AN INTEGRATED APPROACH TO FATIGUE CRACKING, RELIABILITY AND INSPECTION OF OFFSHORE STRUCTURES

BY

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SUMMARY

This thesis describes an integrated approach to fatigue cracking, reliability and inspection of offshore structures. The basis of the approach is statistical in nature and draws on recent experimental data and field measurements. It is intended as a working tool for those engaged in design, structural appraisal and sub-sea inspection of steel jacket structures.

A review of current practice has been made and the requirements of an integrated approach are established. An approach is proposed comprising a series of compatible models dealing with fatigue cracking, the reliability of cracked joints and the inspection of welds for fatigue cracks. The primary linking parameter is the distribution of fatigue crack size which is considered as a time dependent variable.

An integral part of the approach is a new statistically-based fatigue crack growth model. This is developed and the parameters involved in the model estimated from an analysis of experiment and oceanographic data. For any fatigue calculation the model allows the corresponding fatigue crack growth distribution to be estimated for any time during, or beyond, the nominal fatigue life. A number of example calculations are included; and using one of these a Bayesian procedure for revising fatigue lives in the light of inspection results is demonstrated.

The effect of fatigue cracking upon the various modes of tubular joint failure is considered using linear statistical models. Example calculations are performed for a typical joint.

An inspection strategy is proposed based on the concept of minimising life costs, including risk costs arising from the consequences of possible structural failure. This allows alternative inspection plans to be evaluated and compared, and a typical example calculation is included.

The approach is discussed in the context of possible alternative approaches and areas for further related research are identified.

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CONTENTS

1. INTRODUCTION AND AIMS

1.1 Introduction

All marine structures must be inspected periodically to ensure that they have sufficient strength to continue in service with an acceptable level of reliability. Ships and most other floating structures can readily be inspected internally whilst still afloat, and externally in a dry dock (in most cases). Fixed offshore structures must however, be inspected on site. The vast majority of these fixed structures are of the steel jacket type [1], for which there is no provision for internal access. All inspection is therefore external and is usually undertaken by divers at great expense. Indeed, it has been estimated [2] that subsea inspection and maintenance costs amount to 2% of the first cost of jacket structures per annum; with the largest share going on inspection. Many structures are designed for a life of 25 years or more, and frequently because of secondary oil recovery, are required for longer. So the total cost of inspection and maintenance taken over the structure's whole life may amount to more than half of the original cost.

The majority of subsea inspection work is directed towards finding fatigue cracks which are most likely to occur in the welded connections at tubular joints [3]. This activity alone may involve costs of £1 million or more on a single structure each year; and in the U.K. North Sea sector alone, inspection is a multi-million pound industry. However, discussions with operating companies and certifying authorities reveal that devising requirements and inspection planning in this area present considerable problems.

Firstly the usual fatigue design procedure for welded joints yields a fatigue life expressed in years, see for example [4] and [5]. It does not indicate at what time a detectable crack will first appear or what size of crack may reasonably be expected at any stage during the fatigue life. In principle a fracture mechanics calculation, e.g. [6], can be performed to obtain this information. However, fatigue crack growth is subject to considerable uncertainties and deterministic crack growth calculations are likely to be of limited value and may even be misleading. Therefore there is a need for an improved procedure which should preferably be based on statistical methods.

The second problem relates to the results of subsea weld inspections, and their interpretation and use. The efficiency of both inspection techniques and the diver/inspector in detecting cracks is subject to uncertainty [7]. Many inspections in fact reveal no defects at all. How should these results be interpreted? Given that a crack must be of a certain size to be detected, it cannot be stated that no cracks exist.

When cracks are found, remedial action is taken and inspection levels are increased. Yet when no cracks at all are found year after year, inspection levels are not reduced, despite the views of some inspection engineers. This is partly due to legislative requirements and partly due to the absence of an appropriate qualitative procedure for revising inspection requirements in the light of inspection results.

A number of structures in the North Sea have been found to contain a significant number of cracks. As a result the fatigue lives of the joints in these structures have been re-assessed and revised downwards. Again there are other structures which after many years show no signs whatsoever of fatigue cracking. Yet their fatigue lives are not re-assessed in the light of this inspection information and a valuable opportunity to re-appraise the structure is missed. This particular problem is assuming greater importance for structures now required to operate beyond their original design fatigue lives because with enhanced recovery techniques their oilfields are not exhausted.

When a crack is found one of the first questions posed is how will the structural integrity or reliability be affected if the crack is left to grow and no repair is made? Obviously the reliability will be reduced but will this reduction be significant? In some cases, because of the level of redundancy, minor members can be lost from the structure without significantly affecting the reliability; but this is not always the case. For critical joints fatigue cracking may cause a significant reduction in overall structural reliability. Samples of such joints are usually inspected but because of the inherent inaccuracy of subsea inspection techniques [7] the reported result will be associated with significant uncertainty. For joints that are not inspected the uncertainty will be even greater. Again the need for a statistical procedure to assess the effect of fatigue cracking upon the structural reliability is clear.

The operator must decide what type of inspection equipment should be used and whether or not structural monitoring is a worthwhile and cost-effective undertaking for a particular structure. A scheme for evaluating the merits of various inspection and structural monitoring equipment is therefore desirable.

Perhaps the largest problem area for the engineer responsible for inspection is deciding what to inspect and when, or how frequently. There are, in the UK, certain statutory requirements to be met [8], which may be interpreted and enforced by a Certifying Authority in a more specific manner [9]. Whilst much has been written on the subject, e.g. [10], [11] [12], there does not appear to be a comprehensive quantitative framework in which to develop an inspection strategy.

This thesis addresses these problems. The approach taken has been to develop relatively simple statistical models endeavouring to make the best use of the data currently available. The models are mutually compatible so that the output from one can be used directly for the input of the next. The procedures developed cover:-

a) Estimation of fatigue crack size at any time during and beyond the fatigue life.

b) The revision of crack size and fatigue life estimates in the light of inspection results

c) Estimation of the effect fatigue cracking has upon tubular joint reliability.

d) Evaluation and comparison of alternative inspection strategies.

The models are intended for use by engineers responsible for the inspection of offshore steel jacket structures but could readily be adapted for other marine, and land-based steel structures.

1 2 Aims

The overall aim of this research is to provide an integrated and coherent approach for tackling the associated problems of fatigue cracking, reliability and inspection of offshore structures.

The specific aims are as follows:

(a) To make a critical review of current practice in fatigue design and analysis, structural assessment and inspection planning for offshore structures.

(b) To establish the requirements for an integrated approach to fatigue cracking, reliability and inspection of offshore structures, and to propose a suitable model.

(c) To develop a statistically based fatigue crack growth model which can incorporate inspection results and to test it using typical examples.

(d) To examine the effect of fatigue cracking upon the reliability of tubular joints.

(e) To develop a compatible inspection and maintenance strategy and to examine its practical application.

(f) To discuss the proposed approach and its practical application in the context of alternative approaches.

2. REVIEW OF CURRENT PRACTICE AND THE PROPOSED APPROACH

2.1 Introduction

This chapter begins by examining briefly the current practice in fatigue analysis, structural assessment and inspection of offshore structures. The review does not cover all the latest research in all these areas in detail, although much is discussed in later chapters, but concentrates on the procedures actually used by design engineers, certifying authorities and inspection engineers. Although they influence one another, fatigue analysis, structural assessment and inspection are treated as distinct and separate activities by the offshore industry for most purposes. They are reviewed as distinct activities here.

The links between these activities are discussed after the review, and the requirements for an integrated approach are established. The proposed integrated approach to fatigue cracking, reliability and inspection is then presented.

2.2 Fatigue

In offshore steel structures fatigue cracks are most likely to occur at the joints of the tubular members. The fatigue lives are calculated for all primary joints and most secondary joints. For many joints in the splash zone and in the upper subsea part of the structures fatigue is the limiting criterion. Two different approaches are widely used in fatigue calculations: a deterministic approach and a spectral approach. The deterministic approach, which is used most frequently, involves the following stages:-

(i) Wave scatter diagrams relating wave height, wave period and frequency of occurrence are established for all directions from oceanographic data for the location of the structure.

(ii) The response of the structure to each of the waves in the scatter diagrams is then considered. This usually involves the use of linear wave theory and Morison's equation.

(iii) In cases where the wave frequency may be close to the natural frequency, the nominal stress range is multiplied by a dynamic application factor

(iv) These individual stress range responses are collected together to form a stress range histogram..

(v) Using formulae derived from acrylic model tests [19] or finite element calculations, e.g. [20] stress concentration factors are estimated. These are used to convert the nominal stress ranges to "hot-spot" stress ranges.

(vi) The "hot-spot" stress range histogram is then used with an S - N curve to calculate annual damage using Miner's Rule.

(vii) The nominal fatigue life is taken as the reciprocal of the annual damage.

This procedure is not considered very satisfactory when the natural frequencies of the principal modes of vibration of the structure coincide with frequencies at which there is significant wave energy [69]. In these circumstances a spectral approach is used.

This differs from the deterministic approach in steps (ii), (iii) and (iv). In the spectral approach these steps are as follows:

(ii) Wave spectra are derived for each sea state and each direction; and the probability of each seastate and direction occurring is derived from the wave scatter diagrams.

(iii) Response amplitude operators are obtained using waves of unit amplitude covering the frequency range and directions of interest. These are used with the wave spectra to calculate a series of response spectra.

(iv) The response spectra are then used to estimate the relationship between stress range and number of cycles per annum.

The spectral approach requires the linearisation of the relationship between wave height and wave force or stress. The linearisation procedure commonly adopted is due to Borgman [55] and this has been found to over-estimate forces; at least on the Christchurch Bay Tower [58].

In both approaches the effects of fabrication processes, marine fouling, the sea water environment and other factors can be considered, e.g. [32] [69] [86]. A number of different S - N curves are currently in use from the American Welding Society, the American Petroleum Institute, British Standards Institution and the Department of Energy amongst others. These and other aspects of the fatigue design of tubular joints are comprehensively discussed in[69].

Fatigue crack growth calculations are not generally made at the design stage for offshore structures, although Det norske Veritas at least do provide rules for such calculations [4]. Until recently suitable data for fatigue crack growth calculations did not exist, but the United Kingdom Offshore Steel Research Project, and similar projects in Europe, have recently provided some data in this area. Crack growth calculations are however, sometimes performed for an existing structure when a fatigue crack is found.

A comprehensive review of the research in the fatigue analysis of offshore structures would be a mammoth task, involving well over a thousand papers, so it is not attempted here. An adequate review is made in the three volumes on the "Design of tubular joints for offshore structures" produced by the U.E.G. in 1985 [69].

2.3 Structural Assessment

The assessment of offshore structures is based largely on traditional safety factors following the procedures well described by Pugsley [59]. However, the partial safety factor approach, e.g. [16], has been introduced by D.N.V. for some limit states [4] but fatigue is not one of them.

Considerable work has been undertaken recently on probabilistic approaches to fatigue [5] [60]. These allow fatigue lives to be expressed as distributed parameters rather than as single nominal values. The probability of a given joint or weld failing before any particular time can then be estimated.

The effect of fatigue cracking upon the reliability of tubular joints has been considered for the case when fatigue crack growth leads to brittle fracture. Some work in this area has been undertaken by Baker [90], and by Smith et al [72]. However, this has involved deterministic fatigue crack growth calculations and cannot therefore be considered as reliability analysis in the conventional sense. The effect of fatigue cracking upon other modes of failure has not, to the author's knowledge, been considered.

For the majority of offshore structures the design fatigue lives are considered adequate if they exceed the structure's nominal operational life by a certain factor (factor of safety on life). Such fatigue lives are generally calculated using conservative assumptions; S - N curves, in particular, implicitly contain a further factor of safety e.g. [21].

2.4 Inspection

In the U.K. sector of the North Sea all structures are required to have a current "Certificate of Fitness". The Department of Energy issue guidance notes which indicate the standards that have to be met. A number of Certifiying Authorities act on behalf of the D. En. and carry out certification work and these include Lloyd's and D.N.V. Inspection programmes are decided directly between the operators and these certifying authorities using the D.En guidelines. D.N.V. have interpreted the guidelines in a partiuclar manner and produced their own rules for inspection [9].

The D.N.V. rules cover many aspects of inspection, including the following:-

(a) Managing inspection programmes

(b) Qualifications of personnel

(c) Approval of equipment

(d) Types of inspection to be undertaken, items to be inspected and sequence of inspection.

(e) Recording and reporting procedures.

They classify inspections into three categories:

Type I - General visual inspection: usually undertaken for the whole structure

Type II - Close visual inspection to detect hidden damage after prior cleaning.

Type III - Close visual and non-destructive testing to detect incipient or hidden damage. Again prior cleaning is undertaken if necessary.

It is required that the whole structure be inspected once during every five year period. Although the percentage of each type of inspection is not stated explicitly, the evaluation of the inspection results is specified in the rules, however it is not clear exactly how this should be done. More detailed information emanating from D.N.V. is given by Sletten et al in reference [11]. This enumerates the characteristics of "significant points" which require detailed inspection and includes the following:

- high member stress
- high interaction ratio
- low estimated fatigue life
- joint complexity
- fabrication inspection problems.

To select critical joints, a weighting procedure is suggested in which each characteristic factor is considered in turn, and a weight attributed according to importance. The joints with the highest overall weighting are then inspected using N.D.T. These are usually the most crucial joints with the shortest fatigue lives.

The N.D.T. technique most commonly used is magnetic particle inspection or M.P.I. which can measure crack length but not crack depth. Crack depth can be measured by ultrasonics but this in practice is found to be unreliable [95].

A typical weld inspection programme may comprise the following:

(a) Cleaning and close visual inspection of all joints designated as significant or critical.

(b) M.P.I. of all these joints.

(c) Photographic recording of all joints inspected.

(d) Visual inspection without cleaning of the entire structure, to check for gross cracks, using an R.O.V.

This type of weld inspection programme is quite widely adopted, e.g. [75], [95], [97].

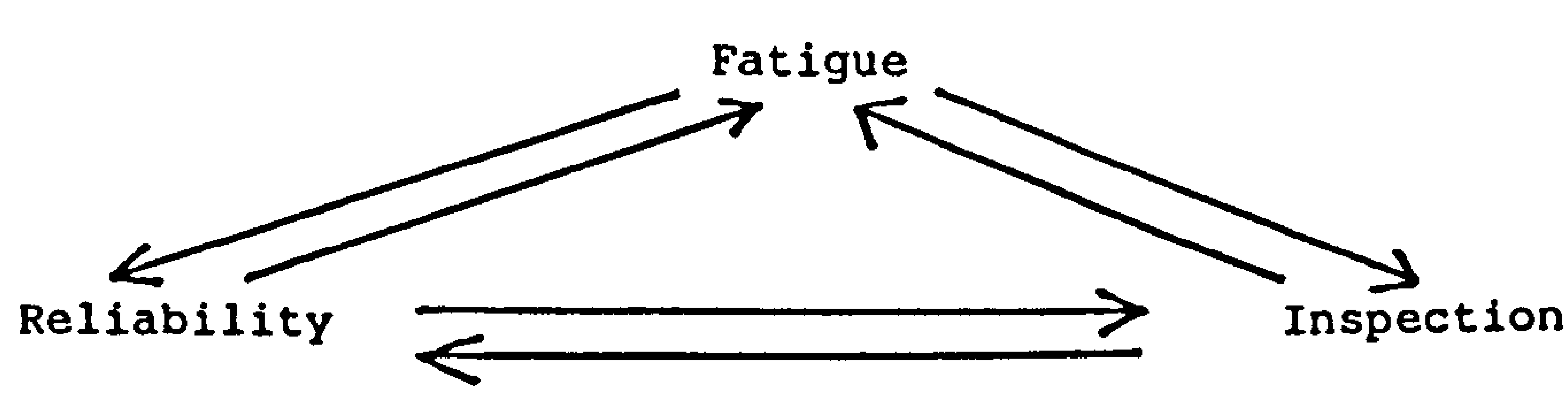
Faulds [95] concludes "that considerable effort is required to establish a less subjective method of arriving at inspection programmes particularly in relationship to weld inspection".

Some work has been done in this area by D.N.V. staff [11] [99] who have developed an optimisation approach. This relates all aspects of inspection planning to a common cost basis. The parameters used in the model are costs of inspection, cost consequences of failure and probabilities of failure. The value in the cost model is minimised subject to a constraint on minimum acceptable reliability. The approach seems logical but there are no means provided for estimating the input parameters.

Marshall [12] has also outlined an approach based on minimising the total life cost, where risk costs (cost consequences of failure multiplied by probability of failure) are included in the considerations. The details of the approach have not been developed in the paper only the broad philosophy. It does not deal explicitly with the problems of what to inspect, when to inspect and how much to inspect, or the revision of fatigue lives in the light of inspection results.

2.5 Requirements for an Integrated Approach

The problem with the current practice is that there are no common linking parameters between fatigue, reliability and inspection. Fatigue lives are expressed in years; reliability, sometimes as a probability, but more frequently as a non-dimensional factor of safety; and inspection is concerned with the size of cracks. To enable interaction to occur, meaningful parameters must be passed between these activities as depicted below.



Suitable linking parameters are now discussed in turn:

(a) crack size - The output from inspection is either a crack (or "defect indication") size estimate, or a report of no cracks found. Clearly then crack size must be selected as a linking parameter or the output of inspection cannot be used in a quantitative manner.

There is no point in making an inspection unless there is some chance of finding a crack. The probability distribution of crack size at any time is therefore a useful parameter linking fatigue calculations and inspection.

At any given time the reliability of a tubular joint will depend on the size of any fatigue crack it may contain. Crack size therefore is a link between fatigue and reliability. Similarly if inspection reveals a large crack the corresponding reliability of the damage joint will be sought and the fatigue life reassessed in light of this result.

(b) Probability of failure - If a joint is of primary importance to a structure, then the possibility of occurrence of a crack in the joint will increase the probability of overall structural failure significantly. Such a joint would warrant close inspection. Probability of failure, and its change with fatigue crack size and time, is therefore an important linking parameter between reliability and inspection. At the design stage it might be thought desirable to improve the fatigue life of such a joint. Probability of failure is therefore a useful link between reliability and fatigue.

(c) Cost - Inspection costs money; but then so does a failure which occurs through lack of inspection and the associated maintenance. In reliability studies, the cost consequences of failure and the associated risk costs, assume an important role. In order to trade off inspection costs against risk cost; cost must be a linking parameter between reliability and inspection.

A further requirement for any integrated approach is a common basis for decision making. The overall objective of the approach here is to contribute towards ensuring that offshore structures can be designed, built and operated economically and safely. Cost and reliability either singly or in combination, therefore provide suitable bases for decision-making.

The question arises as to whether the approach should be deterministic or statistical in character? Now fatigue is subject to considerable uncertainties [5], [28], [35] and it is therefore desirable to treat it in a probabilistic or, preferably, statistical manner. Reliability is intrinsically a statistical problem. Inspection of welds subsea is subject to substantial uncertainties [7], [30], [66] and hence it should also be treated in a statistical way. As has been discussed, many of the linking parameters, to be useful, have to be expressed in a statistical form. Clearly then there is a requirement for a statistically based approach.

There are a number of desirable features for any statistical model (or set of statistical models). The principal amongst these are now discussed in the context of the approach under consideration.

(i) Unbiasedness - Every model should represent reality without bias if possible. Repeated use of the model should not give estimates which are either systematically higher or lower than the real value. This can lead to unknown and unintended conservatism and waste of resources. It can also lead to unintended risk-taking and catastrophic failures [59]. Neither is desirable.

(ii) Precision - Even in statistical models precise estimates are desirable. However, in this particular case, given both the intrinsic uncertainties and lack of complete knowledge of the physical processes involved, the precision of estimates will not be high. But as they are statistical models, uncertainties should be quantified, albeit not completely.

(iii) Use of Data - The models should make the best use of the most appropriate and relevant data.

(iv) Flexibility - The models should be capable of easy adaption to suit a range of situations and changing needs. The models should be capable of incorporating new knowledge and data as they become available.

Finally, and perhaps the over-riding requirements of any approach, are that it should be easy to understand, easy to use and yield useful results.

2.6 The Proposed Approach

The proposed approach comprises a series of compatible models designed to meet the requirements established in the previous section. The models are all statistical in nature, and use data generated by research and field measurements offshore. The models are now described in turn:

(a) Fatigue crack growth model - For every fatigue life calculation this model allows the equivalent crack size distribution to be estimated at any time during the fatigue life of the joint. The model has an intrinsically linear nature which gives it statistical robustness and allows it to be handled in a very general way using Taylor series e.g. [56]. The model is essentially simple with just three independent distributed parameters. However, additional linear terms can be included, if necessary, to cater for other factors which, as a result of research, may be found to influence crack growth.

The crack size distribution outputs from this model can be used directly in the reliability and inspection models. In addition inspection results can be incorporated into the model using a Bayesian approach, to revise crack size distributions and fatigue lives.

(b) Reliability of Cracked Joints - This model allows the effect of a fatigue crack upon the capability of tubular joint to resist various modes of failure to be estimated. The modes of failure considered cover all those likely to be affected by a growing fatigue crack. The model allows the variation in reliability for the various modes of failure to be examined as a fatigue crack grows with time.

It is again of the linear statistical type described above and has the same attributes. The probability of failure is calculated through the convolution integral. In structural reliability jargon, the model, although simple, corresponds to a "Level-3" approach, e.g. [78].

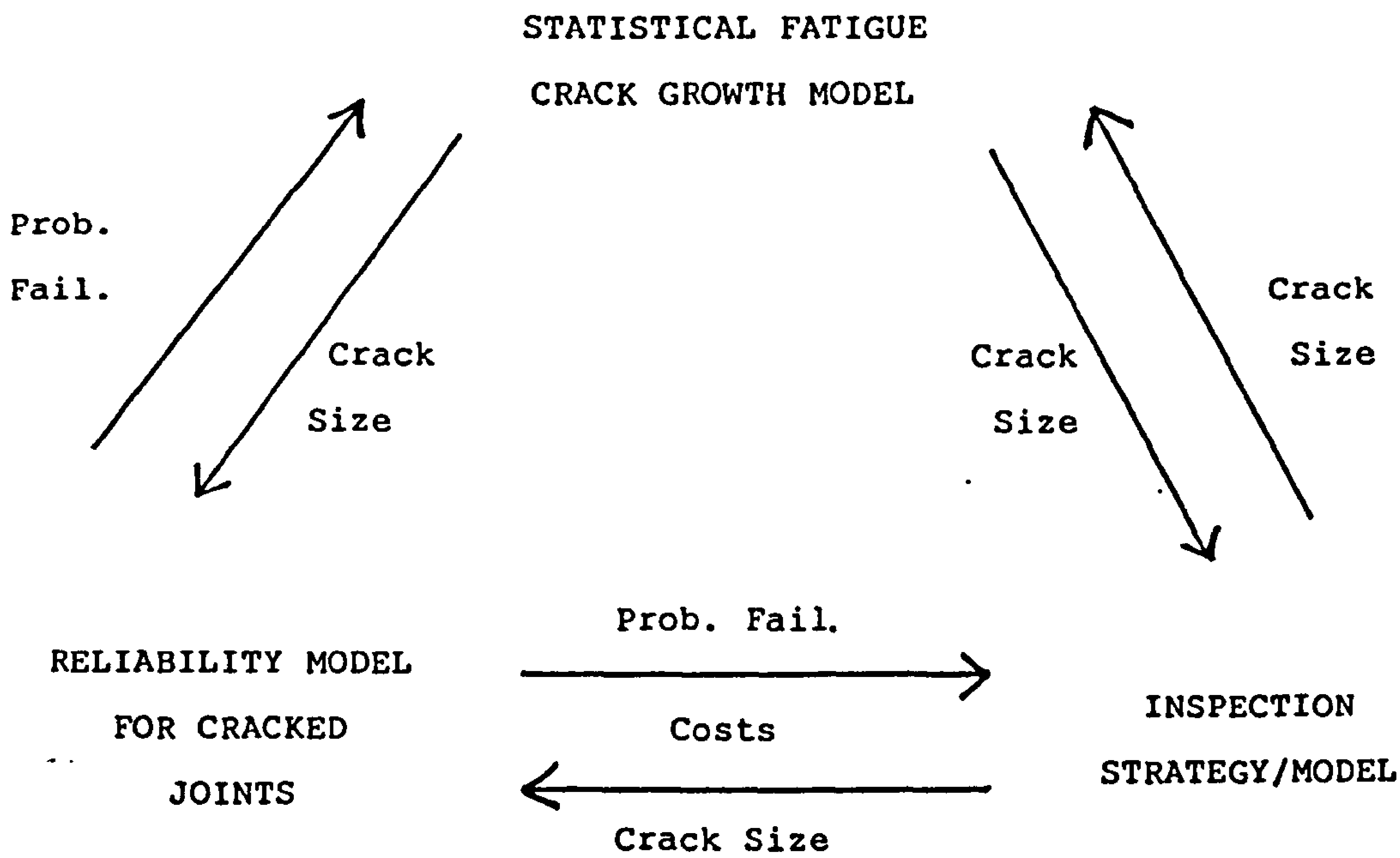
The inputs to the model can be both from the crack growth model and the results of inspection. The outputs can go directly to the inspection model and can also be used to assess fatigue constrained tubular joint designs.

(c) Inspection model - This allows alternative inspection plans to be compared on a cost basis. The model is based on a strategy which involves an economic trade-off of inspection costs, repair/maintenance costs and risk costs. The thinking behind the model is similar in some respects to that used by Marshall [12] and Sletten and colleagues [11] in that it seeks to minimise through-life costs. It is different in that it considers time dependent aspects explicitly; and the value of inspection results for re-appraising other, non-inspected joints in the structure.

The inputs to the inspection model are probabilities of failure from the reliability model and crack size distributions from the fatigue model. The outputs are life costs associated with alternative inspection/maintenance plans.

The inspection strategy can be adapted to estimate the allocation of resources required to maintain an acceptable minimum level of safety in the most economical manner.

The overall form of the approach is depicted overleaf.



All the models require other external inputs not catalogued above and these are discussed in later chapters. Efforts have been made to use the most relevant and realistic data to avoid the problems of bias.

The approach is flexible and any of the models can be substituted by alternative models if required provided the linking parameters are the same (or similar).

The approach can be used at the design stage, if minimum through life costs are a design objective, for the fatigue design of joints. This however, is not considered in this thesis, which concentrates on the post-installation stage and the problems facing the inspection engineer.

3. A NEW STATISTICAL FATIGUE CRACK GROWTH MODEL

3.1 Introduction

The conditions under which an offshore structure operates are subject to considerable uncertainty. The response of the structure to cyclic loading, usually estimated by fatigue calculations, is also subject to considerable uncertainty. This produces two effects. One, a need for general conservatism at the design stage and two, the requirement to inspect representative parts of the structure at intervals during its life to check for fatigue cracks. Unfortunately the fatigue calculations give no indication of what should be found on inspection at any intermediate stage during the structure's life.

To tackle this problem a crack growth model is required which can be related to the fatigue life of the structure. Fracture mechanics have been used extensively in the development of such models. Unfortunately most of these are of a deterministic nature and require well-defined inputs. In the case of offshore structures the important input parameters are not well defined in practice and the use of deterministic crack-growth models may well provide misleading results. There are a few probabilistic and statistically based crack growth models and these are reviewed in this chapter. Unfortunately none of them meets the requirements under consideration satisfactorily and so a new statistical crack growth model has been developed. This new model is based on the well known Paris equation [13] and is specifically applicable to the joints in offshore structures.

The chapter begins by addressing briefly the various sources of uncertainties of fatigue crack growth as they arise through the design, building and operating life of a typical offshore structure. The following section looks critically at the various calculation methods which have already been used to predict crack growth in the face of uncertainty. A new statistically based model is then developed. In the final section the assumptions underlying the new model are discussed.

3.2 Sources of uncertainty

This section catalogues the various sources of uncertainty which affect fatigue and fracture mechanics calculations for offshore structures. The uncertainties are of several different types:

a) Some quantities have an intrinsically random nature; for example structural steel at the microscopic level and ocean waves at a macroscopic level.

b) Systematic errors exist in the mathematical models used to describe the fatigue process and perform the associated calculations.

c) Estimation errors occur when parameters are predicted from sparse samples of data rather than the whole population.

d) Errors involved in approximating the future by extrapolating the past.

By looking at the whole fatigue process from the first design concept until final collapse, it is readily seen how the uncertainty increases with time.

One of the first design decisions is the choice of structural steel. This is usually specified in terms of minimum strength properties which must be achieved including yield stress, ductility and toughness, e.g. [4]. The toughness can either be expressed in terms of Charpy Impact test values or crack tip opening displacement (CTOD). Empirical relationships have been developed to relate these measures of toughness [14]; however, whatever measure is used, specimens of nominally identical material show considerable scatter, not only in their toughness [15] but also in their yield strength [16] and fatigue crack growth rate parameter C from Paris' equation [17]. Purely on the basic material considerations therefore there is uncertainty concerning not only the rates at which cracks will grow, but also the critical crack size at which brittle fracture may take place and the ultimate load required to cause plastic collapse in the case of ductile failure. Fortunately in this area considerable data has been collected as a result of the United Kingdom Offshore Steels Research Project and similar efforts abroad.

For most fatigue calculation purposes the stress range histogram or spectra are obtained by assuming simple deterministic relationships between wave height or energy spectra and wave induced stress. There is considerable data available on wave height histories for various locations around the world. However, the a priori estimation of the long term wave spectra for any new offshore location is extremely difficult and this is discussed in the next chapter.

The nominal wave induced stress in a structural member is calculated using wave particle kinematics and Morrison's equation. There are uncertainties associated not only with the wave particle kinematics of real waves and the force coefficients in Morrison's equation, but also as a result of ignoring vortex shedding and lift forces.

When the structure is being designed the "hot spot" stress at the joints must be calculated from the nominal stresses, obtained using design loads, by means of geometric stress concentration factors. These, whether they be obtained by experiment or theoretical calculations, show considerable scatter giving local stresses which may differ by up to 100% [18, 19, 20, 21].

When the structure is under construction and the tubular members are joined together the metallurgical properties change as a result of the welding process. The precise extent of the change is difficult to quantify [22]. The welding process also gives rise to residual stresses in the welded region which will increase the rate of crack growth at least initially. Residual stresses can be reduced by heat treatment, grinding or peening. However, whether or not stress-relieving is undertaken the final state of "built-in" stress, upon which the crack growth rate will depend, is not known exactly. The actual profile of the weld will also affect the local stress field; and these profiles even after grinding, if this is undertaken, will show considerable variation from one weld to the next. This is another significant source of uncertainty in fatigue life estimation.

After fabrication all welds should be inspected, both visually and in critical regions by N.D.T. Welded joints are rarely perfect and inspection is not 100% reliable both due to the inspector and the techniques [23]. There is variety in both type and size of weld defects [24, 25, 26] which exist in offshore structures after they have been accepted as certifiable. This is a major source of uncertainty.

After fabrication the structure must be transferred to a barge, towed out, launched and piled in position. These activities involve loads, and particularly displacements which are difficult to quantify. Quite possibly elastic yielding occurs in highly stressed regions causing "shakeout" (or "shakedown") of residual stresses. Perhaps, equally possibly, extraneous loads due to launching and piling may cause cracking. The net result is a structure, which, when finally installed may have significantly different residual stress and defect distributions from those at the time of final post-fabrication inspection. Kallaby and Price [27] suggest a ten per cent allowance on fatigue lives for these effects - a guess in the face of uncertainty?

A fracture mechanics calculation for a joint in a structure assumes an initial defect size, local stress field and fracture toughness for the point under examination. However all these parameters are subject to spatial variations throughout the structure which may be correlated in a complex manner. If a significant defect exists in a region of low local fracture toughness then cracking may occur even though the local stress is not a "hot-spot" maximum.. There is no information, to the author's knowledge on the <u>spatial</u> distribution of defects, of a given size, and fracture toughness in relation to local hot-spot stress concentrations.

The fatigue and crack growth calculations usually performed assume linear cumulative damage relationships which are independent of the relative order of stress of cycles of different magnitudes. However, it is well known [e.g. 28] that because of the large plastic zone generated at the crack tip as a result of a large stress cycle the damage caused by subsequent lower stress cycles is less than would occur under constant amplitude loading. Little work has been done in this area in the context of offshore structures although this problem has been considered in the aerospace industry [e.g. 29]. The associated problem of cycle-counting for a broad band spectral loading has been addressed by Wirsching [5] who produced a correction factor, using the rainflow method [e.g. 6], for the equivalent narrow band process.

The offshore environment will affect the rate of crack growth. Some work has been done on stress corrosion cracking and effects of cathodic protection and this is reviewed briefly by Schutz [31]. During the life of the structure marine fouling will accumulate and can increase drag coefficients by over 100% [32]. There is still considerable uncertainty concerning not only the effects of the marine environment but also the way in which these effects should be modelled.

Given this catalogue of uncertainties it is not surprising that fatigue and fracture mechanics calculations give very imprecise estimates of crack size and fatigue life. It therefore seems eminently sensible to use the results of periodic weld inspections that are made throughout the life of the structure and which are required by certifying authorities, to revise and improve the estimation of fatigue life. To do this sensibly a crack growth model which takes some account of uncertainties must be used.

3.3 Existing statistical approaches to fatigue life and crack growth prediction

Despite the uncertainties enumerated above comparatively little work has been done on statistically based procedures to predict crack growth in offshore structures. Wirsching and his colleagues (see reference [5] for further references) have done considerable work on probabilistic/statistical approaches to fatigue life prediction for offshore structure using Miner's cumulative damage approach. Others [33, 34] have also worked in this area. Engesvik [35] has reviewed many of the uncertainties affecting the fatigue capacity of welded joints and, drawing on considerable data and the results of Monte-Carlo simulation, has estimated distributions for many of the parameters.

In the nuclear power pressure-vessel field Besuner and Tetelman [36] have used what they call probabilistic fracture mechanics to assess the fatigue lives of components. Their procedure is based on the well-known Paris equation:

$$\frac{da}{dN} = C(\Delta K)^m = C(YS\sqrt{\pi a})^m \qquad -(1)$$

Where
- C = experimentally determined material crack growth constant
- ΔK = range of stress intensity factor
- m = experimentally determined exponent
- Y = crack shape parameter, assumed to a constant in this case
- S = cyclic stress range
- a = crack size
- N = number of stress cycles

This is integrated to give:-

$$N_x = \frac{a_o^{(1-m/2)} - a_x^{(1-m/2)}}{CS^m Y^m \pi^{m/2} \ (m/2-1)} \qquad -(2)$$

Where N_x = number of cycles to obtain crack size a_x.

It is assumed that all the variation in crack growth rate can be expressed by C, and that m can be made constant, and that $a_o << a_x$. Then taking logarithms yields:-

$$\text{Log } N_x = -\log C - m \log S - m \log Y - \log\{\pi^{m/2}(m/2-1)\} + (1-m/2)\log a_o \qquad -(3)$$

The parameters C, S and a_o are then considered to be distributed log normally and the resultant log normal distribution of N_x is obtained. This simple "log-linearising" approach allows the effect of the initial defect or crack size to be taken into account when calculating the fatigue life. However, because of the necessary assumption for linearisation, $a_x >> a_o$, the number of cycles required to reach intermediate crack sizes cannot be estimated.

Kozin and Bogdanoff [37], working principally in the aeronautical field, have developed a probabilistic model for fatigue crack growth based on Markov chains e.g. [38]. The model uses a "semi-Markov" process as the states are not independent and "history" effects are included. This model does allow the variation in time required to reach a given crack size to be predicted. However, it requires a particular form of relationship between variance of time to reach a given crack length, crack length itself and the stress range. In addition for practical purposes, such as correlation with inspection results, it is more useful to have the variation in crack size at a given time; rather than the variation of time to reach a given crack size.

Lin and Yang [39] have also worked on Markov chain models and more recently on a cumulant-neglect closure model [40] This latter has been presented in a Gaussian form, which is approximate, in order to estimate time to reach a given crack size. The model, in essence, considers crack size to be the product of a deterministic calculation and a stochastic parameter; the latter being defined in terms of a truncated series expansion of a log-characteristic functional. The model, at least as presented in [40], takes no account of the variations in the distributions of the different parameters which make up the Paris crack growth model. It is not in its present form suitable, at least in the author's view, for application to the tubular joints of offshore structures.

One method of obtaining the distribution of crack size as a function of time is to rearrange equation(2) to give:-

$$a_x = \frac{1}{\left[a_o^{(1-m/2)} - N_x C S^m Y^m \pi^{m/2} (m/2-1)\right]^{\frac{1}{(m/2-1)}}} \qquad -(4)$$

Distributions can be assumed for the various variables on the R.H.S. and a Monte-Carlo simulation used to find the distribution of a_x for any number of cycles N_x. To simplify this exercise m is taken as a fixed quantity, frequently 3 or 4, and all the variation in crack growth rate is accounted for by the variable C which will have a correspondingly greater coefficient of variation (standard deviation/mean) [34, 41, 42]. However, a Monte-Carlo simulation is time consuming on a computer particularly when a simulation is needed at a range of intervals during the fatigue life to obtain a comprehensive picture of crack growth with time. More importantly as demonstrated in the Appendix 1, equation (4) lacks statistical robustness when any of the terms in the denominator are considered as variables with significant coefficients of variation. The terms C, S^m and a_o, as will be seen later, all have significant coefficients of variation. In this case the mean value of a_x depends heavily on the higher moments of the distributions chosen for these parameters, particularly as the crack size becomes significant. Now the choice of distribution is a little arbitrary when the amount of data is small, and whilst the mean, standard deviation and perhaps the third central moments will be the same or similar, the higher moments will usually be significantly different. It is also notoriously difficult to estimate higher moments accurately from small samples of data.

