DAMAGE SURVIVABILITY OF PASSENGER SHIPS IN A SEAWAY

by

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A Thesis Submitted for the Degree of Doctor of Philosophy

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1996

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To the woman I love.

" O once I lov'd a bonie lass,
Ay, and I love her still!
And whilst that virtue warms my breast,
I'll love my handsome Nell. "

–Robert Burns

Summary

This thesis addresses the formulation and assessment of the damage survivability of passenger/RO-RO vessels from a fundamental point of view, whilst accounting for water ingress and taking fully into consideration vessel and flood water dynamics. Following the mathematical formulation, a numerical code was developed in the time domain capable of predicting the dynamic behaviour of the damaged vessel in a realistic environment. The developed model describes a free drifting vessel undergoing extreme motions in six degrees-of-freedom under the action of wind, current and waves of arbitrary direction whilst subjected to progressive flooding. The position and attitude of the vessel are updated continuously in time and consequently all the terms in the vessel/flood water systems. A semi-analytical method is also presented, for the evaluation of the flooding rate as a function of the relative position of the water level on either side of a damage opening.

This treatise begins with a thorough review of the available literature concerning models and methods proposed to date to assess damaged stability and survivability of passenger ships aiming to identify the strength and weaknesses of the existing theories and determine the key factors involved in the degradation of a vessel's ability to survive damage. Having validated the numerical code to the extent that confidence was gained of its ability to simulate the dynamic behaviour of the damaged vessel meaningfully and to predict her resistance to capsize with acceptable accuracy, a top-down approach was pursued, leading from a comprehensive model - including most of the critical features highlighted by the preliminary investigation - to a simplified one in which only the most relevant elements are retained if the significance of which had been demonstrated through a sensitivity analysis designed for this purpose, thus aiding in the transition of complex models to becoming useful engineering "tools".

The mathematical/numerical models described in this thesis represent the most advanced treatment to date of the ability of a damaged vessel to resist capsize in a seaway. The most surprising conclusion of the investigation presented herein is that the damage survivability of a passenger/RO-RO ships can be predicted with sufficient engineering accuracy with the simplest of models, deriving from the fact that at the final stages before capsize, the "fate" of the vessel is governed by quasi-static forces.

Acknowledgments

Many people I have to thank for having helped me in one way or another with my work throughout these three years. Among others, some have played an often crucial role and I would like their names to share the merits of the successful accomplishments of this research with me.

Prof. Dracos Vassalos, *deus ex machina* and supervisor of this investigation. To him goes all my deepest gratitude for his constant support, encouragement and friendship.

Dr. Osman Turan, whose ideas and concepts I merely further developed. I am especially indebted to him for his inestimable assistance during the development and validation of CASSANDRA.

Russell Smith, for his substantial, substantive and frequently inspiring aid during the elaboration of the mathematical and numerical theory.

David Clelland, Jim Gentles, Elaine Harmon, John McAllister and Bill McMillan of the IMPACT Project team, for having initiated me to the pains, pleasures and mysteries of computing.

Michele Francioni and Charlie Keay for their help in the collection and analysis of the experimental data.

Nigel, Lesley, Lisa and Julie - to whom this work is dedicated - for their love and support, because these years meant for me much more than what a post-graduate degree could ever signify.

I would also like to thank:

Vincenzo Armenio, Francesca Bondini, Tracey Brian, Tugrul Çömlekçi, Giorgio Contento, Elisa Fontana, Alberto Francescutto, Rama Gatiganti, Jan deKat, B. S. Lee, Nicky Massey, Gianluca Oppo, Francesco and Linda Papoff, Alpesh Patel, Angela Rawson, Martin Renilson, Andrew Tuite, Naoya Umeda, Ruth Wilson, Kate Wooding and all the crew of Ailish.

Finally, I want to express my boundless gratefulness and love to my family.

Nomenclature

1. General:

L_{PP}	:	Length of the vessel (between perpendiculars)
L_{OA}	:	Length of the vessel (over all)
B	:	Breadth of the vessel
T	:	Draught of the vessel (design, upright condition)
D	:	Depth of the vessel to the uppermost continuous deck
D_{bd}	:	Depth to bulkhead deck
D_{db}	:	Depth to double bottom
Δ	:	Displacement
∇	:	Underwater volume
A_w	:	Water plane area
C_B	:	Block coefficient
C_M	:	Midships coefficient
C_P	:	Prismatic coefficient
V_S		Ship service speed
KG	:	Height of the centre of gravity of the intact ship over the keel
KM	:	Height of the metacentre over the keel
GM	:	Vertical distance between the centre of gravity of the intact ship and the metacentre
GM_f	:	Flooded GM
CG	:	Centre of gravity
f, F	:	Freeboard

Hs	:	Significant wave height
T_0, T_z	:	Zero crossing period
ζ_A	:	Wave amplitude
k	:	Wave number
ω	:	Circular frequency
T_n T_N	:	Natural period of oscillation
φ	:	Roll angle
ρ	:	Density of sea water
g	:	Gravitational acceleration
$O\ XYZ$	:	Earth-fixed system of reference (this symbol at the bottom right of a vector signifies that the vector co-ordinates are given with respect to these axes)
$Gs\ x'y'z'$	:	Ship-fixed system of reference (this symbol at the bottom right of a vector signifies that the vector co-ordinates are given with respect to these axes)
$o\ xyz$	:	Ship geometry system of reference (this symbol at the bottom right of a vector signifies that the vector co-ordinates are given with respect to these axes)
$o_d\ x_d y_d z_d$	:	Forces system of reference (this symbol at the bottom right of a vector signifies that the vector co-ordinates are given with respect to these axes)
$O\ \hat{X}\hat{Y}Z$	:	System of reference employed to express Euler's angles

2. Equations of Motion:

Gs	:	Centre of gravity of the intact ship
m_s	:	Mass of the intact ship
$\vec{x}_{Gs}$	:	Co-ordinates of Gs in the inertial system of reference (if a prime

sign appears at the right-hand side of this symbol, its co-ordinates are given with respect to the ship-fixed co-ordinate system)

$\vec{v}_{Gs}$: Velocity of Gs in the inertial system of reference (if a prime sign appears at the right-hand side of this symbol, its co-ordinates are given with respect to the ship-fixed co-ordinate system)

Gw : Centre of gravity of the flood water

m_w : Mass of the flood water

$\vec{x}_{Gw}$: Co-ordinates of Gw in the inertial system of reference (if a prime sign appears at the right-hand side of this symbol, its co-ordinates are given with respect to the ship-fixed co-ordinate system)

$\vec{v}_{Gw}$: Velocity of Gw in the inertial system of reference (if a prime sign appears at the right-hand side of this symbol, its co-ordinates are given with respect to the ship-fixed co-ordinate system)

G : Centre of gravity of the damaged ship

m : Mass of the damaged ship

$\vec{x}_G$: Co-ordinates of G in the inertial system of reference (if a prime sign appears at the right-hand side of this symbol, its co-ordinates are given with respect to the ship-fixed co-ordinate system)

$\vec{v}_G$: Velocity of G in the inertial system of reference (if a prime sign appears at the right-hand side of this symbol, its co-ordinates are given with respect to the ship-fixed co-ordinate system)

$\vec{F}$: Force vector expressed in an inertial system of reference (if a prime sign appears at the right-hand side of this symbol, its co-ordinates are given with respect to the ship-fixed co-ordinate system)

$\vec{M}_o$: Vector of the force moments around the origin of an inertial system of reference (if a prime sign appears at the right-hand side of this symbol, its co-ordinates are given with respect to the ship-fixed co-ordinate system)

$\vec{x}$: Absolute position vector in an inertial system of reference (if a prime sign appears at the right-hand side of this symbol, its co-ordinates are given with respect to the ship-fixed co-ordinate

		system)
$\vec{r}$	:	Position vector relative to the origin of a moving system of reference (if a prime sign appears at the right-hand side of this symbol, its co-ordinates are given with respect to the ship-fixed co-ordinate system)
$\vec{v}$	:	Velocity vector in an inertial system of reference (if a prime sign appears at the right-hand side of this symbol, its co-ordinates are given with respect to the ship-fixed co-ordinate system)
V	:	Volume occupied by the mass system
ρ	:	Mass system density
ρ_s	:	Density of the intact ship
$\vec{\alpha}$	:	Rotation vector
$\vec{\Omega}$	:	Vector of the rate of rotation
φ, ϑ, ψ	:	Euler's angles
$[D]$	:	Transformation matrix between inertial and ship-fixed system of reference
$\vec{i}, \vec{j}, \vec{k}$	:	Unit vectors along the three axes of $Gs\,x'y'z'$
$\dot{q}'_i\ i = 1..6$	:	Co-ordinates of the generalised velocity vector of the vessel with respect to the ship-fixed system of reference
$I_{x'x'}, I_{x'y'}, \ldots$	:	Moments of inertia of the intact ship
$\vec{F}'^*$	:	Generalised external excitation vector
$\vec{F}'^*_w$	:	Generalised flood water excitation vector
$[M]$	:	Intact ship mass matrix
$[M_w]$	:	Flood water mass matrix
$[\dot{M}_w]$	:	Matrix of the rate of change of inertia

3. Analysis of Forces:

$\vec{F}'_E$	:	Generalised excitation vector
$\vec{F}'_H$	:	Generalised hydrodynamic reaction vector
$\vec{F}'_R$	:	Generalised restoring vector
$\vec{F}'_G$	:	Generalised gravitational vector
ζ	:	Irregular wave elevation
A_j	:	Wave amplitude components
ω_j	:	Circular frequency components
k_j	:	Wave number components
ε_j	:	Random phase components
$S(\omega)$	:	Wave spectrum
T_{rep}	:	Repeat period
$V_{19.5}$	:	Wind speed at 19.5 m above the still water level
ω_0	:	Peak frequency
α	:	Phillips constant
γ	:	Peakness parameter
τ	:	Shape parameter
Φ	:	Linear velocity potential
h	:	Water depth
ϕ_I	:	Incident wave potential
ϕ_D	:	Diffracted wave potential
ϕ_{Rk}	:	Radiated wave potentials
ξ_k	:	Motion response amplitudes
$\vec{F}_{Rad}$	:	Generalised radiated wave reaction vector
$a_{lk}(\omega)$	:	Added mass coefficients
$b_{lk}(\omega)$	:	Wave damping coefficients

$H(\omega)$	:	Frequency response function
$S_{F_k}(\omega)$	:	Wave force spectrum
$RAO^2_{F_k}(\omega)$	:	Wave force response amplitude operator
$\beta_k(\omega)$	:	Wave force phase angle operator
$\vec{F}'_{Wave}$	:	Generalised first order wave excitation vector
$\vec{F}'_{Drift}$	:	Generalised second order wave excitation vector
$\vec{F}'_{Wind}$	:	Generalised wind excitation vector
$\vec{F}'_{Current}$	:	Generalised current excitation vector
$[D^*]$	:	Transformation matrix between ship-fixed and force system of reference
$\hat{\varphi}, \hat{\vartheta}, \hat{\psi}, \hat{z}$	:	Moving average values of the instantaneous motion of the vessel in the system $O\ XYZ$
A_I	:	Incident wave amplitude
A_R	:	Reflected wave amplitude
$C_{wx_d} C_{wy_d} C_{wn_d}$	:	Second order wave excitation coefficients
α_w	:	Wave encounter angle
$C_{Wx_d} C_{Wy_d} C_{Wn_d}$	:	Wind excitation coefficients
V_W	:	Absolute wind velocity
V_{Wr}	:	Relative wind velocity
α_W	:	Wind direction off the bow
α_W^A	:	Absolute wind direction
α_W^R	:	Relative wind direction
$S_W(\omega)$	:	Wind spectrum
$\overline{V}_W$	:	Wind mean velocity
$C_{cx_d} C_{cy_d} C_{cn_d}$	:	Current excitation coefficients
V_c	:	Absolute current velocity
V_{cr}	:	Relative current velocity

α_c^A	:	Absolute current direction
α_c^R	:	Relative current direction
α_c	:	Current encounter angle
$U_k\ \dot{U}_k$	:	Linear and angular velocities and accelerations in the direction of the $o_d\ x_d y_d z_d$ axis
$[A]$	:	Added mass matrix expressed with respect to $o_d\ x_d y_d z_d$ (if a prime sign appears at the right-hand side of this symbol, its elements are expressed with respect to the ship-fixed co-ordinate system)
$[C]$	:	Radiating wave damping coefficients matrix expressed with respect to $o_d\ x_d y_d z_d$ (if a prime sign appears at the right-hand side of this symbol, its elements are expressed with respect to the ship-fixed co-ordinate system)
$h(t)$	:	Unit impulse response function
$X(\omega)$	:	Fourier transform of $x(t)$
$\delta(t)$	:	Dirac's delta function
$F_{jk}^{Rad}(t)$	:	Net hydrodynamic force (or moment) acting on the hull in the j^{th} mode, due to an arbitrary small motion of the ship in the k^{th} mode
ω_{MAX}	:	Hydrodynamic coefficients vanishing frequency
b_e	:	Equivalent linearised viscous roll damping coefficient
b_F	:	Viscous roll damping component due to friction
b_E	:	Viscous roll damping component due to eddies shedding
b_L	:	Viscous roll damping component due to lift
b_W	:	Viscous roll damping component due to wave making
b_{BK}	:	Viscous roll damping component due to bilge keels
N_r	:	Viscous yaw damping hydrodynamic derivative
$[B'_{Viscous}]$	:	Viscous damping matrix

4. Flooding:

Q	:	Volume flow rate
H, h	:	Vertical height over the opening
A	:	Area of the opening
C	:	Flooding coefficient
p	:	Hydrostatic pressure
v	:	Flow velocity
p_{atm}	:	Atmospheric pressure

5. Numerical Analysis:

$[A]$	:	Time varying generalised mass matrix
$[B]$	:	Time varying generalised damping matrix
$\vec{F}$	:	Generalised excitation vector
$\vec{q}$	:	Position of the origin of the ship-fixed system of reference with respect to the inertial one (a ^ sign indicates the same quantity averaged over fifteen seconds)
$[D]^R$	:	Euler's angles transformation matrix between the ship-fixed and the inertial systems of reference
$[R]$	:	Generalised transformation matrix between the ship-fixed and the inertial systems of reference
$\vec{p}$	:	Dummy variable representing the generalised velocity vector with respect to the ship-fixed system of reference
$[C],[E],[G]$	:	Dummy matrices
u_i, f_i	:	Exact solutions of a m^{th} order system of first order initial value

		problems and functions representing their first derivatives, respectively
w_i	:	Approximate solutions of a m^{th} order system of first order initial value problems (a ^ sign indicates the intermediate values of the same quantity)
α_j	:	Initial values
$RTOL_j$	:	Assumed tolerance vector; containing the value of relative tolerance limits for each unknown
$h\max$, $h\min$	:	Maximum and minimum time steps
h	:	Time step
$k_{i,j}$	:	Intermediate RKF approximation step functions
R_j	:	Error estimates
TOL_j	:	Actual tolerance vector
δ	:	Step size factor
K'_{jk}, $\dot{K}'_{jk}$	:	Kernel functions and their time derivatives
Δt and Δw_j	:	Any of the increments involved in the evaluation of $k_{i,j}$
$conv_{jk}$	:	Approximate value of the convolution integrals
$conv^*_{jk}$	:	Approximate value of the convolution integrals up to the last evaluated solution
T_d	:	Convolution integrals tail

Contents

1. Introduction

General Remarks

The experience gained from the losses of ferries like the *European Gateway* and the *Herald of Free Enterprise* put transient motion first in the agenda of ship survivability researchers. The rapid capsize and sinking of the *Estonia* did not come as a surprise. Bearing in mind that the primary objective of damage stability criteria for passenger vessels is to ensure that there is sufficient time to evacuate passengers and crew, the need to model correctly the events immediately succeeding an accident becomes indisputable.

In the sequence of events leading a passenger ship to capsize, numerous questions can be raised on the importance of issues usually regarded as of second order. From the sloshing of the flood water in the vehicle deck to the vessel's motion in the horizontal plane, there is a large variety of matters that are usually neglected in the models proposed so far to predict passenger vessels survivability. Perhaps with the exception of the effect of forward thrust from the propellers and that of the impulsive excitation caused by the impact of a piercing body with the vessel - topics that have been excluded from this research - the arguments presented in this study respond to the need of investigating problems related with the highly non-linear transient phenomena following a collision. In this light, this project finds justification as part of the extensive venture undertaken by the Department of Ship and Marine Technology of the University of Strathclyde to investigate damaged ship stability and survivability, and ideally follows the elaboration of previous mathematical models and numerical tools to explore this research field.

The first assignment to attend to, in the quest for an effective prediction of the behaviour of a ship during a flooding transient, is the identification of those features that could have some weight on the evolution of the phenomena under examination and have been disregarded or over-simplified in other models. To overcome this initial obstacle, it is customary to scan all the available literature published in recent years on the subject, keeping a critical eye open and bearing in mind the physics of the problem. The outcome of such a process is a clearer picture of what are possible requirements of approaching the problem from a fundamental point of view. A list of subjects that deserve an often compelling attention can thus be made, of which only some will eventually enter the circle of topics worth including in the final model. Naturally, the methodology of selection adopted to reduce the initially proposed model to a final refined one, requires a clear, effective and stringent structure.

One of the possible routes to accomplish this task is adopting a so called top-down approach. The research presented in this thesis was conducted in the spirit of this conception the essential properties of which are briefly explained next.

Damage Survivability

The problem of assessing the ability of a vessel to survive damage has been neglected for a long time by the research community as a result of the very small interest demonstrated in the past by most of the parties involved in the design and operation of merchant vessels, as well as by the pertinent regulatory bodies. This often underestimated issue periodically became the centre of attention of the maritime community only when dramatic tragedies resulting in the loss of many lives shook the dormant public opinion of the western world. Ironically, the development of craft designed for the transport of passengers has historically been oriented so far to the optimisation of almost every feature regarding a ship's performance - speed, capacity, manoeuvrability, comfort - other than her safety. This trend, which was accidentally broken for the first time with the introduction of multihulls, led to the almost universal adoption of the RO-RO monohull type as the most profitable solution to the short sea shipping needs of commercial transport of passengers and cars.

The roll on roll off ship was derived from naval vessels used for the transport and quick landing of tanks and troops during military operations. These vessels were designed to offer sufficient stability for the purpose they were intended, having acknowledged that a certain amount of risk could be accepted in such a context. Successively, thanks to the huge economical benefits following the introduction of this craft for use by the merchant navy, the criticisms about its inherent weaknesses were hushed away until the obvious stability problems linked to the presence of large uninterrupted vehicle decks were virtually forgotten. Even the repeated accidents involving shifting of cargo were not deemed a sufficiently alarming signal to justify the introduction of a proper regulation regarding the design and operation of those vessels that were meant for the transport of passengers, until the first inevitable tragedy had its course.

Fundamentally, the two greatest dangers linked with the presence of a large open space inside a seagoing vessel - characteristic feature of a RO-RO ship - are either the great increase in the damaged displacement resulting in sinkage, or the large loss of stability resulting in capsize following progressive flooding due to damage. To solve this problem, such spaces were located where the sea water would not be able to reach them, at least in a perfectly calm sea; vehicle decks were thus to be placed above the mean water line. This kind of thinking apparently lead to a suitable solution. It is in fact well known that the sea has the unfortunate habit to be very rarely calm and that most of the accidents during which a ship would be damaged often occur in considerably rough seas. In these conditions, a damage opening would let water on the vehicle deck even for moderate freeboards, thanks to the ship motion and the presence of waves. Once water has even only partially flooded the vehicle deck, the possible outcomes are basically three: if the ship is inherently very stable, the vehicle deck is well above the waterline and the sea conditions are not extreme, the water would flow out again causing no harm; if the ship is uncommonly stable but the elevation of the vehicle deck over the water is such that the flooding will continue, the vessel will progressively ship water until virtually sunk; finally, if none of the above conditions is met, the ship will loose her dynamic stability and capsize during the flooding transient. This last scenario has been observed in most of the major disaster involving RO-RO passenger vessels.

It is important to underline how the most likely way to loose a RO-RO vessel entails transient dynamics. Among physicists these two words are synonyms of unpredictability once referred to a highly non-linear system such as a damaged ship at sea. For this kind of systems, the influence of even small changes in some of the parameters playing a part in determining the sequence of events taking place during a transient, can be the cause of dramatic alterations of the final result. A typical example is the effect of slight changes in the position of the centre of gravity of the intact ship on the probability of the ship heeling on either side after the first few seconds of flooding. At this stage, it is probably unnecessary to state that the proper assessment of every possible influence of the many parameters concerning the flooding transient of a damaged Ro-Ro vessel is not just accessory but a basic necessity. This is even more importantly so, if the interest of research would focus on the evaluation of ephemeral quantities like the time to capsize.

Following these considerations, the problem of passenger vessels survivability was approached in this study trying to explore the effect of as many possible parameters as possible, according to a well structured methodology.

Research Philosophy

" Everything should be made as simple
as possible but no simpler "

–Albert Einstein

The general philosophy of this research cannot be described better than the way these words of Albert Einstein's do. In fact, if a physical system is known to be particularly sensitive to even the smallest change of the various parameters at stake, it is of vital importance to simulate its behaviour as accurately as possible. At the same time, the limits of this accuracy are imposed by the means available to seek it. In the case of numerical models these limits are embodied by an increasingly longer time to evaluate solutions as the mathematical model complexity intensifies.

Unmistakably, the only two ways to go around this rock are either employing more sophisticated tools, or simplifying the theoretical model to the bare essentials - or both. At this point, the objection of an attentive scientist would be that since the first assumption was that the physical system under examination is highly sensitive to any change of external and internal conditions, the choice of simplifying the mathematical model cannot be endured without risking a complete failure to achieve the primary purpose. Almost simultaneously, the attentive engineer would shiver at the thought of expensive days of computation on an extremely expensive super-computer, hunting for an ethereal second order ghost. As usual, the right is in the middle although possession has to be attained passing through both extremes.

It is a well known fact that when the factors affecting an event increase to a number greater than one, their interdependence becomes itself a tricky business to sort out. It might in fact happen that given some initial conditions, the importance of some effects becomes totally unimportant. In such cases, expending energies trying to accurately assess the influence of these parameters, is nothing but a tedious exercise. In other circumstances, the same effects can instead be of primary importance in determining the final result and should therefore be treated with extreme respect. A typical example of such a situation can be recognised in the consequences of water sloshing on a ship stability depending on her size. Much in the same way, the variation of the added mass and damping coefficients with a vessel's attitude can be appreciated especially if its underwater geometry changes drastically as the heeling angle and displacement increase. In other words, this phenomenon is particularly important for multihulls as opposite to conventional ferries.

A light at the end of the tunnel starts finally to become more and more visible to both scientist and engineer. In fact, if a tool is available to ascertain the relative significance of some features of the simulation model, given certain initial conditions, a specific model configuration can then be selected accordingly, optimising accuracy and time expenditure. Probably, the simplest approach to the solution of this problem is that in which a coarse preliminary investigation with a *full* model is performed to examine the sensitivity of the particular system in question to all possible factors, after which all features deemed unimportant are excluded and the examination is restored with the remaining, on an intensive/extensive basis. In this idea resides the essence of a top-down approach.

An ideal tool to dwell in the application of this method would possibly be able to provide the researcher with the means to switch flexibly from one model to the next, so that no significant effort should be expended to adapt to every possible situation. The main intent of this project is therefore to develop a mathematical and numerical model to simulate the dynamic behaviour of a damaged ship freely drifting in a realistic environment, accounting for as many pertinent factors the state of the art would allow. This instrument would permit the examination of the qualitative changes in the results obtained following exclusions and/or combinations of the various parameters, to end up with a manageable but still significant method of assessment of dangerous characteristics of a variety of passenger vessels.

Structure of the Thesis

The thesis can be subdivided in three main parts. In Chapter 3 a report of the findings of a thorough literature review is presented. A short synopsis of some of the papers examined can be found in Appendix E. Chapters 5 to 7 embrace the development of the theory behind the numerical model produced, including the elaboration of suitable equations of motion which correctly incorporate the mass variation of flood water, and the generation of a semi-analytical model to simulate the flooding mechanism of compartments open to the sea. In Chapter 6, the forces operating on the system are analysed and introduced in the mathematical formulation. This is then translated into an appropriate numerical scheme the details of which are given in Chapter 8.

Finally in Chapter 9 a validation and sensitivity analysis are performed to demonstrate the application of the investigation method suggested. A thorough discussion of the results obtained leads to a few suggestions on the direction that future research on the subject should take and to draw some key conclusions. Most of the material concerning either the user guidance to the use of the software produced or the details and results of the experimental and numerical testing performed, are described in the appendices.

2. Aims

The principal aim of this research work is to develop comprehensive mathematical and numerical models of a damaged vessel subjected to progressive flooding in a realistic environment, that accounts correctly for the vessel/flood water dynamics and incorporates recent advances in nonlinear dynamics as well as experience gained in damaged stability and survivability research. The specific tasks of this project can be outlined as follows:

- to perform an extensive critical review of previous research in order to highlight virtues and limitations of the various models used to date to deal with the issue of damaged passenger vessels survivability;

- to formulate a rigorous mathematical model of the dynamic behaviour of the damaged vessel/flood water system from a fundamental point of view;

- to develop a numerical code to accommodate all the innovative features of the theoretical model whilst offering a flexible instrument of analysis and application to practice;

- to undertake a parametric investigation aimed at validating the devised model and at establishing the effect and relative importance of each key feature included in it with the view to offering recommendations concerning its transition to a practical engineering "tool".

3. Review of Recent R&D Work

General Remarks

In this chapter a critical review of the relevant research and development work in the subject of damage stability and survivability of passenger/RO-RO vessels is presented, spanning approximately the last fifteen years, aiming to elucidate the state of the art by meeting the following specific objectives:

- Identify the capabilities and limitations in dealing with the subject, pertaining to:

  - mathematical modelling and model testing of the behaviour of damaged ships at sea;
  - modelling of water ingress through the ship;
  - computational tools to assess water accumulation and sloshing;
  - proposed design concepts and devices for enhancing resistance to capsize;
  - modelling of cargo shift;
  - damage collision statistics.

- Identify ideas and proposals addressing the development of damage stability criteria and recommendations regarding future directions in research and development work on the stability and safety of passenger/RO-RO vessels.

- Select an appropriate number of parameters influencing the dynamic behaviour of a damaged vessel in need of attention, on the ground of which a general approach to the problem could be given shape.

Introduction

The R&D review presented herewith, is not meant to be exhaustive of all the work addressing the damage stability and survivability of RO-RO vessels. This would have been an enormous task. The last fifteen years have seen this subject becoming progressively a focus of research, development and application on an international scale, particularly in the wake of recent well publicised disasters. As a result, virtually hundreds of publications have found their way in international journals, proceedings of conferences, meetings and workshops, a clear indication of the strife in the shipping community to provide solutions to the RO-RO survivability problem. Having said this, many areas of interest, have been the centre of attention for many years earlier, such as water sloshing in tanks.

The main purpose of this review instead, is to ensure that research trends, results and limitations are well represented by the large choice of reports contemplated. Generally, one readily forms the impression that not all topics have been given the same amount of attention by the scientific and engineering community. For instance, there have been many proposed solutions to the problem of RO-RO survivability in terms of new design types and concepts but not much effort has been expended toward the development of a theoretical model for the water ingress to support and verify the various attempts to simulate numerically the ship dynamic behaviour when progressive flooding takes place. All recommendations rely on the results of a few, valuable experimental results, but there is but a few studies focused at understanding the problem of the shifting of discrete cargo. At a first glance, it would appear that there has been a rush toward providing solutions for apparently simpler issues, at the expense of all others. A closer examination reveals that this may not be the case. Of the aspects addressed in the following, few were considered to be of relevance to RO-RO survivability. One good example concerns the water ingress problem. Even though procedures to approximate this have been used by some, none was well reported, documented and discussed. Therefore, when Strathclyde published the first results on the effect of progressive flooding on the damage survivability of RO-RO vessels, the importance of developing suitable methods to evaluate the rate of water ingress in a damaged compartment of a RO-RO vessel became apparent. As a result, this particular aspect of the problem is currently being explored in considerable detail.

The format adopted in reporting the findings of the R&D review undertaken comprises a brief review on the development of residual stability standards, followed by a critical survey of the most important publications available in the public domain by addressing the main aspects of damage stability and survivability of passenger/RO-RO vessels whilst summarising the key trends and findings. This is followed by a final section in which conclusions are drawn and the opportune basis for the formulation of the general approach to the problem in question are set. Together with the customary reference numbers, an E followed by a number is used in this chapter to indicate abstracts of the main publications reviewed, a complete list of which can be found in Appendix E.

Brief Review of Residual Stability Standards

In general, it is true to state that the ship damage stability problem has not received much research attention in the past mainly because a meaningful treatment of it, particularly one involving progressive flooding in a random seaway, was perceived to be too difficult an undertaking. However, the question of survivability standards of a ship following damage was first considered towards the end of the nineteenth century by several Select Committees of the House of Commons in the UK. In 1890, the first Committee on watertight subdivision recommended that passenger vessels over 425 ft (129.5m) in length should be capable of floating with any two adjacent compartments open to the sea. It is interesting to mention that the 1890 Committee also recommended one compartment standard for cargo ships but this was not implemented due to lack of support. Following the loss of *Titanic* in 1912, the second Bulkhead Committee was set up to consider in depth the question of damage survivability. A year later, the first International Convention for the Safety of Life at Sea took place in London. This Convention laid down an empirical system for deciding on the appropriate bulkhead spacing, consistent with the number of compartments which could be flooded without submerging the so-called "margin line", located at 3" (76 mm) below the bulkhead deck. Again, it is interesting to note that the 1890 Committee used a percentage of depth rather than 3" as the margin line. The number of adjacent compartments considered to be flooded (between one and three) was based on an empirical relationship involving ship length and type. The work, interrupted by the Fist World War, was finally completed at the 1929 SOLAS Convention which also laid down the first details on the extent of damage.

For the first time, there was a quantifiable standard for ships aimed to ensure that realistic damage could be sustained without the prospect of immediate progressive flooding and eventual loss of the ship. However, there were no specific requirements for damage stability criteria. The 1932 Supplement to the 1928 "Instructions as to the Survey of Passenger Steamships" went one stage further by imposing restrictions on the amount of heel permissible after flooding and following counter flooding ($\leq 7°$). It is interesting to note again that the effect on stability of flooding in way of the bulkhead deck was also to be calculated and considered on its merits. The extent of damage and damage stability criteria on the final condition after damage and equalisation was detailed at the 1948 SOLAS Convention introducing also the requirement for a positive residual GM. The UK Department of Transport formulated for the first time construction rules for passenger ships in 1952 in order to implement the 1948 Convention. The first specific criterion on residual stability standards was introduced at the 1960 SOLAS Convention with the requirement for a minimum residual GM (0.05m). This represented an attempt to introduce a margin to compensate for the upsetting environmental forces. "Additionally, in cases where the Administration considered the range of stability in the damaged condition to be doubtful, it could request further investigation to their satisfaction". Although this was a very vague statement, it was the first attempt to legislate on the range of stability in the damaged condition. It is interesting to mention that a new regulation on "Watertight Integrity above the Margin Line" was also introduced reflecting the general desire to do all that was reasonably practical to ensure survival after severe collision damage by taking all necessary measures to limit the entry and spread of water above the bulkhead deck.

At about the same time as the 1974 SOLAS Convention was introduced, the International Maritime Organisation (IMO), published the much discussed Resolution A265 (VIII). These regulations used a probabilistic approach to assessing damage location and extent drawing upon statistical data to derive estimates for the likelihood of particular damage cases. The Equivalent Regulations raised new damage stability criteria addressing equilibrium as well as recommending a minimum GM of 0.05m to ensure sufficient residual stability during intermediate stages of flooding. The 1980 Passenger Ship Construction Regulations introduced requirements on the range of the residual stability curve as well as on the stability of the vessel at intermediate stages of flooding.

The loss of the *Herald of Free Enterprise* in 1987 drew particular attention to RO-RO ferries in which the absence of watertight subdivision above the bulkhead deck is a particular feature. The implications of this feature were highlighted by the Court of Inquiry which observed that the SOLAS Conventions and UK Passenger Ship Construction rules had been aimed primarily at conventional passenger ships in which there is normally a degree of subdivision above the bulkhead deck, albeit of unspecified ability to impede the spread of flood water. In response to this, the Department of Transport issued Consultative Document No 3 in 1987 which outlined a level of residual stability that required all existing ro-ro ferries to demonstrate compliance with the 1984 Passenger Ship Construction Regulations. This standard had previously formed the basis of a submission by a number of Governments to IMO which considered the question of passenger ship stability in some detail. This was the fore-runner to SOLAS '90. The much deliberated SOLAS '90 introduced new higher standards of residual stability, to be applied to all passenger ships including RO-RO vessels and, for the first time, cargo ships, requiring the following in the final condition after damage:

- a minimum range of 15 degrees beyond the angle of equilibrium which should not exceed 12 degrees for two-compartment flooding and 7 degrees for one compartment flooding;
- a minimum area of 0.015 m rad. under the residual GZ curve;
- a minimum residual GM of 0.05m with a maximum GZ of at least 0.10m, increased as necessary to meet certain stipulated heeling moments due to wind heeling, passenger crowding and lifeboat launching.

The need to evaluate the adequacy of the various standards in terms of providing sufficient residual stability to allow enough time for the orderly evacuation of passengers and crew in realistic sea states has prompted the Department of Transport to set up the RO-RO Research Programme comprising two phases. Phase I addressed the residual stability of existing vessels and the key reasons behind capsizes. To this end, theoretical studies were undertaken into the practical benefits and penalties of introducing a number of devices for improving the residual stability of existing RO-RO vessels including Flare, Structural Sponsons and Buoyancy Bags. Retractable transverse barriers aiming to restrict the floodwater on the vehicle deck were also considered. In addition, model experiments have been undertaken by the British Maritime Technology Ltd and the Danish Maritime Institute, in order to gain an insight into the dynamic behaviour of a damaged vessel in realistic environmental conditions and of the progression of flood water through the ship. Phase II was set up with the following objectives in mind:

- to confirm the findings of Phase I in respect of the ability of a damaged vessel to resist capsize in a given sea state;
- to carry out damaged model tests, in which the enhancing devices assessed in Phase I would be modelled to determine the improvements in survivability achieved in realistic sea-going conditions;
- to confirm that damage in the amidships region is likely to lead to the most onerous situation in respect of the probability of capsize;

- to undertake theoretical studies into the nature of the capsize phenomenon, with a view to extrapolating the model test results to RO-RO passenger ships of different sizes and proportions.

In view of the above, it is worth noting that even though the mandatory damage stability criteria not improved substantially since their implementation in 1929, passenger ship design has altered substantially in terms of passenger capacity, interior arrangements regarding the location of sleeping cabins and entertainment areas, higher superstructures as well as smaller machinery spaces containing progressively more powerful engines. None of these changes in ship design is reflected in current damage stability criteria. This is an additional strong reason supporting the claimed inadequacy of the existing criteria to provide an acceptable safety standard for modern passenger ships. The intermediate effect of recent positive steps forward, for example SOLAS '9O, on existing ships is already being experienced. For example, a considerable number of ships which failed to satisfy the amended regulations have had to be modified structurally to provide extra buoyancy and residual stability. These compulsory modifications have paved the way for new research aimed at identifying optimum solutions to enhancing RO-RO safety commensurate with economic and operational needs.

Critical Survey of Technical Papers

1. Model Tests

A damage stability safety standard relates to the ability of a ship, measured by means of a representative characteristic property or properties, to resist capsize when damaged in a seaway. Invariably, this requires the establishment of relationships between ship design and environmental parameters and limiting stability parameters to be used as a basis for the development of survival criteria. Establishing such relationships, however, is a difficult matter as capsizing of a ship is a highly non-linear problem depending on a plethora of parameters. Since it is unlikely that appropriate damage stability characteristics can be derived on the basis of full-scale data alone [2.17 E60],[2.4 E70] (accidents or full-scale experiments), there remain two principal methods for establishing the aforementioned relationships, namely: model experiments and theoretical studies. In the strenuous international effort towards the comprehension of the mechanism of capsize of damaged RO-RO vessels, great attention has been paid to model experiments. This is a perfectly understandable and thoroughly proper choice as in this field, theory which is not backed up by experimental evidence is not normally valued.

Pioneering damage stability experiments relevant to the above were carried out in the early 1970s, [2.1 E77]. From these experiments it can be concluded that capsizing is critically dependent on parameters used to define environmental conditions. The model used was a typical passenger/vehicle ferry and experiments were carried out for different sea states, loading conditions and freeboards. In each case where capsizing occurred, the primary cause was considered to be the accumulation of water on the main deck due to the spillage of water and roll motion. It was also found from these experiments that wave height is a very significant factor affecting capsizing. These findings suggest that significant increases in initial ship stability are required in order to resist the tendency to capsize as freeboard decreases and wave height increases.

Apart from validation purposes, the reasonably extensive experimental programmes undertaken during the last ten years - primarily the result of the strong wish of the UK Department of Transport to give the problem of RO-RO survivability a workable solution [2.28 E58], [2.29 E59] - have provided important clues to the researchers. From the first set of experiments carried out at BMT Ltd. [2.29 E40], the following results are worth highlighting:

- during the intact tests with fixed amounts of water on deck, the ship did not capsize;

- there is a clear dependence of the resistance to capsize of the vessel on wave conditions, flooded freeboard and GM;
- the time for capsize to occur depends on the above parameters, but it always revealed to be less than 30 minutes (the whole run time was about 1 hour in real scale);
- a sufficient quantity of water on the vehicle deck is needed for capsize to take place;
- the vessel behaviour with the opening facing the oncoming waves is widely different from that with the damaged side in the opposite direction;
- the mechanism of water ingress is crucial to determine the vessel dynamics;
- static stability calculations for a flooded vessel may give a misleading impression of its dynamic behaviour in relation to capsize.

A comparison of these results with those obtained by DMI, [2.24 E48], on a different ship model showed an encouraging agreement. Another innovation with respect to the study by Bird and Browne was the introduction of non-dimensional plots on the GM_f-H_S plane, aiming to disclose operational areas in which the model is likely to capsize. Using these parameters in a non-dimensional form would accommodate the influence of freeboard and also enable generalisation of the results to different ship dimensions. As indicated in the foregoing, experimental effort in the second Phase of the UK RO-RO Research focused on investigating the efficiency of a number of devices proposed to reduce the danger of capsize. The BMT Ltd. model experiments, [2.50 E6], revealed that a centre casing (barrier) increases the chance of capsizing as water cannot cross the deck and remains on the side where the damage opening is. On the other hand, side casings decrease the chances of capsizing, even though to a limited extent. Experiments also prove that heeling towards the damage side almost guarantees capsizing. A smaller damage opening has virtually no effect on the capsize trend when the damage faces the waves but a big improvement is observed when the damage breach is away from the waves. The author suggests that waves coming towards the ship create a *pump action* at the damage opening as they hit the ship.

Although results on the effect of different permeability in the engine room are quite limited, they are somewhat difficult to explain. With a damage freeboard of 0.25 metres, the ship could survive an H_S/F ratio of 21, i.e. a significant wave height of 5.25 m. For the same GM, but with 1.0 metres freeboard, the ship survives only a ratio of 3. These results appear to contradict other experimental and theoretical results and the commonly accepted finding that the ship chances of survival increase as the damaged freeboard increases. Along a similar line, the author suggests that putting dummy vehicles on the car deck to alter its permeability would eliminate the danger of capsize at 0.25 metres freeboard, while at 1.0 metres freeboard this will not be the case for values of H_S/F greater than 3. In contrast, however, DMI experimental results, [2.52 E8], in which vehicles were present on the deck, show only a slight improvement in the vessel behaviour when compared to those in which no vehicles were used. Clearly, the effect of permeability - as well as the discrepancies on the effects and the interaction between an increasing freeboard and a changing permeability - are in need of further clarification. Experimental evidence suggests that flares and structural sponsons could improve damage survivability only marginally while air bags offer significant improvements, rendering the ship almost incapsizable. However, adding air bags also increases the GM significantly, causing high accelerations, which could create cargo shifting and discomfort or injuries to the passengers. Additionally, damage to inflatable bags during collision will have to be considered as it could cause dangerous or excessive heeling.

DMI testing provided results which are essentially similar to those just summarised. In particular, they show that a centre casing is a dangerous design feature in terms of ship survivability as it prevents water on deck going to the undamaged side, therefore creating asymmetrical flooding which leads to capsizing of the ship. Full height transverse bulkhead doors proved to be the most effective devices regarding the ship's survivability. However, although transverse half height bulkhead doors are very effective at low wave heights, they become ineffective or even worsen the chance of survivability at high waves as water flows over the bulkheads and becomes trapped in the contiguous compartments. It is interesting also to mention that damage openings smaller than the standard one, surprisingly worsen the stability of the damaged ship and increase the required damaged GM considerably, compared to ships with a normal damage opening. This is due to the flood water not being able to flow out easily, which leads to a substantial increase of the amount of water trapped on deck. When the B/5 side tanks are introduced instead of a centre casing, the required GM decreased considerably. When transverse bulkhead doors are added to the side tanks, the ship becomes almost incapsizable even in the worst sea states. The flares and side structural sponsons made hardly any difference on the ship survivability while side air bags improved the chance of survival drastically. Finally, additional tests proved that mid-damage is more dangerous than forward damage.

Notwithstanding the great importance of the experience gained by employing experimental techniques, what must be reinforced here is the notion that the role of experimental testing in research is not contradictory to that of theoretical tools, but complementary. A natural consequence of this is that more experimental work will be needed and welcome in the future, as an invaluable means to acquire the necessary physical understanding. However, theoretical investigations on the dynamic stability of damaged passenger vessels are not to be overlooked. Currently, there is a lack of accurate models endeavouring in this direction. It is obvious that a theoretical study is long overdue, as theoretical investigations are more flexible and much cheaper, and can provide a considerable amount of information for a limited expenditure of effort. Developments along this line are considered in the following section.

2. Mathematical Modelling of a Damaged Ship Motion

One of the fields in which people were most creative, when considering ways to assess the survivability chances of a RO-RO vessel following damage at sea, is the sphere of the mathematical models to be used in numerical time simulation studies. With the exception of very few cases in which frequency analysis was employed [4.16 E55], [3.6 E56] or those where a totally empirical formulation was preferred [2.51 E7], all researchers trying to recreate the dynamic response of a damaged hull, approached this task by having a digital computer integrating all different kinds of non-linear equations of motion. Only a limited number, however, of the proposed models, succeeded in producing results that could be proven to match those obtained by authoritative experimental testing.

A representative work of the empirical formulation family is that undertaken by DNV Technica which is summarised next. The equation resulting from this study is defined in the form of a functional relation between a non-dimensional limiting GM and parameters like wave frequency (ω), wave height (H) and slope, ship freeboard (F), etc. as follows:

$$GM_N = L_N \left(a_1 H_N + a_2 H_N^2 + a_3 H_N^3 + \ldots\right)$$

a_i's are coefficients which have to be determined from experiments, L_N is the non-dimensional vehicle deck length and H_N is the non-dimensional wave height defined by the expression

$$H_N = \left(\omega^2 \, HB/4gF\right)$$

The proposed formulation is straight forward offering a simple criterion as well as useful suggestions concerning the choice of key influencing factors to be used in formulating survival criteria. However, such formulation is derived from a very simple theoretical background and needs coefficients determined from better sources than those available to the author at the time. One of the problems, therefore, is how to derive the required coefficients accurately and meaningfully as more advanced theoretical research is required to achieve this purpose. Finding out the exact relations between the parameters which are important for ship survivability is equally paramount. In fact, as it is widely accepted that ship dynamics plays a very important role, ignoring the effect of ship motions and water accumulation on deck will probably lead to wrong conclusions, especially considering that there is no exact knowledge of what the effect of these phenomena is. These omissions lead to some discrepancies in the results shown in this work. For instance, experimental results (Figure 3 in the paper) show that the worst case occurs at 0.076 metres freeboard, however using the non-dimensional form presented by the author this proves to be, in fact, the best case (Figure 4). The author suggests that this empirical form is not sensitive to the low freeboard values or it is "very sensitive to the uncertainties". In this respect, the lack of information on some significant areas suggests caution in adopting this approach as it stands or indeed other empirical approaches.

Existing and current experimental and theoretical studies are at the stage of shedding more light on the capsizing phenomena. This target can be attained reasonably quickly following concerted effort on the subject so that more quantitative findings can be obtained.

A second family of models which will not be discussed in detail here is that encompassing the so called static and quasi-static approaches. All early attempts to investigate the intermediate stages of flooding belong to this group. The philosophy behind these methods is elucidated by the analysis of the work of the author already mentioned above: John Spouge. In 1986 he proposed a theory to explain the capsize of the "European Gateway", giving it the name of *Transient Asymmetric Flooding*, [2.4 E70]. This idea was tested in a time simulation in which the static equilibrium of the ship - as a result of the reciprocal elimination of the forces and moments due to the shipped water and those arising from the hull restoring ability - is supposedly attained at each time step, and the water ingress is modelled by "established methods of hydraulics". External excitation forces were neglected and the parameters ruling the motion of the centre of gravity of the flood water were tuned according to the documented evidence of the accident. Other authors following a similar route are Sen [3.5 E68] and Dand, [2.17 E60].

Subsequently, once full dynamics had been introduced in the ship motion model, recourse to this concept was not necessary any longer to explain the transient roll induced by rapid water ingress. The coupling between ship motions and water flooding rate is now correctly understood and adopted, [3.15 E11], [3.10 E41]. Leaving aside studies in which the frequency domain approach strongly limited the complexity of the model employed, in most of the fully dynamic time domain simulations attempted, the emphasis was put on different aspects of the complex behaviour of this ensuing dynamic system. One of the most evident parameters distinguishing the multitude of formulations suggested, is the number of degrees of freedom deemed to be important. In this respect, five major classes can be identified:

- single degree of freedom (roll motion), [3.10 E41];
- two degrees of freedom (roll and sway), [4.3 E75];
- motion in the transverse plane (sway, heave and roll), [4.25 E9], [3.15 E11], [3.11 E27], [3.8 E52];
- antisymmetric motions (sway, roll and yaw), [4.16 E55];
- complete set of equations (six-degrees-of-freedom), [3.18 E1], [3.9 E53], [3.7 E57], [3.2 E73].

The reasons put forward to justify one assumption or the other, are various and, to a great extent, quite arbitrary. No well structured analysis has yet proved that the coupling between any two modes of motion can be disregarded.

Besides the problem of limiting the number of equations to be solved, a second main distinction must be made between those scientists employing finite differences or Markers and Cells methods to include water sloshing on deck, [4.26 E10], [3.9 E53], [3.7 E57], [4.3 E75], and those who prefer to exclude this effect, [3.2 E73], [3.11 E27], [3.10 E41], [3.15 E11] or treat it with empirical and semi-empirical models, [4.1 E78], [4.6 E71], [4.25 E9], [3.18 E1]. Leaving the discussion on the decision to make on this matter to the section dedicated to it, one general, practical implication will be briefly discussed here. Considering the resources put to use to achieve the target of a speedy and accurate simulation, the various elements influencing the process must be given the space they deserve but no more than that. Space discretisation techniques in the realm of computer integration require a massive number of operations to be computed at each time step and this cannot be justified if the effect they reproduce is not of primary importance.

The manifold non-linear effects included in the various mathematical models adopted, encompass the following:

- non-linear exciting and restoring forces;
- water ingress and time varying inertia terms;
- non-linear viscous damping and hydrodynamic coefficients as a function of frequency and vessel attitude;
- non-linear terms arising from the adoption of a system of reference which moves with the body;

Of the papers reviewed, a selection of the most interesting must include the works by Dillingham [4.3 E75], Lee [4.26 E10], [3.2 E73], Petey [3.9 E53], Turan [3.11 E27], Vassalos and Turan, [2.36 E31] and Rakhmanin [4.25 E9]. Dillingham tackled the problem of simulating the dynamic behaviour of a fishing vessel with an open deck by including shallow water sloshing using Glimm's method. The two-degrees-of-freedom motion model employed includes features like impulse response functions and non-linear viscous roll damping. The amount of water flooding the deck is varied according to the relative position of the bulwark and the freeing ports with respect to wave and the flood water level. It is not clear if the beam wave used is regular or irregular, although some of the results refer to constant wave frequency to report that a steady state could not be reached. In spite of the fact that such a model is considered to be accurate, the derived results are quite disappointing. The effect of water sloshing is reasonably well reproduced and in agreement with some experimental results but that created by the time variation of the mass of water is not terribly well reported. The importance of a sound model of the water ingress is again underlined here but the energy based explanation of the roll enhancing power of water flowing in and out of the deck is not very well illustrated. Surprisingly enough, the size of the scuppers and the presence of flaps on them is deemed unimportant, while a slight camber of the deck produces a notable reduction in roll. No capsize is reported, whatsoever.

Lee concentrates on the influence of the variation of exciting forces and hydrodynamic coefficients with the vessel attitude finding that larger motion and asymmetrical response with the ship heading should be expected, whilst Petey underlines the importance of parametric excitation in quartering and following seas. The latter work also takes into account shallow water sloshing and flood water mass variation, although no information is given either on the transient flooding response calculated or on the theory behind the model used.

Turan has developed a two-dimensional model including controlled water flooding and regular wave excitation, in which the sloshing is disregarded with the water level assumed to be parallel to the mean free surface. The ship restoring ability is simulated by integration over the underwater volume at each time step and viscous damping is calculated through Ikeda's method [10.7]. This model was later developed to accommodate a more realistic water ingress model, adopted from hydraulics, and irregular waves by Vassalos and Turan. The results of this latter model have been adequately validated with the experimental evidence deriving from the investigations carried out during the UK RO-RO Research Programme.

Recently, Rakhmanin proposed a new set of equations that seem to offer good potential. This new formulation for the problem of determining the motion of a damaged vessel includes fluid sloshing and mass variation through an infinite number of linear differential equations to be added to a modified version of the canonical equations of motion with three-degrees-of-freedom. This is given next in order to demonstrate some of its interesting features.

$$(M+A_{22})\,\ddot{\xi}_2+B_{22}\,\dot{\xi}_2+A_{24}\,\ddot{\xi}_4+B_{24}\,\dot{\xi}_4-\gamma\sum_{n=1}^{\infty}\beta_n\,\frac{\omega^2}{g}\,q_n(t)=F_2^c\cos(\omega\,t)+F_2^s\sin(\omega\,t)$$

$$(M+A_{33})\,\ddot{\xi}_3+B_{33}\,\dot{\xi}_3+C_{33}\,\xi_3=F_3^c\cos(\omega\,t)+F_3^s\sin(\omega\,t)$$

$$A_{42}\,\ddot{\xi}_2+B_{42}\,\dot{\xi}_2+(I_x+A_{44})\,\ddot{\xi}_4+C_{44}\,\xi_4-\gamma\,S_0\,\eta_g\,q_{OF}(t)+\gamma\sum_{n=1}^{\infty}\beta_n\left(1+\frac{\delta_n}{\beta_n}\cdot\frac{\omega^2}{g}\right)q_n(t)=$$
$$=F_4^c\cos(\omega\,t)+F_4^s\sin(\omega\,t)$$

Apart from the absence of the roll damping term - which could be a misprint in the paper - these equations are interesting for the explicit inclusion of terms expressing the mass variation of the flood water ($\gamma\,S_0\,\eta_g\,q_{OF}(t)$) due to water inflow and outflow through the damage opening as a result of ship motions and waves. Note that this term is only retained in the roll equation. Its form appears to be that of an oscillator tuned to the mass variation, thus acting as an additional excitation term, rather than an implicit damper. The infinite sums express the effect of water sloshing. However, this model still awaits validation and a full understanding of *it* is often made difficult by the lack of clarity in the exposition.

None of the approaches utilised so far has taken into consideration all of the above listed effects. In particular, one point consistently missing in almost all the approaches, concerns the time derivatives of the varying mass of the flood water. This improper exclusion must be underlined as a possible gross approximation. Overlooking this aspect of the problem could perhaps be blamed on the assumption that the canonical equations of motion - derived from Newton's second law with the system mass considered to be a time constant - are still holding for the peculiar dynamic system embodied by a damaged ship undergoing progressive flooding.

3. Water Ingress

There is not much work done towards the development of a sound model for water ingress and this is not really surprising, even leaving aside the relatively recent need to focus in this direction. One of the major drawbacks of such a phenomenon is that it is not properly defined, nor easy to outline. This fact naturally contribute to make the eventual attempts to emulate this type of events drown in a sea of uncertainties. What is really meant by modelling water ingress? One could say this consists of estimating the amount of water flooding a space internal to a ship - or one capable to hold water on board (for instance, the working decks of fishing vessels) - and its time history, which is probably a good enough interpretation. However, the point in question is slightly different. The trouble concerning the modelling of water ingress derives from the wide range of manifold scenarios that can lead to it. In other words, how does water flood these spaces? Is it a sensible enough choice to restrict the attention to side damages only, especially since most of the recent RO-RO capsizes were not caused by side collision? What is the influence of wave dynamics with respect to the rate of flooding, and what is the influence of ship velocity and damage position?

In the research work at the University of Strathclyde limits were chosen arbitrarily. Of course, the effect of forward speed on the rate of water inflow was not considered and thus attention was focused on a fairly circumscribed - but yet very complicated - set of possible scenarios. The variable parameters drafting the boundaries of this assortment of cases are: side damage openings of variable geometrical shape (rectangular or trapezoidal), size and location, wave parameters and ship motions and, ultimately, the internal and external geometry on the vessel. As already anticipated, the results of the preliminary examination of the past research on the subject brought to surface nothing or little more. All numerical simulations of damaged ship motions, allegedly adopted "established methods of hydraulics", i.e. engineering formulae derived for the measurement of stationary flow characteristics. The limits of this approach in the case of a strongly time-dependent phenomenon - like the water ingress through a breach on the side of a free floating body - are self-evident. Although this theory cannot be applied to calculate the rate of flooding of a ship as it is, it can be adapted to the task if adequate changes are made.

Maybe, the only attempt in this direction alternative to that of the University of Strathclyde, was that by Glosten Associates, inc. [11.8]. Although recognition must be awarded to the authors for having tried to develop a generalised formula for modelling the problem of water accumulation, there are a number of limitations reducing the practical importance of their work. In fact, if the aim of their work was to establish or simulate the effect of water accumulating on a vessel deck, this has been done by excluding all possible effects due to ship motions. Water accumulation on a vessel deck has been already observed and described in other contexts, where full dynamic properties of the vessel were considered. Moreover, the statistical properties of water accumulation on the deck are not terribly important when the primary objective is to assess the dynamic behaviour of a vessel. In this case the actual time variation of the amount of water plays a much more significant role and thus the need for a proper model of water ingress becomes clear.

4. Water Sloshing

The literature on water sloshing in tanks is infinitely large and varied. In this review however, greater attention has been awarded to those publications directly related to the interaction between ship motion and free surface liquids, in order to limit the quantity of papers to be examined. Generally, the problems connected to water sloshing in a tank are treated by numerical integration in the time domain of the differential equations describing the fluid behaviour (Laplace, [4.26 E10], [4.23 E29], [4.17 E50], [4.3 E75] or Navier-Stokes, [4.22 E28]), with appropriate boundary conditions on the tank walls and at the water free surface. These conditions are themselves functions of time, as the tank is oscillating according to prescribed or excited motions. These two methods are fundamentally different, since the first approach assumes the motion of the tank as known and independent from the sloshing water loads, [4.22 E28], [4.17 E50], while in the second the tank behaviour is calculated according to the external excitation as well as that due to the water inside it, in a time stepping sequence, [4.26 E10], [4.23 E29], [4.3 E75]. Although this methodology is quite elegant, considering its inherent adherence to the physical reality of the phenomenon studied, its shortcomings are not irrelevant and make it unattractive from the point of time constraints. One of the main reasons why this fully hydrodynamic approach has to be dismissed lies on the enormous consumption of computer time required. The simulation of the dynamic behaviour of a vessel with only one compartment flooded, assuming the liquid is irrational and a reasonably dense grid, can take as long as a few days to estimate on a large microcomputer. This is clearly unacceptable, especially bearing in mind that often more than one compartment should be taken into consideration. A second consideration concerns the time variation of the mass of the water flooding a damaged compartment of a RO-RO vessel. It has been demonstrated by different authors that the study of transient flooding is crucial when trying to assess the survivability of this type of vessels, thus the time history of the amount of water in each damaged compartment cannot be overlooked. At present, the established methods to calculate the dynamic behaviour of liquids with a free surface in enclosed spaces cannot handle mass variation adequately, [4.3 E75].

Nevertheless, even if sloshing cannot be treated efficiently using CFD techniques in the context of time simulation of the transient motion of damaged passenger ferries, the dynamic effect of water free to move on the vehicle deck or in a damaged compartment, has to be taken into consideration. From experimental tests and accident records, sloshing does not seem to play a cardinal role in determining the response of a damaged RO-RO vessel, although some interaction between ship motion and water dynamics can be postulated. This is probably because of the typical dimensions of this family of vessels - the effect of water sloshing is expected to be different in the case of, for example, fishing vessels. It has been shown theoretically and experimentally, that the damping introduced by the presence of a free surface can be considerable. Evidence of this can be found in the use of anti-rolling tanks which have been devised to stabilise vessels especially prone to large roll motion. The theory regarding the design of such devices is somewhat simpler than that developed to model sloshing, and can be adapted to suite the need to introduce the roll damping effect due to the motion of a liquid on board, [4.16 E55], [4.6 E71], [4.1 E78]. Existing data, however, are mostly focused on particular cases, [4.15 E54] and in need of considerable work aimed at consolidating the considerable body of results and in structuring the output according to relevant variables.

5. New Design Concepts

During the years elapsed since the capsize of the "European Gateway", the proposals for new design concepts have mushroomed. In the following some of these, ranging from attempts to reduce the permeability of wing tanks, to introducing "lifebelts" and perforated decks, are reported, analysed and commented upon. One is led to believe that more than half of the people concerned with RO-RO survivability have worked like gerbils at the wheel to find a solution to the problem, even before having properly defined it. Probably this is an inevitable effect of the pressure stemming from the public anxiety and concern toward safety of life at sea. Most of the proposed solutions reflect the classical concept of subdividing the ship's internal spaces, in a way that buoyancy and stability are still preserved after some of the resulting compartments are flooded, [2.7 E67], [2.20 E64], [2.19 E63]. The approaches to achieve this result - and optimise it to compromise with the commercial need to keep the RO-RO concept alive - vary, ranging from designs based on probabilistic assumptions [2.41 E22], to more practically oriented solutions which could also be applied to existing vessels, [2.33 E37].

Some authors detach from this general philosophy, to explore some more innovative ideas. Of these, it is worth mentioning those contemplating means of removing the water flooding the vehicle decks, by dumping it overboard. A specific example is given by passive and controllable freeing ports, [2.22 E47]. Another theoretically promising device seems to be the automatic inflatable side sponsons ring, [2.21 E45]. A blow of fresh air comes finally from some proposals to abdicate the monohull concept for the inherently safer multi-hulls, [2.6 E66].

Aside from considerations deriving from the fact that the efficacy of most of these solutions is evaluated with methods based on static stability calculations, [2.21 E45], [2.22 E47], attention is focused here on which characteristics are required from the ideal ferry survivability enhancing system, [2.42 E20], [2.6 E66]. This fairly hard task is in fact considered at least useful, to the proper development of new design concepts and absolutely essential for the evaluation of recent proposals. From the papers published on the subject, the main attributes of the perfect device appear to be:

- effectiveness in improving the ship <u>dynamic</u> response to situations in which damage deteriorates the initial buoyancy and stability characteristics of the vessel without altering the original proportions of these attributes excessively;
- feasibility of application to existing ships;
- least possible impingement of the original safety from other hazards;

- economic viability, i.e. low initial costs and preservation of the present commercial value of this vessel type;
- simplicity and independence from the human action.

Clearly, the order of the above listed qualities does not reflect their importance. Such a marvel is probably not attainable, but some of these points are undeniably paramount and cannot be renounced. It is left to the intelligence of the responsible parts to compromise these occasionally conflicting requirements. Of the main devices proposed so far, the following summarises virtues and defects:

- <u>retractable transverse subdivisions;</u>
  their effectiveness depend on their number and position, although they seem to offer great benefits to the damage stability when properly placed. The main disadvantage concerns the loss of revenue due to relatively modest payload reduction and considerable delay in the turnaround time;
- <u>longitudinal subdivisions below the bulkhead deck;</u>
  this internal arrangement does not offer a substantial improvement in the stability characteristics of the vessel;
- <u>buoyant wings above the bulkhead deck;</u>
  a great increase in the damage stability can be achieved by this implementation but accessibility of these spaces for the storage of cargo can be a major problem when water- tightness and turnaround time are borne in mind;
- <u>buoyant "lifebelt" with perforated decks;</u>
  although restricted to new designs only, the "lifebelt" concept offers a way to drastically improve RO-RO survivability. In the further development needed for this idea to become a feasible alternative, other type of hazards should ultimately have be considered, starting with fire confinement problems arising from the adoption of perforated decks;
- <u>freeing ports</u>.

scuppers were only evaluated qualitatively and this concept needs to be further developed.

6. Shifting of Cargo

Most of the published literature on cargo shifting concentrates on the safety factors concerning the lash loads that the securing devices have to sustain during transportation. Even if this aspect of the problem can be of some interest, the fact that most of short freight ferries do not secure their cargo at all, makes the modelling of cargo shifting somewhat complicated. Some methodologies have been proposed to calculate the forces acting on discrete cargo as a result of the ship attitude and acceleration, [12.1 E72]. Nevertheless, not much is known about the final result of exceeding the limits beyond which cars and lorries would move. In simple terms, nothing is known concerning either the impulsive loads exerted by the shifting cargo to the ship or the final probable arrangement that the crushed vehicles (and their centre of gravity) would assume, [3.16 E13].

On the basis of the above, it would be possible to offer recommendations to researchers concerning which direction to follow. However, this should be done bearing in mind that although ship accelerations can be calculated precisely, the distribution of the cargo on a ferry is usually random. This leads to the requirement of a probabilistic approach to provide the values of the limits and figures just mentioned.

7. Damage Collision Statistics

IMO introduced the concept of probability in the world of ship survivability many years ago and research in this direction has been persistent and coherent to date. The same cannot be said, however, about the relative legislation which has encountered many difficulties in gaining the understanding and trust of many in the marine business. Perhaps, it is due to this unpopularity that papers on damage statistics are quite rare to find and often vague. Of the various publications available on this issue, the paper by Abicht [12.4] has been chosen to describe the actual state-of-art. This is because it presents one of the most uncomplicated and yet formal methods for evaluating damage statistics. Moreover, it offers one of the most recent revisions of the probabilistic theory of collision damage and its assumptions, which is readily available to the public. The formulae proposed by this author are of great value for their simplicity and adherence to the damage statistical data. It is worth summarising them here briefly. The joint probability density of the damage location and extent ratios is approximated with a plane within appropriate limits:

$$p_x(\xi, \eta) = \frac{30}{11}(\xi - 16\eta + 3)$$

where $\xi = x/L$ and $\eta = y/L$; x is the distance of the forward end of the damage opening from the aft end of the ship and y is the longitudinal length of the damage. L is the ship length. The following limits apply: $0 \le \xi \le 1$ and $0 \le \eta \le 0.25$. Of course $\eta \le \xi$. An expression for the probability density of damage penetration is given as a function of the damage extent ratio as follows

$$p_y(\eta, \tau) = (1.5\,\tau)^{20\eta} \cdot \exp[20\,\eta\,(1 - 1.5\,\tau)]$$

A limit to the maximum value of the penetration ratio is set to 2/3. On the basis of the above described formulae the probability of flooding of any kind of compartment (wing or central) is worked out in a very elegant and formally correct way. The explanation of how this is done is very clearly exposed and will not be reported here. In this section some work on collision resistance has been accounted for, e.g. [12.9 E4], [12.6 E17], [12.5 E39], [12.3 E44], although this topic is not directly related to damage statistics. The reason for such inclusion has to be traced to the logical deduction that the extent of a damage on a ship side depends on the side structure. The study reported in the paper by Sen, Cocks and Pawlowski, is an appreciable attempt to tie two connected aspects of collision damage together, namely collision dynamics and structural response mechanics. The next rational step would be to generalise the procedure for the ultimate purpose of checking and legitimating the actual damage statistics, using data on the average structural characteristics of each vessel type. In simple words, this would provide an answer to the questions of whether it is sensible to assume that all commercial ships will suffer the same collision damage irrespective of their specific structural configuration and of the need for a better way to evaluate the mean values of the damage size on a representative ro-ro vessel instead of using data which are generally valid for different ship types.

Finally, a general comment is made regarding the lack of consistent treatment of the results deriving from risk analysis studies and the lack of a formal method of assessing the survival risk of a damaged vessel. One of the ways to appreciate this truth is by contemplating the following example. From the investigation carried out by Lloyds Register of Shipping on behalf of the UK Department of Transport, [12.2 E43] - in which risk analysis is assessed in terms of hazards (frequency of an incident) and consequences (number of fatalities, extent of damage, pollution etc.) - evidence is provided that:

- the danger of heavy loss of life in ferries is larger than in any other means of transport;
- fire hazard presents a comparable risk of loss of life to that of capsize and sinking, on a world-wide basis, but this is not strictly true for ships crossing the Channel.

Incidentally, the fire accident frequency on the car deck appears to be the lowest when compared to all other areas within a passenger ro-ro ferry. This should be kept in mind when asserting the fire hazard implications of retractable transverse bulkheads to enhance stability, [2.42 E20]. A different point of view is expressed in [12.1 E62], where an interesting result is given in terms of individual passenger risk. While people travelling by car face one chance in ten thousand to die, those who use a ferry seem to have much better odds: one in two million. Looking at the problem of ro-ro safety in this way could distort the perspective quite substantially.

Conclusions

From an attentive analysis of the above, a preliminary set of topic can be extracted which would form the basis on which a comprehensive model to investigate the dynamic behaviour of a damaged vessel at sea can be built. The mathematical models developed so far have tackled different aspects of this problem, each highlighting a particular issue which could have some weight on the overall outcome of each specific damage case. None have so far attempted to assign the influence of the interaction of all these parameters nor assessed the relative importance of each of them. Probably, the best way to address the task of setting appropriate basis for a new model sweeping the largest possible range of correlated effects is to start from an existing model which has been proved suitable to provide meaningful answers to some of the questions of interest, and successively add all those effects originally left out of it. In the case of damaged ship motion, this approach is only partly befitting the purpose, since radical changes seem to be opportune in the mathematical formulation of the equations of motion. Having acknowledged this, the model developed by Turan still ideally represents a good starting point to proceed with the development of a more complete simulation tool.

Of the effects omitted in this model a few emerge for their potential significance in the context of such a non-linear phenomenon as transient flooding. A shortlist of these should certainly include:

- 3D equations of motions to enable motion coupling, wave variable headings and vessel drifting;
- second-order wave forces;
- wind forces, including random gusts;

- current forces with variable stream velocity and direction;

- non-linear restoring forces;

- inertial water sloshing forces;

- hydrodynamic coefficients variable with the vessel attitude;

- a sound model, attained by means of experimental and theoretical investigations, of the water inflow and outflow as a function of wave particulars and damage location and extent.

The mathematical model would also have to accommodate convolution integrals to allow for significant variation of the potential damping terms with frequency.

Finally, it should be stated that the choice of leaving forward speed and impact simulation out of the above list, as well as the effects of cargo shifting is absolutely arbitrary. However, aiming for a complete model that includes forward speed and all the maneuverability related non-linear terms (in addition to other problems like including the effect of side velocity and temporary partial obstruction of the damage opening by the colliding object in the water ingress mechanism) would mean budgeting for a much longer time to develop a feasible model. Obviously this does not imply that these effects are not important but only that they are left to coming researchers to contemplate.

4. Adopted Approach

General Remarks

The general developments delineated in the previous chapter and the priority areas identified through the critical review, reasonably indicate a way forward to achieving the principal aim of this research. Undertaking the task of developing the theory necessary to formulate the problem in rigorous mathematical terms and thus translate it into a flexible numerical "tool" requires a clear plan of action and distinct, well defined stages in accomplishing it. In this chapter an overview of the general approach followed is given, aiming to clarify the methodology adopted during each phase of this study and provide both a framework and justification for the structure of the following chapters.

Once the problem of describing the behaviour of a damaged vessel at sea had been accurately defined and the strength and weaknesses of the previous attempts to solve it had been recognised, it became clear that an approach was needed which, starting from fundamentals, would produce a model encompassing all of the most relevant features which influence the phenomenon studied. To achieve this, the basis of a theoretical formulation had to be sought in the revision of the equations of motions used to describe the vessel/flood water system. This first and important step led, as a necessary consequence, to the analysis of the external excitation forces applied to the system and the examination of the flooding mechanism. Having devised appropriate methods to accommodate these features in the model, the emphasis finally shifted to the question of forging a numerical scheme befitting the needs of the practical application of the developed theory.

As the final aim of this research is to enhance the insight on the effect and relative importance of the many parameters determining the behaviour of a damaged ship at sea and to offer suggestions concerning the most suitable way to accomplish the development of a practical engineering "tool" to study her ability to survive, a parametric investigation followed the validation of the devised model, to allow the fulfillment of these purposes along the guidelines of the top-down philosophy explained in the introduction. The results from this last phase led to the formulation of suitable conclusions and inevitably to recommendations for future research.

Theoretical Formulation

Equations of Motion

One of the most surprising findings of the analysis of the recent research work on the subject of damaged ship stability and survivability is that no trace can be found of equations of motions derived systematically for such a dynamic system. To simulate correctly the motion of a mass varying system, it is in fact necessary to account for additional terms deriving from the product of the non-zero rate of variation of the mass and the velocity of the moving system. These terms are instead neglected in most of the equations of motions used so far, which seem to be derived from orthodox equations devised for a constant mass system: the intact ship. The first essential step in the direction of the development of a sound mathematical theory to investigate the stability of damaged ships is thus to formulate new equations that would correctly take into account the presence of the flooding water in the damaged compartments.

If the water flooding the damaged compartments of a ship is considered as a part of the dynamic system excited by those external forces (wave, wind etc.) that would normally be considered exciting the vessel alone, all the effects due to the presence of such water will automatically be taken into account without the need to consider any additional external term. Of the effects that can be simulated this way, it is worth listing at least one other than the forces due to the rate of variation of the system's mass: the inertial water sloshing forces. These forces would derive from the acceleration of the flooding water in the compartments and have to be recognised as distinct and diverse entities from the gravitational excitation deriving from the relative motion of the flooding water.

Since new equations have to be drawn from basic principles it is advisable to aim at obtaining a generalised set which could successively be simplified if this were deemed necessary. In addition, the equations of motion need to address all six degrees-of-freedom, thus incorporating the effect of planar motions on the damaged vessel whilst incorporating second order effects such as wave drift forces as well as environmental excitation from wind and currents that augments vessel drifting, particularly in beam direction. These last aspects of the problem appears to be particularly important with respect to the wave heading which will certainly affect the evolution of the flooding phenomenon and ultimately the survival of the vessel.

Forces and Moments

Once a suitable set of equations of motion had been obtained, an accurate analysis of the external forces acting on the dynamic system in consideration as well as the hydrodynamic and hydrostatic reaction forces is needed describing as accurately as possible the forcing terms deriving from the environment and/or the system itself. Unlike the equation describing the behaviour of a damaged vessel, the external forces have been given considerable attention by various researchers in the past. As a result, the introduction of wind, current, second-order wave forces and viscous effects can be accomplished with sufficient accuracy for studying the dynamic behaviour of a damaged ship. Considering first order wave forces and hydrodynamic reaction forces, on the other hand, necessitates the use of an innovative procedure that would allow for the large variation of the underwater geometry of the vessel during the flooding process. Furthermore, since the response of the damaged vessel cannot be expected to be at one well defined frequency (only linear systems would respond in such fashion to a sinusoidal excitation) an additional allowance must be granted to take care of the variation of the wave radiation damping with frequency.

Again, a well structured theory is already available to include multi-frequency response in the determination of the effects of wave radiation. The best method to allow for an accurate evaluation of the first order wave forces and the hydrodynamic reaction as functions of the vessel's attitude appears to be the adoption of a database of predetermined values among which the correct quantities could be chosen or interpolated. This procedure would in fact allow for a fast and easy-to-check technique which will not encroach upon the accuracy of the overall results. This is justifiable on the basis of the relatively slower variation of the ship's heel, trim and sinkage as opposed to her instantaneous motions.

Finally, and most importantly, a particular emphasis has to be put on the importance of a stringent modelling of buoyancy and gravitational forces. In fact, although the other external forces are important in determining the onset of flooding and its initial stages, it is mainly buoyancy and gravity that will rule the process evolution during the latter phases, until the vessel reaches a final condition. As far as the buoyancy forces are concerned, there is not a universally accepted method of evaluation since some researchers extend the integration of the hydrostatic pressure to the instantaneous wave elevation and others prefer to limit it to the mean water level to be consistent with the evaluation of the wave radiation and diffraction forces. In addition, additional questions can be raised on which particular wave profile to be adopted if the first assumption is deemed appropriate. The gravitational forces offer less reason for doubts, but are strictly linked to the particular method chosen to simulate the behaviour of the flood water inside the ship. In this respect, it is worth noting that unlike the case of an intact ship, gravity plays here a fundamental role in the active excitation of the vessel in roll.

Flooding Mechanism

Probably, one of the most important innovations introduced by the research presented here is the creation of the first model to simulate the flooding mechanism using a semi-empirical approach. In all the past investigations, attempts to link the flooding of internal compartments to the relative position of wave and damage opening have been very limited. Most of these have been relying on models derived for civil engineering hydraulics to evaluate the inflow and outflow of water but none has demonstrated the validity of such an approach in case of fully dynamic circumstances. The methodology proposed here follows similar lines but proceeds to formulate specific equations for the rate of flooding through the damage opening, and also to provide the first flooding coefficients expressly measured experimentally for use with the derived equations.

The correct modelling of the inflow/outflow of water through the damage opening is naturally of primary importance for a reliable simulation of the progress of events during the flooding process. Quantities like the time to capsize depend in fact critically on the evolution of this phenomenon. On the basis of the assumption that a simplified technique can be acceptably accurate if supported by experimental evidence, the formulation attempted here does not diverge drastically from those employed by other researchers. However, it attempts to offer credibility to the adopted approach by providing a solid theoretical and experimental background. Since the overall aim of the thesis is not to ascertain all the features of the flooding phenomenon and propose a generalised model to reproduce it, the investigation on this subject will not examine every detail of the problem but focus instead on its fundamental nature. The task of fine tuning the coefficients proposed is left for future research.

Numerical Analysis

The equations derived to describe the dynamic system under examination contain several features which make integration a delicate matter to undertake. Apart from the presence of convolution integrals, the varying nature of the hydrodynamic coefficients and the non-linear character of most of the excitation forces introduced, demand an innovative numerical scheme to be devised specifically for the purpose. To complicate the situation even further, the dependence of the water inflow and outflow on the ship's motion, makes the system prone to numerical instabilities and rules out global methods that are generally applied to deal with Volterra's equations. It is, therefore, important to build on strong foundations and be aware of the vital role of accuracy to forge a numerical "tool" that will assure reliable results without compromising on execution speed.

Ideally, the starting block for the construction of such an integration instrument must be a highly trustworthy and stable scheme such as one of the Runge-Kutta methods. These methods not only guarantee numerical stability but also allow for a reasonably easy introduction of the convolution terms and the flooding mechanism. If a variable time-step is also introduced, the likelihood of a reasonable computation speed is increased as well.

Parametric Investigation

Validation

Once the mathematical and numerical models had been finalised, it will be necessary to attempt to verify the correctness of the predictions obtained through it by comparing the results of the numerical simulations with published experimental results. Having taken the model developed by Vassalos and Turan [2.36] as an ideal starting point, it is appropriate to utilise the same set of tests used there as a means of comparison for the validation of the new methodology. The code developed will therefore be tested by using a typical RO-RO ferry, a typical internal arrangement and the most probable damage scenario. The purpose of the numerical test will be to verify that the capability of the new program to predict capsize within the ascertained boundaries is acceptable and that the most important quantities evaluated will match those measured during the experiments. An additional goal will be to assess the relative performance of the new top-down approach compared with the old bottom-up approach.

Sensitivity Analysis

To demonstrate the applicability of the method proposed and its potential at being employed to establish the influence of each feature included in it, their reciprocity and relative importance, it is necessary to examine a particular case and draw appropriate conclusions. To minimise the number of runs required, the same ship, damaged conditions and general arrangement used for the validation of the code will be applied here, while the value of the parameters of interest will be varied around their typical values to appraise their effect on the overall outcome of the simulations. Since the number of innovations introduced in the new model is substantial, only a coarse grid of cases will be analysed to limit the time necessary to undertake this first sensitivity analysis, but extra runs can be planned if necessary to investigate the influence of those parameters that the dynamic system will demonstrate to be highly receptive to their changes.

5. Formulation of Equations of Motion for a Damaged Ship with Progressive Flooding

General Remarks

The dynamic behaviour of damaged vessels has been assessed, so far, by solving the canonical equations used in the study of wave induced ship motions. In doing this, the effect of the rate of variation of the mass of flood water on the system performance was taken into account only by updating the total inertia of the ship with that of the water accumulating in each damaged compartment as the time elapsed. If the effect of transient flooding has to be modelled accurately these equations of motion cannot be applied any longer since they were derived from Newton's equations assuming constant mass. In this chapter equations of motion will be derived for a damaged ship subjected to flooding by considering the ship and flood water as one dynamic system. These equations are developed in six degrees of freedom, to be used as a basis for developing a time simulation program.

Problem Formulation

It is worth stressing from the beginning that the dynamic effect of the flood water on the system response is assumed here to be only due to the relative motion of the centre of gravity of the water (Gw) with respect to the centre of gravity of the intact ship (Gs) and that this relative motion is postulated to be known. This is in full agreement with the experimental and numerical results available at the moment, since the only forces deemed unimportant, according to this hypothesis, are those due to the dynamic pressure of the sloshing water on the compartment walls. Of course, such a premise is defensible only if sloshing is totally disregarded or, better, treated in a semi-empirical way. Given the nature of the problem, this is not a major deficiency, though.

In fact, if the sloshing effect can be seen, as far as it concerns ship motions, as the consequence of the difference in phase and amplitude of the motion of Gw and that of Gs, this difference will be small for large vessels when the quantity of water is sufficiently large to have an appreciable effect on the system behaviour. This is because the natural frequency of the sloshing water is directly proportional to the tank beam as well as to the inverse of the square root of the water depth; $T_N = 2\pi \Big/ \left(\frac{g\pi}{b}\tanh\left(\frac{\pi h}{b}\right)\right)^{1/2}$ [1.8]. Clearly, for large ships the beam will be large and the vessel natural frequency in roll will be low, thus it will be unlikely for the flood water to be excited in resonance conditions. Indeed, when the water volume is sufficiently large to alter the ship behaviour, small differences are expected in the ship and water motions phases. This consideration implies that the water level inside the tank remains parallel to the horizon (like when one slowly rotates a glass full of water).

An alternative way to include the eventual phase/amplitude difference between the motion of Gw and that of Gs, is by experimental data. If the kinematic quantities describing the behaviour of Gw can be expressed as function of the ship motions (or even as function of the excitation forces characteristics), employing experimentally derived coefficients, an empirical formula can then be developed and thus used in conjunction with the following relationships to complete and integrate them. This method is similar to the one adopted by some authors to simulate the effect of anti-roll tanks [4.5]. Given the lack of available data in this direction, though, the previous methodology will be complied with in the next chapters, even if research is taking place to broaden our knowledge in this field and its results could be used in future within this theoretical approach.

If the dynamic behaviour of the water in the compartment (sloshing dynamic pressure) is found to be considerable and can not be neglected, these equations cannot be efficiently used any longer and the system must be treated as two separate worlds interacting. The variation of the water inertia during transient flooding must then be taken into consideration in the derivation of the equations describing the fluid behaviour and these must be employed in place of Laplace's or Navier-Stokes' in the CFD routine calculating sloshing. This, of course, goes well beyond the simplified approach adopted here.

All hydrodynamic coefficients are traditionally referred to in a system of reference which exploits the ship's symmetries i.e. to a system of axis fixed to the ship and normally positioned as in fig. 1, where Gs is the centre of gravity of the intact ship. Adherence to this system in developing equations of motion is essential in order to be able to use published data and adopt the usual definition of hydrodynamic coefficients.

In dealing with the damaged ship problem, though, a system of axis as specified above is only partly a good choice. It is indeed easy to observe that every symmetry of the underwater body will be lost as soon as the ship undergoes large heel in response to the flooding of her compartments. It is thus clear that the *traditional* definition of added mass and damping can not be sensibly applied here any longer. Even in this case the system of reference illustrated below still retains some of its attractiveness.

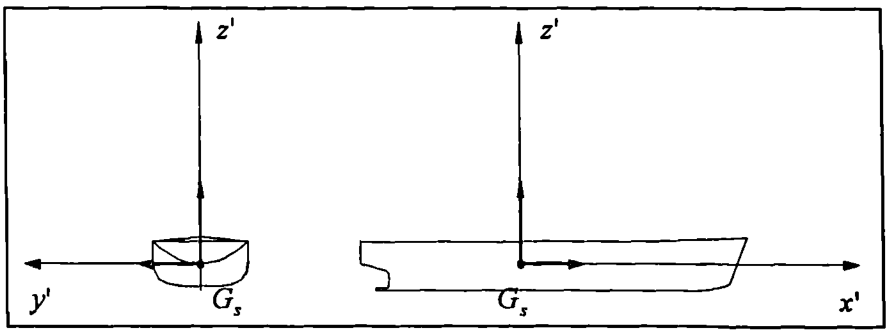

Fig. 1

Traditional system of reference for the formulation of equations of motion.

The choice of the particular system of reference to use for developing a set of equations describing the motion of a body is quite arbitrary. Systems of axis other than this could be chosen but it was felt that the selected one was more apt to the structure of the numerical program these equations were written for and therefore easier to use. Secondly, even if symmetries are generally lost from the hydrodynamics point of view, they still survive when it comes to consider the mass distribution of the intact ship. This means that benefits of some simplification of the equations will take place in the end by adopting this set of axis.

A further and last point is that, once the terms involving the mass of flood water are neglected, the remaining equations could easily be cross-checked with those derived by other authors. All this made the choice of a conventional ship-fixed, Gs centred system of reference preferable to others.

Before starting with the actual derivation of the equations, it must be noted that such system is not inertial, and that the centre of gravity of the system (ship + water), G, will not generally coincide with Gs (indeed G is function of time, for its position varies in response to the quantity of shipped water). This implies:

1. taking into account the accelerations induced by a moving system of reference, i.e. introducing extra terms which would not appear in an inertial system;

2. developing the equations with respect to a co-ordinate system which will not be parallel to the principal axis of inertia of the dynamic system (ship + water), nor centred in its real CG. Consequently additional terms will be arising for this reason. On the other hand, the influence of the relative position of Gw on the ship motion will be better stressed this way and therefore more readily dealt with;

3. having to include those terms which are functions of the first derivative of the mass in applying Newton's second law since the inertia terms of the system will be changing with time. This does not really depend on the specific system of reference chosen, but it is a point worth highlighting here considering that yet more terms will appear in the equations of motion due to this reason.

Against this background, a fresh start will be made here considering a body moving in space as illustrated in fig. 2:

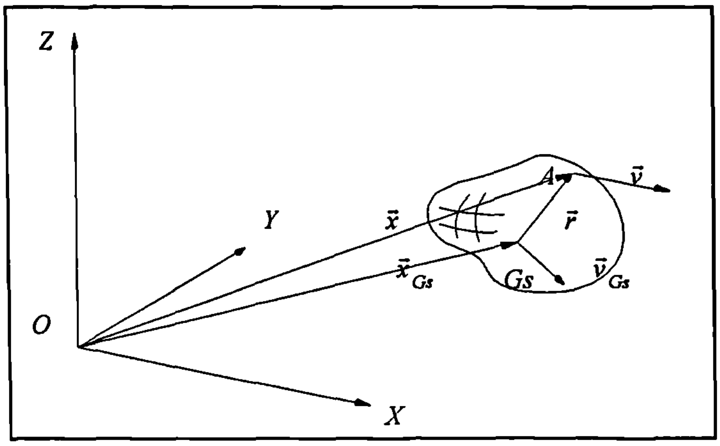

Fig. 2

Rigid body moving in space, described by the earth fixed system of reference $O\ XYZ$.

The general expression for Newton's second law is:

(1.1) $$\vec{F} = \frac{d}{dt}\iiint_V \rho\, \vec{v}\, dV$$

and the equivalent expression for moments is:

(2.1) $$\vec{M}_o = \frac{d}{dt}\iiint_V \vec{x} \wedge \rho\, \vec{v}\, dV$$

where $\vec{F}$ represents the summation of all external forces applied to the system and $\vec{M}_o$ is the moment of these forces with respect to the co-ordinate system origin O (the system $O\ XYZ$ is inertial, i.e. if it moves, it moves with constant velocity on a straight line and it does not rotate; it is generally considered fixed to the earth in this kind of problems). V is the volume occupied by the body, ρ is the density of the system, $\vec{v}$ is the velocity of point A with respect to the inertial co-ordinate axis and $\vec{x}$ is its position vector.

The dynamic system (ship + flood water) will be treated assuming that all the mass of the flood water is concentrated in its centre of gravity $Gw(t)$ and, since this is expected to be lying within the ship boundaries, V can be considered to be a rigid volume moving but not changing shape with time and ρ a function of time and position, which tends to infinity in close proximity of Gw such that:

$$\lim_{R\to 0}\iiint_{V'}\rho(\vec{x},t)\,dV = m_w(t)$$

where V' is a sphere of radius R centred at $Gw(t)$ and $m_w(t)$ is the mass of the flood water. In the following pages the notation below will be referred to (the time dependency is emphasised with respect to a ship-fixed reference):

Gs, m_s, $\vec{x}_{Gs}$, $\vec{v}_{Gs}(t)$: centre of gravity, mass of the intact ship (displacement) and co-ordinates of Gs and its velocity, respectively;

$Gw(t)$, $m_w(t)$, $\vec{x}_{Gw}(t)$, $\vec{v}_{Gw}(t)$: same quantities for the flood water;

$G(t)$, $m(t)$, $\vec{x}_G(t)$, $\vec{v}_G(t)$: same quantities for the dynamic system (ship and flood water).

Clearly, the following relations hold:

$$m(t)\,\vec{x}_G(t) = m_s\,\vec{x}_{Gs} + m_w(t)\,\vec{x}_{Gw}(t)$$

$$m_s = \iiint_V \rho_s\,dV$$

$$m_s \, \vec{x}_{Gs} = \iiint_V \rho_s \, \vec{x} \, dV$$

$$m(t) = m_s + m_w(t)$$

and equation **(1.1)** can be re-written as:

$$\vec{F} = \frac{d}{dt} \iiint_V \rho \, \vec{v} \, dV = \frac{d}{dt} \iiint_V \rho_s \, \vec{v} \, dV + \frac{d}{dt}\left[m_w(t) \, \vec{v}_{Gw}\right].$$

Now,

$$\vec{v} = \vec{v}_{Gs} + \frac{d}{dt}\vec{r} \quad \Rightarrow$$

$$\begin{aligned}
\frac{d}{dt} \iiint_V \rho_s \, \vec{v} \, dV &= \frac{d}{dt}\left[\iiint_V \rho_s \, \vec{v}_{Gs} \, dV + \iiint_V \frac{d}{dt}(\rho_s \, \vec{r}) \, dV\right] = \\
&= \frac{d}{dt}\left[\vec{v}_{Gs} \iiint_V \rho_s \, dV + \frac{d}{dt} \iiint_V \rho_s \, \vec{r} \, dV\right] = \\
&= \frac{d}{dt}\left[\vec{v}_{Gs} \, m_s\right]
\end{aligned}$$

since, according to the definition of Gs:

$$\frac{d}{dt} \iiint_V \rho_s \, \vec{r} \, dV = 0$$

and

$$\iiint_V \rho_s \, dV = m_s$$

as mentioned above. This yields:

$$\vec{F} = \frac{d}{dt}\left[m_s\ \vec{v}_{Gs}\right] + \frac{d}{dt}\left[m_w(t)\ \vec{v}_{Gw}\right]$$

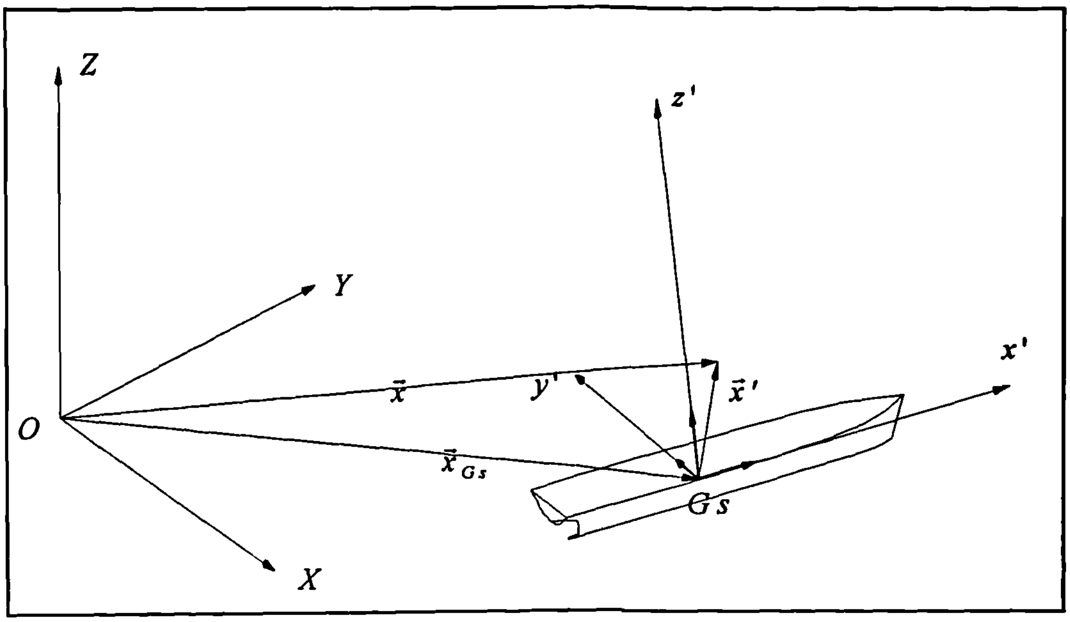

Fig. 3

Ship and earth fixed co-ordinate systems.

and expanding the R.H.S.:

$$\vec{F} = m_s\ \dot{\vec{v}}_{Gs} + \dot{m}_w\ \vec{v}_{Gw} + m_w\ \dot{\vec{v}}_{Gw} \tag{3.1}$$

All vector quantities are expressed with respect to the inertial system of reference. Consider now the two systems $Gs\,x'y'z'$ and $O\,XYZ$ (fig. 3). $Gs\,x'y'z'$ is translating and rotating with respect to the earth-fixed system $O\,XYZ$ and, since it is fixed to the ship, its velocity of translation will be $\vec{v}_{Gs}$. Let $\vec{\Omega}$ be the vector of instantaneous infinitesimal rate of rotation of the ship around x', y' and z' axes:

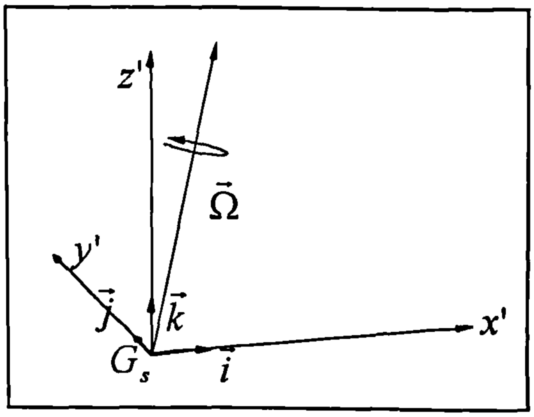

Fig. 4

Rate of rotation vector as seen from the point of view of an observer moving with the ship.

$$\vec{\Omega} = \begin{pmatrix} \dot{q}'_4 \\ \dot{q}'_5 \\ \dot{q}'_6 \end{pmatrix}_{Gs\,x'y'z'} .$$

At a given instant, the system $Gs\,x'y'z'$ is rotating with this same angular velocity ,

thus if $\vec{\alpha} = \begin{pmatrix} \varphi \\ \vartheta \\ \psi \end{pmatrix}$ is the vector of rotations that the earth-fixed system has to perform

around its axis to become parallel to the earth-fixed one (the components of vector $\vec{\alpha}$ are known as Euler's angles), the two systems are linked through the relations:

$$(\vec{x})_{O\,XYZ} = (\vec{x}_{Gs})_{O\,XYZ} + [D]^T (\vec{x}')_{Gs\,x'y'z'} \qquad \text{and} \qquad (\vec{x}')_{Gs\,x'y'z'} = [D]\left((\vec{x})_{O\,XYZ} - (\vec{x}_{Gs})_{O\,XYZ}\right)$$

where $[D]$ is the matrix:

$$[D] = \begin{bmatrix} \cos\vartheta\cdot\cos\psi - \sin\varphi\cdot\sin\vartheta\cdot\sin\psi & \cos\vartheta\cdot\sin\psi + \sin\varphi\cdot\sin\vartheta\cdot\cos\psi & -\cos\varphi\sin\vartheta \\ -\cos\varphi\cdot\sin\psi & \cos\varphi\cdot\cos\psi & \sin\varphi \\ \sin\vartheta\cdot\cos\psi + \sin\varphi\cdot\cos\vartheta\cdot\sin\psi & \sin\vartheta\cdot\sin\psi - \sin\varphi\cdot\cos\vartheta\cdot\cos\psi & \cos\varphi\cdot\cos\vartheta \end{bmatrix}$$

and obviously $\dot{\vec{\alpha}} = \vec{\Omega}$.

In defining $\vec{\alpha}$ we see that this quantity represents the finite rotation that the system $O\,XYZ$ must perform around its axis to become parallel to $Gs\,x'y'z'$. Since the sequence of rotations

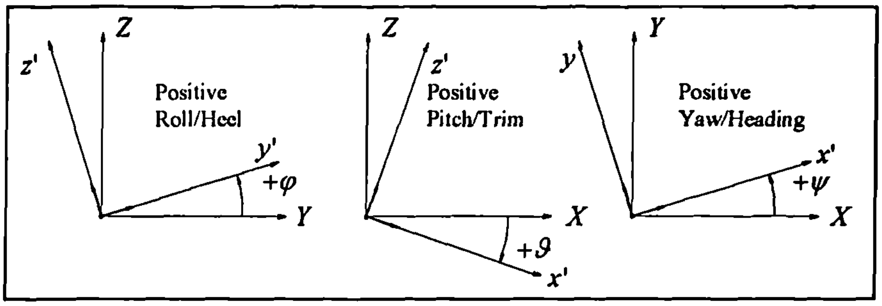

Fig. 5

Conventional right-handed reference positive rotations.

is of vital importance (the following convention is adopted here to derive $[D]$: ψ (yaw), then φ (roll) and finally ϑ (pitch)) $\vec{\alpha}$ cannot be considered as a proper vector and this has to be taken into account when integrating the equations of motions. For details on the derivation of matrix $[D]$ see Appendix A.

Going back to the problem of expressing equation **(3.1)** in terms of $Gs\,x'y'z'$ co-ordinates, having defined the way $Gs\,x'y'z'$ moves with respect to $O\,XYZ$, the relations between velocities expressed in $Gs\,x'y'z'$ co-ordinates and those expressed in $O\,XYZ$ co-ordinates, can now be written as shown next.

It can be seen from fig. 7 that for infinitesimal rotations the following relations hold:

$$\begin{aligned} d\vec{i} &= \vec{j}\,dq'_6 - \vec{k}\,dq'_5 \\ d\vec{j} &= \vec{k}\,dq'_4 - \vec{i}\,dq'_6 \\ d\vec{k} &= \vec{i}\,dq'_5 - \vec{j}\,dq'_4 \end{aligned} \quad \Rightarrow \quad \begin{aligned} \dot{\vec{i}} &= \vec{\Omega} \wedge \vec{i} \\ \dot{\vec{j}} &= \vec{\Omega} \wedge \vec{j} \\ \dot{\vec{k}} &= \vec{\Omega} \wedge \vec{k} \end{aligned}$$

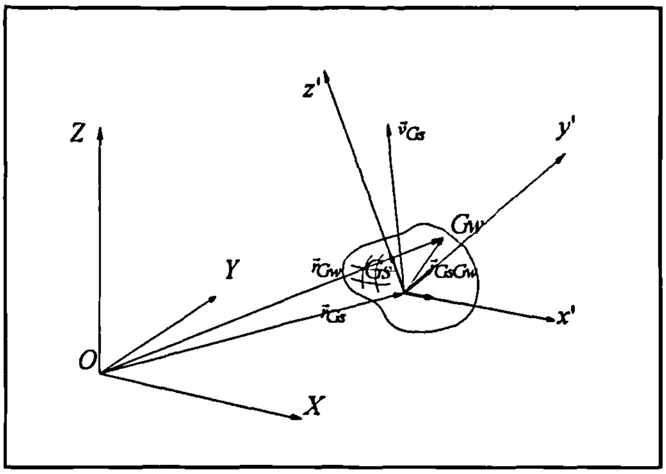

Fig. 6

Moving and inertial systems of reference for a rigid body.

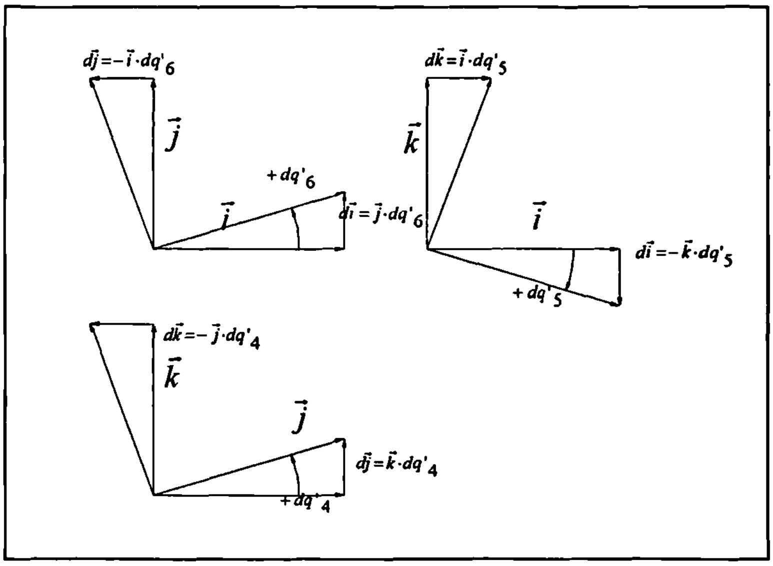

Fig. 7

Relationships between the unit vectors of a moving system and their derivatives with respect to time.

It is important to observe that the relationships linking the time derivatives of the unit vectors with the unit vectors themselves (just shown above) are universally valid and rigorous since time derivatives are by definition the limit the ratio $\Delta\vec{i}/\Delta t$ tends to, when both $\Delta\vec{i}$ and Δt becomes infinitesimal, thus assuming linearity to determine these relations is in no way restrictive and they can be used to describe both linear and non-linear dynamic systems.

Consider now a vector $\vec{\tilde{v}} = \left(\tilde{v}_x\,\vec{i} + \tilde{v}_y\,\vec{j} + \tilde{v}_z\,\vec{k}\right)$ (note that $\vec{\tilde{v}}$ is not necessarily a velocity vector in this context). Performing the absolute time derivative will lead to:

$$\dot{\vec{\tilde{v}}} = \dot{\tilde{v}}_x\,\vec{i} + \dot{\tilde{v}}_y\,\vec{j} + \dot{\tilde{v}}_z\,\vec{k} + \tilde{v}_x\,\dot{\vec{i}} + \tilde{v}_y\,\dot{\vec{j}} + \tilde{v}_z\,\dot{\vec{k}} \quad \Rightarrow$$

$$\textbf{(4.1)} \qquad \left(\frac{d\vec{\tilde{v}}}{dt}\right)_{O\,XYZ} = \left(\frac{d\vec{\tilde{v}}}{dt}\right)_{Gs\,x'y'z'} + \left(\vec{\Omega} \wedge \vec{\tilde{v}}\right)_{Gs\,x'y'z'}$$

Applying this relation to the vector $\vec{r}_{GsGw} = \vec{r}_{Gw} - \vec{r}_{Gs}$ (see fig. 6) yields:

$$\left(\frac{d}{dt}\left(\vec{r}_{Gw} - \vec{r}_{Gs}\right)\right)_{O\,XYZ} = \left(\frac{d}{dt}\vec{r}_{GsGw}\right)_{Gs\,x'y'z'} + \left(\vec{\Omega} \wedge \vec{r}_{GsGw}\right)_{Gs\,x'y'z'} \quad \Rightarrow$$

$$\boxed{\left(\vec{v}_{Gw}\right)_{O\,XYZ} = \left(\vec{v}_{Gs}\right)_{Gs\,x'y'z} + \left(\vec{v}\,'_{Gw}\right)_{Gs\,x'y'z'} + \left(\vec{\Omega} \wedge \vec{x}\,'_{Gw}\right)_{Gs\,x'y'z'}}$$

The double application of formula **(4.1)** to vector $\vec{\tilde{v}}$ gives:

$$\left(\frac{d^2\vec{\tilde{v}}}{dt^2}\right)_{O\,XYZ} = \left(\frac{d^2\vec{\tilde{v}}}{dt^2}\right)_{Gs\,x'y'z'} + \left(\dot{\vec{\Omega}} \wedge \vec{\tilde{v}}\right)_{Gs\,x'y'z'} + \left(\vec{\Omega} \wedge \left(\vec{\Omega} \wedge \vec{\tilde{v}}\right)\right)_{Gs\,x'y'z'} + \left(2\,\vec{\Omega} \wedge \left(\frac{d\vec{\tilde{v}}}{dt}\right)\right)_{Gs\,x'y'z'}$$

i.e. in the case of $\vec{r}_{GsGw}$:

$$\left(\dot{\vec{v}}_{Gw}\right)_{O\,XYZ}-\left(\dot{\vec{v}}_{Gs}\right)_{O\,XYZ}=\left(\dot{\vec{v}}'_{Gw}\right)_{Gs\,x'y'z'}+\left(\dot{\vec{\Omega}}\wedge\vec{r}'_{Gw}\right)_{Gs\,x'y'z'}+ \\ +\left(\vec{\Omega}\wedge\left(\vec{\Omega}\wedge\vec{r}'_{Gw}\right)\right)_{Gs\,x'y'z'}+\left(2\,\vec{\Omega}\wedge\left(\vec{v}'_{Gw}\right)\right)_{Gs\,x'y'z'} \tag{5.1}$$

Applying **(4.1)** to $\vec{v}_{Gs}$ gives:

$$\boxed{\left(\dot{\vec{v}}_{Gs}\right)_{O\,XYZ}=\left(\dot{\vec{v}}_{Gs}\right)_{Gs\,x'y'z'}+\left(\vec{\Omega}\wedge\vec{v}_{Gs}\right)_{Gs\,x'y'z'}}$$

thus substituting the above in **(5.1)**, the latter becomes:

$$\boxed{\begin{aligned}\left(\dot{\vec{v}}_{Gw}\right)_{O\,XYZ}&=\left(\dot{\vec{v}}_{Gs}\right)_{Gs\,x'y'z'}+\left(\vec{\Omega}\wedge\vec{v}_{Gs}\right)_{Gs\,x'y'z'}+\\&+\left(\dot{\vec{v}}'_{Gw}\right)_{Gs\,x'y'z'}+\left(\dot{\vec{\Omega}}\wedge\vec{x}'_{Gw}\right)_{Gs\,x'y'z'}+\\&+\left(\vec{\Omega}\wedge\left(\vec{\Omega}\wedge\vec{x}'_{Gw}\right)\right)_{Gs\,x'y'z'}+2\left(\vec{\Omega}\wedge\vec{v}'_{Gw}\right)_{Gs\,x'y'z'}\end{aligned}}$$

The three relations found (boxes) can be used to express **(3.1)** in $Gs\,x'y'z'$ co-ordinates. Note that the free vector $\vec{v}_{Gs}$ can be expressed in $Gs\,x'y'z'$ co-ordinates as:

$$\left(\vec{v}_{Gs}\right)_{Gs\,x'y'z'}=\dot{q}'_1\,\vec{i}+\dot{q}'_2\,\vec{j}+\dot{q}'_3\,\vec{k}$$

thus **(3.1)** finally becomes:

$$\vec{F}'=m_s\left(\dot{\vec{v}}_{Gs}+\vec{\Omega}\wedge\vec{v}_{Gs}\right)+\dot{m}_w\left(\vec{v}_{Gs}+\vec{v}'_{Gw}+\vec{\Omega}\wedge\vec{x}'_{Gw}\right)+ \\ +m_w\left(\dot{\vec{v}}_{Gs}+\vec{\Omega}\wedge\vec{v}_{Gs}+\dot{\vec{v}}'_{Gw}+\dot{\vec{\Omega}}\wedge\vec{x}'_{Gw}+\vec{\Omega}\wedge\left(\vec{\Omega}\wedge\vec{x}'_{Gw}\right)+2\,\vec{\Omega}\wedge\vec{v}'_{Gw}\right)$$

and substituting $\begin{pmatrix}\dot{q}'_1\\ \dot{q}'_2\\ \dot{q}'_3\end{pmatrix}$ to $\vec{v}_{Gs}$, $\begin{pmatrix}\ddot{q}'_1\\ \ddot{q}'_2\\ \ddot{q}'_3\end{pmatrix}$ to $\dot{\vec{v}}_{Gs}$, $\begin{pmatrix}\dot{q}'_4\\ \dot{q}'_5\\ \dot{q}'_6\end{pmatrix}$ to $\vec{\Omega}$ and $\begin{pmatrix}\ddot{q}'_4\\ \ddot{q}'_5\\ \ddot{q}'_6\end{pmatrix}$ to $\dot{\vec{\Omega}}$:

$$
\begin{aligned}
\begin{pmatrix}F_1'\\ F_2'\\ F_3'\end{pmatrix} &= m_s\left[\begin{pmatrix}\ddot{q}'_1\\ \ddot{q}'_2\\ \ddot{q}'_3\end{pmatrix}+\begin{pmatrix}\dot{q}'_4\\ \dot{q}'_5\\ \dot{q}'_6\end{pmatrix}\wedge\begin{pmatrix}\dot{q}'_1\\ \dot{q}'_2\\ \dot{q}'_3\end{pmatrix}\right]+\dot{m}_w\left[\begin{pmatrix}\dot{q}'_1\\ \dot{q}'_2\\ \dot{q}'_3\end{pmatrix}+\begin{pmatrix}\dot{x}'_{Gw}\\ \dot{y}'_{Gw}\\ \dot{z}'_{Gw}\end{pmatrix}+\begin{pmatrix}\dot{q}'_4\\ \dot{q}'_5\\ \dot{q}'_6\end{pmatrix}\wedge\begin{pmatrix}x'_{Gw}\\ y'_{Gw}\\ z'_{Gw}\end{pmatrix}\right]+\\
&+m_w\left[\begin{pmatrix}\ddot{q}'_1\\ \ddot{q}'_2\\ \ddot{q}'_3\end{pmatrix}+\begin{pmatrix}\dot{q}'_4\\ \dot{q}'_5\\ \dot{q}'_6\end{pmatrix}\wedge\begin{pmatrix}\dot{q}'_1\\ \dot{q}'_2\\ \dot{q}'_3\end{pmatrix}+\begin{pmatrix}\ddot{x}'_{Gw}\\ \ddot{y}'_{Gw}\\ \ddot{z}'_{Gw}\end{pmatrix}+\begin{pmatrix}\ddot{q}'_4\\ \ddot{q}'_5\\ \ddot{q}'_6\end{pmatrix}\wedge\begin{pmatrix}x'_{Gw}\\ y'_{Gw}\\ z'_{Gw}\end{pmatrix}+\begin{pmatrix}\dot{q}'_4\\ \dot{q}'_5\\ \dot{q}'_6\end{pmatrix}\wedge\left[\begin{pmatrix}\dot{q}'_4\\ \dot{q}'_5\\ \dot{q}'_6\end{pmatrix}\wedge\begin{pmatrix}x'_{Gw}\\ y'_{Gw}\\ z'_{Gw}\end{pmatrix}\right]+\right.\\
&\left.\quad+2\begin{pmatrix}\dot{q}'_4\\ \dot{q}'_5\\ \dot{q}'_6\end{pmatrix}\wedge\begin{pmatrix}\dot{x}'_{Gw}\\ \dot{y}'_{Gw}\\ \dot{z}'_{Gw}\end{pmatrix}\right]
\end{aligned}
$$

i.e. equations **(9.1)**:

$$
\begin{aligned}
F_1' &= m_s\left(\ddot{q}'_1+\dot{q}'_5\dot{q}'_3-\dot{q}'_6\dot{q}'_2\right)+\dot{m}_w\left(\dot{q}'_1+\dot{x}'_{Gw}+\dot{q}'_5 z'_{Gw}-\dot{q}'_6 y'_{Gw}\right)+\\
&+m_w\left(\ddot{q}'_1+\dot{q}'_5\dot{q}'_3-\dot{q}'_6\dot{q}'_2+\ddot{x}'_{Gw}+\ddot{q}'_5 z'_{Gw}-\ddot{q}'_6 y'_{Gw}+\dot{q}'_5\dot{q}'_4 y'_{Gw}+\right.\\
&\qquad\left.-\dot{q}'^2_5 x'_{Gw}-\dot{q}'^2_6 x'_{Gw}+\dot{q}'_6\dot{q}'_4 z'_{Gw}\right)+\\
&+m_w\left(2\,\dot{q}'_5\dot{z}'_{Gw}-2\,\dot{q}'_6\dot{y}'_{Gw}\right)\\
F_2' &= m_s\left(\ddot{q}'_2+\dot{q}'_6\dot{q}'_1-\dot{q}'_4\dot{q}'_3\right)+\dot{m}_w\left(\dot{q}'_2+\dot{y}'_{Gw}+\dot{q}'_6 x'_{Gw}-\dot{q}'_4 z'_{Gw}\right)+\\
&+m_w\left(\ddot{q}'_2+\dot{q}'_6\dot{q}'_1-\dot{q}'_4\dot{q}'_3+\ddot{y}'_{Gw}+\ddot{q}'_6 x'_{Gw}-\ddot{q}'_4 z'_{Gw}+\dot{q}'_6\dot{q}'_5 z'_{Gw}+\right.\\
&\qquad\left.-\dot{q}'^2_6 y'_{Gw}-\dot{q}'^2_4 y'_{Gw}+\dot{q}'_4\dot{q}'_5 x'_{Gw}\right)+\\
&+m_w\left(2\,\dot{q}'_6\dot{x}'_{Gw}-2\,\dot{q}'_4\dot{z}'_{Gw}\right)\\
F_3' &= m_s\left(\ddot{q}'_3+\dot{q}'_4\dot{q}'_2-\dot{q}'_5\dot{q}'_1\right)+\dot{m}_w\left(\dot{q}'_3+\dot{z}'_{Gw}+\dot{q}'_4 y'_{Gw}-\dot{q}'_5 x'_{Gw}\right)+\\
&+m_w\left(\ddot{q}'_3+\dot{q}'_4\dot{q}'_2-\dot{q}'_5\dot{q}'_1+\ddot{z}'_{Gw}+\ddot{q}'_4 y'_{Gw}-\ddot{q}'_5 x'_{Gw}+\dot{q}'_4\dot{q}'_6 x'_{Gw}+\right.\\
&\qquad\left.-\dot{q}'^2_4 z'_{Gw}-\dot{q}'^2_5 z'_{Gw}+\dot{q}'_5\dot{q}'_6 y'_{Gw}\right)+\\
&+m_w\left(2\,\dot{q}'_4\dot{y}'_{Gw}-2\,\dot{q}'_5\dot{x}'_{Gw}\right)
\end{aligned}
$$

These equations - which reduce to Abkowitz's [1.1 - page I-7], when $m_w = 0$ - should be solved with respect to the unknown $\vec{v}_{Gs}$ which can then be expressed in earth-fixed co-ordinates and therewith integrated with respect to time to find the ship motions.

