if the worker is loss averse, and must be considered in relation to the benefit of paying the lower wage: asymmetric reference-dependent reciprocity on the part of workers influences the marginal considerations of the firm.

As established by Theorem 1, the worker's optimal effort function $\tilde{e}(w, r, \lambda)$ is continuous in the wage, but that there is a kink at $w = r$ for a loss averse worker. Hence the firm's payoff function, being otherwise smooth, will inherit this property. The fact that the profit function is continuous but has a kink at $w = r$ enables to derive the optimal wage setting rule, which is explained intuitively below before its formalisation in the statement of Theorem 2. For $w \neq r$ define the marginal profit as

$$\Psi(w; q, r, \lambda) \equiv \frac{d\pi(w; q; \tilde{e}(w, r, \lambda))}{dw} = \frac{\partial y(q, \tilde{e}(w, r, \lambda))}{\partial e} \frac{\partial \tilde{e}(w, r, \lambda)}{\partial w} - s'(w),$$

and note that the profit function is concave.[18] Subject to the optimal wage contract

[18]This is established by noting that profit is continuous, and Assumptions F1 and F2 and the results of Theorem 1 imply $\partial \Psi(w; q, r, \lambda)/\partial w < 0$ for all $w \neq r$.

being profitable, the optimal wage setting rule is characterised by two productivity thresholds, q_l and q_u. The upper threshold q_u is derived such that, if $q > q_u$ then profit is increasing in the wage at wages just above the reference wage so concavity implies the optimal wage must exceed r and will be characterised by $\Psi(w; q, r, \lambda) = 0$ (in which $\tilde{e}(w, r, \lambda) = \tilde{e}(w, r)^+$ since $w > r$). The lower threshold q_l is derived such that, if $q < q_l$ then profit is decreasing in the wage for wages just smaller than the reference wage, which by concavity implies the optimal wage will be below r and will be characterised by $\Psi(w; q, r, \lambda) = 0$ (in which $\tilde{e}(w, r, \lambda) = \tilde{e}(w, r, \lambda)^-$ since $w < r$). For $q_l \leq q \leq q_u$ it will be the case that for all wages below the reference wage $\Psi(w; q, r, \lambda) > 0$ and for all wages above the reference wage $\Psi(w; q, r, \lambda) < 0$, and therefore profit will be maximised with a wage equal to the reference wage: any worker whose reference wage is r and whose productivity lies in this range will be paid the same wage, giving rise to what is defined as a "range of rigidity". If the optimal wage contract is unprofitable, which will be the case if the match productivity falls below a threshold that is influenced by the reference wage, the firm will end the employment relationship.

Theorem 2. *For any given r, the firm's optimal wage $\tilde{w}(r, q, \lambda)$ is a continuous function of q and r and is characterised by*

$$\tilde{w}(r, q, \lambda) = \begin{cases} \tilde{w}(r, q)^+ & \text{if } q > q_u(r) \\ r & \text{if } q_l(r, \lambda) \leq q \leq q_u(r) \\ \tilde{w}(r, q, \lambda)^- & \text{if } q < q_l(r, \lambda), \end{cases}$$

so long as $q \geq \underline{q}(r, \lambda)$, the firm's reservation productivity, where

$$\begin{aligned} q_l(r, \lambda) &= \{q : \lim_{\epsilon \to 0} \Psi(r - \epsilon; q, r, \lambda) = 0\}, \\ q_u(r) &= \{q : \lim_{\epsilon \to 0} \Psi(r + \epsilon; q, r, \lambda) = 0\}, \\ \underline{q}(r, \lambda) &= \max\{0, q : \pi(\tilde{w}(r, q, \lambda); q, \tilde{e}(\tilde{w}(r, q, \lambda), r, \lambda)) = 0\} \end{aligned}$$

(all singletons), and where $q_l(r, 1) = q_u(r)$ and $\partial q_l(r, \lambda)/\partial \lambda < 0$ implying $q_l(r, \lambda) < q_u(r)$ for all $\lambda > 1$. The optimal wage $\tilde{w}(r, q)^+ > r$, is implicitly defined by $\Psi(w; q, r, \lambda) = 0$ in which $\tilde{e}(w, r, \lambda) = \tilde{e}(w, r)^+$; and $\tilde{w}(r, q, \lambda)^- < r$, is implicitly defined by $\Psi(w; q, r, \lambda) \leq 0$, with equality if $w > \underset{\sim}{w}(r, \lambda)$, in which $\tilde{e}(w, r, \lambda) = \tilde{e}(w, r, \lambda)^-$. Moreover,

a) for all $q \in [\underline{q}(r, \lambda), \infty] \setminus [q_l(r, \lambda), q_u(r)]$, $\tilde{w}(r, q, \lambda)$ is increasing in q;

b) for all $q \in [\underline{q}(r, \lambda), \infty]$, $\tilde{w}(r, q, \lambda)$ is increasing in r;

c) for all $q \in [\underline{q}(r, \lambda), q_l(r, \lambda))$ (if non-empty), $\tilde{w}(r, q, \lambda)$ is increasing in λ.

Finally, the productivity thresholds defining the wage setting rule satisfy: $q_u'(r) > 0$ *and* $\partial q_l(r,\lambda)/\partial r > 0$*; and the reservation productivity satisfies:* $\partial \underline{q}(r,\lambda)/\partial r > 0$ *and* $\partial \underline{q}(r,\lambda)/\partial\lambda \geq 0$*, where the final inequality is strict if* $\tilde{w}(r, \underline{q}(r,\lambda), \lambda) < r$*.*

Proof. See Appendix B.1. ■

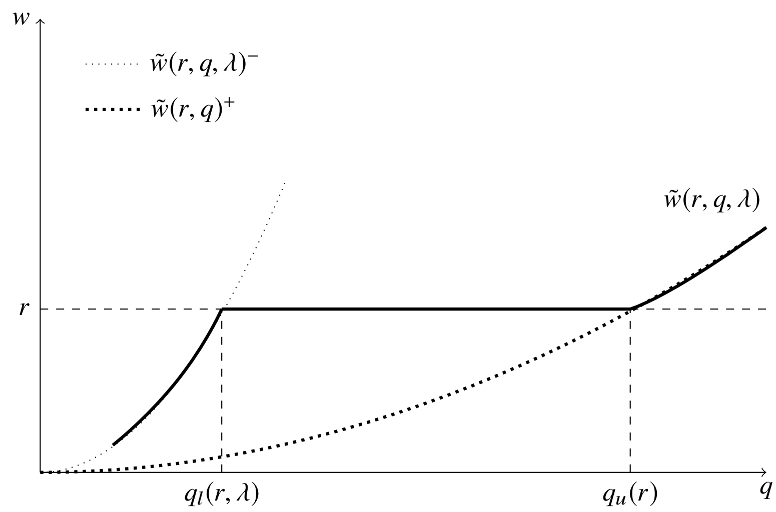

Figure 1.2:
The Wage Setting Rule.

Theorem 2 elucidates the features of the firm's optimal wage setting rule when facing a loss averse worker, which is illustrated in Figure 1.2. The optimal wage is non-decreasing in the match productivity and if the worker is loss averse there is a range of match productivity within which the wage is not adjusted. The lower and upper thresholds of this range depend on the worker's reference wage. If the match productivity is $q_u(r)$ the worker will be paid their reference wage which is perceived to be fair and so will exert normal effort. If the match productivity exceeds $q_u(r)$ then the firm will find it profitable to pay above the reference wage since such a wage, seen as a gift, will be positively reciprocated with supra-normal effort. If the match productivity is slightly less than $q_u(r)$ the firm would look to reduce the wage, but understands that paying a wage below the reference wage will be seen as unfair and generate a reduction in effort, the value of which exceeds the wage cut despite the relatively low match productivity. This is true for all $q \in [q_l(r,\lambda), q_u(r)]$, identifying the *range of rigidity*, which is non-empty if the worker is loss averse. If the match productivity is below $q_l(r,\lambda)$ then the very low match productivity warrants a wage below the reference wage despite inciting sub-normal effort as the worker perceives this wage as unfair.

If a worker has a greater degree of loss aversion then the effect of negative reciprocity borne from paying a wage below the reference wage is stronger, resulting in a greater reduction in effort which is more costly to the firm. As such, for a more loss averse worker, the lower threshold of the range of rigidity is lower since the firm is unwilling to suffer the relatively high cost of negative reciprocity: $\partial q_l(r, \lambda)/\partial \lambda < 0$. However, if the match productivity is low enough that the firm wishes to pay a wage below the reference wage, that wage will be higher the more loss averse a worker is, as the firm has an incentive to attenuate some of the effect on effort from stronger reciprocity: $\partial \tilde{w}(r, q, \lambda)^{-}/\partial \lambda > 0$.[19] Negative reciprocity not only tempers the firm's incentive to cut the wage, it also reduces the extent to which the wage is cut. Finally, the more loss averse a worker is, the higher the match productivity the firm requires from the employment relationship for it to be profitable: $\partial \underline{q}(r, \lambda)/\partial \lambda > 0$. This follows since the firm pays a higher wage in an attempt to mitigate the effect of negative reciprocity on effort, which is nevertheless lower due to negative reciprocity being stronger. Hence, profit is reduced and the reservation productivity above which the firm becomes profitable increases.

The effect of a worker feeling entitled to a higher wage (i.e. a higher reference wage) increases the optimal wage offered by the firm and increases both the threshold above which the wage is higher than the reference wage, and the threshold below which the wage is less than the reference wage. In addition, since for any match productivity the firm receives lower profit, the range of match productivity over which the employment relationship is entered into will shrink.

The features of the wage setting rule highlighted in Theorem 2 imply that when a worker is loss averse a range of rigidity exists and it is larger for individuals that are more loss averse. For a worker that is not loss averse ($\lambda = 1$) there is no range of rigidity and the wage responds smoothly to productivity.

1.4 Adaptation, Loss Aversion and the Employment Contract

This section explores the implications of capturing reciprocity and reference-dependence in the employment relationship in a dynamic environment that is subject to uncertainty and in which the worker adapts their reference wage to the wage they have been paid in the past.[20]

[19] This implication has been theoretically derived and empirically corroborated by Holden and Wulfsberg (2014), who show that "even if the wage is cut, the resulting wage will be higher than if the wage-setting process had been completely flexible". In contrast with their theoretical model, our theory attributes this result to the worker's extent of negative reciprocity.

[20] This latter assumption, which is going to be thoroughly discussed in Chapter 3 is inspired by the literature that suggests reference points are influenced by previous contractual arrangements. According

Consider the following assumption over the evolution of the worker's reference wage:

A1. $r_t = w_{t-1}$, r_0 given.

A worker's reference wage is therefore assumed to be (endogenously) determined by the wage they were paid in the previous employment period. The worker is therefore characterised by what can be defined as "symmetric perfect adaptation" of the reference wage, which henceforth will be referred simply as *adaptation*.[21]

The timing of the model is as follows. At the beginning of period $t = 0$ the productivity characterising the match, q_0, is observed, as is the exogenously given reference wage of the worker, r_0. Knowing these, the firm then decides whether to offer a wage contract to the worker and start the employment relationship, where again it is assumed that any offer will be accepted by the worker. If an employment relationship is established, then at the end of the first employment period the match productivity for the subsequent period, q_1 (which is independent of q_0), is randomly drawn from the same distribution F. In addition, the worker adapts their reference wage to the wage paid in the initial period of employment. At the beginning of period $t = 1$, after observing q_1, and inferring the worker's reference wage $r_1 = \tilde{w}_0$, the firm considers whether it wants to continue the employment relationship and, if so, whether to adjust the wage of the worker in light of the change in the match productivity. The timing of the employment relationship is illustrated in Figure 1.3.

The firm therefore faces a two-period dynamic optimisation problem under uncertainty, which consists of choosing a sequence of optimal wages $\{\tilde{w}_0, \tilde{w}_1\}$ that maximises the sum of its expected discounted profits from the employment relationship. Letting δ represent the firm's discount factor, this can be formalised by the following sequence

to Kahneman et al. (1986) when workers enter a firm there is a shift in their feelings of entitlement and the most recent negotiated wage is adopted as the standard of fairness. This sort of adaptation is believed to be an active behavioural feature of workers' perceptions of fairness, supported by the anthropological evidence surveyed by Bewley (2007) and by laboratory and field experiments of the employment relationship (see, for instance, Chemin and Kurmann (2014) and Koch (2016)). In contract theory the idea of "contracts as reference points" has been analysed by Hart and Moore (2008) and further explored by Herweg and Schmidt (2015). The laboratory experiments of Fehr, Hart, and Zehnder (2011, 2014) and Bartling and Schmidt (2015) provide strong support for this hypothesis, which also reflects the idea that past experience and adaptation play a significant role in the process of individuals' reference point formation (see Herz and Taubinsky (2016) and Smith (2015) for evidence of this hypothesis, and Stommel (2013) for a review of the literature).

[21]As an anticipation to the framework analysed in Chapter 3, Section 1.5 will briefly discuss the potential implications of other forms of reference wage adaptation in the context of the model developed in this chapter.

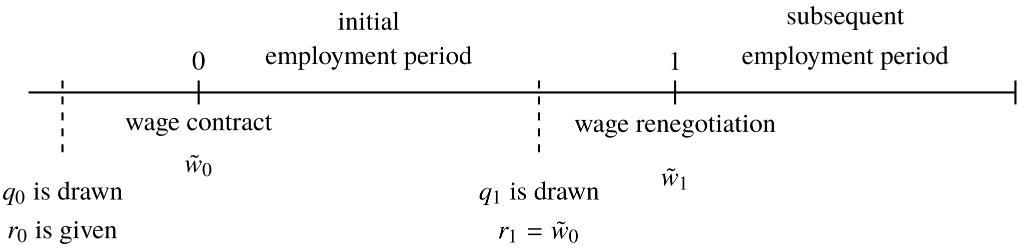

Figure 1.3:
Employment Relationship Time-line.

problem:

$$\begin{aligned} J_0(r_0, q_0) &= \max_{\{w_0, w_1\}} \mathbb{E}_0 \left[\sum_{t=0}^{1} \delta^t \pi(w_t; q_t, e_t) \right] \\ \text{s.t.} \quad r_1 &= w_0, \\ e_t &= \tilde{e}(w_t, r_t, \lambda), \\ r_0, q_0 & \quad \text{given} \end{aligned} \tag{1.6}$$

where $J_0(r_0, q_0)$ is the firm's value function of the two-period employment relationship. Note that whilst the firm is forward looking, since there is no link between the initial and the subsequent employment period from the perspective of the worker, they will choose productive effort as to maximise their per-period utility, in accordance with the derivation of the optimal effort function established in Theorem 1.[22]

The remainder of the section is divided in two parts. First the analysis will illustrate the wage and effort dynamics by considering a myopic firm ($\delta = 0$), and highlight that wage rigidity, both downward and upward, may occur as a result of the worker's reference wage adaptation combined with loss aversion. Then the case of a forward-looking firm ($\delta > 0$) will be considered, in which the analysis will characterise the optimal employment contract that solves the firm's problem in (1.6), and explore its properties.

1.4.1 Wage and Effort Dynamics

Consider a myopic firm that ignores the worker's adaptation of the reference wage from the initial employment period into the next. As such, the firm will adopt the standard wage setting rule as derived in Theorem 2. That is, in each period $t = 0, 1$ the optimal

[22]Forward-looking behaviour on the side of workers could be incorporated with expectations-based reference points, for instance à la Kőszegi and Rabin (2006). However, the evidence previously cited strongly suggests that workers' reference wages are backward looking.

wage takes the form

$$\tilde{w}_t = \tilde{w}(r_t, q_t, \lambda) = \begin{cases} \tilde{w}(r_t, q_t)^+ & \text{if } q_t > q_u(r_t) \\ r_t & \text{if } q_l(r_t, \lambda) \leq q_t \leq q_u(r_t) \\ \tilde{w}(r_t, q_t, \lambda)^- & \text{if } q_t < q_l(r_t, \lambda) \end{cases}$$

so long as $q_t \geq \underline{q}(r_t, \lambda)$. Otherwise, the firm will not offer, or renegotiate, the employment contract and the employment relationship will be over.

To illustrate the dynamics of firm's wage setting behaviour and worker's reciprocity, the analysis considers different scenarios that the firm may face during the employment relationship, assuming that in both the initial and subsequent employment periods the match productivity exceeds the firm's reservation productivity $\underline{q}(r_0, \lambda)$ and $\underline{q}(r_1, \lambda)$ respectively, reserving commentary on when this is not the case as a postscript.

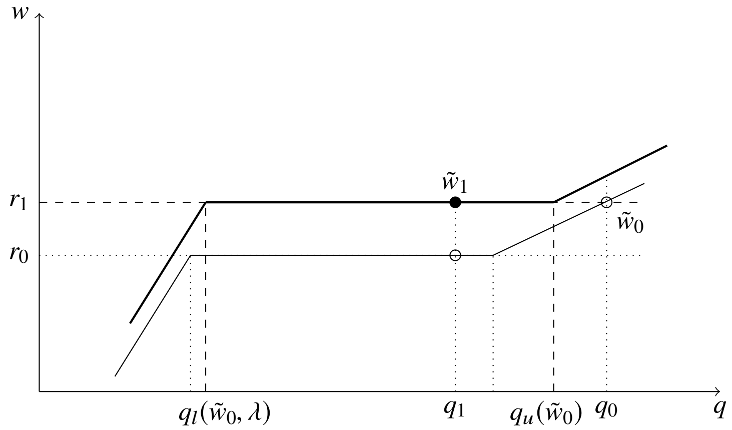

Figure 1.4:
Adaptation and Downward Wage Rigidity when $\tilde{w}_0 > r_0$.

Suppose that, in the initial employment period, $q_0 > q_u(r_0)$ so that the firm optimally offers a wage that exceeds the worker's reference wage $\tilde{w}_0 = \tilde{w}(r_0, q_0)^+$, and the worker will therefore exert supra-normal effort $\tilde{e}(w_0, r_0)^+ > \tilde{e}_n$. As the employment relationship passes from the initial period into the next, the worker adjusts their feelings of entitlement, adapting their reference wage to their initial wage: $r_1 = \tilde{w}_0$. This "shifts" the wage-setting rule, as illustrated in Figure 1.4: the reference wage increases, the lower threshold increases, and the upper threshold increases to somewhere between its previous value and the initial match productivity.[23]

[23]If the production function exhibits constant returns to effort, $q_u(\tilde{w}_0)$ increases to exactly q_0. To see this, consider the following. It has been established by Theorem 2 that $\partial q_l / \partial r > 0$ and $q_u' > 0$. Recall that $q_u(r)$ is the value of q where $\lim_{\epsilon \to 0} \Psi(r + \epsilon; q, r, \lambda) = 0$. If $\tilde{w}_0 = \tilde{w}_0^+$ then the first-order condition is satisfied with equality at the optimal wage in the initial contract: $\Psi(\tilde{w}_0; q_0, r_0, \lambda) = 0$. Next, recall (from the preliminaries in the proof of Theorem 2) that $\partial \Psi / \partial r \geq 0$. Then, in $t = 1$, since the worker's reference wage increases to $\tilde{w}_0$, it implies that $\lim_{\epsilon \to 0} \Psi(\tilde{w}_0 + \epsilon; q_0, \tilde{w}_0, \lambda) \geq 0$, and therefore, since $\partial \Psi / \partial q > 0$,

If $q_1 > q_0 (\geq q_u(\tilde{w}_0))$ then the match productivity in the subsequent employment period exceeds the upper threshold of the wage setting rule, implying that the firm will optimally increase the worker's wage to benefit from the gift being reciprocated with supra-normal effort. If $q_1 = q_0$ then $\tilde{w}_1 \geq \tilde{w}_0$. However, whilst a wage of $\tilde{w}_0$ was positively reciprocated by the worker when compared to a reference wage of r_0, after adaptation the worker has an updated sense of entitlement meaning effort with this wage is merely normal, and so the firm optimally pays at least this wage in the subsequent contract. Hence, the analysis formally establishes that, due to reference wage adaptation, reciprocity is essentially a temporary phenomenon.[24] This adjustment of effort over time induced by the worker's adaptation of the reference wage is defined as dynamic "re-normalisation" of effort, which will play an important role in the analysis of the next section.

In the event of $q_1 < q_0$, whether the wage is renegotiated depends on by how much the match productivity reduces. Only if $q_1 < q_l(\tilde{w}_0, \lambda)$ the firm will then optimally implement a wage cut, which is nevertheless more muted, in order to partially mitigate the resulting negative reciprocity response of the worker. As such, a fall in match productivity over time ($q_1 < q_0$) is not necessarily followed by a wage cut: the worker's adaptation to a wage consistent with a match productivity of q_0 implies that, if the match productivity only moderately decreases (i.e. $q_1 \in [q_l(\tilde{w}_0, \lambda), q_u(\tilde{w}_0)]$, the firm will optimally keep the wage equal to the worker's reference wage.[25] The negative effect of what would now be perceived as an unfair wage, borne through negative reciprocity, will be larger than the benefit of paying the lower wage and hence the firm will avoid inciting such negative reciprocity, by freezing the wage. The theory developed in this chapter therefore formally demonstrates that downward wage rigidity is an inherent feature of the employment contract in a dynamic environment, and establishes as its key drivers are the worker's adaptation of the reference wage ($r_1 = \tilde{w}_0$), and the relatively large cost to the firm of negative reciprocity that stems from loss aversion ($\lambda > 1$).

In the case where the initial match productivity is such that $q_0 < q_l(r_0, \lambda)$ the firm will pay an initial wage below the worker's reference wage $\tilde{w}_0 = \tilde{w}(r_0, q_0, \lambda)^-$,

the value of q that regains equality of this expression with 0, which is precisely $q_u(r_1) = q_u(\tilde{w}_0)$, will not exceed q_0. In fact, if $\partial^2 y/\partial e^2 = 0$, then $\partial \Psi/\partial r = 0$ and $\lim_{\epsilon \to 0} \Psi(\tilde{w}_0 + \epsilon; q_0, \tilde{w}_0, \lambda) = 0$, so it must be that $q_u(\tilde{w}_0) = q_0$.

[24]This implication of the model is consistent with the evidence reported by field research (experiments and surveys) that the positive effects of a wage gift on morale and effort are believed to be weak and only temporary by firms' managers. It also supports the interpretation of this evidence according to which positive reciprocity quickly disappears as workers get used to the wage they receive (see, for instance, Campbell (1997), Bewley (1999), Gneezy and List (2006) and Cohn et al. (2014)). Evidence of this type of reciprocity dynamics has recently been provided by the laboratory experiment of Sliwka and Werner (2017).

[25]Indeed, reductions in the match productivity may be associated with an increase in the wage (if they are small and in the range $[q_u(\tilde{w}_0), q_0]$, which is non-empty only with decreasing returns to effort).

despite the implied negative reciprocity $\tilde{e}(w_0, r_0, \lambda)^- < \tilde{e}_n$. In the following period, the worker's reference wage reduces to the initial wage level, and the firm's wage setting rule adjusts accordingly. In this case, the lower threshold of the range of rigidity reduces to somewhere between q_0 and $q_l(\tilde{w}_0, \lambda)$ with the implication that the employment relationship will exhibit *upward* wage rigidity if the subsequent match productivity only moderately increases (i.e. $q_1 \in [q_0, q_u(\tilde{w}_0, \lambda)]$), since within this range the firm will not raise the wage above the initial $\tilde{w}_0$. This particular case is illustrated in Figure 1.5 The intuition is that in the initial employment period the firm essentially "over-paid" the worker in the optimal trade-off of wage *versus* negative reciprocity; hence in the subsequent period the firm will increase the wage only if the match becomes substantially more productive (i.e. only if $q_1 > q_u(\tilde{w}_0, \lambda)$) and the benefits of positive reciprocity warrant a wage rise.

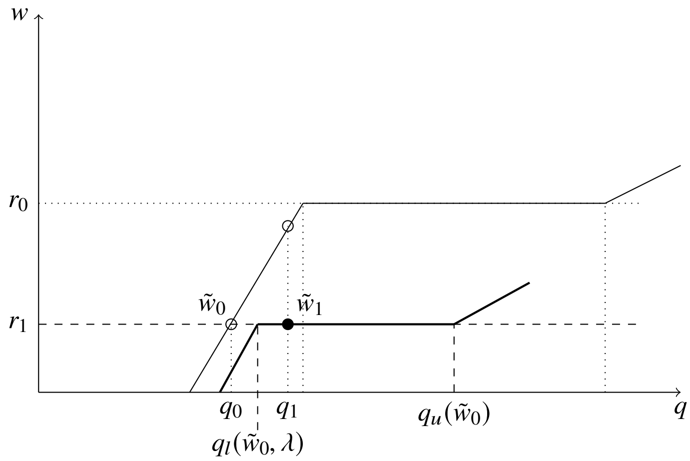

Figure 1.5:
Adaptation and Upward Wage Rigidity when $\tilde{w}_0 < r_0$.

Finally, if, in the initial employment period the match productivity is such that $q_l(r_0, \lambda) \leq q_0 \leq q_u(r_0)$ then the worker will be paid their reference wage, which consequently does not change between periods. As such, the wage setting rule in the subsequent employment period is exactly the same, implying that there may be both downward and upward wage rigidity if the subsequent productivity draw is such that $q_l(r_0, \lambda) \leq q_1 \leq q_u(r_0)$.

Next consider the evolution of the firm's reservation productivity between periods; the initial determining hiring behaviour, and the subsequent capturing the firm's layoff decision. If $q_0 < \underline{q}(r_0, \lambda)$ then no contract is offered. Otherwise a contract is offered and, due to the adaptation of the reference wage, when the firm comes to renegotiate the contract the lay-off reservation productivity will depend on the initial wage. It has been established in Theorem 2 that $\partial \underline{q}(r, \lambda)/\partial r > 0$. This implies, in particular, that if the firm has initially paid a wage that exceeds the worker's reference wage, then, because of adaptation, in the subsequent period the reservation productivity will increase, implying the firm will re-contract only over a reduced subset of the support of the match productivity distribution. As such, it is not inconceivable for an employment relationship to be characterised by the same match productivity in both periods but, whilst the worker is hired and paid a wage above their reference wage in the initial period, the firm doesn't renegotiate the contract in the subsequent period. Due to workers' re-normalisation of effort induced by adaptation of the reference wage, the value of the subsequent period employment contract is lower if a worker was initially paid a wage in excess of their reference wage. Hence, there is a higher probability that the subsequent period match productivity will not be high enough to make the relationship profitable, increasing the likelihood of the worker being laid off. The implications of these results for layoff decisions and unemployment are discussed in Section 1.5.

1.4.2 The Optimal Employment Contract

If a firm is forward looking it will consider the link between the initial and the subsequent wage negotiation, that comes from the worker's adaptation of their reference wage. This influences the worker's future effort response as it is relative to the reference wage $r_1 = \tilde{w}_0$ that subsequent wage offers will be evaluated, therefore also influencing the continuation value of employment relationship to the firm.

Consider the functional equation corresponding to the firm's sequence problem (1.6), which characterises the optimisation problem of a forward-looking firm setting the initial wage:

$$J_0(r_0, q_0) = \max_{w_0} \{\pi(w_0; q_0, \tilde{e}(w_0, r_0, \lambda)) + \delta \mathbb{E}_0[J_1(w_0, q_1)]\}, \tag{1.7}$$

$$\text{where } J_1(r_1, q_1) = \max_{w_1} \pi(w_1; q_1, \tilde{e}(w_1, r_1, \lambda)). \tag{1.8}$$

The expected continuation value of the employment relationship $\mathbb{E}_0[J_1(w_0, q_1)] = \int_{\underline{q}(w_0,\lambda)}^{\infty} J_1(w_0, q_1)\, dF(q_1)$ now also depends on the initial wage contract. Recognising that the reservation productivity for this contract, below which the firm would lay off the worker, may fall anywhere in the support of the distribution of match productivity, this can be expressed as[26]

$$\mathbb{E}_0[J_1(w_0,q_1)] = \int_{\underline{q}(w_0,\lambda)}^{\max\{\underline{q}(w_0,\lambda),q_l(w_0,\lambda)\}} J_1(w_0,q_1)^- \, dF(q_1) + \int_{\max\{\underline{q}(w_0,\lambda),q_l(w_0,\lambda)\}}^{\max\{\underline{q}(w_0,\lambda),q_u(w_0)\}} J_1(w_0,q_1)^= \, dF(q_1) + \int_{\max\{\underline{q}(w_0,\lambda),q_u(w_0)\}}^{\infty} J_1(w_0,q_1)^+ \, dF(q_1). \quad (1.9)$$

This expression highlights that the firm faces different realisations of future profit when setting the initial wage contract w_0, depending on whether the subsequent match productivity q_1 is below, within or above the range of rigidity defined by $q_l(w_0,\lambda)$ and $q_u(w_0)$. Attentive observation of equation (1.9) allows us to infer two important insights. When setting the wage in the initial employment period the firm influences: i) the level of the expected value of future profit $J_1(w_0,q_1)$; and ii) the range of the distribution of the future match productivity within which the firm will subsequently cut or freeze the wage at time 1, and lay off the worker by ceasing an unprofitable match.

Let $\Phi(w_0,\lambda) \equiv \frac{\partial}{\partial w_0}\int_{\underline{q}(w_0,\lambda)}^{\infty} J_1(w_0,q_1)\, dF(q_1)$ be the marginal effect of a wage increase in the initial wage contract on the expected future profit in period 1:[27]

Proposition 1. *For all $\lambda \geq 1$,*

$$\Phi(w_0,\lambda) = \int_{\underline{q}(w_0,\lambda)}^{q_l(w_0,\lambda)} \frac{\partial y(q_1,\tilde{e}_1)}{\partial e}\frac{\partial \tilde{e}(\tilde{w}_1,w_0,\lambda)^-}{\partial r}\, dF - \int_{q_l(w_0,\lambda)}^{q_u(w_0)} s'(w_0)\, dF + \int_{q_u(w_0)}^{\infty} \frac{\partial y(q_1,\tilde{e}_1)}{\partial e}\frac{\partial \tilde{e}(\tilde{w}_1,w_0)^+}{\partial r}\, dF < 0.$$

Proof. See Appendix B.1. ∎

When setting the initial wage contract in a dynamic environment a forward-looking firm will account for an additional expected future cost: a higher initial wage influences

[26] $J_1(w_0,q_1)^{-;=;+}$ represents the continuation value of the employment relationship when $w_1 < w_0; w_1 = w_0; w_1 > w_0$, in which effort is given by $\tilde{e}(w_1,w_0,\lambda)^-; \tilde{e}_n; \tilde{e}(w_1,w_0)^+$.

[27] From this point on two innocuous assumptions are imposed in order to ease notational burden. That is, the parameter of the model are assumed to be such that: 1) any contract offered by a firm is not constrained by the lower bound on effort, i.e. the wage always exceeds $\underline{w}(r,\lambda)$, which implies that unless the optimal wage is equal to the reference wage the firm's first-order condition is satisfied with equality; and 2) in the second employment period the firm's reservation productivity $\underline{q}(w_0,\lambda)$, which determines layoffs, is always less than the lower threshold of the range of rigidity $q_l(w_0,\lambda)$.

the worker's feelings of entitlement in the subsequent renegotiation, which consequently influences the worker's effort and the value of the contract to the firm. A marginal increase in the initial wage lowers the expected value of profit because if, relative to the initial wage, the firm wants to lower the wage then the effect of negative reciprocity is greater; if it wishes to freeze the wage then the wage paid is simply higher; and if it wants to increase the wage then the effect of positive reciprocity is lower.

Denote by $\tilde{w}_0$ the optimal wage contract of a forward-looking firm. So long as $w_0 \neq r_0$ the necessary first-order condition that captures this inter-temporal tradeoff is

$$\frac{\partial y(q_0, \tilde{e}_0)}{\partial e} \frac{\partial \tilde{e}(w_0, r_0, \lambda)}{\partial w} - s'(w_0) - \delta |\Phi(w_0, \lambda)| = 0. \tag{1.10}$$

The optimal wage contract will balance the net marginal value in the initial period of an increase in the wage with the expected discounted marginal cost that stems from adaptation to this wage in the subsequent period.

As it is noted in the proof of the following theorem that presents the properties of the optimal employment contract, in order to proceed with the analysis it is required to impose one additional condition. In particular, it is conjectured that

$$\left| \frac{\partial \Psi(w_0, q_0, r_0, \lambda)}{\partial w} \right| > \delta \left| \frac{\partial \Phi(w_0, \lambda)}{\partial r} \right| .$$

That is, the magnitude of the "current direct effect" of a change in the wage on marginal profit in the initial contract, $|\partial \Psi / \partial w|$, is always larger than the "discounted expected future indirect effect" on marginal profit that results from the initial wage becoming the reference wage, captured by $\delta |\partial \Phi / \partial r|$.[28]

Theorem 3. *Consider a forward-looking firm for whom $\delta > 0$. For any given r_0, the firm's optimal wage characterising the initial employment contract is a continuous*

[28]Note that these effects could work in opposite directions as the sign of the second derivative of the expected future profit function with respect to the initial wage, i.e. $\partial \Phi / \partial r$, remains undetermined. The reason is because a higher wage in period 0 will raise the worker's reference wage in period 1, increasing the *ex ante* probability of terminating the employment relationship, and therefore decreasing the support of the distribution over which the firm would optimally implement a wage cut and bear the negative effect of sub-normal effort. As such, the conjecture simply imposes the condition that this latter effect is sufficiently small, so that the current direct effect of wage changes will dominate the expected future indirect effect through the influence of the initial wage on the reference wage. Nevertheless, note that the concavity of profit established in Theorem 2 implies the assumed inequality will hold if the firm is sufficiently impatient.

function of q_0 *and* r_0 *given by*

$$\tilde{w}_0 = \hat{w}(r_0, q_0, \lambda, \delta) = \begin{cases} \hat{w}(r_0, q_0, \lambda, \delta)^+ & \text{if } q_0 > \hat{q}_u(r_0, \lambda, \delta) \\ r_0 & \text{if } \hat{q}_l(r_0, \lambda, \delta) \le q_0 \le \hat{q}_u(r_0, \lambda, \delta) \\ \hat{w}(r_0, q_0, \lambda, \delta)^- & \text{if } q_0 < \hat{q}_l(r_0, \lambda, \delta), \end{cases}$$

so long as $q_0 \ge \underline{\hat{q}}(r_0, \lambda, \delta)$*, the forward-looking firm's reservation productivity governing hiring, where*

$$\begin{aligned} \hat{q}_l(r_0, \lambda, \delta) &= \{q_0 : \lim_{\epsilon \to 0} \Psi(r_0 - \epsilon; q_0, r_0, \lambda) - \delta|\Phi(r_0, \lambda)| = 0\}, \\ \hat{q}_u(r_0, \lambda, \delta) &= \{q_0 : \lim_{\epsilon \to 0} \Psi(r_0 + \epsilon; q_0, r_0, \lambda) - \delta|\Phi(r_0, \lambda)| = 0\} \\ \underline{\hat{q}}(r_0, \lambda, \delta) &= \max\{0, q_0 : J_0(r_0, q_0) = 0\}. \end{aligned}$$

The optimal wage $\hat{w}(r_0, q_0, \lambda, \delta)^+ > r_0$*, is implicitly defined by* $\Psi(w_0; q_0, r_0, \lambda) - \delta|\Phi(w_0, \lambda)| = 0$*, in which* $\tilde{e}(w_0, r_0, \lambda) = \tilde{e}(w_0, r_0)^+$*; and* $\hat{w}(r_0, q_0, \lambda, \delta)^- < r_0$*, is implicitly defined by* $\Psi(w_0; q_0, r_0, \lambda) - \delta|\Phi(w_0, \lambda)| = 0$*, in which* $\tilde{e}(w_0, r_0, \lambda) = \tilde{e}(w_0, r_0, \lambda)^-$*. Moreover, for all* $q_0 \in [\underline{\hat{q}}(r_0, \lambda, \delta), \infty] \setminus [\hat{q}_l(r_0, \lambda, \delta), \hat{q}_u(r_0, \lambda, \delta)]$*,* $\hat{w}(r_0, q_0, \lambda, \delta)$ *is a continuous and increasing function of* q *and* r*.*

Proof. See Appendix B.1. ∎

Compare the optimal wage contract set by a myopic firm $\tilde{w}(r_0, q_0, \lambda)$ with the one set by a forward-looking firm $\hat{w}(r_0, q_0, \lambda, \delta)$, as respectively derived in Theorem 2 and Theorem 3. Intuitively, since a forward-looking firm perceives an additional expected future cost of raising the current wage due to the worker's adaptation of the reference wage, it will have an incentive to compress the initial wage for a newly hired worker, relative to that of a myopic firm in an otherwise identical employment relationship.[29] The following Proposition confirms this intuition.

Proposition 2. *Consider a firm for whom* $\delta > 0$ *facing a worker whose* $\lambda \ge 1$*. Then, for any given* r_0 *and* q_0 $\hat{w}(r_0, q_0, \lambda, \delta) \le \tilde{w}(r_0, q_0, \lambda)$*, with a strict inequality whenever* $w_0 \neq r_0$*.*[30]

[29]The insight behind this intuition was analysed by Elsby (2009), who attributes the incentive to actively compress wages entirely to the firm's anticipation of downward wage rigidity in the expected continuation value of the employment relationship. As we will show next, our theory identifies the worker's adaptation and re-normalisation of effort as the main drivers of active wage compression.

[30]Indeed, implicit differentiation of the wage setting rule reveals

$$\frac{\partial \hat{w}(r_0, q_0, \lambda, \delta)}{\partial \delta} = -\frac{\Phi}{\partial \Psi / \partial w + \delta \partial \Phi / \partial r} < 0,$$

so the more a firm cares about the future the lower will be the initial wage offered.

Proof. See Appendix B.1. ■

Notice that for Proposition 2 to hold it is not necessary that a worker is loss averse ($\lambda > 1$). Loss aversion is the behavioural force underlying the asymmetry in a worker's reference-dependent reciprocity, which combined with adaptation generates downward/upward wage rigidity in the second employment period. But it is adaptation *per se*, and not wage rigidity, that gives rise to wage compression. This is transparent in the first-order condition characterising a forward-looking firm's optimal wage contract when $\lambda = 1$, which is

$$\frac{\partial y(q_0, \tilde{e}_0)}{\partial e}\frac{\partial \tilde{e}(w_0, r_0, \lambda)}{\partial w} - s'(w_0) - \delta \left| \int_{\underline{q}(w_0,1)}^{\infty} \frac{\partial y(q_1, \tilde{e}_1)}{\partial e}\frac{\partial \tilde{e}(w_1, w_0, \lambda)}{\partial r}\, dF \right| = 0. \quad (1.11)$$

Even if wage rigidity is not expected to be a feature of the employment relationship, there is still an additional marginal cost $|\Phi(w_0, 1)| > 0$ that has to be borne by a forward-looking firm setting the initial wage. This result is based on the adaptation of the reference wage which leads to a "re-normalisation" of effort in the course of the employment relationship: as shown in Section 1.4.1, reciprocity is a temporary phenomenon, since a firm cannot obtain the same amount of reciprocity for two consecutive employment periods by paying a worker the same wage. Equation (1.11) reveals a higher initial wage will result in a higher reference wage in the subsequent employment period, which, in expectation, reduces the worker's extent of reciprocity in the future. As such, even in the absence of wage rigidity, a forward-looking firm has an incentive to compress the initial wage, in order to offset the effect of the re-normalisation of effort that stems from the worker's adaptation of the reference wage.

The remainder of the analysis will investigate the effect of loss aversion on the nature of the employment contract, by considering its effects on the initial wage $\hat{w}(r_0, q_0, \lambda, \delta)$, and on the reservation productivity $\underline{\hat{q}}(r_0, \lambda, \delta)$ determining the firm's hiring decision. Recall that loss aversion influences the strength of negative reciprocity whenever the firm finds itself in a position where it optimally pays a wage below the worker's reference wage; which could be the case in each of the two employment periods.

Consider first the effect of loss aversion on the optimal initial wage contract $\tilde{w}_0$. For a more loss averse worker the firm has a stronger incentive to reduce the gap between the wage paid and the reference wage, to attenuate the stronger effects of negative reciprocity whenever $\tilde{w}_t < r_t$. In the initial employment period where the reference wage is given, this puts upward pressure on the initial wage (as established in Section 1.3). Define this as the *current direct effect* of loss aversion, denoted by $\partial\Psi/\partial\lambda > 0$. However, due to the worker's adaptation in the subsequent employment period, a greater extent of loss aversion puts downward pressure on the initial wage, since a lower initial wage will translate into a lower reference wage, reducing the magnitude of the expected negative reciprocity. Define this effect as the *expected indirect effect* of loss aversion, denoted by $\partial\Phi/\partial\lambda < 0$.[31] The relative importance of these two effects determines the overall incidence of loss aversion on the initial wage contract.

Proposition 3. *Consider a firm for whom $\delta > 0$. For any given r_0,q_0 and $\lambda' > \lambda$*

a) if $q_0 \geq \hat{q}_l$, then $\hat{w}(r_0, q_0, \lambda', \delta) < \hat{w}(r_0, q_0, \lambda, \delta)$;

b) if $q_0 < \hat{q}_l$, then $\hat{w}(r_0, q_0, \lambda', \delta) \gtreqless \hat{w}(r_0, q_0, \lambda, \delta) \Leftrightarrow \partial\Psi/\partial\lambda + \delta\partial\Phi/\partial\lambda \gtreqless 0$

Proof. See Appendix B.1. ■

Proposition 3 states that if an employment contract is characterised by a sufficiently high initial match productivity (i.e. $q_0 \geq \hat{q}_l$), then the firm will pay a lower initial wage to a worker with a higher degree of loss aversion. Since a worker will be paid at least their reference wage, there is no current direct effect of loss aversion, and the firm optimally sets a lower wage to reduce the magnitude of the expected negative reciprocity in the following employment period. The incentive for wage compression is therefore reinforced by the anticipation of stronger negative reciprocity in the event of a future wage cut. On the other hand, if the firm is facing an employment relationship such that the optimal contract in the initial period calls for a wage below the reference wage (i.e. if $q_0 < \hat{q}_l$), the overall effect of a higher degree of loss aversion depends on the relative magnitudes of the two aforementioned counteracting effects, and how much the firm cares about the future. If the current direct effect dominates the expected indirect effect, then a firm hiring a more loss averse worker will set a higher initial wage; otherwise, and if the firm is also not too impatient, the initial wage will be lower.[32] Thus, more wage rigidity is not necessarily associated with greater wage compression, because a firm facing a more loss averse worker may find it optimal to *increase* the initial wage to attenuate the effect of negative reciprocity. The incentive for wage compression is driven by adaptation; wage rigidity may either strengthen or dampen this incentive.

Finally, consider the effect of loss aversion on the determination of the firm's reservation productivity governing hiring $\underline{\hat{q}}(r_0, \lambda, \delta)$. There are two channels through

[31]The conclusion that $\partial\Phi/\partial\lambda < 0$ is subject to the qualification that the firm's lay-off reservation productivity $\tilde{q}(w_0, \lambda)$ doesn't increase too much with the degree of loss aversion: as it has been shown in Theorem 2 the firm's reservation productivity in the second period of employment is increasing in λ, implying that a more loss averse worker faces a greater *ex ante* probability of being laid off, which reduces the probability of the firm having to enact a wage cut, partially offsetting the greater expected cost of doing so.

[32]In Theorem 1 it has been established that $\partial^2\tilde{e}^-/\partial w\partial\lambda > 0$, that is, the negative effect of loss aversion on effort is stronger for wages just below the reference wage than when substantial reductions below the reference wage are considered. As such, the current effect will be larger the closer is the match productivity to the lower threshold $\hat{q}_l$.

which loss aversion could influence the value of the employment relationship to the firm. First, there is the usual direct negative effect on effort, which exacerbates an employed worker's negative reciprocity response, and is present in both the initial and subsequent employment period. Second, there is an indirect effect that comes from the optimal initial contract becoming the worker's reference wage: if the initial wage is increasing in λ this provides a compounding negative effect on expected effort which lowers profit; whilst if the initial wage is decreasing in λ there is a partially offsetting positive effect on expected effort which increases profit. The following proposition establishes that if a firm is considering contracting with a more loss averse worker, the reservation productivity determining hiring unambiguously increases, independently of how the initial wage contract adjusts.

Proposition 4. *Consider a firm for whom* $\delta > 0$*, with the initial reservation productivity characterised by* $\underline{\hat{q}}(r_0, \lambda, \delta) = \max\{0, q_0 : J_0(r_0, q_0) = 0\}$*. Then for any given* r_0*,* q_0 *and* $\lambda' > \lambda$*,* $\underline{\hat{q}}(r_0, \lambda', \delta) > \underline{\hat{q}}(r_0, \lambda, \delta)$*.*

Proof. See Appendix B.1. ■

The firm's hiring reservation productivity is based on the calculation of the expected value of the two-period employment relationship. Since the initial wage contract is set to satisfy the optimality condition in (1.10), a higher degree of loss aversion influences profit only through its negative impact on the worker's effort whenever $\tilde{w}_t < r_t$.

Therefore, the analysis formally suggests that independently of whether wage rigidity reinforces or tempers the incentive to compress initial wages, the anticipation of stronger negative reciprocity and wage rigidity unambiguously reduces a firm's incentive to hire.

1.5 Further Implications

The Role of the Reference Wage. One of the key features of the dynamic analysis developed in this chapter is the assumption defining the worker's adaptation of the reference wage (Assumption A1: $r_1 = \tilde{w}_0$). Although being consistent with a large body of evidence, this assumption abstracts from several other aspects of workers' reference wage formation: i) adaptation can be slow and may not be symmetric; and ii) workers' perceptions of fairness at each negotiation date, namely r_0 and r_1, may be influenced by more than just past wages.

This chapter has considered what can be defined as *complete symmetric adaptation* of the reference wage. That is, it takes only one employment period for the worker to adapt to the wage they are paid (complete); and this adaptation process is the same independently of whether the worker received a wage gift or an unfair wage in their first employment period (symmetric). The implication of this assumption is that both positive and negative reciprocity are only temporary phenomena, lasting for one employment period only absent further changes in the wage. However, consider the following, more general, adaptation rule:

$$r_t = \begin{cases} \alpha^+ \tilde{w}_{t-1} + (1-\alpha^+) r_{t-1} & \text{if } \tilde{w}_{t-1} > r_{t-1} \\ \alpha^- \tilde{w}_{t-1} + (1-\alpha^-) r_{t-1} & \text{if } \tilde{w}_{t-1} \leq r_{t-1}, \end{cases}$$

where r_0 is given, and $\alpha = \{\alpha^-, \alpha^+\} \in [0, 1]$ denotes the worker's speed of adaptation. If $\alpha = \alpha^- = \alpha^+$ and $\alpha \in (0, 1)$, then adaptation would be *partial*, as it will take more than one employment period to the worker to completely adapt their reference wage to the previous wage (but it will still be symmetric). On the other hand, if $\alpha^- < \alpha^+$ adaptation would be *asymmetric*, that is, the worker will adapt more rapidly to wage gifts than to unfair wages over time.[33] Indeed, this chapter considered the case in which $\alpha = \alpha^- = \alpha^+ = 1$. Exploring the implications of asymmetries in reference wage adaptation may help explain evidence that suggests negative reciprocity is not only stronger, but also more persistent than positive reciprocity. Moreover, asymmetric adaptation may exacerbate the firm's cost of negative reciprocity when facing a loss averse worker, neutralising its ability to reduce the likelihood and extent of the expected negative reciprocity through wage compression.

Alternative formulations of the worker's reference wage that may differ depending on whether the worker is a new hire (at time 0) or an incumbent (at time 1) could also be considered in this framework. For instance a *newly hired* worker's reference wage r_0 could be influenced by the state of the labour market (as in Akerlof (1982) and Summers (1988)); by the most recent wage contract paid in the *previous* employment relationship (as considered in Koenig, Manning, and Petrongolo (2016)); or by the wage of incumbent workers employed by the same firm (as the "equal treatment" hypothesis of Snell and Thomas (2010) would suggest). On the other hand an *existing/incumbent* worker's reference wage r_1 might be influenced by the wage of his peers outside the firm (as in Keynes (1936), Bhaskar (1990) or Driscoll and Holden (2004)); by expectations (as in Eliaz and Spiegler (2014)); or by the firm's ability to pay (as in Danthine and Kurmann (2007)). For example, if a worker considers the firm's ability to pay, they may revise their perceptions of fairness accordingly and accept a lower wage in periods in which their firm is experiencing adverse economic conditions, without inducing a

[33]This latter consideration is supported by the experimental evidence on reference point adaptation provided by, for instance, Arkes, Hirshleifer, Jiang, and Lim (2008, 2010): individuals (in this case workers) adapt more rapidly to gains (gifts) than to losses (unfair wages).

loss of morale and negative reciprocity. Investigating this insight could shed new light on the importance of employers' information disclosure for wage dynamics, through its influence on workers' perceptions of fairness as discussed by Kahneman et al. (1986) and Bewley (1999) among others.

The model developed in this chapter provides a tractable framework within which to consider these issues and understand their relative importance in employment relationships. These and several other aspects concerning reference wage formation will be thoroughly discussed and analysed in Chapter 3 of the thesis.

History Dependence, Wage Dynamics, Hiring and Lay Offs. The dynamic analysis of Section 1.4 highlights two additional important insights of the theory developed in this chapter that can be informative to ongoing theoretical and empirical research concerned with labour market fluctuations. Both insights are driven by the worker's adaptation of the reference wage—which carries the information contained in the wage contract $\hat{w}(r_0, q_0, \lambda, \delta)$ from the initial employment period into the next—and by the worker's asymmetric reference-dependent reciprocity.

First, it has been shown that the anticipation of the costs related to negative reciprocity and wage rigidity influence a forward-looking firm's behaviour at the time of hiring. This analysis has implications for understanding the effects of wage rigidity for job creation and wage dynamics. In the literature concerned with labour market fluctuations, much attention has been devoted to the effect of wage rigidity of *newly hired* workers on firms' job creation incentives (see for instance the discussion in Elsby, Michaels, and Ratner (2015) and references therein). In contrast, the analysis has shown that independently of what happens to the wage of newly hired workers, it is the anticipated negative reciprocity and the expected wage rigidity in the second employment period that reduce a firm's incentive to hire. This result suggests that incorporating the behavioural mechanisms into a richer macroeconomic framework could potentially enhance the understanding of the effects of wage rigidities of existing/incumbent workers on job creation and unemployment. In particular, wage rigidity could matter for job creation to the extent that it negatively influences the continuation value of an employment relationship once workers become incumbent and adapt their perception of the fair wage to their initial wage contract. This analysis and its related implications are the subject of Chapter 2

Second, the model reveals that the initial conditions which characterise the state of the economy when a new employment contract is offered (captured by the information contained in the two state variables r_0 and q_0) persist into the employment relationship and influence the subsequent wage setting and layoff decisions made by the firm. For instance, a worker who receives a higher initial wage contract due to favourable economic conditions (high q_0) is more likely to be paid a higher wage in the subsequent period than an otherwise identical worker hired at a lower match productivity. Moreover, all else equal, the worker employed at a higher wage also faces a greater ex-ante probability of being laid off. This insight has implications for labour market models that endogenise the job destruction rate through the derivation of a layoff reservation productivity similar to $\underline{q}(r_1, \lambda)$ (e.g. à la Mortensen and Pissarides (1994)). It also has implications for empirical research that attempts to capture persistent effects of labour market conditions at the time of hiring for the subsequent path of workers' wages during the employment relationship, and on their probability of being laid off (e.g. Beaudry and DiNardo (1991); Schmieder and von Wachter (2010)). In the model presented here this persistence depends on whether information about the state of the labour market is incorporated in the reference wage by the worker at the time of hiring.

1.6 Conclusion

Inspired by evidence from anthropological and experimental research on labour markets, this chapter has advanced a microeconomic theory of wage setting behaviour based on contractual incompleteness, fairness, reciprocity, and reference dependence and loss aversion in the evaluation of wage contracts by workers. This approach has permitted rigorous formalisation of several aspects of wage setting and the employment relationship within a realistic and tractable model. By establishing a clear link between assumptions and conclusions, the theory developed in this chapter provides novel insights that explain the observed asymmetry and dynamics of workers' reciprocity, and identify the sources of wage rigidity, wage compression and hiring incentives.

A worker's effort response to wage changes is formally characterised as being reference dependent, where positive and negative reciprocity are defined as relative deviations from normal effort, and loss aversion is identified as the psychological foundation for the stronger intensity of negative reciprocity. Although this feature has been the subject of conjectures by others (e.g. Campbell and Kamlani (1997), Mas (2006), Bewley (2007) and Fehr et al. (2009)), the model developed here is the first that formally derives a link between reference dependence, loss aversion and the asymmetric nature of reciprocity from a worker's optimal behaviour. Moreover, by analysing the implications of this theory in a two-period employment relationship, it has been established that the worker's adaptation of the reference wage and the relatively large cost of negative reciprocity are the key drivers of wage rigidity; and that the reference-dependent nature of reciprocity, combined with adaptation, leads to a dynamic "re-normalisation" of effort, consistent with the recent experimental findings of Sliwka and Werner (2017). As such the model stands as a general and realistic micro-foundation for wage rigidity.

By analysing the consequences of these wage and effort dynamics, the chapter draws new conclusions about their implications for a forward-looking firm's incentive to compress the initial wage (Elsby, 2009); and for the expected value of the employment relationship, which in turn influences hiring decisions. It has been shown that the primary behavioural mechanism that generates wage compression is the worker's re-normalisation of effort due to adaptation, even absent downward wage rigidity. In fact, the presence of wage rigidity may strengthen or dampen the wage compression incentive. Nevertheless, independently of how the initial wage adjusts, the anticipation of stronger negative reciprocity and the expectation of wage rigidity unambiguously reduces the expected value of the employment relationship, raising the reservation productivity above which the firm will hire a worker.

The framework developed in this chapter lends itself as a tractable benchmark model for the analysis of reference-dependent reciprocity, adaptation and wage rigidity, and their effect on wage setting and hiring behaviour. As discussed in Section 1.5, two main extensions have been identified. First, exploring the insights of the model within a richer macroeconomic framework can potentially shed new light on the effects of expected wage rigidity in long-term employment relationships for job creation and wage dynamics. As discussed in Elsby et al. (2015) this aspect is not yet settled in the theory of labour market fluctuations, and has drawn particular attention in light of recent cross-country experiences following the Great Recession (Elsby et al., 2016). Second, it will be interesting to analyse the predictions of the model under different specifications of a worker's reference wage. The choice of the initial wage as the only determinant of an incumbent worker's reference wage is motivated by the fact that this hypothesis is the most corroborated in the empirical literature. However, like every model based on reference dependence, predictions are sensitive to the choice of reference point. Investigating if, and how, the conclusions derived might change is the natural next step.

Therefore, a promising line of research lies in developing and combining these two extensions. The remainder of the thesis unfolds around this route, providing a first step towards a richer unified framework.

Chapter 2

Asymmetric Reference-dependent Reciprocity and the Theory of Equilibrium Unemployment

2.1 Introduction

Workers' and firms' attitudes toward wage setting have fundamental implications for the cyclical behaviour of the labour market and for the determination of unemployment in equilibrium. Grounded in the job flows approach and following the seminal work of Shimer (2005), advances in the theory of labour market fluctuations have placed particular emphasis on the role of rigidities in the wage determination of newly hired workers to explain the observed volatility of vacancies and unemployment. This literature has also shifted away from the view that downward wage rigidity in existing jobs may be an important driver of large and persistent unemployment fluctuations. Although undoubtedly present, this latter rigidity is irrelevant for the volatility of job creation (Pissarides, 2009), and it appears unlikely to be the main driver of the extraordinary long duration of unemployment (Elsby et al., 2016).

However, more recent evidence has shown that wage offers made to newly hired workers are substantially pro-cyclical (e.g. Haefke et al. (2013)); and that the existing theoretical framework cannot simultaneously accommodate the empirical volatilities of both the wage component of the user cost of labour and the vacancy-unemployment ratio (Kudlyak, 2014). Hence, although the existing literature on the subject is particularly dense (see Mortensen and Nagypál (2007), Rogerson and Shimer (2011) and Elsby et al. (2015) for surveys), it is not yet clear whether the emphasis on new hires' wage cyclicality is well placed, or what impact the wage rigidity of incumbent workers has on the cyclical volatility of job creation.

This chapter aims to shed light on these aspects by adopting a different, more in-depth approach. Given the evident connection between microeconomic behaviour within employment relationships and the resulting macroeconomic implications for wage dynamics, job creation and unemployment, contemporary macroeconomic theory would greatly benefit from the development of more micro-founded frameworks that can describe in detail the actual behavioural incentives governing these outcomes. Inspired by recent prominent attempts to bridge this gap (e.g. Snell and Thomas (2010), Danthine and Kurmann (2010), Eliaz and Spiegler (2014) and Kuang and Wang (2017)), the present chapter incorporates the microeconomic theory of wage setting behaviour developed in Chapter 1 into a canonical search and matching model à la Pissarides (1985, 2000).

This approach yields the following main contributions. First the macroeconomic implications of asymmetric reference-dependent reciprocity, reference wage adaptation and optimal wage setting behaviour for the determination of equilibrium unemployment are explored. Here, it is established the existence of a unique, but distinct, steady-state equilibrium outcome for each initial value of new hires' wage entitlements in the labour market.

Then, the chapter focuses on how asymmetric reciprocity and reference wage adaptation can influence firms' wage setting and job creation decisions, in ways that enhance the understanding of the *amplitude* and co-movement of vacancies and unemployment over the business cycle. First, by appealing to the reciprocity effects of wage changes on firms' output, the analysis demonstrates that the cyclicality of the hiring wage is *irrelevant* for the volatility of the vacancy-unemployment ratio. Moreover, by considering the impact of workers' wage entitlements, their intrinsic motivation and their negative reciprocity on the present value of a new match, the analysis offers a *qualitative* and *quantitative* assessment of how these novel behavioural aspects can result in increased volatility of job creation. Finally, when uncertainty around the evolution of a job match productivity is introduced, it is shown that the expected downward rigidity in the wage of incumbent workers—that is, the anticipation of stronger negative reciprocity in the event of a wage cut—negatively influences the expected present value of new employment relationships, dampening hiring incentives and increasing the volatility of both job creation and unemployment.

Building on the theory developed in Chapter 1, wage setting behaviour is formalised as a two-stage game where firms (the first movers) make take-it-or-leave-it wage offers to workers (the second movers). Workers evaluate wage contracts with respect to a reference "fair" wage and are loss averse. Heterogeneity among workers is imposed only on the basis of their employment status and reference wage. Newly hired workers arrive at firms with an exogenously-given reference wage. Incumbent workers are instead characterised by adaptation: their reference wage is endogenously determined by the wage they were paid in the previous employment period. Employed workers' optimal choice of effort, in light of the wage paid by firms, yields a wage-effort relationship where loss aversion implies a kink at the reference wage, characterising their asymmetric reference-dependent reciprocity (see Chapter 1). Asymmetric reciprocity combined with adaptation of the reference wage influence firms' optimal wage policy. A firm that is setting the optimal wage faces an inter-temporal trade-off at the margin between the benefit of a higher wage today, i.e. higher effort, *versus* the cost associated with employing a worker with a higher reference wage in the future due to adaptation. In addition, the discontinuity at the margins induced by the kinked wage-effort relationship is such that for a range of possible initial conditions at the start of an employment relationship, a firm is better off by paying a new hire their reference wage. Within this framework optimal wage setting yields a range of wage solutions, the realisation of which is shown to depend upon new hires' (exogenous) wage entitlements at the start of the job.

By embedding these theoretical results into an otherwise standard search and matching framework, the model generates a unique, but distinct, *path-dependent* steady-state equilibrium, which can be ranked in terms of wage levels, new hires' morale and reciprocity, and vacancy and unemployment rates. This result hinges crucially on workers' asymmetric reference-dependent reciprocity and it is a natural implication of firms' optimal wage setting behaviour. The discontinuity that affects firms' marginal trade-off in their maximisation problem translates into three possible combinations of wages and effort in the steady state. This in turn affects the anticipated present value of profit from a new employment relationship, and ultimately determines the firms' optimal job creation decisions. Importantly, the determination of which steady state will characterise the labour market depends crucially on the level of the new hires' reference wage at the time of hiring, and not on their reservation wage. In fact, as long as there exists a wedge between a workers' reservation wage and their reference wage, the results derived in this chapter will hold.[1]

[1]It can be useful to clarify the conceptual distinction between a worker's *reservation* wage and a worker's *reference* wage. The reservation wage is the wage below which a worker would optimally turn down a job offer, stay unemployed and continue to search for jobs. The reference wage instead is a concept that captures a worker's perception of what is a fair wage, that is, the wage level relative to which the worker evaluates the fairness of a wage contract. While the possibility that the two might coincide should not be ruled out, this chapter considers the case where they do not; thus, for simplicity, the worker's reservation wage is set to be zero. In fact, so long as the worker's reservation wage is not binding—i.e. so long as a wage offer exceeds the reservation wage—firms' anticipation of how workers evaluate wage contracts relative to a reference "fair" wage generates additional "behavioural" constraints to their wage setting and hiring decisions.

This framework is used to contribute to an important, yet unsettled, debate in the theory of labour market fluctuations. In a highly influential work Shimer (2005) pointed out the *quantitative* failure of the canonical search and matching model to replicate the observed volatility of vacancies and unemployment over the business cycle. Despite providing a theoretical puzzle to solve, the understanding of the determinants of fluctuations in vacancies and unemployment has become particularly important in the recent years, in light of the different cross-country labour market experiences observed in the aftermath of the Great Recession (see Elsby et al. (2016)). To date, the literature has developed two main streams of thought.

On one hand, following the insight of Shimer, a large body of theoretical work has emphasised the role of wage *rigidity*, or wage *stickiness*, as a potential resolution to the puzzle (see Mortensen and Nagypál (2007) and Rogerson and Shimer (2011) for surveys).[2] Essentially, by making the hiring wage less responsive to shocks, changes in aggregate productivity will generate larger fluctuations in firms' present value of profit from a new match, and hence in vacancies and unemployment. However this theoretical position has not received much support from recent empirical evidence, which has shown that wage offers to newly hired workers are instead substantially procyclical.[3] Moreover, by applying the notion of the user cost of labour, and showing that this can be even more cyclical than the hiring wage, Kudlyak (2014) draws the conclusion that wage rigidity and wage formation cannot contribute to solving the puzzle. On the other hand, in response to Shimer's argument, several authors (e.g. Mortensen and Nagypál (2007), Pissarides (2009), Kennan (2010)) have pointed out that it is the actual size of the match surplus gained by firms that is relevant for the volatility of job creation: only if this surplus is sufficiently small will slight changes in productivity generate large fluctuations in the present value of profit, and therefore in vacancy creation (see, for instance, Elsby et al. (2015) and Ljungqvist and Sargent (2016) for a complete exposition of this argument).

Building on these perspectives, the contribution of this chapter rests on three main results. First, in contrast to the existing theoretical literature, it is shown that in the presence of reference-dependent reciprocity the cyclicality of the hiring wage is irrelevant for the volatility of vacancies and unemployment. Essentially, as a consequence of

[2] Wage rigidity is defined as the *acyclical* behaviour of wages, i.e. when wages do not adjust to shocks (downward/upward or both); while wage stickiness is defined as a *less than proportional cyclicality* of wages with respect to aggregate productivity, i.e. when the wage-productivity elasticity is less than one (Pissarides, 2009). In the search and matching literature these terms have been used interchangeably, sometimes referring to the latter and sometimes to the former (e.g. Hall (2005b), Mortensen and Nagypál (2007), Michaillat (2012)).

[3] See the survey in Pissarides (2009) and the recent evidence from Martins et al. (2012), Carneiro et al. (2012), Haefke et al. (2013) and Stüber (2017).

optimal wage setting behaviour, any wage change triggered by a change in productivity will also generate a counteracting change in workers' effort at the margins, which in turn positively (or negatively) affects output and leaves a room for the productivity shock to be fully absorbed by firms' anticipated present value of profit. As such the present framework produces theoretical results that are consistent with the empirical estimates of the volatilities of both the hiring wage and the vacancy-unemployment ratio.

Second, the analysis investigates how the novel behavioural aspects introduced in the present framework can affect firms' anticipated present value of a new employment relationship and, therefore, the elasticity of labour market tightness. The results are: i) higher wage entitlements in the market increase the volatility of vacancies and unemployment, either by dampening workers' reciprocity, or by making labour more costly; ii) higher workers' intrinsic motivation, subsumed by the notion of normal effort, decreases volatility by increasing firms' profit margins; and iii) whenever firms expect to hire workers with relatively high reference wages, the anticipation of a greater extent of negative reciprocity, that is, the anticipation of a greater cost of hiring "de-moralised" workers, acts to reduce firms' expected value of a new match, increasing volatility. To assess the quantitative relevance of these behavioural mechanisms, a simple calibration exercise is undertaken in which new hires are assumed to be heterogenous on the basis of their initial reference wage. Here it is shown that there exists a variety of combinations of the behavioural parameters that could achieve the observed elasticity of market tightness; and that, for plausible parameter values, the framework developed in this chapter can simultaneously deliver plausible elasticities of both the hiring wage and labour market tightness, overcoming one of the issues mentioned by Kudlyak (2014).[4]

Third, in contrast with Pissarides (2009) and in support of the qualitative insight of Eliaz and Spiegler (2014), by introducing uncertainty around the evolution of a job match productivity at the time of hiring, it is shown that the expected downward rigidity in the wage of incumbent workers *does* matter for job creation. Eliaz and Spiegler (2014) emphasise the effect of future downward wage rigidity—due to a random degree of contractual incompleteness—on the firms' expected duration of a new match. On the other hand, in this chapter it is shown that even if incumbent workers' wage rigidity does not generate endogenous layoffs, it is the relatively large expected cost of implementing wage cuts—that is, the anticipation of stronger negative reciprocity by incumbent workers—which negatively influences firms' expected present value of new employment relationships, dampening hiring incentives and increasing the volatility of both job creation and unemployment.

[4]See Section A.2.4 of Appendix A.2 for a close comparison with the analysis of Kudlyak (2014).

The remainder of the chapter is organised as follows. Section 2.2 extends the theory in Chapter 1 to an infinite horizon environment, and derives workers' optimal effort decision and firms' optimal wage setting policy. Section 2.3 embeds this model into a canonical search and matching framework without uncertainty; and Section 2.4 characterises the resulting steady-state equilibria and analyses their comparative statics and transitional dynamics properties. Section 2.5 studies the qualitative and quantitative implications of this framework for the theory of labour market fluctuations and Section 2.6 introduces uncertainty. Section 2.7 provides some concluding remarks. Additional material referred to throughout the chapter is included in Appendix A.2.

2.2 The Employment Relationship and Wage Setting Behaviour

This section describes the key features of a representative infinite-horizon employment relationship between a worker and a firm in a deterministic environment. The worker's optimal effort function and the firm's optimal wage setting policy are characterised and discussed.

Consider a worker-firm employment relationship that starts in some initial period denoted by τ. The firm's instantaneous profit function π_t in each $t \geq \tau$ is given by the difference between the per-worker value of output y_t (the price of which is normalised to 1) and the wage paid w_t. The value of output y_t is assumed to be a function of a match productivity q, an aggregate productivity p and the level of effort e_t chosen by the worker. Both q and p are parametric and time-invariant.

At the beginning of each employment period $t \geq \tau$ the firm learns the worker's reference wage, denoted by r_t, and decides on the wage to offer to the worker. Subsequently the worker evaluates the wage offer received relative to their reference wage and decides on a level of effort e_t. Wage setting behaviour is therefore formalised as a two-stage game where firms have complete and perfect information and make take-it-or-leave-it wage offers to workers: the worker (the second mover) chooses optimal effort for any given wage offer by the firm relative to their reference wage; the firm (the first mover), infers this optimal effort response and chooses the optimal wage, taking the worker's reference wage as given (the only state variable of the firm's problem).

Following the logic of backward induction, the reminder of this section considers first the maximisation problem of the worker, which will characterise their optimal effort response, and then the maximisation problem of the firm, which will characterise the optimal wage setting policy.

2.2.1 Workers

At the beginning of each period t a worker can be in one of three states: unemployed, if $t < \tau$; employed as a new hire, if $t = \tau$; or employed as an incumbent, if $t > \tau$. This can be formalised by assuming that at the beginning of each t a worker is characterised by one of three employment status $C_t = \{U, N, E\}$ where

$$C_t = \begin{cases} U & \forall t < \tau \\ N & \forall t = \tau \\ E & \forall t > \tau, \end{cases}$$

and in which U stands for *unemployed*, N stands for *newly hired*, and E stands for *existing* worker or *incumbent*. A worker is assumed to evaluate wage contracts with respect to a reference "fair" wage. The worker's reference wage at the beginning of each t depends on their employment status C_t, that is

$$r_t = r_t(C_t) \equiv r_{C,t} \quad \forall t \geq \tau.$$ [5]

Hence:

A1. $\forall t \geq \tau$, and $C_t = \{N, E\}$,

$$r_{C,t} = \begin{cases} r_{N,t} \geq 0 \text{ given} & \text{if } t = \tau \\ r_{E,t} = w_{t-1} & \text{if } t > \tau. \end{cases} \tag{2.1}$$

Assumption A1 imposes a crucial distinction between newly hired and incumbent workers, entirely captured by their reference wage: a newly hired worker is assumed to have a non-negative exogenously-given reference wage, while an incumbent's reference wage is assumed to be determined (endogenously) by the wage they were paid in the previous employment period. Incumbent workers are therefore characterised by reference wage adaptation (as in Chapter 1). The evolution of a typical worker's employment status and reference wage over time is illustrated in Figure 2.1.

The workers' problem

Following the theory developed in Chapter 1, the instantaneous utility function for all $t \geq \tau$ of employed workers with employment status $C_t = \{N, E\}$ takes the following

[5] Indeed this assumption holds only for employed workers. Unemployed workers are not engaged yet in employment relationships and do not have to evaluate wage offers until they are matched.

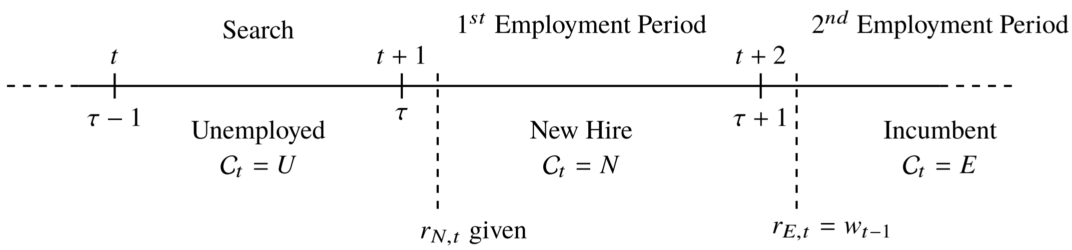

Figure 2.1:
The Evolution of Workers' Employment Status and Reference Wages

form:

$$u(e_t; w_t, r_{C,t}) = m(w_t) + \xi n(w_t|r_{C,t}) + b(e_t) - c(e_t) + M(e_t; w_t, r_{C,t}),$$

where $m(\cdot)$ captures the effect of absolute wage levels on the worker's utility; $b(e)$ represents the worker's intrinsic psychological benefit of being productive, and $c(e)$ their intrinsic psychological and physical cost of productive activity; $M(e_t; w_t, r_{C,t}) \equiv g(e_t)n(w_t|r_{C,t})$ is the morale function, and $n(w_t|r_{C,t}) \equiv \mu(\cdot)$, where $\mu(\cdot)$ is a piece-wise linear gain-loss utility:

$$\mu(m(w_t) - m(r_{C,t})) = \begin{cases} \eta[m(w_t) - m(r_{C,t})] & \text{if } w_t \geq r_{C,t} \\ \lambda\eta[m(w_t) - m(r_{C,t})] & \text{if } w_t < r_{C,t} \end{cases} \tag{2.2}$$

in which $\eta > 0$ is a scaling parameter and $\lambda \geq 1$ is the worker's degree of loss aversion. Since the this chapter aims to derive closed-form analytical solutions, to proceed with the analysis consider the following set of assumptions:

W1. $m(x) = \ln x$.

W2. $b(e) = be$, $c(e) = e^2/2$.

W3. $g(e) = e$.

W4. $\eta = 1$, $\lambda \geq 1$.

These functional forms are consistent with assumptions W1-W4 of Chapter 1. Note that a worker's psychological and physical cost of effort is assumed to be quadratic.

Definition 1. *For any given sequence of optimal wage offers $\{w_t\}_{t=\tau}^{\infty}$ set by the firm, in each $t \geq \tau$ a worker with employment status $C_t = \{N, E\}$ chooses a sequence of levels of effort $\{e_t\}_{t=\tau}^{\infty}$ that maximises their utility, given their evaluation of the wage in relation to their reference wage $r_{C,t}$.*

In an employment relationship starting in period $t = \tau$, the employed worker's problem can be written as a discrete-time infinite-horizon sequence problem:

$$\begin{aligned} W_\tau(w_\tau, r_\tau, C_\tau) &= \max_{\{e_t\}_{t=\tau}^{\infty}} \sum_{t=\tau}^{\infty} \psi^{t-\tau} u(e_t; w_t, r_{C,t}) \\ \text{s.t.} \quad & w_t \text{ given } \forall t \geq \tau \\ & r_{C,t} = \begin{cases} r_{N,t} \geq 0 \text{ given} & \text{if } t = \tau \\ r_{E,t} = w_{t-1} & \text{if } t > \tau. \end{cases} \end{aligned} \tag{WP}$$

where $W_\tau(w_\tau, r_\tau, C_\tau)$ is the value function of the employment relationship of a newly hired worker with employment status $C_\tau = N$; the factor $\psi \in (0, 1)$ is given by $\psi \equiv \delta(1 - \rho)$, where $\delta \in (0, 1)$ is the discount factor and $\rho \in (0, 1)$ is an exogenous job destruction probability; w_t and $r_{C,t}$ are the two state variables and e_t is the control variable. Notice that the worker's choice of effort in each employment period does not affect the evolution of the state variables.