The difficulties above also afflict the use of extended level 2 reliability methods e.g. [43] in the stochastic solution of equation (4).

3.4 A new statistical crack growth model

To avoid these problems equation (4) can be inverted and a reciprocal function of crack size considered.

$$\frac{1}{a_x^{(m/2-1)}} = \frac{1}{a_o^{(m/2-1)}} - N_x C S^m Y^m \pi^{m/2} (m/2-1) \qquad -(5)$$

This equation can be written more simply as:-

$$r = d - TML \qquad -(6)$$

Where $r = 1/a_x^{(m/2-1)}$

$d = 1/a_o^{(m/2-1)}$

T = represents the age of the structure in years (Time)

$M = CY^m \pi^{m/2} (m/2-1)$

$L = \Sigma n_i S_i^m$ where n_i is the number of cycles at stress range S_i per annum.

This equation is much easier to handle because it is linear in the derived variables r, d, M and L. r represents a "reciprocal" function of the crack size; when a distribution for r is found the corresponding distribution for the crack size a_x can readily be obtained numerically. The term d is a reciprocal function of initial crack or "defect" size. Values of d can readily be obtained from raw data on initial defect sizes. The terms L and M represent the "loading" and "material" responses respectively, and once again values can be obtained from oceanographic and experiment data.

This equation can be used for a Monte-Carlo simulation after distributions have been fitted to the derived variables, d, M and L. However, apart from the problem of computer time mentioned above, largely arbitrary distribution assumptions will have to be made for these variables. The preferred alternative is to handle this equation using a Taylor Series approach. The Taylor Series has already been used in marine structural reliability [42] and also in maritime engineering economics [51] to develop simple statistical models.

From equation (6) closed form expressions can be obtained for the mean, variance and higher moments of r as demonstrated below.

For any variable y which is a function of independent variables x_1, x_2, x_3 --- x_n the Taylor series yields:

$$y = f(\mu_1, \mu_2 \cdot \mu_3 ----) + \sum_{i=1}^{n} \left(\frac{dy}{dx_i}\right)_{\mu_i} (x_i - \mu_i)$$

$$+ \frac{1}{2!} \sum_{j=1}^{n} \sum_{i=1}^{n} \left(\frac{d2y}{dx_i dx_j}\right)_{\mu_i \mu_j} (x_i - \mu_i)(x_j - \mu_j) + \text{higher order terms}$$

Where μ denotes a mean value

In the case of equation (6), where the terms on the R.H.S. are all independent, this yields:

$$r = \mu_d - T\mu_m\mu_L + (d-\mu_d) - T\{\mu_m(L-\mu_L) + \mu_L(M-\mu_m) + (L-\mu_L)(M-\mu_m)\}$$

The expected or mean value of r is then:

$$E(r) = \mu_r = \mu_d - T\mu_m\mu_L \qquad -(7)$$

and the variance is given by:

$$E[(r-\mu_r)^2] = \sigma_r^2 = E[((d-\mu_d) - T\{\mu_m(L-\mu_L) + \mu_L(M-\mu_m) + (L-\mu_L)(M-\mu_m)\})^2]$$

$$= \sigma_d^2 + T^2(\mu_m^2\sigma_L^2 + \mu_L^2\sigma_m^2 + \sigma_m^2\sigma_L^2)$$

where σ_x denotes the standard deviation of x.

This can be re-expressed in terms of coefficients of variation

$$V_r = \frac{1}{\mu_r} [\mu_d^2V_d^2 + T^2\mu_m^2\mu_L^2 (V_L^2 + V_m^2 + V_m^2V_L^2)]^{\frac{1}{2}} \qquad -(8)$$

where $V_x = \sigma_x/\mu_x$ is coefficient of variation of x.

The third, fourth, etc. higher central moments can be obtained by expanding $E[(r - \mu_r)^n]$ when n = 3, 4, etc in a similar manner

This yields the coefficient of skewness ($\Theta_r = E(r - \mu_r)^3/\sigma_r^3$ - normalised third central moment, and sometimes denoted by $\sqrt{\beta_1}$)

$$\Theta_r = \frac{1}{V_r^3\mu_r^3}\Big[\mu_d^3V_d^3\Theta_d - T^3\mu_m^3\mu_L^3\,(V_L^3\Theta_L + V_m^3\Theta_m$$

$$+ V_m^3V_L^3\Theta_m\Theta_L + 3V_m^3V_L^2\Theta_m + 3V_m^2V_L^3\,\Theta_L + 6V_L^2V_m^2)\Big] \quad -(9)$$

Where $\Theta_x = E[(x-\mu_x)^3]/\sigma_x^3$

and finally the coefficient of kurtosis ($K_r = E[(r-\mu_r)^4]/\sigma_r^4$ - the non-dimensionalised fourth central moment which is sometimes denoted by β_2):-

$$K_r = \frac{1}{V_r^4\mu_r^4}\Big[\mu_d^4V_d^4K_d - T^4\mu_L^4\mu_m^4\Big\{V_m^4K_m + V_L^4K_L + V_L^4V_m^4K_LK_m$$

$$+ 4(V_m^4V_L^3\Theta_LK_m + V_L^4V_m^3\Theta_mK_L)$$

$$+ 6(V_m^4V_L^2K_m + V_L^4V_m^2K_L + V_m^2V_L^2)$$

$$+ 6\frac{\mu d^2}{\mu_L^2\mu_m^2T^2}\,(V_d^2V_m^2 + V_d^2V_L^2 + V_d^2V_m^2V_L^2)$$

$$+ 12(V_m^3V_L^2\Theta_m + V_L^3V_m^2\Theta_L + V_L^3V_m^3\Theta_L\Theta_m)\Big\}\Big] \quad -(10)$$

where $K_x = E[(x-\mu_x)^4]/\sigma_x^4$.

The derivation of these expressions is given in Appendix II.

To obtain the crack size distribution at any time during the fatigue life the following steps are employed:

1. Estimate the mean, coefficient of variation, coefficient of skewness (and perhaps the coefficient of kurtosis) for the derived variables d, M and L using basic data This is discussed further in the next chapter where appropriate data are presented.

2. Using equations 7, 8, 9 (and perhaps 10) determine the corresponding moments for r as functions of time T.

3. From an examination of these moments select suitable distributions to represent r at a series of times T during the fatigue life.

4. For each distribution of r at time T calculate numerically (or analytically if feasible) the corresponding crack size distribution.

This procedure is illustrated in Figure 1.

The index m from the crack growth equation is the same in the index in the fatigue equation: NS^m = Constant.

A constant value of 3, 4 or 5 is usually chosen as appropriate, although any real number in this range can be used. The advantage of this approach compared with the Monte Carlo method is that all the calculations can be performed to give a whole series of crack size distributions covering the entire fatigue life on a desktop micro-computer in a few seconds. Also, only one distribution has to be assumed compared with the Monte-Carlo method where distributions must be assumed for every basic variable. Finally the contributions of each variable to the uncertainty and shape of the final distribution can easily be investigated by parametric variations.

Before continuing a few points should be noted about equations (7) - (10) and the moments of probability distributions. The vast majority of distributions currently used are uniquely defined in terms of the first four moments [44], certainly all those generally used in engineering. The Weibull, Gamma and log-normal distributions are defined by the first three moments and the normal distribution by only the first two moments. Another point worth noting is that even if all the variables d, M and L are assumed to be normally distributed, and therefore not skewed, the distribution of r will still be slightly skewed. This is clear from an examination of the final term in equation (9).

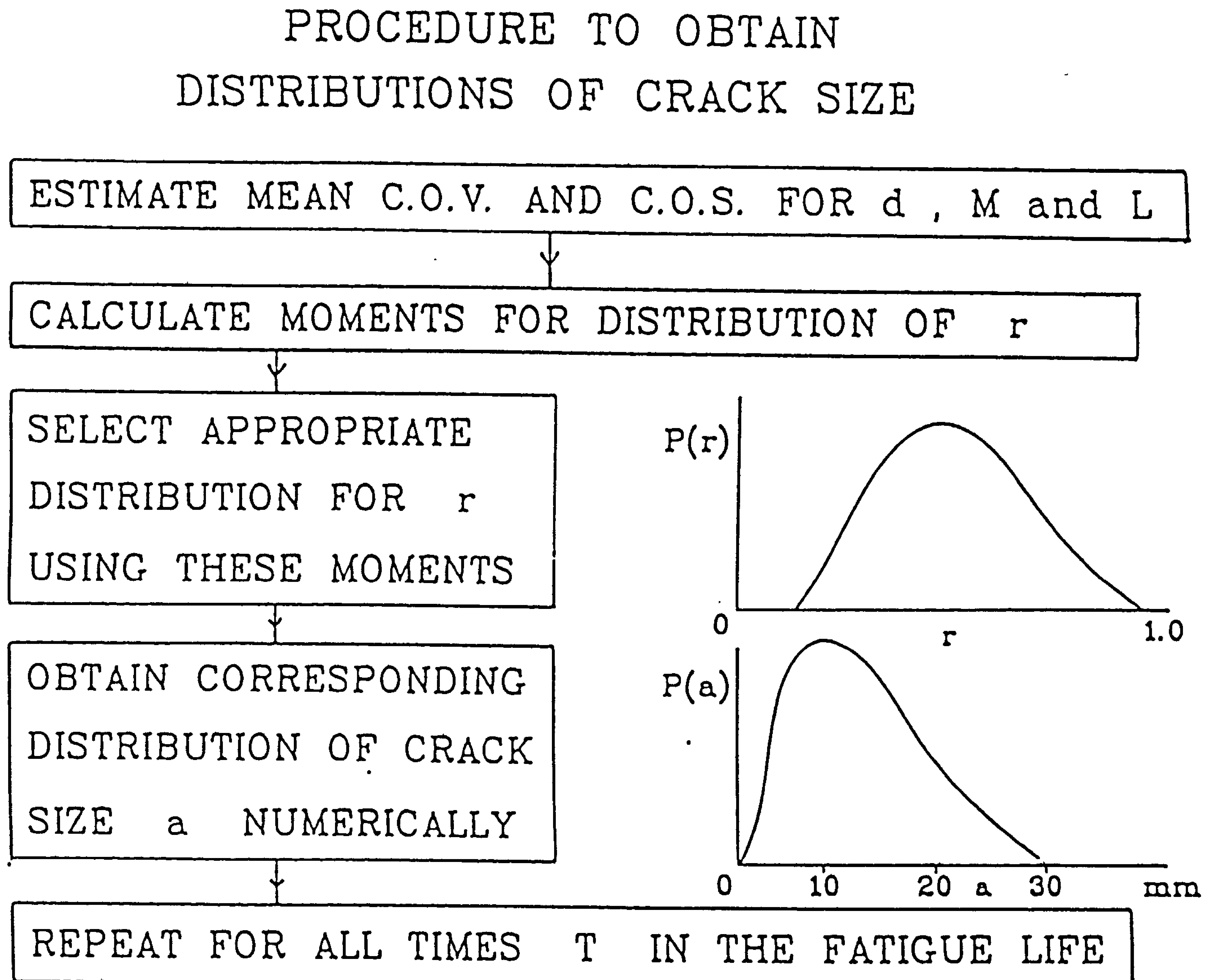


Figure 1: Diagram illustrating procedure for obtaining fatigue crack size distribution at any time during the fatigue life.

3.4 Extending the new model to include more variables

The model described above assumes that all the uncertainties related to crack growth can be gathered together under three headings and expressed as three variables, d, M and L. In many cases it may be difficult to determine the moments of the distributions of these variables directly. However the approach can be extended to cope with this problem in most circumstances.

Consider, for example, the loading parameter L. This is directly related to the hot spot stress range which in turn is related to the waveheight:

$$\text{i.e.} \quad L = \sum_{i=1}^{q} n_i S_i^{\,m} = \sum_{i=1}^{q} n_i \times (A \times H_i^{\,p})^m$$

$$= A^m \times \sum_{i=1}^{q} n_i \times H_i^{\,mp} \qquad -(11)$$

Where A = transfer function relating hot spot stress to waveheight

H = waveheight

p = index, usually between 1 and 2.

Now the transfer function A is unlikely to be precisely determined. It usually contains errors due to modelling assumptions and uncertainties related to its estimation. Consequently it should be treated as a variable. The waveheight, of course, must also be treated as a variable. The term L then is the sum of the products A^m and $H_i^{\,mp}$ of at least two variables.

This presents no difficulties provided L (and the same applies to d and M) can be represented as the sum (or difference) and/or product of a number of linear variables (x_1, x_2, etc.). So for the case above:

$$L = A^m \times \sum_{i=1}^{1} n_i \times H_i^{\,mp} = x_1 \times x_2$$

where m and p are both fixed constants and the statistical moments of A^m and $\Sigma n_i H_i^{\,mp}$ can be found.

The corresponding moments for L can then be found using the relationships below which have been derived using a Taylor Series expansion in Appendix II.

For Y = A + B

$$\mu_y = \mu_A + \mu_B \qquad -(12)$$

$$\mu_y^2 V_y^2 = \mu_A^2 V_A^2 + \mu_B^2 V_B^2 \qquad -(13)$$

$$\mu_y^2 V_y^3 \Theta_y = \mu_A^3 V_A^3 \Theta_A + \mu_B^3 V_B^3 \Theta_B \qquad -(14)$$

$$\mu_y^4 V_Y^4 K_y = \mu_A^4 V_A^4 K_A + \mu_B^4 V_B^4 K_B + 6\mu_A^2 \mu_B^2 V_A^2 V_B^2 \qquad -(15)$$

and for Z = C x D

$$\mu_z = \mu_c \mu_D \qquad -(16)$$

$$V_z^2 = V_c^2 + V_D^2 + V_c^2 V_D^2 \qquad -(17)$$

$$\mu_z^3 V_z^3 \Theta_z = \mu_c^3 \mu_D^3 \left(V_c^3 \Theta_c + V_D^3 \Theta_D + V_c^3 V_D^3 \Theta_c \Theta_D + 3V_C^3 V_D^2 \Theta_c + 3V_D^3 V_c^2 \Theta_D + 6V_c^2 V_D^2\right) \qquad -(18)$$

$$\begin{aligned}\mu_z^4 V_z^4 K_z = \mu_c^4 \mu_D^4 \Big[& V_c^4 K_c + V_D^4 K_D + V_c^4 V_D^4 K_c K_D \\ & + 4\left(V_c^4 V_D^3 K_c \Theta_D + V_D^4 V_c^3 K_D \Theta_C\right) \\ & + 6\left(V_c^4 V_D^2 K_c + V_D^4 V_c^2 K_D + V_c^2 V_D^2\right) \\ & + 12\left(V_c^3 V_L^2 \Theta_c + V_D^3 V_c^2 \Theta_D + V_c^3 V_D^3 \Theta_c \Theta_D\right)\Big]\end{aligned} \qquad -(19)$$

By using these relationships d, M and L can all be expressed, if necessary as linear functions of a number of independent variables. This allows more of the uncertainties outlined in section 2 to be represented in the crack growth model. It also allows terms catering for modelling error and bias to be introduced.

3.6 Discussion of the new crack growth model

The statistical fatigue crack growth model presented above makes a number of assumptions which warrant further discussion. Principal amongst these are the following:-

(a) The crack shape parameter does not vary with crack shape or size.

(b) The lower limit of stress intensity at which no crack growth occurs, the stress intensity threshold, ΔK_{TH}, can be taken as zero.

(c) The "hot spot" stress is independent of crack size.

(d) The crack growth index m from Paris' equation is constant.

(e) The irreversible linear cumulative damage approach in which a broad-banded stress range spectrum is approximated by arbitrarily ordered blocks of constant stress range is valid.

(f) Crack growth rate is independent of chord wall thickness.

These assumptions are now discussed in turn.

The crack shape parameter theoretically varies according to crack shape and stress field geometry, e g. [28]. However, for the type of crack shapes found to occur in the tubular members of offshore structures, the variation in magnitude is not very large [45]. The principal problem however, is that the crack shape is not known à priori, and, as experiment results show that actual crack shapes vary significantly under similar testing conditions ([46] and [47]) it is difficult to estimate the crack shape parameter. For this reason it is frequently assumed to be constant and, indeed, this is the recommended practice of Det Norske Veritas [4] for the design of offshore structures. In equation (6) all the statistical uncertainty in the crack shape parameter is subsumed by the material response parameter M which is of course independent of crack size. However it is worth pointing out that the model can readily cope with a crack shape parameter of the form:-

$$\text{Constant 1} \times [\text{thickness/crack depth}]^{\text{constant 2}}$$

This form has been proposed by Dover and Dharmarasan [45] based on experimental results, albeit a limited number.

The assumption that the threshold stress intensity is zero has been made for three reasons. The proposed model would require an iterative solution otherwise. The effect of the value of threshold stress intensity upon calculated crack size has been found to be very small [46]. This is supported by experimental data collected on Forties Bravo platform which shows that the majority of fatigue damage occurs in storms [48]. Finally there appears to be little data on the value of the threshold stress intensity in welded tubular joints. The model can however, easily include a minimum stress range threshold, corresponding to the endurance limit sometimes used with S.N. curves, e.g. [49].

Ideally the stress intensity at the crack tip should be recalculated for every increment in crack growth. This is not however, feasible. For a start the crack will probably not be regular in shape [57] and the stress intensity will vary along the crack edge. The local stress field will be considerably influenced by the residual stress field in a welded section. This latter is subject to variation due to "shake-out" or "shakedown" and the non-homogeneity of the local microstructure. In practice none of these factors can be assessed accurately and so the originally calculated hot spot stress, in the absence of any cracks, is used. The other parameters are estimated in the face of this assumption. This practice is adopted by Det Norske Veritas for offshore structures.

The crack growth index m is assumed to be constant because otherwise the crack growth model becomes statistically intractable. This arises because m and C are correlated and there is not sufficient data, at least for tubular joints, to estimate the joint probability density function with any confidence.

For structural and high strength steel Gurney [50] established empirically a relationship of the following type:-

$$C = \text{Constant } A/(\text{Constant } B)^m \qquad -(20)$$

Therefore in assuming m to be constant at a particular value the corresponding value for C obtained from the analysis of experiments is affected. All the uncertainty in m is thus subsumed by the uncertainty in C which in consequence is greater. There is no loss in generality therefore, just in precision.

There have been numerous studies of the applicability of linear cumulative damage models to fatigue cracking under random loading. Recent work in tubular joints has been summarised by Shutz [51], and the consensus of opinion is that in the absence of more detailed data on particular non-linear effects, the linear cumulative damage model is acceptable.

With the current state of knowledge concerning fatigue cracking in offshore structures, and the related dearth of data, there is little alternative but to accept this approach.

Recent work summarised by Gurney [52] and adopted by the D.En. [53] in their fatigue design recommendation, shows fatigue life to be dependent upon chord thickness. From a fracture mechanics standpoint this means that the crack growth constant C is a function of thickness. A different value of C for each thickness of material can of course be used in the model above. However, in the relatively small sample of data the author has analysed, and which is described in the next chapter, the thickness effect was not clearly discernible. So C is assumed to be independent of thickness in this thesis.

4. ESTIMATION OF PARAMETERS FOR THE CRACK GROWTH MODEL

4.1 Introduction

In the crack growth model developed in the previous chapter there are three independent distributed parameters notably d, M and L. These must be estimated before the model can be used. Ideally large samples of data obtained from field measurements or full-scale experiments conducted under realistically simulated conditions should be used to obtain these estimates. Unfortunately such data do not exist, and compromises have had to be made.

To estimate the moments of the distributions of the variables d and M the U.K.O.S.R.P. tubular joint fatigue experiment results, as reported by Clayton [47], have been used. The analysis involved is described in the next two sections of this chapter.

To estimate the moments of the distribution of the loading parameter L data purchased from U.K. Meteorological Office has been used together with published data. The analysis involved is described in a subsequent section of this chapter.

The chapter concludes with a discussion of the statistical estimators obtained and a comparison with other published estimators obtained from comparable data.

4.2 Material response parameter M

The United Kingdom Offshore Steels Research Project (UKOSRP) included a number of fatigue tests on tubular joints representative of offshore jacket structures. These tests were undertaken at various centres in the UK and included both in-plane and out-of-plane bending as well as axial loading. The results of these tests, some 50 in all, have been collected and presented by Clayton [op. cit.]. The crack depth as well as the crack length was measured in 17 cases at various times during the fatigue test usually from the onset of the first visible crack.

All the tests were undertaken at constant amplitude loading, and the experimentally determined hot-spot stress range is quoted in each case.

This data has been analysed in the context of the Paris equation which may be written as:-

$$dN = \frac{1}{CY^m S^m \pi^{m/2} a^{m/2}} da$$

hence:

$$\int_{N_i}^{N_{i+1}} dN = \frac{1}{CY^m S^m \pi^{m/2}} \int_{a_i}^{a_{i+1}} a^{-m/2} da$$

and

$$(N_{i+1} - N_i) = \frac{1}{CY^m S^m \pi^{m/2}} \cdot \frac{(a_{i+1}^{1-m/2} - a_i^{1-m/2})}{1-m/2} \qquad -(21)$$

where (N_i, a_i) and (N_{i+1}, a_{i+1}) are corresponding pairs of number of cycles and crack size. This assumes that S and Y do not change with crack size. Unfortunately, as has already been noted. there is in practice no alternative, as the variation in S and Y with crack size depends on the crack shape and crack path. These latter are not known a priori and indeed were not measured during the fatigue tests analysed here. However, the variation in Y and S is usually relatively small, and the uncertainty introduced by assuming constant values is included in the variation of C. Y is assumed equal to 1.0 throughout and S is taken as the initial hot spot stress range reported for each experiment. Any departure of the mean value of Y from 1.0 is accommodated by a bias in C.

Hence

$$C = \frac{1}{S^m \pi^{m/2} (1-m/2)} \frac{(a_{i+1}^{1-m/2} - a_i^{1-m/2})}{(N_{i+1} - N_i)} \qquad -(22)$$

Now the exponent m is usually determined empirically by plotting $\log \frac{da}{dN}$ versus $\log (YS\sqrt{\pi a})$.

Whence:

$$\log \frac{da}{dN} = \log C + m \log (YS\sqrt{\pi a}) \qquad -(23)$$

Now in general because of the scatter of experiment results between test specimens the parameters C and m are both found to be distributed variables. However, to simplify matters m can be assumed to be an appropriate constant leaving all the uncertainty to be subsumed in the variability of C. That approach is adopted here. Furthermore, in order to tie-in with fatigue curves [21, 4] integer values of 3 or 4 are chosen for m.

Then $$C = \frac{0.3592}{S^3} \cdot \frac{(a_i^{-1/2} - a_{i+1}^{-1/2})}{(N_{i+1} - N_i)} \quad \text{for } m = 3 \qquad -(24)$$

and $$C = \frac{0.1013}{S^4} \cdot \frac{(a_i^{-1} - a_{i+1}^{-1})}{(N_{i+1} - N_i)} \quad \text{for } m = 4 \qquad -(25)$$

There are two sources of variation in C values:

a) Variations in C due to fluctuations in the rate of crack growth from the onset of cracking to final failure in each individual test specimen.

b) Variations in the mean rate of crack growth from one test specimen to the next.

Now for practical purposes the variation in C due to a, is much less significant than that due to b, and can be ignored. See Virkler's data [54]. In fact it is realistic to determine the mean value of C for each joint tested and assess only the variation in these values across all the specimens. That approach is used here.

When calculating C for pairs of adjacent (a_i, N_i) values account must be taken of the size of the interval. If the interval is small and the measurement of crack size not precise the value of C obtained is subject to a greater error than when the interval is large. This point is illustrated in Figure 2. To account for this a weighting procedure is introduced so that all contributions to the meanvalue have the same expected error.

A suitable procedure is derived in Appendix III and the following expressions for estimating the mean value C (C_{mean}) are obtained.

$$C_{mean} = \frac{0.1013}{S^4} \cdot \frac{\sum_{i=1}^{n-1} (a_{i+1} - a_i)}{\sum_{i=1}^{n-1} (N_{i+1} - N_i) a_{i+1} a_i} \quad \text{for } m = 4 \qquad -(26)$$

and

$$C_{mean} = \frac{0.3592}{S^3} \cdot \frac{\sum_{i=1}^{n-1} a_{i+1}^{3/4} a_i^{1/4} - a_{i+1}^{1/4} a_i^{3/4}}{\sum_{i=1}^{n-1} (N_{i+1} - N_i) a_{i+1}^{3/4} a_i^{3/4}} \quad \text{for } m = 3 \quad -(27)$$

These expressions have been used to calculate values of C_{mean} for many of the fatigue experiments reported by Clayton. Not all the fatigue experiment data was used for one or more of the following reasons:

(i) Too few crack depth (or length) measurements leading to an unreliable estimate of C mean. Four pairs of points was considered to be the minimum acceptable.

(ii) Chord wall thickness less than 10mm. In thin material crack behaviour is found to be significantly different [52].

(iii) Variable amplitude loading - not consistent with the other results.

(iv) First crack length measurement of over 100mm. Clearly crack growth was not the experimenter's main priority and results are therefore suspect.

(v) First crack length measurement 1mm or less Resolution of cracks of this size is considered unrealistic by the author.

The data used are presented in Appendix V.

This left sixteen fatigue experiments that could be used to estimate C when crack depth (a) was used to characterise crack size. From these sixteen values of C_{mean} the "overall C_{mean}" has been calculated together with statistical central moments μ_2, μ_3 and μ_4 These later have been expressed in their non-dimensional form in Table 1, i.e coefficient of variation (V), coefficient of skewness (θ or $\sqrt{\beta_1}$) and coefficient of kurtosis (K or β_2).

Now crack size can be measured by length, or even the square root of the crack area, as well as depth. In fact crack length is easier to measure than crack depth, particularly during in-service inspections subsea. There are 24 suitable experiments where crack length was measured. The values of "overall C_{mean}" and the corresponding statistical parameters are again presented in a non-dimensional form in Table 1.

When the square root of crack area was used as a parameter only sixteen sets of data were available and the corresponding results are also given in Table 1.

To obtain the corresponding M values the "overall C_{mean}" values are multiplied by $(m/2-1)\pi^{m/2}$.

It is worth noting that the method used here to estimate C is different from that usually employed when C and m are determined jointly. The usual procedure involves taking logarithms of Paris equation yielding:-

$$\log\left(\frac{da}{dN}\right) = m \log(\Delta K) + \log C \qquad -(28)$$

There are many ways of calculating da/dN, see for example Virkler et al [54]. A classical least squares regression of log (ΔK) on log (da/dN) then yields estimates for m and C.

This method has the disadvantage that C and m depend to some extent on the method used to estimate (da/dN) from the sample data. That is why the alternative method developed above has been employed.

Crack Size Parameter		"C" Statistics			
		Mean	V_c	Θ_c	K_c
Depth	m = 3	3.06×10^{-14}	1.12	1.30	7.85
	m = 4	4.44×10^{-17}	1.18	3.02	6.04
Square Root of Cross Section Area	m = 3	1.39×10^{-14}	1.07	1.78	2.69
	m = 4	9.88×10^{-18}	1.02	1.52	3.22
Length	m = 3	6.37×10^{-15}	0.93	2.33	4.80
	m = 4	2.33×10^{-18}	0.94	2.50	5.67

TABLE 1: Statistics for the distributions of the parameter C from Paris' equation when crack size is measured by various parameters.

4.3 Estimating initial defect size

The distribution of defect depths in the welded connections of offshore structures has been considered by Burdekin and Townend [26], Rodrigues et al [24] and Rogerson and Wong [25]. All collected post-fabrication weld inspection data and concluded that the weld defect heights followed a Weibull distribution. Burdekin and Townend take into account the reliability of the inspection technique (ultrasonic) but do not mention the effects of repair of sub-standard welding. Rogerson and his colleagues (including Rodrigues) on the other hand take into account the effects of repair but make no mention of inspection technique reliability. The latter quote a 3-parameter Weibull distribution as having the best fit, which implies that there will always be defects of some minimum size: in this case 0.1mm depth.

Whilst all the authors have favoured a Weibull distribution, the mean and the other statistical parameters for the various sets of data are markedly different. Little is said about the spatial distribution of defects and nothing about their location relative to the "hot-spot" regions. Clearly a defect which is in a "hot spot" region is more critical than a defect somewhere else in the weld. It has been observed [57] that a series of small but distinct defects occurring close together may rapidly coalesce under cyclic loading to form a more substantial crack. This is not catered for in the simple Paris equation, although it is obviously important, particularly if length is used as the crack parameter.

In none of the UKOSRP fatigue experiments reported was the initial crack or defect size measured. However, by the reverse extrapolation of the crack growth data using Paris' equation, the initial cracksize can be estimated. This is illustrated in Figure 3. By re-arranging equation (21) and letting $N_{i+1} = 0$ cycles and $a_{i+1} = a_o$, the initial crack size, the following expression is obtained:

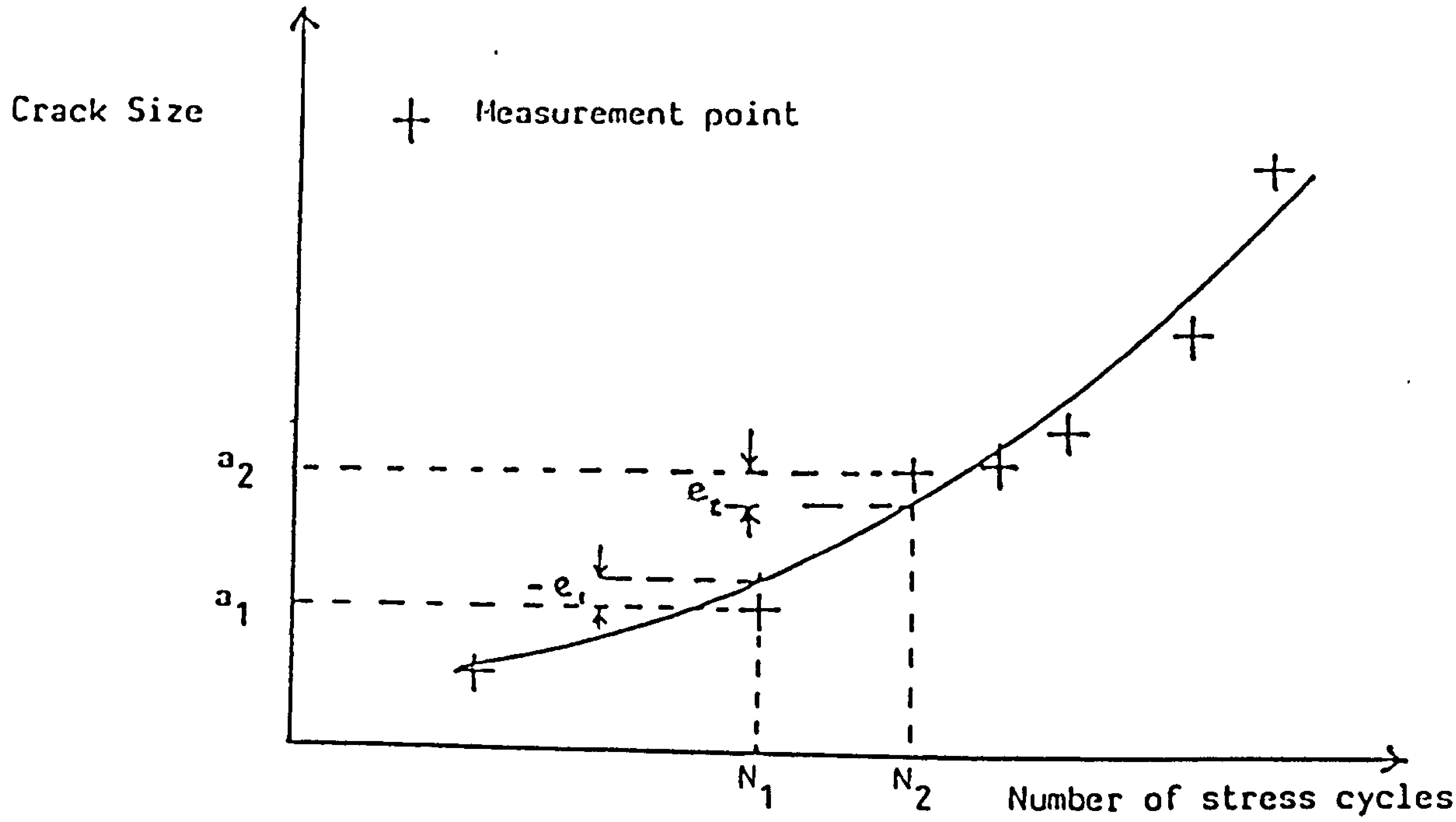


Figure 2. Errors in the measurement of crack size

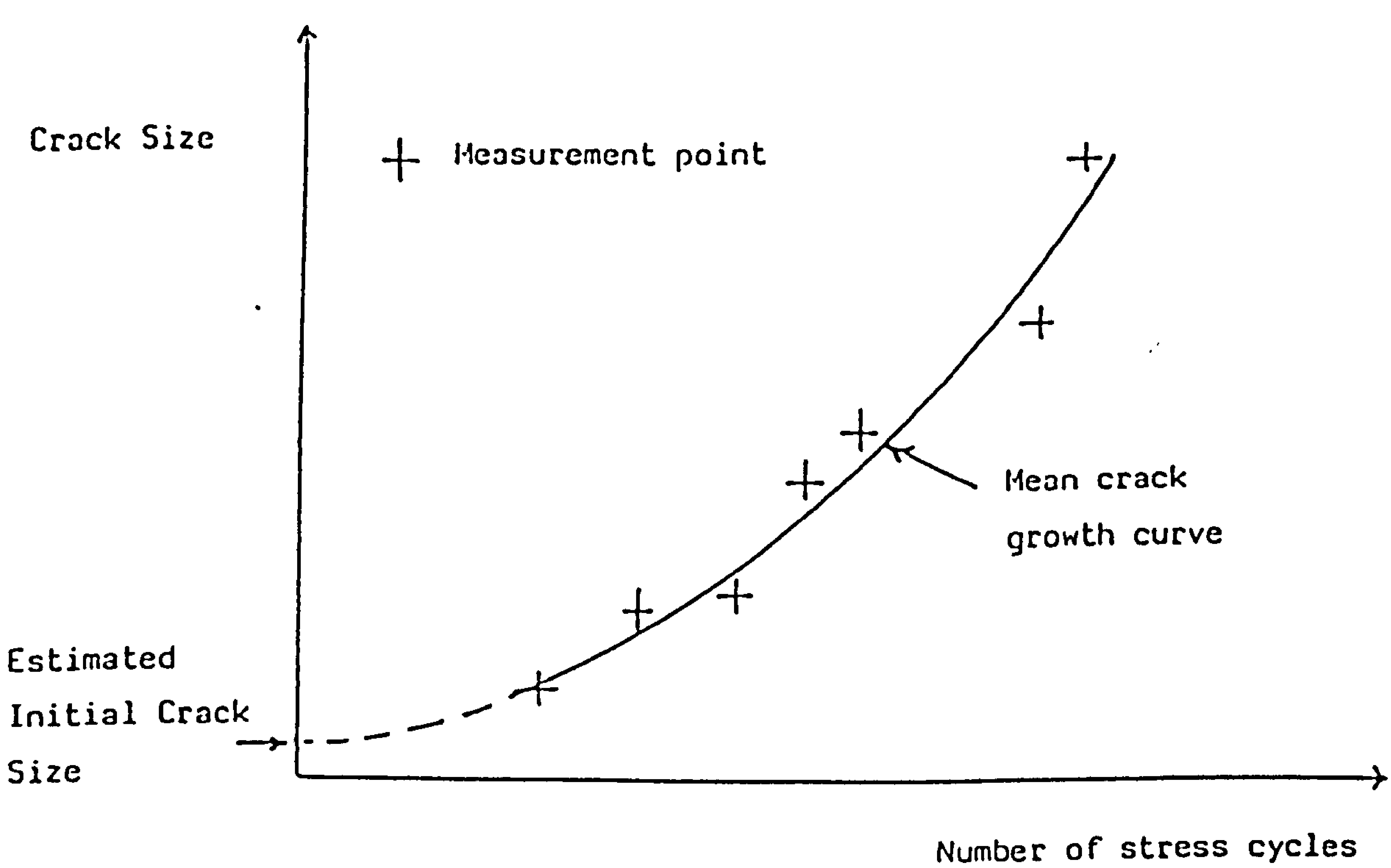


Figure 3. Reverse extrapolation to obtain initial crack size

$$a_o = \left[a_i^{1-m/2} + CY^m S^m \pi^{m/2} (m/2 - 1)\ N_i \right]^{\frac{1}{1-m/2}} \quad -(29)$$

Any pair of values a_i, N_i co;uld be used in the above equation to obtain an estimate of a_o but obviously values obtained close to the start of crack growth are likely to provide the best estimators. In order to obtain estimators of approximately equal variance a weighting function (w.f.) is required which can be applied to each pair of values (a_i, N_i) that contribute to the estimation of the initial crack size. Such a weighting function is developed below.

Now equation (29) can be re-written as:

$$a_o = a_i \left[1 + CY^m S^m \pi^{m/2} a_i^{m/2-1}\ N_i (m/2-1) \right]^{\frac{1}{1-m/2}} \quad -(30)$$

and the R.H.S. of this equation can be expanded by the binomial theorem to give:-

$$a_o = a_i \left[1 + \frac{1}{1-m/2}\ CY^m S^m \pi^{m/2} a_i^{m/2-1}\ N_i (m/2-1) + \text{higher order terms} \right]$$

Hence $a_i - a_o = CY^m S^m \pi^{m/2} N_i a_i^{m/2}$ + higher order terms -(31)

For points a_i close to a_o which are likely to provide the best estimators, the higher order terms can be neglected to a first approximation:

$$\text{i.e.} \quad a_i - a_o = CY^m S^m \pi^{m/2} N_i a_i^{m/2} \quad -(32)$$

Now for any pair of values (a_i, N_i) it is possible to postulate a value of C which will exactly predict the correct value of a_o. This value of C will probably be different from the C_{mean} discussed above, being either somewhat smaller or larger. It is reasonable to assume, to a first approximation, that the C values for all pairs of (a_i, N_i) are symmetrically distributed about C_{mean} with constant variance.