Before linearising the equations above and the analysis of the force $\vec{F}'$, similar relations for the angular equation **(2.1)** will first be derived. The previous simplifying hypothesis gives:

$$\vec{M}_o = \frac{d}{dt}\iiint_V \vec{x} \wedge \rho_s\, \vec{v}\, dV + \frac{d}{dt}\left[\vec{x}_{Gw} \wedge m_w(t)\, \vec{v}_{Gw}\right]$$

In addition, from fig. 3 it may be seen that $\vec{v} = \vec{v}_{Gs} + \frac{d}{dt}\vec{x}'$ and $\vec{x} = \vec{x}_{Gs} + \vec{x}'$, thus:

$$\frac{d}{dt}\iiint_V \vec{x} \wedge \rho_s\, \vec{v}\, dV = \frac{d}{dt}\iiint_V \vec{x} \wedge \rho_s\, \vec{v}_{Gs}\, dV + \frac{d}{dt}\iiint_V \vec{x} \wedge \rho_s \frac{d}{dt}\vec{x}'\, dV =$$

$$= \frac{d}{dt}\left[-\vec{v}_{Gs} \wedge \iiint_V \rho_s\, \vec{x}\, dV + \iiint_V \vec{x} \wedge \rho_s \frac{d}{dt}\vec{x}'\, dV\right] =$$

$$= \frac{d}{dt}\left[m_s\, \vec{x}_{Gs} \wedge \vec{v}_{Gs} + \vec{x}_{Gs} \wedge \frac{d}{dt}\iiint_V \rho_s\, \vec{x}'\, dV + \iiint_V \vec{x}' \wedge \rho_s \frac{d}{dt}\vec{x}'\, dV\right] =$$

$$= \frac{d}{dt}\left[m_s\, \vec{x}_{Gs} \wedge \vec{v}_{Gs} + \iiint_V \vec{x}' \wedge \rho_s \frac{d}{dt}\vec{x}'\, dV\right]$$

since $\iiint_V \rho_s\, \vec{x}\, dV = m_s\, \vec{x}_{Gs}$ and $\iiint_V \rho_s\, \vec{x}'\, dV = 0$. Note that the second term on the R.H.S. of the equation above, is the angular momentum of a rigid body with respect to its centre of gravity Gs. Given that $\frac{d}{dt}\vec{x}' = \vec{\Omega} \wedge \vec{x}'$, then:

$$\iiint_V \vec{x}' \wedge \rho_s \frac{d}{dt}\vec{x}'\, dV = \iiint_V \rho_s \left(\vec{x}' \wedge \left(\vec{\Omega} \wedge \vec{x}'\right)\right) dV$$

thus:

$$\vec{M}_o = \frac{d}{dt}\left[m_s\, \vec{x}_{Gs} \wedge \vec{v}_{Gs} + \iiint_V \rho_s \left(\vec{x}' \wedge \left(\vec{\Omega} \wedge \vec{x}'\right)\right) dV + m_w(t)\, \vec{x}_{Gw} \wedge \vec{v}_{Gw}\right] \tag{6.1}$$

The integral in the R.H.S. can be written as:

$$\iiint_V \rho_s \left(\vec{x}' \wedge \left(\vec{\Omega} \wedge \vec{x}'\right)\right) dV =$$

$$= \left(I_{x'x'}\, \dot{q}'_4 - I_{x'y'}\, \dot{q}'_5 - I_{x'z'}\, \dot{q}'_6\right) \vec{i} +$$
$$+ \left(-I_{y'x'}\, \dot{q}'_4 + I_{y'y'}\, \dot{q}'_5 - I_{y'z'}\, \dot{q}'_6\right) \vec{j} +$$
$$+ \left(-I_{z'x'}\, \dot{q}'_4 - I_{z'y'}\, \dot{q}'_5 + I_{z'z'}\, \dot{q}'_6\right) \vec{k} = [I]_s\, \vec{\Omega}$$

where:

$$I_{x'x'} = \iiint_V \rho_s \left(y'^2 + z'^2\right) dV$$

$$I_{y'y'} = \iiint_V \rho_s \left(z'^2 + x'^2\right) dV$$

$$I_{z'z'} = \iiint_V \rho_s \left(x'^2 + y'^2\right) dV$$

$$I_{x'y'} = \iiint_V \rho_s\, x'\, y'\, dV$$

$$I_{y'z'} = \iiint_V \rho_s\, y'\, z'\, dV$$

$$I_{z'x'} = \iiint_V \rho_s\, z'\, x'\, dV\,.$$

The terms above are evaluated using $Gs\,x'y'z'$ co-ordinates. This result can be obtained by expanding the double cross-product as follows:

$$\vec{\Omega} \wedge \vec{x}' = \begin{pmatrix} \dot{q}'_4 \\ \dot{q}'_5 \\ \dot{q}'_6 \end{pmatrix} \wedge \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \dot{q}'_5 z' - \dot{q}'_6 y' \\ \dot{q}'_6 x' - \dot{q}'_4 z' \\ \dot{q}'_4 y' - \dot{q}'_5 x' \end{pmatrix} \quad \Rightarrow$$

$$\vec{x}' \wedge \left(\vec{\Omega} \wedge \vec{x}'\right) = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \wedge \begin{pmatrix} \dot{q}'_5 z' - \dot{q}'_6 y' \\ \dot{q}'_6 x' - \dot{q}'_4 z' \\ \dot{q}'_4 y' - \dot{q}'_5 x' \end{pmatrix} = \begin{pmatrix} \dot{q}'_4 \left(y'^2 + z'^2\right) - \dot{q}'_5 x' y' - \dot{q}'_6 z' x' \\ -\dot{q}'_4 x' y' + \dot{q}'_5 \left(z'^2 + x'^2\right) - \dot{q}'_6 y' z' \\ -\dot{q}'_4 z' x' - \dot{q}'_5 y' z' + \dot{q}'_6 \left(x'^2 + y'^2\right) \end{pmatrix}.$$

Equation **(6.1)** can be expanded to give:

(7.1)
$$\begin{aligned} \vec{M}_o &= m_s \frac{d}{dt}\left(\vec{x}_{Gs} \wedge \vec{v}_{Gs}\right)_{O\,XYZ} + [I]_s \frac{d}{dt}\vec{\Omega}_{O\,XYZ} + \\ &+ \dot{m}_w \left(\vec{x}_{Gw} \wedge \vec{v}_{Gw}\right)_{O\,XYZ} + m_w \frac{d}{dt}\left(\vec{x}_{Gw} \wedge \vec{v}_{Gw}\right)_{O\,XYZ} \end{aligned}$$

To express **(7.1)** in $Gs\,x'y'z'$ co-ordinates, the following should be noted:

$$\left(\vec{x}_{Gs}\right)_{O\,XYZ} = \left(\vec{0}\right)_{Gs\,x'y'z'}$$

$$\left(\vec{x}_{Gw}\right)_{O\,XYZ} = \left(\vec{x}'_{Gw}\right)_{Gs\,x'y'z'}$$

$$\left(\vec{v}_{Gw}\right)_{O\,XYZ} = \left(\vec{v}_{Gs}\right)_{O\,XYZ} + \left(\vec{v}'_{Gw}\right)_{Gs\,x'y'z'} + \left(\vec{\Omega} \wedge \vec{x}'_{Gw}\right)_{Gs\,x'y'z'}$$

and $\left(\vec{v}_{Gs}\right)_{O\,XYZ} = \left(\dot{q}'_1\,\vec{i} + \dot{q}'_2\,\vec{j} + \dot{q}'_3\,\vec{k}\right)$ in $Gs\,x'y'z'$ co-ordinates. $\Rightarrow$

(8.1)
$$\begin{aligned} \vec{M}'_{Gs} &= [I]_s \frac{d}{dt}\vec{\Omega}_{O\,XYZ} + \dot{m}_w \left(\vec{x}'_{Gw} \wedge \vec{v}_{Gs} + \vec{x}'_{Gw} \wedge \vec{v}'_{Gw} + \vec{x}'_{Gw} \wedge \left(\vec{\Omega} \wedge \vec{x}'_{Gw}\right)\right) + \\ &+ m_w \left(\frac{d}{dt}\left(\vec{x}'_{Gw} \wedge \vec{v}_{Gs}\right) + \frac{d}{dt}\left(\vec{x}'_{Gw} \wedge \vec{v}'_{Gw}\right) + \frac{d}{dt}\left(\vec{x}'_{Gw} \wedge \left(\vec{\Omega} \wedge \vec{x}'_{Gw}\right)\right)\right) \end{aligned}$$

Expanding the cross-products and finding their derivatives:

$$\vec{x}'_{Gw} \wedge \vec{v}_{Gs} = \begin{pmatrix} x'_{Gw} \\ y'_{Gw} \\ z'_{Gw} \end{pmatrix} \wedge \begin{pmatrix} \dot{q}'_1 \\ \dot{q}'_2 \\ \dot{q}'_3 \end{pmatrix} = \begin{pmatrix} y'_{Gw}\dot{q}'_3 - z'_{Gw}\dot{q}'_2 \\ z'_{Gw}\dot{q}'_1 - x'_{Gw}\dot{q}'_3 \\ x'_{Gw}\dot{q}'_2 - y'_{Gw}\dot{q}'_1 \end{pmatrix}$$

$$\frac{d}{dt}\left(\vec{x}'_{Gw} \wedge \vec{v}_{Gs}\right) = \begin{pmatrix} \dot{y}'_{Gw}\dot{q}'_3 + y'_{Gw}\ddot{q}'_3 - \dot{z}'_{Gw}\dot{q}'_2 - z'_{Gw}\ddot{q}'_2 \\ \dot{z}'_{Gw}\dot{q}'_1 + z'_{Gw}\ddot{q}'_1 - \dot{x}'_{Gw}\dot{q}'_3 - x'_{Gw}\ddot{q}'_3 \\ \dot{x}'_{Gw}\dot{q}'_2 + x'_{Gw}\ddot{q}'_2 - \dot{y}'_{Gw}\dot{q}'_1 - y'_{Gw}\ddot{q}'_1 \end{pmatrix}$$

$$\vec{x}'_{Gw} \wedge \vec{v}'_{Gw} = \begin{pmatrix} x'_{Gw} \\ y'_{Gw} \\ z'_{Gw} \end{pmatrix} \wedge \begin{pmatrix} \dot{x}'_{Gw} \\ \dot{y}'_{Gw} \\ \dot{z}'_{Gw} \end{pmatrix} = \begin{pmatrix} y'_{Gw}\dot{z}'_{Gw} - z'_{Gw}\dot{y}'_{Gw} \\ z'_{Gw}\dot{x}'_{Gw} - x'_{Gw}\dot{z}'_{Gw} \\ x'_{Gw}\dot{y}'_{Gw} - y'_{Gw}\dot{x}'_{Gw} \end{pmatrix}$$

$$\frac{d}{dt}\left(\vec{x}'_{Gw} \wedge \vec{v}'_{Gw}\right) = \begin{pmatrix} \dot{y}'_{Gw}\dot{z}'_{Gw} + y'_{Gw}\ddot{z}'_{Gw} - \dot{z}'_{Gw}\dot{y}'_{Gw} - z'_{Gw}\ddot{y}'_{Gw} \\ z'_{Gw}\ddot{x}'_{Gw} \qquad - x'_{Gw}\ddot{z}'_{Gw} \\ x'_{Gw}\ddot{y}'_{Gw} \qquad - y'_{Gw}\ddot{x}'_{Gw} \end{pmatrix}$$

$$\vec{x}'_{Gw} \wedge \left(\vec{\Omega} \wedge \vec{x}'_{Gw}\right) = \begin{pmatrix} x'_{Gw} \\ y'_{Gw} \\ z'_{Gw} \end{pmatrix} \wedge \left[\begin{pmatrix} \dot{q}'_4 \\ \dot{q}'_5 \\ \dot{q}'_6 \end{pmatrix} \wedge \begin{pmatrix} x'_{Gw} \\ y'_{Gw} \\ z'_{Gw} \end{pmatrix}\right] = \begin{pmatrix} x'_{Gw} \\ y'_{Gw} \\ z'_{Gw} \end{pmatrix} \wedge \begin{pmatrix} z'_{Gw}\dot{q}'_5 - y'_{Gw}\dot{q}'_6 \\ x'_{Gw}\dot{q}'_6 - z'_{Gw}\dot{q}'_4 \\ y'_{Gw}\dot{q}'_4 - x'_{Gw}\dot{q}'_5 \end{pmatrix} =$$

$$= \begin{pmatrix} \dot{q}'_4 y'^2_{Gw} - \dot{q}'_5 x'_{Gw} y'_{Gw} - \dot{q}'_6 x'_{Gw} z'_{Gw} + \dot{q}'_4 z'^2_{Gw} \\ \dot{q}'_5 z'^2_{Gw} - \dot{q}'_6 y'_{Gw} z'_{Gw} - \dot{q}'_4 y'_{Gw} x'_{Gw} + \dot{q}'_5 x'^2_{Gw} \\ \dot{q}'_6 x'^2_{Gw} - \dot{q}'_4 z'_{Gw} x'_{Gw} - \dot{q}'_5 z'_{Gw} y'_{Gw} + \dot{q}'_6 y'^2_{Gw} \end{pmatrix}$$

$$\frac{d}{dt}\left(\vec{x}'_{Gw} \wedge \left(\vec{\Omega} \wedge \vec{x}'_{Gw}\right)\right) =$$

$$
= \begin{pmatrix}
\ddot{q}'_4 y'^2_{Gw} + 2\,\dot{q}'_4 y'_{Gw} \dot{y}'_{Gw} - \ddot{q}'_5 x'_{Gw} y'_{Gw} - \dot{q}'_5 \left(\dot{x}'_{Gw} y'_{Gw} + x'_{Gw} \dot{y}'_{Gw}\right) + \\
\qquad - \ddot{q}'_6 x'_{Gw} z'_{Gw} - \dot{q}'_6 \left(\dot{x}'_{Gw} z'_{Gw} + x'_{Gw} \dot{z}'_{Gw}\right) + \ddot{q}'_4 z'^2_{Gw} + 2\,\dot{q}'_4 z'_{Gw} \dot{z}'_{Gw} \\
\ddot{q}'_5 \left(z'^2_{Gw} + x'^2_{Gw}\right) + 2\,\dot{q}'_5 \left(z'_{Gw} \dot{z}'_{Gw} + x'_{Gw} \dot{x}'_{Gw}\right) - \ddot{q}'_6 y'_{Gw} z'_{Gw} - \dot{q}'_6 \left(\dot{y}'_{Gw} z'_{Gw} + y'_{Gw} \dot{z}'_{Gw}\right) + \\
\qquad - \ddot{q}'_4 y'_{Gw} x'_{Gw} - \dot{q}'_4 \left(\dot{y}'_{Gw} x'_{Gw} + y'_{Gw} \dot{x}'_{Gw}\right) \\
\ddot{q}'_6 \left(x'^2_{Gw} + y'^2_{Gw}\right) + 2\,\dot{q}'_6 \left(x'_{Gw} \dot{x}'_{Gw} + y'_{Gw} \dot{y}'_{Gw}\right) - \ddot{q}'_4 z'_{Gw} x'_{Gw} - \dot{q}'_4 \left(\dot{z}'_{Gw} x'_{Gw} + z'_{Gw} \dot{x}'_{Gw}\right) + \\
\qquad - \ddot{q}'_5 z'_{Gw} y'_{Gw} - \dot{q}'_5 \left(\dot{z}'_{Gw} y'_{Gw} + z'_{Gw} \dot{y}'_{Gw}\right)
\end{pmatrix}
$$

The term $[I]_s \dfrac{d}{dt}\vec{\Omega}_{O\,XYZ}$ has to be expanded as follows. Since $Gs\,x'\,y'\,z'$ is close to the principal and central system of axis for the intact ship, $[I]_s$ can be approximated by:

$$
[I]_s = \begin{bmatrix} I_{x'x'} & 0 & 0 \\ 0 & I_{y'y'} & 0 \\ 0 & 0 & I_{z'z'} \end{bmatrix} \text{ and } [I]_s\,\vec{\Omega} = I_{x'x'}\,\dot{q}'_4\,\vec{i} + I_{y'y'}\,\dot{q}'_5\,\vec{j} + I_{z'z'}\,\dot{q}'_6\,\vec{k} \quad \Rightarrow
$$

$$
\frac{d}{dt}\left([I]_s\,\vec{\Omega}\right)_{O\,XYZ} = [I]_s\,\dot{\vec{\Omega}}_{Gs\,x'y'z'} + \left(I_{x'x'}\,\dot{q}'_4\,\vec{\Omega}\wedge\vec{i} + I_{y'y'}\,\dot{q}'_5\,\vec{\Omega}\wedge\vec{j} + I_{z'z'}\,\dot{q}'_6\,\vec{\Omega}\wedge\vec{k}\right) =
$$

$$
= \begin{pmatrix} I_{x'x'}\,\ddot{q}'_4 \\ I_{y'y'}\,\ddot{q}'_5 \\ I_{z'z'}\,\ddot{q}'_6 \end{pmatrix} + I_{x'x'}\,\dot{q}'_4 \begin{pmatrix} 0 \\ \dot{q}'_6 \\ -\dot{q}'_5 \end{pmatrix} + I_{y'y'}\,\dot{q}'_5 \begin{pmatrix} -\dot{q}'_6 \\ 0 \\ \dot{q}'_4 \end{pmatrix} + I_{z'z'}\,\dot{q}'_6 \begin{pmatrix} \dot{q}'_5 \\ -\dot{q}'_4 \\ 0 \end{pmatrix} =
$$

$$
= \begin{pmatrix} I_{x'x'}\,\ddot{q}'_4 + \left(I_{z'z'} - I_{y'y'}\right)\dot{q}'_5\,\dot{q}'_6 \\ I_{y'y'}\,\ddot{q}'_5 + \left(I_{x'x'} - I_{z'z'}\right)\dot{q}'_6\,\dot{q}'_4 \\ I_{z'z'}\,\ddot{q}'_6 + \left(I_{y'y'} - I_{x'x'}\right)\dot{q}'_4\,\dot{q}'_5 \end{pmatrix}
$$

which is the well known form of Euler's equations. Substituting all the above in **(8.1)** gives:

$$\begin{pmatrix} M'_1 \\ M'_2 \\ M'_3 \end{pmatrix} = \begin{pmatrix} I_{x'x'}\,\ddot{q}'_4 + \left(I_{z'z'} - I_{y'y'}\right)\dot{q}'_5\,\dot{q}'_6 \\ I_{y'y'}\,\ddot{q}'_5 + \left(I_{x'x'} - I_{z'z'}\right)\dot{q}'_6\,\dot{q}'_4 \\ I_{z'z'}\,\ddot{q}'_6 + \left(I_{y'y'} - I_{x'x'}\right)\dot{q}'_4\,\dot{q}'_5 \end{pmatrix} +$$

$$+\dot{m}_w \left[\begin{pmatrix} y'_{Gw}\,\dot{q}'_3 - z'_{Gw}\,\dot{q}'_2 \\ z'_{Gw}\,\dot{q}'_1 - x'_{Gw}\,\dot{q}'_3 \\ x'_{Gw}\,\dot{q}'_2 - y'_{Gw}\,\dot{q}'_1 \end{pmatrix} + \begin{pmatrix} y'_{Gw}\,\dot{z}'_{Gw} - z'_{Gw}\,\dot{y}'_{Gw} \\ z'_{Gw}\,\dot{x}'_{Gw} - x'_{Gw}\,\dot{z}'_{Gw} \\ x'_{Gw}\,\dot{y}'_{Gw} - y'_{Gw}\,\dot{x}'_{Gw} \end{pmatrix} + \right.$$

$$\left. + \begin{pmatrix} \dot{q}'_4 \left(y'^2_{Gw} + z'^2_{Gw}\right) - \dot{q}'_5\,x'_{Gw}\,y'_{Gw} - \dot{q}'_6\,z'_{Gw}\,x'_{Gw} \\ -\dot{q}'_4\,x'_{Gw}\,y'_{Gw} + \dot{q}'_5 \left(z'^2_{Gw} + x'^2_{Gw}\right) - \dot{q}'_6\,y'_{Gw}\,z'_{Gw} \\ -\dot{q}'_4\,z'_{Gw}\,x'_{Gw} - \dot{q}'_5\,y'_{Gw}\,z'_{Gw} + \dot{q}'_6 \left(x'^2_{Gw} + y'^2_{Gw}\right) \end{pmatrix} \right] +$$

$$+m_w \left[\begin{pmatrix} \dot{y}'_{Gw}\,\dot{q}'_3 + y'_{Gw}\,\ddot{q}'_3 - \dot{z}'_{Gw}\,\dot{q}'_2 - z'_{Gw}\,\ddot{q}'_2 \\ \dot{z}'_{Gw}\,\dot{q}'_1 + z'_{Gw}\,\ddot{q}'_1 - \dot{x}'_{Gw}\,\dot{q}'_3 - x'_{Gw}\,\ddot{q}'_3 \\ \dot{x}'_{Gw}\,\dot{q}'_2 + x'_{Gw}\,\ddot{q}'_2 - \dot{y}'_{Gw}\,\dot{q}'_1 - y'_{Gw}\,\ddot{q}'_1 \end{pmatrix} + \begin{pmatrix} y'_{Gw}\,\ddot{z}'_{Gw} - z'_{Gw}\,\ddot{y}'_{Gw} \\ z'_{Gw}\,\ddot{x}'_{Gw} - x'_{Gw}\,\ddot{z}'_{Gw} \\ x'_{Gw}\,\ddot{y}'_{Gw} - y'_{Gw}\,\ddot{x}'_{Gw} \end{pmatrix} + \right.$$

$$\left. + \begin{pmatrix} \ddot{q}'_4 \left(y'^2_{Gw} + z'^2_{Gw}\right) + 2\,\dot{q}'_4 \left(y'_{Gw}\,\dot{y}'_{Gw} + z'_{Gw}\,\dot{z}'_{Gw}\right) - \ddot{q}'_5\,x'_{Gw}\,y'_{Gw} - \dot{q}'_5 \left(\dot{x}'_{Gw}\,y'_{Gw} + x'_{Gw}\,\dot{y}'_{Gw}\right) + \\ - \ddot{q}'_6\,x'_{Gw}\,z'_{Gw} - \dot{q}'_6 \left(\dot{x}'_{Gw}\,z'_{Gw} + x'_{Gw}\,\dot{z}'_{Gw}\right) \\ \ddot{q}'_5 \left(z'^2_{Gw} + x'^2_{Gw}\right) + 2\,\dot{q}'_5 \left(z'_{Gw}\,\dot{z}'_{Gw} + x'_{Gw}\,\dot{x}'_{Gw}\right) - \ddot{q}'_6\,y'_{Gw}\,z'_{Gw} - \dot{q}'_6 \left(\dot{y}'_{Gw}\,z'_{Gw} + y'_{Gw}\,\dot{z}'_{Gw}\right) + \\ - \ddot{q}'_4\,y'_{Gw}\,x'_{Gw} - \dot{q}'_4 \left(\dot{y}'_{Gw}\,x'_{Gw} + y'_{Gw}\,\dot{x}'_{Gw}\right) \\ \ddot{q}'_6 \left(x'^2_{Gw} + y'^2_{Gw}\right) + 2\,\dot{q}'_6 \left(x'_{Gw}\,\dot{x}'_{Gw} + y'_{Gw}\,\dot{y}'_{Gw}\right) - \ddot{q}'_4\,z'_{Gw}\,x'_{Gw} - \dot{q}'_4 \left(\dot{z}'_{Gw}\,x'_{Gw} + z'_{Gw}\,\dot{x}'_{Gw}\right) + \\ - \ddot{q}'_5\,z'_{Gw}\,y'_{Gw} - \dot{q}'_5 \left(\dot{z}'_{Gw}\,y'_{Gw} + z'_{Gw}\,\dot{y}'_{Gw}\right) \end{pmatrix} \right]$$

i.e., expanding each term:

$$\begin{aligned} M'_1 = {} & I_{x'x'}\,\ddot{q}'_4 + \left(I_{z'z'} - I_{y'y'}\right)\dot{q}'_5\,\dot{q}'_6 + \\ & + \dot{m}_w \left(y'_{Gw}\,\dot{q}'_3 - z'_{Gw}\,\dot{q}'_2 + y'_{Gw}\,\dot{z}'_{Gw} - z'_{Gw}\,\dot{y}'_{Gw} + \dot{q}'_4 \left(y'^2_{Gw} + z'^2_{Gw}\right) - \dot{q}'_5\,x'_{Gw}\,y'_{Gw} - \dot{q}'_6\,z'_{Gw}\,x'_{Gw}\right) + \\ & + m_w \left(\dot{y}'_{Gw}\,\dot{q}'_3 + y'_{Gw}\,\ddot{q}'_3 - \dot{z}'_{Gw}\,\dot{q}'_2 - z'_{Gw}\,\ddot{q}'_2 + y'_{Gw}\,\ddot{z}'_{Gw} - z'_{Gw}\,\ddot{y}'_{Gw}\right) + \\ & + m_w \left(\ddot{q}'_4 \left(y'^2_{Gw} + z'^2_{Gw}\right) + 2\,\dot{q}'_4 \left(y'_{Gw}\,\dot{y}'_{Gw} + z'_{Gw}\,\dot{z}'_{Gw}\right) - \ddot{q}'_5\,x'_{Gw}\,y'_{Gw} - \dot{q}'_5 \left(\dot{x}'_{Gw}\,y'_{Gw} + x'_{Gw}\,\dot{y}'_{Gw}\right)\right) + \\ & + m_w \left(-\ddot{q}'_6\,x'_{Gw}\,z'_{Gw} - \dot{q}'_6 \left(\dot{x}'_{Gw}\,z'_{Gw} + x'_{Gw}\,\dot{z}'_{Gw}\right)\right) \end{aligned}$$

$$M'_2 = \cdots$$

This set of equations will be referred to as equations **(10.1)**.

Equations **(9.1)** and **(10.1)** form a system of six differential equations in the six unknowns $\dot{q}'_1$, $\dot{q}'_2$, $\dot{q}'_3$, $\dot{q}'_4$, $\dot{q}'_5$ and $\dot{q}'_6$. The remaining terms are:

$\vec{F}'=(F'_1, F'_2, F'_3)$	: sum of all forces applied to the system (ship + flood water);
$\vec{M}'_{Gs}=(M'_1, M'_2, M'_3)$	: sum of all moments of the forces applied to the system (ship + flood water), with respect to the centre of gravity of the intact ship;
$m_s, m_w, \dot{m}_w$	: respectively mass of the intact ship, mass of the flood water and rate of flooding of the damaged compartments;
$\vec{x}'_{Gw}=(x'_{Gw}, y'_{Gw}, z'_{Gw})$	: position of the centre of gravity of the flood water with respect to the ship-fixed co-ordinate system;
$\vec{v}'_{Gw}=(\dot{x}'_{Gw}, \dot{y}'_{Gw}, \dot{z}'_{Gw})$	: velocity of the centre of gravity of the flood water;
$\dot{\vec{v}}'_{Gw}=(\ddot{x}'_{Gw}, \ddot{y}'_{Gw}, \ddot{z}'_{Gw})$	: acceleration of the C.G. of the flood water;
$I_{x'x'}, I_{y'y'}, I_{z'z'}$	: principal moments of inertia of the intact ship with respect to its centre of gravity.

All vector quantities above are expressed with respect to the ship-fixed system of reference $Gs\,x'y'z'$.

Note that equations **(9.1)** and **(10.1)** are highly non-linear. Naturally, they could be integrated in the present form, yet linearisation will provide a simpler set to use so that computing time would reduce greatly. In order to proceed with such operation a few assumptions must be made, but before this equations **(9.1)** and **(10.1)** need to be rearranged in the following way. First all acceleration terms ($\ddot{q}'_i$, $i=1..6$) as well as all velocity terms at first order ($\dot{q}'_i$, $i=1..6$) are factorised, then the remaining terms are grouped. The result will be:

$$
\begin{aligned}
F_1' = {} & (m_s+m_w)\,\ddot{q}'_1 + m_w\, z'_{Gw}\,\ddot{q}'_5 - m_w\, y'_{Gw}\,\ddot{q}'_6 + \\
& + \dot{m}_w\,\dot{q}'_1 + \left(\dot{m}_w z'_{Gw} + 2\, m_w \dot{z}'_{Gw}\right)\dot{q}'_5 - \left(\dot{m}_w y'_{Gw} + 2\, m_w \dot{y}'_{Gw}\right)\dot{q}'_6 + \\
& + \dot{m}_w \dot{x}'_{Gw} + m_w \ddot{x}'_{Gw} + \\
& + (m_s+m_w)\left(\dot{q}'_5\,\dot{q}'_3 - \dot{q}'_6\,\dot{q}'_2\right) + m_w\left(\dot{q}'_5\,\dot{q}'_4\, y'_{Gw} - \dot{q}'^{\,2}_5\, x'_{Gw} - \dot{q}'^{\,2}_6\, x'_{Gw} + \dot{q}'_6\,\dot{q}'_4\, z'_{Gw}\right)
\end{aligned}
$$

$$
F_2' = (m_s+m_w)\,\ddot{q}'_2 + m_w x'_{Gw}\,\ddot{q}'_6 - m_w z'_{Gw}\,\ddot{q}'_4 + \cdots
$$

$$
\begin{aligned}
M'_1 = {} & -m_w z'_{Gw}\,\ddot{q}'_2 + m_w y'_{Gw}\,\ddot{q}'_3 + \left[I_{x'x'} + m_w\left(y'^2_{Gw} + z'^2_{Gw}\right)\right]\ddot{q}'_4 - m_w x'_{Gw}\, y'_{Gw}\,\ddot{q}'_5 - m_w x'_{Gw}\, z'_{Gw}\,\ddot{q}'_6 + \\
& - \left(\dot{m}_w z'_{Gw} + m_w \dot{z}'_{Gw}\right)\dot{q}'_2 + \left(\dot{m}_w y'_{Gw} + m_w \dot{y}'_{Gw}\right)\dot{q}'_3 + \\
& + \left(\dot{m}_w\left(y'^2_{Gw} + z'^2_{Gw}\right) + 2\, m_w\left(y'_{Gw}\,\dot{y}'_{Gw} + z'_{Gw}\,\dot{z}'_{Gw}\right)\right)\dot{q}'_4 + \\
& - \left(\dot{m}_w x'_{Gw}\, y'_{Gw} + m_w\left(\dot{x}'_{Gw}\, y'_{Gw} + x'_{Gw}\,\dot{y}'_{Gw}\right)\right)\dot{q}'_5 + \\
& - \left(\dot{m}_w z'_{Gw}\, x'_{Gw} + m_w\left(\dot{x}'_{Gw}\, z'_{Gw} + x'_{Gw}\,\dot{z}'_{Gw}\right)\right)\dot{q}'_6 + \\
& + \dot{m}_w\left(y'_{Gw}\,\dot{z}'_{Gw} - z'_{Gw}\,\dot{y}'_{Gw}\right) + m_w\left(y'_{Gw}\,\ddot{z}'_{Gw} - z'_{Gw}\,\ddot{y}'_{Gw}\right) + \\
& + \left(I_{z'z'} - I_{y'y'}\right)\dot{q}'_5\,\dot{q}'_6
\end{aligned}
$$

$$
M'_2 = -m_w x'_{Gw}\,\ddot{q}'_3 + m_w z'_{Gw}\,\ddot{q}'_1 + \left[I_{y'y'} + m_w\left(z'^2_{Gw} + x'^2_{Gw}\right)\right]\ddot{q}'_5 - m_w y'_{Gw}\, z'_{Gw}\,\ddot{q}'_6 + \cdots
$$

The equations above are now put in a form which allows a better understanding of the various terms. Considering the first equation above, it may be noticed that the last line contains all second order velocity terms, whilst the last but one contains terms representing the force and moment due to the flood water. Since the behaviour of the Gw is supposed to be known, these terms could be rightly moved to the L.H.S. of the equations. Cross product terms involving the flood water CG and the ship velocities are present in the second line. The first assumption made here is that Gw velocity terms are of the same order as those of the ship. This assumption holds either considering the water moving in phase with the ship, or by implementing the accuracy of the model with an empirical method to simulate the water behaviour in the flooded compartments.

The typical linear assumption will then be applied to all second order cross terms containing elements of the ship and/or the flood water centre of gravity velocities vectors, by neglecting them. This is supported by the fact that the ship velocity vector is typically small for the type of vessels and the scenarios considered, especially if no forward speed is taken into consideration. Ignoring the last line and all the other second order velocity terms, the first equation above becomes:

$$F_1'-\dot{m}_w\dot{x}'_{Gw}-m_w\ddot{x}'_{Gw}=\left(m_s+m_w\right)\ddot{q}'_1+m_w z'_{Gw}\ddot{q}'_5-m_w y'_{Gw}\ddot{q}'_6+\dot{m}_w\dot{q}'_1+\dot{m}_w z'_{Gw}\dot{q}'_5-\dot{m}_w y'_{Gw}\dot{q}'_6$$

similarly the remaining **(9.1)** and **(10.1)** equations become:

$$F_2'-\dot{m}_w\dot{y}_{Gw}-m_w\ddot{y}_{Gw}=\left(m_s+m_w\right)\ddot{q}'_2+m_w x'_{Gw}\ddot{q}'_6-m_w z'_{Gw}\ddot{q}'_4+\dot{m}_w\dot{q}'_2+\dot{m}_w x'_{Gw}\dot{q}'_6-\dot{m}_w z'_{Gw}\dot{q}'_4$$

$$F_3'-\dot{m}_w\dot{z}_{Gw}-m_w\ddot{z}_{Gw}=\left(m_s+m_w\right)\ddot{q}'_3+m_w y'_{Gw}\ddot{q}'_4-m_w x'_{Gw}\ddot{q}'_5+\dot{m}_w\dot{q}'_3+\dot{m}_w y'_{Gw}\dot{q}'_4-\dot{m}_w x'_{Gw}\dot{q}'_5$$

$$\begin{aligned}M'_1-&\left(\dot{m}_w\left(y'_{Gw}\dot{z}'_{Gw}-z'_{Gw}\dot{y}'_{Gw}\right)+m_w\left(y'_{Gw}\ddot{z}'_{Gw}-z'_{Gw}\ddot{y}'_{Gw}\right)\right)=\\&-m_w z'_{Gw}\ddot{q}'_2+m_w y'_{Gw}\ddot{q}'_3+\left[I_{x'x'}+m_w\left(y'^2_{Gw}+z'^2_{Gw}\right)\right]\ddot{q}'_4-m_w x'_{Gw}y'_{Gw}\ddot{q}'_5-m_w x'_{Gw}z'_{Gw}\ddot{q}'_6+\\&-\dot{m}_w z'_{Gw}\dot{q}'_2+\dot{m}_w y'_{Gw}\dot{q}'_3+\dot{m}_w\left(y'^2_{Gw}+z'^2_{Gw}\right)\dot{q}'_4-\dot{m}_w x'_{Gw}y'_{Gw}\dot{q}'_5-\dot{m}_w z'_{Gw}x'_{Gw}\dot{q}'_6\end{aligned}$$

$$\begin{aligned}M'_2-&\left(\dot{m}_w\left(z'_{Gw}\dot{x}'_{Gw}-x'_{Gw}\dot{z}'_{Gw}\right)+m_w\left(z'_{Gw}\ddot{x}'_{Gw}-x'_{Gw}\ddot{z}'_{Gw}\right)\right)=\\&-m_w x'_{Gw}\ddot{q}'_3+m_w z'_{Gw}\ddot{q}'_1+\left[I_{y'y'}+m_w\left(z'^2_{Gw}+x'^2_{Gw}\right)\right]\ddot{q}'_5-m_w y'_{Gw}z'_{Gw}\ddot{q}'_6-m_w y'_{Gw}x'_{Gw}\ddot{q}'_4+\\&-\dot{m}_w x'_{Gw}\dot{q}'_3+\dot{m}_w z'_{Gw}\dot{q}'_1+\dot{m}_w\left(z'^2_{Gw}+x'^2_{Gw}\right)\dot{q}'_5-\dot{m}_w y'_{Gw}z'_{Gw}\dot{q}'_6-\dot{m}_w x'_{Gw}y'_{Gw}\dot{q}'_4\end{aligned}$$

$$\begin{aligned}M'_3-&\left(\dot{m}_w\left(x'_{Gw}\dot{y}'_{Gw}-y'_{Gw}\dot{x}'_{Gw}\right)+m_w\left(x'_{Gw}\ddot{y}'_{Gw}-y'_{Gw}\ddot{x}'_{Gw}\right)\right)=\\&-m_w y'_{Gw}\ddot{q}'_1+m_w x'_{Gw}\ddot{q}'_2+\left[I_{z'z'}+m_w\left(x'^2_{Gw}+y'^2_{Gw}\right)\right]\ddot{q}'_6-m_w z'_{Gw}x'_{Gw}\ddot{q}'_4-m_w z'_{Gw}y'_{Gw}\ddot{q}'_5+\\&-\dot{m}_w y'_{Gw}\dot{q}'_1+\dot{m}_w x'_{Gw}\dot{q}'_2+\dot{m}_w\left(x'^2_{Gw}+y'^2_{Gw}\right)\dot{q}'_6-\dot{m}_w z'_{Gw}x'_{Gw}\dot{q}'_4-\dot{m}_w y'_{Gw}z'_{Gw}\dot{q}'_5\end{aligned}$$

which can be put in the matrix form:

$$\text{(11.1)} \qquad \vec{F}'^{*} - \vec{F}'^{*}{}_{w} = \left[M + M_w\right]\ddot{\vec{q}}' + \left[\dot{M}_w\right]\dot{\vec{q}}'$$

where:

$$\vec{F}'^{*} = \begin{pmatrix} \vec{F}' \\ \vec{M}'_{Gs} \end{pmatrix},$$

$$\vec{F}'^{*}{}_{w} = \begin{pmatrix} \dot{m}_w \dot{x}'_{Gw} + m_w \ddot{x}'_{Gw} \\ \dot{m}_w \dot{y}'_{Gw} + m_w \ddot{y}'_{Gw} \\ \dot{m}_w \dot{z}'_{Gw} + m_w \ddot{z}'_{Gw} \\ \dot{m}_w \left(y'_{Gw}\dot{z}'_{Gw} - z'_{Gw}\dot{y}'_{Gw}\right) + m_w \left(y'_{Gw}\ddot{z}'_{Gw} - z'_{Gw}\ddot{y}'_{Gw}\right) \\ \dot{m}_w \left(z'_{Gw}\dot{x}'_{Gw} - x'_{Gw}\dot{z}'_{Gw}\right) + m_w \left(z'_{Gw}\ddot{x}'_{Gw} - x'_{Gw}\ddot{z}'_{Gw}\right) \\ \dot{m}_w \left(x'_{Gw}\dot{y}'_{Gw} - y'_{Gw}\dot{x}'_{Gw}\right) + m_w \left(x'_{Gw}\ddot{y}'_{Gw} - y'_{Gw}\ddot{x}'_{Gw}\right) \end{pmatrix},$$

$$[M] = \begin{bmatrix} m_s & 0 & 0 & 0 & 0 & 0 \\ 0 & m_s & 0 & 0 & 0 & 0 \\ 0 & 0 & m_s & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{x'x'} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{y'y'} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{z'z'} \end{bmatrix},$$

$$\left[M_w\right] = \begin{bmatrix} m_w & 0 & 0 & 0 & m_w z'_{Gw} & -m_w y'_{Gw} \\ 0 & m_w & 0 & -m_w z'_{Gw} & 0 & m_w x'_{Gw} \\ 0 & 0 & m_w & m_w y'_{Gw} & -m_w x'_{Gw} & 0 \\ 0 & -m_w z'_{Gw} & m_w y'_{Gw} & m_w\left({y'}^2_{Gw} + {z'}^2_{Gw}\right) & -m_w x'_{Gw} y'_{Gw} & -m_w z'_{Gw} x'_{Gw} \\ m_w z'_{Gw} & 0 & -m_w x'_{Gw} & -m_w x'_{Gw} y'_{Gw} & m_w\left({z'}^2_{Gw} + {x'}^2_{Gw}\right) & -m_w y'_{Gw} z'_{Gw} \\ -m_w y'_{Gw} & m_w x'_{Gw} & 0 & -m_w z'_{Gw} x'_{Gw} & -m_w y'_{Gw} z'_{Gw} & m_w\left({x'}^2_{Gw} + {y'}^2_{Gw}\right) \end{bmatrix}$$

and

$$\left[\dot{M}_w\right] = \begin{bmatrix} \dot{m}_w & 0 & 0 & 0 & \dot{m}_w z'_{Gw} & -\dot{m}_w y'_{Gw} \\ 0 & \dot{m}_w & 0 & -\dot{m}_w z'_{Gw} & 0 & \dot{m}_w x'_{Gw} \\ 0 & 0 & \dot{m}_w & \dot{m}_w y'_{Gw} & -\dot{m}_w x'_{Gw} & 0 \\ 0 & -\dot{m}_w z'_{Gw} & \dot{m}_w y'_{Gw} & \dot{m}_w\left({y'}^2_{Gw} + {z'}^2_{Gw}\right) & -\dot{m}_w x'_{Gw} y'_{Gw} & -\dot{m}_w z'_{Gw} x'_{Gw} \\ \dot{m}_w z'_{Gw} & 0 & -\dot{m}_w x'_{Gw} & -\dot{m}_w x'_{Gw} y'_{Gw} & \dot{m}_w\left({z'}^2_{Gw} + {x'}^2_{Gw}\right) & -\dot{m}_w y'_{Gw} z'_{Gw} \\ -\dot{m}_w y'_{Gw} & \dot{m}_w x'_{Gw} & 0 & -\dot{m}_w z'_{Gw} x'_{Gw} & -\dot{m}_w y'_{Gw} z'_{Gw} & \dot{m}_w\left({x'}^2_{Gw} + {y'}^2_{Gw}\right) \end{bmatrix}.$$

When more than one compartment are involved, the expressions above can be readily shown to become:

$$
\vec{F}'^{*}_{w} = \begin{pmatrix}
\sum_{j=1}^{N} \left(\dot{m}_{wj} \dot{x}'_{Gwj} + m_{wj} \ddot{x}'_{Gwj} \right) \\
\sum_{j=1}^{N} \left(\dot{m}_{wj} \dot{y}'_{Gwj} + m_{wj} \ddot{y}'_{Gwj} \right) \\
\sum_{j=1}^{N} \left(\dot{m}_{wj} \dot{z}'_{Gwj} + m_{wj} \ddot{z}'_{Gwj} \right) \\
\sum_{j=1}^{N} \left(\dot{m}_{wj} \left(y'_{Gwj} \dot{z}'_{Gwj} - z'_{Gwj} \dot{y}'_{Gwj} \right) + m_{wj} \left(y'_{Gwj} \ddot{z}'_{Gwj} - z'_{Gwj} \ddot{y}'_{Gwj} \right) \right) \\
\sum_{j=1}^{N} \left(\dot{m}_{wj} \left(z'_{Gwj} \dot{x}'_{Gwj} - x'_{Gwj} \dot{z}'_{Gwj} \right) + m_{wj} \left(z'_{Gwj} \ddot{x}'_{Gwj} - x'_{Gwj} \ddot{z}'_{Gwj} \right) \right) \\
\sum_{j=1}^{N} \left(\dot{m}_{wj} \left(x'_{Gwj} \dot{y}'_{Gwj} - y'_{Gwj} \dot{x}'_{Gwj} \right) + m_{wj} \left(x'_{Gwj} \ddot{y}'_{Gwj} - y'_{Gwj} \ddot{x}'_{Gwj} \right) \right)
\end{pmatrix},
$$

$$
\left[M_w \right] = \begin{bmatrix}
\sum_{j=1}^{N} m_{wj} & 0 & 0 & 0 & \sum_{j=1}^{N} m_{wj} z'_{Gwj} & -\sum_{j=1}^{N} m_{wj} y'_{Gwj} \\
0 & \sum_{j=1}^{N} m_{wj} & 0 & -\sum_{j=1}^{N} m_{wj} z'_{Gwj} & 0 & \sum_{j=1}^{N} m_{wj} x'_{Gwj} \\
0 & 0 & \sum_{j=1}^{N} m_{wj} & \sum_{j=1}^{N} m_{wj} y'_{Gwj} & -\sum_{j=1}^{N} m_{wj} x'_{Gwj} & 0 \\
0 & -\sum_{j=1}^{N} m_{wj} z'_{Gwj} & \sum_{j=1}^{N} m_{wj} y'_{Gwj} & \sum_{j=1}^{N} m_{wj} \left(y'^{2}_{Gwj} + z'^{2}_{Gwj} \right) & -\sum_{j=1}^{N} m_{wj} x'_{Gwj} y'_{Gwj} & -\sum_{j=1}^{N} m_{wj} z'_{Gwj} x'_{Gwj} \\
\sum_{j=1}^{N} m_{wj} z'_{Gwj} & 0 & -\sum_{j=1}^{N} m_{wj} x'_{Gwj} & -\sum_{j=1}^{N} m_{wj} x'_{Gwj} y'_{Gwj} & \sum_{j=1}^{N} m_{wj} \left(z'^{2}_{Gwj} + x'^{2}_{Gwj} \right) & -\sum_{j=1}^{N} m_{wj} y'_{Gwj} z'_{Gwj} \\
-\sum_{j=1}^{N} m_{wj} y'_{Gwj} & \sum_{j=1}^{N} m_{wj} x'_{Gwj} & 0 & -\sum_{j=1}^{N} m_{wj} z'_{Gwj} x'_{Gwj} & -\sum_{j=1}^{N} m_{wj} y'_{Gwj} z'_{Gwj} & \sum_{j=1}^{N} m_{wj} \left(x'^{2}_{Gwj} + y'^{2}_{Gwj} \right)
\end{bmatrix}
$$

and

$$
\left[\dot{M}_w \right] = \begin{bmatrix}
\sum_{j=1}^{N} \dot{m}_{wj} & 0 & 0 & 0 & \sum_{j=1}^{N} \dot{m}_{wj} z'_{Gwj} & -\sum_{j=1}^{N} \dot{m}_{wj} y'_{Gwj} \\
0 & \sum_{j=1}^{N} \dot{m}_{wj} & 0 & -\sum_{j=1}^{N} \dot{m}_{wj} z'_{Gwj} & 0 & \sum_{j=1}^{N} \dot{m}_{wj} x'_{Gwj} \\
0 & 0 & \sum_{j=1}^{N} \dot{m}_{wj} & \sum_{j=1}^{N} \dot{m}_{wj} y'_{Gwj} & -\sum_{j=1}^{N} \dot{m}_{wj} x'_{Gwj} & 0 \\
0 & -\sum_{j=1}^{N} \dot{m}_{wj} z'_{Gwj} & \sum_{j=1}^{N} \dot{m}_{wj} y'_{Gwj} & \sum_{j=1}^{N} \dot{m}_{wj} \left(y'^{2}_{Gwj} + z'^{2}_{Gwj} \right) & -\sum_{j=1}^{N} \dot{m}_{wj} x'_{Gwj} y'_{Gwj} & -\sum_{j=1}^{N} \dot{m}_{wj} z'_{Gwj} x'_{Gwj} \\
\sum_{j=1}^{N} \dot{m}_{wj} z'_{Gwj} & 0 & -\sum_{j=1}^{N} \dot{m}_{wj} x'_{Gwj} & -\sum_{j=1}^{N} \dot{m}_{wj} x'_{Gwj} y'_{Gwj} & \sum_{j=1}^{N} \dot{m}_{wj} \left(z'^{2}_{Gwj} + x'^{2}_{Gwj} \right) & -\sum_{j=1}^{N} \dot{m}_{wj} y'_{Gwj} z'_{Gwj} \\
-\sum_{j=1}^{N} \dot{m}_{wj} y'_{Gwj} & \sum_{j=1}^{N} \dot{m}_{wj} x'_{Gwj} & 0 & -\sum_{j=1}^{N} \dot{m}_{wj} z'_{Gwj} x'_{Gwj} & -\sum_{j=1}^{N} \dot{m}_{wj} y'_{Gwj} z'_{Gwj} & \sum_{j=1}^{N} \dot{m}_{wj} \left(x'^{2}_{Gwj} + y'^{2}_{Gwj} \right)
\end{bmatrix}
$$

where the index j points each individual flooded compartment and N is their total number. In equation **(11.1)** the *damping* nature of the matrix of the rate of change of inertia $\left[\dot{M}_w \right]$ becomes evident. It must be noted, however, that this does not necessarily mean that the flooding water will smother the ship motions. It is in fact clear that the sign of the terms in $\left[\dot{M}_w\right]$ can easily be negative, and that this would in fact exacerbate motions rather than reduce them.

Of course, similar considerations apply to the matrix $\left[M_w\right]$. Indeed, when the sign of $\left[\dot{M}_w\right]$ is negative, $\left[M_w\right]$ becomes smaller and so does the total inertia of the system. It can be expected that a reduced inertia will ease the resistance of the system to be excited by the external forces and therefore, in so doing, meet and enhance the effect of a negative $\left[\dot{M}_w\right]$.

This double nature of the action of flooding water results in a very important conclusion: considering the effect of the varying mass of flood water on the system behaviour only by updating the total inertia of the ship with that of the water accumulating in each damaged compartment is wrong and could lead to underestimating or overestimating the ship motions.

The above equation **(11.1)** and associated matrices is the set of formulae which will be adopted in the time simulation. In the following chapters the terms $\vec{F}'$ and $\vec{M}'_{Gs}$ will be expanded and commented on, and details will be given on the methodology leading to a numerical algorithm which will use the equations just specified to compute the quantities of interest, namely the ship position in the inertial system $O\ XYZ$ and the roll/heel angle time histories.

6. Analysis of the Forces and Moments Acting on a Damaged Ship

General Remarks

In the previous chapter equations of motion for a damaged ship subjected to progressive flooding were derived. It is now necessary to clarify what the vectors $\vec{F}'$ and $\vec{M}'_{Gs}$ represent and to explain how to calculate them. Equations **(9.1)**, **(10.1)** and **(11.1)** are expressed in ship fixed co-ordinates. It is therefore in this reference system that the terms $\vec{F}'$ and $\vec{M}'_{Gs}$ will have to be studied. In the most general case:

$$\vec{F}'^{*} = \begin{pmatrix} \vec{F}' \\ \vec{M}'_{Gs} \end{pmatrix} = \vec{F}'_E + \vec{F}'_H + \vec{F}'_R + \vec{F}'_G + \vec{F}'_O$$

where:

$\vec{F}'_E$ is the generalised excitation vector. This arises from the action of wind, current and waves. It is a function of time, the position of the ship in the inertial system, its heading, etc.;

$\vec{F}'_H$ is the generalised hydrodynamic reaction vector. The forces and moments due to the propagating and the standing parts constituting the radiating wave, are included in this term, as well as non-linear viscous components of roll damping;

$\vec{F}'_R$ is the generalised restoring vector. It is generated by the faculty of buoyancy of a vessel and as such it depends on the volume of water displaced by this. It is mainly a function of $\vec{x}'_{Gs}$ and $\vec{\alpha}$;

$\vec{F}'_G$ is the gravitational force vector;

$\vec{F}'_O$ are other forces and moments (thrust, rudder, resistance etc.). These are usually neglected when dealing with the damaged ship problem. Note that if the investigation focused on the influence of forward motion after a ship is damaged, all the non-linear terms in the equations of motion should be retained. However, this is not a part of the current investigation.

All the quantities above will be analysed one by one in the following pages.

Irregular Waves and Spectral Techniques

So far, in attempting to investigate the behaviour of a damaged ship in a seaway, mathematical equations were formulated to simulate the ship's motion, which conform to the actual case in a realistic way. Therefore, the sea model adopted will also have to be as realistic as possible. This means that spectral techniques will have to be employed to provide a statistically meaningful representation of a confused sea. To keep the model uncomplicated, though, a few assumptions need to be made, starting with the normally adopted assumption that a two dimensional irregular wave propagating along the X axis of the inertial system $O\,XYZ$, is a good enough compromise between completeness and simplicity.

Such a wave is represented by the equation:

(1.2) $$\zeta(t)=\sum_{j=1}^{N} A_j \cdot \cos\left(\omega_j t - k_j x + \varepsilon_j\right)$$

where $\zeta(t)$ is the irregular wave elevation, A_j, ω_j and k_j are the component wave amplitudes, circular frequencies and wave numbers, respectively, and ε_j are normally uniformly distributed random phase angles. Naturally, x and t represent the abscissa of the position considered along the X axis of the earth system and the time variable, respectively. N is the total number of component waves, taken as large as possible.

Regarding this, it must be said that an infinite number of component waves should in fact be utilised to ensure that the irregular wave profile would not ever repeat itself, i.e. [5.9]:

(1.2 a) $$\zeta(t)=\int_{0}^{\infty} \cos\left(\omega t - k(\omega) x + \varepsilon(\omega)\right) \cdot \sqrt{2 S(\omega) d\omega}$$

in which $S(\omega)$ is a wave spectrum. In this context it needs to be ensured that the wave realisation will not repeat itself as long as it is necessary for the simulation considered. Therefore, N needs to be large enough to satisfy the relations:

$$N=(\omega_{\max}-\omega_{\min})/\Delta\omega \qquad \text{and} \qquad T_{rep}=2\pi/\Delta\omega$$

where T_{rep} is the repeat period which must be larger than the total duration necessary for the simulation, $\Delta\omega$ is the circular frequency interval used to derive the component waves characteristics from the given spectrum and $\omega_{\max}$ and $\omega_{\min}$ are the boundaries of the frequency range taken into consideration. This is chosen to be between 0.2 to 2 rad/sec to give a good enough portrait of most spectra.

Taking 30 minutes as the repeat period (experiments have shown that if a ship does not capsize during the first half hour, it is unlikely that it will capsize at all [2.8][2.9]) the frequency interval will be about 0.003 rad/sec and $N = 600$. These values are in fact generally regarded as those referring to the time duration upper limit for a physically reasonable representation of a sea state, given the ergotic hypothesis will not normally stand for longer. Equation **(1.2)** will thus change to:

(1.2 b) $$\zeta(t) = \sum_{j=0}^{600} \sqrt{2\, S(\omega_j)\, 0.003} \cdot \cos\left(\omega_j\, t - k_j\, x + \varepsilon_j\right)$$

where $\omega_j = 0.2 + j\,0.003$,

$k_j = f(\omega_j)$ f representing the dispersion relation and

$\varepsilon_j = 2\pi\, R_N$ in which R_N is a random number generator.

The wave spectra considered here are the Pierson-Moscowitz (P-M) spectrum and the Joint North Sea Wave Project spectrum (JONSWAP), which are commonly regarded as a good representation of open sea (unlimited fetch conditions) and short crested seas (limited fetch conditions) respectively.

The P-M spectrum is given by the expression:

(2.2) $$S(\omega) = \frac{A}{\omega^5} \cdot \exp\left(-B/\omega^4\right)$$

where:

$A = 8.1 \cdot 10^{-3} \cdot g^2$ and

$B = 0.74 \cdot \left(g/V_{19.5}\right)^4$.

In the equation above, $V_{19.5}$ is the wind speed at 19.5 m above the still water level in m/sec.

The following relation can be found by imposing the first derivative of the spectrum with respect to ω to be zero:

$$\omega_0 = \left(\frac{4}{5}B\right)^{1/4}$$

which is the peak frequency (i.e. the frequency at which the spectrum attains its maximum), thus:

$$B = \frac{5}{4}\omega_0^{\,4}$$

which can be used to find an alternative presentation for the P-M spectrum:

(2.2 a) $$S(\omega) = \alpha \cdot \frac{g^2}{\omega^5} \cdot \exp\left[-\frac{5}{4}\left(\frac{\omega_0}{\omega}\right)^4\right], \quad \alpha = 8.1 \cdot 10^{-3} \quad \text{(Phillip's constant)}$$

Expressions for the significant wave height and the mean zero crossing period can be found as follows:

$$H_s = 4\,\sigma = 4\,\sqrt{m_0} = 4\,\sqrt{\int_0^\infty S(\omega)\,d\omega} = 4\,\sqrt{\frac{\alpha\,g^2}{5\,\omega_0^4}} \cong \frac{1.58}{\omega_0^2} \quad \text{and}$$

$$T_z = \sqrt{\frac{m_0}{m_2}} = \sqrt{\frac{\int_0^\infty S(\omega)\,d\omega}{\int_0^\infty \omega^2\,S(\omega)\,d\omega}} = \sqrt{\frac{\frac{\alpha\,g^2}{5\,\omega_0^4}}{\left(\frac{4}{5}\pi^5\right)^{\frac{1}{2}} \cdot \frac{\alpha\,g^2}{(2\pi\,\omega_0)^2}}} = \frac{1}{\omega_0}\sqrt{2/\sqrt{5\pi}} \cong \frac{0.71}{\omega_0}$$

Hence this spectrum is completely identified by one of the following quantities: T_z, H_s, ω_0 or $V_{19.5}$.

The expression **(2.2 a)** is modified as follows to obtain the JONSWAP spectrum [1.4]:

(3.2) $$S(\omega)=\alpha\cdot\frac{g^2}{\omega^5}\cdot\exp\left[-\frac{5}{4}\left(\frac{\omega_0}{\omega}\right)^4\right]\cdot\gamma^{\exp\left[-\frac{(\omega-\omega_0)^2}{2\cdot\tau^2\cdot\omega_0^2}\right]}$$

in which γ is the peakness parameter, assumed equal to 3.3 for the mean JONSWAP spectrum, although it may vary from 1.0 to 7.0 approximately. γ is defined as the ratio between the height of the spectral peak to that of the corresponding P-M wave spectrum. It is a function of the wind duration and the stage of the growth or decay of the storm and can be found, given H_s and T_z in tables such as those developed by Houmb and Overvik in 1976 [5.3 pp. 135] or those in [5.5]. The shape parameter, τ, is the ratio between the area under the spectrum on either side of ω_0 and the maximum spectral ordinate. Normally $\tau = 0.07$ for $\omega \le \omega_0$ and $\tau = 0.09$ for $\omega > \omega_0$. These are again mean, empirical values, generally valid for the North Sea.

The constant α becomes here: $\alpha = 7.6\cdot 10^{-2}\cdot\left(g\frac{X}{V_{10}^2}\right)^{-\frac{2}{9}}$ if the fetch, X, and the wind velocity at 10 m above the calm water, V_{10}, are known. An alternative empirical formula to evaluate α in terms of the significant wave height, γ and the peak frequency is: $\alpha = 603.9\cdot\left(\frac{H_s\,\omega_0^2}{4\pi^2\,g}\right)^{2.036}\cdot\left(-0.298\log(\gamma)+1.0\right)$.

It is to be pointed out here, that this coefficient is radically different from Phillip's constant, and that the formulae used to evaluate it are only valid within the fetch and sea state ranges used to develop these. It is important to underline the fact that the P-M and the JONSWAP spectra are similar in appearance but not related, meaning that the former cannot be considered as a particular case of the latter.

The peak frequency can be expressed as a function of X as: $\omega_0 = 2\pi\cdot\left(g^2/X\,V_{10}\right)^{\frac{1}{3}}$

or else as: $\omega_0 = \frac{2\pi}{T_z\cdot\left(0.327\cdot\exp(-0.315\,\gamma)+1.17\right)}$ when the use of mean zero crossing period is preferred.

Unlike the P-M, two quantities are needed to specify a JONSWAP spectrum, assuming all other coefficients can be found via the first two. This is the reason why this particular spectrum is often referred to as a two-parameters spectrum. In the formulae above, γ, α and ω_0 can be found either if both wind velocity and fetch are known, or through H_s and T_z. For the purposes of this investigation, the characteristics of either spectrum will be assessed, using significant wave heights and zero crossing periods, giving priority to the first in the case of a P-M spectrum.

Note finally, that significant wave height and zero crossing period are related to each other through statistical records, if, for instance, mean values for a given sea state are accepted. This practically reduces the parameters needed to describe a given JONSWAP spectrum to only one, as in the case of P-M spectra.

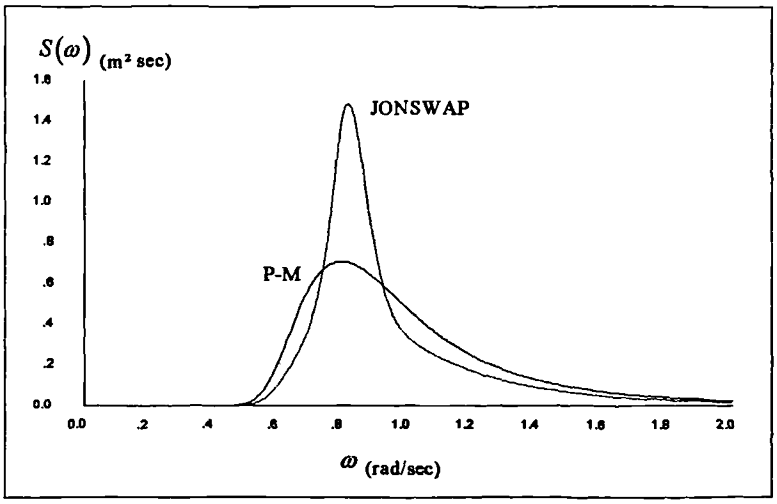

Fig. 1

The figure above shows a comparison between a P-M and a mean JONSWAP spectra, for a given significant wave height of 2.5 metres and a mean zero-crossing period of 6.0 seconds.

Excitation Forces and Moments

First Order Wave Forces and Moments

1. First order wave forces and CFD theory: diffraction, radiation and shallow waters wave potentials.

Undisturbed wave potential.

Each regular component wave used in the spectral definitions illustrated in the previous section, can be described by a linear velocity potential of the form:

$$\Phi_I = -\frac{g\,A_j}{\omega_j}\cdot\frac{\cosh\left[k_j\,(z+h)\right]}{\cosh\left(k_j\,h\right)}\cdot\sin\left(\omega_j\,t-k_j\,x\right) \tag{4.2}$$

where h is the water depth at a given location and z the vertical position of the point where Φ is evaluated. Formula **(4.2)** is expressed with respect to $O\,XYZ$. The wave number, k_j, is linked to the circular frequency through the dispersion relation: $\frac{\omega_j^2}{g}=k_j\,\tanh\left(k_j\,h\right)$ and $k_j=\frac{2\pi}{\lambda}$, where λ is the wave length. Alternatively, the following equivalent expression can be used:

$$\Phi_I = \mathrm{Re}\left[\phi_I\,(x,y,z)\cdot\exp\left(-i\omega_j\,t\right)\right]$$

(5.2)

$$\phi_I = -i\frac{g\,A_j}{\omega_j}\cdot\frac{\cosh\left[k_j\,(z+h)\right]}{\cosh\left(k_j\,h\right)}\cdot\exp\left(i\,k_j\,x\right)$$

Formulae (4.2) and (5.2) represent the incident wave potential for limited water depth, but indeed there is no restriction on the value of h. These relations apply to a greater range of cases than the traditionally used *infinite depth* potentials - which can be seen as special cases of the ones employed here - and were chosen in this context for this very reason. The equations above are solutions of the following boundary value problem:

$$\nabla^2 \phi = 0$$

(Laplace's equation; it describes the behaviour of an irrotational and incompressible fluid.)

$$g\zeta(x,y,t) + \frac{\partial \Phi(x,y,0,t)}{\partial t} = 0$$

(Linearised dynamic boundary condition at the free surface; it expresses the necessity for the pressure at the water surface to be equal to the atmospheric one. Although this condition should be applied at the actual water surface - i.e. at points belonging to the instantaneous wave elevation ζ - it is assumed that the wave amplitude is small enough to allow it to be approximated with the mean free water surface.)

$$\frac{\partial \Phi(x,y,0,t)}{\partial z} = \frac{\partial \zeta(x,y,t)}{\partial t}$$

(Linearised kinematic boundary condition at the free surface; it implies that the vertical velocity of the wave elevation must be equal to that of the water particles. Again this relation should be satisfied at $z = \zeta$, but the above mentioned linearisation is applied here as well.)

which can be combined, to yield

$$\frac{\partial \phi(x,y,0)}{\partial z} - \frac{\omega_j^2}{g}\phi(x,y,0) = 0$$

(Complete linear boundary condition at the free surface.)

$$\frac{\partial \phi(x,y,-h)}{\partial z}=0$$

(Kinematic linear boundary condition at the bottom; this implies that the water particles vertical velocity is zero at the seabed, which is considered horizontal.)

Of the assumptions leading to the above formulation of the problem, the following two are worthy of note: the water is considered irrotational (thus viscosity is neglected) and the wave amplitudes are assumed small. The components of the generalised excitation vector due to the incident wave only, computed by making use of the linearised dynamic pressure $p=-\rho\frac{\partial \Phi_I}{\partial t}$, are known as the Froude-Krylov (F-K) forces and moments. These, of course, do not take into account viscous components. Also, given the *small amplitude* hypothesis just mentioned, the integration of the dynamic pressure is performed up to the mean water level.

Diffracted wave potential.

The validity of the above potential stands when the wave is undisturbed, i.e. if there is no large submerged or surface piercing object in the close vicinity of the location where the wave is observed. Of course this is not the case when it comes to consider a ship, given that the dimensions of seagoing vessels are large enough to change the fluid velocity potential characteristics radically.

To tackle the problem of evaluating wave forces on large structures, diffraction techniques are normally employed, in association with Computational Fluid Dynamics (CFD) numerical procedures. In doing this, viscous forces are still neglected and the diffracted wave velocity potential is found by requiring specific boundary conditions to be satisfied on the sea bed, the water surface and the vessel wetted body.

The boundary value problem describing diffraction of waves by a fixed object can be represented as follows (equations **(6.2)**):

$$\Phi = \mathrm{Re}\left[\phi(x,y,z)\cdot\exp(-i\omega_j\, t)\right]$$
$$\phi = \phi_I + \phi_D$$
(Assumed linear form for the total potential)

$$\nabla^2\phi = 0$$
(Laplace's equation.)

$$\frac{\partial\phi(x,y,0)}{\partial z} - \frac{\omega_j^2}{g}\phi(x,y,0) = 0$$
(Complete linear boundary condition at the free surface.)

$$\frac{\partial\phi(x,y,-h)}{\partial z} = 0$$
(Kinematic linear boundary condition at the bottom.)

$$\frac{\partial\phi(x,y,z)}{\partial n} = 0 \qquad \text{on } S(x,y,z) = 0$$
(Linearised kinematic boundary condition at the hull surface; this implies that the water particles velocity normal to the underwater surface of the hull is nil.)

where n represents the outward unit vector normal to the surface of the underwater part of the hull, which is defined by $S(x,y,z) = 0$.

The undisturbed velocity potential presented above satisfies Laplace's equation and all conditions specifying this problem except the last one. Bearing this in mind, the substitution of the general form for the total potential in the formulae above produces:

$$\nabla^2\phi_D = 0$$
(Laplace's equation.)

$$\frac{\partial\phi_D(x,y,0)}{\partial z} - k_j\tanh(k_j\, h)\,\phi_D(x,y,0) = 0$$
(Complete linear boundary condition at the free surface, taking into account the dispersion relation.)

$$\frac{\partial\phi_D(x,y,-h)}{\partial z} = 0$$
(Kinematic linear boundary condition at the bottom.)

$$\frac{\partial \phi_D(x,y,z)}{\partial n} = -\frac{\partial \phi_I(x,y,z)}{\partial n} = h(x,y,z)$$ (Linearised kinematic boundary condition at the hull surface.)

on $S(x,y,z)=0$

$h(x,y,z)$ can be shown to be:

$$h(x,y,z) = -\frac{g\,A_j\,k_j}{\omega_j}\cdot\left[n_x\frac{\cosh\left[k_j\,(z+h)\right]}{\cosh(k_j\,h)} + i\,n_z\frac{\sinh\left[k_j\,(z+h)\right]}{\cosh(k_j\,h)}\right]\cdot\exp(i\,k_j\,x)$$

and n_x, n_z are the components of n in the x and z directions.

In addition to the above, the following condition assures that the diffracted wave potential will approach the form of an outgoing progressive wave at infinity:

$$\lim_{r'\to\infty}\phi_D(r',\theta,z) = C(\theta)\,r'^{-1/2}\cdot\frac{\cosh\left[k_j(z+h)\right]}{\cosh(k_j\,h)}\cdot\exp(i\,k_j\,r')$$

in which $r' = \sqrt{x^2+y^2}$ $\theta = \arctan(y/x)$ and $C(\theta)$ denotes a function of θ only.

The boundary value problem established and described by the foregoing formulae admits solutions of the form:

$$\phi_D(x,y,z) = \frac{1}{4\pi}\cdot\iint_S f(\xi,\eta,\varsigma)\cdot G(x,y,z;\xi,\eta,\varsigma)\,dS$$

where ξ,η,ς are the co-ordinates of a point on the underwater surface, $f(\xi,\eta,\varsigma)$ is the strength of the sources distributed over the wetted hull and $G(x,y,z;\xi,\eta,\varsigma)$ is the Green function for the problem that characterises the potential at point (x,y,z) due to a source at (ξ,η,ς).

This solution has no close form and must be evaluated numerically by discretising the vessel hull as in fig. 2, for instance, distributing a finite number of sources at the centroid of each panel.

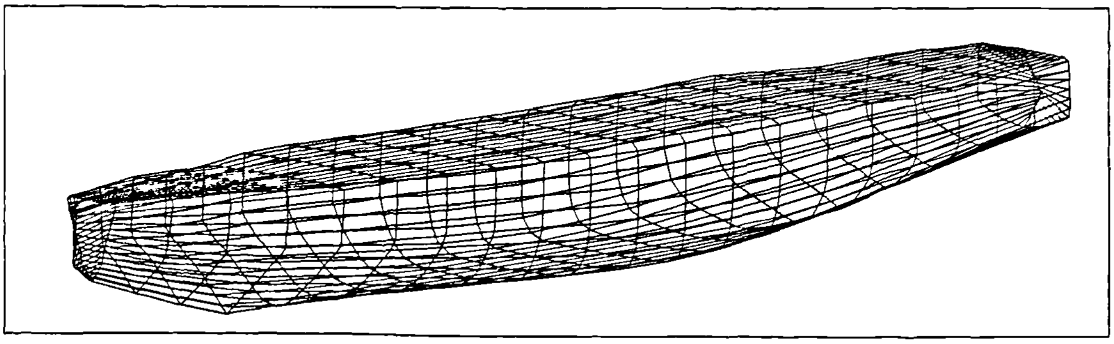

Fig. 2

Complete panellised geometry of a typical hull.

Once the velocity potential for the diffracted wave is found, the wave forces and moments acting on the vessel can be calculated by integrating the total dynamic pressure $p = -\rho \dfrac{\partial \Phi}{\partial t}$ over the underwater body. The generalised vector containing these, will be referenced as $\vec{F}_W$. The considerations about viscosity and linearity mentioned for F-K forces, apply here as well.

Radiated wave potentials.

Diffraction theory assumes the body to be fixed in space. This is adequate for many marine applications, like oil concrete platforms and underwater reservoirs, but certainly does not apply to floating vessels. The ship motion response to the wave excitation will introduce a second disturbance to the incident wave potential, in addition to diffraction. This will result in extra velocity potentials to be added to ϕ_I and ϕ_D; the components due to a body oscillating at the free surface of the fluid are called radiated wave potentials, and their number reflects the possible modes of motion of the object in question.

The radiated wave potentials are responsible for those forces known as hydrodynamic reactions. Particularly, when an object oscillates at the water surface, two distinct wave sets can be observed: a standing wave and a propagating one. The standing wave is in phase with the body movements, whilst the propagating wave presents a phase shift of about 90 degrees. These two particular fluctuations are shown to be connected with what is commonly known as added mass and wave damping forces, respectively. In this section a brief overview will be given of the general theory regarding wave radiation, while a complete treatise on the hydrodynamic reaction forces can be found further on in this chapter.

The general form of the BVP describing radiation of waves from a moving body can be obtained by altering the general set of equations outlining the diffraction problem **(6.2)**, taking into account new boundary conditions on the wetted hull, due to the ship motions. Considering in particular that diffracted and incident wave potentials already satisfy equations **(6.2)**, the radiated wave will have to be a solution of the following BVP [1.2]:

$$\Phi = \mathrm{Re}\left[\phi\left(x,y,z\right)\cdot\exp\left(-i\omega_j\, t\right)\right]$$

$$\phi = \phi_I + \phi_D + \sum_{k=1}^{6}\xi_k\, \phi_{Rk}$$

(Assumed linear form for the total potential; note that linear dependence of the radiated wave potentials on the motion response amplitudes ξ_j is assumed.)

$$\nabla^2\phi_{Rk} = 0 \quad k = 1,2..,6$$

(Laplace's equation.)

$$\frac{\partial\phi_{Rk}\left(x,y,0\right)}{\partial z} - k_j\,\tanh\left(k_j\, h\right)\phi_{Rk}\left(x,y,0\right) = 0$$
$$k = 1,2..,6$$

(Complete linear boundary condition at the free surface.)

$$\frac{\partial\phi_{Rk}\left(x,y,-h\right)}{\partial z} = 0 \quad k = 1,2..,6$$

(Linear kinematic boundary condition at the bottom.)

$$\frac{\partial \phi_{Rk}(x,y,z)}{\partial n} = -i\,\omega_j\, n_k \quad k = 1,2,3$$

$$\frac{\partial \phi_{Rk}(x,y,z)}{\partial n} = -i\,\omega_j\,(\vec{r} \wedge \vec{n})_{k-3} \quad k = 4,5,6$$

(Kinematic boundary condition at the hull surface; this requires that the water particles velocity normal to the hull is equal to that of the point of the hull where this quantity is calculated.)

on $S(x,y,z) = 0$

where $\vec{r}$ is the vector describing the position of a point with respect to the centre of gravity of the floating object. Again the *radiation condition* applies, adding the following equations to those above:

$$\lim_{r' \to \infty} \phi_{Rk}(r',\theta,z) \propto r'^{-1/2} . \frac{\cosh[k_j(z+h)]}{\cosh(k_j\, h)} \cdot \exp(i\, k_j\, r') \quad k = 1,2..,6$$

The above formulation presupposes the implicit notion that the ship motion response can be expressed by:

$$q_k(t) = \mathrm{Re}[\xi_k \exp(-i\,\omega_j\, t)] \quad k = 1,2..,6.$$

Therefore the corresponding velocities and accelerations will be:

$$\textbf{(7.2)} \qquad \begin{aligned} U_k(t) &= \mathrm{Re}[-i\,\omega_j\,\xi_k \exp(-i\,\omega_j\, t)] \\ \dot{U}_k(t) &= \mathrm{Re}[\omega_j^2\,\xi_k \exp(-i\,\omega_j\, t)] \end{aligned} \qquad k = 1,2..,6$$

where the frequency of response is the same as the excitation frequency. This assumption is not necessarily true for non-linear systems, in as much as it is a sensible one for a linear correlation. As well as imposing excitation and response frequencies to be equal, this also implies that the vessel oscillations are small.

Bearing in mind the limits imposed by such a resolution, the same approach will be adopted in the context of extreme vessel behaviour, as presented in these pages. Although this might sound drastic, experimental results [10.12] showed good agreement with this formulation even for relatively large amplitudes of oscillation, for underwater geometry as complex as that of twin hull vessels. This is judged a sensible enough reason to trust the validity of such an approach, even when applied to the investigation of resistance of damaged ships to capsize. The functional dependency of the radiation forces and moments on the vessel attitude and its varying geometry will be treated by means of a database-based numerical approach, as described later.

Once the radiating potentials are calculated solving the above set of equations, the forces and moments due to their presence can then be evaluated through the usual integration of the linearised dynamic pressure. A possible way to express this, is:

$$\vec{F}_{Rad} = -\rho \,\mathrm{Re}\left[\sum_{k=1}^{6}\left(-i\,\omega_j \xi_k \exp\left(-i\,\omega_j\, t\right)\iint_S \begin{pmatrix} \vec{n} \\ \vec{r} \wedge \vec{n} \end{pmatrix} \phi_{Rk}\, dS\right)\right]$$

The integral above can be rewritten taking account of the above boundary conditions at the ship wetted surface, to yield:

$$F_{Rad\,l} = \mathrm{Re}\left[\sum_{k=1}^{6}\left(\xi_k \exp\left(-i\,\omega_j\, t\right) f_{lk}\right)\right] \quad l = 1,2..,6$$

$$f_{lk} = -\rho \iint_S \frac{\partial \phi_{Rl}}{\partial n}\, \phi_{Rk}\, dS$$

The coefficients f_{lk} are complex and functions of ω_j, hence they can be put in the form:

$$f_{lk} = \omega_j^2 \, a_{lk}(\omega_j) - i \, \omega_j \, b_{lk}(\omega_j)$$

thus the force $F_{Rad\,l}$ becomes:

(8.2) $$F_{Rad\,l} = -\sum_{k=1}^{6}\left[a_{lk}(\omega_j)\,\dot{U}_k - b_{lk}(\omega_j)\,U_k\right]$$

having used equations **(7.2)**.

Coefficients $a_{lk}(\omega_j)$ and $b_{lk}(\omega_j)$ are those known as added mass and wave damping and represent the energy subtracted from the system by the standing wave and the propagating wave caused by the oscillation of the floating object, respectively. An important part of the system damping is due to viscous effects and must be added to the linear inviscid component described here. This will be discussed later in this chapter.

An additional possible alteration of the original potential can be caused by the forward speed of the ship (or indeed velocities and accelerations in any direction, other than those due to the response to the wave excitation). This is again not interesting as far as this dissertation is concerned and will therefore be disregarded.

2. Application of spectral techniques to generating random wave forces realisations

The forces and moments calculated using the methods presented above, are based on regular waves. In other words, they refer to monochromatic excitation. If the same process is repeated for a reasonably dense range of frequencies assuming a wave amplitude of unit value, continuous curves can be fitted through the points obtained, representing the response amplitude and phase angle operators of each force or moment considered (fig. 3, 4).

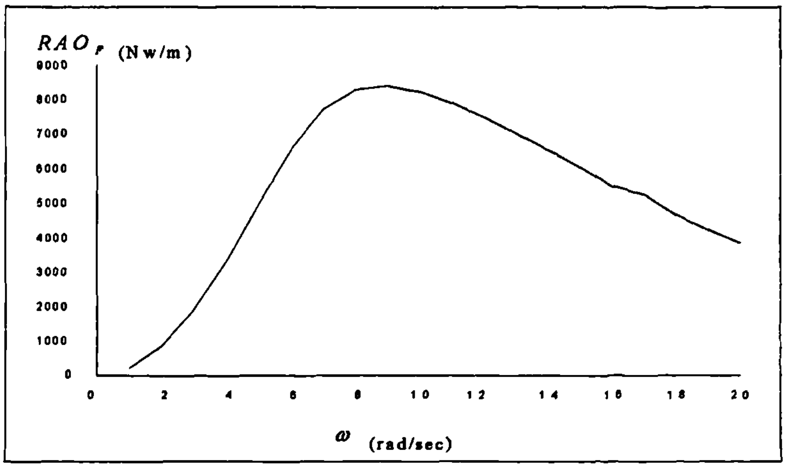

Fig. 3

Response amplitude operator for sway force.

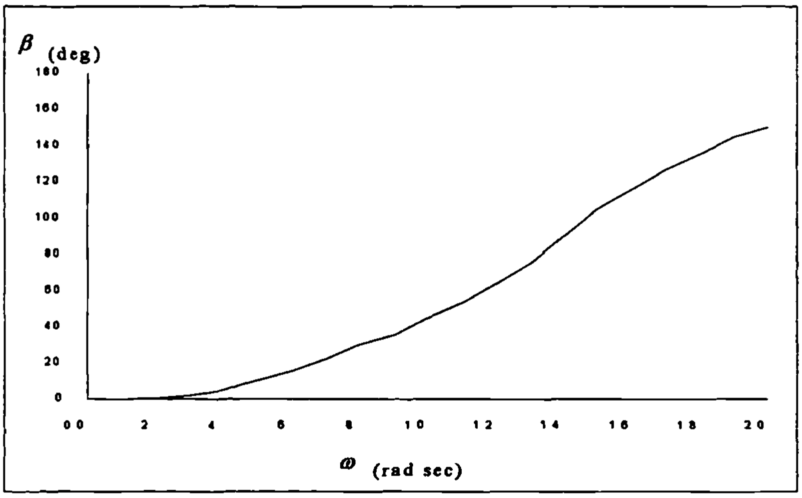

Fig. 4

Phase angle operator for sway force.

Often, the relation between wave and force amplitudes is expressed by a complex *transfer function* or *frequency response function*, $H(\omega) = A - iB$, defined so that the force amplitude ratio corresponds to the modulus of $H(\omega)$, and the phase angle has the same value as the arc tangent of the slope of $H(\omega)$ in the complex plane. In the following comments, the approach which makes use of force response amplitude and phase angle operators will be preferred to that adopting the complex transfer functions, but some basic results of the latter will be recorded where necessary.

Since a linear relation subsists between wave amplitudes and those of the forces and moments acting on the body, a force spectrum for each mode k, can be defined as follows (see fig. 5):

$$S_{F_k}(\omega) = RAO_{F_k}^2(\omega)\, S(\omega)$$

From this and the phase angle operator, force realisations related to the wave realisation expressed by **(1.2)** or **(1.2 a)**, can be obtained. The formula expressing the relation between a random force realisation and the corresponding wave realisation is:

(9.2 a) $$F_k^{Wave}(t) = \int_0^{\infty} \cos\left(\omega\, t - k(\omega)\, x + \varepsilon(\omega) - \beta(\omega)\right) \cdot \sqrt{2\, S_{F_k}(\omega)\, d\omega}$$

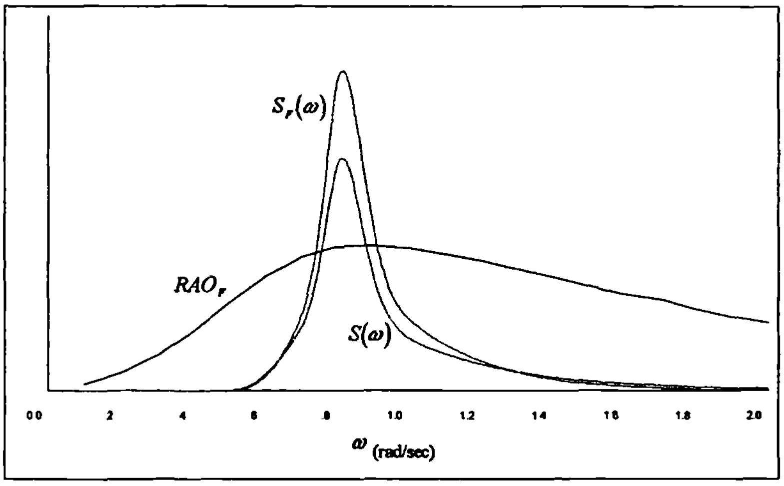

Fig. 5

Building the wave forces spectra. Ordinate axis not to scale.

with respect to the reference system $O\,XYZ$ or, for a finite number of wave components:

(9.2 b) $$F_k^{Wave}(t)=\sum_{j=0}^{600}\sqrt{2\,S_{F_k}(\omega_j)\,0.003}\cdot\cos\left(\omega_j\,t-k_j\,x+\varepsilon_j-\beta_k(\omega_j)\right)$$

having assumed the same circular frequency interval and range and the same random phase angles ε_j as in the wave realisation (see section "Irregular Wave and Spectral Techniques").

Note that if a complex transfer function notation is adopted, the above formula can be shown to correspond to the general form:

$$x(t)=\int_{-\infty}^{\infty}y(\omega\,t)\,d\omega$$

where

$$y(t)=x_0\,\mathrm{Re}\left[H(\omega)\exp(i\,\omega\,t)\right]$$

and $y(t)$ represents the output of a linear system characterised by $H(\omega)$ to the harmonic excitation $x_0\,\mathrm{Re}\left[\exp(i\,\omega\,t)\right]$. The proof can easily be obtained by expanding the exponential in terms of its sine and cosine components and remembering the definition of $H(\omega)$. The superposition principle is then employed to extend the result to the simultaneous action of multiple sinusoidal excitation sources. In general the notation:

(10.2) $$y(t)=H(\omega)\,x_0\exp(i\,\omega\,t)$$

is preferred, once it is understood that only the real part of the expression is of practical interest.

The fact that the same random phase angles used to simulate the wave history, have been retained in the calculations of the force time record, deserves some comments. When dealing with regular waves, a phase lag between the wave occurrence and the force associated to it, is generally readily accepted. For irregular waves, the use of random phase angles to evaluate wave and force realisations, relaxes the perception of the importance of the time relation between these two events. In the case of a damaged ship, though, the wave profile and the vessel motion (which is primarily due to the wave excitation) are both used to evaluate the water flow through the damage opening, which, in turn, influences the ship behaviour itself. It is thus clear that the relation expressed by β_j is much more important in this case than in other contexts.

In the approach indicated above, the assumption that the wave motion has been going on for a long period of time is essential for the applicability of the superposition principle. That is, the *frequency response function* gives the *steady state* response of a system to a sine wave input. In the case of radiation forces, a different and more appropriate method to take account of the frequency variation in the time domain, will be adopted. This is described in the next part of this chapter.

Following the necessary transformations as described in Appendix A, equations **(9.2 a)** and **(9.2 b)** will be rearranged as follows:

$$\textbf{(11.2 a)}\quad \vec{F}'_{Wave}(t) = \left[D^*\right] \int_0^\infty \cos\left(\omega\, t - k(\omega)\, x_{O\,XYZ} + \varepsilon(\omega) - \beta(\omega)\right) \sqrt{2\, \vec{S}^F_{o_d\, x_d y_d z_d}(\omega)\, d\omega}$$

$$\textbf{(11.2 b)}\quad \vec{F}'_{Wave}(t) = \left[D^*\right] \sum_{j=0}^{600} \sqrt{2\, \vec{S}^F_{o_d\, x_d y_d z_d}(\omega_j)\, 0.003} \cos\left(\omega_j\, t - k_j\, x_{O\,XYZ} + \varepsilon_j - \beta_k(\omega_j)\right)$$

where the matrix product of the 3×3 rotation matrix $\left[D^*\right]$ is performed on the two different components of the generalised excitation vector (force and moments terms) separately. This notation will be adopted hereafter.

Equation **(11.1)** can be now re-written as (equation **(12.2)**):

$$\left[M + M_w\right] \cdot \ddot{\vec{q}}' + \left[\dot{M}_w\right] \cdot \dot{\vec{q}}' = \vec{F}'_{Wave} + \vec{F}'_{Drift} + \vec{F}'_{Wind} + \vec{F}'_{Current} + \vec{F}'_H + \vec{F}'_R + \vec{F}'_G - \vec{F}'^*_w$$

having represented the excitation force by its components:

$$\vec{F}'_E = \vec{F}'_{Wave} + \vec{F}'_{Drift} + \vec{F}'_{Wind} + \vec{F}'_{Current}$$

In the next sections expressions for $\vec{F}'_{Drift}$, $\vec{F}'_{Wind}$ and $\vec{F}'_{Current}$ will be given to complete the derivation of the generalised excitation vector.

3. Damaged ship case: variable geometry and position dependency

The first order regular wave forces and moments and the hydrodynamic coefficients described in the foregoing, depend on a number of factors such as: water depth, wave frequency, angle of encounter, heel, trim and draught. The dependency on wave (or response) amplitude is assumed to be linear. In determining the forces and moments acting on the ship at any moment in time, the ship position on the *XY* plane will also play an important part. For instance, this will introduce a phase shift in the force and moment realisation as the irregular wave proceeds (see equation **(9.2 a)**). To take account of some of the above mentioned dependencies and in absence of a non-linear approach to solving the BVP described earlier in the time domain, a numerical approach employing a discrete database will be presently adopted. This will be described next, but first it is necessary to underline the following:

- In order to limit the dimension of the hyper-surfaces representing the forces and moments acting on the ship as functions of all the parameters listed above, the water depth is assumed to be constant. Clearly, it could be important to investigate the influence of the variation of water depth, assuming the ship drifting in an area where the sea floor might - say - rise or fall abruptly or be at a slope such that it cannot be considered flat. Nevertheless this is considered an unnecessary complication and will not be considered here.

- Wave frequency is taken care of, using convolution (see part on "Hydrodynamic Reaction Forces and Moments") and the spectral techniques already introduced; for the other factors, an important observation must be made here. It would be conceptually wrong to update wave forces and hydrodynamic coefficients at each time step as a function of the ship instantaneous position and attitude. The use of words like heading, heel, trim and draught above, is not accidental.

This consideration descends naturally from the hypothesis of linearity assumed to derive these hydrodynamic quantities (see for example the theory on radiated wave potentials) which implies an equilibrium position of the hull around which oscillations take place. The boundary conditions are satisfied on this fixed geometry and forces and moments are integrated over it. In the case of a ship undergoing progressive flooding, this assumption can only be justified within limited lengths of time, i.e. just until the mean ship position stays reasonably close to that of instantaneous equilibrium. The updating of first order wave forces and hydrodynamic coefficients should therefore be performed by screening the mean rather than the instantaneous ship motion. Of course, in a numerical procedure this is only possible by discrete steps.

The previous consideration leads naturally to choosing a computational structure which can be described laconically by the designation of *database*. A database approach - as it is intended in this context - can be visualised as a structured almanac of data (which will be addressed to as *data library*) that can be browsed through systematically by a selective routine. The data library used here describes the variation of the first order wave forces and hydrodynamic reaction as a function of frequency, ship heading, heel, trim and sinkage and can be produced in terms of the following parameters:

$$\left\{ \begin{array}{ll} S_{F_k}\left(\omega, \hat{\varphi}, \hat{\vartheta}, \hat{\psi}, \hat{z}\right) & a_{jk}\left(\omega = \infty, \hat{\varphi}, \hat{\vartheta}, \hat{z}\right) \\ \beta_k\left(\omega, \hat{\varphi}, \hat{\vartheta}, \hat{\psi}, \hat{z}\right) & b_{jk}\left(\omega, \hat{\varphi}, \hat{\vartheta}, \hat{z}\right) \end{array} \right.$$

where the variables $\hat{\varphi}, \hat{\vartheta}, \hat{\psi}, \hat{z}$ are moving average values of the instantaneous motion of the vessel in the system $O\ XYZ$ evaluated over an appropriate time length. It is to be stressed that the relations shown above are not to be considered as continuous functions when representing the data library in question. This remark, implies that it is indeed necessary to interpolate through the values given in the library to achieve a stepwise linear continuity of the four hyper-surfaces a_{jk}, b_{jk}, S_{F_k} and β_k. This is especially obvious considering - for instance - the number of frequencies required to evaluate integrals **(9.2 b)** and **(15.3 b)**.

The discretisation of the data library opens a dilemma regarding the accuracy of the description of the wave forces and hydrodynamic coefficients dependency on the ship mean attitude. In a nut-shell, this means that one cannot know what the shape of each of the surfaces above is *a priori*, consequently choosing which points would be most significant for representing the whole surface can prove somehow puzzling. In reality, this consequence can be minimised if the intervals between two consecutive values of the independent variables are small. This assertion is justified by the understanding that the outline of force and coefficient surfaces will not generally vary at a great rate with $\hat{\varphi}, \hat{\vartheta}, \hat{\psi}$ or $\hat{z}$. Of course, this implies that the density of the grid of $\hat{\varphi}, \hat{\vartheta}, \hat{\psi}$ and $\hat{z}$, should be weighted against the actual derivatives of a_{jk}, b_{jk}, S_{F_k} and β_k with respect to $\hat{\varphi}, \hat{\vartheta}, \hat{\psi}$ and $\hat{z}$. In addition, the range of characteristic values that the independent variables are most likely to assume will have to be taken account of.

Quantity			Range	Units
$\hat{\varphi}$	heel	:	-20, -10, 0, 10, 20	deg.
$\hat{\vartheta}$	trim	:	-2, 0, 2	deg.
$\hat{\psi}$	heading	:	0, 45, 90, 135, 180, 225, 270, 315	deg.
$\hat{z}$	sinkage	:	0.0, 1.5, 3.0	m.
ω	frequency	:	0.2, 0.7, 1.2, 1.7, 2.2	rad/sec

Table 1

From a preliminary analysis, the values in Table 1 seemed to be a good compromise between accuracy and the need to limit the database dimensions (and thus the computing time necessary to estimate it). The frequency values were determined so that they would cover the range in which most of the energy of the wave spectra considered is contained. As already anticipated, the interpolation through the values read from the data library is chosen to be linear. Again this can be justified by the need to meet simplicity and accuracy of calculations.

Having chosen to adopt this database approach, the computational procedure to update the quantities which depend on the vessel mean underwater geometry will then consists of an averaging algorithm to calculate the mean ship motion, multiple conditional switches to choose the appropriate values of a_{jk}, b_{jk}, S_{F_k} and β_k, according to the values of $\hat{\varphi}, \hat{\vartheta}, \hat{\psi}$ and $\hat{z}$, an interpolation routine and the data library itself. The proper evaluation of the time moving averages of the vessel motions involves choosing the right size of *time filter* to use. By *time filter*, it is meant the time length over which the mean motion is calculated. This length of time immediately precedes each time step. Analysing the results from former studies [3.10], it was found that the mean motion can be approximated accurately enough, employing a time filter equal to the wave period. In this context - since irregular waves are employed - the time averages will be weighed over a time duration equal to the period corresponding to the cut-off frequency (lowest frequency below which a spectrum energy is negligible) of the wave spectra considered. This yields $\Delta t_{filter} = 15.0$ seconds approximately, assuming one common filter for all six mean responses.

Obviously, the structure of this procedure implies that a delay equal to the time filter length needs to be allowed before the coefficients are updated, following the time simulation start. In fact, only when the simulation time exceeds the time filter, there will be enough points to allow for the first calculation of the time averages. As a consequence of this, the vessel will necessarily have to oscillate around the initial upright position for the first fifty seconds to avoid the need to update a_{jk}, b_{jk}, S_{F_k} and β_k when the values of $\hat{\varphi}, \hat{\vartheta}, \hat{\psi}$ and $\hat{z}$ cannot be calculated properly. The easiest way to ensure that this condition is respected is to delay the start of the flooding routine for a time span longer than the time filter. Note that, although this does not necessarily guarantee that the ship heading will not change too quickly during the initial fifteen seconds (this quantity does not depend directly on the flooding mechanism), it still avoids large coupling of the ship motions into yaw due to an asymmetrical underwater geometry. This is - together with the slow varying nature of the heading of a ship without forward speed - a reasonable enough reason for expecting no need to update the above parameters with $\hat{\psi}$ within the first fifteen seconds of simulation.

In conclusion, assuming the structure of the multiple conditional switches needed to "browse" through the database and the interpolation routine are elementary enough not to need any in-depth description - and leaving the description of the process of building the database from the geometrical data of the ship to Appendix A - this section will be closed by the following important remark. Apart from the better evaluation of the forces acting on a ship progressively changing in attitude, the choice of employing a file library of force amplitudes and hydrodynamic coefficients to describe the variation of these quantities with the ship attitude is particularly valuable since it implies an improved numerical efficiency. This point can be readily appreciated once this consideration is borne in mind: the calculation of a database library can be time consuming - especially if its dimensions (i.e. the number of combination of the values that each independent variable can assume) are large - but it is performed only once for each ship shape. The same database will then be used in every simulation for the same vessel. In this way the time saved by eluding the calculations of wave forces and hydrodynamic coefficients every time the simulation program is used, offers justifiable compensation. The time necessary to evaluate the database described above through the 3D CFD program described in Appendix A (for a complete hull form approximated by 860 panels, corresponding to representative underwater geometry of 250 to 400 panels on average) is typically about 24 hours, using a medium-high speed PC.

Second Order Wave Drift Forces and Moments

As all horizontal constant, steady or slowly varying forces, the second order wave forces have a limited importance on the dynamic behaviour of a damaged vessel but cannot be neglected *a priori*. This is valid much in the same way for current forces and to a lesser extent for wind loading too. Physically, drift forces are caused by the unbalanced pressure acting over the part of a floating body which is periodically washed by the wave surface. As such, it is a non-linear phenomenon and could be treated rigorously only if second and higher order terms were included in the derivation of the incident wave potential which could then be integrated up to the instantaneous water elevation. In practice some authors (Maruo [1.8]) consider them as a result of that part of the wave that is reflected by the water piercing body and in this light they derived some useful expressions setting the limits and the general form of the equation describing these forces.

Maruo's formula can be written as:

$$\left|\vec{F}_{Drift}\right| = \frac{\rho_{Water}\, g}{2} A_R^2 \qquad (13.2)$$

where the direction of the force is always the same as that of the incident wave. Note that the proportionality of the drift forces with the square of the reflected wave amplitude justify the *second order* designation of this drift force. The reflected wave amplitude A_R must of course be greater than or equal to zero but smaller than the incident wave amplitude. When the wave frequency is high - in which case the amplitude is usually small - most of the incident wave will in fact be reflected and $A_R = A_I$. If the wave length is long enough, the transmitted wave will grow more and more at the expenses of A_R and thus of the total drift force.

Naturally the extent up to which either of these two cases prevails depends much on the geometry of the floating object and the reflected wave amplitude can be expected to be a function of the principal dimensions of the vessel as well as the incident wave parameters. Another important aspect of this problem is the variable underwater geometry of a damaged ship undergoing progressive flooding. In absence of any model for estimating the changes of the drift coefficients with the heel angle and deeming the importance of such variation reasonably small, this effect has been neglected here. The same considerations and assumptions are considered valid for the current forces as well.

As well as a drift force, a yaw drift moment can be expected. This arises irrespective of the eventual symmetries of the vessel, whenever the ship heading differs from the wave direction or its perpendicular. The reason for the existence of such moment is that rarely the projection of centre of pressure of the drift force on the vertical plane perpendicular to the wave direction, will coincide with that of the centre of rotation of the ship.

In both cases - having excluded a rigorous approach since the beginning - the relation linking second order wave forces to the incident wave amplitude will be similar to **(13.2)**. In this context, the expressions below were adopted following the approach suggested by Gatiganti in [6.7]:

(14.2)
$$\vec{F}_{Drift} = \begin{pmatrix} C_{wx_d}\, B \dfrac{\rho_{Water}\, g}{2} A_I^2 \\ C_{wy_d}\, L_{PP} \dfrac{\rho_{Water}\, g}{2} A_I^2 \\ 0 \\ 0 \\ 0 \\ C_{wn_d}\, L_{PP}^2 \dfrac{\rho_{Water}\, g}{2} A_I^2 \end{pmatrix}$$

where

$$\begin{cases} \left|C_{wy_d}\right| = abs\left(-1.1414 - 0.10235 \dfrac{L_{PP}}{B} + 12.559 \dfrac{\sqrt{B\,T}}{\lambda} - 132.01 \dfrac{(B\,T)^2}{\lambda^4} + 0.06374 \dfrac{\lambda}{\sqrt{B\,T}} \right) \\ \left|C_{wy_d}\right| \le 1.0 \end{cases}$$

and

$$\begin{cases} \left|C_{wy_d}\right| = 1.0 & if \quad \lambda/L_{PP} < 0.3 \\ \left|C_{wy_d}\right| = 0.0 & if \quad \lambda/L_{PP} > 1.5 \end{cases}.$$

The dependence of the lateral drift force coefficient on the wave direction can be expressed by:

$$\begin{cases} C_{wy_d} = \sin^2\alpha_w \left|C_{wy_d}\right| & if \quad 0 \le \alpha_w < \pi \\ C_{wy_d} = -\sin^2\alpha_w \left|C_{wy_d}\right| & if \quad \pi \le \alpha_w < 2\pi \end{cases}.$$

where α_w is the wave encounter angle, defined positive anticlockwise from following to head sea. The relation between this quantity and the ship heading in the earth-fixed system of reference is:

$$\begin{cases} \alpha_w = -\psi \\ 0 \le \alpha_w < 2\pi \end{cases}.$$

This empirical method gives the lateral (sway) drift coefficient as a function of wave and ship characteristics. The expressions describing C_{wy_d} are based on experimental data covering a wide range of real and model scale ships in the upright condition. Unfortunately, no similar generalised coefficients are readily available for the longitudinal force or the yaw moment. These are hence derived from the results of experimental measurements collected by English and Wise for the Wimpey Sealab [8.1], understanding that these results cannot be generalised to every hull form and that the coefficients given below should be considered only a rough approximation, just adequate for the purposes of this research. The added resistance and yaw moment coefficients can be represented by the following expressions:

$$\begin{cases} \left|C_{wx_d}\right| = 0.62\,\sin^2\left(\dfrac{\pi\,\lambda}{1.6\,L_{PP}}\right) \\ \left|C_{wx_d}\right| = 0.0 \quad if \quad \lambda/L_{PP} > 1.6 \end{cases}$$

$$\begin{cases} C_{wx_d} = \cos^2\alpha_w \left|C_{wx_d}\right| \quad if \quad -n\dfrac{\pi}{2} < \alpha_w < n\dfrac{\pi}{2} \\ C_{wx_d} = -\cos^2\alpha_w \left|C_{wx_d}\right| \quad if \quad n\dfrac{\pi}{2} < \alpha_w < n\dfrac{3\,\pi}{2} \end{cases} \quad \forall\, n \in I$$

and

$$\begin{cases} \left|C_{wn_d}\right| = 0.16\,\sin^2\left(\dfrac{\pi\,\lambda}{L_{PP}}\right) \\ \left|C_{wn_d}\right| = 0.0 \quad if \quad \lambda/L_{PP} > 1.0 \end{cases} \qquad C_{wn_d} = -\sin(2\,\alpha_w)\left|C_{wn_d}\right|$$

As it can be easily noticed, the system of reference in which the above formulae were developed is neither the earth-fixed system adopted in this investigation, nor that in which the equations of motion were produced. In fact, equation **(14.2)** is expressed with respect to $o_d\ x_d y_d z_d$ (cf. Appendix A) and thus this co-ordinate system need to be converted into $Gs\,x'y'z'$ before the introduction of **(14.2)** into the equations of motions. This yields:

(15.2)
$$\vec{F}'_{Drift} = \left[D^*\right]\cdot\begin{pmatrix} C_{wx_d}\ B\ \dfrac{\rho_{Water}\ g}{2}\ A_I^2 \\ C_{wy_d}\ L_{PP}\ \dfrac{\rho_{Water}\ g}{2}\ A_I^2 \\ 0 \\ 0 \\ 0 \\ C_{wn_d}\ L_{PP}^2\ \dfrac{\rho_{Water}\ g}{2}\ A_I^2 \end{pmatrix}$$

Second order drift forces are calculated for regular waves (constant mean forces and moments) or for irregular waves (slowly varying drift waves and moments - see [6.2]) in a discretised manner. According to the theory, mean drift forces and moments can be evaluated every time that a peak or trough of the wave realisation takes place at the location of the centre of gravity of the vessel. The absolute value of these extremes is then used instead of A_I to calculate $\vec{F}'_{Drift}$. The forces obtained are finally assumed constant over the period between the occurrence of two local minima/maxima. This procedure obviously implies a constant drift force for regular waves and a slowly oscillating one for irregular ones, as long as the ship attitude remains constant. It must be noticed though that the varying nature of the ship heading will have an influence over the drift coefficients and hence a variation of the drift forces with time can be expected for regular waves too.

Wind Forces and Moments

Unlike the other two loadings acting prevalently in the horizontal plane, wind forces have a direct influence on the ship heeling and need to be modelled with some accuracy. This is all in all truer in case of extreme wind velocities, when the magnitude of these forces can be quite large. Generally wind loading is treated empirically and the method used here [7.1] has been developed for merchant ships on the basis of regression analysis over an extensive quantity of experimental data. As such it is believed sufficiently reliable for most of the cases of interest of this study. The two sole shortcomings of the formulation offered by Isherwood are the assumption of constant wind speed and the absence of a sound and validated method for estimating the wind loading variation with the heel angle. Leaving the discussion on these two points to later, a brief summary of the theory is given next.

Isherwood method consists of a basic set of equations linking the wind lateral and longitudinal forces and the yawing moment to relative wind speed and ship geometry parameters. This set can be represented by:

$$
(16.2) \qquad
\begin{aligned}
F_{x_d\,Wind} &= -C_{Wx_d}\ A_T\ \frac{\rho_{Air}\,V_{Wr}^2}{2} \\
F_{y_d\,Wind} &= C_{Wy_d}\ A_L\ \frac{\rho_{Air}\,V_{Wr}^2}{2} \\
F_{n_d\,Wind} &= C_{Wn_d}\ A_L\ L_{OA}\ \frac{\rho_{Air}\,V_{Wr}^2}{2}
\end{aligned}
$$

where

$$
V_{Wr} = \sqrt{(u_W - \dot{x}_{Gs})^2 + (v_W - \dot{y}_{Gs})^2}
$$

and

$$
\begin{cases}
u_W = |V_W| \cos\alpha_W^A \\
v_W = |V_W| \sin\alpha_W^A
\end{cases}
$$

with reference to the wind characteristics as in fig. 6. The remaining variables and constants in **(16.2)** are:

ρ_{Air} : density of air taken as 0.0012 tonnes/m^3;

A_T : transverse projected area of the part of the model above the water in the upright condition;

A_L : lateral projected area of the part of the model above the water in the upright condition;

and, indeed the wind loading coefficients. These are function of the wind direction off the bow α_W and other ship related parameters according to the following formulae:

$$C_{W x_d} = A_0 + A_1 \frac{2\,A_L}{L_{OA}^2} + A_2 \frac{2\,A_T}{B^2} + A_3 \frac{L_{OA}}{B} + A_4 \frac{S}{L_{OA}} + A_5 \frac{C}{L_{OA}} + A_6\, M$$

$$C_{W y_d} = B_0 + B_1 \frac{2\,A_L}{L_{OA}^2} + B_2 \frac{2\,A_T}{B^2} + B_3 \frac{L_{OA}}{B} + B_4 \frac{S}{L_{OA}} + B_5 \frac{C}{L_{OA}} + B_6 \frac{A_{SS}}{A_L}$$

$$C_{W n_d} = C_0 + C_1 \frac{2\,A_L}{L_{OA}^2} + C_2 \frac{2\,A_T}{B^2} + C_3 \frac{L_{OA}}{B} + C_4 \frac{S}{L_{OA}} + C_5 \frac{C}{L_{OA}}$$

where the coefficients A_i, B_i, and C_i are tabulated as a function of $|\alpha_W|$ and this is in relation with the ship's heading according to the following expressions:

$$\begin{cases} \alpha_W = \psi - \alpha_W^R - \pi \\ -\pi < \alpha_W \le \pi \end{cases}$$

and

$$\begin{cases} C_{W y_d} = C_{W y_d} \quad and \quad C_{W n_d} = C_{W n_d} \quad if \quad 0 < \alpha_W < \pi \\ C_{W y_d} = -C_{W y_d} \quad and \quad C_{W n_d} = -C_{W n_d} \quad if \quad -\pi < \alpha_W < 0 \\ C_{W y_d} = C_{W n_d} = 0 \quad if \quad \alpha_W = 0 \quad or \quad \alpha_W = \pi \end{cases}$$

where

$$\begin{cases} \alpha_W^R = \arctan\left(\dfrac{v_W - \dot{y}_{Gs}}{u_W - \dot{x}_{Gs}}\right) \quad if \quad u_W - \dot{x}_{Gs} > 0 \\ \alpha_W^R = \pi + \arctan\left(\dfrac{v_W - \dot{y}_{Gs}}{u_W - \dot{x}_{Gs}}\right) \quad if \quad u_W - \dot{x}_{Gs} < 0 \quad and \quad v_W - \dot{y}_{Gs} \ge 0 \\ \alpha_W^R = -\pi + \arctan\left(\dfrac{v_W - \dot{y}_{Gs}}{u_W - \dot{x}_{Gs}}\right) \quad if \quad u_W - \dot{x}_{Gs} < 0 \quad and \quad v_W - \dot{y}_{Gs} < 0 \\ \alpha_W^R = \dfrac{\pi}{2} \quad if \quad u_W - \dot{x}_{Gs} = 0 \quad and \quad v_W - \dot{y}_{Gs} > 0 \\ \alpha_W^R = -\dfrac{\pi}{2} \quad if \quad u_W - \dot{x}_{Gs} = 0 \quad and \quad v_W - \dot{y}_{Gs} > 0 \\ \alpha_W^R = 0 \quad if \quad u_W - \dot{x}_{Gs} = 0 \quad and \quad v_W - \dot{y}_{Gs} = 0 \end{cases}$$

Finally:

A_{SS} is the lateral projected area of the superstructure above the sheer-line;

S is the lenght of the perimeter of the lateral projection of the ship excluding the waterline and slender bodies like masts;

C is the distance from the bow of the centroid of the lateral projected area;

M is the number of distinct groups of masts and kingposts seen in the lateral projection.

As it has been often pointed out [7.3] the dependence of the wind loading coefficients on the heel angle is usually not modelled properly and very little literature is available on the subject. Even less information is available on the dependence of the yaw moment on the ship trim, but this effect can be safely excluded from the sphere of interest of this investigation. Generally, the variation of the wind forces and moments with roll and heel can be considered null for the lateral and longitudinal forces as well as for the yaw moment, since the extention of the projected areas concerned does not change significantly. Experimental evidence teaches that the same cannot be said in the case of the heeling moment.

This quantity can be evaluated by chosing a proper lever over which the lateral wind force is considered to be acting. Although the vertical distance between the centroids of the lateral projected areas of the ship above and below the water level can be a good first approximation, in reality this value results to be underestimated. From experimental tests [7.2] it appears that a better value can be obtained by multiplying the height of the geometrical centroid of the lateral projected area above the waterline by a coefficient which is in the range of 1.5 for merchant ships:

$$h_{Upright} = 1.5\, h_{aw} - h_{uw}$$

The relation above is given with reference to the ship geometry system of co-ordinates ($o\,xyz$). One reason to partially explain the necessity for such a correction can be individuated in the shape of the wind velocity profile. Once the upright wind heeling lever has been found, the variation of this with the heel angle can be approximated by the following relation:

$$h_\varphi = \left(0.3 + 0.7\,\cos^2(\varphi)\right) h_{Upright}$$

having assumed the lateral wind force constant with φ. Naturally, the heeling arm does not go to zero for heel angles approaching 90 degrees since part of the ship will always be subjected to the inclining action of wind. In the end, the heeling moment will be found as:

$$\text{(17.2)} \qquad F_{m_d\,Wind} = -\left(0.3 + 0.7\,\cos^2(\varphi)\right)\left(1.5\,h_{aw} - h_{uw}\right) C_{Wy_d}\; A_L\, \frac{\rho_{Air}\, V_{Wr}^2}{2}$$

and thus the generalised wind excitation vector in the ship-fixed co-ordinate system will be:

$$\text{(18.2)} \qquad \vec{F}'_{Wind} = \left[D^*\right]\cdot \begin{pmatrix} -C_{Wx_d}\; A_T\, \dfrac{\rho_{Air}\, V_{Wr}^2}{2} \\ C_{Wy_d}\; A_L\, \dfrac{\rho_{Air}\, V_{Wr}^2}{2} \\ 0 \\ -\left(0.3 + 0.7\,\cos^2(\varphi)\right)\left(1.5\,h_{aw} - h_{uw}\right) C_{Wy_d}\; A_L\, \dfrac{\rho_{Air}\, V_{Wr}^2}{2} \\ 0 \\ C_{Wn_d}\; A_L\; L_{OA}\, \dfrac{\rho_{Air}\, V_{Wr}^2}{2} \end{pmatrix}.$$

Although the influence of dynamic wind loading is generally regarded as scarcely important for stability analysis, a wind spectrum has been used here to simulate gusts superimposed to the mean wind velocity. The resulting time realisation can be used in

(18.2) above to find the corresponding time varying forces. This methodology is not absolutely correct since Isherwood coefficients to calculate wind forces and moments were derived for steady constant winds, so one could not use it in conjunction with wind velocity spectral analysis. Once again, however, in absence of better options, this hybrid procedure was judged sufficiently reasonable and trustworthy. The spectrum used is the Davenport [1.9] which can be represented by:

$$S_W(\omega) = 4K \frac{\overline{V}_W^2}{\omega} \frac{X_D^2}{\left(1 + X_D^2\right)^{4/3}}$$

$$X_D = 600\,\omega / \left(\pi\, \overline{V}_W\right)$$

where $\overline{V}_W$ is the mean wind velocity at 10 metres above the sea and $K = 0.003$. The time history of the wind velocity can thus be expressed as:

$$V_W = \overline{V}_W + \int_0^{\infty} \cos(\omega\, t + \varepsilon(\omega))\, \sqrt{2\, S_W(\omega)\, d\omega}$$

which becomes:

$$V_W = \overline{V}_W + \sum_{j=0}^{50} \sqrt{2\, S_W(0.03\, j)\, 0.03} \cdot \cos\left(0.03\, j\, t + \varepsilon_{W\,j}\right)$$

in discrete terms.

Finally it is worth underlining at this point, that the mean wind velocity and the significant wave height can be considered indipendent variables during the relatively short time periods taken into consideration here, although care should be taken of the joint probability of these two events which are indeed correlated. The same consideration stands for the relative direction of wind and wave.

Current Forces and Moments

The most evident property of current forces is being essentially restrain to the horizontal plane. This feature brings as a consequence the limited direct effect of current over the transverse stability of a vessel - as already mentioned above - and thus allow for a less strictly accurate evaluation of the forces resulting from it. Furthermore, currents have the characteristic of varying very slowly with time. Typically, the frequency of fluctuations is in the range of 1/30 secs. to virtually zero, with an average value of 1/10 mins. For this reason the absolute current speed is treated as a constant in this context.

Having said this, it is important to stress that the forces in the horizontal plane due to the viscosity of the water are present even if the absolute velocity of the surface current is close to zero. This is a natural consequence of the planar motions of the ship, which induce a relative speed of the water over the wetted hull which is hardly ever null.

Against this background - having excluded any significant variation of the viscous forces with heel, trim and sinkage - an empirical approach is adopted here, which is based on three fairly large set of experimental results, analysed and put in the form of parametrical relations by C. Morris in 1986 [8.2]. According to Morris, the current loading on a typical ship geometry can be expressed through relations whose structure is akin to that of the aerodynamic wind loading. These can be represented by:

$$\text{(19.2)} \qquad \vec{F}_{Current} = \begin{pmatrix} C_{cx_d} \; L_{PP} \; T \dfrac{\rho_{Water} V_{cr}^2}{2} \\ C_{cy_d} \; L_{PP} \; T \dfrac{\rho_{Water} V_{cr}^2}{2} \\ 0 \\ 0 \\ 0 \\ C_{cn_d} \; L_{PP}^2 \; T \dfrac{\rho_{Water} V_{cr}^2}{2} \end{pmatrix}$$

where

$$V_{cr}=\sqrt{\left(u_c-\dot{x}_{Gs}\right)^2+\left(v_c-\dot{y}_{Gs}\right)^2}$$

is the relative current velocity, having defined u_c and v_c as:

$$\begin{cases} u_c=|V_c|\cos\alpha_c^A \\ v_c=|V_c|\sin\alpha_c^A \end{cases}.$$

The absolute current velocity and direction are as in fig. 6.

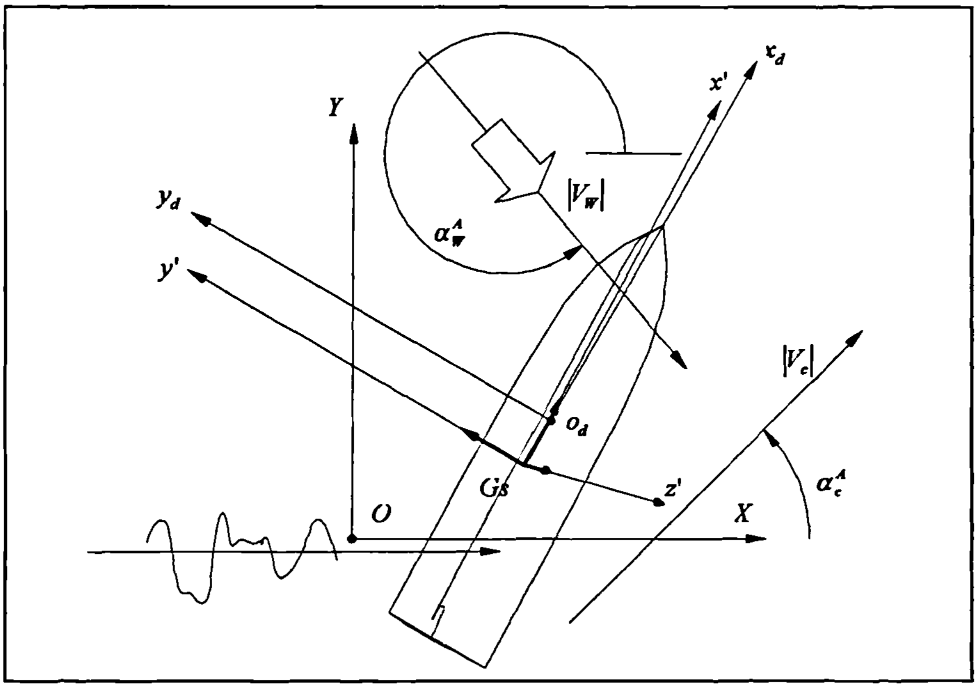

Fig. 6

Plan view of the systems of reference showing the wind and current related parameters.

The coefficients to be used in **(19.2)** are given as functions of the main ship parameters and the angle of attack of the water stream:

$$C_{cx_d} = \cos\alpha_c \left|1.264\cdot10^{-4}\ \alpha_c^* - 0.08466\frac{B}{T} - 0.0368\frac{L_{PP}}{T} + 0.06545\frac{L_{PP}}{B}\right|$$

$$C_{cy_d} = \sin\alpha_c \left|1.765\cdot10^{-3}\ \alpha_c^* - 1.549\frac{B}{T} + 0.09031\frac{L_{PP}}{T} + 5.897\,C_B\right|$$

$$C_{cn_d} = -\sin 2\,\alpha_c \left|1.291\cdot10^{-3}\ \alpha_c^* - 0.02154\frac{B}{T} + 0.003413\frac{L_{PP}}{T} - 0.04181\,C_B\right|$$

where

$$\begin{cases} \alpha_c^* = \alpha_c \dfrac{180}{\pi} & if \quad 0<\alpha_c<\pi \\ \alpha_c^* = 360-\alpha_c \dfrac{180}{\pi} & if \quad \pi<\alpha_c<2\,\pi \end{cases}$$

and α_c is the current encounter angle. This is linked to the ship heading in the earth-fixed system of reference through the expression:

$$\begin{cases} \alpha_c = \alpha_c^R - \psi \\ 0\le\alpha_c<2\,\pi \end{cases}$$

where α_c^R is the direction relative current speed, defined similarly to α_W^R.

As for the other forces current forces have to be translated into the ship-fixed system of reference, yelding:

$$\text{(20.2)} \qquad \vec{F}'_{Current} = \left[D^*\right]\cdot\begin{pmatrix} C_{cx_d}\ L_{PP}\ T\dfrac{\rho_{Water}\ V_{cr}^2}{2} \\ C_{cy_d}\ L_{PP}\ T\dfrac{\rho_{Water}\ V_{cr}^2}{2} \\ 0 \\ 0 \\ 0 \\ C_{cn_d}\ L_{PP}^2\ T\dfrac{\rho_{Water}\ V_{cr}^2}{2} \end{pmatrix}.$$

Hydrodynamic Reaction Forces and Moments

Fluid Velocity Potential Components

As far as the hydrodynamic reaction forces and moments are concerned, the traditional, small amplitude linear approach is adopted here, as anticipated in the foregoing. In the context of this investigation, however, the coefficients introduced in equation **(8.2)** are updated to suit the varying underwater geometry of the hull. This piece-wise linear model is believed to be a good approximation of reality, since the small amplitudes assumption can be considered valid around the mean position and attitude of the ship even for extreme behaviour cases such as that describing a damaged ship being flooded. Another point of some weight is the dependence of hydrodynamic reaction forces and moments on frequency. This is taken into consideration using convolution techniques as illustrated in this section. Before exploring the particulars of this procedure, though, it is worth briefly describing the transformations necessary to adapt the coefficients output by the numerical program used to evaluate the linear velocity potentials - DWAVE [5.7] - to the system of reference used in the equations of motion. The resulting matrices are those that will have to be used in the calculation of the kernels of the convolution integrals.

The familiar expression describing the part of hydrodynamic reaction due to the radiating wave potentials is:

(8.2) $$F_{Rad\,l} = -\sum_{k=1}^{6}\left[a_{l\,k}(\omega_j)\,\dot{U}_k - b_{l\,k}(\omega_j)\,U_k\right]$$

in which U_k and $\dot{U}_k$ are the linear and angular velocities and accelerations in the direction of the $o_d\ x_d y_d z_d$ axis. Preserving this structure, $\vec{F}_{Rad}$ can be expressed as:

$$\vec{F}_{Rad} = -[A]\ddot{\vec{q}}_d - [C]\dot{\vec{q}}_d = -\vec{F}_A - \vec{F}_C \qquad \text{if} \qquad \begin{aligned}\vec{F}_A &= [A]\ddot{\vec{q}}_d \\ \vec{F}_C &= [C]\dot{\vec{q}}_d\end{aligned}$$

or

$$\vec{F}'_{Rad} = -[A']\ddot{\vec{q}}' - [C']\dot{\vec{q}}' = -\vec{F}'_A - \vec{F}'_C \qquad \text{if} \qquad \begin{aligned}\vec{F}'_A &= [A']\ddot{\vec{q}}' \\ \vec{F}'_C &= [C']\dot{\vec{q}}'\end{aligned}$$

in which $[C]$ is the positive defined radiating wave damping coefficients matrix - expressed with respect to $o_d\ x_d y_d z_d$ - and $[A]$ the added mass matrix - again positive defined and expressed with respect to the same coordinates system. Apart from the fact that the second expression above refers to $Gs\, x' y' z'$ instead of $o_d\ x_d y_d z_d$, it describes precisely the same quantity, thus:

$$\vec{F}'_{Rad} = \vec{F}_{Rad} \qquad \text{or} \qquad \begin{aligned}\vec{F}'_A &= \vec{F}_A \\ \vec{F}'_C &= \vec{F}_C\end{aligned}$$

regardless of the system of reference. The formula to convert the hydrodynamic coefficient matrices calculated by DWAVE to the ship fixed reference system is very similar to the one used for the wave forces and it can be derived as follows. Considering the added mass component only:

$$\vec{F}'_A = [A']\ddot{\vec{q}}' \qquad \text{and} \qquad \vec{F}_A = [A]\ddot{\vec{q}}_d$$

therefore

$$[A']\ddot{\vec{q}}' = [A]\ddot{\vec{q}}_d.$$

Now:

$$\ddot{\vec{q}}' = \begin{bmatrix} [D^*] & 0 \\ 0 & [D^*] \end{bmatrix} \ddot{\vec{q}}_d = [D^6]\ddot{\vec{q}}_d$$

where $[D^*]$ is the matrix describing the transformation from the system of reference of the diffraction program to the ship-fixed one (see Appendix A, equation **(3.A)**), thus:

$$[A]\ddot{\vec{q}}_d = [A'] \cdot [D^6]\ddot{\vec{q}}_d$$

i.e.

(1.3 a) $$[A'] = [A] \cdot [D^6]^T$$

bearing in mind the general properties of the rotation matrices. Similar reasoning leads to:

(1.3 b) $$[C'] = [C] \cdot [D^6]^T$$

for the radiating wave damping coefficients.

1. Convolution techniques.

A method alternative to that of measuring the *steady state* response of a system to a sine input to assess its dynamic behaviour is that of surveying its *transient* response to an impulse disturbance. The response to a unit impulse at $t = 0$ can be represented by a *unit impulse response function*, $h(t)$. Note that the unit-impulse response function satisfies the condition $h(t) = 0$ for $t < 0$ since the system is supposed to be inert before the disturbance is applied.

Since there is complete equivalence between the frequency response and impulse response methods - meaning that both give the same information about the system dynamic behaviour characteristics -, relationships linking the two functions are expected to hold. The Fourier transform method expressing an aperiodic function with an integral involving a series of sine and cosine terms is employed in order to define this bond.

More specifically, it is possible to demonstrate that under certain conditions, an arbitrary function of time $x'(t)$ can be expressed as:

(2.3)
$$x'(t) = 2\int_0^{\infty} A(\omega)\cos(\omega\, t)\, d\omega + 2\int_0^{\infty} B(\omega)\sin(\omega\, t)\, d\omega$$

where

(3.3)
$$A(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x'(t)\cos(\omega\, t)\, dt$$
$$B(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x'(t)\sin(\omega\, t)\, dt$$

The terms $A(\omega)$ and $B(\omega)$ are the components of the *Fourier transform* of $x'(t)$ and the equation is a representation of $x'(t)$ by a *Fourier integral* or *inverse Fourier transform*. An alternative form to express the equations above is as follows:

(3.3 a)
$$X'(\omega) = A(\omega) - i\, B(\omega) = \frac{1}{2\,\pi}\int_{-\infty}^{\infty} x'(t)\exp(-i\,\omega\, t)\, dt$$

which is the *Fourier transform* of $x'(t)$. This can be shown to yield:

(2.3 a)
$$x'(t) = \int_{-\infty}^{\infty} X'(\omega)\exp(i\,\omega\, t)\, d\omega$$

Prerequisites to the above expression **(2.3)** - or indeed **(2.3 a)** - is that $x'(t)$ is piece-wise continuous over any finite interval, its derivatives exist and are finite and the following condition is satisfied:

(4.3) $$\int_{-\infty}^{\infty} \left| x'(t) \right| \, dt < \infty .$$

This implies that the function $x'(t)$ must decay to zero when $|t| \to \infty$, although there are means to relax this condition.

Consider for the moment a stable system, i.e. a system for which, if the response is zero before the impulse is applied, it will decay afterwards to its initial value. For such systems the unit impulse response function will satisfy condition **(4.3)**. Of course, the same can be said for the unit impulse which can be represented by:

$$x(t) = \delta(t)$$

where the Dirac's delta function $\delta(t)$ is defined so that it is zero everywhere except for $t = 0$ and $\int_{-\infty}^{\infty} \delta(t) \, dt = 1$. This allows for the evaluation of the Fourier transforms of both $h(t)$ and $x(t)$. It can be shown that:

(5.3)

$$X(\omega) = \frac{1}{2\,\pi} \int_{-\infty}^{\infty} x(t) \exp(-i\,\omega\,t) \, dt = \frac{1}{2\,\pi} \int_{-\infty}^{\infty} \delta(t) \exp(-i\,\omega\,t) \, dt = \frac{1}{2\,\pi}$$

.

whilst

(6.3) $$Y(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} h(t)\exp(-i\,\omega\,t)\,dt\,.$$

Given these results, the input and output of this system can be expressed through the Fourier integrals:

$$x(t) = \int_{-\infty}^{\infty} X(\omega)\exp(i\,\omega\,t)\,d\omega$$

$$y(t) = \int_{-\infty}^{\infty} Y(\omega)\exp(i\,\omega\,t)\,d\omega$$

For a linear system, the relationship between input and output can be expressed - in the frequency domain - by **(10.2)** in the case of a sinusoidal excitation. This can be expected to be valid for each sinusoidal component of the Fourier integrals above, thus:

$$Y(\omega)\exp(i\,\omega\,t)\,d\omega = H(\omega)\,X(\omega)\exp(i\,\omega\,t)\,d\omega \quad\Rightarrow\quad Y(\omega) = H(\omega)\,X(\omega)$$

If this result is used in **(6.3)** together with **(5.3)**, it yields:

(7.3) $$H(\omega) = \int_{-\infty}^{\infty} h(t)\exp(-i\,\omega\,t)\,dt$$

thus also

(8.3) $$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\exp(i\,\omega\,t)\,d\omega$$

as for **(2.3 a)** and **(3.3 a)**. This is equivalent to saying that the frequency response function of a system and its impulse response function form a Fourier transform pair.