The optimal effort choice

Denote the solution to the worker's problem (WP) by $\tilde{e}_t = \tilde{e}(w_t, r_{C,t}, \lambda)$, which determines the optimal level of effort at each $t \geq \tau$ for given values of the state variables w_t and $r_{C,t}$. The employed worker's optimal effort is therefore characterised by the following first-order condition, which is both necessary and sufficient for an optimum:

$$\Omega(e_t; w_t, r_{C,t}, \lambda) \equiv \frac{\partial u(e_t, w_t, r_{C,t})}{\partial e} = b - e_t + \mu(\ln w_t - \ln r_{C,t}) = 0. \tag{2.3}$$

The following proposition is the infinite-horizon analog of Theorem 1, Chapter 1, characterising the optimal effort policy of a worker with employment status $C_t = \{N, E\}$.

Proposition 5. *For all $t \geq \tau$, and for any given wage offer w_t relative to their reference wage $r_{C,t}$, workers with employment status $C_t = \{N, E\}$ are characterised by the following optimal effort function:*

$$\tilde{e}_t = \tilde{e}(w_t, r_{C,t}, \lambda) = \begin{cases} \tilde{e}_n + \ln w_t - \ln r_{C,t} & \equiv \tilde{e}^+_{C,t} & \text{if } w_t > r_{C,t} \\ \tilde{e}_n & \equiv \tilde{e}_n & \text{if } w_t = r_{C,t} \\ \tilde{e}_n - \lambda[\ln r_{C,t} - \ln w_t] & \equiv \tilde{e}_{C,t}(\lambda)^- & \text{if } w_t < r_{C,t} \end{cases} \tag{2.4}$$

where $\tilde{e}_n = b$ denotes "normal" effort; and $\tilde{e}^+_{C,t} > \tilde{e}_n > \tilde{e}_{C,t}(\lambda)^-$.

Proof. See Appendix B.2. ■

The optimal effort function defined by (2.4) captures an employed worker's *asymmetric reference-dependent reciprocity*, and retains the same qualitative properties of the optimal effort function derived in Chapter 1.[6]

2.2.2 Firms

At the beginning of each t a firm can be in one of the following three stages: the vacancy posting stage, if $t < \tau$, searching for unemployed workers with employment status $C_t = U$; the first operating period of production, if $t = \tau$, employing a newly hired worker with employment status $C_t = N$; or the second (or subsequent) operating period of production, if $t > \tau$, employing an incumbent worker with employment status $C_t = E$.

The firms' problem

For all $t \geq \tau$ the instantaneous profit function of an operating firm takes the following form:

$$\pi(w_t; y_t) = y(\tilde{e}_t) - s(w_t),$$

where $s(w_t)$ is the per-worker labour cost, $y(\tilde{e}_t)$ is the per-worker value of output, and $\tilde{e}_t = \tilde{e}(w_t, r_{C,t}, \lambda)$ is the worker's optimal effort choice given by (2.4), inferred by the firm at the beginning of each employment period. Consistent with assumptions F1-F2 of Chapter 1, it is assumed that:

F1. $y(e) = pqe$.

F2. $s(w) = w$.

Note that output is linear in the worker's effort.

Definition 2. *The firm's wage setting problem in each $t \geq \tau$ consists of choosing a sequence of wages $\{w_t\}_{t=\tau}^{\infty}$ that maximises its profit, taking as given the employed worker's reference wage $r_{C,t}$ and their optimal effort responses, defined by the sequence $\{\tilde{e}_t\}_{t=\tau}^{\infty}$.*

Before stating the problem formally, two observations are worth noting. First, the operating firm's instantaneous profit can be rewritten as a function of w_t and $r_{C,t}$ only, after substituting the worker's optimal effort function:

$$\pi(w_t; r_{C,t}) = y(\tilde{e}(w_t, r_{C,t}, \lambda)) - w_t.$$

[6]The linearity of $\tilde{e}_t$ with respect to the gain-loss function $\mu(\ln w_t - \ln r_t)$ is due to the quadratic form assumed for $c(e)$ (Assumption W2).

Second, the firm's problem is essentially a stationary dynamic optimisation problem: in each $t \geq \tau$ the problem reduces to the choice of the optimal wage $w_t = r_{C,t+1}$, given a worker's reference wage $r_{C,t}$. All other variables are parametric and time invariant by definition.

Hence, for a given initial period τ, which defines the beginning of the employment relationship, an operating firm's wage setting problem can be formalised by the following stationary sequence problem:

$$\begin{aligned} J(r_\tau, C_\tau) &= \max_{\{w_t\}_{t=\tau}^{\infty}} \sum_{t=\tau}^{\infty} \psi^{t-\tau} \pi(w_t; r_{C,t}) \\ \text{s.t.} \quad r_{C,t} &= \begin{cases} r_{N,t} \geq 0 \text{ given} & \text{if } t = \tau \\ r_{E,t} = w_{t-1} & \text{if } t > \tau. \end{cases} \end{aligned} \tag{FP}$$

where $J(r_\tau, C_\tau)$ is the firm's value function of the employment relationship with a newly hired worker of employment status $C_\tau = N$; the factor $\psi \in (0, 1)$ is a combination of $\delta \in (0, 1)$ and $\rho \in (0, 1)$ as previously defined; $r_{C,t}$ is the state variable and $r_{C,t+1} = w_t$ is the control variable. For any initial condition given by the newly hired worker's reference wage $r_N \geq 0$ in τ, at the beginning of each $t \geq \tau$ the firm's problem consists of setting the optimal wage w_t, and hence the next period incumbent worker's reference wage $r_{C,t+1}$, for a given reference wage $r_{C,t}$. As such, both the objective function and the firm's instantaneous profit function do not explicitly depend on time.

The optimal wage setting policy

The relevant functional equation corresponding to the firm's problem in (FP) can therefore be written in the following recursive form:

$$J(r_C) = \max_{w \geq 0} \{\pi(r_C, w) + \psi J(w)\}, \tag{2.5}$$

where r_C corresponds to the current period worker's reference wage and w corresponds to the current period wage and the following period worker's reference wage. According to this formulation the operating firm's problem consists of choosing not a sequence $\{w_t\}_{t=\tau}^{\infty}$, but a *time-invariant policy function* $\tilde{w} = \tilde{w}(r_C, \lambda)$, which determines what the next period worker's reference wage $r_{C,t+1}$ (the control) will be for a given worker's reference wage $r_{C,t}$ (the state) in the current employment period.

The operating firm's optimal wage policy is characterised by the following first-order condition (Euler equation), which is both necessary and sufficient to characterise an optimum:[7]

$$\frac{\partial \pi(r_C, w)}{\partial w} + \psi J'(w) = 0,$$

so long as $w \neq r_C$. This condition is sufficient to solve for the optimal policy $\tilde{w}$ if the form of the $J(\cdot)$ function were known. However, since this function is determined recursively as part of the optimisation problem, an additional condition is needed, that is obtained by applying the equivalent of the Envelope Theorem for dynamic programming. Thus, differentiating (2.5) with respect of the worker's reference wage r_C yields

$$J'(r_C) = \frac{\partial \pi(r_C, w)}{\partial r}.$$

Combining the two latter conditions, and using the fact that $\pi(r_C, w) = y(\tilde{e}(w, r_C, \lambda)) - w$, the condition that characterises the optimal wage policy is given by

$$\Upsilon(w; r_C, \lambda) \equiv \frac{\partial y(\tilde{e})}{\partial e} \frac{\partial \tilde{e}(w, r_C, \lambda)}{\partial w} - 1 + \psi \frac{\partial y(\tilde{e}')}{\partial e} \frac{\partial \tilde{e}(w', r_C', \lambda)}{\partial r} = 0 \qquad (2.6)$$

for all $w \neq r_C$. The intuition behind condition (2.6) is the following. Since the worker's effort is increasing in the wage w and decreasing in the reference wage r_C, the firm will choose the optimal wage such that the marginal benefit in terms of positive reciprocity—or less negative reciprocity—is equalised to the marginal cost of paying a higher wage in the current employment period, net of the additional expected discounted marginal cost of employing a worker with a higher reference wage in the subsequent employment period.[8]

Suppose that a worker is characterised by a "relatively low" reference wage $r_C = r'$ such that, for some values of q, p and other parameters determining profit, the firm optimally pays a wage *gift* above the worker's reference wage, denoted by $\tilde{w}_L^+ > r'$. Now consider instead a worker with a "relatively high" reference wage $r_C = r'' > r'$ such that the firm optimally pays a wage below their reference wage, perceived as *unfair*, denoted by $\tilde{w}_H^- < r''$. Due to workers' asymmetric reference-dependent reciprocity (when $\lambda > 1$), the wage paid to a worker with $r'' > r'$ will be greater than the wage paid to a worker with r', all other parameters being equal: $\tilde{w}_H^- > \tilde{w}_L^+$ if $\lambda > 1$. The intuition behind this preliminary conjecture is that the firm employing the worker characterised by r'' will have to compensate for their stronger negative reciprocity, which is triggered by the wage $\tilde{w}_H^- < r''$ being perceived as unfair. Finally, consider a worker with a

[7]The technical details which ensure that this condition is sufficient, as well as others relevant for the characterisation of the firm's optimal wage policy are discussed in the proof of Proposition 6.

[8]This condition is the infinite-horizon analog of the first-order condition (1.10), Chapter 1, and extends the insight discussed therein in relation to a firm's wage compression incentive to a more general set up without uncertainty.

"moderate" reference wage $r_C = r''' \in (r', r'')$, such that it is optimal for the firm to pay them the *fair* wage $\tilde{w}_M^{=} = r'''$. If there exist a range of reference wages r''', say, between r_L and r_H, under which this is the optimal wage policy, then there also exists a range of optimal wages $\tilde{w}_M^{=} = r''' \in [r_L, r_H]$.

As formally established by the following proposition, this range exists in the presence of asymmetric reference-dependent reciprocity ($\lambda > 1$).

Proposition 6. *For all $t \geq \tau$ and for any given worker's reference wage r_C, the time-invariant optimal wage policy of operating firms employing a worker of employment status $C_t = \{N, E\}$, characterised by asymmetric reference-dependent reciprocity $\tilde{e}(w_t, r_{C,t}, \lambda)$ with $\lambda > 1$ and adaptation $r_{C,t+1} = w_t$, is given by*

$$\tilde{w}_t = \tilde{w}(r_{C,t}, \lambda) = \begin{cases} pq(1-\psi) & \equiv \tilde{w}_L^{+} & \text{if } r_{C,t} < r_L \\ r_{C,t} & \equiv \tilde{w}_M^{=} & \text{if } r_{C,t} \in [r_L, r_H] \\ \lambda pq(1-\psi) & \equiv \tilde{w}_H(\lambda)^{-} & \text{if } r_{C,t} > r_H(\lambda) \end{cases} \tag{2.7}$$

where

$$r_L \equiv \left\{ r_{C,t} : \lim_{\epsilon \to 0} \Upsilon(r_{C,t} + \epsilon; r_{C,t}, \lambda) = 0 \right\}$$
$$r_H(\lambda) \equiv \left\{ r_{C,t} : \lim_{\epsilon \to 0} \Upsilon(r_{C,t} - \epsilon; r_{C,t}, \lambda) = 0 \right\}.$$

Moreover, $r_H(\lambda) = \lambda r_L$, which implies that if $\lambda = 1$ then $r_H(1) = r_L$ and $\tilde{w}_H(1)^{-} = \tilde{w}_L$.

Proof. See Appendix B.2. ■

In support of the preceding discussion, Proposition 6 shows that the firm's wage setting policy crucially depends on the level of a worker's reference wage. If a worker's reference wage is relatively low, the firm will pay them a relatively low wage, which will then be perceived as a gift $\tilde{w}_L^{+} > r_{C,t}$; if a worker's reference wage is relatively high, the firm will pay them a relatively high wage, which will however be perceived as unfair $\tilde{w}_H(\lambda)^{-} < r_{C,t}$; while if a worker has a relatively moderate reference wage $r_{C,t} \in [r_L, r_H]$, then the firm will pay them their fair wage $\tilde{w}_M^{=} = r_{C,t}$. These results are illustrated in Figure 2.2 below.

Hence Proposition 6 establishes the existence of a range of worker's reference wages within which it is optimal for the firm to pay them their reference wage. This result hinges crucially on the worker's asymmetric reference dependent reciprocity that stems from their extent of loss aversion $\lambda > 1$. Whenever a firm is facing a worker with a moderate reference wage $r_{C,t} \in [r_L, r_H]$, the marginal benefit of setting a lower, but unfair, wage $w_t < r_{C,t}$ will not be sufficient to offset the marginal cost generated by the

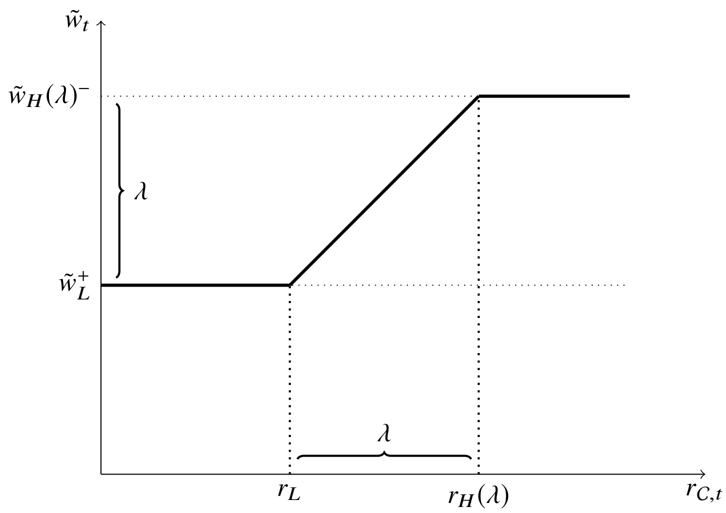

Figure 2.2:
Optimal Wage Setting Policy

worker's negative reciprocity; similarly, the marginal benefit derived from the worker's positive reciprocity, generated by the wage $w_t > r_{C,t}$ being perceived as a gift, will not be enough to offset the marginal cost of paying a higher wage and having to employ a worker with a higher reference wage. In fact, if reciprocity were symmetric, i.e. if $\lambda = 1$, these trade-offs at the margins would disappear and the optimal wage set by the firm would be unique for any given $r_{C,t}$.

Note that the optimal wage setting policy (2.7) established by Proposition 6 also implies that, for any given $r_{C,t} = r_N$ at the start of an employment relationship in period $t = \tau$, the solution to the firm's first-order condition (2.6) automatically characterises the steady-state equilibrium wage of the worker-firm employment relationship. This conclusion relies on three main assumptions of the environment modelled in this section: i) the firm's output is linear in effort, i.e. the production function exhibits constant returns to effort (Assumption F1); ii) the worker's psychological/physical cost of effort are quadratic (Assumption W2)—i.e. the optimal effort function is linear in its gain-loss component $\mu(\cdot)$; and iii) an employed worker is characterised by a complete symmetric adaptation of the reference wage (Assumption A1). Assumption i) and ii) imply that the first-order condition characterising the optimal wage is a function of w_t and the model's parameters only, i.e. the optimal wage policy $\tilde{w}$ does not explicitly depend on the current level of $r_{C,t}$, but only on its initial level in period $t = \tau$; while assumption iii) is such that for any given optimal wage $\tilde{w}_t$ at $t = \tau$, from $t > \tau$ onwards it will be the case that $r_{C,t} = \tilde{w}_t$, implying that $r_{C,t} = r_E \in [\tilde{w}_L^+, \tilde{w}_H(\lambda)^-]$ for all $t > \tau$, which characterises a steady-state. This latter argument is summarised by the following

Corollary to Proposition 6.[9]

Corollary 1. *In worker-firm employment relationships that start in $t = \tau$, where firms' output is linear in workers' effort (Assumption F1); workers' cost of effort is quadratic (Assumption W2); and in which employed workers in $t > \tau$ are characterised by complete symmetric adaptation of their reference wage: $r_{C,t} = r_E = w_{t-1}$ (Assumption A1); a firm's optimal wage setting policy also characterises the steady-state equilibrium wage, which is entirely determined by a worker's initial reference wage at the start of the employment relationship $r_{C,t} = r_N$. Hence $\tilde{w}_t = \tilde{w} \in [\tilde{w}_L^+, \tilde{w}_H(\lambda)^-]$ for all $t \geq \tau$.*

The implications of Corollary 1 are discussed more in detail in Section 2.4.

2.3 A Search and Matching Framework

This section develops a canonical search and matching framework where worker-firm employment relationships and wage setting behaviour are modelled following the set up developed in Section 2.2.

2.3.1 Search Frictions and Unemployment Dynamics

Consider a labour market with a continuum of infinitely lived identical firms and a continuum of measure one of infinitely lived workers who differ only with respect to their employment status $C_t = \{U, N, E\}$. Firms maximise the present discounted value of per-worker profit and workers maximise the present discounted value of utility as expressed by problems (FP) and (WP) respectively in Section 2.2.

The unemployment rate is u_t, the employment rate is n_t and the vacancy rate is v_t.[10] The number of job matches taking place per unit time is m_t, where $m_t = m(u_t, v_t)$ is a standard matching function assumed to be increasing in both arguments, concave, and linearly homogeneous. Since the purpose of this chapter is to derive explicit form/analytical solutions of the relevant equilibrium outcomes, the matching function is assumed to be a standard Cobb-Douglas:

M1. $\bar{m}(u, v) = \bar{m} u^{\sigma} v^{1-\sigma}$.

[9]Notice that by relaxing either F1 or A1 the model can potentially generate endogenous persistence in reciprocity and wage dynamics. Since this chapter is mainly concerned with the steady-state properties of the model, the analysis of a richer out-of-steady-state dynamics implied by considering more general assumptions is not pursued. Nevertheless, the dynamic implications that result from relaxing Assumption A1 are thoroughly analysed in Section 3.5.1, Chapter 3. Relaxing assumption F1, i.e. considering decreasing returns to effort, will also generate similar results.

[10]Since workers are assumed to be a continuum of measure one, the labour force is constant and fixed normalised to $L_t = 1$. Hence u_t, n_t and v_t can also be interpreted as the number of unemployed workers, employed workers or vacancies in the market in period t respectively.

The parameter $\bar{m}$ captures the efficiency of matching and $\sigma \in (0,1)$ is the match elasticity. Let the tightness of the labour market be defined by $\theta_t = v_t/u_t$. The probability that a vacant job is matched with a worker is $\bar{m}(u_t, v_t)/v_t = \bar{m}\theta^{-\sigma} \equiv h(\theta_t),\ h'(\theta_t) < 0$, whilst the probability of an unemployed worker making contact with a vacancy is $\bar{m}(u_t, v_t)/u_t = \bar{m}\theta^{1-\sigma} \equiv f(\theta_t),\ f'(\theta_t) > 0$. When the labour market is tight, i.e. θ_t is large, from the perspective of firms it is harder to find workers and hence the probability of matching with an unemployed worker is lower; on the other hand, from the workers' perspective the probability of matching with an open vacancy is greater if θ_t is large, since the number of vacancies relative to the number of unemployed workers is greater. These aspects synthesise the main assumptions that generate the search, or congestion, externality which underlies the theory of equilibrium unemployment based on search frictions (see for instance Pissarides (2000)).

The parameters of the model (and in particular, the employed workers' normal effort $\tilde{e}_n$) are assumed to be such that every worker-firm match is mutually advantageous: all the unemployed workers of employment status $C_t = U$ that are matched with firms are hired.[11] As such, $f(\theta_t)$ represents the job-finding rate. On the other hand, employed workers of employment status $C_t = \{N, E\}$ move into unemployment at a rate $\rho \in (0,1)$, which has been defined as the exogenous job-destruction rate.

Hence, $\rho(1 - u_t)$ captures the number of workers that enter unemployment in each t and $\bar{m}(u_t, v_t) = f(\theta_t)u_t$ captures the number of workers that are matched and hired, leaving unemployment in each t. The evolution of mean unemployment can therefore be expressed by the difference between these flows, in and out of unemployment:

$$\Delta u_{t+1} = \rho(1 - u_t) - f(\theta_t)u_t,\ u_0 \text{ given.} \tag{2.8}$$

[11]By this it is implicitly assumed that firms' zero-profit condition at the time of hiring is always satisfied, and that any wage offer is such that the value to workers of being employed is greater or equal to the value of being unemployed. The former assumption has been widely used in the literature (see for instance Pissarides (1987, 2000)), whilst the latter is a simplification which implies that unemployed workers matched with firms will accept any wage offer (as for instance in Michaillat (2012)). In the present framework this assumption is useful to maintain the focus on firms' job creation decisions, and to ensure that the condition determining unemployed workers' reservation wage is always satisfied. Note that in this context, a worker could accept an employment contract that pays a wage they perceive to be unfair. In fact it is not inconceivable that a worker might prefer to be employed at a wage perceived as unfair, rather than remaining unemployed. Being unemployed is a social status that bears many negative aspects beyond those that can be offset by accounting for the benefits of some unemployment insurance or leisure (e.g. scarring). Moreover, as long as there realistically exists a wedge between a worker's return from being unemployed and the value of being employed, the results derived hereafter will hold. The conditions that need to be satisfied by the parameters of the model in order for this assumption to hold are derived in Appendix A.2 (see the Subsection A.2.1 for the effort condition, and A.2.2 for the reservation wage condition). Finally note that a numerical simulation of the model reveals that if the firms' zero profit condition is satisfied, i.e. if workers' normal effort is sufficiently high, then the workers' reservation wage condition is also always satisfied.

The difference equation (2.8) represents what is known in the literature as the unemployment dynamics equation (Pissarides, 2000).

2.3.2 Value Functions and Job Creation

The environment described so far is consistent with a labour market in which each period t is characterised by a number of firms searching for unemployed workers $C_t = U$; a number of firms employing newly hired workers $C_\tau = N$; and a number of firms employing incumbent workers $C_t = E$.

The value of a vacancy to the firm is denoted by V_t; the value of a job filled by a newly hired worker by $J(r_{N,t})$; and the value of a job filled by an incumbent worker by $J(r_{E,t})$. The value of a job filled by a newly hired worker, in any $t = \tau$, satisfies

$$J(r_{N,t}) = y(\tilde{e}_t) - \tilde{w}_t + \delta\left[(1-\rho)J(r_{E,t+1}) + \rho V_{t+1}\right], \quad \forall t = \tau, \tag{2.9}$$

while the value of a job filled by an incumbent worker, in any $t > \tau$, satisfies

$$J(r_{E,t}) = y(\tilde{e}_t) - \tilde{w}_t + \delta\left[(1-\rho)J(r_{E,t+1}) + \rho V_{t+1}\right], \quad \forall t > \tau. \tag{2.10}$$

Finally, let κ be a time-invariant cost of posting a vacancy. The value of a vacancy for a firm, that is facing the probability of matching and hiring an unemployed worker in $t < \tau$ to start an employment relationship in $t + 1 = \tau$, can be expressed as

$$V_t = -\kappa + \delta\left[h(\theta_t)J(r_{N,t+1}) + (1 - h(\theta_t))V_{t+1}\right], \quad \forall t < \tau. \tag{2.11}$$

That is, the value of a vacancy in each t is given by the difference between the current cost of posting a vacancy κ and the expected discounted profit gained by the firm in the following period $\delta J(r_{N,t+1})$ given the probability $h(\theta_t)$ of matching and hiring a newly hired worker with reference wage $r_{N,t+1}$, plus the value of posting a vacancy in the following period if matching does not occur.

To derive a condition governing firms' job creation decisions, following the standard approach in the literature, it is assumed that there is free entry, so that firms will enter in the market and exploit all profit opportunities from hiring unemployed workers, driving the value of vacancies to zero, i.e. $V_t = 0\ \forall t$.[12] Hence, rearranging equation (2.11) and making use of the *free-entry condition* yields the job creation condition, which characterises the optimal vacancy posting decision of firms, subsumed in the value of

[12]This condition holds both in and out of steady state (Pissarides, 2000, p.29).

θ_t that satisfies the following equation:

$$\frac{\kappa}{h(\theta_t)} = \delta J(r_{N,t+1}). \tag{2.12}$$

This condition requires that firms will hire workers until the cost κ of posting a vacancy, multiplied by the expected duration of the vacancy $1/h(\theta_t)$, equals the expected present discounted value of a new employment relationship $\delta J(r_{N,t+1})$ with a newly hired worker with reference wage r_N. The left-hand side captures the effective cost of creating a vacancy while the right-hand side captures the expected benefit.

2.4 Equilibrium Analysis

The main purpose of this section is to characterise the steady-state equilibrium of the model and to analyse its properties. Before providing a formal characterisation however, let us define the equilibrium path of the labour market, and recap the key aspects concerning workers' and firms' optimal decisions as well as the evolution of the two state variables of the model, namely $r_{C,t}$ and u_t.

The control variables of the system are: 1) the employed workers' optimal effort choice $\tilde{e}_t$, which maximises utility given their matched firm's optimal wage offer $\tilde{w}$ evaluated relative to their reference wage $r_{C,t}$; 2) the optimal time-invariant wage policy set by firms $\tilde{w}$, which maximises their present discounted value of profit taking as given their employed workers' reference wage $r_{C,t}$ for all $t \geq \tau$; and 3) the equilibrium level of vacancies, subsumed by labour market tightness θ_t, that satisfies the job creation condition, taking as given the expected present discounted value of an employment relationship for any given r_N, where the optimal wage paid and the employed workers' reciprocity are endogenously determined by their respective optimisation problems.

The state variables of the system are the employed workers' reference wage $r_{C,t}$ and the unemployment rate u_t. The laws of motion describing their evolution are reproduced here for clarity of exposition:

$$\begin{aligned} r_{C,t+1} &= w(r_{C,t}, \lambda), \quad r_0 \text{ given}, \\ u_{t+1} &= u_t + \rho(1-u_t) - f(\theta_t)u_t, \quad u_0 \text{ given}. \end{aligned}$$

The equilibrium path of the labour market can therefore be defined as follows.

Definition 3. *Given initial unemployment u_0 and employed workers' reference wage r_0, an equilibrium path is a sequence of wages, reference wages, effort levels, market tightness and unemployment rates $\{w_t, r_{C,t}, e_{C,t}, \theta_t, u_t\}$ such that firms maximise the* *present discounted value of profits as formalised by* (FP)*; workers of employment status* $C_t = \{N, E\}$ *maximise the present discounted value of utility as formalised by* (WP)*; employed workers' reference wages are given by* (2.1)*; labour market tightness satisfies* (2.12)*; and unemployment is given by* (2.8).

2.4.1 Characterisation of the Steady-State Equilibria

The steady-state equilibrium of the labour market can be defined as follows.

Definition 4. *A steady-state equilibrium is an equilibrium path* $\mathcal{E} \equiv \{w_t, r_{C,t}, e_{C,t}, \theta_t, u_t\}$ *in which* $w_t = \tilde{w}^*$, $r_{C,t} = r_C^*$, $e_{C,t} = \tilde{e}_C^*$, $\theta_t = \tilde{\theta}^*$ *and* $u_t = u^*$ *for all* t.

To fully characterise the steady-state equilibrium of the model this section derives first the steady-state levels of wages, reference wages and effort, and subsequently uses these results to derive the steady-state levels of market tightness and unemployment.

Proposition 7. *In a labour market where employment relationships are formed with workers of employment status* $C_t = \{N, E\}$ *characterised by asymmetric reference-dependent reciprocity given by* (2.4)*; firms' set wages according to* (2.7)*; and employed workers reference wages are given by* (2.1)*; there exists a range of steady-state equilibrium wages*

$$\tilde{w}^* = \tilde{w}_C^* = \tilde{w}^*(r_0, \lambda) \in [\tilde{w}_L^+, \tilde{w}_H(\lambda)^-]$$

paid to newly hired and incumbent workers. Employed workers steady-state equilibrium levels of reference wages and effort are given by:

$$\begin{aligned} r_N^* &= r_N = r_0,\ r_0 \text{ given} \qquad & \tilde{e}_N^* &= \tilde{e}^*(\tilde{w}^*, r_N^*, \lambda) = \{\tilde{e}_N^+, \tilde{e}_n, \tilde{e}_N(\lambda)^-\} \\ r_E^* &\in [r_L, r_H(\lambda)] \qquad & \tilde{e}_E^* &= \tilde{e}^*(\tilde{w}^*, r_E^*, \lambda) = \tilde{e}_n. \end{aligned}$$

Proof. See Appendix B.2. ■

Proposition 7 establishes that if employed workers are characterised by asymmetric reference-dependent reciprocity ($\lambda > 1$), there exists a range of steady-state equilibrium wages paid to workers for the whole duration of the employment relationship. Depending on the equilibrium level of newly hired workers' reference wage, which is given by the initial state r_0, their steady-state equilibrium level of effort exerted will either be in the form of positive reciprocity $\tilde{e}_N^* = \tilde{e}_N^+$, negative reciprocity $\tilde{e}_N^* = \tilde{e}_N(\lambda)^-$, or normal effort $\tilde{e}_N^* = \tilde{e}_n$. On the other hand, independently of the absolute level of the steady-state wage, incumbent workers perceive their wage as fair due to adaptation $r_E^* = \tilde{w}^*$, and therefore exert normal effort $\tilde{e}_E^* = \tilde{e}_n$ in the steady-state (i.e. the dynamic re-normalisation of effort discussed in Chapter 1). Hence, there exists a range of steady-state equilibrium incumbent workers' reference wages, and a corresponding unique steady-state level of effort, which is independent of the absolute level of the optimal wage paid, and therefore it cannot be influenced by the firms' wage policy.

Proposition 7 fully characterises the steady-state equilibrium levels of wages, reference wages and effort of a representative employment relationship. As such, from equation (2.9), the steady-state value of a job filled by a newly hired worker can be written as

$$J(r_N) = y(\tilde{e}_N^*) - \tilde{w}^* + \psi J(r_E) \tag{2.13}$$

for all $\tilde{w}^* \in [\tilde{w}_L^+, \tilde{w}_H(\lambda)^-]$ and $\tilde{e}_N^* = \{\tilde{e}_N^+, \tilde{e}_n, \tilde{e}_N(\lambda)^-\}$; while from equation (2.10), the steady-state value of a job filled by an incumbent worker can be written as

$$J(r_E) = y(\tilde{e}_n) - \tilde{w}^* + \psi J(r_E). \tag{2.14}$$

To distinguish between the three equilibrium outcomes established in Proposition 7, denote the steady-state values of jobs to firms as $J(r_C)^+$, $J(r_C)^-$ and $J(r_C)^=$ for all $C = \{N, E\}$, where the super-scripts are indicative of new hires' positive reciprocity, negative reciprocity or normal effort respectively.[13]

Lemma 1. *Due to incumbent workers' adaptation $r_E^* = \tilde{w}^*$, depending on the level of newly hired workers' reference wage r_N, the following holds:*

$$J(r_N) = \begin{cases} J(r_N)^+ & > & J(r_E)^+ & \text{if } r_N < r_L \\ J(r_N)^= & = & J(r_E)^= & \text{if } r_N \in [r_L, r_H] \\ J(r_N)^- & < & J(r_E)^- & \text{if } r_N > r_H(\lambda). \end{cases} \tag{2.15}$$

Moreover, it follows that $J(r_N)^- < J(r_N)^= < J(r_N)^+$.

Proof. See Appendix B.2. ■

Lemma 1 implies that the value of a job filled by a newly hired worker does not always necessarily equal the value of a job filled by an incumbent.[14] In fact, even though an employment relationship starts with some degree of positive or negative reciprocity, the value of the job to the firm once a worker becomes incumbent will either shrink or increase due to the dynamic re-normalisation of effort, which is a direct consequence of reference wage adaptation (see Chapter 1). In addition, the steady-state value of

[13]Thus, for instance, $J(r_N)^-$ is the value of a job with a newly hired worker exerting negative reciprocity in the first employment period and $J(r_E)^-$ is the corresponding value of a job with that worker once they have become incumbent. Also note that since all workers are identical, only one of these three possible situations will characterise the steady-state equilibrium.

[14]In the canonical model there is no such distinction, since wages are re-negotiated at each period and there is no link between one employment period and the next.

jobs in which new hires exert negative reciprocity is lower than it would be if workers were exerting normal effort or positive reciprocity. As such, the present framework endogenously generates a systematic difference between the output produced by newly formed and existing matches.

It is useful at this stage to introduce some additional notation, that will ease the notational burden in the remainder of the analysis. Denote by $\overline{w}^* = \{\overline{w}_L^+, \overline{w}_M^=, \overline{w}_H(\lambda)^-\}$ the steady-state equilibrium present discounted value of the wage from a new match:

$$\begin{aligned}\overline{w}^* &\equiv \tilde{w}_N^* + \sum_{t=\tau+1}^{\infty} \psi^{t-\tau} \tilde{w}_E^*, \quad \forall \tilde{w}_C^* = \tilde{w}^* \in [\tilde{w}_L^+, \tilde{w}_H^*(\lambda)^-] \\ &\equiv \frac{\tilde{w}^*}{1-\psi};\end{aligned} \tag{2.16}$$

and denote by $\overline{y}^*(\tilde{e}_N^*) = \{\overline{y}(\tilde{e}_N^+), \overline{y}(\tilde{e}_n), \overline{y}(\tilde{e}_N(\lambda)^-)\}$ the corresponding steady-state equilibrium present discounted value of output from a new match:

$$\begin{aligned}\overline{y}^*(\tilde{e}_N^*) &\equiv y(\tilde{e}_N^*) + \sum_{t=\tau+1}^{\infty} \psi^{t-\tau} y(\tilde{e}_E^*), \quad \forall \tilde{e}_N^* = \{\tilde{e}_N^+, \tilde{e}_n, \tilde{e}_N(\lambda)^-\} \\ &\equiv y(\tilde{e}_N^*) + \frac{\psi}{1-\psi} y(\tilde{e}_n).\end{aligned} \tag{2.17}$$

By rearranging equations (2.13) and (2.14) and using the present values of wages and output just defined, the steady-state equilibrium job creation condition is obtained:

$$\frac{\kappa}{h(\theta)} = \delta\left[\overline{y}^*(\tilde{e}_N^*) - \overline{w}^*\right]. \tag{JC}$$

Note that the right-hand side of (JC), that is, the expected benefit of opening a vacancy, could be different depending on whether a new match is characterised by positive/negative reciprocity or normal effort in the first employment period (as established by Lemma 1). Hence, the number of vacancies posted in the steady state crucially depends on the level of newly hired workers' reference wages $r_N = r_0$. The solution to (JC), which characterises the steady-state equilibrium level of market tightness, is denoted by $\tilde{\theta}^* = \tilde{\theta}^*(r_0, \lambda)$ and can be written explicitly as

$$\tilde{\theta}^* = \left(\frac{\bar{m}\delta}{\kappa}\left[\overline{y}^*(\tilde{e}_N^*) - \overline{w}^*\right]\right)^{\frac{1}{\sigma}}.$$

The condition for the steady-state equilibrium unemployment can be obtained by rearranging equation (2.8) for $\Delta u_{t+1} = 0$:

$$\rho(1 - u) = f(\theta)u, \tag{BC}$$

implying that, in the steady state, the flows into and out of unemployment are equalised. Denote the solution to this equation by $u^* = u^*(r_0, \lambda)$, which takes the following familiar form:

$$u^* = \frac{\rho}{\rho + \bar{m}\tilde{\theta}^{*1-\sigma}}.$$

The equilibrium solutions are expressed as functions of the newly hired workers' reference wage r_0 and their extent of loss aversion λ in order to emphasise the importance of these two parameters for the steady-state determination.

Proposition 8. *In a labour market where employment relationships are formed with workers of employment status $C_t = \{N, E\}$ characterised by asymmetric reference-dependent reciprocity given by* (2.4)*; firms' set wages according to* (2.7)*; and employed workers reference wages are given by* (2.1)*; there exists a unique steady-state equilibrium labour market tightness $\tilde{\theta}^*$ that satisfy the job creation condition* (JC)*, which can take a unique distinct value $\tilde{\theta}^* = \{\tilde{\theta}_l^*(\lambda), \tilde{\theta}_m^*, \tilde{\theta}_h^*\}$ depending on the level of newly hired workers' reference wage $r_N = r_0$:*

$$\tilde{\theta}^* = \tilde{\theta}^*(r_0, \lambda) = \begin{cases} \left[\frac{\bar{m}\delta}{\kappa}\left(\overline{y}(\tilde{e}_N^+) - \overline{w}_L^+\right)\right]^{\frac{1}{\sigma}} & \equiv \tilde{\theta}_h^* \quad \text{if } r_0 < r_L \\ \left[\frac{\bar{m}\delta}{\kappa}\left(\overline{y}(\tilde{e}_n) - \overline{w}_M^=\right)\right]^{\frac{1}{\sigma}} & \equiv \tilde{\theta}_m^* \quad \text{if } r_0 \in [r_L, r_H] \\ \left[\frac{\bar{m}\delta}{\kappa}\left(\overline{y}(\tilde{e}_N(\lambda)^-) - \overline{w}_H(\lambda)^-\right)\right]^{\frac{1}{\sigma}} & \equiv \tilde{\theta}_l^*(\lambda) \quad \text{if } r_0 > r_H(\lambda), \end{cases} \tag{2.18}$$

where $\tilde{\theta}_l^*(\lambda) < \tilde{\theta}_m^* < \tilde{\theta}_h^*$ and if $\lambda = 1$ then $\tilde{\theta}_l^*(1) = \tilde{\theta}_m^* = \tilde{\theta}_h^*$. Therefore for each possible $\tilde{\theta}^*$ there exists a unique steady-state equilibrium unemployment rate u^* that satisfies condition (BC)*, which can take a unique distinct value $u^* = \{u_L^*, u_M^*, u_H^*(\lambda)\}$ as follows:*

$$u^* = u^*(r_0, \lambda) \equiv \begin{cases} u_L^* & \text{if } r_0 < r_L \\ u_M^* & \text{if } r_0 \in [r_L, r_H] \\ u_H^*(\lambda) & \text{if } r_0 > r_H(\lambda), \end{cases} \tag{2.19}$$

where $u_H^*(\lambda) > u_M^* > u_L^*$ and if $\lambda = 1$ then $u_H^*(1) = u_M^* = u_L^*$.

Proof. See Appendix B.2. ■

Proposition 8 formally establishes the existence of a unique, but distinct, steady-state equilibrium of labour market tightness and unemployment depending on the value of the reference wage $r_N = r_0$ characterising newly hired workers' wage entitlements in the labour market. In fact, r_0 determines the optimal steady-state equilibrium wage $\tilde{w}^*$ for the entire duration of any employment relationship, as well as new hires' reciprocity response $\tilde{e}_N^*$ in the first employment period. Since these two elements influence the present discounted values of output $\overline{y}^*(\tilde{e}_N^*) = \{\overline{y}(\tilde{e}_N^+), \overline{y}(\tilde{e}_n), \overline{y}(\tilde{e}_N(\lambda)^-)\}$ and the wage $\overline{w}^* = \{\overline{w}_L^+, \overline{w}_M^=, \overline{w}_H(\lambda)^-\}$ characterising a new employment relationship, it is natural that job creation, vacancies and unemployment will depend on which of these three outcomes firms are expecting to experience in the steady state. Note that this result is also a natural implication of the optimal wage setting policy implemented by firms that anticipate to hire workers characterised by asymmetric reference-dependent reciprocity ($\lambda > 1$). As stated in the proposition, if reciprocity were symmetric ($\lambda = 1$) the steady-state equilibrium would be uniquely characterised, independently of the new hires' reference wage level.

These equilibria can be ranked in terms of their macroeconomic outcomes as well as on the basis of the microeconomic behaviour characterising workers and firms. Consider first the steady-state equilibrium for all $r_0 < r_L$, and define the corresponding tuple as

$$\mathcal{L}^* \equiv \{\tilde{w}_L^+, r_N^*, r_L, \tilde{e}_N^+, \tilde{e}_n, \tilde{\theta}_h^*, u_L^*\},$$

which is labelled as the "low" equilibrium, where the adjective refers to the unemployment rate. A labour market that is stationed at this equilibrium is characterised by a relatively high labour market tightness and a relatively low unemployment rate, and all employed workers are paid the same relatively low wage $\tilde{w}_L^+$. However, while incumbents perceive this wage as fair $r_E^* = r_L = \tilde{w}_L^+$ and exert normal effort $\tilde{e}_E^* = \tilde{e}_n$, newly hired workers perceive this wage as a gift, since $r_N^* < \tilde{w}_L^+$, their morale is relatively high and therefore exert supra-normal effort $\tilde{e}_N^* = \tilde{e}_N^+ > \tilde{e}_n$.

On the other hand, consider the equilibrium characterising the steady-state for all $r_0 > r_H(\lambda)$, and define the corresponding tuple as

$$\mathcal{H}^*(\lambda) \equiv \{\tilde{w}_H(\lambda)^-, r_N^*, r_H(\lambda), \tilde{e}_N(\lambda)^-, \tilde{e}_n, \tilde{\theta}_l^*(\lambda), u_H^*(\lambda)\},$$

which is labelled as the "high" equilibrium. If the labour market is stationed at this equilibrium, labour market tightness will be relatively low, unemployment relatively high and employed workers will be paid the same relatively high wage $\tilde{w}_H(\lambda)^-$. In contrast with $\mathcal{L}^*$, newly hired workers perceive the wage as unfair, since $r_N^* > \tilde{w}_H(\lambda)^-$, their morale is relatively low and therefore exert sub-normal effort $\tilde{e}_N^* = \tilde{e}_N(\lambda)^- < \tilde{e}_n$ in the form of negative reciprocity.

Finally, if the labour market is stationed at the "moderate" equilibrium, defined over the tuple

$$\mathcal{M}^* \equiv \{\tilde{w}_M^=, r_N^*, \tilde{e}_n, u_M^*\},$$

depending on the level of the steady-state wage $\tilde{w}_M^= = r_N^*$, the equilibrium labour market tightness and unemployment can be anywhere between the two previously considered equilibria. All workers will perceive the wage they are paid as fair, independently of their employment status $C = \{N, E\}$, implying that in this equilibrium there is no distinction between new hires and incumbents, whom will therefore exert normal effort $\tilde{e}_N^* = \tilde{e}_E^* = \tilde{e}_n$.

The steady-state labour market equilibria characterised in this section can therefore be ranked in terms of wage levels, new hires' morale and reciprocity, and vacancy and unemployment rates. If $\mathcal{E} = \mathcal{L}^*$, the equilibrium wage is relatively low, new hires exert positive reciprocity (high morale), the vacancy rate is high and the unemployment rate is low; if $\mathcal{E} = \mathcal{H}^*(\lambda)$ the equilibrium wage is relatively high, new hires exert negative reciprocity (low morale), the vacancy rate is high and unemployment is low; while if $\mathcal{E} = \mathcal{M}^*$, new hires exert their intrinsically motivated normal effort while the equilibrium wage and the unemployment rate are positively correlated and lie between the two previously described equilibria. Nevertheless, due to reference wage adaptation, in any of the three identified steady states incumbent workers always perceive the wage they are paid as fair and exert normal effort. These equilibria are illustrated in Figure 2.3, which follows the literature by plotting the solution for the equilibrium labour market tightness in the (w, θ) space (Figure 2.3a), and the corresponding equilibrium unemployment in the (v, u) space (Figure 2.3b).

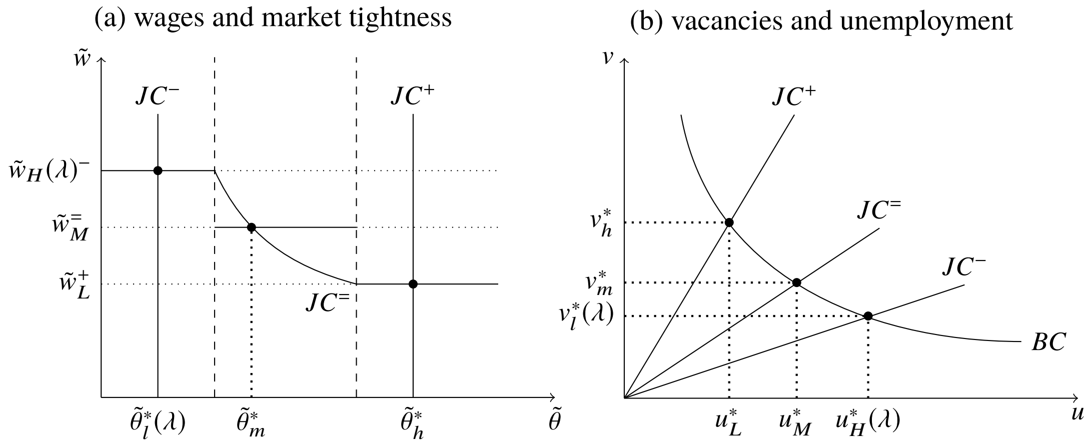

Figure 2.3:
The Steady-state Equilibria

By looking at Figure 2.3a it is also possible to visually identify the main differences between the present framework and a canonical search and matching model. In the canonical model there is a unique and upward-sloping wage setting curve and a unique and downward-sloping job creation curve, the intersection of which uniquely determines the steady-state equilibrium level of labour market tightness. On the other hand, the present framework generates a range of wage setting curves $\tilde{w}^* \in [\tilde{w}_L^+, \tilde{w}_H(\lambda)^-]$ as established by Proposition 7, which are independent of labour market tightness and are therefore plotted as horizontal lines in the (w, θ) diagram.[15] In addition, for the reasons discussed in this section, there also exist three distinct job creation curves associated with each of the possible equilibrium wage characterising the steady state. Importantly, none of these job creation curves depend on the absolute equilibrium wage level, with the exception of the one determining the moderate equilibrium $JC^=$, for which the steady-state wage equals new hires' reference wage. For these reasons, the job creation curves JC^- and JC^+ determining the high and low equilibria are plotted as vertical lines in the (w, θ) diagram, while $JC^=$ is downward-sloping within the range $[\tilde{w}_L^+, \tilde{w}_H(\lambda)^-]$.

The behavioural mechanisms underlying these results and their strictly related implications will be formally analysed in the remainder of the chapter.

2.4.2 Comparative Statics

The following exercise investigates the comparative statics properties of the derived steady-state equilibrium. The results of this section are useful in assessing the qualitative importance of the mechanisms that underlie how the model responds to exogenous shocks.

The analysis is divided in two main parts. Following a standard approach in the literature (see Shimer (2005), Elsby et al. (2015)), Subsection 2.4.2 analyses the model's comparative statics with respect to changes in aggregate productivity p and in the job-destruction rate ρ. This is to draw attention to the similarities as well as the most distinctive differences of the current framework as against the standard predictions of the canonical model. Subsection 2.4.2 analyses how changes in the two

[15]The result that wage setting is independent of labour market tightness is due to the fact that the workers' reservation wage condition is always satisfied, hence it is not binding for optimal wage setting (see also footnote 11, Section 2.3, and Section A.2.2, Appendix A.2), and that newly hired workers' reference wage is exogenous. In fact, for these reasons the vacancy rate and the unemployment rate do not affect the optimal effort choice of workers, neither the wage setting behaviour of firms. Given the existing evidence of the impact of labour market conditions for employed workers wages (e.g. Bils (1985), Beaudry and DiNardo (1991) and Schmieder and von Wachter (2010)), one way to make unemployment realistically relevant for wage setting behaviour in this model would be to assume that newly hired workers' reference wages are negatively correlated with the unemployment rate, or some equivalent measure of the state of the labour market. The investigation of the consequences of this hypothesis for the theoretical predictions of the model is beyond the purpose of this chapter. Nevertheless it can be inferred that even in the presence of a negative relationship between the optimal wage and labour market tightness, due to workers' asymmetric reference-dependent reciprocity, the key results derived in this section will still hold.

additional (exogenous) behavioural parameters introduced by the wage setting model, i.e. newly hired workers' reference wages r_N and their extent of loss aversion λ, affect the steady-state equilibrium outcomes.

Aggregate productivity and job-destruction rate

The following proposition establishes how equilibrium wages, reference wages, effort, market tightness and unemployment are affected by changes in aggregate productivity p and job-destruction rate ρ.

Proposition 9. *For all $\mathcal{E} = \{\mathcal{L}^*, \mathcal{M}^*, \mathcal{H}^*(\lambda)\}$, labour market tightness $\tilde{\theta}^*(p,\rho)$ is increasing in p and decreasing in ρ, while unemployment $u^*(p,\rho)$ is decreasing in p and increasing in ρ. Moreover:*

a) if $\mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}$, the wage $\tilde{w}^*_C(p,\rho) = \{w^+_L, w_H(\lambda)^-\}$ is increasing in both p and ρ, implying that new hires' effort $\tilde{e}^*_N(p,\rho) = \{\tilde{e}^+_N, \tilde{e}_N(\lambda)^-\}$ and incumbents' reference wage $r^*_E(p,\rho) = \{r_L, r_H(\lambda)\}$ are also increasing in p and ρ.

b) if $\mathcal{E} = \mathcal{M}^*$, then $\tilde{w}^*_C = \tilde{w}^=_M$, $\tilde{e}^*_N = \tilde{e}_n$ and $r^*_E = r_N$, which are unaffected by changes in p and ρ.

Proof. See Appendix B.2. ∎

For what concerns the macroeconomic outcomes of the model, higher productivity p increases job creation and reduces unemployment.[16] However, the second part of Proposition 9 highlights differences in the microeconomic behaviour of workers and firms respectively, depending on whether a) the labour market is stationed at a low or high equilibrium, or b) at a moderate equilibrium. In case a) higher productivity p implies higher wages in equilibrium: newly hired workers' effort increases and incumbent workers' reference wages are also higher due to adaptation. On the other hand in case b) changes in aggregate productivity p have no effect on wages, reference wages and effort. In fact, in this equilibrium both new hires and incumbents are paid their reference wage $\tilde{w}^=_M = r^*_N$, which is independent of productivity, implying that new hires' effort is normal $\tilde{e}^*_N = \tilde{e}_n$, and the reference wage of incumbents also does not change. This result has implications for the steady-state elasticity of labour market tightness with respect to productivity (see Section 2.5).

[16]As explained by Pissarides (2000), this is not a desirable property of a model in long-run equilibrium, where wages should fully absorb productivity changes and there should exist a balanced-growth equilibrium with constant unemployment. In the canonical model, one way to make the unemployment rate independent of aggregate productivity is to assume that workers' "unemployment income" depends on their "permanent income" (see Pissarides, 1987). However since the comparative statics results in this section should be considered as approximations of the short-run dynamic adjustment of the model following a shock this issue shall not be addressed here.

A higher job-destruction rate ρ reduces job creation and, by definition, increases the number of employed workers that are exogenously laid off. This leads to lower vacancies and higher unemployment. As such, in the current framework exogenous changes to the job-destruction rate generate negatively correlated movements in vacancies and unemployment, which is in line with stylised facts of business cycles, and overcomes one of the shortcomings of the canonical search and matching model as pointed out by Shimer (2005). To see this, first note that, as in the canonical model, in addition to shifting the Beveridge curve (BC) outwards a higher ρ also increases the probability that employment relationships will be exogenously terminated, i.e. it decreases their expected duration: the expected value from a new match will therefore be lower, reducing job creation incentives. However, the aforementioned distinctive result with respect to job creation relies on the microeconomic properties of the wage setting model implemented here.

The standard approach in the literature is to assume that workers and firms bargain over wages to split the surplus generated by a match according to the generalised Nash bargaining solution, where the threat points are the value of unemployment to a worker and the value of a vacancy to a firm. In this context, a higher job-destruction rate increases the flow of workers into unemployment, which in turn negatively affect workers' threat point and bargaining position. As a consequence, firms expect to pay lower wages in equilibrium and therefore have an incentive to post more vacancies, increasing job creation. For plausible parameterisations of the model, it has been shown that this latter incentive dominates over the one induced by the decrease in the expected duration of a new match, resulting in positively correlated movements in vacancies and unemployment (Elsby et al., 2015).

In contrast to these predictions, the second part of Proposition 9 establishes that: a) if the labour market is stationed at the low or high equilibrium, a higher ρ increases equilibrium wages, new hires' effort and incumbents reference wages; while b) if the labour market is stationed at the moderate equilibrium, changes in ρ have no effect on these variables. Since in case b) employed workers are paid their reference wage $\tilde{w}_M^{=} = r_N^*$, which is exogenous and independent of the job-destruction rate, changes in ρ do not affect the equilibrium wage. The greater flow of workers into unemployment and the decrease in the expected duration of new matches unambiguously reduce job creation incentives and raise unemployment. In case a) the result that wages are increasing in ρ may seem surprising. The intuition behind this conclusion follows directly from firms' wage compression incentive discussed in Chapter 1. In fact, in the intertemporal trade-off faced by firms, as highlighted by (2.6), a higher ρ reduces the weight placed on the expected discounted marginal cost of employing an incumbent worker who will have a higher reference wage in the future. As such the wage compression incentive is eased, and firms will optimally pay newly hired workers a higher wage to elicit additional positive reciprocity or to dampen negative reciprocity. This effect, combined with the reduced expected duration of new matches and the increase in the inflow rate of workers into unemployment (outward shift in the Beveridge curve (BC)), decreases the expected present value of a new employment relationship: there are fewer vacancies in the market; the job-finding rate is lower; the job-destruction rate is higher; and unemployment is higher.

New hires' reference wages and loss aversion

Consider first the effect of the new hires' reference wage $r_N = r_0$ on the steady-state equilibrium outcomes of interest.

Proposition 10. *For all $\mathcal{E} = \{\mathcal{L}^*, \mathcal{M}^*, \mathcal{H}^*(\lambda)\}$, labour market tightness $\tilde{\theta}^*(r_N)$ is decreasing in r_N, implying that unemployment $u^*(r_N)$ is increasing in r_N. Moreover:*

a) if $\mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}$, the wage $\tilde{w}^*_C = \{w^+_L, w_H(\lambda)^-\}$ and incumbents' reference wage $r^*_E = \{r_L, r_H(\lambda)\}$ are unaffected by changes in r_N, implying that new hires' effort $\tilde{e}^*_N(r_N) = \{\tilde{e}^+_N, \tilde{e}_N(\lambda)^-\}$ is decreasing in r_N.

b) if $\mathcal{E} = \mathcal{M}^*$, the wage $\tilde{w}^*_C = \tilde{w}^=_M = r_N$ and incumbents' reference wage $r^*_E = r_N$ are increasing in r_N, implying that new hires' effort $\tilde{e}^*_N = \tilde{e}_n$ is unaffected by changes in r_N.

Proof. See Appendix B.2. ■

Recall that $r_N = r^*_N$ captures the reference wage that characterises newly hired workers at the start of an employment relationship, and that, in the present context, is taken as parametric and determined exogenously. With respect to the macroeconomic outcomes of the model, a higher wage entitlement of newly hired workers decreases job creation and increases unemployment, as it was anticipated in Section 2.4.1. Although this result is valid for all the steady-state, the underlying microeconomic mechanisms are different.

If the economy is stationed at the low or high equilibrium, hence as long as $r_N < r_L$ and $r_N > r_H(\lambda)$, no matter how high are newly hired workers' wage entitlements, the equilibrium level of wages and reference wages of incumbent workers are unaffected. However, higher r_N negatively influence newly hired workers' effort for any given optimal wage $\tilde{w}^* = \{w^+_L, w_H(\lambda)^-\}$: either by reducing the perceived gain from a wage gift, hence resulting in lower positive reciprocity; or by increasing the perceived loss from an unfair wage, therefore resulting in greater negative reciprocity. These effects act to reduce the expected value of a new employment relationship to firms. Hence, consistent with a unique equilibrium wage, there exist a range of labour market tightness and unemployment which monotonically depend on the level of new hires' wage entitlements.

On the other hand, if the economy is stationed at the moderate equilibrium, higher newly hired workers' reference wages r_N imply higher wages and reference wages in equilibrium since $r_E^* = \tilde{w}_M^= = r_N$; but the equilibrium level of effort $\tilde{e}_N^* = \tilde{e}_n$ remains unaffected, since any equilibrium wage is perceived as fair by both incumbents and new hires. As such, the higher equilibrium wage coupled with a higher reference wage increase the cost of employing a worker, leading to fewer vacancies and higher unemployment in equilibrium.

To conclude, consider the effect of changes in workers' strength of negative reciprocity, captured by λ, on the steady-state outcomes of a labour market that is stationed at the high equilibrium (since this is the only equilibrium that depends on λ).

Proposition 11. *If $\mathcal{E} = \mathcal{H}^*(\lambda)$, labour market tightness $\tilde{\theta}^* = \tilde{\theta}_l^*(\lambda)$ is decreasing in λ, implying that unemployment $u^* = u_H^*(\lambda)$ is increasing in λ. Moreover, the wage $\tilde{w}^* = w_H(\lambda)^-$ and incumbents' reference wage $r_E^* = r_H(\lambda)$ are increasing in λ, while new hires effort $\tilde{e}_N^* = \tilde{e}_N(\lambda)^-$ is decreasing in λ.*

Proof. See Appendix B.2. ■

A greater λ implies that a worker paid a wage below their reference wage places a larger weight on the perceived loss in utility, triggering stronger negative reciprocity in the first employment period. Anticipating this behaviour firms will set higher wages for any given new hires' reference wage $r_N > r_H(\lambda)$ for which they optimally pay the unfair steady-state wage $\tilde{w}_H^*(\lambda) < r_N$. This, in turn, implies a higher reference wage of incumbent workers. The resulting equilibrium outcome involves lower new hires' effort, as a consequence of greater negative reciprocity. The combination of higher equilibrium wages and lower effort contribute to reduce firms' expected value of new employment relationships, resulting in fewer vacancies and even higher unemployment in equilibrium. Hence, if the labour market is populated by newly hired workers who are characterised by asymmetric reference-dependent reciprocity, and with relatively high reference wages, a greater extent of loss aversion leads to a steady-state equilibrium where both wages and unemployment are higher.[17]

[17]By introducing uncertainty over the evolution of the match productivity q, Section 2.6 shows that the result established in Proposition 11 holds independently of newly hired workers' reference wage level in the labour market.

2.4.3 Transitional Dynamics

The question of interest of this section is the following: starting from arbitrary initial conditions, u_0 and r_0, to which steady-state equilibrium will the labour market converge?

First note that the model maintains the fundamental transitional dynamic properties of a canonical search and matching model: the wage and labour market tightness are jump variables, whilst unemployment is a backward-looking, predetermined variable. As established in the preceding sections, for any given initial condition $r_0 = r_N$ the optimal wage set by firms $\tilde{w}$ instantaneously characterises the steady-state equilibrium wage for both newly hired and incumbent workers. As such firms have all the information that is needed to calculate the expected steady-state value of a new employment relationship, which also serves to make their vacancy posting decision $\tilde{\theta}$ a forward-looking jump variable.

The two novel variables introduced by the framework developed in this chapter are the employed workers' reference wage and their effort decisions for all $C_t = \{N, E\}$. Their corresponding laws of motion can be expressed as follows:

$$r_{C,t+1} = \tilde{w}(r_{C,t}, \lambda), \quad r_0 \text{ given} \tag{2.20}$$

$$\tilde{e}(w_{t+1}, r_{C,t+1}, \lambda) = \tilde{e}(w_t, r_{C,t}, \lambda) - \mu(\ln r_{C,t+1} - \ln r_{C,t}), \quad r_0 \text{ given} \tag{2.21}$$

where, by exploiting the dynamic properties of the model, equation (2.21) is a convenient way of expressing the employed workers' optimal effort as a first-order difference equation (in which $\tilde{e}(w_{t+1}, r_{C,t+1}, \lambda) = \tilde{e}_n$ for any given $r_{C,t}$). Consider newly hired workers first. From their perspective, the reference wage and effort are jump variables: for any given arbitrary initial condition r_0 and u_0, they instantaneously jump to their steady-state levels: $r_N^* = r_0$ and $\tilde{e}_N^* = \{\tilde{e}_N^+, \tilde{e}_n, \tilde{e}_N(\lambda)^-\}$, as established by Proposition 7. Now consider incumbent workers: from their perspective, the reference wage and effort are backward-looking predetermined variables that need one employment period to reach their steady-state levels, unless the optimal wage setting policy is $\tilde{w}^* = \tilde{w}_M^=$, for which $r_E^* = \tilde{w}_M^=$ and $\tilde{e}_E^* = \tilde{e}_n$. Given these premises, the following proposition establishes the transitional dynamics of the model.

Proposition 12. *In a labour market in which employment relationships are formed with workers of employment status* $C_t = \{N, E\}$ *characterised by asymmetric reference-dependent reciprocity given by* (2.4)*; firms' set wages according to* (2.7)*; and post vacancies according to* (2.12)*: employed workers reference wages and effort adjust according to* (2.20) *and* (2.21)*, and unemployment adjusts according to* (2.8)*. Hence, starting from any initial* u_0*:*

a) *if* $r_0 < r_L$*, then* $\tilde{w}^* = \tilde{w}_L^+$*,* $\tilde{\theta}^* = \tilde{\theta}_h^*$*,* $r_N^* = r_0$*,* $\tilde{e}_N^* = \tilde{e}_N^+$*,* $r_{E,t} \nearrow r_L$*,* $\tilde{e}_{E,t} \searrow \tilde{e}_n$*, and* $u_t \rightarrow u_L^*$*;*

b) *if* $r_0 \in [r_L, r_H]$*, then* $\tilde{w}^* = \tilde{w}_M^= = r_E^*$*,* $\tilde{\theta}^* = \tilde{\theta}_m^*$*,* $\tilde{e}_{C,t} = \tilde{e}_n$*, and* $u_t \rightarrow u_M^*$*;*

c) *if* $r_0 > r_H(\lambda)$*, then* $\tilde{w}^* = \tilde{w}_H(\lambda)^-$*,* $\tilde{\theta}^* = \tilde{\theta}_l^*(\lambda)$*,* $r_N^* = r_0$*,* $\tilde{e}_N^* = \tilde{e}_N(\lambda)^-$ $r_{E,t} \searrow r_H(\lambda)$*,* $\tilde{e}_{E,t} \nearrow \tilde{e}_n$*, and* $u_t \rightarrow u_H^*(\lambda)$*.*

Proof. See Appendix B.2. ∎

As established by Proposition 12, starting from a given and fixed u_0, there exist a unique, but distinct, steady-state equilibrium path for each initial value of new hires' reference wage. Which equilibrium path will characterise the transitional dynamics crucially depends on the initial reference wage r_0. The results of Proposition 12, in particular a) and c), are illustrated in Figure 2.4.[18] Note that the initial condition for unemployment has been purposely placed between the low and high equilibria to show that its dynamic paths can be entirely different. The conclusions of Proposition 12 are valid for any given and fixed u_0.

Consider case a) and suppose that the initial condition are such that the labour market is populated by workers with relatively low initial reference wages $r_0' < r_L$. The optimal wage paid $\tilde{w}_L^+ > r_0$ will be perceived as a gift by workers, whom will exert positive reciprocity $\tilde{e}_{C,0}^+ > \tilde{e}_n$. This makes the expected present value of a new employment relationship relatively high, giving firms an incentive to post relatively more vacancies and hire more workers: labour market tightness will jump to its high equilibrium level $\tilde{\theta}_0 = \tilde{\theta}_h^*$ as shown in Figure 2.4b. In the following period, this triggers the adjustment dynamics of vacancies and unemployment typical of search and matching models, and as shown in Figure 2.4d unemployment monotonically decreases towards the low steady state u_L^*. Moreover, due to adaptation, the reference wage of incumbent workers increases towards its steady state level r_L, as shown in Figure 2.4a, and therefore their effort decreases to normal as shown in Figure 2.4c.

A rather different dynamic path exists if the labour market is populated by workers that have a relatively high initial reference wage $r_0'' > r_H(\lambda)$, i.e. case c). In this situation optimal wage setting implies a relatively high equilibrium wage $\tilde{w}_H(\lambda)^- < r_0$—required to partially offset employed workers' negative reciprocity $\tilde{e}_{C,0}(\lambda)^- < \tilde{e}_n$. The expected present value of a new employment relationship is now relatively low, dampening firms' incentives to hire: labour market tightness jumps to its low steady-state level $\tilde{\theta}_0 = \tilde{\theta}_l^*(\lambda)$ as shown in Figure 2.4b. In the following period unemployment monotonically increases towards the high steady-state $u_H^*(\lambda)$, whilst incumbents' reference wage decreases towards $r_H(\lambda)$ and their effort increases to $\tilde{e}_n$.

[18]See Appendix A.2, Section A.2.3, for details on the construction of the effort phase diagram.

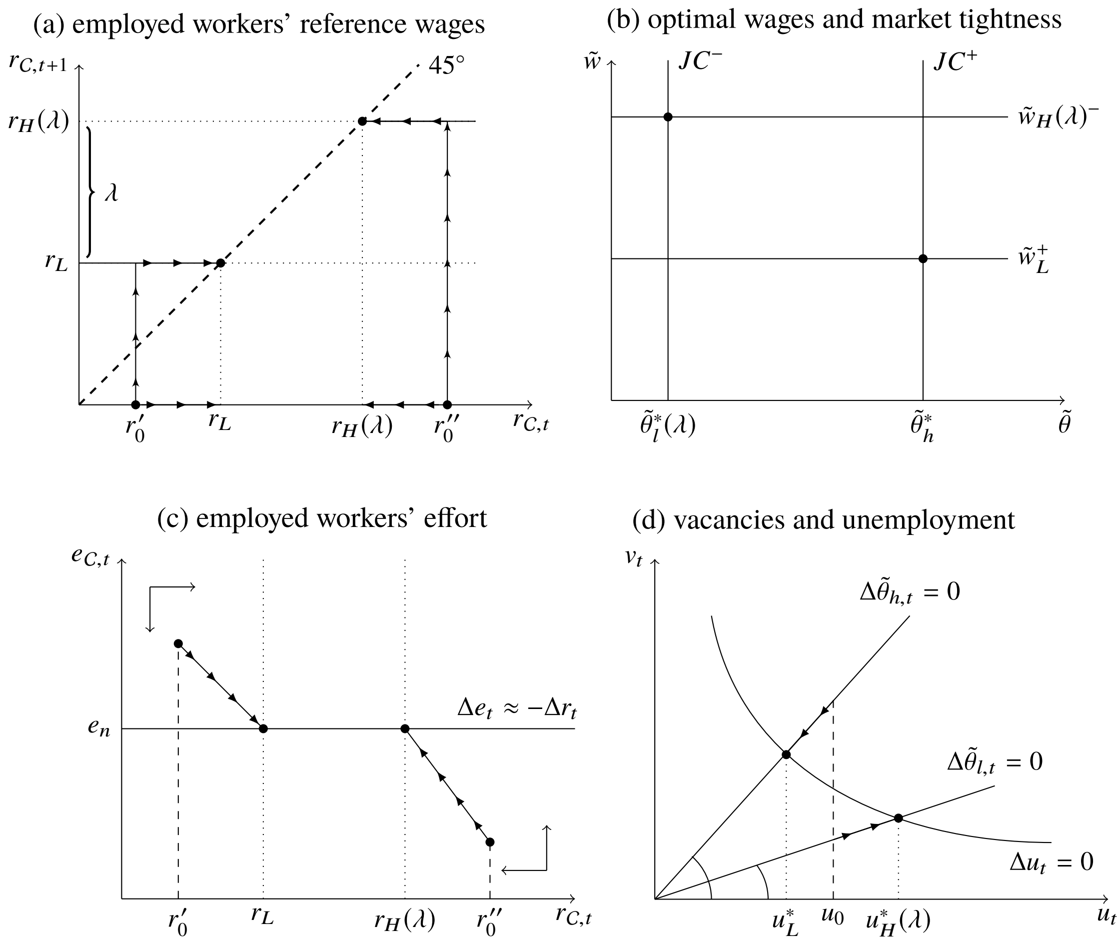

Figure 2.4:
Transitional Dynamics

If the labour market were populated by workers with relatively moderate initial reference wages $r_0 \in [r_L, r_H]$, as in case b), there would be no transitional dynamics for reference wages and effort levels: all employment relationships start with the optimal wage $\tilde{w}_M^= = r_0$ being perceived as fair and workers exerting normal effort in production.

This simple transitional dynamics analysis has shown that, depending on workers' wage entitlements at the time of hiring, the labour market can be characterised by two very different, and even opposite, dynamic paths: one in which workers are paid relatively higher wages and unemployment converges (increases in Figure 2.4d) towards a relatively high equilibrium; and another in which workers are paid relatively lower wages and unemployment converges (decreases in Figure 2.4d) towards a relatively low equilibrium.

2.5 The Volatility of Vacancies and Unemployment

This section implements the framework developed in this chapter to derive and analyse the steady-state elasticities of labour market tightness with respect to aggregate productivity. The section is structured as follows. First, Section 2.5.1 engages in a concise discussion of the relevant literature on labour market fluctuations. Then Sections 2.5.2 and 2.5.3 provide both qualitative and quantitative analyses of how the novel behavioural mechanisms considered in this chapter can potentially affect the volatility of the vacancy-unemployment ratio. In so doing, this section contributes to the labour market literature that aims to explain the *amplitude* and co-movement of vacancies and unemployment fluctuations.

2.5.1 A Concise Discussion of the Relevant Literature

The analysis of the steady-state elasticity of labour market tightness with respect to productivity (referred to as the elasticity of market tightness henceforth) is commonly used as a good approximation of the volatility of vacancies and unemployment when the labour market is hit by exogenous shocks to aggregate productivity (Mortensen and Nagypál, 2007; Elsby et al., 2015). Moreover, as shown by Shimer (2005), this elasticity is particularly important for the assessment of the quantitative implications of the model dynamics: a greater elasticity of market tightness implies that job creation is more responsive to exogenous shocks in productivity.[19]

In a highly influential paper Shimer (2005) calibrates a canonical search and matching model and shows that the model cannot quantitatively account for the high volatility of the vacancy-unemployment ratio observed in U.S. data over the period 1951–2003 (see Amaral and Tasci (2016) for a comparable exercise on a set of OECD countries). This *quantitative* failure has been labelled as "the unemployment volatility puzzle" (Pissarides, 2009). Shimer's insight is that the wage response to shocks in productivity predicted by the model is too large, i.e. the elasticity of the wage with respect to productivity is close to unity, offsetting almost all the effect of the productivity shock on job creation. Hence, introducing a degree of wage stickiness will improve the model's explanatory power. Subsequent to Shimer (2005) the literature attempting to solve the puzzle has flourished, and two main streams of thought have been developed.

On one hand, following the suggestion of Shimer (2005) and starting with the contribution of Hall (2005b), a large body of literature has placed considerable emphasis

[19]A greater elasticity of market tightness increases the magnitude of the rotation of the JC curves in Figure 2.3b whenever any of the possible equilibria is perturbed by a change in p. In fact the greater this elasticity the larger will be the amplitude (volatility) and co-movement of unemployment and vacancies in the (u, v) space for any given change in p.

on the role of the cyclicality of wages by proposing alternative, and often *ad hoc*, wage determination mechanisms that can generate some form of wage rigidity, i.e. acyclicality, or wage stickiness, i.e. less than proportional cyclicality. For surveys of this literature see, for instance, Mortensen and Nagypál (2007) and Rogerson and Shimer (2011). Given the emphasis on the cyclical behaviour of job creation, these models have stressed the importance of rigidities in *newly hired* workers' wages, which, as shown by Pissarides (2009), is the relevant wage affecting hiring decisions in the canonical model. However, the more recent empirical literature has challenged the theory underlying these models by providing evidence that wages offered to newly hired workers are instead substantially pro-cyclical (Martins et al., 2012; Carneiro et al., 2012; Haefke et al., 2013; Stüber, 2017). Building on these findings, Kudlyak (2014) has shown that it is not the hiring wage that is the relevant price of labour for firms, but rather, it is the user cost of labour, i.e. the opportunity cost of delaying hiring decisions. By providing estimates of this measure, and showing that it can be even more pro-cyclical than the hiring wage, Kudlyak (2014) concludes that wage rigidity is not relevant to address the unemployment volatility puzzle.

On the other hand, a different perspective in response to Shimer's critique has been pursued by Mortensen and Nagypál (2007) and Pissarides (2009) among others. These authors have argued that the literature has put too much emphasis on the role of newly hired workers' wage cyclicality. Even if hiring wages were more sticky, for this to have a substantive effect on the size of the elasticity of market tightness, the present value of the wage would also need to be sufficiently high relative to the firm's present value of output from a new match (Elsby et al., 2015). As such, what matters for job creation is the size of the present value of the *profit margin* from a new employment relationship (Kennan, 2010), i.e. the difference between the present values of output and the wage: only if this margin is small enough will slight changes in productivity generate large fluctuations in the anticipated profits from new matches, and hence in vacancy creation and unemployment. This perspective, which Ljungqvist and Sargent (2016) summarised under the concept of the "fundamental surplus", downplays the role of new hires' wage rigidity and shifts the focus to the size of the surplus generated by new employment relationships. Nevertheless, a question remains: what are the aspects that make firms' profit margins from new employment relationships sufficiently small as to generate a realistically large amplitude in vacancies and unemployment fluctuations?

To understand the key insights of these arguments more clearly, denote the elasticity of any variable x with respect to productivity p as $\varepsilon_x = \frac{p}{x}\frac{dx}{dp}$, and consider the elasticity of market tightness in the context of the framework developed in this chapter, for all possible steady-state equilibria that have been established in Section 2.4:

Lemma 2. *For all $\mathcal{E} = \{\mathcal{L}^*, \mathcal{M}^*, \mathcal{H}^*(\lambda)\}$, the elasticity of labour market tightness $\tilde{\theta}^*$ with respect to productivity p takes the form:*

$$\varepsilon_{\tilde{\theta}^*} = \frac{1}{\sigma} \frac{\overline{y}^*(\tilde{e}_N^*) - \varepsilon_{\tilde{w}^*} \overline{w}^* \cdot \Theta}{\overline{y}^*(\tilde{e}_N^*) - \overline{w}^*}, \tag{2.22}$$

where $\overline{y}^*(\tilde{e}_N^*)$ and $\overline{w}^*$ are the present discounted values of output and the wage as defined by (2.17) *and* (2.16) *respectively, $\varepsilon_{\tilde{w}^*}$ is the elasticity of the wage $\tilde{w}^* \in [\tilde{w}_L^+, \tilde{w}_H(\lambda)^-]$ with respect to productivity p and Θ is a function of the parameters of the model (to be defined shortly).*

Proof. See Appendix B.2. ■

Lemma 2 derives an equation for the elasticity of market tightness that is directly comparable with the literature (see for instance Pissarides (2009), equation (20), p.1352; or Elsby et al. (2015), equation (11), p.590).[20] In the canonical model $\Theta = 1$, which implies that the size of the elasticity of market tightness hinges crucially on the size of the elasticity of wages $\varepsilon_{\tilde{w}^*}$. Indeed if $\varepsilon_{\tilde{w}^*} = 1$ wages are perfectly proportional to changes in productivity and the elasticity equation (2.22) collapses to $\varepsilon_{\tilde{\theta}^*} = 1/\sigma$, implying that its size depends crucially on the match elasticity $\sigma \in (0, 1)$. As it has been shown in the literature cited above, for values of $\sigma \in [0.235, 0.72]$ the model fails to generate the target elasticity of $\varepsilon_{\tilde{\theta}^*} = 7.56$[21] (see Mortensen and Nagypál (2007), Pissarides (2009) and Kudlyak (2014)). This numerical exercise reproduces, in essence, the analysis underlying the insight of Shimer (2005): by implementing a wage setting mechanism that yields an elasticity of wages with respect to productivity lower than unity $\varepsilon_{\tilde{w}^*} < 1$, i.e. by introducing some sort of wage rigidity/stickiness, the size of $\varepsilon_{\tilde{\theta}^*}$ will increase, improving the explanatory power of the model.

