Essays in Macroeconomic Interdependence, Business Cycles and Nowcasting in a Multi-Country Context

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Declaration

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Abstract

This thesis employs a multi-country approach and builds upon the existing literature on the Bayesian Panel Vector Autoregressions (PVARs) as its foundation for analysing empirical macroeconomic interdependence, business cycles synchronisation and economic forecasting. The contribution is provided in three essays.

The first essay (Chapter 2) examines macroeconomic interdependency of main macroeconomic variables in terms of dynamic, static, and cross-sectional homogeneity features by using a PVAR model. In order to accurately measure these features, a stochastic search specification selection (S^4) prior algorithm is employed to investigate their interdependencies within the G-7 countries. The results indicate that while cross-sectional homogeneity is of little significance among the G-7, dynamic and static interdependencies are of great importance. In brief, the S^4 algorithm is beneficial for classifying each type of the panel structure of macro-financial interlinkages. This essay also compares the inflation forecasting performance of the S^4 algorithm with the original factor shrinkage prior of Canova and Ciccarelli (2009) and finds that the PVARs with the S^4 algorithm give a better point forecasting performance, particularly in the short-term forecast horizons. Regarding the density forecasts, the PVARs with the S^4 prior outperform the PVARs with the factor shrinkage prior for all the G-7 in the short-term horizons, whereas in the long-term horizons, although the PVARs with the factor shrinkage prior give an improved performance, they still only forecast better for two of the seven countries, namely Canada and Japan.

The second essay (Chapter 3) investigates the economic interdependencies between the ASEAN+3 and the US as well as between the ASEAN+3 members themselves through the lens of business cycle synchronisation, by using a Bayesian panel Markov-switching VAR approach (The PMS-VAR model). The main reason for investigating this phenomenon is that the increasing level of regional economic integration of the ASEAN+3 has led to a discussion over the past decade about whether or not the ASEAN+3 is decoupling from the US economy. The results provide evidence that the business cycles of the ASEAN+3 economies are much more synchronised with each other than any of them are with the US economy, especially for real economic variables. However, for financial variables, the results indicate that after the US subprime crisis of 2008 the synchronisations of the ASEAN+3 and the US have become more substantial, particularly of their stock price indices and exchange rates.

The third essay (Chapter 4) studies recent literature on nowcasting. Upon study, there is a substantial gap to be found regarding investigation into whether or not multi-country nowcasting models can give predictive gains, no doubt due to the historical issue of over-parameterisation, and this thesis meets the challenge of filling that gap. These models are helpful when considering the role of interdependence among a particular group of economies and have potential to help in the assessment of nowcasts of several different GDPs. Therefore, this chapter focuses mainly on comparing nowcasting performance between multi-country models - large Bayesian VARs, Panel VARs and a multi-country dynamic factor model, and individual-country models - MF-BVARs, MF-DFM, with mixed-frequency approaches, applied to the four largest European economies during both normal periods and the Covid-19 pandemic. The results show that country-specific models outperform the other models when it comes to nowcasts for almost all countries, especially the pandemic period.

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Chapter 1

Introduction

1.1 Motivation

In recent decades, the importance of macroeconomic interdependence has increased due to the growing economic interconnectedness of regions and countries through globalisation. These terms - 'interconnectedness', 'interlinkages and transmission channels', 'international business cycle synchronisation', etc. - have emerged in the literature to acknowledge that regions and economies can no longer be treated in isolation and that spillover effects are important when analysing and evaluating macroeconomic policies. More importantly, these phenomena become even more complex when policymakers perform economic forecasting under VUCA - volatility, uncertainty, complexity and ambiguity. Therefore, multi-country models are absolutely essential as quantitative instruments for investigating the transmission channels of interlinkages and assessing economic impacts as well as forecasting.

In this regard, works by Kose et al. (2003) and Canova and Ciccarelli (2009, 2013) have greatly influenced the development of the multi-country approach for modellers, which is commonly referred to as panel vector autoregressive models (panel VARs). Many of the useful features in panel VAR models are evidenced by adding a panel structure to the standard VAR model - the panel structure's properties mean that a panel VAR can measure dynamic and static interdependencies as well as cross-sectional heterogeneity, something that is impossible with a standard VAR model. Nevertheless, when considering a large number of countries and variables, it is inevitable that a trade-off exists between the usefulness of the panel structure of VARs and the fundamental issue of over-parameterisation - a large number of parameters that need to be estimated from the available short time-series data. Consequently, Bayesian estimation with prior algorithms is greatly required to resolve the problem, namely using Bayesian panel VARs for the multi-country approach.

Regarding Bayesian estimation methods, many shrinkage priors have been developed over recent decades in the literature, however, that vast majority of these have constructed the shrinkage of parameters for country-specific models with many predictors and large-scale VAR models but not for panel VARs (see Banbura et al, 2010, Giannone et al, 2018, Koop and Korobilis, 2019, Chan, 2021, Huber et al., 2021, etc.). Far fewer researchers, Canova and Ciccarelli (2013) and Korobilis (2016) being the only two of note, have chosen to construct such shrinkage priors for panel VARs in order to account for the features of the panel structure of the data. There are many reasons for this discrepancy, the main one being the huge abundance of big data available on the developed economies, particularly the US, which enables economists to better predict through nowcasting and forecasting. However, macroeconomic interdependencies, phenomena that can only be observed and measured by using panel structures in multi-country models, are worth studying because they offer significant potential for developing both theoretical and empirical evidence to analyse synchronisation of international business cycles, interlinkages, interconnectedness, and other related interdependent phenomena.

1.2 Contributions and Unifying Themes

Building on the aforementioned motivation, which emphasises the usefulness of panel structures and the limitation of the recent panel VAR approach, this thesis extends the existing panel VAR literature by investigating various macroeconomic issues associated with globalisation that are of relevance to the purposes of policymakers. Firstly, the thesis refines and applies the panel VAR models to forecast and analyse macroeconomic interdependencies by employing Bayesian estimation with prior algorithms. Secondly, the thesis develops the existing panel VAR framework to analyse international business cycle synchronisation. Finally, the thesis seeks to improve nowcasting performance through multi-country approaches, as opposed to relying solely on country-specific approaches. The thesis consists of three essays, each with self-contained contributions that are related and relevant to essays in a multi-country context.

Chapter 2 is titled "Macroeconomic Interdependence and Multi-Country Forecasts". The ability of panel VAR models to analyse macroeconomic interdependencies and heterogeneity across countries is limited by the fundamental problem of the curse of dimensionality, or over-parameterisation, which is inherent in panel VAR models due to them including many countries, variables and lags all at once. This chapter, therefore, employs Bayesian estimation to the panel VAR models with the stochastic search specification selection (S^4) prior algorithm in order to identify features of interdependency – static and dynamic patterns – including heterogeneity across countries. Additionally, a Bayesian panel VAR model with the S^4 prior can produce parameters that are driven by the data, rather than being driven by the choice of parameters themselves. In this regard, this chapter applies this approach to the G-7 countries for investigating the patterns of macroeconomic interdependencies and heterogeneities among them from the period of January 1999 through to December 2018. Six restrictions in the panel VAR models are imposed to examine which one of them can potentially classify the most accurate panel structures of interlinkage between the G-7 economies. The results show that both dynamic and static interdependencies are greatly significant while homogeneity restriction should not be imposed across all the G-7 economies. Specifically, Canada and Japan have a unique occurrence of interdependencies and homogeneity features which align with the findings of Carstensen and Salzmann (2017). Moreover, the chapter also applies the panel VAR Models with the S^4 prior to inflation forecasts, comparing these with the factor shrinkage prior of Canova and Ciccarelli (2009). The results indicate that the S^4 prior produces a better performance in both point and density forecasting of inflations, especially in the short-term forecast horizons. Meanwhile, the factor shrinkage prior retains significance in that it still gives a better density forecasting performance in the long-term horizons, albeit only for some of the G-7.

Chapter 3 is titled "Business Cycles for the US and the ASEAN+3: A Bayesian Panel Markov-Switching VAR Approach". The current stylised fact shows evidence that the Asian countries contribute the largest share of the world's economic growth due to globalisation and their increasing level of regional economic integration. This chapter examines whether or not the ASEAN+3 economies are decoupling from the US economy and therefore builds on the Bayesian panel Markov-Switching (PMS) VAR model, which incorporates interconnectedness with a nonlinearly time-varying transition mechanism. The chapter investigates the synchronisation of the business cycles for the US and the ASEAN+3 over the forty-year period from 1980 through to 2019. The estimation of the PMS-VAR Model adopts a simulation-based Bayesian approach with hierarchical priors to deal with an over-parameterisation problem. Additionally, the Bayes factor is used for determining the number of regimes from all possible models. Results not only show a synchronisation of business cycles but also present the regimes' persistence on the smoothed probabilities as well as their identification on the posterior densities. In summary, the findings show evidence that the business cycles of the ASEAN+3 economies exhibit greater synchronisation among themselves for real variables compared to their synchronisation with the US economy. Regarding financial variables, the results indicate that, after the US subprime crisis of 2008, each ASEAN+3 member's synchronisation with the US has become more substantial, particularly that of their stock price indices and exchange rates.

Chapter 4 is titled "Nowcasting GDP using Multi-Country Models". The recent increase in literature on nowcasting GDP has been centred mainly on country-specific models, issuing a challenge to researchers who wish to investigate whether or not multi-country nowcasting models provide predictive improvement by utilising their panel structure properties. This chapter, therefore, develops a mixed-frequency approach towards multi-country models (both of the panel VAR models with different prior algorithms and the multi-country dynamic factor models) that constructs nowcasting GDP in order to compare their predictive performance to country-specific models. Regarding the priors, this essay applies four different algorithms to the multi-country models, namely the Minnesota prior with marginal data density, the stochastic search variable selection (SSVS), the stochastic search specification selection (SSSS or S^4) and the Minnesota-type adaptive hierarchical algorithm (MNG). The empirical application of nowcasting GDP is conducted on the four largest economies in Europe. The dataset for each country is divided into two sets: the first set consists of four main macroeconomic variables, while the second set comprises eleven economic indicators. The results are obtained by evaluating the performance of GDP nowcasts before the Covid-19 pandemic period and extending the analysis to include the pandemic period. The findings indicate that only a single country is able to exploit the predictive gain of GDP nowcasting from multi-country contexts. However, for the remaining countries, the significance of country-specific models in GDP nowcasting remains substantial. Moreover, upon closer examination during the pandemic period, the results indicate that more economic indicators from each country can improve the accuracy of that country's GDP nowcasting performance.

The overriding theme of all three essays is centred around a multi-country approach. In the same way that this thesis explores these different perspectives, so too are researchers and policymakers afforded different choices when it comes to adopting multi-country models to reach their specific objectives. Therefore, these essays introduce to the reader the principle that different types of multi-country models are appropriate for different purposes, as evidenced by their empirical applications. Put differently, there is no single model that can perfectly meet all purposes for policymakers. Rather than specifically developing new algorithms for Bayesian econometric models, this thesis focuses on augmenting and extending the existing multi-country models to effectively build them on the specific interests and objectives of the study.

What is explored throughout the three chapters is the role that multi-country models play in the analysis of economic phenomena, a role that should be suitably designed so that it performs well regardless of the economic phenomenon in question. This thesis contributes to the analysis of macroeconomic interdependence, interconnectedness, and forecasting within a multi-country context. Chapter 2 presents the effectiveness of a panel VAR model in identifying different types of macroeconomic interdependencies. This is accomplished by employing the stochastic search specification selection algorithm on the panel VAR model.

Additionally, the algorithm generates highly accurate forecasts of inflation, both in terms of point estimates and density forecasts. Chapter 3 introduces a distinct method for analysing economic interlinkages or interdependencies by defining interconnectedness as the switching of regimes within Markov processes. This approach functions well when the purpose of researchers and policymakers is to analyse international business cycles. Also, this chapter sheds some further light on interesting issues through the examination of business cycles between emerging economic markets in Asian countries and the US. Chapter 4 is a discussion on the limitations of panel VAR models when they are applied to nowcast GDP for the specific multi-country application of the study. Despite employing various prior algorithms to deal with the challenge of over-parameterisation, these panel VAR models still have inherent limitations. Consequently, this chapter proposes a more effective approach, emphasising country-specific models that incorporate foreign economic variables for accurate GDP nowcasting rather than relying on multi-country models.

Each chapter is structured to provide the reader with the motivation and essential information needed to comprehend the objectives and proposed methods with an introduction that enables the reader to understand the principles and methodologies as outlined in the thesis.

Chapter 2

Macroeconomic Interdependence and Multi-Country Forecasts

2.1 Introduction

What will the global economy over the next decade look like? How will the economy of each country face policy uncertainty, technology transformation and increasing social complexity around the world? It is essential that policy-makers realise the importance of these questions and design effective policies to overcome the various dimensions of these problems. However, due to their inherent unpredictability, it can be very difficult to know exactly how to deal with these uncertain circumstances. They might exist because globalisation and rapid technological progress in recent decades, especially after the global financial crisis in 2008, have led to an increase in levels of economic interdependence. This means that, due to the volatility of one country's economy, others that have close macro-financial linkages can easily experience a pass-through effect and vice-versa, particularly when it comes to GDP, employment rate and inflation.

As globalisation causes an increase in economic independence, multi-country models are needed as instruments for assessing the economic impacts and also forecasting macroeconomic variables. These have led to an increasing amount of literature on investigating appropriate models in recent years. Particularly appropriate for the purposes of this study, multi-country models study the transmission of shocks across countries; i.e. Canova and Ciccarelli (2009), Koop & Korobilis (2016, 2019), Dées and Güntner (2017), Feldkircher et al. (2020), etc. Multi-country VAR models, therefore, are essential economic models and very useful for capturing these shocks.

In general, one such type of these useful models is the panel vector autoregressive model (PVAR). PVAR models are popular econometric tools for jointly analysing multivariate time series data in a multi-country setting. This is because their panel structures are very useful for analysing the cross-country linkages, interactions and interdependencies within macroeconomic variables. Moreover, PVAR models are able to substantially capture static and dynamic interdependencies as well as heterogeneities across countries. Fundamentally, PVAR models are an extension of vector autoregressive models (VARs), in which all economic variables are assumed to be endogenous and interdependent from a single country to several countries by augmenting with the lagged endogenous variables of other countries. Nevertheless, the fundamental problem of the curse of dimensionality or over-parameterisation in PVAR models still remains due to a large number of parameters to be estimated from short time-series data. Therefore, this chapter aims to employ PVAR models in various restrictions by adapting the stochastic search specification selection (S^4) algorithm (Koop & Korobilis, 2016) with empirical application of the G-7 countries in order to deal with the problem of over-parameterisation and to compare the performance of PVAR models with their feature restrictions, which are cross-sectional homogeneity (CSH), dynamic interdependency (DI) and static interdependency (SI). Additionally, the chapter also compares forecasting performances of PVARs with S^4 algorithm to PVARs with other restrictions. Furthermore, the general form of PVAR models and their feature restrictions are discussed in more detail.

2.2 The Framework of Panel VARs (PVARs)

Bearing in mind the aforementioned discussion on the differences between VAR and PVAR models, the choice of which one to use in this chapter becomes clear - PVAR models are much more suitable for studying cross-country linkages due to their inclusion of essential cross-sectional information. This section shows how PVAR models incorporate cross-sectional information. Let $\mathbf{y}_{i,t}$ be an M-dimensional vector of endogenous variables for country $i = 1, \ldots, N$ and observed at time $t = 1, \ldots, T$ and let $\mathbf{y}_t = (\mathbf{y'}_{1t}, \ldots, \mathbf{y'}_{Nt})'$ be an $K = M \times N$-dimensional vector of dependent variables. Therefore, the PVAR model for countries i associated p lags can be written as:

$$\mathbf{y_{it}} = \mathbf{A}_{i1}\mathbf{y}_{it-1} + \ldots + \mathbf{A}_{ip}\mathbf{y}_{it-p} + \mathbf{B}_{i1}\mathbf{y}_{-i,t-1} + \ldots + \mathbf{B}_{ip}\mathbf{y}_{-i,t-p} + \epsilon_{it}, \qquad (2.1)$$

where $\mathbf{A}_{ip}$ is $M \times M$-dimensional matrices of autoregressive coefficients for country i, $\mathbf{B}_{ip}$ captures the impact of other countries' lagged dependent variables with $M \times (N-1)M$. Finally, $\epsilon_{it} \sim N(\mathbf{0}_M, \mathbf{\Sigma}_{it})$ is an M-dimensional vector of disturbances with a variance-covariance matrix (correlation between countries) - $cov(\epsilon_{it}, \epsilon_{jt}) = \mathbf{\Sigma}_{ij} \neq 0$ for each $i, j = 1, \ldots, N$ and $i \neq j$.

From equation (2.1), the PVAR model can be rewritten as follows:

$$\mathbf{y}_{it} = \mathbf{\Phi}_i \mathbf{x}_{it} + \mathbf{\Lambda}_i \mathbf{x}_{-it} + \epsilon_{it}, \tag{2.2}$$

where $\mathbf{\Phi}_i = (\mathbf{A}_{i1}, ..., \mathbf{A}_{ip})'$ is a $M \times Mp$ -dimensional matrix of coefficients for domestic endogenous variables, $\mathbf{x}_{it} = (\mathbf{y}'_{it-1}, \ldots, \mathbf{y}'_{it-p})'$ and $\mathbf{\Lambda}_i = (\mathbf{B}_{i1}, ..., \mathbf{B}_{ip})'$ is $M \times Mp(N-1)$ -dimensional matrix of coefficients of other countries lagged dependent variables, $\mathbf{x}_{-it} = (\mathbf{y}'_{-it-1}, \ldots, \mathbf{y}'_{-it-p})'$. Equation 2.2 illustrates the standard PVAR models which are commonly found in the literature, called unrestricted PVAR models (Feldkircher et al., 2020). Let $K = N \times M$, therefore $\mathbf{\Sigma}_t$ is the full variance-covariance matrix of dimension $K \times K$. Additionally, let $\mathbf{\Sigma}_t$ be the full variance-covariance matrix of dimension $K \times K$ for $\varepsilon_t = (\varepsilon_{t,1}, \ldots, \varepsilon_{t,N})'$ with $\epsilon_{it} \sim N(\mathbf{0}_M, \mathbf{\Sigma}_{it})$ which can be represented by

$$\mathbf{\Sigma}_t = \begin{bmatrix} \mathbf{\Sigma}_{11} & \ldots & \mathbf{\Sigma}_{1N} \\ \vdots & \ddots & \vdots \\ \mathbf{\Sigma}_{N1} & \ldots & \mathbf{\Sigma}_{NN} \end{bmatrix} \tag{2.3}$$

Moreover, the block-diagonal element $\mathbf{\Sigma}_{ii}$ represents the variance-covariance matrix of endogenous variables within country i with dimension $M \times M$ while the block-off diagonal elements $\mathbf{\Sigma}_{ij}$ and $\mathbf{\Sigma}_{ji}$ refer to covariance between variables in country i and country j with dimension $M \times M$ for $i \neq j$. Consequently, the entire variance-covariance matrix of PVARs model is a $K \times K$ dimensional matrix, $\mathbf{\Sigma}_t$.

2.2.1 Cross-Sectional Homogeneity (CSH)

For the sake of simplicity, the concept of cross-sectional heterogeneity is that the coefficients of own lagged endogenous variables for country i differ from country j, implying that $\mathbf{\Phi}_i \neq \mathbf{\Phi}_j$. Conversely, if the coefficients of lagged endogenous variables for country i and country j are similar, it is labeled as cross-sectional homogeneity; $\mathbf{\Phi}_i = \mathbf{\Phi}_j = \mathbf{\Phi}$, for $i \neq j$. In other words, either the responses to dynamic movements in one country are identical or non-identical to other countries. In addition, this feature possibly includes a case of different block-diagonal elements; $\mathbf{\Sigma}_{it} \neq \mathbf{\Sigma}_{jt}$, for $i \neq j$. Nevertheless, this assumption depends on the specific application of panel data in the sense that units and countries are selected.

2.2.2 Dynamic Interdependency (DI)

Fundamentally, dynamic interdependency is defined as the coefficients of foreign lagged endogenous variables being non-zero elements, meaning that $\mathbf{\Lambda}_i \neq \mathbf{0}$. This means that the domestic variables of country i are affected by the dynamic (lagged) movement of foreign variables of country j. In other words, there are dynamic cross-country spillover effects from one country to other countries. The assumption made by DI is therefore more feasible, more realistic and useful for analysing economic impacts across countries in the context of globalisation. However, this assumption can be ruled out because there are asymmetric effects between any two given economies, particularly smaller ones. For instance, the money policy shock of the USA can easily transmit to smaller economies but not vice versa. It depends directly on the international linkages - through trade and financial channels - of each economy.

2.2.3 Static Interdependency (SI)

Finally, static interdependency deals with non-zero elements of the covariance matrix, meaning that $\mathbf{\Sigma}_{ij} \neq \mathbf{0}$. It implies that shocks from one country enable to transmit contemporaneously to other countries. In other words, static interdependency is the contemporaneous relations between economies and their respective variables. In the theoretical and empirical literature on economics, many examples of contemporaneous effects can be found in policies, behavioural variables and stock/financial indexes. Although static interdependency is an assumption that is allowed to exist, it is still ultimately determined by stylised facts in the panel data.

2.3 The Problems of High-Dimensionality in PVARs

In order to understand the over-parameterisation problem, a comparison of a number of restricted and unrestricted parameters in PVAR models and a number of observations in panel data are needed. In this regard, the number of parameters in a PVAR model can be calculated. This chapter focuses on the G-7 countries: Canada, France, Germany, Italy, Japan, the United Kingdom and the United States. The monthly data on three macroeconomic variables - taken from January 1999 to December 2018 - will be used in this study in order to ascertain the existence of economic interdependency. These three variables are (i) industrial production index, (ii) consumer price index, and (iii) the 10-year government bond yield. Therefore, this results in 240 monthly observations for three variables across these seven major economies.

Given three macroeconomic variables ($M = 3$) for the G-7 countries ($N = 7$) and one lagged endogenous variable ($p = 1$), the number of coefficients of every individual equation (one endogenous variable) are $M \times (Np) = 3 \times (7) = 21$. Each country has $M \times (NMp) = 3 \times (21) = 63$ coefficients and the full model is $NM \times (NMp) = 441$ coefficients. Moreover, the error covariance matrix can be defined as $N(N-1)/2 = 21$ restrictions for PVARs. This leads to the fundamental problem of the over-parameterisation in PVAR models due to a large number of parameters to be estimated under short time-series data - the total number of estimated parameters exceeds the available sample size. Therefore, Stochastic

Search Specification Selection (S^4) prior of Koop & Korobilis, (2016) is adopted to deal with this problem and also to examine DI, CSH, and SI in PVARs. In addition, the chapter compares the forecasting performance between (S^4) prior and Canova & Ciccarelli's factor shrinkage prior (2009) for predicting inflation.

2.4 The Stochastic Search Specification Selection (S^4) Algorithm

As previously mentioned the curse of dimensionality or over-parameterisation of PVAR models, many techniques for reducing the high dimensionality of the parameters are strongly required. Love and Zicchino (2006) applied their model without imposing heterogeneity and dynamic interdependencies. In contrast, Ciccarelli et al, (2013) restricted the model with dynamic interdependencies. These assumptions can be successfully applied to a small open economy by assuming that foreign variables are exogenous. Other methods for alleviating over-parameterisation from estimation are suggested by Canova and Ciccarelli (2009), Ciccarelli et al. (2013) and Canova et al. (2013), which construct a common feature of country-specific and variable-specific groups by employing a dynamic factor structure to re-parameterise the full system of PVAR models. They assume that country-specific coefficients are determined by sets of both global and local-specific factors. This technique is very useful for controlling coefficients over time in PVAR models with parsimonious patterns. Koop and Korobilis (2019) extend the technique proposed by Canova and Ciccarelli (2009) to manipulate stochastic volatility in terms of static interdependency and dynamic interdependency by modelling the variance-covariance matrix with an identical low-dimensional factor pattern and introducing hierarchical shrinkage prior in the time-varying variance-covariance matrix, respectively. In addition, in order to successfully forecast crucial macroeconomic variables, especially in density forecasts, various pieces of literature mention that the models should restrict for heteroscedasticity in the error variances.

Another approach to reducing the high dimensionality of the parameters is proposed by Koop and Korobilis (2016). The authors adapt a stochastic search variable selection (SSVS) prior from George, Sun, and Ni (2008) for investigating whether coefficients across countries are identical or not, (i.e. whether the cross-sectional homogeneities (CSH) are similar) and whether coefficients in dynamic and static relations across countries are zero (i.e. whether the dynamic interdependency (DI) and static interdependency (SI) across countries exist). This approach relies on a data-driven method to select the restrictions in PVAR models by introducing latent binary indicators, called the Stochastic Search Specification Selection (S^4) algorithm. This method not only reduces the high dimensionality of the parameters but also initiates a stochastic approach to model uncertainty by using Markov chain Monte Carlo (MCMC) simulation for estimation in PVAR models.

Instead of using equation (2.1), the study uses an equivalent but more compact form for the PVAR in order to easily rearrange coefficients of the original PVAR matrix **A** and **B**. The unrestricted PVARs are now written more compactly as

$$Y_t = Z_t\alpha + \epsilon_t, \tag{2.4}$$

where $\varepsilon_t \sim N(0, \Sigma)$ for $t = 1, \ldots, T$ (no-autocorrelation), α is a $\left[P(MN)^2 \times 1\right]$ vector of all coefficients of endogenous variables in the VAR model and Z_t is a matrix of p lagged endogenous variables ($Z_t = I_{NM} \otimes \mathbf{y}_t$). Therefore, the unrestricted VAR is

$$Z_t = \begin{bmatrix} z_t & 0 & \dots & 0 \\ 0 & z_t & \ddots & 0 \\ \vdots & \ddots & z_t & 0 \\ 0 & \dots & 0 & z_t \end{bmatrix} \tag{2.5}$$

The basic idea of the SSVS algorithm in George et al. (2008) can be shown as follows:

$$\alpha_j|\gamma_j \sim (1-\gamma_j)N(0, c \times \tau_j^2) + \gamma_j N(0, \tau_j^2); \tag{2.6}$$

where α_j is the j^{th} element of α and $\gamma_j \in \{0,1\}$ is the unknown coefficient estimated from the data. The SSVS has a hierarchical prior with a mixture of two normal distributions, c and τ_j^2 for each of which a different parameter can be applied. If $\gamma_j = 0$, α_j is shrunk to zero. Bayesian estimation of this algorithm can use a Bernoulli prior for γ_j. In this regard, DI, SI and CSH restrictions can be defined as $\gamma = \{\gamma^{DI}, \gamma^{SI}, \gamma^{CSH}\}$ and use Gibbs sampler algorithm for the S^4 algorithm by drawing the block of lagged coefficients (γ^{DI}), the block of the error covariance matrix (γ^{SI}) and the block of identical and nonidentical dynamic movements of endogenous variables for each country (γ^{CSH}). In accordance with the Gibbs sampler algorithm, the estimated probability is imposed - that the appropriate element of γ is to be greater than one half.

In order to apply the S^4 prior, all coefficients in the PVAR are indexed for each of the M macro variables of country i, j for $i, j = 1, \dots, N$. In other words, coefficients are indexed for all possible pairs. For example, given $k, l = 1, ..., M$ index of each of the M macro variables of each country, α_{kl}^{ij} is a scalar which denotes the variable l of country j, which appears on the equation of variable k of country i. Subsequently, we need to pick the relevant scalar of the corresponding matrices and test whether all of them are jointly zero or not.

1. CSH prior:

$$\begin{aligned} vec(A_{ii}) \sim & (1-\gamma_{ij}^{CSH})N(A_{jj}, \xi_{ij}^2 \times \underline{c}^{CSH} \times I_{M^2}) \\ & + \gamma_{ij}^{CSH} N(A_{jj}, \xi_{ij}^2 \times I_{M^2}); \end{aligned} \tag{2.7}$$

$$\xi_{ij}^{-2} \sim Gamma(1, \underline{\theta}^{CSH}); \tag{2.8}$$

$$\gamma_{ij}^{CSH} \sim Bernoulli(\pi_{ij}^{CSH}), \forall j \neq i; \tag{2.9}$$

$$\pi_{ij}^{CSH} \sim Beta(1, \underline{\varphi}); \tag{2.10}$$

for $i = 1, ..., N$, $j = 1, ..., N-1$ and $i \neq j$ such that A_{ii} and A_{jj} are not the same matrix. If $\gamma_{ij}^{CSH} = 0$, then the coefficients of own lagged endogenous variables for country i and for country j are similar. This means that the homogeneity restriction, $A_{ii} \approx A_{jj}$, holds.

2. DI prior:

$$vec(A_{ij}) \sim (1-\gamma_{ij}^{DI})N(0, \tau_{ij}^2 \times \underline{c}^{DI} \times I_{M^2}) + \gamma_{ij}^{DI} N(0, \tau_{ij}^2 \times I_{M^2}); \tag{2.11}$$

$$\tau_{ij}^{-2} \sim Gamma(1, \underline{\theta}^{DI}); \tag{2.12}$$

$$\gamma_{ij}^{DI} \sim Bernoulli(\pi_{ij}^{DI}), \forall j \neq i; \tag{2.13}$$

$$\pi_{ij}^{DI} \sim Beta(1, \underline{\varphi}); \tag{2.14}$$

for $i = 1, ..., N$ $j = 1, ..., N-1$ and $i \neq j$ such that not applying to A_{ii} and A_{jj}. If $\gamma_{ij}^{DI} = 0$, then the coefficients on the lags of all country j variables in the VAR for country i are set to zero.

3. SI prior:

$$\begin{aligned} vec(\Psi_{ij}) \sim &(1-\gamma_{ij}^{SI})N(0,\kappa_{ij}^2 \times \underline{c}^{SI} \times I_{M^2}) \\ &+ \gamma_{ij}^{SI} N(0,\kappa_{ij}^2 \times I_{M^2}); \end{aligned} \tag{2.15}$$

$$\kappa_{ij}^{-2} \sim Gamma(1, \underline{\theta}^{SI}); \tag{2.16}$$

$$\gamma_{ij}^{SI} \sim Bernoulli(\pi_{ij}^{SI}); \tag{2.17}$$

$$\pi_{ij}^{SI} \sim Beta(1, \underline{\varphi}); \tag{2.18}$$

for $i = 1, ..., N$, $j = 1, ..., N-1$ and $i \neq j$ and $i > j$. If $\gamma_{ij}^{SI} = 0$, then the block of the PVAR error covariance matrix relating to the covariance between countries i and j is set to zero.

Additionally, the error covariances prior is shown as

$$\Psi_{kl}^{ii} \sim \begin{cases} N(0, \underline{\kappa}_2^2), & \text{if} \quad k \neq l \\ Gamma(\underline{\rho}_1, \underline{\rho}_2) & \text{if} \quad k = l \end{cases} \tag{2.19}$$

where $k, l = 1, ..., M$ index each of the M macro variables of country $i = 1, ..., N$. The study imposes a matrix $\Gamma = \prod_{i=1}^{N-1} \prod_{j=i+1}^{N} \Gamma_{ij}$ where Γ_{ij} represents the matrices $K \times K$ for using CSH restriction indicators (γ_{ij}^{CSH}). The prior hyperparameters of the model are $\underline{c}^{DI}, \underline{c}^{CSH}, \underline{c}^{SI}, \underline{\theta}^{DI}, \underline{\theta}^{CSH}, \underline{\theta}^{SI}, \underline{\varphi}, \underline{\kappa}_2^2, \underline{\rho}_1, \underline{\rho}_2$, which are built on Koop and Korobilis (2016). The method of the MCMC algorithm is shown in Appendix A.1.

2.5 Model Selection

In terms of model selection, the study adopts two main methods - the logarithm of the marginal likelihood (ML) and the deviance information criterion (DIC) as described below.