Hence s.d. $(a_i - a_o) \approx Y^m S^m \pi^{m/2} N_i a_i^{m/2}$ s.d. (C)

$$\text{or} \quad \frac{1}{N_i a_i^{m/2}} \text{ s.d. } (a_i - a_o) \approx \text{constant} \qquad -(33)$$

where s.d. denotes standard deviation.

Then to obtain estimators of constant variance for a_o from the data set of (a_i, N_i) experiment values the weighting function required is approximately given by:

$$\text{w.f.} = \frac{N_i^{-1} \; a_i^{-m/2}}{\sum_{i=1}^{m} N_i^{-1} \; a_i^{-m/2}} \qquad (34)$$

Using this weighting function means that only the first two or three crack size measurements have a significant effect upon the estimated value of a_o.

The initial crack or defect size has been estimated using equation (29) and the weighting function above, applied to all pairs of points in each data set, for the 16 experiments where crack size is characterized by crack depth.

Ideally the value of C_{mean} for each data set should be used to calculate the initial defect size for that data set. However this yields pairs of values of d and C_{mean} which are correlated; albeit the correlation coefficients are in practice found to be low. The crack growth model specifically requires statistically independent parameters and hence "overall C_{mean}" values as quoted in Table 1 are employed in order to yield sensibly independent parameter estimates. In fact the values obtained for d in this way are not significantly different.

When calculating the mean value of the initial crack size from the results obtained from the sixteen sets of data, account must be taken of the number of data points in each set and the proximity of the initial crack size measurement to the start of the fatigue experiment. Obviously when several crack size measurements are made as soon as visible cracks are discernible the corresponding defect estimate is likely to be more precise than when only two or three measurements are made, and only then when the cracks are large. To combine results from different data sets, a non-dimensional weighting function (N.W.F.) is needed. For this purpose following the form of equations (33) and (34), the parameter below is used:-

$$\text{N.W.F.} = \frac{N_2}{N_i} \frac{t^{m/2}}{a_i^{m/2}} \qquad -(35)$$

where N_2 = number of cycles for a through crack to develop

t = 1mm, nominal minimum detectable crack size.

Then for each set of data the associated cumulative non-dimensional weighting function (C.N.W.F.) is given by:-

$$\text{C.N.W.F.} = N_2 \sum_{i=1}^{p} \frac{1}{N_i \, a_i^{m/2}} \qquad -(36)$$

where p = number of crack measurement made in the data set.

For each data set the C.N.W.F. is calculated and used with the corresponding mean value when estimating the overall mean and associated statistical moments for all the data sets taken together. The corresponding values are shown in Table 2. Also shown in Table 2 are the reciprocal functions of initial crack size which are used in the statistical crack growth model. The associated overall mean values and coefficients of variation, skewness and kurtosis are given at the bottom of this table. Corresponding results for crack size characterised by length and $\sqrt{al}$ are also given. In the latter case only 13 data sets were used. This was because an upper limit of 750mm^2 was introduced for the first measured value of the product "al" to avoid gross errors when extrapolating back to the crack area at zero cycles.

To illustrate the range of the estimates found for d and C_{mean} Table 3 is included with the values obtained from the sixteen sets of data when crack size is characterised by depth and m = 3.

Crack Size Parameter		Index m	Apparent Initial Crack Size Statistics			
			Mean	V_d	Θ_d	K_d
Depth	a_o	3	1.25	0.55	1.27	6.70
	$1/\sqrt{a_o}$	3	0.922	0.26	0.93	2.16
	a_o	4	1.57	0.42	2.01	7.25
	$1/a_o$	4	0.714	0.29	-0.03	4.08
Square Root of Cross Section Area						
	$\sqrt{a_o \ell_o}$	3	5.79	0.48	1.78	8.63
	$1/(a_o \ell_o)^{\frac{1}{4}}$	3	0.464	0.22	1.07	3.69
	$\sqrt{a_o \ell_o}$	4	6.56	0.40	2.22	7.69
	$1/\sqrt{a_o \ell_o}$	4	0.168	0.25	-1.08	3.07
Length*	ℓ_o	3	10.94	0 73	1.16	4.03
	$1/\sqrt{\ell_o}$	3	0 382	0.44	1 14	3.51
	ℓ_o	4	12 01	0 79	1 67	7.36
	$1/\ell_o$	4	0 176	1.09	2 17	6.82

TABLE 2: Statistics for the distribution of apparent initial crack size when crack size is measured by various parameters.

* During some experiments very large numbers of crack length measurements were made; many more than necessary. The cumulative non-dimensional weighting function, C.N.W.F. equation (36) Chapter 4, gives very large weights to these results which as a consequence unduly influence the estimated mean. To avoid this problem the fourth root of the C.N.W.F. has been used instead to calculate the values quoted here. This reduces the impact of particularly large numbers of measurements whilst still attaching some importance to the number of measurements.

Test Specimen Reference Number	C_{mean}	a_o
37.7	6.17×10^{-15}	0.34
37.9	2.36×10^{-14}	0.49
37.10	1.53×10^{-15}	0.85
37.13	2.46×10^{-14}	2.12
38.5	2.49×10^{-14}	1.57
38.8	7.25×10^{-14}	0.79
41.1	1.36×10^{-14}	1.71
41.3	2.27×10^{-15}	0.31
41.4	1.18×10^{-14}	0.65
42.2	9.58×10^{-14}	1.68
42.3	1.54×10^{-14}	2.20
43.2	9.72×10^{-14}	3.97
43.4	6.51×10^{-15}	0.45
44.3	1.65×10^{-14}	0.91
37.1	9.74×10^{-15}	2.73
43.1	8.36×10^{-14}	1.39

TABLE 3: Values of C_{mean} and equivalent initial crack size for each of 16 tubular joint fatigue tests where crack size is characterised by depth

4.4 The loading parameter L

The cyclic loading which causes fatigue in offshore structures is caused primarily by waves and is calculated, usually, by means of Morison's equation. For members which are small in diameter compared to the mean significant waveheight (H) drag force loading prediminates and the resultant cyclic strain amplitude is proportional to the waveheight squared. For larger members the inertia force dominates and strain is proportional to waveheight. This has been found from full scale experiments [48] and is accepted by the offshore industry [65].

Assuming the stress is directly related to the strain, then:-

$$S^m = A^m H^{mp}$$

$$\text{and} \qquad L = A^m \Sigma\, n_i H_i^{\,mp} \qquad -(37)$$

where $p = 2$ when the drag force dominates and
$p = 1$ when the inertia force dominates.

The term A includes many factors about which there is uncertainty: the accuracy of the wave theory modelling, the choice of C_D and C_I even without marine fouling are not precisely established. The estimation of hot spot stress from nominal stress however, is already largely catered for in the uncertainty associated with C. Using experiment data from tubular joint fatigue experiments Iwasaki and Wylde [66] have performed regression analyses of cycles to failure on both measured and calculated "hot spot" stresses. They found the residual errors to be very similar. Now C has been estimated using measured stresses. If C had been estimated using the "hot spot" stresses calculated from the applied forces used in the experiment, the residual error would not have been much greater. Hence the uncertainty involved in going from member force to calculated "hot spot" stresses will be largely accounted for in the uncertainty associated with C. The term A therefore involves the uncertainties in transferring from oceanographic data to nominal member stresses.

The uncertainty in L can readily be estimated using the equations of the last chapter to combine the uncertainties in A and $\Sigma n_i H_i^{mp}$, provided the uncertainties in both are known.

In order to estimate the uncertainties in the latter, waveheight records for 5 locations around the U.K. shown in Figure 4, have been analysed on a year by year basis for periods up to 30 years. For each year at each location the product $\Sigma n_i H_i^{mp}$ (denoted by H' for convenience) has been found where n_i is the number of waves in waveheight band i.

As waveheight records only are available, the relationship between waveheight and period given by Fang and Hogben [67] has been used to estimate corresponding wave periods:

$$T_{mean} = 3.925 + 1.439\, H_{mean} \qquad -(38)$$

The values obtained for each location vary significantly from year to year and a set of typical results in shown in Table 4. The coefficients of variation, skewness and kurtosis have been calculated for all the locations and are presented in Table 5 together with non-dimensionalised mean values. These results show some interesting characteristics. As would be expected the variation in mean value for the different locations is considerable and illustrates the importance of obtaining reliable wave data for the specific location of a structure. The drag force dominated regime (p=2) shows significantly greater random variation than the inertia force regime, and also higher skewness. For similar reasons the higher value of m shows the same trend. Clearly the scatter and skewness increase with the power to which H is raised.

In most practical cases only two or three years' data will be available for the design location and the estimated mean value of H' (H'_{mean}) will have appreciable uncertainty. In fact for n years of sample data the mean value will have the following statistical coefficients:-

$$V_{H'_{mean}} = \frac{V_{H'}}{\sqrt{n}} \text{ and } \Theta_{H'_{mean}} = \frac{\Theta_{H'}}{\sqrt{n}} \qquad -(39)$$

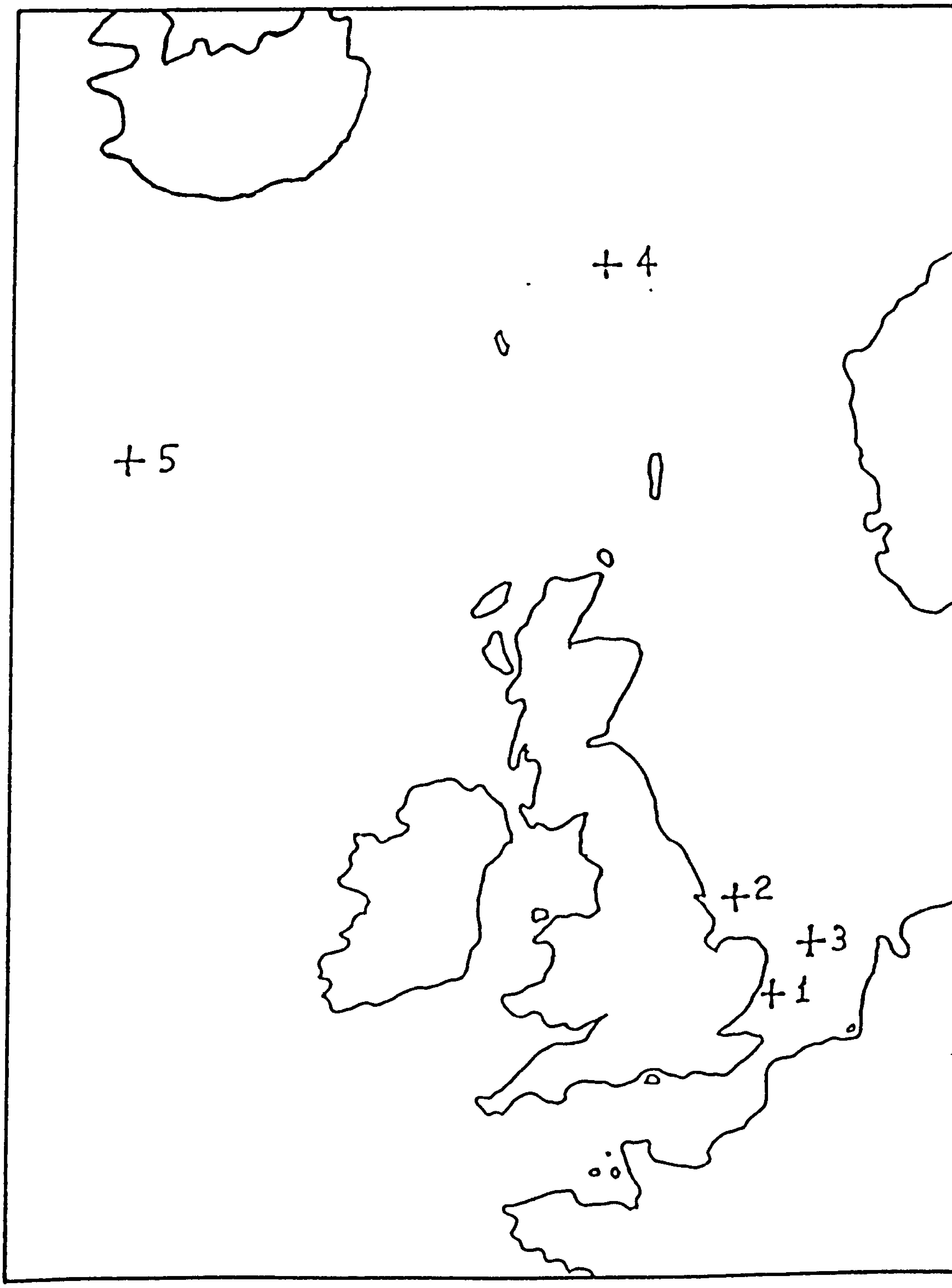


Figure 4 Location of weather ships recording wave height data.

Year	$\Sigma n_i H_i^{mp}$			
	mp=3	mp=6	mp=4	mp=8
1	0.808	0.534	0.725	0.320
2	0.964	0.885	0.928	0.630
3	0.905	0.538	0.830	0.334
4	1.160	0.849	1.119	0.495
5	0.894	0.469	0.779	0.219
6	1.020	1.236	1.122	1.089
7	1.001	0.726	0.934	0.500
8	0.878	1.431	1.025	1.633
9	0.937	1.358	1.047	1.742
10	0.661	1.430	0.727	2.883
11	0.476	0.144	0.350	0.043
12	0.533	0.214	0.419	0.085
13	0.651	0.306	0.539	0.138
14	0.827	0.437	0.700	0.235
15	0.992	0.872	0.965	0.738
16	1.061	1.037	1.069	0.886
17	1.112	1.247	1.181	1.114
18	0.827	0.576	0.764	0.346
19	1.046	0.732	0.992	0.419
20	0.666	0.292	0.536	0.140
21	1.070	1.061	1.054	1.032
22	0.820	0.406	0.689	0.203
23	0.900	0.511	0.796	0.256
24	1.647	1.369	1.653	0.994
25	1.592	2.348	1.815	2.848
26	1.563	1.977	1.747	1.830
27	1.990	3.973	2.505	5.700

TABLE 4: Variation in the non-dimensionalised loading $\Sigma n_i H^{mp}$ from year to year.

The actual distribution of H' cannot sensibly be characterised from only two or three years of data. However, typical non-dimensional moment parameters such as those given in Table 5 may be used to characterise the form of the distribution, F. Then it is plausible to write that:

$$H' = H'_{mean} F \qquad -(40)$$

where H'_{mean} and F are statistically independent variables, and F has a mean value of 1.0.

Now the wave height parameter (H') will be sampled in each and every year of the fatigue life. If the coefficients of variation and skewness of F are denoted by V_F and Θ_F respectively, then corresponding coefficients for a period T years into the fatigue life will be:

$$V_{F_T} = \frac{V_F}{\sqrt{T}} \text{ and } \Theta_{F_T} = \frac{F}{\sqrt{T}} \qquad -(41)$$

In effect then, there are two sources of uncertainty; one due to the intrinsic random nature of the wave process, which is represented by F, and the other due to the limited set of measurements used to estimate the mean magnitude of the process, which is represented by variable H'_{mean}. The distribution of F will also have some sampling uncertainty. In principle this could be estimated using statistical prediction analysis [68] however, this requires a knowledge of the distribution function which unfortunately is unknown. If for a new location in the U.K. area the overall mean values for V, Θ and K are used to estimate F, the error is likely to be comparatively small. The error involved in estimating H'_{mean} is likely to be much larger; particularly when wind speed measurements are used to make wave height forecasts.

The distribution moments of A are more difficult to assess. Kenley [48] presents graphically data relating measured wave heights to measured strains on a tubular member for the Forties platform. From these the intrinsic random uncertainty is estimated

Location	m	Inertia Regime ($\rho=1$)				Drag Regime ($\rho=2$)			
		Mean	c.o.v.	$\theta(\sqrt{\beta_1})$	$K(\beta_2)$	Mean	c.o.v.	$\theta(\sqrt{\beta_1})$	$K(\beta_2)$
1	3	1.0	0.35	1.14	4.04	1.0	0.80	2.04	7.90
	4	1.0	0.47	1.44	5.19	1.0	1.22	2.38	9.03
2	3	3	0.27	0.46	2.92	14	0.50	1.32	4.67
	4	5	0.33	0.88	4.12	37	0.73	1.22	3.37
3	3	5	0.15	0.39	2.86	33	0.57	1.61	4.54
	4	9	0.25	0.88	3.21	123	1.05	1.98	5.41
4	3	15	0.49	0.42	2.78	224	0.82	0.96	3.45
	4	39	0.62	0.60	2.04	1207	0.99	1.46	5.07
5	3	58	0.38	0.03	1.76	3131	0.82	1.69	6.19
	4	216	0.48	0.43	2.31	44344	1.28	2.67	10.81
Mean Values	3		0.33	0.49	2.87		0.70	1.52	5.35
	4		0.43	0.85	3.53		1.05	1.94	6.74

TABLE 5: Statistical parameters for annual wave loading term $n_i H^{mp}$ for five different offshore locations.

to have c.o.v. of around 25%. Skewness cannot sensibly be estimated from the data presented. However, because of modelling error and bias estimated values of A will have greater uncertainty than this. Wirsching [5] examines these uncertainties and from his data values of V_{A^3} of around 0.6 and Θ_{A^3} of around 1.1 are inferred. Wirsching also obtained a concensus view among practitioners in the U.S.A. that the stress range S was over-estimated and the bias was around 0.7. That is the actual value of S^m is 0.7^m times smaller than the estimated value.

In some cases different values of m may be required at a lower stress level, e.g. D.En.'T' curve [op.cit.] or a stress range endurance limit may be employed. In the latter case those waves producing a stress range below the endurance limit are not included in the summation $\Sigma n_i H_i^{mp}$. In the former case the wave data can be divided into two groups, one for each m value. The D.En.'T' curve is defined by:

$$\log(N) = 12.16 - 3\log(S) \text{ for } S > 52.4 \text{ Nmm}^{-2}$$

$$\log(N) = 15.61 - 5\log(S) \text{ for } S < 52.4 \text{ Nmm}^{-2}$$

The corresponding loading can then be written in the following manner:

$$L = A^3 \left[\frac{\mu_A^{5/3}}{10^{4.67}} H_1' + H_2' \right] \qquad -(42)$$

where the first term in the brackets corresponds to the waves at the lower stress range and the second term to the higher stress range. This expression involves a few approximations. H_1' and H_2' are assumed statistically independent, which they are not, and the transfer function A is assumed not to vary with wave height, which it does. The first assumption is necessary in the context of the method, but is unlikely to have a significant effect upon the results, as for most fatigue lives H_2' is found to be the dominant term. The second assumption concerns not just the case above equation (42) but also equation (37). Ideally for each wave height band a different distribution of A should be used. With the current state of knowledge, however, this is not realistic.

In this thesis A is taken to include a modelling error uncertainty because its uncertainty is assumed independent of wave height.

4.5 Discussion on the estimation of parameters

Consider first the estimation of M and d for which the same sample of data [47] has been used. The value of m has been selected as an integer. Ideally the value of m which gives the best fit should be used. However, a value of m = 3 gives a reasonable fit and corresponds to the exponent used in new Department of Energy 'T' fatigue curve [21] which is based, in part, on the same data.

The values quoted in Tables 1 and 2 are based on relatively small samples of data. These data could be augmented by results obtained elsewhere in Europe [61] and in other parts of the world. The resulting values may then be significantly different.

In fact values obtained for C using large samples of data by Johnston [14] in the U.K. and Bokalrud and Karlsen [34] in Norway are almost an order of magnitude larger but with similar variance. However, the data involved in these cases is not from experiments on tubular joints, but from small steel test pieces. As a check on the procedure described in this chapter, manual calculations were performed to determine the C value required to go from an assumed 1 mm depth initial defect to a through crack in the corresponding number of cycles quoted for each fatigue experiment. The average of the C values for m = 3 was 5.39 $\times 10^{-14}$ which is somewhat higher than the 3.06 $\times 10^{-14}$ obtained using the more refined procedure described above, but still of the same order of magnitude. Perhaps there is an intrinsic difference in the crack growth rate in a tubular joint compared with that in a small sample of the parent metal. This proposition is more plausible when it is borne in mind that the crack shape parameter is assumed to be 1.0 in the above, which may be thought a low value. It would, however, be foolish to draw a firm inference from what is a relatively small sample of data.

The mean values obtained for the depth of initial defects lie in between those obtained by Burdekin and Townend [op.cit.] and those obtained by Rogerson and his colleagues [op.cit].

It must be remembered though that the initial crack size estimated here is a "perceived" or "apparent" size rather than an actual measurement. It is the apparent starting point for fatigue crack growth if Paris' equation is used to characterise crack growth where m is fixed (3 or 4) and C is estimated from subsequent crack measurements. It is therefore consistent with, and an integrated element of, the crack growth model.

There are other procedures to estimate initial defect size employed in the aircraft industry [62] [63] and suggested for offshore structures [64]. These all rely on waiting until the fatigue crack reaches a certain size, 0.03 inches in the case of [62] and through thickness for [64]. Reverse extrapolation to the apparent initial defect size is then by a deterministic reverse extrapolation procedure. Usually a particular distribution is fitted to the time to reach the given crack size. Yang and Manning [62] favour the Weibull distribution and Swift and Connolly [64] the log-normal distribution. The transformation to an initial defect distribution is then made via a simple crack growth model. This approach assumes all the variability in fatigue life may be attributed to the distribution of the initial defect size. This sweeping assumption is at odds not only with data presented here but also with much work undertaken elsewhere, e.g. [5], [14], [35] and [54]. Notwithstanding this the initial defect distribution presented graphically by Swift and Connolly compares well with the results obtained here and presented in Table 2.

Turning now to the estimation of L, this is clearly a complex variable which contains many sources of uncertainty. The data for estimating these uncertainties is sparse. In addition procedures for estimating fatigue stress ranges vary, and methods involving either wave height histograms or spectral techniques are employed in deterministic calculations [69]. The modelling errors associated with each technique, outside the dynamically sensitive region, have not, to the author's knowledge, been properly quantified. Perhaps the most comprehensive attempt to date to quantify the uncertainties related to fatigue offshore has been the four year A.P.I. sponsored project, "Probability Based Fatigue Design Criteria for Offshore Structures", the results of which are summarised by Wirsching [5]. This was concerned with fatigue modelling rather than fatigue crack growth modelling, however, some aspects are similar and the uncertainties in the transfer function A^m as presented by Wirsching, and already quoted, are adopted in this thesis.

The à priori estimation of the sea state in the vicinity of the structure is likely to be the largest source of uncertainty. The range of values H'_{mean} in Table 5 bears testimony to this fact. If sea states are estimated from wind data rather than wave data the uncertainty if anything is likely to be greater. Clearly wave data for the location at which a structure is to be sited for a period of several years would be valuable in reducing uncertainty. In this thesis the data presented in Table 5 are used and assumed to be typical. In summary, then, the loading parameter L is estimated in the following way:-

$$L = A^m \times H'_{mean} \times F$$

where A^m, H'_{mean} and F are variables as defined in the previous section.

Finally it is worth remarking that all the variables d, M and L are found to be subject to substantial uncertainty thus reinforcing the need not only for more data, but for a statistical based fatigue crack growth model.

5. A TYPICAL CRACK GROWTH CALCULATION AND THE USE OF INSPECTION RESULTS

5.1 Introduction

The test of any engineering model is how well it works in practice. This chapter therefore is concerned with the practical application of the crack growth model developed in Chapter 3 using the parameters that have been estimated in Chapter 4.

Generally in fracture mechanic calculations, crack size is characterised by depth. However, whilst crack depth can readily be measured onshore using N.D.T. techniques [e.g. 23], subsea only crack length can be measured with any accuracy [e.g. 95]. So if use is to be made of inspection results to reassess fatigue lives of joints in jacket structures, crack length must be used to characterise fatigue crack size.

Crack size can also be represented by the cross section area of a fatigue crack. In the next chapter it will be seen that crack area is an important parameter when considering reliability. In practice the square root of crack area is used in the crack growth equation as it is a linear measure. The corresponding crack area is then found directly afterwards.

In this chapter the crack growth model is applied in turn to all these crack size measures. This both illustrates the versatility of the model and provides suitable examples for subsequent discussion.

During their lives offshore structures are subject to periodic inspections for fatigue cracks in welded joints. When cracks or "significant defects" [10] are found action is usually taken and fatigue lives may be reassessed (albeit in a way not necessarily directly related to the crack size).

However, on the majority of occasions no significant cracks are found and the fatigue lives are not re-evaluated. An opportunity to re-assess the state of the structure is thus wasted. This

Chapter describes a simple Bayesian [e.g. 70] procedure for revising the fatigue crack size distribution in the light of inspection results and making a corresponding adjustment to the nominal fatigue life. The effect of structural monitoring on estimated fatigue crack size distribution is also examined.

The final section discusses the practical problems of representing uncertainties with probability density functions and other points of interest which arise from the examples.

5.2 Fatigue Crack Depth Calculation

The most frequently used measure of crack size in fracture mechanics calculations is crack depth. The first example therefore uses crack depth to characterise crack size.

Consider a T-joint comprising a 1000 mm diameter brace meeting a chord of 32 mm thickness, that has been designed for a fatigue life of 20 years using the D. En. 'T' for air [53] but ignoring the change in slope:-

$$\text{i.e.} \quad \log(N) = 12.16 - 3\log(S)$$

Now When Miner's cumulative damage rule is used:-

$$\log \Sigma N_i S_i^3 = 12.6 \quad \text{for 20 years}$$

$$\text{hence} \quad \log \Sigma n_i S_i^3 = 12.6 - \log 20$$

$$\text{and} \quad \Sigma n_i S_i^3 = 7.23 \times 10^{10} \quad \text{per annum.}$$

Using the bias value quoted by Wisching [5] the actual value of S is assumed to be 0.7 times the value used in the fatigue design calculations. This yields:-

$$\mu_L = 0.7^3 \times 7.23 \times 10^{10} = 2.48 \times 10^{10}.$$

Again using the data compiled by Wisching the uncertainty in the transfer function from wave height to nominal member stress is assumed to be characterised by:-

$$V_{A3} = 0.6 \qquad \Theta_{A3} = 1.1$$

It is assumed that the distribution of the mean wave height parameter H'_{mean}, from which the stress ranges for the fatigue calculation have been calculated, has the following characteristics:

$$V_{H'_{mean}} = \frac{0.7}{\sqrt{2}} \quad \text{and} \quad \Theta_{H'_{mean}} = \frac{1.5}{\sqrt{2}}$$

That is, it has been estimated from two years of sample data.

The wave height parameter H' is assumed to have a distribution whose form F is characterised by:-

$$V_{F_T} = \frac{0.7}{\sqrt{T}} \quad \text{and} \quad \Theta_{F_T} = \frac{1.52}{\sqrt{T}}$$

That is the form F is the average of the distribution shapes found for the parameter $\Sigma n_i H_i^6$ (m=3, p=2) for the five off-shore locations from which wave data has been analysed.

When crack size is characterised by depth, Table 1 suggests the following values for m = 3.

For M:- $\mu_M = 0.5 \times \pi^{1.5} \times 3.06 \times 10^{-14} = 8.52 \times 10^{-14}$

$V_M = 1.12$

$\Theta_M = 1.30$

and for d:

$\mu_d = 0.922$

$V_d = 0.26$

$\Theta_d = 0.93$

Using the equations given in Chapter 3 μ_r, V_r and Θ_r are calculated at times during and beyond the nominal fatigue life. These are presented in graphical form to a base of time in years in Figure 5. This shows that the distribution of r has a mean value which declines in a linear manner with time. As crack depth is the square root of the inverse of r (for m = 3) it clearly increases rapidly in later years. In fact the behaviour of this function of r corresponds closely to the usual deterministic fracture mechanics calculation. However, the magnitude will be different as conservative rather than mean values are usually assumed in deterministic calculations.

From Figure 5 it is clearly seen that uncertainty (as measured by coefficient of variation) grows with time. That is the distribution of crack size becomes wider spread as time goes on. It is also interesting to note that the distribution of r changes from having a positive skew to a negative skew during the fatigue life.

A three-parameter log-normal distribution [71] is fitted to the statistics of r for each year in the fatigue life following the procedure described in Chapter 3 and illustrated in Figure 1. The log normal distribution has been chosen because it allows large coefficients of skewness to be obtained with a unimodal bell-shaped distribution. To obtain negative skewness a simple parametric transformation is used, with truncation of the distribution tail, which gives rise to a slight discontinuity in the results. However, this does not alter the nature of the results in any significant way. The problem of selecting a suitable distribution from amongst the standard types available is discussed at the end of this chapter.

The probability of exceeding a certain crack size a_d is calculated directly from the probability distribution fitted to r. The probabilities of crack depth exceedance for several values of crack depth have been calculated at intervals during the life and these are tabulated in Table 6 and presented graphically in Figure 6.

It is interesting to note that the probabilities of exceedance for large crack sizes are relatively similar. This is indicative of the rapid growth of large cracks. At the nominal fatigue life there is about a 0.001 % chance of a through crack. It is interesting to note that if the modelling bias is not included in the calculation of S^3 this probability rises to 4.0 %. From an examination of Table 6 the bias is seen in effect to constitute

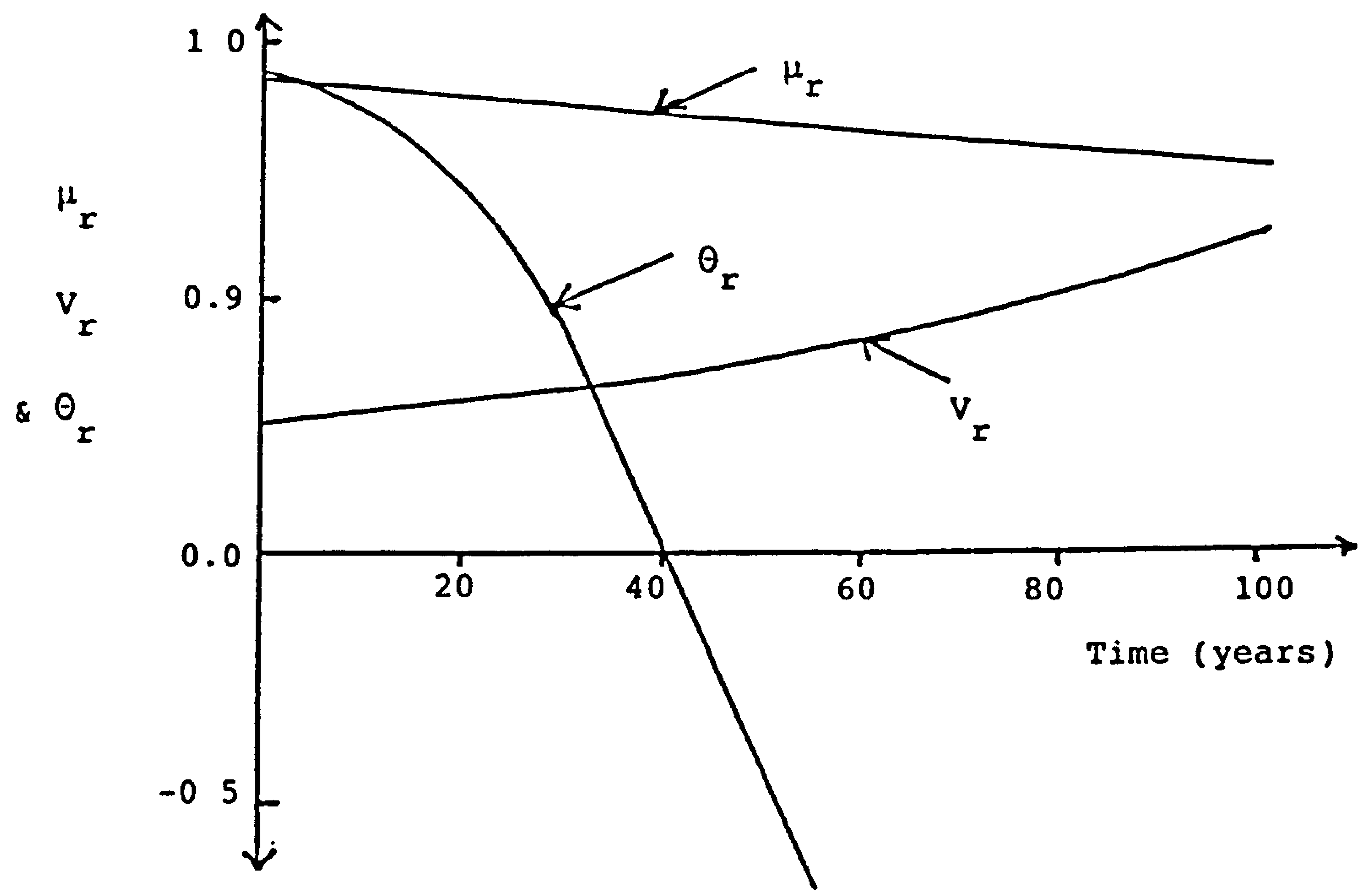


Figure 5: Changes in mean and coefficients of variation and skewness of r with time when crack size is characterised by depth

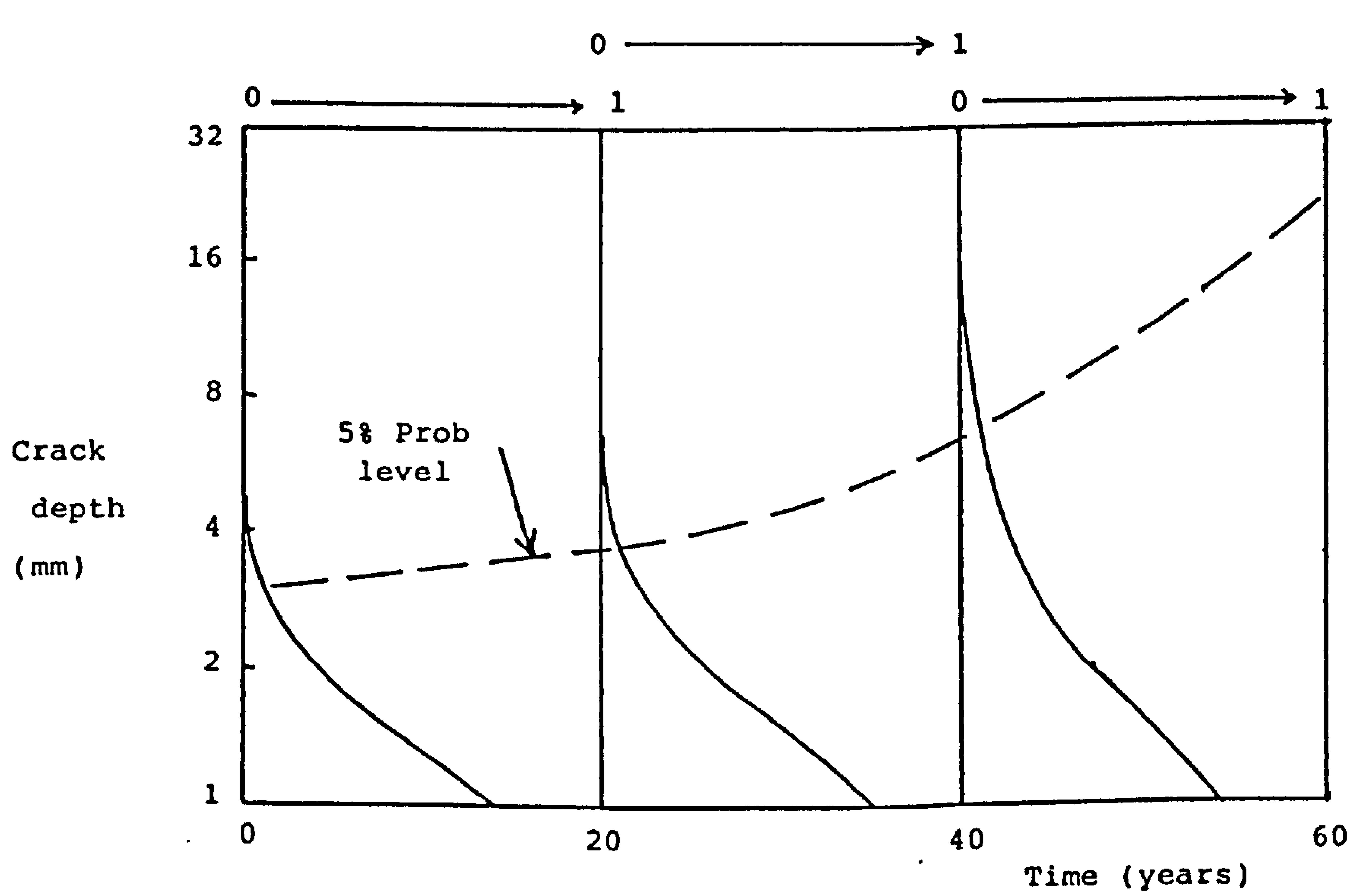


Figure 6: Probabilities of crack depth exceedance at times during the fatigue life.

a factor of safe of about 3 on the fatigue life (at least in this case). It is well accepted that when uncertainties are high; factors of safety must also be high to achieve the same target reliability.

5.3 Fatigue Crack Area Calculation

In the next chapter it will be seen that fatigue crack area as well as fatigue crack depth is important when considering the effect of fatigue crack growth upon the reliability of the joints of tubular members. In this section the fatigue crack area is considered for the joint described in the previous section.

