Modelling the Probability Distribution of the Time Series of Stock Price Changes in the UK Market

By

Mohammed Fawzy Omran

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University of Strathclyde

Department of Accounting and Finance

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Dedication

I would like to dedicate my thesis to my wife, Amal, and son, Rami, for their continual moral support and patience during a long period of study.

Abstract

The thesis considers different aspects of the probability distribution of the time series of stock price changes in the UK market. It places particular emphasis on the character of the volatility of the series. Chapter 2 documents some preliminary findings about changes in the FT-ALL share price index. These findings are: (1) its distribution has fat tails; (2) the BDS test rejects the hypothesis of identically, and independently distributed price changes; (3) the BDS test applied to the GARCH(1,1) residuals, adjusted according to de Lima (1995b), indicates that Autoregressive Conditional Heteroscedasticity explains most of the nonlinearity in the FT-ALL price changes.

The hypothesis of constant variance is rejected for the FT-ALL series using the Loretan and Phillips test, reported in chapter 3. An intervention model along the lines of Box and Tiao (1975) is used to model possible shifts in the variance of the FT-ALL price changes during the 1973 oil crisis and the 1987 market crash. The model allows for slow decay in the shocks effects and a different level of volatility after both crises. The results suggest that the reaction of the UK market to both crises differs only with regard to the slow decay of the shocks. The null hypothesis of constant variance is "accepted" for the residuals from the intervention model. This "acceptance" is due to the filtering of the effects of the 1973 and 1987 crisis from the FT-ALL series.

The hypothesis that GARCH volatility persistence becomes insignificant when the volume of trade is included is examined in chapter 4. In a test covering the price behaviour of 57 UK companies over the period from 4/1/1988 to 28/2/1994, it is found that although the parameter estimates of the GARCH model becomes insignificant when volume is used in the conditional variance of price changes, the autocorrelations of the squared residuals still exhibit a highly significant GARCH pattern. It is argued that the GARCH-volume model of Lamoureux and Lastrapes (1990b) suffers from a multicollinearity problem, apart from the possible simultaneity bias which could lead to an inconsistent estimate of the parameter for volume. It is found that unexpected volume reduces volatility persistence. This reduction can be attributed to the strong association in the timing of innovational outliers in the price changes and unexpected volume found in the study. The results are consistent with the market depth hypothesis of Bessembinder and Seguin (1993).

The GARCH model with the conditional normal, Student's t and generalized error distributions is estimated for the UK FT-ALL price changes in chapter 5. The model also considers seasonal and leverage effects. The time period for the study is chosen so as to avoid including the 1987 crash. The results suggest: (1) volatility persistence is low after the 1987 crash; (2) the ARMA and ARCH effects, along with the seasonal effects of Monday and holidays, explain a significant part of the departure from normality; (3) there is a need for leptokurtic distribution such as the Student's t; and (4) there is no evidence for a leverage effect in the FT-ALL series. That is, positive and negative surprises tend to affect volatility in the same way.

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Chapter 1 : Introduction

1. Introduction

The departure of stock price changes from normality has been well documented over the last three decades since the seminal works of Mandelbrot and Fama in the mid 1960's [see Mandelbrot (1963a) and Fama (1963 and 1965a)]. The enthusiasm of academics and practitioners alike to find out more about the stochastic process generating stock prices has never waned. Practitioners are interested in finding out how to improve their share selection and portfolio management skills [see Lofthouse (1994)], whilst academics are interested in developing a better understanding of how asset prices are determined in the capital market. This understanding can tell them, for example, what the theoretical models for derivative securities should consider. The well-used Black-Scholes formula for the valuation of European call option prices is based on the assumption that continuously compounded returns are normally distributed with a constant variance. Contrast this assumption, however, with the well documented findings that stock returns have fatter tails than the normal distribution [see Bollerslev *et al.* (1993) for a review].

Bookstaber and McDonald (1987) describe two approaches to the study of the distribution of stock price changes in finance. The first begins by describing the economic theory that gives rise to stock price changes, and the second searches for a distribution function that empirically fits stock price changes. Examples of research using the first approach are Epps and Epps (1976), Tauchen and Pitts (1983), and Andersen (1995). Such studies give support to a model of price changes based on a mixture of distributions, and emphasise the market process and the relationship between price volatility and volume of trade. The second approach generally start by observing that the empirical distribution of stock price changes is leptokurtic compared with the normal distribution. The next step is to search for a density function that can account for this leptokurtosis [see for example, Fama (1963 and 1965a), Mandelbrot (1963a), Blattberg and Gonedes (1974), and Hall *et al.* (1989)].

The influential paper of Fama (1965a) suggested that stock price changes can be safely approximated as uncorrelated despite the observation that they exhibit very small though significant autocorrelations. However, Mandelbrot (1963a) and Fama (1965a) recognized that there is a tendency for large price changes to be followed by large price changes, and small price changes to be followed by small price changes of either sign. This observation went unnoticed until the seminal work of Engle (1982). Engle presented a model which can account for the fact that although stock price changes can be characterized as uncorrelated in the mean, they are correlated in the variance. Based on Engle's approach price changes can be modelled so as to generate the clustering of volatility observed empirically by Mandelbrot and Fama.

Many papers have appeared after Engle's 1982 work with a common message-stock price changes can be adequately approximated by a variant of the ARCH models [see Engle (1995), Bollerslev *et al.* (1992), Bera and Higgins (1992), and Bollerslev *et al.* (1993)]. There is no longer any doubt that stock price changes cannot be described as being independent of each other. As a result, the interest in the distribution of stock price changes has shifted to studying the properties of its conditional distribution. The standard ARCH model, and its generalized form (GARCH) of Bollerslev (1986), assume for convenience that the conditional distribution of stock price changes is normally distributed. It is commonly found that the conditionally normal GARCH model does not explain the fat tails of stock price changes [see Bollerslev (1987)]. Several probability distributions are suggested to be used as the conditional density function of the GARCH model. These include the Student's t in Bollerslev (1987), the stable Paretian in McCulloch (1985), and the generalized error distribution in Nelson (1991).

The thesis considers different aspects of the modelling of the probability distribution of the time series of the UK stock price changes with particular emphasis being placed on the character of the volatility of the series. A literature review along with an exploratory analysis of the UK price changes can be found in chapter 2. The main findings of the chapter are: (1) the distribution of the FT-ALL price changes has heavier tails than the normal distribution; (2) the hypothesis of independently, and identically distributed (i.i.d.) price changes is rejected using the BDS test [see Brock *et al.* (1987)]; (3) the hypothesis of i.i.d. is not rejected for the residuals from a GARCH(1,1) model, suggesting that conditional heteroscedasticity can explain most of the nonlinearity of the FT-ALL price changes. This evidence is consistent with the results of Hsieh (1991) from the USA market, and Abhayanker *et al.* (1995b) and Paudyal *et al.* (1993) from the UK market. However, my analysis contributes by applying the recent adjustment suggested by de Lima (1995b) to the residuals from the GARCH model before conducting the BDS test. As is well recognised [see Hsieh (1991)], the asymptotic distribution of the BDS test is not known when it is applied to the residuals from the GARCH model. However, de Lima (1995b) shows that asymptotic normality of the BDS test carries over the residuals from the GARCH model after a simple adjustment.

The hypothesis that the unconditional variance of the FT-ALL price changes is constant over time is tested in chapter 3 using the recent test suggested by Loretan and Phillips (1994). Their test does not assume the existence of the fourth moment of the data, and therefore, it is more consistent with the findings of fat tails in the financial time series [see Fama (1963 and 1965a), and Mandelbrot (1963)]. Since the critical values for the test depends on the existence of moments, the null hypothesis of finite second moment along with the null hypothesis of finite fourth moment are tested using the Loretan and Phillips (1994) estimators[1]. The results suggest that although the second moment of the data seems to be finite, there is doubt about the existence of the fourth moment. This evidence is consistent with the evidence of Longin (1993), Loretan and Phillips (1994), Hiemstra and Jones (1995) and Abhyanker *et al.* (1995a) from the US market. Also consistent with the US evidence

[1]The Loretan and Phillips (1994) estimators were criticised by Mittnik and Rachev (1993) and Pagan (1995). The criticism is discussed in chapter 3.

of Pagan and Schwert (1990a) and Loretan and Phillips (1994), the null hypothesis of constant variance over time is rejected for the FT-ALL series.

The plot of the unconditional variance of rolling sub-samples of the data is checked for variance constancy. The results can only be considered as indicative since graphical inspection cannot be regarded as a formal test for variance constancy [see Pagan and Schwert (1990a)]. The results suggest that there were two shifts in the variance: during the 1973 oil crisis and the 1987 market crash. This is consistent with the finding of Schwert (1989) from the US market that "the "OPEC oil shock" (1973-1974) caused an increase in the volatility of stock and bond returns".

An intervention model on the line of Box and Tiao (1975) is used to model and test for statistical significance of possible shifts in the variance around the 1973-74 oil crisis and the 1987 market crash. The model is flexible in the sense that it allows for a possible slow decay in the shocks effects and a different level of volatility after both crisis. The parameters of the model are estimated jointly using the BHHH routine (see Berndt *et al.* (1974)) with the conditional Student's t density. It is found that the slow decay of the shocks effects differ between the two crisis, with the oil crisis having a longer effect on volatility than the market crash. However, there is no evidence that either of the two crisis had a lasting effect on the volatility of the stock market.

The Loretan and Phillips (1994) test for variance constancy is applied to the residuals form the intervention model. The result suggests that the null of constant variance should be "accepted". This acceptance is due to the filtering of the effects of the oil crisis and market crash from the data. It is concluded that the stock market is subject to abrupt changes in volatility during some exceptional periods. Outside these periods, however, stock price changes can be described as covariance stationary.

The issue of whether GARCH modelling captures the temporal dependence in volume of trade for individual stocks in the UK market is examined in chapter 4. Lamoureux and Lastrapes (1990b) offered results suggesting that for the purpose of forecasting the conditional variance of stock price changes the volume of trade is sufficient to replace the entire history of the past squared price changes. This issue is important since it can explain the observed volatility clustering in stock prices [see Diebold (1986), and Stock (1987 and 1988)]. Whilst it is widely accepted that GARCH models can account for volatility clustering there is less agreement on its causes[2]. Bessembinder and Seguin (1992) added more insight on the volume-volatility relationship by decomposing the volume of trade into its expected and unexpected components, and studying the effects that these have on the volatility of price changes. Bessembinder and Seguin (1992) explained unexpected volume as shocks to the trading activity. They found that unexpected volume has a greater influence on

[2]Due to the lack of an immediate economic rationale behind ARCH models, Hall *et al.* (1989) note that ARCH models should be interpreted on the same lines as ARMA models. That is they are "a convenient and parsimonious representation of the behaviour of time series data", see Hall *et al.* (1989), pp. 344.

the variance of US stock returns than expected volume.

The objective of the chapter is to combine the methodologies of Lamoureux and Lastrapes (1990b) and Bessembinder and Seguin (1992) to investigate the volume-volatility relationship in the UK stock market. The results are consistent with those of Lamoureux and Lastrapes (1990b) in that the parameter estimates of the GARCH model become insignificant when volume of trade for individual stocks is used in the conditional variance of price changes. However, the autocorrelations of the residuals from the model are found to display highly significant ARCH effects. I argue that the GARCH-Volume model of Lamoureux and Lastrapes (1990b) suffers from a multicollinearity problem between volume of trade and the past conditional volatility[3].

It is found that unexpected volume reduces volatility persistence. Since unexpected volume is not serially correlated by construction (the residuals from an ARMA model), the results cannot be attributed to unexpected volume capturing the serial dependence in the information flow rate. The results are more consistent with the interpretation of Bessembinder and Seguin (1992 and 1993) of unexpected volume as shocks to the trading activity. Moreover, the evidence is consistent with their

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[3]There is a possible simultaneity bias in Lamoureux and Lastrapes model if volume and price changes are correlated. Lamoureux and Lastrapes (1990b) assumed that volume of trade is weakly exogenous in the sense of Engle *et al.* (1983). This assumption could lead to inconsistent parameter estimate of volume of trade. However, it would not affect the inference about volatility persistence. More details about weak exogeneity and the problems of relaxing the assumption are provided in chapter 4.

finding that positive unexpected volume has a greater affect on volatility than negative unexpected volume. Bessembinder and Seguin (1993) use this evidence to argue in favour of the hypothesis that volatility is affected by existing market depth for each security. That is volatility is greater when unexpected volume is positive, i.e. there is more trading activity than expected in the market in terms of number and/or size of transactions for the security.

Finally, I present evidence that the price changes-volume relationship is due to a strong association in the timing of innovational outliers in both series. This association is responsible for the noted reduction in the persistence of the GARCH model when volume is included in the variance of price changes. The results in general suggest that unexpected volume can help in forecasting the future volatility of stock price changes. This is contrary to the conclusion drawn by Lamoureux and Lastrapes (1994) that contemporaneous volume and squared price changes are not useful instruments in predicting the future volatility.

The GARCH model of Bollerslev (1986) with different conditional densities is used to model the FT-ALL returns in chapter 5. It is well documented that although the GARCH model with conditional normal distribution generates some degree of unconditional kurtosis, it is typically less than adequate to fully account for the fat tails of stock returns [see Bollerslev (1987)]. The study models the conditional mean and variance of FT-ALL returns using two distributions which allow for the leptokurtic behaviour of stock returns; the Student's t and generalized error distributions. The study also considers several factors which might affect the UK stock returns. These include possible ARMA or ARCH effects, seasonal effects corresponding to Monday, holidays, January, and the turn of the month, as well as any asymmetries in the UK stock returns due to leverage effects. It is found that the Student's t distribution offers a better fit to the conditional distribution of FT-ALL returns than the normal and generalized error distributions. The results also suggest that the ARMA, ARCH, Monday, and holidays explain a significant part of the departure of the FT-ALL returns from normality. In the UK, the average of returns on Monday is found to be negative. Also, Mondays' returns have a higher volatility than those of other trading days. In addition, the day following the closure of the market for holidays is characterised by significantly positive returns. This suggests that the negative Monday returns cannot be due to the market being closed on the preceding two days since returns after holidays tend to be positive.

The January effect is found to be significant under the assumption of conditionally normally distributed returns. But this assumption may not be appropriate given the fat tails property of stock returns, and its use could lead to the wrong inferences being drawn. There is no evidence of a leverage effect in the UK stock returns, suggesting that positive and negative surprises tend to affect the return volatility in the same way. Finally, the results indicate that starting the sample period after the 1987 crash leads to a significant reduction in the volatility persistence of the UK returns. However, this does not result in any reduction in the departure of the returns distribution from normality.

The thesis proceeds as follows. Chapter 2 briefly reviews the literature on the probability distribution of financial time series, and offers some preliminary evidence on the distribution of stock price changes in the UK market. The chapter also discusses the possibility that structural changes in the unconditional variance of stock price changes can explain both the departure from normality, and the high volatility persistence observed empirically. Chapter 3 investigates the assumption of the covariance stationarity for the UK stock price changes. Chapter 4 investigates the relationship of the volume-volatility for individual stocks in the UK market. Chapter 5 models the conditional mean and variance of the UK equities returns using the GARCH model with conditional Student's t and generalized error distributions. Chapter 6 presents some concluding comments and provides some suggestions for further research on the issues covered in the thesis.

Chapter 2

Literature Review and Exploratory Data Analysis[1]

1. Introduction

The chapter briefly discusses some major probability distributions suggested in the literature as a basis for modelling the price behaviour of securities, and provides a preliminary analysis of the properties of the UK stock price changes. It also contributes to the UK studies of nonlinearity [see for example, Paudyal *et al.* (1993) and Abhayanker *et al.* (1995b)] by incorporating the recent adjustment suggested by de Lima (1995b) for tests of nonlinearity using the BDS test statistic [see Brock *et al.* (1987)]. de Lima (1995b) suggests a simple adjustment to the GARCH residuals before applying the test. This adjustment overcomes the problem that the asymptotic normality of the BDS test is not valid when it is applied to the GARCH residuals [see Hsieh (1991)].

2. Normal Distribution

The assumption that the distribution of asset price changes is normal is usually based on the following reasoning - if price changes from one transaction to another are independent and identically distributed (i.i.d.) variables, then the sum of them

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[1]Some parts of this chapter are forthcoming in the Journal of Applied Economics Letters titled "Nonlinear Dependence and Conditional Heteroscedasticity in Stock Returns: UK Evidence".

over a fixed number of transactions will converge to the normal distribution as the number of transactions in the sum increases (the central limit theorem, CLT). Bachelier (1900) and Osborne (1959) used the central limit theorem in their discussion to support the normality assumption. As noted by Brock and de Lima (1995), the central limit theorem would still be applicable for weakly nonstationary and weakly dependent price changes as long as the number of variates in the sum goes to infinity[2]. Figure 1 shows the histogram of the changes in the natural logarithm of the daily FT-ALL stock index from 2/1/1970 to 31/12/91[3]. The FT-ALL price changes are standardized by subtracting the mean and dividing by the standard deviation. The figure also shows the density function of the standard normal distribution. Apparently the FT-ALL distribution is more peaked in the middle and has more outliers in the tails compared with the normal distribution.

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[2]However, Brock and de Lima (1995) argue that the use of the central limit theorem for weakly dependent and weakly nonstationary data "... is not very useful as a discriminator across the class of potential data generating process".

[3]The data set contains 5555 daily prices. Krushna Paudyal and Pradeep Yadav kindly provided the data.

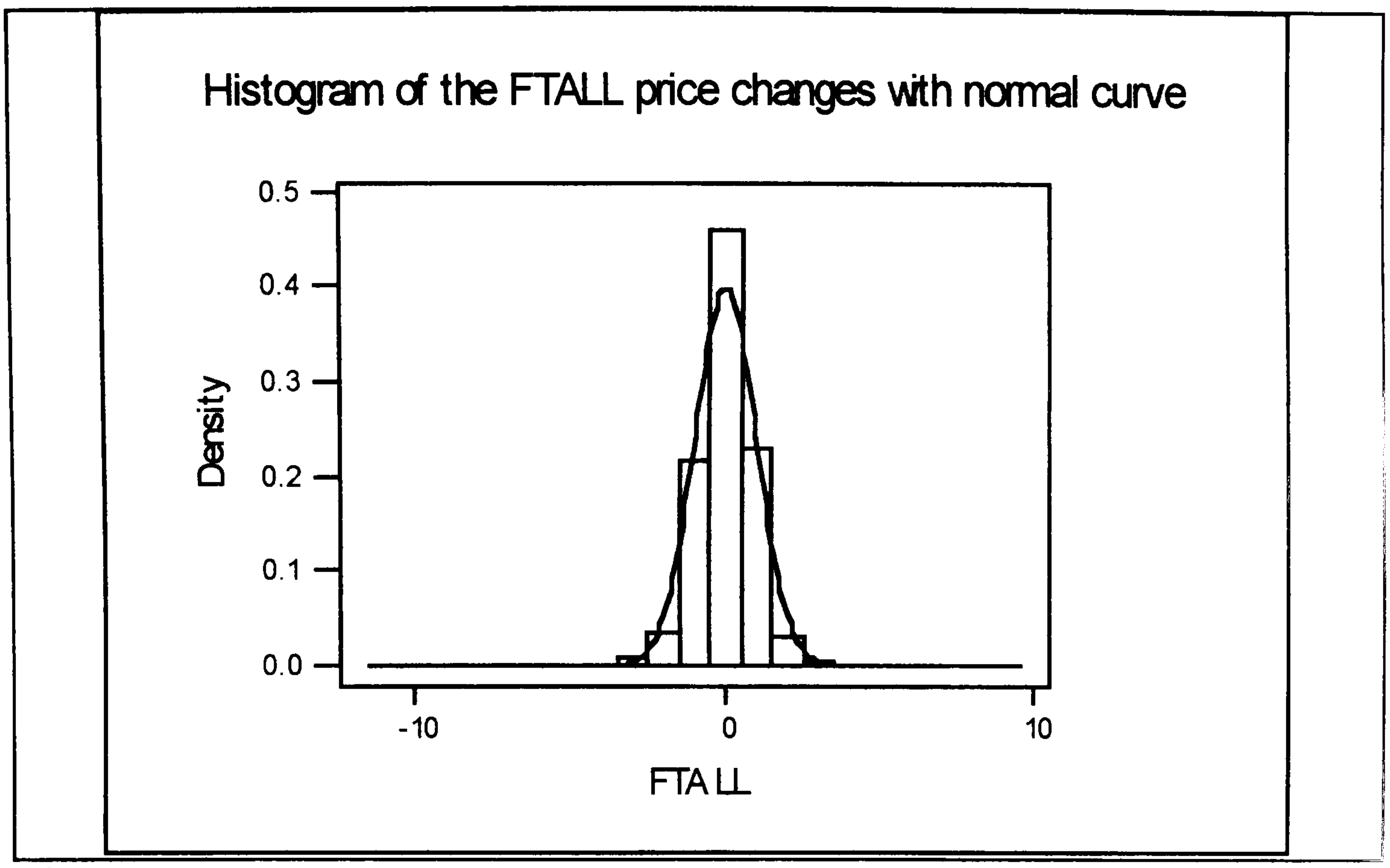

Figure 1

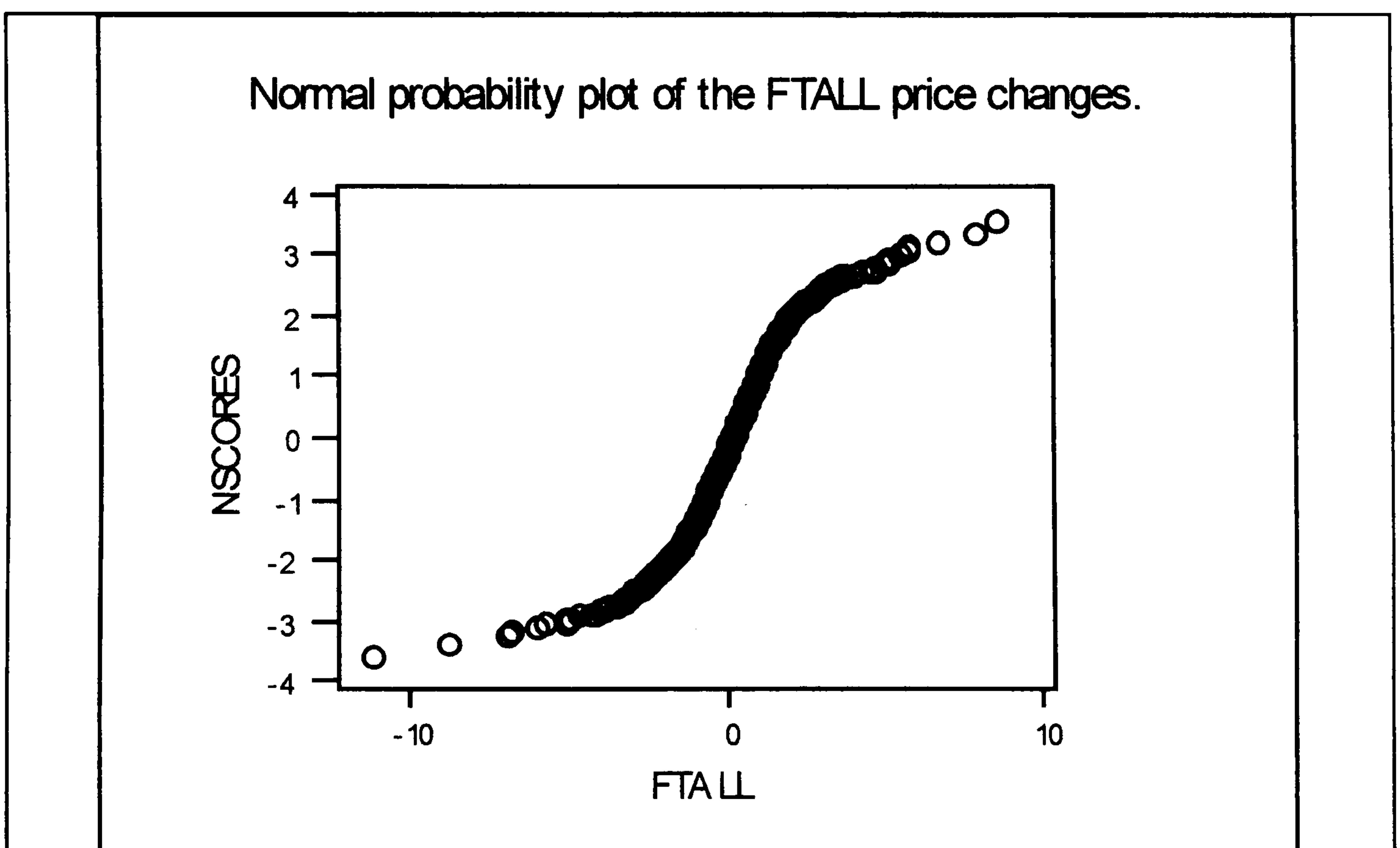

Figure 2

Figure 2 illustrates the normal probability plot for the FT-ALL price changes over the period of the study. The horizontal axis of the graph shows the FT-ALL price changes and the vertical axis shows the n-scores values. The n-scores values are the z values derived from the unit normal distribution at different fractile points for a sample size equivalent to that of the FT-ALL price changes[4]. If the FT-ALL price changes follow the normal distribution, they will fall on an approximately straight line. The graph takes the shape of an elongated S, as do the graphs of the data sets on the USA stocks examined in Fama (1965a). It indicates that there are more outliers in the tails of the distribution relative to the normal distribution. It is clear that the FT-ALL price changes cannot be viewed as normally distributed.

3. Stable Paretian Distribution (SPD)

Mandelbrot (1963a) argued that previous researchers in this area had neglected the observed departure from normality, especially the observed leptokurtosis in the distribution of asset price changes. As an alternative, he proposed the stable Paretian distributions. This family of distributions is usually defined by its characteristic function, since its density function is not known explicitly except for a few special cases, of which the best known are the Cauchy and the normal distributions. The logarithm of the characteristic function for the stable family is:

$$\log f(t) = i\delta t - \gamma |t|^{\alpha} (1 + i\beta (t/|t|) \tan(\pi\alpha/2)), \qquad (1)$$

[4]The n-scores are the inverse of the cumulative distribution function of the estimated fractile (i - 3/8)/(n + 1/4), where i=1,...,n, and n is the sample size [see Ryan and Joiner (1976) and Ryan (1990)].

where t is a real number and i is the imaginary number. Four main parameters are needed to specify the characteristic function: the location parameter δ (the mean if α is greater than 1), the scale parameter γ, the measure of skewness β (zero in symmetric distributions), and the most important parameter of all, the characteristic exponent α. This last parameter relates to the probability mass in the tails of the distribution, and can take a value in the range 0 to 2. The smaller the value it takes, the thicker the tails and vice versa. Consequently, the distribution has moments only of order $k =< \alpha < 2$. When $\alpha=2$, we have the normal distribution, which, of course, has moments of all orders.

"By definition, a stable Paretian distribution is any distribution that is stable or invariant under addition" [Fama (1963)]. Therefore the distribution of the sum of i.i.d. stable variables is stable with the same α and β as the individual variables in the summand (the distribution is closed under addition). Mathematically, the log characteristic function of the sum of n i.i.d. stable variables is given by[5]

$$n\log f(t) = i(n\delta)t - (n\gamma)|t|^{\alpha}(1 + i\beta(t/|t|)\tan(\pi\alpha/2)), \tag{2}$$

Equation 2 shows that the distribution of the sum of i.i.d. stable variables has the same α and β as the component distribution. The location and scale parameters in equation 2 are n times the component values. Stability under addition requires α

[5]See Fama (1963 and 1965b).

and β to be constant under addition. The property of stability also applies to the case in which the n stable variables have different locations and scale parameters. The log characteristic function then is given by

$$\sum_{j=1}^{n} \log f_j(t) = i\left(\sum_{j=1}^{n} \delta_j\right) t - \left(\sum_{j=1}^{n} \gamma_j\right) |t|^{\alpha} \left(1 + i\beta (t/|t|) \tan(\pi\alpha/2)\right). \quad (3)$$

Where log $f_j(t)$ is the log characteristic function of the jth stable variable in the sum. The expression shows that the location and the scale parameters of the resulting sum are equal to the sum of the δ_j and γ_j parameters, but that α and β are the same as the values in the component distributions.

Another important property of the SPD is that it is the only possible limiting distribution for sums of i.i.d. variables (generalized central limit theorem, GCLT)[6]. Thus, if the individual variables in the sum do not have finite variances, then the limiting distribution of their sum, if the sum has a limiting distribution, is a SPD with $\alpha < 2$. Let x_{it} denote the natural logarithm of the high frequency price relatives for security i for day t (the high frequency logarithmic returns). Then the low frequency logarithmic returns, y_{iT}, are given by

[6]See Gnedenko and Kolmogorov (1954) for details.

$$y_{iT} = \sum_{t=1}^{T} x_{it}. \quad (4)$$

The returns y_{iT} are the continuously compounded returns over a period of T consecutive days. If the x_{it}'s are i.i.d. with finite variance, the distribution of y_{iT} converges to the normal as T goes to infinity (CLT)[7]. However, if the x_{it}'s are i.i.d. with infinite variance, the distribution of y_{iT} converges (if at all) to the SPD with an $\alpha < 2$ (GCLT)[8]. On the other hand, if x_{it}'s are stably distributed with $\alpha = \alpha^* < 2$, then the distribution of y_{iT} will be stably distributed with $\alpha = \alpha^*$ for all T.