Ignoring the empirical estimates of $\varepsilon_{\tilde{w}^*}$ for the time being, consider the extreme solution proposed by Hall (2005b), in which wages are entirely acyclical, i.e. $\varepsilon_{\tilde{w}^*} = 0$. In such a case equation (2.22) can be re-expressed as follows:

$$\varepsilon_{\tilde{\theta}^*} = \frac{1}{\sigma} \frac{1}{1 - (\overline{w}^* / \overline{y}^*(\tilde{e}_N^*))}, \tag{2.23}$$

where in the canonical model $\overline{w}^* / \overline{y}^*(\tilde{e}_N^*) = \tilde{w}^*/p$. It is clear from this expression that, even if wages are entirely rigid, the size of the elasticity of market tightness depends crucially on the wage-output ratio determining the firms' profit margin—the fundamental surplus—from new employment relationships. As pointed out in the

[20] The discussion that follows draws on Pissarides (2009) and Elsby et al. (2015).

[21] This figure corresponds to the regression coefficient in a simple regression with labour market tightness as the dependent variable and productivity as the independent variable.

previous discussion of the literature, the higher the wage relative to the value of output from a new match, i.e. the closer the $\tilde{w}^*/p$ ratio is to one, the lower the profit margin and therefore the greater the size of the elasticity of market tightness (Elsby et al., 2015).

To conclude this brief *excursus* around the determinants of the size of $\varepsilon_{\tilde{\theta}^*}$, notice that the empirical literature has estimated the cyclicality of hiring wages to be around 1.[22] This finding supports the aforementioned conclusions reached by Kudlyak (2014), that the volatility of the hiring wage is not useful to explain the high volatility of vacancies and unemployment observed in the data; and that the free entry condition cannot simultaneously accommodate the empirical volatilities of the wage component of the user cost of labour and of the vacancy-unemployment ratio.

How does the framework developed in this chapter contribute to the arguments highlighted above? Building on the literature just discussed, the following section will analyse the qualitative properties of the model and whether the behavioural mechanisms considered can provide a novel perspective on the channels through which vacancies and unemployment fluctuations can be amplified. Subsequently, Section 2.5.3 will provide a quantitative assessment of these mechanisms, by performing a simple calibration exercise in which the empirical estimate of the elasticity of market tightness is used as one of the main targets.

2.5.2 Qualitative Implications

This section analyses the qualitative properties of the steady-state elasticities of market tightness for all the equilibria characterised in Section 2.4. First, the analysis will consider the role of new hires' wage cyclicality; and subsequently, it will consider the role of the novel behavioural aspects introduced in this chapter, in particular new hires' wage entitlements and asymmetric reciprocity.

The role of the cyclicality of wages

Consider the implications of the cyclicality of wages for the size of the elasticities of market tightness implied by the model.

Proposition 13. *Consider the elasticity of labour market tightness with respect to productivity $\varepsilon_{\tilde{\theta}^*}$ as established in Lemma 2:*

[22] For instance, Carneiro et al. (2012) provide an estimate of $\varepsilon_{\tilde{w}^*} = 1.07$ and Haefke et al. (2013) an estimate of $\varepsilon_{\tilde{w}^*} = 0.8$; while using the estimates provided in Pissarides (2009), Kudlyak (2014) computes a combined elasticity (of the user cost of labour) of $\varepsilon_{\tilde{w}^*} = 1.5$.

a) if $\mathcal{E} = \mathcal{M}^*$,

$$\varepsilon_{\tilde{w}^*} = 0,$$

which implies that wages are acyclical;

b) if $\mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}$,

$$\Theta = 0 \quad \forall \varepsilon_{\tilde{w}^*},$$

which implies that the cyclicality of wages $\varepsilon_{\tilde{w}^*}$ is irrelevant for the size of $\varepsilon_{\tilde{\theta}^*}$.

Proof. See Appendix B.2. ■

Proposition 13 highlights some noteworthy features of the steady-state elasticities derived within the framework developed in this chapter. First consider case a): $\mathcal{E} = \mathcal{M}^*$. In a labour market that is populated by workers with moderate reference wages, due to the reasons outlined in Section 2.4.2, the wage of newly hired workers does not respond to changes in productivity. This case reproduces the result generated by the modification based on "norms" proposed, and analysed, by Hall (2005b). Although the wage setting model implemented in this chapter provides a micro-founded rationale—based on fairness and loss aversion—that endogenously generates rigidity in the wage of newly hired workers, its conclusions remain nevertheless vulnerable to the same critique put forward by the empirical evidence discussed.

Next consider case b): $\mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}$. If the labour market is populated by workers with relatively low, or relatively high, reference wages at the start of an employment relationship, the cyclicality of wages with respect to productivity is irrelevant for the determination of the size of the elasticity of market tightness. Note that this conclusion holds for any value of $\varepsilon_{\tilde{w}^*}$, hence irrespectively of the specific value implied by its empirical estimate. To understand the intuition behind this statement, consider the function Θ, which takes the following form:

$$\Theta \equiv -\left[\frac{\partial y(\tilde{e}_N^*)}{\partial e}\frac{\partial \tilde{e}_N^*}{\partial w} - \psi \frac{\partial y(\tilde{e}_E^*)}{\partial e}\frac{\partial \tilde{e}_E^*}{\partial w} - 1\right] = 0. \tag{2.24}$$

Notice that the expression inside the square brackets is equivalent to the first-order condition $\Upsilon(w; r_C, \lambda) = 0$ given by (2.6), which characterises the firms' optimal wage policy for all $w \neq r$. Hence, $\Theta = 0$.

The interpretation of this result is as follows. For any given change in aggregate productivity, firms anticipate setting a wage that will optimally balance the inter-temporal trade-off between the marginal cost of a higher wage, and the marginal benefit, on output y, generated by the workers' reciprocity (to satisfy the first-order condition (2.6)). As such, it does not matter how responsive new hires' wages are to productivity, since firms optimally exploit this response by inducing a counteracting response in workers' effort, which positively (or negatively) affects output and leaves room for the impact of the change in productivity to be reflected in firms' present value of output. Using a terminology more familiar with the literature discussed above (e.g. Haefke et al. (2013)), when aggregate productivity increases, firms are able to turn the additional surplus received by workers, in the form of a higher wage, into an additional surplus that they receive, in the form of higher effort exerted by newly hired workers. This is the reason why, in the framework set out here, the elasticity of the wage of new hires' with respect to productivity is irrelevant for job creation; and changes in aggregate productivity are fully absorbed by firms' present value of profit. Note that this conclusion remains valid even if reciprocity were symmetric ($\lambda = 1$) with the labour market being characterised by a unique steady-state equilibrium $\mathcal{E}^* = \mathcal{L}^* = \mathcal{H}^*(1)$. Moreover, at this stage, it is also clear why in the canonical model $\Theta = 1$: by neglecting the impact of wage changes on workers' effort, the term in the square brackets of equation (2.24) equals -1, capturing only the marginal cost of a higher wage on firms' profits.

This qualitative result is particularly important for two reasons. First it reinforces the argument summarised in Elsby et al. (2015), that besides the extent of cyclicality of new hires' wages, it is the anticipated present value of the firms' profit margin that matters for the size of the elasticity of labour market tightness. Second, it shows that, in the presence of reference-dependent reciprocity, the extent of new hires' wage cyclicality is in fact irrelevant for the volatility of vacancies and unemployment, unless the wage paid to new hires is entirely rigid, i.e. when $\mathcal{E} = \mathcal{M}^*$. In addition notice that the framework developed here falls into the class of models in which the wage component of the user cost of labour, as defined by Kudlyak (2014), is equal to the wage.[23] As such, this framework is consistent with any empirical estimate of the cyclicality of the relevant price for labour at the time of hiring, be it the hiring wage, the average wage, or the wage component of the user cost of labour.

Investigating whether the statements of Proposition 13 hold under more general assumptions around the workers' cost of effort and the firms' production function (i.e. relaxing assumptions W2 and F1) is left to further research.

Reference wages, effort, loss aversion and the size of the profit margin

Motivated by the conclusion of the preceding section, and by the arguments put forward in the literature discussed, this section considers how the novel behavioural aspects introduced in this chapter affect firms' anticipated present value of the profit margin

[23] See Section A.2.4 in Appendix A.2 for a closer comparison with Kudlyak's argument.

from a new match, subsumed by the wage-output ratio $\overline{w}^*/\overline{y}^*(\tilde{e}_N^*) < 1$. In fact, as previously highlighted in the discussion of equation (2.23), anything that makes this ratio closer to 1 will naturally increase the size of the elasticity of market tightness. These novel behavioural aspects are: the reference wage characterising newly hired workers r_N; the employed workers' optimal effort in equilibrium $\tilde{e}_C^*$; and their extent of loss aversion $\lambda > 1$ (which is only relevant in the high equilibrium $\mathcal{E} = \mathcal{H}^*(\lambda)$).

The wage-output ratio for each of the three distinct steady-state equilibria can be expressed analytically as:

$$\frac{\overline{w}^*}{\overline{y}^*(\tilde{e}_N^*)} = \begin{cases} \dfrac{1}{\tilde{e}_N^+ + \frac{\psi}{1-\psi}\tilde{e}_n} & \text{if } \mathcal{E} = \mathcal{L}^* \\[2ex] \dfrac{r_N}{pq\tilde{e}_n} & \text{if } \mathcal{E} = \mathcal{M}^* \\[2ex] \dfrac{\lambda}{\tilde{e}_N(\lambda)^- + \frac{\psi}{1-\psi}\tilde{e}_n} & \text{if } \mathcal{E} = \mathcal{H}^*(\lambda). \end{cases} \tag{2.25}$$

Consider first the moderate equilibrium $\mathcal{E} = \mathcal{M}^*$, in which newly hired workers are paid their fair wage and as a consequence exert normal effort for the entire duration of the employment relationship. Since wages are fully rigid, the size of the elasticity of market tightness depends crucially on two elements: the intrinsically motivated level of effort $\tilde{e}_n$; and the reference wage of new hires r_N. Higher normal effort raises firms' profit margin since it implies a higher value of output for a given r_N; while a higher initial reference wage decreases the profit margin since it implies a higher equilibrium wage for a given $\tilde{e}_n$.

Next consider the low and high equilibria $\mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}$. In the low steady state newly hired workers perceive the wage they are paid as a gift, and therefore exert supra-normal effort in production. In contrast, in the high steady-state newly hired workers perceive that they are paid an unfair wage and therefore exert sub-normal effort. The effect of $\tilde{e}_n$ on the wage-output ratio is the same as for the moderate equilibrium: higher normal effort increases the optimal effort response of workers, which in turn increases the profit margin of firms by raising the anticipated value of output from a new match. Also the initial reference wage r_N has the same negative effect as in the moderate equilibrium, but the underlying behavioural mechanism is qualitatively different. If the labour market is at the low equilibrium, a higher r_N decreases the amount of positive reciprocity exerted by newly hired workers for any given $\tilde{w}_L^+$: i.e. the gap $\mu(\ln \tilde{w}_L^+ - \ln r_N)$ which determines the gains in workers' utility shrinks. While if the labour market is at the high equilibrium, a higher r_N increases the amount of newly hired workers' negative reciprocity for any given $\tilde{w}_H(\lambda)^-$: i.e. the gap $\mu(\ln \tilde{w}_H(\lambda)^- - \ln r_N)$ which determines the losses in the workers' utility widens. Both these effects act to reduce the anticipated value of output from a new match, decreasing the firms' profit margin and therefore increasing the size of the respective steady-state elasticities.

Finally, consider the high equilibrium only, and recall that workers' degree of loss aversion $\lambda > 1$ determines the strength of their negative reciprocity response when they feel they have been treated unfairly in the first employment period. As such a higher λ reduces the present value of effort exerted by newly hired workers. However, in exploiting the optimal trade-off between the wage and negative reciprocity, firms matched with more loss averse workers anticipate the reduction in effort that will occur, and will therefore pay a higher steady-state wage to partially offset the higher negative reciprocity, therefore mitigating the losses in utility perceived by workers for any given r_N.

Proposition 14. *If $\mathcal{E} = \mathcal{H}^*(\lambda)$, then $\overline{w}^*_N/\overline{y}(\tilde{e}^*_N)$ is increasing in λ, which implies that the elasticity of labour market tightness $\varepsilon_{\tilde{\theta}^*} = \varepsilon_{\tilde{\theta}^*_l}$ is increasing in λ.*

Proof. See Appendix B.2. ■

Proposition 14 states that a higher anticipated negative reciprocity of newly hired workers unambiguously acts to reduce the expected profit margin of firms, therefore raising the volatility of vacancies and unemployment. This result implies that in labour markets where the wage entitlements of newly hired workers are relatively high, and firms anticipate a greater cost of setting an optimal wage that is perceived to be unfair—in terms of low morale and effort—the volatility of vacancy and unemployment will be higher.

By exploiting the richer microeconomic foundations of the present model, this analysis suggests the existence of additional behavioural channels that might affect the fundamental surplus relevant to firms' hiring decisions. The initial level and subsequent evolution of employed workers' wage entitlements, their intrinsic motivation captured by normal effort, and their extent of asymmetric reference-dependent reciprocity, are all qualitatively relevant aspects influencing the firms' anticipated present value of new employment relationships, and hence the cyclical behaviour of job creation, vacancies and unemployment. However, are these aspects also quantitatively relevant?

2.5.3 Quantitative Assessment

This section performs a calibration exercise in order to evaluate the quantitative relevance of the novel behavioural aspects introduced in this chapter.

The standard calibration approach in the literature is to assign values to structural/exogenous parameters, based on labour market data and/or empirical estimates, in order to assess the quantitative performance of the model, in particular in terms of whether its key endogenous outcomes can match their empirical counterparts. In the literature of labour market fluctuations, in which the search and matching framework has become the workhorse of economic analysis, several models have been evaluated on the basis of their potential to quantitatively account for the observed volatility of vacancies and unemployment fluctuations, i.e. for the size of the elasticity of labour market tightness.

Given the relatively high number of unobservable exogenous parameters (degrees of freedom) introduced by the theoretical framework developed in this chapter, the following calibration strategy and subsequent quantitative analysis will be slightly different to what performed in other studies. That is, instead of using debatable proxy measures to assign values to the unobservable parameters of the model—such as normal effort $\tilde{e}_n$ and new hires' reference wages r_N—and to subsequently calculate the steady-state elasticity of market tightness, the following approach will use the empirical estimate of this volatility measure as a calibration target. Hence, the analysis will study a combination of the behavioural parameters that can potentially deliver a target elasticity of $\varepsilon_{\tilde{\theta}^*} = 7.56$; then, it will evaluate their quantitative plausibility and relevance by appealing to the resulting relationships generated by these values with respect to other endogenous outcomes of the model.

A model for calibration

Before proceeding with the analysis, this section provides a version of the model in which new hires are assumed to be heterogenous with respect to their reference wage at the start of an employment relationship. The advantage of this assumption is that the path-dependence of the model's steady state will crucially depend on the *distribution* of reference wages among new hires. This enables to perform a unique calibration and subsequent model evaluation which exhibits all the properties of the three distinct steady states analysed in the preceding section.

Hence, consider the following assumption with respect to the distribution of wage entitlements r_N in the labour market:

C1. $r_N = \{r_{N_1}, r_{N_2}, r_{N_3}\}$ is distributed according to $\overline{\varphi} \equiv (\overline{\varphi}_1, \overline{\varphi}_2, \overline{\varphi}_3)$, where

$$\begin{aligned} r_{N_1} &< r_L \\ r_{N_2} &\in [r_L, r_H] \quad \text{and} \quad \sum_{i=1}^{3} \overline{\varphi}_i = 1. \\ r_{N_3} &> r_H(\lambda) \end{aligned}$$

Assumption C1 imposes heterogeneity among new hires with respect to their wage entitlements: the labour market is populated by a fraction $\overline{\varphi}_1$ of workers with relatively low reference wages $r_{N_1} < r_L$; a fraction $\overline{\varphi}_2$ of workers with relatively moderate reference wages $r_{N_2} \in [r_L, r_H]$; and a fraction $\overline{\varphi}_3$ of workers with relatively high reference wages $r_{N_3} > r_H(\lambda)$.[24] Moreover notice that $\sum_{i=1}^{3} \overline{\varphi}_i r_{N_i}$ represents both the *average* and the *expected* reference wage characterising potential new hires in the labour market.

Without the need of any additional formal proof, it is possible to express the steady-state job creation condition of the model with free entry as

$$\frac{\kappa}{\bar{m}\theta^{-\sigma}} = \delta \sum_{i=1}^{3} \overline{\varphi}_i J(r_{N_i}),$$

where indeed: $J(r_{N_1}) > J(r_{N_2}) > J(r_{N_3})$ and

$$\begin{aligned} J(r_{N_1}) &= J(r_N)^{+} &> J(r_E)^{+} \\ J(r_{N_2}) &= J(r_N)^{=} &= J(r_E)^{=} \\ J(r_{N_3}) &= J(r_N)^{-} &< J(r_E)^{-} \end{aligned}$$

for all $t \geq \tau$, as established by Lemma 1. Similarly, the expected present discounted values of output and wages from a new employment relationship can be analytically expressed as

$$\sum_{i=1}^{3} \overline{\varphi}_i \overline{y}^*(\tilde{e}^*_{N_i}) = pq\left[\sum_{i=1}^{3} \overline{\varphi}_i \tilde{e}^*_{N_i} + \frac{\psi}{1-\psi}\tilde{e}_n\right] \quad \text{and} \quad \sum_{i=1}^{3} \overline{\varphi}_i \overline{w}^*_i = \sum_{i=1}^{3} \overline{\varphi}_i \frac{\tilde{w}^*_i}{1-\psi}$$

respectively. As such, the steady-state elasticity of labour market tightness with respect to productivity characterising this set up is given by:

$$\varepsilon_{\tilde{\theta}^*} = \frac{1}{\sigma} \frac{\sum_{i=1}^{3} \overline{\varphi}_i \overline{y}^*(\tilde{e}^*_{N_i})}{\sum_{i=1}^{3} \overline{\varphi}_i \left[\overline{y}^*(\tilde{e}^*_{N_i}) - \overline{w}^*_i\right]},$$

which gives one of the key expressions to be used in the following calibration exercise.

[24]Note that Assumption C1 concerns all workers that can potentially become new hires in the period in which firms post vacancies, and not only those new hires that are successfully matched in their first period of production.

Calibration strategy

The calibration strategy implemented in this quantitative analysis aims to evaluate the combination of the exogenous behavioural parameters which is required to replicate the observed volatility of vacancies and unemployment fluctuations. To enhance comparability with the literature, where possible, the conventional parameters and targets are chosen following the calibration performed by Pissarides (2009). The calibration proceeds as follows: first the conventional parameters of the model are chosen in accordance with the standard approach in the literature; then, the remaining behavioural parameters are calibrated so as to achieve the desired target elasticity of market tightness.

Conventional parameters. The time period is given by a quarter. The elasticity of the matching function with respect to unemployment σ is set equal to 0.5 as in Pissarides (2009). This value is at the lower bound of the range of estimates provided by Petrongolo and Pissarides (2001), i.e. $\sigma \in [0.5, 0.7]$, and it is in the middle of the range of values used in the literature, i.e. $\sigma \in [0.235, 0.72]$ (see Kudlyak (2014)). The discount factor $\delta = 0.996$ is set to match a quarterly interest rate of 0.004, and the exogenous job destruction rate ρ is set equal to 0.036 (see Pissarides (2009) and Shimer (2012)). The aggregate productivity parameter p is normalised to unity (standard); while the idiosyncratic match productivity parameter q is normalised to 100 in order to ensure a non-negative wage utility, i.e. so that $m(\tilde{w}_i) = \ln \tilde{w}_i \geq 0$. The remaining conventional parameters—namely, the efficiency of matching $\bar{m}$ and the cost of posting a vacancy κ—are calibrated to match an average job finding probability of 0.594, and an average vacancy-unemployment ratio of 0.72 (as in Pissarides (2009)).[25] Notice that this calibration yields a steady-state probability that a vacant job is matched with a worker of $h(\theta) = 0.7 \cdot (0.72)^{-0.5} = 0.825$.

Behavioural parameters. The behavioural parameters of the model are: the employed workers' normal effort $\tilde{e}_n$; their degree of loss aversion λ, which also affects their extent of negative reciprocity in the event of an unfair wage; their wage entitlements at the start of the employment relationship r_{N_i}; and the relative frequencies $\overline{\varphi}_i$ with which these entitlements are distributed among workers. The loss aversion parameter λ is set to be equal to 2, which implies that the negative effect of an unfair wage is two times bigger than the positive effect of a wage gift on workers' morale and reciprocity. This parameter value is based on the experimental analysis of Abdellaoui, Bleichrodt, and Paraschiv (2007) and lies below the median of the range of loss aversion parameters

[25]The relatively high number which results from the calibration of the vacancy cost κ is essentially a product of the non-conventional normalisation of the idiosyncratic match productivity q. However note that none of these two parameters are crucial for the determination of the elasticity of market tightness.

$\lambda \in [1.43, 4.8]$ estimated in the literature (see Abdellaoui et al. (2007) for a review).

Table 2.1: Parameter Values, Quarterly Calibration

Parameter	Value	Description	Source/Reason
Conventional			
σ	0.500	Elasticity of matching	Literature
ρ	0.036	Exogenous job destruction Rate	Literature
δ	0.996	Discount factor	Interest rate = 0.004
$\bar{m}$	0.700	Efficiency of matching	Job finding probability
κ	44.40	Vacancy cost	v/u ratio
Behavioural			
λ	2.000	Loss aversion parameter	Abdellaoui et al. (2007)
$\tilde{e}_n$	0.082	Normal effort	1% above $\underline{e}_n$
$\frac{\lvert\tilde{w}_i - r_{N_i}\rvert}{r_{N_i}}$	1.34%	% deviation $\forall i = \{1, 3\}$	Calibrated
Normalisations			
p	1.000	Aggregate productivity	Standard
q	100.0	Idiosyncratic match productivity	$\ln \tilde{w}_i \geq 0$
$\overline{\varphi}_i$	$0.33\overline{3}$	Fraction of r_{N_i}, $\forall i = \{1, 2, 3\}$	Uniform
Targets			
θ	0.720	Average v/u (tightness)	Pissarides (2009)
$\bar{m}\theta^{1-\sigma}$	0.594	Average job finding probability	Pissarides (2009)
$\varepsilon_{\tilde{\theta}^*}$	7.560	Average elasticity of θ w.r.t. p	Literature

At this stage notice that it is possible to find several combinations of the remaining behavioural parameters that could deliver the desired target elasticity of 7.56. Hence, the framework developed in this chapter shows that there exists a richer set of potential channels through which the volatility of vacancies and unemployment could be amplified. Nevertheless, the calibration strategy proceeds as follows. Workers' normal effort $\tilde{e}_n$ is set to be 1% above the minimum effort required to ensure that firms' zero-profit condition when hiring a worker with a relatively high reference wage, i.e. when $r_{N_3} > r_H(\lambda)$, is always satisfied (see Section A.2.1 of Appendix A.2 for the analytical derivation of this condition). That is, $\tilde{e}_n = \underline{e}_n(r_{N_i}, \lambda) \cdot [1 + 1\%]$ where

$$\underline{e}_n(r_{N_i}, \lambda) \equiv \max\{\tilde{e}_n : J(r_{N_i}) = 0, \forall r_{N_3} > r_H(\lambda)\}.$$

This yields a value for normal effort of $\tilde{e}_n \approx 0.081 + 0.001 \approx 0.082$. Next, the distribution of reference wages among workers is arbitrarily set to be uniform:

$$\overline{\varphi}_1 = \overline{\varphi}_2 = \overline{\varphi}_3 = 1/3.$$

Since it might be extremely hard to find an empirical counterpart of these frequencies, this parameterisation takes a neutral and agnostic stand by giving an equal weight to each possible scenario.

Finally, new hires' reference wages are calibrated to match an elasticity of labour market tightness of $\varepsilon_{\tilde{\theta}^*} \approx 7.56$. This is performed by simultaneously setting the following conditions: i) the percentage deviations of the relatively low r_{N_1} and relatively high r_{N_3} reference wages from their respective steady-state equilibrium wages are assumed to be the same in absolute magnitude:

$$\frac{|\tilde{w}_1 - r_{N_1}|}{r_{N_1}} = \frac{|\tilde{w}_3 - r_{N_3}|}{r_{N_3}};$$

and ii) the moderate reference wage is assumed to be the average of the high and low reference wage:

$$r_{N_2} = \frac{r_{N_1} + r_{N_3}}{2}.$$

This calibration yields $|\tilde{w}_i - r_{N_i}|/r_{N_i} \approx 1.34\%$ and a moderate reference wage of $r_{N_2} = 6.004 = \tilde{w}_{\overline{M}}$. Hence, firms face the same probability $h(\theta) \cdot \overline{\varphi}_i = 0.825 \cdot 0.\overline{3}$ of being matched with a worker for which it is optimal to either pay a wage 1.34% above their reference wage; to pay a wage 1.34% below their reference wage; or to pay them their reference wage. All parameter values are summarised in Table 2.1.

Calibration results and discussion

The calibration results concerning the main steady-state outcomes of interest are reported in Tables 2.2 and 2.3. The first three rows of each table display the results for a representative worker-firm employment relationship in which the employed worker is characterised by a relatively low, moderate or high reference wage respectively. The last row displays the expected labour market values of the endogenous outcomes in the steady state. This numerical exercise also enables to see more clearly some of the qualitative properties of the model that have been discussed throughout the chapter.

As shown in Table 2.2, the fraction of workers with the relatively low reference wage $r_{N_1} \approx 3.9$ are paid a steady-state equilibrium wage $\tilde{w}_1 \approx 4.0$ and exert supra-normal effort $\tilde{e}_{N_1} \approx 0.095 > \tilde{e}_n$ in their first employment period. This corresponds to a (calibrated) wage gift of 1.34% which triggers an endogenous positive reciprocity response of +16.3%, calculated as the percentage deviation of new hires' optimal effort from their normal level $\tilde{e}_n$ in the first employment period. In this case the value of output produced is $y(\tilde{e}_{N_1}) \approx 9.5$, implying that the wage paid to these workers corresponds to 42% of the output produced in the first employment period. On the contrary, the fraction of workers with the relatively high reference wage $r_{N_3} \approx 8.1$ are paid a steady-state equilibrium wage $\tilde{w}_3 \approx 8.0$ and exert sub-normal effort $\tilde{e}_{N_3} \approx 0.055 < \tilde{e}_n$ in their first employment period. This corresponds to a (calibrated) unfair wage 1.34% below their wage entitlement, which triggers an endogenous negative reciprocity response of −33.1%. Moreover notice that in this case the value of output is $y(\tilde{e}_{N_3}) \approx 5.5$, implying that the wage paid to these workers corresponds to 146% of the output produced in the first employment period. These results imply that firms expect to pay new hires a

Table 2.2: Newly Hired Workers Statistics

Steady State	Outcomes					
	$\tilde{w}_i$	as % of $y(\tilde{e}_{N_i})$	r_{N_i}	w-r gap	$\tilde{e}_{N_i}$	% deviation from $\tilde{e}_n$
Low	4.0	42.0%	3.9	+1.34%	0.095	+16.3%
Moderate	6.0	73.6%	6.0	-	0.082	-
High	8.0	146.0%	8.1	−1.34%	0.055	−33.1%
Expected	5.99	77.7%	6.00	-0.31%	0.077	−5.6%

steady-state wage that is just 0.31% below the expected reference wage in the labour market. However, this generates an expected sub-normal effort response in the first employment period, corresponding to 5.6% less of normal effort.

Table 2.3 displays the present discounted values of output and wages in the market, the resulting value of a new employment relationship to firms, and related elasticities of new hires' wages and market tightness. From these results it is clear that the value of a job filled by a newly hired worker with a high reference wage is very low relative to the one filled by a worker with either a low or a moderate reference wage. The main drivers of this outcome are: the relatively low normal effort (which is 1% above the minimum required for the job to be profitable), combined with optimal wage setting, according to which firms pay a relatively higher (here twice as high) steady-state wage in order to offset the greater cost of new hires' negative reciprocity. However, in expectation,

Table 2.3: Present Values and Elasticities

Steady State	Outcomes				
	$\overline{y}(\tilde{e}_{N_i})$	$\overline{w}_i$	$J(r_{N_i})$	$\varepsilon_{\tilde{w}^*}$	$\varepsilon_{\tilde{\theta}^*}$
Low	206.1	100.0	106.1	1.0	3.886
Moderate	204.7	150.7	54.03	0.0	7.578
High	202.0	200.0	2.027	1.0	199.3
Expected	204.3	150.2	54.04	0.67	7.560

the value of a new employment relationship is "reasonably" large, consistent with a steady-state elasticity of labour market tightness of 7.56 as required by the calibration.

Finally notice that for the frequencies of the distribution of wage entitlements in the market considered, the calibration performed in this section delivers an elasticity of the wage paid to newly hired workers of $\varepsilon_{\tilde{w}^*} \equiv \sum_{i=1}^{3} \overline{\varphi}_i \varepsilon_{\tilde{w}^*_{N_i}} = 0.67$. While this measure does not match its empirical estimate (which is in a range of [0.8, 1.07], see footnote 22), two important points are worth noting. First, as the qualitative analysis has demonstrated, for those employment relationships in which new hires have either low or high reference wages, the elasticity of the wage with respect to productivity is entirely irrelevant for the size of $\varepsilon_{\tilde{\theta}^*}$. Secondly, an alternative calibration could be performed, in which the relative frequencies $\overline{\varphi}_i$ are calibrated to achieve a measure of $\varepsilon_{\tilde{w}^*}$ consistent with evidence.[26]

For instance, consider an alternative calibration in which the frequencies $\overline{\varphi}_i$ are set to deliver an elasticity of new hires' wages of 0.9. That is: $\overline{\varphi}_1 = 44.96\%$, $\overline{\varphi}_2 = 10\%$ and $\overline{\varphi}_3 = 45.04\%$.[27] The steady-state outcomes of this exercise are shown in Table 2.4 (since the outcomes for each single equilibrium are obviously unaffected). Indeed,

Table 2.4: Alternative Calibration

Expected Outcomes		
	$\tilde{w}_i$	as % of $y(\tilde{e}_{N_i})$
Wages	5.98	79.7%
	r_{N_i}	w-r gap
Reference Wages	6.01	−0.42%
	$\tilde{e}_{N_i}$	% deviation from $\tilde{e}_n$
Effort	0.075	−7.6%
	$\overline{y}(\tilde{e}_{N_i})$	$\overline{w}_i$
Present Values	204.1	150.1
	$\varepsilon_{\tilde{w}^*}$	$\varepsilon_{\tilde{\theta}^*}$
Elasticities	0.9	7.560

given a larger fraction of workers with relatively high reference wages, the expected

[26]Notice first that the maximum value of $\varepsilon_{\tilde{w}^*}$ that can be achieved in the framework considered in this section is bounded above by 1; and that this can be delivered by any reference wage distribution $\overline{\varphi} \equiv (\overline{\varphi}_1, \overline{\varphi}_2, \overline{\varphi}_3)$ in which $\overline{\varphi}_2 = 0$. However, this is a direct implication of logarithmic utility, i.e. $m(w) = \ln w$, imposed by assumption W1. For instance, if $m(w) = w^{\varsigma}/\varsigma$, with $\varsigma \in (0, 1)$, it can be shown that the steady-state wage takes the form

$$\tilde{w}^* = \begin{cases} [pq(1-\psi)]^{\frac{1}{1-\varsigma}} & \text{if } r_N = r_{N_1} \\ r_{N_2} & \text{if } r_N = r_{N_2} \\ [\lambda pq(1-\psi)]^{\frac{1}{1-\varsigma}} & \text{if } r_N = r_{N_3}. \end{cases}$$

As such, in this case the maximum value of $\varepsilon_{\tilde{w}^*}$ that can by achieved is bounded above by the factor $\frac{1}{1-\varsigma}$, which is greater than 1 for any $\varsigma \in (0, 1)$.

[27]These have been found by combining the following conditions: i) $\sum_{i=1}^{3} \overline{\varphi}_i \varepsilon_{\tilde{w}^*_{N_i}} = 0.9$; ii) $\sum_{i=1}^{3} \overline{\varphi}_i = 1$; and iii) $\overline{\varphi}_1$ is such that $\varepsilon_{\tilde{\theta}^*} \approx 7.560$. All other parameters kept the same as in the benchmark calibration.

wage-reference wage gap in the labour market is now larger in absolute magnitude, implying a lower expected effort in the first employment period (−7.6% of normal effort), but a larger share of output received by workers (79.7% of output). On the other hand, the expected present discounted values of output and wages are essentially robust to the change in the relative frequencies $\overline{\varphi}_i$. This alternative calibration delivers a (targeted) elasticity of new hires' wages with respect to productivity of $\varepsilon_{\tilde{w}^*} = 0.9$ and a (targeted) elasticity of labour market tightness of $\varepsilon_{\tilde{\theta}^*} = 7.56$. Hence, the present framework can simultaneously accommodate the empirical volatilities of both the hiring wage and of the vacancy-unemployment ratio, overcoming one of the issues mentioned by Kudlyak (2014) (see Section A.2.4, Appendix A.2, for the analog theoretical analysis in support of this result).

To conclude, though tackling the unemployment volatility puzzle from an alternative, un-conventional, perspective, the analysis of this section has demonstrated that for plausible values of the behavioural parameters introduced by the wage setting model of Chapter 1, the framework developed in this chapter can replicate some of the key empirical estimates discussed in the literature. This finding enhances the quantitative relevance of the behavioural aspects considered, and suggests a promising route for a richer dynamic stochastic simulation of the model developed here.

2.6 Uncertainty, Negative Reciprocity and Job Creation

The analysis of Section 2.5 has established that the cyclicality of newly hired workers' wages is irrelevant for the volatility of job creation. However, if for the reasons exposed in Chapter 1 the wage of incumbent workers is expected to be rigid, what is the role of expected downward wage rigidity for firms' hiring decisions?

In his influential paper Pissarides (2009) has answered this question by showing that it is irrelevant. Essentially, his argument is that even if the wage of incumbent workers were entirely rigid, firms will be able to internalise these future rigidities in the equilibrium wage negotiated with their newly hired workers at the start of the employment relationship, leaving the volatility of job creation unaffected by the expected rigidity of wages in subsequent employment periods. This theoretical result is general and holds true also under various modifications of the canonical model put forward to address the unemployment volatility puzzle (which, as discussed, have focused on the cyclicality of newly hired workers' wages).

Within a framework based on reference dependence, incomplete contracts and fairness, Eliaz and Spiegler (2014) challenge Pissarides' conclusion with the following *qualitative* insight. In a model where there is uncertainty about the evolution of aggregate productivity and wage rigidity of incumbent workers generates ex-post inefficient layoffs, the latter can negatively affect the expected present value of a new employment relationship by reducing its expected duration (essentially working as an additional discount factor). As such, Eliaz and Spiegler (2014) conclude that expected wage rigidity of incumbent workers can increase the volatility of job creation.

Despite its logical appeal, the prediction that expected wage rigidity dampens hiring incentives by reducing the expected duration of a match is not supported by the available evidence (e.g. Hall (2005a) and Shimer (2012)). The observed stability of the unemployment inflow rate during the more recent recessions downplays the role of job duration in determining hiring decisions (see Hall's comments to Eliaz and Spiegler (2014) in the same volume). In addition, Eliaz and Spiegler (2014) are unable to determine whether the extent to which the labour contract is incomplete—which is their relevant measure of the disproportionate drop in output in the event of future wage cuts—unambiguously increases the volatility of vacancies and unemployment. And finally, as shown by Moscarini, the volatility of the vacancy-unemployment ratio in the model of Eliaz and Spiegler (2014) is maximised when reference-dependence and wage rigidity do not play any role (see comments from Moscarini in the same volume).

The contribution made in what follows is to provide an alternative and complementary perspective to the qualitative insight advanced by Eliaz and Spiegler (2014). To do so the framework developed in this chapter is extended by introducing uncertainty around the evolution of an employed worker's match productivity throughout the employment relationship. In this context it is shown that even if incumbent workers' wage rigidity does not generate endogenous layoffs, firms' expectations of the relatively large cost of implementing wage cuts in the event of a low realisation of future match productivity—that is, the anticipation of stronger negative reciprocity by incumbent workers—negatively influence the expected present value of new employment relationships, dampening hiring incentives and increasing the volatility of job creation and unemployment.

2.6.1 Additional Assumptions

For the purpose of this analysis, employed workers are assumed to be characterised by a time-variant match productivity

$$q_t = q_t(C_t) \equiv q_{C,t},$$

which evolves stochastically from the start of a job following a first-order Markov process, the characteristic of which will be defined below. As such, firms' instantaneous profit function is modified accordingly as

$$\pi(w_t; r_{C,t}, q_{C,t}) = y(q_{C,t}, \tilde{e}_t(w_t, r_{C,t}, \lambda)) - s(w_t).$$

Firms' wage setting problem in each $t \geq \tau$ consists of choosing a sequence of wages $\{w_t\}_{t=\tau}^{\infty}$ that maximises their present value of profit, taking as given their employed worker's reference wage $r_{C,t}$, their match productivity $q_{C,t}$, and their optimal effort responses, defined by the sequence $\{\tilde{e}_t\}_{t=\tau}^{\infty}$. This is formalised by the following sequence problem

$$\begin{aligned} J(r_\tau, q_\tau, C_\tau) &= \max_{\{w_t\}_{t=\tau}^{\infty}} \mathbb{E}_\tau \left[\sum_{t=\tau}^{\infty} \psi^{t-\tau} \pi(w_t; r_{C,t}, q_{C,t}) \right] \\ \text{s.t.} \quad & r_{C,t+1} = w_t \ \forall t \geq \tau, \\ & r_{C,\tau}, \ q_{C,\tau} \text{ given,} \end{aligned} \tag{U-FP}$$

where the only difference with respect to (FP) of Section 2.2 is that firms are uncertain about the evolution of their employed workers match productivity from $t > \tau$ onwards. The relevant functional equation corresponding to the firms' problem (U-FP) can be written as:

$$J(r_C, q_C) = \max_w \left\{ \pi(r_C, w, q_C) + \psi \mathbb{E}\left[J(w, q'_C) | q_C \right] \right\} \tag{2.26}$$

where r_C and q_C correspond to the current period workers' reference wage and match productivity, and $w = r'_C$ and q'_C correspond to the following period workers' reference wage and match productivity. Denote the firms' optimal policy function as $\tilde{w} = \tilde{w}_C = \tilde{w}(r_C, q_C, \lambda)$, for all $C = \{N, E\}$, which determines the next period workers' reference wage $r_{C,t+1}$ (the control variable) for a given reference wage $r_{C,t}$ and match productivity $q_{C,t}$ (the state variables). The first-order condition corresponding to problem (2.26) takes the following form:

$$\Upsilon(w; r_C, q_C, \lambda) \equiv \frac{\partial \pi(r_C, w, q_C)}{\partial w} + \psi \Phi(w, \lambda) = 0, \quad \forall w \neq r_C, \tag{2.27}$$

where $\Phi(w, \lambda) \equiv \frac{\partial}{\partial r} \mathbb{E}\left[J(w, q'_C) | q_C \right]$ captures the marginal effect of a higher wage in the current period on the expected continuation value of the employment relationship. This condition represents the key inter-temporal trade-off faced by firms when setting the wage, and it is the stochastic infinite-horizon analogous of the previously analysed first-order conditions (1.10), Chapter 1, and (2.6), Section 2.2.

Before proceeding with the formal characterisation of firms' optimal wage setting policy and steady-state job creation condition, consider the following set of assumptions:

U1. $r_{C,\tau} = 0$.

U2. $q_{C,t}$ is a martingale with the following characteristics:

$$q_{C,t} = \begin{cases} q_N \text{ given} & \text{if } t = \tau \\ q_N + \varepsilon & \text{if } t > \tau, \end{cases} \qquad \varepsilon \sim \mathcal{N}(0, \nu^2).$$

U3. $e_n \geq \underline{e}_n(r_{C,t}, q_{C,t}, \lambda)$, where

$$\underline{e}_n(r_{C,t}, q_{C,t}, \lambda) \equiv \max\{e_n : J(r_{C,t}, q_{C,t}) = 0, \forall r_{C,t}, q_{C,t} \text{ and } \forall t \geq \tau\}.$$

Assumption U1 implies that newly hired workers' reference wage $r_{C,\tau} = r_N$ is normalised to zero. There are two main reasons for imposing this assumption: i) as it can be deduced from the analysis of Section 2.4, in this case the labour market is characterised by procyclical wages of newly hired workers; and ii) new hires always exert positive reciprocity, which enables to isolate the potential impact of negative reciprocity on the expected continuation value of a new employment relationship with a prospective incumbent worker. This assumption can also be interpreted as capturing the idea that new hires arrive at firms with the lowest possible wage entitlement.[28]

Assumption U2 implies that employed workers' match productivity changes stochastically once they become incumbent, but then remains constant for the entire duration of the employment relationship. This makes incumbents' match productivity q_E a random variable with expected value $\mathbb{E}[q_E] = q_N$ and cumulative distribution function $F(q_E) = \Pr[q_N + \varepsilon \leq q_E]$, which is henceforth denoted by $F_{q_E|q_N}$. The main purpose of this assumption is to keep the model tractable enough to analytically characterise its equilibrium outcomes. Another way to achieve this would be to impose a two-period employment relationship (as, for instance, in Chapter 1 or in Eliaz and Spiegler (2014)). This assumption introduces a form of uncertainty faced by firms at the time of hiring, whom might have to re-adjust their labour cost in the future, in the event of exogenous unanticipated changes in per-worker profits. Another interpretation is that it takes time for firms to learn the quality of a new match, which is revealed as workers' tenure increases (à la Jovanovic (1979)).

Finally, assumption U3 ensures that employed workers' normal effort, which is parametric, is such that the value to the firm of an employment relationship is always non-negative, independently of the realisation of the match productivity, for all $t \geq \tau$. Hence incumbent workers are never endogenously laid off and additions into unemployment remain determined by the time-invariant exogenous job-destruction rate $\rho \in (0, 1)$.

[28]This also resembles the assumption of "modest aspirations" considered by Eliaz and Spiegler (2014).

This assumption is useful to isolate the effect of anticipated negative reciprocity and expected wage rigidity on firms' job creation incentives only. The motivation for this choice also comes from a strand of empirical evidence (e.g. Hall (2005a) and Shimer (2012)) according to which unemployment fluctuations are largely explained by fluctuations in the job-finding rate.[29]

2.6.2 Wage Setting Behaviour Under Uncertainty

The introduction of a stochastic variation into employed workers' match productivity does not directly affect their optimal choice of effort. For any given sequence of optimal wage offers $\{w_t\}_{t=\tau}^{\infty}$ set by firms, the employed workers' problem consists of choosing a sequence of levels of effort $\{e_t\}_{t=\tau}^{\infty}$ that maximises their present discounted value of utility, given their evaluation of the wage w_t in relation to their reference wage $r_{C,t}$. Hence employed workers do not directly observe the change in $q_{C,t}$, and their optimal choice of effort remains characterised by the asymmetric reference-dependent reciprocity as given by (2.4). On the other hand the stochastic change in $q_{C,t}$ has implications for the optimal wage setting policy of firms. In fact, as explored in Chapter 1, they now have to consider how the optimal wage in $t = \tau$, which becomes the reference wage in $t > \tau$, will influence workers' reciprocity responses, and therefore the expected continuation value of the employment relationship, for any possible realisation of the match productivity in the future.

Given the structure imposed by assumption U2 it is useful to think about firms' wage setting problem as essentially being divided into two separated optimisation problems. Since an incumbent worker's match productivity remains constant after it is revealed, a firm can: i) derive the optimal wage policy for any possible realisation of $q_C = q_E$, taking as given $r_C = r_E$ for all $t > \tau$; then ii) use the obtained "state-contingent" optimal wage policy to calculate the continuation value of an employment relationship with an incumbent worker $J(w, q'_C) = J(r_E, q_E)$ for any possible realisation of q_E, the expectation of which is crucial for the characterisation of the optimal wage policy in

[29]The analysis of Elsby, Michaels, and Solon (2009) confirms this empirical finding. However, in contrast with the sharp conclusions of Hall (2005a) and Shimer (2012) who argue that the job-separation rate is quantitatively irrelevant, Elsby et al. (2009) emphasize a minor, yet significant, role of the unemployment inflow rate in explaining the cyclical behaviour of unemployment.

$t = \tau$. This two-step structure can be expressed recursively as:

$$J(r_N, q_N) = \max_w \left\{ \pi(r_N, w, q_N) + \psi \int J(w, q_E)\, dF_{q_E|q_N} \right\},\ \forall t = \tau, \tag{2.28}$$

$$\text{where } J(r_E, q_E) = \max_w \left\{ \pi(r_E, w, q_E) + \psi J(w, q_E) \right\},\ \forall t > \tau, \tag{2.29}$$

$r'_E = w$ and $r_N = 0$ given. The following proposition establishes the solution to this two-step recursive problem.

Proposition 15. *For any given worker's reference wage r_C and match productivity q_C, the optimal wage policy of a firm employing a worker with employment status $C = \{N, E\}$, characterised by asymmetric reference-dependent reciprocity $\tilde{e}(w_t, r_{C,t}, \lambda)$ with $\lambda > 1$ and adaptation $r_{C,t+1} = w_t$, is characterised as follows:*

a) for all $t > \tau$ and $C = E$, the optimal wage $\tilde{w}_E = \tilde{w}(r_E, q_E, \lambda)$ is given by:

$$\tilde{w}_E = \begin{cases} pq_E(1-\psi) & \equiv \tilde{w}_E^+ \quad \text{if } q_E > q_u(r_E) \\ r_E & \equiv \tilde{w}_E^= \quad \text{if } q_E \in [q_l, q_u] \\ \lambda p q_E(1-\psi) & \equiv \tilde{w}_E(\lambda)^- \quad \text{if } q_E < q_l(r_E, \lambda) \end{cases} \tag{2.30}$$

where

$$q_u(r_E) \equiv \{q_E : \Upsilon(r_E + \epsilon; r_E, q_E, \lambda) = 0\}$$
$$q_l(r_E, \lambda) \equiv \{q_E : \Upsilon(r_E - \epsilon; r_E, q_E, \lambda) = 0\};$$

b) for all $t = \tau$ and $C = N$, the optimal wage $\tilde{w}_N = \tilde{w}(r_N, q_N, \lambda)$ is (implicitly) given by:

$$\tilde{w}_N = \frac{p\left[q_N - \psi\left(\int^{q_l(\tilde{w}_N,\lambda)} \lambda q_E\, dF_{q_E|q_N} + \int_{q_u(\tilde{w}_N)} q_E\, dF_{q_E|q_N}\right)\right](1-\psi)}{1 - \psi\left[1 - \left(F(q_u(\tilde{w}_N)) - F(q_l(\tilde{w}_N, \lambda))\right)\right]} \tag{2.31}$$
$$\equiv \tilde{w}_N(\lambda)^+ \geq r_N = 0 \quad \forall r_N, q_N.$$

Proof. See Appendix B.2. ■

Part a) of Proposition 15 implies that depending on the realisation of q_E, and for any given reference wage $r_E = \tilde{w}_N$, firms' may optimally implement a wage raise $\tilde{w}_E^+ > \tilde{w}_N$ if q_E is sufficiently high ($> q_u$), a wage cut $\tilde{w}_E(\lambda)^- < \tilde{w}_N$ if q_E is too low ($< q_l$), or a wage freeze $\tilde{w}_E^= = \tilde{w}_N$ for intermediate values of q_E ($\in [q_l, q_u]$). This characterises the solution to the first step (2.29). In contrast with the deterministic environment analysed in Section 2.2, the stochastic change in match productivity and the resulting (potential) wage re-negotiation are such that incumbent workers may exert positive, negative reciprocity or normal effort in $t = \tau + 1$. These considerations influence the expected continuation value of an employment relationship in $t = \tau$, and therefore also affect the optimal wage paid to newly hired workers. Although an explicit solution is not provided, as established in part b) of Proposition 15 (and as anticipated in the discussion of assumption U1), it can be deduced that the optimal wage paid to newly hired workers is always perceived as a gift, independently of the initial conditions in $t = \tau$. This characterises the solution to the second step (2.28).

The qualitative properties of a wage setting policy of this sort, as established by Proposition 15, have been extensively discussed in Chapter 1 in a two-period setting and are therefore not repeated.

2.6.3 Characterisation of the Steady-state Job Creation Condition

This section provides the characterisation of the model's steady-state equilibrium job creation condition.

Expected equilibrium wages, effort and output

To begin with, consider incumbent workers. For any given $r_E = \tilde{w}_N$ incumbent workers are paid the optimal wage $\tilde{w}_E \in [\tilde{w}_E(\lambda)^-, \tilde{w}_E^+]$ as given by (2.30), depending on the realisation of q_E. Since from $t > \tau$ the environment is deterministic, for the same reasons outlined in Corollary 1, $\tilde{w}_E$ will also characterise the incumbent workers' steady-state wage $\tilde{w}_E^*$. Hence, the expected present discounted value in period $t = \tau$ of the equilibrium wage paid to incumbent workers, from any $t > \tau$ onwards and for any possible realisation of q_E, can be expressed as:

$$\mathbb{E}_\tau \left[\sum_{t=\tau+1}^{\infty} \psi^{t-\tau} \tilde{w}^*(r_E, q_E, \lambda) \right] = \frac{\psi}{1-\psi} \int \tilde{w}^*(r_E, q_E, \lambda)\, dF_{q_E|q_N}.$$

Moreover it can be deduced that in any $t = \tau + 1$ incumbent workers will exert either negative reciprocity $\tilde{e}_{E,t}^* = \tilde{e}_{E,t}(\lambda)^-$ (if $\tilde{w}_E^* < r_{E,t}$ since $q_E < q_l(r_E, \lambda)$), either positive reciprocity $\tilde{e}_{E,t}^* = \tilde{e}_{E,t}^+$ (if $\tilde{w}_E^* > r_{E,t}$ since $q_E > q_u(r_E)$), or normal effort $\tilde{e}_{E,t}^* = \tilde{e}_n$ (if $\tilde{w}_E^* = r_{E,t}$ since $q_E \in [q_l, q_u]$). While in any $t > \tau + 1$, incumbent workers' effort will converge to normal $\tilde{e}_{E,t}^* = \tilde{e}_n$ due to reference wage adaptation $r_{E,t} = \tilde{w}_E^*$. As such, the expected present discounted value in period $t = \tau$ of the equilibrium output produced by firms employing incumbent workers from any $t > \tau$ onwards and for any possible realisation of q_E, can be expressed as:

$$\mathbb{E}_\tau\left[\sum_{t=\tau+1}^{\infty}\psi^{t-\tau}y(q_E,\tilde{e}_E^*)\right]=\psi\int y(q_E,\tilde{e}_E^*)+\frac{\psi}{1-\psi}y(q_E,\tilde{e}_n)\,dF_{q_E|q_N}.$$

Next consider newly hired workers. For any q_N and $r_N = 0$, which are known to firms, the expected steady-state equilibrium wage paid to them is $\tilde{w}_N^* = \tilde{w}_N(\lambda)^+$ as given by (2.31), and the expected steady-state value of output produced is $y(q_N, \tilde{e}_N^*) = y(q_N, \tilde{e}_N^+)$.

Steady-state value functions

The steady-state equilibrium value of a job filled by a newly hired worker can therefore be expressed as:

$$J(r_N,q_N)=y(q_N,\tilde{e}_N^+)-\tilde{w}_N(\lambda)^++\delta\left[(1-\rho)\int J(r_E,q_E)\,dF_{q_E|q_N}+\rho V\right]\qquad(2.32)$$

where $r_E = \tilde{w}_N(\lambda)^+$ and $\int J(r_E, q_E)\,dF_{q_E|q_N}$ is the expected continuation value of the employment relationship with a worker becoming incumbent in the following period, for any possible realisation of q_E:

$$\int J(r_E,q_E)\,dF_{q_E|q_N}=\int^{q_l(r_E,\lambda)}J(r_E,q_E)^-\,dF_{q_E|q_N}$$
$$+\int_{q_l(r_E,\lambda)}^{q_u(r_E)}J(r_E,q_E)^=\,dF_{q_E|q_N}+\int_{q_u(r_E)}J(r_E,q_E)^+\,dF_{q_E|q_N}.$$

The superscripts $-$, $=$ and $+$ denote as usual the case of a wage cut accompanied by negative reciprocity; the case of a wage freeze accompanied by normal effort; and the case of a wage rise accompanied by positive reciprocity respectively. In fact, depending on the realisation of q_E, the value of a job filled by an incumbent worker takes the following form:

$$J(r_E,q_E)=\begin{cases}y(q_E,\tilde{e}_E^+)+\dfrac{\psi}{1-\psi}y(q_E,\tilde{e}_n)-\dfrac{\tilde{w}_E^+}{1-\psi} & \text{if } q_E>q_u(r_E)\\[2ex] \dfrac{y(q_E,\tilde{e}_n)}{1-\psi}-\dfrac{\tilde{w}_E^=}{1-\psi} & \text{if } q_E\in[q_l,q_u]\\[2ex] y(q_E,\tilde{e}_E(\lambda)^-)+\dfrac{\psi}{1-\psi}y(q_E,\tilde{e}_n)-\dfrac{\tilde{w}_E(\lambda)^-}{1-\psi} & \text{if } q_E<q_l(r_E,\lambda).\end{cases}\qquad(2.33)$$

Finally, the steady-state value of a vacancy can be expressed as

$$V = -\kappa + \delta \left[h(\theta) J(r_N, q_N) + (1 - h(\theta)) V \right]. \tag{2.34}$$

Steady-state job creation condition under uncertainty

Following the same approach of Section 2.4 and the expressions just derived above, denote by $\overline{w}_C^*(\lambda)$ the steady-state equilibrium expected present discounted value of the wage from a new employment relationship:

$$\begin{aligned} \overline{w}_C^*(\lambda) &\equiv \tilde{w}_N^* + \mathbb{E}_\tau \left[\sum_{t=\tau+1}^{\infty} \psi^{t-\tau} \tilde{w}_E^* \right] \\ &\equiv \tilde{w}_N(\lambda)^+ + \frac{\psi}{1-\psi} \int \tilde{w}_E^* \, dF_{q_E|q_N}; \end{aligned} \tag{2.35}$$

and by $\overline{y}_C^*(\tilde{e}_C^*(\lambda))$ the corresponding steady-state equilibrium expected present discounted value of output:

$$\begin{aligned} \overline{y}_C^*(\tilde{e}_C^*(\lambda)) &\equiv y(q_N, \tilde{e}_N^*) + \mathbb{E}_\tau \left[\sum_{t=\tau+1}^{\infty} \psi^{t-\tau} y(q_E, \tilde{e}_E^*) \right] \\ &\equiv y(q_N, \tilde{e}_N^+) + \psi \int y(q_E, \tilde{e}_E^*) + \frac{\psi}{1-\psi} y(q_E, \tilde{e}_n) \, dF_{q_E|q_N}. \end{aligned} \tag{2.36}$$

Next, by imposing the free-entry condition $V = 0$, and rearranging equations (2.34) and (2.32) using the expected present value of the equilibrium wage (2.35) and output (2.36) just defined, the steady-state equilibrium job creation condition determining firms' vacancy posting decision can be expressed as follows:

$$\frac{\kappa}{h(\theta)} = \delta \left[\overline{y}_C^*(\tilde{e}_C^*(\lambda)) - \overline{w}_C^*(\lambda) \right]. \tag{U-JC}$$

Firms will hire workers until the cost of posting a vacancy, multiplied by the expected duration of the vacancy, equals the expected present discounted value of a new employment relationship, expressed as the difference between the expected present value of output produced and the expected present value of the wage paid to workers.

The job creation condition (U-JC) differs from the deterministic version (JC), Section 2.4.1, in two fundamental ways. First, for any given $r_N = 0$, q_N and $F_{q_E|q_N}$, the right-hand side of (U-JC) is unique, implying that there exists a unique steady-state equilibrium labour market tightness. As previously anticipated this result is a natural implication of Assumption U1. Second, even though new hires always exert positive reciprocity in equilibrium, the job creation condition explicitly depends on λ. Note that in this context λ captures the expected cost of wage cuts in the future continuation value of a new employment relationship, i.e. the relative strength of the expected negative reciprocity response of incumbent workers in the event of a future wage cut; and not the anticipated relative large cost of employing newly hired workers at a wage below their reference wage (as it was the case in the analyses of Sections 2.4 and 2.5).

The solution to (U-JC) is denoted by $\tilde{\theta}^*(\lambda)$ and takes the following explicit form:

$$\tilde{\theta}^*(\lambda) = \left(\frac{\bar{m}\delta}{\kappa}\left[\overline{y}_C^*(\tilde{e}_C^*(\lambda)) - \overline{w}_C^*(\lambda)\right]\right)^{\frac{1}{\sigma}}. \tag{2.37}$$

The remainder of the analysis uses the results derived in this section to qualitatively address the following questions:

A) what is the effect of expected wage rigidity and anticipated negative reciprocity of incumbent workers on the steady-state equilibrium level of wages and labour market tightness?

B) What is the effect of expected wage rigidity and anticipated negative reciprocity of incumbent workers on the cyclical behaviour of job creation and on the volatility of vacancies and unemployment?

2.6.4 The Steady-state Effects of Expected Negative Reciprocity

This section investigates the effect of expected wage rigidity and anticipated negative reciprocity on the steady-state equilibrium levels of wages and labour market tightness (therefore addressing Question A)).

Consider first the steady-state wages that firms expect to pay in equilibrium. The expected wage of new hires is $\mathbb{E}[\tilde{w}_N^*] \equiv \tilde{w}_N(\lambda)^+$, while the expected wage of incumbents is $\mathbb{E}[\tilde{w}_E^*] \equiv \int \tilde{w}_E^* \, dF_{q_E|q_N}$. The following proposition establishes how an increase in the relative strength of expected negative reciprocity of incumbent workers, that is, an increase in the expected cost of wage cuts, affects these values.

Proposition 16. *For any given $r_N = 0$, q_N and $F_{q_E|q_N}$, a higher λ decreases the expected equilibrium wage paid to newly hired workers $\mathbb{E}[\tilde{w}_N^*]$; while it increases the expected equilibrium wage paid to incumbents $\mathbb{E}[\tilde{w}_E^*]$.*

Proof. See Appendix B.2. ■

A higher λ implies that it is optimal for firms to set a lower wage in $t = \tau$ for newly hired workers to reduce the magnitude of the expected negative reciprocity response in the event of a wage cut. Hence, expected wage rigidity reinforces firms' incentive to compress wages at the start of the employment relationship.

For what concerns the expected wage to be paid to incumbent workers, the anticipation in $t = \tau$ of stronger negative reciprocity in $t > \tau$, involves two opposing effects: a higher λ puts upward pressure on the wage in the event of a future wage cut, since firms will have an incentive to set a higher wage in order to partially offset workers' negative reciprocity; however, as noted above, a higher λ also implies a lower wage in $t = \tau$, which puts downward pressure on the reference wage in $t > \tau$, and hence on the wage paid to incumbents in the event of a wage freeze. As established by Proposition 16, the former effect dominates, implying that the expectations of stronger negative reciprocity increase the expected equilibrium wage of incumbent workers.

Next, consider the effect of the expected relative strength of negative reciprocity λ on the equilibrium labour market tightness $\tilde{\theta}^*(\lambda)$. Total differentiation of (2.37) with respect to λ yields:

$$\frac{d\tilde{\theta}^*(\lambda)}{d\lambda} = \frac{\tilde{\theta}^*(\lambda)}{\sigma\left[\overline{y}_C^*(\tilde{e}_C^*(\lambda)) - \overline{w}_C^*(\lambda)\right]} \cdot \left[\frac{d\overline{y}_C^*(\tilde{e}_C^*(\lambda))}{d\lambda} - \frac{d\overline{w}_C^*(\lambda)}{d\lambda}\right]$$

The sign of this derivative depends crucially on the effect of λ on the present values of both output and wages from a new employment relationship (i.e. the term in square brackets).

Lemma 3. *The expected present value of the steady-state equilibrium wage* $\overline{w}_C^*(\lambda)$ *characterising new employment relationships is unaffected by changes in* λ.

Proof. See Appendix B.2. ■

Lemma 3 implies that firms' anticipation of stronger negative reciprocity does not affect the expected present value of the optimal wage they will pay to employed workers. Essentially, the expected marginal decrease in the wage of newly hired workers, and hence in the reference wage of those workers from $t > \tau$ onwards in the event of a wage freeze, is entirely offset by firms' optimal wage policy accounting for the expectations of a marginal increase in the wage they will have to pay to incumbents in the event of a wage cut in $t = \tau + 1$ (which also implies a higher reference wage from $t > \tau + 1$ onwards).[30]

Lemma 4. *The expected present value of the steady-state equilibrium output* $\overline{y}_C^*(\tilde{e}_C^*(\lambda))$ *characterising new employment relationships is decreasing in* λ.

[30]This result echoes the theoretical point of Pissarides (2009), according to which forward-looking firms are able to internalise any potential future negative effect of wage rigidity into the initial wage contract.

Proof. See Appendix B.2. ■

Lemma 4 implies that a higher λ reduces the expected present value of output that firms will expect to produce from new employment relationships. That is, a higher λ implies a lower expected effort in the event of a wage cut in $t = \tau + 1$, and a greater probability of having to enact a costly wage freeze. This insight is the main driver of the results derived in this section and it stands as an additional channel through which expected wage rigidity can affect firms' present value of an employment relationship.[31] Based on these results, the following proposition formally establishes the effect of λ on the steady-state equilibrium labour market tightness.

Proposition 17. *For any given $r_N = 0$, q_N and $F_{q_E|q_N}$, the steady-state equilibrium labour market tightness $\tilde{\theta}^*(\lambda)$ is decreasing in λ.*

Proof. See Appendix B.2. ■

Proposition 17 uses the results derived by Lemma 3 and 4 to establish that the anticipation by firms of stronger negative reciprocity by employed (incumbent) workers in the event of future wage cuts will reduce job creation. Hence, Proposition 17 generalises the result established by Proposition 11, Section 2.4, and shows that expected downward wage rigidity does have detrimental effects for job creation and unemployment. Even in a labour market populated by newly hired workers with relatively low wage entitlements (in this section normalised to zero), the expectation of incurring a relatively large cost of implementing wage cuts in the future reduces the expected present value of new employment relationships, resulting in fewer vacancies and therefore in higher unemployment in the steady state.

2.6.5 Expected Negative Reciprocity and the Volatility of Job Creation

The purpose of this section is to investigate the effect of expected wage rigidity and anticipated negative reciprocity on the volatility of vacancies and unemployment (therefore addressing Question B)). In order to proceed, the following proposition derives an expression for the elasticity of labour market tightness with respect to productivity that characterises this version of the model with uncertainty.

[31] Note that this channel is absent in Pissarides (2009) and in virtually any other search and matching model developed to date, with the exception of Eliaz and Spiegler (2014). However, by considering endogenous layoffs, Eliaz and Spiegler (2014) are unable to derive an unambiguous prediction as the one established by Lemma 4.

Proposition 18. *The elasticity of labour market tightness $\tilde{\theta}^*(\lambda)$ with respect to productivity p takes the following form*

$$\varepsilon_{\tilde{\theta}^*(\lambda)} = \frac{1}{\sigma}\frac{\overline{y}^*_C(\tilde{e}^*_C(\lambda)) + \Lambda(\varepsilon_{\overline{w}^*_C}, \overline{w}^*_C(\lambda))}{\overline{y}^*_C(\tilde{e}^*_C(\lambda)) - \overline{w}^*_C(\lambda)}. \tag{2.38}$$

where $\overline{y}^*_C(\tilde{e}^*_C(\lambda))$ and $\overline{w}^*_C(\lambda)$ are the expected present values of output and the wage as defined by (2.36) *and* (2.35)*, and $\Lambda(\varepsilon_{\overline{w}^*_C}, \overline{w}^*_C(\lambda))$, to be defined shortly, is a function of $\overline{w}^*_C(\lambda)$ and its elasticity $\varepsilon_{\overline{w}^*_C}$ with respect to productivity p.*

Proof. See Appendix B.2. ■

Proposition 18 derives an equation for the elasticity of market tightness when there is uncertainty about the evolution of $q_{C,t}$. This expression is directly comparable with the one derived for the deterministic version of the model, given by (2.22), and with the literature (see Section 2.5.1). The impact of the cyclicality of wages on the size of this elasticity is captured by the function $\Lambda(\varepsilon_{\overline{w}^*_C}, \overline{w}^*_C(\lambda))$.

Corollary 2. *The function $\Lambda(\varepsilon_{\overline{w}^*_C}, \overline{w}^*_C(\lambda))$ takes the form*

$$\begin{aligned}\Lambda(\varepsilon_{\overline{w}^*_C}, \overline{w}^*_C(\lambda)) \equiv -\varepsilon_{\tilde{w}^+_N}\tilde{w}_N(\lambda)^+ \cdot \Theta_N \\ - \frac{\psi}{1-\psi}\int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} \varepsilon_{\tilde{w}^-_E}\tilde{w}_E(\lambda)^- \cdot \Theta^-_E \, dF_{q_E|q_N} \\ - \frac{\psi}{1-\psi}\int_{q_u(\tilde{w}_N(\lambda)^+)} \varepsilon_{\tilde{w}^+_E}\tilde{w}^+_E \cdot \Theta^+_E \, dF_{q_E|q_N}\end{aligned} \tag{2.39}$$

where:

$$\begin{aligned}\Theta_N &\equiv -\left[\frac{\partial y(\tilde{e}_N)}{\partial e}\frac{\partial \tilde{e}^+_N}{\partial w} - 1 + \psi\Phi(\tilde{w}_N, \lambda)\right] &&= -\Upsilon(\tilde{w}_N; r_N, q_N, \lambda) &&=0;\\ \Theta^-_E &\equiv -\left[(1-\psi)\frac{\partial y(\tilde{e}_E)}{\partial e}\frac{\partial \tilde{e}_E(\lambda)^-}{\partial w} - 1\right] &&= -\Upsilon(\tilde{w}_E(\lambda)^-; r_E, q_E, \lambda) &&=0; \text{ and}\\ \Theta^+_E &\equiv -\left[(1-\psi)\frac{\partial y(\tilde{e}_E)}{\partial e}\frac{\partial \tilde{e}^+_E}{\partial w} - 1\right] &&= -\Upsilon(\tilde{w}^+_E; r_E, q_E, \lambda) &&=0.\end{aligned}$$

Hence,

$$\Lambda(\varepsilon_{\overline{w}^*_C}, \overline{w}^*_C(\lambda)) = 0, \ \forall\varepsilon_{\overline{w}^*_C};$$

which implies that $\varepsilon_{\overline{w}^*_C}$ is irrelevant for the size of $\varepsilon_{\tilde{\theta}^*(\lambda)}$.

Proof. See Appendix B.2. ■

Corollary 2 generalises the irrelevance result established by Proposition 13, Section 2.5, to an environment in which there is uncertainty around the evolution of a job match productivity. That is, the cyclicality of the expected present value of the wage characterising new employment relationships is irrelevant for the size of the volatility of the vacancy-unemployment ratio. Essentially, for any given increase in aggregate productivity, firms' wage setting policy is such that the inter-temporal trade-off between the marginal cost of a higher wage—and a higher reference wage in the future—and the marginal benefit on output achieved as a result of an increase in workers' effort, is optimally balanced. Note that this is achieved by firms' optimal wage setting for both newly hired and incumbent workers, i.e. for any given q_N and q_E.[32] As such, changes in aggregate productivity p are fully absorbed by firms' expected present discounted value of profit $J(r_N, q_N)$.

The elasticity of labour market tightness with respect to aggregate productivity can therefore be expressed as a function of the wage-output ratio:

$$\varepsilon_{\tilde{\theta}^*(\lambda)} = \frac{1}{\sigma} \frac{1}{1 - [\overline{w}^*_C(\lambda)/\overline{y}^*_C(\tilde{e}^*_C(\lambda))]}. \tag{2.40}$$

Proposition 19. *For any given $r_N = 0$, q_N and $F_{q_E|q_N}$, the elasticity of labour market tightness $\varepsilon_{\tilde{\theta}^*(\lambda)}$ is increasing in λ.*

Proof. See Appendix B.2. ■

The statement of Proposition 19 establishes that the anticipation by firms of stronger negative reciprocity from incumbent workers, that is, the anticipation of a greater cost of implementing wage cuts in the future due to employed (incumbent) workers' asymmetric reference-dependent reciprocity, unambiguously increases the size of the elasticity of labour market tightness with respect to aggregate productivity. This result generalises the statement of Proposition 14, Section 2.5, to a labour market in which newly hired workers are characterised by a relatively low (here normalised to zero) reference wage.

The key insight stemming from Proposition 19 is that the expectations of downward wage rigidity reduce the firms' expected present value of the surplus they obtain from new employment relationships. This result complements and extends the insight of Eliaz and Spiegler (2014) by showing that, even if the anticipation of wage rigidity does not reduce the expected duration of a match, it can unambiguously increase the volatility of job creation. Moreover, while Eliaz and Spiegler (2014) attribute the relatively large cost of implementing wage cuts to a random realisation of a parameter capturing the

[32] In fact, as it is demonstrated in the proof of Corollary 2, the functions Θ_N, Θ^-_E and Θ^+_E are equivalent to the negative of the first-order conditions characterising $\tilde{w}_N(\lambda)^+$ for all q_N, $\tilde{w}_E(\lambda)^-$ for all $q_E < q_l(r_E, \lambda)$, and $\tilde{w}^+_E$ for all $q_E > q_u(r_E)$ respectively. The underlying logic is equivalent to the one explained in Section 2.5.2.

incompleteness of the employment contract, the present framework has shown that it is the asymmetric and reference-dependent nature of workers' reciprocity—in particular, the greater strength of negative reciprocity in the event of a wage cut—that is the relevant concern affecting firms' hiring decisions. In fact, if reciprocity were symmetric, i.e. $\lambda = 1$, the range of rigidity $[q_l, q_u]$ would disappear, the wage of incumbent workers would be expected to be pro-cyclical for all realisations of q_E and the size of the elasticity of market tightness $\varepsilon_{\tilde{\theta}^*(\lambda)}$ would be smaller. In addition note that, due to the irrelevance result established in Corollary 2, the statement of Proposition 19 holds independently of how much pro-cyclical the wage paid to newly hired workers is, and is therefore fully consistent with the empirical evidence on its cyclicality.

To conclude, it has been qualitatively demonstrated that new hires' wage cyclicality is irrelevant for the size of the elasticity of market tightness, and that when there is uncertainty around the evolution of a job match productivity, the expectations of downward wage rigidity in long-term employment relationship can dampen hiring incentives and increase the volatility of vacancies and unemployment fluctuations.

2.7 Conclusions

Inspired by recent prominent attempts to provide more realistic micro-foundations for the macroeconomic analysis of wage and unemployment fluctuations (e.g. Snell and Thomas (2010), Danthine and Kurmann (2010), Eliaz and Spiegler (2014) and Kuang and Wang (2017)) this chapter has incorporated the theory of wage setting behaviour developed in Chapter 1 into a canonical search and matching model à la Pissarides (1985, 2000). This approach has allowed a novel and more in-depth analysis of the underlying behavioural incentives that determine wage setting behaviour and job creation, also in light of the current debate around the role of expected wage rigidity for the volatility of the vacancy-unemployment ratio.

It has been shown that employed workers' asymmetric reference-dependent reciprocity, coupled with their reference wage adaptation, generates a unique, but distinct, steady-state equilibrium outcome for each initial value of new hires' wage entitlements in the labour market. These outcomes are characterised by different comparative statics properties and can be ranked in terms of wage levels, new hires' morale and reciprocity, and unemployment rates.

This framework was then used to contribute to the theory of labour markets concerned with the amplitude and co-movement of vacancies and unemployment fluctuations over the business cycle (see, for instance, Shimer (2005); Pissarides (2009); Kudlyak (2014)). The contribution of this analysis rests on three main results. First, in contrast to existing theoretical models, in the presence of reference-dependent reciprocity the cyclicality of the price of labour at the time of hiring is shown to be irrelevant for the size of the volatility of vacancies and unemployment. Then, it is shown how workers' wage entitlements, their intrinsic motivation and their degree of loss aversion can influence firms' expected present value of the surplus from a new employment relationship, which has been identified by the literature as the crucial determinant of the amplitude of cyclical fluctuations in job creation and unemployment. A calibration exercise, aimed at evaluating the quantitative relevance of these novel behavioural mechanisms, has shown that the framework developed in this chapter can simultaneously accommodate plausible estimates of the volatilities of both the hiring wage and the vacancy-unemployment ratio. Finally, it has been shown that if there is uncertainty around the evolution of a job match productivity throughout the employment relationship, the expectation of downward wage rigidity and of the relatively large cost of implementing wage cuts—due to employed (incumbent) workers' asymmetric reference-dependent reciprocity—increases unemployment in the steady state, and reduces the firms' expected present value of new employment relationships. This result complements and extends the insight of Eliaz and Spiegler (2014) by showing that, even if the anticipation of wage rigidity does not generate ex-post inefficient lay-offs, it can unambiguously increase the volatility of job creation.