In order to estimate crack area the parameter used to characterise crack size is the square root of the nominal enclosing rectangle i.e. $\sqrt{a_d \ell}$. The appropriate statistical parameters in this case when m = 3 (as quoted in Tables 1 and 2) are as follows:-

$$\mu_m = 0.5 \times \pi^{1.5} \times 1.39 \times 10^{-14} = 3.87 \times 10^{-14}$$

$$V_m = 1.07$$

$$\theta_m = 1.78$$

$$\mu_d = 0.464$$

$$V_d = 0.22$$

$$\theta_d = 1.07$$

The statistics for the "loading" parameter L, of course remain the same. The calculation proceeds in exactly the same way as the crack depth calculation of the previous section. The resulting statistics for r are presented graphically in Figure 7 as a function of time. The corresponding probabilities of crack area exceedance are presented graphically in Figure 8 and are tabulated in Table 7.

The crack area statistics have a very similar nature to the crack depth statistics. The principal difference is the more marked

Crack depth mm	Crack depth Exceedance Probabilities 1	2	4	8	16	32
Year 0	0.681	0.178	0.0070	0.0000	0.0000	0.0000
4	0.692	0.192	0.0089	0.0000	0.0000	0.0000
10	0.706	0.216	0.0139	0.0001	0.0000	0.0000
20	0.724	0.260	0.0311	0.0011	0.0000	0.0000
30	0.733	0.303	0.0636	0.0083	0.0009	0.0001
60	0.710	0.340	0.0165	0.0930	0.0606	0.0444
90	0.732	0.394	0.2277	0.1505	0.1114	0.0900

TABLE 6: Crack depth exceedance probabilities for a nominal 20 year fatigue life

Crack area (mm^2)	Crack area Exceedance Probabilities 30	100	1000	10,000	100,000
Year 0	0.401	0.041	0.0000	0.0000	0.0000
4	0.417	0.047	0.0000	0.0000	0.0000
10	0.442	0.061	0.0000	0.0000	0.0000
20	0.478	0.095	0.0001	0.0000	0.0000
30	0.499	0.140	0.0034	0.0001	0.0000
60	0.496	0.220	0.0656	0.0317	0.0210
90	0.539	0.285	0.1168	0.0698	0.0523

TABLE 7: Crack area exceedance probabilities for a nominal 20 year fatigue life.

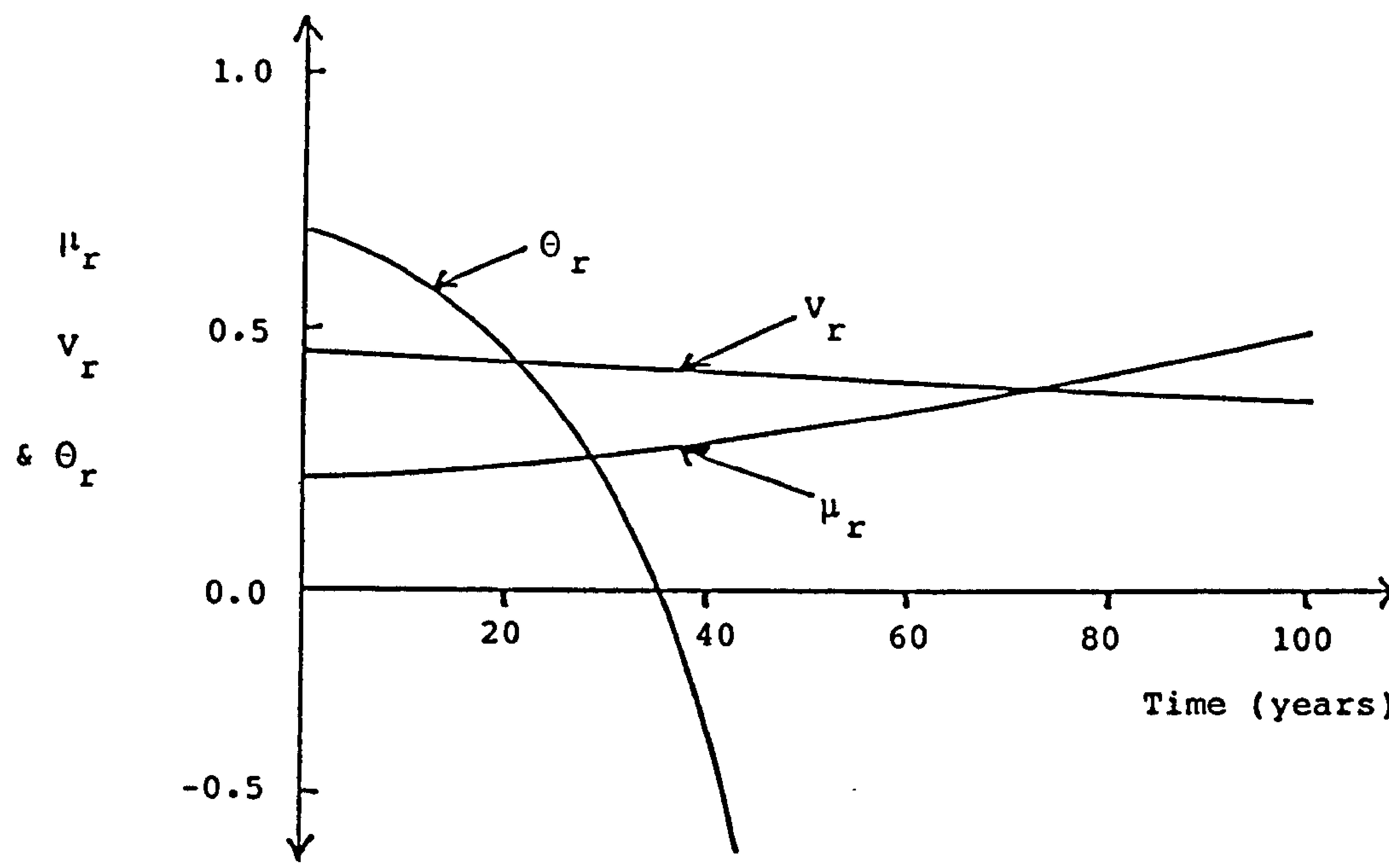


Figure 7: Changes in mean and coefficients of variation and skewness of r with time when crack area is used as a measure of crack size.

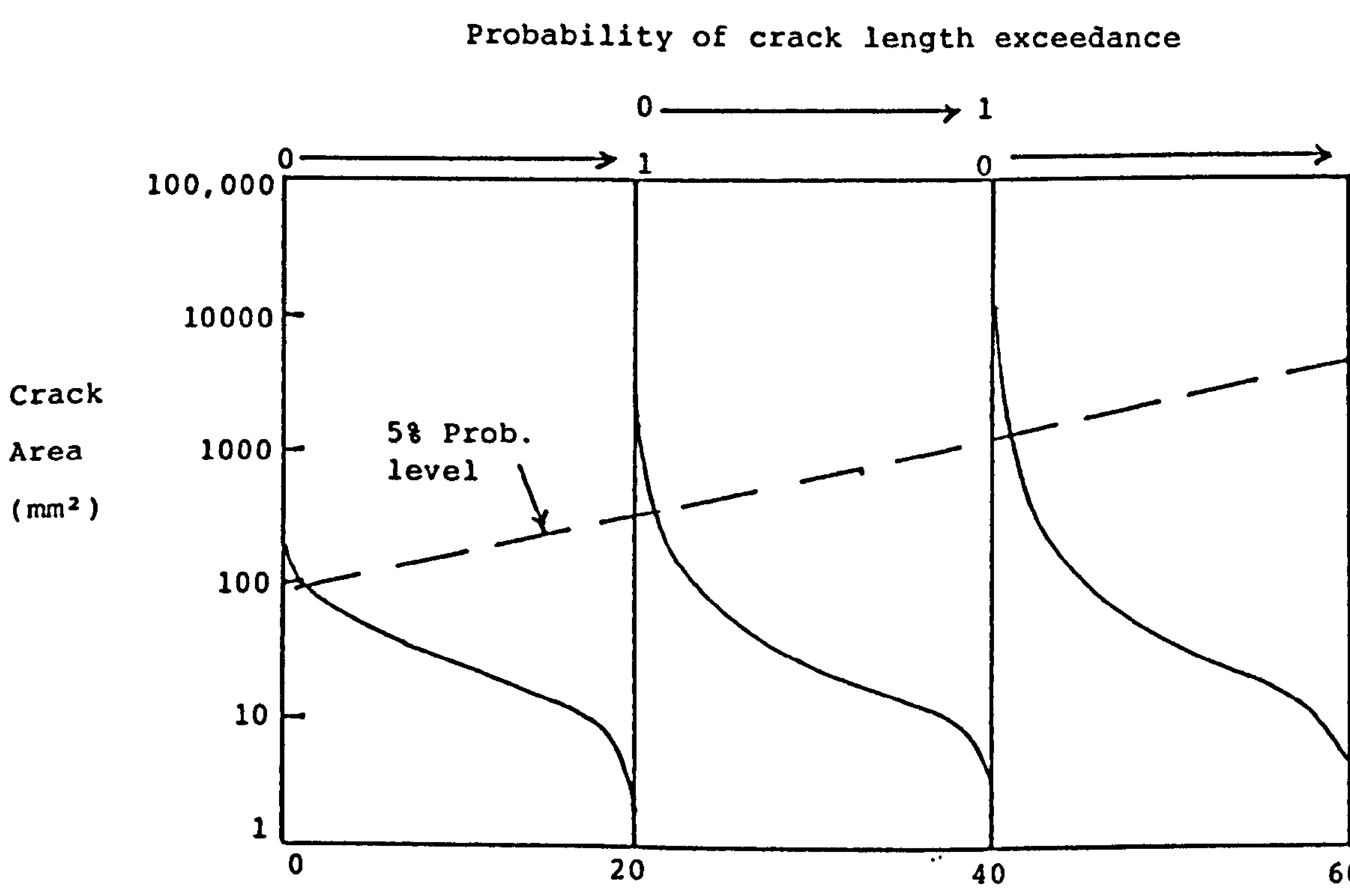


Figure 8: Probabilities of crack area exceedance at times during the fatigue life.

increase in magnitude for crack area towards the end of the fatigue life.

In order to calculate the reliability of a fatigue cracked tubular joint the moments of the distributions of crack area are needed for regular intervals during and beyond the fatigue life. For this purpose a log-normal distribution has again been utilised. It has been fitted to the first three moments of the crack area which in turn have been found numerically from the corresponding log normal distribution of r. The corresponding fatigue crack area statistics are given in Table 8, together with the "characteristic" values. This latter is defined as the value which has only a 5% chance of being exceeded. From Table 8 it is clearly seen that the skewness of the crack area distribution increases rapidly towards the end of the actual ife. In this region the moment based estimation of the log normal distribution has low efficiency, as discussed at the end of the chapter. However, up to the nominal fatigue life of 20 years the skewness is much lower and the efficiency is higher.

The moments of the distribution of crack depth are also required for the reliability calculations of the next chapter. These are presented here in Table 9 for comparison purposes. Again not until well beyond the end of the nominal fatigue life does the skewness become large.

5.4 Crack length calculation and the use of inspection results

As noted at the beginning of this chapter only crack length can reliably be assessed by subsea inspection. So, in this section the crack length calculation for the joint considered in the previous sections is undertaken. The appropriate statistics in this case, from Tables 1 and 2 are as follows:

For M:- $\mu_m = 0.5 \times \pi^{1.5} \times 6.37 \times 10^{-15} = 1.77 \times 10^{-14}$

$V_m = 0.93$

$\theta_m = 2.33$

FATIGUE CRACK AREA STATISTICS				
Time (years)	μ (mm^2)	V	θ	Characteristic (mm^2)
0	28	0.77	1.13	93
10	31	0.80	1.22	108
20	35	0.87	1.42	138
30	41	1.01	1.86	204
60	93	2.46	5.31	2058
90	1108	7.45	8.83	100530+

TABLE 8: Variation in fatigue crack area with time for a joint with a nominal 20 year fatigue life.

FATIGUE CRACK DEPTH STATISTICS				
Time (years)	μ (mm)	V	θ	Characteristic (mm)
0	1.42	0.51	1.30	2.80
10	1.51	0.54	1.47	3.07
20	1.63	0.61	1.94	3.53
30	1.84	0.79	3.66	4.37
60	2.99	2.27	7.25	23.95
90	3.48	2.40	6.32	32.0+

TABLE 9: Variation in fatigue crack depth with time for a joint with a nominal 20 year fatigue life.

for d: $\mu_d = 0.382$

$v_d = 0.44$

$\theta_d = 1.137$

The calculations are performed in exactly the same way as already described and the corresponding results for crack length exceedance are presented graphically in Figure 9 and in tabular form in Table 10.

Now the critical welded joints of steel structures are inspected at intervals throughout their operational life. When no cracks are found despite extensive and careful surveys of several welds, there may be a case for reducing the frequency or extent of inspection. To do this rationally a number of questions must be answered. Given the fatigue life of the joint, what is the chance that a significant crack will exist at the time of inspection? If no crack is found what is the probability that there was one but it was not detected? Given that inspection programmes are related to fatigue lives, how can fatigue lives be revised if no cracks are found?

The first question can be answered by doing the type of calculation already presented and the other two questions are now examined in turn.

Moncaster [7] has undertaken some experiments with divers to determine the efficiency of M.P.I. crack detection underwater. He states "Isolated cracks of perhaps 30 mm in length can be detected with almost certainty ..."; and from Table 8 of [7] the detection rate for 10 mm cracks is about 60%. At 5 mm length he suggests detection rates of about 10%. A recent study on behalf of U.K. D.En. [66] suggests cracks of 5 mm length can often be detected, but that detection rates are independent of crack lengths. This latter remark does not agree with experience in air for the nuclear power or aircraft industries [e.g. 73, 41].

For the purposes of the example which follows the probability of not detecting a crack, given one may exist, is assumed to follow the distribution shown in Figure 10. This has been constructed on the basis of the information in references [7] and [73] and is only tentative. Suppose now an inspection is made after 10 years of the joint discussed in the previous section, and no crack is found. The crack size distribution, based on the design model, can be modified in a Bayesian manner using the following steps:-

(i) Construct an a priori probability density curve from the crack length exceedance probabilities (as given in Table 10);

(ii) apply Bayes' theorem [e.g. 70] to find the posterior distribution of crack size given no crack is found;

(iii) find the corresponding distribution of r;

(iv) calculate the moments of the distribution of r to yield revised values μ_r^*, V_r^* and Θ_r^*.

This process is illustrated in Figure 11 and involves numerical techniques. It is possible to convert the "crack non-detection" probability to a base of r, and then to apply Bayes' theorem in the r domain to determine r*. This is preferable when the crack length distribution itself is not actually required for the time of inspection, as it avoids numerical errors. This procedure is illustrated in Figure 12.

The new values (μ_r^*, V_r^* and Θ_r^*) can then be used as the starting point for a revised crack growth calculation, beginning from the time of the inspection, and will replace the initial defect parameters, μ_d, V_d and Θ_d in the equations of Chapter 3. Table 11 shows some of the original and modified values for the example.

A revised fatigue life can be estimated as the time when the probability of a very long crack (say, half the circumference) exceeds a certain value. Logically, this value should be the

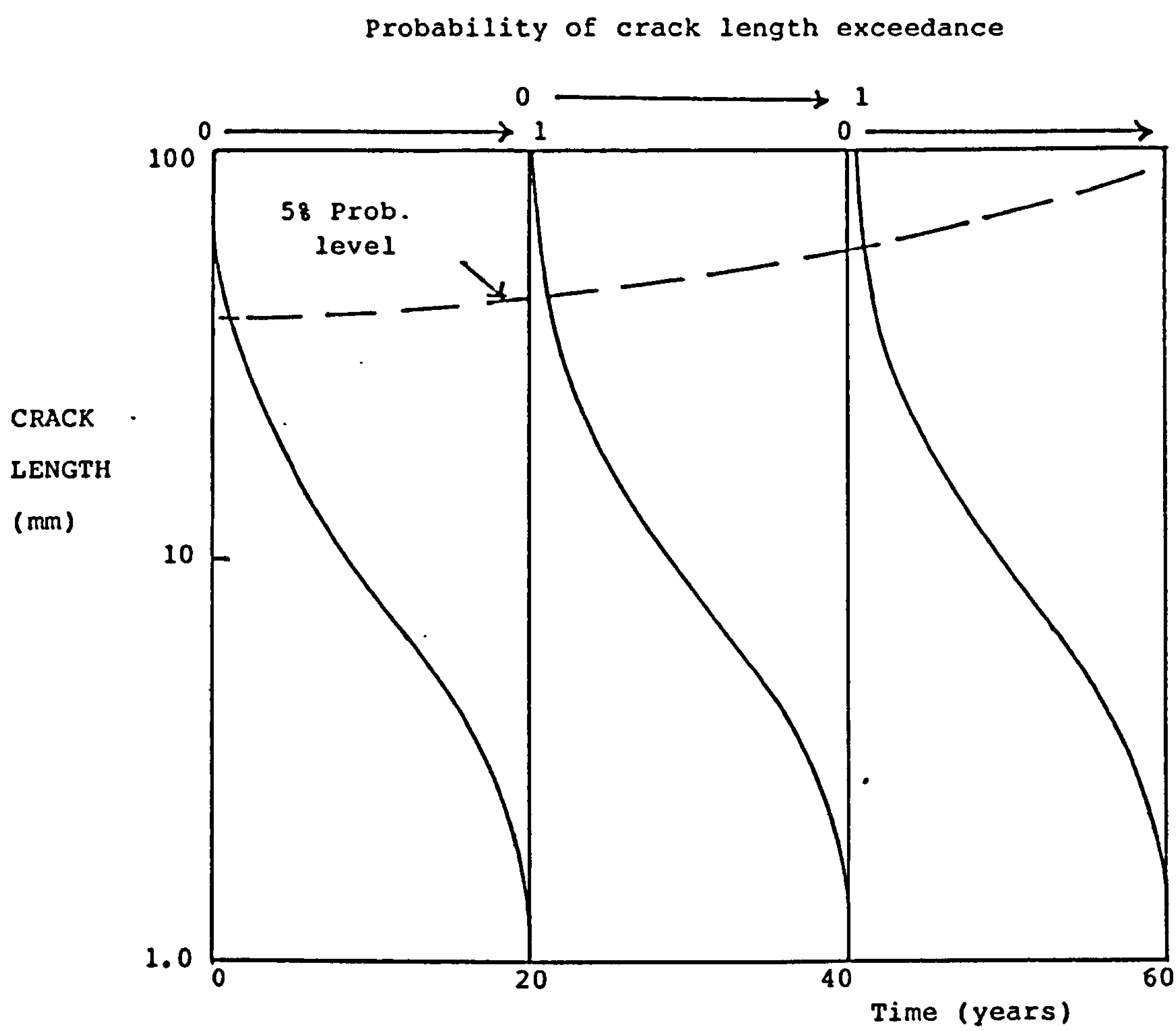


Figure 9: Probabilities of crack length exceedance at times during the fatigue life

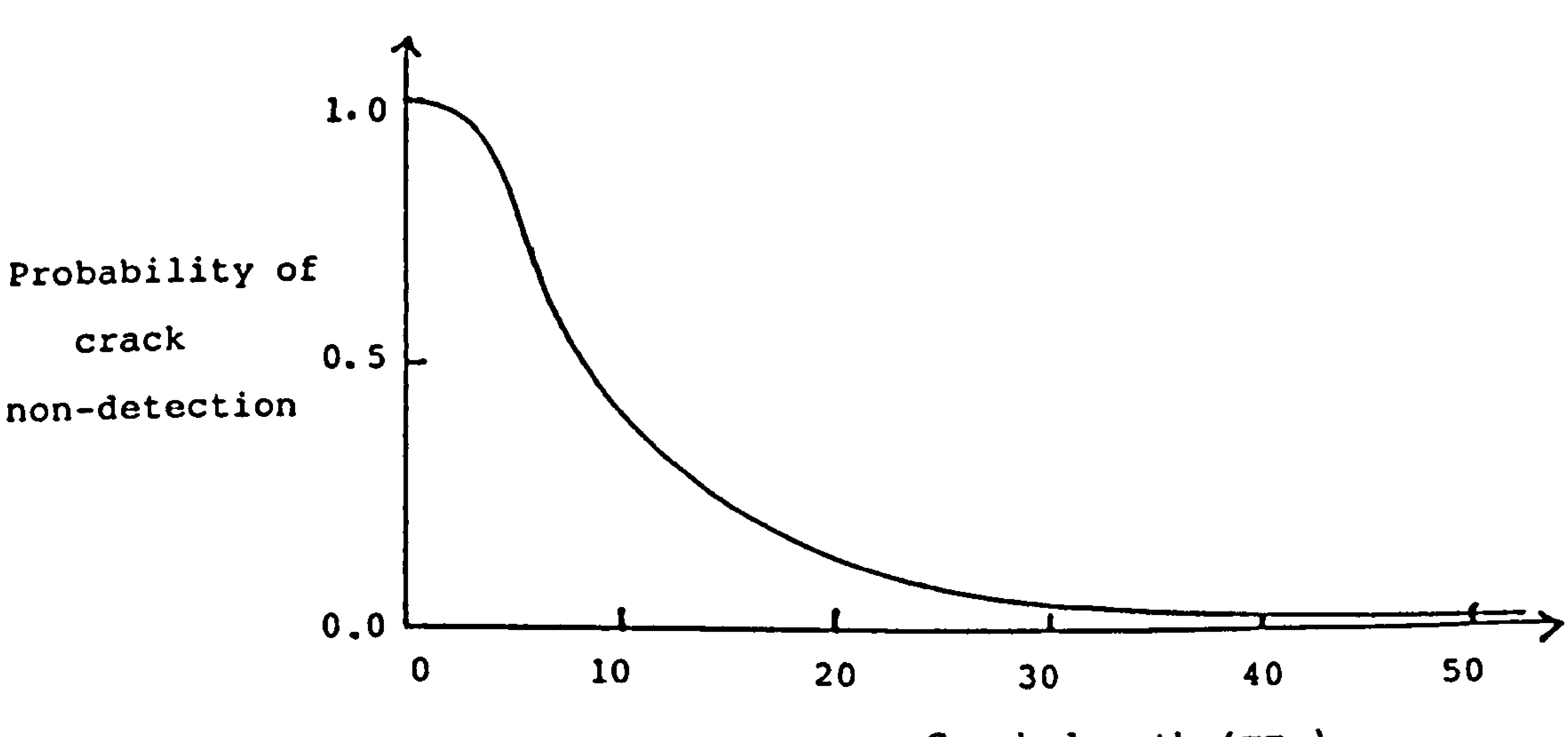


Figure 10: The probability of missing a crack and its variation with crack length

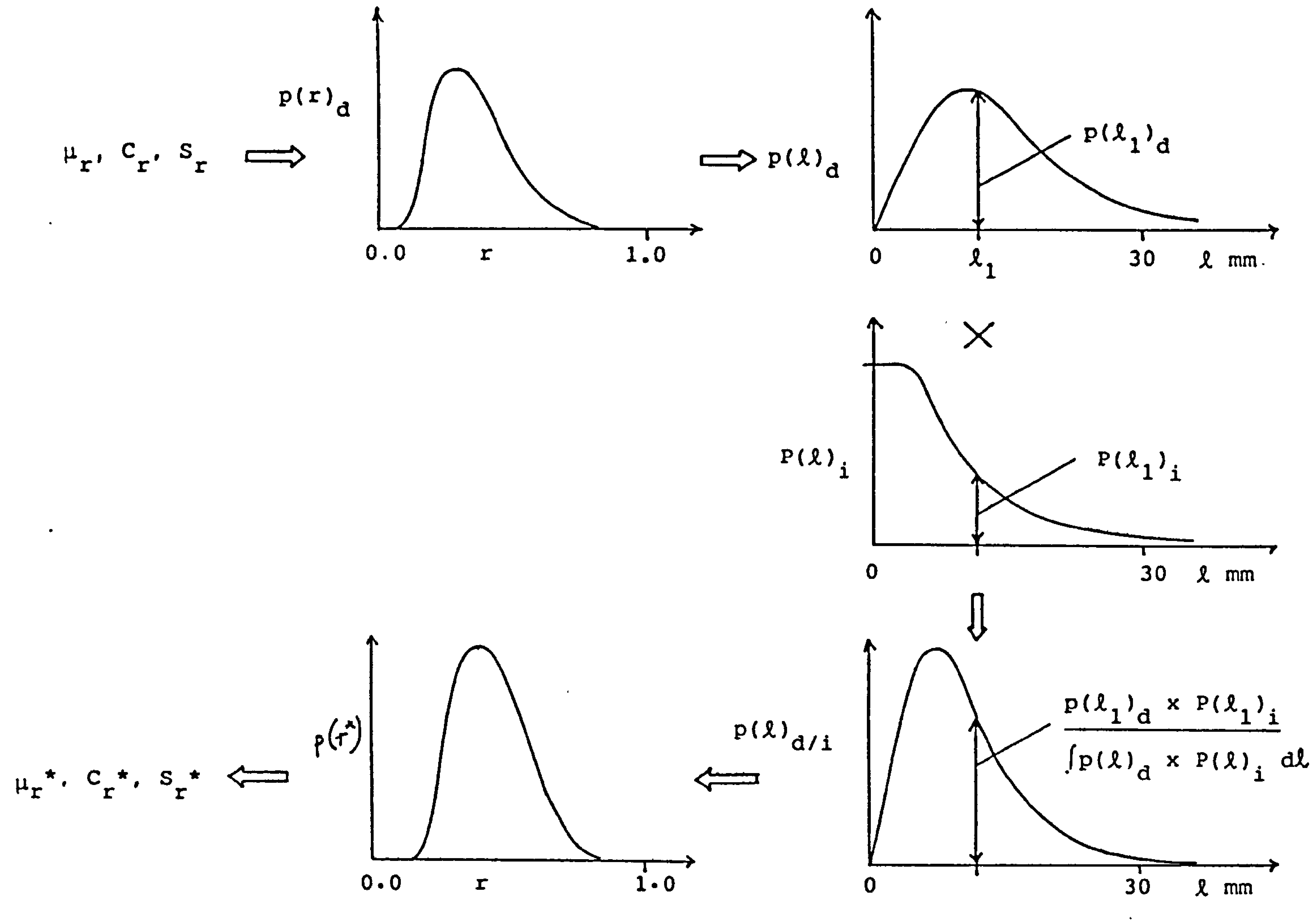


Figure 11: Illustration of the procedure for revising crack length distribution in light of inspection finding no cracks.

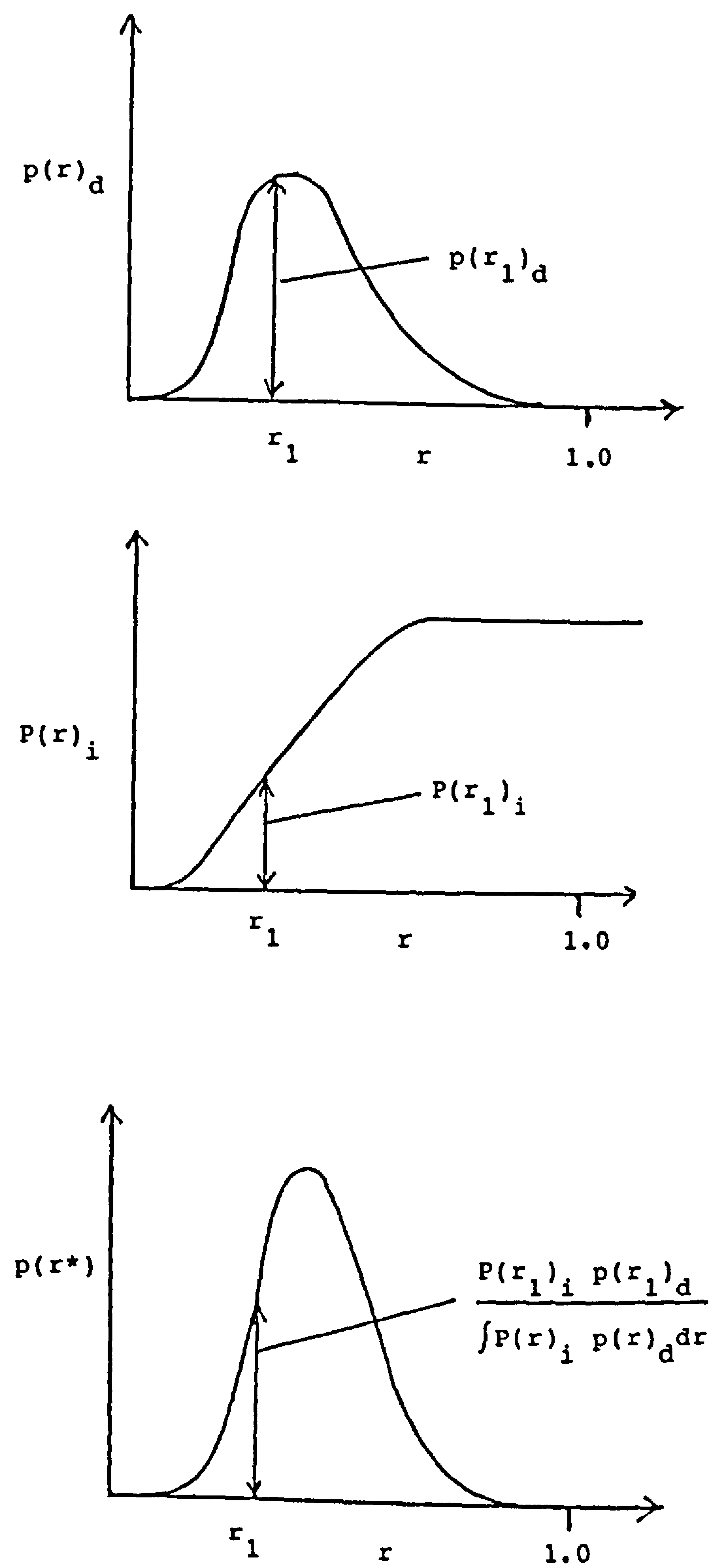


Figure 12: Illustration of the procedure for revising crack length distribution in the r domain alone.

	Crack length Exceedance Probabilities						
Crack length (mm)	5	10	20	50	100	500	1000
Year 0	0.706	0.395	0.158	0.032	0.008	0.0004	0.0002
4	0.710	0.401	0.163	0.034	0.009	0.0005	0.0002
10	0.716	0.411	0.171	0.037	0.010	0.0007	0.0003
20	0.725	0.426	0.185	0.044	0.014	0.0011	0.0005
30	0.736	0.442	0.201	0.053	0.018	0.0019	0.0009
60	0.753	0.484	0.253	0.091	0.041	0.0094	0.0060
90	0.762	0.514	0.302	0.141	0.083	0.0333	0.0256

TABLE 10: Crack length exceedance probabilities for a joint with a nominal fatigue life of 20 years.

	Crack Length Exceedance Probabilities		
Crack length (mm)	20	100	1000
Year 0	0.049 (0.158)	0.0017 (0.0082)	0.0000 (0.0002)
4	0.051 (0.163)	0.0019 (0.0090)	0.0000 (0.0002)
10	0.055 (0.171)	0.0022 (0.0104)	0.0000 (0.0003)
20	0.062 (0.185)	0.0029 (0.0136)	0.0002 (0.0005)
30	0.071 (0.201)	0.0041 (0.0180)	0.0003 (0.0009)
60	0.106 (0.253)	0.0131 (0.041)	0.0023 (0.0060)
90	0.151 (0.302)	0.0491 (0.0832)	0.0136 (0.0256)

TABLE 11: Crack length exceedance probabilities revised after no cracks found on 10th year inspection. Original values are in brackets for comparison purposes.

same probability as that which occurs at the end of the nominal fatigue life in the original crack growth calculations. In the example here the nominal fatigue life could be extended by about 20 years. There is little point in inspecting or re-inspecting any joint until there is a realistic probability that a crack may be found. This probability can be calculated at any time during the life if the crack detection probability function for the equipment to be used is known or can be estimated. The crack detection probability is complementary to the type of curve shown in Figure 11 starting at zero probability and rising to 100% as crack size increases.

When a crack is found, the Bayesian approach described here can be used provided the reliability of the inspection result can be quantitatively estimated.

In addition to regular inspections many structures are equipped with monitoring instrumentation. This allows the uncertainty in the perceived crack size to be reduced even when no inspections have occurred.

Consider the situation after 10 years on a structure where the wave height and direction are monitored, together with the cyclic strains in representative members. The loading parameter L for the ten year period can then be estimated quite accurately and V_L and Θ_L become negligibly small. The corresponding uncertainty in r is reduced and the "expected" probability of large crack length exceedance is much reduced as shown in Table 12 and illustrated by the broken line in Figure 13.

This assumes that the loading actually monitored is close to the estimated mean or "expected" value. In practice the monitoring may show the loading to be higher or lower than the "expected" value; which means the crack size exceedance probabilities may either be somewhat higher or lower than those recorded in Table 12. The important point is that because the loading is known more precisely the probable size of any possible crack can be more closely defined. Now the object of subsea weld inspection, which itself is subject to uncertainty, is to obtain more precise know-

Crack Length mm	Crack Size Exceedance Probabilities		
	Monitored	Not monitored	Ratio
5	0.731	0.731	1.00
10	0.431	0.432	1.00
20	0.180	0.182	0.99
50	0.036	0.037	0.97
100	0.0064	0.0091	0.70
200	0.0013	0.0021	0.62
500	0.00016	0.00035	0.46
1000	0.00004	0.00011	0.36

TABLE 12: The effect of Structural Monitoring upon the "expected" crack length exceedance probabilities after 20 years

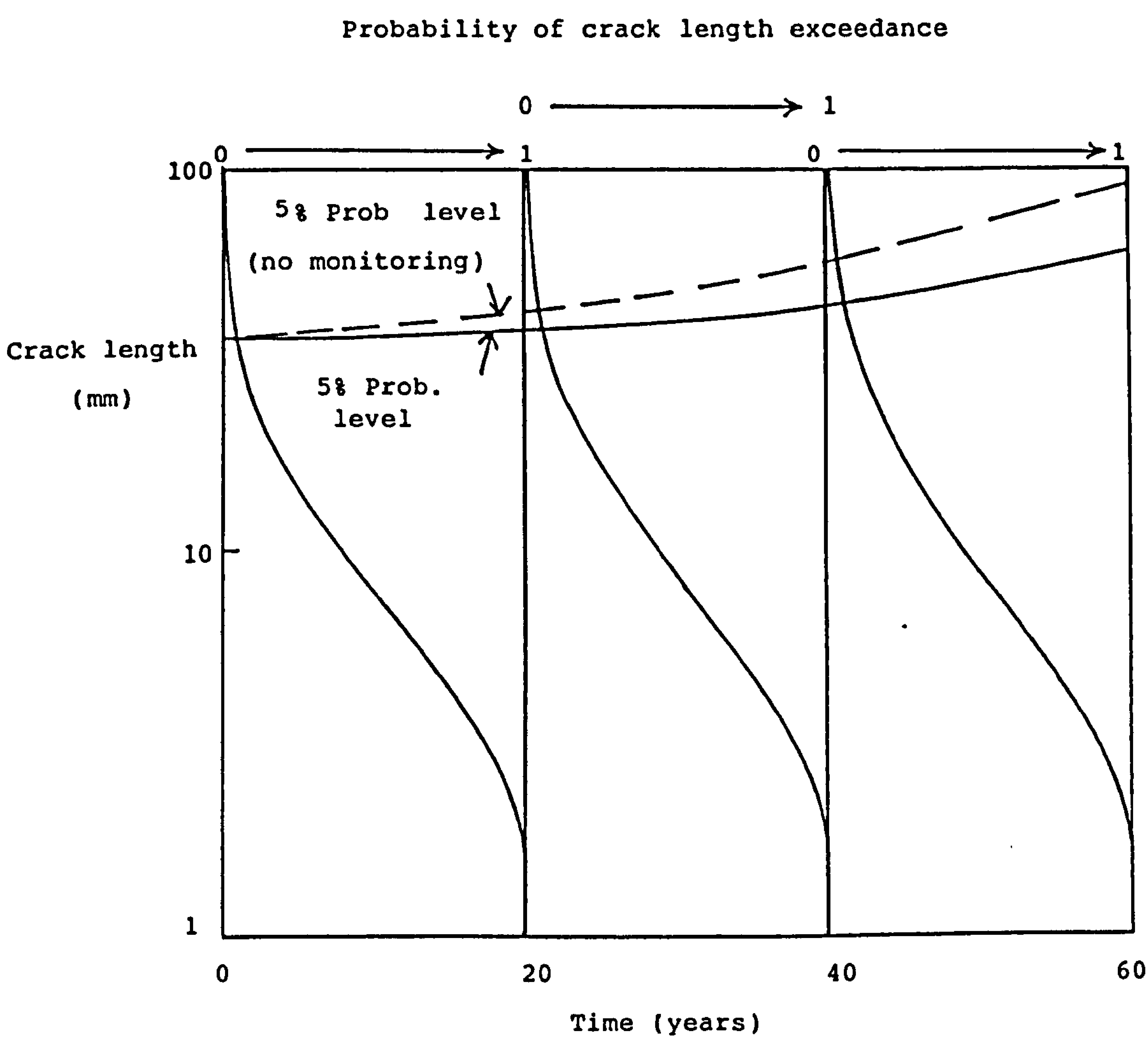


Figure 13: Expected probabilities of crack area exceedance at times during the fatigue life when structural monitoring is employed. Broken line shows 5% probability level when no monitoring is employed.

ledge of the state of the structure. Clearly to obtain the same level of knowledge about the structure less inspection is required when environmental and/or structural monitoring has been employed. Logically the monitoring could be "traded-off" against reduced inspection requirements and there is probably a good economic case for doing this. Of course, crack depth and crack area uncertainties can also be reduced as a result of structural and environmental modelling.