It is important to note that the family of SPD's is the only one which is closed under addition. This property was used by Fama (1965b) to generalize the Markowitz's portfolio model to the case where price changes are assumed to follow a non-normal SPD. Also, Fama (1971) generalizes the CAPM model to a market where price changes may have a non-normal SPD.

Stability under addition and the GCLT properties of the SPD is often used to test the hypothesis that returns follow a non-normal SPD against the alternative that they follow the mixture of normals distribution (MND) [see for example Officer

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[7]This central limit theorem is called the Lindeberg-Levi limit theorem, see Feller (1968), pp. 244, Davidson (1994), pp. 366, Brock and de Lima (1995), pp. 20, and Greene (1997), pp. 122.

[8] There are some exceptions to this general rule. For example, Feller (1971) gives an example of a central limit theorem with infinite variance. He shows that some distributions with infinite variance lie in the domain attraction of the normal distribution, see Feller (1971) pp. 260 and pp. 312-313.

(1972), Fielitz and Rozelle (1983), and Hall *et al.* (1989)]. These hypotheses can be tested by estimating α for the entire sample and for non-overlapping sums of the data. If the estimate of α increases over the larger sums towards a value of 2, then the underlying distribution follows the MND [Fama and Roll (1971)]. On the other hand, if the estimate of α tends to be equal across the sums, this suggests that returns follow a non-normal SPD. It is also possible that the underlying distribution follows a mixture of SPD's with different scales.

Fama and Roll (1971) suggest a simple method for estimating α of the symmetric SPD. Their estimator is based on just 4 sample fractiles. Formally,

$$z_f = 0.827 \left[\frac{x_f - x_{1-f}}{x_{0.72} - x_{0.28}}\right], \tag{5}$$

$$\alpha_f = G(f, z_f). \tag{6}$$

where z_f is an estimator of the f fractile of the standardized SPD with characteristic exponent α. Accordingly, G is a function that uniquely maps the estimated fractile, z_f, and fractile f into the characteristic exponent α. Also, $x_{0.72}$ and $x_{0.28}$ are the estimates of 0.72 and 0.28 fractiles of the returns distribution. Fama and Roll (1968) presented tables for the cumulative density function of standardized symmetric SPD's for 12 values of α in the range 1 to 2 under different distributional assumptions. Once the estimated fractile z_f is calculated, it is compared to these tables to get the value of α.

Fama and Roll's (1971) method offers very similar results to those derived using much more complex methods. Fielitz and Rozelle (1983), for example, compared the results obtained by using the Press (1972) method of moments with the Fama-Roll method, and found that both methods produced very close results. They report that "The mean difference between the Press α estimates and the Fama-Roll α estimates for the 50 stock distributions studied here is - 0.0018 with standard deviation 0.0715. The mean absolute value difference is 0.0490 with standard deviation 0.0517". In another study Leitch and Paulson (1975) compared the results they got using a method which assumes asymmetry, and is based on minimizing the modulus of the difference between the theoretical and empirical functions of the data, and the results obtained by using the Fama-Roll method. Their results suggest that "agreement is generally very good for estimates of α and γ" for the twenty stocks studied. They also compared the results of estimating α with and without restricting β to zero, and found that β has a very small effect on α as well as finding that this effect decreases as α approaches 2.

The empirical results on whether speculative price changes follow the SPD are mixed. Mandelbrot (1963a), Fama (1963 and 1965a), Mcfarland *et al.* (1982), Cornew *et al.* (1984), and So (1987) offer evidence in favour of the SPD, whereas Blattberg and Gonedes (1974), Upton and Shannon (1979), Akgiray and Booth (1988), Hall *et al.* (1989) and Lau *et al.* (1990) offer evidence against it.

The Fama and Roll (1971) method is used to examine whether the FT-ALL price changes, during the period from 2/1/1970 to 31/12/1991, lie in the domain of attraction of the normal distribution. The method is based on the Lindeberg-Levi CLT which states that the distribution of the sum of daily price changes approaches the normal distribution as the number in the sum increases. This is only true if daily price changes are i.i.d. variables with finite variance. If price changes lie in the domain attraction of the normal distribution, then the estimate of the characteristic exponent, α, for the sums of daily price changes should be closer to 2 than for the entire sample [Fama and Roll (1971)]. The problem with this methodology is that the sample size decreases as the sum size increases which makes the estimates of α subject to greater sampling error [see Hall *et al.* (1989)]. Monte Carlo results of Fama and Roll (1971) suggest that estimates of α are free of bias with a small downward bias for sample sizes of less than 99 observations. The sample of the FT-ALL price changes under investigation contains 5555 observations which makes a sum size of 40 feasible. Adjacent daily price changes are summed up into groups of non overlapping sums of 2, 10, 20 and 40 days, and the characteristic exponents are estimated using the Fama and Roll method (1971). The results are presented in table 1.

Table 1: the estimates of α of the FT-ALL price changes at different sum sizes

	Sum size of daily price changes				
	1 day	2 days	10 days	20 days	40 days
α	1.77	1.72	1.76	1.81	1.73

α is a measure of the total probability in the tails of the distribution of FT-ALL price changes. α is estimated using equation 5 where the f fractile is set equal to 0.96.

The table shows that α's do not tend to increase over larger sums towards a value of 2. As previously mentioned, α is a measure of the total probability in the tails of the distribution. The range for α is $0<\alpha\leq 2$, with $\alpha=2$ implying the normal distribution. The lower the value that α takes, the thicker the tails of the distribution. The results suggest that α for the distribution of the sums of 40 daily price changes is far less than 2 indicating that the tails of the distribution are much thicker than the normal distribution. However, a major drawback of the Fama and Roll (1971) method is that it does not provide standard errors which makes it difficult to statistically assess the discrepancy between the empirical value found for α and its theoretical value under the normal distribution. Therefore, these results can be best viewed as indicative, and a more recent test suggested by Loretan and Philips (1994) will be used in chapter 3.

One possible explanation for the estimate of α being different from 2 at sum size of 40 days is that daily price changes are not independent of each other [see Hsu *et al.* (1974)]. In other words, the existence of a linear or nonlinear dependence structure in the data may cause the observed departure form normality. To investigate this possibility Hsu *et al.* (1974) suggest randomizing the entire sample before the sums are taken and α is estimated. If the daily price changes are independent, the α's pattern of the original data should be quite similar to the α's pattern of the randomized set. Any difference will indicate the effect of the linear or nonlinear dependence on the distribution of price changes.

The possibility that data dependence is responsible for the deviation from normality is examined by randomizing daily price changes first before taking the sums, and computing the estimates of the f fractile of the data, Z_f, using equation 5. This exercise is repeated 100 times to get a vector of 100 Z_f at each sum size. Then, the average and standard deviation, σ, of Z_f are computed at each sum size. The average estimate of Z_f is then matched to the tables for the cumulative density function for the standardized symmetric SPD in Fama and Roll (1968) to get the corresponding estimate of α. Finally, the standard error of the average Z_f at each sum size are computed using $\sigma/\sqrt{n}$, where σ is the standard deviation of Z_f, and n is the number of the repeated randomizations. These standard errors are used to test the hypothesis that the average estimate of Z_f is not significantly different from 2.477, the value taken by the normal distribution. The results are summarized in table 2.

Table 2: Estimates of the average fractile, Z_f, for daily price changes in randomized order at different sums. The t-stats test the hypothesis that the average estimated fractile is not significantly different from 2.477, the value taken by the normal distribution and corresponds to a characteristic exponent of 2. The f used is 0.96.

	Sum size			
	2	10	20	40
Average fractile t-stat. corresponding α	2.7568** (39.97) 1.77	2.6259** (9.31) 1.86	2.5689** (4.60) 1.91	2.5085 (1.13) 1.96

(*) significantly different from 2.477 (an α of 2) at 5% level of significance.
(**) significantly different from 2.477 (an α of 2) at 1% level of significance.

The results indicate that the estimated average fractile, Z_f, is significantly different from 2.477, the value taken by the normal distribution, at each sum size except at sum size 40. This suggests that the distribution of the sums of 40 daily price changes may be assumed to be normally distributed, and we can conclude that the distribution of randomized daily price changes lie in the domain of attraction of the normal distribution. The estimate of the characteristic exponent α at sum size 40, is equal to 1.73 for the daily price changes when summed in chronological order which is far less than the average α for the sum of the randomized price changes (1.96). This indicates that the order of the chronological data is important and this would not be the case for i.i.d. random variables. However, given the limitations of the Fama and Roll (1986) methodology [the non availability of standard errors, and smaller sample sizes at larger sum sizes], the null hypothesis of i.i.d. will be further examined using the powerful test suggested by Brock *et al.* (1987) [see page 37].

Another approach to studying the distribution of stock price changes is to make direct inferences about the tail behaviour of their distribution without making any assumptions about the form of the underlying distribution, an approach described by DuMouchel (1983) as "Letting the tails speak for themselves" [see also Hill (1975), Smith (1987), and Dekkers *et al.* (1989)]. Jansen and de Vries (1991) used a method developed by Hill (1975) to study the probability mass in the distributions of returns for ten American companies and two returns indices. They concluded that at least the first two moments exist. Their results were also confirmed by Longin (1993), Loretan and Phillips (1994), Hiemstra and Jones (1995), and Abhyanker *et al.* (1995a). The results of these studies suggest that the fourth moment of stock returns does not exist for either US aggregate nor individual stock returns[9].

4. Student's t distribution

The fact that there is no explicit density function to the SPD, except for a few cases limits its use in economic analysis. Other distributions which have finite variance and can account for the leptokurtic behaviour observed empirically have consequently been suggested as alternatives. Blattberg and Gonedes (1974), for example, suggest the Student's t distribution as an alternative to the SPD. The density function for the Student's t distribution with location parameter m, scale parameter $H>0$, and degrees of freedom, $d>0$, is:

[9]The studies by Loretan and Philips (1994), Abhaynker *et al.* (1995a), and Longin (1993) examined aggregate US returns. The study by Hiemstra and Jones (1995) examined 1,952 US ordinary common stocks returns.

$$f(y|m,H,d) = \frac{d^{(d/2)}}{B(\frac{1}{2},\frac{d}{2})} [d+H(y-m)^2]^{-(d+1)/2}\sqrt{H}, \quad (7)$$

where y is the logarithmic return and B(.,.) is the beta function. The mean of y is equal to m for d>1, and the variance is equal to $H^{-1}d/(d-2)$ for d>2. If d=1, the Student's t distribution is the Cauchy distribution, and therefore the mean and variance do not exist. The advantage of Student's t distribution is that the classical central limit theorem is still applicable. Therefore, price changes converge to the normal distribution as the interval of time over which price changes are measured increases. Another important implication is that it can account for the fat tails observed empirically in stock price changes. Also, it can account for the cluster of price changes around the mean if it is properly standardized. This is achieved by dividing the deviations of y from its mean by its standard deviation rather than the square root of the scale parameter H [see Blattberg and Gonedes (1974)].

Blattberg and Gonedes (1974) presented evidence that the Student's t distribution offers a better fit of the daily rates of return of the 30 securities in the Dow-Jones Industrial average over the period 1957-62 than the SPD. Their estimates of the degrees of freedom of the Student's t distribution for most of the securities examined in their work are over 25 for monthly intervals which makes them very close to the normal distribution. Also they pointed out that their results do not "mean that the rates of return do, in fact, follow a Student model. It only indicates that the latter provides a better empirical fit than the stable model. The Student model has fat tails as does the stable model, but converges to normality for larger sums (larger sums of daily rates of return). The stable model does not converge to normality." Their results were supported by Praetz (1972) who offered evidence that the Student's t distribution offers a more accurate representation to the Sydney share price indices than the SPD. However, Tucker and Pond (1988) present evidence that the mixed jump process distribution offers a better fit than either the SPD or the Student's t distribution. The mixed jump distribution models the total asset price changes as having two components, the first is the standard Brownian motion which corresponds to normal rate of changes, and the second is a jump process which corresponds to abnormal rate of changes [see Merton (1976)].

5. Mixture of Normals Distribution (MND) and Subordinated Stochastic Process (SSP)

An alternative to the SPD is the MND. If a variable, y_t follows a MND, it can be expressed as $y_t=x_t z_t$, where z_t is normally distributed with mean zero and variance 1, and x_t is a positive random variable. The distribution of y_t will be a discrete MND if x_t is a discrete random variable, and continuous MND if x_t is a continuous random variable. In addition, specifying a distribution for the variable x_t would lead to a different distribution to y_t. Blattberg and Gonedes (1974) showed that the symmetric SPD and Student's t can also be represented as a MND [see also Mandelbrot and Taylor (1967) and Mandelbrot (1973)]. However, the distribution of y_t conditional on a given realization for x_t is normally distributed.

The MND is based on the notion that changes in speculative prices, conditional on their variances, are i.i.d. random variables which follow a normal distribution. A possible explanation for this model is that information evolves unevenly through time, and so the variance of price changes may be greater during days when information becomes available than those when there is no new information arriving in the market.

Clark (1973) introduced the SSP model with finite variances. Let the sequence P_{t1}, P_{t2},.....,P_t present realizations of a stochastic process at time t. The sequence of P's is indexed by the t's rather than the integers 0,1,2,... The t's are realizations of a stochastic process with positive increments T(t). If T(t) is a positive stochastic process, then a new process can be formed, P(T(t)). The distribution of ΔP(T(t)), the series of daily price changes, is subordinate to the distribution of ΔP(t), the price changes on individual trades. These latter constitute the evolution of the stock price P(T(t)), and T(t) is the directing process which determines the speed of evolution. T(t) can be regarded as a clock which evolves according to economic time rather than calender time[10]. It then follows that specifying stochastic processes for each of ΔP(t) and T(t) would affect the distribution of the subordinate process ΔP(T(t)). Clark (1973) shows that if P(t) follows a normal distribution with independent increments, directed by T(t) which follows a lognormal distribution with independent increments, then ΔP(T(t)) will follow a lognormal-normal distribution.

[10]See Stock (1987 and 1988)

In Clark's model the distribution of daily price changes, $\Delta P(T(t))$, is subordinate to the distribution of price changes on individual trades $\Delta P(t)$, and is directed by the random rate of information arrival to the market on a single day $T(t)$. It is assumed that there are more trades on the day on which $T(t)$ is high, and fewer when it is low. In other words, prices evolve at a higher rate whenever new information arrives at the market. Clark (1973) used the cumulative volume of trade up to time t as a proxy to the directing process $T(t)$. His results show that the distribution of daily price changes conditional on volume of trade is less leptokurtic and closer to the normal distribution than the unconditional distribution. Clark's model can also be regarded as daily price changes following an MND where the amount of information arriving to the market on a single day is the mixing variable. Empirical evidence in favour of the mixture of normals distribution is also provided by Morgan (1976), Epps and Epps (1976), Westerfield (1977), Tauchen and Pitts (1983) and Harris (1986).

Another area of application for the MND involves modelling of outliers as contaminated data. Thus, "observations are generated by mixture of two normal distributions, one of which has a small weight but a large variance, and is considered as a random contaminator" [So (1987)]. The probability density function for this distribution is given by:

$$f_{a,h}(y) = (1-a)\left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{-(y-\mu)^2/2\sigma^2} + a\left(\frac{1}{h\sigma\sqrt{2\pi}}\right) e^{-(y-\mu)^2/2(h\sigma)^2}, \quad (8)$$

where y is the logarithmic returns with mean μ, a is the fraction of the wider normal distribution, and h is the multiplier of the standard deviation parameter, σ, of the narrow normal distribution. The difficulty in using this discrete MND lies in figuring out the value of a. In a similar spirit, one can argue that the distribution of price changes can be a mixture of three normal distributions, each depending on the motive of traders at a particular time. Three types of traders with different motivations can be defined: information traders who have private and valuable information, liquidity traders who would like to smooth consumption over time, and noise traders who act as if they had information[11]. However, the actual number of distributions in the mixture of normals is an empirical question.

Kon (1984) investigated the number of distributions in the mixture of normals by using the likelihood ratio test to select the number of normal distributions in the range 1 to 5 for the sample of 30 Dow-Jones stocks. He found that a mixture of four normal distributions fits 7 stocks, a mixture of three normal distributions for 11, and a mixture of two normal distributions for the remaining 12 stocks. Booth and Glassman (1987) used the method of Quandt and Ramsey (1978) to estimate a

[11] Such a definition of noise traders is not consistent with rationality. However, noise traders can be defined as people who mimic the information traders in order to influence price changes or to limit the market impact of their orders. One valuable piece of information they have is their knowledge that they do not have information, see Black (1986).

mixture of two distributions for four exchange rates. They also compared the fit of the mixture distribution to the fit of the normal, SPD and Student's t. They concluded that both the Student's t and the mixture of two normals produced similar and better results than the SPD and the normal distribution. They also estimated a in equation 8 and found it to be 25% for the Canadian dollar and German mark and 45% for the British pound and Japanese yen. The value of h in equation 8 takes the value 2.30 for the Canadian dollar, 2.76 for the German mark, 3.63 for the British pound and 4.84 for the Japanese yen.

6. ARCH Models

Another application of the mixture of normals distribution was proposed by Engle (1982), in his Autoregressive Conditional Heteroscedasticity (ARCH) model. In an ARCH model, observed price changes are unconditionally distributed as a MND, and have a conditional normal distribution based on its past realizations. ARCH models can account for the fat tails observed in returns distributions as well as the clustering of volatility observed initially by Mandelbrot (1963). Mandelbrot reported that although price changes seem to be independent of each other, there is a tendency for large (small) price changes to be followed by large (small) price changes with unpredictable sign. This volatility cluster is apparent when the FT-ALL price changes are plotted against time (figure 3). The figure clearly supports Mandelbrot's observation of volatility clustering. There are some periods which are characterized by larger price changes than is typically the case.

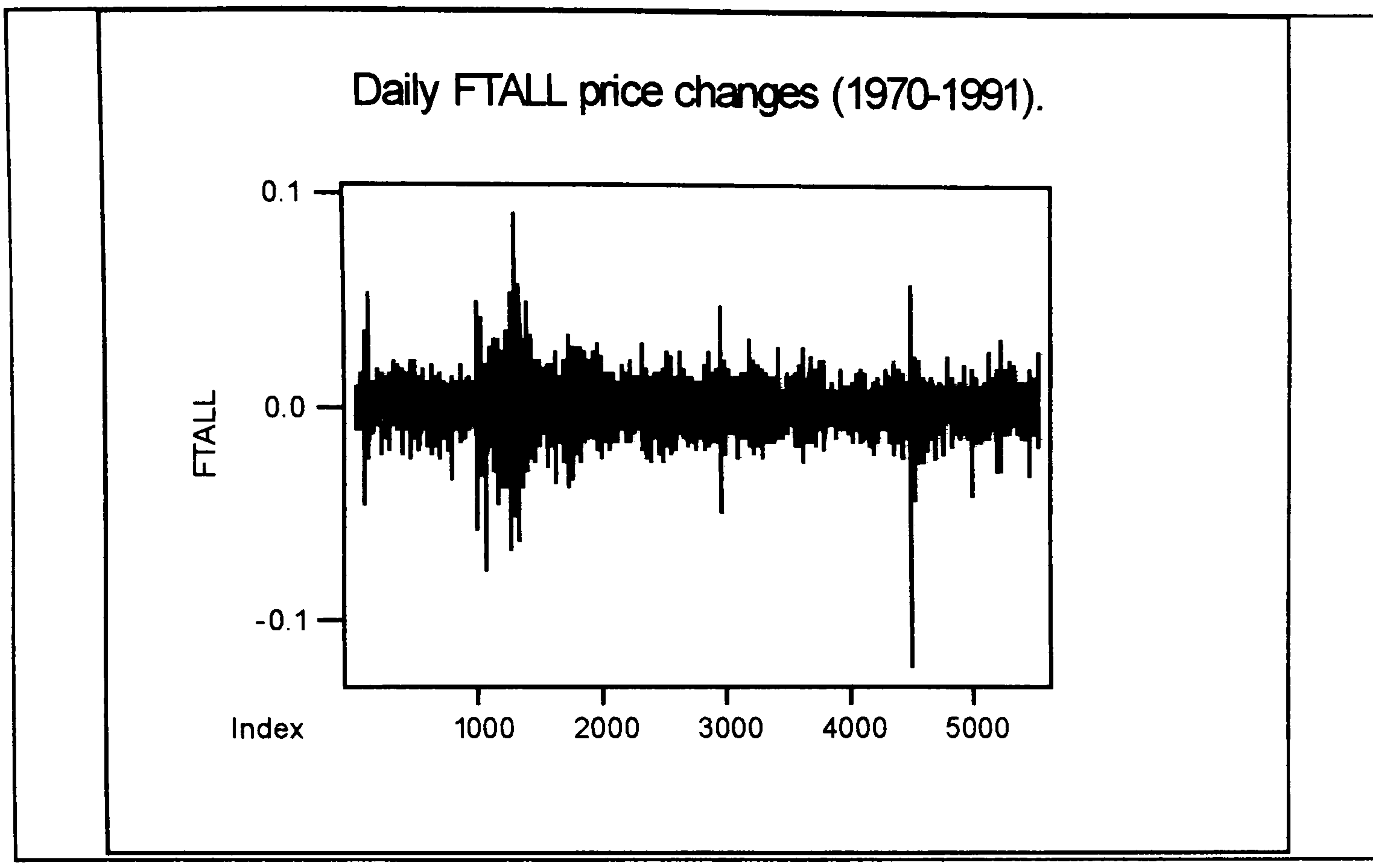

Figure 3

Bollerslev (1986) generalized the ARCH model of Engle to the GARCH model on the same lines as the extension of AR models to ARMA models. This generalization allows for low order GARCH models to capture the long memory in the conditional volatility of stock price changes observed empirically. The GARCH model of orders p and q for a variable u_t, denoted as GARCH(p,q), can be described as follows:

$$u_t = \sigma_t z_t, \tag{9}$$

$$\text{where } z_t \sim iid\ D(0,1), \tag{10}$$

and

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2. \qquad (11)$$

Note that u_t is the outcome of the conditional volatility of u_t, forecastable from its past, multiplied by the realization of an i.i.d. random variables, z_t, with a distribution function D. Thus, the conditional distribution, given the history of u_t, is D with mean and variance parameters 0 and σ_t^2. Further, the conditional volatility, σ_t^2, is a function of the squared past values of u_t, and past conditional volatilities, σ_{t-j}^2. To ease the identification of the order, p and q, of the model, equation 11 can be rewritten as follows :

$$u_t^2 = \alpha_0 + \sum_{i=1}^{m} (\alpha_i + \beta_i) u_{t-1}^2 - \sum_{j=1}^{p} \beta_j v_{t-j} + v_t, \qquad (12)$$

where

$$v_t = u_t^2 - \sigma_t^2 \ , \ m = \max(p,q) \quad \alpha_i \equiv 0 \ for \ i > q \ , \ \beta_j \equiv 0 \quad for \quad j > p. \qquad (13)$$

It then follows that the autocorrelations and partial autocorrelations of u_t^2 will mimic the behaviour of the same functions for an ARMA(m,p) process with autoregressive parameters $\alpha_i + \beta_i$, i=1,...,m, and moving-average parameters $-\beta_j$, j=1,...,p [see

Bollerslev (1988)]. Thus, the standard Box and Jenkins methodology (1976)[12] can be used to identify the orders of the GARCH(p,q) model [see Bollerslev (1986) pp. 313, Bollerslev (1987) pp. 544, Engle and Bollerslev (1986) pp. 24, Najand and Yung (1991) pp. 615, and Balke and Fomby (1992) pp. 16]. Thus, for example, under the null hypothesis of no GARCH effects, the sample autocorrelations for u_t^2 are asymptotically normal with mean 0 and variance 1/T, and so this hypothesis can be tested routinely.

The estimated autocorrelations and partial autocorrelations can be used to identify a tentative GARCH(p,q) in the usual way. For example, the sample partial autocorrelations of a GARCH(0,q) model should show signs of a cut-off point at lag q. They can also be used to identify any model inadequacy and suggest an alternate specification to the GARCH model [see Akgiray (1989) pp. 70]. This can be done by checking the sample autocorrelations and partial autocorrelations of the squared residuals from the fitted GARCH(p,q).

There are also automatic model identification techniques which can be used to identify the orders of the GARCH model such as the Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC)[13]. As noted by Maddela (1992) the serial correlation pattern of the residuals should also be checked for any model inadequacy. This is because these automatic techniques do not ensure that the

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[12]See Cuthbertson *et al.* (1992), pp. 93-94.

[13]See Mills (1990), pp. 138.

residuals of the model are uncorrelated. The BIC and AIC criteria are similar to the adjusted R-squared or minimum variance of the residuals criteria. They choose the model which balances the minimum variance of the residuals, regardless of whether the residuals are uncorrelated or not, with the number of parameters in the model.

A probability density function is needed to specify the stochastic process z_t in equation 9. The normal distribution is one possibility, and was the first one suggested by Engle (1982). Recent empirical findings, however, suggest that the GARCH model with normal density cannot fully account for the leptokurtosis of stock price changes, [see Baillie and Bollerslev (1989), Bollerslev *et al.* (1992), Bera and Higgins (1992) and Diebold and Lopez (1994)]. Bollerslev (1987) suggests the use of the Student's t distribution to represent innovations in the GARCH model. The advantage of using the Student's t is that it accommodates the normal distribution as a limiting form. Bollerslev estimated the GARCH model with conditional Student's t distribution for stock price indexes and exchange rates. He found that the theoretical kurtosis derived from the estimated model was very close to the sample kurtosis of the residuals, suggesting that the Student's t distribution fitted the data adequately.

Nelson (1991) introduced the Exponential GARCH model to account for a possible leverage effect in stock returns, i.e. possible effects of negative past surprises on conditional volatility. He also used a conditional generalized error distribution (GED) to represent the innovations in the EGARCH model. Again, the GED accommodates the normal distribution, and allows for distributions with either thicker or thinner tails than the normal. He found the tails of the GED to be not thick enough to account for the kurtosis of the data. Baillie and Bollerslev (1989) and Bollerslev *et al.* (1993) present evidence that the Student's t offers a better fit than the GED to exchange rates and stock indices. In contrast, Taylor (1994) presented evidence that the GED is superior to the Student's t distribution for the DM/$ returns. Other conditional distributions which have been employed with ARCH related models include a stable Paretian distribution (McCulloch (1985)), a normal poisson distribution (Jorion (1988)), and normal-lognormal mixture (Hsieh (1989)).

There is little doubt that GARCH models offer good approximations for the underlying stochastic process generating price changes [see Bera and Higgins (1992), Bollerslev *et al.* (1992), Bollerslev *et al.* (1993), and Diebold and Lopez (1994)]. Most empirical results suggest that volatility persistence, as measured by the GARCH model, is substantial in financial data [see Bollerslev (1987), Baillie and Bollerslev (1989), Engle and Mustafa (1992) and Bollerslev *et al.* (1993)]. It was on this basis that Engle and Bollerslev (1986) introduced the Integrated-GARCH (IGARCH) model in which shocks to volatility do not die out over time. In other words, a shock to the conditional volatility remains important for forecasting the conditional volatility for all future horizons.

Another feature of GARCH models is that convolutions of their unconditional distribution appear to converge empirically to the normal distribution[14]. Diebold (1988) demonstrated that if a variable follows an AR-ARCH or ARCH process, the limiting distribution of its sum is an unconditional normal distribution. Empirically, Diebold found that the monthly nominal log dollar spot exchange rates against seven currencies, are very close to the normal distribution, and the ARCH effects although still present in the data are considerably less than in the daily series. Ghose and Kroner (1995) generalized Diebold's results to the case where a variable follows an IGARCH process [see also Groenendijk *et al.* (1995)]. They showed that its limiting distribution is a non-normal SPD with a characteristic exponent less than two. Their empirical results indicate that under aggregation, the characteristic exponent increased to two for the stationary GARCH process, while it remained constant for IGARCH process. As noted by Diebold and Lopez (1994), "This seems to bode poorly for the IGARCH model, because actual series displaying GARCH effects seem to approach normality when temporally aggregated."

Another class of models were then proposed to accommodate the possibility that a shock to the conditional volatility persists for a longer period than in the GARCH model, but eventually dies out in contrast to the IGARCH model. In the Long Memory Stochastic Volatility (LMSV) model of de Lima and Crato (1994)[15] and Fractionally Integrated GARCH (FIGARCH) model of Baillie, Bollerslev, and

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[14]The term convolution refers to the distribution of the sum of random variables, see Davidson (1994), pp.161.

[15]See also Harvey (1993a)

Mikkelsen (1993), a shock to the conditional volatility dies out at a slow hyperbolic rate rather than the exponential rate of the GARCH model.

7. Nonlinear Dependence in Stock Price Changes.

The models discussed in this chapter assume that stock price changes, either unconditionally (e.g. Cauchy) or conditionally (e.g. ARCH), are identically, and independently distributed (i.i.d.) random variables. As noted by Hsieh (1991), it is difficult to interpret the distribution of stock price changes and its implications if the i.i.d. assumption is violated. For example, if stock price changes are i.i.d. and follow the Cauchy distribution, then the probability of observing a large price change such as that associated with the 1987 crash is small but nonzero. However, such an interpretation is so dependent on the assumption of i.i.d. that it becomes important to examine whether stock price changes can be described as i.i.d. random variables.

Brock *et al.* (1987) developed a test of the null hypothesis of i.i.d. for a univariate time series against an unspecified alternative. Generally referred to as the BDS test it has been shown to be robust to the nonexistence of fourth moments which may characterises stock price changes[16]. Hsieh (1991) points out that the robustness of the BDS test to the nonexistence of fourth moments is one of the advantages of the test over other tests of nonlinearity such as those proposed by Tsay (1986b) and Hinich and Patterson (1985).

