LINEAR SYSTEMS MODELLING OF TWO-DIMENSIONAL PIEZOELECTRIC STRUCTURES

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ABSTRACT

A new, two-dimensional model is presented for predicting the behaviour of tall, thin piezoelectric transducer structures, typical of those encountered in many ultrasonic phased array assemblies. Based on linear systems theory, the model may be represented in block diagram format, utilising feedback and feedforward loops to model mechanical and piezoelectric cross coupling, in addition to secondary and tertiary piezoelectric generation. As a result, the model permits a ready understanding of the electrical, mechanical and piezoelectric interactions which take place when two principal vibrational modes are present within a transducer structure.

Comprehensive experimental and theoretical results are shown to produce close agreement, and compare favourably with alternative modelling strategies. It is considered that this novel approach will be of considerable benefit to the analysis and understanding of the transduction process within such structures.

LIST OF SYMBOLS

This list contains the more important symbols used in the mathematical expressions, and system diagrams of this thesis. Less common or rarely occurring terms are defined in the text when required.

s Complex Laplace operator (Laplace quantities are denoted by the bar symbol)

C_o Transducer static capacitance

h (h_{33}) Piezoelectric charge constant for the thickness dimension.

ϕ Piezoelectric voltage constant for the lateral dimension.

Y_{33} Young's modulus of elasticity relating stress and strain in the thickness dimension.

Y_{11} The modulus of elasticity in the lateral dimension.

Y_{13} The cross coupled modulus of elasticity.

ϵ_{33} The electrical permittivity.

$\bar{\xi}_N$ Laplace transform of particle displacement in the Nth dimension.

Zc_N Mechanical impedance of the transducer in the Nth dimension.

R_F Transducer front face reflection coefficient.

R_B Transducer rear face reflection coefficient.

R_L Transducer left face reflection coefficient.

R_R Transducer right face reflection coefficient.

T_F Transducer front face transmission coefficient.

T_B Transducer rear face transmission coefficient.

T_L Transducer left face transmission coefficient.

T_R Transducer right face transmission coefficient.

$\bar{F}_F$ Laplace transform of force emanating or applied to the front face.

$\bar{F}_B$ Laplace transform of force emanating or applied to the rear face.

$\bar{F}_L$ Laplace transform of force emanating or applied to the left face.

$\bar{F}_R$ Laplace transform of force emanating or applied to the right face.

$\bar{A}_T$ Laplace transform of the direct feedback factor for the thickness dimension.

$\bar{A}_W$ Laplace transform of the direct feedforward factor for the lateral dimension.

$\bar{A}_{13}$ Laplace transform of the cross-coupled feedforward factor from the lateral to the thickness dimension.

$\bar{A}_{31}$ Laplace transform of the cross-coupled feedback factor from the thickness to the lateral dimension.

$\bar{A}_{EQ}$ Laplace transform of the equivilent two-dimensional feedback factor.

$\bar{V}_{3}$ Laplace transform of the voltage measured across the transducer.

$\bar{Q}$ Laplace transform of electronic charge.

$\bar{Z}_{T}$ Laplace transform of transducer operational impedance

ψ_{13} Mechanical cross-coupling factor which converts lateral displacement to thickness force.

ψ_{31} Mechanical cross-coupling factor which converts thickness displacement to lateral force.

$\bar{K}_{1}$ Reverberation function for the thickness dimension.

$\bar{K}_{3}$ Reverberation function for the lateral dimension.

$\bar{M}$ Mechanical cross-coupling function.

CHAPTER I

INTRODUCTION

The piezoelectric effect is evident in a variety of both naturally occuring and synthetic materials, and is widely applied to the generation and detection of acoustic signals. A primary consideration, and one which greatly influences the vibrational characteristics of such electromechanical transducers, is shape, or more accurately, the relationship between the various physical dimensions. The present investigation primarily considers piezoelectric transducers which are tall, thin and long. That is, the type of device which is found in a large number of transducer array assemblies. Such transducers are assumed to vibrate in two principal dimensions; the thickness dimension and the lateral or width dimension. The length, which is large compared to the other dimensions, is assumed to make an insignificant contribution to the behaviour of the device, and may be ignored safely.

1.1 ULTRASONIC ARRAYS

The desire to dynamically adjust the spatial characteristics of an ultrasonic beam, has led to the development of transducer arrays. These arrays find application in such diverse fields as;

- **Biomedicine :** Tissue characterisation, real time medical imaging and hyperthermia. [14][25][48]

- <u>Underwater Systems</u> : Sonar, navigation and communication systems. [39]

- <u>Non-destructive Evaluation</u> : Inspection of aircraft, offshore structures, pipelines etc. [64]

The use of arrays is advantageous in such applications since the acoustic beam may be steered or focussed as a result of the firing sequence. Similarly, in reception, the received signals from each element may be temporally delayed and summed to receive from a particular angle or provide a focal point. Two types of ultrasonic array are in common use; phased arrays, where the beam may be dynamically steered and focussed; and linear arrays in which the beam is shifted by switching between different elements or groups of elements.

Some common piezoelectric materials for array fabrication are; Lead Zirconate-Titanate (PZT), Lithium Niobate, Lead MetaNiobate, Barium Titanate and Polyvinylidene -flouride (PVDF). In general, PZT derivatives are probably the most widely used. [67][33][5]

The geometrical shape and construction of an array may assume many differing forms. For example, GEC Research [15] have developed a circular passive listening array (500kHz) for short range underwater surveillance. Medical arrays [29], (1-10MHz) generally possess a linear geometry, for operation in a phased or switched mode. Low frequency (<30kHz) towed sonar arrays [59], are designed to operate as straight line sources/detectors. However the effects of drag tend to impose a slight curve in the line of elements.

Very high frequency arrays (above 5MHz), exhibit their own unique constructional difficulties, since the size of the transducing elements becomes very small. For example a 5MHz array operating into water would require an element centre to centre spacing of $150\mu m$. This problem has resulted in the development of monolithic type arrays [66]. Borrowed from surface acoustic wave (SAW) technology, the complete array is formed on one piezoelectric slab and the individual elements are defined by an appropriate electrode pattern. Although possessing definite advantages in terms of manufacture, arrays of this type suffer from the problems of mechanical and electrical cross-coupling.

The overall shape of the sound field generated by an array depends upon a number of variables. For example, in the majority of applications, the 'ideal' array would possess the following qualities.

1. The transducer dimension in the steering direction should be small compared to the wavelength generated. That is, the device should act as a line source and be omnidirectional in the plane of steer.

2. The element spacing for a phased array should be accurately known and is often required to be less than or equal to one half wavelength, in order to avoid grating lobes in the field.

3. The firing sequence should be generated as accurately as possible, especially when dynamic focussing is required.

4. The magnitude of the signal applied to each element should be controllable. This permits weighting or apodisation to be applied which can improve the beam characteristics, usually at the expense of spatial resolution.

5. The in-circuit operational characteristics of each transducer in the array should be identical in terms of centre frequency, efficiency and damping

6. Electrical and mechanical interelement cross coupling should be minimised as far as possible.

However under practical conditions, none of these criteria are physically realisable. For example, the behaviour of each transducer in the array is often not identical. Since device operation is affected by slight variations in driving circuit parameters and variations in the mechanical and piezoelectric properties of the device. Furthermore, the existence of mechanical [31] and electrical [7] cross coupling between the elements results in degraded beam characteristics for individual elements, and a subsequent degradation of array performance. In general, cross coupling has the effect of increasing the apparent aperture width, and thus narrowing the transmitted acoustic beam.

In order to determine the extent to which these and other factors influence the operation of an array, a mathematical foundation is necessary to describe the non-ideal behaviour of the array and its constituent elements.

The type of arrays of interest to this investigation are those which possess cuboidal elements which are long, thin and tall. Furthermore, the theory may be extended to provide approximate mathematical models for other transducer geoemetries.

1.2 MODELLING OF TRANSDUCERS

Various models have been developed to describe the behaviour of piezoelectric transducers. For example Mason's equivalent circuit model [42], describes the electromechanical operation of a thickness mode transducer in terms of an electrical circuit. The radiated field patterns generated by such a transducer have also been theoretically analysed [65]. In general, the operational characteristics of uni-dimensional transducers, which exhibit plane wave propagation in one dimension only, have been comprehensively documented in the literature.

However, a significant number of applications involve the use of transducers which do not exhibit uni-dimensional behaviour. Consequently, there is clearly a requirement for models which describe the operation of devices which operate in more than one dimension, such as the tall, thin, two-dimensional transducers found in phased and linear arrays.

A number of modelling and prediction techniques have been applied to this end, and these are described fully in chapter II. It is however important at this point to emphasize the qualities required of a good transducer model.

1. The majority of piezoelectric transducer applications involve some form of transient excitation. Any model must therefore be amenable to transient analysis.

2. The electrical and mechanical loading conditions must be incorporated, since these often exert considerable influence on transducer behaviour.

3. The physical operation of the device should be readily apparent upon inspection of the model.

4. In the case of multidimensional transducers, the effects of interdimensional coupling should be clearly evident from the model.

Existing modelling strategies (see chapter II) for tall, thin array transducers, which operate in two dimensions, suffer from one or more of the following limitations.

1. They permit only the approximate prediction of resonant frequencies, and do not evaluate the complete behaviour of the device. They are thus of limited use for transient analysis.

2. They do not provide a good physical understanding of the nature of piezoelectric behaviour in two dimensions.

3. They often include approximations derived from one dimensional models, and are thus of limited accuracy for two dimensional structures.

4. They are mathematically complex and often expensive to implement in terms of computing costs.

1.3 AIMS AND CONTRIBUTIONS OF THE THESIS

A new model is developed to describe the operation of a two-dimensional piezoelectric transducer. Linear systems theory is employed to yield intuitively satisfying block diagrams which accurately describe transmission, reception and operational impedance. The concept of feedback is utilised to model secondary and tertiary piezoelectric action, in addition to inter-dimensional mechanical cross coupling. The model is considered valid for configuration or width/thickness ratios of greater than 2 or less than 0.5. Overall, the new technique possesses the following significant advantages.

1. It is valid over a wide range of operating frequencies, and as such may be applied to both CW and transient modes of operation.

2. The influence of mechanical and piezoelectric cross coupling may be clearly isolated.

3. The model uses only real physically realisable parameters, hence improving understanding of two dimensional behaviour.

4. Arbitrary conditions of electrical and mechanical loading may be considered with ease.

5. The model may be readily extended to incorporate

structures which possess multiple layers.

6. The technique is particularly amenable to implementation in software.

For these reasons, the model is thought to improve current understanding of two-dimensional behaviour, and offers an accurate and meaningful modelling tool for simulation and transducer design.

Furthermore, it is considered that the thesis offers the following additional contributions to array system design.

1. The factors which affect secondary piezoelectric action in each dimension are clearly defined. This results in improved understanding of the resonant behaviour of the transducer.

2. Interdimensional mechanical and piezoelectric cross coupling are explained and their influence upon factors such as efficiency and resonant frequency is described.

3. The effective efficiency of the two-dimensional transducer is evaluated with respect to variations in the material parameters for each dimension. Consequently it is possible to identify methods for

improving the efficiency of the transducer.

4. A technique is presented which permits the evaluation of multiple layers in either of the two principal dimensions. This allows assesment of bondline and matching layer quality, in addition to mechanical cross coupling in array structures.

5. Manufacturing methods are developed for the reliable and repeatable fabrication of phased arrays.

The simulations generated by the model are verified using measured impedance spectra, time domain voltages and transducer wave profiles generated in both solids and liquids

Chapter III, part 1 describes the development of the system equations from the relevant mechanical and piezoelectric relationships. The block diagram models are presented in part 2, where the operation of the device is explained. In chapter IV the model is extended to incorporate multiple layers in each of the two principal dimensions. The model is experimentally verified in chapter V, and is then used in chapter VI to design and evaluate two 16 element phased arrays.

CHAPTER II

A REVIEW OF MODELLING TECHNIQUES FOR PIEZOELECTRIC ARRAY TRANSDUCERS

2.1 INTRODUCTION

This chapter contains a review of currently available modelling strategies, which are applicable to the tall, thin piezoelectric transducers used in linear and phased array assemblies. There are three fundamental sections which are presented in some detail.

Firstly, various uni-dimensional models are presented in order to provide a framework within which the two-dimensional models may be evaluated. It will be shown that the techniques used for one-dimensional modelling may also be applied to the two-dimensional case. The electromechanical equivalent circuit of Mason [42] is described, along with that of Krimholtz, Leedom and Mathaei [37][38], in addition to the linear systems approach of Hayward [16].

Secondly, a number of two-dimensional approaches are illustrated. These include :

- The prediction of resonant and anti-resonant frequencies. This may be performed with some accuracy for given dimensional constraints. Although this does not constitute a model in the full sense, it can provide some useful information regarding the behaviour of a transducing element.

- The representation of two-dimensional behaviour by the use of a modified uni-dimensional thickness mode analogy, over a given frequency interval and for a range of configuration ratios.

- Finite element techniques, which may be applied in order to simulate with a great deal of accuracy, the complete vibrational characteristics of transducers which exhibit multi-dimensional wave behaviour.

Each of the modelling strategies illustrated will be assessed according to factors such as the amount of physical insight gained, and the ease with which the technique may be implemented.

In the final section of the chapter a new two-dimensional model is proposed which is believed to offer significant advantages over existing methods.

2.2 UNIDIMENSIONAL MODELLING STRATEGIES

The process of piezoelectric transduction may be described by a series of linear equations, making it possible to construct a relevant model.

There are two fundamental techniques which are employed in the modelling of uni-dimensional piezoelectric transducers. The first involves the application of specific boundary conditions in order to solve the relevant differential equations. Zero or finite stress boundary conditions are assumed, with the transducer excited under continuous wave conditions. This resonator type theory [44], has been been widely used in the calibration and measurement of transducers and transducer parameters. [49][26]

The second modelling technique utilises a complete solution of the differential equations with arbitrary boundary conditions. This results in a mathematical model for the device, which may to some extent be likened to its physical operation. This type of analysis is more versatile and is therefore more suited to system design applications. [10][18][38][42] A number of different models have been developed, based on the resultant defining equations, including electrical networks, block diagrams and various algorithmic methods which improve the computational efficiency in terms of processing time and storage space required. [19][20][40][41]

The principles involved in the derivation of the equations, and their related models, are now briefly illustrated for two of the more physically meaningful uni-dimensional thickness mode models. That is, the electrical equivalent circuit of Mason [42] and Hayward's linear systems block diagram model. [18]

2.2.1 THE ELECTROMECHANICAL EQUIVALENT CIRCUIT

In order to derive the equivalent circuit model of the thickness mode device, the following assumptions are applied to its operation,

- The transducer only operates in the thickness mode, all other modes are of negligible effect.

- The transducer and its surrounding media are loss free.

- The force and displacement functions are uniform and parallel to the surface of the transducer.

Mason [42] used the fundamental piezoelectric relationships :-

$$\Gamma_3 = Y_{33} S_3 - h_{33} D_3 \qquad 2.1$$

$$E_3 = -h_{33} S_3 + D_3 / \epsilon_{33} \qquad 2.2$$

Equation (2.1) represents the indirect piezoelectric effect where,

Γ_3 is the mechanical stress in the thickness dimension.

Y_{33} is the modulus of elasticity (Young's modulus) relating stress and strain in the thickness direction.

S_3 is the mechanical strain in the thickness direction.

h_{33} is the piezoelectric constant relating stress and electrical displacement for the thickness direction.

D_3 is the electrical displacement

and, Equation (2.2) represents the direct piezoelectric effect. Where,

E_3 is the thickness direction electrical field strength.

h_{33} relates the electrical field to strain in the thicknes direction.

ϵ_{33} is the electrical permittivity measured at constant strain.

The appropriate wave equation is applied in conjunction with the relevant boundary conditions to yield three coupled equations, which describe fully the operation of the device in terms of three dependant and three independant variables. These three equations are realised in the form of the three port electromechanical equivalent circuit model illustrated in figure 2.1. Its operation may be summarised in the following manner.

The transducer is considered to have two mechanical terminals which represent the front and rear surfaces. Any mechanical loading upon the device may then be considered as an impedance connected to the relevant terminal. The two mechanical ports are linked by a lossless transmission line, depicting mechanical waves travelling through the transducer from the front face to the rear face, and vice versa. These travelling mechanical waves generate a secondary charge on the device, as a result of piezoelectricity, this effect, generally referred to as secondary piezoelectric action, is simulated by the negative capacitance, while force to voltage conversion is represented by an ideal transformer. The model is exact in accordance with the simplifying assumptions previously stated and is applicable both to transient and continuous wave excitation.

The transmission line concept is intuitively satisfying from a mechanical point of view, and this model has found widespread acceptance in transducer system analysis. Despite this, the Mason model suffers from several inherent disadvantages, mainly centred around the negative capacitance, which has no real electrical analogy. For example, the analysis of arbitrary electrical or mechanical loading often demands the use of complicated network reductions. As an alternative, a number of simplified versions have been developed by either constraining the frequency range or interpreting only simple cases of mechanical and electrical loading.

Examples of these include, Kossof [32] and Thurston [67] who developed simplified models to operate in the region of the first resonance and Redwood [52] and Cook [8], who analysed the transient behaviour of the device under constrained mechanical conditions.

The transmission line model of Krimholtz, Leedom and Matthaei [38] (KLM model) has been widely used in the design of thickness mode [10] and array transducers [30], as outlined in section 2.3.2. The KLM model, whilst retaining the intuitively satisfying transmission line approach, replaced the distributed nature of piezoelectric coupling with a single coupling point at the centre of the transmission line. A coupling transformer with a frequency dependant turns ratio and a series reactance are utilised to account for this discrepancy in the representation of piezoelectric coupling.

The model is illustrated in figure 2.2, where the length of the transmission line L corresponds to the thickness of the transducer. Zc and Vc are the acoustic impedance and velocity respectively. Figure 2.2 shows the true three port nature of the model. The acoustic forces F_1 and F_2 are developed directly across the transmission line, and not, as in Masons circuit, across both the line and the secondary side of the transformer. This fact permits the separation of the acoustic and electrical quantities, and as a result it is possible to model a large number of mechanical layers by simply cascading the acoustic ports. [63]

Unfortunately the KLM model does not offer the same versatility when considering the electrical loading conditions. This may be accredited to the complex characteristics of the frequency dependant transformer and series reactance. Subsequently, any understanding of secondary piezoelectric action is obscured, and in this respect the model suffers from the same inherant drawbacks as that of Mason.

2.2.2 THE LINEAR SYSTEMS MODEL.

The block diagram approach may be adequately described by considering the method presented by Hayward [16]. Again the technique is based on a set of linear equations, resulting directly from the analysis of the fundamental piezoelectric relationships. A general Laplace transform solution is proposed for the propagating mechanical wave, the relevant boundary conditions applied, and a set of three linear equations are obtained which describe the electrical behaviour of the transducer, the transmission behaviour of the transducer and finally, its behaviour as a receiver.

The model is represented as a linear systems block diagram and the concept of feedback is applied to describe and simplify the effects of secondary piezoelectric action. In order to illustrate the technique, the block diagram model for a piezoelectric transmitter is presented in figure 2.3.

The transmitting device has two mechanical output ports and one electrical input port. At any point in the diagram it is possible to identify a quantity which relates directly to the physical operation of the device. Each block represents either an inter-disciplinary conversion from one of these quantities to another (ie. mechanical displacement to electrical charge), or alternatively a block may represent the transfer of a single quantity over some kind of boundary. (ie. internal force to external force.) The main advantage in this approach lies in the fact that the block diagram may be easily analysed and manipulated thereby permitting an improved understanding of the nature of all aspects of piezoelectric transduction.

The disadvantage in this method is that it does not facilitate the analysis of multilayered structures, the method however, has since been extended by Jackson [27][28], to incorporate the effect of multiple layers, although this tends to obscure some of the physical insight into the operation of the device.

2.3 TWO-DIMENSIONAL TRANSDUCER MODELS

The operation of a piezoelectric transducer which supports two principal vibrational modes may also be described by a set of linear equations. The two particular modes of importance to array design are the thickness and the lateral or width modes. The configuration ratio, or width to thickness ratio of the device, has an important influence on its behaviour, and both modes tend to be active simultaneously for configuration ratios (G) of between 0.2 and 5. The great majority of modelling strategies for these devices generally impose some dimensional constraint, within which the model may be assumed accurate. This will be clearly stipulated in addition to any other assumptions, for each technique under consideration.

In general terms, two-dimensional transducer models may be classified in two categories. Firstly, those which provide some means of visualising the physical operation of the device, however limited. These tend to either, constrain the frequency range; prohibit the variation of some design parameter, such as electrical load impedance; or simply, they only predict one aspect of transducer performance, such as the resonant frequency. Alternatively, more exact two-dimensional models, whilst providing more accurate results are invariably shrouded in complicated mathematical analysis and as a result often provide little or no insight into the operation of the device.

The following sections describe a number of these modelling methods with particular reference to the following factors.

o The amount of insight gained into the physical behaviour of the device.

- The ability to consider arbitrary electrical and mechanical load configurations.
- The ease with which the technique may be implemented.
- The ease with which the results may be interpreted.

2.3.1 THE COUPLED MODES THEORY

Onoe and Tiersten [50] first attempted to characterise one aspect of two-dimensional transducer behaviour in 1963. They applied a method formerly used to determine the frequency spectra of purely elastic bodies, to the analysis of piezoelectric devices. Assuming that the vibrational modes may be treated as two, one degree-of-freedom systems, coupled by a single frequency invariant mode coupling factor, they obtained the following biquadratic coupled mode equation.

$$(fa^2 - f^2).(fb^2 - f^2) = \Gamma.fa^2.fb^2 \qquad 2.3$$

where fa and fb are the eigenfrequencies of the two uncoupled systems, and Γ is a dimensionless coupling factor. This equation is plotted in figure 2.4 on a log(fH) Vs log(G) coordinate system, where H is the thickness dimension and G the configuration ratio. The solid lines give the frequencies of the coupled system whereas the dashed lines refer to the uncoupled frequencies. In order to select a suitable value for Γ it is necessary to obtain the four limiting frequencies, two for very large and two for very small values of configuration ratio G. It may also be shown that if f_1 and f_2 are the limiting frequencies as G becomes small, and f_3 and f_4 are the limiting frequencies as G becomes large, then the relationship $f_1/f_4 = f_3/f_2$ must also be true in order to preserve the internal consistency of equation 2.3.

Fabian [13] and Meeker [45] have verified the accuracy of the coupled modes technique with detailed experimental studies of frequency spectra for varying configuration ratios. The method is adequate for predicting the resonant frequencies of mechanically free devices, which are not connected to a finite electrical loading network. The problem of incorporating arbitrary mechanical and electrical loading cannot be solved with such a simple analysis, furthermore the technique does not promote any degree of intuitive understanding into the nature of transducer behaviour, and subsequently the usefulness of the technique is severely limited.

2.3.2 THE MODIFIED UNI-DIMENSIONAL MODEL.

Another useful technique which may be applied to the characterisation of two-dimensional transducers is the method involving modified coefficients [30]. In this technique the stress generated by the lateral mode of vibration is assumed to be negligible, thereby allowing a reduction in the number of constitutive equations. These equations may then be solved in exactly the same manner as the uni-dimensional case, and the appropriate model derived. The technique is now illustrated for the set of piezoelectric relationships which relate to the tall, thin transducer. That is,

$$\Gamma_1 = Y_{11} S_1 + Y_{13} S_3 - h_{13} D_3 \qquad 2.4$$

$$\Gamma_3 = Y_{13} S_1 + Y_{33} S_3 - h_{33} D_3 \qquad 2.5$$

$$E_3 = -h_{13} S_1 - h_{33} S_3 + D_3 / \epsilon_{33} \qquad 2.6$$

where Γ_1 and Γ_3 represent stress in the lateral and thickness dimensions respectively. Similarly S_1 and S_3 denote strain in the respective dimensions and E_3 and D_3 are the electric field strength and the electrical displacement for the thickness direction. Y_{mn} are the relevant stiffnesses, measured at constant electrical displacement, h_{mn} the relevant piezoelectric constants and ϵ_{33} is the relative permittivity of the transducer.

For the sake of consistency, this form of the constitutive relationships will be used as the basis for all of the modelling strategies considered. Although these are not the particular relationships used by Kino and DeSilets in the original derivation of this method, the principles of the analysis are identical.

If the stress in the lateral direction is negligible ($\Gamma_1 = 0$) then equation 2.4 may be written in the following manner :-

$$S_1 = \frac{h_{13} D_3 - Y_{13} S_3}{Y_{11}} \qquad 2.7$$

Substitution of equation 2.7 into equations 2.5 and 2.6 provides the following two relationships.

$$\Gamma_3 = Y_{13}' S_3 - h_{33}' D_3 \qquad 2.8$$

$$E_3 = -h_{33}' S_3 + D_3 / \epsilon_{33}' \qquad 2.9$$

where

$$Y_{33}' = Y_{33} \left[1 - \frac{Y_{13}^2}{Y_{11} Y_{33}} \right] \qquad 2.10$$

$$h_{33}' = h_{33} - \frac{h_{13} Y_{13}}{Y_{13}} \qquad 2.11$$

and

$$1/\epsilon_{33}' = 1/\epsilon_{33} - h_{13}^2 / Y_{11} \qquad 2.12$$

Equations 2.8 and 2.9 are identical to those for a uni-dimensional thickness mode device (2.1 and 2.2), with the modified coefficients defined in equations 2.10, 2.11 and 2.12. As a result, any analysis or modelling strategy, which has been applied to thickness mode devices, is also applicable to the two-dimensional transducer.

The method however, is inaccurate in the assumption used to obtain equation 2.9. That is, the stress in the lateral dimension is not negligible. Due to this simplifying assumption, the resonance which occurs at the lateral mode frequency is absent from any simulated response generated by this method. Furthermore, as a result of this absence, the diminished coupling to the thickness direction causes the main resonance frequency to deviate by a small amount from the expected value.

The dimensional constraint limits the configuration ratio to a maximum of around 0.5 in order to achieve maximum accuracy, and in keeping with the majority of two-dimensional models the accuracy decreases rapidly above G = 0.5. There is an additional major disadvantage with this method, since it does not facilitate any understanding of two-dimensional operation.

Given its limitations, the technique is still fairly powerful since the analysis and description of one-dimensional behaviour is common in the literature. In particular, the method is not confined to one specific type of transducer model, thus a wide variety of modelling options may be considered. In order to provide more accurate results, Kino and DeSilets [30] have combined the method with Onoe and Tiersten's coupled modes theory, to predict the frequency of the lateral mode vibration.

2.3.3 FINITE ELEMENT ANALYSIS OF MULTI-DIMENSIONAL TRANSDUCERS.

The finite element method has been used for many years to determine the vibratory characteristics of an arbitrarily shaped body. Allik and Hughes [1] initially applied the method to the analysis of piezoelectric materials. The method may be applied to virtually any shape of piezoelectric transducer, to yield both the electromechanical coupling coefficient, and the eigenvalues which define the resonant and anti-resonant frequencies for a particular mode of vibration. The mathematical analysis utilised by the technique will be omitted from this review due to its complexity. (additional information regarding the mathematical foundation may be obtained from reference [4]) The method is based upon the division of the transducer into a theoretically infinite number of infinitesimally small elements, however in practice a finite number of small cells are used. The physical basis for the technique involves the various energy quantities which are associated with each cell, for example, the total energy in each cell may comprise,

- o Elastic energy (kinetic and potential).
- o Dielectric energy
- o Thermodynamic energy

o Electromechanical coupling energy.

Each of the techniques referenced utilises some or all of these energies.

There are two fundamental approaches which are used in the finite element analysis of piezoelectric devices. Firstly, the complete set of piezoelectric constitutive relationships may be used as the basis for the analysis, including any set of reduced relationships for a particular case. For example, in the case of a long, tall and thin array transducer, the length mode of vibration may be neglected which reduces the order of the matrices involved. [34]

The second technique utilises only the elastic relationships in order to form the finite element equations, and the piezoelectric effect is introduced as a perturbation on the elastic behaviour. This is valid since the elastic behaviour of the device is the dominant factor, even for transducers with large values of electromechanical coupling. Boucher et al [6] have implemented this method for a study of the resonant behaviour of a piezoelectric cube and excellent experimental agreement was demonstrated. The application of this second technique permits a reduction in the matrix order for any specific case, hence allowing a generous computational saving.