5.5 Discussion of crack growth examples and calculation procedures

The results obtained here are dependent to some extent on the choice of distribution function to represent r. The results in the central region of the distribution either side of the mean value will not be significantly affected, at least until well past the end of the nominal fatigue life. The results in the extreme tails, however, are very much a reflection of the distribution function chosen. This is also true for much of the distribution when the crack size becomes very large, however, at this stage in time the probability of failure is so high it ceases to be of interest for practical designs. The problem of poorly defined distribution tails afflicts many practical statistical problems [e.g. 80] and in particular structural reliability [e.g. 43].

Another problem is that the efficiency of moments as estimators for the parameters of the log-normal distribution, decreases with increasing skewness [e.g. 71]. However, the distribution of r is not highly skewed in the region of interest which is a mitigating factor. The crack size distributions themselves (see Tables 8 and 9) are highly skewed but only well beyond the nominal fatigue lives.

Before selecting the log-normal distribution many others were examined. In fact all those based on Pearson's curves [e.g. 74] together with Gram-Charlier and Edgeworth Series, and the log-normal series [e.g. 44]. None of these distributions, unfortunately, meet all the requirements listed below:

(a) always "bell" shaped;

(b) positive range only;

(c) capable of assuming large positive or negative skewness without change of origin;

(d) efficiently estimated by first 3 or 4 central moments;

(e) cumulative distribution and probability density functions easily calculated or well tabulated.

Two distributions met all requirements but the range of skewness was not sufficient. These were the Beta distribution [70] and the mixtures of truncated normal distributions [51]. The Weibull distrubution [e.g. 71] met the requirements well, except for the fact that the negative skew can not exceed a coefficient of much more than -1.0 without a change of origin. There is not, at least to the author's knowledge, any distribution which meets all of the above requirements. The log-normal distribution was chosen as it could be adapted, albeit with a not totally satisfactory truncation at one end when negative skewness is needed, to meet the requirements above.

Another problem that afflicts the calculations is that the inspection results can only be used to revise crack length distributions, at least with existing N.D.T. techniques. Unfortunately crack depth and crack area, as will be seen in the next Chapter, are more significant in terms of structural reliability. Of course, crack area and crack depth are correlated to crack length, however the correlation coefficient is low, and a plot of measured crack length against measured crack depth shows considerable scatter - see Clayton [47]. Therefore, predicting revised crack depths and crack areas after subsea weld inspections for crack lengths introduces additional uncertainties, which cannot be well estimated with the limited sample of data available [47].

Perhaps the most interesting aspect revealed by these examples is that for every fatigue calculation an equivalent crack growth calculation may be made using the model proposed here. This is equally true for dynamically sensitive structures provided the uncertainty in the loading term L ($\Sigma n_i S_i^m$ p.a.) can be estimated. Such estimates have been made by Wirsching and others (see references in [5]) in connection with fatigue life calculations.

6. THE RELIABILITY OF FATIGUE CRACKED JOINTS

6.1 Introduction

Reliability concepts were introduced into structural engineering by Freudental [81] and have developed rapidly since then and now form the basis of many codes of practice [e.g. 82]. Their introduction into the marine structural design field was well advocated by Caldwell [83] and their use for T.L.P. structures has been studied by Faulkner et al [84]. Considerable work on fixed offshore structures has been done by Baker and colleagues [e.g. 85] in the U.K. and by many others in the U.S.A. and elsewhere e.g. [87, 88]. However, reliability methods are not generally explicitly used in the design of offshore structures because of complications concerning identification of coupled failure modes in complex systems [e.g. 43]. It is, however, readily possible to study a single structural connection and that is undertaken here.

Currently the ultimate limit collapse of a tubular structure is considered separately from the fatigue analysis. Yet any structure whose members are designed with a finite fatigue live is subject to possible fatigue cracking. The presence of these fatigue cracks will obviously affect the capability (resistance to withstand loading) of the tubular members in which they occur. Clearly then the structural reliability of the tubular members is much higher in the early years of life than it is towards the end of the fatigue life. This leads one to pose several questions.

Is the structural reliability significantly reduced in the final years of the designed fatigue life? If at the end of the nominal fatigue life the structure is still required for future service, and inspection reveals no cracks, for how much longer can it safely be used? Is the most likely mode of failure going to change during the fatigue life?

As has already been mentioned, in jacket structures fatigue cracks are most likely to occur at the joints of the tubular members. So this chapter addresses itself to joint failures, and in particular those modes of failure which will be affected by cracking. The following types of joint failure have been identified from tests [69]:-

1) Plastic failure of the chord;
2) Cracking and gross separation of chord and brace;
3) Cracking of the bracing;
4) Local buckling;
5) Shear failure of the chord between adjacent bracings;
6) Lamellar tearing of thick chord walls under brace tension loading.

There have not, at least to the author's knowledge, been any reported static tests to failure on tubular joints containing fatigue cracks. Little theoretical work appears to have been done on this problem, apart from the examination of the effect of fatigue upon possible fracture, e.g. [72].

Because of the uncertainties of predicting the exact location, shape and size of fatigue cracks, a precise analysis is not attempted here. Instead a simple analysis of the effect of cracking upon the resistance to the types of failure above is undertaken.

In the case of plastic failure of the chord, clearly the crack will reduce the cross section area available to carry a tensile load. To a first approximation then, loss of strength is proportional to loss of cross section area. Similarly under bending load loss of strength will be approximately proportional to loss of plastic section modulus.

Local buckling of the chord wall is usually associated with a high punching shear stress. Clearly loss of area in the attachment between chord and brace will cause a higher punching shear stress and a smaller fraction of the chord wall will be subject to the load causing local buckling. Again to a first approximation loss of punching shear strength, or resistance to chord wall buckling, may be considered proportional to crack area. Similar arguments can be applied to lamellar tearing of the chord walls under brace tension loading.

The problem of fracture of either the brace or chord is closely related to the size of any existing crack or defect. Obviously as the fatigue life progresses, fatigue cracking and the probability of a fracture failure increases. Crack depth, or defect height, is widely accepted as the key parameter in this case [72], [15] and [26].

Shear failure of the chord between adjacent bracings is not considered here. It seems unlikely, at least to the author, that such a failure would be affected by cracking between chord and brace.

The chapter first examines loss of axial cross section area, and hence axial strength, with crack growth in a statistical manner for a tubular joint. Loss of plastic section modulus and bending strength, is similarly considered. In both cases the time scale is related to the calculated nominal fatigue life and the variation of probability of failure with time under typical loading is estimated.

Attention is then turned to the problem of fracture. The increase in crack depth and the associated probability of fracture is examined as a function of time for the same tubular joint.

A final section discusses the modelling employed and the relative importance of the various potential modes of failure and how this varies during the life.

6.2 Reduction in axial strength and reliability

The axial strength of a tubular joint depends on the cross-section area in the region of the connection. Clearly a reduction in cross-section area can be equated directly with loss of strength, provided of course fracture does not intervene. The ultimate punching shear strength can be taken as:-

Brace circumference x Chord thickness x yield stress

$$\text{i.e. } R_A = \pi D t \sigma_y \qquad -(43)$$

Provided of course, there is no failure in the wall of the brace.

As fatigue cracking occurs the shear area is reduced and hence the axial resistance decreases with time (T).

$$\text{i.e. } R_A(T) = (\pi D t - a_a(T))\, \sigma_y \qquad -(44)$$

where $a_a(T)$ = the crack area

In this expression the terms D, t, $a_a(T)$ and σ_y are all subject to some variability or uncertainty. The diameter of the chord D, however, is usually kept within quite fine tolerances and the coefficient of variation is very small compared with that of the other terms and so it is considered as a constant here. The chord wall thickness is also subject to quality control but the c.o.v. is several times larger. In addition there may be loss of thickness with time, as a result of corrosion. It is interesting to note that if suitable data exists on corrosion rates then a time dependent chord thickness parameter, t(T) can readily be introduced into the model above. However, here t is considered as a time independent random variable. The term σ_y is also considered as a variable. The fatigue cracked joint discussed in Chapter 5 is now used as an example. Baker has investigated the variation in material properties [76] and the values adopted here are similar to those he has used for offshore structures [77]:

viz: μ_{σ_y} = 380N/mm^2 V_{σ_y} = 0.05 Θ_{σ_y} = 1.0

μ_t = 32mm V_t = 0.01 Θ_t = 1.0

The maximum annual axial loading or demand (D_A) on the structure was considered to have a Type 1 Extreme Value distribution [71], [78] with a coefficient of variation of 0.20. Using Rackwitz and Fiessler's transformation [79], the mean value of the demand is chosen to give a safety index $\beta = 3.00$ in the first year of the structure's life. Now as a result of marine fouling the maximum loading may increase substantially within a few years [32]. Again whilst the model used here can readily cope with a time-dependent loading, the mean loading is considered as time independent.

For each year of the structure's life, the moments of the distribution of resistance $R_A(t)$ have been calculated using the expressions given in Chapter 3. A log-normal distribution has been fitted to these moments to represent $R_A(t)$.

The à priori probability of failure in each year due to axial loading has been calculated by means of the convolution integral.

$$P_F = P_r(D_A > R_A(t)) = \int_{-\infty}^{+\infty} (1 - P_{D_A}(x))\, p_{R_A}(x)dx \qquad -(45)$$

The results of these calculations are given in Table 13 which gives the mean and the characteristic value of the resistance in a non-dimensional form $R_A(t)$ together with the reliability index and probability of failure per annum. It is interesting to note that safety index β and probability of failure do not change significantly during the nominal fatigue life but increase dramatically towards the end of the actual life.

6.3 Reduction in bending strength and reliability

Bending strength can be considered in a similar manner to that above. The loss of area will affect the ultimate moment of resistance of the chord wall at the point of attachment of the brace. Similarly if the crack was in the brace the effect would be the same. The analysis given below, like that above, is very simple and does not consider joint flexibility, load redistribution, etc. (see for example [69]) and is offered to provide an insight into the problem rather than a final solution.

Consider a cracked area in a joint as shown in Figure 14(a). The plastic neutral axis for the section will fall beneath the centroid by a distance y sin $\alpha_1/2$ and the plastic section modulus will be given by the following expressions:

$$SM = \tfrac{1}{2}\int_{-\alpha_1/2}^{\pi/2-\alpha_1} D^2t\,(\sin\alpha + \sin\alpha_1/2)\,d\alpha + \tfrac{1}{2}\int_{\alpha_1/2}^{\pi/2} D^2t\,(\sin\alpha - \sin\alpha_1/2)\,d\alpha$$

which yields

$$SM = D^2t\,(\cos\alpha_1/2 - \tfrac{1}{2}\sin\alpha_1) \qquad -(46)$$

Now a more convenient and tractable expression, at least for the purposes here, for the plastic section modulus is:-

$$SM = D^2t - \tfrac{1}{2}\,Da_a$$

where a_a = crack area = $D\,\alpha_1\,t$

i.e $SM = D^2t\,(1 - \alpha_1/2) \qquad -(47)$

It is interesting to compare the coefficients of D^2t from equations (46) and (47) for various angles

α_1	$(\cos\alpha_1/2 - \frac{1}{2}\sin\alpha_1)$	$(1 - \alpha_1/2)$
11.25	0 896	0.901
22.5	0 804	0.789
45	0.607	0.570

It would appear then that the approximate equation over-estimates the plastic section modulus. However, the assumption concerning the distribution of crack area was the most pessimistic possible. A more realistic assumption as illustrated in Figure 14(b) would give a larger value of SM. Given that the actual shape is not

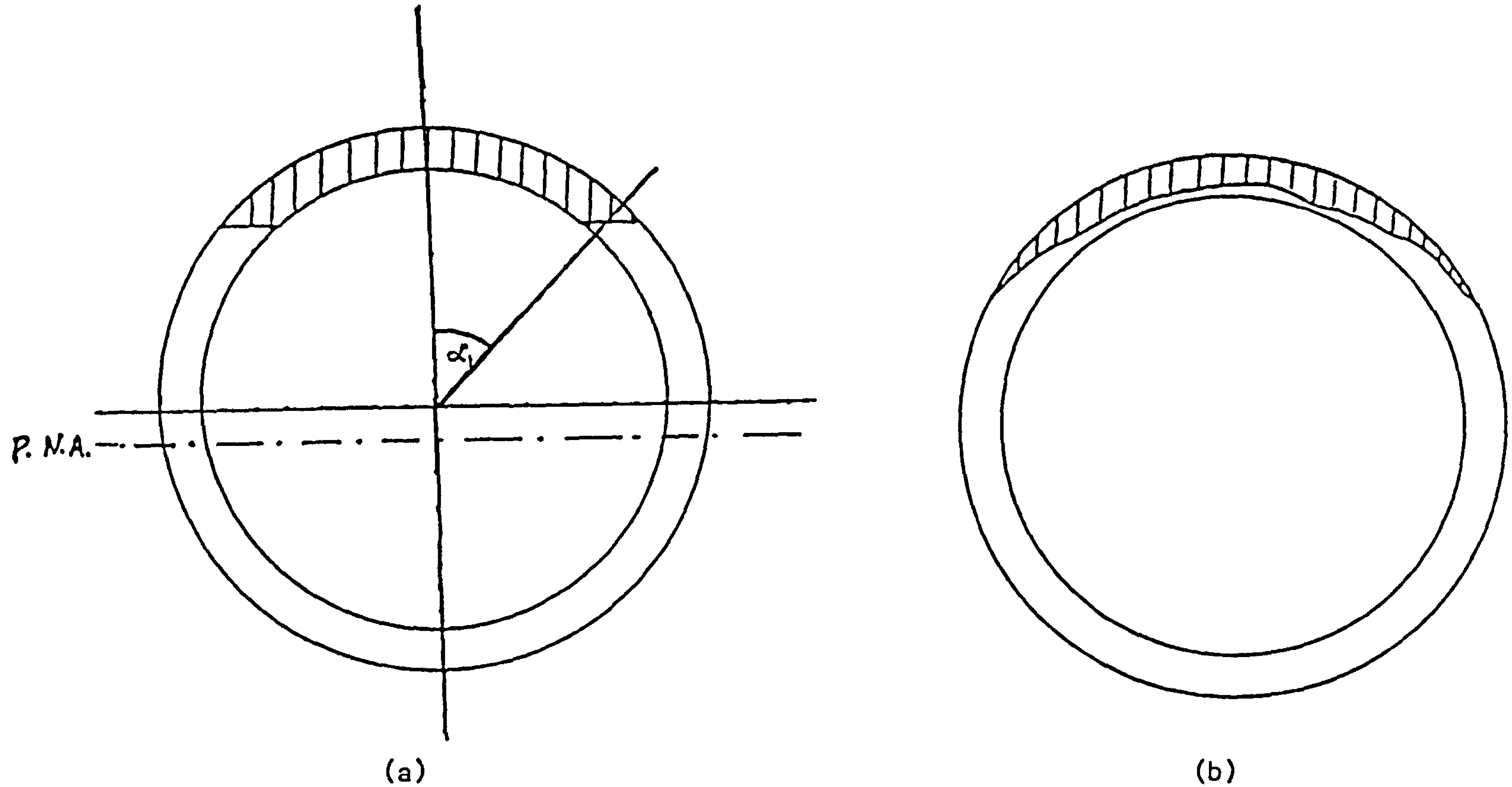


Figure 14: (a) Idealised crack in a section of a tubular member
(b) Actual crack.

known and the general level of uncertainties concerning crack size the case for the adoption of the approximate equation (47) seems very plausible and so it is used here.

It is perhaps appropriate at this point to comment on the choice of bending axis, which was the most pessimistic possible. Generally cracking, if it occurs, will be in a region of high stress concentration and high nominal stress, i.e. remote from the axis of bending if bending loads are significant. If linear wave theory is assumed to apply then for a horizontal brace parallel to the direction of the oncoming seas the wave induced bending moment about any diameter will be the same. For other members, provided the most severe seas are not all from one direction, the wave induced bending moments about in-plane and out-of-plane axes will be of a similar order.

For the purposes of the analysis here the expression below is used to characterise the ultimate moment of resistance at the joint:

$$R_m(t) = D(Dt - \tfrac{1}{2} a_a (T)) \sigma_y \qquad -(48)$$

As in the previous section, D is assumed constant, and t and σ_y are considered as variables. Exactly the same type of analysis as above, has been carried out and the results are presented in Table 14. It is perhaps worth pointing out that even if a plastic hinge occurs at one joint a collapse mechanism is unlikely to exist since residual, albeit reduced, load carrying capacity remains for the members in question.

In many cases there will be a combination of axial and bending loads and the strength of the tubular joint will be assessed using an empirical interaction equation. The simplest of which has the following form [77]:

$$(P/P_u) + (BM/BM_u) = 1.0 \qquad -(49)$$

where P_u and BM_u represent the ultimate strengths under axial and bending loads alone. In terms of the simple analysis above this may be written:

AXIAL RESISTANCE STATISTICS				
Time (years)	μ	Characteristic	β	Pf
0	1.000	0.8451	3.000	1.35×10^{-3}
10	1.000	0.8451	3.000	1.35×10^{-3}
20	1.000	0.8450	3.000	1.35×10^{-3}
30	1.000	0.8449	3.000	1.35×10^{-3}
60	0.999	0.8455	2.998	1.36×10^{-3}
90	0.989	0.8134	0.848	1.98×10^{-1}

TABLE 13: Variation of axial resistance with fatigue crack growth and its effect upon the safety index and probability of failure

BENDING RESISTANCE STATISTICS				
Time (years)	μ	Characteristic	β	Pf
0	1.000	0.8451	3.000	1.35×10^{-3}
10	1.000	0.8451	3.000	1.35×10^{-3}
20	1.000	0.8450	3.000	1.35×10^{-3}
30	1.000	0.8449	3.000	1.35×10^{-3}
60	0.999	0.8440	2.997	1.37×10^{-3}
90	0.983	0.7607	0.447	3.27×10^{-1}

TABLE 14: Variation of bending resistance with fatigue

$$\frac{P}{(\pi Dt - a_a(T))\, \sigma_y} + \frac{BM}{D(Dt - a_a(T)/2)\, \sigma_y} = 1.0$$

or $$\frac{P}{\pi(Dt - a_a(T)/\pi)\, \sigma_y} + \frac{BM}{D(Dt - a_a(T)/2)\, \sigma_y} = 1.0$$

i.e $$\frac{1}{\sigma_y\,(Dt - a_a(T)/k)} \left[\frac{P}{\pi} + \frac{BM}{D}\right] = 1.0 \qquad -(50)$$

where $2 < k < \pi$ and the precise value of k depends on the relative magnitudes of P and M. Clearly then the deterioration of strength and the increasing probability of failure with time under combined loading will lie somewhere between the sets of values given in Tables 13 and 14.

6.4 Crack growth and fracture resistance

The fracture toughness of steels is usually measured with a Charpy impact value although increasingly Crack Opening Displacemet (C O.D.) or the associated Crack Tip Opening Displacement (C.T.O.D) are used [69], [72]. The advantage of the latter measurements is that they can be used directly to assess the maximum size of defect or crack a structure can sustain before fracture [75], [80]. The size of a critical defect is usually determined by two factors: its initial size immediately after fabrication, and the growth that occurs in service. Generally structures are inspected post-fabrication to ensure that no defects of a critical size, which may cause fracture, exist. However, subsequent fatigue can cause small cracks to grow to a size at which fracture may occur. The expression generally adopted [80] for this assessment is:

$$2\bar{a} = \frac{\delta_c E}{\pi\,(\sigma_1 - 0\ 25\sigma_y)} \qquad \text{for } \sigma_1 > 0.5\,\sigma_y \qquad -(51)$$

where $\bar{a}$ = the critical defect size and may be adjusted for the shape and location of the crack

δ_c = critical crack opening displacement

σ_l = applied stress including the effect of stress concentration and residual stress due to welding

The precise shape of any crack or defect which is likely to occur is difficult to predict à priori and the measured aspect ratios of fatigue cracks show considerable scatter about a mean value of 1/15 (a_d/ℓ) The precise crack shape parameter to be used is difficult to define and a conservative assumption of an infinitely long surface crack is often made [15] The critical crack size measure used is then the depth.

As the crack depth increases the defect shape parameter increases, as do the stress concentration factors for bending and tensile loading [69] These factors and the variation in applied stress in the thickness direction tend to cancel out and are assumed to do so here. In which case crack depth a_d directly replaces $\bar{a}$ in equation (51) which may be re-written as:-

$$\delta = a_d \left[\frac{\sigma + \sigma_y}{\sigma_y} - 0.25 \right] \frac{2\pi \sigma_y}{E} \qquad -(52)$$

where δ is the crack opening displacement at any time during the life and the residual stress is taken as σ_y. Failure occurs when $\delta_c - \delta < 0$:

$$\text{i.e.} \quad \delta_c - a_d \left[\sigma + 0.75\, \sigma_y \right] \frac{2\pi}{E} = \text{MARGIN} \qquad -(53)$$

For the analysis here the terms σ, δ_c, a_d and σ_y are taken as variables. Young's modulus E has a comparatively low c.o.v.

The term σ, the applied stress, is taken as the "demand" on the joint and is assumed to be represented by a Type 1 extreme distribution with c.o.v. of 0.2. The mean value of σ is chosen to give an initial value of the safety index β of 3.0 as in previous cases. The statistics of the other variables have been chosen as follows:

	mean	V	Θ	
σ_y	380	0.05	1.0	see [77]
δ_c	0.2	0.30	0.0	see [75]

The moments for the distribution of a_d have already been calculated using the crack growth model in Chapter 5.

As in previous cases the variation in reliability and safety index β have been calculated at each stage during, and beyond the nominal fatigue life. The results of these calculations are presented in Table 15.

Equation (52) does implicitly include a factor of safety of about 3 [69]. However, the effect of this is removed in the selection of the loading stress, to give a safety index of 3.0 based on this equation.

Figure 15 shows graphically how the probability of failure due to each of the three modes considered, axial, bending and fracture varies with time as a result of fatigue cracking.

6.5 Discussion of the modelling and results

Before looking at the results a few comments concerning the modelling are in order. The models used are based on simple mechanics and are directly related to current design methods. Little is known about the actual residual strength of fatigue cracked tubular joints as no experimental programmes, to the author's knowledge, have been undertaken. In the face of this uncertainty complex models are difficult to justify. Modelling error, which undoubtedly exists to some extent, has not been taken into account explicitly. However, when comparative trends are being examined modelling bias may be less important.

The results obtained are based on limited samples of data. With more, or different, data the results may alter. They may also

FRACTURE RESISTANCE STATISTICS		
YEAR	β	Pf
0	3.000	1.35×10^{-3}
10	2.965	1.51×10^{-3}
20	2.892	1.91×10^{-3}
30	2.689	3.58×10^{-3}
60	1.933	2.66×10^{-2}
90	1.470	7.07×10^{-2}

TABLE 15: Effect of fatigue cracking upon fracture resistance as represented by safety index and probability of failure.

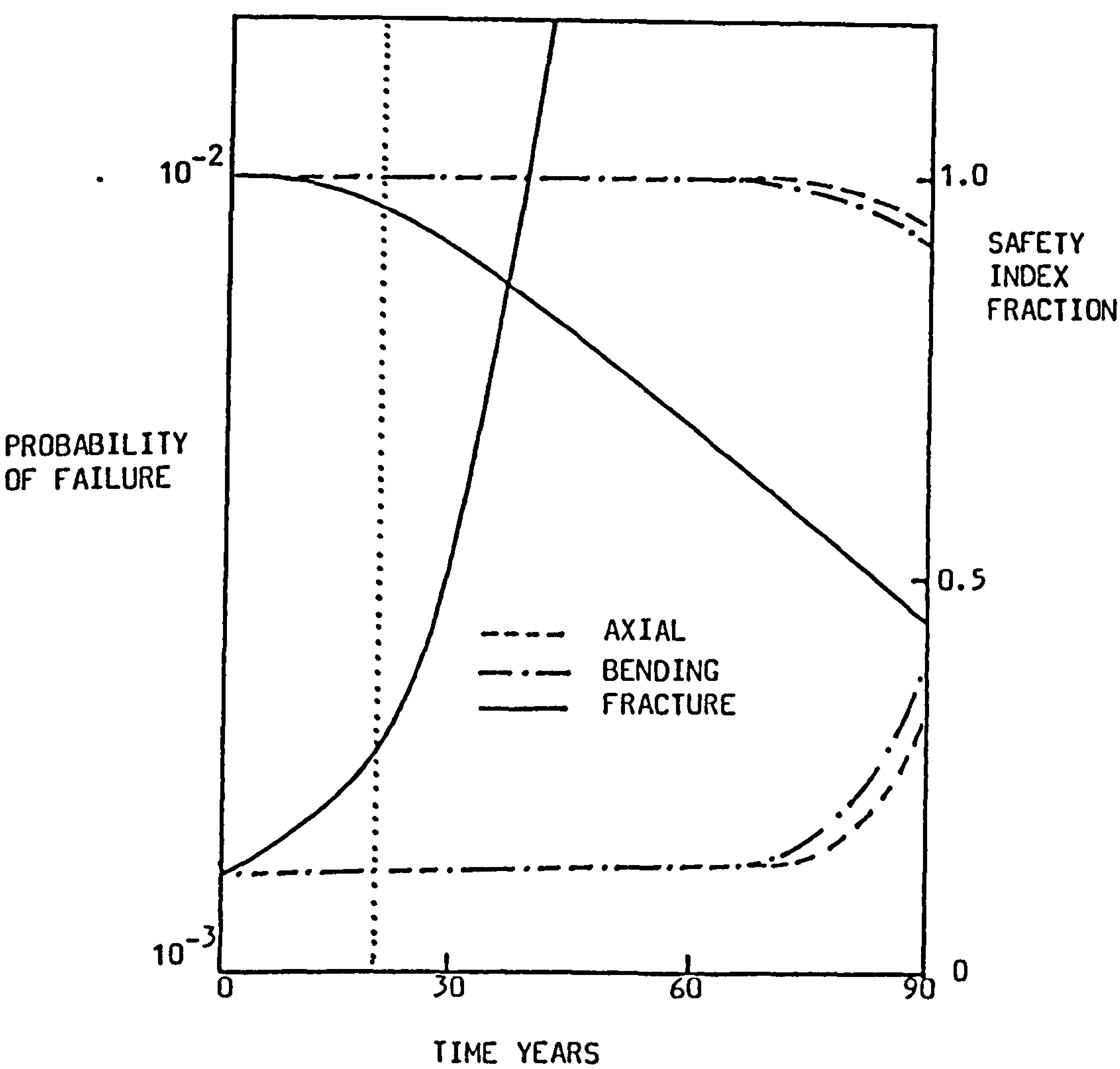


FIGURE 15: Variation in Probability of Failure and Safety Index with Crack Growth

alter with another choice of distribution function. However, the relative trends will remain the same.

The results do show some interesting trends. There is little loss of section properties during normal fatigue life and, in this case at least, the reliability does not reduce significantly until after twice the nominal fatigue life. At this stage, the bending reliability reduces rapidly and the axial reliability almost as rapidly. Whilst the general trends will remain the same they will undergo a time shift if the uncertainty surrounding the crack growth calculation alters. In the case of fracture it is interesting to note that the reliability reduces much more rapidly with time. This suggests it is desirable to have a relatively higher safety index for this mode of failure for the intact design condition.

A point which requires emphasis is that the reliabilities examined here are a priori perceived reliabilities. Sub-sea inspection may well allow the absence of significant cracks to be confirmed affecting the perceived crack size and hence enhancing the perceived reliability. The role of inspection and structural monitoring is very important in this connection as was seen in the previous chapter.

7. A TECHNO-ECONOMIC APPROACH TO UNDERWATER INSPECTION STRATEGY

7.1 Introduction

The Department of Energy [7] give the following guidance concerning in service inspection of offshore structures:

> "In-service inspections of fixed installations should be planned by an experienced engineer who has examined the design characteristics, the records of severe environmental and other loads to which the structure may have been exposed and any available records of structural behaviour such as settlement, tilt, distortion or abnormal response, etc. ..."

and further appropriate advice is given on what considerations should be made before drawing up the inspection schedules. However, no detailed advice is given on precisely what should be inspected, and when, this being left to the "experienced engineer" in consultation with the surveyor from the certifying authority.

Det norske Veritas, as a certifying authority, give further guidance in their Rules [9] on the preparation of inspection schedules and intervals of inspection, but they speak of "significant-area/items" without defining precisely how these can be identified, and give no rational reason for their choice of inspection intervals. This is not, of course, to say that they are in any way inadequate - only that there is no clear rationale. Other certifying authorities' publications are no more specific.

Some attention has, however, been given to the problem of developing a rational inspection and maintenance strategy. Sletten et al [11] suggest a resource allocation approach which takes into account the probabilities of failure, the corresponding cost consequences, the cost of inspections and the detection rate for defects. Whilst this paper is a significant step forward it does not consider the time-dependent aspect of the problem nor does it deal with such problems as appropriate intervals for cleaning to reduce the loading due to marine growth. Marshall [12] also discusses strategies for inspection and repair and whilst he proposes a rational techno-economic basis fof dealing with the decision as to whether or not to repair a particular item, he does not provide a rational basis on which a whole inspection and maintenance programme can be constructed.

7.2 Background

Risk Cost and Failure

The probabilistic approach to structural safety has received considerable attention in civil engineering, naval architecture and offshore engineering. In essence this approach acknowledges that no structure is indestructible and that all structures have some risk of failing. The stronger a structure is designed to be and the more carefully it is constructed the more expensive it is, but the less likely it is to fail. A risk cost can be obtained by looking at the cost consequences of failure and multiplying by the probability of its occurring. This is the basis on which an insurance assessor would calculate an insurance premium. For any structure the expected life cost can be expressed as:

$$\text{Life Cost} = \underset{\text{(Design and Construction)}}{\text{Initial Cost}} + \underset{\text{Costs}}{\text{Operating}} \qquad -(54)$$

where, for the subsea part of an offshore structure:

$$\begin{matrix}\text{Operating} \\ \text{Costs}\end{matrix} = \begin{matrix}\text{Inspection, Repair} \\ \text{Maintenance Costs}\end{matrix} + \begin{matrix}\text{Risk} \\ \text{Costs}\end{matrix} \qquad -(55)$$

or, written in algebraic form:

$$\text{Cost} = \sum_{i=1}^{m_1} C_{Ii} + \sum_{k=1}^{m_1} C_{R/Mk} + \sum_{j=1}^{m_1} Pf_j \, Cf_j \qquad -(56)$$

where C_{Ii} is the cost of inspection of item i

$C_{R/Mk}$ is the cost of repair or maintenance of item k

Pf_j is the probability of failure of item j

Cf_j is the cost consequences of failure of item j

Figure 16 shows how the expected life cost varies with the probability of failure. The objective of the designer is obviously to try to minimise the expected life cost, but because of the uncertainties involved, he tends to the left hand side of the diagram where the risks are lower.

Once the structure is built, the initial cost is fixed. The operating costs, although related to the original strength of the structure, will depend very much on the policies of inspection, repair and maintenance which are adopted. Clearly, if little maintenance and no repairs are carried out, the structure will deteriorate rapidly and the risk costs will become very high. On the other hand, excessive inspection and maintenance, which are very expensive, cannot completely eliminate the risk of a failure. Clearly a compromise is necessary to achieve realistic and perhaps even optimum operating costs. The approach to be described in this chapter is aimed at achieving this optimum level of operating costs, but before that is considered a quick look at the cost consequences of failure is appropriate.

Consequences of Failure

These may be considered in economic terms, although there are those [89] who feel that some of the consequences of failure, such as loss of life, result in intangible costs. However, actuaries and the insurance markets have generally overcome these difficulties and there is no reason why one should not draw on their expertise and follow their example.

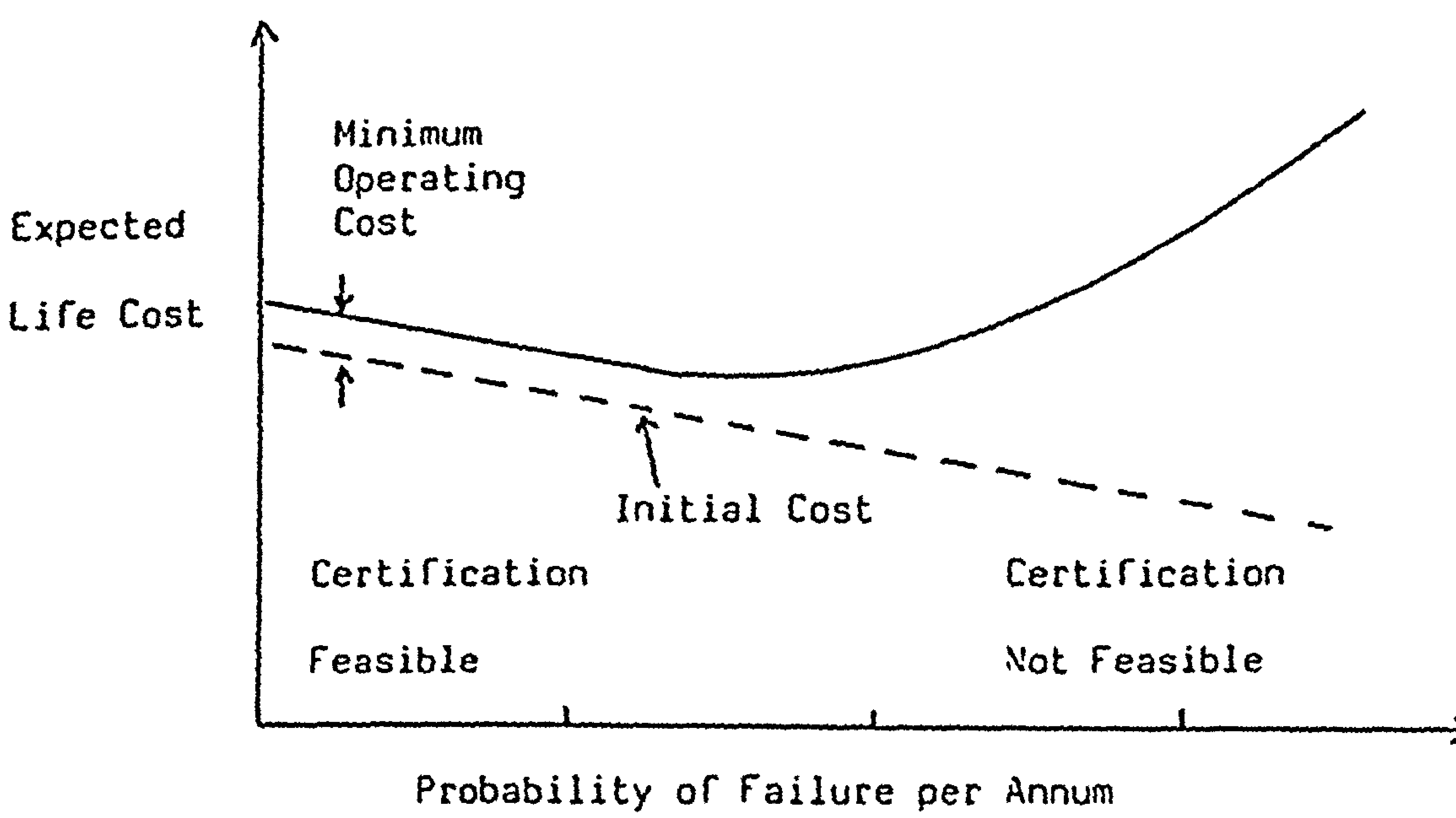


Figure 16: Variation in expected life cost with designed probability of failure.

Failure, in the context under discussion, can conveniently be divided into three categories:

a) Minor Failure - e.g. a redundant bracing member breaking off. The cost of removing the debris (C_{DEB}) and repairing the brace (C_{REP}) will be a consequent cost. Between the time of the failure and the replacement of the brace the structure will be weaker and more liable to other independent types of failure. This added risk may also be viewed as a consequent cost and expressed as: (Sum of increases in probability of failure) multiplied by (consequences of failure).

Hence, cost of failure:

$$C_{f1} = C_{DEB} + C_{REP} + \sum_{j=1}^{n_1} (P_{f1j} - P_{foj}) C_{fj} \qquad -(57)$$

b) Serious Failure – this is one which requires production or work operations to be shut down or significantly disrupted. In this case the cost of failure is given by:-

$$C_{f2} = C_{REP} + C_{p/d} \times T_d + C_{IN} + \sum_{j=1}^{n_1} (P_{f2j} - P_{foj}) \times C_{fj} \qquad -(58)$$

where $C_{p/d}$ = cost of lost production per day

T_d = number of days between failure and repair being completed and production restarted

C_{IN} = incidental costs, e.g. possible pollution.

c) Complete structural collapse – the costs here have to be estimated by considering a scenario of the consequences, which will be far-ranging and will include such items as loss of public confidence and company image - C_{f3}.

The above breakdown is appropriate for a simple type of steel jacket structure. It does not take into account the effect of possible interaction between the modes of failure as this would introduce considerable complication and is thought likely to be of secondary importance. A similar breakdown could be produced for any other type of structure.

7.3 The Proposed Approach

The Basis

Structural failures can, for the purposes of this discussion, be divided into two categories:

a) Failures which occur irrespective of the age and level of maintenance applied to the structure: for example collision with an uncontrolled ship, or toppling over through the impact of a freak wave.

b) Failures whose probability of occurrence are directly related to levels of maintenance and whose likelihood may be estimated from the results of inspection and design data.

The object of structural inspection and maintenance is to control the probabilities of occurrence of these latter types of failure. Consider what happens to a steel structure as time goes by: cracks develop and grow, anodes waste away, steel corrodes, fouling increases, the seabed changes its contours and the chances increase of the structure having suffered collision or impact damage. This gradual change of state results in gradual increases in the probabilities associated with various types of failure and a corresponding increase in the associated risk costs. The basis of this new approach is an examination of how the risk costs grow with time and how, with appropriate inspection and subsequent maintenance, they can be controlled.

