Multi-criteria decision support system for the best ship maintenance strategy

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Submitted for the degree of Master of Philosophy

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June 2018

Declaration of Authenticity and Author's Rights

This thesis is the result of the author's original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.

Signed:

Date: June 2018

Acknowledgements

I would like to express my gratitude and sincere thanks to my supervisor Dr Iraklis Lazakis of the Department of Naval Architecture, Ocean and Marine Engineering at the University of Strathclyde. He always keeps providing me with the opportunity to research this study and for supervising my work throughout a whole year.

My sincere appreciation also goes to Korea-UK Global Engineer Education Program for Offshore Plant at Department of Naval Architecture and Ocean Engineering of INHA UNIVERSITY. It provided the first step to study at Strathclyde University and keep supporting me. It was great to help to endure a difficult time during my study.

Finally, I would like to thank my colleagues at the lab for their advice and encouragement during my work. This project would have been impossible without them.

Sincerely thanks to all.

Dosoo Kwon

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1. Introduction

Merchandise transported in the European Union/European Economic Area (EU/EEA) zone by ships reached a level of 90% of its external trade and another 40% was carried internally among its member-states. Fifty thousand seafarers employed in the maritime sector.

A close relationship of maritime transportation and growth of countries' GDP. In this regard, for each job created at sea, the equivalent of 5 works are created on land-based related activities (e.g. shore-based companies, consultancy activities, workshops, shipyards, etc.) [1] However, downtime originating from the application of the minimum requirements for maintenance and subsequently the occurrence of unexpected failures such as required repair cost, ship production loss and so on. In the case of ship maintenance applied improperly, it has a potential threat to people (both crew & passengers), environment, assets, carried cargo.

In addition, major ship accidents can be partially attributed to lack of maintenance or even inappropriate maintenance procedures followed both onboard and onshore (including all stakeholders.) [2]

As such, the maritime industry takes charge of the transport of huge commodities around the world, and a new approach to improving the reliability and efficiency of ships has been recently studied. Ship maintenance, which accounts for up to 30% of the operating cost of a ship, is related to financial aspects in terms of loss of unexpected ship repair and operation possibilities and subsequent income. In order to consider implementing an overall maintenance strategy, several parameters that are essential for the overall maintenance on board are taken into account. These parameters include the maintenance approach of the shipping company, the cost of spare parts for the vessel, and the cost of crew training, etc. Moreover, If the proper maintenance sequence does not apply on board, side effects can lead to too many inspections (sup. Engineers, cargo/Class surveyors, etc.), additional spare parts and not well-planned logistics for the availability of the correct spare parts and machinery.

When it comes to synthesizing opinions of decision makers, human opinions often collide due to collective decision making in the fuzzy conditions. A major issue of fuzzy multiple attribute group decision making is gathering adverse thought. Generally, the attribute weights (importance) by each expert can be different. For example, the final decision is affected by the evaluation of outstanding specialists in the group of judgment (i.e. chief engineer or skilled expert, as compared to other experts). Consequently, considering attributes based on the importance of each expert is an effective way to select the best maintenance method.

In this respect, the aim and objectives of the project and the approach used to undertake this study is presented in Chapter 2.

More detail, a number of different ways to tackle ship maintenance are applied and then figure out the best maintenance approach through the problem of ship maintenance in Chapter 3.

Section 3.1 the 'Critical Review - Maintenance approaches' section gives a background review of various ongoing maintenance methods and approaches.

Section 3.2 represents Fuzzy set theory (FST). FST by Zadeh (1965) is developed by this circumstance. He suggested that a fundamental element of human thinking is a fuzzy set rather than a figure. FST is a more suitable tool for humans than rigorous mathematical rules and equations by handling inaccurate data and vague expressions.

The AHP method which is employed to compute the relative importance of each expert and experts' weighting factors and entire attribute and alternative is introduced in Section 3.3. Furthermore, FAHP is offered to overcome ambiguity in decision making.

The Multiple Attribute Decision Making (MADM) method is introduced in Section 3.4. MADM approach is a way to help engineering and management decisions when evaluating and selecting what you want from an infinite figure of alternatives distinguished by multi-attribution.

Chapter 4 then presents an FMADM approach using FST and AHP. The problem of selection of the best maintenance is a fuzzy multi-attribute group decision making regarding fuzzy evaluation and various expert judgements in terms of the technical aspect. Decision makers often face the problem of selecting an alternative from a given number of solutions. Selected alternatives are the best alternative to meet a specific purpose in advance. The classic MADM methodology effectively addresses problems caused by inaccurate information. This classical method is not effective in delivering inaccuracies and ambiguities in unplanned and planned processes. The decision data for selecting the best maintenance approach to the problem of ship maintenance for the MADM problem is usually fuzzy, crisp or mixed. Thus, the decision model can handle all types of data.

In Chapter 5, as a practical case study, the best choice of maintenance approach is employed to demonstrate the applicability of the suggested methods.

Lastly, the final section summarizes and concludes this article.

Therefore, this research is dedicated to finding reasonable decision models to address the issues mentioned above. It enables to make the development of MADM methods involving multiple decision makers who can work in the fuzzy conditions.

2. Project definition

2.1. Aim and objectives

The overall aim of this research is to develop and examine a multi-criteria decision support system and suggest solutions for the best ship maintenance strategy. More specific objectives are as follows:

a) To review various methods of ship maintenance and consider their advantages and disadvantages.
b) To apply the state-of-the-art decision-making method.
c) To select the best maintenance method in circumstances where the problem is uncertain and ambiguous.
d) To suggest complicate parameters to make a rather reliable model. i.e. experts from the diverse area, a large number of attributes and alternatives
e) Examine case studies related to the ship maintenance problem

2.2. Approach

This chapter introduces the process of how to achieve the aim of research. The approach can be summarised in four tasks as follows:

a) Suggest the various solutions toward the problem of ship maintenance
    a. Available maintenance alternatives such as Corrective maintenance, Preventive maintenance, Condition Based Maintenance, etc.
b) Survey and aggregate the opinions of decision makers
c) Collect and establish decision matrix takes into account the beta coefficient and weighting of attributes and experts.
d) Implement the proposed aggregation in the MATLAB environment to make a ranking model.
e) Compare the results of calculating and rank the alternatives.
f) Put forward recommendations for future work.

3. Critical review

3.1. Maintenance approaches

In this section, various maintenance approaches applied to the problem of ship maintenance so far are introduced. Maintenance strategies include the investigation, repairs, replacements and inspections [3]. There are generally three categories of maintenance, and predictive maintenance is again divided into three methods.

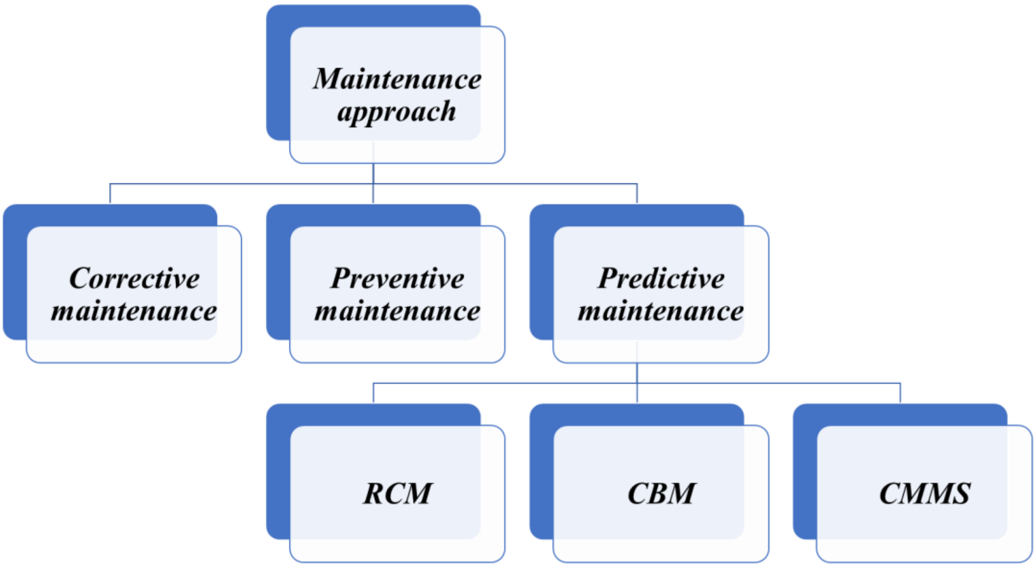


Figure 1. Category of maintenance approach

3.1.1. Corrective maintenance

Failure Based Maintenance (FBM) is performed especially when faults or failures occur, or when measures are not performed to discover the onset of failure or to prevent breakdown. Hence, FBM is corrective maintenance [3]. Expenses related to maintenance are generally high, nevertheless purely random failures and low failure costs (e.g. no down-time), this may be a cost-effective method [4].

Examples of corrective maintenance include:

- Emergency repairs, repairs to faults that can seriously affect the operation of the equipment.
- Interruption of service, restoring services that are down. This includes measures to repair failed items or machinery.
- Overhauling, which means that the failed item is reconstructed to its former condition that meets the appropriate criteria.
- Repair, repairing things that are broken to an operational state.

- Performance, maintenance designed to recover something with optimum performance such as a slow running software service maintenance
- Quality, which includes maintenance to prevent deterioration of machinery performance and crew discomfort caused by fluid leakage from the mechanical component.

Corrective maintenance the most fundamental solution of ship maintenance. Furthermore, a procedure of this maintenance made up of a procedure to figure out the trouble, identify the causation, make a plan, perform and assess appropriate solution [5]. More detailed procedures are described below.

- Acknowledgement of a failed case.
- Accurate identification of the failed component or item.
- Categorization of a failure event.
- Elimination of the initial cause of the breakdown case.
- The decision of the foundation procedure.

The main drawbacks of corrective measures are the high application of maintenance action which is not planned, improper maintenance work and more frequent inventory replacement than necessary [6]. Whereas, this maintenance approach has advantages when it accompanies minimal repairs and cost on spares. Corrective maintenance has been considered classical maintenance, but in recent years various studies have been conducted to overcome the limitations.

Recently, various studies on corrective maintenance have been conducted. Wang (2014) proposes a new calibration maintenance plan for engineering equipment. The expansion of FMECA constitutes a number of failure modes and the Failure Propagation Model describes the causal relationship between failures. In addition, the proposed method considers not only failure probability but also the probability of failure detection and severity [7]. Shabrina (2018) extracts knowledge and experience from operators based on knowledge transformation and creates e-Learning content for correct corrective maintenance activities that are fixing the bearings on the machine spindle [8].

3.1.2. Preventive maintenance

Preventive maintenance (PM) intend to comply with scheduled guidelines from time-to-time to prevent machinery/components breakdown. The essential aim of this maintenance is to avoid the consequences of machinery/components failure. Regarding this, PM is minimizing the full cost occurred onboard such as the cost of investigation and repair (e.g. overhaul), and component downtime.

More specifically, preventive maintenance offers a number of important benefits including

- Extended life of machinery
- Reduced unintended downtime due to machinery breakdown
- Possible failure as time elapses or steady use

- No need for maintenance and inspection
- Reduced errors in routine tasks
- Advanced reliability of machinery
- Reduced repair costs due to unexpected machinery breakdown
- Less injury risk

Contrarily, this maintenance has disadvantages such as the limited number of maintenance personnel onboard and it is also taken into account the characteristics of different type of vessels.

Assets eligible for preventive maintenance include the following substance:

- Having a crucial functional operation
- Failure cases that can reduce the frequency of faults with regular maintenance
- Possible failure as time elapses or steady use

Differently, inappropriate applications for preventive maintenance include:

- Irregular failures that are not related to maintenance (i.e. circuit board)
- Assets that do not provide critical functionality

The concept of preventive maintenance is dynamic. Factory managers consider proactive maintenance to be a service interruption for days or hours. Assuming the maintenance personnel want more equipment to be operational for more time, the amount of preventive maintenance applied to the equipment can increase if the maintenance personnel are able to diagnose faults that cause the equipment to fail [28].

Condition-based PM, which was developed by Lawrence Mann Jr (1995), uses sensor-based monitoring of machine conditions to predict when equipment failures may appear. This maintenance approach is compared to the classic statistical reliability (S-R) based PM method, which is using reliability and statistical analysis of machinery breakdown and analyses the profits of condition-based PM [9]. Liu (2014) investigates dynamic preventive maintenance policies. Unlike traditional cost-based preventive maintenance policies, maintenance strategies are performed from a value perspective and component values are modelled as a function of the reliability distribution. This maintenance system is implemented when the stability falls below a certain threshold [10]. Imad (2015) calculated the optimal replacement time for critical components and then reduced the overall cost by suggesting an actual preventive maintenance scheme. The author also developed a control chart to monitor the time between failures (TBF) based on the calculated failure rate (ROCOF) [11]. Huang (2015) takes a two-step approach that simultaneously considers the time and use of a repairable product and considers periodic preventive maintenance to develop a two-dimensional warranty policy for the repairable product [12]. Sheu (2015) proposes optimal preventive maintenance for multi-state systems. This study proposes a recursive approach to efficiently calculate the time-dependent distribution of a multi-state system and finds the optimal PM schedule that minimizes the average cost rate for each type of repair [13].

3.1.3. Predictive maintenance

Predictive maintenance (PdM) approaches in the maritime sector include the following maintenance; Reliability Centred Maintenance (RCM), Condition Based Maintenance (CBM), Computerized Maintenance Management System (CMMS). PdM differs from preventive maintenance because it depends on the actual state of the equipment, not the average or expected life data, to predict when maintenance is needed. This approach is contrived to ascertain the state of the equipment in operation to predict when maintenance should be done. *Figure 2* shows a simple layout of correlation of each predictive maintenance [6], [14].

