Foreign Reserves Strategic Asset Allocation: A Bayesian Approach

Timbul Budi Santoso

Supervisors:

Professor Jonathan Fletcher

Professor Krishna Paudyal

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Business School
University of Strathclyde
United Kingdom

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Declaration

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Glasgow, 20 December 2016

Timbul Budi Santoso

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Abstract

Despite massive global foreign reserves accumulation since the late 1990s, a study on strategic asset allocation from a central bank perspective remains relatively limited. Many countries have built up foreign reserves greater than their trading and financing sufficiency; consequently, there is increased public awareness as to how central banks manage their reserves. This research addresses the issue of asset structure of foreign reserves portfolios; currency composition and strategic asset allocation. This thesis provides an alternative strategic asset allocation framework that focuses more on liquidity and safety objectives, without sacrificing return objective of the existing central banks' portfolios. This research contributes to the theoretical literature that combines mean-variance Bayesian framework with the spirit of the transaction theory of central bank foreign reserves.

First, using the unconditional Bayesian approach with external transaction constraints, I investigate risk reduction benefits to both minimum-variance and optimal portfolios for a given existing return for central banks' currency allocation. Our results show that before imposing currency weight constraints, all the diversification strategies considered provide significant benefits, regardless of the current benchmark used. The results when the trade constraints and debt constraints are imposed reveal that there is the potential for diversification benefits to be obtained from the currency portfolio optimisation. However, the choice of the current benchmark, and both trade and debt constraints, play an important role in the decision as to with which diversification strategy central bank reserves managers should proceed.

Second, using the similar objective functions and diversification benefits measures, the framework is then applied to define asset allocation policy for government bond portfolio. The results from the optimal portfolio for a given existing benchmark return show that there is significant risk minimization benefit by adding developed market longer maturity, quasi-government, emerging and

inflation-linked government bonds altogether to the current benchmark portfolio. The benefits could not be found if each asset class is added directly to the current portfolio policy. From the same spanning strategies as the optimum portfolio, the global minimum variance portfolio analysis shows that the diversification benefits are mainly driven by United States Treasury Bills.

Third, I investigate risks minimization of government bonds portfolio for a given benchmark returns in various investment policies and risk preferences. This research examines the important role of budget constraints, liquidity buffer allocation, global financial crisis investment opportunities, investing in ultra-long government bond and non-bond investment. Analysis on the impact of the budget provides policy implications that central bank should not to tranche their reserves, but express liquidity requirements in the form of constraints on the portfolio optimisation framework. Investigation on the role of liquidity allocation confirms earlier findings that the risk reduction benefits are mainly driven by US Treasury Bills. The impact of global financial crisis since 2008 emphasises the need for central banks to diversify their foreign reserves benchmark beyond its current setting. We could not find benefits for the central bank to invest in ultra-long maturity government bonds. Lastly, we found that non-government bond investments provide significant benefits when it compares to the existing benchmark, and incremental benefits from the well-diversified government-related bond portfolio.

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CHAPTER ONE

This chapter starts by giving the background of the study. Contextualizing and the importance of the thesis are explained. Motivations for each chapter are broken down into more detail research questions. Key conclusions reached in exploring these questions and the main contributions of the thesis are outlined. Finally, main results and the structure of the thesis are presented.

1. INTRODUCTION

A remarkable figure shows that global foreign reserves increased from USD4 trillion in 2000 to over USD12 trillion by the end of 2014 (Figure 1.1); the majority of which was collected in the previous five years. More specifically, the steep increase of the official reserves accumulation was dominated by upper middle income countries (Figure 1.2), including China, Japan, and Brazil. If we look at accumulation by region, this was led by central banks from East Asia and Pacific (e.g. China, Japan), and Europe and Central Asia (e.g. Germany, Switzerland, Russia Federation, and South Korea) countries.

This enormous size of funds held by central banks around the globe attracts research attention to the foreign reserves management topics. As global reserves increase significantly, it becomes more prominent in news-making related to international markets (Scheherazade and Blitz, 2007). Additionally, investment choices made by central banks have a potential economic impact on global financial markets (Economist, 2008; Higgins and Klitgaard, 2004). At the same time, major supranational institutions including World Bank (WB), Bank of International Settlement (BIS), the International Monetary Fund (IMF) and Organization for Economic Co-operation and Development (OECD) have begun to ask questions as to how central banks should manage their foreign reserves (Mitchell, Piggott, and Kumru, 2008; Musalem and Palacois, 2004).

Figure 1.1 World FX Reserve (USD, Billion)

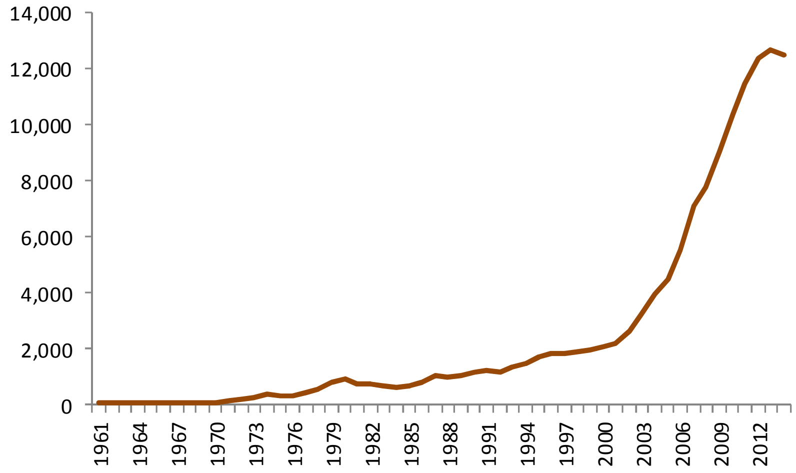


Figure 1.2 FX Reserve by Income Group (USD, Billion)

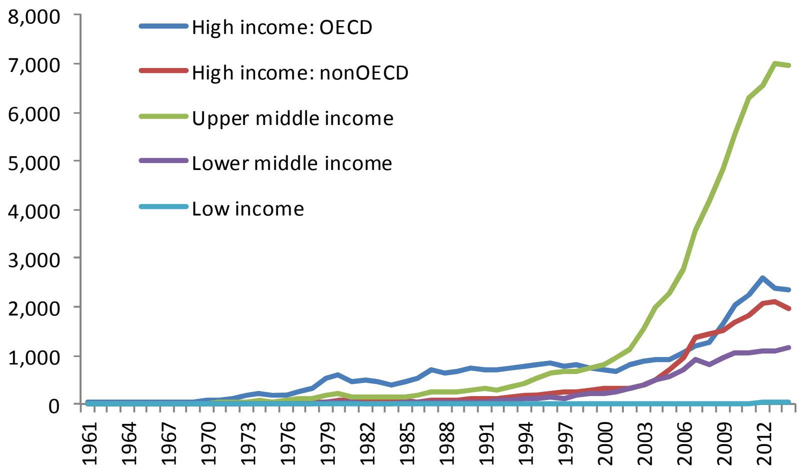
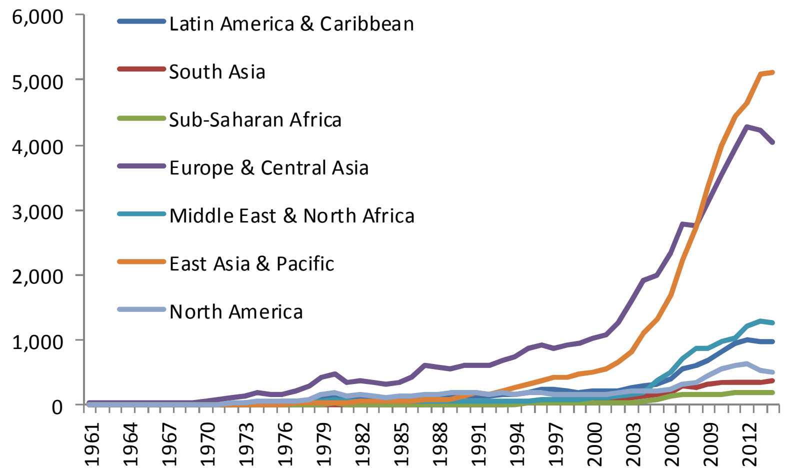


Figure 1.3 FX Reserve by Regional (USD, Billion)



Source: World Bank

The study of foreign reserve management is inseparable from the objectives for holding the reserves. Truman and Wong (2006), Borio, Ebbesen, Galati, and Heath (2008) and Romanyuk (2012) explain the rationales as to why countries accumulate foreign exchange reserves. These are comprised of the following: 1) supporting and maintaining self-confidence in monetary and exchange rate policy, as well as the capability to intervene in the domestic currency market. They can also be used as a preventive external vulnerability measure during crisis periods, when access to external borrowing may be stopped; 2) providing an emergency fund in case of disaster; 3) providing assurance that a country is capable of servicing its external liabilities, and by doing so reduces the likelihood of financial crises. Furthermore, the accumulation of foreign exchange reserves may increase a country's credit rating and therefore lower its external borrowing costs; 4) resisting currency appreciation in order to support their export; and 5) generating income from the international financial investments.

It is important to note, however, that once a government intentionally or as a result of another policy has accumulated the foreign exchange reserve, the options for utilising the fund are limited. They might use the reserves to repay external debt, gradually sell it for local currency, or wait until its own currency is under pressure and do so. Aside from these options, the government has become a foreign investor. Foreign exchange reserve management policy is designed to fulfil the common features: reserves must be held in safe and prudent instruments; these must be liquid, because the need to use reserve may arise suddenly; and, in relation to the extent by which the reserves exceed their liquidity requirement, the prospective for generating income cannot be overlooked; although bounded by liquidity and safety constraints (Bakker and Herpt, 2007; Romanyuk, 2012). Therefore, there are three known multi-objectives for foreign exchange reserves management: liquidity, security, and profitability.

The enormous size and the complexity in the nature of the national foreign reserves require a systematic methodology in defining multi-objective investment strategies. Roger (1993) emphasises that the important and special function of foreign reserves is liquidity, which includes funding the everyday transaction and intervention. Such needs determine the necessity for the liquidity management of the reserves. With respect to the risk feature, Beck and Rahbari (2011) provide an example of the importance of the safety aspect of the reserves. They proposed a theoretical model in the presence of sudden stops[1] and provide empirical analysis to show the importance of such attention to the sudden stop risks.

Central banks' foreign exchange reserves are mostly held in the short-to medium-term major government bonds, which is the common strategy to address the safety objective of the reserve portfolio. This approach has been proven to work well during the last three decades; a period of relatively high and positive yield curves. Nowadays, however, global interest rates are a record low. As such, the reserve portfolios of the central banks are exposed to substantial interest-rate

[1] Foreign reserve portfolios are subject to sudden reversals in foreign capital flows and the central bank uses its foreign reserves to service the short term foreign denominated debt that is not rolled over in this events.

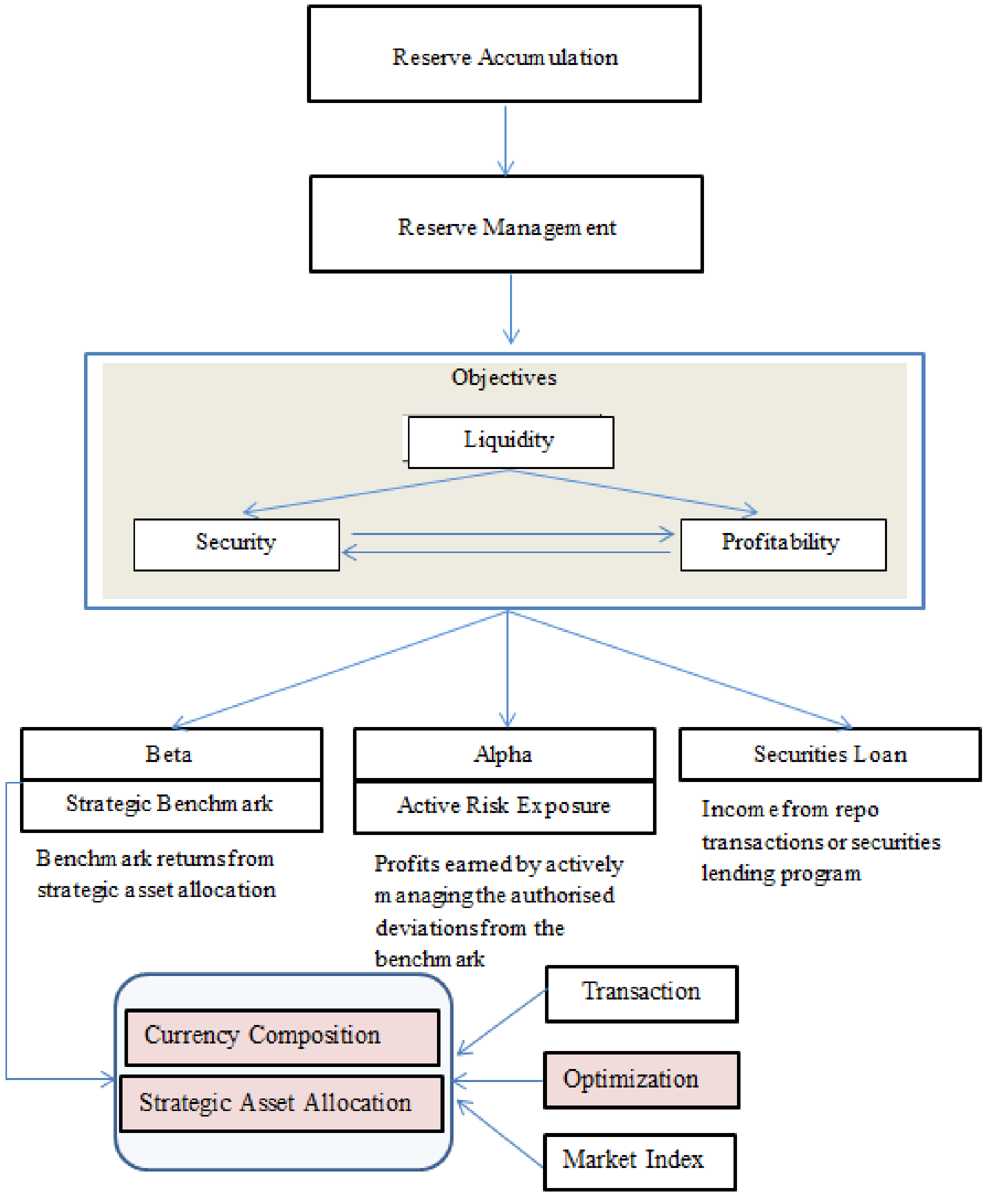
risk. Therefore, portfolios should be managed to ensure the risk-return related to interest rates hikes is well-mitigated.

The other and the most popular aspect of investment is of course return (Briere, Mignon, Oosterlinck, and Szafarz (2016) and Bakker and Herpt (2007)). In recent years, many countries have built up foreign exchange reserves beyond the level that adequate for trading and financing activities. Furthermore, increasing public awareness as to how central bank manages their national wealth has put higher pressure on the return aspect of the reserve management. Manchev (2009) argues that liquidity and safety should be the main objectives for central bank reserve portfolios. Hence, I believe it appropriate to measure diversification benefits using the risk reduction measures rather than return enhancement for the central banks' foreign reserves portfolio, which will be implemented in this thesis.

1.1. Contextualising the Research

This research project has a particular motivation into how the central banks should allocate their foreign reserves in order to sustain diversification benefits in a multiple objectives portfolio framework. In the scope of this research project, currency composition, broader government securities and non-government-related investment are investigated. The choice of the importance of reserve objectives and spanning strategy approximate to what can be believed as stages of central bank reserves management practice; the approach to which has been shifting towards market and credit risk over the decades.

Figure 1.4 Basic Scope of Foreign Reserves Management



Source: adapted from Litterman in Bakker and Herpt (2008), p.178

Figure 1.4 provides general scope foreign reserves practices by central banks. Foreign reserves study can be categorised into the optimal size of the reserve and how to manage it once the reserves have been accumulated. Reserve management can be classified into currency composition and asset allocation of foreign reserves. In practical terms, the asset allocation of foreign reserves management can be related to various aspects; strategic asset allocation, choosing model portfolio, or defining benchmark portfolio policy and active portfolio management by managing the authorised deviation from the benchmark. Additional income may also be gained from securities lending and repo markets. Our focus is on strategic asset allocation from a central bank reserve management perspective.

Strategic asset allocation is the primary stage in the hierarchy of investment decisions. The most crucial point is to select a benchmark for foreign reserves. The benchmark is a model portfolio which expresses the preferred investment policy which is translated through investment objectives and risk tolerances. Manchev (2009) argues that for the central bank reserve portfolio, the benchmark has a similar contribution to the risk characteristics of a portfolio. The benchmark is the most important; a number of studies[2] show that over 90 percent of a portfolio's return is determined by its benchmark.

The mainstream benchmark policy for currency composition is based on the transaction theory and mean-variance theory. Foreign reserve asset allocation policy relies mainly on published global bond market index data (e.g., World Government Bond Index or Group G7 government bond index), with weights proportional to market capitalization or GDP figures (Leon and Vela, 2011). On the other hand, for some more advanced reserves managers may prefer using

2 See for instance Ibbotson, Roger G. and Paul D. Kaplan, "Does Asset Allocation Policy Explain 40, 90, or 100 Percent of Performance?", Financial Analyst Journal, January/February, 2000.

optimisation techniques. Besides the main objective to replicate market holdings, published indices provide transparency and traceability. Portfolio optimisation aims to achieve a superior performance for a given set of constraints within a quantitative framework. Mean-variance spanning test has proven an appropriate approach to develop optimal asset structure, constructed on the unconditional expected return and risk of the equity portfolios. From a central bank point of view, modifications of a standard optimisation approaches open-up the opportunities as to how central banks should allocate their wealth in a multiple-objective perspective.

1.2. Why Is This Topic Important To Be Investigated?

The evolution of foreign exchange reserves has gone through different stages in history, and reserve management has always been a serious concern for central banks. More specifically, since the Asian crisis in 1997, the world has witnessed substantial foreign exchange reserves accumulation; so much so that perhaps one could expect that many countries now dismiss the threat of reserves insufficiency. This positive development, however, carries other challenges for central banks in managing their country's reserves; beyond contemporary liquidity or external transaction purposes.

The set of central banks' reserve management objectives requires a multi-facet process. In the first place, central banks are conservative and very risk-averse investors. This risk preference predetermines that protection of the value of reserve as the top priority. As such, their investments tend towards the safest instruments, with the consequences of low expected return on their portfolios (Fisher and Lie (2004) and Beck and Rahbari (2011)). Second, in line with the missions to cover day-to-day transactions and interventions, an appropriate percentage of reserves must maintain minimum risk with high degree of liquidity. The purpose of which is to mitigate the impact of negative shocks in the economy (Romanyuk, 2012). Third, given the massive foreign reserves accumulation, it is also possible for central banks to assign a certain portion of their reserves for higher risk-return investment, while maintaining the overall reserve management conservativeness. Parallel to the last argument, Bri'ere et al. (2015), Berkelaar, Coche, and Nyholm (2010) and Borio et al. (2008) claim that many central banks are seeking higher returns especially for countries which has an excess reserve.

Ben-Bassat (1980) was one among few pioneer studies to determine the optimal reserve portfolio in a mean-variance framework. The later mean-variance studies in reserve management area use Black-Litterman (Black and Litterman, 1992) and stochastic programming methods to optimise their strategic asset allocation and to mitigate conventional mean-variance optimisation weaknesses[3]. Fernandes et al. (2012) combine the Black-Litterman model and the re-sampling approach of Michaud and Michaud (2008) to propose a foreign reserve portfolio. Petrovic (2009) employs the Black-Litterman model to central banking reserve

[3] Despite being the most popular method, mean-variance portfolio optimisation entails several shortcomings. First, the assumption of normality of asset or portfolio returns which ultimately hold the mean-variance together. Second, high sensitivity to the inputs as shown by Black and Litterman (1992), minor variation in expected returns can lead to enormous assets weight reallocation. Third, corner solutions or the presence of extreme portfolio weights (He and Litterman, 2002). Fourth, excessive risk taking portfolio in long-term (Pastor and Stambaugh, 2009).

management practices. Claessens and Kreuser (2007) construct a framework for strategic foreign reserve management. They provide a framework that combines risk-return objectives of the reserve portfolio with macroeconomic, macro-prudential and sovereign debt aspects. Their study also provides institutional guidance in creating benchmarks policy, evaluation and portfolio reporting.

Leon and Vela (2011) implemented a long-term-dependence and non-loss-constrained version of the Black-Litterman model to develop central bank strategic asset allocation. Zhang, Chau, and Xie (2012) incorporated behavioural portfolio management within the mean-variance mental accounting framework, and the Black-Litterman model is used to estimate assets return. Romanyuk (2010) outlines asset-liability management (ALM) and identifies risks, portfolio allocation, and asset-liability strategies within reserves management. A later study by Romanyuk (2012) examines how to interpret the three collective objectives of foreign reserve management (liquidity, security and profitability) into the objective function as the modelling framework to capture the objective of reserve management of the Bank of Canada.

Reserve managers of central banks encounter greater challenges than those typically faced by private fund managers. An increasing demand for accountability and transparency contributes another pressure to provide higher returns, while maintaining the other objectives of providing liquidity and preserving capital value of the reserves. Such pressures may be conflicting at times, and as such, raise more challenges for reserves managers. As a public institution, reserve management must meet international disclosure standards. The bureaucracy structure for the reserve may lead to full or partial portfolio separation to meet their specific objective, resulting in obstacles for assessing the global risk position of the reserves. Furthermore, constraints on the permissible invested assets and challenging risk preferences, including non-loss constraints, complicate reserves management. In addition, international financial markets have changed significantly since the financial crisis in 2008. A consequence of which is that major bond market returns are reducing near zero interest rates. Those factors impose greater challenges for the management of foreign reserve investments.

The literature demonstrates the increasing popularity of the Black-Litterman and stochastic programming methods to form a strategic asset allocation for central bank reserve management. On the other side, the Bayesian technique shows that it can handle portfolio constraints and has proved successful when applied in the stock market, insurance and pension fund[4] portfolio optimisation. Furthermore, Schöttle, Werner, and Zagst (2010) argue that the Black-Litterman approach is only a special case of Bayesian model.

The discussion from this sub-section shows that despite its attractiveness, the Bayesian approach is relatively lacking in popularity behind the other counterparts. Given the advantages of the Bayesian model and successful

[4] Bayesian model applications in stock market (see, e.g. Black and Litterman, 1992; Jorion, 1986; Kandel, McCulloch, and Stambaugh, 1995; Li, Sarkar, and Wang, 2003; Pastor, 2000), in hedge funds (see (Bessler, Holler, and Kurmann, 2012)) and in insurance and pension funds (see, e.g. Andreu, Gargallo, Salvador, and Sarto, 2011; Puustelli, Koskinen, and Luoma, 2008; Streftaris and Worton, 2008).

applications in other areas of portfolio analyses, it provides confidence that it is possible to develop a beneficial model for strategic reserves management in a Bayesian framework. This thesis is one among few studies to provide strategic asset allocation policy for a central bank portfolio based on Bayesian approach.

1.3. What Does This Research Do?

Countries hold foreign exchange reserves for different motivations, which include foreign exchange market intervention, providing liquidity and generating income. To guarantee its reserves meet these objectives, reserve managers could employ strategic models to determine the optimal allocation of reserves over some investment horizon. An objective function is a key element of such model framework, and its specification is more challenging especially related to the special purpose of foreign reserves as insurance in crises. To this end, the author proposes how to translate the three objectives of holding reserves into an objective function for strategic reserve management.

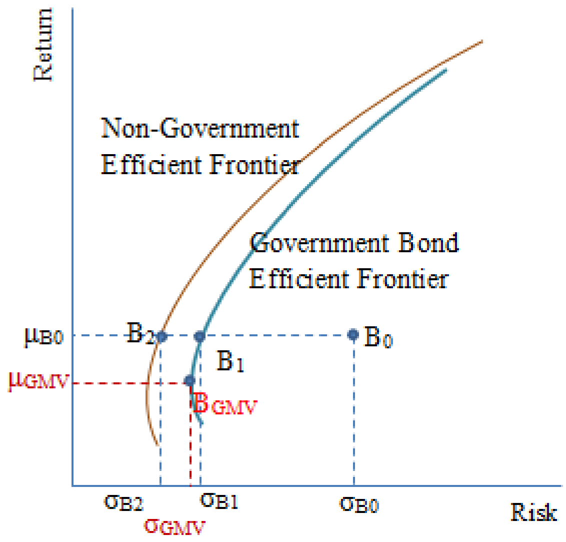
This research investigated the potential diversification strategies to provide an alternative solution for the currency composition and strategic asset allocation of a central bank's reserves portfolios. The process of the construction of central banks' strategic asset allocation must be framed by the sufficiency of the external transactions need at all time, which will be controlled by foreign trade and debt constraints. As a starting point, to find optimal currency allocation, this research follows the ideas of Briere et al. (2015) in defining the benchmark strategy[5].

We propose the construction of the central bank's asset allocation policy that must be framed as an appropriate liquidity policy at all the time. Data of the official (central banks and sovereign wealth funds) investment in the US securities shows that maximum 30 percent the global official reserves are held in the treasury bills (less than one-year maturity). This figure is therefore utilised and adopted to provide an assurance of the liquidity sufficiency. Hence, I assume the benchmark investment policy used by central banks is to allocate 30 percent of their reserves in the treasury bills and 70 percent in the G7 1-5 year government bonds. Multi asset weight constraints are defined as asset allocation constraint to resemble the nature of the central banks' investment policy.

The optimisation begins from the existing benchmark policy. Several diversification strategies are then combined with multi-weight constraints to assess whether significant risk reduction benefits may be achieved. The benefits of the diversifications are framed in the objective function illustrated in Figure 1.5 as follow:

[5] Briere et al. (2015) investigate three benchmark strategies to allocate central banks reserve assets: 1) investment in US government bond 1 to 5 years; 2) investment in G4 government bonds 1 to 5 years in the proportion of 63 percent in USD, 22 percent in EUR, 4 percent in GBP, and 4 percent in JPY which based on the actual allocation of central banks in the G4 currencies, and 3) investment in G4 government bonds 1 to 5 years with the proportion referred to the relative-weight of the currency in the SDR basket: 47 percent in USD, 34 percent in EUR, 12 percent in GBP and 7 percent in JPY.

Figure 1.5 Objective function framework



Source: author

The optimal currency composition deals with objective function as shown in Figure 1.5. The existing central bank's reserves investment is B_0 with μ_{B_0} expected portfolio returns and portfolio risk of σ_{B_0}. First I seek the optimum portfolio B_1 to reduce the portfolio risk σ_{B1} subject to a given existing portfolio return μ_{B_0} by currency reallocation among the existing benchmark currencies themselves or investing in broader universe while imposing trade and debt constraints. Second, portfolio risk minimization to the global minimum variance B_{GMV} for the same investment strategy as the first step is analyzed.

A similar approach, as applied to the currency composition of the foreign reserve, is now implemented for defining strategic asset allocation. In order to capture the liquidity requirements and to limit the holding of some asset classes, a multi-weight constraint is therefore applied. In order to guarantee the central bank's liquidity need, 30 percent of the reserves have to be allocated in the treasury bills all the time. To reflect central banks' conservativeness, it is required that a minimum of 25 percent be invested in the existing bond market index and liquid assets. In this framework, at least 55 percent of the current assets policy will continue to be held in the optimised portfolio. Both risk reduction for a given expected return and to the minimum variance will be addressed in this analysis.

The next step is to analyse the risk reduction benefits for the central bank from investing beyond government-related assets. This non-government diversification strategy will be analysed in two measures; i.e. the reduction of the optimum portfolio risk σ_{B_2} from the current benchmark risk σ_{B_0} and incremental benefits which reflect the different of the risk of portfolio B_2 and B_1.

In short, this thesis comprises of three empirical chapters. The first empirical chapter is devoted to solving the foreign reserves risk minimization issues at currency allocation level, with a focus on the external transaction requirements. The second empirical chapter emphasises maintaining sufficient liquidity at all times, while minimising portfolio risks at the structure of assets classes level. The third empirical chapter takes the portfolio analysis further to accommodate differing risk preferences of central banks, including investment beyond government-related securities.

In addition to the above general motivation of the whole thesis, there are more specific research questions for each empirical chapter, as detailed below: Chapter 3 concentrates on the safety objectives and at the same time ensures the central bank's external transaction needs will be fulfilled by the optimal portfolio.

I measure the safety objective as portfolio risk reduction for a given existing return and risk reduction to the global minimum variance portfolio (GMV). In this chapter, two research questions were asked, i.e. do foreign currency diversifications offer potential diversifications benefits for a given existing expected return for central bank reserve portfolios? And whether 2) central bank's currency diversifications provide risk reduction benefits to minimum variance portfolio?

Chapter 4 focuses on the safety objectives without sacrificing the existing liquidity and/or expected return profiles of the central banks' portfolios. Similar to the currency composition framework, the asset allocation framework measures the safety objective as portfolio risk reduction for a given existing return and risk reduction to the global minimum variance portfolio. The first research question in this chapter addresses as to whether there are any risk reduction benefits for central banks when investing in longer maturity bonds, and credit constraints are relaxed to include quasi-government bonds. The second research question arises because of increasing popularity in literature that emerging market assets offer diversification benefits. Therefore, it is important to raise a question as to whether investing in emerging government bond markets provides significant benefits when portfolio safety is the primary concern of the central bank. The third question is to provide a solution as to whether a central bank will gain benefits if inflation-linked government bonds are included in their investable assets. Existing literatures show that the inclusions of such asset classes are well-studied on an individual basis; however, the impact on government bond portfolios from a multi-objective central bank point of view, remains largely unknown.

Chapter 5 investigates if the risk reduction benefits found in the second empirical chapter are sensitive to varying the investment policy, risk appetite, or sample period of the central bank. The first research question in this empirical chapter is to examine the impact of budget constraints to the diversification benefits. This question is raised to address the doubts whether central banks should or should not tranche reserves portfolio to satisfy their multiple objectives. The second question is to study whether asset allocation in terms of differing liquidity changes the risk reduction benefits for central bank reserve management. Central banks with different reserves size, or the same bank in different economy conditions, may require a different portion of their reserve invested in liquidity assets. I therefore, need to investigate if the risk reduction benefits are sensitive to the central banks' liquidity allocation.

The next question is to investigate if the government bond risk reduction diversification benefits are mainly driven by the low yield environment and fewer benefit are observed in high-yielding bond markets. Since I could not find diversification benefits on the longer dated bond strategy, which restricted up to 10 years bond maturity, this question aims to examine whether central banks should consider ultra-long bond maturity in order to achieve risk reduction benefits. The last question is to investigate the benefits and incremental benefits of the risk reduction from relaxing central bank investment restrictions beyond quasi-government issuer securities. As such, the investment opportunity set is expanded to include gold, mortgage-backed securities (MBS), asset-backed securities (ABS), investment grade corporate bond, and world equity.

Since the scope of reserve management is fairly broad, it is important to keep in mind the following will not be addressed. The theory employed in this paper is based on modern portfolio theory. It is important to remember that, even though I focus more on the risk aspect, this is not a study of risk management. Portfolio theory which always tries to seeking minimum portfolio risk is at the heart of this research. Some elements of risk management might be included since we are dealing with a risk reduction of a portfolio, but the reader should not expect this to be a key element.

Analysis regarding the optimal size of foreign reserves is not addressed in this thesis; the level of the central bank reserves are assumed to be given exogenously, i.e. I do not address the financing or how the foreign reserve are to be accumulated. Additionally, I do not consider taxes, transaction costs and portfolio hedging. This is because I want to focus more on how central banks allocate their reserve assets to satisfy multiple objectives in a unified framework. The data used is relatively extensive and I do not see a reason to include additional securities, which would make the analysis intractable. The assets included are a group of G7, developed and emerging market government bond indices, semi-government, supranational and government agency bonds, inflation-linked government bonds. Gold, ABS, MBS, equity index and investment grade corporate bonds are also included for broader investible assets. Thus, I will not deal with options, futures or other derivatives. I use the longest available data in Thomson DataStream which is spanned from December 1984 to December 2014. I obtained all data from the same source in order to make it more equivalent.

1.4. Findings

The main objective of this chapter is to provide alternative frameworks for central banks to allocate currency compositions and asset allocation of their reserves. Risk reduction for a given existing expected return, and risk reduction to the minimum variance portfolio are implemented in this study to concentrate on the safety objective of the foreign reserve. The Bayesian method of Li et al. (2003) is combined with two currency constraints. For example, trade constraints and debt constraints are employed to analyse the potential benefits.

Our analyses on the risk reduction for a given existing benchmark return of the foreign reserves currency composition, show that before imposing currency weights constraints, all the diversification strategies considered provide significant benefits regardless of the current benchmark used. The results when the trade constraints and debt constraints are imposed, reveal that there is potential for diversification benefits to be obtained from the currency portfolio optimisation. However, the choice of the current benchmark and both trade and debt constraints play an important role in the decision as to which diversification strategy central bank reserves managers should select to proceed.

Empirical evidence related to the risk minimization for a given benchmark returns, before constraining asset allocations, show that there are significant benefits. Imposing trade constraints and debt constraint on the optimisations change the significance of the benefits. Broader currency diversifications provide the bigger mean of the benefits. However, for the original benchmark which allocates more than 60 percent to USD, significant benefits at the 5th percentile of the posterior distributions could be attained by broader currency investment into selected developed and emerging market country currencies. This occurs when trade constraints are imposed, however, none of the considered strategies is able to provide benefits after imposition of debt constraints. Central bank that uses the original benchmark which resembles Special Drawing Rights-International Monetary Fund (SDR-IMF) allocation, significant risk reduction benefits at the 5th percentile of the posterior distributions could be achieved by reallocate the existing benchmark currencies and by broader currency investment into selected developed market currencies from imposing trade constraint and broader spanning into selected emerging market currencies from imposing debt constraints.

A similar objective function to the earlier chapter, but it is now applied for defining an alternative strategic asset structure of foreign exchange reserves. The study concentrates on liquidity with the portfolio objective to reduce portfolio risks, without compromising the return aspects. In this framework, the more complicated asset weights constraints of equality, lower bounds and upper bounds constraints are applied. Our results on the minimum variance analyses show that longer bond maturity diversification strategies offer significant benefit to the government bond portfolio across the three constraints.

However, if a central bank aims to reduce portfolio risk for a given existing target return; the results could not reject the null hypotheses, even before imposing asset allocation constraints. When emerging market and inflation-linked bonds were introduced, our finding shows that introducing longer-dated bonds, emerging market country rating and inflation-linked government bonds altogether improve the investment opportunity set and deliver significant risk reduction benefits. The inclusion of inflation-linked and emerging market government bonds cannot be spanned directly to the current benchmark, but it needs to be added together in order to deliver significant diversification benefits.

The third empirical chapter investigates if the risk reduction benefits found in the second empirical chapter are sensitive to the different conditions that may lead an adjustment on the central bank's liquidity requirement for the precautionary, changes investment policies, different investment opportunities or sample period, and impact and marginal impact of non-government bonds risk preferences.

Our findings show that significant diversification benefits could only be achieved if it is fully invested or budget constraints equal to one. After imposing asset allocation constraints, none of the spanning strategies considered provides significant benefits for the lower budget constraints. Analysis of the impact of liquidity buffer allocation confirmed that US Treasury Bills drive one of the main sources of risk reduction. Allowing the optimised portfolio to hold US treasury bills at least as its weights in the benchmark offers significant risk reductions. These findings provide practical implications. The size of the reserves relative to their economy, risk aversion, or economic conditions which could have an impact on their liquidity portions, have important role in the risk minimization analysis.

Our analysis on the impact of budget and liquidity constraints has policy implications for central bank reserves management. Our results show that the requirement to provide more cash results in a higher uncertainty of the mean risk reduction. This implies that central bank should not segregate foreign reserves to cover short-term liabilities from the investable reserves. Segregation of reserves may introduce inefficiencies as it increases the difficulty of optimising the risk-return profile for the whole reserves. Central banks, therefore, should not to tranche their reserves, but define and express liquidity requirements and objectives in the form of constraints and objective functions in the portfolio optimisation framework.

Lastly, we found that the inclusion of non-government bond investments, which include gold, ABS, MBS, investment grade corporate bonds and world equity asset classes, provides significant risk reduction benefits. These benefits were both significant when compared to the existing benchmark, and incremental in terms of benefits from government-related bond diversification strategies.

1.5. Main Contributions

The overall empirical contribution of this research can be summarised as the alternative strategic asset allocation for central bank reserve portfolios, taking account of the multiple objectives: liquidity, safety and return. The current state of the liquidity aspect of the central bank is assumed to be sufficient and hence be adopted for the requirement of the framework. While maintaining central bank liquidity needs at all the time, my proposed framework offers two measures of diversification benefits. First, in relation to risk reductions for a given existing benchmark return, and second, the risk reduction of the minimum variance portfolio.

This thesis extends the contribution that the importance of the constraints of Papaioannou, Portes, and Siourounis (2006) and Wu (2007) remains valid under the Bayesian analysis. The first empirical chapter suggests there are potential benefits for the central bank to reallocate its currency composition either between current benchmark currencies or into broader currencies. Imposing trade constraints and debt constraints however, change the significance at the 5th percentile of the posterior distribution of the benefits. The choice of the current benchmark and trade and debt constraints play an important role in deciding which diversification strategy central bank reserves managers should select to proceed. These results suggest that the trade and debt constraints for currency allocation reflect the individual country exposure might need adjustment if the central bank needs to diversify their currency portfolio.

Second empirical chapter implies that the first-two priority of the objective for the central bank; liquidity and safety objective is an appropriate alternative approach for risk minimization framework. These results confirmed Manchev (2009) arguments that central banks portfolio objectives should focus on liquidity and security rather than return objective. Whilst preserving central bank liquidity need, first I minimised portfolio risk for a given existing benchmark return. Our Bayesian analysis suggests that central banks need to conduct portfolio switching 30 percent of their foreign reserve in new government related asset classes. The new asset classes included in the new strategic asset allocation are longer dated G7 (5 to 10 years), developed market (1 to 10 years) and SSA bond index together with emerging market and inflation-linked government bond market, and retain 70 percent of the reserves in the existing portfolio in order to reduce significant portfolio volatility whilst maintaining the existing benchmark expected returns.

Our finding on the various setting analyses intensified the important for the central bank to diversify the foreign reserve portfolio beyond its current policy setting. Our findings on the budget and the portions of liquidity aspects imply that central banks should not to tranche their reserves, but define and express liquidity requirements and objectives in the form of constraints and objective function on the optimisation framework.

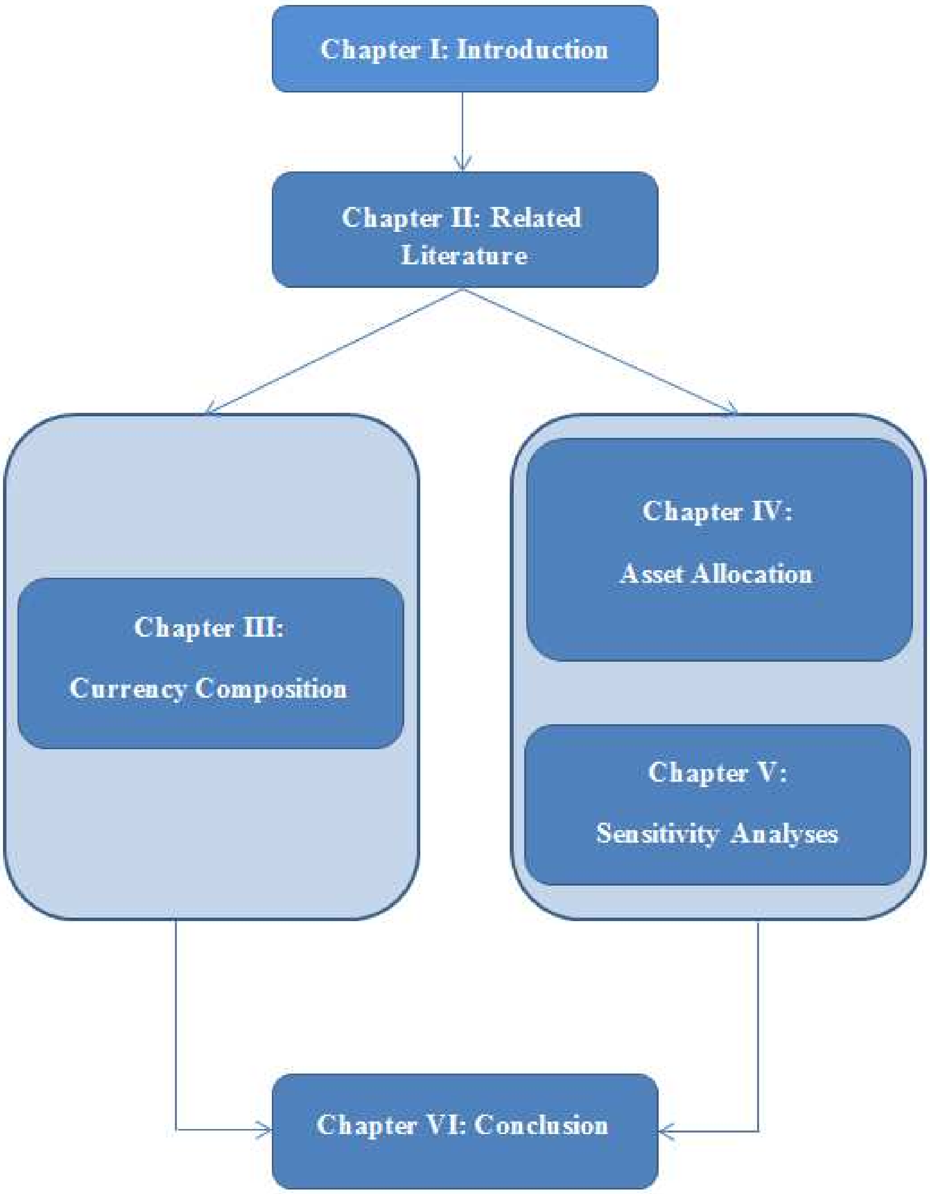
The purpose of storing the foreign reserves and the mandate central banks have in managing their reserves vary among central banks and therefore the preference between two measures might also differ amongst them. I would argue that reducing the risk for a given existing target return is more desirable for central bank reserve managers than the second measure. Foreign reserves diversification ideas resulting from the first measure also easier to communicate to the stakeholders since the benefits resulted from the frameworks without giving up the existing expected returns. However, it might be worth thinking about should central bank follow the minimum variance strategy since the empirical findings show minimum variance portfolio offer significant benefits across all constraints.

To my best knowledge, this thesis is the first study to propose the use of Bayesian model in the reserve management literature. This research contributes to the literature in portfolio management area in several ways: (1) it combines mean-variance Bayesian spanning test of Li et al. (2003) and portfolio constraints of Jagannathan and Ma (2003) for foreign reserve management topics; (2) conventional models are customized to the case where the investor faces multiple portfolio objectives. (3) This thesis complements and extends the evidence in Briere et al. (2015), Hunter and Simon (2004), Zhang et al. (2012), and Hanson, Liljeblom, and Loflund (2009) among others by incorporating the impact of a greater number of portfolio constraints and using Bayesian approach to evaluate statistical significance of the diversification benefits. This research also provides an alternative approach that is designed to help top executives at central banks reserve management to define their new benchmark policy for currency structure and asset allocation.

1.6. Thesis Structure

This section purposes to describe the structure of the thesis. Overall, our thesis consists of six chapters, which are divided into a number of sub-sections.

Figure 1.6 Thesis Structure



Chapter One: Introduction

Chapter one aims to introduce the thesis topic and the research problem addressed throughout the analysis. The contribution of our research to the existing literature regarding reserve management and portfolio choice in general is highlighted. Furthermore, the general methodology, main findings and contributions of the thesis are presented.

Chapter Two: Related Literature

This chapter reviews the previous studies in the foreign reserve management literature. It begins by outlining the importance of foreign reserve funds for the central bank, domestic economy and international financial markets. The purpose of which is to give the reader a basic understanding of the central banks' practices in reserve management.

Subsequently, I review the classical mean-variance framework of modern portfolio theory to provide the reader with a basic knowledge of its assumptions, applications and the definition of risk. Furthermore, the existing frameworks are reviewed. This includes defining an objective function from the multiple goals of the reserves management, combining with asset weights constraint and sensitivity analysis to the different risk preferences. The purpose is to give the reader an understanding of what can be used from the previous studies and how I can apply the strategic asset allocation frameworks of a central bank reserve management in practice.

Chapter Three: Optimal Currency Composition of Foreign Exchange Reserve Portfolio

The first empirical chapter investigates the potential diversification strategies of a central bank's reserve currency compositions. This research considers two currencies benchmark investment strategies and examines the impact of trade and debt constraints for the portfolio risk reduction benefits.

Chapter Four: Central Banks' Government Diversification Benefits: Multi-constraint Mean-variance Spanning Test

In the second empirical chapter, the study focuses on the safety needs without compromising liquidity and return aspects from asset allocation perspectives. I measure the risk reduction portfolio both for a given level of the existing return and risk reduction to the global minimum portfolio.

Chapter Five: Risk Minimization on Multi-weight Constraint Bond Portfolio in Various Setting

This empirical chapter investigates the importance of various factors and evaluates if diversification benefits are sensitive to the varying conditions that may lead an adjustment on the central bank's risk preferences and investment policy. Some different settings considered in this analysis to evaluate the role of budget constraints, liquidity buffer allocation, and global financial crisis investment opportunities; longer than 10 year bond investment and non-government bond investment preferences for the risk minimization reserve portfolio.

Chapter Six: Conclusion

The overall conclusion is drawn with respect to the empirical analysis and aims to answer the questions of the thesis and offers benchmark asset allocation policy for reserves management practices. Furthermore, limitations of the research are outlined and future improvements are proposed.

CHAPTER TWO

This chapter aims to identify current knowledge gaps from the previous studies, how important are these gaps, and what might fill them. The second purpose is to justify the importance for my own research by providing evidences leading to the proposed central banks' foreign reserves asset allocation framework and specific research questions that will be investigated.

2. RELATED LITERATURE

The purpose of chapter two is twofold. First, this chapter aims to identify current knowledge gaps from the previous studies, how important are these gaps, and what might fill them. Moreover, second, to justify the need for my own research by providing evidence leading to the design of the proposed central banks' foreign reserves asset allocation framework and specific research questions that will be investigated.

The chapter starts with the question "what are the main theories that lie beneath the study of foreign reserve currency composition and foreign reserve assets allocation?" There are two leading theories – the transaction theory and the mean-variance theory of currency composition; and two main approaches to constructing reserve asset allocation policy – the market index and portfolio optimisation. Discussion of the development of each theory and summary of the important empirical studies in this field will be the main theme of the beginning of this chapter.

The remainder of the chapter proceeds as follows. Section 1 describes the existing literature of the current states and trends in reserve management. Section 2 and 3 give a background of the use of portfolio weight constraints and portfolio tranching in central banks portfolio practices. Section 4 describes asset class selection of both government securities and broader investment universe spanning strategy, and section 5 concludes.

2.1. Central Bank Foreign Reserve Allocation

There are two main theories of the foreign reserves composition. One theory stated as the transaction theory, claims that market transaction activities are the major factor in determining the central bank's foreign reserves currency structure. This theory believes that the currencies which form the largest portion of a nation's foreign reserves portfolio are those that are the most essential in accommodating various foreign transaction of the country. These include foreign currency intervention (purchases and sales of foreign currencies), financing of foreign trade and the settlement of foreign debt obligations. The transaction theoriy therefore suggests that the preferred currency allocation is likely to be independent of the optimal distribution of the foreign reserves across currencies (Dooley, Lizondo, & Mathieson, 1989).

On the other hand, the exact opposite argument is proposed by the mean-variance theories. This is based on the model of general mean-variance optimal portfolio selection developed by (Markowitz, 1952) which also the foundation of Capital Asset Pricing Model (Markowitz, 2014). Mean-variance theory argues that the allocation of wealth embedded the risk and returns associated with holding foreign reserves assets in different currencies are the main factors in foreign reserves structure. Consequently, the theory suggests that rational central bankers construct some optimal allocation of reserve assets that minimise reserve portfolio risk for a given level of return, or vice versa; maximise the return for a given level of the reserves portfolio risk (Ben-Bassat, 1980).

The main assumption of the mean-variance framework is that the returns are normally distributed. The mean-variance portfolio decision models require certain ex-ante input parameters are assumed to be known to investors. The investor is required to provide estimates of the expected returns and covariances of all the assets in the investment universe considered. In practice, however, investors often lack the knowledge as to these values. As such they take their ex-post estimates from samples of the assets' past performance. If ex-post estimates are obtained when the underlying probability distribution of asset weights in an optimal portfolio are unknown, there is an estimation risk problem ((Barry, 1974, 1978), (Bawa, Brown, and Klein, 1979), (Dhingra (1980, 1983), Klein and Bawa (1976, 1977) and Klein, Rafsky, Sibley, and Willy (1978)). Furthermore, mean-variance optimisation is very sensitive to the inputs. Chopra (1993) and Chopra and Ziemba (1993) shows that minor changes in the input parameters can result in the very different asset allocation of the optimal portfolio. Following those drawbacks, I will discuss some of the common techniques for mitigating estimation errors.

One of the most popular methods to mitigate estimation error is by using the Bayesian technique. A great number of general Bayesian and shrinkage approaches have been used to estimate the inputs to mean-variance optimisation. For the expected returns see, for example, Jobson and Korkie (1981), Frost and Savarino (1986), and Jorion (1991, 1986). For the covariance matrix see, for example, Ledoit and Wolf (2003, 2004). For both expected returns and covariance matrix see, for example Wang (1998) and Li, Sarkar, and Wang (2003). The basic idea underlying these types of estimators is the bias-variance trade-off; whereby sacrificing some bias one can obtain a more efficient estimator that is less sensitive to changes in the data.

Bayesian predictive distribution provides a natural method to express investment opportunities in the presence of parameter uncertainty (Avramov and Chao, 2014; Avramov and Zhou, 2010). An additional benefit of the Bayesian approach is that it can be applied when the central bank faces portfolio restrictions such as budget, short selling, asset allocation or the combination of those constraints. Most mean-variance strategic asset allocation for foreign reserve portfolios are based on Black-Litterman (1992) for example Zhang, Chau, and Xie (2012).

Despite the popularity of the mean-variance theory in constructing currency composition policy of foreign reserve investments in term of portfolio optimisation or diversification, there are a number of drawbacks associated with the data availability and practicality of the mean-variance approach. Based on those difficulties, Dooley et al. (1989) concluded that the transaction theory, as an alternative theory, should provide a more practical framework to allocate the reserve compositions of central banks.