As an example, consider a crack in a joint which has grown by a small amount, say "a". The structure is now marginally weaker than hitherto, and the reduction in the reliability of the joint can be calculated as described in the previous chapter. Making certain assumptions and knowing the degree of redundancy of the joint involved, the reduction in overall reliability can be calculated. The corresponding increases in probability of failure for the categories of failure discussed can then be estimated. These are denoted here by δP_{f1}, δP_{f2} and δP_{f3}. The corresponding increase in the risk cost is then given by:

$$\delta C_r = \delta P_{f1} \times C_{f1} + \delta P_{f2}(C_{f2} - C_{f1}) + \delta P_{f3}(C_{f3} - C_{f2} - C_{f1}) \quad -(59)$$

where C_{f1}, C_{f2} and C_{f3} are the associated cost consequences of failure. The costs subtracted from the second and third terms are to avoid double counting. Clearly a structure which fails overall also fails locally and shuts down production.

Now if the crack is allowed to grow the probability of failure will continue to rise and so will the associated risk cost. It is therefore possible to examine how risk cost increases with time, and crack growth. This can be done using results of the type presented in Figure 15 of the last chapter together with estimates of cost consequences of failure. Figure 17 shows the typical form of the curve obtained.

It is worth pointing out that a similar exercise can be carried out for the effect of marine growth on a brace, which increases the loading on the structure as it accumulates. This makes failure more likely and causes the associated risk cost to

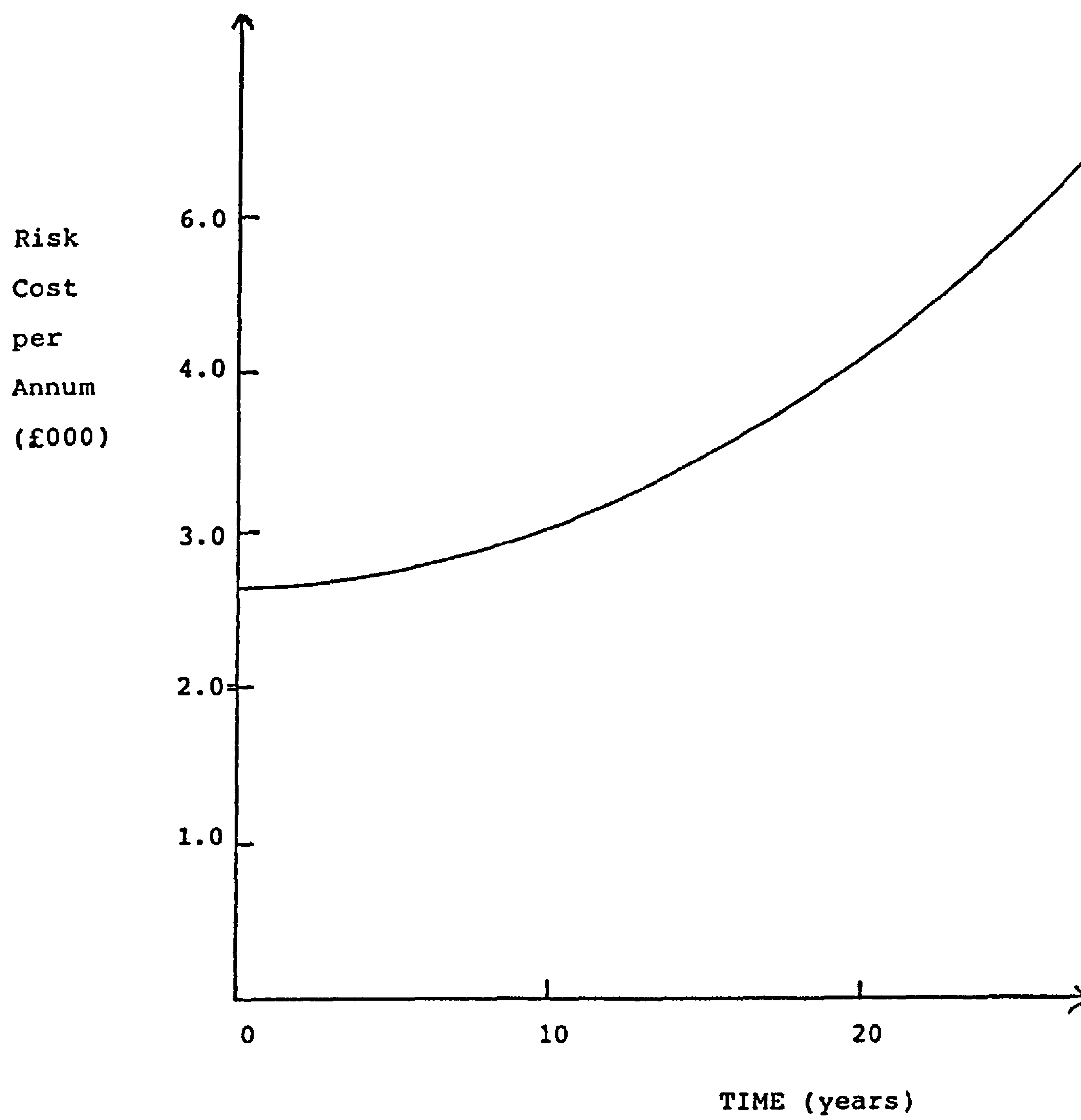


Figure 17: Risk cost variation with time and growing crack size for a joint with a nominal fatigue life of 20 years.

rise. In fact, all the time-dependent effects which determine the reliability of the structure can, with a few assumptions be treated in this manner. A whole series of curves like the one shown in Figure 17 can be produced, one being associated with each type of degradation and each joint or critical region of the structure.

When to Inspect

Returning to the joint; until an inspection is made the possible existence of a particular size of crack can only be predicted, not confirmed. There is no point in making an inspection at a stage when the crack is most unlikely to be of sufficient size to be detectable. However inspection must occur before the risk costs become too great and allowance must be made for the fact there there is a time delay between inspection and any consequent maintenance or repair. From a techno-economic standpoint the optimum time to make the first inspection is when the anticipated risk cost can be reduced by an amount equal to the cost of inspection and consequent maintenance. This is illustrated in Figure 18 where it is assumed that the structure can be returned to its original as-built state, but not to one of higher reliability.

The risk considered here is for the time interval between the maintenance activity now under consideration and the following maintenance of the same item. Choosing the optimum length for this interval is discussed later. Consideration of the value of the inspection result in predicting the state of the surrounding structure has been left aside at this stage, but this is also discussed later.

When to maintain or repair

Of course, upon inspection the state of the structure may be very different from that predicted by the model. If the structure is in a worse condition than anticipated, then maintenance or repair activities should be undertaken as soon as possible. But if it is in a much better condition than anticipated then no action will be taken unless the possible reduction in risk cost C_{vi} is greater than the cost of maintenance or repair $C_{M/Ri}$, i.e.:

if $C_{vi} < C_{M/Ri}$ then no repair or maintenance

if $C_{vi} > C_{M/Ri}$ then proceed with repair or maintenance.

This point is illustrated in Figure 19.

However, the attitude usually taken is that all faults found should be repaired or corrected if there is a strong chance that they will have to be relocated and repaired later in the life of the structure at possibly proportionally greater expense. Also illustrated in Figure 19 is how the results of inspection could be used to modify the predicted growth of risk costs. This will affect the choice of subsequent inspection/maintenance intervals.

Varying the Inspection/Maintenance Interval

Clearly the more frequent the inspections, and the consequent maintenance activities, the lower the probability of a failure occurring during the period between inspections. This is illustrated in Figure 20 which shows that the average risk cost over the life of the structure is lower with more frequent inspection/maintenance. However, this lower average risk cost would be offset by the additional inspection/maintenance costs. It must be borne in mind that such a diagram is only useful for the remaining life of the structure. For example, if the structure has survived to time T (Figure 20) under the lower maintenance policy then all the risk costs between times 0 and T no longer exist. True, the structure was at greater risk in this interval but that does not affect the risk cost once the period is passed. This means that the inspection programme should be reassessed at regular intervals and adapted to recognise that the risks of the past need no longer be met.

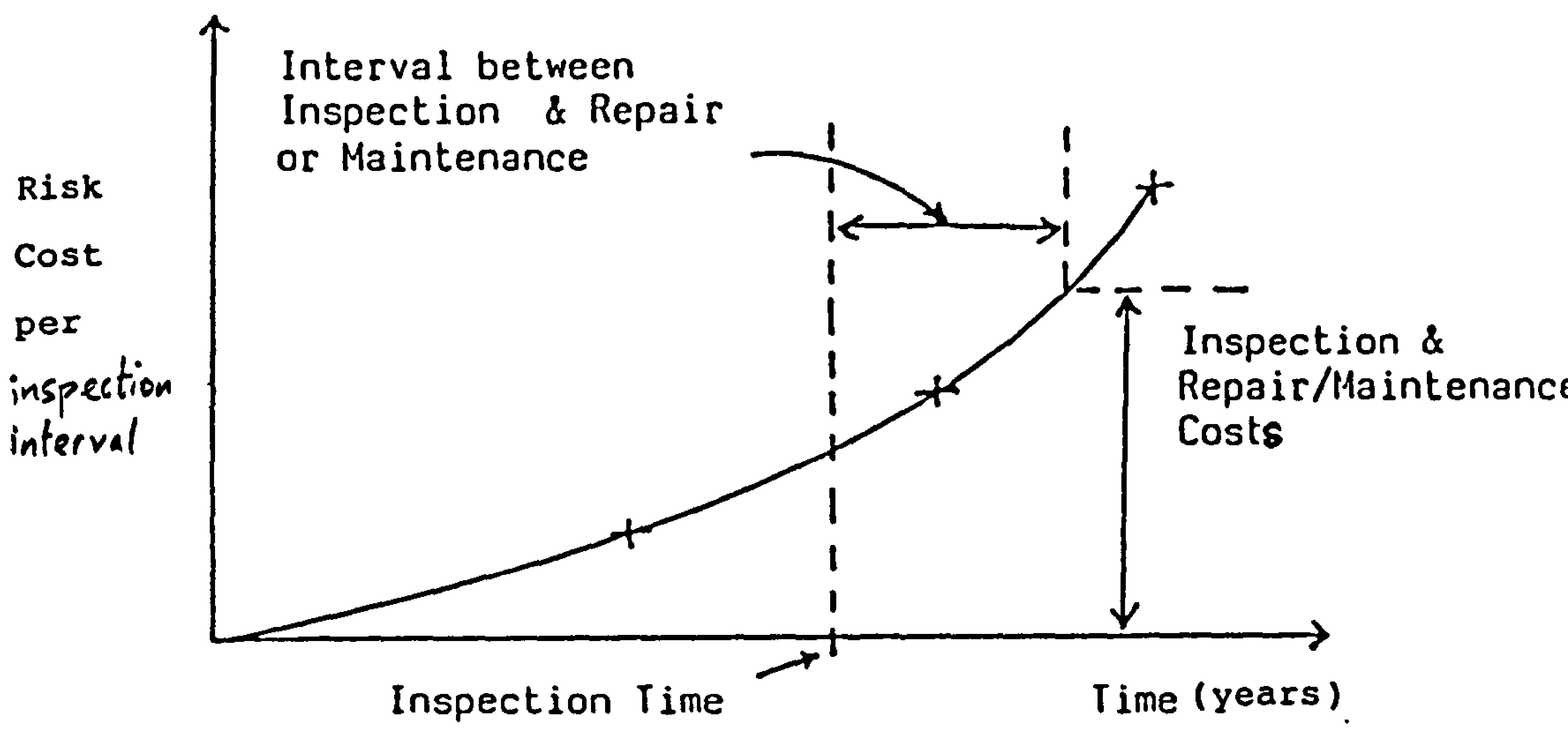


Figure 18: Interval between inspection and subsequent major repair.

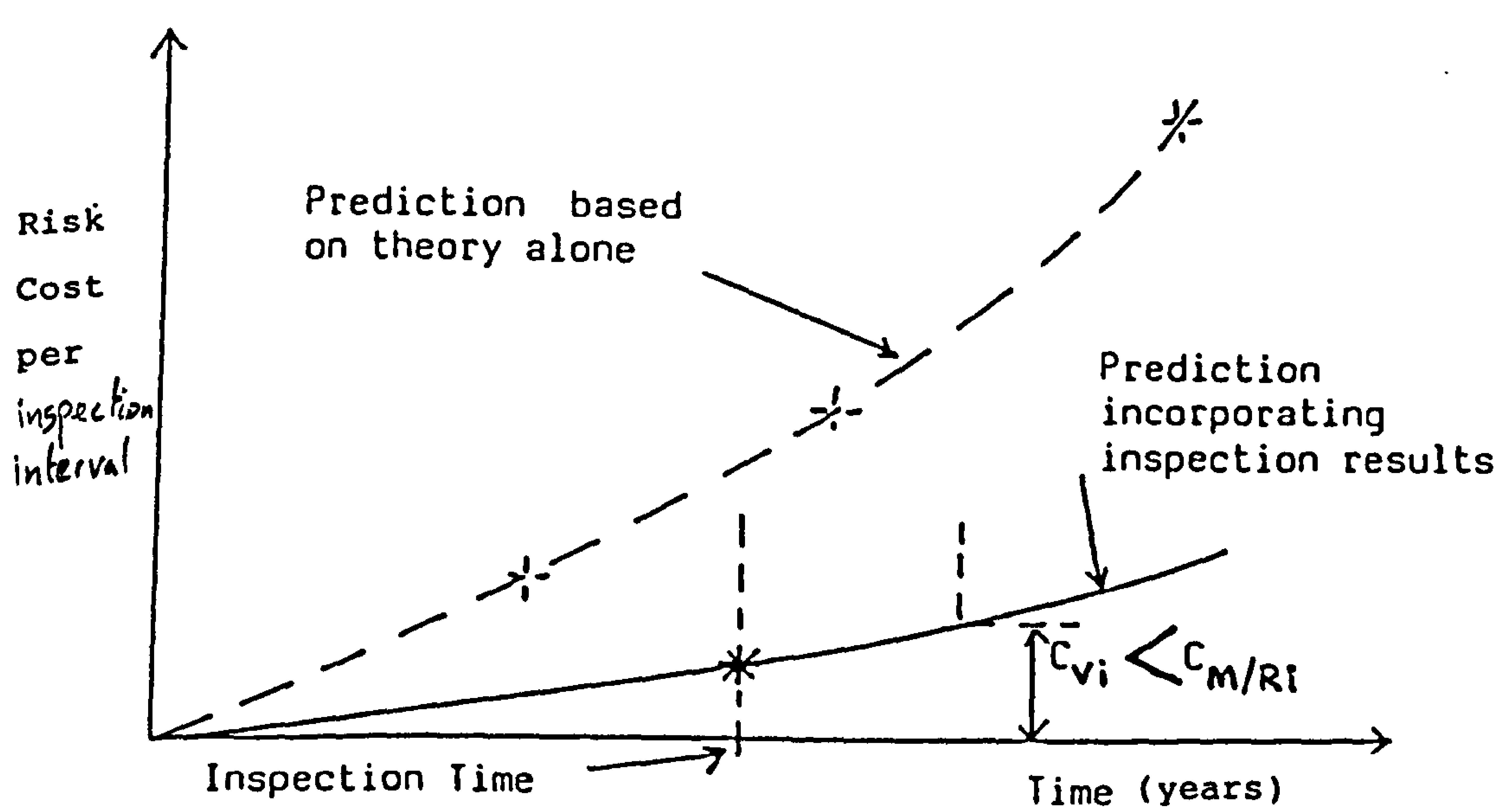


Figure 19: Revised risk cost curve in the light of inspection results and its effect upon maintenance decisions.

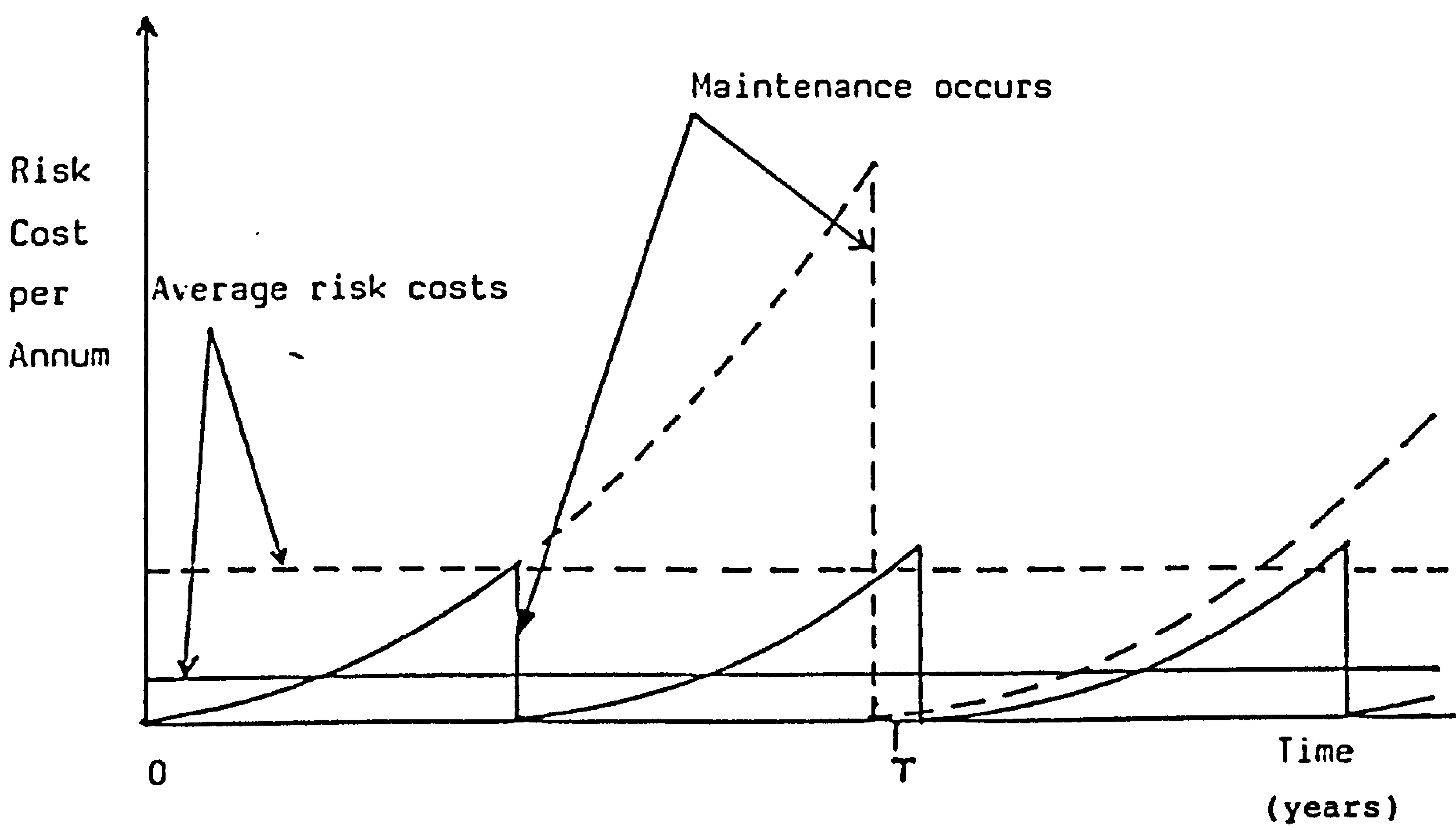


Figure 20: Variations in risk cost behaviour with inspection interval.

Value of Inspection Results for Predicting State of Surrounding Structure

The more frequent the inspection the more precise the predictions about the state of the surrounding structure, on average, throughout the life of the structure. There is a definite cost-benefit in having more precise information, and a sum relating to this considerable cost-benefit must be included in the final cost function to be minimised.

To arrive at the sum involved requires consideration of the uncertainties concerning the inspection/maintenance requirements of adjacent areas of the structure. This is discussed further later in this chapter.

The Cost Equation

In summary, from a techno-economic standpoint the optimum inspection/ maintenance intervals are those which minimise the net present value of the cost expression below at any time in the life of the structure:

$$\text{Optimum Cost} = \text{Min} \sum_{i=1}^{n_1+1} \int_{o}^{t_i} \sum_{j=1}^{3} P_{fij}(T) \times C_{fij}(T)\, dT$$

$$+ \sum_{i=1}^{n_1} C_{Mi}(T) + \sum_{i=1}^{m_1} [C_{Ii}(T) - C_{Pi}(T)] \quad \ldots \qquad -(60)$$

where:

m_1 = number of inspections

n_1 = $P_{m_1} \times m_1$ = expected number of maintenance activities

P_{m_1} = probability of maintenance following inspection

T = measured time from completion of each maintenance activity which returns the structural element to its original state.

t_i = time interval between subsequent maintenance activities which returns the structural item to its original state

C_I = cost of inspection of item:

C_M = cost of maintenance of item:

C_P = cost benefit of inspection result from i in predicting state of surrounding structure.

The term P_m (probability of maintenance being required) is introduced because maintenance will not always follow an inspection, particularly if the structural degradation is found to be less than expected. This probability can be assessed from the estimated crack size distribution and distribution of crack non-detection which was discussed in Chapter 5.

$$\text{Prob(No crack found)} = \int_0^\infty p(\text{crack size a})\, P(\text{Non-detection/a})\,da \quad -(61)$$

and

$$\text{Prob(Crack found)} = 1 - \text{Prob (No crack found)} \quad -(62)$$

Equation (60) assumes that the structure can be returned to its original state by maintenance. This is not always the case and the equation can readily be modified as necessary.

7.4 Consideration of Uncertainties

Many of the sources of uncertainty in the inspection strategy proposed have been discussed in previous chapters. These include the estimation of crack size and the affect of fatigue cracking upon reliability. However, there are many others and the principal ones are discussed here.

In the previous chapter the reduction in reliability due to fatigue cracking was estimated for a single joint. Quite what effect the failure of a particular joint has upon the overall structural reliability depends upon the degree of redundancy of the structure and the relative importance of the joint. To the author's knowledge a few analytical studies have been undertaken on structures with specific members removed to assess the effects of a local failure. However, the cost involved in such analyses is usually very high and they are not undertaken for all members in a structure and certainly not for all potential modes of failure.

Simplified reliability studies can be undertaken to estimate upper and lower bounds on the probability of failure for a complex system of elements such as a jacket structure. The current state of this art is well described by Baker [90]. Unfortunately this still involves considerable analysis and the bounds are very wide. One approach proposed by Sletten et al [10] is to rank members in order of relative importance on the basis of available analysis and engineering judgement. In the current state of knowledge this seems a reasonable proposition.

The cost consequences of failure are also difficult to predict. The cost of reinstating a tubular member or joint can be predicted from past experience albeit not with any great accuracy [91]. The cost may be of the order £1M+ and will depend on how long, if at all, production has to be shut down to affect a repair. The cost of a complete failure is both much greater and more difficult to estimate. It will vary with location, operating company and governments involved, and will depend on the perceived consequences of the risk [92]. Marshall [12] uses a value of $100M in an illustrative example. For many North Sea structures it could be at least an order of magnitude higher even without serious oil pollution.

Another way to tackle this problem is to place limiting values on acceptable reductions in reliability due to structural degradation. This is what is done implicitly, if not explicitly, for existing structures. In terms of the cost equation of the model proposed here that means that the cost consequences of failure tend to infinity as a certain minimum level of reliability is approached.

The costs of inspection and maintenance are also subject to uncertainty and these have been studied in detail by Bolland [93]. Detailed inspection of welds is undertaken by divers using magnetic particle inspection tèchniques: These divers usually operate from a diving support vessel, although occasionally they are supported from the platform. The charter cost of a diving support vessel varies with size and whether or not a saturation diving spread is included as well as an air diving spread. Charter rates also vary from month to month and usually peak in July when daily hire rates for a large vessel can exceed £30K. Diving can only be undertaken when the significant wave height is less than about 1.5m for air diving and about 3.5m for saturation diving. Thus the weather introduces a further uncertainty into the total cost of inspection.

Because of the time involved in setting up and decommissioning after inspection several welds are examined on each occasion. One weld more or less can then be considered on a marginal basis provided vessel time is available. The time required to inspect a weld will depend on its depth beneath the sea surface, the amount of cleaning required and the length of the weld.

As an example the cost of cleaning and undertaking M.P.I. inspection on 20 node welds involving 900mm diameter bracing members is estimated to be between £250K and £500K [3], but may indeed be higher.

The cost of repairing fatigue cracks is not usually great if the cracks are small. In fact they are ground out: which takes about the same time as a nodal weld inspection. Normal practice is to repair all small cracks when they are found. For larger cracks and failed joints either a welded repair is undertaken using a watertight habitat or a grouted clamp is used to reinforce the joint [91]. The cost of this later operation may be of the order of £2M or more. The time involved to affect a repair after its first discovery varies considerably with the circumstances and in particular with the weather conditions [94].

The effect of these uncertainties accumulate to give the optimum first inspection interval considerable variance as illustrated in Figure 21. In reality however, in the North Sea at least, inspection only occurs during the summer. So an optimum inspection interval on a continuous time scale is not necessary, rather a choice between two or more distinct inspection years is needed.

7.5 Inspection Results and their Uses

The use of inspection results to modify estimated fatigue lives and crack growth distributions for inspected joints has been considered in Chapter 5. However, the inspection results can also be used to give an indication of the state of the remaining, uninspected joints in the structure. The nature of the problem of drawing inferences from inspection results is indicated by Figure 22.

There will be uncertainty involved both in the process of inspection and in the process of interpretation of inspection results. The problem of not being able to detect a crack below a certain size has been discussed in Chapter 5, but even when a crack is detected its size is difficult to assess accurately, e.g. [72] [95]. As has already been mentioned most underwater inspection uses M.P.I. which measures crack length alone. However, the A.C. potential drop technique has been used offshore to measure crack depth [96] and may well be used more extensively in the future after further development. At the moment, crack depth must usually be assessed by estimation from crack length.

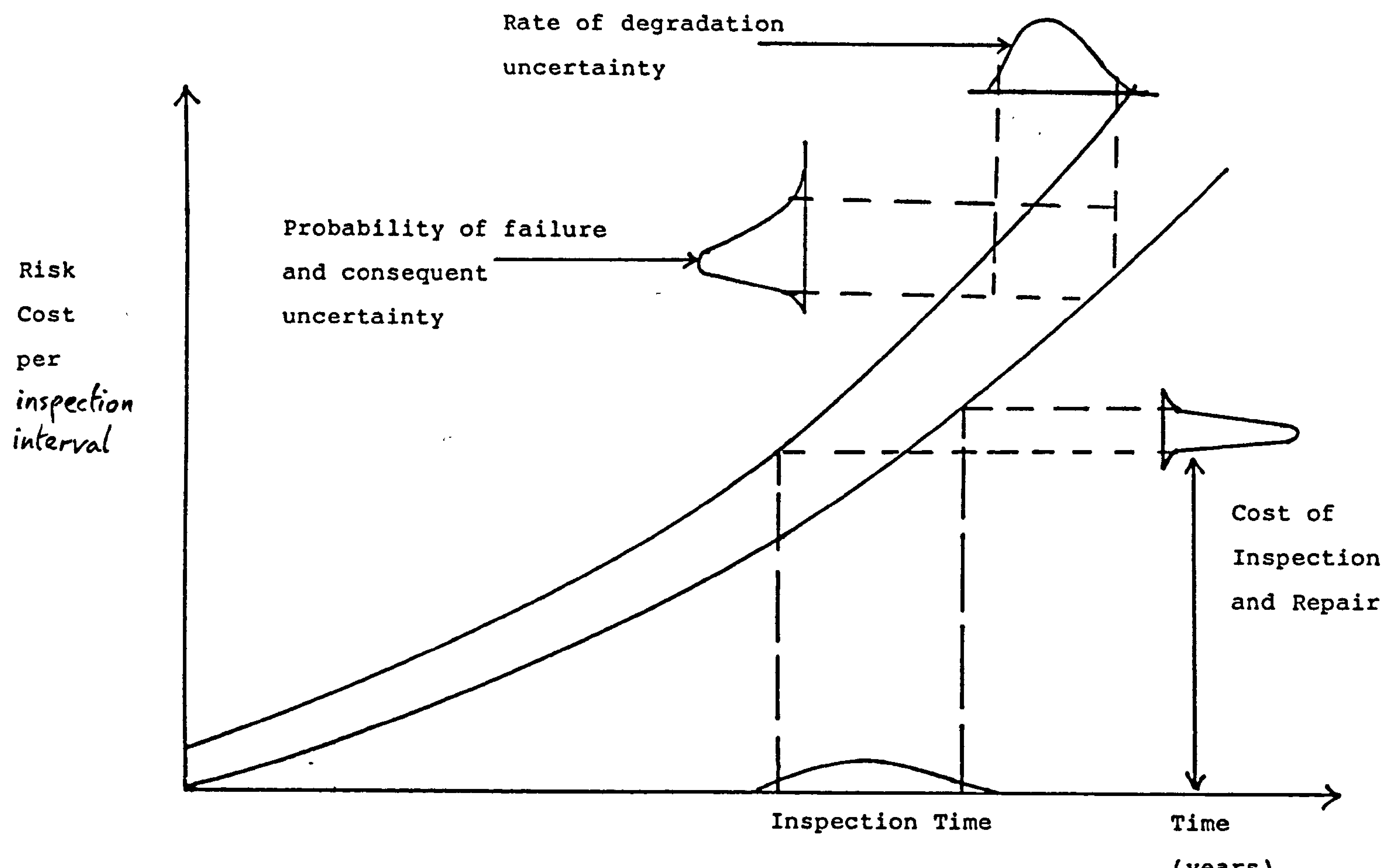


Figure 21: Schematic diagram showing the effect of various sources of uncertainty upon the "optimum" inspection time interval.

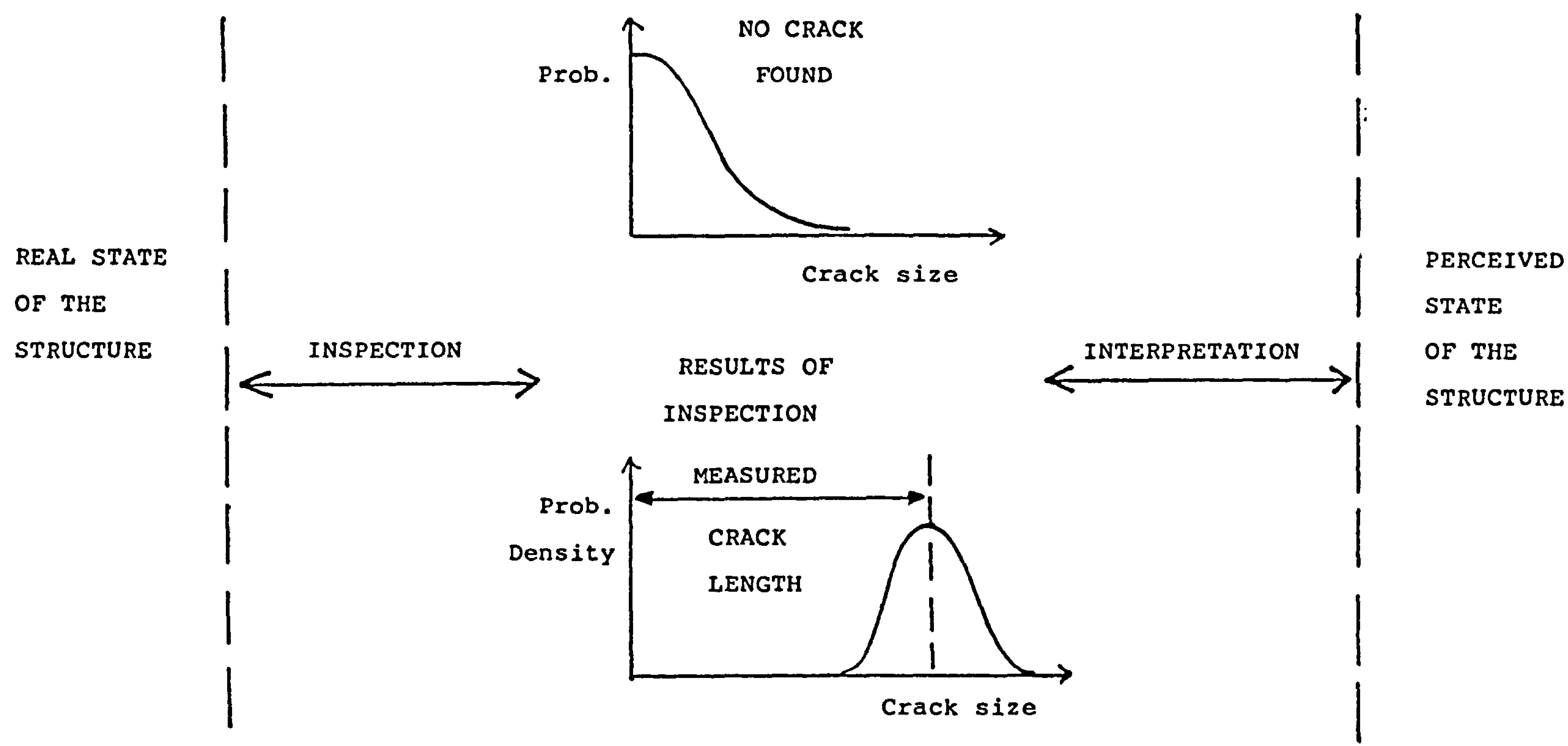


Figure 22: Sources of uncertainty in assessing the state of a structure through inspection results.

This introduces further uncertainty. Through cracks can, of course, be readily assessed by members flooding.

When a single inspection result is obtained very little can be discerned about the general state of the structure, as the initial size of that particular crack is unknown. However, when, say 20 inspection results are obtained, much more can be inferred. This is because these inspection results are in effect a sample from a distribution with a significantly lower variance than the original distribution of predicted crack size. Reduction in uncertainty occurs in the following areas:-

(i) The loading prediction - At the design stage the loading could only be estimated. However, at the time of inspection a given loading history will have occurred about which there is no uncertainty. The inspection sample is then from a structure which has been subject to a particular loading. This loading depends on the wave spectra of the preceeding years, which is of course the same for the whole structure. It also depends on the transfer function A which, whilst not identical for all joints, will have had the same sources of uncertainty à priori.

(ii) The material response - The effect of the environment on the material response will actually have occurred. It is no longer the subject of uncertainty at least for the period up to the time of inspection. The effects of variable amplitude loading, residual stresses, cathodic protection shadows and stress intensity thresholds will all have been experienced.

(iii) Initial defect distribution - If the initial defect distribution was estimated from a large, and not necessarily completely homogeneous, sample of data then the actual defect distribution for the platform is likely to be of lower variance. This is because the platform was built under particular conditions in a particular yard. This type of problem has been investigated by Baker for the material properties of steel [76]. Of course if the initial defect distribution has been estimated from post-fabrication weld inspection on the structure itself, this will not apply.

Consider a structure containing a large number of identical joints with fatigue lives of, say, 20 years which are all inspected after 10 years. The distribution of measured crack sizes (A) would have a much lower variance than the distribution of predicted crack size (B) for the reasons given above. This is illustrated in Figure 23.

When only a sample of say 20 joints are inspected the actual crack size distribution cannot be precisely defined and a distribution of intermediate variance (C) results, as depicted in Figure 23. As the sample size increases the distribution (C) converges from (B) towards (A).

Distribution (C) then represents an estimate of the revised crack size distribution for all the joints. The important difference between this distribution and the original distribution (A) is that there is less area in the right hand tail. That is the perceived probability of a large crack is reduced. This in turn will affect the perceived reliability of a joint and the corresponding risk cost.