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[16]See Brock and de Lima (1995), pp. 23 and Hsieh (1991), pp. 186.

The BDS test makes use of the idea of the correlation integral. Let $\{a_t: t=1, \ldots. T\}$ be a sequence of observations which are i.i.d. Form m-dimensional vectors, a_t^m, to be the set of adjacent values $a_t^m = (a_t, a_{t+1}, \ldots, a_{t+m-1})$. These are called "m-histories". A pair of such vectors are within ε of each other if this is true for each pair of corresponding terms [see Granger and Teräsvirta (1993)]. Define the correlation integral, $C(\varepsilon,m)$, as[17]

$$C(\epsilon,m) = \frac{\sum_{1 \le t \ne s \le T} I(a_t^m, a_s^m; \varepsilon)}{T\,(T-1)} \tag{14}$$

where I(.) is the heaviside step function which maps positive arguments into one, and nonpositive arguments into zero [see Ramsey *et al.* (1990)]. Thus $I(a_t^m, a_s^m; \varepsilon)$ is equal to one if $\|a_t^m - a_s^m\| < \varepsilon$, and zero otherwise[18]. $\| \ \|$ is the sup norm. The correlation integral is a measure of total number of pairs (a_t^m, a_s^m) that are within ε distance of each other. It is a measure of the concentration of m consecutive observations, a_t^m. Brock *et al.* (1987) show that under the null hypothesis $\{a_t\}$ is i.i.d., $C(\varepsilon,m) \to C(\varepsilon,1)^m$ with probability one, as $T \to \infty$. A simple interpretation of that can be found in de Lima (1995b)[19]. Since $C(\varepsilon,m)$ is an estimator of $\Pr\{|a_t^m - a_s^m| < \varepsilon\}$, and $C(\varepsilon,1)$ is an estimator of $\Pr\{|a_t - a_s| < \varepsilon\}$, under the null hypothesis of i.i.d., $\Pr\{|a_t^m - a_s^m| < \varepsilon\} \approx (\Pr\{|a_t - a_s| < \varepsilon\})^m$. This is based on the idea that the joint

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[17] See Brock and Baek (1991).

[18] See Brock *et al.* (1991), pp. 42.

[19] See also Scheinkman (1990) pp. 41 and Brock and Baek (1991) pp. 698

distribution is equal to the product of the marginal distributions for i.i.d. random variables.

The BDS test statistic is

$$S(\varepsilon,m) = \sqrt{\frac{T}{\sigma^2(\varepsilon,m)}}\,[C(\varepsilon,m)-(C(\varepsilon,1))^m] \tag{15}$$

for some single choice of m and ε. $\sigma^2(\varepsilon,m)$ is the asymptotic variance of $T^{1/2}(S(\varepsilon,m))$ under the null of i.i.d[20]. Under the null hypothesis of i.i.d., $S(\varepsilon,m)$ has a limiting standard normal distribution.

Brock *et al.* (1991) used monte carlo methods to evaluate the choice of m and ε on the asymptotic normality of $S(\varepsilon,m)$. Their results suggest that m should not be more than 5, and ε should be in the range of 0.5 to 2 times the standard deviation of the series under study. Brock *et al.* (1991) warned against relying on asymptotic normality for values of T/m of less than 200 observations. Their results indicate that the BDS statistic fails to achieve normality at small ε for higher dimensions because of the few observations available. In summary, their results suggest that asymptotic normality of $S(\varepsilon,m)$ holds well for sample sizes of at least 1000 observations, and for values of ε between 1/2 and 2[21].

Hsieh (1991) used Monte Carlo simulations in an extensive study to examine

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[20] The formula for computing $\sigma^2(\varepsilon,m)$ can be found in Hsieh (1989).

[21] See Brock *et al.* (1991), pp. 48-53.

the finite sample distribution of the BDS statistic. He conducted three sets of simulations. In the first set, the objective was to examine how well the asymptotic distribution approximates the finite sample distribution of the BDS statistic. The simulation experiment was based on generating 1000 i.i.d. observations, computing the BDS, and repeating the procedure 2000 times. He used different density functions to generate the i.i.d. observations, among them the Normal, Student t, and Cauchy. He used $m=2$ and 5, and $\varepsilon=0.25$, 0.5, 1, 1.5, and 2, in computing the BDS statistic. He then computed the percentage of the BDS statistics, in the 2000 replications, rejecting the null hypothesis of i.i.d. when it is true. The results show that the rejections are far less than 5% of the replications[22].

The second set of simulations conducted by Hsieh (1991) measured the ability of the BDS statistic to reject the null of i.i.d. for a number of time series models. The results suggest that the BDS statistic can easily detect linear dependence for autoregressive or moving average models of order one except in the case when the first order autocorrelation is less than 0.2. This result is important since it is well documented that financial data exhibit a significant small first order autocorrelation [see Bollerslev *et al.* (1993)]. Accordingly, linear dependence in the data should be removed before employing the BDS statistic to test for nonlinearity.

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[22]This is based on the 5% level of significance. However, this conclusion is not valid for two other distributions used in Hsieh (1991), which are the Uniform and Bimodal distributions. Hsieh (1991), however, notes that "very little financial data look like these distributions." For more details, see table 1 and 2 in Hsieh (1991), pp. 1850-1.

The rest of the results of Hsieh (1991) suggest that BDS statistic has no trouble in rejecting the null of i.i.d. for regime changes, nonlinear moving average, threshold autoregressive, and chaotic (Mackey-Glass equation) process. With regard to ARCH type models, Hsieh (1991) results indicate that BDS statistic can more easily detect the ARCH and GARCH models than the EGARCH models.

The third set of simulations in Hsieh (1991) addressed the issue of the asymptotic distribution of the BDS statistic when applied to the residuals, from an autoregression of order one, moving average of order one, nonlinear moving average, GARCH or EGARCH models. The results indicate that the asymptotic distribution still approximates the finite sample distribution of the BDS statistic with the same degree of accuracy for the autoregression of order one, the moving average of order one, and the nonlinear moving average. With regard to the GARCH and EGARCH residuals, the results show that the BDS statistic may reject too infrequently.

The results of Hsieh (1991) are consistent with the simulation findings in Brock *et al.* (1991) which suggest that the BDS test has power against wide class of uncorrelated but not independent processes as well as against nonstationary alternatives. Also consistent with Hsieh (1991) are the results reported by Brock *et al.* (1991) indicating that the BDS test is still powerful when it is applied to the residuals from a wide class of models, except for those of the GARCH variety[23].

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[23]See Brock *et al.* (1991), pp. 53-81.

The empirical results of applying the BDS to stock price changes reject the null hypothesis of i.i.d [see Hsieh (1991) and de Lima (1995a)]. As noted by Brock and de Lima (1995), "the main issue in finance does not seem to be the inability to detect departures from linearity because rejections of linearity are so frequent. The main issue is to find reasons for the rejections". As mentioned earlier, the BDS test is robust to moment condition failure. There are two possible causes for the rejections of i.i.d., nonstationarity or conditional heteroscedasticity[24].

Hsieh (1991) presents results that conditional heteroscedasticity rather than nonstationarity is the main cause of the rejection of the i.i.d. hypothesis. In UK based studies, Paudyal *et al.* (1993) found that EGARCH(1,1) model considerably reduce, but do not fully remove, the nonlinear dependence in hourly FTSE returns. Similar evidence was found by Abhayanker *et al.* (1995b) using the FTSE returns over frequencies of 1, 5, 15, 30, and 60 minutes[25]. However, de Lima (1995a) presents evidence indicating that simulated data generated by GARCH models fail to explain the behaviour of the BDS test for the S&P500 index returns. de Lima (1995a) concludes his study that "nonstationarities should not be ruled out as an explanation for the behaviour of stock returns.... The test [BDS] statistics clearly indicate the existence of a shift in the distribution of stock returns around the stock market

[24]The rejection of i.i.d. is also consistent with the view that stock price changes are generated by nonlinear stochastic systems or economic models with chaotic dynamics, see Baumol and Benhabib (1989) and Hsieh (1991).

[25] The study of Abhayanker *et al.* (1995b) used the GARCH(1,1) model rather than the EGARCH(1,1) model used in the Paudyal *et al.* (1993) study.

"crash" of October 1987".

The BDS statistic is applied to the daily FT-ALL price changes under investigation in this chapter. I examine whether conditional heteroscedasticity is the main cause of any nonlinear dependence observed in price changes. The difference between the current study and the studies of Paudyal *et al.* (1993) and Abhaynker *et al.* (1995b) in the UK market is in the use of the adjustment recently suggested by de Lima (1995b), to the residuals from a GARCH model before applying the BDS test statistic. As is well known[26], the asymptotic distribution of the BDS test statistic cannot be used when the test is applied to the residuals from a GARCH model. de Lima (1995b) proves that the asymptotic distribution remains valid if it is applied to the natural logarithm of the squared residuals form a GARCH model.

Since the BDS test statistic is applied to linearly filtered data, the linear dependence in price changes is removed using an autoregressive process with the lag truncation length set up to the one which has the minimum value for the Bayesian Information Criterion (BIC). The BIC value is calculated for truncation length in the range from 0 to 10. Table 3 has the BIC values for different lag truncations.

Table 3: the BIC values for the Autoregressive process at lag truncations p=0,1,2,...,10

P	0	1	2	3	4	5	6	7	8	9	10
BIC	.138	.105	.106	.106	.107	.108	.110	.112	.113	.113	.112

[26]See Hsieh (1991) pp. 1870, and Brock and de Lima (1995) pp.16.

According to the BIC criterion the lag truncation length of the autoregressive process should be set up to 1. The BDS test statistic is then applied to the linearly filtered price changes where m is set in the range from 2,..,10, and ε is set equal to 0.5, 1, and 1.5. The values of m and ε were chosen because of the simulation results of Brock *et al.* (1991) which show that the asymptotic normality of the BDS statistic can be relied on for values of ε in the range 0.5 to 2 times the standard deviation of the series under study, and any values for m as long as T/m is more than 200 observations. The results are reported in table 1 in the appendix to this chapter. The results strongly reject the null hypothesis of i.i.d. for the linearly filtered price changes.

Lee *et al.* (1993) raised the issue of whether the detection of nonlinearity in financial time series could be due to either neglected nonlinear structure in the mean or ARCH effects. One way to test whether conditional heteroscedasticity is responsible for the rejection of i.i.d. hypothesis is to apply the BDS test statistic to the residuals from a GARCH model [see Paudyal *et al.* (1993) and Abhayanker *et al.* (1995b)]. The trouble is that we cannot depend on the asymptotic distribution of the BDS statistic when applied to the residuals from a GARCH model [see Hsieh (1991)]. Hsieh (1991) overcomes this problem by using critical values of the BDS statistic for simulated EGARCH process[27]. However, as noted earlier, de Lima (1995b) proves that the asymptotic distribution of the BDS statistic remains valid if the test is applied to the natural logarithm of the squared residuals from a GARCH

[27]The simulation is based on 2000 replications, each with 1000 observations.

model. This is because the BDS statistic is valid if it is applied to data generating process that is additive in the error term [see de Lima (1995b)]. The GARCH process models the error term in a multiplicative form, $u_t=\sigma_t z_t$, where u_t is a random variable following the GARCH process, z_t is i.i.d. random variable, and σ_t is the conditional standard deviation. The standardized residuals from this model are $z_t=u_t/\sigma_t$. It follows that
$\ln(z_t^2)=\ln(u_t^2)-\ln(\sigma_t^2)$, and therefore the asymptotic distribution of the BDS statistic remains valid if it is applied to $\ln(z_t^2)$.

I examine whether conditional heteroscedasticity is responsible for the rejection of the i.i.d. hypothesis by applying the BDS test to the residuals, and the natural logarithm of the squared residuals from a GARCH(1,1) model. As mentioned earlier, the GARCH(1,1) process has been shown to provide an adequate representation for stock market prices [see Akgiray (1989) and Bollerslev *et al.* (1992)]. The results are provided in tables 2 and 3 in the appendix to the chapter.

The results of applying the BDS test to the residuals from the GARCH(1,1), table 2, suggest that the null hypothesis of i.i.d. should be "accepted" at the 5%. Therefore, it seems that conditional heteroscedasticity is the cause of the rejection of the null of i.i.d. for the linearly filtered price changes. Table 3 which details the results from applying the BDS test to the natural logarithm of the squared residuals from the GARCH model also supports this interpretation. However, there are some differences between the results in tables 2 and 3. The null of i.i.d. is "accepted" for every dimension, m, and different values for ε in table 2, while it is rejected for m equal 9 and 10, and $\varepsilon=0.50$ in table 3. Although the results are fairly similar whether the BDS test is applied to the residuals or the natural logarithm of the squared residuals, it is better, in summary, to apply the de Lima (1995b) adjustment to the residuals since the asymptotic distribution of the BDS statistic is not valid for the residuals from a GARCH model[28]. Finally, the results suggest that if we are interested in modelling nonlinearity, our attention should be on conditional heteroscedasticity rather than conditional mean dependence.

8. Nonstationarity in Stock Price Changes

Another plausible explanation for the source of the thick tails of the distributions of stock price changes is the possibility that long time series of price changes are not stationary. Hsu *et al.* (1974) offered evidence that shifts in the scale parameter can cause the behaviour of stock market prices to be consistent with the stable Paretian distribution. They studied whether monthly returns on 20 USA firms from January 1926 to December 1960 confirm well to the SPD. They estimated the characteristic exponent and scale parameters using Fama and Roll's (1971) method. Using a χ^2 goodness of fit test, they did not reject an SPD for the returns for 19 out of the 20 stocks studied. They said that "it is generally agreed that the stock market has been much more stable since World War II than it was prior to this period".

[28]The author extended the analysis of nonlinearity and conditional heteroscedasticity reported in this section to a sample of common stocks in the UK market. The results are similar to those reported here that conditional heteroscedasticity is the main cause of the rejection of the i.i.d. hypothesis [see Omran (1996a)].

They investigated this possibility by dividing the sample into two periods, the years prior to 1941 and those from 1941 to 1960, and estimated the α and the scale parameter for the 20 firms for the two sub-periods. They found that α increased from a pre-war value of 1.4 to a post-war value of 2, and the scale parameter decreased by 40% from its pre-war value. They then used the standardized range test, range to standard deviation, to test for normality for the 20 stocks for the two sub-periods. Whilst monthly returns could be reasonably described as normally distributed in the latter period, this was not the case for the former period. On this basis it was concluded that the probability distribution of stock returns is non-stationary in the scale parameter over time, and that within sub-periods of homogenous activity, the normal or mixture of normals distribution provides a reasonable approximation to the data.

In terms of ARCH models, Diebold (1986) suggested that the apparent need for IGARCH models may arise because of shifts in regimes of the unconditional variance. Lamoureux and Lastrapes (1990a) show that the apparent persistence in variance may be a result of a misspecification in the variance equation. They demonstrate that volatility persistence as measured by the GARCH model is reduced when dummy variables are introduced into the variance equation to take account of structural changes in the unconditional variance. Similarily, Siomonato (1992) offers evidence that by allowing for structural breaks in the unconditional variance the degree of volatility persistence in the GARCH process is reduced. Moreover, Simonato's (1992) results suggested that GARCH effects became statistically insignificant when structural breaks were allowed in the unconditional variance. de Lima (1995a) provides evidence that suggests there was a shift in the distribution of stock returns around the 1987 crash. Also, Pagan and Schwert (1990a) and Loretan and Phillips (1994) provide evidence against the assumption of covariance stationarity, suggesting that the unconditional variance of stock returns is not constant over time. The hypothesis of constant unconditional variance for the FT-ALL price changes will be dealt with in more detail in the next chapter.

9. Conclusions

This chapter has reviewed the major probability distributions proposed for the analysis of stock price changes in the finance literature. An exploratory data analysis has been carried out to investigate the properties of the UK FT-ALL price changes and whether these properties are consistent with the evidence largely drawn from the US. It was found: (a) the distribution of the FT-ALL price changes is leptokurtic, (b) the FT-ALL price changes are not independent of each other which is likely to contribute to an even greater departure from i.i.d. normality. The latter finding is reached using the Fama and Roll (1968) method for estimating the characteristic exponent for i.i.d. stable random variables. Unfortunately, the Fama and Roll (1968) method suffers from two major drawbacks: non availability of standard errors, and the sample size decreases as the sum of daily price changes increases. Therefore, the null hypothesis of i.i.d. was tested using the recent test proposed by Brock *et al.* (1987).

The null hypothesis of i.i.d. is rejected for linearly filtered FT-ALL price changes. It is found that the main cause of the rejection of i.i.d. is conditional heteroscedasticity. The adjustment to the residuals form a GARCH model proposed by de Lima (1995b) is used before applying the BDS test statistic. The BDS test applied to the adjusted residuals suggests that the i.i.d. hypothesis should be rejected at the 5% level for m equal to 9 and 10, and ε equal to 0.50. Apart from these values, the results are largely consistent with those obtained from applying the BDS test to the GARCH residuals without adjustment.

The rest of the thesis deals with three empirical issues in more detail. The issue of structural changes in the probability distribution of the FT-ALL price changes is examined in chapter 3. This is important, since all the probability distributions discussed in this chapter are based on the assumption of stationarity. The volatility-volume relationship for individual stocks in the UK market is investigated in chapter 4. Finally, the GARCH model, with the Student's t and generalized error distributions, is used to model the FT-ALL price changes in chapter 5.

Appendix

Table 1: The BDS statistic applied to the linearly filtered price changes. m is the m histories, ε is 0.5, 1, and 1.5 times the standard deviation of the series, and BDS is the BDS statistic computed using equation 15.
* indicates significance at 5% level.

m	ε	BDS
2	.50	14.791*
3	.50	20.064*
4	.50	23.426*
5	.50	27.335*
6	.50	31.561*
7	.50	36.936*
8	.50	42.427*
9	.50	47.634*
10	.50	54.637*
2	1.00	17.534*
3	1.00	22.357*
4	1.00	25.242*
5	1.00	28.109*
6	1.00	30.836*
7	1.00	33.962*
8	1.00	37.131*
9	1.00	40.814*
10	1.00	45.056*
2	1.50	21.246*
3	1.50	26.102*
4	1.50	28.595*
5	1.50	30.579*
6	1.50	32.314*
7	1.50	34.116*
8	1.50	35.852*
9	1.50	37.752*
10	1.50	39.860*

Table 2: The BDS statistic applied to the residuals from a GARCH(1,1) for the price changes. m is the m histories, ε is 0.5, 1, and 1.5 times the standard deviation of the series, and BDS is the BDS statistic computed using equation 15.
* indicates significance at 5% level.

m	ε	BDS
2	.50	-.917
3	.50	.494
4	.50	.749
5	.50	1.350
6	.50	1.405
7	.50	1.874
8	.50	1.568
9	.50	.828
10	.50	1.574
2	1.00	-.525
3	1.00	.731
4	1.00	1.142
5	1.00	1.393
6	1.00	1.348
7	1.00	1.570
8	1.00	1.510
9	1.00	1.383
10	1.00	1.432
2	1.50	-.132
3	1.50	.847
4	1.50	1.174
5	1.50	1.357
6	1.50	1.207
7	1.50	1.236
8	1.50	1.030
9	1.50	.850
10	1.50	.814

Table 3: The BDS statistic applied to the natural logarithm of the squared residuals from a GARCH(1,1) for the price changes. m is the m histories, ε is 0.5, 1, and 1.5 times the standard deviation of the series, and BDS is the BDS statistic computed using equation 15.
* indicates significance at 5% level.

m	ε	BDS
2	.50	.133
3	.50	.625
4	.50	.976
5	.50	1.068
6	.50	1.127
7	.50	1.277
8	.50	1.634
9	.50	2.294*
10	.50	2.565*
2	1.00	.509
3	1.00	.840
4	1.00	.906
5	1.00	.971
6	1.00	1.114
7	1.00	1.328
8	1.00	1.419
9	1.00	1.450
10	1.00	1.389
2	1.50	.406
3	1.50	.798
4	1.50	.711
5	1.50	.614
6	1.50	.749
7	1.50	.907
8	1.50	1.021
9	1.50	1.020
10	1.50	.869

Chapter 3

Testing for Covariance Stationarity in the UK ALL Equity Index

1. Introduction

The possibility that the fat tails property of stock price changes could be partly due to non-stationarity in its variance was discussed in chapter 2. The evidence of Pagan and Schwert (1990a) and Loretan and Phillips (1994) indicates that US stock returns series cannot be assumed as covariance stationary. Pagan and Schwert (1990a) proposed several non-parametric tests for covariance stationarity, and applied them to the US stock returns. These tests were subsequently criticized by Loretan and Phillips (1994) on the ground that they rely on the assumption of finite fourth unconditional moments. Loretan and Phillips (1994) raised the possibility that the finding of Pagan and Schwert (1990a) "is merely the byproduct of a "thick tail" phenomenon in the data generating process, or is indeed due to failure of covariance stationarity."

Loretan and Phillips (1994) provided an asymptotic theory for tests of covariance stationarity which considers the possibility of fourth moment condition failure. Their test is appealing given the findings of fat tails in financial time series [see chapter 2]. The test is the same as the post-sample prediction test of Pagan and Schwert (1990a), except that the critical values for the former are derived for the cases when the fourth moment is infinite.

This chapter investigates the proposition that the unconditional variance of the UK FT-ALL price changes is constant over time, using the test for variance constancy recently suggested by Loretan and Philips (1994). It also examines the plot of the unconditional variance of price changes computed from rolling periods of the data. It is well documented that stock price changes exhibit a significant small first order autocorrelation and highly significant ARCH effects[1]. Hamilton (1994) gives the unconditional variance of a variable which follows the ARMA and/or ARCH process and requires the estimation of the underlying process. The AR(1)-GARCH(1,1) model is used to approximate the underlying stochastic process of FT-ALL price changes[2]. The residuals of the model are checked for model adequacy using the Mcleod and Li (1983) test statistic, and the constancy of the unconditional variance derived from the model is inspected graphically.

The graphical approach has the advantage of pointing out possible periods of exceptional circumstances in the data. However, it has the disadvantage of not providing a formal test statistic that can be used to judge the constancy of the

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[1]See for example, Bollerslev *et al.* (1993), Akgiray (1989) and Engle (1995).

[2]Recall that the AR(1) model was chosen using the BIC criterion in chapter 2. In addition, the results of chapter 2 suggest that the GARCH(1,1) model explains most of the nonlinearity of the FT-ALL series.

unconditional variance on an objective basis. Therefore, the intervention analysis of Box and Tiao (1975) is applied to the data to test for the statistical significance of variance shifts in price changes around the exceptional periods identified by the graphical analysis. The intervention model used is flexible in the sense that it allows for a possible shift in the variance, a slow decay in the effect of the shift, and a different level of variance after the intervention. Finally, the Loretan and Philips (1994) test is re-applied to the standardized residuals obtained from the intervention model. The purpose is to test whether the periods identified by the graphical analysis and tested for significance using the intervention model, have a significant impact on our initial decision of "accepting" or rejecting the null hypothesis of constant variance.

The data set used in the analysis is the same one used in chapter 2, which contains 5555 observations for the FT-ALL index during the period from 2/1/1970 to 31/12/1991. As in chapter 2, the term price changes refer to the changes in the natural logarithm of the FT-ALL index.

The results of the empirical work suggest that the null hypothesis of constant variance should be rejected. The graphical analysis offers dramatic evidence of a failure of covariance stationarity around the time of the 1973-74 oil crisis and the 1987 market crash. In fact, most of the outliers of the FT-ALL price changes during the period of study happened around these two periods. The results of the intervention model suggest that there were statistically significant shifts in the variance around the oil crisis and the market crash. The results also indicate that the shift in the variance was greater around the market crash than in the oil crisis. However, it took less time for the shift to die out after the market crash than in the oil crisis. In both cases, the variance eventually returned to its pre-event level, indicating that neither crisis had a lasting effect on the volatility of price changes. The results of Loretan and Philips (1994) test applied to the residuals from the intervention model indicate that the hypothesis of constant variance cannot be rejected. Therefore, it is concluded that the stock market is subject to abrupt changes in volatility during some exceptional periods. Outside these periods, however, stock price changes, as typified by the FT-ALL price index, can be described as being characterized by covariance stationary.

The chapter is organized as follows. The second section discusses the variance constancy test of Loretan and Philips (1994), and applies it to the FT-ALL price changes during the period from 2/1/1970 to 31/12/1991. The third section discusses and applies the graphical inspection methodology. The fourth section applies the intervention analysis of Box and Tiao (1975) to the data. The fifth section applies the Loretan and Philips (1994) test to the residuals from the intervention model. The final section provides some concluding comments.

2. Testing for the constancy of the unconditional variance.

Loretan and Phillips (1994) provided an asymptotic theory for tests of covariance stationarity which considers the possibility of fourth moment condition failure. The Loretan and Phillips test is the same as the post-sample prediction test of Pagan and Schwert (1990a) except that the critical values of the test are derived for the cases when the fourth moment is infinite. The test starts by splitting the sample into two parts according to $n=n_1+n_2$ with $n_1=k_n n_2$. For example if $k_n=1$, then the sample is divided into two equal parts. The null hypothesis of constant unconditional variance in the two parts can be stated as:

$$H_0:\ E\mu_2^{(1)} = E\mu_2^{(2)}\ , \tag{1}$$

where

$$\mu_2^{(1)} = n_1^{-1} \sum_{t=1}^{n_1} u_t^2\ , \tag{2}$$

$$\mu_2^{(2)} = n_2^{-1} \sum_{t=n_1+1}^{n} u_t^2\ . \tag{3}$$

where u_t denote the residuals from an AR(p) process for price changes $\{y_t\}$ given by

$$y_t = \sum_{i=1}^{p} \hat{b}_i y_{t-i} + u_t. \tag{4}$$

As in Loretan and Phillips (1994), the affixes "(1)" and "(2)" are used to signify quantities that correspond to the first and second subera, respectively. Putting $d=\mu_2^{(1)}-\mu_2^{(2)}$, the null hypothesis of constant variance in the two eras can be restated as H_0: $E(d)=0$. The next step is to determine a measure of variation for d to examine whether d is significantly different from zero. The kernel-based estimate of the "long run" variance of u_t^2 is given by

$$\upsilon^2 = \gamma_0 + 2\sum_{j=1}^{L} (1-j/(L+1))\gamma_j. \tag{5}$$

where γ_j is the j-th serial covariance of u_t^2 and L is a suitable lag truncation number. υ^2 is robust to serial correlation in the second moments of u_t^2 (ARCH effects) since they are considered in the computation [see Loretan and Phillips (1994)]. The test statistic for the constancy of the variance in the two eras is

$$V_k(d) = ((1+k_n)\upsilon^2)^{-1/2} n_1^{1/2} d. \tag{6}$$

The critical values for the test depends on the maximal finite moment of the data, denoted as α. In particular whether α exceeds 4, lies in the range [2,4], or is less than 2. If $\alpha > 4$, then $n^{1/2}d$ weakly converges to the normal distribution with mean 0 and variance $(1+k_n)\upsilon^2$. When $2 < \alpha \leq 4$, the test is consistent but with critical values provided by Loretan and Phillips (1994)[3]. When $\alpha < 2$, the data lie in the domain of attraction of the stable distribution with characteristic exponent α.

[3]For details, see Loretan and Phillips (1994) sections 2 and 4.

Therefore, the variance is infinite, and the test statistic is not consistent. This is not surprising since the test is based on comparing the variances in the two eras, which in case of $\alpha < 2$ are poor estimates of the population dispersion[4].

The maximal moment exponent is estimated using the Loretan and Philips (1994) estimators. The left and right tail estimators for α are given by

$$\alpha_L(s) = 1/(s^{-1} \sum_{j=1}^{s} \ln(-u_j) - \ln(-u_{s+1})). \tag{7}$$

$$\alpha_R(s) = 1/(s^{-1} \sum_{j=1}^{s} \ln(u_{n-j+1}) - \ln(u_{n-s})). \tag{8}$$

where u_t's are ordered such that $u_1 < u_2 < \dots < u_n$, and s is some-integer. It is assumed that n is sufficiently large and s/n is small enough that $u_{n-s} > 0$ and $-u_{s+1} > 0$. The asymptotic distribution of $\alpha_L(s)$ and $\alpha_R(s)$ is given by theorem 2 of Hall (1982)[5]:

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[4]Loretan and Phillips (1994) discuss a heterogeneity test between two eras of the data using the scale dispersion coefficient. The reader is also referred to the discussion about the scale parameter as a measure of variation for the i.i.d. stable variables in chapter 2.

[5]A criticism of Loretan and Phillips (1994) estimator is that its standard errors are computed based on the assumption that the data generating process is i.i.d. variables. The simulation results reported in Pagan (1996) indicate that the standard error of α can be larger for data generated from a GARCH process than for i.i.d. variables [see also Brock and de Lima (1995)].

$$(\alpha_s - \alpha)\, s^{1/2} \sim AN(0, \alpha_s^2)\,. \qquad (9)$$

In finite samples, the point estimate and standard errors of α may be affected by the choice of s. Jansen and de Vries (1991) and Loretan and Phillips (1994)[6] conducted monte carlo experiments to examine the influence of s on the statistical inference. The results of both studies suggest that "using too many observations such that some do not belong to the tail, but rather to the centre of the distribution, is more harmful than not using all the available information" [Jansen and de Vries (1991)]. DuMouchel (1983) and Loretan and Phillips (1994), report that s should not exceed 10% of the data to avoid including observations belonging to the centre of the distribution in the sample. Also, Loretan and Phillips (1994) advised that α should be estimated for a variety of values of s.