Once the relationship between frequency response and the impulse response functions has been ascertained, all that is left to state the equivalence of these two methods, is the generalisation of the calculation of the system response to a random input using the impulse response function. To do this, the arbitrary input $x(t)$ is decomposed to a close occurrence of a series of impulses. The response at time t to the single impulse $x(\tau)$, occurred at time τ, will be proportional to the impulse magnitude $x(\tau)\,d\tau$ and the unit-impulse response function $h(t-\tau)$ (linearity assumption). The total response at time t will therefore be given by the *convolution integral*:

$$(9.3) \qquad y(t) = \int_{-\infty}^{t} h(t-\tau)\, x(\tau)\, d\tau .$$

So far it has been established that, if a system is stable and its response to an external input can be described by a linear correlation, then the input-output relationship can be expressed by a method which is equivalent to the *frequency response function* one: the *impulse response function method.* This was employed by Cummins [10.2] to take account of the hydrodynamic reaction forces correctly and extend the application of time domain equations to a random input.

In Cummins' work, the input of the system is represented by the ship's generalised velocity vector, whilst the output is the fluid velocity potential induced by the ship motion (radiated wave potential). This is then employed to estimate the hydrodynamic reaction yielding the following expression in the case of zero forward speed:

$$(10.3) \qquad -F_{jk}^{Rad}(t) = m_{jk}\, \ddot{x}_k(t) + \int_{0}^{t} K_{jk}(t-\tau)\, \dot{x}_k(\tau)\, d\tau$$

where $F_{jk}^{Rad}(t)$ is the net hydrodynamic force (or moment) acting on the hull in the j[th] mode, due to an arbitrary small motion of the ship in the k[th] mode. The following relations are shown to hold, linking m_{jk} and the kernels K_{jk} to the sinusoidal oscillation added mass and hydrodynamic damping coefficients $a_{jk}(\omega)$ and $b_{jk}(\omega)$, similar to those in **(8.2)** [10.3]:

$$a_{jk}(\omega) = m_{jk} - \frac{1}{\omega}\int_0^t K_{jk}(\tau)\sin(\omega\,\tau)\,d\tau$$

$$b_{jk}(\omega) = \int_0^t K_{jk}(\tau)\cos(\omega\,\tau)\,d\tau$$

and more importantly

(11.3)
$$\begin{aligned} K_{jk}(t) &= \frac{2}{\pi}\int_0^\infty b_{jk}(\omega)\cos(\omega\,t)\,d\omega \\ \lim_{\omega\to\infty} a_{jk}(\omega) &= m_{jk} \end{aligned}.$$

The only assumption made is that the ship motions are small enough to allow the boundary conditions to be satisfied on the initial position of the hull. The system of reference used here differs from the one adopted to derive the equations of motions (see Chapter 5) since this is assumed earth-fixed and with its origin lying at the free calm water surface above the CG of the vessel. Within the limits of the assumption of piece-wise linearity - see the theory on variable geometry and position dependency of the first order wave forces and hydrodynamic coefficients - this system of reference only differs from $o_d\ x_d y_d z_d$ because of the position of its origin. Remembering that for free vectors the position of the centre of the particular system of coordinates they refer to does not influence the value of their components, expressions **(10.3)** and **(11.3)** can thus be rewritten in $o_d\ x_d y_d z_d$ terms as:

(12.3)
$$F_{jk}^{Rad}(t) = -m_{jk}\,\ddot{q}_k^d(t) - \int_0^t K_{jk}(t-\tau)\,\dot{q}_k^d(\tau)\,d\tau$$

and

(13.3)
$$K_{jk}(t) = \frac{2}{\pi}\int_0^\infty b_{jk}(\omega)\cos(\omega\, t)\, d\omega$$
$$\lim_{\omega\to\infty} a_{jk}(\omega) = m_{jk}$$

since both velocities and accelerations are free vectors. The coefficients $a_{jk}(\omega)$ and $b_{jk}(\omega)$ are now the same as those in **(8.2)**. To transform these equations to the ship-fixed system of reference relations **(1.3)** are employed to yield:

(14.3)
$$F'^{Rad}_{jk}(t) = -m'_{jk}\,\ddot{q}'_k(t) - \int_0^t K'_{jk}(t-\tau)\,\dot{q}'_k(\tau)\, d\tau$$

and

(15.3)
$$K'_{jk}(t) = \frac{2}{\pi}\int_0^\infty b'_{jk}(\omega)\cos(\omega\, t)\, d\omega$$
$$\lim_{\omega\to\infty} a'_{jk}(\omega) = m'_{jk}$$

or in matrix notation:

(14.3 a)
$$\vec{F}'_{Rad} = -[A'_\infty]\,\ddot{\vec{q}}' - \int_0^t [K'(t-\tau)]\,\dot{\vec{q}}'(\tau)\, d\tau$$

once it is understood that:

(16.3)
$$\int_0^t [K'(t-\tau)]\,\dot{\vec{q}}'(\tau)\, d\tau = \sum_{k=1}^{6}\int_0^t K'_{jk}(t-\tau)\,\dot{q}'_k(\tau)\, d\tau$$

Before proceeding with some comment on Cummins' work, the discrete form of the first relation in **(15.3)** will be given next:

(15.3 a) $$K'_{jk}(t)=\frac{2}{\pi}\sum_{n=0}^{100} b'_{jk}\left(n\frac{\omega_{MAX}}{100}\right)\cos\left(n\frac{\omega_{MAX}}{100}t\right)\frac{\omega_{MAX}}{100}$$

where the range of frequencies determined by the vanishing value ω_{MAX} is generally a function of the characteristics of $b'_{jk}(\omega)$ (this implies that the energy dissipated by the radiating wave at frequencies higher than ω_{MAX} is negligible) and the number of frequency components has been assumed equal to 100 after examining the inverse transformation of the kerenel functions and having made sure that the integral of these over the convolution time range would be reasonably close to zero.

One of the compensations brought up by Cummins work was to relate directly the hydrodynamic reaction forces to the vessel motions history, thus freeing them from the instinctive belief that they are functions of the excitation frequencies. There is however a further virtue which must be recognised in Cummins' work. When a body is forced to oscillate at the free surface, waves will be generated and these will propagate as the time increases. The effect of each radiated wave will be felt by the floating object starting at the moment when this is generated, but it will continue to do so for all subsequent times. This phenomenon introduces the concept of *memory effect* and is correctly modelled by the introduction of convolution integrals in the equations of motion.

Viscous Damping

The convolution techniques illustrated above take care of the added inertia of the system and the damping contribution due to the generation of a propagating set of waves. Apart from this, however, the viscosity of the fluid through which the vessel is oscillating, is responsible for additional damping terms. Of these, the only ones that are significant in this context in terms of relative magnitude to the respective radiation terms and effect on the vessel's planar motion, are those concerning roll, surge, sway and yaw. Having already found a formulation to express the influence of surge and sway velocities on the viscous components of the planar forces and moments when the nature of the generalised current force vector was analysed, attention will be focused here only on the non-linear viscous roll damping and that term generally known as the hydrodynamic derivative N_r.

A characteristic of the viscous roll damping term is its dependence on the roll amplitude as well as frequency. This is the source of the non-linear nature of this moment. According to Ikeda and Himeno [10.7], the total roll damping moment for a regular oscillation of frequency ω and amplitude ϕ_A can be expressed in a linearised form as:

(17.3) $$M'_3(\dot{q}'_3) = b_e\, \dot{q}'_3 \quad \text{where} \quad b_e(U,\omega,\phi_A) = b_F + b_E + b_L + b_W + b_{BK}$$

The equivalent linear damping coefficient b_e is expressed in terms of its components which are: skin friction damping, eddies shedding damping, lift damping, radiating wave damping and bilge keels damping, respectively. b_e is generally dependent on the ship forward velocity U as well as on ω and ϕ_A. In the cases considered by this investigation however, the forward velocity is considered to be zero and thus the lift component will consequently disappear. Moreover, the radiating wave component - which indeed is not a viscous component - has already been accounted for previously in the chapter, and it can, therefore, be omitted here. Following these simplifications, the expression for b_e can be re-written as:

(18.3) $$b_e(\omega,\phi_A) = b_F + b_E + b_{BK}.$$

In the latter expression the dependence on roll amplitude is roughly linear with a fairly gentle positive slope for most ship types and configurations. Indeed, the skin friction damping is basically constant with ϕ_A although the same is not exactly correct for b_E and definitely inaccurate for b_{BK}. Nonetheless, for the purpose of this research, a fixed roll amplitude will be used to evaluate the roll viscous damping. Bearing in mind the assumption of stepwise linearity mentioned above, this is in fact an equitable position. Moreover, if the roll amplitude employed for the evaluation of b_e is chosen as the mean amplitude of oscillation around the instantaneous heel value, the error produced will be reasonably small. Of course, this would imply having to take into account the variation of the viscous roll damping with heel, trim and draught of the vessel. A further approximation is thus introduced, assuming that this variation will not be considerable if a coefficient calculated for the initial upright condition of the ship, using a slightly larger roll amplitude, is employed as a constant; ϕ_A is then assigned a representative value $\tilde{\phi}_A$ equal to 20 deg.

Unlike the roll amplitude dependence, the viscous damping variation with frequency is much larger. Normally, this would be a sufficient reason to seek for a reliable method to include the frequency relationship describing $b_e(\omega)$ in the simulation model correctly. Although at first this could appear possible and necessary, it is in fact impracticable and readily dispensable. This is because of the limited effect of roll viscous damping in the frequency ranges removed from the ship natural frequency. Bearing this in mind, the viscous roll damping can therefore be considered constant with frequency as well, as long as it is evaluated correctly for the ship natural frequency which can be easily estimated through the expression:

$$\omega_n = \sqrt{\frac{g\,\Delta\,\overline{GM}}{I'_{xx}}}.$$

As far as the yaw motion is concerned, an empirical expression for the viscous moment is given by Clarke et al. [10.8] based on an extensive multiple regression analysis:

$$M'_6(\dot{q}'_6) = N_r\,\dot{q}'_3.$$

In the expression above, the hydrodynamic derivative N_r is given in non-dimensional form as:

$$N_r = N'_r \frac{1}{2} \rho L^4 V_S$$

where V_S is the ship service speed and

$$N'_r = \pi \left(\frac{T}{L}\right)^2 \left(0.25 + 0.039 \frac{B}{T} - 0.56 \frac{B}{L}\right).$$

The foregoing considerations finally lead to:

$$\vec{F}'_{Viscous} = [B'_{Viscous}] \cdot \dot{\vec{q}}'$$

where all the terms of the matrix $[B'_{Viscous}]$ are zero except for $B'_{4,4}$ and $B'_{6,6}$ that are equal to $b_e(\omega_n, \tilde{\phi}_A)$ and N_r, respectively. Introducing the hydrodynamic reaction forces and damping terms in equations **(12.2)** will thus give:

$$[M + M_w + A'_\infty] \cdot \ddot{\vec{q}}' + [\dot{M}_w + B'_{Viscous}] \cdot \dot{\vec{q}}' + \int_0^t [K'(t-\tau)] \dot{\vec{q}}'(\tau)\, d\tau =$$
$$= \vec{F}'_{Wave} + \vec{F}'_{Drift} + \vec{F}'_{Wind} + \vec{F}'_{Current} + \vec{F}'_R + \vec{F}'_G - \vec{F}'^{*}_w$$

which will be referenced as **(19.3)**.

Gravitational and Restoring Forces and Moments

The Generalised Gravitational Vector

In this part of the chapter on the external forces acting on a damaged ship, the vector $\vec{F}'_G$ will be assessed. Although this vector is generally regarded as a dull constant vertical force applied at the centre of gravity of the system and pointing downwards - and hence unable to actively perturb the ship motions - it upsurge to the role of dynamic exciting force in the case of a damaged ship.

As any other thing on earth, ships are subjected to the gravitational acceleration field. The general expression describing the forces and moments resulting from this is:

$$(1.4) \qquad \vec{F}_G = \begin{pmatrix} \iiint_V \rho\, \vec{g}\, dV \\ \iiint_V \rho\, \vec{x} \wedge \vec{g}\, dV \end{pmatrix} \qquad \text{where } \vec{g} = -g \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

in an earth-fixed system of reference, where g is approximately equal to 9.81 m/sec^2 and $\vec{x}$ is the vector giving the position of any point belonging to the system. The triple integral is to be evaluated over the volume occupied by the system and the density ρ describes the distribution of mass within this volume (cf. equations **(1.1)** and **(2.1)**). When applied to a rigid body, the integral above can be effectively reduced to a vertical force applied to its centre of gravity. In the case of a damaged ship - having considered ship and flood water as one dynamic system - this is still the case but the varying nature of the position of the centre of gravity of the flood water introduces some peculiarities. Apart from this, a further point to be accurately considered is the change caused by operating in a non-inertial ship-fixed system. Assuming $O\,XYZ$ as inertial system and $Gs\,x'y'z'$ as the ship-fixed one, the derivation of the generalised gravitational vector $\vec{F}'_G$ will follow the general outline of the derivation of the equations of motion.

Having supposed the flood water concentrated in one or more discrete points having variable mass and moving according to some known law within the ship outer boundaries, the integral **(1.4)** can be re-written as:

$$\vec{F}_G = \begin{pmatrix} \iiint_V \rho\, \vec{g}\, dV \\ \iiint_V \rho\, \vec{x} \wedge \vec{g}\, dV \end{pmatrix} = \begin{pmatrix} \iiint_V \rho_s\, \vec{g}\, dV + \sum_{j=1}^{N} m_{wj}(t)\, \vec{g} \\ \iiint_V \rho_s\, \vec{x} \wedge \vec{g}\, dV + \sum_{j=1}^{N} m_{wj}(t)\, \vec{x}_{Gwj}(t) \wedge \vec{g} \end{pmatrix} \Rightarrow$$

(2.4)
$$\vec{F}_G = \begin{pmatrix} \left(m_s + \sum_{j=1}^{N} m_{wj}(t)\right) \vec{g} \\ \left(m_s\, \vec{x}_{Gs} + \sum_{j=1}^{N} m_{wj}(t)\, \vec{x}_{Gwj}(t)\right) \wedge \vec{g} \end{pmatrix} = \begin{pmatrix} m(t)\, \vec{g} \\ m(t)\, \vec{x}_G(t) \wedge \vec{g} \end{pmatrix}$$

Once the espression for the gravitational vector is put in this form, the dynamic excitation character of it can be recognised in the time dependence of m_{wj} and $\vec{x}_{Gwj}$. Of course, the type of function describing the variation of these quantities with time will depend on the sort of flooding mechanism and dynamic water behaviour model employed, but it can be safely assumed that its nature will generally be oscillatory.

The next step - previous the introduction of **(2.4)** in the equations of motion - is the translation of the vector $\vec{F}_G$ into its expression with respect to the ship-fixed co-ordinates. This will produce:

(3.4)
$$\vec{F}'_G = \begin{pmatrix} \left(m_s + \sum_{j=1}^{N} m_{wj}(t)\right) [D] \cdot \vec{g} \\ \left(\sum_{j=1}^{N} m_{wj}(t)\, \vec{x}'_{Gwj}(t)\right) \wedge [D] \cdot \vec{g} \end{pmatrix} = \begin{pmatrix} m(t)\, \vec{g}' \\ m_w(t)\, \vec{x}'_{Gw}(t) \wedge \vec{g}' \end{pmatrix}$$

given that:

$$\left(\vec{x}_{Gs}\right)_{O\,XYZ} = \left(\vec{0}\right)_{Gs\,x'y'z'}$$

$$\left(\vec{x}_{Gw}\right)_{O\,XYZ} = \left(\vec{x}'_{Gw}\right)_{Gs\,x'y'z'}.$$

The integral above will prove to be responsible for most of the system excitation - and thus the final degradation of its stability - once the value of the mass of the flood water exceeds a limit value marking the final loss of restoring ability of the vessel. For this reason particular attention and accuracy comparable to that bestowed on the evaluation of the gravitational forces have to be granted to the calculation of the restoring forces. The non-linear model adopted to estimate restoring forces in this investigation is illustrated next.

Restoring Forces

$\vec{F}'_R$ is the restoring force arising from the buoyancy capability of a vessel and it is accounted for by direct integration of the underwater volume of the hull.

The expression that first comes to mind when considering restoring forces contains these terms which are directly proportional to the vessel displacement vector. In such an expression the balance between gravitational and buoyancy forces has already taken place and this vector - which is defined here as the result of the integration of the hydrostatic pressure over the underwater body - is considered separately from all the other external excitation vectors. Obviously such an approximation is not feasible for the description of a transient during which the underwater geometry of the floating body changes rapidly and significantly as in the case of a damaged ship.

The need for an accurate and rigorous method to estimate the restoring term leads to the basic Archimedean theory. According to this, the force applied to the floating body is proportional to the mass displaced by the vessel and it is applied in the centroid of the underwater volume - the centre of buoyancy. Although this formulation seems to be neat and trouble-free, there is in fact a point worth discussing briefly before describing the methodology employed to apply it. This issue regards the presence of waves.

In the linear theory adopted for the wave excitation forces and the hydrodynamic reaction, the integration of the dynamic pressure was performed up to the mean water level. This is in accordance with the assumptions adopted in deriving the wave velocity potentials. As far as the hydrostatic pressure is concerned, no hypothesis of linearity was adopted to derive it and this could induce a strong temptation to extend the domain of integration of the underwater volume up to the instantaneous wave elevation in an attempt to improve accuracy. This is incorrect, however, as the time-varying pressure distribution due to the disturbance of the free surface has already been accounted for by the Froude-Krylov and diffracted wave forces. It is readily proven that such procedure would generate, in the extreme case of long wave lengths, the contradictory result of Froude-Krylov forces and the oscillatory part of the hydrostatic force neutralising each other in a region close to the water surface. Clearly, this implies that - considering only the linear incident wave potential - the influence of the perturbation would be felt with more strength away from the mean water surface. This is obviously in direct contrast with both experimental evidence and common sense.

The reason that give rise to such a contradiction is that estimating the underwater volume up to the instantaneous water surface means, in fact, assuming that the hydrostatic pressure goes to zero at the wave surface rather than at the mean water level. This dissents with the dynamic condition at the water surface employed to derive the wave velocity potential, which states that the sum of static and dynamic pressure should, indeed, be zero at the boundary with atmosphere. The scenario becomes even more complex in the case of a time varying domain of integration - as for a ship whose mean underwater body varies significantly with time - in which case making different assumptions for hydrostatic and hydrodynamic forces might lead to unpredictable results. Nevertheless, since some might still be tempted to give credit to this proceeding, the calculation of the restoring forces is supported here both including the instantaneous wave elevation in the evaluation of the underwater volume and limiting it to the mean water level, in the attempt of proving with facts the inadequacy of the first method.

For obvious reasons, the expression for $\vec{F}'_R$ cannot be simple. Generally speaking the restoring vector is a function of the instantaneous position of the ship and, for this characteristic, it can be represented by:

$$\text{(4.4)} \qquad \vec{F}_R = -\begin{pmatrix} \rho_{Water} V_{Underwater} \vec{g} \\ \rho_{Water} V_{Underwater} \vec{x}_B \wedge \vec{g} \end{pmatrix} \qquad \text{where} \quad V_{Underwater} = V_{Ship} \cap V_{Sea}$$

This expression derives from the integration of the hydrostatic pressure ($p_{Hydrostatic} = -\rho g\, z$ in the earth-fixed system of reference) over the wetted surface of the hull, once the Gauss-Green theorem has been applied as follows:

$$\vec{F} = \iint_S p\,\vec{n}\, dS = -\iiint_V \vec{\nabla} p\, dV = -\begin{pmatrix} \iiint_V \frac{\partial p}{\partial x} dV \\ \iiint_V \frac{\partial p}{\partial y} dV \\ \iiint_V \frac{\partial p}{\partial z} dV \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \iiint_V \rho g\, dV \end{pmatrix} = -\rho V \vec{g}$$

where $\vec{n}$ points into the underwater volume. Similarly, for the moments equation:

$$\vec{M} = \iint_S p\,\vec{x} \wedge \vec{n}\, dS = \begin{pmatrix} \iint_S p\left(y\, n_z - z\, n_y\right) dS \\ \iint_S p\left(z\, n_x - x\, n_z\right) dS \\ \iint_S p\left(x\, n_y - y\, n_x\right) dS \end{pmatrix} = \begin{pmatrix} \iint_S p \begin{pmatrix} 0 \\ 0 \\ y \end{pmatrix} \cdot \vec{n}\, dS - \iint_S p \begin{pmatrix} 0 \\ z \\ 0 \end{pmatrix} \cdot \vec{n}\, dS \\ \iint_S p \begin{pmatrix} z \\ 0 \\ 0 \end{pmatrix} \cdot \vec{n}\, dS - \iint_S p \begin{pmatrix} 0 \\ 0 \\ x \end{pmatrix} \cdot \vec{n}\, dS \\ \iint_S p \begin{pmatrix} 0 \\ x \\ 0 \end{pmatrix} \cdot \vec{n}\, dS - \iint_S p \begin{pmatrix} y \\ 0 \\ 0 \end{pmatrix} \cdot \vec{n}\, dS \end{pmatrix}$$

and applying $\iint\limits_{S} \vec{F}\cdot\vec{n}\, dS = -\iiint\limits_{V} \vec{\nabla}\cdot\vec{F}\, dV$ to each double integral above:

$$\vec{M} = \begin{pmatrix} -\iiint\limits_{V} \dfrac{\partial(-\rho g z y)}{\partial z}\, dV + \iiint\limits_{V} \dfrac{\partial(-\rho g z^2)}{\partial y}\, dV \\ -\iiint\limits_{V} \dfrac{\partial(-\rho g z^2)}{\partial x}\, dV + \iiint\limits_{V} \dfrac{\partial(-\rho g z x)}{\partial z}\, dV \\ -\iiint\limits_{V} \dfrac{\partial(-\rho g z x)}{\partial y}\, dV + \iiint\limits_{V} \dfrac{\partial(-\rho g z y)}{\partial x}\, dV \end{pmatrix} = \begin{pmatrix} \iiint\limits_{V} \rho g y\, dV \\ -\iiint\limits_{V} \rho g x\, dV \\ 0 \end{pmatrix} =$$

$$= -\iiint\limits_{V} \rho\, \vec{x} \wedge \vec{g}\, dV = -\rho V\, \vec{x}_B \wedge \vec{g}$$

where $\vec{x}_B$ is the centroid of the underwater body. Expression **(4.4)** becomes:

(5.4) $$\vec{F}'_R = -\begin{pmatrix} \rho_{Water} V_{Underwater} \vec{g}' \\ \rho_{Water} V_{Underwater} \vec{x}'_B \wedge \vec{g}' \end{pmatrix}$$

in non-inertial co-ordinates. Note that the underwater volume $V_{Underwater}$ should still be calculated with reference to the earth-fixed co-ordinate system. This implies that volumes above the *Z=0* plane should be considered negative.

In conclusion, equation **(19.3)** can be expressed as **(6.4)**:

$$\left[M + M_w(t,\vec{q}) + A'_\infty\left(\hat{\vec{q}}\right)\right]\cdot\ddot{\vec{q}}' + \left[\dot{M}_w(t,\vec{q}) + B'_{Viscous}\right]\cdot\dot{\vec{q}}' + \int_0^t \left[K'\left(t-\tau,\hat{\vec{q}}\right)\right]\dot{\vec{q}}'(\tau)\, d\tau =$$
$$\vec{F}'_{Wave}(t,\vec{q}) + \vec{F}'_{Drift}(t,\vec{q}) + \vec{F}'_{Wind}\left(t,\vec{q},\dot{\vec{q}}\right) + \vec{F}'_{Current}\left(t,\vec{q},\dot{\vec{q}}\right) + \vec{F}'_R(t,\vec{q}) + \vec{F}'_G(t,\vec{q}) - \vec{F}'^{*}_w(t,\vec{q})$$

In the above matrix expression the dependence of the various terms on time and instantaneous and mean values of displacement and velocity is made explicit. This is a useful way to isolate those terms which are slowly varying and can therefore be considered as pseudo-constants in the numerical integration of **(6.4)**. In Chapter 8 the numerical scheme developed to solve this equation will be presented in details, but first some comments are necessary to clarify the nature of the function $M_w(t, \vec{q})$.

7. Flooding of Damaged Compartments

General Remarks

Flooding of damaged compartments is a highly complicated matter to tackle and it needs very strong assumptions to be made, if easy-to-use equations are desired. If a theoretical approach to the problem applying the concept of velocity potential was attempted, this would present a great number of difficulties, not least the fact that a velocity potential for the water sloshing inside the compartments would be necessary to model outflow, as well as that governing the dynamics of the water flowing in. Notoriously, neither of these is readily available, unless employing CFD techniques. Clearly, once this route is abandoned for its evident impracticability, other effects would have to be overlooked or treated empirically. One of these is, for instance, the diffraction of the incident wave by the ship and how this essentially change the fluid flow characteristics depending on the angle of encounter. Preliminary experimental investigations showed, in fact, that the *shielding* due to the presence of ship, when the damage is on the lee side, has some influence on the time record of the flood water. On the basis of the foregoing considerations, any effort to use the undisturbed wave potential only to describe the flow pattern of the water going through a damage opening, will have to be considered partial and incomplete.

Problem Formulation

Having discarded every expectation of using potential theory in approaching this problem, the first, obvious attempt will be refreshing concepts and formulae from the engineering hydraulics books ([11.2] to [11.6]) and trying to adapt them to the specific case investigated. At first glance, in the case of water flooding through a damage, the most suitable flow configurations with a well established theoretical background appear to be the water flow through orifices and over notches, or - although only marginally - some of the open channels flows. All the formulations regarding these applications assume steady flow conditions and calm water surface; all of them rely on the application of Bernoulli's equation (and the continuity equation in the case of open channels), corrected by an empirical coefficient to take account of all energy losses.

Disregarding open channels, attention will be initially given to the formulations adopted for the remaining flow configurations. The general form of the equations used to evaluate the volume rate of flow trough orifices is:

(1.5) $$Q = CA\sqrt{2gH}$$

where Q is the volume flow rate, A is the area of the opening, and the term $\sqrt{2gH}$,

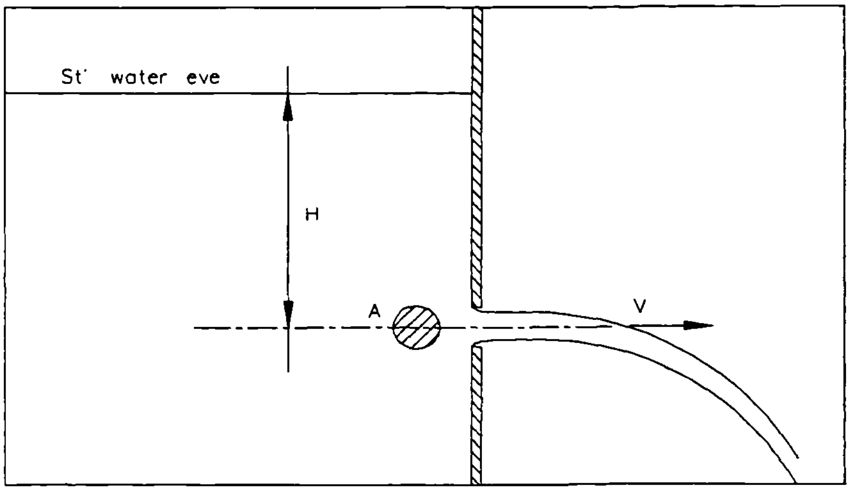

Fig. 1

Flow through an orifice.

derived from the equation $h + \frac{p}{\rho g} + \frac{v^2}{2g} = const$, applied between the still water surface and the orifice position, represents the theoretical velocity of water jet. The formula above assumes zero velocity of the water at the free surface in the reservoir and atmospheric pressure there and at the orifice centroid. The coefficient C takes account of the reduction in velocity of the out-flowing volume, due to viscous resistance, as well as of the energy losses caused by the contraction of the jet section. This coefficient is usually expressed as: $C = C_v C_a$

In the case of flow over a notch a slightly different formula applies:

(2.5) $$Q = CLH\sqrt{2gH}$$

where L is the width of the notch. The main differences consist of the definition of H and the values that C assumes depending on the shape of the notch, its dimensions, the channel geometry, etc.

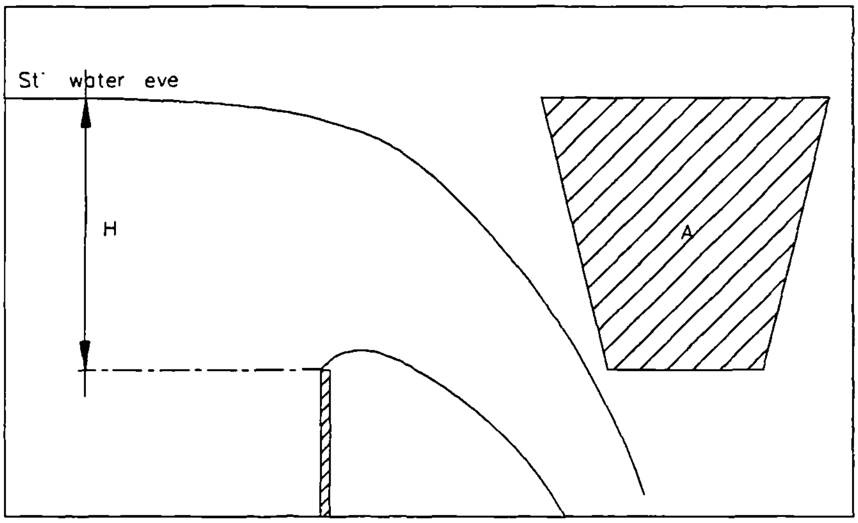

Fig. 2

Flow over a trapezoidal notch.

In this context, the pressure head is measured from the water level to the crest of the notch, and the coefficient C can be expressed, for instance, for a rectangular notch without side contraction, as:

$$C = \frac{2}{3}(0.627 + 0.018\frac{H}{p}) \qquad (\text{I.M.F.T.}) \tag{3.5}$$

where p is the distance of notch crest from the bottom of the channel. An exhaustive and authoritative treatise of all the empirical and semi-empirical formulae proposed to evaluate C can be found in the paragraph ' Thin-Plate Weirs and Notches ' in [11.4].

Although the formulae above are quite attractive for their simplicity they do not supply an adequate answer to the problem investigated here. Indeed, they are not even representative of all the possible flooding modes that a ship can experience. Water ingress in a damaged compartment of a vessel is expected to depend upon many different parameters including the velocity pattern of the water outside the compartment, the sloshing of the shipped water, the vessel motions and the relative vertical position of the two water levels (on the inner and the outer sides of the

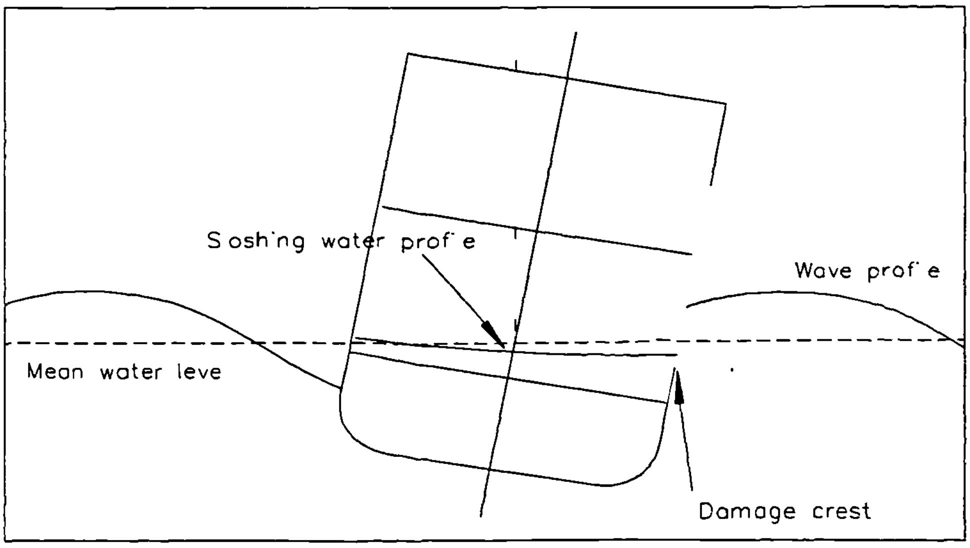

Fig. 3

Typical ship flooding scenario.

damage window), the location and extent of the damage, the internal arrangement of the compartment, the shape of the damage opening, the ship speed and so on - just to name some.

Some papers describe previous attempts to simplify this scenario [2.11]. Most refers to the technical inquiries following ship losses and all seem to employ some version of the above mentioned formulae, corrected with an empirical coefficient especially adapted to the case in consideration. In this context a general formulation is sought, valid for most flooding scenarios and including as many dynamic effects involved as possible.

Comparing fig. 3 to the previous ones, the first difference clearly showing is unquestionably that water is often present on both sides of the opening in the case of a ship flooding, whilst this does not occur in the situations described by figs. 1 and 2. This simple observation, leads to considering other civil engineering applications of Bernoulli's equation, namely submerged orifices and channels fed by reservoirs through uncontrolled inlets (figs. 4 and 5).

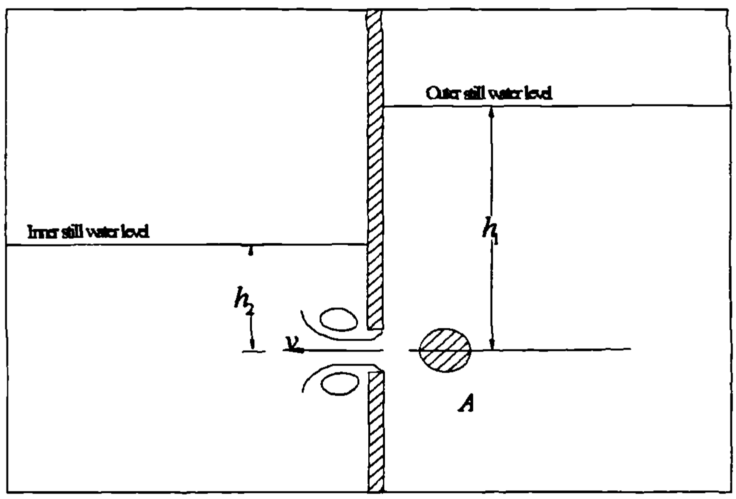

Fig. 4

Flow through a submerged orifice.

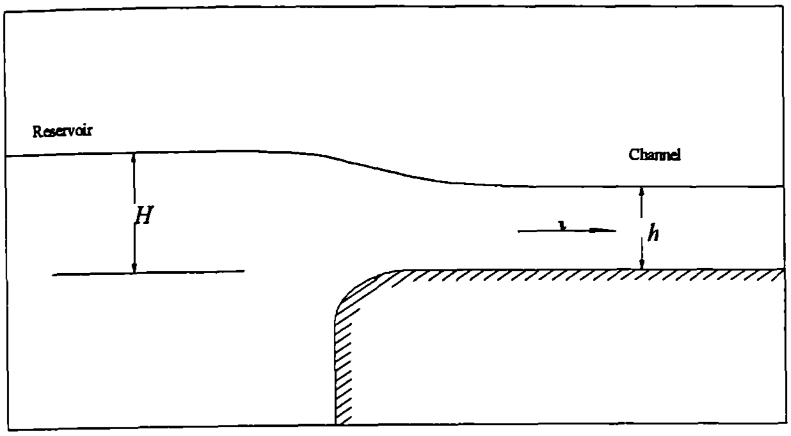

Fig. 5

Flow from a reservoir to an open channel.

The formulae describing the flow rate for the above cases, are:

(4.5) $$Q = CA\sqrt{2g(h_1 - h_2)}$$ for the submerged orifice, and

(5.5) $$Q = Cbh\sqrt{2g(H - h)}$$ in the second example, where b is the channel breadth.

Equations **(4.5)** and **(5.5)** can be found applying Bernoulli between an upstream section where the water can be considered still and thus the pressure hydrostatic, and a second section at which the mean velocity is calculated. The total pressure head is considered to keep theoretically the same for both sections. Again, frictional losses are taken care of, by introducing the coefficient C.

It is now time to consider a simplified picture of flooding of ship compartments and generalise the theory used so far for steady condition cases to a fully dynamic one. Clearly, a first, major assumption needs to be made in order to do this; that is accepting that theory developed for time independent phenomena can be indeed modified and adapted to a dynamic case and still produce results which are reasonably close to reality.

Consider a vessel the inner space of which is open to the sea as in fig. 3. Ignoring for a moment the real dynamics of the water particles - inside and outside the compartment - assume the picture can be simplified by supposing the two water levels

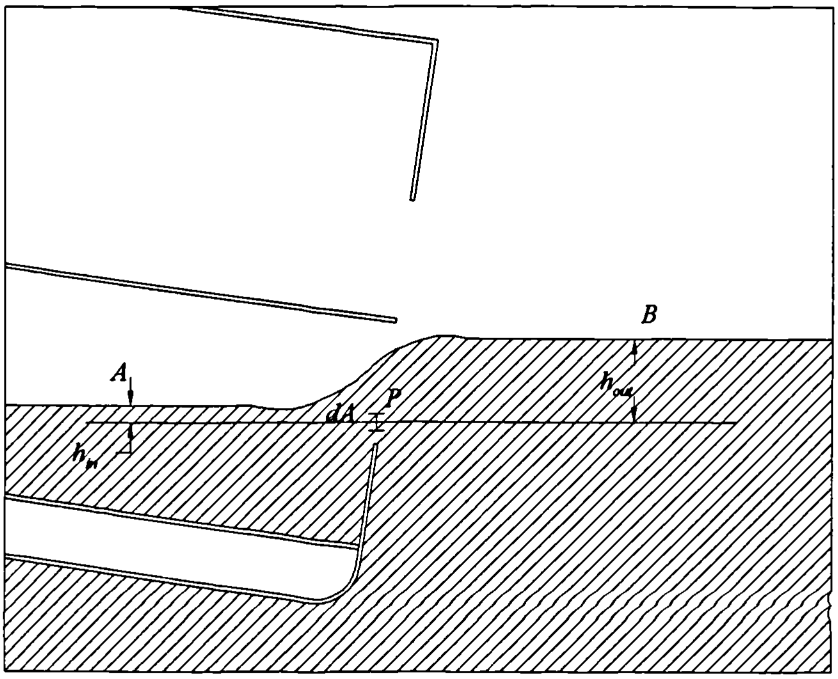

Fig. 6

Ship flooding main parameters.

as still and horizontal. The resemblance to the static cases just illustrated is evident. If Bernoulli's equation is applied at sections A and B (fig. 6 above), considering the total pressure head maintained constant and the velocity zero in the reservoir at point P the flow rate can be found as follows:

$$H_{in} = h_{in} + \frac{p_{atm}}{\rho g} + \frac{v^2}{2g} \qquad \text{and} \qquad H_{out} = h_{out} + \frac{p_{atm}}{\rho g} + 0$$

$$H_{out} = H_{in} \quad \Rightarrow \quad h_{out} + \frac{p_{atm}}{\rho g} + 0 = h_{in} + \frac{p_{atm}}{\rho g} + \frac{v^2}{2g} \qquad \text{i.e.}$$

$$v = \sqrt{2g(h_{out} - h_{in})}$$

and the flow rate through the horizontal layer around P:

(6.5 a) $$dQ = C\sqrt{2g(h_{out} - h_{in})}dA$$

This formula will be referred to as the *pressure head* equation. The total flow rate can be found by integrating **(6.5 a)** over the damage opening height. Two observations are worth noticing at this point. Firstly, the formula just derived reduces to the general form of those used for free discharging orifices and notches when either h_{out} or h_{in} is negative, if the following limits are set:

(6.5 b) $$\begin{cases} h_{in} = 0 & if \quad h_{in} \le 0 \\ h_{out} = 0 & if \quad h_{out} \le 0 \end{cases}$$

This will take care of those situations in which water is present only on one side of the damage. Of course, when h_{out} is less than h_{in}, the flow becomes negative, and water is expected to flow out of the compartment and into the sea. To accommodate for this the *pressure head* equation becomes:

(6.5)
$$dQ = C \cdot sign(h_{out} - h_{in}) \cdot \sqrt{2g|h_{out} - h_{in}|} \cdot dA$$

$$\begin{cases} h_{in} = 0 & if \quad h_{in} \le 0 \\ h_{out} = 0 & if \quad h_{out} \le 0 \end{cases}$$

The second point deserving attention concerns C and how this parameter can be altered to suit a more realistic, dynamic case in which incident, diffracted and radiating waves are present, as well as sloshing of the flood water. Before tackling this point, an alternative formulation for the flow through the elementary area dA, will be introduced. Some authors [11.8] have derived equations describing the flow through a damage opening building on slightly different basis from those adopted in this dissertation. The resulting expression is:

(7.5) $$dQ = C\left[\sqrt{2gh_{out}} - \sqrt{2gh_{in}}\right]dA$$

The main assumption necessary to obtain **(7.5)** is that the flows into and out of the compartment can be evaluated independently at the same location by using a free discharge kind of expression. The rates so found can thus be superimposed to find the net flow. Note that this approach implies that the pressure at P is considered atmospheric everywhere over the damage window (i.e. even where water is present on both sides, in which case this supposition can not be accepted without some perplexity), since this was one of the hypothesis used to derive **(1.5)** and **(2.5)**. This assumption is added to considering the water velocity superposition valid, therefore it makes the pressure head equation preferable, in principle, to formula (7.5), which will be referred to as the *velocity superposition* equation hereafter. Limits as in **(6.5 b)** also apply here. Since all the hypothesis made so far to derive **(7.5)** and **(6.5)** are quite arbitrary - thanks to the time varying nature of the phenomenon studied - the most appropriate formula to use should be chosen on the basis of the results of the experimental investigation necessary to tune the flooding coefficient.

About the coefficient C and the problem of modeling unsteady flow conditions, a lot is still to be learned. The main idea is that C can be considered as a function of some relevant parameters which do not appear in the time independent formulations and that the expression for this function can be found empirically. Q is expected to vary as a function of:

- the velocity pattern of the water outside the compartment, including the influence of diffraction, radiation and the ship speed;
- the dynamic behaviour of the shipped water and how this is affected by the internal arrangement of the compartment;
- the vessel motions and the relative vertical position of the two water levels, as well as the location and extent of the damage;

- the shape of the damage opening.

Of the parameters contemplated above, the last two are taken care of, to a certain extent, within the formulations described so far when h_{out}, h_{in} and dA are functions of time. Although further analysis will have to be carried out to improve the understanding of such relationships, prominence will be given here to the variation of C with the external and internal water characteristics.

To assess the action of waves and sloshing, the experimental route seems to be the only choice available at present. Naturally, adopting this investigation technique implies selecting a finite quantity of well defined variables (i.e. parameters capable of describing the state of the fluid on either side of the opening), which will be used to set and run tests. The results of these will create the skeleton on which the function C can be modelled. It is postulated here that the following quantities are adequately chosen and sufficient to give a complete picture of the water configurations inside and outside the compartment, assuming zero forward speed of the ship:

Water outside the ship

H_s	=	Significant wave height;
T_z	=	Mean zero crossing period;
ψ	=	Angle of encounter (see previous chapter for definition);

Water inside the compartment

b	=	Breadth of the damaged compartment;
KG	=	Vertical position of the centre of gravity of the intact ship;
GM	=	Metacentric height of the intact ship;
KF	=	Vertical position of the compartment floor.

These parameters can be put in non-dimensional form using the ship's main dimensions and, therefore, applied to define the functional dependency of C. At the moment, very little testing has been carried out, mainly examining the influence of those quantities characterising the behaviour of the water outside the ship. Although the need for a much more accurate investigation is called here again, some observations and findings achieved so far can be helpful in directing future research. Of the few results available it is worth mentioning that, once the position of the water levels over the damage opening is accurately estimated, the rate of flooding does not seem to depend upon the wave height. This is not surprising, if one bears in mind that establishing the exact position of the water levels in question somehow gives an estimation of the effect of diffraction, radiation and sloshing. Besides, the velocity patterns of the water particles being very changeable, the effect of these appears to be evened out as the time progresses.

Although C seems to be independent from H_s, this is no more the case when the variation of C with the wave frequency is considered. A general trend of the coefficient changing with the non-dimensional quantity $\frac{\omega^2 B}{2\pi g}$ is apparent. Unfortunately the range of frequencies investigated here (see Appendix C) is not sufficiently broad to allow reliable curves to be drawn, covering the spectrum peak frequencies usually encountered by a vessel at sea. New experiments with a larger scale model could obviate to this inconvenient deficiency.

Finally, a remarkable fall in the mean value of C was observed in the time records as soon as water started being present on both sides of the damage opening. As this is a clear sign of the importance of water sloshing in the compartment on the flooding dynamics, two coefficients have been proposed which are intended to be used according to the particular case encountered. The mean value for the flooding coefficient when pure inflow occurs (water only present on one side of the damage window) is about 1.4, whilst in the case of water present on both sides, it falls down to 0.8.

When the direction of the oncoming waves is away from the damage, the corresponding values of the flooding coefficient seem to be closer to each other and laying between those calculated for the "wave into damage" case.

Once a value for C has been established the rate of flooding of a given compartment - and thus the time history of the quantity of water flooding it - can be found according to the following expression:

$$\dot{m}_{wj}(t,\vec{q}) = C \int_{Damage} sign(h_{out} - h_{in}) \sqrt{2\, g\, |h_{out} - h_{in}|}\; dA$$

as long as the damaged compartment is open to the sea. Obviously the matrix $M_w(t,\vec{q})$ can therefore be calculated integrating $\dot{m}_{wj}(t,\vec{q})$ in time.

8. Numerical Integration of the Equation of Motion in the Time Domain

General Remarks

A set of equations describing the behaviour of a ship undergoing progressive flooding has been derived and particulars have been given on the nature of the forces and moments involved. In order to calculate those quantities of interest for the assessment of the damage survivability of a vessel - ship motions, velocities and accelerations; but also water accumulating in the compartments, time to capsize etc. - a numerical algorithm is needed which will integrate the equations provided. The theory concerning the development and validation of this computational technique will be discussed in the present chapter.

Problem Formulation

Before attempting to find a numerical scheme to integrate **(6.4)**, it is advisable to simplify the notation used to represent this equation. The following substitutions accomplish this task without neglecting to display the dependence of the various terms on time, $\vec{q}\,'$, $\vec{q}$ and their derivatives.

$$\left[M + M_w(t,\vec{q}) + A'_\infty\left(\hat{\vec{q}}\right)\right] = \left[A(t,\vec{q})\right]$$

$$\left[\dot{M}_w(t,\vec{q}) + B'_{Viscous}\right] = \left[B(t,\vec{q})\right]$$

$$\vec{F}'_{Wave}(t,\vec{q}) + \vec{F}'_{Drift}(t,\vec{q}) + \vec{F}'_{Wind}(t,\vec{q},\dot{\vec{q}}) + \vec{F}'_{Current}(t,\vec{q},\dot{\vec{q}}) + \vec{F}'_R(t,\vec{q}) + \vec{F}'_G(t,\vec{q}) - \vec{F}'^{*}_w(t,\vec{q}) = \vec{F}(t,\vec{q},\dot{\vec{q}})$$

thus equation

$$\left[M + M_w(t,\vec{q}) + A'_\infty(\hat{\vec{q}})\right]\cdot\ddot{\vec{q}}' + \left[\dot{M}_w(t,\vec{q}) + B'_{Viscous}\right]\cdot\dot{\vec{q}}' + \int_0^t \left[K'(t-\tau,\hat{\vec{q}})\right]\dot{\vec{q}}'(\tau)\,d\tau =$$
$$\vec{F}'_{Wave}(t,\vec{q}) + \vec{F}'_{Drift}(t,\vec{q}) + \vec{F}'_{Wind}(t,\vec{q},\dot{\vec{q}}) + \vec{F}'_{Current}(t,\vec{q},\dot{\vec{q}}) + \vec{F}'_R(t,\vec{q}) + \vec{F}'_G(t,\vec{q}) - \vec{F}'^{*}_w(t,\vec{q})$$

becomes

(1.6) $$\left[A(t,\vec{q})\right]\cdot\ddot{\vec{q}}' + \left[B(t,\vec{q})\right]\cdot\dot{\vec{q}}' + \int_0^t \left[K'(t-\tau)\right]\dot{\vec{q}}'(\tau)\,d\tau = \vec{F}(t,\vec{q},\dot{\vec{q}}).$$

where the slowly varying nature with $\hat{\vec{q}}$ of some of the terms is considered understood and omitted from the notation.

Equation **(1.6)** is a system of six second order, non-homogeneous integral-differential equations with variable coefficients and convolution integrals of the unknown $\dot{\vec{q}}'$. Note that the principal quantity of interest is $\vec{q}$ (position of the ship-fixed system of reference with respect to the inertial one) and that the matrices $[A]$ and $[B]$ and the vector $\vec{F}$ are directly dependent on it and its first derivative. In order to complete this set of equations an additional relation linking $\vec{q}$ and $\dot{\vec{q}}'$ is needed. This can be found considering the definition of $\dot{\vec{q}}'$ (cf. Chapter 5).

The free vector $\vec{v}_{Gs}$ can be represented by:

$$\left(\vec{v}_{Gs}\right)_{Gs\,x'y'z'} = \dot{q}'_1\,\vec{i} + \dot{q}'_2\,\vec{j} + \dot{q}'_3\,\vec{k}$$

which can be transformed to earth-fixed co-ordinates simply by applying the rotation matrix $[D]^T$ to it:

$$(\vec{v}_{Gs})_{O\,XYZ} = [D]^T \cdot \begin{pmatrix} \dot{q}'_1 \\ \dot{q}'_2 \\ \dot{q}'_3 \end{pmatrix} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix}.$$

Once integrated, this vector results in the position of the centre of gravity of the intact ship in the inertial system; that is in the first three elements of vector $\vec{q}$.

The same considerations do not hold exactly for the equivalent rotational vector. As already anticipated in Chapter 5, $\vec{\alpha}$ is not a proper vector, consequently the sequence of rotations must be taken into account when integrating the rate of rotation vector:

$$\left(\vec{\Omega}\right)_{Gs\,x'y'z'} = \dot{q}'_4\,\vec{i} + \dot{q}'_5\,\vec{j} + \dot{q}'_6\,\vec{k}\,.$$

Adopting the general notation introduced to derive the rotation matrix $[D]$ (see Appendix A), it is thus necessary to express $\vec{\Omega}$ in the same system of reference around which the Euler's angles are measured, then integrate it to find their new value. To achieve this, note that the earth-fixed system is first rotated by ψ around its Z axis, then by φ around the newly found $\hat{X}$ axis and finally by ϑ around $\tilde{Y}$ to become parallel to the ship-fixed one. The three axes around which the Euler's angles are measured are thus generally non-orthogonal.

The vector $\vec{\Omega}$ can be represented in this system of reference as:

$$\left(\vec{\Omega}\right)_{O\,\hat{X}\tilde{Y}Z} = \begin{pmatrix} \dot{\varphi} \\ \dot{\vartheta} \\ \dot{\psi} \end{pmatrix}.$$

To express $\dot{\psi}\,\vec{k}$ in the ship-fixed co-ordinates, it is necessary to apply the whole rotation matrix $[D]$ to this vector. This operation yields:

$$\left(\dot{\vec{\psi}}\right)_{Gs\,x'y'z'} = \begin{bmatrix} \cos\vartheta & 0 & -\sin\vartheta \\ 0 & 1 & 0 \\ \sin\vartheta & 0 & \cos\vartheta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{bmatrix} \cdot \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \dot{\psi} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \dot{\psi} \begin{pmatrix} -\cos\varphi \cdot \sin\vartheta \\ \sin\varphi \\ \cos\varphi \cdot \cos\vartheta \end{pmatrix}$$

Unlike $\dot{\psi}$, the second rate of rotation $\dot{\varphi}$ take place around the $\hat{X}$ axis thus only the last two rotations are needed to convert this vector to $Gs\,x'y'z'$ co-ordinates:

$$\left(\dot{\vec{\varphi}}\right)_{Gs\,x'y'z'} = \begin{bmatrix} \cos\vartheta & 0 & -\sin\vartheta \\ 0 & 1 & 0 \\ \sin\vartheta & 0 & \cos\vartheta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{bmatrix} \cdot \dot{\varphi} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \dot{\varphi} \begin{pmatrix} \cos\vartheta \\ 0 \\ \sin\vartheta \end{pmatrix}$$

Finally, using only the last rotation matrix to transform $\dot{\vartheta}$:

$$\left(\dot{\vec{\vartheta}}\right)_{Gs\,x'y'z'} = \begin{bmatrix} \cos\vartheta & 0 & -\sin\vartheta \\ 0 & 1 & 0 \\ \sin\vartheta & 0 & \cos\vartheta \end{bmatrix} \cdot \dot{\vartheta} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \dot{\vartheta} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Using these results, it is possible to give an expression for $\vec{\Omega}$ in the ship-fixed system of reference as a function of the rate of change of the Euler's angles:

$$\left(\vec{\Omega}\right)_{Gs\,x'y'z'} = \begin{pmatrix} \dot{q}'_4 \\ \dot{q}'_5 \\ \dot{q}'_6 \end{pmatrix} = \begin{bmatrix} \cos\vartheta & 0 & -\cos\varphi \cdot \sin\vartheta \\ 0 & 1 & \sin\varphi \\ \sin\vartheta & 0 & \cos\varphi \cdot \cos\vartheta \end{bmatrix} \begin{pmatrix} \dot{\varphi} \\ \dot{\vartheta} \\ \dot{\psi} \end{pmatrix}$$

where the matrix above is not a proper rotational matrix. Inverting this relationship to find $\dot{\varphi}$, $\dot{\vartheta}$ and $\dot{\psi}$ as function of $\dot{q}'_4$, $\dot{q}'_5$ and $\dot{q}'_6$ finally produces:

$$\begin{pmatrix} \dot{\varphi} \\ \dot{\vartheta} \\ \dot{\psi} \end{pmatrix} = \begin{bmatrix} \cos\vartheta & 0 & \sin\vartheta \\ \tan\varphi\cdot\sin\vartheta & 1 & -\tan\varphi\cdot\cos\vartheta \\ -\sec\varphi\cdot\sin\vartheta & 0 & \sec\varphi\cdot\cos\vartheta \end{bmatrix} \begin{pmatrix} \dot{q}'_4 \\ \dot{q}'_5 \\ \dot{q}'_6 \end{pmatrix} = [D]^R \cdot \begin{pmatrix} \dot{q}'_4 \\ \dot{q}'_5 \\ \dot{q}'_6 \end{pmatrix}$$

which has singularities for $\varphi = \dfrac{\pi}{2} + n\pi$, i.e. when the first and the last Eulerian rotation take place around the same axis - naturally this eventuality is of no concern to the problem in question since the ship is considered capsized for roll angles larger than 40 degrees. These quantities can now be integrated with respect to time to find the Euler's angles; i.e. the last three elements of $\vec{q}$.

In terms of $\dot{\vec{q}}'$ and $\dot{\vec{q}}$, the expressions found can be re-written as:

$$\dot{\vec{q}} = [R(\vec{q})]\cdot\dot{\vec{q}}'$$

where

$$[R(\vec{q})] = \begin{bmatrix} [D]^T & 0 \\ 0 & [D]^R \end{bmatrix}.$$

Renaming $\dot{\vec{q}}' = \vec{p}$ for simplicity of notation, equations **(1.6)** become:

$$\begin{cases} \dot{\vec{q}} = [R(\vec{q})]\cdot\vec{p} \\ \dot{\vec{p}} = [A(t,\vec{q})]^{-1}\cdot\vec{F}(t,\vec{q},\vec{p}) - [A(t,\vec{q})]^{-1}\cdot[B(t,\vec{q})]\cdot\vec{p} - [A(t,\vec{q})]^{-1}\cdot\int_0^t [K'(t-\tau)]\vec{p}(\tau)\,d\tau \end{cases}$$

having separated the first derivatives of the functions $\vec{p}$ and $\vec{q}$ from the remaining terms. Note that to allow this transformation, the inverse of $[A]$ has to be evaluated.

Given the complex nature of this matrix, this operation can only be achieved employing numerical methods every time the values of its elements change. This requirement comes as a result of the direct dependence of the generalised inertia matrix on t and $\vec{q}$. Once again, to simplify the notation let:

$[A]^{-1} \cdot [B] = [C]$ to find:

$$(2.6) \quad \begin{cases} \dot{\vec{q}} = [R(\vec{q})] \cdot \vec{p} \\ \dot{\vec{p}} = [A(t,\vec{q})]^{-1} \cdot \vec{F}(t,\vec{q},\vec{p}) - [C(t,\vec{q})] \cdot \vec{p} - [A(t,\vec{q})]^{-1} \cdot \int_0^t [K'(t-\tau)] \vec{p}(\tau)\, d\tau \end{cases}$$

Once put in this form, expression **(2.6)** represents a system of twelve first order equations which can be solved through any standard numerical scheme. However, the presence of the convolution integrals suggests the choice of a one step method as the simplest available. A variation of the Runge-Kutta-Fehlberg formula is adopted here.

The reasons that lead to such choice are multiple. Firstly, Runge-Kutta schemes are widely used and extensively validated formulae which allow solutions of systems of first order differential equations with great accuracy (RKF is a fourth order method, i.e. bearing a local truncation error proportional to the fifth order of the time step). Moreover, the Runge-Kutta-Fehlberg scheme employed here is an adaptive modification of the original method which permits one to keep the local absolute error under control by modifying the size of the time step. Other valuable characteristics of this scheme are the limited number of function evaluations per time step which makes it faster in comparison with other adaptive methods of the same kind (see [13.13]) and - last but not least - the implicit simplicity of one step methods which greatly reduces the complexity implied in dealing with convolution terms. The following algorithm illustrates the adaptive form of the Runge-Kutta-Fehlberg method used here.

To approximate the solution of the mth order system of first order initial-value problems

$$\dot{u}_j = f_j(t, u_1, u_2, \dots, u_m); \quad j = 1, 2, \dots, m$$

(3.6)

$$a \le t \le b; \quad u_j(a) = \alpha_j; \quad j = 1, 2, \dots, m$$

with local truncation error within a given tolerance (the quantities w_j represent the approximated solutions of **(3.6)** at time t) do:

(alg 1.6)

start

input a, b, m, α_j, $RTOL_j$ (tolerance vector; containing the value of relative tolerance limits for each unknown), $h\max$ (maximum step size), $h\min$ (minimum step size; can be put equal to zero), h (initial time-step; a small value should be chosen);

(assign initial values)

set $t = a$;

for $j = 1, 2, \dots, m$ **do**

  set $w_j = \alpha_j$;

end

(output first point of solution)

output t;

for $j = 1, 2, \dots, m$ **do**

  output w_j;

end

(evaluate k functions)

100 **for** $j = 1, 2, \dots, m$ **do**

  set $k_{1,j} = h\, f_j(t, w_1, w_2, \dots, w_m)$;

end

for $j = 1, 2, \dots, m$ **do**

set $k_{2,j} = h\, f_j\left(t + \frac{1}{4}h, w_1 + \frac{1}{4}k_{1,1}, w_2 + \frac{1}{4}k_{1,2}, \ldots, w_m + \frac{1}{4}k_{1,m}\right)$;

end

for $j = 1, 2, \ldots, m$ **do**

set

$$k_{3,j} = h\, f_j\Big(t + \tfrac{3}{8}h, w_1 + \tfrac{3}{32}k_{1,1} + \tfrac{9}{32}k_{2,1}, w_2 + \tfrac{3}{32}k_{1,2} + \tfrac{9}{32}k_{2,2}, \ldots, w_m + \tfrac{3}{32}k_{1,m} + \tfrac{9}{32}k_{2,m}\Big);$$

end

for $j = 1, 2, \ldots, m$ **do**

set $k_{4,j} = h\, f_j\left(t + \frac{12}{13}h, w_1 + \frac{1932}{2197}k_{1,1} - \frac{7200}{2197}k_{2,1} + \frac{7296}{2197}k_{3,1}, \ldots, w_m + \ldots + \frac{7296}{2197}k_{3,m}\right)$;

end

for $j = 1, 2, \ldots, m$ **do**

set

$$k_{5,j} = h\, f_j\Big(t + h, w_1 + \tfrac{439}{216}k_{1,1} - 8\, k_{2,1} + \tfrac{3680}{513}k_{3,1} - \tfrac{845}{4104}k_{4,1}, \ldots, w_m + \ldots - \tfrac{845}{4104}k_{4,m}\Big);$$

end

for $j = 1, 2, \ldots, m$ **do**

set

$$k_{6,j} = h\, f_j\Big(t + \tfrac{1}{2}h, w_1 - \tfrac{8}{27}k_{1,1} + 2\, k_{2,1} - \tfrac{3544}{2565}k_{3,1} + \tfrac{1859}{4104}k_{4,1} - \tfrac{11}{40}k_{5,1}, \ldots, w_m + \ldots - \tfrac{11}{40}k_{5,m}\Big);$$

end

(find error and step-size factor)

for $j = 1, 2, \ldots, m$ **do**

set $R_j = \left|\frac{1}{360}k_{1,j} - \frac{128}{4275}k_{3,j} - \frac{2197}{75240}k_{4,j} + \frac{1}{50}k_{5,j} + \frac{2}{55}k_{6,j}\right| / h$ (error estimates);

if $|w_j| \le 1$ **then**

(previously calculated value of w_j smaller than unity; use absolute tolerance limit)

set $TOL_j = RTOL_j$;

else

(use relative tolerance limit)

set $TOL_j = RTOL_j \left|w_j\right|$;

endif

end

set $Check = \max_{1 \le j \le m}\left(\dfrac{R_j}{TOL_j}\right)$;

set $\delta = 0.84\left(1/_{Check}\right)^{\frac{1}{4}}$ (step size factor);

if $Check \le 1$ **then** (approximation accepted)

set $t = t + h$;

for $j = 1, 2, \ldots, m$ **do**

set $w_j = w_j + \frac{25}{216}k_{1,j} + \frac{1408}{2565}k_{3,j} + \frac{2197}{4104}k_{4,j} - \frac{1}{5}k_{5,j}$;

end

(output solution)

output t, h;

for $j = 1, 2, \ldots, m$ **do**

output w_j;

end

end if

(calculate next time-step)

if $\delta \le 0.1$ **then**

set $h = 0.1\,h$;

else

if $\delta \ge 4.0$ **then**

set $h = 4.0\,h$;

else

set $h = \delta\,h$;

end if

```
end if
if h > hmax then
   set h = hmax;
else
   if h < hmin then
      output 'minimum h exceeded';
      stop
   end if
end if
(check for end of simulation)
if t ≥ b then
   output 'simulation is over';
   stop
end if
if t + h > b then
   set h = b - t;
end if
goto 100
```

Clearly, most of the computational effort is put in calculating the functions $k_{i,j}$. In the case of equations **(2.6)** m is equal to twelve and the functions f_j can be re-written as:

$$
(4.6)\quad \begin{cases} f_1^n = \sum_{j=1}^{6}\left(R_{1j}^{n-1}\, w_{j+6}^{n-1}\right) \\ \quad\dots \\ f_7^n = \sum_{j=1}^{6}\left\{A^{-1}{}_{1j}^{n-1}\, F_j^{n-1} - C_{1j}^{n-1}\, w_{j+6}^{n-1} - A^{-1}{}_{1j}^{n-1}\left[\sum_{k=1}^{6}\left(\int_{t^0}^{t^{n-1}} K'_{jk}\left(t^{n-1}-\tau\right) w_{k+6}(\tau)\, d\tau\right)\right]\right\} \\ \quad\dots \end{cases}
$$

where

$$A^{-1}{}_{ij}^{n-1} = A^{-1}{}_{ij}\left(t^{n-1}, w_1^{n-1}, w_2^{n-1}, \ldots, w_6^{n-1}\right),$$

$$C_{ij}^{n-1} = C_{ij}\left(t^{n-1}, w_1^{n-1}, w_2^{n-1}, \ldots, w_6^{n-1}\right),$$

$$R_{ij}^{n-1} = R_{ij}\left(w_4^{n-1}, w_5^{n-1}, w_6^{n-1}\right)$$

and

$$F'^{n-1}_j = F'_j\left(t^{n-1}, w_1^{n-1}, w_2^{n-1}, \ldots, w_{12}^{n-1}\right).$$

Equation **(4.6)** represents a semi-discretised form of **(2.6)** at the n^{th} calculation step. The following substitutions apply:

$$u_1 = q_1,\quad u_2 = q_2,\quad u_3 = q_3,\quad u_4 = q_4,\quad u_5 = q_5,\quad u_6 = q_6,$$
$$u_7 = p_1,\quad u_8 = p_2,\quad u_9 = p_3,\quad u_{10} = p_4,\quad u_{11} = p_5,\quad u_{12} = p_6;$$

also note that:

$$n = 1, 2, \ldots$$
$$w_j^0 = \alpha_j, w_j^1 \cong u_j\left(t^1\right), \ldots, w_j^n \cong u_j\left(t^n\right),$$
$$h = 0, h^1, h^2, \ldots$$

and

$$t^0 = a, t^1 = t^0 + h^1, t^2 = t^0 + h^1 + h^2 = t^1 + h^2, \ldots, t^n = t^{n-1} + h^n.$$

The convolution integrals need to be evaluated as function of previously calculated discrete quantities. Note that the kernels are continuous functions of time according to the expression (cf. **(15.3 a)**):

$$K'_{jk}\left(t^{n-1}-\tau\right) \cong \frac{2}{\pi}\sum_{m=0}^{100} b'_{jk}\left(m\frac{\omega_{MAX}}{100}\right)\cos\left(m\frac{\omega_{MAX}}{100}\left(t^{n-1}-\tau\right)\right)\frac{\omega_{MAX}}{100}$$

where $b'_{jk}(\omega)$ are slowly varying functions of time treated here as pseudo-constants. Clearly, the kernels assume different values at every new evaluation of the functions $k_{i,j}$.

In order to retain some of the accuracy offered by the RKF method just shown, it is necessary to transform the integrals in **(4.6)** using an higher order numerical quadrature scheme. Considering the variable nature of the time step and the possibility of obtaining the first derivatives of the integrands for each solution point evaluated, an integration method based on Hermite interpolation seems to be the best option to achieve this purpose. Hermite interpolation [13.12 page 190] fits a third order polinomial through two given points. There, the first derivatives of this function have the same values as those of the curve that it is simulating. In this respect, the method is essentially a cubic-spline type of interpolation, where the value of the derivatives at each node are supposed known. Whenever the underlying RK method is of order greater or equal to three, the use of Hermite interpolation reduces it to three. In the case under examination though, this kind of interpolation is used to evaluate integral quantities and therefore its application might not effectively reduce the order of the original RKF method.

The form of a Hermite polynomial is:

$$u(\theta) = (1-\theta)\,y_0 + \theta\,y_1 + \theta(\theta-1)\left((1-2\,\theta)(y_1-y_0) + (\theta-1)\,h\,f_0 + \theta\,h\,f_1\right)$$

where y_0 and y_1 are the values of the function at the two nodes, whilst f_0 and f_1 represent those of its first derivatives at each node, respectively. h represents the distance between the abscissae of the two points and $0 \le \theta \le 1$, so that $x = x_0 + \theta\,h$ and $x_0 \le x \le x_1 = x_0 + h$. Integrating the expression above within these limits will produce:

$$\int_{x_0}^{x_1} y(x)\, dx = \int_0^1 h\, u(\theta)\, d\theta = \frac{h}{12}\left(6\left(y_0 + y_1\right) + h\left(f_0 - f_1\right)\right)$$

If a C^2 curve is known by a succession of $N+1$, irregularly spaced points belonging to it and its first derivatives, an approximation of the area under such a curve can then be expressed by the sum:

$$\sum_{i=0}^{N-1}\left(\frac{h^{i+1}}{12}\left(6\left(y^i + y^{i+1}\right) + h^{i+1}\left(f^i - f^{i+1}\right)\right)\right)$$

To apply the expression above to the convolution integrals in **(4.6)**, an explicit analytical form for the integrands is needed. This can be derived as follows:

$$\int_{t^0}^{t^{n-1}} y(\tau)\, d\tau = \int_{t^0}^{t^{n-1}} K'_{jk}\left(t^{n\ 1} - \tau\right) w_{k+6}(\tau)\, d\tau \quad \therefore \quad y(\tau) = K'_{jk}\left(t^{n-1} - \tau\right) w_{k+6}(\tau) \quad \Rightarrow$$

$$\begin{aligned}\frac{\partial y(\tau)}{\partial \tau} &= \frac{\partial K'_{jk}\left(t^{n-1} - \tau\right)}{\partial \tau} w_{k+6}(\tau) + K'_{jk}\left(t^{n-1} - \tau\right) \frac{\partial w_{k+6}(\tau)}{\partial \tau} = \\ &= \dot{K}'_{jk}\left(t^{n-1} - \tau\right) w_{k+6}(\tau) + K'_{jk}\left(t^{n-1} - \tau\right) f_{k+6}(\tau)\end{aligned}$$

where

$$\dot{K}'_{jk}\left(t^{n-1} - \tau\right) = \frac{\partial K'_{jk}\left(t^{n-1} - \tau\right)}{\partial \tau} \cong \frac{2}{\pi}\sum_{m=0}^{100} m\frac{\omega_{MAX}}{100}\, b'_{jk}\left(m\frac{\omega_{MAX}}{100}\right) \sin\left(m\frac{\omega_{MAX}}{100}\left(t^{n-1} - \tau\right)\right)\frac{\omega_{MAX}}{100}$$

Thus the convolution integrals in **(4.6)** become:

$$\int_{t^0}^{t^{n-1}} K'_{jk}\left(t^{n-1} - \tau\right) \dot{q}'_k(\tau)\, d\tau =$$

$$= \sum_{i=0}^{n-2} \frac{h^{i+1}}{12} \left(6\left(K'_{jk}\left(t^{n-1}-t^{i}\right) w^{i}_{k+6} + K'_{jk}\left(t^{n-1}-t^{i+1}\right) w^{i+1}_{k+6}\right) + \right.$$

$$\left. + h^{i+1}\left(\dot{K}'_{jk}\left(t^{n-1}-t^{i}\right) w^{i}_{k+6} + K'_{jk}\left(t^{n-1}-t^{i}\right) f^{i}_{k+6} - \dot{K}'_{jk}\left(t^{n-1}-t^{i+1}\right) w^{i+1}_{k+6} - K'_{jk}\left(t^{n-1}-t^{i+1}\right) f^{i+1}_{k+6}\right)\right)$$

A second modification of **(4.6)** is necessary to calculate functions $k_{i,j}$ for $i = 2, 3, \ldots, 6$. Assuming Δt and Δw_j represent any of the increments involved in the evaluation of $k_{i,j}$ (the increments become zero when $i = 1$) and that the quantities $\hat{w}^n_j$ are the intermediate values of w_j when $t^{n-1} \le \hat{t}^n \le t^n$, $w^{n-1}_j \le \hat{w}^n_j \le w^n_j$ (so that $\hat{t}^n = t^{n-1} + \Delta t$, $\hat{w}^n_j = w^{n-1}_j + \Delta w_j$), expression **(4.6)** can be shown to become:

(5.6)
$$\begin{cases} f^n_1 = \sum_{j=1}^{6}\left(\hat{R}^n_{1j}\, \hat{w}^n_{j+6}\right) \\ \ldots \\ f^n_7 = \sum_{j=1}^{6}\left\{\hat{A}^{-1n}_{1j}\left[\hat{F}^n_j - \left(\sum_{k=1}^{6} conv_{jk}\right)\right] - \hat{C}^n_{1j}\, \hat{w}^n_{j+6}\right\} \\ \ldots \end{cases}$$

where, of course

$$\hat{A}^{-1n}_{ij} = A^{-1}_{ij}\left(\hat{t}^n, \hat{w}^n_1, \hat{w}^n_2, \ldots, \hat{w}^n_6\right),$$

$$\hat{C}^n_{ij} = C_{ij}\left(\hat{t}^n, \hat{w}^n_1, \hat{w}^n_2, \ldots, \hat{w}^n_6\right),$$

$$\hat{R}^n_{ij} = R_{ij}\left(\hat{w}^n_4, \hat{w}^n_5, \hat{w}^n_6\right),$$

$$\hat{F}^n_j = F_j\left(\hat{t}^n, \hat{w}^n_1, \hat{w}^n_2, \ldots, \hat{w}^n_{12}\right) \qquad \text{and}$$

$$conv_{jk} = \sum_{i=0}^{n-2} \frac{h^{i+1}}{12}\left[6\left(K'_{jk}\left(\hat{t}^n - t^i\right) w^i_{k+6} + K'_{jk}\left(\hat{t}^n - t^{i+1}\right) w^{i+1}_{k+6}\right) + \right.$$
$$\left. + h^{i+1}\left(\dot{K}'_{jk}\left(\hat{t}^n - t^i\right) w^i_{k+6} + K'_{jk}\left(\hat{t}^n - t^i\right) f^i_{k+6} - \dot{K}'_{jk}\left(\hat{t}^n - t^{i+1}\right) w^{i+1}_{k+6} - K'_{jk}\left(\hat{t}^n - t^{i+1}\right) f^{i+1}_{k+6}\right)\right] +$$
$$+ \frac{\Delta t}{12}\left[6\left(K'_{jk}(\Delta t)\, w^{n-1}_{k+6} + K'_{jk}(0)\, \hat{w}^n_{k+6}\right) + \right.$$

$$+\Delta t\left(\dot{K}'_{jk}(\Delta t)\, w^{n-1}_{k+6} + K'_{jk}(\Delta t)\, f^{n-1}_{k+6} - \dot{K}'_{jk}(0)\, \hat{w}^{n}_{k+6} - K'_{jk}(0)\, f^{n}_{k+6}\right)\Big]$$

Note that $\dot{K}'_{jk}(0) = 0$; note as well that the unknown terms f^{n}_{k+6} are present on both sides of the equal sign in **(5.6)**. To isolate them on the right hand side, some manipulations are thus necessary. The convolution sum above can be re-written as:

$$conv_{jk} = conv^{*}_{jk} - \frac{\Delta t^2}{12} K'_{jk}(0)\, f^{n}_{k+6}$$

where

$$\begin{aligned} conv^{*}_{jk} = \sum_{l=0}^{n-2} \frac{h^{l+1}}{12} \Big[& 6\left(K'_{jk}(\hat{t}^{n} - t^{l})\, w^{l}_{k+6} + K'_{jk}(\hat{t}^{n} - t^{l+1})\, w^{l+1}_{k+6}\right) + \\ & + h^{l+1}\left(\dot{K}'_{jk}(\hat{t}^{n} - t^{l})\, w^{l}_{k+6} + K'_{jk}(\hat{t}^{n} - t^{l})\, f^{l}_{k+6} - \dot{K}'_{jk}(\hat{t}^{n} - t^{l+1})\, w^{l+1}_{k+6} - K'_{jk}(\hat{t}^{n} - t^{l+1})\, f^{l+1}_{k+6}\right)\Big] + \\ & + \frac{\Delta t}{12}\Big[6\left(K'_{jk}(\Delta t)\, w^{n-1}_{k+6} + K'_{jk}(0)\, \hat{w}^{n}_{k+6}\right) + \Delta t\left(\dot{K}'_{jk}(\Delta t)\, w^{n-1}_{k+6} + K'_{jk}(\Delta t)\, f^{n-1}_{k+6}\right)\Big] \end{aligned}$$

therefore equations **(5.6)** become:

$$\begin{cases} f^{n}_{1} = \sum_{j=1}^{6}\left(\hat{R}^{n}_{1j}\, \hat{w}^{n}_{j+6}\right) \\ \dots \\ f^{n}_{7} - \frac{\Delta t^2}{12}\sum_{k=1}^{6} \hat{E}^{n}_{1k}\, f^{n}_{k+6} = \sum_{j=1}^{6}\left\{\hat{A}^{-1\,n}_{1j}\left[F^{n}_{j} - \left(\sum_{k=1}^{6} conv^{*}_{jk}\right)\right] - \hat{C}^{n}_{1j}\, \hat{w}^{n}_{j+6}\right\} \\ \dots \end{cases}$$

where

$$\hat{E}^{n}_{ik} = \sum_{j=1}^{6} \hat{A}^{-1\,n}_{ij}\, K'_{jk}(0)$$

The last six equations in the new expression for **(5.6)** can be put in matrix form and solved with respect to the unknowns f^n_{k+6}. Such operation yields (equation **(6.6)**):

$$\begin{cases} f_1^n = \sum_{j=1}^{6}\left(\hat{R}^n_{1j}\ \hat{w}^n_{j+6}\right) \\ \quad \dots \\ f_7^n = \sum_{i=1}^{6}\left\{\hat{G}^{-1}{}^n_{1i}\sum_{j=1}^{6}\left[\hat{A}^{-1}{}^n_{ij}\left(\hat{F}^n_j - \left(\sum_{k-1}^{6} conv^*_{jk}\right)\right) - \hat{C}^n_{ij}\ \hat{w}^n_{j+6}\right]\right\} \\ \quad \dots \end{cases}$$

where

$$\left[\hat{G}^n\right] = [I] - \frac{\Delta t^2}{12}\left[\hat{E}^n\right].$$

The equations above contain convolution terms which take account of the motion history of the vessel since it is left free to move - i.e. since the simulation is started. In fact, the peculiar form of the convolution integrals determine a summation of previously calculated terms which grows in size as the integration proceeds. Naturally, this is a great limitation since it means larger and larger portions of memory to be reserved to the storage of already computed terms as the simulation time elapses and obviously an increasingly slower execution. Since in reality the kernels die out to zero as one moves back in time from the moment considered, sufficient accuracy can be assumed in limiting the range of integration to fifteen seconds prior the integration time.

In terms of computing procedure reducing the convolution time means employing the following form of the convolution sum within the set of equations **(6.6)**:

$$conv^*_{jk} = \sum_{i=n-\bar{n}}^{n-2} \frac{h^{i+1}}{12}\left[6\left(K_{jk}\left(\hat{t}^n - t^i\right) w^i_{k+6} + K_{jk}\left(\hat{t}^n - t^{i+1}\right) w^{i+1}_{k+6}\right) + \dots\right.$$

where the integer $\tilde{n}$ can be obtained - for instance - according to the following algorithm:

set $\tilde{n} = {}^{T_d}\!/_{h\text{min}}$;

100 set $T = \sum_{i=n-\tilde{n}}^{n-1} h^i$;

if $T > T_d$ **then**

set $\tilde{n} = \tilde{n} - 1$;

goto 100

end if

output $\tilde{n}$;

T_d being equal to fifteen seconds. The next chart - **(alg 2.6)** - summarises all the principal changes necessary to adapt **(alg 1.6)** to calculate $k_{i,j}$ in accordance with expression **(6.6)**. Some of the operations involved in the calculation of these quantities are indicated in detail in the algorithms following the next (**(alg 3.6)** and **(subroutine 1.6)**).

(alg. 2.6)

start

input a, b, α_j, $RTOL_j$, $h\text{max}$, $h\text{min}$, h;

(assign initial values)

set $t = a$;

for $j = 1, 2, \ldots, 12$ **do**

set $w_j = \alpha_j$;

end

(initialise convolution sum parameters)

set $n = 0,\ t^n = t,\ h^n = 0$;

for $j = 1, 2, \ldots, 12$ **do**

  set $w_j^n = w_j$;

  set $f_j^n = 0$;

end

(output first point of solution)

output t;

for $j = 1, 2, \ldots, 12$ **do**

  output w_j;

end

(initialise moving average vector)

initialise $\hat{\bar{q}}$;

(calculate pseudo-constant matrices)

100 **find** $\left[A'_\infty\left(\hat{\bar{q}}\right)\right]$,

for $l = 0, 2, \ldots, 600$ **do**

  find $b'_{jk}\left(\omega_l, \hat{\bar{q}}\right)$;

end

(evaluate k functions)

do **(alg 3.6)**

(find error and step-size factor)

for $j = 1, 2, \ldots, 12$ **do**

  set $R_j = \left|\frac{1}{360}k_{1,j} - \frac{128}{4275}k_{3,j} - \frac{2197}{75240}k_{4,j} + \frac{1}{50}k_{5,j} + \frac{2}{55}k_{6,j}\right| / h$;

if $|w_j| \le 1$ **then**

set $TOL_j = RTOL_j$;

else

set $TOL_j = RTOL_j\,|w_j|$;

endif

end

set $Check = \max\limits_{1 \le j \le m}\left(\dfrac{R_j}{TOL_j}\right)$;

set $\delta = 0.84\left(1/Check\right)^{\frac{1}{4}}$;

if $Check \le 1$ **then**

(evaluate solutions and accelerations at the newly accepted point)

set $\Delta t = h$;

for $j = 1, 2, \ldots, 12$ **do**

set $\Delta w_j = \frac{25}{216}k_{1,j} + \frac{1408}{2565}k_{3,j} + \frac{2197}{4104}k_{4,j} - \frac{1}{5}k_{5,j}$;

end

call **(subroutine 1.6)**

(update convolution sum parameters)

set $t = t + h$, $n = n + 1$, $t^n = t$, $h^n = h$;

for $j = 1, 2, \ldots, 12$ **do**

set $w_j = w_j + \Delta w_j$;

set $w_j^n = w_j$;

set $f_j^n = f_j$;

end

(output solution)

output t, h;

for $j = 1, 2, \ldots, 12$ **do**

  output w_j;

end

(calculate moving averages of solution)

compute $\hat{\bar{q}}$;

(cut convolution tail)

200 **if** $\sum_{i=0}^{n} h^i > T_d$ **then**

  for $i = 0, 1, \ldots, n-1$ **do**

    set $h^i = h^{i+1}, t^i = t^{i+1}$;

    for $j = 1, 2, \ldots, 12$ **do**

      set $w_j^i = w_j^{i+1}$;

      set $f_j^i = f_j^{i+1}$;

    end

  end

  set $n = n - 1$;

  goto 200

end if

end if

(calculate next time-step)

if $\delta \le 0.1$ **then**

  set $h = 0.1\,h$;

else

  if $\delta \ge 4.0$ **then**

    set $h = 4.0\,h$;

  else

    set $h = \delta\,h$;

```
      end if
   end if
   if h > hmax then
      set h = hmax;
   else
      if h < hmin then
         output 'minimum h exceeded';
         stop
      end if
   end if
   (check for end of simulation)
   if t ≥ b then
      output 'simulation is over';
      stop
   end if
   if t + h > b then
      set h = b − t;
   end if
   goto 100
```

(alg 3.6)

```
   set Δt = 0.0;
   for j = 1, 2, ..., 12 do
      set Δw_j = 0.0;
   end

   call (subroutine 1.6)
```

for $j = 1, 2, \ldots, 12$ **do**

  set $k_{1,j} = h f_j$

end

set $\Delta t = \frac{1}{4} h;$

for $j = 1, 2, \ldots, 12$ **do**

  set $\Delta w_j = \frac{1}{4} k_{1,j};$

end

call **(subroutine 1.6)**

for $j = 1, 2, \ldots, 12$ **do**

  set $k_{2,j} = h f_j$

end

set $\Delta t = \frac{3}{8} h;$

for $j = 1, 2, \ldots, 12$ **do**

  set $\Delta w_j = \frac{3}{32} k_{1,j} + \frac{9}{32} k_{2,j};$

end

call **(subroutine 1.6)**

for $j = 1, 2, \ldots, 12$ **do**

  set $k_{3,j} = h f_j$

end

set $\Delta t = \frac{12}{13} h;$

for $j = 1, 2, \ldots, 12$ **do**

  set $\Delta w_j = \frac{1932}{2197} k_{1,j} - \frac{7200}{2197} k_{2,j} + \frac{7296}{2197} k_{3,j};$

end

call **(subroutine 1.6)**

for $j = 1, 2, \ldots, 12$ **do**

  set $k_{4,j} = h f_j$

end

set $\Delta t = h$;

for $j = 1, 2, \ldots, 12$ **do**

  set $\Delta w_j = \frac{439}{216} k_{1,j} - 8 k_{2,j} + \frac{3680}{513} k_{3,j} - \frac{845}{4104} k_{4,j}$;

end

call **(subroutine 1.6)**

for $j = 1, 2, \ldots, 12$ **do**

  set $k_{5,j} = h f_j$

end

set $\Delta t = \frac{1}{2} h$;

for $j = 1, 2, \ldots, 12$ **do**

  set $\Delta w_j = -\frac{8}{27} k_{1,j} + 2 k_{2,j} - \frac{3544}{2565} k_{3,j} + \frac{1859}{4104} k_{4,j} - \frac{11}{40} k_{5,j}$;

end

call **(subroutine 1.6)**

for $j = 1, 2, \ldots, 12$ **do**

  set $k_{6,j} = h f_j$

end

(subroutine 1.6)

for $j = 1, 2, \ldots, 6$ **do**

  set $q_j = w_j + \Delta w_j$;

end

compute $\left[R(\vec{q})\right]$, $\left[D^{*}\left(\vec{q}, \hat{\vec{q}}\right)\right]$;

for $j = 1, 2, \ldots, 6$ **do**

  set $f_j = \sum_{j-1}^{6} R_{jk}\left(w_{k+6} + \Delta w_{k+6}\right) = \dot{q}_j$;

end

compute $\left[\dot{M}_w(t + \Delta t, \vec{q})\right]$, $\left[M_w(t + \Delta t, \vec{q})\right]$, $\vec{F}'\left(t + \Delta t, \vec{q}, \dot{\vec{q}}\right)$;

set $[A] = [M + M_w + A'_\infty]$, $[B] = \left[\dot{M}_w + B'_{Viscous}\right]$;

invert $[A]$;

set $[C] = [A]^{-1} \cdot [B]$;

set $K'_{jk} = \frac{2}{\pi} \sum_{l=0}^{100} b'_{jk}\left(l \frac{\omega_{MAX}}{100}\right) \frac{\omega_{MAX}}{100}$;

for $i = 0, 1, \ldots, n$ **do**

  set $K'^{i}_{jk} = \frac{2}{\pi} \sum_{l=0}^{100} b'_{jk}\left(l \frac{\omega_{MAX}}{100}\right) \cos\left(l \frac{\omega_{MAX}}{100}\left(t^n + \Delta t - t^i\right)\right) \frac{\omega_{MAX}}{100}$;

  set $\dot{K}'^{i}_{jk} = \frac{2}{\pi} \sum_{l=0}^{100} l \frac{\omega_{MAX}}{100} b'_{jk}\left(l \frac{\omega_{MAX}}{100}\right) \sin\left(l \frac{\omega_{MAX}}{100}\left(t^n + \Delta t - t^i\right)\right) \frac{\omega_{MAX}}{100}$;

end

set $[E] = [A]^{-1} \cdot [K']$

set $[G] = [I] - \frac{\Delta t^2}{12}[E]$

invert $[G]$

set

$$
\begin{aligned}
conv^{*}_{jk} = \sum_{i=0}^{n-1} \frac{h^{i+1}}{12}\Big[& 6\left(K'^{i}_{jk}\, w^{i}_{k+6} + K'^{i+1}_{jk}\, w^{i+1}_{k+6}\right) + \\
& + h^{i+1}\left(\dot{K}'^{i}_{jk}\, w^{i}_{k+6} + K'^{i}_{jk}\, f^{i}_{k+6} - \dot{K}'^{i+1}_{jk}\, w^{i+1}_{k+6} - K'^{i+1}_{jk}\, f^{i+1}_{k+6}\right)\Big] + \\
& + \frac{\Delta t}{12}\Big[6\left(K'^{n}_{jk}\, w^{n}_{k+6} + K'_{jk}\left(w^{n}_{k+6} + \Delta w_{k+6}\right)\right) + \\
& + \Delta t\left(\dot{K}'^{n}_{jk}\, w^{n}_{k+6} + K'^{n}_{jk}\, f^{n}_{k+6}\right)\Big]
\end{aligned}
$$

for $l = 1, 2, \ldots, 6$ **do**

set

$$
f_{l+6} = \sum_{i=1}^{6}\left\{ G^{-1}{}_{li} \sum_{j=1}^{6}\left[A^{-1}{}_{ij}\left(F_j - \left(\sum_{k=1}^{6} conv^{*}_{jk}\right)\right) - C_{ij}\left(w_{j+6} + \Delta w_{j+6}\right)\right]\right\}
$$

end

Some statements in the algorithms above indicate complex operations that have been mostly described in the previous chapters. An exception to this is represented by the inversion of matrices $[A]$ and $[G]$. Given the dimension of these matrices a direct method seems to be preferable to iterative ones. Indeed, since the accuracy of the inversion is the major priority in this context, an algorithm based on LU decomposition with scaled partial pivoting was chosen in order to minimise the effects of round-off errors. For the same reason, double precision arithmetic needs to be employed throughout the inversion procedure; this will also somehow improve the likelihood of avoidance of ill-conditioned systems. More information relative to the general structure of the algorithms above can be found in [13.11].

Note that the evaluation of the coefficients dependent on the ship attitude takes place outside (**subroutine 1.6**). This is consistent with the assumption that these can be treated as pseudo-constants. In other words, the variation of the moving averages of the ship motions can be assumed slow enough over each time step, so that all the parameters depending on them do not change sensibly. In terms of computation this assumption yields to a faster and smoother numerical integration.

9. Validation and Sensitivity Analysis

General Remarks

Once a theoretical model has been developed to simulate the behaviour of a physical system, two further steps are still required to be able to grant it reasonable confidence. Firstly, its predictions have to be validated against reliable experimental results. Then an investigation is needed to assess the influence of the different parameters involved as they vary within their respective ranges. The scope of the latter operation is to measure the effectiveness of each element of the devised model versus its costs (for instance, in terms of the computational time required or of the uncertainties at stake) and identify where simplifications are needed, possible and worth performing. This mode of action represents the core of a top-down approach, as it moves from a complex model to a simpler one.

In this chapter, two purposely designed, although limited, series of tests are presented, which form an adequate foundation onto which the body of the proposed numerical "tool" - CASSANDRA - can be shaped and its reliability substantiated. On the basis of these, innovations and improvements of the new theory are introduced and its capabilities compared with those of a simpler model.

Experimentation strategy

For validation purposes, the results of a well known and established investigation [2.36] were chosen as the best candidate for representing a proper basis of comparison for the predictions of the new model. The reason behind this resolution is that these findings are the outcome of both numerical and experimental tests, run by two independent bodies during early research on the resistance of damaged RO-RO vessels to capsize. This study is also particularly interesting as it employs a time simulation program similar to the one presented here, although significantly different from CASSANDRA.

Coarsely simplifying (an exhaustive description of this code and the theory behind it can be found in [3.11]) , this 2D model makes use of constant hydrodynamic coefficients and a basic hydraulic method which applies the concept of net inflow to reproduce the coupled roll, heave and sway behaviour of a damaged vessel. As it can be seen from the results shown in Fig. 3, this model can represent reality meaningfully for medium to low values of KG, but it fails to do so in the higher range of GM's. This response can be probably blamed on the inherent limitations imposed by assuming a net inflow model which does not allow for any discharge of flood water back into the sea.

Ahead of illustrating the comparison between these two numerical models and their adherence to the experimental evidence, the vessel's particulars, the compartment and damage geometry and the matrix of variables used in the series of tests performed using the new theory are given next. The first series of tests was planned to verify that the capsize prediction of the new model compared well with the experimental boundary survivability line, making allowance for possible discrepancies for values of the GM factor (defined as 10.0 $GM_f\, C_b\, T/B^2$) greater than 0.1. In the second series, a systematic variation of some of the parameters governing the main features implemented in CASSANDRA was arranged, so that their influence could be examined both against the performance of the 2D simulation program and that of CASSANDRA itself when operating as a simplified model.

The analysis of the outcome of these series, their interpretation and discussion follow the exposition of the data employed. Graphic representation of the results of each test can be found in Appendix D.

Vessel's Particulars and Environmental Parameters

The vessel geometry adopted in all series of numerical experiments is the same used by BMT in their set of model tests and virtually identical to that of the "Herald

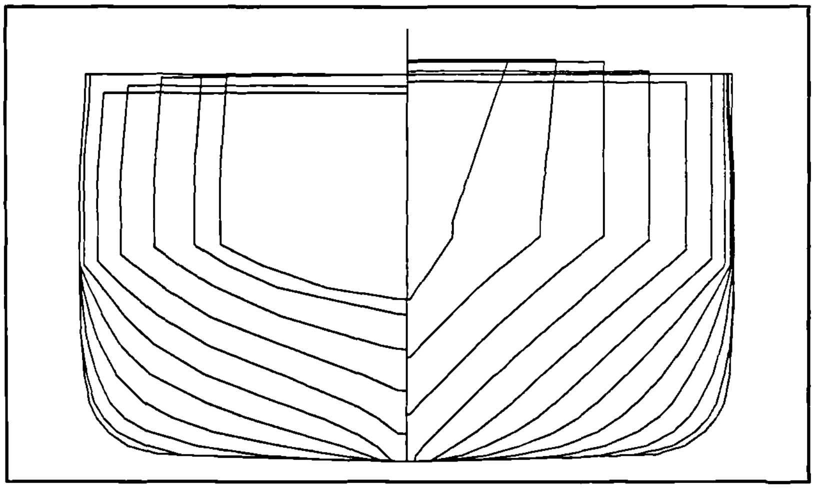

Fig. 1

Hull form.

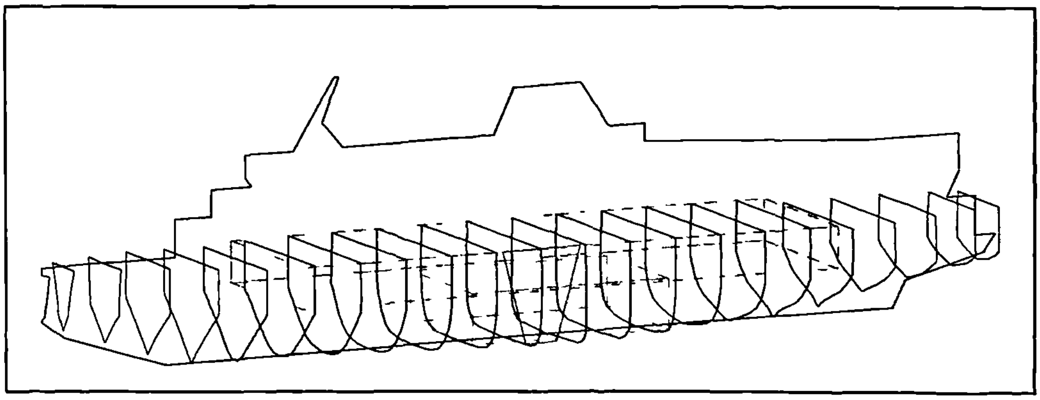

Fig. 2

Damaged compartments particulars.

of Free Enterprise". The main particulars are given in Table 1, while the body plan is shown in Fig. 1.

L_{OA}	(length over all)	=	131.9	m
L_{PP}	(length between perpendiculars)	=	126.1	m
B	(moulded breadth)	=	22.7	m
T	(design draught)	=	5.625	m
D	(depth to uppermost continuous deck)	=	12.6	m
D_{bd}	(depth to bulkhead deck)	=	7.87	m
D_{db}	(depth to double bottom)	=	1.2	m
Δ	(displacement)	=	8807	tonnes
C_b	(block coefficient)	=	0.53	
C_p	(prismatic coefficient)	=	0.56	
C_m	(midships coefficient)	=	0.93	
KG	(design KG)	=	10.10	m
KM	(intact KM)	=	11.62	m
Service speed		=	20.0	kn
Number of passengers		=	1326	
Number of cars		=	350	

Table 1

Only one damage scenario was considered, which sees a large compartment (comp. 1) and the car deck (comp. 2) open to the sea through a 100% standard trapezoidal SOLAS damage opening amidships. The open deck extends longitudinally for 75.0 metres from ordinate 5 (-41.8 m from the midships) forward and transversely from side to side. Its depth is 5.53 m above the bulkhead deck. The lower compartment occupies the space between double bottom and bulkhead deck for a length of 27.7 metres, from -19.0 m forward (fig. 2). This arrangement yields a damaged freeboard of 0.7 metres which is kept constant for all the tests performed.

The damaged compartments are initially completely empty and the water is let flooding them twenty seconds after the simulation has started. This artifice - which somehow imitates reality - allows the dynamic and numerical transients to be completely separated from the flooding one, so that the effects of the latter can be fully appreciated, because they are totally independent from the particular initial condition chosen.

Run N^0	GM factor	KG (m)	Hs/freeboard	Hs (m)	Result
201	0.02	11.91	0.5	0.35	capsize
202	0.02	11.91	1.5	1.05	capsize
203	0.06	11.23	1.0	0.7	no capsize
204	0.06	11.23	2.0	1.4	capsize
205	0.1	10.54	1.5	1.05	no capsize
206	0.1	10.54	2.5	1.75	capsize
207	0.14	9.86	2.0	1.4	no capsize
207b	0.14	9.86	2.0	1.4	no capsize
208	0.14	9.86	3.5	2.45	capsize
208b	0.14	9.86	3.5	2.45	capsize
209	0.18	9.17	2.0	1.4	no capsize
210	0.18	9.17	3.0	2.1	capsize
210b	0.18	9.17	3.0	2.1	no capsize
211	0.18	9.17	4.0	2.8	capsize
211b	0.18	9.17	4.0	2.8	no capsize
212	0.18	9.17	5.0	3.5	capsize
212b	0.18	9.17	5.0	3.5	capsize

Table 2

The series of numerical simulations run to verify the capsize prediction capability of the code, was obtained by varying the ship KG and the sea state parameters. In table

Significant Wave Height Hs (m)	Zero Crossing Period T_0 (sec)
0.0 - 0.99	4.5
0.99 - 1.99	5.5
1.99 - 2.99	6.0
2.99 - 3.99	6.25
3.99 - 4.55	6.5

Table 3

2 above, the complete schedule of runs is listed with their results, while table 3 provides the zero crossing periods used to generate the JONSWAP spectra with a

Run N^0	Heading (deg)	Wind Speed (m/sec)	Wind Direction (deg)	Current Speed (m/sec)	Current Direction (deg)
101	90.0	0.0	0.0	0.0	0.0
102	"	"	"	"	"
103	"	"	"	"	"
104	"	"	"	"	"
105	"	"	"	"	"
106	"	"	"	"	"
107	"	15.5	180.0	"	"
108	"	"	"	"	"
109	"	"	0.0	"	"
110	"	0.0	"	3.0	180.0
111	"	"	"	"	"
112	"	"	"	"	0.0
113	0.0	"	"	0.0	"
114	"	"	"	"	"
115	45.0	"	"	"	"
116	"	"	"	"	"
117	0.0	"	"	"	"
118	"	"	"	"	"
119	90.0	"	"	"	"
120	"	"	"	"	"
121	"	"	"	"	"
122	"	"	"	"	"
123	"	"	"	"	"
124	"	"	"	"	"
125	"	"	"	"	"
126	"	"	"	"	"

Table 4a

Run N^0	Restoring up to Wave Elevation	Flood Water Forces	Variable Hydrodynamic Coefficients	Damage Opening	2D/3D Simulation
101	On	On	On	Closed	3D
102	"	"	"	Open	"
103	Off	Off	Off	Closed	"
104	"	"	"	Open	"
105	"	"	"	Closed	2D
106	"	"	"	Open	"
107	On	On	On	Closed	3D
108	"	"	"	Open	"
109	"	"	"	Open	"
110	"	"	"	Closed	"
111	"	"	"	Open	"
112	"	"	"	Open	"
113	"	"	"	Closed	"
114	"	"	"	Open	"
115	"	"	"	Closed	"
116	"	"	"	Open	"
117	Off	"	"	Closed	"
118	"	"	"	Open	"
119	"	"	"	Closed	"
120	"	"	"	Open	"
121	On	Off	"	"	"
122	"	On	Off	"	"
123	Off	"	"	"	"
124	"	Off	On	"	"
125	On	"	Off	Closed	"
126	"	"	"	Open	"

Table 4b

peak enhancement factor of 3.3. The suffix b following the run numbers signifies that the test has been performed calculating the restoring forces up to the mean water level rather than the instantaneous wave elevation.

The features introduced with the new model that are of interest for the sensitivity analysis are several and it is often not possible to investigate their effect avoiding drastic changes in the structure of the simulation code. One of these is - for instance - the number of degrees of freedom of the dynamic system taken into consideration. In other cases, it is preferable to compare the results of CASSANDRA with those of a modified version of the 2D simulation program that allows for flood water to flow out into the sea. This methodology permits to compare the influence of the variation of the hydrodynamic coefficients with frequency, once the initial heading of the vessel is set to be at 90 degrees with the direction of the waves and it remains reasonably close to this value for the duration of the simulation.

Table 4 gives record of the various parameter used in the numerical simulations carried out to analyse the effect created by wind, current and second order drift forces, as well as those produced by the forces due to the flood water acceleration and its mass rate of variation. Other issues at stake were the variation of the hydrodynamic coefficients with the vessel attitude and the calculation of the restoring forces up to the instantaneous wave elevation. All runs of the second series were performed for constant values of Hs and KG, which were set to 2.10 m and 9.86 m, respectively. These yield a GM factor of 0.14 and an Hs/F ratio of 3.00, which fix a point on the boundary line between safe and unsafe capsize zones as outlined by the model experiments. This peculiar choice was made to amplify the possible effects of the various parameters investigated.

Analysis of Results and Discussion

Validation

The output of the tests carried out assuming buoyancy proportional to the vessel's underwater volume integrated up to the instantaneous wave profile (Fig. 3) shows a trend which is close to that of the old numerical model. The results of the numerical simulations performed with the restoring terms calculated up to the mean water level (Fig. 3b) compare instead very well with the capsize boundary line produced by BMT on the basis of experimental evidence, even for high values of GM_f. Before commenting on these important findings, a remark on the general effectiveness of the validation procedure presented here is deemed appropriate.

Although the number of runs executed in this first series is quite limited, the consistent recurrence of capsize/no-capsize cases within the expected areas of the chart allows sufficient confidence to be granted to the new numerical code. Mean

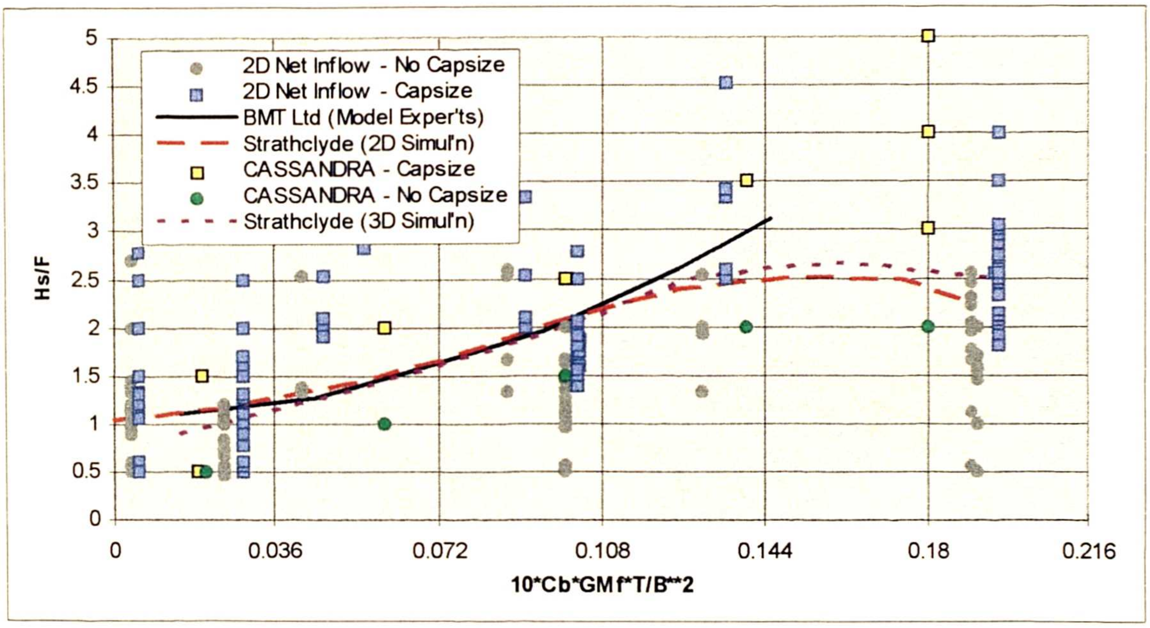

Fig. 3

Comparison between numerical and experimental results.

capsize boundary lines can thus be sketched, which closely reproduce the experimental one considering the uncertainty in play. Obviously it is still not possible to generalise these results to the whole variety of possible scenarios that can be generated by changing the many parameters taken into consideration by the new model. On the other hand, this preliminary validation offers the opportunity for two significant observations to be made. For a start, it is important to underline the

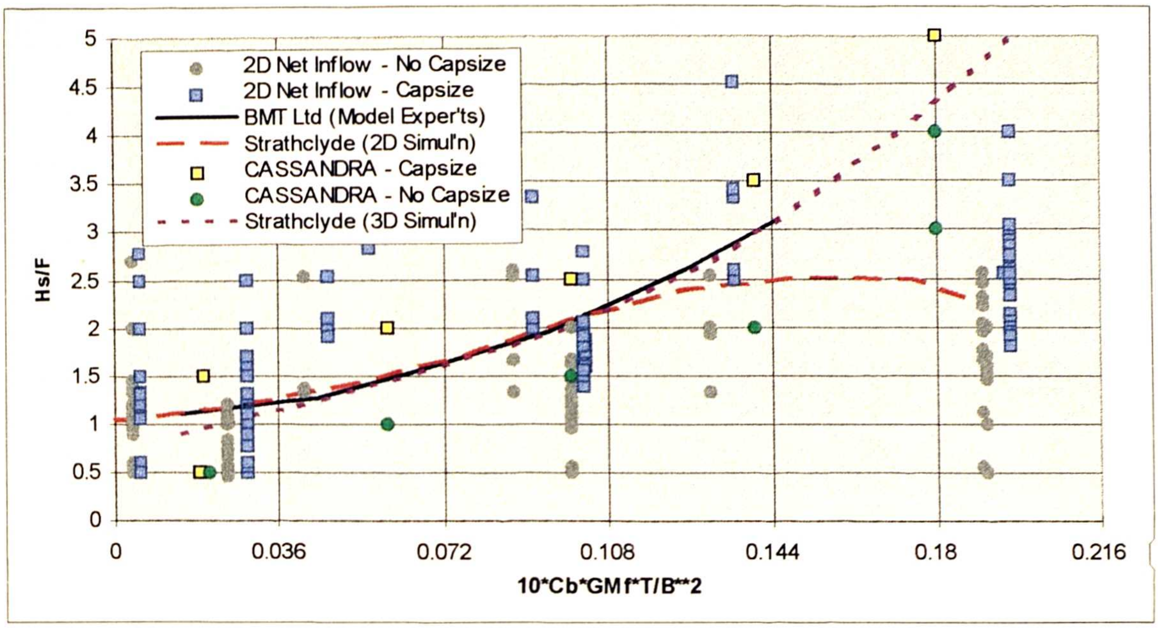

Fig. 3b

Comparison between numerical and experimental results (b series).

evident improvement obtained on the prediction of the previous numerical model, principally due to the changes in the flooding routine. Secondly, having assessed the quality of CASSANDRA's predictions - although only for this particular damage scenario - it is now possible to investigate the significance of each singular part of the code on its general performance.

A few comments are also necessary, on the significance of the differences observed between the two methods of calculations of the buoyancy forces, to stimulate thinking prior to concentrating on the second set of runs. Comparing the output of runs 207 to 212 and 207b to 212b, it is possible to obtain an idea of the reasons that make the capsize boundary curve rise and meet the extrapolated extension of the experimental one in the high range of GM_f when the wave profile is neglected from the evaluation of the underwater volume of the hull. It is in fact possible to observe that the vertical motion of the ship's CG is roughly the same in all cases, and that the sea water floods the vehicle deck only once the roll/heel angle exceeds 5.0 degrees, submerging the bulkhead deck. Obviously, the substantial difference between the two models must therefore lie in the prediction of larger roll motion by one of them. This is all the more logical once it has been recognised that the event of capsize is strictly connected to the flood water exceeding a limit value, mainly depending on the loading condition of the vessel.

Having observed a distinct difference between the final results of these two methods of simulating the forces and moments due to the buoyancy characteristics of the vessel, and having hinted at a possible explanation for such a behaviour, it is now indispensable to highlight a fundamental point, before assessing this phenomenon in greater detail. This regards the uncertainties at stake in trying to validate the model over the whole range of employed KGs, by comparing the capsize boundary curves obtained with the experimental one. It is in fact clear that no final and authoritative conclusion can be drawn from this simple test, considering that the region of validity of the experimental curve does not cover the higher GM_f values used in the numerical simulations. It is thus necessary to proceed to a more detailed verification of the prediction of the vessel motion by the model.

Sensitivity Analysis

Restoring Forces

The prediction of larger roll oscillation when the wave elevation is taken into account in the calculation of the Archimedean forces and moments, can be generally observed both for an intact and a damaged ship either in beam or following sea (cf. tests 101 - 104, 113, 114, 117 - 120, 125, 126). No significant difference is instead observed in the pitch motion in following sea, probably since the vessel length is comparatively much larger than the mean wave length.

It is reasonable to expect the ship to ride the wave when at an angle of about 90 degrees with its direction, if the wave profile is large in comparison to the ship dimensions (long waves). This does not necessarily mean, though, that the phase angle between wave slope and roll angle would be anywhere close to zero. If the right conditions are met, this contingency would bring as a consequence the almost complete disassociation of heave motion from the flooding mechanism and thus justifies the observation made above about the trends of the capsize boundary curves.

A proof that the inclusion of the wave profile in the calculation of restoring forces brings as a necessary consequence a much larger roll amplitude can be found as follows. According to Bhattacharyya [1.3], the amplitude of the roll excitation due to the unbalanced buoyancy caused by a regular wave hitting the vessel beam on, is:

$$M = \rho g \nabla k \zeta_A GM_T = 4\pi^2 \rho \nabla \frac{\zeta_A}{T} GM_T$$

where ζ_A and T are the wave amplitude and period, respectively. For an irregular wave, the magnitude of this moment can be assessed by replacing the wave amplitude with half the significant wave height, and the wave period with the zero crossing period, obtaining an estimate of the significant moment amplitude. Using the conditions of runs 207 and 207b, this yields an approximate value of the roll moment of 27.8 MNm, which compares very well with the value of the wave excitation due to buoyancy, output by numerical simulation 207 (29.3 MNm). Naturally, the equivalent value measured for test 207b is reasonably close to zero - the value of 1.6 MNm being the effect of the non-linear nature of the evaluation of the buoyancy forces - and yet the moment due to the first order wave potential is in both cases in the range of 4.0 to 5.0 MNm.

The above observations should be interpreted in the following way. The wave excitation calculated by the program, when the wave profile is taken into consideration in the definition of the underwater volumes of the vessel, is in agreement with accredited theory. Nevertheless, its magnitude is by far larger than the one of the moment caused by the wave velocity potential and could well cause significant changes in the estimated roll motion. In the case of following sea, a diminished restoring ability of the vessel should instead be considered responsible for the moderate increase of the roll motion.

Slightly different tests are now needed to investigate the validity of these two divergent kinds of output and the phase angle between the estimated roll motion and encountered waves. Since the only available record regarding the wave is its elevation at the ship position, a direct comparison of the roll realisation with the encountered wave slope is difficult to perform, but some interesting information can be obtained by comparing the wave elevation with the vertical motion of the centre of gravity. Tests 103 and 125 show two radically different histories of the vertical motion of the intact centre of gravity. When these are compared with the time realisations of the wave elevation, two considerations spring to mind: firstly, the correlation between the two signals is by far better in test 103 than in test 125. Secondly, the heave motion - the phase of which with the wave elevation seems to be between 90 and 180 degrees in both cases - is unnaturally small for test 125, considering that the wave mean frequency is near to the vessel natural frequency calculated according to the expression [1.3]:

$$T_n = 2\pi\sqrt{\frac{m_v}{\rho\, g\, A_w}} = 2\pi\sqrt{\frac{(8807.0 + 17795.0)}{1.025 \times 9.81 \times 2199.0}} = 6.89 \text{ sec}$$

The natural conclusion that these observations lead to, is that the inclusion of the wave elevation in the calculation of the restoring force has the effect of damping heave and as such it can be expected to produce discrepancies in the roll response as well. Considering the premises behind the formulation of this methodology, this does not really come as a surprise (cf. Chapter 6).

Indeed, if a direct comparison of the roll record of the test series 100 is performed with the respective time histories available from the experimental investigation by BMT, the following table can be generated, which adds some other useful information towards the understanding of the role of the wave elevation in the calculation of restoring. As it can be seen, of the experimental runs taken into consideration, only No. 13 conditions are close enough to those of one of the numerical simulations performed (207 and 207b) to allow for meaningful comparison of their results. If a non-dimensional roll factor is defined as:

$$\text{Roll factor} = \overline{\varphi}\frac{g\,T_0^2}{2\pi^2 H_s}$$

where $\overline{\varphi}$ is the mean roll amplitude, it can then be used to compare the output of these tests.

Run No	freeboard	GM	GMf	GM factor	Hs	Hs/freeboard	Roll Amplitude	To	Roll factor
26	1	2.16	0.83	0.05	4.86	4.86	2.5	6.5	10.80
130	0.75	2.16	1.18	0.07	4.77	6.36	3	6.5	13.21
225	1	2	1.33	0.08	5.2	5.20	6	6.5	24.23
142	0.5	2.16	1.76	0.10	2.96	5.92	2.7	6	16.32
83	0.75	2.45	2.07	0.12	4.82	6.43	4.3	6.5	18.73
87	0.5	2.45	2.05	0.12	3.31	6.62	4.5	6.2 5	26.39
13	0.5	2.59	2.18	0.13	1.06	2.12	0.25	5.5	3.55
207	0.7			0.14	1.4	2.00	3.4	5.5	36.51
207b	0.7			0.14	1.4	2.00	0.75	5.5	8.05

Table 5

This would immediately make clear that the introduction of the wave elevation in the calculation of the roll moment due to buoyancy exacerbates the roll response, bringing the value of the roll factor from 8.05 (which at least is of the same order as the 3.55 of the experimental record) to an excessively large 36.51.

In conclusion, one can state that the evidence described above gives additional credit to the theoretical hypothesis that sees the evaluation of the vessel buoyancy up to the mean water level as the right one to adopt. On the basis of such indication, the integration of the underwater volume up to the instantaneous wave elevation is thus abandoned as it is inconsistent with the theory behind the evaluation of the linear wave potentials and because it gives rise to results which do not reproduce reality.

Wind Forces

Comparing the output of tests 101 and 107, and 102, 108 and 109, the effect of wind on the vessel behaviour can be easily assessed. The roll time history of test 107 shows a constant list of about 1.15 degrees for a wind speed of 15.5 m/sec (correlated with a value of H_s=3.5 m), which results in a insubstantial enhancement of the surviving capability of the vessel if the wind blows onto the damaged side (109) and a corresponding degradation of the same for the opposite wind direction (108). In fact, this effect can only be appreciated when comparing the time to capsize in each case.

These results are hardly surprising if the wind rolling moment is examined. Comparing the moment due to a mean wind speed of 30 knots with the one caused by the most probable corresponding wave (3.5 m Hs), the great difference between the two is evident. With a factor of 0.1 with respect to the amplitude of wave induced moment, the wind heeling moment can safely be excluded from the list of crucial factors for the capsize of typical RO-RO vessels.

Current Forces

Current forces do not affect the ability of a vessel to resist capsize following damage. Although a small constant heel angle is in fact due to the indirect action of current (test 110), this is of the order of 0.6 deg for a current stream of 3.0 m/sec running in the opposite direction of the wave. This effect - which has been observed experimentally and reproduced numerically here for the first time - is probably due to the interaction of the lateral motion of the ship with the wave forces and entirely not affecting the flooding mechanics thanks to its size. It is essential to underline here though, that the influence of such a current on the wave steepness - which is expected to be quite substantial - has not been taken into account.

Looking at the time histories of the vertical motion of simulations 111 and 112, it is possible to note another side effect of introducing a current stream in either direction with respect to the wave. When the two directions are in fact opposite to each other, the resulting encountered frequency becomes higher (111) and thus closer to the natural frequency of the damaged vessel. This results in a larger heave motion following the flooding of the lower compartment. If the direction of the current is instead the same as that of the wave (112), the encountered frequency will become closer to the one of the intact ship. Therefore a larger heave motion is to be expected prior to the opening of the side damage. Again, with the heave motion having a relatively small importance on the rate of flooding of the upper compartment, even this collateral effect of current does not influence the vessel's capability of surviving a side damage. Furthermore, although a similar occurrence can be observed in the output of the roll motion of the ship, this is nevertheless minimal and does not lead to any significant change of the time to capsize.

Finally, one last observation has to be made on the drift effects of wind, compared to that of the current. As it can be seen by the track of the planar motion of the ship in the presence of wind or current opposite to the wave direction or in the absence of both, the effect of wind on the ship drift is by far smaller than that of the current. Although an extremely high current velocity has been employed here to emphasise its effects on the ship's heel, it is clear from the comparison of the three paths that even a fraction of current velocity employed would have caused the ship to stop drifting in the direction of the wave, whilst the action of a 30.0 knots wind scarcely alter the movement to the vessel in the positive x direction. Last but not least, no significant effect of wind or current has been observed on the vessel yaw, underlining the limited importance of the longitudinal asymmetry of ship on the yawing moment due to these environmental parameters.

Second Order Wave Drift Forces

Having assessed the effect of wind and current on the ship motion, it is now essential to underline the consequences of the action of the second order drift forces. These forces do not have a direct effect on the roll motion of a vessel, but dictate its movement in the horizontal plane and - most importantly - its heading, once her steering ability has been lost. As such, their effect is important and cannot be overlooked in a 3D simulation. It is in fact evident that if the flooding of the vehicle deck is mainly dependent on the ship roll (in case of side damage) and this changes drastically according to the angle of encounter of the ship with the wave, a vessel damaged when facing head or following sea will resist capsize longer than if the damage was opened with sea at the beam.

Experimentally, it has been observed that a ship without steering ability, will inevitably tend to turn until her heading is roughly around 90.0 deg with the direction of the wave. This feature is mostly simulated correctly by CASSANDRA, unless the initial heading of the vessel is zero. For instance, it is interesting to note how a heading of 45.0 deg (quartering sea) increases steadily up to about 90.0 to stabilise around this point both for a damaged and an intact case (115, 116). It is possible to see how capsize only occurs after the vessel has reached 80.0 deg; that is more than two minutes later than if the damage was open with sea at the beam (102).

In the case of following sea (113, 114), the ship keeps her heading throughout the duration of the simulation, without any significant oscillation. This event seems to be the outcome of the compound effect of a small second order wave drift yawing moment and an unbalanced wave enhanced buoyancy excitation that contrast each other as the vessel moves from the initial position. This result obviously contradicts the experimental evidence and cannot be but interpreted as yet another limitation of including the wave profile in the calculation of the buoyancy forces.

One last comment is finally necessary to attract attention on the scarcity of experimental records of the planar motion of the vessel in consideration. The acquisition of this information - which would be a precious term of comparison to assess the validity of the computed results - should be part of a research study aimed at appraising the various aspects of all the second order wave forces and moments, including issues like their dependence on the ship heading.

Variable Hydrodynamic Coefficients

In this section the effects of the variation of the hydrodynamic coefficients with the vessel attitude and the frequency of response will be examined. Unlike the other features implemented in CASSANDRA, there is no direct way of assessing the importance of having added mass and non-viscous damping change with the vessel trim, heel and sinkage or the weight on the overall result of having employed convolution integrals to accommodate the variation of these coefficients with frequency. Therefore, the only information available for this purpose is that resulting from the comparison of the motion output of runs for which the hydrodynamic coefficients were kept constant with other simulations performed with variable added mass and damping.

Comparing simulation 102 with 122 a slightly larger roll amplitude and a more significantly larger heave motion can be observed in the case of variable added inertia and damping coefficients. However, this trend is not maintained when comparing simulations 104 and 124, that is when all the other features present in the model have been switched off to isolate the effect of the variable coefficients on their own. Indeed the difference between the motion output in the two cases is inconsequential and this evidence can be taken as a good enough proof of the little weight of the variation of the hydrodynamic coefficients with the ship attitude, at least in the case under examination.

Clearly, this simple check is indeed far from being exhaustive of the problem, since the effect of variable hydrodynamic coefficients can be observed only if the variation of the latter with the ship heel, trim and sinkage is large. If some of the hyper-surfaces describing the roll and heave added inertia and damping are given a closer look, they reveal that a significant change takes place only if the sinkage varies or for large heel angles. The reason for the small influence of the variation of the hydrodynamic coefficients on the ship motion can thus be explained easily. In fact, if the time history of the vertical motion of the centre of gravity and the roll angle are observed, it can be seen that the ship usually settles on a mean sinkage after damage which keeps reasonably constant and mainly depends on the size of compartment 1. Therefore, if the sinkage does not vary once the flooding of the compartment below the vehicle deck has been completed, there is little surprise if all the quantities depending on this follow the same course.