In this manner then, the reliability of all the joints not inspected can be revised and the value of the information in the inspection sample can be assessed. Inspection therefore has a

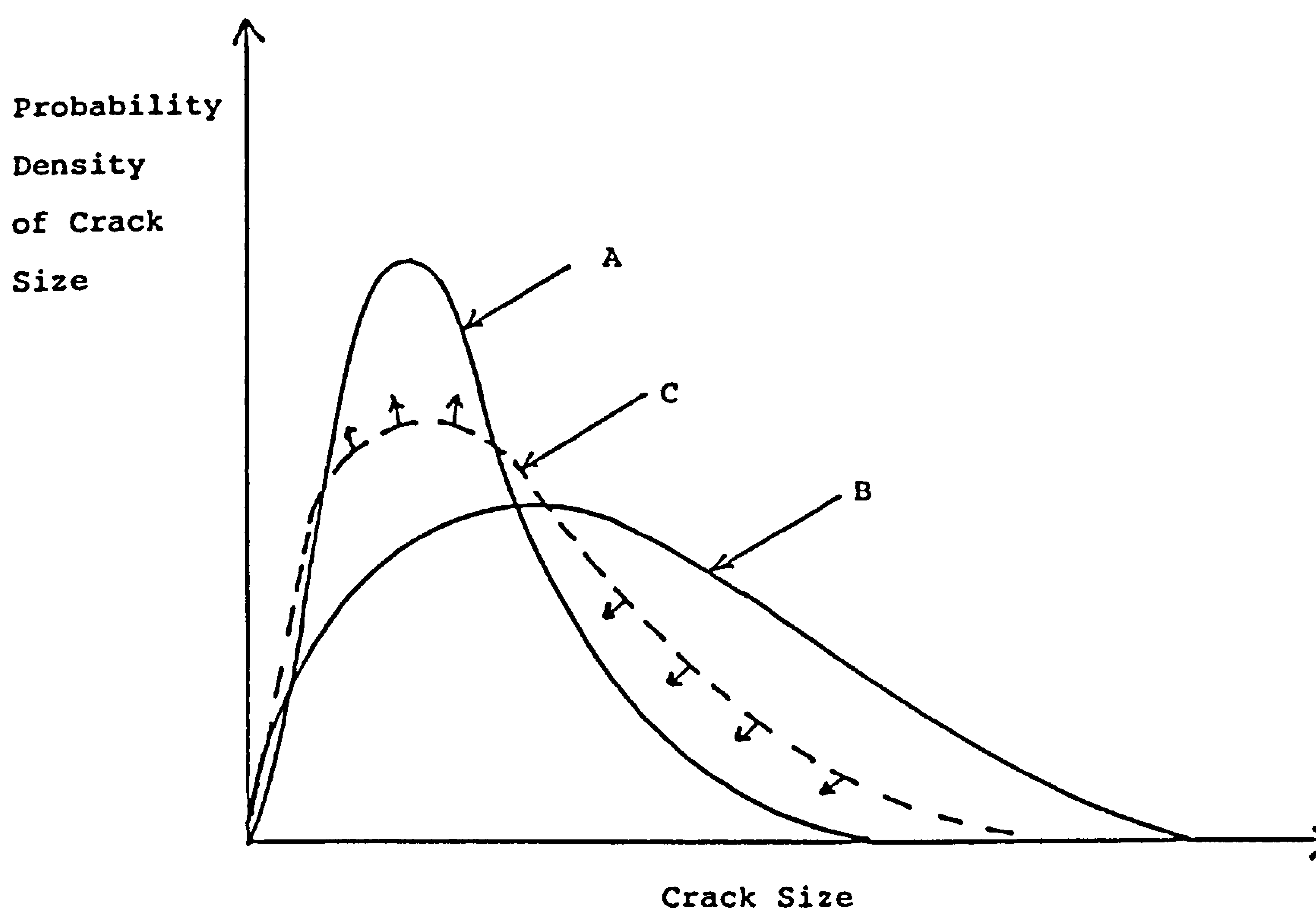


Figure 23: Distribution C converges from the a priori predicted distribution of crack size A, towards the actual distribution of crack size at the time of inspection B, as the number of inspection results increase.

cost benefit not just to the joints inspected but also to the joints not inspected. However, there is a non-linear relationship between the number of joints inspected and the additional cost benefits. As the inspection sample size increases the law of diminishing returns sets in and the value of each addititional inspection result becomes progressively smaller.

The discussion above concerns the expected results of inspection before the inspection takes place. The actual results may in fact be different due to bias. However, the extent of this is not known a priori when the decision to inspect must be made and therefore there is no option but to deal with the expected outcome. In practice, because of design conservatism, there is usually some negative bias and inspected cracks are usually smaller than anticipated. But this is not always the case [e.g. 97] especially where design errors have been found retrospectively.

Now in practice not all joints will have the same fatigue lives. This will be true both for the inspected joints and those which are not inspected. A scheme therefore which allows the revising of the crack size distribution of a joint of an arbitrary fatigue life is needed. This must also recognise that the crack size distributions for the inspected joints may be different. This problem is discussed in detail in Appendix IV and leads to the following crack size distribution revision procedure:

(a) For all joints inspected calculate the moments of the distributions of r (denote by r_o here) as discussed in Chapter 3 and illustrated in Chapter 5, for the time of the proposed inspection.

(b) Calculate moments for revised distributions where the uncertainties are reduced to those anticipated for the actual distributions of crack size at the time of inspection. As a first approximation the variances in d and M may be considered unaltered and those in L reduced to zero.

(c) The revised distribution is altered to allow for the uncertainties in crack length measurement and crack detection. As the accuracy with which a crack can be measured is unknown, this revision is a somewhat arbitrary increase in variance. The distribution so obtained corresponds to Λ in the discussion above as is denoted here by r_1.

(d) Fit log-normal distributions to these two sets of moments and estimate the corresponding means and variances for the values of $\ln r_o$ and $\ln r_1$, i e $\mu_{\ln r_o}$, $\mu_{\ln r_1}$, $\sigma_{\ln r_o}{}^2$ and $\sigma_{\ln r_1}{}^2$.

(e) Calculate the following mean and variance updating factors (which are developed in Appendix IV):-

$$F\mu = \frac{n\mu_{non} + \sigma_{non}{}^2}{n^2 + \sigma_{non}{}^2} \qquad -(61)$$

$$F\sigma^2 = \frac{\sigma_{non}{}^2}{n^2 + \sigma_{non}{}^2} + \frac{1}{n}\,\sigma_{non}{}^2 \qquad -(62)$$

$$\text{where } \mu_{non} = \sum_{i=1}^{n}\left(\mu_{\ln r_{1i}} - \mu_{\ln r_{oi}}\right)$$

$$\sigma_{non}^{2} = \sum_{i=1}^{n} \left(\sigma^2_{\ln r_{1i}} \Big/ \sigma^2_{\ln r_{oi}} \right)$$

and n = number of joints to be inspected.

(f) For all joints not inspected calculate $\mu_{\ln r_o}$ and $\sigma_{\ln r_o}$ as described in (a) and (c) above.

(g) For all joints not inspected calculate revised mean and variances for natural logarithm of r as below:

$$\mu_{\ln r_o'} = F\mu \times \mu_{\ln r_o} \qquad -(63)$$

$$\sigma^2_{\ln r_o'} = F\sigma^2 \times \sigma^2_{\ln r_o} \qquad -(64)$$

(h) The factors $F\mu$ and $F\sigma^2$ are explained in Appendix IV Using the revised parameters $\mu_{\ln r_o'}$ and $\sigma^2_{\ln r_o'}$ calculate revised moments for r_o'.

(i) Use the revised moments of r_o' to characterise crack size at the time of inspection, for the joints which are not inspected, and use this as the starting point for subsequent calculations.

This procedure of necessity makes a number of assumptions, which are considered plausible, and these are discussed in Appendix IV.

The question arises should the crack growth rate be modified in the light of the inspection results? This will depend to a large extent on the weather experienced up to the time of inspection, that which was anticipated at the design stage, and that which is predicted for the future. If the weather up until the time of inspection was milder than usual then inspected cracks are likely to be smaller and revision of crack growth rate would be most unwise. On the other hand if the weather was much more severe than anticipated and still inspected cracks were smaller than anticipated, there would be a case for reducing the estimated growth rate. However it must be borne in mind that weather in one five or six year period may be very different to the next as seen in Table 5.

7.6 A Practical Example and Other Inspection Problems

To illustrate the procedures described in this Chapter consider a practical example of a typical inspection problem. A five year old jacket structure has met the statutory requirements for inspection [8] for the first five year period. During this period it was decided to do all weld inspections in the first year because of the absence of a thorough post-fabrication weld survey and also to take advantage of unexpected production down-time. No weld defects were found during this first subsea survey of 60 welds. The inspection planning for the second five years period is underway and it has been agreed that a sample of 20 welds should be inspected. The question arises as to whether it would be better to inspect these welds at the beginning or the end of the five year period? That is inspection in year 6 or year 10. It is assumed that the next weld inspections will probably occur in year 14 in the subsequent five year period.

In order to perform the necessary calculations a number of assumptions must be made for this hypothetical case. These are enumerated and discussed below:

(a) It is assumed that all the joints inspected will be repaired if necessary by grinding out any cracks/weld defects found. The proportion of cracks requiring repair is assumed to be that expected to be found at the time of inspection using the non-detection/detection probability curve in Figure 10 (Chapter 5). It is further assumed that by such action the structure is returned to its original condition.

This is thought reasonable despite the fact that some cracks may be missed, because some initial defects which have passed the post-fabrication inspection will be found and ground out. It is assumed these factors will cancel one another.

(b) Crack detection and crack length measurement uncertainties will increase the variance of the distribution of actual crack sizes when it is transformed into the distribution of measured crack sizes. Unfortunately the accuracy with which crack length can be measured is unknown. It is assumed that the actual crack size distribution corresponds to that which occurs when $V_L = \Theta_L = 0.0$ and the distribution moments for d and L remain unaltered. To account for crack detection uncertainties the c.o.v. of the actual crack size distribution is increased by 2 %. This assumption though somewhat arbitrary is thought reasonable in the circumstances.

(c) The joints in the inspection sample are assumed to have a spatial distribution covering the whole region of the structure where significant joints are located. It is also assumed that they cover the whole range of critical joint types. In short it is assumed that they are an unbiased sample from the population [98].

(d) In Chapter 6 it was seen that for a typical joint enjoying the same initial reliability in the face of all possible modes of failure that resistance to fracture reduced most rapidly with increasing crack size. In fact the loss of reliability for the other modes of failure was comparatively negligible in the years up to and around the nominal fatigue life. Here it is assumed that fracture is the only likely mode of failure and the other modes are ignored.

(e) The problem of assessing the effect of the failure of a single joint upon the overall reliability of a redundant structure has already been discussed. Here it is assumed that the increase in probabilities of a gross failure and a catastrophic failure are directly related to the increase in probability of a joint failure. The relationships are assumed constant for all joints and are expressed in Table 16.

(f) It is assumed that as a result of inspection the rate of crack growth is not modified. That is the inspection of the structure only allows a re-appraisal of the structural integrity at the time of inspection and not a revision of the prediction of the rate of degradation.

(g) The 20 year nominal fatigue life joints are assumed to be the same as those described in Chapter 4 and used for reliability calculations in Chapter 5. The 40 year joints only differ in as much as the mean of the loading terms is $\mu_L = 1.24 \times 10^{-10}$ instead of 2.48×10^{-10}.

(h) A range of numerical values including costs have had to be assumed and these are summarised in Table 16. All the values are thought plausible and represent as far as the author can discern from discussion and published sources what could be typical for the North Sea.

The calculations follow the procedures outlined in the earlier sections of this Chapter and the results are presented in Table 17. These are presented in cost terms and are expressions of equation (60). It is seen that a 10th year inspection comes out on top. However, it would be foolish to read too much into these particular results as they depend very much on the cost values assumed initially. The objective here was to demonstrate the practicality of the method and illustrate the nature of the assumptions necessary rather than draw inference from a particular set of results.

Also shown in Table 17 are the perceived probabilities of joint failure under the alternative inspection options. As indicated earlier another option, instead of cost based criteria, is to go for a minimum perceived probability of failure using a given amount of resources. This has the advantage of avoiding difficult cost estimation, however, it is a more arbitrary criterion, in the author's view. The problem of the optimum allocation of inspection resources has been considered by Mjelde, Lotsburg and Lee [99]. However, their model has no provision for the feedback of inspection results and the re-appraisal of

Data for Inspection Examnple

Cost of inspection for 20 welds	£200K
Cost of repair per weld (grinding)	£10K
Cost of minor failure	£2m
Cost of loss production per day	£1m
Shut down time for gross failure	30 days
Cost of repair, and other costs, for gross failure	£75m
Cost of catastrophic failure	£1b

Summary of Numerical Assumptions

Increase in probability of gross failure	$10^{-1} \times \delta P_{fL}$
Increase in probability of catastrophic failure	$10^{-3} \times \delta P_{fL}$
Opportunity cost of capital - indexed	10%
Number of significant joints in the structure	400
Number with 20 year nominal fatigue life	200
Number with 40 year nominal fatigue life	200
Increase in variance between actual crack size and measured crack size distribution due to crack length measurement inaccuracies	2 %

All 20 sample joints have a nominal 20 year fatigue life

TABLE 16: Summary of data and assumptions used in comparison of 6th and 10th year inspections.

	Year 6	Year 10
Inspection costs	-£182K	-£124K
Probability of finding a crack	0.408	0.412
Defect grinding costs	-£82K	-£56K
Reduction in risk costs for welds inspected - up to year 14	+£103K	+£70K
Reduction in risk costs for welds not inspected - up to year 14		
20 year fatiuge life	+£502K	+£477K
40 year fatigue life	+ £32K	+ £34K
Total Cost (N.P.V. at year 5)	+£373K	+£401K

Increase in perceived probability of joint failure at 14th year

	Year 6	Year 10
Inspected joints	8.2×10^{-5}	0.6×10^{-5}
Other joint with 20 year life	15.5×10^{-5}	7.8×10^{-5}
Other joint with 40 year life	5.9×10^{-5}	4.4×10^{-5}
Without inspection - 20 year life	34.1×10^{-5}	
Without inspection - 40 year life	6.2×10^{-5}	

TABLE 17: Results from analysis of expected outcome of inspection in years 6 and 10.

structural integrity. It does not involve a crack model and uses a distributed fatigue life to assess reliability.

There is a range of inspection problems that can be investigated using the procedures developed in this chapter and these include the following:

(i) The value of extra inspection results and their effect on perceived reliability can be studied using the relationships developed in Appendix IV.

(ii) The value of more accurate crack detection and measurement methods can be assessed. At the moment the accuracy is not well defined for any method but research in this area is currently being undertaken at University College London.

(iii) The effect of leaving cracks un-repaired can also be examined.

(iv) If marine fouling is not removed from structures the loading may increase dramatically. The effect of structural cleaning on predicted crack size can be examined.

Apart from the need to make assumptions concerning costs, the efficiency of inspection techniques and the effect of joint cracking on overall reliability; a number of other problems may be faced when attempting to apply the methods described here.

Inspection results may show a spatial variation which is not predicted by the design calculations. In principle this could be handled by a regression analysis of the least squares [e.g. 100] or robust type [101], where the x, y and z co-ordinates of each inspection result are the coefficients of independent variables. However, this would require a large quantity of inspection results to provide a fruitful analysis.

Spatial variations in the inspection results have been found to occur due to loads not considered at the design stage. For example mud piling on horizontal conductor guide frames has caused severe cracking. Caissons not designed to carry structural loads have failed at their connections to the jacket because of their high relative stiffness compared with the structural members on the designed load path. These and other load effects not considered at the design stage must be carefully monitored and their effects filtered out for the approach described here to function properly.

Another problem which occurs in practice is deciding what is and what is not a fatigue crack. Corrosion and stress corrosion cracking can occur in joints, and weld discontinuities can be interpreted as cracks. On these matters a conservative approach is prudent. All "defect indications" should be assumed to be fatigue cracks unless it can be established both that they are not, and that they will not act as an intiial defect for fatigue cracking.

The interaction between modes of failure and failures at different joints has been ignored. It is not clear what effect this will have on the reliabililty of actual structures but for comparative decisions about inspection strategies it is not thought, at least by the author, to be of primary importance.

8. DISCUSSION

8.1 Review of the Proposed Approach

It is appropriate to discuss the proposed approach in the context of the requirements established in Chapter 2. Perhaps the first question that should be posed is how accurate are the results obtained from the various models? A precise answer is difficult. It depends on the validity of the assumptions made explicitly, and implicitly, in each of the models. The principal assumptions are now reviewed starting with those of the fatigue model:

(a) Paris equation – this is the basis of the fatigue crack growth model. The Paris equation has been derived empirically from a wide variety of data and has been shown to explain a large range of data more precisely than many other models [13]. The Paris equation is recommended for the fatigue analysis of offshore structures by D.N.V. rules [4] and other codes of practice (see [69]).

(b) Data – it is assumed that the data obtained from the analysis of constant stress amplitude experiments in air, can be applied to tubular joints in sea water, subject to random loading at a much lower frequency. Experiments have shown that with cathodic protection, which is normally applied to offshore structures, fatigue crack growth rates do not significantly alter in sea water [86]. Some experiments have been undertaken using typical wide band random loading and these support the cumulative damage approach used here [103]. The data sample used here was small, however further, perhaps more appropriate data can of course be analysed in the manner described in Chapter 4.

(c) Linear modelling - The crack growth model is linear and the crack growth exponent m treated as a constant. The effects of these assumptions are though, reflected in the parameter estimations used. Threshold stress intensity and variations in crack shape parameter have been ignored. However, both effects could be included, at least in an approximate way consistent with the linear model, if suitable data were available.

(d) Log-normal distribution - All distributions of crack size and functions of crack size, are assumed to be log-normal. The log-normal distribution approaches the normal distribution in the limit as the standard deviation of the logarithm of the variable tends to zero. Therefore based on the central limit theorem (e.g. [105]) there is some heuristic argument for picking the log-normal distribution to represent a derived variable which is a linear combination of sums and products of other independent variables. The disadvantage of the log-normal distribution is that it must be truncated when used to represent negative skewness. However, another distribution could be used, if a more appropriate one can be found, as the approach does not depend on the selection of the log-normal. For another distribution, though, some of the procedures in Chapter 7 and Appendix IV would have to be modified slightly.

(e) Hot spot stress range estimation - This contains a number of assumptions. It is assumed that wave height distributions can be forecast from historical data; that a simple linear or quadratic relationship exists between wave height and nominal member stress: and that the nominal member stress can be related linearly to the hot spot stress. These assumptions however are not intrinsic to the model, which allows the hot spot stress range to be estimated in any manner, rather they are a reflection of lack of knowledge of the physical mechanisms involved. On the other hand the assumption that the hot spot stress range is independent of crack size is intrinsic to the model. But in the absence of detailed knowledge on this point there is no sensible alternative.

In summary, for the crack growth model, the level of accuracy will be affected both by the assumptions made and by the intrinsic random nature of the process modelled. However, the accuracy will be of the same order as for "probabilistic" fatigue life calculations and, of course, considerably more information is generated.

The assumptions and limitations of the reliability modelling are considered now:

(f) No experiment data - There have not been, at least to the author's knowledge, any published results of experiments undertaken to determine the residual strength of fatigue cracked joints. However, some research, for which details are not available, has apparently been done [106]. Work has been done in the aircraft industry in this field [107], but that has concentrated on cracked aluminium plates. The consequences of this is that the theoretical predictions may have an unknown bias.

(g) Linear modelling - The models used to predict the structural behaviour of a cracked joint were simple and parameters with comparatively small variance were considered as constants. In the cases under discussion more complex models are difficult to justify, especially in the absence of experiment data.

(h) Data - The reliability models use both well established data obtained from large samples [76] [77] and the output from the crack growth model. The latter brings with it the errors associated with the uncertainties and assumptions of the crack growth model.

It is difficult to assess the accuracy of the reliability model in the absence of experiment data. This difficulty afflicts many reliability studies, particularly those on large structural systems [43]. The accuracy of the reliability models used here are not likely to be any worse, or any better, than other reliability models of offshore structural elements.

The inspection strategy contains many assumptions, most of which have been discussed in Chapter 7. As it relies on inputs from both the crack growth and reliability models it implicitly contains the inaccuracies and assumptions associated with these models. In addition the following are thought to be particularly significant:

(i) Estimates of probability of major failure - In a redundant structure the effect of a crack in any one joint upon the overall reliability is difficult to assess currently. In this area engineering judgements and assumptions must be made. These of course may involve considerable unknown and unintended errors.

(j) Crack measurements - Subsea cracks are difficult to detect, and difficult to measure accurately. Unfortunately the uncertainties involved in crack detection and measurement are not well quantified. Assumptions must therefore be made which lead to errors in the statistical procedures. This situation will be vastly improved when crack depth can be measured accurately and reliably.

(k) Costs - Costs form a major input to the inspection strategy. These and their uncertainties are difficult to quantify: particularly the cost consequences of a major failure.

(l) Sampling - When a sample of joint weld inspection results is used to infer the state of other joints in the structure, several assumptions are implicit. The sample is assumed to be random, without bias, and representative of the structure. This implies, amongst other things, that the loading on the structure has not been abnormally high or low in any particular area and that there is no damage from non-anticipated sources. In practice this is not always the case.

Again it is difficult to quantify the accuracy of the outputs using this inspection strategy. However, when it is used in a comparative way, to evaluate alternative inspection plans, these inaccuracies effect all alternatives similarly. The comparisons, therefore are much less suceptible to error. This is particularly true when the inspection strategy is used to identify and compare different ways of ensuring a certain level of through life reliability for all joints. In this case overall failure and the associated cost consequences are not considered.

There are other features of the approach which warrant discussion in terms of the requirements established in Chapter 2 and these are considered below:

(i) Updating - The approach is designed to allow new data to be introduced into the models. Inspection results may be used to revise fatigue lives, re-assess reliabilities and alter inspection plans. The results of structural monitoring can also be used in this way. New data from other sources, the effect of marine growth on structural loading for example, can be included and its influence allowed to permeate through the models. The most important point about updating is that it reduces the likelihood of bias.

(ii) Flexibility - The models comprising the approach can be substituted by other models if required. This may be necessary if any of the simple models proposed here is found to be incapable of sufficient adaption in the light of new research. Indeed there are already alternative models which could be used in some cases.

In reliability the extended level-2 approach [43] could be used instead of the simple linear models proposed here. However, although this allows more complex models to be adopted it does require distribution functions to be assumed, often in a somewhat arbitrary way, for all variables. In some cases, see for example Appendix I, these assumptions can significantly affect the result. In many cases, though, the extended level-2 methods are both reliable and the most reasonable choice. They are widely accepted [42] [43] [78] in structural engineering and now form the basis of some codes of practice [42] [43] [84].

For inspection planning the strategy proposed could be substituted by the methods put forward by Sletten, Fjeld, Lotsberg and colleagues [10] [11] [99]; although these are more limited in scope. However, in many respects the approaches are complimentary and elements from each could be combined.

It would be appropriate to compare the approach developed here with other approaches to the same set of problems. However, little work has been done on linking the elements of fatigue, reliability and inspection in the marine field. Marshall [12] has outlined an approach in very broad terms, as discussed in Chapter 2. It is similar, but not as extensive or detailed as the approach described here. The same is true of the approach developed by Sletten et al [10] [11]. The work of Swift and Connolly [64], referred to in Chapter 4, relates inspection and fatigue crack growth, but does not consider reliability explicitly.

Detailed work has, however, been undertaken in the nuclear power industry, e.g. [41] [108] [109] and in the aircraft industry, e.g. [110] [111] [112]. In the nuclear power industry the concern has been with initial defects being below a certain size to avoid brittle fracture, as well as defects growing by fatigue to a critical size. Much, but not all, of this work has been deterministic in nature. Some aspects are similar to the proposed approach. Crack size is considered a key parameter for instance and Paris equation, or a modified form, is used. But much of the modelling is very different; updating fatigue lives, drawing inferences from sample inspection are not apparently considered. Direct comparison with the method proposed here is difficult: but there does not appear to be any conflict of views about how to approach the problem.

Extensive work has been undertaken in this area in the aircraft field with fatigue cracks starting at rivet holes in aluminium plate being the problem rather than cracks in welded steel joints. Assessment of inspection reliability is much more advanced in this area, e g. [73] than for subsea work. Again whilst the basic thinking is similar and crack size is a key parameter, the detail of the modelling is often different so direct comparison is difficult.

The author has not found in his examination of the literature any approaches in the aircraft or nuclear fields directly analogous to the approach proposed here.

Finally it is worth emphasising the particular strengths of the proposed approach. These are listed below:

- For every fatigue calculation an equivalent crack growth distribution can be found for any time in the life of the structure.

- All inspection results can be used to revise fatigue life estimates both for inspected and uninspected joints.

- Alternative inspection plans can be compared in a rational manner.

- The cost benefits of structural and environmental monitoring can be estimated.

- An assessment can be made of the reliability of any tubular joint at any time during its life.

8.2 Areas for Further Research

Further research is needed both to reduce the number of assumptions made in the proposed approach and to quantify more precisely the effect of those assumptions which remain. A better understanding and quantification of the underlying physical processes is required. Research in the following areas is particularly important to the integrated approach proposed in this thesis:

(i) The development of techniques for detecting small fatigue cracks reliably and for measuring crack depth and crack length accurately is required. The associated study of crack detection probabilities and measurement uncertainties for typical diver/ inspectors under realistic conditions is also necessary.

(ii) Experiments to determine the effect of fatigue cracking upon the residual capability of tubular joints to resist various modes of failure would be particularly useful to the approach.

(iii) The development of a distribution function of a uni-modal type, capable of adopting both substantial positive and negative skew without change of origin and having moments as efficient unbiased estimators would be of benefit. The distribution function would need to have a range of zero to plus infinity, a shape that is heuristically plausible for a function of crack size and, preferably, it would be simple to manipulate numerically.

(iv) A better understanding of the relationship between wave height and the loading of both clean and marine fouled tubular members of offshore structures is needed.

(v) A considerable quantity of research is currently being undertaken in the field of fatigue crack growth. That concerned with fatigue crack shape, grow rate, effect of the environment, initial crack size, stress intensity variation and stress intensity threshhold is particularly important here.

In addition more work needs to be undertaken using actual, and reliable, inspection data to assess clearly the extent of the practical value of the integrated approach developed in this thesis.

9. CONCLUSIONS

On the basis of the research undertaken and described in this thesis the following conclusions are drawn:

(a) The requirement for an integrated approach to fatigue cracking, reliability and inspection of offshore structures has been established and a suitable approach proposed.

(b) A new statistically based fatigue crack growth model has been developed and tested for use in this approach.

(c) The effect of fatigue cracking upon the reliability of tubular joints has been examined in a time dependent manner.

(d) An inspection strategy has been developed which allows the rational comparison of alternative inspection plans on the basis of life costs and/or maintaining an acceptable level of reliability. This approach has been tested using a practical example.

(e) The proposed approach is a step forward because it provides a linking of fatigue, reliability and inspection that makes use of all inspection results and other relevant data collected during the life of the structure.

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NOMENCLATURE

a = crack size

a_o = initial crack depth

a_a = crack area

a_d = crack depth

a_x = crack size after N_x crack cycles

$\bar{a}$ = critical defect size

A = wave height to hot spot stress transfer function

BM = bending moment

BM_u = ultimate bending moment

C = crack growth constant

C_{DEB} = cost of removing debris

C_{fx} = cost of failure of event x

C_{IN} = incidental cost of failure

C_{Ii} = cost of inspection of item i

$C_{p/d}$ = cost of lost oil production per day

C_{REP} = cost of repair

$C_{R/Mk}$ = cost of repair/maintenance of item k

C_{vi} = reduction in risk cost

$d = a_o^{1-m/2}$ - initial "defect" size

D = tubular member diameter

D_A = axial loading or demand - variable

e = crack measurement error

E = Young's modulus

F = distribution function

H = wave height

$H' = \Sigma n_i H_i^{mp}$

k = constant

K_x = coefficient of kurtosis

ℓ = crack length

ℓ_o = initial crack length

L = "loading" parameter

m = crack growth index

m_1 = number of maintenance activities

M = "material" response parameter

n = number of stress cycles per annum

n_1 = number of inspected items

N = number of stress cycles

N_x = number of stress cycles to reach crack size a_x

p = constant

P = axial load

Pf = probability of failure

Pu = ultimate axial load

p(x) = probability of x

r = "reciprocal" of crack size

R_A = axial resistance or capability - variable

S = cyclic stress range

SM = plastic section modulus

t = tubular member thickness

T = time in years

T_d = number od days lost production

T_{mean} = mean wave period

V_x = coefficient of variation of x

Y = crack shape parameter

α = angle

$\beta = - \Phi^{-1}$ (Pf)

δ = crack opening displacement

δ_c = critical crack opening displacement

μ_x = mean value of x

σ = stress

σ_y = yield stress

σ^2 = variance

θ_x = coefficient of skewness of x

Φ = cumulative normal distribution function

N.B. Some terms are defined separately within the Appendices.

APPENDIX I

The Statistical Instability of the Integrated Form of Paris' Equation

The integrated form of the Paris equation is given by equation (4) of Chapter 3 which may be written as follows:-

$$a_x = \frac{a_0}{\left[1.0 - \pi^{m/2}(m/2-1)\,C\,N_x Y^m S^m a_0^{(m/2-1)}\right]^{\frac{1}{m/2-1}}} \qquad \text{-(A1)}$$

where a_o, C and S^m are variables with considerable variance.

Consider the case for which m = 4; equation (A1) may then be re-written:-

$$a_x = \frac{a_0}{\left[1.0 - \pi^2 C N_x Y^4 S^4 a_0\right]} \qquad \text{-(A2)}$$

This in turn may be written:-

$$y = \frac{x_1}{(1.0 - K x_1 x_2 x_3)} \qquad \text{-(A3)}$$

where $K = \pi^2 Y^m N_x$ and x_1, x_2, x_3 represent a_o, C and S^4 respectively.

Now repeated differentiation of y with respect to x_1 yields:-

$$\frac{\delta n y}{\delta x_1^n} = \frac{n!\,(K x_2 x_3)^{n-1}}{(1.0 - K x_1 x_2 x_3)} \qquad \text{-(A4)}$$

Similarly, repeated differentiation of y with respect to x_2 and x_3 yields:-

$$\frac{\partial^n y}{\partial x_2^n} = \frac{n!\,(Kx_1x_3)^n}{(1.0 - Kx_1x_2x_3)^n}$$

-(A5)

and $$\frac{\partial^n y}{\partial x_3^n} = \frac{n!\,(Kx_1x_2)^n}{(1.0 - Kx_1x_2x_3)^n}$$

-(A6)

Consider now the differentiation of equation (A5) with respect to x_3:

$$\frac{\partial^{n+1} y}{\partial x_2^n \partial x_3} = \frac{(1-Kx_1x_2x_3)^n n!\,n\,x_3^{n-1}(Kx_1)^n + n!\,(Kx_1x_3)^n n\,Kx_1x_2(1-Kx_1x_2x_3)^{n-1}}{(1 - Kx_1x_2x_3)^n}$$

$$= \frac{n!\,n\left\{x_3^{n-1}(Kx_1)^n - Kx_1x_2x_3^n(Kx_1)^n + Kx_1x_2(Kx_1x_3)^n\right\}}{(1 - Kx_1x_2x_3)^{n+1}}$$

i.e. $$\frac{\partial^{n+1} y}{\partial x_2^n \partial x_3} = \frac{n!\,n\,x_3^{n-1}(Kx_1)^n}{(1 - Kx_1x_2x_3)^{n+1}}$$ -(A7)

A further differentiation yields:

$$\frac{\partial^{n+2} y}{\partial x_2^n \partial x_3^n} = \frac{n!\left\{n(n-1)(Kx_1)^n x_3^{n-2} + 2(Kx)^{n+1} x_2 x_3^{n-1}\right\}}{(1 - Kx_1x_2x_3)^{n+2}}$$

-(A8)

Now each term in (A8) has the form

$$y_1 = \frac{K_1 x_i^p}{(1 - K_2 x_i)^q} \qquad \text{-(A9)}$$

where K_1 and K_2 are terms not involving x_i and p and q are integer constants.

Then:

$$\frac{\partial y_1}{\partial x_i} = \frac{(1-K_2 x_i)^q p K_1 x_i^{p-1} + K_1 x_i^p q K_2 (1-K_2 x_i)^{q-1}}{(1-K_2 x_i)^{2q}}$$

$$\frac{\partial y_1}{\partial x_i} = \frac{pK_1 x_i^{p-1} + K_1 K_2 x_i^p (q-p)}{(1-K_2 x_i)^{q+1}}$$

-(A10)

Now (A10) will always be positive provided q > p, which from inspection of (A4), (A5) (A6), (A8) and (A10) is seen always to be the case, and $(1 - K_2 x_i) > 0$. Clearly subsequent differentiations of (A10) with respect to x_i (= x_1, x_2 or x_3) will yield positive (or zero) terms.

Therefore in general the following expression is obtained.

$$\frac{\partial^{(n+m+p)} y}{\partial x_1^n \partial x_2^m \partial x_3^p} = \frac{\text{positive function } (x_1, x_2, x_3)}{(1 - K x_1 x_2 x_3)^{n+m+p}}$$

where n, m, p = 0, 1, 2, 3 etc.

Consider now the Taylor series expansion of y about the mean values μ_1, μ_2 and μ_3 of x_1, x_2 and x_3.

viz.

$$y = \frac{\mu_1}{(1 - K\mu_1\mu_2\mu_3)} + \sum_{i=1}^{3} \left(\frac{\partial y}{\partial x_i}\right)_{\mu_i} (x_i - \mu_i)$$

$$+ \frac{1}{2!} \sum_{i=1}^{3} \sum_{j=1}^{3} \left(\frac{\partial 2 y}{\partial x_i \partial x_j}\right)_{\mu_i \mu_j} (x_i - \mu_i)(x_j - \mu_j)$$

$$+ \frac{1}{3!} \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} \left(\frac{\partial 3 y}{\partial x_i \partial x_j \partial x_k}\right)_{\mu_i \mu_j \mu_k} (x_i - \mu_i)(x_j - \mu_j)(x_k - \mu_k)$$

+ higher order terms.

The expection of y - E(y) - will clearly include at least all higher order terms of the type:

$$\frac{\partial 2(n+m+p) y}{\partial x_2^{2n} \partial x_3^{2m} \partial x_1^{2p}} \qquad \text{where } n, m, p = 0, 1, 2, 3 \ldots$$

even when x_1, x_2 and x_3 are independent, which is the case considered here, as these involve the even moments which are always non-zero.

An examination of equations (A2) and (A3) reveals that when crack size becomes significant the denominator becomes substantially less than 1.0. It is clear from equation (A11) that at this stage the higher order derivatives become substantial. It is also clear that they will make a significant contribution to the expected value of y, when the corresponding higher moments are finite. Unfortunately estimates of higher moments from small samples of data are notoriously unreliable.

Frequently distributions are chosen in somewhat arbitrary manner. For example Swift and Connolly [64] choose a log-normal distribution to represent initial defect size whereas Rogerson and Wong [25] use a Weibull distribution. Now a log-normal and a Weibull distribution with the same means and a c.o.v. of 0.523 have coefficients of skewness and kurtosis of 2.17 and 8.6, and 0.63 and 3.25 respectively. These higher moments are found to have significantly different effects upon the estimated mean crack size when equation (4) is used, and crack size has grown by a substantial amount. In fact for typical distributions of d, M and L these higher moments are found to dominate the estimation of mean crack size long before this reaches the through thickness value.

APPENDIX II

1. Moments of the Distribution of "r" for the Fatigue Crack Growth Model

The crack growth model proposed in Chapter 3 is given by equation (6), viz.:

$$r = d - TML$$

This may be expanded using the Taylor series quoted in Appendix I to yield:-

$$r = \mu_d - T\mu_L\mu_M + \left(\frac{\partial r}{\partial d}\right)(d-\mu_d) + \left(\frac{\partial r}{\partial M}\right)(M-\mu_M) + \left(\frac{\partial r}{\partial L}\right)(L-\mu_L) + \left(\frac{\partial r}{\partial M\,\partial L}\right)(M-\mu_M)(L-\mu_L) \qquad \text{-A11)}$$

where all higher order terms are zero and the expansion is at the mean values of d, M and L.

i.e. $$r = \mu_d - T\mu_L\mu_M + (d-\mu_d) - T\left\{\mu_L(M-\mu_M) + \mu_M(L-\mu_L) + (L-\mu_L)(M-\mu_M)\right\} \qquad \text{-(A12)}$$

Consider now the expectation of r. Now if the terms are uncorrelated, which is assumed to be the case, then:-

$$E[r] = \mu_d - T\mu_L\mu_M \qquad \text{-(A13)}$$

Now the higher central moments of r are given by:

$$E\left[(r-\mu_r)^n\right] = E\left[\left(r - E(r)\right)^n\right] \qquad \text{-(A14)}$$

From equation (A12) the following expression is obtained directly:

$$E\left[(r-\mu_r)^n\right] = E\left[\left[(d-\mu_d) - T\left\{\mu_L(M-\mu_M) + \mu_M(L-\mu_L) + (L-\mu_L)(M-\mu_M)\right\}\right]^n\right] \qquad \text{-(A15)}$$

The variance (i.e. n = 2) has already been considered in Chapter 3, so the third central moment is the first considered here.

$$E\left[(r-\mu_r)^3\right] = E\left[(d-\mu_d)^3 - T^3\left\{\mu_L^3(M-\mu_M)^3 + \mu_M(L-\mu_L)^3 + (L-\mu_L)^3(M-\mu_M)^3 + 3\mu_M(M-\mu_M)^2(L-\mu_L)^3 + 3\mu_L(L-\mu_L)^2(M-\mu_M)^3 + 6\mu_L\mu_M(L-\mu_L)^2(M-\mu_M)^2\right\}\right]$$

+ other cross product terms whose expectation is zero. -(A16)

$$\mu_3(r) = \mu_3(d) - T^3\left\{\mu_L^3\mu_3(M) + \mu_M^3\mu_3(L) + \mu_3(L)\mu_3(M) + 3\mu_M\sigma_M^2\mu_3(L) + 3\mu_L\sigma_L^2\mu_3(M) + 6\mu_L\mu_M\sigma_L^2\sigma_M^2\right\} \qquad \text{-(A17)}$$

where $\mu_3(x)$ denotes the third central moment of x.

This expression can be non-dimensionalised by noting:

$$\sigma_x^2 = V_x^2\mu_x^2 \quad \text{and} \quad \mu_3(x) = \theta_x\sigma_x^3 = \theta_x V_x^3\mu_x^3 \qquad \text{-(A18)}$$

where V_x and $_x$ are the coefficients of variance and skewness respectively. Using this notation equation (A17) can be re-written as below:

$$\theta_r V_r^3 \mu_r^3 = \theta_d V_d^3 \mu_d^3 - T^3 \{ \mu_L^3 \theta_M V_M^3 \mu_M^3 + \mu_M^3 \theta_L V_L^3 \mu_L^3 + \theta_L V_L^3 \mu_L^3 \theta_M V_M^3 \mu_M^3 + 3 \mu_M^3 V_M^2 \theta_L V_L^3 \mu_L^3 + 3 \mu_L V_L^2 \mu_L^2 \theta_M V_M^3 \mu_M^3 + 6 \mu_L^3 \mu_M^3 V_L^2 V_M^2 \} \quad \text{-(A19)}$$

This yields:

$$\theta_r = \frac{1}{V_r^3 \mu_r^3} \left[\theta_d V_d^3 \mu_d^3 - T^3 \mu_M^3 \mu_L^3 \left(V_L^3 \theta_L + V_M^3 \theta_M + V_M^3 V_L^3 \theta_M \theta_L + 3 V_M^3 V_L^2 \theta_M + 3 V_M^2 V_L^3 \theta_L + 6 V_M^2 V_L^2 \right) \right]$$

which is equation (9) of Chapter 3.