A number of authors have applied the finite element technique for the analysis of two-dimensional array elements, with excellent results. Sato et al [54][55] have demonstrated the effect of variations in configuration ratio upon the coupling coefficient and the radiated field, although their field profile computations were later improved by Selfridge et al [57]. Naillon [46] demonstrated the complex behaviour of a two-dimensional array transducer using an animated sequence, which outlined the physical deformation undergone by the device. Their analysis included partially electroded structures in addition to fully electroded structures. Coursant et al [9] used the finite element approach to characterise the resonance and electromechanical coupling factors for a variety of different ceramics.

In general the accuracy of the finite element method is solely dependant upon the size of the elemental cells used (usually referred to as the mesh size). The use of a smaller mesh size results not only in increased accuracy but also increased computational expense. Hence a tradeoff exists between the degree of accuracy required and the computational expense permitted.

The analysis is most useful when the electro-mechanical coupling coefficient or the resonant frequency are of interest. It is however, somewhat limited as a general transducer model for the following reasons

o No attempt is made to suggest some physical interpretation of the behaviour which is predicted.

- o Arbitrary mechanical and electrical loading is difficult to incorporate in the modelling strategy.
- o A working knowledge of complex finite element techniques is an essential prerequisite to using the technique in a useful way.
- o The method can be extremely computationally expensive.
- o It is difficult to establish useful cause and effect relationships.

The following section describes a new model for piezoelectric array transducers which operate in two dimensions. The model is believed to offer significant advantages over existing techniques for this type of device.

2.4 A NEW TWO-DIMENSIONAL TRANSDUCER MODEL.

Reffering to uni-dimensional modelling strategies, each of the techniques possesses distinct advantages for any particular application. For example, the KLM model permits simple analysis of layered structures while the linear systems approach facilitates a more general physical understanding of device behaviour. All of the existing methods for modelling two-dimensional transducer behaviour suffer from the following inherent drawbacks :

1. None of them provide a simple physical description of

the nature of two-dimensional operation as applied to array transducers.

2. None of them describe the cause and effects of inter-dimensional secondary piezoelectric action.

3. None of them permit a ready understanding of the mechanical and piezoelectric cross coupling which exists between the two modes of vibration.

In consideration of these factors, it is apparent that the applicationn of linear systems theory to the modelling and analysis of two-dimensional piezoelectric structures would offer significant advantages over present methods. For example, when compared to the finite element technique, a two-dimensional systems approach would permit an equivalent degree of accuracy and computational efficiency, in addition to an excellent intuitive understanding of device behaviour. Since an electrical equivalent circuit is not available for two-dimensional transducers, even the somewhat limited insight offered by such techniques does not exist. In order to improve the quality of present transducer arrays it is imperative that the physical mechanisms controlling the operation of the device are understood. This may be best achieved by isolating each aspect of the transduction process and establishing simple cause and effect relationships for it's behaviour. The linear systems technique readily permits such simplifying manipulations and is thus a prime contender for this application.

The following 3 chapters describe the derivation of a linear systems model which predicts the behaviour of two-dimensional transducers, subject to the following simplifying assumptions.

1. The two modes are both essentially one-dimensional plane wave vibrations.

2. The coupling between the two modes is entirely dependant upon the elastic and piezoelectric properties of the material.

3. The transducer and surrounding media are loss free.

4. The force, displacement and electric field variations in the device are planar.

5. Apart from the thickness and lateral mode longitudinal vibrations, all other modes are negligible.

The application of these assumptions permits the development of a linear systems model which provides the following advantages.

1. All the elements of the block diagram may be obtained directly from the defining equations. Each block describes a particular process including conversion between inter-disciplinary as well as

inter-dimensionary quantities.

2. The interactive coupling among all the processes within the system is clearly illustrated by the interconnections between blocks.

3. The model may be re-arranged in order to highlight any particular input/output relationship. Consequently the physical process behind each block or group of blocks may be determined, and its effect on the overall transduction system may be assessed.

4. The model may be partitioned to seperate the thickness and lateral modes of vibration, thus providing an enhanced understanding of two-dimensional interaction. Subsequently, it is possible to model a single mode of vibration using the same facility.

5. Block diagram reduction is generally a relatively straightforward process, and as a result the determination of the overall transfer function is less susceptible to mathematical error.

Finally, the block diagram method is essentially a frequency domain technique and requires a knowledge of linear systems theory, hence familiarity with such concepts as transfer functions and feedback analysis are desirable.

CHAPTER III

THE DEVELOPMENT OF LINEAR SYSTEMS MODELS FOR TRANSMISSION, RECEPTION AND ELECTRICAL IMPEDANCE

PART 1

THEORETICAL DEVELOPMENT

3.1 INTRODUCTION

This chapter will present the thoeretical development of a new technique for the modelling of tall, thin piezoelectric transducers. Linear systems theory is employed to yield accurate models, which describe the operation of such devices, as transmitters and receivers of ultrasound. In addition, a transfer function is derived, relating to the electrical impedance (or admittance) function of the transducer, which may also be represented in block diagram format. The first part of the chapter outlines the mathematical derivation of the relevant system equations. These are then utilised in the second part, to generate, and subsequently analyse the related block diagram models. The system equations are derived in a completely general fashion, and as a result, the effects of arbitrary electrical and mechanical loading may be readily identified.

The geometry of the transducer is illustrated in figure 3.1, where poling is in the z-direction (thickness direction). The device is assumed to operate via a Thevenin equivilent electrical load impedance denoted by Z_e. In order to simplify the mathematical derivation, the following assumptions are proposed, concerning the behaviour of the device.

1. Plane wave only propagation exists in both the thickness and lateral dimensions. This approximation

may be considered valid while the configuration ratio (G) of the device remains either below 0.5 or above 2.

2. The transducer and its surrounding media are loss free, thereby permitting the use of purely real mechanical impedances. The effects of the finite active surface of the transducer, upon the nature of its mechanical impedance function, has been investigated by Kino et al [30]. He concluded that the use of purely real impedances was perfectly adequate for the great majority of practical situations.

3. The operation of the transducer is linear. This assumption precludes such effects as cavitation and plastic deformation from the analysis. Thus the set of equations used as the basis for the model, describe the behaviour of the device under purely linear operating conditions.

3.1.1 THE ACOUSTIC WAVE EQUATIONS.

In order to represent the behaviour of the tall thin piezo-electric transducer in a two-dimensional linear systems format, it is necessary to first obtain two coupled wave equations, which describe the propagation of acoustic waves within the device. Consider the fundamental constitutive relations for the device. The $_1$ suffix refers to the lateral (x) direction, and the $_3$ suffix to the thickness (z) direction.

$$
\begin{bmatrix} \Gamma_1 \\ \Gamma_3 \\ E_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{13} & -h_{13} \\ Y_{13} & Y_{33} & -h_{33} \\ -h_{13} & -h_{33} & 1/\epsilon_{33} \end{bmatrix} \begin{bmatrix} S_1 \\ S_3 \\ D_3 \end{bmatrix} \qquad 3.1
$$

Γ_1 is the stress in the lateral direction in Newtons per square metre.

Γ_3 is the stress in the thickness direction in Newtons per square metre.

E_3 is the electric field strength in the thickness direction expressed in volts per metre.

S_1 is the strain or fractional change in length in the lateral dimension.

S_3 is the strain or fractional change in length in the thickness dimension.

D_3 is the electrical displacement in the thickness direction in coulombs per square metre.

Y_{11} is the elastic stiffness coefficient for the lateral dimension. It is measured at constant electrical displacement and expressed in Newtons per square metre.

Y_{33} is the elastic stiffness coefficient for the thickness dimension.

Y_{13} is the elastic stiffness coefficient relating quantities of stress and strain in the thickness and lateral dimensions.

h_{33} is the piezoelectric constant relating the stress to the applied charge density, or the electric field to the applied mechanical strain for the thickness dimension. It is usually measured under conditions of constant electrical displacement and is expressed in either Newtons per Coulomb or volts per metre

h_{13} is the piezoelectric constant relating the stress in the lateral direction to the applied charge density, or electrical field strength in the thickness to the mechanical strain in the lateral direction.

ϵ_{33} is the absolute permittivity of the transducer material in the thickness direction, measured at constant applied strain, in Farads per metre.

Furthermore,

$$S_1 = \frac{\delta \xi_1}{\delta x}$$

and

$$S_3 = \frac{\delta \xi_3}{\delta z}$$

Where ξ_1 and ξ_3 are the mechanical displacements of an arbitrary point inside the transducer.

Partial differentiation with respect to both x and z of each line in Equation 3.1 yields the following six relationships.

$$\frac{\delta \Gamma_1}{\delta x} = Y_{11} \frac{\delta^2 \xi_1}{\delta x^2} + Y_{13} \frac{\delta^2 \xi_3}{\delta x \delta z} - h_{13} \frac{\delta D_3}{\delta x} \qquad 3.2$$

$$\frac{\delta \Gamma_3}{\delta z} = Y_{13} \frac{\delta^2 \xi_1}{\delta x \delta z} + Y_{33} \frac{\delta^2 \xi_3}{\delta z^2} - h_{33} \frac{\delta D_3}{\delta z} \qquad 3.3$$

$$\frac{\delta \Gamma_1}{\delta z} = Y_{11} \frac{\delta^2 \xi_1}{\delta x \delta z} + Y_{13} \frac{\delta^2 \xi_3}{\delta z^2} - h_{13} \frac{\delta D_3}{\delta z} \qquad 3.4$$

$$\frac{\delta \Gamma_3}{\delta x} = Y_{13} \frac{\delta^2 \xi_1}{\delta x^2} + Y_{33} \frac{\delta^2 \xi_3}{\delta x \delta z} - h_{33} \frac{\delta D_3}{\delta x} \qquad 3.5$$

$$\frac{\delta E_3}{\delta x} = -h_{13}\frac{\delta^2 \xi_1}{\delta x^2} - h_{33}\frac{\delta^2 \xi_3}{\delta x \delta z} + 1/\epsilon_{33}\frac{\delta D_3}{\delta x} \qquad 3.6$$

$$\frac{\delta E_3}{\delta x} = -h_{13}\frac{\delta^2 \xi_1}{\delta x \delta z} - h_{33}\frac{\delta^2 \xi_3}{\delta z^2} + 1/\epsilon_{33}\frac{\delta D_3}{\delta z} \qquad 3.7$$

A number of points or conditions may be noted with regard to these six equations,

1. The electrical displacement is a function of the x direction only, or alternatively, there is no free charge within the transducer, therefore Gauss' law yields

$$\frac{\delta D_3}{\delta z} = 0$$

2. The conducting electrodes cover the full extent of the lateral dimension and any fringing effects are considered negligible. Subsequently, the electric field strength is independant of the x-coordinate, or

$$\frac{\delta E_3}{\delta x} = 0$$

3. The mechanical stress profile in each of the two dimensions is considered planar, and as a result the stress in the x direction is independant of the z-coordinate and vice versa. That is,

$$\frac{\delta \Gamma_1}{\delta z} = 0$$

$$\frac{\delta \Gamma_3}{\delta x} = 0$$

Applying point 2 above to equation 3.6 results in the following relationship.

$$\frac{\delta D_3}{\delta x} = \epsilon_{33} \left[h_{13} \frac{\delta^2 \xi_1}{\delta x^2} + h_{33} \frac{\delta^2 \xi_3}{\delta x \delta z} \right] \qquad 3.8$$

When equation 3.8 is substituted into 3.5 and condition 3 above is applied, the following differential equation is obtained,

$$\frac{\delta^2 \xi_3}{\delta x \delta z} = - \frac{\delta^2 \xi_1}{\delta x^2} \left[\frac{Y_{13} - h_{13} h_{33} \epsilon_{33}}{Y_{33} - h_{33}^2 \epsilon_{33}} \right] \qquad 3.9$$

Equations 3.8 and 3.9 may then be combined to yield,

$$\frac{\delta \Gamma_1}{\delta x} = \frac{\delta^2 \xi_1}{\delta x^2} \left[(Y_{11} - h_{13}^2 \epsilon_{33}) - \frac{(Y_{13} - h_{13} h_{33} \epsilon_{33})^2}{Y_{33} - h_{33}^2 \epsilon_{33}} \right] \qquad 3.10$$

Similarly, applying point 1 to equation 3.4 gives

$$\frac{\delta^2 \xi_1}{\delta x \delta z} = - \frac{Y_{13}}{Y_{11}} \cdot \frac{\delta^2 \xi_3}{\delta z^2} \qquad 3.11$$

and finally, equation 3.11 may be combined with 3.3 and condition 3 above to yield,

$$\frac{\delta \Gamma_3}{\delta z} = \frac{\delta^2 \xi_3}{\delta z^2} \left[Y_{33} - \frac{Y_{13}^2}{Y_{11}} \right] \qquad 3.12$$

By applying Newtons' second law relating force and acceleration, to an infinitesimal volume element within the transducer, it may be shown that,

$$\frac{\delta^2 \xi_1}{\delta t^2} = v_1^2 \frac{\delta^2 \xi_1}{\delta x^2} \qquad 3.13$$

and

$$\frac{\delta^2 \xi_3}{\delta t^2} = v_3^2 \frac{\delta^2 \xi_3}{\delta z^2} \qquad 3.14$$

where

$$v_1^2 = \frac{1}{\rho} \left[(Y_{11} - h_{13}^2 \ \epsilon_{33}) - \frac{(Y_{13} - h_{13} h_{33} \epsilon_{33})^2}{Y_{33} - h_{33}^2 \ \epsilon_{33}} \right]$$

and

$$v_3^2 = \frac{1}{\rho}\left[(Y_{33} - \frac{Y_{13}^2}{Y_{11}}\right]$$

and ρ is the density of the transducer material.

Equations 3.13 and 3.14 are two standard wave equations which describe planar wave propagation in each of the two principal dimensions within the transducer. The velocities of acoustic wave propagation in the lateral and thickness dimensions are given by v_1 and v_3 respectively. It may be shown also [42] that the acoustic wave velocity in a purely thickness mode thin disk transducer is given by

$$v_T^2 = Y_{33}/\rho$$

Consequently, comparing this with the expression for v_3, the reduction in the velocity of the acoustic waves which travel in the thickness dimension of a two-dimensional transducer, may be directly attributed to elastic cross coupling through Y_{13}.

Similarly, the acoustic velocity for the isolated lateral or width mode has been shown by Redwood [52] to be

$$v_L^2 = (Y_{11} - h_{13}^2\ \epsilon_{33})/\rho$$

A comparison with the expression for v_1 indicates that the reduction in the lateral acoustic velocity is dependant upon both Y_{13} and h_{13}. (The lateral mode is termed piezoelectrically stiffened, since its effective elastic stiffness depends upon the piezoelectric constant h_{13}.)

3.1.2 THE TWO-DIMENSIONAL SYSTEMS EQUATIONS

Solution of the acoustic wave equation may be simplified by utilising the Laplace transform technique. Two suitable plane wave solutions to equations 3.13 and 3.14 are given by,

$$\bar{\xi}_1 = \bar{A}_1 e^{-sx/v_1} + \bar{B}_1 e^{+sx/v_1} \qquad 3.15$$

$$\bar{\xi}_3 = \bar{A}_3 e^{-sz/v_3} + \bar{B}_3 e^{+sz/v_3} \qquad 3.16$$

In this notation A_1 and B_3 are functions of mechanical displacement describing forward and backward wave propagation in the lateral and thickness dimensions respectively. s is the Laplace complex variable and Laplace quantities are denoted by the bar symbol.

The fundamental piezoelectric constitutive relationships may be rewritten in the following format.

$$\bar{\Gamma}_1 = Y_{11} \frac{\delta \bar{\xi}_1}{\delta x} + Y_{13} \frac{\delta \bar{\xi}_3}{\delta z} - h_{13} \bar{D}_3 \qquad 3.17$$

$$\bar{\Gamma}_3 = Y_{13} \frac{\delta \bar{\xi}_1}{\delta x} + Y_{33} \frac{\delta \bar{\xi}_3}{\delta z} - h_{33} \bar{D}_3 \qquad 3.18$$

$$\bar{E}_3 = - h_{13} \frac{\delta \bar{\xi}_1}{\delta x} - h_{33} \frac{\delta \bar{\xi}_3}{\delta z} - \frac{\bar{D}_3}{\epsilon_{33}} \qquad 3.19$$

By combining equations 3.19 and 3.17, the expression for the stress in the lateral direction may be rewritten as

$$\bar{\Gamma}_1 = (Y_{11} - h_{13}^2 \epsilon_{33}) \frac{\delta \bar{\xi}_1}{\delta x} + (Y_{13} - h_{13} h_{33} \epsilon_{33}) \frac{\delta \bar{\xi}_3}{\delta z} - h_{13} \epsilon_{33} \bar{E}_3 \qquad 3.20$$

In order to fully develop the systems model, equations 3.18, 3.19 and 3.20 should be expressed in terms of force instead of stress, voltage instead of electric field strength and electronic charge instead of elecrrical displacement. The force relationships may be obtained by performing a surface integration over the relevant lateral or thickness surfaces. That is,

$$\text{LATERAL FORCE} = \int_0^L \int_0^H \bar{\Gamma}_1 \, \delta z \delta y$$

$$\text{THICKNESS FORCE} = \int_0^L \int_0^W \bar{\Gamma}_3 \, \delta x \delta y$$

or

$$\bar{F}_1 = (Y_{11}-h_{13}{}^2\epsilon_{33})\int_0^L\int_0^H \frac{\delta\bar{\xi}_1}{\delta x}\,\delta z\delta y + (Y_{13}-h_{13}h_{33}\epsilon_{33})\int_0^L\int_0^H \frac{\delta\bar{\xi}_3}{\delta z}\,\delta z\delta y$$

$$- \; h_{13}\,\epsilon_{33}\int_0^L\int_0^H \bar{E}_3\,\delta z\delta y$$

and

$$\bar{F}_3 = Y_{13}\int_0^L\int_0^W \frac{\delta\bar{\xi}_1}{\delta x}\,\delta x\delta y + Y_{33}\int_0^L\int_0^W \frac{\delta\bar{\xi}_3}{\delta z}\,\delta x\delta y$$

$$- \; h_{33}\int_0^L\int_0^W \bar{D}_3\,\delta x\delta y$$

where L, W and H are the length, width and thickness of the transducer as shown in figure 3.1. Performing the surface integration results in the following two relationships.

$$\bar{F}_1 = s(Y_{11} - h_{13}^2 \ \epsilon_{33}) \frac{HL}{v_1} \left[-\bar{A}_1 e^{-sx/v_1} + \bar{B}_1 e^{+sx/v_1} \right]$$

$$+ (Y_{13} - h_{13} h_{33} \epsilon_{33}) \ L \left[\bar{A}_3 (e^{-sT_3} - 1) + \bar{B}_3 (e^{+sT_3} - 1) \right]$$

$$- h_{13} \epsilon_{33} L \ \bar{V}_3$$

$$\bar{F}_3 = \frac{sY_{33} WL}{v_3} \left[-\bar{A}_3 e^{-sz/v_3} + \bar{B}_3 e^{+sz/v_3} \right]$$

$$+ \ Y_{13} L \left[\bar{A}_1 (e^{-sT_1} - 1) + \bar{B}_1 (e^{+sT_1} - 1) \right]$$

$$+ \ h_{33} \bar{Q}$$

where V_3 is the voltage developed across the electrodes of the transducer, and Q is the electronic charge resident upon either of the electrodes. These equations may be written in the following more general format.

$$\bar{F}_1 + \phi \ \bar{V}_3 = sZc_1 \left[-\bar{A}_1 e^{-sx/v_1} + \bar{B}_1 e^{+sx/v_1} \right]$$

$$+ \ \psi_{31} \left[\bar{A}_3 (e^{-sT_3} - 1) + \bar{B}_3 (e^{+sT_3} - 1) \right] \qquad 3.21$$

$$\bar{F}_3 + h_{33}\bar{Q}_3 = sZc_3\left[-\bar{A}_3 e^{-sz/v_3} + \bar{B}_3 e^{+sz/v_3}\right]$$

$$+ \psi_{13}\left[\bar{A}_1(e^{-sT_1} - 1) + \bar{B}_1(e^{+sT_1} - 1)\right] \qquad 3.22$$

Equations 3.21. and 3.22 are Laplace transform relationships which describe the electromechanical behaviour of the piezoelectric element. Referring to these, the following physically meaningful parameters may be defined,

$$\phi = \frac{h_{13}\,Co\,L}{W}$$

where Co is the static or clamped capacitance of the device. The factor ϕ corresponds to the piezoelectric voltage constant relating the voltage which appears across the electrodes, to the force incident upon a lateral face, as defined by Redwood [52] for a uni-dimensional lateral mode. T_1 and T_3 are the transit times for waves of force to traverse the lateral and thickness dimensions respectively. For example T_1=W/v_1 Furthermore,

$$\psi_{31} = (Y_{13} - h_{13}h_{33}\epsilon_{33})\,L$$

$$\psi_{13} = Y_{13}L$$

ψ_{13} and ψ_{31} are thus conversion factors which account for mechanical cross coupling between the two dimensions. That is, ψ_{31} may be considered as a transfer function which converts thickness displacement to lateral force. Similarly ψ_{13} is a transfer function which converts mechanical displacement in the lateral direction to a force in the thickness direction.

Finally, the mechanical impedances are given by,

$$Zc_1 = Zc_L \frac{v_L}{v_1}$$

$$Zc_3 = Zc_T \frac{v_T}{v_3}$$

Where Zc_T is the mechanical impedance for a purely thickness mode transducer, and Zc_L the mechanical impedance for a lateral mode device. An increase in the effective acoustic impedance associated with each vibrational mode is thus apparent, compared with their uni-dimensional counterparts. The magnitude of the increase is directly proportional to the ratio of the uni-dimensional velocity and the two-dimensional velocity.

Considering now equation 3.19, it is required to express the electric field strength in terms of voltage and the electrical displacement in terms of electronic charge. This may only be achieved by performing a three-dimensional volume integration throughout the volume of the transducer. The integration may be written as follows,

$$\int_0^H\int_0^L\int_0^W \bar{E}_3\,\delta x\delta y\delta z = -h_{13}\int_0^H\int_0^L\int_0^W \frac{\delta\bar{\xi}_1}{\delta x}\,\delta x\delta y\delta z$$

$$-h_{33}\int_0^H\int_0^L\int_0^W \frac{\delta\bar{\xi}_3}{\delta z}\,\delta x\delta y\delta z + \frac{1}{\epsilon_{33}}\int_0^H\int_0^L\int_0^W \bar{D}_3\,\delta x\delta y\delta z$$

On performing the integration, this expression reduces to,

$$\bar{V}_3 - \bar{Q}/Co = -\phi/Co\left[\bar{A}_1(e^{-sT_1} - 1) + \bar{B}_1(e^{+sT_1} - 1)\right]$$

$$-h_{33}\left[\bar{A}_3(e^{-sT_3} - 1) + \bar{B}_3(e^{+sT_3} - 1)\right] \qquad 3.23$$

It is interesting to note that in the absence of lateral vibration ($\bar{A}_1=0, \bar{B}_1=0$) then equations 3.22 and 3.23 are identical to those given by Hayward [18] for a uni-dimensional thickness mode transducer. Similarly, in the absence of thickness vibrations ($\bar{A}_3=0, \bar{B}_3=0$) then equations 3.21 and 3.23 reduce to those given by Redwood [52] for an isolated lateral mode.

Equations 3.21, 3.22 and 3.23 are now in a form where they may be used, in conjuction with the relevant boundary conditions, to provide transfer function relationships which may be used as a basis for the systems model.

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3.1.3 THE BOUNDARY CONDITIONS

In order to solve fully the describing relationships, it is required to develop expressions which represent $\bar{A}_1$, $\bar{B}_1$, $\bar{A}_3$ and $\bar{B}_3$. These expressions may only be obtained upon application of a complete set of boundary conditions. This section describes such a set of conditions and outlines the method by which $\bar{A}_1$, $\bar{B}_1$, $\bar{A}_3$ and $\bar{B}_3$ may be found. For the sake of continuity, some of the more involved mathematical operations are presented in appendix A.

Consider that the transducer is mechanically loaded on all four major faces by semi-infinite non-piezoelectric elastic media as shown in figure 3.2. These elastic loading media may be characterised by the parameters illustrated in table 3.1, where the total particle displacements and forces are given by the relationships.

$$\bar{\xi}_{11} = \bar{A}_{11}\, e^{-sx/v_L} + \bar{B}_{11}\, e^{+sx/v_L}$$

$$\bar{\xi}_{12} = \bar{A}_{12}\, e^{-sx/v_R} + \bar{B}_{12}\, e^{+sx/v_R}$$

$$\bar{\xi}_{33} = \bar{A}_{33}\, e^{-sz/v_F} + \bar{B}_{33}\, e^{+sz/v_F}$$

$$\bar{\xi}_{34} = \bar{A}_{34}\, e^{-sz/v_B} + \bar{B}_{34}\, e^{+sz/v_B}$$

and,

$$\bar{F}_{11} = sZ_L \left[-\bar{A}_{11} e^{-sx/v_L} + \bar{B}_{11} e^{+sx/v_L} \right]$$

$$\bar{F}_{12} = sZ_R \left[-\bar{A}_{12} e^{-sx/v_R} + \bar{B}_{12} e^{+sx/v_R} \right]$$

$$\bar{F}_{33} = sZ_F \left[-\bar{A}_{33} e^{-sz/v_F} + \bar{B}_{33} e^{+sz/v_F} \right]$$

$$\bar{F}_{34} = sZ_B \left[-\bar{A}_{34} e^{-sz/v_B} + \bar{B}_{34} e^{+sz/v_B} \right]$$

The system is constrained at each boundary by two specific conditions, namely continuity of normal particle displacement and continuity of normal force. Considering for the moment, the continuity of particle displacement, these may be written and subsequently evaluated in the following manner.

Firstly, at the left hand boundary,

$$\bar{\xi}_{11} \Big|_{x=0} = \bar{\xi}_{1} \Big|_{x=0}$$

$$\Rightarrow \qquad \bar{A}_{11} + \bar{B}_{11} = \bar{A}_1 + \bar{B}_1$$

Similarly, at the right hand boundary,

$$\bar{\xi}_{12}\Big|_{x=W} = \bar{\xi}_{1}\Big|_{x=W}$$

$$\Rightarrow \qquad \bar{A}_{32} + \bar{B}_{33} = \bar{A}_{3} + \bar{B}_{3}$$

at the front boundary,

$$\bar{\xi}_{33}\Big|_{z=0} = \bar{\xi}_{3}\Big|_{z=0}$$

$$\Rightarrow \qquad \bar{A}_{12}\, e^{-sW/v_R} + \bar{B}_{12}\, e^{+sW/v_R} = \bar{A}_{1}\, e^{-sT_1} + \bar{B}_{1}\, e^{+sT_1}$$

and finally at the back boundary.

$$\bar{\xi}_{34}\Big|_{z=H} = \bar{\xi}_{3}\Big|_{z=H}$$

$$\Rightarrow \qquad \bar{A}_{34}\, e^{-sH/v_B} + \bar{B}_{34}\, e^{+sH/v_B} = \bar{A}_{3}\, e^{-sT_2} + \bar{B}_{3}\, e^{+sT_2}$$

Similarly, considering the continuity of normal forces, at the left hand boundary,

$$\bar{F}_{11}\Big|_{x=0} = \bar{F}_{1}\Big|_{x=0}$$

$$\Rightarrow \quad sZ_L\,(\bar{B}_{11} - \bar{A}_{11}) = sZc_1\,(\bar{B}_1 - \bar{A}_1) - \phi\,\bar{V}_3$$

$$+ \quad \psi_{31}\left[\bar{A}_3\,(e^{-sT_3} - 1) + \bar{B}_3\,(e^{+sT_3} - 1)\right]$$

at the right hand boundary,

$$\bar{F}_{12}\Big|_{x=W} = \bar{F}_{1}\Big|_{x=W}$$

$$\Rightarrow sZ_R\,(\bar{B}_{12}\,e^{sW/v_R} - \bar{A}_{12}\,e^{-sW/v_R}) = sZc_1\,(\bar{B}_1\,e^{sT_1} - \bar{A}_1\,e^{-sT_1})$$

$$- \phi\,\bar{V}_3 + \psi_{31}\left[\bar{A}_3\,(e^{-sT_3} - 1) + \bar{B}_3\,(e^{+sT_3} - 1)\right]$$

at the front boundary,

$$\bar{F}_{33}\Big|_{z=0} = \bar{F}_{3}\Big|_{z=0}$$

$$\Rightarrow \qquad sZ_F\,(\bar{B}_{33} - \bar{A}_{33}) = sZc_3\,(\bar{B}_3 - \bar{A}_3) - h\,\bar{Q}$$

$$+ \quad \psi_{13} \left[\bar{A}_1\,(e^{-sT_1} - 1) + \bar{B}_1\,(e^{+sT_1} - 1) \right]$$

and at the back.

$$\bar{F}_{34}\Big|_{z=H} = \bar{F}_3\Big|_{z=H}$$

$$\Rightarrow sZ_B\,(\bar{B}_{34}\,e^{sH/v_B} - \bar{A}_{34}\,e^{-sH/v_B}) = sZc_3\,(\bar{B}_3\,e^{sT_3} - \bar{A}_3\,e^{-sT_3})$$

$$- h\,\bar{Q} \quad + \quad \psi_{13} \left[\bar{A}_1\,(e^{-sT_1} - 1) + \bar{B}_1\,(e^{+sT_1} - 1) \right]$$

For the sake of brevity $h = h_{33}$ from this point onwards.