Empirical studies on the determinants of reserves currency composition and asset allocation have been hampered by the fact that foreign reserves data is confidential in most countries. The reserves of less than 40 percent emerging countries are included in the Currency Composition of Official Foreign Exchange

Reserves (COFER) database from the International Monetary Fund (IMF) (Beck and Rahbari, 2011).

The first empirical study that used the transaction theory was Heller and Knight (1978). Their study explained variations in the currency shares of a country's official reserves in US dollar, pound sterling, French franc, Deutsche mark and other reserves currencies. This study provided the first empirical evidence that central bank's transaction needs to play a key element in the determinant of currency compositions. Using data of 76 countries during the period from 1970 to 1976, their empirical findings showed that countries added the portion of their reserves held as a given reserve currency if they fixed their exchange rate to that currency, or if the country of issuing reserve currency was a major trading partner.

A later study by Dooley et al. (1989) tried to explain that the currency reserve held by a single country is a function of their trade and debt servicing payments. Their empirical examination used confidential data from the International Monetary Fund (IMF) on the currency composition of the reserve for individual countries over the period from 1975 to 1985. Similar to the previous study, this empirical research found that trade flows and rate arrangement played a significant role in the currency composition decisions.

In a later period, a study by Eichengreen and Mathieson (2000) proposed three important findings. First, as the economy grows larger relative to other countries, its currency should also increase in proportion. Second, as a country increases trade with the rest of the world, other countries tend to hold that country's currency in their reserves portfolio for foreign trade payment purposes. Third, the existing domination of the USD as the reserve currency was found to be an important factor to ensure the dominance of a major reserve currency. With regards to those findings, they concluded that the sharp shifts of USD from its dominance position in reserves currency were very unlikely.

Only five empirical studies of the mean-variance theory of the currency composition appeared to have been published. During the study period 1972 to 1987, Ben-Bassat (1980) show that risk and return consideration are important factors in the Central Bank of Israel's foreign reserve currency composition, and the developing countries. Similarly, Dellas and Yoo (1991) found that risk and return played an important factor in the determination of South Korea's foreign reserve currency allocation during 1980 and 1987.

Papaioannou, Portes, and Siourounis (2006) propose a dynamic mean-variance framework and compute the optimal level of the world reserve portfolio using different methods to estimate mean returns and covariance matrices. The authors also impose different constraints to reflect transaction considerations. They find that the reference currency is very important and the optimal world reserve portfolio suggests that the share of the Euro is lower than the actual share published in the IMF COFER database.

The most recent studies available on the central banks' currency composition are those of Beck and Rahbari (2011) and Kim and Ryou (2011).

Beck and Rahbari (2011) develop optimal reserve portfolio in a minimum variance framework with the presence of sudden stop in capital flows. Their study provides interesting findings; (1) optimal reserve portfolios are dominated by US dollar as base currency; (2) during the sudden stop, US dollar performs as a safe haven currency, increasing the optimal portion in reserve portfolios; (3) US dollar shares should decline when the debt-to-reserve ratio declines, and (4) the denomination of foreign currency debt has little importance for optimal reserve portfolios.

Following the recent debate regarding the need for the central bank to diversify their reserve, Kim and Ryou (2011) study the mean-variance efficiency of the foreign reserve portfolios. They implement likelihood ratio procedure and examine the efficiency of the reserve portfolio from 18 countries. Their findings suggest that the domination of the US dollar as an international reserve currency has not declined, despite the recent US dollar depreciation during the 2008-2009 financial crisis.

The common stage following a decision on currency composition is to decide how to allocate the foreign reserve, or to decide asset allocation or benchmark policy. In contrast to the study currency of which relatively well documented, literatures on central banks reserve asset allocation are very limited. The mainstream benchmark asset allocation for foreign reserve relies mainly on published global bond market index data with weights proportional to market capitalization or GDP figures (Leon and Vela, 2011). However, Brennan, Kobor, and Rustaman (2011) argue that central banks practitioners suggest that reserve management should be moving away from the bond market indexes as the benchmark allocation policies given by these indexes tend to overweight more indebted countries.

Research on strategic asset allocation relatively less documented than currency composition in reserve management literature. Berkelaar, Coche, and Nyholm (2010), Bernadell, Cardon, Coche, Diebold, and Manganelli (2004), Bakker and Herpt (2007), and Joia and Coche (2010) edit publications on the various aspects of foreign reserve management in which some parts focus on the strategic asset allocation of central bank's reserve portfolio. Cardon and Coche (2004) propose a management framework for the central bank strategic asset allocation where asset allocation decision can be performed by a three-layer governance consisting of an oversight committee, investment committee and portfolio management. Fisher and Lie (2004) provide a reserves strategic asset allocation framework considering various assets classes. These include government bonds, non-government bonds, equities and currencies while securing liquidity sufficiency for trade and intervention needs.

Zhang, Chau, and Xie (2012) suggest a behavioural finance application with the Black-Litterman (B-L) model to determine central banks' strategic asset allocation. This is applied to the case of Chinese central bank to develop optimal asset allocation, whereby they argue that their framework is suitable for the reserve management allocation with multiple objectives. The Black-Litterman model is embedded into behavioural variables that may influence the attitude of reserve managers' toward risk-return, and assumes that central banks have two sub-portfolios. One is a safety portfolio which governed by the precautionary motive and has lower expected return. This portfolio satisfies safety and liquidity objectives. The other is investment sub-portfolio which focuses more on the return objective. These two sub-portfolios are then combined to construct an aggregate portfolio.

The more recent study on the strategic asset allocation for China's foreign reserves by Zhang, Zhang, and Zhang (2013) is using copula opinion approach. Regime-switching copula is used to examine the dynamic dependence amongst risky assets. They found that when the central bank objective mainly focuses on minimising portfolio risk, their reserves allocation encourages the flight to safety. Conversely, when higher risks level is permissible to get a higher return, it would discourage the flight to safety. Therefore, the authors suggest that China's central bank should mitigate its flight to safety after 2008 and increase their investment in short-term bank deposits, long-term treasury bonds and euro bonds. Previous works in strategic asset allocation for reserve management literatures is relatively limited; some of which utilise BL-Bayesian approach to derive strategic asset allocation. None of them extends the conventional Bayesian to solve multiple goals of central banks' portfolio problems.

The most recent study by Bri'ere et al. (2015) implement a geometric test of mean-variance efficiency, finds that introducing currencies which has a low correlation to the USD significantly reduce portfolio risk. Expected portfolio return is improved by spanning into mortgage-backed securities, corporate bonds, and equities. In general, those literatures argue that the return aspect of the foreign reserves becomes more essential in central banks' reserve management; more specifically those who have excess reserves. They find that relaxing various constraints can obtain a better expected portfolio return for the same level of risk.

Most foreign reserve asset allocation literatures analyse return objective, and limited studies look at the safety aspect as a portfolio objective. More specifically, despite general view safety aspect is the most important objective for central banks most of reserve management practices and literatures address them using portfolio constraints. In this thesis, I will evaluate portfolio safety features from the portfolio constraints and the objective function perspectives.

The recent global financial market meltdown highlighted the need for central banks to refocus from return back to liquidity objective. Unfortunately, liquidity risk is more challenging to measure than the other financial risks; for example, currency or interest rate risk. Market liquidity is a shadowy concept with several features that cannot be taken by a single measure (Amihud, 2002). Widening of bid-offer and/or credit spreads, a sharp rise in the correlation of the risky assets, a reluctance of institutions to trade with each other, and a complete bid disappearance from brokers' screen are some indicators of market illiquidity. In academia, liquidity issues have traditionally been measured in terms of transaction costs which are reflected by bid-offer spreads, while practitioners refer to the inability to liquidate a trade position when needed (Longstaff, 2001). Additionally, there is a big literature on market microstructure on liquidity measure[6]. However, it appears to be no single agreement which one is the most appropriate liquidity measure from a central banks reserves management point of view.

The further complication arises given the fact that portfolio management literature has mainly focused on risk and return, relative to liquidity. The commonly employed mean-variance and value-at-risk (VaR) metric and their variations also unable to reveal clear information related to the liquidity aspect of the reserve portfolio[7]. Liquidity management in central banks practice involves dividing reserves portfolios into more and less liquid assets and imposing constraints on these tranches, requiring a minimum quantity of highly liquid securities to be held at all times (Romanyuk, 2012).

The United Kingdom employs an asset allocation model that explicitly trades off liquidity and return (Treasury, 2010). The challenge of incorporating liquidity risk into strategic asset allocation is that: illiquidity issues are usually observed in real time, which does not lend itself easily to being packaged in estimated terms over investment frameworks. Also, liquidity or illiquidity from

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[6] see (Chordia, Roll, and Subrahmanyam, 2000; Deuskar, Gupta, and Subrahmanyam, 2010; Goyenko, Holden, and Trzcinka, 2009; Korajczyk and Sadka, 2008) for a discussion of related issue.

[7] The problem with VaR approach for the case of illiquidity is that when everyone relies on this framework, during times of market volatility, the risk limits of some investors are hit, who then sell their assets at the same time which increases market volatility and covariances, and then risk limits of more investor are hit, who then sell, and so on, creating a vicious cycle of asset price crash, higher volatility and market panic (Persaud, 2000; Romanyuk, 2012).

central bank perspective might not be a continuous but rather an “on-off” variable that is not easily adapted to particular portfolio allocation models. The potential needs for liquidity are difficult to predict, and extremely so during crises. As such, I implement the ideas proposed by Romanyuk (2012) that highly liquid assets should be held all the time by imposing asset allocation constraints in order to provide sufficient liquidity when needed.

Given the advantages of the Bayesian model discussed above and its successful applications in the stock market, insurance and pension fund portfolio optimisation, it opens up the opportunity to develop a useful model for strategic reserves management in Bayesian framework. In this research, Bayesian approach combined with multi-weight constraints are utilised. The main motivation of the application with multi-weight constraints is to capture the spirit of the transaction theory; providing an alternative strategic asset allocation which resembles central bank reserve portfolio. The proposed framework aims to offer an alternative currency composition and asset allocation policy when central banks are most concerned with portfolio safety, without compromising their liquidity and profitability objectives.

2.2. Portfolio Weight Constraints

Institutional investors often implement portfolio weight limits of assets or groups of assets to prevent extreme asset allocations that may result from model inaccuracies or to meet with their investment mandate. Jagannathan and Ma (2003) provide a theoretical explanation for such practices. They show that imposing negative weight constraints are equal to reducing the estimated security covariances, whereas upper bounds are comparable to increasing the corresponding covariances. For example, assets that have high covariance with others tend to receive negative portfolio weights. Therefore, when their covariance is reduced (which is identical to the effect of imposing no short-selling constraints), these negative weights reduce in magnitude. Likewise, an asset that has low covariances with other assets tends to get over-weighted. Hence, by increasing the corresponding covariances the impact of these over-weighted assets decrease.

Using a minimum-variance portfolio strategy with elastic portfolio weight constraints, Behr, Guettler, and Miebs (2013) reassess the findings of Jagannathan and Ma (2003). They show evidence that incorporating portfolio weight constraints is favourable for the optimisation problem if the input parameters are error-free. On the one side, weight constraints guarantee that portfolio weights are not mainly driven by sampling error inherent in historical data parameter estimates, which then leads to the concentrated portfolio[8]. On the other side, weight constraints in their approach reduced the sampling error and loss of sample information in portfolio optimisations.

Analysing the work of Jagannathan and Ma (2003) in a more specific setting, Roncalli (2010) find that imposing portfolio weight constraints on the global minimum variance portfolio is similar to use a shrinkage estimate of the

8 See (Chopra & Ziemba, 1993; Chopra, 1993; Green & Hollifield, 1992) studies for the literature concerning concentrated portfolio in mean-variance setting.

covariance matrix. The impact on the mean-variance and tangency portfolio however is largely less known. More specifically, the impact of the portfolio weight constraints on mean-variance portfolio on the risk minimization of the mean-variance portfolio for a given expected returns is unknown.

Ling (2011) propose an active portfolio model with an objective function to minimise the probability of big losses occurring in the optimum portfolio subject to multiple weights constraints. Analysing the mainstream mean-variance tracking error model with the proposed model, the author finds that both efficient frontiers are not intersecting. The expected excess return of the proposed model is not larger than the classical mean-variance portfolio for any tracking error in the same setting. Ling (2011) findings are not surprising given the facts that more assets restricted in multiple weights constraints are smaller than that of portfolio with single or fewer weights constraints.

However, Kolm et al. (2014) argue that extreme attention need to be taken in designing and implementing portfolio weight constraints. The authors provide an example, if the weight constraints are too ‘‘tight,’’ the portfolio allocation will be completely determined by the constraints instead of the forecasted expected returns and their covariances. Therefore defining and imposing multiple portfolio weights constraints for the reserves portfolio with multiple objectives need extra considerations more specifically to capture unique nature of the central bank investments.

Taking into account the above mentioned studies, I apply multiple weights constraints for both risk minimization and return maximisation of the mean-variance portfolio for a given level of return and a minimum level of risk respectively, while satisfying central banks' liquidity requirements. The main purpose of the multiple assets weight is to approximate what can be thought as stages of reserves management, based on central bank practice towards certain asset classes. The design of the asset weights constraint requires extra carefulness, given the facts that the effects of the restriction inherent in defining optimal strategic asset allocation. To capture central bank conservativeness, the proposed optimum mean-variance portfolio will only be allowed to change new asset allocation policy for less than half of the reserve portfolio.

2.3. Portfolio Tranches

Large reserves accumulation following currency crises in several Asian and Latin American countries in the late 1990's is one motivation for some central banks to structure and divide their foreign reserves into two or more tranches. While a higher level of reserves reduces the likelihood of currency crisis, the price for holding massive amounts of liquidity is costly. This is because central banks prefer to invest their reserves in very liquid and safe assets which therefore earn low returns. Portfolio tranching allows reserve to be divided according to the central banks' specific requirements; for example for liquidity and investment objectives and other policy requirements. Based on the theoretical model of sudden stops and central bank liquidity management developed by Caballero and

Panageas (2005), Osorio (2007) provides a method to lower the opportunity costs of holding large amounts of foreign reserves by breaking reserves into two tranches. Osorio (2007) argues that the proposed structure would allow the central bank to invest a portion of its reserves more efficiently during normal periods without creating high liquidation and transaction costs in events of the crisis.

Reveiz (2004) provides an example of portfolio tranches at Banco de la Republica, the Central Bank of Colombia. He explains that the central bank uses three separate portfolios which are working capital, intermediate and stable portfolio tranche. Working capital tranche portfolio is to cover intervention needs. Intermediate tranche or passive portfolio is held in the United States, German and Japanese government bonds that do not allow any active management. Additionally, the size of working capital and intermediate portfolio in aggregate has to be sufficient to cover one-year intervention with a 99 percent of confidence. Lastly, the stable tranche portfolio can take an active risk by deviating from the benchmark within predetermined ex-ante tracking error limits.

A later study from the Banco de la Republica, Colombia by Garcia-Pulgarin, Gomez-Resrepo, and Vela-Baron (2015) explores an alternative strategic asset allocation framework. The purpose of which is to maximise the risk-adjusted returns while maintaining the objectives of liquidity and safety of a foreign reserves' portfolio. The authors argue that the overall portfolio should be separated into two tranches; safety and wealth tranches. The safety portfolio is comprised of liquid, default-free and low volatility assets, where the safety and liquidity objectives are the objectives function. The wealth tranche aims to maximise the return with a wider asset universe and a longer horizon. They argue that while maintaining the safety and liquidity needs of a traditional reserves portfolio, their historical and forward looking analysis found evidence that the framework is able to deliver better reserves portfolio performance.

An interesting survey by IMF in 2012 (Morahan and Mulder, 2013) shows that more than 80 percent of central banks in their sample make use of the popular practice of reserve tranching. Interestingly, advanced countries make less use of tranching than middle and low-income countries. Morahan and Mulder (2013) argue that this preference to tranche their portfolios reflects the fact that low and middle income countries may need reserve for frequent intervention purposes. Intervention needs and central banks' explicit liabilities are the most important factors in defining the relative size of the tranche portfolio, demonstrating the importance of immunisation in reserve management policies. Some other factors shaping the relative size are target return, potential demand for liquidity and size of the government's short term liabilities.

Most of the previous studies support the idea for central banks to divide their reserve portfolio into tranches based on return enhancement objective. Strategic asset allocation proposed by (Garcia-Pulgarin et al., 2015; Osorio, 2007; Reveiz, 2004) are based on a framework to maximise the return of the portfolio for a given level of risk, while satisfying central banks' liquidity objectives. None of these studies is looking at the different objective framework for example risk minimization portfolio. To fill this gap, this is our opportunity to evaluate if separating foreign reserve into tranches is the right choice for the central bank who believes portfolio safety is the main objective for reserve portfolio.

2.4. Asset Class Selection for Reserves Portfolio

Asset class selection is one important step in portfolio choice problems for investors in general, including foreign reserves. In this sub-section I discuss the potential diversification benefits for central banks' reserve portfolio, amongst the existing asset classes and beyond. Asset classes issued by government and quasi-government will be discussed first and followed by non-government issued securities in order to find diversification opportunities.

2.4.1. Government bond securities

A central bank may be inspired to diversify their reserve investments for a number of motives. Portfolio risk reduction and return enhancement are the most popular objectives of diversification. Government bond market can be classified into high-grade sovereign bond (a group of G7 and other advanced countries), quasi-government (supranational, semi-government, and a government agency - SSA), emerging market and inflation-linked government bond market. Another popular strategy to diversify bond portfolios is pursued by lengthening bond maturity investments. Those asset classes will be discussed to address the potential benefits of international diversification for central bank portfolios.

2.4.1.1. Longer bond maturity

Historically, in a positive yield curve environment, investors have always been compensated in higher returns over time by investing in longer dated securities. This argument can be explained by liquidity preference (Bodie, Kane, and Marcus, 2008). Liquidity preference theory says that longer maturity treasury bonds are subject to greater interest risk than short-term bonds. As a result, investors in long-term bond bonds entail a risk premium to compensate them for the risk. This theory is also derived from the fact that shorter bonds are generally more liquid than the longer bonds. The preference to hold lower liquidity bonds will only occur if those bonds offer higher expected returns.

The motivation to examine longer-dated bond strategy because investors have typically been rewarded in higher returns by investing in longer dated bond (Berkelaar, Coche, and Nyholm, 2010). More specifically, Johnson-Calari et al. (2007) document a study about trends in reserves management by central banks, point out the opportunities to relax duration and credit risks constraints on the strategic asset allocation to improve the risk-return performance through diversification. The recent ultra-low yield phenomena, however, raises doubts as to the significant diversification benefits from investing in longer maturity bonds. More importantly, I would like to know whether it is still the case when central bank values more on safety than portfolio profitability.

2.4.1.2. *Developed market government bond*

The first and the most basic reason of portfolio spanning from high-grade government bond markets would be to achieve overall portfolio risk reduction due to the expectation that government bonds market issued from different countries are less-than-perfectly correlated. Beyond portfolio volatility reduction, the international investment could also be driven by return enhancement motives.

Barr and Priestley (2004) analysed monthly returns in United States, United Kingdom, German, Canadian, and Japanese government bond markets between 1986 and 1996, found that the ultimate diversification benefits have not been realised in the global bond markets. Abad, Chulia, and Gomez-Puig (2010) discovered that European government bonds are less sensitive to world risk factors, and are only partially integrated with German bond market, both studies suggesting diversification opportunities in the international bond markets.

When analysing the prospective benefits of international diversification for developed market government bond portfolios, Brennan et al. (2011) decompose the returns on G7 sovereign bond into local and global factors. The authors find that on average 75 to 80 percent of bond returns are determined by global factors, whereas 20 to 25 percent of bond returns are more related to local factors. Interestingly, while the government and the government-related bond market are integrated to a relatively high degree, there is still some potential room for diversification. More specifically, this research would like to examine central bank portfolios risk reduction benefits from investing in more diversified developed government bond markets.

2.4.1.3. Semi-government, supranational and government-agency (SSA) bond markets

There was an increasing central bank focus on profitability that increases exposure into riskier asset classes of their reserves portfolio. In 2006 Reserve Management Survey (Pringle and Carver, 2006) detected an increasing move into riskier assets, with more investment particularly in a government agency securities. Similarly a Bank of International Settlement (BIS) survey in 2007 (Borio, Ebbesen, Galati, and Heath, 2008) confirms comparable results, with notable movements of central banks' investment towards agency paper to increase the risk/return profile of their reserves portfolio.

The old-fashioned asset classes which include treasury bills, bank deposits, highly rated government securities and supranational bonds were still making up the most dominant portion of reserve portfolio. Nevertheless, reserve managers' appetite for higher credit risk has been increasing. Gradual increased risk appetite for central banks investments are typical to issuers which closely linked to government or has a guarantee from the government. These type of securities is included semis/states/landes/provincials and a government agency, or state-owned-enterprise and supranational (such as IFC, EIB, ADB, AFDB) bonds.

These moves taken by central banks to gradually relax their credit limit motivate me to examine the benefits of quasi-government bonds for government bonds portfolio. This research measures the diversification benefits in term of portfolio risk reduction for a given target return, and return enhancement from the minimised benchmark risk.

2.4.1.4. Emerging market government bond markets

From the equity markets perspective, De Santis (1997) and Harvey (1995) demonstrate that the efficient frontier shifted when emerging market stocks are included. However, De Roon, Nijman, and Werker (2001) find that the diversification benefits disappear when short-sale restriction and transaction costs are imposed. It is possible that the result will be similar for emerging market government bonds, since some recent studies on emerging market credit spread have documented significant co-movement in spread changes. Cifarelli and Paladino (2006) documented convincing evidence of emerging sovereign bonds co-movement spread; it changes more within geographical area than between geographical area.

However, very few studies analyse the role of emerging market government bonds for developed market government bond portfolios from a central bank perspective. In a similar approach to Li et al. (2003), who work on emerging market equity portfolio, this study applies their Bayesian spanning test method to search diversification benefits for foreign reserve portfolio to invest in emerging market government securities.

2.4.1.5. Inflation-linked government bond market

An inflation-linked bond is a debt security which generates cash-flows associated with the evolution of a given price index in order to provide protection with the real value of the investment (Selves and Stamirowski, 2011). Different from the nominal sovereign bonds which offers investors certain nominal rates of return, inflation-linked bonds assure a real rate of return as a result of the price index movements. The risk and return characteristics of the index-linked differ from those of conventional government bonds, while still offering the same credit exposure.

Today, dominated by sovereign issuers, the global government inflation-linked bond market is worth above USD1200 billion and the main issuers are the United States, United Kingdom, France and Italy (Selves and Stamirowski, 2011). The general mechanism of inflation-linked securities is used to ensure protection against inflation. The prices of inflation-linked bonds are quoted in real terms. Settlement values and cash-flows then are adjusted for accrued price index changes. This mechanism makes inflation-linked bonds completely equivalent to a nominal bond.

Implementing unconditional and conditional spanning test, Mamun and Visaltanachoti (2006) conclude that inflation-linked bond creates a meaningful asset class for a diversified portfolio which contain stocks, treasury bills, treasury bonds, corporate bonds and real estate. The findings of Mamun and Visaltanachoti (2006) are consistent with economic theory and similar to Kothari and Shanken

(2004), but different from those of Hunter and Simon (2005). In this study, Hunter and Simon (2005) conclude that treasury bills may be a sensible substitute for indexed bonds, especially in periods without any inflation shocks. Such findings are contradictory to Campbell and Viceira (2001), who point out that it is possible to replicate the return of long-term real bonds with short-term nominal bonds using a rollover strategy. However, they note that it is high risk because this strategy is exposed to the real interest rate variations.

The more recent study of Huang and Zhong (2013) using data from 1970 to 2010 finds that the commodity, real estate and inflation-protected securities asset classes are not substitutes for each other. Furthermore, the diversification benefits of each asset class change substantially over time. Therefore, all those three asset classes should be included in investors' portfolios. Another empirical analysis by Bri'ere and Signori (2009) shows the dynamics of conditional volatilities and correlations for inflation protected bonds, nominal bonds and equities asset classes for the period 1997–2007. Bri'ere and Signori (2009) argue that, although inflation-linked bonds once had positive diversification power, they are now highly correlated with nominal bonds and have reached similar volatility levels. As a result, the two asset classes are practically substitutable. This seems to be due to more stable inflation expectations and a more liquid inflation-linked bond market. Although diversification was a valuable reason for introducing inflation-linked bonds in a global portfolio before 2003, Bri'ere and Signori (2009) argue this is no longer the case.

Unfortunately, as to my knowledge, only one study examines the inclusion of the linkers in a sovereign bond portfolio which make this strategy difficult to compared and analysed. Utilising the French market for the inflation-linked bonds, Selves and Stamirowski (2011) analyse the impact of containing inflation-linked instruments in a nominal government bond portfolio. Their theoretical analyses suggest constructing a bond portfolio which includes nominal and index-linked bonds. Empirical evidence of this study reports that the case for including inflation-linked in a bond portfolio is reduced to the classical CAPM model.

These contradictions between theory and inadequate empirical study on the inclusion of the government inflation-linked bond to the government bond portfolio, calls for the further examination of the benefits; more specifically from a central banks' perspective. The objective of this asset class selection analysis is to examine the reduction of portfolio risks from various investment strategies for a central bank foreign reserve portfolio.

2.4.2. Non-government asset classes

As some central banks continue to pursue higher return from their reserves portfolio, an enthusiastic discussion on the merits and consequences of investing beyond government securities has developed. Adding non-government risk to a reserve portfolio should result in long-term outperformance for the similar government risk profile. In this section some asset classes which could potentially be considered by central banks to be included in reserve portfolios other than government bonds are reviewed. Gold, asset-backed and mortgage-backed securities, high grade corporate bond, and equity are considered asset classes to be included for the new benchmark policy in this study.

2.4.2.1. Gold

The investment roles of gold in financial markets were believed to be important for many decades. Gold is often perceived to provide potential diversification benefits within broad investment portfolios, given its low correlation with other asset classes (Hillier, Draper, and Faff, 2006). Analysing the daily data from 1976-2004, the authors find that gold has some hedging capability, particularly during periods of high volatility and provides a significantly better performance than standard portfolios.

The central bank is one of the major holders of gold as a tradable financial asset, and the bulk of reserves were invested in gold. In the study of gold holding and trading during 1979-2010, Aizenman and Inoue (2013) find that central bank continues maintaining a passive stock of gold, regardless of gold real price movements. However, in recent periods they observe the synchronisation of gold sales by central banks. More specifically, Wooldridge (2006) finds that central banks' gold holding declined from about 60 percent of total reserves in 1980 to a record low of less than 10 percent in 2005.

The management of gold reserves has evolved over time. Initially gold was segregated from other reserve assets and physical holdings of gold were left steadied even as prices changed and reserves accumulated. Wooldridge (2006) explains that starting in the late 1980's, some central banks sold part or even all of their gold holding. The sharp increase in the gold prices in 2006 helped to boost gold's share of reserves above 10 percent in 2006. However, physical holdings of gold continue contracted further at a rate of 2 percent per year.

The prolonged debates about the role of gold for investors in general, and the findings of some studies which reveal share of gold have become smaller in central banks' portfolios are the main motivation to examine the benefits of gold in the proposed framework. The objective of gold asset class selection analysis is to examine the reduction of portfolio risks for a given current benchmark target returns: without compromising the importance of the liquidity aspects of central bank's foreign reserves.

2.4.2.2. High-grade corporate securities (includes Asset-backed securities (ABS) and Mortgage-backed securities (MBS))

As central banks' reserve managers became more comfortable managing market and credit risks, the allocation of their bond portfolios changed towards longer-term bonds and securities with lower credit rating. Even so, the allocation to securities with a short term maturity and high rated government bonds remains high. This move shows that reserve managers' enthusiasm to take on market and credit risk has certainly increased.

Central banks continue to invest their foreign reserves largely in assets with low credit and liquidity risks. Government bonds remain the largest asset class in their reserve portfolios. Wooldridge (2006) shows that official holdings of US government securities accounted to 95 percent in 1989 and decreased to 73 percent in 2005. Furthermore, he explained that AAA rated agency securities are the most actively traded securities after the government bonds.

Nonetheless, central banks' reserve manager appetite for market, credit and liquidity risk has been increasing. About 50 percent of the 56 respondents of the Central Banking Publications survey of reserve managers informed of an increase in the amount invested in non-government higher-risk securities (Pringle and Carver, 2006). This survey also reveals that a significant minority of central banks held asset-backed securities, mortgage-backed securities and corporate bonds. The US annual survey of foreign portfolio holdings of US securities confirms that in 2005 official institutions have increased their exposure to mortgage-backed and asset-backed securities and corporate bonds. The three asset classes together accounted for about 9 percent of official institutions' exposure of US securities (Wooldridge, 2006).

The increasing popularity amongst central bank's reserve managers and their enthusiasm to take on market and credit risk have certainly inspired the researcher to examine such moves further especially when they are most concerned with portfolio risk. This thesis evaluates whether a central bank government bond portfolio will get diversification benefits in term of volatility reduction for a given target return, and return optimisation from the minimised risk; without sacrificing liquidity aspects of the foreign reserves.

2.4.2.3. Equity market

A recent survey from International Monetary Fund shows surprising figure; 18 percent of central bank reserve managers are exposed to the equity markets, which is not an asset class traditionally associated with reserve portfolios (Morahan and Mulder, 2013). A number of years back equity was a greatly avoided asset class by central bank reserves portfolios. With the equity risk premium relatively high, a normalisation of the world's growth prospects could result in significant pick-up in equity prices, offsetting the risk of declining bond returns.

While one third of advanced countries are by now invests in equities, none of the low income central bank countries surveyed invest in this asset class. The percentage of low income countries invested in securitized products is, however, relatively close to that of advanced countries. Middle income countries are even more invested in these products than advanced countries, but have barely set a foot on the path of equity and REIT investment (Morahan and Mulder, 2013). Another publication by BIS Quarterly Review shows that equities accounted for less than 2 percent of the reserves portfolio at end-2004 (Wooldridge, 2006). These big changes on central banks risk appetite towards equity investments certainly motivate the researcher to examine its benefits for reserve portfolio especially when central banks are most concerned with the risk of the overall reserve portfolio.

2.5. Conclusions

As discussed, two theories have been used as the foundation of the development of theoretical and empirical models in previous studies of foreign exchange reserve currency composition; the mean-variance and the transaction theory. Similarly, there is also two approaches for defining strategic asset allocation; market index and portfolio optimisation. This chapter has presented an overview of the two theories and a summary of the previous empirical research based on these theories. Empirical studies on the subject have been relatively limited due to difficulties in obtaining required data. The aim of this chapter has been to use the existing literature to establish an empirical examination of the foreign reserve management.

Existing literature shows that mean-variance theory is less popular than the transaction theory despite its strong theoretical foundation in the modern portfolio literature. Among the mean-variance theory itself, the Bayesian approach is relatively lacking in popularity behind the other counterparts, for example Black-Litterman approach. Given the advantages of the Bayesian model which easily handle a greater number of portfolio constraints and its successful applications in stock market, insurance and pension fund portfolio optimisation, it provides confidence that it is possible to develop a beneficial model for strategic reserves management in a Bayesian framework.

This study is the one among few types of researches to use Bayesian approach for foreign reserves portfolio optimisation. More specifically, we build an empirical framework based on mean-variance Bayesian approach. Different from standard Bayesian model, this thesis takes into account the spirit of the transaction theory of the central bank reserve portfolio by imposing multi-weight constraints on the optimisation. This research concentrates on the decision regarding the currency composition and assets structure of foreign reserves to satisfy multiple goals of the central bank reserve management i.e. security, liquidity and profitability.

Existing literature on the assets selection for reserve management does not provide a clear suggestion as to what assets central banks should invest. Very limited studies are addressing asset selection in a risk minimization framework, and put a strong emphasis on the existing liquidity profiles. These issues call for the examination of the benefits of longer government bond maturity, emerging market government bond, inflation-linked government bonds, and non-government investment strategy from a central bank perspective. The objective of this asset class selection analysis is to examine the reduction of portfolio risks from various investment strategies for a central bank foreign reserve portfolio, when the main emphasis is to provide liquidity.

In the later chapters, the dissertation will seek to develop the existing literature in the direction discussed above. Empirical evidence obtained in the three empirical chapters will be useful to clarify the relevance of the theories and to shed light on the issue of potential diversification of foreign reserve portfolio from central banks perspective.

CHAPTER THREE

In this chapter, the optimal currency compositions are investigated to minimize portfolio risk. For a risk minimization portfolio, all the diversification strategies suggest significant benefits to the two original benchmarks and across transaction constraints. However, if the objective of the framework is to reduce portfolio risk for a given existing target return, imposing trade constraints and debt constraint change the significance of the benefits at the 5th percentile of the posterior distributions, which could be attained from the existing benchmark.

3. OPTIMAL CURRENCY COMPOSITION OF FOREIGN EXCHANGE RESERVE PORTFOLIO

Abstract

The objective of this chapter is to investigate whether current currency composition is efficient and to examine whether holding broader foreign currencies provides risk reduction benefits for central bank reserve portfolios. I measure the reduction of portfolio risk as the benefits of diversification using Bayesian approach combined with debt and trade constraints. Findings on the risk minimization for a given level of return and the results from the analyses on minimum variance portfolio before asset allocation constraints are imposed show similarity. These strategies provide potential significant risk reduction benefits for central banks to reallocate their currency composition among the existing benchmark currency, or to invest in broader currencies regardless of the original benchmark currently being used and across constraints. The results for the risk reduction for a given benchmark return reveal that there are potential benefits that could be obtained. However, the choice of the existing benchmark and trade and debt constraints play an important role in the decision as to which diversification strategy central bank reserves managers should pursue.

3.1. Introduction

The accumulation of national foreign exchange reserves in some countries has risen significantly in last few decades, and the rate of accumulation is expected to continue to increase in the future. When the supremacy of the pound sterling declined in the 1950s, the USD came to be the world's main trading currency. It has also been the major currency held by central banks in their foreign reserves. Currently more than 63 percent of such reserves are denominated in USD.

In recent years, however, the prolonged dominance of the USD has come into question. The large US current account deficit, massive external debt, and the increased volatility of the USD exchange rate have put pressure on central banks to reduce investment in USD denominated assets. This issue was amplified since the financial crisis of 2008. Interest rates of main reserve assets are close to zero, resulting in a low yield environment for central banks' foreign reserve investments. On the domestic side, increasing public awareness on how central banks manage their national wealth has put higher pressure on the return aspect of the reserve management. Such factors have imposed additional pressure on central banks to shift their reserve holdings from USD to other currencies.

This condition becomes more complicated since the outlook for other currencies is no less risky either. The euro, strong contender to the dominance of the US dollar (Chinn and Frankel, 2008), had to combat for its survival in the shade of the Eurozone crisis. China promotes the creation of an international reserve currency with a wider use of special drawing right (SDRs)[9] especially in international trade and commodity pricing (Zhou, 2009) which will broaden the use of CNY in international trade and potentially become a reserve currency (Lee, 2014). The outlook for the global economy is heading toward slowest growth since the great depression which will also have a negative impact to the emerging market currencies. In this circumstance, the use of a dominant currency such as the US dollar as an international reserve currency heightens the needs for sound and prudent of foreign reserves management (Ryan, 2009).

To date, the existing literature of foreign reserve currency composition provides two approaches, i.e. the transactional approach and the mean-variance approach (Roger, 1993). Heller and Knight (1978) claim that home country's exchange rate regime and its trade patterns are associated with their reserve currency compositions, and conclude that transaction activity plays an important role in determining currency composition. These findings are supported by the later study of Dooley, Lizondo, and Mathieson (1989) and Chinn and Frankel (2007, 2008).

While the previous literature find evidence for international transaction demand as the main driver for reserve currency composition, the international version of Markowitz (1952) type portfolio theory offers a potential alternative solution for the reserve currency composition problem. Ben-Bassat (1980)

[9] The special drawing right (SDR) was first introduced in 1969 to provide additional liquidity to the global financial system. Its value is based on a basket of four international currencies including the USD, EUR, JPY and GBP. During the 2008/09 financial crisis, allocations were made to avoid a liquidity shortage: a New SDR allocation was started on 28 August 2009, and a special allocation on 9 September 2009, raising the amount of SDRs to SDR 204.1 billion from SDR 21.4 billion.

suggests a solution using mean-variance optimisation in terms of a basket of import currencies. Based on 1976 to 1980 data, he compares optimal to actual reserve portfolio and finds evidence that for emerging countries, portfolio objectives as the main factor in deciding currency composition, but not those of developed economies. Using trade data and reserve composition (Dellas and Yoo, 1991) examine mean-variance optimisation and consumption capital asset pricing model (CCAPM) import-based version for South Korea case. They find that the mean-variance approach performs well in explaining reserve currency composition; more specifically the share of US Dollar as the main currency.

Institutional investors, including central banks generally impose portfolio weight constraints, including short-sale restriction and require the weight of certain type of assets to follow the investment policy. Imposing portfolio weights in mean-variance portfolio selection has been conducted by Frost and Savarino (1988), Grauer and Shen (2000), and Jagannathan and Ma (2003). Those studies find that imposing portfolio weight constraints does not only create a portfolio to follow their investment policies of what it should look like, but also reduces the impact of estimation error and delivers a better portfolio performance.

Papaioannou et al. (2006) and Wu (2007) are two among few studies who implement asset weight constraints in the reserve management area. Inspired by these studies, I will utilise portfolio weight constraints to capture the spirit of the transaction theory of foreign reserve allocation. Two currency compositions constraint: trade constraints which reflect the payment needs for foreign trade and debt constraints which articulate public debt repayment to the foreign investors.

Fletcher (2014) argues that Bayesian approach of Wang (1998) and Li et al. (2003) is easier to implement than the use of classical statistics to assess portfolio efficiency (Basak et al., 2002), or test mean-variance spanning in the presence of short sales constraints (DeRoon and Nijman, 2001). The most relevant attraction of Bayesian approach for my study is that it can be applied when investor faces portfolio restrictions such as budget equal to one, short selling, strategic asset allocation or the combination of those constraints.

Driven by the interest to fill the gap in portfolio management that focused on foreign reserve portfolio, and to offer an alternative solutions for central banks' currency composition, this research uses the Bayesian approach of Wang (1998) and Li et al. (2003) combines with multiple weights constraints of Jagannathan and Ma (2003). This study will enhance our understanding of diversification benefits on the currency composition of the foreign exchange reserves. This study examines the effect of trade constraints and debt constraints on the existence of currency diversification benefits for central banks' foreign reserves portfolio.

When the central bank focuses mainly on risk reduction, risk minimization frameworks for unconstrained portfolio encourage the flight to safety (Zhang, Zhang, and Zhang, 2013). However, for the case of constrained asset weight and when central bank emphasises most on liquidity sufficiency, the impact of the benefits and currency composition remain unknown. The questions of whether: 1) foreign currency diversifications provide risk reduction benefits for a given expected return of the current benchmark for central banks reserve portfolio? And whether Moreover, whether 2) central bank’s currency diversifications provide risk reduction benefits to minimum variance portfolio remain unanswered.

This work also has a practical implication especially for central bank reserves management. It would help to provide a new perspective on setting and evaluating their currency compositions in order to minimise portfolio volatility. The economic interpretation of our measure is straightforward, and it tells directly how much risk will decrease by switching their investment from the existing to the new benchmark policy. It is different from other studies which suggest the benefits in average terms. The posterior distribution of the benefits of the Bayesian analyses provides the mean, the minimum, and other measures of distribution that could be gained in a certain level of significance.

Starting from the two benchmarks, which mimic the actual composition of the central bank investment and the currency composition in the SDR, the optimisations to minimise portfolio risk are conducted by imposing shares of external transactions. Empirical evidence on the minimum variance portfolio shows that there are potential significant risk reductions benefits for central banks to reallocate their currency composition among the existing benchmark or broader currency diversifications, regardless the original benchmark currently being used and across debt and trade constraints.

My findings related to the risk minimization for a given benchmark returns show that there are significant benefits before currency weights constraints are enacted. Imposing trade constraints and debt constraint on the optimisations change the significance of the benefits. In general, the more currencies included in diversifications provide bigger the mean of the benefits. However, for the portfolio which utilises actual composition of the central banks' investment, the benefits at the 5th percentile of the posterior distributions could be attained by broader currency investment into CAD, CHF, AUD and NZD and selected emerging market currencies from imposing trade constraint. Such benefits, however, could not be achieved after imposing debt constraints. A central bank that uses the original benchmark which replicates SDR composition, significant risk reduction benefits at the 5th percentile of the posterior distributions could be achieved by reallocating benchmark currency shares and by broader currency investment into selected developed market currency from imposing trade constraint and broader spanning into selected emerging market currencies from imposing debt constraints. In short, the choice of the current benchmark and both trade constraints and debt constraints play an important role in the decision in which diversification strategy central bank reserves managers should proceed.

The contribution of the empirical chapter to the existing literature is that it complements Papaioannou et al. (2006) and Wu (2007) findings that transaction constraints are important in determining central bank's currency composition. This thesis extends the contribution that the importance of the constraints remains valid under the Bayesian analysis. The empirical findings show that in general the new optimal portfolios suggest allocating more USD and less EUR compares to the current benchmark portfolio currency. Our findings suggest that each individual central bank should consider relaxing their currency investment policy beyond USD, EUR, GBP and JPY and adjust the transaction constraints associated with the individual country exposure. It is desirable and feasible to adopt Bayesian approach combined with external transaction constraints as an alternative framework for determining central banks' foreign currency structure.

The remainder chapter is organised as follows. In section 2, it discusses the main set-up of the portfolio optimisation framework and data description and investment strategy. In section 3 I present estimation results for risk reduction for a given level of benchmark returns and risk reduction to the minimum portfolio from the currency diversification and transaction constraints considered. Section 5 concludes.

3.2. Research Methods and Data

3.1.1. Mean-variance tests of diversification benefits

The goal of the portfolio choice problem is to seek minimum risk for a given level of return and to seek maximum expected returns, for a given level of risk (see Fabozzi, Gupta, and Markowitz (2002) for a review). The mean-variance framework assumes a single period investment horizon which means that individuals are risk-averse and are only concerned with the expected return and variance of the portfolio at the end of the period. The mean-variance approach is valid if asset returns are normally distributed or agents have quadratic utility function.

This research presents risk minimization portfolio problem for a given benchmark returns subject to short-sale constraint and budget equal to one, followed by the introduction of strategic asset allocation constraints to the framework, and then applying the same constraints and strategies, risk minimization to the global minimum variance portfolio are examined. Applying mean-variance model of Markowitz (1952), when the central bank has restrictions on their investment policy, the objective of mean-variance portfolio for the same expected benchmark returns is to:

$$Min\ x'Cx \tag{3.1}$$

$$\text{Subject to: } x'u = E(r_p) \tag{3.2}$$

$$x'e = 1 \tag{3.3}$$

$$x_i \geq 0, i = 1,2,3,\ldots,N \tag{3.4}$$

moreover, for the global minimum variance portfolio, it assumes that central bank as an investor selects a set of vector risky assets to:

$$Min\ x'Cx \tag{3.5}$$

$$\text{Subject to: } x'e = 1 \tag{3.6}$$

$$x_i \geq 0, i = 1,2,3,\ldots,N \tag{3.7}$$

where u is a $(N, 1)$ vector of expected returns of N assets, C is the (N, N) covariance matrix, and e is a $(n, 1)$ vector of ones. Furthermore, if the central bank faces additional asset allocation constraints, equality, lower, upper bound or combination of those constraints can be incorporated to solve the problems.

This method emphasises the investment efficiency in term of risk and returns. However, managing foreign currency reserves must first guarantee international transaction and monetary policy operation activities. Therefore, I propose two types of optimal portfolio constrained by data of the top five countries of global trade and top four governments of global debt. Following Zhang, Ding, and Zhang (2010), the corresponding currency is set to be half of the foreign trade and debt share of that currency. Fractions of the sum of the major import and export and public debt and the corresponding currency constraints are stated in Table 3.1.

Table 3.1 Benchmark and currency composition constraint

The table shows the existing benchmark and the currency composition constraint for broader currency investment strategy. The current benchmark consists of 2 benchmark currencies; actual composition of central bank investment in G4 currencies and currency composition in the SDR. Two currency constraints are used, trade constraint represents global trade and debt constraints which reflect global debt position. All figures are in percent.

Currency	Current Benchmark		Trade Constraints		Public Debt Constraints	
	Actual CB Investment[1)]	SDR-IMF[2)]	Share[3)]	Min. Allocation	Share[4)]	Min. Allocation
USD	63	47	30	15.0	50	25.0
EUR	22	34	23	12.0	19	9.5
GBP	4	12	5	2.5	7	3.5
JPY	4	7	10	5.0	24	12.0
CNY			31	15.5		

Source: compiled by author

1) This share corresponds to the IMF actual currency composition of the central bank investment in the G4 currencies. http://data.imf.org/?sk=E6A5F467-C14B-4AA8-9F6D-5A09EC4E62A4. These figures are those of June 2015.

2) This share corresponds to the currency composition in the IMF's Special Drawing Right (SDR). https:// www.imf.org/external/np/exr/faq/sdrallocfaqs.htm

3) This share reflects top 5 countries in global trade, source World Trade Organization, https://www.wto. org/english/res_e/statis_e/wts2016_e/WTO_Chapter_05.pdf

4) This share reflects top 5 central government debt position, source World Bank, http://databank.worldbank. org/data/views/reports/ReportWidgetCustom.aspx?Report_Name =Table-C2.-Gross-Central-Gov.-Debt-Position&Id=46819dee27&ht=1520

For currency composition constraints, this study applies lower-bound constraints:

$$x_i \geq \bar{x}, i = 1,2,3, \ldots, N \tag{3.8}$$

Lower bound constraint $x_i \geq \bar{x}$ applied to satisfy the requirement that each currency is enough to cover import or repay government debt. For example, from Table 3.1 shows that for trade constraints, foreign reserve currency need to be allocated at least 15percent in USD, 12 percent in EUR, 2.5 percent in GBP, 5 percent in JPY and 15.5 percent in CNY. In order to ensure that the reserves is sufficient to service public debt, it needs minimum 25 percent, 9.5 percent, 3.5 percent and 12 percent to be held in currency USD, EUR, GBP and JPY respectively.

This research uses Bayesian inference framework to investigate diversification benefits for foreign reserve currency composition subject to portfolio multi-constraints portfolio. Define K as the number of benchmark currencies and N as the number of currencies added to the benchmark portfolio.

Two different measures of the diversification benefits in term of portfolio risks reduction are employed. The first one follows Kandel et al. (1995), Wang (1998) and Li et al. (2003) that is for the same expected returns of the existing K benchmark, the variance of N portfolio is smaller than the variance of K benchmark portfolio. This research calculates the difference of variance between the portfolios to measure the magnitude of the diversification benefits. The advantage of this measure is that intuitively it tells how far the inefficiency of the existing central bank's benchmark portfolio and provides an interpretation of the risk reduction through diversification while maintaining the existing expected returns.

Follow Li et al. (2003) measures of diversification benefits in term of variance reduction for the same target returns of the benchmark, risk reduction $\emptyset$ is the delta of the standard deviation of the K efficient frontier and the current benchmark, and defined as:

$$\emptyset = \left(1 - \sqrt{\frac{x'Cx}{x_b'Cx_b}} \right); \text{subject to } x'u = x_b{}'u \tag{3.9}$$

where x is the $(N, 1)$ optimal weights of the portfolio of the N test assets, x_b is the corresponding $(N, 1)$ portfolio of the K benchmark assets and u is the expected returns. The risk reduction benefits $\emptyset$ measure in equation 3.9 subject to the condition that the optimal portfolio returns $x'u$ is equal to the return of the existing benchmark $x_b{}'u$.

My second measure of the reduction of portfolio risks follows the work of Stambaugh (1997) and Li et al. (2003) and is the reduction in standard deviation when a central bank switches part of their reserves to the global minimum variance portfolio that includes N assets from the existing benchmark as:

$$\emptyset_{GMV} = 1 - \sqrt{\frac{x'Cx}{x_b{}'Cx_b}} \tag{3.10}$$

If there are no diversification benefits to the optimal mean-variance or minimum-variance portfolio, one could expect both $\emptyset = 0$ and $\emptyset_{GMV} = 0$. The $\emptyset$ measure is more relevant for a central bank to reduce its reserves portfolio without suffering from the existing expected benchmark returns, while $\emptyset_{GMV}$ measure is more relevant when the goal is to minimize their foreign exchange reserve risk.

Furthermore, Li et al. (2003) argue that since global minimum variance portfolio does not require expected returns, the estimated weights may be more accurate and relatively stable over the different period. On the other hand, global minimum variance portfolio may be not the best choice for some central banks that have a concern on return objective. This is because this framework might result in lower expected returns in order to achieve the global minimum variance of the efficient frontier.

A further attraction of the Bayesian approach is that it can be applied when the central bank faces portfolio restrictions such as budget, short selling, asset allocation or the combination of those constraints. Define u and C as the sample moments of the expected returns and covariance matrix, and R as the (T, N) matrix of returns on N assets. The posterior probability density function is given by:

$$p(u, C|R) = p(u|C, u_s, T)^{\circ} p(C|C_s, T) \tag{3.11}$$

where $p(u|C, u_s, T)$ is the conditional distribution of a multivariate normal $(u_s, \left(\frac{1}{T}\right) C)$ distribution and $p(C|C_s, T)$ is the marginal posterior distribution that has an inverse Wishart $(TC, T - 1)$ distribution (Zellner, 1971).

Wang (1998) proposes a Monte Carlo method to approximate the posterior distribution of return enhancement, which can also be applied for the measure of risk reduction $\emptyset$. Following approach of Wang (1998), first, the researcher draws a random C matrix from inverted Wishart $(TC, T$-1$)$ distribution. Second, a random vector of returns u is drawn from a multivariate normal distribution $(u_s, \left(\frac{1}{T}\right) C)$, where C is from the first step. Third, given the u and C from steps 1 and 2, the diversification benefits measures of $\emptyset$ from equation $\emptyset = 1 - \sqrt{\frac{x{\prime}Cx}{x_b{\prime}Cx_b}}$ are estimated. Fourth, repeating steps 1 to 3 for 10,000 times as in Wang (1998) and Li et al. (2003) to generate the approximate posterior distribution of $\emptyset$. The posterior distribution of $\emptyset$ then is used to evaluate the magnitude of the diversification benefits and the statistical significance of these benefits. The average values from the posterior distribution provide the average diversification benefits in term of reduction in mean-variance and in global minimum variance portfolio. As Wang (1998) proposes, the standard deviation of posterior distribution served as standard errors.