Consider now the fourth central moment. This is given by:

$$\begin{aligned} E[(r-\mu_r)^4] = E\Big[& (d-\mu_d)^4 + T^4 \{ \mu_L^4 (M-\mu_M)^4 + \mu_M^4 (L-\mu_L) + (L-\mu_L)^4 (M-\mu_M)^4 \} \\ & + 4 (L-\mu_L)^3 (M-\mu_M)^3 T^3 \{ T\mu_L (M-\mu_M) + T\mu_M (L-\mu_L) \} \\ & + 6 (d-\mu_d)^2 T^2 \{ \mu_L^2 (M-\mu_M)^2 + \mu_M^2 (L-\mu_L)^2 + (M-\mu_M)^2 (L-\mu_L)^2 \} \\ & + 6 T^2 \mu^2 (M-\mu_M)^2 \{ T^2 \mu_M^2 (L-\mu_L)^2 + T^2 (L-\mu_L)^2 (M-\mu_M)^2 \} \\ & + 6 T^2 \mu_M^2 (L-\mu_L)^2 T^2 (L-\mu_L)^2 (M-\mu_M)^2 \\ & + 12 T^3 \mu_M \mu_L (M-\mu_M)^2 (L-\mu_L)^2 \{ T\mu_M (L-\mu_L) + T\mu_L (M-\mu_M) \\ & + T (M-\mu_M)(L-\mu_L) \} \Big] \end{aligned}$$

+ other terms whose expectation is zero -(A20)

The fourth moment can be expressed in a non-dimensional form:

$$K_x = \frac{E[(x-\mu_x)^4]}{\mu_x^4} \qquad \text{-(A21)}$$

Using equations (A18) and (A21) equation (A20) can be re-expressed as below.

$$\begin{aligned} K_r = \frac{1}{V_r^4 \mu_r^4} \Big[& \mu_d^4 V_d^4 K_d - T^4 \mu_L^4 \mu_M^4 \Big\{ V_M^4 K_L + V_L^4 K_L + V_L^4 V_M^4 K_L K_M \\ & + 4\left(V_M^4 V_L^3 \theta_L K_M + V_L^4 V_M^3 \theta_M K_L\right) \\ & + 6\left(V_M^4 V_L^2 K_M + V_L^4 V_M^2 K_L + V_M^2 V_L^2\right) \\ & + \frac{6\mu_d^2}{\mu_L^2 \mu_M^2 T^2}\left(V_d^2 V_M^2 + V_d^2 V_L^2 + V_d^2 V_M^2 V_L^2\right) \\ & + 12\left(V_M^3 V_L^2 \theta_M + V_L^3 V_M^2 \theta_L + V_L^3 V_M^3 \theta_L \theta_M\right) \Big\} \Big] \end{aligned}$$

which is equation (10) of Chapter 3.

2. Moments for the Sum and Product of Distributed Variables

In the case of the sum Y of two variables A and B, the Taylor series is as follows:

$$\begin{aligned} y &= \mu_A + \mu_B + \left(\frac{\partial Y}{\partial A}\right)(A-\mu_A) + \left(\frac{\partial Y}{\partial B}\right)(B-\mu_B) \\ &= \mu_A + \mu_B + (A-\mu_A) + (B-\mu_B) \end{aligned} \qquad \text{-(A22)}$$

hence

$$E(y) = \mu_A + \mu_B \qquad \text{-(A23)}$$

and:

$$E\left[(y-E(y))^n\right] = E\left[((A-\mu_A)+(B-\mu_B))^n\right] \qquad \text{-(A24)}$$

which for n = 2 yields:

$$\sigma_Y^2 = E\left[(A-\mu_A)^2+(B-\mu_B)^2+2(A-\mu_A)(B-\mu_B)\right]$$
$$= \sigma_A^2 + \sigma_B^2 \qquad \text{-(A25)}$$

when A and B are statistically independent.

The corresponding non-dimensional form is:-

$$\mu_Y^2 V_Y^2 = \mu_A^2 V_A^2 + \mu_B^2 V_B^2$$

Similarly for n = 3:

$$\mu_3(Y) = E\left[(A-\mu_A)^3 + (B-\mu_B)^3 + 3(A-\mu_A)(B-\mu_B)^2 + 3(B-\mu_B)(A-\mu_A)^2\right]$$

$$= \mu_3(A) + \mu_3(B) \qquad \text{-(26)}$$

and $$\mu_Y^3 V_Y^3 \theta_Y = \mu_A^3 V_A^3 \theta_A + \mu_B^3 V_B^3 \theta_B$$

Finally for n = 4:

$$\mu_4(Y) = E\left[(A-\mu_A)^4 + (B-\mu_B)^4 + 4(A-\mu_A)^3(B-\mu_B) + 4(B-\mu_B)^3(A-\mu_A) + 6(A-\mu_A)^2(B-\mu_B)^2\right]$$

$$= \mu_4(A) + \mu_4(B) + 6\sigma_A^2\sigma_B^2 \qquad \text{-(27)}$$

and $$\mu_Y^4 V_Y^4 K_Y = \mu_A^4 V_A^4 K_A + \mu_B^4 V_B^4 K_B + 6\mu_A^2\mu_B^2 V_A^2 V_B^2$$

Consider now the product Z of two variables C and D.

The corresponding Taylor Series yields:

$$Z = \mu_C \mu_D + \left(\frac{\partial Z}{\partial C}\right)(C-\mu_C) + \left(\frac{\partial Z}{\partial D}\right)(D-\mu_D) + \left(\frac{\partial^2 Z}{\partial C \partial D}\right)(D-\mu_D)(C-\mu_C)$$

$$= \mu_C \mu_D + \mu_D (C-\mu_C) + \mu_C (D-\mu_D) + (D-\mu_D)(C-\mu_C) \qquad -(A28)$$

hence $E(Z) = \mu_Z = \mu_C \mu_D$

and $\sigma_Z^2 = E\left[(Z-\mu_Z)^2\right] = E\left[\mu_D^2 (C-\mu_C)^2 + \mu_C^2 (D-\mu_D)^2 + (D-\mu_D)(C-\mu_C)^2\right]$

+ other terms with zero expectation

$$= \mu_D^2 \sigma_C^2 + \mu_C^2 \sigma_D^2 + \sigma_D^2 \sigma_C^2 \qquad -(A29)$$

which in non-dimensional form is:

$$\frac{\sigma_Z^2}{\mu_Z^2} = V_Z^2 = \frac{\mu_D^2 \sigma_C^2 + \mu_C^2 \sigma_D^2 + \sigma_D^2 \sigma_C^2}{\mu_C^2 \mu_D^2} \qquad -(A30)$$

i.e. $V_Z^2 = V_C^2 + V_D^2 + V_C^2 V_D^2$

In a similar way the third central moment may be obtained:

$$\mu_{(3)} Z = E\Big[\mu_D^3 (C-\mu_C)^3 + \mu_C^3 (D-\mu_D)^3 + (D-\mu_D)^3 (C-\mu_C)^3$$
$$+ 3\mu_D (C-\mu_C)^3 (D-\mu_D)^2 + 3\mu_C (D-\mu_D)^3 (C-\mu_C)^2$$
$$+ 6\mu_C \mu_D (D-\mu_D)^2 (C-\mu_C)^2\Big]$$

+ other terms with zero expectation $\qquad$ -(A31)

In non-dimensional form this is:

$$\mu_Z^3 V_Z^3 \theta_Z = \mu_C^3 \mu_D^3 \Big(V_C^3 \theta_C + V_D^3 \theta_D + V_C^3 V_D^3 \theta_C \theta_D + 3 V_C^3 V_D^2 \theta_C^2 + 3 V_D^3 V_C^2 \theta_D + 6 V_C^2 V_D^2 \Big)$$

Finally, consider the case of the fourth moment of the product

$$E\left[(Z-\mu_Z)^4\right] = \mu_{(4)}Z = E\Big[\mu_D^4 (C-\mu_C)^4 + \mu_C^4 (D-\mu_D)^4 + (D-\mu_D)^4 (C-\mu_C)^4$$

$$+ 4\mu_D (C-\mu_C)^4 (D-\mu_D)^3 + 4\mu_C (D-\mu_D)^4 (C-\mu_C)^3$$

$$+ 6\mu_D^2 (C-\mu_C)^4 (D-\mu_D)^2 + 6\mu_C^2 (D-\mu_D)^4 (C-\mu_C)^2$$

$$+ 12 \mu_C^2 \mu_D (C-\mu_C)^2 (D-\mu_D)^3 + 12 \mu_D \mu_C (C-\mu_C)^3 (D-\mu_D)^3$$

$$+ 6 \mu_C^2 \mu_D^2 (C-\mu_C)^2 (D-\mu_D)^2 + 12 \mu_D^2 \mu_C (C-\mu_C)^3 (D-\mu_D)^2 \Big]$$

+ other terms with zero expectations. -(A32)

This may be non-dimensionalised to yield:

$$\mu_Z^4 V_Z^4 K_Z = \mu_C^4 \mu_D^4 \Big[V_C^4 K_C + V_D^4 K_D + V_C^4 V_D^4 K_C K_D$$

$$+ 4\left(V_C^4 V_D^3 \theta_D K_D + V_D^4 V_C^3 \theta_C K_C \right)$$

$$+ 6\left(V_C^4 V_D^2 K_C + V_D^4 V_C^2 K_D + V_C^2 V_D^2 \right)$$

$$+ 12\left(V_C^3 V_D^2 \theta_C + V_D^3 V_C^2 \theta_D + V_C^3 V_D^3 \theta_C \theta_D \right) \Big]$$

which is equation (19) of Chapter 3.

APPENDIX III

Weighting Functions for the Estimation of C

In Chapter 3 the following expression (equation 22) has been obtained for the crack growth constant C in Paris' equation:

$$C = \frac{1}{S^m \pi^{m/2}(1-m/2)} \cdot \frac{\left(a_{i+1}^{1-m/2} - a_i^{1-m/2}\right)}{(N_{i+1} - N_i)}$$

For any particular constant stress amplitude fatigue experiment this has the form:-

$$C = \frac{Const.}{(N_{i+1} - N_i)} \cdot \left(a_{i+1}^{1-m/2} - a_i^{1-m/2}\right) \qquad \text{-(A32)}$$

Now whilst the number of cycles in a fatigue experiment can be counted accurately the crack size is more difficult to measure, as illustrated in Figure 2, and is subject to error. It is reasonable to write therefore:-

$$C = \frac{Const.}{(N_{i+1} - N_i)} \cdot \left[(a_{i+1} + e_2)^{1-m/2} - (a_i + e_1)^{1-m/2}\right] \qquad \text{-(A33)}$$

where e_1 and e_2 are random errors in measurements.

This may be expanded by the binomial theorem to give:-

$$C = \frac{Const.}{(N_{i+1} - N_i)} \cdot \left[a_{i+1}^{1-m/2}\left(1 + (1-m/2)\frac{e_2}{a_{i+1}}\right) - a_i^{1-m/2}\left(1 + (1-m/2)\frac{e_1}{a_i}\right)\right] \qquad \text{-(A34)}$$

where the first term only in the expansion is considered on the assumption that a >> e

Re-arranging this equation gives:-

$$C = \frac{Const.}{(N_{i+1} - N_i)} \cdot \left[a_{i+1}^{1-m/2} - a_i^{1-m/2} + (1-m/2)\left(e_2 a_{i+1}^{-m/2} - e_1 a_i^{-m/2}\right)\right] \qquad \text{-(A35)}$$

and therefore the error term has the form:-

$$\text{Error} = \frac{Const.(1-m/2)}{(N_{i+1} - N_i)} \cdot \frac{e}{a^{m/2}} \qquad \text{-(A36)}$$

$$\text{where } e = \frac{e_2 a_i^{m/4}}{a_{i+1}^{m/4}} - \frac{e_1 a_{i+1}^{m/4}}{a_i^{m/4}} \quad \text{and} \quad a = \sqrt{a_i a_{i+1}} \qquad \text{-(A37)}$$

If e_2 and e_1 are assumed to be normally distributed with constant variance which seems plausible, e will also be so distributed for any given values of a_i and a_{i+1}. The error term then becomes:

$$\text{Error} = \frac{Const.\ (1-m/2)}{(N_{i+1} - N_i)\, a_i^{m/4} a_{i+1}^{m/4}} \cdot e \qquad \text{-(A38)}$$

So, to obtain estimators for C of constant variance the weighting function (W.F.) below should be used.

$$\text{W.F.} = \frac{(N_{i+1} - N_i)\, a_{i+1}^{m/4} a_i^{m/4}}{\sum_{i=1}^{n-1} (N_{i+1} - N_i)\, a_{i+1}^{m/4} a_i^{m/4}} \qquad \text{-(A39)}$$

where n is the number of pairs of values (a_i, N_i) in any set of experiment data.

Using equation (A39) with equation (22) of Chapter 3, the following estimator for the mean value of C for any fatigue crack experiment is obtained.

$$C_{mean} = \frac{1}{S^m \pi^{m/2}(1-m/2)} \cdot \sum_{i=1}^{n-1} \frac{\left(a_{i+1}^{1-m/2} - a_i^{1-m/2}\right)}{\left(N_{i+1} - N_i\right)} \cdot \frac{\left(N_{i+1} - N_i\right) a_{i+1}^{m/4} a_i^{m/4}}{\sum_{i=1}^{n-1} \left(N_{i+1} - N_i\right) a_{i+1}^{m/4} a_i^{m/4}} \qquad \text{-(A40)}$$

Hence:

$$C_{mean} = \frac{1}{S^m \pi^{m/2}(1-m/2)} \cdot \frac{\sum_{i=1}^{n-1} \left(a_{i+1}^{1-m/2} - a_i^{1-m/2}\right) a_{i+1}^{m/4} a_i^{m/4}}{\sum_{i=1}^{n-1} \left(N_{i+1} - N_i\right) a_{i+1}^{m/4} a_i^{m/4}} \qquad \text{-(A41)}$$

which for m = 3 yields:

$$C_{mean} = \frac{0.3592}{S^3} \cdot \frac{\sum_{i=1}^{n-1} a_{i+1}^{3/4} a_i^{1/4} - a_{i+1}^{1/4} a_i^{3/4}}{\sum_{i=1}^{n-1} \left(N_{i+1} - N_i\right) a_{i+1}^{3/4} a_i^{3/4}}$$

and for m = 4

$$C_{mean} = \frac{0.1013}{S^4} \cdot \frac{\sum_{i=1}^{n-1} \left(a_{i+1} - a_i\right)}{\sum_{i=1}^{n-1} \left(N_{i+1} - N_i\right) a_{i+1} a_i}$$

These are respectively equations (26) and (27) of Chapter 4.

APPENDIX IV

Structural Re-appraisal and Cost Benefits of Inspection Results

The expected distribution of measured fatigue crack lengths will depend both on the crack detection efficiency and on the precision of crack size measurement. This will vary according to the type of equipment used and the proficiency of the diver-operator [7] [73]. Crack detection efficiency has been discussed in Chapter 5. Crack measurement precision is still the subject of investigation. However, from discussions with operators it appears reasonable to assume that measurement precision is independent of crack length. That is the standard deviation of measurements about the actual crack length are constant; and the corresponding coefficient of variation decreases with increasing crack length.

Now applying Bayes' Theorem [e.g 70], the probability density of crack length measured, given that a crack is found, may be written as:-

$$p(l_m|det) = \frac{p(l_m|l)\ p(det|l)\ p(l)}{\int_0^\infty p(det|l)\ p(l)\ dl \int_0^\infty p(l_m|l)\, p(l|det)\, dl} \quad (A42)$$

or in the r domain:

$$p(r_m|det) = \frac{p(r_m|r)\ p(det|r)\ p(r)}{\int_0^\infty p(det|r)\ p(r)\ dr \int_0^\infty p(r_m|r)\, p(r|det)\, dr}$$

-(A43)

where $p(\ell_m|\ell)$ = prob density of ℓ_m (measured length) given ℓ (actual length)

$p(det|\ell)$ = prob. density of finding a crack given its actual length is ℓ.

$p(\ell)$ = prob. density of actual crack length at the time of inspection

The terms in the r domain have a corresponding meaning.

When r is small (i.e ℓ is large) the error in crack length measurement is small. This is important because the left hand tail of the distribution of r is significant in terms of the joint's reliability and as a result perceived reliability is less affected if measurement error is constant and independent of length. In addition if a large number of measurements are made and the measured values are distributed about the actual values without bias· then as the sample size n increases the effect of measurement errors upon the measured crack size distribution (r or ℓ) asymtotically approaches zero

On this basis, and in the absence of any data on crack detection precision, it is assumed that the effect of crack measurement accuracy can be handled by a small, nominal, increase in variance the magnitude of which depends on sample size. Equation (A43) can then be rewritten:

$$p(r_m|det) = F_{mn} \frac{p(r)\ p(det|r)}{\int_0^\infty p(det|r)\, p(r)\, dr} \qquad \text{-(A44)}$$

where F_{mn} is a factor which has the effect of increasing the variance of $p(r_m/det)$ by an amount which varies inversely as the square root of sample size.

Consider now those inspections on which no cracks are found. These cannot be considered as samples from the actual crack size distribution at the time of inspection, as in the case above, because however many of these results are obtained the actual crack size distribution cannot be discerned. However, these results can be used to draw some inferences about the actual crack size distribution as has been discussed and illustrated in Chapter 5. Based on the work presented in Chapter 5 and given that no crack is found, the revised distribution of crack size r* is:-

$$p(r^*) = \frac{p(r_o)\left(1 - p(det|r)\right)}{\int_0^\infty \left(1 - p(det|r)\right) p(r)\, dr} \qquad \text{-(A45)}$$

where $p(r_o)$ is the a priori distribution of r at time t = 0

Before an inspection is made it is not known how many, if any, cracks will be found. An estimate is therefore needed and this can be based on the expected actual crack size distribution and the probability of crack detection.

$$P(det) = \int_0^\infty p(det|r)\, p(r)\, dr \qquad \text{-(A46)}$$

where P(det) is the probability of finding a crack.

The expected distribution of crack size from which an inspection result will be drawn can then be obtained by combining equations (A44), (A45) and (A46)

$$p(r_1) = P(det) \frac{F_{mn}\, p(r)\, p(det|r)}{\int_0^\infty p(r)\, p(det|r)\, dr} + \left(1 - P(det)\right) \frac{p(r_o)\left(1 - p(det|r)\right)}{\int_0^\infty p(r)\left(1 - p(det|r)\right) dr}$$

i.e. $$p(r_1) = F_{mn}\, p(r)\, p(det|r) + p(r_o)\left(1 - p(det|r)\right) \qquad \text{-(A47)}$$

where $p(r_1)$ is the expected distribution of an inspection result prior to the inspection

Consider now a structure on which n identical joints are about to be inspected. These will be independent events drawn from the probability distribution given by equation (A47). Now the variance of this distribution is known, or is assumed to be known as it can be estimated using the procedure described above. The actual mean value, as opposed to the expected mean value, however,, is not known as it depends on the actual loading and environmental conditions experienced up to the time of inspection. It is expected, however, that the mean value will be estimated, after inspection, with a precision which depends on the number of inspection results. This problem has been treated by Cox and Hinkley [102]. for example, and the mean value of r, μ_r, has a variance given by:

$$\sigma^2_{\mu_{\ln r}} = \frac{\sigma^2_{\ln r_1} \, \sigma^2_{\ln r_0}}{\sigma^2_{\ln r_1} + n\,\sigma^2_{\ln r_0}} \qquad \text{-(A48)}$$

where r_1 and r_o are both assumed to be log-normally distributed and n is the number of inspection results in the sample. In this case $p(r_o)$ is the prior distribution, and $p(r_1)$ the sampling distribution, and Bayes' theorem has been applied.

The corresponding variance of the predicted crack size function, r_o', for an identical joint which will not be inspected is given by:-

$$\sigma^2_{\ln r_o'} = \frac{\sigma^2_{\ln r_1} \, \sigma^2_{\ln r_o}}{\sigma^2_{\ln r_1} + n\,\sigma^2_{\ln r_o}} + \sigma^2_{\ln r_1} \qquad \text{-(A49)}$$

The discussion so far has concentrated on the expected outcome of inspection, and the corresponding expected mean values will be the same as the à priori mean values unless bias has been found to exist. The actual inspection results may well be different, and after inspection the predicted mean value of r_0' for an identical joint not in the inspection sample, also changes.

This is given by([102]):-

$$\mu_{\ln r_0'} = \frac{\mu_{\ln r_0} \cdot \sigma^2_{\ln r_1} + n\,\sigma^2_{\ln r_0} \cdot \overline{\ln r_1}}{\sigma^2_{\ln r_1} + n\,\sigma^2_{\ln r_0}} \qquad \text{-(A50)}$$

where $\overline{\ln r_1}$ is the mean value of the actual inspection results

In fact after inspection the variance of $p(r_1)$ can be modified as the actual fraction of cracks found on inspections made will be known. This is at odds with the assumption of constant variance made above; but, before inspection, the assumption is reasonable.

For a real structure the fatigue lives will not all be the same; this is likely to be true both for the joints inspected and those not inspected. The equations above must therefore be modified to be of practical value.

Equation (A49) can be rewritten as follows:-

$$\frac{\sigma^2_{\ln r_0'}}{\sigma^2_{\ln r_0}} = \frac{\sigma^2_{\ln r_1} / \sigma^2_{\ln r_0}}{n + \sigma^2_{\ln r_1} / \sigma^2_{\ln r_0}} + \frac{\sigma^2_{\ln r_1}}{\sigma^2_{\ln r_0}} \qquad \text{-(A51)}$$

The term $\sigma^2_{\ln r_o'} / \sigma^2_{\ln r_o}$ can be calculated for each joint to be inspected. The magnitude of this term may vary somewhat but it seems reasonable, to the author at least, to use the mean value of these terms from all the joints to be inspected as an estimator in equation (A51)

i.e.

$$\frac{\sigma^2_{\ln r_o'}}{\sigma^2_{\ln r_o}} \approx F_{\sigma^2} = \frac{\displaystyle\sum_{i=1}^{n} \frac{\sigma^2_{\ln r_{1i}}}{\sigma^2_{\ln r_{oi}}}}{n^2 + \displaystyle\sum_{i=1}^{n} \frac{\sigma^2_{\ln r_{1i}}}{\sigma^2_{\ln r_{oi}}}} + \frac{1}{n} \sum_{i=1}^{n} \frac{\sigma^2_{\ln r_{1i}}}{\sigma^2_{\ln r_o}} \qquad \text{-(A52)}$$

In a similar manner equation (A50) may also be rewritten:

$$\frac{\mu_{\ln r_o'}}{\mu_{\ln r_o}} \approx F_{\mu} = \frac{\displaystyle\sum_{i=1}^{n} \frac{\sigma^2_{\ln r_{1i}}}{\sigma^2_{\ln r_{oi}}} + n \sum_{i=1}^{n} \frac{\mu_{\ln r_{1i}}}{\mu_{\ln r_{oi}}}}{n^2 + \displaystyle\sum_{i=1}^{n} \frac{\sigma^2_{\ln r_{1i}}}{\sigma^2_{\ln r_{oi}}}} \qquad \text{-(A53)}$$

Equations (A52) and (A53) allow the mean variance for the non-inspected joints to be updated as a result of inspection:

i.e.

$$\mu_{\ln r_o'} \approx F_{\mu} . \mu_{\ln r_o} \qquad \text{-(A54)}$$

and

$$\sigma^2_{\ln r_o'} \approx F_{\sigma^2} . \sigma^2_{\ln r_o} \qquad \text{-(A55)}$$

where r_o', the revised crack size term, and r_o, the original crack size terms, are both assumed to be log-normally distributed.

Now, before the planned inspection the expected value of $F\mu$ is unity as discussed above. But the expected value of $F\sigma^2$ is less than 1.0 and becomes progressively smaller, up to a certain limit, as the number of joints to be inspected increases. This expected reduction in variance corresponds to an expected increase in perceived reliability for joints not inspected. Which in turn results in an expected reduction in risk costs. This last is the cost benefit of the inspection programme for the joints <u>not</u> inspected and corresponds to the final term in equation (60) of Chapter 7.

Based on the foregoing procedures have been devised for (a) assessing the cost benefits of inspection before it takes place; and (b) revising the moments of crack size distributions in the light of inspection results. The steps involved in each case are enumerated below:

(a) <u>Assessing Cost Benefits of Inspection</u>

1. Calculate the distribution r_o for each joint in the structure.

2. Calculate a revised distribution for r for the time of inspection, assuming the actual loading and environmental effects are mean (or expected) values.

3. Calculate the expected inspection distributions of r_1 for all joints in the inspection programme using equation (A47) above.

4. Assuming all r terms are log-normally distributed calculate $F\sigma^2$ using equation (A52) and estimate the expected revised variance for each uninspected joint.

5. Calculate the moment of the corresponding revised crack size distribution r_o' for each joint.

6. Calculate the reduction in perceived reliability using both the original distribution moments of r_o and the revised distribution moments of r_o' as starting points, for each joint, for each year, in the time period up to the next inspection.

7. Calculate the corresponding risk costs for each year in the period between the planned inspections and find the difference. Calculate the related net present value for each joint the sum over all joints.

This procedure yields the expected cost benefit of inspection for those joints which are not inspected. For the joints that are to be inspected the procedure is similar from step 5. onwards. The revised distribution used, however, is that obtained by the procedure described in Chapter 5, for the case where no cracks are found on inspection. This is because even when a crack is found and ground out, another crack in the same joint may be missed. It is assumed in the procedure here that all cracks found will be ground out.

An important assumption is implicit in the above procedure. The reduction in variance applied to crack length can also be applied to crack depth; as crack depth is the key factor in joint reliability This is reasonable when the sample size is large and unfortunately there is no alternative until techniques for measuring crack depth subsea are further developed and generally adopted.

(b) Revising the Moments for Crack Size Distribution in Light of Inspection Results

Steps 1 and 2 are identical to those for procedure (a) above.

3. For joints where no cracks are found the procedure of Chapter 5 is used to calculate revised moments for r, i e. μ_{r*}. V_{r*} and θ_{r*}.

4 For joints where cracks are found the measured crack size is used as the mean value of r, and the variance obtained in step 2 as the corresponding variance. Assuming a log-normal distribution the corresponding revised non-dimensional moments are calculated.

5. For joints which have not been inspected, the factors $F\mu$ and $F\sigma^2$ are calculated using the results from steps 1 , 3, and 4. above. The original values of $\mu_{\ln r_o}$ and $\sigma^2_{\ln r_o}$ are then revised using equations (A54) and (A55), and the non-dimensional moments for the corresponding revised log-normal distributions are calculated.

The resulting non-dimensional moments for crack size distributions can then be used as starting points for subsequent calculations after the year of inspection and up to the time of the next inspection. The procedures are thus iterative with the perceived state of the structure being reviewed after every inspection both for inspected and non-inspected joints.

The procedures described above are approximate, but this is inevitable as there is not sufficient data available to define the problem precisely.

APPENDIX V

Data Used to Estimate the Parameter C and Apparent Initial Defect Size

The fatigue crack growth data below are some of those which have been collected and presented by Clayton [47]. In the first 16 sets crack depth (a) has been measured as well as crack length (L).

All linear measurements are in millimetres and the hot spot stress range (S.R.) is in MPa. The number of stress cycles (N) are expressed as multiples of 10^5.

REF. =37.70 S. R. =129.0
DIA. =457.0 THK. =16.0

N	a	L
59.14	6.0	60.0
60.64	7.0	63.0
61.69	7.0	68.0
63.36	0.0	80.0
64.53	7.0	82.0
66.13	7.0	84.0
72.20	10.0	107.0
75.34	0.0	125.0
79.28	10.0	148.0
80.82	9.3	158.0
84.64	0.0	205.0
86.13	10.6	231.0
87.20	0.0	244.0
88.02	0.0	272.0
88.46	0.0	275.0
92.19	12.6	330.0
94.47	0.0	372.0
96.70	0.0	387.0
98.38	0.0	391.0
100.14	16.0	396.0

REF. =37.90 S. R. =120.0
DIA. =457.0 THK. =16.0

N	a	L
23.90	0.0	25.0
32.60	0.0	45.0
34.50	1.6	57.0
41.90	2.1	74.0
45.60	1.6	89.0
48.00	2.8	103.0
50.70	1.6	124.0
51.40	2.0	127.0
52.90	6.5	138.0
54.60	5.6	141.0
56.80	8.1	147.0
66.00	9.9	194.0
68.60	15.6	207.0
72.60	16.0	258.0

REF. =37.10 S. R. =183.0
DIA. =457.0 THK. =16.0

N	a	L
6.06	0.0	5.0
10.80	8.7	117.0
11.30	8.4	125.0
11.80	11.8	138.0
16.10	13.5	216.0
18.80	14.2	311.0
23.20	16.0	320.0
27.10	0.0	423.0
29.00	16.0	446.0
31.80	16.0	460.0
33.20	16.0	492.0
37.90	16.0	540.0

REF. =37.13 S. R. =169.0
DIA. =457.0 THK. =16.0

N	a	L
3.37	0.0	30.0
5.95	5.0	120.0
6.18	6.5	210.0
7.09	10.3	300.0
10.56	16.0	330.0

REF. =38.50 S. R. =146.0
DIA. =457.0 THK. =16.0

N	a	L
10.70	0.0	15.0
12.60	0.0	19.0
13.02	0.0	41.0
15.07	8.0	130.0
16.80	10.3	186.0
19.80	16.0	238.0

REF. =38.80 S. R. =269.0
DIA. =457.0 THK. =16.0

N	a	L
2.27	0.0	25.0
2.79	2.1	65.0
3.18	6.3	75.0
3.42	9.0	84.0
3.69	16.0	220.0

REF. =41.10 S. R. = 77.0
DIA. =914.0 THK. =32.0

N	a	L
21.90	0.0	18.0
56.40	4.2	54.0
70.90	0.0	58.0
72.50	6.2	69.0
88.00	7.5	86.0
103.50	8.5	99.0
125.40	10.6	143.0
129.00	11.2	161.0
130.00	12.4	171.0
149.00	15.3	216.0
160.90	14.8	236.0
167.00	13.5	275.0
176.00	17.7	302.0
180.50	20.6	328.0
183.60	17.9	337.0
189.00	25.2	398.0
194.00	29.6	430.0
195.60	25.1	447.0
204.00	31.5	488.0
208.30	31.5	531.0

REF. =41.30 S. R. =262.0
DIA. =914.0 THK. =32.0

N	a	L
3.32	0.0	15.0
8.09	9.4	242.0
10.57	28.0	443.0
12.28	29.4	635.0
16.70	31.4	640.0

REF. =41.40 S. R. =164.0
DIA. =914.0 THK. =32.0

N	a	L
12.30	0.0	30.0
17.80	4.2	51.0
20.10	7.3	168.0
21.00	9.6	176.0
21.80	6.4	293.0
24.80	10.3	300.0
29.30	17.7	360.0
31.10	24.6	394.0
34.60	31.5	465.0

REF. =42.20 S. R. = 92.0
DIA. =914.0 THK. =31.5

N	a	L
17.80	1.5	20.0
19.94	5.2	55.0
24.77	6.8	95.0
25.26	5.5	100.0
25.45	2.6	108.0
26.48	4.7	110.0
28.87	3.7	112.0
30.98	3.8	118.0
32.17	4.5	120.0
33.68	2.7	138.0
35.45	7.4	140.0
38.32	6.5	170.0
39.43	5.6	184.0
39.97	0.0	197.0
43.83	13.1	287.0
45.58	12.4	323.0
47.94	31.5	366.0

REF. =42.30 S. R. = 94.0
DIA. =914.0 THK. =32.0

N	a	L
35.30	7.1	66.0
39.30	7.6	72.0
43.10	0.0	86.0
43.70	0.0	90.0
45.30	10.3	94.0
47.60	11.2	166.0
52.10	12.3	176.0
59.50	0.0	208.0
60.20	0.0	219.0
61.70	16.0	224.0
65.10	19.6	240.0
73.40	25.0	316.0
77.40	29.3	335.0
78.40	30.4	352.0
83.30	31.5	393.0

REF. =43.20 S. R. = 90.0
DIA. =914.0 THK. =32.0

N	a	L
4.92	0.0	7.0
6.11	0.0	11.0
7.75	0.0	30.0
8.14	0.0	32.0
9.51	3.7	55.0
10.80	5.1	119.0
12.10	13.1	183.0
13.60	23.9	239.0
14.40	26.9	264.0
18.70	32.0	304.0

REF. =43.40 S. R. =147.0
DIA. =914.0 THK. =32.0

N	a	L
26.90	0.0	25.0
30.10	0.0	34.0
31.90	0.0	37.0
33.60	6.5	49.0
34.20	5.9	51.0
34.80	6.1	52.0
39.90	8.5	70.0
44.60	12.0	87.0
46.40	14.3	97.0
47.80	15.0	109.0
50.40	16.0	124.0
53.00	17.2	135.0
57.80	23.0	170.0
63.40	31.0	228.0
65.80	32.0	253.0

REF. =44.30 S. R. =188.0
DIA. =914.0 THK. =32.0

N	a	L
7.20	0.0	25.0
7.55	3.0	49.0
10.50	7.8	68.0
10.90	8.7	83.0
11.50	8.2	93.0
13.50	8.6	180.0

REF. =37.10 S. R. =271.0
DIA. =457.0 THK. =16.0

N	a	L
1.24	8.0	20.0
1.44	9.5	140.0
2.10	13.5	300.0
2.59	0.0	430.0
2.95	16.0	480.0

REF. =43.10 S. R. =279.0
DIA. =914.0 THK. =32.0

N	a	L
0.18	1.5	20.0
1.42	7.9	168.0
1.71	21.0	220.0
1.91	29.0	275.0
2.12	320.0	380.0

REF. =21.10 S. R. =171.0
DIA. =457.0 THK. =16.0

N	a	L
6.77	0.0	41.0
6.96	0.0	48.0
7.07	0.0	51.0
7.40	0.0	56.0
7.90	0.0	80.0
8.07	0.0	83.0
8.19	0.0	100.0
8.53	0.0	103.0
8.92	0.0	106.0
9.30	0.0	1.2
9.62	0.0	147.0
10.04	0.0	163.0
10.36	0.0	250.0
10.68	0.0	328.0

REF. =21.20 S. R. =263.0
DIA. =457.0 THK. =16.0

N	a	L
1.57	0.0	8.0
1.93	0.0	15.0
2.08	0.0	17.0
3.45	0.0	20.0
3.81	0.0	23.0
4.05	0.0	40.0
4.41	0.0	71.0
4.56	0.0	72.0
4.65	0.0	73.0
5.25	0.0	84.0
5.41	0.0	92.0
5.49	0.0	98.0
5.75	0.0	108.0
6.00	0.0	116.0
6.25	0.0	121.0

REF. =21.30 S. R. =131.0
DIA. =457.0 THK. =16.0

N	a	L
27.27	0.0	5.0
29.50	0.0	6.0
30.90	0.0	7.0
32.90	0.0	8.0
34.70	0.0	9.0
38.40	0.0	20.0
42.10	0.0	21.0
44.00	0.0	24.0
47.70	0.0	33.0
49.70	0.0	34.0
51.80	0.0	35.0
53.30	0.0	38.0
55.30	0.0	42.0
57.20	0.0	50.0
59.00	0.0	57.0
61.00	0.0	75.0
63.04	0.0	83.0
64.70	0.0	92.0
66.70	0.0	107.0
68.76	0.0	136.0
70.40	0.0	148.0
72.50	0.0	172.0
74.45	0.0	190.0
76.20	0.0	202.0

REF. =22.20 S. R. =175.0
DIA. =457.0 THK. =16.0

N	a	L
2.15	0.0	5.0
2.58	0.0	9.0
3.45	0.0	17.0
3.86	0.0	19.0
4.37	0.0	32.0
5.55	0.0	57.0
6.07	0.0	59.0
6.67	0.0	87.0
7.25	0.0	128.0
7.77	0.0	132.0
8.95	0.0	135.0

REF. =24.10 S. R. =180.0
DIA. =457.0 THK. =16.0

N	a	L
2.24	0.0	3.0
3.40	0.0	12.0
4.43	0.0	15.0
5.56	0.0	16.0
7.01	0.0	17.0
7.60	0.0	19.0
8.11	0.0	21.0
8.48	0.0	23.0
9.60	0.0	27.0
10.40	0.0	31.0
10.60	0.0	33.0
11.50	0.0	37.0
12.40	0.0	41.0
12.86	0.0	43.0
13.40	0.0	49.0
14.20	0.0	52.0
14.77	0.0	57.0
15.30	0.0	63.0
16.10	0.0	74.0
16.70	0.0	81.0
17.20	0.0	92.0
18.04	0.0	100.0
18.60	0.0	105.0
19.16	0.0	111.0

REF. =37.10 S. R. =271.0
DIA. =457.0 THK. =16.0

N	a	L
1.24	8.0	20.0
1.44	9.5	140.0
2.10	13.5	300.0
2.59	0.0	430.0
2.95	16.0	480.0

REF. =37.50 S. R. =190.0
DIA. =457.0 THK. =16.0

N	a	L
6.50	0.0	19.0
6.80	0.0	37.0
7.40	0.0	65.0
8.80	0.0	100.0
10.60	0.0	130.0

REF. =40.20 S. R. =265.0
DIA. =457.0 THK. =16.0

N	a	L
12.20	0.0	7.0
18.10	0.0	17.0
20.80	0.0	24.0
34.20	12.5	40.0
40.60	16.2	91.0