Loretan and Phillips (1994) left and right tail estimators are used to estimate the maximal moment exponent, α, for the FT-ALL price changes. The maximal moment exponent α is estimated for s equal to 20, 50, 100, 200 and 300. The choice of s is based on Loretan and Phillips (1994) suggestion that s should not exceed 10%

[6]Loretan and Phillips (1994) summarized the results of the simulations conducted by Loretan (1991).

of the data, and α should be estimated for variety of values of s[7].

Table 1 has the estimates of the maximal moment exponent α, and its standard errors for the left and right tails of the residuals from an AR(1) for the FT-ALL price changes[8]. The table also reports the t statistics testing the null hypothesis of the existence of the fourth moment, $\alpha=4$, against the alternative of fourth moment condition failure, $\alpha<4$.

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[7]Mittnik and Rachev (1993) criticise the Loretan and Phillips (1994) estimator on the basis that different values for s produce significantly different values for α [see also Omran (1996b)]. However, Brock and de Lima (1995) report that "for reasonably sized samples, Loretan's (1991) simulations indicate that $\hat{\alpha}_s$ is a robust estimator of α if s does not exceed 10% of the sample size".

[8]As noted earlier, the AR(1) model was chosen according to the BIC criterion in chapter 2. However, Hiemstra and Jones (1995) found that "the difference between the estimates for the AR-filtered and unfiltered series are small and do not affect the conclusion that fourth moments do not exist in the majority of return series." Their study was based on return series of 1,952 ordinary common stocks.

Table 1: Point estimates of the maximal moment exponents for different values of s, where s is the number of order statistics. The t-statistics are for the null hypothesis of finite fourth moment against the alternative of infinite fourth moment.

	Left		Right	
s	α_s	t-stat $H_0{:}\alpha=4$ $H_a{:}\alpha<4$	α_s	t-stat $H_0{:}\alpha=4$ $H_a{:}\alpha<4$
20	2.628 (0.588)	-2.33*	2.875 (0.643)	-1.75*
50	3.199 (0.452)	-1.77*	2.979 (0.421)	-2.43*
100	3.098 (0.310)	-2.91*	3.010 (0.301)	-3.29*
200	2.800 (0.198)	-6.06*	2.838 (0.201)	-5.78*
300	2.793 (0.161)	-7.50*	2.676 (0.154)	-8.60*

Standard errors are in brackets. * statistically significant at 5% level.

The table shows that the point estimates of the maximal moments exponents are all less than 4; they range from 2.676 to 3.01 for the right tail, and from 2.628 to 3.199 for the left tail. The table also shows that the null hypothesis of existence of the fourth moment, $\alpha=4$, is rejected in favour of the alternate hypothesis of fourth moment condition failure, $\alpha<4$. Accordingly, it can be concluded that the fourth moment of the FT-ALL price changes does not exist. The point estimates of α are found to be greater than 2 suggesting that the variance is finite. The results suggest that although the tails of the FT-ALL price changes are heavier than those of the normal distribution, they are not heavy enough to lie in the domain attraction of the stable Paretian distribution with $\alpha<2$.

Since the second moment of the distribution of the FT-ALL price changes seems to be finite, the second moment based tests of covariance stationarity are consistent with critical values which depend on the maximal moment exponent. These critical values are provided by Loretan and Phillips (1994). The variance constancy test of equation 6 is applied to the FT-ALL series. Two decisions have to be made about the lag truncation L, and the value k_n. The lag truncation L is set equal to 12, the value chosen by Loretan and Phillips (1994) for daily returns. The value of k_n is set equal to 0.5, 1, 1.5. The results are presented in table 2.

Table 2: The variance constancy test of Loretan and Phillips (1994)

k_n	0.5	1	1.5
$V_k(d)$	4.219	2.830	2.501

The 99% critical values of the test statistic reported in Loretan and Phillips (1994) are 2.07 for $\alpha=2.5$, 2.15 for $\alpha=3.0$, and 2.24 for $\alpha=3.5$. These α's values are chosen since they are close to those obtained in the current study which lie in the range [2,628, 3.199]. Evidently the sample prediction test statistic for different values of k_n is larger than the 99% critical values suggesting that the hypothesis of constant variance in the two sub-samples should be rejected. Accordingly, the proposition that the FT-ALL price changes were covariance stationary during the period from 2/1/1970 to 31/12/1991 is rejected.

3. Graphical inspection of the unconditional variance of the data.

Mandelbrot (1963a) suggested the graphical examination of the constancy of the second moment of stock returns as a way for checking the assumption of covariance stationarity. Mandelbrot's approach has the advantage of pointing out possible periods of exceptional circumstances in the data. For example, the plot of the recursive estimates of the monthly US stock return variance during the period 1835-1987 in Pagan and Schwert (1990a) revealed "dramatic evidence of a failure of covariance stationarity around the time of the Great Depression." Pagan and Schwert (1990a) consider Mandelbrot's idea to be useful though it has the disadvantage of no formal test statistic associated with it which makes it difficult to assess the constancy or the existence of moments. Therefore, the results of graphical inspection should be taken as indicative and more formal tests should be applied to verify the results.

In this section, the constancy of the unconditional variance of price changes is checked graphically over time. The unconditional variance of stock price changes is computed using the formula provided by Hamilton (1994) for a variable which follows the ARMA and/or ARCH process. This formula requires the estimation of the parameters of the underlying ARMA and/or ARCH process. The AR(1)-GARCH(1,1) model is used to approximate the price changes process. This model was chosen because of its successful record in explaining the time series properties of stock returns[9]. In addition, the AR(1) process was picked up as an appropriate model to the FT-ALL price changes using the BIC criterion in chapter 2. Also,

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[9]The GARCH(1,1) model has been shown to provide an adequate fit for stock and currency prices, see Engle (1995). Also, Akgiray (1989) demonstrated that an AR(1)-GARCH(1,1) model can provide a reliable basis for explaining the nature of returns over time.

chapter 2 results suggest that the GARCH(1,1) model accounts for almost all of the nonlinearity in the FT-ALL price changes. Nevertheless, the adequacy of the AR(1)-GARCH(1,1) model is further examined using the Mcleod and Li (1983) test statistic. The constancy of the unconditional variance derived from the model is then inspected graphically.

The methodology employed involves estimating the AR(1)-GARCH(1,1) on "Rolling Periods" of the data, and checking whether the unconditional variance is stable over different rolling periods. The rolling periods used for estimation are based on a window size of 528 observations for estimation with a moving block of 66 observations. For example, after estimating the model to the first 528 observations, the first 66 observations are removed from the sample and a new 66 observations are added to it. The data set contains 5555 observations, which creates 77 periods for estimation. The AR(1)-GARCH(1,1) model to be estimated is as follows:

$$Y_t = b_0 + b_1 Y_{t-1} + u_t, \tag{10}$$

Equation 10 represents the mean of Y_t as an expected component which follows an autoregression of order 1 with an unexpected component, u_t, which is given by:

$$u_t = \sqrt{h_t}\, Z_t \quad ; \quad Z_t \sim IIDN(0,1), \tag{11}$$

Equation 11 shows that u_t has a mean of zero and variance of h_t. The standardized residuals of u_t, Z_t, represents the shocks to the system and are identically, independently and normally distributed with mean of zero and variance of one. The variance of u_t has the following GARCH(1,1) representation:

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1}. \qquad (12)$$

Equation 12 shows that the conditional variance of u_t is a function of a constant term, α_0, u_{t-1}^2 and h_{t-1}. While the conditional variance of Y_t is changing over time according to equation 12, its unconditional variance is constant if the process is covariance stationary. The unconditional variance is given by:

$$\sigma^2 = \frac{\alpha_0}{(1-\alpha_1-\beta_1)}. \qquad (13)$$

The necessary and sufficient conditions for the stationarity of the GARCH(1,1) process is that $\alpha_1+\beta_1<1$ in equation 13[10]. The BHHH iteration routine [see Berndt *et al.* (1974)] is used to estimate the parameters of equations 10 and 12 simultaneously. The stability of the unconditional variance is checked graphically over different rolling periods. If the price changes series is covariance stationary, the

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[10]Nelson (1990) shows that the GARCH(1,1) process can be strict stationary without being covariance stationary. Strict stationarity does not require the existence of the moments of the data.

estimate of the unconditional variance based on equation 13, should be independent of time. Therefore, a small change of 66 observations in the constituents of a sample of 528 observations, should not drastically change the parameters of equations 10 and 12.

The Mcleod and Li (1983) test statistic is used to check for model adequacy of the AR(1)-GARCH(1,1) model over different rolling periods. Mcleod and Li (1983) show that the sample autocorrelations of the squared residuals have asymptotic variance of T^{-1}, and that the Ljung and Box (1978) Q-statistic calculated from them is asymptotically distributed as χ^2. The test statistic is as follows:

$$Q = T(T+2) \sum_{k=1}^{p} (T-k)^{-1} r_k^2 . \qquad \textbf{(14)}$$

where r_k is the sample autocorrelation at lag k and p is the number of lags. The Q statistic is distributed as χ_p^2 [see Mcleod and Li (1983)]: if r_k is not close to zero, the statistic will be inflated and this will lead to the rejection of the hypothesis of uncorrelated squared residuals. The Q statistic is checked for the squares of the raw data and the squares of the residuals from the AR(1)-GARCH(1,1) model at lag 10 for each of the 77 rolling periods. The results are set out in table 1 in the appendix to this chapter.

The table shows that the FT-ALL price changes have highly significant Q statistics, suggesting the existence of statistically significant autocorrelations in its squares. The Q statistics for the squared residuals from the AR(1)-GARCH(1,1) model are not significant at 5% level except for very few cases. This implies that the AR(1)-GARCH(1,1) is an adequate model over different rolling periods. It must be emphasized that although the GARCH(1,1) model seems to successfully account for the autocorrelations in the squared of price changes, it is only an approximate or an assumption for the true data generating process. Therefore, the results of the graphical inspection should be viewed on the basis that the GARCH(1,1) is at best an approximate for the true data generating process. Figure 1 shows the estimates of the unconditional variance, σ^2, derived from equation 13.

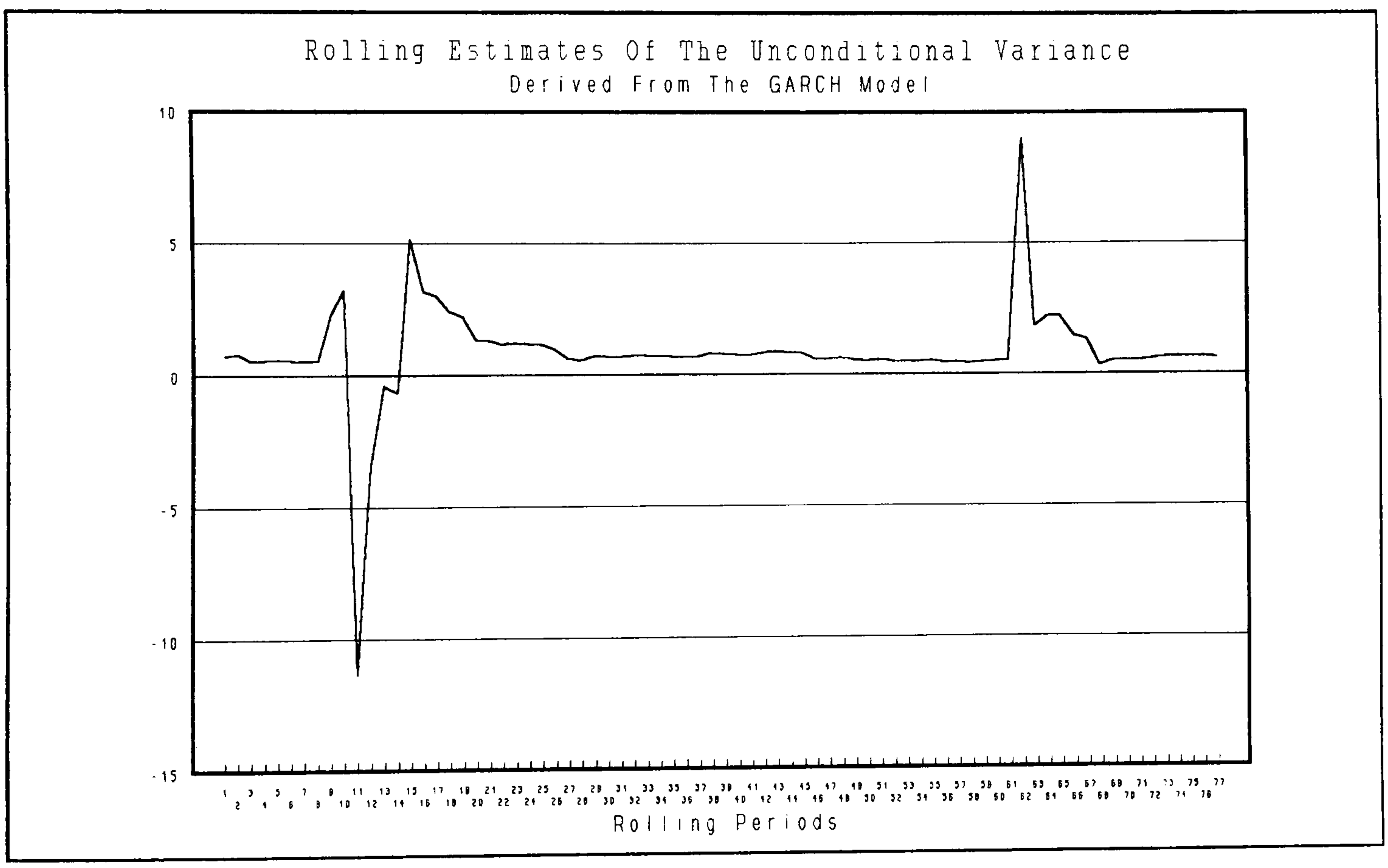

Figure 1

The figure demonstrates an unusual result that the unconditional variance of the FT-ALL price changes, derived from the parameter estimates of the AR(1)-

GARCH(1,1) model, was negative in periods 11, 12, 13 and 14 which correspond to the 1973-74 oil crisis. Another feature to notice is that the unconditional variance was much higher in period 62 than at any other, and this is the period that includes the 1987 crash. As mentioned earlier, the necessary and sufficient requirement for the covariance stationarity of the GARCH(1,1) model is that $\alpha_1+\beta_1<1$. Table 3 has the estimates of $\alpha_1+\beta_1$ for the rolling periods 11,12,13,14, and 62.

Table 3: Volatility persistence ($\alpha_1+\beta_1$) in rolling periods 11,12,13,14, and 62.

Rolling period	Dates	$\alpha_1+\beta_1$
11	14/8/72 - 17/9/74	1.004
12	16/11/72 - 19/12/74	1.008
13	22/2/73 - 26/3/75	1.014
14	30/5/73 - 1/7/75	1.013
62	14/10/87 - 19/1/88	0.991

The table shows that the requirement for the covariance stationarity of the estimated GARCH(1,1) model that $\alpha_1+\beta_1<1$ is violated around the 1973-74 oil crisis. It also follows that the source of the negative estimates of the unconditional variance during the rolling periods 11, 12, 13, and 14 is that $\alpha_1+\beta_1>1$.

It deserves mentioning that during the 1973-74 period, the decline in stock market prices was over an extended period of time. The FT-ALL index declined from a level of 218.82 on 2/1/73 to 66.89 level on 31/12/74. This constitutes a percentage decline of 69.43% over a two year period, and is quite different to the experience of the 1987 crash where the decline occurred over a much shorter period. Therefore, it seems that there was a long memory component in volatility during the oil crisis of the 70's which led to the observed volatility persistence taking higher value than 1. Table 2 in the appendix shows the date and percentage decline in the FT-ALL price changes outside 3 standard deviation from the mean of the data. The table shows that the period from 5/12/1973 to 22/8/1975, and the period around the 1987 crash were the most volatile periods in the series in terms of outliers.

4. Intervention analysis

One possible explanation for the concentration of outliers during the two periods of the oil crisis and market crash, is that these observations were from a distribution with a higher variance than the rest of the data. In other words there was a shift in the unconditional variance of the FT-ALL price changes during these two periods. This suggests that there were structural changes in the process being modelled during the 1973-74 oil crisis, and 1987 market crash. However, as mentioned earlier, the results of graphical inspection are to be interpreted as being no more than indicative.

To overcome the problem that graphical inspection does not have a formal test statistic associated with it, intervention analysis in the spirit of Box and Tiao (1975) is used to model possible shifts in the variance of price changes around the oil crisis and the market crash. Intervention was termed by Box and Tiao (1975) to refer to the behaviour of specific observations of a time series whose times are specified. Box and Tiao (1975) offer examples of an intervention, of which two are shown in figures 2 and 3. Figure 2 shows a pulse variable which is the simplest form of intervention analysis. A pulse variable models an intervention lasting only for one or a limited number of observations. In other words, the event has only an instantaneous effect on the variable of interest [see Mills (1990), pp. 235-280]. The pulse variable is widely known and used in finance as the 0,1 dummy variable. For example, Lastrapes (1989) and Lamoureux and Lastrapes (1990a) among others incorporated a pulse variable in the GARCH specification to examine their effects on the volatility persistence of the GARCH process.

However, the pulse variable may not be the most suitable form of representing the effect of an economic event since it represents a sudden shift forward in the variable of interest and a sudden shift backward to its pre-event level. A more realistic approach is presented in figure 3, which I refer to as response to a pulse model. The model shows the case of an immediate positive shift followed by a decay and possibly a lasting residual effect. This model is more flexible since the pulse model is nested in it. Therefore, it is easy to test the hypothesis of a transient effect on the variable of interest. A response to a pulse model is applied to the FT-ALL price changes.

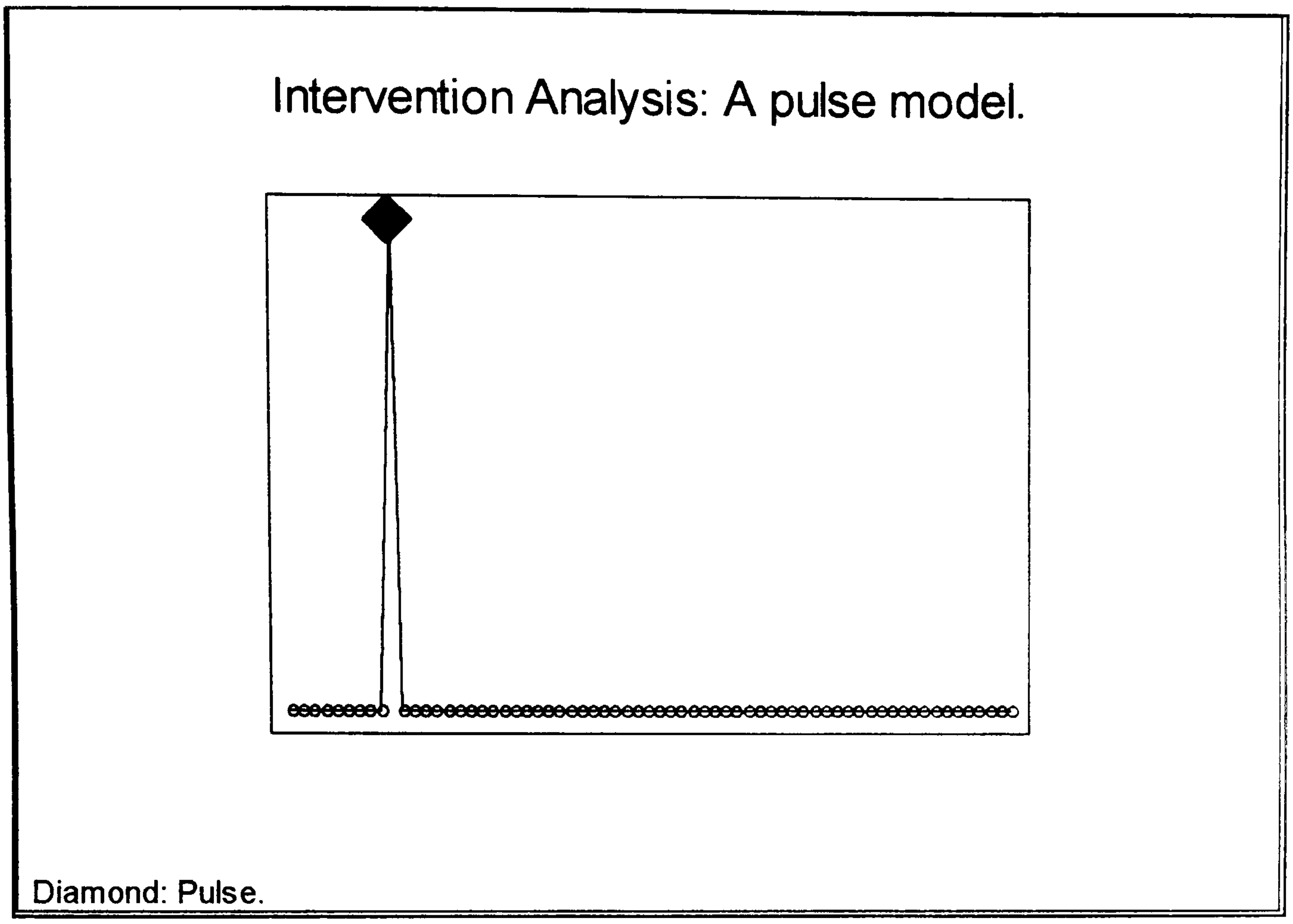

Figure 2

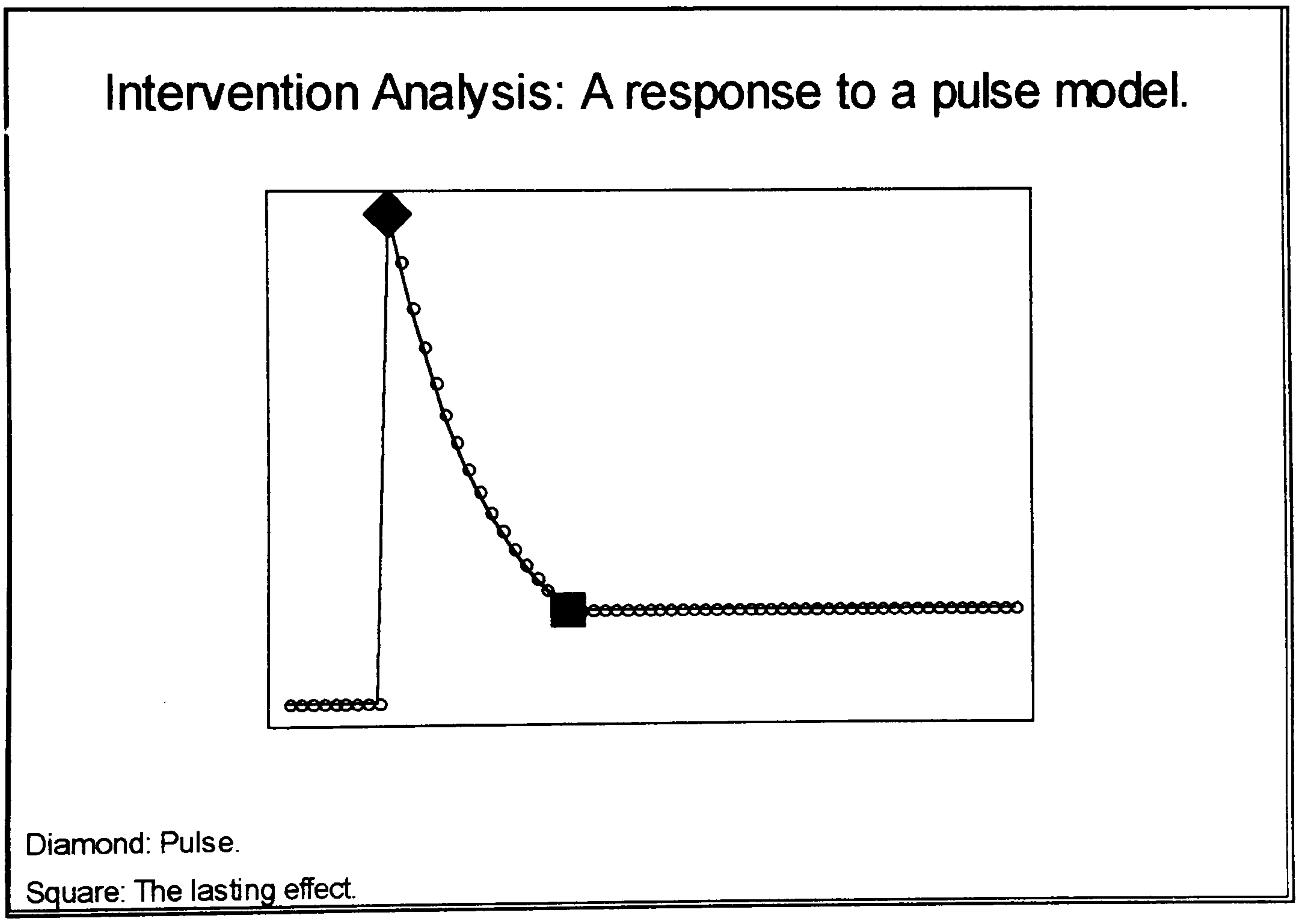

Figure 3

The model allows for possible shifts in the variance of price changes around the oil crisis and the market crash. Price changes, Y_t, are modelled as follows:

$$Y_t = \sigma_t \epsilon_t, \tag{15}$$

where ε's are the residuals which are i.i.d. variables with a conditional Student's t distribution with mean zero and variance one. The variance of price changes is:

$$\sigma_t^2 = \omega_0 + (\omega_1 + \delta_1) Z_{1t} + (\delta_1 + b_1(\sigma_{t-1}^2 - \omega_0 - \delta_1)) X_{1t} +$$

$$(\omega_2 + \delta_2) Z_{2t} + (\delta_2 + b_2\ (\sigma_{t-1}^2 - \omega_0 - \delta_2)) X_{2t}. \tag{16}$$

where

X_{1t} = 1 if t > 5/12/1973, (the start of the oil crisis) and if t < 19/10/1987 (the market crash), and zero otherwise.

Z_{1t} = 1 if t = 5/12/1973 (the start of the oil crisis), and zero otherwise.

X_{2t} = 1 if t > 19/10/1987 (the crash day), and zero otherwise.

Z_{2t} = 1 if t = 19/10/1987, and zero otherwise.

Equation 16 models the variance of price changes as constant over time and equal to ω_0 apart from the days of the 1973 oil crisis and the 1987 market crash. At the start of the oil crisis, the variance jumps by the magnitude of $(\omega_1 + \delta_1)$, and then ω_1 dies out exponentially at a rate b_1 to a new level of volatility equal to $(\omega_0 + \delta_1)$.

The level of volatility after the oil crisis can be different than the level of volatility before the oil crisis according to the magnitude and sign of δ_1. Then, the variance of price changes stays at this level till the crash day of 1987 on which it jumps up by a magnitude equal to $(\omega_2+\delta_2)$, and on the following day ω_2 starts to die out exponentially at a rate b_2 to a level equal to $(\omega_0+\delta_2)$. As with the oil crisis, the post crash level of volatility can be different from the pre crash level according to the sign and magnitude of δ_2.

The intervention model is estimated using the BHHH routine[11] and using the Student's t distribution as a conditional density function. The density function for the Student's t is given by:

$$f(u_t) = \frac{\Gamma[(d+1)/2]}{\pi^{-1/2}\Gamma(d/2)}(d-2)^{-1/2}\sigma_t^{-1}[1+\frac{Y_t^2}{\sigma_t^2(d-2)}]^{-(d+1)/2}$$

where Y_t and σ_t^2 are given by equations 15 and 16. $\Gamma(.)$ is the gamma function, and d refers to the degrees of freedom. The latter shapes the t distribution, and therefore decides how fat the tails of the distribution are, and is restricted to be greater than 2 to ensure a finite variance. When d goes to infinity, the resulting distribution is normal. The results are in table 4. Part (a) of the table has the parameters estimates of equation 16 while part (b) has the parameters estimates of equation 16 under the restrictions that $\delta_1=\delta_2=0$ as will be explained later.

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[11]See Berndt *et al.* (1974), and Harvey (1990), chapter 4.

Table 4(a): The parameter estimates of the intervention model. The dependent variable is Y_t as in equation 15, multiplied by 100 to get the %price changes.

	ω_0	ω_1	δ_1	b_1	ω_2	δ_2	b_2	d
Coeff	0.676	5.01	-0.029	0.998	29.48	-0.087	0.931	9.75
T-stat	20.49	8.92	-0.62	4218	2.43	-1.92	82.01	s.e.= 0.973
The Log Likelihood				-7453.775				

Table 4(b): The parameter estimates of the intervention model under the restrictions that $\delta_1=\delta_2=0$.

	ω_0	ω_1	δ_1	b_1	ω_2	δ_2	b_2	d
Coeff	0.638	4.94	-	0.998	30.62	-	0.929	9.65
T-stat	34.11	9.32	-	4958	2.34	-	76.33	s.e.= 0.949
The Log Likelihood				-7455.602				

Table 4(a) shows that the parameter estimates of the pulse variable for the oil crisis, ω_1, and for the market crash, ω_2, are significant at the 5% level. The estimates of the decay rate of the oil crisis, b_1, and the market crash, b_2, are also significant at the 5% level. However, the results provide no support for the notion that either the oil or crash crises had any lasting effect on volatility. Neither the estimates of δ_1 nor δ_2 are statistically significant at the 5% level. This indicates that the baseline volatility of the stock market is constant over the period of study. In fact this can be tested by restricting δ_1 and δ_2 to zeros in equation 16, and using the likelihood ratio (LR) to test the null hypothesis that $\delta_1=\delta_2=0$ versus the alternative hypothesis that at least one of the two coefficients is not equal to zero[12]. The likelihood ratio has a χ^2 distribution with 2 degrees of freedom.