There are some problems with PdM. First, many systems today contain embedded monitoring systems, but data is often not stored. That is, you can typically use up to two months of history to diagnose a failure, but when the expiration date is reached, the data is overwritten. In other words, the total time history of device operation is often unavailable. Second, the sampling frequency is often inadequate. For example, hourly measurements are useless for rapidly changing diesel engine parameters such as speed or load. Lastly, the quality of the data is often inadequate, especially if the data operator has to manually enter the data. For instance, fault registration is often not entered directly into the maintenance management system, which can result in incorrect date/time during a registration [15].

T Tinga (2017) suggests the cylinder liner model and printed circuit boards (PCB) model develop prognostic models. Next, the application should monitor the appropriate parameters for the use and loading of the system. Internal radar surveillance systems cannot use PCBs because they only store data for a limited period of time. For cylinder liners, they are replaced with predefined maintenance intervals, but the actual state is not evaluated at replacement. For this reason, perfect verification of the radar PCB model has not been successful so far [15].

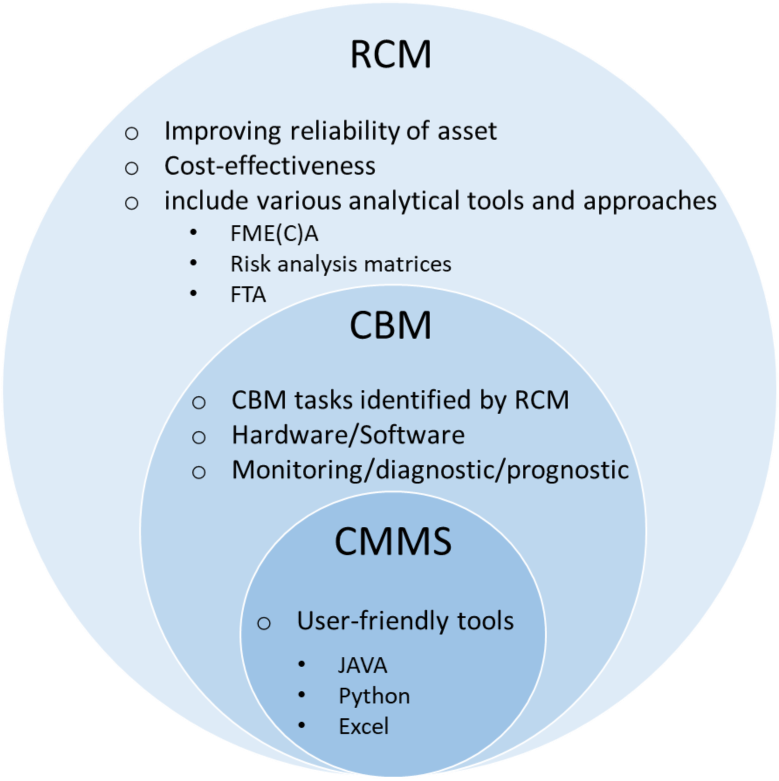


Figure 2. The layout of correlation of each predictive maintenance

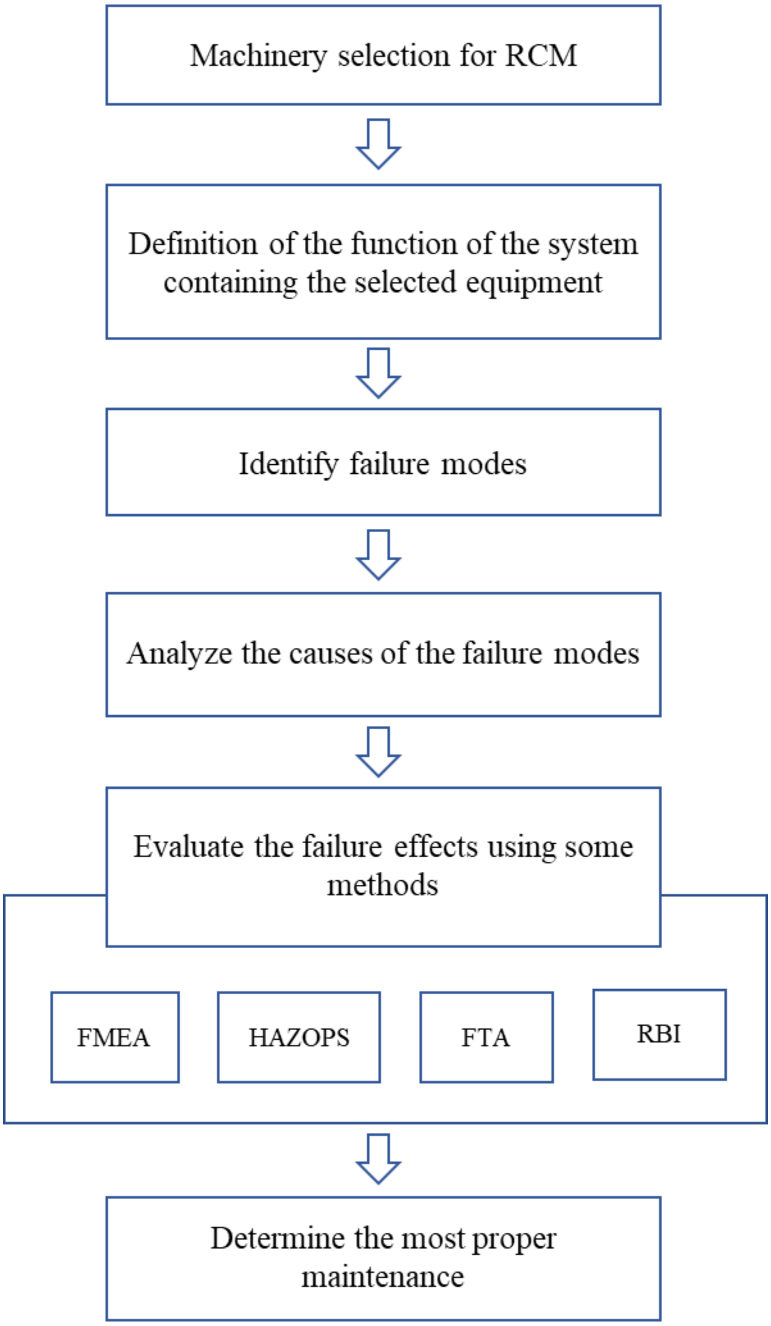
3.1.3.1. Reliability Centred Maintenance

Reliability Centred Maintenance (RCM) is a maintenance analysis method that systematically allocates optimal preventive maintenance tasks to items at an optimal frequency to maintain the ability to perform required functions over a period of time. On a standards-based basis, RCM is characterized by IEC 60300-3-11 (IEC, 2010) as a methodical approach to recognize efficient preventive maintenance operations on items and set maintenance work interval according to the specific procedures [16].

There are several concepts that are essential to reliability centred maintenance as follows.

- Preserving system activity
- Analysing failure modes may influence the system
- Prioritization the failure modes
- Control failure modes by selecting the applicable effective tasks

Moreover, the process of performing RCM is depicted in *Figure 3*.



FMEA: Failure modes and effects analysis; HAZOPS: A hazard and operability study; FTA: Fault tree analysis; RBI: Risk-based inspection;

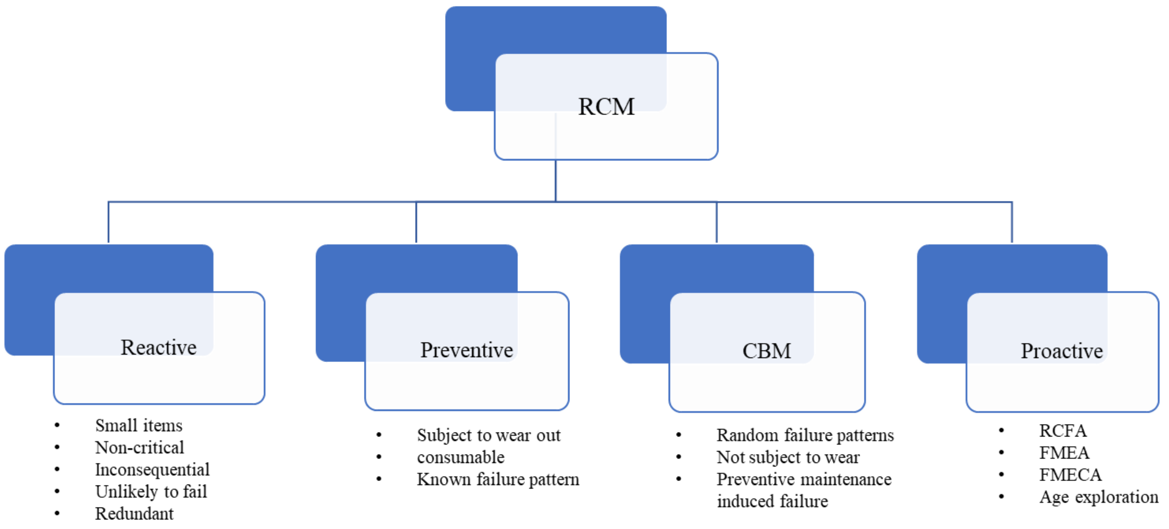
Figure 3. The procedure of performing RCM

The following tools and expertise are used to perform the RCM analysis.

- Design, engineering and systems operating knowledge
- Condition monitoring technology
- Risk-based decision making
- Failure Mode, Impact and Critical Analysis (FMECA)

FMECA consists of two separate analyzes: failure mode and impact analysis (FMEA) and critical analysis (CA). While FMEA analyzes the various failure modes and their impact on the system, the CA classifies or prioritizes the importance according to the degree of failure and the impact of the failure. The existing failure data can be used to perform the CA's ranking process.

The RCM program is divided into four components as shown in *Figure 4*.



RCFA: Root Cause Failure Analysis

Figure 4. Component of RCM

RCM is generally acknowledged as the most economical way by maintenance experts to create an optimum maintenance strategy. The RCM typically prioritizes maintenance operations based on some indicator of equipment characteristics and importance [17]. Additionally, RCM provides technical consideration of the asset and improved asset reliability as well as cost effectiveness.

When it comes to evaluating the benefit of RCM, the following additional benefits are overlooked by considering cost alone.

- Enhancing system availability
- optimising spare parts inventory
- Classifying unidentified failure modes
- Finding a meaningful failure scenario that was not previously known
- Training operational engineer

However, this maintenance method needs a lot of participants (facilitator, technical managers, and engineers) and substantial time for application. While RCM is insufficient to evaluate the cost influence of various maintenance methodologies, it does not assure that the maintenance efforts focus on the most admissible element and failures [18].

Thus, considering assets in RCM (called Reliability-Centred Asset Maintenance (RCAM)), it establishes to concentrate the consequent advancement and utilization of mathematical models for practically related features and failures [19].

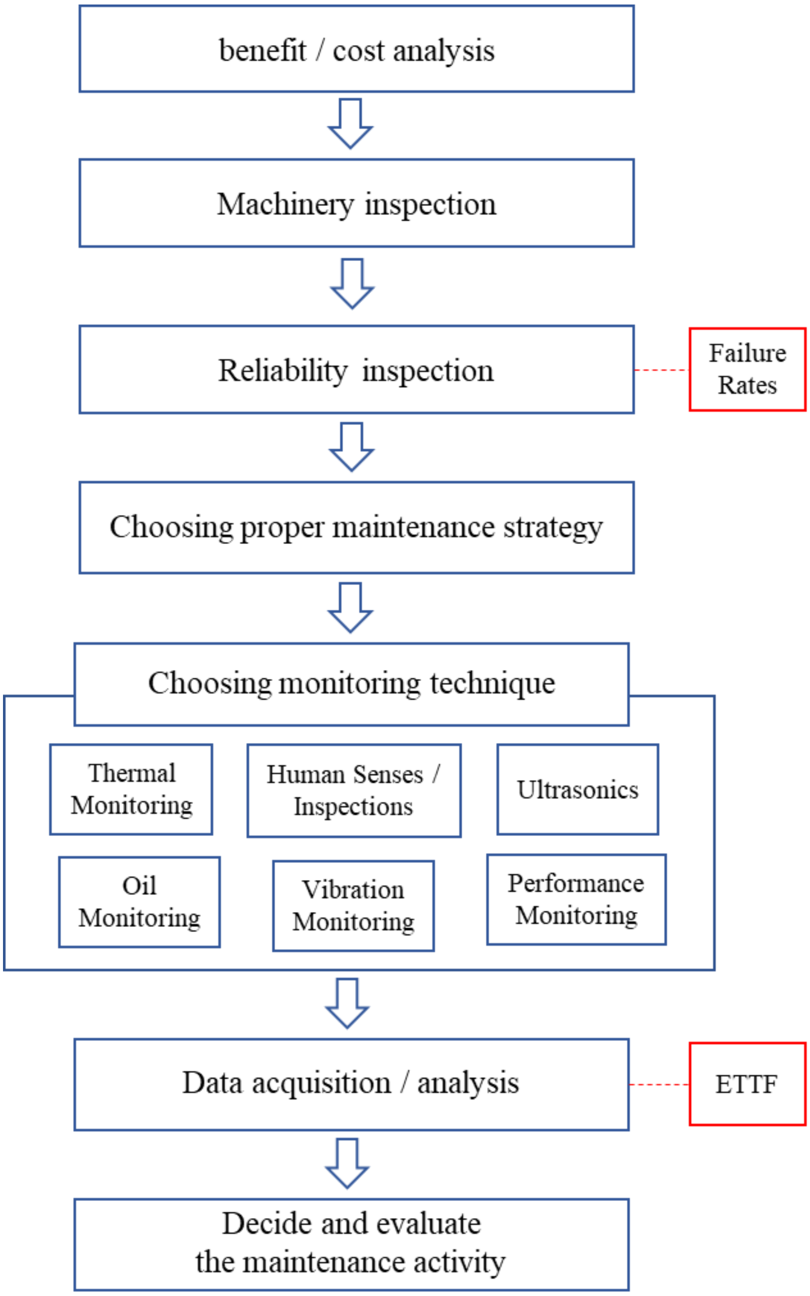
In recent studies, Yssaad (2015) experimentally decided the most important component through the choosing of the Failure mode, effects and criticality analysis (FMECA) method in the study of rational RCM Optimization for power distribution systems [20]. E. Ruijters (2016) proposes a new framework for integrating RCM with fault tree analysis. The author supports system reliability, availability, average failure time, maintenance and failure costs over time, various maintenance procedures separated by various cost components, and reliability measurements [21]. G. Gupta (2016) presents a SWOT analysis of 19 different RCM frameworks to make strategic decisions to implement RCM in different organizations. The various 19 RCM cases are grouped into three groups based on qualitative or theoretical, quantitative and pragmatic approaches, respectively [22].