The framework developed in this chapter highlights additional theoretical aspects that will benefit from further research. For instance, The path-dependent nature of the steady-state equilibrium of the model, and the crucial importance of new hires' wage entitlements for its determination, points toward two research routes in particular. First, gaining insight on how wage entitlements are formed, and whether firms' information about these is complete, can potentially enhance the understanding of the determinants of equilibrium unemployment. For instance, it will be interesting to explore alternative, endogenous or exogenous, reference wage formation processes, and to analyse their related implications in the context of the framework developed here. Second, it could be relevant to systematically study whether and how workers' wage entitlements can be "manipulated" downwards (for instance, during recessions), either by firms, government policies or both. Doing so will not only allow firms to produce more output at the same cost—due to an increase in morale and effort among workers—but will also allow governments to achieve a more desirable equilibrium rate of unemployment.

In conclusion, the approach in this chapter has highlighted the existence of behavioural aspects characterising employment relationships that can influence firms' expected surplus from a new match, and has provided a benchmark model to analyse their implications that is transparent and directly comparable with the existing literature.

These behavioural aspects can be exogenous to the specific economic environment, or can be influenced by the labour market institutional and social context. Gaining further insights in this direction can potentially contribute to enhance the understanding of the cyclical behaviour of labour markets.

Chapter 3

Reference Wage Formation: A General Framework and Two Applications

3.1 Introduction

From the development of reference-dependent preferences and their application in the analysis of the employment contract and wage negotiations, the concept of a worker's reference wage has become an essential ingredient of micro- and macro-economic theories of the labour market. The reference wage however does not necessarily indicate a single monetary wage level relative to which a worker compares a wage offer. This chapter advances the idea that the reference wage is an artefact of economic models that captures a broader set of feelings, entitlements, information and norms about what should be a fair remuneration. It is a useful representation of a worker's perception of what is a "fair" wage. Before tacitly converging towards this broader concept, the labour market literature has adopted a variety of terms such as: the "fair wage" (Marshall, 1890; Hicks, 1963; Akerlof and Yellen, 1990); "perceptions of entitlement" (Kahneman et al., 1986); the "wage norm" (Okun, 1981; Lindbeck and Snower, 1986); the "frame of reference" (Bewley, 1999); or, more recently, the "reference frame for fairness judgements" (Fehr et al., 2009).

Strictly related with the notion of the reference "fair" wage are the concepts of fairness and norms, now ubiquitous in every economic model that aims to provide a more realistic account of employed workers' reciprocity and firms' wage setting behaviour. However, the theoretical literature on labour markets and wage determination has largely focused on the consequences of *deviations* of wage offers from a specific, exogenous or endogenous, reference "fair" wage. That is, on the effects of wage gifts and unfair wages on workers' effort. Assumptions regarding the relevant reference wage vary extensively among models, which consequently provide a range of predictions that depend crucially on the choice of the reference wage and its determinants. These assumptions are either grounded on some selected strand of evidence, also stemming from other behavioural disciplines, or are alternatively formulated by appealing to the theorist's intuition. Despite this prominent and crucial role, the existing literature does not yet offer a systematic approach aimed at a comprehensive understanding of the workers' reference wage formation process and determinants in the context of labour markets.

The aim of this chapter is to fill this gap by providing a rigorous and portable theoretical framework to think about reference wage formation; and to show its advantages for the analysis of reciprocity and wage dynamics in employment relationships, and for the cyclical behaviour of job creation and unemployment. In doing so, the chapter makes several contributions.

The first part of the chapter, Section 3.3, is dedicated to a survey of the labour market literature on reference wage formation, which is complemented by a concise discussion of other behavioural sub-disciplines concerned with reference point formation. Workers' perceptions of fairness with respect to wage contracts depend on complex interactions between someone's own experience, existing social norms and the surrounding institutional context. These perceptions can therefore be determined by previously received wage contracts, the wage paid to other peers, the type of employment contract, and/or the state of the labour market. Importantly, perceptions of fairness can change over time and their evolution and shape does not necessarily have to be "symmetric". These features can be grouped into three main properties of reference wage formation: i) there is an intrinsic tendency of workers to adapt their reference wage over time; ii) this process is strictly dependent on the readily available information, which can be influenced/manipulated by intentional disclosure practices (e.g. firm internal communication), exogenous public events (e.g. news), and institutional and social devices (e.g. labour unions); and iii) there appears to be asymmetries in reference wage formation: workers adapt more rapidly to experienced gains than to experienced losses, and disadvantageous pay inequality has a larger effect on fairness judgements and morale than equity or advantageous inequality.

The second part of the chapter, Section 3.4, analytically formalises these principles into a general and portable framework. This formulation deals with several aspects of reference wage formation. The relevant information affecting a worker's reference wage is categorised into two types of determinants: "entitlement" determinants—which can constitute a reference wage by themselves—and "shift" determinants—which, instead, influence/shift the reference wage. The framework also considers how workers evaluate wage contracts in the presence of multiple references/determinants, and what can be inferred from studying the dynamics of a generalised reference wage rule. On the basis of this set up, the key element that completes the theory is the postulate of a general behavioural principle that captures the *unconscious cognitive process* through which workers form their perception of what is a fair wage. By appealing to the social identity approach introduced in economics by Akerlof and Kranton (2000, 2005, 2008, 2010) it is assumed that the relevant set of determinants for fairness evaluations depend on how a wage contract is framed; and that framing is influenced by a worker's *social identity*: a concept that ties together the worker's social categories with their related fair wage norms. This approach generates a unique conceptual framework that can be applied in a variety of economic and social contexts, where the most important task left to the theorist/analyst is to identify the relevant set of social categories that characterise the environment of analysis, and to specify the sets of reference wage determinants corresponding to each category.

The final part of the chapter, Section 3.5, illustrates how this approach can be used to extend the theory of reciprocity and wage setting behaviour developed throughout the thesis. The focus is on two main applications. The first is concerned with the microeconomic implications of a general asymmetric adaptation rule for wage and reciprocity dynamics (using the model of Chapter 1). Among other results, the analysis provides a behavioural explanation for the observed persistence of wage dynamics, and a rationale for why negative shocks generate a more persistent response. As such, the theory gives a unified and coherent explanation for the conjectured asymmetry in the *intensity* (loss aversion) and *persistence* (asymmetric adaptation) of negative *versus* positive reciprocity. The second application analyses the implications of relative wage comparisons between newly hired and incumbent workers for the cyclicality of wages, job creation and unemployment (using the search and matching framework developed in Chapter 2). Here it is shown that the *endogenous* downward rigidity (or the "muted" adjustment) of the wage of existing workers generates entitlement effects in the reference wage of new hires due to social comparison. This spill-over effect, in turn, influences firms' optimal wage setting and hiring behaviour and generates *hysteresis* in wages, vacancies and unemployment. That is, temporary cyclical shocks have permanent effects on the steady state equilibrium. This result leads to two further insights. On one hand, in the case of large economic cycles, the model is consistent with procyclical wages and high and persistent unemployment, a cyclical pattern that fits well the puzzling experience of the U.S. labour market during the Great Recession (see Elsby et al. (2016)). On the other hand, the existence of hysteresis highlights the failure of a labour market—comprised of rational payoff-optimising agents—to fully return, after a temporary shock, to the original steady-state equilibrium. Therefore, this theoretical result suggests that exogenous market interventions—also aimed at "manipulating" workers wage entitlements—might be effective to achieve more desirable labour market outcomes, and appoints the approach developed here as a tractable framework to analyse these issues from a novel behavioural perspective.

The interpretation of the literature and the theory of reference wage formation developed here are grounded on the concepts of reference dependence and loss aversion in the evaluation of wage contracts by workers. As such, before diving into the main themes of this chapter, the next section will provide a concise summary of these behavioral aspects, drawing from the theoretical framework developed in Chapter 1. Finally, a summary of the key contributions made is given in Section 3.6; while additional material referred to throughout the chapter is included in Appendix A.3.

3.2 Conceptual Framework

As formalised in Chapter 1, workers are assumed to be reference dependent and loss averse, which implies that they evaluate wage contracts with respect to a reference "fair" wage and that unfair wages below the reference wage generate a disproportional decrease in payoff. These behavioural aspects have been formalised by assuming the following worker's instantaneous utility function (or payoff):

$$u(e_t; w_t, r_t) = m(w_t) + \xi n(w_t|r_t) + b(e_t) - c(e_t) + M(e_t; w_t, r_t), \tag{3.1}$$

where $m(\cdot)$ captures the effect of absolute wage levels on utility; $b(e)$ and $c(e)$ represent the worker's intrinsic psychological and physical benefits and costs of productive activity. The function $M(e_t; w_t, r_t) \equiv \zeta e n(w_t|r_t)$ is the *morale function*, which measures the worker's evaluation of the productive effort they undertake in light of the fairness of the wage they are paid; and $n(w_t|r_t) \equiv \mu(m(w_t) - m(r_t))$ captures the worker's gain-loss utility in the spirit of Kahneman and Tversky (1979) and Tversky and Kahneman (1991), where $\mu(\cdot)$ is a piece-wise linear gain-loss function:

$$\mu(m(w_t) - m(r_t)) = \begin{cases} \eta[m(w_t) - m(r_t)] & \text{if } w_t \geq r_t \\ \lambda\eta[m(w_t) - m(r_t)] & \text{if } w_t < r_t \end{cases} \tag{3.2}$$

in which $\eta > 0$ is a scaling parameter that represents the importance of gain-loss utility for the worker and $\lambda \geq 1$ is the worker's degree of loss aversion. In the context of this chapter, the function (3.2) will be referred to as the worker's *experienced* gain-loss utility. This is to emphasise the fact that gains and losses in utility are the result of "contemporaneous" fairness evaluations of the wage contract set by the firm, and not of "prospective" evaluations of potential wage offers. Assumptions W1-W4, Chapter 1, hold throughout unless otherwise specified.

The theory developed in Chapter 1 aimed at understanding workers' and firms' optimal behaviour in a wage setting context. That is, the worker's optimal choice of effort $\tilde{e}_t$ for any given wage offer w_t relative to a unique, exogenous or endogenous, reference wage r_t; and the firm's optimal wage setting policy $\tilde{w}_t$ given the worker's optimal effort response and their reference wage. In contrast, the first part of this chapter will focus solely on the determination of the reference wage r_t, and on how a worker's payoff $u(e_t; w_t, r_t)$ will be influenced by the experienced gain-loss utility $\mu(m(w_t)-m(r_t))$ subject to several properties characterising the reference wage formation process.

3.3 The Reference Wage in the Existing Literature

This section surveys the existing labour market literature on reference wage formation (Section 3.3.1) and discusses some key insights that stem from other behavioural science sub-disciplines concerned with reference point formation (Section 3.3.2). The aim of this wide-ranging survey is to identify more clearly which are the most likely determinants of a worker's reference wage, and how they affect the reference wage formation process.

3.3.1 Labour Market Literature

The following survey is divided into two main sections. First, theoretical models of the labour market with an element of reference dependence are discussed. As it will become clearer from the discussion, although the reference wage is a concept widely used in the theoretical literature, there is no complete general treatment of the reference wage formation process. Then, the survey is complemented by discussing the existing empirical evidence on reference wages in labour markets, drawing from a wide range of empirical approaches.

Theoretical models and reference wage assumptions

In this section theoretical models of the labour market are classified on the basis of their most distinctive assumptions about the determinants of a worker's reference wage. These models may differ in many ways depending on the modelling approach, the economic perspective (micro, macro or both) and the questions they aim to investigate. The following survey pulls them together around the concept of the reference wage.

External labour market conditions. The first formal models to incorporate the notion of a reference "fair" wage are those belonging to the efficiency wage tradition in the spirit of Akerlof (1982), Summers (1988) and Akerlof and Yellen (1990). These models have predated the dissemination of reference-dependent preferences in economics, and have postulated a wage-effort relationship around the assumption of a "fair" wage. Despite assuming various determinants of the reference wage, the key distinctive feature of these models is the assumption that contemporaneous labour market conditions external to the firm—such as the unemployment rate, or the market clearing wage—negatively influence workers' wage entitlements, easing the efficiency wage constraint. This approach, and in particular the partial gift-exchange model of Akerlof (1982), has also been incorporated into dynamic general equilibrium macroeconomic models, in which external labour market conditions influencing the reference wage have been assumed to be given by: the income from self-employment and the unemployment rate in the Real Business Cycle model of Danthine and Donaldson (1990); the value of unemployment benefits in a Search and Matching framework analysed by Wesselbaum (2013).

The purpose of this assumption has been to highlight the role of aggregate labour market conditions on workers' wage entitlements and fairness considerations.

Most recent wage contract. A different stream of theoretical models has investigated the consequences of assuming the reference wage being determined by the value of workers' most recent wage contract or their past wage. As it will be discussed in the next section, the main reference cited in support of this assumption is the seminal work of Kahneman et al. (1986) and Bewley (1999) whom provide experimental and anthropological evidence of the importance of past market transactions for the formation of wage entitlements and fairness judgements by workers. Collard and de la Croix (2000) are the first to consider the worker's past wage as the reference wage in a general equilibrium Real Business Cycle model; Danthine and Kurmann (2004) incorporate this assumption into a New Keynesian DSGE model; while Ball and Moffitt (2002) explore the impact of past wages on "wage aspirations" in a standard model of the Phillips curve (e.g. à la Blanchard and Katz (1997)). More descriptive approaches that considered the implications of the past wage as the reference wage, along with other prominent insights from behavioural economics, are the microeconomic models of wage setting proposed by Holden and Wulfsberg (2009), Elsby (2009) and Benjamin (2015).[1] The theory developed in Chapter 1 can also be placed under this class of

[1]Although in a conceptually different manner the past wage, couched with the notion of a reference point, has been implemented in the Search and Matching literature by Boitier (2015) and Koenig et al. (2016) as an alternative to the workers' *outside option* in the generalised Nash bargaining solution; and in the job search literature by DellaVigna, Lindner, Reizer, and Schmieder (2016) as the reference income level, in order to explain cyclical variations in search effort.

models.

This assumption has proven to be extremely useful in enhancing the explanatory power of these models, in particular by generating various sorts of rigidity, stickiness, or "sluggishness" in the adjustment of wages to exogenous shocks.

Wage of other workers. Following on from the insight of Keynes (1936), among the most popular candidates for a worker's reference wage is the wage paid to other workers. Within this broad definition, the theoretical literature can be classified in three main strands. As initially proposed by Keynes, some papers have assumed that workers compare their wage relative to those of other workers—the relevant reference group—employed in other firms (e.g. Bhaskar (1990), Akerlof and Yellen (1990), Akerlof, Dickens, and Perry (2000) and Driscoll and Holden (2004)). In the context of the chapter, this type of relative comparison will be referred to as *external* social comparison. On the other hand, building on the early insights of Hamermesh (1975) and Frank (1984), the paper of Cabrales, Calvó-Armengol, and Pavoni (2008) analyses the implications of a firm's internal pay structure for wage dynamics and skill segregation, by assuming that workers compare wage offers relative to the wage of other peers with similar productivity employed in the same firm. This relative comparison will be referred to as *internal* social comparison. As the evidence presented in the following section will confirm, both forms of social comparison are found to be important for workers' reference wage formation. Finally the original contributions of Thomas (2005) and Snell and Thomas (2010) analyse the implications for wage and unemployment fluctuations of the "equal treatment" hypothesis, according to which firms cannot pay discriminate between new hires and incumbents. Despite being imposed as an *ad hoc* constraint, this assumption aims to capture the idea that social comparison may also exists between unemployed workers being hired and existing workers already employed in the same hiring firm.

The implications of these assumptions vary within the literature. For instance, external social comparison generates coordination failures and multiple rational-expectations equilibria in Bhaskar (1990); internal social comparison provides an explanation for productivity-unrelated wage increases in the model of Cabrales et al. (2008); while the "equal treatment" hypothesis generates downward rigidity in the wage of newly hired workers (Snell and Thomas, 2010).

Firm's ability to pay. Motivated by the inability of the aforementioned efficiency wage approach to generate wage rigidity in general equilibrium models, Danthine and Kurmann (2006, 2007) have advanced the hypothesis that workers' reference wage is influenced by their firms' ability to pay, captured by a measure of the firm's output per employee. This "internal reference" perspective also resembles the notion of rent sharing: a worker employed in a firm that is making relatively more profit will expect to be paid a relatively higher wage, that is, in the model of Danthine and Kurmann (2007), a higher per-worker output implies a higher worker's reference wage.[2]

In addition to generating wage *stickiness* to aggregate demand shocks and being consistent with evidence that finds firm performance being a quantitatively significant predictor of wages, this assumption enables to consider the role of firm-specific information about profitability on workers' reference wage formation, and the related consequences for wage dynamics and unemployment.

Lagged expectations. In a recent contribution Eliaz and Spiegler (2014) analyse a search and matching model à la Mortensen and Pissarides (1994) where workers have reference-dependent preferences and employed workers' reference wage is given by the expected equilibrium wage paid to existing workers. More specifically, following the lagged expectations approach of Kőszegi and Rabin (2006), an employed worker's reference wage is determined by their rational expectations conditional on the information acquired in the previous employment period.

This lagged structure generates wage rigidity in the model of Eliaz and Spiegler (2014). Another advantage of this assumption is that it can potentially capture realistic situations in which even a wage raise may be perceived as unfair, if the resulting wage level is below what was expected by the worker.

Multiple references. Indeed all the elements outlined above might be relevant for a worker's reference wage formation process. The following set of theoretical models have used a multiple reference approach to characterise the reference wage, by considering various combinations of the aforementioned determinants. For instance, among other factors related to external labour conditions, Skott (2005) assumes that the reference wage is determined by workers' current real wage and by the external relative comparison with the wage of other workers. Danthine and Kurmann (2010) analyse a reference wage equation which depends on three determinants: external labor market conditions, captured by the aggregate wage and the employment level; a measure of firm-internal labour productivity representing rent-sharing considerations; and past wages capturing the notion of wage entitlement on the part of workers. Chouliarakis and Correa-Lopez (2012) consider a combination of the market clearing wage with the wage of other workers and introduce a backward-looking element by assuming that only a random fraction of workers update their entitlements on current market conditions. On the other hand Knell and Stiglbauer (2012) consider four distinct cases: workers' past wage; the wage of workers employed in another sector; a price indexation

[2]See Section A.1.1, Appendix A.1, for an overview of Danthine and Kurmann's (2007) model and assumptions.

mechanism; and a "wage leadership" framework according to which the leading sector reference wage is unaffected by past and current market conditions while the follower take the wage set by the leader as the reference wage. Finally Kuang and Wang (2017) consider a combination of a worker's past wage; their outside option, i.e. the average wage paid in the market; and the wage level in the steady-state.

The advantage of a multiple reference approach is to enable a better understanding of the relative importance of each reference wage determinant for the outcomes of the model. In addition, some of those models generate structural wage equations where the relevance of each reference wage determinant can be estimated and quantified with labour market data (e.g. as in Danthine and Kurmann (2010)).

Evidence on reference wage determinants

This section surveys the empirical literature concerned with the investigation of what determines workers' reference wage entitlements in the labour market. This literature consists of several research approaches that span from survey interviews, panel data analysis and laboratory and field experiments.

Past wages and adaptation. The first piece of evidence supporting the idea that past wage contracts serve as a reference for fairness judgements in the labour market comes from the seminal telephone survey experiment of Kahneman et al. (1986). This finding has been rapidly coupled with the psychological notion of adaptation, or habituation, popular in social psychology (see discussion in the next section; and Kahneman and Thaler (1991) and Baucells and Sarin (2010) for a review of this early literature).

Adaptation to past wage contracts is also supported by several anthropological studies (see the survey of Bewley (2007) and other references cited in this section). In every firm interviewed, regardless of the country or industry of origin, compensation managers believe that past wage contracts are important determinants of workers' wage entitlements, and that workers rapidly adapt to what they are paid: "Even generous pay raises do not increase morale or productivity, because workers quickly get used to increases and grow to believe they have a right to them."(Bewley, 2007, p.162). Nevertheless, this entitlement adaptation might not be as quick in the event of wage cuts. As reported in these studies, managers fear a more intense and *persistent* drop in their workers' effort following wage cuts, suggesting that it takes more time for workers to adapt to wages that are perceived to be unfair.

Another strand of evidence comes from panel data analysis of the wage-job satisfaction relationship using British (Clark, 1999; Smith, 2015) and German (Grund and Sliwka, 2007) panels. All these studies report evidence of a significant positive relationship between job satisfaction and wage changes between waves (rather than absolute wage levels). Hence, this evidence stands in support of the hypothesis of adaptation of wage entitlements.

In the context of experimental studies, indirect evidence in support of this hypothesis comes from the field experiment of Gneezy and List (2006), and the laboratory experiments of Clark, Masclet, and Villeval (2010), Gächter and Thöni (2010) and Koch (2016) among others. In particular, based on the results of their experiment, Gneezy and List (2006) are the first to conjecture that the observed temporary effect of a wage-gift on workers' effort could be explained by an adaptation of the reference point. This conjecture, and the hypothesis of reference wage adaptation, has been directly tested in the field experiment of Chemin and Kurmann (2014), and in the laboratory experiment of Sliwka and Werner (2017). The distinctive feature of both these studies is that the authors purposely investigate the effects of increasing and decreasing wage profiles on workers' effort in a dynamic environment. Chemin and Kurmann (2014) find that a wage rise of 45% did not have a significant effect on field-workers' un-monitored effort; while a subsequent wage reduction back to the initial wage level has lead to a significant drop in effort of about about 30% less relative to the rate before the wage rise was implemented. On the other hand, analysing several wage profiles offered to individuals working on a real-effort task, Sliwka and Werner (2017) find that the positive effect on effort of a wage increase are only temporary and that, absent subsequent increases in the wage, working performance converges back towards the level associated with a constant wage.

Taken together this evidence identifies past wage contracts as one of the most plausible candidate as a determinant of the reference wage, and points toward the existence of a dynamic adaptation in workers' wage entitlements over time.[3]

Relative wage comparisons. Evidence in support of the role of social comparison for workers formation of wage entitlements in the labour market comes from a variety of empirical approaches.

The first strand of evidence comes from field surveys to compensation managers from a set of firms and industries in the U.S. (e.g. Bewley (1999)), the U.K. (e.g. Kaufman (1984)) and other European countries (e.g. Du Caju et al. (2015)). These studies provide anthropological evidence in support of all the three types of social comparison hypotheses previously discussed. For instance Blinder and Choi (1990), Campbell and Kamlani (1997), Bewley (1999) and Galuscak et al. (2012) find that firm's *internal pay structure* is particularly relevant to workers fairness considerations and relative wage comparisons.[4] While Agell and Lundborg (1995, 2003) and Agell and Bennmarker

[3]This evidence also justifies the extensive use of the assumption of reference wage adaptation implemented in Chapters 1 and 2.

[4]"This structure is created in large part to achieve internal equity, which is both uniformity in

(2007) for Sweden and Du Caju et al. (2015) for a group of 14 European countries find evidence in support of the hypothesis that it is the wage of other workers employed in other firms that forms the relevant entitlement for wage comparisons. Moreover, Kaufman (1984), Bewley (1998, 1999) and more recently Galuscak et al. (2012) report that the internal pay structure is also relevant for social comparisons between new hires and incumbent workers; a finding that supports the "equal treatment" hypothesis of Snell and Thomas (2010) that was previously discussed. Taken together this evidence does not seem to give a clear cut conclusion on what type of social comparison is predominant for workers' reference wage formation. Nevertheless, several of these studies documented that external social comparison—be it between new hires and incumbents or only between incumbents—is more prevalent in markets and industries characterised by a higher degree of unionisation or coverage of collective agreements (see Bewley (1998), Agell and Lundborg (2003), Agell and Bennmarker (2007), Galuscak et al. (2012) and Du Caju et al. (2015)). These findings are consistent with Bewley's (1998) conjecture, that the presence of unions, as well as more centralised forms of bargaining, facilitates the dissemination of *information* about wages and working conditions across firms and industries, therefore making relative wage comparisons, with peers employed in other firms, more likely to take place.

Another strand of evidence comes from more recent laboratory and field experiments, the majority of which have been designed to test the hypothesis of internal social comparison. These are: the real-effort gift-exchange experiment of Charness and Kuhn (2007), the three-person gift-exchange experiment of Gächter and Thöni (2010), the team production experiment of Bartling and von Siemens (2011) and the labour supply experiment of Bracha, Gneezy, and Loewenstein (2015). While Gächter and Thöni (2010) and Bracha et al. (2015) find evidence in support of the role of social comparison for workers' effort and labour supply decisions, this relationship is less significant in the studies of Charness and Kuhn (2007) and Bartling and von Siemens (2011). However, as the authors point out, this result might be caused by the chosen experimental design. As such, evidence from the laboratory appears to be inconclusive. Instead, the field experiments of Card, Mas, Moretti, and Saez (2012), Cohn, Fehr, Herrmann, and Schneider (2014) and Breza, Kaur, and Shamdasani (2016) find supportive evidence of internal social comparison for workers' job satisfaction, effort and morale respectively. Interestingly, in virtually every laboratory or field experiment workers' social comparisons have been found to be *asymmetric*: independently of the choice variable analysed by the experimenters, individuals were more responsive to dis-

the application of the rules setting pay and a set of beliefs about fair relations between pay and its determinants."(Bewley, 1998, p.477).

advantageous pay inequality, while they were almost never responsive to advantageous inequality. Hence experimental evidence suggests that workers consider the wage of their peers as a relevant determinant of their reference wage mostly when they feel they are relatively underpaid.

Experimental evidence in support of external social comparison is reported in Clark et al. (2010). They perform a standard gift-exchange experiment between an employer and an employee where information about relative wages (those of other employees in other firms participating in the same experiment) was given. Indeed they find that wage comparison has a significant effect on employees performance relative to a benchmark in which information about relative wages was not given.

Finally, another piece of evidence in support of relative wage comparisons is reported in Smith (2015). By using data from the British Household Panel Survey from 1991 to 2007, Smith investigates the relationship between workers' job satisfaction and pay growth and finds that the median wage growth is important in explaining this relationship: any reduction in earnings growth below the median reduces satisfaction, relative to those with above-median earnings growth. This evidence is interpreted by Smith as supportive of the social comparison hypothesis.

Labour market conditions and firm's performance. Most of the evidence discussed thus far has been capable of testing directly, or at least gaining an explicit insight on, the role of two main determinants of workers' reference wage: their own past wage contract and the wage contract paid to their peers. This was facilitated by the fact that the reference wage and its determinants were expressed in the same unit measure, i.e. a monetary value (or a substitute for it in laboratory experiments). However it is more troublesome to empirically identify a reference wage determinant when this is expressed in a different unit measure. Evidence on the role of external market conditions—such as the unemployment rate—or firm's performance—such as output per worker—must therefore be interpreted within a clear conceptual framework.

For instance, consider the popular assumption in the efficiency wage tradition according to which the reference "fair" wage is a decreasing function of the unemployment rate: $r = r(\cdot, u)$, $\partial r(\cdot, u)/\partial u < 0$. Here the role of the unemployment rate is not to form an entitlement by itself, but rather to influence the formation of the entitlement, generating a shift in the reference wage. In fact, an increase in the unemployment rate will generate a downward shift in the reference wage, with the strictly related consequence that a firm will be able to pay a lower wage without this being perceived as unfair (obviously so long as $w \geq r$).[5] As such, the evidence presented here will be interpreted around the following conjecture: whenever a piece of information makes a wage cut

[5]This conceptual framework is implicit in most of the theoretical models cited above.

more acceptable, or equivalently a wage rise less perceived as a gift, this information is considered to be an influential determinant of a worker's reference wage.

In light of this premise, the remainder will discuss evidence on the role of firms' performance and external labour market conditions. First consider firms' performance. The field surveys to compensation managers of Kaufman (1984), Blinder and Choi (1990) and others discussed in Bewley (2007) have found that when workers are aware of a period of economic crisis, and in particular, when they are informed about a decline in their employer's firm performance and profitability, wage cuts are not seen as unfair and they become more acceptable.

This finding, and in particular the role of the firm's performance during a recessionary episode, is also corroborated by the telephone survey experiment of Kahneman et al. (1986). Simulating a scenario characterised by high unemployment and in which a firm could replace existing workers with other equally productive workers at a lower wage, 68% (N=195) of the respondents thought that it was acceptable to receive a 5% wage cut when information about poor firm performance was given, while only 23% (N=195) of the respondents found that option acceptable when knowing that firm performance was good.

There are other two empirical exercises that report some indirect evidence in support of the role of firm performance for wage entitlements. Danthine and Kurmann (2010) estimate a DSGE model in which the reference wage is assumed to be determined by the worker's past wage, the firm's ability to pay (per-worker output), and external market conditions (wage paid by other firms and employment rate). The result of their estimation attributes substantial importance to past wages (consistent with evidence on adaptation) and, by a lesser extent, to firms' performance; while external labour market conditions do not appear to matter much (see also Section A.1.1, Appendix A.1). By implementing a different methodology, the aforementioned empirical investigation of Smith (2015) also highlights a significant interaction between firm performance and the wage-job satisfaction relationship estimated. Her findings suggest that if firm performance declines wage cuts are more acceptable.

For what concerns external market conditions the available evidence on their role for reference wage formation is scant and even less direct. First of all, as it appears in the studies of Kahneman et al. (1986) and Danthine and Kurmann (2010) just discussed, the state of the labour market does not seem to be important for workers' fair wage considerations, as was instead postulated in the efficiency wage literature. However, external labour market conditions seem to matter more when the internal pay structure linking new hires' reference wage with the wage of incumbents is less binding, i.e. in firms characterised by a high workforce turnover and offering short-term/temporary contracts (see the seminal work of Bewley (1999), as well as more recent field surveys by Galuscak et al. (2012) and Du Caju et al. (2015)). The intuition that justifies this finding is that temporary workers have less chances to know each other and do not think of their jobs as careers (Bewley, 1999). As such, internal pay equity matters less and workers are more willing to accept lower wages in times of high unemployment. This insight is also indirectly supported by the observation that newly hired workers' wages are more flexible in the secondary than in the primary sector (Bewley, 1999, 2007).

To sum up, the labour market evidence discussed here lends support to the assumption that past wage contracts and social comparison are two key determinants of the reference wage, and it also provides insights on the relative importance of internal *versus* external social comparison. In addition the discussion of this evidence has highlighted the social and institutional contexts within which firm performance and labour market conditions might be relevant for workers' reference wage formation process.

3.3.2 The Reference Point: Insights from Behavioural Sciences

This section discusses some theoretical and empirical insights on individual's reference point formation which stem from other behavioural sub-disciplines such as social psychology, organisational psychology, organisational behaviour, decision-making, management science and behavioural economics. This discussion should not be considered a thorough review of the literature. Rather, it is an overview of some key aspects of reference point formation that will be useful as guidelines to organise the labour market literature discussed above. An excellent review of this wide-ranging literature can be found in Stommel (2013).

Behavioural research on reference-dependent preferences has focused on three main aspects of reference point formation: i) how reference points are determined; ii) how reference points can be manipulated or influenced by exogenous stimuli; and iii) how do individuals evaluate outcomes when there are multiple reference points available. The remainder of the section will unfold around these three main themes.

Determinants and entitlements

Psychological studies of decision making identify three main candidates that determine an individual's reference point in experienced utility: adaptation, social comparison and expectations.

Adaptation. In its most general definition, adaptation—or habit formation—is a dynamic mechanism according to which individuals update their reference point over time to past information. As acknowledged by Baucells and Sarin (2013) habit formation has a long-standing tradition in economic modelling of consumption theory, spanning from Duesenberry (1952) to Rozen (2010). Adaptation in the reference point has been usually formalised following the models of Constantinides (1990) (continuous time) and Wathieu (1997) (discrete time), where an individual's reference point r in period t is given by a convex combination of the experienced outcome x and reference point r in period $t-1$. That is, for instance:

$$r_t = (1-\alpha)r_{t-1} + \alpha x_{t-1} \quad r_0 \text{ given},$$

where $\alpha \in [0,1]$ represents the "speed of adaptation" to the last outcome; and $(1-\alpha)$ measures the "constancy of tastes" (Wathieu, 1997, p.1554). Evidence of this dynamic mechanism is pervasive in virtually every decision-making or experienced utility context (surveys can be found in Kahneman and Thaler (1991) and Baucells and Sarin (2010)). In addition there appears to be an asymmetric adjustment of the reference point to past experienced outcomes, depending on whether those outcomes have been coded as gains or losses. This asymmetry, defined by Frijda (1988) as "the law of hedonic asymmetry", implies that: "Pleasure is always contingent upon change and disappears with continuous satisfaction. Pain may persist under persisting adverse conditions."(p.353). Hence the law of hedonic asymmetry suggests the existence of an intrinsic tendency to adapt more rapidly to those experienced outcomes that generated gains than to those that generated losses. This conjecture is supported by the recent laboratory experiments of Arkes et al. (2008, 2010) and Baucells et al. (2011), whom provide evidence in support to *asymmetric adaptation* of reference points in a financial context.

Contracts as reference points. In relation to reference point adaptation, behavioural contract theory has recently advanced the hypothesis that ex-ante contracts serve as entitlements for future renegotiations (Hart and Moore, 2008; Herweg and Schmidt, 2015). This hypothesis is particularly relevant in the presence of incomplete contracts, where one of the contracting parties have discretion with respect to their performance, and can potentially withdraw part of it—the "consummate performance"—if the outcome of renegotiation falls short of what they were entitled to, i.e. the previously signed contract (Hart and Moore, 2008). Building on the theoretical framework put forward by Hart and Moore (2008), this hypothesis has been corroborated in the laboratory experiments of Fehr et al. (2011, 2014), Bartling and Schmidt (2015) and Herz and Taubinsky (2016).

Social comparison. Following Goodman (1974), behavioural studies of pay evaluation have found experimental evidence in support to the hypothesis that individuals use the pay of others as a relevant reference for both fairness judgements and satisfaction (e.g. Loewenstein and Bazerman (1989), Blau (1994), Ordóñez, Connoly, and Coughlan (2000) and McDonald, Nikiforakis, Olekalns, and Sibly (2013)). Interestingly, as reported by the experimental evidence on labour markets previously discussed, all these investigations find evidence in support of *asymmetric social comparison*, that is, disadvantageous pay inequality have a stronger negative effect on satisfaction and it is perceived as unfair relative to pay equality and advantageous inequality.

Expectations. A class of theories of reference-dependent preferences advanced the idea that reference points are determined by individuals' expectations (see for instance Loomes and Sugden (1986), Gul (1991), Shalev (2000), and Kőszegi and Rabin (2006)). Among these models, which differ in the relevant notion of expectations, the most popular and widely implemented has been the portable framework developed by Kőszegi and Rabin (2006), in which the reference point is endogenously determined by the rational "expectations a person held in the recent past". While this formulation has been successful in explaining empirical regularities, for instance, in consumption-savings decisions (Kőszegi and Rabin, 2009) and life-cycle consumption (Pagel, 2017), its empirical validity in other contexts, such as labour markets and employment relationships, has been harder to assess (see Hack and Lammers (2015) for a review of this hypothesis in the behavioural science literature). Expectations-based reference points have been found to play a role in the real-effort experiment of Abeler, Falk, Goette, and Huffman (2011) and in the exchange experiment of Marzilli Ericson and Fuster (2011). However, while reference points are easier to identify and manipulate in controlled laboratory experiments, it is not possible in these settings to fully understand how expectations and reference points would actually be influenced in the field: "Though we show that expectations are an important determinant of reference points, we cannot rule out that other factors, such as social norms, aspirations, salience, and history, may also influence the reference point."(Marzilli Ericson and Fuster, 2011, p.1901). In addition, evidence from the field, such as in the context of flexible labour supply (e.g. taxi drivers), appears to be mixed (e.g. Crawford and Meng (2011) (supportive) and Farber (2015) (not supportive)).

Summing up, the literature discussed in this section highlights three fundamental determinants of individuals' reference point formation: past experience/transactions, social comparisons and expectations. However, while expectations might in theory be relevant, as Kőszegi and Rabin (2006) suggest psychological and economic judgement is needed in choosing the appropriate notion of expectations, which can indeed be influenced by the history of transactions and social comparisons. In addition, none of the evidence discussed in the present and preceding sections support the idea that workers' reference "fair" wage in the context of worker-firm employment relationships is determined solely by expectations. If so, all the aforementioned determinants should nevertheless be considered as elements of the relevant information set influencing these expectations. Finally, several theoretical and empirical studies suggest that there exist asymmetries in the way in which past experience and social comparisons influence reference points: entitlement effects that generate gains disappear relatively quickly, whilst entitlement effects that lead to losses are more pronounced as well as more persistent.

Information, anchors and norms

The aforementioned determinants appear to be the main source of entitlement in individuals' reference point formation. However, behavioural decision theory concerned with bilateral negotiation also acknowledged that the way in which individuals frame negotiation outcomes heavily depends on the readily available information (Neale and Bazerman, 1992); and that by manipulating this information, the negotiating parties can influence the other side's reference point: either by inducing norms or by providing anchors (Kahneman, 1992).

According to Kahneman and Miller's (1986) Norm Theory, a norm is a mental representation of the relative frequency distribution with which a potential reference point will dominate the experienced utility from an outcome. To clarify this concept consider the following example. A wage offer of £6 per hour to work as a waitress may initially be perceived as unfair—i.e. coded as a loss—if the only readily available information to the job applicant is a salary of £8 per hour, e.g. the one perceived by a friend employed in another restaurant (hence forming a reference entitlement induced by social comparison). However, if the employer reveals that £6 per hour is the wage paid to the majority of waiters in the market—i.e. the relative frequency distribution of the £6 wage offer has increased in the mental accounting of the job applicant—the £6 salary may now be viewed as the *norm*, forming a reference salary and making the transaction fair, or at least more acceptable. In this situation the information revealed by the employer influenced the reference point of the job applicant by making an outcome being perceived as the norm.

The recent laboratory experiment of Herz and Taubinsky (2016) provide some direct evidence that observed market outcomes generate an entitlement norm which affects individuals' perceptions of fairness (alongside their personal transaction payoff experienced in the past). In particular they find that in a two-phase experiment, the market experiences of responders from the first part of the experiment had a significant and persistent impact on what offers were considered acceptable in the second part: "[. . .] a key implication of our results on observational experience is that informing buyers (workers) about other prices (wages) should change the prices (wages) that are perceived as acceptable, even when such information is payoff irrelevant. Consequently, increased information dispersion should have the effect of homogenizing fairness norms."(p.36). This finding echoes the previous discussion on the role of labour unions and collective bargaining as two institutions that facilitate the dissemination of information about wage rates and other working conditions, making relative wage comparisons more likely and inducing homogeneous fair-wage norms and entitlements in the labour market.

Strictly related with norms are the concepts of anchors and anchoring effects. An anchor can be defined as a stimulus, or a piece of information, that increases the normality of a possible outcome (Kahneman, 1992). Building on the previous example suppose that the job applicant was aware that the minimum legal wage for waiters is £4 per hour. Even though this information does not induce any specific mental accounting of the frequency distribution of wage offers in the market, it provides an *anchor* which makes the £6 per hour more acceptable (e.g. the wage offer may be perceived as less unfair, fair, or even as a gift). As such, even if an anchor does not form a reference point by itself, nevertheless it can influence the way in which outcomes are framed. Importantly, the efficacy of anchoring effects depends on the implicit categorisation of the given anchor as a member of the set of relevant stimuli (Kahneman, 1992). The minimum wage of a bartender—or of a waiter in another country—might not produce any anchoring effect on the job applicant's evaluation of the £6 wage offer.

The laboratory experiment conducted by Falk, Fehr, and Zehnder (2006) provide an interesting perspective of the anchoring effects of minimum wage policies. In an experimental labour market game, they find that after the introduction of a minimum wage law, workers' reservation wages significantly increased, with the majority of workers feeling entitled to wages even higher than the legal minimum. In addition, this entitlement effect was persistent even after the minimum wage policy was removed, as workers' reservation wages remained higher than before its introduction.

These insights and their related empirical findings can be couched together under Baucells and Sarin's (2013) conceptualisation of *reframing*. Information disclosure in the form of norms and anchors can be thought as exogenous stimuli "that make us see reality in a different way, reset our expectations, or create new comparisons." The present section has shown that norms and anchors can be induced by firms, economic policy as well as by information dispersion and the observational experience of workers in the market.

Multiple reference points

In situations in which there are multiple relevant reference points available, an individual's evaluation of an outcome is different depending on which reference point is adopted. For instance, a wage offer of £6 per hour might be coded as a gain compared to the worker's most recent wage contract of £4 per hour; or it might be coded as a loss compared to the wage paid to their peers of £8 per hour. How do individuals evaluate outcomes in the presence of multiple reference points?

Behavioural decision theory has usually couched the adoption of one reference point or another with framing effects (Kahneman, 1992), and has developed two main models of retrospective evaluation of outcomes under multiple reference points: the "integrated" mechanism and the "segregated" mechanism (see for instance Ordóñez et al. (2000); Baucells et al. (2011)).

The *integrated* mechanism is the simplest model and postulates that individuals use a single reference point which is a weighted combination of two or more other salient reference points. That is, under the integrated mechanism the single reference point r can take the form of a weighted average of a series of multiple reference points $(r_1, r_2, \ldots, r_N)$. Hence, an individual's experienced gain-loss utility can be expressed as

$$\mu(w - r), \quad \text{with } r = \sum_{n=1}^{N} \bar{\beta}_n r_n,$$

where $\mu(w - r)$ is the gain-loss value function and $\bar{\beta}_n$ is a weighting function. Drawing from the existing empirical evidence on multiple reference points, Ordóñez et al. (2000) conjecture that the presence of several reference points makes the integration mechanism more likely.

On the other hand, according to the *segregated* mechanism, an individual compares each relevant reference point to their outcome separately. Hence there are multiple comparisons that have different relative prevalence in the individual's experience, and the final judgement of fairness or satisfaction is determined by which gain-loss experience ultimately prevails. Under the segregated mechanism, in the presence of multiple reference points $(r_1, r_2, \ldots, r_N)$ an individual's experienced gain-loss utility can be expressed as:

$$\sum_{n=1}^{N} \bar{\gamma}_n \mu(w - r_n), \quad \text{with } \sum_{n=1}^{N} \bar{\gamma}_n = 1$$

where $\mu(w - r_n)$ is a gain-loss value function and $\bar{\gamma}_n$ is the relative prevalence of the experienced gains and/or losses on the individual's utility. In the frequency model of Kahneman (1992), the final experienced utility is the result of the frequencies—the relative values of $\bar{\gamma}_n$—with which gains and losses are perceived. According to Ordóñez et al. (2000), whom find evidence in support to this, the segregated mechanism is more likely to be relevant when the number of available reference points is small.

While behavioural research has converged toward a dual view based on the integrated *versus* segregated mechanisms in theory, the empirical literature concerned with how multiple reference points compete and combine is yet in its infancy. Hence it is hard to draw solid and evidence-based conclusions regarding the relative importance of the two mechanisms described in this section.

3.3.3 Summarising Discussion

Drawing from this wide-ranging literature discussion it is now possible to outline several noteworthy features of reference wage formation. Workers' perceptions of fairness over wage contracts depend on a complex interaction of several factors and determinants. Theoretical models of the labour market have been insightful in exploring the implications of assuming various reference wage determinants, but it is hard to argue on the sole basis of intuition and anecdotal evidence which specification is more plausible. Thanks to a variety of empirical approaches adopted by economists, and to developments in behavioural decision research, the understanding of the main principles underlying the determination of workers' reference wages has substantially improved.

The *prima facie* candidates to form a fair wage entitlement are an individual's experienced transactions in the past and relative social comparisons with others. Translating this into the context of employment relationships and labour markets, wage entitlements appear to be primarily formed by a worker's most recent wage contract and the wage contract paid to their peers, either employed in the same firm—internal social comparison—or elsewhere—external social comparison. As such, if expectations are to play any role in reference wage formation, they are likely to be influenced mainly by these two determinants.

In addition to these entitlements there are other factors that could potentially affect the way in which a wage contract is perceived by workers. These factors are typically indicators capturing information of market conditions, such as the state of the labour market or the employer's ability to pay and firm performance. Importantly, while these indicators do not form an entitlement by their own, as they are expressed in a different unit measure than wages, they have been shown to influence a worker's wage entitlement and subsequent evaluation of their employer's wage policy.

Other important aspects of reference wage formation are the fact that perceptions of fairness can change over time, they are strictly dependent on readily available information, and they can be shaped by the relevant social/institutional context in place. These dynamic and context-dependent features can be grouped into three main properties of reference wage formation: i) there is an intrinsic tendency of workers to adapt their wage entitlements over time—i.e. reference wage adaptation; ii) the reference wage formation process can be influenced by information disclosure and reframing—due to the acknowledgement/revelation of wage norms or relevant anchors; and iii) institutional and social devices—such as labour unions, collective agreements and temporary (short-term) *versus* permanent (long-term) contracts—can potentially shape the information set that is relevant for workers' fairness evaluations.

Finally, a large variety of empirical approaches have shown that there appears to be asymmetries in individuals' (workers') reference point (wage) formation process. In particular, wage entitlements adapt more rapidly following experienced gains than following experienced losses; and disadvantageous pay inequality in social comparisons has a substantially larger effect on fairness judgements and morale than equity or advantageous pay inequality.

The challenge of economic theory would be to formalise all these aspects into a unified general framework that can be tractable enough to analytically study reference wage formation and its implications for firms' wage setting and employment decisions. The remainder of the chapter provides a first step into this direction.

3.4 A General Framework

Drawing from the key principles highlighted in Section 3.3.3, this section develops a general framework to think about, and formally model, workers' reference wage formation. To begin with, Section 3.4.1 will formulate a general reference wage rule for any possible reference wage determinant and will subsequently analyse its main static and dynamic properties. Then, by drawing from the social identity approach in economics, Section 3.4.3 will provide an organising framework to analytically characterise a worker's reference wage and its determinants.

3.4.1 Type of Determinants and General Rules

In the theoretical framework that follows, what are referred to as the *determinants* of a reference wage should be considered as an abstract representation of all the possible and potentially relevant information that could influence a worker's reference wage formation process. More specifically, it is assumed that there exist two types of determinants: "entitlement determinants" and "shift determinants". An *entitlement* determinant is typically expressed in the same unit measure of a wage offer and therefore it can potentially constitute a reference wage, a fair-wage norm or a feeling of entitlement by itself. Examples are past wage contracts or the minimum legal wage in the market. A *shift* determinant instead is a piece of information that could potentially influence—i.e. shift—a worker's reference wage. However, since it is not typically expressed in the same unit measure of wages, it cannot form an entitlement by itself. Examples are the unemployment rate or a measure of firm performance.

Denote by $\mathcal{X}$ the set of all possible entitlement determinants, where x in $\mathcal{X}$ denotes one of its typical elements: $\mathcal{X} \equiv \{x : x \text{ is an entitlement determinant}\}$; and denote by $\mathcal{S}$ the set of all possible shift determinants, where s in $\mathcal{S}$ denotes one of its typical elements: $\mathcal{S} \equiv \{s : s \text{ is a shift determinant}\}$. Then the universal set of determinants of a worker's reference wage r is defined by $\Omega = \mathcal{X} \cup \mathcal{S}$. As such, the universal set of determinants Ω represents the set of all possible and potentially relevant information that could influence a worker's reference wage formation process. On top of this fundamental distinction, it is assumed that in economic situations characterised by multiple determinants available, workers tend to *segregate* over entitlement determinants while they tend to *integrate* over shift determinants. In addition, consistent with the behavioural literature on adaptation, it is assumed that the the reference wage is subject to a dynamic adaptation of its entitlement component. These aspects are going to be formalised in what follows.

Segregation of entitlement determinants

Consider a worker i's payoff and experienced gain-loss utility as given by (3.1) and (3.2) respectively, and assume as a general case that for a given set of entitlement determinants $\mathcal{X}$ the worker is facing a situation characterised by multiple reference wages $r_{i,t} = r_i(x_t)$ for all x in $\mathcal{X}$. The worker i's experienced gain-loss utility corresponding to a wage contract $w_{i,t}$ under segregation of entitlement determinants is assumed to be given by

$$\sum_{x \in \mathcal{X}} \gamma_{i,x} \mu_i(m(w_{i,t}) - m(r_i(x_t))), \quad \text{with} \sum_{x \in \mathcal{X}} \gamma_{i,x} = 1, \tag{3.3}$$

where $\gamma_{i,x}$ is the subjective relative prevalence of $\mu_i(m(w_{i,t}) - m(r_i(x_t)))$ in worker i's utility $u_i(e_{i,t}; w_{i,t}, r_i(x_t))$. This assumption implies that the wage $w_{i,t}$ will be perceived as fair, unfair or as a gift depending on the extent and relative prevalence of each experienced gain-loss utility generated by each of the multiple reference points available, in this general case, for all x in $\mathcal{X}$.

Example 1. *Consider a worker i that has just been hired into a firm at an hourly wage contract $w_{i,t} = £6$. The current minimum legal wage is $\underline{\omega} = £4$, and the worker's peers already employed in the firm are paid the wage $w_t^{-i} = £8$. If this information is readily available, worker i's experienced gain-loss utility (assuming $m(w) = w$ for simplicity)* *is given by*

$$\gamma_{i,\underline{\omega}}[£6 - £4] + \gamma_{i,w^{-i}}\lambda[£6 - £8], \quad \text{with } \gamma_{i,\underline{\omega}} + \gamma_{i,w^{-i}} = 1.$$

It is straightforward to notice that whether the worker perceives a gain or a loss in utility, in this particular example, will strictly depend on the following conditions: if $\gamma_{i,\underline{\omega}}/\gamma_{i,w^{-i}} > \lambda$*, then* $w_{i,t}$ *will be perceived as a gift; if* $\gamma_{i,\underline{\omega}}/\gamma_{i,w^{-i}} = \lambda$*, then* $w_{i,t}$ *will be perceived as fair; while if* $\gamma_{i,\underline{\omega}}/\gamma_{i,w^{-i}} < \lambda$*, then* $w_{i,t}$ *will be perceived as a unfair. Since loss aversion implies that* $\lambda > 1$*, it can be noticed that if the worker assigns equal weights to these comparisons, i.e. if* $\gamma_{i,\underline{\omega}} = \gamma_{i,w^{-i}}$*, then the wage contract* $w_{i,t} = £6$ *will be perceived as a unfair, i.e. "losses loom larger than gains".*

Integration of shift determinants

Consider a general case in which a worker i's reference wage at time t is a function of one entitlement determinant x in $\mathcal{X}$ and several shift determinants s in $\mathcal{S}$. The worker i's reference wage under integration of shift determinants is assumed to be given by the following expression:

$$r_{i,t} = x_t \cdot \left[1 + \sum_{s \in \mathcal{S}} \beta_{i,s} \left(\frac{s_t - s_{t-1}}{s_{t-1}} \right) \right], \tag{3.4}$$

where $\beta_{i,s} \in [-1, 1]$ is a scaling parameter capturing the sign and relative importance of the shift determinant s in worker's i reference wage formation process.

This assumption implies that for a given entitlement x_t, the reference wage in period t will either be higher or lower—or equivalently, increase or decrease—depending on the sign, magnitude and relative importance of the observed percentage change in the relevant shift determinants s in $\mathcal{S}$, between periods t and $t - 1$.

Example 2. *Consider a worker i that at the end of period* $t - 1$ *feels entitled to the wage he is currently paid:* $w_{i,t-1} = £6$*. Then suppose that at the beginning of t the economy falls into a recession. The worker observes a 4% increase in unemployment, i.e.* $\Delta u_t/u_{t-1} = 4\%$ *and comes to know from their managers that the firm performance has decreased by 10% due to, for instance, a decrease in the idiosyncratic match productivity, i.e.* $\Delta q_t/q_{t-1} = -10\%$*. Logical reasoning suggests that:* $\beta_{i,u} \in [-1, 0]$ *and* $\beta_{i,q} \in [0, 1]$*. As such, worker i's reference wage at the beginning of period t is given by*

$$r_{i,t} = £6 \cdot \left[1 - |\beta_{i,u}| 4\% - \beta_{i,q} 10\% \right].$$

If the wage paid to worker i remains unchanged at the beginning of period t, they will perceive a gain in utility, since $w_{i,t} = w_{i,t-1} > r_{i,t}$ *will now be perceived as a gift. Moreover notice that in this situation the firm might well be able to set a lower wage* $w_{i,t} < w_{i,t-1}$ *without incurring the adverse effects of low morale and negative reciprocity—as discussed in Chapter 1—indeed, so long as* $w_{i,t} \geq r_{i,t}$.

Adaptation of entitlements

Consider a worker i's reference wage in period t that is given by the entitlement determinant x in $\mathcal{X}$ only. If the worker adapts equivalently to perceived gains and losses over time, i.e. if dynamic adaptation is symmetric over gains and losses, then the process of reference wage adaptation with respect to the entitlement determinant x_t is assumed to be given by the following convex combination:

$$r_{i,t} = (1 - \alpha_i)r_{i,t-1} + \alpha_i x_t, \quad r_{i,0}, \text{ given,} \tag{3.5}$$

where $\alpha_i \in [0, 1]$ is worker i's subjective, and time-invariant, *speed of adaptation*. Indeed, the higher α_i the more weight is given on the entitlement determinant x_t, over the past reference wage $r_{i,t-1}$, in the worker's reference wage $r_{i,t}$. More loosely, a higher speed of adaptation implies a "faster" adaptation of the worker's reference wage $r_{i,t}$ to the entitlement determinant x_t. Moreover, if $\alpha_i = 0$ there is *no adaptation* and $r_{i,t} = r_{i,t-1}$, while if $\alpha_i = 1$ adaptation to x_t is defined as *complete*, i.e. $r_{i,t} = x_t$ at the beginning of period t.

Social comparison: forming or shifting wage entitlements?

Denote the wage paid to other workers from a worker i's perspective as w_t^{-i}. One of the main insights stemming from the literature on social comparison previously discussed is that w_t^{-i}, if known, should be considered as an entitlement determinant, i.e. w_t^{-i} is an element of $\mathcal{X}$. The simplest form of relative wage comparison can be expressed as:

$$r_{i,t} = w_t^{-i}; \tag{3.6}$$

which implies that worker i's reference wage in period t is entirely determined by the wage paid to their reference group of workers in period t. Specification (3.6) has been commonly assumed in the theoretical literature based on social preferences, and it has been extensively investigated by the empirical research on relative wage comparisons discussed in Section 3.3. This specification can be defined as an entitlement to *pay equality*: a worker feels entitled to be paid the same wage paid to their peers (their relevant reference group).

However, there might be situations in which even if worker i does not feel entitled to be paid the same wage as their reference group of workers—due to, for instance, differences in seniority or job category within the same firm—a change in the wage of their peers will still influence their current feelings of entitlement.

Drawing from the framework developed thus far, this situation corresponds to the case in which w_t^{-i} is a shift determinant, i.e. w_t^{-i} is an element of $\mathcal{S}$. In this case relative wage comparison can be expressed as:

$$r_{i,t} = x_t \cdot \left[1 + \beta_{i,w^{-i}} \left(\frac{w_t^{-i} - w_{t-1}^{-i}}{w_{t-1}^{-i}}\right)\right]; \tag{3.7}$$

where $x_t \neq w_t^{-i}$ is an arbitrary reference wage entitlement of worker i in period t; and $\beta_{i,w^{-i}} \in [0,1]$ is the scaling parameter capturing the importance of social comparison on $r_{i,t}$. Hence, even if worker i does not feel to be paid the same wage of their peers, the observed percentage increase (decrease) in w_t^{-i} from period $t-1$ to t will generate an upward (downward) shift in worker i's reference wage in period t.[6]

Specification (3.7) also nests two additional special cases. First, if worker i was feeling entitled to be paid the same wage of their peers in period $t-1$, that is if $x_t = w_{t-1}^{-i}$—note that this could be plausible if i and their peers were paid the same wage in $t-1$—then the reference wage rule (3.7) collapses to:

$$r_{i,t} = (1 - \beta_{i,w^{-i}})x_t + \beta_{i,w^{-i}} w_t^{-i}; \tag{3.8}$$

that is, i's reference wage is a weighted average of their entitlement and the wage paid to their peers in period t. For instance, if i's entitlement was given by their past wage, i.e. $x_t = w_{i,t-1} (= w_{t-1}^{-i})$, then the reference wage (3.8) captures another common assumption in the theoretical literature (e.g. equation (6-7), p.271, in Akerlof and Yellen (1990)). Second, notice that if $\beta_{i,w^{-i}} = 1$ in (3.8), that is if worker i only cares about the wage of their peers, the reference wage rule in which w_t^{-i} is considered a shift determinant, collapses to (3.3), in which w_t^{-i} is an entitlement determinant.

[6]The social comparison rule (3.7) could potentially capture the spill-over effects of minimum/living wage policies towards those workers that were not initially part of the workforce for which the policy was initially designed. Embedding this specification into a richer framework, such as those developed in Chapters 1 and 2, is an interesting venue for future research.

3.4.2 Properties of the Reference Wage Rule

By combining together the reference wage formation rules formalised by (3.3), (3.4) and (3.5) it is possible to express a worker i's experienced gain-loss utility subject to a general reference wage rule, for any possible combination of entitlement determinant x in $\mathcal{X}$ and shift determinant s in $\mathcal{S}$, as follows:

$$\begin{aligned} &\sum_{x\in\mathcal{X}} \gamma_{i,x}\mu_i(m(w_{i,t}) - m(r_{i,t}(x_t))), \quad \text{with} \sum_{x\in\mathcal{X}} \gamma_{i,x} = 1, \\ \text{where} \quad & r_{i,t} = r_{i,t}(x_t) = \left[\alpha_i x_t + (1-\alpha_i) r_{i,t-1}\right] \cdot \left[1 + \sum_{s\in\mathcal{S}} \beta_{i,s}\left(\frac{s_t - s_{t-1}}{s_{t-1}}\right)\right], \\ & r_{i,0}, \text{ given;} \end{aligned} \tag{3.9}$$

and where, as previously defined, $\gamma_{i,x}$ captures the subjective relative prevalence of $\mu_i(m(w_{i,t}) - m(r_i(x_t)))$ for any x in $\mathcal{X}$ on worker i's utility; $\beta_{i,s} \in [-1, 1]$ is a scaling parameter capturing the sign and relative importance of the shift determinant s in $\mathcal{S}$ in worker's i reference wage formation process; and $\alpha_i \in [0, 1]$ is worker i's subjective speed of adaptation. The remainder of this section will study some distinctive properties of this formulation.

Preliminary dynamic analysis

The general reference wage rule (3.9) is a non-autonomous, non-homogenous first-order linear difference equation. This dynamical system represents the most general description of a worker's reference wage over time.

Consider the general solution of (3.9) (the i subscript is dropped to ease notation). From the theory of dynamical systems, this is given by

$$r_t = \left[\prod_{l=0}^{t-1} A_l\right] \cdot r_0 + \sum_{h=0}^{t-1} \cdot \left[\prod_{l=h+1}^{t-1} A_l\right] \cdot B_h, \quad r_0 \text{ given;} \tag{3.10}$$

where:

$$A_l \equiv (1-\alpha)\left[1 + \sum_{s\in\mathcal{S}} \beta_{i,s}\left(\frac{s_l - s_{l-1}}{s_{l-1}}\right)\right] \quad \text{and} \quad B_h \equiv \alpha\left[1 + \sum_{s\in\mathcal{S}} \beta_{i,s}\left(\frac{s_h - s_{h-1}}{s_{h-1}}\right)\right] \cdot x_h.$$

Without any further assumption regarding the dynamic behaviour of entitlement and shift determinants, the dynamic path of r_t is highly unpredictable. At this level of generality it is in fact complicated to study whether a worker's reference wage tends to an equilibrium state (steady state), and to characterise its related stability properties. In particular, it can be induced from (3.10) that the dynamic behaviour of the shift determinants (subsumed in the time-variant coefficient A_l) is crucial in determining the time path of r_t, i.e. whether it is asymptotically stable, while the dynamic behaviour of any entitlement determinant (subsumed in the time-variant coefficient B_h) will be crucial in determining the existence of a steady state.

Nevertheless, suppose that any entitlement or shift determinant (which are considered as exogenous) that enter the reference wage will tend to some steady-state equilibrium. That is, assume that the sequences $\{s_t\}_{t=0}^{\infty}$ and $\{x_t\}_{t=0}^{\infty}$ are monotone and bounded. This property is common in many economic analyses or models involving a dynamical system. By imposing this simple structure, the following proposition highlights one important dynamic property of a worker's reference wage.

Proposition 20. *If the sequences $\{s_t\}_{t=0}^{\infty}$ and $\{x_t\}_{t=0}^{\infty}$ are monotone and bounded for all s in $\mathcal{S}$ and x in $\mathcal{X}$, they will converge to their steady states $\bar{s}$ and $\bar{x}$; therefore, there exists a unique steady-state equilibrium reference wage:*

$$r^* = \begin{cases} \bar{x} & \text{if } \alpha \in (0, 1] \\ r_0 & \text{if } \alpha = 0. \end{cases}$$

Moreover, r^* is globally asymptotically stable, and starting from any r_0, the dynamical system for r_t described by (3.9) *monotonically converges to r^*.*

Proof. See Appendix B.3. ■

The key equilibrium property established by Proposition 20 is the following: if there is dynamic adaptation, the steady-state level of a worker's reference wage is entirely determined by the steady-state equilibrium value $\bar{x}$ of the relevant entitlement determinant x in $\mathcal{X}$; while if there is no adaptation, the initial condition characterising a worker's reference wage r_0 fully determines its steady-state equilibrium level. Moreover, it is straightforward to deduce that a higher speed of adaptation α implies a faster speed of convergence.

The noteworthy aspect of the prediction established in Proposition 20 is that even if changes in shift determinants may affect the *short-run* dynamic adjustment of a worker's reference wage (e.g. changes in the unemployment rate, firm performance or other economic indicators), they do not matter for its *long-run* determination. Essentially, workers' perceptions of fairness in the long run are entirely determined by entitlements, independently of whether they are past wage contracts, relative pay comparisons or other existing wage norms in the market. Indeed, while general, this prediction is based on an incomplete partial model. In fact, there can be many interesting dynamic paths characterising a worker's reference wage which might not be necessarily stable and monotonically convergent. Moreover, as it is most likely the case, both entitlement and shift determinants could be *endogenous* with respect to the reference wage, requiring the study of a more complex dynamical system of difference equations.

The economic context of analysis appears to be crucial to provide a more complete characterisation of a worker's reference wage dynamic properties. This conclusion will become more evident in the two applications developed in Section 3.5 of this chapter.

Asymmetries in adaptation and social comparison

Based on the main arguments developed throughout the discussion of the literature, it is assumed that there exist two types of asymmetries in a worker's reference wage formation process: asymmetries in adaptation and asymmetries in social comparison.

Consider reference wage adaptation first. If a worker i adapts more rapidly to experienced gains than to experienced losses, based on the adaptation rule (3.5) this asymmetry can be captured by the following asymmetric adaptation rule:

$$r_{i,t} = \left[(1-\alpha_i^+)\mathbb{A}^+ + (1-\alpha_i^-)\mathbb{A}^-\right] r_{i,t-1} + \left[\alpha_i^+\mathbb{A}^+ + \alpha_i^-\mathbb{A}^-\right] x_t, \quad r_{i,0}, \text{ given}, \tag{3.11}$$

where $\alpha_i^+ > \alpha_i^-$; and $\mathbb{A}^{+,-}$ is an indicator variable such that: $\mathbb{A}^+ = 1$ if $w_{i,t-1} \geq r_{i,t-1}$, and zero otherwise; and $\mathbb{A}^- = 1$ if $w_{i,t-1} < r_{i,t-1}$, and zero otherwise. That is, if worker i received a wage gift in period $t-1$, their reference wage will adapt "more rapidly" to their entitlement in period t, than if they had been paid an unfair wage. Hence, the following reference wage adaptation rules can be defined:

$$\begin{aligned}
\textit{symmetric partial adaptation :} \quad & \alpha_i^+ = \alpha_i^- = \alpha_i \text{ with } \alpha_i \in (0,1);\\
\textit{symmetric complete adaptation:} \quad & \alpha_i^+ = \alpha_i^- = \alpha_i \text{ with } \alpha_i = 1;\\
\textit{asymmetric partial adaptation:} \quad & \alpha_i^+ > \alpha_i^- \text{ with } \alpha_i^+, \alpha_i^- \in (0,1);\\
\textit{asymmetric complete adaptation:} \quad & \alpha_i^+ > \alpha_i^- \text{ with } \alpha_i^+ = 1 \text{ and } \alpha_i^- = 0;
\end{aligned}$$

where the adjective *partial* implies that it will take more than one employment period to the worker to completely adapt their reference wage to the entitlement determinant x_t. The *symmetric complete* adaptation rule with respect to past wage contracts, i.e. when $x_t = w_{i,t-1}$, is the most common assumption imposed in the literature and it is also the one analysed in Chapters 1 and 2.

Next consider social comparison. If social comparison forms an entitlement in a worker i's reference wage, as formalised by (3.6), asymmetric social comparison can be captured by:

$$r_{i,t} = x_t\mathbb{C}^+ + w_t^{-i}\mathbb{C}^-, \tag{3.12}$$

where $\mathbb{C}^{+,-}$ is an indicator variable such that: $\mathbb{C}^+ = 1$ if $w_{i,t} \geq w_t^{-i}$, and zero otherwise; and $\mathbb{C}^- = 1$ if $w_{i,t} < w_t^{-i}$, and zero otherwise. Hence, a worker feels entitled to be paid the same wage paid to their peers only when this is higher than what the worker is currently paid, that is, only when there is a disadvantageous pay inequality.

On the other hand, if social comparison enters the reference wage as a shift determinant, as formalised by (3.7), asymmetric social comparison can be captured by:

$$r_{i,t} = x_t \cdot \left[1 + \beta^{+}_{i,w^{-i}} \left(\frac{w^{-i}_t - w^{-i}_{t-1}}{w^{-i}_{t-1}} \right) \mathbb{B}^{+} + \beta^{-}_{i,w^{-i}} \left(\frac{w^{-i}_t - w^{-i}_{t-1}}{w^{-i}_{t-1}} \right) \mathbb{B}^{-} \right]; \tag{3.13}$$

where $\beta^{+}_{i,w^{-i}} > \beta^{-}_{i,w^{-i}}$; and $\mathbb{B}^{+,-}$ is an indicator variable such that: $\mathbb{B}^{+} = 1$ if $w^{-i}_t \geq w^{-i}_{t-1}$, and zero otherwise; and $\mathbb{B}^{-} = 1$ if $w^{-i}_t < w^{-i}_{t-1}$, and zero otherwise. Essentially, a worker i's reference wage is more sensitive to observed wage raises perceived by their reference group between period t and $t-1$, than to observed wage cuts. Hence, the following social comparison rules can be defined:

symmetric imperfect social comparison: $\beta^{+}_{i,w^{-i}} = \beta^{-}_{i,w^{-i}} = \beta_{i,w^{-i}}$ with $\beta_{i,w^{-i}} \in (0,1)$;
symmetric perfect social comparison: $\beta^{+}_{i,w^{-i}} = \beta^{-}_{i,w^{-i}} = \beta_{i,w^{-i}}$ with $\beta_{i,w^{-i}} = 1$;
asymmetric imperfect social comparison: $\beta^{+}_{i,w^{-i}} > \beta^{-}_{i,w^{-i}}$ with $\beta^{+}_{i,w^{-i}}, \beta^{-}_{i,w^{-i}} \in (0,1)$;
asymmetric perfect social comparison: $\beta^{+}_{i,w^{-i}} > \beta^{-}_{i,w^{-i}}$ with $\beta^{+}_{i,w^{-i}} = 1$ and $\beta^{-}_{i,w^{-i}} = 0$.

The case of *asymmetric perfect* social comparison whenever $x_t = w_{i,t-1} = w^{-1}_{t-1}$, that is, whenever a worker is entitled to his past wage contract which was the same of their reference peer group, collapses to an expression equivalent to (3.12).

To conclude, Sections 3.4.1 and 3.4.2 provided a formal characterisation and analysis of a worker's experienced gain-loss utility subject to a general reference wage rule for any possible and given determinant x, s in Ω. However, to become suitable for a more complete analysis of employment relationships and labour markets, this framework requires a theory of how, when and why different determinants affect the reference wage formation process: how do workers form their reference wage? How is the set of readily available information relevant for fairness evaluations determined? This is the subject of the following section.

3.4.3 Framing: Social Categories, Fair Wage Norms and Identities

The framework developed in Section 3.4.1 has shown that the adoption of an entitlement determinant over another, or the consideration of information about the state of the economy, can radically change the way in which a wage contract is perceived by workers. In addition, the analysis of Section 3.4.2 has established that the dynamics of a worker's reference wage formation process can be different depending on the behaviour of the relevant entitlement and shift determinants.

This inherent sensitivity of workers' experienced gain-loss utility to the specific form of reference wage used for fairness evaluations could lead to different predictions with respect to reciprocity, wage setting behaviour and other related labour market outcomes. This is also a common characteristic of every other economic theory based on reference-dependent preferences.

The complex interactions of determinants that could influence a worker's perception of fairness in the dynamic and ever adjusting social and economic environment of labour markets does not easily lend itself to the development of a rigorous and general theory of reference wage formation. Nevertheless, it seems feasible to postulate an underlying behavioural principle governing the process of reference wage formation, which can then be applied in various, different economic and social contexts. The purpose of this section is to conceptualise, and subsequently formalise, this underlying principle.

Conceptualising framing and identity

According to theories of reference-dependent preferences, the coding of an outcome as a gain or a loss depends on how the outcome is framed by the individual (Kahneman, 1992). As such, framing is defined as the cognitive process through which an individual codes an outcome. In the context of employment relationships and workers' fairness evaluations of the wage they are paid, it is the location of the reference wage that affects the process of coding a wage offer as a gain or a loss: whether a wage is perceived as a gift, fair, or unfair depends on how it is located relative to the reference wage.

However, how are reference wages formed? What are their relevant determinants? The theory of reference wage formation put forward in this chapter is grounded around two main behavioural principles:

I) the set of reference wage determinants relevant for fairness evaluations depends on the way in which workers frame the wage contracts they are paid;

II) this process of framing is influenced by factors such as the social context, the institutional settings and workers' experiences, i.e. by workers' social identity.

These two principles are inherently linked and are assumed to happen simultaneously and unconsciously whenever a worker is evaluating a wage contract. As such, this chapter advances the idea that the process of reference wage formation crucially depends on a worker's social identity and self-categorisation. This formulation of the process of framing applied in the context of reference wage formation is inspired by, and draws from, the social identity approach introduced into economics by Akerlof and Kranton (2000, 2005): "Economists have recently adapted from psychology the idea that utility depends upon how a situation is framed (Kahneman and Tversky, 1979).

Identity describes one special way in which people frame their situation." (Akerlof and Kranton, 2005, p.13).

The social identity approach in sociology refers to the joint contribution of social identity theory and self-categorisation theory. In their conceptual framework, Akerlof and Kranton define *identity* as a word that brings together the concepts of identity, social categories and norms.[7] Hence, identity refers to an individual's *self-image* and *social categories*. Associated with social categories are particular *norms* of behaviour. Norms are social rules, standards of behaviour and beliefs about how people in these categories should behave. A similar concept is defined by the ideal: the exemplary characteristics and behaviour associated with a social category. Social categories influence individuals' decisions because different norms for behaviour are associated with different social categories: "social categories and norms are automatically tied together." (Akerlof and Kranton, 2010, p.11).

According to this conceptual framework Akerlof and Kranton argue that the incorporation of identity and norms into economics generates a theory of decision making where the social context matters: "identities and norms derive from the social setting." (Akerlof and Kranton, 2010, p.11). Utility functions can change as norms of behaviour and social categories differ across space and time. This is indeed in contrast with the tradition, where utility functions are not situation dependent but fixed (Akerlof and Kranton, 2005).

This section builds on Akerlof and Kranton's framework and incorporates the social identity approach into the study of employment relationships and reference wage formation. A worker's experienced utility depends on how the wage contract they are paid is evaluated relative to a reference "fair" wage. The reference wage is an abstract representation of what should be a fair remuneration, and can be influenced by a various set of entitlement and shift determinants. The theory proposed here advances the idea that the process through which the relevant set of information, entitlements and norms contribute to the formation of the reference wage corresponds to the process of framing; and that this is strictly dependent on the social and economic context in which workers' relative wage comparisons and fairness considerations are undertaken. Hence, it is assumed that a worker's reference wage depends on their identity.

Whether a wage contract is considered fair or unfair depends on the worker's *social categories* and self-categorisation, on the specific social and institutional context, and

[7]In their first publication Akerlof and Kranton (2000) use the word "prescription" instead of "norms" or "social norms" to not create confusion with their previous usage in economics. However in their more recent publications (Akerlof and Kranton, 2005, 2008, 2010) the word *norms* is instead used extensively. This chapter will adopt the more recent convention, which also facilitates the connection with the broader notion of perceptions of fairness and its determinants, defined in this context as *fair wage norms*.

on the related *fair wage norms* associated with these. For instance, a newly hired worker (social category) who has just started their job will have a different set of fair wage norms (determinants) influencing framing and the formation of their reference wage relative to an incumbent worker (social category) who has an established long-term relationship with their employer. Likewise, a worker applying for a job in a highly centralised and unionised market (institutional/social context) will be influenced by a different set of fair wage norms (determinants) than a worker applying in a very decentralised labour market.[8]

Formalising framing and identity with set theory

This section develops a formal framework that aims to capture a worker's framing and reference wage formation process. In the spirit of Akerlof and Kranton's identity framework, the following concepts are defined:

Social categories: a worker i can belong to one or more social categories c in C, where C is the set of all possible social categories that can be assigned to a worker. Hence, $C_{i,t} \subseteq C$ denotes the set of social categories that has for elements the social categories $c_{i,t}$ in $C_{i,t}$ assigned to worker i in period t.