The likelihood ratio for the restriction of $\delta_1=\delta_2=0$ takes the value of 3.65 which is less than the critical value of 5.99 at the 5% level of significance. Therefore, the evidence does not allow the rejection of the null hypothesis and on this basis it seems sensible to conclude that the baseline volatility is constant over the period of study. In other words, the variance of the FT-ALL price changes was constant during the period of study apart from the periods corresponding to the oil crisis and market crash. The degrees of freedom for the conditional distribution (the unrestricted model) are 9.75 which indicates that the distribution of price changes has fatter tails than those of the normal distribution[13].

It is also interesting to compare the parameter estimates of the intervention model for the oil crisis and market crash. This comparison could suggest whether the response of the stock market to different crises is similar, and if so it could suggest how the stock market might respond to a new crisis. The similarity between the two

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[12] The likelihood ratio (LR) test statistic is -2(log likelihood under the restrictions - log likelihood without the restrictions). The LR has a χ^2 distribution with r degrees of freedom, where r is the number of restrictions.

[13]If the degrees of freedom are more than 25, then the resulting distribution can be approximated by the normal. see Blattberg and Gonedes (1974)

crises is that in both cases, there was no lasting effect on the volatility after the event. The major differences between the response of volatility to both crises are in the pulse magnitude and the rate of exponential decay. In case of the market crash, the pulse magnitude is 29.48 which is much higher than the pulse magnitude for the oil crisis which is 5.01[14]. However, the oil crisis shock died out at a much slower rate than in the market crash. The rate of decay for the oil crisis is 0.998 while it is 0.931 for the market crash, indicating that the oil shock took a lot longer time to die out. This result is consistent with the results in table 3 on the estimates of volatility persistence of the GARCH model during the oil crisis and market crash. Recall that volatility persistence derived from the GARCH model indicated that the persistence of volatility in case of oil crisis was much greater than in the market crash. The result about the lower rate of decay for the market crash is consistent with Schwert's (1990) finding for the USA that "the stock market returned to relatively normal levels of volatility quickly at the end of 1987." His results were later supported by the empirical evidence presented by Engle and Lee (1993).

In summary, it seems that even though the initial response of the market to the start of the oil crisis was smaller than in the 1987 crash, it took longer time for the shock of the former to die out than for the latter. In neither case was there a lasting effect on volatility.

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[14]These estimates of the pulses magnitudes are of course related to the level of the series and its changes at the time of the two crises. The UK stock market fell by 3.30% on the starting day of the oil crisis, and 9.55% on the crash day [see table 2 in the appendix].

5. The constancy of variance test applied to the residuals from the intervention model.

The results of the intervention analysis support the identification by the graphical analysis of the oil crisis and market crash as exceptional periods. However, the question remaining is whether these two periods were responsible for the rejection of the hypothesis of constant variance for the overall period 1970-1991. The Loretan and Philips (1994) test methodology as described in section 2 is re-applied to the standardised residuals from the intervention model (the unrestricted model of equation 16) to consider this issue. If the results change from those of the same test applied to the raw data, then the difference can be attributed to the effects of the oil crisis and market crash on the volatility of price changes. Table 5 lists the estimates of the maximal moment exponents for the raw data and the residuals. In addition, the null hypothesis of finite fourth moment, $\alpha=4$, is tested against the alternative of fourth moment condition failure, $\alpha<4$.

Table 5: Point estimates of the maximal moment exponents for the raw data and the residuals from the intervention model. s is the number of order statistics.

	Left		Right	
s	α_s raw	α_s Residuals	α_s raw	α_s Residuals
20	2.628* (0.588)	5.281 (1.18)	2.875* (0.643)	4.641 (1.038)
50	3.199* (0.452)	5.078 (0.718)	2.979* (0.421)	4.955 (0.701)
100	3.098* (0.310)	4.410 (0.441)	3.010* (0.301)	4.147 (0.415)
200	2.800* (0.198)	4.073 (0.288)	2.838* (0.201)	3.915 (0.277)
300	2.793* (0.161)	3.473* (0.201)	2.676* (0.154)	3.516* (0.203)

Standard errors are in brackets.
"*" indicates that the null hypothesis $\alpha=4$ is rejected in favour of the alternative hypothesis that $\alpha<4$ at the 5% level.

The table shows that the maximal moment exponents have generally increased for the residuals than for the raw data. While the hypothesis of the existence of the fourth moment is rejected for the raw data at all values of s, the same hypothesis is not rejected for the residuals except at s=300.

The variance constancy test discussed in section 2 is re-applied to the residuals from the intervention model. The results are reported in table 6. The table also includes the results of the test applied to the raw data for comparison.

Table 6: Variance constancy test applied to the raw data and the residuals from the intervention model.

k_n	0.5	1	1.5
$V_k(d)$ raw	4.219*	2.830*	2.501*
$V_k(d)$ Residuals	0.720	-0.595	1.09

* indicates significance at the 1% level.

The 99% critical values of the test statistic applied to the raw data are discussed in section 2. Since the hypothesis that the maximal moment exponent is equal to 4 was not rejected for the residuals except for s=300, the critical values for the test statistic are given by the normal distribution. The critical value for the test statistic when $\alpha=3.5$ (which is the case when s=300) is 2.24. Clearly the variance constancy statistic for different values of k_n is not significant at the 1% contrary to the results of the same statistic applied to the raw data. Therefore, the hypothesis of variance constancy is not rejected for price changes adjusted for the effects of the two exceptional periods associated with the oil crisis and market crash.

6. Conclusions

The chapter examined the proposition that the variance of the FT-ALL price changes is constant over time. The variance constancy test of Loretan and Phillips (1994) is applied to the FT-ALL price changes during the period from 2/1/1970 to 31/12/91. The null hypothesis of constant variance is rejected at the 5% level. There is evidence that the fourth moment of the FT-ALL price changes is not finite. A graphical analysis of the unconditional variance derived from an AR(1)-GARCH(1,1) is conducted over different rolling periods of the data. The results not surprisingly point out to the existence of two exceptional periods in the data: the 1973-74 oil crisis and the 1987 market crash. The volatility persistence of the GARCH(1,1) over the oil crisis indicates the failure of the proposition of covariance stationarity during this period.

An intervention model on the line of Box and Tiao (1975) is applied to the FT-ALL price changes. The results suggest the existence of statistically significant shifts in the variance of price changes around the oil crisis and market crash. Also, the effect of the oil crisis persisted for a longer time period than the effect of the market crash. However, in both cases the volatility of price changes returned to its pre-event level indicating that there was no lasting effect from both crisis. In fact the null hypothesis of constant baseline volatility over the period of study was not rejected using the likelihood ratio test.

In addition, the results of the variance constancy test applied to the residuals from the intervention model suggest that the null hypothesis of constant variance should not be rejected. Also, the null hypothesis of finite fourth moment is not rejected except for s equal to 300. Therefore, it can be concluded that the rejection of the hypothesis of a constant variance and the hypothesis of a finite fourth moment in case of the raw data was due to the oil crisis and the market crash. In summary, the results suggest that the stock market is subject to abrupt changes in volatility during some exceptional periods. Outside these periods, however, stock price changes can be reasonably described as covariance stationary.

Appendix
Table 1

McLeod and Li (1983) statistic on the price changes squared and the residuals squared of the AR(1)-GARCH(1,1) over different rolling periods. RP stands to the rolling periods, Y stands to price changes, and Z stands to the residuals.

RP	Q^2		RP	Q^2		RP	Q^2	
	Y	Z		Y	Z		Y	Z
1	96.5*	5.81	27	166*	16.2	53	81.9*	4.35
2	89.7*	7.28	28	56.1*	16.3	54	95.4*	4.18
3	16.1*	4.43	29	51.9*	27.6*	55	82.3*	2.21
4	10.8	4.04	30	44.1*	27.7*	56	85.2*	5.87
5	45.1*	8.22	31	39.1*	28.2*	57	79.1*	12.5
6	48.3*	7.25	32	28.6*	17.1	58	107*	23.1
7	57.9*	14.3	33	26.8*	11.9	59	64.2*	22.1
8	84.2*	18.2	34	23.8*	12.2	60	48*	20.9
9	124*	1.95	35	25.4*	9.05	61	42.2*	12.7
10	113*	2.76	36	20.5	2.92	62	257*	.88
11	114*	3.78	37	16.3	3.09	63	256*	1.04
12	115*	3.65	38	194*	12.9	64	257*	1.19
13	379*	2.05	39	205*	16.5	65	257*	1.19
14	294*	1.49	40	223*	17	66	257*	1.27
15	257*	.995	41	235*	15.1	67	258*	1.30
16	259*	1.54	42	187*	11.6	68	259*	154*
17	391*	6.37	43	165*	9.4	69	262*	4.35
18	403*	5.91	44	169*	7.77	70	20.2	5.70
19	437*	4.79	45	173*	8.29	71	23.6*	6.27
20	442*	8.84	46	13.7	3.71	72	27*	8.61
21	147*	8.41	47	13.5	5.77	73	39.7*	8.10
22	81.7*	6.33	48	13.2	7.22	74	38.2*	9.64
23	77.3*	8.91	49	23.1	8.56	75	39.2*	9.45
24	87.3*	11.2	50	29.3*	6.42	76	35.1*	7.91
25	102*	11	51	42.9*	4.22	77	46.7*	9.20
26	137*	19.9	52	63.7*	4.31			

* statistically significant at 1% level.

Table 2
Date and Percentage Change outside 3σ from the mean in FT ALL price changes.

Date	Δ%
26/5/70	-4.46
28/5/70	3.53
19/6/70	5.22
22/1/73	-3.33
5/12/73	-3.30
6/12/73	-5.64
7/12/73	4.89
14/12/73	-5.54
27/12/73	4.12
1/3/74	-7.56
27/3/74	-3.33
14/8/74	-4.57
16/8/74	-3.32
25/9/74	-3.37
3/10/74	-3.65
15/10/74	3.56
13/11/74	-3.74
2/1/75	-6.62
8/1/75	3.81
13/1/75	5.14
16/1/75	3.42
21/1/75	3.26
22/1/75	4.41
23/1/75	4.94
24/1/75	8.94
27/1/75	5.94
29/1/75	5.37
30/1/75	6.94
31/1/75	-3.92
4/2/75	-3.49
6/2/75	4.20
7/2/75	6.05
10/2/75	8.25
11/2/75	-4.34
13/2/75	3.72

Date	Δ%
17/2/75	-5.16
26/2/75	4.89
27/2/75	3.89
5/3/75	3.31
11/3/75	-6.26
21/3/75	-3.29
24/3/75	-3.89
16/4/75	5.24
17/4/75	5.67
22/4/75	5.33
5/5/75	-3.65
1/7/75	4.86
22/8/75	3.36
2/6/76	-3.43
8/10/76	-3.68
4/11/76	3.38
8/11/76	-3.57
22/11/76	-3.29
23/9/81	-3.40
28/9/81	-4.78
29/9/81	4.71
19/10/87	-9.55
20/10/87	-12.11
21/10/87	5.66
22/10/87	-5.50
26/10/87	-7.42
30/10/87	3.58
3/11/87	-3.70
4/11/87	-3.65
9/11/87	-3.49
11/11/87	4.12
12/11/87	3.78
30/11/87	-4.26
29/12/87	-3.31
16/10/89	-4.14

Chapter 4

Heteroscedasticity in the UK Stock Market Prices: Unexpected Volume versus GARCH Effects[1]

1. Introduction

The evidence presented in chapters 2 and 3 suggests that the AR(1)-GARCH(1,1) model offers an adequate fit for the UK FT-ALL price changes. Chapter 2 offered evidence that most of the nonlinearity of the FT-ALL price changes can be explained by the AR(1)-GARCH(1,1) model. The results of the Mcleod and Li (1983) test in chapter 3 suggest that the squared residuals of the AR(1)-GARCH(1,1) model can be described as uncorrelated. This is consistent with the conclusions of Akgiray (1989), based on the results of a study of USA stock returns, that the AR(1)-GARCH(1,1) model provides an empirically reasonable model for stock returns. The widespread applicability of Autoregressive Conditional Heteroscedasticity models in describing stock market prices has led to a growing interest in identifying its origin [see Bollerslev *et al.* (1992)]. Engle, in his seminal work in 1982, presented his ARCH models as a better approximation to reality than that provided by homoscedastic models. However, he recognized that ARCH

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[1]This chapter was presented at the British Accounting Association national conference in Cardiff in March 1996, and the sixteenth annual International Symposium on Forecasting in Istanbul in June 1996.

might be a manifestation of model misspecification resulting from omitted variables or structural changes. It has long been argued that the variance of stock price changes is not constant through time but is related to the volume of trade[2]. Clark (1973) suggested that stock prices could be modelled as a subordinated stochastic process with their variance evolving at different rates according to the amount of information becoming available in the market. The implication is stock price changes follow a mixture of distributions in which the rate of information arrival to the market is the mixing variable. Diebold (1986) suggested that ARCH might capture the stochastic properties of this mixing variable [see also Stock (1987 and 1988)].

Lamoureux and Lastrapes (1990b) examined the validity of Diebold's contention by testing whether the ARCH effects disappear when volume of trade is used as a proxy for the random rate of information arrival to the market. They discovered that lagged squared disturbances have little if any effect on the conditional variance when volume of trade is used as an explanatory variable in the conditional variance. They concluded that "ARCH is a manifestation of the daily time dependence in the rate of information arrival to the market for individual stocks." Hiemstra and Jones (1994) examined whether the non-linear causality from volume to stock price changes found in their study is due to volume working as a proxy for information arrival to the market. They found that after controlling for the effects of volatility clustering in stock price changes using

[2] See Karpoff (1987) for a survey.

a suitable EGARCH model, there is still a significant nonlinear relationship between volume and stock price changes. In a UK based study, Board and Sutcliffe (1993) used the number of trades [bargains] during a day, rather than volume of trade, as a proxy for the number of information arrivals in modelling the conditional variance of the FT-ALL returns. Board and Sutcliffe (1993) results suggest that accounting for the effects of the number of trades on the conditional variance of the FT ALL returns did not remove the ARCH effects.

Bollerslev *et al.* (1992) argue that a simultaneity problem may bias Lamoureux and Lastrapes results as contemporaneous correlation between volume and price changes is documented in the literature [see Karpoff (1987)]. Lamoureux and Lastrapes (1990b) considered volume of trade to be weakly exogenous in the sense of Engle *et al.* (1983)[3]. Weak exogeniety implies that although the parameter estimate on volume of trade is inconsistent, inference about volatility persistence of the GARCH model is still valid. However, Lamoureux and Lastrapes (1994) relaxed the assumption of exogenous volume, and tested a mixture model for volume of trade and price changes where both variables are assumed to be generated by independent, and identically distributed shocks to the system and a single common factor, the speed of information arrival to the market. This common factor was assumed to be serially correlated. Therefore, volume and price

[3] Engle *et al.* (1983), pp. 278, suggest that "...a variable z_t in a model is defined to be weakly exogenous for estimating a set of parameters λ if inference on λ conditional on z_t involves no loss of information."

change volatilities were serially correlated. In contrast to the results of their 1990 study, the evidence presented by Lamoureux and Lastrapes in their 1994 study suggests that "...accounting for serial dependence in the information-arrival process does not eliminate GARCH persistence in variance". They also report that contemporaneous volume and squared price changes are not very useful instruments for predicting the future volatility of price changes.

A drawback of Lamoureux and Lastrapes (1994) methodology is that maximum likelihood method cannot be used to estimate the joint density of price changes and volume because of the serial correlation in the common factor. Lamoureux and Lastrapes (1994) used two step approach. In the first step they extracted, for each observation in the sample, the value of the common factor (the rate of information flow) that sets the observed values of the squared price changes and volume as close as possible to the respective conditional means implied by the model being suggested. Lamoureux and Lastrapes (1994) report that "... Although ignoring the time series properties of the data in this step is inefficient, full-information extraction procedures are likely to be computationally burdensome and intractable".

In the second step Lamoureux and Lastrapes (1994) test the null hypothesis that price changes adjusted for the values of the common factor estimated in the first step, do not exhibit persistence in variance. The drawback of this exercise is that: (1) it depends on the efficiency of the extraction technique of the first step in estimating the values of the common factor; (2) the asymptotic properties of the test are unknown. Lamoureux and Lastrapes (1994) used simulations based on random drawings with replacement to derive the critical values of the test. However, as they rightly pointed out, these simulations may not be representative since they were conducted on one company out of a sample of ten [see Lamoureux and Lastrapes (1994) pp. 258].

Bessembinder and Seguin (1992) interpreted volume of trade differently from that of the information flow hypothesis of Lamoureux and Lastrapes (1990b and 1994). They used volume of trade as a measure of trading activity. They partitioned volume into expected and unexpected. The unexpected component is interpreted as a measure of shocks to the daily activity [see also Bessembinder and Seguin (1993)]. Bessembinder and Seguin (1992) showed that unexpected volume has a larger significant affect on the conditional variance of price changes. This is consistent with the results of Schwert (1989), Ying (1966) and Gallant *et al.* (1992). Schwert (1989) found evidence that there is a strong correlation between "shocks" to volume and volatility. Ying (1966) presented evidence that a large increase in volume of trade is usually associated with a large rise or fall in price. Gallant *et al.* (1992) found that the days with high volume of trade are associated with high price volatility.

The purpose of this chapter is to explore the issue of whether GARCH modelling captures the effects of the temporal dependence in the volume of trade for individual stocks in the UK market. The empirical results suggest that:

(a) although the parameter estimates of the GARCH model become insignificant when volume of trade is used in the conditional variance of price changes, the autocorrelations of the squared residuals still exhibit a highly significant GARCH pattern. It is argued that the Lamoureux and Lastrapes (1990b) model suffers from a multicollinearity problem;

(b) unexpected volume of trade interpreted as a measure of trading activity reduces the volatility persistence of the GARCH model; and

(c) the relevance of the unexpected volume of trade is due to a strong association in the timing of innovational outliers in both the price changes and volume of trade series, a factor that may contribute to the noted reduction in volatility persistence of the GARCH model.

One implication of the results is that volume of trade can help in forecasting the future conditional volatility of stock price changes as long as our emphasis is on prediction rather than explaining the joint dynamics of price-volume. This forecast of future volatility will be only a function of the past squared price changes since the ***expected*** value of the future unexpected volume of trade is zero. This is because the unexpected volume of trade is the residual from a suitable ARMA model, and therefore it is distributed with a mean of zero and constant variance. The role of unexpected volume in the model is that it filters the effects of shocks in trading activity on the volatility persistence of price changes. This implication is inconsistent with the finding of Lamoureux and Lastrapes (1994) that volume and squared price changes are noisy predictors of the future conditional volatility.

The chapter is organised as follows: section 2 summarizes the literature on the price-volume relationship, section 3 discusses the data and the methodology, section 4 reports the empirical results, and section 5 provides a discussion of the conclusions of the study.

2. The price change and volume of trade relationship

Karpoff (1987) surveys most of the work done on the price change and volume relationship. His survey offers strong support for the view that there is a positive correlation between volume and absolute price changes, and a positive correlation between volume and price changes in equity markets. One explanation for the positive relationship[4] between volume and absolute price changes comes from literature on the distribution of speculative price changes. Mandelbrot (1963a) and Fama (1963 and 1965a) report that changes in speculative prices appear to be uncorrelated with each other, and symmetrically distributed with greater frequency close to the mean and out in the extreme tails (leptokurtic) then is consistent with a normal distribution. They

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[4] There are several other theoretical explanations for the relationship between volume and price changes, see for example, Karpof (1987), Copeland (1976), and Hiemstra and Jones (1994).

postulate that speculative price changes are best described by a non-normal symmetric stable distribution. Such distributions allow for the leptokurtic behaviour observed in the empirical distributions but their second and higher moments do not exist.

The competitive alternative is to view the distribution of price changes as a mixture of normals with a changing variance. Since daily price changes are the simple sum of within day price changes, and because the number of within day price changes, n_t, is random, daily price changes follow a mixture of normals where n_t is the mixing variable. If we assume that prices evolve when new information arrives at the market, then n_t will be the number of information arrivals in the market on a certain day. The positive correlation between volume and price changes in Clark's model arises because the variances of price changes and volume of trade are positively correlated with the unobserved random number of information arrivals to the market, n_t.

Clark's empirical results indicate that the distribution of daily price changes conditional on the volume of trade is less leptokurtic and closer to the normal distribution than the unconditional distribution. The empirical work of Morgan (1976) and Westerfield (1977) provide support for the position taken by Clark. Epps and Epps (1976) and Tauchen and Pitts (1983) also offer support for Clark's explanation of the positive correlation between volume and absolute price changes[5].

The ARCH model of Engle (1982) and Generalized ARCH (GARCH) model of Bollerslev (1986) model the distribution of price changes as a mixture of distributions in which the variance is a function of its past. A possible interpretation for the success of GARCH models in modelling stock price changes is the information flow hypothesis of Clark (1973). If Clark's assumption that the mixing variable, n_t, the number of information arrival forms an independent sequence is relaxed, then it is possible that GARCH is capturing the temporal dependence in this variable. To explain how ARCH dynamics can capture the effect of the information arrival to the market, the daily stock price changes, R_t, are decomposed in the following way [see Bera and Higgins (1992)]:

$$R_t = \sum_{i=1}^{n_t} x_i \quad where \quad x_i \sim IID\ N(0 \cdot \sigma^2) \tag{1}$$

Here, the x_i's are the intraday equilibrium price changes, and n_t is the random number of information arrivals to the market on day t. Equation 1 indicates that daily price changes are generated by a subordinated stochastic process in which R_t is

[5] The studies of Epps and Epps (1976) and Tauchen and Pitts (1983) differed from that of Clark's. Epps and Epps (1976) examined the distribution of transaction to transaction data conditional on the volume of transaction. Their results are consistent with the mixture of distributions hypothesis and will be consistent with Clark's model if n_t has a finite mean. Tauchen and Pitts (1983) investigated the joint distribution of price changes and volume of trade.

subordinate to x_i and n_t is the directing process. It is assumed that n_t is independent of the x_i's but is serially correlated. Because of the randomness of n_t, the Central Limit Theorem does not apply, and R_t is distributed as a mixture of normals. Equation 1 can be rewritten as:

$$R_t = \sigma \sqrt{n_t} z_t \quad where \quad z_t \sim IID\ N(0,1) \tag{2}$$

Thus

$$R_t | n_t \sim N(0, \sigma^2 n_t) \tag{3}$$

From equation 3, daily price changes conditional on the number of information arrivals at the market, n_t, is normally distributed with zero mean and variance that reflects the intensity of information arrivals [n_t]. As noted by Andersen (1995), "...this representation is closely related to the idea of time deformation as the return variance is driven by an event time scale (information arrivals) rather than a calender time scale" [see also Stock (1987 and 1988)]. The auto-covariances of the squared price changes are given by:

$$Cov(R_t^2\ ,\ R_{t-j}^2) = \sigma^4\ Cov(n_t\ z_t^2\ ,\ n_{t-j}\ z_{t-j}^2) \tag{4}$$

$$= \sigma^4 Cov(n_t, n_{t-j}) \quad (5)$$

Since $\{z_t\}$ are independently distributed, equations 4 and 5 show the auto-covariances of price changes [ARCH effects] are due to the serial dependence in the number of information arrivals [n_t]. To summarize this section, relaxing Clark's assumption that the number of within day information arrivals, n_t, is independent, suggests that it may be possible for GARCH models to capture the temporal dependence in n_t.

3. Data and Methodology

The data set comprises daily price changes and volume on 60 British companies. The data was obtained from Datastream International. Volume of trade is the number of shares traded for a particular stock on a particular day[6]. Volume is chosen since it is the same variable used by Lamoureux and Lastrapes (1990b), and therefore there is a scope for comparison between the studies.

The sample is chosen from the population of FTSE 100 stocks which comprises

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[6]A drawback of Datastream International is that it keeps no history about the number of shares representing the capital of the company. Therefore, it is not feasible to construct a variable like turnover (volume divided by number of shares in issue), which has been used in some studies in US.

the 100 biggest companies in the UK according to market capitalization. These companies account for about 70% of the total market capitalization of all UK equities. A review is carried out every quarter by the Stock Exchange to replace those companies which have lower market capitalization than companies excluded from the index at the time[7]. The criterion for choosing the companies for the sample was simply whether a particular company was continuously represented in the FTSE 100 from 4/1/1988 to 28/2/1994. While 60 companies met this requirement, only 57 companies were included in the final sample since three companies had many missing values in the volume series.

In the first stage of the analysis, the following model is estimated for each stock in the sample:

$$R_t = b_0 + b_1 R_{t-1} + \epsilon_t \tag{6}$$

$$where \quad \epsilon_t \sim N(0, h_t) \tag{7}$$

$$h_t = \alpha_0 + \alpha_1 \epsilon^2{}_{t-1} + \beta_1 h_{t-1} + \omega_1 V_t \tag{8}$$

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[7] For more on the rules of deletion and insertion of companies in FTSE 100, the reader is referred to the publications of the International Stock Exchange of the United Kingdom entitled "The FTSE 100 Share Index", January 1988.

where R_t is $100*\log_e(P_t/P_{t-1})$, and P_t is the stock price at time t. Equation 6 allows for an autoregression of order 1 in the mean of price changes since most of the data on price changes used in this study exhibit a small but significant first order autocorrelation. Equation 8 models the variance of the unexpected price changes, ε_t, as a GARCH(1,1) process, and the volume of trade, V_t. The sum of the α_1 and β_1 measures the persistence in volatility and lies between 0 and 1.

Following the same methodology as Lamoureux and Lastrapes (1990b)[8], I start by estimating the restricted model of equation 8 by setting the coefficient of volume to be zero, and fitting a GARCH(1,1) model to the ε_t. The GARCH(1,1) model has been shown to be a parsimonious representation of conditional variance [Bollerslev (1986) and Akgiray (1989)]. The parameters of the model, equations 6 and 8, are estimated jointly using the BHHH maximization routine [see Berndt *et al.* (1974)].

In the second stage, the unrestricted model of equation 8 is employed. If volume of trade is serially correlated, and works as a proxy for information arrivals to the market, then it can be anticipated that $\omega_1 > 0$, and the persistence in volatility as measured by the sum of the α_1 and β_1 becomes negligible.

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8 Lamoureux and Lastrapes (1990b) report that allowing for a first order autoregression in the mean of the data did not change their results.

4. The Empirical Results

The discussion of the empirical results is divided into two sections. Fifty companies are examined in section *A*, and the remaining seven in section *B*. The reason for this division is that the data on the seven companies in the second section behave differently from the rest of the sample. The volume data for these companies have one or two unusual observations which affect their sample statistics. The reason behind the occurrence of these few outliers is discussed in section *B*.

Section *A*

Table 1 shows the results of estimating the restricted model of equations 6 to 8 where ω_1 is set equal to zero[9]. Note that price changes are measured as 100 multiplied by the ratio of the natural log of the stock price at time t and the stock price at time (t-1). Volume of trade is measured in millions of units.

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[9] The results of the AR(1) estimation for the mean equation are not reported since the variance is the focus of this study. Also, the full results of the GARCH estimation are not reported because of the space required.

Table 1: The results of estimating the GARCH(1,1) model to stock price changes. The equation estimated is, $h_t=\alpha_0+\alpha_1\varepsilon_{t-1}^2+\beta_1 h_{t-1}$. Q_{10}^2 is the Mcleod and Li (1983) statistic at lag 10 for the squared price changes (R), and for the GARCH(1,1) squared residuals (U). The statistic is distributed as χ^2 with 10 degrees of freedom. The critical value of χ_{10}^2 is 18.3 at the 5% level.

Company	$\alpha_1+\beta_1$	Q_{10}^2R	Q_{10}^2U	Company	$\alpha_1+\beta_1$	Q_{10}^2R	Q_{10}^2U
Allied lyons	.68	41.4*	2.02	Imperial Chemicals Industries	.942	43.1*	7.72
A.British Foods	.857	29.5*	4.66	Land Securities PLC	.918	67.4*	5.72
BAA	.584	80.5*	5.62	Marks & Spencer	.182	75*	7.70
Argyll Group	.599	25.6*	8.85	Lioyds Bank	.963	47.5*	7.01
BASS	.992	46.9*	9.80	National Westminster Bank	.984	114*	19.0*
Blue Circle Ind.	.969	710*	5.83	RTZ Corp'n	.974	211*	9.98
ASDA	.933	182*	3.17	Rank Organization	.959	89.4*	8.87
BOC Group	.937	29.8*	4.11	RMC Group	.963	180*	5.70
Barclays	.979	117*	5.88	Scottish & Newcastle Brews	.755	2.27	.333
B.A.T	.991	0.843	0.940	United Biscuits	.769	139*	1.52
Boots	.978	25.8*	9.30	Unilever PLC	.959	67.8*	13.3
British Airways	.765	32.2*	3.44	Prudential Corp'n	.891	112*	7.15
British Gas	.822	35.9*	14.5	Reed International	.968	69.1*	8.05
British Petroleum	.989	22.3*	5.13	Reuters Holdings PLC	.957	147*	8.41
British Telecom	.883	20.2*	5.68	Royal Insurance	.995	168*	5.55
Cable & Wireless	.979	16.2	9.21	Rolls-Royce	.967	104*	21.2*
Glaxo Holdings PLC	.991	93.9*	16.9	Redland	.981	364*	0.05
Grand Metropolitan	.981	210*	3.65	Reckitt & Colman	.197	29.7*	18.4*
Courtaulds	.940	92.8*	36.1*	Sainsbury	.975	19.1*	4.52
General Accident	.996	77.9*	3.58	Tesco PLC	.985	87.1*	11.5
Guardian Royal Exchange	.996	178*	6.44	Thorn EMI PLC	.973	18.5*	9.00
General Electric Corporation	.949	18.5*	7.85	TSB Group	.974	31.7*	3.58
Coats Vyella PLC	.851	36.7*	5.81	Royal Bank of Scotland Group	.990	94.3*	4.17
Guinness	.992	18.2	4.07	Legal & General	.952	142*	3.68
Hansons Trust	.977	64.9*	11.3	Ladbroke Group PLC	.969	245*	9.69

(*) statistically significant at 5% level.