Regarding the calculation of the logarithm of the marginal likelihood (ML), a modification of the harmonic mean estimator (HME) of Newton-Raftery (1994) is used, known as the GD method of Gelfand and Dey (1994). This algorithm proposes that for any probability density function f that has support contained in the support of the posterior density, the reciprocal of ML is defined as

$$\begin{aligned}\mathbb{E}\left[\frac{f(\boldsymbol{\theta})}{p(\boldsymbol{\theta})p(\mathbf{y}|\boldsymbol{\theta})}|\mathbf{y}\right] &= \int \frac{f(\boldsymbol{\theta})}{p(\boldsymbol{\theta})p(\mathbf{y}|\boldsymbol{\theta})}p(\boldsymbol{\theta}|\mathbf{y})d\boldsymbol{\theta} \\ &= \int \frac{f(\boldsymbol{\theta})}{p(\boldsymbol{\theta})p(\mathbf{y}|\boldsymbol{\theta})}\frac{p(\boldsymbol{\theta})(p(\mathbf{y}|\boldsymbol{\theta})}{p(\mathbf{y})}d\boldsymbol{\theta} = p(\boldsymbol{y})^{-1}\end{aligned} \tag{2.20}$$

Therefore, Gelfand and Dey propose the following estimator for the marginal likelihood $p(\mathbf{y})$:

$$\widehat{p}_{GD}(\mathbf{y}) = \left\{\frac{1}{L}\sum_{l=1}^{L}\frac{f(\boldsymbol{\theta}_l)}{p(\mathbf{y}|\boldsymbol{\theta}_l)p(\boldsymbol{\theta}_l)}\right\}^{-1} \tag{2.21}$$

where $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_L$ are collected from all samples of the posterior distribution. Although Eq. 2.21 is applicable to any density function f, the estimator $\mathbb{E}[\widehat{p_{GD}}(\mathbf{y})]$ remains biased; for instance, $\mathbb{E}\left[\widehat{p_{GD}}\left(\mathbf{y}\right)\right] \neq p\left(\mathbf{y}\right)$ in general. Nevertheless, the accuracy of this estimator depends crucially on the choice of the tuning function f. In this study, a method, outlined by Geweke (1999), was adopted, in which f is chosen as a normal approximation to the posterior density with a tail truncation determined through asymptotic arguments. Geweke demonstrates that if the tuning function f exhibits tails lighter than those of the posterior density, the estimation in Eq. 2.21 attains finite variance. Specifically, let θ^* and Σ^* denote the posterior mean and covariance matrix, respectively, with f denoting $N(\theta^*, \Sigma^*)$. For more details on the approximation of θ^* and Σ^*, one can refer to the work of Liu and Liu (2012). Note that a higher ML indicates a better model performance.

For the second method, the criteria are based on the deviance information criterion (DIC), which is a hierarchical Bayesian method for model comparison; a generalisation of the Akaike information criterion (AIC). The DIC can be defined as

$$DIC = D(\overline{\theta}) + 2p_D \tag{2.22}$$

in which the first term is interpreted as the measurement of goodness-of-fit and the second term as the complexity. It can measure the fit as the deviance $D(\theta) = -2log(p(y|\theta))$ where y is the data, θ is the unknown parameters of the model and the $p(y|\theta)$ are the likelihood function. The complexity can be measured by estimating the effective number of parameters as $p_D = E_{\theta|y}[D] - D(E_{\theta|y}[\theta]) = \overline{D} + p_D$. In other words, the complexity is the posterior mean deviance minus deviance evaluated at the posterior mean of the parameters. It can be concluded that models with a smaller DIC are better (see Spiegalhalter et al; 2002).

2.6 Empirical Application

This section presents the results of empirical applications employing a panel VAR model with the stochastic search specification selection (S^4) algorithm as a prior on the G-7 economies. Section 2.6.1 shows the macroeconomic interdependencies among the G-7 countries and Section 2.6.2 presents a comparison of forecasting performance using different priors.

2.6.1 Macroeconomic Interdependence

Regarding the most recent period of economic globalisation, the G-7 is recognised as the world's largest intergovernmental organisation, within which any member of the G-7 has close economic ties with any other member. Despite their status, there is a noticeable gap in the literature on the G-7 economies (Canada - 'CAN', France - 'FRA', Germany - 'GER', Italy - 'ITA', Japan - 'JAP', the United Kingdom - 'UK' and the United States - 'US') when it comes to the study of the both static and dynamic interdependency features, including cross-sectional homogeneity, and what is more there has been a reluctance to use panel VAR models for this due to the issue of over-parameterisation. As explained in section 2.4, The Stochastic Search Specification Selection (S^4) Algorithm, this study negates this issue by employing the S^4 algorithm and therefore explicitly contributes evidence of interesting interdependency features by exploiting them, using the full panel structure of a panel VAR model. Additionally, there are three main macroeconomic variables, which are (i) industrial production index, (ii) consumer price index, and (iii) the 10-year government bond yield, each using monthly data from the period of January 1999 through to December 2018. In addition, the monthly data is transformed to be approximately stationary. This means that there are two endogenous variables - the industrial production index and the consumer price index, are transformed to the first difference of their logarithm, while the 10-year government bond yield is not. Regarding the number of restrictions, there are $N(N-1) = 42$ possible restrictions of DI, $N(N-1)/2 = 21$ for SI and $N(N-1)/2 = 21$ for CSH.

According to features of PVAR models, the study specifies six restrictions in PVARs as follows:
Model 1: The full model with DI, SI and CSH restriction search.
Model 2: The full model with DI and SI restriction search (no search for CSH, ${\gamma_{jk}}^{CSH} = 1$).
Model 3: The full model with DI restriction search (no search for SI and CSH, ${\gamma_{jk}}^{SI}, {\gamma_{jk}}^{CSH} = 1$).
Model 4: The full model with CSH restriction search (no search for DI and SI, ${\gamma_{jk}}^{DI}, {\gamma_{jk}}^{SI} = 1$).
Model 5: The full model with SI restriction search (no search for DI and CSH, ${\gamma_{jk}}^{DI}, {\gamma_{jk}}^{CSH} = 1$).
Model 6: The full unrestricted PVAR model without any restriction search (non-informative prior).

To measure the best performance of PVAR models with various restrictions, the study employs the log of the marginal likelihood (ML) and the deviance information criterion (DIC). A higher ML value indicates a better performance of the model whereas a lower DIC value indicates a better performance of the model, respectively. It can clearly be seen that using S^4 prior brings the huge benefit of enabling the researcher to classify the panel structure of macro-financial linkages between the G-7 economies. From the results in Table 2.1, the ML and DIC suggest that there exists a significant relationship between the G-7 economies in terms of dynamic and static interdependencies in the model (M2). As a result of this sample application, it is important to allow the impact of the dynamic movement of foreign lagged endogenous variables (DI) and the influence of contemporaneous shocks (SI) from one country to other countries. Nevertheless, when evaluating the ML and DIC values of Models M2 and M3, where the values appear quite similar, additional consideration is prudent. This similarity may imply that imposing restrictions on SI might not be as crucial as imposing restrictions on DI. Despite this finding, the study still incorporates both DI and SI restrictions, as indicated in Table 2.1.

Table 2.1: Model Performance

Method	M1	M2	M3	M4	M5	M6
ML	-20.37	**-12.00**	-12.01	-23.27	-15.37	-15.36
DIC	32.73	**18.12**	18.13	34.66	18.29	18.26

Furthermore, it is worth noting that this result can be discussed in relation to the work of Canova and Ciccarelli (2009), which compared different specifications of the panel VAR model using the factorisation method for the G-7 countries. Although the findings of this study did not account for time variations as Canova and Ciccarelli (2009) did, this result aligns with Canova and Ciccarelli's findings in the sense that international dynamic (lagged) interdependence (DI), country-specific components (SI) and dynamics of own lagged endogenous variables for each country (non-homogeneity) are still preferred for this application.

Nevertheless, the M2 model's approach only suggests that there exists significant dynamic and static interdependencies - not the full panel structures. Taking into account the full panel structures, it is useful to also address each DI, SI and CSH restriction in order to study the relationship between the G-7 economies. On the other hand, the M4 model's approach of assuming that all parameters of one's own country are similar to those of other countries appears to be the least effective, as the ML and DIC indicated. Therefore, all possible results are shown in Tables 2.2 - 2.4.

Next, there are 42 possible restrictions for DI, 21 for SI and 21 for CSH respectively. These possible restrictions were examined by imposing an estimated probability that the appropriate element is to be greater than one half[1]. This means that Table 2.2 and Table 2.3 show the existence of interlinkages in terms of dynamic and static patterns, respectively, for the G-7 economies. However, the results did not find interlinkages for certain pairs of countries (for example, France and Germany in Table 2.2 or Germany and Canada in Table 2.3) and therefore these pairs are not included in the tables.

For DI restrictions in Table 2.2, examination shows that 26 of the 42 possible restrictions are not imposed. It can clearly be seen that there are significant dynamic interdependencies among the G-7 economies, implying that the coefficients of foreign lagged endogenous variables are non-zero elements. In other words, there are dynamic cross-country spillover effects from one country (labeled as 'From') to other countries (labeled as 'To'). For example, Italy, the UK and the US exhibit the dynamic interdependency pattern with all the other countries in the G-7, indicating that the endogenous variables of Italy, the UK and the US are affected by the dynamic (lagged) movement of all endogenous variables from

[1]This means that the study imposed $\gamma = \{\gamma^{DI}, \gamma^{SI}, \gamma^{CSH}\} > 0.5$.

all the other countries in the G-7. However, this pattern seems to be less common for Canada and Japan. This implies that Canada is exclusively affected, in terms of dynamic interdependency, by Japan, and vice versa. Nevertheless, Canada and Japan have an impact, in terms of the dynamic interdependency feature, on all the other countries in the G-7.

Table 2.2: Dynamic Interdependency (DI)

No.	To	From	No.	To	From
1	FRA	CAN	14	UK	GER
2	FRA	JAP	15	UK	ITA
3	GER	FRA	16	UK	US
4	GER	US	17	UK	CAN
5	GER	CAN	18	UK	JAP
6	GER	JAP	19	US	FRA
7	ITA	FRA	20	US	GER
8	ITA	GER	21	US	ITA
9	ITA	UK	22	US	UK
10	ITA	US	23	US	CAN
11	ITA	CAN	24	US	JAP
12	ITA	JAP	25	CAN	JAP
13	UK	FRA	26	JAP	CAN

For SI restrictions in Table 2.3, the examination shows that 11 of the 21 possible restrictions are not imposed. This means that there are significant static interdependencies among the G-7 economies as well. In other words, there are contemporaneous shocks along countries. For example, France, represented in the C1 column, demonstrates the static interdependency pattern with all of the other G-7 countries apart from the US in the C2 column, indicating that France is contemporaneously affected by these countries. Another example illustrates that Germany is contemporaneously affected by Italy, the UK, and Canada, but not the US, France and Japan. Additionally, it is worth noting that Japan, Italy and the UK are not contemporaneously affected by other countries in the G-7[2].

Table 2.3: Static Interdependency (SI)

No.	C1	C2	No.	C1	C2
1	FRA	GER	7	GER	UK
2	FRA	ITA	8	GER	CAN
3	FRA	UK	9	US	CAN
4	FRA	CAN	10	US	JAP
5	FRA	JAP	11	CAN	JAP
6	GER	ITA			

Finally, considering the CSH restrictions[3] in Table 2.4, examination shows that 11 of the 21 possible restrictions are not imposed as the homogeneity. This implies that coefficients of own lagged endogenous variables for one country differ from another. In other words, some countries from C1 listed in Table 2.4 - France, Germany, Italy, the UK and the US - do not share homogeneity with any other countries listed in C2. For example, it can be noticed that Italy exhibits notable differences when compared to the UK, the US, Canada, and Japan. However, though the results of the CSH may be meaningful in certain cases, they may not universally apply. Therefore, the insights gained from the model selection process in the previous section are valuable for evaluating the significance of these CSH restrictions across different scenarios.

[2] The study defines the SI restriction as a comparison between the block of error covariance matrices of country i and country j, indicating contemporaneous linkages between countries, rather than between individual variables within these countries. This is because the latter restriction might lead to a huge model space.

[3] To understand this more clearly, the cross-sectional homogeneity restriction refers to a comparison of parameters between any given country and the other countries in a VAR model, in which one country contains only its own variables - no linkages between countries.

Table 2.4: Cross Sectional Homogeneity (CSH)

No.	C1	C2	No.	C1	C2
1	FRA	ITA	7	ITA	CAN
2	FRA	US	8	ITA	JAP
3	GER	ITA	9	UK	US
4	GER	US	10	US	CAN
5	ITA	UK	11	US	JAP
6	ITA	US			

In summary, the results indicate a unique occurrence of static interdependency (SI) and cross-sectional homogeneity (CSH) features in Canada and Japan. It is worth noting that these findings align with the work of Carstensen and Salzmann (2017), which suggests that the business cycles of Canada and Japan are mainly driven by their own domestic shocks.

2.6.2 Forecasting

In terms of evaluating a point forecast accuracy, the mean squared forecast error (MSFE) is used, defined as

$$MSFE = \frac{\sum_{T_0}^{T-h}[\hat{y}_{T_0+h} - y^o_{T_0+h}]^2}{T-h-T_0+1} \tag{2.23}$$

where h is the number of time periods over which forecasting is performed. Naturally, the smaller the MSFE, the better the out-of-sample forecast performance.

When encountering parameter and estimation uncertainties, a predictive distribution is used. Therefore, density forecast accuracy is determined by the average log predictive likelihood (LPL) (for more details, see Geweke and Amisano, 2010). The LPL can be defined as:

$$LPL_{t+h} = \frac{1}{T-h-T_0+1}\sum_{T_0}^{T-h} \ln P(y_{t+h} = y^o_{t+h}|\mathcal{F}_t) \tag{2.24}$$

where y^0_{t+h} is the observed value of the random variable y_{t+h}. $\mathcal{F}_t$ denotes the information available at time t and $P(y_{t+h}|\mathcal{F}_t)$ is the predictive density based on the information available at time t, constructed by using a kernel density estimation for drawing forecasts of period $t+h$. This means that a larger LPL value implies a better density forecast performance.

In addition, PVARs are formulated with both the S^4 prior algorithm - PVARs(S^4), and the factor shrinkage prior - PVARs(CC) (see Appendix A.1-A.2), for evaluating the forecasting performance of inflation. The predictions are built by recursive monthly forecasting for h=1, 3, 9 and 12 for period 2011M1 to 2018M12 from the sample period 1999M1 to 2010M12. Additionally, the unrestricted ordinary least square estimate of the full-dimensional vector autoregressive (VAR) model (noninformative prior) with one lag is used as a benchmark model for comparison. When evaluating the point forecasts, the MSFE's results suggest that the S^4 prior algorithm outperforms the factor shrinkage prior, especially in the short-term forecast horizons as shown in Tables 2.5 through 2.8.

Table 2.5: MSFE for each model relative to OLS: Forecast Horizon h=1

Model	FR	DE	IT	UK	US	CA	JP
PVARs(S^4)	**0.659**	**0.885**	**0.771***	**0.724**	**0.419**	**0.600**	2.392
PVARs(CC)	1.259*	3.431	1.280	4.255	0.933*	1.332	2.448

Note: The best model is indicated in bold. * denotes rejection of the null of equal forecast accuracy against the benchmark of the OLS model at the 0.10 significant level using a two-sided (Diebold & Mariano, 1995) test.

Table 2.6: MSFE for each model relative to OLS: Forecast Horizon h=3

Model	FR	DE	IT	UK	US	CA	JP
PVARs(S^4)	**0.456**	**0.616**	**0.531**	**0.477**	**0.311**	**0.435**	1.462
PVARs(CC)	0.714*	1.881	0.768*	2.169	0.496	0.919	**0.994**

Note: The best model is indicated in bold. * denotes rejection of the null of equal forecast accuracy against the benchmark of the OLS model at the 0.10 significant level using a two-sided (Diebold & Mariano, 1995) test.

Table 2.7: MSFE for each model relative to OLS: Forecast Horizon h=6

Model	FR	DE	IT	UK	US	CA	JP
PVARs(S^4)	**0.791**	**0.984**	**0.751**	**0.802**	0.561	**0.579**	1.730
PVARs(CC)	0.831	1.906	0.841*	1.984	0.561	1.088*	**0.771**

Note: The best model is indicated in bold. * denotes rejection of the null of equal forecast accuracy against the benchmark of the OLS model at the 0.10 significant level using a two-sided (Diebold & Mariano, 1995) test.

Table 2.8: MSFE for each model relative to OLS: Forecast Horizon h=12

Model	FR	DE	IT	UK	US	CA	JP
PVARs(S^4)	0.877	1.022	0.751	**0.823**	0.673	**0.777**	1.347
PVARs(CC)	**0.718**	1.277	**0.739**	1.053	**0.625**	1.261	1.348

Note: The best model is indicated in bold. * denotes rejection of the null of equal forecast accuracy against the benchmark of the OLS model at the 0.10 significant level using a two-sided (Diebold & Mariano, 1995) test.

With respect to the density forecasts, Tables 2.9 through 2.12 show the average predictive likelihoods resulting from the posterior predictive density at the observed information y_{t+h}. As mentioned in Eq. 2.22, a higher LPL value means a better performance of the models. The results indicate that the panel VAR models with the stochastic search specification selection (S^4) outperform their density forecasts for all countries in the G-7, especially for the short-term forecast horizons (h1 and h3). However, those with the factor shrinkage prior (PVARs(CC)) outperform their density forecasts for only some of the G-7 (i.e., the US and Japan), especially for the longer-term forecast horizons (h6 and h12).

Table 2.9: LPL of inflation forecasts: Forecast Horizon h=1

Model	FR	DE	IT	UK	US	CA	JP
PVARs(S^4)	**0.963**	**0.864**	**0.757**	**0.863**	**0.636**	**0.619**	**0.526**
PVARs(CC)	0.713	0.319*	0.641	0.433*	0.519	0.479*	0.450

Note: The best model is indicated in bold. * denotes rejection of the null of equal forecast accuracy against the benchmark of the OLS model at the 0.10 significant level using a two-sided (Diebold & Mariano, 1995) test.

Table 2.10: LPL of inflation forecasts: Forecast Horizon h=3

Model	FR	DE	IT	UK	US	CA	JP
PVARs(S^4)	**0.889**	**0.802**	**0.722**	**0.778**	**0.587**	**0.590**	**0.467**
PVARs(CC)	0.661	0.317*	0.615	0.417	0.504*	0.469	0.454*

Note: The best model is indicated in bold. * denotes rejection of the null of equal forecast accuracy against the benchmark of the OLS model at the 0.10 significant level using a two-sided (Diebold & Mariano, 1995) test.

Table 2.11: LPL of inflation forecasts: Forecast Horizon h=6

Model	FR	DE	IT	UK	US	CA	JP
PVARs(S^4)	**0.728**	**0.655**	**0.648**	**0.642**	**0.503**	**0.541**	0.386
PVARs(CC)	0.587	0.318	0.577	0.392*	0.475	0.442	**0.462**

Note: The best model is indicated in bold. * denotes rejection of the null of equal forecast accuracy against the benchmark of the OLS model at the 0.10 significant level using a two-sided (Diebold & Mariano, 1995) test.

Table 2.12: LPL of inflation forecasts: Forecast Horizon h=12

Model	FR	DE	IT	UK	US	CA	JP
PVARs(S^4)	**0.609**	**0.508**	**0.576**	**0.485**	0.411	**0.479**	0.327
PVARs(CC)	0.515	0.265	0.565	0.362	**0.430**	0.413	**0.451**

Note: The best model is indicated in bold. * denotes rejection of the null of equal forecast accuracy against the benchmark of the OLS model at the 0.10 significant level using a two-sided (Diebold & Mariano, 1995) test.

2.7 Conclusion

In order to examine macroeconomic interdependence in terms of homogeneity and spillover effects, panel vector autoregressive models (PVARs) are very useful. This is because of some features of PVARs such as dynamic interdependency (DI), static interdependency (SI), and cross-sectional homogeneity (CSH). However, there is the fundamental problem of over-parameterisation in PVAR models. Therefore, this chapter employed the stochastic search specification selection (S^4) algorithm to deal with the problem with empirical application of the G-7 economies with respect to main macroeconomic variables. The results of the study show the importance of dynamic and static interdependencies among the G-7 economies. Moreover, using the stochastic search specification selection (S^4)

algorithm is beneficial for classifying the panel structure of macro-financial linkages between the G-7 economies. However, a limitation of the study is treating coefficients as matrices of DI, SI, and CSH for each country, without combining individual variables and these countries together. This approach is taken because combining the variables and countries might otherwise result in a vast model space; the magnitude of this restriction could however be beneficial for an in-depth study of the relationships in further research. Lastly, but of no less significance, when comparing the performance of inflation point forecasts between using the S^4 algorithm and the factor shrinkage prior of Canova and Ciccarelli (2009), the PVARs with the S^4 prior algorithm give a better forecasting performance, particularly in the short-term forecast horizons. For the density forecasts, the PVARs with the S^4 prior outperform for all countries in the G-7 in the short-term horizons while the PVARs with the factor shrinkage prior give a better forecasting performance for only some of the countries in the longer-term forecast horizons.

Chapter 3

Business Cycles for the US and the ASEAN+3: A Bayesian Panel Markov-Switching VAR Approach

3.1 Introduction

Over the past two decades, there has been a gradual growth of regional economic integration among Asian countries into the global economy. In this regard, the current stylised fact is that the economic growth of Asian countries contributes the largest share of the world's economic growth. One reason for this is that China, the largest of all Asian countries with the largest population of any country in the world, has become the world's second largest economy (Benjamin and Sato, 2022). Additionally, the higher levels of regional economic integration of Southeast Asian countries, particularly the Association of Southeast Asian Nations with China, South Korea and Japan (ASEAN+3)[1], might be a driver of enhancing their potential to play a significant role in the global economy and the decoupling of the US economy. Therefore, Asia's growing global economic weight makes it a fascinating area of study.

There has been a rapidly growing amount of literature related to the ASEAN - particularly in the field of business cycle synchronisation and the economic interdependencies among them - for more than a decade, especially after the Asian financial crisis and the global financial crisis. (see Park and Shin, 2009; He and Liao, 2012; Lam and Yetman, 2013; Leduc and Spiegel, 2013; Park, 2017; Davidson, 2022). Interestingly, it is not only the growing intra-regional trade in Asia that influences Asia's rising share of global flows and networks, but the relationship between Chinese and Asian stock markets which have been highly correlated during recent periods of turmoil, i.e. US-China tariff wars, the Shanghai stock market collapse and the COVID-19 pandemic, as stated by Shi (2022). In addition, Shu et al. (2018) propose that Chinese spillovers on currencies and equities in Asian countries are nearly equivalent in size to the US spillovers. These pieces of empirical evidence from the literature indicate that regional connectedness in Asian economies has been taken to a higher level in both regional trade and financial integration.

In order to consider the co-movements of international business cycles, multi-country vector autoregressive models are introduced to identify their interdependencies. For example, Canova and Ciccarelli (2009) develop multi-country vector autoregressive (VAR) models with time-varying parameters and avoid the curse of dimensionality by using coefficient factorisation. These factor structures

[1] The Association of Southeast Asian Nations (ASEAN) is a regional grouping consisting of 10 members; Brunei, Cambodia, Indonesia, Laos, Malaysia, Myanmar, the Philippines, Singapore, Thailand, and Vietnam.

provide (i) variations in the coefficients across countries and variables (common effect), (ii) variations in the country-specific coefficients and (iii) variations in the variable-specific coefficients. Koop and Korobilis (2016) propose panel VAR models with the stochastic search specification selection (S^4) algorithm to examine the international transmission of macroeconomic and financial variables for considering cross-section homogeneities and dynamic and static interdependencies between countries. However, in a dimension of multilateral linkages, a large panel VAR approach might lead to over-parameterisation problems when extracting interdependencies between individual countries. Alternatively, a Bayesian panel Markov-switching (PMS) VAR approach, proposed by Billio et al. (2016), can provide not only a tool for the analysis of business cycles, classified by regimes such as recession and expansion phases, but also include heterogeneity in cross-country panel data in order to examine the synchronisation of business cycles. In addition, the model assumes interconnections by incorporating a time-varying transition mechanism with the transition matrix of the country-specific Markov chains. This assumption implies spillover effects are nonlinearly included in the model.

For a number of reasons mentioned above, it is worth examining the regional economic integration in the ASEAN+3 by investigating business cycle synchronisation in the context of regional and global synchronisation. To be more specific, this study fills a gap in related literature by considering the important role of ASEAN+3 and the US in synchronising economic activity within the ASEAN+3. In this study, we build on Billio et al. (2016) and extend their PMS-VAR model by allowing other variables with regime-dependent parameters and other modelling restrictions.

Two main contributions of this study are providing evidence for the integration of all the ASEAN economies into one united ASEAN economy and shedding some light on its integration, to different degrees, with four significant partner countries - China, Japan, Korea and the US. The finding of this study can play a constructive role in encouraging ASEAN's members as a group to conduct inter-state relations and strengthen economic and trade cooperation with other countries. These relationships have the potential to not only contribute significantly to their economic cooperation but also to regional stability as a whole.

3.2 Literature Review

What many economists are most curious about is the comparative health of an economy over time. Typically, the periodic ups and downs in economic activity are defined as economic expansion and economic contraction or recession, respectively. These fluctuating states of an economy are known as the 'business cycle' or 'economic cycle'; this phenomenon is commonly measured with reference to movements in the Gross Domestic Product (GDP) of a country or region. There has long been an interest in studying business cycles in many aspects including 'business cycle synchronisation'. The study of the interaction between different business cycle phases of different economies has become of more interest since the spreading of globalisation. For example, Kose, Prasad and Terrones (2003) found that globalisation increases the degree of synchronisation of business cycles. Therefore, understanding the impacts of these international economic fluctuations is important, especially for the development of business cycle models.

Many factors, both domestic and international, contribute to change in the business cycle. It is interesting to understand the business cycle through transmission channels of international factors. Knowing this helps policymakers to create a suitable policy regarding the economic situation. For example, when the instability and volatility of the economic activity of a country is heavily influenced by international factors, policies targeting only domestic issues might be inefficient. Regarding the study of the synchronisation of business cycles, this study briefly categorises previous studies into three areas of interest. The first group of studies is relevant to finding empirical evidence for the synchronisation of specific groups or countries. The second group aims to investigate determinants of business cycle synchronisation or examine economic variables for considering transmission channels and their effects. The third and last group serves the purposes of the first two groups by developing various explicit modelling techniques, in particular time-series econometric approaches, in order to determine suitable methods for future research.

There are different approaches to identifying transmission channels of this time-series international business cycle. Up until now, there is no consensus on this identification. Burns and Mitchell (1946) originally identified business cycles by considering the duration of a complete specific cycle - adding the length of its expansion and recession phases. Notwithstanding a lack of consensus, the prediction models of business cycles can be divided into two types according to the characteristics of data - 1) large-scale data and 2) small-scale data. Regarding large-scale data models, some literature explores business cycle synchronisation by applying large-scale econometric approaches and assessing trade multipliers (see Ball, 1973; Hickman, 1983; and Hickman and Filatov, 1983). Meanwhile, some literature employs international input-output models for linking economic interdependencies such as Yamazawa, Nohara, and Osada (1986). However, an obvious disadvantage is that large-scale models can lead to over-parameterisation which can result in worse performance in terms of model prediction.

In this regard, the latter model type - small-scale time-series econometric models - plays a role in solving this problem. Some recent research, for example, analyses international dependence, business cycle synchronisation, and transmission by applying recursive correlation, vector autoregression models, state-space models, factor models and regime-switching models (Barry and Guille, 1976; Dellas, 1986; Gerlach, 1988; Ahmed et al., 1993; Canova and Dellas, 1993). Moreover, Billio et al. (2016) propose an interesting model based on a Bayesian approach, namely a Panel Markov-switching (PMS) VAR model. This model is not only useful for examining the synchronisation and the heterogeneity of business cycles for a regional/multi-country analysis, but also alternatively provides an interconnection mechanism by incorporating a time-varying transition mechanism with the transition matrix of the country-specific Markov chains. These mainly help a reduction of high dimensionality problems and also apply non-linearity to the model.

Various studies investigate and try to identify a source of drivers and transmission channels of international business cycles; most of these focus on developed countries, for example, Canova and Marrinan (1998) propose that regarding the US, German and Japanese economies, contemporaneous shocks determine the transmission of international business cycles in the short run whereas production and consumption interdependencies account for the transmission of technology shocks and consumption shocks in the medium run, respectively. Gregory et al. (1997) employed the Kalman filter and a dynamic factor model for the G-7 countries to measure world business cycles by considering output, consumption and investment. They found that world and country-specific factors played a significant role in economic fluctuations during the period 1970 through 1993. Kose, Prasad and Terrones (2003) studied sixty countries that included both developed and developing economies by using a Bayesian factor model and proposed empirical evidence on business cycle synchronisation that interdependencies appear to be stronger for developed economies than for developing economies. In addition, their result of factors-driven business cycles is similar to Gregory et al. (1997). Lumsdaine and Prasad (2003) constructed the common component of the world business cycle by using time-varying weights based on GARCH models with the industrial production index for 17 OECD economies over the period of 1963-1994. The study found that industrial production growth fluctuation for all countries has strongly positive correlations with the common component of international fluctuations constructed by time-varying weights.

In the context of Asian economies, researchers are now paying more attention to the interdependencies between Asian countries and the two most powerful countries in 2022, the US and China, than they did during the previous two decades. Recent work by Davidson (2022) showed macro-financial interdependencies among the Asia Pacific countries and between each Asia Pacific country and the US by adopting large Bayesian Panel VARs with the stochastic search specification selection approach of Koop and Korobilis (2016). In this respect, the study proposed that deepening regional integration is not a driver of decoupling from the US. Instead, regional and global shocks deeply affected the interdependencies of the ASEAN nations and the US before the financial crisis but after the crisis different linkages, such as the Asian financial crisis, became more influential on their relationships. In more detail, the US financial shocks have played a substantial role in the fortunes of the Asia Pacific economies in the past decade, meanwhile US macroeconomic shocks played a less important role in Asia Pacific economies because of rising intra-regional trade. By using a two-stage least square method, Cheng et al. (2020) suggested that the business cycle tendency between China and its emerging markets is more synchronised through trade intensity and foreign direct investment (FDI) channels but less through economic synchronisation between China and its advanced economic partners. In addition, Richard and Ran (2022) found evidence that there is a significant inflation transmission from China to Australia and China to ASEAN economies by employing a Markov-switching VAR approach. This means that the effect of inflation dynamics in China also passes through to other regional economies. In a related previous paper, Benjamin and Sato (2022) shown that China's currency, the renminbi (RMB) still has a higher weight within currency baskets of Asian countries than shown in Richard and Ran's paper. Moreover, they report that there is an attempt to promote international uses of the renminbi among trading partners which might be a key factor in increasing levels of synchronisation in Asian countries. In addition, in the microeconomic aspect, Di Giovanni, Levchenko, and Mejean (2018) demonstrated that international business cycle transmission can be explained by linkages of multinational firms.

Bearing in mind the wealth of evidence for the interconnectedness between the ASEAN+3 and the US, this study will explain the phenomenon through the use of the Panel Markov-Switching VAR model, as shown in the next section.