A similar argument can be used to justify the little sensitivity of the model to variations of the mean heel angle. This quantity usually does not vary beyond 10.0

deg, until a critical mass of water on the vehicle deck has been accumulated, after which the vessel proceeds to a rapid capsize. This means that the ship never reaches the range of heel values for which the hydrodynamic coefficients start feeling the effect of a different underwater geometry. Obviously this is true only for the particular vessel under consideration here and possibly, for those of similar shape. This last observation leads then to a new call for further investigation involving a different and more complex hull geometry, which could better show if the variation of added inertia and damping coefficients should indeed be taken under consideration or not.

Finally, a world has to be said on the effects of the variation of the damping coefficients with frequency and the use of convolution integrals. The direct comparison of the results from the two models (simulations 103, 104, 105 and 106) seems to show that although they output slightly different responses (the main disagreement being on the frequency and amplitude of the roll record) this result does not really change much in the prediction of the ship capability to survive capsize. For the purpose of this investigation, it is therefore admissible to reduce the hydrodynamic coefficients to simple constants as long as they refer to the damaged equilibrium position and the natural frequency of the damaged ship in roll. Again it is nevertheless important to underline that this simplification could introduce misleading results for unconventional vessels or if an improper range of frequencies is chosen to describe the added inertia and damping coefficients. Besides, if the interest of research shifts from the resistance to capsize to quantities that are more likely to be affected by the non-linear response of the vessel - one example of which is the time to capsize - the variation of the hydrodynamic coefficients with the vessel's attitude might well be important.

<u>Flood Water Inertia Forces and $\left[\dot{M}_w\right]$</u>

Other parameters influencing the survivability of a vessel to capsize, are the forces and moments due to the time variation of the mass of the flood water and the velocity and acceleration of its centre of gravity within the compartments. These forces were introduced in Chapter 5 as a result of the particular model adopted here to simulate the interaction between the vessel and a discrete number of concentrated time varying masses of flood water and represent the dynamic inertial action of the water accumulated in the compartments as opposed to that due to gravity. In the notation used in Chapter 5, these forces were referred to as $\vec{F}'^{*}_{w}$ and could be thought as a simplified sloshing term. In addition to these, a flood water "damping" matrix $\left[\dot{M}_{w}\right]$ was found to introduce yet another term in the equations of motion for a damaged ship, which was not present in any other model previously developed and put to use. Given the absolute novelty of the theory involved, assessing the significance of these forces in the context of this investigation is therefore at the same time extremely important and difficult.

To accomplish this task, simulations 121 and 123 were run trying to isolate the eventual effects on the vessel motion of these terms which from here on are going to be improperly referenced as "sloshing forces". Furthermore, the time history of the sloshing forces was recorded for every run, to allow a direct comparison of the latter with those forces usually regarded as of primary importance: gravity, buoyancy and first order wave forces. A superficial comparison of the roll and heave motion output for runs 121, 123, 102 and 104, reveals little of the effects of the sloshing forces on the general behaviour of a damaged vessel since the phenomenon of capsize is principally controlled by gravity and buoyancy forces, given the dimensions of the ship. A much more significant picture can instead be drawn by the comparison of the sloshing roll moment with all other sources of excitation.

The graphs of the roll moment for simulations 102, 109 and 123 show an essential fact that could alone justify the presence of the sloshing forces in a damaged ship model: their magnitude is of the same proportions as that of first order wave forces and, more importantly of the same size as the gravity forces in a fully dynamic situation (i.e. untill a critical mass of water on the vehicle deck has been reached, after which the degradation of the vessel's stability is rapid but happens without significant oscillations). More generally, it can be stated that in the limits of the assumptions of this model, both gravitational and inertial forces due to the flood water affect the behaviour of the vessel during the dynamic flooding transient, but that the final stages of flooding and the event of capsize itself can be mainly ascribed to the action of gravity alone.

The importance of these findings is certainly great, once it has been appreciated that the mechanism that leads the ship to reach the critical mass of flood water - i.e. the mechanism that determines the survival time of the vessel - is principally ruled by dynamic terms: wave, wind, current and sloshing, and that the dynamic action of the flood water is definitively not second to any of the others. A simple way to convince oneself of this, is to compare the wind heeling moment with the sloshing forces for simulation 109. Again, the sloshing moment is close to that of gravity and generally of greater magnitude than the wind heeling excitation. In conclusion, it must be underlined that although all of the above could be a strong indication of the consequential role of the sloshing forces, there is currently no way of comparing the records produced by these computational simulations with experimental evidence. Once again, future research should be promoted keeping in mind the need for experimental testing designed to the purpose of validating the theory presented here. These conclusions, not only highlight the need for a better model, but also justify and support the research work presented in this thesis.

10. Discussion

General Remarks

The principal purpose of this research was to produce flexible and reliable means to evaluate the influence of those non-linear phenomena generally regarded as the most influential in the dynamic transient occurring during the progressive flooding of a passenger ship. This instrument would then be used to assess the feasibility of simplifying the mathematical model developed to increase its numerical capacity, adopting a top-down approach. To this extent and considering the great complexity of the task undertaken, this research has been altogether fruitful and successful. The mathematical model developed satisfies indeed all the requisites sought but not only this. It also demonstrated factually that it is possible to achieve reasonably good accuracy in the prediction of the dynamic response of a damaged vessel subject to progressive flooding without compromising on speed or generality.

The problems encountered in the development and validation of the model presented were manifold and complex and occasionally their solution was not entirely satisfying. This is not to be regarded as a deficiency of this study though. Since the subject is so vast and normally the topics treated had received no previous attention, it is hardly surprising if sometimes the answer to some of the many questions raised only partially clarified the matter or offered just an approximate solution. It is in fact understandable that research work has to go on to provide successive refinements leading to more and more accurate conclusions, much in the same way as design does. Besides, considering that a large number of suitable and adequate solutions were found to most of the principal issues at stake, the successfulness of this study cannot be deemed but complete. Which difficulties have been met then and how were they solved? And what are the contributions and innovations of this study? These questions present the subject matter of this chapter.

Equations of Motion

All the models adopted to date to simulate the behaviour of a damaged vessel, were more or less simplified versions of the canonical equations of motions of a rigid body with a constant mass. For obvious *reasons this model is not suitable for a system such* as a damaged ship at sea, as it does not take account of any of the effects due to the movements of the centre of gravity of the system within its boundaries nor the rate of variation of its mass. A further weakness of some of the equations traditionally used in seakeeping was a over-linearisation of their terms due to the assumption that the two frames of reference employed (inertial and ship-fixed) could be considered overlapping as long as the ship motion was restricted to pure oscillations around the equilibrium position. This position implies that all the forces included in the model had to be linear and oscillatory, which means that none of the second order effects that are of interest for this study could be rightfully covered using such an approach.

The inadequacy of the existing equations of motions lead to the development and introduction of mildly linearised, six degrees-of-freedom, seakeeping/manoeuvrability equations. In this theory, the mass of the flood water in each damaged compartment can vary with time but it is considered concentrated in its centre of gravity, which is free to move within the compartment's confines. All the effects linked to the rate of variation of the mass of the system and the motion of the centres of gravity of the flood water are therefore correctly simulated within the limits of the assumptions of the model, as they derive directly from the application of Newton's law of motion to the system comprising both the ship and flood water. Furthermore, the flexibility of this model allows for a swift incorporation of other effects such as cargo shifting and water sloshing, as long as the law controlling the motion of the variable centres of gravity is known *a priori*.

The newly introduced equations of motion were developed with no assumption of linearity in the ship-fixed system of reference, but they were successively stripped of all higher order terms thanks to the assumption that the rate of rotation of the vessel would not be significantly large since forward thrust and rudder action are removed from the model. This single simplification is not crucial and should be relieved if these effects are to be included in the mathematical model, nevertheless it permitted the rearrangement of matrix equations which opportunely stress the effects of each new term deriving from the insertion of varying mass particles in the system. For instance, one of the possible implications of the introduction of the matrix of the rate of variation of the flood water mass is an additional time varying damping term which can, at times, change its sign, promoting the ship motion rather than inhibiting it. This approach also allowed both to estimate and emphasise the role and magnitude of inertial forces and moments due to the moving flood water. These resulted to be at times of the same order as the first order wave forces, that is of primary importance in determining the succession of events leading to capsize.

Together with the introduction of a great flexibility in the handling of forces due to movable masses on board, new limits and even new unknowns were brought up by the hypothesis adopted to formulate the new equations of motion. For example, the moment of inertia of the system was inevitably simplified and the movements of the centres of gravity of the flood water were calculated assuming zero phase between the ship motion and that of the water free surface. This last approximation, which is reasonably accurate in the case of large vessels subjected to large scale flooding, cannot stand alone if the displacement of the ferry falls below certain limits or when the amount of flood water is small. In such cases, a different treatment would be needed to define the behaviour of the sloshing water. For example, this could be achieved either by an empirical method or employing full CFD techniques, once the necessity for such a great increase in the complexity of the model had been ascertained through experimental or experiential evidence.

Hydrodynamic Coefficients

One of the effects that this research was meant to investigate is the change of hydrodynamic coefficients and first order wave forces with the vessel's attitude. The reason for deeming such an effect important in the context of damaged ship stability are obvious, given the often large differences between the intact and damaged equilibrium positions. Moreover, some researchers [3.2] already highlighted in the past the possible effects of large underwater asymmetries on the hydrodynamic coefficients and ultimately on the vessel's roll.

Indeed, the importance of added inertia and damping is great in determining the series of events leading to the point of no return signaled by the reaching of a critical amount of flood water on the vehicle deck, although once this threshold had been reached, the resulting dynamic outcome effectively excludes any interference from the hydrodynamic coefficients. The only important stage during which these quantities need to be evaluated with a reasonable accuracy is thus when the dynamic equilibrium is attained after the flooding of lower compartments, when the hydrodynamic coefficients become virtually constant. The two major achievements of this study, as far as the hydrodynamic coefficients are concerned, are therefore to have demonstrated that it is possible and numerically feasible to include the non-linear variation of these terms in a numerical scheme, although this might be sometimes redundant.

To accommodate the variation of the RAOs, phase angles, added inertia and hydrodynamic damping terms with the vessel's attitude without breaching the linear assumptions adopted to calculate them, heel, trim and sinkage need to be extracted out of the vessel's motions. This implied the estimation of a feasible time filter to calculate the time averages of the ship motion without over-filtering. The natural choice proved to be using the inverse of the cut-off frequency of the wave spectrum within a downstream averaging routine, although this arrangement introduces an unwanted time delay between the instantaneous and the mean motions, which is a function of the time filter. Possible solution of this problem are difficult to devise since centred algorithms would require an evaluation of all those solution points yet unknown because they are succeeding the time step under consideration. Indeed, the difference between actual and mean motion can be kept under control varying the length of the time filter so that it could be considered negligible.

Further problems encountered in adjusting the hydrodynamic coefficients to suit the underwater geometry of the hull, are linked to the necessity of interpolating between the discrete points of a database library representing the coefficients multidimensional surfaces as function of heel, trim, sinkage and frequency. This database approach has been introduced for the first time in the numerical simulation of ship motion during this research and demonstrated to be a fast and effective way to include fully non-linear coefficients in the equations determining the vessel's behaviour in a seaway. As such it must be regarded as a major achievement of this study and a method worth retaining in future models and refining to achieve a better efficacy.

Being a pioneering attempt, the database utilised in CASSANDRA presents a few flaws, of which the most important is the ineptitude of the interpolation routine to handle discontinuities in the hyper-surfaces. Generally this is not a great problem, for added inertia, damping or RAO terms are generally smooth, continuous functions of the parameters under consideration, but it becomes a singular dilemma in the case of phase angles when these quantities suddenly jump from 0 to 2π and *vice versa*. To deal with this problem the database library has been manually conditioned to restore the continuity of the phase angles utilised for the validation and sensitivity analysis performed in this study, but an alternative methodology is strongly needed to generalise the interpolating algorithm so that it could automatically manage discontinuous functions. An additional necessity to aid this goal, is increasing the library dimensions to provide a better definition of the functions it represents, to be able to identify eventual discontinuities more easily. Obviously, this operation needs to be carefully planned not to create problems arising from too great an amount of data to store in memory.

The number of points representing the hydrodynamic coefficients hyper-surfaces is indeed a tricky figure to define also for reasons other than eventual discontinuities in the functions. For example, perplexities have been raised about the accuracy of definition of the multidimensional surfaces and the method of interpolation chosen to work out the values of these quantities at intermediate points. The procedure employed to describe the hydrodynamic coefficients as stepwise continuous functions currently applies multiple linear interpolation so that a reasonably high number of known points would be necessary to achieve a good degree of accuracy, if the shape of the coefficients curves was wildly changing with the variable parameters. Given that the hydrodynamic coefficients surfaces are generally good mannered (provided that a fair number of points is used to describe them where peaks and troughs take place) a greatly improved speed of calculation and the possibility to easily generalise the algorithm to any set of data, became the reasons that justify such stepwise linearisation process.

Since the current method proved to be satisfactory and proficient in rendering the effects caused by variable hydrodynamic quantities for the case under examination, possible future improvements would mainly regard the generalisation of the structure of the database library to suit any potential underwater geometry, rather than a transition from linear to high order interpolation or multi-chromatic regression analysis. Also, a more flexible definition of the boundaries of the hyper-surfaces would be appreciable to allow larger variation of parameters like heel and sinkage, as well as user friendly tools to optimise the definition of the coefficients curves if this process becomes necessary.

Finally, it is opportune to note that the number of frequency components used for convolution integrals and first order wave forces were tailored on the set of data concerning response operators and damping curves of the ship model used for the validation of the code. This operation implies checking the shape of the kernel functions and recalculating their inverse transformation to compare them with the original curves. The same process could be repeated to make sure that the number of frequencies used is sufficient for ships other than the one analysed here, but a reasonable confidence can be granted to assuming that the same number could be applied to any other case, although the same cannot be asserted for the frequency range. Again, a more flexible way to change these parameters according to the vessel geometry is needed if complete freedom of choice and flexibility of the numerical "tool" are sought.

Drift, Current and Wind

The absence of a generalised method to assess the magnitude of the second order wave longitudinal force and yaw moment, demanded substantial approximations to be made. The drift forces are a secondary source of excitation and do not directly affect much the lateral stability of either damaged or intact ships. As such, their estimation does not require a high level of precision, although assuming that experimental data collected for one particular ship type could be used for any other type, could be considered a far fetching presupposition. Indeed, some improvements are needed to optimise and validate the algorithm which calculates these forces and specifically designed experimental tests should be planned to broaden the knowledge on the subject.

A particular point to be investigated is the behaviour of these forces with the angle of encounter between vessel and wave direction. It has been observed numerous times during experiments in waves, how angles of attack close to head or following sea are extremely unstable equilibrium positions as far as the heading of a ship without steering capability is concerned. The moment that forces a ship to turn around until it reached beam on position is deemed to result from second order forces, given their characteristic slow oscillations and the non-zero mean value. In the model adopted in this study, the variation of the second order wave yaw moment with the angle of attack is sinusoidal with zero values for both following and head sea. This means that the weak instability introduced by the positive slope at these locations, is usually overridden by other secondary effects with the result that these locations are wrongly estimated to be directionally stable. Possible solutions to this problem obviously concern a more stringent study of the functional dependency of this moment on the angle of attack.

Since the subject of wind loading has been given much more attention by the scientific community, the assumptions made in this case to adapt the empirical method employed to the needs of this investigation were less and much milder. Of these, the only two worth recalling in this discussion to hint at possible future development on the subject, are the approximate formula used to calculate the wind heeling moment lever and the evaluation of wind gust forces by extending a constant wind excitation method to a full dynamic case. In the first instance, the literature on the subject is generally hazy and/or specific to particular cases, so that the model adopted might need to be further refined. As for the second, a robust validation is now needed to assure that this new methodology is indeed correctly representative of the physical reality of the phenomenon it simulates. Finally, it is worthwhile underlining that the innovative way of treating wind gust loading has been introduced for the first time during this investigation and that as such, it deserves to be included in the list of the contributions of this study to the research in the field of ship dynamics.

The routines developed to estimate the action of current forces as a consequence of a current stream or just as viscous damping resulting from the planar velocity of the ship, were one of the most successful features introduced in the numerical code. In fact, not only the expected effects were simulated with a good accuracy by means of a simple theoretical approach, but also phenomena such as the rise of a small static heel angle as a result of the interaction between opposite current and wave forces were unpredictably forecast by the program, although only for relatively high current speeds. The modalities of this latter occurrence are still to be investigated, it is therefore essential to stress out once again that even if this success remains one of the many achievements of this study, further research in this field has to be promoted and motivated.

Buoyancy and Gravity

The principal progress ensured by the strategy devised in this context to deal with buoyancy and gravity forces and moments, is the elimination of the traditional concept of *restoring terms*. In fact, when neither of the two components that guarantee the equilibrium of the static ship can be safely considered linear any more, the only reasonable route to follow to restore some clarity in the way theory handles these forces, is to separate them back to individual entities. One of the advantages of this procedure can be appreciated in terms of a better understanding of the behaviour of each of these two terms when large trim and heeling angles, typical of the flooding transient and capsize of a damaged ship, make them both highly non-linear. In this way, the gravitational element clearly appears to be a function of the motion of the centre of gravity of the system comprising the vessel and the flood water, whilst buoyancy is shown to depend greatly on the large variation of the underwater volume of the ship's hull, thus ultimately on the vessel's motion.

Flooding Mechanics

The question of flooding mechanics has been one of the greatest challenges in the study of damaged ship survivability ever since the numerical capabilities of modern computers allowed the first simulations in the time domain. The fundamental reason behind this matter was the almost total absence of a suitable model to simulate flooding of compartments open to the sea. Following some pioneering experimental studies [11.7], a sound theory has been developed during this research, which supports a semi-empirical model now already generally accepted as one of the best approximations to the extraordinarily complex flow dynamics occurring at the damage opening. One of the merits of the structure of this method it to permit the generalisation of the same basic theory to a wide range of damage conditions, including lower compartments as well as the vehicle deck, and a flexible definition of the damage location and extent. The modelling of the flooding mechanism through this innovative and effective method is also capable of simulating the accumulation of water on the vehicle deck without recurring to any concept founded on energy balance.

The hydraulic model provided needs, however, an empirical estimation of the flooding coefficient, which is supposedly a function of various environmental and geometrical parameters. Obviously, the major limit of this approach pertain thus to the limited experimental investigation carried out to support it. Due to the novelty of the subject, a restricted set of parameters were in fact taken into consideration during the analysis of the experimental data and evaluation of the flooding coefficient. These proved to have a relatively little effect on the value of the coefficient, while some of the side observations made, indicated other factors to merit a more in-depth evaluation.

Finally, when expanding the flooding model to water flowing between internal compartments became an appealing feature, a new need for further investigation arose. In fact, the experimental testing presented here does not provide any useful information for cases other than the direct ingress of water from the sea into a damaged compartment adjacent to the ship side. For such a reason, no cross flooding between compartments has been implemented in the numerical routine, although it is clear that the theoretical principles utilised in this context are the same that should be applied to adapt the present methodology to any other case of interest, once the necessary experimental data became available.

Numerical Method

Fortunately, the theory on numerical analysis is well developed so that only a few but essential transformations were necessary to incorporate the convolution integrals in a variable step Runge-Kutta method without loosing on the accuracy of this fourth order numerical scheme. Of these, the most innovative feature was the introduction of a numerical integration based on a Hermite interpolation to evaluate these integrals in the time domain. Traditionally, Volterra type equations are solved with matrix inversions over a discretisation of the whole span of time that the simulation is planned for. Obviously, this way of proceeding is not feasible when some of the quantities that appear in the equations depend on the solution at time just preceding that for which they are calculated; for instance, gravitational and buoyancy terms.

A different process was thus introduced, which exploits the property of the kernels to fade to zero going back in time from the instant the new solution is required. This approach required an evaluation of the measure of time strictly necessary to cover all the effective influence of the memory effect with good approximation - the convolution tail. Such an evaluation has been accomplished on purely empirical basis by observing the trends of a suitable number of graphs of the integrands and allowing for eventual discrepancies whenever the impulse functions might vary - even drastically - from those employed here.

One of the advantages of the structure of the new numerical method devised, is the different treatment reserved to proper variables as opposed to slowly varying quantities or pseudo-constants. The latter were in fact calculated only once the new solution had been accepted within the accuracy limits, and utilised to calculate the next step solution, while all other variables - including the convolution integrals - were evaluated for each intermediate approximation used by the RKF method to evaluate solutions and error estimates. Of the slowly varying quantities, the most excellent example are the hydrodynamic coefficients which vary as a function of the moving averages of the ship motion, although velocity and acceleration of the centres of gravity of the flood water in each compartments - which should be considered as proper variables - were evaluated outside the RK scheme to stabilise the numerical derivatives.

The numerical integrator provided, has been so far reliable, accurate and stable but not particularly fast. This last technical detail does not however impinge on the successfulness of the method, since optimisation features like sine and cosine numerical functions to be substituted by pre-calculated tables of values could substantially speed up the code.

Numerical Validation

Not many problems were encountered in the validation of CASSANDRA, the turnout of which went beyond all most optimistic expectations. Although only a limited number of tests were performed, an excellent correlation was indeed found with the experimental results. As in the case of most new methodologies, the need for specifically designed experimentation and further research to complete the validation of this program is still great, but confidence in the future comes easy when the first results are so encouraging.

As for the sensitivity analysis, only a small number of tests were performed to assess the influence of some of the many parameters that can be varied within the program and priority was granted this time around, to those that were investigated for the first time during this research. It is nevertheless essential to invite coming researchers to compare the consequences of the variation of other parameters whose effects are better known, with the simulation output by this numerical tool, so that a cross-check could be performed on the capability of CASSANDRA to predict these phenomena correctly.

The most substantial achievement of the sensitivity analysis is to have confirmed that the capsize mechanism is predominantly governed by the buoyancy and gravitational forces for this size and typology of vessels, although the action of all the other excitation forces determines the time and mode of the water accumulation on deck. The cardinal consequence of this conclusion is therefore that, as far as the prediction of capsize goes, even the simplest model including buoyancy, gravity, a correct flooding mechanism and a secondary oscillatory disturbance, can succeed with reasonable accuracy, even if it will not give sufficient information on the dynamic transient leading to capsize.

11. Recommendations for Future Work

General Remarks

As already anticipated above in the discussion, no research project is ever perfectly conclusive and complete, as there will always be new questions arising from the findings generated to answer the issues of the original inquiry. This study is no exception. The copious number of solutions provided hinted at a new set of problems that call for fresh resources to be put to work to provide suitable explanations. In this section a brief list is supplied, of some suggested directions for the new investigations to take, with the hope that they will skillfully and devotedly persevere in the quest for knowledge.

Sloshing Forces

- the sloshing model should be improved and validated through experiments and CFD calculations;

At present, the model does not predict the dynamic motion of the flood water in the damaged compartments rigorously, and indeed this is not strictly necessary unless the dimensions of the vessel or the amount of flood water are relatively small. If the influence of the oscillations of the flood water on the vehicle deck proved to be important for future studies, the mathematical model developed in this analysis could still be used, as long as the additional motion of the flood water was superimposed onto that associated with the vessel's heeling. One way to achieve this could be using frequency response curves and phase angles to be read out of a database library, much in the same way as for the potential damping. The data employed for building this library could be collected through experimental or numerical CFD testing. The same techniques should finally be applied to validate the current method for large displacement ships.

Hydrodynamic Coefficients

- the significance of variable hydrodynamic coefficients should be further assessed using more sensitive hull geometries (multihulls);
- the hydrodynamic coefficients database might need to be expanded in terms of frequency, heel, trim and sinkage ranges and dimensions to improve accuracy and flexibility;
- a new interpolation routine is necessary to handle discontinuities;

After verifying that the variation of the hydrodynamic coefficients does truly affect the dynamics of transient flooding, the structure of the database library might need some modifications to make it more flexible and user friendly. Ideally, no manual operation should be necessary to adapt the data describing the multidimensional surfaces to suit the interpolation routines, although it is really hard to imagine how such an achievement could be accomplished without rendering the database dimensions too big for any machine to handle. In fact, if a dense enough array of points describing the curves was available, discontinuities could be easily located and therefore automatically removed. Unfortunately, the number of elements of vectors with five dimensions (cf. the wave forces operators), increases quite dramatically as soon as any of the five indices goes up, leaving very little space for manoeuvring if the others need to be increased too. Obviously, a growth in density of the database would imply an improved accuracy as well; that is to say an additional desirable effect. Although such an improvement of the algorithm can indeed be appealing in itself, nonetheless it is strongly advisable not to blindly raise the order of interpolation to achieve it, since this would probably only slow down the execution of the program.

Flooding Mechanics

- the hydraulic model for the flooding mechanism needs a more extensive experimental investigation to assess the influence of other parameters on the flooding coefficient;
- internal compartments cross-flooding and damage opening during collision should be included in the model to assess the efficacy of some of the means to improve resistance to capsize;
- the mechanism of water accumulation on the vehicle deck should be further investigated;

The theory developed for simulating the flooding mechanism relies on a reasonably accurate evaluation of an empirical function known as *flooding coefficient*. So far the experimental investigations carried out to give shape to this quantity have been limited and failed to determine which parameters truly influence it. A strong call for further research in this direction is therefore a necessary step towards the achievement of suitable knowledge and understanding of the subject.

The routine implemented in the simulation program presented, does not take into account any other flooding modality but that of compartments open to sea through a damage opening which is considered free from any obstruction as soon as it has been burst open. Therefore, the state of the art does not allow internal compartments to be flooded through connecting openings, nor the study of features like cross flooding and freeing ports. Also, the temporary blocking of the damage opening by the perforating body soon after the impact has not been simulated yet. All these innovations can easily be introduced in the routine by generalising the same theory used for the cases already included.

Finally, a theoretical investigation should be performed to reveal the causes that induce water to accumulate on the vehicle deck. This phenomenon that has been simulated numerically, still needs research to clarify the parameters that promote/discourage it. The usefulness of such investigation is hopefully clear considering the possible use of its findings in the attempt to create means of improving the ship resistance to capsize.

Drift Forces

- the evaluation of the second order wave forces and the function modelling their dependence on the angle of encounter should be improved;
- the interaction of drift forces with the non-linear effects of the viscous and buoyancy components in the longitudinal direction and around the vertical axis should be assessed;

Although the second order drift forces have been thoroughly studied by many, there is still a compelling need for a reasonably simple but comprehensive empirical method to calculate this excitation without resorting to complex CFD techniques. So far, the only attempt in this direction was made for the sway force only, and produced a good coefficient function based on regression analysis, which can be effectively used for many types of monohulls. Unfortunately, the same was not achieved for surge force and yaw moment. Furthermore, the dependency of these forces upon the angle of encounter has been simulated with approximate functions that are only partially satisfying. A new research project is therefore needed, to assess these issues and produce generalised functions that could be confidently used for any kind of ship.

An additional argument of interest related to the drift forces, is the interaction of the first and second order, non-linear planar forces and moments. These forces, that govern the vessel's motion in the horizontal plane, are at present working reasonably correctly, although the incomplete calibration of some of them leads to a peculiar behaviour for initial following and head seas. The model should thus be further validated and the causes of such effects removed.

Code Development

- the code should be optimised to increase its speed and flexibility;
- forward thrust and rudder forces might be introduced in the model, as well as all the second order terms of the equations of motions, manoeuvrability coefficients, forward resistance, non-linear viscous damping and eventual changes in the flooding mechanics;
- the mechanics of collision should be included to simulate the effect of transient flooding and initial impulsive forces;

CASSANDRA has been coded trying to balance clarity and linearity of programming with the powerful necessity for optimising the speed of calculation. At present the serial code can run on average a complete simulation of a damaged condition, for a total simulation time of 900 seconds in about 9 hours. Many are the possible modifications that can be introduced to optimise the speed of calculation, from translating the program to parallel code, to adopting tables of sine and cosine functions to save the time of calculation of these through their in-built routines. Just as important as the optimisation of the execution speed is however the flexibility and user-friendliness of a program. CASSANDRA has been modelled so that a minimum number of files and operations is needed to initiate the execution, so that the program can be run at minimum time and easily adapted to run in batch mode. The flexibility of the code is already remarkable and indeed very little space for improvement is left, with the sole exception of the making of the database library.

Apart from these considerations, the model could be further expanded by introducing those features left out of the aims of this study, such as forward thrust, rudder forces, collision mechanics and so on. However, it is important to highlight that if these topics proved to deserve attention and should be included in the model, so should also all the associated effects. This implies that many of the simplifications adopted in the development of the theory behind this simulation code should be revised and opportunely amended before any changes were performed.

12. Conclusions

On the basis of the work carried out during this research and presented in the foregoing, successful achievement of the aims can be confidently stated. A robust and comprehensive mathematical model has been developed to examine the non-linear behaviour of damaged vessels in a seaway subjected to progressive flooding. This model proved to be capable of allowing the estimation of the sensitivity of this physical system to the effect of several variables with accuracy. Furthermore the following specific conclusions can be drawn:

- A rigorous mathematical formulation describing the damaged vessel motion in six degrees-of-freedom has been developed which includes all the effects due to the mass variation of the flooding water. This innovative approach to the problem has been attempted here for the first time and proved to give encouraging results. One of these is that the simplified sloshing forces modelled in this way revealed to play an important role in the evolution of the transient leading to capsize;

- An original methodology was employed to implement variable hydrodynamic coefficients in the model to allow the dependence of these terms on frequency and vessel's attitude to be taken into account. This non-linear feature confirmed to be of little concern in the case of a typical monohull ferry, although a larger influence cannot be ruled out *a priori* for other ship types;

- Wind, current and second-order wave forces were included in the external excitation. Their effect does not seem to be important in determining the capsizability of a vessel, but it is determinant in affecting the events taking place during the flooding transient and, ultimately, the survival time;

- Buoyancy and gravitational excitation are confirmed to be the two major factors determining the ship's destiny in a damaged condition. These quantities were modelled as fully non-linear terms, adopting procedures that should be further developed and generalised;

- A semi-empirical model of the water inflow and outflow as a function of the relative level of wave and accumulated water has been established and verified theoretically. Some of the dynamic effects governing this phenomenon were investigated experimentally and specific coefficients were provided to finish and fine-tune the method. The rate of flooding of the damaged compartments showed to be possibly the first and most important feature in determining the dynamic progression of events following the opening of the damage hole.

References and Bibliography

1. General

[1.1] Abkowitz M. A.: " **Stability and motion Control of Ocean Vehicls** " - Massachussets Institute of Technology Press - 1969.

[1.2] Newman, J. N.: " **Marine Hydrodynamics** " - The Massachsetts Institute of Technology - 1977.

[1.3] Bhattacharyya, R.: " **Dynamics of Marine Vehicles** " - John Wiley and Sons. - 1978.

[1.4] Bishop, R. E. D. and Price W. G.: " **Hydroelasticity of Ships** " - Cambridge University Press - 1979.

[1.5] Clayton, B. R. and Bishop, R. E. D.: " **Mechanics of Marine Vehicles** " - E. & F. N. Spon Ltd. - 1982.

[1.6] Chakrabarti S. K.: " **Hydrodynamics of Offshore Structures** " - Computational Mechanics Publications - 1987.

[1.7] " **Principles of Naval Architecture** " - SNAME - 1989.

[1.8] Faltinsen, O. M.: " **Sea Loads on Ships and Offshore Structures** " - Cambridge University Press - 1990.

[1.9] Chakrabarti S. K.: " **Non-Linear Methods in Offshore Engineering** " - Elsevier- 1990.

2. Damage Stability

[2.1] Bird, H. and Browne, R. P.: " **Damage Stability Model Experiments** ". - RINA Spring Meeting - 1973.

[2.2] IMO Resolution A.265 (VIII): "**Regulations on Subdivision and Stability of Passenger Ships as Equivalent to Part B of Chapter II of the International Convention for the Safety of Life at Sea 1960** ", IMO, London, 1974.

[2.3] Elsimillawy, N. and Miller, N. S.: " **Time Simulation of Ship Motions: A Guide to the Factors Degrading Dynamic Stability** " - Trans. SNAME - 1986.

[2.4] Spounge, J. R.: " **The Technical Investigation of the Sinking of the RO/RO Ferry ' EUROPEAN GATEWAY '** ". -Trans. RINA - 1986.

[2.5] Barwell, D. R. M.: "**Passenger Ro-Ro Ferry Safety - An Owners View**", The First H. Kummerman Foundation International Conference on Ro-Ro Safety and Vulnerability: The Way Ahead, London, RINA, December 1987.

[2.6] Crawford, J. A. Gilfillan, A. W. and Mackie, G. C.: "**The Designer Dilemma**", The First H. Kummerman Foundation International Conference on

Ro-Ro Safety and Vulnerability: The Way Ahead, London, RINA, December 1987.

[2.7] (5) Aston, J. G. L. and Rydill, L. J.: **"Improving the Safety of Ro-Ro Ships"**, The Naval Architect, April 1987.

[2.8] Wardelmann, H.: **" The New SOLAS Amendments - Agreed and Possible "** - RO-RO '88, Göteborg - June 1988.

[2.9] Rusaas, S.: **" Survival Capability Class: Increased Safety, but does it Destroy the RO-RO Concept? "** - RO-RO '88, Göteborg - June 1988.

[2.10] Boltwood, D. T.: **" RO-RO Ship Survivability: Comments on Damage Stability Modelling "** - RO-RO '88, Göteborg - June 1988.

[2.11] Spencer, J. S. and Gilbert, R. R.: **" RO-RO Stability "** - RO-RO '88, Göteborg - June 1988.

[2.12] Pawlowski, M. and Winkle, I. E.: **" Capsize Resistance Through Flooding - A New Approach to RO-RO Safety "** - RO-RO '88, Göteborg - June 1988.

[2.13] Byrne, D.: **" Practical Solutions to Improved Survivability of RO-RO Ferries "** - RO-RO '88, Göteborg - June 1988.

[2.14] Vossnack, E.: **" Putting a Lifebelt around the Ship "** - RO-RO '88, Göteborg - June 1988.

[2.15] Paffett, J.: **" RO-RO Safety and Vulnerability: The Way Ahead "** - The Naval Architect - January 1988.

[2.16] Jakic, K.: " **Flotation and Stability of RO-RO Vessels in the Damaged Conditions** " - International Shipbuilding Progress, No. 34 - 1987.

[2.17] Dand, I. W.: " **Hydrodynamic Aspects of the Sinking of the Ferry ' HERALD OF FREE ENTERPRISE '** ". - Trans. RINA - 1988.

[2.18] Hua, J.: " **A Theoretical Study of the Capsize of the Ferry ' HERALD OF FREE ENTERPRISE'** " - The Royal Inst. of Technology (KTH) Div. of Naval Architecture - 1988.

[2.19] Woodyard, D.: "**Ro-Ro Safety Challenges Mechanical Engineers**", Chartered Mechanical Engineer, Vol. 35, No. 2, 1988.

[2.20] Vossnack, E.: "**Buoyancy in the Wings**", The Naval Architect, May 1988.

[2.21] Judd, P. H.: "**RO-RO Passenger Ferry Survivability Study - Hull Form and Superstructure**", International Symposium on the Safety of Ro-Ro Passenger Ships, RINA, April 1990.

[2.22] Lloyd, C. J.: "**Research into Enhancing the Stability and Survivability Standards of Ro-Ro Passenger Ferries - Overview Study**", International Symposium on the Safety of Ro-Ro Passenger Ships, RINA, April 1990.

[2.23] Lloyd, C. J.: "**Research into Enhancing the Stability and Survivability Standards of Ro-Ro Passenger Ferries - Internal Arrangements**", International Symposium on the Safety of RO-RO Passenger Ships, RINA, April 1990.

[2.24] Pucill, F and Velschou, S.: "**RO-RO Passenger Ferry Safety Studies - Model Test of a Typical Ferry**", International Symposium on the Safety of Ro-Ro Passenger Ships, RINA, April 1990.

[2.25] Jin Hao and Yuan Don Lei: "**Model Tests on the Rolling Behaviour under Damaged Conditions**", STAB '90 Naples, September 1990.

[2.26] Rogan, A. J. and White, N. J.: "**A Study to Compare the Residual Standards of Stability after Damage of Existing RO-RO Passenger Ferries**", International Symposium on the Safety of Ro-Ro Passenger Ships, RINA, April 1990.

[2.27] Gilbert, R. R. and Card, J. C.: " **The New International Standard for Subdivision and Damage Stability of Dry Cargo Ships** " - Marine Technology, Vol. 27 -1990.

[2.28] DMI 88116 : "**RO-RO Passenger Ferry Safety Studies Model Test for F10 - Final Report of Phase I**", DMI Project Report to the UK Department of Transport, 1990.

[2.29] Dand, I. W.: " **Experiments with a Floodable Model of a RO/RO Passenger Ferry** ". - MT Project Report to the Department of Transport, BMT Fluid Mechanics Ltd - 1990.

[2.30] Rawson, K. J.: "**An Overview of RO-RO Safety**", The Second H. Kummerman Foundation International Conference on Ro-Ro Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

[2.31] Gilbert, R. and Sheehan, D.: "**Pros and Cons of Opting for an Existing Standard**", The Second H. Kummerman Foundation International Conference on Ro-Ro Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

[2.32] Aston, J. G. L. and Rydill, L. J.: **"Assessing the Safety of RO-RO Ships"**, The Second H. Kummerman Foundation International Conference on Ro-Ro Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

[2.33] Brown, J. G.: **"Buoyant Wing Spaces - Economic Compliance with SOLAS '90"**, The Second H. Kummerman Foundation International Conference on Ro-Ro Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

[2.34] Sen, P. and Wimalsiri, W. K.: **"RO-RO Cargo Ship Design and IMO Subdivision Regulations"**, The Second H. Kummerman Foundation International Conference on Ro-Ro Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

[2.35] Vassalos, D. and Turan, O.: **"Location and Extent of Flooding - A Dynamic Analysis"**, RO-RO '92, Göteborg, May 1992.

[2.36] Vassalos, D. and Turan, O.: **"Development of Survival Criteria for RO/RO Passenger Ships - A Theoretical Approach"** - Final Report on the RO/RO Damage Stability Programme, Phase II, Department of Ship and Marine Technology, University of Strathclyde - 1992

[2.37] Rowson, K. J.: **"Why Do We Bother"**, RINA International Conference on LIFE SAVING AT SEA, London, December 1993.

[2.38] Vassalos, D. and Turan, O.: **"The Question of Subdivision Revisited"**, RINA International Conference on LIFE SAVING AT SEA, London, December 1993.

[2.39] Miller, R.: "**Flood Control Doors as Practical Retrofit Options on RO-RO Decks**", RINA International Conference on LIFE SAVING AT SEA, London, December 1993.

[2.40] Vassalos, D. and Turan, O.: "**The Impact of SOLAS '90 on the Design and Safety of RO-RO Vessels**", RO-RO '94, Göteborg, April 1994.

[2.41] Pawlowski, M.: "**A New Concept of RO-RO Ships Subdivision for Enhanced Safety in the Damaged Condition**" - RO-RO '94, Göteborg, April 1994.

[2.42] Allan, T.: "**The Practical Implication of SOLAS '90 on Existing RO-RO Passenger Ships**", RO-RO '94, Göteborg, April 1994.

[2.43] Vassalos, D and Turan, O: "**The Influence of Design Constraints on the Damage Survivability of Ro-Ro Vessels**" - Proceedings of the 5th IMDC '94-STG, TU Delft, Vol. I, May 1994, pp 381-402.

[2.44] Vassalos, D and Turan, O: "**A Realistic Approach to Assessing the Damage Survivability of Passenger Ships**", Transactions SNAME, Vol. 102, 1994, 34pp.

[2.45] Vassalos, D.: "**Damage Survivability: Searching for the Right Question**" - Tecnica Italiana, Vol. 59, No. 3, September 1994.

[2.46] Vassalos, D and Letizia, L: "**Behaviour of a RO-RO Vessel during Transient Flooding in a Random Sea**" - Proceedings of the International Conference on Ship and Marine Research NAV '94, Rome, vol. 2 session IX - 1994.

[2.47] Pawlowski, M.: "**Damage Stability Assessment: State of the Art**", STAB '94, Melbourne, USA, Nov. 1994.

[2.48] Vassalos, D and Turan, O: "**Damage Scenario Analysis: A Tool for Assessing the Damage Survivability of Passenger Ships**", STAB '94, Melbourne, USA, Nov. 1994.

[2.49] Vassalos, D.: " **Capsizal Resistance Prediction of a Damaged Ship in a Random Sea** ", RINA Symposium on RO-RO SHIPS' SURVIVABILITY, London, Nov. 1994.

[2.50] Dand, I. W.: " **Factors Affecting the Capsize of Damaged RO-RO Vessels in Waves** ", RINA Symposium on RO-RO SHIPS' SURVIVABILITY, London, Nov. 1994.

[2.51] Spouge, J. R.: " **A Technique to Predict the Capsize of a Damaged RO-RO Ferry** ", RINA Symposium on RO-RO SHIPS' SURVIVABILITY, London, Nov. 1994.

[2.52] Velschou, S. and Schindler, M.: " **RO-RO Passenger Stability Studies: A Continuation of Model Tests for a Typical Ferry** ", RINA Symposium on RO-RO SHIPS' SURVIVABILITY, London, Nov. 1994.

3. Equations of Motion

[3.1] Goldenstein, H.: " **Classical Mechanics** ", Addison-Wesley, 1969.

[3.2] Lee, C. M. and Kim, K-H.: " **Prediction of Motion of Ships in Damaged Condition in Waves** ", STAB '82 Tokyo, October 1982.

[3.3] Manacorda, T.: " **Appunti di Meccanica Razionale** " - Libreria Scientifica G. Pellegrini, Universitá di Pisa - 1987 - (ital.).

[3.4] Meriam, J. L. and Kraige, L. G.: " **Engineering Mechanics** " - J. Wiley and Sons - 1987.

[3.5] Sen, P. and Konstantinis, C.: " **A Time Simulation Approach to the Assessment of Damage Survivability of RO/RO Cargo Ships** " - Trans. SNAME - 1987.

[3.6] Falzarano, J. M. and Troesh, A. W.: "**Application of Modern Geometric Methods for Dynamical Systems to the Problem of Vessel Capsizing with Water on Deck**", STAB '90 Naples, September 1990.

[3.7] Trincas, G.: "**Safety for Damaged Vessels as Probability of Non-Capsizing in Following Seas**", STAB '90 Naples, September 1990.

[3.8] Rakhmanin, N.: "**Stability of Damaged Ships During Ship Motion in Waves**", STAB '90 Naples, September 1990.

[3.9] Petey, F.: "**Determination of Capsizing Safety of Damaged Ships by Means of Motion Simulation in Waves**", STAB '90 Naples, September 1990.

[3.10] Vredeveldt, A. W. and Journèe, J. M. J.: "**Roll Motions of Ships Due to Sudden Water Ingress, Calculations and Experiments**", The Second H. Kummerman Foundation International Conference on Ro-Ro Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

[3.11] Turan, O.: " **Dynamic Stability Assessment of Passenger Ship Using a Time Simulation Approach** " - Ph.D. Thesis, Department of Ship and Marine Technology, University of Strathclyde - 1993.

[3.12] Kan, M. and Taguchi, H.: " **Chaos and Fractals in Non-Linear Roll and Capsize of a Damaged Ship** " - International Workshop on Physical and Mathematical Modelling of Vessel's Stability in a Seaway, OTRADNOYE '93, Kaliningrad - May 1993.

[3.13] Hamamoto, M., Kim, Y. and Matsuda, A.: " **Dynamic Stability of a Ship in Quartering Seas** " - Proceedings of the Seminar on Stability of Ships and Offshore Structures, Pusan, - October 1994.

[3.14] Hamamoto, M., Fujino, M. and Kim, Y.: " **Dynamic Stability of a Ship in Quartering Seas** " - Proceedings of the 5th International Conference on Stability of Ships and Ocean Vehicles STAB '94, Melbourne, Fl., Volume 4 - November 1994.

[3.15] Vermeer, H.: "**Mathematical Modelling of Motions and Damage Stability of RO-RO Ships in the Intermediate Stages of Flooding**" - STAB '94, Melbourne, USA, Nov. 1994.

[3.16] Hua, J. and Rutgersson, O.: "**A Study of the Dynamic Stability of a RO-RO Ship in Waves**", STAB '94, Melbourne, USA, Nov. 1994.

[3.17] Makov, Y. and Sevastianov, N. B.: "**Physical Modelling of Damaged Vessels Capsising under the Influence of Wind and Waves**", The Naval Architect, April 1995.

[3.18] Letizia, L. and Vassalos, D.: "**Formulation of a Non-Linear Mathematical Model for a Damaged Ship Subjected to Progressive Flooding**", International Symposium on Ship Safety in a Seaway, Kaliningrad, Russia, May 1995.

4. Sloshing

[4.1] Goodrich, G. J.: **"Development and Design of Passive Roll Stabilisers"**, RINA Spring Meeting , 1968.

[4.2] Faltinsen, O. M.: " **A Numerical Non-Linear Method of Sloshing in Tanks with Two-Dimensional Flow** " - Journal of Ship Research, Vol. 22, No. 3 - September 1978.

[4.3] Dillingham, J.: " **Motion Studies of a Vessel with Water on Deck** " - Marine Technology, Vol. 18, No. 1, pp. 38-50 - 1981.

[4.4] Çaglayan, I. and Storch, R. L.: " **Stability of Fishing Vessels with Water on Deck: A Review** " - Journal of Ship Research, Vol. 26, No. 2 - June 1982.

[4.5] Adee, B. H. and Çaglayan, I.: " **The Effect of Free Water on Deck on the Motions and Stability of Vessels** " - Proceedings of the 2nd International Conference on Stability of Ships and Ocean Vehicles STAB '82, Tokyo, - October 1982.

[4.6] Vassalos, D., Lee B.S.: " **An Experimental Investigation of ORELIA Double Deck Stabilising Tank** " - Internal Report, University of Strathclyde - 1983.

[4.7] Demirbilek, Z.: " **Energy Dissipation in Sloshing Waves in a Rolling Rectangular Tank - I. Mathematical Theory** " - Ocean Engineering, Vol. 10, No. 5 - 1983.

[4.8] Demirbilek, Z.: " **Energy Dissipation in Sloshing Waves in a Rolling Rectangular Tank - II. Solution Method and Analysis of Numerical Technique** " - Ocean Engineering, Vol. 10, No. 5 - 1983.

[4.9] Demirbilek, Z.: " **Energy Dissipation in Sloshing Waves in a Rolling Rectangular Tank - III. Results and Applications** " - Ocean Engineering, Vol. 10, No. 5 - 1983.

[4.10] Mikelis, N. E. Miller, J. K. and Taylor, K. V.: " **Sloshing in Partially Filled Liquid Tanks and its Effect on Ship Motions: Numerical Simulations and Experimental Verification** " - RINA Trans. - 1984.

[4.11] Hamlin, N. A. et al.: " **Liquid Sloshing in Slack Ship Tanks - Theory, Observations and Experiments** " - Trans. SNAME - 1986.

[4.12] Fenwick, J.: " **An Investigation into Numerical Sloshing Simulations Using the SALE Algorithm** " - Internal Report, University of Strathclyde - 1987.

[4.13] Pantazopoulos, M. S.: " **Three Dimensional Sloshing of Water on Decks** " - Marine Technology, Vol. 25, No. 4, pp. 253-261 - 1988.

[4.14] Mizoguchi, S.: " **Analysis of Shipping Water with the Experiments and the Numerical Calculations** " - J.S.N.A., Japan, Vol. 163 - 1988.

[4.15] Rakitin, V. Natchev, R. and Tzetanov, T.: "**Series Stand Tests with Passive Stabilising Tanks**", STAB '90 Naples, September 1990.

[4.16] Francescutto, A. and Armenio, V.: "**On the Stability of Antisymmetric Motions of a Ship Equipped with Passive Antirolling Tanks**", STAB '90 Naples, September 1990.

[4.17] Pantazopulos, M. S.: "**Sloshing of Water on Deck of Small Vessels**", STAB '90 Naples, September 1990.

[4.18] Cardo, A. et al.: "**Experimental Study of the Effect of Water on Deck on the Stability of a Fishing Vessel**", NAV '92, Genoa, July 1992.

[4.19] Armenio, V.: " **Comportamento Dinamico dei Liquidi a Superficie Libera Imbarcati a Bordo delle Navi** " - Ph.D. Thesis, Universitá di Napoli, Universitá di Trieste - 1992. - (ital.)

[4.20] Contento, G.: " **Liquid Sloshing in a Rectangular Tank in Roll Motion: A Comparison between Numerical and Experimental Results** " - Internal Report No. 5, Universitá di Trieste - 1992.

[4.21] Armenio, V. et al.: " **Time Domain Simulation of Roll Motion of a Ship with Liquids with Free Surface** " - Design of Marine and Offshore Structures, Computational Mechanics Publications - 1992.

[4.22] Armenio, V.: " **Numerical Simulation of Large Amplitude Sloshing of a Viscous Liquid in Rectangular Containers** " - AIMETA'93, Settimo Convegno Italiano di Meccanica Computazionale - 1993.

[4.23] Armenio, V. et al.: " **Dynanic Effects of Liquids On Board on the Stability of a Fishing Vessel** " - IMAM'93, Varna - 1993.

[4.24] Francescutto, A. and Contento, G.: " **An Experimental Study of the Coupling between Roll Motion and Sloshing in a Compartment** " - ISOPE'94, Osaka, No. 94 -YI1-4 - 1994.

[4.25] Rakhmanin, N. and Zhivitsa, S.: "**Prediction of Motion of Ships with Flooded Compartments in a Saeway**", STAB '94, Melbourne, USA, Nov. 1994.

[4.26] Lee, A. and Adee, B.: "**Numerical Analysis of Vessel's Dynamic Responses with Water Trapped on Deck**", STAB '94, Melbourne, USA, Nov. 1994.

[4.27] Amagai, K. Kimura, N. and Ueno, K.: " **On the Practical Evaluation of Shallow Water Effect in Large Inclinations for Small Fishing Boats** " - STAB '94, Melbourne USA - 7-11 November 1994.

[4.28] Armenio, V. et al.: " **Numerical Prediction of Roll Motion of a Ship with Liquids on Board in Regular Waves from Different Directions** " - STAB '94, Melbourne USA - 7-11 November 1994.

5. Sea Wave Theory and First Order Wave Forces

[5.1] Lamb, H.: " **Hydrodynamics** " - Cambridge University Press, No. 60 - 1932.

[5.2] Michel, W. H.: " **Sea Spectra Simplified** " - Trans. SNAME - April 1967.

[5.3] Garrison, C. J. and Seetharama, R.V.: " **Interaction of Waves with Submerged Objects** " - Journal of Waterways, Harbors and Coastal Engineering Division A.S.C.E., Vol. 97, pp. 259-277 - May 1971.

[5.3] Garrison, C. J. and Chow, P. Y.: " **Wave Forces on Submerged Bodies** " - Journal of Waterways, Harbors and Coastal Engineering Division A.S.C.E., pp. 375-392 - August 1972.

[5.4] Hallam, M. G. Heaf, N. J. and Wootton, L. R.: " **Dynamics of Marine Structures: methods of calculating the dynamic response of fixed structures subject to wave and current action** " - CIRIA Underwater Engineering Group, Report UR 8 - October 1978.

[5.5] Newland, D. E.: " **Random Vibrations and Spectral Analysis** " - Longman - 1984.

[5.6] Hogben, N. Dacuna, N. M. C. and Olliver, G. F.: " **Global Wave Statistics** " - BMT Ltd. - 1986.

[5.7] Pizer, D.: " **DWAVE User Manual** " - Strathclyde University, Dept. of Ship and Marine Technology - 1987.

[5.8] de Kat, J. O. and Paulling, J. R.: " **The Simulation of Ship Motions and Capsizing in Severe Seas** " - Trans. SNAME, vol. 97 - 1989.

[5.9] Naito, S.: " **Generation of an Irregular Wave and its Randomness** " - Seminar on Stability of Ships and Offshore Sructures, Pusan - October 1993.

[5.10] Comlekci, T.: " **SECTR and SMTR Quick Reference - V2.0** " - University of Strathclyde - 1995.

6. Second Order Wave Drift Forces

[6.1] Newman, J. N.: " **The Drift Force and Moment on Ships in Waves** " - Journal of Ship Research, pp. 51-60 - March 1967.

[6.2] Hsu, F. H. and Blenkarn, K. A.: " **Analysis of Peak Mooring Force Caused by Slow Vessel Drift Oscillation in Random Sea** " - Proceedings of the 2nd OTC, paper n° 1159, pp. I 135-146 - 1970

[6.3] Faltinsen, O. M. and Løken, A. E.: " **Drift Forces and Slowly Varying Forces on Ships and Offshore Structures in Waves** " - Norwegian Maritime Research N° 1, pp. 2-6 - 1978.

[6.4] Chakrabarti, S. K.: " **Steady and Oscillating Drift Forces on Floating Objects** " -Journal of the Waterway, Port, Coastal and Ocean Division, pp. 205-228 - May 1980.

[6.5] Ogilvie, F. T.: " **Second Order Hydrodynamic Effects on Ocean Platforms** " -Proceedings of the International Workshop on Ship and Platform Motions, University of California, Berkeley - 1983.

[6.6] Faltinsen, O. M. and Zhao, R.: " **Slow Drift Motions of a Moored Two Dimensional Body in Irregular Waves** " - Journal of Ship Research, pp. 93-106 - June 1989.

[6.7] Gatiganti, R. M.: " **Extimation of Second Order Wave *Drift Forces*** " - Department of Ship and Marine Technology, University of Strathclyde - *1993*.

[6.8] Takagi, K.: " **Wave Induced Steady *Lateral* Force and *Turning* Momenton a Ship in Waves** " - *Seminar* on Stability of Ships *and* Offshore Sructures, Pusan - October 1993.

7. Wind Forces

[7.1] Isherwood, R. M.: " **Wind Resistance of Merchant Ships** " - Trans. RINA - 1972.

[7.2] Blendermann, W.: " **Wind Loads on Moored and Manoeuvring Vessels** " - 12th International Symposium on Offshore Mechanics and Artic Engineering, Glasgow, Vol. 1, pp. 183-189 - 1993.

[7.3] McTaggart, K. and Savage, M.: " **Wind Heeling Loads on a Naval Frigate** " - STAB '94, Melbourne USA - 7-11 November 1994.

8. Current Forces

[8.1] *English, J. W. and Wise, D. A.:* " ***Hydrodynamic* Aspects of Dynamic Positioning** " - Joint Meeting of RINA, pp. 53-72 - December 1975.

[8.2] Morris, C.: " **Methods for Initial Extimation of Vessel Hydrodynamic and Aerodynamic Loads** " - Department of Ship and Marine Technology, University of Strathclyde - 1986.

9. Gravitational and Restoring Forces

[9.1] Di Bella, A.: " **Statica della Nave** " - Pubblicazioni Scientifiche di Ingegneria, Genova - 1965 - (ital.).

[9.2] " **Strathclyde University Ship Stability - User Manual: Version 1.0** " - University of Strathclyde - 1988.

10. Hydrodynamic Reaction Forces

[10.1] Tick, L. T.: " **Differential Equations with Frequency Dependent Coefficients** " - Journal of Ship Research, Vol. 3, No. 2 - 1959.

[10.2] Cummins, W. E.: " **The Impulse Response Function and Ship Motions** " - Schiffstechnik, Bd. 9, Heft 47 - June 1962.

[10.3] Ogilvie, T. F.: " **Recent Progress Toward the Understanding and Prediction of Ship Motion** " - Proc. of the Fifth Symposium on Naval Hydrodynamics, Bergen - September 1964.

[10.4] Kotic, J. and Lurye, J.: " **Some Topics in the Theory of Coupled Ship Motions** " - Proc. of the Fifth Symposium on Naval Hydrodynamics, Bergen - September 1964.

[10.5] Timman, R.: " **Applied Mathematics in Ship Hydrodynamics** " - Schiffstechnik - 1973.

[10.6] Perez y Perez, L.: " **A Time Domain Solution to the Motions of a Steered Ship in Waves** " - Journal of Ship Technology, Vol. 18, No. 1 - 1974.

[10.7] Himeno, Y.: " **Prediction of Ship Roll Damping - State of the Art** " - Dept. of Naval Architecture and Marine Engineering, University of Michigan Report No. 239, Ann Arbor, Mich. - 1981.

[10.8] Clarke, D. Gedling, P. and Hine, G.: " **The Application of Manouvering Criteria in Hull Design using Linear Theory** " - Trans. RINA, Vol. 125 - 1983.

[10.9] Maimun, A.: " **Stability of Fishing Vessels in an Astern Sea Shallow Water Environment** " - Ph.D. Thesis, Department of Ship and Marine Technology, University of Strathclyde - June 1993.

[10.10]Jo, H. J. and Kim B. W.: " **Non-Linear Response of a Semi-Submersible with Non-Linear Restoring Forces** " - Seminar on Stability of Ships and Offshore Sructures, Pusan - October 1993.

[10.11]Lee, S. K. Hasegawa, K. and Kim, S. J.: " **The Calculation of Zig-Zag Maneuver in Regular Waves with Use of the Impulse Response Functions** " - Seminar on Stability of Ships and Offshore Sructures, Pusan - October 1993.

[10.12]Vassalos, D.: " **Development of Downflooding and Survival Criteria for High Speed Twin Hull Craft** " - Final Report on the HSTHC Stability Programme for the UK Department of Transport, Marine Directorate - Dicember 1993.

[10.13]Hua, J. and Palmquist, M.: " **A Description of SMS - A Computer Code for Ship Motion Simulation** " - The Royal Inst. of Technology (KTH) Div. of Naval Architecture - 1995.

11. Flooding

[11.1] Vennard, J. K. and Street, R. L.: " **Elementary Fluids Mechanics** " - John Wiley and Sons. - 1975.

[11.2] Giles, R. V.: " **Meccanica dei Fluidi e Idraulica** " - Collana Shaum, Gruppo Editoriale Fabbri - 1975 - (ital.).

[11.3] Hughes, W. F. and Brighton, J. A.: " **Fluidodinamica** " - Collana Shaum, Gruppo Editoriale Fabbri - 1978 - (ital.).

[11.4] Ackers, P. et al.: " **Weirs and Flumes for Flow Measurements** " - John Wiley and Sons. - 1978.

[11.5] Marchi, E. and Rubatta, A.: " **Meccanica dei Fluidi** " - UTET - 1981 - (ital.).

[11.6] Featherstone, R. E. and Nalluri, C.: " **Civil Engineering Hydraulics** " - Granada - 1982.

[11.7] Letizia, L.: " **An Experimental Investigation into the Rate of Flooding of a Damaged Hull** " - Department of Ship and Marine Technology, University of Strathclyde - 1993.

[11.8] The Glosten Associates, inc.: " **Water Accumulation on the Deck of a Stationary Ship** " - SNAME Ad Hoc RO-RO Safety Panel, Annex A, File No. 94209.01 - 8 February 1995.

12. Structural Resistance, Cargo Shifting and Damage Statistics

[12.1] Andersson, P. Allenström, B. and Niileksela, M.: **"Safe Stowage and Securing of Cargo on Board Ships"**, Mariterm AB 1982, Göteborg, 1982.

[12.1] Spounge, J. R.: **"The Safety of RO-RO Passenger Ferries"**, RINA Spring Meeting, June 1988.

[12.2] Aldwinckle, D. S. and Prentice, D.: **"The Safety Record and Risk Analysis Of Ro-Ro Passenger Ferries"**, International Symposium on the Safety of Ro-Ro Passenger Ships, RINA, April 1990.

[12.3] Sen, P. and Cocks, J. A.: **"Collision Resistance"**, International Symposium on the Safety of Ro-Ro Passenger Ships, RINA, April 1990.

[12.4] Abicht, W.: **"The Probability of Compartment and Wing Compartment Flooding in the Case of Side Damage - New Formulae for Practical Application"**, STAB '90 Naples, September 1990.

[12.5] Sen, P. Cocks, J. A. and Pawlowski, M.: **"Integrated Collision Analysis for Ships"**, The Second H. Kummerman Foundation International Conference on Ro-Ro Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

[12.6] Paik, J. K. and Pedersen, P. T.: **"Modelling the Internal Mechanics in Ship Collision"**, Technical University of Denmark, June 1994.

[12.7] Pedersen, P. T.: **" Ship grounding and Hull-Girder Strength "** - Marine Structures, Elsevier Science Limited, England - 1994.

[12.8] Pedersen, P. T. and Simonsen, B. C.: **" Dynamics of Ship Running Aground "** - Technical University of Denmark - 1994.

[12.9] Pedersen, P. T.: " **Collision and Grounding Mechanics** ", WEMT '95, Ship Safety and Protection of the Environment, May 1995.

13. Numerical Methods, Mathematics and Computing Science

[13.1] Jerri, A. J.: " **Introduction to Integral Equations with Applications** " - Dekker - 1985.

[13.2] Linz, P.: " **Analytical and Numerical Methods for Volterra Equations** " - SIAM - 1985.

[13.3] Price, W. J., Twizell, E. H. and Wang, Y.: " **The Application of Padé Approximant in the Time Domain Simulation of the Non-Linear Motion of a Semi-Submersible Excited by Wave** " - Proceedings of PRAD's 87, Trondheim, Norway - June 1987.

[13.4] Haberman, R.: " **Elementary Applied Partial Differential Equations** " - Prentice Hall - 1987.

[13.5] Sloan, D. M.: " **Introduction to Numerical Methods for Students of Science and Engineering** " - University of Strathclyde - 1988.

[13.6] Thomas, G. B. J. and Finney R, L.: " **Calculus and Analytic Geometry** " - Addison Wesley - 1988.

[13.7] Kreyszig, E.: " **Advanced Engineering Mathematics** " - J. Wiley and Sons - 1988.

[13.8] Devaney, R. L.: " **An Introduction to Chaotic Dynamical Systems** " - Addison Wesley - 1989.

[13.9] Grimshaw, R.: " **Nonlinear Ordinary Differential Equations** " - Blackwell Scientific Publications - 1990.

[13.10]Sandefur, J. T.: " **Discrete Dynamical Systems, Theory and Applications** " - OUP - 1990.

[13.11]Press, W. H. Teukolsky, S. A. Vetterling, W. T. and Flannery, B. P.: " **Numerical Recipes in Fortran** " - Cambridge University Press - 1992.

[13.12]Etter, D. M.: " **Structured Fortran 77 for Engineers and Scientists** " - Benjamin/Cummings Publishing Company Inc. - 1993.

[13.13]Burden, R. L. and Faires, J. D.: " **Numerical Analysis** " - PWS Publishing Company - 1993.

[13.14]Hairer, E. Nørsett S. P. and Wanner, G.: " **Solving Ordinary Differential Equations I - Non-Stiff Problems** " - Springler-Verlag - 1993.

[13.15]" **FTN77/486 Reference Manual** " - University of Salford - 1995.

Appendix - A

Building the Database - Users Manual

In this appendix a brief description is given of the procedure to build the data library containing the first order wave forces amplitude and phase operators and the hydrodynamic coefficients, used by the simulation program CASSANDRA. Elements of the theory involved are introduced at each passage, whenever this is necessary to foster the understanding of the programs which generate the information necessary to build this data library. The purpose of each of these programs is explained as their use becomes necessary in the process leading from ship geometry and mass properties data to the final library format. Input, output and some other features of each program are illustrated as well. More detailed information about each program of this suite are given in the references provided.

In the second section of this appendix, details are given on the various systems of reference used throughout this work and on the transformations necessary to express forces and moments in the system of reference adopted for the equations of motion.

Calculation of the Wave Forces Operators and Hydrodynamic Coefficients using a 3D CFD Code.

The information necessary to initiate the process that will produce the database library is contained in two files named *filename.SUS* (ship geometry file) and *filename.CON* (mass properties data file). Examples of these two files are given next; more information can be found in [9.2] and Appendix B (*hellas.sus*):

filename.SUS

```
PRIDE OF BRUGES   CM44606/18/02   (MET)
 126.100   22.700   12.600   1.0000
 22
18  -66.800    0
 0.000   0.200
 0.000   0.500
 0.000   1.000
 0.000   1.500
 0.000   2.000
 0.000   3.000
 0.000   4.000
 0.000   5.000
 0.000   5.600
 0.300   5.605
 2.760   6.000
 4.880   6.750
 6.450   7.500
 6.450   7.924
 6.450   8.000
 6.450  10.050
 6.223  13.400
 0.000  13.400
19  -63.050    0
 0.000   0.200
 0.000   ....

 ....    7.800
 1.580   8.224
 1.580   8.300
 2.500  11.000
 3.500  13.900
 0.000  13.900
```

Damaged compartment section:

```
B
3 3 1 1 1
 4  -1.00
11  -19.00    2
 0.000   1.200
 9.300   1.200
 9.780   1.500
10.270   2.000
10.770   3.000
11.050   4.000
11.210   5.000
11.310   6.000
11.350   6.750
11.350   7.870
```

0.000 7.870
11 -12.61 2
0.000 1.200
10.300 ....

.... 7.870
0.000 7.870

Lateral profile section:

C
37
0.00 0.00
47.72 0.00
63.05 4.65
66.35 6.98
65.77 ....

.... 8.73
-67.51 6.40
-56.45 3.49
-54.70 0.00
0.00 0.00

Lateral profile of superstructure:

D
24
0.00 13.96
46.56 13.96
46.56 18.91
41.32 18.91
41.32 22.40
35.50 22.40
36.37 ....

.... 17.46
-62.27 13.96
0.00 13.96

Transverse profile section:

E
0.00
50
0.000 0.000
7.800 0.199
7.830 ....

.... 0.200
-7.800 0.199
0.000 0.000

filename.CON

-3.48 0.0 8.9 8807.0 5.625

```
0.0  0.0  0.0   0.0
0.0  0.0 270.0  34.67 0.0
47.23  0.0  10    10   0.0
1   0.0  5.5  0.35  20.0
5.625 5.625 0.0
0.0  1
```

These file formats are ready to be used by the stability program SUSS but the geometry definition of the hull needs to be processed before it can be used by the CFD program DWAVE. To this purpose, it is in fact necessary to transform the sectional definition of the vessel in an array of connected panels covering the hull. This task can be achieved as follows.

1. SECTR

This program (SECtion TRanslator) converts a standard ship geometry file (*filename.SUS* or *filename.SEC*) in file formats that can be read either by AutoCAD® or MultiSURF® (*filename.DXF* or *filename.MSF*). The latter programs can be employed to define the complete array of panels (see fig. 2 in Chapter 6) which will then be used by a third program (SMTR) to prepare the geometry files utilised by DWAVE. In this appendix the panellisation of the hull is described as achieved by utilising MSURF.

The geometry file *filename.SUS* describes the hull through transverse sections which refer to a system of reference ($o\,xyz$) centred at the intersection between the keel and the centre plane, with its axis oriented so that x is positive forward, y is positive to port and z is positive upward (fig. 2). In the format conversion the co-ordinate system will not be changed. This implies that the *filename.MSF* model will also refer to $o\,xyz$ and so will the subsequently created panels.

The list of commands to input in SECTR is:

```
def
dir \SECTRworkdirectory
read filename sus
ssel all
vsel y0cut
write filename msf
```

SECTR and SMTR are both compiled to run in a Microsoft® Windows environment. This allow the previous list of commands to be cut and paste from any Windows editor to the program window. The result will be a sequential and uninterrupted execution of the list. Although the advantages of this procedure can be unclear in the case of SECTR, they become evident when the list of directions is much longer as in the case of SMTR. Note that *filename.SUS* must be in \SECTRworkdirectory\. The output file *filename.MSF* will also be in that directory. It must be moved or copied to C:\MSURF\ before editing. Once this has been done, MSURF can be run.

2. MultiSURF

Unlike SECTR, MSURF can only be run from MS-DOS®. It is advisable to change directory to C:\MSURF\ not to have problems with the editing of files. Once the program is running, *filename.MSF* can be loaded. The file created by SECTR contains the complete hull surface defined from the first to the last section as well as all the points and sections necessary to define it and the ship's lateral profile for visual reference. The definition of the bow and transom surfaces is omitted. These must be therefore added manually within MSURF (see reference manual). Once this has been accomplished, the symmetric part of the hull is created automatically by pressing F5. Note that for a consistent orientation of the panels, it is necessary to ensure that the newly created surfaces - bow and transom - are oriented in the same way as the hull. It is possible to check the orientation of each surfaces within the window options of MSURF. Also, the number of transverse division of the three surfaces has to be the same for all of them to ensure continuity between each two consecutive parts of the hull.

The main purpose of MSURF - aside from adding stern and bow to the hull geometry - is to define the panels array. Again this task is automatically achieved by saving the model as a 3D *filename.PAT* file. The format of this file can then be converted into a *filename.DXF* file by the MSURF suite program PAT2DXF. Finally *filename.DXF* - which contains the master model of the hull panellisation - is ready for the program SMTR. Before this program is employed, though, it is advisable to run SUSS in order to identify the positions that the ship CG will assume in the system of reference $o\ xyz$ as the heel angle varies, as well as the values of displacement for each draught considered.

3. SUSS

The purpose of running SUSS (Strathclyde University Ship Stability) is to convert the information contained in *filename.CON* about the mass distribution of the vessel, in that set of data included in the control file used by DWAVE (*filename.GIN*), which change with heel and draught. The format of the general information file (*filename.GIN*) is described below. All the values of the static stability related parameters contained in this file refer to the particular condition the ship has reached. For instance, if the ship is upright but its draught is larger than that of the intact position, the mass M will be larger and the values of XGG, YGG, ZGG, IXX, IYY, IZZ, WPA, GMR and GMP will - in general - be different. If the vessel is heeled and trimmed, these differences are even more evident. The first consequence of this observation is that there will be as many *filename.GIN* files as the number of combinations of different values of heel, trim and draught. To simplify this scenario, only heel and draught variation are taken into account in this respect, assuming that the variation in trim will not affect sensibly the values of M, XGG etc.

SUSS needs both *filename.SUS* and *filename.CON* as input files. By choosing option 5 the upright hydrostatics (*filename.HYD*) will be output for the range of draughts defined in *filename.CON.* If option 9 is chosen, the program will balance the ship according to the position of the centre of gravity and the displacement given, and the *filename.HYD* will thus contain information on heeled hydrostatic data this time. When running option 9, it is advisable to let the free trim switch in *filename.CON* equal to 1 (free trim hydrostatics) in order to be able to trace the right displacements.

Within SUSS, the position of the centre if gravity for each of the various heel angles considered, is found by iteration, i.e. by moving CG in the transverse plane until the wanted value of $\hat{\varphi}$ has been reached. For practical reasons, the position of CG is changed just by increasing the value of YGG in filename.CON. This methodology deserves a justification. In principle the position of the centre of gravity cannot be completely defined by a heel angle only. This will dictate the position of the centre of buoyancy and the CG will necessarily have to lay on the same vertical passing through the CB - if equilibrium is granted - but its location along this line can vary arbitrarily (for instance it will depend on the geometry and position of the flooded compartments in case of damage). The intersection of this vertical line with the plane passing through the initial CG and parallel to oxy is just a first approximation of where the new centre of gravity will be. Given the multitude of cases which would lead to the same underwater geometry this is deemed the simplest and most general but realistic assumption.

The variation of the quantities of interest as a function of draught is ascertained in a similar way. The displacement at each value of T is evaluated by choosing option 5. This is then input into a new *filename.CON* and option 9 is run to find all the requested parameters for the various heel angles.

The stability program will output the water plane area, GMT and GML (i.e. GMR and GMP). The mass moment of inertia need to be approximated by the formulae:

$$IXX_{heeled} = IXX_{upright} + M\ YGG_{heeled}{}^{2}$$

and

$$I_{new\ draught} = I_{old\ draught} \left({M_{new\ draught}} \middle/ {M_{old\ draught}} \right).$$

The first of the formulae above expresses the variation of the mass moment of inertia with respect to the x axis passing through the initial CG, with the heel angle only. The values of IYY and IZZ can be considered approximately invariant with $\hat{\varphi}$. It is also assumed that the intrinsic moment of inertia with respect to the new position of the centre of gravity is roughly equal to that pertaining to the old (upright) one. The second equation gives the variation of any of IXX, IYY or IZZ with the displacement only. It is recommended to calculate this at the upright position and then work out the heeled IXX in relation to the new displacement using the first formula.

When all the parameters of interest have been found as outlined above, the set of *filename.GIN* files can finally be edited manually. Further information on SUSS can be found in [9.2].

4. SMTR

Once the complete panellisation of the hull is available, the underwater portions resulting from the intersection of the calm water plane with the ship geometry must be specified for each combination of $\hat{\varphi}$, $\hat{\vartheta}$ and $\hat{z}$. To achieve this task, a program called SMTR (Surface Model TRanslator) was written, which rotates, translates and cuts the hull described in the file containing the complete geometrical definition of the panels as output by ACAD or MSURF. SMTR finally produces a new geometry file and output in a *filename.DXF* or *filename.H3D* format.

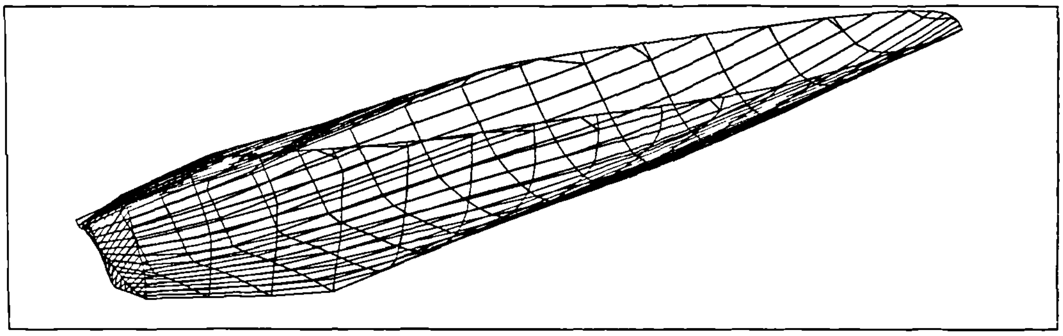

Fig. 1

Panels covering the underwater body for a heel angle of 30 deg., a trim angle of 10 deg. and a draught of 5 m. These values are unrealistically high and are adopted here only to produce a picture where the effect of rotations are easily perceived.

The first is used to visually check the new geometry on ACAD (fig. 1), whilst the latter will be one of the input files used by DWAVE to produce all the wave force amplitudes and hydrodynamic coefficients necessary to build the database.

SMTR is a very flexible tool which allows for any sequence of translations and rotations around the centre of a fixed system of reference. This has its centre lying on the still water plane and has its z axis pointing upwards. Initially, this system is defined so that it coincides with the reference used to define the ship sections (see *filename.SUS*). It must be noted though, that after each transformation these two reference systems will not overlay each other any longer. As a result of this, attention must be paid to the position acquired by the centre of rotation assumed by the program (centre of $o_d\ x_d y_d z_d$ - see fig. 2) with respect to the centre of gravity of the ship, before any further rotation is performed.

An additional point concerns the sequence of rotations. In order to comply with the convention used in the derivation of the equations of motion, the series of rotations must be heel first followed by trim. In brief, the whole sequence of transformations can be described as:

- A series of translations to bring the initial CG of the ship to coincide with the centre of the fixed system of reference;
- A rotation around the x axis, representing the heel angle;
- A rotation around the y axis, representing the trim angle;
- A series of translations to bring the CG of the ship back to its initial position;
- A final translation in the negative z direction representing the final draught (initial draught plus sinkage).

This translates in terms of directions to input into the program as:

```
def
trident

dir \luca\tugrul\work
acws
```

```
read mbru075m dxf
elinvert all

trident
point -3.48 0 8.9
translate 3.48 0 -8.9
heel 0
trim 0
translate -3.48 0 8.9
draught 5.625
trsurface
cutsurface
trpoint
write p00p0s00 h3d cut
```

producing the following output on screen:

```
SURFACE MODEL TRANSLATOR V2.0 (1.2.1995)

SMTR> def
default settings chosen
SMTR> trident

SMTR> dir \luca\tugrul\work
the directory for files is: \luca\tugrul\work\
SMTR> acws
anticlockwise heel and trim transformations set

SMTR> read mbru075m dxf
read file: \luca\tugrul\work\mbru075m.dxf
  6 POLYLINEs are read
 432 nodes are generated
 860 elements are generated
total surface area: 7421.470703
SMTR> elinvert all

SMTR> trident
SMTR> point -3.48 0 8.9
P0 (x,y,z) =   -3.480000,    0.000000,    8.900000
SMTR> translate 3.48 0 -8.9
translate (dx,dy,dz) =    3.480000,    0.000000,   -8.900000
SMTR> heel 0
heel =    0.000000
SMTR> trim 0
trim =    0.000000
```

```
SMTR> translate -3.48 0 8.9
translate (dx,dy,dz) =   -3.480000,    0.000000,    8.900000
SMTR> draught 5.625
draught =    5.625000
SMTR> trsurface
transforming node coordinates ... finished.
total transformed model surface area: 7421.470703
SMTR> cutsurface
532 elements above z=0
  92 elements intersected by z=0
 236 elements below z=0
cut model has  227 nodes and  364 elements
total cut model surface area: 2727.759521
total cut model volume: 8334.172852
SMTR> trpoint
P0 (x,y,z) =   -3.480000,    0.000000,    8.900000
P1 (x,y,z) =   -3.480000,    0.000000,    3.275000
SMTR> write p00p0s00 h3d cut
write file: \luca\tugrul\work\p00p0s00.h3d
```

The list of commands above is complete for only one value of draught, heel and trim. Indeed, similar commands have to be executed for each combination of the parameters in question. Note that only positive heel angles should be taken into account. This is to limit the computation time required. In fact, it must be observed that forces and moments for opposite heel angles only differ in sign from those calculated for angles of encounter which are symmetrical with respect to the $o_d\ x_d z_d$ plane. This allow us to exclude all these cases from the work load of DWAVE. Eventually, the missing data is automatically created by CASSANDRA.

It can be observed as well, that all the rotations are performed around the centre of the system of reference of the equations of motions - i.e. the centre of gravity of the intact ship - thus the position of the centre of gravity corresponding to each heeled position will be generally transformed accordingly. In order to input the correct co-ordinates of the position of the latter CG in the corresponding *filename.GIN* file, their expression with respect to the ship geometry system of reference $o\ xyz$ must be translated to that referring to $o_d\ x_d y_d z_d$. Once the first values have been established by the stability program SUSS, the task of translating them to the second can be achieved within SMTR and the resulting data input manually in the *filename.GIN* files. The transformed co-ordinates of the CG are those of point P1 (x,y,z) in the screen output of SMTR. Clearly, given this sequence of operations to attain

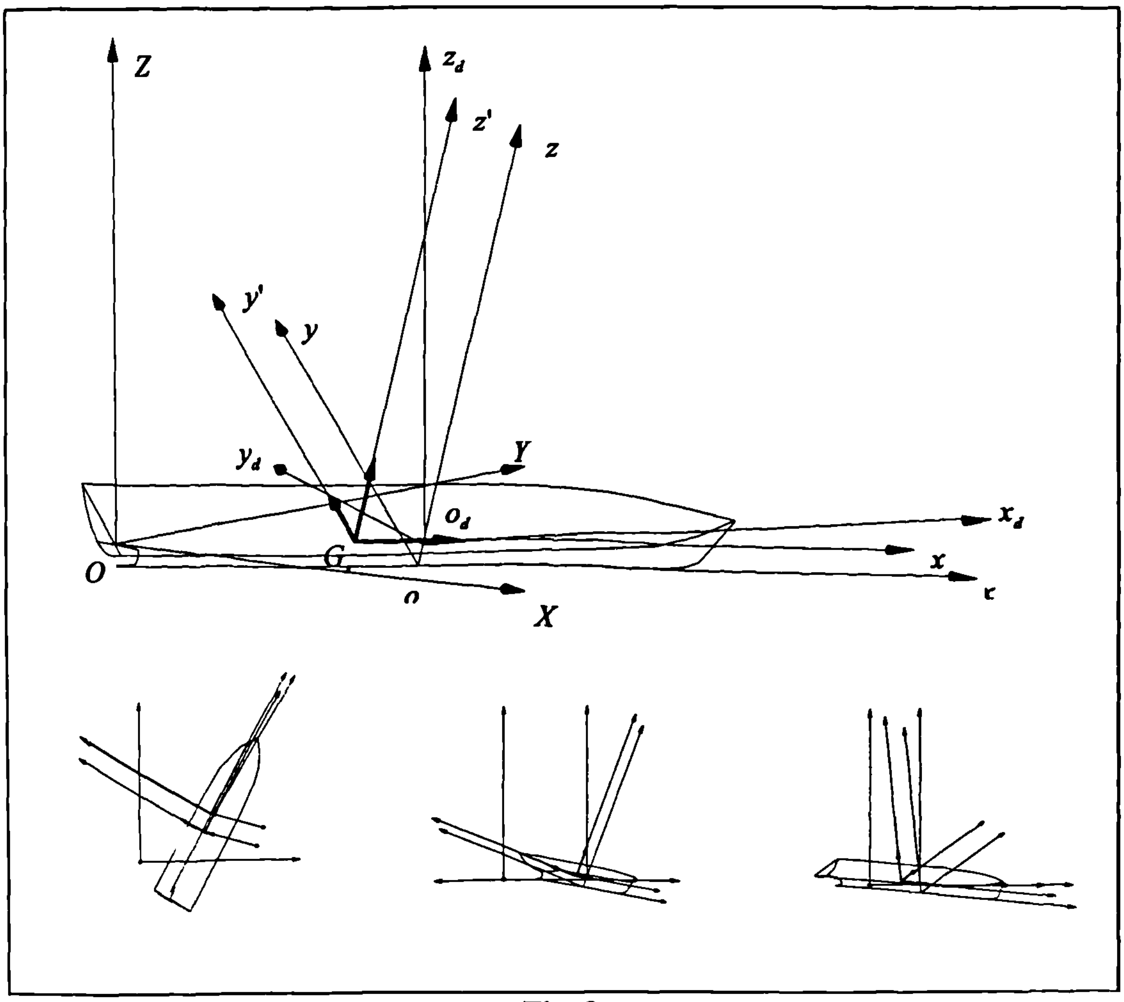

Fig. 2

All systems of reference employed, for heel=20 deg., trim=5 deg. and draught=3 m. The vessel is supposed to have drifted by 30 m. in the Y direction and 50 m. in the X direction from the origin of $O\ XYZ$. The heading reached is 60 deg. The three images at the bottom show orthogonal projections on the co-ordinate planes of the earth-fixed system $O\ XYZ$.

the final expression of XGG, YGG etc., it is necessary to run the stability program before SMTR. Also, attention must be paid to match the right sign of YGG with that of the corresponding heel angle (for positive heel angles, the sign of YGG must be negative).

For further information regarding SECTR and SMTR see [5.10].

5. DWAVE

DWAVE is a 3D CFD program developed at the Department of Ship and Marine Technology of the University of Strathclyde to calculate the first order regular wave forces and moments acting on an arbitrary shaped body. Following the general theory outlined in first section of Chapter 6, this code produces also hydrodynamic coefficients - including coupling terms - assuming unit motion.

The two input files needed to run this program essentially contain information about the ship underwater geometry (*filename.H3D*) and general information including the mass properties of the floating body and some control parameters (*filename.GIN*). The making of the first of these files has been already discussed in detail. The general information file contains the array of frequencies and ship headings used by DWAVE to determine forces, moments and hydrodynamic coefficients. Its format is the following:

filename.GIN

```
-3.48 0.0 3.28                         XGG YGG ZGG
0                                      ISYM
0.0 0.0 0.0                            XTRANS YTRANS ZTRANS
250.                                   D
3                                      IHYD
5                                      NWAVE
  0.2 0.7 1.2 1.7 2.2                    WAVE PARAMETERS
8                                      NWAVEANGLES
  0. 45. 90. 135. 180. 225. 270. 315.    WAVE ANGLES
8807000.                               M
800531000. 1.04E10 1.04E10             IXX IYY IZZ
2199.                                  WPA
2.90 230.4                             GMR GMP
                                       (EOF)
```

where

XGG YGG ZGG — are the co-ordinates of the centre of gravity of the ship in metres;

ISYM	is the longitudinal symmetry switch: it is 1 for symmetrical underwater geometry and 0 for asymmetrical ones. Note that the symmetry plane is parallel to the plane containing the z_d and x_d axis (see fig. 2). This variable must be 0 in this context;
XTRANS YTRANS ZTRANS	are the co-ordinates of the centre of symmetry in *o xyz* co-ordinates. They are usually set to zero;
D	is the water depth in metres;
IHYD	is the switch defining which parameter will be used to describe the frequency of excitation. It can assume the following values: 1 wave period (secs) 2 wave frequency (Hz) 3 circular frequency (rads/sec) 4 deep water wave number (m^{-1}) 5 wave length (m) 6 shallow water wave number (m^{-1}) For the purpose of building the library in question, this must be set to 3.
NWAVE	is the number of wave parameters;
WAVE PARAMETERS	is the list of wave parameters;
NWAVEANGLES	is the number of wave encounter angles;
WAVE ANGLES	is the list of wave encounter angles in degrees;

M is the ship's displacement in kg;

IXX IYY IZZ are the ship's moments of inertia in kg m^2;

WPA is the water plane area (m^2);

GMR GMP are the transverse and longitudinal GM co-ordinates respectively (GMT and GML, in metres).

In the example above, all numerical data refer to an intact ship at the upright condition.

As already mentioned, unless stated differently, all the co-ordinates above refer to the system of reference used by SMTR ($o_d\ x_d y_d z_d$). The convention for the wave encounter angles as input in the *filename.GIN* files is such that the following relationship holds between the values of the ship heading angle $\hat{\psi}$ and those of the wave encounter angle α_w:

$$\begin{cases} \alpha_w = -\hat{\psi} \\ 0 \le \alpha_w < 2\pi \end{cases}.$$

A brief comment is finally necessary about the definition of the phase angle between the wave elevation and the wave force β. Due to a slightly different notation for the wave potential, this is obtained as:

$$\begin{cases} \beta = \arctan\left(\left(-\mathrm{Re}[H(\omega)] \right) \big/ \mathrm{Im}[H(\omega)] \right) \\ 0 \le \beta < 2\pi \end{cases}$$

which differs from what usually agreed as a convention. This angle is output together with the real and imaginary parts of the wave force and its amplitude by the modified version of DWAVE employed for this investigation.

A more complete description of DWAVE can be found in [5.7].

6. O2D and database library structure

The program O2D (Output to Database) transfers the data contained in the DWAVE output to the set of files embodying the database library. The execution of this program is straight forward and will be disregarded but some comments are necessary to explain the structure of the database library and how this can be used by the time simulation program.

The database library consists of a certain number of files containing the first order wave forces RAOs, phase angles, added mass and damping coefficients. To reduce the size of each single file, their filename choice is restricted to a standard stem used to identify which particular ship attitude the data contained in the file correspond to and a fixed extension indicating if the data regards RAOs, phase angles, added mass or damping coefficients. This standard (as far as the stem is concerned) has to be introduced during the execution of SMTR and the identification of the particular vessel the library refers to, is taken care of by the subdirectory name. For instance: P10N2S15.OUT in subdirectory \BRUGES\ contains the output of DWAVE for the vessel "Pride of Bruges" at (Positive 10) 10 degrees of heel, (Negative 2) -2 degrees trim and (Sinkage 15) 1.5 metres sinkage, whilst P20P2S00.AM in the same directory contains the added mass matrices for the same vessel at 20 degrees of heel, 2 degrees trim and 0.0 metres sinkage.

The extensions corresponding to RAOs, phase angles, added mass and damping coefficients are .WF, .PH, .AM and .DA, respectively. The format of each of these files is given next:

filename.AM

```
FREQ(I)
AM(1,1,I), AM(1,2,I), AM(1,3,I), AM(1,4,I), AM(1,5,I), AM(1,6,I)
AM(2,1,I), AM(2,2,I), AM(2,3,I), AM(2,4,I), AM(2,5,I), AM(2,6,I)
AM(3,1,I), AM(3,2,I), AM(3,3,I), AM(3,4,I), AM(3,5,I), AM(3,6,I)
AM(4,1,I), AM(4,2,I), AM(4,3,I), AM(4,4,I), AM(4,5,I), AM(4,6,I)
AM(5,1,I), AM(5,2,I), AM(5,3,I), AM(5,4,I), AM(5,5,I), AM(5,6,I)
AM(6,1,I), AM(6,2,I), AM(6,3,I), AM(6,4,I), AM(6,5,I), AM(6,6,I)
FREQ(I+1)
AM(1,1,I+1), AM(1,2,I+1), AM(1,3,I+1), ...
AM(2,1,I+1), ...
...
```

filename.DA

```
FREQ(I)
DA(1,1,I), DA(1,2,I), DA(1,3,I), DA(1,4,I), DA(1,5,I), DA(1,6,I)
DA(2,1,I), DA(2,2,I), DA(2,3,I), DA(2,4,I), DA(2,5,I), DA(2,6,I)
DA(3,1,I), DA(3,2,I), DA(3,3,I), DA(3,4,I), DA(3,5,I), DA(3,6,I)
DA(4,1,I), DA(4,2,I), DA(4,3,I), DA(4,4,I), DA(4,5,I), DA(4,6,I)
DA(5,1,I), DA(5,2,I), DA(5,3,I), DA(5,4,I), DA(5,5,I), DA(5,6,I)
DA(6,1,I), DA(6,2,I), DA(6,3,I), DA(6,4,I), DA(6,5,I), DA(6,6,I)
FREQ(I+1)
DA(1,1,I+1), DA(1,2,I+1), DA(1,3,I+1), ...
DA(2,1,I+1), ...
...
```

filename.WF

```
FREQ(I)
WAVE_ANGLE(J)
WF(1,I,J)
WF(2,I,J)
WF(3,I,J)
WF(4,I,J)
WF(5,I,J)
WF(6,I,J)
WAVE_ANGLE(J+1)
WF(1,I,J+1)
WF(2,I,J+1)
...
FREQ(I+1)
WAVE_ANGLE(J)
WF(1,I+1,J)
WF(2,I+1,J)
WF(3,I+1,J)
WF(4,I+1,J)
WF(5,I+1,J)
WF(6,I+1,J)
WAVE_ANGLE(J+1)
WF(1,I+1,J+1)
WF(2,I+1,J+1)
....
```

filename.PH

```
FREQ(I)
WAVE_ANGLE(J)
PH(1,I,J)
PH(2,I,J)
PH(3,I,J)
PH(4,I,J)
PH(5,I,J)
PH(6,I,J)
WAVE_ANGLE(J+1)
PH(1,I,J+1)
PH(2,I,J+1)
...
FREQ(I+1)
WAVE_ANGLE(J)
PH(1,I+1,J)
PH(2,I+1,J)
PH(3,I+1,J)
PH(4,I+1,J)
PH(5,I+1,J)
PH(6,I+1,J)
WAVE_ANGLE(J+1)
PH(1,I+1,J+1)
PH(2,I+1,J+1)
....
```

where the meaning of the variables involved is trivial.

Note that all the values for heel, trim, sinkage and wave angle are as input in SMTR (the list of all the values assumed by these parameters to complete the library is given in Chapter 6).

Systems of Reference and Forces and Moments Transformations

In the derivation of the equations of motion a ship-fixed system of reference ($Gs\,x'\,y'\,z'$) is adopted, which is linked to the earth-fixed system through the relations:

$$(\vec{x})_{O\,XYZ} = (\vec{x}_{Gs})_{O\,XYZ} + [D]^T (\vec{x}')_{Gs\,x'y'z'}$$

$$(\vec{x}')_{Gs\,x'y'z'} = [D]\left((\vec{x})_{O\,XYZ} - (\vec{x}_{Gs})_{O\,XYZ}\right)$$

where $[D]$ is the matrix:

$$[D] = \begin{bmatrix} \cos\vartheta\cdot\cos\psi - \sin\varphi\cdot\sin\vartheta\cdot\sin\psi & \cos\vartheta\cdot\sin\psi + \sin\varphi\cdot\sin\vartheta\cdot\cos\psi & -\cos\varphi\sin\vartheta \\ -\cos\varphi\cdot\sin\psi & \cos\varphi\cdot\cos\psi & \sin\varphi \\ \sin\vartheta\cdot\cos\psi + \sin\varphi\cdot\cos\vartheta\cdot\sin\psi & \sin\vartheta\cdot\sin\psi - \sin\varphi\cdot\cos\vartheta\cdot\cos\psi & \cos\varphi\cdot\cos\vartheta \end{bmatrix}$$

The matrix $[D]$ can be obtained in the following way. Consider the system $O\,XYZ$ rotating around its Z axis by an angle ψ (see fig. 6 in the Chapter 5). A vector $\vec{X}$ will be represented in the new system by $\hat{\vec{X}}$, the relationship between the two being:

$$\hat{\vec{X}} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \vec{X}$$

as it can be easily checked.

A second system of reference can now be obtained by rotating the new system around its $\hat{X}$ axis. The vector $\hat{\vec{X}}$ in this system will become:

$$\tilde{\vec{X}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{bmatrix} \cdot \hat{\vec{X}}$$

if the second rotation angle is φ. A third and last rotation of ϑ around $\tilde{Y}$ will finally yield $O\,\tilde{X}\tilde{Y}\tilde{Z}$, and thus $O\,XYZ$, to coincide with $O\,X'Y'Z'$. The relative formula for this transformation is:

$$\vec{X}' = \begin{bmatrix} \cos\vartheta & 0 & -\sin\vartheta \\ 0 & 1 & 0 \\ \sin\vartheta & 0 & \cos\vartheta \end{bmatrix} \cdot \tilde{\tilde{X}}$$

and therefore:

$$\vec{X}' = \begin{bmatrix} \cos\vartheta & 0 & -\sin\vartheta \\ 0 & 1 & 0 \\ \sin\vartheta & 0 & \cos\vartheta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{bmatrix} \cdot \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \vec{X}.$$

Clearly systems $O\,X'Y'Z'$ and $Gs\,x'y'z'$ will be parallel and only differ because their origins are not in the same place if angles ϑ, φ and ψ are representative of the pitch, roll and yaw respectively. This means that:

$$[D] = \begin{bmatrix} \cos\vartheta & 0 & -\sin\vartheta \\ 0 & 1 & 0 \\ \sin\vartheta & 0 & \cos\vartheta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{bmatrix} \cdot \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Apart from position vectors - which are not free vectors - the transformation of vector co-ordinates from system to can be described by expression:

(1.A) $$\vec{X}_{Gs\,x'y'z'} = [D]\,\vec{X}_{O\,XYZ}\,.$$

It must be underlined here that the sequence of rotations is not casual. It is clear that since the yaw/heading angle is usually referred to a vertical axis and can assume quite large values, the choice of the first rotation should comply with this requirement. Roll and pitch are normally measured starting again from the vertical position. Of course the likelihood of observing them both with respect to such reference cannot be expected to be a reasonable wish since, once the system of axis is rotated around one of the two horizontal axis, the next rotation will take place in a plane that is no more perpendicular to the still water level. The choice made is roll first, then pitch, because rotations around the longitudinal axis are certainly much more pronounced than those in the longitudinal plane from the point of view of damage stability.

The program SMTR employs similar transformation matrices, but the angles used there are $\hat{\vartheta}$ and $\hat{\varphi}$, whilst the ship heading $\hat{\psi}$ is introduced by DWAVE as already shown. The forces and moments calculated by DWAVE are expressed with respect to the system $o_d\ x_d y_d z_d$ which differs from $O\ XYZ$ only because it is rotated of $\hat{\psi}$ and its centre is somehow offset. If a free vector in $o_d\ x_d y_d z_d$ co-ordinates has to be expressed with respect to $O\ XYZ$, only the rotation around the vertical is important and the formula which applies is therefore:

$$\vec{X}_{O\,XYZ} = \begin{bmatrix} \cos\hat{\psi} & -\sin\hat{\psi} & 0 \\ \sin\hat{\psi} & \cos\hat{\psi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \vec{X}_{o_d\ x_d y_d z_d}$$

or

$$\text{(2.A)} \qquad \vec{X}_{O\,XYZ} = \left[\hat{D}\right] \vec{X}_{o_d\ x_d y_d z_d}$$

As anticipated at the beginning of Chapter 6, all the forces and moments acting on the ship have to be expressed in ship-fixed co-ordinates. These quantities can be represented by free vectors and the complete transformation to convert them from the system of co-ordinates $o_d\ x_d y_d z_d$ to $Gs\,x'y'z'$ can be readily derived using expression **(1.A)** in conjunction with **(2.A)**:

$$\vec{X}_{Gs\,x'y'z'} = [D] \cdot \left[\hat{D}\right] \vec{X}_{o_d\ x_d y_d z_d}$$

or

$$\text{(3.A)} \qquad \vec{X}_{Gs\,x'y'z'} = \left[D^{*}\right] \vec{X}_{o_d\,x_d y_d z_d}$$

where

$$\left[D^{*}\right] = \left[D\right] \cdot \left[\hat{D}\right]$$

Note that matrix $\left[D^{*}\right]$ is yet again a rotation matrix, therefore its inverse will coincide with its transpose. Expression **(3.A)** will have to be applied to **(9.2 a)** or **(9.2 b)** as well as to the hydrodynamic coefficients (see part on "Hydrodynamic Reaction Forces and Moments") prior to introducing them into **(11.1)**.

Appendix - B

The two parts of this appendix are meant to give robust references to the future users and eventual developers of the simulation program CASSANDRA. In the first half, a step by step introduction to the simple operations necessary to run this program is provided. All the input and output files are described and discussed as well as the meaning of possible error messages. The second part of the appendix contains a synthetic description of the program's functions and routines along with a diagram of the program structure. More information on the latter topic can be found in Chapter 8.

CASSANDRA - Users Manual

The program CASSANDRA was developed in standard FORTRAN 77 and compiled to run on Silicon Graphic® workstation machines using IRIX® 5.3. Although the source code has been kept as simple as possible to improve portability, a few changes might be needed to adapt it to PC compilers. At present the program needs 32 MB RAM memory and performs a 900 seconds simulation of a damage scenario in about 20 hours on an Indy standard machine.

In order to run a simulation, three basic input files are needed together with the database library discussed in Appendix A. These are described next. At present all the files embodying the library should be held in a subdirectory called */Library* whilst *troy.mmx*, *horse.spc* and *hellas.sus* reside in the same directory as the executable.

hellas.sus

The file *hellas.sus* contains information about the ship external and internal geometry. The format and system of reference of this file are basically the same as any other *filename.sus* file (see[9.2]) although some sections have been appended to provide the wind excitation routines with data on the longitudinal and transversal profiles of the vessel. This file is of the form:

```
TITLE
LENGTH, BREADTH, HEIGHT, SCALE
NS(1,1)
NP(1,1,1), XP(1,1,1), SYM(1,1,1)
YP(1,1,1,1), ZP(1,1,1,1)
...
YP(1,1,1,NP(1,1,1)), ZP(1,1,1,NP(1,1,1))
...
NP(1,1,NS(1,1)), XP(1,1,NS(1,1)), SYM(1,1,NS(1,1))
YP(1,1,NS(1,1),1), ZP(1,1,NS(1,1),1)
...
YP(1,1,NS(1,1),NP(1,1,NS(1,1))), ZP(1,1,NS(1,1),NP(1,1,NS(1,1)))
```

Damaged compartment section:

```
B
DUMMY, NC, NA(2), NA(3), NA(4)
NS(2,1), PERM(2,1)
NP(2,1,1), XP(2,1,1), SYM(2,1,1)
YP(2,1,1,1), ZP(2,1,1,1)
...
YP(2,1,1,NP(2,1,1)), ZP(2,1,1,NP(2,1,1))
....
NP(2,1,NS(2,1)), XP(2,1,NS(2,1)), SYM(2,1,NS(2,1))
YP(2,1,NS(2,1),1), ZP(2,1,NS(2,1),1)
...
YP(2,1,NS(2,1),NP(2,1,NS(2,1))), ZP(2,1,NS(2,1),NP(2,1,NS(2,1)))
...
NS(NC+1,NA(4)), PERM(NC+1,NA(4))
NP(NC+1,NA(4),1), XP(NC+1,NA(4),1), SYM(NC+1,NA(4),1)
YP(NC+1,NA(4),1,1), ZP(NC+1,NA(4),1,1)
...
YP(NC+1,NA(4),1,NP(NC+1,NA(4),1)), ZP(NC+1,NA(4),1,NP(NC+1,NA(4),1))
....
NP(NC+1,NA(4),NS(NC+1,NA(4))), XP(NC+1,NA(4),NS(NC+1,NA(4))), SYM(...)
YP(NC+1,NA(4),NS(NC+1,NA(4)),1), ZP(NC+1,NA(4),NS(NC+1,NA(4)),1)
...
YP(NC+1,NA(4),NS(NC+1,NA(4)),NP(NC+1,NA(4),NS(NC+1,NA(4)))), ZP(...)
```

Lateral profile section:

```
C
N_PROF
```

XPROF(1), ZPROF(1)
...
XPROF(N_PROF), ZPROF(N_PROF)

Lateral profile of superstructure:

D
N_SPROF
XSPROF(1), ZSPROF(1)
...
XSPROF(N_SPROF), ZSPROF(N_SPROF)

Transverse profile section:

E
XTPROF
N_TPROF
YTPROF(1), ZTPROF(1)
YTPROF(N_TPROF), ZTPROF(N_TPROF)
(EOF)

An example of this file can be found in Appendix A.

File *hellas.sus* consists of five parts. The ship hull and damaged compartments are defined in the first and second part, respectively. The ship hull is described through transverse two-dimensional section along its length. The internal damaged compartments are also described using transverse sections, but these are now used to give shape to up to three continuous appendages per compartment which can correctly simulate eventual discontinuities (sudden rise of the compartment floor, engines, etc.) once considered part of the same enclosed space (see Fig. 1). Hull and damaged compartments are stored in memory using the same set of arrays. The ship hull is considered the first compartment and it is assumed to be made of one appendage only. The maximum number of damaged compartments is set to three and each appendage (including the hull) can be made of up to 32 section containing a maximum of 60 points each.

It is important that all the damaged compartments are defined within the external boundaries described in the first section of the file (damaged compartments cannot be piercing or external to the ship's hull) thus, the definition of the external geometry might well need to be extended above the bulkhead deck. In addition, the succession of points describing each appendage section must outline an anticlockwise path if one looks at the section from the bow; all the transverse sections must be defined consistently with this convention. Longitudinal profiles follow the same rule but the point of view is the starboard side instead. For example, hull sections are typically described as a series of points along the port half station (positive y axis), the first point being on the keel and the last at the intersection between the centreline and the deck. Finally, it is necessary to define at least one hull section beyond both stations at the fore and aft perpendiculars. It is indeed advisable to increase further the number of these to improve the geometry definition of such complex regions.

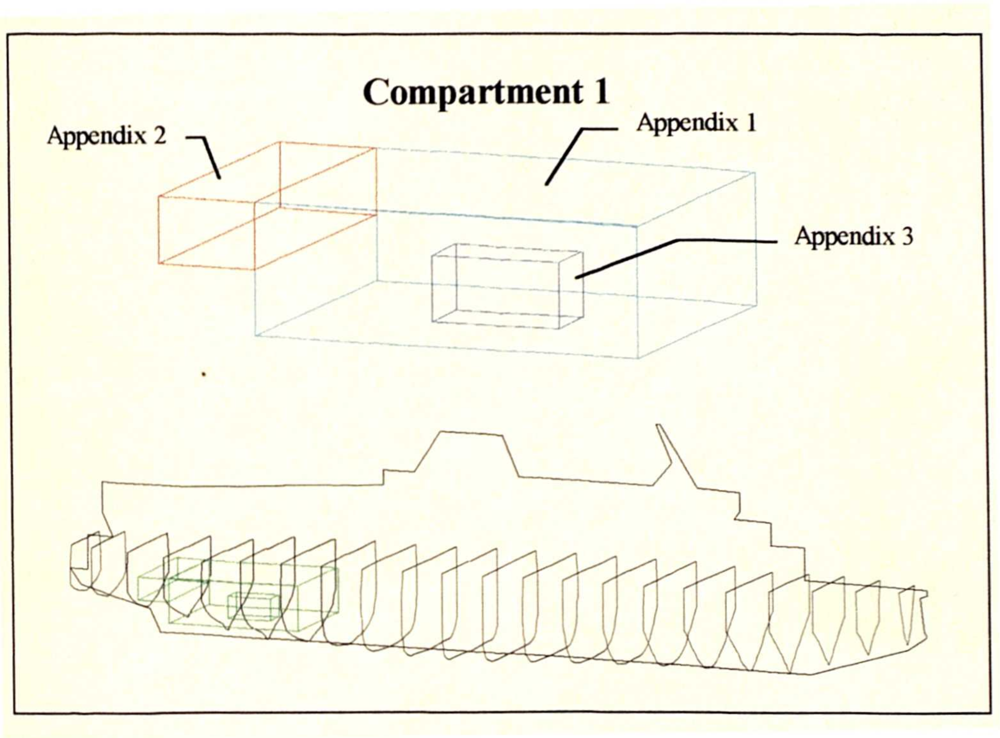

Fig. 1

Typical geometry of a ER compartment. The decomposition of a complex discontinuous compartment into three continuous appendages is illustrated in the picture. The third appendage simulate the main engine and has permeability set equal to zero.

The third, fourth and fifth section of *hellas.sus* contain the complete longitudinal profile of the vessel (including the underwater profile and superstructures), the profile of superstructures, and the transversal profile of the ship, respectively. Each profile can contain up to 200 points. A description of the variables used in the file's template follows.

TITLE	CHARACTER*80	Identification tag for the file.
LENGTH	REAL	Length between perpendiculars (m).
BREADTH	REAL	Ship's breadth (m).
HEIGHT	REAL	Depth of the ship up to the uppermost continuous deck (m).
SCALE	REAL	Scale factor. All the geometrical data entered by *hellas.sus* are multiplied by this parameter. It must be set it equal to 1.0 unless model dimensions are supplied.
NC	INTEGER	Number of damaged compartments (excluding the hull).
NA(1:4)	INTEGER	Number of appendages in compartments 1 to 4 (this variable is automatically set to 1 if the index is 1; i.e. for the hull).
NS(1:4,1:3)	INTEGER	Number of sections in appendage 1 to 3 of compartment 1 to 4.
PERM(1:4,1:3)	REAL	Permeability of appendage 1 to 3 of compartment 1 to 4. Damaged empty compartments have a permeability of -1.0, whilst the hull has a permeability of 1.0. A permeability value of 0.0 implies a totally obstructed compartment. Positive values of this parameter are invalid for damaged

		appendages.
NP(1:4,1:3,1:32)	INTEGER	Number of points in section 1 to 32 of appendage 1 to 3 of compartment 1 to 4.
SYM(1:4,1:3,1:32)	INTEGER	Symmetry switch. This parameter must be set to 0 for the hull sections, 1 for symmetric sections of the damaged appendages or 2 for asymmetric sections of the damaged appendages.
XP(1:4,1:3,1:32)	REAL	x co-ordinate of section 1 to 32 of appendage 1 to 3 of compartment 1 to 4 (m).
YP(1:4,1:3,1:32,1:60)	REAL	y co-ordinate of point 1 to 60 of section 1 to 32 of appendage 1 to 3 of compartment 1 to 4 (m).
ZP(1:4,1:3,1:32,1:60)	REAL	z co-ordinate of point 1 to 60 of section 1 to 32 of appendage 1 to 3 of compartment 1 to 4 (m).
DUMMY	INTEGER	This variable is not used by CASSANDRA.
N_PROF	INTEGER	Number of points in the longitudinal profile.
XPROF(1:200)	REAL	x co-ordinate of point 1 to 200 of the longitudinal profile (m).
ZPROF(1:200)	REAL	z co-ordinate of point 1 to 200 of the longitudinal profile (m).
N_SPROF	INTEGER	Number of points in the longitudinal profile of superstructure.

XSPROF(1:200)	REAL	x co-ordinate of point 1 to 200 of the longitudinal profile of superstructure (m).
ZSPROF(1:200)	REAL	z co-ordinate of point 1 to 200 of the longitudinal profile of superstructure (m).
XTPROF	REAL	x co-ordinate of the transversal profile (m).
N_TPROF	INTEGER	Number of points in the transversal profile.
YTPROF(1:200)	REAL	y co-ordinate of point 1 to 200 of the transversal profile (m).
ZTPROF(1:200)	REAL	z co-ordinate of point 1 to 200 of the transversal profile (m).

troy.mmx

The file *troy.mmx* contains general information about ship and environment. An example of this file is given next, followed by the file template and its description.

```
8807.0
800531.0 1.04E7 1.04E7
-3.48 0.0 8.9
5.625
3.0  2.0
0.0 0.0 90.0
0.0 20.0
0.0 900.0
0.0 0.0 0.0
900.0 900.0 900.0
12 15.0
1.5 12.6
1.25 7.87
7.87 13.0
0.0 0.0
```

```
0.0 45.0
0
0.0 0.0
0.53
0.93
0.0 0 0
12.2 20.0
1 1 1
```

```
DISPLACEMENT
IXX,IYY,IZZ
XG,YG,ZG
DRAUGHT
IN_X, IN_Y
IN_ROLL, IN_PITCH, IN_YAW
SPEED, SERVICE_SPEED
TMIN,TMAX
IN_WATER(2), IN_WATER(3), IN_WATER(4)
FLOOD_ST(2), FLOOD_ST(3), FLOOD_ST(4)
DAM_LPOS, DAM_ANGLE
ZLOW(1), ZHIG(1)
ZLOW(2), ZHIG(2)
ZLOW(3), ZHIG(3)
ZLOW(4), ZHIG(4)
WIND_SPEED, WIND_DIRECTION
N_MASTS
CURRENT_SPEED, CURRENT_DIRECTION
BLOCK_COEFF
MIDSHIP_COEFF
BBK, XBK1, XBK2
DAMP_ROLL_PERIOD, DAMP_ROLL_AMPLITUDE
RES_SWITCH,WAT_SWITCH,COE_SWITCH
(EOF)
```

DISPLACEMENT	REAL	Intact ship displacement (tonnes).
IXX,IYY,IZZ	REAL	Intact ship principal and central moments of inertia (tonnes m^2).
XG,YG,ZG	REAL	Co-ordinates of the centre of gravity of the intact ship with respect to the ship geometry system of reference (m).
DRAUGHT	REAL	Ship intact draught (m).
IN_X, IN_Y	REAL	Initial position of the centre of gravity of the

		intact ship in the horizontal plane. These co-ordinates are given with respect to the earth-fixed system of reference (m).
IN_ROLL	REAL	Initial roll angle (deg).
IN_PITCH	REAL	Initial pitch angle (deg).
IN_YAW	REAL	Initial heading. A heading angle of 0.0 deg indicates following sea, whilst 90.0 deg is beam sea from port and so on - see *Chapter 6 (deg).*
SPEED	REAL	Initial forward speed of the vessel. Note that since no wave-making resistance is included in the model, only small initial speed can be handled correctly (kn).
SERVICE_SPEED	REAL	Service speed of the vessel (kn).
TMIN,TMAX	REAL	Start and finish time for the simulation (sec).
IN_WATER(2:4)	REAL	Initial amount of water in compartment 2 to 4 (tonnes).
FLOOD_ST(2:4)	REAL	Start time for the flooding of compartment 2 to 4 (sec).
DAM_LPOS	INTEGER	Number of the hull station indicating the longitudinal position of the damage opening.
DAM_ANGLE	REAL	Angle of the slope of the sides of the damage

		opening. Zero value indicates a rectangular opening, positive angles describe trapezoids pointing downward (deg).
ZLOW(1:4), ZHIG(1:4)	REAL	Lowest and highest z co-ordinates of the damage opening on compartment 1 (hull) to 4 (m).
WIND_SPEED	REAL	Wind speed (m/sec).
WIND_DIRECTION	REAL	Wind direction. An angle of 0.0 deg indicates wind coming from the same direction of the waves - see Chapter 6 (deg).
N_MASTS	INTEGER	Number of masts, cranes, antennas etc.
CURRENT_SPEED	REAL	Current speed (m/sec).
CURRENT_DIRECTION	REAL	Current direction. An angle of 0.0 deg indicates current coming from the same direction of the waves - see Chapter 6 (deg).
BLOCK_COEFF	REAL	Ship block coefficient.
MIDSHIP_COEFF	REAL	Midships coefficient.
BBK	REAL	Breadth of bilge keels (m).
XBK1, XBK2	INTEGER	Longitudinal position of the two ends of the bilge keels measured from the aft perpendicular in tenths of L_{PP}.

DAMP_ROLL_PERIOD	REAL	Roll period used for the calculation of the viscous roll damping - see Chapter 6 (sec).
DAMP_ROLL_AMPLITUDE	REAL	Roll amplitude used for the calculation of the viscous roll damping - see Chapter 6 (deg).
RES_SWITCH	INTEGER	Switch to incorporate the wave elevation in the evaluation of the buoyancy forces (RES_SWITCH=1). Setting this parameter to zero limits the calculation of the underwater volume of the vessel to the mean water level. RES_SWITCH=0 results in a much faster simulation run.
WAT_SWITCH	INTEGER	Switch to include the simplified sloshing excitation and the matrix $\left[\dot{M}_w\right]$ in the equation of motion (WAT_SWITCH=1) - see Chapter 5. Setting this parameters to zero simplifies the equations of motion so that sloshing forces and the effects of the rate of flooding will be neglected. Switching off the sloshing forces does not speed up the simulation significantly.
COE_SWITCH	INTEGER	Switch to allow the hydrodynamic coefficients and the first order wave forces RAOs to vary with the ship attitude (COE_SWITCH=1). Setting this parameter to zero results in the above quantities being kept constant and equal to those calculated for the intact ship, for the whole duration of the simulation. Keeping constant coefficients does not speed up the simulation significantly.

horse.spc

This file contains the information necessary to generate a sea spectrum. At present CASSANDRA is capable of creating Pierson-Moscowitz, JONSWAP, mono or multi-chromatic spectra. For each of these the set of data required is slightly different and so is the format of *horse.spc*. The first line of this file always holds an integer (OPTION) that indicates the type of spectrum required. On the basis of this, the file template will be:

- Poly-harmonic spectra (regular waves, pulsing waves, etc.):

```
OPTION=1
NUMBER
FREQUENCY(1)
AMPLITUDE(1)
...
FREQUENCY(NUMBER)
AMPLITUDE(NUMBER)
(EOF)
```

where

NUMBER	INTEGER	Number of spectrum components (peaks).
FREQUENCY(1:NUMBER)	REAL	Wave frequency for component 1 to NUMBER (rad/sec).
AMPLITUDE(1:NUMBER)	REAL	Wave amplitude for component 1 to NUMBER (m).

- Pierson-Moscowitz Spectra:

```
OPTION=2
HS
```

```
(EOF)
```

where

HS	REAL	Significant wave height (m).

- JONSWAP spectra:

```
OPTION=3
GAMMA
ZCRP
HS
(EOF)
```

where

GAMMA	REAL	Peak enhancement factor (usually in the range 3.0 to 3..3).
ZCRP	REAL	Zero crossing period (sec).
HS	REAL	Significant wave height (m).

Examples of these files are:

Mono-chromatic	Pierson-Moscowitz	JONSWAP
1 1 1.5 1.0	2 2.0	3 3.3 4.4 2.0

The output of CASSANDRA consists of a series of standard two column files containing the time histories of a number of quantities of interest. The format of these files is elementary and can be easily read by most graphical packages. The first column always reports the value of time. The content of the second column varies depending on the file extension as follows:

output.x	x position of the centre of gravity of the intact ship in the earth-fixed system of reference (m).
output.x_mp	Moving average value of the x position of the centre of gravity of the intact ship in the earth-fixed system of reference (m).
output.x_wf	First order wave force in the x direction of the ship-fixed system of reference (N).
output.x_rf	Buoyancy force in the x direction of the ship-fixed system of reference (N).
output.x_gf	Gravity force in the x direction of the ship-fixed system of reference (N).
output.x_wn	Wind force in the x direction of the ship-fixed system of reference (N).
output.x_dr	Drift force in the x direction of the ship-fixed system of reference (N).
output.x_cu	Current force in the x direction of the ship-fixed system of reference (N).
output.x_sl	Sloshing force in the x direction of the ship-fixed system of reference (N).
output.y	y position of the centre of gravity of the intact ship in the earth-fixed system of reference (m).
output.y_mp	Moving average value of the y position of the centre of gravity of the intact

ship in the earth-fixed system of reference (m).

output.y_wf	First order wave force in the y direction of the ship-fixed system of reference (N).
output.y_rf	Buoyancy force in the y direction of the ship-fixed system of reference (N).
output.y_gf	Gravity force in the y direction of the ship-fixed system of reference (N).
output.y_wn	Wind force in the y direction of the ship-fixed system of reference (N).
output.y_dr	Drift force in the y direction of the ship-fixed system of reference (N).
output.y_cu	Current force in the y direction of the ship-fixed system of reference (N).
output.y_sl	Sloshing force in the y direction of the ship-fixed system of reference (N).
output.z	z position of the centre of gravity of the intact ship in the earth-fixed system of reference (m).
output.z_mp	Moving average value of the z position of the centre of gravity of the intact ship in the earth-fixed system of reference (m).
output.z_wf	First order wave force in the z direction of the ship-fixed system of reference (N).
output.z_rf	Buoyancy force in the z direction of the ship-fixed system of reference (N).
output.z_gf	Gravity force in the z direction of the ship-fixed system of reference (N).
output.z_wn	Wind force in the z direction of the ship-fixed system of reference (N).

output.z_dr	Drift force in the z direction of the ship-fixed system of reference (N).
output.z_cu	Current force in the z direction of the ship-fixed system of reference (N).
output.z_sl	Sloshing force in the z direction of the ship-fixed system of reference (N).
output.roll	Roll motion (deg).
output.heel	Heel angle (deg).
output.r_wf	First order wave roll moment (Nm).
output.r_rf	Buoyancy force roll moment (Nm).
output.r_gf	Gravity roll moment (Nm).
output.r_wn	Wind roll moment (Nm).
output.r_dr	Drift roll moment (Nm).
output.r_cu	Current roll moment (Nm).
output.r_sl	Sloshing roll moment (Nm).
output.ptch	Pitch motion (deg).
output.trim	Trim angle (deg).
output.p_wf	First order wave pitch moment (Nm).

output.p_rf	Buoyancy force pitch moment (Nm).
output.p_gf	Gravity pitch moment (Nm).
output.p_wn	Wind pitch moment (Nm).
output.p_dr	Drift pitch moment (Nm).
output.p_cu	Current pitch moment (Nm).
output.p_sl	Sloshing pitch moment (Nm).
output.yaw	Yaw motion (deg).
output.head	Heading angle (deg).
output.h_wf	First order wave yaw moment (Nm).
output.h_rf	Buoyancy force yaw moment (Nm).
output.h_gf	Gravity yaw moment (Nm).
output.h_wn	Wind yaw moment (Nm).
output.h_dr	Drift yaw moment (Nm).
output.h_cu	Current yaw moment (Nm).
output.h_sl	Sloshing yaw moment (Nm).
output.wc1	Amount of water in compartment 1 (tonnes).

output.wc2 Amount of water in compartment 2 (tonnes).

output.wc3 Amount of water in compartment 3 (tonnes).

output.yw1 y co-ordinate of the centre of gravity of the flood water in compartment 1 with respect to the ship-fixed system of reference (m).

output.yw2 y co-ordinate of the centre of gravity of the flood water in compartment 2 with respect to the ship-fixed system of reference (m).

output.yw3 y co-ordinate of the centre of gravity of the flood water in compartment 3 with respect to the ship-fixed system of reference (m).

output.wav Wave elevation at the position of the centre of gravity of the intact ship in the earth-fixed system of reference (m).

output.wa0 Wave elevation at the origin of the earth-fixed system of reference (m).

Some of the above quantities are output on screen as well to allow monitoring of the execution. In addition to this, the program produces the following error messages which are also sent to screen.