Table 1 shows that the Mcleod-Li statistics[10] at lag 10 for the squared price changes are highly significant suggesting the existence of ARCH effects except for 4 companies out of the 50 examined. The results of estimating the GARCH model suggest that volatility persistence as measured by the sum of α_1 and β_1 is generally very high, and is higher than 0.90 for 37 of the companies examined. Mcleod-Li statistics at lag 10 indicate that the squared standardized residuals do not show any significant GARCH pattern other than for 4 out of the 50 companies, suggesting that the data sets for these companies requires different GARCH specifications. For example, for Reckitt and Colman, β_1 is zero but there is a significant Mcleod-Li statistic at lag 10 indicating that GARCH(1,1) is not the appropriate model. Another company, Marks and Spencer, has a non-significant β_1 while the Mcleod-Li statistic is very small and far from being significant, indicating that an ARCH(1) model may be adequate for this data set.

Before attempting to include volume of trade in the variance of stock price changes, it is worthwhile to investigate whether there is any trend in volume of trade. Figures 1 and 2 show the time series plot of volume of trade for two companies, Allied Lyons and BAA. There is no indication of trends in the volume of trade of either of these companies. The time series plots were also checked for more companies and none indicated a strong time trend. This is possibly because the companies considered in the study are the largest British companies and the trading patterns for their shares tend to

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[10] The reader is referred to chapter 3 for a discussion of the Mcleod-Li Q statistic.

be well established. The figures also show that there are some huge outliers in volume of trade. Table 2 shows the results of estimating the unrestricted model of equation 8.

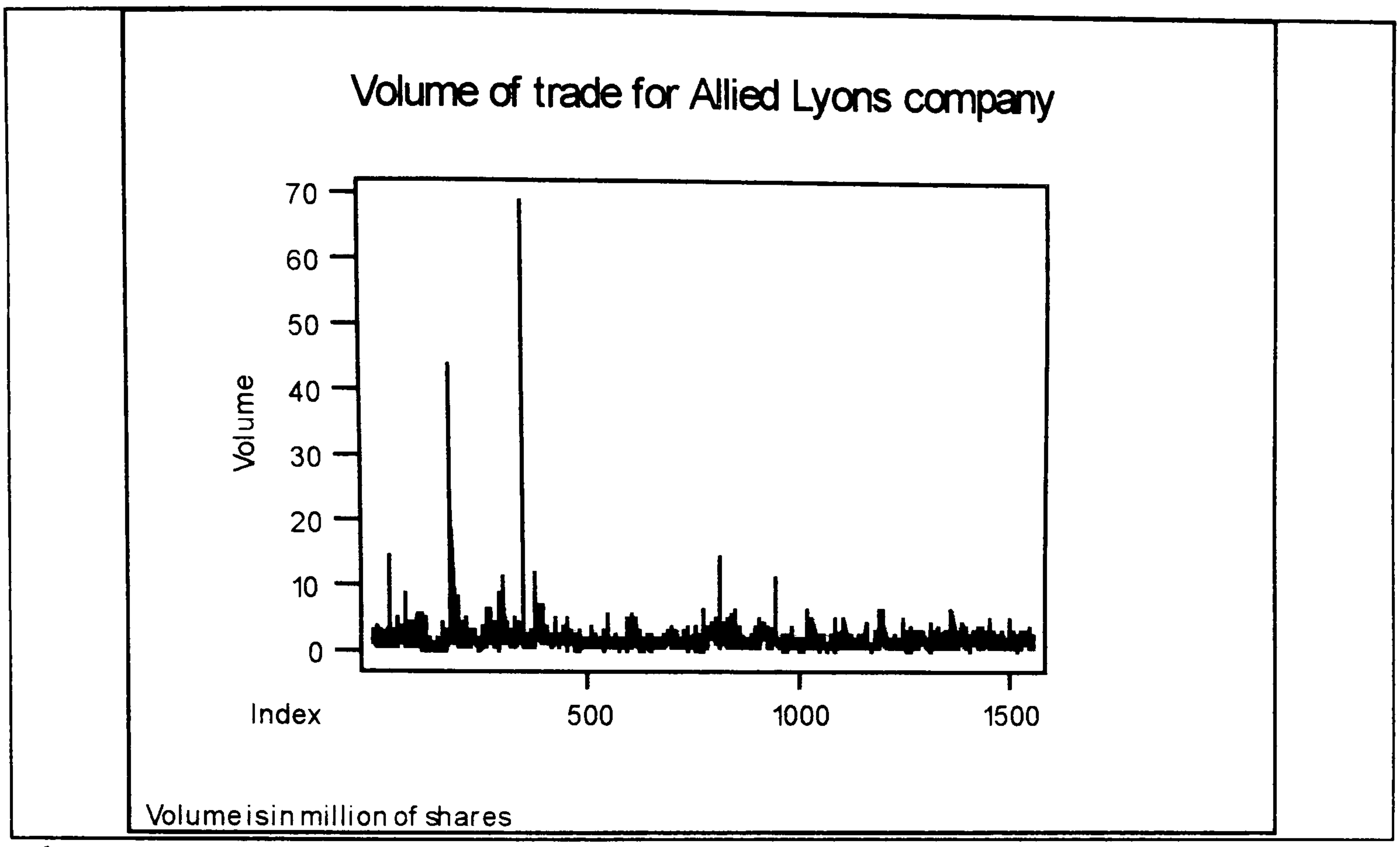

Figure 1

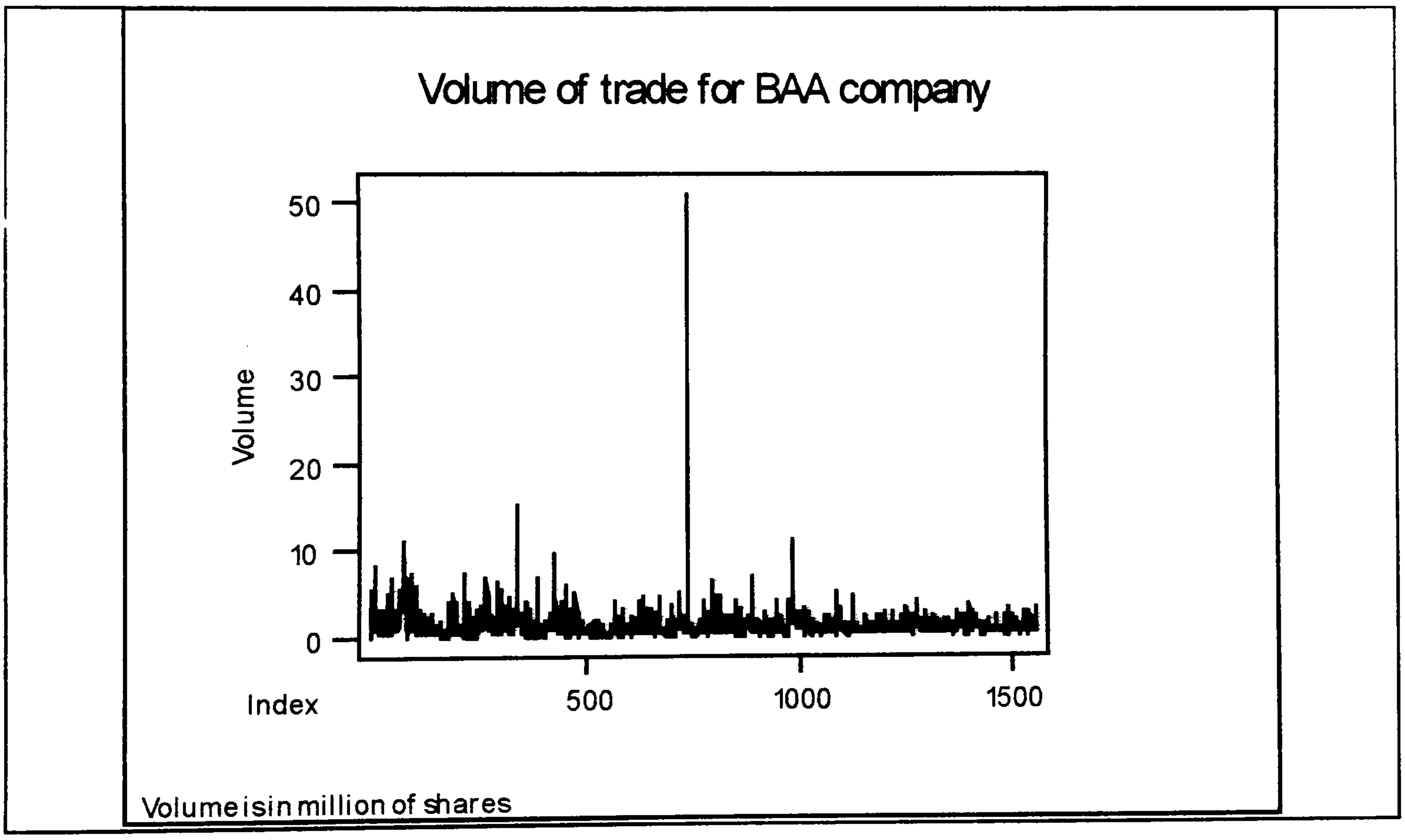

Figure 2

Table 2: The results of estimating the GARCH(1,1)-Volume model. The equation estimated is, $h_t=\alpha_0+\alpha_1\varepsilon_{t-1}^2+\beta_1 h_{t-1}+\omega_1 V_t$. Q_{10}^2 is the Mcleod-Li statistic at lag 10 for the squared residuals. The statistic is distributed as χ^2 with 10 degrees of freedom. The 5% critical value of χ_{10}^2 is 18.3.

Company	ω_1	$\alpha_1+\beta_1$	Q_{10}^2	Company	ω_1	$\alpha_1+\beta_1$	Q_{10}^2
Allied Lyons^x	.795* (13.45)	.036	72.0*	Imperial Chemicals Industries	.445* (16.41)	.047	48.8*
A.British Foods	1.518* (13.68)	.072	24.8*	Land Securities PLC	1.05* (13.14)	.017	39.6*
BAA	.738* (15.38)	.119	63.5*	Marks & Spencer	.536* (14.05)	.052	35.7*
Argyll Group	.568* (11.10)	.045	5.66	Lioyds Bank^x	.717* (10.97)	.059	54.0*
BASS	.804 (14.50)	.087	53.3*	National Westminster Bank	.672* (15.74)	.127	106*
Blue Circle Ind.	1.01* (12.44)	.227	162.0*	RTZ Corp'n	.775* (13.10)	.438	185.0*
ASDA	.741* (25.24)	.051	178.0*	Rank Organization^x	2.251* (17.49)	.104	85.2*
BOC Group	1.19* (12.84)	.04	123.0*	RMC Group	3.604* (15.88)	.103	115.0*
Barclays	.634* (14.58)	.131	67.2*	Scottish & Newcastle Brews	1.229* (20.07)	.088	72.5*
B.A.T^x	.314* (14.85)	.048	40.6*	United Biscuits	1.081* (15.70)	.101	45.8*
Boots	.765* (13.39)	.038	35.3*	Unilever PLC	.464* (8.34)	.179	53.1*
British Airways	.725* (18.60)	.086	30.8*	Prudential Corp'n	.583* (11.88)	.076	66.8*
British Gas	.197* (11.48)	.037	55.5*	Reed International	1.27* (13.75)	.078	59.5*
British Petroleum	.214* (12.43)	0.0	21.4*	Reuters Holdings PLC	2.53* (18.74)	.145	112.0*
British Telecom	.208* (11.18)	0.0	51.8*	Royal Insurance	.018* (2.20)	.991	5.12
Cable & Wireless	.480* (15.02)	.005	35.4*	Rolls-Royce	.638* (13.44)	.064	115.0*
Glaxo Holdings PLC	.438* (10.78)	.028	192*	Redland	1.80* (15.58)	.081	129.0*
Grand Metropolitan	.422* (13.87)	.089	139*	Reckitt & Colman	1.25* (13.05)	.119	11.2
Courtaulds PLC	1.26* (14.08)	.087	105*	Sainsbury	.637* (11.24)	.012	26.7*
General Accident	1.52* (15.86)	.099	131*	Tesco PLC	.497* (11.17)	.035	48.9*
Guardian Royal Exchange	1.34* (12.58)	.283	87.3*	Thorn EMI PLC^x	1.36* (14.86)	.010	10.6
General Electric Corporation	.290* (11.56)	.015	20.3*	TSB Group	.681* (12.88)	.105	26.3*
Coats Vyella PLC	1.288* (20.01)	.093	60.4*	Royal Bank of Scotland Group	.900* (14.51)	.266	40.2*
Guinness^x	.460* (18.18)	.099	44.1*	Legal & General Group	1.61* (11.58)	.163	107.0*
Hansons Trust	.208* (12.14)	.052	98.9*	Ladbroke Group PLC	1.03* (16.23)	.124	113.0*

t-statistics are in brackets. (*) significant at 5% level of significance. ^{x} The maximization routine failed to improve the objective function. Different starting values have been tried without success.

The table shows that the coefficient of volume is highly significant for all companies. Also, volatility persistence becomes negligible for most stocks, contrary to the results reported in table 1. The estimate of α_1 is still significant for 39 companies, while the estimates of β_1 are insignificant for 47 companies out of 50. The results so far are consistent with those of Lamoureux and Lastrapes (1990b). However, the analysis is taken a step further than that reported in their paper by checking the square of the standardized residuals for serial correlation. The Mcleod-Li statistics at lag 10, reported in the last column of table 2, show that there is still a highly significant GARCH pattern in the squared standardized residuals of the model for all but 4 companies (the Argyll group, Royal Insurance, Thorn EMI and Reckitt and Colman). The results indicate that although volatility persistence becomes negligible when volume of trade is included in the variance of stock price changes, ARCH effects are still present in the residuals of the model.

A possible explanation of the results lies in the complex structure of equation 8 which includes past values of both conditional volatility, h_{t-1}, and volume of trade, V_t, as explanatory variables. The complication arises because h_{t-1} is itself function of V_{t-1}. Moreover, V_t is highly correlated with its past, V_{t-1}, V_{t-2}..., which can lead to a multicollinearity problem between the explanatory variables used, h_{t-1} and V_t. This suggests that the serial dependence in volume of trade and past conditional volatility have similar information content. Therefore, either of them can be used in the conditional variance of price changes but it does not appear that both are needed.

The previous conjecture is supported by the results of Lamoureux and Lastrapes (1990b) in which the parameter estimates and standard errors of β_1 in equation 8, become zero when V_t is included for 17 out of 20 stocks. I find similar result for 30 out of the 50 stocks examined. However, the autocorrelations of the squared residuals of the model are highly significant. This means that the volume of trade does not adequately model the linear dependence in conditional volatility. Some GARCH modelling is still necessary.

The next step in the analysis focus on removing the serial dependence from volume of trade, and using unexpected volume (or volume surprises) in place of volume in equation 8. Unexpected volume is interpreted as shocks to the trading activity as in Bessembinder and Seguin (1992). Recall that according to the information flow hypothesis of Lamoureux and Lastrapes (1990b), volatility persistence of price changes is due to the serial dependence in volume, the proxy used for the rate of information arrivals. Since unexpected volume does not have any serial correlation by construction, any reduction in volatility persistence cannot be due to the serial correlation in volume. Unexpected volume is defined to be the residuals from an ARMA model.

The sample autocorrelations and partial autocorrelations are used to identify the orders of the ARMA model [Box and Jenkins (1976)]. I begin by fitting a tentative model, and then I examine the sample autocorrelations and partial autocorrelations for ten lags of the residuals to judge the model adequacy. The model is accepted as adequate if the residuals do not exhibit a significant autocorrelations and partial autocorrelations pattern, and the Ljung-Box statistic is insignificant at the 5% level. This procedure is repeated until an adequate model is achieved. I then re-estimate equations 6 to 8, replacing the volume of trade in equation 8 with the innovations in volume of trade, V_t^* (the residuals of the fitted ARMA model). Specifically, the variance equation will now take the following form:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_{t-i}\epsilon^2_{t-i} + \sum_{j=1}^{p} \beta_{t-j} h_{t-j} + \omega_2 V_t^* \tag{9}$$

Equation 9 models the variance of the unexpected price changes, ε_t, as a GARCH(p,q) process where q is the order of the autoregressive conditional heteroscedasticity, and p is the order of the moving average presentation of conditional volatility, and the unexpected volume of trade, V_t^*. The sum of the α_i and β_j measures the persistence in volatility and lies between 0 and 1. The BHHH maximization routine is used in estimating the parameters of equations 6 and 9 simultaneously. Sample autocorrelation and partial autocorrelation functions of the squared price changes are used to identify the orders p and q of the GARCH model in the same way as the specification of the ARMA model for volume of trade [see Bollerslev (1988)].

Appropriate ARMA models are fitted to the volume of trade of individual stocks to obtain the innovations in volume of trade, V_t^*[11]. In all cases, F tests support the claim that the estimated models are significant at the 5% level. The adjusted R^2 values lie between 3.6% and 25.2% implying that volume of trade can be more accurately predicted using its own past for some companies than for others. Table 3 shows the Ljung-Box statistics at lag 10 on the residuals and the Mcleod-Li statistics at lag 10 for the squared residuals from the ARMA model.

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[11] The parameter estimates of the ARMA model are not reported since the focus of this study is in obtaining white noise residuals.

Table 3: The Ljung and Box Q_{10} statistic for the residuals, and Mcleod and Li Q_{10}^2 statistic for the squared residuals from the ARMA model for the volume of trade.

Company	Q_{10}	Q_{10}^2	Company	Q_{10}	Q_{10}^2
Allied Lyons	11.22	1.41	Imperial Chemicals	3.72	.136
A. British Foods	9.55	13.7	Land Securities PLC	15.33	6.91
BAA	6.98	.957	Marks & Spencer	10.59	5.88
Argyll Group	2.86	18.7*	Lioyds Bank	12.13	11.9
BASS	2.62	4.18	National Westminster	6.78	17.6
Blue Circle	17.9	13.1	RTZ Corp'n	.228	2.38
ASDA	7.86	24.2*	Rank Organization	13.57	22.2*
BOC Group	12.42	1.77	RMC Group	4.94	6.84
Barclays	2.44	2.30	Scottish & Newcastle	11.07	.276
BATS	14.55	5.50	United Biscuits	2.90	51.2*
Boots	12.80	8.27	Uniliver PLC	12.6	59.6*
British Airways	11.37	2.87	Prudential Corp'n	16.01	2.21
British Gas	2.81	2.21	Reed International	4.80	7.33
British Petroleum	11.00	25.7*	Reuters Holdings PLC	14.29	150.0*
British Telecom	9.61	1.46	Royal Insurance	2.29	.173
Cable & Wireless	10.78	35.3*	Rolls-Royce	17.61	11.2
Glaxo Holdings PLC	4.94	6.13	Redland	1.77	12.4
Grand Metropolitan	1.40	78.8*	Reckitt & Colman	10.43	25.3*
Courtaulds PLC	12.36	54.2*	Sainsbury	2.07	3.67
General Accident	0.24	24.9*	Tesco PLC	6.77	4.41
Guardian Royal Exchange	1.57	8.96	Thorn EMI PLC	2.83	2.95
General Electric	4.38	17.9	TSB Group	6.98	1.36
Coats Vyella PLC	13.96	44.2*	Royal Bank of Scotland	8.89	6.34
Guinness	9.28	.262	Legal & General Group	5.59	4.02
Hansons Trust	11.43	22.3*	Ladbroke Group PLC	.564	45.8*

(*) Indicates significance at 5% level of significance.

The Ljung-Box statistics show that the residuals, V_t^*, are cleared of any linear dependence at the 5% level. The Mcleod-Li statistics for the squared residuals indicate that there is no significant GARCH pattern for 35 out of 50 stocks examined at the 5% level. The sample autocorrelation and partial autocorrelation functions are checked for the squared volume of trade for ten lags to identify whether the Mcleod-Li statistics are significant because of the existence of a significant pattern in the squared residuals or due to chance correlation. It was found that only 3 out of 15 sets have a significant pattern, indicating the presence of strong GARCH effects.

The next step in the analysis is to estimate equation 9 where volume is replaced by the unexpected volume obtained from the fitted ARMA. Since the unexpected volume can be negative or positive, I have to decide whether to leave it unchanged in the variance of price changes. This will be acceptable as long as the other parts of the variance are large enough to ensure positive variance. Alternatively, the unexpected volume can be transformed by taking its absolute value or square. However, equation 9 was estimated with the unexpected volume unchanged, absolute and squared for four random data sets. The highest maximum likelihood value was given by leaving the unexpected volume unchanged. This suggests that if unexpected volume is negative the conditional volatility is less than it otherwise would be. This result is consistent with the Bessembinder and Seguin (1993) results, from the futures market, that positive unexpected volume is associated with greater volatility than negative unexpected volume.

Bessembinder and Seguin (1993) relate this finding to the hypothesis that volatility is affected by existing market depth. Their explanation is quoted below:

> **A negative volume shock implies that fewer orders than expected were brought to the market, and some of the market-making capital is underutilized. When more than the expected number of orders arrives, a capital shortage can arise. If a shortage of capital has more deleterious effects on depth than a surplus, market depth during positive volume shocks will be smaller than depth during negative volume shock.**

Equation 9 is estimated for each stock in the sample with unexpected volume unchanged. Recall that the hypothesis tested in the second stage of analysis was that the volatility persistence of the GARCH model captures the serial correlation in the rate of information arrival to the market. Thus, if volume of trade, the proxy used for the rate of information arrival to the market, is serially correlated, then volatility persistence should become negligible when volume of trade is included in the variance of stock price changes. Since the unexpected volume of trade, V_t^*, is not serially correlated by construction, then a reduction in the GARCH volatility persistence cannot be attributed to V_t* capturing the serial correlation in the rate of information arrival.

As mentioned earlier, the Box-Jenkins (1976) methodology is used to identify the orders of the GARCH model on the lines suggested by Bollerslev (1988). However, the GARCH(1,1) specification is used if there is not a clear pattern observed in the data. A higher order model is estimated if the Mcleod-Li statistic combined with examination of the autocorrelations and partial autocorrelations of the squared residuals indicate model inadequacy. The results are reported in table 4.

Table 4: The results of estimating the GARCH(p,q)-Unexpected Volume model. p and q are the orders of the moving average and autoregressive specifications of the GARCH model respectively. α's+β's is the measure of volatility persistence. ω_2 is the coefficient of the unexpected volume. Q_{10}^2 is the Mcleod and Li (1983) statistic at lag 10 for the squared residuals. The statistic is distributed as χ^2 with 10 degrees of freedom. The critical value for χ_{10}^2 is 18.3 at the 5% level.

Company	(p,q)	α's+β's	ω_2	Q_{10}^2	Company	(p,q)	α's+β's	ω_2	Q_{10}^2	Company	(p,q)	α's+β's	ω_2	Q_{10}^2
Allied Lyons	(1,2)	.583	.465* (15.37)	7.22	Boots	(0,4)	.216	.630* (13.86)	12.4	Guardian Royal Exchange	(1,1)	.757	.799* (11.16)	17.4
A. British Foods	(0,3)	.144	1.031* (20.34)	8.92	British Airways	(1,1)	.587	.588* (16.73)	14.4	General Electric Corp'n	(0,2)	.074	.122* (18.81)	10.5
BAA	(1,1)	.772	.299* (16.41)	11.8	British Gas	(0,2)	.110	.052* (19.64)	23.4*	Coats Vyella PLC	(1,1)	.573	.814* (19.89)	16.7
Argyll Group	(1,1)	.358	.509* (11.66)	9.05	British Petroleum	(1,1)	.749	.138* (12.35)	18.7*	Guinness	(0,3)	.175	.100* (26.62)	16.9
BASS	(1,5)	.414	.692* (18.14)	23.1*	British Telecom	(0,6)	.148	.125* (17.78)	10.2	Hansons Trust	(1,6)	.415	.178* (14.66)	15.1
Blue Circle	(1,6)	.526	.911* (11.32)	9.14	Cable & Wireless	(1,2)	.393	.343* (20.74)	21.1*	Imperial Chemicals	(0,3)	.196	.188* (24.39)	18.4*
ASDA	(1,1)	.904	.223* (24.50)	12.2	Glaxo Holdings	(0,6)	.404	.323* (12.80)	30.6*	Land Securities	(0,4)	.229	.548* (14.20)	14.1
BOC Group	(1,6)	.40	.962* (15.69)	11.4	Grand Metropolitan	(0,6)	.330	.319* (15.26)	12.1	Marks & Spencer	(0,3)	.295	.344* (15.61)	14.4
Barclays	(2,1)	.877	.308* (12.52)	12.6	Courtaulds PLC	(2,1)	.702	1.00* (15.67)	16.7	Lioyds Bank	(0,4)	.212	.423* (14.08)	14.9
B.A.T Industries	(1,1)	.611	.253* (16.25)	16.8	General Accident	(2,1)	.774	.955* (14.19)	21.3*	National Westminster	(1,4)	.674	.554* (15.42)	12.9

t-statistics are in brackets. (*) significantly different at 5%.

x Q_9^2 is 10.7 which is not significant at 5% level of significance.

Table 4 (*Cont.*): The results of estimating the GARCH(p,q)-Unexpected Volume model. The results of estimating the GARCH(p,q)-Unexpected Volume model. p and q are the orders of the moving average and autoregressive specifications of the GARCH model respectively. α's+β's is the measure of volatility persistence. ω_2 is the coefficient of the unexpected volume. Q_{10}^2 is the Mcleod and Li (1983) statistic at lag 10 for the squared residuals. The statistic is distributed as χ^2 with 10 degrees of freedom. The critical value of χ_{10}^2 is 18.3 at the 5% level.

Company	(p,q)	α's + β's	ω_2	Q_{10}^2	Company	(p,q)	α's+β's	ω_2	Q_{10}^2
RTZ Corp'n	(1,6)	.580	.800* (12.44)	22.0*	Rolls-Royce	(0,3)	.469	.307* (15.46)	15.3
Rank Organization	(0,6)	.283	.889* (17.62)	10.7	Redland	(0,5)	.408	.514* (15.34)	14.3
RMC Group	(1,5)	.374	3.37* (15.32)	13.1	Reckitt & Colman	(0,1)	.135	.957* (14.48)	13.7
Scottish & Newcastle Brews	(0,5)	.245	.467* (36.70)	3.07	Sainsbury	(0,5)	.169	.451* (14.98)	17.4
United Biscuits	(0,4)	.233	.506* (20.60)	17.4	Tesco PLC	(0,4)	.211	.226* (17.46)	19.1*
Unilever PLC	(0,6)	.273	.411* (11.75)	8.67	Thorn EMI PLC	(0,2)	.086	.942* (19.0)	8.42
Prudential Corp'n	(0,5)	.396	.312* (14.30)	9.21	TSB Group	(0,2)	.131	.405* (22.22)	15.3
Reed International	(0,4)	.287	.668* (18.32)	17.7	Royal Bank of Scotland	(0,2)	.365	.338* (20.09)	32.8*
Reuters Holdings	(2,1)	.747	1.80* (16.51)	14.2	Legal & General	(0,6)	.430	.907* (13.73)	12.1
Royal Insurance	(1,1)	.955	.326* (10.42)	7.06	Ladbroke Group	(0,4)	.450	.430* (14.95)	16.9

t-statistics are in brackets. (*) significantly different at 5%.

The table shows volatility persistence has dropped considerably from the results of the first stage analysis, though it is not negligible like its counterparts in table 2. This suggests that the inclusion of unexpected volume leads to a reduction but not elimination of the GARCH volatility persistence. However, this reduction cannot be attributed to the unexpected volume capturing the serial dependence in the rate of information arrival. It seems that the Bessembinder and Seguin (1992 and 1993) interpretation of unexpected volume as shocks to trading activity is more consistent with the results. Mcleod-Li statistics suggest that the standardized residuals of the model do not exhibit any significant GARCH pattern for 40 stocks at 5% level and for 47 stocks at 1% level.

As mentioned earlier, the observed reduction in volatility persistence cannot be attributed to unexpected volume capturing the serial dependence in the rate of information arrivals. A possible explanation is that innovational outliers happen in both volume of trade and stock price changes simultaneously. This explanation is supported by the finding of Ying (1966), Schwert (1989), and Gallant *et al.* (1992) that there is a strong correlation between shocks to volume and volatility. Thus, conditioning stock price changes on unexpected volume removes the effect of innovational outliers on stock price changes which leads to a reduction in volatility persistence[12].

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[12] See also Tsay (1986a) for the effect of additive and innovational outliers on model specification.