3.3 Econometric Framework

3.3.1 A Panel Markov-Switching VAR Model

This section introduces initial features of a panel Markov-Switching model (PMS-VAR) developed by Billio et al. (2016). These features consist mainly of two components, which are interconnections and endogenous transitions. To be more specific, an assumption of the model is that it allows the transitions of country-specific Markov chains to be dependent on their own past history and on the history of Markov chains for other countries for capturing the features of interconnections. Moreover, an estimation of the model can be done using a Bayesian framework with hierarchical priors and stochastic restriction of parameters. A general specification of the PMS-VAR model can be shown as follows:

Let $\mathbf{y}_{it}$ be a vector of endogenous variables with dimensions $M \times 1$ for $t = 1, ..., T$, and $i = 1, ..., N$ countries. A general specification of the PMS-VAR model can be shown as follows;

$$\mathbf{y}_{it} = \mathbf{a}_i(s_{it}) + \sum_{j=1}^{N} \sum_{p=1}^{P} A_{ijp}(s_{it}) \mathbf{y}_{jt-p} + \mathbf{D}_i(s_{it}) \mathbf{z}_t + \varepsilon_{it}, \quad \varepsilon_{it} \sim \mathcal{N}_M(\mathbf{0}, \Sigma_i(s_{it})) \tag{3.1}$$

where $\mathbf{z}_t$ is a vector of (common) exogenous variables with $G \times 1$, $\mathbf{a}_i(s_{it})$, $A_{ijp}(s_{it})$, $\mathbf{D}_i(s_{it})$, and $\Sigma_i(s_{it})$ are parameters which follow Markov chains and $\mathcal{N}_M$ is an M-variate normal distribution. We denote $\{s_{it}\}$ as unit-specific and independent K -states Markov chain processes; $\{1, ..., K\}$.

In this regard, time-varying transition probabilities of $\{s_{it}\}$ are $\mathbb{P}(s_{it} = k | s_{it-1} = l, V_t, \alpha_i^{kl}) = p_{it,kl}$ with $k, l \in \{1, ..., K\}$. The V_t is a set of G_ν common endogenous covariates to all chains and the α_i^{kl} is a unit-specific vector of parameters.

In general, the parameters of a panel VAR model can vary across units and across time which implies dynamic interdependencies. From Eq.(3.1), dynamic interdependencies can be defined by setting $A_{ijp}(s_{it}) \neq 0$ for $i \neq j$.

Next, let $\xi_{it} = (\xi_{i1t}, ..., \xi_{iKt})'$ be a vector of indicators representing parameter shifts of Markov chains where $\xi_{ikt} = \mathbb{I}(s_{it} = k)$, such that;

$$\mathbb{I}(s_{it} = k) = \begin{cases} 1 & \text{if} \quad s_{it} = k, \\ 0 & \text{otherwise.} \end{cases} \tag{3.2}$$

for $i = 1, ..., N$, $t = 1, ..., T$, and $k = 1, ..., K$. This means that ξ_{it} presents information of the realisation of the i-th unit-specific Markov chains over the sample period. With regard to the chains, we can use these indicators for writing shifting parameters from Eq.(3.1) as

$$\mathbf{a}_i(s_{it}) = \sum_{k=1}^{K} \mathbf{a}_{i,k}\xi_{ikt}, \quad A_{ijp}(s_{it}) = \sum_{k=1}^{K} A_{ijp,k}\xi_{ikt},$$
$$\mathbf{D}_i(s_{it}) = \sum_{k=1}^{K} D_{ik}\xi_{ikt}, \qquad \Sigma_i(s_{it}) = \sum_{k=1}^{K} \Sigma_{ik}\xi_{ikt}$$

where $\mathbf{a}_{i,k} = (a_{i1,k}, ..., a_{iM,k})'$ are column vectors of VAR intercepts for unit i and regime-specific k with $M \times 1$ dimension, $A_{ijp,k}$ are matrices of VAR autoregressive coefficients for unit i, j and regime-specific k with $M \times M$ dimension, D_{ik} are matrices of exogenous coefficients for unit i and regime-specific k with $M \times G$ dimension and Σ_{ik} are covariance matrices for unit i and regime-specific k with $M \times M$ dimension.

The flexibility of this typical PMS-VAR model may lead to an over-parameterisation problem. To simplify the problem, this study assumes firstly that there are no dynamic interdependency restrictions among the same variables across units; i.e. $A_{ijp,k} = A_{ip,k}\mathbb{I}(i = j) + \mathbb{O}_{M\times M}(1 - \mathbb{I}(i = j))$, with as Canova and Ciccarelli (2009) and also no static interdependencies, $\mathbb{E}(\varepsilon_{it}\varepsilon_{jt}^{'}) = \mathbb{O}_{M\times M}$ with $\mathbb{O}_{n\times m}$. Secondly, the parameters driven by the regime-switching variables (s_{it}) following the Markov chains are (i) the intercepts, $\mathbf{a}_i(s_{it})$ and (ii) the variances, $\Sigma_i(s_{it})$, whereas the exogenous coefficients and autoregressive coefficients are assumed to be constant across regimes, $\mathbf{D}_{i,k} = D_i, \forall k, i$ and $A_{ip,k} = A_{ip}, \forall k$, respectively. Taking into account the presence of across unit dependencies, the model is assumed to have interaction mechanisms through a hierarchical prior restriction which is explained in the next section.

The PMS-VAR model from Eq.(3.1) given the previous restrictions can rewrite as follows;

$$\mathbf{y}_{it} = \mathbf{a}_i(s_{it}) + \sum_{p=1}^{P} A_{ip}\mathbf{y}_{it-p} + \mathbf{D}_i\mathbf{z}_t + \varepsilon_{it}, \qquad \varepsilon_{it} \sim \mathcal{N}_M(\mathbf{0}, \Sigma_i(s_{it})) \tag{3.3}$$

The assumption of the regime-switching variables for intercepts of the autoregressive model is used by McCulloch and Tsay (1994). In addition, Billio et al. (2016) reparameterise the model following Frühwirth-Schnattere (2006) by partitioning the set of regressors $(1, y^{'}_{it-1}, ..., y^{'}_{it-P}, z^{'}_t)$ into $1 + K + G$ subsets as $\overline{x}_{i0t} = (y^{'}_{it-1}, ..., y^{'}_{it-P}, z^{'}_t)'$ and $\overline{x}_{ikt} = 1$ with $k = 1, ..., K$. This means that $\overline{x}_{i0t}$ is an M_0- dimensional vector of regressors with the regime-invariant coefficients $(M_0 = MP+G)$ and $\overline{x}_{ikt}$ is M_K- dimensional vector of regressors with the regime-variant coefficients $(M_K = 1, \forall k)$. Therefore, we can rewrite the equation (3.3) as:

$$\mathbf{y}_{it} = X_{i0t}\gamma_{i0} + \xi_{i1t}X_{i1t}\gamma_{i1} + ... + \xi_{iKt}X_{iKt}\gamma_{iK} + \varepsilon_{it}, \quad \varepsilon_{it} \sim \mathcal{N}_M(\mathbf{0}, \Sigma_i(s_{it})) \tag{3.4}$$

where $X_{i0t} = (I_M \otimes \overline{x}^{'}_{i0t})$ with $\overline{x}_{i0t} = (y^{'}_{it-1}, ..., y^{'}_{it-P}, z^{'}_t)'$ and $X_{ikt} = (\overline{x}_{ikt} \otimes I_M)$ are determined by the regime-invariant and the regime-variant, respectively. In addition, $\gamma_{i0} = \text{vec}((A_{i,1}, \ldots, A_{iP}, D_i)^{'})'$ and $\gamma_{ik} = \mathbf{a}_{i,k}$.

3.3.2 A Markov Chain State Transition Approach

Transition probabilities of Markov chains can be defined by following Kaufmann (2015) as:

$$\mathbb{P}(s_{it} = k | s_{it-1} = l, V_t, \alpha_i) = H(V_t, \alpha_i^{kl}), \qquad k, l = 1, ..., K \tag{3.5}$$

and

$$H(V_t, \alpha_i^{kl}) = \frac{\exp((V_t - c_i)'\alpha_{1i}^{kl} + \alpha_{0i}^{kl})}{\sum_{k=1}^{K} \exp((V_t - c_i)'\alpha_{1i}^{kl} + \alpha_{0i}^{kl})} \tag{3.6}$$

where $\alpha_i^{kl} = (\alpha_{0i}^{kl}, \alpha_{1i}^{kl\prime})'$ and c_i is a vector of threshold parameters which might be defined it as the average of V_t. In this regard, let K be the reference state and assume $\alpha_{1i}^{Kl} = \mathbf{0}$ and $\alpha_{0i}^{Kl} = 0$ for all $l = 1, ..., K$. Moreover, denote $\alpha_i = \text{vec}((\alpha_i^{11}, ..., \alpha_i^{KK}))$ as the collection of parameters of the sequence of transition matrices for the ith unit.

To determine the transition probabilities of each regime within a business cycle, restrictions can be imposed by setting a minimum recession duration, depending on monthly or quarterly variables following Billio et al. (2016) instead of using a logit or probit model to determine the transition probabilities, such as Harding and Pagan (2011) and Amisano and Tristani (2013).

$$\mathbb{P}(s_{it} = k | s_{it-1} = l, s_{it-2}, V_t, \alpha_i) = \begin{cases} H(V_t, \alpha_i^{kl}) & \text{if} \quad s_{it-2} = 1, \\ 1 & \text{if} \quad s_{it-2} \neq 1, k = 1, l = 1, \\ 0 & \text{if} \quad s_{it-2} \neq 1, k \neq 1, l = 1, \\ H(V_t, \alpha_i^{kl}) & \text{if} \quad s_{it-2} \neq 1, \forall k \ \& \ l \neq 1. \end{cases} \tag{3.7}$$

3.3.3 Interconnection in the PMS-VAR Model

From the previous section, the study assumes that dependence among chains follow a Markov process through a set of common covariates V_t. In other words, the set can incorporate observable variables with the state value of the N unit-specific Markov chains used in the PMS-VAR Model. For achieving parsimony in parameterisation, let η_t be an auxiliary variable defined as $\eta_t = (\eta_{1t}, \ldots, \eta_{Kt})'$ resulting from the aggregation of past values of the unit-specific chains. Each element of η_t is assumed to be the weighted average as:

$$\eta_{kt} = \sum_{i=1}^{N} \omega_{it} \mathbb{I}(s_{it-1} = k) \tag{3.8}$$

where $\omega_{it} \geq 0$ and the sum of weights (ω_{it}) can be greater than one. However, the study assumes $\sum_{i=1}^{N} \omega_{it} = 1$ in order to interpret η_{kt} as probabilities. In addition, the unit-specific weights (ω_{it}) can be assumed, for example, from the relative GDP growth rate or size of the i-th unit at time $t-1$, with regard to the GDP growth rate or size of other units. This means that if $k = 1$ is assumed to be a recession regime, η_{1t} presents a probability measured by the relative economic size of the proportion of countries that are in the recession regime. Moreover, the weights (ω_{it}) can also be measured by imposing an aggregate of the hidden states. In summary, this process explains how to capture the features of interconnections.

In brief, the PMS-VAR model is well-suited for analysing business cycle synchronisations at a multi-country level because, although it assumes the absence of dynamic interdependency restrictions, it employs a hierarchical prior specification to illustrate dependence between units using latent variables and therefore doesn't impose direct linkages between variables in VAR equations, which could result in overparameterisation. Although the model isn't explicitly designed for investigating business cycle synchronisation channels through sets of economic variables, this approach enables the representation of interdependence between units through latent variables and interconnected mechanisms.

3.3.4 Bayesian Inference

The estimation of the PMS-VAR Model adopts a simulation-based Bayesian approach with hierarchical priors in order to avoid over-parameterisation concerns. This study employs the specification of priors following Canova and Ciccarelli (2009), namely the dependence between panel units through common latent variables, as follows:

$$\gamma_{i0} \sim \mathcal{N}_{MM_0}(\lambda_0, \underline{\Sigma}_{i0}), \quad i = 1, ..., N \tag{3.9}$$

$$\lambda_0 \sim \mathcal{N}_{MM_0}(\underline{\lambda}_0, \underline{\Sigma}_0), \tag{3.10}$$

$$\gamma_{ik} \sim \mathcal{N}_{MM_K}(\lambda_k, \underline{\Sigma}_{ik}), \tag{3.11}$$

$$\lambda_k \sim \mathcal{N}_{MM_K}(\underline{\lambda}_k, \underline{\Sigma}_k), \quad k = 1, ..., K \tag{3.12}$$

Regarding different units, we assume conditional independence across units - $\text{cov}(\gamma_{i0}, \gamma_{j0}|\underline{\lambda}_0) = O_{MM_0 \times MM_0}$, and $\text{cov}(\gamma_{ik}, \gamma_{jk}|\underline{\lambda}_k) = O_{MM_K \times MM_K}$) for $i \neq j$. For the inverse covariance matrix (Σ_{ik}^{-1}), it assumes to follow the independent Wishart prior defined as:

$$\Sigma_{ik}^{-1} \sim \mathcal{W}_M(\underline{\nu}_{ik}, \Upsilon_k), \quad i = 1, ..., N, k = 1, ..., K \tag{3.13}$$

$$\Upsilon_k^{-1} \sim \mathcal{W}_M(\underline{\nu}_k, \underline{\Upsilon}_k), \quad k = 1, ..., K \tag{3.14}$$

Additionally, the covariance matrix, $\text{cov}(\Sigma_{ik}^{-1}, \Sigma_{jk}^{-1}|\underline{\Upsilon}_k^{-1}) = O_{MM^2 \times MM^2}$ for all $i \neq j$.

Finally, we also apply a hierarchical prior specification for the transition matrices from Eq.(3.5). In this regard, the parameters of the l-th row, $\mathbf{p}_{it,l} = (p_{it,1l}, \ldots, p_{it,Kl})$ for $l = 1, \ldots, K$, of the ith transition matrix at time t are assumed as:

$$\alpha_i^{kl} \sim \mathcal{N}_{G_\nu+1}(\psi, \underline{\Upsilon}_i) \quad i = 1, ..., N, k = 1, ..., K-1 \tag{3.15}$$

$$\psi \sim \mathcal{N}_{G_\nu+1}(\underline{\psi}, \underline{\Upsilon}) \tag{3.16}$$

At this stage, we choose some priors for the PMS-VAR Model following Billio et al. (2016) by setting $\underline{\lambda}_0 = \mathbf{0}$, $\underline{\Sigma}_{i0} = I_{MM_0}$, $\underline{\Sigma}_0 = 10I_{MM_0}$, $\underline{\Sigma}_{ik} = I_{MM_K}$, $\underline{\lambda}_k = \mathbf{0}$, $\underline{\Sigma}_k = 10I_{MM_K}$ for $k = 1, \ldots, K$, $\underline{\nu}_k = 5$, $\underline{\nu}_{ik} = 5$, $\underline{\Upsilon}_k = 10I_M$,$\underline{\psi} = \mathbf{0}$, $\underline{\Upsilon}_i = I_{G_\nu+1}$, and $\underline{\Upsilon} = 10I_{G_\nu+1}$.

From Fruhwirth-Schnatter (2006), the Bayes factor is a fundamental tool for examining competing models, which considers a ratio of predictive likelihood for one model over all possible models, in order to be used for model selection. In this case,

$$BF = \frac{p(\mathbf{y}|K=3)}{p(\mathbf{y}|K=2)} \tag{3.17}$$

where $p(\mathbf{y}|K=3) = K! \prod_{t=1}^{T-1} \prod_{i=1}^{N} p(\mathbf{y}_{it+1}|\mathbf{y}_{it}, K=3)$ with $p(\mathbf{y}_{it+1}|\mathbf{y}_{it}, K=3)$ is one-step ahead predictive density for $\mathbf{y}_{it+1}$ conditional on information up to time t and $K = 3$ regimes. If BF is greater than 1, then the PMS-VAR model with three regimes is relatively more plausible than the PMS-VAR model with two regimes. In addition, this study imposes the following restrictions on the intercept of the GDP growth as $a_{i1,1} < 0$ and $a_{i1,1} < a_{i1,2} < a_{i1,3}$, $i = 1, \ldots, N$ in order to identify the regimes. The study labels regime 1 as recession; regime 2 as characterised by slow and recovery growth, and regime 3 as an expansion phase. The section on data and model specification provides insight into how the model can be identified through the Bayes factor.

In order to summarize the Gibbs sampler algorithm, let $\mathbf{y}_{it} = \text{vec}((\mathbf{y}_{i1}, \ldots, \mathbf{y}_{iT}))$, $\mathbf{y} = \text{vec}((\mathbf{y}_1, \ldots, \mathbf{y}_N)')$ and $\xi = \text{vec}((\Xi_1, \ldots, \Xi_N))$ with $\Xi_i = (\xi_{i1}, \ldots, \xi_{iT})$. In addition, we define $\gamma = \text{vec}((\gamma_1, \ldots, \gamma_N))$ where $\gamma_i = \text{vec}((\gamma_{i0}, \gamma_{i1}, \ldots, \gamma_{iK}))$ and $\Sigma = (\Sigma_1, \ldots, \Sigma_N)$ with the transition probability parameter vector, $\alpha = \text{vec}((\alpha_1, \ldots, \alpha_N))$.

To conclude the Gibbs sampler, the following steps are:

1. Draw a new regime-independent parameter,γ_{i0}, from the corresponding Normal posterior distribution.

2. Draw new regime-dependent parameters,γ_{ik}, from the corresponding Normal posterior distribution.

3. Draw a new regime-dependent inverse variance-covariance matrix, Σ_{ik}^{-1}, from the corresponding Wishart posterior distribution.

4. Draw new parameters , α_i, in the l-th row of the transition matrix from the corresponding Normal posterior distribution where $\alpha_i^l = \text{vec}((\alpha_i^{1l}, \ldots, \alpha_i^{K-1l}))$.

5. Draw a new parameter of the third stage of the hierarchical structure, λ_0, from the corresponding Normal posterior distribution.

6. Draw new parameters of the third stage of the hierarchical structure, λ_k, from the corresponding Normal posterior distribution.

7. Draw new parameters, Υ_k^{-1}, from the corresponding Wishart posterior distribution.

In order to sample the hidden states, the study employs the forward-filtering backward sampling algorithm, a multi-move Gibbs sampler developed by Carter and Kohn (1994), Sheppard (1994), and Krolzig (1997), also known as the FFBS algorithm (see, e.g., Fruhwirth-Schnatter, 2006). This algorithm is specifically applied to unit-specific chains within the panel, with conditioning on the sampled values of other chains, as in Eq. 3.7. Additionally, factorisation is applied to simulate the full conditional distribution of the hidden states, meaning that this factorisation between a forward-filtering and a backward-sampling or the FFBS algorithm can be used for the hidden states of the i-th bivariate chain s_{it} of the panel. When employing data-dependent priors for hidden states, the generation of allocation variables should exclude draws that result in the impropriety of the posterior. In this particular prior setting, the set of non-problematic groupings for the i-th unit is represented as $\mathcal{S}_i = \mathcal{S}_{i,\nu} \cap \mathcal{S}_{i,\sigma} = \mathcal{S}_{i,\sigma}$. Specifically, if the set of allocation variables $\xi_{i1:T}$ fails to assign at least two observations to each component of the dynamic mixture, the entire set $\xi_{i1:T}$ is rejected, and a new set is drawn until an appropriate set is obtained. For additional information, refer to chapter 11 of Fruhwirth-Schnatter's (2006) work, where one can find a detailed description of the steps in the FFBS algorithm.

3.4 Data and Model Specification

This study would ideally analyse the US and the ASEAN+3, which consists of the 10 members of the Association of Southeast Asian Nations (ASEAN) plus the three major East Asian economies (China, South Korea and Japan). However, due to the lack of availability of quarterly data, only the five founding members of ASEAN - Indonesia, Malaysia, the Philippines, Singapore and Thailand - will be used. These five countries together initiated a multilateral negotiating forum for strengthening economic integration among themselves. For each country, quarterly data is used and four dependent variables are considered: GDP growth, term spread, changes in stock price index and changes in exchange rate. In addition, the oil price index is included as an exogenous variable. For more details, see Appendix B.1-B.2 of the data description as well as the data sources and transformations.

The selection of these four endogenous variables is primarily based on macroeconomic indicators representing real economic activity: GDP growth, the foreign currency market represented by foreign exchange rates and the interaction between real and financial sectors indicated by both term spreads and stock price indices. Despite the existence of numerous other economic variables illustrating interdependencies between countries, these specific variables are perfectly sufficient for the requirements of this study; they are chosen due to their simultaneous determination, making them suitable for investigating business cycle synchronisation using the PMS-VAR model (Billio et al., 2016). However, the potential for including additional economic variables remains open for future studies, providing an opportunity to illustrate further linkage channels in business cycle synchronisation. Nevertheless, it's crucial to consider that such additions could lead to overparameterisation, necessitating careful consideration for modelling specification.

The crucial assumption of the PMS-VAR model is that there are interconnection mechanisms between the economies of these countries. This study specifies the set of common endogenous covariates V_t as the η_{1t} indicator that is a weighted average of the number of ASEAN+3 countries and the US in the recession regime (regime 1) at time $t-1$; $\mathbb{I}(s_{it-1} = 1)$ and 0 otherwise. This means that when $k = 1$, η_{1t} represents the proportion of countries that are in the recession phase with equal weights ω_{it}. In equation 3.8, this mechanism is used to specify the interconnectedness and impose the recession regime by including two quarters of negative growth.

In addition, a number of regimes should be considered using the Bayes factor, as suggested by Fruhwirth-Schnatter (2006). In this study, there will be two possible outcomes identified through the use of three possible regimes; $K = 2$ for all countries (expansion and recession) or $K = 3$ for all counties (expansion, recovery/slow growth and recession). A number of autoregressive lags are also considered, varying from 1 to 4, to examine the Bayes factor[2]. From the result of the Bayes factor, $K = 3$ regimes with 2 lags is used in this study, with regime 1 labelled as recession periods, regime 2 labelled as slow/recovery growth periods and regime 3 labelled as expansion periods.

[2]In the study, the Bayes factor is higher than 1 and is at its highest when $p = 2$.

3.5 Empirical Results

In this section, empirical results from the PMS-VAR model as described in Section 3.3 and 3.4 are presented. In this regard, there are four main macroeconomic variables for each country - GDP growth, term spread, stock price growth and exchange rate growth, the data sources and transformations of which are summarised in Appendix B.2.

3.5.1 Regime Identification on Posterior Density of Intercepts and Volatilities

The results focus on two important parameters - intercepts and volatilities, which depend on the time-varying regime-dependent parameters (s_{it}). To analyse the results from the PMS-VAR model, a graphical analysis is very useful for investigating characteristics of business cycles through both parameters for each country. In addition, the parameters are drawn by using the posterior densities. According to Figures 3.1-3.8, the intercepts and the volatilities are represented by $a_{i1,k}$ and $\sigma_{i1,k}$ (the square root of the diagonal elements of the variance-covariance), respectively with country $i = 1, \ldots, 9$ and regime $k = 1, \ldots, 3$. Additionally, this study compares the approximate posterior densities of the intercepts and volatilities so as to examine whether and how individual countries differ during regimes of recession, slow growth and expansion.

At the beginning of the results, Figure 3.1 shows the posterior densities for the intercepts of GDP growth in each country. There are three interesting findings with regard to this figure. Firstly, it can be noted that during a recession regime ($a_{i1,1}$) the intercepts of most countries are in a negative range of the posterior densities and not concentrated on the zero bound, except China. This implies that the results of the PMS-VAR model can be well-defined according to the correspondences between the negative range of intercepts and empirical facts. In more detail, during the recession periods, while the GDP growth of ASEAN countries as well as Japan, South Korea and the US are all negative, China's GDP growth is concentrated around the zero bound, likely due to the effect of rising globalisation of China. Secondly, during the recession periods, it is noticeable that the posteriors of the intercepts can be divided into two groups. The first group consists of five countries; Indonesia, Malaysia, the Philippines, Thailand and South Korea, whose posteriors of the intercepts are less than negative three percent - clearly seen in Figure 3.1. Also noticeable in Figure 3.1 is that Japan, Singapore and the US have markedly higher posteriors, much closer to negative three percent or even higher, indicating that these three economies have a smaller impact on the recession period. These three economies make up the second group. A particularly arresting result is that Thailand and South Korea are most likely to be in the worst economic situation in these recessionary periods. Last but not least, when comparing the posteriors of slow growth and expansion regimes ($a_{i1,2}$ and $a_{i1,3}$) between ASEAN+3 and the US, the posteriors of ASEAN+3 are likely to be coincident but slightly higher than the US. In addition, China has the highest GDP growth among ASEAN+3. Overall, it can be concluded that the posteriors of the GDP growth's intercepts of the members of ASEAN+3 (except China) have heavily concentrated on a negative range and these figures are highly coincident during recession periods.

Additionally, the study takes into account the variations of GDP growth of each country in different regimes. The results of the posterior densities of the GDP volatilities are shown in Figure 3.2. It can be seen that the countries can be categorised into three groups with regard to the characteristics of volatility during the recession regime periods - 1) high volatility including Indonesia, Malaysia and the US (the right tail of their red lines are flat), 2) low volatility including Thailand, Singapore and China and 3) undefined pattern including the Philippines, South Korea and Japan. Regarding periods of slow growth and expansion, the volatility behaviours of ASEAN+3, except Malaysia are similar. Furthermore, focusing on the expansion periods, it seems that the volatility of the US is substantially higher than the rest.

For the term spread's intercepts, the posterior densities are shown in Figure 3.3. It can be clearly seen that the posteriors of all countries are similar; they are concentrated around the zero bound. However, it can also be seen that the posteriors of China during slow growth and expansion periods (green and red lines) are slightly negative. This implies that the long-term interest rate in China is slightly less than the short-term interest rate during both periods. This finding could be of interest to researchers wishing to engage in further study, as normally the long-term interest rate is supposed to be higher than the short-term interest rate, due to high risk of holding a long-term asset.

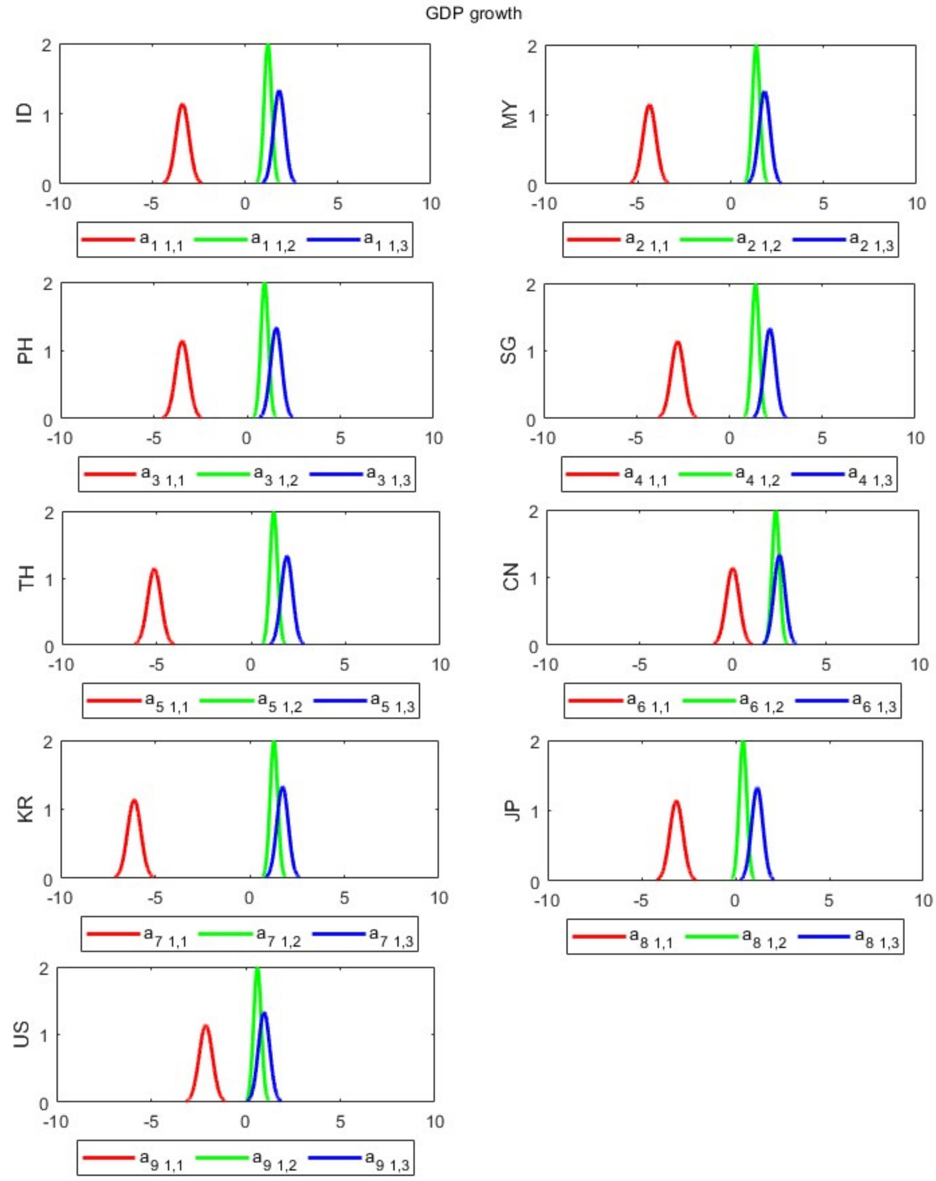
In terms of the volatilities, Figure 3.4 shows the posterior densities of the term spread's volatility in different regimes. The term spread's volatilities of all countries have similar patterns, particularly during the recession periods. Moreover, the posterior densities of the volatility in expansion periods are quite flat, meaning that the term spreads are substantially volatile for most countries except Indonesia. In addition, Singapore and the US have the highest volatility of the term spread in the expansion regime.

Regarding stock price growth, the posterior densities of the stock price growth's intercepts can be shown in Figure 3.5. This figure indicates that the intercepts of ASEAN+3 and the US (except Indonesia and China) are remarkably similar. To be more specific, when recessionary periods occur, the stock price growths of almost all these countries are negatively affected by the recession shock. The ones that experience the negative effect from the shock, ranked from largest to smallest, are Thailand, the US, South Korea, Japan, Malaysia, Singapore and the Philippines. In contrast, surprisingly, the stock price growth's intercept of Indonesia seems to be non-negative and China's intercept is around the zero bound during recession periods. For slow growth and expansion periods, the intercepts of ASEAN+3 and the US are substantially similar. In Figure 3.6, the posterior densities of the stock price growth's volatilities of ASEAN+3 (except China) are quite similar, meanwhile volatilities of China and the US fluctuate greatly during the recession regime.

Turning to the intercepts of exchange rate growth, Figure 3.7 shows the posterior densities of the intercepts in three different regimes. There are three types of changes in the value of exchange rate, which are depreciation, appreciation and changeless exchange rate; these are related to three patterns of the posterior densities: changes in a positive bound, changes in a negative bound and around a zero bound, respectively. During the recession periods, there are five countries whose money value depreciates (their intercepts change within a positive bound), namely Indonesia, Malaysia, Thailand, South Korea, and the US. Conversely, only Japan experiences an exchange rate appreciation. In addition, for the Philippines, Singapore and China, their value of money is likely to change less or their posterior densities are around the zero bound. Regarding slow growth and expansion periods, the posteriors of all countries are similar at around the zero bound.