3.1.3.2. Condition Based Maintenance

CBM is an approach applied to diminish the ambiguity of maintenance works and is performed according to the requirements signified by the condition of machinery.

CBM proposes maintenance works formed on the data congregated by condition monitoring. The CBM strives to prevent unneeded maintenance by performing maintenance operations only in the case of proof of abnormal behaviours are found. [23] The purpose of the CBM is to make smaller the total inspection expense and repair by accumulating and describing irregular or constant data which has a connection with the operational status of crucial units of the resource [24]. Nevertheless, CBM is not applicable to all retained assets and should only be applied if condition monitoring techniques are useful and cost-effective [25].

The implementation of the CBM goes through a series of steps, as shown in *Figure 5*.



ETTF: Estimated Time to Failure

Figure 5. The procedure of performing CBM

Two main classifications of CBM is represented by proactive maintenance and predictive maintenance. Proactive maintenance looks for ways to anticipate and handle possible errors in the future. Predictive maintenance monitors the condition indicators and intervenes during the failure period to prevent malfunctions, as described in the previous section. The layout of categories of CBM is depicted in *Figure 6*.

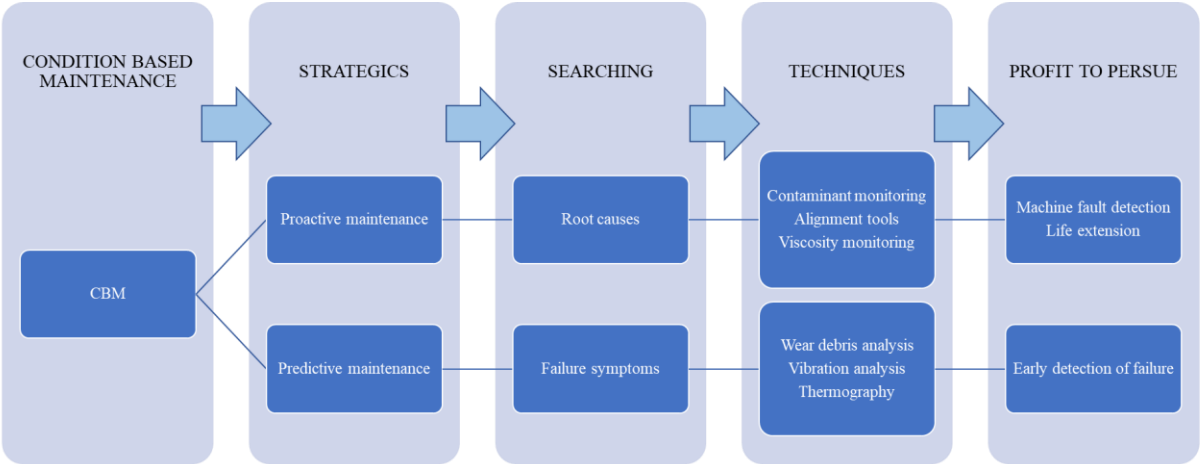


Figure 6. The layout of CBM's two classification process

Time-based preventive maintenance (PM) has a delimitation to extent the components lifetime. In a majority of cases, the component has been changed by a useful lifetime of many hours. [26] Preventive maintenance operations which are based on time intervals can spend many assets. [27] In other words, factories or buildings are often maintained more than necessary [28]. In this respect, asset monitoring can take action before it can seriously affect the performance of the organization. Accordingly, the CBM offers an alternative to the PM assumption of age-related failure modes [26].

Coetzee (1999), contrarily, advocated a holistic procedure to maintenance functions to implement CBM. The holistic approach includes the assessment of maintenance assets, including building structures [29]. Shonet (2003) noted that in order to effectively implement CBM, performance metrics for building components and systems should be developed. When evaluating building components, the physical achievement of them, the failure frequency of them, and finally the actual preventive maintenance (PM) performed on the system, should be evaluated [30]. Peng (2009) suggests a CBM program performed into the aspects of diagnostics and prognostics. Nowadays, CBM is not employed for less important machine parts despite its apparent advantages due to its expense. Nevertheless, it can be applied to where increased reliability and safety are required, and will be more widely applied in the future [31]. P Do (2015) addresses Proactive Condition Based Maintenance (CBM), which takes into account both complete and incomplete maintenance tasks for degraded systems. The author is investigating the impact of incomplete maintenance work and suggesting an adaptable maintenance policy that can help you choose the optimal maintenance task [32]. P Mehta (2015) suggests a way to prevent fatal errors by combining information from two or more sensors. The author also describes the intelligence in the CBM system using the Bayesian probabilistic decision framework and the data generated during validation [33].

3.1.3.3. Computerized Maintenance Management System

CMMS is an application that colligates different maintenance methodologies and computerized planned maintenance systems. CMMSs can manage maintenance information effectively and efficiently [34]. The basic idea is to consolidate all significant data into one central database which interconnects the various shipping company departments with the ship itself [6]. CMMS enables ship officers to use their shipboard computer applications without complex computer systems. Additionally, it supports on-board engineers overcome the complicatedness of their day-to-day business processes and maintain reasonable system costs. CMMSs are also software programs based on a computer for adjustment and connection which is employed to manage resource usage and work actions and to control extensive data on the labour force, inventory, restoration programs [35]. CMMS converts maintenance information into appropriate data for decision-making. Regarding this, it requires an effective control mechanism for restoring and managing information [36]. In this regard, a computerized management system can substantially enhance the opportuneness and correctness of storing and retrieving the necessary data. Otherwise, in terms of companies, applying CMMS has a few drawbacks. The installation of CMMS may cause opposition to adjust and requires education for those who handle delicate system. Also, due to the high degree of equipment dependency, a failure can interrupt the entire process [37].

On the other hand, in a minority case of CMMSs, it demonstrates inoperable or inefficient. Although these systems are often revised, they indicate incomplete operation. One of the prevalent reason for this failure is that the computer system creator has no experience managing the maintenance department. Designing a proper and effective computer maintenance management system requires the designer to have a proper understanding of the maintenance function and its purpose in the maintenance organization. Applying the concept of quality management has proven that maintenance work can be improved and applied more cost-effectively. Modern maintenance management must extend beyond equipment repair and service to the long-term performance aspects of customer service systems. The viability of the entire organization depends on effective maintenance policies, planning and operations. Thus, the implementation of quality control programs by maintenance can lead to improved maintenance productivity and cost savings [38].

The goal of the maintenance manager is to adopt a management system that optimizes the use of valuable resources needed to maintain facilities and equipment. The system should provide an integrated process so that the administrator can control the maintenance of all equipment and maintenance-capable equipment from acquisition to disposal. The system should do the following:

- Maintain maintenance inventory
- Record and maintain work history
- Include job tasks and frequency
- Accepting all work practices
- Effective interface and communication with related systems and support systems
- Provide feedback information for analysis

- Reduce costs through effective maintenance plans

The latest CMMS meets these requirements and supports the equipment maintenance manager through job receipt, planning, control, performance, evaluation and reporting. These systems also maintain historical information about administrative use. Administrators must assess management data requirements and set up electronic data requirements before acquiring new CMMSs or adding/replacing existing systems. The following items contain details about the functions that can be included in the latest CMMS.

- Enter and trace where the equipment works
- Module for tracking labour resource
- Modules that allow the operator to keep accurate and detailed records of each device
- Safety program
- Allow the operator to track inventory movements
- Utility consumption, distribution, usage, weighing, user assignment and cost
- Maintenance history of facilities and machinery
- Accumulate the key performance indicator (KPI) data for maintenance program evaluation.

CMMS is coming more and more essential in the industry area [39]. Implemented CMMS intends to diminish the total time out and frequency of machines breakdown as enhancing the performance and efficiency of maintenance operations. Currently, precise information is important in making decisions that assure the dependable working of the supplies [40]. In terms of the software program, CMMS came out to be achieved in an industry in 1997 [41]. However, the first mention of CMMS is given by Gilbert et al. (1985), "The development of computerized preventive maintenance systems has improved maintenance personnel and material planning" [42]. In Fernandez et al. (2003), CMMS was devised, advanced, customized about disc brake pad manufacturers [43]. And a system that improves the RCM process integrated with CMMS was proposed by Gabbar et al. (2003) [44]. Moreover, in the study of Labib (2004), The intelligent model of CMMS makes meaningful data collected as a form of decision supporter [45]. Carnero, MC and JL Novés developed the evaluation system for selecting computerized maintenance management software in an industrial plant using multi-criteria approaches [46].

Table 1. Features, pros and cons of maintenance methodologies in this study

Maintenance method	Features	Advantages	Disadvantages
Corrective maintenance	The action performed because of apparent failure or deficiencies occurring	Minimal repairs, minimum cost on spares	Improper operation of maintenance work, high substitution component list
Preventive maintenance	Scheduled inspections, performances are recorded	Minimize cost elements	Extensive use of resources, the limited number of personnel onboard
Predictive maintenance – RCM	Being very detailed but also quite complex in structure	Technical consideration of the asset in question, improved asset reliability, cost-effectiveness	A number of participants needed, substantial time for implementation
Predictive maintenance – CBM	Performed when equipment is predicted to fail	Identifying efficiency losses and safety-critical defects, reduce overall maintenance cost in the long term	only applied to a fraction of the merchant and passenger worldwide fleet
Predictive maintenance – CMMS	Combine key elements of IT nowadays	Data collecting, real-time reliability and criticality analysis	Needs highly trained and skilled personnel

RCM: Reliability Centred Maintenance; CBM: Condition Based Maintenance; CMMS: Computerized Maintenance Management System

3.2. Fuzzy set theory

Zadeh first suggested fuzzy set theory in order to design fuzzy factors within a numerical area [47]. Zebda refers that the term 'fuzzy' is the status which vagueness or uncertainty occur [48].

Assuming X is the universe of an object with an element x, where A is a fuzzy set which is a fuzzy sub-set of X. Membership of x in conventional set A is considered to be the characteristic function μ_A, from X to (0,1)

$$\mu_A(x) = \begin{cases} 1, & if\ x \in A \\ 0, & if\ x \notin A \end{cases}$$

For a fuzzy set A of the universe X, the grade of membership of x in A is specified as :

$$\mu_A(x) \in [0,1]$$

where $\mu_A(x)$ = the membership function.

The value of $\mu_A(x)$ has a value for every x from 0 to 1, signifying the extent of membthe ership function $\mu_A(x)$, and this range differs from the crisp set. The more the value of $\mu_A(x)$ is close to one (1), the more x pertains to the A. In this respect, the fuzzy set A does not have clear bounda ary. Each crisp subset of X can correspond to a one-to-one feature function, and the fuzzy set is an extension of a crisp set because the membership function is an extension of the feature operation. A fuzzy set element is an ordered pair that indicates the element set value and the membership level. The following equation shows this.

$$\mathrm{A} = \{(x, \mu_A(x)) | x \in X\}$$

Where $\mu_A(x)$ is referred to in A as the membership level of x. Addtionaly, μ_A: X → N indicates a function from X to a scope N. Normally N has a real number in the closed interval [0,1] where 0 and 1 represent to whole membership and non-membership respectively. When N is consisted of two points, 0 and 1, A is non-fuzzy, and the membership function is the same as the characteristic function of a crisp set [49].

Therefore, the fuzzy set theory requires a soft threshold to determine the intermediate estimate $\mu_A(x)$ [50]. The membership function μ_A defines a ductility threshold to allow soft and practical evaluation of the measure x of the sustainability indicator [51].

Furthermore, the most common set operators for the fuzzy set A and the fuzzy set B defined as follows [52]. The union operation of them is:

$$\mu_{A\cup B}(x) = max[\mu_A(x), \mu_B(x)]$$

Their intersection operation is:

$$\mu_{A\cap B}(x) = min[\mu_A(x), \mu_B(x)]$$

And the complement operation is:

$$\mu_{\tilde{A}}(x) = 1 - \mu_A(x)$$

Furthermore, the fuzzy relation is an especially practical mathematic formulate. A fuzzy relation is defined as:

$$\mathrm{R} = \{(x, y, \mu_R(x, y)) | x \in X, y \in Y\}$$

3.2.1. Fuzzy number

Regarding the normalization process, triangular fuzzy numbers are proposed as follow.

If $\tilde{y}_{ij} = (a_{ij}, b_{ij}, c_{ij}) (i = 1, \ldots, n,\ j = 1, \ldots, m)$ are triangular fuzzy numbers, then the normalization procedure is carried out as

$$(\tilde{y}_{ij})_N = \left((a_{ij})_N, (b_{ij})_N, (c_{ij})_N\right) = \left(\frac{a_{ij} - a_j^{Min}}{\Delta_{Min}^{Max}}, \frac{b_{ij} - a_j^{Min}}{\Delta_{Min}^{Max}}, \frac{c_{ij} - a_j^{Min}}{\Delta_{Min}^{Max}}\right)$$

Where $i = 1, \ldots, n,\ j \in \Omega_b$

$$(\tilde{y}_{ij})_N = \left((a_{ij})_N, (b_{ij})_N, (c_{ij})_N\right) = \left(\frac{c_{ij} - c_j^{Max}}{\Delta_{Max}^{Min}}, \frac{b_{ij} - c_j^{Max}}{\Delta_{Max}^{Min}}, \frac{a_{ij} - c_j^{Max}}{\Delta_{Max}^{Min}}\right)$$

Where $i = 1, \ldots, n,\ j \in \Omega_c$

where Ω_b and Ω_c indicates the sets of benefit attributes and cost attributes, respectively and

$$c_j^{Max} = Max c_{ij},\ a_j^{Min} = Min a_{ij}$$

$$\Delta_{Min}^{Max} = c_j^{Max} - a_j^{Min},\ \Delta_{Max}^{Min} = a_j^{Min} - c_j^{Max}$$

In the following basic concept about fuzzy numbers with left and right scores are represented below.