A simple procedure is applied to detect the correlation in the timing of innovational outliers in both volume and price changes. Let DR_t be a dummy variable which takes the value 1 if $R_t < (\mu_R - n\sigma_R)$ or $R_t > (\mu_R + n\sigma_R)$, and 0 otherwise, where R_t is the stock price change, μ_R and σ_R are its unconditional mean and variance respectively, and n takes the values 2, 3, and 4. Let DV_t be a dummy variable which takes the value 1 if $V_t < (\mu_v - n\sigma_V)$ or $V_t > (\mu_v + n\sigma_V)$, and 0 otherwise, where V_t is the volume of trade for a particular stock, μ_v and σ_V are its unconditional mean and variance respectively, and n takes the values 2,3, and 4. On the same line, DV_t^* is a dummy variable which takes the value 1 if $V_t^* < (\mu_{V*} - n\sigma_{V*})$ or $V_t^* > (\mu_{V*} + n\sigma_{V*})$, and 0 otherwise, where V_t^* is the unexpected volume of trade for a particular stock, μ_{V*} and σ_{V*} are its unconditional mean and variance respectively, and n takes the values 2,3, and 4.

Each of these dummy variables takes the values 1 if an observation is identified to be a number of standard deviation away from its mean. The next step is to compute the correlation coefficient between DR_t & DV_t, and DR_t & DV_t^*, for different values of n of 2, 3, and 4. The value of the correlation coefficient between DR_t & DV_t, for example, depends on the covariance of the two dummies which is, $Cov(DR_t, DV_t) = E(DR_t.DV_t) - E(DR_t).E(DV_t)$, where E is the expectation operator. Note that $(DR_t.DV_t)$ takes the value 1 if $DR_t = DV_t = 1$, and therefore a big value for $E(DR_t.DV_t)$ indicates that the outliers of both series tend to happen at the same time. On the other hand, if the timing of the outliers of both series are independent of each other, then the correlation coefficient should be zero since

$E(DR_t.DV_t)=E(DR_t).E(DV_t)$. At the same time if the correlation coefficient tends to increase for larger values of n, then it can be deduced that the timing of large outliers in both series tend to be more associated with each other. Table 5 shows the results.

Table 5: The correlation coefficient between the price changes and each of volume and unexpected volume. The columns entitled Timing V_t and V_t^* contain the corr(DR_t,DV_t) and corr(DR_t,DV_t^*) respectively, for n= 2, 3, and 4.

Company	Corr of R_t &		Timing(n=2)		Timing (n=3)		Timing (n=4)	
	V_t	V_t^*	V_t	V_t^*	V_t	V_t^*	V_t	V_t^*
Allied Lyons	0.061	0.065	0.234	0.208	0.348	0.328	0.460	0.460
A. British Foods	0.085	0.088	0.098	0.099	.089	0.089	0.113	0.124
BAA	-0.008	-0.021	0.140	0.161	0.116	0.120	0.129	0.138
Argyll Group	0.007	-0.0009	0.189	0.171	0.271	0.322	0.461	0.317
BASS	0.048	0.052	0.235	0.250	0.306	0.319	0.386	0.400
Blue Circle	0.212	0.216	0.199	0.177	0.159	0.126	0.108	0.129
ASDA	0.047	0.045	0.251	0.277	0.274	0.158	0.172	0.195
BOC Group	-0.075	-0.086	0.135	0.162	0.205	0.168	0.189	0.215
Barclays	-0.027	-0.043	0.308	0.302	0.339	0.381	0.355	0.421
B.A.T	0.263	0.268	0.244	0.224	0.341	0.355	0.317	0.576
Guardian	.160	.160	.210	.199	.269	.222	.498	.520
General Electric	.141	.138	.163	.136	.099	.108	.186	.193
Coats Vyella	.044	.044	.137	.147	.115	.112	.094	.080
Guinness	.100	.100	.132	.132	.123	.130	.330	.351
Hansons Trust	.120	.122	.180	.237	.215	.220	.331	.331

Company	Corr R_t &		Timing(n=2)		Timing(n=3)		Timing(n=4)	
	V_t	V_t^*	V_t	V_t^*	V_t	V_t^*	V_t	V_t^*
Boots	.014	.040	.270	.264	.312	.269	.254	.285
British Airways	.083	.095	.224	.235	.245	.256	.312	.323
British Gas	.047	.046	.207	.220	.268	.302	.407	.446
British Petroleum	-.042	-.019	.235	.210	.418	.355	.478	.520
British Telecom	.103	.108	.220	.245	.185	.185	.375	.351
Cable & Wireless	-.013	.017	.224	.231	.328	.355	.390	.446
Glaxo Holdings	.083	.104	.169	.220	.057	.059	-.002	-.002
Grand Metropolitan	.053	.050	.216	.209	.258	.258	.119	.130
Courtaulds	.115	.130	.129	.151	.135	.147	.298	.398
General Accident	.109	.093	.114	.123	-.014	.033	-.002	-.002
Imperial Chemicals	.174	.158	.245	.240	.471	.442	.219	.355
Land Securities	.065	.071	.216	.191	.254	.229	.274	.147
Marks & Spencer	.031	.033	.262	.268	.420	.326	-.003	.137
Lioyds Bank	.104	.128	.115	.110	.082	.101	.189	.189
National Westminster Bank	.078	.075	.189	.192	.130	.157	.147	.173

Table 5 (*Cont.*): The correlation coefficient between the price changes and each of volume and unexpected volume. The columns entitled Timing V_t and V_t^* contain the corr(DR_t,DV_t) and corr(DR_t,DV_t^*) respectively, for n= 2, 3, and 4 .

Company	Corr R_t		Timing(n=2)		Timing(n=3)		Timing(n=4)	
	V_t	V_t^*	V_t	V_t^*	V_t	V_t^*	V_t	V_t^*
RTZ Corp'n	.055	.050	.179	.241	.167	.127	.124	.137
Rank Organization	.109	.110	.226	.215	.299	.311	.247	.346
RMC Group	.116	.114	.184	.204	.159	.153	.313	.330
Scottish & Newcastle Brews	.016	.020	.264	.264	.569	.569	.598	.598
United Biscuits	.213	.236	.235	.212	.283	.269	.388	.487
Unilever PLC	.020	.022	.199	.211	.067	.073	-.001	-.002
Prudential Corp'n	.075	.087	.076	.086	.173	.186	.201	.221
Reed International	.101	.107	.246	.254	.214	.179	.163	.171
Reuters Holdings PLC	-.054	-.043	.277	.316	.283	.256	.316	.343
Royal Insurance	.028	.013	.155	.141	.157	.080	.129	.129

Company	Corr		Timing(n=2)		Timing(n=3)		Timing(n=4)	
	V_t	V_t^*	V_t	V_t^*	V_t	V_t^*	V_t	V_t^*
Rolls-Royce	.070	.082	.204	.214	.108	.160	.124	.140
Redland	.174	.182	.224	.224	.207	.179	.242	.118
Reckitt & Colman	.174	.173	.100	.089	.124	.174	.201	.189
Sainsbury	-.089	-.086	.260	.222	.250	.250	.276	.332
Tesco PLC	.045	.055	.197	.114	.116	.130	-.002	-.002
Thorn EMI PLC	.095	.106	.257	.237	.316	.290	.384	.384
TSB Group	.208	.223	.245	.189	.162	.162	.154	.147
Royal Bank Of Scotland Group	.018	.023	.217	.228	.123	.116	.194	.219
Legal & General Group	.143	.147	.249	.238	.195	.199	.401	.444
Ladbroke Group PLC	.075	.048	.218	.215	.262	.211	.103	.108

The first two columns present the correlation coefficient between stock price changes, and volume and unexpected volume respectively. This coefficient does not seem to be high, indicating a weak linear relationship between the two variables. For example, the correlation coefficient between price changes and volume for Allied Lyons company is 0.061 indicating that regressing price changes on volume of trade will yield an R^2 of 0.004. However, the correlation coefficients between DR_t & DV_t for the same company is 0.234, 0.348, and 0.460 for n equal to 2, 3, and 4 respectively. This suggests that there is a strong association in the timing of outliers for this company, and this association gets higher for the more unusual observations (n=4). Allied Lyons company is analyzed because it is the first company in table 5. Examining other companies suggests that in general the correlation between the dummies for outliers gets bigger as n increases[13]. The same picture is true for the correlation between DR_t and DV_t^*.

The association in the timing of outliers could explain the reduction in the volatility persistence of the GARCH model when volume of trade is introduced in the conditional volatility. In other words, conditioning price changes on volume of trade may have removed some of the outliers' affects on the volatility persistence of the GARCH model. This is consistent with the results of Balke and Fomby (1992) that accounting for outliers' effects in financial time series leads to a reduction in

[13]There are some companies which are exceptions to this conclusion. For example, the second company in the table, Associated British Foods (ABF). The correlation coefficient between price changes and volume is 0.085 suggesting an R^2 of 0.007. The correlation coefficients between DR_t & DV_t are 0.098, 0.089, and 0.113 for n equal to 2, 3, and 4 respectively.

volatility persistence.

The observed strong association in the timing of outliers in price changes and volume of trade is consistent with the idea of a limited market depth for each security. The market can clear reasonable quantities of stocks at the market prices but for large quantities outside this range the price changes necessary to clear the market will be a function of the volume of trade. Of course, prices can also fall or rise significantly with a very low level of trade if there is new information in the market requiring such adjustment. As a matter of fact, the seven companies analyzed in the next section have one or two unusual outliers in volume of trade which are not associated with huge volatility.

Section *B*

This section analyzes the data sets for seven companies which behave differently from the rest of the data. One feature of the fifty companies analyzed so far is the existence of highly significant autocorrelation and partial autocorrelation patterns in their volume series. However, seven companies do not show such a significant pattern in their volumes of trade. When I examine these data sets in detail, I find that there is one huge outlier in 6 of the 7 companies and two in the other company, and these almost destroy the pattern in the serial correlation structure of volume of trade. Table 6 displays some descriptive statistics for these companies.

Table 6: Some Descriptive Statistics on the Price Changes & Volume for 7 companies.

μ and σ are the mean and standard deviation, respectively. Out refers to the outlier magnitude.

Companies	Volume(000,000)				Price Changes%		
	μ	σ	Out	Out/σ	Out	σ	Out/σ
Sears	4.408	8.448	309.3598	36.619	-3.884	1.943	-1.998
Pearson	1.039	3.183	115.4341	36.265	-0.655	1.450	-0.451
Standard Chartered	0.940	2.115	60.6210	28.662	-1.853	1.780	-1.041
Enterprise Oil PLC	1.120	5.976	232.3965	38.888	-6.920	1.719	-4.025
Cadbury Schwepps	2.124	6.659	254.2343	38.179	-7.111	1.522	-4.672
Sun Alliance	1.683	3.304	118.5155	35.870	-3.750	1.711	-2.191
Commercial Union							
(Volume)	1.305	4.745	133.627	28.161	1.071	1.377	0.778
(Adjusted Volume)	1.219	3.355	127.755	38.078	0.0	1.377	0.0

The table shows the size of the volume outliers, and how many times they are bigger than the volume's standard deviation. It also shows the price change on the corresponding day and how many times it is greater than the standard deviation of price changes. A comparison of the figures of the table shows that the huge volumes of trade were not associated with large changes in the stock prices, except for two companies: Enterprise Oil and Cadbury Schwepps. These volume outliers were carefully researched to check their validity. It was found that they are not recording errors and they had, in some cases, been expected by the market. For example, a

USA insurance company, Chubb, sold a huge part of its stake in Sun Alliance in the UK market as a part of an agreed shift of strategic stakes. Consequently, price changes did not respond dramatically despite the huge volume of trade recorded. Accordingly, there is a strong argument for contending that these data sets need to be treated differently from the first 50 sets.

The first and second stages of analysis are as before. In the first stage, the AR(1)-GARCH(1,1) is fitted to the data, and in the second the volume of trade is added to the variance of price changes. The results are presented in table 1 of the appendix to this chapter. The results are completely in line with the results for the first 50 companies, in that the GARCH parameters become insignificant for most companies when the volume of trade is introduced into the variance equation.

To consider the effect of huge outliers that have been identified in volume of trade, the volume of trade is regressed on a dummy variable which takes the value 1 on the day of the outlier and zero otherwise[14]. Then the sample autocorrelation and partial autocorrelation functions are used to decide the orders of the ARMA models. The results are in table 2 in the appendix. They show that the Ljung-Box statistics at lags 5 and 10 for adjusted volume are highly significant at any level of significance in contrast with the Ljung-Box statistics on the volume of trade. Ljung-Box statistics indicate that the standard residuals from the ARMA models are uncorrelated. Also,

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[14] For one company, Commercial Union Assurance Co., the volume of trade is regressed on two dummies since there are two outliers in the data set.

Mcleod-Li statistics for the squared residuals show that there is no GARCH effect for 5 out of the 7 companies examined.

The GARCH-unexpected volume model of equation 9 is fitted to the data, and the Box and Jenkins (1976) methodology is used to identify the orders p and q of the GARCH models. The results are consistent with the results of the first section. The GARCH parameters are highly significant as well as the coefficient on the unexpected volume of trade, ω_2. The residuals are cleared from GARCH effects for 5 out of 7 companies at 5% level and 6 out of 7 at 1% level.

There is one company whose results differ from the rest. It is the Sun Alliance and London Insurance Co. Volatility persistence as measured by the sum of α_1 and β_1 is 1 in table 1, the AR(1)-GARCH(1,1) model, indicating that the unconditional variance does not exist and the series is non-stationary. This result does not change when the volume of trade is added to the GARCH variance equation. In fact, ω_1, the coefficient on volume of trade is far from being significant. Although, the coefficient of the unexpected volume, ω_2, is highly significant in table 3, volatility persistence is still high at 0.895. These results indicate that the effects of shocks to volatility tend to persist longer for this data set than for the others, even after taking into account the effect of the unexpected volume of trade.

Finally table 4 in the appendix shows the correlations, in the level and outlier's timing, of price changes with volume, adjusted volume, and adjusted volume residuals. Again, the results are consistent with those of section 1. There seems to be a high correlation in the timing of innovational outliers in volume and price changes.

5. Conclusions

The chapter's empirical results, based on data drawn from the UK market, are different from those of Lamoureux and Lastrapes (1990b). The results are consistent with theirs in that volatility persistence, as measured by the GARCH model, becomes negligible when volume of trade is introduced in the variance equation of price changes. However, when the autocorrelation functions of the squared standardized residuals are checked, it is found that they exhibit a highly significant GARCH pattern.

When unexpected volume is used in the variance of price changes in place of volume, it is found that the GARCH volatility persistence is reduced. Since unexpected volume is not serially correlated, this result cannot be attributed to volume capturing the effects of the serial dependence in volatility, the proxy used by Lamoureux and Lastrapes (1990b) for the information flow rate. It seems that my results are more consistent with the interpretation of Bessembinder and Seguin (1992 and 1993) of unexpected volume as shocks to trading activity. Moreover, evidence is uncovered which supports the finding of Bessembinder and Seguin (1993) that positive unexpected volume has a greater affect on volatility than negative unexpected volume. Bessembinder and Seguin (1993) argues that this evidence supports the hypothesis that volatility is affected by existing market depth for a security.

It is argued that the relevance of the unexpected volume of trade is due to a strong association in the timing of innovational outliers in both the price changes and volume of trade series. This factor may also contribute to the noted reduction in persistence in the GARCH component. The results suggest that unexpected volume can help in forecasting the future conditional volatility as long as our emphasis is on forecasting rather than explaining the joint dynamics of volume and price changes. This finding is inconsistent with that of Lamoureux and Lastrapes (1994) that contemporaneous volume and squared price changes are noisy instruments for predicting the future conditional volatility.

At least one research area remains to be studied in more details. If unusual values for volume and price changes tend to happen at the same time, then it is possible that there is a threshold for volume of trade and volatility. In other words, if volume exceeds a specific level, then it triggers a big movement in the price of the stock. Therefore, a threshold model for price changes and volume may be a useful way to proceed. Connecting this model to Bessembinder and Seguin's market depth argument could prove to be useful. An interesting point is if the threshold for volume of trade for one company is higher than another, then it is possible that the former has more depth than the latter. In other words, it takes less volume to get big movements in prices in the case of the latter.

Appendix

Table 1: The results of estimating the GARCH(1,1) model, $h_t=\alpha_0+\alpha_1\varepsilon_{t-1}^2+\beta_1 h_{t-1}$, and the GARCH(1,1)-Volume model, $h_t=\alpha_0+\alpha_1\varepsilon_{t-1}^2+\beta_1 h_{t-1}+\omega_1 V_t$. Q_{10}^2 is the Mcleod and Li (1983) statistic at lag 10 for the price changes (R), for the GARCH(1,1) residuals (U), and for the GARCH(1,1)-Volume residuals (E). The statistic is distributed as χ^2 with 10 degrees of freedom. The critical value of χ_{10}^2 is 18.3 at the 5% level.

Company	GARCH(1,1)			GARCH(1,1)+Volume		
	$\alpha_1+\beta_1$	Q_{10}^2Y	Q_{10}^2U	ω_1	$\alpha_1+\beta_1$	Q_{10}^2E
Sears	.923	198*	17.6	.664* (13.27)	.059	96.3
Pearson	.968	264*	14.3	1.47* (17.73)	.293	254.0*
Standard Chartered	.983	223*	8.28	2.65* (19.05)	.148	144.0*
Enterprise Oil PLC	.047	67.0*	.984	2.01* (25.48)	.075	30.9*
Cadbury Schwepps	.951	473*	8.71	.908* (14.50)	.015	92.2*
Sun Alliance & London Ins.	1.0	68.5*	2.46	.002 (.665)	1.0	2.48
Commercial Union Assurance Co.	.950	65.8*	6.85	.995* (10.66)	.054	66.8*

t-statistics in brackets. (*) indicates significance at 5% level.

Table 2 : The Ljung and Box Q statistics on the volume, adjusted volume, and the residuals from the ARMA model. Also reported the Mcleod-Li statistics for the squared residuals from the ARMA model for volume.

Company	Volume		Adjusted Volume		Q_{10}	Q_{10}^2
	Q_5	Q_{10}	Q_5	Q_{10}		
Sears	8.25	14.9	332.0*	438*	.517	18.2
Pearson	2.06	3.13	74.0*	110.0*	.693	.080
Standard Chartered	4.47	5.39	36.0*	45.2*	9.81	.111
Enterprise Oil PLC	.813	.912	150.0*	263.0*	.615	.709
Cadbury Schwepps	4.67	6.65	646.0*	853.0*	5.58	284.0*
Sun Alliance & London Ins.	19.3*	24.7*	357.0*	436.0*	8.41	3.10
Commercial Union Assurance Co.	6.44	8.87	583.0*	886.0*	11.19	50.8*

(*) indicates significance at 5% level.

Table 3 : The results of estimation of the GARCH(p,q)-unexpected volume model.p and q are the orders of the moving average and autoregressive specifications of the GARCH model respectively. α's+β's is the measure of volatility persistence. ω_2 is the coefficient of the unexpected volume. Q_{10}^2 is the Mcleod-Li statistic at lag 10 for the squared residuals. The statistic is distributed as χ^2 with 10 degrees of freedom. The critical value for χ_{10}^2 is 18.3 at the 5% level.

Company	(p,q)	α's+β's	ω_2	Q_{10}^2
Sears	(1,5)	.437	.603* (15.32)	26.7*
Pearson	(0,5)	.420	.515* (22.07)	21.6*
Standard Chartered	(0,5)	.646	.898* (15.72)	14.9
Enterprise Oil PLC	(0,3)	.184	.969* (36.39)	11.1
Cadbury Schwepps	(0,6)	.364	.361* (19.68)	11.2
Sun Alliance & London Ins.	(2,1)	.895	.573* (16.47)	8.60
Commercial Union Assurance Co.	(0,3)	.217	.805* (16.49)	15.3

t-statistics in brackets. (*) indicates significance at 5% level.

Table 4: The correlation coefficient between the price changes and each of volume and unexpected volume. The columns entitled Timing V_t and V_t^*contain the corr(DR_t,DV_t) and corr(DR_t,DV_t^*) respectively, for n= 2, 3, and 4. AV_t is the volume series adjusted by taking the effect of the outliers out using suitable dummy variables. AV_t^* is the residuals series of the ARMA model for volume of trade.

Company	Correlation R_t &			Timing(n=2)			Timing(n=3)			Timing(n=4)		
	V_t	AV_t	AV_t^*	V_t	AV_t	AV_t^*	V_t	AV_t	V_t^*	V_t	AV_t	AV_t^*
Sears	.011	.146	.143	.147	.333	.268	-.002	.253	.243	-.001	.130	.137
Pearson	.048	.145	.138	.073	.064	.026	.159	.079	.087	.211	.211	.211
Standard Chartered	.035	.079	.078	.077	.127	.127	-.006	.057	.050	-.003	-.004	-.004
Enterprise Oil PLC	-.076	.131	.126	.163	.283	.259	.351	.319	.185	.377	.213	.213
Cadbury Schwepps	-.031	.302	.320	.252	.369	.310	.418	.396	.438	.470	.551	.551
Sun Alliance & London Ins.	.014	.147	.142	.141	.171	.199	-.004	.072	.091	-.002	.242	.254
Commercial Union Assurance Co.	.041	.137	.136	-.008	.087	.075	-.003	.028	.033	-.001	-.003	-.003

Chapter 5

Modelling the Conditional Distribution of the UK ALL Equity Index

1. Introduction

Although the Autoregressive Conditional Heteroscedasticity model with conditional normal innovations generates some degree of unconditional kurtosis, it is typically less than adequate to fully account for the fat tails of stock price changes [see Baillie and Bollerslev (1989), Bollerslev *et al.* (1992), Bera and Higgins (1992), and Diebold and Lopez (1995)]. Other distributions which allow for fatter tails that have been employed with ARCH related models include the stable Paretian distribution (McCulloch (1985)), the Student's t distribution (Bollerslev (1987)), the normal poisson distribution (Jorion (1988)), the normal-lognormal mixture (Hsieh (1989)), and the generalized error distribution (Nelson (1991)).

This chapter models the conditional mean and variance of the UK FT-ALL price changes (returns hereafter) using two distributions, Student's t and generalized error distribution (GED). The FT-ALL Share Index returns from 4/1/1988 to 28/2/1994 are analyzed. The period is chosen to exclude the crash period from the analysis. The rationale for this is provided by the results presented in chapter 3 suggesting that there were two significant shifts in the variance of the FT-ALL price changes around, the oil crisis in 1973-74 and the 1987 market crash. Therefore, it seems interesting to study the properties of the UK stock prices during a period of more normal activity. The chapter attempts to analyze the data set as comprehensively as possible by considering several factors which may affect stock returns. These include any ARMA effects in the mean or/and the variance of returns; allowing for the days or periods that appear to generate abnormal returns, such as the Monday, January, turn of the month and holiday effects; and finally any asymmetries in stock returns due to the leverage effect.

2. The Data and the Statistical Properties of the UK FT-ALL Returns

The data are extracted from Data Stream and contain 1558 daily values for the FT-ALL share index (p_t) from 4/1/1988 till 28/2/1994. The returns, (y_t), are measured as $100*\log_e(p_t/p_{t-1})$. Table 1 contains a range of descriptive statistics for the returns series, y_t.

Table 1: Sample Statistics on Daily FT-ALL Percentage Returns (1557 observations)

Mean	Variance	Skewness	Excess Kurtosis	χ^2	Jarque-Bera	Q_{10}	Q_{20}
.041*	.597	.157*	3.921*	255.68*	1003.8*	39.1*	51.6*
		(2.53)	(31.62)				

(*) significant at 5% level.

The table shows that the skewness coefficient is statistically significant from zero at the 5% level but not at the 1% level. However, it is the departure from normality which is the main feature of the data. The excess kurtosis is highly significant indicating that the empirical distribution is heavy tailed. The Chi-square[1] and Jarque-Bera[2] tests for normality[3] indicate a huge departure from normality. The Ljung and Box (1978) Q-statistics at lags 10 and 20 are highly significant suggesting that the null hypothesis of uncorrelated returns should be rejected. Table 2 reports the first ten autocorrelations and partial autocorrelations for the returns series.

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[1]The test is

$$\sum_{i=1}^{k} \frac{(a_i - e_i)^2}{e_i} \sim \chi^2_{k-1}$$

where k is the number of intervals in which the data is classified (k=22), a_i is the number of actual observed frequency in each class, and e_i is the number of expected frequency in each interval from a normal distribution [see Shapiro *et al.* (1968)].

[2] The test is

$$J\text{-}B = (T/6)\, b_1 + (T/24)\,(b_2 - 3)^2$$

where

$$\sqrt{b_1} = \sigma_y^{-3} \sum (y - \bar{y})^3 / T$$

and

$$b_2 = \sigma_y^{-4} \sum (y - \bar{y})^4 / T$$

[3] The Chi-square test is distributed as χ_{21}^2 i.e. the standardized residuals are divided into 22 classes. The Jarque-Bera test is distributed as χ_2^2 in large samples [see Harvey (1993b)].

Table 2: The Sample Autocorrelations and Partial Autocorrelations of Returns

k	1	2	3	4	5	6	7	8	9	10
ρ_k	.117*	-.0005	.009	.058*	.034	0.0	-.024	.046	.023	.058*
ϕ_k	.117*	-.01	.011	.056*	.021	-.005	-.024	.049	.009	.056*

(*) significant at 5% level.

As is well known, stock returns often exhibit a significant small first order autocorrelation [see Bollerslev *et al.* (1993)]. Besides, there are significant autocorrelations and partial autocorrelations at lags 4 and 10 at the 5% level of significance. This suggests that an ARMA(0,1) process might provide a good approximation of the data[4].

ARMA(1,0) and ARMA(0,1) models are fitted and compared: it is found that both models seem to fit the data well. The standard errors of the two models take the value 0.7659 for the ARMA(1,0) and 0.7658 for the ARMA(0,1) suggesting that both models explain almost the same variation in y_t. Therefore, it is not possible to choose between these models if the objective criteria employed is the minimization of the sum of the squared residuals.

The same conclusion is reached if choice of the model is based on their ability

[4]An ARMA(m,p) process refers to AutoRegressive Moving Average process where m is the number of autoregressive parameters and p is the number of moving average parameters [see Hamilton (1994)].

to clear the serial correlation from the residuals. In fact, the autocorrelations and partial autocorrelations of both models behave in the same way. For example, both models fail to take into account the small but statistically significant autocorrelations and partial autocorrelations at lags 4 and 10, leaving their values around the ones in table 2. Therefore, I decided to approximate this process using an ARMA(1,0) model. To generate an uncorrelated sequence of returns, u_t, the ordinary least squares residuals are obtained from the following regression:

$$y_t = b_0 + b_1\, y_{t-1} + u_t, \tag{1}$$

To check that the regression residuals (u_t) are uncorrelated and do not exhibit ARCH effects, the Ljung-Box and Mcleod-Li statistics are computed for the regression residuals for lags 10 and 20. The results are presented in table 3.

Table 3: The Ljung-Box statistic (Q), and Mcleod-Li statistic (Q^2) for the regression residuals (u_t).

Q_{10}	Q_{20}	Q_{10}^2	Q_{20}^2
17.7	28.7	52.1*	88.7*

(*) significant at 5% level.

The Ljung and Box statistics at lags 10 and 20 are not significant, indicating that the null of uncorrelated residuals can be "accepted." However, the Mcleod-Li statistics on the squares of the residuals show a different picture. They are highly statistically significant indicating the presence of significant autocorrelations (ARCH effects) in the squared residuals. The GARCH model of Bollerslev (1986) is used to model the FT-ALL returns. A GARCH process of orders p and q, denoted as GARCH(p,q) can be described as follows:

$$u_t = \sigma_t z_t \quad where \quad z_t \sim iid\ D(0,1), \tag{2}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 . \tag{3}$$

Where u_t are the regression residuals which are identically, and independently distributed variables with a conditional D distribution with mean zero and variance one. As explained in chapter 2, the sample autocorrelations and partial autocorrelations can be used to identify the orders p and q of the GARCH model. Table 4 shows the autocorrelations and partial autocorrelations of u_t^2 for ten lags.

Table 4: The autocorrelations and partial autocorrelations of u_t^2

k	1	2	3	4	5	6	7	8	9	10
ρ_k	.111*	.094*	.039	.023	.030	.026	.059*	.015	.029	.059*
ϕ_k	.111*	.083*	.020	.009	.022	.022	.049	.0008	.017	.050

(*) Indicates significance at 5% level.

The table shows that there are four significant autocorrelations at lags 1, 2, 7 and

10. The partial autocorrelation function has a cut off point at lag 2 suggesting that u_t seems to follow a GARCH(0,2) process.

3. A Conditional Model for FT-ALL Returns

It was proposed in the previous section that an ARMA(1,0)-GARCH(0,2) is a good candidate to model the FT-ALL share returns. It is also desirable to allow for the Monday, holiday, January and turn of the month effects given the evidence that has been accumulated to suggest their importance. French (1980), Lakonishok and Levi (1982), Rogalski (1984), Jaffe and Westerfield (1985), Condoyanni *et al.* (1987), Board and Sutcliffe (1988), and Choy and O'Hanlon (1989) have all found evidence that returns on Monday tend to be negative. This is the case whether returns are measured from the Friday close to the Monday opening (the Weekend effect) or from the Friday close to the Monday close (the Monday effect). Monday returns are also differentiated by a higher variance than the variance of returns that is found on any other trading day [see Mills (1993)]. Fama (1965a) pointed out that the variance of returns would be greater during the weekends and holidays if the variance were proportional to the actual number of calender days rather than trading days.