Fair wage norms: for each social category c in C, there exists a set denoted by $\mathcal{P}_c$ which has for elements the reference wage determinants x, s in Ω, i.e. the fair wage norms, that influence the reference wage of the typical worker belonging to the social category c. Hence, $\mathcal{P}_{c_{i,t}}$ denotes the set of fair wage norms corresponding to the social category $c_{i,t}$ in $C_{i,t}$ assigned to worker i in period t; and the indexed family of sets $\mathcal{P}_{i,t} = \{\mathcal{P}_{c_{i,t}}\}_{c_{i,t} \in C_{i,t}}$ corresponds to the family of fair wage norms that characterise worker i in period t.[9]

Identities: identity is a concept that brings together social categories and fair wage norms. It refers to the worker i's self-image and assigned social categories $c_{i,t}$ in $C_{i,t}$ in period t, tied together with the corresponding set of fair wage norms $\mathcal{P}_{c_{i,t}}$. Formally, the identity $\mathcal{I}_{i,t}$ of a worker i in period t is defined as the union of the indexed family of fair wage norms $\mathcal{P}_{i,t}$ corresponding to worker i's set of assigned social categories $C_{i,t}$ in period t:

$$\mathcal{I}_{i,t} = \bigcup_{c_{i,t} \in C_{i,t}} \mathcal{P}_{c_{i,t}}. \tag{3.14}$$

[8]Note that workers can also belong to more than just one social category.

[9]Note that the indexed family of sets $\mathcal{P} = \{\mathcal{P}_c\}_{c \in C}$, which has for elements all the possible sets of fair wage norms $\mathcal{P}_c$ for each possible social category c in C that can be assigned to a worker, is equivalent to the universal set of determinants Ω.

Hence, $\mathcal{I}_{i,t}$ is the set consisting of all x in $\mathcal{X}$ and s in $\mathcal{S}$ that belong to $\mathcal{P}_{c_{i,t}}$ for at least one $c_{i,t}$ in $C_{i,t}$: $\mathcal{I}_{i,t} = \{x, s : x \in (\mathcal{X} \cap \mathcal{P}_{i,t}), s \in (\mathcal{S} \cap \mathcal{P}_{i,t})\}$. A worker i's identity can therefore be written as a function of social categories and fair wage norms: $\mathcal{I}_{i,t} = \mathcal{I}(C_{i,t}, \mathcal{P}_{i,t})$.

Reference wage: the reference wage is an artefact that captures the relevant set of information, perceptions and norms of what should be a fair wage. Given the premises postulated in this section, the reference wage $r_{i,t}$ of a worker i in period t is assumed to be a function of their identity: $r_{i,t} = r_{i,t}(\mathcal{I}_{i,t})$.

The formation of the identity set, as captured by (3.14), which groups together different sets of fair wage norms depending on a worker's social categories, stands as a formal representation of the process of framing and reference wage formation: the unconscious cognitive process through which a worker forms their perception of what is a "fair" wage. Through self-categorisation worker i acknowledges their assigned social categories $c_{i,t}$ in $C_{i,t}$ and therefore their identity $\mathcal{I}_{i,t}$. In this process they also acquire the corresponding sets of fair wage norms $\mathcal{P}_{c_{i,t}}$ in $\mathcal{P}_{i,t}$ which are going to form the relevant set of determinants of their reference wage $r_{i,t}$.

Based on this formulation, for a given set of social categories and corresponding fair wage norms, experienced gain-loss utility in period t of a worker of identity $\mathcal{I}_{i,t}$ subject to a general reference wage rule can be expressed as follows:

$$\begin{aligned}
&\sum_{x \in (\mathcal{X} \cap \mathcal{I}_i)} \gamma_{i,x} \mu_i(m(w_{i,t}) - m(r_{i,t}(\mathcal{I}_{i,t}))), \quad \text{with} \sum_{x \in (\mathcal{X} \cap \mathcal{I}_i)} \gamma_{i,x} = 1, \\
\text{where} \quad &r_{i,t}(\mathcal{I}_{i,t}) = \left[\alpha_i x_t + (1 - \alpha_i) r_{i,t-1}(\mathcal{I}_{i,t-1})\right] \cdot \left[1 + \sum_{s \in (\mathcal{S} \cap \mathcal{I}_i)} \beta_{i,s} \left(\frac{s_t - s_{t-1}}{s_{t-1}}\right)\right], \\
&r_{i,0}, \text{ given;}
\end{aligned} \tag{3.15}$$

where the parameters $\gamma_{i,x}$, $\alpha_i \in [0, 1]$ and $\beta_{i,s} \in [-1, 1]$ are defined as in Section 3.4. The set of equations in (3.15) captures in a compact and rigorous form the most general representation of a worker's reference wage and gain-loss utility, and provides a formulation that is readily portable in a variety of economic settings. Within this framework, the most important task left to the theorist/analyst is to identify, based on observation, the particular set of social categories that characterises workers in the labour market under consideration, and then to specify the corresponding sets of fair wage norms (as also suggested by Akerlof and Kranton (2005, 2010)). Indeed, this procedure requires a certain amount of judgement and could involve various degrees of abstraction, which, nevertheless, are not germane to the usual "art" of theory modelling. As such, the framework developed in this chapter lends itself as a portable model to analyse these aspects in a systematic and standardised manner once they have been appropriately identified. The remainder of the chapter illustrates how this procedure can be used to extend the theory of reciprocity and wage setting behaviour developed in this thesis.

3.5 Two Applications

This section implements the theoretical framework developed in Section 3.4 to analyse the implications of the two most relevant features of reference wage formation discussed: adaptation and social comparison. More precisely, Section 3.5.1 will study the microeconomic implications of a more general adaptation rule for wage and reciprocity dynamics, in a worker-firm employment relationship where wage setting behaviour is modelled based on the theory developed in Chapter 1. Then Section 3.5.2 will consider the wage entitlement effects of social comparison between newly hired and incumbent workers and will analyse, through the framework developed in Chapter 2, its implications for the cyclical behaviour of wages, job creation and unemployment.

3.5.1 Asymmetric Partial Adaptation and Persistence

The purpose of the following analysis is to study the dynamics of reciprocity and wage setting behaviour when a worker-firm employment relationship is hit by an exogenous shock and the worker's reference wage evolves over time according to an asymmetric partial adaptation rule. Before doing so, the remainder of the section will first derive the optimal wage setting policy of the firm, and then will characterise the steady-state and transitional dynamics properties of the model.

Consider a representative employment relationship between a worker and a firm which starts at some initial period τ. The firm's and worker's payoffs and the structure of the wage setting environment are identical to those formalised in Chapter 1, extended to an infinite horizon without uncertainty as in Section 2.2, Chapter 2. Assumptions W1-W4 and F1-F2 of Chapter 2 hold throughout.

Asymmetric partial adaptation

To study the implications of asymmetric adaptation on wage and reciprocity dynamics it is assumed that:

A1. $\forall t \geq \tau$,

$$r_t = \mathbb{A}^+ \left[(1-\alpha^+)r_{t-1} + \alpha^+ w_{t-1}\right] + \mathbb{A}^- \left[(1-\alpha^-)r_{t-1} + \alpha^- w_{t-1}\right], \qquad (3.16)$$

r_τ given, where $\alpha^+ > \alpha^-$; $\alpha^{+,-} \in (0, 1)$; and

$$\mathbb{A}^+ = \begin{cases} 1 & \text{if } w_{t-1} \geq r_{t-1} \\ 0 & \text{otherwise,} \end{cases} \qquad \mathbb{A}^- = \begin{cases} 1 & \text{if } w_{t-1} < r_{t-1} \\ 0 & \text{otherwise.} \end{cases}$$

Assumption A1 implies that for any given initial reference wage r_τ at the start of the employment relationship, reference wage formation follows an *asymmetric partial adaptation* rule: the worker will adapt their reference wage more rapidly to wage gifts than to unfair wages over the course of the employment relationship. Note that the asymmetric adaptation rule given by (3.16) is a special case of the more general rule (3.11) formalised in Section 3.4.2, where in this analysis the entitlement determinant x_t is given by the worker's most recent wage contract, i.e. $x_t = w_{t-1}$.

Before proceeding, it is worth clarifying some terminology that it will be used henceforth. Recall that a higher speed of adaptation α implies that the worker places more weight on the most recent wage contract w_{t-1} over the most recent reference wage r_{t-1} in their reference wage formation process. In addition, it has been established by Proposition 20, Section 3.4.2, that if the relevant entitlement determinant w_t eventually converges to a steady state w^* (as it will be shown in this section), the speed of adaptation α also determines the speed of convergence of the reference wage r_t to w^*. As such it is possible to deduce that in a worker-firm employment relationship converging to a steady state, a higher (lower) speed of adaptation is equivalent to a "faster" ("slower") dynamic adaptation of the reference wage throughout the course of the employment relationship.

Optimal wage setting under asymmetric partial adaptation

This section characterises the firm's optimal wage setting policy when the worker's reference wage adapts over time according to (3.16). Since in each employment period $t \geq \tau$ the worker maximises their utility for any given sequence of wage offers relative to their reference wage, Assumption A1 does not influence the characterisation of the worker's optimal choice of effort:

$$\tilde{e}_t = \tilde{e}(w_t, r_t, \lambda) = \tilde{e}_n + \mu(\ln w_t - \ln r_t). \tag{3.17}$$

As such, let $r_{t+1} = g(r_t, w_t, \alpha)$ be given by (3.16), the functional equation corresponding to the firm's maximisation problem can be written as

$$J(r) = \max_{w \geq 0} \; \{pq\tilde{e}(w, r, \lambda) - w + \psi J(g(r, w, \alpha))\}. \tag{3.18}$$

The first-order necessary and sufficient condition characterising the optimal wage policy $\tilde{w}_t = \tilde{w}(r_t, \alpha, \lambda)$ for all $\alpha = \{\alpha^-, \alpha^+\}$ and $w \neq r$ is given by

$$\Upsilon(w; r, \alpha, \lambda) \equiv pq\frac{\partial \tilde{e}(w, r, \lambda)}{\partial w} - 1 + \psi pq\frac{\partial \tilde{e}(w', r', \lambda)}{\partial r}\frac{\partial g(w, r, \alpha)}{\partial w} = 0, \tag{3.19}$$

which captures the (familiar) inter-temporal trade-off between the marginal benefit of inducing higher effort today *versus* the marginal cost of paying a higher wage today and employing a worker with a higher reference wage in the future.[10]

Proposition 21. *For all $t \geq \tau$ and for any given worker's reference wage r_t, the optimal wage policy of a firm employing a worker characterised by asymmetric reference-dependent reciprocity* (3.17) *with $\lambda > 1$ and asymmetric partial adaptation* (3.16)*, is given by*

$$\tilde{w}_t = \tilde{w}(r_t, \alpha, \lambda) = \begin{cases} \tilde{w}(r_t, \alpha^+)^+ & \text{if } r_t < r_L(\alpha^+) \\ r_t & \text{if } r_t \in [r_L, r_H] \\ \tilde{w}(r_t, \alpha^-, \lambda)^- & \text{if } r_t > r_H(\alpha^-, \lambda) \end{cases} \tag{3.20}$$

where

$$r_L(\alpha^+) \equiv \left\{ r : \lim_{\epsilon \to 0} \Upsilon(r + \epsilon; r, \alpha^+, \lambda) = 0 \right\}$$
$$r_H(\alpha^-, \lambda) \equiv \left\{ r : \lim_{\epsilon \to 0} \Upsilon(r - \epsilon; r, \alpha^-, \lambda) = 0 \right\}.$$

The optimal wage policy $\tilde{w}(r_t, \alpha^+)^+ > r_t$ is implicitly defined by $\Upsilon(w; r, \alpha, \lambda) = 0$ in which $\tilde{e}(w, r, \lambda) = \tilde{e}(w, r)^+$ and $\alpha = \alpha^+$; and $\tilde{w}(r_t, \alpha^-, \lambda)^- < r_t$ is implicitly defined by $\Upsilon(w; r, \alpha, \lambda) = 0$ in which $\tilde{e}(w, r, \lambda) = \tilde{e}(w, r, \lambda)^-$ and $\alpha = \alpha^-$. Moreover,

a) for all $r_t \in [r_L, r_H]$, $\tilde{w}(r_t, \alpha, \lambda)$ is increasing and linear in r; and for all other $r_t \notin [r_L, r_H]$, $\tilde{w}(r_t, \alpha, \lambda)$ is increasing and concave in r;

b) for all $r_t < r_L(\alpha^+)$ and $r_t > r_H(\alpha^-, \lambda)$, $\tilde{w}(r_t, \alpha, \lambda)$ is decreasing in α^+ and α^- respectively;

c) for all $r_t > r_H(\alpha^-, \lambda)$, $\tilde{w}(r_t, \alpha, \lambda)$ is increasing in λ.

Proof. See Appendix B.3. ∎

The firm's optimal wage policy under asymmetric partial adaptation maintains the same trigger-policy structure and main properties of the dynamic wage setting policies analysed in Chapters 1 and 2. However it also displays two additional features. First, in

[10]This condition is the infinite-horizon analog of the first-order conditions (1.10), Chapter 1, and (2.6), Chapter 2, subject to asymmetric partial adaptation of the reference wage.

part a), it is established that even if the firm's output is linear in effort (Assumption F1, Chapter 2), whenever the worker is paid a wage above or below their reference wage, the optimal wage policy is increasing in the reference wage, but at a decreasing rate. This non-linear relationship is due to the inter-temporal trade-off captured by (3.19), coupled with the partial adaptation mechanism of the reference wage.

Second, in part b), it is established that if the worker's speed of adaptation is higher, the firm will set a lower wage, independently of whether this is perceived by the worker as a gift or as unfair. The intuition behind this result is the following. A higher speed of adaptation implies that the weight on the entitlement effect of any wage set in period t will be larger in the worker's reference wage in $t + 1$. Hence, part of the positive reciprocity induced by a wage gift in the current period will disappear in the future, all else equal, due to the worker feeling entitled to be paid a relatively higher wage; similarly, part of the negative reciprocity triggered by the current wage being perceived as unfair will disappear, since the worker will feel entitled to a relatively lower wage in the future than the one in the current period. In the optimal inter-temporal trade-off in period t between the current marginal benefit of inducing higher positive reciprocity—or, of partially offsetting negative reciprocity—and the expected future marginal cost of employing a worker with a higher reference wage, a higher speed of adaptation increases the weight of the latter/future effect, giving to the firm an incentive to set a lower wage in period t. As such, this result stands as a generalised explanation for the firm's wage compression incentive analysed and discussed in Chapter 1. The optimal wage setting policy (3.20) is illustrated in Figure 3.1.

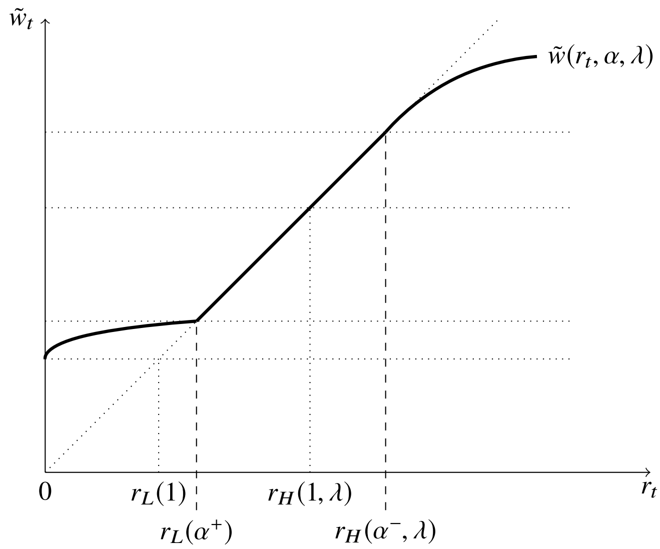

Figure 3.1:
Optimal Wage Setting Policy

Moreover notice that since adaptation is asymmetric, the incentive to compress the wage will be lower whenever the optimal policy requires to set a wage below the worker's reference wage. In fact in this situation a lower speed of adaptation implies that the negative reciprocity induced by an unfair wage will be more persistent than the positive reciprocity induced by a wage gift, as the weight on the entitlement effect of an unfair wage is smaller and the worker's adaptation to it will be slower. Hence, the expected indirect marginal cost of employing a worker with a relatively higher reference wage in the future due to adaptation is less important in the firm's inter-temporal trade-off, augmenting the incentive to set a higher wage to partially offset the current direct effect of negative reciprocity.

Equilibrium analysis: steady state and transitional dynamics

The following analysis will characterise the steady-state equilibrium and the transitional dynamic properties of the model. These will in turn be useful to discuss the main dynamic implications of reciprocity and wage setting behaviour in the following section.

To begin with, consider the equilibrium laws of motion of the worker's reference wage (the state variable) and optimal effort:

$$r_{t+1} = (1-\alpha)r_t + \alpha \tilde{w}_t(r_t), \quad r_\tau \text{ given,} \tag{3.21}$$

$$\tilde{e}_{t+1} = \tilde{e}_t + \mu(\ln \tilde{w}_{t+1}(r_{t+1}) - \ln r_{t+1}) - \mu(\ln \tilde{w}_t(r_t) - \ln r_t), \quad r_\tau \text{ given;} \tag{3.22}$$

where equation (3.22) is a convenient way of expressing employed workers' optimal effort as a first-order difference equation (some of the functions' arguments have been omitted to ease notation). The equilibrium path of the worker-firm employment relationship can be defined as follows.

Definition 5. *Given an initial worker's reference wage r_τ, an equilibrium path for a worker-firm employment relationship starting at $t = \tau$ is a sequence of wages, reference wages and effort levels $\{w_t, r_t, e_t\}_{t=\tau}^{\infty}$ such that the firm maximizes the present discounted value of profit as formalised by the functional equation* (3.18)*; the employed worker's reference wage evolves according to* (3.16)*; and the worker's effort is given by* (3.17).

First, consider the steady states of the model.

Definition 6. *A steady-state equilibrium is an equilibrium path $\mathcal{E} \equiv \{w_t, r_t, e_t\}$ in which $w_t = \tilde{w}^*$, $r_t = r^*$, $e_t = \tilde{e}^*$ for all t.*

Proposition 22. *In a worker-firm employment relationship where the worker is characterised by asymmetric reference-dependent reciprocity* (3.17) *with $\lambda > 1$ and asymmetric partial adaptation* (3.16)*; and the firm sets the wage according to* (3.20)*; there* *exists a range of steady-state equilibrium wages and reference wages*

$$\tilde{w}^* = \tilde{w}^*(r^*, \alpha, \lambda) = r^*$$

given by

$$\tilde{w}^* = \tilde{w}^*(r^*, \alpha, \lambda) = \begin{cases} pq(1-\psi\alpha^+) & \equiv \tilde{w}_L(\alpha^+)^+ & \textit{if } r_\tau < r_L(\alpha^+) \\ r^* & \equiv \tilde{w}_M^= & \textit{if } r_\tau \in [r_L, r_H] \\ \lambda pq(1-\psi\alpha^-) & \equiv \tilde{w}_H(\alpha^-, \lambda)^- & \textit{if } r_\tau > r_H(\alpha^-, \lambda) \end{cases} \quad (3.23)$$

in which $\tilde{w}_L(\alpha^+)^+ = r_L(\alpha^+)$, $\tilde{w}_M^= = r_\tau$, $\tilde{w}_H(\alpha^-, \lambda)^- = r_H(\alpha^-, \lambda)$*; and a unique steady-state level of effort*

$$\tilde{e}^*(\tilde{w}^*, r^*, \lambda) = \tilde{e}_n \quad \forall r_\tau.$$

Proof. See Appendix B.3. ■

Proposition 22 establishes the existence of a range of steady-state equilibrium levels of wages and reference wages, and a unique steady-state level of effort. The main reasons behind this result are the same as those described in the steady-state characterisation of Section 2.4, Chapter 2. However there is one important difference generated by the presence of asymmetric partial adaptation.

First notice that the equilibrium levels of the wage and reference wage, defining the boundaries of the range of equilibria, depend explicitly on the speed of adaptation $\alpha = \{\alpha^-, \alpha^+\}$. That is, the higher the speed of adaptation, the lower the wage paid to the employed worker in the steady state. The intuition behind this prediction relies on the wage compression incentive described above, which is driven by the worker dynamic re-normalisation of effort. A lower speed of adaptation gives more time to the firm to exploit the worker's reciprocity by setting relatively higher wages in every period (at a decreasing rate). However, since effort will eventually reach its normal level in the steady state, a firm that has exploited this slower adaptation, by setting relatively higher wages, will also end up paying a higher steady-state wage.

By comparing this result with the one established in Proposition 7, Chapter 2, in which adaptation is both symmetric and complete, i.e. $\alpha^- = \alpha^+ = \alpha = 1$, it is possible to deduce without any additional proof that partial adaptation raises the equilibrium wage level in the market for any given initial reference wage r_τ. In addition, since adaptation is also asymmetric, the asymmetry in the worker's reciprocity due to loss aversion (i.e. $\lambda > 1$) is no longer a necessary condition for the existence of a range of steady-state equilibria in which $\tilde{w}^* = r_\tau$. In fact, even if $\lambda = 1$, from the results established in Propositions 21 and 22 it can be deduced that a range of equilibria will exist so long as $\alpha^+ > \alpha^-$, since $r_L(\alpha^+) < r_H(\alpha^-, 1)$. It is therefore possible to conclude that asymmetric partial adaptation has implications for the steady-state equilibrium levels of the wage and reference wage in worker-firm employment relationships.

Next consider the equilibrium transitional dynamics of the model.

Proposition 23. *In a worker-firm employment relationship where the worker is characterised by asymmetric reference-dependent reciprocity* (3.17) *with* $\lambda > 1$ *and asymmetric partial adaptation* (3.16)*; and the firm sets the wage according to* (3.20)*; the range of steady-state equilibria established by Proposition 22 are locally asymptotically stable, and starting from any*

a) $r_\tau < r_L(\alpha^+)$, *then* $r_t \nearrow r^* = r_L(\alpha)$, $\tilde{w}_t \nearrow \tilde{w}^* = \tilde{w}_L(\alpha^+)^+ = r^*$, *and* $\tilde{e}_t \searrow \tilde{e}_n$*;*

b) $r_\tau \in [r_L, r_H]$, *then* $\tilde{w}_t = \tilde{w}^* = \tilde{w}_M^= = r^* = r_\tau$, *and* $\tilde{e}_t = \tilde{e}_n$*;*

c) $r_\tau > r_H(\alpha^-, \lambda)$, *then* $r_t \searrow r^* = r_H(\alpha, \lambda)$, $\tilde{w}_t \searrow \tilde{w}^* = \tilde{w}_L(\alpha^-, \lambda)^- = r^*$, *and* $\tilde{e}_t \nearrow \tilde{e}_n$.

Proof. See Appendix B.3. ■

Proposition 23 establishes the existence of multiple equilibrium paths for the optimal wage, effort and reference wage in a worker-firm employment relationship. In line with the main prediction of Proposition 12, Chapter 2, which equilibrium path will characterise the transitional dynamics and the resulting steady-state level of the wage and reference wage crucially depends on the initial value of r_τ. The following figure illustrates the transitional dynamics of a worker's reference wage for the two possible equilibrium paths a) and c).

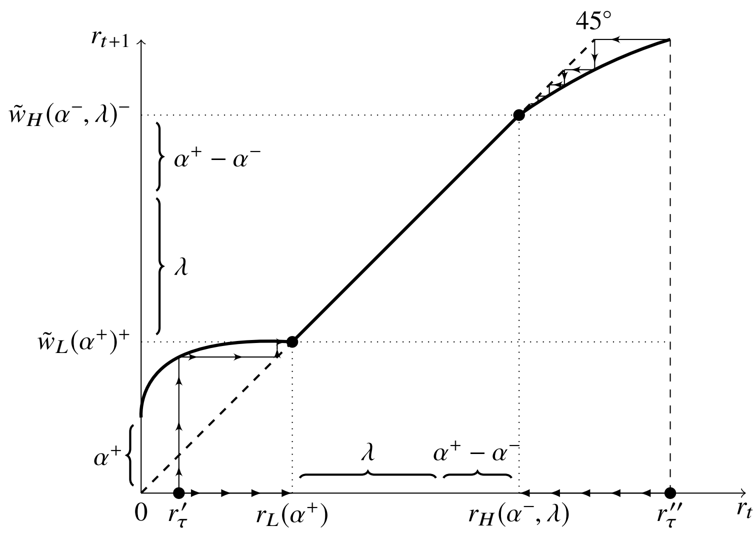

Figure 3.2:
Range of Equilibria and Transitional Dynamics of the Reference Wage

However, in contrast with the results established in Chapter 2, asymmetric partial adaptation implies that the convergence to the steady state will take longer than one employment period only; and that the speed of this convergence, determined by $\alpha = \{\alpha^-, \alpha^+\}$, will be different depending on whether the worker is paid a wage gift or an unfair wage in their initial employment period $t = \tau$.

If the worker starts the job with a relatively low reference wage (e.g. $r'_\tau < r_L(\alpha^+)$), hence perceiving the wage contract as a gift, they will adapt more rapidly to it in the following employment period. As noted before, this higher speed of adaptation implies that the firm's optimal wage policy will be more compressed, and that the steady-state wage will be relatively lower. This result is reinforced by the prediction that due to the higher speed of adaptation, the worker's effort will converge to its normal equilibrium level more rapidly. Hence, the firm will be able to exploit the benefits of positive reciprocity, by implementing a series of wage rises over time, but only over a relatively shorter time span. On the other hand, if the worker feels entitled to a relatively high wage at the start of the employment relationship (e.g. $r''_\tau > r_H(\alpha^-, \lambda)$), hence they perceive the initial wage contract as unfair, the dynamic adaptation will be slower; the firm's optimal wage policy less compressed; and the steady-state wage relatively higher. In fact, due to the lower speed of adaptation it will take longer for the worker's effort to converge to its normal level, implying that negative reciprocity will be more persistent throughout the employment relationship. In this case, the firm will optimally reduce the worker's wage entitlement and their negative reciprocity over time by implementing a series of wage cuts, but this process will take longer.

Asymmetric endogenous persistence in reciprocity and wage dynamics

This section builds on the results established in the preceding analysis to illustrate the out-of-steady-state dynamics of the model in response to unanticipated parametric shifts in productivity p. The focus of the analysis will be on the different equilibrium adjustment paths of the wage, reference wage and effort when a firm optimally changes the wage in response to positive and negative productivity shocks of the same magnitude.

Consider a worker-firm employment relationship in which $r_\tau < r_L(\alpha^+)$, characterised by the steady state:

$$\widetilde{\mathcal{E}} \equiv \{\tilde{w}^*, r^*, \tilde{e}^*\}, \quad \text{in which} \quad \tilde{w}^* = \tilde{w}_L(\alpha^+)^+, \quad r^* = r_L(\alpha^+), \quad \text{and} \quad \tilde{e}^* = \tilde{e}_n;$$

where as previously established $\tilde{w}^* = r^* = pq(1 - \psi\alpha^+)$. Then suppose that in some period t_0 there is an unanticipated permanent shock ε_{t_0} to productivity p, which shifts toward a new level denoted by $\hat{p} = p \pm \varepsilon_{t_0}$ (i.e. the shock is symmetric).

The following analysis will consider both a positive and a negative shock. Moreover, for illustrative purposes, it will be assumed that ε_{t_0} is large enough so that the model does not generate endogenous downward wage rigidity in response to negative shocks. Imposing this assumption enables the reference wage and effort dynamics to be compared when a firm optimally changes the wage in response to both positive and negative shocks, which obviously will not be possible if there is downward wage rigidity.[11]

To begin with, consider the new steady-state equilibria towards which the employment relationship will converge. If the shock is positive, that is if $\hat{p} = p + \varepsilon_{t_0}$, the employment relationship will be characterised by the steady state:

$$\widehat{\mathcal{E}}^{+} \equiv \{\hat{w}^{*}, \hat{r}^{*}, \hat{e}^{*}\}, \quad \text{in which} \quad \hat{w}^{*} = \hat{w}_{L}(\alpha^{+})^{+}, \quad \hat{r}^{*}, = \hat{r}_{L}(\alpha^{+}) \quad \text{and} \quad \hat{e}^{*} = \tilde{e}_{n};$$

where $\hat{w}^{*} = \hat{r}^{*} = \hat{p}q(1 - \psi\alpha^{+})$. On the other hand, if the shock is negative, that is if $\hat{p} = p - \varepsilon_{t_0}$, the steady-state equilibrium will be:

$$\widehat{\mathcal{E}}^{-} \equiv \{\hat{w}^{*}, \hat{r}^{*}, \hat{e}^{*}\}, \quad \text{in which} \quad \hat{w}^{*} = \hat{w}_{H}(\alpha^{-}, \lambda)^{-}, \quad \hat{r}^{*} = \hat{r}_{H}(\alpha^{-}, \lambda), \quad \text{and} \quad \hat{e}^{*} = \tilde{e}_{n};$$

where $\hat{w}^{*} = \hat{r}^{*} = \lambda\hat{p}q(1 - \psi\alpha^{-})$. What are the implications of asymmetric partial adaptation for the adjustment dynamics of the wage, reference wage and effort?

In the case of a positive shock, at impact the firm implements a wage raise which is perceived as a gift by the worker, whom positively reciprocates by exerting supra-normal effort:

$$t_0 : \quad \hat{w}_{t_0}(r^{*}, \alpha^{+})^{+} > r^{*}, \quad \text{and} \quad \hat{e}^{+}_{t_0} > \tilde{e}_{n}, \quad \text{if} \quad \hat{p} = p + \varepsilon_{t_0}.$$

In the subsequent employment periods the worker will feel entitled to be paid a higher wage due to the dynamic adaptation of the reference wage, while the firm will continue to implement a series of wage raises that are perceived as gifts, optimally exploiting the worker's positive reciprocity over time. However, following the dynamic properties of the wage and reference wage established in the preceding section, this combined dynamics of wage gifts and adaptation will eventually terminate at the new steady-state equilibrium $\widehat{\mathcal{E}}^{+}$, which is reached in some period t'. Hence, in the employment periods after t_0 the worker's perceived gain in utility generated by the series of wage rises will gradually decrease over time due to the dynamic adaptation of the reference wage, until it will eventually disappear in the steady state. As a consequence, the worker's positive

[11] The dynamic implications of reciprocity and wage setting behaviour when there is downward wage rigidity have been largely analysed in Chapter 1. For details of the condition that the magnitude of the shock needs to satisfy for this assumption to hold, see Section A.3.1, Appendix A.3.

reciprocity will also decrease over time, until effort eventually converges to its normal level when the new steady state is reached in period t'. The speed of convergence to $\widehat{\mathcal{E}}^+$ is determined by the worker's speed of adaptation α^+.

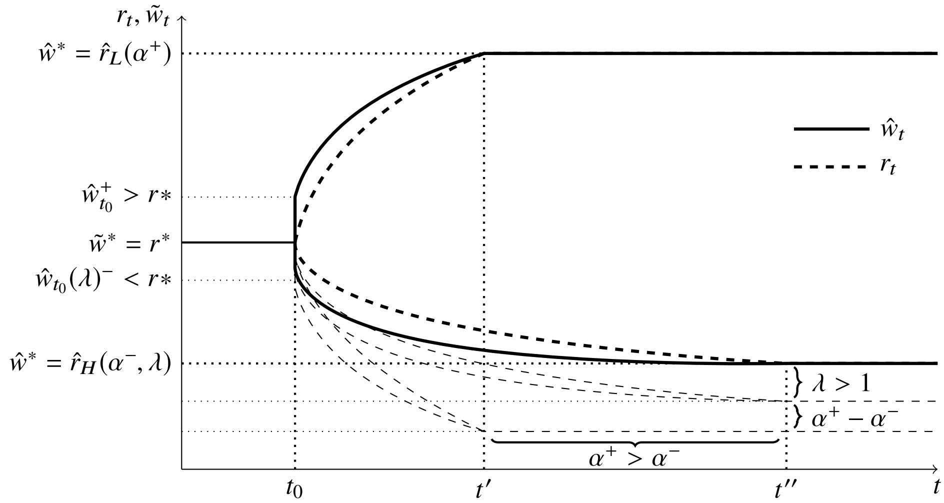

Figure 3.3:
Asymmetric Intensity and Persistence in Wage and Reference Wage Dynamics

In the case of a negative shock, at impact the firm implements a wage cut which is perceived as unfair by the worker, whom negatively reciprocates by exerting sub-normal effort:

$$t_0: \quad \hat{w}_{t_0}(r^*, \alpha^-, \lambda)^- < r^*, \quad \text{and} \quad \hat{e}_{t_0}(\lambda)^- < \tilde{e}_n, \quad \text{if} \quad \hat{p} = p - \varepsilon_{t_0}.$$

In the subsequent employment periods the worker will gradually adapt their entitlement to a lower wage while the firm will continue to implement a series of unfair wage cuts in order to optimally balance the inter-temporal trade-off between reducing current negative reciprocity and employing a worker with a lower reference wage in the future. Hence, in this case, optimal wage setting in response to a negative shock triggers the combined dynamics of unfair wage cuts and adaptation, which will eventually terminate at the new steady-state equilibrium $\widehat{\mathcal{E}}^-$, reached in some period t''. In fact, in the employment periods after t_0 the worker's perceived loss in utility, generated by the series of unfair wage cuts, will decrease over time due to reference wage adaptation, and will disappear in the steady state. The worker's negative reciprocity will also decrease over time, with effort converging back to its normal level when the new steady state is reached in period t''. In this case, the speed of convergence to $\widehat{\mathcal{E}}^-$ is determined by the worker's speed of adaptation α^-. The adjustment dynamics of the wage and the reference wage just described are illustrated in Figure 3.3; while the corresponding effort dynamics are illustrated in Figure 3.4.

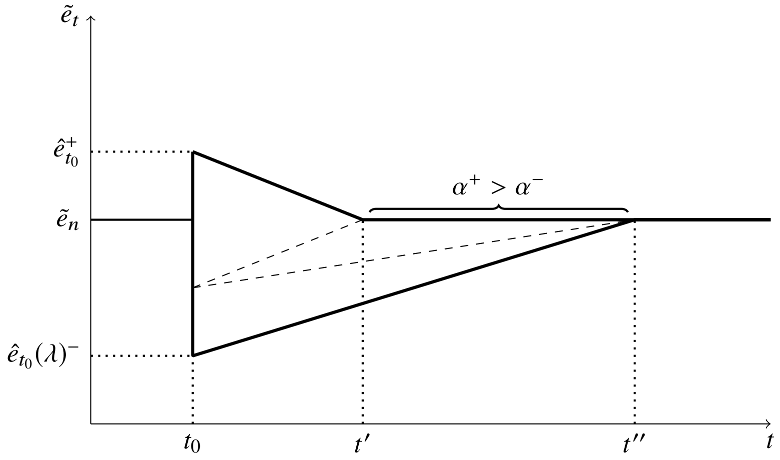

Figure 3.4:
Asymmetric Intensity and Persistence in Reciprocity Dynamics

Based on this analysis, the model developed in this section enables to highlight several noteworthy features of optimal wage setting behaviour and reciprocity dynamics in the presence of asymmetric partial adaptation of the reference wage.

First note that even though the shock is characterised by a one-off permanent change in productivity (i.e. no persistence of the shock ε_{t_0}), the model endogenously generates *persistence* in wage and effort dynamics. This is driven by the assumption that reference wage adaptation is partial, i.e. $\alpha^+, \alpha^- \in (0,1)$. Second, note that even if the shock is symmetric, this persistence is asymmetric: the wage and effort dynamics generated by a negative shock persists longer than the one generated by a positive shock of the same magnitude, that is $t'' > t'$. This is driven by the assumption that reference wage adaptation is asymmetric, i.e. $\alpha^+ > \alpha^-$. As such, the theoretical analysis presented in this section provides a rationale for why negative reciprocity is more persistent than positive reciprocity. Finally, in addition to asymmetric persistence, the model also generates an asymmetry in the *intensity* of wage and effort adjustments at impact in period t_0. As illustrated in Figure 3.3, the optimal wage adjustment to a negative productivity shock is relatively more muted, in absolute terms, than the wage adjustment to a positive productivity shock of the same magnitude. This result is driven by two main behavioural factors: i) the firm has an incentive to set a relatively higher wage than otherwise, in order to partially offset the worker's stronger negative reciprocity response to an unfair wage cut (since $\lambda > 1$, see Chapter 1); ii) such incentive is reinforced by the slower reference wage adaptation to this relatively higher wage by the worker (since $\alpha^- < \alpha^+$, see discussion above).

Moreover note that the intensity of the worker's effort response could also be asymmetric at impact. However, this will depend on the following condition:

$$\frac{|\hat{e}_{t_0}(\lambda)^{-} - \tilde{e}_n|}{|\hat{e}^{+}_{t_0} - \tilde{e}_n|} > 1 \quad \text{only if} \quad \frac{|\ln w_{t_0}(\lambda)^{-} - \ln r^*|}{|\ln w^{+}_{t_0} - \ln r^*|} > \frac{1}{\lambda}.$$

Hence, negative reciprocity will be stronger than positive reciprocity following a symmetric productivity shock only if the unfair wage cut in period t_0 is sufficiently large. This particular case corresponds to the one illustrated in Figure 3.4. It is worth noting, however, that for wage raises and wage cuts of the same absolute magnitude in t_0, this condition is always satisfied (since $\lambda > 1$). As such the model predicts stronger intensity (loss aversion) and persistence (asymmetric adaptation) of negative reciprocity relative to positive reciprocity.

Summarising discussion

This section analysed the implications of asymmetric partial adaptation of the reference wage for wage and effort dynamics, in a microeconomic model of wage setting behaviour formalised on the basis of the theory developed in Chapter 1. Asymmetric partial adaptation has implications for both the steady-state equilibrium of the model and the out-of-steady-state dynamic adjustment to exogenous shocks.

In terms of steady states, it has been shown that if adaptation is partial, the equilibrium wage paid to the worker will be higher, relative to the case in which it takes just one employment period for workers to adapt their wage entitlements (e.g. as in Chapter 2). In fact, a slower speed of adaptation implies a higher steady-state equilibrium wage. This conclusion relies on the firm's optimal wage setting behaviour facing an inter-temporal trade-off between triggering higher effort today *versus* employing a worker with a higher reference wage in the future. Since a slower speed of adaptation eases the future entitlement effect, the firm has an incentive to set a relatively higher wage in each employment period to induce higher effort from the worker.

In terms of out-of-steady-state dynamics, the theory developed in this section presents several important predictions. First, the model provides a behavioural explanation for the observed persistence of wage dynamics, as well as providing a rationale for why the response of wages and reciprocity to negative shocks might be more persistent. This rationale is based on the principle of hedonic asymmetry applied to perceptions of fairness in the context of wage setting behaviour and employment relationships. Since it takes longer for workers to adapt to wage cuts that are perceived as unfair—which in addition have a stronger effect on workers' morale and effort—firms that need to adjust their labour costs in the face of negative shocks will optimally implement a series of small wage cuts over a prolonged time span, rather than impose a large and unique wage reduction. This result is based on the assumption of firms having complete and perfect information—i.e. the firm knows that $\lambda > 1$ and that $\alpha^+ > \alpha^-$. However, since in some employment relationships this assumption may be unrealistic, nevertheless the above prediction has a *normative* interpretation. If asymmetric partial adaptation is a good description of the way in which workers adapt their wage entitlements over time, then firms will be better off by implementing a series of small wage cuts over a longer time period when they face the incentive to do so, rather then imposing a one-off large cut. The model of this section lends itself as a useful, tractable, framework for the analysis of these issues.

Another set of key predictions concern the dynamics of a worker's reciprocity facing a series of wage changes over time. First of all it has been shown that even in the presence of a more general adaptation rule, a worker's reciprocity response to wage changes is only a temporary phenomenon: any increase (decrease) in effort triggered by a wage raise (cut) that is perceived as a gift (unfair) will eventually disappear over time, due to the worker feeling entitled to the wage they are paid. Hence, the model preserves the key distinctive property of the theory set out in Chapter 1, namely the dynamic re-normalisation of effort. This prediction is consistent with evidence on the wage-effort relationship coming from a variety of empirical approaches, which have been extensively discussed throughout the thesis; and is in stark contrast with theories of reciprocity in labour markets in which workers' effort in the steady state is a function of absolute wage levels. Moreover, the present analysis has enabled the exploration of two additional features of the dynamics of reciprocity. If adaptation is partial, the dynamic re-normalisation of effort will not be immediate: the persistence of any extent of positive or negative reciprocity strictly depends on the worker's speed of adaptation $\alpha = \{\alpha^-, \alpha^+\} \in (0, 1)$. Moreover, if adaptation is asymmetric, the theory predicts that negative reciprocity will be more persistent than positive reciprocity. As such, the analysis developed in this section combined with the key insights of the theory developed in Chapter 1 provide a unified and coherent explanation for the conjectured asymmetry in *intensity* and *persistence* of negative *versus* positive reciprocity: negative reciprocity is stronger than positive reciprocity due to workers' loss aversion; while it is more persistent due to the slower adaptation to wage cuts that are perceived as unfair.[12]

[12]This conjecture stems from the evidence reported by a variety of empirical approaches, including surveys to compensation mangers (e.g. Bewley (1999)) and laboratory and field experiments (e.g. Cohn et al. (2014)). See also the discussion of Section 1.2, Chapter 1, and the references provided therein.

3.5.2 Social Comparison and Wage and Unemployment Hysteresis

This section illustrates how the general framework developed in Section 3.4 can be incorporated into the theoretical framework set out in Chapter 2. In so doing, the following analysis will consider the effects of relative wage comparisons between newly hired and incumbent workers on the cyclical behaviour of the model, and will show that this form of social comparison generates asymmetric fluctuations and hysteresis in wages, job creation and unemployment.

The section is organised as follows. First, the concept of hysteresis is defined. Then, the relevant social categories and fair wage norms characterising a stylised labour market are designed and subsequently the steady-state equilibrium of the labour market is derived. Finally, the cyclical behaviour of the model is analysed, by considering the impact of two unanticipated permanent shocks to productivity of opposite sign representing an economic cycle.

Hysteresis defined

In the context of this analysis a dynamic system is said to be hysteretic whenever temporary disturbances have permanent equilibrium effects. Based on this loose definition, hysteresis can be defined more formally following the general theory of systems with hysteresis (see, for instance, Cross, Grinfeld, and Lamba (2009)).

Consider an input-output system that is in a steady-state equilibrium in period t^*. The system has scalar input $p^* = p_{t^*}$ and scalar output $n^* = n_{t^*}$. Then suppose that the input p_t changes from p_{t^*} to some value $p' = p_{t'}$ and then back to p_{t^*} (hence, in an economic context, suppose that p_{t^*} undergoes a symmetric cycle characterised by an expansion and a recession phase of the same magnitude). The system can be defined *hysteretic* if for each p_{t^*} there are values $p_{t'}$, such that, after the excursion (i.e. the cycle), the output n_t does not return to n_{t^*}, but to some different value $n_{t''}$. This phenomenon is known as *remanence*. To return the output to its original value $n^* = n_{t^*}$, the input needs to be changed by an additional amount called *coercive force*. These concepts are illustrated in Figure 3.5 below.

Notice that this definition is different from the one most commonly used in economics, according to which hysteresis refers to the substantial persistence of deviations from equilibrium after the impact of a shock, i.e. the presence of a unit root in a linear dynamic system (as, for instance, in the models of unemployment hysteresis developed by Blanchard and Summers (1986) and Skott (2005)). Reviews of the concept of hysteresis and its application in economics can be found in Røed (1997), Göcke (2002), Cross et al. (2009) and Cross (2014).

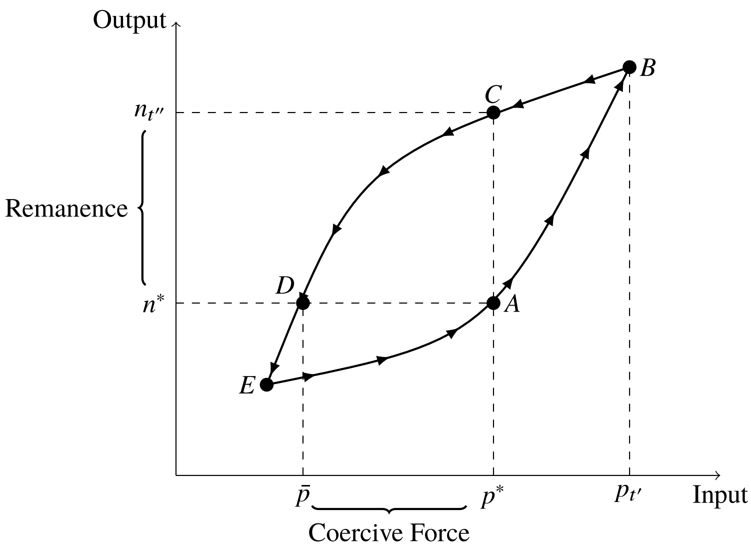

Figure 3.5:
A Hysteresis Loop

Labour market environment: social categories and norms

Consider the benchmark search and matching framework developed in Chapter 2, in which wage setting behaviour is modelled following the theory developed in Chapter 1. Assumptions W1-W4, F1-F2 and M1 of Chapter 2 hold throughout.

Building on the general framework developed and formalised in Section 3.4, it is assumed that in each period t workers can be characterised by the following social categories and related fair wage norms:

S1. Social Categories:

$$C = \{\text{unemployed, new hire, incumbent, social comparison}\}.$$

S2. Fair Wage Norms:

$$\begin{aligned} \text{unemployed} \equiv U, &\quad \mathcal{P}_U = \emptyset; \\ \text{new hire} \equiv N, &\quad \mathcal{P}_N = \{\underline{\omega}\}; \\ \text{incumbent} \equiv E, &\quad \mathcal{P}_E = \{w_{-1}\}; \\ \text{social comparison} \equiv S, &\quad \mathcal{P}_S = \{w^{-i}\}; \end{aligned}$$

where

$$\Omega = \{\underline{\omega}, w_{-1}, w^{-i}\} \quad \text{with} \quad \underline{\omega}, w_{-1}, w^{-i} \in \mathcal{X}.$$

According to Assumption S1, this environment can be considered as a stylised labour market in which workers differ on the basis of their employment status, (as in Chapter 2) and in which the social category S is representative of any social or institutional category that fosters relative wage comparison between workers.[13] In addition, Assumption S2 specifies the corresponding sets of fair wage norms influencing the reference wage of a typical worker belonging to each c in C where: $\underline{\omega}$ is the minimum legal wage in the market, and it is assumed to form an entitlement determinant in the reference wage of a typical new hire $c = N$; w_{-1} is the wage contract paid in the previous employment period and it is assumed to form an entitlement determinant in the reference wage formation of a typical incumbent $c = E$; while w^{-i} is the wage paid to other workers in the market, which forms an entitlement determinant in the reference wage of a typical worker belonging to the social category $c = S$.

In line with this set up, consider the following assumption about the evolution of a representative worker's identity and related fair wage norms over time (recall that τ denotes the starting period of an employment relationship):

S3. Identities:

$$\begin{array}{lll} \text{if } t < \tau & C_t = \{U\} & \mathcal{I}_t = \emptyset; \\ \text{if } t = \tau & C_t = \{N, S\} & \mathcal{I}_t = \{\underline{\omega}, w_t^{-i}\}; \\ \text{if } t > \tau & C_t = \{E\} & \mathcal{I}_t = \{w_{t-1}\}. \end{array}$$

Assumption S3 formalises the premise which motivated the analysis of this section. That is, relative wage comparisons affect the reference wage formation process of newly hired workers only—along with the minimum legal wage in the market—while incumbent workers use the most recent wage contract as their unique entitlement determinant. Hence, reference wages evolve over time according to the following rule:

S4. Reference Wage Formation:

$$r_t(\mathcal{I}_t) = \begin{cases} \underline{\omega}\mathbb{C}^+ + w_t^{-i}\mathbb{C}^- & \text{if } t = \tau \\ w_{t-1} & \text{if } t > \tau \end{cases} \tag{3.24}$$

where

$$\mathbb{C}^+ = \begin{cases} 1 & \text{if } \underline{\omega} > w_t^{-i} \\ 0 & \text{otherwise,} \end{cases} \qquad \mathbb{C}^- = \begin{cases} 1 & \text{if } \underline{\omega} \leq w_t^{-i} \\ 0 & \text{otherwise.} \end{cases}$$

Note that the reference wage equation (3.24) also implies an asymmetric social comparison rule, that is, newly hired workers' reference wage in period $t = \tau$ will be determined by the wage paid to other employed workers w_t^{-i} only if this wage exceeds

[13] For instance the social category S could be representative of workers applying for jobs in a unionised industry which discloses information about wages across firms; or equivalently S could represent the social category assigned to workers employed in firms featuring standardised internal pay structure procedures.

the current minimum legal wage in the market $\underline{\omega}$.[14]

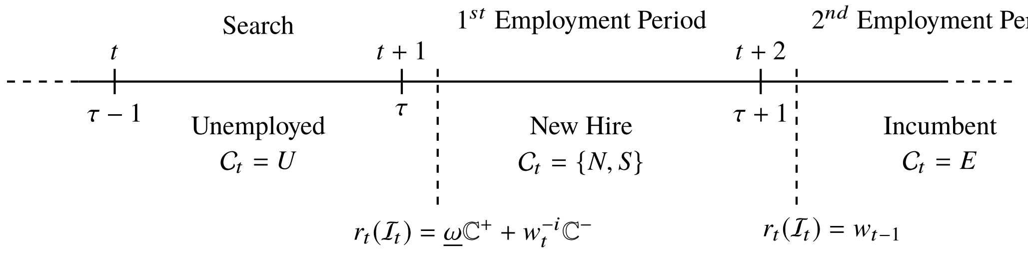

Figure 3.6:
The Evolution of Identity and Reference Wages

Assumptions S1-S4 entirely characterise the the social and institutional context, and the resulting reference wage formation process, of the stylised labour market that is analysed next. The dynamics of a typical worker's identity and their reference wage is illustrated in Figure 3.6. Finally, consider the following notation, which is adopted in order to ease the notational burden:

$$\begin{aligned} &\text{if } t = \tau \quad \mathcal{I}_t \equiv i \\ &\text{if } t > \tau \quad \mathcal{I}_t \equiv j. \end{aligned} \tag{3.25}$$

Hence, i denotes newly hired workers with identity $\mathcal{I}_t = \{\underline{\omega}, w_t^{-i}\}$ and j denotes incumbent workers with identity $\mathcal{I}_t = \{w_{t-1}\}$.

Steady-state characterisation

This section derives the steady-state equilibrium of the labour market in the presence of social comparison. The results established here will form the basis for the analysis of cyclical behaviour of the model.

Referring to the framework developed in Chapter 2, the key equation characterising

[14]This particular assumption (asymmetric social comparison) is not crucial for any of the results derived in this section.

the model with free entry (i.e. $V_t = 0$ for all t) are:

$$J(r_t) = \max_{w_t \geq 0} \; \{y(\tilde{e}_t) - w_t + \psi J(r_{t+1})\} \quad \forall t \geq \tau \tag{3.26}$$

$$\text{s.t. } \tilde{e}_t = \tilde{e}_n + \mu(\ln w_t - \ln r_t) \tag{3.27}$$

$$r_t = \begin{cases} r_{i,t} & = \underline{\omega}\mathbb{C}^+ + w_{j,t}\mathbb{C}^- \quad \forall t = \tau \\ r_{j,t} & = w_{j,t-1} \qquad\qquad\quad \forall t > \tau \end{cases} \tag{3.28}$$

$$\frac{\kappa}{\bar{m}\theta_t^{-\sigma}} = J(r_{i,t}) \tag{3.29}$$

$$\Delta u_{t+1} = \rho(1 - u_t) - \bar{m}\theta_t^{1-\sigma} u_t \tag{3.30}$$

$$r_0, u_0 \quad \text{given,}$$

where (3.26) is the firms' value of an employment relationship with a newly hired worker for all $t = \tau$, and with an incumbent worker for all $t > \tau$; the effort function (3.27) captures the employed workers' asymmetric reference-dependent reciprocity; the reference wage rule (3.28) is equivalent to (3.24) but it is expressed using the simplified notation (3.25) set out above; equation (3.29) is the job creation condition of the model under the additional assumption of instantaneous production at the time of hiring;[15] and (3.30) captures the standard law of motion of unemployment.

The equilibrium path of the labour market and the corresponding steady state can therefore be defined as follows.

Definition 7. *Given initial unemployment u_0 and employed workers' reference wage r_0, an equilibrium path is a sequence of wages, reference wages, effort levels, market tightness and unemployment rates $\{w_t, r_t, e_t, \theta_t, u_t\}$ such that firms maximise the present discounted value of profits as formalised by the functional equation* (3.26)*; employed workers' effort is given by* (3.27)*; employed workers' reference wage evolves according to* (3.28)*; labour market tightness satisfies* (3.29)*; and unemployment is given by* (3.30).

Definition 8. *A steady-state equilibrium is an equilibrium path $\mathcal{E} \equiv \{w_t, r_t, e_t, \theta_t, u_t\}$ in which $w_t = \tilde{w}^*$, $r_t = r^*$, $e_t = \tilde{e}^*$, $\theta_t = \tilde{\theta}^*$ and $u_t = u^*$ for all t.*

The following proposition characterises the unique steady-state equilibrium of the model when the initial reference wage in the market r_0 is given by the current minimum legal wage $\underline{\omega}$, under the mild (and arguably realistic) condition that this is weakly lower than the optimal steady-state wage.

Proposition 24. *If $r_0 = \underline{\omega} \leq pq(1 - \psi)$ there exists a unique steady-state equilibrium characterised by the following outcomes.*

[15]Note that while this assumption enables a more intuitive discussion of the cyclical behaviour of the labour market in the next section, it does not affect the main qualitative predictions of the model.

a) Wages:

$$\tilde{w}^* = \tilde{w}_i^* = \tilde{w}_j^* = pq(1 - \psi).$$

b) Reference wages and effort levels:

$$r^* = r_i^* = r_j^* = r_L;$$

$$r_i^* = \tilde{w}_j^* \mathbb{C}^-, \qquad \tilde{e}_i^* = \tilde{e}_n;$$
$$r_j^* = \tilde{w}_j^*, \qquad \tilde{e}_j^* = \tilde{e}_n.$$

c) Labour market tightness and unemployment rate:

$$\tilde{\theta}^* = \left(\frac{\bar{m}}{\kappa}\left[\overline{y}^*(\tilde{e}^*) - \overline{w}^*\right]\right)^{\frac{1}{\sigma}}, \qquad u^* = \frac{\rho}{\rho + \bar{m}\tilde{\theta}^{*1-\sigma}};$$

where

$$\overline{y}^*(\tilde{e}^*) = \frac{pq\tilde{e}_n}{1 - \psi} \quad \textit{and} \quad \overline{w}^* = \frac{\tilde{w}^*}{1 - \psi}.$$

Proof. See Appendix B.3. ■

Proposition 24 highlights a distinctive result with respect to the steady-state properties of the model under reference wage adaptation as established in Propositions 7 and 8, Chapter 2. That is, independently of the initial reference wage level that characterises the labour market, due to relative wage comparisons newly hired workers will always feel entitled to be paid the same wage paid to incumbent workers. As such, since the firms' optimal wage setting policy implies that new hires and incumbents are paid the same steady-state equilibrium wage, new hires perceive this as fair and therefore always exert their normal effort in production. Hence, in the presence of social comparison there is no distinction between new hires and incumbent workers in terms of reference wages and effort levels in the steady state; and as such, the value of an employment relationship with a new hire is the same as the one with an incumbent: $J(r_i) = J(r_j)$.

Cyclical behaviour: asymmetric fluctuations and hysteresis

This section analyses the cyclical properties of the model developed thus far and shows that social comparison can generate asymmetric fluctuations and hysteresis in wages, job creation and unemployment.

In any economic model of the labour market which does not feature hysteresis, a symmetric economic cycle—i.e. an expansion and a subsequent recession of the same magnitude—should not affect the steady-state equilibrium at which the labour market will eventually converge after the resolution of the cycle.[16] Otherwise, according to the definition provided above, the labour market will be hysteretic. Based on this premise, what follows will evaluate the effects of two unanticipated permanent changes in aggregate productivity p, which are opposite in sign but of the same absolute magnitude.[17]

Formally, consider a symmetric economic cycle characterised by an expansionary episode in some period t', in which aggregate productivity increases from p to $p_{t'}$; and a subsequent recessionary episode at t'' in which aggregate productivity decreases from $p_{t'}$ back to $p_{t''} = p$. The following analysis will focus on two types of symmetric cycles of differing magnitudes:

Definition 9. *A moderate symmetric economic cycle between t' and t'' is an economic cycle in which $\frac{p_{t'}-p}{p} \in (0, \lambda-1]$ and $p_{t''} = p$. A large symmetric economic cycle between t' and t'' is an economic cycle in which $\frac{p_{t'}-p}{p} \geq (\lambda - 1)$ and $p_{t''} = p$.*

This *a priori* distinction is important in order to identify the main forces that drive hysteresis in the model. To understand this, notice that for the reasons explained in Chapter 1, depending on the magnitude of the exogenous decrease in productivity, employed workers might experience either downward wage rigidity or "muted" wage cuts, which, in the context of the richer model developed here, will have different implications for other related labour market outcomes. The technical details on which the following analysis is based can be found in Section A.3.2, Appendix A.3.

Moderate cycles. Consider an increase in aggregate productivity in period t' so that $p_{t'} \in (p, \lambda p]$. Existing firms will find it optimal to raise their employed workers' (incumbents in period t') wages from $\tilde{w}^*$ to $\tilde{w}_{j,t'} = \tilde{w}'_j$, where:

$$\begin{aligned} \tilde{w}'_j &= p_{t'} q(1-\psi) \\ &> pq(1-\psi) \\ &= \tilde{w}^*. \end{aligned}$$

As a consequence, due to social comparison, newly hired workers in period t' will now feel entitled to receive the equilibrium wage paid to incumbents: i.e. new hires are characterised by the reference wage $r_{i,t'} = \tilde{w}_{j,t'}$. Therefore, while the increase

[16] This is in fact an inherent feature of the canonical search and matching framework, and of any other model of the labour market based on the *natural* rate of unemployment hypothesis (see, for instance, Cross (2014)).

[17] This qualitative approach in the study of temporary cyclical shocks is in the spirit of Pissarides (1985, 2000), whom evaluate the model cyclical behaviour by studying the effects of a series of unanticipated permanent shocks to aggregate productivity. This also facilitates the comparison of the cyclical properties of the present framework with those of the canonical search and matching model.

in aggregate productivity implies that more workers are hired at a higher optimal equilibrium wage $\tilde{w}_{i,t'} = \tilde{w}'_j > \tilde{w}^*$, social comparison implies that this wage is perceived as fair (and not as a gift) by the new hires, whom will exert their normal effort $\tilde{e}_{i,t'} = \tilde{e}_n$.[18]

These responses are illustrated in Figure 3.7 by an upward shift of the wage curve and a rightward shift of the job creation curve (reaching the equilibrium B). As a result, wages and labour market tightness immediately jump to their new steady-state levels: $\tilde{w}_{i,t'} = \tilde{w}_{j,t'} = \tilde{w}'$ and $\tilde{\theta}_{t'} = \tilde{\theta}'$; while unemployment gradually converges towards a lower steady state u' following the law of motion given by (3.30).

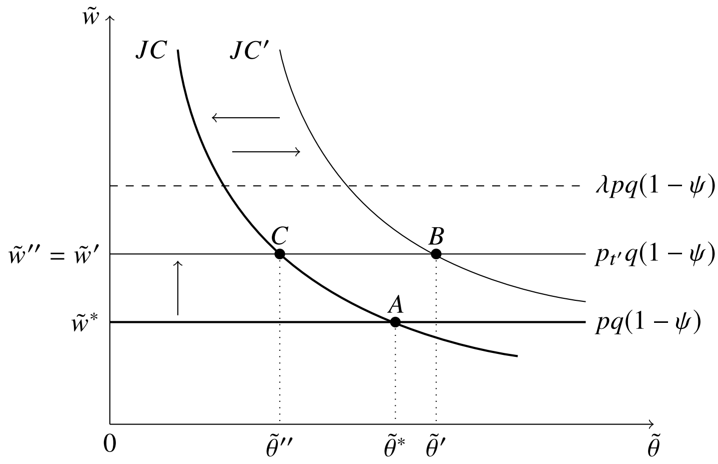

Figure 3.7:
Moderate Cycle: Downward Wage Rigidity and Hysteresis

Next consider a recessionary episode in some period t'' in which aggregate productivity decreases back to its initial level p. Optimal wage setting implies that existing firms employing incumbent workers in period t'' will implement wage freezes in order to avoid the costs of negative reciprocity. Hence, incumbent workers experience downward wage rigidity, i.e. they are paid the same wage as in the previous steady state: $\tilde{w}_{j,t''} = \tilde{w}''_j = \tilde{w}'_j$. Moreover, as a consequence of social comparison, new hires in period t'' will now feel entitled to be paid that wage. That is, incumbents' downward wage rigidity generates a spill-over effect on new hires' wage entitlements, whom will now have a higher reference wage with respect to the one characterising the initial steady state: $r_{i,t''} = \tilde{w}_{j,t''} = \tilde{w}'_j > r^*_i$. As such, optimal wage setting implies that firms hiring workers in period t'' will pay them their reference wage $\tilde{w}_{i,t''} = r_{i,t''}$, and will experience normal effort in production $\tilde{e}_{i,t''} = \tilde{e}_n$. The logic behind this result is explained by the theory developed in Chapters 1 and 2: in the trade-off between a lower equilibrium wage and negative reciprocity, firms find it optimal to pay newly

[18]Hence, if there is social comparison, firms do not benefit from new hires' positive reciprocity in the first employment period, as it would have been the case if the reference wage of new hires was given by the minim legal wage in the market $\underline{\omega}$ only (e.g. as in Chapter 2 for the case of $r_N < r_L$).

hired workers their reference wage as to avoid low morale and sub-normal effort in the first employment period. Therefore, while the present discounted value of output from new employment relationships starting in period t'' is the same as the one in the initial steady state: $\overline{y}''(\tilde{e}'') = \overline{y}^*(\tilde{e}^*) = pq\tilde{e}_n/(1-\psi)$; the present discounted value of the wage is now higher due to the wage entitlement effect generated by social comparisons between new hires and incumbents: $\overline{w}'' = \tilde{w}'/(1-\psi) > \overline{w}^* = \tilde{w}^*/(1-\psi)$. Essentially, this is due to the combination of downward rigidty in the wage of incumbent workers and relative wage comparisons of new hires.

The spill-over effect from downward wage rigidity of incumbents to reference wages of new hires implies that after a decrease in aggregate productivity back to its initial level p, firms will post fewer vacancies and pay higher wages than in the initial steady state prior to the beginning of the cycle. These responses are illustrated in Figure 3.7 by a wage curve that does not shift and by a leftward shift of the job creation curve to its initial position (reaching the equilibrium C). As a result, while the steady-state wage paid to workers in the market does not change $\tilde{w}_{i,t''} = \tilde{w}_{j,t''} = \tilde{w}'' = \tilde{w}'$, labour market tightness immediately jumps to a new (relatively lower) steady-state $\tilde{\theta}_{t''} = \tilde{\theta}'' < \tilde{\theta}^*$ and unemployment gradually converges towards a new (relatively higher) steady state $u'' > u^*$.

Hence, during moderate economic cycles in which incumbent workers experience downward wage rigidity—due to firms' fairness concerns in the presence of asymmetric reference-dependent reciprocity—social comparison between new hires and incumbents generates spill-over effects to perceived wage entitlements, which in turn generate hysteresis in wages, vacancies and unemployment.

Large cycles. Consider a large increase in productivity in period t'' so that $p_{t'} > \lambda p$. The qualitative properties of the model adjusting to this positive shock are the same as those described in the expansionary phase of the moderate cycle. That is, incumbent workers receive a wage raise from $\tilde{w}^*$ to $\tilde{w}_{j,t'} = \tilde{w}'_j$, where:

$$\begin{aligned}\tilde{w}'_j &= p_{t'}q(1-\psi)\\ &> pq(1-\psi)\\ &= \tilde{w}^*;\end{aligned}$$

newly hired workers are paid the wage $\tilde{w}_{i,t'} = \tilde{w}_{j,t'} = r_{i,t'}$ which is perceived as fair so that $\tilde{e}_{i,t'} = \tilde{e}_n$; and firms post more vacancies in the market due to the increase in aggregate productivity, so that $\tilde{\theta}_{t'} = \tilde{\theta}' > \tilde{\theta}^*$. The only difference is that wages and market tightness now jump to an even higher steady-state levels than in the moderate cycle (as it is illustrated in Figure 3.8), while unemployment gradually converges to an even lower steady state.

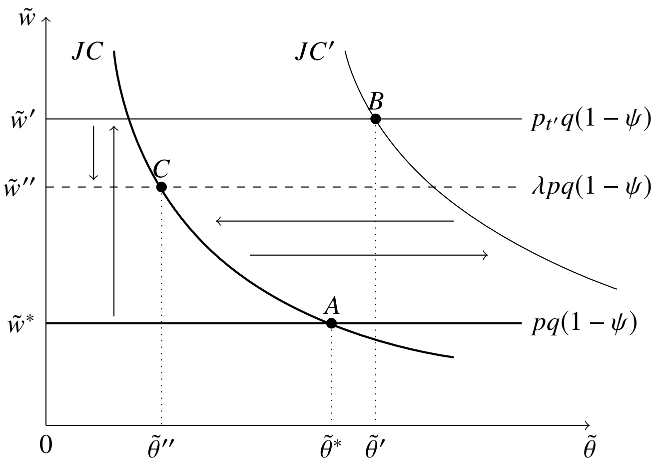

Figure 3.8:
Large Cycle: Muted Wage Cuts and Hysteresis

Then, consider a recessionary episode in some period t'' in which aggregate productivity decreases back to its initial level p. Since this recession is now larger in magnitude than in the moderate cycle, optimal wage setting implies that existing firms employing incumbent workers in period t'' will implement optimal wage cuts from $\tilde{w}'$ to $\tilde{w}_{j,t''} = \tilde{w}_j''$, incurring the cost of negative reciprocity. Hence:

$$\begin{aligned}\tilde{w}_j'' &= \lambda p_{t''}q(1-\psi)\\ &= \lambda pq(1-\psi)\\ &< p_{t'}q(1-\psi)\\ &= \tilde{w}_j',\end{aligned}$$

where the third line follows from the fact that $p_{t'} > \lambda p$. However, as thoroughly explained in Chapter 1, these wage cuts are "muted", that is, due to asymmetric reference-dependent reciprocity, the resulting optimal wage below the reference wage of incumbent workers is relatively higher than if they were not loss averse (i.e. if $\lambda = 1$). In the optimal trade-off between the wage *versus* negative reciprocity, firms have an incentive to partially offset the drop in workers' effort by setting a relatively higher wage. Therefore, although incumbent workers receive a wage cut, the optimal equilibrium wage once the productivity shock reverses in period t'' is yet higher than

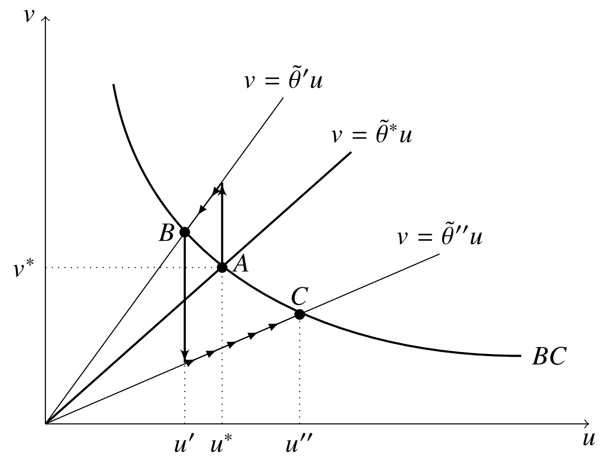

Figure 3.9:
Unemployment Dynamics and Hysteresis

the one characterising the steady-state before the beginning of the cycle:

$$\begin{aligned}\tilde{w}''_j &= \lambda p_{t''} q(1-\psi)\\ &= \lambda pq(1-\psi)\\ &> pq(1-\psi)\\ &= \tilde{w}^*,\end{aligned}$$

where the third line follows from the fact that $\lambda > 1$. Hence, as a consequence of social comparison, new hires in period t'' will feel entitled to be paid the same steady-state wage paid to incumbents, which implies a relatively higher reference wage than the one characterising the initial steady state: $r_{i,t''} = \tilde{w}_{j,t''} = \tilde{w}'_j > r^*_i$. Moreover, as in the moderate cycle case, optimal wage setting implies that firms hiring in period t'' will pay new hires their reference wage $w_{i,t''} = r_{i,t''}$; and the new hires in turn will then exert their normal effort $\tilde{e}_{i,t''} = \tilde{e}_n$.

These outcomes imply that after a decrease in aggregate productivity back to its initial level p, the present discounted value of output from new employment relationships is the same as the one in the initial steady state: $\overline{y}'' = \overline{y}^*$; but the present discounted value of the wage is still higher: $\overline{w}'' = \tilde{w}'/(1-\psi) > \overline{w}^* = \tilde{w}^*/(1-\psi)$. This is due to the muted downward adjustment of incumbent workers' wages, combined with the wage entitlement spill-over that they generate on new hires' reference wage as a consequence of social comparison. Hence, firms will post fewer vacancies and pay higher wages than in the initial steady state prior to the beginning of the cycle: the steady-state wage paid to employed workers in the market decreases from $\tilde{w}'$ to $\tilde{w}'' > \tilde{w}^*$, labour market tightness jumps to a new (relatively lower) steady-state level $\tilde{\theta}_{t''} = \tilde{\theta}'' < \tilde{\theta}^*$

and unemployment gradually converges towards a new (relatively higher) steady state. These responses are illustrated in Figure 3.8 by a wage curve that shifts downwards, but that is still above the initial wage curve, and by a leftward shift of the job creation curve to its initial position (reaching the equilibrium C). The adjustment dynamics of unemployment, which is qualitatively similar for both moderate and large cycles, is illustrated in Figure 3.9.

Hence, even during large economic cycles in which the wage of both new hires and incumbents is endogenously pro-cyclical, social comparison, coupled with firms' fairness concerns in the presence of asymmetric reference-dependent reciprocity, generate hysteresis in wages, vacancies and unemployment. This result is particularly important as it could potentially explain, at least in part, the cyclical patterns in wage and unemployment fluctuations observed during the Great Recession, which, as documented by Elsby et al. (2016) for the U.S., has been characterised by more procyclical wages and an extraordinary longer duration of unemployment spells (e.g. due to less hiring).

Summarising Discussion

This section has analysed the implications of relative wage comparisons between newly hired and incumbent workers for the cyclical behaviour of wages, job creation and unemployment by incorporating the framework developed in Section 3.4 into the benchmark search and matching model developed in Chapter 2. This approach enabled a tractable and transparent analysis, which delivered several noteworthy results.

First, it has been shown that relative wage comparisons between newly hired and incumbent workers generate hysteresis in wage and unemployment dynamics. This result contributes to the theory of labour market fluctuations and provides a rigorous explanation for why temporary cyclical shocks may have permanent effects on the steady-state equilibrium unemployment rate. It is also worth to emphasise that the underlying mechanism generating hysteresis in unemployment is not a product of assumptions that impose *ad hoc* wage rigidities. Rather, it is the product of firms' and workers' optimising behaviour under plausible, evidence-based, assumptions with respect to fairness concerns, reciprocity and wage entitlements in the labour market. The endogenous downward wage rigidity (or the "muted" wage cuts) in existing matches generates entitlement effects into the reference wage of newly hired workers due to social comparison; this, in turn, influences firms' expected present value of the optimal wage that they will have to pay when hiring new workers, which ultimately affect their vacancy posting decisions.

This spill-over effect also provides an additional channel through which wage rigidity featuring existing employment relationships can influence the job creation decisions of other firms, responding to the call put forward by Elsby et al. (2016): "[. . .] future theoretical research needs to analyze the nature of implicit contracts in long-term employment relationships and consider how contracts for new workers interact with ongoing contracts for incumbent workers."(p.S275) (recall the analysis and discussion of this issue, in the context of the literature, developed in Section 2.6, Chapter 2). While a similar spill-over mechanism has been already analysed through the "equal treatment" hypothesis in a series of papers by Thomas (2005), Snell and Thomas (2010) and Martins, Snell, and Thomas (2010), the model developed here is different in many respects. In fact, even though motivated by a large body of empirical evidence, the equal treatment hypothesis—which suggests that new hires should be paid the same wage paid to incumbent workers—is an *ad hoc* constraint imposed on firms in this class of models. On the other hand, in the model developed here, equal treatment arises from firms' optimal wage setting with reference-dependent and loss averse workers whom care about relative wages. As such, the present framework can be considered a microfoundation of the equal treatment hypothesis. Moreover, the model developed in this section is consistent, in the case of large economic cycles, with procyclical downward wage adjustments and large increases in unemployment due to less hiring; a cyclical pattern that fits well the recent evidence concerning the U.S. labour market during the Great Recession.