In order to simplify the subsequent mathematical derivation we now apply the assumption that the transducer is only mechanically excited at one lateral face ($\bar{B}_{12}$=0) and at one thickness face ($\bar{B}_{34}$=0).

The combination of the two boundary conditions at each of the faces results in a set of four linear simultaneous equations which may be written in the following matrix format,

$$
\begin{bmatrix}
s(Zc_1-Z_R)e^{-sT_1} & -s(Zc_1+Z_R)e^{+sT_1} & -\psi_{31}(e^{-sT_3}-1) & -\psi_{31}(e^{-sT_3}-1) \\
s(Zc_1+Z_L) & -s(Zc_1-Z_L) & -\psi_{31}(e^{-sT_3}-1) & -\psi_{31}(e^{-sT_3}-1) \\
-\psi_{13}(e^{-sT_1}-1) & -\psi_{13}(e^{-sT_1}-1) & s(Zc_3-Z_3)e^{-sT_3} & -s(Zc_3+Z_3)e^{+sT_3} \\
-\psi_{13}(e^{-sT_1}-1) & -\psi_{13}(e^{-sT_1}-1) & s(Zc_3-Z_3) & -s(Zc_3+Z_3)
\end{bmatrix}
\begin{bmatrix}
\bar{A}_1 \\ \bar{B}_1 \\ \bar{A}_3 \\ \bar{B}_3
\end{bmatrix}
=
\begin{bmatrix}
-\phi\,\bar{V}_3 \\
-\phi\,\bar{V}_3 - 2sZ_L\,\bar{A}_{11} \\
-h\,\bar{Q} \\
-h\,\bar{Q} - 2sZ_F\,\bar{A}_{33}
\end{bmatrix}
$$

where,

$$R_F = \frac{(Zc_3 - Z_F)}{(Zc_3 + Z_F)} \quad , \qquad R_B = \frac{(Zc_3 - Z_B)}{(Zc_3 + Z_B)}$$

$$R_L = \frac{(Zc_1 - Z_L)}{(Zc_1 + Z_L)} \quad , \qquad R_R = \frac{(Zc_1 - Z_R)}{(Zc_1 + Z_R)}$$

$$T_F = \frac{2Zc_3}{(Zc_3 + Z_F)} \quad , \qquad T_B = \frac{2Zc_3}{(Zc_3 + Z_B)}$$

$$T_L = \frac{2Zc_1}{(Zc_1 + Z_L)} \quad , \qquad T_R = \frac{2Zc_1}{(Zc_1 + Z_R)}$$

R_F, R_B, R_L and R_R are reflection coefficients for internal waves of force incident upon the relevant transducer faces. In a similar manner, the corresponding transmission coefficients are denoted by T_F, T_B, T_L and T_R.

The matrix equation (3.25) may now be solved for the functions $\bar{A}_1$, $\bar{B}_1$, $\bar{A}_3$ and $\bar{B}_3$. The solution, presented fully in appendix A, results in the following four expressions,

$$\bar{A}_1 = \frac{\left[\frac{R_L e^{-sT_1}}{s(Zc_1+Z_R)} - \frac{1}{s(Zc_1+Z_L)}\right]\left[\phi\,\bar{V}_2 + \frac{\psi_{31}}{sZc_3}\left(2sT_F\,\bar{K}_F\,Z_F\,\bar{A}_{33} - \bar{K}_3\,h\,\bar{Q}\right)\right] + (1 - R_L)\left[1 + \frac{T_R\,\psi_{13}\,\psi_{31}\,(e^{-sT_1} - 1)\,\bar{K}_3}{2s^2 Zc_1\,Zc_3}\right]\bar{A}_{11}}{(1 - R_L R_R\,e^{-2sT_1}).\,\bar{M}}$$

$$\bar{B}_1 = \frac{\left[\dfrac{e^{-sT_1}}{s(Zc_1+Z_R)} - \dfrac{R_R e^{-2sT_1}}{s(Zc_1+Z_L)} \right] \left[\phi\, \bar{V}_3 + \dfrac{\psi_{31}}{sZc_3} (2sT_F \bar{K}_F Z_F \bar{A}_{33} - \bar{K}_3 h\, \bar{Q}) \right] + (1 - R_L) R_R e^{-2sT_1} \left[1 + \dfrac{T_R \psi_{13} \psi_{31} (1 - e^{sT_1}) \bar{K}_3}{2s^2 R_R Zc_1 Zc_3} \right] \bar{A}_{11}}{(1 - R_L R_R e^{-2sT_1}) .\ \bar{M}}$$

$$\bar{A}_3 = \frac{\left[\frac{R_F e^{-sT_3}}{s(Zc_3+Z_B)} - \frac{1}{s(Zc_3+Z_F)} \right] \left[h\,\bar{Q} + \frac{\psi_{13}}{sZc_1} (2sT_L \bar{K}_L Z_L \bar{A}_{11} - \bar{K}_1 \phi \bar{V}_3) \right] + (1 - R_F) \left[1 + \frac{T_3 \psi_{13} \psi_{31} (e^{-sT_3} - 1)\bar{K}_1}{2s^2 Zc_1 Zc_3} \right] \bar{A}_{33}}{(1 - R_F R_B e^{-2sT_3}) . \bar{M}}$$

$$\bar{B}_3 = \frac{\left[\dfrac{e^{-sT_3}}{s(Zc_3+Z_B)} - \dfrac{R_B e^{-2sT_3}}{s(Zc_3+Z_F)}\right]\left[h\,\bar{Q} + \dfrac{\psi_{13}}{sZc_1}\left(2sT_L\,\bar{K}_L\,Z_L\,\bar{A}_{11} - \bar{K}_1\,\phi\,\bar{V}_3\right)\right] + (1 - R_F)R_B e^{-2sT_3}\left[1 + \dfrac{T_3\,\psi_{13}\,\psi_{31}\,(1 - e^{sT_3})\,\bar{K}_1}{2s^2 R_B\,Zc_1\,Zc_3}\right]\bar{A}_{33}}{(1 - R_F R_B\,e^{-2sT_1}).\,\bar{M}}$$

The following quantities may now be defined from equations 3.26 - 3.29,

$$\bar{K}_F = (1 - e^{-sT_3})(1 - R_B e^{-sT_3})/(1 - R_F R_B e^{-2sT_3})$$

$$\bar{K}_B = (1 - e^{-sT_3})(1 - R_F e^{-sT_3})/(1 - R_F R_B e^{-2sT_3})$$

$$\bar{K}_L = (1 - e^{-sT_1})(1 - R_R e^{-sT_1})/(1 - R_L R_R e^{-2sT_1})$$

$$\bar{K}_R = (1 - e^{-sT_1})(1 - R_L e^{-sT_1})/(1 - R_L R_R e^{-2sT_1})$$

These are the appropriate reverberation factors for each face as defined by Hayward et al [16]. In addition,

$$\bar{K}_1 = [T_L \bar{K}_L + T_R \bar{K}_R]/2$$

$$\bar{K}_3 = [T_F \bar{K}_F + T_B \bar{K}_B]/2$$

are defined as reverberation functions for each dimension. Finally,

$$\bar{M} = \left[1 - \frac{\psi_{13} \psi_{31} \bar{K}_1 \bar{K}_3}{s^2 Zc_1 Zc_3} \right]$$

3.1.4 THE ELECTRICAL IMPEDANCE FUNCTION

Having satisfied the boundary conditions and obtained expressions for $\bar{A}_1$, $\bar{B}_1$, $\bar{A}_3$ and $\bar{B}_3$, it is now possible to evaluate equation 3.23. The electrical, or operational impedance of a device is generally defined as the ratio of voltage across its terminals to the current flowing between the same terminals. In the case of the piezoelectric transducer this is measured under conditions of zero external acoustic excitation. Subsequently, the functions $\bar{A}_{11}$ and $\bar{A}_{33}$ are set to zero in equations 3.26-3.29. This results in the following voltage-charge relationship,

$$\bar{V}_3\left[1 + \bar{A}_W - \bar{A}_{13}\right] - \frac{\bar{Q}}{Co}\left[1 - \bar{A}_T + \bar{A}_{31}\right]$$

$$= \frac{(1-R_L)\bar{K}_L}{\bar{M}}\left[\frac{\phi}{Co} - \frac{\psi_{13} h \bar{K}_3}{sZc_3}\right]\bar{A}_{11}$$

$$+ \frac{(1-R_F)\bar{K}_F}{\bar{M}}\left[h - \frac{\psi_{13}\phi \bar{K}_1}{sCoZc_1}\right]\bar{A}_{33} \qquad 3.30$$

Incorporating the relationship between electronic charge and current,

$$\bar{I} = s.\bar{Q}$$

the expression for the electrical impedance may be obtained.

$$\bar{Z}_T = \frac{\bar{V}_3}{\bar{I}_3} = \frac{1}{sCo}\left[\frac{1 - \bar{A}_T + \bar{A}_{31}}{1 + \bar{A}_W - \bar{A}_{13}}\right] \qquad 3.31$$

where

$$\bar{A}_W = \frac{\phi^2 \bar{K}_1}{sZc_1 Co\bar{M}}$$

$$\bar{A}_T = \frac{h^2 \; Co \; \bar{K}_3}{sZc_3 \bar{M}}$$

$$\bar{A}_{13} = \frac{\psi_{13} h \; \phi \; \bar{K}_1 \bar{K}_3}{s^2 Zc_1 Zc_3 \bar{M}}$$

$$\bar{A}_{31} = \frac{\psi_{31} h \; \phi \; \bar{K}_1 \bar{K}_3}{s^2 Zc_1 Zc_3 \bar{M}}$$

and $\bar{I}_3$ is the current flowing in the transducer.

The form of this impedance function and the quantities $\bar{A}_W$, $\bar{A}_T$, $\bar{A}_{13}$ and $\bar{A}_{31}$ will be analysed, and the physical significance of individual parameters assessed in a subsequent section.

3.1.5 THE RECEIVER TRANSFER FUNCTION

The development of a transfer function type relationship between the applied forces and the resultant recieved voltage is presented in this section. The transducer is assumed to be electrically loaded by a Thevenin equivilent impedance denoted by $\bar{Z}_E$, and the circiut is illustrated in figure 3.3.

The expressions for $\bar{A}_1$, $\bar{B}_1$, $\bar{A}_3$ and $\bar{B}_3$ given by equations 3.26-3.29 are now applied to equation 3.23, resulting in the following complete voltage charge relationship.

$$\bar{V}_3\left[1 + \bar{A}_W - \bar{A}_{13}\right] - \frac{\bar{Q}}{Co}\left[1 - \bar{A}_T + \bar{A}_{31}\right]$$

$$= \frac{(1-R_L)\bar{K}_L}{\bar{M}}\left[\frac{\phi}{Co} - \frac{\psi_{13}h\,\bar{K}_3}{sZc_3}\right]\bar{A}_{11}$$

$$+ \frac{(1-R_F)\bar{K}_F}{\bar{M}}\left[h - \frac{\psi_{13}\phi\,\bar{K}_1}{sCoZc_1}\right]\bar{A}_{33} \qquad 3.32$$

The relationship between the amount of electronic charge which is piezoelectrically generated by the transducer and the voltage developed across it may be written as,

$$\bar{Q} = -\bar{V}_3/s\bar{Z}_E \qquad 3.33$$

Equation 3.33 is then substituted into equation 3.32 to reveal,

$$\bar{V}_3\left[1 + \bar{A}_W - \bar{A}_{13} - \frac{1}{sCo\bar{Z}_E} - \frac{\bar{A}_T}{sCo\bar{Z}_E} + \frac{\bar{A}_{31}}{sCo\bar{Z}_E}\right]$$

$$= \frac{(1-R_L)\bar{K}_L}{\bar{M}}\left[\frac{\phi}{Co} - \frac{\psi_{31}\, h\, \bar{K}_3}{sZc_3}\right]\bar{A}_{11}$$

$$+ \frac{(1-R_F)\bar{K}_F}{\bar{M}}\left[h - \frac{\psi_{13}\, \phi\, \bar{K}_1}{sCoZc_1}\right]\bar{A}_{33}$$

which may be rearranged to give,

$$\bar{V}_3 = \frac{\dfrac{(1-R_L)\bar{K}_L}{\bar{M}}\left[\dfrac{\phi}{Co} - \dfrac{\psi_{13}\, h\, \bar{K}_3}{sZc_3}\right]\bar{A}_{11} + \dfrac{(1-R_F)\bar{K}_F}{\bar{M}}\left[h - \dfrac{\psi_{13}\, \phi\, \bar{K}_1}{sCoZc_1}\right]\bar{A}_{33}}{\left[1 + \bar{U}\,[(\bar{A}_W - \bar{A}_{13}) - (\bar{A}_T - \bar{A}_{31})/sCo\bar{Z}_E]\right]} \qquad 3.34$$

where

$$\bar{U} = \frac{sCo\bar{Z}_E}{1 + sCo\bar{Z}_E}$$

The functions $\bar{A}_{11}$ and $\bar{A}_{33}$ denote incident mechanical waves which strike the left and front faces respectively. It is advantageous to describe these excitation functions in terms of force instead of particle displacement.

The expressions for the incident waves of force may be written as,

$$\bar{F}_{11} = sZ_L \left[- \bar{A}_{11}\, e^{-sx/v_L} + \bar{B}_{11}\, e^{+sx/v_L} \right]$$

$$\bar{F}_{33} = sZ_F \left[- \bar{A}_{33}\, e^{-sz/v_F} + \bar{B}_{33}\, e^{+sz/v_F} \right]$$

Considering only the incident waves at x = 0 and z = 0, then

$$\bar{A}_{11} = - \frac{\bar{F}_{11}}{sZ_L} \qquad 3.35$$

$$\bar{A}_{33} = - \frac{\bar{F}_{33}}{sZ_F} \qquad 3.36$$

Application of 3.35 and 3.36 provide the following comprehensive relationship describing the general behaviour of a two-dimensional piezoelectric reciever.

$$\bar{V}_3 = \frac{\dfrac{\bar{U}}{M}\left[\dfrac{[T_L\bar{K}_L\bar{F}_L + T_R\bar{K}_R\bar{F}_R]}{sZc_1}\left[\dfrac{\phi}{Co} - \dfrac{\psi_{13} h \bar{K}_3}{sZc_3}\right]\bar{A}_{11} + \dfrac{[T_F\bar{K}_F\bar{F}_F + T_B\bar{K}_B\bar{F}_B}{sZc_3}\left[h - \dfrac{\psi_{13}\phi \bar{K}_1}{sCoZc_1}\right]\bar{A}_{33}\right]}{\left[1 + \bar{U}[(\bar{A}_W - \bar{A}_{13}) - (\bar{A}_T - \bar{A}_{31})/sCo\bar{Z}_E]\right]}$$

3.37

The form of equation 3.37 will be analysed and interpreted in a subsequent section.

3.1.6 THE TRANSMITTER TRANSFER FUNCTION

In order to provide a complete description of two-dimensional device operation, the remaining force-voltage or transmission relationships are now derived. As in the case of the electrical impedance, any external mechanical excitation of the device is assumed to be zero. $(\bar{A}_{11}=\bar{A}_{33}=0)$

Since the boundary conditions are identical, apart from the absence of $\bar{A}_{11}$ and $\bar{A}_{33}$, then the solutions for $\bar{A}_{1}$, $\bar{B}_{1}$, $\bar{A}_{3}$ and $\bar{B}_{3}$ may be substituted readily into equations 3.21 and 3.22.

Considering the force emitted from the left face of the device (x=0), this gives,

$$\bar{F}_{L} = \frac{Z_{L}\bar{K}_{L}}{(Z_{L}+Zc_{1})M}\left[\phi\,\bar{V}_{3} - \frac{\psi_{31}\bar{K}_{3}h\,\bar{Q}}{sZc_{3}}\right]$$

also, the force output from the right hand face (x=W) may be written as,

$$\bar{F}_{R} = \frac{Z_{R}\bar{K}_{R}}{(Z_{R}+Zc_{1})M}\left[\phi\,\bar{V}_{3} - \frac{\psi_{31}\bar{K}_{3}h\,\bar{Q}}{sZc_{3}}\right]$$

Similarly, in the thickness dimension, the force emitted from the front face (z=0) is,

$$\bar{F}_F = \frac{Z_F \bar{K}_F}{(Z_F + Zc_3)\bar{M}} \left[h \bar{Q} - \frac{\psi_{13} \bar{K}_1 \phi \bar{V}_3}{sZc_1} \right]$$

and finally, the force from the back face (z=H),

$$\bar{F}_B = \frac{Z_B \bar{K}_B}{(Z_B + Zc_3)\bar{M}} \left[h \bar{Q} - \frac{\psi_{13} \bar{K}_1 \phi \bar{V}_3}{sZc_1} \right]$$

The relationship between charge on the transducer and the voltage across it may be written as,

$$\bar{Q} = \frac{\bar{V}_3}{s\bar{Z}_T}$$

where $\bar{Z}_T$ is the electrical impedance of the device as previously defined. Applying this to the above expressions for force reveals,

$$\frac{\bar{F}_L}{\bar{V}_3} = \frac{Z_L \bar{K}_L}{(Z_L + Zc_1)\bar{M}} \left[\phi - \frac{\psi_{31} \bar{K}_3 h}{s^2 Zc_3 \bar{Z}_T} \right] \qquad 3.38$$

$$\frac{\bar{F}_R}{\bar{V}_3} = \frac{Z_R \bar{K}_R}{(Z_R + Zc_1)\bar{M}} \left[\phi - \frac{\psi_{31} \bar{K}_3 h}{s^2 Zc_3 \bar{Z}_T} \right] \qquad 3.39$$

$$\frac{\bar{F}_F}{\bar{V}_3} = \frac{Z_F \bar{K}_F}{(Z_F + Zc_3)\bar{M}} \left[\frac{h}{sZ_T} - \frac{\psi_{13} \bar{K}_1 \phi}{sZc_1} \right] \qquad 3.40$$

$$\frac{\bar{F}_B}{\bar{V}_3} = \frac{Z_B \bar{K}_B}{(Z_B + Zc_3)\bar{M}} \left[\frac{h}{sZ_T} - \frac{\psi_{13} \bar{K}_1 \phi}{sZc_1} \right] \qquad 3.41$$

Equations 3.38-3.41 are thus transfer functions which relate the force emanating from any face to the voltage generated across the transducer.

As in the case of the piezoelectric reciever, the device is assumed to be driven via a Thévenin equivilent load impedance Z_E. Figure 3.4 represents this situation, where V_s is the driving voltage and V_3 is the voltage apearing across the transducer. By employing voltage division techniques, in conjuction with the previously defined electrical impedance of the transducer, equations 3.38-3.41 may be extended to incorporate the effects of electrical loading. The relationship between V_s and V_3 may be written as,

$$\bar{V}_3 = \frac{\bar{V}_S \bar{Z}_T}{\bar{\bar{Z}}_E + \bar{\bar{Z}}_T}$$

This may now be used along with equations 3.38-3.41 to describe completely the transmission behaviour of a two-dimensional transducer which is driven via a general Thevenin equivilent load impedance.

In summary, the first part of this chapter has presented the derivation of three fundamental linear systems transfer function type relationships, describing transmission, reception and electrical impedance. The following sections utilise these relationships in order to develop and analyse block diagram models for the three cases.

CHAPTER III

THE DEVELOPMENT OF LINEAR SYSTEMS MODELS FOR TRANSMISSION, RECEPTION AND ELECTRICAL IMPEDANCE

PART 2

BLOCK DIAGRAM REPRESENTATION

3.2 INTRODUCTION

This section describes the development and analysis of block diagram models for the electrical impedance, reception and transmission characteristics of two-dimensional transducers. The models are structured to highlight the actual electrical, mechanical and piezoelectric interactions which take place within the device. Furthermore, the effects of secondary piezoelectric action in each of the two dimensions is illustrated, and comparisons are made with the relevant uni-dimensional cases.

3.2.1 THE ELECTRICAL IMPEDANCE BLOCK DIAGRAM MODEL

The electrical impedance function derived in section 3.1.4 may be represented usefully in a linear systems block diagram format. In so doing, the physical operation of the device is illustrated, and improved insight is offered into the nature of two-dimensional secondary piezoelectric action.

Equation 3.31 may be represented as shown in figure 3.5, where the transform quantities $\bar{A}_W$, $\bar{A}_T$, $\bar{A}_{13}$ and $\bar{A}_{31}$ are defined in the following manner.

$\bar{A}_T$ is a feedback factor which defines the amount of secondary charge which is generated as a result of secondary piezoelectric action in the thickness dimension only. This is similar to the feedback factor defined by Hayward [17] for the uni-dimensional thickness mode device. In the two dimensional case however, it is modified by the mechanical cross coupling denoted by $\bar{M}$. Furthermore, a reduction in acoustic velocity and an increase in acoustic impedance occurs as a result of the finite boundary conditions.

$\bar{A}_{w}$ is a feedforward factor which defines the amount of secondary charge which is generated as a result of secondary piezoelectric action in the lateral dimension only. Again, apart from the mechanical cross-coupling , reduction in acoustic velocity and increase in acoustic impedance, $\bar{A}_{w}$ corresponds to a feedforward factor describing secondary action for an isolated lateral mode.

$\bar{A}_{13}$ is a feedforward factor which defines the amount of tertiary charge which is generated as a result of lateral-to-thickness cross coupled secondary piezoelectric action. $\bar{A}_{13}$ thus describes piezoelectric cross-coupling from the lateral to the thickness dimension.

$\bar{A}_{31}$ is a feedback factor which defines the amount of tertiary charge which is generated as a result of thickness-to-lateral cross coupled secondary piezoelectric action. $\bar{A}_{31}$ thus describes piezoelectric cross coupling from the thickness to the lateral dimension.

Figure 3.5 illustrates the electrical admittance of the transducer rather than its impedance, as this simplifies the physical understanding of the model, and allows useful comparisons to be made with the uni-dimensional thickness mode systems model. In order to expand upon this block diagram, consider firstly the feedback loop given by $[\bar{A}_T - \bar{A}_{31}]$. This expression may be written as follows,

$$[\bar{A}_T - \bar{A}_{31}] = \left[h^2Co - \frac{\psi_{31} h \phi \bar{K}_1}{sZc_1} \right] \left[\frac{\bar{K}_3 / sZc_3}{1 - \left(\dfrac{\psi_{31}\psi_{31}K_1K_3}{s^2 Zc_1 Zc_3} \right)} \right] \qquad 3.42$$

Equation 3.42 is similar in form to the standard cascaded feedback/feedforward system illustrated in figure 3.6, and which possesses a transfer function denoted by,

$$\frac{\text{output}}{\text{input}} = \frac{G_1(s)}{[1 - G_1(s).H_1(s)]} \cdot [G_2 + H_2(s)] \qquad 3.43$$

By comparing equations 3.42 and 3.43, the following identities may be constructed,

$$G_1(s) = \bar{K}_3/sZc_3$$

$$G_2 = h^2Co$$

$$H_1(s) = \frac{\psi_{13}\psi_{31}\bar{K}_1}{sZc_1}$$

and,

$$H_2(s) = -\frac{\psi_{31}h\phi\bar{K}_1}{sZc_1}$$

Applying the above relationships to the block diagram structure given in figure 3.6 results in that of figure 3.7.

At this point it should be noted that both the input and output quantities to figure 3.7 consist of electronic charge. Bearing this in mind, it is necessary to rearrange the block diagram to ensure the continuity of physical parameters around the loop. For example, immediately after the first summing point, a charge quantity forms the input to the particle displacement block denoted by $\bar{K}_3/sZc_3$. This clearly has no physical significance. The diagram must therefore be manipulated to provide a real physical parameter at this and every other point. Block rearrangement may be performed by applying the following two steps.

1. The common factor h may be removed from the blocks representing G_2 and $H_2(s)$, and placed prior to the first summing point. This effectively converts the incoming quantity of electronic charge to mechanical force. The force in turn is multiplied by the reverberation factor $\bar{K}_3$, and then divided by sZc_3 to

give a function of particle displacement at point 'a' in figure 3.7.

2. Considering the blocks representing $H_1(s)$ and $H_2(s)$, the factor $\psi_{31}\bar{K}_1/sZc_1$ may be identified in both, and thus may be removed and replaced as a single block between points 'a' and 'b'.

The reorganised block diagram is given in figure 3.8, where real physical quantities now exist as inputs and outputs to every block. The actual physical significance of each quantity is explained shortly.

A similar procedure may be applied to the feedforward loop in figure 3.5. This results in the block diagram given by figure 3.9. Once again the existence of real physical parameters at each point in the diagram is apparent. The complete block diagram model representing the electrical admittance may now be drawn, as depicted in figure 3.10. It is now possible to explain the behaviour of the admittance function in terms of the physical parameters involved.

A quantity of charge is initially deposited upon the electrodes of the device, as a result of the applied voltage and the static capacitance Co. This primary charge then produces two distinct piezoelectrically generated forces. Firstly, a function of primary lateral force occurs as a result of the lateral piezoelectric constant ϕ/Co. Secondly, primary thickness force is generated via the piezoelectric constant h.

The reverberation characteristics are represented by $\bar{K}_1$ and $\bar{K}_3$ for the lateral and thickness dimensions respectively. Consequently the blocks $\bar{K}_1/sZc_1$ and $\bar{K}_3/sZc_3$, convert the primary lateral and thickness forces to reverberating particle displacements, in the respective dimensions.

Each of these particle displacements, which may be identified at points 'c' and 'd' in figure 3.10, now follows two distinct paths. Consider initially, the lateral particle displacement at point 'c' in the figure. Firstly it is converted via ϕ to a function of laterally generated secondary charge, which is in turn fed back in a positive fashion, to the summing point marked 'B'. Secondly, the primary lateral displacement at point 'c' gives rise to a reverberating component of secondary thickness displacement, as a result of mechanical cross coupling described by $\psi_{13}\bar{K}_3/sZc_3$. This in turn is responsible for tertiary charge, which is fed back into the forward path via the block hCo. The secondary thickness displacement also introduces via ψ_{31}, a secondary force component in the lateral direction which is shown to modify the original lateral force.

The primary thickness particle displacement at point 'd' undergoes a similar interaction, to produce secondary and tertiary charge components. That is, the block denoted by hCo produces a secondary function of charge, while the block $\psi_{31}\bar{K}_1/sZc_1$ converts the primary thickness displacement to lateral displacement. This component of secondary lateral displacement is responsible for the generation of a tertiary charge component via ϕ, and also a secondary thickness force which is fed back via ψ_{13} as illustrated in figure 3.10.

Finally, the resultant charge now residing on the device electrodes is converted to current by virtue of the differential operator s.

The exact nature of such two-dimensional transducer impedance has not been analysed in the literature. Consequently, the following section presents a detailed explanation of the impedance characteristics, with particular emphasis upon inter-dimensional mechanical and piezoelectric coupling and its effect on secondary and tertiary charge generation.

3.2.2 THE ELECTRICAL IMPEDANCE CHARACTERISTICS

Since the generation of secondary and tertiary charge components plays a vital part in describing the overall electrical impedance of the transducer. It is useful to determine which parameters exercise most influence upon this process. For example, it is apparent that no supplementary charge is produced under the following conditions.

- o When both of the piezoelectric coupling factors ϕ and hCo are zero. Although this is of course impossible to achieve in practice, in some cases the values are so small that their effects may be neglected. For example, Lithium Niobate ($LiNbO_3$) possesses a relatively low value for ϕ, while polyvinylidene-flouride (PVDF) exhibits very little secondary charge generation in either dimension due to its very small capacitance. The available material constants are given in table 3.2 for a variety of commonly used piezoelectric materials.

- When the functions $\bar{K}_1$ and $\bar{K}_3$ are simultaneously equal to zero. In this case, no force at all is generated either into the transducer, or into the load media. This null in the reverberation factors occurs as a result of destructive interference within the transducer cavity. In practice, it is virtually impossible for both reverberation functions to be exactly zero at one particular frequency.