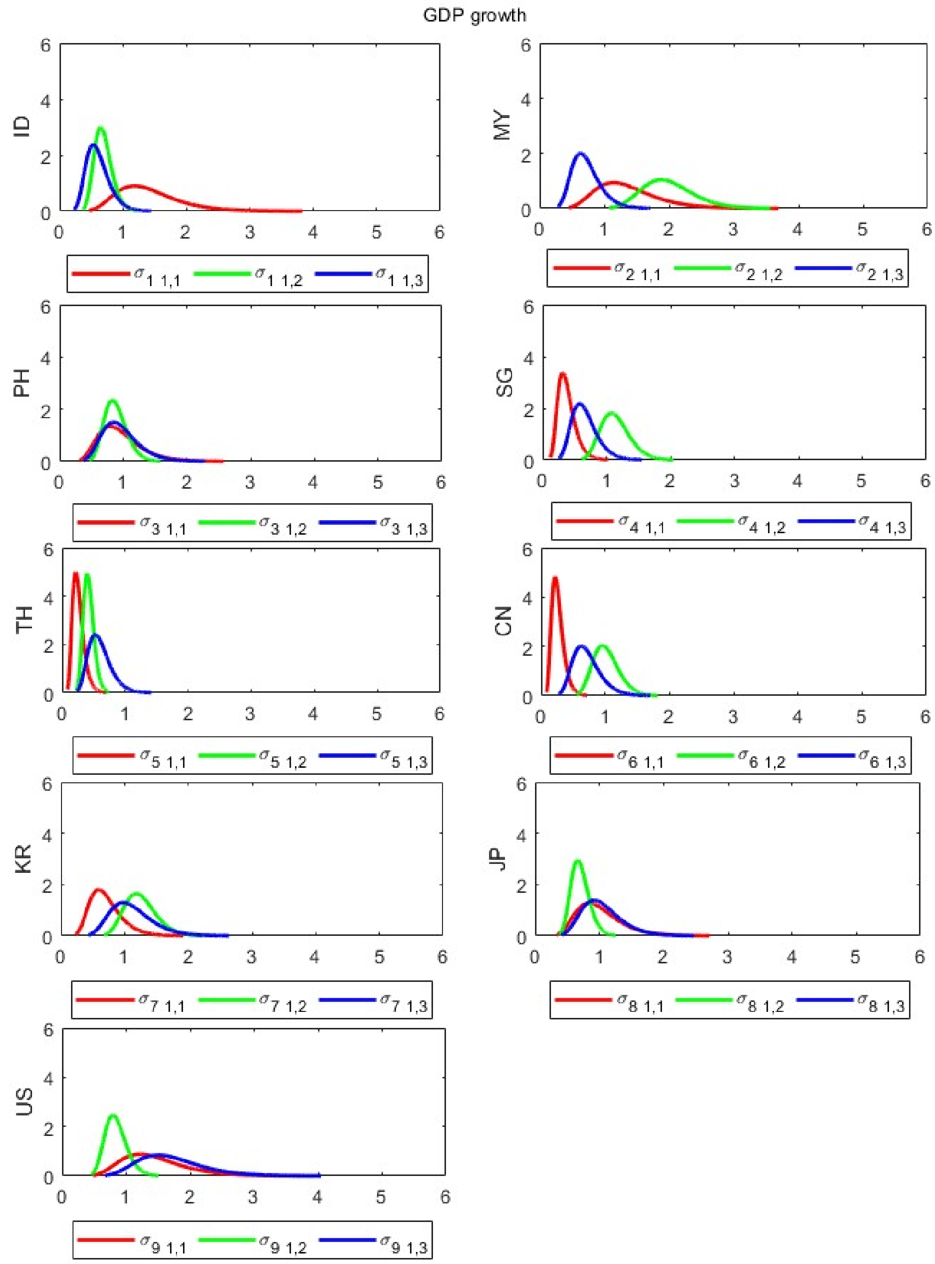
Figure 3.8 indicates the posterior densities of the exchange rate volatilities. It can be quite clearly seen that during recession periods Malaysia, Singapore, China, Japan and the US are substantially volatile whilst other countries seem to be at around the zero bound. Additionally, during slow growth and expansion periods all countries are more likely to display the same pattern, which is at around the zero bound.

Figure 3.1: The kernel density estimate of the posterior density of GDP growth's intercepts



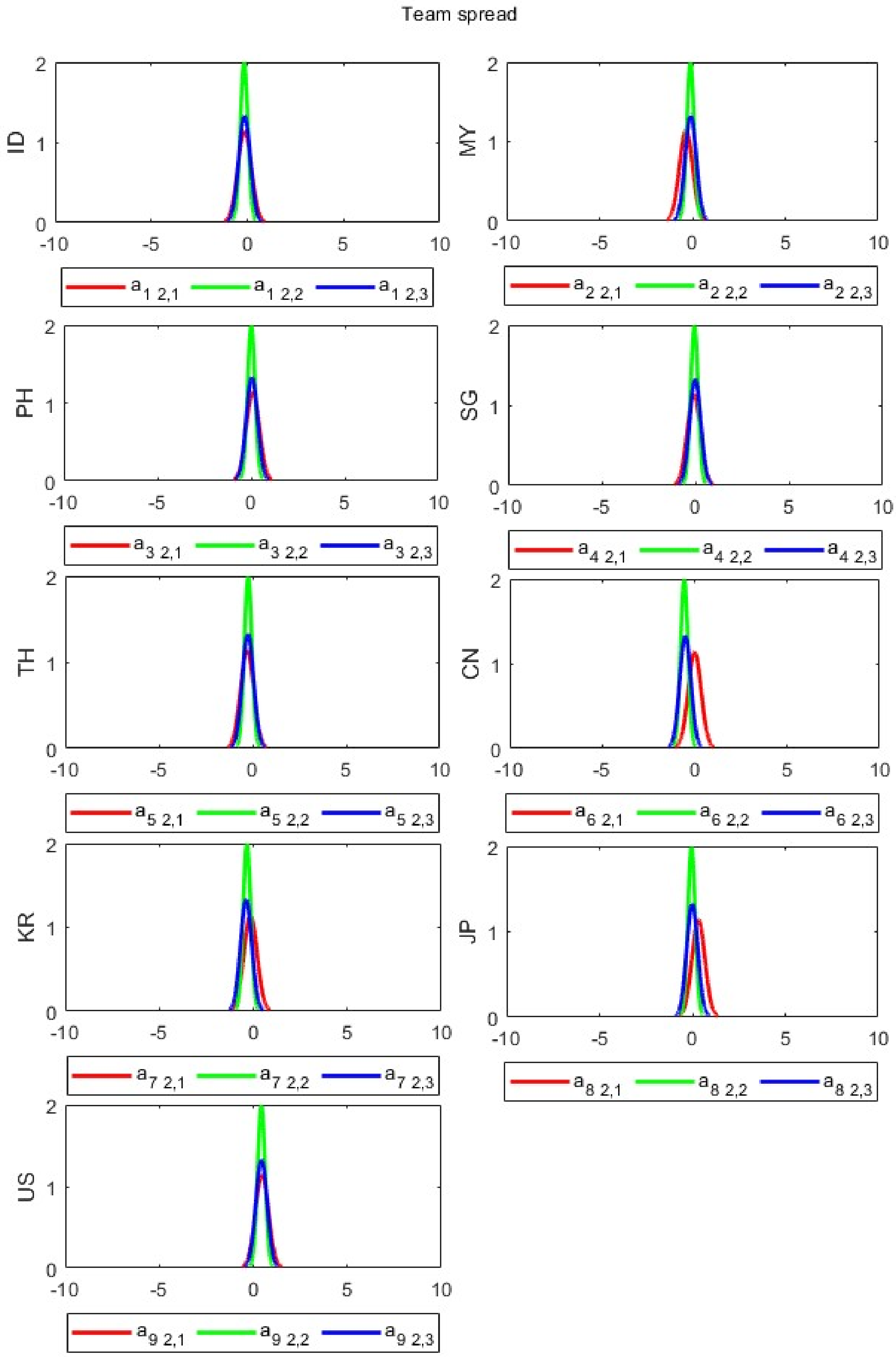
Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United States, respectively. The intercepts and volatilities of the posterior densities labeled as $a_{i1,k}$ and $\sigma_{i1,k}$, respectively with country $i = 1, \ldots, 9$ and regime $k = 1, \ldots, 3$. The first regime (recession) is in red, the second (slow growth) regime is in green and the third regime (expansion) is in blue.

Figure 3.2: The kernel density estimate of the posterior density of GDP growth's volatilities



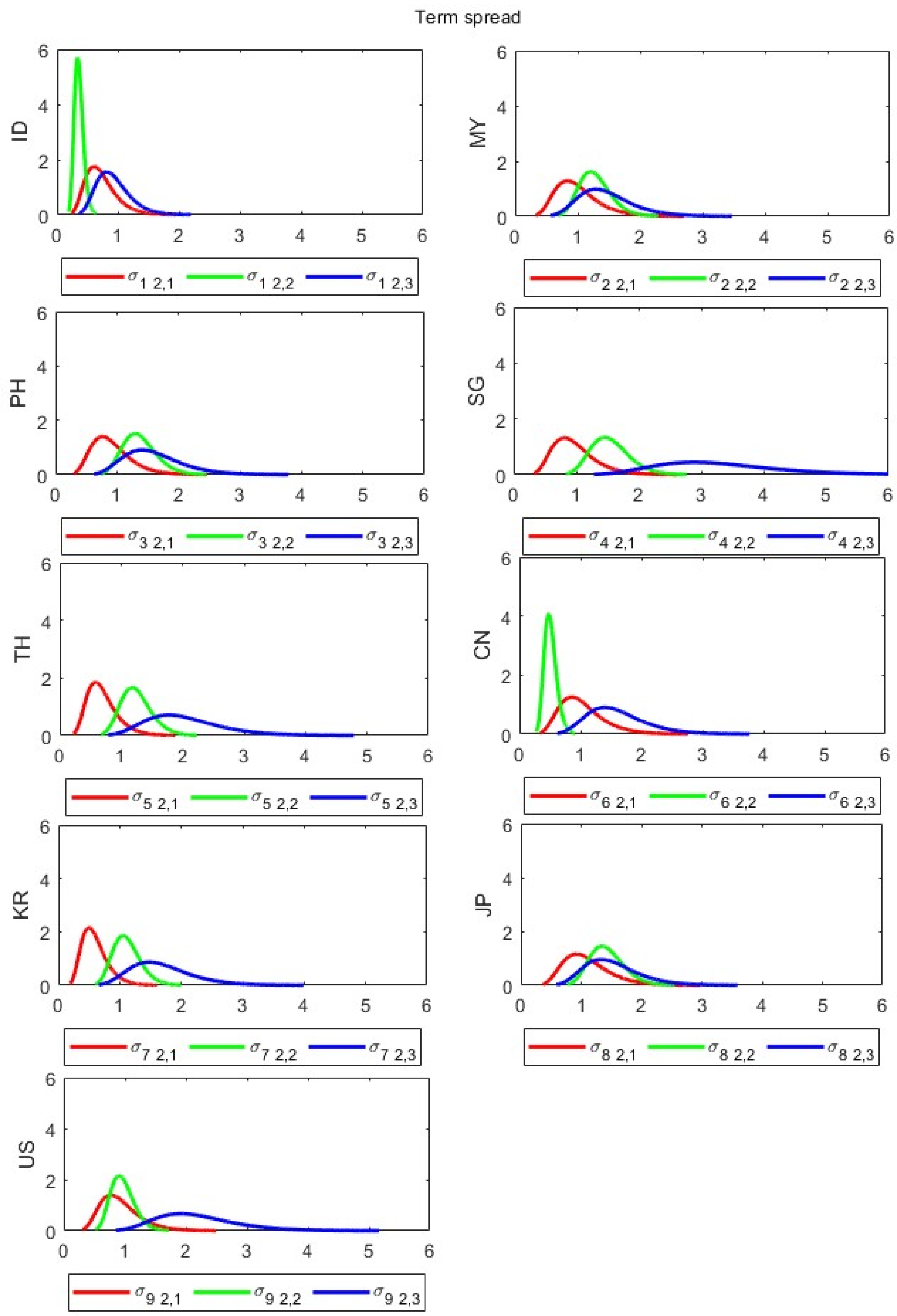
Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United States, respectively. The intercepts and volatilities of the posterior densities labeled as $a_{i1,k}$ *and* $\sigma_{i1,k}$*, respectively with country* $i = 1, \ldots, 9$ *and regime* $k = 1, \ldots, 3$*. The first regime (recession) is in red, the second (slow growth) regime is in green and the third regime (expansion) is in blue.*

Figure 3.3: The kernel density estimate of the posterior density of term spread's intercepts



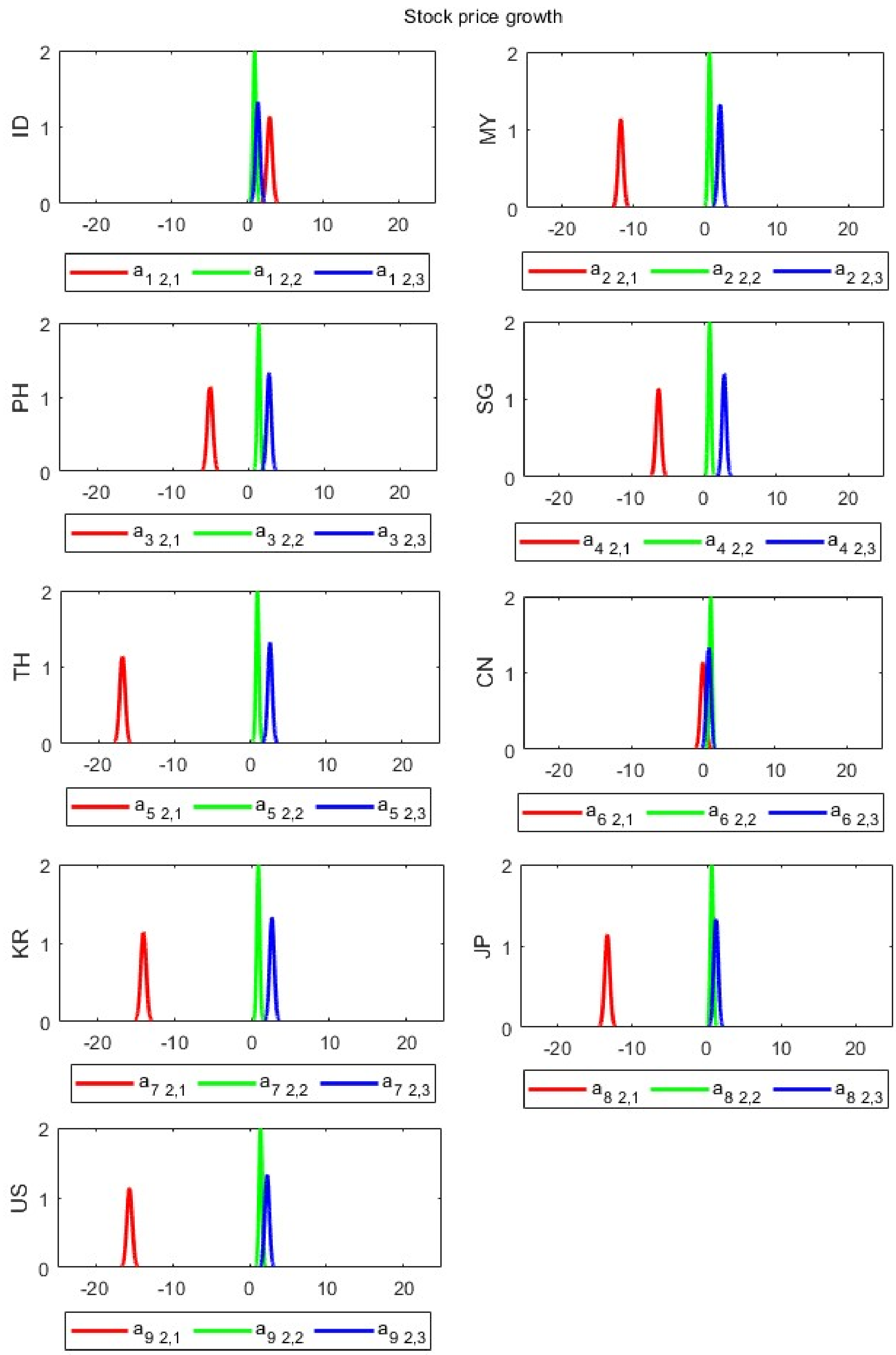
Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United States, respectively. The intercepts and volatilities of the posterior densities labeled as $a_{i1,k}$ and $\sigma_{i1,k}$, respectively with country $i = 1, \ldots, 9$ and regime $k = 1, \ldots, 3$. The first regime (recession) is in red, the second (slow growth) regime is in green and the third regime (expansion) is in blue.

Figure 3.4: The kernel density estimate of the posterior density of term spread's volatilities



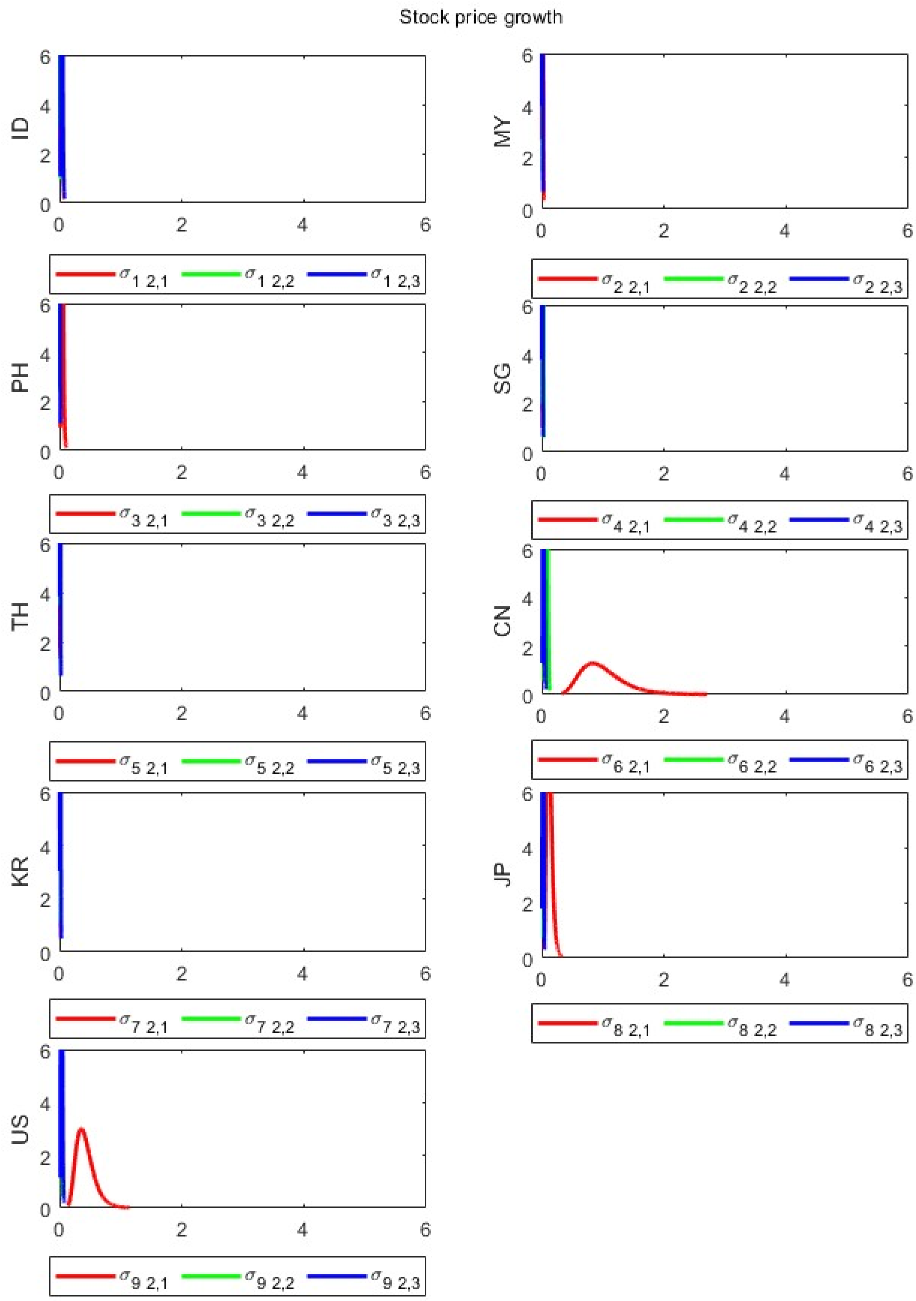
Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United States, respectively. The intercepts and volatilities of the posterior densities labeled as $a_{i1,k}$ and $\sigma_{i1,k}$, respectively with country $i = 1, \ldots, 9$ and regime $k = 1, \ldots, 3$. The first regime (recession) is in red, the second (slow growth) regime is in green and the third regime (expansion) is in blue.

Figure 3.5: The kernel density estimate of the posterior density of the stock price growth's intercepts



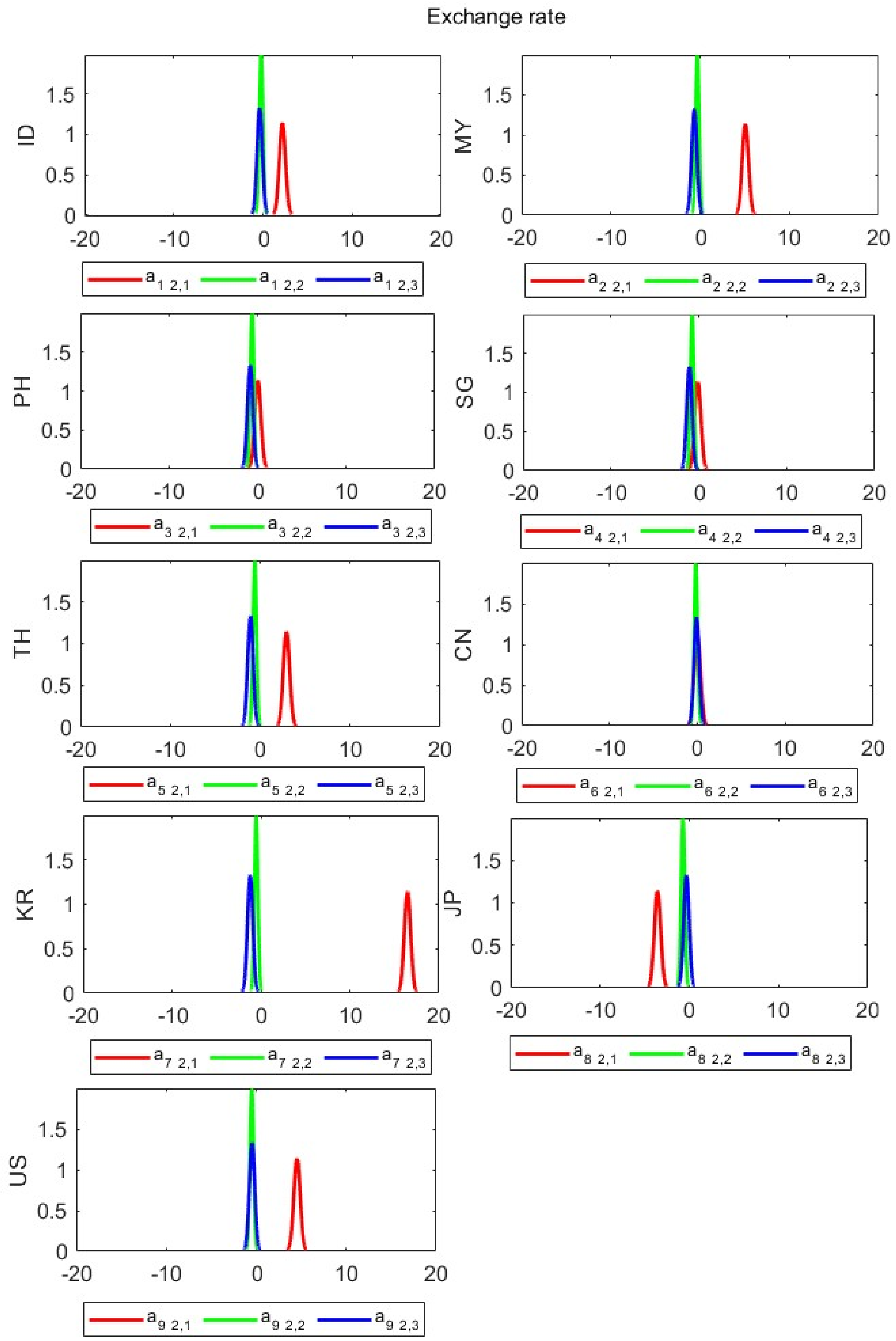
Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United States, respectively. The intercepts and volatilities of the posterior densities labeled as $a_{i1,k}$ *and* $\sigma_{i1,k}$*, respectively with country* $i = 1, \ldots, 9$ *and regime* $k = 1, \ldots, 3$*. The first regime (recession) is in red, the second (slow growth) regime is in green and the third regime (expansion) is in blue.*

Figure 3.6: The kernel density estimate of the posterior density of the stock price growth's volatilities



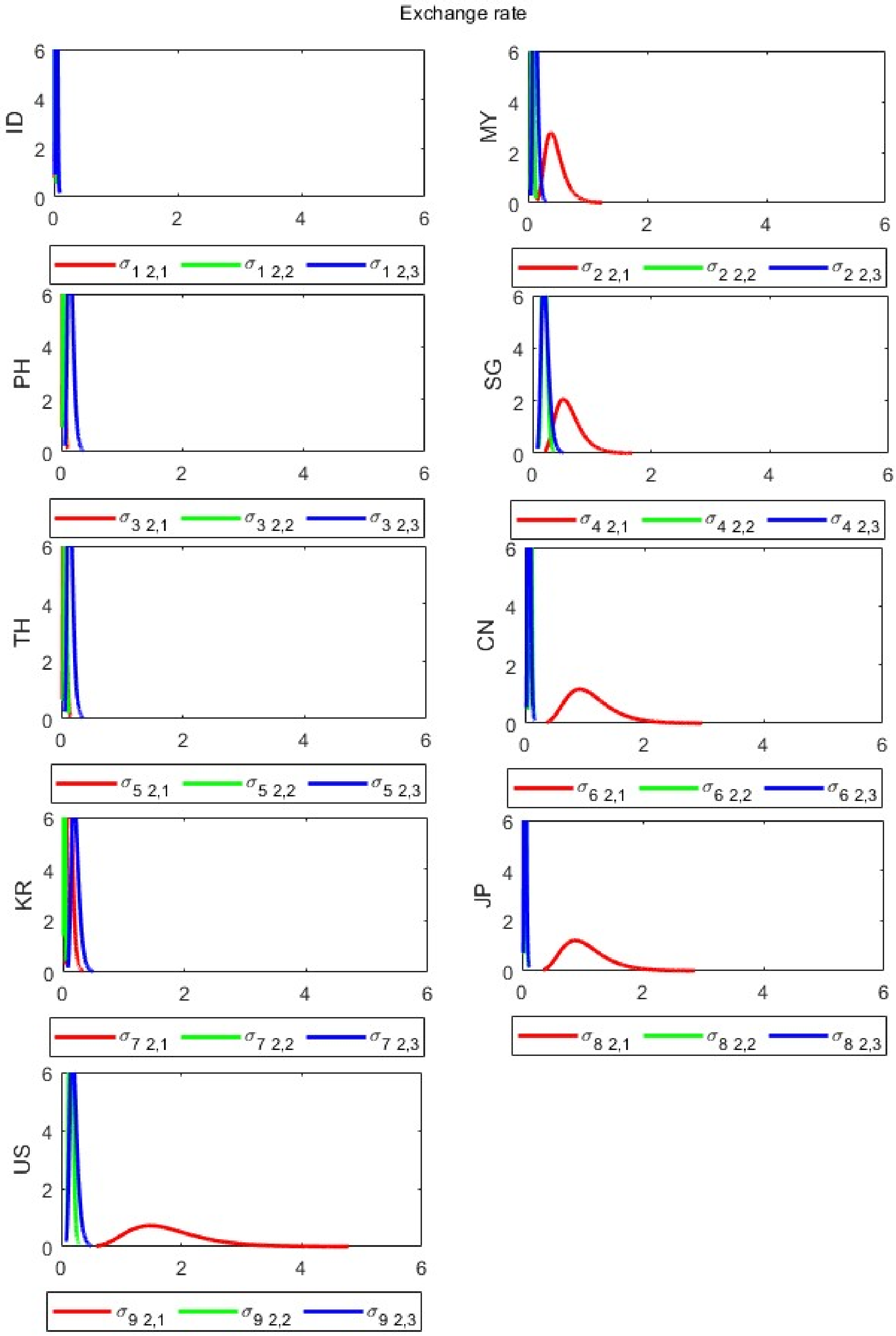
Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United States, respectively. The intercepts and volatilities of the posterior densities labeled as $a_{i1,k}$ and $\sigma_{i1,k}$, respectively with country $i = 1, \ldots, 9$ and regime $k = 1, \ldots, 3$. The first regime (recession) is in red, the second (slow growth) regime is in green and the third regime (expansion) is in blue.

Figure 3.7: The kernel density estimate of the posterior density of the exchange rate growth‘s intercepts



Note: The labels “ID”, “MY”, “PH”, “SG”, “TH”, “CN”, “KR”, “JP” and “US” stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United States, respectively. The intercepts and volatilities of the posterior densities labeled as $a_{i1,k}$ and $\sigma_{i1,k}$, respectively with country $i = 1, \ldots, 9$ and regime $k = 1, \ldots, 3$. The first regime (recession) is in red, the second (slow growth) regime is in green and the third regime (expansion) is in blue.

Figure 3.8: The kernel density estimate of the posterior density of the exchange rate growth's volatility



Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United States, respectively. The intercepts and volatilities of the posterior densities labeled as $a_{i1,k}$ and $\sigma_{i1,k}$, respectively with country $i = 1, \dots, 9$ and regime $k = 1, \dots, 3$. The first regime (recession) is in red, the second (slow growth) regime is in green and the third regime (expansion) is in blue.

3.5.2 Synchronisation of Business Cycles and Persistence of Regimes

This section looks in detail at the interconnectedness, or lack thereof, between the ASEAN+3 economies and the US economy. As mentioned in section 3.4, the analysis focuses on the macroeconomic variables are collected from secondary sources, such as the IFS database, over the forty-year period of Q1, 1980 - Q4, 2019. For more details on data sources, see Appendix B.2

As the PMS-VAR model generates smoothed probabilities of various different regimes for each economy, it is necessary to produce two different visual representations in order to analyse the results satisfactorily. Therefore, Figures 3.9 and 3.10 are produced and shown in this section. Figure 3.9 shows the smoothed probabilities of the recession regime of the US and the proportion of countries in ASEAN+3 in recession periods standardised between 0 and 1 or V_t[3]. Meanwhile, Figure 3.10 shows the smoothed probabilities of the recession regimes by specific economies. These figures not only provide interesting results of business cycle synchronisation between ASEAN+3 and the US but also indicate the synchronisation of each economy with the US as well as among ASEAN+3 economies themselves during recession periods.