' Minimum time step exceeded.' The RKF scheme has not been able to integrate the equations within the accuracy required. Restart the program.

' Singular matrix in ludcmp.' One of the matrix inversion has failed. Check the input data and restart the program.

'Zero underwater volume. Exiting the program.'	All the ship hull sections are out of the water. Check input data paying special attention to the definition of the ship geometry and the database library.
'Roll period zero in JAPROL.'	Check that DAMP_ROLL_PERIOD in *troy.mmx* has not been given zero value.

CASSANDRA output the following warning signals if some of the calculated quantities seem suspicious:

'WARNING: negative area in STATION.'	The program has interpolated a negative area in the routine that calculates the ship underwater volume. If the value of the area is not small, check the geometry definition of the hull.
'WARNING: negative area in STATION2.'	Same as above but for the damaged compartments. Check the definition of the geometry of the compartments' appendages.
'WARNING: negative area in STATION3.'	Same as above but in the routine interpolating ten regular sections along the length of the ship to calculate the viscous roll damping. If the area is not small, check the definition of the hull.
'WARNING: area coefficient out of range in EDDY.'	One of the sectional coefficients calculated to enter the viscous roll damping routines is out of the range of validity of the equations. If this warning does not occur repeatedly, just

carry on with the computation.

Finally, if the program exits normally, one of the following messages should be output to the screen:

```
'Simulation completed. The ship has capsized.'
'Simulation completed. The ship has sunk.'
'Simulation completed. The ship has not capsized nor sunk.'
```

CASSANDRA - Program Code Documentation

In this section, a brief account is given of the subroutines and functions that are used by the program CASSANDRA. Their purpose is explained and a synthetic scheme representing the program structure is included in the end.

SUBROUTINES AND FUNCTIONS INDEX

File	File #	Subroutine
cas12.f	1	CASSANDRA
		OUTPUT
		READ_GEOMETRY
rkf16.f	2	EQUATIONS
		INTEGRATOR
		RKF
mtx2.f	3	INVERT
		LUBKSB
		LUDCMP
spe4.f	4	READ_WFR_RAO_LIBRARY
		SEA_SPECTRUM
		TIME_REALISATION_PARAMETERS
		WAVE_FORCE_RAO

		WAVE_FORCE
		TIME_REALISATION
coe3.f	5	COEFFICIENTS
		READ_COEFF_LIBRARY
res5.f	6	STATION
		TRIANG
		VOLUMES
		A1_SIMPLINT
wat6.f	7	DAMAGE
		DTRIANG
		FLOOD_WATER
		FLOODING
		STATION2
		TRACE_FLOOD_WATER
		VOLUMES2
		WATER_MATRICES
		DA1_SIMPLINT
win1.f	8	WIND_EXCITATION
		WIND_SPECTRUM_AND_PARAMETERS
		GUST
cur1.f	9	CURRENT_EXCITATION
dri1.f	10	DRIFT_COEFFICIENTS
		DRIFT_FORCES
vrd1.f	11	BK

EDDY

FRICT

HOKAN1

JAPROL

LAG3

LIFT

STATION_PARAMETERS

STATION3

WAVE

SUBROUTINES AND FUNCTIONS DOCUMENTATION

In the following, the sign // indicates that the subroutine is at the end of a call branch. Also, the numbers at the end of subroutines and functions names indicates the file that holds them. Subroutine G05CCF and function G05CAF provide random numbers and are fully described in the NAg library documentation.

CASSANDRA(1)

Purpose : This program produces a time simulation of the motion of a vessel in intact or damaged conditions. Wave, wind and current forces are taken into account as well as progressive flooding of the compartments opened to the sea.

OUTPUT(1)//

Purpose : This subroutine is used by the routine INTEGRATOR to write the intermediate results of the integration at each time step T on the screen and on files.

READ_GEOMETRY(1)//

Purpose : To read the ship data file and create a mirror image of the symmetric ordinates.

EQUATIONS(2)

Purpose : This subroutine calculates the values of the right-hand side of the integral-differential equations solved by subroutine INTEGRATOR.

INTEGRATOR(2)

Purpose : This subroutine integrates a set of twelve integral-differential equations using a modified version of the Runge-Kutta-Fehlberg scheme. This is an adaptive one-step scheme in which the time step length is controlled by comparing the estimated local truncation error with a given tolerance. The convolution integrals are evaluated though a quadrature scheme based on Hermite interpolation.

RKF(2)

Purpose : This subroutine evaluates a set of six approximate values of the solution at intermediate locations within a time step, according to the original Runge-Kutta-Fehlberg scheme. It also calculates an estimated local truncation error for each unknown function.

INVERT(3)

Purpose : This subroutine finds the inverse of matrix a and output it as y destroying the original matrix.

LUBKSB(3)//

Purpose : Solves the set of n linear equations A*x=b. Here a is input not as the matrix A but rather as its LU decomposition, determined by the routine ludcmp. indx is input as the permutation vector returned by ludcmp. b(1:n) is input as the right-hand side vector b, and returns with the solution vector x. a, n, np and indx are not modified by this routine and can be left in place for successive calls with different right-hand sides b. This routine takes into account the possibility that b will begin with many zero elements, so it is efficient for use in matrix inversion.

LUDCMP(3)//

Purpose : Given a matrix a(1:n,1:n), with physical dimension np by np, this subroutine replaces it by the LU decomposition of a row-wise permutation of itself. a and n are input. a is output arranged as the sum of L and U; indx(1:n)is an output vector that records the row permutation effected by the partial pivoting; d is output as +/- 1 depending on whether the number of rows interchanged was even or odd, respectively. This routine is used in combination with lubksb to solve linear equations or invert a matrix.

READ_WFR_RAO_LIBRARY(4)//

Purpose : This subroutine reads the wave force RAO and phase angles operator library from files.

SEA_SPECTRUM(4)//

Purpose : This subroutine is used to create a sea spectrum which can be chosen to be a poly-harmonic, a Pierson-Moskowitz or a JONSWAP spectrum. The frequency range goes from 0.2 to 2.0 rad/s in steps of 0.003 rad /s. This means that the repeat time for a realisation of the sea from the spectra is about 30 minutes.

TIME_REALISATION_PARAMETERS(4)

Purpose : This routine calculates values for the random phase angles and the wave numbers necessary for the generation of time histories of the wave elevation and the first order wave forces.

WAVE_FORCE_RAO(4)//

Purpose : This subroutine interpolates the first order wave force RAOs and phase angle operators read from the appropriate files of the database library.

WAVE_FORCE(4)

Purpose : This routine calculates the value of the random first order wave force at a location X along the first axis of the earth-fixed co-ordinate system at time T.

TIME_REALISATION(4)//

Purpose : This function calculates the value of a random realisation in the time domain, given a spectrum and a phase angle operator (wave elevation, force etc.) at a location X along the first axis of the earth-fixed co-ordinate system at time T.

COEFFICIENTS(5)//

Purpose : This subroutine interpolates the added mass and radiation damping matrices read from the appropriate files of the database library.

READ_COEFF_LIBRARY(5)//

Purpose : This subroutine reads the added mass and radiation damping coefficients library from files.

STATION(6)

Purpose : This routine calculates the immersed area and first order moments of a hull section given by the N points (X,Y,Z) expressed with respect to the ship geometry co-ordinate system. It has the capability to include the wave elevation in the calculations.

TRIANG(6)//

Purpose : This routine calculates the area and the centroid of the area of the closed contour defined by the arrays X and Y by splitting it into triangles. As long as the contours are closed (first and last point of each closed line are the same) and consistently defined (successive points always describe the contours anticlockwise), this subroutine can handle any figure (convex, concave, non-connected).

VOLUMES(6)

Purpose : This subroutine calculates the underwater volume and the co-ordinates of the centre of buoyancy of the vessel with respect to the ship-fixed system of reference. The wave elevation is taken into account for the definition of the dominion of integration.

A1_SIMPLINT(6)//

Purpose : Function to integrate area beneath a set of range values (Y), along the domain (X). The spacing interval may be even or uneven.

DAMAGE(7)//

Purpose : This routine creates a suitable number of horizontal layers along the damage window of each damaged compartment, and calculates their areas. The parameter LAYERHEIGHT controls the width of the area around each station. It is initially set to 0.1 metres but it can be increased by the program if the number of layers becomes excessive.

DTRIANG(7)//

Purpose : This is the DOUBLE PRECISION version of subroutine TRIANG.

FLOOD_WATER(7)

Purpose : This routine calculates the amount of flood water and the position, velocity and acceleration of its centre of gravity with respect to (Gs x'y'z'), for each damaged compartment defined in the ship geometry file.

FLOODING(7)

Purpose : Code to evaluate the water inflow in compartment J. This code calculates the velocity of water at several horizontal stations along the damage window of compartment J. The water flow rate through the area around the station is then evaluated as the velocity times the area itself. The integral of these values over the damage will give the total flow rate in compartment J, once multiplied by a corrective coefficient.

Note that no arrangement is made for cross-flooding, so this code is not valid for internal compartments. Also, the compartment length is supposed to be large enough to allow for the damage breadth to lie completely within the space between the compartment bulkheads. If this is not the case in reality, it is necessary to define the neighbouring compartments as two appendages of the same compartment. A last restriction worth some note is that the damaged compartments are assumed to be defined so that their volume is completely enclosed by the ship hull. This allows the routine to consider only the points belonging to one of the ship stations to define the damage window layers.

STATION2(7)

Purpose : This routine calculates the immersed area and first order moments of a damaged compartment section given by the N points (X,Y,Z) expressed with respect to the ship geometry co-ordinate system.

TRACE_FLOOD_WATER(7)

Purpose : This routine searches for the correct amount of water flooding the damaged compartments and the co-ordinates of its centre of gravity by iterations. The water surface is considered horizontal at any time.

VOLUMES2(7)

Purpose : This subroutine calculates the volume of the water flooding a damaged compartment and the co-ordinates of its centre of gravity with respect to the ship-fixed system of reference.

WATER_MATRICES(7)//

Purpose : This routine calculates the elements of the matrices MW, MW_DOT and those of the vector WATER_FORCE_PRIME.

DA1_SIMPLINT(7)//

Purpose : This is the DOUBLE PRECISION version of function A1_SIMPLINT.

WIND_EXCITATION(8)

Purpose : To calculate wind forces and moments acting on a vessel. This subroutine is based on the "Isherwood method" (Ref. Isherwood, R.M. 'Wind Resistance of Merchant Ships',Trans. RINA 1972, pp.327-338.) and calculates lateral and longitudinal forces as well as the heeling and yawing moments.

WIND_SPECTRUM_AND_PARAMETERS(8)

Purpose : This subroutine is used to create a Davenport wind spectrum with a frequency range that goes from 0.0 to 1.5 rad/s in steps of 0.03 rad /s. It also calculates values for the random phase angles necessary for the generation of the irregular wind gusts time realisation and some of the geometrical parameters needed to calculate the wind forces on the vessel.

GUST(8)//

Purpose : This function calculates the value of a random wind gust as a function of time, according to a Davenport spectrum.

CURRENT_EXCITATION(9)//

Purpose : To calculate current forces and moments acting on a vessel. This subroutine is based on experimental data and it calculates the viscous resistance of the water to the planar motions (surge, sway and yaw) only, if the current velocity is set to zero.

DRIFT_COEFFICIENTS(10)//

Purpose : To calculate the drift forces and moments coefficients to be used in subroutine DRIFT_FORCES.

DRIFT_FORCES(10)

Purpose : To calculate drift forces and moments acting on a vessel. Drift forces are calculated for generally irregular waves (slowly varying drift waves and moments - see [6.2]) using drift coefficients as in " Estimation of Second Order Wave Drift Forces " by Rama Mohan Gatiganti - University of Strathclyde, 1993. This method gives the lateral (sway) drift coefficient as function of wave and ship characteristics. No similar coefficients are readily available for the longitudinal force or the yaw moment. These are thus assumed to be reasonably similar to those found experimentally for the "WIMPEY SEALAB" (English J.W. and Wise D.A., "Hydrodynamic Aspects of Dynamic Positioning", RINA Transactions, 1975).

BK(11)

Purpose : To calculate the roll damping contribution due to the presence of BILGE KEELS.

EDDY(11)

Purpose : To calculate the damping contribution due to the shedding of eddies.

FRICT(11)//

Purpose : To calculate the frictional component to the viscous roll damping.

HOKAN1(11)

Purpose : Lagrange 3 points interpolation.

JAPROL(11)

Purpose : Vessel roll damping program top level routine. Empirical formulation developed by Ikeda [10.7].

LAG3(11)//

Purpose : Server routine used by HOKAN1.

LIFT(11)//

Purpose : To calculate the lift contribution to the roll damping.

STATION_PARAMETERS(11)

Purpose : To calculate the parameters needed for the evaluation of the viscous roll damping for a set of 11 regular stations along the ship length between perpendiculars.

STATION3(11)

Purpose : This routine calculates the area of a section under the still water surface and finds its maximum beam.

WAVE(11)//

Purpose : To calculate the wave making contribution to the roll damping.

PROGRAM STRUCTURE

The following scheme is intended to give an overall view of the subroutines interdependence and the program structure. The green box indicates that all the subroutines within work in a loop that is called at each time step. All the remaining subroutines are instead called just once during the execution of the program. The red and blue * signs indicate exits and pauses, respectively. Proper subroutine names are in black characters whilst functions are in red. Finally, the blue lines indicate direct communication links (calls) between subroutines and functions.

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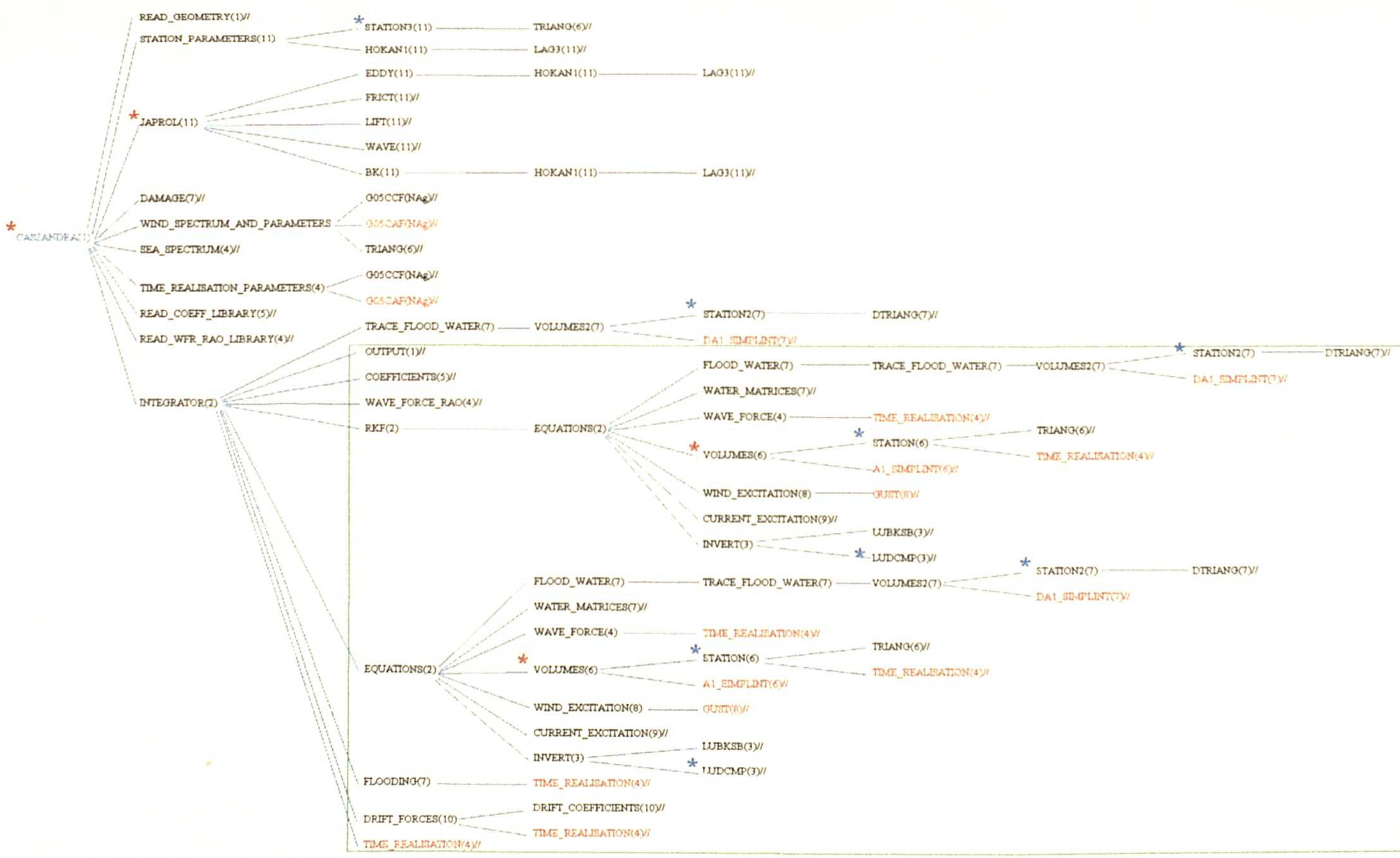


CASSANDRA(1)
READ_GEOMETRY(1)//
STATION_PARAMETERS(11)
STATION3(11)
TRIANG(6)//
HOKAN1(11)
LAG3(11)//
JAPROL(11)
EDDY(11)
HOKAN1(11)
LAG3(11)//
FRICT(11)//
LIFT(11)//
WAVE(11)//
BK(11)
HOKAN1(11)
LAG3(11)//
DAMAGE(7)//
WIND_SPECTRUM_AND_PARAMETERS
G05CCF(NAg)//
G05CAF(NAg)//
TRIANG(6)//
SEA_SPECTRUM(4)//
TIME_REALISATION_PARAMETERS(4)
G05CCF(NAg)//
G05CAF(NAg)//
READ_COEFF_LIBRARY(5)//
READ_WFR_RAO_LIBRARY(4)//
INTEGRATOR(2)
TRACE_FLOOD_WATER(7)
VOLUMES2(7)
STATION2(7)
DTRIANG(7)//
DA1_SIMPLINT(7)//
OUTPUT(1)//
COEFFICIENTS(5)//
WAVE_FORCE_RAO(4)//
RKF(2)
EQUATIONS(2)
FLOOD_WATER(7)
TRACE_FLOOD_WATER(7)
VOLUMES2(7)
STATION2(7)
DTRIANG(7)//
DA1_SIMPLINT(7)//
WATER_MATRICES(7)//
WAVE_FORCE(4)
TIME_REALISATION(4)//
VOLUMES(6)
STATION(6)
TRIANG(6)//
TIME_REALISATION(4)//
A1_SIMPLINT(6)//
WIND_EXCITATION(8)
GUST(8)//
CURRENT_EXCITATION(9)//
INVERT(3)
LUBKSB(3)//
LUDCMP(3)//
EQUATIONS(2)
FLOOD_WATER(7)
TRACE_FLOOD_WATER(7)
VOLUMES2(7)
STATION2(7)
DTRIANG(7)//
DA1_SIMPLINT(7)//
WATER_MATRICES(7)//
WAVE_FORCE(4)
TIME_REALISATION(4)//
VOLUMES(6)
STATION(6)
TRIANG(6)//
TIME_REALISATION(4)//
A1_SIMPLINT(6)//
WIND_EXCITATION(8)
GUST(8)//
CURRENT_EXCITATION(9)//
INVERT(3)
LUBKSB(3)//
LUDCMP(3)//
FLOODING(7)
TIME_REALISATION(4)//
DRIFT_FORCES(10)
DRIFT_COEFFICIENTS(10)//
TIME_REALISATION(4)//
TIME_REALISATION(4)//

Appendix - C

Flooding Coefficient Experiments

When an experiment is to be prepared, two main aspects are of primary importance; firstly objectives must be clear and secondly the best possible way to achieve them has to be found. Sometimes though, what might appear clearly important at the beginning can be found not so consequential once the tests are run and what seemed trivial becomes unfortunately fundamental for the validity and value of the results obtained. In the following, a series of tests are presented, which were run in 1993 with the purpose of finding an appropriate flooding mechanism for damaged vessels in presence of regular waves. The analysis of the data collected has been performed taking account of the developments of the theory involved as already introduced in Chapter 7. The benefit obtained from the results of this study can only be qualitative as some of the parameters that were chosen previous the experimentation, eventually resulted to be removed from the ranges of interest. Nevertheless, the methods and tools employed here proved to be perfectly suitable for the purpose they were developed for and some of the observations made should be considered as an indispensable guideline for the planning of future investigations.

The Experimental Set-Up

The flooding rate experiments described here were conducted using a 2D model whose main dimensions are reported in the table below:

L_{OA}	=	1.40 m.
B	=	0.30 m.
T	=	0.13 m.
Displacement	=	57.431 kg.
Length of flooded compartment	=	0.89 m.
Breadth of flooded compartment	=	0.25 m.
KG_1	=	0.081 m.
KG_2	=	0.087 m.
KG_3	=	0.092 m.

Assuming a scale of 1:100 most of the values above are comparable to those of a bulk carrier although the draught is far from anything existing in reality. The reason for choosing such an unrealistic value are purely technical and dictated by the necessity of keeping the experimental apparatus as simple as possible. This could have been avoided if a larger scale was adopted, which was not a suitable option at the time, given the limiting dimensions of the tank utilised. Of course such an incongruity has a great effect on the final results if these are to be related to the full scale case by non-dimensional factors. Also, the variation of KG is very limited and didn't allow any reliable conclusion to be drawn about the importance of this parameter. Again the causes of such oversight are to be searched in the limits imposed by the scale adopted. Figure 1 gives a general view of the model set-up.

The model was tested in beam sea with the damage opening facing the oncoming wave. The position of the damage was then inverted to investigate the *shielding* effect of the ship on the flooding rate. In this respect it must be noted that the great draught of the model might have amplified this phenomenon. This is on the whole truer since slack mooring were necessary to limit the model drift and because of the model extending across the whole breadth of the towing tank. The damage location is central and its dimensions and shape dictated by the SOLAS 74 recommendations. The model is open at the top and the compartment floor lies at 2.4 cm below the damage crest which is initially at the intact water level. This feature permitted the observation of a distinctively different behaviour of the flooding coefficient depending on the position of the water level inside the tank with respect to the damage crest.

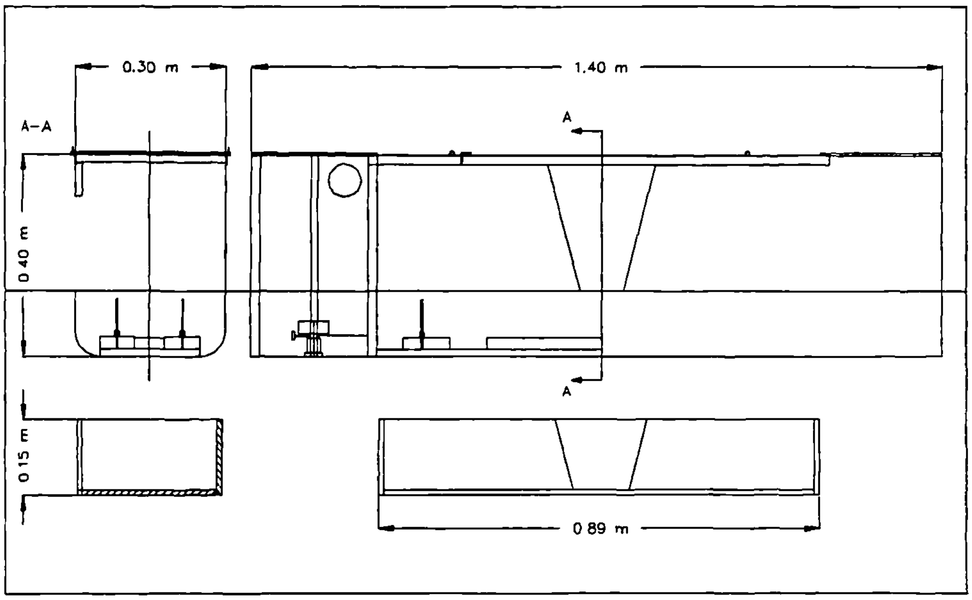


Fig. 1

Main dimensions of the model and the tray employed in the tests.

The survey of the variation of the mass of the flood water was performed by four load cells on which the tray representing the internal compartment was placed. Other quantities recorded during the tests were the level of water just outside the damage (measured by a wave probe fixed to the model), the level of water inside the compartment from the bottom of the tray at a quarter of its inner breadth from each side (wave probes x and z in the figure below; wave probe y was not activated) and finally the model motions (roll/heel, heave/sinkage, drift/sway obtained by the readings of three linear variation of distance transducers - lvdt - as in figure) and the wave profile upstream.

The model was made of one sheet of aluminium wrapped around four shaped bulkheads made of ply-wood which subdivided the hull in three compartments. The floor of the main compartment consist of a 12 mm aluminium plate on which the four

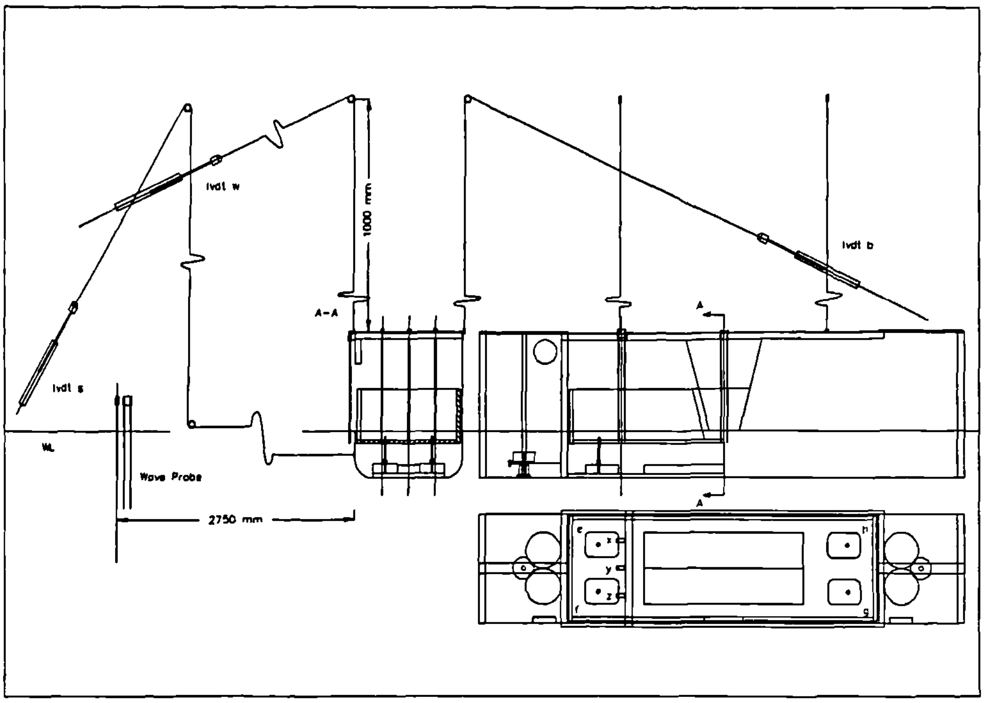


Fig. 2

Experimental set-up.

load cells were bolted. It provides the stiffness necessary for assuring correct measurements. The gap between the perspex tray and the model walls was made water-proof with slack tape and tank sealing rubber. The remaining two sections at the ends of the model provide buoyancy and space for vertical rails accommodating the movable ballast.

Finally the tank dimensions were:

Length	=	25.0 m.
Breadth	=	1.50 m.
Water depth	=	0.63 m.

Testing

The rig that has just been described, was designed and utilised for the first time during this set of experiments. To verify its efficacy a few intact tests with fixed amount of flood water were run, prior the damage was open. These were necessary to check the possible loss of information due to the different direction between the acceleration of the system consisting of the flood water and the compartment tray and the direction along which the cells are sensitive to variations of the load. The results of such trials revealed that, although the cells record could not be put in direct relation with the instantaneous water variation thank to its sensitivity to the model motions (instantaneous flooding rate and ship motions are supposed to have frequencies), the mean value of this signal was indeed giving a good estimation of the amount of water on deck. This was still true even for relatively large roll amplitudes if the mean heel angle was taken into account in the calibration of the signal. This observation led to opportune changes in the following analysis of the results.

Once the damage was open, a free flooding test (flooding in absence of wave) was run after the above described series. This test permitted to adjust the results of the following tests in waves to obtain desirable consistency among them. Other useful information were the asymptotic value of the flooding coefficient at zero frequency and the value of the amount of water in the compartment at static equilibrium. The results of this test are particularly precious because of the utter simplicity of the condition examined.

Ninety-nine tests were run with the above described rig in regular waves. The series comprises most of the possible permutations of five different frequencies, three amplitudes and three GM values and both the "wave into damage" and "wave away from damage" arrangements. The complete list of frequencies and amplitudes is given below:

Frequencies	=	0.4, 0.6, 0.8, 1.0 and 1.2 Hz
Amplitudes	=	1.0, 2.0, 3.0 cm.

Visual observations during the course of the experiments and the results of the following analysis indicated that the tests at frequencies 0.4, 0.6 and 0.8 Hz gave the most reliable readings although at 0.4 hz no flooding occurred for four rig configurations at 1.0 cm amplitude. The excessive wave dynamic loading at 1.0 and 1.2 Hz caused large sloshing and drift. This resulted in time records hard to analyse for the almost total absence of a steady state. The absence of flooding at 0.4 hz frequency and 1.0 cm wave amplitude can be explained by the small phase angle between incident wave and model motions.

Another useful observation regard the delay of the beginning of flooding with some of the investigated parameters. The shape of the curve of the rate of flooding is roughly the same for all the wave frequency considered, whilst flooding starts with an increasing delay, the frequency decreasing. This behaviour is not observed when considering the wave amplitude as varying parameter.

Water accumulating beyond the static equilibrium value could be observed for both high frequencies and amplitudes. This last observation leads to supposing a lesser importance of the dynamics of the flood water with respect to those of the incident wave when the steepness of the latter increases. This point deserves further attention.

Data Analysis

The first task to be attended to in order to analyse the eleven signals recorded during the experimental testing is to derive smooth curves from the load cells readings. This operation is necessary to filter the noise due to the varying acceleration of the tray-water system. The curves obtained represent the mean amount of flood water as a function of time. The averaging method used is similar to that described in Chapter 6 and employs a filter length equal to the frequency of the oncoming wave. Once the curves describing the time history of the water on deck are available a simple numerical derivative routine produces the curves of the rate of flooding (see figures).

Having derived the measured rate of flooding, it is necessary to find an estimation of the integral:

$$Q_{Estimated}(t) = \int_{Damage} sign(h_{out} - h_{in}) \sqrt{2\,g\,|h_{out} - h_{in}|}\; dA$$

This is possible once the relative positions over the damage crest of the water levels inside and outside the hull are known. There are two ways to appraise these values. One is by direct measurements (thank to the wave probes inside the compartment and that in front of the damage opening) and the second - which will be referenced as the calculation method - is through a lengthy and elaborated manipulation of the oncoming wave, the vessel motions and the flood water signals. Three different integrals were calculated, corresponding to curves 1, 2 and 3 in the graphs at the end of this appendix. Integral 1 was obtained using the calculation method, integral 2 only employs measured quantities and finally integral 3 was derived using the measured water level outside the ship and the calculated water level inside the tank. As it can be observed, the three methods produce similar results for the free flooding test but those employing the calculation method for estimating the water levels fail to yield satisfactory curves once the waves are present. For this reason, only integral 2 was taken into consideration in the analysis of the flooding coefficient trends with wave frequency and amplitude. Note that the behaviour of C for different loading conditions was totally disregarded in the analysis, having already verified the insignificance of the changes introduced by the limited variation of KG.

The remaining sets of figures show the time history of the water levels on the inside and outside of the damage opening with reference to the crest of the cut-out and finally the flooding coefficient:

$$C(t) = \frac{Q_{Measured}(t)}{Q_{Estimated}(t)}$$

The coefficient has been estimated only during the steady state of flooding. Comparing the two series of graphs it is also possible to observe a conspicuous change of the mean value of C as soon as the water inside the compartment reaches the lower limit of the damage opening - that is as soon as water is present on both sides of the aperture. For this reason two mean values of the coefficient were calculated and their trends plotted against wave amplitude and frequency. The flooding coefficient calculated when water is on one side of the damage only is generally larger than the other and will be referred to as the unilateral flooding coefficient - the other being labelled bilateral.

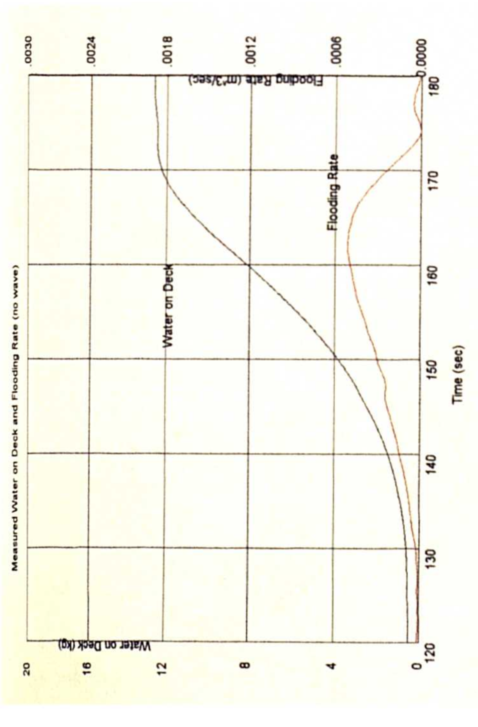
The plots of C_u and C_b against wave amplitude showed no particular trend and little variation of the value of the coefficients within the limits of measurements error. This property should be further investigated but it has been accepted as experimental evidence in this context. Different conclusions can be drawn for the dependence of the flooding coefficient with the wave circular frequency ω. The graphs show an attempt to draft a second order curve through the few available data and its agreement with them - although not perfect - can be said encouraging. Successively, the circular frequency was replaced by the non-dimensional quantity $\frac{B}{\lambda}=\frac{\omega^2 B}{2\pi g}$ and a linear regression proposed to extent the results of the analysis to the real scale world. Unfortunately the typical range of peak frequencies normally encountered by a vessel at sea seems to be quite removed from that where most of the experimental data lie. Future investigation should indeed concentrate on the behaviour of the flooding coefficient in presence of irregular waves corresponding to values of $\frac{B}{\lambda}$ between 0.5 and 2.0. This can be comfortably achieved if large scale models were employed.

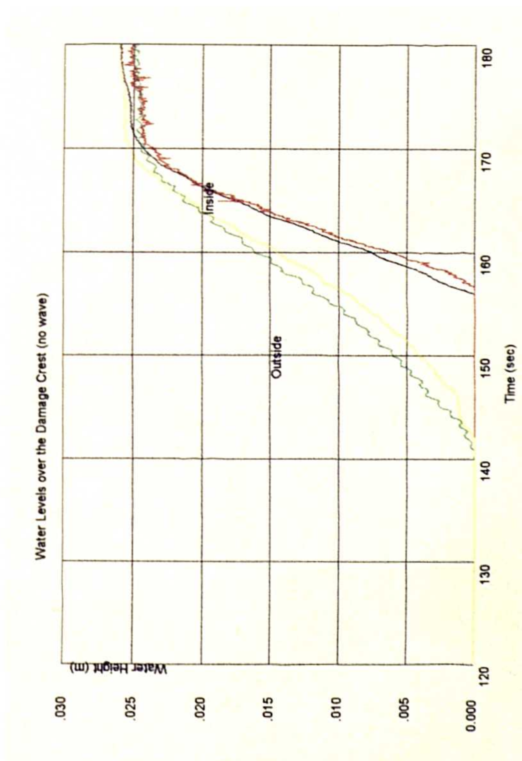
Finally, the behaviour of the coefficients was investigated for different wave directions and the most remarkable observation is that the gap between C_u and C_b seems to reduce once the ship is concealing the damage opening from the wave. This result is to be accepted with caution though, since the unrealistic draught of the model might have amplified the phenomenon. Moreover it is interesting that the general trend of both coefficients with ω can be detected again for this new set of tests and compare very well with the precedent ones'.

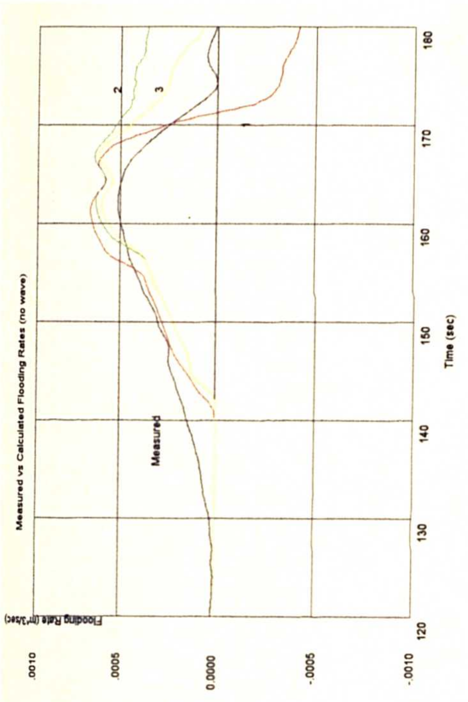
Conclusions

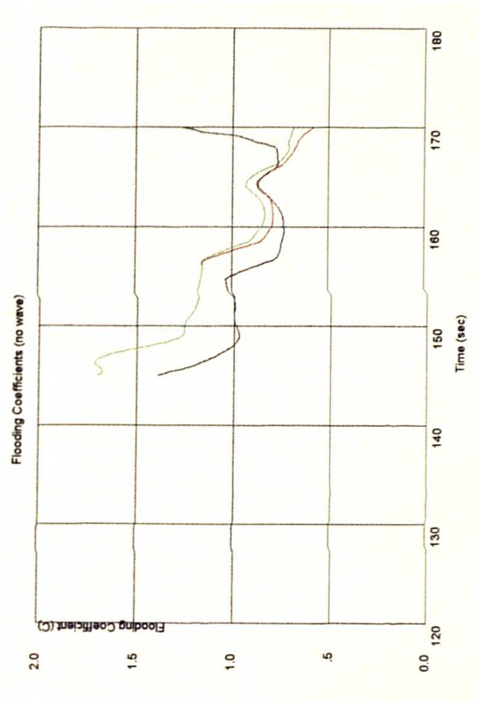
The results achieved by this limited investigation - although not quantitatively immediately applicable to the real world - are possibly an important step forward in the quest for knowledge and understanding of a terribly complex phenomenon as the flooding of the internal compartments of a vessel is. Apart from the general observations about the experimental techniques and the analysis method - which can be a useful guideline for future research in this direction - a more important datum consist of the few hints about the behaviour of C. One for all, its mean value which seems to oscillate between *1.4 and 0.8*.

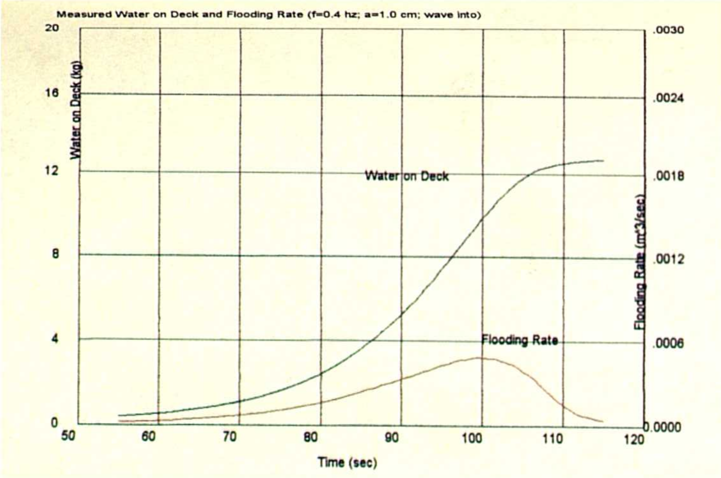
A last word has to be spent on possible the scale effects. Having already highlighted the urgent need for some larger scale investigation in order to cover the whole range of frequencies of interest, it is probably worth advising to put some effort in studying the eventual effect of viscosity on the values of the flooding coefficient. However important this might seem though, the dynamic effects due to the wave are expected to be great enough to overcome viscous losses.

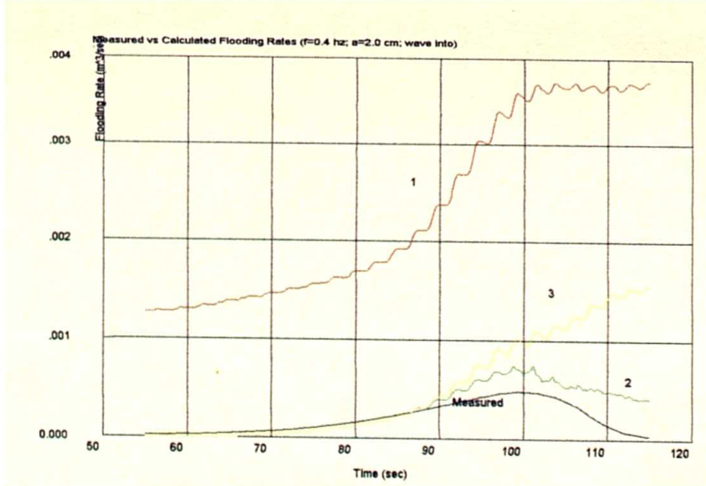

Measured Water on Deck and Flooding Rate (no wave)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)
20
16
12
8
4
0
.0030
.0024
.0018
.0012
.0006
0.0000
120
130
140
150
160
170
180

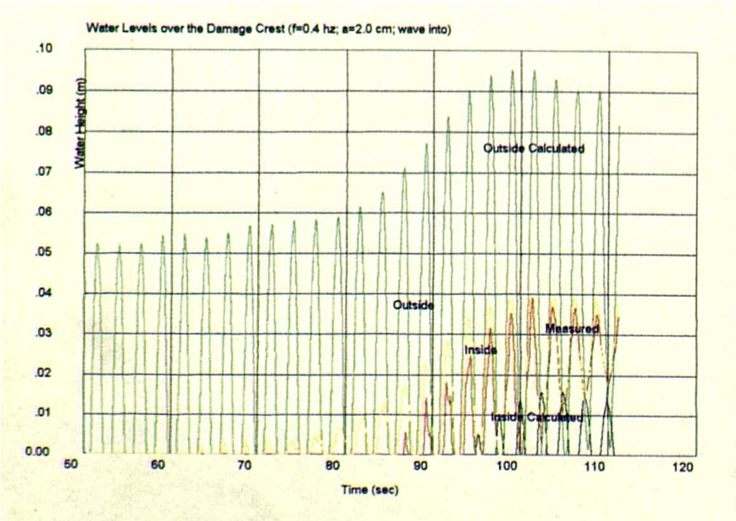

Water Levels over the Damage Crest (no wave)
Water Height (m)
Inside
Outside
Time (sec)
.030
.025
.020
.015
.010
.005
0.000
120
130
140
150
160
170
180

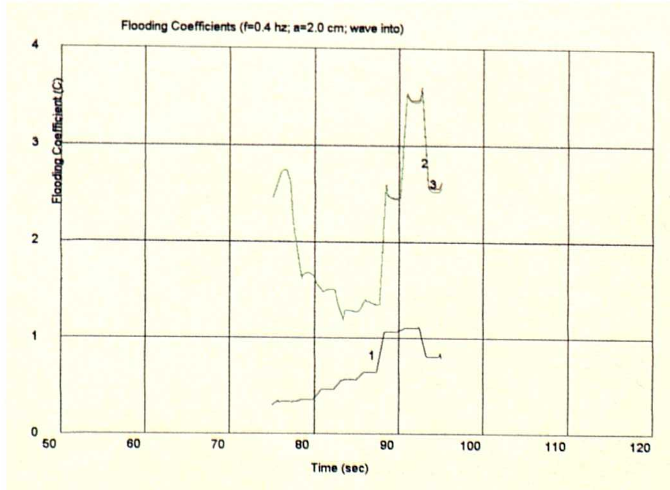

Measured vs Calculated Flooding Rates (no wave)
Flooding Rate (m^3/sec)
Measured
2
3
Time (sec)
.0010
.0005
0.0000
-.0005
-.0010
120
130
140
150
160
170
180


Flooding Coefficients (no wave)
Flooding Coefficient (C)
Time (sec)
2.0
1.5
1.0
.5
0.0
120
130
140
150
160
170
180

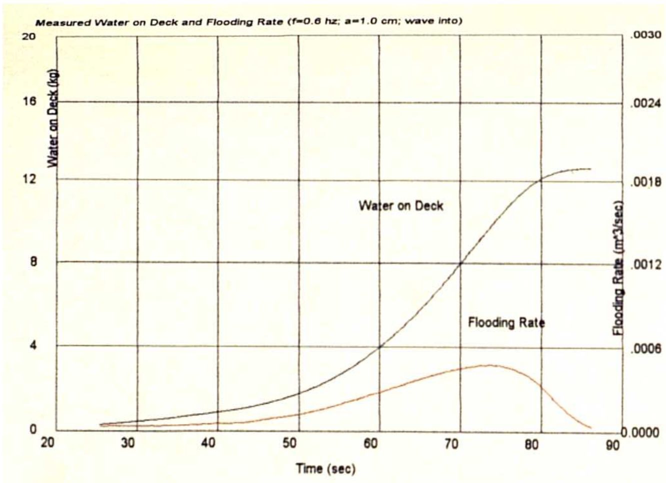

Measured Water on Deck and Flooding Rate (f=0.4 hz; a=1.0 cm; wave into)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)

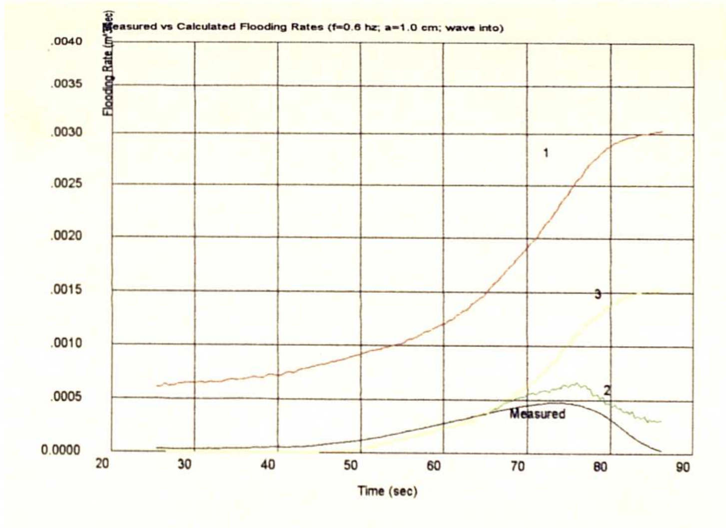

Measured vs Calculated Flooding Rates (f=0.4 hz; a=2.0 cm; wave into)
Flooding Rate (m^3/sec)
1
2
3
Measured
Time (sec)

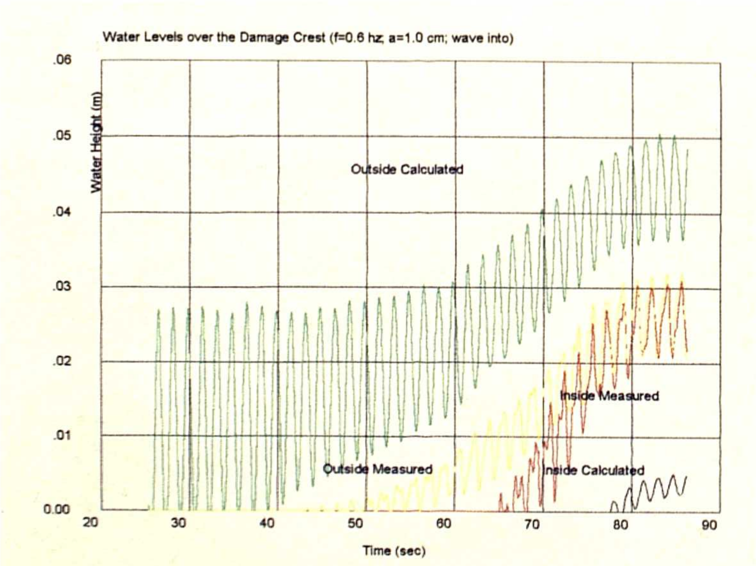

Water Levels over the Damage Crest (f=0.4 hz; a=2.0 cm; wave into)
Water Height (m)
Outside Calculated
Outside
Inside
Measured
Inside Calculated
Time (sec)

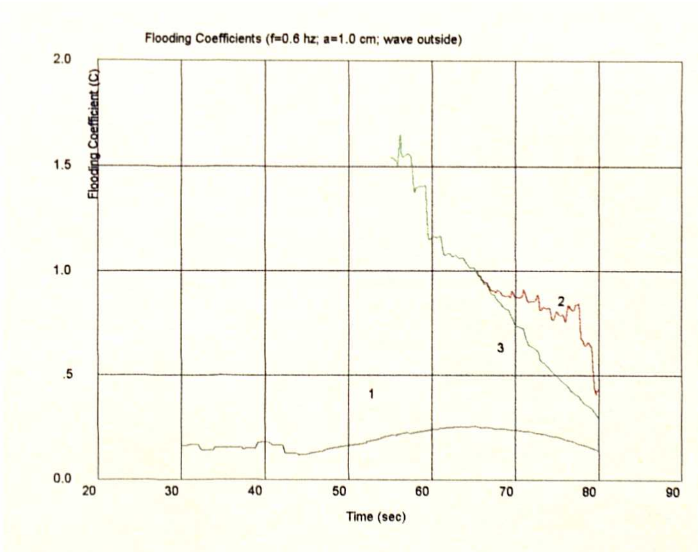

Flooding Coefficients (f=0.4 hz; a=2.0 cm; wave into)
Flooding Coefficient (C)
1
2
3
Time (sec)

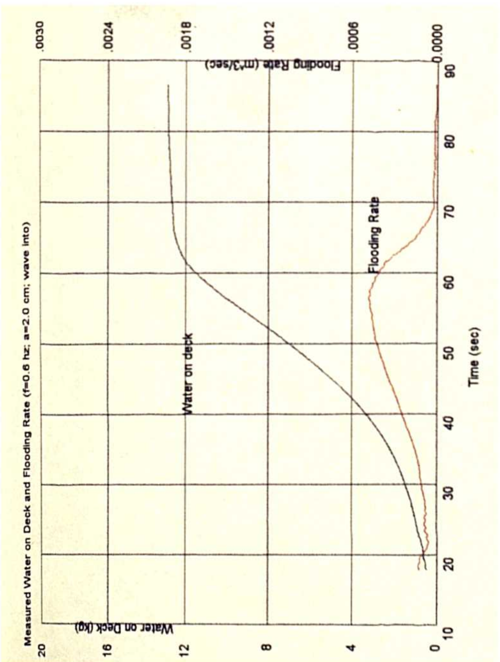
30a

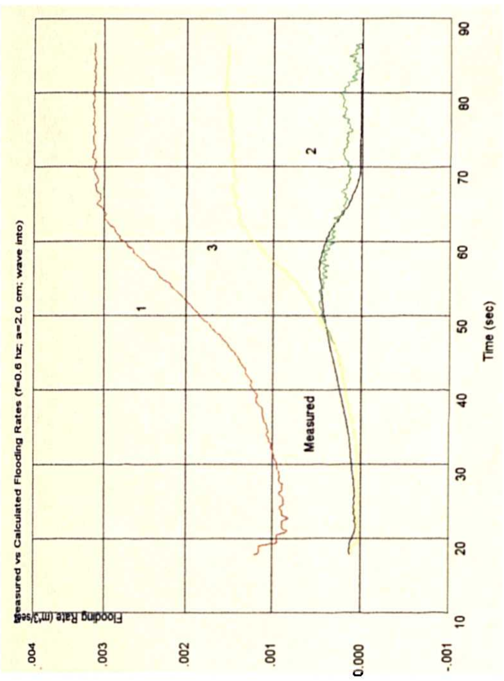

Measured Water on Deck and Flooding Rate (f=0.6 hz; a=1.0 cm; wave into)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)

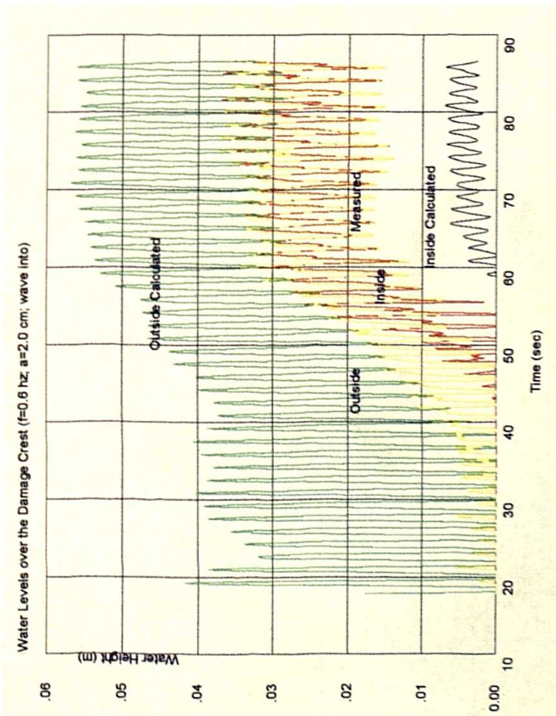

Measured vs Calculated Flooding Rates (f=0.6 hz; a=1.0 cm; wave into)
Flooding Rate (m^3/sec)
1
2
3
Measured
Time (sec)

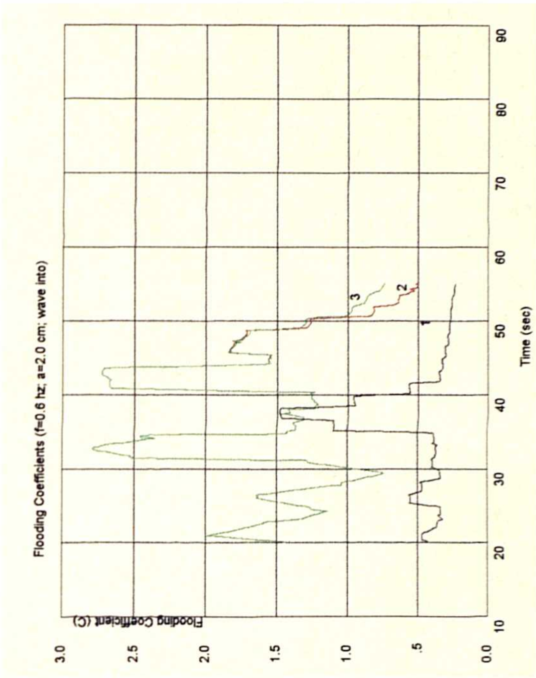

Water Levels over the Damage Crest (f=0.6 hz; a=1.0 cm; wave into)
Water Height (m)
Outside Calculated
Inside Measured
Outside Measured
Inside Calculated
Time (sec)

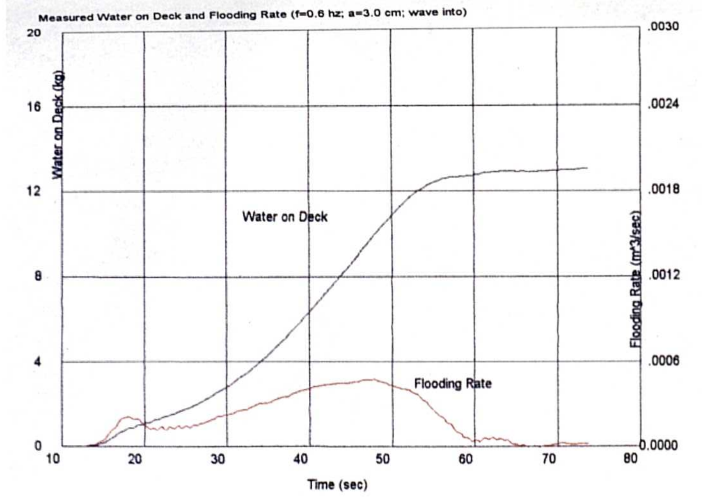

Flooding Coefficients (f=0.6 hz; a=1.0 cm; wave outside)
Flooding Coefficient (C)
1
2
3
Time (sec)

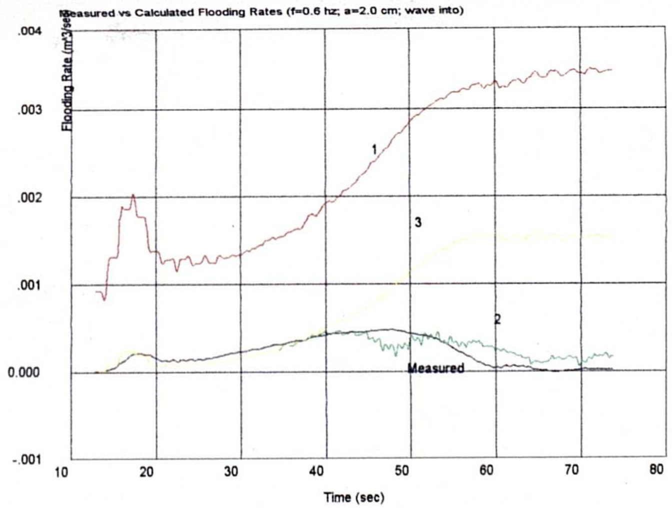

Measured Water on Deck and Flooding Rate (f=0.6 hz; a=2.0 cm; wave into)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on deck
Flooding Rate
Time (sec)

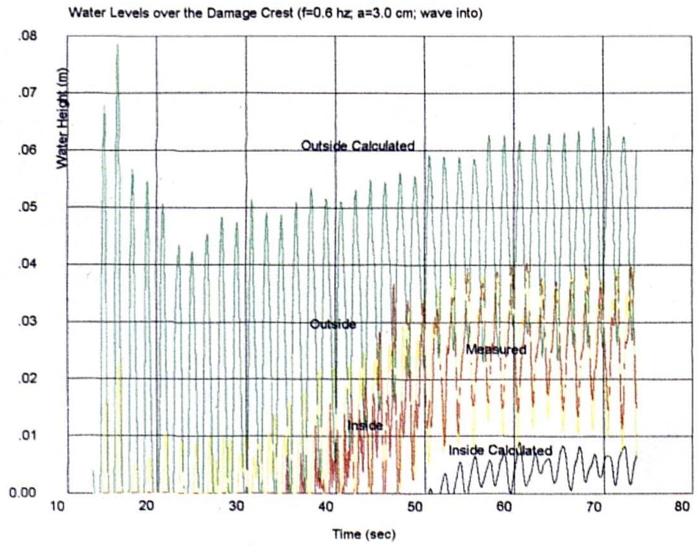

Measured vs Calculated Flooding Rates (f=0.6 hz; a=2.0 cm; wave into)
Flooding Rate (m^3/sec)
1
2
3
Measured
Time (sec)

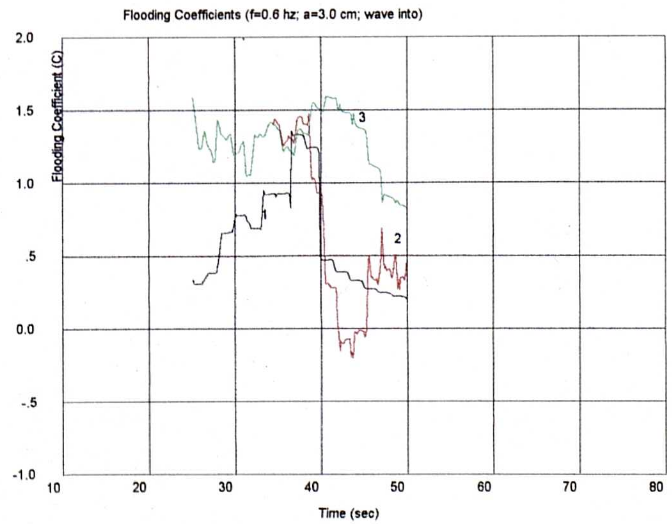

Water Levels over the Damage Crest (f=0.6 hz; a=2.0 cm; wave into)
Water Height (m)
Outside Calculated
Outside
Inside
Measured
Inside Calculated
Time (sec)

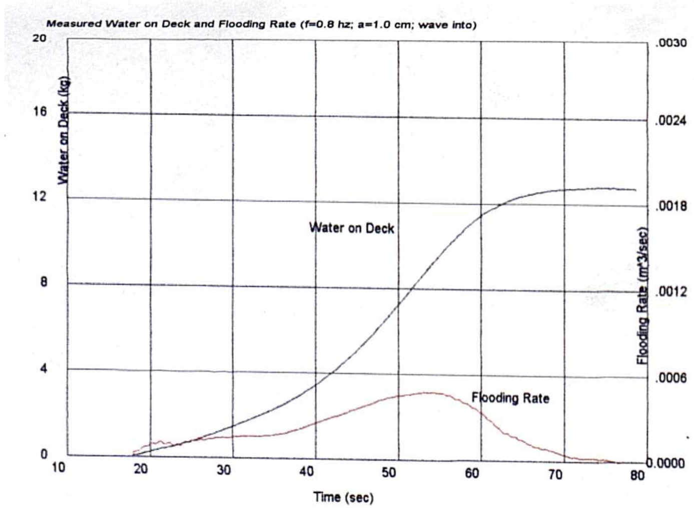

Flooding Coefficients (f=0.6 hz; a=2.0 cm; wave into)
Flooding Coefficient (C)
1
2
3
Time (sec)

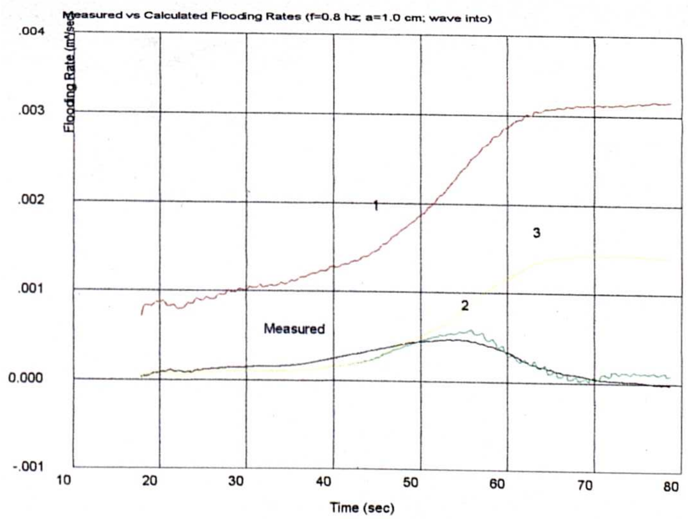

Measured Water on Deck and Flooding Rate (f=0.6 hz; a=3.0 cm; wave into)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)

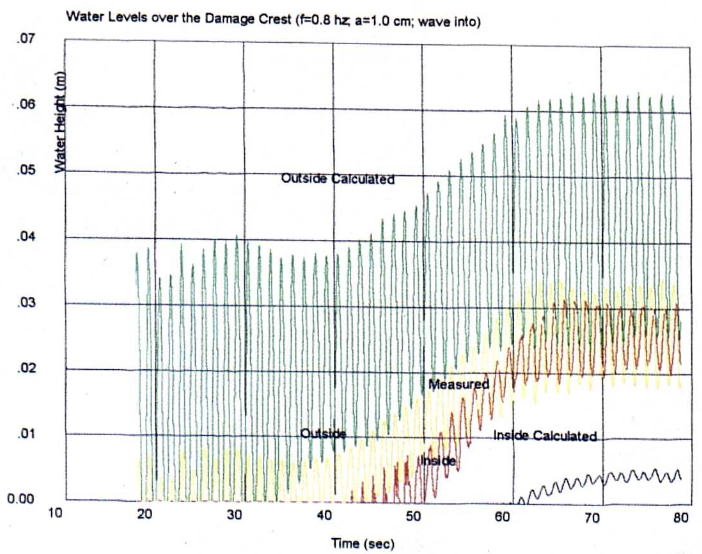

Measured vs Calculated Flooding Rates (f=0.6 hz; a=2.0 cm; wave into)
Flooding Rate (m^3/sec)
1
2
3
Measured
Time (sec)

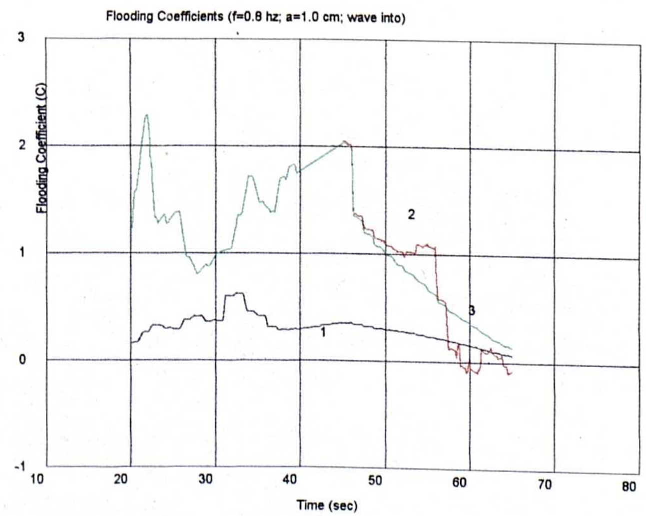

Water Levels over the Damage Crest (f=0.6 hz; a=3.0 cm; wave into)
Water Height (m)
Outside Calculated
Outside
Measured
Inside
Inside Calculated
Time (sec)

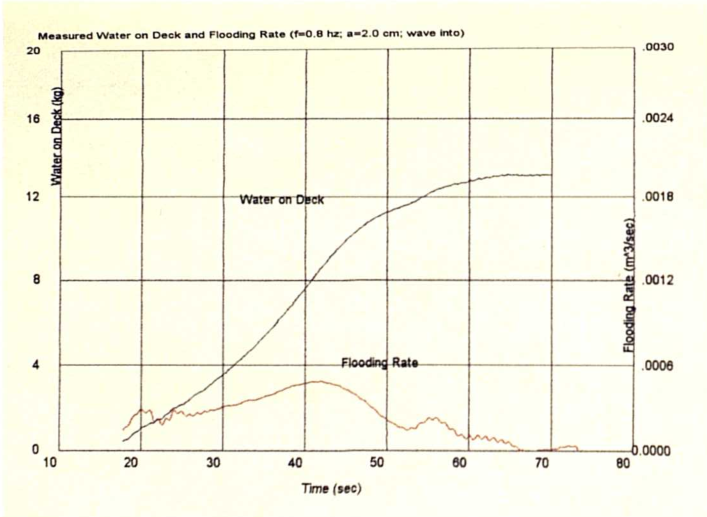

Flooding Coefficients (f=0.6 hz; a=3.0 cm; wave into)
Flooding Coefficient (C)
1
2
3
Time (sec)

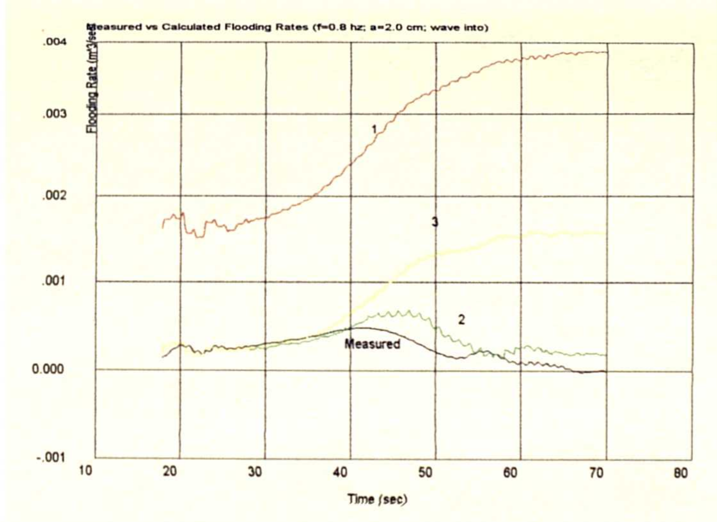

Measured Water on Deck and Flooding Rate (f=0.8 hz; a=1.0 cm; wave into)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)

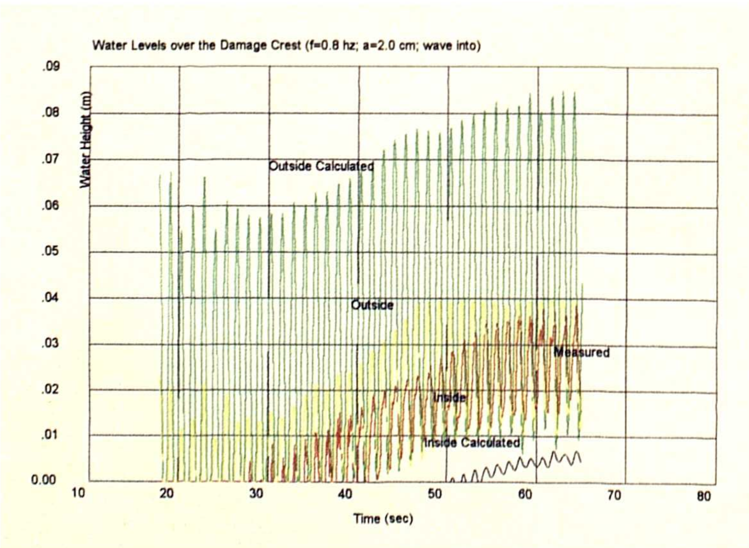

Measured vs Calculated Flooding Rates (f=0.8 hz; a=1.0 cm; wave into)
Flooding Rate (m³/sec)
1
2
3
Measured
Time (sec)

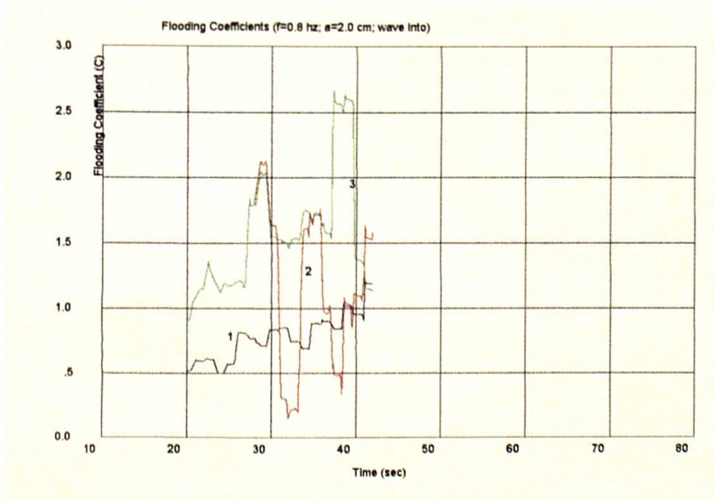

Water Levels over the Damage Crest (f=0.8 hz; a=1.0 cm; wave into)
Water Height (m)
Outside Calculated
Measured
Outside
Inside Calculated
Inside
Time (sec)

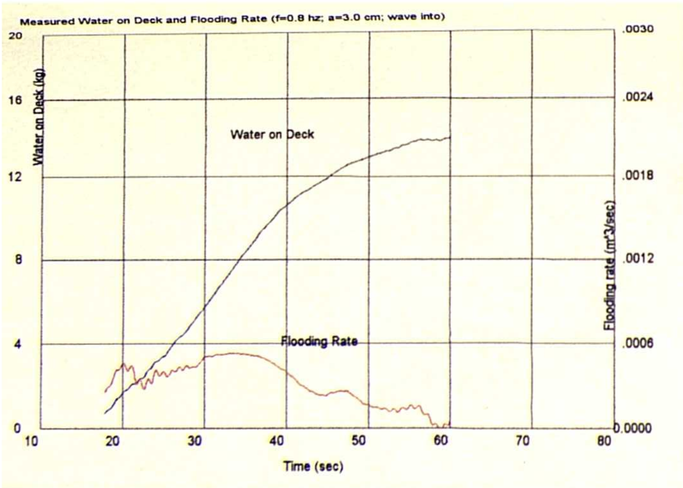

Flooding Coefficients (f=0.8 hz; a=1.0 cm; wave into)
Flooding Coefficient (C)
1
2
3
Time (sec)


Measured Water on Deck and Flooding Rate (f=0.8 hz; a=2.0 cm; wave into)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)

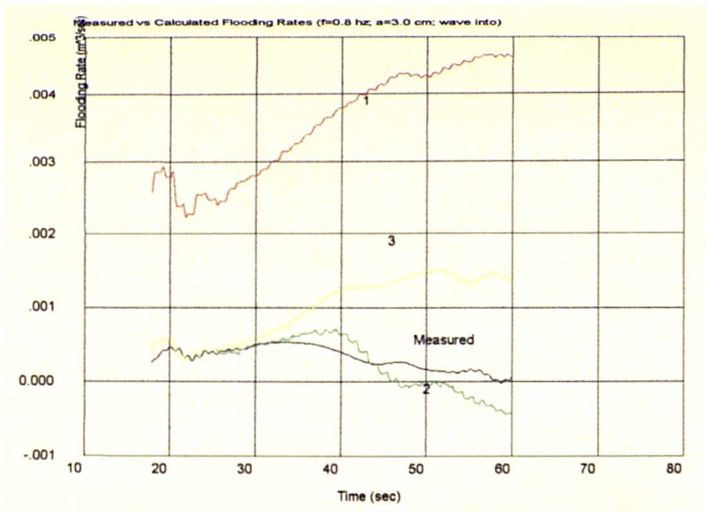

Measured vs Calculated Flooding Rates (f=0.8 hz; a=2.0 cm; wave into)
Flooding Rate (m^3/sec)
1
2
3
Measured
Time (sec)


Water Levels over the Damage Crest (f=0.8 hz; a=2.0 cm; wave into)
Water Height (m)
Outside Calculated
Outside
Measured
Inside
Inside Calculated
Time (sec)

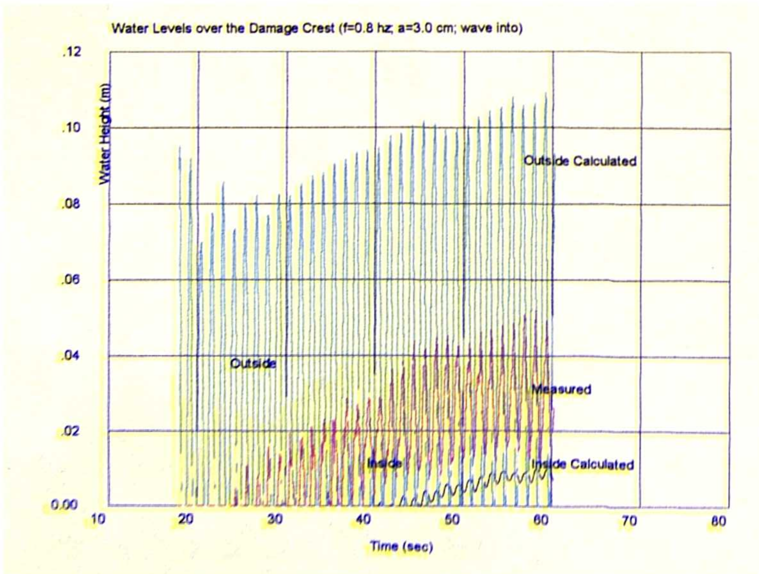

Flooding Coefficients (f=0.8 hz; a=2.0 cm; wave into)
Flooding Coefficient (C)
1
2
3
Time (sec)



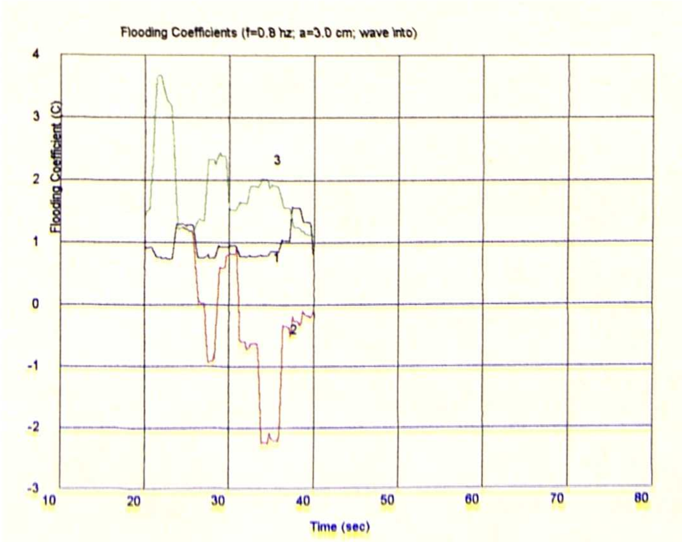
Measured Water on Deck and Flooding Rate (f=0.8 hz; a=3.0 cm; wave into)
Water on Deck (kg)
Flooding rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)



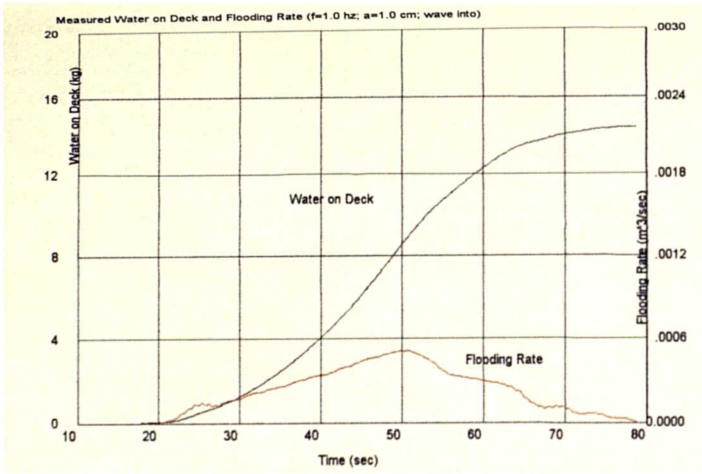
Measured vs Calculated Flooding Rates (f=0.8 hz; a=3.0 cm; wave into)
Flooding Rate (m^3/sec)
1
2
3
Measured
Time (sec)

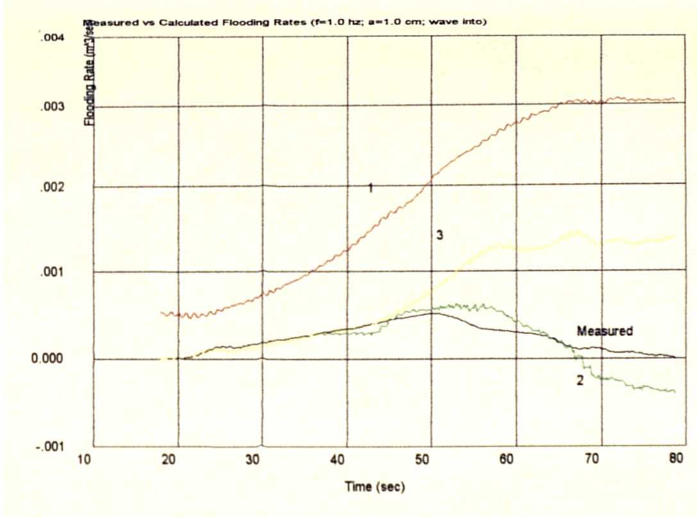


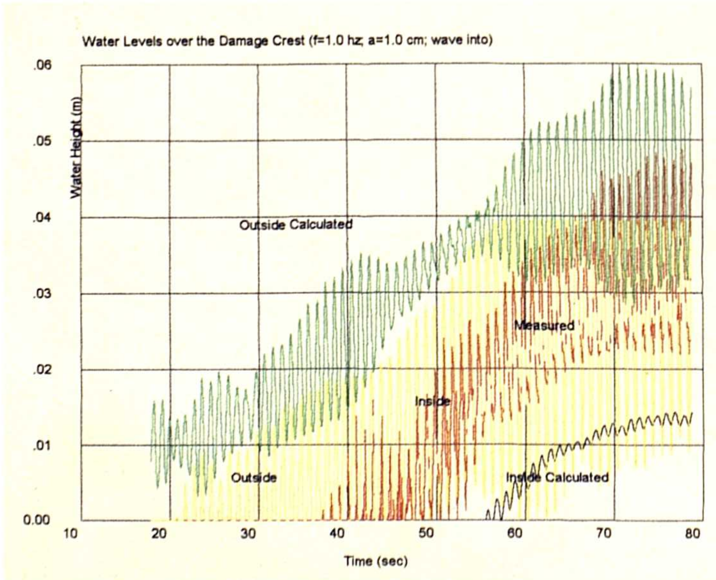
Water Levels over the Damage Crest (f=0.8 hz; a=3.0 cm; wave into)
Water Height (m)
Outside Calculated
Outside
Measured
Inside
Inside Calculated
Time (sec)

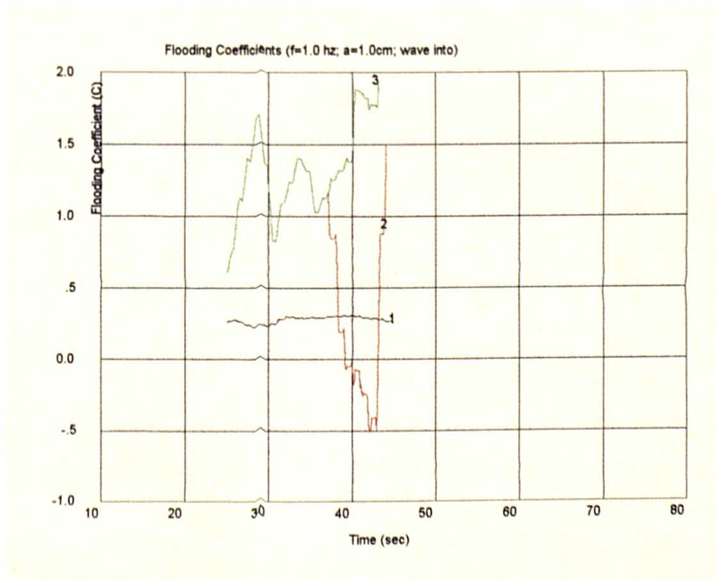


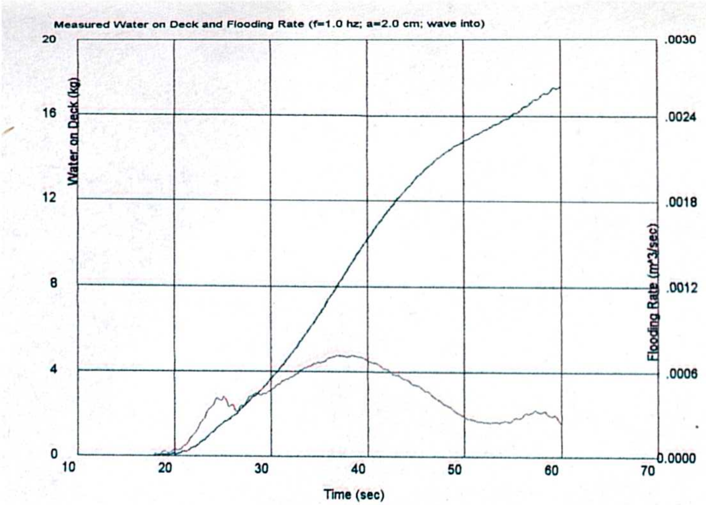
Flooding Coefficients (f=0.8 hz; a=3.0 cm; wave into)
Flooding Coefficient (C)
1
2
3
Time (sec)

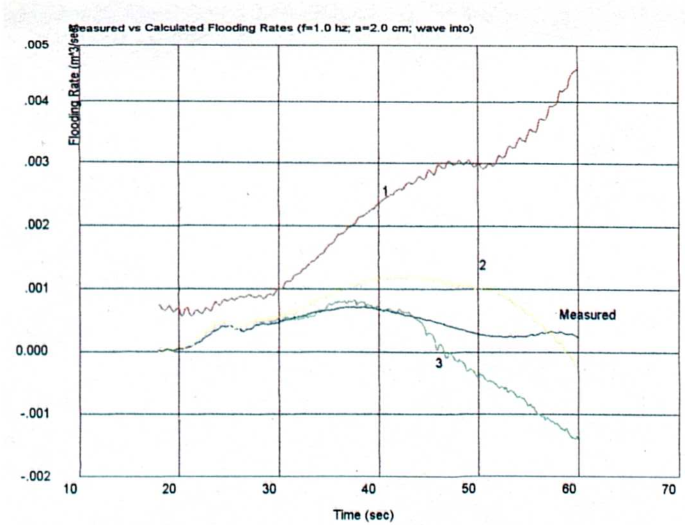

Measured Water on Deck and Flooding Rate (f=1.0 hz; a=1.0 cm; wave into)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)

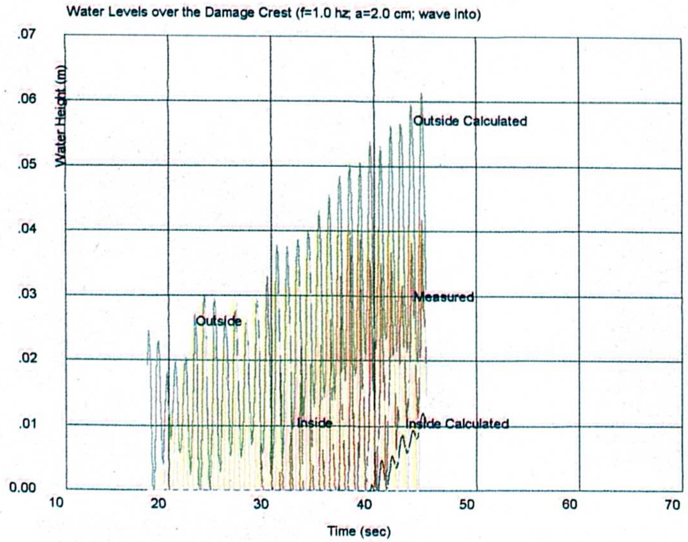

Measured vs Calculated Flooding Rates (f=1.0 hz; a=1.0 cm; wave into)
Flooding Rate (m^3/sec)
1
2
3
Measured
Time (sec)

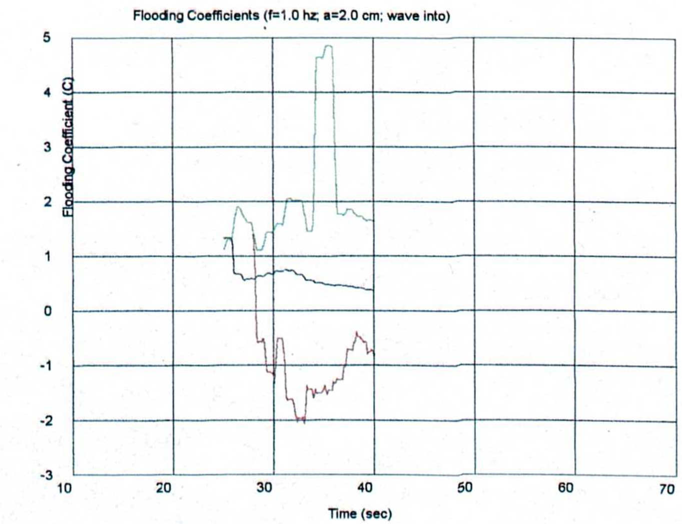

Water Levels over the Damage Crest (f=1.0 hz; a=1.0 cm; wave into)
Water Height (m)
Outside Calculated
Measured
Inside
Outside
Inside Calculated
Time (sec)


Flooding Coefficients (f=1.0 hz; a=1.0cm; wave into)
Flooding Coefficient (C)
1
2
3
Time (sec)

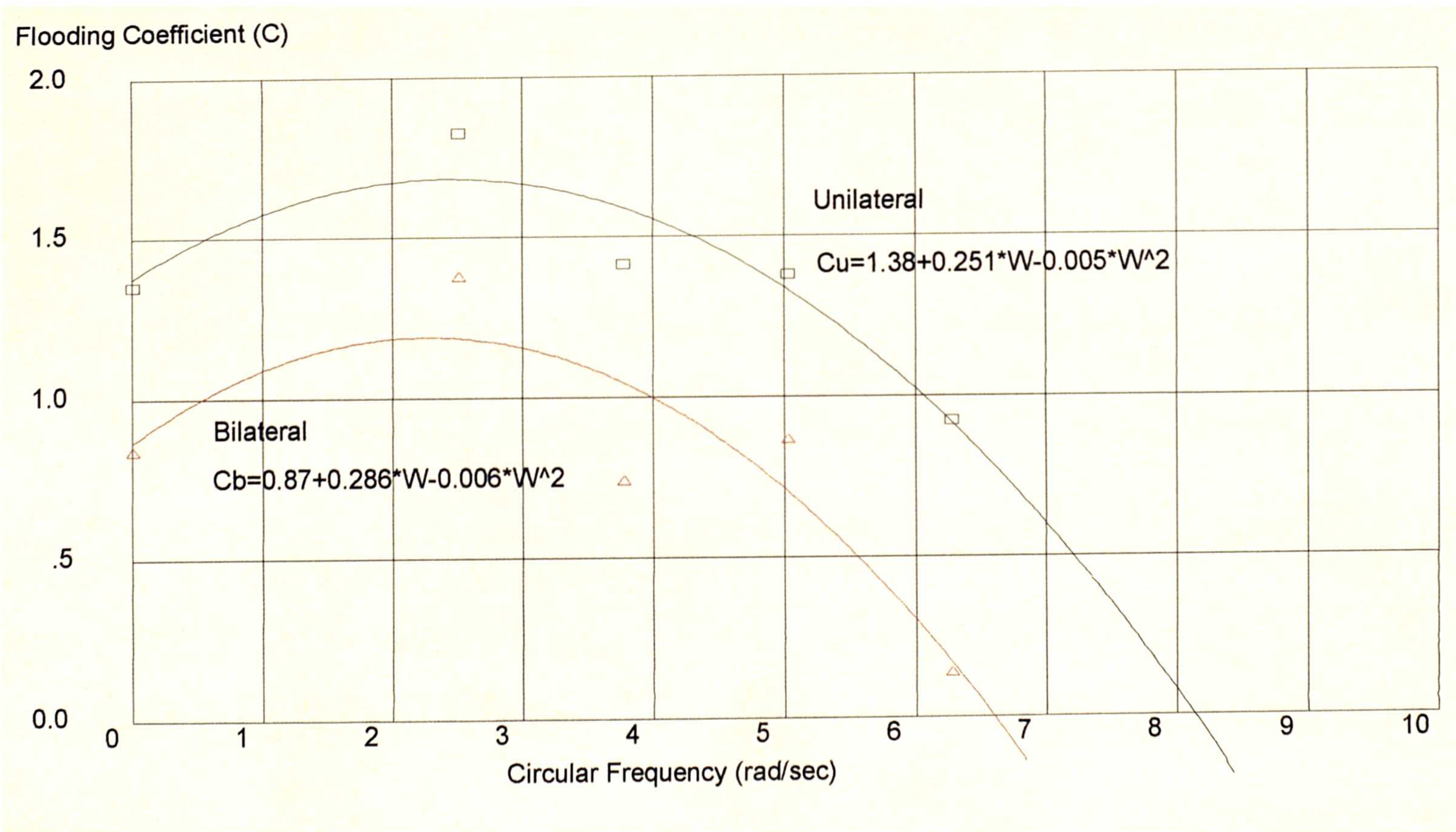

Measured Water on Deck and Flooding Rate (f=1.0 hz; a=2.0 cm; wave into)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Time (sec)


Measured vs Calculated Flooding Rates (f=1.0 hz; a=2.0 cm; wave into)
Flooding Rate (m^3/sec)
1
2
3
Measured
Time (sec)

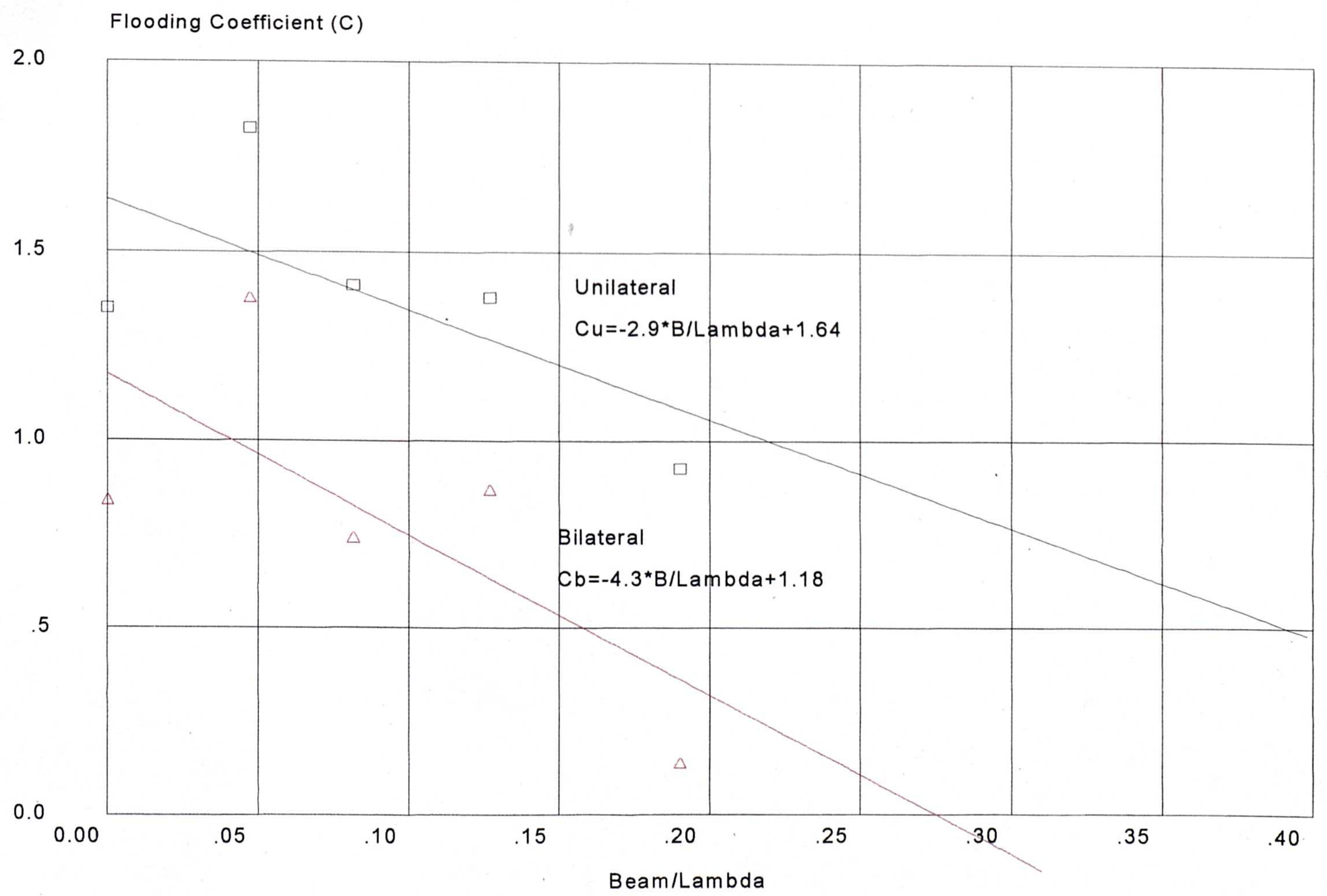

Water Levels over the Damage Crest (f=1.0 hz; a=2.0 cm; wave into)
Water Height (m)
Outside Calculated
Measured
Outside
Inside
Inside Calculated
Time (sec)

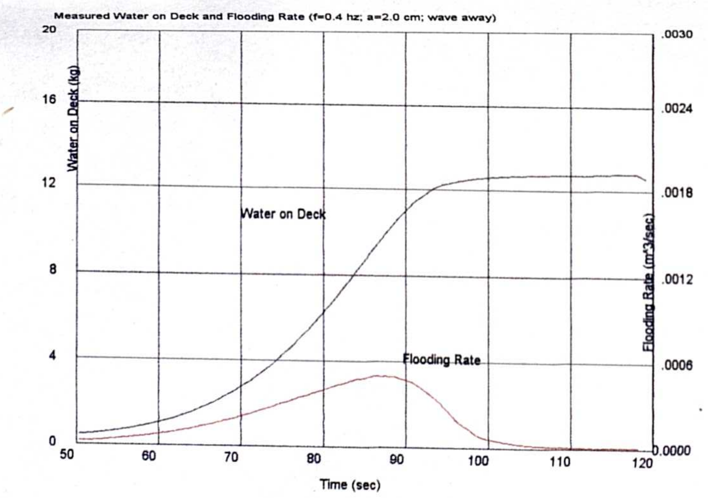

Flooding Coefficients (f=1.0 hz; a=2.0 cm; wave into)
Flooding Coefficient (C)
Time (sec)

Flooding Coefficients Variation with Frequency (Model Scale)

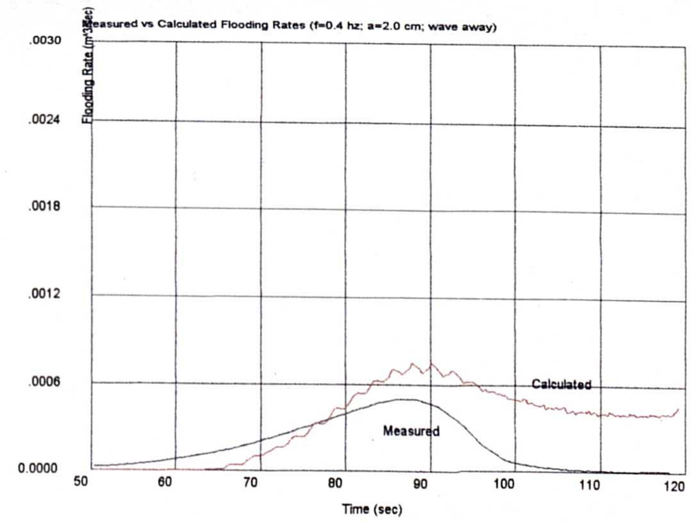


Flooding Coefficients vs Non-Dimensional Frequency

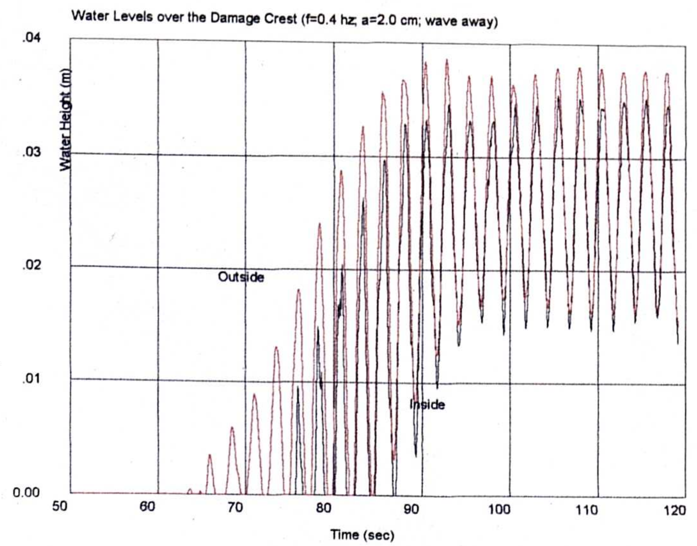




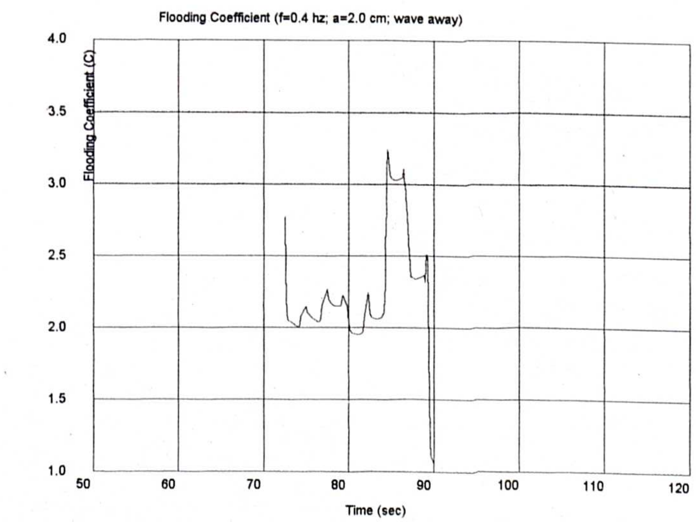
Measured Water on Deck and Flooding Rate (f=0.4 hz; a=2.0 cm; wave away)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)



Measured vs Calculated Flooding Rates (f=0.4 hz; a=2.0 cm; wave away)
Flooding Rate (m^3/sec)
Calculated
Measured
Time (sec)

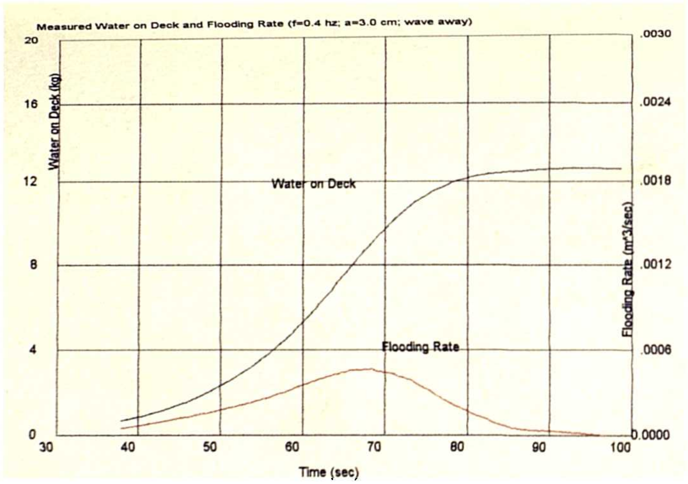


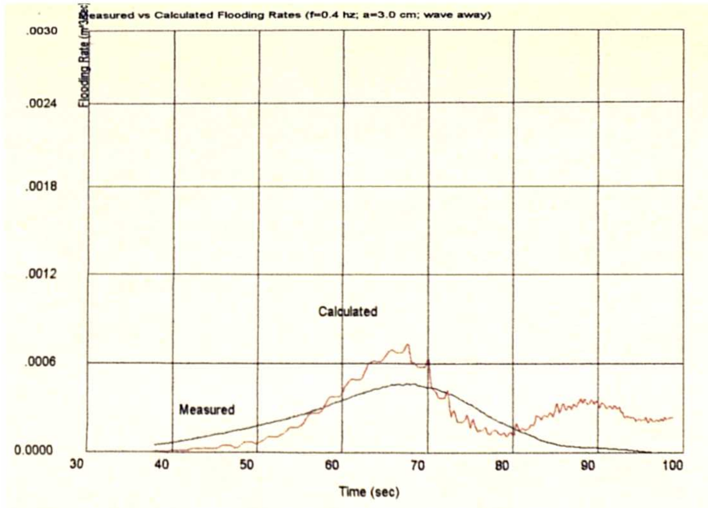
Water Levels over the Damage Crest (f=0.4 hz; a=2.0 cm; wave away)
Water Height (m)
Outside
Inside
Time (sec)

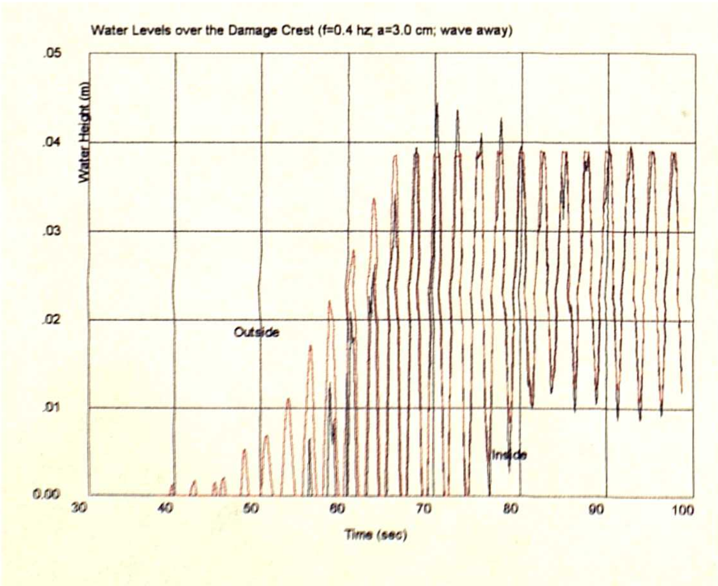


Flooding Coefficient (f=0.4 hz; a=2.0 cm; wave away)
Flooding Coefficient (C)
Time (sec)

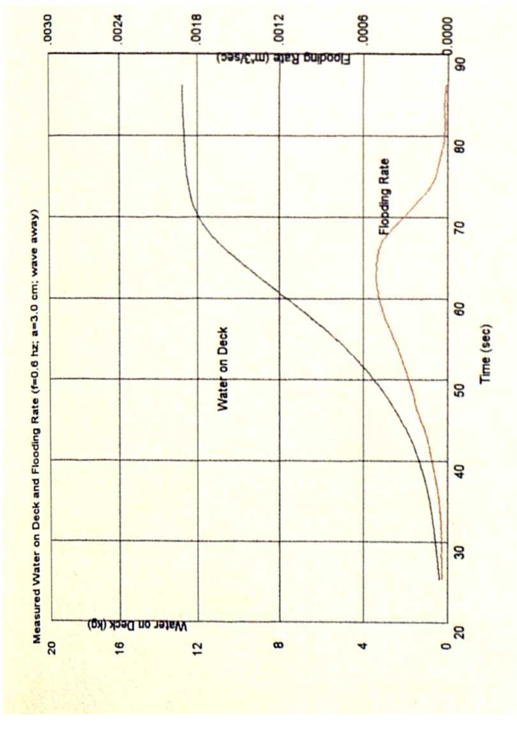
312

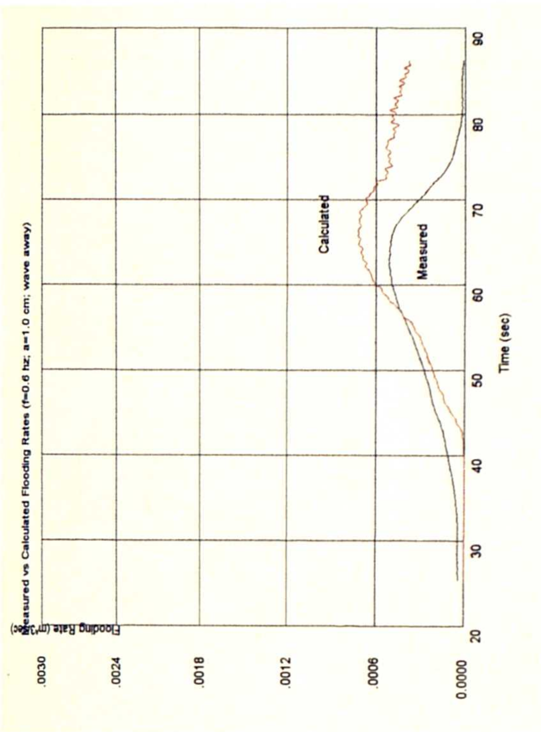

Measured Water on Deck and Flooding Rate (f=0.4 hz; a=3.0 cm; wave away)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)

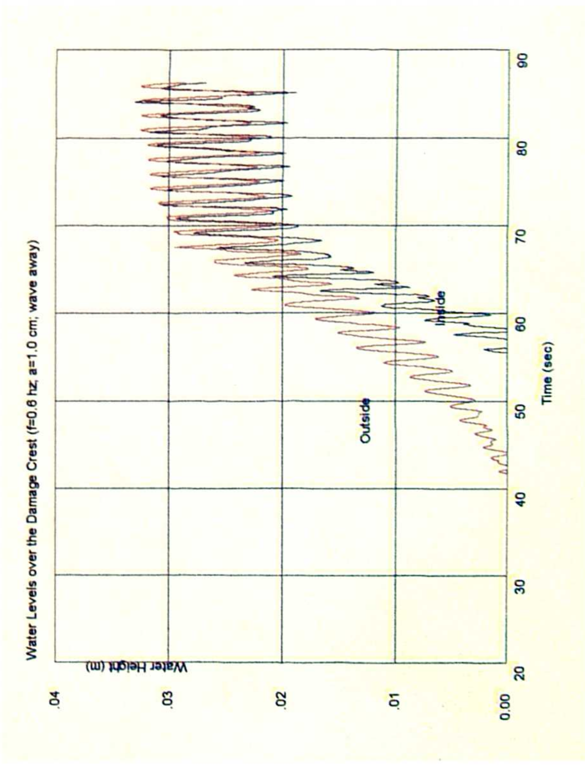

Measured vs Calculated Flooding Rates (f=0.4 hz; a=3.0 cm; wave away)
Flooding Rate (m^3/sec)
Calculated
Measured
Time (sec)

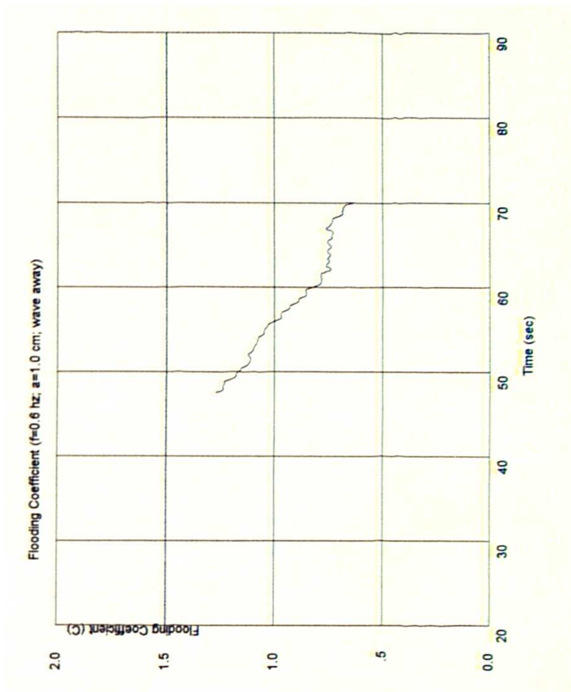

Water Levels over the Damage Crest (f=0.4 hz; a=3.0 cm; wave away)
Water Height (m)
Outside
Inside
Time (sec)

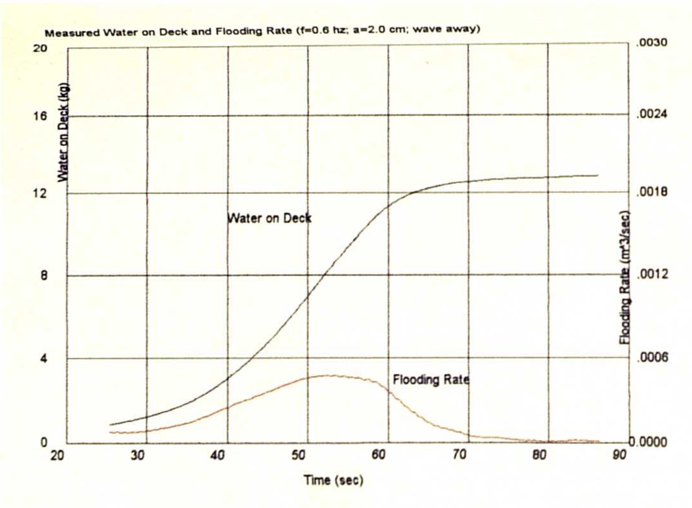

Flooding Coefficient (f=0.4 hz; a=3.0 cm; wave away)
Flooding Coefficient (C)
Time (sec)

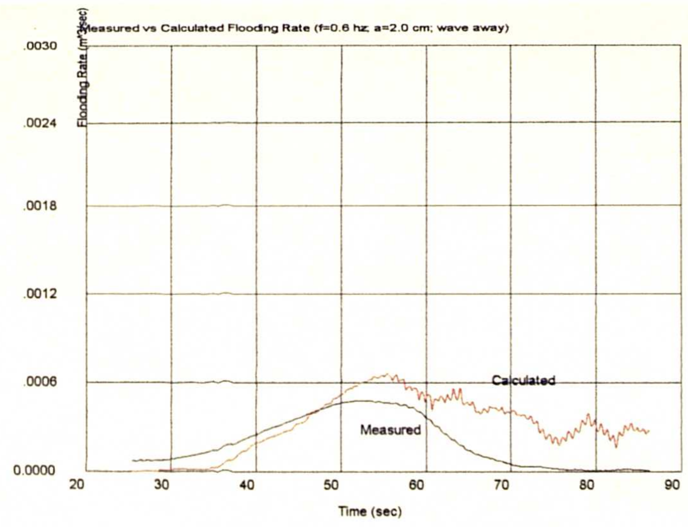

Measured Water on Deck and Flooding Rate (f=0.6 hz; a=3.0 cm; wave away)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)
20
16
12
8
4
0
.0030
.0024
.0018
.0012
.0006
0.0000
20
30
40
50
60
70
80
90

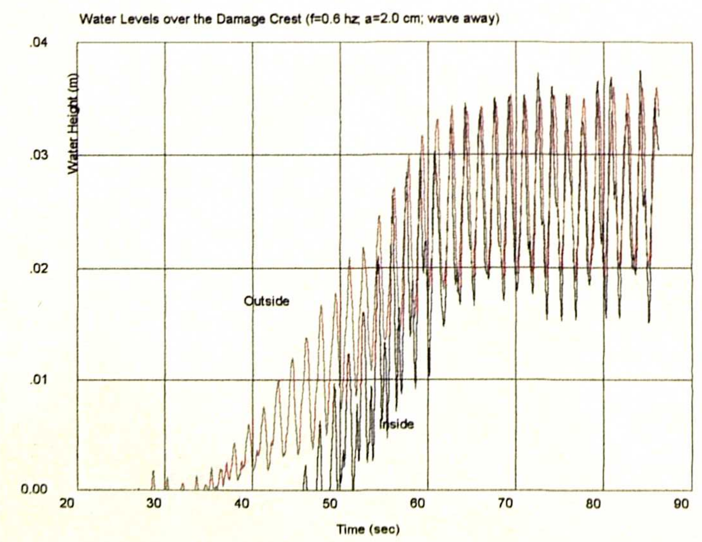

Measured vs Calculated Flooding Rates (f=0.6 hz; a=1.0 cm; wave away)
Flooding Rate (m^3/sec)
Calculated
Measured
Time (sec)
.0030
.0024
.0018
.0012
.0006
0.0000
20
30
40
50
60
70
80
90

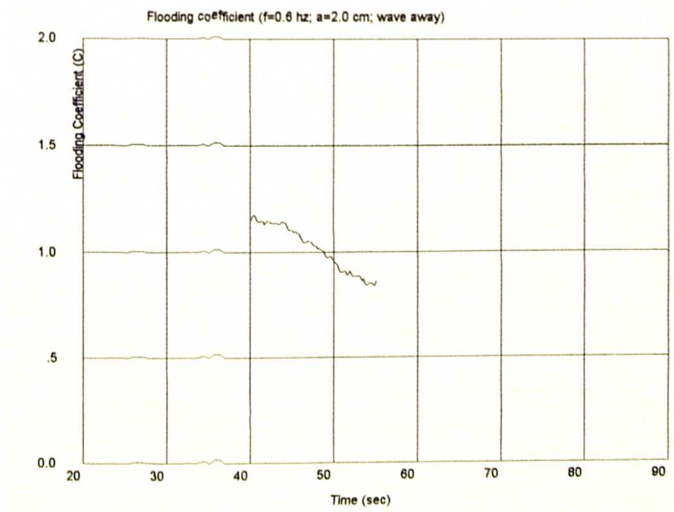

Water Levels over the Damage Crest (f=0.6 hz; a=1.0 cm; wave away)
Water Height (m)
Outside
Inside
Time (sec)
.04
.03
.02
.01
0.00
20
30
40
50
60
70
80
90

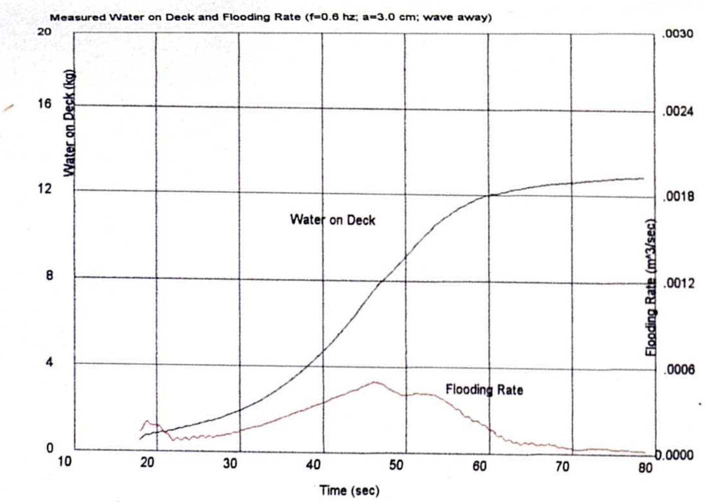

Flooding Coefficient (f=0.6 hz; a=1.0 cm; wave away)
Flooding Coefficient (C)
Time (sec)
2.0
1.5
1.0
.5
0.0
20
30
40
50
60
70
80
90

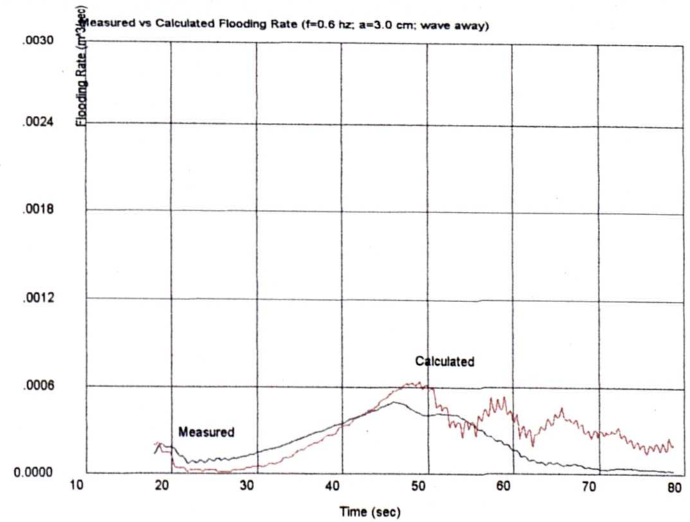

Measured Water on Deck and Flooding Rate (f=0.6 hz; a=2.0 cm; wave away)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)

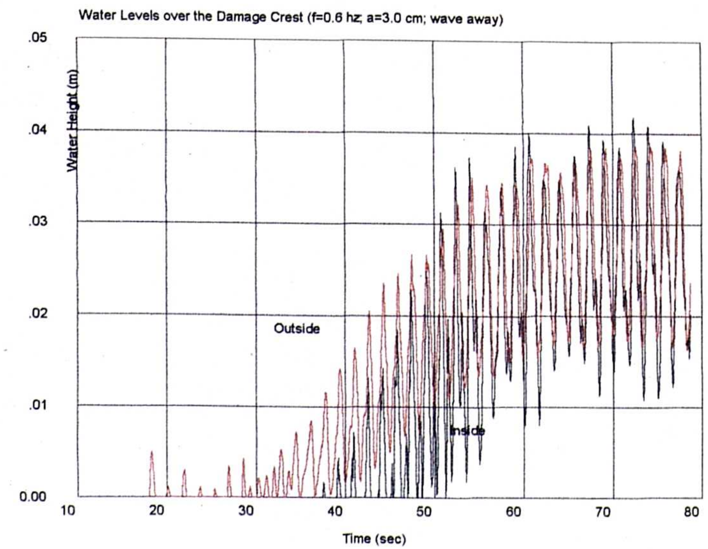

Measured vs Calculated Flooding Rate (f=0.6 hz; a=2.0 cm; wave away)
Flooding Rate (m^3/sec)
Calculated
Measured
Time (sec)

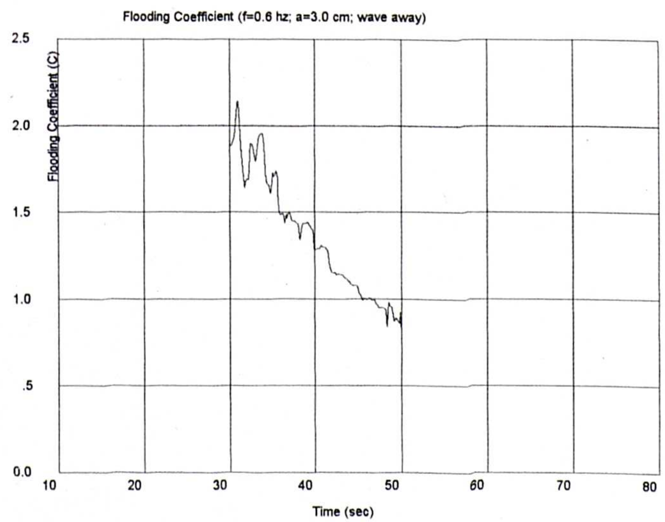

Water Levels over the Damage Crest (f=0.6 hz; a=2.0 cm; wave away)
Water Height (m)
Outside
Inside
Time (sec)