- When the operating frequency is high. Since particle displacement is inversely proportional to frequency, secondary and tertiary charge production, which are a direct result of particle displacements, both diminish with increasing frequency.

- When the transmission coefficients T_F, T_B, T_L and T_R are equal to zero. This implies that the loading media on all four faces are totally rigid, resulting in no mechanical displacement. Such a situation is clearly impractical.

o When the factors $\bar{A}_T$, $\bar{A}_W$, $\bar{A}_{13}$ and $\bar{A}_{31}$ combine in such a manner as to produce zero supplementary charge. This case is outlined in a subsequent section.

The nominal transducer parameters used to obtain all the curves presented in this chapter are those for PZT-5A, given in table 3.2. A commonly encountered transducer configuration is that of the lossless resonator, which is mechanically free on all four faces. The spectral magnitude and phase characteristics of the electrical impedance function for this configuration are outlined in figures 3.11a and 3.11b. The magnitude is expressed in Ohms and the phase in degrees. (All subsequent phase information in this thesis is expressed in degrees). The dimensions of the device are 2mm, 1mm and 20mm, for the thickness, width and length dimensions respectively. It should be noted that this thickness dimension corresponds to a thickness mode resonance of just over 1MHz, in a thin disc transducer. Upon close inspection, the following features may be clearly identified.

o The phase angle of the impedance is always $\pm 90°$ ($\pm \pi/2$). That is, it always constitutes either a pure inductive or capacitive reactance.

o The magnitude plot of the impedance function contains a number of frequencies which exhibit maximum amplitudes, denoted by 'A', 'B' and 'C' in figure 3.11a. These maxima are termed the mechanical resonant frequencies, or anti-resonant frequencies of the device. Each of the mechanical resonant frequencies may be attributed to either thickness or lateral reverberation within the device. For example, mechanical resonances 'A' and 'C' correspond to the fundamental and first overtone for the thickness dimension. Point 'B' denotes the fundamental mechanical resonance as a result of lateral reverberation. It is interesting to note that the overtones of mechanical resonance in the thickness dimension do not occur at odd multiples of the fundamental as in the uni-dimensional case. [17]

o The magnitude response also contains two sets of minima which relate to thickness and lateral behaviour. These minima, usually referred to as electrical resonances, invariably occur at a lower frequency than the corresponding mechanical resonances. The positions of the electrical resonances in figure 3.11a are denoted by the labels 'D', 'E' and 'F'. 'D' and 'F' result from vibrations in the thickness direction, while electrical resonance 'E' results from lateral reverberation.

o The mechanical and electrical resonances for both dimensions occur at lower frequencies than their uni-dimensional counterparts. For example, a 2mm thick, thickness mode transducer with the material properties outlined in table 3.2, would possess a mechanical resonant frequency of just above 1MHz. The two-dimensional mechanical resonance in the thickness direction occurs at a lower frequency.

It should be noted that this theory applies only to the ideal lossless mechanically free transducer, which behaves as a pure reactance. In practice electrical and mechanical dissipation in the device obscure the definition of the resonances. For example, there may be as many as three frequencies of interest near both the electrical and mechanical resonances, corresponding to the frequencies of maximum and minimum magnitude, resistance and reactance. However, for the purpose of the present analysis it is sufficient to consider the loss free case.

The two-dimensional impedance behaviour may be summarised in the following manner.

(i) The electrical impedance contains two resonant modes which correspond to the two principal dimensions.

(ii) The resonance due to the thickness dimension is more prominent than the resonance occurring as a result of the lateral dimension.

(iii) The frequencies of electrical and mechanical resonance for both dimensions are reduced when compared to their respective uni-dimensional counterparts.

3.2.3 THE EQUIVALENT TWO-DIMENSIONAL FEEDBACK FACTOR

In order to investigate further the behaviour of a two-dimensional transducer, it is useful to compare its operation to that of a purely thickness mode device. The transfer function of the closed loop feedback system, which represents the operational impedance of such a uni-dimensional, thickness mode transducer is given by,

$$\bar{Z}_T = [1/sCo].[1 - \bar{A}_{EQ}] \qquad 3.44$$

where $\bar{A}_{EQ}$ represents the total amount of charge feedback or secondary action. Equation 3.44 may be compared with the two-dimensional impedance function given by equation 3.31. Giving,

$$[1/sCo] \frac{1 - \bar{A}_T + \bar{A}_{31}}{1 + \bar{A}_W - \bar{A}_{13}} = [1/sCo].[1 - \bar{A}_{EQ}]$$

This yields the following equality.

$$1 - \bar{A}_{EQ} = \frac{1 - \bar{A}_T + \bar{A}_{31}}{1 + \bar{A}_W - \bar{A}_{13}}$$

which may be rearranged to give,

$$\bar{A}_{EQ} = \frac{\bar{A}_T + \bar{A}_W - \bar{A}_{13} - \bar{A}_{31}}{1 + \bar{A}_W + \bar{A}_{13}} \qquad 3.45$$

In this manner the two-dimensional electrical impedance function may be represented as an equivalent uni-dimensional transfer function relationship. It should be emphasized that this equivalent uni-dimensional representation is no less accurate than the original two-dimensional model, it is merely an alternative method of analysing the describing equation for electrical impedance.

In the notation of equation 3.44, the electrical impedance is effectively split into two distinct components. Firstly, 1/sCo depicts the ideal pure capacitive reactance which is characteristic of a non-piezoelectric insulator positioned between two parallel conducting electrodes. Secondly the term $-[1/sCo].[\bar{A}_{EQ}]$ describes the modification which results from piezoelectrically generated charge components. In the two-dimensional case, $\bar{A}_{EQ}$ describes the amount of supplementary charge which is generated as a result of secondary and tertiary piezoelectric action in each dimension. $\bar{A}_{EQ}$ will henceforth be referred to as the two-dimensional equivalent feedback factor.

Figures 3.12 and 3.13 illustrate the spectral behaviour of $\bar{A}_{EQ}$ under mechanically unloaded and loaded conditions respectively. Figure 3.14 shows the mechanically free, uni-dimensional thickness mode feedback factor, simulated for the same material constants and thickness. The mechanical loading portrayed in figure 3.13 corresponds to an acoustic impedance of 10MRayls positioned at all four faces of the device. A careful study of these simulations reveals a number of interesting features.

- o In all cases, points of minimum feedback may be identified between each resonant peak. In the two unloaded resonator configurations, these minima actually correspond to points of zero feedback, whereas under mechanically loaded conditions, they fall to a finite minimum value.

- o The resonant peaks evident in figure 3.12 correspond to the mechanical resonances which were identified in the electrical impedance characteristic given in figure 3.11. These peaks occur at a lower frequency than in the uni-dimensional case shown in figure 3.14.

- o When the feedback factor possesses a magnitude of unity, and a phase of zero, then the supplementary charge which is fed back, subtracts exactly from the primary charge generated as a result of static capacitance. Under these conditions, zero net charge resides on the conducting surfaces of the device, resulting in a null in the electrical impedance function. This is the condition described previously as electrical resonance. The condition $\bar{A}_{E0}$ = +1 is satisfied exactly for the mechanically free transducer, but is not satisfied for the loaded case. This results in a minimum, as opposed to a null, in the electrical impedance function. It should also be noted that this condition occurs at a higher frequency for the uni-dimensional case shown in figure 3.14.

- Close examination of figure 3.12 and 3.13 reveals that the zero frequency value of the two-dimensional equivalent feedback factor is independant of mechanical load. This is in agreement with the uni-dimensional thickness mode feedback factor. [17]

3.2.4 THE ELECTROMECHANICAL COUPLING COEFFICIENT

In the case of a uni-dimensional thickness mode device, the total amount of zero frequency secondary action is equal to the square of the electromechanical coupling coefficient kt. [18] In order to illustrate this statement, consider the following expression for the operational impedance of a thickness mode transducer.

$$\bar{Z}_T = [1/sCo][1 - \bar{A}_{EQ}]$$

Where $\bar{A}_{EQ}$ now corresponds to the thickness mode secondary action feedback factor given by,

$$\bar{A}_{EQ} = \frac{h^2 Co \bar{K}_T}{sZc_T}$$

$\bar{K}_T$ is the thickness mode reverberation function and Zc_T is the acoustic impedance for the thickness direction. It has been shown by Hayward [18] that under conditions of arbitrary mechanical loading, then

$$\lim_{s \to 0} \frac{\bar{K}_T}{s} = T_T$$

Where T_T is the transit time for mechanical waves to traverse the thickness of the device. Applying this observation to the expression for $\bar{A}_{EQ}$ gives,

$$\lim \bar{A}_{E0} = \frac{h^2 C_0 T_T}{Z c_T}$$

$$= \frac{h^2 \epsilon_{33} A d}{\rho v_T^2 A d}$$

Where A is the surface area of the transducer, d is the thickness, and v_T is the longitudinal thickness mode wave velocity. As before ρ is the density of the transducer material. This expression may be simplified further. Yielding,

$$\lim_{s \to 0} \bar{A}_{E0} = \frac{h^2 \epsilon_{33}}{Y_{33}}$$

which is equal to the square of the laterally clamped electromechanical coupling coefficient kt. Considering the above analysis, a measure of the efficiency of a two-dimensional device is thus possible by evaluating the two-dimensional equivalent feedback factor $\bar{A}_{E0}$, at zero frequency.

In order to mathematically evaluate the two-dimensional equivalent feedback factor at zero frequency, it is first necessary to obtain expressions for the zero frequency values of $\bar{A}_T$, $\bar{A}_W$, $\bar{A}_{13}$ and $\bar{A}_{31}$. This may be achieved in the following manner.

Noting firstly that,

$$\lim_{s \to 0} \frac{\bar{K}_1}{s} = T_1 \quad \text{and} \quad \lim_{s \to 0} \frac{\bar{K}_3}{s} = T_3$$

where T_1 and T_3 are the transit times for acoustic waves to cross the lateral and thickness dimensions respectively. Then,

$$\lim_{s \to 0} \bar{A}_T = \frac{h_{33}{}^2 C_0 T_3}{Zc_3\left[1 - \dfrac{\psi_{13}\psi_{31}T_1T_3}{Zc_1Zc_3}\right]}$$

$$= \frac{h_{33}{}^2 \epsilon_{33} WLH}{Zc_3 v_3 H\left[1 - \dfrac{\psi_{13}\psi_{31}HW}{Zc_1 v_1 Zc_3 v_3}\right]}$$

$$= \frac{h_{33}{}^2 \epsilon_{33}}{Y_{33}\left[1 - \dfrac{Y_{13}(Y_{13} - h_{13}h_{33}\epsilon_{33})}{Y_{33}(Y_{11} - h_{13}{}^2 \epsilon_{33})}\right]}$$

and,

$$\lim_{s \to 0} \bar{A}_W = \frac{\phi^2 T_1}{CoZc_1\left[1 - \dfrac{\psi_{13}\psi_{31}T_1T_3}{Zc_1Zc_3}\right]}$$

$$= \frac{h_{13}^2 \epsilon_{33}^2 L^2 WH}{Zc_1 v_1 \epsilon_{33} LW\left[1 - \dfrac{\psi_{13}\psi_{31}HW}{Zc_1 v_1 Zc_3 v_3}\right]}$$

$$= \frac{h_{13}^2 \epsilon_{33}}{(Y_{11} - h_{13}^2 \epsilon_{33})\left[1 - \dfrac{Y_{13}(Y_{13} - h_{13}h_{33}\epsilon_{33})}{Y_{33}(Y_{11} - h_{13}^2 \epsilon_{33})}\right]}$$

Similarly,

$$\lim_{s \to 0} \bar{A}_{13} = \frac{\psi_{13}h_{33}\phi T_1 T_3}{Zc_1Zc_3\left[1 - \dfrac{\psi_{13}\psi_{31}T_1T_3}{Zc_1Zc_3}\right]}$$

$$= \frac{Y_{13}h_{33}h_{13}\epsilon_{33}HWL^2}{Zc_1 v_1 Zc_3 v_3\left[1 - \dfrac{\psi_{13}\psi_{31}HW}{Zc_1 v_1 Zc_3 v_3}\right]}$$

$$= \frac{Y_{13}h_{33}h_{13}\epsilon_{33}}{Y_{33}(Y_{11} - h_{13}^2 \epsilon_{33})\left[1 - \dfrac{Y_{13}(Y_{13} - h_{13}h_{33}\epsilon_{33})}{Y_{33}(Y_{11} - h_{13}^2 \epsilon_{33})}\right]}$$

and finally,

$$\lim_{s \to 0} \bar{A}_{31} = \frac{\psi_{31} h_{33} \phi T_1 T_3}{Zc_1 Zc_3 \left[1 - \dfrac{\psi_{13}\psi_{31} T_1 T_3}{Zc_1 Zc_3} \right]}$$

$$= \frac{(Y_{13} - h_{13}h_{33}\epsilon_{33})h_{33}h_{13}\epsilon_{33}HWL^2}{Zc_1 v_1 Zc_3 v_3 \left[1 - \dfrac{\psi_{13}\psi_{31}HW}{Zc_1 v_1 Zc_3 v_3} \right]}$$

$$= \frac{(Y_{13} - h_{13}h_{33}\epsilon_{33})h_{33}h_{13}\epsilon_{33}}{Y_{33}(Y_{11} - h_{13}{}^2\epsilon_{33}) \left[1 - \dfrac{Y_{13}(Y_{13} - h_{13}h_{33}\epsilon_{33})}{Y_{33}(Y_{11} - h_{13}{}^2\epsilon_{33})} \right]}$$

In each case it should be noted that the units of the zero frequency expressions for $\bar{A}_T$, $\bar{A}_w$, $\bar{A}_{13}$ and $\bar{A}_{31}$ are dimensionless. Consequently, numerical evaluation, using the material constants given in table 3.2 for PZT-5A, yields the following dimensionless constants.

$$\lim_{s \to 0} \bar{A}_T = 0.320$$

$$\lim_{s \to 0} \bar{A}_W = 0.044$$

$$\lim_{s \to 0} \bar{A}_{13} = -0.052$$

$$\lim_{s \to 0} \bar{A}_{31} = -0.063$$

Examination of these values reveals that the most efficient charge generation mechanism in a two-dimensional transducer, is via direct secondary action in the thickness dimension ($\bar{A}_T$). The contributions made by direct lateral secondary action ($\bar{A}_W$), and cross-coupled tertiary generation ($\bar{A}_{13}, \bar{A}_{31}$) are much less significant. It should also be noted that the zero frequency values of $\bar{A}_T$ and $\bar{A}_W$, the direct secondary generation factors for the thickness and lateral dimensions respectively, exhibit an increase of more than 25% when compared to the uni-dimensional thickness and lateral mode values of 0.231 and 0.032 respectively. This increase is due exclusively to the mechanical cross coupling between the dimensions described by $\bar{M}$.

The effective electromechanical coupling coefficient of a two-dimensional transducer may be determined by the value of the two-dimensional equivalent feedback factor at zero frequency. That is,

$$k_{EFF}^2 \quad \lim_{s \to 0} \quad \frac{\bar{A}_T + \bar{A}_W - \bar{A}_{13} - \bar{A}_{31}}{1 + A_W - A_{13}}$$

The zero frequency values of $\bar{A}_T$, $\bar{A}_W$, $\bar{A}_{13}$ and $\bar{A}_{31}$ may be substituted into equation 3.46 to give,

$$k_{EFF}^2 = 0.44$$

The value of kt^2 for a PZT-5A thickness mode device is 0.231, thus, the two-dimensional device as a whole is 75% more efficient in transduction than its uni-dimensional thickness mode counterpart. This is a result of mechanical and piezoelectric cross coupling from the lateral mode. In order to demonstrate the effects of mechanical coupling through Y_{13}, and piezoelectric coupling through h_{13}, the following section investigates variations in the feedback factors, feedforward factors and the total effective two-dimensional coupling coefficient.

3.2.5 THE EFFECT OF VARIATIONS IN Y_{13} AND h_{13} UPON THE ELECTROMECHANICAL COUPLING COEFFICIENT

Variations in the mechanical cross coupling coefficient Y_{13}, and the piezoelectric coefficient h_{13}, have a distinct effect upon the value of the two-dimensional electromechanical coupling coefficient. This section presents a number of graphical interpretations of these effects. By varying Y_{13} and h_{13} in the expressions for the zero frequency values of $\bar{A}_T$, $\bar{A}_W$, $\bar{A}_{13}$ and $\bar{A}_{31}$, it is possible to identify which factors bear most influence on the value of k_{EFF}.

(a) Variations in the zero frequency value of $\bar{A}_T$

The expression for the zero frequency value of the direct thickness dimension feedback factor $\bar{A}_T$ may be written as,

$$\bar{A}_T(0) = \frac{h_{33}^2 \epsilon_{33}}{Y_{33}\left[1 - \dfrac{Y_{13}(Y_{13} - h_{13}h_{33}\epsilon_{33})}{Y_{33}(Y_{11} - h_{13}^2 \epsilon_{33})}\right]}$$

The quantity $\bar{A}_T(0)$ is plotted against the mechanical cross coupling coefficient Y_{13} in figure 3.15, and against the piezoelectric coefficient h_{13} in figure 3.16. The material parameters are those quoted for PZT-5A in table 3.2. It is apparent from figure 3.15 that under conditions of zero mechanical cross coupling ($Y_{13} = 0$), then $\bar{A}_T(0)$ corresponds to the square of the electromechanical coupling coefficient kt, for a thickness mode device, regardless of the value of h_{13}.

The value of $\bar{A}_T(0)$ increases with increasing Y_{13}, it is therefore desirable to maximise mechanical cross coupling in order to achieve greater efficiency from the device. Figure 3.16 illustrates the variation of $\bar{A}_T(0)$ with the piezoelectric coefficient h_{13} while Y_{13} is kept constant at the nominal value. As h_{13} becomes larger in magnitude, $\bar{A}_T(0)$ also increases. This effect is a direct result of the increase in piezoelectric stiffening in the lateral dimension due to h_{13}. It should be noted for example, that when the mechanical coupling to the lateral dimension via Y_{13} is zero, then variations in h_{13} do not affect the value of $\bar{A}_T(0)$. In relative terms, variations in Y_{13} affect the value of $\bar{A}_T(0)$ to a greater extent than h_{13}.

(b) Variations in the zero frequency value of $\bar{A}_W$

The expression for the zero frequency value of the direct lateral direction feedforward factor $\bar{A}_W$ may be written as,

$$\bar{A}_W(0) = \frac{h_{13}^2 \epsilon_{33}}{(Y_{11} - h_{13}^2 \epsilon_{33}) \left[1 - \dfrac{Y_{13}(Y_{13} - h_{13} h_{33} \epsilon_{33})}{Y_{33}(Y_{11} - h_{13}^2 \epsilon_{33})} \right]}$$

Figure 3.17 and figure 3.18 illustrate the variation of $\bar{A}_W(0)$ with h_{13} and Y_{13} respectively. Inspection of figure 3.17 reveals that when h_{13} equals zero then the zero frequency value of the direct lateral feedforward factor also equals zero. This is to be expected since all piezoelectric behaviour in the lateral direction is controlled by h_{13}. The value of $\bar{A}_W(0)$ also increases with increasing mechanical coupling to the thickness dimension through Y_{13}. In common with the direct thickness feedback factor, $\bar{A}_W(0)$ has a limiting value, when Y_{13} is zero, equal to the electromechanical coupling coefficient for an isolated lateral mode. The zero frequency value of $\bar{A}_W(0)$ may thus be increased by inreasing h_{13} and Y_{13}, but in general its value is virtually an order of magnitude smaller than $\bar{A}_T(0)$.

(c) Variations in the zero frequency value of $\bar{A}_{13}$ and $\bar{A}_{31}$.

The piezoelectric cross coupling factors $\bar{A}_{13}$ and $\bar{A}_{31}$ are defined at zero frequency as follows,

$$\bar{A}_{13}(0) = \frac{Y_{13}h_{33}h_{13}\epsilon_{33}}{Y_{33}(Y_{11} - h_{13}{}^2\epsilon_{33})\left[1 - \dfrac{Y_{13}(Y_{13} - h_{13}h_{33}\epsilon_{33})}{Y_{33}(Y_{11} - h_{13}{}^2\epsilon_{33})}\right]}$$

$$\bar{A}_{31}(0) = \frac{(Y_{13} - h_{13}h_{33}\epsilon_{33})h_{33}h_{13}\epsilon_{33}}{Y_{33}(Y_{11} - h_{13}{}^2\epsilon_{33})\left[1 - \dfrac{Y_{13}(Y_{13} - h_{13}h_{33}\epsilon_{33})}{Y_{33}(Y_{11} - h_{13}{}^2\epsilon_{33})}\right]}$$

The quantity $\bar{A}_{13}(0)$ is plotted against Y_{13} and h_{13} in figures 3.19 and 3.20 respectively, while $\bar{A}_{31}(0)$ is plotted against Y_{13} and h_{13} in figures 3.21 and 3.22. The following points are of interest. Firstly, both $\bar{A}_{13}(0)$ and $\bar{A}_{31}(0)$ are always negative in value as a result of the negative sign associated with h_{13}. Secondly, the magnitude of both factors increases with any increase in either Y_{13} or h_{13}. Thirdly, if h_{13} is equal to zero, then both of the cross coupling factors are zero. However, if Y_{13} is zero then $\bar{A}_{13}(0)$ is also zero while $\bar{A}_{31}(0)$ is finite. Finally, for any given value of h_{13} and Y_{13}, piezoelectric coupling from the thickness to the lateral dimension is greater than from the lateral to the thickness.

(d) The effective electromechanical coupling coefficient

The effects upon the total effective electromechanical coupling coefficient of varying Y_{13} and h_{13} are illustrated in figure 3.23 and 3.24. When Y_{13} is equal to zero then k_{EFF} corresponds to the electromechanical coupling coefficient under conditions of zero mechanical coupling. The piezoelectric cross coupling is still active. The condition whereby the piezoelectric cross coupling is zero is given in figure 3.24 at the point where h_{13} is zero. This condition corresponds to a two-dimensional transducer which is piezoelectrically inactive in the lateral dimension. In common with $\bar{A}_T(0)$, $\bar{A}_W(0)$, $\bar{A}_{13}(0)$ and $\bar{A}_{31}(0)$, any increase in either h_{13} or

Y_{13} results in an increase in k_{EFF}. That is, the efficiency is improved.

In conclusion, a number of general points are apparent regarding the electromechanical efficiency of a two-dimensional transducer.

- An increase in Y_{13}, the mechanical coupling coefficient serves to increase the efficiency of the device.

- Similarly, an increase in the lateral piezoelectric coefficient also increases the efficiency

- The direct thickness feedback factor is the dominant quantity in device efficiency. This may be substantiated by referring to the impedance plot given in figure 3.11, where the thickness resonances are clearly greater in extent than the lateral resonances.

- The efficiency of the device may also be increased by varying the other material parameters. For example, increasing ϵ_{33} and h_{33} increases the efficiency while reducing Y_{11} and Y_{33} increases efficiency.

3.3.1 THE BLOCK DIAGRAM MODEL FOR A TWO-DIMENSIONAL PIEZOELECTRIC RECEIVER

This section presents the development of a full block diagram representation for a two-dimensional piezoelectric receiver. The model is derived and analysed in two distinct parts. Firstly, the forward path is examined. This represents primary piezoelectric generation in each dimension, in addition to mechanical cross coupling effects. Secondly, the feedback behaviour of the system, which represents secondary and tertiary piezoelectric action is analysed and related to the forward path in order to provide the complete model. As with the electrical impedance model, the receiver model will be structured to provide an intuitive understanding of the underlying physical processes taking place within the device.

The two-dimensional receiver transfer function given by equation 3.37 may be written as,

$$\bar{V}_3 = -\bar{U}\,\bar{G} \,/\, (1 + \bar{U}\,\bar{H}) \qquad 3.51$$

where

$$\bar{G} = \frac{[T_L\bar{K}_L\bar{F}_L + T_R\bar{K}_R\bar{F}_R]}{sZc_1\bar{M}}\left[\frac{\phi}{Co} - \frac{\psi_{13} h \bar{K}_3}{sZc_3}\right]\bar{A}_{11}$$

$$+ \frac{[T_F\bar{K}_F\bar{F}_F + T_B\bar{K}_B\bar{F}_B}{sZc_3\bar{M}}\left[h - \frac{\psi_{13}\phi \bar{K}_1}{sCoZc_1}\right]\bar{A}_{33}$$

$$\bar{H} = [(\bar{A}_W - \bar{A}_{13}) - (\bar{A}_T - \bar{A}_{31})/sCo\bar{Z}_E]\Big]$$

$$\bar{U} = \frac{sCo\bar{Z}_E}{1 + sCo\bar{Z}_E} \quad \text{as defined in section 3.1.5.}$$

Equation 3.51 is in the form of a closed loop feedback system, and as such may be represented by the block diagram illustrated in figure 3.25. The upper loop A-B corresponds to secondary and tertiary piezoelectric action as a result of thickness vibrations, while the lower loop represents secondary and tertiary action in the lateral dimension. It should be noted that secondary and tertiary piezoelectric action in the thickness direction diminishes with increases in Z_E. Lateral secondary and tertiary generation are not affected to the same extent.

3.3.2 ANALYSIS OF THE FORWARD PATH

As stated previously, the block diagram will be analysed in two parts. The first of these is the forward path or primary piezoelectric behaviour corresponding to -UG, which is illustrated in figure 3.26. Consider firstly the upper branch of the forward path, which represents the response of the device to incident thickness forces striking the front and back faces. Components of the forces $\bar{F}_f$ and $\bar{F}_b$ are transmitted into the transducer via the transmission coefficients T_f and T_b, these in turn reverberate according to the reverberation factors $\bar{K}_f$ and $\bar{K}_b$. The block $1/sZc_3$ then converts these reverberating forces into a function of internal particle displacement in the thickness direction, which may be identified at point 'D'. The lower section of the diagram which represents primary lateral behaviour, may be described in an analogous manner, and results in an equivalent lateral particle displacement at point 'E' in the diagram. The cross-coupled feedback loops describe purely mechanical interaction between the lateral and thickness modes of vibration. Considering the upper feedback loop, the factor ψ_{31} converts particle displacement in the thickness direction into lateral force. This internally generated lateral force component then reverberates according to $\bar{K}_1$, and is converted via $1/sZc_1$, into lateral particle displacement at point 'P'. This cross-coupled lateral displacement is in turn fed across, to enhance the total resultant lateral particle displacement at point 'E'', and also back into the upper forward path through $\psi_{13}\bar{K}_3/sZc_3$, to modify the original thickness particle displacement. The initial lateral particle displacement in the lower forward path also undergoes this conversion, cross-coupling and re-conversion process which is mathematically described by the mechanical coupling factor $\bar{M}$, defined in section 3.1.3.

The points 'D'' and 'E'' hence represent functions of total particle displacement due to the incident forces striking the left, right, front or back faces, taking into account the mechanical cross-coupling between the dimensions. In the absence of secondary and tertiary interaction, a voltage is generated across the transducer in the following manner. Firstly, the piezoelectric conversion factors h and ϕ/Co convert the thickness and lateral particle displacements at 'D'' and 'E'' respectively, to functions of voltage, which are then added at summing point 'S'. The summed voltage is then modified by the electrical load attenuation factor $\bar{U}$, to produce a total resultant voltage which may be measured across the device.

3.3.3 ANALYSIS OF THE FEEDBACK LOOPS.

The two feedback loops shown in figure 3.25 describe secondary and tertiary piezoelectric action and the associated mechanical interactions. They are similar in nature to the feedback and feedforward loops analysed in section 3.20 for the electrical impedance model. An expanded version of the upper loop is shown in figure 3.27 while the lower loop is illustrated in figure 3.28. The two loops combine to describe the modification of voltage across the transducer as a result of both lateral and thickness secondary and tertiary piezoelectric action.

Commencing at point A in figure 3.27, the first block encountered is $1/s\bar{Z}_E$. This gives rise to a secondary function of charge which in turn is translated into thickness force at the first summing point. The thickness force is in turn converted via $\bar{K}_3/sZc_3$ into a function of particle displacement which reverberates internally in the thickness direction. It then follows two distinct paths. Firstly, it is converted via h into a secondary component of voltage which is fed back in a positive fashion to the forward path at point B. Secondly, the thickness function of particle displacement gives rise to a reverberating lateral displacement, as a result of mechanical cross-coupling. This process is described by the block $\psi_{31}\bar{K}_1/sZc_1$. The function of particle displacement is responsible for the generation of a tertiary voltage component, via the block ϕ/Co. It also introduces an additional thickness component of force, as a result of mechanical cross coupling. This is described by the block ψ_{13} in figure 3.27, where it is shown to modify the original thickness force.