The analysis of this section also contributes to the long-standing tradition of modelling hysteresis in labour markets (see, for instance, Røed (1997) and Cross (2014) for surveys). The contribution to this literature is two-fold. First, the model set out here provides a novel explanation for the existence of hysteresis in wage and unemployment dynamics, in the context of a modern search and matching framework. The analysis also shows that the negative, permanent effects of a temporary shock into equilibrium unemployment do not necessarily arise from the "curses" left over by a recessionary episode (Cross, 2014, p.137). In fact in the illustration considered above, the inherent source of hysteresis is rooted in the wage and reference wage dynamics characterising the initial expansionary episode preceding the recession. Following the expansion incumbent workers receive wage rises; these in turn raise the reference "fair" wage of incumbents (due to adaptation) and of new hires (due to social comparison), constraining the firms' ability to adjust wages in the future. This contributes to further exacerbate the negative impact of the recession on job creation and unemployment. The "ratchet effect" on the wage of incumbent workers, generated endogenously in the form of downward wage rigidity—or in the form of "muted" downward wage adjustments—is transmitted through relative wage comparison to the wage entitlements of new hires, and ultimately translates into an "over-shooting" of the unemployment rate above the initial steady-state level before the cycle had started. This prediction, according to which "wage hysteresis" is beneficial for employed workers, but unfavourable for the unemployed, echoes the tenets of the models of hysteresis based on the insider-outsider approach to labour markets (see for instance Lindbeck and Snower (1987), and the collection of papers in Lindbeck and Snower (1989)).[19]

The second contribution rests on a slightly more technical ground. It has been shown that the asymmetric reference-dependent nature of employed workers' reciprocity—which stems from utility maximisation—generates a trigger-policy structure which governs firms' optimal wage setting behaviour—which stems from profit maximisation—where the "triggers" take the form of productivity thresholds determining the so called *range of rigidity*—which is non-empty if $\lambda > 1$ (see also Chapters 1 and 2). In addition, the consideration of workers' reference wage adaptation in the context of a dynamic labour market is such that these thresholds are not static, but rather, evolve over time, therefore being themselves history dependent. As such, if viewed under the general theory of systems with hysteresis, the dynamic model analysed in this section can be considered the analog of a Preisach system in economics, in which the Preisach weight function is non-empty if workers are loss averse, and evolves endogenously over time due to reference wage adaptation and social comparisons.[20]

In light of the illustration analysed in this section, the permanent effect on the steady-state equilibrium unemployment rate (i.e. the *remanence*) generated by temporary cyclical shocks highlights the failure of the labour market—comprised of rational and payoff-optimising agents—to fully return to the initial steady-state equilibrium. This theoretical result poses questions on whether the labour market needs an additional exogenous change (i.e. a *coercive force*) in its key determinants (inputs), in order to restore the steady-state unemployment back to its original rate. As such, the model suggests that exogenous interventions, such as economic policy or firm-level managerial practices, might be effective to achieve more desirable labour market outcomes. While the model developed here is yet too abstract for the analysis of these issues, it nevertheless lay down the foundations for a promising research agenda, which brings together the modern microfoundations approach to macroeconomic theory with the

[19] However, notice that the theory developed in this section differs from the insider-outsider approach with respect to several aspects, such as, the nature of workers' effort decisions, the wage setting process, the modelling approach to labour market fluctuations, and the underlying behavioural factors which generate wages and unemployment hysteresis. These differences imply novel explanations, predictions (as it has been illustrated) and potentially novel policy implications (the analysis of which is left to further research).

[20] A Preisach model is the mathematical generalisation of a system exhibiting hysteresis. The model takes the name from Ferenc Preisach, a German physicist whom first proposed a model of magnetic hysteresis in 1935 (Mayergoyz, 1986). See Cross et al. (2009) and the references therein for a discussion of Preisach systems and their applications to economics.

long-standing hypothesis of hysteresis in labour markets.

3.6 Conclusions

The understanding of workers' perceptions of what is a "fair" wage has become increasingly crucial for the analysis of employment relationships and labour markets. However, there is no established systematic approach aimed at an in-depth theoretical investigation of the reference wage formation process and its determinants. Building on a large body of research that has explored the concepts of *fairness* in labour relations (Fehr et al., 2009); *reference dependence* in behavioural decision theory (Kahneman, 1992); and social *norms* and *identity* from sociology (Akerlof and Kranton, 2000); this chapter developed a general, and portable, analytical framework for the analysis of reference wage formation in employment relationships. Within this formulation, the reference wage is an artefact that captures a broad set of feelings, entitlements, information and norms about what should be a fair remuneration, i.e. it is a useful representation of the notion of a "fair" wage.

Worker's perceptions of fairness depend on a complex interaction of experience, history, social norms and the institutional context. Moreover, shaped by these interactions, the reference wage formation process and the information set affecting its determinants evolve over time and exhibit asymmetries: losses loom larger—and persist longer—than gains; and disadvantageous pay inequalities are perceived as particularly unfair.

This state-contingent, and context-dependent, nature of fairness perceptions does not easily lend itself to the development of a self-contained theory of reference wage formation. To overcome this issue, the present chapter has advanced a general behavioural principle, which captures the unconscious cognitive process through which workers form their reference wage: the elements of the information set affecting fairness evaluations are determined by how a wage contract is framed, which, in turn, is influenced by the workers' *social identity*—a concept that ties together social categories with their corresponding fair wage norms. This approach has generated a unique conceptual framework that is portable to a variety of economic, institutional and social settings. The task left to the theorist/analyst is then to design a stylised labour market consisting of social categories and fair wage norms that appropriately characterise the environment of analysis.

By applying this framework into the benchmark models developed throughout the thesis, this chapter analysed the implications of two particularly salient features of reference wage formation: the dynamic and asymmetric adaptation of wage entitlements to past wage contracts; and the spill-over effect stemming from relative wage comparisons between new hires and existing incumbent workers. These applications have proven to be insightful and transparent in their analysis of some inherent aspects of long-term employment relationships and labour markets. More precisely, it has been shown that asymmetric partial adaptation generates *asymmetric endogenous persistence* in wage and reciprocity dynamics—also providing a coherent explanation for the observed asymmetry in the *intensity* and *persistence* of negative *versus* positive reciprocity; and that relative wage comparisons generate *hysteresis* in the cyclical behaviour of wages, vacancies and unemployment.

This general approach to reference wage formation, along with its two selected applications, could be implemented and extended in several ways. First, the analytical framework of Section 3.4 could be used to construct partial equilibrium models of workers' optimal effort responses to wage changes from which to draw empirically testable hypotheses. These will help to assess the importance of the reference wage formation mechanisms put forward in this chapter (e.g. asymmetric adaptation/social comparison; segregated *versus* integrated mechanisms), also on the basis of their relationship with the choice of social categories and fair wage norms. For instance, the framework can be used to model an existing labour market environment in order to subsequently test its predictions in the field; or, alternatively, it can be used to design an hypothetical labour market with a more distinct selection of social categories and fair wage norms, which will be easier to analyse and test in controlled laboratory experiments.

Another potential extension would be to incorporate the microeconomic model with asymmetric partial adaptation analysed in Section 3.5.1 into a richer macroeconomic framework such as the benchmark search and matching model developed in Chapter 2. Here the endogenous asymmetric persistence in wage and effort dynamics could potentially provide a complementary channel—in addition to relaxing the free-entry condition—through which to explain the observed persistence in vacancy dynamics (see Elsby et al. (2015) for a discussion of this issue). From a wider perspective, the general framework developed here and the benchmark search and matching model set out in Chapter 2 could be considered as two self-contained blocks which can be combined together in a variety of ways. Doing so would allow a transparent theoretical analysis of labour market environments characterised by different reference wage formation processes—which will depend on the relevant social categories, fair wage norms, and identity dynamics describing these markets—within a unique and rigorous analytical framework. Based on the existing state of the literature this type of comparative analysis may be difficult to perform, especially due to differences in the modelling of wage setting behaviour and reciprocity, which can obfuscate whether the main results are driven exclusively by the choice of what determines the reference wage.

In conclusion, while it seems unfeasible to completely remove the degree of freedom imposed by the choice of the reference wage, the theory advanced in this chapter provides an organising framework to think about reference wage formation, fostering richer and more context-dependent analyses of employment relationships and labour markets.

Conclusion

One of the most important objectives of macroeconomics concerns the understanding of the amplitude and persistence of unemployment fluctuations in the business cycle. To address this, advances in the theory of labour markets have placed particular emphasis on the role of nominal and real wage rigidities. Virtually every modern macroeconomic model of the business cycle—be it based on efficiency wages, search and matching frictions, other market imperfections, or a combination of these—requires a degree of wage rigidity to be built-in to sufficiently explain unemployment fluctuations. However the recent theoretical and empirical literatures have identified some challenges in reconciling the cyclical co-movement of wages, vacancies and unemployment. It is not yet clear whether the emphasis on new hires' wage cyclicality is well placed, or what impact the wage rigidity of incumbent workers has on the cyclical behaviour of unemployment and vacancies. These challenges suggest that contemporary macroeconomic theory would greatly benefit from an in-depth understanding of wage setting behaviour and of its consequences for workers' productivity and firms' hiring decisions.

The main contribution of this thesis is in the development of such in-depth theoretical approach. By drawing insights from behavioural economics, psychology and sociology, the theories developed throughout Chapters 1, 2 and 3 have placed a considerable emphasis on the behavioural aspects that influence firms' wage setting behaviour and workers' attitudes in the production process, abstracting from potential legislative and institutional constraints. This approach demonstrated that asymmetry and irreversibility—stemming from asymmetric reciprocity and downward wage rigidity—are inherent features of employment relationships; has enabled a transparent identification of their sources; and provided a tractable analysis of their consequences for labour markets fluctuations.

Chapter 1 formalised a theory of reciprocity and wage setting behaviour in a worker-firm employment relationship, in which the worker evaluates wage contracts with respect to a reference "fair" wage and is assumed to be loss averse. The chapter provided a formal characterisation of a worker's "asymmetric reference-dependent reciprocity", and analysed its implications for optimal wage setting and the employment contract. The analysis has established that the worker's adaptation of their reference wage and the relatively large cost of negative reciprocity that stems from loss aversion are the key fundamental drivers of downward wage rigidity; and that the reference-dependent nature of reciprocity, combined with the adaptation of the reference wage, leads to a "re-normalisation" of effort during the course of the employment relationship; formally demonstrating that reciprocity is essentially a temporary phenomenon. As such, the theory developed in Chapter 1 provides a solid and realistic microfoundation for downward wage rigidity and highlights several inherent features of reciprocity and wage setting behaviour in employment relationships. Specifically: i) loss aversion is identified as the main driver of negative reciprocity being stronger than positive reciprocity; ii) a firm has an incentive to compress wage contracts even absent downward wage rigidity; and iii) the anticipation of stronger negative reciprocity reduces the expected value of the employment relationship.

Building on these results, Chapter 2 analysed the macroeconomic implications of the theory developed in Chapter 1 through the lens of a canonical search and matching model. The novel theoretical predictions derived are the outcome of two fundamental behavioural mechanisms: the inter-temporal marginal trade-off faced by firms when choosing the optimal wage; and their consideration, at the time of hiring, of the workers' asymmetric reference-dependent reciprocity response. The analysis has established the existence of three distinct path-dependent equilibria, the realisation of which crucially depends on the level of new hires' wage entitlements in the labour market. Moreover, by drawing a clear distinction between newly hired and incumbent workers, this framework has been used to contribute to the theory of labour market fluctuations concerned with the amplitude and co-movement of vacancies and unemployment. This contribution rests on two main theoretical results. First, it has been shown that due to cyclical changes in positive and negative reciprocity from new hires, the cyclicality of the hiring wage is irrelevant to the volatility of job creation. Second, by introducing uncertainty around the evolution of a job-match productivity, it has been shown that the expectation of downward rigidity in the wage of incumbent workers negatively influences firms' expected surplus from new employment relationships, dampening hiring incentives and increasing the volatility of both job creation and unemployment.

These predictions, and their implications, were based on the assumption that new hires' reference wages are exogenously given, while incumbents adapt the reference wage to their most recent wage contract. Whilst several other candidates for the relevant reference wage might be considered, the existing literature lacks a systematic approach to the reference wage formation process and determinants, from which other possible assumptions could have been drawn.

Motivated by this gap, Chapter 3 provided an in-depth investigation of the concept of the reference "fair" wage, and developed a general and portable analytical framework to think about reference wage formation. Recognising that perceptions of fairness and fair wage norms are state contingent and context dependent, and inspired by the social identity approach in economics, this chapter advanced a general behavioural principle to model workers' reference wage formation process. This approach has enabled the formalisation, under a common theoretical framework, of several properties of the reference wage, such as: the intrinsic tendency of workers to adapt their reference wage over time; the role of the information set influencing this process, and how it can be exogenously, or endogenously, influenced; and the presence of asymmetries: workers adapt more rapidly to wage gifts than to unfair wages, and disadvantageous pay inequality has a substantially larger effect on fairness judgements and morale.

This framework has been subsequently used to explore the implications of the two most salient features of reference wage formation: the partial and asymmetric adaptation of wage entitlements to past contracts; and the spill-over effect that stems from relative wage comparisons between newly hired and incumbent workers. Incorporating these aspects into the models developed in Chapters 1 and 2, delivered insightful and transparent predictions in the context of reciprocity and wage dynamics in long-term employment relationships and labour market fluctuations. In particular it has been shown that asymmetric partial adaptation of the reference wage by workers can generate asymmetric endogenous persistence in wage and reciprocity dynamics, providing a coherent explanation for the observed asymmetry in the intensity(loss aversion) and persistence(partial adaptation) of negative *versus* positive reciprocity. Moreover, it has been shown that considerations by firms of relative wage comparisons between new hires and existing workers can generate hysteresis in the cyclical behaviour of wages, vacancies and unemployment.

In what follows, the thesis concludes with a discussion of some limitations and potential extensions to this line of research. The theory of "asymmetric reference-dependent reciprocity" formalised in Chapter 1, combined with workers' reference wage adaptation in a dynamic setting—as in Section 1.4.1, Chapter 1, and Section 3.5.1 Chapter 3—captures, within a unique framework, several features of reciprocity that have been documented in anthropological and experimental research: i) the higher strength of negative reciprocity responses, i.e. asymmetric intensity due to loss aversion; ii) the temporary nature of reciprocity, i.e. the dynamic re-normalisation of effort due to adaptation; and iii) the extended duration of negative reciprocity responses, i.e. asymmetric persistence due to asymmetric partial adaptation. While the potential sources of these features—in particular i) and ii)—have only been the subject of conjectures by others (e.g. Campbell and Kamlani (1997), Mas (2006), Bewley (2007) and Fehr et al. (2009)), the existing literature has provided direct experimental tests only for i) (e.g. Chemin and Kurmann (2014)) and ii) (e.g. Sliwka and Werner (2017)). As such, in future research it would be interesting to test these three hypotheses all together in a laboratory or field experiment. An investigation of this kind would shed light on the nature of reciprocity in employment relationships, the implications of which could then be analysed using the models developed in the thesis.

The search and matching framework developed in Chapter 2 (also implemented in the Section 3.5.2, Chapter 3) focused on analytical solutions and steady-state comparative statics analyses. This approach yielded a transparent and tractable framework which enabled several insightful results and predictions to be drawn. Nevertheless it will be interesting to extend this framework to consider more general assumptions, some of which, as discussed, are likely to generate a richer out-of-steady-state dynamics. Developing a richer macroeconomic framework will enhance the generality of the results that have been established, and will provide a solid benchmark to evaluate the model performance against the data and to perform more standard quantitative analyses such as stochastic dynamic simulations.

Finally, perhaps the most important limitation of the framework developed for the analysis of reference wage formation in Chapter 3 is the presence of a certain degree of freedom in the choice of the relevant workers' identities. Unfortunately, as discussed throughout the chapter, the very nature of fairness and norms does not easily lend itself to the formalisation of a self-contained theory of reference wage formation. This limitation can be overcome in two ways. First, by induction, the relevant set of social categories and fair wage norms can be designed based on the theorist's observation, introspection, and analysis of the empirical evidence available. Then, through a process of abstraction, the framework of Chapter 3 is instructive for the way in which such evidence should be interpreted and incorporated into reference-dependent preferences. The models developed in Chapters 1 and 2 can then be used to draw the associated theoretical predictions. The second way would work in the opposite direction by initially specifying a broader set of social categories and fair wage norms and then, thorough a deductive process, draw theoretical predictions—in terms of reciprocity, wage setting, or even job creation and unemployment—which can be subsequently validated using standard hypothesis tests. As such, despite not being the ultimate theory of reference wage formation, the theoretical investigation of Chapter 3 provides a tractable and portable analytical framework which can be considered as a first step towards a more systematic approach to the analysis of the reference wage and its determinants.

The models developed in Chapters 2 and 3 can be considered as two distinct analytical blocks which can be combined together in a variety of ways. The resulting combinations can then be used to study the implications of alternative specifications about workers' wage entitlements for the cyclical behaviour of reciprocity, wages, vacancies and unemployment. Moreover, by appealing to the concepts of framing and anchoring discussed in Chapter 3, the resulting model can be used to design novel "behavioural" policies aimed at "manipulating" workers' perceptions of fairness in order to achieve more efficient, and welfare improving, labour market outcomes.

To conclude, the central theme underlying the contribution of this thesis is that asymmetry and irreversibility are inherent features of labour markets, generated by the rational behaviour of payoff-optimising agents. The theoretical approach undertaken to investigate these aspects has led to the development of a tractable and transparent framework for the analysis of their implications in the theory of wages and unemployment fluctuations. This theoretical investigation has also uncovered the possibility that economic policies aimed at shaping workers' wage entitlements could deliver more desirable labour market outcomes. As such, this thesis lays down the foundations for a promising, more empirically- and quantitatively-oriented, research agenda aimed at understanding the norms of fairness and reciprocity in employment relationships, and their consequences for the macroeconomics of labour markets.

Appendix A

Additional Material

A.1 Additional Material for Chapter 1

A.1.1 An Overview of Relevant Wage Setting Models

This section provides a concise overview of the wage setting models mentioned in the introduction section of Chapter 1. The objective is to highlight the main conceptual differences—in terms of assumptions and results—between these models and the theory developed in Chapter 1. When possible, to enhance comparability, notation will be set consistent with that adopted throughout the thesis.

Elsby (2009)

Elsby (2009) develops an infinite-horizon dynamic model of wage setting behaviour under uncertainty, featuring an ongoing employment relationship where downward wage rigidity binds. This model is used to show that the firm's anticipation of downward wage rigidity in the expected continuation value of the employment relationship generates an incentive to actively compress wage rises. This prediction is subsequently tested using micro-data from the U.S. and Great Britain. Elsby (2009) finds evidence in support of firms' wage compression, formally demonstrating that the existing literature has overstated the costs of downward wage rigidity to firms, in terms of higher levels aggregate wage growth.

Consider the reduced-form effort function assumed by Elsby. For expositional purposes, all variables are in real terms:

$$e = \ln\left(\frac{w}{z}\right) + c \cdot \ln\left(\frac{w}{r}\right)\mathbf{1}^{-}, \tag{E1}$$

where: w is the wage; $r = w_{-1}$ is the wage paid in the previous employment period; $\mathbf{1}^{-}$ is an indicator that takes a value of 1 if the worker receives a wage cut, i.e. if $w < w_{-1}$ and 0 otherwise; z is a measure of real unemployment benefits (constant overtime); and the parameter $c > 0$ varies the productivity cost to the firm of a wage cut. Hence this effort function has a kink at $w = w_{-1}$ if $c > 0$, reflecting the worker's resistance to wage cuts: the marginal productivity loss of a wage cut exceeds the marginal productivity gain of a wage increase by a factor $1 + c > 1$. The instantaneous per-worker profit of a firm is given by $\pi = qe - w$.

The model of Elsby (2009) captures a discrete-time infinite-horizon ongoing employment relationship, abstracting from hiring or layoffs: the worker-firm match is assumed to be already formed, and always profitable. As such, the functional equation corresponding to the firm's wage setting problem is expressed as

$$J(r,q) = \max_{w} \left\{ q \left[\ln\left(\frac{w}{z}\right) + c \ln\left(\frac{w}{r}\right) \mathbf{1}^{-} \right] - w + \delta \int J(r',q')\, dF(q'|q) \right\} \qquad \text{(E2)}$$

where the prime $'$ represents forward values; $r' = w$ and q is an idiosyncratic technology shock with the purpose to add uncertainty to the model.

To enhance comparability (and without loss of generality for the conclusion to follow) it is possible to express Elsby's model in terms of a two-period employment relationship in which r_0 is exogenous, $r_1 = w_0$, and the idiosyncratic technology shocks q_0 and q_1 are independent draws from the cumulative distribution function F, as it is assumed in the analysis of Section 1.4.

$$J(r_0,q_0) = \max_{w_0} \left\{ q_0 \left[\ln\left(\frac{w_0}{z}\right) + c \ln\left(\frac{w_0}{r_0}\right) \mathbf{1}^{-} \right] - w_0 + \delta \int J(w_0,q_1)\, dF(q_1) \right\}$$

$$\text{where } J_1(r_1,q_1) = \max_{w_1} \left\{ q_1 \left[\ln\left(\frac{w_1}{z}\right) + c \ln\left(\frac{w_1}{r_1}\right) \mathbf{1}^{-} \right] - w_1 \right\}$$

Notice that q_0 and q_1 are assumed to be always profitable, ruling out the possibility of no employment contract in the first period or job destruction at the beginning of the second period. It is now straightforward to compare Elsby's model with the the forward-looking firm's optimisation problem formalised in (1.7-1.8), Section 1.4.1, Chapter 1.

The first-order necessary and sufficient conditions for this problem for all $w_t \neq r_t$ are given by

$$(1 + c\mathbf{1}^{-})q_1 \frac{1}{w_1} - 1 = 0, \qquad \text{(A.1)}$$

which characterises the optimal wage in period $t = 1$, denoted by $\tilde{w}_1 = \tilde{w}(r_1, q_1, c)$ and

$$(1 + c\mathbf{1}^{-})q_0 \frac{1}{w_0} - 1 + \delta \cdot \frac{\partial}{\partial w_0} \int J(w_0,q_1)\, dF(q_1) = 0 \qquad \text{(A.2)}$$

which characterises the optimal initial wage contract in period $t = 0$, denoted by $\tilde{w}_0 = \hat{w}(r_0, q_0, c, \delta)$. As in Elsby, denote the marginal effect of the wage in period 0 on the future profits of the firm in period 1 by $D(w_0, c) \equiv \frac{\partial}{\partial w_0} \int J(w_0,q_1)\, dF(q_1)$. In this simplified two-period version of the model, this takes the following analytical form:

$$D(w_0,c) = \int^{q_l(w_0,c)} -c \cdot \frac{q_1}{w_0}\, dF(q_1) \; + \; \int_{q_l(w_0,c)}^{q_u(w_0)} \frac{q_1}{w_0} - 1\, dF(q_1),$$

which is then directly comparable with the analog (but different) expression derived in Proposition 1 of Chapter 1. The first term on the right-hand side captures the expected marginal cost of enacting a wage cut in period 1, and the second term captures expected marginal benefits of enacting a wage freeze in period 1. Notice that, in contrast to the theory of Chapter 1, in Elsby's model future wage freezes generate expected marginal benefits—since effort is a function of absolute wage levels—and future wage raises do not generate any expected marginal benefits—i.e. there is no positive reciprocity.

Based on this set up and analysis, one of the key theoretical predictions drawn by Elsby (2009) is that the expectation of downward wage rigidity in period 1—that is, if $c > 0$—generates an incentive for the firm to set a relatively lower initial wage (wage compression) than if downward wage rigidity was absent—that is, if $c = 0$. This prediction is illustrated by performing two simple comparative statics exercises with respect to the parameter c, which captures the disproportional fall in the worker's effort generating downward wage rigidity, and the parameter δ, which captures the firm's discount factor.

In fact,

$$\text{if} \quad c = 0, \quad \text{then} \quad D(w_0, 0) = 0 \quad \text{and} \quad \hat{w}(r_0, q_0, 1, \delta) = \tilde{w}(r_0, q_0, 1) = q_0$$

That is, in the absence of expected downward wage rigidity, there will be no expected marginal effects of the initial wage into the future profits of the firm in period 1. The first-order condition characterising the initial wage contract will be given by (A.2) when $c = 0$, which is simply

$$q_0 \frac{1}{w_0} - 1 = 0.$$

By comparing this condition with the first-order condition 1.11 derived in Chapter 1 when $\lambda = 1$, i.e. in the absence of downward wage rigidity, it is possible to highlight the main qualitative difference between Elsby's model and the theory developed in the first chapter of this thesis. In fact,

$$\text{if} \quad \lambda = 1, \quad \text{then} \quad \Phi(w_0, 1) < 0 \quad \text{and} \quad \hat{w}(r_0, q_0, 0, \delta) < \tilde{w}(r_0, q_0, 0).$$

That is, even if wage rigidity is not expected to be a feature of the employment relationship, there is still an additional expected marginal cost $|\Phi(w_0, 1)| > 0$ that has to be borne by a forward-looking firm setting the initial wage.

The key driver of these different predictions is the worker's optimal effort response characterising reciprocity in the two models. Recall that this has been assumed in Elsby (2009), while it has been derived from the worker's optimising behaviour in the theory of Chapter 1. In the reduced-form effort function (E1) above, whenever the worker is paid a fair wage, or a wage gift (i.e. whenever $w_0 \geq r_0$ and $\mathbf{1}^- = 0$), their optimal effort is a function of the absolute level of the wage. Despite being perhaps inconsistent with the evidence discussed in this thesis, such effort function implies that:

i) a higher wage in period 0 increases the worker's effort in period 1 in the event of a wage freeze, since effort is a function of absolute wage levels; and

ii) absent any form of wage rigidity and expected negative reciprocity, there is no additional expected marginal cost on the future profit of the firm when setting the wage in period 0.

On the other hand, the effort function derived in Chapter 1, which captures a worker's dynamic re-normalisation of effort under the adaptation of the reference wage, implies that:

i) a higher wage in period 0 increases the firm's labour cost in period 1 in the event of a wage freeze, since reciprocity is only temporary (effort will be normal in the event of $\tilde{w}_1 = r_1 = \tilde{w}_0$); and

ii) absent any form of wage rigidity and expected negative reciprocity, there is an additional marginal cost in period 0, since any optimal wage in period 1 will now be reciprocated by a relatively lower amount of effort than in period 0.

As such, a higher initial wage will result in a higher reference wage in the subsequent employment period, which, in expectation, reduces the worker's extent of reciprocity in period 1, for any given $r_1 = w_0$ and q_1. Hence, even in the absence of expected downward wage rigidity, a forward-looking firm still has an incentive to compress the initial wage. The implications that stem from this result are thoroughly analysed in Chapter 1.

Danthine and Kurmann (2007)

Danthine and Kurmann (2007) develop a New Keynesian macroeconomic model in which workers' preferences exhibit reciprocity à la Rabin (1993) from whence gift exchange à la Akerlof (1982) can be derived. Their analysis focuses on the macroeconomic implications of gift exchange when workers' reference wages are also influenced by firms' ability to pay ("internal reference"), the relative importance of which determines the extent of wage flexibility subject to demand and technology shocks.

Consider a typical worker's utility as assumed by Danthine and Kurmann (2007):

$$u = u(c, e) + \overline{\zeta}\tilde{s}(e, w)$$

where $u(c, e) = \log c + e^{\overline{\theta}}$; c is consumption; e is effort; w is the wage and $\overline{\theta} > 1$. Their model closely resembles Rabin's (1993) formulation of the utility of a worker in a reciprocity game, in which $\tilde{s}(e, w)$ is defined as the product of the respective gifts of the worker and the firm:

$$\overline{\zeta}\tilde{s}(e, w) = \overline{\zeta}d(e, \cdot)g(w, \cdot);$$

where $d(e, \cdot)$ is the gift of the worker towards the firm, $g(w, \cdot)$ is the gift of the firm towards the worker, and $\overline{\zeta}$ captures the relative importance of reciprocity considerations into the worker's utility. The first-order condition (equation (1), p.862) characterising a worker's optimal effort response $\tilde{e}$ is

$$-\frac{\partial u(c, e)}{\partial e} = \overline{\zeta}\frac{\partial d(e, \cdot)}{\partial e}g(w, \cdot), \tag{DK1}$$

which gives the amount of effort a worker is willing to supply in response to a certain wage offer. This expression can be considered the analog of the first-order condition 1.4, derived in Chapter 1.

The typical firm's production function is assumed to be given by

$$y = y(en) = q(en)^{\overline{\alpha}}, \tag{DK2}$$

with $\overline{\alpha} \in (0, 1)$, and where q is technology and n is the level of employment. Notice that the firm's production function exhibits decreasing returns to labour. This is a fundamental assumption for the derivation of the results in Danthine and Kurmann (2007), as they also note in footnote 10, p.863. As such, the typical (myopic) firm's problem takes the form of

$$\max_{w,n} \overline{\varrho}y(en) - wn. \tag{DK7}$$

where the parameter $\overline{\varrho}$ captures the inverse of the optimal markup (real marginal cost) that the typical firm applies as a result of its monopolistic position in the product market. In perfect competition $\overline{\varrho} = 1$, whilst in monopolistic competition $\overline{\varrho} < 1$.

In order to highlight the qualitative properties of the typical worker's effort choice and of the typical firm's wage setting behaviour, and compare it with the theory developed in Chapter 1, the analysis proceeds by deriving the model in accordance with Danthine and Kurmann's (2007) assumptions.

Hence, the gift of the worker is assumed to take two possible forms

$$d(e, \cdot) = qn^{\overline{\alpha}-1}\left[e^{\overline{\alpha}} - e_r^{\overline{\alpha}}\right] \tag{DK3}$$

$$d(e, \cdot) = (e - e_r). \tag{DK4}$$

where e_r is a reference effort level, considered as a weighted average of a maximum and minimum effort level. On the other hand, the gift of the firm towards the worker is

$$g(w, \cdot) = \log w - \log r,$$

where r is the worker's reference wage. The main novelty introduced by the model of Danthine and Kurmann (2007) is that the worker's reference wage r is essentially specified as a weighted average of the outside option $\overline{w}^{\overline{n}} z^{1-\overline{n}}$ (as it is standard in efficiency wage models) and an "internal reference", namely a measure of the firm's profitability $(y/n)^{\nu}$ (which is the new element) with $\nu < 1$ (this latter restriction is also necessary for the working of the model, the absence of which will imply a degenerate version in which the workers' optimal effort is zero). Denote by φ and $(1 - \varphi)$ the relative weights of the reference wage determinants, and assume an income tax rate given by $(1 - \tau)$, the gift to the firm is assumed to take the following form (equation (5), p. 864):

$$g(w, \cdot) = \log(tw) - \left\{\varphi \log\left[\tau(y/n)^{\nu}\right] + (1-\varphi)\log\left[\tau \overline{w}^{\overline{n}} z^{1-\overline{n}}\right]\right\}. \tag{DK5}$$

Notice that if $\varphi = 0$ the reference wage only depends on the outside option, and the model essentially becomes a standard efficiency wage model, for instance, à la Akerlof (1982).

By implementing the functional forms of the worker's gifts as given by (DK3) and (DK4) respectively into the first-order condition (DK1), the model of Danthine and Kurmann (2007) yields the following two forms of optimal effort functions. If $d(e, \cdot) = (e - e_r)$, then

$$\tilde{e} = \left[\frac{\overline{\zeta}}{\overline{\theta}}(\log w - \log r)\right]^{\frac{1}{\overline{\theta}-1}}; \tag{DK6a}$$

whilst if $d(e, \cdot) = qn^{\overline{\alpha}-1}[e^{\overline{\alpha}} - e_r^{\overline{\alpha}}]$, then

$$\tilde{e} = \left[\frac{\overline{\zeta}\overline{\alpha}}{\overline{\theta}} qn^{\overline{\alpha}-1}(\log w - \log r)\right]^{\frac{1}{\overline{\theta}-\overline{\alpha}}}; \tag{DK6b}$$

where $(\log w - \log r)$ is given by (DK5). These two equations are essentially the analytical forms of the effort conditions expressed by their equation (6), p.864.

At this stage it is possible to identify a very important difference between these effort functions and the worker's optimal effort (1.5) derived in Chapter 1 (which captures what has been defined as a worker's asymmetric reference-dependent reciprocity). In fact, in the model of Danthine and Kurmann (2007), whenever a worker is paid their reference wage $w = r$, their optimal effort is zero. This qualitative feature seems odd, and it is in stark contrast with the empirical evidence discussed throughout this thesis. Moreover, as such, Danthine and Kurmann's (2007) model cannot possibly capture negative reciprocity, since whenever a worker is paid a wage that is perceived as unfair $w < r$, their optimal effort will be negative, implying negative levels of output. Since this result does not have any sensible economic interpretation in the model, it has been therefore ruled out *a priori*.

Danthine and Kurmann's (2007) is a macroeconomic model. As such, they aggregate workers and firms assuming homogeneity and then they solve for the steady-state equilibrium. One of the key results is that firms find it optimal to pay a constant wage gift to workers:

$$g^* = \frac{1 - \varphi \nu}{\bar{\theta} - 1}.$$

Moreover, if $d(e, \cdot)$ is given by (DK4) workers will exert constant "positive reciprocity"; while if $d(e, \cdot)$ is given by the more complex (DK3), workers' optimal effort varies with y/n, i.e. in times of high productivity workers are willing to provide a higher gift to the firm (all else equal). This result points to another fundamental difference between the model of Danthine and Kurmann (2007) and the one developed in Chapter 1. That is, since firms' gifts are always constant and positive, the workers' gifts will also always be positive and above the "reference effort level" (i.e. normal effort). As such Danthine and Kurmann's (2007) model only captures positive reciprocity: any variation in worker's effort, which could result from changes in parameters or from shocks, essentially corresponds to variations in positive reciprocity. The theory of reciprocity developed in their model therefore rests within the gift-exchange paradigm formalised by Akerlof (1982).

Another aspect in which the model can be compared concerns firms' optimal wage setting behaviour, and its ability to endogenously generate wage rigidity. First recall that wage rigidity corresponds to the acyclicality of wages with respect to changes in external market conditions, such as employment n (demand shock) or technology q (technology shock). While wage stickiness corresponds to the less than proportional adjustment of wages with respect to changes in these parameters. Hence, it is worth noting that Danthine and Kurmann's (2007) framework generates wage rigidity in response to exogenous shocks (with no distinction with respect to their magnitudes), only when the following three conditions hold simultaneously:

a) worker's effort is constant, i.e. $d(e, \cdot) = (e - e_r)$;

b) the reference wage is determined by firms' performance only, i.e. if $\varphi = 1$;

c) the economy is hit by a technology shock, i.e. exogenous changes in q.

In all the other possible cases analysed, Danthine and Kurmann's (2007) model only generates various forms of wage stickiness depending on the different combinations of parameters and shocks considered. Also note that in some calibration exercise, wages are found to be even counter-cyclical following demand shocks.

Danthine and Kurmann (2010)

In this paper Danthine and Kurmann (2010) incorporate the theory of reciprocity formalised in Danthine and Kurmann (2007) into a DSGE model, which is subsequently estimated on U.S. data. In so doing, some ingredients of the model have been modified, but the underlying modelling approach to workers' effort has essentially remained unchanged.

To see this, consider the utility of the typical "household", which is a simpler version of the worker's utility assumed in Danthine and Kurmann (2007):

$$u = \log(c_t - \vartheta c_{t-1}) + \log(1 - L_t) - L_t \left[\frac{e_t^2}{2} - \tilde{s}(w_t, e_t)\right]; \tag{DK1}$$

in which $\vartheta \geq 0$ is a habit parameter; L_t is the fraction of hours worked (the total amount of which is normalised to 1); and e_t corresponds the effort level per hour worked. The optimality condition characterising workers' effort choice is

$$e_t = \frac{\partial \tilde{s}(w_t, e_t)}{\partial e} \tag{DK2}$$

where $\tilde{s}(w_t, e_t)$ is the product of the respective gifts of a worker and the firm. As Danthine and Kurmann (2010) note, unlike Rabin (1993), they only consider the case in which $\tilde{s}(w_t, e_t) > 0$ (see footnote 6, p.839). That is, they only consider workers reciprocating the firms' gift with higher than normal effort, and not the "perverse equilibrium" in which workers' punish unfair wage offers with lower than normal effort. As such, like in their earlier paper, negative reciprocity is not considered.

The respective gifts are specified as deviations of effort and wages from some reference or norm levels:

$$d(e_t, \cdot) = (e_t^{\overline{\alpha}} - e_n^{\overline{\alpha}})$$
$$g(w_t, \cdot) = (\log w_t - \log r_t).$$

Hence, $d(\cdot)$ and $g(\cdot)$ are both concave. Within this context, it is assumed that the normal level of effort is $e_n = 0$ (which is the effort level that workers will exert when paid the reference wage). This assumption is even stronger than the one in Danthine and

Kurmann (2007), and again gives the odd prediction that when workers are fairly paid, they optimally exert zero effort. As such, the gift of the worker is

$$d(e_t, \cdot) = e_t^{\overline{\alpha}}; \tag{DK7}$$

while the gift of the firm is assumed to be given by the difference between the consumption utility attributed to the wage, $\log[(1-\tau_t)w_t]$, and the reference wage r_t.

In Danthine and Kurmann (2010) the worker's reference wage is assumed to be a function of three main components: the firm-internal labour productivity $\log[(1-\tau_t)l_t y_t/n_t]$; the worker's outside option $\log[(1-\tau_t)\overline{w}_t\overline{n}_t]$; and past wages $\log[(1-\tau_t)[s\overline{w}_{t-1}+(1-s)w_{t-1}]]$. Here $\overline{w}_{t-1}$ is the aggregate past wage and τ_t is the state contingent tax rate. Notice that if $s = 1$, the "social norm" case, the worker considers the past aggregate wage as the relevant determinant; while if $s = 0$, the "personal norm" case, the worker considers their own past wage within the firm as the relevant determinant. As such, the gift of the firm is:

$$\begin{aligned} g(w_t, \cdot) = \log[(1-\tau_t)w_t] \\ - \varphi_1 \log[(1-\tau_t)l_t y_t/n_t] - \varphi_2 \log[(1-\tau_t)\overline{w}_t\overline{n}_t] \\ - \varphi_3 \log[(1-\tau_t)[s\overline{w}_{t-1}+(1-s)w_{t-1}]] \end{aligned} \tag{DK8}$$

The strategy of this encompassing specification is to let the data speak, therefore they do not impose any conditions on the parameters φ_1, φ_2, φ_3 and s.

Then the model is estimated and analysed under various shocks (the methodology being consistent with the standard DSGE approach; details are omitted here). Some key results are the following. First, the model does not generate wage rigidity. However, it generates wage stickiness, the extent of which depends on the importance of past wages and rent sharing, as opposed to the outside option, in the workers' reference wage. In fact, past wages and rent sharing contribute to reduce the flexibility of wages to shocks (sometimes even inducing countercyclical behaviour), whilst the outside option increases the flexibility of wages. Moreover, the estimation results suggest that, for an estimate of $s = 1$, the most important determinants of the reference wage are past wages ($\varphi_3 = 0.68$), and firms' performance ($\varphi_1 = 0.27$), while external market conditions only play a minor role ($\varphi_2 = 0.05$). Given the implausibility of the estimate of s, Danthine and Kurmann (2010) perform a robustness check by setting $s = 0.25$. In this exercise it is found that the only important determinant of workers' reference wages is their own past wage ($\varphi_3 = 0.914$), with the parameter on labour market conditions slightly higher ($\varphi_2 = 0.086$); while firms' performance is found to be irrelevant ($\varphi_1 = 0$). These results are entirely consistent with the assumptions and qualitative predictions of the theory developed in Chapter 1.

Eliaz and Spiegler (2014)

In this paper Eliaz and Spiegler (2014) introduce a reduced-form reference-dependent production function into a canonical search and matching framework à la Mortensen and Pissarides (1994), in which workers do not have bargaining power and their non-market payoff is proportional to productivity. Their objective is to qualitatively analyse the role of reference-dependence, contractual incompleteness and fairness on firms' layoff and hiring decisions and hence on the volatility of vacancies and unemployment.

Workers get the wage w if employed and the payoff zq if unemployed, where z captures unemployment benefits and q is productivity. Moreover they are assumed to be reference dependent. Newly hired workers are assumed to have "modest aspirations", i.e. they feel entitled to be paid the lowest admissible wage $r_0 = 0$. Instead incumbent workers' reference wage is given by their lagged expectations of the wage they would get in equilibrium (à la Kőszegi and Rabin (2006)), i.e. $r_1 = \mathbb{E}_0[w_1]$.

The firm's payoff is $\pi = y_t - w_t$, in which y_t is a reference-dependent production function:

$$y_t = \begin{cases} q_t & \text{if } w_t \geq r_t \\ \gamma_t q_t & \text{if } w_t < r_t; \end{cases} \tag{ES2}$$

where $\gamma_t \in [0, 1]$ is a random parameter representing the fraction of output loss due to worker demoralisation when their wage falls below the reference wage. It captures the effect of wage disappointment on workers' output, or implicitly, as they say, the extent to which the labour contract is incomplete. It is assumed that γ_t is i.i.d. according to a cumulative distribution function G that has no mass point in $[0, 1)$. Moreover, $G(\gamma) < 1$ for every $\gamma < 1$ (see p. 165). Taken as such, Eliaz and Spiegler's (2014) model essentially imposes, *ad hoc*, a disproportional change into the representative firm's production function whenever the employed worker ends up being paid an unfair wage. The firm's production function displays a step change at $w = r$, the extent of which is randomly determined by the parameter γ_t.

Nevertheless, in Appendix C, Eliaz and Spiegler (2014) provide a microfoundation for their assumed reference-dependent output. Their assumptions and microeconomic model can be presented as follows. A worker is assumed to be committed to a minimal level of effort; then, on top of that, it is assumed that they choose a level of discretionary effort $e_t \in \{0, 1\}$—not observable by the firm and where $e_t = 1$ is referred to "normal effort". As such, the worker's output is assumed to take the following form

$$y_t = q_t[\gamma_t + (1 - \gamma_t)e_t]$$

where γ_t is an indicator of the "completeness of the labour contract", so that $1 - \gamma_t$ captures the importance of discretionary effort in the output function (see p.195).

Employed workers maximise expected discounted payoffs, where the payoff is given by

$$w_t - e_t \cdot \mathbf{1}[w_t < r_t]. \tag{ES23}$$

> "The interpretation is that when the worker's wage is below his reference point, he perceives this as unfair treatment; his intrinsic motivation is damaged, and he strictly prefers not to exert his normal effort. Otherwise, the worker is indifferent between $e_t = 0$ and $e_t = 1$, and we assume that he chooses the latter."(p.195; notations consistent with this thesis)

Hence by denoting Eliaz and Spiegler's (2014) normal effort with $\tilde{e}_n = 1$, it is possible to write employed workers' optimal effort choice in a form that is analogous to that derived in Chapter 1:

$$\tilde{e}_t = \begin{cases} \tilde{e}_n & \text{if } w_t \geq r_t \\ 0 & \text{if } w_t < r_t. \end{cases}$$

It is worth noting that the fact that workers exert their normal effort whenever they receive a wage gift or a fair wage, i.e. whenever $w_t \geq r_t$, has been assumed *a priori*, rather than being derived from the worker's optimising behaviour (see quote above).

At this stage, despite being derived from different modelling approaches, it is now straightforward to notice the key fundamental differences between Eliaz and Spiegler's (2014) effort function with the worker's asymmetric reference-dependent reciprocity derived in in Chapter 1. In fact Eliaz and Spiegler (2014) do not capture positive reciprocity, i.e. supra-normal effort; moreover, they only capture an extreme form of negative reciprocity: even a very small wage cut will induce the worker to exert zero discretionary effort (but notice that they are still committed, by assumption, to a "minimal level of effort"). Thus, the resulting effect of such (supposedly ephemeral) wage cut on output will instead be determined randomly by the draw of γ_t, which is independent of the actual extent of the wage cut. This leads to the following behavioural interpretation: the more a contract is complete (the higher γ_t), the lower is the adverse effect of wage cuts on output.

A.2 Additional Material for Chapter 2

A.2.1 Condition on Normal Effort

This section derives an explicit expression for the level of normal effort $\tilde{e}_n$ that ensures a firm zero-profit condition is always satisfied, i.e. $J(r_C) \geq 0$. Define the minimum level of normal effort that ensures the value of a new employment relationship is non-negative as

$$\underline{e}_n(r_N, \lambda) \equiv \max\{\tilde{e}_n : J(r_N) = 0, \forall r_N \geq 0\}.$$

Lemma 5. *The minimum level of normal effort takes the following form*

$$\underline{e}_n(r_N, \lambda) = \begin{cases} (1-\psi)\big[1 - \big(\ln(pq(1-\psi)) - \ln r_N\big)\big] & \forall r_N < r_L \\ r_N/pq & \forall r_N \in [r_L, r_H] \\ (1-\psi)\lambda\big[1 - \big(\ln(\lambda pq(1-\psi)) - \ln r_N\big)\big] & \forall r_N > r_H(\lambda). \end{cases}$$

Proof. The steady-state value of a new employment relationship for all $\overline{y}^*(\tilde{e}_N^*) = \{\overline{y}(\tilde{e}_N^+), \overline{y}(\tilde{e}_n), \overline{y}(\tilde{e}_N(\lambda)^-)\}$ and $\overline{w}^* \in [\overline{w}_L^+, \overline{w}_H(\lambda)^-]$ can be written as

$$\begin{aligned} J(r_N) &= \overline{y}^*(\tilde{e}_N^*) - \overline{w}^* \\ &= y(\tilde{e}_N^*) + \frac{\psi}{1-\psi} y(\tilde{e}_n) - \frac{\tilde{w}^*}{1-\psi} \end{aligned}$$

where $y(\tilde{e}_N^*) = pq\tilde{e}_n + pq\mu(\ln w^* - \ln r_N)$ and $y(\tilde{e}_n) = pq\tilde{e}_n$. Substituting these values into the above equation, the zero-profit condition $J(r_N) = 0$ that needs to be satisfied can be written as

$$\frac{pq\tilde{e}_n}{1-\psi} + pq\mu\big(\ln \tilde{w}^* - \ln r_N\big) - \frac{\tilde{w}^*}{1-\psi} = 0,$$

where $\mu(\cdot) = 0$ for all $\tilde{w}^* \in (\overline{w}_L^+, \overline{w}_H(\lambda)^-)$. Next, solving for normal effort yields

$$\tilde{e}_n = (1-\psi)\left[\frac{\tilde{w}^*}{pq(1-\psi)} - \mu\big(\ln \tilde{w}^* - \ln r_N\big)\right]$$

and substituting $\tilde{w}^*$ for each case yields the expressions used in Lemma 5. ∎

Note that since $J(r_N)^- < J(r_N)^= < J(r_N)^+$ and $\frac{\partial J(r_C)}{\partial \tilde{e}_n} > 0$, it is straightforward to infer that if the condition for $J(r_N)^-$ is satisfied, then all the others will be automatically satisfied.

A.2.2 Workers' Value Functions and Reservation Wage Condition

This section defines, and subsequently derives, an expression for the unemployed workers' reservation wage. Consider a steady-state equilibrium. An incumbent worker's steady-state value of being employed can be written as:

$$W(\tilde{w}^*, r_E) = u(\tilde{e}_E^*; \tilde{w}^*, r_E) + \delta\,[\rho U + (1-\rho)W(\tilde{w}^*, r_E)]\,; \tag{A.3}$$

A newly hired worker's steady-state value of being employed can e written as:

$$W(\tilde{w}^*, r_N) = u(\tilde{e}_N^*; \tilde{w}^*, r_N) + \delta\,[\rho U + (1-\rho)W(\tilde{w}^*, r_E)]\,; \tag{A.4}$$

whilst an unemployed worker's steady-state value of being unemployed can be written as:

$$U = u(z) + \delta\,[f(\theta)W(\tilde{w}^*, r_N) + (1 - f(\theta))U]\,; \tag{A.5}$$

where

$$\begin{aligned}
u(\tilde{e}_E^*; \tilde{w}^*, r_E) &= \ln \tilde{w}^* + b\tilde{e}_n - \tilde{e}_n^2/2;\\
u(\tilde{e}_N^*; \tilde{w}^*, r_N) &= \ln \tilde{w}^* + \mu(\ln \tilde{w}^* - \ln r_N) + b\tilde{e}_N^* - (\tilde{e}_N^*)^2/2 + \tilde{e}_N^* \cdot \mu(\ln \tilde{w}^* - \ln r_N);\\
u(z) &= \ln z;
\end{aligned}$$

and in which z captures the value of unemployment benefit and leisure. In addition denote the present discounted value of an employed worker's utility by

$$\begin{aligned}
\overline{u}^*(\tilde{w}^*) &\equiv u(\tilde{e}_N^*; \tilde{w}^*, r_N) + \sum_{t=\tau+1}^{\infty} \psi^{t-\tau} u(\tilde{e}_E^*; \tilde{w}^*, r_E)\\
&\equiv u(\tilde{e}_N^*; \tilde{w}^*, r_N) + \frac{\psi}{1-\psi} u(\tilde{e}_E^*; \tilde{w}^*, r_E).
\end{aligned}$$

Rearranging the workers' Bellman equations above using this notation yields:

$$W(\tilde{w}^*, r_E) = \frac{u(\tilde{e}_E^*; \tilde{w}^*, r_E) + \delta\rho U}{1-\psi} \tag{A.6}$$

$$W(\tilde{w}^*, r_N) = \overline{u}^*(\tilde{w}^*) + \frac{\delta\rho}{1-\psi} U \tag{A.7}$$

$$U = \frac{u(z) + \delta f(\theta) W(\tilde{w}^*, r_N)}{1 - \delta(1 - f(\theta))} \tag{A.8}$$

In the canonical search and matching model it is assumed that an unemployed worker accepts a job only if $W(\tilde{w}^*, r_N) \geq U$. Hence, the reservation wage is defined as follows:

$$\underline{w} = \max\{0, w : W(w, r_N) = U\}.$$

To find an expression for the reservation wage $\underline{w}$ the analysis will proceed following the standard approach in the literature. First, substitute this wage into (A.7) and use the definition of $\underline{w}$ to write

$$\overline{u}^*(\underline{w}) = U \cdot \left[\frac{1-\psi-\delta\rho}{1-\psi}\right]. \tag{A.9}$$

where the left-hand side of this equation is given by

$$\overline{u}^*(\underline{w}) \equiv u(\tilde{e}_N^*; \underline{w}, r_N) + \sum_{t=\tau+1}^{\infty} \psi^{t-\tau} u(\tilde{e}_E^*; \underline{w}, r_E),$$

which captures the worker's present discounted value of utility from a job that pays the reservation wage $\underline{w}$. Next substitute $W(w, r_N)$ evaluated at $\underline{w}$ from (A.7) out of (A.8) and rearrange it to obtain:

$$U = u(z) \cdot \left[\frac{(1-\psi)}{\breve{\psi}(\theta)}\right] + \overline{u}^*(\underline{w}) \cdot \left[\frac{(1-\psi)\delta f(\theta)}{\breve{\psi}(\theta)}\right] \tag{A.10}$$

where

$$\breve{\psi}(\theta) = (1-\psi)(1-\delta) + \delta f(\theta)(1-\psi-\delta\rho)$$

Finally making use of (A.9) to substitute for U in (A.10), and rearranging, the following expression is obtained:

$$\overline{u}^*(\underline{w}) = u(z) \cdot \left[\frac{1-\psi-\delta\rho}{(1-\psi)(1-\delta)}\right]. \tag{A.11}$$

This equation gives a condition that a worker's reservation wage $\underline{w}$ needs to satisfy.

By either setting $U = 0$ in (A.9), or $z = 0$ in (A.11) (as for instance in Michaillat (2012)), the condition that needs to be satisfied by the reservation wage will collapse to $\overline{u}^*(\underline{w}) = 0$. This implies that the reservation wage is the wage such that the anticipated present discounted value of utility from a new employment relationship, denoted here by $\overline{u}^*(w)$, must be non-negative. This condition is satisfied in the present framework by any non-negative $m(w) = \ln w$. Numerical simulations of the model reveal that if the firms' zero profit condition is satisfied, i.e. if workers' normal effort satisfies the condition derived in Lemma 5, then the workers' reservation wage condition (A.11) will be always satisfied.

A.2.3 Effort Phase Diagram: Details

To construct the transitional dynamics of employed workers' effort consider (2.21) (omitting functions' arguments). Starting from any r_0, employed workers' effort for all $t \geq 0$ is given by

$$\tilde{e}_t = \tilde{e}_n + \mu(\ln w_t - \ln r_t),$$

and it is known from the steady-state equilibrium results that $r_{t+1} = \tilde{w}_t = \tilde{w}^*$ and that $\tilde{e}_{t+1} = \tilde{e}_n$ for all $t \geq 0$. Hence, the equation above can be rearranged as:

$$\tilde{e}_{t+1} = \tilde{e}_t - \mu(\ln r_{t+1} - \ln r_t), \quad \forall t \geq 0.$$

Define $\Delta e_t = \tilde{e}_{t+1} - \tilde{e}_t$ and $\Delta r_t = r_{t+1} - r_t$. It is possible to deduce that $\Delta e_t < 0$ whenever $\Delta r_t > 0$ and that $\Delta e_t > 0$ whenever $\Delta r_t < 0$. It follows that $\Delta e_t = -\Delta r_t$, which gives a unique demarcation curve, and implies that $\Delta e_t = 0$ only if $\Delta r_t = 0$.

A.2.4 Comparison with Kudlyak (2014)

Kudlyak's analysis. In her influential paper Kudlyak argues that the relevant measure of the price of labour is not the average wage or the hiring wage, but rather what she defines as the *user cost of labour*: the difference between the expected present value of wages paid to a worker hired in t and the expected present value of wages paid to a worker hired in $t+1$. Using notations consistent with Chapter 2, the wage component of the user cost of labour as defined and derived by Kudlyak is:

$$UC_\tau^W = w_{\tau,\tau} + \mathbb{E}_\tau \left[\sum_{t=\tau+1}^{\infty} \psi^{t-\tau}(w_{\tau,t} - w_{\tau+1,t}) \right]. \tag{K2}$$

The user cost of labour in period τ is the sum of the hiring wage in τ and the expected present value of the differences between wages paid from the next period onward in the match that starts in τ and the match that starts in $\tau+1$. Two special cases in which the user cost equals the wage are (i.e. for which $UC^W = w$):

i) Nash bargaining period by period, for which $w_{\tau,t} = w_{\tau+1,t} = w_t \; \forall t \geq \tau + 1$;

ii) complete rigid wages, so that $w_{\tau,t} = w_{\tau+1,t} = w \; \forall t \geq \tau + 1$.

The vacancy component of the user cost of labour is defined along the same logic as:

$$UC_\tau^V = \frac{\kappa}{h(\theta_\tau)} - \psi \mathbb{E}_\tau \left[\frac{\kappa}{h(\theta_{\tau+1})} \right], \tag{K4}$$

that is, the difference between the costs associated with vacancy opening to hire a worker in τ and in $\tau + 1$. The user cost of labour in a search and matching model can then be defined as $UC_\tau = UC_\tau^W + UC_\tau^V$.

Given free entry (and instantaneous production in the same period of vacancy posting), Proposition 1 (Kudlyak, 2014, p.59) establishes that firms create jobs in τ as long as the marginal benefit of adding a worker equals the user cost of labour (see Kudlyak (2014, Appendix) for the derivation of this expression):

$$y_\tau = \underbrace{\left(w_{\tau,\tau} + \mathbb{E}_\tau\left[\sum_{t=\tau+1}^{\infty} \psi^{t-\tau}(w_{\tau,t} - w_{\tau+1,t})\right]\right)}_{UC_\tau^W} + \underbrace{\left(\frac{\kappa}{h(\theta_\tau)} - \psi\mathbb{E}_\tau\left[\frac{\kappa}{h(\theta_{\tau+1})}\right]\right)}_{UC_\tau^V}; \qquad \text{(K6)}$$

where in Kudlyak (2014), as it is in the canonical model, $y_t = p_t$. To quantitatively analyse the role of the wage component of the user cost of labour for the volatility of vacancies and unemployment, Kudlyak rewrites equation (K6) in terms of elasticities with respect to productivity. Following her steps, consider the steady-state version of (K6):

$$p = w + (1 - \psi)\frac{\kappa}{\bar{m}\theta^{-\sigma}},$$

which, after total differentiation with respect to p, yields:

$$1 = \varepsilon_{UC^W,p}\frac{UC^W}{p} + \varepsilon_{UC^V,p}\frac{UC^V}{p}. \qquad \text{(K7)}$$

Algebraic manipulation of the expression above yields the key equation derived by Kudlyak (Kudlyak, 2014, equation (9) p.60):

$$0 < \frac{1 - \varepsilon_{UC^W,p}}{\sigma\varepsilon_{\theta,p} - \varepsilon_{UC^W,p}} < 1, \qquad \text{(K9)}$$

where $\varepsilon_{\theta,p}$ is the elasticity of market tightness with respect to p and σ is the match elasticity. At this stage, Kudlyak makes the following point: this equation holds if $\varepsilon_{UC^W,p} < 1 < \sigma\varepsilon_{\theta,p}$ or $\sigma\varepsilon_{\theta,p} < 1 < \varepsilon_{UC^W,p}$. Given that $\varepsilon_{\theta,p} = 7.56$ and that $\sigma \in [0.232, 0.72]$ as estimated in the existing literature, for the above equation to hold it should be the case that the elasticity of the wage component of the user cost of labour is less than one, i.e. $\varepsilon_{UC^W,p} < 1$ (Essentially Shimer's argument). However, by using her combined estimate of $\varepsilon_{UC^W,p} = 1.5$, Kudlyak demonstrates that the unemployment volatility puzzle cannot be explained by wage rigidity, since the free entry condition of the model cannot simultaneously accommodate the empirical volatilities of the wage component of the user cost of labour (1.5) and of the vacancy-unemployment ratio

(7.56). Since this result does not depend on any particular wage setting mechanism, Kudlyak concludes that the unemployment volatility puzzle cannot be explained by wage formation:

> "Importantly, the conclusion does not depend on a particular wage formation. [...] Consequently, the solution for the unemployment volatility puzzle cannot be explained by a wage formation. This is so because, even if a particular wage formation delivers wage rigidity theoretically, it should generate the empirical elasticity of the wage component of the user cost of labor."(p.60)

The framework of Chapter 2. Before providing a comparison with the conclusions of Kudlyak, recall that the instantaneous value of output is a function of the worker's effort $y = y(\tilde{e}^*) = pq\tilde{e}^*$; and that, due to the optimal wage policy given by (3.23), the hiring wage also equals the average wage and the wage component of the user cost of labour, $UC^W_\tau = \tilde{w}^*_t \; \forall t \geq \tau$.

To derive a comparable expression with that of Kudlyak's Proposition 1 (i.e. the steady-state version of equation (K6)), let us follow her strategy by calculating the difference between hiring this period *versus* hiring the next period, for all $\mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}$ (since the predictions of the model when $\mathcal{E} = \mathcal{M}^*$, i.e. complete wage rigidity, are not comparable with Kudlyak). Hence, the following expression characterises the equivalent of the steady-state version of (K6) in the context of Chapter 2:

$$\underbrace{(1-\psi)y(\tilde{e}^*_N) + \psi y(\tilde{e}_n)}_{output} = \underbrace{\tilde{w}^*}_{UC^W} + \underbrace{(1-\psi)\frac{\kappa}{\delta\bar{m}\tilde{\theta}^{*-\sigma}}}_{UC^V} .$$

Note that in the present framework the "benefit side" of the job creation condition now accounts for a systematic difference in productivity between newly hired $y(\tilde{e}^*_N)$ and incumbent workers $y(\tilde{e}_n)$: new hires can either exert positive or negative reciprocity in their first employment period; while incumbents will always exert normal effort. This systematic difference is absent in Kudlyak, but it has been recognised by her to be important (see Kudlyak (2014, footnote 13, p.59)). Total differentiation of the equation above with respect to p yields (after collecting $\frac{\partial \tilde{w}^*}{\partial p}$ as the common factor from both sides):

$$(1-\psi)q\tilde{e}_N^* + \psi q\tilde{e}_n = \varepsilon_{UC^W,p}\frac{UC^W}{p}\underbrace{\left(1-(1-\psi)\frac{\partial y}{\partial e}\frac{\partial \tilde{e}_N^*}{\partial w}\right)}_{\Theta\equiv-\Upsilon(w,r_C,\lambda)=0} + \varepsilon_{UC^V,p}\frac{UC^V}{p}$$

$$(1-\psi)q\tilde{e}_N^* + \psi q\tilde{e}_n = \varepsilon_{UC^V,p}\frac{UC^V}{p}.$$

Hence, for the same reasons highlighted in Section 2.5.2 of Chapter 2, since a change in the wage also induces a change in the value of output due to a change in workers' effort—i.e. in the "benefit side" of the job creation condition—the elasticity of the wage component of the user cost of labour, i.e. the hiring and average wage in this model, disappears from this expression. Following the steps of Kudlyak, the above equation can be rearranged as

$$0 < \frac{1}{\delta\sigma\varepsilon_{\theta,p}} < 1.$$

However notice that, in contrast with Kudlyak, given the values for $\delta \approx 0.996$, $\sigma \in [0.232, 0.72]$ and $\varepsilon_{\theta,p} = 7.56$, the expression above always holds. As such, in the framework set out in Chapter 2, the elasticity of the wage component of the user cost of labour is irrelevant for the determination of the size of the elasticity of labour market tightness (as formally established by Proposition 13, Section 2.5.2; and Corollary 2 of Proposition 18, Section 2.6.5). Moreover, as it is shown in the calibration exercise performed in Section 2.5.3, the model can simultaneously deliver plausible elasticity measures of both the hiring wage and market tightness, by appealing to the behavioural mechanisms which affect the benefits side of job creation, namely workers' reference wage entitlements and their optimal effort decisions under reference-dependent preferences.

A.3 Additional Material for Chapter 3

A.3.1 Condition on the Productivity Shock

This section derives a condition for the magnitude of ε_{t_0} such that the model developed in Section 3.5.1, Chapter 3 does not endogenously generate downward wage rigidity in response to negative shocks. To begin with, as established in Proposition 21, consider the firm's optimal wage policy at impact in period t_0:

$$\hat{w}_{t_0} = \hat{w}(r_{t_0}, \alpha, \lambda) = \begin{cases} \hat{w}(r_{t_0}, \alpha^+)^+ & \text{if } r_{t_0} < \hat{r}_L(\alpha^+) \\ r_{t_0} & \text{if } r_{t_0} \in [\hat{r}_L, \hat{r}_H] \\ \hat{w}(r_{t_0}, \alpha^-, \lambda)^- & \text{if } r_{t_0} > \hat{r}_H(\alpha^-, \lambda) \end{cases}$$

where $r_{t_0} = r^* = r_L(\alpha^+) = pq(1 - \psi\alpha^+)$ and

$$\hat{r}_L(\alpha^+) \equiv \hat{p}q(1 - \psi\alpha^+)$$
$$\hat{r}_H(\alpha^-, \lambda) \equiv \lambda\hat{p}q(1 - \psi\alpha^-).$$

It is useful at this stage to re-express the wage setting policy just derived, making it contingent on the level of p rather than the level of r. That is:

$$\hat{w}_{t_0} = \hat{w}(r_{t_0}, \alpha, \lambda) = \begin{cases} \hat{w}(r_{t_0}, \alpha^+)^+ & \text{if } \hat{p} > \hat{p}_u(r_{t_0}, \alpha^+) \\ r_{t_0} & \text{if } \hat{p} \in [\hat{p}_l, \hat{p}_u] \\ \hat{w}(r_{t_0}, \alpha^-, \lambda)^- & \text{if } \hat{p} < \hat{p}_l(r_{t_0}, \alpha^-, \lambda) \end{cases}$$

where $r_{t_0} = r^* = r_L(\alpha^+) = pq(1 - \psi\alpha^+)$ and

$$\begin{aligned} \hat{p}_u(r_{t_0}, \alpha^+) &\equiv \left\{p : \lim_{\epsilon \to 0} \Upsilon(r + \epsilon; r, \alpha^+, \lambda) = 0\right\} \\ &\equiv \frac{r_{t_0}}{q(1 - \psi\alpha^+)} = \frac{pq(1 - \psi\alpha^+)}{q(1 - \psi\alpha^+)} = p \end{aligned} \tag{A.12}$$

$$\begin{aligned} \hat{p}_l(r_{t_0}, \alpha^-, \lambda) &\equiv \left\{p : \lim_{\epsilon \to 0} \Upsilon(r + \epsilon; r, \alpha^-, \lambda) = 0\right\} \\ &\equiv \frac{r_{t_0}}{\lambda q(1 - \psi\alpha^-)} = \frac{pq(1 - \psi\alpha^+)}{\lambda q(1 - \psi\alpha^-)} = \frac{p(1 - \psi\alpha^+)}{\lambda(1 - \psi\alpha^-)}. \end{aligned} \tag{A.13}$$

Hence, in order to ensure that the model does not generate downward wage rigidity—i.e. the firm will not freeze the wage in response to negative shocks—it must be that after a negative shock:

$$\hat{p} < \hat{p}_l(r_{t_0}, \alpha^-, \lambda).$$

In order to find the value of the shock ε_{t_0} such that this condition is satisfied, substitute for both sides of the equation above using the fact that $\hat{p} = p - \varepsilon_{t_0}$ and the expression for $\hat{p}_l(r_{t_0}, \alpha^-, \lambda)$ as given by (A.13):

$$p - \varepsilon_{t_0} < \frac{p(1 - \psi\alpha^+)}{\lambda(1 - \psi\alpha^-)},$$

and solve for ε_{t_0}. After some algebra, the necessary condition that needs to be satisfied by the shock is therefore

$$\varepsilon_{t_0} > \frac{p[(\lambda - 1) + \psi(\alpha^+ - \lambda\alpha^-)]}{\lambda(1 - \psi\alpha^-)}.$$

A.3.2 Details on Section 3.5.2: Cyclical Analysis and Hysteresis

This section develops more formally the firms' optimal wage setting policy on which the analysis performed in Section 3.5.2 is based. To begin with, consider the firms' functional equation for all $t \geq \tau$:

$$J(r_t) = \max_{w_t \geq 0} \left\{ p_t q \tilde{e}_t(w_t, r_t, \lambda) - w_t + \psi J(r_{t+1}) \right\}.$$

The first-order necessary and sufficient condition which characterises the optimal wage setting policy is given by:

$$\Upsilon(w_t; r_t, p_t, \lambda) \equiv p_t q \frac{\partial \tilde{e}_t}{\partial w_t} - 1 + \psi p_t q \frac{\partial \tilde{e}_t}{\partial r_t} = 0 \quad \forall w \neq r.$$

At this stage, it is useful to express the firms' optimal wage setting policy in terms of aggregate productivity thresholds for what concerns incumbent workers j, i.e. for all $t > \tau$, and it terms of reference wage thresholds for what concerns newly hired workers i, i.e. for all $t = \tau$. Hence, the optimal wage setting policy for incumbent workers is given by:

$$\tilde{w}_{j,t} = \begin{cases} p_t q(1 - \psi) & \text{if } p_t > p_u(r_{j,t}) \\ r_{j,t} & \text{if } p_t \in [p_l, p_u] \\ \lambda p_t q(1 - \psi) & \text{if } p_t < p_l(r_{j,t}, \lambda), \end{cases} \tag{A.14}$$

where

$$p_u(r_{j,t}) \equiv \left\{p_t : \lim_{\epsilon \to 0} \Upsilon(r_{j,t} + \epsilon; r_{j,t}, p_t, \lambda) = 0\right\} \equiv \frac{r_{j,t}}{q(1-\psi)}$$
$$p_l(r_{j,t}, \lambda) \equiv \left\{p_t : \lim_{\epsilon \to 0} \Upsilon(r_{j,t} - \epsilon; r_{j,t}, p_t, \lambda) = 0\right\} \equiv \frac{r_{j,t}}{\lambda q(1-\psi)},$$

and the optimal wage setting policy for new hires' is given by:

$$\tilde{w}_{i,t} = \begin{cases} p_t q(1-\psi) & \text{if } r_{i,t} < r_L(p_t) \\ r_{i,t} & \text{if } r_{i,t} \in [r_L, r_H] \\ \lambda p_t q(1-\psi) & \text{if } r_{i,t} > r_H(p_t, \lambda), \end{cases} \tag{A.15}$$

where

$$r_L(p_t) \equiv \left\{r_{i,t} : \lim_{\epsilon \to 0} \Upsilon(r_{i,t} + \epsilon; r_{i,t}, p_t, \lambda) = 0\right\} \equiv p_t q(1-\psi)$$
$$r_H(p_t, \lambda) \equiv \left\{r_{i,t} : \lim_{\epsilon \to 0} \Upsilon(r_{i,t} - \epsilon; r_{i,t}, p_t, \lambda) = 0\right\} \equiv \lambda p_t q(1-\psi).$$

Starting from the initial steady state as characterised in Proposition 24, an increase in aggregate productivity in period t' from p to $p_{t'}$ (of any magnitude) implies that:

$$\begin{aligned} p_{t'} &> p \\ &= pq(1-\psi)/q(1-\psi) \\ &= r_j^*/q(1-\psi) \\ &= p_u(r_j^*) \end{aligned}$$

where the third line follows from the fact that in the steady state $r_j^* = pq(1-\psi)$. Hence, $p_{t'} > p_u(r_j^*)$ and according to (A.14), incumbent workers receive a wage raise to $\tilde{w}_{j,t'} = p_t' q(1-\psi)$. This wage raise generates a reference wage entitlement to newly hired workers due to social comparison, since $r_{i,t'} = \tilde{w}_{j,t'}$. As such:

$$\begin{aligned} r_{i,t'} &= p_t' q(1-\psi) \\ &= r_L(p_{t'}), \end{aligned}$$

which implies that, according to (A.15), newly hired workers are paid their reference wage $\tilde{w}_{i,t'} = r_{i,t'}$ and exert normal effort $\tilde{e}_{i,t'} = \tilde{e}_n$. As such, in the steady state: $\tilde{w}_i' = r_i' = \tilde{w}_j' = r_j' = p_{t'} q(1-\psi)$ and $\tilde{e}_i' = \tilde{e}_j' = \tilde{e}_n$.

Next consider a decrease in aggregate productivity in period t'' from $p_{t'}$ back to p of the same magnitude of the initial increase in period t' from p to $p_{t'}$. Depending on this initial magnitude, optimal wage setting in the recessionary episode of the cycle will differ.

Consider first the case of $\frac{p_{t'}-p}{p} \in (0, \lambda - 1]$ and $p_{t''} = p$, i.e. the moderate cycle scenario. Since $p_{t'} \leq \lambda p$, this implies that

$$\begin{aligned} p_{t''} &< p_{t'} \\ &= p_{t'}q(1-\psi)/q(1-\psi) \\ &= r'_j/q(1-\psi) \\ &= p_u(r'_j), \end{aligned}$$

where the third line follows from the fact that in the steady state, after the expansion, $r'_j = p_{t'}q(1-\psi)$. Moreover, note that

$$\begin{aligned} p_{t''} &\geq p_{t'}/\lambda \\ &= p_{t'}q(1-\psi)/\lambda q(1-\psi) \\ &= r'_j/\lambda q(1-\psi) \\ &= p_l(r'_j, \lambda), \end{aligned}$$

where the first line is a direct construct of the moderate cycle. Hence $p_{t''} \in [p_l, p_u)$ which implies that, according to (A.14), incumbent workers are paid their reference wage $\tilde{w}_{j,t''} = r'_j = \tilde{w}'_j = p_{t'}q(1-\psi)$, i.e. they experience downward wage rigidity following a decrease in aggregate productivity. As a consequence, new hires' reference wage in period t'' is given by $r_{i,t''} = \tilde{w}_{j,t''}$. As such:

$$\begin{aligned} r_{i,t''} &= p_{t'}q(1-\psi) \\ &> p_{t''}q(1-\psi) \\ &= r_L(p_{t''}), \end{aligned}$$

and

$$\begin{aligned} r_{i,t''} &= p_{t'}q(1-\psi) \\ &\leq \lambda p_{t''}q(1-\psi) \\ &= r_H(p_{t''}, \lambda), \end{aligned}$$

since as stated above $p_{t'} \leq \lambda p_{t''}$. Hence $r_{i,t''} \in (r_L, r_H]$ which implies that, according to (A.15), newly hired workers are again paid their reference wage $\tilde{w}_{i,t''} = r_{i,t''}$ and exert normal effort $\tilde{e}_{i,t''} = \tilde{e}_n$. As such, in the steady state: $\tilde{w}''_i = r''_i = \tilde{w}''_j = r''_j = p_{t'}q(1-\psi)$

and $\tilde{e}_i'' = \tilde{e}_j'' = \tilde{e}_n$. It is therefore straightforward to notice that despite aggregate productivity is back at its original level $p_{t''} = p$, the steady-state equilibrium wage is now higher $\tilde{w}'' > \tilde{w}^*$.

Finally consider first the case of $\frac{p_{t'}-p}{p} > (\lambda - 1)$ and $p_{t''} = p$, i.e. the large cycle scenario. Since $p_{t'} > \lambda p$, this implies that

$$\begin{aligned} p_{t''} &< p_{t'} \\ &= r_j'/q(1-\psi) \\ &= p_u(r_j'), \end{aligned}$$

but also, and in contrast with the moderate cycle case, that

$$\begin{aligned} p_{t''} &< p_{t'}/\lambda \\ &= r_j'/\lambda q(1-\psi) \\ &= p_l(r_j', \lambda), \end{aligned}$$

Hence $p_{t''} < p_l(r_j', \lambda)$ which implies that, according to (A.14), incumbent workers receive a wage cut to $\tilde{w}_{j,t''} = \lambda p_{t''} q(1-\psi)$. Again, due to social comparison new hires' reference wage in period t'' is given by $r_{i,t''} = \tilde{w}_{j,t''}$. As such:

$$\begin{aligned} r_{i,t''} &= \lambda p_{t''} q(1-\psi) \\ &= r_H(p_{t''}, \lambda). \end{aligned}$$

Hence $r_{i,t''} = r_H(p_{t''}, \lambda)$ which implies that, according to (A.15), newly hired workers are paid their reference wage $\tilde{w}_{i,t''} = r_{i,t''}$ and exert normal effort $\tilde{e}_{i,t''} = \tilde{e}_n$. As such, in the steady state: $\tilde{w}_i'' = r_i'' = \tilde{w}_j'' = r_j'' = \lambda p_{t''} q(1-\psi)$ and $\tilde{e}_i'' = \tilde{e}_j'' = \tilde{e}_n$. It is therefore straightforward to notice that despite aggregate productivity is back at its original level $p_{t''} = p$, the steady-state equilibrium wage is now higher $\tilde{w}'' > \tilde{w}^*$.