Chen first presents the left and right scores approach rank fuzzy numbers [53]. Let fuzzy number as $\widetilde{N}$, the left scores indicate the intersection of a fuzzy number $\widetilde{N}$ with the fuzzy minimum and the right scores mean the junction of a fuzzy number with the fuzzy maximum [54]. Accordingly, the left and right scores of a fuzzy number $\widetilde{N}$ are described as

$$R_s(\widetilde{N}) = \sup_x [\mu_{\widetilde{N}}(x) \wedge \mu_{max}(x)]$$

$$L_s(\widetilde{N}) = \sup_x [\mu_{\widetilde{N}}(x) \wedge \mu_{min}(x)]$$

These equations above can be simplified as below

$$R_s(\widetilde{N}) = \frac{(c_{ij})_N}{1 + (c_{ij})_N - (b_{ij})_N}$$

$$L_s(\widetilde{N}) = \frac{(b_{ij})_N}{1 + (b_{ij})_N - (a_{ij})_N}$$

3.3. Analytical Hierarchy Process (AHP)

Goossens (2015) investigated a maintenance policy selection by using the AHP, focussing on naval ships. In this study, the applicability appears to provide an idea-structured method that provides insight by guiding choices and facilitating discussion rather than making actual decisions on different types of assets of the same criteria [55]. Emovon (2018) presents two MCDM method for the selection of appropriate maintenance strategies for ship machinery systems, which is Delphi-AHP and Delphi-AHP-PROMETHEE (Preference Ranking Organisation Method for Enrichment Evaluations). In this paper, AHP was used as a tool for determining the final ranking and weight of the maintenance strategy alternatives in relation to the decision criteria, and as a tool for assessing the weights of the decision criteria, respectively [56]. Delphi method and AHP are also applied with TOPSIS to determine the suitability of the approach a case study of the sea water pump of the central cooling system of a marine diesel engine [57].

AHP, one kind of multiple criterion decision processes, utilizes hierarchic structure to express problems and develops alternatives priorities on the basis of the user judgement [58].

(1) Pairwise comparison

Considering hierarchy entries, all accompanying entries at the lower hierarchy are compared in the matrices as below:

$$\mathrm{A} = \begin{bmatrix} 1 & \frac{I_1}{I_2} & \cdots & \frac{I_1}{I_n} \\ \frac{I_2}{I_1} & 1 & \cdots & \frac{I_2}{I_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{I_n}{I_1} & \frac{I_n}{I_2} & \cdots & 1 \end{bmatrix}$$

Where A = pairwise comparison matrix,

I_1 = Importance of entry 1,

I_2 = Importance of entry 2,

I_n = Importance of entry n.

(2) Evaluate the relative importance

The eigenvalue method is applied to compute the relative importance of the entries for every pairwise comparison matrices. The relative importance (I) of the matrix M is given by the equation below:

$$(M - \lambda_{max}\mathrm{U}) \times I = 0$$

Where λ_{max} = maximum eigenvalue of M,

U = unit matrix.

(3) Checkout the consistence

At this stage, the consistence properties for the matrix are examined to confirm that a decision maker's judgment is consistent. This requires some pre-parameter. The consistence index (CI) is measured as follows:

$$\mathrm{CI} = \frac{\lambda_{max} - n}{n - 1}$$

The consistence index of a nonchalantly generated inverse matrices is necessarily invoked in a random index (RI) by a reciprocal force. The mean RI for the matrices of order 1–15 was produced applying an example size of 100 (Nobre et al., 1999). Saaty (1980) represents the random indexes table of order 1–15 matrices. And then, the Consistency Ratio (CR) is finally measured. When CR has a value less than 0.1, the analysis is generally constant, in this way, the obtained importance is employed. The formula for CR is:

$$\mathrm{CR} = \frac{CI}{RI}$$

(4) Acquire the whole evaluation

At the final phase, the relative importance of the factors of judgment are gathered to get a full assessment of the alternatives as follows:

$$I_i^s = \sum_{j=1}^{j=m} I_{ij}^s I_j^a \quad ,i = 1,2,\ldots,\mathrm{n}$$

Where I_i^s= total importance of site i,

I_{ij}^s= importance of alternative i combined to attribute j,

I_j^a= importance of attribute j,

m = number of attribute,

n = number of site.

T. Evangelos and Lin Chi-Tun (1996) explained how fuzzy operations are used on fuzzy numbers. In their study, triangular fuzzy numbers (which have fuzzy numbers of low, modal and high values) are employed since they are more uncomplicated than trapezoidal fuzzy numbers. The triangular estimation of the fuzzy procedure is a reasonable way to fuzzifying decision matters [59].

3.3.1. Fuzzy Analytical Hierarchy Process (FAHP)

Regardless of the recognition of AHP, it is generally condemned because it does not adequately address the intrinsic uncertainties and inaccuracies related the perceptions of the correct number of decision makers [60]. Because ambiguity and uncertainty are ordinary features in numerous decision-making problems, fuzzy AHP (FAHP) methods should be allowed to permit ambiguity or unclearness [61]. In other words, since FAHP is able to capture human emotions because decision-makers are more likely to make interval determinations than to express judgment in the form of a single numerical value such as the traditional AHP approach [62]. This ability exists when a crisp judgment is transformed into a fuzzy judgment. More specifically, in a general classical theory, approved membership functions to work within the scope of real numbers between 0 and 1 (one). In other words, in the fuzzy set $S = \{(x, \mu_s(x)), x \in R\}$, x is on the real line R: $-\infty < x < +\infty$ and $\mu_s(x)$ is continued mapping from R to the closed distance [0,1] [63]. One of the major features of fuzzy is to group individuals into unclearly determined bounded classes [64]. The fuzzy numbers can describe the uncertain comparison judgments. A shape of this fuzzy number A $= (n_1, n_2, n_3)$ known as triangular fuzzy a number, where n_1 refers to the smallest value, n_2 is the most probable modal value, and n_3 is the largest value. The triangular fuzzy number is explained in *Figure 7*. In addition, the membership function of a fuzzy number $\mu_A(x): R \rightarrow [0,1]$ is equivalent to following equation.

$$\mu_{the\ A}(x) = \begin{cases} (x - n_1) \ / \ (n_2 - n_1), & n_1 \leq x \leq n_2 \\ (n_3 - x) \ / \ (n_3 - n_2), & n_2 \leq x \leq n_3 \\ 0 & otherwise \end{cases}$$

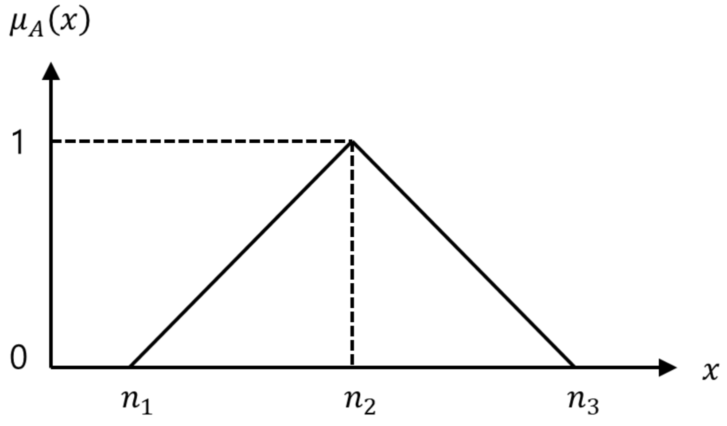


Figure 7. Fuzzy triangular number

To compare pairs of alternatives for each criterion or alternative, as mentioned for the traditional AHP, the triangular fuzzy comparison matrix is defined as follows:

$$\tilde{A} = \left(\tilde{a}_{ij}\right)_{n\times n} = \begin{bmatrix} (1,1,1) & (s_{12}, m_{12}, l_{12}) & \cdots & (s_{1n}, m_{1n}, l_{1n}) \\ (s_{21}, m_{21}, l_{21}) & (1,1,1) & \cdots & (s_{2n}, m_{2n}, l_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (s_{n1}, m_{n1}, l_{n1}) & (s_{12}, m_{12}, l_{12}) & \cdots & (1,1,1) \end{bmatrix}$$

Where $\tilde{a}_{ij} = \left(s_{ij}, m_{ij}, l_{ij}\right) = \tilde{a}_{ij}^{-1} = \left(1 / l_{ji}, 1 / m_{ji}, 1 / s_{ji}\right)$,

$i = 1, \dots, n$,

$j = 1, \dots, n$,

$i \neq j$.

In order to make it clear the numbers on the matrix, n_1, n_2, and n_3 are expressed as s, m, and l, respectively. Regarding this, s represents the smallest value of A and l imply the largest value and m stands for the modal value. If $s = m = l$, then it is a non-fuzzy number according to the custom [65].

The principal operations of two triangular fuzzy numbers $\widetilde{N}_1$ and $\widetilde{N}_2$ are as bellows [66]

Addition of triangular fuzzy numbers:

$\widetilde{N}_1 \oplus \widetilde{N}_2 = (s_1 + s_2, m_1 + m_2, l_1 + l_2)$,

Multiplication of triangular fuzzy numbers:

$\widetilde{N}_1 \otimes \widetilde{N}_2 = (s_1 s_2, m_1 m_2, l_1 l_2)$,

Multiplication with a constant:

$\lambda \otimes \widetilde{N}_1 = (\lambda s_1, \lambda m_1, \lambda l_1)$, $\lambda > 0$, $\lambda \in R$,

Minus triangular fuzzy number:

$\ominus \widetilde{N}_1 = (-s_1, -m_1, -l_1)$,

The inverse of a triangular fuzzy number:

$1 / \widetilde{N}_1 = (1 / s_1, 1 / m_1, 1 / l_1)$.

3.4. Multiple Criteria Decision Making (MCDM)

Decision making is that selecting the most reasonable solution among all of the alternatives. In general, numerical data is incompetent in real since the ambiguity of decision data. Applying of fuzzy approach to the decision-making problem is very effective because human judgment, including preferences, can often not be expressed with ambiguous and precise numbers. The Multiple Attribute Decision Making (MADM) methodology is generally adopted to find a solution to a variety of problems. This approach demand decision-makers for giving qualitative/quantitative assessments to determine the relative importance of each criterion associated with the comprehensive objectives and the performance of alternatives in relation to every criteria [67].

The MCDM consists of a defined set of alternatives that have weights in accordance with the importance of each alternative. Based on this, the decision maker selects or ranks the alternatives. In the paper of Zanakis (1998), various methods applied to the decision problem was introduced such as simple additive weighting (SAW), multiplicative exponential weighting (MEW), the analytic hierarchy process (AHP), technique for order preference by similarity to ideal solution (TOPSIS) and so on [68]. Meanwhile, many useful fuzzy MCDM methods have been developed by Liang (1999) [69]. Fuzzy MCDM can be used to evaluate the significance of criteria and the evaluation of alternatives to each criterion conforming to the fuzzy linguistic variables. Chin and Klein (1997) are introduced some of the fuzzy linguistic models [70].

4. Methodology

4.1. Introduction

In this thesis, FMAGDM approach is comprised of the rating, aggregation and the selection stage as given in *Figure 8*

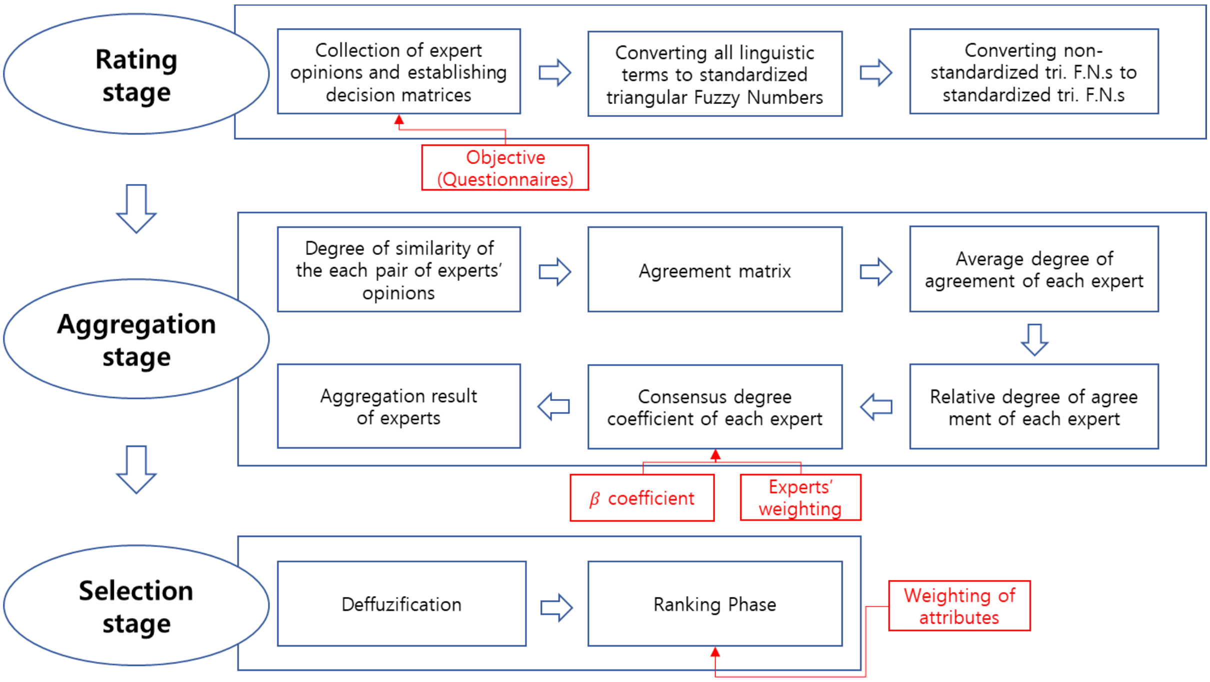


Figure 8. The framework of the method on this study

In the first state, each expert evaluates alternatives for each subjective attribute. This evaluation is typically a linguistic term, fuzzy data format. This kind of qualitative data can be better modelled with fuzzy numbers. This state aims to convert the fuzzy data to a standardized triangular fuzzy number. In the second state, all performance ratings are aggregated for each alternative in each subjective attribute after the weights of the attributes and the importance of the experts are assigned. Finally, the ambiguous element of the aggregated decision matrix for the expert group are defuzzified at the defuzzification stage of the final state. The result of this step is a decision matrix and contains only crisp data. The alternatives are rated in the TOPSIS method.