Hodrick and Zhang (2014) provide further interpretations of Li et al. (2003) results. Hodrick and Zhang (2014) use the value of the 5th percentile to judge the statistical significance of the benefits. This measure also provides the interpretation of the average of the actual benefits, how big are the diversification benefits in economic terms using simple formula as follow:

$$\bar{o}_e = (1 - \emptyset)^2 \qquad (3.12)$$

where $\bar{o}_e$ indicates the actual measures of risk reduction benefits in economic terms the squared of the difference from one. For example, the mean value of the risk reduction is 0.0725 indicates that the mean of the benefits resulted from new optimal portfolio for a certain spanning strategy is 86 percent or 14% lower from the existing portfolio risk ($1 - \sqrt{0.86} = 0.0725$). Similarly, measure could also be applied for the other distribution measures, such as 5th percentile, median, etc.

In addition, when the researcher runs steps one to four, it also gives the approximate posterior distribution of weights in the optimal portfolios. The posterior distribution resulted from previous steps can be used to examine the statistical significance of the average weights. The posterior distribution has the attractive feature as Britten-Jones (1999), and Kan and Smith (2008) provide the distribution theory of the optimal portfolio weights when there are no constraints.

However, Li et al. (2003) argue that the distribution theory for the constrained case portfolio is unknown.

3.2.1. Data description and investment strategy

In order to calculate returns of each currency, two types of datasets are needed, i.e. the exchange rate of the foreign currency to US dollar and the interest rate of the currency-issuing country. To focus on the currency effect, I assume that foreign reserves are exclusively invested in the short-term instrument, which is 3-month time deposit in this case (except for the Australia which available from November 1989). To comprehend the benefits of diversification, an adequate number of currencies are to be included in the optimisation. We select 13 currencies; therefore, we need 13 corresponding interest rates of these countries and 12 foreign exchange rates (EUR, GBP, JPY, CAD, CHF, CAD, AUD, NZD, CNY, BRL, INR, KRW, and THB) to the US dollar. The horizon of the datasets is from 31 December 1985 to 31 December 2014 (except for the Brazilian Real (BRL) which only available between January 1991 and December 2014) and all the data are in monthly frequency.

Currency returns are calculated from the combination of the interest rate and exchange rate returns:

$$r_{i,t} = S_{i,t} + I_{i,t} \tag{3.13}$$

Where $I_{i,t}$ the interest is rate of currency i and $S_{i,t}$ is return of the exchange rate i to the US dollar.

Table 3.2 reports summary statistics of the potential currency diversifications for central banks' foreign reserve portfolios. We classify the selected currency into three categories; Group G4, Group D4 and Group EM5. Group of G4 consists of 4 currencies from the existing constituent of the benchmarks i.e. USD, EUR, GBP and JPY. Group of D4 consists of selected developed economy currencies i.e. CAD, CHF, AUD and NZD. Group of EM5 consist of selected currencies of the emerging economies i.e. CNY, KRW, BRL and THB. The table shows the mean, standard deviation, minimum and maximum monthly returns (percent) and its correlations with benchmark assets.

Table 3.2 Summary statistics of broader investment universe assets

The table reports summary statistics of the monthly returns of the eligible currencies. G4 currencies consist of USD, EUR, GBP and JPY. D4 currencies include CAD, CHF, AUD and NZD. EM5 include CNY, INR, BRL and THB. The table shows the short-term interest rate in the respected currency, mean, standard deviation, minimum and maximum monthly returns (percent) and its correlations with the current benchmark currencies.

Currency		Interest rate	Mean	Stdev	Min	Max	Correlation			
							USD	EUR	GBP	JPY
G-4	USD	3m deposit	0.3419	0.2221	0.0158	0.8537	1	0.0304	0.0959	0.0095
	EUR	3m deposit	0.2782	3.0749	-8.4912	12.6573	0.0304	1	0.7040	0.4086
	GBP	3m deposit	0.5175	2.8724	-7.9996	14.3090	0.0959	0.7040	1	0.3250
	JPY	3m deposit	0.0498	3.2361	-14.2935	11.3567	0.0095	0.4086	0.3250	1
D-4	CAD	3m deposit	0.3875	2.1236	-7.5069	14.8267	0.0064	0.3412	0.3069	0.0090
	CHF	3m deposit	0.2488	3.3554	-12.2542	12.7343	0.0878	0.8911	0.6366	0.4922
	AUD	2m deposit	0.2278	3.4683	-8.3923	19.7868	0.0990	0.4005	0.3684	0.0799
	NZD	3m deposit	0.5886	3.4905	-11.9863	15.5598	0.1640	0.4868	0.3994	0.1758
EM-5	CNY	3m deposit	0.4122	3.2432	-1.9224	50.5541	0.0482	-0.0191	0.0051	-0.0286
	KRW	3m deposit	0.2329	3.3919	-16.5678	27.2319	0.0168	0.2721	0.1863	0.1262
	INR	3m deposit	0.7828	2.1953	-6.1152	21.2280	0.0064	0.1229	0.0728	0.0240
	BRL	3m deposit	0.5721	5.1657	-16.6053	64.0983	-0.0607	0.0129	-0.0027	0.0693
	THB	3m deposit	0.4204	2.9406	-17.7174	29.3878	0.0623	0.2863	0.2004	0.2062

Benchmark currencies mean returns are ranging from 0.05 percent (JPY) to 0.52 percent (GBP). JPY is the most volatile currencies amongst other benchmark currencies, and US dollar is the most stable currency relative to the other benchmark currencies and the rest of the currencies. In general, benchmark currencies have a lower correlation among the other currencies as shown by correlation smaller than 0.5 which indicate potential diversification benefits for the existing benchmark itself.

Group of D4 currencies return are between 0.22 percent and 0.59 for the currency AUD and NZD returns respectively. Currencies CHF, AUD and NZD have relatively similar risk level at around 3.5 percent higher than that of the benchmark currencies. All the D4 currencies have very low correlation to USD and JPY, but are a higher correlation to EUR and GBP. The means, standard deviations and correlation of the D4 currency returns to the existing benchmark do not provide a clear indication as to whether these additional currencies provide risk reduction diversification benefits.

Group of emerging market currencies returns are from 0.23 percent to 0.78 percent for KRW and INR correspondingly. Standard deviations are spread between 2.2 percent (INR) and 5.2 percent (BRL). Different from the correlation amongst D4 to benchmark currencies, EM5 currencies have relatively low correlation figures with the benchmark which I could expect that this currency basket will provide diversification benefits.

These potential benefits relative to the existing benchmark portfolio resulting from that currency diversifications strategy, however, are unclear for an investor who considers safety as the main portfolio objective. The means, standard deviations and correlation of the currency returns to the benchmark currencies in this table; do not provide a clear indication as to whether these strategies provide greater risk reduction benefits than the existing benchmark portfolio risk.

3.3. Empirical Results

The dominance of the US dollar in the international reserve caused academia and practitioners perceive that central banks foreign reserve portfolio are under-diversification. The search for alternative reserve currencies includes Euro, commodity currency such as Canadian dollar, Australia dollar, and New Zealand dollar and emerging market currency such as China Yuan, Brazilian Real and Korean Won. Current global conditions, however, do not simply agree with the argument to diversify away from the US dollar dominance.

The euro, the most popular contender to the dominance of the dollar, had to combat for its survival and the Eurozone crisis (Chinn and Frankel, 2008). Global economic outlook is heading toward its slowest growth since the great depression and China is no longer experience double-digit growth. These current conditions may suggest reserve managers to against the diversification. These circumstances, however, heightens the needs for sound and prudent of foreign reserves management (Ryan, 2009). To capture these aspects for foreign reserve currency allocation, my first measure is the reduction in portfolio risk for a given existing benchmark returns.

3.3.1. Risk minimization for a given expected return currency composition

3.3.1.1. No-currency constraint allocation

Morahan and Mulder (2013) and Briere et al. (2015) explain that USD, EUR, GBP and JPY serve as the reference for the SDR and mostly used by the central bank to diversify their foreign reserve. The previous study from the IMF (Medeiros and Nocera, 1988) reports that the SDR plays a key role in central banks' currency allocation. These two studies motivate me to investigate further whether the choice of currency benchmark has an impact on the central bank diversification benefits.

Table 3.3 reports the posterior distribution of portfolio risk reduction in switching from the existing benchmark to the new benchmark allocation while the central bank maintains the current state of expected returns. This table contains the mean, standard deviation, actual measure, and 5th, 50th and 95th percentile of the posterior distribution of the risk reduction. This table shows that when a central bank faces only short-sale restrictions and budget constraint equal to one, all the currency diversification strategies show the average potential diversification benefits between 0.34 percent to 0.60 percent and 0.03 percent to 0.22 percent at the 5th percentile. The average of the new optimal portfolio risk is less than half of the existing portfolio volatility.

Table 3.3 Posterior distribution of risk reduction of currency composition for a given existing benchmark returns

This table reports the posterior distribution of risk reduction benefits ∅ (percent) for a given expected benchmark returns for short-sale and budget constraints portfolios. The summary statistics are the mean, standard deviation, the average actual risk reduction, and 5th, 50th, and 95th percentile.

Diversification Strategy	Mean	Stdev	ō	Percentile		
				5th	50th	95th
Benchmark #1 Actual Investment (USD 63%, EUR 22%, GBP 4%, & JPY 4%)						
G 4	0.3386	0.2281	0.4375	0.0298	0.3034	0.7198
DM 4	0.4315	0.2342	0.3232	0.0299	0.4645	0.7336
EM 5	0.4708	0.2422	0.2801	0.0313	0.5123	0.7439
Benchmark #2 Composition in the SDR(USD 47%, EUR 34%, GBP 12%, & JPY 7%)						
G 4	0.3754	0.2457	0.3901	0.0452	0.3331	0.8147
DM 4	0.4886	0.2330	0.2615	0.1100	0.4923	0.8349
EM 5	0.5902	0.2020	0.1679	0.2184	0.6239	0.8429

This table shows that before constraining currency allocation, both the existing benchmarks are not efficient. There are potential benefits by readjusting currency weights from the existing currency benchmark's constituents. For the benchmark that resembles actual central banks investment currency composition, there are potential benefits of 0.34 percent on average or at 95 percent chances to benefits greater than 0.03 percent from readjusting the currency compositions among USD, EUR, GBP and JPY themselves. Spanning into broader currencies provides greater risk reduction benefits for reserve portfolio. Investing in selected developed countries and emerging market currencies deliver 0.43 percent and 0.47 percent with a standard deviation of 0.23 percent and 0.24 percent benefits. At the 5th percentile of the distribution all the three strategies deliver relatively similar magnitudes at 0.03 percent.

The use of the currency composition in the SDR provides even bigger benefits if the central banks reallocate their currency compositions. The potential benefits from readjusting the currency compositions among the existing currency benchmark on average are 0.37 percent or at 95 percent chances to gain greater than 0.05 percent. Investing in broader currencies doubled the benefits at the same confidence level. Diversifying into selected developed countries and emerging market currencies provide 0.11 percent and 0.22 percent at the 5^{th} percentile of the posterior risk reduction distribution.

Interestingly, the use of the first benchmark shows that the more currencies have been included in the optimisation delivers greater both the mean and the standard deviation of the benefits. For the second benchmark, however, the more currencies considered will deliver higher mean and lower standard diversification of the benefits. This indicates that the probability for the central bank to achieve a mean of the benefits is greater for the more diversified currency reserve portfolio when composition in the SDR is used for the original benchmark. In terms of portfolio efficiency, however, it shows that actual investment is closer to the efficient portfolio than the benchmark suggested by currency composition in the SDR compositions.

Table 3.4 shows the posterior distribution of the average and standard deviations of the optimal currency weights for the existing benchmark returns resulting from the spanning strategies considered. When there is only budget and short-sales constraint taken into consideration, spanning from the first benchmark requires more than 82 percent of the reserves to be allocated into currency USD

Table 3.4 Currency weights in the efficient portfolio for a given benchmark return in the non-currency allocation portfolio

This table reports the posterior distribution of the average and standard deviations of the weights in the efficient portfolio with the same target return resulted from 3 spanning tests considered against 2 benchmarks. The portfolio weights are restricted to budget equal to one and constrained to non-negativity.

Currency	Benchmark #1 (USD 63%, EUR 22%, GBP 4%, & JPY 4%)						Benchmark #2 (USD 47%, EUR 34%, GBP 12%, & JPY 7%)					
	Group G4		Developed4		Emerging5		Group G4		Developed4		Emerging5	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
CNY					0.0108	0.0227					0.0262	0.0403
KRW					0.0191	0.0303					0.0142	0.0304
INR					0.0055	0.0156					0.0334	0.0548
BRL					0.0043	0.0098					0.0179	0.0258
THB					0.0024	0.0109					0.0182	0.0378
CAD			0.0128	0.0333	0.0085	0.0248			0.0485	0.0806	0.0266	0.0530
CHF			0.0128	0.0392	0.0100	0.0308			0.0353	0.0706	0.0215	0.0492
AUD			0.0286	0.0415	0.0194	0.0324			0.0190	0.0414	0.0104	0.0277
NZD			0.0022	0.0085	0.0002	0.0017			0.0190	0.0418	0.0102	0.0284
USD	0.8223	0.0839	0.8393	0.1043	0.8377	0.1022	0.6955	0.1415	0.7266	0.1507	0.7324	0.1609
EUR	0.0863	0.0924	0.0152	0.0498	0.0112	0.0378	0.1408	0.1515	0.0488	0.0978	0.0253	0.0631
GBP	0.0483	0.0651	0.0059	0.0182	0.0017	0.0082	0.0934	0.1212	0.0528	0.0894	0.0336	0.0645
JPY	0.0432	0.0529	0.0833	0.0669	0.0692	0.0583	0.0702	0.0887	0.0500	0.0710	0.0302	0.0493

for across strategies considered. For the second benchmark, however, it requires smaller allocation for the USD at around 70 percent. In general the table shows that the current benchmarks are underweight USD and overweight EUR relative to the optimal allocation. The difference between the optimal currency allocation and the existing composition may explain the magnitude of the risk reduction benefits.

3.3.1.2. Currency weights constraints

In this sub-section, I analyse the influences of weight constraints on the choice of currency portfolio. Two sets of trade constraints and debt constraints are imposed to the portfolio optimisations. Trade constraints correspond to the currency shares in top-5 countries global trade while debt constraints relate to the currency shares of the top-5 central government public debt. We have reported that currency diversification is beneficial for central banks' reserve portfolio in Bayesian approach, but these benefits acquired when no constraints are imposed on the share of the currency.

Taking those two ad-hoc weight constraints will make this study resemble the reality more closely. One major function of a nation's foreign reserves is to ensure the international payment such as foreign trade and debt repayment. These two constraints of minimal allocation are set up following Papaioannou et al. (2006) and Wu (2007).

3.3.1.2.1. Trade constraints

Trade constraint in this analysis is defined as the minimum allocation in the new optimal reserve portfolios to be held 15 percent in USD, 12 percent in EUR, 2.5 percent in GBP, 5 percent in JPY, and 15.5 percent in CNY currency. Since the investment in G4 and DM4 strategies do not constitute holding CNY currency, the currency constraints apply for USD, EUR, GBP and JPY currencies. In both diversification strategies we put zero minimum weight for the CNY currency or practically similar towards no-short sale constraints. Applying complete all the 5 currencies imposed only for the EM5 diversification strategies.

Table 3.5 presents potential risk reduction benefits from adjusting currency weights for the same currency benchmark expected returns. For a central bank that uses the actual investment data currency allocation as the benchmark, there are potential benefits of 0.12 percent on average. However, the benefits at 95 percent chances are zero. Imposing trade constraints into D4 diversification strategy decreases the chances for the central bank to gain the benefits even though the average risk reduction benefit is relatively greater at 0.20 percent. Spanning further the emerging currency provides diversification benefits at 0.13 percent and at 95 percent probability we could expect benefits to be greater than 0.07 percent.

When central bank reserve portfolio use currency composition in the SDR as the benchmark, the table shows that on average there are 0.26 percent risk reduction diversification benefits and 95 percent chances the benefits will be greater than 0.03 percent from reallocating currency compositions from the current benchmark. Interestingly, when additional developed countries currencies are included it increases the mean of the benefits but fail to provide risk reduction benefits greater than zero at the 5th percentile of the posterior distributions. As expected, greater benefits could be obtained through incorporating emerging market currencies. This strategy will increase the mean of the risk reduction benefits. More specifically, at the 0.95 probabilities, selected emerging market currency diversification could offer at least 0.08 percent risk reduction.

Table 3.5 Posterior distribution of risk reduction for a given existing benchmark returns in trade constrained portfolio

This table reports the posterior distribution of risk reduction benefits for a given benchmark returns for the trade constrained portfolios. The trade constraint is defined that the minimum allocation of the USD is 15%, EUR 12%, GBP 2.5%, JPY 5% and CNY 15.5% in the optimal portfolio. The summary statistics are the mean, standard deviation, the average actual risk reduction and 5th, 50th, and 95th percentile.

Diversification Strategy	Mean	Stdev	$\bar{o}$	Percentile		
				5th	50th	95th
Benchmark #1 Actual Investment (USD 63%, EUR 22%, GBP 4%, & JPY 4%)						
G 4	0.1265	0.1137	0.7630	0.0000	0.1030	0.3288
DM 4	0.3766	0.2442	0.3887	0.0442	0.3354	0.8137
EM 5	0.1295	0.0331	0.7578	0.0745	0.1299	0.1831
Benchmark #2 Composition in the SDR (USD 47%, EUR 34%, GBP 12%, & JPY 7%)						
G 4	0.2581	0.1740	0.5504	0.0276	0.2285	0.5726
DM 4	0.3600	0.1655	0.4096	0.0807	0.3726	0.5981
EM 5	0.3089	0.1504	0.4776	0.0000	0.3532	0.4838

The most interesting finding in this analysis is the role of trade constraints on risk minimization for a given benchmark return shows the different impact on the original benchmarks. The requirement to hold minimum 15.5 percent of the reserve in CNY currency for the existing benchmark that requires more USD allocation provides smaller average benefits but increase the likelihood to obtain significant benefits at 95 confidence level. When this strategy has been applied for

Table 3.6 Currency weights in the efficient portfolio for a given benchmark return in trade constrained portfolio

This table reports the posterior distribution of the average and standard deviations of the weights in the efficient portfolio with the same target return resulted from 3 spanning tests considered against 2 benchmarks. The portfolio weights are restricted to budget equal to one and constrained to non-negativity and trade constraints. The trade constraints are defined that the minimum allocation of the USD is 15%, EUR 12%, GBP 2.5%, JPY 5% and CNY 15.5% in the optimal portfolio.

Currency	Benchmark #1 (USD 63%, EUR 22%, GBP 4%, & JPY 4%)						Benchmark #2 (USD 47%, EUR 34%, GBP 12%, & JPY 7%)					
	Group G4		Developed4		Emerging5		Group G4		Developed4		Emerging5	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Trade Constraints (USD 15.0%, EUR 12.0%, GBP 2.5%, JPY 5.0%, & CNY 15.5%)												
CNY					0.1550	0.0000					0.1554	0.0031
KRW					0.0000	0.0001					0.0153	0.0352
INR					0.0038	0.0076					0.0419	0.0676
BRL					0.0035	0.0048					0.0270	0.0430
THB					0.0000	0.0000					0.0194	0.0464
CAD			0.6953	0.1409	0.0000	0.0000			0.0444	0.0764	0.0271	0.0610
CHF			0.1369	0.1500	0.0000	0.0000			0.0177	0.0452	0.0158	0.0607
AUD			0.0953	0.1219	0.0000	0.0000			0.0180	0.0401	0.0083	0.0287
NZD			0.0725	0.0896	0.0000	0.0000			0.0157	0.0374	0.0076	0.0252
USD	0.7251	0.0476	0.7341	0.0465	0.6427	0.0089	0.6064	0.0920	0.6277	0.1028	0.4423	0.1388
EUR	0.1585	0.0593	0.1290	0.0290	0.1200	0.0000	0.2012	0.1039	0.1422	0.0597	0.1283	0.0441
GBP	0.0514	0.0392	0.0351	0.0259	0.0250	0.0000	0.0994	0.0967	0.0585	0.0672	0.0441	0.0574
JPY	0.0652	0.0252	0.0576	0.0191	0.0500	0.0000	0.0930	0.0631	0.0757	0.0474	0.0677	0.0451

benchmark with smaller USD share, it increases the average of the benefits but fails to provide significant benefits at 5th percentile if the posterior distribution. One explanation might be due to the standard deviation of the posterior distribution benefits. Imposing minimal CNY currency allocation reduces the standard deviation in the first benchmark significantly, but not in the second one.

Trade constraints analysis implies that central bank which currently allocates USD currency more than 60 percent in the benchmark, there is significant risk reduction benefits to broader their currency investment into selected developed and selected emerging market currencies. For a central bank that allocate less than 50 percent in their benchmark, diversification benefits could be obtained through reallocating current currency composition and investment into selected developed market currencies.

Table 3.6 shows the posterior distribution of the average and standard deviations of the optimal currency weights for the existing benchmark returns resulting from the spanning strategies considered when trade constraints are imposed. Reallocate currency composition from the first benchmark requires variation of the USD weights, between 64 percent and 73 percent of the total reserves. For the second benchmark, the allocation of the currency USD varies between 44 percent and 62 percent. Similar to the earlier analysis before imposing transaction constraints, the current benchmarks are underweight USD and overweight EUR relative to the optimal allocation. When there is a requirement to allocate certain share into CNY currency, all the strategies provide it through the cost of the reduction of the USD allocation. The difference between the optimal currency allocation and the existing composition may also explain the magnitude of the risk reduction benefits.

3.3.1.2.2. Debt constraints

Table 3.7 Posterior distribution of risk reduction for a given existing benchmark returns in debt constrained portfolio

This table reports the posterior distribution of risk reduction benefits for a given expected benchmark returns for debt constraints portfolio. The debt constraints are defined that the minimum allocation of the USD is 25%, EUR 9.5%, GBP 3.5%, JPY 12% in the optimal portfolio. The summary statistics are the mean, standard deviation, the average actual risk reduction and 5th, 50th, and 95th percentile.

Diversification Strategy	Mean	Stdev	$\bar{\delta}$	Percentile		
				5th	50th	95th
Benchmark #1 Actual Investment (USD 63%, EUR 22%, GBP 4%, & JPY 4%)						
G 4	0.0454	0.0667	0.9114	0.0000	0.0000	0.1835
DM 4	0.2969	0.1656	0.4943	0.0000	0.3427	0.4898
EM 5	0.1384	0.0730	0.7424	0.0000	0.1621	0.2242
Benchmark #2 Composition in the SDR (USD 47%, EUR 34%, GBP 12%, & JPY 7%)						
G 4	0.1718	0.1693	0.6859	0.0000	0.1250	0.4845
DM 4	0.2920	0.1670	0.5013	0.0000	0.3182	0.5177
EM 5	0.3906	0.1407	0.3714	0.1128	0.4326	0.5377

Table 3.7 presents the posterior risk reduction benefits from adjusting currency weights for the same currency benchmark expected returns when debt constraints are imposed. For the central bank that the current benchmark uses the actual currency allocation, G4 and DM 4 investment strategy provide potential benefits of 0.04 percent and 0.30 percent on average. However, the benefits at 95 percent chances are zero. The median of the risk reduction distribution is zero indicates that relatively low probability that reallocate USD, EUR, GBP and JPY composition provide greater risk benefits than that of the original currency allocation. Emerging currency spanning strategy provides smaller average

diversification benefits at 0.14 percent and unable to offer benefits at the 5^{th} percentile.

When central bank reserve portfolio use currency composition in the SDR as the benchmark, this table shows that on average there are 0.17 percent risk reduction benefits and 95 percent chances the benefits will zero resulted from reallocating currency compositions from the current benchmark itself and broader DM 4 currencies investment. When E5 diversification strategy has been implemented, it increases the mean of the benefits to 0.39 percent and provides risk reduction benefits greater than 0.11 percent at the 5^{th} percentile of the posterior distributions.

These results show the actual allocation of the central bank's investment as the benchmark is relatively more efficient compare the use of currency share in the SDR if the central bank only investing in the big four currencies; USD, EUR, GBP and JPY. When more currencies have been considered into foreign reserves portfolios, there are potential benefits could be obtained.

The debt constraints analysis shows that for the central bank which allocates foreign currency resemble the actual investment in the current benchmark; there is no potential risk reduction benefits could be obtained even after considering broader currencies investments. For the central banks that currently allocate USD currency less than 50 percent in their benchmark, diversification benefits could be obtained through broader currency investment into selected emerging market currencies.

Table 3.8 Currency weights in the efficient portfolio for a given benchmark return in trade constrained portfolio

This table reports the posterior distribution of the average and standard deviations of the weights in the efficient portfolio with the same target return resulted from 3 spanning tests considered against 2 benchmarks. The portfolio weights are restricted to budget equal to one and constrained to non-negativity and trade constraints. The debt constraints are defined that the minimum allocation of the USD is 25%, EUR 9.5%, GBP 3.5%, JPY 12% in the optimal portfolio.

Currency	Benchmark #1 (USD 63%, EUR 22%, GBP 4%, & JPY 4%)						Benchmark #2 (USD 47%, EUR 34%, GBP 12%, & JPY 7%)					
	Group G4		Developed4		Emerging5		Group G4		Developed4		Emerging5	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Debt Constraints (USD 25.0%, EUR 9.5%, GBP 3.5%, & JPY 12.0%)												
CNY					0.0201	0.0327					0.0315	0.0455
KRW					0.0061	0.0230					0.0124	0.0313
INR					0.0202	0.0385					0.0338	0.0587
BRL					0.0102	0.0245					0.0168	0.0268
THB					0.0050	0.0199					0.0116	0.0324
CAD			0.0168	0.0453	0.0145	0.0355			0.0499	0.0833	0.0233	0.0543
CHF			0.0246	0.0412	0.0024	0.0177			0.0201	0.0530	0.0072	0.0281
AUD			0.0110	0.0256	0.0038	0.0152			0.0227	0.0474	0.0084	0.0259
NZD			0.0025	0.0078	0.0023	0.0116			0.0172	0.0403	0.0067	0.0242
USD	0.6328	0.1076	0.6636	0.0525	0.6592	0.0748	0.5335	0.1112	0.5672	0.1102	0.5733	0.1184
EUR	0.1809	0.1181	0.1185	0.0503	0.0981	0.0220	0.2188	0.1406	0.1209	0.0738	0.1037	0.0392
GBP	0.0673	0.0454	0.0428	0.0251	0.0384	0.0177	0.1121	0.1048	0.0725	0.0776	0.0486	0.0423
JPY	0.1215	0.0071	0.1203	0.0012	0.1200	0.0006	0.1367	0.0343	0.1297	0.0268	0.1227	0.0132

Table 3.8 reports the optimal currency weights for risk minimization for a given existing benchmark returns resulting from three spanning strategies considered when debt constraints are imposed. Reallocate currency composition from the first benchmark requires small variation of the USD weights, between 63 percent and 66 percent of the total reserves. For the second benchmark, the allocation of the currency USD varies between 53 percent and 57 percent. Similar to that of unconstraint, the current benchmarks are underweight USD and overweight EUR relative to the optimal allocation. The difference between the optimal currency allocation and the existing composition may also explain the magnitude of the risk reduction benefits.

Our analyses on the risk minimization for a given existing benchmark return of the country’s foreign reserves currency composition show that before imposing currency weight all the currency diversification strategy considered provide significant benefits regardless of the current benchmark used. Diversification benefits when the trade constraints and debt are imposed reveal that there is potential diversification benefits could be obtained from the currency portfolio optimisation. However the choice of the current benchmark and the currency constraints play an important role in the decision in which diversification strategy central bank reserves managers should proceed.

3.3.2. Risk minimization to the minimum variance portfolio

3.3.2.1. Non-currency constraint allocation

Table 3.9 reports the posterior distribution of portfolio risk reduction to the minimum variance portfolio in switching from the existing benchmark allocation to the new currency allocation. This table shows that when central bank faces short-sale restrictions and budget constraint equal to one, all the currency diversification strategies provide average diversification benefits between 0.74 percent and 0.85 percent with relatively low standard deviation of 0.008 and 0.01, and 0.71 percent and 0.83 percent at the 5th percentile and the average of the new optimal portfolio risk is less than half of the existing portfolio risk.

Table 3.9 Posterior distribution of risk reduction to the minimum variance portfolio for short-sale and budget constraints

This table reports the posterior distribution of risk reduction benefits ∅ (percent) to the minimum variance portfolio for the short-sale and budget constraints portfolio. The summary statistics are the mean, standard deviation, the average actual risk reduction and 5th, 50th, and 95th percentile.

	Mean	Stdev	ō	Percentile		
				5th	50th	95th
Benchmark #1 (USD 63%, EUR 22%, GBP 4%, & JPY 4%)						
G 4	0.7367	0.0138	0.0693	0.7134	0.7371	0.7587
DM 4	0.7380	0.0138	0.0686	0.7145	0.7384	0.7602
EM 5	0.7414	0.0136	0.0669	0.7185	0.7417	0.7630
Benchmark #2 (USD 47%, EUR 34%, GBP 12%, & JPY 7%)						
G 4	0.8449	0.0082	0.0241	0.8311	0.8450	0.8581
DM 4	0.8458	0.0081	0.0238	0.8319	0.8460	0.8588
EM 5	0.8478	0.0080	0.0232	0.8341	0.8481	0.8605

Table 3.9 shows that for non-currency allocation constraints, the two existing currency benchmarks are not efficient. There were massive potential benefits from adjusting currency weights from the same currency benchmark's constituents. For the central bank that uses average actual currency composition as a benchmark, there are potential benefits of 0.74 percent on average or at 95 percent chances to benefits greater than 0.71 percent from readjusting the currency compositions among USD, EUR, GBP and JPY themselves. Spanning

Table 3.10 Currency weights in the minimum variance portfolio for budget and short-sale constraints

This table reports the posterior distribution of the average and standard deviations of the weights in the minimum variance efficient portfolio resulted from 3 spanning tests considered against 2 benchmarks. The portfolio weights are restricted to budget equal to one and constrained to non-negativity

Currency	Benchmark #1						Benchmark #2					
	Group G4		Developed4		Emerging5		Group G4		Developed4		Emerging5	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
No Short-Sale												
CNY					0.0024	0.0026					0.0023	0.0026
KRW					0.0013	0.0020					0.0013	0.0020
INR					0.0079	0.0052					0.0079	0.0051
BRL					0.0044	0.0023					0.0044	0.0022
THB					0.0005	0.0014					0.0005	0.0014
CAD			0.0097	0.0055	0.0072	0.0052			0.0096	0.0054	0.0071	0.0053
CHF			0.0000	0.0001	0.0000	0.0001			0.0000	0.0001	0.0000	0.0001
AUD			0.0000	0.0002	0.0000	0.0001			0.0000	0.0002	0.0000	0.0001
NZD			0.0000	0.0000	0.0000	0.0000			0.0000	0.0000	0.0000	0.0000
USD	0.9940	0.0038	0.9854	0.0062	0.9723	0.0081	0.9940	0.0038	0.9854	0.0062	0.9724	0.0080
EUR	0.0024	0.0028	0.0011	0.0020	0.0008	0.0017	0.0024	0.0029	0.0010	0.0020	0.0008	0.0017
GBP	0.0002	0.0008	0.0001	0.0005	0.0001	0.0005	0.0002	0.0008	0.0001	0.0005	0.0001	0.0005
JPY	0.0035	0.0032	0.0038	0.0032	0.0032	0.0030	0.0035	0.0031	0.0039	0.0032	0.0032	0.0030

into broader currencies provides relatively the same risk reduction benefits for reserve portfolio. Investing in selected developed countries and emerging market currencies both offer the average gain of 0.74 percent and standard deviation of 0.01 percent. At the 5^{th} percentile of the distribution, the two strategies deliver relatively similar magnitudes at 0.71 percent.

Similar to the benchmark that resembles actual central bank allocation on the G4 investments, the use of the currency composition in the SDR as the benchmark provides bigger benefits if the central banks want to reallocate their currency compositions. The potential benefits from readjusting the currency compositions among the existing currency benchmark are on average 0.84 percent and at 95 percent chances to benefits greater than 0.83 percent. Diversify into selected developed countries and emerging market currencies both provide 0.83 percent at the 5^{th} percentile of the risk reduction distribution.

The impact of the use of benchmark relatively minor when the constraints are restricted to no-short selling and budget equal to one. Both benchmarks deliver relatively similar level of the benefits. The more interesting findings, however, for the risk reduction to the GMV portfolio the benefits from the broader currency are relatively similar to that of G4 currencies investment. This finding suggests that for central banks that have an objective to minimise the portfolio risk, they need to concentrate their investment in the USD, EUR, GBP and JPY currency.

Table 3.9 shows the posterior distribution of the average and standard deviations of the optimal currency weights of the risk reduction portfolio from the spanning strategies considered before trade and debt constraints are imposed. When there is only budget and short-sales constraint taken into consideration, spanning from both benchmarks requires more than 97 percent of the reserves to be allocated into currency USD for across strategies. These findings are drawn before imposing transaction constraints for the currency compositions. In order to make this study more realistic from central bank perspectives, trade and debt constraints will be applied in the next sub-section.

3.3.2.2. Transaction constraints currency allocation

3.3.2.2.1. Trade Constraints

Table 3.11 presents the distribution of risk reduction to the minimum variance portfolio from the three strategies of currency diversification. Adjusting the weight among the current currency that uses the actual composition of the central bank's investment offer potential risk reduction benefits of 0.35 percent on average, and 0.34 percent at the 5th percentile of the posterior distribution. These benefits are similar when broader investment into selected developed countries currencies is taken. Spanning further the emerging currency provides even lower benefits at 0.13 percent and at 95 percent probability we could expect benefits to be greater than 0.07 percent. It shows that the requirement to hold minimum 15.5 percent of the reserve in CNY currency reduces potential risk reduction diversification benefits.

When central bank reserve portfolio use currency composition in the SDR as the benchmark, this table shows that on average there are 0.62 percent risk reduction benefits and 95 percent chances the benefits will be greater than 0.34 percent from reallocating currency compositions among the existing currencies

Table 3.11 Posterior distribution of risk reduction to the minimum variance in trade constrained portfolio

This table reports the posterior distribution of risk reduction benefits to the minimum variance portfolio for the trade constraints on the currency compositions. The trade constraints are defined that the minimum allocation of the USD is 15%, EUR 12%, GBP 2.5%, JPY 5% and CNY 15.5% in the optimal portfolio. The summary statistics are the mean, standard deviation, the average actual risk reduction and 5^{th}, 50^{th}, and 95^{th} percentile.

	Mean	Stdev	$\bar{o}$	Percentile		
				5^{th}	50^{th}	95^{th}
Benchmark #1 (USD 63%, EUR 22%, GBP 4%, & JPY 4%)						
G 4	0.3485	0.0076	0.4245	0.3360	0.3485	0.3610
DM 4	0.3484	0.0075	0.4245	0.3360	0.3484	0.3607
EM 5	0.1300	0.0330	0.7570	0.0741	0.1306	0.1824
Benchmark #2 (USD 47%, EUR 34%, GBP 12%, & JPY 7%)						
G 4	0.6164	0.0060	0.1471	0.6065	0.6164	0.6260
DM 4	0.6164	0.0060	0.1472	0.6065	0.6165	0.6261
EM 5	0.4876	0.0194	0.2625	0.4557	0.4880	0.5188

themselves. The magnitude of the benefits is similar to selected developed countries currencies strategy. Similar to the results from the first benchmark, the requirement to hold minimum 15.5 percent of the reserve in CNY currency reduces diversification benefits. Including emerging market currencies will reduce the mean and increase the standard deviation of the risk reduction benefits.

Table 3.12 shows the posterior distribution of the average and standard deviations of the optimal currency weights for the trade allocation of the risk reduction portfolio. Imposing trade constraints, spanning from the two benchmarks requires more 80 percent and 60 percent of the reserves to be allocated into currency USD for the G4 and DM4 strategies. When it requires

Table 3.12 Currency weights of the trade constrained minimum variance portfolio

This table reports the posterior distribution of the average and standard deviations of the weights in the trade constrained minimum variance portfolio resulted from 3 spanning tests considered against 2 benchmarks. The portfolio weights are restricted to budget equal to one and constrained to non-negativity and trade constraints. The trade constraints are defined that the minimum allocation of the USD is 15%, EUR 12%, GBP 2.5%, JPY 5% and CNY 15.5% in the optimal portfolio.

Currency	Benchmark #1						Benchmark #2					
	Group G4		Developed4		Emerging5		Group G4		Developed4		Emerging5	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Trade Constraints (USD 15.0%, EUR 12.0%, GBP 2.5%, JPY 5.0%, & CNY 15.5%)												
CNY					0.1550	0.0000					0.1554	0.0031
KRW					0.0000	0.0001					0.0153	0.0352
INR					0.0039	0.0077					0.0419	0.0676
BRL					0.0035	0.0047					0.0270	0.0430
THB					0.0000	0.0000					0.0194	0.0464
CAD			0.0000	0.0000	0.0000	0.0000			0.0444	0.0764	0.0271	0.0610
CHF			0.0000	0.0000	0.0000	0.0000			0.0177	0.0452	0.0158	0.0607
AUD			0.0000	0.0000	0.0000	0.0000			0.0180	0.0401	0.0083	0.0287
NZD			0.0000	0.0000	0.0000	0.0000			0.0157	0.0374	0.0076	0.0252
USD	0.8050	0.0000	0.8050	0.0000	0.6426	0.0091	0.6064	0.0920	0.6277	0.1028	0.4423	0.1388
EUR	0.1200	0.0000	0.1200	0.0000	0.1200	0.0000	0.2012	0.1039	0.1422	0.0597	0.1283	0.0441
GBP	0.0250	0.0000	0.0250	0.0000	0.0250	0.0000	0.0994	0.0967	0.0585	0.0672	0.0441	0.0574
JPY	0.0500	0.0000	0.0500	0.0000	0.0500	0.0000	0.0930	0.0631	0.0757	0.0474	0.0677	0.0451

providing at least 15.5 percent of the reserve to be invested in CNY, the USD proportions decreased to 64 percent and 44 percent respectively.

3.3.2.2.2. Debt constraints

Table 3.13 presents the distribution of risk reduction to the minimum variance portfolio from the three strategies of currency diversification. Reallocating the weight among the current currency that uses the actual composition of the central banks' investment as benchmark offer potential risk reduction benefits of 0.21 percent on average, and 0.18 percent at the 5th percentile of the posterior distribution. Similar to that of trade constraints, these benefits are similar when it diversified broader investment into selected developed countries and emerging market currencies.

When central bank reserve portfolio use currency composition in the SDR as the benchmark, this table shows that on average there are 0.54 percent risk reduction benefits and 95 percent chances the benefits will be greater than 0.52 percent from reallocating currency shares among the existing currencies themselves. The results are similar for broader currency diversifications. This result suggests that for central banks that need to achieve minimum portfolio risk, they maintain their investment policy in the current four currencies is sufficient. Since there is no significant difference between G4 diversification and broader investment strategy, imply that central bank should continue their current policy.

Table 3.13 Posterior distribution of debt constrained risk reduction to the minimum variance portfolio

This table reports the posterior distribution of risk reduction benefits to the minimum variance portfolio for the debt constrained portfolio. The debt constraints are defined that the minimum allocation of the USD is 25%, EUR 9.5%, GBP 3.5%, JPY 12% in the optimal portfolio. The summary statistics are the mean, standard deviation, the average actual risk reduction and 5th, 50th, and 95th percentile.

	Mean	Stdev	$\bar{\delta}$	Percentile		
				5th	50th	95th
Benchmark #1 (USD 63%, EUR 22%, GBP 4%, & JPY 4%)						
G 4	0.2107	0.0185	0.6229	0.1798	0.2110	0.2406
DM 4	0.2102	0.0182	0.6237	0.1801	0.2103	0.2404
EM 5	0.2118	0.0183	0.6212	0.1816	0.2120	0.2418
Benchmark #2 (USD 47%, EUR 34%, GBP 12%, & JPY 7%)						
G 4	0.5350	0.0112	0.2162	0.5165	0.5352	0.5532
DM 4	0.5350	0.0112	0.2162	0.5162	0.5352	0.5532
EM 5	0.5359	0.0112	0.2154	0.5173	0.5361	0.5540

Table 3.14 shows the average and standard deviations of the optimal currency weights for the trade allocation of the risk reduction portfolio when debt constraints are imposed. Spanning from the first benchmarks requires the allocation of the USD currencies between 63 percent and 66 percent. Different from previous spanning strategies which reduce the USD share, the additional currencies in this table require the lesser EUR for the more diversified portfolios. The findings are relatively similar when it is applied for the second benchmark.

These findings suggested that the role of the USD as the base currency in our analyses may be a key to the results we obtain, which contribute it a big portion in the optimal portfolios. These proportions are not far from the actual central bank investment data. Since USD returns do not vary massively, there is a “bias” to hold the asset with low variance. One possible interpretation is that the

Table 3.14 Currency weights in debt constrained minimum variance portfolio

This table reports the posterior distribution of the average and standard deviations of the weights in debt constrained minimum variance portfolio from 3 spanning tests considered against 2 benchmarks. The debt constraints are defined that the minimum allocation of the USD is 25%, EUR 9.5%, GBP 3.5%, and JPY 12.5% and negative weights are restricted in the optimal portfolio.

Currency	Benchmark #1						Benchmark #2					
	Group G4		Developed4		Emerging5		Group G4		Developed4		Emerging5	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Debt Constraints (USD 25.0%, EUR 9.5%, GBP 3.5%, & JPY 12.0%)												
CNY					0.0201	0.0327					0.0315	0.0455
KRW					0.0061	0.0230					0.0124	0.0313
INR					0.0202	0.0385					0.0338	0.0587
BRL					0.0102	0.0245					0.0168	0.0268
THB					0.0050	0.0199					0.0116	0.0324
CAD			0.0168	0.0453	0.0145	0.0355			0.0499	0.0833	0.0233	0.0543
CHF			0.0246	0.0412	0.0024	0.0177			0.0201	0.0530	0.0072	0.0281
AUD			0.0110	0.0256	0.0038	0.0152			0.0227	0.0474	0.0084	0.0259
NZD			0.0025	0.0078	0.0023	0.0116			0.0172	0.0403	0.0067	0.0242
USD	0.6328	0.1076	0.6636	0.0525	0.6592	0.0748	0.5335	0.1112	0.5672	0.1102	0.5733	0.1184
EUR	0.1809	0.1181	0.1185	0.0503	0.0981	0.0220	0.2188	0.1406	0.1209	0.0738	0.1037	0.0392
GBP	0.0673	0.0454	0.0428	0.0251	0.0384	0.0177	0.1121	0.1048	0.0725	0.0776	0.0486	0.0423
JPY	0.1215	0.0071	0.1203	0.0012	0.1200	0.0006	0.1367	0.0343	0.1297	0.0268	0.1227	0.0132

USD's high proportion reflects its status as the "safe haven" currency. Another explanation is that central banks do in fact use the dollar as the reference currency for expressing returns on alternative assets. Our preferred interpretation, however, is that taking the dollar as the risk-free asset corresponds to the case in which the domestic currency is pegged to the dollar, so there is zero volatility with respect to the dollar. This is consistent, for example, with the results of Dellas and Yoo (1991) for Korea and Papaioannou et al. (2006) for the global foreign reserve cases.

3.4. Conclusions

The main objective of this chapter is to provide alternative frameworks for to allocate foreign reserve currency compositions. Risk reduction for a given existing expected returns and risk reduction to the minimum variance portfolio are implemented in this study to concentrate on the safety objective of the foreign reserve. Bayesian method of Li et al. (2003) is combined with currency constraints to capture the transaction motives of the central bank, i.e. trade constraints and debt constraints are employed to analyse diversification benefits.

To analyse the currency compositions we replicate the actual central banks investment in the G4 currency and G4 currency composition in the SDR as the original currency benchmark. Depart from those benchmarks the optimisation to minimise portfolio risk by imposing shares of external transactions. Empirical evidence on the minimum variance portfolio show that there are potential significant risk reductions benefits for central banks to reallocate their currency composition among the existing benchmark or broader currency diversifications regardless the original benchmark currently being used and across debt and trade constraints.

My findings related to the risk minimization for a given benchmark returns show that before imposing portfolio constraints, there are significant diversification benefits. Imposing trade constraints and debt constraint on the optimisations change the significant of the benefits. In general, greater currency diversifications provide bigger the mean of the benefits. However, for the original benchmark which allocates USD more than 60 percent, broader currency investment could attain significant benefits at the 5^{th} percentile of the posterior distributions into selected advanced countries and selected emerging market currency from imposing trade constraint and none from imposing debt constraints. Central bank that uses the original benchmark which allocates USD less than 50 percent, the significant risk reduction benefits at the 5^{th} percentile of the posterior distributions could be achieved by reallocate benchmark currency shares and by broader currency investment into currencies of the selected developed economy from imposing trade constraint and broader spanning into selected emerging market currencies from imposing debt constraints.

Minimising foreign reserve portfolio risk for a given existing benchmark expected return framework generally more desirable than the second one. Foreign reserves diversification ideas resulting from the first measure also easier to communicate to the public and other stakeholders since the new model portfolio do not sacrifice the existing expected returns. However, it is interesting that the empirical findings reveal minimum variance portfolio offer significant benefits across all constraints. Might be worth thinking about should the central bank follow the GMV strategy.

The empirical findings show that in general, the new optimal portfolios suggested allocating more USD and less EUR comparing to the current composition in the benchmark portfolio. Our findings suggest that central bank should consider relaxing their investment policy beyond USD, EUR, GBP and JPY currency. However, the choice of the current benchmark and both trade and debt constraints play an important role in the decision in which diversification strategy central bank reserves managers should proceed. It is desirable and feasible to adopt Bayesian approach combined with external payment constraints as an alternative framework for determining central banks' foreign currency structure.

CHAPTER FOUR

This chapter investigates asset allocation in the optimal portfolio risk minimization framework. First, risk minimization for a given expected benchmark return is examined. The findings show that there are significant risk reduction benefits by adding developed market longer maturity bonds, quasi-government, emerging market and inflation-linked bonds altogether to the existing benchmark policy. Second, using the same diversification strategy as the earlier step, risk minimization of the minimum variance portfolio was conducted. The second framework shows that the portfolio risks are mainly driven by US treasury bills.

4. GOVERNMENT BOND DIVERSIFICATION BENEFITS

Abstract

This study investigates whether longer maturity bonds, including emerging government debt and inflation-linked government bonds, provide diversification benefits to the central bank's benchmark portfolios. I measure the reduction of portfolio risk as the diversification benefits using Bayesian approach combined with multiple assets weight constraints. For the minimum variance portfolio, all of the three strategies offer significant risk reduction benefits to the government bond portfolio across multi-constraints. However, for *the central bank that aims to reduce portfolio risk for a given existing expected return, the inclusion of emerging government and inflation-linked government bonds provides significant risk reduction benefits to the existing benchmark. This is the case if those assets classes are added together with longer maturity bonds of G7, developed market (DM), semi-government, supranational and government agency and emerging market government bonds. Our results also show that asset allocation policy have a greater role than the asset class selection.*

4.1. Introduction

Management of foreign exchange reserves is one of a key task undertaken by central banks in order to maintain their credibility. The exchange rate and monetary policy will determine a variety purposes for which such foreign reserve assets may be used; ranging from exchange rate management to external debt management. Foreign reserves management ensures that the capacity to intervene in the foreign exchange markets exists when needed, while simultaneously minimising the costs of holding reserves. Its importance has increased over the past decade as a result of the upsurge of the global reserves accumulation.

As described in Choo (2007) and Claessens and Kreuser (2007), central banks have traditionally invested their foreign reserves in a portfolio of highly liquid assets that have very minimal exposure to market and credit risk. However, as the size of foreign reserves has increased to a record level, the likelihood that they are all needed for financial contingency decreases. At the same time, stakeholder awareness toward risk-return is increasing, and further attention is being directed as to the ways by which central banks manage their foreign reserves. This background highlights that when setting benchmarks for central banks, some areas for risk-return enhancement are required to be identified. However, the evidence from the previous literature is limited and inconclusive concerning the benefits of government bond diversification, since most studies in modern portfolio theory area only consider equity or bond-equity portfolios. Thus, a gap in the literature exists towards what investment opportunity set should be considered from a central bank perspectives.

Central banks have typically relied mainly on published bond market index data, while some depend on modern portfolio theory for determining the strategic allocation of their reserves (Jorion and Rolfo, 1992). Despite the fact that the published indices provide transparency and traceability, their composition may not fit perfectly with central bank portfolio objectives. The most popular framework for portfolio optimisation is the Markowitz (1952) mean-variance analysis. The key assumption in this framework is that the returns are normally distributed. The mean-variance portfolio decision models require certain ex-ante input parameters that are assumed to be known to investors. Investors are required to provide estimates of the expected returns and covariances of all the assets in the investment universe considered. In practice however, this leads to estimation risk problem[10]. Furthermore, mean-variance optimisation is very sensitive to the inputs[11].

Following those drawbacks, a number of general Bayesian and shrinkage approaches have been used to overcome these issues. Bayesian predictive distribution provides a natural method to express investment opportunities in the presence of parameter uncertainty (Avramov and Chao, 2014; Avramov and Zhou, 2010). An additional benefit of the Bayesian approach is that it can be

[10] Further discussion regarding estimation risk see Barry (1974, 1978), Bawa, Brown, and Klein (1979), Dhingra (1980, 1983), Klein and Bawa (1976, 1977) and Klein, Rafsky, Sibley, and Willy (1978).

[11] See Chopra (1993) and Chopra and Ziemba (1993) for further discussion about this issue.

applied when the central bank faces portfolio restrictions such as budget, short selling, asset weights or the combination of those constraints. Hence, I believe that Bayesian technique is the promising method to be used in this mean-variance analysis.

The assets weight constraints of Jagannathan and Ma (2003) provide inspiration to extend the mean-variance framework, in addition to the imposition of short-sale constraints. Extending further from the earlier chapter, multiple assets weight constraints are applied. The objective of these constraints is threefold; to guarantee that liquid assets will be held at all times, there is a minimum holding of the current benchmark assets, and the maximum investment of certain asset classes is limited. Literatures provides empirical evidence of the benefits from relaxing bond maturity, including emerging market exposure, and investing in inflation-linked government bonds as independent asset classes for an individual or private type of investors. However, such study has not yet been extended to examine the effects when they are included together in a central banks' portfolio. As such, the following points remain open to question: 1) should central bank relax the bond maturity constraints and investing in a broader spectrum of government bond market? 2) Do emerging market government bond and inflation-linked government bonds provide benefits for central bank's government bonds portfolio? The findings in this chapter offer a resolution.

Some may argue that the ultimate benefits of international diversification are in terms of the risk reduction rather than returns enhancement. Clements (1997) proposes that the main reason to invest in the foreign market is not to replicate the global market portfolio holding or to increase returns, but is to reduce volatility. Additionally, Manchev (2009) from Central Bank of Bulgaria claims that liquidity and safety should be the main objectives for central bank reserve portfolios. It seems appropriate to measure diversification benefits of central banks' government bond portfolio using the risk reduction measures rather than return enhancement. This objective framework with more focus on liquidity and safety will be adopted in this thesis.