These resultant secondary and tertiary voltages are finally fed back into the forward path at point 'B'. A similar investigation of figure 3.28 reveals an equivalent mode of operation for the lateral feedback loop. The feedback behaviour of the two-dimensional receiver is thus a combination of secondary effects, tertiary effects and mechanical cross coupling.

The complete receiver block diagram model is presented in figure 3.29. The transducer is represented as a two-dimensional five port system, which utilises the concept of feedback to explain such effects as secondary and tertiary piezoelectric action, in addition to mechanical cross coupling between the dimensions. Each node in the diagram represents a real physical quantity involved in the transduction process, while each block represents the relevant conversion between these quantities.

3.4.1 THE BLOCK DIAGRAM MODEL FOR A TWO-DIMENSIONAL PIEZOELECTRIC TRANSMITTER

As shown in section 3.1.6 the two-dimensional piezoelectric transmitter may be described by the following four equations.

$$\bar{F}_L = \frac{Z_L \bar{K}_L}{(Z_L + Zc_1)\bar{M}} \left[\phi \bar{V}_3 - \frac{\psi_{31} \bar{K}_3 h \bar{Q}}{sZc_3} \right] \qquad 3.52$$

$$\bar{F}_R = \frac{Z_R \bar{K}_R}{(Z_R + Zc_1)\bar{M}} \left[\phi \bar{V}_3 - \frac{\psi_{31} \bar{K}_3 h \bar{Q}}{sZc_3} \right] \qquad 3.53$$

$$\bar{F}_F = \frac{Z_F \bar{K}_F}{(Z_F + Zc_3)\bar{M}} \left[h \bar{Q} - \frac{\psi_{13} \bar{K}_1 \phi \bar{V}_3}{sZc_1} \right] \qquad 3.54$$

$$\bar{F}_B = \frac{Z_B \bar{K}_B}{(Z_B + Zc_3)\bar{M}} \left[h \bar{Q} - \frac{\psi_{13} \bar{K}_1 \phi \bar{V}_3}{sZc_1} \right] \qquad 3.55$$

Equations 3.52-3.55 may be represented simultaneously by the block diagram given in figure 3.30. The similarity in mechanical cross coupling between the receiver and the transmitter is clearly evident from the cross-coupled feedback loops shown in figure 3.30 and figure 3.26. However the inputs to the transmitter model, $\bar{Q}$ and $\bar{V}_3$ are highly dependant upon the nature of the electrical driving network.

Consider, for example, the transmitter configuration illustrated in figure 3.31. This represents the transducer, excited by an ideal voltage source $\bar{V}_s$, via the Thevenin equivalent impedance $\bar{Z}_E$, of the driving network. $\bar{Z}_T$ is the operational or electrical impedance of the transducer as defined in equation 3.31, section 3.1.4. The Laplace transform of the total current flowing in the system is therefore,

$$\bar{I} = \bar{V}_s/(\bar{Z}_E + \bar{Z}_T)$$

Similarly, the voltage transform across the transducer may be expressed as,

$$\bar{V}_3 = \bar{V}_s.\bar{Z}_T/(\bar{Z}_E + \bar{Z}_T)$$

Thus the electrical behaviour of the loaded piezoelectric transmitter may be represented by the block diagram shown in figure 3.32. The transmission behaviour is highly dependant upon the impedance characteristics of both the transducer and the electrical load $\bar{Z}_E$. That is, the amount of charge generated on the device, and the voltage which is developed across it, are directly responsible for piezoelectric generation in the thickness and lateral dimensions respectively. To illustrate this more fully, the following points should be noted.

- For a given value of $\bar{Z}_E$, the voltage which is developed across a two-dimensional transmitter is a minimum at any point of electrical resonance as defined previously. As a result, thickness forces are maximised and primary, piezoelectrically generated lateral forces are minimised, although mechanical coupling still leads to a lateral output force being generated.

- At frequencies of mechanical resonance, that is, maximum electrical impedance, force generation in the lateral dimension is enhanced due to the increased voltage developed across the device, while thickness generation is reduced, as a result of the reduced current.

- Apart from any loading effects, the amount of charge deposited on the electrodes of the transducer, decreases with frequency, and thus force generation in the thickness direction also decreases.

- Any increase in the electrical load impedance $\bar{Z}_E$ reduces the force generation in both dimensions.

The diagrams given in figure 3.30 and 3.32 may be readily combined to give figure 3.33, which is a complete systems block diagram model for a two-dimensional piezoelectric transmitter, operating via a Thevenin equivalent load impedance $\bar{Z}_E$. Utilising a similar mode of analysis to that used for the receiver model, the physical operation of the transmitting device may be described in the following manner.

- The interaction of the driving network and the electrical impedance of the transducer results in a quantity of electronic charge being deposited on the electrodes of the device , and a resultant voltage being developed across it.

- The piezoelectric conversion factora h and ϕ convert this charge and voltage into internal thickness and lateral force components respectively.

- The force components in the lateral and thickness dimensions are cross coupled through the factor $\bar{M}$ in a similar manner to the particle displacements at 'D' and 'E' in the receiver model.

- Finally the resultant reverberating lateral and thickness forces are fed out of the device according to the mechanical loading on each face. The output forces are represented $\bar{F}_F$, $\bar{F}_B$, $\bar{F}_R$ and $\bar{F}_L$ in figure 3.33.

The two-dimensional transmitter model is thus a five port system which relates the driving signal to the four principal output forces. The transmission behaviour depends heavily upon the electrical characteristics of the transducer and driving network.

3.2.5 THE MECHANICALLY LOADED TWO-DIMENSIONAL TRANSDUCER

Mechanical loading has a pronounced effect on the behaviour of a piezoelectric transducer [3][32]. For example, in transmission or reception, the amount of force which is transferred to or from the acoustic load media, depends greatly on the relationship between its mechanical impedance , and that of the transducer.

In order to appreciate the nature of the loaded two-dimensional transducer characteristics, it is useful to study the behaviour of the equivalent feedback factor $\bar{A}_{E0}$ under different mechanical load conditions. By doing this it is possible to identify the extent to which mechanical loading affects secondary and tertiary charge generation, and hence the transduction performance of the transducer. A number of separate loading configurations are considered, including combinations of different loading on each dimension.

The magnitude characteristics of $\bar{A}_{E0}$ are presented in figures 3.34a, 3.35a and 3.36a for conditions of light, medium and heavy damping respectively. Light damping corresponds to an acoustic load impedance on all four faces of 5MRayls, medium damping to 15MRayls, while heavy damping is denoted by a load impedance of 25MRayls. A comparison of the three plots reveals the following key features.

1. The total amount of secondary and tertiary

piezoelectric charge generation decreases with increasing mechanical load. This is due to the increase in mechanical energy which is being transferred to the load media, and hence less secondary force exists within the device.

2. In the case of light mechanical damping, the maxima and minima are less well defined than in the mechanically free case, but the general shape of the curves is retained.

3. Under conditions of medium mechanical damping, the resonant maxima and minima become extremely rounded, hence obscuring their spectral definition.

4. Heavy mechanical damping severely restricts the amount of secondary and tertiary charge which is generated, other than at low frequencies where it is limited by the finite zero frequency value of the equivalent feedback factor.

5. The total amount of secondary action decreases with increasing frequency. This is similar to the behaviour of the mechanically free transducer.

The phase characteristics of the equivalent feedback factor under respective conditions of light, medium and heavy mechanical damping are presented in figures 3.34b, 3.35b and 3.36b. As in the case of the mechanically free device, the phase characteristics determine the effective polarity of the supplementary secondary and tertiary charge which is generated. That is, whether supplementary charge is generated in a constructive or destructive manner. Hence, the phase characteristics determine the relative positions of the various electrical and mechanical resonances in the impedance function. The phase response becomes increasingly more complicated with increasing mechanical loading and under heavily loaded conditions it varies continuously with frequency.

These complex phase characteristics bear important ramifications when considering the magnitude of the electrical and mechanical resonances in the impedance function of the transducer. The secondary and tertiary charge quantities are no longer purely real functions and hence the condition for electrical resonance is no longer strictly satisfied. This leads to the following modified condition for electrical resonance.

$$| 1 - \bar{A}_{EO} | \text{ is a minimum.}$$

This condition results in electrical resonances whose characteristics depend on the mechanical loading upon the device.

The electrical impedance magnitude and phase characteristics of a two-dimensional transducer which is mechanically loaded by light, medium and heavy damping are given in figures 3.37, 3.38 and 3.39. The variation in extent and position of the electrical and mechanical resonances with increasing damping is clearly evident. As damping increases, the maxima and minima decrease in frequency and eventually disappear, and for load impedances greater than 30MRayls, the shape of the impedance function tends towards that of a pure capacitance. It should be noted that the low frequency portion of the electrical impedance spectra have been truncated, to improve the resolution, this technique will be employed throughout the remainder of the thesis.

3.2.6 COMBINATIONS OF RIGID/LIGHT MECHANICAL LOADING.

Rigid loading is said to exist when a transducer is loaded by a mechanical impedance which is much greater than the characteristic impedance of the device. Rigid loading implies that there is zero net displacement at the surface of the transducer. Since piezoelectric materials in general possess high values of acoustic impedance, this is clearly impractical. The situations which are illustrated in this section are included for the sake of completeness, and to illutrate more fully the interaction between the dimensions. The following conditions are described.

1. Rigid loading upon all four faces.

Figure 3.40 illustrates the electrical impedance for this case. Since $T_F=0$, $T_B=0$, $T_R=0$ and $T_L=0$ and hence $\bar{K}_1$ and $\bar{K}_3$ are zero, then no secondary or tertiary charge generation occurs in either dimension. The impedance is identical to that of the static capacitance Co.

2. **Rigid lateral loading/light thickness loading.**

This corresponds to the case where $T_R=T_L=0$ and $K_1=0$, thus there are no lateral mode vibrations. This situation may be illustrated by referring to figures 3.41 and 3.42 which represent the equivalent feedback factor $\bar{A}_{EQ}$ and the electrical impedance $\bar{Z}_T$ respectively. In this case $\bar{A}_{EQ}$ is similar in form to $\bar{A}_T$ for a uni-dimensional thickness mode device, since $\bar{K}_1=0$ and hence the mechanical coupling factor $\bar{M}$ is equal to unity at all frequencies. It should be noted that the acoustic velocity is lower than that of the thickness mode, due to the modified boundary conditions used to derive the two-dimensional model.

3. **Rigid thickness loading/light lateral loading.**

This is similar to case (2) above with $T_F=T_B=\bar{K}_3=0$. $\bar{A}_{EQ}$ and $\bar{Z}_T$ are illustrated in figures 3.43 and 3.44. There are no thickness mode reverberations and the factor $\bar{A}_W$ is similar to that of an isolated lateral mode. Again the acoustic velocity is reduced as a result of the finite boundary conditions.

3.4 CONCLUDING REMARKS

Block diagram models for a two-dimensional transducer have been developed. The models, which are exact within the constraints of the initial simplifying assumptions, represent the transducer in transmission, reception, or as an electrical impedance. They are wideband, and are valid under conditions of arbitrary electrical and mechanical loading. The physical significance of each element in the diagrams, has been utilised in order to illustrate the complex relationships between electrical and mechanical parameters, in addition to inter-dimensional quantities. The factors which control secondary and tertiary piezoelectric generation have been clearly defined in relation to the overall transfer function. It is condsidered that this approach to modelling two-dimensional transduction offers fundamental advantages over other 'approximate' and 'exact' methods, since the general behaviour of the transducer under any specific operating conditions may be identified virtually upon inspection of the relevant block diagram. This is not possible for example, when using 'exact' finite element methods.

The technique of using feedback and feedforward factors to determine the effective electromechanical efficiency of the two-dimensional device has been developed. This allows representation of the device efficiency in an equivalent uni-dimensional manner, while the accuracy of the two-dimensional analysis is maintained.

There remains however, one disadvantage with this type of model. The transfer function relationships, when derived in this form, do not lend themselves readily to the analysis of multi-layered structures. The following chapter presents an alternative derivation of the systems model, which permits the evaluation of multiple layers and multiple transducer structures.

CHAPTER IV

A TWO-DIMENSIONAL LATTICE TYPE MODEL FOR THE ANALYSIS OF MULTI-LAYERED PIEZOELECTRIC STRUCTURES

4.1 INTRODUCTION

The two-dimensional linear systems model, when expressed in block diagram format, offers excellent intuitive insight into the electrical, mechanical and piezoelectric interactions within a two-dimensional transducer. However it is not readily applicable to the analysis of structures which possess multiple layers in either of the two principal dimensions. For this reason it is useful to represent the two-dimensional device in a notation which permits the analysis of such configurations. In this chapter a new five port lattice type model is developed. The model is based upon the lattice techniques used by Jackson [27][28], to model a thickness mode piezoelectric transducer. By representing two-dimensional behaviour in such a manner, it is possible to obtain a single general model for the device which may be used for simultaneous transmission and reception. Subsequently the addition of further mechanical or piezoelectric layers in the thickness or lateral direction is a relatively straightforward process.

It should be noted however that this type of model provides extremely limited insight into the nature of the transduction process, and is primarily intended for the efficient mathematical analysis of multi-layered structures. [21]

4.2 THE TWO-DIMENSIONAL LATTICE EQUATIONS

Consider the cellular configuration shown in figure 4.1, which depicts a structure which is layered in two dimensions. Each cell represents a cross-sectional view of either a piezoelectric or a non-piezoelectric layer. As in chapter III the coordinate system for each cell runs from left to right, for the x-coordinate and bottom to top for the z-coordinate. For example, the x-coordinate of the N-th cell may vary between x=0 and $x=W_N$, similarly the z-coordinate of the N-th cell varies between z=0 and $z=H_N$. The mechanical forces in each cell are defined as shown in figure 4.1.

Referring to equations 3.21, 3.22 and 3.23 in the previous chapter and relating these to the N-th cell, the two-dimensional equations may be written in the following format.

$$\bar{F}_{1N} + \phi_N \bar{V}_{3N} = sZc_1(N) \left[-\bar{A}_{1N}\, e^{-sx_N/v_{1N}} + \bar{B}_{1N}\, e^{+sx_N/v_{1N}} \right]$$

$$+ \psi_{31}(N) \left[\bar{A}_{3N} (e^{-sT_{3N}} - 1) + \bar{B}_{3N} (e^{+sT_{3N}} - 1) \right] \qquad 4.1$$

$$\bar{F}_{3N} + h_N \bar{Q}_N = sZc_3(N) \left[-\bar{A}_{3N}\, e^{-sz_N/v_{3N}} + \bar{B}_{3N}\, e^{+sz_N/v_{3N}} \right]$$

$$+ \psi_{13}(N) \left[\bar{A}_{1N} (e^{-sT_{1N}} - 1) + \bar{B}_{1N} (e^{+sT_{1N}} - 1) \right] \qquad 4.2$$

$$\bar{V}_{3N} - \bar{Q}_N/Co_N = -\phi_N/Co_N\left[\bar{A}_{1N}(e^{-sT_{1N}} - 1) + \bar{B}_{1N}(e^{+sT_{1N}} - 1)\right]$$

$$- h_N\left[\bar{A}_{3N}(e^{-sT_{3N}} - 1) + \bar{B}_{3N}(e^{+sT_{3N}} - 1)\right] \qquad 4.3$$

$\bar{F}_{1N}$ is the Laplace transform of the total force propagating in the lateral dimension, and $\bar{F}_{3N}$ is the transform of the total force travelling in the thickness dimension of the N-th cell. $\bar{A}_{1N}$, $\bar{B}_{1N}$, $\bar{A}_{3N}$ and $\bar{B}_{3N}$ are Laplace quantities which represent forward and backward functions of particle displacement as defined in chapter III. $\psi_{13}(N)$ and $\psi_{31}(N)$ are the mechanical cross-coupling coefficients, and $Zc_1(N)$ and $Zc_3(N)$ are the respective acoustic impedances in the lateral and thickness dimensions. The remaining quantities are as previously defined with the N suffix relating to the relevant cell.

The forward and backward particle displacements may be represented in terms of the mechanical forces shown in figure 4.1 in the following manner. For the lateral dimension;

$$\bar{A}_{1N} = -\bar{F}_{RN}/sZc_1(N) \qquad 4.4$$

$$\bar{B}_{1N} = [\bar{F}_{LN}/sZc_1(N)]\, e^{-sT_{1N}} \qquad 4.5$$

Also for the thickness dimension,

$$\bar{A}_{3N} = - \bar{F}_{FN}/sZc_3(N) \qquad 4.6$$

$$\bar{B}_{3N} = [\bar{F}_{BN}/sZc_3(N)]\, e^{-sT_{3N}} \qquad 4.7$$

The delay terms $e^{-sT_{1N}}$ and $e^{-sT_{3N}}$ in equations 4.5 and 4.7 are necessary since $\bar{B}_{1N}$ and $\bar{B}_{3N}$ are defined at x = 0 and z = 0 as in chapter III, whereas $\bar{F}_{LN}$ and $\bar{F}_{BN}$ are defined at $x = W_N$ and $z = H_N$.

Equations 4.4 to 4.7 may be used to provide the following expressions for the total mechanical particle displacement in each dimension.

$$\bar{\xi}_{1N} = \left[- \bar{F}_{RN}\, e^{-sT_{1N}} + \bar{F}_{LN}\, e^{+sT_{1N}} \right] / \; sZc_1(N) \qquad 4.8$$

$$\bar{\xi}_{3N} = \left[- \bar{F}_{FN}\, e^{-sT_{3N}} + \bar{F}_{BN}\, e^{+sT_{3N}} \right] / \; sZc_3(N) \qquad 4.9$$

Equations 4.4 to 4.7 may also be substituted into equations 4.1, 4.2 and 4.3 to give the following describing equations.

$$\bar{F}_{1N} = \bar{F}_{RN}\, e^{-sx_N/v_N} + \bar{F}_{LN}\, e^{+sx_N/v_N} - \phi_N \bar{V}_{3N}$$

$$+ \frac{\psi_{31}(N)}{sZc_3\,(N)} \left[(\ 1 - e^{-sT_{3N}})(\ \bar{F}_{FN} + \bar{F}_{BN}\ ) \right] \qquad 4.10$$

$$\bar{F}_{3N} = \bar{F}_{FN}\, e^{-sz_N/v_N} + \bar{F}_{BN}\, e^{+sz_N/v_N} - h_N \bar{Q}_N$$

$$+ \frac{\psi_{13}(N)}{sZc_1\,(N)} \left[(\ 1 - e^{-sT_{1N}})(\ \bar{F}_{RN} + \bar{F}_{LN}\ ) \right] \qquad 4.11$$

$$\bar{V}_{3N} = - \frac{\phi_N}{sCoZc_3(N)} \left[(\ 1 - e^{-sT_{1N}})(\ \bar{F}_{RN} + \bar{F}_{LN}\ ) \right]$$

$$- \frac{h_N}{sZc_3(N)} \left[(\ 1 - e^{-sT_{3N}})(\ \bar{F}_{FN} + \bar{F}_{BN}\ ) \right] + \frac{\bar{Q}_N}{Co_N} \qquad 4.12$$

The centre cell (N=0) is now considered in detail in order to solve the boundary conditions, and provide a set of general describing equations.

Firstly, applying the boundary condition of force continuity between cells 1 and 0 results in the following expression.

$$\bar{F}_{R1} e^{-sT_{11}} + \bar{F}_{L1} + \frac{\psi_{31}(1)}{sZc_3(1)} \left[(1 - e^{-sT_{31}})(\bar{F}_{F1} + \bar{F}_{B1}) \right] - \phi_1 \bar{V}_{31}$$

$$= \bar{F}_{L0} e^{-sT_{10}} + \bar{F}_{R0} + \frac{\psi_{31}(0)}{sZc_3(0)} \left[(1 - e^{-sT_{30}})(\bar{F}_{F0} + \bar{F}_{B0}) \right] - \phi_0 \bar{V}_{30} \qquad 4.13$$

Continuity of particle displacement across the 1-0 interface yields,

$$\bar{F}_{L1} = \frac{Zc_1(1)}{Zc_1(0)} \left[-\bar{F}_{R0} + \bar{F}_{L0} e^{-sT_{10}} \right] + \bar{F}_{R1} e^{-sT_{11}} \qquad 4.14$$

Equations 4.13 and 4.14 may be combined to give the rightward travelling force at the left hand boundary. That is,

$$\bar{F}_{R0} = T_{10} \bar{F}_{R1} e^{-sT_{11}} + R_{01} \bar{F}_{L0} e^{-sT_{10}}$$

$$+ \frac{T_{10} \phi_0 \bar{V}_{30}}{2} - \frac{T_{10} \phi_1 \bar{V}_{31}}{2}$$

$$+ \frac{T_{10} \psi_{31}(1)}{2sZc_3(1)} \left[(\bar{F}_{F1} + \bar{F}_{B1})(1 - e^{-sT_{31}}) \right]$$

$$- \frac{T_{10} \psi_{31}(0)}{2sZc_3(0)} \left[(\bar{F}_{F0} + \bar{F}_{B0})(1 - e^{-sT_{30}}) \right] \qquad 4.15$$

where

$$T_{10} = \frac{2Zc_1(0)}{(Zc_1(1) + Zc_1(0)}$$

and

$$R_{01} = \frac{(Zc_1(0) - Zc_1(1)}{(Zc_1(0) + Zc_1(1)}$$

T_{10} represents a transmission coefficient for waves of force travelling from cell 1 to cell 0, while R_{01} represents a reflection coefficient for waves of force travelling from cell 0 to cell 1.

Equation 4.15 is the general describing equation for force interactions at the left hand lateral interface of the cell. Incorporating the relevant boundary conditions at the 0-2 interface, the equation describing the leftward travelling force at the right hand boundary may thus be expressed as,

$$\bar{F}_{L0} = T_{20}\,\bar{F}_{L2}\,e^{-sT_{12}} + R_{02}\,\bar{F}_{R0}\,e^{-sT_{10}}$$

$$+ \frac{T_{10}\,\phi_0\,\bar{V}_{30}}{2} - \frac{T_{20}\,\phi_2\,\bar{V}_{32}}{2}$$

$$+ \frac{T_{20}\,\psi_{31}(2)}{2sZc_3(2)} \left[(\bar{F}_{F2} + \bar{F}_{32})(1 - e^{-sT_{32}}) \right]$$

$$- \frac{T_{20}\,\psi_{31}(0)}{2sZc_3(0)} \left[(\bar{F}_{F0} + \bar{F}_{30})(1 - e^{-sT_{30}}) \right] \qquad 4.16$$

In a similar manner, the boundary conditions may be applied at the interfaces between cell 0 and cell 3, and cell 0 and cell 4. This results in the following expressions for the forward travelling force at the back boundary, and the backward travelling force at the front boundary.

$$\bar{F}_{30} = T_{30}\,\bar{F}_{33}\,e^{-sT_{33}} + R_{03}\,\bar{F}_{F0}\,e^{-sT_{30}}$$

$$+ \frac{T_{30}\,h_0\,\bar{Q}_0}{2} - \frac{T_{30}\,h_3\,\bar{Q}_3}{2}$$

$$+ \frac{T_{30}\,\psi_{13}(3)}{2sZc_1(3)}\left[(\bar{F}_{R3} + \bar{F}_{L3})(1 - e^{-sT_{13}})\right]$$

$$- \frac{T_{30}\psi_{31}(0)}{2sZc_3(0)}\left[(\bar{F}_{R0} + \bar{F}_{L0})(1 - e^{-sT_{10}})\right] \qquad 4.17$$

$$\bar{F}_{F0} = T_{40}\,\bar{F}_{F4}\,e^{-sT_{34}} + R_{04}\,\bar{F}_{30}\,e^{-sT_{30}}$$

$$+ \frac{T_{40}\,h_0\,\bar{Q}_0}{2} - \frac{T_{40}\,h_4\,\bar{Q}_4}{2}$$

$$+ \frac{T_{40}\,\psi_{13}(4)}{2sZc_1(4)}\left[(\bar{F}_{R4} + \bar{F}_{L4})(1 - e^{-sT_{14}})\right]$$

$$- \frac{T_{40}\psi_{13}(0)}{2sZc_1(0)}\left[(\bar{F}_{R0} + \bar{F}_{L0})(1 - e^{-sT_{10}})\right] \qquad 4.18$$

The effects of electrical loading may be included by assuming that the transducer is connected to an electrical load impedance Z_{EN} as shown in figure 4.2, where the excitation voltage applied to the device is given by V_{SN}. The amount of electrical charge produced on the electrodes of the transducer is thus given by,

$$\bar{Q}_N = \frac{[\bar{V}_{SN} - \bar{V}_{3N}]}{s\bar{\bar{Z}}_{EN}}$$

This expression for charge may be incorporated in equations 4.17, 4.18 and 4.12 to give the following three relationships for the centre cell.

$$\bar{F}_{30} = T_{30}\bar{F}_{33}e^{-sT_{33}} + R_{03}\bar{F}_{F0}e^{-sT_{30}} + \frac{T_{30}h_0\bar{V}_{30}}{2s\bar{\bar{Z}}_{E0}} - \frac{T_{30}h_3\bar{V}_{33}}{2s\bar{\bar{Z}}_{E3}}$$

$$- \frac{T_{30}h_0\bar{V}_{30}}{2s\bar{\bar{Z}}_{E0}} + \frac{T_{30}\psi_{13}(0)}{2sZc_1(0)}\left[(\bar{F}_{R0} + \bar{F}_{L0})(1 - e^{-sT_{10}})\right]$$

$$+ \frac{T_{30}h_3\bar{V}_{33}}{2s\bar{\bar{Z}}_{E3}} + \frac{T_{30}\psi_{13}(3)}{2sZc_1(3)}\left[(\bar{F}_{R3} + \bar{F}_{L3})(1 - e^{-sT_{13}})\right] \qquad 4.19$$

$$\bar{F}_{FO} = T_{40}\bar{F}_{F4}e^{-sT_{34}} + R_{04}\bar{F}_{B0}e^{-sT_{30}} + \frac{T_{40}h_0\bar{V}_{B0}}{2s\bar{Z}_{E0}} - \frac{T_{40}h_4\bar{V}_{B4}}{2s\bar{Z}_{E4}}$$

$$- \frac{T_{40}h_0V_{30}}{2s\bar{Z}_{E0}} + \frac{T_{40}\psi_{13}(0)}{2sZc_1(0)}\left[(\bar{F}_{R0} + \bar{F}_{L0})(1 - e^{-sT_{10}})\right]$$

$$+ \frac{T_{40}h_4V_{34}}{2s\bar{Z}_{E4}} + \frac{T_{40}\psi_{13}(4)}{2sZc_1(4)}\left[(\bar{F}_{R4} + \bar{F}_{L4})(1 - e^{-sT_{14}})\right] \quad 4.20$$

$$\frac{\bar{V}_{30}}{\bar{U}_0} = - \frac{\phi_0}{sCo_0Zc_3(0)}\left[(1 - e^{-sT_{10}})(\bar{F}_{R0} + \bar{F}_{L0})\right]$$

$$- \frac{h_0}{sZc_3(0)}\left[(1 - e^{-sT_{30}})(\bar{F}_{F0} + \bar{F}_{B0})\right] + \frac{\bar{V}_{B0}}{sCo_0\bar{Z}_{E0}} \quad 4.21$$

Equations 4.13, 4.14, 4.19, 4.20 and 4.21 thus describe the general behaviour of a two-dimensional piezoelectric transducer, operating into any piezoelectric or mechanical layer which may be attached to any major face. Close inspection of these relationships reveal that any one of the forces propagating inside the centre transducer is dependant upon:

1. The thickness forces in the layers at the left and right hand faces.

2. The lateral forces in the layers positioned at the front and rear faces.

3. The voltage appearing across a piezoelectric layer positioned at any of the four faces.

4. The excitation voltage applied to a layer on either the front or the rear face.

5. The input forces applied to the layer from the front, back, left or right faces.

6. Other forces inside the centre transducer.

These observations suggest that a closed form solution to such a system of equations would be extremely difficult to obtain and would perhaps offer no additional insight into transducer behaviour. Subsequently, the equations may be represented in a variable matrix format, and numerical inversion techniques used to obtain the relevant set of solutions.