Focusing on the first decade of the sample (from 1980 to 1989) as shown in Figure 3.10, it can clearly be seen that the recession probability of the US economy is quite high during the fourth quarter of 1987. This result corresponds to the

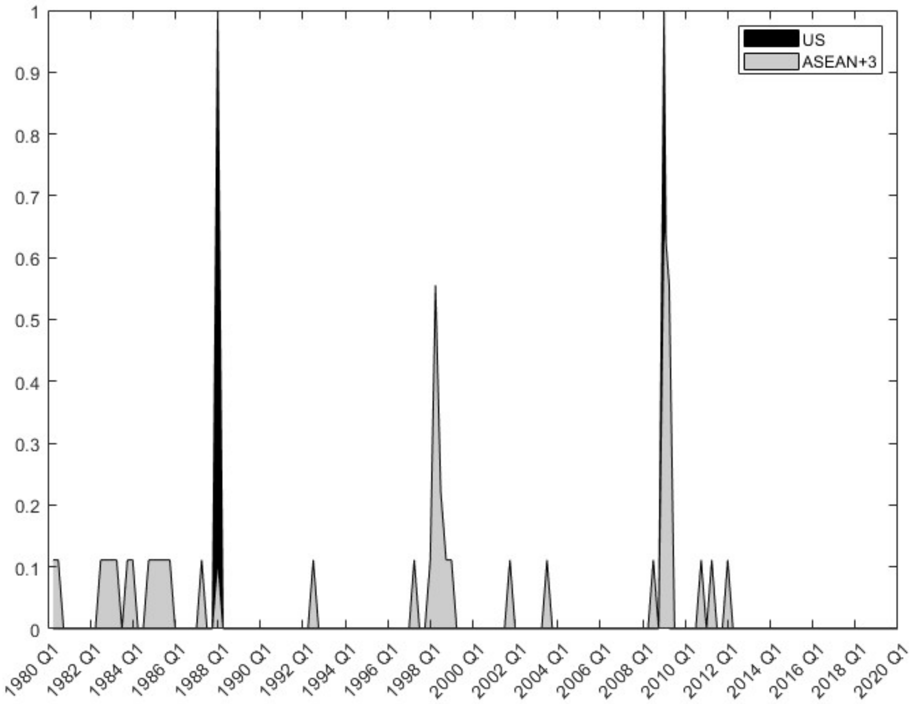
[3] One can refer to Eq. 3.8, which addresses unit-specific weights (ω_{it}) in this study. Here, there is an assumption of assigning equal weight for ASEAN countries to the US. This assumption arises from the absence of an official framework for aggregating ASEAN variables, in contrast to the Eurostat regulation governing the EU. The economic integration level of ASEAN relies on agreements and associations rather than a structured economic union like the EU. Nonetheless, it is reasonable to assume that the US economy has an almost uniform effect on all the ASEAN countries.

time of the unexpected crash of stock markets around the world that occurred in October 1987, namely ‘Black Monday’. Meanwhile, Figure 3.9 clearly shows that ASEAN+3 on the whole (seen as a single economic entity) had a much lower recession probability than the US at this time and in the following few quarters. This means that only some of the ASEAN+3 economies faced economic crisis, whereas others were relatively unaffected. Figure 3.10 indicates that the recession probabilities of the Philippines and Malaysia were quite high in the fourth quarter 1987. This can imply that the Philippine and Malaysian economies were strongly involved with or affected by this collapse of global stock markets. In addition, from 1980 to 1985 the recession probabilities of Malaysia, the Philippines and Thailand were also quite high. This might have been caused by the effect of the oil price crisis in 1979 - the global oil supply disruption caused by the Iranian Revolution. The crisis led to the US and other advanced economies tightening their monetary policies in order to control double-digit inflation. As a result, these measures induced several economies to enter their own recessions during that period (see Kose et al., 2020).

Turning to the second decade of the sample (from 1990 to 1999), it is noticeable from Figure 3.9 that the ASEAN+3 economy was more volatile than the US, especially from 1995 to 1997. Figure 3.10 indicates that there were high recession probabilities for Indonesia, Malaysia, South Korea and Thailand from 1997 - early 1999. This result corresponds with empirical evidence of the Asian financial crisis, which was the result of high volatility of exchange rates as a consequence of monetary policy mismanagement and the burned of foreign debt as well as financial contagion effects in ASEAN+3. Overall, the result can imply a high level of economic synchronisation among ASEAN+3 economies and a lower level of economic synchronisation with the US economy.

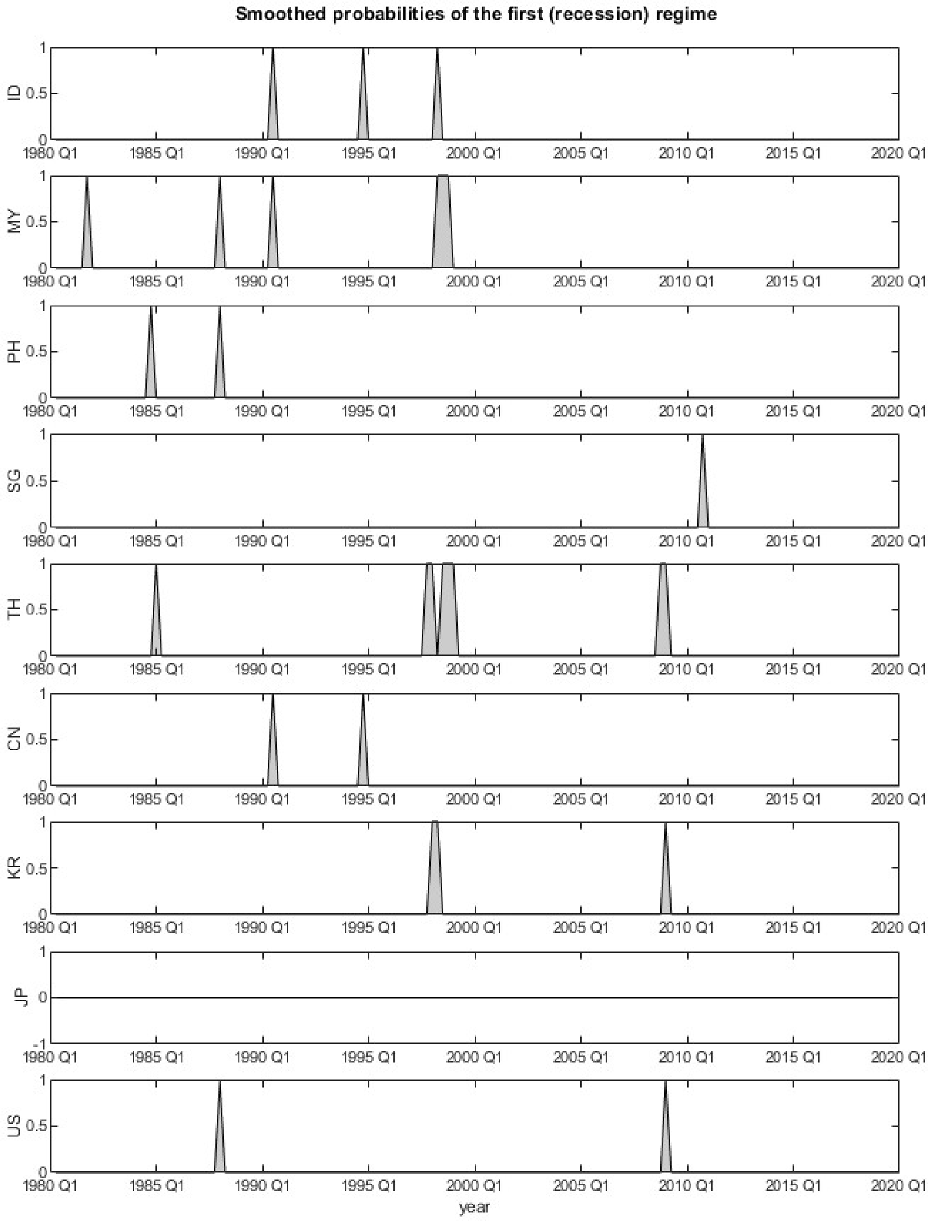
With regard to the third decade of the sample (from 2000 to 2009), Figure 3.9 shows that the recession probability of the US economy was substantially higher than ASEAN+3, especially in 2008. This corresponds directly with evidence of the US subprime crisis, a large decline in US home prices after the collapse of a housing bubble, which led to the global financial crisis of 2008 - 2009. These events significantly contributed to the Great Recession. Regarding Figure 3.10, it is important to note that the recession probabilities of Thailand and South Korea were also quite high during 2008. Equally worthy of note, the recession probability of Singapore was quite high in 2010. This means that the effect of the US recession from the 2008 financial crisis on these countries is less inevitable, particularly Singapore's economy, which fell into recession for the only time in the four decades of this sample period. This might be because of the spillover effects of the global economic slowdown, initially caused by the US subprime crisis. Additionally, it can also imply that the Singaporean, Thai and South Korean economies were highly interconnected with the US economy, especially in terms of synchronisation of international financial markets.

Figure 3.9: The probabilities of the ASEAN+3 and US economies in the recession regime



Note: The black line represents the US smoothed probability of state 1, namely the recession regime, and the light grey shows the fraction of ASEAN+3 countries in the recession periods which are standardised between 0 and 1.

Figure 3.10: Smoothed probabilities of the first (recession) regime

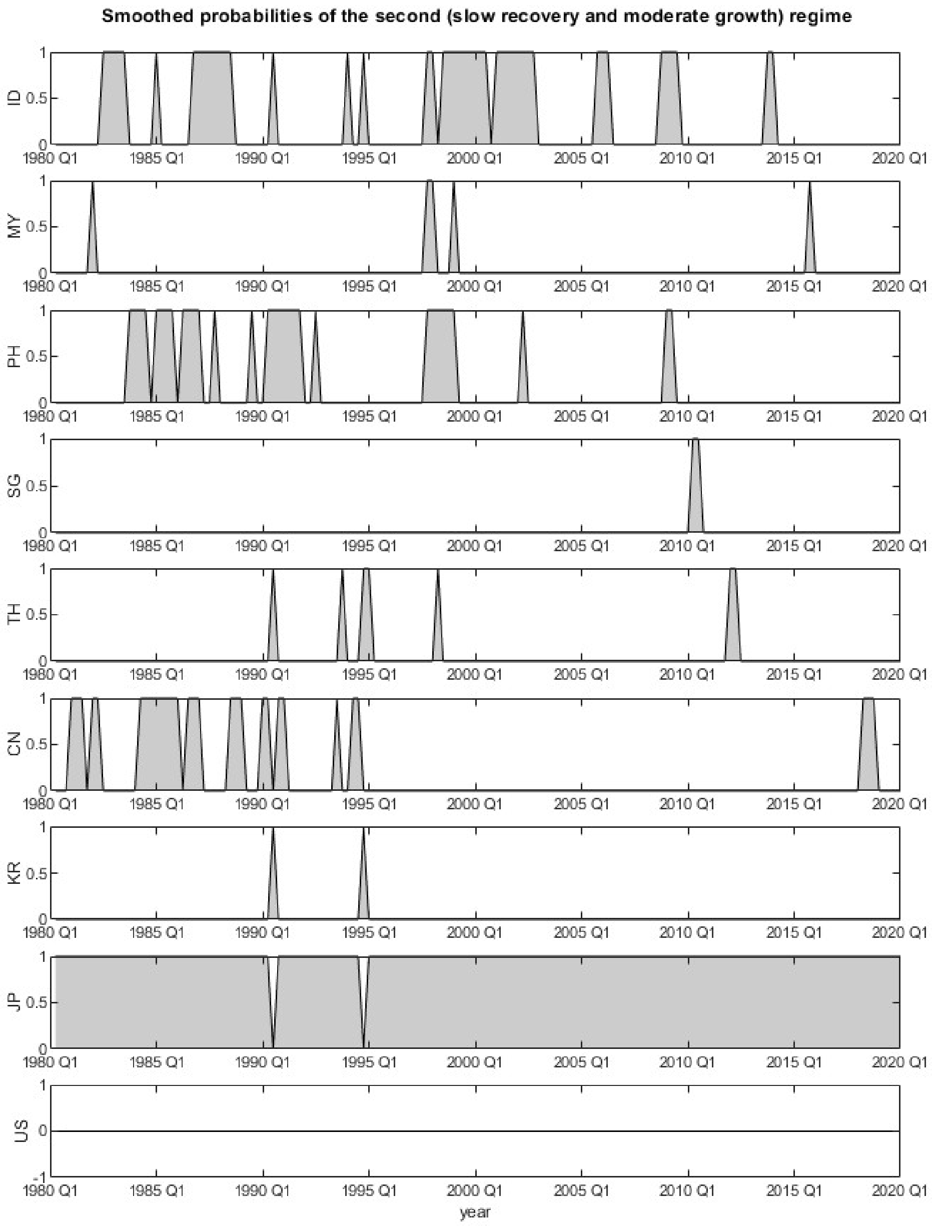


Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United states, respectively.

In order to analyse persistence of regimes, Figures 3.11 and 3.12 show the smoothed probabilities of the second (slow recovery and moderate growth) regime and the smoothed probabilities of the third (expansion) regime, respectively. Upon considering the high smoothed probabilities illustrated across figures 3.10 - 3.12, it can be clearly seen that there is persistence of regime for all the economies. It is very noticeable, however, that Japan's economy, whilst showing persistence of regime like all the other economies studied over this period, was firmly placed in a different regime from the rest - the second regime of slow recovery and moderate expansion. Meanwhile, the US and the rest of the ASEAN+3 economies - Indonesia, Malaysia, the Philippines, Singapore, Thailand, China and South Korea - went through four decades fraught with instability and turbulence - from recessions (Figure 3.10) to expansion (Figure 3.12).

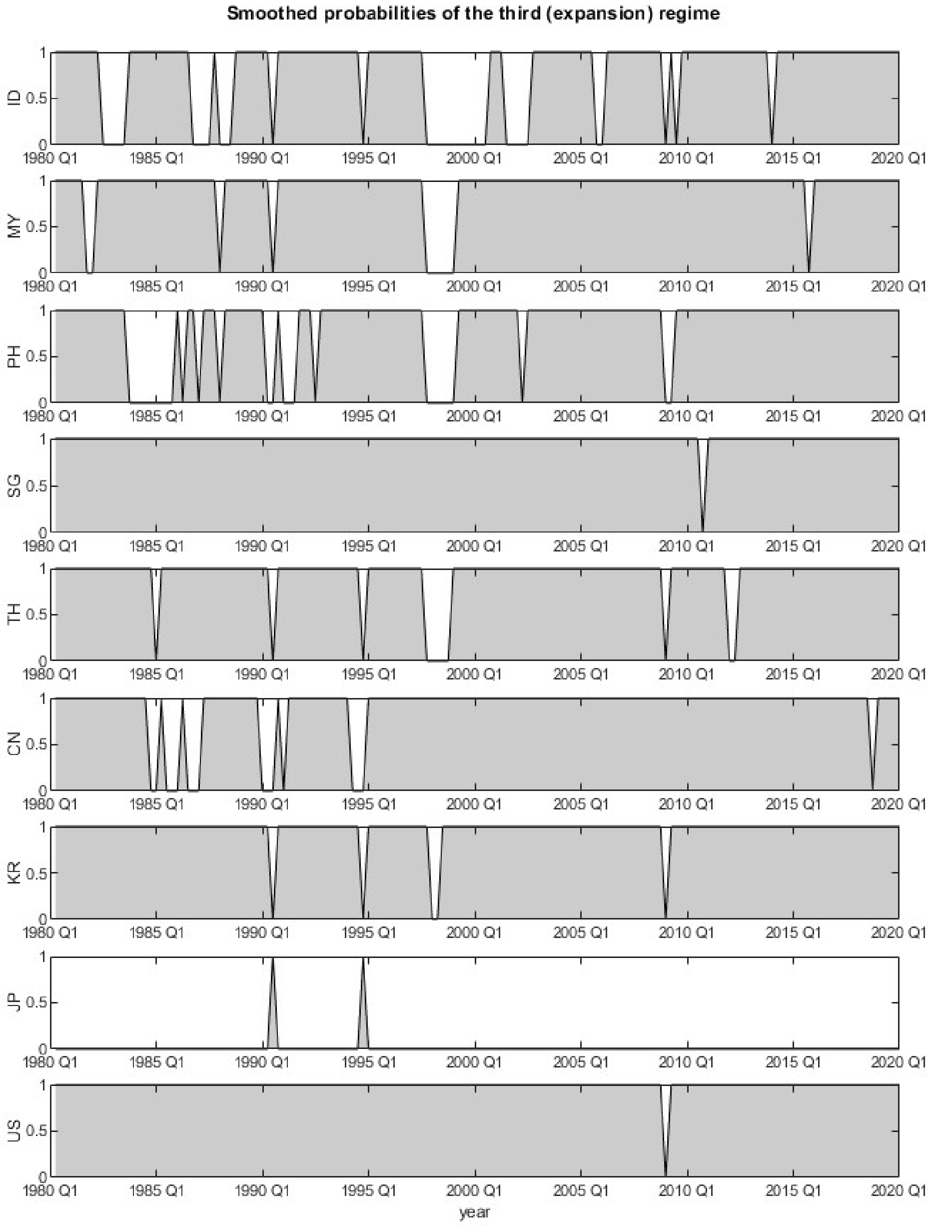
This might be because of the structure of the Japanese economy. Japan's economy experienced an asset bubble and burst from 1980 to 1990. As a result, the Japanese economy has suffered from economic stagnation. Coupled with this, the country's aging population has led to a negative impact on labour force and productivity. Moreover, Japan's economy is greatly dependent on oil imports for energy needs but overall inflation still remains low because of low spending of Japanese households. These might be, therefore, causes of slow and prolonged economic growth and deflation of Japan's economy.

Figure 3.11: Smoothed probabilities of the second (slow recovery and moderate growth) regime



Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United states, respectively.

Figure 3.12: Smoothed probabilities of the third (expansion) regime



Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United states, respectively.

3.5.3 Features of Business Cycles through Posterior Mean Distributions for the Time-Varying Intercept

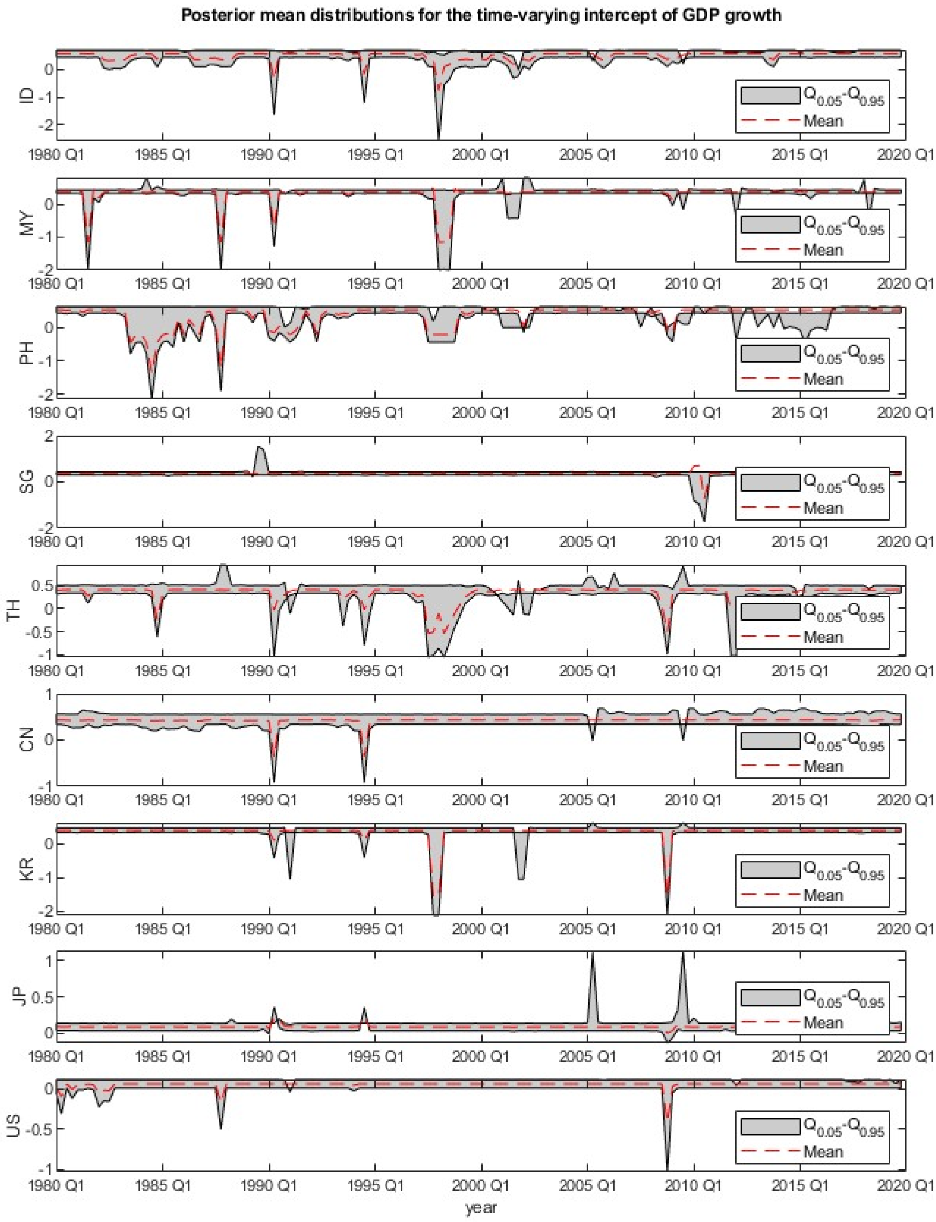
To analyse heterogeneity in business cycle patterns, it is useful to address the posterior mean distributions for the time-varying intercept, which can be computed as follows:

$$\widehat{a_{im}(s_{it})} = \frac{1}{N} \sum_{n=1}^{N} \sum_{k=1}^{K} a_{im,k}^{(n)} \xi_{i,k,t}^{(n)} \tag{3.18}$$

in which N is the number of iterations following MCMC and K represents the number of regimes. In addition, the posterior distributions can be calculated in terms of 5% and 95% quantiles, as shown in Figure 3.13.

Following the above equation, Figure 3.13 shows the result of the posterior distributions of the time-varying intercepts of GDP growth for each country. This means that the PMS-VAR model can also provide the posteriors of time-varying intercepts throughout the sample period by adopting the above equation in order to analyse the heterogeneity in business cycle patterns. In this section, the important results from Figure 3.13 can be summarised as follows. During the first two decades of the sample period (1980 - 1999), most economies in ASEAN+3 were highly volatile compared to the US economy in terms of GDP growth intercept, particularly during the Asian financial crisis of 1997. During this period, the posteriors were less than or equal to -2% in cases of Malaysia, Indonesia and South Korea, while at the same time Thailand was less than or equal to -1%. In addition, during the US subprime shock, all of the ASEAN+3 economies except Japan experienced negative impacts on their GDP growth with the highest impact being on South Korea (the posterior is -2%) and the next being Singapore and Thailand, respectively.

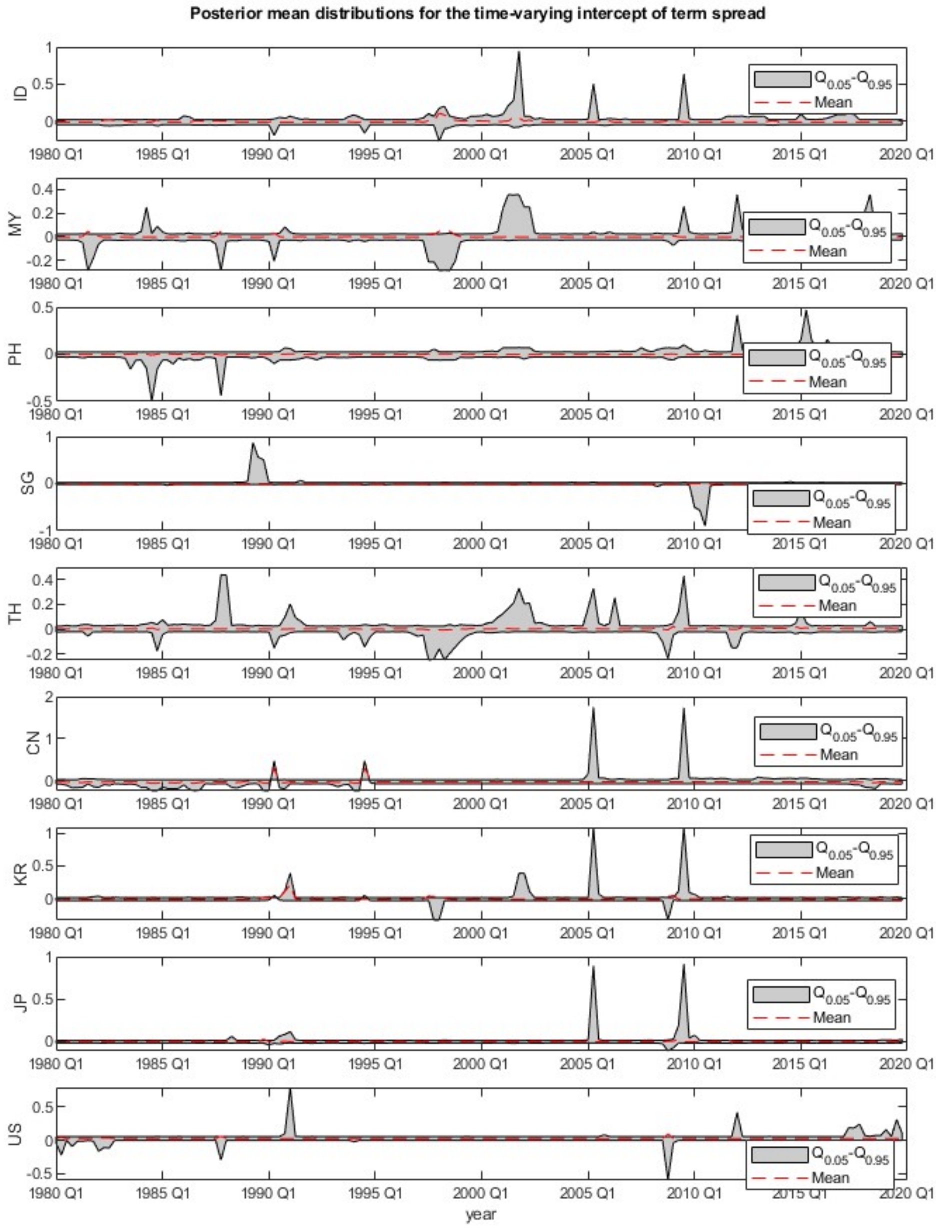
Figure 3.13: Posterior mean distributions for the time-varying intercept of GDP growth



Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United States, respectively.

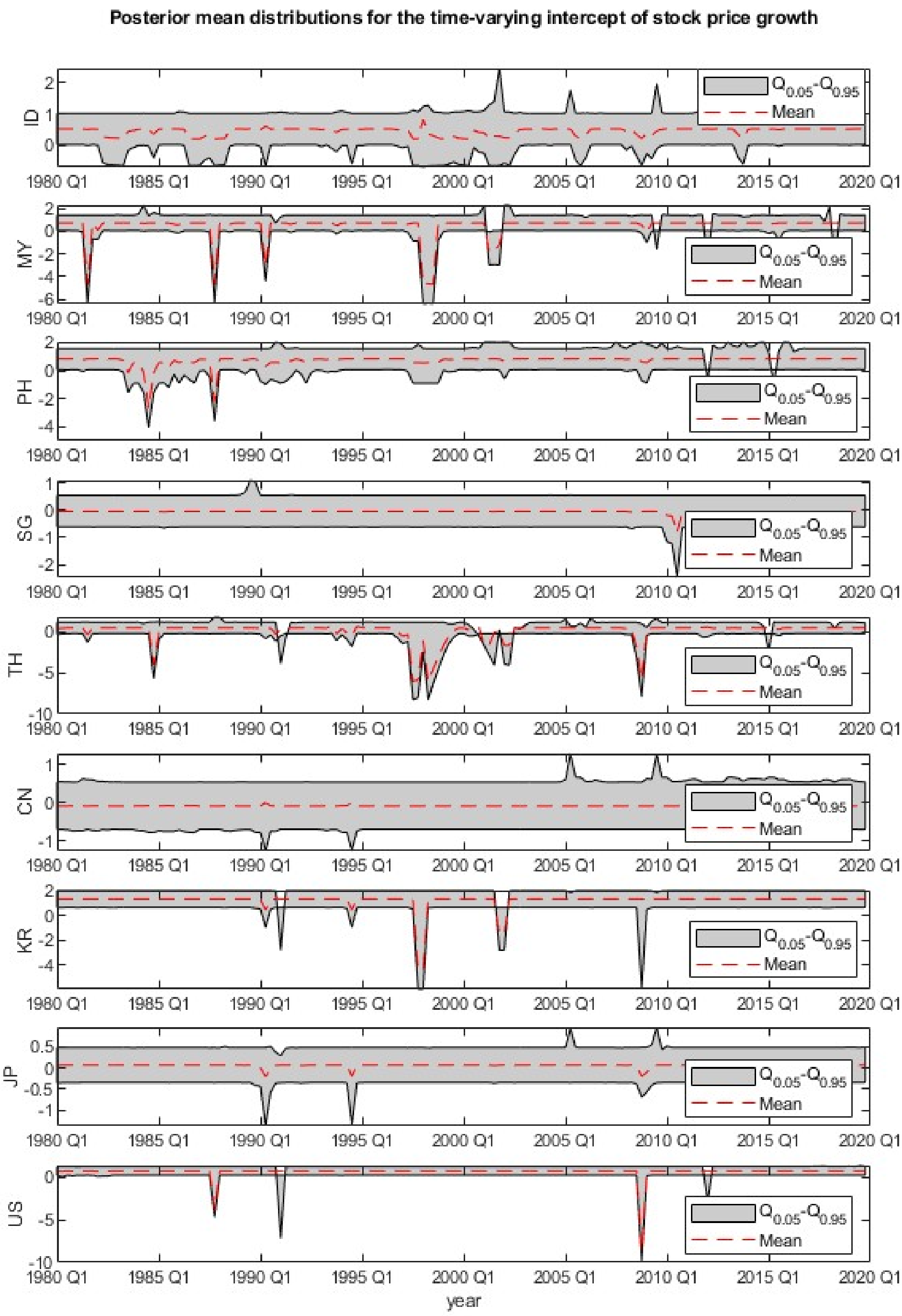
Turning to financial variables, the posteriors of the time-varying intercepts of term spread in Figure 3.14 can indicate that the posterior patterns for Singapore and the US are highly aligned and the posterior patterns for most economies in ASEAN+3 coincide with each other. However, for 1997 to 2002, there is a broad range of posteriors for Indonesia, Malaysia and Thailand, no doubt due to the Asian financial crisis. In terms of stock price growth, Figure 3.15 shows that during the US subprime crisis the posteriors for the US hit -10% while the posteriors for Thailand and South Korea are around minus 6-7%. Interestingly, the posteriors for Singapore, China and Japan have noticeably different patterns from the other economies. Regarding exchange rate growth, Figure 3.16 indicates that after the US subprime crisis in 2008, there is a broad range of posteriors of exchange rate growth intercepts, especially Indonesia, Malaysia, the Philippines, Thailand and China.

Figure 3.14: Posterior mean distributions for the time-varying intercept of term spread



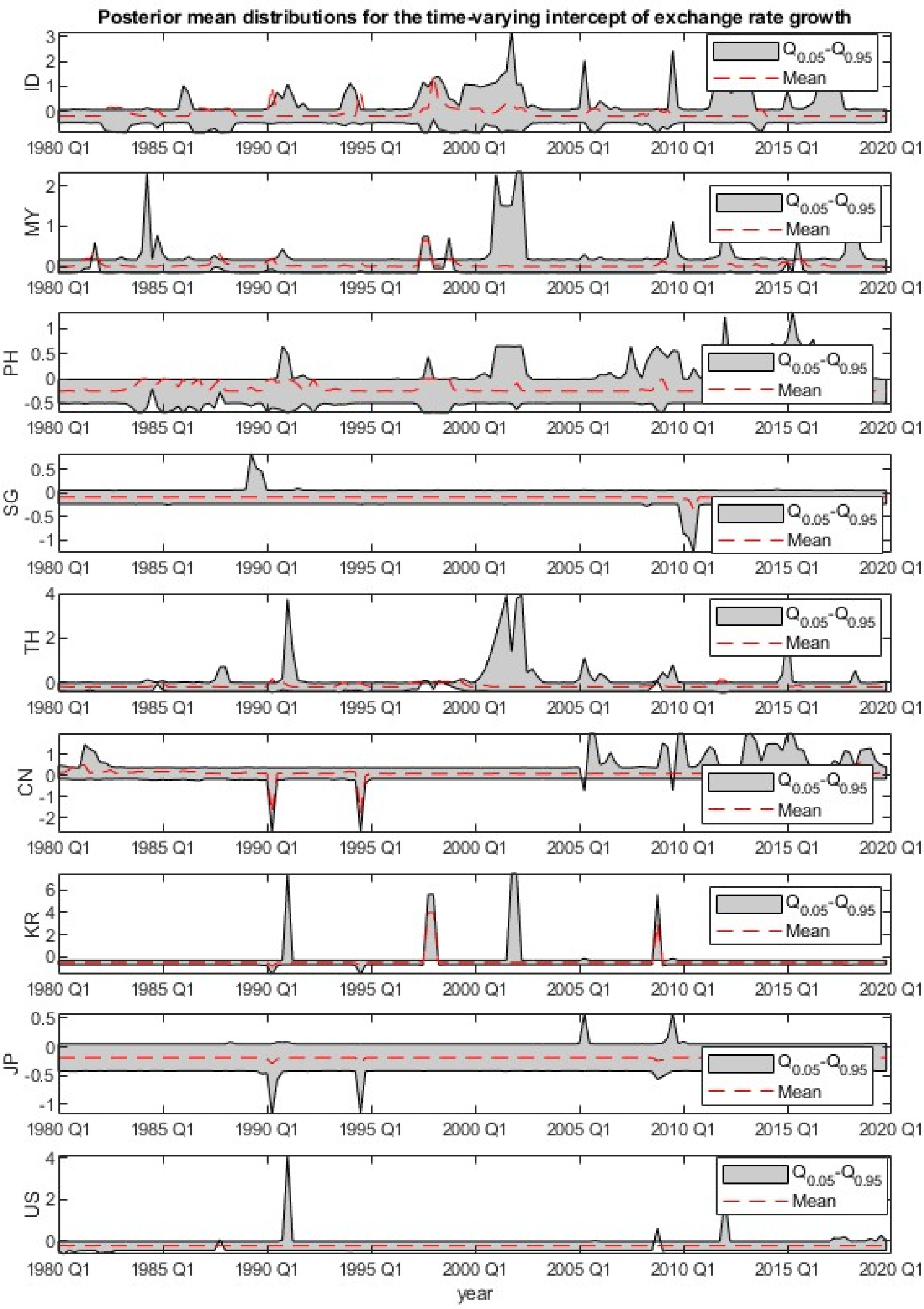
Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United States, respectively.