There are four main results in our empirical analysis on government bond diversification benefits. First, our analysis shows that the results are strong when central banks' main goal is to minimise foreign reserve risks. However, this is not the case if the same expected return is to be maintained. For a central bank that is concerned with portfolio risk minimisation of the global minimum variance portfolio, all three strategies offer significant diversification benefit to the government bonds portfolio across multi-constraints. However, if the objective of the central banks' reserve management is to minimise portfolio risk while maintaining the existing target returns, this study could not find empirical evidence to support the argument that diversification using the longer dated bonds provides significant risk reduction benefits. On the other hand, inflation-linked government bonds can provide significant diversification benefits. However, this is only the case if these are added together with a group of G7 government bonds, developed market government bonds, semi-government, supranational and government-agency bond markets, and emerging market country rating assets.

Second, despite the massive risk reduction benefits that resulted from the minimum variance portfolio results, because they are driven mostly by treasury bills it is perhaps less interesting. More interesting are the risk reduction benefits for a given expected returns as this research finds here. Furthermore, global minimum variance portfolio may be not the best choice for central banks since this framework might result in a lower expected return. Third, by applying Bayesian mean-variance framework with multi-weight asset constraints, this research shows that asset allocation has a bigger impact on risk reduction than asset class selection; when short-sale restrictions have already been considered. Fourth, imposing asset allocation constraints can provide more diversified portfolios that theory suggests it should, but with the cost of lower its performance.

The results look interesting and provide direct implication for central banks in managing their foreign exchange reserve. In this empirical chapter, our framework's objectives address liquidity and safety problems faced by the central bank in general. Two risk minimizations have been addressed; first risk minimization of the global minimum variance portfolio and risk minimization for a given expected benchmark return. Our analysis provides some alternatives for how much risk reduction central bank want to pursue, which spanning strategy should be chosen, and the new optimal portfolio weights they should hold in addressing their liquidity, safety and surplus goals.

The remainder of the chapter proceeds as follows. Section 2 describes concept the research methods to make it operational. Section 3 describes the data that will be followed by the description of the central bank benchmark portfolio and test assets. Section 4 presents and discusses the results, and summary of the chapter in Section 5.

4.2. Research Method

This chapter focuses on the objective of risk minimization for a given expected benchmark return when the central bank has restrictions on their investment policy, including no short-selling and a budget constraint equal to one as given by the equation (3.1) in Chapter Three. This research measures diversification benefits in term of volatility reduction for the same target returns as benchmark return, similar to the earlier chapter.

4.2.1. Asset allocation constraints

Table 4.1 shows the survey data of US dollar denominated assets held by global central banks and sovereign wealth funds by maturity and sovereignty from the year 2007 to 2013. This survey is intended to illustrate how official reserve managers reacted to the recent challenges in the management of their portfolios. Capital preservation is typically highlighted as a portfolio constraint in the central banks' investment policy. Based on portfolio theory, capital preservation should be tied to the market value of the portfolio as a whole over the appropriate investment horizon, and not based on any single security or accounting period.

Table 4.1 Foreign official holdings of US Dollar assets by instrument and original maturity (billion USD)

This table indicates foreign official (e.g., Central Bank and Sovereign Wealth Funds) holdings of US Dollar denominated assets by instrument and original maturity from 2007 to 2013. Figures for Treasury, agency, equity, onshore deposits and money market instruments and corporate long-term debt and ABS are from Department of Treasury, Federal Reserve Bank of New York, Foreign Portfolio Holdings of US Securities. Figures for offshore deposits (short-term others) are from Table CM-I-1. Section I – Liabilities to Foreigner (sub-section foreign official institutions) Reported by Banks in the United States. Long-term is defined by original maturity. IMF data from COFER, IFS. World Bank data from http://data.worldbank.org/indicator/ FI.RES.TOTL.CD.

	2007	2008	2009	2010	2011	2012	2013
Short-term instruments	1,100	1,097	1,145	911	1,001	954	996
Treasury	159	226	572	454	414	366	363
Agency	80	130	34	27	16	5	3
Others[1)]	861	741	539	430	571	583	630
Long-term instruments	2,567	3,119	3,266	3,861	4,409	4,772	5,026
Treasury	1,452	1,684	2,054	2,617	3,103	3,489	3,648
Agency	750	966	794	721	635	543	452
Others[2)]	99	106	107	97	104	110	127
Equity[3)]	266	363	311	426	567	630	799
Total holdings	3,667	4,216	4,411	4,772	5,410	5,726	6,022
Share of short-term holdings	**30%**	**26%**	**26%**	**19%**	**19%**	**17%**	**17%**
Share of treasury+agency	67%	71%	78%	80%	77%	77%	74%
World FX reserve	1,999	2,782	2,682	2,995	10,205	10,952	11,683
Share of claims in USD (source: COFER, IMF)[4)]	64%	64%	62%	62%	62%	61%	61%
Global FX Reserve (source: World Bank)[5)]	7,113	7,773	9,043	10,387	11,496	12,367	12,617

1) Others (short-term) comprise of short-term corporate debt, deposits and money market paper in the United States, and offshore USD deposits.
2) Others (long-term) comprise of long-term corporate debt; corporate asset backed securities (ABS).
3) Includes common stock, fund shares, preferred stock, and other type of equity.
4) Source: Currency Composition of Official Foreign Exchange Reserves (COFER), International Financial Statistics, IMF. COFER data for individual countries are strictly confidential. The data are only possible to be disseminated in aggregate form. At present, 147 reporters consist of IMF member countries, non-member countries, and other foreign exchange reserve holding entities. COFER data started to be published from 1995 with relatively minimal coverage at the beginning.
5) Total reserves comprise holdings of monetary gold, special drawing rights, reserves of IMF members held by the IMF, and holdings of foreign exchange under the control of monetary authorities. The gold component of these reserves is valued at year-end (December 31) London prices. Data are in current U.S. dollars

Most central bank reserves managers understand the theory but have only partly put it into practice.

Over the past decade, Johnson-Calari et al. (2007) explain that typical central banks invested their foreign reserves in fixed income government securities with portfolio duration of around 1 to 2 years and individual securities with maturities up to ten years. This largely reflects the low tolerance for capital losses that would result if bond yield rises. In this stage, central banks mostly accepted that capital preservation could be measured on a portfolio basis and they generally extended the accounting and performance period to one year. However, most central banks still diverge from the financial industry. For example, a refusal to apply the same principle of credit risk as they have to interest rate risk. In this case, default risk on any individual security is unacceptable no matter the impact on the total portfolio value and additional returns due to investment in higher credit risk sectors. Hence, the spirit of central banks conservativeness will be adopted in this study in defining benchmark portfolios.

In order to compare official holdings of US dollar denominated asset to the total reserve, this research also presents global foreign exchange reserve data from IMF and World Bank. This table also indicates that during 2007-2013, more than 50 percent of global reserves are invested in US dollar denominated assets. Total IMF-reported foreign exchange reserves are lower than that of World Bank due to IMF data just only started from 1995 with relatively minimal coverage at the beginning. At present, there are 147 reporters consisting of IMF member countries, non-member countries, and other foreign exchange reserve holding entities.

Table 4.1 shows that in 2007 identified US dollar reserves were 30 percent invested in securities of less than one-year original maturity. From this table, it appears that official reserve managers turned more than 10 percent of its liquidity buffer portfolio into a profit centre in 2013. This change indicates that the traditional consideration of keeping reserves has declined by some degree. However, since those data only identify original maturity, officials holding of short term securities are probably understated. For instance, buying a US treasury note that has a remaining life of 11-months is classified as long-term securities holding in those definitions. This survey shows similarity to Kotz and Strauss-Kahn (2007) argument that central bank must hold at least 20 percent of gross reserve in highly liquid securities to cover contingency plans. Hence, an allocation of 30 percent of reserves to be invested in treasury bills assets will be adopted and utilised in this study.

In terms of the sovereignty, this table also shows that quasi-government investments remain the most favourable asset class for official reserve managers. The only exception was in during financial crisis 2010 when the portion of treasury and agency reached 80 percent of official reserve. Quasi-government portion gradually increased and back to pre-crisis level at around 70 percent of the total reserve in 2013. The researcher believes that this preference for a central bank to invest most of their reserve in quasi-government securities will not change dramatically in the near future.

4.2.1.1. Equality constraint

Equality constraint is a specialised linear constraint that enforces membership among group of assets in a portfolio must satisfy the following:

$$A_{Eq} * x = b_{Eq};\ \ b_{Eq} = 0.3 \qquad (4.1)$$

where A_{Eq} is the matrix of group membership indexes (n_{Eq}-by-n matrix), x is the assets weight (n vector), n is the number of assets in the universe and n_{Eq} is the number of equality constraints. This constraint ensures liquidity sufficiency at all the time so that 30 percent of the reserves have to be kept in the treasury bills or other securities which has remaining life less than 12 months.

4.2.1.2. Lower-bound constraint

The lower-bound constraint is a linear inequality constraint for portfolio weights. This constraint regulates an asset or a group of assets to satisfy the following:

$$A_{Lb} * x \geq b_{Lb};\ \ \ b_{Lb} = 0.15 \text{ and } 0.25 \qquad (4.2)$$

where A_{Lb} is the matrix of group membership indexes (n_{Lb}-by-n matrix), x is the portfolio (n vector), n is the number of assets in the universe and n_{Lb} is the number of lower-bound constraints. The objective of this constraint is to satisfy the requirement that at least 15 percent have to be invested in US treasury bills and 25 percent in the current benchmark index i.e. G7 1-5 year bond index. The

Table 4.2 Asset allocation constraint

The table shows the currency allocation constraint for benchmark and test assets for broader investment strategy. The shaded area indicates the eligible assets considered for each optimisation strategy. The benchmark portfolio consists of 15 percent US Treasury Bills, 15 percent of G7 non-US Treasury Bills and 70 percent of G7 government bond one to five year maturity. Test assets consist of two strategies; bond spanning and broader investment universe. Asset allocation constraints for test asset are 30 percent invested in treasury bills with at least 15 percent have to be kept in US Treasury Bills and minimum 25 percent of the total portfolios have to be invested in the government bond market index. Semi-government, supranational and government agency (SSA), emerging government bond (EM) and index-linked government bond asset classes are constrained maximum 10 percent, maximum 10 percent and maximum 15 percent respectively, maximum 5percent for gold, no more than 10 percent for the MBS and ABS combined, and maximum 5 percent for the combination of corporate bond and world equity.

Portfolio	Benchmark			Government Bond					Non-Government Bond				
	G7 Tbills		G7 1-5	G7 5-	DM 1-	SSA	EM	Inflation-	Gold	MBS	ABS	Corporate	
	US	ex-US	year	10 year	10 year			Linked				Bond	Equity
Benchmark	15%	15%	70%										
Test Asset	30%												
Longer Maturity													
G7													
DM													
SSA													
Emerging Market													
Regional	Min 15%		Min 25%										
Credit Rating						Max 10%							
Inflation-Linked													
Global Inflation							Max 10%						
G7 Inflation								Max 15%					
Non-Government Bond									Max 5%	Max 10%		Max 5%	

the objective of both the equality and the lower-bound constraint is to maintain central bank conservativeness, in a way that the new assets suggested from the new optimal allocation are less than half the total reserve portfolio.

4.2.1.3. Upper-bound constraint

Given a linear inequality constraint matrix A and vector $\bar{x}$, every weight x_i in a portfolio must satisfy the following:

$$A_{Ub} * x \leq b_{Ub}; \quad b_{Ub} = 0.1;\ 0.1;\ 0.15 \tag{4.3}$$

where A_{Ub} is the matrix of group membership indexes (n_{Ub}-by-n matrix), x is the portfolio (n vector), n is the number of assets in the universe and n_{Ub} is the number of lower-bound constraints. This constrain is intended to limit certain asset class holding includes 10 percent in SSA; 10 percent in emerging market; and 15 percent in inflation linked government bonds asset classes.

4.2.2. Data

Data for this study consists of government bonds returns and are derived from Thomson DataStream between December 1985 and December 2014. This research uses bond indices monthly returns from developed countries that have the most liquid government bond market in local currency denomination with the longest period data available. Government bond market indices in this study are classified into three asset classes: a group of G7-country (G7), developed market (DM) and semi-government, supra-national and government-agency (SSA). The group of G7 government developed countries includes United States (US), United Kingdom (UK), Germany (GE), France (FR), Japan (JP), Canada (CA) and Italy

Table 4.3 Data grouping for asset class formation

The table shows asset class construction from the individual bond market index. Bond index in this study classified into six asset classes: Benchmark, group of G7-country (G7), developed market (DM), semi-government, supra-national and government agency (SSA), emerging market government (EM) and inflation-linked government bond (IL). Group of G7 government developed countries includes United States (US), United Kingdom (UK), Germany (GE), France (FR), Japan (JP), Canada (CA) and Italy (IT). Government bond issued by Netherland (NE), Austria (OE), Belgium (BE), Switzerland (SW), Denmark (DE), Australia (AU) and New Zealand (NZ) are a member of DM asset class. Those G7 and DM index classified into a 1-5 year, 5-7 year, 7-10 year, 5-10 and 1-10 year maturity. SSA asset class is formed by Barclays US Supranational bond, Barclays Euros Supranational bond maturity 1-7 years, Barclays Agency Bond maturity 1-5 years, Bank of America Merril-Lynch Canada Province and Municipal 1-10 years, and Australia semi-government bond maturity 2-10 years. Emerging market asset classes use geographical (Asia, Europe, Middle East and Africa/EMEA, and Latin America/Latam) bond market and the country rating (B, BB and BBB) bond index. IL asset class uses Global Government inflation-indexed and inflation-linked bonds issued by individual government G7 country.

Asset Class	Constituent/country	Maturity	No. of index	Notes
Benchmark				
Govt bond market	G7 1-5	1-5 yr	1	single index
UST Bills	United States (US)	6 month	1	single index
Non-US Bills	United Kingdom (UK), Germany (GE), France (FR), Italy (IT), Japan	6 month	6	equally weighted
Group of G7 Government Bond				
G7 5-7	US, UK, GE, FR, IT, JP, CA	5-7 yr	7	equally weighted
G7 7-10	US, UK, GE, FR, IT, JP, CA	7-10 yr	7	equally weighted
G7 5-10	US, UK, GE, FR, IT, JP, CA	5-10 yr	14	equally weighted
Developed Market Government Bond				
DM 1-5	Netherland (NE), Austria (OE), Belgium (BE), Switzerland (SW), Denmark (DE), Australia (AU), and New Zealand (NZ)	1-5 yr	7	equally weighted
DM 5-7	NE, OE, BE, SW, DE, AU, NZ	5-7 yr	7	equally weighted
DM 7-10	NE, OE, BE, SW, DE, AU, NZ	7-10 yr	7	equally weighted
DM 5-10	NE, OE, BE, SW, DE, AU, NZ	5-10 yr	14	equally weighted
DM 1-10	NE, OE, BE, SW, DE, AU, NZ	1-10 yr	28	equaly weighted
Semi Government, Supranational & Government Agencies Bond				
SSA	US Supranational, Euros Supranational, Agency bond, Canada Povince & Municipal, and Australia semi-govt	1-10 yr	5	equally weighted
Emerging Market Government Bond				
EM Country Rating	B, BB, BBB	1-10 yr	3	single index
EM Geographic	Asia, EMEA, LATAM	1-10 yr	3	single index
Inflation-Linked Government Bond				
Global IL	Global IL government bond	1-10 yr	1	single index
G7 IL	US, UK, GE, FR, IT, JP, CA	1-10 yr	7	equally weighted

(IT). Government bonds issued by Netherland (NE), Austria (OE), Belgium (BE), Switzerland (SW), Denmark (DE), Australia (AU) and New Zealand (NZ) are members of the developed market asset class. The researcher grouped those G7 and developed market government bonds into 1 to 5 year, 5 to 7 year and 7 to 10-year maturities.

SSA asset class is formed by Barclays US Supranational bond, Barclays Euros Supranational bond maturity 1 to 7 years, Barclays Agency Bond maturity 1 to 5 years, Bank of America Merrill Lynch Canada Provinces and Municipal 1 to 10 years, and Australia semi-government bond maturity 2 to 10 years (available between January 1988 and December 2014). This research uses two categories for emerging market asset classes; the first is geographical (Asia, Europe, Middle East and Africa/EMEA, and Latin America/Latam) bond market and the second is country rating (B, BB and BBB) bond index (available from December 2004 to December 2014). Inflation linked bonds in this research include Global Government inflation-indexed and inflation-linked bonds issued by the individual government of the group of the G7 country that are available between December 1985 and December 2014.

Since the central bank's reserves portfolio are valued in US Dollar, end of month currency exchange rates are needed to convert domestic currency index returns into USD. Exchange rate data are originally from Bank of England (BoE) and are used to convert the local currency developed market bonds price index and non-USD G7 treasury bills into USD. Bank of America Merrill Lynch G7

Government Bond 1 to 5-year index, serves as government bond market proxy. Note that all the analysis is in USD.

4.2.2.1. Benchmark assets

Central banks are usually conservative in setting their benchmarks due to various reasons. Central banks are unique in a sense that their credibility depends on their independence from government, to avoid a potential conflict of interest. This conflict is most often cited in the context of monetary policy, but it may also exist in the area of foreign currency reserves management where government want to maximise short-term earning for budgetary purposes. The government also prefer for a stable contribution from central banks to the budget. It is not unusual for governments to pressure central banks for higher investment returns but declines to accept higher risks. This conflict typically will be translated by central banks to retreat to low risk-return investment strategies. The aim of doing so is to avoid year-to-year volatility in government remittance, even though at the cost of foregoing higher portfolios performance.

Technical details of the benchmark portfolios formation are explained in the appendix. The benchmark portfolio consists of three assets. The first is a monthly return on the US treasury bills. The second sets are the equally weighted group G7 non-USD treasury bills equivalent issued by the government of UK, Germany, France, Italy, Japan, and Canada. The third are the return on G7 government bond short-medium-term maturity as the representation of government bond market. Taking account for the reserve management conservatism, the existing benchmark portfolio contains 30 percent of the investable reserves invested in securities with a maturity of less than one year, split equally into US treasury bills and non-US bills, and 70 percent of the portfolio invested in the bond market index.

This research defines the benchmark portfolio that takes into account central bank liquidity needs and limits its exposure to interest rate and market risks. This research applies lower-bound constraints in the optimisation to ensure that a certain portion of the portfolios is sufficient to cover central bank liquidity needs at all times. In order to confirm that the spanned portfolios meet such liquidity criteria, it requires 30 percent of the foreign reserves portfolio to be invested in securities with maturities less than one year, with at least 15 percent in US treasury bills and at least 25 percent of the portfolio will be kept in group of G7 government bond 1 to 5 year maturity index. This framework also confirms the conservatism of the central bank, since the new asset allocation maintains at least 55 percent of the existing benchmark portfolio. Regarded as conservative investors, central banks are more likely to reject drastic change in their reserves composition, which is also another consideration in this framework.

4.2.2.2. Test assets

For the empiricism, this research proposes the allocation process under several different scenarios and assumptions to approximate to what can be thought as stages of reserves management practices, reflecting the shifting attitude of central banks towards credit and market risk. The researcher constructs several test assets which are divided into three categories; longer bond maturity, emerging market, and index-link government bonds. This is to allow corresponding research questions, as these assets are implemented in various strategies. This research introduced 11 scenarios of the longer dated bonds, 8 spanning strategies of the emerging market bonds inclusion, and 12 portfolios of the addition of the inflation-linked bonds to the existing central bank benchmark portfolio.

a. Test assets for longer bond maturity

   Historically, in positive yield curve environments and in relation to the liquidity premium hypotheses as discussed in Bodie, Kane, and Marcus (2008), fixed income investors have always been compensated in higher returns over time by investing in longer duration. This argument is the main motivation to assess investing in longer maturity bonds on government bond portfolios. More specifically, Johnson-Calari et al. (2007) document a study about trends in reserve management that relaxing bond duration will improve central bank's risk-return performance. The construction of 11 portfolios of longer dated bonds approximates central bank investment policy. This is first investigated by investing in longer-dated maturity within a group of G7 government bonds; 5 to 7-year, 7 to 10-year, and 5 to10-year maturity buckets. Second, the selected advanced economy government bonds; 1 to 5-year, 5 to 7-year, 7 to 10 year, and 1 to 10-year maturity; are included. Lastly, in the reserve management survey, Pringle and Carver (2006) and Borio et al. (2008) identify an increasing central bank investment in government agency security. Hence, semi-government, supranational and government-agency bond asset classes are also added in this analysis.

b. Test assets for emerging market strategy

Central banks typically avoided any class of issuer that could give rise to default risk, no matter how small the probabilities of defaults actually were. Eligible asset classes were thus widened, but only to allow issuers judged to have virtually zero default risk. At this stage, central bank defines default risk from the perspective of the issuer, which differs from the probability of default that is commonly used by rating agencies. They first perceive the level of sovereignty (government, state/province, an agency with or without explicit government indemnity, supranational or corporate) before the probability of default frameworks of the rating agencies.

The inclusion of the broader definitions of government securities to the existing government bond portfolio may provide additional diversification benefits. Cifarelli and Paladino (2006) document convincing evidence of emerging sovereign bonds co-movement spread, it changes more within geographical area than between geographical area. However, very few studies analysing the role of emerging market government bonds for developed market government bond portfolios from a central bank perspectives

Stages of the inclusion of emerging market government bonds strategies are indicated from the 8 portfolio scenarios. First, by adding emerging market bond index directly to the existing benchmark. Second, by adding emerging market together with a group of G7 bond assets. Third, by adding emerging market with the group of G7 and developed market government bonds simultaneously. Fourth by adding emerging market, G7, developed market and semi-supra-agency assets altogether to the existing benchmark. Scenarios five to eight involve repeating the previous scenarios for the different emerging market index.

c. Test assets for inflation-linked strategy

Bri'ere and Signori (2009) argue that, although inflation-linked bonds once had significant diversification benefits, they are now highly correlated with nominal bonds and have reached similar volatility levels. As a result, they argue that the two asset classes are practically substitutable. Although it was a valuable reason for introducing inflation-linked bonds in a global portfolio before 2003, Bri'ere and Signori (2009) argue this is no longer the case.

The diversification benefits of the inclusion of index-linked government bond will be examined using two different indexes; global government inflation link and G7 government inflation linked index. 12 portfolios are constructed to reflect stages of the index-linked bond spanning to the current benchmark. First, by adding index-linked bond directly to the existing benchmark. Second, by adding index-linked bond together with G7 bond assets. Third, by adding index-linked with the G7 and developed market simultaneously. Fourth by adding index-linked, G7, developed market and semi-supra-agency. Fifth and sixth by adding index-linked with G7, developed market, semi-supra-agency and emerging market geographic or country rating approach. Scenarios seven to twelve are constructed by repeating scenario one to six for the different index-linked government bonds.

Table 4.2 shows the construction of the weights constraints for the test assets. Asset allocation decisions in general, need to consider all elements of risk and the central banks' policy objectives in managing their reserves. Asset allocation constraints are proposed to address the typical central banks' risk appetite that can be considered as low tolerance toward risk and conservative. Hence, this exercise tries to limit the interest rate and market risks exposure by setting up the total allocation of new assets to be less than 45 percent of the total portfolio. In this framework, one may see that the portfolio optimisation will only be given less than half the portion of the total reserves portfolio. This framework is intended to reflect conservatism and the central bank will likely aim to avoid radical change compare to the status quo; even though the new portfolio may offer significant improvements.

In addition to the total test assets allocation, this research also sets portfolio weight constraints at the asset class level to ensure that our framework is consistent with the risk preference of central banks. Further, this individual asset class weight constraint can be seen as a preventive measure to avoid excessive holdings of new and unfamiliar instruments and market; particularly emerging and index-linked government bonds. This research defines constraints of maximum 10 percent to be held in SSA, maximum 10 percent held in emerging market government bond and maximum 15 percent to be held in index-linked government bonds markets.

4.3. Empirical Results

4.3.1. Descriptive statistics

Table 4.4 reports summary statistics for the bond market index and the first four asset classes; treasury bills, G7, developed market and semi-supra-agency government bonds index. These will be used to examine central banks' foreign reserves diversification benefits by relaxing bond maturity and credit constraints to quasi-government. Average monthly return on the G7 1 to 5-year maturity index was 0.49 percent over the entire period, and the monthly volatility was 1.52 percent. Mean return as well as the volatility varies substantially across developed market government bond indexes as shown in Table 4.4. Interestingly, a non-US treasury bill provides lowest volatility returns than US Treasury Bills. Non-US Treasury Bills have the expected returns and volatility of 0.23 percent and 0.13 percent respectively, lowest amongst other asset classes.

Table 4.4 Summary statistics of benchmark assets and government bond portfolios

The table reports summary statics of the monthly returns of the government bond benchmark assets and 16 quasi-government bond indices between December 1985 and November 2013. Benchmark assets consist of the market, UST Bills and Non-US Bills. Test assets include a group of G7 government bond market, developed government bond market, semi-government, supranational and government agency bond market, emerging government bond market and index-linked government bond market. The table shows the mean, standard deviation, minimum and maximum monthly returns (percent) and its correlations with each benchmark assets.

Asset Class		Mean	Standard Deviation	Minimum	Maximum	Correlation		
						Bond market	UST Bills	Non US Bills
Benchmark								
Bond Market index (G7 1-5 year)		0.4857	1.5185	-4.2092	5.1026	1	0.0948	0.0630
US Treasury Bills		0.3014	0.2056	0.0025	0.7492	0.0948	1	0.9294
G7 ex-US TBills		0.2346	0.1318	-0.0126	0.5698	0.0630	0.9294	1
Lengthening Duration								
Group G7	5-7 year	0.0876	0.9864	-2.8021	2.8961	0.4287	-0.0781	-0.0333
	7-10 year	0.1517	1.2417	-3.4884	4.3097	0.4039	-0.0783	-0.0454
	5-10 year	0.1196	1.1096	-3.1452	3.6029	0.4165	-0.0785	-0.0402
Developed Mkt (DM)	1-5 year	-0.0356	0.4522	-1.1234	1.3960	0.3024	-0.0601	0.0222
	5-7 year	0.0888	0.9642	-2.5490	2.7860	0.2989	-0.1001	-0.0375
	7-10 year	0.1318	1.0703	-2.6752	3.7107	0.2764	-0.1131	-0.0623
Semis, Supranational and Govt. Agencies (SSA)		0.2162	0.9416	-3.5064	3.2574	0.4965	-0.0089	-0.0272
Emerging Market								
Geographics	Sovereign B rated	-0.0315	2.3366	-26.6681	8.8721	0.0428	-0.0007	-0.0712
	Sovereign BB rated	0.0969	1.6280	-18.3602	12.2182	0.1426	-0.0734	-0.1218
	Sovereign BBB rated	0.0336	1.3233	-11.6660	9.3027	0.2317	-0.0479	-0.0883
Country Rating	Asia	0.0882	1.7390	-18.1024	13.2688	0.1364	-0.0530	-0.1070
	Europe, Midle East & Africa	0.0332	1.4234	-16.1205	6.3711	0.1512	-0.0493	-0.1013
	Latin America	0.0246	1.6421	-16.9598	8.2663	0.1710	-0.0213	-0.0784
Index-linked Bond								
Global Government Inflation-Linked Bond		0.3307	1.6957	-11.9180	7.7225	0.5030	-0.1200	-0.1944
G7 Government Inflation-Linked Bond		0.1923	1.5214	-11.3113	9.3810	0.4695	-0.1075	-0.1784

Those short-dated assets' risk and returns are dominated by their currency performance over the sample period. From the perspective of the central bank as a US investor, it can be seen that holding non-US bills is reasonably comparable to holding foreign currency portfolios. Most developed market bond indexes exhibit negative correlation with the market index. As such, one may see the potential for diversification benefits through the reduction of risk, rather than return enhancement. The mean return and volatility are typically higher in longer dated indexes than in short-dated bucket. The means, standard deviations and correlation of assets' returns with to the market index in this table, however, do not provide a clear indication as to whether longer maturity bonds offer diversification benefits beyond those offered by benchmark portfolio.

The indices used to examine diversification benefits by investing in emerging market government bonds are categorised by country credit rating and geography. The average returns of emerging market government debt range between -0.03 percent (Asia) and 0.10 percent (Europe, Middle East and Africa). Emerging market government debt returns are relatively similar to those of developed market government bonds. However, the down-side risk of sovereign emerging market asset classes, as shown by the minimum return of -26.67 percent, was almost ten times higher than the developed market bonds index (-2.67 percent). In contrast to developed markets, correlations of sovereign emerging market returns to the government bond market index are all positive, which may offer return improvement to foreign reserve portfolios. The main reason as to why the difference in correlation between developed and emerging markets exists is that aside from global economic conditions, emerging markets have country specific situations influencing their growth potential. Additionally, given their developing nature, emerging economies usually enjoy higher growth rates than developed markets as they accelerate to catch up with advanced markets. For this reason, investing in emerging markets will not only diversify portfolio risk, but can also add a source of higher expected return.

The average returns of index-linked bonds are 0.33 percent and 0.19 percent, and the standard deviations are 1.70 percent and 1.52 percent. The means and standard deviations of index-linked government bonds returns in Table 4.4 provide a clearer early indication as to whether index-linked government bond markets offer diversification benefits for central banks' foreign reserves management beyond those offered by developed government bond markets. During the sample period, index-linked bonds generally have higher means, but lower standard deviations and correlations with developed and emerging market government bond market indices. This low correlation suggests that central banks may benefit from investing in index-linked bonds.

In the following sections, this research examines the diversification benefit in term of risk reduction. The researcher considers risk reduction in two sources; first the reductions of risk for a given benchmark return, and second the risk reduction to the global minimum-variance portfolio.

4.3.2. Risk reduction for a given existing benchmark returns

The sharp increase in central banks' foreign reserves has been an important factor influencing the growth in attention as to how central banks manage their foreign reserves. However, that is not the only factor. In general, the amplified focus on transparency and accountability has also raised pressure on central banks to better optimise the risk and return of their foreign reserves funds. From the perspective of a central banks' portfolio manager, the increase in reserves has provided extra flexibility. Thus, given that the focus has shifted more towards the risk and return, the pressure to keep the reserves all for liquidity purposes has become lesser. To capture these aspects, my first measure is the reduction in government bond portfolio standard deviation when central banks switch their investment from benchmark assets to the longer maturity, emerging market and inflation-linked government bonds for the same level of expected benchmark returns.

4.3.2.1. Longer bond maturity diversification

Liquidity preference theory (Bodie et al., 2008) says that longer bonds are subject to greater interest risk than short-term bonds. As a result, investors in long-term bond bonds require a risk premium to compensate them for the risk. This theory is also derived from the fact that shorter bonds are more liquid than the longer bonds. The preference to hold lower liquidity bond will only occur if those bonds offer higher expected returns. The recent ultra-low yield phenomena, however, raises doubts about the benefit of lengthening bond portfolio duration.

The motivation to examine longer dated bond strategy because investors have typically been rewarded in higher returns by investing in longer-dated bond (Berkelaar, Coche, and Nyholm, 2010). More specifically, Johnson-Calari et al. (2007) document a study about trends in reserves management by central banks. They point out there are opportunities to relax duration and credit risks constraints on the strategic asset allocation, which improves the risk-return performance through diversification. The first part of my analysis is to consider diversification benefits from investing in a longer-dated government bond. Subsequently, the assets considered a move down the credit curve to quasi-government for the short-sale constrained reserves portfolio; the aim of which is to minimise portfolio risk for a given expected benchmark returns.

Table 4.5 reports the posterior distribution of portfolio risk reduction in switching from the efficient frontier of the existing K government bond benchmarks to the efficient frontier portfolio and longer government bonds maturity, while the central bank maintains the current state of expected returns. Table 4.5 contains the mean, standard deviation, actual measure, 5th, 50th, and 95th percentile of the posterior distribution of the risk reduction. This table shows that when central bank faces short-sale restrictions and budget constraint equal to one, all the longer bond maturity strategies show positive signs of diversification

Table 4.5 Posterior distribution of risk reduction of the longer bond maturity strategy for the same return as benchmark portfolio

This table reports the posterior distribution of risk reduction benefits for a given expected benchmark returns from two different constraints and 11 diversification scenario by adding test assets to the benchmark portfolio. In Panel A and B, the actual efficiency gain is based on 10000 Monte Carlo simulation as in Li, et al. (2003) with and without asset allocation constraints for short-sales constrained investors. The first-3 frontier is spanned by lengthening portfolio duration within G7 government bond markets up to 10-years maturity. The next strategy is diversifying to a longer-dated selected developed market government bond (DM) 1 to 10 year maturity. The last-4 portfolio is spanned by SSA asset class on top of DM strategy. The summary statistics are the mean, standard deviation, the average actual risk reduction, and 5th, 50th, and 95th percentile.

Frontier of Benchmark +		Mean	Stdev	ō	Percentile		
					5th	50th	95th
Panel A. Short-sales Constraints							
G7	5-7 year	0.3221	0.2395	0.4596	0.0103	0.2763	0.7366
	7-10 year	0.3245	0.2398	0.4563	0.0103	0.2791	0.7377
	5-10 year	0.3289	0.2400	0.4504	0.0103	0.2840	0.7358
G7+DM	5-7 year	0.3317	0.2362	0.4466	0.0177	0.2875	0.7367
	7-10 year	0.3375	0.2379	0.4390	0.0155	0.2997	0.7375
	5-10 year	0.3419	0.2371	0.4331	0.0190	0.3038	0.7379
	1-10 year	0.3467	0.2361	0.4268	0.0242	0.3083	0.7393
G7+DM+SSA	5-7 year	0.3700	0.2362	0.3969	0.0276	0.3491	0.7407
	7-10 year	0.3693	0.2404	0.3978	0.0270	0.3412	0.7437
	5-10 year	0.3753	0.2368	0.3902	0.0306	0.3528	0.7434
	1-10 year	0.3801	0.2351	0.3843	0.0369	0.3595	0.7428
Panel B. Asset Allocation Constraints							
G7	5-7 year	0.0524	0.0759	0.8980	0.0000	0.0061	0.2294
	7-10 year	0.0407	0.0643	0.9202	0.0000	0.0000	0.1888
	5-10 year	0.0598	0.0806	0.8839	0.0000	0.0152	0.2328
G7+DM	5-7 year	0.0796	0.0881	0.8472	0.0000	0.0442	0.2443
	7-10 year	0.0653	0.0764	0.8736	0.0000	0.0291	0.2092
	5-10 year	0.0889	0.0893	0.8302	0.0000	0.0581	0.2448
	1-10 year	0.1093	0.0978	0.7933	0.0000	0.0819	0.2743
G7+DM+SSA	5-7 year	0.1053	0.0930	0.8005	0.0000	0.0831	0.2611
	7-10 year	0.1079	0.0945	0.7958	0.0000	0.0867	0.2617
	5-10 year	0.1142	0.0944	0.7847	0.0000	0.0986	0.2626
	1-10 year	0.1337	0.1012	0.7505	0.0000	0.1181	0.2905

benefits between 0.01 percent to 0.04 percent at the 5^{th} percentile. The average of the new optimal portfolio risk is less than quarter of the existing portfolio risk, but they failed to deliver greater than zero benefits at the 5^{th} percentile of the posterior distribution.

Strategic asset allocation constraints for longer bond diversification strategy are defined as; 30 percent of bills with a minimum of 15 percent in US treasury bills; at least 25 percent needs to be invested in the bond market index, and a maximum of 10 percent of the portfolios can be invested in SSA asset class. The table shows that additional asset allocation constraints change the location of the posterior distribution of the risk minimization benefits toward zero and provide smaller diversification benefits. Longer bond maturity strategies deliver zero diversification benefits at the 5^{th} percentile of the posterior distribution, and the average of the new optimal portfolio risk is about 70 to 90 percent from the existing portfolio volatility. These results show that all the longer bond and quasi-government diversification fail to reject the hypothesis after imposing asset allocation constraints.

Interesting findings are the sharp drop in mean and standard deviation of the benefits from before to after imposing asset allocation constraints. These sharp drops are due to asset allocations which limit the holding of the certain assets that could be seen too strict from a standard portfolio optimisation. This strict limit, however, is a vital risk tool to regulate the portfolio to ensure consistency with central banks' investment policies. Imposing stringent constraints, however, significantly makes the benefits of constrained asset allocation portfolio disappear.

Table 4.5 reports the posterior distribution of the average and standard deviations of the optimal asset weights for the same existing government bond benchmark target return, resulting from the longer dated diversification strategies considered. When there are only budget and no short-sales constraint taken into consideration, US T-Bills and G7 government bonds 1-5 year require weights amounting to almost 50 percent and 40 percent of the total portfolio respectively. Taking more restrictive asset allocation constraints into account, total weights for n assets is 7-8 percent and benchmark assets weights are more than 90 percent of the total portfolio. The weights of n assets is much smaller than was allowed and resulted in a portfolio that relatively identical to benchmark. This provides further evidence that longer bond maturity may not able to deliver significant risk reduction benefits.

One possible reason of why longer dated bond investment failed to offer diversification benefits for government bond portfolio is because during 1985-2013 bond markets have often experienced what are termed flight-to-quality and flight-to-liquidity. Bernanke and Gertler (1995) describe flight-to-quality phenomena as when market participants suddenly want to decrease their investment exposure to securities bearing credit risk and move to default-free assets. In recent years, however, a related but different phenomenon has been observed in the global financial markets: flight-to-liquidity.

Table 4.6 Portfolio weights in the efficient portfolio for the same return as benchmark portfolio of the longer bond maturity strategy.

This table reports the posterior distribution of the average and standard deviations of the weights in the efficient portfolio with the same target return resulted from 9 spanning tests considered. Test assets included in this scenario are emerging market government bond indices based on country rating (B, BB and BBB) and geographical area (Asia, Europe, Middle East and Africa (EMEA), and Latin America), G7 government maturity 5-7 and 7-10 years, DM bond index 1-5 year, 5-7 year and 7-10 year and SSA. The last three are the benchmark assets; US Treasury Bills, non-US bills and G7 government bond index 1-5 year. The portfolio weights are restricted to budget equal to one, constrained to non-negativity and asset allocation.

Bond Index	5-7 year		7-10 year		5-10 year	
	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel A. Group of G7 Bond						
Panel A.1. No Short-sales			Mean	Stdev		
G7 7-10 year			0.0473	0.0712	0.0312	0.0624
G7 5-7 year	0.0528	0.0797			0.0237	0.0584
UST Bills	0.4832	0.2793	0.4933	0.2783	0.4943	0.2759
Non US Bills	0.0656	0.1044	0.0673	0.1047	0.0623	0.1021
G7 1-5 year	0.3984	0.2079	0.3921	0.2092	0.3886	0.2076
Panel A.2. Asset Allocation						
G7 7-10 year			0.0682	0.0798	0.0231	0.0534
G7 5-7 year	0.0784	0.0797			0.0646	0.0785
UST Bills	0.2236	0.0719	0.2094	0.0690	0.2272	0.0715
Non US Bills	0.0764	0.0719	0.0906	0.0690	0.0728	0.0715
G7 1-5 year	0.6216	0.0797	0.6318	0.0798	0.6123	0.0840

Bond Index	5-7 year		7-10 year		5-10 year		1-10 year	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel B. Developed Market Government Bonds								
Panel B.1. No Short-sales								
DM 7-10 year			0.0229	0.0491	0.0116	0.0364	0.0075	0.0293
DM 5-7 year	0.0229	0.0491			0.0162	0.0420	0.0116	0.0356
DM 1-5 year							0.0145	0.0442
G7 7-10 year			0.0349	0.0631	0.0216	0.0518	0.0224	0.0529
G7 5-7 year	0.0349	0.0631			0.0134	0.0438	0.0136	0.0445
UST Bills	0.5181	0.2636	0.5181	0.2636	0.5187	0.2635	0.5271	0.2561
Non US Bills	0.0498	0.0940	0.0498	0.0940	0.0456	0.0910	0.0369	0.0836
G7 1-5 year	0.3743	0.2023	0.3743	0.2023	0.3729	0.2016	0.3665	0.1976

Bond Index	5-7 year		7-10 year		5-10 year		1-10 year	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel B. Developed Market Government Bonds								
Panel B.2. Asset Allocation								
DM 7-10 year			0.0306	0.0515	0.0107	0.0341	0.0047	0.0228
DM 5-7 year	0.0325	0.0532			0.0271	0.0491	0.0101	0.0327
DM 1-5 year							0.0512	0.0637
G7 7-10 year			0.0599	0.0742	0.0154	0.0445	0.0159	0.0450
G7 5-7 year	0.0696	0.0794			0.0607	0.0770	0.0489	0.0707
UST Bills	0.2396	0.0694	0.2234	0.0700	0.2459	0.0674	0.2674	0.0597
Non US Bills	0.0604	0.0694	0.0766	0.0700	0.0541	0.0674	0.0326	0.0597
G7 1-5 year	0.5979	0.0810	0.6094	0.0760	0.5862	0.0810	0.5692	0.0801
Panel C. Supranational/Semi-Government/Agency Bond Index								
Panel C.1. No Short-sales								
SSA	0.0742	0.1199	0.0727	0.1182	0.0690	0.1169	0.0689	0.1158
DM 7-10 year			0.0208	0.0454	0.0088	0.0312	0.0053	0.0238
DM 5-7 year	0.0205	0.0455			0.0146	0.0393	0.0110	0.0343
DM 1-5 year							0.0124	0.0400
G7 7-10 year			0.0193	0.0501	0.0125	0.0392	0.0130	0.0399
G7 5-7 year	0.0204	0.0517			0.0091	0.0352	0.0088	0.0343
UST Bills	0.5267	0.2607	0.5289	0.2634	0.5378	0.2577	0.5451	0.2515
Non US Bills	0.0426	0.0859	0.0422	0.0857	0.0385	0.0829	0.0311	0.0753
G7 1-5 year	0.3155	0.2124	0.3160	0.2145	0.3097	0.2099	0.3044	0.2052
Panel C.2. Asset Allocation								
SSA	0.0530	0.0477	0.0532	0.0478	0.0536	0.0476	0.0514	0.0475
DM 7-10 year			0.0322	0.0510	0.0090	0.0298	0.0045	0.0218
DM 5-7 year	0.0323	0.0512			0.0261	0.0473	0.0107	0.0333
DM 1-5 year							0.0536	0.0635
G7 7-10 year			0.0618	0.0714	0.0113	0.0368	0.0118	0.0374
G7 5-7 year	0.0607	0.0707			0.0549	0.0702	0.0397	0.0614
UST Bills	0.2437	0.0654	0.2425	0.0655	0.2455	0.0648	0.2699	0.0561
Non US Bills	0.0563	0.0654	0.0575	0.0655	0.0545	0.0648	0.0301	0.0561
G7 1-5 year	0.5540	0.0940	0.5527	0.0948	0.5451	0.0943	0.5284	0.0921

Longstaff (2004) and DeSantis (2014) describe that in a flight to liquidity, some market participants abruptly prefer to hold highly liquid assets such as US treasury bonds rather than less liquid securities. When financial market experienced flight-to-quality or flight-to-liquidity, it is likely that investors do not want to know anything about whether a particular asset is risky or not. They just want to disengage. Moreover, the reason investors hold a liquid asset is that because this asset enables investors to disengage as fast as possible. Given these arguments, one may expect that investors do not require higher premiums to hold higher risk instruments. Hence relaxing constraints to a longer bond duration and lower credit quality to quasi-government bond is not suitable for foreign reserves portfolio risk reduction.

4.3.2.2. Emerging market diversification

The next part of the analysis examines reserves portfolio risk minimization for the same or higher return as the benchmark; in this case when central banks switch their investment to the combination of benchmark and emerging government bonds when short-sale constraints are already in place. From the equity markets perspective, De Santis (1997) and Harvey (1995) demonstrate that the efficient frontier shifted when emerging market stocks are included. De Roon, Nijman, and Werker (2001) find that the diversification benefits disappear when short-sale restriction and transaction costs are imposed. It is possible that the result will be similar to emerging market government bonds, since some recent studies on emerging market credit spread have documented significant co-movement in spread changes. Cifarelli and Paladino (2006) document convincing evidence of emerging sovereign bonds co-movement spread; it changes more within geographical area than between geographical area. In this sub-section, however, this research extends the analysis to the impact of additional multi-asset weights when the non-negativity weights restrictions in any of the assets are already in place.

Table 4.7 reports the statistics of the distribution of risk reduction for a given expected returns for emerging market government bonds diversification. This table indicates only when emerging market government bonds included together with G7, developed market government bonds or SSA are able to offer risk reduction benefits. All the emerging market diversifications strategy delivers risk reduction benefits to the current benchmark portfolio.

However, when it is added together with the other asset classes, both regional and country rating strategy delivers the biggest mean of risk reduction of around 0.47 percent and greater than 0.07 percent at the fifth percentile. The smallest benefits could be attained through adding emerging market index directly to the current benchmark. This strategy could provide the average benefits of 0.40 percent and 0.07 percent at the 5th percentile of the posterior distribution.

Strategic asset allocations for the inclusion of emerging government bonds is similar to that of longer-dated bonds, and add 10 percent holding limit is imposed in emerging market. The results show that imposing asset allocation in addition to short-sale constraints reduces half of the benefits. Ehling and Ramos (2003) find that the choice of using the industrial or geographic approach of emerging market assets does have an impact on the equity portfolio performance. However, when it is applied to fixed income portfolios, my result show that the choice of credit rating and geographic approach of emerging government bond provides similar risks reduction benefits. Both spanning strategies provide on average 0.25% volatility reduction. At the 5th percentile of the posterior

Table 4.7 Posterior distribution of risk reduction for the same return as the benchmark of the emerging government bond strategy

This table reports the posterior distribution of risk reduction for a given expected benchmark return. Panel A for the budget and short-sale constraints on portfolio weight and Panel B for the asset allocation constraints. First diversification strategy is spanned the emerging market directly to the benchmark portfolio, the next are spanned with the inclusion of G7, developed government bond market and SSA bond index. The summary statistics are the mean, standard deviation, the average of actual risk reduction measure, 5^{th}. 50^{th} and 95^{th} percentile,

Frontier of Benchmark +	Mean	Stdev	$\bar{\delta}$	Percentile		
				5^{th}	50^{th}	95^{th}
No Short-Sale						
EM Geographics (EMG)	0.3927	0.2504	0.3688	0.0165	0.3994	0.7494
G7 + EMG	0.4183	0.2441	0.3384	0.0328	0.4264	0.7533
G7+DM+EMG	0.4398	0.2348	0.3138	0.0553	0.4559	0.7550
G7+DM+SSA+EMG	0.4579	0.2298	0.2939	0.0683	0.4814	0.7562
EM Country Rating (EMR)	0.3975	0.2497	0.3630	0.0188	0.4057	0.7510
G7 + EMR	0.4303	0.2393	0.3246	0.0378	0.4498	0.7529
G7+DM+EMR	0.4508	0.2337	0.3017	0.0622	0.4720	0.7565
G7+DM+SSA+EMR	0.4688	0.2279	0.2822	0.0756	0.4996	0.7576
Asset Allocation Constraint						
EM Geographics (EMG)	0.1388	0.1123	0.7417	0.0000	0.1376	0.3138
G7 + EMG	0.1812	0.1195	0.6704	0.0000	0.1966	0.3407
G7+DM+EMG	0.2307	0.1270	0.5918	0.0000	0.2502	0.3948
G7+DM+SSA+EMG	0.2462	0.1278	0.5683	0.0065	0.2712	0.4082
EM Country Rating (EMR)	0.1578	0.1221	0.7092	0.0000	0.1642	0.3364
G7 + EMR	0.2031	0.1262	0.6351	0.0000	0.2272	0.3652
G7+DM+EMR	0.2497	0.1343	0.5630	0.0003	0.2783	0.4184
G7+DM+SSA+EMR	0.2678	0.1327	0.5361	0.0097	0.3041	0.4268

distribution, it shows that this strategy might deliver greater than zero risk reduction benefits, and the average of the new optimal portfolio risk is only about 54 percent from the existing policy. More specifically, it is only when emerging market country rating is added together with longer G7, developed market government bonds and SSA has statistically significant diversification benefits greater than zero at 0.05 significance level.

Table 4.8 presents the posterior distribution of the average and standard deviations of the optimal weights in the efficient portfolio for a given expected returns resulted from the emerging market government bonds spanning strategies. Test assets included in this scenario are emerging market government bond indices based on country rating (B, BB and BBB) and regional classifications (Asia, EMEA, and Latam), G7 government maturity 5 to 7 and 7 to 10 years, developed market bond index 1 to 5 year, 5 to 7 year and 7 to 10 year and SSA bond index.

The average weight of emerging market assets for geographic and country rating approaches relatively similar at more than eight percent, slightly lower than the 10 percent limit for both non-negativity and asset allocation restrictions. Adding emerging markets to the investible government bonds portfolio strategies significantly increases the holding of n assets to 25 percent; more than half of the

Table 4.8 Portfolio weights in the efficient portfolio for the same return as benchmark portfolio of the emerging market strategy

This table reports the posterior distribution of the average and standard deviations of the weights in the efficient portfolio with the same target return resulted from 9 spanning tests considered. Test assets included in this scenario are emerging market government bond indices based on country rating (B, BB and BBB) and geographical area (Asia, Europe, Middle East and Africa (EMEA), and Latin America), G7 government maturity 5-7 and 7-10 years, DM bond index 1-5 year, 5-7 year and 7-10 year and SSA. The last three are the benchmark assets; US Treasury Bills, non-US bills and G7 government bond index 1-5 year. The portfolio weights are restricted to budget equal to one, constrained to non-negativity and strategic asset allocation.