It is also well documented that average monthly returns tend to be higher in January than any other calender month [see Draper and Paudyal (1996), Clare *et al.* (1995), Clark *et al.* (1992), and Thaler (1987a)]. There is also evidence that most of the advances in the stock market prices happen around the turn of the month [see Ariel (1987) and Thaler (1987b)]. An attempt is made to take account of the possible effects of these anomalies when the mean and variance of the FT-ALL returns are estimated. The conditional mean and variance of y_t are estimated as follows[5]:

$$y_t = b_0 + b_1 y_{t-1} + b_2 MON + b_3 HOL + b_4 JAN + b_5 TURN + u_t \quad (4)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + a_1 MON + a_2 HOL + a_3 JAN + a_4 TURN \quad (5)$$

Where MON is a dummy variable which takes the value 1 if the trading day is Monday and zero otherwise. HOL is a holiday dummy variable which takes a value equivalent to the number of days the market was closed since the last trading day and zero otherwise [as in Hsieh (1989)]. JAN is a January dummy which takes the value one for every day in January and zero otherwise. TURN is a dummy variable which takes

[5]Datastream International does not provide total volume of trade for the companies of the FT-ALL index. Therefore, unexpected volume could not be used in the variance of the FT-ALL returns on the same line as in chapter 4. However, Datastream International provides the total market value of the shares traded for the FT-ALL index on a particular day. A plot of the total market value indicates that it is nonstationary variable in the sense that it has a strong time trend. Differencing this variable yields a stationary variable with significant ARCH effects. The total market value is the sum of volume multiplied by the stock price for all companies in the FT-ALL. Therefore, it is possible that any trends in stock prices can be transformed to the total market value even if volume does not exhibit time trends. The same is true with regard to the ARCH effects in the differences of total market values. The ARCH effects can be transformed from the differences in prices to the differences in the total market values.

the value one for the first half month of the trading day given that the month is not January.

The ARMA(1,0)-GARCH(0,2) with the seasonal effects model is estimated using two conditional distributions, the Student's t and the GED. The density function for the Student's t distribution is given by

$$f(u_t) = \frac{\Gamma[(d+1)/2]}{\pi^{-1/2}\Gamma(d/2)} (d-2)^{-1/2} \sigma_t^{-1} [1+\frac{u_t^2}{\sigma_t^2(d-2)}]^{-(d+1)/2} \qquad (6)$$

where u_t and σ_t are given by equations 4 and 5. $\Gamma(.)$ is the gamma function, and d refers to the degrees of freedom. The latter shapes the Student's t distribution, and therefore decides how fat the tails of the distribution are, and is restricted to be greater than 2 to ensure a finite variance. When d goes to infinity, the resulting distribution becomes normal[6]. For small values of d, the distribution is leptokurtic in the sense that there are more observations close to the mean of the distribution, and out in the extreme tails compared with the normal distribution.

The density function for the GED standardized to have a mean zero and variance one is given by

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[6]In practice, if d > 25, then the resulting distribution can be approximated by the normal, see Blattberg and Gonedes (1974).

$$f(u_t) = \frac{\nu}{\sigma_t(\lambda . 2^{(\nu+1)/\nu}\Gamma(1/\nu))} . e^{-1/2|\frac{u_t}{\sigma_t . \lambda}|^{\nu}} \quad (7)$$

where

$$\lambda = [\frac{2^{-2/\nu}\Gamma(1/\nu)}{\Gamma(3/\nu)}]^{1/2} \quad (8)$$

ν is a positive parameter governing the thickness of the tails of the distribution. When ν is less than 2, the distribution has thicker tails than the normal, whereas for ν greater than 2, the distribution has thinner tails than the normal. When ν is equal to 2, the resulting density function is the standard normal.

4. The Empirical Results

4.1 The Student's t and GED Distributions

The BHHH maximization routine is used to obtain estimates of the conditional mean and variance of equation 4 and 5 simultaneously using the conditional Student's t and GED. Before discussing the results of the estimation, the standardized residuals are checked to identify any model inadequacy. Table 5 shows the skewness and excess kurtosis for the standardized residuals [residuals hereafter] from equations 4 and 5 [u_t/σ_t].

Table 5: The Skewness and Excess Kurtosis of Residuals

	Skewness	Excess Kurtosis
Student t	0.0737	2.631*
GED	0.0640	2.450*

(*) Indicates significance at 5%.

The table shows that the estimated skewness coefficient is not significantly different from zero. The estimated excess kurtosis is significantly different from zero suggesting that the tails of the conditional distribution are heavier than those of the normal distribution. However, the excess kurtosis is less than the excess kurtosis of the regression residuals from equation 1. Also, the autocorrelations and partial autocorrelation functions and the Ljung-Box statistic of the residuals, not reported here, are in line with the diagnostic tests of the regression residuals in table 3, and show that there is no significant pattern in the residuals.

Table 6 reports the autocorrelations and partial autocorrelations estimate for the squared residuals from the conditional Student's t and GED for the first 5 lags and the Mcleod-Li statistics at lags 10, 20, 30 and 50. For comparison, the same statistics are also reported for the squared residuals from the regression in equation 1.

Table 6: The autocorrelations (ρ_k) and partial autocorrelations (ϕ_k) of the squared residuals. Q^2 is the Mcleod-Li statistic for the squared residuals. The same statistics are also reported for the squared residuals (u_t^2) from the regression in equation 1 for comparison.

	k	1	2	3	4	5	Q_{10}^2	Q_{20}^2	Q_{30}^2	Q_{50}^2
t	ρ_k	.030	-.005	.037	.015	.020	14.5	26.6	37.8	48.3
	ϕ_k	.030	-.005	.037	.013	.020				
GED	ρ_k	.020	-.007	.036	.015	.021	13.9	25.7	37.2	48.1
	ϕ_k	.020	-.007	.036	.013	.021				
u_t^2	ρ_k	.111*	.094*	.039	.023	.030	52.1*	88.7*	97.3*	110*
	ϕ_k	.111*	.083*	.020	.009	.022				

* indicates significance at the 5% level.

The table shows that there are no significant autocorrelations and partial autocorrelations for the residuals from either the conditional Student's t or GED. Besides, none of the Mcleod-Li statistics are significant. Contrast these results with the same statistics for the squared residuals from equation 1 which have highly significant Mcleod-Li statistics. Based on the results, it seems safe to conclude that the ARMA(1,0)-GARCH(0,2) model provides an adequate fit for the data. The next step is to check and compare the estimates of the models. Tables 7 and 8 show these estimates. The tables are divided into 4 columns. The first column has the estimates of equations 4 and 5 under the assumption that returns follow the Student's t distribution in table 7 and GED in table 8. The second column contains the estimates of equations 4 and 5 when the conditional distribution is restricted to the normal. The third column reports the parameter estimates of equations 4 and 5 when the leverage effect is considered, as it will be explained later. The fourth column provides the parameter estimates of equations 4 and 5 under the assumption that the unconditional distribution of returns follows either the Student's t or GED.

Table 7: The estimates of the conditional mean and variance of equations 4 and 5 using the conditional Student's t distribution

	1	2	3	4
	d=?	d=40	d=? & Leverage	d=? & Uncond.
b_0	.054 (1.94)	.044 (1.47)	.055* (1.98)	.041* (2.29)
b_1 (y_{t-1})	.086* (3.40)	.104* (3.88)	.087* (3.48)	-
b_2 (MON)	-.146* (-2.96)	-.168* (-3.16)	-.147* (-2.97)	-
b_3 (HOL)	.152* (2.21)	.147* (2.24)	.149* (2.18)	-
b_4 (JAN)	.086 (1.23)	.128* (1.73)	.085 (1.22)	-
b_5 (TURN)	-.001 (-.03)	.015 (.40)	-.0002 (-.006)	-
α_0	.413* (10.95)	.403* (14.63)	.413* (10.94)	.578* (21.20)
α_1 (u^2_{t-1})	.055* (2.12)	.067* (3.42)	.031 (.83)	-
α_2 (u^2_{t-2})	.145* (3.57)	.149* (5.13)	.143* (3.53)	-
Φ	-	-	.049 (.96)	-
a_1 (MON)	.136* (1.98)	.141* (2.98)	.137* (2.00)	-
a_2 (HOL)	-.043 (-.32)	-.074 (-.72)	-.045 (-.34)	-
a_3 (JAN)	.092 (.928)	.061 (.79)	.091 (.92)	-
a_4 (TURN)	.009 (.200)	-.002 (-.06)	.010 (.22)	-
d	8.36 s.e.=1.29	40	8.49 s.e.=1.36	6.57 s.e.=.82
Log Likelihood	-1711.30	-1727.67	-1710.75	-1742.78

(*) Indicates significance at 5% level. t-statistics in brackets.

Table 8: Estimates of the conditional mean& variance of equations 4 and 5 using the conditional GED

	1	2	3	4
	ν=?	ν=2	ν=? & Leverage	ν=? & Uncond.
b_0	.057* (2.01)	.039 (1.28)	.058* (2.07)	.037* (2.11)
b_1 (y_{t-1})	.092* (3.54)	.117* (4.18)	.093* (3.64)	-
b_2 (MON)	-.163* (-3.20)	-.183* (-3.27)	-.163 (-3.18)	-
b_3 (HOL)	.167* (2.50)	.142* (2.20)	.164* (2.43)	-
b_4 (JAN)	.086 (1.22)	.145* (1.93)	.085 (1.20)	-
b_5 (TURN)	-.007 (-.19)	.024 (.59)	-.006 (-.18)	-
α_0	.410* (11.96)	.410* (16.27)	.411* (12.05)	.584* (24.50)
α_1 (u^2_{t-1})	.072* (2.82)	.089* (5.28)	.029 (.77)	-
α_2 (u^2_{t-2})	.151* (4.15)	.156* (6.01)	.148* (4.15)	-
Φ	-	-	.080 (1.70)	-
a_1 (MON)	.153* (2.57)	.155* (3.70)	.160* (2.67)	-
a_2 (HOL)	-.068 (-.55)	-.085 (-.87)	-.068 (-.55)	-
a_3 (JAN)	.074 (.76)	.045 (.61)	.072 (.76)	-
a_4 (TURN)	.001 (.02)	-.007 (-.20)	.0009 (.02)	-
ν	1.53* s.e.=.059	2	1.54* s.e.=.063	1.39* s.e.=.046
Log Likelihood	-1725.35	-1743.26	-1724.00	-1762.39

(*) Indicates significance at 5% level. t-statistics in brackets.

The first column in tables 7 and 8 shows the results of estimating the conditional mean and variance of returns simultaneously using the Student's t and GED. Concerning the mean equation, the results show that the autoregressive part in the mean of the process is highly significant. The coefficient on the Monday dummy is also highly significant and negative in line with the evidence reported in the literature [Jaffe and Westerfield (1985) and Thaler (1987b)][7]. Also in line with this evidence, the days following the market close for holidays have significantly positive returns. However, in contrast with the results of other studies [see Thaler (1987a)], the January effect is not significant, and the turn of the month is far from being significant. In the variance equation, the ARCH parameters are found significant. Also, in line with the literature, the market is more volatile on Monday than on any other trading day. Holidays, January, and turn of the month do not have any significant affect on the variance.

Next I consider the estimates of the values of d and ν. The estimated value of 8.36 for d suggests that the conditional distribution is far away from being normal. The estimate for ν is well below 2, the value taken by the normal distribution, suggesting that the tails of the conditional distribution are thicker than for those of a normal distribution. Since the Student's t and GED are not nested, it is generally difficult to test whether one

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[7]Equations 4 and 5 were estimated allowing for three other dummy variables to take account of the Tuesday, Wednesday and Thursday effects besides the Monday effect with a Student's t conditional distribution. The results, not reported here, showed that these dummy variables do not have any significant effect on either the mean or the variance of the process.

of them provides a better fit than the other. However, the log likelihood for the Student's t is 14.05 higher than the log likelihood for the GED. This suggests that the Student's t may provide a better fit to the conditional distribution of the residuals than the GED.

It is important to note that volatility persistence as measured by the sum of α_1 and α_2 is equal to 0.20 and 0.223 for the Student's t and GED respectively. Since the sum of α_1 and α_2 is well below unity, the fitted model is second order stationary and at least the second moment exists[8]. The unconditional variances of y_t [9] are 0.516 and 0.527 under the Student's t and GED specifications respectively. These estimates are close to the sample variance of y_t which is 0.595 [1554 observations].

The finding of a low volatility persistence goes against the results of almost all other studies where it has been found to be very high and close to one, suggesting the possibility of integration in the variance [see Bollerslev *et al.* (1992) for a review]. One possible explanation for this finding of low volatility persistence is that the sample is

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[8]Bollerslev (1986) shows that a sufficient and necessary condition for the existence of the second moment and stationarity of the process is that the sum of the estimated ARCH dynamics has to be less than unity.

[9]The unconditional variance is given by

$$\sigma_y = \frac{\alpha_0}{(1 - \alpha_1 - \alpha_2)}$$

chosen in such a way to avoid the inclusion of the 1987 crash. This is consistent with the results of Engle and Mustafa (1992) who found that the degree of volatility persistence of the S&P 500 returns was markedly lower after the 1987 crash[10].

The second columns of tables 7 and 8 show the results of estimating equations 4 and 5 when the conditional distribution is restricted to be normal by setting d equal to 40 in the Student's t distribution, and ν equal to 2 in the GED. The likelihood ratio tests, distributed as χ_1^2, are 32.74 and 35.82 for the Student's t and GED respectively, suggesting that the conditional normal distributions are unacceptable. There is one significant change in the results from the first column. The coefficient on the January effect in the mean is statistically significant at 5% level[11] and has the right sign. This suggests that January effect is not significant under the more appropriate distributional assumption. This also suggests that failure to model the fat tails property of returns distributions can lead to spurious results.

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[10]Engle and Mustafa (1992) measured the degree of volatility persistence by two methods. They estimated the GARCH volatility persistence implied by the S&P 500 index option prices by combining the Black-Scholes pricing formula with a GARCH process. They then backed this by estimating the volatility persistence of a GARCH model from the historical returns. They found similar results from both methods that volatility persistence was significantly lower after the 1987 crash.

[11]It is one tail test since the January effect is expected to be positive indicating an advance in returns during January [see Thaler (1987a)].

4.2 The Leverage Effect and FT-ALL Returns.

Pagan and Schwert (1990b) and Engle and Lee (1993) found evidence of asymmetric response in the conditional volatility of US stock prices. A negative surprise increases volatility more than a positive surprise. Black (1976) argues that a reduction in the stock price will lead to an increase in the debt to equity ratio measured in terms of market values which might cause an increase in the riskiness of the firm's stocks, and subsequently higher return volatility. The standard GARCH model of Bollerslev (1986) does not allow for asymmetric effects of returns surprises on volatility. That is, negative and positive returns surprises of the same size are assumed to produce the same amount of volatility. As noted by Engle and Ng (1993), "If a negative return shock causes more volatility than a positive return shock of the same size, the GARCH model underpredicts the amount of volatility following bad news and overpredicts the amount of volatility following good news." Engle and Ng (1993) found the model of Glosten *et al.* (1993) provides a good basis for dealing with the asymmetric effects of returns surprises on volatility. Glosten *et al.* (1993) modified the GARCH model of equation 3 as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \omega\,(u_{t-1}^2)\,(I_{t-1}), \qquad (9)$$

$$where \quad I_{t-1} = 1 \; if \; u_{t-1} \geq 0 \quad , \; I_{t-1} = 0 \; if \; u_{t-1} < 0.$$

If the leverage effect holds, then it is anticipated that ω is negative since a past positive surprise will reduce the variance of the current returns and vice versa. The Glosten *et al.* (1993) model is used to test for the leverage effect in the FT-ALL returns. The leverage component is added to the variance of y_t in equation 5. Columns 3 in table 7 and 8 present the results. It turns out that ω is insignificant and has the wrong sign, suggesting that the leverage effect does not exist in the data set.

4.3 The Unconditional Distribution

Praetz (1972) and Blatteberg and Gonedess (1974) have offered evidence that the Student's t distribution provides a good fit for the unconditional distribution of price changes. To investigate this possibility, the coefficients b_1, b_2, b_3, b_4, b_5, α_1, α_2, a_0, a_1, a_2, a_3 and a_4 are restricted to zero in the conditional mean and variance of equations 4 and 5. In effect, this implies estimating the unconditional distribution of y_t with a constant mean, b_0, and constant variance, α_0. The results are presented in columns 4 of tables 7 and 8.

The results show that d in the Student's t distribution is 6.57, less than the value 8.36 found for the conditional distribution. Also, the estimate ν in the GED is 1.39, less than 1.53 found for the conditional distribution. These results cast some doubt on any financial models which assume that daily returns are normally distributed. Even during periods free from significant disturbances, such as the 1987 crash, returns still cannot be assumed to be normally distributed. The likelihood ratio for the restriction to the unconditional distribution, distributed as χ_{11}^2, is 62.96 and 74.08 for the Student's t and GED respectively, suggesting that the ARMA, ARCH, and various seasonal effects have a significant role in accounting for the departure of the returns distributions from normality, besides explaining some of the variation in both of the conditional mean and variance of returns.

Finally, a comparison of the log likelihood under the Student's t with its corresponding value under the GED suggests that the log likelihood is higher in all cases for the Student's t than for the GED. This is consistent with the results of Baillie and Bollerslev (1989) from the comparison of the fit of the Student's t and GED to six daily spot exchange rates.

5. Conclusions

The chapter attempted to model the conditional mean and variance of the FT-ALL returns using two conditional distributions, the Student's t and GED. It is shown that an ARMA(1,0)-GARCH(0,2) model provides an adequate fit for the FT-ALL returns in the period from 4/1/1988 to 28/2/1994. The study also presented evidence suggesting that the ARMA, ARCH, Monday, and the holiday effects explain a significant part of the departure of the FT-ALL returns from normality. It is found that the distribution of the residuals is too leptokurtic to be approximated with a normal distribution. The Student's t was found to be superior to the GED in modelling the FT-ALL returns judged by a higher likelihood value for the former. No evidence was found for a leverage effect in the UK FT-ALL returns, suggesting that positive and negative surprises tend to affect the subsequent returns volatility in the same way. The estimate of the January effect on the mean of FT-ALL returns is significant, but only under the inappropriate assumption of conditionally normally distributed returns. This result is in line with Baillie and DeGennaro (1990) finding that failure to consider the fat tails property of stock returns leads to the possibility of wrong inferences being drawn.

Examining the estimated auotcorrelations and partial autocorrelations of the squared residuals from an AR(1) model for the FT-ALL returns indicate the existence of significant but not persistent ARCH effects. For example there are only two highly significant autocorrelations at lags 1 and 2. Contrast this with the findings of many studies [see Diebold and Lopez (1995) for a review] that there are significant autocorrelations in the squared returns for long lags indicating the possibility of long memory in the variance. Regarding volatility persistence, it was found that the sum of the ARCH parameters is far away from unity, suggesting that the fitted model is second order stationary, and that at least the second moment exists. This result also suggests that volatility persistence is very low in the post crash UK market. This is consistent with the finding of Engle and Mustafa (1992) of low volatility persistence in the S&P 500 returns after the 1987 crash.

Chapter 6: Conclusions

The thesis has considered various different aspects of the probability distribution of the time series of the stock price changes in the UK market, with particular emphasis being placed on the character of the volatility of the series. Chapter 2 reviewed the major theoretical probability distributions suggested in the literature, and carried out an exploratory data analysis to investigate the properties of the UK FT-ALL price changes. The chapter's findings are: (a) the distribution of the FT-ALL price changes is leptokurtic, (b) the FT-ALL price changes are not independent of each other which leads to more departure from i.i.d. normality. The latter conclusion is reached using the Fama and Roll's method for estimating the characteristic exponent for i.i.d. stable random variables. However, the Fama and Roll's method suffers from two major drawbacks: non availability of standard errors, and the sample size decreases as the sum of daily price changes increases. Therefore, the null hypothesis of i.i.d. was tested using the recent test proposed by Brock *et al.* (1987).

The null hypothesis of i.i.d. is rejected for linearly filtered FT-ALL price changes. It is found that the main cause of the rejection of i.i.d. is conditional heteroscedasticity. This result is consistent with the evidence of Hsieh (1991) from the US market and Abhayanker *et al.* (1995b) and Paudyal *et al.* (1993) from the UK market. The chapter improves on the results of previous studies by applying an adjustment suggested by de Lima (1995b) to the residuals from the GARCH model before conducting the BDS test. As is well established [see Hsieh (1991)], the asymptotic distribution of the BDS test is not known when it is applied to the residuals from the GARCH model. de Lima (1995b) shows that asymptotic normality of the BDS test carries out to the residuals from the GARCH model after a simple adjustment. The results of the BDS test applied to the residuals adjusted in the way proposed by de Lima (1995b) suggests that the i.i.d. hypothesis should be rejected at the 5% level for m equal to 9 and 10, and ε equal to 0.50. Apart from these values, the results are largely consistent with those obtained from applying the BDS test to the GARCH residuals without adjustment. That is conditional heteroscedasticity explains most of the nonlinearity of the FT-ALL price changes.

The proposition that the variance of the FT-ALL price changes is constant over time was examined in chapter 3. The variance constancy test of Loretan and Phillips (1994) was applied to the FT-ALL price changes during the period from 2/1/1970 to 31/12/91. The null hypothesis of constant variance is rejected at the 5% level. There is evidence that the fourth moment of the FT-ALL price changes is not finite. A graphical inspection of the unconditional variance derived from an AR(1)-GARCH(1,1) is conducted over different rolling periods of the data. The results suggest the existence of two exceptional periods in the data: the 1973-74 oil crisis and the 1987 market crash.

An intervention model on the line of Box and Tiao (1975) was applied to the FT-

ALL price changes to model and test for the statistical significance of variance shifts around the 1973 oil crisis and the 1987 market crash. The results suggest the existence of statistically significant variance shifts around the oil crisis and market crash. Also, the effect of the oil crisis persisted for longer time period than the effect of the market crash. However, in both cases the volatility of price changes returned to the pre-event level, indicating that there was no lasting effect from either crisis.

The results of the variance constancy test applied to the residuals from the intervention model suggest that the null hypothesis of constant variance should be accepted. In addition, the null hypothesis of finite fourth moment is not rejected except for s equal to 300. Therefore, it can be concluded that the rejection of the hypothesis of constant variance and the hypothesis of finite fourth moment in case of the raw data was due to the oil crisis and the market crash. In summary, the results suggest that the stock market is subject to abrupt changes in volatility during some exceptional periods. Outside these periods, however, stock price changes can be described as covariance stationary.

The results, in general, suggest that there is a need for models which allow for discrete shifts in the unconditional variance of stock returns at unpredictable points in time. One example of such models is the SWitching ARCH (SWARCH) model of Hamilton and Susmel (1994)[1]. This model allows for several different ARCH models and

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[1]See also Hamilton (1988 and 1989), and Hamilton (1994) chapter 22.

for the economy switching from one model to another following a Markov chain. Therefore, this model can account for extremely high volatility periods such as the 1973-74 oil crisis and the 1987 market crash. At the same time, the model allows for periods of usual levels of activity. Bollerslev *et al.* (1993) note that since the 1987 crash "could happen at any time but with very low probability, the behaviour of risk averse agents will take this into account". Therefore, it seems that the SWARCH model may prove useful in modelling the volatility of stock returns.

It seems useful as well to compare the volatility forecasting ability of the SWARCH model with a variant of the GARCH models, and the Fractionally Integrated GARCH model (FIGARCH) [see Bollerslev, and Mikkelsen (1993)]. These three models have different economic implications. The GARCH and FIGARCH models differ in the time in which each model allows for volatility shocks to persist. But both models do not allow for a change in regimes which the SWARCH model takes into account. Accordingly, the comparison between these models may provide an empirical insight on the appropriateness of the use of these models in finance. For example, for predicting short term volatility for option prices.

The issue of whether GARCH modelling captures the temporal dependence in volume of trade for individual stocks in the UK market was examined in chapter 4. The chapter's empirical results differ from those of Lamoureux and Lastrapes (1990b) using

US data. The results for the UK are consistent with theirs in that volatility persistence, as measured by the GARCH model, becomes negligible when volume of trade is introduced in the variance equation of price changes. However, when the autocorrelation functions of the squared standardized residuals are checked, it is found that they exhibit a highly significant GARCH pattern.

When unexpected volume is used in the variance of price changes in place of volume, it is found that the GARCH volatility persistence is reduced. Since unexpected volume is not serially correlated, this result cannot be attributed to volume capturing the effects of the serial dependence in volume, the proxy used by Lamoureux and Lastrapes (1990b) for the information flow rate. The results seem to be more consistent with the interpretation of Bessembinder and Seguin (1992 and 1993) of unexpected volume as shocks to trading activity. Moreover, evidence is uncovered which supports the findings of Bessembinder and Seguin (1993) that positive unexpected volume has a greater affect on volatility than negative unexpected volume. Bessembinder and Seguin (1993) argue that this evidence supports the hypothesis that volatility is affected by existing market depth for a security.

It is argued that the relevance of the unexpected volume of trade is due to a strong association in the timing of innovational outliers in both the price changes and volume of trade series. This factor may also contribute to the noted reduction in persistence in the GARCH component. The results suggest that unexpected volume can help in forecasting the future conditional volatility as long as our emphasis is on forecasting rather than explaining the joint dynamics of volume and price changes. This is inconsistent with the evidence presented by Lamoureux and Lastrapes (1994) which suggests that contemporaneous volume and squared price changes are noisy instruments for predicting the future conditional volatility.

At least one research area remains to be studied in more detail. If outliers for volume and price changes tend to happen simultaneously, then it is possible that there is a threshold for volume of trade and volatility. For example, if volume exceeds a specific level, then it triggers a big movement in the price of the stock. However, if volume does not exceed this threshold, then it appears that there is no relationship between price changes and volume of trade. Therefore, a threshold model for price changes and volume may be a useful way to proceed. This model can be useful in comparing the response of the prices of different stocks to high trading activity as implied by large trading volume. If it takes more volume to induce more price volatility for one stock than another, then it may be concluded that there is more market depth for the former than the latter, i.e. the former stock can take more trading shocks than the latter before reflecting these shocks into price volatility.

Chapter 5 attempted to model the conditional mean and variance of the UK FT-

ALL returns using two conditional distributions, the Student's t and GED. Motivated by chapter's 3 evidence about the possibility of variance shifts in price changes around the oil crisis and the 1987 crash, and the recent evidence from the US [Diepold and Lopez (1995) and de Lima (1995a)] that the presence of ARCH effects in the S&P 500 returns depends on the period under study, the chapter studied the time series of FT-ALL returns post the 1987 crash.

The estimated auotcorrelations and partial autocorrelations of the squared residuals from an AR(1) model indicate the existence of significant but not persistent ARCH effects. For example there are only two highly significant autocorrelations at lags 1 and 2. This result is inconsistent with the findings of many studies [see Diebold and Lopez (1995) for a review] that there is significant autocorrelations in the squared returns for long lags indicating the possibility of long memory in the variance.

It is shown that an ARMA(1,0)-GARCH(0,2) model offers an adequate fit to the FT-ALL index returns in the period from 4/1/1988 to 28/2/1994. With regard to volatility persistence, it is found that the sum of the ARCH parameters is far away from unity, suggesting that volatility persistence is very low in the post crash UK market. The results are in line with those of Diepold and Lopez (1995) and de Lima (1995a) that the existence of strong ARCH effects depends on the period under study. It is also consistent with the finding of Engle and Mustafa (1992) of low volatility persistence in the S&P

500 returns after the 1987 crash. In addition, low volatility persistence indicates that the fitted model is second order stationary, and that at least the second moment exists. This is consistent with the evidence from chapter 3 that outside the periods of the oil crisis and the 1987 crash, stock price changes can be described as covariance stationary.

The study also presented evidence suggesting that the ARMA, ARCH, Monday, and the holiday effects explain a significant part of the departure of the UK FT-ALL returns from normality. Nevertheless, the distribution of the standardized residuals has fatter tails than those of the normal distribution. It is found that the Student's t offers a better fit for the standardized residuals than the normal and GED. There is no statistical evidence for the existence of a leverage effect in the FT-ALL returns, suggesting that positive and negative surprises tend to affect the subsequent returns volatility in the same way. The estimate of the January effect on the mean of FT-ALL returns is significant, but only under the inappropriate assumption of conditionally normally distributed returns. This result is in line with Baillie and DeGennaro's (1990) finding that a failure to consider the fat tails property of stock returns leads to the possibility of wrong inferences being drawn.

The results of chapter 5 can be extended by studying the returns behaviour for individual stocks. In other words, whether individual stocks behave differently from aggregate stock series. For example, the results of chapter 4 suggest that volatility persistence of the GARCH model is high for many of the 57 companies examined. Contrast this result with the finding of chapter 5 of low volatility persistence in the FT-ALL returns after the 1987 crash. Note that the sample period is the same for the studies reported in both chapters 4 and 5: the period runs from 4/1/1988 to 28/2/1994. However, the 57 companies studied in chapter 4 are the biggest in the UK in terms of market capitalization. On the other hand, the FT-ALL index contains more than 800 companies including small companies. Can the inclusion of small companies be responsible for the low volatility persistence observed in the FT-ALL returns? A preliminary analysis of the FT-SE index returns during the same period suggests that this is unlikely. In fact, the behaviour of the autocorrelations and partial autocorrelations (not reported) for the FT-ALL and FT-SE returns are very similar and both indicate low volatility persistence after the crash.

Finally, the analysis carried out in the thesis was often influenced by the fact that outliers (or better called unusual values) dominated the behaviour of the stock price data examined. As noted by Loretan and Phillips (1994), there is a need for theoretical models of rational economic behaviour which can explain and predict outliers activity in stock prices. Currently, there is no agreement about the definition of outliers. Most of the statistical techniques for outliers' identification depend on an arbitrary cut off point [see for example Balke and Fomby (1992)]. Although, the techniques are statistically elegant, there is a need for theoretical models which can offer an economic explanation for the observations identified as outliers.

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