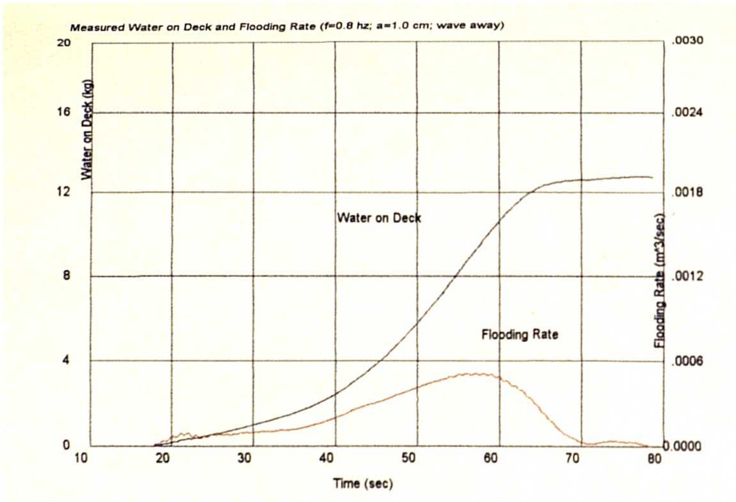

Flooding coefficient (f=0.6 hz; a=2.0 cm; wave away)
Flooding Coefficient (C)
Time (sec)

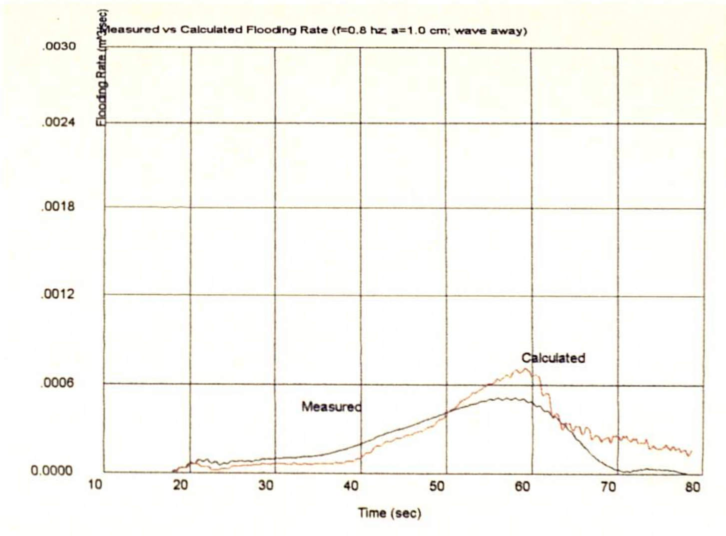

Measured Water on Deck and Flooding Rate (f=0.6 hz; a=3.0 cm; wave away)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)

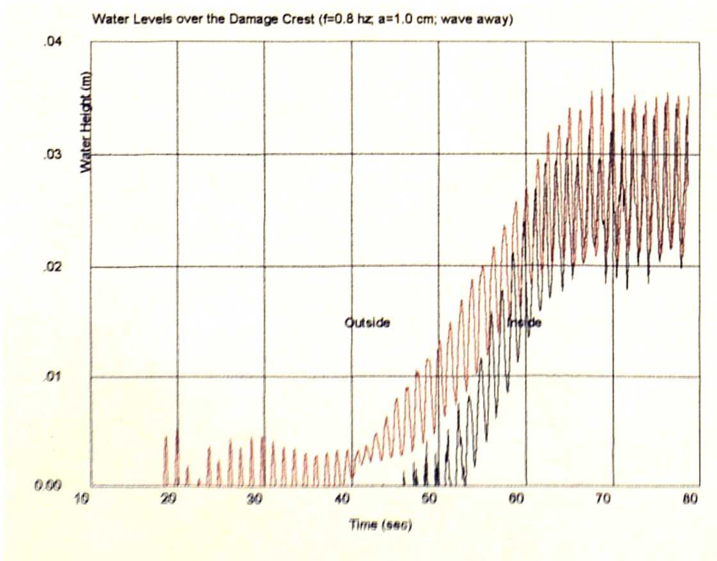

Measured vs Calculated Flooding Rate (f=0.6 hz; a=3.0 cm; wave away)
Flooding Rate (m^3/sec)
Calculated
Measured
Time (sec)

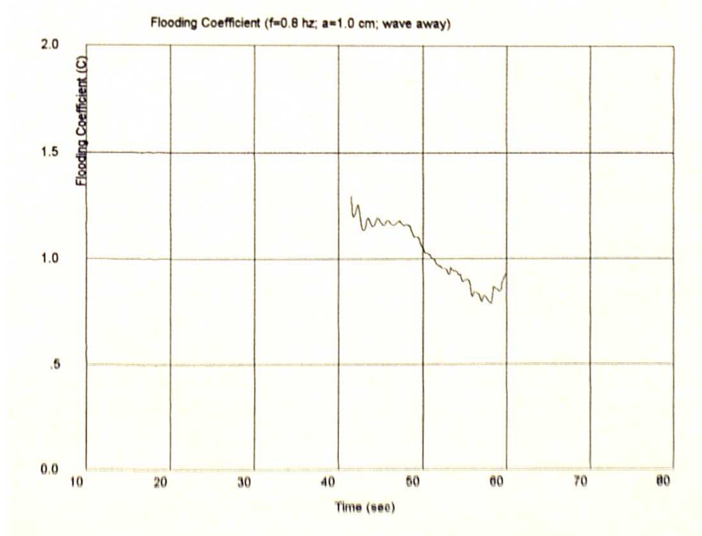

Water Levels over the Damage Crest (f=0.6 hz; a=3.0 cm; wave away)
Water Height (m)
Outside
Inside
Time (sec)


Flooding Coefficient (f=0.6 hz; a=3.0 cm; wave away)
Flooding Coefficient (C)
Time (sec)

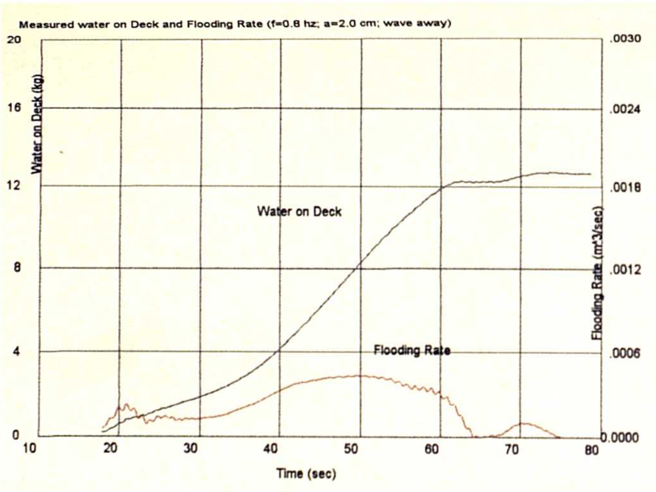

Measured Water on Deck and Flooding Rate (f=0.8 hz; a=1.0 cm; wave away)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)

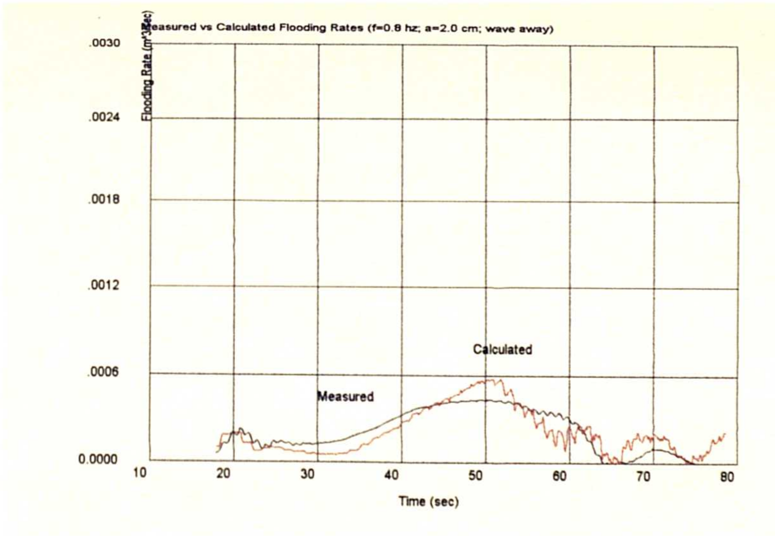

Measured vs Calculated Flooding Rate (f=0.8 hz; a=1.0 cm; wave away)
Flooding Rate (m^3/sec)
Calculated
Measured
Time (sec)

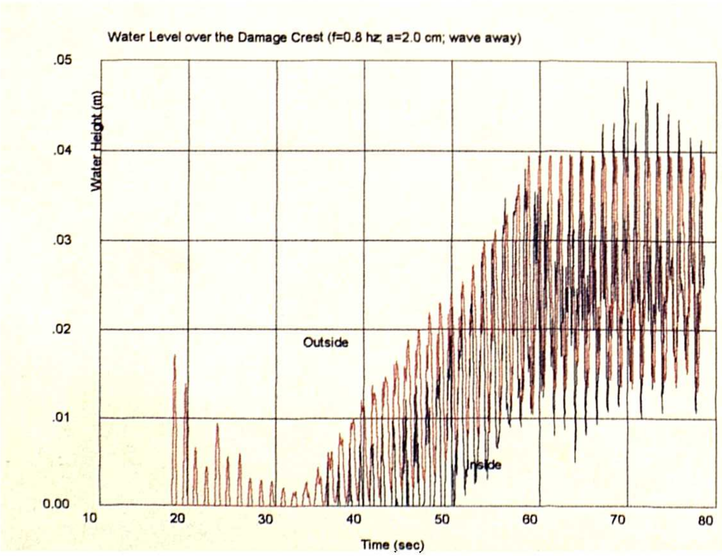

Water Levels over the Damage Crest (f=0.8 hz; a=1.0 cm; wave away)
Water Height (m)
Outside
Inside
Time (sec)

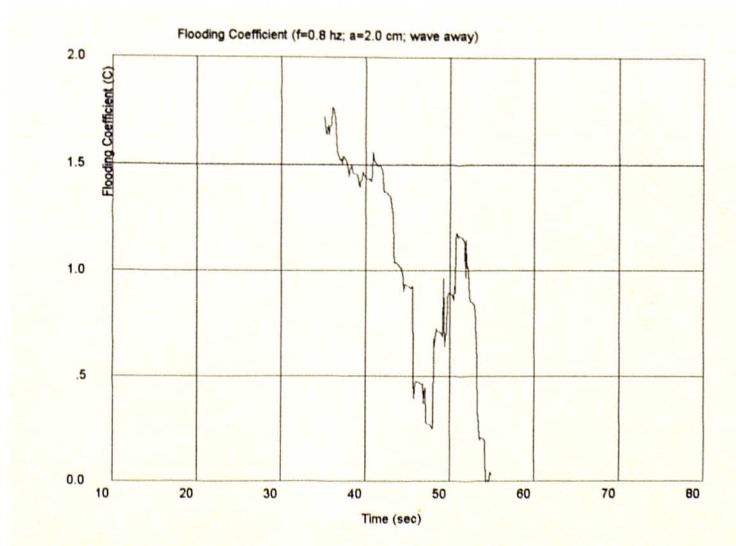

Flooding Coefficient (f=0.8 hz; a=1.0 cm; wave away)
Flooding Coefficient (C)
Time (sec)

39

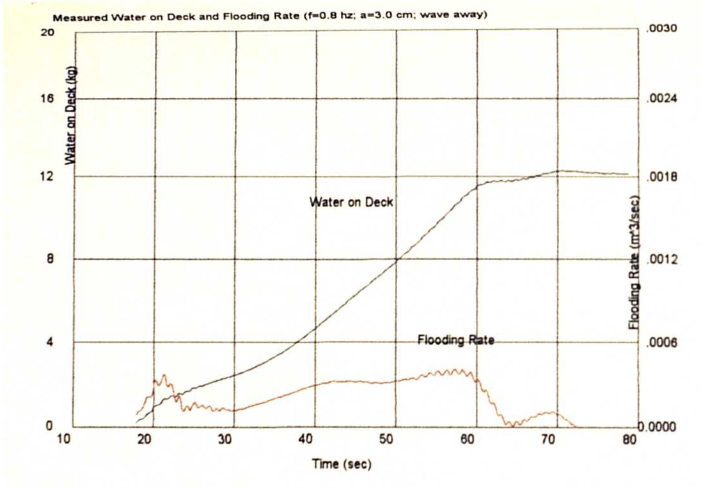

Measured water on Deck and Flooding Rate (f=0.8 hz; a=2.0 cm; wave away)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)


Measured vs Calculated Flooding Rates (f=0.8 hz; a=2.0 cm; wave away)
Flooding Rate (m^3/sec)
Calculated
Measured
Time (sec)

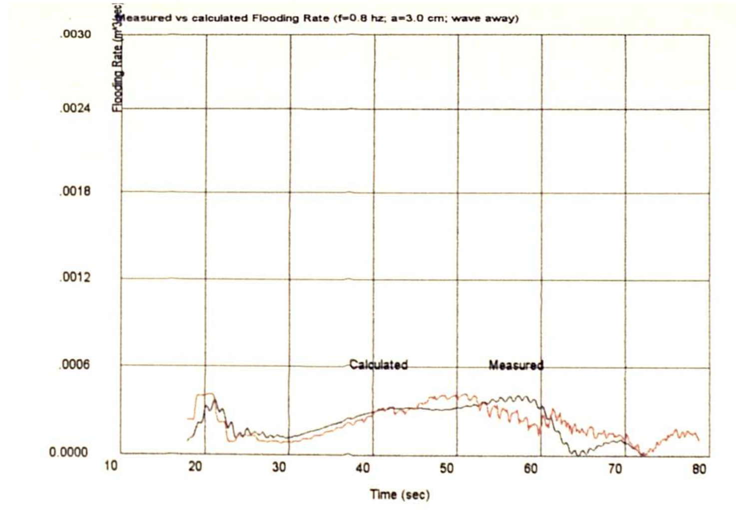

Water Level over the Damage Crest (f=0.8 hz; a=2.0 cm; wave away)
Water Height (m)
Outside
Inside
Time (sec)


Flooding Coefficient (f=0.8 hz; a=2.0 cm; wave away)
Flooding Coefficient (C)
Time (sec)

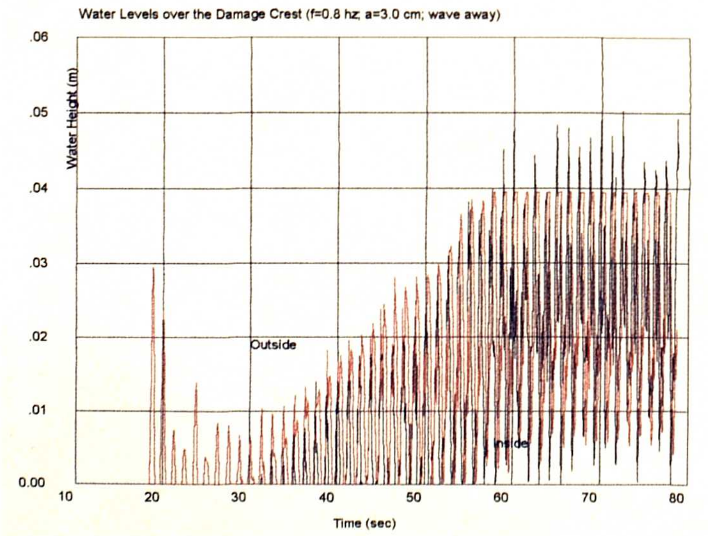
321



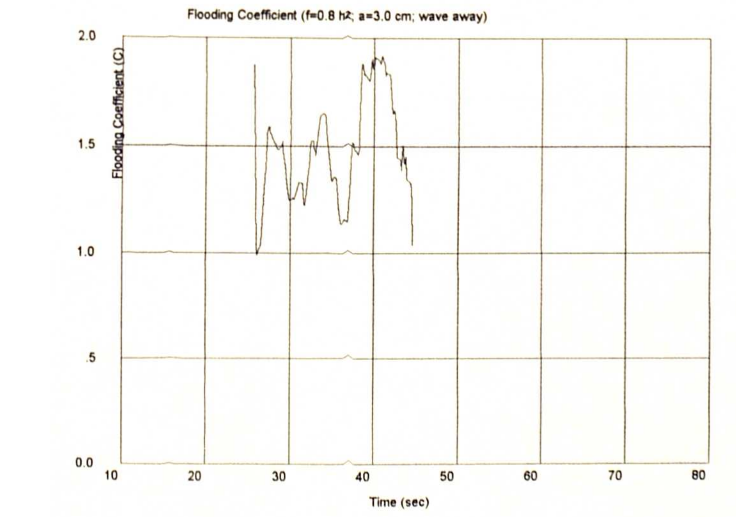
Measured Water on Deck and Flooding Rate (f=0.8 hz; a=3.0 cm; wave away)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)



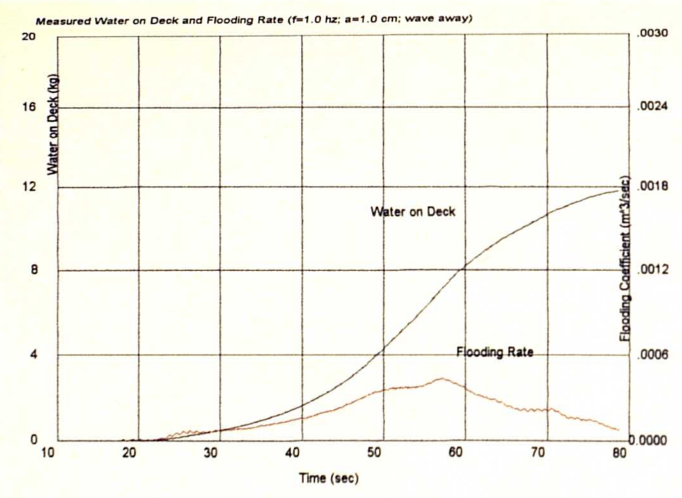
Measured vs calculated Flooding Rate (f=0.8 hz; a=3.0 cm; wave away)
Flooding Rate (m^3/sec)
Calculated
Measured
Time (sec)

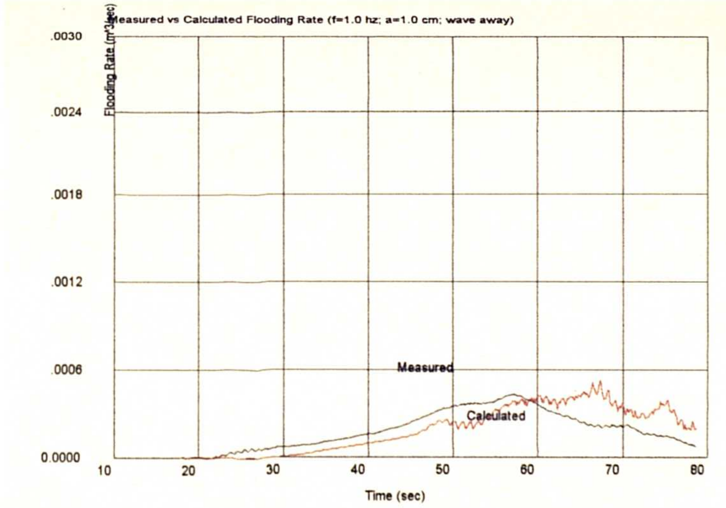


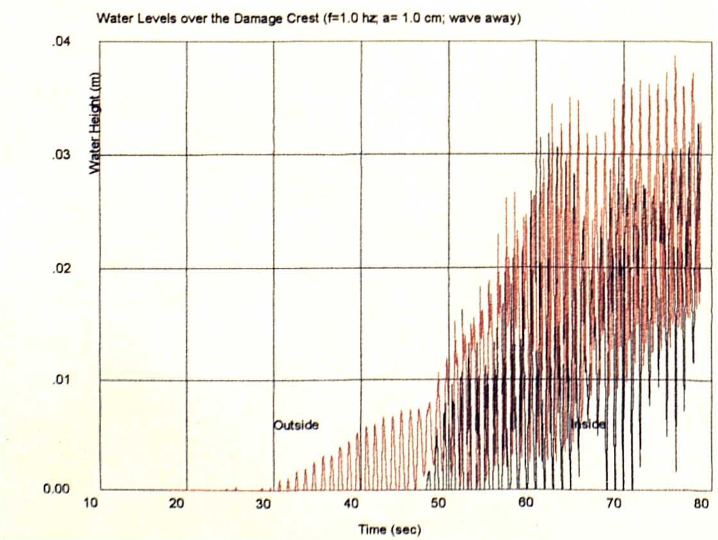
Water Levels over the Damage Crest (f=0.8 hz; a=3.0 cm; wave away)
Water Height (m)
Outside
Inside
Time (sec)

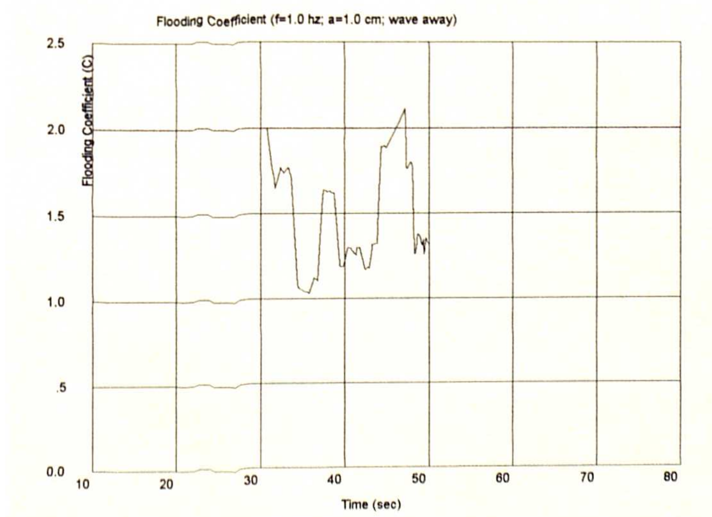


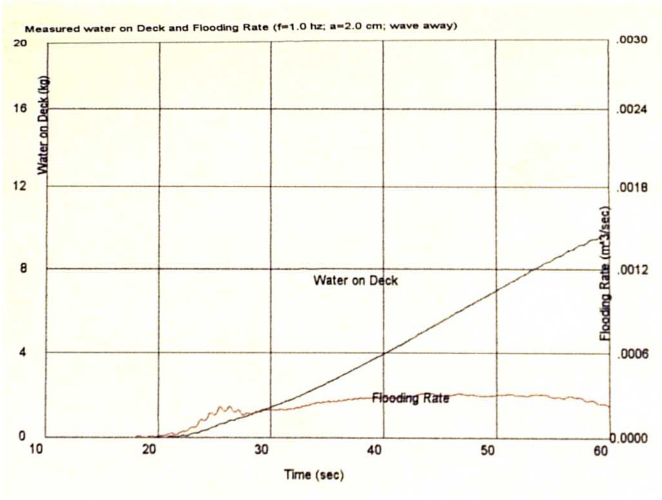
Flooding Coefficient (f=0.8 hz; a=3.0 cm; wave away)
Flooding Coefficient (C)
Time (sec)

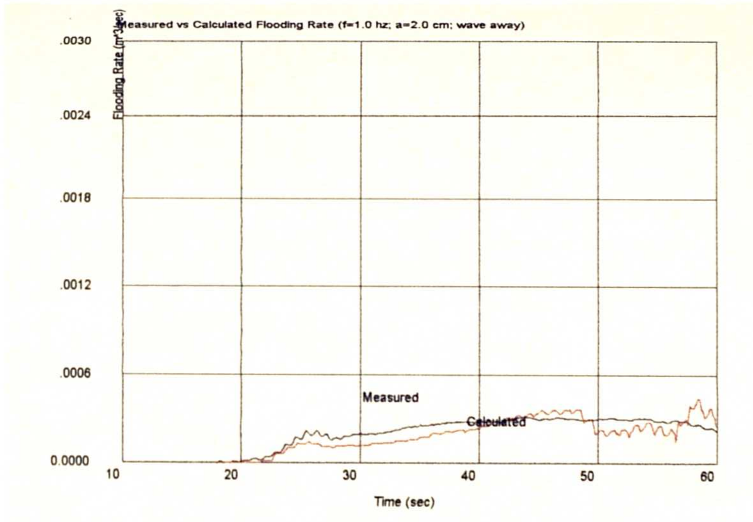

Measured Water on Deck and Flooding Rate (f=1.0 hz; a=1.0 cm; wave away)
Water on Deck (kg)
Flooding Coefficient (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)

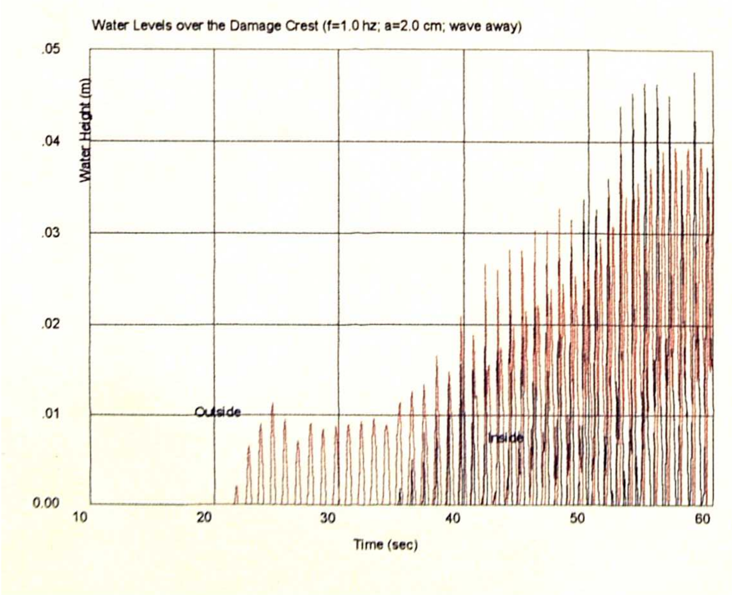

Measured vs Calculated Flooding Rate (f=1.0 hz; a=1.0 cm; wave away)
Flooding Rate (m^3/sec)
Measured
Calculated
Time (sec)

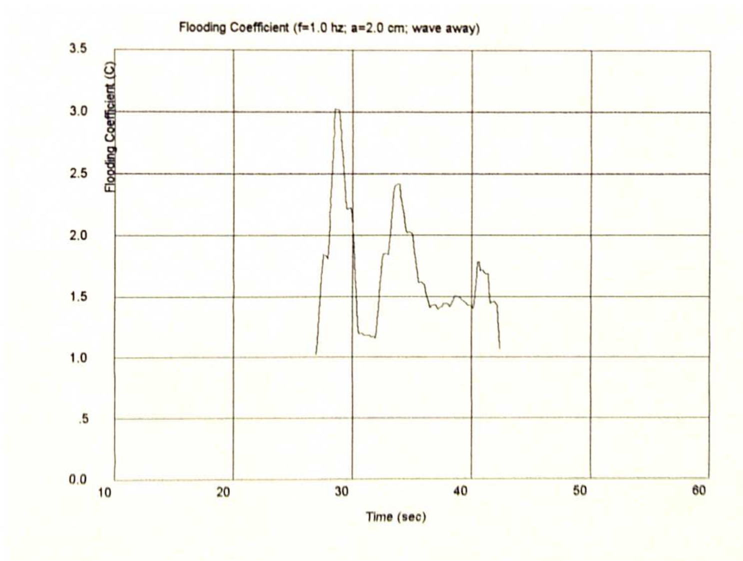

Water Levels over the Damage Crest (f=1.0 hz; a= 1.0 cm; wave away)
Water Height (m)
Outside
Inside
Time (sec)

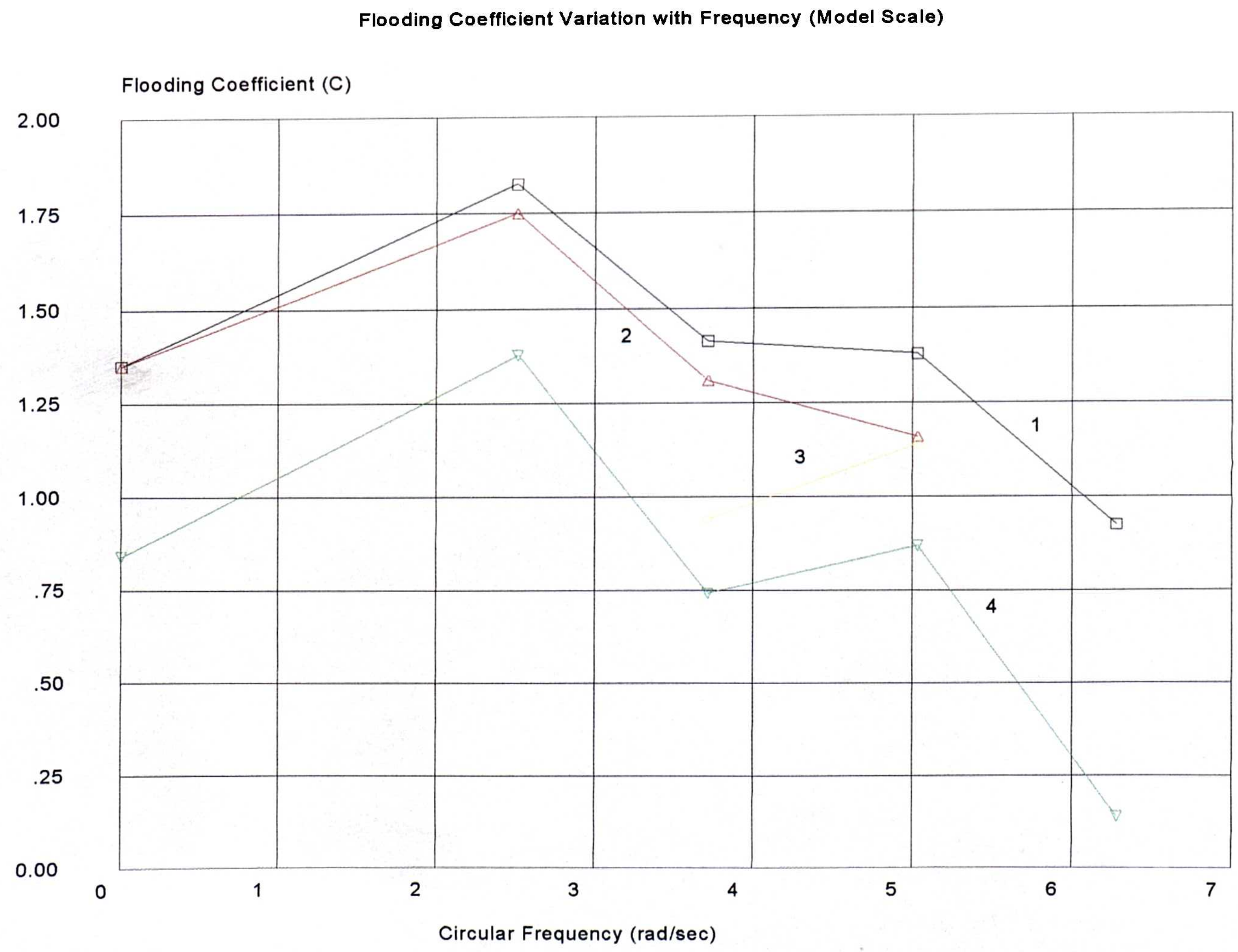

Flooding Coefficient (f=1.0 hz; a=1.0 cm; wave away)
Flooding Coefficient (C)
Time (sec)


Measured water on Deck and Flooding Rate (f=1.0 hz; a=2.0 cm; wave away)
Water on Deck (kg)
Flooding Rate (m^3/sec)
Water on Deck
Flooding Rate
Time (sec)


Measured vs Calculated Flooding Rate (f=1.0 hz; a=2.0 cm; wave away)
Flooding Rate (m^3/sec)
Measured
Calculated
Time (sec)


Water Levels over the Damage Crest (f=1.0 hz; a=2.0 cm; wave away)
Water Height (m)
Outside
Inside
Time (sec)


Flooding Coefficient (f=1.0 hz; a=2.0 cm; wave away)
Flooding Coefficient (C)
Time (sec)


Flooding Coefficient Variation with Frequency (Model Scale)
Flooding Coefficient (C)
2.00
1.75
1.50
1.25
1.00
.75
.50
.25
0.00
0
1
2
3
4
5
6
7
1
2
3
4
Circular Frequency (rad/sec)

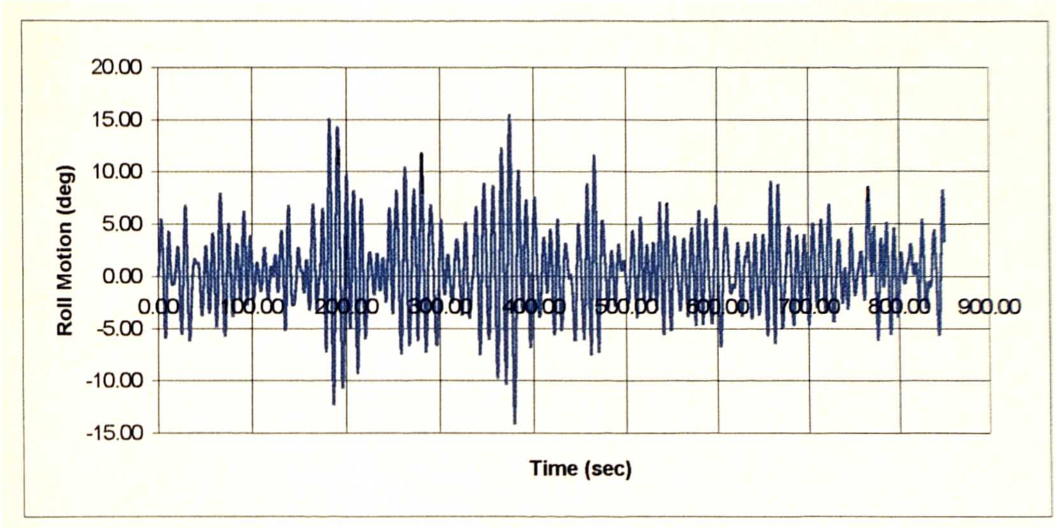
Appendix - D

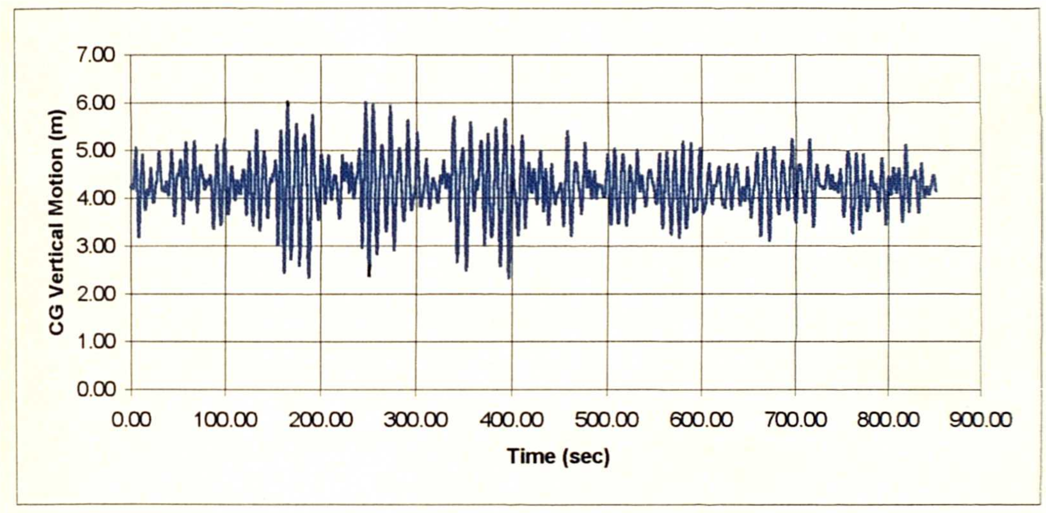
Numerical Experiments

Graphs of the quantities of interest for the code validation and the sensitivity analysis are collected in this appendix (see Chapter 9).

Simulation No. 101

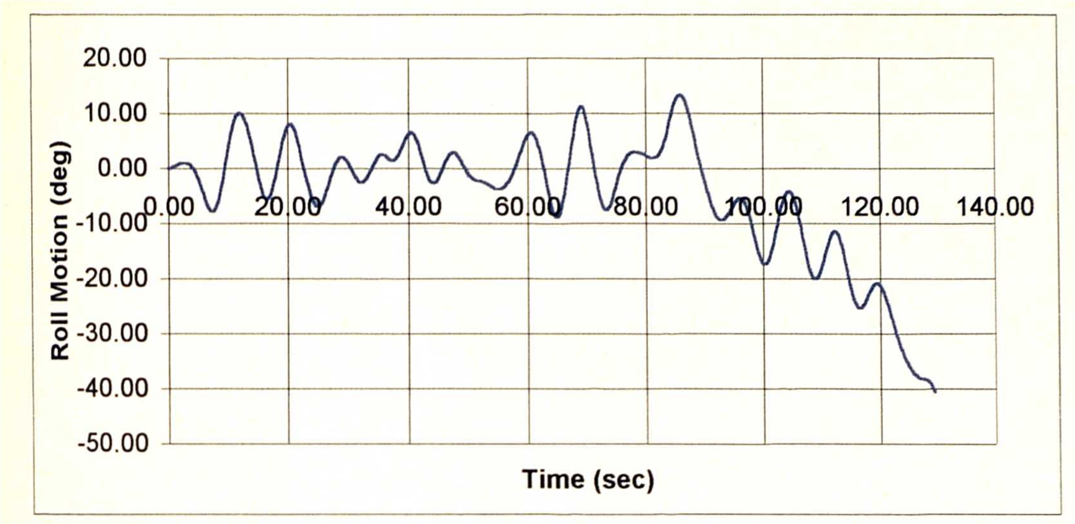
Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

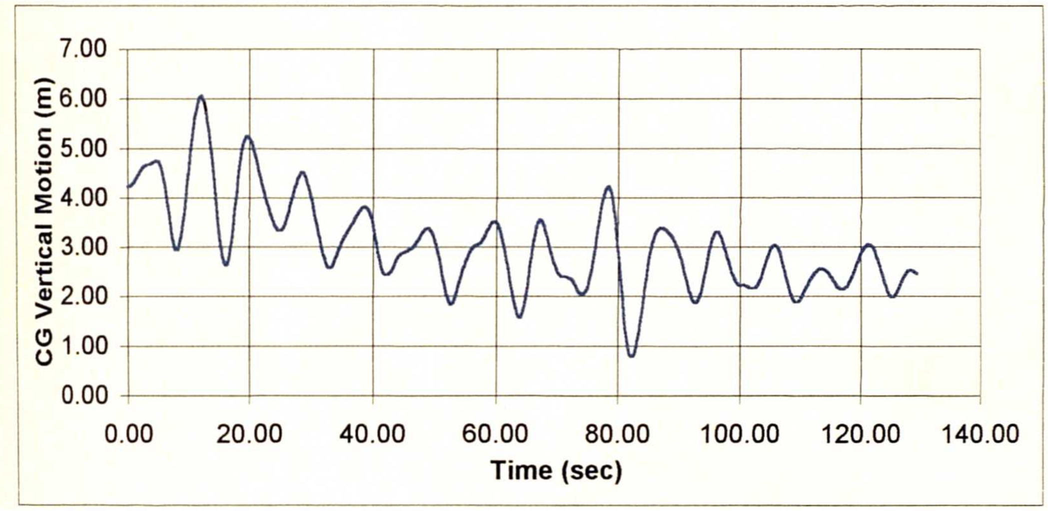


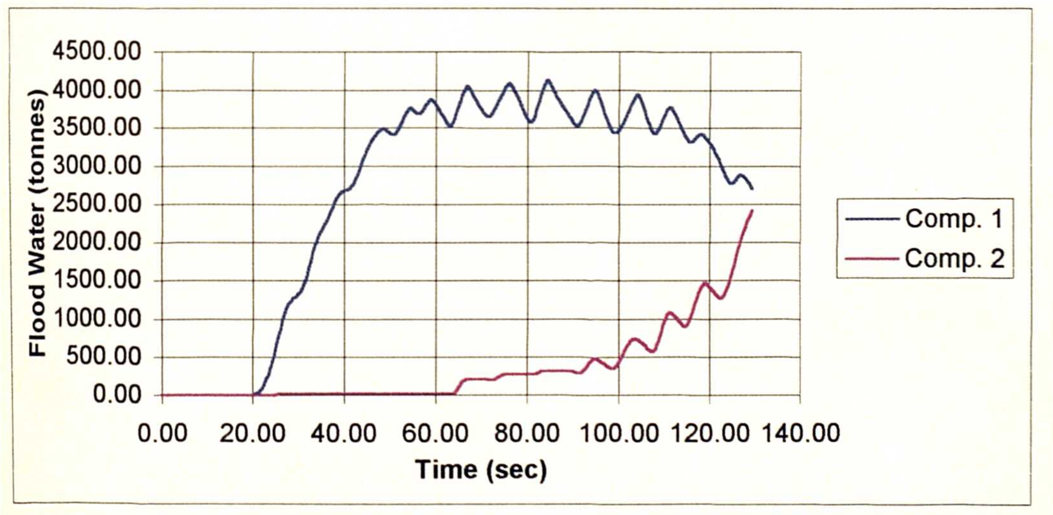


Simulation No. 102

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

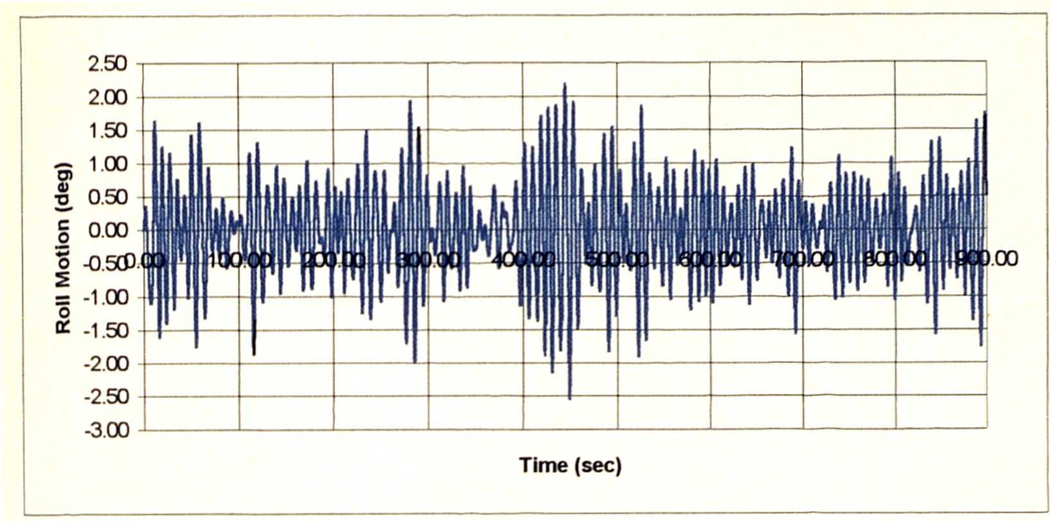


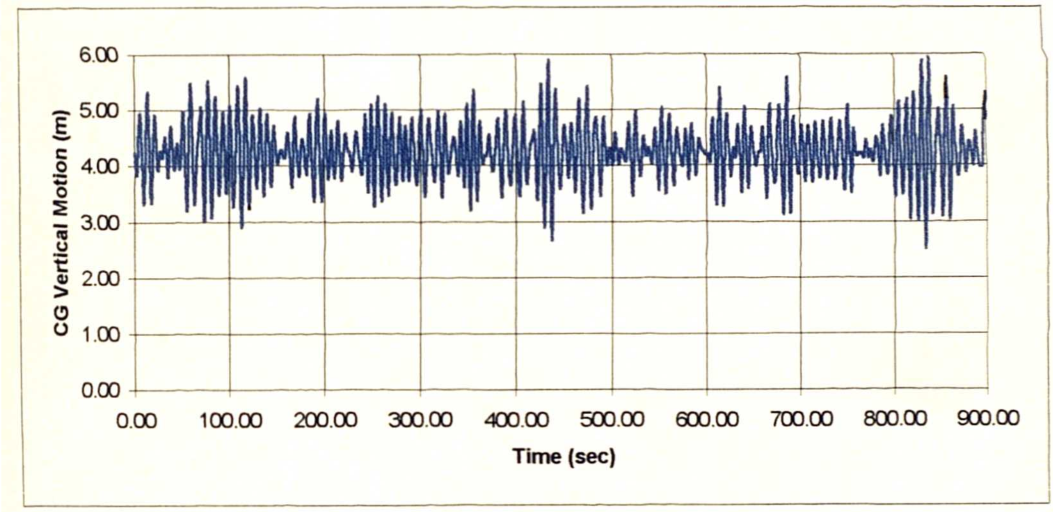


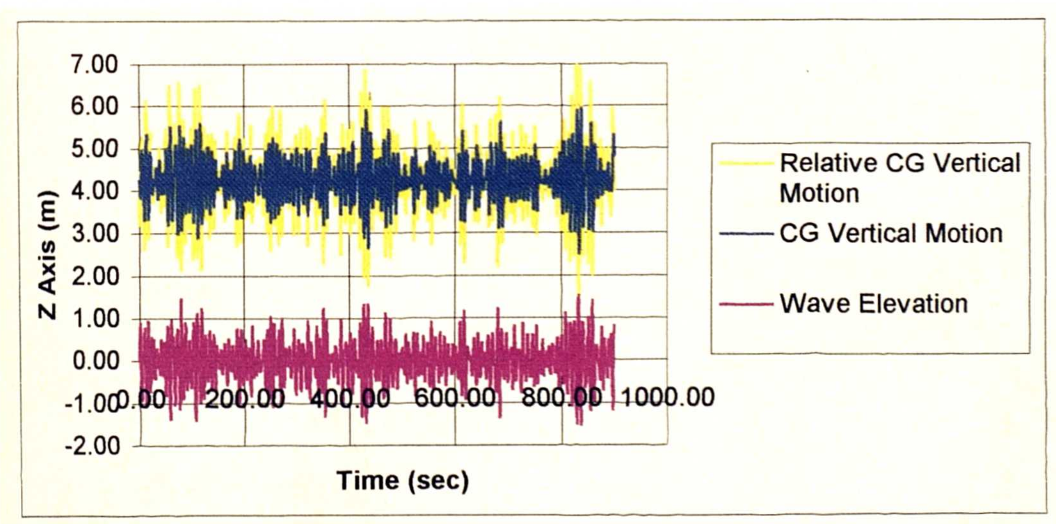


Simulation No. 103

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

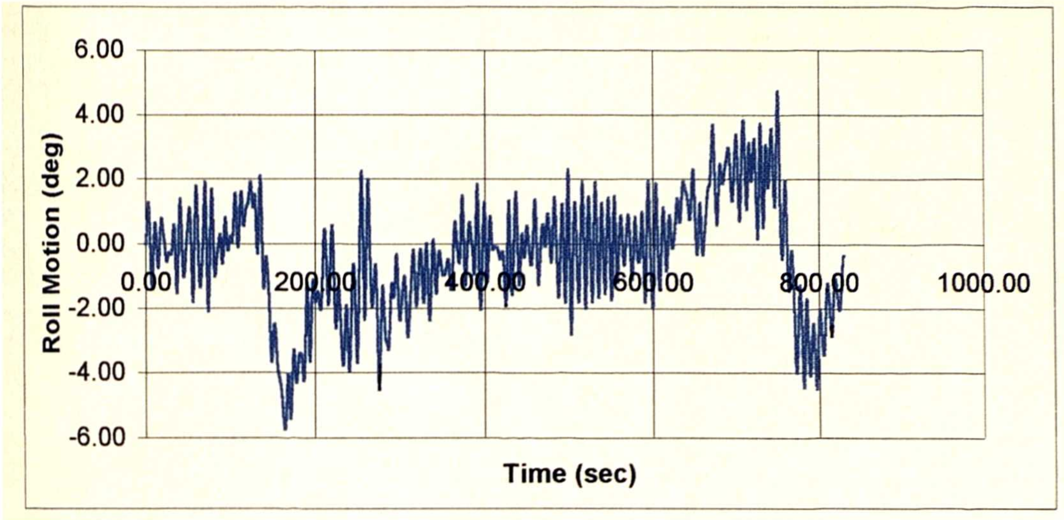


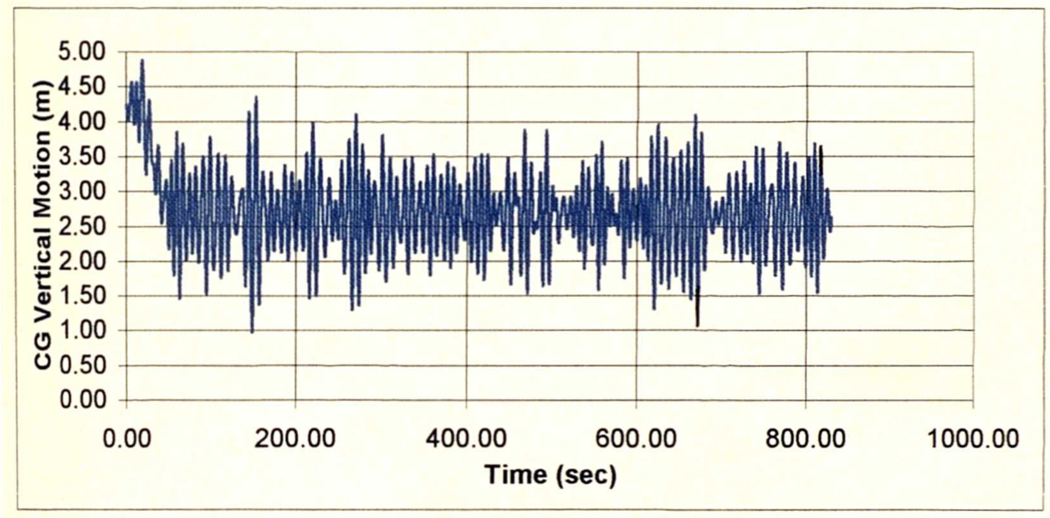


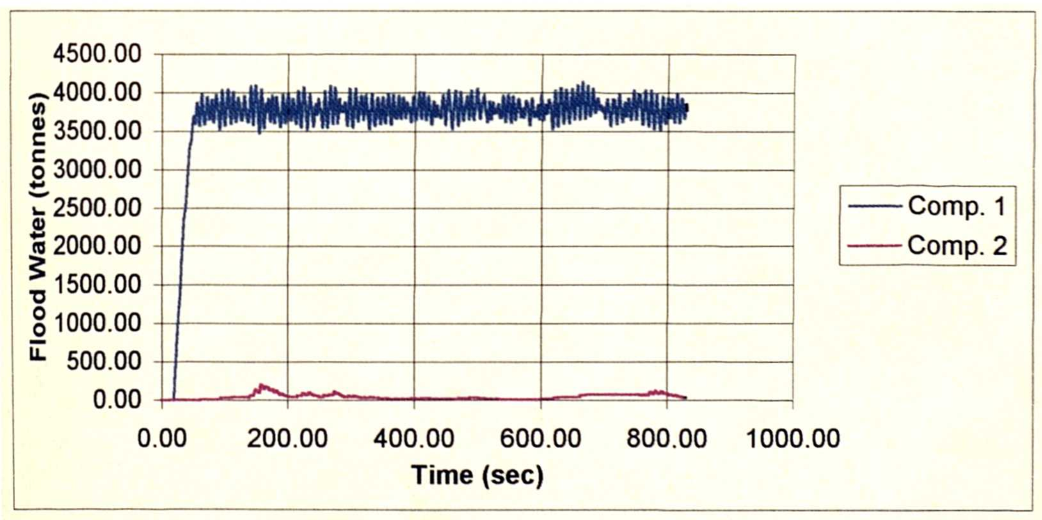


Simulation No. 104

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

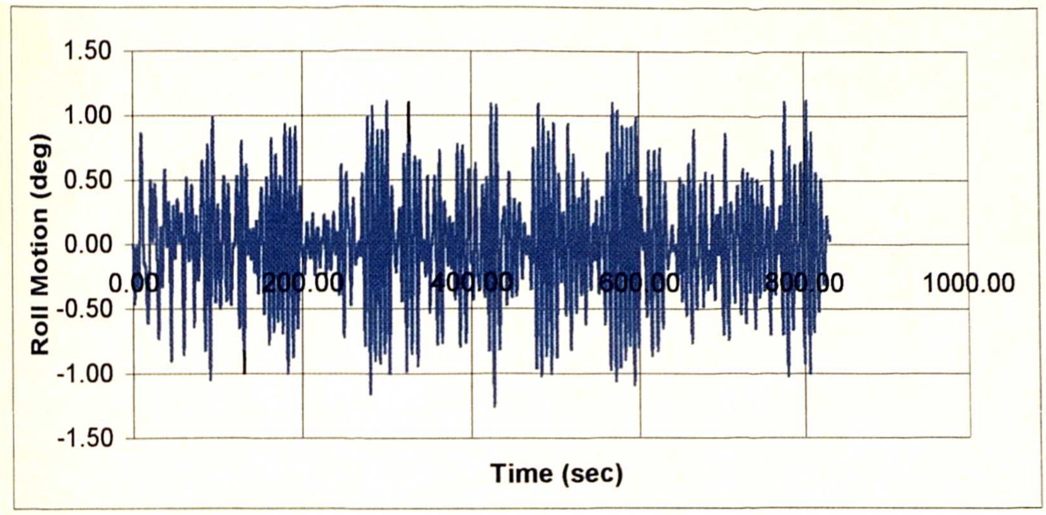


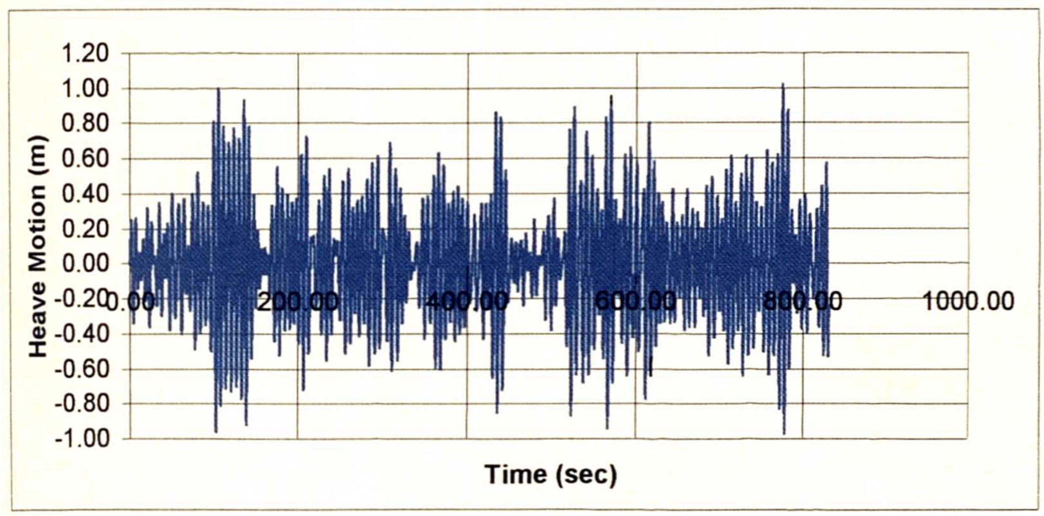




Simulation No. 105

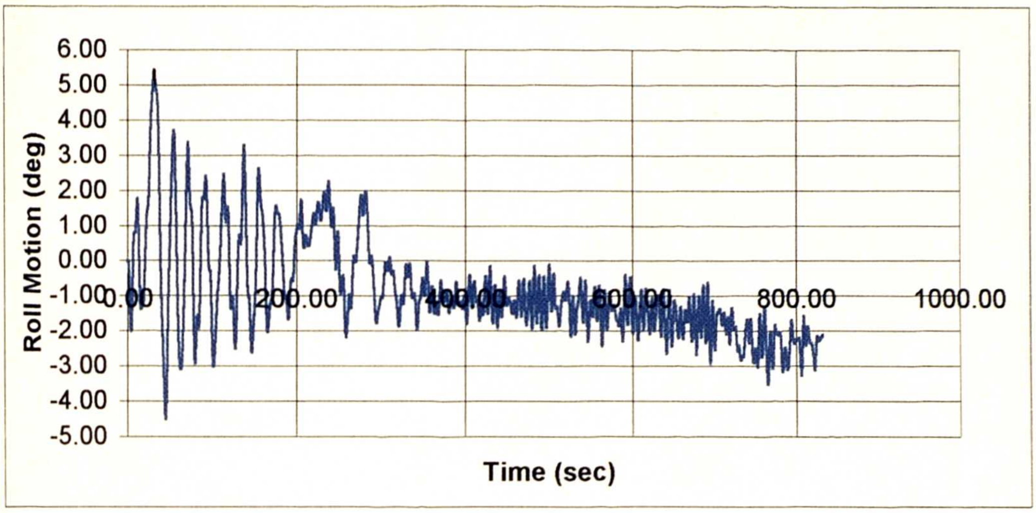
Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

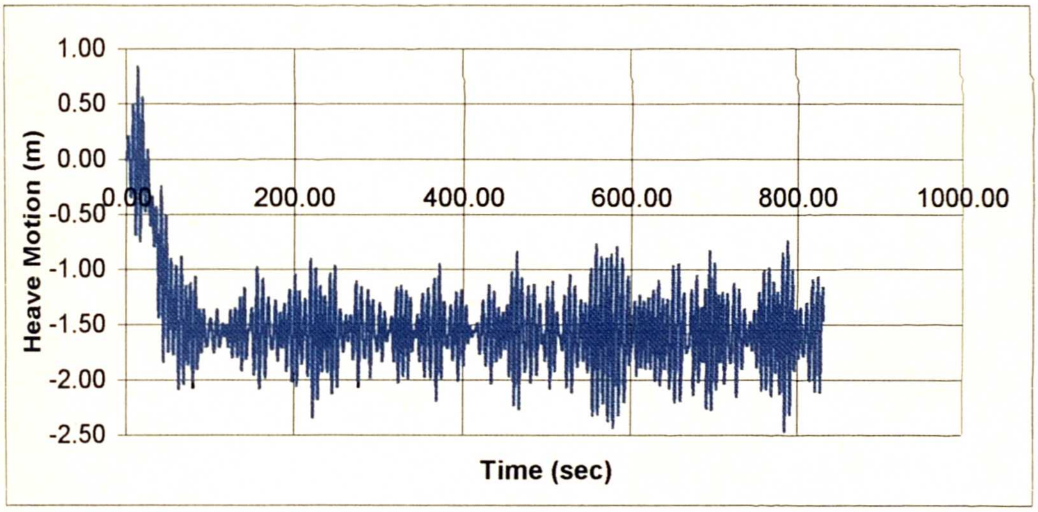


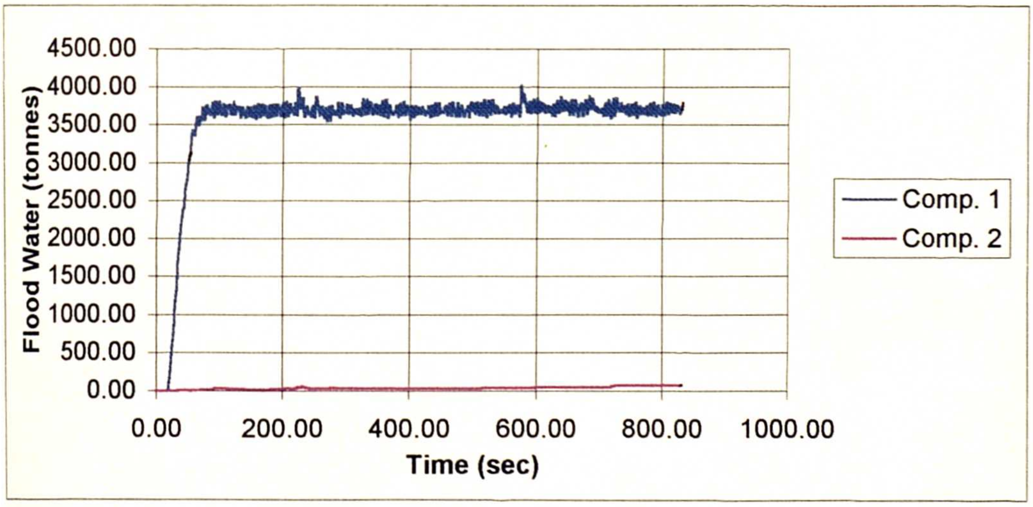


Simulation No. 106

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

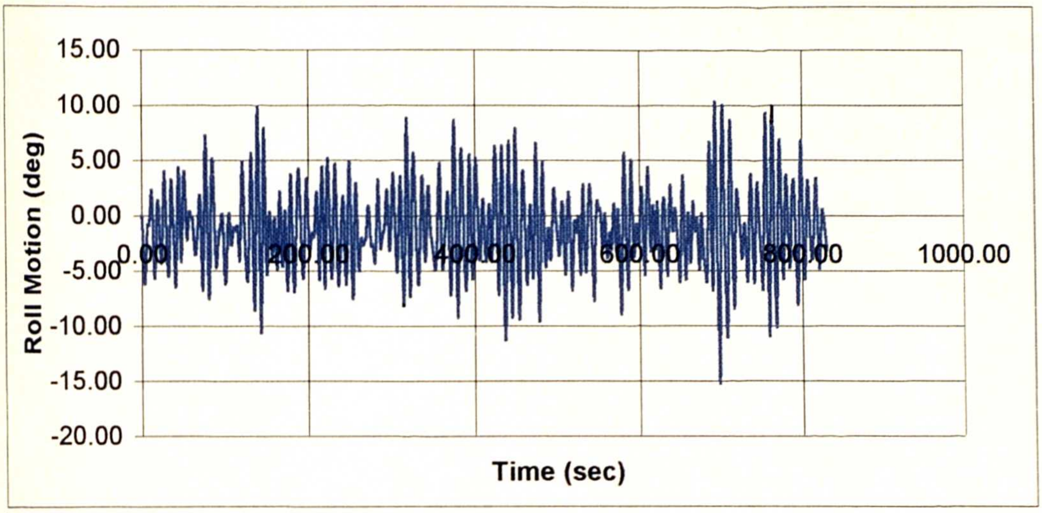


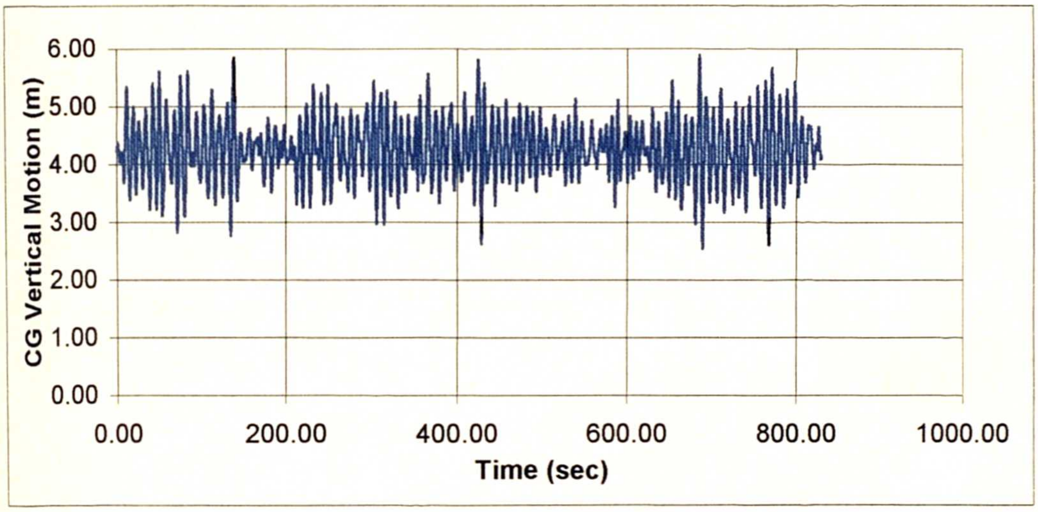




Simulation No. 107

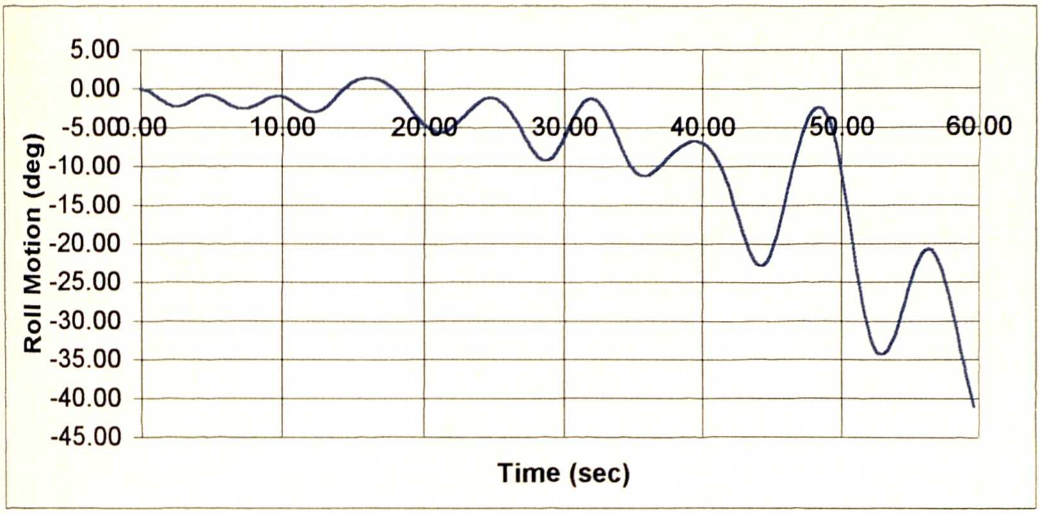
Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

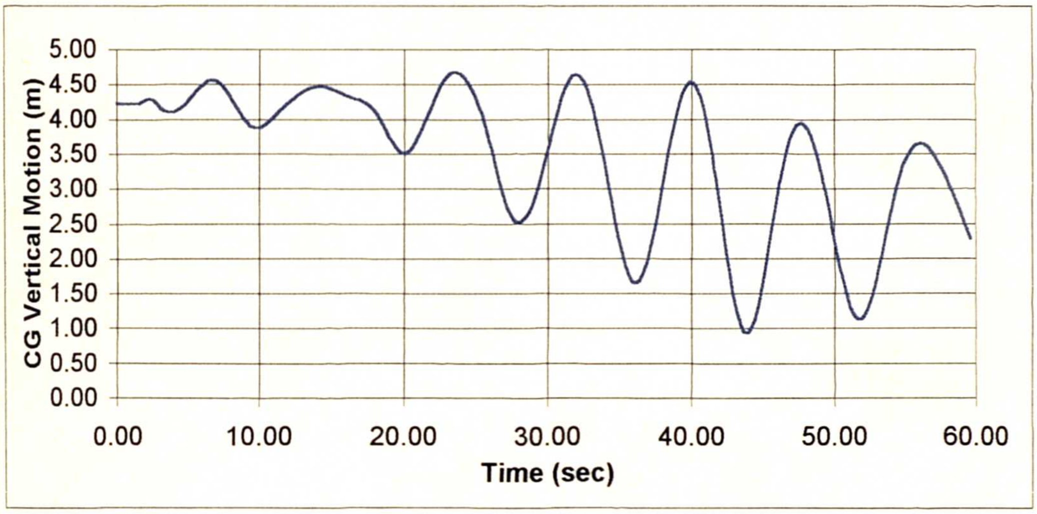


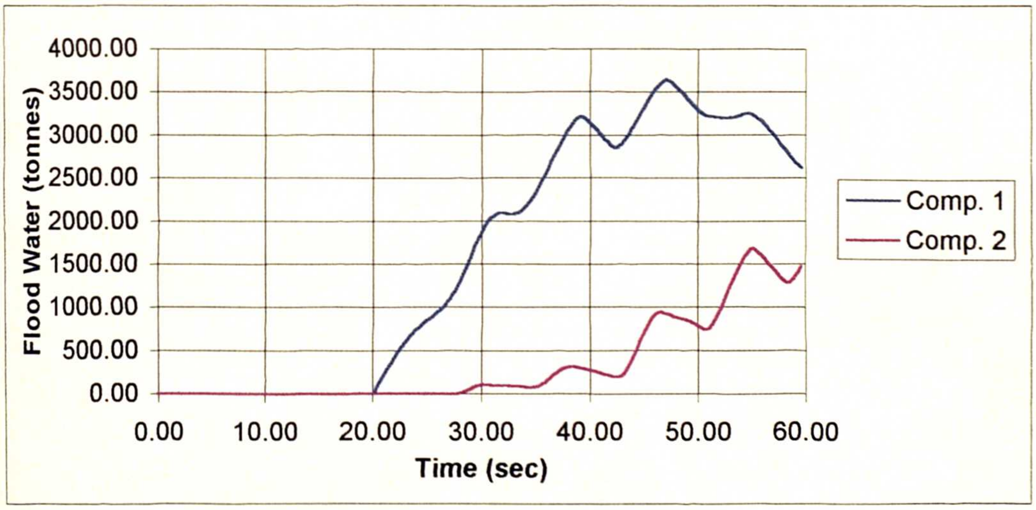


Simulation No. 108

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

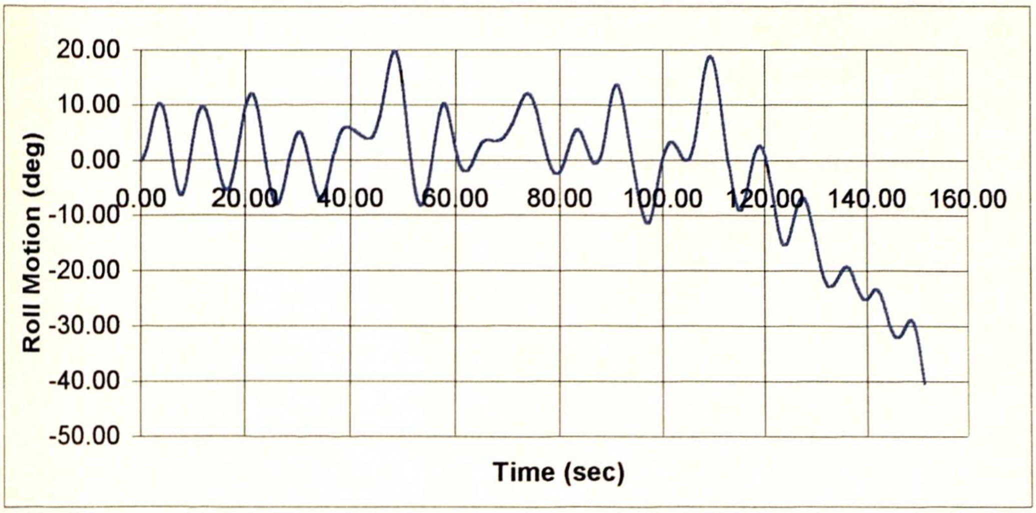


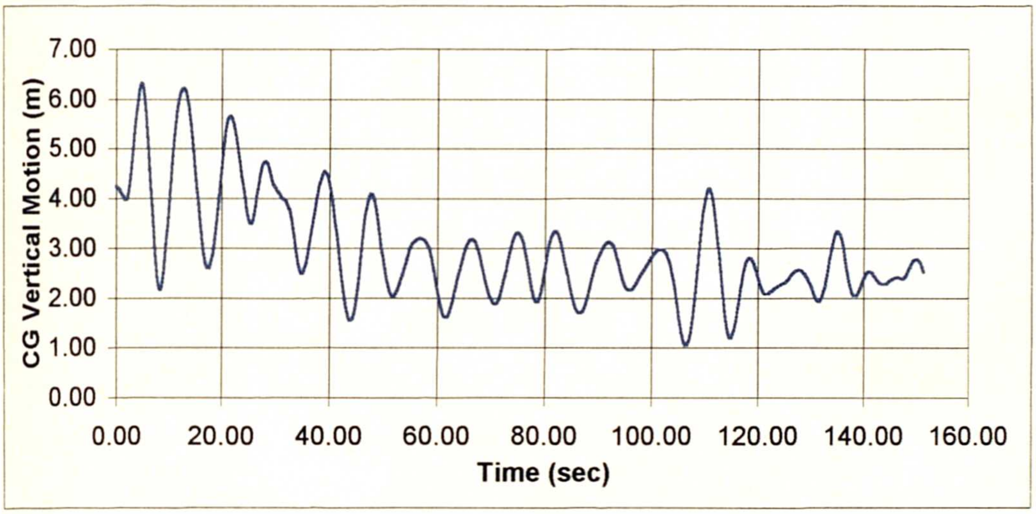


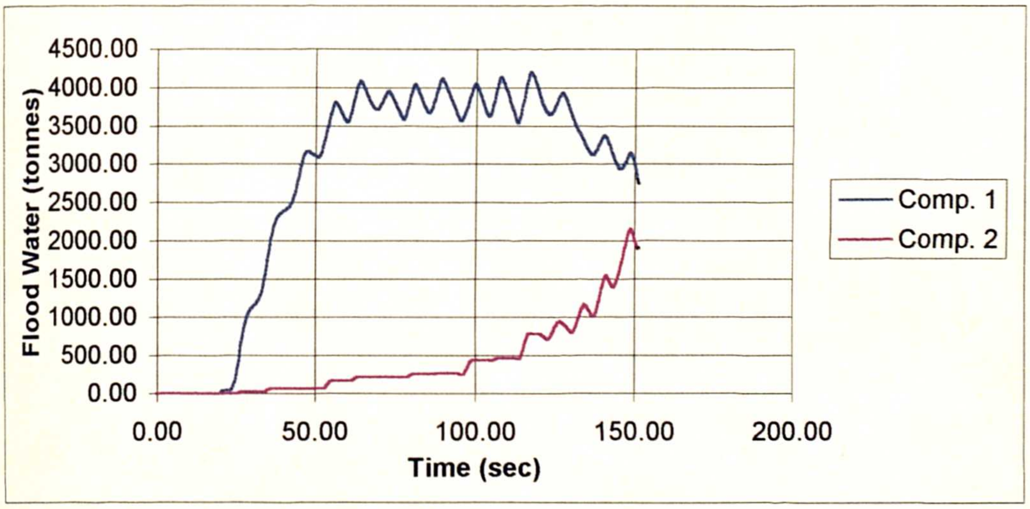


Simulation No. 109

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

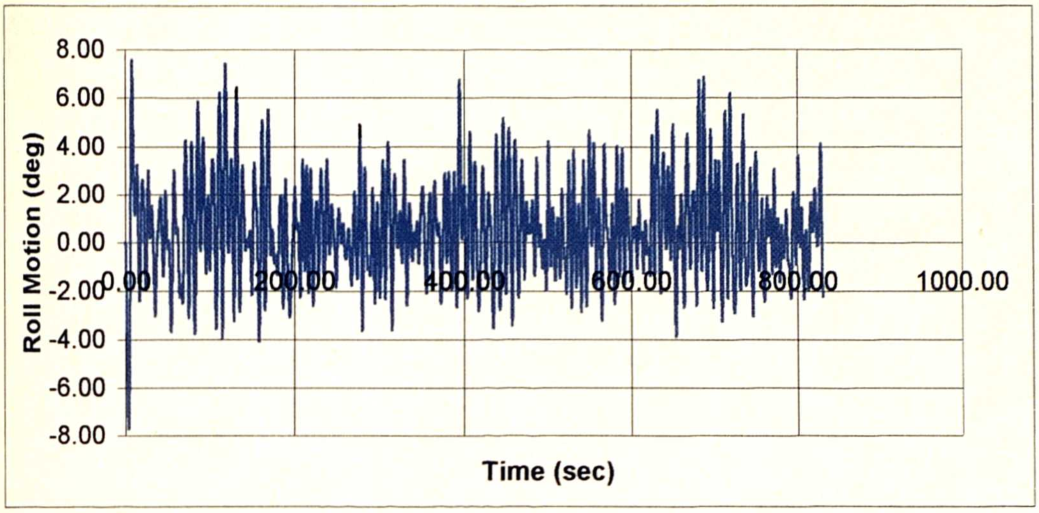


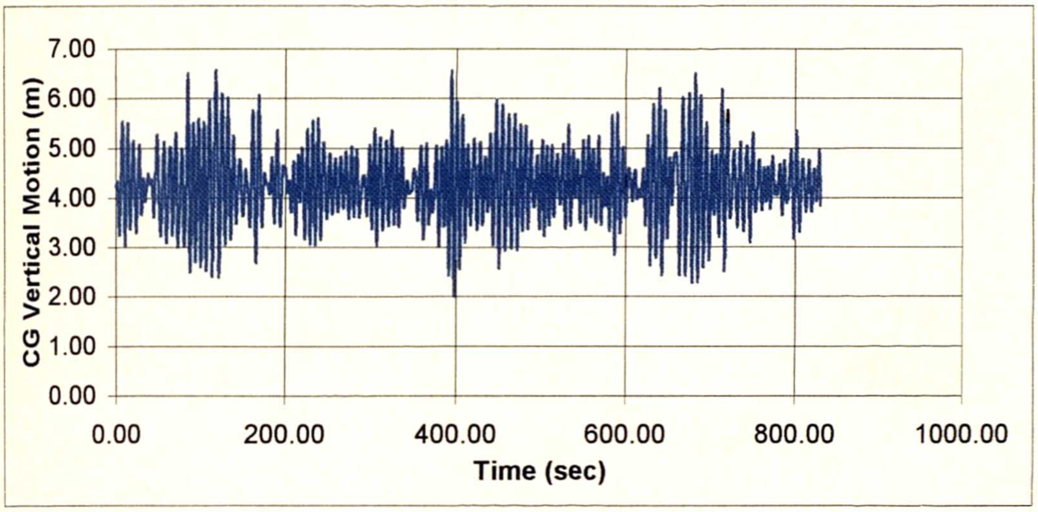




Simulation No. 110

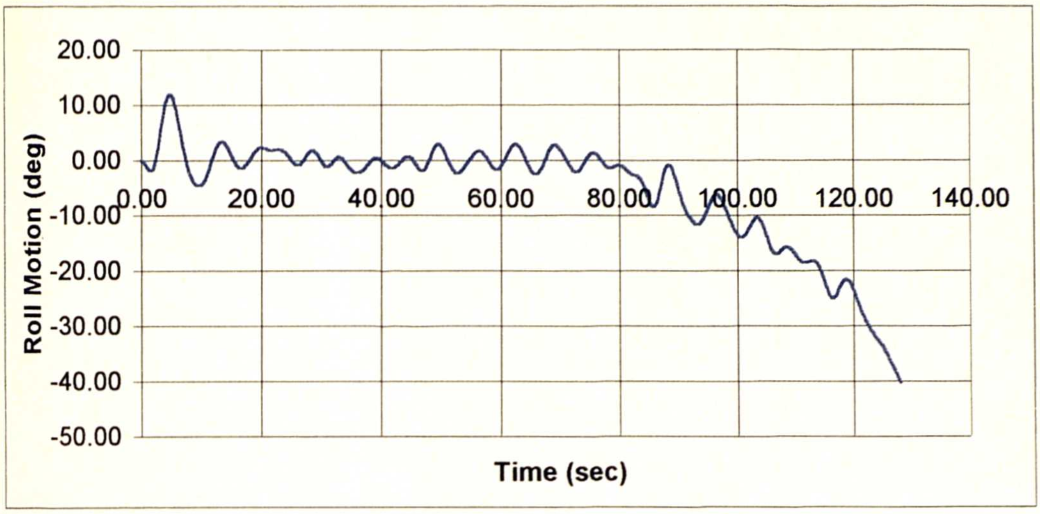
Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

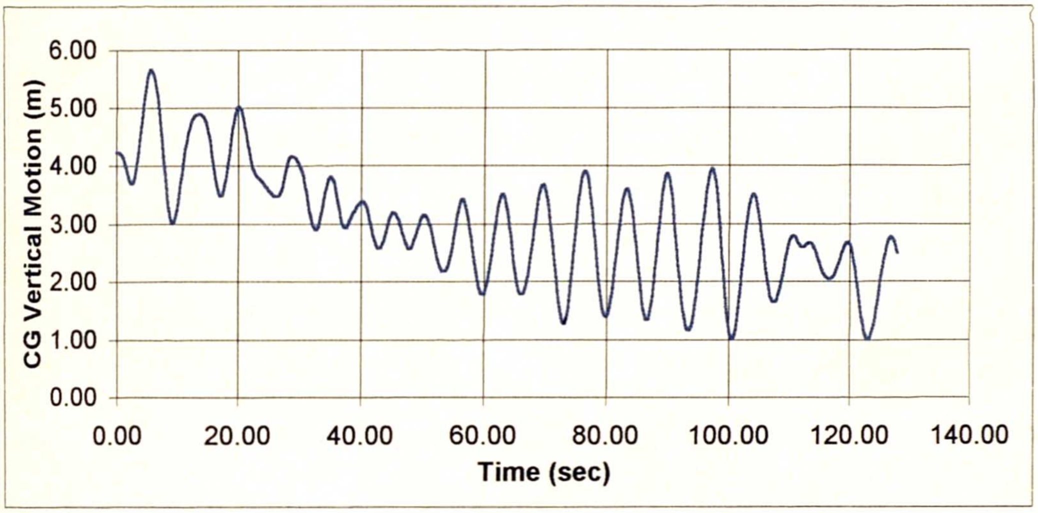


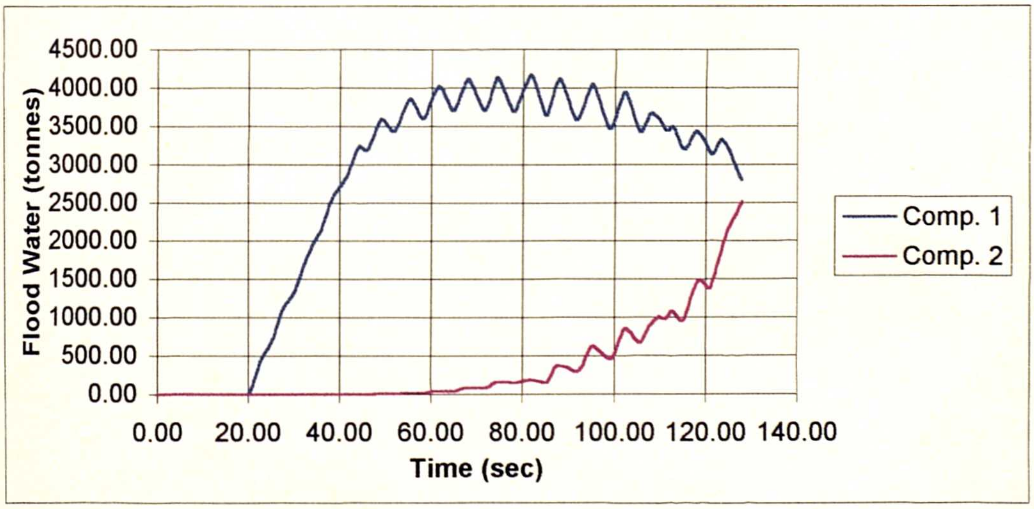


Simulation No. 111

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

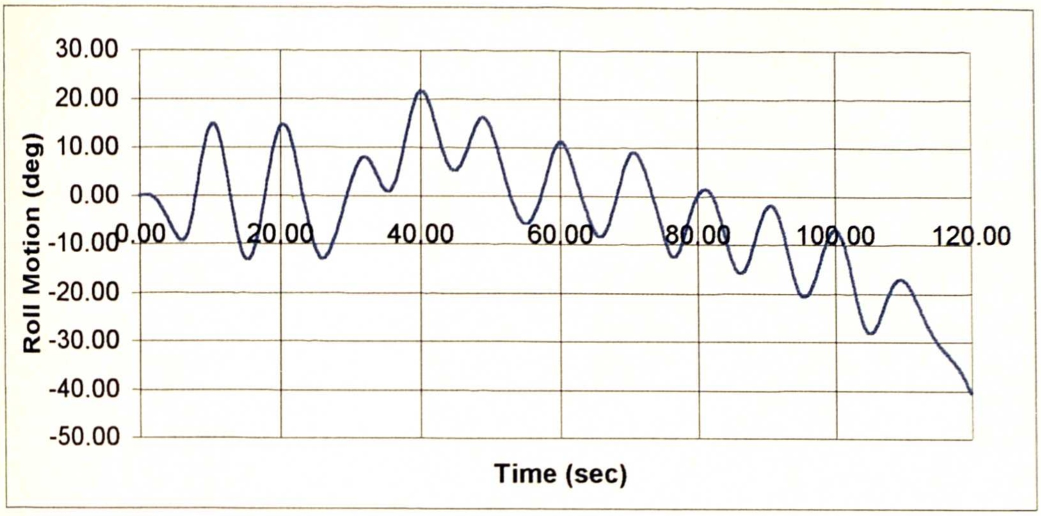


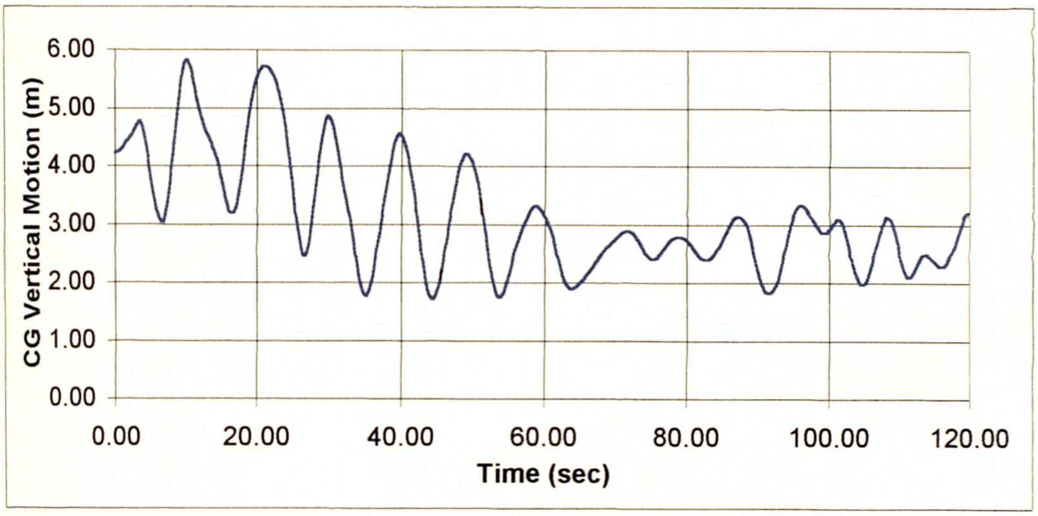


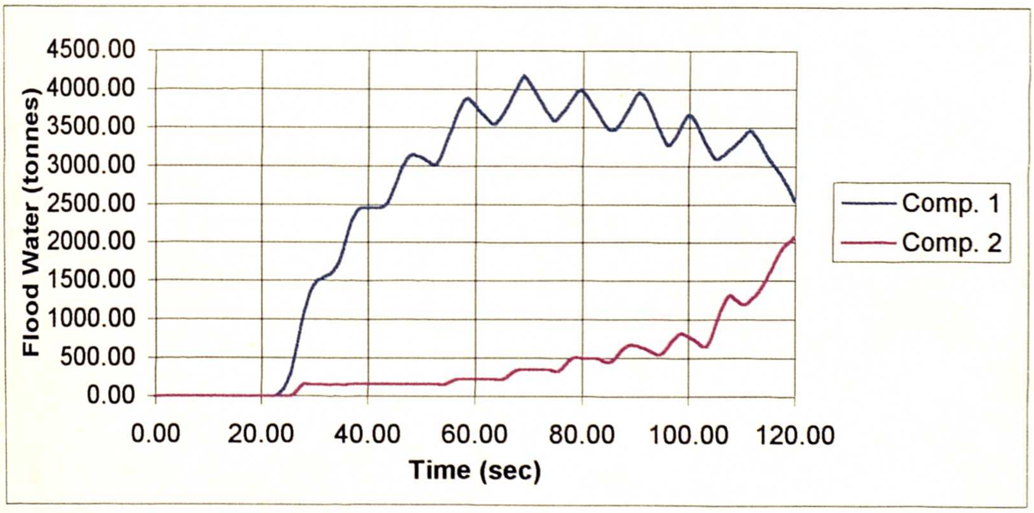


Simulation No. 112

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

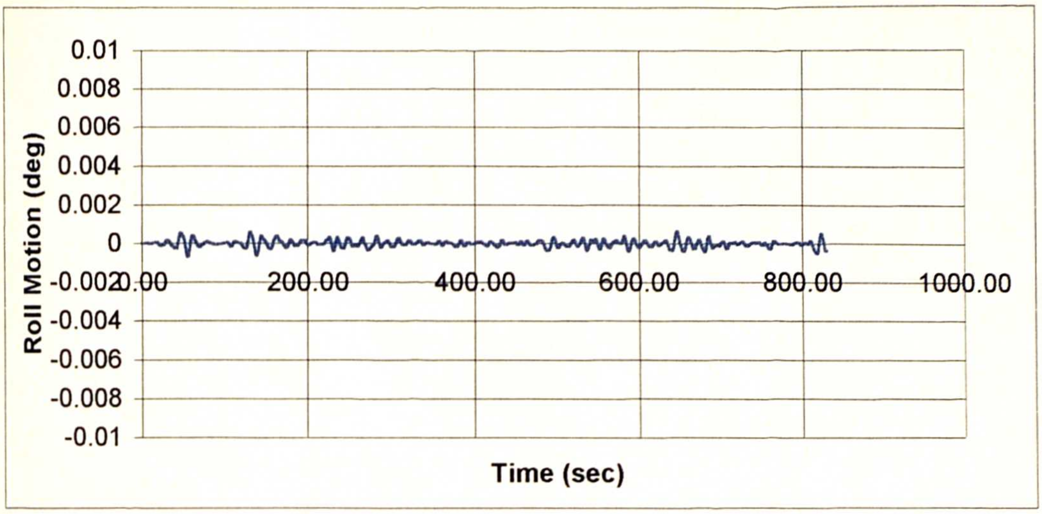


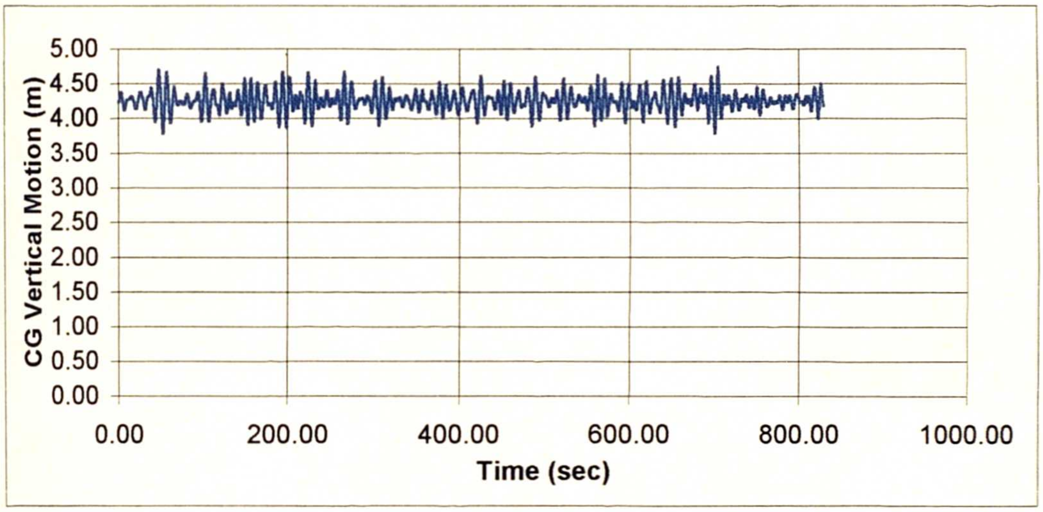


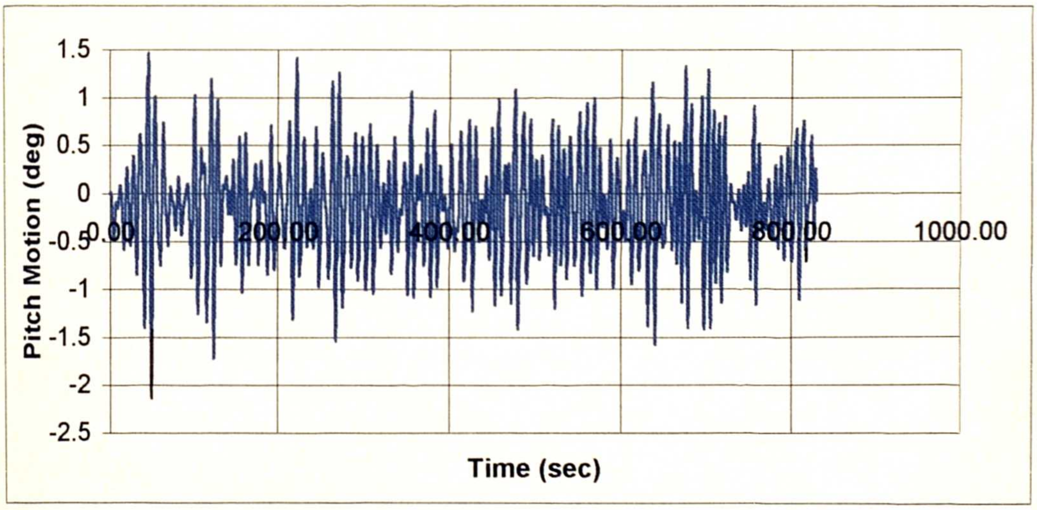


Simulation No. 113

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

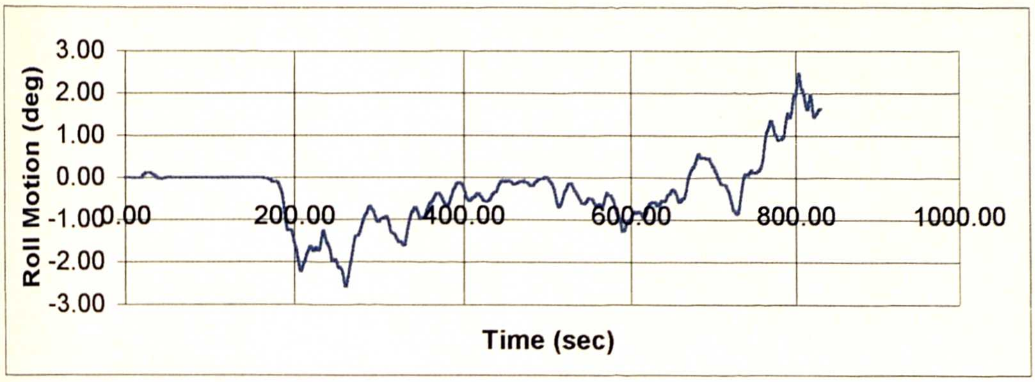


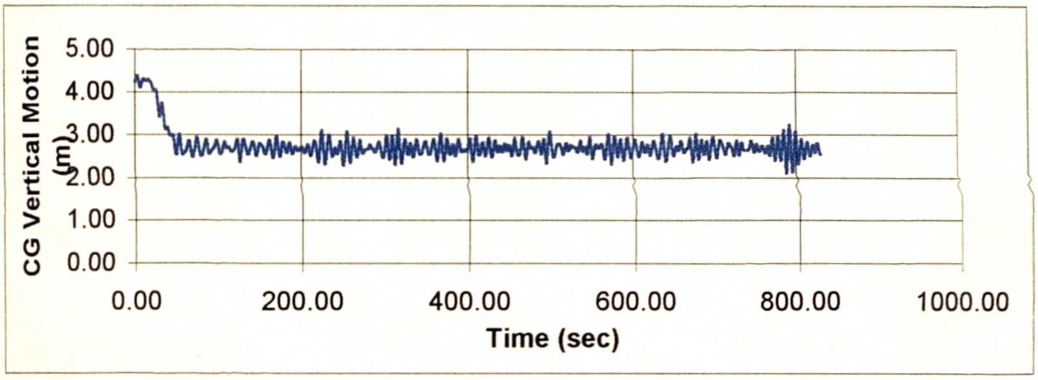


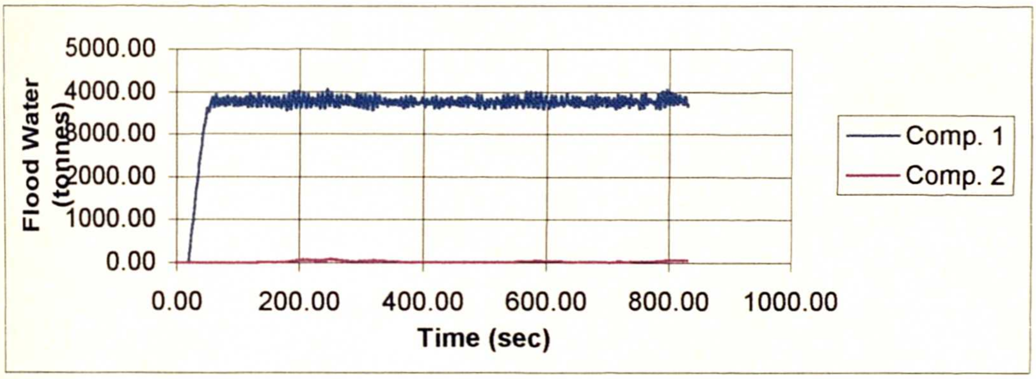


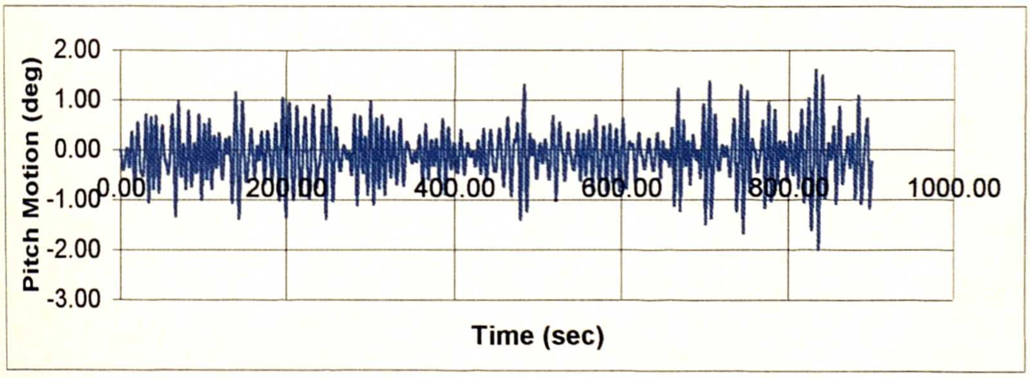
Simulation No. 114

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m



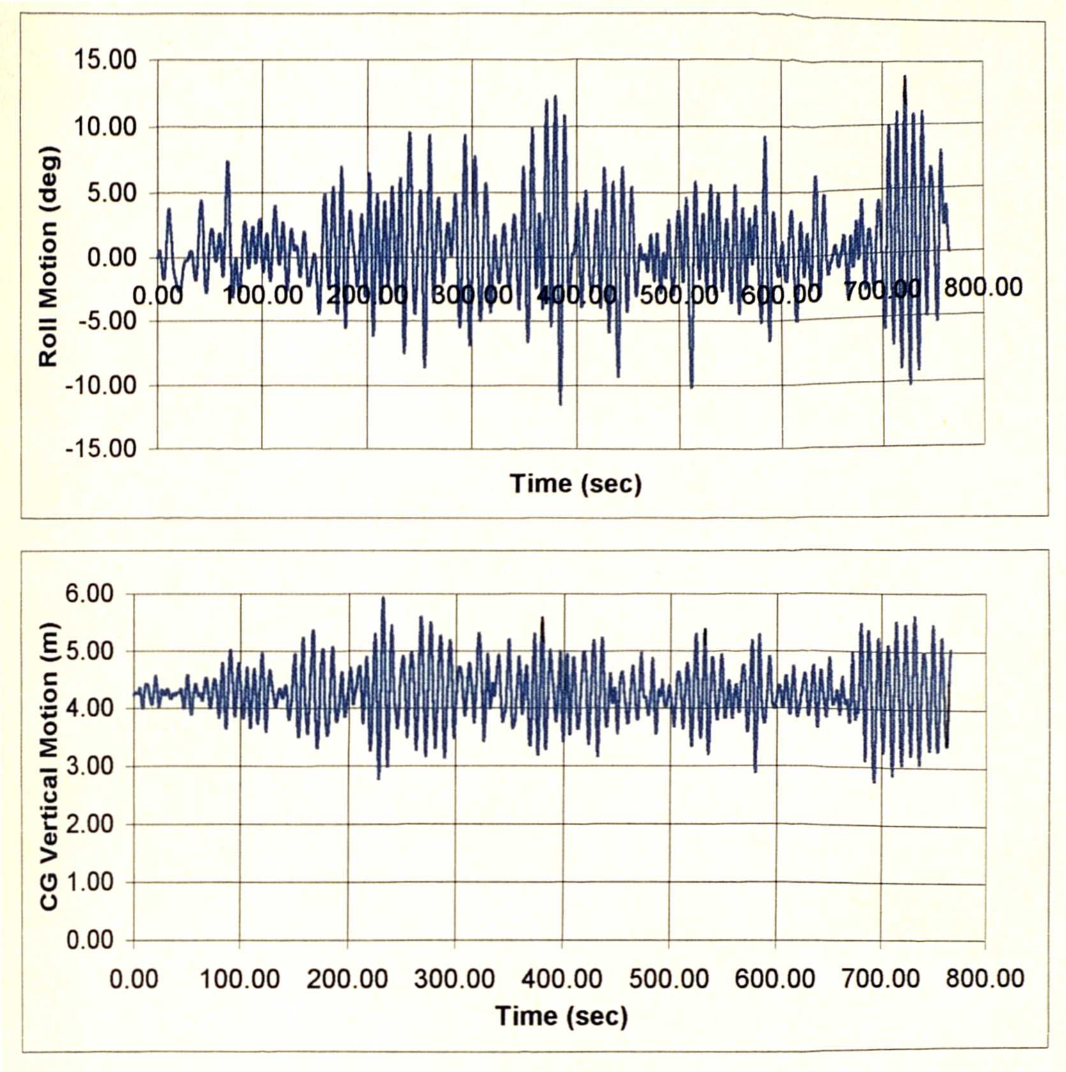






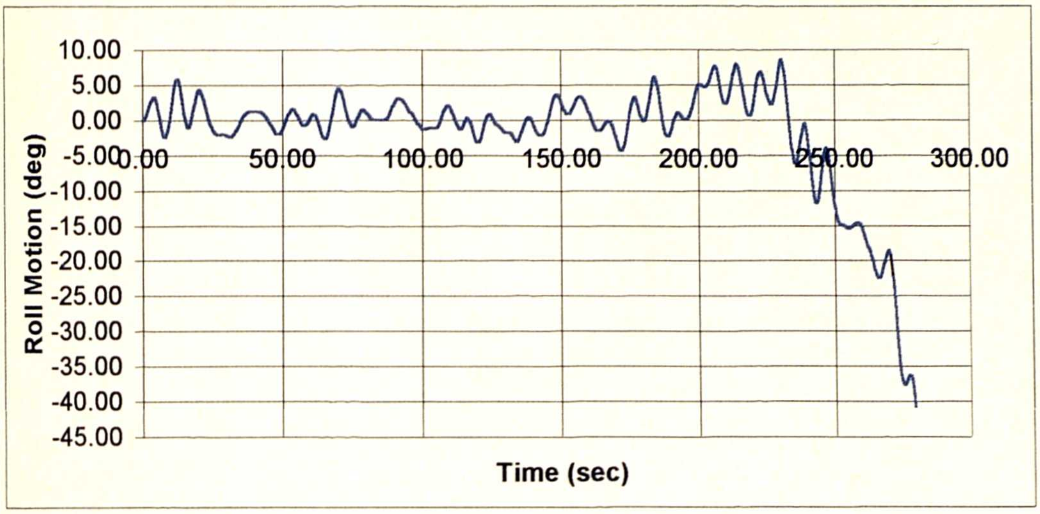
Simulation No. 115

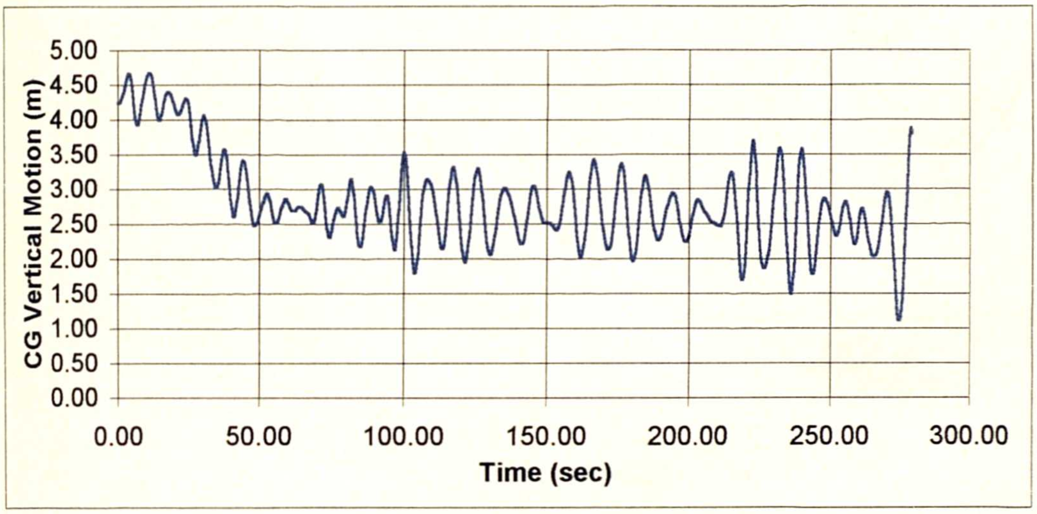
Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

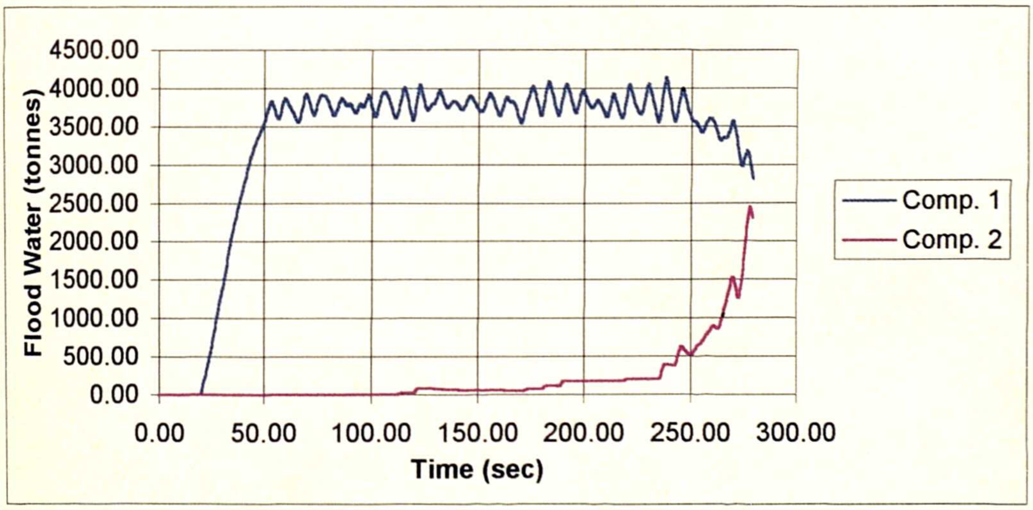


Simulation No. 116

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

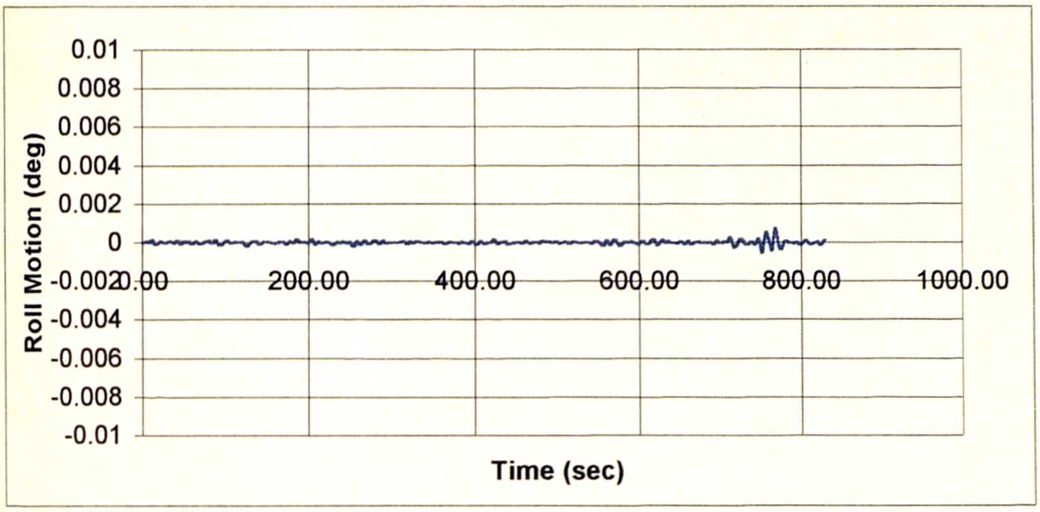


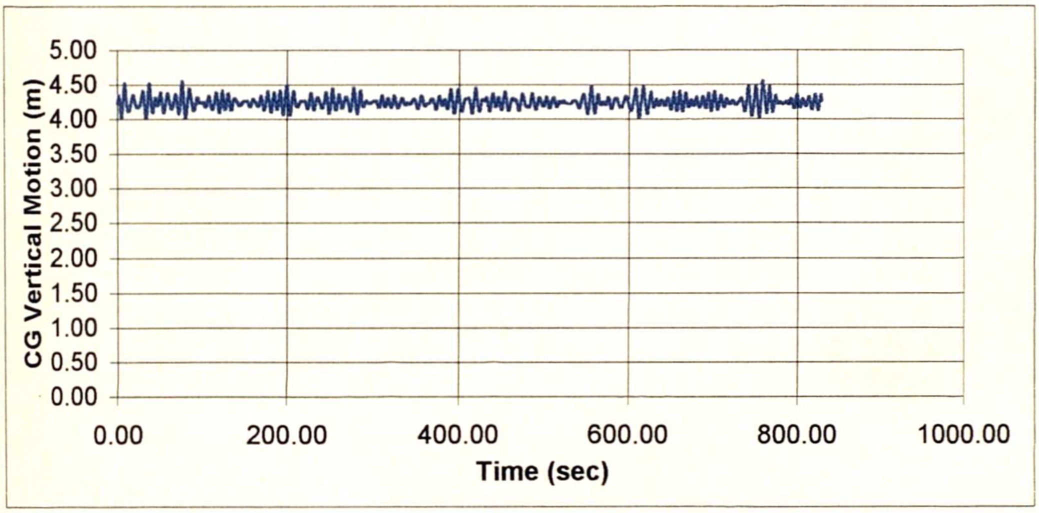


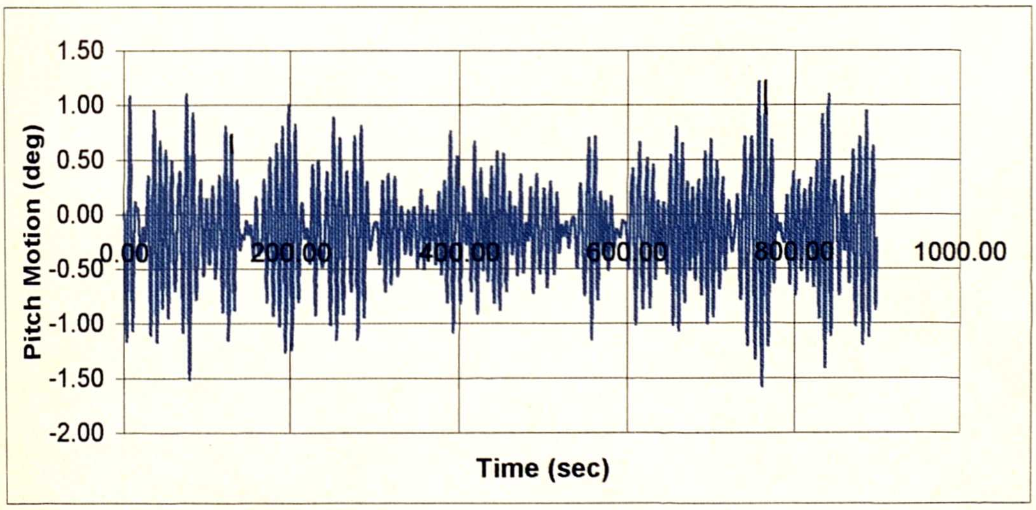


Simulation No. 117

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

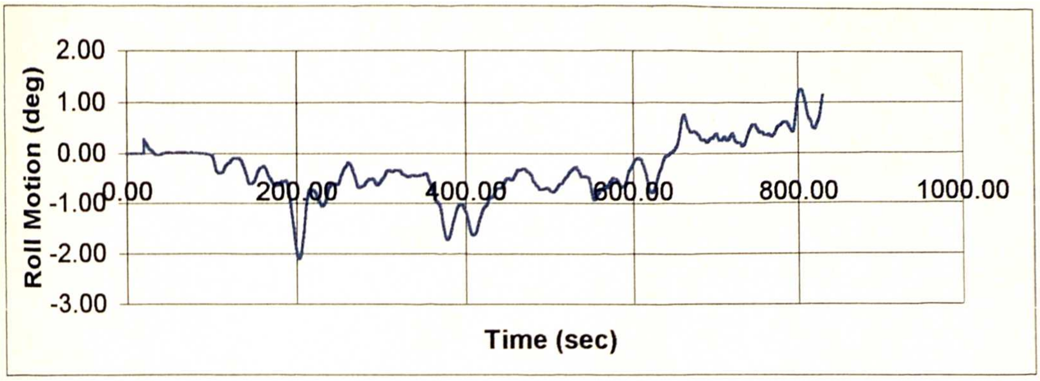


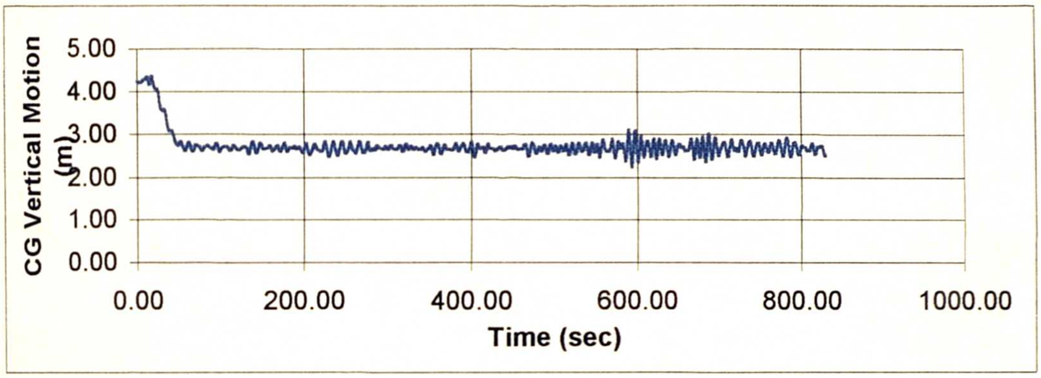


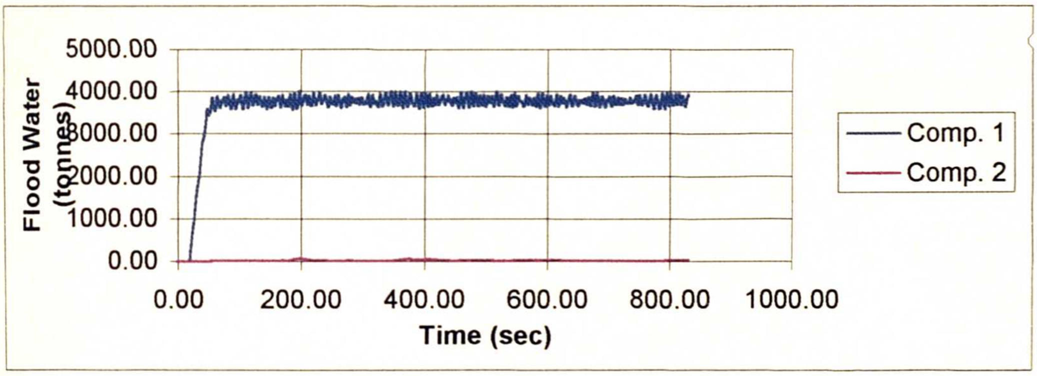


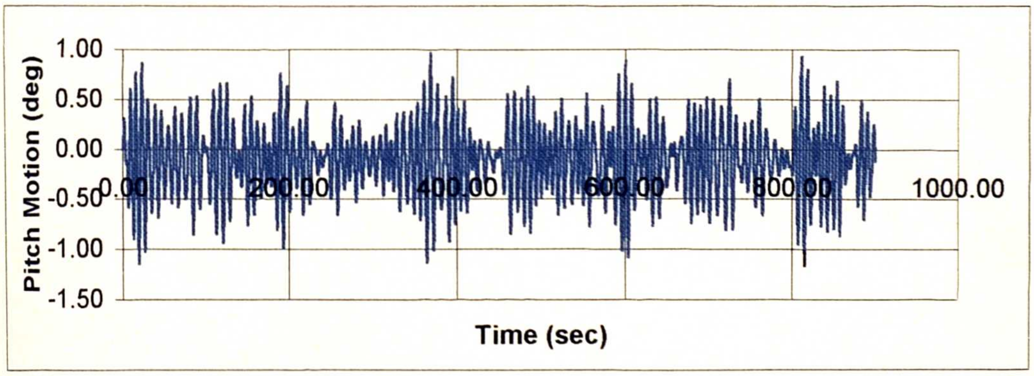
Simulation No. 118

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m



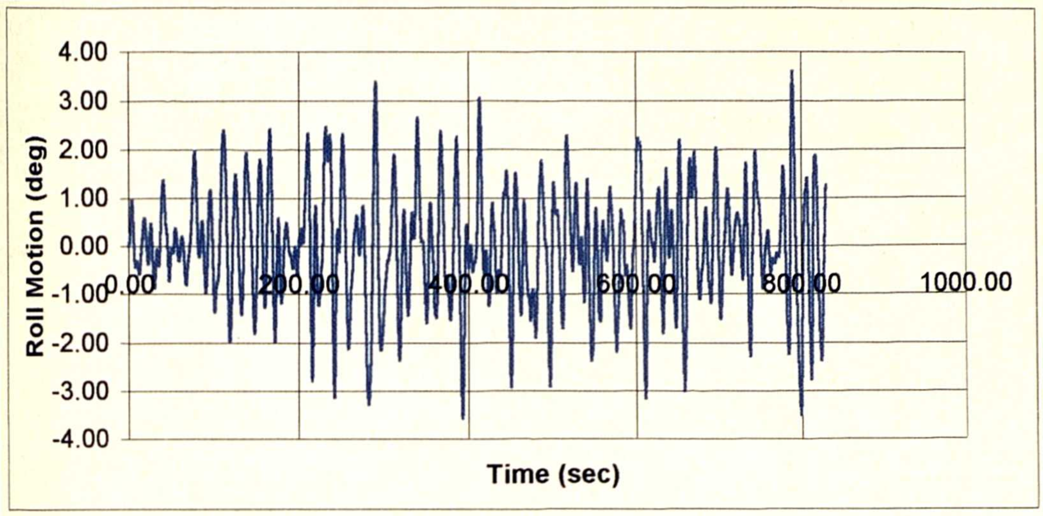


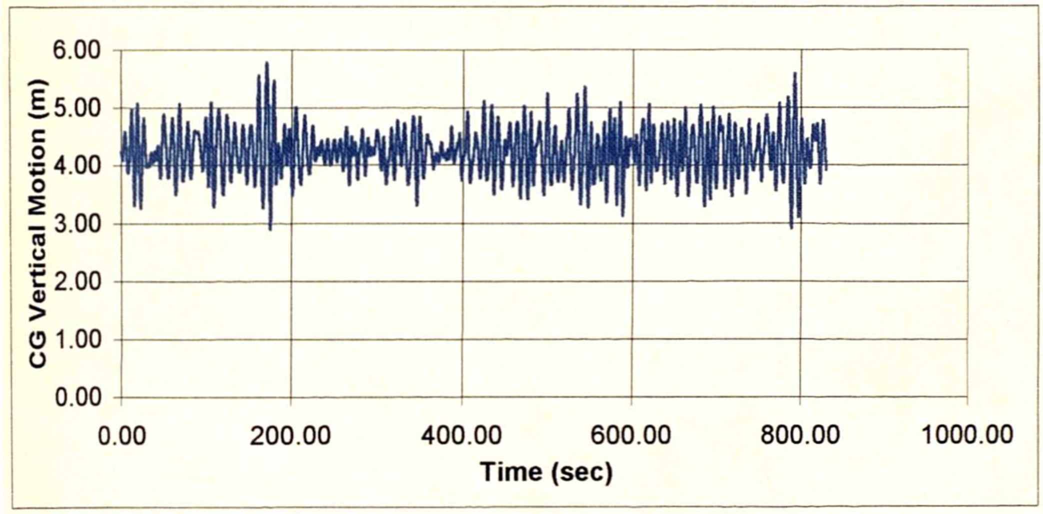




Simulation No. 119

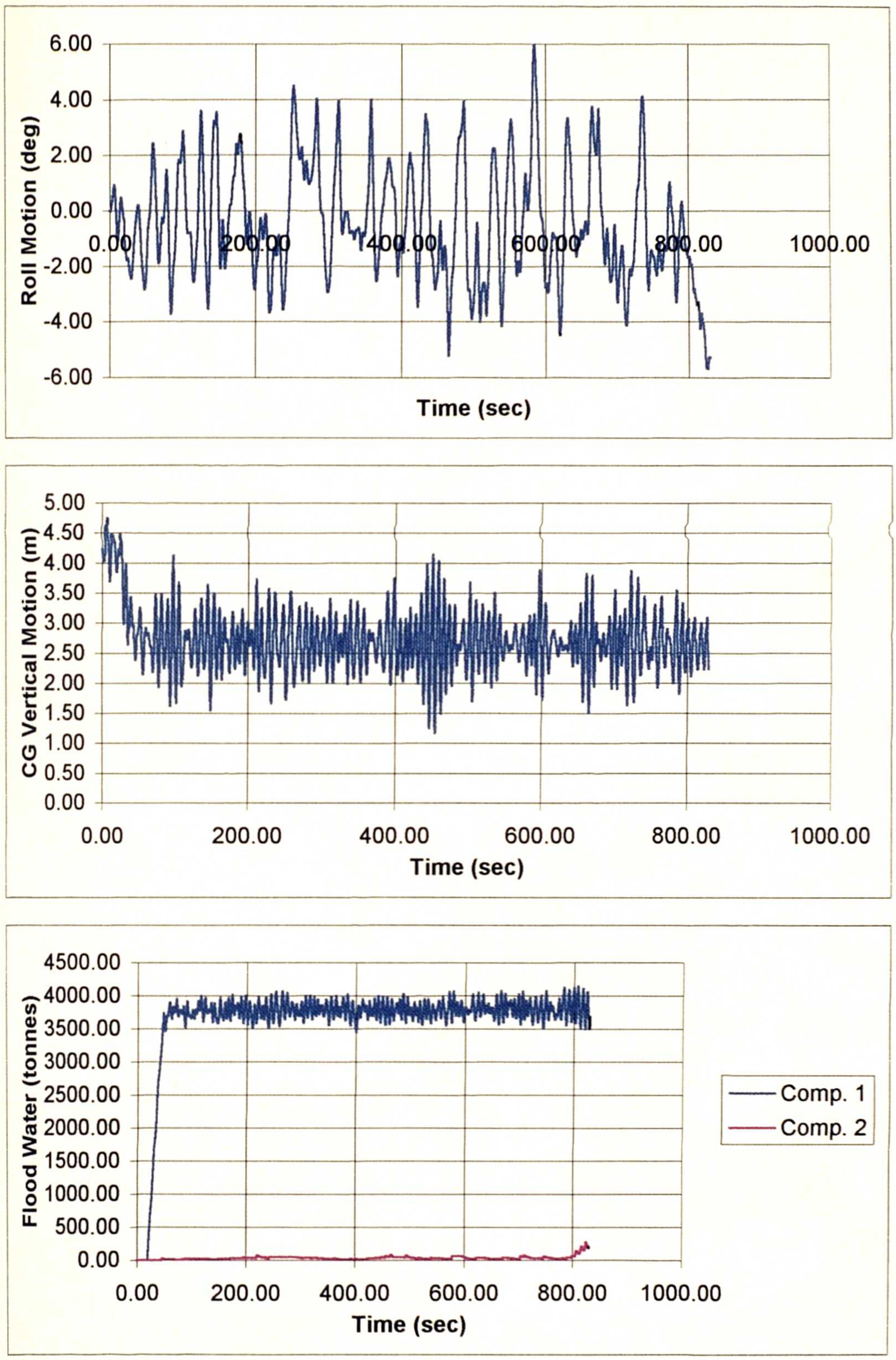
Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m