4.3 THE LATTICE MODEL WITH NO LAYERS.

The form of the matrix representation will be modified as the number of layers changes, and the simplest case is obviously that of a transducer with no mechanical or piezoelectric layers attached, as illustrated in figure 4.3. This situation may be described by the following matrix equation.

$$[\bar{c}] = [\bar{A}].[\bar{c}] + [\bar{B}].[\bar{r}]$$

which may be rewritten as,

$$[\bar{c}] = [I-\bar{A}]^{-1}.[\bar{B}].[\bar{r}] \qquad 4.22$$

where

$$[\bar{c}]^T = [\ \bar{F}_{RO}\ ,\ \bar{F}_{LO}\ ,\ \bar{F}_{FO}\ ,\ \bar{F}_{BO}\ ,\ \bar{V}_{3O}\ ]$$

and

$$[\bar{r}]^T = [\ \bar{F}_1\ ,\ \bar{F}_2\ ,\ \bar{F}_3\ ,\ \bar{F}_4\ ,\ \bar{V}_{EO}\ ]$$

The matrix $[\bar{A}]$ and the matrix $[\bar{B}]$ are readily obtained from the describing equations. In this case the vector $[\bar{c}]$ contains the resultant force components inside, and the voltage appearing across, the transducer. The vector $[\bar{r}]$ represents the input forces which are applied to each face of the device in addition to the excitation voltage applied via the Thevenin impedance $\bar{Z}_{EO}$.

A relationship must now be obtained which describes the four output forces in terms of the internal forces $\bar{F}_{FO}$, $\bar{F}_{BO}$, $\bar{F}_{LO}$ and $\bar{F}_{RO}$, in addition to the voltage $\bar{V}_{3O}$. This may be achieved by relating equations 4.15, 4.16, 4.19 and 4.20 to cells 1, 2, 3 and 4 respectively. (In this case cells 1-4 consist simply of semi-infinite mechanical load media positioned at each transducer face.) For example, the output force at the right hand face is equivilent to the rightward travelling force in cell-2. Similarly, the front face output force is identical to the forward travelling force in cell-3. Thus the expressions for the four output forces illustrated in figure 4.3, may be written as,

$$\bar{F}_{O1} = \bar{F}_{L1} = T_{O1}\bar{F}_{LO}e^{-sT_{10}} + R_{10}\bar{F}_{1}$$

$$+ \frac{T_{O1}\psi_{31}(O)}{2sZc_{3}(O)}\left[\bar{F}_{FO} + \bar{F}_{3O}\right]\left[1 - e^{-sT_{30}}\right] - \frac{T_{O1}\phi_{O}\bar{V}_{3O}}{2} \qquad 4.23$$

$$\bar{F}_{O2} = \bar{F}_{R2} = T_{O2}\bar{F}_{RO}e^{-sT_{10}} + R_{20}\bar{F}_{2}$$

$$+ \frac{T_{O2}\psi_{31}(O)}{2sZc_{3}(O)}\left[\bar{F}_{FO} + \bar{F}_{3O}\right]\left[1 - e^{-sT_{30}}\right] - \frac{T_{O2}\phi_{O}\bar{V}_{3O}}{2} \qquad 4.24$$

$$\bar{F}_{O3} = \bar{F}_{F3} = T_{O3}\bar{F}_{FO}e^{-sT_{30}} + R_{30}\bar{F}_{3}$$

$$+ \frac{T_{O3}\psi_{13}(O)}{2sZc_{1}(O)}\left[\bar{F}_{RO} + \bar{F}_{LO}\right]\left[1 - e^{-sT_{10}}\right] - \frac{T_{O3}h_{O}[\bar{V}_{2O} - \bar{V}_{3O}]}{2} \qquad 4.25$$

$$\bar{F}_{04} = \bar{F}_{34} = T_{04}\bar{F}_{30}e^{-sT_{30}} + R_{40}\bar{F}_{4}$$

$$+ \frac{T_{04}\psi_{13}(0)}{2sZc_{1}(0)}\left[\bar{F}_{R0} + \bar{F}_{L0}\right]\left[1 - e^{-sT_{10}}\right] - \frac{T_{04}h_{0}[\bar{V}_{30} - \bar{V}_{30}]}{2} \qquad 4.26$$

These may be written in matrix format as,

$$[\bar{d}] = [\bar{E}][\bar{c}] + [\bar{G}][\bar{r}] \qquad 4.27$$

where the vector

$$[\bar{d}]^{T} = [\ \bar{F}_{01}\ \ \bar{F}_{02}\ \ \bar{F}_{03}\ \ \bar{F}_{04}\ ]$$

The vectors $[\bar{c}]$ and $[\bar{r}]$ remain unchanged, and the matrices $[\bar{E}]$ and $[\bar{G}]$ are obtained from equations 4.23 to 4.26. The matrix equations 4.22 and 4.27 thus comprise a complete description of the transducer configuration.

4.4 THE LATTICE MODEL WITH MECHANICAL AND PIEZOELECTRIC LAYERS

(i) Layers in the thickness dimension.

Consider the layered configuration shown in figure 4.4 where two two-dimensional transducer structures are positioned between three mechanical layers in the thickness direction. Each layer possesses both finite width and finite thickness. Layers 2 and 4 are piezoelectric, while layers 1, 3 and 5 are non-piezoelectric.

Relating this configuration to the matrix equations 4.22 and 4.27 yields a $[\bar{c}]$ vector with order 25 and an $[\bar{A}]$ matrix of order 25*25. The order of the $[\bar{d}]$ and $[\bar{r}]$ vectors and also the $[\bar{E}]$ and $[\bar{G}]$ matrices depend upon which faces undergo external force excitation and also upon which outputs are of interest. For the configuration shown in figure 4.4, the only input force shown is that striking the front face, while the only force output of interest is that emanating from the front face. There are two input voltages and two output voltages which are also determined. Thus $[\bar{d}]$ is order 1, $[\bar{r}]$ is order 3, $[\bar{E}]$ is order 25*1 and $[\bar{G}]$ is 3*1.

(ii) Layers in the lateral direction.

The same matrix technique may be applied to the case illustrated in figure 4.5, where the layers are positioned side by side. Five layers are shown in the diagram, two are piezoelectric and three are non piezoelectric. The $[\bar{c}]$ vector and the square $[\bar{A}]$ matrix are of the same order as the thickness case, although they are obviously different in form.

The $[\bar{d}]$ vector has order 5 since there are five output forces, the $[\bar{r}]$ vector has order 7 since, there are two input voltages and five input forces. The $[\bar{B}]$ matrix is 7*25, the $[\bar{E}]$ matrix is 5*25 and the $[\bar{G}]$ matrix is 7*5. Again the entries in each of these matrices may be determined from the describing equations.

(iii) Layers in both directions.

The general two-dimensional layered configuration may be theoretically analysed by the two-dimensional lattice model. However, this is beyond the scope of the thesis and it is also considered that the results would be difficult to verify experimentally.

4.5 CONCLUDING REMARKS

This chapter has presented a lattice type technique for analysing two-dimensional multi-layered piezoelectric transducer systems. The complete set of describing equations for the transducer were shown to be extremely difficult to solve analytically. As a result, a matrix representation has been developed to describe the system, and may easily be applied to layered structures in either the thickness or lateral directions. The technique is primarily intended for the efficient mathematical evaluation of layered two-dimensional piezoelectric structures, and does not offer any improved insight into the transduction process.

In subsequent chapters, simulated and experimental results are presented to confirm the accuracy of both the block diagram technique presented in chapter III, and the lattice method presented in this chapter.

CHAPTER V

SIMULATION AND EXPERIMENTAL VERIFICATION

5.1 INTRODUCTION

This chapter describes the experimental verification of the two-dimensional linear systems model. Single transducers and multi-layered piezoelectric structures are considered, simulated and measured responses compared, and the accuracy of the technique assessed with respect to an alternative modelling strategy.

The behaviour of a particular piezoelectric structure may be conveniently characterised by measurement of its electrical or operational impedance as a function of frequency. Performing this measurement is a relatively simple matter, involving the use of a vector impedance meter, which measures the spectral magnitude and phase of the impedance associated with the specimen.

An alternative method of verifying the model is by measurement of the voltage which appears across the transducer when configured as a transmitter. A step function of voltage is applied to the transducer, via some electrical load network, and the resulting transducer voltage is monitored.

The effect of adding multiple layers to the transducer structure is assessed in section 5.6. Electrical impedance spectra and time domain voltage responses, are presented for a variety of layered structures incorporating single and multiple transducers in the thickness dimension. The close agreement which is evident in the case of thickness layers, serves to confirm the accuracy of the new two-dimensional lattice model. The addition of layers in the lateral direction is also considered in this section, however, the same accuracy is not obtained and the reasons for this are discussed.

In order to fully validate the new model, it is necessary to assess the behaviour of a two-dimensional transducer in transmission and reception. In section 5.7, force profile and pulse-echo responses are measured and compared to their simulated equivalents. This involves the use of single and multiple layered transducer structures operating as transmitters and receivers into both solid and liquid media.

5.2 TRANSDUCER MECHANICAL LOSS

In order to accurately reproduce the behavioural characteristics of a transducer structure under arbitrary conditions, it is necessary to consider the effects of mechanical loss. Inherent in the development of the two-dimensional model is the assumption that the transducer, layers and surrounding media are loss free. The model therefore requires further development to include the effects of mechanical wave absorption in the structure and propagating media. Transducer mechanical loss has been treated in the literature [12][62], but the following simplified approach is adopted for the present investigation.

The resonant characteristics of the transducer are controlled by $\bar{K}_F$, $\bar{K}_B$, $\bar{K}_L$ and $\bar{K}_R$, as described in chapter III, thus any mechanical loss mechanism should be related to these reverberation factors. Consequently, the following expression may be postulated for the lossy reverberation factor $\bar{K}_{FL}$.

$$\bar{K}_{FL} = \frac{(1 - e^{-sT_B} e^{-(\alpha H+\beta w)})(1 - R_B e^{-sT_B} e^{-(\alpha H+\beta w)})}{(1 - R_F R_B e^{-2sT_B} e^{-2(\alpha H+\beta w)})}$$

Where H is the transducer thickness and w is the frequency in radian measure. α is a loss factor which describes the amount of distance dependant loss due to mechanical absorption in the structure, and β defines the amount of frequency dependant loss which is applied. This approach, although by no means rigorous, provides acceptable results for the low loss structures under consideration.

In the simulations, the frequency dependant loss factor β remains at a constant value of 10^{-9} throughout. This has very little effect on the results. The distance related attenuation however is scaled to permit use of a relative loss factor between 0 and 10 in the simulations. It was found that a relative loss factor of 5 was adequate for the PZT-5A transducers. The loss factors which must be applied to other materials are given in due course.

5.3 TRANSDUCER MATERIAL PARAMETERS

In order to preserve the expected accuracy of any modelling technique, it is necessary to determine the accuracy of the parameters upon which the simulations are based. The manufacturing process for synthesising piezoelectric materials is often the weakest link in the reliable and repeatable fabrication of ultrasonic probes and arrays. For example the material parameters Y_{11}, Y_{13} and Y_{33} may vary by as much as 20% from their nominal specified values. This alone may have a dramatic effect on the resonant behaviour of a transducer. Similarly, the piezoelectric constants h_{13} and h_{33} may vary by as much as 10%. Such variations in device parameters differ from manufacturer to manufacturer and even from batch to batch. It is important therefore to identify some straightforward technique which will allow accurate determination of the material parameters.

A number of these parameters (Y_{33}, h_{33}, ρ and ϵ_{33}) may be measured in a matter of seconds using the computerised thickness mode calibration system developed by Hayward [23]. The remaining parameters (Y_{11}, Y_{13} and h_{13}) are however more difficult to obtain. Complex measurement techniques do exist for their determination, but the use of these was considered beyond the scope of the present work. Subsequently, it is considered that the following approach yields sufficiently accurate results.

Firstly, it was found that a 10% variation in the piezoelectric constant h_{13} did not result in a significant difference in transducer behaviour. Upon this basis it was deemed sufficient to use the nominal value of h_{13}, irrespective of variations in the other parameters. Secondly, since the mechanical stiffness parameter Y_{11} is closely related to Young's modulus Y_{33}, the measured variation in Y_{33} is also applied to Y_{11}. Finally, according to simulation, the cross-coupled mechanical stiffness coefficient Y_{13}, was found to be consistently 10-15% lower than the nominal value for PZT-5A, although this was very difficult to obtain accurately. Throughout the simulations the value of Y_{13} is assumed to be 10% lower than the nominal values quoted. The material parameters for the layers were obtained in a similar manner.

The experimental results in the remainder of this chapter were obtained from transducers which were cut from two thickness mode slabs. The measured properties, and the parameters used in the simulations for the transducers and the various layer materials are given in appendix B.

5.4 MEASUREMENT AND SIMULATION OF ELECTRICAL IMPEDANCES

In this section a variety of experimentally measured electrical impedance spectra are compared with their simulated equivalents. A selection of different configuration ratios are used in this way to verify the accuracy of the model. The relative accuracy of an alternative modelling technique is assessed by comparing simulations generated by the modified uni-dimensional approach.

The effects of mechanical loading on the device faces is illustrated by presenting a selection of measured electrical impedance spectra for various mechanical loading configurations. The experimental apparatus and procedure are detailed in appendix B.

The impedance spectra shown in figure 5.1a correspond to that of an oil-damped (Zc=1.5) PZT-5A device with thickness 1.96mm, length 50mm and configuration ratio (G=width/thickness) of 0.14. Upon inspection, the resonant behaviour of the device appears to be uni-dimensional, since there is no trace of the lateral mode, which has a fundamental resonant frequency around 5.5MHz. It should be emphasized however that this device was cut from a thickness mode slab with a thickness mode electrical resonance of 1MHZ. The fundamental electrical resonance in figure 5.1a occurs at 740kHz. This 25% shift in resonant frequency indicates that the device may certainly not be modelled as a uni-dimensional thickness mode transducer.

The simulations for this particular case may be described in the following manner.

1. Figure 5.1b shows the simulation generated by the modified uni-dimensional model. The general shape is fairly consistent, however, the points of electrical and mechanical resonance are clearly not in close agreement.

2. The theoretical electrical impedance spectra given in figure 5.1c are generated by the full two-dimensional model. The positions of the electrical and mechanical resonances are correct and the overall shape is in excellent agreement.

Figure 5.2a shows the measured impedance spectra for a similar PZT-5A device with a configuration ratio of 0.23. In this case, the fundamental lateral mode resonance occurs at 3.25MHz, although it may only be distinguished in the phase spectrum due to its small magnitude. The simulated impedance spectra by the modified thickness mode technique is given in figure 5.2b, while the full 2-d model is used to provide the spectra shown in figure 5.2c.

The modified thickness mode model clearly has incorrect values for electrical and mechanical resonance. Applying the 2-d model, the lateral mode resonance at 3.25MHz is predicted, in addition to the correct frequencies of electrical and mechanical resonances for the thickness direction.

A configuration ratio of 0.35 is used to provide the measured impedance spectra shown in figure 5.3a. All other properties of the device remain as before. In this case, the lateral mode mechanical resonance occurs at 2.15MHz. The simulated responses are shown in figures 5.3b and 5.3c. The modified thickness mode simulation given in figure 5.3b, is now decreasing in accuracy, since the lateral mode which is unpredicted by this method, becomes more significant. The full two-dimensional modelling technique is used to provide figure 5.3c. The agreement is excellent, both in terms of the positions of the resonances and also the prediction of the lateral mode.

A further increase in configuration ratio to 0.42 results in the experimental plots given in figure 5.4a. The corresponding simulations are given in figures 5.4b and 5.4c. As before, the thickness mode technique, does not predict the thickness resonance particularly well, and offers no insight into lateral mode behaviour, now apparent at 1.85MHz. The agreement offered by the two-dimensional model is excellent, including the accurate prediction of the lateral mode electrical and mechanical resonances.

Finally, in figure 5.5a, the configuration ratio is increased to 0.51 while the thickness and material parameters are kept constant. In this case, which is defined as the limiting case to preserve accuracy for the 2-d model, the lateral mode appears to have weakened somewhat compared to the previous example. This is a direct result of its proximity in frequency to the fundamental thickness mode resonance. This slight weakening is not predicted by the 2-d simulation shown in figure 5.5c, although the positions of the lateral mode electrical and mechanical resonances are still correct. Figure 5.5b illustrates the modified thickness mode model. The limited accuracy of the technique is again apparent.

In order to illustrate the effects of mechanical loading upon the electrical impedance spectra, consider the experimental and simulated results presented in figures 5.6 to 5.9 inclusive. These correspond to the following mechanical load configurations;

1. Air loading - all faces

   Figure 5.6a illustrates the electrical impedance characteristic of a PZT-5A transducer with length dimension 20mm, thickness dimension 1.96mm and width 0.85mm. (G = 0.434) The device is air loaded on all four faces. Due to the very large magnitude which occurs at mechanical resonance in an air-loaded device, the y-axis is truncated to allow improved resolution of the remaining features in the graph. The x-axis extends to 2.2MHz for a similar reason. The two-dimensional model is used to provide the simulation given in figure 5.6b. The agreement is excellent. The artefact which occurs at the lateral mode electrical resonance is thought to be the result of a slight lack of parallelism in the lateral dimension of the transducer. Obviously this effect is not predicted.

2. Tungsten-filled-epoxy loading - left face only

The left face of a PZT-5A transducer is loaded with a tungsten/epoxy mix possesing an acoustic imedance of 4MRayls, while the other three faces are loaded with transformer oil (Zc = 1.5). The transducer is 1.96mm thick, 20mm long and 0.8mm wide (G = 0.408). The magnitude and phase characteristics of its electrical impedance are given in figure 5.7a. The extent of the lateral mode reverberation is clearly reduced, while there is also a slight decrease in the thickness direction resonances, particularly the harmonic at 3MHz. Simulation of this configuration via the two-dimensional systems model is presented in figure 5.7b. All of the important features of the measured spectra are again evident in the simulation, including the slight reduction in the thickness resonances.

3. Tungsten-filled-epoxy loading - back face only

In this case a PZT-5A transducer of length 50mm, thickness 1.96mm and width 0.76 mm was backed with tungsten-filled-epoxy (Zc = 4) The remaining faces are loaded with transformer oil. Figure 5.8a illustrates the experimental impedance spectra while figure 5.8b shows the simulated version. The theoretical impedance spectra for a similar transducer with all faces immersed in oil is shown in figure 5.8c for reference purposes. The reduction in extent of the thickness direction resonances is readily apparent, while excellent agreement between thoery and experiment is maintained.

4. Lead loading - back face

The effect of heavy mechanical damping on the back face is illustrated in figure 5.9a. In this case the transducer is identical to that used to provide the air loaded result of figure 5.6. The lead possesses an acoustic impedance of approximately 24 MRayls while the epoxy bondline between the transducer and the lead is approximately 50 microns. As expected the resonant peak for thickness vibrations is severely rounded and there is a slight reduction in the lateral mechanical resonant maximum. The simulated impedance spectra for this configuration are given in figure 5.9b. The agreement is fairly good, although the theoretical curve does not exhibit the jagged peaks prominent in the experimental result of figure 5.9a.

The foregoing impedance spectra serve to demonstrate the accuracy attainable by the two-dimensional linear systems approach. The effects of mechanical loading upon various transducer faces have been demonstrated experimentally, and accurate simulations have been presented for the cases considered.

5.5 MEASUREMENT AND SIMULATION OF TRANSMITTER VOLTAGES

In this section a variety of experimental and simulated time domain voltage responses are presented. The effects of different electrical loading configurations on the transmitter voltage are shown. Each of the following voltage measurements was performed in accordance with the procedure detailed in appendix B.

Firing circuit #1

Firing circuit #1 consists of a VMOS switching FET which is used to excite the transducer via a blocking capacitor C_B. The circuit is given in figure 5.10, and its operation may be summarised as follows:

With the VMOS in the off state the transducer and blocking capacitor charge from HT via the resistor Rc. When the VMOS is driven into the on state, one side of the blocking capacitor is connected to ground via the on resistance of the VMOS (3Ω). The voltage across the transducer therefore immediately becomes negative by the amount $(C_B/(Co+C_B))*HT$, where Co is the static capacitance of the transducer. The turn-on time of the VMOS switch is less than 30ns. The resonant behaviour of the transducer then causes the measured voltage to oscillate in a manner which is a function of the transduction characteristics, mechanical and electrical loading. (A comprehensive treatment of transducer pulsers and firing circuits is available in reference [22]).

Figure 5.11a illustrates experimentally the voltage which appears across an air loaded PZT-5A transducer which is 20mm long, 1.96mm thick and 0.85mm wide (G=0.434), when excited in such a manner. The response is severely rounded, and does not exhibit the exponential behaviour associated with thickness mode devices [18].

Simulation of the time domain response of the transducer also involves modelling the firing circuit and measuring apparatus. The oscilloscope probe which was used to monitor the voltage shown in figure 5.11a was a Tektronix model P6106A, which possesses an input impedance of 10MΩ in parallel with 8pF, when operating into a Tektronix 7633 storage oscilloscope. The VMOS switch, is essentially considered to provide a negative going pulse with a finite turn on time, which is applied to the blocking capacitor C_B. The simulated response, presented in figure 5.11b affords excellent agreement with the measured waveform.

A similar experiment was performed for a transducer with thickness 1.96mm, length 20mm and width 0.45 (G=0.229). The measured and modelled waveforms shown in figure 5.12a and 5.12b, again demonstrate excellent agreement.

Firing circuit #2

A commonly encountered modification to firing circuit #1 is the addition of the pulse shaping resistor Rp, as shown in figure 5.13. This has the effect of superimposing a slow exponential rise on the reverberating waveform. For example, figures 5.14a and 5.14b illustrate the measured and simulated responses for the above transducer with the configuration ratio 0.434. The frequency and extent of the reverberation are correct, as is the exponential rise time of the waveform. Figures 5.15a and 5.15b show the equivalent measured and simulated responses for the transducer with configuration ratio 0.229.

Firing circuit #3

Another important firing circuit configuration is described by the addition of an inductor in parallel with the transducer as outlined in fig 5.16. The inclusion of this component, which may serve to improve efficiency at the expense of bandwidth, or remove the effects of low frequency radial modes in thin disk transducers, has a very marked influence on the transducer voltage characteristic. For example, when Lp is equal to 20.6μH then the voltage which appears across a transducer of thickness 1.96mm, length 20mm and width 0.85mm is as shown in figure 5.17a. Modelling the configuration, including the tuning inductor, with the two-dimensional systems model yields the wavefom given in figure 5.17b. When the value of Lp is increased to 180μH, and the same transducer is used, the measured waveform is shown in figure 5.18a, with the simulated response given by figure 5.18b. Agreement is excellent.

5.5.1 THE MECHANICALLY LOADED TRANSMITTER VOLTAGE

In order to demonstrate the effects of mechanical loading on the time domain response, consider the experimentally measured waveforms shown in figure 5.19a and 5.20a. Figure 5.19a represents the transducer voltage generated by firing circuit #1 when applied to the lead-backed transducer described in 5.2(iv). Figure 5.20a shows the transducer voltage in response to firing circuit #2. In both cases the ring time of the waveform has been severely reduced due to the heavy mechanical damping. The simulated waveforms for these two cases are given in figure 5.19b and 5.20b. It should be noted that failure to simulate the jagged peaks in the impedance spectra, does not appear to affect the accuracy of the time domain simulation, excellent agreement is obtained.

5.6 MEASUREMENT AND SIMULATION OF MULTI-LAYERED STRUCTURES

The fabrication of ultrasonic probes and arrays for practical purposes inevitably involves the addition of mechanical layers to the basic transducing element. For example, a bondline of finite thickness is produced when a transducer is bonded to a backing material. This may have a detrimental effect on the behaviour of the structure [24]. Single or multiple front face layers may be employed to increase the efficiency of the device [10]. Layers may also be introduced by couplant when transmitting into a solid material such as steel, or they may simply be used for mechanical protection or strengthening. This section provides a number of experimental and theoretical results for structures which possess multiple mechanical layers. The theoretical simulations are generated by the two-dimensional lattice type model described in chapter IV. For layers in the thickness direction, the simulations are shown to be very good, however when the structure possesses layers in the lateral direction the technique may become less acurate.

5.6.1 LAYERS IN THE THICKNESS DIRECTION

(A) RESULTS AND SIMULATIONS

Layered structures which are stacked in the thickness direction are considered in this section. Three distinct configurations of structures comprising three layers, are employed to verify the theory. Measurements of electrical impedance magnitude are performed in the frequency domain, in addition to time domain transmitter voltages. In both cases, the measured and simulated responses demonstrate close agreement. Furthermore, the general operational features applying to this type of layered structure are discussed in section 5.6.1 (B).

(i) LAYERED STRUCTURE #1

The first layered structure which is investigated has the general form shown in figure 5.21. Layers 1 and 3 comprise PZT-5A ceramic possessing a thickness of 1.00mm, and a width of 0.70. (G=0.7), while layer 2 is a 60μm epoxy bondline. The complete structure was immersed in transformer oil for measurement of its electrical impedance characteristics. Figure 5.22a illustrates the impedance spectrum of layer 1. Four main resonances are labelled in the figure, these correspond to the following.

1. The thickness vibration across all three layers.

2. The thickness vibration within layer 1

3. A periodic mode which is generated between layer 1 and layer 3 since they are both piezoelectric.

4. The lateral vibration in layer 1

Although the configuration ratio for layers 1 and 3 is outwith the 'accuracy' limit of the two-dimensional model, the technique is still used to simulate these layers. Layer 2 is simulated as a pure thickness mode layer due to its large configuration ratio (11.6). The theoretical result is shown in figure 5.22b, where reasonably close agreement may be observed for the main resonances. There is some error in the prediction of the lateral mode resonance in layer 1 due to its configuration ratio of 0.7.

Measurement of the time domain voltage response to firing circuit #2 is given in figure 5.23a. The simulated equivalent is presented in figure 5.23b. The shape of the waveform, which may be seen to comprise two main frequency components, is predicted fairly accurately by the model.

(ii) LAYERED STRUCTURE #2

The second thickness layered structure has the same, three layered format given in figure 5.21. However, in this case, layer 1 comprises PZT-5A with thickness 1.96mm, width 0.69mm, and length 15mm. Layer 2 is a measured 70um epoxy bondline. Layer 3 consists of aluminium with thickness 1.58mm and width 0.69mm. The structure is immersed in a bath of transformer oil to enable measurement of the impedance magnitude characteristic given in figure 5.24a. Once again the resonances may be identified as follows.

1. The thickness vibration across all three layers.

2. The thickness vibration within layer 1

3. The lateral vibration in layer 1

4. The harmonic of resonance '2' above

The 'periodic' resonance in the previous case no longer exists, since layer 3 is now non-piezoelectric.

The simulation of this impedance spectrum is shown in figure 5.24b. The agreement between theory and experiment for this case is excellent. The transmitter time domain voltage response produced when the structure is excited by firing circuit #2 is given in figure 5.25a. The waveform, which again comprises two major frequency components, is predicted accurately by the model. (figure 5.25b)

(iii) LAYERED STRUCTURE #3

The third three layered system in the thickness dimension comprises

layer 1: PZT-5A, thickness-1.96mm, width-0.6mm.

layer 2: epoxy bondline, thickness-15μm.

layer 3: steel, thickness-0.8mm.

The complete structure is 20mm long. The impedance magnitude spectra shown in figure 5.26a were measured in a bath of transformer oil. In common with the previous case, the impedance characteristic exhibits four main resonances. Namely,

1. The thickness vibration across all three layers.

2. The thickness vibration within layer 1

3. The lateral vibration in layer 1

4. The first harmonic of resonance '2'

The periodic resonance exhibited by structure 1 is again non-existent since only the first layer is piezoelectric. The two-dimensional simulation of structure #3 yields the results given in figure 5.26b. The transmitter voltage generated by structure #3 when excited by firing circuit #2, is given in figure 5.27a, and the simulated equivalent is presented in figure 5.27b. In both cases the simulated and experimental graphs are in close agreement.

5.6.1 (B) DISCUSSION

In order to analyse and explain the variations in the resonant characteristics of the three layered structures considered, their major differences may be listed as follows;

1. In each case the thickness of the bondline is slightly different. This may be attributed to the finish or smoothness of the third layer. For example, the bondline thickness of 20μm in structure #3 is thinner than the other two because the steel layer was highly polished prior to fabrication of the structure. Correspondingly, structure #2 possesses the thickest bondline since the aluminium was not finished to the same standard.

2. The acoustic impedance of the material corresponding to layer 3 is different in each case. Three situations are represented whereby layer 3 has an acoustic impedance which is less than, equal to, and greater than that of the transducer or layer 1.

3. Layer 3 of structure #1 possesses piezoelectric properties whereas the third layers in the other two cases are non-piezoelectric and operate in a purely mechanical fashion.