Asset Class	EM Geographics (EMG)		G7 + EMG		G7+DM+EMG		G7+DM+ SSA+EMG		EM Country Rating (EMR)		G7 + EMR		G7+DM+EMR		G7+DM+ SSA+ EMR	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel A. No Short-sales																
EM Asia	0.0271	0.0557	0.0245	0.0520	0.0242	0.0508	0.0232	0.0486								
EM EMEA	0.0340	0.0674	0.0333	0.0646	0.0336	0.0640	0.0317	0.0602								
EM LATAM	0.0259	0.0576	0.0244	0.0545	0.0257	0.0547	0.0242	0.0519								
EM B									0.0259	0.0456	0.0271	0.0456	0.0271	0.0448	0.0251	0.0421
EM BB									0.0278	0.0578	0.0255	0.0527	0.0261	0.0521	0.0245	0.0492
EM BBB									0.0349	0.0726	0.0301	0.0650	0.0287	0.0621	0.0302	0.0633
SSA							0.0542	0.0971							0.0536	0.0956
DM 7-10 year					0.0073	0.0278	0.0051	0.0221					0.0071	0.0269	0.0049	0.0220
DM 5-7 year			0.0000	0.0000	0.0111	0.0333	0.0103	0.0315			0.0000	0.0000	0.0104	0.0324	0.0098	0.0304
DM 1-5 year			0.0000	0.0000	0.0208	0.0498	0.0165	0.0435					0.0206	0.0497	0.0171	0.0441
G7 7-10 year			0.0183	0.0469	0.0126	0.0388	0.0075	0.0294			0.0195	0.0487	0.0129	0.0391	0.0075	0.0289
G7 5-7 year			0.0217	0.0501	0.0134	0.0400	0.0087	0.0316			0.0217	0.0497	0.0141	0.0404	0.0088	0.0311
US Bills	0.4596	0.3062	0.5069	0.2894	0.5348	0.2689	0.5520	0.2599	0.4575	0.3098	0.5151	0.2870	0.5450	0.2676	0.5543	0.2664
Non US Bills	0.1077	0.1069	0.0667	0.0963	0.0353	0.0756	0.0295	0.0680	0.1095	0.1066	0.0655	0.0955	0.0342	0.0744	0.0315	0.0707
G7 1-5 year	0.3457	0.2317	0.3043	0.2177	0.2812	0.2017	0.2371	0.1969	0.3445	0.2326	0.2956	0.2141	0.2739	0.2005	0.2326	0.1993

Asset Class	EM Geographics (EMG)		G7 + EMG		G7+DM+EMG		G7+DM+ SSA+EMG		EM Country Rating (EMR)		G7 + EMR		G7+DM+EMR		G7+DM+ SSA+ EMR	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel B. Asset Allocation																
EM Asia	0.0243	0.0392	0.0217	0.0369	0.0259	0.0387	0.0255	0.0385								
EM EMEA	0.0244	0.0387	0.0296	0.0405	0.0248	0.0379	0.0248	0.0381								
EM LATAM	0.0219	0.0380	0.0219	0.0369	0.0251	0.0384	0.0262	0.0389								
EM B									0.0388	0.0430	0.0470	0.0434	0.0458	0.0426	0.0473	0.0422
EM BB									0.0201	0.0349	0.0187	0.0329	0.0203	0.0336	0.0205	0.0333
EM BBB									0.0132	0.0301	0.0108	0.0273	0.0108	0.0271	0.0111	0.0273
SSA							0.0507	0.0468							0.0492	0.0462
DM 7-10 year					0.0047	0.0224	0.0036	0.0194					0.0052	0.0244	0.0042	0.0210
DM 5-7 year					0.0086	0.0303	0.0086	0.0294					0.0083	0.0300	0.0082	0.0291
DM 1-5 year					0.0719	0.0724	0.0721	0.0696					0.0742	0.0736	0.0752	0.0713
G7 7-10 year			0.0128	0.0414	0.0099	0.0352	0.0070	0.0283			0.0124	0.0407	0.0101	0.0362	0.0072	0.0288
G7 5-7 year			0.0656	0.0728	0.0395	0.0614	0.0281	0.0513			0.0681	0.0741	0.0407	0.0619	0.0299	0.0524
US Bills	0.1180	0.0668	0.1894	0.0907	0.2652	0.0696	0.2677	0.0667	0.1090	0.0698	0.1836	0.0936	0.2629	0.0727	0.2652	0.0697
Non US Bills	0.1820	0.0527	0.1106	0.0869	0.0348	0.0567	0.0323	0.0577	0.1910	0.0597	0.1164	0.0879	0.0371	0.0637	0.0348	0.0587
G7 1-5 year	0.6294	0.0422	0.5483	0.0916	0.4897	0.0916	0.4534	0.1039	0.6279	0.0416	0.5431	0.0905	0.4846	0.0939	0.4472	0.1045

45 percent limit. More specifically, this strategy requires 8 percent of the total portfolio to be invested in emerging market government bonds, regardless the emerging market approach used. However, none of the average weights of the emerging market assets, both country credit rating and geographic diversifications are more than two standard errors.

These results demonstrate that the choice of credit rating and regional approach of emerging government bond provides similar risks reduction benefits. It shows both in the absence and in the presence of asset allocation constraints, no empirical evidence is found to support the argument that geographic diversification dominates country rating diversification. Our findings are different from Ehling and Ramos (2006), finding that the choice of index and constraints does matter. Ehling and Ramos (2006) study the performance of geographical and industrial diversification in the Eurozone equity markets. In an unconstraint case, they did not find empirical evidence to support the argument that one of those strategies provides better portfolio performance. With short-sell constraints, however, their analysis shows that the tangency portfolio of geographic diversification is not attainable by industry diversification. Applying in the government bond portfolio, this research shows different findings that the choice of geographic or country rating diversification is not essential for multi-weight constraints portfolio when non-negativity constraints are already considered.

4.3.2.3. Inflation-linked diversification

Inflation protected securities issued by the government might be the most desirable asset classes for central banks since they are the least risky of all assets because, theoretically, they are immune to both inflation and default risks. Mamun and Visaltanachoti (2006) conduct an empirical analysis to show that introducing inflation-indexed securities to a diversified portfolio of treasury bills, treasury bonds, equities, corporate bonds and real-estates asset classes provides investors with diversification benefits. Their findings are derived in different economic and inflationary conditions, and confirm the prediction that inflation-linked bonds are important for investors who are exposed to inflation risks. Cartea, Saul, and Toro (2012) solve an optimal portfolio choice problem to measure the benefits of Treasury Inflation Indexed Securities (TIPS) to investors who are concerned about inflation. By comparing optimal portfolio with and without TIPS in the presence of other asset classes such as equities, commodities, and real estate, they show that the introduction of a real riskless asset completes the investor asset space. Their analysis distinguishes between buy-and-hold long-term investors for whom TIPS fully displace nominal risk-free assets and short-term investors for whom TIPS improve the investment opportunity set of real returns. Finally, this research shows how gains from inflation-linked treasuries are alleviated by the availability of alternative assets that co-vary with inflation, such as gold and real estate.

However, the more recent study of Huang and Zhong (2013) using data from 1970 to 2010 finds that the commodity, real estate and inflation-protected securities asset classes are not substitutes for each other, and that diversification benefits of each asset class change substantially over time. Therefore, all those three asset classes should be included in investors' portfolios. Another empirical analysis by Hunter and Simon (2005) argue that US indexed bond does not deliver diversification benefits and does not enhance the mean-variance efficiency of a diversified portfolio. One possible explanation was given by Bri'ere and Signori (2009) when they study the dynamics of conditional volatilities and correlations for inflation-protected bonds, nominal bonds and equity asset classes for the period 1997–2007. Bri'ere and Signori (2009) argue that, although inflation-linked bonds once had positive diversification power, they are now highly correlated with nominal bonds and have reached similar volatility levels.

As a result, the two asset classes are practically substitutable. This seems to be due to more stable inflation expectations and a more liquid inflation-linked bond market. Although diversification was a valuable reason for introducing inflation-linked bonds in a global portfolio before 2003, Bri'ere and Signori (2009) argue this is no longer the case. This contradiction between theory and empirical evidence calls for attention to examine the benefits of inflation-indexed government bond for government bonds portfolio. This last asset class selection analysis is to examine the reduction of portfolio risks for a given current benchmark target returns from inflation-linked government bonds diversification.

For a central bank which faces only short-sale constraint in their investment policy, there are eight out of 12 spanning strategies (global government index-linked (GIL), GIL+G7 5-7 year and G7 government inflation linked+G7 5-7 year) able to reject H_0 as indicated by its benefit greater than zero at the 5th percentile of the posterior distribution. The average of risk reduction of the no short-sale and budget constraints optimization is just below 0.50 percent or only about 26 to 36 percent from the existing portfolio risk. The minimum of the reduction of risk is 0.02 percent at the fifth percentile posterior distribution for adding global government inflation linked together with group of G7 government bonds market to the benchmark government bond portfolio. Both global government and G7 government inflation-linked bond offer the similar risk reduction benefit for no short-sales constrained government bond portfolios.

The asset allocation constraint for index-linked government bond strategy is defined similarly to that of emerging market strategy and limit to 15 percent holding on inflation-linked bond. Imposing asset allocations in addition to short-sale constraint significantly reduce the power of the diversification benefits. The reductions of the benefit change their significance toward zero and only two out of 12 strategies provide benefits greater than zero at 5 percent confidence level. The average of risks reduction magnitude of this strategy is 0.30 to 0.31 percent and the benefits could be greater than 0.01 percent at the 5th percentile of the posterior distribution.

Table 4.9 Posterior distribution of risk reduction for the same benchmark returns of the index-linked government bond strategy.

This table reports the posterior distribution of risk reduction ∅ (percent) for a given target returns resulted from the two different constraints and six index-linked government bond diversification strategies. This study compares two different index-linked bonds; Global and G7 Government index linked bonds. The first strategy is spanned directly from the benchmark portfolio, the next are spanned with the inclusion of the longer dated G7, developed government bond market, SSA and both country rating and geographic emerging market approaches. The summary statistics are the mean, standard deviation, the average of actual risk reduction measure, and the 5th, 50th, and 95th percentile.

Frontier of Benchmark +	Mean	Stdev	ō	Percentile		
				5th	50th	95th
Panel A. Short-Sale Constraints						
GGIL	0.2217	0.2861	0.6057	0.0021	0.0562	0.8475
GGIL+G7	0.3324	0.3093	0.4457	0.0037	0.2378	0.8646
GGIL+G7+DM	0.3722	0.3115	0.3941	0.0053	0.3156	0.8721
GGIL+G7+DM+SSA	0.3953	0.3079	0.3656	0.0062	0.3684	0.8728
GGIL+G7+DM+SSA+EMG	0.4524	0.3011	0.2998	0.0100	0.4678	0.8762
GGIL+G7+DM+SSA+EMR	0.4637	0.2992	0.2876	0.0117	0.4875	0.8781
G7IL	0.2217	0.2851	0.6057	0.0021	0.0557	0.8408
G7IL+G7	0.3422	0.3079	0.4327	0.0041	0.2668	0.8647
G7IL+G7+DM	0.3825	0.3098	0.3813	0.0056	0.3394	0.8728
G7IL+G7+DM+SSA	0.4097	0.3090	0.3485	0.0070	0.3942	0.8729
G7IL+G7+DM+SSA+EMG	0.4472	0.3003	0.3056	0.0106	0.4582	0.8763
G7IL+G7+DM+SSA+EMR	0.4580	0.3004	0.2937	0.0119	0.4811	0.8774
Panel B. Asset Allocation Constraints						
GGIL	0.0102	0.0251	0.9797	0.0000	0.0000	0.0785
GGIL+G7	0.1105	0.1316	0.7912	0.0000	0.0262	0.3471
GGIL+G7+DM	0.1975	0.2048	0.6441	0.0000	0.1095	0.5262
GGIL+G7+DM+SSA	0.2103	0.2067	0.6237	0.0000	0.1428	0.5279
GGIL+G7+DM+SSA+EMG	0.2596	0.2145	0.5481	0.0000	0.2694	0.5444
GGIL+G7+DM+SSA+EMR	0.2683	0.2160	0.5354	0.0000	0.2909	0.5487
G7IL	0.0070	0.0172	0.9860	0.0000	0.0000	0.0387
G7IL+G7	0.1210	0.1392	0.7726	0.0000	0.0322	0.3624
G7IL+G7+DM	0.2063	0.2084	0.6300	0.0000	0.1300	0.5304
G7IL+G7+DM+SSA	0.2160	0.2095	0.6147	0.0000	0.1524	0.5296
G7IL+G7+DM+SSA+EMG	0.2970	0.1525	0.4941	0.0051	0.3336	0.4812
G7IL+G7+DM+SSA+EMR	0.3109	0.1530	0.4749	0.0125	0.3539	0.4900

Both inflation-linked indexes are unable to provide benefits if they were spanned directly to the current benchmark policy. Different from to the choice of emerging market index, this study finds the difference between the use of global government and Group G7 inflation-linked index. The G7 index-linked government bonds provide diversification benefits only when it added together with group of G7, developed market, SSA and emerging market (either regional or country rating) government bonds index, but not when global government inflation linked index is used.

Table 4.10 reports the posterior distribution of the mean and standard deviations of asset weights in the efficient portfolio for a given target return resulted from the inclusion of inflation-linked government bonds to the current benchmark. Test assets included in this scenario are G7 government inflation-linked bond, G7 government maturity 5 to 7 and 7 to 10 years, developed market government bond index 1 to 5 year, 5 to 7 year and 7 to 10 year, SSA, and three emerging market government bond indices based on geographic and country rating classifications.

When short-sale constrained central bank pursues risk minimization portfolio while maintaining the existing expected returns, it requires 3 percent global bond index and 4 percent G7 index-linked when it combined with emerging market geographical approaches, and requires three percent when it is invested together with emerging market country rating bond markets.

Table 4.10 Portfolio weights in the efficient portfolio for the same return as benchmark portfolio of the index-linked government bond strategies

This table reports the posterior distribution of the average and standard deviations of the weights of optimal portfolio resulted in the efficient portfolio with the same target return as the existing benchmark of the 10 test assets considered. Test assets included in this scenario are G7 government inflation-linked bond, G7 government maturity 5-7 and 7-10 years, DM bond index 1-5 year, 5-7 year and 7-10 year and also SSA, emerging market government bond indices based on geographic and country rating. The last three are the benchmark assets; US Treasury Bills, non-US bills and G7 government bond index 1-5 year. The portfolio weights are constrained for budget constraint equal to one and to non-negativity, and constrained to asset allocation.

Spanning Strategy	GGIL		GGIL+G7		GGIL+G7+DM		GGIL+G7+ DM+SSA		GGIL+G7+DM+ SSA+EMG		GGIL+G7+DM+ SSA+EMR	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel A. Global Inflation Linked												
Short-Sale Constraints												
Global Inflation-linked	0.1144	0.1391	0.0884	0.1222	0.0811	0.1153	0.0742	0.1105	0.0404	0.0828	0.0384	0.0797
EM Asia									0.0202	0.0556		
EM EMEA									0.0264	0.0709		
EM LATAM									0.0226	0.0620		
EM B rating											0.0277	0.0546
EM BB rating											0.0203	0.0572
EM BBB rating											0.0223	0.0675
SSA							0.0697	0.1519	0.0555	0.1325	0.0520	0.1270
DM 7-10 year					0.0976	0.1944	0.0896	0.1853	0.0895	0.1780	0.0902	0.1782
DM 5-7 year					0.0234	0.0881	0.0198	0.0794	0.0180	0.0745	0.0175	0.0727
DM 1-5 year					0.0297	0.0945	0.0260	0.0874	0.0223	0.0763	0.0217	0.0754
G7 7-10 year			0.0540	0.1199	0.0329	0.0949	0.0242	0.0790	0.0215	0.0752	0.0205	0.0715
G7 5-7 year			0.0828	0.1660	0.0406	0.1196	0.0304	0.1029	0.0299	0.0978	0.0287	0.0945
US Bills	0.1727	0.2212	0.1697	0.2252	0.1542	0.2181	0.1498	0.2160	0.1433	0.2183	0.1431	0.2199
G7 Non US Bills	0.2793	0.2895	0.3005	0.3167	0.2661	0.3167	0.2726	0.3200	0.3060	0.3333	0.3189	0.3400
G7 1-5 year	0.4336	0.2893	0.3045	0.2899	0.2743	0.2810	0.2439	0.2729	0.2043	0.2503	0.1988	0.2455

Spanning Strategy	GGIL		GGIL+G7		GGIL+G7+DM		GGIL+G7+ DM+SSA		GGIL+G7+DM+ SSA+EMG		GGIL+G7+DM+ SSA+EMR	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Asset Allocation Constraints												
Global Inflation-linked	0.0126	0.0371	0.0511	0.0649	0.0409	0.0640	0.0409	0.0606	0.0291	0.0536	0.0255	0.0506
EM Asia									0.0139	0.0317		
EM EMEA									0.0135	0.0303		
EM LATAM									0.0124	0.0303		
EM B rating											0.0241	0.0372
EM BB rating											0.0107	0.0278
EM BBB rating											0.0066	0.0224
SSA					0.0183	0.0467	0.0183	0.0362	0.0142	0.0327	0.0136	0.0320
DM 7-10 year					0.1076	0.0747	0.1076	0.1476	0.1282	0.1541	0.1351	0.1583
DM 5-7 year					0.0087	0.0292	0.0087	0.0453	0.0085	0.0434	0.0078	0.0410
DM 1-5 year					0.0138	0.0207	0.0138	0.0530	0.0128	0.0495	0.0128	0.0498
G7 7-10 year			0.0176	0.0595	0.0157	0.0277	0.0157	0.0561	0.0127	0.0500	0.0129	0.0508
G7 5-7 year			0.0893	0.1380	0.0161	0.0514	0.0161	0.0606	0.0134	0.0540	0.0143	0.0558
US Bills	0.2120	0.0733	0.2221	0.0745	0.2228	0.0701	0.2228	0.0745	0.2158	0.0738	0.2154	0.0739
G7 Non US Bills	0.0880	0.0072	0.0779	0.0067	0.0772	0.0701	0.0772	0.0087	0.0842	0.0127	0.0846	0.0467
G7 1-5 year	0.6874	0.0371	0.5420	0.1816	0.4789	0.1214	0.4789	0.2006	0.4413	0.1989	0.4366	0.1977

Spanning Strategy	GGIL		GGIL+G7		GGIL+G7+DM		GGIL+G7+ DM+SSA		GGIL+G7+DM+ SSA+EMG		GGIL+G7+DM+ SSA+EMR	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel B. Group G7 Inflation-Linked Bond												
Short-Sale Constraints												
G7 Inflation-linked	0.1241	0.1518	0.1024	0.1344	0.0946	0.1286	0.0830	0.1208	0.0470	0.0932	0.0445	0.0904
EM Asia									0.0206	0.0567		
EM EMEA									0.0227	0.0664		
EM LATAM									0.0215	0.0609		
EM B rating											0.0270	0.0534
EM BB rating											0.0186	0.0550
EM BBB rating											0.0208	0.0657
SSA							0.0674	0.1486	0.0598	0.1401	0.0524	0.1271
DM 7-10 year					0.0962	0.1911	0.0898	0.1817	0.0863	0.1729	0.0887	0.1732
DM 5-7 year					0.0248	0.0908	0.0224	0.0842	0.0195	0.0775	0.0187	0.0757
DM 1-5 year					0.0317	0.0955	0.0275	0.0874	0.0233	0.0789	0.0225	0.0773
G7 7-10 year			0.0600	0.1267	0.0354	0.0983	0.0279	0.0867	0.0220	0.0741	0.0222	0.0752
G7 5-7 year			0.0853	0.1687	0.0425	0.1210	0.0302	0.1008	0.0288	0.0958	0.0289	0.0957
US Bills	0.1707	0.2174	0.1669	0.2216	0.1518	0.2156	0.1471	0.2121	0.1462	0.2164	0.1427	0.2188
G7 Non US Bills	0.2669	0.2802	0.2923	0.3145	0.2590	0.3140	0.2714	0.3188	0.2946	0.3306	0.3089	0.3361
G7 1-5 year	0.4383	0.2876	0.2931	0.2868	0.2641	0.2776	0.2333	0.2697	0.2078	0.2516	0.2041	0.2483

Spanning Strategy	GGIL		GGIL+G7		GGIL+G7+DM		GGIL+G7+ DM+SSA		GGIL+G7+DM+ SSA+EMG		GGIL+G7+DM+ SSA+EMR	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Asset Allocation Constraints												
G7 Inflation-linked	0.0113	0.0264	0.0533	0.0653	0.0436	0.0618	0.0421	0.0610	0.0866	0.0643	0.0859	0.0637
EM Asia									0.0188	0.0344		
EM EMEA									0.0274	0.0386		
EM LATAM									0.0209	0.0355		
EM B rating											0.0376	0.0385
EM BB rating											0.0179	0.0316
EM BBB rating											0.0136	0.0284
SSA							0.0173	0.0352	0.0393	0.0432	0.0384	0.0426
DM 7-10 year					0.1053	0.1461	0.1070	0.1466	0.0879	0.0750	0.0926	0.0762
DM 5-7 year					0.0095	0.0466	0.0079	0.0427	0.0080	0.0290	0.0072	0.0271
DM 1-5 year					0.0160	0.0577	0.0146	0.0541	0.0047	0.0221	0.0046	0.0219
G7 7-10 year			0.0191	0.0624	0.0198	0.0643	0.0163	0.0575	0.0063	0.0269	0.0064	0.0274
G7 5-7 year			0.0876	0.1365	0.0169	0.0626	0.0147	0.0579	0.0168	0.0412	0.0177	0.0429
US Bills	0.2141	0.0735	0.2221	0.0745	0.2228	0.0745	0.2239	0.0746	0.2719	0.0522	0.2730	0.0516
G7 Non US Bills	0.0859	0.0567	0.0779	0.0207	0.0772	0.0015	0.0761	0.0072	0.0281	0.0522	0.0270	0.0516
G7 1-5 year	0.6887	0.0264	0.5400	0.1815	0.4889	0.2004	0.4801	0.2008	0.3831	0.1301	0.3782	0.1278

If asset allocation constraints are imposed, adding G7 inflation-linked government bonds lowers index-linked assets holding at nine percent when it combined with emerging market regional asset class, and two percent when it combined with emerging market country rating bond markets. Different from longer bond maturity and emerging market strategy, the average weights of index-linked assets are higher than two standard errors from zero, consistent with the results of Britten-Jones (1999) and Li et al. (2003). Our results show that the posterior distribution of index-linked assets weights is less noisy when asset allocation constraints imposed for the efficient portfolio for a given expected returns.

Different to the emerging market diversification findings that the choice of country rating or geographic diversification is not important, this research finds evidence that the choice of global inflation index leads to different diversification benefits on government bond risk reduction portfolio. G7 government inflation index delivered significant risk reduction benefits when it added together with EM and other longer government bonds.

4.3.3. Risk reduction to the minimum variance portfolio

In this section, this research examines diversification benefit in term of risk reduction to the global minimum-variance portfolio, resulting from bond maturity extension, emerging market and inflation-linked government bonds spanning strategies.

4.3.3.1. Longer bond maturity diversification

Table 4.11 reports the posterior distribution of risk minimisation to the global minimum variance that resulted from longer dated government bonds portfolios when short-sale and budget constraint equal to one are already in place. This table indicates that relaxing all bond maturity and credit constraints strategies provide significant diversification benefits in a relatively similar magnitude of risk reduction. The average of risk reduction benefits that resulted from the new optimal portfolio is about 0.73 percent or only six percent from the original portfolio risk.

As expected, adding more restrictive weight constraints to the short-sale constrained optimisation shift the location of the posterior distribution of risk reduction toward zero and reduces the risk reduction to the average of 0.16 to 0.22 percent, much lower compared to that of no short-sale constraints. Variability of the reduction of risk when asset allocation constraints are imposed also doubled from 0.01 percent to 0.02 percent. The 5th percentile of the posterior distribution shows that the new portfolio will deliver risk reduction by a magnitude ranging from 0.15 percent to 0.24 percent; or only 20 percent of the original portfolio risk. The biggest diversification benefits of longer dated bonds amongst eleven strategies is realised when bond maturity 5-7 year bucket issued by G7, developed market government and SSA bond markets; consistent with that of no short-sales and budget constraint optimisation.

Table 4.11 Posterior distribution of risk reduction of the longer bond maturity strategies to the GMV Portfolios

This table reports the posterior distribution of risk reduction to the GMV portfolio from 11 longer bond diversifications scenario for short-sale constrained reserves portfolios. The first-3 frontier is diversifying by longer bond within G7 government bond market up to 10 years maturity. The next strategy is diversifying to longer-dated selected developed market government bond (DM) one to 10-year maturity. The last-4 portfolio is diversifying to SSA asset class on top of DM strategy. The summary statistics are the mean, standard deviation, the average actual risk reduction measure and the 5^{th}, 50^{th} and 95^{th} percentile.

Frontier of Benchmark +		Mean	Stdev	ō	Percentile		
					5^{th}	50^{th}	95^{th}
Panel A. Short-sales Constraints							
G7	5-7 year	0.7495	0.0130	0.0627	0.7275	0.7499	0.7702
	7-10 year	0.7494	0.0131	0.0628	0.7275	0.7497	0.7702
	5-10 year	0.7497	0.0129	0.0627	0.7280	0.7500	0.7703
G7+DM	5-7 year	0.7496	0.0133	0.0627	0.7271	0.7499	0.7711
	7-10 year	0.7493	0.0132	0.0629	0.7270	0.7496	0.7704
	5-10 year	0.7497	0.0129	0.0626	0.7279	0.7500	0.7704
	1-10 year	0.7497	0.0131	0.0626	0.7275	0.7501	0.7705
G7+DM+SSA	5-7 year	0.7507	0.0129	0.0621	0.7289	0.7512	0.7711
	7-10 year	0.7505	0.0131	0.0622	0.7287	0.7506	0.7716
	5-10 year	0.7508	0.0130	0.0621	0.7287	0.7511	0.7716
	1-10 year	0.7505	0.0127	0.0623	0.7292	0.7508	0.7709
Panel B. Asset Allocation Constraints							
G7	5-7 year	0.2390	0.0269	0.5791	0.1956	0.2389	0.2832
	7-10 year	0.1808	0.0181	0.6710	0.1504	0.1808	0.2102
	5-10 year	0.2391	0.0265	0.5789	0.1953	0.2394	0.2829
G7+DM	5-7 year	0.2388	0.0262	0.5795	0.1966	0.2383	0.2826
	7-10 year	0.2026	0.0260	0.6358	0.1602	0.2023	0.2460
	5-10 year	0.2387	0.0263	0.5797	0.1957	0.2381	0.2832
	1-10 year	0.2680	0.0256	0.5358	0.2259	0.2681	0.3107
G7+DM+SSA	5-7 year	0.2521	0.0262	0.5594	0.2097	0.2520	0.2958
	7-10 year	0.2524	0.0259	0.5588	0.2093	0.2524	0.2948
	5-10 year	0.2523	0.0260	0.5591	0.2091	0.2523	0.2956
	1-10 year	0.2824	0.0256	0.5150	0.2406	0.2824	0.3239

It is worth to note that for central banks that have an objective to minimise portfolio risk to the minimum variance portfolio, switching from the benchmark portfolio to a longer bonds government maturity and incorporating quasi-government bonds to their investible assets, is able to offer diversification benefits before and after imposing asset allocation constraints. The smallest risk reduction benefit of longer dated bond strategy is when central bank adds the only group of G7 government bond maturity 7 to 10 year bucket into reserves portfolio with an average of risk reduction 0.18 percent and a minimum of 0.15 percent at the 5th percentile of the posterior distribution. The risks reduction will benefit most for central bank when they included G7 government 5 to 7 year, developed market 5 to 7 year and SSA bond index altogether in their government bond portfolio. This strategy offers risk reduction of average 0.28 percent or about 50 percent of the original risk level.

Table 4.12 reports the means and standard deviations of the asset weights in the global minimum variance portfolios. Results for short-sale constrained central bank shows that the weights of almost all longer dated bonds are zero, only G7 5 to7 year bond index has positive weight at one percent. This indicates that the driver of the results here is the fact that central banks are now allowed to invest more in US T-Bills. The standard deviation of the positive weight assets shows that the posterior relatively stable at less than two percent and much lower than the mean.

When asset allocation constraints are taken into consideration, developed market 1 to 5 years, G7 5 to 7 years and SSA show significant positive weights at 16 percent, 4 percent and 9 percent respectively. Benchmark asset weights are at around 70 percent, or 15 percent higher than the obligatory holding. Optimal

Table 4.12 Portfolio weights in the GMV portfolio of the longer bond maturity strategy

This table reports the posterior distribution of the average and standard deviations of the assets weights in the GMV portfolio resulted from longer bond portfolio diversification. Test assets included in this scenario are G7 government maturity 5-7 and 7-10 years, DM bond index 1-5 year, 5-7 year and 7-10 year and SSA. The last three are the benchmark assets; US Treasury Bills, non-US bills and G7 government bond index 1-5 year. The portfolio weights are constrained for budget constraint equal to one, constrained to non-negativity, and constrained to asset allocation.

Bond Index	5-7 year		7-10 year		5-10 year	
	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel A. Group of G7 Bond						
Panel A.1. No Short-sales						
G7 7-10 year			0.0165	0.0054	0.0027	0.0062
G7 5-7 year	0.0191	0.0056			0.0160	0.0082
UST Bills	0.0002	0.0012	0.0006	0.0027	0.0001	0.0012
Non US Bills	0.9597	0.0124	0.9643	0.0117	0.9605	0.0123
G7 1-5 year	0.0210	0.0085	0.0186	0.0082	0.0206	0.0085
Panel A.2. Asset Allocation						
G7 7-10 year			0.1000	0.0000	0.0000	0.0000
G7 5-7 year	0.3894	0.0262			0.2385	0.0274
UST Bills	0.1958	0.1272	0.1500	0.0000	0.2676	0.0354
Non US Bills	0.1042	0.1071	0.1500	0.0000	0.0324	0.0354
G7 1-5 year	0.3106	0.0262	0.6000	0.0000	0.4615	0.0274

Bond Index	5-7 year		7-10 year		5-10 year		1-10 year	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel B. Developed Market Government Bonds								
Panel B.1. No Short-sales								
DM 7-10 year			0.0013	0.0031	0.0002	0.0014	0.0002	0.0014
DM 5-7 year	0.0012	0.0032			0.0011	0.0030	0.0010	0.0030
DM 1-5 year							0.0002	0.0013
G7 7-10 year			0.0153	0.0063	0.0026	0.0060	0.0028	0.0061
G7 5-7 year	0.0176	0.0068			0.0148	0.0086	0.0144	0.0087
UST Bills	0.0001	0.0009	0.0004	0.0021	0.0001	0.0009	0.0001	0.0008
Non US Bills	0.9598	0.0125	0.9642	0.0117	0.9601	0.0123	0.9603	0.0123
G7 1-5 year	0.0212	0.0085	0.0188	0.0083	0.0210	0.0085	0.0210	0.0085

Bond Index	5-7 year		7-10 year		5-10 year		1-10 year	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel B. Developed Market Government Bonds								
Panel B.2. Asset Allocation								
DM 7-10 year			0.0036	0.0104	0.1567	0.0296	0.0991	0.0065
DM 5-7 year	0.0080	0.0152			0.0000	0.0000	0.0000	0.0000
DM 1-5 year							0.0000	0.0000
G7 7-10 year			0.2346	0.0295	0.0000	0.0000	0.0075	0.0152
G7 5-7 year	0.1361	0.0219			0.0841	0.0348	0.1789	0.0336
UST Bills	0.1533	0.0112	0.2684	0.0355	0.3000	0.0000	0.2544	0.0400
Non US Bills	0.1467	0.0112	0.0316	0.0355	0.0000	0.0000	0.0456	0.0400
G7 1-5 year	0.5559	0.0131	0.4618	0.0274	0.4592	0.0103	0.4144	0.0304
Panel C. Supranational/Semi-Government/Agency Bond Index								
Panel C.1. No Short-sales								
SSA	0.0204	0.0129	0.0201	0.0130	0.0199	0.0128	0.0198	0.0128
DM 7-10 year			0.0018	0.0035	0.0001	0.0009	0.0001	0.0009
DM 5-7 year	0.0017	0.0035			0.0016	0.0033	0.0015	0.0033
DM 1-5 year							0.0002	0.0013
G7 7-10 year			0.0101	0.0074	0.0013	0.0038	0.0013	0.0038
G7 5-7 year	0.0100	0.0073			0.0089	0.0076	0.0088	0.0075
UST Bills	0.0002	0.0013	0.0002	0.0013	0.0002	0.0014	0.0001	0.0011
Non US Bills	0.9559	0.0120	0.9558	0.0120	0.9560	0.0122	0.9561	0.0121
G7 1-5 year	0.0118	0.0093	0.0121	0.0095	0.0120	0.0094	0.0121	0.0094
Panel C.2. Asset Allocation								
SSA	0.0530	0.0477	0.0991	0.0065	0.0991	0.0062	0.0994	0.0041
DM 7-10 year			0.0075	0.0152	0.0000	0.0000	0.0000	0.0000
DM 5-7 year	0.0323	0.0512			0.0078	0.0156	0.0000	0.0000
DM 1-5 year							0.1595	0.0259
G7 7-10 year			0.1789	0.0336	0.0000	0.0000	0.0000	0.0000
G7 5-7 year	0.0607	0.0707			0.1790	0.0340	0.0401	0.0304
UST Bills	0.2437	0.0654	0.2544	0.0400	0.2546	0.0397	0.3000	0.0000
Non US Bills	0.0563	0.0654	0.0456	0.0400	0.0454	0.0397	0.0000	0.0000
G7 1-5 year	0.5540	0.0940	0.4144	0.0304	0.4141	0.0304	0.4010	0.0102

weights after imposing asset allocation constraints for longer dated bond strategy might also deliver longer portfolio duration since 30 percent of the total portfolios have positive weights on the test assets. The impact on portfolio duration is expected to be moderate since they long position on the moderate and mimicking bond market bucket maturity.

4.3.3.2. Emerging market diversification

My next analysis is to examine the reduction of foreign reserves portfolio uncertainty when central banks sell some of their benchmark government bonds holding to fund their emerging government bonds purchase when short-sales is not allowed, and budget constraint equal to one have already in place.

Table 4.13 reports that all the emerging market diversifications strategy delivers significant risk reduction, the average of benefits are around 0.75 percent or less than five percent of the original portfolio risks. The reduction in portfolio risk does not have a big variation as indicated by the standard deviation of 0.01 and relatively small difference between 5th and 95th percentile of the posterior distribution of risk reduction. Both emerging government market classification; regional and country rating provide similar risk reduction benefit for government bond portfolios.

Asset allocation drastically reduces the diversification benefit to 0.11-0.41 percent. Variability of the reduction of risk when asset allocation constraints are added doubled to 0.02 percent from 0.01 percent. The minimum reduction in

Table 4.13 Posterior distribution of risk reduction of the emerging government bond strategy to GMV Portfolios

This table reports the posterior distribution of risk reduction $\emptyset_{GMV}$ (percent) to GMV portfolio resulted from emerging market diversification using geographic and country rating index. First scenario is spanned directly from the benchmark portfolio, the next are spanned with the inclusion of G7, developed government bond market and SSA bond index. The summary statistics are the mean, standard deviation, the average of actual risk reduction measure, and 5^{th}, 50^{th} and 95^{th} percentile.

Frontier of Benchmark +	Mean	Stdev	$\bar{\delta}$	Percentile		
				5^{th}	50^{th}	95^{th}
Panel A. Short-Sale Constraints						
EM Regional (EMG)	0.7543	0.0128	0.0604	0.7328	0.7545	0.7749
G7 + EMG	0.7562	0.0128	0.0594	0.7350	0.7565	0.7765
G7+DM+EMG	0.7571	0.0127	0.0590	0.7356	0.7574	0.7772
G7+DM+SSA+EMG	0.7579	0.0125	0.0586	0.7369	0.7581	0.7778
EM Country Rating (EMR)	0.7546	0.0129	0.0602	0.7331	0.7551	0.7757
G7 + EMR	0.7563	0.0128	0.0594	0.7348	0.7567	0.7767
G7+DM+EMR	0.7571	0.0126	0.0590	0.7359	0.7574	0.7775
G7+DM+SSA+EMR	0.7579	0.0128	0.0586	0.7363	0.7582	0.7787
Panel B. Asset Allocation Constraints						
EM Regional (EMG)	0.1147	0.0071	0.7838	0.1030	0.1148	0.1263
G7 + EMG	0.3289	0.0233	0.4504	0.2909	0.3287	0.3672
G7+DM+EMG	0.3852	0.0227	0.3780	0.3483	0.3851	0.4225
G7+DM+SSA+EMG	0.3969	0.0228	0.3637	0.3592	0.3969	0.4343
EM Country Rating (EMR)	0.3358	0.0218	0.4412	0.2993	0.3363	0.3712
G7 + EMR	0.3539	0.0241	0.4175	0.3135	0.3540	0.3932
G7+DM+EMR	0.4082	0.0234	0.3503	0.3699	0.4083	0.4465
G7+DM+SSA+EMR	0.4171	0.0233	0.3398	0.3779	0.4174	0.4550

portfolio volatility is 0.10 percent at the fifth percentile posterior distribution when emerging market geographic is added directly to the existing benchmark. This new optimal portfolio provides less than 50 percent of the existing portfolio volatility.

Table 4.14 Portfolio weight in the GMV portfolio of the emerging market strategy

This table reports the posterior distribution of the means and standard deviations of the optimal weights in the GMV portfolio. Test assets included in this scenario are emerging market government bond indices based on country rating (B, BB and BBB) in Panel B and geographical area (Asia, Europe, Middle East and Africa (EMEA), and Latin America) in Panel A, G7 government maturity 5-7 and 7-10 years, DM bond index 1-5 year, 5-7 year and 7-10 year and SSA. The last three are the benchmark assets; US Treasury Bills, non-US bills and G7 government bond index 1-5 year. The portfolio weights are constrained for budget constraint equal to one, constrained to non-negativity, and asset allocation constraints.

Asset Class	EM Geographics (EMG)		G7 + EMG		G7+DM+EMG		G7+DM+ SSA+EMG		EM Country Rating (EMR)		G7 + EMR		G7+DM+EMR		G7+DM+ SSA+ EMR	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel A. No Short-sales																
EM Asia	0.0065	0.0081	0.0049	0.0072	0.0058	0.0078	0.0061	0.0080								
EM EMEA	0.0259	0.0118	0.0259	0.0111	0.0263	0.0116	0.0252	0.0120								
EM LATAM	0.0013	0.0038	0.0013	0.0039	0.0016	0.0043	0.0018	0.0046								
EM B									0.0018	0.0033	0.0025	0.0038	0.0028	0.0040	0.0025	0.0038
EM BB									0.0192	0.0118	0.0185	0.0111	0.0199	0.0115	0.0185	0.0117
EM BBB									0.0122	0.0134	0.0097	0.0122	0.0101	0.0124	0.0117	0.0130
SSA							0.0162	0.0123							0.0167	0.0121
DM 7-10 year					0.0003	0.0018	0.0002	0.0014					0.0003	0.0016	0.0002	0.0012
DM 5-7 year					0.0013	0.0037	0.0020	0.0042					0.0014	0.0038	0.0019	0.0040
DM 1-5 year					0.0051	0.0069	0.0051	0.0066					0.0061	0.0074	0.0061	0.0070
G7 7-10 year			0.0001	0.0013	0.0001	0.0013	0.0001	0.0009			0.0000	0.0007	0.0000	0.0008	0.0000	0.0005
G7 5-7 year			0.0193	0.0068	0.0162	0.0092	0.0100	0.0088			0.0189	0.0069	0.0151	0.0092	0.0088	0.0085
US Bills	0.9084	0.0209	0.9251	0.0165	0.9209	0.0155	0.9184	0.0154	0.9117	0.0207	0.9269	0.0163	0.9225	0.0152	0.9197	0.0150
Non US Bills	0.0249	0.0074	0.0034	0.0067	0.0004	0.0025	0.0006	0.0025	0.0240	0.0074	0.0037	0.0069	0.0003	0.0021	0.0005	0.0024
G7 1-5 year	0.0330	0.0117	0.0200	0.0096	0.0218	0.0088	0.0143	0.0098	0.0311	0.0120	0.0197	0.0100	0.0215	0.0090	0.0135	0.0098

Asset Class	EM Geographics (EMG)		G7 + EMG		G7+DM+EMG		G7+DM+ SSA+EMG		EM Country Rating (EMR)		G7 + EMR		G7+DM+EMR		G7+DM+ SSA+ EMR	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel B. Asset Allocation																
EM Asia	0.0074	0.0187	0.0127	0.0197	0.0336	0.0275	0.0325	0.0267								
EM EMEA	0.0916	0.0198	0.0713	0.0281	0.0354	0.0293	0.0333	0.0287								
EM LATAM	0.0010	0.0067	0.0159	0.0227	0.0310	0.0280	0.0342	0.0283								
EM B									0.0907	0.0115	0.0931	0.0099	0.0865	0.0125	0.0831	0.0135
EM BB									0.0092	0.0115	0.0069	0.0099	0.0134	0.0125	0.0166	0.0135
EM BBB									0.0000	0.0008	0.0001	0.0010	0.0001	0.0010	0.0003	0.0021
SSA							0.0956	0.0112							0.0903	0.0165
DM 7-10 year					0.0000	0.0000	0.0000	0.0000					0.0000	0.0000	0.0000	0.0000
DM 5-7 year					0.0000	0.0000	0.0000	0.0000					0.0000	0.0000	0.0000	0.0000
DM 1-5 year					0.1797	0.0223	0.1702	0.0124					0.1751	0.0228	0.1708	0.0153
G7 7-10 year			0.0000	0.0000	0.0000	0.0000	0.0000	0.0000			0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
G7 5-7 year			0.1183	0.0300	0.0326	0.0259	0.0053	0.0118			0.1146	0.0287	0.0420	0.0268	0.0101	0.0162
US Bills	0.0007	0.0132	0.1435	0.0404	0.3000	0.0036	0.3000	0.0004	0.2996	0.0108	0.1695	0.0395	0.3000	0.0001	0.3000	0.0032
Non US Bills	0.2993	0.0121	0.1565	0.0540	0.0000	0.0000	0.0000	0.0001	0.0004	0.0018	0.1305	0.0039	0.0000	0.0001	0.0000	0.0002
G7 1-5 year	0.6000	0.0000	0.4817	0.0300	0.3877	0.0084	0.3290	0.0103	0.6000	0.0000	0.4854	0.0287	0.3829	0.0082	0.3287	0.0129

Table 4.14 reports the average and standard deviations of the asset weights resulted from the inclusion emerging market geographical area and country rating index in the global minimum portfolio. The emerging market weights on both approaches range between three and four percent, much lower than 10 percent limit. In the case of asset allocation constraints, both emerging market strategies show significant increase holding to the 10 percent holding limit. However, most of the average weights in emerging market assets lower than two standard errors from zero as it reported by Britten-Jones (1999) and Li et al. (2003).

Our results show that the posterior distribution of the assets weight resulted from emerging market diversification are relatively stable before imposing asset allocation constraints and more noisy after asset allocation constraints are implemented.

4.3.3.3. Inflation-linked diversification

The third part of the global minimum variance portfolio analysis is when central bank objective to maximise benefits of diversification by allowing to invest in inflation-protected government bonds. Table 4.15 reports the posterior distribution of the risk reduction to the minimum variance portfolio when index-linked bonds are considered in their investible assets. The magnitude of average of risk reduction that resulted from inflation-linked diversifications, for before imposing asset weights constraints, is around 0.75 percent when global government inflation-linked bonds are used, and 0.88 percent if using G7

Table 4.15 Posterior distribution of risk reduction of the index-linked government bond strategies to the GMV portfolio

This table reports the posterior distribution of risk reduction ∅ (percent) to the GMV portfolio resulted from index-linked government bond diversification for short-sale constrained reserves portfolio. This study compares two different index-linked diversifications; Global and G7 Government index linked bonds. The first strategy is spanned directly from the benchmark portfolio; the next are spanned with the inclusion of the longer dated G7, developed government bond market, SSA and both country rating and geographic EM approaches. The summary statistics are the mean, standard deviation, the average of actual risk reduction measure, and 5th, 50th, and 95th percentile.

Frontier of Benchmark +	Mean	Stdev	ō	Percentile		
				5th	50th	95th
Panel A. Short-Sale Constraints						
GGIL	0.7589	0.0125	0.0581	0.7377	0.7594	0.7790
GGIL+G7	0.7605	0.0124	0.0574	0.7392	0.7607	0.7803
GGIL+G7+DM	0.7612	0.0125	0.0570	0.7402	0.7614	0.7811
GGIL+G7+DM+SSA	0.7616	0.0123	0.0568	0.7405	0.7619	0.7814
GGIL+G7+DM+SSA+EMG	0.7628	0.0125	0.0562	0.7418	0.7633	0.7826
GGIL+G7+DM+SSA+EMR	0.7628	0.0125	0.0563	0.7418	0.7631	0.7831
G7IL	0.7599	0.0126	0.0577	0.7387	0.7603	0.7799
G7IL+G7	0.8820	0.0061	0.0139	0.8720	0.8821	0.8917
G7IL+G7+DM	0.8847	0.0059	0.0133	0.8748	0.8849	0.8941
G7IL+G7+DM+SSA	0.8848	0.0059	0.0133	0.8750	0.8850	0.8943
G7IL+G7+DM+SSA+EMG	0.8850	0.0060	0.0132	0.8749	0.8853	0.8945
G7IL+G7+DM+SSA+EMR	0.8850	0.0060	0.0132	0.8751	0.8852	0.8945
Panel B. Asset Allocation Constraints						
GGIL	0.1078	0.0140	0.7960	0.0844	0.1079	0.1310
GGIL+G7	0.3145	0.0260	0.4700	0.2719	0.3147	0.3572
GGIL+G7+DM	0.3710	0.0238	0.3957	0.3314	0.3713	0.4096
GGIL+G7+DM+SSA	0.3804	0.0242	0.3839	0.3397	0.3805	0.4195
GGIL+G7+DM+SSA+EMG	0.4391	0.0235	0.3146	0.3994	0.4395	0.4770
GGIL+G7+DM+SSA+EMR	0.4500	0.0238	0.3025	0.4109	0.4500	0.4890
G7IL	0.1422	0.0125	0.7358	0.1211	0.1424	0.1626
G7IL+G7	0.3586	0.0170	0.4114	0.3302	0.3588	0.3864
G7IL+G7+DM	0.5440	0.0098	0.2080	0.5280	0.5440	0.5598
G7IL+G7+DM+SSA	0.5440	0.0098	0.2080	0.5278	0.5440	0.5599
G7IL+G7+DM+SSA+EMG	0.4721	0.0228	0.2787	0.4342	0.4724	0.5089
G7IL+G7+DM+SSA+EMR	0.4792	0.0229	0.2712	0.4407	0.4792	0.5167

inflation-linked bond index. The smallest reduction of 0.74 percent appeared when inflation-linked bonds added directly to the existing benchmark portfolio.

The asset weight constraints for the index-linked government bond strategy are defined similarly to that of emerging market strategy. In this case, though, an added constraint on inflation-linked bonds is that they can be invested for no more than 15 percent. As expected, the asset allocation constraints shift the location of the posterior distribution of risk reduction toward zero. The average risk reduction decreased to 0.11-0.48 percent. The smallest benefits were obtained when global government inflation-linked bonds were added directly to the existing benchmark portfolio and the greatest when G7 inflation linked bond are added together with EM and the other longer maturities bond indexes.

Variation of the reduction of risk when asset allocation constraints are added into optimisation has also increased significantly from 0.01 percent to 0.02 percent. The biggest benefit occurred when G7 government index-linked bond included together with longer dated and quasi-government, and country rating diversifications. These two strategies yield average risk reduction of 0.47 percent and could be expected to gain more than 0.43 percent risk reduction at the 5th percentile of the posterior distribution.

Table 4.16 reports the average and standard deviations of the asset weights of the inclusion global or group of G7 index-linked government bonds in the global minimum variance portfolio. The average weight shows some variations when applying these scenarios on country rating and geographical approaches. Both adding global and G7 inflation-linked government bond diversifications

Table 4.16 Portfolio weight in the GMV portfolio of the Index-linked government bond strategy

This table reports the posterior distribution of the average and standard deviations of the optimal weights in the GMV portfolio of the inflation-linked diversification. Test assets included in this scenario are G7 government inflation-linked bond, G7 government bond 5-7 and 7-10 years, DM government bond 1-5 year, 5-7 year and 7-10 year, SSA, and also EM government bond indices based on geographic and country rating. The last three are the benchmark assets; US Treasury Bills, non-US bills and G7 government bond index 1-5 year. The portfolio weights are constrained for budget constraint equal to one, constrained to short-sale, and asset allocation.