In this study, three experts from each group of the crew, shipping company and regulator are collected to take into consideration the characteristics of expert groups in various fields that were not tried in previous studies. Besides, a simple but clear result is obtained by applying for the triangular fuzzy number, and scale 7 is used to provide various options for the experts to evaluate. Most of all, two types of attribute/assessment are applied, which are subjective and objective for attributes, linguistic and crisp for assessments. This application enables a reliable assessment of experts on the alternatives to the problem of ship maintenance.

4.2. Rating stage

At this stage, experts are required to make assessments for alternatives in respect of the subjective attributes in order to construct the decision matrices. Regarding attributes, subjective and objective attributes are the two kinds of attributes. In terms of subjective attributes, the experts' assessment for them towards an alternative contains factors such as uncertainty, vagueness and subjectivity. Accordingly, these estimations can be expressed as linguistic words. Since the existing quantitative explanations do not properly explain the complicated or vague situation, the linguistic variable concept is applied to dealing with. The objective attributes, otherwise, are defined by a numerical value which can be regarded as standard one. Attributes can be divided by cost attribute and benefit attribute when it comes to subjective (linguistic) attribute. The benefit attribute has a positive meaning, while the cost attribute has a negative meaning. In other words, the cost attribute means that the lower the value of the attribute, the higher the evaluation. To the next, determining the problem If the matrices include information of fuzzy that can be indicated in linguistic words, first convert the linguistic term to a fuzzy number using the proper transform scale. To convert the linguistic variables to the fuzzy numbers, the rule for numerical estimation developed by Chen and Hwang (1992) is used. Selected Scale corresponds to all the linguistic variables in the attribute row of the decision matrices. Besides, the interpretation of these linguistic variables is represented by fuzzy numbers on this Scale. As mentioned in '4.3.1 Fuzzy Analytical Hierarchy Process (FAHP)', this study is using the triangular fuzzy number. *Table 2* shows the Scale 7 with its linguistic variable and fuzzy number.

Table 2. Assessment by linguistic variables (Scale 7 form Chen and Hwang's study)

Linguistic variable	Fuzzy number
Very low (VL)	(0,0,0.2)
Low-very low (LVL)	(0,0,0.1,0.2)
Low (L)	(0,0.2,0.4)
Fairly low (FL)	(0.2,0.35,0.5)
Medium (M)	(0.3,0.5,0.7)
Fairly high (FH)	(0.5,0.65,0.8)
High (H)	(0.6,0.9,1)
High-very high (HVH)	(0.7,0.9,1)
Very high (VH)	(0.8,1,1)

4.3. Aggregation stage

Consider the degree of importance of experts as $\mathrm{w}e_k$ ($k = 1,2,\ldots,m$), weight one (1) is appointed to the most important decision maker among experts. In comparison with the most important one, the other experts have their own relative weights expressed as re_l. Hence, the degree of importance $\mathrm{w}e_k$ is defined as:

$$\mathrm{w}e_k = \frac{re_k}{\sum_{k=1}^{m} re_k}$$

Let A and B be two standardised triangular fuzzy numbers, they are represented as:

$$\mathrm{A} = (a_1, a_2, a_3), \text{ where } 0 \leq a_1 \leq a_2 \leq a_3 \leq 1,$$

$$\mathrm{B} = (b_1, b_2, b_3), \text{ where } 0 \leq b_1 \leq b_2 \leq b_3 \leq 1.$$

The similarity function S can calculate the degree of likeness between the two standardised triangular fuzzy numbers as follows.

$$\mathrm{S(A,B)} = 1 - \frac{|a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3|}{3}$$

As the similarity between the two standardised triangular fuzzy numbers increases, the value of similarity function also enlarges.

Meantime, $\mathrm{S(A,B)} = \mathrm{S(B,A)}$ is needs to be remembered.

The agreement matrix (AM) is constructed as a next step after all experts evaluate the similarity degrees.

$$\mathrm{AM} = \begin{bmatrix} 1 & S_{12} & \cdots & S_{1v} & \cdots & S_{1n-1} & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2v} & \cdots & S_{2n-1} & S_{2n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ S_{u1} & S_{u2} & \cdots & S_{uv} & \cdots & S_{un-1} & S_{un} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ S_{m1} & S_{m2} & \cdots & S_{mv} & \cdots & S_{mn-1} & 1 \end{bmatrix}$$

Where $S_{uv} = \mathrm{S}(R_u, R_v)\ if\ u \neq v$ and $S_{uv} = 1\ if\ u = v$.

To the next, $\mathrm{AA}(E_u)$ of expert E_u ($u = 1,2,\ldots,m$), the average degree of agreement is calculated by the AM. The equation of $\mathrm{AA}(E_u)$, which is given by:

$$\mathrm{AA}(E_u) = \frac{1}{m-1}\sum_{\substack{r=1 \\ u\neq v}}^{m} S(X_u, X_v).$$

In the same way as previous, compute the relative degree of agreement $\mathrm{RA}(E_u)$ and the consensus degree coefficient agreement $\mathrm{CC}(E_u)$ using the AA and RA, respectively. The equation of $\mathrm{RA}(E_u)$ and $\mathrm{CC}(E_u)$ are shown as follows:

$$\mathrm{RA}(E_u) = \frac{\mathrm{AA}(E_u)}{\sum_{r=1}^{m} \mathrm{AA}(E_u)},$$

$$\mathrm{CC}(E_u) = \beta we_k + (1-\beta)\mathrm{RA}(E_u).$$

Where $u = 1,2,\ldots,m$

In the equation of $\mathrm{CC}(E_u)$, $\beta(0 \leq \beta \leq 1)$ reflects the influence of facilitator towards the decision-making procedure. Regarding that, the importance of we_k through $\mathrm{RA}(E_u)$ is showed by β. When it comes to refer the consensus degree coefficient of experts, $\mathrm{CC}(E_u)$ is an effective way to estimate the relative value (importance) of each expert evaluation.

Following the previous phase, the aggregated fuzzy evaluation result is obtained at last as follows:

$$\mathrm{R} = \mathrm{CC}(E_1) \otimes R_1 \oplus \mathrm{CC}(E_2) \otimes R_2 \oplus \cdots \oplus \mathrm{CC}(E_{m-1}) \otimes R_{m-1} \oplus \mathrm{CC}(E_m) \otimes R_m$$

where $\otimes$ = the fuzzy multiplication operator,

$\oplus$ = the fuzzy addition operator.

Kaufmann (1991) noted that multiplying or adding different fuzzy numbers can still be fuzzy numbers [71].

4.4. Selection stage

In selection stage, to select the best one of the problem among all the alternatives, whole aggregated triangular fuzzy numbers should be deffuzified in order that each element of the gathered matrices of decision is transformed to numerical numbers.

To obtain the total score of the fuzzy numbers concerning the fuzzy scoring method developed by Chen and Hwang (1992), the fuzzy maximization and minimization group must first be considered.

$$\mu_{max}(X) = \begin{cases} x, & for\ 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

$$\mu_{min}(X) = \begin{cases} 1-x, & for\ 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

The right score of the fuzzy number $\widetilde{N}$ is defined as:

$$R_s(\widetilde{N}) = \sup_x [\mu_{\widetilde{N}}(x) \wedge \mu_{max}(x)]$$

The left score of $\widetilde{N}$ is defined as:

$$L_s(\widetilde{N}) = \sup_x[\mu_{\widetilde{N}}(x) \wedge \mu_{min}(x)]$$

According to the right and left scores of $\widetilde{N}$, the total score of $\widetilde{N}$ is obtained by the following equation.

$$T_s(\widetilde{N}) = \frac{[R_s(\widetilde{N}) + 1 - L_s(\widetilde{N})]}{2}$$

Once the defuzzification stage is completed in this way, the selection phase of the FMAGDM approach, the ranking subordinate step, proceeds. In this study, the Technique Ordered Preference by Similarity to Ideal Solution (TOPSIS) method is applied.

TOPSIS has been strongly applied to a lot of problem area and decision-making. TOPSIS was originally designed to reflect the expert opinion, but the existing TOPSIS did not do it well. As a result, the method is developed to solve the FMAGDM problems. For example, Tsaur, Chang and Yen (2002) transform a FMAGDM problem to a numerical value and resolve the non-Fuzzy multi-criteria decision-making problem applying the TOPSIS method [72]. Chen and Tzeng (2004) make changes a FMAGDM problem to a non-FMAGDM utilizing fuzzy integral. [73] They apply a grey relation grade to describe the relative closeness among alternatives instead of adopting the interval of each alternative. Additionaly, Chu (2002) and Chu and Lin (2003) convert a FMAGDM problem into a numeric value and figure out the crisp MADM problem applying the TOPSIS approach [74], [75]. In real life, data (attributes) are often not deterministic due to inadequate or inaccessible information and are generally inaccurate, so, TOPSIS for fuzzy data needs to be extended [76]. The linguistic expression of the fuzzy theory is considered a natural expression of judgment. Assessments from experts refer to the capability of the appliance of fuzzy set theory (FST) to represent the preferences of decision makers. FST helps to measure the vagueness of idea related to the subjective assessment of human. Decision-makers (experts) usually feel more comfortable making linguistic assessments than assessments with crisp value [77]. This occurs because the fuzzy nature of the comparison process cannot explicitly describe preferences [78]. Yang and Wu (2008) developed the TOPSIS method based on the principle that the chosen alternative must get the smallest separation (in the Euclidean sense) from the Positive Ideal Solution (PIS) and the furthest separation (in the Euclidean sense) from the Negative Ideal Solution (NIS) [79], [80]. TOPSIS is uncomplicated and easy to do or understand in Multi-Criteria Decision Making (MCDM) since it can give crisp values by decision makers. Extended TOPSIS into a fuzzy environment used a fuzzy language value (described by a fuzzy number) as a replacement for a decisive value given directly in the rating [81]. This TOPSIS is a practical method and is suitable for human thinking in a real environment. The weighting of criteria is given by experts [78].

In order to execute the TOPSIS rating technique, the normalized grades are computed to convert diverse attribute dimension to dimensionless attribute, allowing comparison through attributes [82]. Using the vector normalisation method for calculating the r_{ji} factor of the normalised decision matrices, is shown as follows

$$r_{ji} = \frac{x_{ji}}{\sqrt{\sum_{j=1}^{N} x_{ji}^2}}$$

where $j = 1,2,\ldots,N$; $i = 1,2,\ldots,\mathrm{K}$ and x_{ji} is the value of alternative j in relation to attribute i.

The group of attribute weights evaluated from the decision maker is aggregated into the normalized decision matrix for the next stage. The weighted normalized decision matrix is estimated as the product of each row of the normalised decision matrix and its corresponding weight of attribute (w_i). The elements of the weighted normalised decision matrix are given as

$$v_{ji} = w_i r_{ji}$$

where $j = 1,2,\ldots,N$; $i = 1,2,\ldots,\mathrm{K}$ and w_i is the ith attribute's weight.

The weighting process is ended by normalizing the relative importance and obtaining the weights. The relative importance is given as $\{r_1, r_2, \ldots, r_K\}$ and the weights are designated as $\{w_1, w_2, \ldots, w_K\}$. The standard normalization is calculated as

$$w_i = \frac{r_i}{\sum_{i=1}^{K} r_i}, i = 1,2,\ldots,K,$$

where $0 \leq w_i \leq 1$

$\sum_{i=1}^{K} w_i = 1$

In the next steps, the positive-ideal solution (A^+) and the negative-ideal solution (A^-) are identified from the perspective of the weighted normalized value as

$$A^+ = \{v_1^+, v_2^+, \ldots, v_i^+, \ldots, v_k^+\},$$

where

$$v_i^+ = \left\{\max_j v_{ji}, i \in J_B;\ \min_j v_{ji}, i \in J_C\right\},$$

and

$$A^- = \{v_1^-, v_2^-, \ldots, v_i^-, \ldots, v_k^-\},$$

where

$$v_i^- = \left\{\min_j v_{ji}, i \in J_B;\ \max_j v_{ji}, i \in J_C\right\},$$

where $J_B =$ the aggregation of benefit criteria

$J_C =$ the aggregation of non-benefit (cost) criteria.

The final ranking is carried out from the measurement of the interval (separation) of each alternative from the positive-ideal and negative-ideal solutions calculated by the n-dimensional Euclidean distance. The distance of each alternative from the positive-ideal solution is defined as

$$S_j^+ = \sqrt{\sum_{i=1}^{K} \left(v_{ji} - v_i^+\right)^2},$$

where $j = 1,2, \dots, N$.

In the same way, the separation of each alternative from the negative-ideal solution is performed by the following formula

$$S_j^- = \sqrt{\sum_{i=1}^{K} \left(v_{ji} - v_i^-\right)^2}$$

where $j = 1,2, \dots, N$.

In the end, the relative closeness of each alternative A_j with respect to A^+ is obtained by the following equation.

$$C_j^+ = \frac{S_j^-}{S_j^+ + S_j^-}$$

where 0≤ C_j^+ ≤1 for $j = 1,2, \dots, N$.

As a consequence, when C_j^+ has the largest value, it is the most reasonable solution among every alternative. The fact that A_j is an ideal alternative means that C_j^+ is close to 1. Contrariwise, in case C_j^+ is closed to 0, it indicates that the alternative is non-ideal.