Appendix B

Proofs

B.1 Proofs to Chapter 1

Proof of Theorem 1. Assumptions W1-W4 hold throughout. When $w = r$, $m(w) = m(r)$ and therefore $\Omega(e; w, r, \lambda) = b'(e) - c'(e)$. By assumption, $\Omega(0; r, r, \lambda) > 0$ and $\partial\Omega(e, w, r, \lambda)/\partial e < 0$, which implies $\tilde{e}(r, r, \lambda) \equiv \tilde{e}_n > 0$. Recalling the definition of $\mu(\cdot)$ in (1.3), when $w > r$, $\Omega(e; w, r, \lambda) = b'(e) - c'(e) + \zeta\eta[m(w) - m(r)]$ with $m(w) - m(r) > 0$. As such $\Omega(\tilde{e}_n; w, r, \lambda) > \Omega(\tilde{e}_n; r, r, \lambda)$ and then the fact that $\partial\Omega(e, w, r, \lambda)/\partial e < 0$ implies $\tilde{e}(w, r)^+ > \tilde{e}_n$ for all $w > r$. When $w < r$, $\Omega(e; w, r, \lambda) = b'(e) - c'(e) + \zeta\lambda\eta[m(w) - m(r)]$ with $m(w) - m(r) < 0$, so $\Omega(\tilde{e}_n; w, r, \lambda) < \Omega(\tilde{e}_n; r, r, \lambda)$ so $\partial\Omega(e, w, r, \lambda)/\partial e < 0$ implies $\tilde{e}(w, r, \lambda)^- < \tilde{e}_n$.

When $w < r$, the wage offered may be such that the worker would optimally choose $e < 0$ but cannot due to the constraint that $e \geq 0$. Define

$$\underset{\sim}{w}(r, \lambda) = \max\{0, w : \Omega(0; w, r, \lambda) = 0\}.$$

Since $\partial\Omega(e, w, r, \lambda)/\partial w > 0$ this identifies the threshold wage at which the worker would choose $e = 0$ and below which they would like to choose $e < 0$ (since $\Omega(e; w, r, \lambda) < 0$ for all $e \geq 0$) but cannot; hence define $\tilde{e}(\underset{\sim}{w}(r, \lambda), r, \lambda)^- \equiv 0$ for all $w \leq \underset{\sim}{w}(r, \lambda)$.

For $w \neq r$, continuity of $\tilde{e}(w, r, \lambda)$ is readily established as $\Omega(e; w, r, \lambda)$ is continuous in all its arguments. Continuity at $w = r$ is established by noting that in the expression for $\Omega(e; w, r, \lambda)$, $\lim_{\epsilon \to 0} m(r + \epsilon) - m(r) = 0$ which implies $\tilde{e}(w, r)^+ \to \tilde{e}_n$ as $w \to r$ from above, and $\lim_{\epsilon \to 0} m(r - \epsilon) - m(r) = 0$ implying $\tilde{e}(w, r, \lambda)^- \to \tilde{e}_n$ as $w \to r$ from below.

When $w \neq r$ and $w > \underset{\sim}{w}(r, \lambda)$ implicit differentiation of the first-order condition reveals

$$\frac{\partial \tilde{e}(w, r, \lambda)}{\partial w} = -\frac{\zeta\mu'(m(w) - m(r))m'(w)}{b''(e) - c''(e)} > 0.$$

Further differentiating this expression (and recalling that $\mu(\cdot)$ is piecewise linear) yields:

$$\frac{\partial^2 \tilde{e}(w, r, \lambda)}{\partial w^2} = -\frac{\zeta\mu'(m(w) - m(r))m''(w)}{b''(e) - c''(e)} < 0.$$

Note from (1.3) that for $w > r$, $\mu'(\cdot) = \eta$ and when $w < r$, $\mu'(\cdot) = \lambda\eta$. To consider the response of effort to the wage above and below the reference wage, the continuity of $\tilde{e}(w, r, \lambda)$ is used to establish that

$$\begin{aligned}
\lim_{\epsilon \to 0} \frac{\partial \tilde{e}(r-\epsilon, r, \lambda)^-}{\partial w} &= -\lim_{\epsilon \to 0} \frac{\lambda \cdot \zeta \eta m'(r-\epsilon)}{b''(\tilde{e}(r-\epsilon, r, \lambda)^-) - c''(\tilde{e}(r-\epsilon, r, \lambda)^-)} \\
&= -\frac{\lambda \cdot \zeta \eta m'(r)}{b''(\tilde{e}_n) - c''(\tilde{e}_n)} \\
&= -\lim_{\epsilon \to 0} \frac{\lambda \cdot \zeta \eta m'(r+\epsilon)}{b''(\tilde{e}(r+\epsilon, r)^+) - c''(\tilde{e}(r+\epsilon, r)^+)} \\
&= \lambda \cdot \lim_{\epsilon \to 0} \frac{\partial \tilde{e}(r+\epsilon, r)^+}{\partial w}.
\end{aligned}$$

Since this implies the effort function kinks to a flatter slope as the wage increases, this result combined with the deduction that $\partial^2 \tilde{e}/\partial w^2 < 0$ for all $w \neq r$, implies $\partial \tilde{e}/\partial w$ is everywhere decreasing in w, i.e. the effort function is concave.

The relationship between $\tilde{e}(w, r, \lambda)$ and r is established by implicit differentiation:

$$\frac{\partial \tilde{e}(w, r, \lambda)}{\partial r} = -\frac{\zeta \mu'(m(w) - m(r)) m'(r)}{b''(e) - c''(e)} < 0.$$

Similarly, the effect of the degree of loss aversion on effort when $w < r$ is

$$\frac{\partial \tilde{e}(w, r, \lambda)^-}{\partial \lambda} = -\frac{\zeta \eta [m(w) - m(r)]}{b''(e) - c''(e)} < 0.$$

Moreover, the effect of the degree of loss aversion on the effort response to the wage (for $w < r$) is

$$\frac{\partial^2 \tilde{e}(w, r, \lambda)^-}{\partial w \partial \lambda} = -\frac{\zeta \eta m'(w)}{b''(e) - c''(e)} > 0.$$

■

Proof of Theorem 2. Throughout the proof it is assumed the worker's productivity and reference wage are such that $q \geq \underline{q}(r, \lambda)$ so the firm will be profitable if it hires the worker, and consider the properties of the threshold productivity at the end. The proof proceeds by first stating some preliminaries, then considering the productivity thresholds, then demonstrating the nature of the optimal wage setting rule.

Preliminaries: First, note that under Assumption F2 and the results of Theorem 1, for $w \neq r$:

$$\begin{aligned}
\frac{\partial \Psi(w; q, r, \lambda)}{\partial q} &= \frac{\partial^2 y(q, e)}{\partial e \partial q} \frac{\partial \tilde{e}(w, r, \lambda)}{\partial w} > 0; \\
\frac{\partial \Psi(w; q, r, \lambda)}{\partial r} &= \frac{\partial^2 y(q, e)}{\partial e^2} \frac{\partial \tilde{e}(w, r, \lambda)}{\partial w} \frac{\partial \tilde{e}(w, r, \lambda)}{\partial r} + \frac{\partial y(q, e)}{\partial e} \frac{\partial^2 \tilde{e}(w, r, \lambda)}{\partial w \partial r} \geq 0,
\end{aligned}$$

after noticing that $\partial^2\tilde{e}/\partial w\partial r = 0$; and

$$\frac{\partial\Psi(w;q,r,\lambda)}{\partial w} = \frac{\partial^2 y(q,e)}{\partial e^2}\left[\frac{\partial\tilde{e}(w,r,\lambda)}{\partial w}\right]^2 + \frac{\partial y(q,e)}{\partial e}\frac{\partial^2\tilde{e}(w,r,\lambda)}{\partial w^2} - s''(w) < 0.$$

In addition,

$$\frac{\partial\Psi(w;q,r,\lambda)}{\partial\lambda} = \frac{\partial^2 y(q,e)}{\partial e^2}\frac{\partial\tilde{e}(w,r,\lambda)}{\partial w}\frac{\partial\tilde{e}(w,r,\lambda)}{\partial\lambda} + \frac{\partial y(q,e)}{\partial e}\frac{\partial^2\tilde{e}(w,r,\lambda)}{\partial w\partial\lambda}$$

which implies that $\partial\Psi/\partial\lambda > 0$ if $w < r$ and $\partial\Psi/\partial\lambda = 0$ if $w > r$. These results also enable to deduce that if $\lambda > 1$, $\Psi(w;q,r,\lambda)$ jumps down at the reference wage, since

$$\begin{aligned}
&\lim_{\epsilon\to 0}\Psi(r-\epsilon;q,r,\lambda) - \lim_{\epsilon\to 0}\Psi(r+\epsilon;q,r,\lambda) = \\
&\qquad \frac{\partial y(q,\lim_{\epsilon\to 0}\tilde{e}(r-\epsilon,r,\lambda)^-)}{\partial e}\lim_{\epsilon\to 0}\frac{\partial\tilde{e}(r-\epsilon,r,\lambda)^-}{\partial w} - s'(r-\epsilon) \\
&\qquad - \frac{\partial y(q,\lim_{\epsilon\to 0}\tilde{e}(r+\epsilon,r)^+)}{\partial e}\lim_{\epsilon\to 0}\frac{\partial\tilde{e}(r+\epsilon,r)^+}{\partial w} - s'(r+\epsilon) \\
&\qquad = \frac{\partial y(q,\tilde{e}_n)}{\partial e}\left[\lim_{\epsilon\to 0}\frac{\partial\tilde{e}(r-\epsilon,r,\lambda)^-}{\partial w} - \lim_{\epsilon\to 0}\frac{\partial\tilde{e}(r+\epsilon,r)^+}{\partial w}\right] \\
&\qquad\qquad = \frac{\partial y(q,\tilde{e}_n)}{\partial e}\lim_{\epsilon\to 0}\frac{\partial\tilde{e}(r+\epsilon,r)^+}{\partial w}[\lambda-1] \geq 0,
\end{aligned}$$

with a strict inequality if $\lambda > 1$. As such, $\Psi(w;q,r,\lambda)$ is everywhere decreasing in w, establishing concavity of the payoff function.

Productivity thresholds: As will be made clear in the remainder of the proof, the threshold $q_l(r,\lambda)$ identifies the critical match productivity below which the firm would want to set the wage below the reference wage, and $q_u(r)$ is the match productivity above which the firm would want to compensate the worker more than the reference wage. The former is the value of q below which profit is decreasing just below the reference wage; the latter is the value of q above which profit is increasing just above the reference wage. Since $\Psi(w,0,r,\lambda) < 0$ when $w > 0$ and $\partial\Psi/\partial q > 0$ there will be a unique value of each productivity threshold.

Next the proof will establish some properties of the thresholds. Implicit differentiation enables to deduce that

$$\frac{\partial q_l(r,\lambda)}{\partial r} = -\lim_{\epsilon\to 0}\frac{d\Psi(r-\epsilon;q,r,\lambda)/dr}{\partial\Psi(r-\epsilon;q,r,\lambda)/\partial q}$$

$$q_u'(r) = -\lim_{\epsilon\to 0}\frac{d\Psi(r+\epsilon;q,r,\lambda)/dr}{\partial\Psi(r+\epsilon;q,r,\lambda)/\partial q},$$

where the numerator in both expressions is given by

$$\begin{aligned}\frac{d\Psi(r\pm\epsilon;q,r,\lambda)}{dr} &= \frac{\partial\Psi(r\pm\epsilon;q,r,\lambda)}{\partial w}+\frac{\partial\Psi(r\pm\epsilon;q,r,\lambda)}{\partial r}\\ &= \frac{\partial^2 y}{\partial e^2}\left[\frac{\partial\tilde{e}^{\pm}}{\partial w}\right]^2+\frac{\partial y}{\partial e}\frac{\partial^2\tilde{e}^{\pm}}{\partial w^2}-s''(w)+\frac{\partial^2 y}{\partial e^2}\frac{\partial\tilde{e}^{\pm}}{\partial w}\frac{\partial\tilde{e}^{\pm}}{\partial r}\\ &= \frac{\partial^2 y}{\partial e^2}\frac{\partial\tilde{e}^{\pm}}{\partial w}\left[\frac{\partial\tilde{e}^{\pm}}{\partial w}+\frac{\partial\tilde{e}^{\pm}}{\partial r}\right]+\frac{\partial y}{\partial e}\frac{\partial^2\tilde{e}^{\pm}}{\partial w^2}-s''(w).\end{aligned}$$

As $\epsilon \longrightarrow 0$ it is possible to infer that

$$\left[\frac{\partial e(r\pm\epsilon,r,\lambda)^{\pm}}{\partial w}+\frac{\partial e(r\pm\epsilon,r,\lambda)^{\pm}}{\partial r}\right]\longrightarrow 0$$

(refer to the expressions of these objects in the proof of Theorem 1), implying $\lim_{\epsilon\to 0} d\Psi(r\pm\epsilon;q,r,\lambda)/dr < 0$. Hence: $\partial q_l(r,\lambda)/\partial r > 0$ and $q_u'(r) > 0$.

Turning next to investigate how the lower threshold depends on the degree of loss aversion, implicit differentiation gives

$$\frac{\partial q_l(r,\lambda)}{\partial\lambda} = -\lim_{\epsilon\to 0}\frac{\partial\Psi(r-\epsilon,q,r,\lambda)/\partial\lambda}{\partial\Psi(r-\epsilon,q,r,\lambda)/\partial q}$$

where the numerator is given by

$$\frac{\partial^2 y}{\partial e^2}\lim_{\epsilon\to 0}\frac{\partial\tilde{e}(r-\epsilon,r,\lambda)^-}{\partial\lambda}\lim_{\epsilon\to 0}\frac{\partial\tilde{e}(r-\epsilon,r,\lambda)^-}{\partial w}+\frac{\partial y}{\partial e}\frac{\partial^2\tilde{e}(r-\epsilon,r,\lambda)^-}{\partial w\partial\lambda} > 0,$$

since it has been established in Theorem 1 that $\partial\tilde{e}^-/\partial\lambda < 0$ and $\partial^2\tilde{e}^-/\partial w\partial\lambda > 0$. Hence, $\partial q_l(r,\lambda)/\partial\lambda > 0$.

From the expression for $\lim_{\epsilon\to 0}\Psi(r-\epsilon;q,r\lambda) - \lim_{\epsilon\to 0}\Psi(r+\epsilon;q,r,\lambda)$ in the preliminaries it is possible to conclude that when $\lambda = 1$ these two objects are equal. This result, combined with the observation that $\lim_{\epsilon\to 0}\tilde{e}(r-\epsilon,r,\lambda)^- = \tilde{e}_n = \lim_{\epsilon\to 0}\tilde{e}(r+\epsilon,r)$ (from Theorem 1) permits the conclusion that $q_l(r,1) = q_u(r)$. Which in turn, along with the fact that $q_l^{\lambda}(r,\lambda) < 0$ implies that $q_l(r,\lambda) < q_u(r)$ for all $\lambda > 1$.

Optimal wage setting: The proof now turn to the optimal wage setting rule, which depends on the match productivity in relation to the productivity thresholds.

If $q \in [0, q_l(r,\lambda))$ then the definition of $q_l(r,\lambda)$ and fact that $\partial\Psi/\partial q > 0$ can be used to deduce that $\lim_{\epsilon\to 0}\Psi(r-\epsilon,q,r,\lambda) < 0$; since $\Psi(w;q,r,\lambda)$ is everywhere decreasing in w, the same is true for all $w \geq r$. As such, the optimising wage must satisfy $w < r$ and will therefore be the solution to

$$\frac{\partial y(q,\tilde{e}(w,r,\lambda)^-)}{\partial e}\frac{\partial\tilde{e}(w,r,\lambda)^-}{\partial w} - s'(w) \leq 0,$$

with equality if $w > \underset{\sim}{w}(r,\lambda)$ (recall from the proof of Theorem 2 that this is either zero, or the wage below which effort takes the boundary value of zero). To account for the fact that the firm may pay the "lowest feasible wage" for a range of match productivity, let $\underset{\sim}{q}(r,\lambda) = \max\{0, q : \Psi(\underset{\sim}{w}(r,\lambda); q, r, \lambda) = 0\}$ (at $\underset{\sim}{q}(r,\lambda)$ the firm would want to pay $\underset{\sim}{w}(r,\lambda)$, and since $\partial\Psi/\partial q > 0$ the same will be true for all $0 \leq q < \underset{\sim}{q}(r,\lambda)$). For all $\underset{\sim}{q}(r,\lambda) < q < q_l(r,\lambda)$ the optimal wage is given by the displayed first-order condition holding with equality, which is denoted by $\tilde{w}(r,q,\lambda)^-$. Implicit differentiation and the deductions in the preliminaries reveal

$$\frac{\partial \tilde{w}(r,q,\lambda)^-}{\partial q} = -\frac{\partial\Psi(w;q,r,\lambda)/\partial q}{\partial\Psi(w;q,r,\lambda)/\partial w} > 0,$$

$$\frac{\partial \tilde{w}(r,q,\lambda)^-}{\partial r} = -\frac{\partial\Psi(w;q,r,\lambda)/\partial r}{\partial\Psi(w;q,r,\lambda)/\partial w} \geq 0$$

$$\frac{\partial \tilde{w}(r,q,\lambda)^-}{\partial \lambda} = -\frac{\partial\Psi(w;q,r,\lambda)/\partial \lambda}{\partial\Psi(w;q,r,\lambda)/\partial w} > 0.$$

If $q \in (q_u(r), \infty]$ then the definition of $q_u(r)$ and the fact that $\partial\Psi/\partial q > 0$ can be used to deduce that $\lim_{\epsilon\to 0}\Psi(r+\epsilon, q, r, \lambda) > 0$; since $\Psi(w;q,r,\lambda)$ is everywhere decreasing in w the same is true for all $w \leq r$ and, as such, the optimising wage must exceed r and will therefore satisfy

$$\frac{\partial y(q, \tilde{e}(w,r)^+)}{\partial e}\frac{\partial \tilde{e}(w,r)^+}{\partial w} - s'(w) \leq 0,$$

Letting $\tilde{w}(q,r)^+$ denote the solution (which is independent of λ), implicit differentiation gives

$$\frac{\partial \tilde{w}(r,q)^+}{\partial q} = -\frac{\partial\Psi(w;q,r,\lambda)/\partial q}{\partial\Psi(w;q,r,\lambda)/\partial w} > 0,$$

$$\frac{\partial \tilde{w}(r,q)^+}{\partial r} = -\frac{\partial\Psi(w;q,r,\lambda)/\partial r}{\partial\Psi(w;q,r,\lambda)/\partial w} \geq 0.$$

If $q \in [q_l(r,\lambda), q_u(r)]$ then the fact that $\partial\Psi/\partial q > 0$ can be used to deduce that $\lim_{\epsilon\to 0}\Psi(r-\epsilon, q, r, \lambda) \geq 0$ and $\lim_{\epsilon\to 0}\Psi(r+\epsilon, q, r, \lambda) \leq 0$. That $\partial\Psi/\partial w < 0$ for all $w \neq r$ then implies $\Psi(w;q,r,\lambda) > 0$ for all $w < r$ and $\Psi(w;q,r,\lambda) < 0$ for all $w > r$, implying profit is maximised if and only if $w = r$.

Finally, if $q < \underline{q}(r,\lambda)$ then then the employment relationship ends. Implicit differentiation of the zero profit condition defining the reservation productivity allows us to deduce that (function arguments are omitted to ease notation):

$$\frac{\partial \underline{q}(r,\lambda)}{\partial r} = -\frac{\frac{\partial \pi}{\partial e}\left[\frac{\partial \tilde{e}}{\partial r} + \frac{\partial \tilde{e}}{\partial w}\frac{\partial \tilde{w}}{\partial r}\right] + \frac{\partial \pi}{\partial w}\frac{\partial \tilde{w}}{\partial r}}{\frac{\partial \pi}{\partial w}\frac{\partial \tilde{w}}{\partial q} + \frac{\partial \pi}{\partial q} + \frac{\partial \pi}{\partial e}\frac{\partial \tilde{e}}{\partial w}\frac{\partial \tilde{w}}{\partial q}}$$
$$= -\frac{\frac{\partial \tilde{w}}{\partial r}\left[\frac{\partial \pi}{\partial e}\frac{\partial \tilde{e}}{\partial w} + \frac{\partial \pi}{\partial w}\right] + \frac{\partial \pi}{\partial e}\frac{\partial \tilde{e}}{\partial r}}{\frac{\partial \tilde{w}}{\partial q}\left[\frac{\partial \pi}{\partial e}\frac{\partial \tilde{e}}{\partial w} + \frac{\partial \pi}{\partial w}\right] + \frac{\partial \pi}{\partial q}} > 0$$

since

$$\left[\frac{\partial \pi}{\partial e}\frac{\partial \tilde{e}}{\partial w} + \frac{\partial \pi}{\partial w}\right] = \left[\frac{\partial y}{\partial e}\frac{\partial \tilde{e}}{\partial w} - s'(\tilde{w})\right] = 0$$

from the first-order condition, $\partial\pi/\partial q, \partial\pi/\partial e > 0$ by Assumption F2, and $\partial e/\partial r < 0$ as established in Theorem 1. In addition,

$$\frac{\partial \underline{q}(r,\lambda)}{\partial \lambda} = -\frac{\frac{\partial \pi}{\partial e}\left[\frac{\partial \tilde{e}}{\partial \lambda} + \frac{\partial \tilde{e}}{\partial w}\frac{\partial \tilde{w}}{\partial \lambda}\right] + \frac{\partial \pi}{\partial w}\frac{\partial \tilde{w}}{\partial \lambda}}{\frac{\partial \pi}{\partial w}\frac{\partial \tilde{w}}{\partial q} + \frac{\partial \pi}{\partial q} + \frac{\partial \pi}{\partial e}\frac{\partial \tilde{e}}{\partial w}\frac{\partial \tilde{w}}{\partial q}}$$
$$= -\frac{\frac{\partial \tilde{w}}{\partial \lambda}\left[\frac{\partial \pi}{\partial e}\frac{\partial \tilde{e}}{\partial w} + \frac{\partial \pi}{\partial w}\right] + \frac{\partial \pi}{\partial e}\frac{\partial \tilde{e}}{\partial \lambda}}{\frac{\partial \tilde{w}}{\partial q}\left[\frac{\partial \pi}{\partial e}\frac{\partial \tilde{e}}{\partial w} + \frac{\partial \pi}{\partial w}\right] + \frac{\partial \pi}{\partial q}},$$

where again the term in square brackets is equal to zero from the first-order condition, $\partial\pi/\partial q, \partial\pi/\partial e > 0$ by Assumption F2; moreover as established in Theorem 1: when $w > r$ $\tilde{e}$ is independent of λ, but when $w < r$, $\partial\tilde{e}/\partial\lambda < 0$. As such, if $\tilde{w}(r, \underline{q}(r,\lambda)) > r$ then $\partial\underline{q}(r,\lambda)/\partial\lambda = 0$, while if $\tilde{w}(r, \underline{q}(r,\lambda)) < r$, $\partial\underline{q}(r,\lambda)/\partial\lambda > 0$. ■

Proof of Proposition 1. If in the first period the match productivity falls short of the reservation productivity with the updated reference wage, i.e. if $q_1 < \underline{q}(w_0, \lambda)$, then contracting with the worker for the final period would be unprofitable and the contract will terminate; the employment relationship only has value for $q_1 \geq \underline{q}(w_0, \lambda)$. As noted,

$$\mathbb{E}_0[J_1(w_0, q_1)] = \int_{\underline{q}(w_0,\lambda)}^{\max\{\underline{q}(w_0,\lambda),q_l(w_0,\lambda)\}} J_1(w_0, q_1)^- \, dF + \int_{\max\{\underline{q}(w_0,\lambda),q_l(w_0,\lambda)\}}^{\max\{\underline{q}(w_0,\lambda),q_u(w_0)\}} J_1(w_0, q_1)^= \, dF + \int_{\max\{\underline{q}(w_0,\lambda),q_u(w_0)\}}^{\infty} J_1(w_0, q_1)^+ \, dF$$

where $J_1(w_0, q_1)^{-;=;+}$ represents the continuation value of the employment relationship if $w_1 < w_0; w_1 = w_0; w_1 > w_0$, in which effort is given by $\tilde{e}(w_1, w_0, \lambda)^-; \tilde{e}_n; \tilde{e}(w_1, w_0)^+$. (In subsequent proofs it is assumed that $\underline{q}(w_0, \lambda) < q_l(w_0, \lambda)$.)

Let $\mathbb{I}^-$ be an indicator variable that takes the value 1 if $\underline{q}(w_0, \lambda) < q_l(w_0, \lambda)$ and is otherwise zero; $\mathbb{I}^=$ an indicator variable that takes the value 1 if $q_l(w_0, \lambda) \leq \underline{q}(w_0, \lambda) \leq q_u(w_0)$ and is otherwise zero; and $\mathbb{I}^+$ an indicator variable that takes the value 1 if $\underline{q}(w_0, \lambda) > q_u(w_0)$ and is otherwise zero. Note that one, and only one, of $\mathbb{I}^-$, $\mathbb{I}^=$ and $\mathbb{I}^+$ is equal to 1. The marginal change in the value of the first-period employment contract (dropping the arguments of the functions characterising the productivity thresholds) is given by

$$\begin{aligned}
\Phi(w_0, \lambda) = &\int_{\underline{q}}^{\max\{\underline{q},q_l\}} \frac{\partial}{\partial r} J_1(w_0, q_1)^- \, dF \\
&+ \mathbb{I}^- \left[\frac{\partial q_l}{\partial r} \lim_{\epsilon \to 0} J_1(w_0, q_l - \epsilon)^- f(q_l) - \frac{\partial \underline{q}}{\partial r} J_1(w_0, \underline{q}) f(\underline{q}) \right] \\
&+ \int_{\max\{\underline{q},q_l\}}^{\max\{\underline{q},q_u\}} \frac{\partial}{\partial r} J_1(w_0, q_1)^= \, dF + \mathbb{I}^- \left[q_u' J_1(w_0, q_u)^= f(q_u) - \frac{\partial q_l}{\partial r} J_1(w_0, q_l)^= f(q_l) \right] \\
&+ \mathbb{I}^= \left[q_u' J_1(w_0, q_u)^= f(q_u) - \frac{\partial \underline{q}}{\partial r} J_1(w_0, \underline{q}) f(\underline{q}) \right] + \int_{\max\{\underline{q},q_u\}}^{\infty} \frac{\partial}{\partial r} J_1(w_0, q_1)^+ \, dF \\
&+ \mathbb{I}^- \left[-q_u' \lim_{\epsilon \to 0} J_1(w_0, q_u + \epsilon)^+ f(q_u) \right] + \mathbb{I}^= \left[-q_u' \lim_{\epsilon \to 0} J_1(w_0, q_u + \epsilon)^+ f(q_u) \right] \\
&+ \mathbb{I}^+ \left[\frac{\partial \underline{q}}{\partial r} J_1(w_0, \underline{q}) f(\underline{q}) \right]
\end{aligned}$$

By definition, $J_1(w_0, \underline{q}) = 0$, and the continuity of the optimal effort function and wage setting rule imply that $\lim_{\epsilon \to 0} J_1(w_0, q_l - \epsilon)^- = J_1(w_0, q_l)^=$ and $\lim_{\epsilon \to 0} J_1(w_0, q_u + \epsilon)^+ = J_1(w_0, q_u)^=$. It then follows for each of the three scenarios: $\mathbb{I}^- = 1; \mathbb{I}^= = 1; \mathbb{I}^+ = 1$, that the terms in square brackets cancel out, so that

$$\Phi(w_0, \lambda) = \int_{\underline{q}}^{\infty} \frac{\partial}{\partial r} J_1(w_0, q_1)\, dF.$$

Next, note that for $q_1 \in [\underline{q}(w_0, \lambda), \infty] \setminus [q_l(w_0, \lambda), q_u(w_0)]$ (i.e. where the wage is not equal to the reference wage)

$$\frac{\partial}{\partial r} J_1(w_0, q_1) = \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^{\pm}}{\partial w}\frac{\partial \tilde{w}}{\partial r} + \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^{\pm}}{\partial r} - s'(w_1)\frac{\partial \tilde{w}}{\partial r}$$

$$= \frac{\partial \tilde{w}}{\partial r}\left[\frac{\partial \tilde{e}^{\pm}}{\partial w}\frac{\partial \tilde{w}}{\partial r} - s'(w_1)\right] + \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^{\pm}}{\partial r} < 0$$

since from the first-order condition the term in square brackets is equal to zero (assuming $\tilde{w}_1 > \underline{w}(w_0, \lambda)$ so the solution is interior); and as established in Theorem 1 that $\partial\tilde{e}/\partial r < 0$. For $q_1 \in [q_l(w_0, \lambda), q_u(w_0)]$ the wage is equal to the reference wage and effort is constant and equal to $\tilde{e}_n$, so it is straightforward to infer that $\frac{\partial}{\partial r} J_1(r_1, q_1) = -s'(w_0) < 0$. As such,

$$\Phi(w_0, \lambda) = \int_{\underline{q}}^{q_l} \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^{-}}{\partial r}\, dF - \int_{q_l}^{q_u} s'(w_0)\, dF + \int_{q_u}^{\infty} \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^{+}}{\partial r}\, dF < 0 \qquad \text{(B.1)}$$

as stated in the proposition. ■

Proof of Theorem 3. The proof is qualitatively similar to the proof of Theorem 2, so the details are largely omitted. However, a key step consists in establishing concavity of the firm's value function $J(r_0, q_0)$ with respect to w_0, which implies to establish that

$$\frac{\partial \Psi(w_0, q_0, r_0, \lambda)}{\partial w} + \delta \frac{\partial \Phi(w_0, \lambda)}{\partial r} < 0.$$

It has been established in the proof of Theorem 2 that $\Psi(w_0; q_0, r_0, \lambda)$ is decreasing in w_0 as $\partial\Psi/\partial w < 0$ for $w \neq r$ and at $w = r$ there is a jump down. Next, recall the expression for $\Phi(w_0, \lambda)$ in (B.1) and note that that both the integrand and the limits of integration depend on w_0. Hence:

$$\begin{aligned}\frac{\partial\Phi(w_0,\lambda)}{\partial r} = &\int_{\underline{q}}^{q_l}\left[\frac{\partial^2 y}{\partial e^2}\left(\frac{\partial\tilde{e}^-}{\partial r}\right)^2 + \frac{\partial y}{\partial e}\frac{\partial^2\tilde{e}^-}{\partial r^2}\right]dF - \frac{\partial\underline{q}}{\partial r}\frac{\partial y}{\partial e}\frac{\partial\tilde{e}^-}{\partial r}f(\underline{q})\\ &+\frac{\partial q_l}{\partial r}\frac{\partial y}{\partial e}\lim_{\epsilon\to 0}\frac{\partial\tilde{e}(w_0-\epsilon,w_0,\lambda)}{\partial r}f(q_l)+\frac{\partial q_l}{\partial r}s'(w_0)f(q_l)\\ &-\int_{q_l}^{q_u}s''(w_0)\,dF\\ &-q_u's'(w_0)f(q_u)-q_u'\frac{\partial y}{\partial e}\lim_{\epsilon\to 0}\frac{\partial\tilde{e}(w_0+\epsilon,w_0,\lambda)f(q_u)}{\partial r}\\ &+\int_{q_u}^{\infty}\left[\frac{\partial^2 y}{\partial e^2}\left(\frac{\partial\tilde{e}^+}{\partial r}\right)^2+\frac{\partial y}{\partial e}\frac{\partial^2\tilde{e}^+}{\partial r^2}\right]dF. \end{aligned} \tag{B.2}$$

Then, notice that from the the expressions for $\partial\tilde{e}/\partial w$ and $\partial\tilde{e}/\partial r$ in the proof of Theorem 2 it follows that

$$\lim_{\epsilon\to 0}\frac{\partial\tilde{e}(w_0\pm\epsilon,w_0,\lambda)}{\partial r} = -\lim_{\epsilon\to 0}\frac{\partial\tilde{e}(w_0\pm\epsilon,w_0,\lambda)}{\partial w}$$

Moreover, when $w \neq r$ the first-order condition holds with equality, which implies that $\frac{\partial y}{\partial e}\frac{\partial\tilde{e}}{\partial r} - s'(w_0) = 0$. By using these two latter results, it is possible to check that

$$\frac{\partial y}{\partial e}\lim_{\epsilon\to 0}\frac{\partial\tilde{e}(w_0\pm\epsilon,w_0,\lambda)}{\partial w} = s'(w_0)$$

which allows several terms to cancel in (B.2). As such,

$$\begin{aligned}\frac{\partial\Phi(w_0,\lambda)}{\partial r} = &\int_{\underline{q}}^{q_l}\left[\frac{\partial^2 y}{\partial e^2}\left(\frac{\partial\tilde{e}^-}{\partial r}\right)^2 + \frac{\partial y}{\partial e}\frac{\partial^2\tilde{e}^-}{\partial r^2}\right]dF\\ &-\int_{q_l}^{q_u}s''(w_0)\,dF+\int_{q_u}^{\infty}\left[\frac{\partial^2 y}{\partial e^2}\left(\frac{\partial\tilde{e}^+}{\partial r}\right)^2+\frac{\partial y}{\partial e}\frac{\partial^2\tilde{e}^+}{\partial r^2}\right]dF\\ &-\frac{\partial\underline{q}}{\partial r}\frac{\partial y}{\partial e}\frac{\partial\tilde{e}^-}{\partial r}f(\underline{q}) \end{aligned} \tag{B.3}$$

It has been established by Theorem 1 that $\partial\tilde{e}/\partial r < 0$ and $\partial^2\tilde{e}/\partial r^2 > 0$, and by Theorem 2 that $\partial\underline{q}/\partial r > 0$. As such, while the first two lines of the expression above are negative, the last line is positive, implying that the sign of the second derivative of the expected future profit function with respect to the initial wage remains undetermined. To proceed it is not necessary to assume $\partial\Phi/\partial r < 0$, but it is conjectured that $\partial\Psi/\partial w + \delta\partial\Phi/\partial r < 0$, which is reasonable given the expressions derived above. Since $\partial\Psi/\partial w_0 < 0$,the inequality will be true for a sufficiently impatient firm. Nevertheless the conjecture essentially implies that $|\partial\Psi/\partial w| > \delta|\partial\Phi/\partial r|$, that is: the direct effect of a change in the wage in the current period will be larger than the expected discounted future effect that comes indirectly through the worker's adaptation of the reference wage. Therefore, even if $\partial\Phi/\partial r$, then $\partial\Psi/\partial w + \delta\partial\Phi/\partial r < 0$.

Under this condition, the proof of the nature of the wage setting rule follows the same steps as the proof of Theorem 2 where Ψ is replaced with $\Psi + \delta\Phi$. ■

Proof of Proposition 2. The proof relies on investigation of the first-order condition of the two optimisation problems, noting from Proposition 1 that $\Phi(w_0, \lambda) < 0$. First it is shown that $\hat{q}_l(r, \lambda, \delta) > \tilde{q}_l(r, \lambda)$. Suppose, by contradiction, that $\hat{q}_l \leq \tilde{q}_l$, then the fact that $\partial\Psi/\partial q > 0$ (see the preliminaries in the proof of Theorem 2) implies

$$0 \equiv \lim_{\epsilon \to 0} \Psi(r - \epsilon; \tilde{q}_l, r, \lambda) \geq \lim_{\epsilon \to 0} \Psi(r - \epsilon, \hat{q}_l, r, \lambda),$$

but then since $\Phi(w, \lambda) < 0$ this yields

$$\lim_{\epsilon \to 0} \Psi(r - \epsilon, \hat{q}_l, r, \lambda) > \lim_{\epsilon \to 0} \Psi(r - \epsilon, \hat{q}_l, r, \lambda) + \delta\Phi(w, \lambda) \equiv 0,$$

yielding a contradiction. That $\hat{q}_u(r, \lambda, \delta) > \tilde{q}_u(r)$ is similarly proved.

Next, compare $\hat{w}(r, q, \lambda, \delta)^{-;+}$ with $\tilde{w}(r, q, \lambda)^{-;+}$ where both functions are defined. It will be shown that $\hat{w}(r, q, \lambda, \delta)^{-} < \tilde{w}(r, q, \lambda)^{-}$ for all $q < \tilde{q}_l(r, \lambda)$. Suppose, by contradiction, that $\hat{w}^{-} \geq \tilde{w}^{-}$. Then the fact that $\partial\Psi/\partial w < 0$ (see the preliminaries in the proof of Theorem 2) implies

$$0 \equiv \Psi(\tilde{w}^{-}; q, r, \lambda) \geq \Psi(\hat{w}^{-}; q, r, \lambda),$$

but then $\Phi(w_0, \lambda) < 0$ implies

$$\Psi(\hat{w}^{-}; q, r, \lambda) > \Psi(\hat{w}^{-}; q, r, \lambda) + \delta\Phi(r, \lambda) \equiv 0,$$

yielding a contradiction. The proof that $\hat{w}(r, q, \lambda, \delta)^{+} < \tilde{w}(r, q, \lambda)^{+}$ for all $q > \hat{q}_u(r, \lambda, \delta)$ is similar and so omitted. ■

Proof of Proposition 3. Consider how the optimal wage changes with the degree of loss aversion. Implicit differentiation of the wage setting rule gives

$$\frac{\partial\hat{w}(r_0, q_0, \lambda, \delta)}{\partial\lambda} = -\frac{\partial\Psi/\partial\lambda + \delta\partial\Phi/\partial\lambda}{\partial\Psi/\partial w + \delta\partial\Phi/\partial r}.$$

It has been conjectured that the denominator is negative, and it has been established in the preliminaries of the proof of Theorem 2 that $\partial\Psi/\partial\lambda = 0$ if $w \geq r$ and $\partial\Psi/\partial\lambda > 0$ if

$w < r$. Hence the proof of the proposition crucially depends on the sign and magnitude of $\partial\Phi/\partial\lambda$. Recalling the definition of $\Phi(w_0, \lambda)$ given (B.1) and noting that $s'(w_0)$ and q_u are independent of λ, it can be deduced that

$$\frac{\partial\Phi(w_0,\lambda)}{\partial\lambda} = \int_{\underline{q}}^{q_l} \left[\frac{\partial^2 y}{\partial e^2}\frac{\partial\tilde{e}^-}{\partial r}\frac{\partial\tilde{e}^-}{\partial\lambda} + \frac{\partial y}{\partial e}\frac{\partial^2\tilde{e}^-}{\partial r\partial\lambda}\right] dF - \frac{\partial\underline{q}}{\partial\lambda}\frac{\partial y}{\partial e}\frac{\partial\tilde{e}^-}{\partial r} f(\underline{q}) + \frac{\partial q_l}{\partial\lambda}\frac{\partial y}{\partial e}\lim_{\epsilon\to0}\frac{\partial\tilde{e}(w_0-\epsilon, w_0,\lambda)}{\partial r} f(q_l) + \frac{\partial q_l}{\partial\lambda} s'(w_0) f(q_l), \quad \text{(B.4)}$$

in which the two terms in the second line cancel out, since, as previously deduced, $\lim_{\epsilon\to0}\partial\tilde{e}(w_0\pm\epsilon, w_0,\lambda)/\partial r = -\lim_{\epsilon\to0}\partial\tilde{e}(w_0\pm\epsilon, w_0,\lambda)/\partial w$ and $\frac{\partial y}{\partial e}\frac{\partial\tilde{e}}{\partial w} = s'(w_0)$ by the first order condition when $w \neq r$. Hence

$$\frac{\partial\Phi(w_0,\lambda)}{\partial\lambda} = \int_{\underline{q}}^{q_l} \left[\frac{\partial^2 y}{\partial e^2}\frac{\partial\tilde{e}^-}{\partial r}\frac{\partial\tilde{e}^-}{\partial\lambda} + \frac{\partial y}{\partial e}\frac{\partial^2\tilde{e}^-}{\partial r\partial\lambda}\right] dF - \frac{\partial\underline{q}}{\partial\lambda}\frac{\partial y}{\partial e}\frac{\partial\tilde{e}^-}{\partial r} f(\underline{q}).$$

From the results established in Theorem 2, i.e. $\partial\tilde{e}^-/\partial\lambda, \partial\tilde{e}^-/\partial r, \partial^2\tilde{e}^-/\partial r\partial\lambda < 0$, it can be deduced that the first term of the expression above is negative, whilst the second term is positive since $\partial\underline{q}/\partial\lambda > 0$. As such, if the layoff reservation productivity doesn't increase too much, that is, if $\partial\underline{q}/\partial\lambda > 0$ is sufficiently small, then $\partial\Phi/\partial\lambda$ is negative, which enables to conclude the statements regarding the wage in the proposition. ∎

Proof of Proposition 4. The reservation productivity governing hiring behaviour in the initial contract is characterised by

$$\hat{\underline{q}}(r_0,\lambda,\delta) = \max\{0, q_0 : \pi(\hat{w}(r_0,q_0,\lambda,\delta), r_0, q_0, \lambda) + \delta\mathbb{E}[J_1(\hat{w}(r_0,q_0,\lambda,\delta), q_1)] = 0\}.$$

Implicit differentiation reveals

$$\begin{aligned}\frac{d\hat{\underline{q}}(r_0,\lambda,\delta)}{d\lambda} &= -\frac{\frac{\partial\pi}{\partial e}\left[\frac{\partial\tilde{e}}{\partial\lambda} + \frac{\partial\tilde{e}}{\partial w}\frac{\partial\hat{w}}{\partial\lambda}\right] + \frac{\partial\pi}{\partial w}\frac{\partial\hat{w}}{\partial\lambda} + \delta\frac{d}{d\lambda}\mathbb{E}_0\left[J_1(\hat{w}(r_0,q_0,\lambda,\delta),q_1)\right]}{\frac{\partial\pi}{\partial w}\frac{\partial\hat{w}}{\partial q_0} + \frac{\partial\pi}{\partial q} + \frac{\partial\pi}{\partial e}\frac{\partial\tilde{e}}{\partial w}\frac{\partial\hat{w}}{\partial q_0} + \delta\frac{d}{dq_0}\mathbb{E}_0\left[J_1(\hat{w}(r_0,q_0,\lambda,\delta),q_1)\right]} \\ &= -\frac{\frac{\partial\hat{w}}{\partial\lambda}\left[\frac{\partial\pi}{\partial e}\frac{\partial\tilde{e}}{\partial w} + \frac{\partial\pi}{\partial w}\right] + \frac{\partial\pi}{\partial e}\frac{\partial\tilde{e}}{\partial\lambda} + \delta\frac{d}{d\lambda}\mathbb{E}_0\left[J_1(\hat{w}(r_0,q_0,\lambda,\delta),q_1)\right]}{\frac{\partial\hat{w}}{\partial q_0}\left[\frac{\partial\pi}{\partial e}\frac{\partial\tilde{e}}{\partial w} + \frac{\partial\pi}{\partial w}\right] + \frac{\partial\pi}{\partial q} + \delta\frac{d}{dq_0}\mathbb{E}_0\left[J_1(\hat{w}(r_0,q_0,\lambda,\delta),q_1)\right]}.\end{aligned} \quad \text{(B.5)}$$

Next, let $\pi(q)^{-;=;+}$ be the profit function when $w_1 < w_0; w_1 = w_0; w_1 > w_0$, in which effort is given by $\tilde{e}(w_1, w_0, \lambda)^-; \tilde{e}_n; \tilde{e}(w_1, w_0)^+$, and note that

$$\frac{d}{d\lambda}\mathbb{E}_0\left[J_1(\hat{w}(r_0, q_0, \lambda, \delta), q_1)\right] = \int_{\underline{q}}^{q_l} \frac{d\pi(q)^-}{d\lambda}\, dF - \frac{d\underline{q}}{d\lambda}\pi(\underline{q})^- f(\underline{q}) + \frac{dq_l}{d\lambda}\pi(q_l)^- f(q_l)$$
$$+ \int_{q_l}^{q_u} \frac{d\pi(q)^=}{d\lambda}\, dF + \frac{dq_u}{d\lambda}\pi(q_u)^= f(q_u) - \frac{dq_l}{d\lambda}\pi(q_l)^= f(q_l)$$
$$+ \int_{q_u}^{\infty} \frac{d\pi(q)^+}{d\lambda}\, dF - \frac{dq_u}{d\lambda}\pi(q_u)^+ f(q_u).$$

Noting that $\pi(\underline{q})^- = 0$ by definition of the layoff reservation productivity, and the other effects on the limits of integration cancel out for the same reasons outlined in throughout the preceding proofs, the expression above reduces to

$$\frac{d}{d\lambda}\mathbb{E}_0\left[J_1(\hat{w}(r_0, q_0, \lambda, \delta), q_1)\right] = \int_{\underline{q}}^{q_l} \frac{d\pi(q)^-}{d\lambda}\, dF + \int_{q_l}^{q_u} \frac{d\pi(q)^=}{d\lambda}\, dF + \int_{q_u}^{\infty} \frac{d\pi(q)^+}{d\lambda}\, dF, \tag{B.6}$$

in which

$$\begin{aligned}
\frac{d\pi(q)^-}{d\lambda} &= \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial \lambda} + \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial w}\frac{\partial \tilde{w}_1}{\partial \lambda} + \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial r}\frac{\partial \hat{w}_0}{\partial \lambda} - s'(\tilde{w}_1)\frac{\partial \tilde{w}_1}{\partial \lambda} \\
&= \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial \lambda} + \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial r}\frac{\partial \hat{w}_0}{\partial \lambda} + \frac{\partial \tilde{w}_1}{\partial \lambda}\left[\frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial w} - s'(\tilde{w}_1)\right] \\
&= \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial \lambda} + \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial r}\frac{\partial \hat{w}_0}{\partial \lambda}
\end{aligned}$$

$$\frac{d\pi(q)^=}{d\lambda} = -s'(\hat{w}_0)\frac{\partial \hat{w}_0}{\partial \lambda}$$

$$\begin{aligned}
\frac{d\pi(q)^+}{d\lambda} &= \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial w}\frac{\partial \tilde{w}_1}{\partial \lambda} + \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial r}\frac{\partial \hat{w}_0}{\partial \lambda} - s'(\tilde{w}_1)\frac{\partial \tilde{w}_1}{\partial \lambda} \\
&= \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial r}\frac{\partial \hat{w}_0}{\partial \lambda} + \frac{\partial \tilde{w}_1}{\partial \lambda}\left[\frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial w} - s'(\tilde{w}_1)\right] \\
&= \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial r}\frac{\partial \hat{w}_0}{\partial \lambda}
\end{aligned}$$

since the term in square brackets equals zero due to the first-order condition determining the wage in the second period of employment $\tilde{w}_1$. Then note that $\partial \hat{w}_0/\partial \lambda$ does not depend on q_1, as such (B.6) can be expressed as

$$\frac{d}{d\lambda}\mathbb{E}_0\left[J_1(\hat{w}(r_0, q_0, \lambda, \delta), q_1)\right] = \int_{\underline{q}}^{q_l} \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial \lambda}\, dF + \frac{\partial \hat{w}_0}{\partial \lambda}\int_{\underline{q}}^{q_l} \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial r}\, dF$$
$$- \frac{\partial \hat{w}_0}{\partial \lambda}\int_{q_l}^{q_u} s'(\hat{w}_0)\, dF + \frac{\partial \hat{w}_0}{\partial \lambda}\int_{q_u}^{\infty} \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial r}\, dF.$$

Then, collecting $\partial\hat{w}/\partial\lambda$ as the common factor and recalling the expression for $\Phi(\hat{w}_0, \lambda)$ as given by equation (B.1), it yields

$$\frac{d}{d\lambda}\mathbb{E}_0\left[J_1(\hat{w}(r_0, q_0, \lambda, \delta), q_1)\right] = \frac{\partial \hat{w}}{\partial \lambda}\Phi(\hat{w}_0, \lambda) + \int_{\underline{q}}^{q_l} \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial \lambda}.$$

Hence the numerator in (B.5) can be expressed as

$$\begin{aligned}\text{Numerator} &= \frac{\partial \hat{w}}{\partial \lambda}\left[\frac{\partial \pi}{\partial e}\frac{\partial \tilde{e}}{\partial w} + \frac{\partial \pi}{\partial w}\right] + \frac{\partial \pi}{\partial e}\frac{\partial \tilde{e}}{\partial \lambda} + \delta\left(\frac{\partial \hat{w}}{\partial \lambda}\Phi(\hat{w}_0, \lambda) + \int_{\underline{q}}^{q_l} \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial \lambda}\, dF\right)\\ &= \frac{\partial \hat{w}}{\partial \lambda}\left[\frac{\partial y}{\partial e}\frac{\partial \tilde{e}}{\partial w} - s(\hat{w}_0) + \delta\Phi(\hat{w}_0, \lambda)\right] + \frac{\partial y}{\partial e}\frac{\partial \tilde{e}}{\partial \lambda} + \delta\int_{\underline{q}}^{q_l} \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial \lambda}\, dF\\ &= \frac{\partial y}{\partial e}\frac{\partial \tilde{e}}{\partial \lambda} + \delta\int_{\underline{q}}^{q_l} \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial \lambda}\, dF\end{aligned}$$

since the term in square brackets equal to zero due to the first-order condition determining the initial wage $\tilde{w}_0$. The same procedure can be used to show that

$$\frac{d}{dq_0}\mathbb{E}_0\left[J_1(\hat{w}(r_0, q_0, \lambda, \delta), q_1)\right] = \frac{\partial \hat{w}}{\partial q_0}\Phi(\hat{w}_0, \lambda),$$

implying that the denominator in (B.5) can be expressed as

$$\begin{aligned}\text{Denominator} &= \frac{\partial \hat{w}}{\partial q_0}\left[\frac{\partial \pi}{\partial e}\frac{\partial \tilde{e}}{\partial w} + \frac{\partial \pi}{\partial w}\right] + \frac{\partial \pi}{\partial q} + \delta\frac{\partial \hat{w}_0}{\partial q_0}\Phi(w_0, \lambda)\\ &= \frac{\partial \hat{w}}{\partial q_0}\left[\frac{\partial y}{\partial e}\frac{\partial \tilde{e}}{\partial w} - s(\hat{w}_0) + \delta\Phi(w_0, \lambda)\right] + \frac{\partial y}{\partial q}\\ &= \frac{\partial y}{\partial q}\end{aligned}$$

As such,

$$\frac{d\underline{q}(r_0, \lambda, \delta)}{d\lambda} = -\frac{\dfrac{\partial y}{\partial e}\dfrac{\partial \tilde{e}}{\partial \lambda} + \delta\displaystyle\int_{\underline{q}}^{q_l} \frac{\partial y}{\partial e}\frac{\partial \tilde{e}^-}{\partial \lambda}\, dF}{\dfrac{\partial y}{\partial q}} > 0,$$

since it has been established in Theorem 1 that $\partial\tilde{e}^-/\partial\lambda < 0$ whenever $w < r$. ■

B.2 Proofs to Chapter 2

Proof of Proposition 5. Consider the first order condition given by (2.3):

$$\Omega(e_t; w_t, r_{C,t}, \lambda) \equiv b - e_t + \mu(\ln w_t - \ln r_{C,t}) = 0.$$

It is straightforward to show that if $w_t = r_{C,t}$, then $\mu(\cdot) = 0$ and the explicit solution to the above equation is $\tilde{e}_t = b$, which corresponds to the definition of normal effort, and is denoted by $\tilde{e}_n = b$. If $w_t > r_{C,t}$, then $\mu(\cdot) > 0$ and the explicit solution to the above equation is given by

$$\tilde{e}_t = b + (\ln w_t - \ln r_{C,t});$$

while if $w_t < r_{C,t}$, then $\mu(\cdot) < 0$ and the explicit solution is

$$\tilde{e}_t = b - \lambda(\ln r_{C,t} - \ln w_t).$$

Substituting $\tilde{e}_n = b$ completes the proof. ■

Proof of Proposition 6. This proof proceeds as follows. First it will be shown that under assumptions W1-W4, F1-F2, A1 and one additional, though innocuous, restriction on the state and control spaces there exists a unique solution to the functional equation (2.5). Then, the proof will characterise the properties of the firm's optimal wage policy, following the steps used in the proof of Theorem 2, Chapter 1.

Preliminaries. Denote the state space by $\mathcal{R}$ and the control space by $\mathcal{W}$. Naturally, $\mathcal{R}$ and $\mathcal{W}$ are both convex subsets of $\mathbb{R}_+$. Throughout the proof it is assumed that $\mathcal{W} = \mathcal{R} = [0, \bar{r}]$, where $\bar{r}$ is sufficiently large, in the sense that all the solutions to the firm's maximisation problem are interior (in particular $\bar{r} > r_H(\lambda)$, to be defined below). Notice that since $\pi(w, r_C)$ is strictly concave in w and decreasing in r_C, it is possible to characterise $\bar{r}$ such that it never binds. Moreover notice that since the firm may want to set the wage w_t either above, equal, or below the worker's reference wage $r_{C,t}$ in each period t, the values of w that are allowed for any given r_C is independent of the actual level of r_C. As such, the set of feasible controls in each t is given by $\mathcal{W}$.

Given these premises, it is possible to establish that the instantaneous profit function $\pi(w, r_C)$ is both bounded and continuous in its domain. This, together with the fact that $\psi \in (0, 1)$, implies that there exists a unique solution to the functional equation given by (2.5); and that at least one optimal wage policy exists (see, for instance, Theorem 4.6, p.79 of Stokey and Lucas (1989)). Finally, since π is concave and $\mathcal{W}$ is convex, to establish uniqueness of the optimal wage policy it remains to be shown that the firm's value function is concave. As such, note that given the optimal effort policy characterised by Proposition 5, it follows that

$$\begin{aligned}\frac{\partial \Upsilon(w; r_C, \lambda)}{\partial w} &= \frac{\partial y(\tilde{e})}{\partial e}\frac{\partial^2 \tilde{e}(w, r_C, \lambda)}{\partial w^2} + \psi\frac{\partial y(\tilde{e}')}{\partial e}\frac{\partial^2 \tilde{e}(w', r_C', \lambda)}{\partial r^2} \\ &= -pq\frac{1}{w^2}(1-\psi) < 0\end{aligned}$$

which implies that $\Upsilon(w; r_C, \lambda)$ is everywhere decreasing in w, establishing concavity of the firm's value function. In addition,

$$\frac{\partial \Upsilon(w; r_C, \lambda)}{\partial \lambda} = \frac{\partial y(\tilde{e})}{\partial e}\frac{\partial^2 \tilde{e}(w, r_C, \lambda)}{\partial \lambda \partial w} + \psi\frac{\partial y(\tilde{e})}{\partial e}\frac{\partial^2 \tilde{e}(w, r_C, \lambda)}{\partial \lambda \partial r}$$

so that if $w < r_C$ then $\partial \Upsilon(w; r_C, \lambda)/\partial \lambda > 0$, and if $w > r_C$ then $\partial \Upsilon(w; r_C, \lambda)/\partial \lambda = 0$. These results enable to deduce that if $\lambda > 1$, $\Upsilon(w; r_C, \lambda)$ jumps down at the reference wage. Hence it is now possible to proceed with the proof following the same approach implemented in the proof of Theorem 2, Chapter 1.

Reference wage thresholds. The threshold r_L is the level of a newly hired worker's reference wage below which a firm would optimally set a wage above their reference wage, and $r_H(\lambda)$ is the level of a newly hired worker's reference wage above which a firm would optimally pay the worker a wage below their reference wage. The former, r_L, is the value of r_C below which the value function is increasing just above the reference wage; the latter, $r_H(\lambda)$, is the value of r_C above which the value function is decreasing just below the reference wage. These thresholds can be explicitly determined by solving respectively for $\lim_{\epsilon\to 0} \Upsilon(r_C + \epsilon; r_C, \lambda) = 0$ and $\lim_{\epsilon\to 0} \Upsilon(r_C - \epsilon; r_C, \lambda) = 0$. Hence,

$$\begin{aligned}\lim_{\epsilon\to 0}\frac{\partial y(\tilde{e})}{\partial e}\frac{\partial \tilde{e}(r_C + \epsilon, r_C, \lambda)}{\partial w} - 1 + \psi\frac{\partial y(\tilde{e})}{\partial e}\frac{\partial \tilde{e}(r_C + \epsilon, r_C, \lambda)}{\partial r} &= 0 \\ \lim_{\epsilon\to 0} pq\left(\frac{1}{r_C + \epsilon}\right) - 1 - \psi pq\left(\frac{1}{r_C + \epsilon}\right) &= 0 \\ \lim_{\epsilon\to 0} pq(1-\psi)\left(\frac{1}{r_C + \epsilon}\right) - 1 &= 0\end{aligned}$$

which solving for r_C yields $r_C = pq(1-\psi) \equiv r_L$. Analogously,

$$\begin{aligned}\lim_{\epsilon\to 0}\frac{\partial y(\tilde{e})}{\partial e}\frac{\partial \tilde{e}(r_C - \epsilon, r_C, \lambda)}{\partial w} - 1 + \psi\frac{\partial y(\tilde{e})}{\partial e}\frac{\partial \tilde{e}(r_C - \epsilon, r_C, \lambda)}{\partial r} &= 0 \\ \lim_{\epsilon\to 0} \lambda pq\left(\frac{1}{r_C - \epsilon}\right) - 1 - \psi \lambda pq\left(\frac{1}{r_C - \epsilon}\right) &= 0 \\ \lim_{\epsilon\to 0} \lambda pq(1-\psi)\left(\frac{1}{r_C - \epsilon}\right) - 1 &= 0\end{aligned}$$

which solving for r_C yields $r_C = \lambda pq(1-\psi) \equiv r_H(\lambda)$. Hence, it is straightforward to show that $r_L < r_H(\lambda)$ and if $\lambda = 1$, $r_L = r_H(1)$.

Optimal wage setting policy. Consider now the optimal time-invariant wage setting policy, which depends on the level employed workers' reference wages r_C in relation to the reference wage thresholds derived above.

If $r_C > r_H(\lambda)$ then by the definition of $r_H(\lambda)$ and since $\partial\Upsilon/\partial w < 0$, the optimal wage must satisfy $w < r_C$ and will therefore be the solution to

$$\Upsilon(w; r_C, \lambda) = \lambda pq\left(\frac{1}{w}\right) - 1 - \psi\lambda pq\left(\frac{1}{w}\right) = 0,$$

given by $\tilde{w} = \lambda pq(1-\psi)$, and denoted by $\tilde{w}_H(\lambda)^-$.

Analogously, if $r_C < r_L$ then by the definition of r_L and since $\partial\Upsilon/\partial w < 0$, the optimal wage must satisfy $w > r_C$ and will therefore be the solution to

$$\Upsilon(w; r_C, \lambda) = pq\left(\frac{1}{w}\right) - 1 - \psi pq\left(\frac{1}{w}\right) = 0,$$

given by $\tilde{w} = pq(1-\psi)$, and denoted by $\tilde{w}_L^+$. Hence it is straightforward to show that $\tilde{w}_L^+ < \tilde{w}_H(\lambda)^-$, and that if $\lambda = 1$ then $\tilde{w}_L^+ = \tilde{w}_H(1)^- = pq(1-\psi)$.

Finally, if $r_C \in [r_L, r_H(\lambda)]$ the fact that $\partial\Upsilon/\partial w < 0$ for all $w \neq r_C$, then $\Upsilon(w; r_C, \lambda) > 0$ for all $w < r_C$ and $\Upsilon(w; r_C, \lambda) < 0$ for all $w > r_C$, implying that $J(r_C)$ is maximised if and only if $w = r_C$, which characterises the solution denoted by $\tilde{w}_M^=$.

Transversality conditions. It is now also possible to verify whether the tranversality condition

$$\lim_{t\to\infty} \psi^t \frac{\partial\pi(r_{C,t}, \tilde{w})}{\partial r} \cdot \tilde{w} = 0,$$

that makes the first-order condition sufficient is satisfied. Consider the optimal wage $\tilde{w} = \tilde{w}_L^+ = pq(1-\psi)$, which is the solution to (2.6) for all $r_C < r_L$. The related transversality condition is:

$$\begin{aligned}
\lim_{t\to\infty} \psi^t \frac{\partial\pi(r_{C,t}, \tilde{w})}{\partial r} \cdot \tilde{w} &= \lim_{t\to\infty} \psi^t(-1) \cdot pq(1-\psi) \\
&= \lim_{t\to\infty} \psi^t(-pq + pq\psi) \\
&= \lim_{t\to\infty} -\psi^t pq + \psi^{t+1} pq \\
&= 0
\end{aligned}$$

The same steps can be applied for all $\tilde{w} \in [\tilde{w}_L^+, \tilde{w}_H(\lambda)^-]$ and are therefore omitted. ■

Proof of Proposition 7. That a range of steady-state equilibrium wages exists follows directly from Proposition 6, as also established by Corollary 1.

The steady-state equilibrium reference wage of new hires is given by $r_N^* = r_N = r_0$ by definition. Therefore it follows that depending on the level of r_N^* in relation to the defined thresholds $r_H(\lambda)$ and r_L, which also determines the steady-state optimal wage $\tilde{w}^* \in [\tilde{w}_L^+, \tilde{w}_H(\lambda)^-]$, the steady-state equilibrium level of effort exerted by new hires can take three distinct values: $\tilde{e}_N^* = \tilde{e}_N^+$ if $r_N^* < r_L$, since in that case $\tilde{w}^* = \tilde{w}_L^+ > r_N^*$; $\tilde{e}_N^* = \tilde{e}_N(\lambda)^-$ if $r_N^* > r_H(\lambda)$, since in that case $\tilde{w}^* = \tilde{w}_H(\lambda)^- < r_N^*$; or $\tilde{e}_N^* = \tilde{e}_n$ if $r_N^* \in [r_L, r_H]$, since in that case $\tilde{w}^* = \tilde{w}_M^= = r_N^*$.

Consider then incumbent workers. The steady-state equilibrium reference wage of incumbent workers is reached when $r_C^* = \tilde{w}(r_C^*, \lambda)$. Since $\tilde{w}(r_C^*, \lambda) = \tilde{w}(r_N^*, \lambda) \in [\tilde{w}_L^+, \tilde{w}_H(\lambda)^-]$, it follows that there exists a range of incumbent workers' reference wages $r_E^* = \tilde{w}(r_N^*, \lambda)$ where, as established by the proof of Proposition 6, the limits of the range are given by $\tilde{w}_L^+ = r_L$ and $\tilde{w}_H(\lambda)^- = r_H(\lambda)$ respectively. Therefore it follows that independently of the level of r_E^* and $\tilde{w}^*$, there exists a unique steady-state level of effort exerted by incumbent workers given by $\tilde{e}_E^* = \tilde{e}_n$, since $r_E^* = \tilde{w}^*$ for all $\tilde{w}^* \in [\tilde{w}_L^+, \tilde{w}_H(\lambda)^-]$ as just established. ■

Proof of Lemma 1. The steady-state value of an employment relationship with a newly hired worker exerting optimal effort $\tilde{e}_N^* = \{\tilde{e}_N^+, \tilde{e}_n, \tilde{e}_N(\lambda)^-\}$ and who is paid the corresponding optimal wage $\tilde{w}^* \in [\tilde{w}_L^+, \tilde{w}_H(\lambda)^-]$ can be written as:

$$J(r_N)^{+,=,-} = y(\tilde{e}_N^*) - \tilde{w}^* + \psi J(r_E)^{+,=,-}$$

where

$$J(r_E)^{+,=,-} = \frac{y(\tilde{e}_n) - \tilde{w}^*}{1-\psi}.$$

Recall that $y(\tilde{e}_n) = pq\tilde{e}_n$ and that $y(\tilde{e}_N^*) = pq\tilde{e}_n + pq\mu(\ln \tilde{w}^* - \ln r_N^*)$, where $\mu(\cdot)$ is the gain-loss function given by (2.2) in the text. Now, using the definitions of the present values of output and wages, $\overline{y}^*(\tilde{e}_N^*) = \{\overline{y}(\tilde{e}_N^+), \overline{y}(\tilde{e}_n), \overline{y}(\tilde{e}_N(\lambda)^-)\}$ and $\overline{w}^* \in [\overline{w}_L^+, \overline{w}_H(\lambda)^-]$ given by (2.17) and (2.16), the value function $J(r_N)^{+,=,-}$ can be expressed as

$$\begin{aligned}
J(r_N)^{+,=,-} &= \overline{y}^*(\tilde{e}_N^*) - \overline{w}^* \\
&= y(\tilde{e}_N^*) + \frac{\psi}{1-\psi} y(\tilde{e}_n) - \frac{\tilde{w}^*}{1-\psi} \\
&= \frac{pq\tilde{e}_n}{1-\psi} + pq\mu\big(\ln \tilde{w}^* - \ln r_N\big) - \frac{\tilde{w}^*}{1-\psi} \\
&= \frac{pq\tilde{e}_n - \tilde{w}^*}{1-\psi} + pq\mu(\ln \tilde{w}^* - \ln r_N) \\
&= J(r_E)^{+,=,-} + pq\mu(\ln \tilde{w}^* - \ln r_N).
\end{aligned}$$

Hence, if $r_N < r_L$ then $\tilde{w}^* = \tilde{w}_L^+$, $\mu(\ln \tilde{w}_L^+ - \ln r_N^*) > 0$ which implies that $J(r_N)^+ >$

$J(r_E)^+$; if $r_N > r_H$ then $\tilde{w}^* = \tilde{w}_H(\lambda)^-$, $\mu(\ln \tilde{w}_H(\lambda)^- - \ln r_N^*) < 0$ which implies that $J(r_N)^- < J(r_E)^-$; and if $r_N \in [r_L, r_H]$ then $\tilde{w}^* = \tilde{w}_M^=$, $\mu(\ln \tilde{w}_M^= - \ln r_N^*) = 0$ which implies that $J(r_N)^= = J(r_E)^=$, as stated in the proposition. Moreover, since $\mu(\ln \tilde{w}_L^+ - \ln r_N^*) > 0 > \mu(\ln \tilde{w}_H(\lambda)^- - \ln r_N^*)$ and $\tilde{w}_L^+ < \tilde{w}_M^= < \tilde{w}_H(\lambda)^-$, it is straightforward to verify that $J(r_N)^+ > J(r_N)^= > J(r_N)^-$. ∎

Proof of Proposition 8. Consider the explicit solution to the job creation condition for all r_N:

$$\tilde{\theta}^* = \left(\frac{\bar{m}\delta}{\kappa}\left[\overline{y}^*(\tilde{e}_N^*) - \overline{w}^*\right]\right)^{\frac{1}{\sigma}}.$$

It is straightforward to verify that if $r_N < r_L$ then $\overline{y}^*(\tilde{e}_N^*) = \overline{y}(\tilde{e}_N^+)$ and $\overline{w}^* = \overline{w}_L^+$ which determine $\tilde{\theta}_h^*$; if $r_N \in [r_L, r_H(\lambda)]$ then $\overline{y}^*(\tilde{e}_N^*) = \overline{y}(\tilde{e}_n)$ and $\overline{w}^* = \overline{w}_M^+$ which determine $\tilde{\theta}_m^*$; and if $r_N > r_H(\lambda)$ then $\overline{y}^*(\tilde{e}_N^*) = \overline{y}(\tilde{e}_N(\lambda)^-)$ and $\overline{w}^* = \overline{w}_H(\lambda)^-$ which determine $\tilde{\theta}_l^*(\lambda)$. Moreover, since $J(r_N)^{+,=,-} = \overline{y}^*(\tilde{e}_N^*) - \overline{w}^*$ and $\tilde{\theta}^*$ is increasing in $J(r_N$, using the results established by Lemma 1 it follows that $\tilde{\theta}_l^*(\lambda) < \tilde{\theta}_m^* < \tilde{\theta}_h^*$. Finally, since u^* is decreasing in $\tilde{\theta}^*$, it follows that $u_H^*(\lambda) > u_M^* > u_L^*$. ∎

Proof of Proposition 9. By appealing to the explicit characterisation of the wage $\tilde{w}^* = r_E^*$ when $\mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}$ it is straightforward to verify that both the wage and incumbents' reference wage are increasing in p and ρ. As such, since new hires' effort $\tilde{e}_N^*$ is increasing in the wage, it follows that it is also increasing in p and ρ. This proves statement a). The proof of statement b) is trivial, since neither the wage, nor the reference wage explicitly depend on p or ρ when $\mathcal{E} = \mathcal{M}^*$.

Aggregate productivity p. Now consider labour market tightness $\tilde{\theta}^*$ for all $\mathcal{E} = \{\mathcal{L}^*, \mathcal{M}^*, \mathcal{H}^*(\lambda)\}$. Total differentiation with respect to p yields:

$$\frac{d\tilde{\theta}^*}{dp} = \frac{\tilde{\theta}^*}{\sigma\left[\overline{y}^*(\tilde{e}_N^*) - \overline{w}^*\right]} \cdot \left[\frac{d\overline{y}^*(\tilde{e}_N^*)}{dp} - \frac{d\overline{w}^*}{dp}\right]$$

where the term in square brackets is given by

$$\frac{q\tilde{e}_n}{1-\psi} \quad \text{if } \mathcal{E} = \mathcal{M}^*,$$

implying that $\tilde{\theta}^*$ is increasing in p if $\mathcal{E} = \mathcal{M}^*$, and by

$$q\tilde{e}_N^* + \frac{\psi}{1-\psi}q\tilde{e}_n + pq\frac{\partial \tilde{e}_N^*}{\partial w}\frac{\partial \tilde{w}^*}{\partial p} - \frac{1}{1-\psi}\frac{\partial \tilde{w}^*}{\partial p} \quad \text{if } \mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}.$$

By collecting $\frac{1}{1-\psi}\frac{\partial \tilde{w}^*}{\partial p}$ as the common factor yields

$$q\tilde{e}_N^* + \frac{\psi}{1-\psi} q\tilde{e}_n + \frac{1}{1-\psi}\frac{\partial \tilde{w}^*}{\partial p}\left((1-\psi)pq\frac{\partial \tilde{e}_N^*}{\partial w}\frac{\partial \tilde{w}^*}{\partial p} - 1\right) \quad \text{if } \mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\},$$

where the term in round brackets i equivalent to the first-order condition $\Upsilon(w; r_C, \lambda) = 0$ characterising the optimal wage for $w \neq r$, implying that $\tilde{\theta}^*$ is increasing in p also if $\mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}$.