Spanning Strategy	GGIL		GGIL+G7		GGIL+G7+DM		GGIL+G7+ DM+SSA		GGIL+G7+DM+ SSA+EMG		GGIL+G7+DM+ SSA+EMR	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel A. Global Inflation Linked												
Short-Sale Constraints												
Global Inflation-linked	0.0423	0.0074	0.0397	0.0071	0.0436	0.0076	0.0425	0.0076	0.0347	0.0089	0.0345	0.0088
EM Asia									0.0039	0.0058		
EM EMEA									0.0095	0.0090		
EM LATAM									0.0008	0.0028		
EM B rating											0.0014	0.0027
EM BB rating											0.0110	0.0079
EM BBB rating											0.0020	0.0053
SSA							0.0116	0.0099	0.0122	0.0102	0.0124	0.0104
DM 7-10 year					0.0143	0.0088	0.0139	0.0083	0.0156	0.0081	0.0161	0.0081
DM 5-7 year					0.0004	0.0020	0.0005	0.0023	0.0004	0.0020	0.0004	0.0020
DM 1-5 year					0.0001	0.0010	0.0000	0.0007	0.0000	0.0006	0.0000	0.0007
G7 7-10 year			0.0000	0.0004	0.0000	0.0006	0.0000	0.0003	0.0000	0.0002	0.0000	0.0002
G7 5-7 year			0.0202	0.0078	0.0094	0.0090	0.0058	0.0076	0.0053	0.0074	0.0049	0.0072
US Bills	0.9139	0.0181	0.9284	0.0137	0.9250	0.0127	0.9206	0.0128	0.9112	0.0143	0.9104	0.0142
G7 Non US Bills	0.0272	0.0069	0.0054	0.0078	0.0007	0.0030	0.0014	0.0040	0.0010	0.0033	0.0010	0.0034
G7 1-5 year	0.0166	0.0108	0.0063	0.0076	0.0065	0.0070	0.0034	0.0056	0.0056	0.0071	0.0060	0.0073

Spanning Strategy	GGIL		GGIL+G7		GGIL+G7+DM		GGIL+G7+ DM+SSA		GGIL+G7+DM+ SSA+EMG		GGIL+G7+DM+ SSA+EMR	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Asset Allocation Constraints												
Global Inflation-linked	0.1500	0.0000	0.1493	0.0034	0.1500	0.0002	0.1500	0.0007	0.1117	0.0147	0.1006	0.0149
EM Asia									0.0159	0.0199		
EM EMEA									0.0589	0.0283		
EM LATAM									0.0252	0.0252		
EM B rating											0.0676	0.0144
EM BB rating											0.0269	0.0169
EM BBB rating											0.0055	0.0110
SSA							0.0890	0.0173	0.0530	0.0226	0.0541	0.0242
DM 7-10 year					0.2085	0.0143	0.1861	0.0094	0.1801	0.0103	0.1838	0.0104
DM 5-7 year					0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DM 1-5 year					0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
G7 7-10 year			0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
G7 5-7 year			0.0000	0.0000	0.0094	0.0156	0.0005	0.0033	0.0000	0.0007	0.0001	0.0014
US Bills	0.1500	0.0000	0.1866	0.0346	0.3000	0.0000	0.3000	0.0000	0.2999	0.0013	0.2999	0.0010
G7 Non US Bills	0.1500	0.0000	0.1134	0.0346	0.0000	0.0000	0.0000	0.0000	0.0001	0.0007	0.0001	0.0006
G7 1-5 year	0.5500	0.0000	0.3874	0.0265	0.3321	0.0075	0.2745	0.0134	0.2551	0.0098	0.2614	0.0143

Spanning Strategy	GGIL		GGIL+G7		GGIL+G7+DM		GGIL+G7+ DM+SSA		GGIL+G7+DM+ SSA+EMG		GGIL+G7+DM+ SSA+EMR	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel B. Group G7 Inflation-Linked Bond												
Short-Sale Constraints												
G7 Inflation-linked	0.0500	0.0082	0.0201	0.0045	0.0205	0.0043	0.0204	0.0043	0.0175	0.0053	0.0445	0.0904
EM Asia									0.0014	0.0026		
EM EMEA									0.0026	0.0039		
EM LATAM									0.0002	0.0011		
EM B rating												
EM BB rating											0.0186	0.0550
EM BBB rating											0.0208	0.0657
SSA							0.0001	0.0009	0.0001	0.0010	0.0524	0.1271
DM 7-10 year					0.0676	0.0143	0.0674	0.0144	0.0673	0.0143	0.0887	0.1732
DM 5-7 year					0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0187	0.0757
DM 1-5 year					0.0000	0.0001	0.0000	0.0001	0.0000	0.0002	0.0225	0.0773
G7 7-10 year			0.0003	0.0019	0.0000	0.0003	0.0000	0.0002	0.0000	0.0003	0.0222	0.0752
G7 5-7 year			0.0153	0.0071	0.0000	0.0005	0.0000	0.0004	0.0000	0.0006	0.0289	0.0957
US Bills	0.8958	0.0197	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1427	0.2188
G7 Non US Bills	0.0316	0.0072	0.9642	0.0075	0.9118	0.0146	0.9120	0.0147	0.9108	0.0146	0.3089	0.3361
G7 1-5 year	0.0226	0.0111	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2041	0.2483

Spanning Strategy	GGIL		GGIL+G7		GGIL+G7+DM		GGIL+G7+ DM+SSA		GGIL+G7+DM+ SSA+EMG		GGIL+G7+DM+ SSA+EMR	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Asset Allocation Constraints												
G7 Inflation-linked	0.1500	0.0000	0.0808	0.0220	0.0436	0.0618	0.0000	0.0002	0.1448	0.0082	0.1429	0.0101
EM Asia									0.0091	0.0149		
EM EMEA									0.0730	0.0242		
EM LATAM									0.0159	0.0200		
EM B rating											0.0574	0.0132
EM BB rating											0.0198	0.0184
EM BBB rating											0.0125	0.0162
SSA							0.0000	0.0000	0.0220	0.0144	0.0285	0.0168
DM 7-10 year					0.1053	0.1461	0.4500	0.0002	0.1848	0.0089	0.1879	0.0090
DM 5-7 year					0.0095	0.0466	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DM 1-5 year					0.0160	0.0577	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
G7 7-10 year			0.0000	0.0000	0.0198	0.0643	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
G7 5-7 year			0.3692	0.0220	0.0169	0.0626	0.0000	0.0000	0.0000	0.0005	0.0001	0.0011
US Bills	0.1500	0.0000	0.1645	0.0416	0.2228	0.0745	0.1522	0.0161	0.2995	0.0029	0.2996	0.0028
G7 Non US Bills	0.1500	0.0000	0.1355	0.0014	0.0772	0.0015	0.1478	0.0099	0.0005	0.0029	0.0004	0.0028
G7 1-5 year	0.5500	0.0000	0.2500	0.0000	0.4889	0.2004	0.2500	0.0000	0.2504	0.0021	0.2510	0.0038

show significant increase of holding index-linked assets to more than 12 percent, slightly lower from the 15 percent limit. Different from lengthening duration and emerging strategy, our results of the average weights assets here are higher than two standard errors, consistent with the results of Britten-Jones (1999) and Li et al. (2003). Our results show that the posterior distribution of index-linked assets weights is less noisy when asset allocation constraints imposed.

Different from portfolio risk reduction to the global minimum variance portfolio, diversification benefits in term of reduction of portfolio risk for the same expected returns as benchmark portfolio relatively vary within strategies and across portfolio constraints. Incremental impacts from imposing different strategies and constraints are more challenging than that of minimum variance framework. Furthermore, minimum variance results are driven more by treasury bills and so perhaps less interesting. More interesting are the risk reduction benefits while maintaining expected returns as some results show here.

4.3.4. Comparing the impact of assets selection versus strategic asset allocation

The results of the emerging market diversification demonstrate that the choice of geographic or country rating diversification does matter; more specifically when asset allocation constraints are imposed. The risk reduction for a given expected benchmark return that resulted from country rating diversification, is not achievable by geographic diversification. This study documents parallel findings to that of Ehling and Ramos (2006), who discover that the choice of index is essential for emerging market equity portfolios. Different from emerging market diversification that the choice of country rating or geographic diversification is important, the choice of inflation-linked government index does matter. This research finds empirical evidence that the choice of global government and G7 government inflation-linked diversification offer different impacts on government bond risk reduction portfolio. It shows that in both short-selling and asset allocation constraints, there is empirical evidence found to support the argument that G7 inflation-index delivers greater benefits than that of global index-linked diversification.

Comparing all the results of the risk reduction and portfolio weights also reveal the role of asset allocation policy for the risk reduction in government bond portfolios. Applying Bayesian mean-variance framework, this research shows that the variability of risk reductions of the strategies are low within the same constraints and are higher across the different constraints. The benefits resulted from longer bond maturity, emerging market and index-linked diversification looks similar within the same constraint. It shows that the magnitude of the posterior risk reduction distributions is less varied in term of mean, standard deviation, 1^{st} and 5^{th} percentile, and median. The impacts of imposing different constraints, however, are more prevalent on the risk reduction impacts than that of asset selections. This indicates that asset allocation constraints give a bigger impact to risk reduction portfolio performance rather than moving to higher credit risk securities when short-sale restrictions are already in place.

Grauer and Shen (2000) find that constraining portfolio weights provide less portfolio risk with the cost of less realised return, in mean-variance portfolio for a given expected returns framework. Our results show, however, multi-weight asset constraints do not provide better risk reduction on short-sale constrained government bonds portfolio. Our results are similar to Jagannathan and Ma (2003), finding that when the no-short-sale restriction is already in place, minimum variance portfolio for a given target returns performs well, for both before and after imposing asset allocation constraints. Our analysis shows that imposing asset allocation constraints provide more diversified portfolios than the theory suggests it should, with the cost of poorer its performance.

4.4. Conclusion

This study examines the effect of asset allocation constraints on the inclusion of longer dated bonds, emerging government bonds and inflation-linked government bonds. The corresponding diversification benefits on the short-sale constrained central banks reserves portfolio are evaluated. An unconditional Bayesian mean-variance framework is used for the analysis. This study measures the benefits using the risk reduction measures of Wang (1998) and Li et al. (2003). This research investigates two sources of risk reduction; first, to reduce portfolio variability without sacrificing the existing benchmark returns, and second, to minimise the risk to the minimum variance portfolio.

Central banks have to deal with specific objectives in their investment choice related monetary and exchange rate policy along with financial stability.

Accordingly, reserves have to be managed in a conservative manner with strong focus on liquidity, and safety. Taking into account those three factors, using Monte Carlo simulation (10,000 iterations), our Bayesian test for relaxing longer bonds maturity restrictions convincingly shows the ability to minimise foreign reserve risks, but not when the existing expected portfolio return is also maintained. For a central bank that has the objective to minimise portfolio risks to the global minimum variance portfolio, all of longer bond maturity diversification strategies offer significant benefit to the government bond portfolio across the three constraints. However, if central bank aims to reduce portfolio risk for the same target return, this research could not reject the null hypotheses even before imposing asset allocation constraints.

The upsurge of foreign reserves accumulation may result in a re-assessment of risk-return characteristics of various investment opportunity sets and may alter central banks' tolerance for market, credit, liquidity and reputational risk. Based on our findings; it should also change the central banks' asset allocation. Our finding shows that the current benchmark is an inefficient portfolio. Hence spanning government bond portfolio with longer-dated bonds, emerging market country rating and G7 inflation-linked government bonds altogether improve the investment opportunity set and deliver significant portfolio risk reduction. The inclusion of inflation-linked government bonds cannot be spanned directly to the current benchmark, but it needs to be added together with the other government bonds index to be able to deliver significant benefits. This investment choice may offer average monthly diversification benefits of 0.30

percent with a standard deviation of 0.15 percent and magnitude of risk reduction at the fifth percentile is 0.013 percent.

In order to obtain above mentioned diversification benefits, the optimization results suggest the central bank to allocate its reserves into 38 percent in G7 1-5 year, 27 percent in US Treasury Bills, 3 percent in Non-US bills, 8.5 percent in G7 inflation-linked bond, 6 percent in emerging market bond index, 10 percent in DM, 2.5 percent in G7 longer maturity bond index, and 0.5 percent in SSA. This strategy requires the central bank to execute a switching 32 percent of the reserve to be diversified beyond the existing asset allocation policy. The majority of the 68 percent, however, should continue to be invested in the existing benchmark policy.

The purpose of holding reserves and the mandate central banks have in managing their reserves vary between central banks and therefore the choice between two measures will differ amongst them. I would argue that reducing the risk for a given existing target return is more desirable for central bank reserve managers than the second measure. Foreign reserves diversification ideas resulting from the first measure are also easier to communicate to the government and other stakeholders, since the benefits are yielded without giving up the existing expected returns. However, it is interesting that the empirical findings reveal the minimum variance portfolio offers significant benefits across all constraints. It could be worth thinking as to whether the central bank should follow the GMV strategy.

Our findings also reveal the role of constraints for the risk reduction in government bond portfolios. Applying Bayesian mean-variance framework, this research shows that asset allocation constraints offer a bigger impact to portfolio performance rather than moving to higher credit risk securities or security selection when short-sale restrictions are already in place. The sharp drops of the benefits are due to asset allocations which limit the holding of the certain assets that could be too strict from a standard portfolio optimisation. This strict limit however is a vital risk tool to regulate the portfolio consistent with the investment policies of central banks. Imposing stringent constraints relative to the conventional portfolio optimisation, however, significantly makes the benefits of constrained asset allocation portfolio be wiped out. The lower standard deviation, however, indicates better chance the benefits could be achieved. Our results are similar to Jagannathan and Ma (2003), finding that when the no-short-sale restriction is already in place, minimum variance portfolio for a given target returns performs well, for both before or after imposing asset allocation constraints.

CHAPTER FIVE

This chapter examines central bank risk reduction benefits for a given expected benchmark return for a variety of investment policy settings. The impact of budget constraints, liquidity buffer allocation, and global financial crisis investment opportunities, non-government bond investment; and sample selection bias test are investigated.

5. OPTIMAL FOREIGN RESERVE ASSET ALLOCATION IN VARIOUS SETTING

Abstract

This study develops various analysis on a risks minimization government bonds portfolio for a given benchmark returns. This research examines the role of budget constraints, liquidity buffer allocation, financial crisis investment opportunities, sample selection bias test, and non-bonds investment policy. Analysis on the impact of budget and liquidity allocation constraints has policy implications that central bank should not to tranche their reserves, but express the external transaction requirements in the form of constraints on the portfolio optimisation framework. The impact of global financial crisis since 2008 emphasises the need for central banks to diversify their foreign reserves benchmark beyond its current setting. Lastly, additional new asset classes beyond quasi-government securities provide significant benefits compare to the existing benchmark and provide significant incremental benefits relative to well-diversified government bond portfolios.

5.1. Introduction

Different economic conditions and size of reserves relative to their economy may change central banks' risk appetite, which will be reflected in their reserve management policy adjustments. Adverse economic conditions may cause central banks to take preventive measures, such as by increasing the allocating of their portfolio to more liquid securities (Romanyuk, 2012). More specifically, some central banks might also have different preference to allocate a separate cash portfolio to cover the day-to-day payment. The bigger excess reserves could be expected to result in more allocation for income generation, or smaller portion for the liquidity and safety purposes (Bakker and Herpt, 2007). These variances and different risk preference in investing in ultra-long bond maturity and different investment opportunities may provide a richer analysis in which cases are more appropriate for a specific central bank and what investment strategy should be pursued.

This empirical chapter investigates whether the risk reduction benefits are sensitive to the different central bank requirements for the precautionary, risk appetite, change investment opportunities or sample period. More specifically the four research questions are raised 1) Does the decision of the central bank to allocate cash to cover day-to-day payment, apart from the bond portfolio, change the benefits? 2) Do precautionary needs to cover intervention, and short-term liabilities change the important central banks' risk reduction benefits? 3) What does financial crisis mean for risk minimization of reserves portfolio? By analysing diversification benefits before and after 2008, this research tries to answer a question: Does different investment opportunities changed the benefits? 4) To answer a question whether the preference to ultra-long government bond offer different benefits for central bank portfolios. Moreover, 5) whether non-government bond investment which includes gold, mortgage-backed securities (MBS), Asset-backed securities (ABS), corporate bond, and world equity offer significant benefits for central bank portfolio? If yes, do the incremental benefits matter relative to the portfolio risk of well-diversified government bonds investment?

The literature shows few studies suggesting the use of mean-variance Markowitz (1952) framework for bond portfolio allocation (see, for example, Korn and Koziol (2006) and Puhle (2008)). There are some reasons to justify the slow development of the usage of mean-variance, and much less is known about Bayesian mean-variance in fixed-income, more specifically in government bond portfolio. Korn and Koziol (2006) and Puhle (2008) argue the reason why Markowitz approach to portfolio selection has not been popular to fixed income due to problems in modelling returns and covariance matrix of bonds. Furthermore, Fabozzi and Fong (1994) claim that if were possible to work out a various bonds covariance matrix, fixed income optimisation could be similar to that of equity portfolios.

The main reason why mean-variance framework has not been applied for fixed-income, according to Korn and Koziol (2006) is that historically bond assets exhibit low volatility, which discouraged the use of the sophisticated method. However, this environment is shifting rapidly in latest years, even in markets where assets have low a probability of default. The turbulence in global markets brought great volatility to fixed-income asset prices. As such, the importance of portfolio optimisation approaches that take into account both risk-return trade-off, and risk diversification across different opportunities has increased.

Previous research, for example Korn and Koziol (2006) and Puhle (2008) proposes the application of mean-variance framework using one factor model for the yield curve. A more recent paper by Caldeira, Moura, and Santos (2012) extend heteroskedastic dynamic factor models to the term structure of interest rates. These estimates are used as inputs to the mean-variance bond portfolio optimisation.

This research extends the Bayesian approach of Wang (1998) and Li et al. (2003) that has been implemented for equity markets, but now applied for government bonds portfolio. It is noteworthy that our approach for bond portfolio optimisation here differs in several respects to existing Bayesian frameworks. Li et al. (2003), for example, examine diversification benefits in term of returns enhancement and risks reduction subject to short-sale constraints for the minimum variance portfolio. The framework here, on the other hand, extends constraints not only short-sale but also multiple weights restriction of Jagannathan and Ma (2003). Furthermore, to address liquidity and safety aspects of the foreign reserves management, our objective here is to minimise government bonds portfolio volatility for the given benchmark returns.

In this study, the researcher considers strategic asset allocation for foreign reserves approach that is based on the risk reduction. The idea of risk minimization implies that among the traditional objectives of reserves management, which is liquidity, safety and returns. This chapter focuses on the safety, and uses liquidity requirements for varying risk choices framework and leave returns objective for the future study. I conduct scenario analysis to address the problem that central banks may have different risk preference in managing their reserves.

More specifically, it is important to examine the role of central banks' risk appetite, sample period, sample selection bias, and other particular constraints in the minimization portfolio volatility analyses. Various scenarios considered in this study include two sub-period analyses, the inclusion of ultra-long government bond to address the potential sample selection bias problem, and to check whether the risk reduction is the result from particular asset-weight constraints. This research performs varying budget and treasury bills allocation constraints. Lastly, in this study I will also look at the opportunity to invest beyond government bonds. The economic interpretation of our measure is straightforward, and it tells directly how much portfolio risk would reduce by switching their investment from the existing policy for the same expected returns as the given benchmark portfolio for different risk preference.

Our study contributes to an increasing popularity of literature that apply Bayesian framework for analysing portfolio diversification benefits; more specifically for foreign reserve portfolio. This analysis is enhanced from the previous empirical chapter by employing techniques for optimal government bond portfolio construction in the presence of varying budget constraints, liquid-asset weight policies, time-varying investment opportunities and test for sample selection bias to address central banks' difference risk tolerance and investment opportunities. The importance to study the varying constraints is to answer the problems that may arise in managing reserve portfolio: The key contribution of this paper is to fill the gap of the portfolio selection problems by including multiple weight constraints in mean-variance Bayesian framework. This study may also relevant for central bank's reserves manager to review their existing investment policy with regards to the subjective risk preference.

Our results show that global inflation-linked government bonds, together with the emerging market country rating and longer dated a group of G7 bonds index, developed market and SSA bond index, spanning is a significant diversification strategy for both short-sale and strategic asset allocation constraints. Furthermore, this analysis shows that the requirement to provide more cash resulting in a higher uncertainty of the risk minimization benefits. The impact of liquidity buffer allocation confirmed that one of the main sources of risk reduction on government bond portfolio is driven by US Treasury Bills. Allowing the portfolio to mimic US Treasury Bills weights to the benchmark offers optimum benefits. However if it exceeded too far from the benchmark holding it may lead to a lower return, which violates the objective to maintain the same expected returns with the current benchmark.

The impact of global financial crisis since 2008 emphasises the need for central banks to diversify their foreign reserves benchmark from its current setting. Lastly, the sample selection bias that might occur in this study, more specifically for longer dated bond strategy, is investigated It is proven that the reason this research could not find diversification benefits on longer dated bond strategy in the earlier empirical chapter is not because of sample selection bias problem. The inclusion of new asset classes including equities and asset-backed securities significantly provide risk reduction diversification benefits for central bank reserve management.

The additional new asset classes beyond quasi-government securities provide significant benefits compared to the existing benchmark, and provide significant incremental benefits compare to well-diversified government bond portfolios. Overall, these results look interesting, and it provides practical implication for different central bank reserve management risk preference. Our analysis provides some risk preference alternatives and their impact on reserves portfolio risk minimization; more specifically in addressing safety and liquidity needs.

The remainder of the chapter is organised as follows. Section 2 describes data and followed by the concept of the research methods used to perform scenario analysis. Section 3 presents and discusses the empirical findings, and concludes in Section 4.

5.2. Data and Research Methods

5.2.1. Data

Data for this study is the same as the data sets of the first empirical chapter, which consist of several government bonds returns between the period December 1985 and December 2014 to represent central banks' foreign exchange reserves allocation policy. Government bond for maturity longer than 10 years will also be included more specifically to investigate if there is any sample selection bias. This research uses two approaches first by including the US more than 10 year government bond return and by constructing equally weight bond return of United States (US), United Kingdom (UK) and Germany (GE). The main reason the researcher uses those bond indexes is because those are the most liquid ultra-long government bond markets. In addition to those data sets, non-bond asset classes which consist of gold, ABS, MBS, corporate bonds and equities are also included.

5.2.2. Research Methods

This chapter focuses on the objective of risk minimization for a given expected benchmark returns when the central bank has restrictions on their investment policy, including no short-selling and budget constraint equal to one as given by the equation (3.1) in Chapter Three. In this study, I concentrate on analysing risk reduction benefits for a given existing benchmark returns.

5.2.2.1. Budget constraint

Budget constraints are particular linear constraints to limit the sum of portfolio weights to fall either or below specific bounds. To study the impact of varying budget constraint, the researcher replaces the condition $x'e = 1$ with:

$$x'e = j, \text{ where } j = 0.90; 0.95; 1.00 \quad (5.1)$$

Here the researcher evaluates three different budget constraint scenarios; first to limit the sum of portfolio weight equal to 90 percent, 95 percent and 100 percent. If the sum of portfolio weight less than 100 percent means that the difference has to be reserved in cash to cover day-to-day transactions. For example, if j = 0.90 then the portfolio manager is only allowed to invest 90 percent of the investable asset and put aside 10 percent of the reserves in cash to cover day-to-day external payments. In this study, I propose this budget constraints framework as an alternative approach to investigating whether the decision to tranche the foreign into liquidity and investment portfolios provides greater benefits than the single portfolio for risk reduction framework.

5.2.2.2. Liquid asset weight constraint

Asset weights are common constraints imposed by institutional investors. Previous studies, for example Frost and Savarino (1998) and Jagannathan and Ma (2003), have seen portfolio weight constraints affected empirically. In the second empirical chapter, it shows that the risk reduction benefits on government bond portfolios are generally driven by treasury bills. In order to confirm whether government bond portfolio risk reduction benefits are sensitive to liquid assets, three varying allocations of treasury bills are examined. In addition, this research maintains the allocation for US treasury bills has to be more than 15 percent as in the previous chapter. To address this problem, lower bound constraint from the equation (4.2) is combined with equality bound constraints:

$$Aeq * x_i = \bar{x}, \qquad \bar{x} = 0{,}2, 0.3, 0.4 \tag{5.2}$$

These constraints are intended to examine whether different allocation from 30 percent, 20 percent and 40 percent of treasury bills will change the benefits.

5.2.2.3. Sub-period

This sub-section will examine whether the risk reduction and the impact of asset allocation documented in the earlier empirical chapter are different following the intensification of the financial crisis in September 2008 after the collapse of Lehman, and the severe Greek fiscal problem announcement on 5 November 2009. It is well known that bond yields since that period were quite different from those in the past due to major central banks implementing ultra-low interest rate policy. Therefore, it is reasonable to suspect that government bond diversification benefits and the impact of asset allocation constraints might have changed as a result of a very low yield environment. In order to study these problems, the researcher examines the risk reduction benefits separately for the period from December 1985 to December 2006 and from January 2007 to December 2014.

5.2.2.4. *Sample selection bias*

In this subsection, this research tries to address the sample selection bias that might occur in this study; more specifically for longer dated bond strategy. The researcher is aware that particular investment policy could be causing a non-random sampling of a population, and causing some members of the asset classes to be less likely to be included in this exercise than others. Our framework for the longer bond is restricted up to 10 years maturity. Therefore, it is realistic to question whether the reason this thesis in earlier chapter could not document diversification benefits evidence on longer dated bond strategy was because of the researcher did not include government bonds that have a maturity longer than 10 years. To answer this issue, I conducted an exercise to include US government bond index longer than 10 years maturity and an equally-weight bond index of longer than 10 year maturity of US, UK and German government bonds.

5.2.2.5. *Non-government diversification*

To limit certain asset class holding in foreign reserve portfolio we apply upper bound constraints as in equation 4.3. In addition to those constraints, additional linear inequality constraint matrix A and vector $\bar{x}$, every weight x_i in a portfolio must satisfy the following:

$$A * x_i \leq \bar{x}, \qquad \bar{x} = 0.05, 0,1, 0.05 \tag{5.3}$$

intended to limit non-government asset class holding, which includes 5 percent in gold; 10 percent in ABS or/and MBS; and 5 percent in a corporate bond or/and equity asset classes.

5.3. Empirical Results

5.3.1. Descriptive statistics

Table 5.1 reports summary statistics for the bond market index and the first four asset classes; treasury bills, group of G7, developed market government and government-related entities bond index that will be used to examine central banks' foreign reserves diversification benefits for a given benchmark return. Average monthly return on the G7 1 to 5 year maturity bond index was 0.49 percent over the entire period, 0.57 percent during the period 1985-2006 and halved to 0.25 percent during financial turmoil with monthly volatility increased slightly to 1.57 percent from 1.50 percent. US Treasury Bills mean return during pre-crisis was 0.39 percent and dropped dramatically to 0.07 percent in a time of financial crisis. Mean returns and volatility of non-US bills relatively stable in the two sub-periods at 0.16 percent and 0.12 percent respectively.

Interestingly, non-US short-term assets have a smaller variation in term of monthly risk and return. Non-US bills have expected returns for the two sub period of 0.19 percent and 0.20 percent with volatility of 2.45 and 2.66 percent, the smallest variation amongst all other asset classes for the respective sub-period. Those short-dated assets risk-return are dominated by their currency performance over the sample period. From the central bank as a US investor perspective, this could be seen that holding non-US Treasury Bills similar to holding foreign currency portfolios. Mean return as well as the volatility, varies substantially across developed market government bond indexes as shown in Table 5.1.

Table 5.1 Summary statistics

The table reports summary statics of the monthly returns of the government bond benchmark assets and test assets for the period of December 1985-December 2006 and January 2007-December 2014. Ultra-long and non-government bonds data cover the period of December 1985-December 2014. Benchmark assets consist of the bond market, US bills and Non-US bills. Test assets include a group of G7 government bond market, developed government bond market, semi-government, supranational and government agency bond market, emerging market, index-linked, ultra-long bond and non-government bonds. The table shows the mean, standard deviation, and its correlations with each benchmark assets for the corresponding time period.

Asset Class		Mean			Standard Deviation			Correlation					
								US Bills		Non US Bills		Bond Market	
		all period	1985-2006	2007-2014	all period	1985-2006	2007-2014	1985-2006	2007-2014	1985-2006	2007-2014	1985-2006	2007-2014
Benchmark													
US Bills		0.3014	0.3896	0.0664	0.2056	0.1569	0.1157	1	1	0.1033	-0.0659	-0.0008	0.1738
Non US Bills		0.1892	0.1853	0.1996	2.5101	2.4566	2.6608	0.1033	-0.0659	1	1.0000	-0.8477	-0.6599
Bond Market (G7 1-5 year)		0.4857	0.5756	0.2462	1.5185	1.4917	1.5707	-0.0008	0.17375	-0.8477	-0.6599	1	1.0000
Lengthening Duration													
Group G7	5-7 year	0.0594	-0.0182	0.2661	2.2736	2.1874	2.4892	0.0238	-0.0991	0.8722	0.9106	-0.5812	-0.4433
	7-10 year	0.1236	0.0393	0.3480	2.3824	2.2922	2.6072	0.0189	-0.1137	0.8261	0.8646	-0.5174	-0.3976
Developed Mkt	1-5 year	-0.0930	-0.1399	0.0322	2.8084	2.4570	3.5929	0.0639	-0.0743	0.9468	0.9659	-0.7211	-0.5006
	5-7 year	0.0333	-0.0377	0.2225	3.0552	2.7604	3.7389	0.0228	-0.1040	0.8852	0.9311	-0.6402	-0.4536
	7-10 year	0.0745	-0.0014	0.2764	3.0183	2.6267	3.8827	0.0320	-0.1033	0.8850	0.9241	-0.6102	-0.4236
Semis, Supranat. & Agencies (SSA)		0.2176	0.2112	0.2347	1.0294	1.1025	0.8079	0.0218	0.0890	-0.0045	0.3779	0.3256	0.2207
Emerging Market													
Country Rating	Sovereign B	-0.0315	0.0409	-0.2241	2.3366	0.5503	4.3921	-0.0111	-0.1212	-0.0017	-0.5349	-0.0458	0.0866
	Sovereign BB	0.0969	0.0293	0.2770	1.6280	0.5657	2.9800	-0.0105	-0.0854	0.0175	-0.5700	-0.0178	0.2994
	Sovereign BBB	0.0336	0.0048	0.1103	1.3233	0.4807	2.4159	-0.0237	-0.0654	0.0160	-0.6102	0.0151	0.4520
Geographics	Asia	0.0882	0.0574	0.1701	1.7390	0.4643	3.2520	-0.0319	-0.0984	0.0331	-0.5601	-0.0771	0.2937
	EMEA	0.0332	0.0035	0.1122	1.4234	0.4289	2.6415	-0.0254	-0.0750	0.0122	-0.5862	0.0073	0.2942
	Latin America	0.0246	0.0291	0.0126	1.6421	0.5746	3.0115	-0.0120	-0.0789	0.0090	-0.6246	-0.0095	0.3366
Index-linked Bond													
Global Govt Inflation-Linked Bond		0.3307	0.2865	0.4483	1.6957	1.2335	2.5528	-0.2499	0.0463	-0.4152	-0.8408	0.4723	0.6211
G7 Govt Inflation-Linked Bond		0.1923	0.1255	0.3700	1.5214	0.6647	2.7048	-0.2863	0.0447	-0.4497	-0.8980	0.4845	0.6533
Ultra-Long Bond													
US Government +10 year		0.2230			2.9179			0.3869		-0.0184		-0.0598	
G3 Government +10 year		0.2242			3.0027			-0.1544		-0.0622		0.5600	
Non-Government Bond													
Gold		0.4678			4.4236			-0.0780		-0.3498		0.3168	
Mortgage-Backed Securities (MBS)		-0.0046			0.9921			-0.0310		-0.2271		0.2737	
Asset-Backed Securities (ABS)		0.0184			0.9846			-0.0032		-0.1133		0.3917	
Investment Grade Corporate Bonds		0.0171			1.2497			-0.0578		-0.1039		0.0590	
World Equity		0.6469			4.4474			0.0303		-0.3821		0.2105	

Most developed market bond indexes exhibit negative correlation with market index, which one may see potential diversification benefits through the reduction of risk rather than return enhancement. The mean return and volatility are typically higher in longer dated indexes than in short-dated bucket. The means, standard deviations and correlation of returns to market index in this table, however, do not provide a clear indication as to whether longer bonds maturity offer diversification benefits beyond those offered by benchmark portfolio.

The indices used to examine diversification benefits by investing in emerging market government bonds are categorised by country credit rating and geographic. The average returns of emerging market government debt range between -0.03 percent (Asia) and 0.10 percent (Europe, Middle East and Africa). During the period of 1985-2006, all emerging market indices exhibit positive average returns, while in the later subperiod emerging Asia indicates substantial negative mean returns of -0.22 percent. Emerging market bonds' volatility increased significantly from 0.50 percent to 2.5-4.4 percent during low yield environment. Emerging market government debt returns show relatively similar to that of developed market government bond. In contrast to a developed market, correlations of emerging market returns to market are all positive which may provide return enhancement to foreign reserve portfolios.

The average returns of index-linked bonds are 0.33 percent and 0.19 percent and the standard deviations are 1.70 percent and 1.52 percent. The means, and standard deviations of index-linked government bonds returns in Table 5.1

provide a clearer early indication as to whether index-linked government bond markets offer diversification benefits for central banks' foreign reserves management beyond those offered by developed government and emerging government bond markets. During the sample period, index-linked bonds generally have higher means but lower both standard deviations and correlation to developed market and emerging market government bond market indices. This low correlation suggests that central banks may benefit from investing in index-linked bonds.

This research conducts various analyses of whether the diversification benefits of longer dated maturity, emerging market, and inflation-linked government bonds and the impact of asset allocation constraints on government bond portfolio, are sensitive to varying subjective central bank risk preferences. First, budget constraints; second, liquidity constraints; third, different time period investment opportunities; fourth, the impact of portfolio risk reduction from investing in government bonds longer than 10 year maturity; and fifth, the impact and marginal impact of non-bond investments.

5.3.2. Impact of budget constraint

In this section, the research will examine whether central banks should or should not put cash aside from investible foreign reserves portfolio. The alternative that some central banks segregate portfolio tranches to cover specific objectives, as Caballero and Panageas (2005) suggest, might be the potential answer for this problem. The cash tranche serves to meet the day-to-day external payment. This portfolio tranche is normally held in demand deposits or time deposit at commercial banks or an international institution such as the Bank for International Settlement. The liquidity tranche serves as a cushion to meet unanticipated and possibly large demand of foreign exchange. As a result, liquidity tranche needs to be held in highly liquid instruments that can be sold in large quantity without having a major impact on market prices. The investment tranche can be considered as the tranche for surplus reserves with a larger focus on generating returns over the long run, which will be left for the future research.

The objective of imposing budget constraints is to limit the sum of portfolio weights to fall below specific bounds, or to require some foreign reserves to be held in cash or demand deposits in commercial banks in order to cover central banks' day-to-day liabilities. Here the researcher examines whether different budget constraints offer different risk reduction benefits for government bonds portfolio. First is to limit the sum of portfolio weight equal to 90 percent, 95 percent, and 100 percent. If the sum of portfolio weight less than 100 percent, this means that the difference has to be reserved in cash assets. For example, if j = 0.95 then the portfolio manager can only invest 95 percent of the investable asset into benchmark and test assets portfolio and put aside five percent in cash equivalent assets.

Table 5.2 reports the posterior distribution of risk reduction for a given expected current benchmark returns when central bank switches their investment

Table 5.2 Posterior distribution of risk reduction for a given benchmark returns from the spanning strategy for different budget constraints

This table reports the posterior distribution of risk reduction for a given expected benchmark returns when investor switches their investment to longer dated, emerging market and inflation-linked government bond for three different budget constraints 100%, 95% and 90%. The first frontier is spanned by longer-dated bonds within G7, DM government and SSA bond market up to 10 years maturity. The next two frontiers are spanned by EM government bond for regional and credit rating bond index. The last four frontiers are spanned by the inclusion of inflation-linked bond using global government and G7 index-linked government bond market. The summary statistics are the mean, standard deviation, the average of actual risk reduction measure, and 5th, 50th and 95th percentile.

Frontier of Benchmark +	No Short-Sale						Asset Allocation					
				Percentile						Percentile		
	Mean	Stdev	$\bar{o}$	5th	50th	95th	Mean	Stdev	$\bar{o}$	5th	50th	95th
Panel A. For the budget constraints 100%												
Longer Bond Maturity	0.3727	0.2326	0.3935	0.0343	0.3479	0.7403	0.1273	0.0970	0.7616	0.0000	0.1150	0.2758
EM Regional	0.4655	0.2275	0.2857	0.0752	0.4932	0.7569	0.2426	0.1217	0.5736	0.0000	0.2662	0.3963
EM Country Rating	0.4374	0.3042	0.3165	0.0089	0.4449	0.8743	0.2485	0.2204	0.5648	0.0000	0.2176	0.5493
Global Inflation + EM Regional	0.4524	0.3011	0.2998	0.0100	0.4678	0.8762	0.2596	0.2145	0.5481	0.0000	0.2694	0.5444
Global Inflation + EM Rating	0.4637	0.2992	0.2876	0.0117	0.4875	0.8781	0.2683	0.2160	0.5354	0.0000	0.2909	0.5487
G7 inflation + EM Regional	0.4472	0.3003	0.3056	0.0106	0.4582	0.8763	0.3118	0.2704	0.4736	0.0030	0.2734	0.7154
G7 inflation + EM Rating	0.4580	0.3004	0.2937	0.0119	0.4811	0.8774	0.3351	0.2684	0.4422	0.0035	0.3295	0.7189
Panel B. For the budget constraints 95%												
Longer Bond Maturity	0.3788	0.2372	0.3859	0.0335	0.7521	0.7521	0.1649	0.1089	0.6973	0.0000	0.1549	0.3265
EM Regional	0.4672	0.2341	0.2838	0.0683	0.7682	0.7682	0.2717	0.1398	0.5304	0.0000	0.2992	0.4435
EM Country Rating	0.4808	0.2299	0.2695	0.0786	0.7701	0.7701	0.2540	0.2222	0.5565	0.0000	0.2163	0.5603
Global Inflation + EM Regional	0.4984	0.2275	0.2516	0.0921	0.7749	0.7749	0.2795	0.2183	0.5191	0.0000	0.3058	0.5604
Global Inflation + EM Rating	0.5001	0.2260	0.2499	0.0930	0.7749	0.7749	0.2805	0.2189	0.5176	0.0000	0.3057	0.5608
G7 inflation + EM Regional	0.4867	0.2312	0.2635	0.0805	0.7764	0.7764	0.2645	0.2214	0.5409	0.0000	0.2680	0.5566
G7 inflation + EM Rating	0.4550	0.3049	0.2970	0.0057	0.4755	0.8825	0.2796	0.2226	0.5190	0.0000	0.3072	0.5612
Panel C. For the budget constraints 90%												
Longer Bond Maturity	0.3816	0.2415	0.3824	0.0320	0.3553	0.7646	0.1978	0.1276	0.6435	0.0000	0.1983	0.3760
EM Regional	0.4701	0.2377	0.2808	0.0704	0.4961	0.7805	0.2944	0.1643	0.4978	0.0000	0.3346	0.4899
EM Country Rating	0.4408	0.3109	0.3128	0.0000	0.4471	0.8862	0.2600	0.2219	0.5476	0.0000	0.2158	0.5722
Global Inflation + EM Regional	0.4567	0.3065	0.2952	0.0000	0.4746	0.8872	0.2899	0.2182	0.5043	0.0000	0.3079	0.5717
Global Inflation + EM Rating	0.4570	0.3063	0.2948	0.0000	0.4773	0.8867	0.2882	0.2162	0.5067	0.0000	0.2994	0.5720
G7 inflation + EM Regional	0.4456	0.3113	0.3074	0.0000	0.4552	0.8871	0.2704	0.2217	0.5323	0.0000	0.2465	0.5687
G7 inflation + EM Rating	0.4613	0.3065	0.2902	0.0000	0.4834	0.8888	0.2891	0.2206	0.5054	0.0000	0.3044	0.5725

from the existing benchmark allocation policy to longer dated government bond for three different budget allocations. When investor faces only short-sale constraints, the inflation-linked and the emerging market government bond strategy may offer risk reduction benefits, but not for the longer bond maturity strategies, across all three budget allocations. Imposing different budget constraints from 100 percent to 90 percent do not significantly change the magnitude of benefits. The mean of risk reduction are around 0.50 percent or less than 30 percent of the initial risk, and standard deviations are about 0.23 percent. At the 5th percentile distribution of risk reduction from investing in equally-weight G7 inflation-linked combined with emerging market regional strategy, the new optimal portfolio delivers 0.09 percent risk reduction or 80 percent from the initial portfolio risk.

Asset allocation constraint does change the significant of the mean of risk reduction benefits. Inflation-linked strategy able to offer risk reduction benefits only if budget constraint equal to one and 95 percent. None of the 90 percent budget constraints is able to offer risk minimization benefits. The average portfolio risk decreases 0.30 percent or around 50 percent from the initial portfolio risk with the risk reduction 0.01 percent at the fifth percentile. These benefits could be achieved either by emerging market or inflation-linked government bonds spanning strategy.

The mean of risk reduction benefits for the 95 percent budget is only being achievable through inflation-linked strategy. None of longer bond and emerging market strategy is able to deliver benefits when central banks allocate

Table 5.3 Assets weight in the risk minimization for a given benchmark return portfolio for different budget constraints

This table reports posterior distribution of the mean and standard deviations of the optimal weights in the efficient portfolio for a given existing benchmark returns for three different budget constraints Panel A. is longer bond maturity strategy that considers test assets consist of G7 government maturity 5-7 and 7-10 years, DM bond index 1-5 year, 5-7 year and 7-10 year and SSA. Benchmark assets are 15% of US Treasury Bills, 15% of non-US bills and 70% of G7 government bond index 1-5 year. Panel B. is Emerging Market diversification which added test assets EM government bond indices based on country rating (B, BB and BBB) and geographical area (Asia, Europe, Middle East and Africa (EMEA), and Latin America) to the longer bond strategy. Panel C is inflation-linked diversification which added test asset of Global or G7 inflation-linked government bond on top of emerging market diversification.

Bond Index	No-Short Sale						Asset Allocation					
	100 percent		95 percent		90 percent		100 percent		95 percent		90 percent	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel A. Longer Bond Maturity												
SSA	0.0689	0.1158	0.0380	0.0690	0.0374	0.0687	0.0514	0.0475	0.0218	0.0384	0.0214	0.0387
DM 7-10 year	0.0053	0.0238	0.0058	0.0254	0.0056	0.0247	0.0045	0.0218	0.0036	0.0229	0.0039	0.0328
DM 5-7 year	0.0110	0.0343	0.0119	0.0355	0.0113	0.0346	0.0107	0.0333	0.0095	0.0318	0.0081	0.0354
DM 1-5 year	0.0124	0.0400	0.0122	0.0401	0.0130	0.0416	0.0536	0.0635	0.0482	0.0604	0.0410	0.0568
G7 7-10 year	0.0130	0.0399	0.0142	0.0419	0.0138	0.0413	0.0118	0.0374	0.0106	0.0348	0.0100	0.0392
G7 5-7 year	0.0088	0.0343	0.0095	0.0360	0.0100	0.0376	0.0397	0.0614	0.0414	0.0630	0.0344	0.0578
UST Bills	0.5451	0.2515	0.5098	0.2455	0.4628	0.2422	0.2699	0.0561	0.2723	0.0605	0.2761	0.0613
Non US Bills	0.0311	0.0753	0.0303	0.0716	0.0278	0.0659	0.0301	0.0561	0.0277	0.0456	0.0239	0.0610
G7 1-5 year	0.3044	0.2052	0.3184	0.1994	0.3182	0.1980	0.5284	0.0921	0.5148	0.0820	0.4812	0.0901

Bond Index	No-Short Sale						Asset Allocation					
	100 percent		95 percent		90 percent		100 percent		95 percent		90 percent	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel B. Emerging Market Government Bond												
Panel B.1. Geographical												
EM Asia	0.0232	0.0486	0.0242	0.0496	0.0231	0.0482	0.0255	0.0385	0.0247	0.0380	0.0222	0.0369
EM EMEA	0.0317	0.0602	0.0334	0.0621	0.0333	0.0627	0.0248	0.0381	0.0252	0.0380	0.0247	0.0381
EM LATAM	0.0242	0.0519	0.0245	0.0525	0.0259	0.0549	0.0262	0.0389	0.0255	0.0386	0.0233	0.0376
SSA	0.0542	0.0971	0.0395	0.0666	0.0399	0.0666	0.0507	0.0468	0.0236	0.0389	0.0240	0.0394
DM 7-10 year	0.0051	0.0221	0.0046	0.0209	0.0048	0.0218	0.0036	0.0194	0.0036	0.0224	0.0041	0.0311
DM 5-7 year	0.0103	0.0315	0.0101	0.0313	0.0093	0.0297	0.0086	0.0294	0.0073	0.0278	0.0069	0.0310
DM 1-5 year	0.0165	0.0435	0.0162	0.0437	0.0164	0.0436	0.0721	0.0696	0.0643	0.0673	0.0546	0.0637
G7 7-10 year	0.0075	0.0294	0.0066	0.0273	0.0068	0.0277	0.0070	0.0283	0.0066	0.0284	0.0064	0.0359
G7 5-7 year	0.0087	0.0316	0.0088	0.0314	0.0089	0.0320	0.0281	0.0513	0.0297	0.0511	0.0232	0.0447
US Bills	0.5520	0.2599	0.5195	0.2559	0.4732	0.2488	0.2677	0.0667	0.2736	0.0601	0.2772	0.0597
Non US Bills	0.0295	0.0680	0.0275	0.0638	0.0246	0.0573	0.0323	0.0577	0.0264	0.0576	0.0228	0.0587
G7 1-5 year	0.2371	0.1969	0.2353	0.1945	0.2337	0.1915	0.4534	0.1039	0.4393	0.0924	0.4106	0.1022
Panel B.2. Country Rating												
EM B	0.0251	0.0421	0.0286	0.0450	0.0317	0.0588	0.0473	0.0422	0.0242	0.0392	0.0243	0.0422
EM BB	0.0245	0.0492	0.0254	0.0499	0.0251	0.0644	0.0205	0.0333	0.0097	0.0275	0.0102	0.0307
EM BBB	0.0302	0.0633	0.0289	0.0619	0.0313	0.0815	0.0111	0.0273	0.0056	0.0217	0.0055	0.0231
SSA	0.0536	0.0956	0.0392	0.0660	0.0604	0.1361	0.0492	0.0462	0.0102	0.0318	0.0058	0.0312
DM 7-10 yr	0.0049	0.0220	0.0042	0.0197	0.0238	0.0812	0.0042	0.0210	0.0113	0.0493	0.0110	0.0535
DM 5-7yr	0.0098	0.0304	0.0094	0.0297	0.0221	0.0842	0.0082	0.0291	0.0060	0.0377	0.0057	0.0403
DM 1-5yr	0.0171	0.0441	0.0170	0.0436	0.0715	0.1505	0.0752	0.0713	0.0991	0.1380	0.0762	0.1179
G7 7-10yr	0.0075	0.0289	0.0064	0.0269	0.0246	0.0800	0.0072	0.0288	0.0156	0.0638	0.0173	0.0744
G7 5-7yr	0.0088	0.0311	0.0084	0.0297	0.0301	0.0997	0.0299	0.0524	0.0095	0.0458	0.0071	0.0413
US Bills	0.5543	0.2664	0.5281	0.2548	0.1029	0.1796	0.2652	0.0697	0.2204	0.0774	0.2215	0.0821
Non US Bills	0.0315	0.0707	0.0274	0.0631	0.2416	0.2964	0.0348	0.0587	0.0830	0.0772	0.0867	0.0815
G7 1-5 yr	0.2326	0.1993	0.2269	0.1908	0.2349	0.2632	0.4472	0.1045	0.4756	0.2007	0.4778	0.2015

Bond Index	No-Short Sale						Asset Allocation					
	100 percent		95 percent		90 percent		100 percent		95 percent		90 percent	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel C. Inflation Linked Government Bond												
Panel C.1. Global Government Inflation-Linked Bond												
GGIL	0.0393	0.0811	0.0430	0.0626	0.0368	0.0784	0.0275	0.0521	0.0238	0.0518	0.0215	0.0532
EM Asia	0.0275	0.0542	0.0222	0.0398	0.0280	0.0549	0.0245	0.0374	0.0228	0.0374	0.0224	0.0388
EM EMEA	0.0199	0.0566	0.0192	0.0437	0.0201	0.0573	0.0107	0.0274	0.0101	0.0275	0.0101	0.0284
EM LATAM	0.0232	0.0685	0.0182	0.0500	0.0238	0.0702	0.0070	0.0229	0.0059	0.0216	0.0055	0.0218
SSA	0.0524	0.1294	0.0413	0.0659	0.0513	0.1266	0.0135	0.0320	0.0098	0.0305	0.0063	0.0298
DM 7-10 year	0.0884	0.1747	0.0195	0.0454	0.0704	0.1483	0.1384	0.1578	0.1116	0.1387	0.0876	0.1192
DM 5-7 year	0.0176	0.0738	0.0088	0.0282	0.0214	0.0827	0.0085	0.0429	0.0068	0.0401	0.0059	0.0393
DM 1-5 year	0.0221	0.0771	0.0039	0.0191	0.0223	0.0775	0.0129	0.0506	0.0109	0.0471	0.0110	0.0498
G7 7-10 year	0.0205	0.0715	0.0051	0.0242	0.0223	0.0748	0.0132	0.0515	0.0131	0.0579	0.0162	0.0681
G7 5-7 year	0.0284	0.0939	0.0075	0.0286	0.0276	0.0935	0.0129	0.0523	0.0099	0.0456	0.0067	0.0379
US Bills	0.1467	0.2236	0.5389	0.2522	0.1117	0.1881	0.2156	0.0738	0.2173	0.0763	0.2204	0.0801
Non US Bills	0.3131	0.3376	0.0271	0.0606	0.2547	0.3001	0.0844	0.0738	0.0853	0.0761	0.0860	0.0794
G7 1-5 year	0.2008	0.2459	0.1953	0.1884	0.2095	0.2561	0.4310	0.1970	0.4385	0.2000	0.4385	0.2005
GGIL	0.0402	0.0826	0.0433	0.0632	0.0386	0.0800	0.0270	0.0517	0.0235	0.0516	0.0223	0.0540
EM B	0.0278	0.0548	0.0222	0.0395	0.0277	0.0553	0.0246	0.0373	0.0225	0.0371	0.0224	0.0389
EM BB	0.0191	0.0548	0.0196	0.0439	0.0200	0.0564	0.0102	0.0269	0.0101	0.0272	0.0098	0.0285
EM BBB	0.0237	0.0702	0.0180	0.0494	0.0235	0.0705	0.0068	0.0227	0.0060	0.0218	0.0061	0.0232
SSA	0.0526	0.1281	0.0421	0.0664	0.0504	0.1254	0.0140	0.0326	0.0094	0.0299	0.0062	0.0297
DM 7-10 yr	0.0884	0.1746	0.0190	0.0436	0.0725	0.1505	0.1390	0.1597	0.1133	0.1407	0.0862	0.1190
DM 5-7yr	0.0185	0.0759	0.0086	0.0281	0.0205	0.0801	0.0078	0.0408	0.0069	0.0402	0.0055	0.0384
DM 1-5yr	0.0206	0.0720	0.0035	0.0181	0.0223	0.0773	0.0124	0.0496	0.0111	0.0477	0.0107	0.0490
G7 7-10yr	0.0208	0.0717	0.0048	0.0230	0.0211	0.0724	0.0131	0.0503	0.0123	0.0547	0.0169	0.0697
G7 5-7yr	0.0290	0.0959	0.0078	0.0297	0.0285	0.0954	0.0128	0.0530	0.0094	0.0447	0.0066	0.0377
US Bills	0.1446	0.2197	0.5399	0.2511	0.1110	0.1895	0.2153	0.0738	0.2187	0.0766	0.2204	0.0800
Non US Bills	0.3172	0.3385	0.0280	0.0629	0.2551	0.3005	0.0847	0.0738	0.0839	0.0762	0.0859	0.0791
G7 1-5 yr	0.1973	0.2452	0.1932	0.1886	0.2087	0.2563	0.4323	0.1967	0.4384	0.1999	0.4389	0.1989

Bond Index	No-Short Sale						Asset Allocation					
	100 percent		95 percent		90 percent		100 percent		95 percent		90 percent	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel C.2. Group G7 Inflation Linked Bond												
G7 IL	0.0402	0.0826	0.0591	0.0797	0.0477	0.0954	0.0297	0.0540	0.0241	0.0527	0.0213	0.0540
EM Asia	0.0278	0.0548	0.0167	0.0410	0.0207	0.0573	0.0129	0.0305	0.0118	0.0300	0.0122	0.0324
EM EMEA	0.0191	0.0548	0.0193	0.0490	0.0231	0.0665	0.0136	0.0302	0.0121	0.0293	0.0110	0.0287
EM LATAM	0.0237	0.0702	0.0172	0.0448	0.0223	0.0614	0.0127	0.0305	0.0112	0.0298	0.0115	0.0320
SSA	0.0526	0.1281	0.0443	0.0683	0.0529	0.1279	0.0139	0.0323	0.0095	0.0305	0.0049	0.0289
DM 7-10 year	0.0884	0.1746	0.0201	0.0461	0.0693	0.1464	0.1250	0.1541	0.1025	0.1352	0.0798	0.1157
DM 5-7 year	0.0185	0.0759	0.0099	0.0303	0.0210	0.0808	0.0080	0.0424	0.0060	0.0372	0.0057	0.0401
DM 1-5 year	0.0206	0.0720	0.0050	0.0225	0.0235	0.0798	0.0124	0.0502	0.0109	0.0486	0.0102	0.0487
G7 7-10 year	0.0208	0.0717	0.0047	0.0230	0.0228	0.0765	0.0140	0.0547	0.0135	0.0607	0.0173	0.0745
G7 5-7 year	0.0290	0.0959	0.0070	0.0284	0.0285	0.0946	0.0128	0.0529	0.0091	0.0437	0.0057	0.0365
US Bills	0.1446	0.2197	0.5173	0.2529	0.1056	0.1809	0.2179	0.0742	0.2200	0.0770	0.2231	0.0812
Non US Bills	0.3172	0.3385	0.0274	0.0624	0.2451	0.2963	0.0821	0.0742	0.0831	0.0767	0.0843	0.0803
G7 1-5 year	0.1973	0.2452	0.2020	0.1928	0.2175	0.2613	0.4450	0.2002	0.4545	0.2055	0.4579	0.2057
G7 IL	0.0439	0.0902	0.0427	0.0894	0.0426	0.0878	0.0264	0.0515	0.0224	0.0508	0.0203	0.0524
EM B	0.0256	0.0522	0.0257	0.0522	0.0265	0.0540	0.0243	0.0371	0.0223	0.0366	0.0217	0.0383
EM BB	0.0187	0.0551	0.0183	0.0541	0.0189	0.0552	0.0107	0.0274	0.0097	0.0268	0.0095	0.0278
EM BBB	0.0221	0.0682	0.0217	0.0679	0.0209	0.0666	0.0065	0.0223	0.0058	0.0217	0.0057	0.0222
SSA	0.0525	0.1279	0.0546	0.1312	0.0525	0.1269	0.0126	0.0310	0.0091	0.0298	0.0053	0.0285
DM 7-10 yr	0.0900	0.1745	0.0817	0.1623	0.0728	0.1496	0.1364	0.1589	0.1133	0.1418	0.0883	0.1208
DM 5-7yr	0.0191	0.0763	0.0219	0.0844	0.0203	0.0796	0.0077	0.0411	0.0058	0.0368	0.0061	0.0410
DM 1-5yr	0.0231	0.0782	0.0222	0.0763	0.0261	0.0844	0.0122	0.0497	0.0109	0.0483	0.0104	0.0498
G7 7-10yr	0.0209	0.0718	0.0228	0.0763	0.0235	0.0770	0.0133	0.0522	0.0127	0.0573	0.0166	0.0710
G7 5-7yr	0.0288	0.0951	0.0300	0.0979	0.0278	0.0926	0.0130	0.0537	0.0087	0.0421	0.0065	0.0384
US Bills	0.1386	0.2163	0.1254	0.2009	0.1084	0.1848	0.2157	0.0739	0.2175	0.0766	0.2203	0.0803
Non US Bills	0.3177	0.3374	0.2757	0.3156	0.2532	0.3010	0.0843	0.0739	0.0853	0.0763	0.0862	0.0794
G7 1-5 yr	0.1991	0.2470	0.2073	0.2524	0.2065	0.2560	0.4369	0.1992	0.4431	0.2032	0.4424	0.2029

more cash outside the portfolio. Diversification benefits could only be achieved by inflation-linked diversification. Interestingly, even though, the average benefits from the new optimal portfolio resulted from G7 inflation-linked bond combined with emerging market rating and global inflation-linked deliver significant benefit at 0.32 percent and 0.31 percent or only 45 percent from the initial benchmark risk, at the fifth percentile of distribution instead of reducing portfolio risk, it increases the portfolio risk for 0.00 or 100.88 percent higher than the initial benchmark portfolio risks. These two strategies bring relatively high diversification benefits uncertainty.