5. Case study

5.1. Introduction

The aim of the fuzzy MCDM problem is selecting the best maintenance method for the problem of ship maintenance. There are five alternatives proposed to a case study of the maintenance problem of a marine diesel engine. These alternatives represent to the five different maintenance methods, those are corrective (X_1), preventive (X_2), reliability centred maintenance (RCM) (X_3), condition based maintenance (CBM) (X_4) and computerized maintenance management system (CMMS) (X_5). The selection of decision among of them is assessed regarding ten different attributes, which are based on six subjective attributes and seven objective attributes. In more detail,

1. Maintenance cost (A_1). The cost of maintaining an item in good condition or good work order status. In other words, the expense of regularly checking and repairing when necessary to keep the machine in good condition. This attribute means the total cost when comparing diverse maintenance solutions,

2. The efficiency of maintenance (A_2). Maintenance efficiency is a measure of the maintenance effort required to provide the required level of performance on the equipment,

3. Reliability (A_3) means the accretion in the system reliability after maintenance approach. Reliability refers to the capability of a system or component to function under fixed conditions for a designated term. In other words, this attribute is the ability of a component or system to perform a function at a specified moment or interval of time. It concentrates on the costs of system failures, spare parts costs, fixing machinery, crew and warranty costs,

4. Management commitment (A_4) should aims to improve quality, performance and cost savings. This attribute represents a participation of senior management teams to support maintenance exertions,

5. Crew training (A_5). This attribute emphasizes a crew training required to gain expertise in using the equipment for maintenance performance. For this attribute, the experts will assess how much crew training is needed for the maintenance method,

6. Operation loss (A_6). The last performance attribute takes into account the range of operating losses that can happen if a particular maintenance approach is chosen,

7. Engine speed (Average RPM) (A_7). RPM (Revolution per minute) refers to the change of engine average speed affects maintenance methods [83]. This study has applied the Normal Continuous Rating (NCR), which enables the engine to operate the most efficiently, economically and with minimal maintenance. The steady state of the NCR is set between 66 rpm and 74 rpm,

8. Exhaust gas temperature (A_8). The sensors of exhaust gas temperature utilized to estimate this parameter are considered the most sensitive features of the whole turbine engine measurements. Exhaust gas temperature instrumentation is regarded as an important criterion for increasing a fuel economy and improving diagnosis and prognosis. This is because the temperature of a turbine blade is a great barometer of the standard life span of the blade [84]. Normal range of exhaust gas temperature in this study is from 66℃ to 74℃,

9. The viscosity of Fuel Oil (A_9). The viscosity of the fuel oil is a critical parameter in the combustion quality inside the diesel engine. It is important to maintain the correct range of fuel oil viscosity to achieve the proper engine efficiency. The normal viscosity ranges from 11 centistokes (cSt) to 15cSt in this study,

10. Scavenging air temperature (A_{10}) and Scavenging air pressure (A_{11}). Marine diesel engines require a proper supply of fresh air. The way in which a sufficient amount of air is fed into the engine's cylinders is called scavenging. The scavenging efficiency is proportional to the fuel combustion and power output of the engine. The turbocharger uses an engine that utilizes exhaust gas to continuously provide fresh air inside the main engine. Normal range of Scavenging air temperature and pressure in this study is from 34℃ to 42℃ and between 1.2kgf/cm^2 and 1.6kgf/cm^2, respectively,

11. Air cooler cooling water inlet temperature (A_{12}) and Air cooler cooling water outlet temperature (A_{13}) The lower the temperature, the less stresses on the piston, piston ring, cylinder liner and cylinder head. However, when much low temperature air comes into the cylinder liner, a sharp thermal shock which can lead the liner crack may occur. Normal range of Air cooler cooling water inlet temperature and outlet temperature in this study is from 36℃ to 40℃ and from 42℃ to 48℃, respectively.

As the objective attributes are A_7, A_8, A_9, A_{10}, A_{11}, A_{12} and A_{13}, which are classified to crisp type of assessment, the assessments of these attribute are not going to be accommodated. In addition, attributes are categorized by their contribution to the problem. This contribution refers whethe tor the attributes have a benefit attribute or a cost attribute. The last column of *Table 3* refers to the type of attribute (subjective or objective attribute). The overall objective is shown as a hierarchical structure as shown in Figure 9 to facilitate understanding. Moreover, the properties of attributes like description, attributes type and assessments type are summed up in *Table 3*.

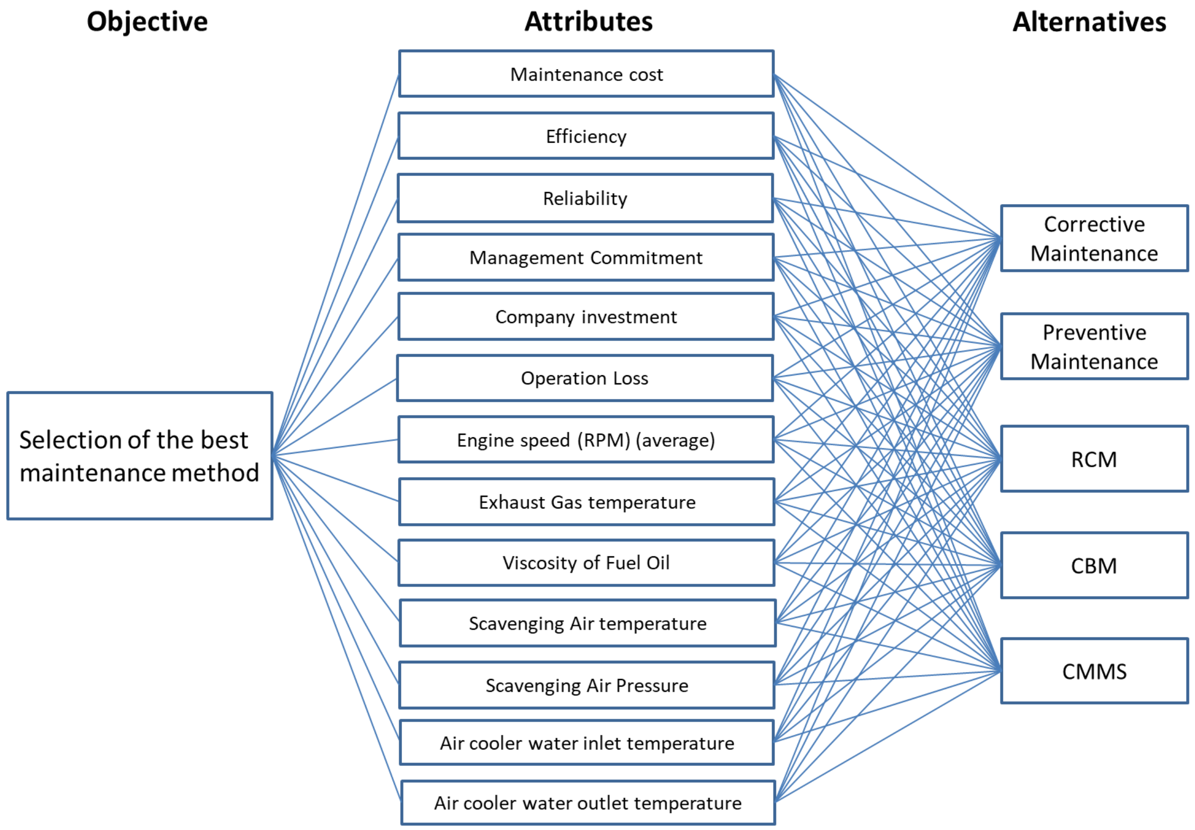


Figure 9. The layout of fuzzy multi attributes ship maintenance decision process

Table 3. characteristics of attributes employed in the case study

Attributes	Description	Type of assessment	Type of attribute	
A_1	Maintenance cost	Linguistic	Cost	Subjective
A_2	Efficiency	Linguistic	Benefit	Subjective
A_3	Reliability	Linguistic	Benefit	Subjective
A_4	Management commitment	Linguistic	Benefit	Subjective
A_5	Crew training	Linguistic	Cost	Subjective
A_6	Operation loss	Linguistic	Cost	Subjective
A_7	Engine speed (Average RPM)	Crisp		Objective
A_8	Exhaust gas temperature	Crisp		Objective
A_9	The viscosity of Fuel Oil	Crisp		Objective
A_{10}	Scavenging air temperature	Crisp		Objective
A_{11}	Scavenging air pressure	Crisp		Objective
A_{12}	Air cooler cooling water inlet temperature	Crisp		Objective
A_{13}	Air cooler cooling water outlet temperature	Crisp		Objective

5.2. Rating stage

At the beginning of the rating stage, assign relative importance factor (RI) to all alternatives. Regarding it, the most important attribute has a value of 100, whilst the other attributes have the same value or lower weighting factors than the highest attribute. Consequently, RI factors are calculated by each assigned attribute as given in the last column of *Table 4*.

For the subjective attributes, linguistic evaluations by experts from the diverse area are converted to fuzzy numbers by using Scale 7 which is from Chen and Hwang's study [85]. The alternatives in this study are assessed by three of the crew in vessel (E_1, E_2, E_3), three of manager of the shipping company (E_4, E_5, E_6) and three of constitutor of IMO (E_7, E_8, E_9) with respect to thirteen attributes consists of six subjective attributes (from A_1 to A_6) and seven objective attributes (from A_7 to A_{13}). With regard to it, the analytical hierarchy process method is employed to measure the RI (relative importance) of each expert and weighting (we) factors for each expert as given in

Table 5. In this regard, each expert has the following characteristics. A crew who worked as second engineer officer (E_1) manage the operation and maintenance for the machinery/component of a vessel following instructions by the chief engineer. Additionaly, he or she supervise the engineering crew such as 3rd engineer, oiler, etc. Two of third engineer officer (E_2, E_3) is the lower ranked experts than a 2nd engineer. They are highly interested in the specific tasks and they evaluate maintenance method using their skills and know-how. On the other hand, three of manager of the shipping company (E_4, E_5, E_6) is in charge with costs for the vessels owned by the company. They also take into account general work of ships and overall management through the ship.

The other three contributors of IMO (E_7, E_8, E_9) are responsible for regulating shipping. Accordingly, they develop and maintain an extensive regulatory framework for shipping including safety, environment, laws, and the efficiency of shipping. In terms of their role in this organization, practical attributes such as maintenance cost (A_1) or operation loss (A_6) and the all of the objective attributes (from A_7 to A_{13}) are easily neglected.

Relative importance assessed by experts has a maximum value as equal to one (1). Then the importance of the others is assigned compared to the highest weighting factor. For example, the second engineer officer (E_1) is assigned the first attribute (maintenance cost) as equal to 1, whilst one of the constitutors of IMO (E_7) is assigned this element as 0.65 for the A_1.

Table 4. Evaluated weights by experts

Attributes	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	**RI**
A_1	90	75	70	100	100	100	60	70	65	**86**
A_2	100	90	80	80	70	75	70	65	55	**80**
A_3	95	95	85	95	90	95	95	95	100	**100**
A_4	60	60	60	90	95	90	75	85	70	**81**
A_5	80	100	95	65	75	85	100	100	95	**94**
A_6	85	80	100	85	65	85	70	80	80	**86**
A_7	80	95	70	80	70	80	50	60	40	**74**
A_8	90	90	80	75	50	60	30	20	20	**61**
A_9	90	90	80	75	50	60	30	20	20	**61**
A_{10}	80	95	80	85	45	65	20	15	15	**59**
A_{11}	80	95	80	85	45	65	20	15	15	**59**
A_{12}	80	80	80	80	50	65	20	15	20	**58**

A_{13}	80	80	80	80	50	65	20	15	20	**58**

Table 5. Weighting factors and weights of attributes and experts

Attributes	RI	W	E_1		E_2		E_3		E_4	
			RI	we_1	RI	we_2	RI	we_3	RI	we_4
A_1	86	0.0901791	0.90	0.12	0.75	0.11	0.70	0.10	1.00	0.14
A_2	80	0.0833848	1.00	0.15	0.90	0.13	0.80	0.12	0.80	0.12
A_3	100	0.1043854	0.95	0.11	0.95	0.11	0.85	0.10	0.95	0.11
A_4	81	0.0846201	0.60	0.09	0.60	0.09	0.60	0.09	0.90	0.13
A_5	94	0.0982088	0.80	0.10	1.00	0.13	0.95	0.12	0.65	0.08
A_6	86	0.0901791	0.85	0.12	0.80	0.11	1.00	0.14	0.85	0.11
A_7	74	0.0772082	0.80	0.13	0.95	0.16	0.70	0.12	0.80	0.13
A_8	61	0.0636195	0.90	0.17	0.90	0.17	0.80	0.16	0.75	0.15
A_9	61	0.0636195	0.90	0.17	0.90	0.17	0.80	0.16	0.75	0.15
A_{10}	59	0.0617665	0.80	0.16	0.95	0.19	0.80	0.16	0.85	0.17
A_{11}	59	0.0617665	0.80	0.16	0.95	0.19	0.80	0.16	0.85	0.17
A_{12}	58	0.0605312	0.80	0.16	0.80	0.16	0.80	0.16	0.80	0.16
A_{13}	58	0.0605312	0.80	0.16	0.80	0.16	0.80	0.16	0.80	0.16

E_5		E_6		E_7		E_8		E_9	
RI	we_5	RI	we_6	RI	we_7	RI	we_8	RI	we_9
1.00	0.14	1.00	0.14	0.60	0.08	0.70	0.10	0.65	0.09
0.70	0.10	0.75	0.11	0.60	0.09	0.65	0.10	0.55	0.08
0.90	0.11	0.95	0.11	0.95	0.11	0.95	0.11	1.00	0.12
0.95	0.14	0.90	0.13	0.75	0.11	0.85	0.12	0.70	0.10

0.75	0.09	0.85	0.11	1.00	0.13	1.00	0.13	0.95	0.12
0.65	0.09	0.85	0.12	0.70	0.10	0.80	0.11	0.80	0.11
0.70	0.12	0.80	0.13	0.20	0.03	0.60	0.10	0.40	0.07
0.50	0.10	0.60	0.12	0.30	0.06	0.20	0.04	0.20	0.04
0.50	0.10	0.60	0.12	0.30	0.06	0.20	0.04	0.20	0.04
0.45	0.09	0.65	0.13	0.20	0.04	0.15	0.03	0.15	0.03
0.45	0.09	0.65	0.13	0.20	0.04	0.15	0.03	0.15	0.03
0.50	0.10	0.65	0.13	0.20	0.04	0.15	0.03	0.20	0.04
0.50	0.10	0.65	0.13	0.20	0.04	0.15	0.03	0.20	0.04

RI: relative importance; w: weights for the attribute; we: weighting factors for each expert

In Table 5, the AHP method is applied in order to calculate the RI of each expert and weighting factors (*we*) for each expert and overall attributes and alternatives. This method is an efficient tool for MCDM by making a difficult hierarchic decision problem at diverse layers.