Figure 3.15: Posterior mean distributions for the time-varying intercept of stock price growth



Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United States, respectively.

Figure 3.16: Posterior mean distributions for the time-varying intercept of exchange rate growth



Note: The labels "ID", "MY", "PH", "SG", "TH", "CN", "KR", "JP" and "US" stand for Indonesia, Malaysia, the Philippines, Singapore, Thailand, China, South Korea, Japan and the United States, respectively.

3.6 Conclusion

This study investigated the synchronisation of business cycles and heterogeneities across the economies of the US and the ASEAN+3 by using a Bayesian Panel Markov-switching (PMS) VAR model. This study exploited the usefulness of this model, which incorporates a set of common covariates as interconnectedness with a time-varying transition mechanism. This reduces the number of high dimensionality problems and greatly helps when analysing business cycles of multiple economies at the same time; a multi-country approach. Regarding linkages of real and financial sectors, this study employed four main macroeconomic variables, GDP growth, term spreads, stock prices and exchange rates. The sample period covers many major economic crises from 1980 to the end of 2019.

In terms of real economic activities, there is evidence that the business cycles of the ASEAN+3 economies are much more synchronised with each other than any of them are with the US economy. This is true for all the ASEAN+3 economies except Japan and Singapore. These results support evidence of an increasing regional interdependence within real sectors between the ASEAN+3, as suggested by Cheng et al. (2020) and Davidson (2022). However, the impacts of the economic crises of the period studied - 1987 and 2008/9 - on the ASEAN+3 economies were different. In terms of financial variables, it is noticeable that after the US subprime crisis, the pattern of stock prices of ASEAN+3 and the US were quite similar, except for China and Japan. Finally, but of no less significance, it is worth mentioning that exchange rates in ASEAN+3, especially in China, have recently been volatile compared to the US. This result is supported by Benjamin and Sato (2022), whose work indicates that the renminbi (RMB) is more influential now than at any other point in the past two decades, due to its higher weight within the currency baskets of most of the major Asian economies.

It is worth noting that the assessment of business cycle synchronisation or independence can also be performed through turning point analysis, rather than the graphical analysis employed in this study. However, this study does not cover the application of this tool. Ideally, turning point analysis involves determining how each country exhibits turning points compared to others by identifying and approximating both downward and upward turns at time 't' for the variable of interest. This enables a comparison of the duration across the business cycle phases of the countries in question, as illustrated by the BB rule of Bry and Boschan (1971) and Monch and Uhlig (2005). Additionally, nonparametric measurements, such as concordance statistics (CS), can be utilised to calculate the proportion of time during which two data series are in the same regime, providing opportunities for further research.

Chapter 4

Nowcasting GDP using Multi-Country Models

4.1 Introduction

In recent decades, there has been an increase in theoretical and empirical literature on real-time forecasting. This real-time forecasting is known as nowcasting and is the prediction of the current state of the economy (Banbura et al., 2013). This increase has occurred because timely estimates and accurate assessment of economic growth are crucial for both policymakers and economic agents, yet official statistics are published with a delay. For instance, the quarterly GDP of the UK is released with an approximate delay of thirty days.

There are two well-known models for nowcasting, one known as the mixed-frequency vector autoregressive model (MF-VAR) and the other known as the mixed-frequency dynamic factor model (MF-DFM). Both of these models exploit information from available monthly data to produce monthly and quarterly nowcasts of macroeconomic variables. The MF-VAR model is based on the vector autoregressive framework while the MF-DFM model is based on the dynamic factor model. In general, MF-DFM models typically have lower dimensionality compared to MF-VAR models because they estimate a reduced set of factors that explain the common variation across variables. This makes them more scalable and computationally efficient when dealing with high-dimensional datasets. Further literature that examines either one or both of these models includes the work of Giannone et al. (2008), Foroni and Marcellino (2013), Banbura et al. (2013), Carriero, Clark and Marcellino (2015), Eraker et al. (2015), Schorfheide and Song (2015), Ghysels (2016), McCracken, Owyang and Sekhposyan (2016), Brave, Butters and Justiniano (2018), Gotz and Hauzenberger (2018), Koop et al. (2020), Gefang et al. (2020), and Cimadomo et al. (2022). In order to utilise the mixed-frequency approach, the Kalman filter and smoother are used to interpolate the missing data of the available series at low frequency in a state-space form (see Marcellino and Sivec; 2021).

However, the aforementioned scholars have mainly centred on nowcasting models for a single economy (a country-specific model) and there are very few pieces of literature focusing on multi-country nowcasting models, which have great potential for helping to monitor nowcasts of GDP for a particular group of economies and examining the role of interdependence among economies. This is a call to researchers who wish to investigate predictive gains from multi-country nowcasting contexts. The main objective of this chapter is, therefore, to compare the nowcasting performance of multi-country models - large Bayesian VARs, panel VARs with different prior algorithms and dynamic factor models - with the nowcasting performance of country-specific models, based on the mixed-frequency approach for the four largest economies in Europe: Germany, the UK, France and Italy.

4.2 Related Literature

Following Kapetanios, Marcellino, Papailias and Mazzi (2020), the contributions of panel VARs can be shown in several ways. Firstly, they are very useful for investigating the similarities of economic structures between countries and the convergence of business cycles in specific countries with panel data, such as Eurozone countries, the G-7 countries and countries in Latin America. For examples of panel VARs used in this way, see Canova and Ciccarelli (2012), Ciccarelli et al. (2013) and Apostolakis and Papadopoulos (2019). Secondly, according to idiosyncratic shocks across units and time, panel VARs are able to analyse the transmission channels of shocks and evaluate their impacts, such as when used by Canova and Ciccarelli (2012), Ciccarelli et al. (2016) and Gnimassoun and Mignon (2016). Lastly, panel VARs are employed to forecast economic variables effectively, especially leading and coincident indicators for different sectors and for different countries, good examples being the work of Canova and Ciccarelli (2009) and Dées and Güntner (2017), respectively. Moreover, Koop and Korobilis (2016 and 2019) developed shrinkage methods for panel VARs in order to deal with the over-parameterisation problem and also adopted the dynamic model averaging (DMA) method of Raftery, Karny and Ettler (2010) for model selection and forecasting.

In terms of using panel VARs with nowcasting and forecasting, Kapetanios, Marcellino, Papailias and Mazzi (2020) employed the panel VARs with high-frequency data by using mixed data sampling (MIDAS) and frequentist estimation techniques for evaluating short-term forecasting performance in panels of EU countries. They proposed that the panel VARs improve the accuracy of point forecasts at longer horizons. Nevertheless, for all cases, they suggested that it is necessary to carefully choose the appropriate specification for the panel VAR models. Next, Koop, McIntyre, Mitchell and Poon (2022) employed mixed-frequency stacked VARs with entropic tilting for gross value added (GVA) nowcasting of the different UK regions. Meanwhile, Forten and Greenaway-McGrevy (2022) used a mixed-frequency panel VAR model with a bias-corrected least squares procedure for regional nowcasting of gross domestic product (GDP) in the United States and state-level GDP nowcasting. In both cases, the usefulness of mixed-frequency panel VARs was exploited by defining panel structures as regional and state-level nowcasting, respectively.

In a recent study by Cascaldi-Garcia et al. (2023), a multi-country model was introduced for the purpose of nowcasting the euro-area aggregate along with its three major economies, namely Germany, France and Italy, individually. Their multi-country nowcasting model was constructed based on a dynamic factor model, allowing for simultaneous estimation of nowcasting. Their indicators were split into three elements – (i) a euro-area component, (ii) a country-specific component and (iii) an idiosyncratic component. Unlike previous literature that predominantly relies on single economy nowcasting models, the multi-country nowcasting model sets itself apart by incorporating multiple economies simultaneously into its framework. This chapter builds upon the framework of Cascaldi-Garcia et al. to present one of several competing nowcasting models for a multi-country approach.

One of these competing models was originally proposed by Schorfheide and Song (2015) - the mixed-frequency vector autoregressive (MF-VAR) model, which is widely recognised as one of the most influential nowcasting models. The MF-VAR model is constructed by employing a state-space form and using a Bayesian approach for its estimation. In addition, the Minnesota prior with the marginal data density (MDD) approximation is used for setting priors and optimising parameters within the MF-VAR model. Schorfheide and Song have made remarkable contributions to nowcasting - their initial framework has enabled many researchers to further develop their own work and significantly enhance nowcasting performance in various ways. Their notable advancements include incorporating time-varying parameters, incorporating stochastic volatility, and utilising different priors. These improvements have played a critical role in the development and refinement of nowcasting models. However, it is important to note that the MF-VAR model was originally developed based on a single-country framework. To fill the gap in the literature, this chapter, therefore, extends their country-specific model by introducing multi-country models augmented with several prior algorithms. This augmentation aims to broaden the model's scope and enhance nowcasting performance through capturing multi-country dynamics.

Several priors have been developed to handle the problem of curse of dimensionality. In this chapter, four significant prior settings are employed, namely (i) The Minnesota prior (Doan, Litterman and Sims; 1984) - this prior directly focuses on shrinking parameters in large VAR models, effectively handling a high number of parameters, (ii) The stochastic search variable selection (SSVS) (George, Sun, and Ni; 2008) - this prior employs a stochastic search algorithm to select relevant variables and shrink irrelevant ones within the model, (iii) The Minnesota-type adaptive hierarchical algorithms (MNG) (Chan; 2021) - this prior utilises adaptive shrinkage techniques based on hierarchical structures, enabling efficient parameter estimation in large VAR models and (iv) The stochastic search specification selection (SSSS) (Koop and Korobilis; 2016) - this prior not only introduces parameter shrinkage but also allows for significant flexibility in capturing panel structures within multi-country models.

Based on the aforementioned related literature, the main objective is to fill the gap in existing research by comparing the nowcasting performance of country-specific models with multi-country models, in order to improve the accuracy of nowcasting. This comparison focuses on the MF-DFM models and the extended MF-VAR (MF-PVAR) models, incorporating the four significant prior settings mentioned above. The fundamentals of these econometric models are presented in the next section.

4.3 Econometric Methodology

This section introduces two main nowcasting models, namely the panel vector autoregressive model (PVAR) and the dynamic factor model (DFM) with the mixed frequency VAR Approach. To provide a clearer understanding, this section begins by elucidating the properties of the PVAR. Subsequently, a methodology for handling mixed frequency data within the PVAR is presented. Furthermore, the section proceeds to put forward the DFM as an interesting competitor to the PVAR. Moving on, the models' specifications and priors in this chapter are provided. Finally, the methods of estimation are presented.

4.3.1 The Panel VAR Model

A panel VAR model is similar to a standard VAR model in that all variables are endogenous, but a panel VAR adds the extra dimension of cross-sectional units. In this case, suppose that there is a cross-section of N countries and G variables for each country which are observed over T time periods. In addition, let $Y_t = (y'_{1,t}, y'_{2,t}, \ldots, y'_{N,t})$, where $t = 1, \ldots, T$, is the vector of dependent variables with $NG \times 1$ in which $y'_{i,t}$ is the vector of dependent variables for country i for $i = 1, \ldots, N$. Therefore, the panel VAR model for country i associated p lags can be written as:

$$y_{i,t} = c_i + A_{i,1}Y_{t-1} + \ldots + A_{i,p}Y_{t-p} + \epsilon_{it}, \tag{4.1}$$

or

$$y_{i,t} = c_i + \sum_{j=1}^{p} A_{i,j}Y_{t-j} + \epsilon_{i,t}, \tag{4.2}$$

where $A_{ij} = (A_1^j, \ldots, A_N^j)$ for $j = 1, \ldots, p$ with $G \times NG$ -dimensional matrices of autoregressive coefficients for country i and c_i is a vector of intercept, $G \times 1$. In addition, $\epsilon_{it} \sim N(0, \Sigma_{it})$ is a G-dimensional vector of disturbances with a variance-covariance matrix, $cov(\epsilon_{it}, \epsilon_{jt}) = \Sigma_{ij} \neq 0$ for $i \neq j$.

According to Canova and Ciccarelli (2013), there are three features to the structure of panel VARs. These are (i) dynamic interdependency (DI), which refers to coefficients of foreign lagged endogenous variables that are non-zero elements. In other words, there are dynamic cross-country spillover effects from one country to other countries (ii) static interdependency (SI), which refers to non-zero elements of the covariance matrix, meaning that $\mathbf{\Sigma}_{ij} \neq 0$, implying that shocks from one country are able to transmit contemporaneously to other countries and (iii) cross-sectional homogeneity (CSH), which is defined as the coefficients of own lagged endogenous variables for country i, not differing from country j.

From equation (4.1), the compact form of the panel VARs can be written as follows:

$$Y_t = X_t' \alpha + \mu_t, \tag{4.3}$$

where $X_t = I \otimes (Y_{t-1}', \ldots, Y_{t-p}', 1)'$ has an $NG \times NGk$ matrix in which $k = NGp + 1$, $\alpha = (vec(A^1)', \ldots, vec(A^p)', vec(c)')'$ and $\mu_t = (\epsilon_{1t}, \ldots, \epsilon_{Nt})'$ with $\mu_t \sim N(0, \Sigma_{ii})$. This equation is known as the unrestricted PVAR, used to analyse the relationships between variables in panel data. However, panel structures can present challenges due to high dimensionality, where the number of variables or parameters involved is large. To overcome these challenges, prior specifications, which impose restrictions on certain parameters in the model, are employed such as the work of Canova and Ciccarelli (2013) and Korobilis (2016).

In other words, in terms of unrestricted PVAR models, these can be viewed as large VARs or standard homoscedastic VAR models, especially with a fixed T and large N, in which they disregard the inherent usefulness of panel structures. Large VARs often suffer from the curse of dimensionality, however there are several studies that propose methods for dealing with this problem, for example Banbura et al. (2010) suggested that by employing the Minnesota prior, a standard Bayesian VAR model can be effectively applied to large panel datasets. Their work demonstrated that large Bayesian VARs tend to outperform small Bayesian VARs in terms of forecast accuracy. In another example, Koop and Korobilis (2019) introduced a hierarchical prior for high-dimensional panel VARs. Their approach involved incorporating a Bayesian dynamic learning procedure to handle the curse of dimensionality. The hierarchical prior was applied to both the VAR coefficients and the error covariance matrix in the panel VAR model. Carriero et al. (2019a) presented a novel Bayesian estimation procedure for large VARs with time-varying volatility and non-conjugate priors. The procedure they proposed relies on a triangularisation of the VAR, which enables the posterior estimation of the VAR's coefficients by drawing them equation by equation, whilst Chan (2021) proposed a novel family of adaptive hierarchical priors known as the Minnesota-type adaptive hierarchical priors (MNG). These priors combine the desirable characteristics of two well-known classes of shrinkage priors: the global-local prior and the Minnesota prior, which offer a parsimonious framework for estimation in high-dimensional models.

4.3.2 The Mixed Frequency VAR Approach

Following Schorfheide and Song (2015), the mixed-frequency VAR (MF-VAR) model can be shown as the state-space model using the Bayesian approach for its estimation. To gain a better understanding of the nature of mixed-frequency data, the following example can be considered. Let $x_{m,t}$ denote the monthly variables that are observed at monthly frequency, e.g., the consumer price index and unemployment rate, and let $x_{q,t}$ denote the unobserved monthly variables published only at quarterly frequency according to "ragged-edges" of the dataset, such as GDP[1]. Let T be the forecast period and let $T_b \leq T$ denote the last period corresponding to the last month of the quarter, in which all quarterly observations are available, and b stand for the balanced sample. Next, the vector of monthly variables, $x_{m,t}$, are observed every month up until period T_b and let $y_{m,t}$ denote the actual observations. Therefore,

$$y_{m,t} = x_{m,t}, \quad t = 1, ..., T_b. \tag{4.4}$$

According to Mariano and Murasawa (2003, 2010), quarterly observations are averages of the constituent months which refer to “intra-quarterly averaging” and are used for data in the natural logarithm. This can be expressed it by assuming that monthly variables in log levels ($\tilde{y}_{q,t}$) are the arithmetic mean of an unobserved monthly variables ($x_{q,t}$) as follows:

$$\tilde{y}_{q,t} = \frac{1}{3}(x_{q,t} + x_{q,t-1} + x_{q,t-2}) = \Lambda_{qz} z_t, \tag{4.5}$$

Note that $\tilde{y}_{q,t}$ - the three-month average - can be observed only for every third month. Let $M_{q,t}$ be a selection matrix in which t represents the data observed in the last month of a quarter and is zero otherwise. The following can, therefore, be written:

$$y_{q,t} = M_{q,t}\tilde{y}_{q,t} = M_{q,t}\Lambda_{qz} z_t, \quad t = 1, \ldots, T_b. \tag{4.6}$$

Additionally, in regard to Equation 4.6, $z_t = [Y_t', \ldots, Y_{t-p+1}']$, in which $Y_t = [y_{q,t}', y_{m,t}']'$ - a vector of dependent variables the same as shown in the panel VAR model.

Let $y_{m,t}$ denote the subset of monthly variables at the current period t that are

[1]This refers to datasets that are unobserved or missing at the end of a sample due to the staggered nature of publication.

reported by the statistical agency and let $M_{m,t}$ be a selection matrix, such that

$$y_{m,t} = M_{m,t} x_{m,t}, \quad t = T_b + 1, \ldots, T. \tag{4.7}$$

Therefore, the state-space equation can be written as:

$$Y_t = M_t \Lambda_z z_t, \quad \text{in which} \quad t = 1, \ldots, T. \tag{4.8}$$

(Measurement equation)

$$z_t = \mu + F z_{t-1} + v_t, \quad \text{in which} \quad v_t \sim N(0, S) \tag{4.9}$$

(State equation)

where M_t is an identity matrix at t corresponding to the observed data and empty otherwise, μ is the intercepts, F contains the AR-coefficients and S is a variance-covariance matrix.

4.3.3 The Dynamic Factor Model

Generally, the dynamic factor model (DFM) can be written as a system of equations, consisting of the measurement equation and the transition equation. The measurement equation links the observed variables to the unobserved common factors and its residuals while the transition equation describes the dynamics of the common factors and residuals. Basically, the DFM can be written as:

$$Y_t = \Lambda . F_t + e_t, \tag{4.10}$$

$$F_t = A.F_{t-1} + \mu_t, \quad \mu_t \sim i.i.dN(0, Q), \tag{4.11}$$

$$e_t = D.e_{t-1} + v_t, \quad v_t \sim i.i.dN(0, R), \tag{4.12}$$

where Y_t is a vector of n stationary variables observed at time t. F_t is an $r \times 1$ vector of unobserved (latent) common factors with zero mean and covariance matrix Q. R is defined as the number of factors. Λ is a factor-loading matrix with a dimension of $N \times r$ and the idiosyncratic disturbances e_t uncorrelated with F_t at all leads and lags. Equations 4.10 to 4.12 can be written in a state-space form and estimated using the Kalman filter and smoother in order to extract common factors and produce a projection for all variables.

4.3.4 Model and Prior Specification

In this paper, nowcasting performance is compared between the country-specific model and the multi-country models, as listed below:

1. The country-specific model;
   (i) MF-BVAR.
   (ii) MF-DFM.
2. The multi-country models;
   (i) MF-Large BVAR,
   (ii) MF-PVAR(SSVS),
   (iii) MF-PVAR(SSSS),
   (iv) MF-PVAR (MNG),
   (v) MF-DFM

For the MF-BVAR and MF-Large BVAR, the prior specifications follow by Schorfheide and Song (2015). This means that the Minnesota prior with the marginal data density (MDD) approximation is employed to deal with the high dimensionality problem.

In terms of the MF-PVAR, three priors are adopted - the stochastic search variable selection (SSVS) prior, the stochastic search specification selection (S^4) and the Minnesota-type adaptive hierarchical algorithms (MNG). The stochastic search variable selection (SSVS) prior was first by George, Sun and Ni (2008).

This prior is an early hierarchical shrinkage prior and the main idea is to separate the coefficients into two groups. The first group of coefficients is regularised towards zero, the second is not. Put differently, the coefficients of MF-PVAR have equal prior weight of importance and the shrinkages of their coefficients are determined by the data. By contrast, the stochastic search specification selection (S^4) algorithm, developed by Koop and Korobilis (2016), takes into account of the panel structures of the parameters. In other words, this prior introduces the shrinkage method by investigating whether coefficients across countries are identical for both the cross-sectional homogeneities and the dynamic interdependencies. In regard to the Minnesota-type adaptive hierarchical algorithms (MNG), Chan (2021) exploits the excellent properties of both the Minnesota prior and recent adaptive hierarchical algorithms (the normal-gamma prior and the horseshoe prior) for shrinkage of a huge number of parameters in large Bayesian VARs. In other words, the adaptive hierarchical priors achieve all VAR coefficients by only shrinking small coefficients to zero and retaining large coefficients intact. Meanwhile, the best feature of the Minnesota prior is that it can incorporate many prior beliefs - i.e., the prior coefficients of higher lags of that variables can be shrunk to zero and the cross-variables (other variables) can be shrunk more aggressively than own lags of those variables. In addition, all models are estimated with six lags except the MF-DFM.

Regarding the MF-DFM, this study builds on the approach proposed by Bok et al., (2018) with the E-M algorithm and the maximum likelihood estimation methodology. In this regard, the blocks' structure is defined using two common factors, a global factor and a local factor. The global factor is extracted from all indicators and the local factor from indicators within the individual country in question, by using the principal component method for initialisation when considering the multi-country models.

4.3.5 Estimation

The estimation of MF-BVAR models, such as MF-PVAR and MF-Large BVAR, is conducted by employing Bayesian methods. These methods incorporate various priors specific to each model. For instance, the prior used in MF-BVAR and MF-Large BVAR is the same - the Minnesota prior with the marginal data density (MDD) approximation. Three different priors, however, are used in the the MF-PVAR model: stochastic search variable selection (SSVS) prior, stochastic search specification selection (S^4), and Minnesota-type adaptive hierarchical algorithms (MNG), the properties of each of these being described in the previous section. Posterior inference is performed using a Gibbs sampler, which is briefly explained in Appendix C.1-C.4.

In contrast, the estimation of MF-DFM models is based on Banbura and Modugno's (2014) approach rather than a Bayesian method. Their method involves modifying the expectation-maximisation algorithm. This modification enables the estimation of parameters by constructing a state-space model on datasets that contain missing data patterns. The purpose of this modification is to effectively handle mixed-frequency data which comprise both observed and unobserved data components. See more details in Appendix C.5.

4.4 Performance Measurement

In terms of evaluating point forecast accuracy, the root mean squared forecast error (RMSFE) is used, defined as:

$$RMSFE = \sqrt{\frac{1}{H}\sum(\hat{y}_{t+h} - y_{t+h})^2}, \tag{4.13}$$

where H is the total number of time periods over which forecasting is performed. Intuitively, the smaller the RMSFE, the better the out-of-sample forecast performance.

In order to evaluate a density forecast, the continuous ranked probability scores (CRPS) are employed, developed by Gneiting and Raftery (2007):

$$\begin{aligned} \mathrm{CRPS}(F, y^0_{T+h}) &= \int_{\infty}^{\infty} \left(F(z) - 1\{y^0_{T+h} \leq z\}\right)^2 dz, & (4.14)\\ &= \mathbb{E}_F|z - y^0_{T+h}| - \frac{1}{2}\mathbb{E}_F|z - y^0_{T+h}|, & (4.15)\\ &= 2\int_0^1 \left(1\{y^0_{T+h} < F^{-1}(q)\} - q\right)\left(F^{-1}(q) - y^0_{T+h}\right) dq. & (4.16) \end{aligned}$$

where F denotes the cumulative distribution function corresponding to the marginal predictive density f for the forecast at period $T+h$, together with the realised value y^0_{T+h}. In addition, $1(.)$ denotes the indicator function. If the condition is verified, the value equals 1 or zero otherwise. For example, with perfect predictive density, the value of $CRPS(F, y^0_{T+h})$ is zero (a mass point of density 1 at $z = y^0_{T+h}$, such that $F(z) = 0$ for $z < y^0_{T+h}$, and $F(z) = 1$ for $z \geq y^0_{T+h}$). This means that a lower score implies a better density forecast performance. Additionally, the Diebold & Mariano test, demonstrated in Chapter 2, can be used to examine the asymptotic test. Nevertheless, this particular study does not encompass this aspect, providing an opportunity for additional investigation.

4.5 Dataset and Forecast setup

4.5.1 Dataset

In this study, the dataset is split into two sets. The first contains four main macroeconomic variables - quarterly GDP, industrial production index, unemployment rate and consumer price index. The second consists of eleven economic indicators - the four macroeconomic variables from the first set plus export of goods, import of goods, the number of new passenger car registrations, turnover index of manufacturing, retail sales, business climate indicator and consumer confidence indicator. Alternatively, these eleven indicators together can be categorised into seven groups; National Account, Labour, Manufacturing, Prices, Trades, Retail sales and Sentiment indicators. It can clearly be seen that the first set has only hard data while the second includes a mix of hard and soft indicators (such as surveys and polls), as shown in Table 4.1. Such soft indicators are included because they can be timely, informative and represent both current and future perceptions of economic agents (e.g., Giannone et.al., 2008 and Bok et.al., 2018). Indeed, just how important these eleven indicators are can be clearly understood from the Bloomberg Website itself, where the majority of them are labelled as 'market moving indicators'. At this point, it is crucial to note that the data does not cover monthly financial indicators (i.e. the four hard indicators mentioned above), as Banbura et.al. (2013) show that such data does not improve the accuracy of nowcasting's performance, either during normal periods or during the Great Recession, as a consequence of their noise.

According to the MF-VARs of Schorfheide and Song (2015), the data for all indicators is transformed into log levels, except for those indicators that are measured using a percentage and are therefore stationary, such as interest rates and employment rates. For a mixed-frequency dynamic factor model, all indicators are transformed to ensure stationarity as per Bok et al., (2018).

It is worth noting that dynamic factor models typically require economic variables to be in a stationary setting. This is because consistent estimation of the model can be achieved using either principal components or maximum likelihood estimation through the EM algorithm, as outlined in studies such as Stock and Watson (2002) and Doz et al. (2012). By contrast, Bayesian estimation does not inherently require economic variables to be stationary. It operates within a probabilistic framework that provides more flexibility for modelling non-stationary data. This flexibility allows for the inclusion of prior beliefs about non-stationary behaviour, enabling the model to adapt to the characteristics of the data. Furthermore, Bayesian estimation produces estimates in the form of probability distributions, allowing for uncertainty of parameters.

Table 4.1: A list of macroeconomic variables in our datasets

Country	Series Name	Unit	Freq	Catagory	Delay (Days)	1^{st} set	2^{nd} set
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
DE	Gross Domestic Product	swda, bil.2015.EUR	q	National Acc.	45	⊠	⊠
DE	Unemployment Rate	sa,%	m	Labor	3	⊠	⊠
DE	Industrial Production Index	swda,2015=100	m	Manufacturing	38	⊠	⊠
DE	Harmonised Index of Consumer Prices	2015=100	m	Prices	13	⊠	⊠
FR	Gross Domestic Product	swda, bil.2014.EUR	q	National Acc.	57	⊠	⊠
FR	Unemployment Rate	sa,%	m	Labor	25	⊠	⊠
FR	Industrial Production Index	swda,2015=100	m	Manufacturing	40	⊠	⊠
FR	Harmonised Index of Consumer Prices	2015=100	m	Prices	15	⊠	⊠
UK	Gross Domestic Product	sa, bil.2016.GBP	q	National Acc.	40	⊠	⊠
UK	Unemployment Rate	sa,%	m	Labor	15	⊠	⊠
UK	Industrial Production Index	sa,2016=100	m	Manufacturing	12	⊠	⊠
UK	Consumer Price Index (CPI)	2015=100	m	Prices	15	⊠	⊠
IT	Gross Domestic Product	swda, bil.2010.EUR	q	National Acc.	45	⊠	⊠
IT	Unemployment Rate	sa,%	m	Labor	30	⊠	⊠
IT	Industrial Production Index	sa,2005=100	m	Manufacturing	40	⊠	⊠
IT	Consumer Price Index (CPI)	2005=100	m	Prices	15	⊠	⊠
DE	Exports of Goods	swda,bil.EUR	m	Trade	40		⊠
DE	Imports of Goods	swda,bil.EUR	m	Trade	40		⊠
DE	New Passenger Car Registrations	NSA, Units	m	Manufacturing	18		⊠
DE	Turnover Index of Mfg	swda,2015=100	m	Manufacturing	35		⊠
DE	Retail Sales Volume Index excl Motor Vehicles	swda,2015=100	m	Retail Sales	30		⊠
DE	Business Climate Indicator	sa,index	m	Sentiment Indicator	-7		⊠
DE	Consumer Confidence Indicator	sa,%	m	Sentiment Indicator	-5		⊠
FR	Total exports incl Military Equipment	swda,bil.EUR	m	Trade	38		⊠
FR	Total imports Incl Military Equipment	swda,bil.EUR	m	Trade	38		⊠
FR	First Registrations of Brand New Passenger Cars	NSA, Units	m	Manufacturing	17		⊠
FR	Turnover Index of Mfg	swda,2015=100	m	Manufacturing	60		⊠
FR	Retail Sales trade index	swda,2015=100	m	Retail Sales	60		⊠
FR	Business Climate Indicator	sa,index	m	Sentiment Indicator	-6		⊠
FR	Consumer Confidence Indicator	sa,%	m	Sentiment Indicator	-4		⊠
UK	Exports of Goods	sa,bil.GBP	m	Trade	40		⊠
UK	Imports of Goods	sa,bil.GBP	m	Trade	40		⊠
UK	New Passenger Car Registrations	NSA, Units	m	Manufacturing	18		⊠
UK	Production Industries Turnover of Mfg	bil.GBP	m	Manufacturing	40		⊠
UK	Retail Trade Turnover Index	swda,2015=100	m	Retail Sales	18		⊠
UK	Business Confidence Index	sa,index	m	Sentiment Indicator	-8		⊠
UK	Consumer Confidence Indicator	sa,%	m	Sentiment Indicator	-8		⊠
IT	Merchandise Exports, fob	swda,bil.EUR	m	Trade	45		⊠
IT	Merchandise Imports, cif	swda,bil.EUR	m	Trade	45		⊠
IT	New Passenger Car Registrations	NSA, Units	m	Manufacturing	15		⊠
IT	Industrial Turnover Index	sa,2015=100	m	Manufacturing	55		⊠
IT	Retail Trade Turnover Index	swda,2015=100	m	Retail Sales	38		⊠
IT	Business Climate Indicator	sa,2010=100	m	Sentiment Indicator	-5		⊠
IT	Consumer Confidence Indicator	sa,%	m	Sentiment Indicator	-5		⊠

4.5.2 Forecast Setup

This study evaluates nowcasts based on the latest data vintage and quarterly averages. This means that the effect of data revision, which can occur at different times for different countries, is not included in the analysis. The sample data covers January 2005 to June 2020 and GDP nowcasts have been generated for two different periods within the sample data. The first has been generated for the period 2010Q1 to 2018Q1, a period chosen specifically to exclude the impact of the COVID-19 pandemic. The second has been generated for the period 2010Q1 to

2020Q2 to allow for additional evaluation of the pandemic[2]. The overall sample is relatively short because some variables are only collected for a short time period. In addition, this chapter follows the information structure as per Schorfheide and Song (2015) by defining +0 month group (I1) as January, April, July and October; +1 month group (I2) as February, May, August, November and +2 month group (I3) as March, June, September, December.