When a central bank needs to allocate more cash up to 10 percent of their portfolio, none of those spanning strategies considered in the scenario offers diversification benefits. Even though, the average risk reduction benefits are higher relative to the previous budget scenarios, all of them have a higher variance of the expected benefits. At the fifth percentile of the distribution all strategies, instead of reducing risk, this spanning strategy increases portfolio risks.

Table 5.3 shows the posterior distribution of the mean and standard deviation of the optimal weight in the mean-variance portfolio for a given benchmark returns for three different budget constraints. Table 5.3 Panel A reports the results of the mean of the optimal weights of the efficient portfolio by investing in longer bond maturity strategy, Table 5.3 Panel B by investing in emerging market strategy, and Table 5.3 Panel C by investing in inflation-linked bond strategy. Those three panels show that imposing asset allocation constraints generally deliver more diversified portfolio with less variation in weights than it delivered before asset allocation constraints are imposed. These results indicate that the requirement to preserve more cash outside the portfolio optimisation may deliver higher uncertainty of the risk reduction benefits of government portfolio to switch their investment from its current benchmark policy. Hence these results imply that the portfolio segregation to cover day-to-day foreign transactions is not necessarily suitable for government bond risk reductions portfolio.

The results show that when more cash is needed to be allocated outside the portfolio, it tends to deliver higher uncertainty of the expected benefits. The impact of varying budget constraint provides an implication that central bank should not isolate cash from the investible portfolio. The idea that some central banks segregate portfolio tranches to cover specific objectives may come from Caballero and Panageas (2005) suggestions. Our results, however, confirm Kotz and Strauss-Kahn (2007) claims that central banks should not tranche their portfolio. These findings show that for the risk-based framework for foreign reserves optimisation, the more cash allocated outside the investible portfolio, the less likely the diversification strategy will deliver risk minimisation benefits.

5.3.3. Impact of liquidity cushion allocation

Central banks face a number of conflicting objectives that arise in reserve management, include liquidity considerations versus storage of national wealth considerations. When an economy faces rapid capital outflows, there tends to be massive pressure to depreciate the currency. Monetary authorities have a limited set of policy choices to safeguard this currency pressure. Dominguez, Hashimoto, and Ito (2012) argue that central banks can (1) tolerate the exchange rate to decline, (2) sell foreign reserves to protect the exchange rate, (3) increase the interest rate in order to discourage capital outflows, (4) impose capital controls, or (5) use a combination of all of the above. Furthermore, Dominguez et al (2012) say that if the pressure is moderate, central banks often tolerate the exchange rate to decline. However, if the pressure is strong, concerns normally arise that depreciation will be excessive and may boost further capital outflows, which could quickly affect in a systemic crisis in the financial institutions. In these circumstances, the central bank typical choice is to use of foreign reserves to absorb capital outflow and to moderate the speed of currency depreciation or to reduce currency fluctuation.

The rationale behind liquidity consideration is that foreign exchange reserves represent a contingency cushion to support an exchange rate policy and to support external liabilities. Bakker and van Herpt (2007) argue this liquidity requirement dictate reserves management at the central bank to preserve their foreign currency assets, even in the short term, and must be held in highly liquid securities. However, Johnson-Calari, Grava, and Kobor, (2007) document that over few years this traditional image of reserves management has shown some changes because of the accumulation of reserves to a record level, mainly in Asia and oil-exporting countries. As the foreign reserves grow larger, they are less likely that all to be needed to cover financial contingency. The traditional consideration of keeping reserves in liquid assets in order to cover for intervention needs has waned to some extent. I could expect that as the size increased, the less portion of the reserve has to be kept in highly liquid assets. Kotz and Strauss-Kahn (2007) claim, however, that minimum of 20 percent level of gross reserve must be held in highly liquid securities to cover contingency plan.

In order to capture these issues and the fact that liquidity needs may vary from one central bank to another, this research examines whether varying liquidity assets constraints change the risk reduction benefits of government bond portfolio. Based on the survey from earlier empirical chapter and Kotz and Strauss-Kahn (2007) claim, hence 20 to 40 percent liquid assets portions of the official reserve will be adopted in this analysis. Here the research measures diversification benefits for three different liquidity allocations to be held 20 percent, 30 percent and 40 percent in treasury bills while maintaining US treasury bills allocation at least 15 percent. Table 5.4 reports the basic statistics of the distribution of risk reduction for a given expected returns for the three different liquid assets allocations.

Table 5.4 presents the posterior distribution of risk reduction for a given expected benchmark returns when reserve portfolio manager switches their investment to test assets for three different liquidity buffer scenarios. Investing in emerging market government bond does deliver diversification benefits only for 40 percent liquid assets allocation scenario for both no short sale and asset allocation constraints. Some of spanning strategies in 20 percent liquidity allocation provide diversification benefits before imposing asset allocation, but the all the benefits at the fifth percentile disappear after asset allocation constraints are imposed. None of the longer maturity diversification strategy

Table 5.4 Posterior distribution of risk reduction for a given benchmark returns for different liquidity buffer

This table reports the posterior distribution of risk reduction for a given expected benchmark returns when investor diversifies their investment for three different liquidity buffers. The first frontier is spanned by longer-dated bonds within G7, DM government and SSA bond market up to 10 years maturity. The next two frontiers are spanned by EM government bond using regional and credit rating bond index. The last four frontiers are spanned by the inclusion of inflation-linked bond using global government and G7 index-linked government bond market. The summary statistics are the mean, standard deviation, the average of actual risk reduction measure, and 5^{th}, 50^{th} and 95^{th} percentile.

Frontier of Benchmark +	No Short-Sale						Asset Allocation					
				Percentile						Percentile		
	Mean	Stdev	$\bar{o}$	5^{th}	50^{th}	95^{th}	Mean	Stdev	$\bar{o}$	5^{th}	50^{th}	95th
Panel A. 30% Treasury Bills												
Longer Bond Maturity	0.3727	0.2326	0.3935	0.0343	0.3479	0.7403	0.1273	0.0970	0.7616	0.0000	0.1150	0.2758
EM Regional	0.4655	0.2275	0.2857	0.0752	0.4932	0.7569	0.2426	0.1217	0.5736	0.0000	0.2662	0.3963
EM Country Rating	0.4374	0.3042	0.3165	0.0089	0.4449	0.8743	0.2485	0.2204	0.5648	0.0000	0.2176	0.5493
Global Inflation + EM Regional	0.4524	0.3011	0.2998	0.0100	0.4678	0.8762	0.2596	0.2145	0.5481	0.0000	0.2694	0.5444
Global Inflation + EM Rating	0.4637	0.2992	0.2876	0.0117	0.4875	0.8781	0.2683	0.2160	0.5354	0.0000	0.2909	0.5487
G7 inflation + EM Regional	0.4472	0.3003	0.3056	0.0106	0.4582	0.8763	0.3118	0.2704	0.4736	0.0030	0.2734	0.7154
G7 inflation + EM Rating	0.4580	0.3004	0.2937	0.0119	0.4811	0.8774	0.3351	0.2684	0.4422	0.0035	0.3295	0.7189
Panel B. 20% Treasury Bills												
Longer Bond Maturity	0.1853	0.2600	0.6637	0.0000	0.0092	0.6692	0.1379	0.1302	0.7432	0.0000	0.0867	0.3668
EM Regional	0.3122	0.1815	0.4731	0.0003	0.3715	0.5225	0.2416	0.1564	0.5752	0.0000	0.2356	0.4615
EM Country Rating	0.3209	0.1812	0.4612	0.0012	0.3846	0.5257	0.2625	0.1637	0.5439	0.0000	0.2737	0.4804
Global Inflation + EM Regional	0.3443	0.1769	0.4299	0.0100	0.4132	0.5338	0.3026	0.1739	0.4864	0.0000	0.3380	0.5169
Global Inflation + EM Rating	0.3461	0.2785	0.4276	0.0000	0.3947	0.7168	0.2756	0.2283	0.5248	0.0000	0.2784	0.5821
G7 inflation + EM Regional	0.3345	0.2846	0.4429	0.0000	0.3863	0.7152	0.2582	0.2291	0.5502	0.0000	0.2442	0.5767
G7 inflation + EM Rating	0.3474	0.2839	0.4259	0.0000	0.4038	0.7190	0.2712	0.2308	0.5312	0.0000	0.2689	0.5813
Panel C. 40% Treasury Bills												
Longer Bond Maturity	0.1149	0.0624	0.7835	0.0000	0.1288	0.1981	0.1085	0.0605	0.7947	0.0000	0.1192	0.1915
EM Regional	0.2572	0.1071	0.5518	0.0305	0.2906	0.3801	0.2284	0.0926	0.5953	0.0304	0.2566	0.3351
EM Country Rating	0.2647	0.1067	0.5407	0.0368	0.2993	0.3853	0.2426	0.0975	0.5737	0.0356	0.2718	0.3543
Global Inflation + EM Regional	0.2822	0.1048	0.5152	0.0530	0.3172	0.3963	0.2700	0.1053	0.5329	0.0455	0.3041	0.3888
Global Inflation + EM Rating	0.3558	0.2733	0.4150	0.0023	0.4019	0.7126	0.2703	0.1947	0.5325	0.0023	0.3244	0.5029
G7 inflation + EM Regional	0.3486	0.2761	0.4243	0.0021	0.3976	0.7139	0.2511	0.1976	0.5608	0.0019	0.2904	0.4987
G7 inflation + EM Rating	0.3620	0.2735	0.4071	0.0022	0.4173	0.7160	0.2688	0.1966	0.5347	0.0021	0.3288	0.5026

across all the liquidity allocations provides greater than zero benefits at the 5th percentile.

The 40 percent treasury bills allocation may deliver 0.26-0.35 percent of risk reduction or less than one third from the initial portfolio risks. Imposing more restrictive asset allocation constraints change the location of risk reduction distribution toward zero but it does not change the significance of the spanning strategy. Imposing asset allocation constraints reduce the average benefits to 0.22 -0.27 percent and 0.02 to 0.03 percent at the 5th percentile of the posterior distribution of the benefits. For this liquidity allocation, only longer bond maturity strategy fails to offer risk reduction benefits even before imposing asset allocations. All the emerging market and inflation-linked spanning strategies are able to provide greater than zero benefits at the 5th percentile.

The least treasury bills allocation scenario shows that it may provide a similar magnitude of the benefits to the 40 percent liquidity allocation, but the uncertainty is higher for both short sale and asset allocation constraints. When more restricted constraints are imposed, it reduced the benefits and wipes all the benefits. Allowing 20 percent liquidity allocation and imposing asset allocation constraints makes none of those spanning strategies provides diversification benefits for foreign reserve risk minimization portfolio.

Table 5.5 Assets weight in the efficient portfolio for a given expected benchmark returns for different liquidity buffer allocation

This table reports the posterior distribution of the average and standard deviations of the optimal weights in the efficient portfolio for a given existing benchmark returns for three different 30%, 20% and 40% Treasury Bills allocation. Panel A. is longer bond maturity strategy that considers test assets consist of G7 government maturity 5-7 and 7-10 years, DM bond index 1-5 year, 5-7 year and 7-10 year and SSA. Benchmark assets are 15% of US treasury Bills, 15% of non-US bills and 70% of G7 government bond index 1-5 year. Panel B. is Emerging Market Government Bond strategy which added test assets EM government bond indices based on country rating (B, BB and BBB) and geographical area (Asia, Europe, Middle East and Africa (EMEA), and Latin America) to the longer bond strategy. Panel C. is Inflation Linked strategy which added test asset of Global or G7 inflation-linked government bond on top of emerging market diversification.

Bond Index	No-Short Sale						Asset Allocation					
	30 percent		20 percent		40 percent		30 percent		20 percent		40 percent	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel A. Longer Bond Maturity												
SSA	0.0625	0.1418	0.0625	0.1418	0.0935	0.0887	0.0182	0.0358	0.0484	0.0484	0.0573	0.0451
DM 7-10 year	0.0141	0.0643	0.0141	0.0643	0.0011	0.0092	0.0150	0.0576	0.0059	0.0249	0.0013	0.0105
DM 5-7 year	0.0095	0.0552	0.0095	0.0552	0.0038	0.0175	0.0089	0.0462	0.0107	0.0329	0.0040	0.0179
DM 1-5 year	0.1118	0.1951	0.1118	0.1951	0.0755	0.0576	0.0704	0.1274	0.0412	0.0624	0.0740	0.0589
G7 7-10 year	0.0182	0.0717	0.0182	0.0717	0.0022	0.0146	0.0183	0.0627	0.0156	0.0420	0.0032	0.0175
G7 5-7 year	0.0193	0.0807	0.0193	0.0807	0.0333	0.0519	0.0159	0.0614	0.0298	0.0578	0.0424	0.0558
UST Bills	0.2289	0.0744	0.2289	0.0744	0.3642	0.0707	0.2254	0.0746	0.1757	0.0485	0.3607	0.0733
Non US Bills	0.0711	0.0744	0.0711	0.0744	0.0358	0.0707	0.0746	0.0746	0.0243	0.0125	0.0393	0.0637
G7 1-5 year	0.4646	0.3091	0.4646	0.3091	0.3906	0.0968	0.5533	0.1948	0.6484	0.1032	0.4179	0.0796

Bond Index	No-Short Sale						Asset Allocation					
	30 percent		20 percent		40 percent		30 percent		20 percent		40 percent	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel B. Emerging Market Government Bond												
Panel B.1. Geographical												
EM Asia	0.0282	0.0525	0.0358	0.0709	0.0243	0.0405	0.0255	0.0385	0.0254	0.0400	0.0233	0.0340
EM EMEA	0.0797	0.0865	0.0857	0.1061	0.0724	0.0682	0.0249	0.0379	0.0217	0.0376	0.0313	0.0377
EM LATAM	0.0403	0.0628	0.0476	0.0803	0.0332	0.0471	0.0260	0.0388	0.0251	0.0400	0.0263	0.0360
SSA	0.0803	0.1073	0.0878	0.1263	0.0615	0.0742	0.0508	0.0468	0.0489	0.0480	0.0528	0.0442
DM 7-10 year	0.0034	0.0191	0.0040	0.0213	0.0018	0.0126	0.0040	0.0205	0.0052	0.0238	0.0021	0.0140
DM 5-7 year	0.0065	0.0261	0.0068	0.0272	0.0041	0.0195	0.0083	0.0291	0.0104	0.0336	0.0043	0.0195
DM 1-5 year	0.0816	0.0703	0.0775	0.0802	0.0871	0.0578	0.0717	0.0699	0.0567	0.0742	0.0858	0.0597
G7 7-10 year	0.0044	0.0234	0.0056	0.0265	0.0021	0.0147	0.0071	0.0286	0.0120	0.0378	0.0029	0.0168
G7 5-7 year	0.0125	0.0366	0.0113	0.0366	0.0136	0.0362	0.0273	0.0504	0.0275	0.0547	0.0221	0.0427
US Bills	0.2729	0.0611	0.1785	0.0490	0.3683	0.0676	0.2664	0.0674	0.1734	0.0544	0.3651	0.0704
Non US Bills	0.0271	0.0611	0.0215	0.0490	0.0317	0.0676	0.0336	0.0407	0.0266	0.0215	0.0349	0.0647
G7 1-5 year	0.3631	0.1393	0.4379	0.1776	0.2999	0.1056	0.4545	0.1048	0.5672	0.1195	0.3491	0.0863
Panel B.2. Country Rating												
EM B	0.0500	0.0492	0.0582	0.0642	0.0425	0.0365	0.0458	0.0420	0.0466	0.0451	0.0438	0.0360
EM BB	0.0324	0.0569	0.0372	0.0718	0.0275	0.0435	0.0210	0.0337	0.0179	0.0340	0.0248	0.0316
EM BBB	0.0629	0.0782	0.0699	0.0983	0.0559	0.0612	0.0112	0.0274	0.0100	0.0273	0.0132	0.0271
SSA	0.0750	0.1013	0.0853	0.1223	0.0609	0.0729	0.0487	0.0465	0.0480	0.0477	0.0497	0.0438
DM 7-10 yr	0.0030	0.0182	0.0041	0.0221	0.0019	0.0135	0.0038	0.0201	0.0055	0.0244	0.0021	0.0136
DM 5-7yr	0.0068	0.0268	0.0071	0.0277	0.0040	0.0191	0.0083	0.0292	0.0098	0.0326	0.0044	0.0201
DM 1-5yr	0.0866	0.0714	0.0812	0.0821	0.0903	0.0589	0.0746	0.0713	0.0620	0.0772	0.0871	0.0605
G7 7-10yr	0.0044	0.0234	0.0057	0.0272	0.0024	0.0157	0.0074	0.0297	0.0116	0.0376	0.0025	0.0153
G7 5-7yr	0.0128	0.0366	0.0120	0.0380	0.0137	0.0357	0.0295	0.0529	0.0305	0.0574	0.0246	0.0442
US Bills	0.2730	0.0611	0.1771	0.0512	0.3688	0.0666	0.2662	0.0687	0.1739	0.0537	0.3642	0.0711
Non US Bills	0.0270	0.0611	0.0229	0.0512	0.0312	0.0666	0.0338	0.0377	0.0261	0.0445	0.0358	0.0537
G7 1-5 yr	0.3661	0.1368	0.4394	0.1736	0.3007	0.1031	0.4496	0.1038	0.5580	0.1217	0.3478	0.0863

Bond Index	No-Short Sale						Asset Allocation					
	30 percent		20 percent		40 percent		30 percent		20 percent		40 percent	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel C. Inflation Linked Government Bond												
Panel C.1. Global Government Inflation-Linked Bond												
GGIL	0.0517	0.0634	0.0611	0.0827	0.0464	0.0474	0.0692	0.0585	0.0753	0.0654	0.0579	0.0472
EM Asia	0.0478	0.0468	0.0550	0.0599	0.0388	0.0335	0.0419	0.0393	0.0430	0.0431	0.0380	0.0330
EM EMEA	0.0285	0.0508	0.0327	0.0633	0.0254	0.0394	0.0205	0.0322	0.0187	0.0335	0.0219	0.0301
EM LATAM	0.0436	0.0642	0.0491	0.0804	0.0387	0.0507	0.0127	0.0274	0.0107	0.0272	0.0159	0.0276
SSA	0.0684	0.0952	0.0799	0.1149	0.0537	0.0669	0.0446	0.0445	0.0436	0.0462	0.0415	0.0416
DM 7-10 year	0.0983	0.0732	0.0964	0.0857	0.1024	0.0583	0.0942	0.0758	0.0826	0.0866	0.1015	0.0614
DM 5-7 year	0.0053	0.0242	0.0062	0.0259	0.0033	0.0174	0.0068	0.0265	0.0083	0.0296	0.0034	0.0176
DM 1-5 year	0.0032	0.0189	0.0037	0.0204	0.0016	0.0123	0.0036	0.0197	0.0050	0.0237	0.0017	0.0123
G7 7-10 year	0.0034	0.0207	0.0042	0.0239	0.0016	0.0131	0.0049	0.0237	0.0082	0.0314	0.0022	0.0144
G7 5-7 year	0.0107	0.0337	0.0108	0.0358	0.0107	0.0322	0.0176	0.0421	0.0202	0.0478	0.0146	0.0363
US Bills	0.2732	0.0613	0.1783	0.0507	0.3717	0.0644	0.2678	0.0673	0.1725	0.0558	0.3666	0.0692
Non US Bills	0.0268	0.0613	0.0217	0.0507	0.0283	0.0644	0.0322	0.0212	0.0275	0.0326	0.0334	0.0587
G7 1-5 year	0.3393	0.1363	0.4009	0.1718	0.2775	0.1020	0.3840	0.1280	0.4845	0.1534	0.3015	0.1014
GGIL	0.0528	0.0640	0.0415	0.0949	0.0324	0.0707	0.0701	0.0589	0.0313	0.0559	0.0208	0.0447
EM B	0.0470	0.0461	0.0352	0.0657	0.0270	0.0483	0.0418	0.0395	0.0266	0.0394	0.0224	0.0346
EM BB	0.0292	0.0504	0.0199	0.0622	0.0147	0.0453	0.0206	0.0325	0.0110	0.0285	0.0107	0.0269
EM BBB	0.0439	0.0656	0.0285	0.0800	0.0204	0.0562	0.0123	0.0272	0.0071	0.0236	0.0066	0.0221
SSA	0.0691	0.0951	0.0426	0.1144	0.0312	0.0835	0.0432	0.0446	0.0155	0.0343	0.0119	0.0297
DM 7-10 yr	0.0989	0.0730	0.2499	0.2741	0.1933	0.2072	0.0937	0.0751	0.1624	0.1934	0.1163	0.1249
DM 5-7yr	0.0053	0.0240	0.0084	0.0517	0.0074	0.0413	0.0062	0.0252	0.0103	0.0524	0.0055	0.0299
DM 1-5yr	0.0034	0.0196	0.0113	0.0574	0.0088	0.0432	0.0039	0.0203	0.0144	0.0577	0.0106	0.0401
G7 7-10yr	0.0035	0.0212	0.0140	0.0633	0.0102	0.0462	0.0048	0.0235	0.0154	0.0608	0.0102	0.0400
G7 5-7yr	0.0105	0.0334	0.0213	0.0829	0.0161	0.0625	0.0182	0.0431	0.0172	0.0671	0.0093	0.0396
US Bills	0.2735	0.0610	0.1761	0.0249	0.2653	0.1216	0.2660	0.0689	0.1732	0.0249	0.2489	0.1200
Non US Bills	0.0265	0.0610	0.0239	0.0250	0.1347	0.1216	0.0340	0.0447	0.0268	0.0250	0.1511	0.1200
G7 1-5 yr	0.3364	0.1349	0.3277	0.3497	0.2386	0.2568	0.3852	0.1277	0.4891	0.2415	0.3756	0.1494

Bond Index	No-Short Sale						Asset Allocation					
	30 percent		20 percent		40 percent		30 percent		20 percent		40 percent	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel C.2. Group G7 Inflation Linked Bond												
G7 IL	0.0472	0.0955	0.0509	0.1078	0.0405	0.0818	0.0298	0.0538	0.0342	0.0577	0.0235	0.0471
EM Asia	0.0169	0.0529	0.0203	0.0625	0.0146	0.0449	0.0137	0.0314	0.0146	0.0328	0.0124	0.0292
EM EMEA	0.0251	0.0654	0.0280	0.0748	0.0220	0.0564	0.0133	0.0300	0.0135	0.0307	0.0126	0.0285
EM LATAM	0.0203	0.0584	0.0240	0.0680	0.0176	0.0499	0.0123	0.0302	0.0130	0.0311	0.0109	0.0278
SSA	0.0370	0.0999	0.0427	0.1153	0.0334	0.0877	0.0133	0.0318	0.0157	0.0345	0.0116	0.0291
DM 7-10 year	0.2085	0.2355	0.2337	0.2675	0.1809	0.2025	0.1254	0.1541	0.1453	0.1854	0.1031	0.1200
DM 5-7 year	0.0083	0.0480	0.0095	0.0556	0.0079	0.0448	0.0071	0.0392	0.0092	0.0508	0.0063	0.0327
DM 1-5 year	0.0111	0.0528	0.0117	0.0589	0.0097	0.0463	0.0121	0.0485	0.0142	0.0573	0.0100	0.0388
G7 7-10 year	0.0131	0.0581	0.0143	0.0658	0.0118	0.0499	0.0126	0.0514	0.0164	0.0639	0.0095	0.0386
G7 5-7 year	0.0176	0.0714	0.0212	0.0848	0.0155	0.0625	0.0123	0.0514	0.0168	0.0680	0.0092	0.0390
US Bills	0.2266	0.0740	0.1769	0.0249	0.2654	0.1215	0.2167	0.0740	0.1742	0.0250	0.2554	0.1210
Non US Bills	0.0734	0.0740	0.0232	0.0250	0.1346	0.1215	0.0833	0.0740	0.0258	0.0250	0.1446	0.1210
G7 1-5 year	0.2951	0.3124	0.3442	0.3614	0.2460	0.2624	0.4481	0.2014	0.5074	0.2458	0.3911	0.1546
G7 IL	0.0438	0.0919	0.0483	0.1032	0.0395	0.0787	0.0278	0.0525	0.0316	0.0562	0.0213	0.0451
EM B	0.0290	0.0555	0.0313	0.0621	0.0246	0.0466	0.0243	0.0372	0.0253	0.0386	0.0219	0.0341
EM BB	0.0159	0.0518	0.0174	0.0583	0.0141	0.0444	0.0109	0.0277	0.0106	0.0278	0.0105	0.0266
EM BBB	0.0203	0.0623	0.0244	0.0739	0.0179	0.0553	0.0066	0.0223	0.0070	0.0233	0.0066	0.0218
SSA	0.0366	0.0991	0.0403	0.1111	0.0327	0.0866	0.0133	0.0317	0.0146	0.0336	0.0112	0.0289
DM 7-10 yr	0.2183	0.2388	0.2486	0.2755	0.1906	0.2045	0.1376	0.1588	0.1576	0.1937	0.1141	0.1244
DM 5-7yr	0.0089	0.0496	0.0098	0.0570	0.0084	0.0451	0.0083	0.0434	0.0101	0.0526	0.0063	0.0324
DM 1-5yr	0.0107	0.0514	0.0116	0.0581	0.0097	0.0456	0.0117	0.0479	0.0143	0.0587	0.0104	0.0404
G7 7-10yr	0.0135	0.0585	0.0143	0.0652	0.0113	0.0480	0.0125	0.0507	0.0161	0.0619	0.0098	0.0382
G7 5-7yr	0.0178	0.0707	0.0212	0.0848	0.0167	0.0643	0.0128	0.0518	0.0157	0.0647	0.0086	0.0375
US Bills	0.2240	0.0741	0.1767	0.0249	0.2631	0.1212	0.2142	0.0737	0.1735	0.0249	0.2516	0.1205
Non US Bills	0.0760	0.0741	0.0234	0.0249	0.1369	0.1212	0.0858	0.0737	0.0266	0.0250	0.1484	0.1205
G7 1-5 yr	0.2852	0.3074	0.3331	0.3561	0.2344	0.2577	0.4343	0.1987	0.4974	0.2444	0.3792	0.1514

Table 5.5 reports the posterior distribution of the average and standard deviations of the portfolio weights for a given existing benchmark returns resulted from seven spanning strategies for different liquidity buffer allocation. Imposing asset allocation constraints for 40 percent liquidity buffers, total weights for the *n* assets is 22 percent and existing benchmark assets weights become 77 percent of the total portfolio. This strategy requires the reserves to be held 2 percent in an inflation-linked government bond, 3.5 percent in emerging market government bond and one percent in SSA, 12-13 percent in developed market and 2 percent in G7 5-10 year maturity bond index.

The results from three different liquidity buffer scenarios confirm that one of the main sources of risk reduction is driven by US treasury bills. Allowing the new optimal portfolio to hold US treasury bills to replicate its weights in the benchmark provide significant benefits. These findings provide practical implications. The size of the reserves relative to their economy, risk aversion, or economic conditions which could have an impact on their liquidity portions have important role for the risk minimization analysis. Smaller the size of the reserve relative to the economy, the more risk averse of a central bank, and negative economic environments could have impact on the bigger liquidity allocation. This bigger liquidity allocation in turn could increase the chance to gain the benefits. For the 40 percent liquidity allocation, significant benefits for foreign reserve portfolios could be attained though emerging market and inflation-linked diversification.

5.3.4. Impact of global financial crisis

It is well known that recent bond yields were quite different from those before the 2010s due to low interest rate policy taken by major central banks following the sovereign debt crisis in some European countries. Therefore, it is reasonable to doubt that government bond diversification benefits and the impact of asset allocation constraints might have changed as prolonged low bond yield environment. It is also likely to question if the results on the second empirical chapter are mainly influenced by low interest rate policy since global financial crisis. To examine these problems, this research measures the risk reduction benefits separately for the period from December 1985 to December 2006 and for the period from January 2007 to December 2014. This research looks at the risk reduction benefits both constrained to be non-negative weight and asset allocations. Table 5.6 presents the basic statistics of the distribution of risk reduction for a given expected returns for the two sub-periods.

Table 5.6 shows that two diversification strategies, longer bond maturity and emerging market regional, fail to deliver benefits for both sub-periods even before imposing asset allocation constraint. Before asset allocation constraints are imposed, during the period of 1985 to 2006, inflation-linked spanning strategy may be able to offer a mean of risk reduction of 0.47 to 0.52 percent with a minimum reduction of 0.06 percent at the fifth percentile of the posterior distribution. These results show that risk reduction benefits are important during both periods of high and low volatility financial market.

Table 5.6 Posterior distribution of risk reduction for a given benchmark return for different sub-period

This table reports the posterior distribution of risk reduction for the same target return as a benchmark when investors diversify their investment for different sub period. The first frontier is spanned by lengthening portfolio duration within G7 government bond market up to 10 years maturity. The next-two strategies is spanned with emerging market government bonds together with longer-dated maturity scenario. The last-4 portfolio is spanned by inflation-linked combined with emerging market strategy. The summary statistics are the mean, standard deviation, the average of actual risk reduction measure, 5th, 50th and 95th percentile.

Frontier of Benchmark +	No Short-Sale						Asset Allocation					
	Mean	Stdev	ō	Percentile			Mean	Stdev	ō	Percentile		
				5th	50th	95th				5th	50th	95th
For the period of 1985-2006												
Longer Bond Maturity	0.3795	0.2483	0.3850	0.0285	0.3472	0.7840	0.1528	0.1267	0.7178	0.0000	0.1222	0.3584
EM Regional	0.1707	0.1491	0.6877	0.0000	0.1484	0.4709	0.1483	0.0959	0.7253	0.0000	0.1431	0.3089
EM Country Rating	0.4726	0.2478	0.2782	0.0642	0.4911	0.8065	0.2386	0.1469	0.5798	0.0005	0.2318	0.4522
Global Inflation + EM Regional	0.5012	0.2463	0.2488	0.0848	0.5276	0.8194	0.2953	0.1592	0.4965	0.0175	0.3172	0.5024
Global Inflation + EM Rating	0.5075	0.2448	0.2426	0.0859	0.5419	0.8176	0.2951	0.1588	0.4969	0.0167	0.3169	0.5019
G7 inflation + EM Regional	0.5107	0.2491	0.2395	0.0828	0.5441	0.8290	0.3128	0.1760	0.4723	0.0132	0.3238	0.5560
G7 inflation + EM Rating	0.5188	0.2443	0.2316	0.0936	0.5521	0.8276	0.3177	0.1743	0.4655	0.0214	0.3286	0.5547
For the period of 2007-2013												
Longer Bond Maturity	0.4021	0.2636	0.3575	0.0340	0.3646	0.8472	0.1379	0.0822	0.7431	0.0000	0.1372	0.2699
EM Regional	0.1733	0.1557	0.6835	0.0000	0.1486	0.5021	0.1482	0.0914	0.7256	0.0000	0.1458	0.3035
EM Country Rating	0.4962	0.2650	0.2538	0.0626	0.5089	0.8716	0.2725	0.1343	0.5292	0.0176	0.3029	0.4489
Global Inflation + EM Regional	0.5073	0.2625	0.2428	0.0689	0.5260	0.8702	0.2660	0.1270	0.5387	0.0162	0.2968	0.4314
Global Inflation + EM Rating	0.5200	0.2632	0.2304	0.0696	0.5491	0.8740	0.2968	0.1400	0.4945	0.0213	0.3330	0.4739
G7 inflation + EM Regional	0.5103	0.2615	0.2399	0.0702	0.5312	0.8714	0.2816	0.1342	0.5162	0.0181	0.3145	0.4551
G7 inflation + EM Rating	0.5184	0.2614	0.2319	0.0734	0.5453	0.8730	0.3049	0.1438	0.4832	0.0250	0.3406	0.4895

Imposing asset allocation constraints reduce the diversification benefits toward zero for both sub-periods. The average benefits, however, different from what this research found in the no short-sale restriction case. The benefits are slightly higher during the first sub-period compare to the more recent one. For the earlier period, diversification benefits are ranging from 0.24 percent for an emerging market regional strategy to 0.32 percent for G7 inflation linked-emerging rating strategy, relatively higher compare to 0.26 and 0.30 percent for the same strategy for 2007-2014 years. If we look at the standard deviation of the benefits, the later period provides more certainty for the benefits as shown by relatively lower standard deviations.

The portfolio risk reduction benefits are relatively the same for the short-sale constrained portfolio and more obvious in the most strategies during low yield environment after imposing asset allocation constraints. The standard deviation of the risk reduction during financial crisis ranged between 0.12 to 0.14 percent, lower than 0.14-0.17 percent for the earlier period.

Table 5.7 reports the posterior distribution of the average and standard deviations of the weights of the optimal portfolio in the efficient portfolio with the same target return as the existing benchmark resulted from seven spanning strategies considered. More specifically this research focuses on the Table 5.7 Panel C1., emerging market country rating approach, the only strategy that has

Table 5.7 Assets weight in the efficient portfolio for a given benchmark return for different sub-period

This table reports the posterior distribution of the average and standard deviations of the optimal weights in the efficient portfolio for a given existing benchmark returns for different sub-period. Panel A. is longer bond maturity strategy that considers test assets consist of G7 government maturity 5-7 and 7-10 years, DM bond index 1-5 year, 5-7 year and 7-10 year and SSA. Benchmark assets are 15% of US Treasury Bills, 15% of non-US bills and 70% of G7 government bond index 1-5 year. Panel B is Emerging Market Government Bond strategy which added test assets EM government bond indices based on country rating (B, BB and BBB) and geographical area (Asia, Europe, Middle East and Africa (EMEA), and Latin America) to the longer bond strategy. Panel C is an Inflation-linked strategy which added test asset of Global or G7 inflation-linked government bond on top of emerging market diversification.

Bond Index	Short-Sale Constraints				Asset Allocation			
	1985-2006		2006-2013		1985-2006		2006-2013	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel A. Longer Bond Maturity								
SSA	0.0571	0.1008	0.0867	0.1645	0.0369	0.0456	0.0735	0.0416
DM 7-10 year	0.0061	0.0274	0.0048	0.0214	0.0055	0.0253	0.0030	0.0163
DM 5-7 year	0.0151	0.0411	0.0070	0.0269	0.0157	0.0416	0.0048	0.0209
DM 1-5 year	0.0160	0.0495	0.0088	0.0325	0.0682	0.0754	0.0276	0.0430
G7 7-10 year	0.0105	0.0367	0.0175	0.0465	0.0094	0.0346	0.0189	0.0454
G7 5-7 year	0.0072	0.0313	0.0091	0.0373	0.0219	0.0502	0.0540	0.0647
UST Bills	0.5247	0.2588	0.5353	0.2591	0.2552	0.0727	0.2771	0.0577
Non US Bills	0.0358	0.0783	0.0281	0.0710	0.0448	0.0727	0.0229	0.0176
G7 1-5 year	0.3275	0.2078	0.3027	0.2073	0.5425	0.0900	0.5183	0.0804
Panel B. Emerging Market								
Geographical								
EM Asia	0.0880	0.1552	0.0145	0.0314	0.0418	0.0467	0.0111	0.0261
EM EMEA	0.0770	0.1550	0.0211	0.0418	0.0167	0.0353	0.0344	0.0387
EM LATAM	0.0518	0.1247	0.0199	0.0386	0.0213	0.0386	0.0257	0.0357
SSA	0.0435	0.0828	0.0758	0.1487	0.0353	0.0449	0.0763	0.0403
DM 7-10 yr	0.0042	0.0209	0.0041	0.0178	0.0061	0.0277	0.0024	0.0162
DM 5-7yr	0.0119	0.0362	0.0060	0.0231	0.0124	0.0361	0.0032	0.0183
DM 1-5yr	0.0115	0.0384	0.0122	0.0352	0.0629	0.0704	0.0456	0.0541
G7 7-10yr	0.0069	0.0281	0.0110	0.0359	0.0092	0.0328	0.0085	0.0324
G7 5-7yr	0.0060	0.0273	0.0102	0.0361	0.0134	0.0374	0.0711	0.0769
US Bills	0.4118	0.2894	0.5745	0.2759	0.2567	0.0757	0.2729	0.0673
Non US Bills	0.0292	0.0702	0.0346	0.0757	0.0433	0.0757	0.0271	0.0147
G7 1-5 yr	0.2581	0.2081	0.2160	0.1988	0.4810	0.0941	0.4218	0.1114
Country Rating								
EM B	0.0924	0.1514	0.0212	0.0310	0.0423	0.0470	0.0481	0.0382
EM BB	0.0332	0.1005	0.0146	0.0328	0.0152	0.0343	0.0161	0.0283
EM BBB	0.0883	0.1573	0.0176	0.0416	0.0230	0.0399	0.0068	0.0208
SSA	0.0425	0.0830	0.0642	0.1387	0.0354	0.0449	0.0715	0.0433
DM 7-10 year	0.0046	0.0222	0.0042	0.0193	0.0054	0.0245	0.0026	0.0161
DM 5-7 year	0.0106	0.0323	0.0064	0.0248	0.0127	0.0358	0.0036	0.0189
DM 1-5 year	0.0133	0.0424	0.0129	0.0377	0.0598	0.0702	0.0411	0.0543
G7 7-10 year	0.0068	0.0275	0.0139	0.0404	0.0099	0.0346	0.0108	0.0371
G7 5-7 year	0.0059	0.0269	0.0130	0.0422	0.0157	0.0405	0.0824	0.0845
US Bills	0.4282	0.2839	0.5700	0.2768	0.2584	0.0708	0.2713	0.0676
Non US Bills	0.0262	0.0641	0.0364	0.0770	0.0416	0.0708	0.0287	0.0279
G7 1-5 year	0.2478	0.1960	0.2257	0.2052	0.4807	0.0956	0.4170	0.1178

Bond Index	No Short-sale				Asset Allocation				No Short-sale				Asset Allocation			
	1985-2006		2007-2013		1985-2006		2007-2013		1985-2006		2007-2013		1985-2006		2007-2013	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
	Panel C1. Global Government Inflation Index								Panel C2. G7 Government Inflation Index							
Geographics																
Inflation-Linked	0.0593	0.0823	0.0362	0.0644	0.0778	0.0662	0.0776	0.0663	0.1212	0.1523	0.0470	0.0769	0.0993	0.0661	0.1003	0.0638
EM Asia	0.0881	0.1517	0.0115	0.0285	0.0451	0.0476	0.0459	0.0473	0.0832	0.1458	0.0109	0.0276	0.0396	0.0467	0.0387	0.0336
EM EMEA	0.0601	0.1330	0.0138	0.0346	0.0163	0.0351	0.0146	0.0336	0.0569	0.1306	0.0129	0.0336	0.0167	0.0355	0.0091	0.0224
EM LATAM	0.0397	0.1065	0.0161	0.0349	0.0198	0.0381	0.0209	0.0382	0.0386	0.1047	0.0152	0.0345	0.0208	0.0388	0.0056	0.0192
SSA	0.0379	0.0783	0.0588	0.1321	0.0314	0.0430	0.0303	0.0426	0.0386	0.0785	0.0576	0.1305	0.0323	0.0436	0.0703	0.0439
DM 7-10 year	0.0137	0.0412	0.0153	0.0411	0.0693	0.0712	0.0692	0.0718	0.0138	0.0399	0.0162	0.0440	0.0508	0.0629	0.0671	0.0681
DM 5-7 year	0.0086	0.0294	0.0068	0.0257	0.0093	0.0305	0.0095	0.0307	0.0087	0.0288	0.0077	0.0278	0.0118	0.0338	0.0043	0.0218
DM 1-5 year	0.0041	0.0206	0.0035	0.0177	0.0045	0.0224	0.0045	0.0222	0.0042	0.0210	0.0043	0.0202	0.0054	0.0248	0.0026	0.0169
G7 7-10 year	0.0060	0.0258	0.0102	0.0347	0.0074	0.0292	0.0070	0.0284	0.0056	0.0250	0.0115	0.0384	0.0099	0.0341	0.0084	0.0340
G7 5-7 year	0.0046	0.0226	0.0133	0.0420	0.0112	0.0341	0.0116	0.0344	0.0050	0.0242	0.0134	0.0428	0.0112	0.0337	0.0758	0.0841
US Bills	0.4477	0.2825	0.5795	0.2786	0.2625	0.0680	0.2616	0.0685	0.4038	0.2784	0.5717	0.2812	0.2637	0.0686	0.2653	0.0768
Non US Bills	0.0251	0.0608	0.0430	0.0826	0.0375	0.0680	0.0384	0.0685	0.0228	0.0591	0.0472	0.0876	0.0363	0.0686	0.0347	0.0188
G7 1-5 year	0.2051	0.1941	0.1921	0.2018	0.4079	0.1195	0.4089	0.1195	0.1975	0.1936	0.1843	0.2011	0.4022	0.1199	0.3335	0.1575
Country Rating																
Inflation-Linked	0.0590	0.0812	0.0339	0.0615	0.0776	0.0663	0.0615	0.0581	0.1212	0.1523	0.0470	0.0769	0.1003	0.0657	0.0845	0.0638
EM B	0.0783	0.1346	0.0181	0.0290	0.0459	0.0473	0.0435	0.0356	0.0832	0.1458	0.0109	0.0276	0.0409	0.0468	0.0387	0.0336
EM BB	0.0310	0.0936	0.0111	0.0286	0.0146	0.0336	0.0110	0.0242	0.0569	0.1306	0.0129	0.0336	0.0146	0.0338	0.0091	0.0224
EM BBB	0.0728	0.1395	0.0113	0.0336	0.0209	0.0382	0.0041	0.0166	0.0386	0.1047	0.0152	0.0345	0.0229	0.0396	0.0056	0.0192
SSA	0.0370	0.0764	0.0607	0.1329	0.0303	0.0426	0.0726	0.0428	0.0386	0.0785	0.0576	0.1305	0.0330	0.0437	0.0703	0.0439
DM 7-10 year	0.0136	0.0404	0.0151	0.0404	0.0692	0.0718	0.0589	0.0624	0.0138	0.0399	0.0162	0.0440	0.0527	0.0641	0.0671	0.0681
DM 5-7 year	0.0083	0.0280	0.0067	0.0252	0.0095	0.0307	0.0035	0.0191	0.0087	0.0288	0.0077	0.0278	0.0118	0.0342	0.0043	0.0218
DM 1-5 year	0.0039	0.0199	0.0035	0.0180	0.0045	0.0222	0.0021	0.0148	0.0042	0.0210	0.0043	0.0202	0.0053	0.0241	0.0026	0.0169
G7 7-10 year	0.0052	0.0234	0.0115	0.0369	0.0070	0.0284	0.0059	0.0279	0.0056	0.0250	0.0115	0.0384	0.0096	0.0334	0.0084	0.0340
G7 5-7 year	0.0051	0.0241	0.0142	0.0438	0.0116	0.0344	0.0799	0.0828	0.0050	0.0242	0.0134	0.0428	0.0114	0.0346	0.0758	0.0841
US Bills	0.4608	0.2827	0.5763	0.2830	0.2616	0.0685	0.2679	0.0729	0.4038	0.2784	0.5717	0.2812	0.2655	0.0664	0.2653	0.0768
Non US Bills	0.0254	0.0609	0.0435	0.0824	0.0384	0.0685	0.0321	0.0473	0.0228	0.0591	0.0472	0.0876	0.0345	0.0664	0.0347	0.0188
G7 1-5 year	0.1997	0.1926	0.1941	0.2020	0.4089	0.1195	0.3570	0.1457	0.1975	0.1936	0.1843	0.2011	0.3977	0.1186	0.3335	0.1575

significant risk reduction benefits for both periods of 1985-2006 and 2007-2014 investment opportunity. Accounting for asset allocation constraints during pre-financial crisis era, total weights for n assets is 30 percent and existing benchmark assets weights become 70 percent of the total portfolio. The weights of the total n assets are much smaller than the 45 percent limit. This strategy requires nine percent of inflation-linked, seven percent of emerging market and three percent of SSA assets holding. The optimized portfolio suggests that n assets holding during 2007-2014 year relatively similar to that of previous period. However, it requires less inflation-index to five percent, and more emerging market and SSA assets to eight and four percent respectively.

Our results show that for government bond portfolio's liquidity and safety framework, diversification benefits are important for before and during financial crisis. The risk reductions are apparent in the most strategies during normal and low yield environment, both before and after imposing asset allocation constraints. Only longer bond maturity and emerging market regional strategy fail to offer diversification benefits, even before asset allocation constraints are imposed.

5.3.5. Impact of ultra-long bond

Extending bond portfolio duration would be a natural step to diversify foreign reserves assets. Since reserves could also mean intergenerational wealth accumulation, it drives central banks to match the long-term liability to future generations with long-dated securities. This advocates that central banks should incorporate asset-liability-matching considering in their investment process, relatively similar to what pension funds and life-insurance companies are doing, even though both the nature and the duration of central banks liabilities to future generations much less clearly defined (Fels, 2007).

Johnson-Calari et al. (2007) convincingly argue that departing from a short duration government portfolio the risk-return trade-off can be significantly improved when duration and credit risk constraints are relaxed. In the first empirical chapter, however, shows that there are no risk reduction benefits by investing in a longer dated bond up to 10 year maturity and moving down the credit curve to quasi-government bonds. Therefore, it is realistic to question whether central banks should relax the maturity constraints beyond 10 years. To answer this issue, this study performs an exercise to include US government bond for more than 10 years maturity and an equally-weight index of longer than 10 year maturity of US, UK and German government bonds. Table 5.8 reports the basic statistics of the distribution of risk reduction for a given expected returns for the inclusion of ultra-long major government bond markets.

This table shows that both government bonds index beyond ten years maturity are able to offer diversification benefits for short-sale constrained but they failed to offer benefits once asset allocation constraints are imposed. The mean of risk reduction for short-sale constrained portfolios are 0.40 percent and 0.39 percent with relatively the same standard deviation of 0.23 percent. When

Table 5.8 Posterior distribution of risk reduction and portfolio weights for a given benchmark returns from the impact of ultra-long bond diversification

Panel A. reports the posterior distribution of risk reduction for a given expected benchmark returns when investor diversifies into a longer bond maturity, and ultra-long government bonds are considered. The first frontier is spanned by US government bond more than 10 year maturity. The second is spanned using equally weight US, UK and Germany government bond more than 10 year maturity. The summary statistics are the mean, standard deviation, the average of actual risk reduction measure, and the 5th, 50th, and 95th percentile. Panel B. presents the posterior distribution of the average and standard deviations of the optimal weights when longer-dated bonds index are included in the optimisation.