A detailed description of the above table shows the evaluation of different expert groups for each attribute in the figures below.

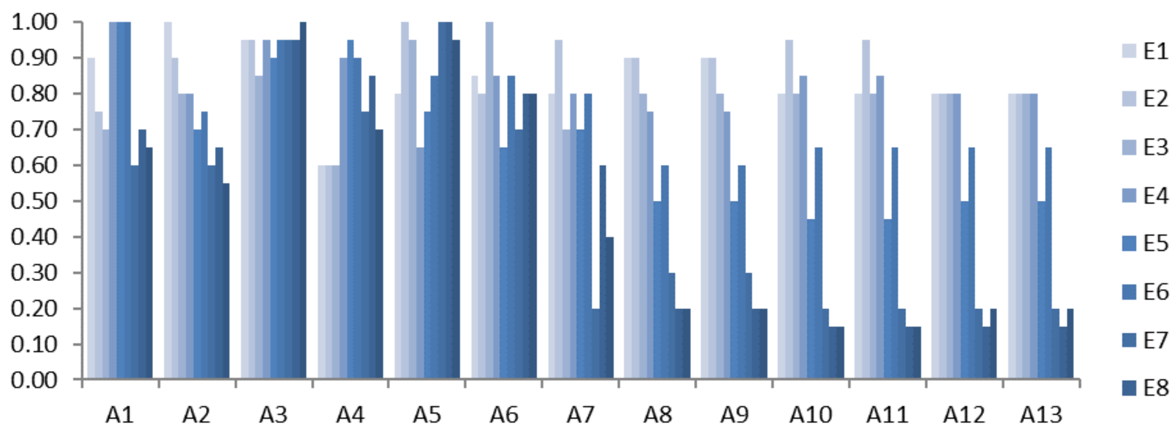


Figure 10. Weights for every attribute by each expert

As *Figure 10* shows, reliability (A_3) generally plays an important role among attributes in all expert group. On the contrary, the attributes with crisp value, which are called the objective attributes, have relatively low importance.

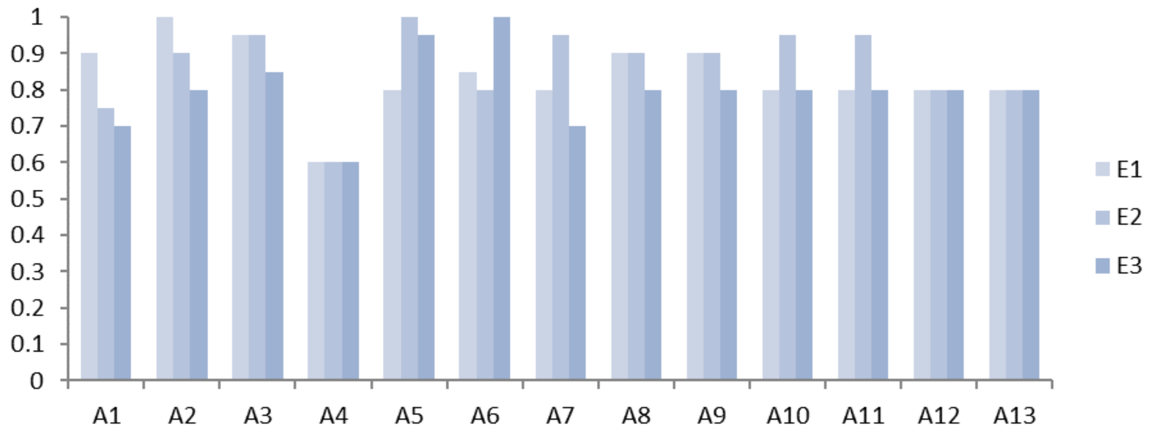


Figure 11. Weights for every attributes by experts of crew group

The selected objective attributes are the main engine parameters that have a major impact on the performance of the engine and are important factors in choosing an alternative to crews who are directly responsible for the operation, maintenance and repair of the marine engine. While all attributes have overall high importance, the fourth attribute (A_4), which represents the assistance of senior management teams, is of minor weight.

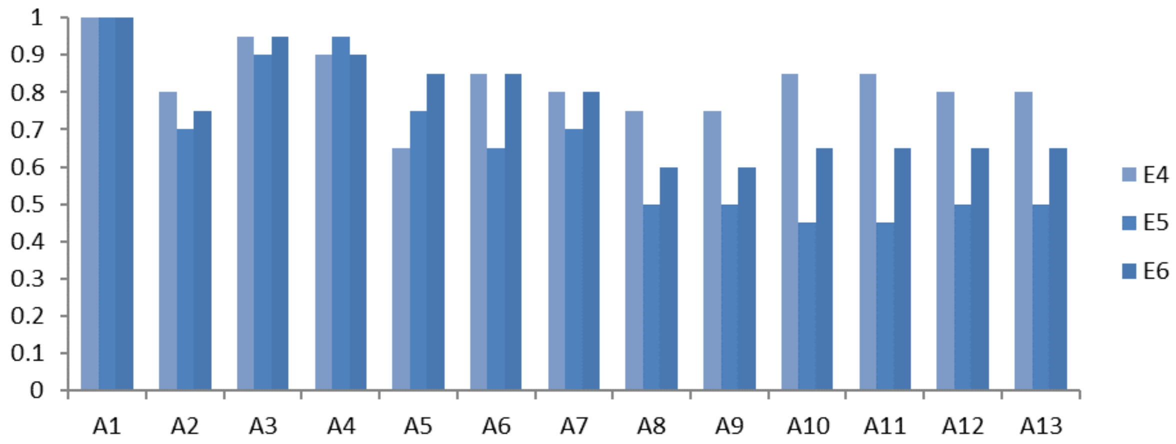


Figure 12. Weights for every attributes by experts of the shipping company group

For shipping companies targeting economic benefit, maintenance costs are the most important factor, as shown in *Figure 12*. In this respect, cost related reliability is also considered a very important attribute. Contrarily, the performance attributes other than the engine speed attribute are different for each expert in this group.

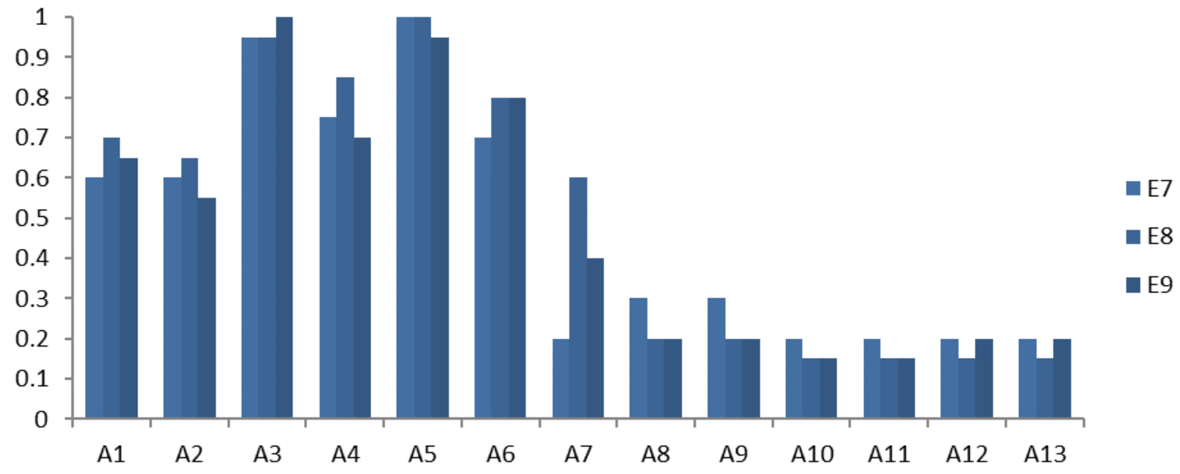


Figure 13. Weights for every attributes by experts of regulator group

In this group, reliability and crew training attributes are overwhelmingly important compared to other properties. To the contrary, properties with crisp values have a significantly lower weight.

As mentioned above, Scale 7 which has nine grade of assessment is employed. These distinct linguistic variables as follows ‘very low (VL)’, ‘low-very low (LVL)’, ‘low (L)’, ‘fairly low (FL)’, ‘medium (M)’, ‘fairly high (FH)’, ‘high (H)’, ‘high-very high (HVH)’ and ‘very high (VH)’ basically means that experts have sufficient distance to evaluate for the subjective attributes. Moreover, these linguistic variables help answers to make a decision easier. The assessments by experts are converted into fuzzy triangular definition employed for the aggregation procedure. Finally, Table 6 shows the initial term of the evaluation of each expert with respect to the maintenance attributes and the whole alternatives.

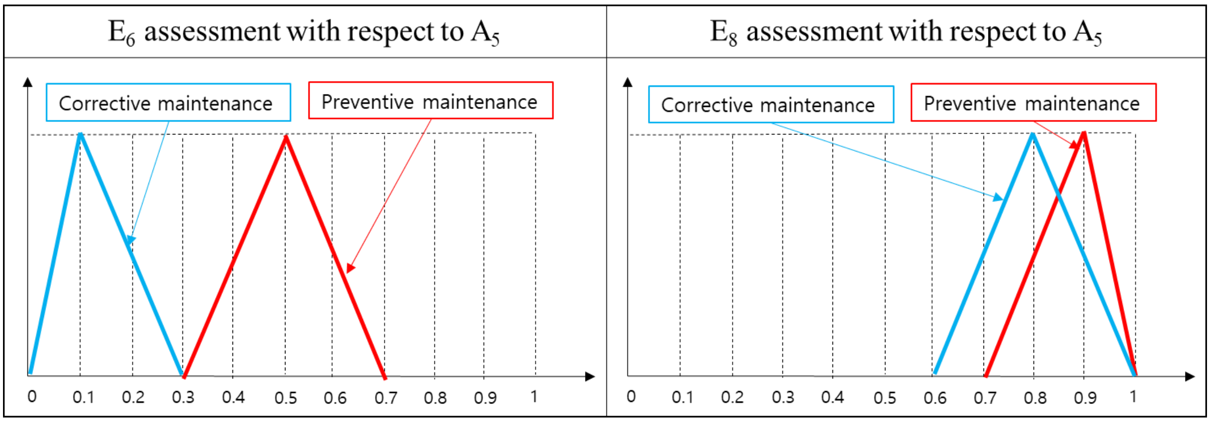


Figure 14. Assessment of E_6 & E_8 with respect to A_5 regarding X_1 & X_2

Table 6. Experts' linguistic answer per attributes and alternatives

		E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
Corrective Maintenance (X_1)	A_1	L	VH	M	H	VH	VH	M	FH	H
	A_2	LVL	L	L	VL	VL	VL	L	LVL	FL
	A_3	LVL	LVL	M	VL	VL	VL	HVH	HVH	VH
	A_4	L	L	FL	VH	VL	M	M	L	M
	A_5	M	M	LVL	VH	VL	M	HVH	HVH	HVH
	A_6	L	VL	L	VL	VL	L	M	L	M
Preventive Maintenance (X_2)	A_1	M	L	H	L	M	M	L	LVL	FL
	A_2	M	M	H	VH	M	M	M	LVL	L
	A_3	M	FH	M	VH	L	H	VH	HVH	HVH
	A_4	M	H	FH	H	M	M	FH	FL	FH
	A_5	FH	VH	FH	M	M	H	VH	VH	HVH
	A_6	FL	FL	FH	VH	M	M	FL	FL	M
Reliability Centered Maintenance (X_3)	A_1	H	M	M	M	L	VL	M	L	FL
	A_2	FH	M	M	M	VH	M	M	FL	M
	A_3	FH	FH	M	L	VH	M	VH	VH	HVH
	A_4	M	FL	M	VL	VH	VH	FL	FL	L
	A_5	H	H	M	VH	VH	M	VH	VH	HVH
	A_6	M	FH	M	M	VH	VH	M	FL	LVL
Condition Based Maintenance (X_4)	A_1	FH	HVH	FH	L	LVL	FL	M	L	LVL
	A_2	FH	HVH	M	FH	FH	M	M	FL	LVL
	A_3	FH	M	M	FH	H	FH	VH	VH	VH
	A_4	FH	L	FH	M	FH	FH	FH	M	LVL
	A_5	H	M	M	M	M	H	VH	VH	HVH
	A_6	M	FH	FH	M	FH	FH	M	M	FL
Computerized maintenance management system (X_5)	A_1	H	HVH	L	FL	L	L	FL	L	LVL
	A_2	H	HVH	M	M	FH	M	M	L	FL
	A_3	H	M	M	FL	M	FH	VH	VH	VH
	A_4	H	L	FL	M	H	FH	M	FL	L
	A_5	HVH	M	M	M	FH	M	VH	VH	HVH
	A_6	M	FH	FL	FH	FH	H	FH	FH	FL

VL(Very low): Fuzzy numbers (0,0,0.2); LVL(Low-very low): Fuzzy numbers (0,0.1,0.2); L(low): Fuzzy numbers (0,0.2,0.4); FL(Fairly low): Fuzzy numbers (0.2,0.35,0.5); M(Medium): Fuzzy numbers (0.3,0.5,0.7); FH(Fairly high): Fuzzy numbers (0.5,0.65,0.8); H(High): Fuzzy numbers (0.6,0.9,1); HVH(High-very high): Fuzzy numbers (0.7,0.9,1); VH(Very high): Fuzzy numbers (0.8,1,1);

Contrariwise, engine parameters can be expressed numerically. The ratings for these performance data, which are objective attributes of the decision problem, are not aggregated as above table. To ensure consistency of alternatives to engine data, the values with respect to each alternative have the same interval between alternatives. Specifically, all maintenance methods are distinguished from each other by the same interval based on preventive maintenance in which maintenance is carried out at a fixed time, that is, maintenance is carried out irrespective of engine parameter variation. For instance, the numeric range of preventive maintenance (X_2) for the steady state of NCR is assumed to be set between 66 rpm and 74 rpm, and the distance between solutions in this attribute is 1. Corrective maintenance, which is expressed in the range 65 to 75 with distance 1 applied, has a value outside of its normal range because maintenance is not initiated until failure occurs. On the other hand, predictive maintenance, which is sensitive to changes in engine state, decreases the interval by 1 in RCM to CMMS, finally reaching 69 to 71 in the most sensitive CMMS.