Accordingly, the fundamental variations in behaviour of the three structures may be attributed directly to the above factors.

For example, it may be shown that the relative resonant strength of the two major vibrational modes is highly dependant upon the thickness of the bondline. Furthermore, the frequencies of electrical and mechanical resonance of the two vibrational modes also vary in sympathy with the bondline thickness. In order to emphasise these effects consider firstly figure 5.28a, which shows the variation in the frequency of electrical resonance in layer 1 of a three layer structure. The three curves in the figure correspond to impedances in layer 3 of 17MRayls (aluminium), 38MRayls (non-piezoelectric ceramic) and 47MRayls (steel). The frequencies are expressed as a percentage of the resonant frequency in layer 1 when the bondline is negligibly thin. (10^{-4} microns)

It may be observed from figure 5.28a that the curvature of the graph decreases with decreasing acoustic impedance. This may be emphasised by referring to figure 5.28b which is a similar plot for a layer 3 impedance of

7MRayls (matching to water). In this case the curve is almost linear for bondline thicknesses between 20 and 100microns. The percentage reduction for a given bondline thickness, is also smaller in this case. Finally, it should be noted that for each case of mechanical loading, the frequency of electrical resonance in layer 1 approaches an identical absolute value when the bondline is made very large.

A relative measure of the resonant strength in a vibrational mode may be obtained by measuring the difference in impedance magnitude between mechanical and electrical resonance. Since the effects of mechanical loading and mechanical loss will dramatically influence this measurement, care must be taken to ensure these factors remain constant during any comparison. This was done to provide figure 5.29a, which shows the ratio of the resonant strength of the transducer (layer 1) to the resonant strength of the vibration across all three layers, as a function of bondline thickness. Three curves are presented corresponding to acoustic impedances in layer 3 of 17, 38 and 47MRayls as before.

As the bondline thickness is increased, the resonance in layer 1 increases in magnitude, while the vibration across all three layers decreases in magnitude. The point where both resonances exhibit equal strength, ('ratio of resonant strength' equal to 1) is clearly marked on the diagram. It may be observed that in order for both resonances to be of equal strength, then the structure with a higher impedance in layer 3 requires a larger bondline. Furthermore, for any given bondline thickness, the smaller the acoustic impedance of layer 3, the larger is the resonance in layer 1. To complete this set of simulations, figure 5.29b illustrates the relationship between the ratio of resonance strengths and the bondline thickness, for the case where layer 3 is a quarter wave layer of impedance 7MRayls. In this case, for any value of bondline thickness, the resonance in the transducer is always stronger than the combined resonance in all three layers.

In the experimental and simulated plots given in figure 5.22a and figure 5.22b respectively, the effect of using piezoelectric devices for both layers 1 and 3 may be clearly observed. There is an additional resonance which is caused by both layers acting as simultaneous transmitters and receivers. That is, layer 1 emits a force which travels towards layer 3 via the bondline, when this force reaches layer 3 a voltage is generated on layer 3 which in turn produces a return force travelling towards layer 1. This resonance is clearly dependant upon the electrical loading upon each transducer. It should be noted that the modelling software in its present form , does not account for variations in electrical loading between different transducers in a layered structure. This may therefore contribute to the discrepancy between simulated and measured responses for layered structure #1, especially in the time domain voltage measurement since the electrical load configuration is relatively complex. The agreement in both impedance and voltage is excellent for the two cases where layer 3 is non-piezoelectric.

5.6.2 LAYERS IN THE LATERAL DIRECTION

In this section the behaviour of transducer structures which possess layers in the lateral direction is investigated. Three distinct configurations of structures comprising three layers are considered. Electrical impedance measurements are presented which do not demonstrate good agreement with the relevant simulations, and the possible reasons for the innaccuracies, are outlined in section 5.6.2 (B).

5.6.2 (A) RESULTS AND SIMULATIONS

(i) LAYERED STRUCTURE #4

All of the lateral layered structures which are investigated have the general form shown in figure 5.30. In this case, layers 1 and 3 comprise PZT-5A ceramic possessing a thickness of 1.96mm, and a width of 0.75mm, (G=0.382) while layer 2 is a 40um epoxy bondline. The complete structure was immersed in transformer oil for measurement of the electrical impedance characteristics (measured across layer 1), which are shown in figure 5.31a. Four main resonances are evident in the spectra, these are labelled in the figure and may be described as follows.

1. The vibration in the thickness dimension of layer 1. The frequencies of electrical and mechanical resonance for this mode of vibration have been shifted upwards in frequency by approximately 150kHz, due to the addition of the layers.

2. The lateral vibration in layer 1.

3. The harmonic of vibration 1. This has also been subject to a shift upwards in frequency.

4. This mode appears to correspond to a lateral vibration through layers 1, 2 and 3.

The simulated electrical impedance spectra are shown in figure 5.31b. Comparison with figure 5.31a indicates a number of major differences between theory and experiment, which may be described in the following manner.

Firstly, the simulated resonance in the thickness dimension does not exhibit any increase in frequency due to the addition of the layers. Secondly, the strength of the lateral reverberation throughout all three layers is significantly smaller than the measured case. Thirdly, In the simulation there are two distinct resonances around 2MHz, corresponding to the lateral mode in layer 1, and an overtone of the lateral vibration in all three layers. In the measured response, only one resonance is apparent, although there is the slightest trace of a second when the phase spectrum is examined closely in the region of 2MHz. Finally, the modelled response does not simulate the features evident at 1.3MHz and 2.6MHz in the measured response.

(ii) LAYERED STRUCTURE #5

This structure possesses the same, three layered format given in figure 5.30. However, in this case, layer 1 comprises PZT-5A with thickness 1.96mm, width 0.9mm, and length 20mm. Layer 2 is a measured 90um epoxy bondline. Layer 3 consists of aluminium with thickness 2.00mm and width 0.8mm. The structure is immersed in a bath of transformer oil to enable measurement of the impedance magnitude characteristic given in figure 5.32a. Once again the resonances may be identified as follows.

1. The thickness vibration within layer 1

2. The lateral vibration in layer 1

3. The harmonic of resonance '1' above

4. The lateral vibration across all three layers.

The simulated electrical impedance response is given in figure 5.32b. In this case, the agreement between thoery and experiment is improved, although the magnitude and shape of resonance '4' remains erroneous. Furthermore, spurious, unpredicted resonant features are evident virtually throughout the measured spectra.

(iii) LAYERED STRUCTURE #6

The third three layered system in the thickness dimension comprises

layer 1: PZT-5A, thickness-1.96mm, width-0.7mm.

layer 2: epoxy bondline, thickness-30um.

layer 3: steel, thickness-2.00mm, width-0.8mm.

The complete structure is 25mm long. The impedance magnitude spectra shown in figure 5.33a were measured in a bath of transformer oil. In common with the previous cases, the impedance characteristic exhibits four main resonances. Namely,

1. The thickness vibration in layer 1.

2. The lateral vibration in layer 1.

3. The first harmonic of resonance '1'.

4. The lateral vibration across all three layers.

The simulated impedance spectra are illustrated in figure 5.33b.

Comparison of the simulated and measured impedance characteristics again highlights the innaccuracy involved when this configuration is simulated. The measured spectra show distinctly different resonant characteristics, in addition to a large number of spurious resonances. In the following section attempts are made to identify the reasons for the poor performance of the model under these conditions, and possible means of improvement are discussed.

5.6.2 (B) DISCUSSION

The results presented in section (A) do not demonstrate good agreement with the theory. A number of fundamental differences exist between the measured and simulated responses. For example,

1. The lateral vibration across all three layers differs in magnitude and shape in each case.

2. The magnitude, and in some cases frequency of the thickness vibration in layer 1 is not accurately predicted.

3. A number of unpredicted resonant features are apparent in each of the measured spectra.

In order to determine the cause of these disagreements it is necessary to examine the geometrical configuration, and consider in detail the physical operation of structures possessing layers in the lateral direction.

Firstly, as a result of adding lateral layers, the configuration ratio of the complete structure may become close to unity. For example, the respective configuration ratios of structures #4, #5 and #6 are 0.786, 0.913 and 0.781. This may cause some interference between the two dimensions, and result in unpredictable behaviour of the device. In a single layered transducer, this type of interference generally occurs when, according to the relative dimensions, the lateral and thickness modes of vibration occur at the same frequency. That is, the transit times for waves of force to cross the thickness and lateral dimensions are approximately equal. In practice, the resonances are not superimposed directly upon one another, instead they disobey the plane wave theory, and the resonant peaks remain slightly apart in frequency. In the case of structures possessing layers in the lateral direction, the properties of the layers will determine to a large extent, at which dimensions the lateral and thickness resonances coincide. It should also be noted that although a value of unity for the configuration ratio is generally considered as the worst case, this depends upon the relative lateral velocities in the layers. For example, in a PZT layer the lateral velocity is less than the thickness velocity, therefore the worst case configuration ratio is actually slightly less than unity. In a multi-layered structure, the lateral velocity of each layer will affect the value of the worst case configuration ratio. In the cases considered here, the bondline will have a pronounced effect on this value, since it possesses a very small lateral acoustic velocity.

This geometrical consideration however is not the only source of error in the simulations, the second effect which may cause the theory to break down is of a significantly more complex nature. In order to illustrate this effect consider the laterally layered structure presented in figure 5.34. The various force functions of interest are illustrated in each layer on the diagram. Assuming layer 1 is piezoelectric and is stimulated by some excitation voltage, then the forces F_{T1} and F_{L1} will be generated immediately in layer 1. The magnitude of F_{T1} will be significantly larger than F_{L1}. Now, according to the two-dimensional lattice theory, F_{L1} is transmitted into layer 2 to produce the force F_{L2} which is mechanically cross-coupled to give F_{T2}. In a similar manner F_{L2} is coupled into layer 3 giving F_{L3} which is which in turn generates F_{T3} via mechanical cross coupling. Due to the relative magnitudes of F_{T1} and F_{L1}, the lateral force which is generated across all three layers should be smaller in magnitude than the thickness force in layer 1. This is in fact the case in the simulations presented in section (A), but does not appear to be experimentally correct.

It is therefore proposed that due to the large magnitude of F_{T1} there is a shear coupling between layers 1 and 2 which may be significantly larger than the longitudinal coupling. For example, when layer 1 expands in the thickness direction, then layer 2 is also forced to expand in this way. The magnitude of this shear coupling will depend upon the relevant material properties of the two adjoining layers. In the cases considered in this investigation, layer 2 comprises a thin epoxy bondline. The thickness of this bond and the material properties of layer 3 will therefore play an important role in determining the extent of this unpredicted mode of mechanical coupling. This is analogous to the effects of bondline thickness and layer 3 acoustic impedance which were demonstrated in section 5.6.1 (B).

The three sets of impedance spectra (#4, #5 and #6) are now examined in the light of this shear coupling theory. The structure which incorporates the thickest bondline, in addition to the lowest acoustic impedance in layer 3 (structure #5), was shown to demonstrate closest agreement with the two-dimensional lattice simulation. Correspondingly, the structure which possessed the thinnest bondline, and the highest acoustic impedance in layer 3 (steel layer), demostrated worst agreement with the simulation. This suggests that the epoxy bondline is not able to efficiently receive the shear coupling from layer 1, and transmit shearwise into layer 3. The epoxy layer would not be expected to efficiently transmit shear vibrations, due to its relatively low mechanical stiffness. On this basis, it is considered that the shear coupling theory offers an adequate explanation of the poor accuracy exhibited by the lattice model for structures possessing layers in the lateral direction.

Finally, it is inevitable that such shear coupling effects would also manifest themselves in the case of layers in the thickness direction. The important difference however, is that the relative magnitudes of the normal and shearing forces at the layer boundaries are reversed. That is, in the case of lateral layers, the shearing force is much larger than the normal force, and the reverse is true for thickness layers. Since the lattice theory only considers interlayer transmission of normal forces, then the model is significantly more accurate for thickness layer. This has been shown in the experimental and simulated impedance spectra.

In order to predict accurately the behaviour of a layered structure in the lateral direction, it is necessary at the present time to employ finite element techniques. However, since the modelling of lateral layered structures is much less important than characterisation of thickness layered structures, this is not considered a significant disadvantage.

5.7 SIMULATION AND MEASUREMENT OF TRANSMISSION AND RECEPTION BEHAVIOUR

The final stage in verifying the accuracy of the new model, consists of simulation and measurement of transducer peformance in transmission and reception. Experimental measurements of transducer force profiles may be effected in a water tank by employing a PVDF membrane hydrophone [60]. In addition, pulse echo responses are obtained both in a water tank and in a block of crown glass. In all cases the transducers are operated under pulsed, transient conditions, and the relevant simulations demonstrate the accuracy of the model. Due to the limitations illustrated in section 5.6.2, structures possessing layers in the lateral direction are not considered. The respective experimental equipment and procedures for both force profile and pulse echo measurements are outlined in appendix B.

5.7.1 FORCE PROFILE SIMULATION AND MEASUREMENT

A selection of measured and simulated force profiles are presented in this section. Three distinct transducer structures are considered and their outputs are measured in a water tank, under conditions of transient excitation. The PVDF membrane hydrophone offers near optimum performance for planar force measurements, however it does suffer from some inherent disadvantages in the present application. For example, the force profiles generated by tall, thin transducers possess a significant degree of curvature in the lateral direction, due to their small width. This results in disruptive radial mode vibrations being induced in the hydrophone membrane. In order to minimise such effects, it is imperative to ensure that the hydrophone and transducer are correctly aligned as shown in appendix B. In addition, the effects of diffraction in the sound field are not simulated, the model however provides fairly accurate theoretical results for the cases considered.

The three transducer structures under consideration may be described in the following manner.

1. Transducer with no layers. The device was cut from a PZT-5A thickness mode slab. It possesses a thickness of 1.96mm, a width of 0.7mm and a length of 50mm

2. Transducer with front face layer of steel. This is the steel layered device whose performance has been assessed in the previous sections.

3. Transducer with front face layer of aluminium. This device has also been previously described.

The measured and simulated results are now presented and discussed for each transducer configuration.

(i) Transducer with no layers

The measured front face output force, transmitted from a single, two-dimensional transducer is presented in figure 5.35a. The hydrophone was placed 250mm from the transducer and was therefore considered to be in the far field region. The hydrophone output, representing the transmitted force profile, was digitised and subjected to a 1024 point fast fourier transform, resulting in the amplitude spectrum shown in figure 5.35b.

The spectral force characteristic possesses one main resonance at 750kHz. The much smaller lateral resonance occurs at 2.15MHz, and the harmonic of the thickness resonance appears to be in the region of 3.3MHz, although definition is poor due to the small amplitude at this frequency. The respective simulations in time and freqeuency are presented in figures 5.36a and 5.36b.

Comparing the modelled and measured spectral characteristics, it should be noticed that the thickness harmonic component in the simulation is substantially larger than in the measured case. This is a result of experimental error, due to the difficulties in measuring non-planar force profiles with the membrane hydrophone. Also, frequency dependant attenuation in the water column between the transducer and the hydrophone introduces some error. The measured lateral mode vibration at 2.15 MHz is thought to be enhanced by edge effects which are mechanically cross coupled from the lateral direction. These effects are not predicted by the model. An additional factor which was not considered in the simulation is the influence of the insulating layer applied to the rear positive electrode. This layer which is applied by aerosol spray, to avoid conduction into the undistilled water in the tank, is assumed to be very thin. Furthermore, it is unlikely to possess a uniform thickness, and its acoustic properties are unknown. For these reasons it was omitted from the simulation.

The important factors to note in a comparison of theory and experiment are the frequency of operation and the effective decay time of the transmitted pulse. In this case, both of these demonstrate close agreement. Finally, bearing in mind the rather crude experimental arrangement, it is considered that the results are very good.

(ii) Transducer with front face layer of steel

The spectral impedance characteristics for this device were presented in section 5.6.1, and showed two main resonances at approximately 500kHz and 1200kHz. The transmitted force profile is given in figure 5.37a, and the corresponding spectral representation is shown in figure 5.37b. The two

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resonances are again evident at these frequencies. The two-dimensional simulations of the transducer force profile in the time and frequency domain are presented in figures 5.38a and 5.38b respectively.

Comparison of the measured and simulated frequency spectra demonstrate excellent agreement both in terms of the frequency and relative magnitude of the two main resonances. The higher frequency lateral and harmonic resonances are obscured in the measured response due to frequency dependant attenuation.

(iii) Transducer with front face layer of aluminium

This is thickness layered structure #2, whose impedance characteristics are presented in figure 5.24. Measured in the far field, the force profile for this device is illustrated in figure 5.39a. The spectral behaviour of the transmitted force is described in figure 5.39b. Two main resonances are again evident, although their relative magnitudes are now reversed, due to the difference in the bondline thicknesses. The respective time and frequency domain simulations are presented in figure 5.40a and 5.40b.

The measured and simulated results for this case do not demonstrate such excellent agreement. The positions of the two main resonant peaks are correctly predicted, however the general shape of the measured profile is not clearly emulated in the simulation. The component at 2.15MHz, corresponding to the transducer lateral resonant frequency, is significantly larger than predicted. Furthermore, there is an additional small amplitude resonance at 1MHz which is not apparent in the simulation. The first of these effects is beleived to be caused by the lateral resonance in the transducer, as outlined above for the single device. The second however may not be explained in the light of the present theory, and hence no explanation is attempted. As before, frequency dependant attenuation in the water column obscures the spectral definition at higher frequencies.

To summarise, the force profile measurements agree with the simulations as far as the main resonant characteristics are concerned. The subsidiary resonances, such as the lateral and the thickness harmonic are less well predicted, and in some cases, other small amplitude spurious resonances occur in the measured force spectrum. The following section section examines pulse echo responses for the three structures considered.

5.7.2 PULSE-ECHO SIMULATION AND MEASUREMENT

A selection of measured and simulated transmit receive responses are presented in this section. The first three cases correspond to the receiver response when the previously described force profiles are reflected back to the transducer from a crown glass block. The remainder comprise pulse echo responses for transducers operating directly into the glass block, via a thin couplant layer. The relevant experimental procedures are given in appendix B.

(i) Pulse-echo response #1

The experimental receiver voltage response to the force profile generated by the single transducer is shown in figure 5.41a. The spectral representation is shown in figure 5.41b, and the relevant time and frequency domain simulations are given in figures 5.42a and 5.42b respectively.

A comparison of the measured and simulated results indicates improved agreement when compared to the transducer force profile theory and experiment. The overall shape of the resonant peak in the frequency domain is more accurate than in the previous case, and the correlation between the simulated and measured time domain receiver voltages is excellent. The improved simulation accuracy is beleived to be partly the result of integration in the receiver, which has the effect of smoothing the response and removing some of the unpredicted edge wave and diffraction effects. However, the principal reason for the apparent improvement may be attributed to the expermental difficulty in measuring the force output.

(ii) Pulse-echo response #2

The steel layered device was employed in a transmit receive mode to provide the receiver voltage given in figure 5.43a, and spectrally in figure 5.43b. In order to obtain these responses, the force profile presented in figure 5.37 was reflected from a glass block and subsequently received by the transducer. The respective time and frequency domain simulations are given in figure 5.44a and figure 5.44b.

The main resonance around 500kHz is reproduced to a high degree of accuracy in the simulation, both in terms of resonant frequency and spectral shape. The subsidiary transducer response at 1200 kHz apears to be corrupted in the measured spectrum. This is possibly a result of slight misalignment between the transducer and the reflecting surface. The simulated and measured time domain waveforms exhibit reasonably good agreement, although the phase changes in the time domain waveform are again slightly erroneous.

(iii) Pulse-echo response #3

In this final water tank experiment, the transducer with the aluminium layer is operated in a pulse-echo mode, reflecting from the glass block. The receiver voltage waveform is given in figure 5.45a, and the spectral characteristics are shown in figure 5.45b. The simulated waveform, and its related frequency response are illustrated in figure 5.46a and figure 5.46b.

The two main resonant peaks are correctly predicted by the model, although the simulated harmonic response at 3Mhz is nonexistent in the experimental measurement, due mainly to error in the placement of the glass block. The measured components at 1MHz and 2.15MHz, which were clearly evident in the force profile, have been attenuated in the receiver response, presumably a result of integration in the receiver. The time domain waveforms do not agree exactly, although their envelope shapes are similar, and the existence of the two main frequency components may be clearly identified.

(iv) Pulse-echo response #4

In this case the single transducer is mounted directly onto the glass block, via a thin couplant layer of natural bee honey. This choice of couplant offers an excellent combination of good adhesive and coupling qualities. Using this technique, the active element may be placed on the block without the need for any mounting apparatus. The full set up is outlined in appendix B.

The measured time domain voltage waveform is presented in figure 5.47a, while the frequency spectrum is shown in figure 5.47b. The simulated time and frequency domain receiver voltage responses are illustrated in figure 5.48a and 5.48b. The couplant layer was simulated as a thickness mode layer with an estimated thickness of 50um, and the acoustic properties of natural honey are presented in appendix B.

Comparison of the measured and simulated responses reveals excellent agreement between theory and experiment. The frequency of resonance is correct, the shape of the spectum is correct and the time domain waveforms exhibit very close agreement. It should be noted that the higher frequency harmonics are effectively removed from the response due to the relatively large amount of mechanical damping due to the acoustic impedance of the crown glass. (15 MRayls)

(v) Pulse-echo response #5

The steel layered transducer was placed on the glass block and the resulting pulse-echo response is given in figure 5.49a and figure 5.49b. The two frequency components may be clearly identified, and are reproduced in the relevant simulations, which are given in figure 5.50a and figure 5.50b.

There is a slight discrepancy in the shape of the predicted transducer response at 1.2MHz. However the positions of the resonances are fairly accurate, and the time domain waveforms are similar. The phase changes in the measured and simulated responses do not coincide, as with the transmit and pulse-echo responses in water.

(vi) Pulse echo response #6

In this experiment the transducer structure incorporating the aluminium layer is placed on the glass block. The resulting transmit receive waveform is illustrated in figure 5.51a, and its frequency domain equivilent is shown in figure 5.51b.

It is very interesting to note that the measured frequency spectrum now exhibits only one resonant peak, around 800kHz, instead of the two resonances which were apparent in the water tank experiments. This may be attributed to the similarity in impedance between the aluminium layer and the crown glass loading medium. The acoustic impedance for crown glass is 15 MRayls, while that of aluminium is 17 MRayls. The transducer (layer 1) does not recognise the aluminium (layer 3) as a finite layer, instead it appears to be an integral part of the glass block, and is effectively transparent to the transducer. This arguement may be reinforced by comparing the pulse echo response into glass via the aluminium layer, with the pulse echo response of the single transducer operating directly into glass (case 4 above). The temporal and spectral characteristics exhibit a significant degree of similarity.

Simulation of the pulse echo response corresponding to the transducer with aluminium layer, operating into a crown glass load, also confirms the transparent layer assumption. The modelled time and frequency domain pulse echo characteristics are given in figure 5.52a and figure 5.52b respectively, illustrating a single resonance, similar to that of the single transducer operating directly into glass.

The experimentally measured pulse echo responses agree in general with the two-dimensional simulations for the various cases considered. The accuracy is improved when compared with the force profile simulations and measurements, due mainly to the integrating effects of the receiver. Furthermore, the higher frequency harmonics are less prominent than in the force profile measurements, again improving the accuracy of the simulation in most cases.

5.8 LATERAL DIRECTIVITY

The angular directivity associated with tall, thin transducers is an important consideration in the design of phased arrays. In order to electronically focus and steer the acoustic beam from an array, the transducing elements must possess a wide lateral beamwidth. The theoretical determination of transducer beamshapes has been investigated fairly extensively in the literature. (see for example [2][43][51][56])

Expressed in polar coordinates, a convenient relationship describing the angular directivity of an array transducer under CW operation is given by the following; [57]

$$P(r,\theta) = \frac{P_o w}{j(\lambda))^{0.5}} e^{2j\pi r/\lambda} \frac{\sin(\pi w/\lambda \sin\theta)}{\pi w/\lambda \sin\theta} \cos\theta$$

$P(r,\theta)$ is the acoustic pressure measured at a distance r from the transducer, and at an angle θ from the normal to the radiating surface. W is the width of the transducer, and λ is the wavelength of the acoustic wave in the propagating medium. This expression exhibits a maximum at $\theta=0°$, and becomes equal to zero at $\theta=90°$. The widest 3dB beamwidths are obtained when the transducer lateral dimension is small compared to the acoustic wavelength λ.

As a typical example, consider the measured and theoretical directivity functions given in figure 5.53. These correspond to the angular directivity of a PZT-5A transducer with a configuration ratio of 0.5, operating into steel. Although the theory is strictly valid for CW, it may be seen that adequate agreement is possible under conditions of transient excitation. The experimental set-up used to obtain the measured response is illustrated in appendix B.

The beam plot presented in figure 5.53 represents the ideal for such a transducer element, since there are no interelement coupling effects or matching layers, which may have a detrimental effect on the transducer beamshape. Figure 5.53 may thus be used as a benchmark in the design of the arrays presented in the following chapter.

5.9 CONCLUDING SUMMARY

Experimental verification of the two-dimensional model has been extensively illustrated in this chapter, demonstrating excellent agreement between theory and experiment under a variety of differing mechanical and electrical conditions.

Firstly, the model was shown to be valid for an unlayered device over a wide range of configuration ratios. In addition, the effect of acoustic loading on different dimensions was demonstrated both theoretically and experimentally. The new model was also shown to compare favourably with the alternative modified thickness mode approach.

Secondly, a selection of practical transmitter firing circuits were employed to excite the devices. This involved modelling of both the transducers and the associated electrical loading conditions. The simulations were shown to be very accurate when compared to the measured voltages.

Thirdly, the behaviour of multi-layered structures was illustrated and simulated using the lattice techniques presented in chapter IV. It was shown that the technique works well for layers in the thickness direction, however agreement was not so good for the laterally layered devices. This is thought to be the result of differences in magnitude between the shearing and normal forces at the layer interfaces. In case of lateral layers, the unpredicted shearing forces dominate, hence impairing the accuracy of the technique for this case.

Fourthly, the operation of two-dimensional structures in transmission and reception was shown to be predictable, although slightly less accurately than the impedance and voltage behaviour. This slight reduction in accuracy is mainly a result of experimental measurement error due to mis-alignment of the transducer and hydrophone/reflector. Other sources of error include diffraction effects, and attenuation in the propagating medium.

Finally, a representative directivity plot was presented for a single transducer operating into steel. Thus, the complete operation of a two-dimensional structure has been accurately characterised, and the modelling technique may be used in the following chapter to design and evaluate the behaviour of two phased arrays which differ in mechanical construction.

CHAPTER VI

A PHASED ARRAY DESIGN STUDY

6.1 INTRODUCTION

In this chapter, the two-dimensional systems model is employed as a fundamental design tool, for the design of two phased arrays. A specialised technique has been developed which attempts to improve reliability and repeatability in the manufacturing stage. This is comprehensively explained, and details of the construction, test and evaluation of two different array designs are presented. The practical behaviour of the individual elements in the arrays is assessed with respect to theoretical simulations, and the effects of constructional variations are discussed. Furthermore, electronic hardware has been developed to drive the arrays and steer the acoustic beam. The design of the hardware is outlined, and experimental beam profile measurements are presented for each array at various angles of steer.

6.2 SPECIFICATION OF THE ARRAY PERFORMANCE

It was proposed to design and manufacture a phased array which would fundamentally operate into any material whose acoustic velocity is greater than that of cast araldite epoxy. This permits direct operation into a variety of solid materials such as steel, aluminium or glass. In addition, evaluation of single element responses could be carried out in a water tank, thus permitting characterisation by force profile measurement. The two arrays considered in the investigation differ only in terms of the mechanical configuration of the front faceplate. The fabrication process is therefore identical until the front face layer is constructed.

In order to use commercially available transducer materials, the array elements were diced from a PZT-5A thickness mode slab, with an electrical resonant frequency of 2.0MHz. Consequently, according to simulation, the electrical resonance in the thickness direction of the resultant two-dimensional transducer with configuration ratio of 0.5, is approximately 1.5MHz. In order to limit the hardware requirements for array control, the arrays contain only 16 elements.

The acoustic velocity in cast epoxy may be as low as 2200m/s, corresponding to a wavelength of 1.44mm at 1.5MHz. An additional consideration at this stage, is the configuration ratio of the array elements. This should preferably be limited to a maximum of 0.5. Since the thickness of a 2MHz thickness mode PZT-5A transducer is approximately 1mm, the width dimension should therefore be less than or equal to 0.5mm. Furthermore, since the array elements are defined by cutting with a diamond wire saw, it is important to bear in mind the diameters of wire which are available. These are 170um, 220um and 330um.