4.6 Empirical Results

In this section, three different sets of empirical results are presented. Firstly, the nowcast evaluation results for the pre-pandemic period (2010Q1 - 2018Q1) of the first dataset are discussed. Secondly, the nowcast evaluation results through the pandemic period (2010Q1 - 2020Q2) of the first dataset are presented. Finally, the results obtained from the evaluation of the second dataset are outlined.

4.6.1 Results of nowcast evaluation, pre-pandemic period of 1st dataset

The results provided in Table 4.2 show the root means square error (RMFSE) for evaluation of point nowcasts of pre-pandemic GDP growth for different countries by each model. It could be considered that there are two groups of categories; (i) country-specific models - based on economic variables of the economies' own indicators and (ii) multi-country models - relying on the variables of all four economies being analysed. In other words, the country-specific models have a domestic perspective whereas the multi-country models have a multinational per-

[2] In this study, an effort is made to incorporate the most up-to-date or latest vintage data by running more than three versions. This ensures that the data remains current throughout the author's writing process, covering the period from October 2019 to November 2020, encompassing both pre-COVID-19 and the extended period during the COVID-19 pandemic. Hence, this study does not account for the impact of data revision, allowing for further research.

spective. With regard to individual-country level, we mainly consider MF-BVAR and MF-DFM. At the multi-country level, we employ the MF-DFM, Large MF-BVAR and MF-PVAR using the three aforementioned priors to deal with the panel structure of their parameters (SSSS, SSVS, MNG), as described in the previous section. Moreover, this nowcast exercise takes into account different monthly information inflows (I1 to I3).

For point nowcast performance in Table 4.2, the results can be summarised as follows. In terms of all information inflows, I2 and I3 information sets can be beneficial to the improvement of nowcasting performance for all countries and all models. This means that when new monthly information is released, these information sets crucially contribute to the precision of nowcasts - the RMSFE of I2 and I3 in almost all models is less than the RMSFE of I1.

When comparing individual- and multi-country levels, it can clearly be seen that the results of the multi-country models give more accurate nowcasts for Germany whereas the country-specific models provide greater accuracy for France, the United Kingdom and Italy. For Germany, large MF-BVAR and MF-DFM models at the multi-country level substantially outperform the competing models. Next, at the country-specific level, it is worth highlighting that the conventional MF-BVAR model provides the most accurate nowcasting for France yet is only slightly better than the MF-DFM at the multi-country level. It is crucial to note here that, for Italy, the MF-DFM at the individual level outperforms the other three competing models that have priors applied (MF-PVAR with SSSS, SSVS and MNG, respectively). However, it is only slightly better than MF-PVAR (MNG) at the multi-country level. Interestingly, regarding the United Kingdom, the MF-BVAR model at the country-specific level proves to be substantially more accurate, except for each Q1 - for these, the UK requires the MF-DFM model for more accurate nowcasting. Therefore, both country-specific models are required. Why this should be is an intriguing prospect for further research.

Table 4.2: RMSFE of GDP growth nowcasts with different models for the 1^{st} dataset

	GDP growth (Through 2018Q1)						
	Country-specific		**Multi-country**				
	MF-BVAR	MF-DFM	Large MF-BVAR	MF-PVAR (SSSS)	MF-PVAR (SSVS)	MF-PVAR (MNG)	MF-DFM
DE							
Mth./Qt.1(I1)	0.778	0.809	0.766	0.785	0.789	0.702	**0.683**
Mth./Qt.2(I2)	0.755	0.722	0.725	0.746	0.731	0.709	**0.664**
Mth./Qt.3(I3)	0.766	0.690	**0.661**	0.680	0.712	0.685	0.686
FR							
Mth./Qt.1(I1)	**0.280**	0.300	0.351	0.337	0.376	0.332	0.316
Mth./Qt.2(I2)	**0.214**	0.237	0.337	0.338	0.352	0.297	0.262
Mth./Qt.3(I3)	**0.251**	0.288	0.339	0.310	0.346	0.329	0.256
UK							
Mth./Qt.1(I1)	0.276	**0.247**	0.418	0.674	0.464	0.558	0.376
Mth./Qt.2(I2)	**0.194**	0.218	0.366	0.568	0.421	0.532	0.319
Mth./Qt.3(I3)	**0.192**	0.245	0.340	0.539	0.423	0.480	0.262
IT							
Mth./Qt.1(I1)	0.426	**0.263**	0.407	0.383	0.371	0.314	0.303
Mth./Qt.2(I2)	0.413	**0.241**	0.389	0.385	0.354	0.277	0.318
Mth./Qt.3(I3)	0.391	**0.259**	0.367	0.346	0.322	0.262	0.318

Note: The best model is indicated in bold.

Table 4.3 summarises the findings for the density nowcasting exercise measured by the continuous ranked probability scores (CRPS). It is important to note that the CRPS for the MF-DFM model cannot be calculated. This is because the MF-DFM model in this study is used for likelihood estimation that generates point nowcasting performance, not density nowcasting performance. The main conclusions from these results can be drawn as follows. Firstly, information inflows might have little impact on the relative accuracy of the predictive densities in the case of the country-specific models. Secondly, the MF-PVARs with SSVS and MNG at the multi-country level can improve the accuracy of density nowcasts for Germany and Italy whereas for France and the United Kingdom it is the MF-BVAR model that is seen to perform better.

Table 4.3: CRPS of GDP growth nowcasts with different models for the 1^{st} dataset

	GDP growth (Through 2018Q1)						
	Country-specific		**Multi-country**				
	MF-BVAR	MF-DFM	Large MF-VAR	MF-PVAR (SSSS)	MF-PVAR (SSVS)	MF-PVAR (MNG)	MF-DFM
DE							
Mth./Qt.1(I1)	0.37	-	0.55	0.38	0.35	0.37	-
Mth./Qt.2(I2)	0.33	-	0.54	0.34	0.25	0.36	-
Mth./Qt.3(I3)	0.33	-	0.53	0.33	0.24	0.36	-
FR							
Mth./Qt.1(I1)	0.15	-	0.53	0.42	0.21	0.46	-
Mth./Qt.2(I2)	0.14	-	0.52	0.38	0.20	0.44	-
Mth./Qt.3(I3)	0.14	-	0.51	0.37	0.20	0.43	-
UK							
Mth./Qt.1(I1)	0.16	-	0.43	0.41	0.26	0.29	-
Mth./Qt.2(I2)	0.15	-	0.22	0.34	0.20	0.31	-
Mth./Qt.3(I3)	0.15	-	0.22	0.39	0.20	0.31	-
IT							
Mth./Qt.1(I1)	0.21	-	0.42	0.40	0.29	0.24	-
Mth./Qt.2(I2)	0.21	-	0.42	0.34	0.24	0.23	-
Mth./Qt.3(I3)	0.22	-	0.41	0.34	0.23	0.23	-

Note: The best model is indicated in bold.

4.6.2 Results of nowcast evaluation, pandemic-inclusive period of 1^{st} dataset

In relation to the previous findings, the analysis initially disregarded the effects of the pandemic. Nevertheless, the subsequent results provide insight into these impacts by expanding the evaluation periods of the first dataset from 2010Q1 through 2020Q2, as shown in Tables 4.4 and 4.5.

Table 4.4 presents the performance of point nowcasts, as indicated by the Root Mean Square Forecast Error (RMSFE). On the other hand, Table 4.5 displays the results of the density nowcasting exercise, measured by the Continuous Ranked Probability Scores (CRPS). To summarise the results, it is evident that the pandemic has had a significant impact on the nowcasting performances of all the models. This is reflected in the higher values of RMSFE in Table 4.4 and CRPS in Table 4.5, indicating reduced accuracy in general during the pandemic period.

These results highlight the adverse impact of the pandemic on the models' predictive abilities. Worthy of note is that the results of the nowcasting performance for each model still align with the patterns observed in Tables 4.2 and 4.3, before the pandemic period.

In the next section, the study focuses on investigating the potential usefulness of incorporating additional economic indicators during the pandemic period, based on these results.

Table 4.4: RMSFE of GDP growth nowcasts with different models for the 1st dataset

	GDP growth (Through 2020Q2)						
	Country-specific		**Multi-country**				
	MF-BVAR	MF-DFM	Large MF-BVAR	MF-PVAR (SSSS)	MF-PVAR (SSVS)	MF-PVAR (MNG)	MF-DFM
DE							
Mth./Qt.1(I1)	2.433	2.521	2.425	2.523	2.522	2.502	**2.314**
Mth./Qt.2(I2)	1.965	1.643	1.746	1.820	1.757	1.724	**1.538**
Mth./Qt.3(I3)	1.871	1.643	**1.532**	1.801	1.832	1.812	1.641
FR							
Mth./Qt.1(I1)	**2.017**	2.110	2.435	2.325	2.514	2.322	2.148
Mth./Qt.2(I2)	**1.714**	1.812	1.989	1.998	2.120	2.011	1.822
Mth./Qt.3(I3)	**1.653**	1.741	1.988	1.872	2.041	2.010	1.695
UK							
Mth./Qt.1(I1)	2.010	**1.875**	2.284	2.322	2.312	2.241	2.143
Mth./Qt.2(I2)	**1.685**	1.723	1.848	2.142	2.015	2.114	1.774
Mth./Qt.3(I3)	**1.547**	1.756	1.812	2.041	1.984	1.963	1.765
IT							
Mth./Qt.1(I1)	2.190	**1.811**	2.152	2.142	2.131	2.022	2.035
Mth./Qt.2(I2)	1.859	**1.782**	1.831	1.829	1.824	1.807	1.798
Mth./Qt.3(I3)	1.793	**1.614**	1.827	1.819	1.801	1.754	1.815

Note: The best model is indicated in bold.

Table 4.5: CRPS of GDP growth nowcasts with different models for the 1^{st} dataset

	GDP growth (Through 2020Q2)						
	Country-specific		**Multi-country**				
	MF-BVAR	MF-DFM	Large MF-VAR	MF-PVAR (SSSS)	MF-PVAR (SSVS)	MF-PVAR (MNG)	MF-DFM
DE							
Mth./Qt.1(I1)	0.54	-	0.67	0.55	0.53	0.54	-
Mth./Qt.2(I2)	0.51	-	0.65	0.50	0.47	0.51	-
Mth./Qt.3(I3)	0.51	-	0.62	0.50	0.46	0.51	-
FR							
Mth./Qt.1(I1)	0.38	-	0.65	0.57	0.39	0.58	-
Mth./Qt.2(I2)	0.32	-	0.63	0.54	0.38	0.56	-
Mth./Qt.3(I3)	0.32	-	0.63	0.54	0.38	0.54	-
UK							
Mth./Qt.1(I1)	0.40	-	0.54	0.51	0.47	0.49	-
Mth./Qt.2(I2)	0.37	-	0.48	0.49	0.45	0.50	-
Mth./Qt.3(I3)	0.37	-	0.48	0.49	0.45	0.50	-
IT							
Mth./Qt.1(I1)	0.47	-	0.58	0.56	0.50	0.49	-
Mth./Qt.2(I2)	0.47	-	0.58	0.53	0.49	0.48	-
Mth./Qt.3(I3)	0.48	-	0.57	0.53	0.49	0.48	-

4.6.3 Results of the 2^{nd} dataset

To incorporate a broader range of indicators into the models, eleven are used for each country in the 2^{nd} dataset (shown in Table 4.1), including the hard and soft data used by most economists and analysed in most of the empirical literature. Table 4.6 summarises the results of point nowcasts from 2010Q1 through the pandemic period (2020Q2). In this context, it is crucial to find out whether larger amounts of information for the four countries can improve accuracy for nowcasting.

In this regard, the focus is only on the MF-BVAR and the MF-DFM at the country-specific level of the I3. This is because the previous results (Tables 4.4 and 4.5) show that nowcasting at the country-specific level is more accurate for the majority of the economies analysed. Moreover, with eleven indicators for four countries and six lags for each of those indicators, the multi-country approach comes with a considerable computational burden. This is primarily due to the substantial number of parameters that need to be computed, resulting from the square of the multiplication of variables, lags, and countries within the multi-country models.

Table 4.6 displays the findings indicating that the second dataset exhibits notably more precise point nowcasts compared to the first dataset throughout the pandemic timeframe. Concerning these enhanced nowcasts, it is evident that the MF-DFM model demonstrates superior accuracy for Germany and Italy, while the MF-BVAR model outperforms other models for France and the United Kingdom.

Table 4.6: RMSFE of GDP growth nowcasts with different models for the 2^{nd} dataset

	GDP growth (Through 2020Q2)			
Country-specific	**DE**	**FR**	**UK**	**IT**
MF-BVAR	1.515	1.514	1.512	1.712
MF-DFM	1.417	1.601	1.594	1.586

4.7 Conclusion

This chapter explored whether the role of interdependency among economies can significantly contribute to predictive gains for GDP nowcasts. In this regard, multi-country models and country-specific models were constructed and compared for the four largest European economies; Germany, France, the United Kingdom and Italy. The focus was on the mixed frequency VAR models, the mixed frequency panel VAR models and the mixed frequency dynamic factor models. In addition, the chapter also considered the significance of different information inflows.

The results suggest that when new monthly information is released, it can improve the precision of GDP nowcasts for all countries and all models. In terms of predictive gains for GDP nowcasts in multi-country contexts, the results show that they can be helpful for predicting Germany's GDP growth in the case of point nowcasts but also for Italy's GDP growth in the case of density nowcasts, though still not quite as helpful as models in the country-specific context. For France, the United Kingdom and Italy, the country-specific models play a crucial role in GDP nowcasts when considering the point nowcasts. This implies that, for these three countries, domestic economic structures are more important for GDP nowcasts. Moreover, when considering the predictive accuracy through the Covid-19 pandemic period, it can be seen that the greater the number of domestic indicators, the more accurate the predictive gains for GDP nowcasts are for periods of economic uncertainty. However, it is important to note that the results might be influenced by different methods of data transformation used, which are necessary to accommodate the specific assumptions required by each model. This introduces a potential limitation to the comparability of the results, as the different transformation approaches may impact the performance and outcomes of the models.

Chapter 5

Conclusions

5.1 Summary and policy implications

The increasing complexity of the global economy necessitates the development of efficient models that accurately predict and analyse economic interdependence, interconnectedness, and international business cycles, as well as capture the interlinkages among regions and countries. Multi-country models, such as a standard panel vector autoregressive model (panel VAR), play a crucial role in understanding the dynamics and statics of economic interdependence (Canova and Ciccarelli, 2013), which refers to the extent to which economies rely on each other. Therefore, a panel VAR model served as a suitable starting point for this thesis.

To further enhance the analysis, the thesis integrated a Markov switching approach with a panel VAR framework, namely a Bayesian panel Markov-switching VAR model, in order to analyse business cycles. This approach allowed for a more comprehensive understanding of the fluctuations and transitions in economic activity over time, capturing the presence of different economic regimes and their impact on the dynamics of business cycles. Furthermore, the thesis employed a mixed-frequency approach in multi-country models for nowcasting purposes.

This involved applying panel VARs with different priors and a dynamic factor model (DFM) so as to effectively utilise data of different frequencies and capture timely information for GDP nowcasting.

In summary, the thesis focused on refining and applying various multi-country models in order to forecast and analyse macroeconomic interdependencies. It employed Bayesian estimation techniques and incorporated empirical applications to provide valuable insights into the dynamics of global economic interconnections. Additionally, the thesis emphasised the importance of empirical results generated from these models. Through rigorous analysis and evaluation, the empirical findings have shed light on the policy implications and usefulness of each model under different specifications. This allows policymakers and researchers to better understand the strengths and limitations of each model and make informed decisions within a multi-country context, based on the empirical evidence.

Chapter 2 proposed a particular approach to investigating the characteristics of macroeconomic interdependencies, including both dynamic and static aspects, as well as heterogeneities among different economies. This is accomplished by utilising a panel Bayesian VAR model along with the stochastic search specification selection (S^4) prior algorithm. This approach also proved valuable in reducing the over-parameterisation problem that arises when conducting forecasts at the multi-country level, enhancing the efficiency and accuracy of the forecasting process. In this regard, empirical application was conducted using the G-7 economies, with respect to main macroeconomic variables in order to investigate the characteristics and assess the forecasting performance of this approach.

The results highlight the significance of both dynamic and static interdependencies among the G-7 economies. More importantly, the findings also suggest that imposing homogeneity across all the G-7 economies may not be appropriate. This implies that each economy within the G-7 group exhibits unique characteristics and behaviours that should be taken into account when formulating economic policies or conducting further analyses. Furthermore, the forecasting performance of the model was evaluated by comparing its inflation forecasts with the factor shrinkage prior introduced by Canova and Ciccarelli (2009), which served as a benchmark. The results indicate that the panel Bayesian VAR model with the S4 algorithm outperforms the factor shrinkage method in both point and density forecasting of inflation, particularly in the short-term forecast horizons. Overall, the approach presented in Chapter 2 provides valuable insights into the characteristics of macroeconomic interdependencies and heterogeneities within specific economies. It not only enhances forecasting accuracy but also deepens our understanding of the complex dynamics within the global economy. By incorporating both dynamic and static aspects, this approach provides a comprehensive framework that enables policymakers to make better-informed decisions and conduct more detailed analyses of macroeconomic interdependencies at the multi-country level.

Chapter 3 focuses primarily on examining the interdependence between Asian economies and the US economy through the analysis of business cycle synchronisation. This investigation was motivated by two important stylised facts: the substantial contribution of Asian economies to global economic growth and the rising level of regional economic integration among the ASEAN+3 (the ten member states of ASEAN plus China, Japan and South Korea). Empirical research was conducted by employing the Bayesian panel Markov-Switching (PMS) VAR model, which incorporates interconnectedness as the nonlinearly time-varying transition mechanism, in order to examine the synchronisation of the business cycles and heterogeneities across the economies of the US and the ASEAN+3.

The study investigated the forty-year period from 1980 to the end of 2019, which includes several major economic crises. The results of the PMS-VAR model reveal distinct characteristics of the posterior density of key macroeconomic variables within each regime, including recession, recovery and slow growth and expansion regimes. The results also provide insights into the persistence of these regimes based on smoothed probabilities and posterior mean distributions, enabling the analysis of business cycle synchronisation across the economies being investigated. The findings shed light on two important aspects of economic activities: real output and financial variables.

In terms of the real outputs studied, the results do not fully support the notion of decoupling between the US and the ASEAN+3 economies. Instead, the findings provide evidence that the business cycles of the ASEAN+3 economies exhibit a higher degree of synchronisation among each other compared to their synchronisation with the US economy, except for Japan and Singapore. This indicates that there is a stronger degree of economic coherence and regional interdependence among the ASEAN+3 economies than between any of them and the US economy, in terms of real business cycle dynamics. However, it is crucial to note that the impacts of the economic crises of 1987 and 2008/2009 on the ASEAN+3 economies were experienced differently, with variations in magnitude, duration, and specific consequences for each economy.

In terms of financial variables, it is worth highlighting that the patterns of stock prices in the ASEAN+3 economies, excluding China and Japan, showed a remarkable similarity to those in the US, particularly in the aftermath of the subprime crisis in 2008/2009. Therefore, these empirical findings provide compelling evidence of the level of business cycle synchronisation between the ASEAN+3 countries and the US, which carries important implications for policymakers.

These implications include fostering inter-state economic relations, strengthening economic cooperation, and formulating effective policy coordination strategies in response to financial market volatilities and potential adverse impacts of future economic crises.

Chapter 4 developed multi-country models, particularly a panel Bayesian VAR (panel VAR) model with various prior algorithms and a dynamic factor model. These models incorporate a mixed-frequency approach which allows for the combination of data inflows with different frequencies. These frameworks are employed to generate GDP nowcasts and investigate whether or not utilising these multi-country models, rather than country-specific models, enhances the accuracy of GDP nowcasting. The analysis focused on the four largest economies in Europe, namely Germany, France, the UK, and Italy. The frameworks were also designed to examine two different sample periods, namely the period before the Covid-19 pandemic and the period during the Covid-19 pandemic. These sample periods provide insights into how these models respond to economic uncertainty. Additionally, two datasets were established - one comprising four main macroeconomic variables for each economy and the other comprising these four variables plus seven economic indicators for each economy - in order to investigate whether the differences between these datasets contribute to the improved accuracy of GDP nowcasting achieved by these models. Through the comprehensive coverage of different dimensions such as models, study periods and economic factors, this framework aims to identify the key factors that contribute to the enhanced accuracy of GDP nowcasting in a multi-country context.

The results indicate that among the countries analysed, only a single country is able to benefit from the enhanced predictive gain in GDP nowcasting achieved through the use of multi-country contexts. Conversely, the remaining countries still rely primarily on country-specific models, suggesting that the benefits of multi-country modelling are not uniformly realised across all economies. These findings are consistent across both the pre-pandemic period and the period during the Covid-19 pandemic, indicating the persistence of these patterns over time. Furthermore, it is important to note that even within country-specific models, the choice of specific model, such as a mixed frequency Bayesian VAR model (MF-BVAR) or a mixed frequency dynamic factor model (MF-DFM), can vary in performance across countries. Therefore, the optimal model for improving nowcasting GDP performance may differ among the countries studied, implying that there is no one model that consistently outperforms others for all countries. Additionally, when considering the inclusion of additional economic indicators in both country-specific models, the results indicate a substantial improvement in the accuracy of GDP nowcasting for all countries during the Covid-19 pandemic period. This highlights the importance of incorporating a broader set of economic variables into country-specific models in order to enhance their predictive accuracy.

These results raise several implications for potential policymakers. They should recognise that the benefits of utilising multi-country models for GDP nowcasting are not universally applicable across all countries. Instead, country-specific models remain crucial for accurate nowcasting in many economies. This means that policymakers should consider the unique characteristics and dynamics of their respective countries when selecting the most appropriate model. Tailoring the choice of model to each country's specific context can lead to better nowcasting results. Next, policymakers should prioritise the inclusion of additional economic indicators in country-specific models, especially during periods of economic uncertainty such as the Covid-19 pandemic. The results suggest that incorporating a broader set of economic variables significantly improves the precision of GDP nowcasting for all countries during such challenging times. Policymakers should explore a diverse range of indicators that capture different aspects of the economy in order to obtain a more comprehensive understanding of economic conditions. By doing so, they can make more informed policy decisions and respond more effectively to emerging challenges.

5.2 Further Research

This thesis has raised several questions that could be further investigated, offering significant possibilities for future research. In Chapter 2, the focus was on exploring the characteristics of interdependencies among economies, utilising the properties of a panel VAR model. To expand on this framework, future research could examine interdependencies among other groups of economies and different types of interlinkages as well as investigate the impact of economic policies on the empirical findings. For example, studying the interdependencies among European countries, Asia-Pacific countries or other regions would provide a more comprehensive understanding of global economic interdependencies. Scholars such as Christou et al. (2017), Caraiani et al. (2023) and Huber et al. (2023) have used different frameworks that would be of great help to future research, as they have all investigated the effects of specific policy interventions and changes in policy regimes by assessing how different policy choices influence macroeconomic interdependencies and heterogeneities among economies.

Another important direction for future research would be the consideration of the dynamic nature of interdependencies over different short-term periods of time. Conducting a time-varying approach and examining how macroeconomic interdependencies evolve during different economic periods, or in response to significant events like economic shocks or sudden downturns, would provide a more comprehensive understanding of the underlying dynamics. The work of Aye et al. (2019) is a good example of this. Furthermore, integrating such a time-varying approach along with stochastic volatility into the modelling framework, as demonstrated by Koop and Korobilis (2019) and Cimadomo et al. (2022) for large Bayesian VARs as well as by Eraslan and Schroder (2023) and Hauzenberger et al. (2023) for mixed-frequency dynamic factor models, has the potential to significantly enhance the accuracy of GDP nowcasting in Chapter 4.

In Chapter 3, the focus was on analysing business cycle synchronisation within a specific group of countries using a Bayesian panel Markov-switching VAR model. Future research could investigate empirical findings of synchronisation for specific countries using the framework introduced by Agudze et al. (2022). Their work incorporates a new dynamic panel model that utilises graphical models with a multi-layer network to analyse various types of interaction effects among hidden chains.

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Appendix A

Chapter 2 Appendix

A.1 Gibbs sampler algorithm for the S^4 algorithm

In this section, a standard procedure of the MCMC with Gibbs sampler can be shown as follows;

1. Sampling $vec(A)$ from

$$(vec(A)|-) \sim N(\Gamma \times \mu_\alpha, D_\alpha), \tag{A.1}$$

where $D_\alpha = (\Sigma^{-1} \otimes X'X + (V'V)^{-1})^{-1}$ and $\mu_\alpha = D_\alpha[(\Sigma^{-1} \otimes X'X)\alpha_{OLS}$, α_{OLS} is the estimate of α by ordinary least square method, and V is a diagonal matrix which has G^2 elements in the diagonal block that equals $\tau_{ij}^2 \times \mathbf{1}$ if $\gamma_{ij}^{DI} = 1$ and equals $\tau_{ij}^2 \times \underline{c}^{DI} \times \mathbf{1}$ if $\gamma_{ij}^{DI} = 0$, while equals $\xi_{ij}^2 \times \mathbf{1}$ if $\gamma_{ij}^{CSH} = 1$ and equals $\xi_{ij}^2 \times \underline{c}^{CSH} \times \mathbf{1}$ if $\gamma_{ij}^{CSH} = 0$, where $\mathbf{1}$ is a $G^2 \times 1$ vector of ones.

2. Sampling τ_{ij}^2 from

$$(\tau_{ij}^2)|-) \sim Gamma(1 + \frac{1}{2}G, \underline{\theta}^{DI} + \frac{1}{2}\sum_{k=1}^{G} \frac{[vec(A_{ij})_k]^2}{(\underline{c}^{DI})^{1-\gamma_{ij}^{DI}}}) \tag{A.2}$$

3. Sampling ξ_{ij}^2 from

$$(\xi_{ij}^2)|-) \sim Gamma(1 + \frac{1}{2}G, \underline{\theta}^{CSH} + \frac{1}{2}\sum_{k=1}^{G} \frac{[vec(A_{ii})_k]^2}{(\underline{c}^{CSH})^{1-\gamma_{ij}^{CSH}}}) \tag{A.3}$$

4. Sampling γ_{ij}^{DI} from

$$(\gamma_{ij}^{DI})|-) \sim Bernoulli(\omega_{ij}^{DI}), \tag{A.4}$$

where $\omega_{ij}^{DI} = \frac{\mu_{2,ij}}{\mu_{1,ij}+\mu_{2,ij}}$ with $\mu_{1,ij} = \phi(vec(A_{ij})|\mathbf{0}, \underline{\tau}_1^2 I_{G^2})\pi_{ij}^{DI}$ and $\mu_{2,ij} = \phi(vec(A_{ij})|\mathbf{0}, \underline{\tau}_2^2 I_{G^2})(1 - \pi_{ij}^{DI})$ and $\phi(x|a, b)$ have the pdf of the Normal distribution with mean a and variance b, evaluated at x.

5. Sampling π_{ij}^{DI} from

$$(\pi_{ij}^{DI})|-) \sim Beta(1 + \sum \gamma_{ij}^{DI}, \underline{\varphi} + \sum(1 - \gamma_{ij}^{DI})), \tag{A.5}$$

6. Sampling γ_{ij}^{CSH} from

$$(\gamma_{ij}^{CSH}|-) \sim Bernoulli(\omega_{ij}^{CSH}), \tag{A.6}$$

where $\omega_{ij}^{CSH} = \frac{\nu_{2,ij}}{\nu_{1,ij}+\nu_{2,ij}}$ with $\nu_{1,ij} = \phi(vec(A_{ii})|vec(A_{jj}), \xi_{ij}^2 \times \underline{c}^{CSH} \times I_{G^2}\pi_{ij}^{CSH}$ and $\nu_{2,ij} = \phi(vec(A_{ii})|vec(A_{jj}), \xi_{ij}^2 \times I_{G^2})(1 - \pi_{ij}^{CSH})$

7. Sampling π_{ij}^{CSH} from

$$(\pi_{ij}^{CSH}|-) \sim Beta(1 + \sum \gamma_{ij}^{CSH}, \underline{\varphi} + \sum(1 - \gamma_{ij}^{CSH})), \tag{A.7}$$

8. Sampling vec(Ψ_{ij}) and Ψ_{kl}^{ii} follow the algorithm of George et al. (2008), which is the $G \times G$ block of the error covariance matrix.

A.2 The PVAR Model with Factor Shrinkage Prior of Canova and Ciccarelli (2009)

The unrestricted PVARs can be shown as:

$$Y_t = Z_t \alpha + \epsilon_t, \tag{A.8}$$

where $\varepsilon_t \sim N(0, \Sigma)$ for $t = 1, \ldots, T$ (no-autocorrelation), α is a $\left[P(MN)^2 \times 1\right]$ vector of all coefficients of endogenous variables in the VAR model and Z_t is a matrix of p lagged endogenous variables. Canova and Ciccarelli (2009) proposed a new prior for dealing with over-parameterisation by factorising each type of parameter - dynamic interdependency and cross-sectional homogeneity - for each country. In order words, this prior extracts one factor for each of the coefficients of its own lags ($\mathbf{\Phi}_i$) and one factor for each of coefficients of other countries' lags ($\mathbf{\Lambda}_i$), as shown by these parameters in Eq.2.2. Therefore, this specification can be shown as:

$$\alpha = \Xi\theta + \upsilon, \tag{A.9}$$

where Ξ is a predetermined factor loading matrix with dimension $K \times s$, θ is a common parameter (factor) vector with $s \times 1$, such that $s \ll K$ and $\upsilon \sim (0, \Sigma \otimes \sigma^2 I)$. This means that $\alpha|\Sigma \sim N(\Xi\theta, \Sigma \otimes \sigma^2 I)$. Additionally, the factor loadings, Ξ, contain zeros and ones, which are associated with the coefficients that are grouped by the same country or by the same variables for different countries. This study assumes $F = 3$, which are (i) a common factor for coefficients of each country (N), (ii) a common factor for coefficients of each variable (M) and (iii) a common factor for all VAR coefficients/a vector of lag-specific factors.