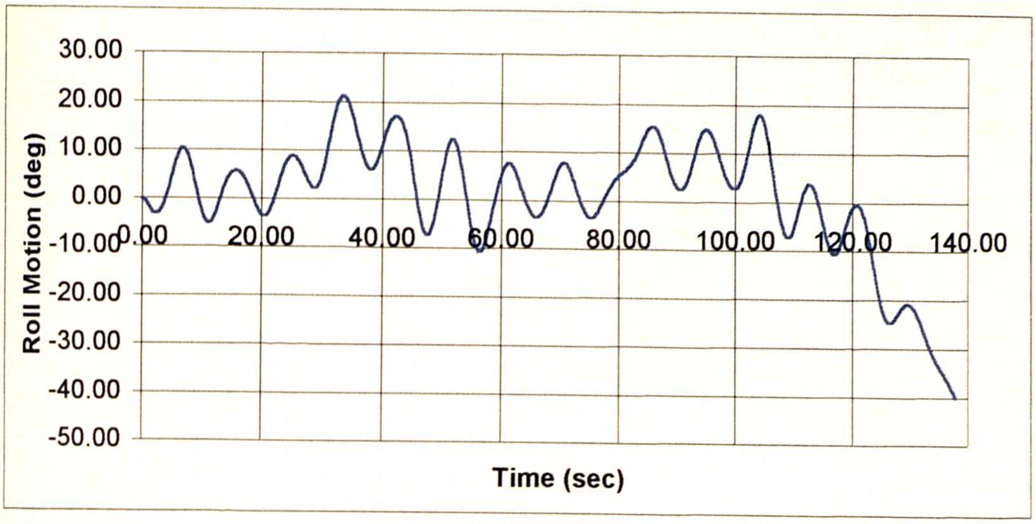
Simulation No. 120

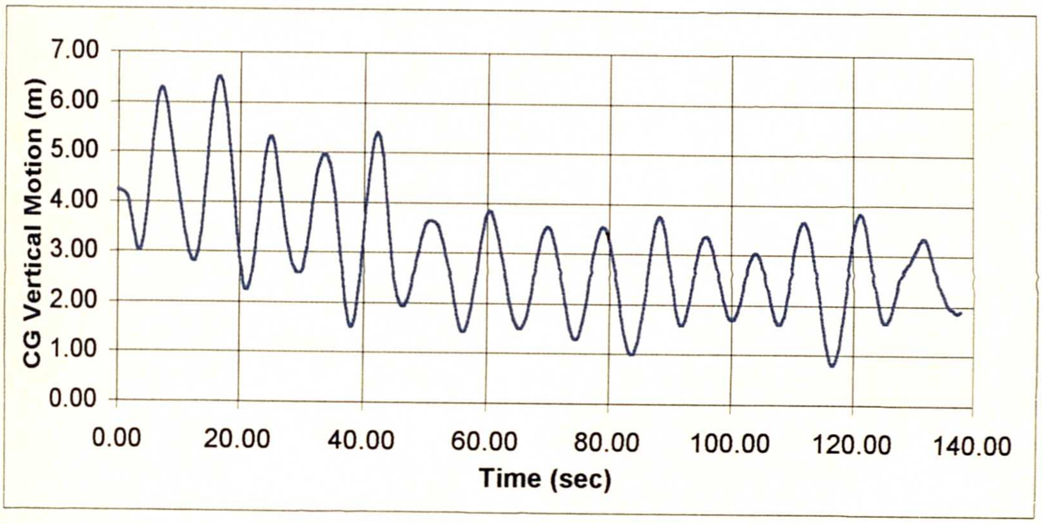
Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

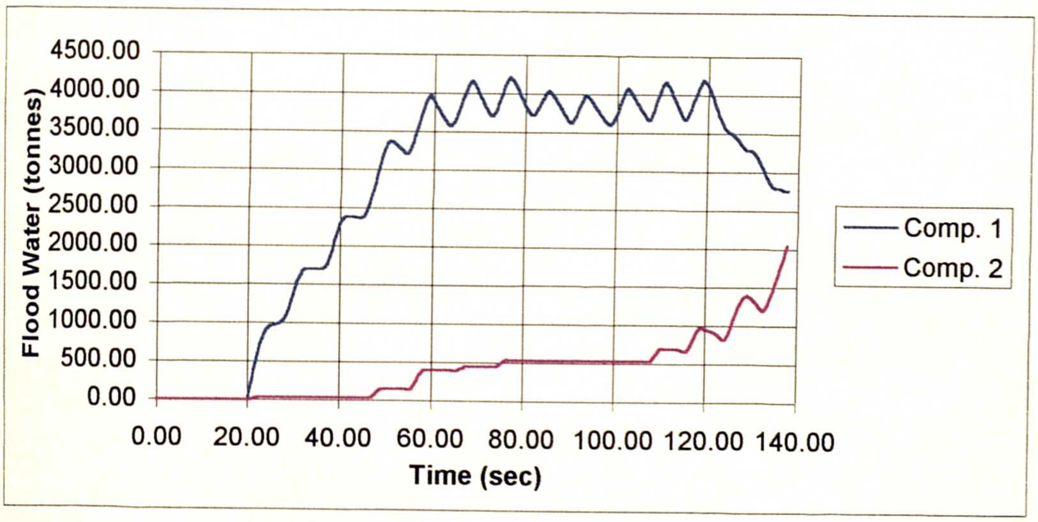


Simulation No. 121

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

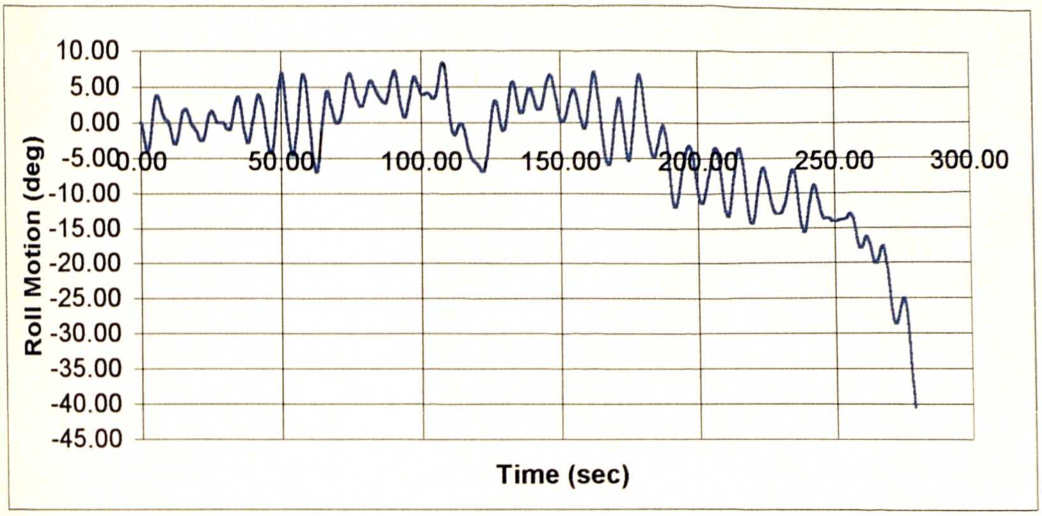


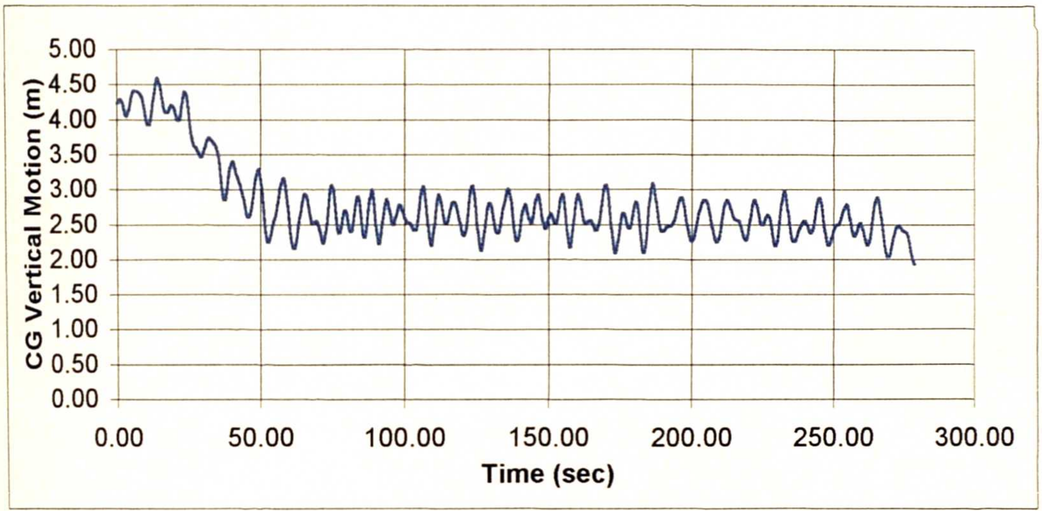


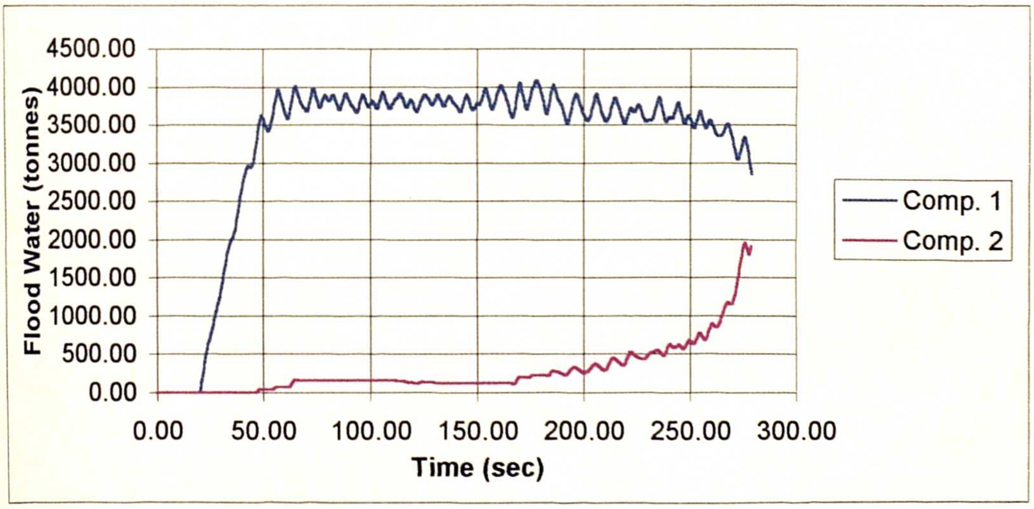


Simulation No. 122

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

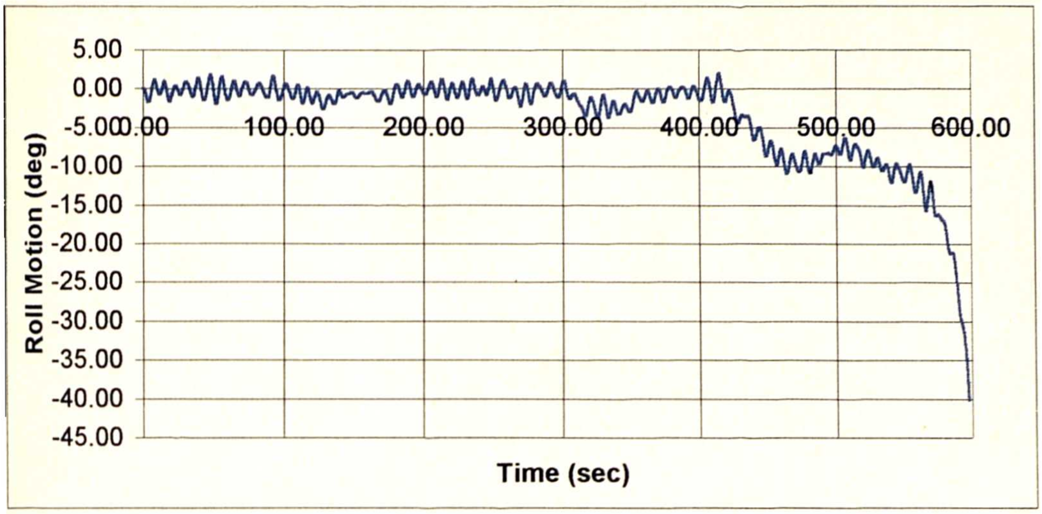


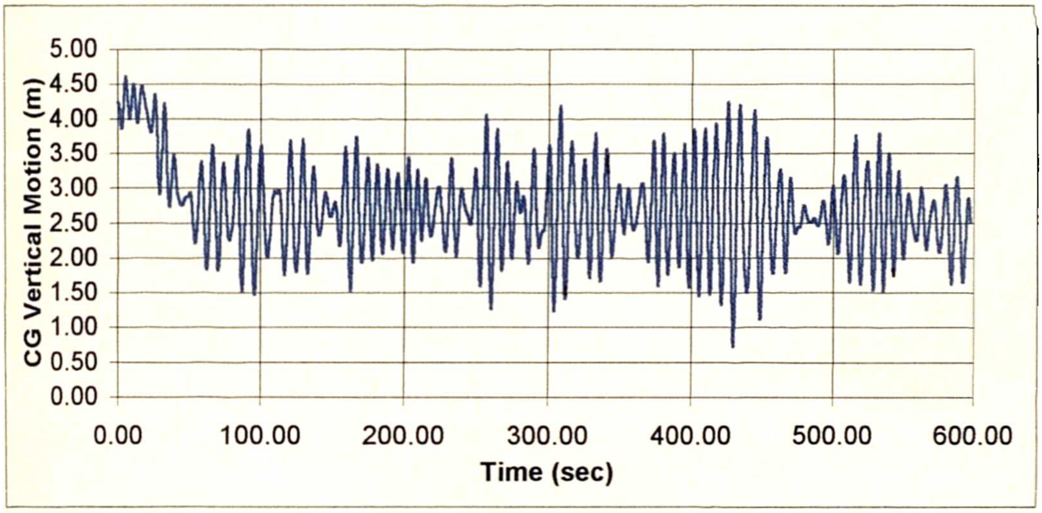


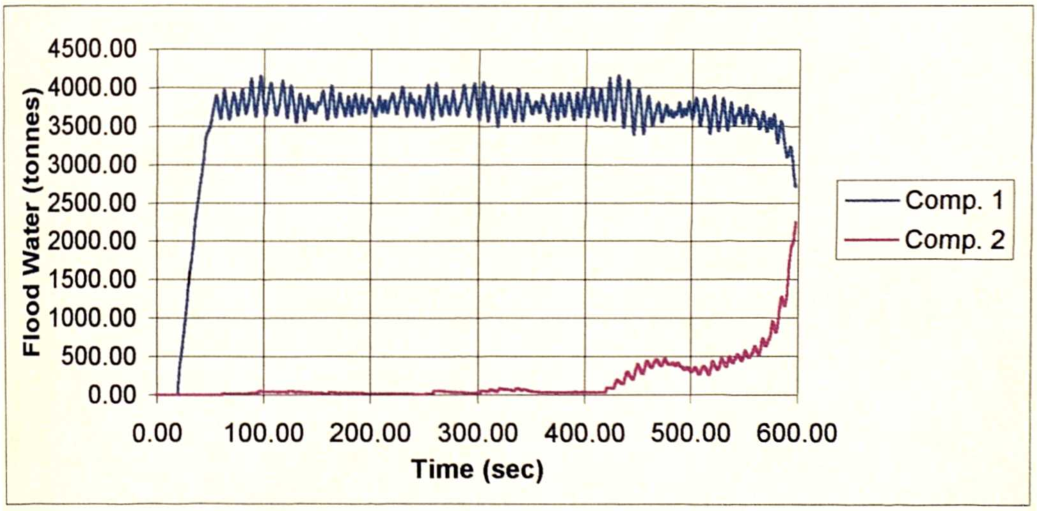


Simulation No. 123

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

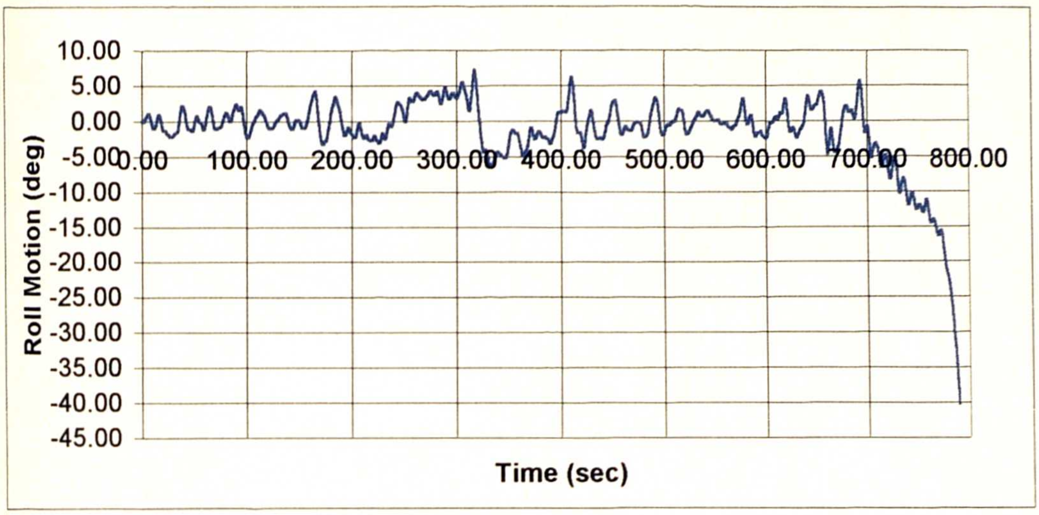


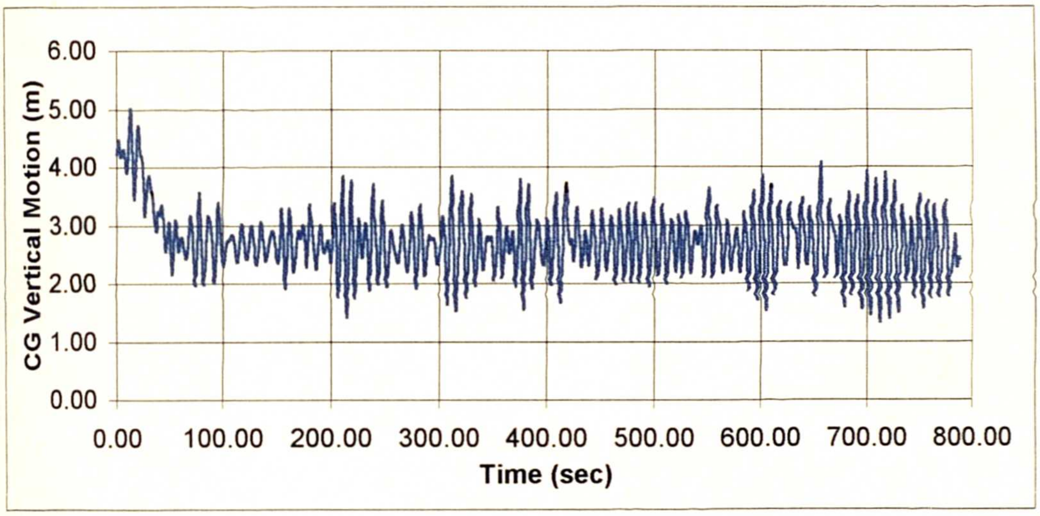


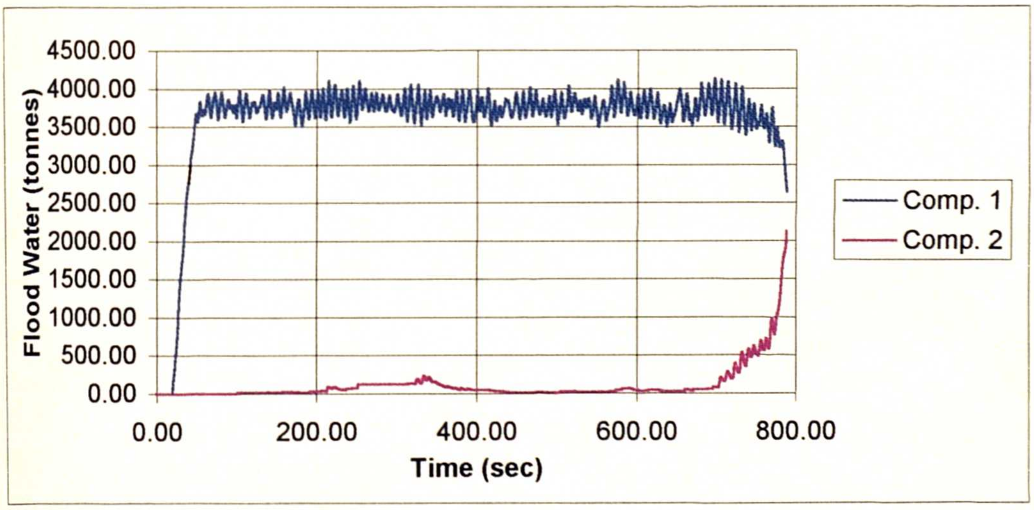


Simulation No. 124

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

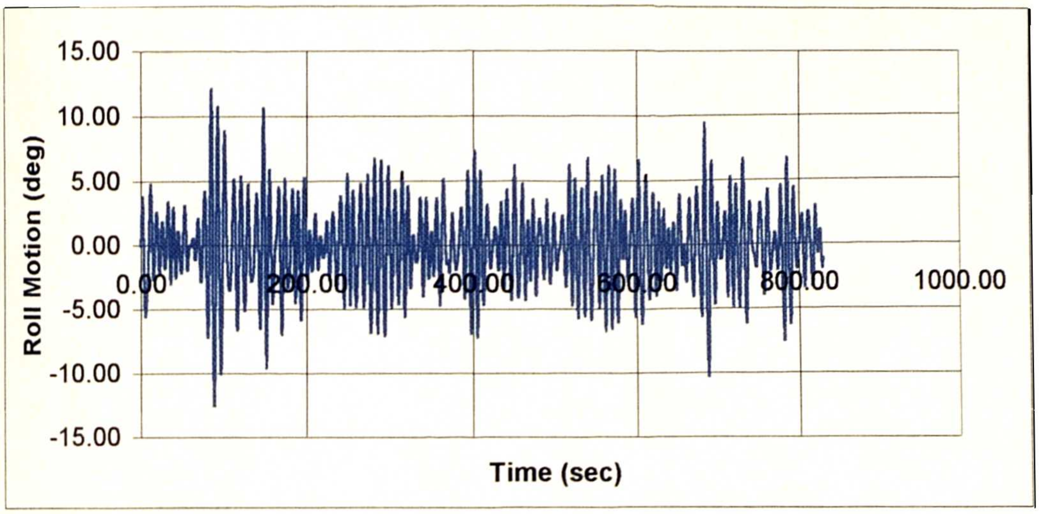


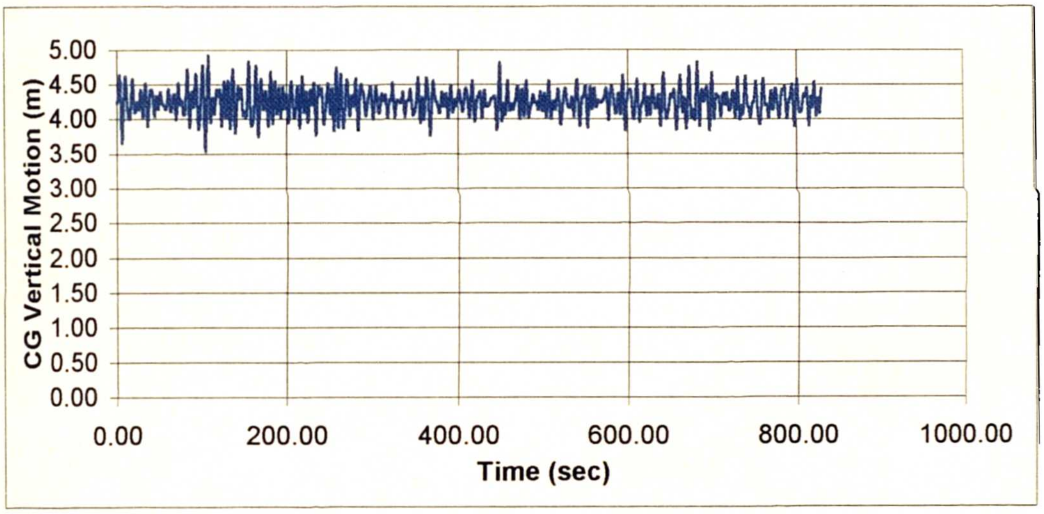


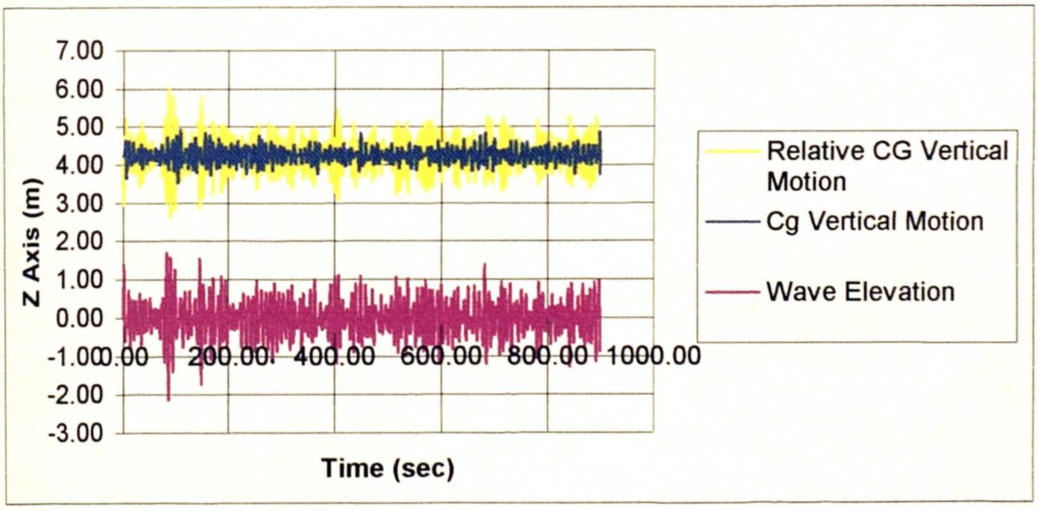


Simulation No. 125

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

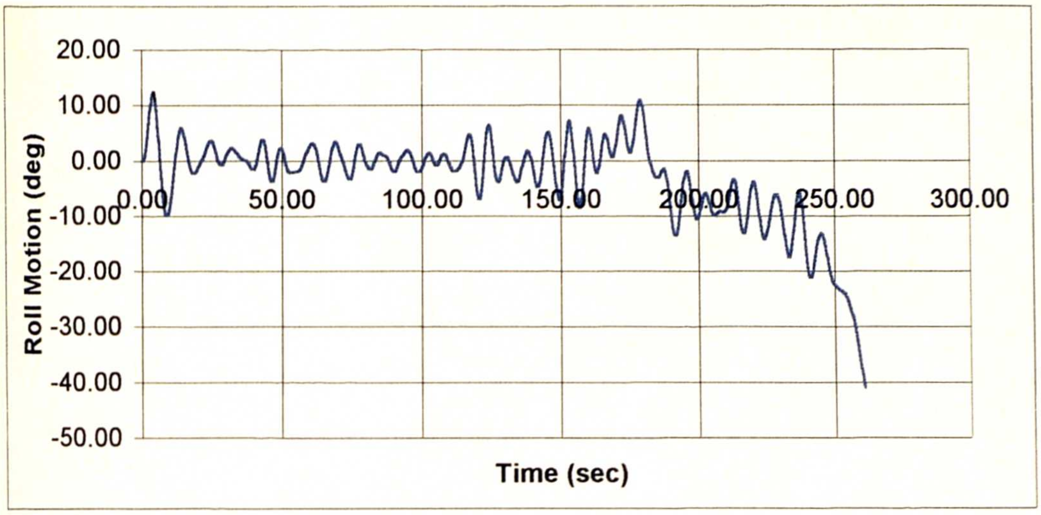


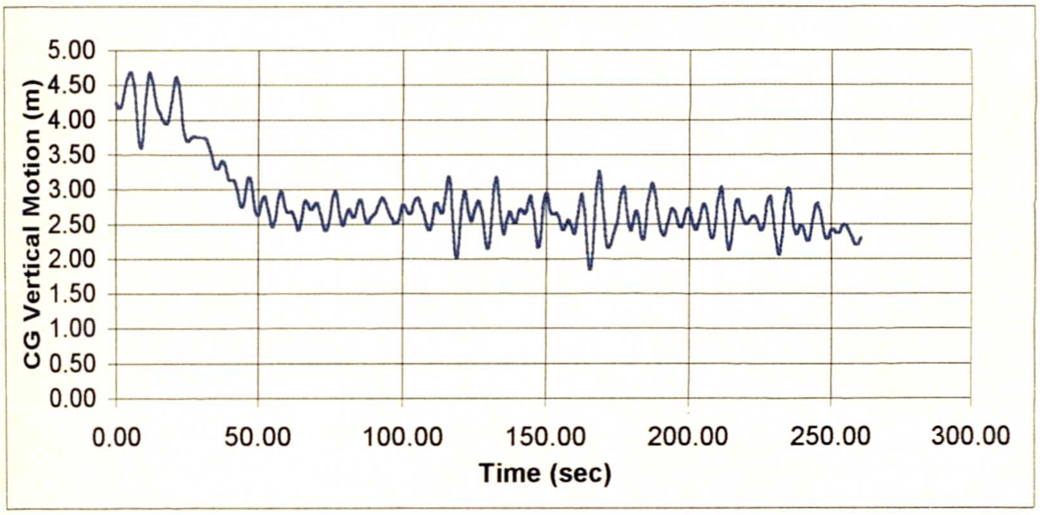


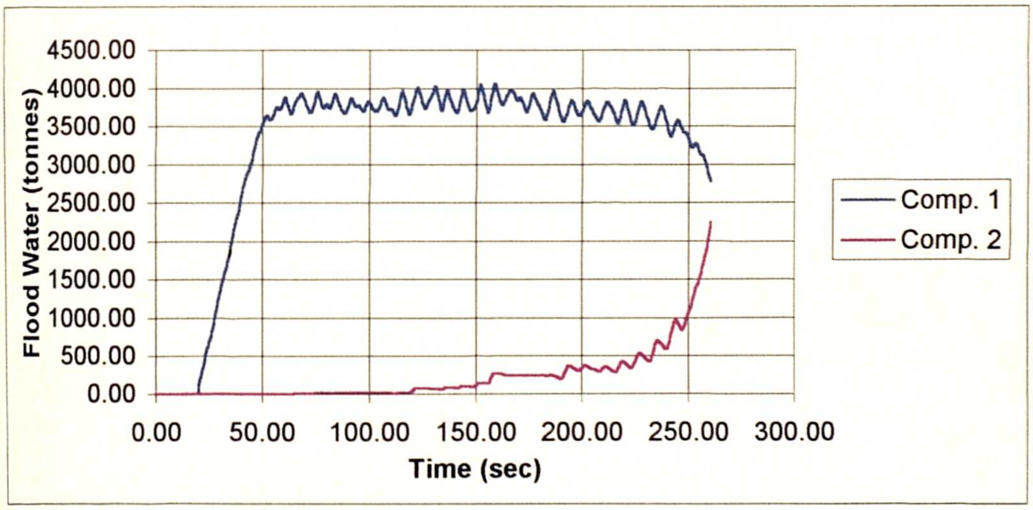


Simulation No. 126

Hs = 2.1 m
KG = 9.86 m
Freeboard = 0.7 m

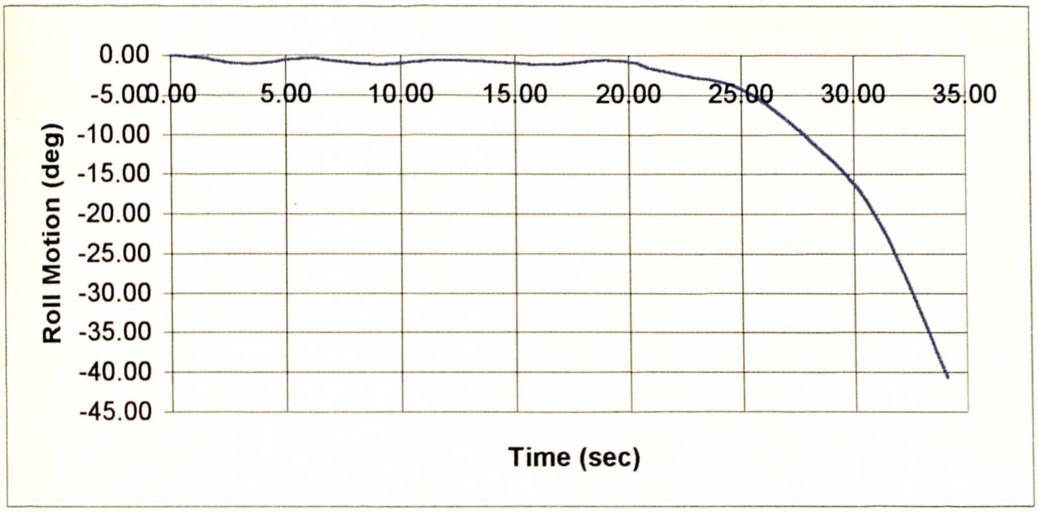


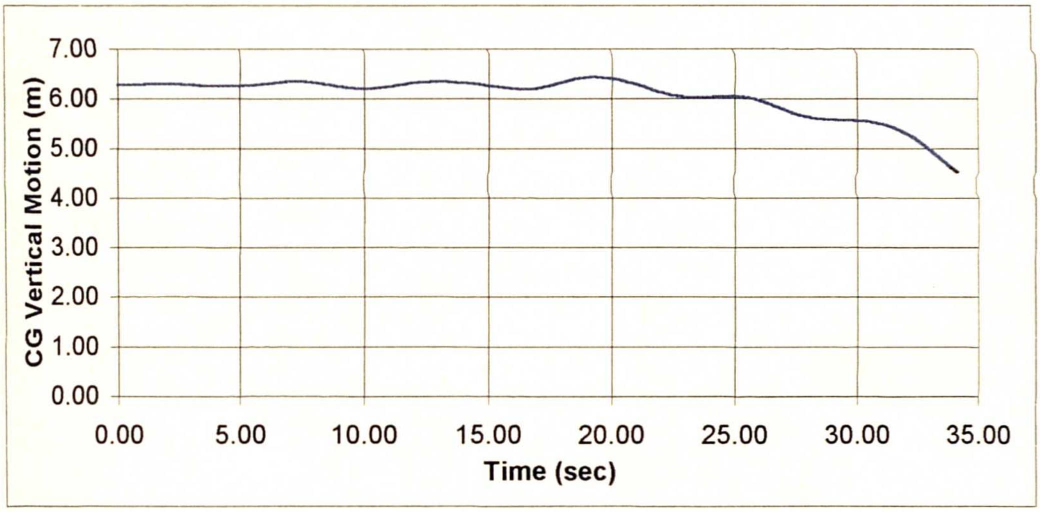


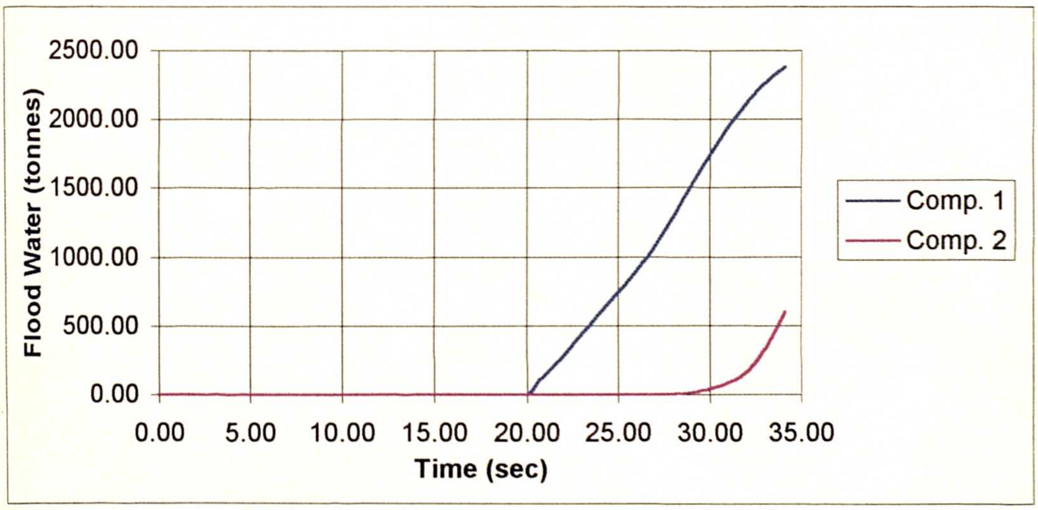


Simulation No. 201

Hs = 0.35 m
KG = 11.91 m
Freeboard = 0.7 m

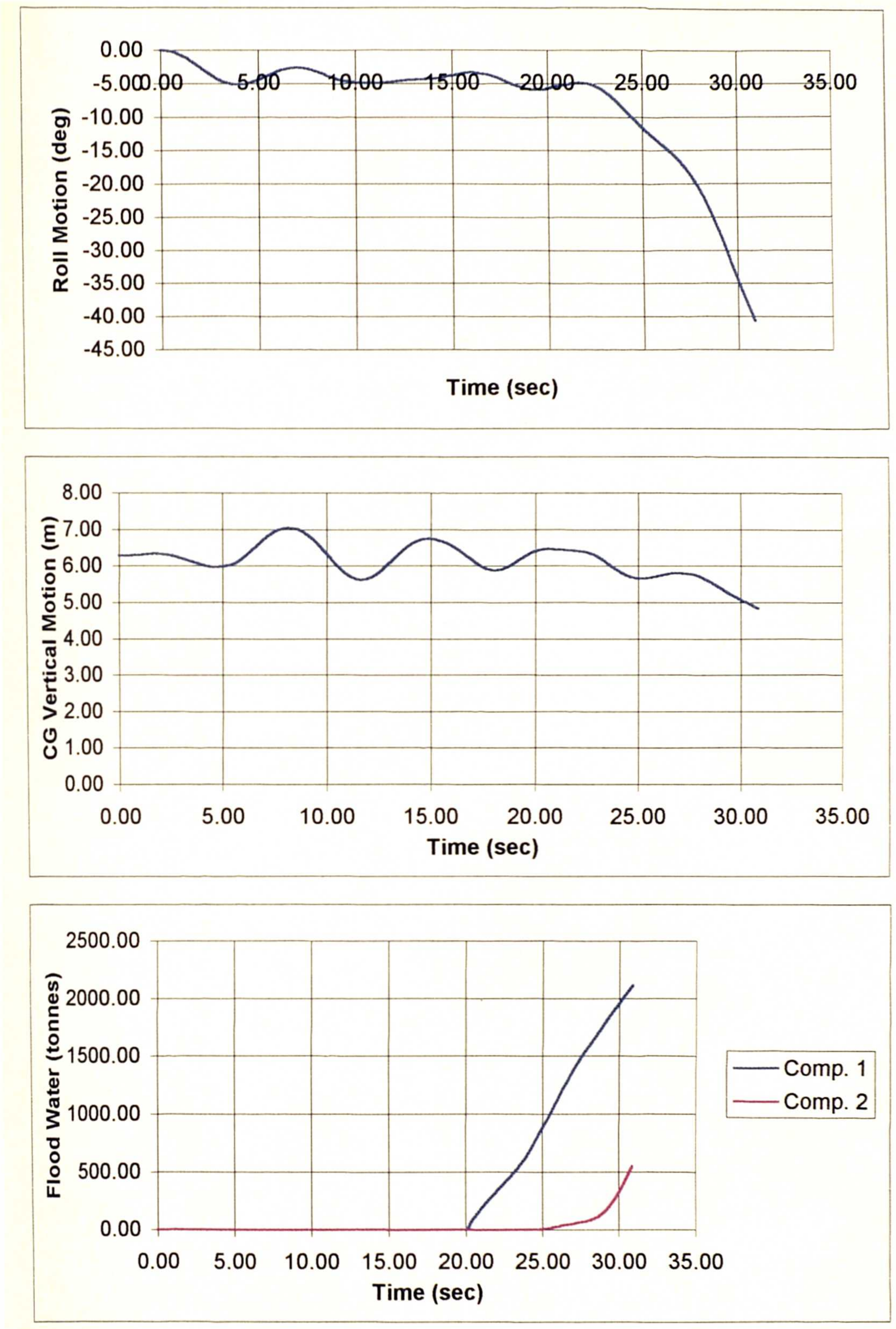






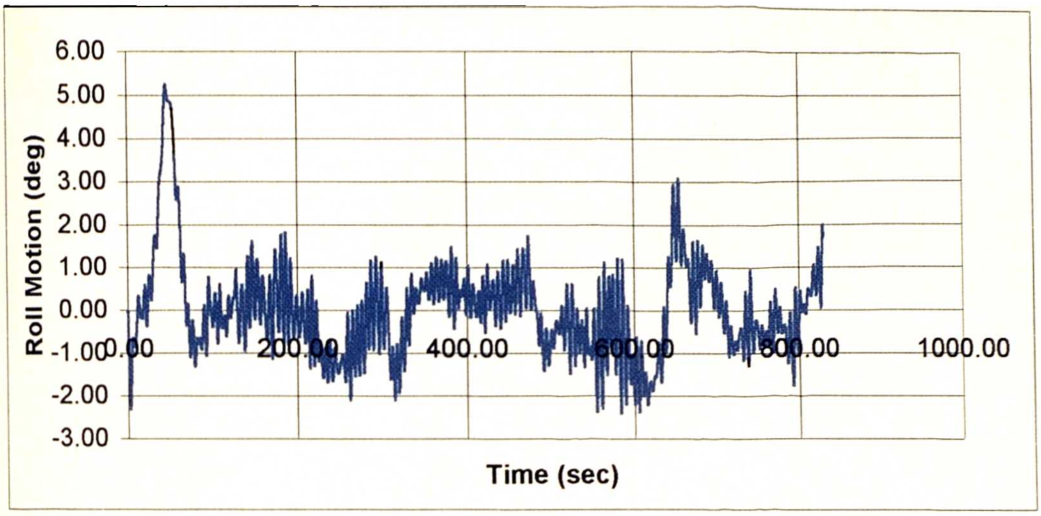
Simulation No. 202

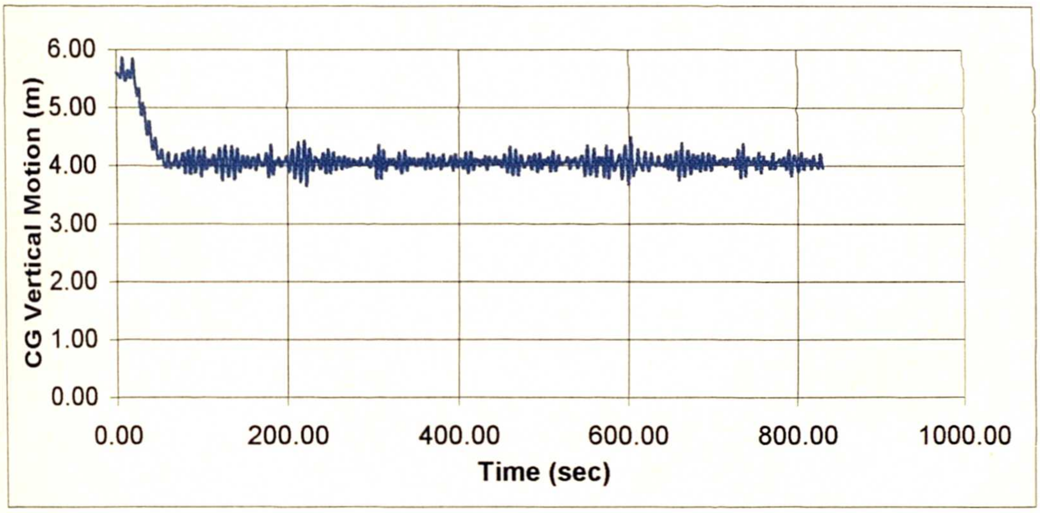
Hs = 1.05 m
KG = 11.91 m
Freeboard = 0.71 m

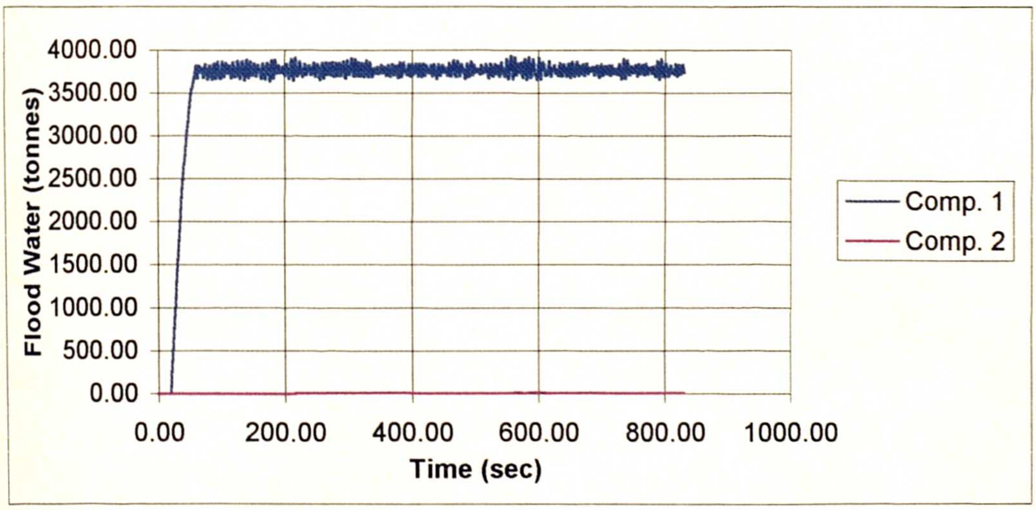


Simulation No. 203

Hs = 0.7 m
KG = 11.23 m
Freeboard = 0.7 m

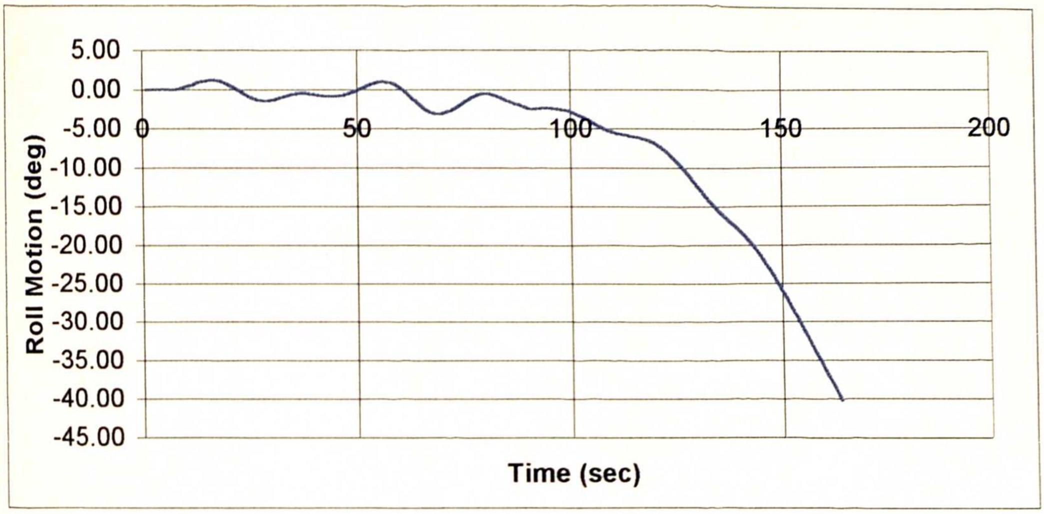


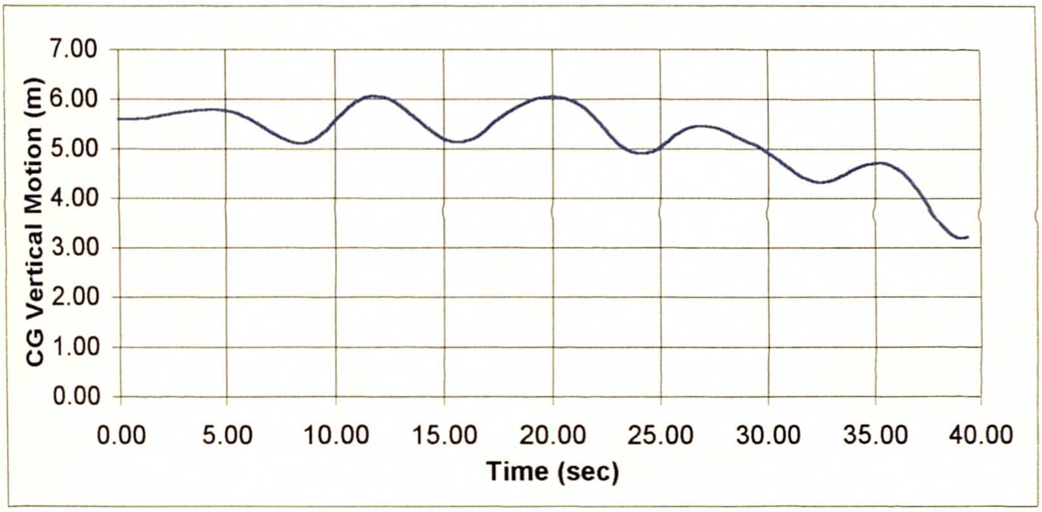


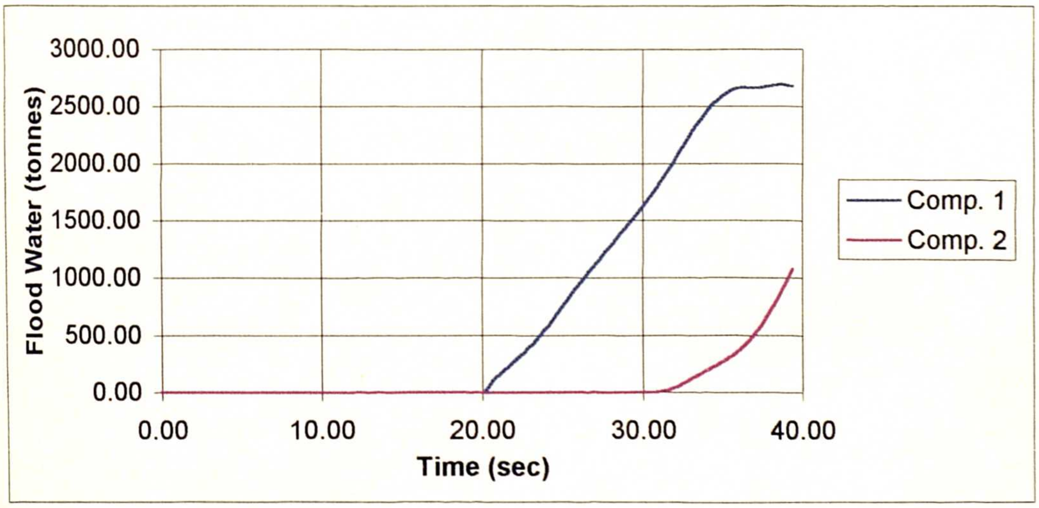


Simulation No. 204

Hs = 1.4 m
KG = 11.23 m
Freeboard = 0.7 m

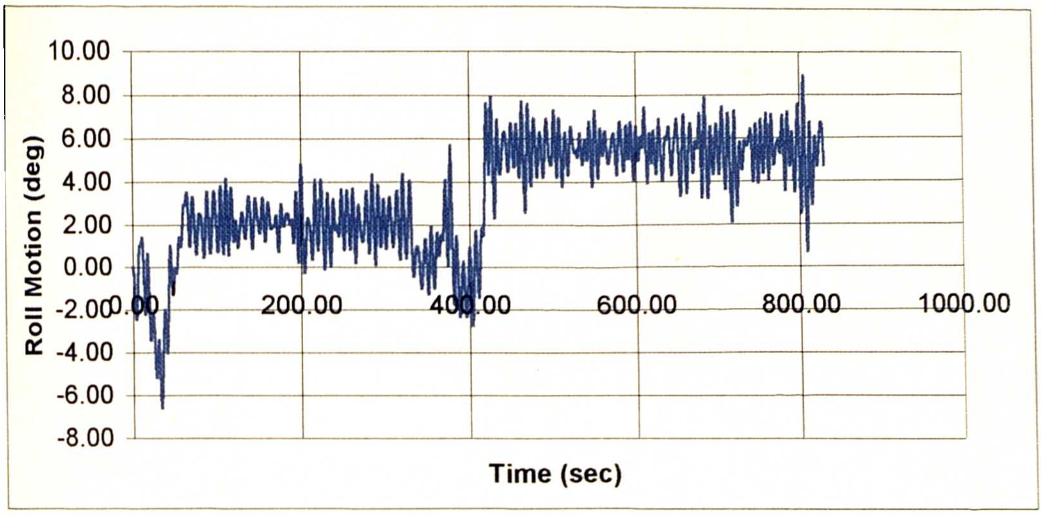


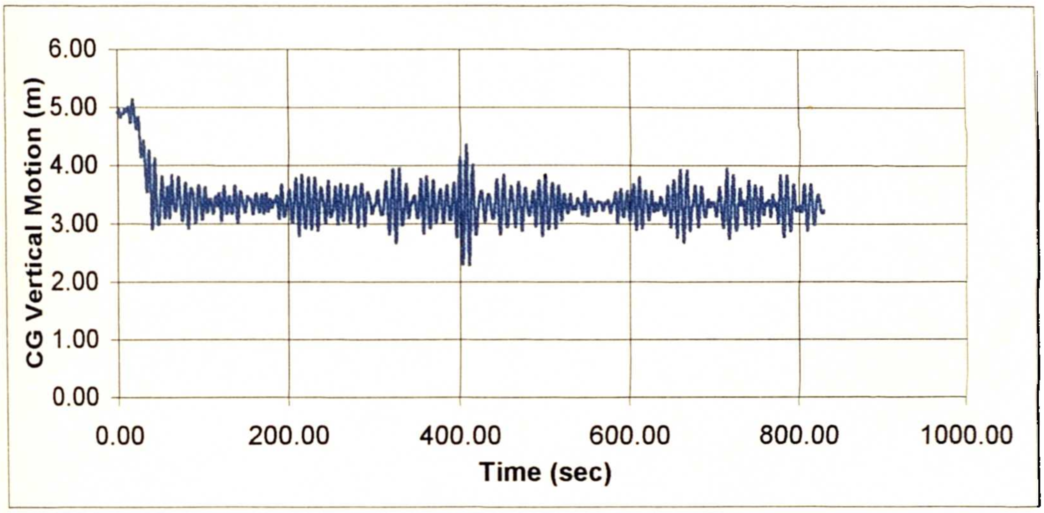


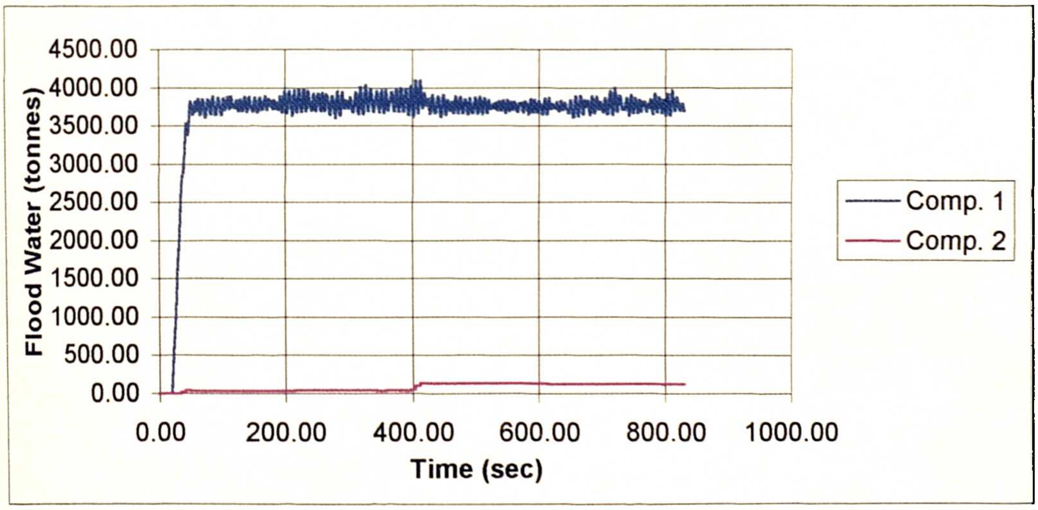


Simulation No. 205

Hs = 1.05 m
KG = 10.54 m
Freeboard = 0.7 m

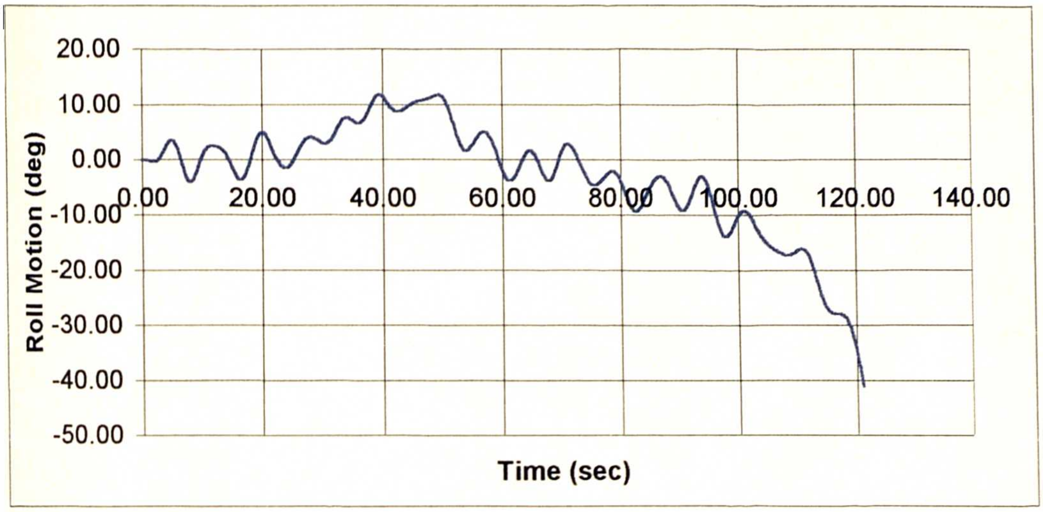


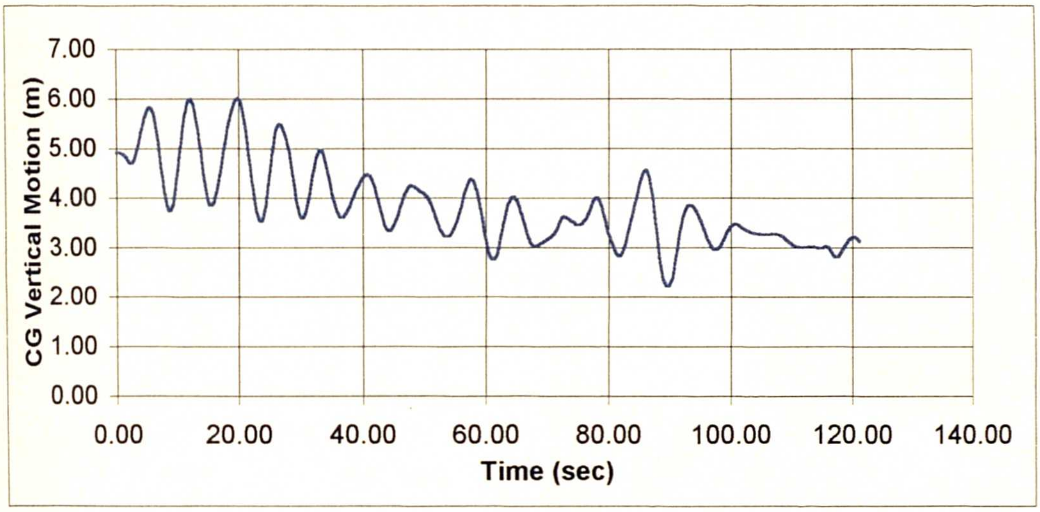


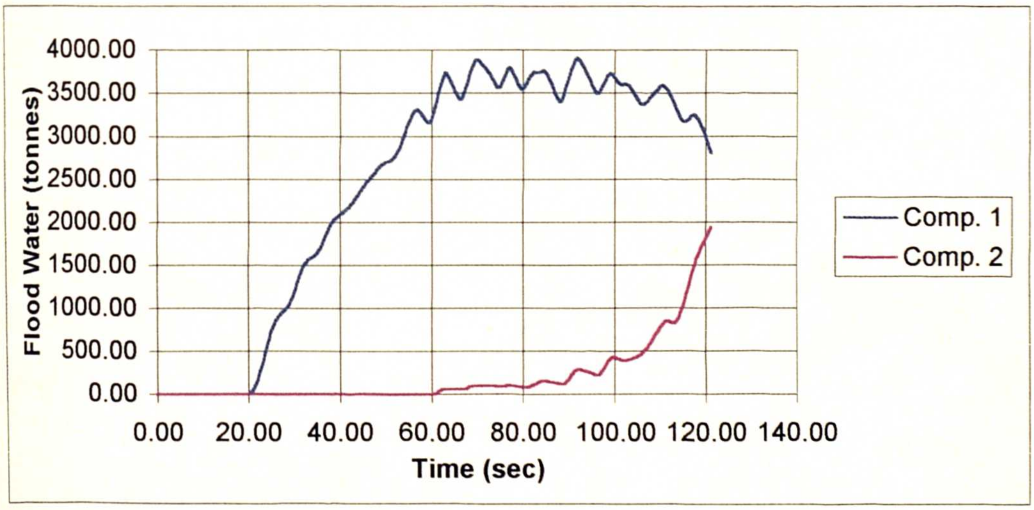


Simulation No. 206

Hs = 1.75 m
KG = 10.54 m
Freeboard = 0.7 m

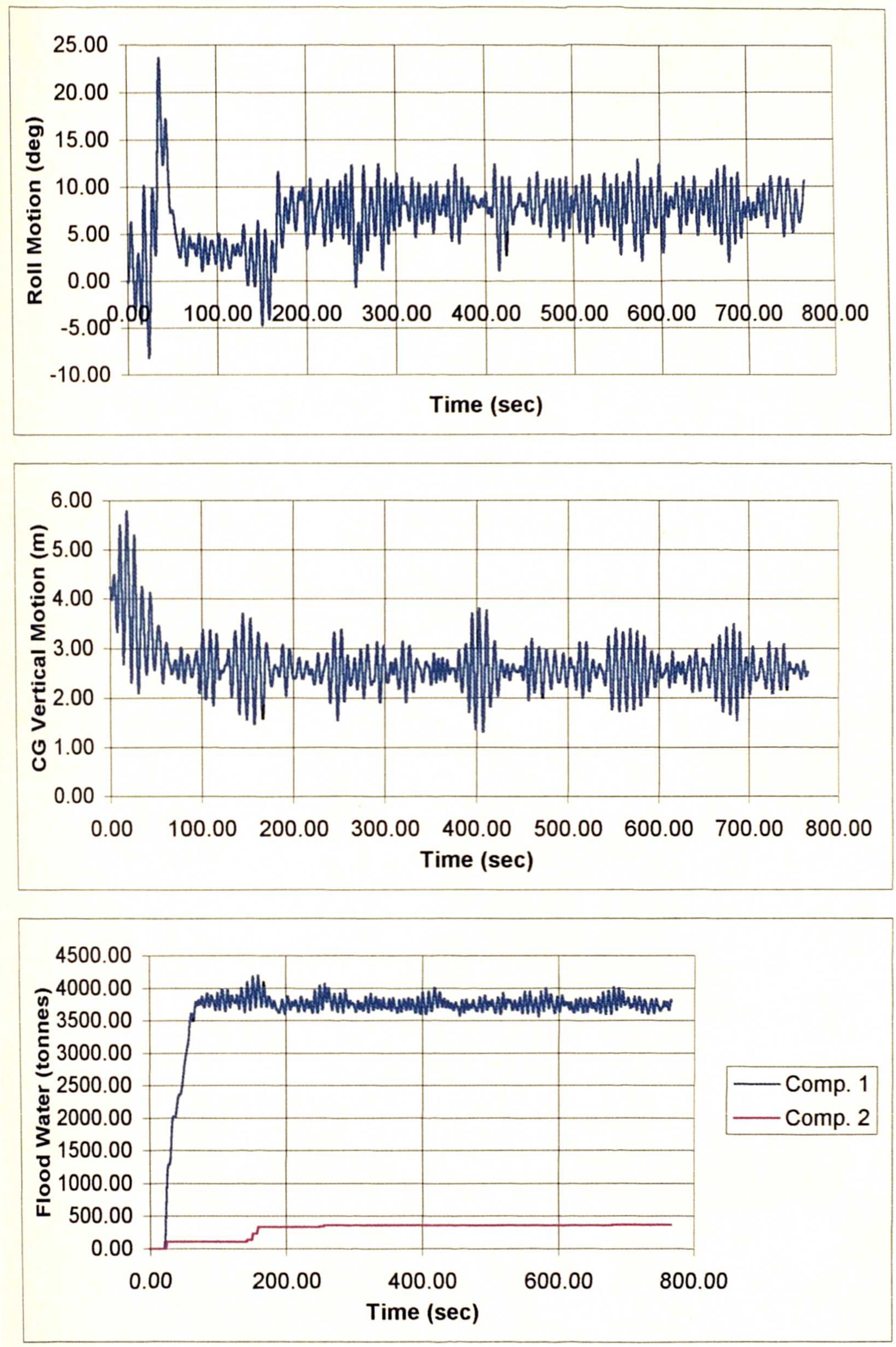






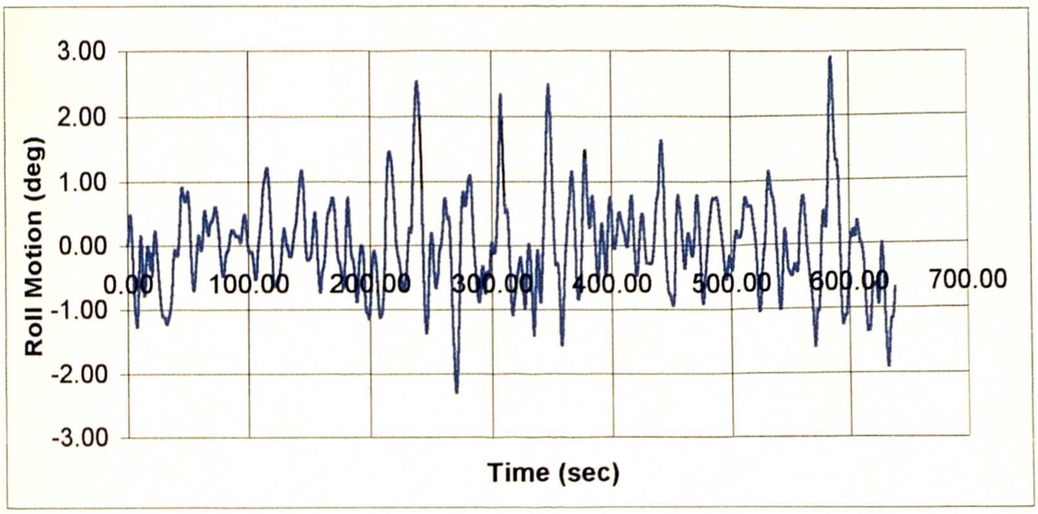
Simulation No. 207

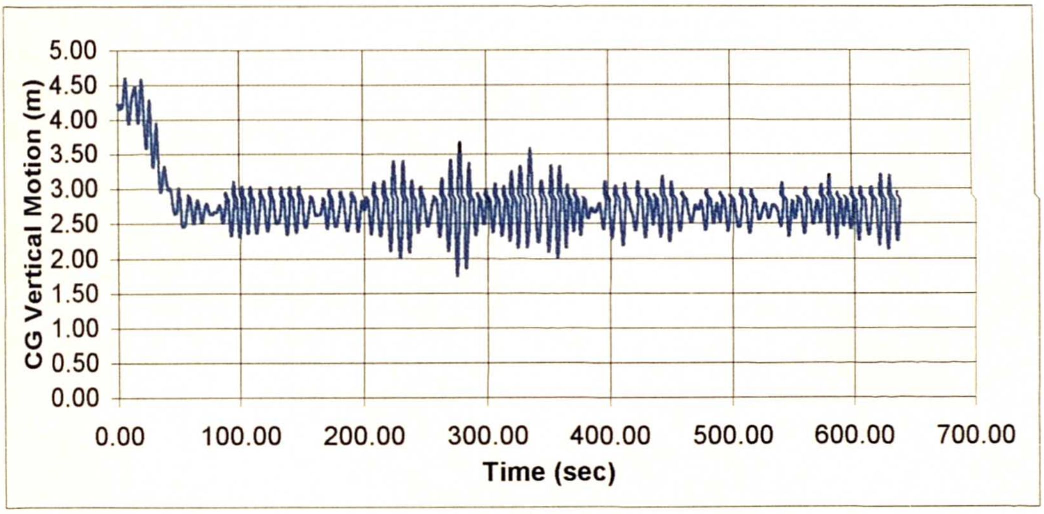
Hs = 1.4 m
KG = 9.86 m
Freeboard = 0.7 m

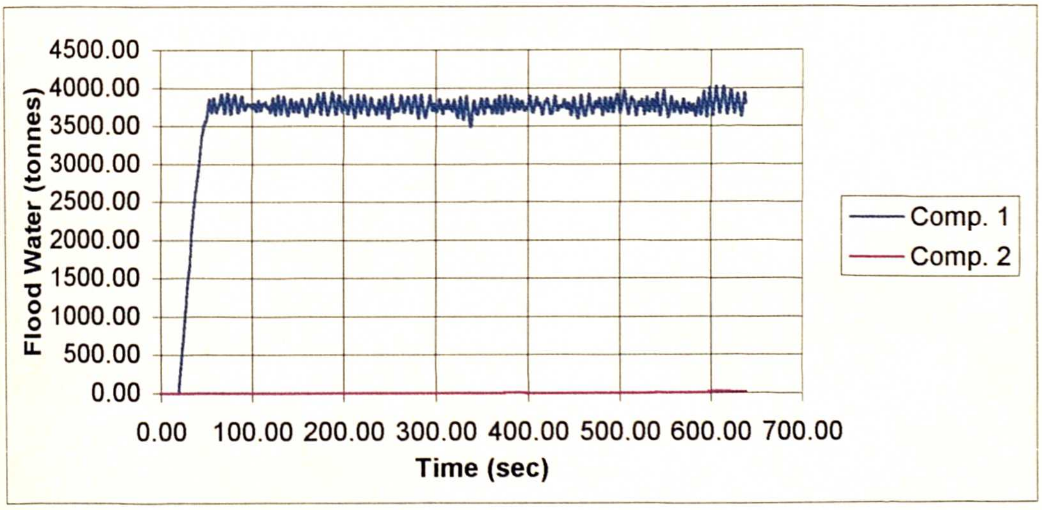


Simulation No. 207b

Hs = 1.4 m
KG = 9.86 m
Freeboard = 0.7 m

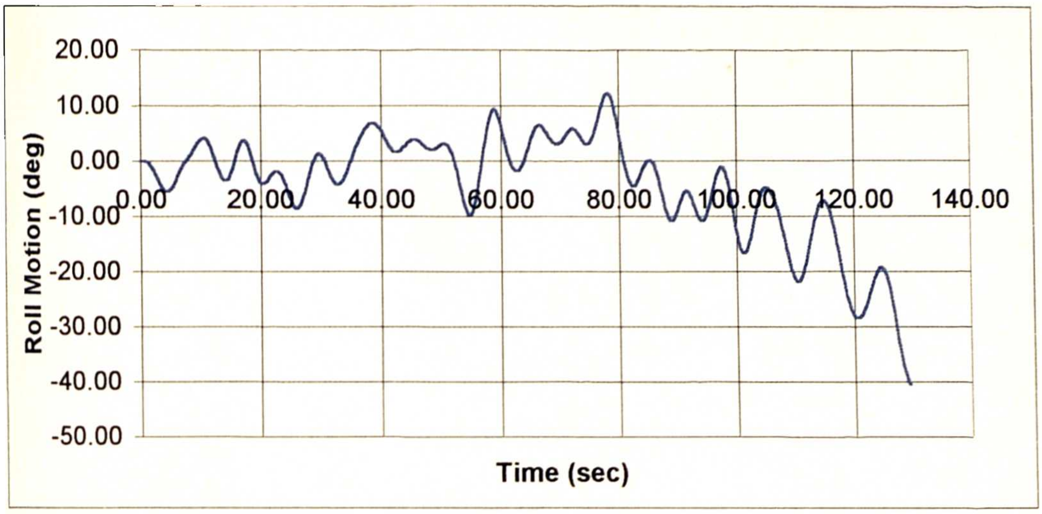


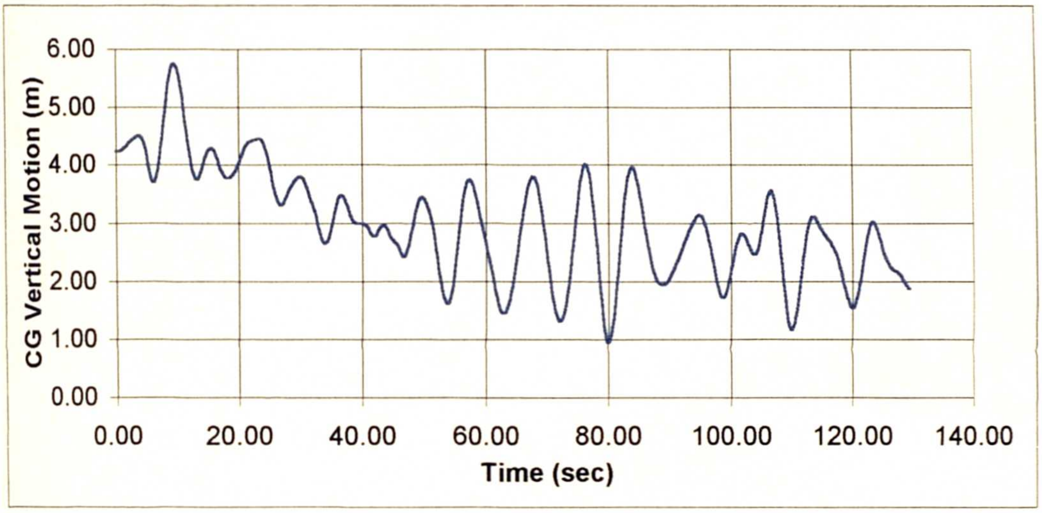


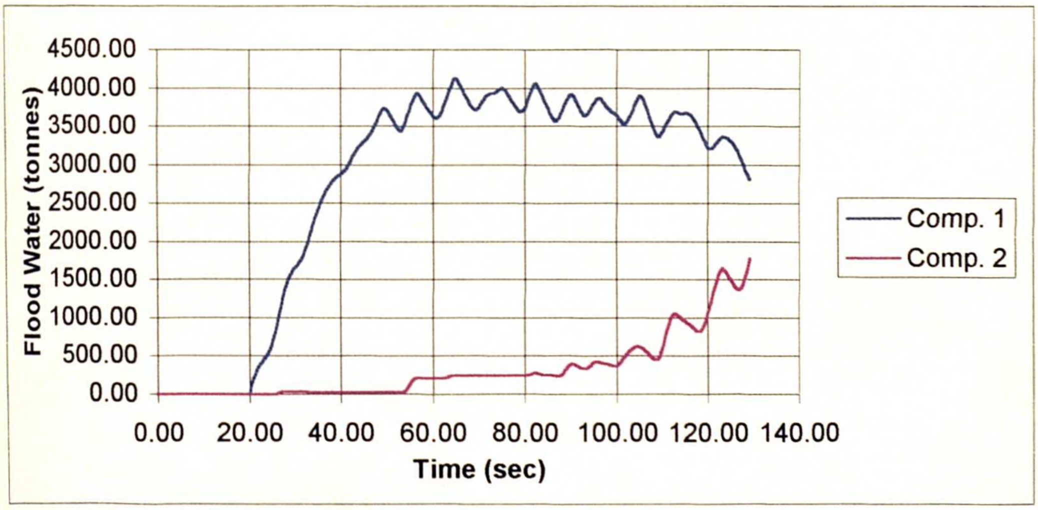


Simulation No. 208

Hs = 2.45 m
KG = 9.86 m
Freeboard = 0.7 m

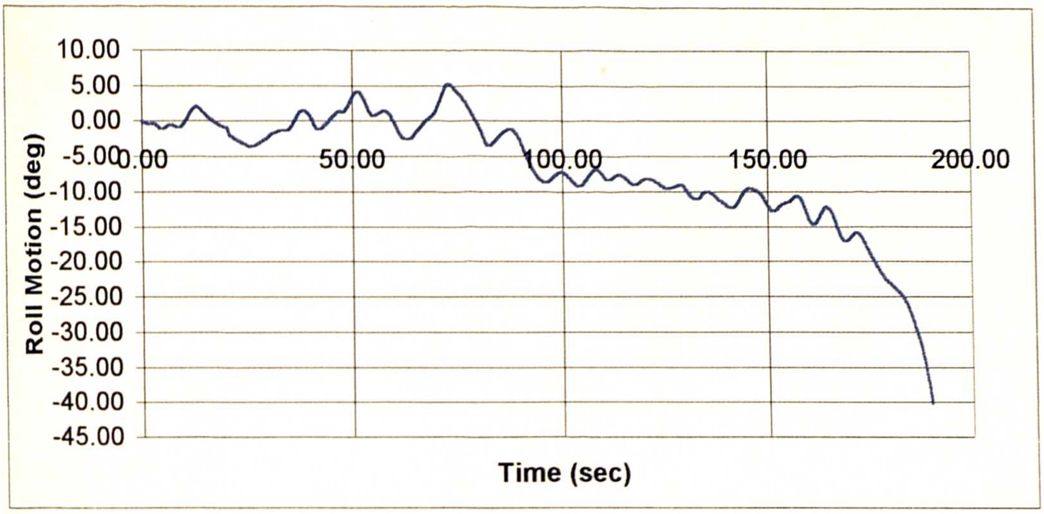


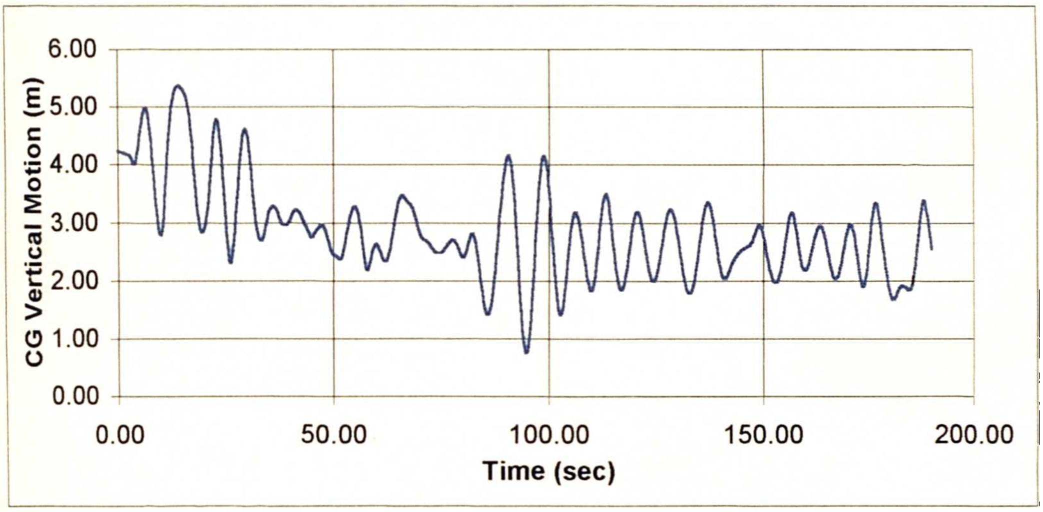


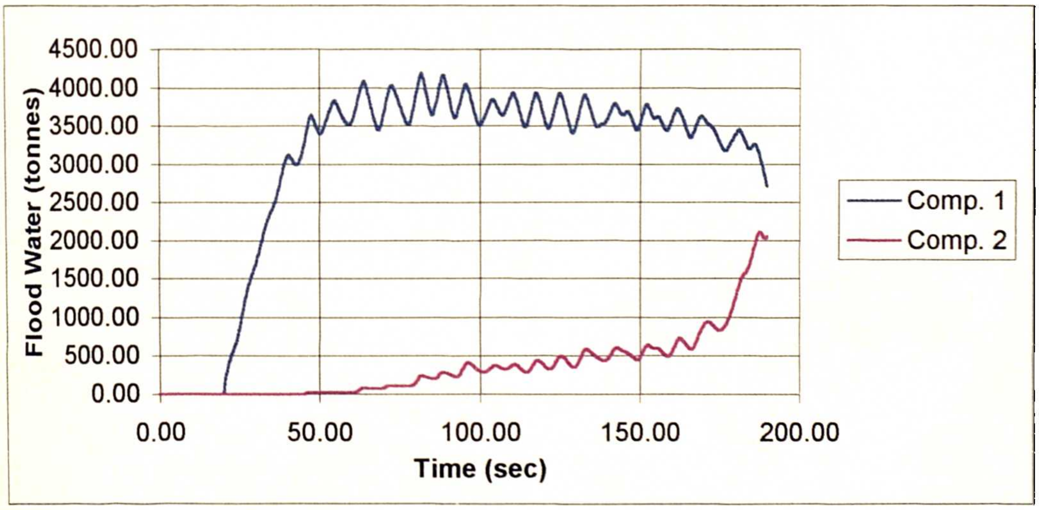


Simulation No. 208b

Hs = 2.45 m
KG = 9.86 m
Freeboard = 0.7 m

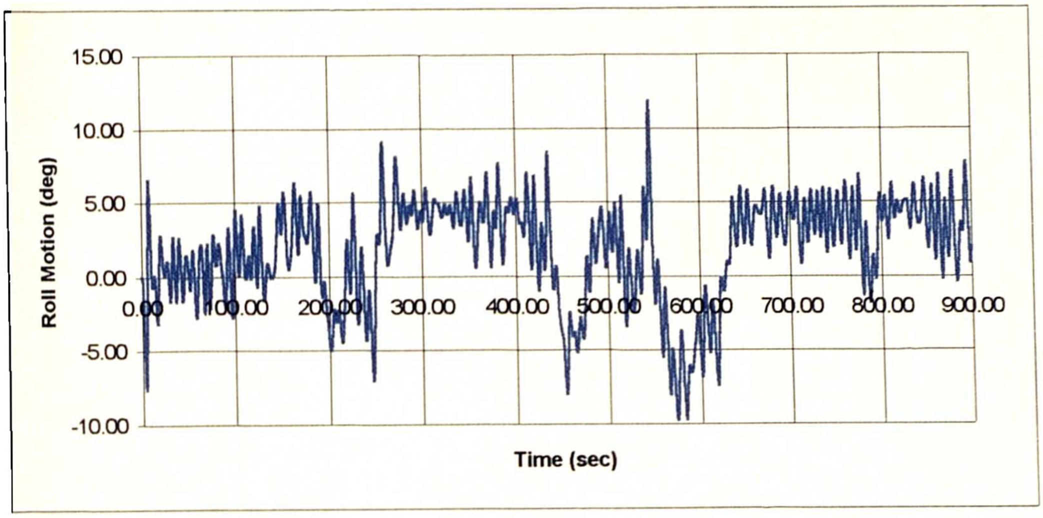


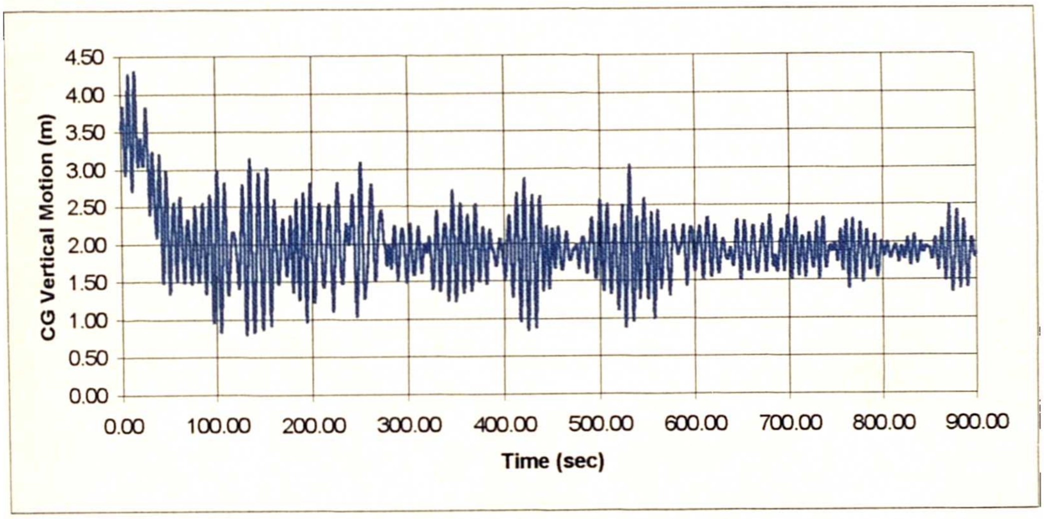


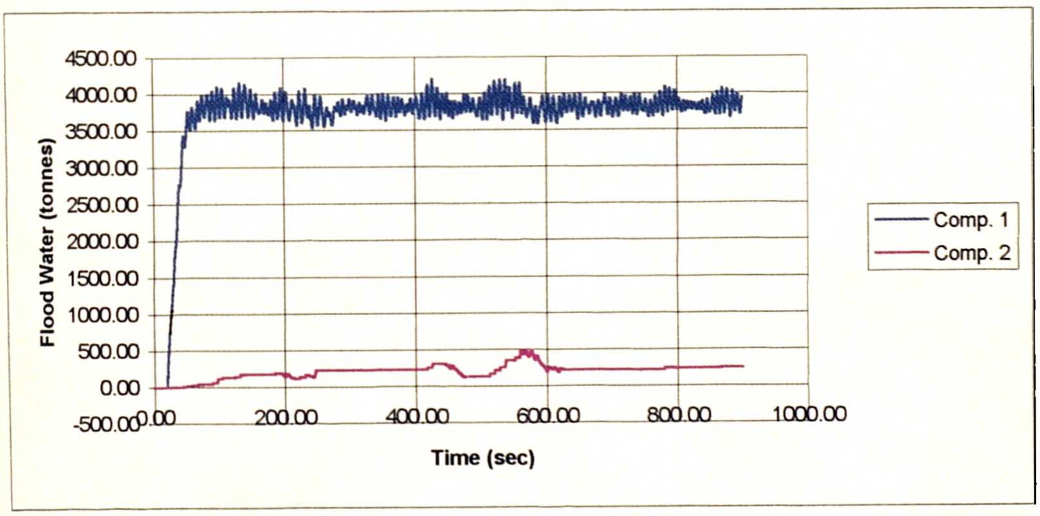


Simulation No. 209

Hs = 1.4 m
KG = 9.17 m
Freeboard = 0.7 m

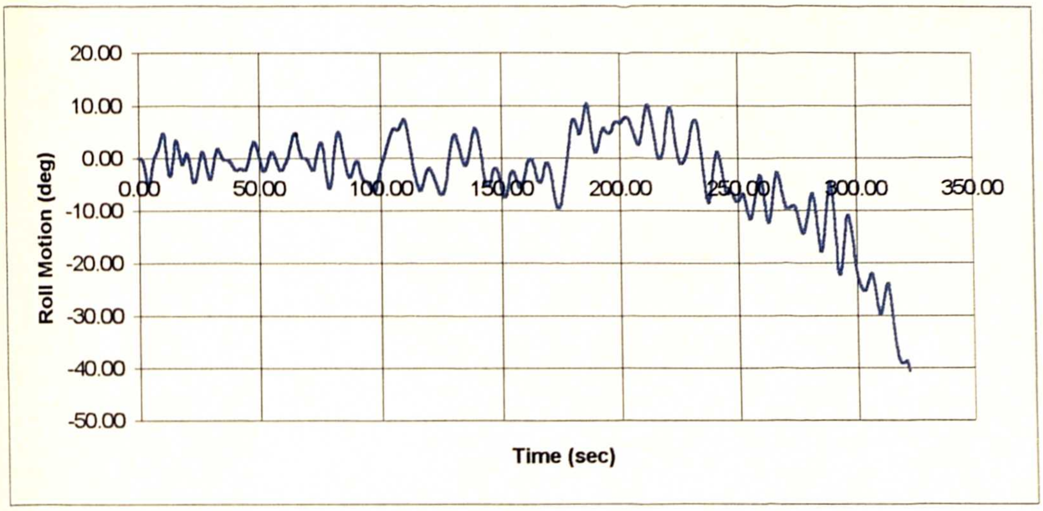


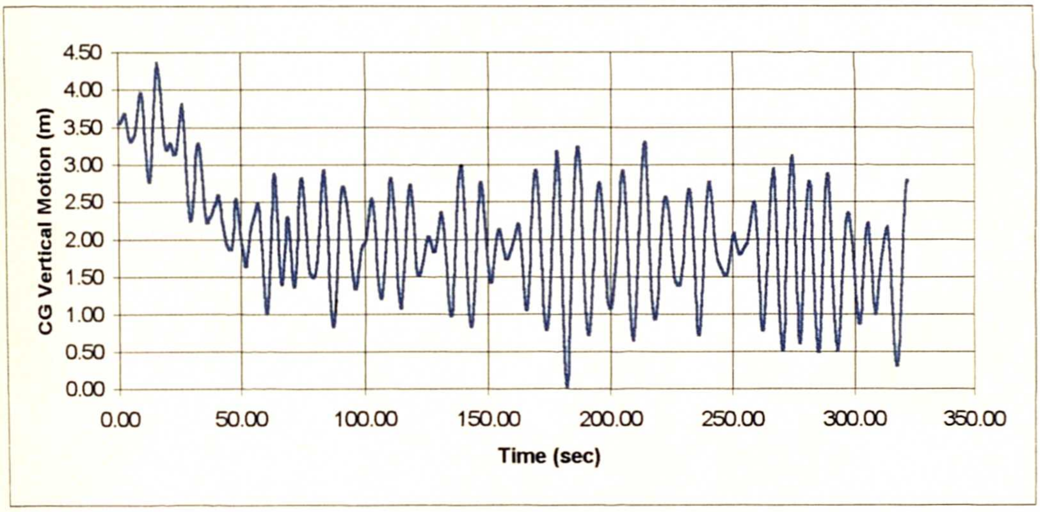


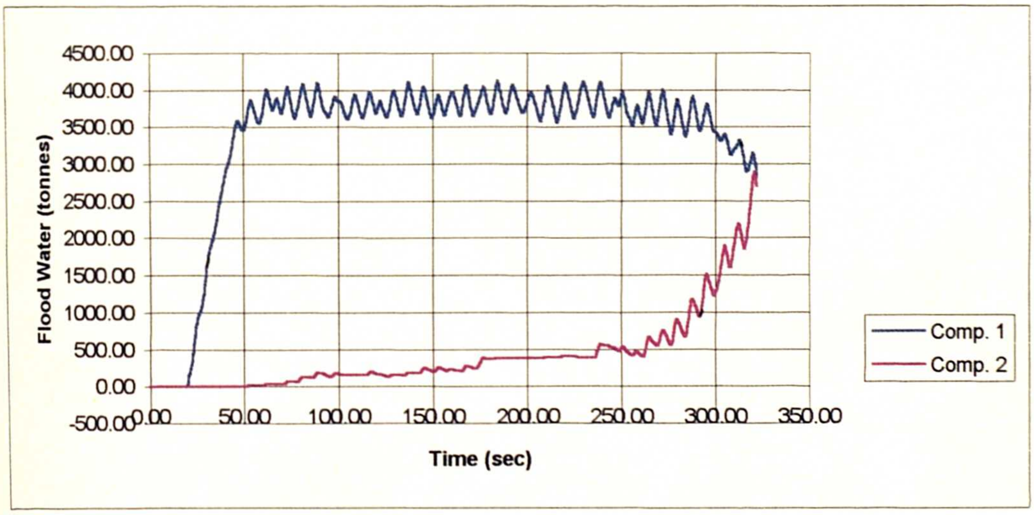


Simulation No. 210

Hs = 2.1 m
KG = 9.17 m
Freeboard = 0.7 m

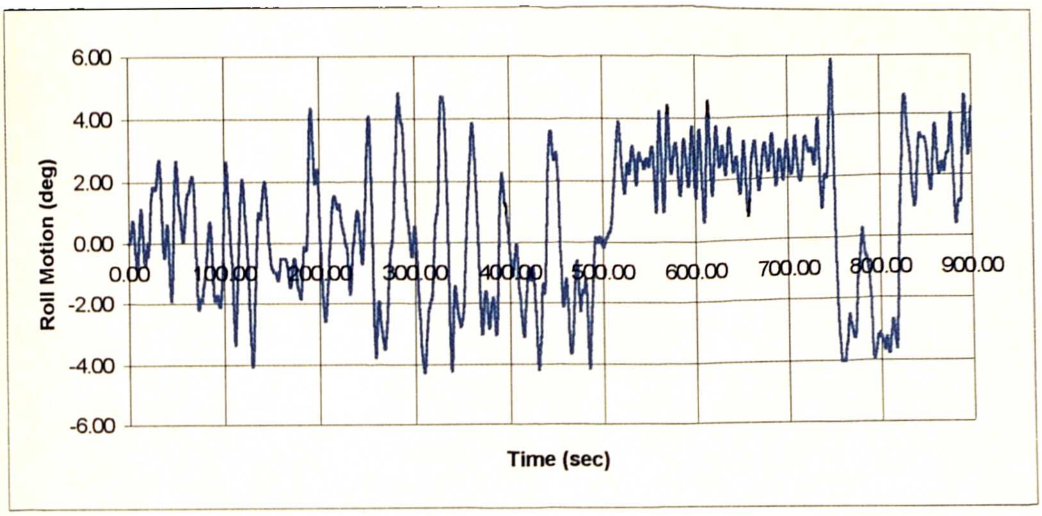


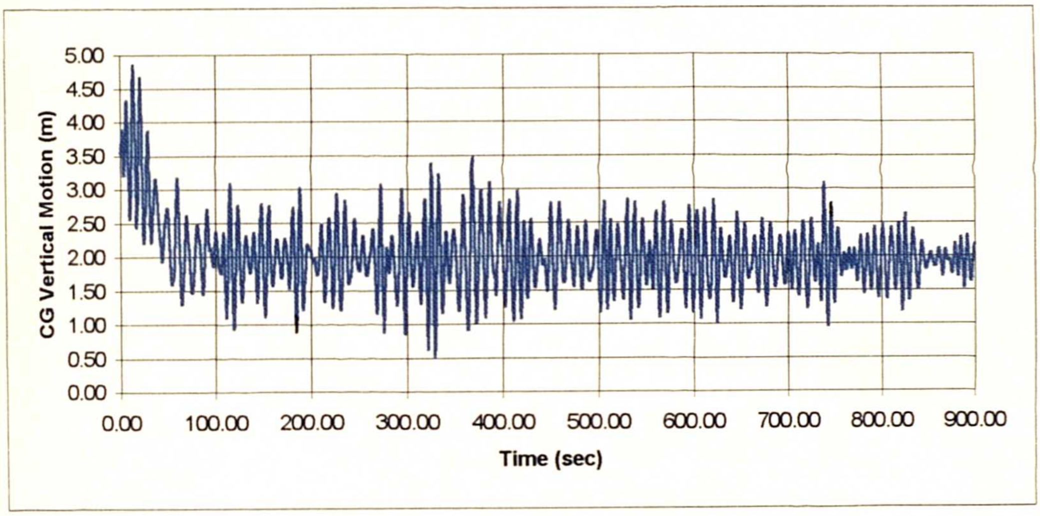


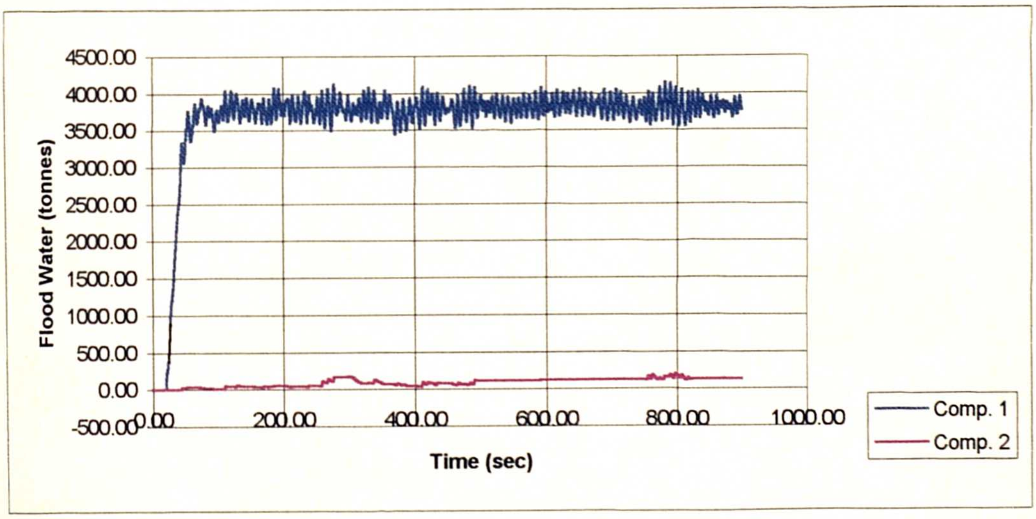


Simulation No. 210b

Hs = 2.1 m
KG = 9.17 m
Freeboard = 0.7 m

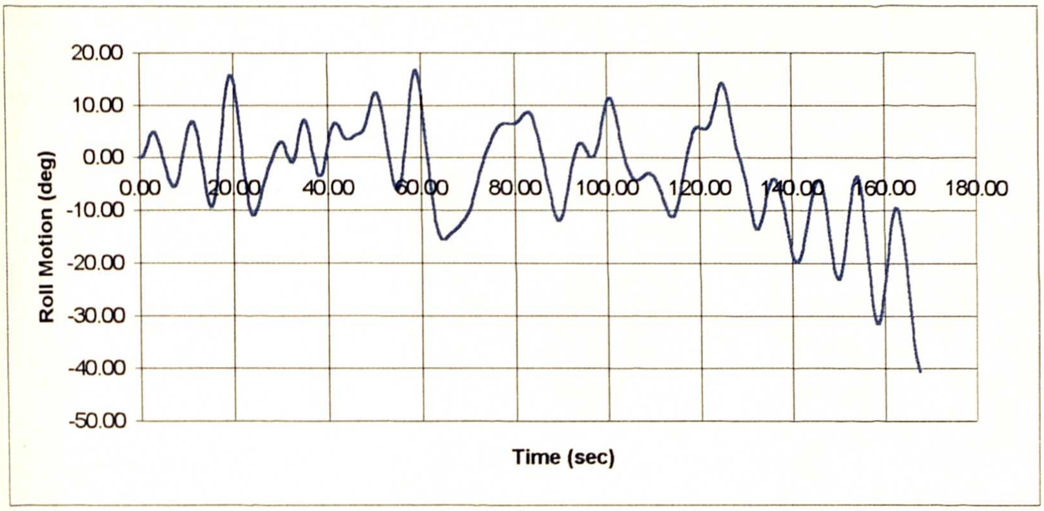


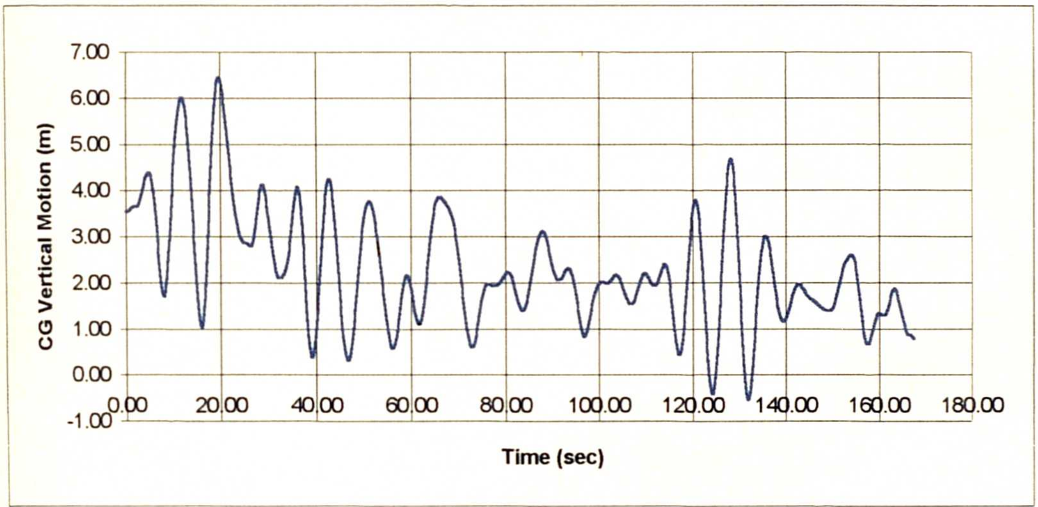


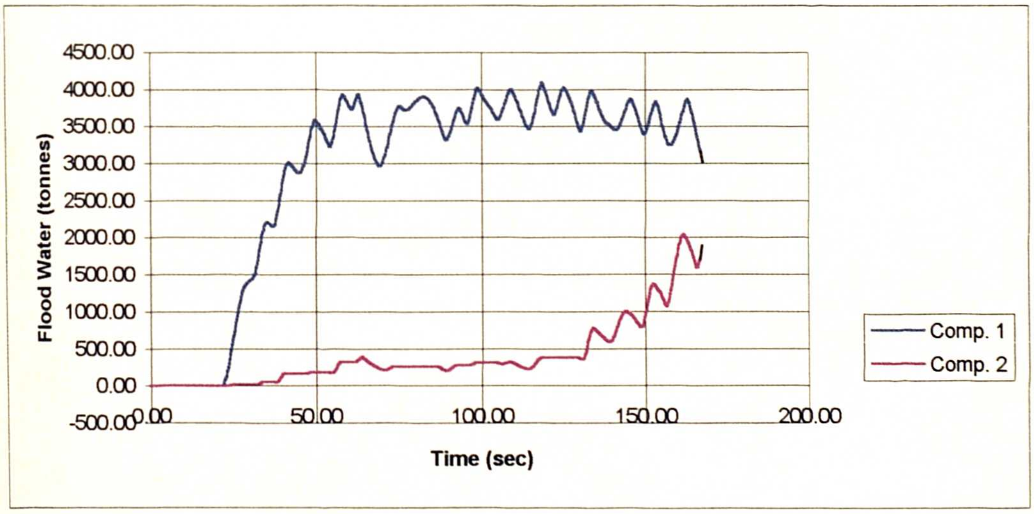


Simulation No. 211

Hs = 2.8 m
KG = 9.17 m
Freeboard = 0.7 m

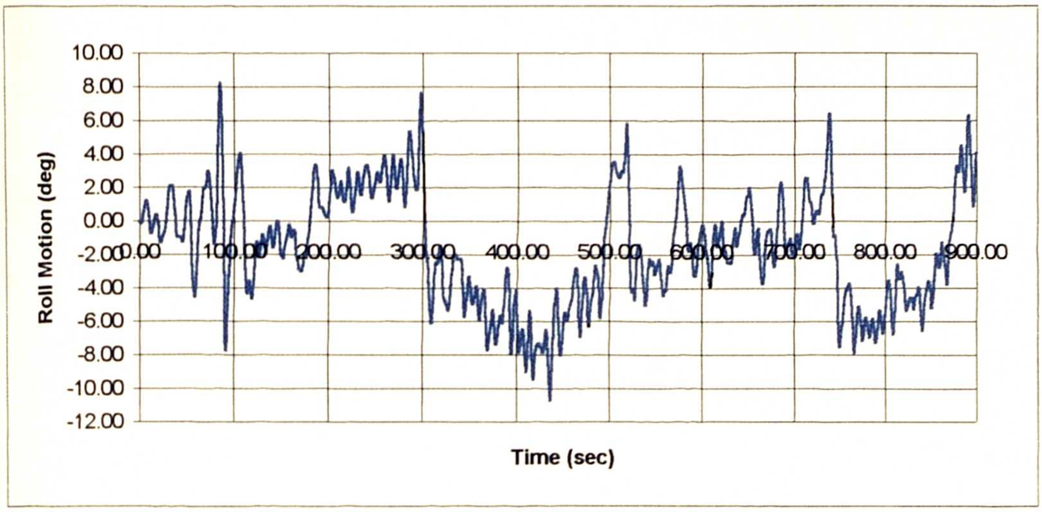


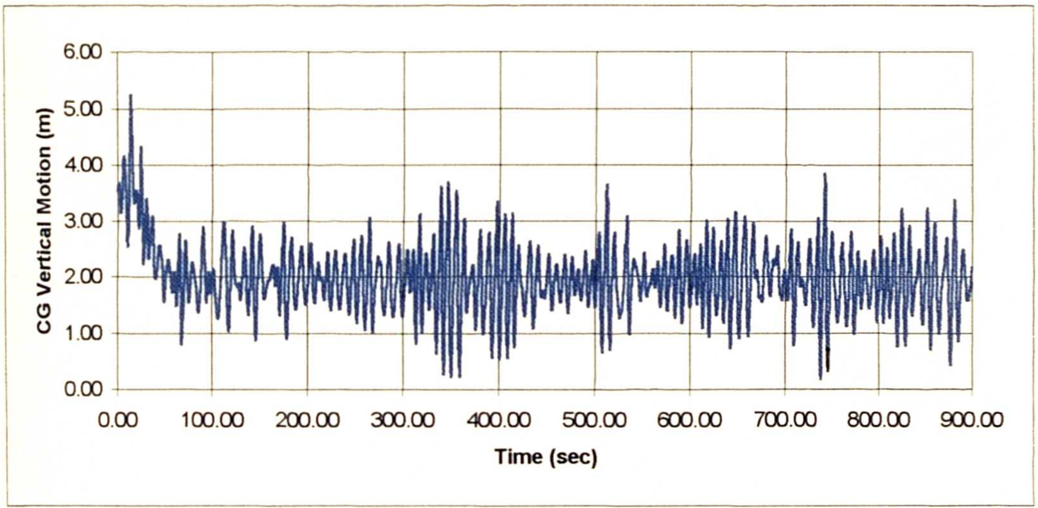


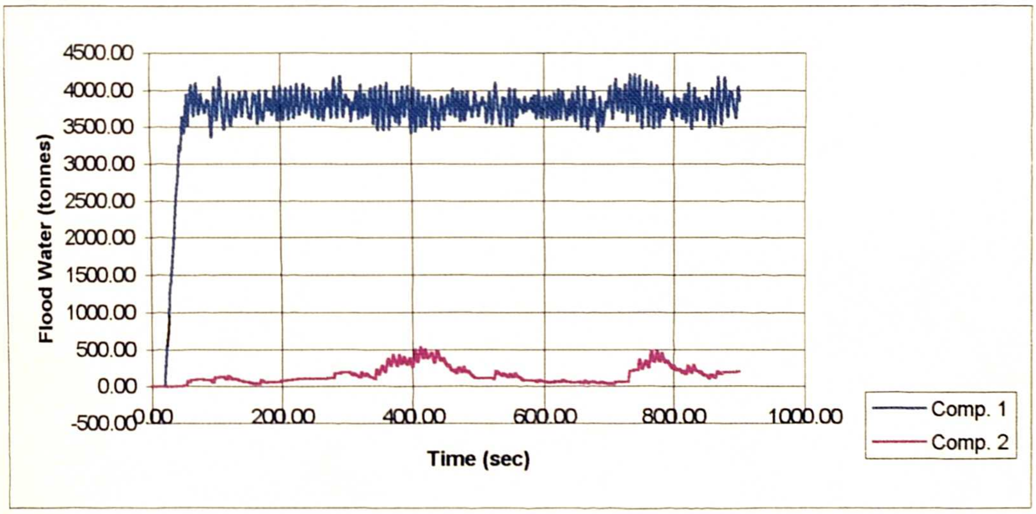


Simulation No. 211b

Hs = 2.8 m
KG = 9.17 m
Freeboard = 0.7 m

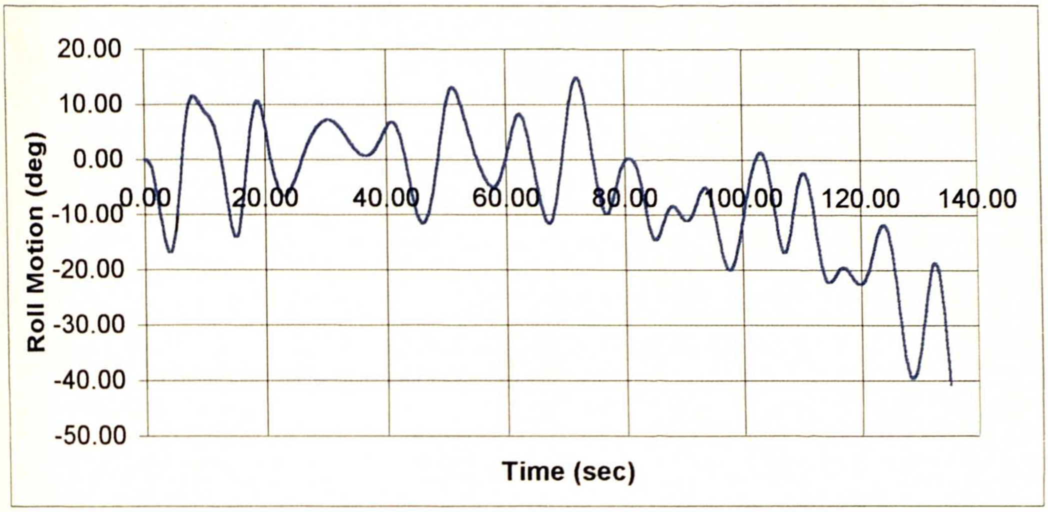


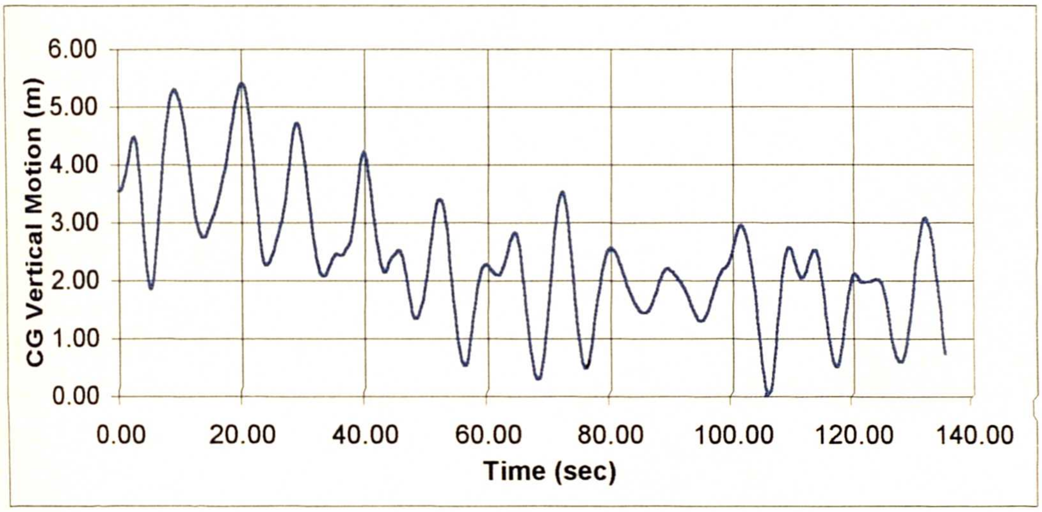


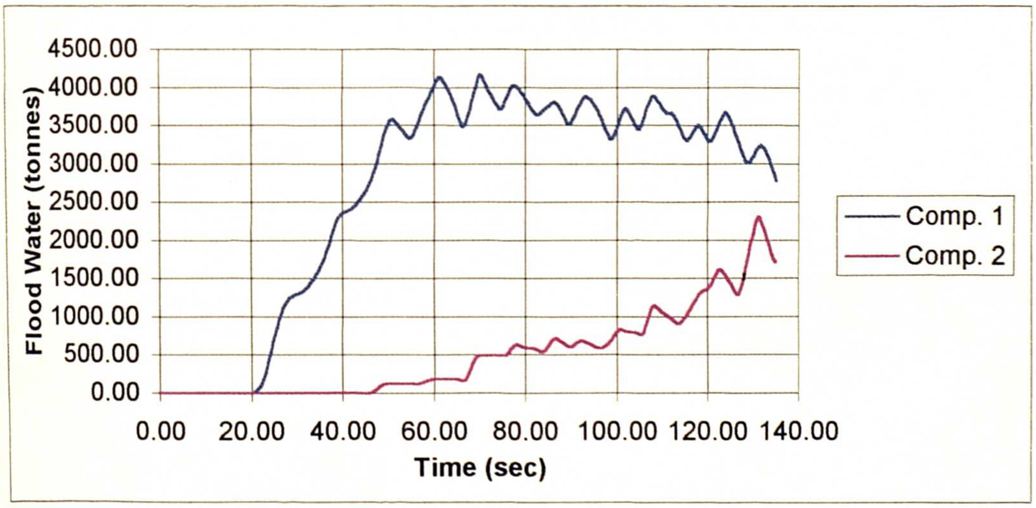


Simulation No. 212

Hs = 3.5 m
KG = 9.17 m
Freeboard = 0.7 m

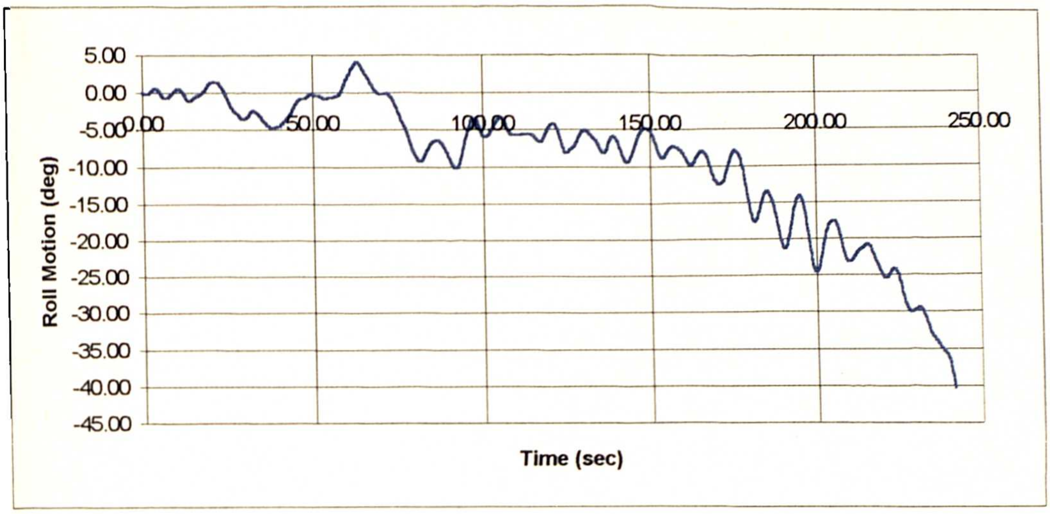


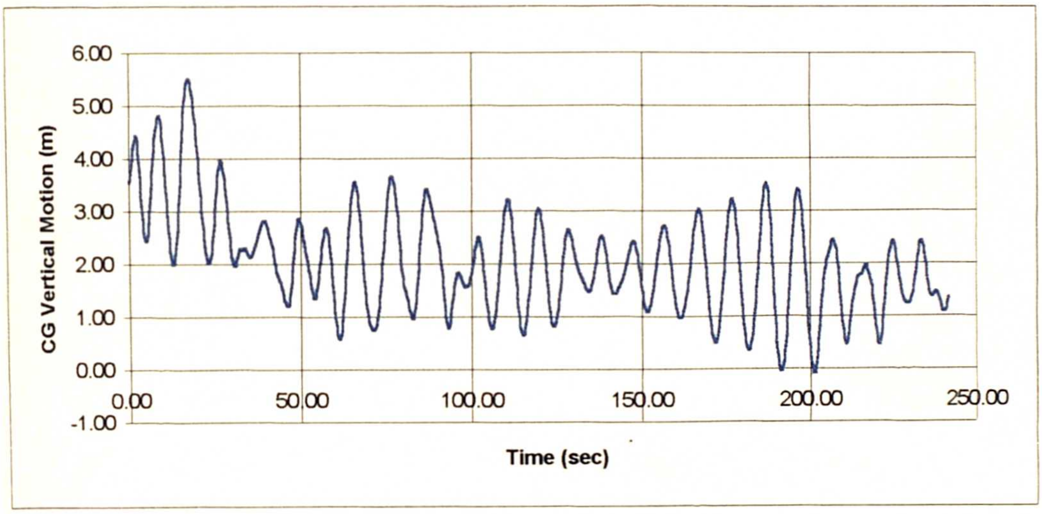


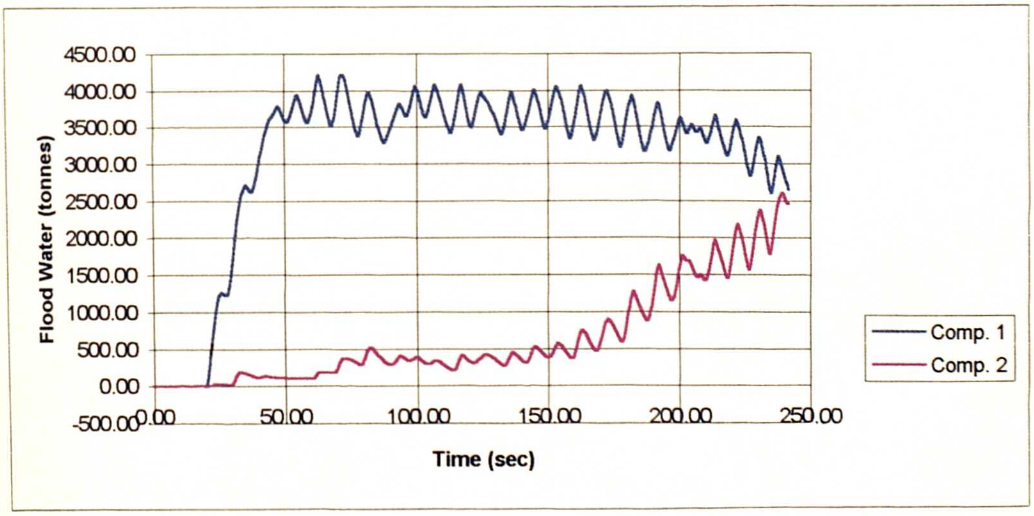


Simulation No. 212b

Hs = 3.5 m
KG = 9.17 m
Freeboard = 0.7 m







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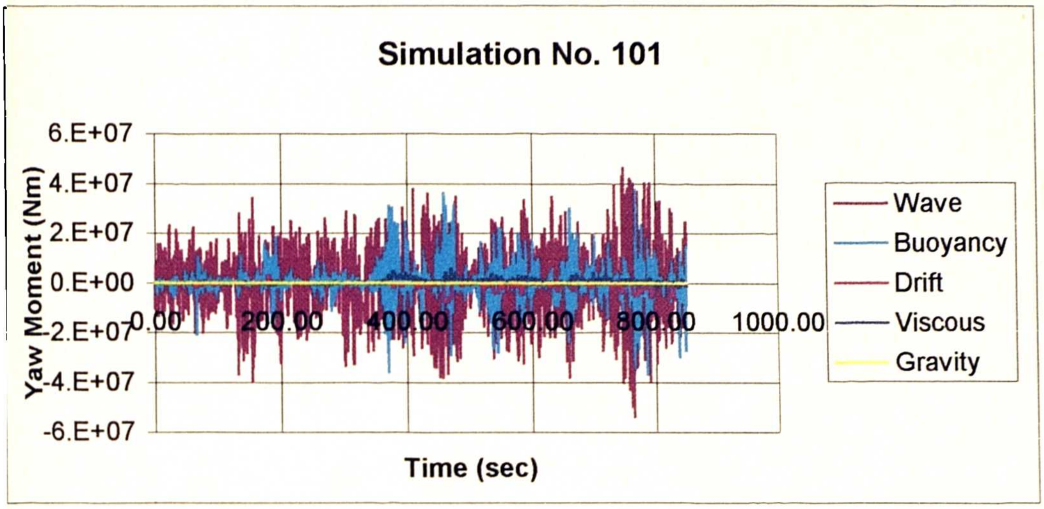