Job-destruction rate ρ. First, note that $\frac{\partial}{\partial \rho}\left\{\frac{1}{1-\psi}\right\} = \frac{\partial}{\partial \rho}\left\{\frac{\psi}{1-\psi}\right\} = -\frac{\delta}{(1-\psi)^2}$. Then, total differentiation of $\tilde{\theta}^*$ for all $\mathcal{E} = \{\mathcal{L}^*, \mathcal{M}^*, \mathcal{H}^*(\lambda)\}$ with respect to ρ yields:

$$\frac{d\tilde{\theta}^*}{d\rho} = \frac{\tilde{\theta}^*}{\sigma\left[\overline{y}^*(\tilde{e}_N^*) - \overline{w}^*\right]} \cdot \left[\frac{d\overline{y}^*(\tilde{e}_N^*)}{d\rho} - \frac{d\overline{w}^*}{d\rho}\right]$$

where the term in square brackets is given by

$$-\frac{\delta}{(1-\psi)^2}[pq\tilde{e}_n - r_N] \quad \text{if } \mathcal{E} = \mathcal{M}^*,$$

implying that $\tilde{\theta}^*$ is decreasing in ρ if $\mathcal{E} = \mathcal{M}^*$, and by

$$-\frac{\delta}{(1-\psi)^2} pq\tilde{e}_N^* + pq\frac{\partial \tilde{e}_N^*}{\partial w}\frac{\partial \tilde{w}^*}{\partial \rho} - \frac{1}{1-\psi}\frac{\partial \tilde{w}^*}{\partial \rho} - \frac{\delta}{(1-\psi)^2}\tilde{w}^* \quad \text{if } \mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}.$$

After collecting $\frac{1}{1-\psi}\frac{\partial \tilde{w}^*}{\partial \rho}$ as the common factor and noticing that the grouped terms are equivalent to the first-order condition for $w \neq r$ as above, it can be concluded that $\tilde{\theta}^*$ is decreasing in ρ also if $\mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}$. ■

Proof of Proposition 10. By appealing to the explicit characterisation of the wage $\tilde{w}^* = r_E^*$ if $\mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}$ it is straightforward to verify that neither the wage nor incumbents' reference wage depend on r_N, as such they are unaffected. As such, it is also straightforward to infer that $\tilde{e}_N^*$ is decreasing in r_N. This proves statement a). The proof of statement b) is also straightforward, since if $\mathcal{E} = \mathcal{M}^*$, then $\tilde{w}^*$ and r_E^* are equal to r_N, implying that they are both increasing in r_N. This also implies that new hires effort $\tilde{e}_N^*$ remains unaffected since

$$\frac{d\tilde{e}_N^*}{dr} = \frac{\partial \tilde{w}_M^=}{\partial r} - \frac{\partial r_N^*}{\partial r} = 0,$$

which prove statement b). Then, total differentiation of $\tilde{\theta}^*$ for all $\mathcal{E} = \{\mathcal{L}^*, \mathcal{M}^*, \mathcal{H}^*(\lambda)\}$

with respect to r_N yields:

$$\frac{d\tilde{\theta}^*}{dr} = \frac{\tilde{\theta}^*}{\sigma\left[\overline{y}^*(\tilde{e}_N^*) - \overline{w}^*\right]} \cdot \left[\frac{d\overline{y}^*(\tilde{e}_N^*)}{dr} - \frac{d\overline{w}^*}{dr}\right]$$

where the term in square brackets is

$$-\frac{1}{1-\psi} \quad \text{if } \mathcal{E} = \mathcal{M}^*,$$

implying that $\tilde{\theta}^*$ is decreasing in r_N if $\mathcal{E} = \mathcal{M}^*$, and

$$pq\frac{\partial \tilde{e}_N^*}{\partial r} \quad \text{if } \mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\},$$

also implying that $\tilde{\theta}^*$ is decreasing in r_N if $\mathcal{E} = \{\mathcal{L}^*, \mathcal{H}^*(\lambda)\}$, since $\frac{\partial \tilde{e}_N^*}{\partial r} < 0$ as noted above. ■

Proof of Proposition 11. By appealing to the explicit characterisation of the wage $\tilde{w}^* = r_E^* = \lambda pq(1-\psi)$ when $\mathcal{E} = \mathcal{H}^*(\lambda)$ it is straightforward to verify that $\tilde{w}_H(\lambda)^-$ and r_E^* are increasing in λ. Then consider new hires effort $\tilde{e}_N^* = \tilde{e}_N(\lambda)^-$. Total differentiation with respect to λ yields

$$\begin{aligned} \frac{d\tilde{e}_N(\lambda)^-}{d\lambda} &= \frac{\partial \tilde{e}_N(\lambda)^-}{\partial \lambda} + \frac{\partial \tilde{e}_N(\lambda)^-}{\partial w}\frac{\partial \tilde{w}_H(\lambda)^-}{\partial \lambda} \\ &= -(\ln r_N^* - \ln \tilde{w}_H(\lambda)^-) + 1 \end{aligned}$$

implying that $\tilde{e}_N^*$ is decreasing in λ (note that due to the logarithmic expression, the term in round brackets must be greater than 1). Finally, consider labour market tightness $\tilde{\theta}^* = \tilde{\theta}_l^*(\lambda)$. Total differentiation with respect to λ yields:

$$\frac{d\tilde{\theta}_l^*(\lambda)}{d\lambda} = \frac{\tilde{\theta}_l^*(\lambda)}{\sigma\left[\overline{y}^*(\tilde{e}_N(\lambda)^-) - \overline{w}_H^*(\lambda)^-\right]} \cdot \left[\frac{d\overline{y}^*(\tilde{e}_N(\lambda)^-)}{d\lambda} - \frac{d\overline{w}_H^*(\lambda)^-}{d\lambda}\right]$$

where the term in square brackets is given by

$$pq\frac{\partial \tilde{e}_N(\lambda)^-}{\partial \lambda} + pq\frac{\partial \tilde{e}_N(\lambda)^-}{\partial w}\frac{\partial \tilde{w}_H(\lambda)^-}{\partial \lambda} - \frac{1}{1-\psi}\frac{\partial \tilde{w}_H(\lambda)^-}{\partial \lambda}.$$

By collecting $\frac{1}{1-\psi}\frac{\partial \tilde{w}_H(\lambda)^-}{\partial \lambda}$ as the common factor yields

$$pq\frac{\partial \tilde{e}_N(\lambda)^-}{\partial \lambda} + \frac{1}{1-\psi}\frac{\partial \tilde{w}_H(\lambda)^-}{\partial \lambda}\left((1-\psi)pq\frac{\partial \tilde{e}_N(\lambda)^-}{\partial w}\frac{\partial \tilde{w}_H(\lambda)^-}{\partial \lambda} - 1\right),$$

where the term in round brackets is equivalent to the first-order condition $\Upsilon(w; r_C, \lambda) = 0$ characterising the optimal wage for $w < r$. Hence, since $\frac{\partial \tilde{e}_N(\lambda)^-}{\partial \lambda} < 0$ it follows that $\tilde{\theta}^*_l(\lambda)$ is decreasing in λ, implying that $u^*_H(\lambda)$ is increasing in λ. ■

Proof of Proposition 12. Trivial. ■

Proof of Lemma 2. From the definition of elasticity, for all $\tilde{\theta}^* = \{\tilde{\theta}^*_l(\lambda), \tilde{\theta}^*_m, \tilde{\theta}^*_h\}$

$$\begin{aligned}
\varepsilon_{\tilde{\theta}^*} &= \frac{p}{\tilde{\theta}}\frac{d\tilde{\theta}}{dp} \\
&= \frac{p}{\tilde{\theta}}\frac{1}{\sigma}\left(\frac{\bar{m}\delta}{\kappa}\left[\overline{y}^*(\tilde{e}^*_N) - \overline{w}^*\right]\right)^{\frac{1}{\sigma}-1}\frac{\bar{m}\delta}{\kappa}\left[\frac{d\overline{y}^*(\tilde{e}^*_N)}{dp} - \frac{d\overline{w}^*}{dp}\right] \\
&= \frac{p}{\tilde{\theta}}\frac{1}{\sigma}\left(\frac{\bar{m}\delta}{\kappa}\left[\overline{y}^*(\tilde{e}^*_N) - \overline{w}^*\right]\right)^{\frac{1}{\sigma}}\left(\frac{\bar{m}\delta}{\kappa}\left[\overline{y}^*(\tilde{e}^*_N) - \overline{w}^*\right]\right)^{-1}\frac{\bar{m}\delta}{\kappa}\left[\frac{d\overline{y}^*(\tilde{e}^*_N)}{dp} - \frac{d\overline{w}^*}{dp}\right] \\
&= \frac{p}{\tilde{\theta}}\frac{\tilde{\theta}}{\sigma}\frac{\kappa}{\bar{m}\delta}\frac{1}{\overline{y}^*(\tilde{e}^*_N) - \overline{w}^*}\frac{\bar{m}\delta}{\kappa}\left[\frac{d\overline{y}^*(\tilde{e}^*_N)}{dp} - \frac{d\overline{w}^*}{dp}\right] \\
&= \frac{1}{\sigma}\frac{1}{\overline{y}^*(\tilde{e}^*_N) - \overline{w}^*}p\left[\frac{d\overline{y}^*(\tilde{e}^*_N)}{dp} - \frac{d\overline{w}^*}{dp}\right],
\end{aligned} \tag{B.7}$$

where for all $\overline{y}^*(\tilde{e}^*_N) = \{\overline{y}(\tilde{e}^+_N), \overline{y}(\tilde{e}_n), \overline{y}(\tilde{e}_N(\lambda)^-)\}$ and $\overline{w}^* \in [\overline{w}^+_L, \overline{w}_H(\lambda)^-]$:

$$\begin{aligned}
\frac{d\overline{y}^*(\tilde{e}^*_N)}{dp} &= \frac{\partial\overline{y}^*(\tilde{e}^*_N)}{\partial p} + \frac{\partial y(\tilde{e}^*_N)}{\partial e}\frac{\partial\tilde{e}^*_N}{\partial w}\frac{\partial\tilde{w}^*}{\partial p} \\
\frac{d\overline{w}^*}{dp} &= \frac{1}{1-\psi}\frac{\partial\tilde{w}^*}{\partial p}.
\end{aligned}$$

By collecting $\frac{1}{1-\psi}\frac{\partial w}{\partial p}$ and rearranging, the last term in equation (B.7) can expressed as

$$p\left[\frac{d\overline{y}^*(\tilde{e}^*_N)}{dp} - \frac{d\overline{w}^*}{dp}\right] = p\frac{\partial\overline{y}^*(\tilde{e}^*_N)}{\partial p} + p\frac{\partial\tilde{w}^*}{\partial p}\frac{1}{1-\psi}\cdot\left[(1-\psi)\frac{\partial y(\tilde{e}^*_N)}{\partial e}\frac{\partial\tilde{e}^*_N}{\partial w} - 1\right].$$

Now, since $\overline{y}^*(\tilde{e}^*_N)$ is linear in p, if follows that $p\frac{\partial\overline{y}^*(\tilde{e}^*_N)}{\partial p} = \overline{y}^*(\tilde{e}^*_N)$. Moreover, by using the definition of elasticity of the wage with respect to productivity, $p\frac{\partial\tilde{w}^*}{\partial p} = \varepsilon_{\tilde{w}^*}\tilde{w}^*$, and by using the definition of the present value of wages as given by (2.16) $\overline{w}^* = \frac{\tilde{w}^*}{1-\psi}$, it follows that:

$$p\left[\frac{d\overline{y}^*(\tilde{e}^*_N)}{dp} - \frac{d\overline{w}^*}{dp}\right] = \overline{y}^*(\tilde{e}^*_N) + \varepsilon_{\tilde{w}^*}\overline{w}^*\cdot\left[(1-\psi)\frac{\partial y(\tilde{e}^*_N)}{\partial e}\frac{\partial\tilde{e}^*_N}{\partial w} - 1\right]$$

Finally, define $\Theta \equiv -\left[(1-\psi)\frac{\partial y(\tilde{e}^*_N)}{\partial e}\frac{\partial\tilde{e}^*_N}{\partial w} - 1\right]$, and substitute the above equation into (B.7), to obtain the expression stated in the lemma. ■

Proof of Proposition 13. The proof of this proposition is almost trivial. For case a) notice that in the neighborhood of these equilibria

$$\varepsilon_{\tilde{w}^*}\overline{w}^* = \varepsilon_{\tilde{w}^*}\frac{\tilde{w}_M^{=}}{1-\psi} = \frac{p}{1-\psi}\frac{\partial \tilde{w}_M^{=}}{\partial p} = 0.$$

For case b) notice that

$$\Theta \equiv -\left[\frac{\partial y(\tilde{e}_N^*)}{\partial e}\frac{\partial \tilde{e}_N^*}{\partial w} - \psi\frac{\partial y(\tilde{e}_E^*)}{\partial e}\frac{\partial \tilde{e}_E^*}{\partial w} - 1\right] = -\left[\frac{\partial y(\tilde{e}_N^*)}{\partial e}\frac{\partial \tilde{e}_N^*}{\partial w}(1-\psi) - 1\right].$$

where the expression in square brackets is equivalent to $\Upsilon(w; r_C, \lambda) = 0$ for all $w \neq r$. Hence $\Theta = 0$. ■

Proof of Proposition 14. When the labour market is stationed at the high equilibrium, the w/y ratio is given by

$$\frac{\overline{w}_N^*}{\overline{y}(\tilde{e}_N^*)} = \frac{\lambda}{\tilde{e}_N(\lambda)^- + \frac{\psi}{1-\psi}\tilde{e}_n}$$
$$= \frac{\lambda}{\lambda\left[\ln \tilde{w}_H(\lambda)^- - \ln r_N\right] + \frac{\tilde{e}_n}{1-\psi}}.$$

Total differentiation with respect to λ yields

$$\frac{d}{d\lambda}\left\{\frac{\overline{w}_N^*}{\overline{y}(\tilde{e}_N^*)}\right\} = \frac{\left[\lambda\left[\ln \tilde{w}_H(\lambda)^- - \ln r_N\right] + \frac{\tilde{e}_n}{1-\psi}\right] - \lambda\left[\left[\ln \tilde{w}_H(\lambda)^- - \ln r_N\right] + \lambda\frac{1}{\tilde{w}_H(\lambda)^-}\frac{\partial \tilde{w}_H(\lambda)^-}{\partial \lambda}\right]}{\left[\lambda\left[\ln \tilde{w}_H(\lambda)^- - \ln r_N\right] + \frac{\tilde{e}_n}{1-\psi}\right]^2}$$

Since $\frac{\lambda}{w_H(\lambda)^-}\frac{\partial w_H(\lambda)^-}{\partial \lambda} = 1$, rearranging the expression above gives

$$\frac{d}{d\lambda}\left\{\frac{\overline{w}_N^*}{\overline{y}(\tilde{e}_N^*)}\right\} = \frac{\tilde{e}_n - \lambda(1-\psi)}{(1-\psi)\left[\lambda\left[\ln \tilde{w}_H(\lambda)^- - \ln r_N\right] + \frac{\tilde{e}_n}{1-\psi}\right]^2} > 0,$$

where the numerator is always positive since $\tilde{e}_n > \underline{e}_n(r_N, \lambda) > \lambda(1-\psi)$ by the minimum-normal effort condition derived in Lemma 5, Appendix A.2, Section A.2.1. ■

Throughout the next proofs: $dF \equiv dF_{q_E|q_N}$.

Proof of Proposition 15. Consider the first-order condition (2.27) characterising the solution to the general recursive problem (2.26). Due to assumption U2 it follows that

$$\Phi(w,\lambda) \equiv \begin{cases} \dfrac{\partial}{\partial r}\displaystyle\int J(w,q_E)\,dF & \text{if } t = \tau \\ \\ \dfrac{\partial J(w,q_E)}{\partial r} & \text{if } t > \tau. \end{cases} \tag{B.8}$$

In the remainder of the proof this result is used to characterise the solution to the two-step recursive problem (2.28-2.29), by first finding the solution to step i), i.e. the optimal wage setting policy for all $t > \tau$, and then the solution to step ii), i.e. the optimal wage setting policy for all $t = \tau$. The reader is referred to the preliminaries of the Proof of Proposition 6 for technical details regarding existence and uniqueness of solutions to the functional equation characterising the first step (2.29).

Step i).

Incumbent workers: preliminaries. Making use of (B.8), the first-order condition for incumbent workers is given by

$$\Upsilon(w; r_E, q_E, \lambda) \equiv \frac{\partial y(q_E,\tilde{e}_E)}{\partial e}\frac{\partial \tilde{e}(w,r_E,\lambda)}{\partial w} - 1 + \psi\frac{\partial J(w,q_E)}{\partial r} = 0 \tag{B.9}$$

so long as $w \neq r_E$. And the corresponding envelope condition is given by

$$\frac{\partial J(r_E,q_E)}{\partial r} = \frac{\partial y(q_E,\tilde{e}_E)}{\partial e}\frac{\partial \tilde{e}(w,r_E,\lambda)}{\partial r}.$$

Combining the two latter conditions gives:

$$\Upsilon(w; r_E, q_E, \lambda) \equiv \frac{\partial y(q_E,\tilde{e}_E)}{\partial e}\frac{\partial \tilde{e}(w,r_E,\lambda)}{\partial w} - 1 + \psi\frac{\partial y(q_E,\tilde{e}_E)}{\partial e}\frac{\partial \tilde{e}(w',r'_E,\lambda)}{\partial r} = 0. \tag{B.10}$$

Note that $\partial\Upsilon(w; r_E, q_E, \lambda)/\partial w < 0$ and that if $w < r_E$, $\partial\Upsilon(w; r_E, q_E, \lambda)/\partial\lambda > 0$, and if $w > r_E$, $\partial\Upsilon(w; r_E, q_E, \lambda)/\partial\lambda = 0$, allowing to conclude that $\Upsilon(w; r_E, q_E, \lambda)$ is everywhere decreasing in w and jumps down at the reference wage. In addition, $\partial\Upsilon(w; r_E, q_E, \lambda)/\partial q > 0$.

Incumbent workers: productivity thresholds. The threshold $q_l(r_E, \lambda)$ identifies the critical match productivity below which a firm would want to set the wage below the reference wage, and $q_u(r_E)$ is the match productivity above which a firm would want to compensate the worker more than the reference wage. The former is the value of q_E below which the value function is decreasing just below the reference wage; the latter is the value of q_E above which the value function is increasing just above the reference wage. As such, they are defined as

$$q_u(r_E) \equiv \left\{ q_E : \lim_{\epsilon \to 0} \Upsilon(r_E + \epsilon; r_E, q_E, \lambda) = 0 \right\}$$
$$q_l(r_E, \lambda) \equiv \left\{ q_E : \lim_{\epsilon \to 0} \Upsilon(r_E - \epsilon; r_E, q_E, \lambda) = 0 \right\}.$$

It can be easily shown that $q_u(r_E) \equiv r_E/p(1-\psi)$ and that $q_l(r_E, \lambda) \equiv r_E/\lambda p(1-\psi)$, which implies that $q_u(r_E) > q_l(r_E, \lambda)$ and that if $\lambda = 1$, $q_u(r_E) = q_l(r_E, 1)$.

Incumbent workers: optimal wage setting policy. The optimal time-invariant wage setting policy for all $t > \tau$ depends on the level of match productivity of incumbent workers q_E in relation to the thresholds derived above.

If $q_E < q_l(r_E, \lambda)$, then the definition of q_l and the fact that $\partial\Upsilon/\partial q > 0$, can be used to deduce that $\lim_{\epsilon\to 0} \Upsilon(r_E - \epsilon; r_E, q_E, \lambda) < 0$; since $\partial\Upsilon/\partial w < 0$, the same is true for all $w \geq r_E$. As such the optimising wage must satisfy $w < r_E$ and will therefore be the solution to

$$\Upsilon(w; r_E, q_E, \lambda) \equiv \lambda p q_E \frac{1}{w} - 1 - \psi \lambda p q_E \frac{1}{w} = 0$$

given by $\tilde{w} = \lambda p q_E(1-\psi)$, and denoted by $\tilde{w}_E(\lambda)^- \equiv \tilde{w}(r_E, q_E, \lambda)^-$.

If $q_E > q_u(r_E)$, then the definition of q_u and the fact that $\partial\Upsilon/\partial q > 0$, can be used to deduce that $\lim_{\epsilon\to 0} \Upsilon(r_E - \epsilon; r_E, q_E, \lambda) > 0$; since $\partial\Upsilon/\partial w < 0$, the same is true for all $w \leq r_E$. As such the optimising wage must satisfy $w > r_E$ and will therefore be the solution to

$$\Upsilon(w; r_E, q_E, \lambda) \equiv p q_E \frac{1}{w} - 1 - \psi p q_E \frac{1}{w} = 0$$

given by $\tilde{w} = p q_E(1-\psi)$, and denoted by $\tilde{w}_E^+ \equiv \tilde{w}(r_E, q_E)^+$.

Finally, if $q_E \in [q_l, q_u]$ the fact that $\partial\Upsilon/\partial q > 0$ can be used to deduce that $\lim_{\epsilon\to 0} \Upsilon(r_E - \epsilon; r_E, q_E, \lambda) \geq 0$ and $\lim_{\epsilon\to 0} \Upsilon(r_E + \epsilon; r_E, q_E, \lambda) \leq 0$. Since $\partial\Upsilon/\partial w < 0$ for all $w \neq r_E$, then $\Upsilon(w; r_E, q_E, \lambda) > 0$ for all $w < r_E$ and $\Upsilon(w; r_E, q_E, \lambda) < 0$ for all $w > r_E$, implying that $J(r_E, q_E)$ is maximised if and only if $w = r_E$, which characterises the solution denoted by $\tilde{w}_E^= \equiv \tilde{w}(r_E, q_E)^=$.

Incumbent workers: expected continuation value. The results above can be used to write an expression for the expected continuation value of an employment relationship at $t = \tau + 1$ for any possible realisation of q_E, which is crucial for the characterisation of the optimal wage of newly hired workers at $t = \tau$.

If $q_E < q_l(r_E, \lambda)$, the incumbent worker is paid the optimal time-invariant wage $\tilde{w}_E(\lambda)^-$ and will exert negative reciprocity $\tilde{e}_E(\lambda)^-$ in the period they receive the wage, and normal effort $\tilde{e}_n$ thereafter due to adaptation, i.e. $r_E = \tilde{w}_E(\lambda)^-$. Hence:

$$J(r_E, q_E) = p q_E \tilde{e}_E(\lambda)^- - \tilde{w}_E(\lambda)^- + \psi J(\tilde{w}_E(\lambda)^-, q_E),$$

where

$$\begin{aligned}
J(\tilde{w}_E(\lambda)^-, q_E) &= pq\tilde{e}_n - \tilde{w}_E(\lambda)^- + \psi J(\tilde{w}_E(\lambda)^-, q_E) \\
&= \frac{pq\tilde{e}_n - \tilde{w}_E(\lambda)^-}{1-\psi} \\
&= \frac{1}{1-\psi} pq\tilde{e}_n - \frac{1}{1-\psi}\tilde{w}_E(\lambda)^-.
\end{aligned}$$

By combining these two expressions and rearranging, and also by using the definition of the instantaneous value of output $y(q_C, \tilde{e}_C) = pq_C\tilde{e}_C$, it follows that

$$\begin{aligned}
J(r_E, q_E) &= pq_E\tilde{e}_E(\lambda)^- + \frac{\psi}{1-\psi} pq\tilde{e}_n - \frac{1}{1-\psi}\tilde{w}_E(\lambda)^- \\
&= y(q_E, \tilde{e}_E(\lambda)^-) + \frac{\psi}{1-\psi} y(q_E, \tilde{e}_n) - \frac{1}{1-\psi}\tilde{w}_E(\lambda)^- \\
&\equiv J(r_E, q_E)^-
\end{aligned}$$

An analogous procedure can be used to show that if $q_E > q_u(r_E)$

$$\begin{aligned}
J(r_E, q_E) &= pq_E\tilde{e}_E^+ + \frac{\psi}{1-\psi} pq\tilde{e}_n - \frac{1}{1-\psi}\tilde{w}_E^+ \\
&= y(q_E, \tilde{e}_E^+) + \frac{\psi}{1-\psi} y(q_E, \tilde{e}_n) - \frac{1}{1-\psi}\tilde{w}_E^+ \\
&\equiv J(r_E, q_E)^+;
\end{aligned}$$

and that if $q_E < q_l(r_E, \lambda)$

$$\begin{aligned}
J(r_E, q_E) &= \frac{1}{1-\psi} pq\tilde{e}_n - \frac{1}{1-\psi}\tilde{w}_E^= \\
&= \frac{1}{1-\psi} y(q_E, \tilde{e}_n) - \frac{1}{1-\psi}\tilde{w}_E^= \\
&\equiv J(r_E, q_E)^=.
\end{aligned}$$

As such the expected continuation value of an employment relationship with a newly hired worker becoming incumbent in the following period can be written as

$$\int J(w, q_E)\, dF = \int^{q_l(w,\lambda)} J(w, q_E)^-\, dF + \int_{q_l(w,\lambda)}^{q_u(w)} J(w, q_E)^=\, dF + \int_{q_u(w)} J(w, q_E)^+\, dF.$$

Step ii).

Newly hired workers: preliminaries. Making use of (B.8), the first-order condition for newly hired workers is given by

$$\Upsilon(w; r_N, q_N, \lambda) \equiv \frac{\partial y(q_N, \tilde{e}_N)}{\partial e}\frac{\partial \tilde{e}(w_N, r_N)}{\partial w} - 1 + \psi\frac{\partial}{\partial r}\int J(w, q_E)\,dF = 0, \qquad \text{(B.11)}$$

which, as it is shown below, can be analysed for the case of $w > r_N$ only, since $r_N = 0$ by assumption U1. In order to characterise the optimal wage paid to newly hired workers, it is crucial to analyse the envelope condition given by $\Phi(w, \lambda) \equiv \frac{\partial}{\partial r}\int J(w, q_E)\,dF$. First notice that since the value function $J(w, q_E)$ is continuous in the wage, the partial derivatives with respect to the limits of the integrals cancel out:

$$\frac{\partial q_l}{\partial r}f(q_l)J(w, q_l)^- - \frac{\partial q_l}{\partial r}f(q_l)J(w, q_l)^= + \frac{\partial q_u}{\partial r}f(q_u)J(w, q_u)^= - \frac{\partial q_u}{\partial r}f(q_u)J(w, q_l)^+ = 0,$$

implying that

$$\frac{\partial}{\partial r}\int J(w, q_E)\,dF = \int \frac{\partial J(w, q_E)}{\partial r}\,dF.$$

Hence, for any possible realisation of q_E, using the results derived in step i), the envelope conditions are given by

$$\begin{aligned}
\frac{\partial J(r_E, q_E)^+}{\partial r} &= \frac{\partial y(q_E, \tilde{e}_E)}{\partial e}\frac{\partial \tilde{e}(\tilde{w}_E, r_E)^+}{\partial r}\\
\frac{\partial J(r_E, q_E)^=}{\partial r} &= -\frac{1}{1-\psi}\\
\frac{\partial J(r_E, q_E)^-}{\partial r} &= \frac{\partial y(q_E, \tilde{e}_E)}{\partial e}\frac{\partial \tilde{e}(\tilde{w}_E, r_E, \lambda)^-}{\partial r}
\end{aligned}$$

which can be used to write an explicit form expression for $\Phi(w, \lambda)$:

$$\begin{aligned}
\Phi(w, \lambda) &\equiv \int^{q_l} \frac{\partial y(q_E, \tilde{e}_E)}{\partial e}\frac{\partial \tilde{e}(\tilde{w}_E, r_E, \lambda)^-}{\partial r}\,dF - \int_{q_l}^{q_u} \frac{1}{1-\psi}\,dF + \int_{q_u} \frac{\partial y(q_E, \tilde{e}_E)}{\partial e}\frac{\partial \tilde{e}(\tilde{w}_E, r_E)^+}{\partial r}\,dF\\
&\equiv -\int^{q_l} \lambda p q_E \frac{1}{w}\,dF - \int_{q_l}^{q_u} \frac{1}{1-\psi}\,dF - \int_{q_u} p q_E \frac{1}{w}\,dF\\
&< 0
\end{aligned} \qquad \text{(B.12)}$$

where indeed $q_u = q_u(w)$ and $q_l = q_l(w, \lambda)$. Finally, by combining the first-order condition (B.11) with the corresponding envelope condition above gives

$$\begin{aligned}
&\Upsilon(w; r_N, q_N, \lambda) \equiv\\
&p q_N \frac{1}{w} - 1 + \psi\left(-\int^{q_l(w,\lambda)} \lambda p q_E \frac{1}{w}\,dF - \int_{q_l(w,\lambda)}^{q_u(w)} \frac{1}{1-\psi}\,dF - \int_{q_u(w)} p q_E \frac{1}{w}\,dF\right) = 0
\end{aligned}$$

which implicitly characterises new hires optimal wage for the case of $w > r_N$. To proceed, rearrange the first-order condition above as

$$w \cdot \left(1 + \int_{q_l(w,\lambda)}^{q_u(w)} \frac{\psi}{1-\psi}\, dF\right) = \left[pq_N - \psi \left(\int^{q_l(w,\lambda)} \lambda pq_E\, dF + \int_{q_u(w)} pq_E\, dF\right)\right]. \tag{B.13}$$

Then, note that

$$\begin{aligned}\frac{\partial \Upsilon(w; r_N, q_N, \lambda)}{\partial w} &= -\frac{1}{w^2} pq_N + \psi \left(\int^{q_l} \lambda pq_E \frac{1}{w^2}\, dF + \int_{q_u} pq_E \frac{1}{w^2}\, dF\right) \\ &= -\frac{1}{w^2}\left[pq_N - \psi\left(\int^{q_l} \lambda pq_E\, dF + \int_{q_u} pq_E\, dF\right)\right]\end{aligned}$$

where, using the expressions for q_u and q_l as derived above, the derivatives with respect to the limits in the first line cancel out since

$$\frac{\partial q_l}{\partial r} f(q_l)\lambda pq_l \frac{1}{w} - \frac{\partial q_l}{\partial r} f(q_l)\frac{1}{1-\psi} + \frac{\partial q_u}{\partial r} f(q_u)\frac{1}{1-\psi} - \frac{\partial q_u}{\partial r} f(q_u) pq_u \frac{1}{w} = 0.$$

Substituting the right-hand side of (B.13) into the expression above gives

$$\frac{\partial \Upsilon(w; r_N, q_N, \lambda)}{\partial w} = -\frac{1}{w}\left(1 + \int_{q_l(w,\lambda)}^{q_u(w)} \frac{\psi}{1-\psi}\, dF\right) < 0 \tag{B.14}$$

establishing concavity of the value function $J(r_N, q_N)$. Moreover note that

$$\frac{\partial \Upsilon(w; r_N, q_N, \lambda)}{\partial \lambda} = -\psi \int^{q_l(w,\lambda)} pq_E \frac{1}{w}\, dF < 0, \tag{B.15}$$

where again, by using the expressions for q_u and q_l, the derivatives with respect to the limits cancel out since

$$\frac{\partial q_l}{\partial \lambda} f(q_l)\lambda pq_l \frac{1}{w} - \frac{\partial q_l}{\partial \lambda} f(q_l)\frac{1}{1-\psi} = 0.$$

Finally, $\partial \Upsilon(w; r_N, q_N, \lambda)/\partial q > 0$.

Newly hired workers: productivity threshold. Define the upper productivity threshold $q_u(r_N)$ as the match productivity above which a firm would want to compensate the worker more than the reference wage as

$$q_u(r_N) \equiv \left\{q_N : \lim_{\epsilon \to 0} \Upsilon(r_N + \epsilon; r_N, q_N, \lambda) = 0\right\}.$$

Since $r_N = 0$ by assumption U1, it follows that $q_u(r_N) = 0$.

Newly hired workers: optimal wage setting. The fact that $q_u = 0$ implies that a firm would always want to pay a newly hired worker wage above reference wage. As such, the optimal wage must satisfy the first-order condition for $w > r_N$, given by (B.11). Although it is not possible to obtain an explicit solution, this condition can be rearranged in a useful expression. Consider the rearrangement in (B.13): by collecting p on the right-hand side and noticing that $\int_{q_l}^{q_u} dF = F(q_u) - F(q_l)$, the first-order condition can be re-written as:

$$w = \frac{p\left[q_N - \psi\left(\int^{q_l} \lambda q_E \, dF + \int_{q_u} q_E \, dF\right)\right](1-\psi)}{1-\psi\left[1-\left(F(q_u)-F(q_l)\right)\right]}.$$

The expression above implicitly characterises the optimal wage paid to newly hired workers, denoted by $\tilde{w}_N(\lambda)^+ \equiv \tilde{w}(r_N, q_N, \lambda)$. ■

Proof of Proposition 16. Consider first the effect of λ on $\mathbb{E}[\tilde{w}_N^*] = \tilde{w}_N(\lambda)^+$. Implicit differentiation reveals:

$$\frac{d\tilde{w}_N(\lambda)^+}{d\lambda} = -\frac{\partial\Upsilon(w; r_N, q_N, \lambda)/\partial\lambda}{\partial\Upsilon(w; r_N, q_N, \lambda)/\partial w} < 0, \tag{B.16}$$

since from the proof of Proposition 15 $\partial\Upsilon(w; r_N, q_N, \lambda)/\partial\lambda < 0$ and $\partial\Upsilon(w; r_N, q_N, \lambda)/\partial w < 0$, implying that $\tilde{w}_N(\lambda)^+$ is decreasing in λ. Then consider the effect of λ on $\mathbb{E}[\tilde{w}_E^*]$, which after substituting $r_E = \tilde{w}_N(\lambda)^+$ can be written as:

$$\int \tilde{w}_E^* \, dF = \int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} \tilde{w}_E(\lambda)^- \, dF + \int_{q_l(\tilde{w}_N(\lambda)^+,\lambda)}^{q_u(\tilde{w}_N(\lambda)^+)} \tilde{w}_N(\lambda)^+ \, dF + \int_{q_u(\tilde{w}_N(\lambda)^+)} \tilde{w}_E^+ \, dF.$$

Differentiation with respect to λ yields:

$$\frac{d}{d\lambda}\int \tilde{w}_E^* \, dF = \int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} \frac{\partial \tilde{w}_E(\lambda)^-}{\partial\lambda} \, dF + \int_{q_l(\tilde{w}_N(\lambda)^+,\lambda)}^{q_u(\tilde{w}_N(\lambda)^+)} \frac{\partial \tilde{w}_N(\lambda)^+}{\partial\lambda} \, dF,$$

where the derivatives with respect to the limits cancel out since $\tilde{w}_E(\lambda)^{-;+}$ evaluated at the productivity thresholds $q_l = \frac{\tilde{w}_N(\lambda)^+}{p(1-\psi)}$ and $q_u = \frac{\tilde{w}_N(\lambda)^+}{p(1-\psi)}$ respectively, as required by differentiation, is equal to $r_E = \tilde{w}_N(\lambda)^+$. That is:

$$\frac{\partial q_l}{\partial r}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial\lambda} f(q_l)\lambda p q_l(1-\psi) - \frac{\partial q_l}{\partial r}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial\lambda} f(q_l)\tilde{w}_N(\lambda)^+ = 0$$

$$\frac{\partial q_l}{\partial \lambda} f(q_l)\lambda p q_l(1-\psi) - \frac{\partial q_l}{\partial \lambda} f(q_l)\tilde{w}_N(\lambda)^+ = 0$$

$$\frac{\partial q_u}{\partial r}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda} f(q_u)\tilde{w}_N(\lambda)^+ - \frac{\partial q_u}{\partial r}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda} f(q_u) p q_u(1-\psi) = 0.$$

It has been proved that $\frac{d\tilde{w}_N(\lambda)^+}{d\lambda} < 0$ and it is straightforward to deduce that $\frac{\partial \tilde{w}_E(\lambda)^-}{\partial \lambda} > 0$. Moreover, since from the proof of Lemma 3 (below):

$$\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda} + \frac{\psi}{1-\psi}\left(\int_{q_l(\tilde{w}_N(\lambda)^+,\lambda)}^{q_u(\tilde{w}_N(\lambda)^+)} \frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda}\, dF + \int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} \frac{\partial \tilde{w}_E(\lambda)^-}{\partial \lambda}\, dF\right) = 0,$$

and since $\frac{d\tilde{w}_N(\lambda)^+}{d\lambda} < 0$, it is possible to deduce that the term in round brackets, which is equivalent to $\frac{d}{d\lambda}\int \tilde{w}_E^*\, dF$, is positive. Hence, $\mathbb{E}[\tilde{w}_E^*]$ is increasing in λ. ■

Proof of Lemma 3. Total differentiation of $\overline{w}_C^*(\lambda)$ with respect to λ yields:

$$\begin{aligned}\frac{d\overline{w}_C^*(\lambda)}{d\lambda} &= \frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda} + \frac{\psi}{1-\psi}\frac{\partial}{\partial \lambda}\int \tilde{w}_E^*\, dF \\ &= \frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda} + \frac{\psi}{1-\psi}\left(\int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} \frac{\partial \tilde{w}_E(\lambda)^-}{\partial \lambda}\, dF + \int_{q_l(\tilde{w}_N(\lambda)^+,\lambda)}^{q_u(\tilde{w}_N(\lambda)^+)} \frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda}\, dF\right),\end{aligned}$$

which is obtained by using the results contained in the proof of Proposition 16. Collecting $\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda}$ as the common factor and rearranging yields:

$$\frac{d\overline{w}_C^*(\lambda)}{d\lambda} = \frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda}\left(1 + \int_{q_l(\tilde{w}_N(\lambda)^+,\lambda)}^{q_u(\tilde{w}_N(\lambda)^+)} \frac{\psi}{1-\psi}\, dF\right) + \frac{\psi}{1-\psi}\int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} \frac{\partial \tilde{w}_E(\lambda)^-}{\partial \lambda}\, dF$$

By using explicit functional forms it can be noted that

$$\frac{\psi}{1-\psi}\int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} \frac{\partial \tilde{w}_E(\lambda)^-}{\partial \lambda}\, dF = \psi \int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} p q_E\, dF$$

and that

$$\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda} = \frac{-\psi \displaystyle\int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} p q_E\, dF}{\left(1 + \displaystyle\int_{q_l(\tilde{w}_N(\lambda)^+,\lambda)}^{q_u(\tilde{w}_N(\lambda)^+)} \frac{\psi}{1-\psi}\, dF\right)}, \tag{B.17}$$

which is obtained by substituting the expressions for $\partial \Upsilon(w; r_N, q_N, \lambda)/\partial \lambda$ and $\partial \Upsilon(w; r_N, q_N, \lambda)/\partial w$ derived in the proof of Proposition 15 (i.e. expressions (B.15) and (B.14)) into the implicit differentiation expression (B.16) derived in the proof of Proposition 16. As

such
$$\frac{d\overline{w}_C^*(\lambda)}{d\lambda} = -\psi \int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} pq_E \, dF + \psi \int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} pq_E \, dF = 0$$

∎

Proof of Lemma 4. Total differentiation of $\overline{y}_C^*(\tilde{e}_C^*(\lambda))$ with respect to λ yields:

$$\begin{aligned}\frac{d\overline{y}_C^*(\tilde{e}_C^*(\lambda))}{d\lambda} = \frac{\partial y_N}{\partial e}\frac{\partial e_N^+}{\partial w}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda} &+ \psi\Bigg(\int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} \frac{\partial y_E}{\partial e}\frac{\partial e_E(\lambda)^-}{\partial r}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda} dF \\ &+ \int^{q_l(\tilde{w}_N(\lambda)^+,\lambda)} \frac{\partial y_E}{\partial e}\frac{\partial e_E(\lambda)^-}{\partial \lambda} + \frac{\partial y_E}{\partial e}\frac{\partial e_E(\lambda)^-}{\partial w}\frac{\partial \tilde{w}_E(\lambda)^+}{\partial \lambda} dF \\ &+ \int_{q_u(\tilde{w}_N(\lambda)^-)} \frac{\partial y_E}{\partial e}\frac{\partial e_E^+}{\partial r}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda} dF \Bigg),\end{aligned}$$

where $y_C \equiv y(q_C, \tilde{e}_C)$ and the derivatives with respect to the limits cancel out since $y(q_E, \tilde{e}_E(\lambda)^{-,+})$ evaluated at the productivity thresholds q_l and q_u respectively, as required by differentiation, is equal to $y(q_E, \tilde{e}_n)$, since $\tilde{w}_E^* = r_E$ at these thresholds. That is:

$$\begin{aligned}\frac{\partial q_l}{\partial r}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda} f(q_l)\frac{pq_l\tilde{e}_n}{1-\psi} - \frac{\partial q_l}{\partial r}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda} f(q_l)\frac{pq_l\tilde{e}_n}{1-\psi} &= 0 \\ \frac{\partial q_l}{\partial \lambda} f(q_l)\frac{pq_l\tilde{e}_n}{1-\psi} - \frac{\partial q_l}{\partial \lambda} f(q_l)\frac{pq_l\tilde{e}_n}{1-\psi} &= 0 \\ \frac{\partial q_u}{\partial \lambda} f(q_u)\frac{pq_u\tilde{e}_n}{1-\psi} - \frac{\partial q_u}{\partial \lambda} f(q_u)\frac{pq_u\tilde{e}_n}{1-\psi} &= 0.\end{aligned}$$

To proceed note that due to the linearity of $\tilde{e}_C$ with respect to the gain-loss function $\mu(\cdot)$: $\frac{\partial e_C^-}{\partial r} = \lambda \frac{\partial e_C^+}{\partial r}$ and that due to the properties of $\mu(\cdot)$: $\frac{\partial e_C^+}{\partial r} = -\frac{\partial e_C^+}{\partial w}$. As such, $\frac{\partial e_C^-}{\partial r} = -\lambda \frac{\partial e_C^+}{\partial w}$. Substituting this equality into the expression for $\frac{d\overline{y}_C^*(\tilde{e}_C^*(\lambda))}{d\lambda}$ and collecting $\frac{\partial e_C^+}{\partial w}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda}$ as the common factor yields

$$\begin{aligned}\frac{d\overline{y}_C^*(\tilde{e}_C^*(\lambda))}{d\lambda} = \frac{\partial e_C^+}{\partial w}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda}&\left[\frac{\partial y_N}{\partial e} - \psi\left(\int^{q_l} \lambda\frac{\partial y_E}{\partial e} dF + \int_{q_u} \frac{\partial y_E}{\partial e} dF\right)\right] \\ &+ \psi\int^{q_l} \frac{\partial y_E}{\partial e}\frac{\partial e_E(\lambda)^-}{\partial \lambda} + \frac{\partial y_E}{\partial e}\frac{\partial e_E(\lambda)^-}{\partial w}\frac{\partial \tilde{w}_E(\lambda)^-}{\partial \lambda} dF\end{aligned}$$

By using the explicit functional forms and substituting expressions (B.17) for $\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda}$ and (B.13) for the term in square brackets, it can be noted that

$$\frac{\partial e_C^+}{\partial w}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial \lambda}\left[\frac{\partial y_N}{\partial e} - \psi\left(\int^{q_l} \lambda\frac{\partial y_E}{\partial e} dF + \int_{q_u} \frac{\partial y_E}{\partial e} dF\right)\right] = -\psi\int^{q_l} pq_E \, dF$$

and that

$$\psi \int^{q_l} \frac{\partial y_E}{\partial e} \frac{\partial e_E(\lambda)^-}{\partial w} \frac{\partial \tilde{w}_E(\lambda)^-}{\partial \lambda} \, dF = \psi \int^{q_l} pq_E \, dF,$$

which substituted into the expression for $\frac{d\overline{y}^*_C(\tilde{e}^*_C(\lambda))}{d\lambda}$ yields

$$\frac{d\overline{y}^*_C(\tilde{e}^*_C(\lambda))}{d\lambda} = \psi \int^{q_l} \frac{\partial y_E}{\partial e} \frac{\partial e_E(\lambda)^-}{\partial \lambda} \, dF < 0,$$

since $\frac{\partial y_E}{\partial e} > 0$ and $\frac{\partial e_E(\lambda)^-}{\partial \lambda} < 0$. ∎

Proof of Proposition 17. Total differentiation of $\tilde{\theta}^*(\lambda)$ with respect to λ yields:

$$\frac{d\tilde{\theta}^*(\lambda)}{d\lambda} = \frac{\tilde{\theta}^*(\lambda)}{\sigma\left[\overline{y}^*_C(\tilde{e}^*_C(\lambda)) - \overline{w}^*_C(\lambda)\right]} \cdot \left[\frac{d\overline{y}^*_C(\tilde{e}^*_C(\lambda))}{d\lambda} - \frac{d\overline{w}^*_C(\lambda)}{d\lambda}\right],$$

The sign of this total derivative crucially depends on the sign of the last term in square brackets. Using the results established by Lemma 3 and 4, that is $\frac{d\overline{w}^*_C(\lambda)}{d\lambda} = 0$ and $\frac{d\overline{y}^*_C(\tilde{e}^*_C(\lambda))}{d\lambda} < 0$, it can be shown that

$$\frac{d\tilde{\theta}^*(\lambda)}{d\lambda} = \frac{\tilde{\theta}^*(\lambda)}{\sigma\left[\overline{y}^*_C(\tilde{e}^*_C(\lambda)) - \overline{w}^*_C(\lambda)\right]} \cdot \frac{d\overline{y}^*_C(\tilde{e}^*_C(\lambda))}{d\lambda} < 0.$$

Hence $\tilde{\theta}^*(\lambda)$ is decreasing in λ. ∎

Proof of Proposition 18. From the definition of elasticity:

$$\begin{aligned} \varepsilon_{\tilde{\theta}^*(\lambda)} &= \frac{p}{\tilde{\theta}^*(\lambda)} \frac{d\tilde{\theta}^*(\lambda)}{dp} \\ &= \frac{1}{\sigma} \frac{1}{\overline{y}^*_C(\tilde{e}^*_C(\lambda)) - \overline{w}^*_C(\lambda)} \cdot p \left[\frac{d\overline{y}^*_C(\tilde{e}^*_C(\lambda))}{dp} - \frac{d\overline{w}^*_C(\lambda)}{dp}\right], \end{aligned} \tag{B.18}$$

The proof is mainly concerned with the analytical characterisation of the following expression:

$$p\left[\frac{d\overline{y}^*_C(\tilde{e}^*_C(\lambda))}{dp} - \frac{d\overline{w}^*_C(\lambda)}{dp}\right] \tag{B.19}$$

where

$$\frac{d\overline{w}^*_C(\lambda)}{dp} = \frac{\partial \tilde{w}_N(\lambda)^+}{\partial p} + \frac{\psi}{1-\psi}\left(\int^{q_l} \frac{\partial \tilde{w}_E(\lambda)^-}{\partial p} \, dF + \int_{q_l}^{q_u} \frac{\partial \tilde{w}_N(\lambda)^+}{\partial p} \, dF + \int_{q_u} \frac{\partial \tilde{w}^+_E}{\partial p} \, dF\right) \tag{B.20}$$

and

$$\frac{d\overline{y}^*_C(\tilde{e}^*_C(\lambda))}{dp} = \frac{\partial y_N}{\partial p} + \frac{\partial y_N}{\partial e}\frac{\partial \tilde{e}^+_N}{\partial w}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial p}$$

$$+\psi\Bigg(\int^{q_l} \frac{\partial y_E}{\partial e}\frac{\partial \tilde{e}_E(\lambda)^-}{\partial w}\frac{\partial \tilde{w}_E(\lambda)^-}{\partial p} + \frac{\partial y_E}{\partial e}\frac{\partial \tilde{e}_E(\lambda)^-}{\partial r}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial p} + \frac{\partial y_E}{\partial p} + \frac{\psi}{1-\psi}\frac{\partial y_n}{\partial p}\,dF$$

$$+\int_{q_l}^{q_u} \frac{1}{1-\psi}\frac{\partial y_n}{\partial p}\,dF$$

$$+\int_{q_u} \frac{\partial y_E}{\partial e}\frac{\partial \tilde{e}^+_E}{\partial w}\frac{\partial \tilde{w}^+_E}{\partial p} + \frac{\partial y_E}{\partial e}\frac{\partial \tilde{e}^+_E}{\partial r}\frac{\partial \tilde{w}_N(\lambda)^+}{\partial p} + \frac{\partial y_E}{\partial p} + \frac{\psi}{1-\psi}\frac{\partial y_n}{\partial p}\,dF\Bigg) \quad \text{(B.21)}$$

where to ease notation $y_C \equiv y(q_C, \tilde{e}_C)$ and $y_n \equiv y(q_C, \tilde{e}_n)$. Notice that the derivatives with respect to the limits cancel out for the same reasons outlined in the proofs of Lemma 3 and 4.

To find an expression for equation (B.19), the proof proceeds as follows. First, collect the terms $\frac{\psi}{1-\psi}\frac{\partial \tilde{w}^+_E}{\partial p}$ and $\frac{\psi}{1-\psi}\frac{\partial \tilde{w}_E(\lambda)^-}{\partial p}$ as the common factors from the second and fourth line inside the round brackets of equation (B.21) to obtain:

$$\frac{\psi}{1-\psi}\int^{q_l} \frac{\partial \tilde{w}_E(\lambda)^-}{\partial p}\cdot\left[(1-\psi)\frac{\partial y_E}{\partial e}\frac{\partial \tilde{e}_E(\lambda)^-}{\partial w} - 1\right]dF$$

and

$$\frac{\psi}{1-\psi}\int_{q_u} \frac{\partial \tilde{w}^+_E}{\partial p}\cdot\left[(1-\psi)\frac{\partial y_E}{\partial e}\frac{\partial \tilde{e}^+_E}{\partial w} - 1\right]dF.$$

Then define $\Theta^-_E \equiv -\left[(1-\psi)\frac{\partial y_E}{\partial e}\frac{\partial \tilde{e}_E(\lambda)^-}{\partial w} - 1\right]$ and $\Theta^+_E \equiv -\left[(1-\psi)\frac{\partial y_E}{\partial e}\frac{\partial \tilde{e}^+_E}{\partial w} - 1\right]$, so that these expressions can be re-written together as

$$-\frac{\psi}{1-\psi}\left(\int^{q_l} \frac{\partial \tilde{w}_E(\lambda)^-}{\partial p}\cdot\Theta^-_E\,dF + \int_{q_u}\frac{\partial \tilde{w}^+_E}{\partial p}\cdot\Theta^+_E\,dF\right). \quad \text{(B.22)}$$

Then, collect the term $\frac{\partial \tilde{w}_N(\lambda)^+}{\partial p}$ as the common factor from both equations (B.20) and (B.21) and write this expression together as

$$\frac{\partial \tilde{w}_N(\lambda)^+}{\partial p}\cdot\left[\frac{\partial y_N}{\partial e}\frac{\partial \tilde{e}^+_N}{\partial w} - 1 + \psi\left(\int^{q_l}\frac{\partial y_E}{\partial e}\frac{\partial \tilde{e}_E(\lambda)^-}{\partial r}\,dF - \int_{q_l}^{q_u}\frac{1}{1-\psi}\,dF + \int_{q_u}\frac{\partial y_E}{\partial e}\frac{\partial \tilde{e}^+_E}{\partial r}\,dF\right)\right],$$

where the term in round brackets is $\Phi(\tilde{w}_N, \lambda)$ as defined by (B.12) in the proof of Proposition 15. Then, define $\Theta_N \equiv -\left[\frac{\partial y_N}{\partial e}\frac{\partial \tilde{e}^+_N}{\partial w} - 1 + \psi\Phi(\tilde{w}_N, \lambda)\right]$, so that the expression above can be expressed as

$$-\frac{\partial \tilde{w}_N(\lambda)^+}{\partial p} \cdot \Theta_N. \tag{B.23}$$

After these arrangements, it is possible to rewrite the term in square brackets of equation (B.19) by putting together equations (B.20) and (B.21) and rearranging them making use of (B.22) and (B.23), to obtain:

$$\begin{aligned}
\left[\frac{d\overline{y}_C^*(\tilde{e}_C^*(\lambda))}{dp} - \frac{d\overline{w}_C^*(\lambda)}{dp}\right] &= \frac{\partial y_N}{\partial p} \\
&+ \int^{q_l} \frac{\partial y_E}{\partial p} + \frac{\psi}{1-\psi}\frac{\partial y_n}{\partial p}\, dF + \int_{q_l}^{q_u} \frac{1}{1-\psi}\frac{\partial y_n}{\partial p}\, dF + \int_{q_u} \frac{\partial y_E}{\partial p} + \frac{\psi}{1-\psi}\frac{\partial y_n}{\partial p}\, dF \\
&- \frac{\partial \tilde{w}_N(\lambda)^+}{\partial p} \cdot \Theta_N - \frac{\psi}{1-\psi}\left(\int^{q_l} \frac{\partial \tilde{w}_E(\lambda)^-}{\partial p} \cdot \Theta_E^-\, dF + \int_{q_u} \frac{\partial \tilde{w}_E^+}{\partial p} \cdot \Theta_E^+\, dF\right).
\end{aligned}$$

Finally, since $y(\cdot)$ is linear in p, it follows that $p\frac{\partial y(\cdot)}{\partial p} = y(\cdot)$, and by using the definition of elasticity of the wage with respect to p, it follows that $p\frac{\partial w}{\partial p} = p\frac{dw}{dp} = \varepsilon_w w$. As such, using the definition of $\overline{y}_C^*(\tilde{e}_C^*(\lambda))$ as given by equation (2.36):

$$\begin{aligned}
&p\left[\frac{d\overline{y}_C^*(\tilde{e}_C^*(\lambda))}{dp} - \frac{d\overline{w}_C^*(\lambda)}{dp}\right] = \\
&\overline{y}_C^*(\tilde{e}_C^*(\lambda)) - \varepsilon_{\tilde{w}_N^+}\tilde{w}_N^+ \cdot \Theta_N - \frac{\psi}{1-\psi}\left(\int^{q_l} \varepsilon_{\tilde{w}_E^-}\tilde{w}_E(\lambda)^- \cdot \Theta_E^-\, dF + \int_{q_u} \varepsilon_{\tilde{w}_E^+}\tilde{w}_E^+ \cdot \Theta_E^+\, dF\right).
\end{aligned}$$

Then, define

$$\begin{aligned}
\Lambda(\varepsilon_{\overline{w}_C^*}, \overline{w}_C^*(\lambda)) \equiv -\varepsilon_{\tilde{w}_N^+}\tilde{w}_N(\lambda)^+ \cdot \Theta_N \\
- \frac{\psi}{1-\psi}\left(\int^{q_l} \varepsilon_{\tilde{w}_E^-}\tilde{w}_E(\lambda)^- \cdot \Theta_E^-\, dF + \int_{q_u} \varepsilon_{\tilde{w}_E^+}\tilde{w}_E^+ \cdot \Theta_E^+\, dF\right)
\end{aligned}$$

the elasticity of labour market tightness takes the form

$$\varepsilon_{\tilde{\theta}^*(\lambda)} = \frac{1}{\sigma}\frac{\overline{y}_C^*(\tilde{e}_C^*(\lambda)) + \Lambda(\varepsilon_{\overline{w}_C^*}, \overline{w}_C^*(\lambda))}{\overline{y}_C^*(\tilde{e}_C^*(\lambda)) - \overline{w}_C^*(\lambda)}.$$

■

Proof of Corollary 2. Notice that $\Theta_E^- \equiv -\Upsilon(w; r_E, q_E, \lambda) = 0$ for all $\tilde{w}_E < r_E$ and that $\Theta_E^+ \equiv -\Upsilon(w; r_E, q_E, \lambda) = 0$ for all $\tilde{w}_E > r_E$, as implied by the first-order condition (B.10) characterising the optimal wage of incumbent workers. In addition notice that

$\Theta_N \equiv -\Upsilon(w; r_N, q_N, \lambda) = 0$ for all $\tilde{w}_N \geq r_N$, as implied by the first-order condition (B.11) characterising the optimal wage of newly hired workers. Hence $\Lambda(\varepsilon_{\overline{w}^*_C}, \overline{w}^*_C(\lambda)) = 0$. ■

Proof of Proposition 19. Total differentiation of $\varepsilon_{\tilde{\theta}^*(\lambda)}$ with respect to λ yields:

$$\frac{d\varepsilon_{\tilde{\theta}^*(\lambda)}}{d\lambda} = \frac{1}{\sigma}\frac{1}{[1-(\overline{w}^*_C(\lambda)/\overline{y}^*_C(\tilde{e}^*_C(\lambda)))]^2}\cdot\frac{d}{d\lambda}\left\{\frac{\overline{w}^*_C(\lambda)}{\overline{y}^*_C(\tilde{e}^*_C(\lambda))}\right\}$$

where

$$\frac{d}{d\lambda}\left\{\frac{\overline{w}^*_C(\lambda)}{\overline{y}^*_C(\tilde{e}^*_C(\lambda))}\right\} = \frac{\dfrac{d\overline{w}^*_C(\lambda)}{d\lambda}\overline{y}^*_C(\tilde{e}^*_C(\lambda)) - \dfrac{d\overline{y}^*_C(\tilde{e}^*_C(\lambda))}{d\lambda}\overline{w}^*_C(\lambda)}{\overline{y}^*_C(\tilde{e}^*_C(\lambda))^2}.$$

The sign of the total derivative crucially depends on the sign of the numerator in the expression above. Using the results established by Lemma 3 and 4, that is $\frac{d\overline{w}^*_C(\lambda)}{d\lambda} = 0$ and $\frac{d\overline{y}^*_C(\tilde{e}^*_C(\lambda))}{d\lambda} < 0$, it can be shown that

$$\frac{d}{d\lambda}\left\{\frac{\overline{w}^*_C(\lambda)}{\overline{y}^*_C(\tilde{e}^*_C(\lambda))}\right\} = \frac{-\dfrac{d\overline{y}^*_C(\tilde{e}^*_C(\lambda))}{d\lambda}\overline{w}^*_C(\lambda)}{\overline{y}^*_C(\tilde{e}^*_C(\lambda))^2} > 0.$$

Hence $\varepsilon_{\tilde{\theta}^*(\lambda)}$ is increasing in λ. ■

B.3 Proofs to Chapter 3

Proof of Proposition 20. First notice that given the premises of the proposition, $\lim_{t\to\infty} \Delta s_t / s_{t-1} = 0$ and $\lim_{t\to\infty} x_t = \bar{x}$. These hold throughout the proof.

Then consider first the case in which $\alpha \in (0, 1]$, i.e. excluding the case of no adaptation, and note that:

$$\lim_{t\to\infty} A_t = (1 - \alpha) \equiv A$$
$$\lim_{t\to\infty} B_t = \alpha \cdot \bar{x} \equiv B$$

Hence using the expression for the general solution as given by (3.10), the equilibrium point for r_t can be characterised as:

$$\begin{aligned} r^* = \lim_{t\to\infty} r_t &= \lim_{t\to\infty} A^t \cdot r_0 + B\left(\frac{A^t - 1}{A - 1}\right) \\ &= \frac{-B}{A - 1}; \end{aligned}$$

since $A \equiv (1 - \alpha) < 1$ for all $\alpha \in (0, 1]$. Substituting the expressions derived for A and B, the steady-state reference wage of the dynamical system for r_t is:

$$r^* = \frac{-\alpha\bar{x}}{1 - \alpha - 1} = \bar{x}.$$

Next, express the dynamical system for r_t, in a given period $t + 1$ for any given initial r_0, as

$$r_{t+1} = (1 - \alpha)r_t + \alpha\bar{x} = g(r_t).$$

Global asymptotic stability is straightforward to verify by noticing that since

$$g'(r^*) = (1 - \alpha) < 1,$$

r^* is asymptotically stable; and also attracting, since there exists $\epsilon > 0$ such that

$$|r_0 - r^*| < \epsilon \quad \text{implies} \quad \lim_{t\to\infty} r_t = r^*.$$

Since the latter is true also for $\epsilon = \infty$, then r^* is globally asymptotically stable.

Finally consider the case of $\alpha = 0$. It can be verified that every initial condition r_0 is a globally stable equilibrium by noticing that $\lim_{t\to\infty} A^t = 1$ and $B = 0$, which implies that $r^* = r_0$. ■

Proof of Proposition 21. The functional equation corresponding to the firm's problem can be written more generally as

$$J(r) = \max_{w \geq 0} \{\pi(r, w) + \psi J(g(r, w, \alpha))\},$$

where $r' = g(r, w, \alpha) = (1 - \alpha)r + \alpha w$ for all $\alpha = \{\alpha^-, \alpha^+\}$. The first-order necessary and sufficient condition characterising the optimal wage policy is given by

$$\frac{\partial \pi(r, w)}{\partial w} + \psi J'(r') = 0, \tag{B.24}$$

while the related envelope condition is given by

$$J'(r') = \frac{\partial g(r, w, \alpha)}{\partial w} J'(g(r, w, \alpha)).$$

Assumptions W1-W4, F1-F2, Chapter 2 and Assumption A1, Chapter 3 hold throughout. Moreover, the same arguments exposed in the preliminaries of the Proof of Proposition 6, Chapter 2, can be used to show that there exists a unique solution to the functional equation above, and that concavity of the firm's value function will also imply a unique wage setting policy. Finally, remainder of the proof will consider the case of $w > r$ only, since the proof for $w < r$ is analogous. Hence, combining the first-order condition with the related envelope condition as outlined above, yields the following condition:

$$\Upsilon(w; r, \alpha, \lambda) \equiv pq \left(\frac{1}{w}\right) - 1 - \psi pq \left(\frac{1}{(1-\alpha)r + \alpha w}\right) \alpha = 0,$$

which characterises the optimal wage setting policy of the firm for all $w > r$.

Given these premises, the proof of the first part of the proposition is analogous to the Proof of Proposition 6, Chapter 2. However, to prove part a) and b), first note that (after some algebra):

$$\begin{aligned}
\frac{\partial \Upsilon(w; r, \alpha, \lambda)}{\partial r} &= \frac{\psi pq\alpha(1-\alpha)}{[(1-\alpha)r + \alpha w]^2} > 0 \\
\frac{\partial \Upsilon(w; r, \alpha, \lambda)}{\partial w} &= -1 - \frac{\psi pq\alpha(1-\alpha)r}{[(1-\alpha)r + \alpha w]^2} < 0 \\
\frac{\partial \Upsilon(w; r, \alpha, \lambda)}{\partial \alpha} &= -\frac{r\psi pq}{[(1-\alpha)r + \alpha w]^2} < 0
\end{aligned}$$

Then consider the effect of r on $\tilde{w}(r, \alpha, \lambda)$ for all $r \notin [r_L, r_H]$, implicit differentiation reveals:

$$\frac{\partial \tilde{w}(r)}{\partial r} = -\frac{\partial \Upsilon(w; r, \alpha, \lambda)/\partial r}{\partial \Upsilon(w; r, \alpha, \lambda)/\partial w} = \frac{\psi pq\alpha(1-\alpha)}{\psi pq\alpha(1-\alpha)r + [(1-\alpha)r + \alpha w]^2} > 0;$$

which implyies that $\tilde{w}(r, \alpha, \lambda)$ is incrasing in r. Moreover, differentiating again with respect to r yields:

$$\frac{\partial^2 \tilde{w}(r)}{\partial r^2} = \frac{-\psi pq\alpha(1-\alpha)\{\psi pq\alpha(1-\alpha) + 2[(1-\alpha) + \alpha\frac{\partial w}{\partial r}][(1-\alpha)r + \alpha w]\}}{\{\psi pq\alpha(1-\alpha)r + [(1-\alpha)r + \alpha w]^2\}^2} < 0, \tag{B.25}$$

which establishes concavity of $\tilde{w}(r, \alpha, \lambda)$ for all $r \notin [r_L, r_H]$. Next, consider the effect of the speed of adaptation α on $\tilde{w}(r, \alpha, \lambda)$. Implicit differentiation reveals:

$$\frac{\partial \tilde{w}(\alpha)}{\partial \alpha} = -\frac{\partial \Upsilon(w; r, \alpha, \lambda)/\partial \alpha}{\partial \Upsilon(w; r, \alpha, \lambda)/\partial w} < 0.$$

Hence, $\tilde{w}(r, \alpha, \lambda)$ is decreasing in α. Finally, to prove part c) consider the case of $w < r$ and note that (after some algebra):

$$\frac{\partial \Upsilon(w; r, \alpha, \lambda)}{\partial w} = -1 - \frac{\lambda\psi pq\alpha(1-\alpha)r}{[(1-\alpha)r + \alpha w]^2} < 0$$
$$\frac{\partial \Upsilon(w; r, \alpha, \lambda)}{\partial \lambda} = \frac{pq[(1-\alpha)r + \alpha w(1-\psi)]}{(1-\alpha)r + \alpha w} > 0.$$

Hence implicit differentiation reveals:

$$\frac{\partial \tilde{w}(\lambda)^-}{\partial \lambda} = -\frac{\partial \Upsilon(w; r, \alpha, \lambda)/\partial \lambda}{\partial \Upsilon(w; r, \alpha, \lambda)/\partial w} > 0.$$

Hence $\tilde{w}(r, \alpha, \lambda)$ is increasing in λ for all $r > r_H(\alpha^-, \lambda)$. ■

Proof of Proposition 22. Consider first the worker's reference wage r_t (the state variable of the system). Any r^* that satisfies $r^* = (1-\alpha)r^* + \alpha\tilde{w}(r^*)$, that is,

$$\frac{\tilde{w}(r^*)}{r^*} = 1, \tag{B.26}$$

is a steady state. Hence, in the steady state it must be that $r^* = \tilde{w}(r^*)$ (the other arguments of $\tilde{w}(\cdot)$ are omitted to ease notations). To establish the existence of a range of steady-state equilibria, note that for all $r_\tau \in [r_L, r_H]$, any reference wage r_τ is already a steady state, since in this range the optimal wage policy is given by $\tilde{w}(r_\tau) = r_\tau$. This also established uniqueness for all $r_\tau \in [r_L, r_H]$. To establish existence of a steady state for all other $r \notin [r_L, r_H]$, note that $\lim_{r\to 0} \tilde{w}(r)/r = \infty$ and $\lim_{r\to\infty} \tilde{w}(r)/r = 0$ due to the concavity of $\tilde{w}(r)$ with respect to r as established by Proposition 21. Moreover $\tilde{w}(r)/r$

is continuous, so there exists r^* such that (B.26) is satisfied. To establish uniqueness in these two cases, differentiating $\tilde{w}(r)/r$ with respect to r gives:

$$\frac{\partial}{\partial r}\left\{\frac{\tilde{w}(r)}{r}\right\} = \frac{\tilde{w}'(r)r - \tilde{w}(r)}{r^2} < 0$$

due to the concavity of $\tilde{w}(r)$, which implies that

$$\tilde{w}'(r)r - \tilde{w}(r) < -\tilde{w}(0)$$

in which $\tilde{w}(0) = pq(1-\psi) > 0$. Hence, since $\tilde{w}(r)/r$ is strictly decreasing, for any $r \notin [r_L, r_H]$, there can only exist a unique r^* that satisfies (B.26) for each case respectively.

Next consider the firm's optimal wage policy $\tilde{w}(r_t)$. For all $r_\tau < r_L(\alpha^+)$, the steady-state equilibrium wage is the policy function that satisfies the first-order condition (B.24) for $w > r$, evaluated at $w^*(r^*) = r^*$. That is

$$pq\left(\frac{1}{\tilde{w}^*(r^*)}\right) - 1 - \psi pq\left(\frac{\alpha^+}{(1-\alpha^+)\tilde{w}^*(r^*) + \alpha^+\tilde{w}^*(r^*)}\right) = 0,$$

which is satisfied by $\tilde{w}^*(r^*) = pq(1-\psi\alpha^+) = r_L(\alpha^+) = r^*$. The same argument can be implemented to show that for all $r_\tau > r_H(\alpha^-, \lambda)$, the steady-state equilibrium wage is the policy function that satisfies

$$\lambda pq\left(\frac{1}{\tilde{w}^*(r^*)}\right) - 1 - \psi\lambda pq\left(\frac{\alpha^-}{(1-\alpha^-)\tilde{w}^*(r^*) + \alpha^-\tilde{w}^*(r^*)}\right) = 0,$$

that is, $\tilde{w}^*(r^*) = \lambda pq(1-\psi\alpha^-) = r_H(\alpha^-, \lambda) = r^*$.

Finally it is straightforward to notice that by the properties of the worker's optimal effort function there exists a unique steady-state level of effort given by: $\tilde{e}(r^*, r^*, \lambda) = \tilde{e}_n$; since in the steady state $\tilde{w}^*(r^*) = r^*$ for all r_τ. ■

Proof of Proposition 23. The following proof will demonstrate statements a) and c) only, since the proof of statement b) is trivial. Consider the evolution of the worker's reference wage which follows the partial adaptation rule for all $\alpha = \{\alpha^-, \alpha^+\}$ (were the other arguments of $\tilde{w}(\cdot)$ are omitted for convenience):

$$r_{t+1} = (1-\alpha)r_t + \alpha\tilde{w}(r_t), \quad r_\tau \text{ given},$$

and let $g(r_t) = (1-\alpha)r_t + \alpha\tilde{w}(r_t)$. Note that $g'(r)$ exists and is always strictly positive, that is $g'(r) > 0$ for all r. Next, consider the law of motion above denoted by $r_{t+1} = g(r_t)$, with one steady state for all $r_\tau < r_L(\alpha)$ and one for all $r_\tau > r_H(\alpha, \lambda)$, both of which satisfy $\tilde{w}(r^*)/r^* = 1$, or equivalently $r^* = g(r^*)$. From the concavity of $\tilde{w}(r)$, as established by Proposition 21, it is possible to write

$$\tilde{w}(r) > \tilde{w}(0) + \tilde{w}'(r)r$$

for any r, with $\tilde{w}(0) = pq(1-\psi) > 0$. Then, since $\tilde{w}(r^*)/r^* = 1$, it follows that $\tilde{w}(r^*)/r^* > \tilde{w}(0) + \tilde{w}'(r^*)$, which implies that $1 > \tilde{w}(0) + \tilde{w}'(r^*)$, and therefore that $\tilde{w}'(r^*) < 1$. This result could have also been obtained by evaluating the expression for $\frac{\partial \tilde{w}(r)}{\partial r}$, as in the proof of Proposition 21, at the fixed point $\tilde{w}(r^*) = r^*$. Therefore

$$\begin{aligned} g'(r^*) &= 1 - \alpha + \alpha \tilde{w}'(r^*) \\ &= 1 - \alpha(1 - \tilde{w}'(r^*)) \\ &< 1, \end{aligned}$$

since $\tilde{w}'(r^*) < 1$, implying that $g'(r^*) \in (0, 1)$ and establishing local asymptotic stability of $r_L(\alpha)$ and $r_H(\alpha, \lambda)$.

Next, note that for any $r_t < r_L(\alpha)$

$$\begin{aligned} r_{t+1} - r_L(\alpha) &= g(r_t) - g(r_L(\alpha)) \\ &= -\int_{r_t}^{r_L(\alpha)} g'(r)dr \\ &< 0, \end{aligned}$$

where the second line uses the Fundamental Theorem of Calculus and the last line follows from the result that $g'(r) > 0$ for all r. Next, the law of motion (3.21) also implies that

$$\begin{aligned} \frac{r_{t+1} - r_t}{r_t} &= \frac{r_{t+1}}{r_t} - 1 \\ &= \frac{(1-\alpha)r_t + \alpha \tilde{w}(r_t)}{r_t} - 1 \\ &= 1 - \alpha + \frac{\alpha \tilde{w}(r_t)}{r_t} - 1 \\ &= \alpha \left(\frac{\tilde{w}(r_t)}{r_t} - 1 \right) \\ &> \alpha \left(\frac{\tilde{w}(r_L(\alpha))}{r_L(\alpha)} - 1 \right) \\ &= 0, \end{aligned}$$

where the fifth line uses fact that $\tilde{w}(r)/r$ is decreasing in r and the last line uses definition of $r_L(\alpha)$. The two results just derived above together establish that for all $r_t < r^* = r_L(\alpha^+)$, $r_{t+1} \in (r_t, r^*)$. Therefore $\{r_t\}_{t=\tau}^{\infty}$ is monotonically increasing and bounded above by $r^* = r_L(\alpha^+)$, and since $r_L(\alpha^+)$ is unique for all $r_\tau < r_L(\alpha^+)$, it follows that $\{r_t\}_{t=\tau}^{\infty}$ monotonically converges to $r^* = r_L(\alpha^+)$, as stated in case a). An identical argument implies that for all $r_t > r^* = r_H(\alpha^-, \lambda)$, $r_{t+1} \in (r^*, r_t)$ and establishes monotonic convergence to $r^* = r_H(\alpha^-, \lambda)$ starting from $r_\tau > r^*$, as stated in case c).

Hence, since $\tilde{w}_t = \tilde{w}(r_t)$ is increasing and concave in r_t with $\tilde{w}'(r) < 1$ it follows that $\{\tilde{w}_t\}_{t=\tau}^{\infty}$ monotonically converges to $\tilde{w}^*(r^*) = r^* \in [r_L, r_H]$. Hence $\{\tilde{w}_t\}_{t=\tau}^{\infty}$ is a monotone and bounded sequence. Finally, denote the gain-loss function $\mu(\ln \tilde{w}(r_t) - \ln r_t)$ with μ_t, and notice that the results established above for the sequences $\{r_t\}_{t=\tau}^{\infty}$ and $\{\tilde{w}_t\}_{t=\tau}^{\infty}$ imply that $\{\mu_t\}_{t=\tau}^{\infty}$ monotonically converges toward 0. Therefore, given the law of motion of the worker's effort (3.22), $\{\tilde{e}_t\}_{t=\tau}^{\infty}$ monotonically converges toward $\tilde{e}(\tilde{w}^*(r^*), r^*, \lambda) = \tilde{e}_n$. ■

Proof of Proposition 24. First note that the model presented in this section maintains the same fundamental properties of the benchmark model set out in Chapter 2. As such, some steps are omitted and the reader is referred to the proof of Proposition 6, Chapter 2, for details.

Reference wage thresholds. The threshold r_L is the level of a newly hired worker's reference wage below which a firm would optimally set a wage above their reference wage, and $r_H(\lambda)$ is the level of a newly hired worker's reference wage above which a firm would optimally pay the worker a wage below their reference wage. These thresholds can be explicitly determined by solving respectively for $\lim_{\epsilon\to 0} \Upsilon(r+\epsilon; r, \lambda) = 0$ and $\lim_{\epsilon\to 0} \Upsilon(r-\epsilon; r, \lambda) = 0$; where

$$\Upsilon(w; r, \lambda) \equiv \frac{\partial y(\tilde{e})}{\partial e}\frac{\partial \tilde{e}(w, r, \lambda)}{\partial w} - 1 + \psi \frac{\partial y(\tilde{e})}{\partial e}\frac{\partial \tilde{e}(w, r, \lambda)}{\partial r} = 0$$

is the first-order necessary and sufficient condition determining firms' optimal wage setting policy for all $w \neq r$ and $t \geq \tau$. As such, $r_L \equiv pq(1-\psi)$ and $r_H(\lambda) \equiv \lambda pq(1-\psi)$.

Optimal wage setting policy. The optimal wage setting policy depends on the level of employed workers' reference wage r_t in relation to the thresholds just derived. Since the proposition considers the case of $r_0 = \underline{\omega} \leq pq(1-\psi)$, from the definition of r_L and the fact that $\partial\Upsilon/\partial r < 0$ can be used to deduce that $\lim_{\epsilon\to 0} \Upsilon(r+\epsilon; r, \lambda) > 0$; moreover since $\partial\Upsilon/\partial w < 0$, the same is true for all $w \leq r$. Hence the optimal wage must satisfy $w > r$, if $\underline{\omega} < pq(1-\psi)$, and will be the solution to

$$\Upsilon(w; r, \lambda) = pq\left(\frac{1}{w}\right) - 1 - \psi pq\left(\frac{1}{w}\right) = 0,$$

given by $\tilde{w}_0 = pq(1-\psi) = r_L$; while for the same logic applied in the proof of Proposition 6, Chapter 2, if $\underline{\omega} = pq(1-\psi)$ the optimal wage will be given by $\tilde{w}_0 = r_0 = pq(1-\psi) = r_L$. Moreover note that for all $t > 0$ and $t \geq \tau$, $r_{i,t} = \tilde{w}_{j,t}$ since $\underline{\omega} \leq w_{j,t} = \tilde{w}_0$ (social comparison); while $r_{j,t} = \tilde{w}_0$ (adaptation). This implies that from $t > 0$ the firms value of an employment relationship with either a newly hired or an incumbent worker is maximised if and only if $\tilde{w} = r$ (see also next paragraph).

Steady-state equilibrium: wages, reference wages and effort. Incumbent workers reference wage for all $t > 0$ is $r_{j,t} = \tilde{w}_0 = r_L$ which implies that $J(r_{j,t})$ is maximised if and only if $\tilde{w}_{j,t} = r_L = \tilde{w}_0$; hence $\tilde{w}_0$ characterises the steady-state equilibrium paid to incumbent workers and it is therefore denoted by $\tilde{w}_j^*$. Similarly, new hires reference wage for all $t > 0$ is $r_{i,t} = \tilde{w}_{j,t} = \tilde{w}_0 = r_L$ which implies that $J(r_{i,t})$ is maximised if and only if $\tilde{w}_{i,t} = r_L = \tilde{w}_0$; hence $\tilde{w}_0$ also characterises the steady-state equilibrium paid to newly hired workers and it is therefore denoted by $\tilde{w}_i^*$. These results can be used to conclude that new hires and incumbents are paid the same wage in the steady state: $\tilde{w}_i^* = \tilde{w}_j^* = \tilde{w}^*$; and that this wage is equal to their reference wage which is also the same in the steady state: $r_i^* = r_j^* = r^* = r_L$. Hence $\tilde{w}^* = r^* = pq(1-\psi)$ for all employed workers, implying that both new hires and incumbents exert normal effort in the steady state: $\tilde{e}_i^* = \tilde{e}_j^* = \tilde{e}_n$.

Steady-state equilibrium: market tightness and unemployment. The steady-state level of market tightness is given by the value of θ which satisfies the job creation condition (3.29) for all $t \geq 0$. Given the results previously established it can be deduced that the value of an employment relationship with a newly hired is the same as the value of an employment relationship with an incumbent in the steady state, hence:

$$J(r^*) = pq\tilde{e}_n - \tilde{w}^* + \psi J(r^*).$$

By rearranging the above equation and denoting the steady-state present discounted value of output with $\overline{y}^*(\tilde{e}^*) = pq\tilde{e}_n/(1-\psi)$ and the steady-state present discounted value of the wage with $\overline{w}^* = \tilde{w}^*/(1-\psi)$, it is possible to solve the job creation condition (3.29) for θ explicitly as stated in the proposition. Then it is possible to use this value to characterise the steady-state unemployment rate by rearranging (3.30) for $\Delta u_{t+1} = 0$. ■

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