	Panel A. Risk Reduction						Asset Allocation					
Long bond maturity +	Mean	Stdev	ō	Percentile			Mean	Stdev	ō	Percentile		
				5th	50th	95th				5th	50th	95th
US 10+ yrs	0.3969	0.2325	0.3637	0.0438	0.3859	0.7441	0.1431	0.0983	0.7342	0.0000	0.1374	0.2917
G3 10+ yrs	0.3897	0.2327	0.3724	0.0405	0.3748	0.7427	0.1407	0.0986	0.7384	0.0000	0.1322	0.2905

Panel B. Asset Weight	US 10+ yrs		G3 10+ yrs		US 10+ yrs		G3 10+ yrs	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
10+ Bond Maturity	0.0181	0.0355	0.0156	0.0344	0.0060	0.0156	0.0055	0.0168
SSA	0.0543	0.1023	0.0653	0.1119	0.0528	0.0472	0.0526	0.0473
DM 7-10 year	0.0043	0.0211	0.0034	0.0189	0.0030	0.0181	0.0029	0.0174
DM 5-7 year	0.0104	0.0328	0.0091	0.0307	0.0100	0.0321	0.0095	0.0312
DM 1-5 year	0.0132	0.0411	0.0125	0.0402	0.0588	0.0644	0.0563	0.0634
G7 7-10 year	0.0056	0.0259	0.0047	0.0243	0.0074	0.0304	0.0076	0.0300
G7 5-7 year	0.0083	0.0332	0.0072	0.0308	0.0430	0.0632	0.0422	0.0621
US Bills	0.5818	0.2567	0.5696	0.2460	0.2650	0.0684	0.2658	0.0660
Non US Bills	0.0312	0.0747	0.0272	0.0705	0.0350	0.0727	0.0342	0.0257
G7 1-5 year	0.2728	0.2087	0.2855	0.2024	0.5191	0.0899	0.5234	0.0889

asset allocation constraints are imposed, it reduces both the mean and standard deviation of risk reduction benefits. Relatively high standard deviation compare to the mean indicates the uncertainty of diversification benefits from the inclusion of bond maturity beyond 10 years. More importantly, the fifth percentile shows those two scenarios deliver zero benefits. These results are different from that of Johnson-Calari et al. (2007) due to some respects; first is the difference of diversification benefits measure. Johnson-Calari et al. (2007) framework focused on return enhancement while in this study the objective of diversification is to minimise the risk for a given expected portfolio returns. Second, it might be due to time-varying investment opportunities. One possible reason why longer dated bond investments did not offer diversification benefits for government bond portfolio because during our observation government bond markets has often experienced flight-to-quality and flight-to-liquidity.

Flight-to-quality occurred when market players suddenly want to decrease their investment exposure to securities bearing credit risk and move to default-free assets (Bernanke and Gertler, 1995), while flight-to-liquidity is when market participants abruptly prefer to hold highly liquid assets such as US Treasury Bonds rather (DeSantis, 2014). Given these arguments, one may expect that investor does not require more premium to hold higher risk instruments hence longer bond strategy and relaxing credit curve to quasi-government bonds market might not be suitable for risk reduction portfolio.

These findings raise some warnings, whether investing in long-dated bonds is an appropriate strategy at this point. The first is that currently the yield curve is extremely flat in most major government bond markets. The second is related to the exceptional amount of liquidity. This excess liquidity has been generated by loose monetary policies around the world in the last several years and to some extent also by the rapid pace of official reserve accumulation. Exceptionally low or negative short-term interest rates have encouraged central banks to move out on the yield curve, consequently leading to flatter yield curve. Thus the yield pick-up to be gained from extending into long-dated bonds is extremely low. After several years of good performance, the risk of an under-performance of long-dated in the period ahead is large, more specifically once central banks started to raise their policy rates. Reserve managers should be aware of this potential risk when looking at the excess returns gained by other central banks that have lengthened their portfolios in the last few years.

These results confirm our findings in the earlier chapter that we could not find empirical evidence that relaxing duration and credit curve constraints to quasi-government able to deliver fewer risk benefits for government bond portfolio. These results are valid after imposing asset allocation constraints. Therefore, this sub-section rejects the hypotheses that the reason I could not document diversification benefits on longer dated bond strategy because of sample selection bias to restrict the investment up to 10 years.

The analysis of the impact of budget constraints, liquidity asset allocation and different time period of investment opportunity shows that the choice of geographic or country rating diversification provides similar magnitudes of diversification benefits. The results are different, however, if emerging market is combined with inflation-linked diversification. It shows that only when country rating diversification included together with global inflation index is able to deliver significant risk reduction across the entire scenario analysis considered. This concludes that it needs global inflation-index to be added together with country rating emerging market, developed government bond 1 to 10 year, group of G7 government bond index 5 to 10 year and SSA bond market to deliver significant benefits for varying budget constraints, different liquidity allocation, and different investment opportunity for both short-sale and asset allocation constraints.

5.3.6. Impact of non-government bond investments

Central banks have traditionally held most of their reserves in gold and government bonds, and within that, mostly in short maturities (Fels, 2007). More central banks are broadening the set of asset classes in which they are prepared to invest. The search for return in a low-yield environment certainly plays an important role, as does in many countries their reserves far exceeds the amount considered necessary for liquidity purposes. Liquidity and safety requirements inherent in reserves management might result in a sub-optimal portfolio when compared to the overall market, as some asset classes must be avoided or restricted in a reserves portfolio. In order to provide higher expected returns, the choice can be sought by the central bank is whether by increasing risk tolerance with the same asset classes or adding new asset classes or risks into the investible assets. A survey among central bank (Pringle and Carver, 2006) shows a general attitude towards riskier assets. Three-quarters of the central banks surveyed said they had introduced new asset classes to their investment. In a few central banks,

Table 5.9 Posterior distribution of risk reduction for a given benchmark returns for non-government bond investments

This table reports the posterior distribution of risk reduction for the same target return as benchmark when investors diversify their investment into non-government asset classes. The frontiers are spanned by investing in single asset classes, gold, ABS, MBS, Corporate Bond, and equity index, dual assets: ABS+MBS and corporate ; and 5 non-bonds asset classes altogether. The summary statistics are the mean, standard deviation, the average of actual risk reduction measure, and 5th, 50th and 95th percentile.

Frontier of Government Bond +	Mean	Stdev	$\bar{\delta}$	Percentile 5th	50th	95th
Panel A. No Short-Sale						
Gold	0.5090	0.2159	0.2411	0.1058	0.5545	0.7649
ABS	0.5077	0.2174	0.2423	0.1107	0.5522	0.7656
MBS	0.4919	0.2256	0.2582	0.0874	0.5354	0.7644
ABS+MBS	0.4983	0.2212	0.2517	0.0927	0.5383	0.7635
Corporate Bond	0.5023	0.2217	0.2477	0.0971	0.5450	0.7672
World Equity	0.5051	0.2192	0.2450	0.1042	0.5464	0.7676
Corporate	0.5222	0.2116	0.2283	0.1184	0.5704	0.7671
All Non Bonds	0.5577	0.1966	0.1956	0.1684	0.6150	0.7709
Panel B. Asset Allocation						
Gold	0.3063	0.1326	0.4812	0.0388	0.3502	0.4547
ABS	0.3053	0.1326	0.4827	0.0375	0.3490	0.4544
MBS	0.2920	0.1407	0.5012	0.0153	0.3315	0.4565
ABS+MBS	0.3093	0.1436	0.4771	0.0211	0.3568	0.4700
Corporate Bond	0.3080	0.1457	0.4788	0.0218	0.3509	0.4753
World Equity	0.3108	0.1445	0.4750	0.0222	0.3550	0.4752
Corporate	0.3320	0.1398	0.4462	0.0435	0.3814	0.4833
All Non Bonds	0.3660	0.1277	0.4020	0.0965	0.4146	0.4968

equities and hedge funds have also been included. In their publication, however, there is no information whether central banks keep maintaining the existing loss tolerance or increase portfolio risk when adding riskier asset classes.

In this section, applying the same framework from the first-two objective function in the earlier section I will also investigate whether central banks' broader investment move beyond quasi-government bond provides significant benefits for a given benchmark returns. A set of broader investment asset classes in this study includes gold, mortgage-backed, asset-backed securities, corporate bonds, and equities in addition to the government bond diversification considered in the earlier chapter and sections.

Table 5.9 reports the posterior distribution of risk minimization for a given benchmark return by including risky assets to the benchmark portfolio. Additional assets considered are Gold, mortgage-backed securities, asset-backed securities, investment grade corporate bond and world equity index. Panel A shows that central banks' broader investment universe diversification offers significant risk reduction benefits for all scenarios and across the constraints. As expected, asset allocation constraints reduce the benefits from that of no asset weight constraints. The means of risk reduction are between 0.30 percent and 0.37 percent. The standard deviations of the benefits for asset constraint indicate that the risk of the benefits to deviate from the suggested value also smaller than that of before asset allocation constraints been imposed.

The table shows that the biggest benefit happens when all non-bond assets are included together with diversified government bond portfolio. This strategy provides average risk reduction of 0.37 percent and could be expected to gain more than 0.09 percent risk reduction at the 5th percentile of the posterior distribution. The other spanning strategies provide potential benefits between 0.01 percent and 0.04 percent at the fifth percentile.

Table 5.10 Risk minimization portfolio weights for a given expected benchmark the impact of non-government bond diversification

This table reports the posterior distribution of the average and standard deviations of the optimal weights in the efficient portfolio for a given existing benchmark returns for non-government bond investments. Panel A. is for short-sale constraints. Panel B. is asset allocation constraints.

	Spanning Strategy															
	Gold		ABS		MBS		ABS+MBS		Corporate Bond		Equity		Corpoate		All Non-Bond	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel A. No Short Sale																
Gold	0.0150	0.0221													0.0108	0.0178
ABS			0.0144	0.0216			0.0337	0.0710							0.0345	0.0543
MBS					0.0364	0.0799	0.0335	0.0767							0.0211	0.0519
Corporate Bond									0.0442	0.0680			0.0122	0.0199	0.0268	0.0613
World Equity											0.0454	0.0685	0.0437	0.0652	0.0119	0.0184
Inflation-linked	0.0363	0.0569	0.0358	0.0562	0.0393	0.0598	0.0360	0.0568	0.0398	0.0594	0.0398	0.0599	0.0370	0.0548	0.0278	0.0444
EM Asia	0.0158	0.0381	0.0156	0.0382	0.0169	0.0398	0.0147	0.0373	0.0142	0.0369	0.0140	0.0368	0.0117	0.0330	0.0088	0.0262
EM EMEA	0.0191	0.0456	0.0185	0.0444	0.0194	0.0468	0.0153	0.0417	0.0145	0.0405	0.0149	0.0409	0.0113	0.0347	0.0073	0.0262
EM LATAM	0.0142	0.0380	0.0144	0.0383	0.0155	0.0413	0.0145	0.0393	0.0152	0.0401	0.0156	0.0408	0.0126	0.0361	0.0078	0.0265
SSA	0.0460	0.0855	0.0474	0.0865	0.0377	0.0799	0.0316	0.0733	0.0453	0.0867	0.0456	0.0872	0.0440	0.0822	0.0291	0.0659
DM 7-10 year	0.0198	0.0440	0.0195	0.0438	0.0211	0.0459	0.0191	0.0430	0.0183	0.0431	0.0177	0.0416	0.0186	0.0418	0.0181	0.0387
DM 5-7 year	0.0082	0.0274	0.0080	0.0266	0.0078	0.0267	0.0079	0.0266	0.0084	0.0274	0.0081	0.0272	0.0067	0.0244	0.0056	0.0210
DM 1-5 year	0.0044	0.0198	0.0049	0.0214	0.0041	0.0198	0.0040	0.0191	0.0047	0.0211	0.0049	0.0216	0.0045	0.0200	0.0040	0.0181
G7 7-10 year	0.0083	0.0286	0.0090	0.0306	0.0068	0.0269	0.0067	0.0257	0.0085	0.0299	0.0080	0.0281	0.0085	0.0288	0.0081	0.0264
G7 5-7 year	0.0062	0.0255	0.0062	0.0260	0.0048	0.0234	0.0046	0.0225	0.0058	0.0246	0.0058	0.0247	0.0059	0.0250	0.0045	0.0207
US Bills	0.6088	0.2488	0.6054	0.2505	0.5755	0.2574	0.5701	0.2560	0.5645	0.2602	0.5690	0.2570	0.5831	0.2543	0.6215	0.2399
G7 Non US Bills	0.0265	0.0618	0.0273	0.0637	0.0275	0.0639	0.0285	0.0646	0.0285	0.0647	0.0273	0.0631	0.0300	0.0642	0.0248	0.0569
G7 1-5 year	0.1714	0.1794	0.1736	0.1814	0.1874	0.1916	0.1797	0.1872	0.1882	0.1878	0.1837	0.1836	0.1700	0.1770	0.1275	0.1559

	Spanning Strategy															
	Gold		ABS		MBS		ABS+MBS		Corporate Bond		Equity		Corpoate		All Non-Bond	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel B. Asset Allocation																
Gold	0.0140														0.0137	0.0157
ABS			0.0137	0.0175			0.0466	0.0441							0.0281	0.0193
MBS					0.0276	0.0238	0.0237	0.0367							0.0452	0.0410
Corporate Bond									0.0346	0.0221			0.0150	0.0172	0.0250	0.0351
World Equity											0.0350	0.0218	0.0264	0.0201	0.0120	0.0181
Inflation-linked	0.0694	0.0563	0.0689	0.0567	0.0676	0.0580	0.0560	0.0528	0.0639	0.0566	0.0653	0.0566	0.0640	0.0533	0.0415	0.0441
EM Asia	0.0208	0.0344	0.0210	0.0346	0.0209	0.0351	0.0201	0.0340	0.0197	0.0342	0.0200	0.0345	0.0189	0.0328	0.0186	0.0310
EM EMEA	0.0316	0.0393	0.0312	0.0393	0.0292	0.0392	0.0262	0.0371	0.0272	0.0380	0.0276	0.0381	0.0295	0.0383	0.0272	0.0355
EM LATAM	0.0233	0.0358	0.0233	0.0358	0.0228	0.0363	0.0237	0.0360	0.0236	0.0364	0.0236	0.0364	0.0231	0.0351	0.0231	0.0334
SSA	0.0447	0.0428	0.0444	0.0428	0.0357	0.0427	0.0298	0.0406	0.0384	0.0427	0.0392	0.0429	0.0401	0.0419	0.0244	0.0373
DM 7-10 year	0.1036	0.0732	0.1031	0.0731	0.0914	0.0725	0.0883	0.0693	0.0864	0.0707	0.0861	0.0710	0.0997	0.0714	0.1034	0.0644
DM 5-7 year	0.0053	0.0232	0.0051	0.0232	0.0062	0.0248	0.0056	0.0233	0.0065	0.0258	0.0064	0.0254	0.0047	0.0221	0.0033	0.0183
DM 1-5 year	0.0035	0.0190	0.0032	0.0180	0.0034	0.0185	0.0033	0.0182	0.0038	0.0201	0.0037	0.0199	0.0032	0.0182	0.0026	0.0160
G7 7-10 year	0.0182	0.0426	0.0184	0.0431	0.0152	0.0391	0.0135	0.0363	0.0177	0.0422	0.0175	0.0414	0.0164	0.0406	0.0132	0.0355
G7 5-7 year	0.0047	0.0235	0.0052	0.0249	0.0045	0.0228	0.0043	0.0221	0.0056	0.0262	0.0056	0.0256	0.0047	0.0235	0.0033	0.0190
US Bills	0.2756	0.0498	0.2752	0.0502	0.2726	0.0517	0.2713	0.0507	0.2722	0.0516	0.2710	0.0521	0.2716	0.0519	0.2726	0.0471
G7 Non US Bills	0.0244	0.0055	0.0248	0.0065	0.0274	0.0075	0.0287	0.0045	0.0278	0.0035	0.0290	0.0045	0.0284	0.0055	0.0274	0.0055
G7 1-5 year	0.3609	0.1177	0.3624	0.1185	0.3755	0.1266	0.3592	0.1249	0.3728	0.1261	0.3700	0.1247	0.3541	0.1200	0.3155	0.1004

Table 5.10 reports the asset weights of the addition of non-bond diversification in the well-diversified government bonds portfolio. The average weight shows some variations when non-bond government assets are included in diversified government bond portfolios. Interestingly there is not much difference in non-bond asset compositions between before and after imposing asset weight constraints. It needs 1 percent of the reserve to be invested in gold, 2.5 percent in ABS, 4.5 percent in MBS, 2.5 percent in corporate bonds and 1 percent in equity in order to obtain the benefits.

However, it is questionable whether it is sensible to extend the investments into gold, mortgage-backed securities, asset-backed securities, corporate bond and equity at this particular point in time. The reason is that risk spreads are extremely tightened and unprecedented amount of excess liquidity. Excess liquidity was created by easy monetary policies around the globe in the last several years and by the rapid increase of central banks reserve accumulation. Negative/low short-term interest rates have encouraged central banks' reserve managers to move out the yield curve, thus leading to flatter curves, and to move out to the risk curve which then compressing risks spreads.

In order to answer the question, I calculated the incremental benefits of non-government investments over well-diversified government bond portfolio. In this analysis the original benchmark is replaced by G7 government inflation-linked, EM government bonds, SSA, selected Developed countries bond market, longer bond maturity from G7 countries, and the original benchmark bond

Table 5.11 Incremental benefits of risk reduction for a given benchmark returns for non-government bond investments

This table reports the posterior distribution of risk reduction for the same target return as benchmark when investors diversify their investment into non-government asset classes. The frontiers are spanned by investing in single asset classes, gold, ABS, MBS, Corporate Bond, and equity index, dual assets: ABS+MBS and corporate ; and 5 non-bonds asset classes altogether. The summary statistics are the mean, standard deviation, the average of actual risk reduction measure, 5^{th}, 50^{th} and 95^{th} percentile.

Frontier of Government Bond +	Mean	Stdev	$\bar{\delta}$	Percentile		
				5^{th}	50^{th}	95^{th}
No Short-Sale						
Gold	0.4870	0.1424	0.2632	0.2125	0.5247	0.6502
ABS	0.4822	0.1435	0.2682	0.2099	0.5173	0.6477
MBS	0.4799	0.1453	0.2705	0.2012	0.5156	0.6488
ABS+MBS	0.4885	0.1416	0.2616	0.2142	0.5248	0.6497
Corporate Bond	0.4895	0.1417	0.2606	0.2129	0.5255	0.6525
World Equity	0.4862	0.1444	0.2640	0.2068	0.5236	0.6529
Corporate	0.5008	0.1384	0.2492	0.2264	0.5435	0.6537
All Non Bonds	0.5218	0.1286	0.2287	0.2599	0.5648	0.6586
Asset Allocation						
Gold	0.1266	0.0312	0.7628	0.0688	0.1299	0.1732
ABS	0.1368	0.0343	0.7451	0.0738	0.1410	0.1868
MBS	0.1290	0.0321	0.7587	0.0703	0.1328	0.1767
ABS+MBS	0.1481	0.0341	0.7257	0.0844	0.1523	0.1971
Corporate Bond	0.1554	0.0371	0.7134	0.0851	0.1609	0.2080
World Equity	0.1555	0.0367	0.7132	0.0855	0.1609	0.2067
Corporate	0.2800	0.0787	0.5183	0.1249	0.3007	0.3704
All Non Bonds	0.2786	0.0793	0.5204	0.1217	0.3002	0.3700

indexes. From that new benchmark, risk reduction portfolio for a given expected return then to be optimised using gold, ABS, MBS, corporate bonds and world equity.

The results of the incremental benefits analyses are shown in the Table 5.11. The table shows that there are significant potential incremental benefits for both before and after constraining asset weights to invest beyond government-related assets. The average risk reduction benefits of 0.49 percent to 0.52 percent could be achieved for central banks that do not have asset allocation constraints in their reserve portfolios.

The more realistic model shows that asset allocation constraints reduce half of the benefits to between 0.12 percent and 0.28 percent. Asset allocation reduces the half variance of the benefits from 0.15 percent to 0.08 percent. This indicates higher chances for central could acquire the incremental benefits from non-government investments relative to that of before constraining on asset weights. These non-bonds investments potentially could achieve between 0.07 percent and 0.13 percent risk reduction at the 5^{th} percentile of the posterior distributions.

Table 5.12 reports the asset weights of the addition of non-bond diversification in the well-diversified government bonds portfolio. Interestingly there are not much different non-bond asset compositions between before and after constraining asset weight. It needs 1 percent of the reserve to be invested in gold, 3.5 percent in ABS, 5.5 percent in MBS, 2.5 percent in corporate bonds and 0.5 percent in equity in order to obtain incremental benefits from the well-diversified bond portfolio.

Our findings support central banks' investment trends toward riskier assets as demonstrated by Pringle and Carver (2006). The inclusion of new asset classes including equities and asset-backed securities do provide diversification benefits for central bank reserve management. From the safety aspect of reserve portfolio, the additional new asset classes beyond quasi-government securities are

Table 5.12 Risk reduction portfolio weights for a given benchmark returns for the incremental benefits of non-government bond diversification

This table reports the posterior distribution of the average and standard deviations of the optimal weights in the efficient portfolio for a given existing benchmark returns for non-government bond investments. Panel A. is for short-sale constraints. Panel B. is asset allocation constraints.

	Spanning Strategy															
	Gold		ABS		MBS		ABS+MBS		Corporate Bond		Equity		Corpoate		All Non-Bond	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel A. No Short Sale																
Gold	0.0085	0.0115													0.0060	0.0095
ABS			0.0263	0.0464			0.0244	0.0447							0.0263	0.0324
MBS					0.0239	0.0455	0.0219	0.0434							0.0158	0.0329
Corporate Bond									0.0311	0.0393			0.0066	0.0102	0.0181	0.0362
World Equity											0.0307	0.0387	0.0308	0.0376	0.0071	0.0098
Inflation-linked	0.0377	0.0447	0.0384	0.0464	0.0403	0.0469	0.0365	0.0436	0.0403	0.0457	0.0394	0.0448	0.0389	0.0427	0.0303	0.0357
EM Asia	0.0127	0.0274	0.0121	0.0266	0.0133	0.0282	0.0118	0.0262	0.0106	0.0250	0.0108	0.0253	0.0091	0.0226	0.0073	0.0193
EM EMEA	0.0169	0.0343	0.0137	0.0317	0.0175	0.0359	0.0139	0.0319	0.0134	0.0315	0.0135	0.0320	0.0105	0.0272	0.0067	0.0205
EM LATAM	0.0106	0.0266	0.0114	0.0277	0.0111	0.0274	0.0103	0.0266	0.0113	0.0275	0.0114	0.0275	0.0088	0.0232	0.0061	0.0188
SSA	0.0357	0.0566	0.0300	0.0529	0.0271	0.0513	0.0231	0.0467	0.0347	0.0561	0.0340	0.0562	0.0314	0.0517	0.0209	0.0409
DM 7-10 year	0.0161	0.0294	0.0154	0.0294	0.0171	0.0315	0.0158	0.0289	0.0139	0.0277	0.0138	0.0279	0.0154	0.0288	0.0154	0.0269
DM 5-7 year	0.0070	0.0208	0.0070	0.0206	0.0068	0.0204	0.0062	0.0188	0.0068	0.0199	0.0068	0.0198	0.0060	0.0185	0.0050	0.0166
DM 1-5 year	0.0037	0.0156	0.0035	0.0151	0.0036	0.0153	0.0033	0.0145	0.0039	0.0153	0.0041	0.0161	0.0035	0.0144	0.0035	0.0141
G7 7-10 year	0.0084	0.0222	0.0078	0.0219	0.0065	0.0209	0.0067	0.0203	0.0079	0.0216	0.0080	0.0217	0.0083	0.0214	0.0084	0.0205
G7 5-7 year	0.0050	0.0189	0.0044	0.0178	0.0036	0.0160	0.0033	0.0150	0.0045	0.0175	0.0049	0.0188	0.0043	0.0171	0.0033	0.0148
US Bills	0.7392	0.1557	0.7230	0.1599	0.7261	0.1587	0.7266	0.1582	0.7187	0.1583	0.7159	0.1597	0.7267	0.1564	0.7430	0.1456
G7 Non US Bills	0.0172	0.0396	0.0189	0.0409	0.0177	0.0398	0.0175	0.0388	0.0180	0.0400	0.0184	0.0400	0.0190	0.0401	0.0155	0.0351
G7 1-5 year	0.0814	0.0925	0.0881	0.0951	0.0852	0.0955	0.0788	0.0917	0.0850	0.0925	0.0882	0.0941	0.0807	0.0905	0.0613	0.0796

	Spanning Strategy															
	Gold		ABS		MBS		ABS+MBS		Corporate Bond		Equity		Corpoate		All Non-Bond	
	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
Panel B. Asset Allocation																
Gold	0.0070	0.0078													0.0105	0.0084
ABS			0.0439	0.0147			0.0660	0.0348							0.0373	0.0111
MBS					0.0376	0.0193	0.0227	0.0294							0.0567	0.0333
Corporate Bond									0.0459	0.0117			0.0069	0.0126	0.0254	0.0283
World Equity											0.0459	0.0116	0.0261	0.0216	0.0054	0.0067
Inflation-linked	0.0966	0.0301	0.0833	0.0331	0.0942	0.0316	0.0703	0.0322	0.0852	0.0335	0.0853	0.0331	0.0202	0.0392	0.0506	0.0305
EM Asia	0.0224	0.0305	0.0226	0.0311	0.0226	0.0312	0.0221	0.0302	0.0195	0.0294	0.0198	0.0297	0.0160	0.0311	0.0187	0.0265
EM EMEA	0.0480	0.0372	0.0415	0.0375	0.0480	0.0380	0.0411	0.0366	0.0422	0.0376	0.0422	0.0374	0.0206	0.0339	0.0402	0.0343
EM LATAM	0.0271	0.0333	0.0313	0.0351	0.0261	0.0333	0.0314	0.0345	0.0321	0.0354	0.0319	0.0353	0.0151	0.0310	0.0300	0.0315
SSA	0.0536	0.0353	0.0399	0.0359	0.0348	0.0355	0.0242	0.0313	0.0445	0.0363	0.0447	0.0364	0.0145	0.0328	0.0171	0.0274
DM 7-10 year	0.1607	0.0290	0.1505	0.0318	0.1551	0.0292	0.1479	0.0288	0.1453	0.0343	0.1452	0.0342	0.2931	0.0943	0.1443	0.0284
DM 5-7 year	0.0002	0.0026	0.0003	0.0033	0.0003	0.0035	0.0003	0.0035	0.0004	0.0044	0.0005	0.0050	0.0051	0.0288	0.0001	0.0023
DM 1-5 year	0.0001	0.0016	0.0001	0.0014	0.0001	0.0018	0.0000	0.0012	0.0001	0.0020	0.0001	0.0017	0.0086	0.0333	0.0000	0.0010
G7 7-10 year	0.0073	0.0208	0.0083	0.0226	0.0053	0.0180	0.0054	0.0175	0.0097	0.0250	0.0094	0.0245	0.0090	0.0367	0.0053	0.0176
G7 5-7 year	0.0001	0.0019	0.0001	0.0019	0.0001	0.0017	0.0001	0.0015	0.0002	0.0026	0.0001	0.0023	0.0050	0.0231	0.0000	0.0013
US Bills	0.2954	0.0177	0.2925	0.0233	0.2938	0.0200	0.2921	0.0216	0.2909	0.0249	0.2906	0.0258	0.1997	0.0695	0.2872	0.0236
G7 Non US Bills	0.0046	0.0022	0.0075	0.0042	0.0062	0.0032	0.0079	0.0042	0.0091	0.0012	0.0094	0.0023	0.1003	0.0077	0.0128	0.0042
G7 1-5 year	0.2770	0.0351	0.2781	0.0372	0.2760	0.0351	0.2686	0.0308	0.2750	0.0354	0.2748	0.0356	0.2597	0.0276	0.2583	0.0216

significant compare to the existing benchmark policy and well-diversified bond portfolios. The benefits are measured using risk reduction for a given existing expected returns.

5.4. Conclusion

Strategic asset allocation is an essential part of central bank reserve management. Its importance is paramount in a time of financial turmoil, based on strategic asset allocation on the risk-based framework. In this paper, risk based framework means that risk minimization for the same expected benchmark returns are employed in this analysis. Five aspects of scenario analysis considered in this study are the impact of budget constraints, liquidity buffer allocation, and global financial crisis investment opportunities, and sample selection bias problems and beyond government bond investments.

This research learnt that after imposing asset allocation constraints, at least one spanning strategy might deliver substantial risk reduction benefits for short-sale constrained government bond portfolio. Global inflation-linked government bond needs to be added together with the emerging market country rating and longer dated group of G7 government bond, developed market government bond and SSA bonds market index to offer government bond portfolio risk reduction across all constraints and scenario analyses considered. Analysis on the different sub-period reveals that risk reduction benefits are important for both before and after the financial crisis. This finding emphasises the need for central banks to diversify their foreign reserves benchmark beyond its current setting.

Our analysis on the impact of budget and liquidity constraints to a risk reduction government bond portfolio has policy implications for central bank reserves management. Our results show that the requirement to provide more cash resulting in a higher uncertainty of the mean risk reduction. This implies that central banks should not segregate foreign reserves to cover short-term liability from investable reserves. Reserves segregations may introduce inefficiencies as it increases the difficulty of optimising the whole reserve allocation. Central banks, therefore, should not to tranche their reserves, but express liquidity requirements in the form of constraints on the portfolio optimisation framework.

Analysis of the impact of liquidity buffer allocation confirmed that one of the main sources of risk reduction is driven by US treasury bills. Allowing the new portfolio to hold US treasury bills at least the same weight as the existing benchmark offers risk reduction benefits. These findings provide practical implications. The size of the reserves relative to their economy, risk aversion, or economic conditions which could have an impact on their liquidity portions, have important role for the risk minimization analysis. Smaller the size of the reserve relative to the economy, the more risk averse of a central bank, and negative economic environments could have an impact on the bigger liquidity allocation. This bigger liquidity allocation in turn could increase the chance to gain the benefits.

This sub-section rejects the hypotheses that the reason I could not find diversification benefits on longer dated bond strategy because of sample selection bias to restrict the investment up to 10 years. Lastly, my findings support central banks investment trends toward riskier assets. The inclusion of new asset classes including equities and asset-backed securities significantly provide risk reduction diversification benefits for central bank reserve management. The additional new asset classes beyond quasi-government securities provide significant benefits compare to the existing benchmark and provide significant incremental benefits compare to well-diversified government bond portfolios.

CHAPTER SIX

This chapter presents summaries and implications of each of three empirical chapters, and of the thesis overall. In addition, limitation of the research and future improvements in the area are suggested.

6. CONCLUSIONS

6.1. Research Conclusions and Implications

6.1.1. Optimal currency composition

Currency composition policy is an important aspect of national foreign reserves management. It is the initial stage in managing foreign reserves where the main objective is risk concern, while maintaining liquidity demand for transactions on foreign trade and debt payment. In chapter three, risk minimization for a given expected benchmark returns and risk reduction to the minimum variance portfolio are examined within Bayesian approach of Li et al. (2003) combine with trade constraints and debt constraints.

Depart from the existing benchmarks the optimisations are conducted to minimise portfolio risk by imposing shares of external transactions. Empirical evidence on the minimum variance portfolio shows that there is potential for significant risk benefits for central banks when they reallocate their currency composition among the existing benchmark, or to invest in broader currency diversifications. Such results are regardless the original benchmark currently being used, and hold across debt and trade constraints.

My findings related to the risk minimization for a given benchmark returns show that there are significant diversification benefits before imposing currency composition constraints. Imposing trade constraints and debt constraint on the optimisations change the significance of the benefits. In general, greater currency diversifications provide bigger means of the benefits. However, for the original benchmark which allocates more than 60 percent to USD, significant benefits at the 5th percentile of the posterior distributions could be attained by broader currency investment into selected developed countries currencies and selected emerging market currencies. This applies when imposing trade constraint, but not from imposing debt constraints. A central bank that uses the original benchmark which allocates USD less than 50 percent, the significant risk reduction benefits at the 5th percentile of the posterior distributions could be achieved by reallocate benchmark currency shares and by broader currency investment into selected developed market currencies from imposing trade constraint and broader spanning into selected emerging market currencies from imposing debt constraints.

The empirical findings show that in general the new optimal portfolios suggest allocating more USD and less EUR compared to the current benchmark portfolio currency. Our findings suggest that central bank should consider relaxing their investment policy beyond USD, EUR, GBP and JPY currency. It is desirable and feasible to adopt Bayesian approach combined with external payment constraints as an alternative framework for determining central banks' foreign currency structure. The transaction constraints currency weight, however, needs to be adjusted reflecting the individual country's foreign trade and debt profiles.

6.1.2. Strategic asset allocation for government bond portfolio

An appropriate strategic asset allocation is an essential aspect of sound and prudent management of foreign exchange reserves. The primary objective of this chapter is to provide alternative asset structure policy where the emphasis is the safety without compromising its liquidity requirements and the existing expected returns. In this chapter, I set up an unconditional Bayesian spanning test framework. This approach allows us to derive an optimal allocation and to investigate the benefits from investing in different asset classes from central bank foreign reserve management perspectives. In this chapter, I look at three government bond diversification strategies: longer maturity, emerging market and inflation-linked bonds. In addition to the risk minimization for a given benchmark return, I also investigate risk minimization to the global minimum variance portfolio.

Our finding shows that the current benchmark policy is an inefficient portfolio, hence spanning government bond portfolio with longer-dated government bonds, sovereign emerging market and inflation-linked government bonds altogether improve the investment opportunity set and deliver significant risk reduction. The inclusion of inflation-linked government bonds cannot be spanned directly to the current benchmark, but it needs to be added together to be able to deliver significant benefits. This investment choice may offer average monthly diversification benefits of 0.33 percent with a standard deviation of 0.14 percent and magnitude of risk reduction at the first, and fifth percentile are zero percent and 0.04 percent respectively. In order to achieve those benefits, it requires reserves portfolio to be held nine percent in global inflation-linked bond index, seven percent in country rating emerging market government bond, three percent in SSA, eight percent in sovereign developed market up to 10 year maturity and three percent in longer dated (5-10 year maturity) group of G7 government bonds while maintain their 70 percent investments in the existing benchmark portfolio.

The purpose of holding reserves and the mandate central banks have in managing their reserves may vary amongst central banks and therefore the preference toward returns will differ amongst them. However, as the accumulation of national reserves increased and tendency that the return criteria become more important for reserves in excess of liquidity needs, I would argue risk reduction for a given existing expected return framework is generally more appropriate than the second one. Foreign reserves diversification ideas resulting from the first measure also easier to communicate to the government and other stakeholders since the benefits yielded without reducing the expected portfolio returns. However, it is interesting that the empirical findings from both currency compositions and asset allocation reveal minimum variance portfolio offer significant benefits across all constraints. Might be worth thinking about should the central bank follow the GMV strategy.

6.1.3. Optimal foreign reserve asset allocation in various setting

Strategic asset allocation is an essential part of central bank reserve management. Its importance is paramount in a time of financial turmoil, based on strategic asset allocation on the robust risk-based framework. In this chapter, risk-based framework means that risk minimisation for the same expected benchmark returns is employed in this analysis. Some different settings considered in this analysis to evaluate the role of budget constraints, liquidity buffer allocation, and global financial crisis investment opportunities; longer than 10 year bond investment and non-government bond investment preferences for the risk minimization reserve portfolio. The findings confirm that inflation-linked government bond needs to be added together with the emerging market country rating and longer dated G7, developed market and SSA bonds market to offer government bond portfolio risk reduction across all constraints and scenario analysis considered. Analysis on the impact of different investment opportunities shows that the diversification is important for both high and low yield bond markets. This emphasises the need for central banks to diversify their foreign reserves benchmark beyond its current setting.

Our analysis on the impact of budget and liquidity constraints to a risk reduction government bond portfolio has policy implications for central bank reserves management. Our results show that the requirement to provide more cash resulting in a higher uncertainty of the mean risk reduction. This implies that central bank should not segregate foreign reserves to cover short-term liability from investable reserves. Reserves segregations may introduce inefficiencies as it increases the difficulty of optimising the whole reserve allocation. Central banks, therefore, should not to tranche their reserves, but express liquidity requirements in the form of constraints on the portfolio optimisation framework.

Analysis of the impact of liquidity buffer allocation confirmed that one of the main sources of risk reduction is driven by US treasury bills. Allowing the optimised portfolio to hold US treasury bills at least as the same as its weights in the benchmark offers risk reductions benefits. These findings provide practical implications. The size of the reserves relative to their economy, risk aversion or economic conditions which could have an impact on their liquidity portions have important role for the risk minimization for a given benchmark return analysis.

Our empirical results support central banks investment trends toward riskier assets as demonstrated by Pringle and Carver (2006). The inclusion of new asset classes including equities and asset-backed securities do provide significant diversification benefits for central bank reserve management. From the safety aspect of reserve portfolio for a given current benchmark returns, the additional new asset classes beyond government-related assets are significantly lower risks level compare to the risk of the existing benchmark policy and provide incremental benefits to the well-diversified government bond portfolios.

6.2. Limitations and Future Improvements

The main reason for the liquidity requirements of foreign reserves to be conservative is the consideration of the likelihood withdrawals from the managed portfolios. The possible sources of withdrawals are summarised in Jeanne and Ranciere (2011) as being from three reasons: international trading needs, financing demands and sudden changes in the capital account. If more information can be obtained and future uncertainty can be reasonably forecasted, the optimisation of the strategic allocation of the reserves should incorporate the liquidity aspect more specific rather than applying the overall liquidity allocation as in this research. Second, further research should look for better ways to incorporate various sources of transaction costs arising from switching from one to another asset classes. It is interesting to examine if the risk reduction benefits I discover in this thesis will be wiped out by transaction costs.

This portfolio risk reduction analyses for central banks' foreign reserve investments assumes static and unconditional framework. For the future research, a study on reserve portfolio diversification benefits can be extended in a number of directions. First, is to use dynamic and conditional framework of asset allocation. Second, instead of focus on risk reduction measure, a number of alternative measures can be used to evaluate the diversification benefits. The returns enhancement (Li et al., 2003), Sharpe ratio (Sharpe, 1966) and certainty equivalent returns (Fletcher, Paudyal, and Santoso, 2016) to measure the benefits could also be an interesting application. Potential topics for future studies includes whether currency and interest rate hedging will increase central bank's portfolio performances. This thesis has focused on asset allocation problem. It would be interesting to extend the analysis to look at the individual bond selection.

My study focuses on strategic asset allocation framework which is only part of the reserves management aspects. Other aspect such as organisational structure of central banks' reserves management and active portfolio management could further enrich the analysis.

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APPENDIX

Appendix 1 Currency return

All currency data in these analyses use USD as base currency, or “X” exchange rate to USD.

United States Dollar (USD): Since USD is the base currency, currency USD returns is solely define from US dollar 3M deposits rate DataStream code S20514.

Euro (EUR) : is defines by EUR currency monthly return DataStream code and Mark/EU Germany 3M deposits rate DataStream code S20532.

Great Britain Pound sterling (GBP): is defines by GBP currency monthly return DataStream code and UK pound sterling 3M deposits rate DataStream code S20508.

Japan Yen (JPY) : defines by JPY currency monthly return DataStream code and Japanese yen 3M deposits rate DataStream code S20963.

Canada Dollar (CAD) : defines by CAD currency monthly return DataStream code and Canadian dollar 3M deposits rate DataStream code S20520.

Swiss Franc (CHF) : defines by CHF currency monthly return DataStream code and Switzerland Franc 3M deposits rate DataStream code S20538.

Australia Dollar (AUD)	:	defines by AUD currency monthly return DataStream code and Australian dollar 2M deposits rate DataStream code S96819.
New Zealand Dollar	:	defines by NZD currency monthly return DataStream code S19587 and New Zealand 90-day deposits rate DataStream code Y70412
China Yuan (CNY)	:	defines by CNY currency monthly return DataStream code S99695 and China Yuan 3M time deposits rate DataStream code Y76069
South Korea Won (KRW)	:	defines by CNY currency monthly return DataStream code S20363 and South Korean Won 3M time deposits rate DataStream code S534EK
India Rupee (INR)	:	defines by INR currency monthly return DataStream code INXRUSD and Indian Rupee 3M time deposits rate DataStream code S4147V
Brasilia Real (BRL)	:	defines by BRL currency monthly return DataStream code S08392 and Brazilian real 3M deposits rate DataStream code S534FC
Thai Baht	:	defines by THB currency monthly return DataStream code S99720 and Thailand baht 3M deposits rate DataStream code S97458.

Appendix 2 Currency Benchmark

Currency Benchmark : benchmark portfolio consists of four currencies i.e. US dollar, Euro, GB pound sterling, and Japanese yen.

Currency Benchmark #1 : to replicate actual currency composition of central banks investment in G4 currency which consist of 63% of USD, 22% of EUR, 4% of GBP and 4% of JPY.

Currency Benchmark #2 : to mimic currency composition in the SDR-IMF portfolio which consist of 47% of USD, 34% of EUR, 12% of GBP and 7% of JPY.

Appendix 3 Benchmark Assets

US treasury bills : I use total return US treasury bills 6 months US$ bid yield, DataStream code S310VH.

Non-USD Bills : Non-USD bills index is formed by equally weighted UK, German, France, Japan, Canada and Italia bills equivalent (issued in their local currency) after converted into USD. In order to convert local currency denominated treasury bills asset returns, I use GBP DataStream code S242W4, EUR DataStream code S242XT, JPY DataStream code S242X6 and CAD DataStream code S242W7 to USD exchange rate data originally sourced from Bank of England (BoE).

UK : I use total return UK treasury bills 6 months bid yield, denominated in GBP, DataStream code S310TK.

Germany : I use JP Morgan Germany treasury bills 6 months total return index, denominated in EUR, DataStream code T99215.

France : I use JP Morgan France treasury bills 6 months total return index, denominated in EUR, DataStream code T99210.

Japan : I use JP Morgan Japan treasury bills 6 months total return index, denominated in JPY, DataStream code T99225.

Canada : I use JP Morgan Canada treasury bills 6 months total return index, denominated in CAD, DataStream code CNTBB6M.

Italy : I use JP Morgan Italy treasury bills 6 months total return index, denominated in EUR.

Market Index : I use Bank of America Merrill Lynch Global Government G7 1 to 5 years US$ Total Return Index, DataStream code T96326 for the proxy market index.

Benchmark Portfolio : benchmark portfolio consists of three assets; US treasury bills, Non-USD bills and market index. When asset allocation constraints are imposed I put constraints that 30 percent have to be invested in treasury bills assets with at least 15 percent in US treasury bills and minimum 25 percent in government bond market 1 to 5 year index.

Appendix 4 Test Assets

I classify my sample of the test assets into three different categories; duration extension strategies, the inclusion of emerging market and index-linked government bonds strategies. In addition, I also divide test assets into five asset classes; Group G7 government (G7), developed government bond market (DM), semi-government, supranational and government agency (SSA) bond market, emerging government (EM) bond market and index-linked government bond (IL).

A. Longer government bond maturity

a. asset index

G7 5-7 year : G7 5-7 index is formed by equally weighted of the 7 members of United States (S07696), United Kingdom (S07689), German (S07641), France (S07635), Japan (S07659), Canada (S07623) and Italia (S07647) government bond 5-7 year maturity index issued in local currency after converted into USD.

G7 7-10 year : G7 7-10 index is formed by equally weighted of the7-member of the G7 group of United States (S07697), United Kingdom (S07690), German (S07642), France (S07636), Japan (S07660), Canada (S07624) and Italia (S07648) government bond 7-10 year maturity index issued in local currency after converted into USD.

DM 1-5 year : DM 1-5 index is formed by equally weighted of the 14 government bond indices issued by 7 advanced economy considered in this study; Netherlands (NL) 1-3 year (S07663) and 3-5 year (S07664); Austria (OE) 1-3 year (S07609) and 3-5 year (S07610); Belgium (BG) 1-3 year (S07615) and 3-5 year (S07615); Switzerland (SW) 1-3 year (S07681) and 3-5 year (S07681); Denmark (DK) 1-3 year (S07627) and 3-5 year (S07627); Australia (AU) 1-3 year (S07603) and 3-5 year (S07603) return index and New Zealand (NZ) government benchmark 2 year (S08943) and 5 year (S08944) bid yield.

DM 5-7 year : DM 5-7 year index is formed by equally weighted of the 7 government bond indices issued by 7 advanced economy countries; NL 5-7 year (S07665); OE 5-7 year (S07611); BG 5-7 year (S07616); SW 5-7 year (S07682); DK 5-7 year (S07628); AU 5-7 year (S07604) and NZ government benchmark 7 year (S08945) bid yield.

DM 7-10 year : DM 7-10 year index is formed by equally weighted of the 7 government bond indices issued by 7 advanced countries; NL 7-10 year (S07666); OE 7-10 year (S07612); BG 7-10 year (S07615); SW 7-10 year (S07683); DK 7-10 year (S07629); AU 7-10 year

(S07605) government bond index and NZ government benchmark yield 10 year (S08946).

SSA index : SSA index is formed by equally weighted of the 7 bond indices; Barclays US aggregate Supranational (LHUSSUP), Barclays Euro Dollar supranational 1-7 year (LHEN1T7), Barclays Agency 1-5 year (Y03986), DEX Capital Canada's province short (T14079), mid-term (T14080) and long-term (T14081) price index, and UBS Australia's semi-government 2-10 year (AUSG2T10) return index.

b. Portfolio strategy:

G7 5-7 : G7 5-7 portfolio strategy is longer bonds strategy spanned by G7 government bond index for maturity 5 to 7 year and the benchmark portfolio.

G7 7-10 : G7 7-10 portfolio strategy is longer bonds strategy that spanned by G7 government bond index for maturity 7 to 10 year and the existing benchmark portfolio.

G7 5-10 : G7 5-10 portfolio strategy is longer bonds strategy that spanned by G7 5-7 and G7 7-10 government bond indexes and the existing benchmark portfolio.

DM 1-5 : DM 1-5 portfolio strategy is a strategy that spanned by DM government bond index for maturity 1 to 5 year and the existing benchmark portfolio.

DM 5-7	:	DM 5-7 portfolio is longer bonds strategy spanned by G7 and DM government bond index for maturity 5 to 7 year and the benchmark portfolio.
DM 7-10	:	DM 7-10 portfolio is longer bonds strategy spanned by G7 and DM government bond index for maturity 7 to 10 year and the benchmark portfolio.
DM 5-10	:	DM 7-10 portfolio is longer bonds strategy spanned by G7 and DM government bond maturity 5-7 year and 7-10 year index and the benchmark portfolio.
DM 1-10	:	DM 7-10 portfolio is longer bonds strategy spanned by G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index and the benchmark portfolio.
SSA	:	SSA portfolio strategy is relaxing credit constraints in addition to longer dated investment that spanned by SSA, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index and the benchmark portfolio.

B. Emerging market government bond

a. Asset index

EM Geographic : consists of three different geographical area; Asia; Europe, Middle East and Africa (EMEA); and Latin America (Latam) government bonds index, DataStream code Z11016, Z11054 and Z11055 respectively.

EM Country rating: consist of emerging market B, BB and BBB rated sovereign bond index, DataStream code Z10960, Z10959 and Z10979.

b. Portfolio strategy

EM Geographic (EMG) : is a spanned strategy by adding three emerging market Geographic index into the existing government bond benchmark.

G7+EMG : is the inclusion of three emerging market Geographic index, G7 5-7 year and G7 7-10 year government bond index to the existing benchmark portfolio.

DM+G7+EMg : is the inclusion of three emerging market Geographic index, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index to the existing benchmark portfolio.

SSA+DM+G7+EMG	: is the inclusion of three emerging market Geographic index, SSA, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index to the existing government bond benchmark.
EM Rating (EMR)	: is a spanned strategy by adding three emerging market countries rating index into the existing government bond benchmark.
G7+EMr	: is the inclusion of three emerging market credit rating index, G7 5-7 year and G7 7-10 year government bond index to the existing benchmark portfolio.
DM+G7+EMR	: is the inclusion of three emerging market credit rating index, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index to the existing benchmark portfolio.
SSA+DM+G7+EMR	: is the inclusion of three emerging market countries rating index, SSA, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index to the existing government bond benchmark.

C. Inflation-linked government bond

a. Asset index

Global government index-linked bond	: I use Bank of America Merrill Lynch Global Government inflation-linked bond returns index, DataStream code: MLGGIL$.
G7 government Index-linked Bond	: is constructed by equally weighted of the index-linked bonds issued by government of the G7 members. I use return index of the United States government inflation-linked 10 year bid yield (TRUT10T), Bank of America Merrill Lynch United Kingdom (MLUKGIL), German (MLG0DIL), France (MLFGVI$), Japan (MLG0YI$), Canada

(BCCANA$) and Italy (MLG0II$) government inflation-linked return index.

b. Portfolio Strategy

Global Govt. IL (GIL) : is the addition of the GIL to the existing government bond portfolio.

GIL+G7 : is the addition of the GIL, G7 5-7 year and G7 7-10 year government bond index to the existing bond benchmark portfolio.

GIL+G7+DM : is the addition of GIL, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index to the existing bond benchmark portfolio.

GIL+G7+DM+SSA : is the addition of GIL, SSA, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index to the existing bond benchmark portfolio.

GIL+G7+DM+SSA+EMG	:	is the addition of GIL, emerging market Asia, emerging market EMEA, emerging market Latam, SSA, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index to the existing bond benchmark portfolio.
GIL+G7+DM+SSA+EMR	:	is the addition of GIL, emerging market B, emerging market BB, emerging market BBB, SSA, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index to the existing bond benchmark portfolio.
G7 Govt. IL (G7IL)	:	is the addition of the G7IL to the existing government bond portfolio.
G7IL+G7	:	is the addition of the G7IL, G7 5-7 year and G7 7-10 year government bond index to the existing bond benchmark portfolio.
G7IL+G7+DM	:	is the addition of G7IL, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index

to the existing bond benchmark portfolio.

G7IL+G7+DM+SSA : is the addition of G7IL, SSA, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index to the existing bond benchmark portfolio.

G7IL+G7+DM+SSA+EMG : is the addition of G7IL, emerging market Asia, emerging market EMEA, emerging market Latam, SSA, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index to the existing bond benchmark portfolio.

G7IL+G7+DM+SSA+EMR : is the addition of G7IL, emerging market B, emerging market BB, emerging market BBB, SSA, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index to the existing bond benchmark portfolio.

D. Non-Government Bond Asset Classes

a. Asset index

Gold	: I use Gold Bullion price index, DataStream code: GOLDBLN.
Mortgage-backed Securities (MBS)	: I use Barclays United States Mortgage-Backed Securities price index, DataStream code: LHMNBCK.
Asset-Backed Securities (ABS)	: I use Barclays aggregate Asset-backed securities return index, DataStream code: LHAGGEA.
Invest. Grade Corporate Bond	: I use Barclays United States Corporate Investment Grade return index, DataStream code: LHCCORP.
World Equity	: I use Morgan Stanley Capital International (MSCI) World $ equity, DataStream code MSWRLD$ for the proxy of equity market index.

b. Portfolio Strategy

Broader investment universe : is the addition of Gold, MBS, ABS, Corporate bond, world equity, IL, emerging market B, emerging market BB, emerging market BBB, SSA, G7 and DM government bond maturity 1-3 year, 3-5 year, 5-7 year and 7-10 year index to the existing bond benchmark portfolio.