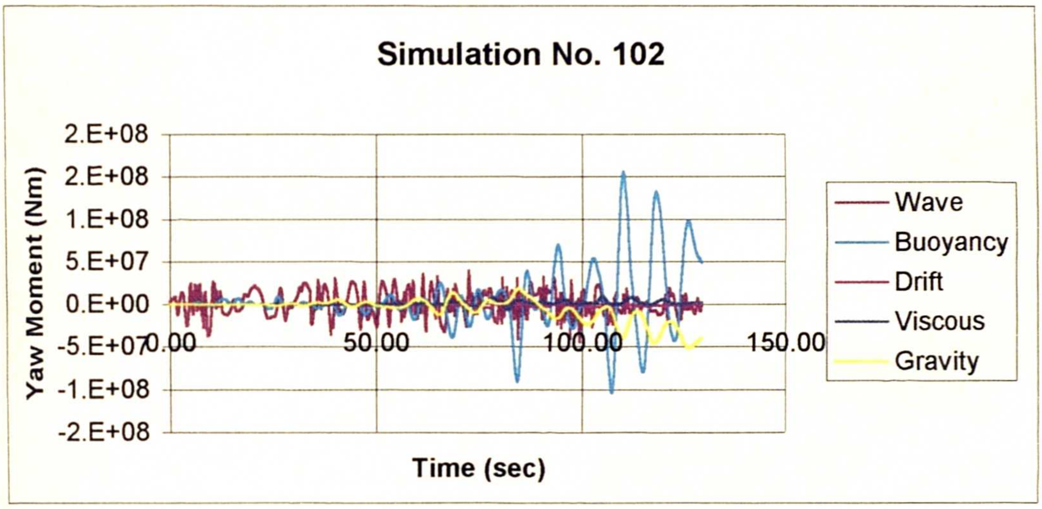

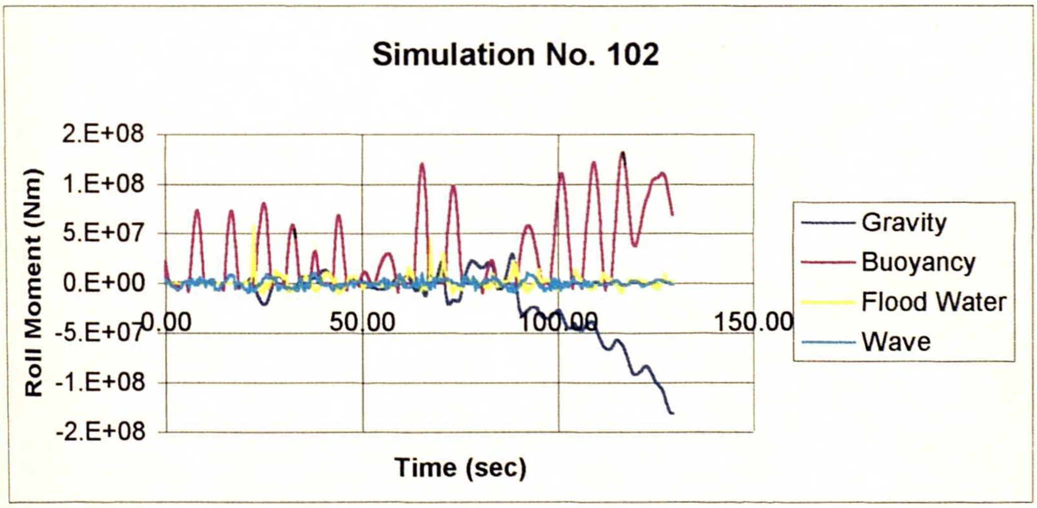

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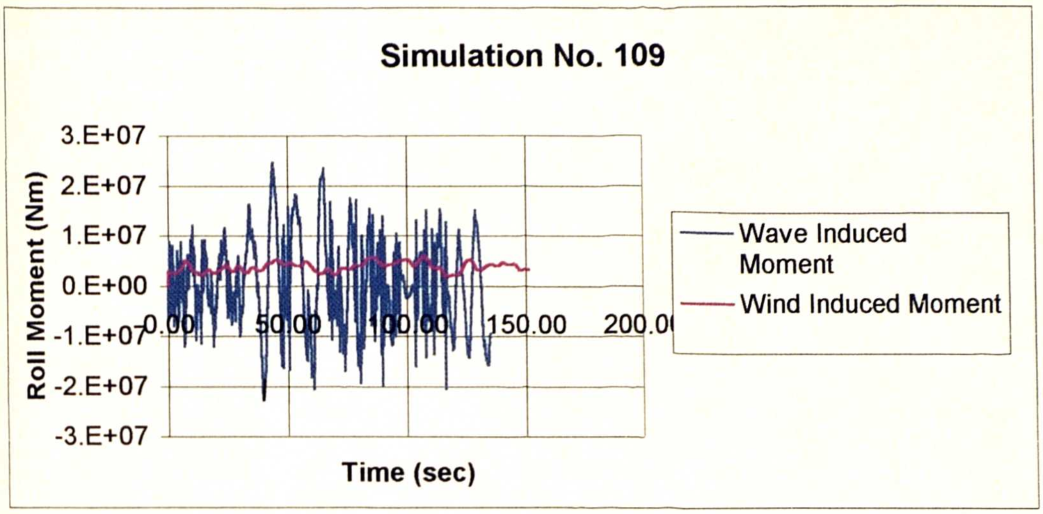

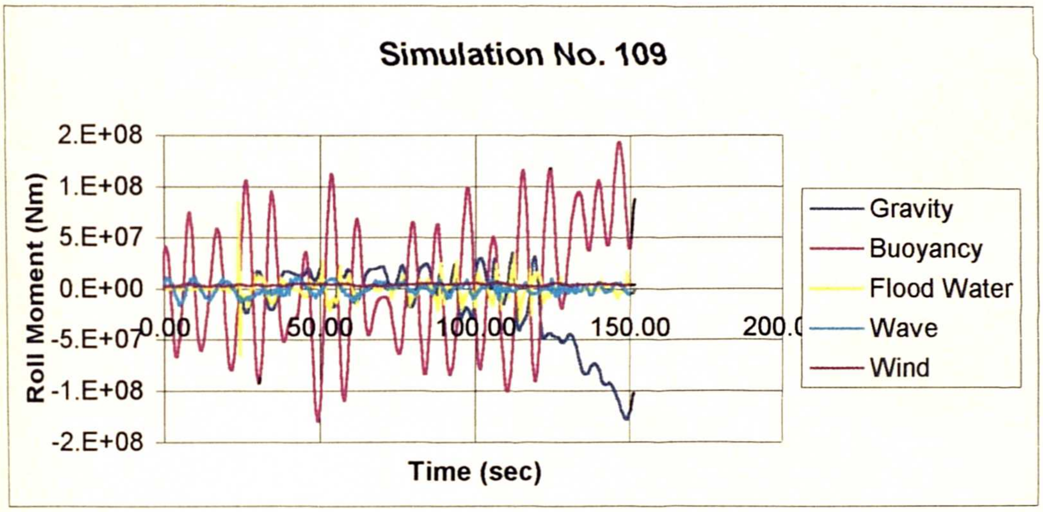

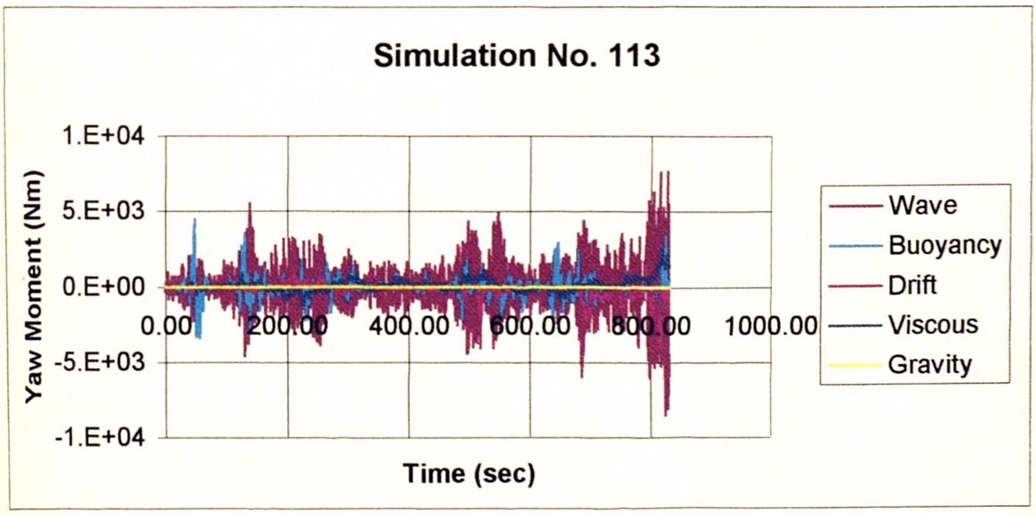

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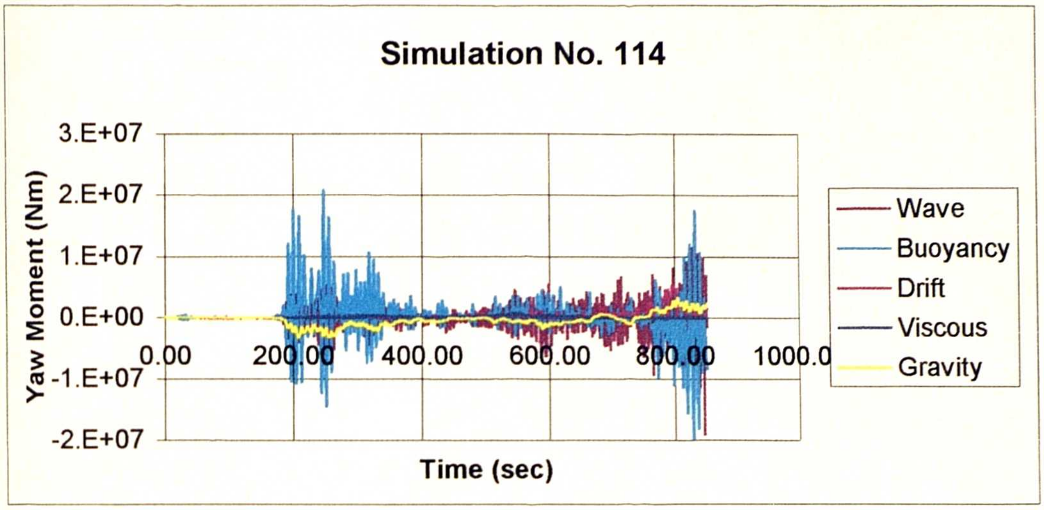

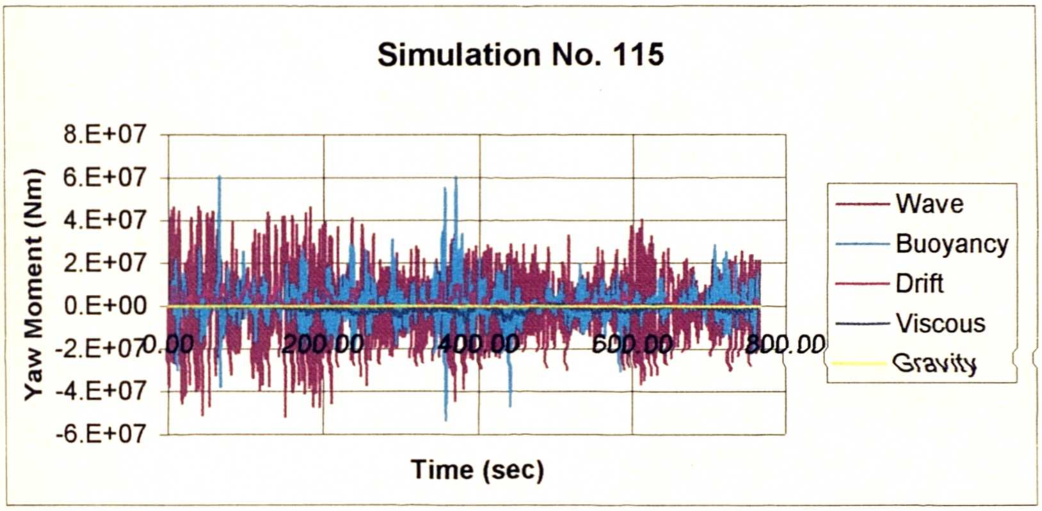

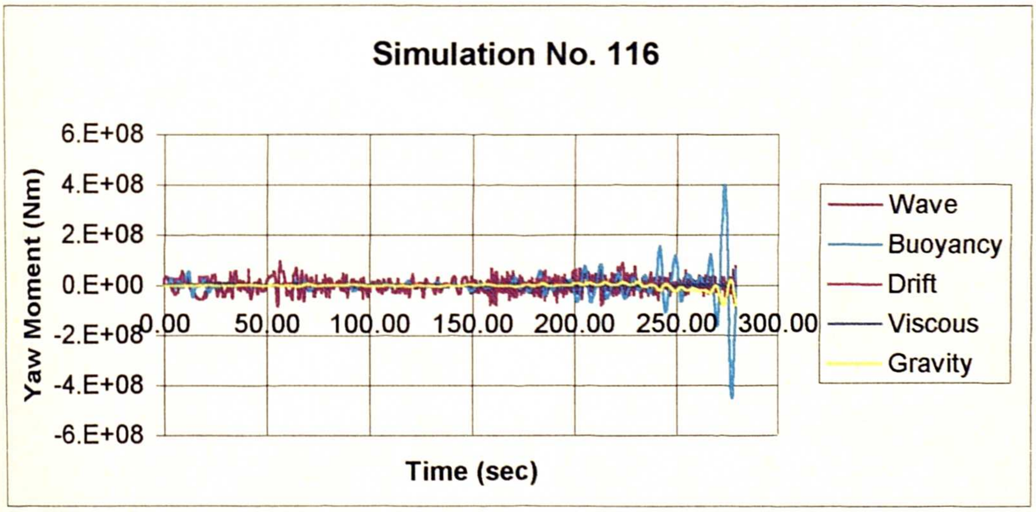

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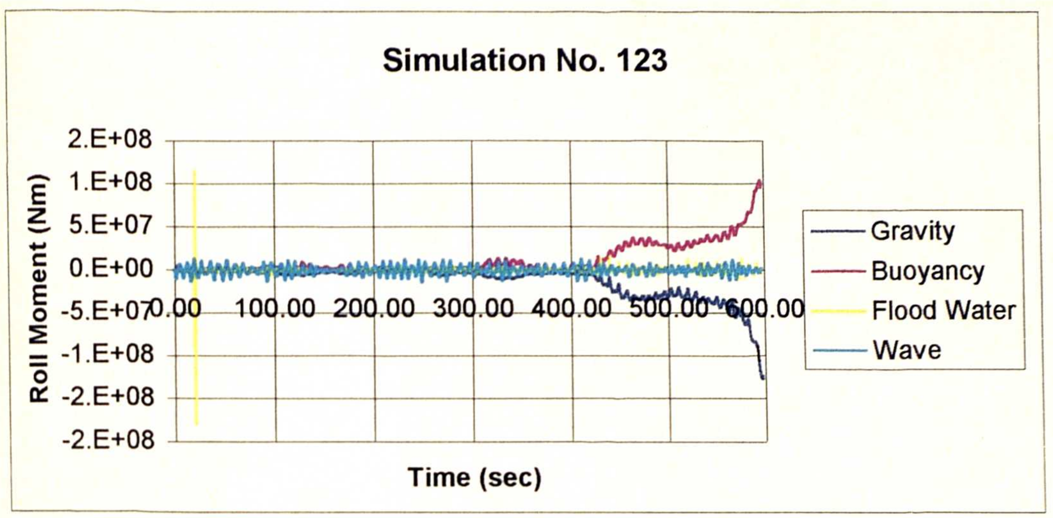
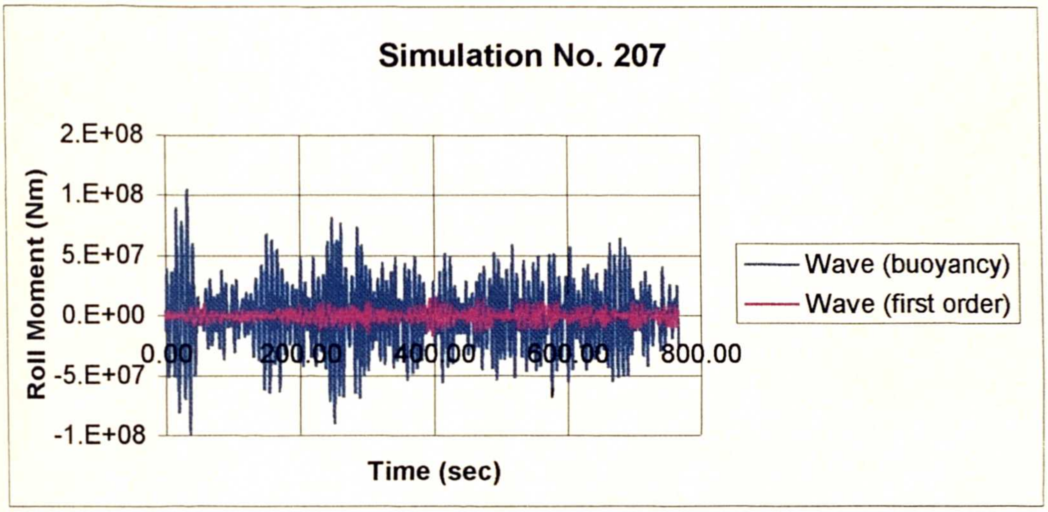
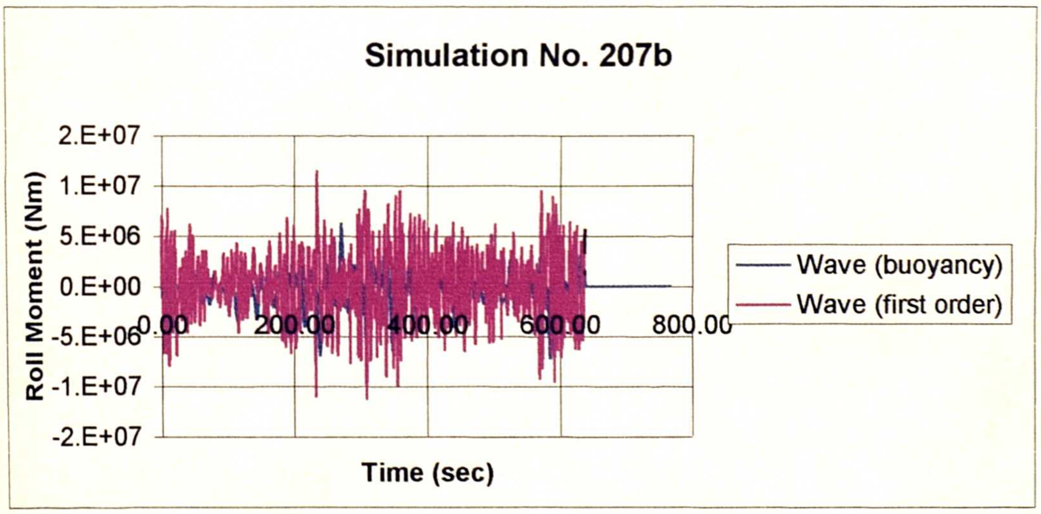
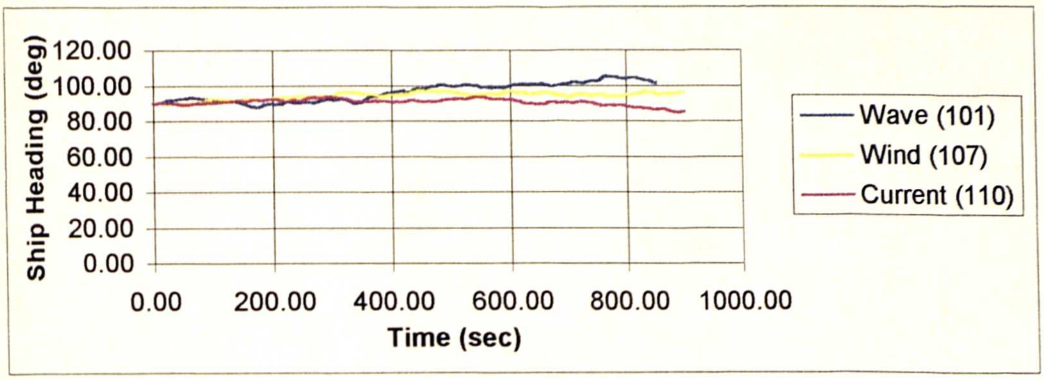
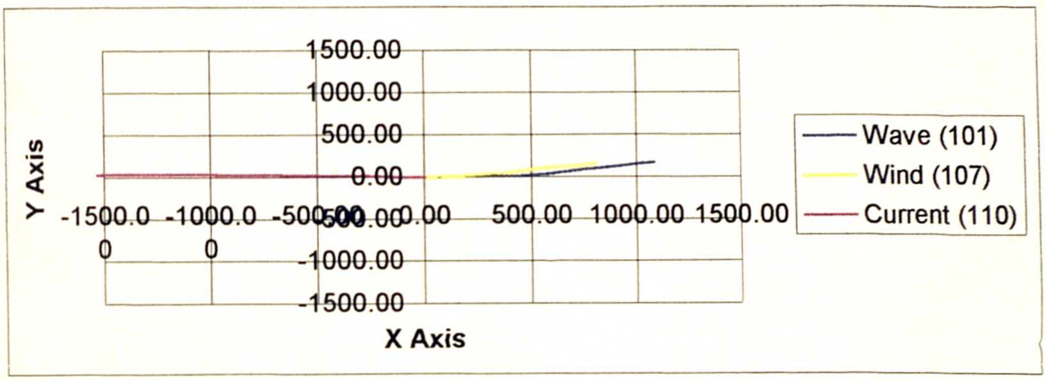


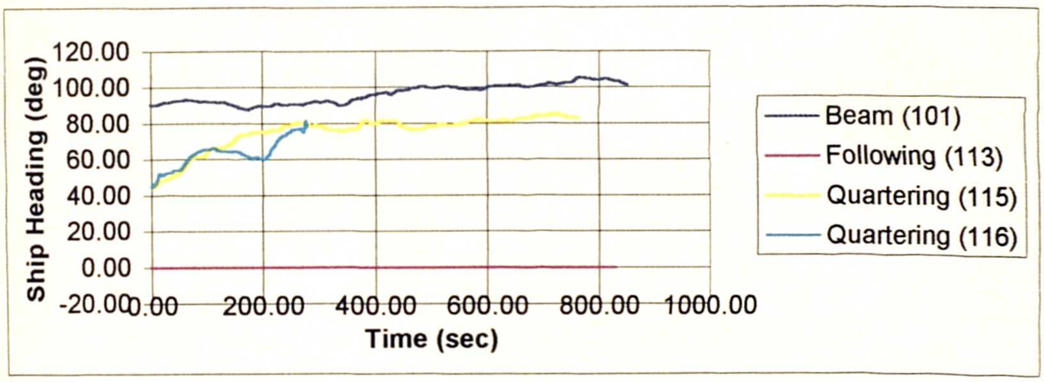


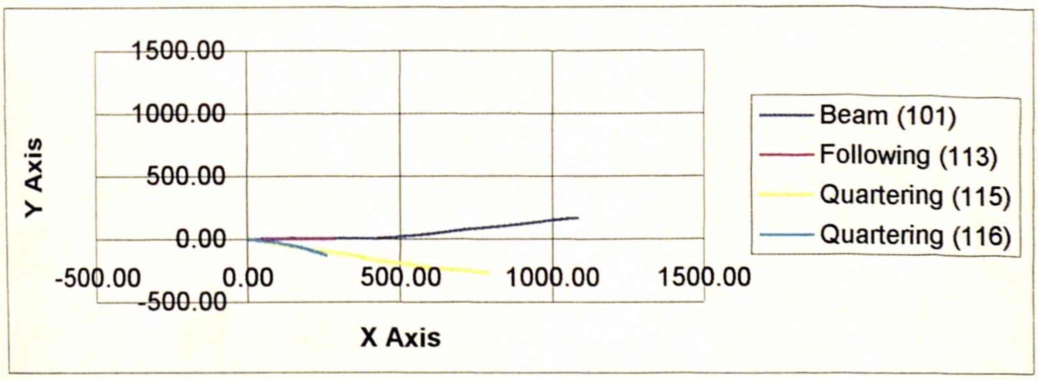


Vessel's Headings and Trajectories

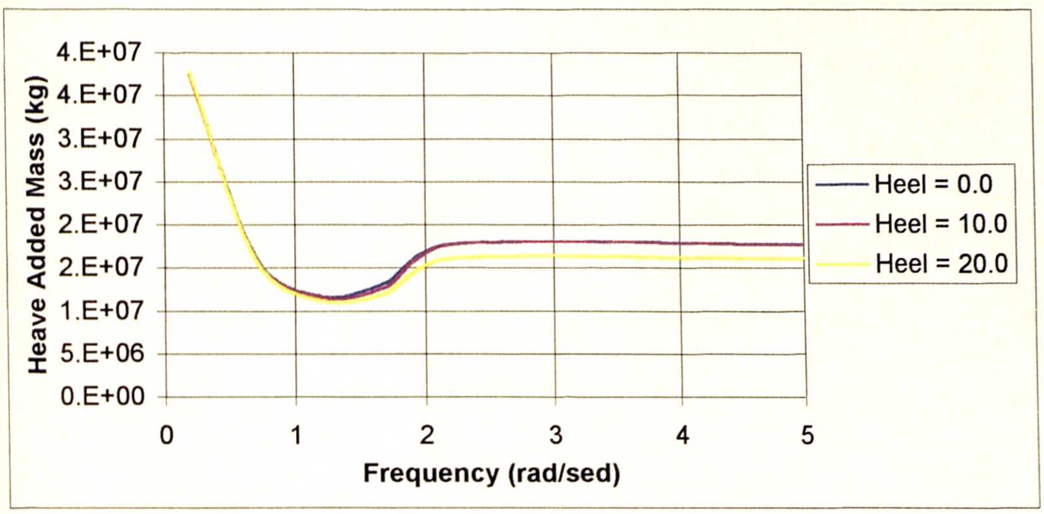


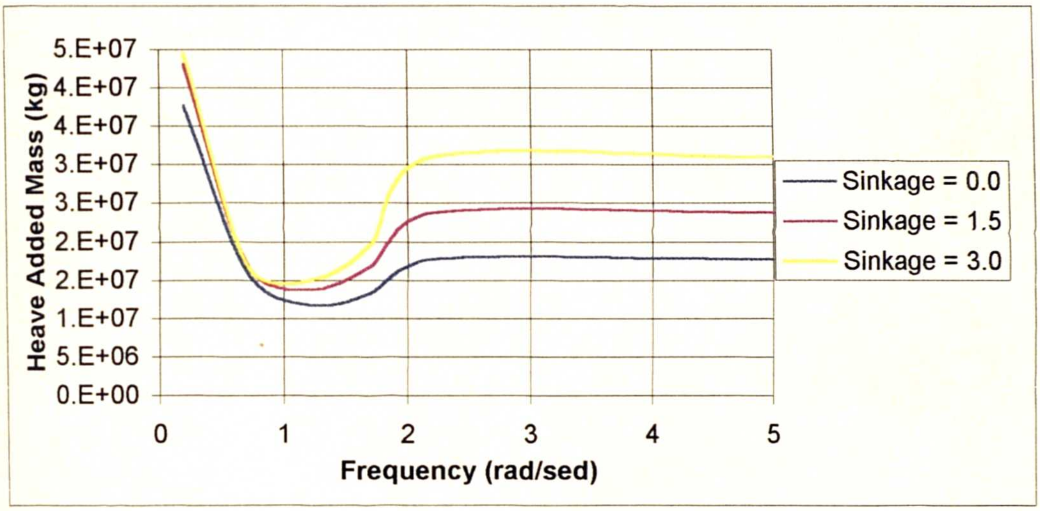


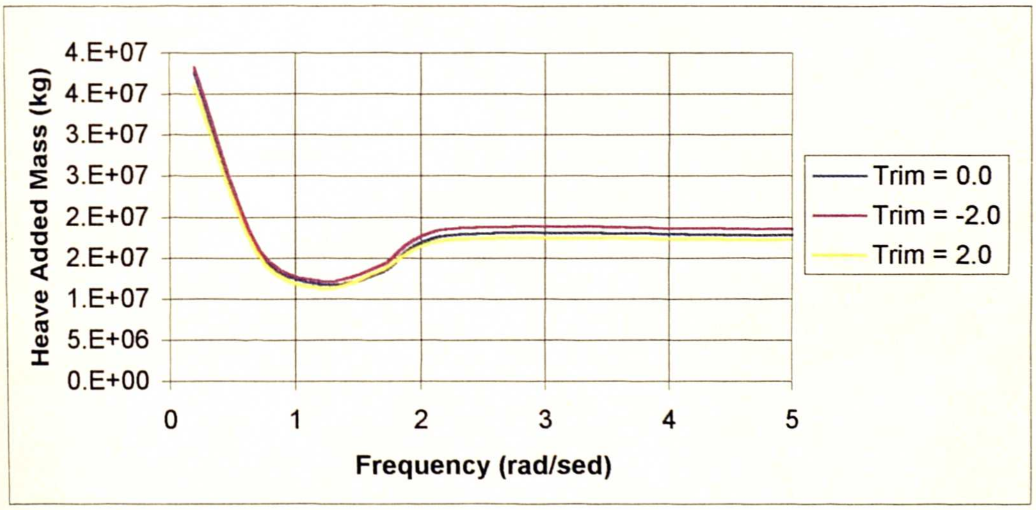




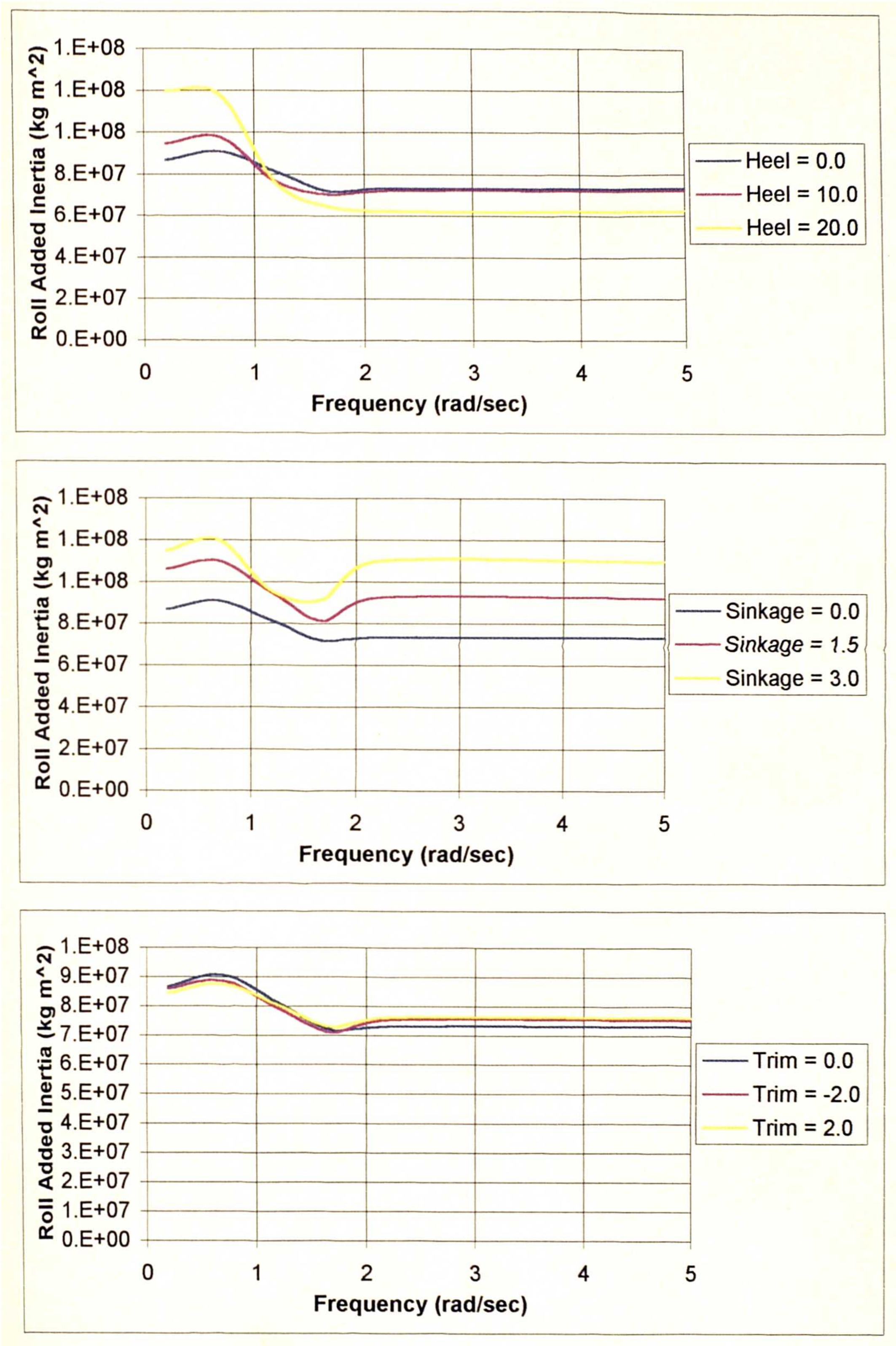
Heave Added Mass Hyper-Surface



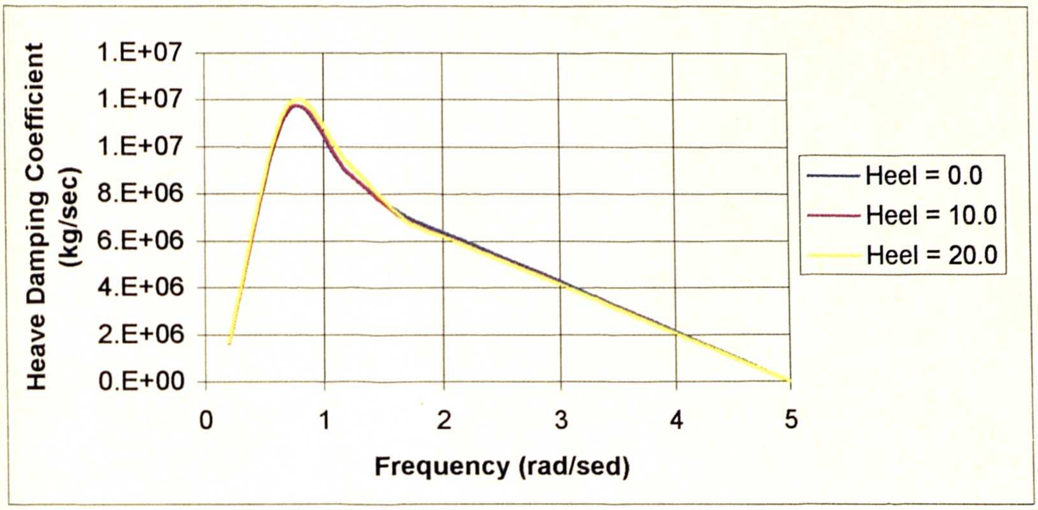


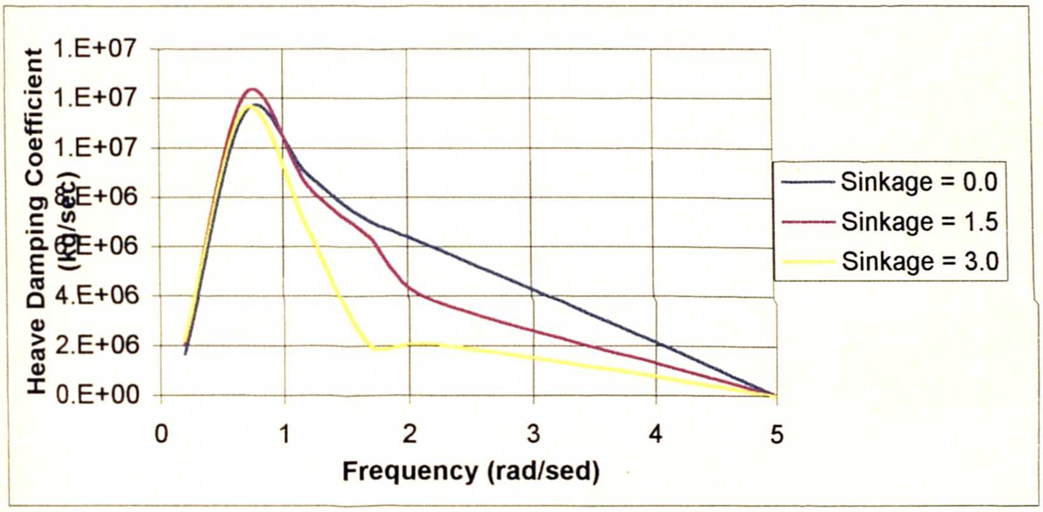


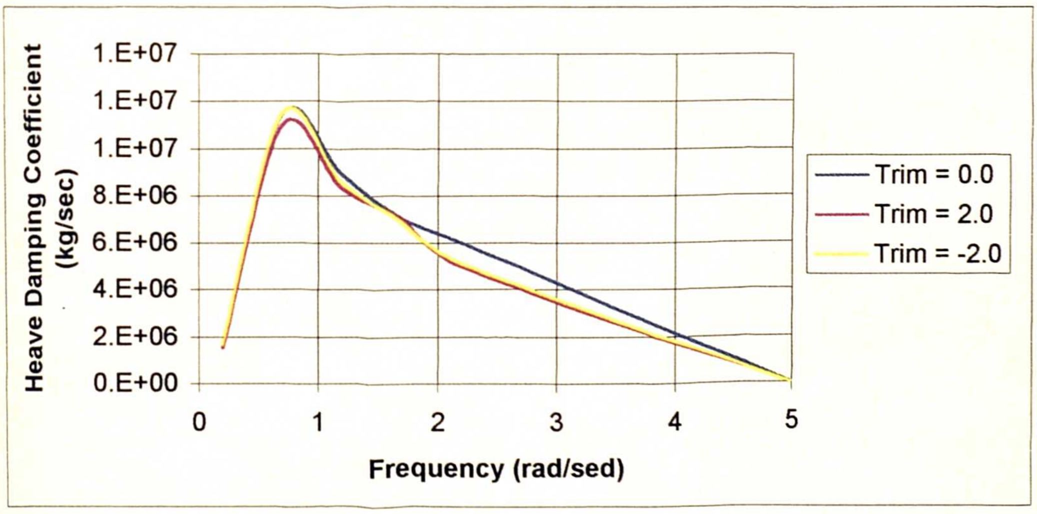
Roll Added Inertia Hyper-Surface



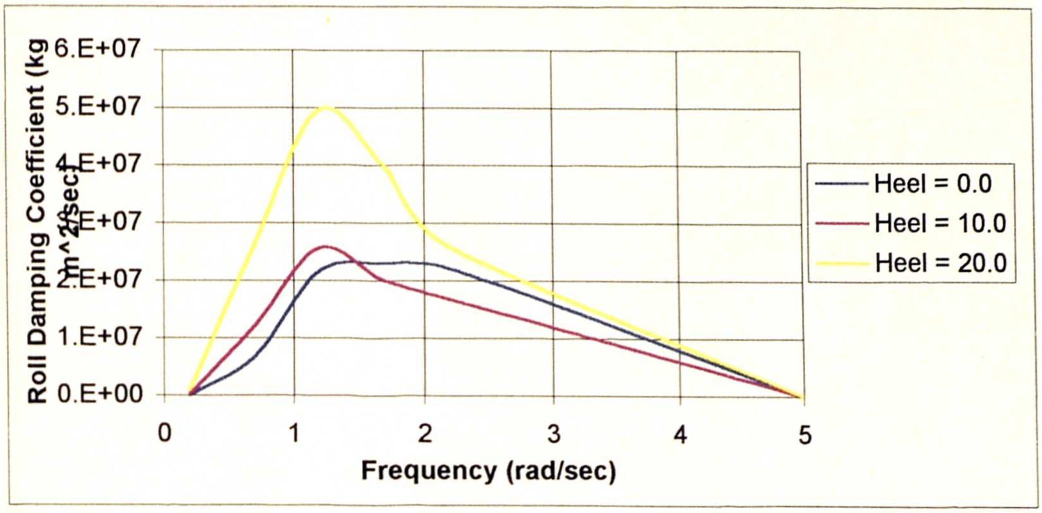
Heave Damping Coefficient Hyper-Surface

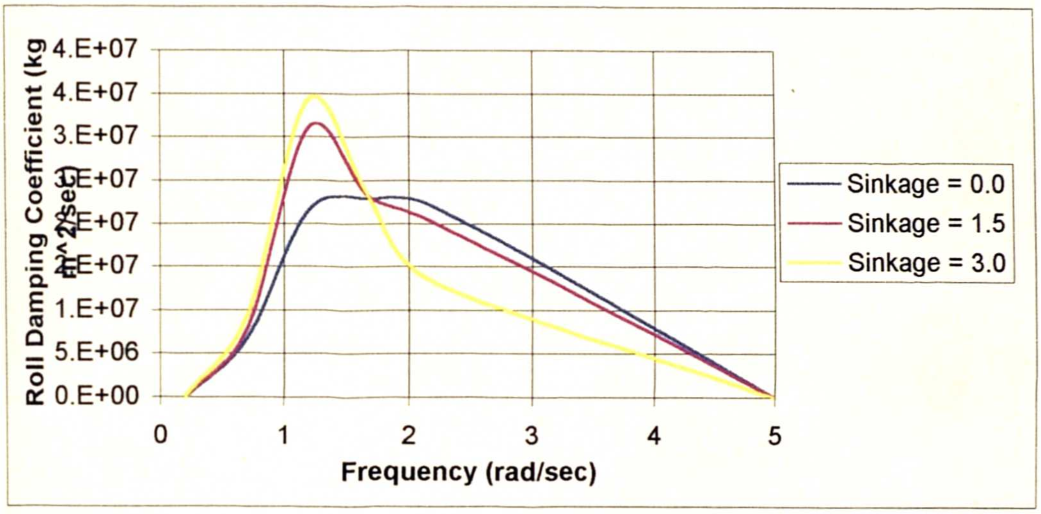


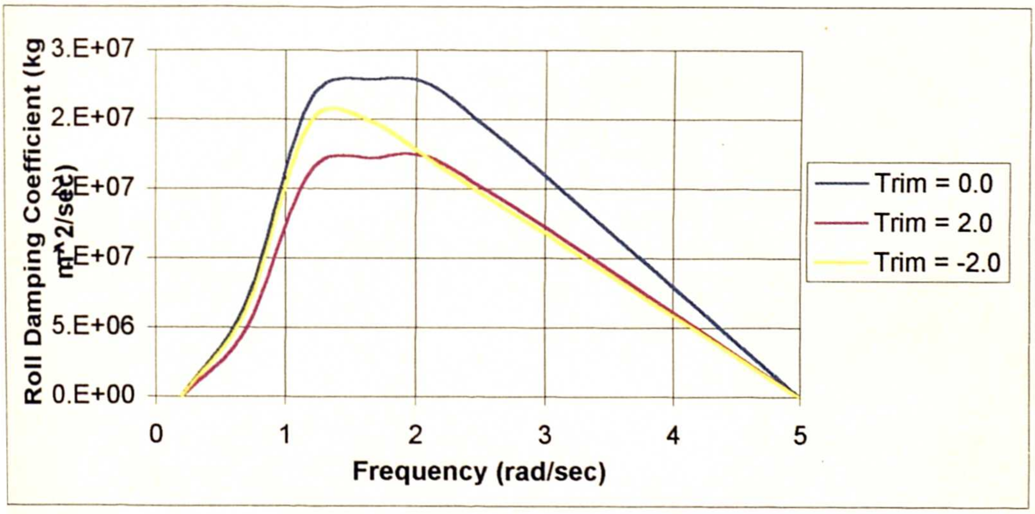




Roll Damping Coefficient Hyper-Surface







Appendix - E

Survey of Relevant References

This appendix contains a brief commentary of some of the technical papers reviewed to generate an awareness of the existing gaps in the mathematical, numerical and experimental treatment of the problem of damage survivability. The numbers in round brackets at the beginning of each reference, refer to the relevant section addressed in Chapter 3. For instance, (6) means that the paper relates to cargo shifting; (4,1) indicate that the contents of the paper are primarily relevant to the sloshing problem but that the paper also is also a useful reference on model tests. The absence of numbers should be interpreted as an indication that the paper pertains to matters of general interest. References are listed in a reverse chronological order.

1] (2) Letizia, L. and Vassalos, D.: **"Formulation of a Non-Linear Mathematical Model for a Damaged Ship Subjected to Progressive Flooding"**, International Symposium on Ship Safety in a Seaway, Kaliningrad, Russia, May 1995.

The dynamic behaviour of damaged vessels has been assessed so far by solving the canonical equations used in the study of wave induced ship motions. In so doing, the effect of the rate of variation of the mass of flood water on the system performance was accounted for only by updating at each time step the total inertia of the ship to accommodate the water accumulating in each damaged compartment. This paper discloses new equations of motion developed to study the response of modern ferries during transient flooding and outlines the inadequacies of the existing models when dealing with time varying inertia. The role of the rate of flooding of a compartment open to the sea is thus put in a new light and the need for an appropriate model to estimate this is emphasised.

[2] (2) Makov, Y. and Sevastianov, N. B.: "**Physical Modelling of Damaged Vessels Capsizing under the Influence of Wind and Waves**", The Naval Architect, April 1995.

Numerical analysis and model experiments have been undertaken to investigate the capsize resistance of damaged vessels (mainly fishing vessels) drifting in the presence of wind and waves. It is worth mentioning here some of the conclusions, particularly because of their similarity to those arrived at with RO-RO vessels:

- water entering a damaged compartment damps rolling to a great extent and so does early submergence of the deck and bulwark;
- the roll motion of a damaged ship is insignificant and varies only slightly with sea state;
- the wind effect is primarily attributable to steady wind rather than to wind gusting;
- the vessel always capsized to leeward (this is very surprising finding and in contradiction to the findings with RO-RO vessels);
- the maximum level of the GZ curve is the most relevant parameter affecting the safety of a damaged ship.

[3] (3) The Glosten Associates, inc.: "**Water Accumulation on the Deck of a Stationary Ship**" - SNAME Ad Hoc RO-RO Safety Panel, Annex A, File No. 94209.01, Feb. 1995.

Following the inaugural meeting of the SNAME Ad Hoc Panel during the Annual Meeting held in November 1994, Glosten Associates inc. developed a probabilistic theory based model to correlate the asymptotic average water depth on the deck of a motionless vessel and the average flow rate onto the deck, with environmental parameters like the significant wave height. Results of extensive numerical simulation in the time domain are compared with experimental results carried out at the Institute for Marine Dynamics, National Research Council Canada (the results of these tests are not available to the public). The mathematical model to calculate the instantaneous flow through the damage opening is based on the concept of the basic weir flow model, assuming an empirical coefficient value of 0.65, equal to the maximum value used in the well known stationary case (civil engineering hydraulics).

[4] (7) Pedersen, P. T.: " **Collision and Grounding Mechanics** ", WEMT '95, Ship Safety and Protection of the Environment, May 1995.

After presenting a statistical method to evaluate the probability of grounding and collision as a function of the ship traffic on a given route, this paper describes modern methods to estimate the extent of structural damage caused to a ship by accidental impact. These are based on an integrated approach, accounting for ship dynamics and inner collision mechanics.

[5] (2) Vassalos, D.: "**Capsizal Resistance Prediction of a Damaged Ship in a Random Sea**", RINA Symposium on RO-RO SHIPS' SURVIVABILITY, London, Nov. 1994.

This paper, in summarising the results of the research undertaken at the University of Strathclyde in association with the Phase II of the UK RO-RO Stability Research Programme, sets out to provide answers to the all important question of RO-RO survivability and to demonstrate the need to address damage survivability by taking full account of the vessel dynamics in realistic operating conditions. An explanation of the new approach adopted to study this problem is presented. A modern RO-RO vessel, representing a ship type most vulnerable to serious flooding, is used to demonstrate the practical applicability of the proposed procedure. The derived results are presented in the form of boundary survivability curves to provide a basis for the development of survival criteria.

[6] (1) Dand, I. W.: "**Factors Affecting the Capsize of Damaged RO-RO Vessels in Waves**", RINA Symposium on RO-RO SHIPS' SURVIVABILITY, London, Nov. 1994.

This paper is based on the work undertaken by BMT Ltd for the UK Department of Transport Phase II Research aiming to develop a theoretical damaged ship capsize model. In this paper, the theoretical model developed by BMT is presented briefly. However, results and conclusions derived do not pertain to this model but are mainly based on the model experiments carried out to assess the efficiency of alternative design configurations. The author differentiates between two different modes of capsize; speed induced capsize (forward speed) and drift induced capsize (zero forward speed) but experimentally derived results focus mainly on drift induced capsize. Overall, it would have been nice to see more results derived directly from the theoretical model as the development of the latter is essential to enable those concerned, to predict the damage survivability of other vessels.

[7] (2) Spouge, J. R.: "**A Technique to Predict the Capsize of a Damaged RO-RO Ferry**", RINA Symposium on RO-RO SHIPS' SURVIVABILITY, London, Nov. 1994.

This paper is based on the work contracted to DNV Technica by the UK Department of Transport to develop a theoretical along the lines explained above. The theoretical model introduced here does not attempt to analyse the behaviour of a damaged ship or to investigate the effects of certain parameters on damage ship survivability. Instead, an empirically derived formula has been developed, based on experimental data and a very simplified roll motion mechanism.

[8] (1) Velschou, S. and Schindler, M.: "**RO-RO Passenger Stability Studies: A Continuation of Model Tests for a Typical Ferry**", RINA Symposium on RO-RO SHIPS' SURVIVABILITY, London, Nov. 1994.

This paper describes the experiments carried out by DMI for The UK Department of Transport Phase II Research to investigate the effect of some external and internal devices as well as damage location on damage survivability. In these experiments B/5 longitudinal side tanks, 2 metres wide longitudinal side tanks, centre casing and transverse partial and full height bulkhead doors on RO-RO decks are considered together with the effect of permeability. Sponsons, hull flare and buoyancy air bags are examined as external devices. These comprehensive experiments provide very useful information on how these devices affect the survivability of a damaged ship. The results are presented as capsize no-capsize boundary curves as functions of GM, GZ positive stability range and area under the GZ curve. These are then compared to the SOLAS '90 regulations. The results obtained show that SOLAS '90 rules are not sufficient to ensure safety and need to be improved considerably.

[9] (4, 2) Rakhmanin, N. *and Zhivitsa*, S.: ***"Prediction of Motion of Ships with* Flooded Compartments in a seaway"**, STAB '94, Melbourne, USA, Nov. 1994.

In this paper a new theoretical approach to the problem of simulating the motion of a damaged vessel is presented, with particular reference to the effect of sloshing and the variation of the mass of flood water. One of the problems presented by this publication is the repeated reference to papers which are not available to the western world. Although the concepts expressed appear to be interesting, a complete appreciation of this work is not actually possible.

[10] (4) Lee, A. and Adee, B.: **"Numerical Analysis of Vessel's Dynamic Responses with Water Trapped on Deck"**, STAB '94, Melbourne, USA, Nov. 1994.

This is an interesting paper addressing the static and dynamic effects of water on deck on the motion and stability of typical fishing vessels. A three-degrees-of-freedom mathematical model is used (coupled sway, heave and roll) and a numerical scheme is employed to study the dynamic behaviour of the vessel involving - in addition to the normal time stepping procedures - a numerical simulation of the non-linear sloshing of water on deck using shallow water wave theory equations solved by Glimm's random choice method. This methodology is developed according to Gillingham's guidelines but incorporating the effect of vessel motions on the water particle's accelerations in accordance to Pantazopoulos' work. On the basis of the derived results the following effects may be evident due to the presence of water on deck:

- for an initially stable vessel with a small amount of water on deck, there is no noticeable quasi-static heel angle. The primary effect of water on deck is to reduce the roll motion response of the vessel;
- for an initially stable vessel with a large amount of water on deck, the determination of the "low frequency" response due to water on deck may be critically important in predicting vessel capsize;
- for an initially marginally stable vessel with water on deck, the effect of water on deck results in the presence of a quasi-static heel angle and three types (harmonic, sub-harmonic and periodic motions) of roll responses.

[11](2) Vermeer, H.: **"Mathematical Modelling of Motions and Damage Stability of RO-RO Ships in the Intermediate Stages of Flooding"** - STAB '94, Melbourne, USA, Nov. 1994.

In this paper a mathematical model is presented to describe the motion behaviour and the associated residual stability of a ship in the time domain after sustaining a collision damage, with particular emphasis placed at the intermediate stages of flooding. A three-degrees-of-freedom model coupled in roll, sway and yaw is utilised at zero speed. The ship motions due to wave excitation as well as sloshing of flood water are ignored - so that only the dynamic transient flooding is simulated - but disregarding these approximations, an in-depth parametric investigation is undertaken and the results show good agreement with model experiments using idealised and representative ferry models. The effect of permeability, vent openings, twin decks and major obstructions, such as propulsion unit in the engine room are accounted for in order to reflect the actual flooding process as closely as possible. The main parameters used in the investigation include:

- damage opening (the most influential parameter);
- initial mass moment of inertia of ship;
- drag coefficient for internal flow (cross-flooding);
- area of air vents;
- roll damping.

The main conclusion of the study presented is that calculations carried out in accordance with existing international legislation do not represent a realistic picture of the actual flooding process of a damaged RO-RO vessel following a high energy collision, mainly due to the fact that the sudden ingress of water in the transient stages initiates and increases the heeling moment of the flood water.

[12](2) Pawlowski, M.: **"Damage Stability Assessment: State of the Art"**, STAB '94, Melbourne, USA, Nov. 1994.

This paper provides a detailed analysis of the probabilistic method dealing with damage stability assessment. The author argues that at the present state of knowledge it is not possible to determine, with a reasonable degree of accuracy criteria for capsizing of ships in waves which are crucial for the assessment of damaged ship safety. He argues, however, that based on the results of damage stability model experiments, it is possible to derive a simplified relationship which takes into account only certain parameters and disregards others. Such a situation is reflected in the present criteria on damage stability. The background to these criteria is briefly discussed. On the other hand, the paper emphasises that relying on model tests only is not sufficient to make much progress and therefore research is essential. Damage stability is a complex problem involving a large number of variables and the results from only one or two given ship shapes are not sufficient to identify the significant parameters governing the problem. Full benefit from test results will not, therefore, be achieved until there is a consistent logical theory which will enable the results to be generalised to other ship forms and sizes. The paper discusses briefly two possibilities of creating such a theory, one based on a simplified model of damaged ship behaviour and the other on a comparative method. Finally, the paper discloses a practical calculation of the factor "s", accounting for the damage stability of a ship in subdivision calculations. There is a need to improve calculations in several points such as intermediate stages of flooding, heeling moments, the minimum possible vertical extent of damage, permeability for dry cargo spaces and - most important of all - the mode of calculation which is overlooked by the rules of all certifying authorities and which has a great impact on the GZ-curve. As these calculations should always be carried out for a freely floating ship, longitudinally balanced at each angle of heel, a new angle of heel, different from those in use, is proposed in the paper consistent with physical considerations.

[13](2, 6) Hua, J. and Rutgersson, O.: "**A Study of the Dynamic Stability of a RO-RO Ship in Waves**", STAB '94, Melbourne, USA, Nov. 1994.

Even though this paper only addresses parametrically excited roll motion of RO-RO ships in regular and irregular waves, some interesting points derive from their results and the consideration of cargo shifting. In particular, the following points are worth mentioning:

- the lashing systems on a RO-RO ship are presently designed on the basis of a 30° limiting roll angle. In practice, however, the simultaneous effect of vertical and horizontal accelerations and of roll motion on the cargo in question should be considered instead as a dynamic stability problem. In addition, owing to the transversely open decks, cargo shifting can result in large angles of heel which in turn may lead to severe consequences, even fatal loss of the ship;
- large B/T ratios, high KGs, fine hull forms and relatively high speeds combine to offer the following relevant characteristic properties for RO-RO vessels dynamics:

- natural frequencies in roll between 0.3 to 0.45 rad/s, whilst conventional ships have natural rod frequencies usually greater than 0.5 rad/s;
- large GM variation in waves;
- strong coupling with sway, heave, pitch and yaw.

[14](2) Vassalos, D and Turan, O: "**Damage Scenario Analysis: A Tool for Assessing the Damage Survivability of Passenger Ships**", STAB '94, Melbourne, USA, Nov. 1994.

This paper presents an approach adopted to assess the damage survivability of a ship which derives from an examination of a number of realisable damage scenarios chosen from accident statistics and IMO recommendations, on the basis of maximising the danger of potential capsize. Damage scenarios analysis refers to the procedure of identifying the "worst damage scenario" by studying the dynamic behaviour of the damaged vessel in a realistic environment using time simulation. The practical applicability of the proposed approach is demonstrated by presenting the results of a case study for a modern car/passenger ferry on the basis of which some revealing conclusions on the damage survivability of passenger ships are drawn and recommendations made.

[15](2) Vassalos, D. and Letizia, L.: "**Behaviour of a RO-RO Vessel during Transient Flooding in a Random Sea**", Proceedings of the International Conference on Ship and Marine Research NAV '94, Rome, Vol. 2, Session IX, October 1994.

Existing approaches for assessing the damage survivability of RO-RO vessels ignore altogether the dynamic response of the vessel and the progression of flood water through the ship in a random sea state and damage survivability is assessed on the basis of the properties of the residual GZ curve which is evaluated in still water conditions. This state of affairs co-exists happily with a continuing focus on residual stability standards addressing the final condition after damage and the lack of availability of suitable theoretical or numerical 'tools' for assessing the survivability of a damaged vessel in realistic operating conditions. This paper challenges the wisdom of using still water residual stability standards by presenting some interesting results describing the dynamic behaviour of modern ferries during transient flooding in random sea states.

[16] (2) Vassalos, D.: **"Damage Survivability: Searching for the Right Question"** - Tecnica Italiana, Vol. 59, No. 3, September 1994.

Safety regulations are to the researcher mundane, to the designer restrictive, to the owner and builder expensive, to the press bureaucratic or inadequate, to the legislator unreasonable one day and insufficient the next. In the meantime vessels and lives are routinely being lost at sea, at an average of one ship a day every single day. In the transition phase from deterministic to probabilistic approaches to assessing the damage survivability of passenger vessels, this paper attempts to draw attention to the real need with regard to the development of practical and meaningful regulations - the need to bridge the gap, yet again, between research and practice, theory and application. The paper addresses the subject of damage survivability by presenting a summary of the recent research work undertaken in the UK, comprising:

- a critical review of subdivision and damage stability requirements;
- the Strathclyde approach to the development of survival criteria;
- computational and model experiment results of realistic damage scenarios;
- proposed solutions.

The paper concludes by arguing that it is possible to achieve a solution to the general problem of damage stability. All that is needed is to ask the right question. This is the ironic situation faced by our profession.

[17] (7) Paik, J. K. and Pedersen, P. T.: **"Modelling the Internal Mechanics in Ship Collision"**, Technical University of Denmark, June 1994.

In this booklet, a complete and detailed report of the theory behind one of the methods of analysis of the structural damage due to ship collision is given. This is based on a modified FEM method, capable to take account of phenomena as yielding, crushing and rupture. The effect of stiffeners is considered as well as that of plating. On the basis of the theory, a computer program has been developed and its predictions compared with experimental results. An illustrative example and the factors influencing the extent of damage in ship collisions are discussed.

[18](2) Vassalos, D and Turan, O: **"A Realistic Approach to Assessing the Damage Survivability of Passenger Ships"**, Transactions SNAME, Vol. 102, 1994, 34pp.

As the profession stands poised to adopt the probabilistic approach of assessing the damage survivability of passenger vessels and discussions at IMO address seriously the harmonisation of residual stability standards, the fundamental question pertaining to the ability of passenger ships to survive damage in a realistic environment, particularly when progressive flooding takes place, still remains unanswered. The paper begins with a brief review on the development of damage stability requirements, highlighting aims and explaining the intentions of the standards laid down and criteria to damage stability and survivability. A full explanation of the new approach presently adopted is then presented. This is based on time simulation of the dynamic behaviour of a damaged vessel in a random wind and wave environment. The mathematical model comprises coupled sway-heave-roll motions whilst taking into consideration progressive flooding and water accumulation as well as the associated instantaneous sinkage and trim. A modern RO-RO vessel, representing a ship type most vulnerable to serious flooding, is used to demonstrate the practical applicability of the proposed procedure. In principle, however, the same procedure could be applied to assessing the damage survivability of general passenger and cargo ships. To this end, a wide-ranging parametric investigation is undertaken aiming to identify and quantify the effect of a number of key parameters on damage survivability with a view to ascertaining the minimum stability requirements to prevent capsize in a given sea state and to establishing relationships between ship design and environmental parameters and inherent stability-related parameters. The derived results are presented in the form of boundary survivability curves which are proposed as a substitute for the existing still-water criteria.

[19] (2) Vassalos, D and Turan, O: **"The Influence of Design Constraints on the Damage Survivability of RO-RO Vessels"** - Proceedings of the 5th IMDC '94-STG, TU Delft, Vol. I, May 1994, pp 381-402.

This paper examines the influence of key design characteristics on the damage survivability of RO-RO vessels in a random sea. The approach adopted in undertaking this study is briefly explained and the results of a parametric investigation, considering a representative vessel, are presented and discussed. The paper focuses on the effects of freeboard and the transverse subdivision above the bulkhead deck by considering the vessel in a number of loading conditions and sea states. Finally, the effect of retrofitting structural sponsons is assessed. Based on the results of the investigation boundary survival curves, involving relationships between ship design and environmental parameters and stability-related parameters are developed as a substitute to still water damage stability criteria. The two key conclusions from this research are: the 0.076m damaged freeboard, required by current regulations is unrealistically low and must be significantly increased before a ship can be considered safe in normal operating conditions; the undivided length of a compartment above the bulkhead deck may have to be as small as 25m to enable the vessel to survive damage in an extreme wave environment.

[20] (5) Allan, T.: **"The Practical Implication of SOLAS '90 on Existing RO-RO Passenger Ships"**, RO-RO '94, Göteborg, April 1994.

In this paper, the author summarises the research work carried out during the UK RO-RO Damage Stability Programme. The practical and economical feasibility of some of the methods suggested to improve RO-RO survivability is assessed and the international action towards the common aim of securing reasonable standards of safety is reported. The author's conclusions agree with the widely expressed necessity of modifications to bring the existing fleet to acceptable standards but, by encompassing the concepts of total safety and economic feasibility, attempts to reduce the workable alternatives considerably. This would contemplate excluding an increase of freeboard by means of rising the level of the bulkhead deck (excessive initial costs and reduction in payload) and transverse movable bulkheads on the vehicle deck (excessive initial costs and increased running costs) which are demonstrated to be the most effective ways to avoid capsize, amongst the proposed and tested devices.

[21](5) Vassalos, D. and Turan, O.: "**The Impact of SOLAS '90 on the Design and Safety of RO-RO Vessels**", RO-RO '94, Göteborg, April 1994.

In the wake of the much deliberated SOLAS '90 rules, owners, operators and designers of RO-RO vessels, *particularly passenger/car ferries, are striving to identify* feasible and viable solutions to satisfy the new requirements for damage stability. Proposals aimed at enhancing the survivability of these vessels include increased freeboard, partial transverse bulkheads on the vehicle deck, flared hull forms and structural sponsons. Some devices have already been implemented on existing vessels and the overall impact on future designs is expected to be large. A key question, however, still remains unanswered:

"To what extent are SOLAS '90 rules safeguarding RO-RO vessels against potential losses in realistic environmental conditions?"

This paper is set to provide answers to this all important question.

[22](5) Pawlowski, M.: "**A New Concept of RO-RO Ships Subdivision for Enhanced Safety in the Damaged Condition**" - RO-RO '94, Göteborg, April 1994.

The author puts forward new concepts of internal arrangements, by which RO-RO ships could become safer without restricting their most valuable design features. This basically consists of having a double hull around the ship up to the second deck above the bulkhead deck. The space between the outer and the inner skins, will have to be sufficiently and conveniently subdivided into watertight compartments, so that buoyancy is guaranteed even in the worse damage condition. An additional safety enhancing characteristic is implemented by means of cross-flooding arrangements between corresponding wing compartments. The idea is to exploit the fact that a large sinkage would prevent capsize. This is achieved by conveying the incoming water to the undamaged side and the bottom spaces, through cross-flooding and down-flooding openings. It is suggested that vehicle decks above the bulkhead deck should not be watertight, so that in the eventuality of water flooding these areas, this would drain to lower and thus safer zones. Such changes in the design of the vehicle decks, would eliminate the accumulation of water on the car deck, which is believed to be one of the most likely and dangerous causes of ship capsize. In the mean time, watertight wing compartments below and especially above the bulkhead deck, will provide the necessary buoyancy. The location of the watertight compartments will have to be decided in a way that sinkage is controlled and ship stability is increased. Similar designs based on this concept are evaluated in the light of the new probabilistic regulations to obtain the maximum attained subdivision index.

This idea seems logical and worth considering. However, as details have not been provided, practical problems could arise after a detailed evaluation, particularly when vessel dynamics is taken into account. For instance, perforated vehicle decks constitute a formidable fire hazard. Economical considerations should be contemplated as well. In fact, this new design would greatly reduce the space available for the paying cargo and, as such, will certainly encounter a strong opposition from the ship owners and operators. Finally, these new design features are only feasible to newly built ships. Of course, replacing the vast passenger fleet is a slow process and measures to improve safety of passengers at sea is an urgent requirement.

[23](4) Francescutto, A. and Contento, G.: **"An Experimental Study of the Coupling between Roll Motion and Sloshing in a Compartment"**, ISOPE '94, Osaka, No. 94 -YI1-4, 1994.

An experimental investigation on the interaction between a free surface, a constant amount of water flooding a compartment and roll motion of a fishing vessel is reported here. The comparison of experimental results with numerical calculations using a non-linear single degree roll motion equation, accounting for the induced sloshing moment through a CFD boundary element method, is performed, showing good agreement. From both experimental evidence and theoretical predictions, the following conclusions are noteworthy:

- The roll amplitude is increased at frequencies lower than the natural frequency with the water frozen. In this case pressure loading on the walls of the tank are dominated by hydrostatic components;
- A similar effect is observed for frequencies higher than the natural frequency, but in this case the dynamic sloshing pressure is prevalent. Large damping in the range between these two zones is clearly evident.

Although this work is useful in supporting the theoretical model developed at the University of Trieste, the choice of the particular model used in this study is of little practical use. In fact, it would be interesting to repeat the experiments and calculations with a different type of ship (e.g. a RO-RO vessel) to verify the importance of sloshing when the dimensions of the damaged compartment and those of the ship are somewhat different.

[24] Rowson, K. J.: **"Why Do We Bother"**, RINA International Conference on LIFE SAVING AT SEA, London, December 1993.

This paper addresses the difficulties faced during the practical application stage of the safety measurements in terms of economical grounds. The author states that adequate basic survivability is coming into effect slowly because of the resistance of owners due to the economical implications involved. Many important aspects of life saving at sea are introduced after tragic accidents. Rowson thinks that it is time for the insurance market to start playing its role by forcing the owners to increase the safety standards as the total cost of losses can be much higher than one assumes. Therefore insurance rates must be rearranged in this way. The author is absolutely right by stating that none of the life saving equipment can be justified economically. Therefore, there must be a law to enforce it as owners will not do so without any compulsory requirements.

[25](2) Vassalos, D. and Turan, O.: **"The Question of Subdivision Revisited"**, RINA International Conference on LIFE SAVING AT SEA, London, December 1993.

This paper addresses the question of subdivision by focusing on the survivability of RO-RO vessels in a random sea. More specifically, it attempts to provide answers to the all important question of whether and how to subdivide the main vehicle deck of a RO-RO vessel. The approach adopted in undertaking this study is briefly explained and the results of a parametric investigation, considering a representative vessel, are presented and discussed. The key conclusion from this investigation is that the undivided length of a compartment on the vehicle deck may have to be as small as 25m to enable the vessel to survive damage in an extreme wave environment.

[26](5) Miller, R.: **"Flood Control Doors as Practical Retrofit Options on RO-RO Decks"**, RINA International Conference on LIFE SAVING AT SEA, London, December 1993.

This paper is based on the investigation on a different aspect of flood control doors (in other words, Partial Transverse Bulkheads) by a leading manufacturing company in RO-RO equipment. They designed and implemented these partial bulkhead doors on the "Spirit of Tasmania" (originally Peter Pan). The author suggests that although it looks simple, fitting flood control doors to provide the required damage stability, it is not, considering the structural, water-tightness and operational aspects of it. In this paper economical effects as well as structural and operational problems are examined in detail. The author agrees with other researchers that the most effective structural arrangement for enhancing the survivability of ferries is by means of transverse bulkheads. Furthermore, these can be optimised for each individual ship depending on the turnaround time etc. Even though emphasis is placed on automation, one man operation is fundamental, while a trained crew is necessary together with spare parts in case of failure. However, this tendency contradicts the opinion of researchers and regulatory bodies when expressed their views during the RO-RO workshop in STAB '94. The prevailing view is that such devices must be as simple as possible and the human involvement required minimal, as failures in automated equipment and human errors have played major roles in the recent well publicised disasters. Full credit must be given to the company as it is important that some leading industrial companies look into suggested solutions and research their practical application.

[27](2) Turan, O.: **"Dynamic Stability Assessment of Passenger Ship Using a Time Simulation Approach"**, PhD Thesis, Department of Ship and Marine Technology, University of Strathclyde, 1993.

In the transition from deterministic to probabilistic approaches to assessing the damage survivability of passenger ships this PhD study seeks to draw attention to the key need in regard to loss prevention - the need to address damage survivability by taking full account of vessel dynamics in realistic environments. The thesis begins by critically reviewing the development of subdivision and damage stability requirements, emphasising the inherent weaknesses in the existing approaches to assessing damage survivability. The approach adopted in the thesis is then described. This is based on time simulation of the dynamic behaviour of the damaged vessel in realistic wind and wave conditions. The mathematical model comprises coupled sway-heave-roll motions in regular beam seas while taking into consideration progressive flooding as well as water accumulation. A series of comprehensive model experiments have been specifically designated and undertaken to investigate the nature and magnitude of couplings in the above modes of motion in upright and inclined conditions. The damage survivability of the vessel is examined by considering a number of damage scenarios, chosen on the basis of maximising the danger of potential capsize (or sinkage) while taking into account actual accident records. The practical applicability of the proposed procedure is demonstrated by means of a parametric investigation aimed at identifying the effect of a number of key parameters on the damage survivability of a modern car/passenger ferry. These include: wave height, wave length, rate of flooding, water accumulation, location and extent of flooding, loading. The results of the investigation are presented and discussed.

[28](4) Armenio, V.: "**Numerical Simulation of Large Amplitude Sloshing of a Viscous Liquid in Rectangular Containers**", AIMETA '93, Settimo Convegno Italiano di Meccanica Computazionale, 1993.

This is a numerical investigation of large amplitude sloshing through the solution of two-dimensional Navier-Stokes equations by means of a MAC-type method. Considering viscosity in the numerical solution of sloshing problems, is essential to model near-resonance conditions and the presence of obstacles (ex. baffles) in the tank. This particular study pertains to high viscosity fluids and its results are not of much use for the purposes of this review. Merit must be accredited to the author, however, for the clear elucidation of the numerical and theoretical background of this work.

[29](4) Armenio, V. et al.: "**Dynamic Effects of Liquids on Board on the Stability of a Fishing Vessel**", IMAM '93, Varna, 1993.

This is a comprehensive report of the work carried out at the University of Trieste in conjunction to the MURST sponsored project "Dynamic Effects Connected to Water on Deck". After a brief review of the methods generally used to treat the effect of liquids with a free surface on the dynamic behaviour of seagoing vessels, the authors illustrate the mathematical model employed in this investigation. This comprises a single-degree-of-freedom, differential equation with constant coefficients describing roll motion, coupled with the sloshing water behaviour by the introduction of an induced moment on the LHS of the equation of motion (apart from the additional variable boundary conditions on the tank walls). Non-linear roll damping is assumed. Two divers methods are used to model the water dynamics, depending on the mean height of the water inside the tank, but in both cases the fluid is considered ideal and incompressible. The results appear to be in good agreement with those obtained by experimental research performed by the Texas A&M University College Station as well as at the University of Trieste, and succeed to predict hydraulic jump and standing wave phenomena in the shallow and deep water cases, respectively. A minor discrepancy is observed near resonance, probably due to the assumption that the fluid is non viscous.

[30] (3) Letizia, L.: **"An Experimental Investigation into the Rate of Flooding of a Damaged Hull"**, Ship and Marine Technology, University of Strathclyde, 1993.

This final year project, was meant to be a small contribution to the work undertaken by the Stability Research Group on the damage survivability of RO-RO vessels by addressing the need to improve the water ingress model used in computer simulation programs. This work begins by critically reviewing some of the above mentioned studies, pointing out the various methods used to model water inflow/outflow, following with the description of the approach applied here to study the flooding mechanism. This relies mainly on experimental results obtained from pioneering tests on a 2D model damaged amidships, in beam regular waves, to investigate the dependence of the flooding rate on wave amplitude and frequency and the model GM. This project, of course, is not meant to give a definitive answer to the problem, but only to set up some general relations useful for further developments on the subject. The mathematical model is willingly kept simple, but still takes into account sway-heave-roll motions, wave dynamics, water accumulation and sloshing effects to some extent.

[31](2) Vassalos, D. and Turan, O.: "**Development of Survival Criteria for RO-RO Passenger Ships - A Theoretical Approach**", Final Report on the RO-RO Damage Stability Programme, Phase II, Department of Ship & Marine Technology, University of Strathclyde, December 1992.

This final report details the theoretical results achieved at the University of Strathclyde, based on the analysis of the dynamic behaviour of a damaged ship in realistic environmental conditions. A three-degrees-of-freedom mathematical model was used, accounting for coupled sway, heave and roll while allowing for water ingress. Modelling of the water ingress was based on a simple formula of water flow through an opening when the pressure head is known, with the water flow coefficient calibrated from model experiments carried out in Phase I of the RO-RO Research Programme. Excellent agreement between experimental and theoretical results provided sufficient justification for embarking on a small-scale parametric investigation which allowed *limiting curves of stability to be derived and relationships* to be established between ship design and environmental parameters and limiting stability parameters. Deriving from the results of this research, there is a strong feeling among the Strathclyde Stability Research Group that the development of survival criteria for damaged passenger ships is within reach.

[32](4) Cardo, A. et al.: "**Experimental Study of the Effect of Water on Deck on the Stability of a Fishing Vessel**", NAV '92, Genoa, July 1992.

In this paper, details are given on the experimental procedure and the set-up used to perform a series of tests to determine the effect of the presence of water on deck on the behaviour of a large scale model of a fishing vessel in roll. In particular, free oscillation tests are initially used to calculate the vessel roll damping; successively modal analysis is employed to explain the results of forced oscillation runs. This is possible, assuming that the vessel-water system can be treated as a non-linear coupled system with two degrees of freedom. The primary significance of this publication derives from the adoption of techniques based on the theory developed for stabilising tanks to explain certain characteristics of sloshing. With a linearised roll damping it is indeed possible to predict the double peaked frequency response curve of roll amplitudes (typical of vessel with free surface water on board) with some accuracy.

[33](2) Vassalos, D. and Turan, O.: "**Location and Extent of Flooding - A Dynamic Analysis**", RO-RO '92, Göteborg, May 1992.

The authors provide here a brief account of their attempts to meet the need to address damage survivability by taking full account of vessel dynamics in realistic environments by considering, through dynamic analysis, the implications of two key variables: damage location and extent.

[34] Rawson, K. J.: "**An Overview of RO-RO Safety**", The Second H. Kummerman Foundation International Conference on RO-RO Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

In this introductory paper to the second Kummerman Foundation conference on RO-RO safety, the chairman summarises the effort of the international community towards improving the general safety of passenger vessels a year after the ratification of SOLAS '90 standards. The great deal of opposition encountered by the UK Department of Transport in proposing retrospective action with respect to the application of these regulations is stated, although their effectiveness in preventing dangerous situations in realistic sea states is questioned. An impressive comparison between the total direct costs of the loss of the "Herald of Free Enterprise" (£90 million) and those expected to cost the British passenger fleet to comply with SOLAS '90 (£80 million) is used as a good argument to back the Department of Transport proposal. The overview provides general trends and issues to conclude by proposing that a sole regulatory body should deal with safety matters and a more flexible form be adopted in the legislation concerning assessment and endorsement of ship safety.

[35] Gilbert, R. and Sheehan, D.: **"Pros and Cons of Opting for an Existing Standard"**, The Second H. Kummerman Foundation International Conference on RO-RO Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

This paper gives some details on the IMO resolution A.265 (VIII) which tackles the problem of passenger vessels safety on probabilistic basis and introduces a simplified version of the same, lately developed by the US. This is proposed to be used as a tool to decide where existing vessels stand and if they necessarily have to be modified to meet the SOLAS '90 requirements, or alternatively, to rise their Attained Subdivision Index to the minimum requested, according to the complete version of A.265. Frankly, the details given do not clarify how this optional regulation would ease the burden that ship owners should have to accept in order to improve the safety of their ferries. Excepting the fact that different risk levels are met by different ships during their service lives (which is indeed something that the so called "Deterministic Approaches" should take account of), it is not clear how the same ship, operating on a given route, could be considered reasonably safe adopting one set of regulations, but unsafe using other (unless admitting that they are not equivalent).

[36] Aston, J. G. L. and Rydill, L. J.: **"Assessing the Safety of RO-RO Ships"**, The Second H. Kummerman Foundation International Conference on RO-RO Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

The authors present in this paper a detailed analysis of the presently adopted regulations and their inadequacies, to advocate the pressing need for more advanced studies in all different aspects implicated in the necessary understanding of those phenomena that rules and regulations are supposed to be created to protect the community from. Although not all arguments presented are original, this paper is a valid and well shaped tool to organise and direct eventual future research on the subject. Shortcomings in the present legislation are found to concern principally the knowledge about *strength* (capability of the vessel to resist and survive the damage) and *risk* models, thus a debatable itemised list of research aims is given.

[37](5) Brown, J. G.: "**Buoyant Wing Spaces - Economic Compliance with SOLAS '90**", The Second H. Kummerman Foundation International Conference on RO-RO Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

A design for a freight ferry adopting buoyant wing spaces is presented in this report, which ensure economical compliance to the newly imposed set of regulations. Aside from the effectiveness of such a device in reducing the danger of rapid capsize, it is proved here that it can be happily exploited to carry payload, even for ferries which do not travel overnight or do not host many passengers. A special feature in the proposed design is a "tiltdeck" which would allow the utilisation of spaces below the bulkhead deck to store additional lane metres "within compliance". This design type is quite interesting, although its claimed effectiveness in ensuring appropriate safety should be investigated further. What is clearly demonstrated by this publication is that rules can be met effectively by new design types, from an economical point of view. However, this does not necessarily mean that the philosophy behind the rules will be appropriately honoured. This is particularly true for regulations based on a *deterministic approach* (like the SOLAS '90), which impose certain limitations to the designer without the compelling need to verify that the design changes, necessary to meet the rules, would indeed be effective in serving the purposes of the rules themselves. Again the need for intelligently formulated regulations to promote good design practice must be stressed.

[38] Sen, P. and Wimalsiri, W. K.: "**RO-RO Cargo Ship Design and IMO Subdivision Regulations**", The Second H. Kummerman Foundation International Conference on RO-RO Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

This work was carried out as part of a post-graduate project at the University of Newcastle-upon-Tyne to investigate the consequences of the newly agreed IMO safety requirements, in terms of design alterations. In the introduction it is mentioned that while the majority of cargo ships will meet the regulation without too much trouble, the same cannot necessarily be said about RO-RO ships. This, however, appears to openly contradict the assertions by Gilbert and Sheehan. Overall this study seems to be a fairly objective and detailed account of vices and virtues of the so called *probabilistic approach*. Amongst the first set, the authors point out that some nominal condition of assessment can produce situations in design whereby small changes in assumptions or small design changes can produce significantly different A values leading to *cliff-edge* effects which are inherently undesirable in any assessment procedure if it is to encourage good design practice. The world is not perfect in a probabilistic light either. The main worth of this paper lies in showing that the probabilistic method of assessing safety can possibly be more flexible for the designers, but it must be improved to ensure its effectiveness and correct use. It is probably worth repeating once more that the need for intelligently formulated regulations to promote good design practice is paramount.

[39](7) Sen, P. Cocks, J. A. and Pawlowski, M.: "**Integrated Collision Analysis for Ships**", The Second H. Kummerman Foundation International Conference on RO-RO Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

This paper reports on the work done at the University of Newcastle-upon-Tyne to produce an integrated system for dealing with collision analysis, taking into consideration the dynamics of the accident as well as the mechanical implications of it on the hull structure. The approach adopted can be classified as semi-analytical and energy based. Using a reasonably detailed model of collision, the dependency of the absorbed energy on parameters such as the angle of encounter, the ship displacement ratio and the colliding speeds is worked out. Successively, this energy is used to evaluate the structural damage effects. This work deserves merit for it tries to treat two connected phenomena that have been usually studied separately, as one. In this respect, it could be taken as a guideline in any future attempt to develop a realistic assessment of collision consequences and damage resistance efficacy.

[40](1) Dand, I. W.: "**Experiments with a Floodable Model of a RO-RO Passenger Ferry**", The Second H. Kummerman Foundation International Conference on RO-RO Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

One of the two series of experiments commissioned by the UK Department of Transport during Phase I of the RO-RO Research Programme, is presented here. BMT Ltd tested one of the models considered, observing its behaviour when damaged in irregular sea and the dependence of it on wave parameters, flooded freeboard and GM. All experiments describe a model of a sister ship to the "Herald of Free Enterprise" ("Pride of Bruge") employing a standard trapezoidal damage opening amidships. The residual freeboard was varied by changing the length of the damaged compartments under the bulkhead deck. This is judged to be conforming to what happens in a real case and could be adopted in future experimental investigations. This report is well presented offering valuable support for further research on the subject.

[41](2) Vredeveldt, A. W. and Journèe, J. M. J.: "**Roll Motions of Ships Due to Sudden Water Ingress, Calculations and Experiments**", The Second H. Kummerman Foundation International Conference on RO-RO Safety and Vulnerability: The Way Ahead, London, RINA, April 1991.

In this paper the authors present a very simple method of simulating the ship dynamic behaviour when sudden flooding of damaged compartments occurs. The influence of waves is neglected and only roll motion is considered. Water ingress and cross-flooding is simulated numerically through semi-empirical formulae and experimental tests on a rectangular pontoon are employed to tune the mathematical model and finally validate the computer integration. Although the experimental research carried out for this report could be of some interest to model the flow of water in cross-flooding ducts in dynamic conditions, the overall value of this project is modest, when the results of the BMT capsize tests are borne in mind. Clearly the effect of waves and the possibility of water flooding the vehicle deck should have been included if some more significant results were to be expected towards the understanding of the capsize mechanism. Some merit should be given to the authors for having recognised the importance of the dynamic effect of a rapid off-centred increase of flood water, but the first time that this hypothesis was put forward must be traced back to the technical investigation over the capsize of the "European Gateway" in 1985.

[42] Rogan, A. J. and White, N. J.: **"A Study to Compare the Residual Standards of Stability after Damage of Existing RO-RO Passenger Ferries"**, International Symposium on the Safety of RO-RO Passenger Ships, RINA, April 1990.

In this article, results are presented of an investigation to assess the capability of ten typical existing RO-RO ferries to meet the IMO standards which came into force in 1990. The research is carried out by using computer packages such as SFOLDS, HYDASC and SIKOB. The results show that most of the ferries analysed do not comply with the new standards and that radical changes in the internal/external geometry would be needed for most of these to conform to an increased depth of the bulkhead deck. Particular emphasis is put on the extended range of positive GZ and whether this requirement should be added to the minimum value of the peak of the GZ curve and its area.

[43] (7) Aldwinckle, D. S. and Prentice, D.: **"The Safety Record and Risk Analysis Of RO-RO Passenger Ferries"**, International Symposium on the Safety of RO-RO Passenger Ships, RINA, April 1990.

This paper introduces a detailed risk analysis carried out by Lloyds Register of Shipping for the Department of Transport. This consists of the collection of accident records during a period extending between 1978 and 1988 and the subsequent creation of a sound risk model and the evaluation of possible measures to enhance the overall safety of RO-RO vessels. This study is vital to understand the importance of passenger ship safety in relation to that of other means of transport, and the weight of all the possible ways that it can be jeopardised (fire, capsize etc.) in comparison to each other.

[44](7, 5) Sen, P. and Cocks, J. A.: "**Collision Resistance**", International Symposium on the Safety of RO-RO Passenger Ships, RINA, April 1990.

This paper summarises the research commissioned by the UK Department of Transport, addressing possible ways of improving ship resistance to collision. A complete model of the mechanics involved in ship collision is presented, the development of which can be found in a number of publications by the same research group. This model was used to verify the efficacy of a number of design alternatives conceived to improve the impact resilience of the sides of a ship and it was found that "even if designing a vessel to resist major strikes is impractical, structures allowing for large deflections are more energy absorbent.". This lead to pointing at longitudinal stiffening associated with relatively weak web frames and/or double skin arrangements as recommended ways to improve collision resistance.

[45](5) Judd, P. H.: "**RO-RO Passenger Ferry Survivability Study - Hull Form and Superstructure**", International Symposium on the Safety of RO-RO Passenger Ships, RINA, April 1990.

In this study, YARD Ltd, using static stability calculations, identify and analyse practical solutions to improve the RO-RO vessel chance of survival when damaged at sea. These can be subdivided into high stability hull forms (flared sides and increased damaged freeboard) and structural and inflatable sponsons. The possible influence of watertight superstructures is considered as well but the impracticability of this solution is deemed to greatly overcome its efficiency. The outcome of this investigation shows that the most effective ways to increase the damage stability of a RO-RO vessel is by introducing inflatable sponsons. This solution, though, needs to be further developed and it is not advisable for new designs. Increasing the damage freeboard by rising the main deck is a very effective method to enlarge the range and area of the GZ curve, whilst flared sides and structural sponsons offer limited aid to this purpose. Towards the utilisation of the latter there could be an increase of the collision resistance of the ship. The distinction between "ship survival" and "passenger survival", made in the introduction, is interesting. Nevertheless, it is restrictive to limit the possible ways to improve the latter, to increasing the time to capsize only. Although the importance of this parameter is paramount, more effective means of escape and life saving should be devised and their value consequently appraised. Caution is recommended on the quantitative results of this study as the effect of dynamics is completely ignored.

[46] Lloyd, C. J.: **"Research into Enhancing the Stability and Survivability Standards of RO-RO Passenger Ferries - Overview Study"**, International Symposium on the Safety of RO-RO Passenger Ships, RINA, April 1990.

[47] (5) Lloyd, C. J.: **"Research into Enhancing the Stability and Survivability Standards of RO-RO Passenger Ferries - Internal Arrangements"**, International Symposium on the Safety of RO-RO Passenger Ships, RINA, April 1990.

In this extensive report, many different internal arrangements are analysed, aiming at ensuring a better ship safety in case of lateral damage. The assessment of their practicability is made by sound consideration of their efficacy in increasing the vessel damage stability, the practical and engineering difficulties implied in the installation and management of these devices as well as the commercial consequences of their adoption. This investigation is also based on static stability calculations and as such it cannot properly assess the dynamic implications of the capsize process. It is interesting though, that its results agree qualitatively with those obtained by experimental studies undertaken to verify the validity of some of the considered stability enhancing devices.

[48](1) Pucill, F and Velschou, S.: "**RO-RO Passenger Ferry Safety Studies - Model Test of a Typical Ferry**", International Symposium on the Safety of RO-RO Passenger Ships, RINA, April 1990.

This paper describes the tests performed, and the results achieved by DMI, in the experimental investigation undertaken by them on behalf of the UK Department of Transport for Phase I of the Research Programme on RO-RO vessel survivability. Like the analogous study carried out by BMT, the capsize probability was investigated as a function of sea state (H_S), damaged freeboard and KG. The results are qualitatively similar to those obtained by Dand and will not be reported here again. A point deserving mentioning is that the two research centres could have agreed on the exposition of the results in a common format. This would have surely eased the comparison.

[49](1) Jin Hao and Yuan Don Lei: "**Model Tests on the Rolling Behaviour under Damaged Conditions**", STAB '90 Naples, September 1990.

This paper reports, to some extent, the results of a series of model tests performed using a damaged ship in regular beam waves at the China Ship Science Research Centre, Wuxi - China. The damage is simulated by circular drilled holes at the bottom and sides of the ship, operated by mechanical valves. These are opened at the beginning of each run, to let water flood the desired compartments to the equilibrium condition. This series of tests include a reasonably large variation of damage scenarios and initial displacements, whilst the wave characteristics are changed only by altering the period, keeping a constant height. Of the results presented, of some interest is the multi-crested shape of the response curves for those cases in which more than one compartment were flooded, which is in good agreement with the theory of multi-degrees-of-freedom linear systems. Unfortunately, details are not given of the internal and external geometry of ship and holds, making it impossible to relate and compare these curves to others. Also, the effect of transient flooding is totally neglected here, thanks to the experimental process.

[50] (4) Pantazopulos, M. S.: **"Sloshing of Water on Deck of Small Vessels"**, STAB '90 Naples, September 1990.

Once again a paper on numerical methods to deal with the problems created by water sloshing on the deck of small vessels. In this publication, the author presents a three-dimensional method to calculate forces and moments due to the mass of water flooding the working deck of a fishing boat. These forces are then compared with those due to wave excitation and the water flow pattern is analysed. In this study the vessel motions are considered known, i.e. no interaction is modelled between water sloshing and vessel motions and the shipped water depth is assumed shallow. The main limits of this work derive from the assumption that the vessel motions can be uncoupled from the *sloshing water effect, but its contributions on the resolution of* shallow water sloshing in three dimensions allowing for temporary deck dryness are of some importance. Swirling of the water when pitch and roll are present is observed and the calculated sloshing induced forces are shown to be significant in comparison with external excitation, for the ship geometry in consideration.

[51] (7) Abicht, W.: **"The Probability of Compartment and Wing Compartment Flooding in the Case of Side Damage - New Formulae for Practical Application"**, STAB '90 Naples, September 1990.

This paper presents a new mathematical model for the statistical distribution of side damages with respect to damage location, damage length and damage penetration, additional to a method for the evaluation of the probability of flooding of wing compartments. A short revision of the inaccuracies and shortcomings resulting from the application of the existing probabilistic method of subdivision is included.

[52](2) Rakhmanin, N.: **"Stability of Damaged Ships During Ship Motion in Waves"**, STAB '90 Naples, September 1990.

The author illustrates an attempt to simulate roll motion of a damaged ship in waves while allowing for water flooding the compartments to a certain extent. The equations of motion used, take account of motions in the transverse plane only and contemplate a non-linear approximate model for the restoring forces. Through this simplified model, the results obtained by Numata (USA 1965, 1970) and Bird (UK 1973) is asserted to be reproduced and boundary curves delimiting safe and unsafe capsize zones in the GZ_m-H_S plane are defined. Due to lack of details and clarity in the reporting, the usefulness of this publication is limited. The simulation results themselves are missing and only values of the limiting Gz_m's are given. No account is released of the methods of calculation or the theory behind the formulae employed and no comment is given on experimentally observed phenomena like extreme response to transient flooding.

[53](2) Petey, F.: **"Determination of Capsizing Safety of Damaged Ships by Means of Motion Simulation in Waves"**, STAB '90 Naples, September 1990.

Yet another numerical study trying to model the dynamic behaviour of damaged ships in a seaway. Using the results of the integration of six-degrees-of-freedom equations of motion in the time domain, the author attempts to assess the probability of capsize of a general cargo vessel type. Particular emphasis is put on the parametric effect experienced when in quartering or following seas. The mathematical model used takes account of water sloshing using an hybrid methodology (Glimm's method for shallow water and his own procedure for larger depths) and its mass variation in time, although little detail is given on the calculation procedure employed to estimate the latter. Transient flooding and sensitivity of the dynamic behaviour of the vessel on initial conditions are explicitly disregarded. Of some interest is the evaluation method to work out the capsize probability, but details on the results of the simulation are lacking. Another interesting comment can be made on the typical time scale for the capsizing of a general cargo ship. The author states: "The probability that the ship in case 4a does not capsize in two days is 9% according to .... In the limit case 4c (KG'=11.76 m) it becomes 91%.". Plenty of time for evacuation anyway.

[54] (4) Rakitin, V. Natchev, R. and Tzetanov, T.: "**Series Stand Tests with Passive Stabilising Tanks**", STAB '90 Naples, September 1990.

This report contains the results of an extensive experimental study on FLUME and FRAME type of stabilising tanks. The information provided could be used to simulate the behaviour of flood water in wing compartments supplied with cross-flooding devices. The approach adopted is straight forward and because of this very attractive. The stabilising moment is investigated accounting for sway and roll. Variation of the mass of water in the tanks is contemplated, although not in a dynamic sense. Other parameters taken into consideration are the tank geometry, the excitation properties and the internal damping of the cross-connecting ducts. No information is given about methods to calculate the latter. The results obtained on amplitude and phase angles of the stabilising moment with respect to the roll excitation, are approximated by polynomial expressions obtained by regression analysis. Unfortunately, no detail is given about these as it was expected from a report of commercial research work.

[55] (4, 2) Francescutto, A. and Armenio, V.: "**On the Stability of Anti-symmetric Motions of a Ship Equipped with Passive Anti-rolling Tanks**", STAB '90 Naples, September 1990.

This paper - which contains an all-comprehensive spanning insight of the problems related to motion instabilities faced by a ship operating in a realistic seaway - concentrates on the effect of the theory on passive, roll stabilising, U-shaped tanks, when applied to coupled equations describing anti-symmetric motion of a dynamic system. Sway, roll and yaw are considered here, in originally fully non-linear equations of motion; the stability of the system with no motion is thus analysed by means of perturbation techniques, by linearising this set of differential equations in the neighbourhood of the origin. Full recognition must be awarded to the authors for the clear presentation of the theory behind this study. Sadly, no much detail is given on the formulation concerning the modelling of the dynamic effect of the devices involved.

[56](2, 4) Falzarano, J. M. and Troesh, A. W.: "**Application of Modern Geometric Methods for Dynamic Systems to the Problem of Vessel Capsizing with Water on Deck**", STAB '90 Naples, September 1990.

In this document, the general theory describing ship motions according to linearised equations of motion supported with non-linear forces is described in considerable detail. A reduced equation accounting for roll only and corrected for the presence of water on deck through the addition of an off-centred weight, is then used to illustrate standard non-linear dynamic analysis techniques. The model used to include water on deck dynamics is not sufficiently accurate for the purpose of this research.

[57](4, 2) Trincas, G.: "**Safety for Damaged Vessels as Probability of Non-Capsizing in Following Seas**", STAB '90 Naples, September 1990.

In this paper a time domain simulation of the dynamic behaviour of a damaged ship undergoing progressive flooding is presented. The six-degrees-of-freedom mathematical model adopted is claimed to account for sloshing through a 2D, finite differences technique (Eguchi and Niho) and progressive flooding using Bernoulli's theory derived formulae. The random wave excitation, hydrostatic forces and hydrodynamic coefficients are updated at each time step with the ship motions making use of a database previously evaluated applying strip theory. From the cloudy illustration of the methods employed, it is not terribly clear how such a complete non-linear model is really shaped. No details are given about important points like the water ingress mechanism and how sloshing interacts with a varying mass of flood water. The updating of forces with the ship attitude is not made clear either. No comment is made on the transient flooding motion - which is demonstrated to be often large and always critical by experiments and experience - and the probability of non-capsizing is calculated from the results obtained, assuming a maximum heel/roll angle of 12 degrees during transient flooding as upper limit (the vessel is considered capsized once this value is exceeded). The computing time allegedly attained is terribly short even for a microcomputer, considering that finite differences methods are used, unless a very coarse grid was employed. Naturally this raises doubts regarding the accuracy of the model; doubts that are shared by the author himself, in this same paper, when drawing the final conclusions.

[58](1) DMI 88116 : **"RO-RO Passenger Ferry Safety Studies Model Test for F10 - Final Report of Phase I"**, DMI Project Report to the UK Department of Transport, 1990.

[59](1) Dand, I. W.: **" Experiments with a *Floodable Model* of a RO-RO Passenger Ferry"**, BMT Project Report to the UK Department of Transport, BMT Fluid Mechanics Ltd, 1990.

[60](1, 2) Dand, I. W.: **" Hydrodynamic Aspects of the Sinking of the Ferry ' HERALD OF FREE ENTERPRISE ' "**, Transactions RINA, 1988.

This is a report of the official investigation on the loss of the "Herald of Free Enterprise", commissioned by the to BMT Ltd by the UK Department of Transport. The sequence of events and the environmental conditions are accurately described as well as the experimental and theoretical study performed to explain the facts. Of the latter, it is worth highlighting the numerical attempt to reproduce the accident. In this a quasi-static model is employed where the changes in trim, heel and draught are updated with the amount of water flooding the compartments. Even with this simplified model, the roll overshoot typical of rapid transient flooding is observed and the importance of the cross-coupling between ship motions and water ingress is emphasised. A minimum amount of water on deck of about 10% of the ship tonnage is deemed necessary for capsize to occur. In the conclusions, the double nature of this capsize is remarked. The author states, "It has been shown that, once sufficient water was on vehicle deck G, a rapid initial capsize would be set in motion, the capsize itself setting the ship to turn uncontrollably.". The implication here is that manoeuvring aspects cannot be separated from seakeeping considerations, when dealing with the non-linear phenomena associated with extreme ship behaviour.

[61](4) Pantazopoulos, M. S.: **"Three Dimensional Sloshing of Water on Decks"**, Marine Technology, Vol. 25, No. 4, pp. 253-261, 1988.

The mathematical model presented in this publication is the same as in the paper submitted by the same author for the STAB '90. More details on the theory and some results are given with respect to simulation performed using a fishing vessel geometry. Faltinsen's findings on the effects of the position of the axis of rotation on the sloshing of water with large depth excited by small oscillations are found in agreement with those calculated here in the shallow water case.

[62](7) Spounge, J. R.: **"The Safety of RO-RO Passenger Ferries"**, RINA Spring Meeting, June 1988.

One of many on risk analysis in the field, this paper attempts to define some of the greatest dangers that passenger RO-RO ferries have to face during their service. One of these is clearly due to the overrun routes RO-RO vessels often have to sail. The observation that the loss of the "European Gateway" was indeed the only disaster - amongst those considered here - which was due to collision with another vessel, is interesting and mitigates the importance of the first conclusion. Flooding through the bow doors had already caused several major accidents ("Herald of Free Enterprise" and "Santa Margherita Dos"; the "Estonia" should have to be added to these now) and so had shifting of cargo ("Heraclion") and weather damage ("Princess Victoria"). Fire is again deemed to be a great hazard, although of the two areas most prone to fire risk, ER and public spaces, the latter is considered to be inherently more dangerous. A comment worth mentioning concerns the monumentally difficult task of transferring possibly 1500 frightened passengers from a large high-sided ferry to lifeboats, life raft and rescue vessels, perhaps in rough weather or at night. Seven years have past since this paper was published but the state of the art on evacuation systems has hardly improved.

[63](5) Woodyard, D.: **"RO-RO Safety Challenges Mechanical Engineers"**, Chartered Mechanical Engineer, Vol. 35, No. 2, 1988.

With this article, the reasons for, and problems connected with the subdivision of a RO-RO vehicle deck are exposed and mechanical engineers are asked to study feasible ways to speed up the operation of movable watertight bulkheads and optimise their stowage without reducing excessively the space available for cargo. Other detected solutions to the problem of improving RO-RO damage survivability, which need further development, include inflatable flotation collars. Apart from the technical contents of this editorial, it is important underlining the great value of divulging information about this issue. It is deemed terribly important to make other engineering branches aware of problems in naval architecture which can be better dealt with by active collaboration.

[64](5) Vossnack, E.: **"Buoyancy in the Wings"**, The Naval Architect, May 1988.

Yet another proposal to make RO-RO ferries safer from capsizing. This time relates to the permeability of side tanks and the efficiency of cross-flooding. The author suggests reducing the space available in the side compartments for flood water to come in, by filling them with empty Polythene drums. Such expedient was used during World War II and proved to be particularly efficient. The presence of the drums should also increase the ship collision resistance which is an additional desirable feature. The suggestion that double skin at 0.13B would improve the ship stability so much that a completely flooded car deck can be survived is considered absolutely unrealistic. Besides, even the insertion of drums in the side compartments would not save the ship from scenarios in which the flood water does not enter the hull through a side damage breach but trough the main door ("Herald of Free Enterprise" and "Estonia").

[65]Barwell, D. R. M.: "**Passenger RO-RO Ferry Safety - An Owners View**", The First H. Kummerman Foundation International Conference on RO-RO Safety and Vulnerability: The Way Ahead, London, RINA, December 1987.

In this note, the technical director of Sealink UK Ltd tries to make clear where owners stand in the quest for the safety of RO-RO vessels. Initially, the emphasis is put on the low occurrence of accidents, which is luckily typical of passenger ships in general - it has been demonstrated that although this fact is true, the deaths toll per accident is so high that RO-RO vessels can be considered even more dangerous than cars. Subsequently, the author points at the human factor as the determining agent responsible for most of collisions and groundings. Some reminiscence of Latin would suggest here a standard *errare umanum est*, but, willing to put it more plainly the question would become: is it sensible to rely entirely on human abilities when thousands of lives are at stake? Would it not be better to improve the inherent safety instead? Thirdly, the question of the high inefficiency of the existing emergency evacuating systems is put forward, with which one can agree unconditionally. Nevertheless, it is more difficult to agree with the final conclusions of the author, who seems to ignore the fact that most of the fire accidents have not resulted in massive loss of life with the same frequency that collisions have, in placing the research of automated means for controlling fire outbursts before that concerning ship stability.

[66](5) Crawford, J. A. Gilfillan, A. W. and Mackie, G. C.: "**The Designer Dilemma**", The First H. Kummerman Foundation International Conference on RO-RO Safety and Vulnerability: The Way Ahead, London, RINA, December 1987.

In this paper the attitude and possibilities of a naval architect approaching the design of a RO-RO ferry are scanned to reveal if commercial, technical, legal and ethical considerations can all be given enough space. However, the value of the critical review of all the possible design options to meet this purpose is limited by the general assumption that the regulations used to compare them are trustworthy. The evaluation of multi-hulls as an alternative, feasible selection merits a note. The safety of this means of water transport is much greater than that of monohulls, but what about the remaining issues? Is a SWATH as economically viable as a traditional RO-RO ferry? The main flaws of multi-hulls in this respect are to be found in their reduced range and the limits in dead-weight.

[67](5) Aston, J. G. L. and Rydill, L. J.: "**Improving the Safety of RO-RO Ships**", The Naval Architect, April 1987.

This is probably one of the earliest attempts to improve RO-RO capabilities to survive a damage greater than those contemplated by the regulations in force at the time. The idea is simple and not particularly imaginative. It consists of flared sides and internal subdivision. This solution is validated on the basis of static stability calculations and presents many flaws, starting with the fact that the initial GM is greatly increased, with evident degradation of the sea-kindliness of the ship. In future, experiments and theory will demonstrate that this solution is not particularly effective in preventing capsize.

[68](7, 2) Sen, P. and Konstantinis, C.: "**A Time Simulation Approach to the Assessment of Damage Survivability of RO-RO Cargo Ships**", Transactions SNAME, 1987.

In this paper the results of a risk analysis are briefly presented, and a new probabilistic method of assessing the survivability of RO-RO vessels is proposed. The simulation program used to evaluate the stability of the intermediate stages of flooding is based on the principle that equilibrium is reached at each time during the flooding of the compartments. The water ingress is simulated through an hydraulic model and the sea is considered calm. Evidently, this method is far from being inclusive of any dynamic effect and, as such, does not reflect the survivability characteristics of the ship.

[69](4) Hamlin, N. A. et al.: "**Liquid Sloshing in Slack Ship Tanks - Theory, Observations and Experiments**", Transactions SNAME, 1986.

This paper accounts for a detailed description of extensive experimental testing carried out to validate a non-linear, analytical model describing sloshing in partially filled tanks. In addition to the results of the experimentation and the model proposed, this publication is particularly valuable for the excellent description of the methods and set-up employed in planning and running the tests. The comparison of the numerical integration and the experimental results shows impressively good matching even when considering the presence of baffles inside the tank.

[70](1, 2) Spounge, J. R.: "**The Technical Investigation of the Sinking of the RO-RO Ferry "EUROPEAN GATEWAY"**", Transactions RINA, 1986.

On December 19, 1982, the RO-RO ferry "European Gateway" capsized in the waters off Felixstowe, after colliding with a similar vessel. The report of the investigation following this accident is resumed in these pages. Apart from the description of the dynamics of the accident and the reasons found to explain it, this paper is particularly known for the introduction of a new concept to get around the impossibility of explaining this capsize with the obsolete free-surface loss of stability model. This new phenomenon introduced was given the name of *Transient Asymmetric Flooding* by the author and is described as an initial, momentary offset of the centre of gravity of the water flooding a damaged compartment which is then reduced at a rate proportional to its value squared, to reach its static equilibrium position at the end. This behaviour of the flood water is blamed on the presence of obstacles which would prevent an instantaneous equalisation of the water surface to the horizontal equilibrium position.

[71](4, 2) Vassalos, D. and Lee, B. S.: "**An Experimental Investigation of ORELIA Double Deck Stabilising Tank**", Internal Report, University of Strathclyde, 1983.

In this study, a mathematical model for simulating the effects of stabilising passive roll tanks is proposed, which can be adapted to model the effect of water sloshing on the vehicle deck of a RO-RO vessel. The basic treatment for the water induced moment, assumes this to be sinusoidal at the same frequency as the exciting wave force but lagging by a phase angle. The amplitude and phase angle can be found experimentally as a function of parameters like frequency, tank dimensions and shape, amount of water, position of the axis of rotation etc. Besides the strong assumption that the sloshing moment can indeed be considered at the same frequency as that of the incident wave, this model is attractive for its simplicity and flexibility.

[72](6) Andersson, P. Allenström, B. and Niileksela, M.: "**Safe Stowage and Securing of Cargo on Board Ships**", Mariterm AB 1982, Göteborg, 1982.

In this report an extensive investigation on the problem of securing cargo is described. The purpose of this research was to provide a wide angle perspective on this problem by incorporating:

- a review of rules, recommendations and other literature available on the subject;
- a few examples of accidents occurred because of cargo shifting;
- a model for calculating the forces acting on the cargo during the transport;
- an analysis of the stress that the securing devices have to sustain, as a result of the above loads.

Although the theory developed for calculating the forces on the cargo due to the accelerations induced by the ship motion can be used for the purposes of investigating the limits beyond which the cargo stowed on the decks of a RO-RO vessel would move, there is still a need to assess the motion of the cargo during and after the shifting has occurred. Apart from this, the absence of any securing system for the vehicles transported by short haul ferries must be taken into account.

[73](2) Lee, C. M. and Kim, K-H.: "**Prediction of Motion of Ships in Damaged Condition in Waves**", STAB '82 Tokyo, October 1982.

The authors propose here a numerical simulation in five-degrees-of-freedom for a ship with large heel and/or trim. The model is based on strip theory and calculations are performed at zero and five knots forward speed. The aim of this study is to investigate the effect of the coupling, arising from the loss of the underwater symmetry, on the ship response. From the results, it appears that a ship which is asymmetrically flooded is likely to undergo larger roll motion. This can happen in head sea as well as for other ship headings. Beam waves coming from the direction opposite to the heel can excite larger roll motion than those incident from the same side.

[74](4, 1) Adee, B. H. and Çaglayan, I.: "**The Effects of Free Water on Deck on the Motions and Stability of Vessel**", STAB '82 Tokyo, October 1982.

This paper describes a series of experiments carried out to verify the theoretical findings of Dillingham's model. Even though this is the declared scope, the experiments are performed in a tank which is forced in sinusoidal motion whilst Dillingham was exciting the ship with a sinusoidal force but with the deck allowed to move according to the ship response whilst keeping the quantity of water constant (in the theoretical simulation water was left free to flow on and off the deck). Basically, this report seems to validate Dillingham's results when a constant amount of water is present in the tank - apart from frequency values close to resonance which is understandable since the water viscosity is not considered in the mathematical model - but there is no trace of any investigation on the time variation of the trapped water inertia.

[75](4, 2) Dillingham, J.: "**Motion Studies of a Vessel with Water on Deck**", Marine Technology, Vol. 18, No. 1, pp. 38-50 - 1981.

This is an exceptionally interesting paper. The two-degrees-of-freedom motion model employed is well described and especially advanced, including water sloshing and a reasonably well structured model of water ingress. Notwithstanding this, an insufficient quantity of results are reported, some of which do seem to be in contradiction with common knowledge. It would have been interesting to view some of the actual impute and output of the numerical program developed in terms of wave and force realisations, and ship and water response, if the author reported them in this publication. The time history of the water on deck would have been useful as well. Notwithstanding these small flaws, this publication has drawn great curiosity and full merit should be granted to the author.

[76] IMO Resolution A.265 (VIII): "**Regulations on Subdivision and Stability of Passenger Ships as Equivalent to Part B of Chapter II of the International Convention for the Safety of Life at Sea 1960** ", IMO, London, 1974.

[77](1) Bird, H. and Browne, R. P.: "**Damage Stability Model Experiments**", RINA Spring Meeting, 1973.

One of the earliest series of tests performed on a damaged vessel, this work was taken as a guideline to answer all the following experimental inquiries. Some of the factors investigated by the authors are still considered most influential in determining a vessel dynamic behaviour and its chances of capsizing. They can be listed as follows:

- freeboard;
- GM;
- compartment characteristics;
- height and direction of waves;

- initial heel.

Of the findings presented in this publication, it is worth reporting here that the results obtained have shown a consistent relationship between combinations of initial stability, residual freeboard and sea state.

[78](4, 2) Goodrich, G. J.: "**Development and Design of Passive Roll Stabilisers**", RINA Spring Meeting, 1968.

This paper suggests that the problem of the damping introduced by the sloshing of water in roll stabilising tanks, can be treated assuming that the system ship and water is closely related to a coupled double spring-mass system. This results in attaching an additional number of differential equations - which depend on the degrees of freedom the CG of the sloshing water is allowed- to those describing the ship motion. In these, the coupling terms would take account of the interaction of the centre of gravity of the water in the tank, with the ship motions. This method is fascinating for its clear and uncomplicated style but still capable of describing the dependence of the sloshing behaviour on parameters like the tank position with respect to the ship centre of gravity. It is clear though, that experimentally derived coefficients (water internal damping) would have to be employed anyway, to make this method manageable.

Further Reading

These additional references include information that can be of interest but their contents were not reported in detail in the foregoing.

[79] Amagai, K. Kimura, N. and Ueno, K.: " **On the Practical Evaluation of Shallow Water Effect in Large Inclinations for Small Fishing Boats** " - STAB '94, Melbourne USA - 7-11 November 1994.

[80]Armenio, V. et al.: " **Numerical Prediction of Roll Motion of a Ship with Liquids on Board in Regular Waves from Different Directions** " - STAB '94, Melbourne USA - 7-11 November 1994.

[81]Pedersen, P. T.: " **Ship grounding and Hull-Girder Strength** " - Marine Structures, Elsevier Science Limited, England - 1994.

[82]Pedersen, P. T. and Simonsen, B. C.: " **Dynamics of Ship Running Aground** " - Technical University of Denmark - 1994.

[83]Kan, M. and Taguchi, H.: " **Chaos and Fractals in Non-Linear Roll and Capsize of a Damaged Ship** " - International Workshop on Physical and Mathematical Modelling of Vessel's Stability in a Seaway, OTRADNOYE '93, Kaliningrad - May 1993.

[84]de Kat, J. O. and Paulling, J. R.: " **The Simulation of Ship Motions and Capsizing in Severe Seas** " - SNAME Trans., Vol. 97 - 1989.

[85]Wardelmann, H.: " **The New SOLAS Amendments - Agreed and Possible** " - RO-RO '88, Göteborg - June 1988.

[86]Rusaas, S.: " **Survival Capability Class: Increased Safety, but does it Destroy the RO-RO Concept?** " - RO-RO '88, Göteborg - June 1988.

[87]Boltwood, D. T.: " **RO-RO Ship Survivability: Comments on Damage Stability Modelling** " - RO-RO '88, Göteborg - June 1988.

[88]Spencer, J. S. and Gilbert, R. R.: " **RO-RO Stability** " - RO-RO '88, Göteborg - June 1988.

[89]Pawlowski, M. and Winkle, I. E.: " **Capsize Resistance Through Flooding - A New Approach to RO-RO Safety** " - RO-RO '88, Göteborg - June 1988.

[90]Byrne, D.: " **Practical Solutions to Improved Survivability of RO-RO Ferries** " - RO-RO '88, Göteborg - June 1988.

[91]Vossnack, E.: " **Putting a Lifebelt around the Ship** " - RO-RO '88, Göteborg - June 1988.

[92]Paffett, J.: " **RO-RO Safety and Vulnerability: The Way Ahead** " - The Naval Architect - January 1988.

[93]Jakic, K.: " **Flotation and Stability of RO-RO Vessels in the Damaged Conditions** " - International Shipbuilding Progress, No. 34 - 1987.

[94]Elsimillawy, N. and Miller, N. S.: " **Time Simulation of Ship Motions: A Guide to the Factors Degrading Dynamic Stability** " - SNAME Trans., Vol. 94 - 1986.

[95]Mikelis, N. E. Miller, J. K. and Taylor, K. V.: " **Sloshing in Partially Filled Liquid Tanks and its Effect on Ship Motions: Numerical Simulations and Experimental Verification** " - RINA Trans. - 1984.

[96]Demirbilek, Z.: " **Energy Dissipation in Sloshing Waves in a Rolling Rectangular Tank - I. Mathematical Theory** " - Ocean Engineering, Vol. 10, No. 5 - 1983.

[97]Demirbilek, Z.: " **Energy Dissipation in Sloshing Waves in a Rolling Rectangular Tank - II. Solution Method and Analysis of Numerical Technique** " - Ocean Engineering, Vol. 10, No. 5 - 1983.

[98] Demirbilek, Z.: " **Energy Dissipation in Sloshing Waves in a Rolling Rectangular Tank - III. Results and Applications** " - Ocean Engineering, Vol. 10, No. 5 - 1983.

[99] Çaglayan, I. and Storch, R. L.: " **Stability of Fishing Vessels with Water on Deck: A Review** " - Journal of Ship Research, Vol. 26, No. 2 - June 1982.

[100] Faltinsen, O. M.: " **A Numerical Non-Linear Method of Sloshing in Tanks with Two-Dimensional Flow** " - Journal of Ship Research, Vol. 22, No. 3 - September 1978.