In consideration of these factors, it was decided to use a centre to centre spacing of 0.7mm, corresponding to approximately half-wavelength spacing for a velocity of 2200m/s at 1.5MHz. Such element spacing is necessary to avoid grating lobes. The elements are diced using the 220um diamond wire, resulting in a transducer width of 0.48mm, as illustrated in figure 6.1.

The array was to be backed by a material of fairly low acoustic impedance and high acoustic loss. The reasons for this are straightforward. Firstly, the light mechanical damping at the rear face permits efficient transfer of acoustic energy into the load medium. Secondly, in order to minimise reflections from the rear face of the backing material, it should possess a high acoustic loss factor. The ring time of the generated pulse was not of considerable importance in the present design.

The gaps between the elements should be filled with a highly lossy material, in order to minimise mechanical cross coupling. Additionally, where a protective faceplate is included on the front of the array, this should be as thin as possible to minimise mechanical cross coupling.

6.3 ELECTRICAL CONNECTION TO THE TRANSDUCERS

As stated in the previous section, the transducer used in the fabrication of the arrays, was a 2MHz (electrical resonance) thickness mode slab, which was diced into elements with a configuration ratio of 0.48. The problem of making electrical contact to each transducer is now addressed. It would be advantageous to avoid having to make connection to the surface of the device, since this influences transmission characteristics, and also limits the minimum possible thickness of the faceplate. Similarly, there is difficulty in providing electrical contact to the rear electrode since this must be firmly attached to the backing material. It must also be emphasised that the electrical connection must be reliable, especially since the electrode is less than half a millimetre wide.

In view of these limitations, it was proposed to use a PZT-5A thickness mode slab with each electrode wrapped over opposite edges, as illustrated in figure 6.2. This allows electrical connection to be made at opposite ends of the resulting array transducers, hence eliminating any interference with the front or rear surfaces. The piezoelectric slab could then be mounted upon the backing block, and a suitable printed circuit board used for electrical connection along each edge of the slab. The printed circuit board also possesses a wrapped over electrode, as illustrated in figure 6.3. The printed circuit board was aligned as shown in figure 6.4, and a conducting joint inserted along the complete length of the array. The joint material comprises conducting, silver loaded epoxy, which has the advantage of contributing to the mechanical strength of the structure as well as providing electrical contact. When the array was diced, the cut was continued into the backing block, and between the tracks of the printed circuit boards. Each element then became electrically isolated from its neighbour, and connection was made to the array by means of the printed circuit board.

It must however be ensured that the addition of this wrap over electrode at each end of the transducer element does not affect the performance. Figures 6.5a and 6.5b illustrate the measured electrical impedance magnitude spectrum for two such PZT-5A transducer elements possessing a width of 0.48mm, a thickness of 0.99mm, and a length of 20mm. However, the transducer used to provide figure 6.5a has the conducting surface removed from each end. There are no significant differences between the two cases, apart from a slight shift in the lateral resonant frequency which is probably due to a small variation in transducer width. It is considered therefore that the transducer with the wrapped over electrode essentially behaves in an identical manner to the standard two-dimensional device.

The technique of using wrapped over electrodes on both the transducer and the printed circuit board, thus provides a reliable and repeatable method of making the neccesary electrical connections, to each individual array element.

6.4 THE MECHANICAL STRUCTURE OF THE ARRAY

As previously mentioned, the backing material should possess a fairly low acoustic impedance and a high loss factor. It was found that a loaded epoxy mix could offer such properties, but that these properties were highly dependant upon a number of variables, some of which are difficult to reproduce from batch to batch. For example, the acoustic impedance, velocity and loss factor depend upon some or all of the following.

1. The amount of filler used to load the epoxy.

2. The properties of the filler used to load the epoxy.

3. The mixing schedule

4. The out-gassing schedule

5. The curing schedule

In order to alleviate the problem of having to reproduce a particular mix of precise specifications, it was proposed to use a standard commercially available epoxy/filler mix. After testing a number of these products, an aluminium loaded epoxy (F2) manufactured by Devcon [11] was selected. When this compound is cured at

30°C for 24 hrs at atmospheric pressure, the acoustic impedance is 3-4 Mrayls, and the acoustic velocity is 2400m/s. The acoustic loss is difficult to quantify, but is sufficiently large so as not to produce a measurable back wall reflection from a 20mm thick backing block under normal operating conditions. Another advantage of using an epoxy mix for the backing is that it may be poured onto the transducer in a mould. This eliminates any detrimental bondline effects and offers improved adhesion, which is vital during the cutting process. Figure 6.6 illustrates the casting technique whereby the thickness mode slab was held in the perspex mould as shown. The loaded epoxy mix was poured into the mould to a depth of 20-30mm. The transducer was clamped tightly along both of the electroded edges, hence preventing any epoxy from impingeing on the side electrodes. When the epoxy was fully cured, the retaining screws were removed, and the assembly immersed in a bath of boiling water. The thermal expansion coefficient of pespex is significantly larger than that of the aluminium loaded epoxy. Consequently, the perspex expanded and detached itself from the block, and the uncut thickness mode transducer was securely mounted on the backing block. This is shown in figure 6.7. The rear face of the block was machined for parallel alignment with the front face.

The next stage in the process was to attach the printed circuit board to the backing block, as discussed previously in section 6.3. This may be performed using a general purpose fast setting cement. Care was taken to ensure correct alignment of the circuit boards on each side of the block. A conducting epoxy joint was then applied along the conducting edge of the board and the transducer. Conducting epoxies for this purpose are commercially available, and in general they must be cured at an elevated temperature of 60-80°C. The mounted array assembly is shown in figure 6.8.

6.5 CUTTING AND HOUSING THE ARRAYS

Until this stage, the manufacture of the two arrays has been identical. In this section, two seperate procedures are described for cutting and housing the arrays. The first array possesses a monolithic faceplate, whereas in the second case, the faceplate is quantised along the length of the array. The relative merits/demerits with respect to performance of each construction are described in subsequent sections.

6.5.1 ARRAY #1 (Monolithic Faceplate)

The cutting and housing process for this array may be conveneintly described by the following list of operations.

1. Having fabricated the array block as described in sections 6.2 - 6.4, the array was diced using the 220um diamond wire saw. The cut was continued into

the backing block, and printed circuit board to a depth of 2 - 3mm, to permit electrical isolation of the tracks on the circuit board, and mechanical isolation in the block. In this case 17 cuts were made, defining 16 transducer elements in the centre of the PZT-5A slab. This left 6.55mm of ceramic at each end of the 25mm long slab as shown in figure 6.9.

2. The electrical wiring connections were made to the printed circuit board, and the wired, diced array then placed in an aluminium housing as shown in figure 6.10. Care was taken at this stage to ensure that the front face of the array was parallel to the front of the array housing. This may be done using an appropriate depth gauge. A small amount of fast curing cement was then used to hold the block in place.

A loaded epoxy mix was produced consisting of araldite MY753 resin and HY956 hardener, mixed with a quantity of glass 'microballoons'. Microballoons are hollow silica spheres with an average diameter of 150μm, they may be used in epoxy to produce an acoustically lossy filler material. This mixture was poured into the array housing up to the level of the front of the ceramic, and allowed to cure. This process results in the epoxy flowing into the sawcuts, carrying with it the floating microballoons, and hence providing a lossy material between the elements of the array. In practice, since the gap between the elements is narrow, the microballoons may become jammed at each end of a cut, which can result in an interelement filler which consists of mainly pure epoxy.

3. When the epoxy/microballoon mixture was fully cured, the front facelate was fabricated. Using a very low viscosity optically clear epoxy (Araldite CY1301+HY1300), a front face layer of 4-5mm was poured onto the array. The assembly was then placed in a vacuum station to outgas. Due to the low viscosity of the epoxy, it was possible to remove all of the air from the front face layer.

4. The final stage in the fabrication is the machining and polishing of the front face layer. With the array housing placed square on a milling machine, the front of the array is automatically aligned with the cutting tool. (since the front of the ceramic is already parallel to the front of the housing).

The front face was machined until the epoxy layer was 1mm thick. The array was then finished by hand on a surface table using successively finer grades of abrasive paper. A final thickness of 366um is desirable for the front face layer as this correspond to a quarter wave for an acoustic velocity of 2200m/s at 1.5MHz. Array #1 was then ready for test and evaluation.

6.5.2 ARRAY #2 (Quantised Faceplate)

In this case each array element possesses a front face layer of epoxy which is seperated from the neighbouring elements by lossy filler material. The fabrication procedure may be described as follows.

1. In order to quantise the front face layer, it is necessary to cast the layer before the array is diced. This was done by forming a convenient receptacle on the front of the block with plastic sticking tape. The tape was be lightly clamped at each side to prevent epoxy from running onto the circuit board, as this results in difficulties when electrical connection is to be made. Using the low viscosity clear epoxy, the front face layer was then poured to a depth of 4-5mm and subsequently outgassed before curing. The array was then ready for cutting as outlined in the previous section. However, in this case, 33 cuts were made and hence the complete 25mm slab was diced. This was done to compare with the first array, which effectively has two 6.55mm wide thickness mode transducers at each end of the array. The 16 elements of interest in the second

array are those positioned in the middle.

2. The diced array assembly was then wired and properly aligned in the aluminium housing. The housing was filled with aluminium loaded epoxy, ensuring that the saw cuts were completely filled.

   After the potting process the array housing was mounted squarely on the milling machine, and the front face was milled until the aluminium loaded epoxy was removed, and the elements may be viewed through the front face layer. This layer was then polished to a thickness of approximately 400µm. The front face of the array in this case consists of alternate bands of clear epoxy (above the elements), and aluminium loaded epoxy (in the gaps).

A comparison of the mechanical structure of the two arrays is given in figures 6.11a and 6.11b, which represent cross sectional views along the length of each array. The two finished arrays were now ready for test and evaluation.

6.6 THE ELECTRONIC HARDWARE

In order to evaluate the behaviour of the two arrays, it was necessary to design and construct an electronic array controller. This consists of 16 channels whose firing sequence may be temporally phased in order to steer the acoustic beam through a range of angles. It should be noted that this equipment offers electronic control in transmission only.

The steering angle θ, of a phased array is given by the relationship,

$$\theta = \sin^{-1} (vt/d)$$

t is the time delay between firing each element, v is the acoustic velocity in the propagating channel, and d is the interelement spacing. It is apparent from the relationship that for large steering angles, t is large, and for small angles t is small. For example, operating into perspex, if $\theta=0°$ then t=13.7ns, and when $\theta=60°$ then t=227ns. If it is assumed that the minimum phase delay is determined by the length of one clock period, then a delay of 13.7ns correesponds to a frequency of 73MHz. In order to operate at angles smaller than 3°, or into materials with a higher acoustic velocity (e.g. steel), then a higher frequency clock is required. This consideration clearly suggests the use of Emitter Coupled Logic (ECL) technology for the timing circuitry.

The question arises of how to vary the clock period to steer at different angles. It was decided to employ the technique of using a voltage controlled oscillator (VCO) to generate a continuously variable clock frequency. This was preferred to dividing a high frequency clock since it permits a continuous scan throughout the angular range.

Sixteen equally delayed firing pulses are generated by decoding a four stage binary counter, which is driven by the master clock. These are applied to the array transducers via the firing circuit given in figure 5.13. The VMOS transistors are available as an array of four devices on one dual-in-line package. [61]. This was advantageous in terms of cost and space savings. Finally, the facility to steer at θ° (no delay) was also included. A block diagram of the array control hardware is presented in figure 6.12.

The main features of the array controller may be summarised in the following manner.

1. 16 channel fast ECL operation.

2. Safe 25V firing voltage

3. Continuously variable delay times from 13ns to 250ns.

4. Relatively straightforward, low cost design.

6.7 EVALUATION OF ARRAY PERFORMANCE

Practical performance of the two arrays is assessed in this section with respect to the following operational characteristics.

1. Transmission and reception behaviour.

2. Single element and steered beam profiles.

3. Element uniformity.

These characteristics are measured and compared with simulations, and the main features of interest are subsequently discussed.

6.7.1 TRANSMISSION AND RECEPTION BEHAVIOUR

The transmission and reception behaviour of a single element in each of the arrays is assessed. Element 9 is used in each case to provide the following results.

1. Force output measurements in water

Force profile measurements were performed in a water tank according to the procedure outlined in appendix B. The measured force output from array #1 is shown in figure 6.13a. The corresponding spectral representation is given in figure 6.13b. Figures 6.14a and 6.14b show the time and frequency domain representations of the force output from array #2.

The time domain response of the arrays is similar, although array #2 exhibits a marginally shorter ring time. The resonant spectral peak in each case is centred around 1.5MHz, and the general spectral shape also demonstrates good agreement. According to the discontinuities on the main resonance it is anticipated that the front face layer of array #2, is thicker than that of array #1. In the specification, the front face layer was to be 366μm thick, however due to the limitations in the fabrication technique, it was considered that this may lie anywhere between 350μm and 450μm. Simulated time and frequency domain force profile responses are presented in figure 6.15a, 6.15b and 6.15c, for front face layer thicknesses of 350μm, 400μm and 450μm respectively.

A comparison of simulation and experiment indicates that array #1 has a front face layer which is close to 400μm thick. However, the front face layer on array #2 has a thickness which is estimated to be nearer 450μm. According to the magnitude of the measured force profiles, array #1 is more efficient than array #2. This point is discussed more fully in section 6.7.4.

2. Pulse-echo measurements in water

The pulse-echo measurement procedure which is detailed in appendix B, was used to measure the transmit/receive characteristics of each array. The results are presented in figure 6.16 and figure 6.17, corresponding to array #1 and array #2 respectively.

The received voltage waveforms are similar in shape, and the double peak caused by the front face layer is clearly evident in each case. Simulations corresponding to respective layer thicknesses of 350, 400 and 450μm are presented in figures 6.18a, 6.18b and 6.18c. Although the double peaks are less prominent in the simulations, the shape of the resonances indicate the relative magnitude of the two components. Again it is confirmed that array #1 has a slightly thinner front face layer than array #2.

3. pulse-echo measurements in glass

Pulse-echo operation of the arrays into a glass block is illustrated in figures 6.19 and 6.20. The honey couplant layer is estimated to be 150μm thick, and the experimental technique is detailed in appendix B.

Under such mechanical loading conditions the effects of the front face layer are less prominent, and in each case the double peaking effect in the spectrum is removed. In addition, the peak output has shifted upwards in frequency to 1.75MHz.

Comparison of the two array responses reveals a high degree of correlation. The pulse envelope shape, centre frequency and spectral shape are all very similar. This configuration may be readily simulated, and the relevant results are shown in figure 6.21. Both time and frequency domain behaviour is fairly accurately reproduced in the simulation.

6.7.2 DIRECTIVITY AND STEERED-BEAM PROFILE MEASUREMENTS

The directional behaviour of a phased array is heavily dependant on the directivity of its constitutive transducers. It is therefore desirable, for each individual transducing element to possess a wide angular directivity. An ideal directivity function for a typical phased array transducer was shown in figure 5.53. This ideal behaviour is however, never attained in the practical array environment due to mechanical and electrical cross coupling effects, which tend to increase the effective radiating aperture.

The directivity of element 9 in each array was measured using the beam profile set-up outlined in appendix B. The results are shown in figure 6.22a and 6.22b, corresponding to array #1 and array #2 respectively. Array #1 possesses a single element -3dB directivity of 30° and a -10dB directivity of 100° as shown in figure 6.22a. Similarly, array #2 has a single element -3dB directivity of 29° and a -10dB directivity of 99°.

The single element directivity is each case is thus severely reduced as a result of inter-element mechanical and electrical cross-coupling. The electrical cross-coupling has been measured and it was found that the instantaneous cross-coupled voltages appearing on elements 8 and 10 were 10 times smaller than the voltage applied to element 9. The voltage induced on the remaining elements was more than 15 times smaller. It is interesting to note that the voltage induced on the majority of the non-adjacent elements was constant. This would indicate that a significant amount of electrical cross-coupling takes place in the wiring harness, as well as in the array assembly. The use of individually screened cables in the wiring may help to reduce such effects.

The single firing circuit is now replaced by the array control hardware, and steered beam directivity functions are measured using the technique presented in appendix B. The arrays are firstly steered at 0°, that is, no delay between firing each element. The associated beam profiles for this case are given in figures 6.23a and 6.23b. Array #1 possesses a -3dB directivity of ± 9° or 18°, array #2 also exhibits a -3dB beamwidth of 18°. The 10° to 60° directivity plots for both arrays are presented in figure 6.24 and 6.25 repectively, and the relevant beamwidths are given in table 6.1.

An additional factor to be considered when evaluating the steered-beam characteristics is the strength of the beam at various angles of steer. This quantity is clearly related to the beam profile of the single transducers. Figure 6.27 illustrates the relative strength of the beam at angles between 0° and 60°. The shape of the curve is similar to the single element directivity plots given in figure 6.22.

6.7.3 COMPARISON OF ELEMENT UNIFORMITY

In order to fully characterise the arrays, the electrical resonant frequency, relative efficiency and electrical impedance at resonance are experimentally evaluated. Resonant frequency and electrical impedance are measured using the vector impedance meter, while relative efficiency is measured with the arrays operating into water. Table 6.2 illustrates these three quantities for each array. It should be noted that the relative efficiency is expressed as a percentage of the peak measured force output from element 9.

The results illustrate a very high degree of uniformity in resonant frequency. The variation in electrical impedance at resonance is also within reason. It should be noted that element 1 of array #2 is not included due to a faulty connection either at the transducer-circuit board conducting epoxy joint, or in the array. The effect of element 1 is ignored in the calculation of the average and percentage variation figures.

The variations in relative efficiency are unexpectedly large, and the possible reasons for this undesirable behaviour are discussed in the following section.

6.7.4 DISCUSSION

The operation of each array has now been comprehensively characterised. The transmission and reception characteristics of element 9 in each array, have shown good agreement with the predicted results. Furthermore, apart from magnitude variations, the behaviour of each element in the arrays is very similar. However it was found that array #1 was more efficient overall than array #2. There are a number of possible reasons for this variation.

Firstly, if the front face layer possesses a thickness equal to exactly one quarter wavelength, then the device will operate more efficiently than a transducer with a front face layer which is greater than or less than $\lambda/4$.

It is considered that array #1 possesses a front face layer which is closer to one quarter wave in thickness than array #2.

Secondly the electromechanical efficiency of the piezoceramic slab used in the fabrication of array #1 is 18% larger than that used to fabricate array #2. Although they should be identical, such a large variation of transducer properties in a single batch of ceramic is common.

Finally, comparing the mechanical structure of the front face of each array, the effective radiating area of array #1 should be larger than that of array #2. This would certainly increase the apparent efficiency of array #1, however, any significant change in effective aperture would also manifest itself in the directivity measurements. Since the directivities of the two arrays are very similar, this effect is not considered to make a major contribution.

It was expected that array #2 would exhibit a narrower effective aperture, and hence wider element beamwidth, than array #1, due to the cutting and filling between the active elements. However, as indicated above, the directivity measurements show that this is not the case. Both the single element and steered beam directivities in fact demonstrate very close agreement when the two arrays are compared.

The single element directivities in array #2 are poor compared to the ideal response, which indicates that the anticipated degree of mechanical isolation is not achieved with the modified cutting and filling process.

The steered-beam directivities illustrate the usefulness of the phased array as a variable angle transmitter of ultrasound. The deterioration in beamwidth and magnitude as a function of steering angle should however be noted. For imaging applications, a narrower beam is often desirable, suggesting the use of an increased number of active elements.

The comparison of element characteristics reveals varying degrees of uniformity for quantities such as electrical resonant frequency, operational impedance and element efficiency. In each array, the electrical resonant frequencies of the elements are extremely consistent. The impedances however demonstrate a greater spread. This may be attributed to slight variations in the width dimension of the elements. Equipment was not available to measure accurately the transducer widths, but it is estimated that the width of the elements may vary by up to as much as 10%.

Finally, the variations in efficiency between elements occurs mainly as the result of slight variations in transducer width, although non-uniformities in the front face layer, and variations in transducer properties such as loss may also affect the efficiency, as well as the impedance. It is interesting to note the minor trend of increased efficiency in the centre of the array. This may be a result of electrical cross coupling, causing all the other elements to radiate in phase at a reduced amplitude.

6.8 SUGGESTIONS FOR IMPROVING THE ARRAY DESIGN

The foregoing evaluation of array behaviour, has identified the principal areas which may be investigated in order to optimise or improve array performance. These consist of the following main considerations.

1. Inter-element electrical cross coupling could be reduced by individually screening the wiring connections to each element. In addition, it may be useful to examine methods of reducing any stray capacitance on the printed circuit board, this would increase overall efficiency and help to reduce electrical cross coupling.

2. Inter-element mechanical cross coupling would be minimised by the use of an air gap between the elements. This however may be impractical from a structural point of view. Investigation of alternative high loss inter-element filler materials is strongly recomended.

3. More accurately calibrated castings and housings would be useful in order to improve the accuracy with which the front face layer thickness may be predicted.

6.9 CONCLUDING REMARKS

In this chapter, the two-dimensional linear systems model has been used to design two prototype arrays. The arrays were intended to operate at 1.5MHz using custom made side-electroded transducers. The complete fabrication process has been described in each case. In addition, the design of a sixteen channel array controller was described, and this was used to produce a steered acoustic beam in a steel test block. Comprehensive evaluative measurements were presented and where appropriate compared with the relevant simulations. Finally, in the light of the measured characteristics, a number of design improvements are suggested.

CHAPTER VII

CONCLUSIONS AND SUGGESTIONS FOR FURTHER INVESTIGATION

7.1 CONCLUDING SUMMARY

A new transducer model has been developed for the simulation of piezoelectric structures which possess two principal modes of vibration. The model, based on linear systems theory, permits enhanced understanding of two-dimensional transduction, through the application of intuitively satisfying block diagram concepts. Secondary and tertiary piezoelectric action and inter-dimensional mechanical and piezoelectric electrical cross coupling have been expressed in terms of feedback and feedforward loops. Consequently, their impact on device operation may be analysed in a straightforward fashion.

The theoretical technique was enhanced in chapter IV to permit evaluation of structures which incorporate multiple layers extending in either of the two principal dimensions. The layered model is formulated in a lattice type representation which may be implemented readily in software. However, in the case of multiple layers, the physical understanding which was apparent in the single transducer case is somewhat impaired.

The major advantages of the new two-dimensional modelling technique may be conveniently summarised as follows.

1. The model encourages an improved understanding of two-dimensional piezoelectric behaviour. The effects of secondary and tertiary piezoelectric charge

generation in both dimensions are clearly illustrated, and the influence of mechanical and piezoelectric cross-coupling may be identified readily.

2. The model is valid over a wide range of operating frequencies, and as such may be applied to both CW and transient modes of operation.

3. The block diagram representation utilises only physically realisable quantities, both within the blocks and on the connecting paths.

4. The model permits straightforward evaluation of arbitrary electrical and mechanical loading configurations.

5. Lattice techniques may be applied in order to simulate the behaviour of multilayered structures

6. The model, with its roots in linear systems theory is particularly amenable to computer implementation.

In the course of this investigation, extensive experimental measurement has illustrated the accuracy of the method for the great majority of practical operating conditions. Finally, in chapter VI, the technique was successfully emloyed in the design of two prototype arrays.

In conclusion, it is considered that the work presented in this thesis represents a significant contribution to the understanding of two-dimensional transducer behaviour. Furthermore, it provides a comprehensive theoretical and experimental basis for a number of areas of further investigation. These are outlined in the following section.

7.2 SUGGESTIONS FOR FURTHER INVESTIGATION

Throughout the course of the development and verification of the two-dimensional linear systems model, a number of areas have been identified which merit further research. The following sections present a brief summary of the more significant improvements which may be implemented.

7.2.1 IMPROVEMENTS IN THE LATERAL LAYER CONFIGURATION

The measured and simulated results presented in chapter V illustrate the limited accuracy of the model for certain configurations of layers in the lateral direction. It would be worthwhile to perform an in depth experimental evaluation of such configurations, in order to determine the limiting factors which influence the accuracy of the present technique. Alternatively, a fresh theoretical analysis of lateral layered structures, including the possibility of shearwise coupling between neighbouring elements, merits rigorous pursuit.

It may however transpire that the analysis is not amenable to implementation using linear systems theory, or indeed that a closed form solution may not be possible in any case. In such a situation, the use of finite element techniques is recommended. Although the underlying physical processes may not be well defined, such a numerical analysis would permit accurate determination of the resultant transducer behaviour.

7.2.2 TRANSMISSION FIELD MODELLING

The present transducer model in transmission predicts the plane wave force component which emanates from the transducer face. In the case of a thickness mode device, it is possible to measure the plane wave force profile, however the small dimensions of the devices considered in this investigation prohibit measurement of this quantity. The force profile which is actually measured consists mainly of the the difracted edge wave.

In addition, it was noted in the course of the water tank force profile measurements, that the shape of the time domain force was highly dependant upon the position of the hydrophone. This effect is espcially noticeable when the hydrophone was moved along the length of the transducer.

The model may be used to predict the transmission field behaviour of two-dimensional transducers. This involves prediction of the plane wave component, combined with the use of diffraction theory, in order to determine the instantaneous pressure amplitude at each point in the field.

It has been shown for thickness mode devices that the electrical and mechanical loading conditions may seriously affect the characteristics of the transmitted field. Similar effects are anticipated for array transducers. A comprehensive study of the field characteristics of two-dimensional radiating structures is thus desirable.

7.2.3 MODELLING OF THICK TRANSDUCERS

The theoretical basis of the two-dimensional approach may also be applied for simulation of thick transducers. For example, cylindrical piezoelectric structures whose diameter is slightly less than their thickness. Such devices offer improved efficiency over thickness mode transducers.

The technique involves the reduction of the two-dimensional radial mode vibrations, to a single equivilent uni-dimensional 'lateral' mode. The model may then be used to simulate the behaviour of the pseudo two-dimensional structure. The two vibrational modes correspond to reverberation in the thickness dimension, coupled to the equivilent lateral mode vibration.

Initial experimental and theoretical comparison has proved encouraging, and the technique is currently being investigated in more detail.

7.2.4 MULTIPLE TRANSDUCER CONFIGURATIONS

At the present time the modelling software permits evaluation of multiple transducer structures. However, a serious limitation exists, since any variation in electrical loading from one transducer to the next is not considered. Furthermore, if the transducers are electrically connected in any way, the electrical impedance characteristic of each transducer will affect the electrical loading on all of the others. Such interaction is not clearly understood, and hence merits further research.

It should be noted that, according to the literature no facility exists for simulation of multiple thickness mode transducer configurations. The two-dimensional lattice model may easily be reduced to a unidilemsional thickness mode model by setting Y_{13} and h_{13} equal to zero. This permits evaluation of layered thickness mode structures possessing multiple transducers. However, as in the two-dimensional case , the influence of electrical connection between the active layers in the structure requires further investigation.

7.2.5 COMPOSITE TRANSDUCERS FOR ULTRASONIC ARRAYS

The use of composite (epoxy/piezoelectric) transducers in ultrasonic arrays may provide a number of significant advantages. The fabrication of such devices involves extensive cutting and filling of a thickness mode transducer, to produce a matrix arrangement of piezoelectric rods which are embedded in epoxy. [47][58]

Composite transducers offer the combined advantages of improved efficiency, lower acoustic impedance, for matching to water, and reduced radial mode effects. These advantages, especially the reduction in radial mode vibrations, are well suited to ultrasonic arrays.

Current technology invokes limitations on the size of the piezoelectric rods which may be used in the matrix, however scope exists for development of modelling and characterisation techniques for such devices. Composite transducers should in the near future find application in monolithic array structures, where lateral vibrations are a significant detrimental feature on array behaviour.

7.2.6 CROSS-COUPLING IN TRANSDUCER ARRAYS

There has been a significant contribution in the literature describing the effects of electrical and mechanical cross coupling [31][36]. The physical mechanisms which initiate such undesirable behaviour are however less well understood, and more importantly, techniques to accurately predict cross coupling are severely limited.

It is not considered that the linear systems modelling technique will provide a definitive solution. However, certain aspects of cross coupling, particularly mechanical, may be analysed. This however, is dependant upon the lateral layer model, and improved accuracy must firstly be ensured.

7.2.7 MECHANICAL LOSS CONSIDERATIONS

Although some mechanical loss was introduced into the model in chapter V, in order to demonstrate the experimental results, The problem of transducer and propagating channel loss was by no means rigorously tackled.

Some of the literature has addressed the problem of such mechanical absorption loss, however the author feels that further investigation in this direction is necessary.

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