Appendix B

Chapter 3 Appendix

B.1 Data Description

This study is essentially interested in the economic cycle of the members of the Association of Southeast Asian Nations (ASEAN). The members of ASEAN consist of ten countries located in Southeast Asia, namely Brunei, Cambodia, Indonesia, Laos, Malaysia, Myanmar, Philippines, Singapore, Thailand and Vietnam.[1] Due to the limitation of data available, there are only five ASEAN countries used here - Indonesia (ID), Malaysia (MY), Philippines (PH), Singapore (SG) and Thailand (TH). However, these five countries are good representatives of the whole ASEAN membership for two reasons: 1) These are the five original members who founded ASEAN on 8th August, 1967 (ASEAN Secretariat, 2020) and 2) the combined GDP of these five countries is approximately 85 percent of the total GDP of all ASEAN states in 2019 (The World Bank Group, 2022). In addition, the study examines the relationship between the economic cycle of ASEAN members and the four big trading partners of ASEAN (China (CN), Korea (KR), Japan (JP) and the United States (US)). These countries are worth examining because the

[1] Despite the fact that there are eleven countries in this region; Timor-Leste, one of Asia's newest nations, has not yet become part of ASEAN.

share of the trading value in goods between ASEAN and these four countries is around 42 percent of the trading value between ASEAN and the whole world (ASEANstats, 2018). Moreover, the trends of the GDP (in log form) of these ASEAN members, ASEAN+3 (ASEAN plus CN, KR and JP) and the world are in an upward parallel trend, while the US trend is also upward but with a slower rate, as presented in the following figure. The figure also shows the fact that the GDP value of the US was equal to the GDP value of ASEAN+3 in 2013 and has been lower than the GDP value of ASEAN+3 since then.

Figure B.1: GDP in Natural logarithm

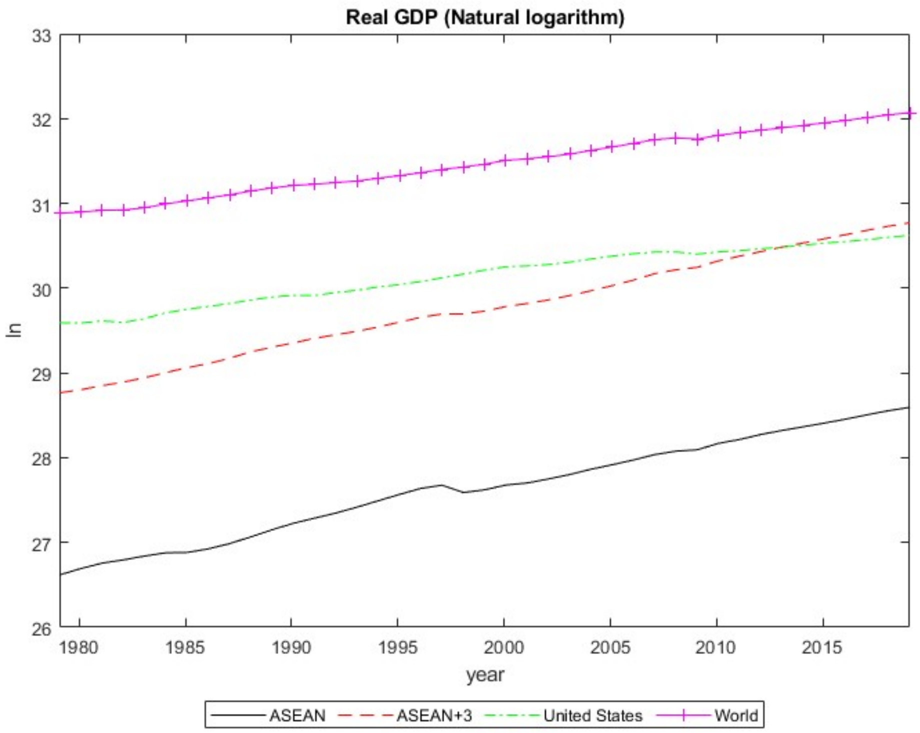


Figure B.2: GDP growth

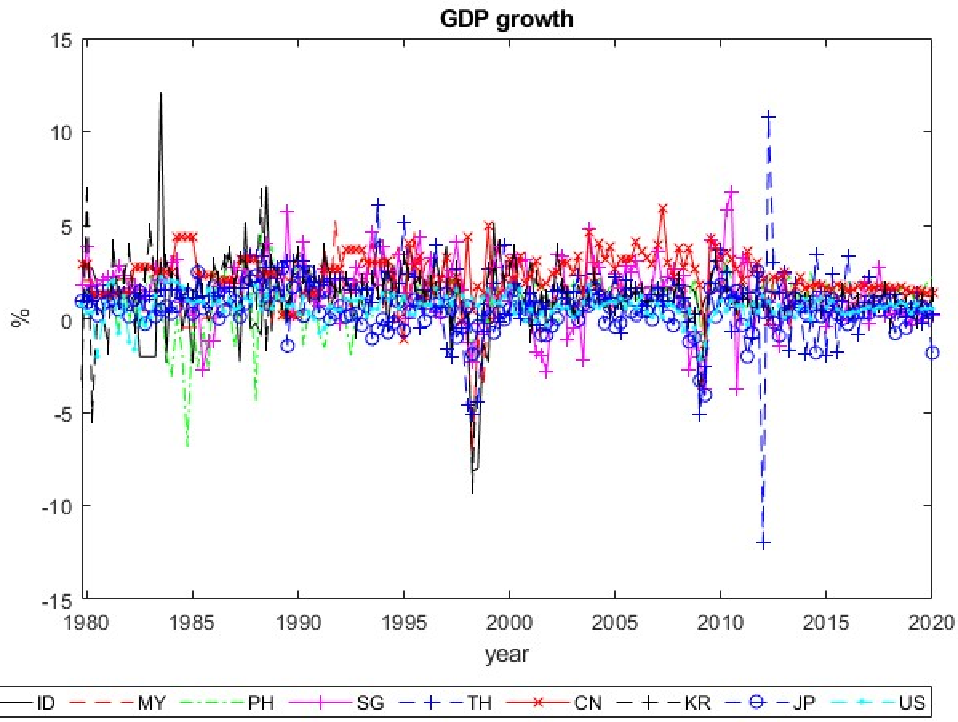


Figure B.3: Term spread

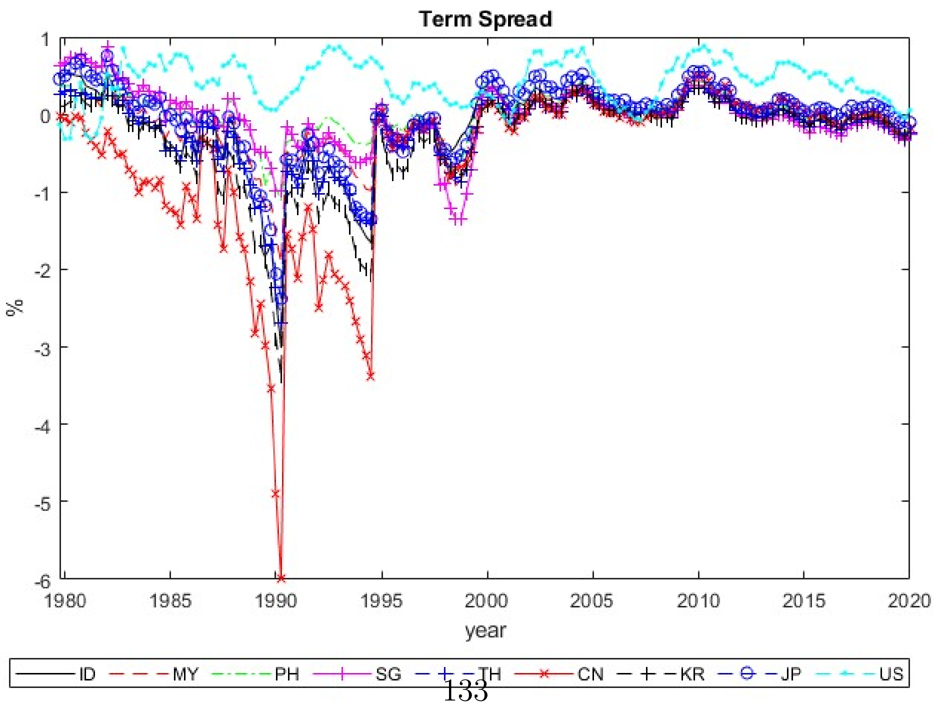


Figure B.4: Stock price growth

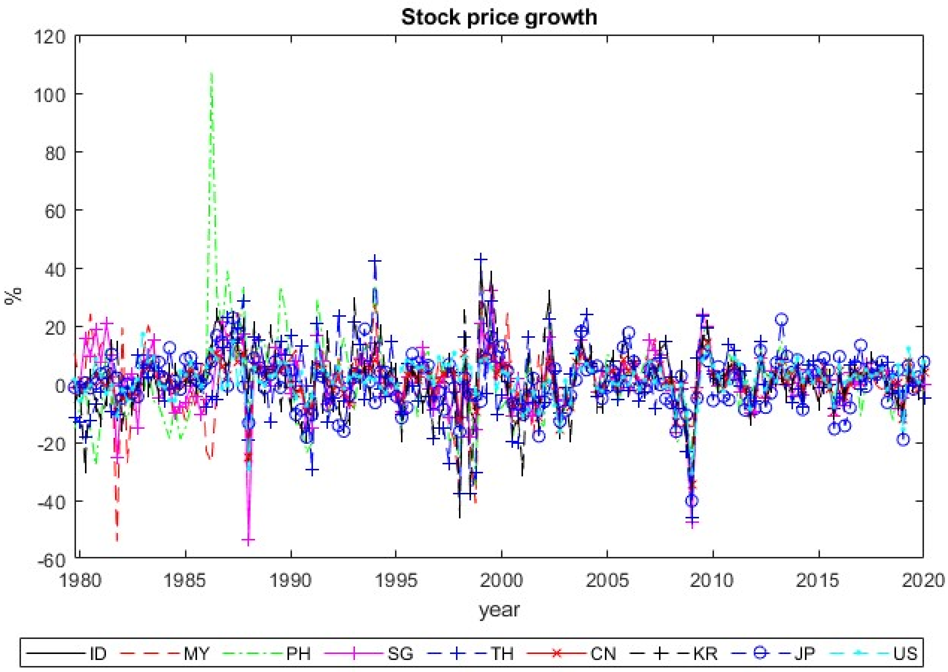
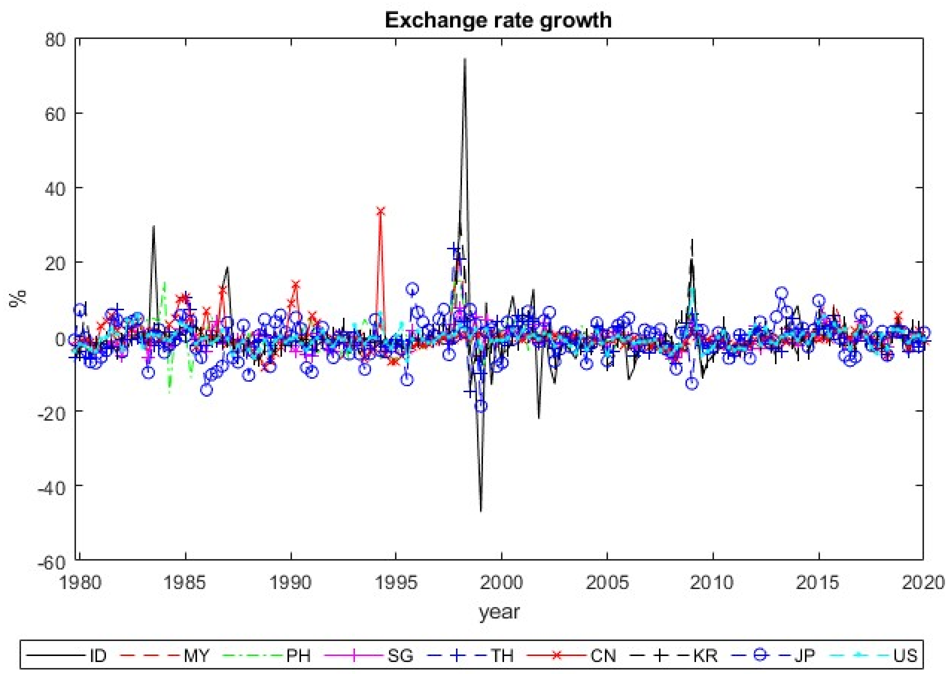


Figure B.5: Exchange rate growth



B.2 Data Sources and Transformations

The four main variables of interest in this study are Real Gross Domestic Product (Real GDP - labelled as $rgdp_{it}$), Interest Rate Term Spread (ts_{it}), Changes in Stock Price (eq_{it}) and Changes in Foreign Exchange Rate with the US dollars (ep_{it}). In addition, the study includes the oil price index as an exogenous variable (oil_{it}). The data set used here is from the global VAR (GVAR) database which includes quarterly macroeconomic variables for 33 economies (accounting for more than 90 percent of world GDP) from 1979Q2 to 2019Q4. However, this study uses quarterly data from 1979Q3 to 2019Q4 (40 years and three quarters - 163 quarters in total); the first quarter of the year 1979 is not included as the data in this year is used for measuring the change in the stock price and changes in the exchange rate in the following year.

Data sources and transformations in the study can be highlighted in the following table.

Table B.1: Data Sources and Transformations

Variables	Description	Source	Trans.
$rgdp_{it}$	Real GDP, seasonally-adjusted	IFS	Δln
ts_{it}	The difference between interest rates on long- and short-term	IFS	levels
eq_{it}	The nominal stock price index deflated by CPI	HA	Δln
ep_{it}	The exchange rate of country i at time t expressed in US dollars deflated CPI of country i.	IFS	Δln
oil_t	The oil price index (a Brent crude oil price in US dollars)	Bloomberg	Δln

Note: IFS=the International Monetary Funds International Financial Statistics (IFS) database. HA=Haver Analytics.

Appendix C

Chapter 4 Appendix

C.1 The Mixed-Frequency PVAR Block

From the measurement equation (4.8), lags of $y_{m,t} = x_{m,t}$ can be separated from lags of $x_{q,t} = \tilde{y}_{q,t}$ and the $Npn_m \times 1$ vector $z_{m,t}$ and $Npn_q \times 1$ vector $z_{q,t}$ defined as

$$z^{i'}_{m,t} = [x^{i'}_{m,t}, \ldots, x^{i'}_{m,t-p+1}], \quad z^{i'}_{q,t} = [x^{i'}_{q,t}, \ldots, x^{i'}_{q,t-p+1}]. \tag{C.1}$$

where i is a country for $i = 1, \ldots, N$. In this case, let $i = 1, \ldots, 4$ and $p = 1, \ldots, 6$. In this regards, the $n_m \times Npn_m$ matrix Φ_{mm}, the $n_m \times Npn_q$ matrix Φ_{mq}, the $n_q \times Npn_m$ matrix Φ_{qm}, and $n_q \times Npn_q$ matrix Φ_{qq} are defined. They can be written as

$$\begin{bmatrix} x^i_{m,t} \\ x^i_{q,t} \end{bmatrix} = \begin{bmatrix} \Phi_{mm} & \Phi_{mq} \\ \Phi_{qm} & \Phi_{qq} \end{bmatrix} \begin{bmatrix} z^i_{m,t-1} \\ z^i_{q,t-1} \end{bmatrix} + \begin{bmatrix} \Phi_{mc} \\ \Phi_{qc} \end{bmatrix} + \begin{bmatrix} \mu^i_{m,t} \\ \mu^i_{q,t} \end{bmatrix} \tag{C.2}$$

Because $t \leq T_b$ monthly series can be observed, these variables, (C.1) and (C.2), are $y_{m,t} = x_{m,t}$ and $z^i_{m,t-1} = y^i_{m,t-p:t-1}$. Let $s^i_t = [x^{i'}_{q,t}, z^{i'}_{q,t-1}]'$ and $\Gamma_s, \Gamma_{zm}, \Gamma_c, \Gamma_u$ be defined for the purpose of rewriting the state-transition equation in a companion form, as

$$s^i_t = \Gamma_s s^i_t + \Gamma_{zm} y^i_{m,t-p:t-1} + \Gamma_c + \Gamma_u u^i_{q,t}, \quad i = 1, \ldots, N. \tag{C.3}$$

The measurement equation of the monthly variables can be written as

$$y^i_{m,t} = \Lambda_{ms} s^i_t + \Phi_{mm} y^i_{m,t-p:t-1} + \Phi_{mc} + u^i_{m,t}, \quad i = 1, \ldots, N. \tag{C.4}$$

Next, the measurement equation of the quarterly variables can be written as

$$y^i_{q,t} = M_{q,t} \Lambda_{qs} s^i_t \quad i = 1, \ldots, N. \tag{C.5}$$

where the matrix of $\Lambda_{qs} s^i_t$ averages $x^i_{q,t}$, $x^i_{q,t-1}$, $x^i_{q,t-2}$ and $M_{q,t}$ is a selection matrix that selects the elements of $\Lambda_{qs} s^i_t$ that are observed in period t. Equations (C.3) - (C.5) are alternative state-space representations that reduce the dimension of the state vector from Nnp to $Nn_q(p+1)$. The lagged $y^i_{m,t-p:t-1}$ is directly entered into both the state and measurement equations. The Kalman filter and simulation smoother can be used straightforwardly due to the observables, which rely on the $t-1$ information. For more information on the Kalman filter algorithm, see the works of Carter and Kohn (1994) and Schorfheide and Song (2015).

C.2 The Stochastic Search Variable Selection (SSVS) prior

The prior can be shown as

$$\alpha_k | \gamma_k \sim (1 - \gamma_k) N(0, \tau_1^2) + \gamma_k N(0, \tau_2^2), \tag{C.6}$$

$$\gamma_k \sim Bernoulli(\pi_k) \tag{C.7}$$

where $\tau_1^2 = 0.001$ $\tau_2^2 = 4$ and $\pi_k = 0.5$ is set for all k, as per Korobilis (2016).

C.3 The Stochastic Search Specification Selection (S^4) prior

DI, SI and CSH restrictions can be defined as $\gamma = \{\gamma^{DI}, \gamma^{SI}, \gamma^{CSH}\}$ and the MCMC by Gibb sampler algorithm used for the SSSS algorithm by drawing of the block of lagged coefficients (γ^{DI}), the block of the error covariance matrix (γ^{SI}) and the block of identical and non-identical dynamic movements of endogenous variables for each country (γ^{CSH}). In accordance with the MCMC algorithm, the estimated probability is imposed - that the appropriate element of γ is to be greater than 0.5.

1. DI prior:

$$vec(A_{ij}) \sim (1-\gamma_{ij}^{DI})N(0, \tau_{ij}^2 \times \underline{c}^{DI} \times I_{M^2}) + \gamma_{ij}^{DI} N(0, \tau_{ij}^2 \times I_{M^2}); \tag{C.8}$$

$$\tau_{ij}^{-2} \sim Gamma(1, \underline{\theta}^{DI}); \tag{C.9}$$

$$\gamma_{ij}^{DI} \sim Bernoulli(\pi_{ij}^{DI}); \tag{C.10}$$

$$\pi_{ij}^{DI} \sim Beta(1, \underline{\varphi}); \tag{C.11}$$

where $i = 1, ..., N$, $j = 1, ..., N-1$ and $i \neq j$, except when applied to A_{ii} and A_{jj}. If $\gamma_{ij}^{DI} = 0$, then the coefficients on the lags of all country j variables in the VAR for country i are set to zero.

2. SI prior:

$$\begin{aligned} vec(\Psi_{ij}) \sim & (1-\gamma_{ij}^{SI})N(0, \kappa_{ij}^2 \times \underline{c}^{SI} \times I_{M^2}) \\ & + \gamma_{ij}^{SI} N(0, \kappa_{ij}^2 \times I_{M^2}); \end{aligned} \tag{C.12}$$

$$\kappa_{ij}^{-2} \sim Gamma(1, \underline{\theta}^{SI}); \tag{C.13}$$

$$\gamma_{ij}^{SI} \sim Bernoulli(\pi_{ij}^{SI}); \tag{C.14}$$

$$\pi_{ij}^{SI} \sim Beta(1, \underline{\varphi}); \tag{C.15}$$

where $i = 1, ..., N$, $j = 1, ..., N-1$, $i \neq j$ and $i > j$. If $\gamma_{ij}^{SI} = 0$, then the block of the PVAR error covariance matrix relating to the covariance between countries i and j is set to zero.

3. CSH prior:

$$\begin{aligned} vec(A_{ii}) \sim & (1-\gamma_{ij}^{CSH}) N(A_{jj}, \xi_{ij}^2 \times \underline{c}^{CSH} \times I_{M^2}) \\ & + \gamma_{ij}^{CSH} N(A_{jj}, \xi_{ij}^2 \times I_{M^2}); \end{aligned} \tag{C.16}$$

$$\xi_{ij}^{-2} \sim Gamma(1, \underline{\theta}^{CSH}); \tag{C.17}$$

$$\gamma_{ij}^{CSH} \sim Bernoulli(\pi_{ij}^{CSH}); \tag{C.18}$$

$$\pi_{ij}^{CSH} \sim Beta(1, \underline{\varphi}); \tag{C.19}$$

where $i = 1, ..., N$, $j = 1, ..., N-1$ and $i \neq j$ such that A_{ii} and A_{jj} are not the same matrix.

Additionally, no restrictions are imposed upon the error covariances prior.

$$\Psi_{kl}^{ii} \sim \begin{cases} N(0, \underline{\kappa}_2^2), & \text{if} \quad k \neq l \\ Gamma(\underline{\rho}_1, \underline{\rho}_2) & \text{if} \quad k = l \end{cases} \tag{C.20}$$

where $k, l = 1, ..., M$ indexes each of the M macro variables of country $i = 1, ..., N$. The study imposes a matrix $\Gamma = \prod_{i=1}^{N-1} \prod_{j=i+1}^{N} \Gamma_{ij}$ where Γ_{ij} represents the matrices $K \times K$ for using CSH restriction indicators (γ_{ij}^{CSH}). The prior hyperparameters of the model are $\underline{c}^{DI}, \underline{c}^{CSH}, \underline{c}^{SI}, \underline{\theta}^{DI}, \underline{\theta}^{CSH}, \underline{\theta}^{SI}, \underline{\varphi}, \underline{\kappa}_2^2, \underline{\rho}_1, \underline{\rho}_2$, which are built on Koop and Korobilis (2016).

C.4 The Minnesota-Adaptive Hierarchical (MNG) prior

Chan (2021) rewrites the structural form of the BVAR as the approach in Carriero, Clark and Marcellino (2019), which imposes the equation-by-equation as a system of n independent regressions. This approach substantially speeds up computations by introducing the reduced-form parameterisation with the time-varying on the diagonal of the error covariance matrices. This prior can be shown as below.

$C_{i,j}$ are defined as positive constants. Then, the following prior on $\theta_i|j$ can be considered as:

$$(\theta_{i,j}|\kappa_1, \kappa_2, \psi_{i,j}) \sim N(m_{i,j}, \kappa_{i,j}\psi_{i,j}C_{i,j}), \tag{C.21}$$

where $\kappa_{i,j} = \kappa_1$ for coefficient on own lags, $\kappa_{i,j} = \kappa_2$ for the coefficients on other lags and $\kappa_{i,j} = 1$ otherwise. For $\psi_{i,j}$, the suitable distribution can be chosen as the global-local priors $\psi_{i,j} \sim F_\psi\psi(i;j)$. Chan (2021) uses the global-local priors by adapting the normal-gamma prior of Huber and Feldkircher (2019) incorporating the Minnesota prior, named as the Minnesota-Type Normal-Gamma prior (MNG).

Regarding $C_{i,j}$, these can be verified as the coefficients of VAR, by defining $\theta_{i,j}$ and $j = 1, ..., k_i$ as follows:

$$C_{i,j} = \begin{cases} \dfrac{1}{l^2}, & \text{for the coefficient on the } l\text{-th lag of variables } i, \\ \dfrac{s_i^2}{l^2 s_j^2}, & \text{for the coefficient on the } l\text{-th lag of variables } j, j \neq i, \\ \dfrac{\kappa_3 s_i^2}{s_j^2}, & \text{for the } j\text{-th element of } \alpha_i, \\ \kappa_4 s_i^2, & \text{for the intercept.} \end{cases} \tag{C.22}$$

C.5 The Dynamic Factor Model (DFM)

Let $y_t^M = (y_{1,t}^M, y_{2,t}^M, ..., y_{n,t}^M)'$ be the monthly stationary variables. Therefore, the factor model can be represented by:

$$y_t^M = \mu + \Lambda f_t + \epsilon_t, \tag{C.23}$$

$$f_t = A_1 f_{t-1} + \upsilon_t, \qquad \upsilon_t \sim i.i.d.N(0, Q), \tag{C.24}$$

$$\epsilon_{i,t} = \alpha_i \epsilon_{i,t-1} + e_{i,t}, \qquad e_{i,t} \sim i.i.d.N(0, \sigma_i^2), \tag{C.25}$$

where f_t is a vector of unobserved (common) factors with the dimension of $r \times 1$ and ϵ_t is a vector of idiosyncratic components with $\mathbb{E}[e_{i,t}e_{j,s}] = 0$ for $i \neq j$. . Let Λ denote the loading factors for the monthly variables. $\mu = (\mu_1, \mu_2, ..., \mu_n)'$ are the unconditional zero means.

Quarterly variables can be incorporated following Mariano and Marasawa (2003) by approximating observed monthly GDP data:

$$y_t^Q = \begin{cases} y_t + 2y_{t-1} + 3y_{t-2} + 2y_{t-3} + y_{t-4}, & \text{for } t = 3, 6, 9, ... \\ \text{unobserved}, & \text{otherwise} \end{cases} \tag{C.26}$$

Furthermore, the factor model of the unobserved monthly real GDP growth variables can be presented as:

$$y_t^Q = \mu_Q + \Lambda_Q f_t + \epsilon_t^Q, \tag{C.27}$$

$$\epsilon_t^Q = \alpha_Q \epsilon_{t-1}^Q + e_t^Q, \qquad e_t^Q \sim i.i.d.N(0, \sigma_Q^2), \tag{C.28}$$

In this regard, a state-space can be represented by defining $\overline{y}_t = (y_t^{M'}, y_t^Q)'$ and $\overline{\mu} = (\mu', \mu_Q)'$:

$$\overline{y}_t = \overline{\mu} + Z(\theta)\alpha_t, \tag{C.29}$$

$$\alpha_t = T(\theta)\alpha_{t-1} + \eta_t, \qquad \eta_t \sim i.i.d.N(0, \Sigma_\eta(\theta)), \tag{C.30}$$

where $\alpha_t = (f_t', \epsilon_{i,t}, ..., \epsilon_{n,t}, \epsilon_t^Q)'$ and all parameters of the model are contained in $\theta = (\mu, \mu_Q, \Lambda, \Lambda_Q, A_1, \alpha_Q, \alpha_1, ..., \alpha_n, \sigma_1, ..., \sigma_n, \sigma_Q)$. Regarding the state-space representation, the parameters (θ) can be estimated by the Expectation Maximisation (EM) algorithm. The basic principle of the EM algorithm is to write the likelihood of both observable and latent variables given the available information, $t = 1, ..., T_v = max_i T_{i,v}$ and maximise the likelihood function in two steps, E-M steps. The log-likelihood can be written in terms of $y = (y_1, y_2, ..., y_{T_v})$, $\Omega_v \subseteq \{y_1, y_2, ..., y_{T_v}\}$ and $f = (f_1, f_2, ..., f_{T_v})$ as $l(y, f; \theta)$. Therefore,

$$\text{E-step:} \qquad L(\theta, \theta(j)) = \mathbb{E}_{(\theta(j))}[l(y, f; \theta|\Omega_v], \tag{C.31}$$

$$\text{M-step:} \qquad \theta(j+1) = \arg\max_\theta L(\theta, \theta(j)). \tag{C.32}$$

In order to clearly understand these, E-step is a procedure of the expectation of the log-likelihood conditional on the data, calculated using the estimates from the previous iteration, $\theta(j)$, whereas M-step is a process of the new parameters, $\theta(j+1)$, estimated through the maximization of the expected log-likelihood (from the previous iteration) with respect to θ.

In order to handle missing observations in $\overline{y}_t$, Banbura and Modugno (2014) proposed the use of selection matrices W_t and W_t^Q. These matrices are diagonal, with W_t having a size of n and W_t^Q having a size of 1. Within these matrices, the elements are set to one for the non-missing values in y_t^M and y_t^Q, respectively.

Next, the E-M algorithm can be concluded as:
M-step: The maximization of the expected likelihood with respect to $\theta(j)$:

- The matrix of loadings for the monthly variables:

$$\text{vec}(\Lambda(j+1)) = (\sum_{t=1}^{T} \mathbb{E}_{\theta(j)}[f_t f_t'|\Omega_v] \otimes W_t)^{-1} \text{vec}(\sum_{t=1}^{T} W_t y_t^M \mathbb{E}_{\theta(j)}[f_t'|\Omega_v] + W_t \mathbb{E}_{\theta(j)}[\epsilon_t f_t'|\Omega_v]), \tag{C.33}$$

- The matrix of loadings for the quarterly variables:

Let $f_t^{(p)} = [f_t', ..., f_{t-p+1}']'$ and $D = \sum_{t=1}^{T} \mathbb{E}_{\theta(j)}[f_t f_t'|\Omega_v] W_t^Q$. The unrestricted $\overline{\Lambda}_Q = (\Lambda_Q \, 2\Lambda_Q \, 3\Lambda_Q \, 2\Lambda_Q \, \Lambda_Q)$ is expressed by

$$\text{vec}(\overline{\Lambda}_Q^{ur}(j+1)) = D^{-1}(\sum_{t=1}^{T} W_t^Q y_t^Q \mathbb{E}_{\theta(j)}[f_t'|\Omega_v]), \tag{C.34}$$

In terms of the restricted $\overline{\Lambda}_Q$, it holds $C\overline{\Lambda}_Q = 0$ with

$$C = \begin{bmatrix} I_r & -\frac{1}{2}I_r & 0 & 0 & 0 \\ I_r & 0 & -\frac{1}{3}I_r & 0 & 0 \\ I_r & 0 & 0 & -\frac{1}{2}I_r & 0 \\ I_r & 0 & 0 & 0 & -I_r \end{bmatrix}$$

Therefore, the restricted $\overline{\Lambda}_Q$ is given by:

$$\overline{\Lambda}_Q(j+1) = \overline{\Lambda}_Q^{ur}(j+1) - D^{-1}C'(CDC')^{-1}C\overline{\Lambda}_Q^{ur}(j+1) \tag{C.35}$$

- The autoregressive coefficients in the factor VAR is given by:

$$A_1(j+1) = (\sum_{t=1}^{T} \mathbb{E}_{\theta(j)}[f_t f_{t-1}'|\Omega_v])(\sum_{t=1}^{T} \mathbb{E}_{\theta(j)}[f_{t-1} f_{t-1}'|\Omega_v])^{-1}, \tag{C.36}$$

- The covariance matrix in the factor VAR is given by:

$$Q(j+1) = \frac{1}{T}(\sum_{t=1}^{T} \mathbb{E}_{\theta(j)}[f_t f_t'|\Omega_v]) - A_1(j+1)\sum_{t=1}^{T} \mathbb{E}_{\theta(j)}[f_{t-1} f_t'|\Omega_v], \tag{C.37}$$

• The autoregressive coefficients in the AR representation for the idiosyncratic component of the monthly variables is given by:

$$\alpha_i(j+1) = (\sum_{t=1}^{T} \mathbb{E}_{\theta(j)}[\epsilon_{i,t}\epsilon_{i,t-1}|\Omega_v])(\sum_{t=1}^{T} \mathbb{E}_{\theta(j)}[(\epsilon_{i,t-1})^2|\Omega_v])^{-1} \quad i = 1, ..., n, \quad \text{(C.38)}$$

• The variance in the AR representation for the idiosyncratic component of the monthly variables is given by:

$$\sigma_i^2(j+1) = \frac{1}{T}(\sum_{t=1}^{T} \mathbb{E}_{\theta(j)}[(\epsilon_{i,t})^2|\Omega_v] - \alpha_i(j+1)\sum_{t=1}^{T} \mathbb{E}_{\theta(j)}[\epsilon_{i,t-1}\epsilon_{i,t}|\Omega_v]) \quad i = 1, ..., n, \quad \text{(C.39)}$$

E-step: The conditional expectations in the M-step, as demonstrated earlier, are obtained by applying the Kalman smoother to the state space equation using the parameters from the previous iteration, $\theta(j)$. Additionally, the initial parameters $\theta(0)$ are obtained by using principal components analysis.