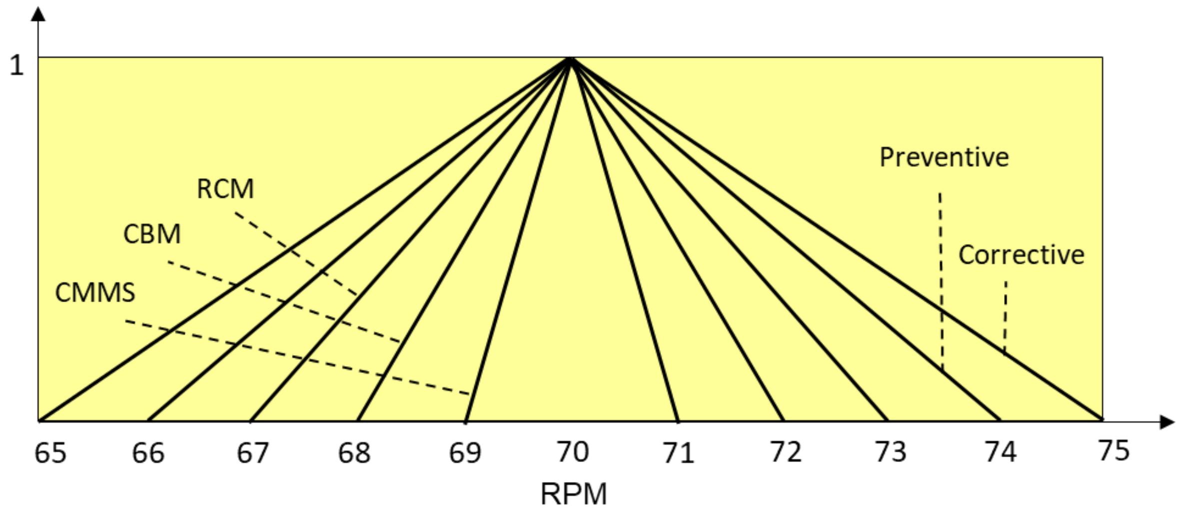


Figure 15. Numerical data for the five maintenance methods about 'Engine speed (A7)'

5.3. Aggregation stage

In the aggregation stage, all answers from experts are aggregated for each attribute and each maintenance approaches. Firstly, the degree of agreement (S) can be measured by using the triangular fuzzy numbers. Based on this, the agreement matrix (AM) is constructed next, followed by the average degree of agreement (AA) for each attribute. To the next, the relative degree of agreement (RA) and the consensus degree coefficient (CC) is computed in order. Finally, the matrix of triangular fuzzy number aggregation result (R) of each expert is constructed. For instance, *Table 7* shows aggregation computations for the operation loss attribute. These aggregations comprise a degree of agreement, the average degree of agreement, the relative degree of agreement, consensus degree of coefficient and results of aggregation. During the entire procedure of this stage, the beta coefficient (β) is assumed to be 0.5. The β means the influence of the facilitator that affects to initial weighting factors for each attribute and relative degree of importance. Meanwhile, the assessment of experts with respect to the 'Management commitment (A_4)' for the third alternative (X_3) is shown in *Figure 16*.

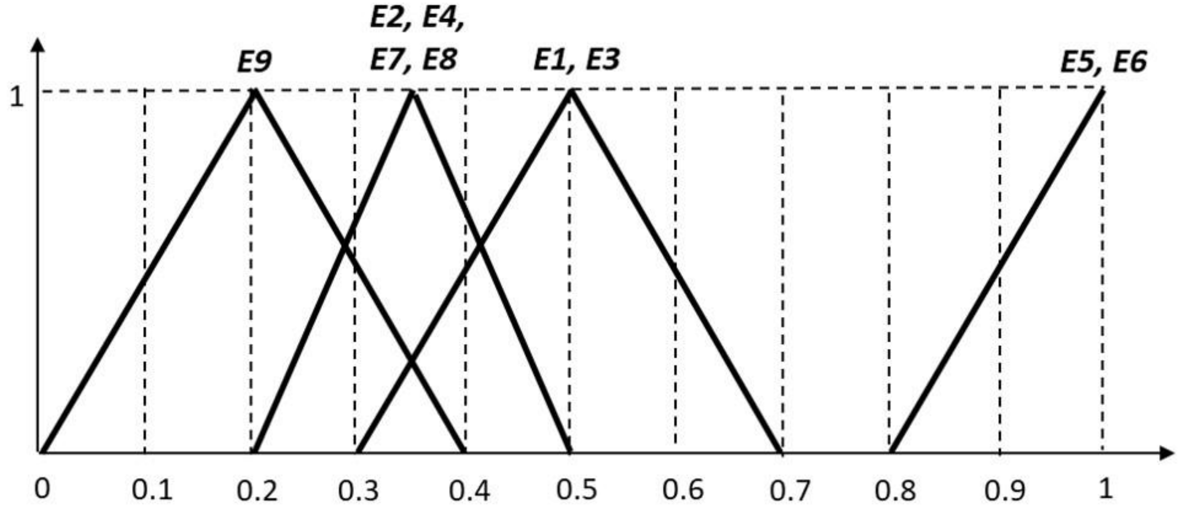


Figure 16. Assessment for RCM with respect to 'Management commitment (A_4)' by 9 experts

Table 7. Final ranking for solutions of operation loss attribute (A_6) (β =0.5)

	X_1	X_2	X_3	X_4	X_5
E_1	(0.5,0.65,0.8)	(0.3,0.5,0.7)	(0,0.1,0.3)	(0,0.1,0.3)	(0,0.1,0.3)
E_2	(0.6,0.8,1)	(0.5,0.65,0.8)	(0,0.2,0.4)	(0,0.1,0.3)	(0,0.1,0.3)
E_3	(0.5,0.65,0.8)	(0.5,0.65,0.8)	(0,0.2,0.4)	(0,0.1,0.3)	(0,0,0.2)
E_4	(0.6,0.8,1)	(0,0.1,0.3)	(0,0.2,0.4)	(0,0.1,0.3)	(0,0.1,0.3)
E_5	(0.5,0.65,0.8)	(0.2,0.35,0.5)	(0,0.1,0.3)	(0,0,0.2)	(0,0,0.2)
E_6	(0.5,0.65,0.8)	(0.2,0.35,0.5)	(0,0.2,0.4)	(0,0.1,0.3)	(0,0,0.2)
E_7	(0.5,0.65,0.8)	(0.3,0.5,0.7)	(0,0.2,0.4)	(0,0.1,0.3)	(0,0.1,0.3)
E_8	(0.3,0.5,0.7)	(0.3,0.5,0.7)	(0,0.2,0.4)	(0,0.2,0.4)	(0,0.1,0.3)
E_9	(0.3,0.5,0.7)	(0.2,0.35,0.5)	(0,0,0.2)	(0,0,0.2)	(0,0,0.2)
Degree of agreement (S)					
S_{12}	0.850	0.850	0.933	1.000	1.000
S_{13}	1.000	0.850	0.933	1.000	0.933
S_{14}	0.850	0.633	0.933	1.000	1.000

S_{15}	1.000	0.850	1.000	0.933	0.933
S_{16}	1.000	0.850	0.933	1.000	0.933
S_{17}	1.000	1.000	0.933	1.000	1.000
S_{18}	0.850	1.000	0.933	0.933	1.000
S_{19}	0.850	0.850	0.933	0.933	0.933
S_{23}	0.850	1.000	1.000	1.000	0.933
S_{24}	1.000	0.483	1.000	1.000	1.000
S_{25}	0.850	0.700	0.933	0.933	0.933
S_{26}	0.850	0.700	1.000	1.000	0.933
S_{27}	0.850	0.850	1.000	1.000	1.000
S_{28}	0.700	0.850	1.000	0.933	1.000
S_{29}	0.700	0.700	0.867	0.933	0.933
S_{34}	0.850	0.483	1.000	1.000	0.933
S_{35}	1.000	0.700	0.933	0.933	1.000
S_{36}	1.000	0.700	1.000	1.000	1.000
S_{37}	1.000	0.850	1.000	1.000	0.933
S_{38}	0.617	0.850	1.000	0.933	0.933
S_{39}	0.850	0.700	0.867	0.933	1.000
S_{45}	0.850	0.783	0.933	0.933	0.933
S_{46}	0.850	0.783	1.000	1.000	0.933
S_{47}	0.850	0.633	1.000	1.000	1.000
S_{48}	0.700	0.633	1.000	0.933	1.000
S_{49}	0.700	0.783	0.867	0.933	0.933
S_{56}	1.000	1.000	0.933	0.933	1.000
S_{57}	1.000	0.850	0.933	0.933	0.933

S_{58}	0.850	0.850	0.933	0.867	0.933
S_{59}	0.850	1.000	0.933	1.000	1.000
S_{67}	1.000	0.850	1.000	1.000	0.933
S_{68}	0.850	0.850	1.000	0.933	0.933
S_{69}	0.850	1.000	0.867	0.933	1.000
S_{78}	0.850	1.000	1.000	0.933	1.000
S_{79}	0.850	1.000	1.000	0.933	1.000
S_{89}	1.000	0.850	0.867	0.867	0.933

Average degree of agreement (AA)

AA(E_1)	0.925	0.860	0.942	0.975	0.967
AA(E_2)	0.831	0.767	0.967	0.975	0.967
AA(E_3)	0.896	0.767	0.967	0.975	0.958
AA(E_4)	0.831	0.652	0.967	0.975	0.967
AA(E_5)	0.925	0.842	0.942	0.933	0.958
AA(E_6)	0.925	0.842	0.967	0.975	0.958
AA(E_7)	0.925	0.879	0.983	0.975	0.975
AA(E_8)	0.802	0.860	0.967	0.917	0.967
AA(E_9)	0.831	0.860	0.900	0.933	0.967

Relative degree of agreement (RA)

RA(E_1)	0.117	0.117	0.109	0.113	0.111
RA(E_2)	0.105	0.105	0.112	0.113	0.111
RA(E_3)	0.114	0.105	0.112	0.113	0.110
RA(E_4)	0.105	0.089	0.112	0.113	0.111

RA(E_5)	0.117	0.115	0.109	0.108	0.110
RA(E_6)	0.117	0.115	0.112	0.113	0.110
RA(E_7)	0.117	0.120	0.114	0.113	0.112
RA(E_8)	0.102	0.117	0.112	0.106	0.111
RA(E_9)	0.105	0.117	0.105	0.108	0.111
Consensus degree coefficient (CC)					
CC(E_1)	0.117	0.117	0.113	0.115	0.114
CC(E_2)	0.107	0.107	0.111	0.111	0.110
CC(E_3)	0.125	0.121	0.125	0.125	0.124
CC(E_4)	0.111	0.103	0.114	0.115	0.114
CC(E_5)	0.103	0.102	0.099	0.099	0.100
CC(E_6)	0.117	0.116	0.114	0.115	0.113
CC(E_7)	0.107	0.108	0.105	0.104	0.104
CC(E_8)	0.106	0.113	0.111	0.108	0.110
CC(E_9)	0.107	0.113	0.107	0.109	0.110
R	0.48,0.65,0.82	0.28,0.44,0.62	0.00,0.16,0.36	0.00,0.09,0.29	0.00,0.06,0.26

5.4. Selection stage

The TOPSIS method is used to gain the whole assessment/rating of the five proposed alternatives through the selection stage. Fuzzy component of the aggregated matrix must be defuzzified to estimate the alternatives. In this regard, *Table 8* shows defuzzified aggregated values, normalised ratings and weighted normalised ratings for decision makers. This process of defuzzification is the beginning stage in this stage. Then, to rank the overall alternatives coming to the defuzzification step.

The TOPSIS method is the conception that the comparison of the positive-ideal and negative-ideal solutions with respect to the diverse alternatives. In terms of ideal solutions, the positive-ideal solution (PIS) comes from the best value of criterions, whereas the negative result derived from the worst value of criterions. The largest factor among of the benefit attribute is simply the PIS and the smallest factor among of the cost attribute. Whereas, the negative ideal solution (NIS) is the reversed structure of the PIS. The separation of each one of the proposed alternatives from S_i^+(PIS) and S_i^-(NIS) is calculated with the C_i^+(final ranking of each alternative) as given in Table 9 after obtaining the PIS and NIS.

Table 8. Deffuzified aggregated values, normalised/weighted normalised ratings

		X_1	X_2	X_3	X_4	X_5
A_1	defuzzified aggregated values	0.679	0.389	0.415	0.470	0.471
	normalised ratings	0.613	0.351	0.375	0.424	0.425
	weighted normalised ratings	0.055	0.032	0.034	0.038	0.038
A_2	defuzzified aggregated values	0.178	0.489	0.549	0.563	0.677
	normalised ratings	0.153	0.421	0.473	0.485	0.583
	weighted normalised ratings	0.013	0.035	0.039	0.040	0.049
A_3	defuzzified aggregated values	0.384	0.689	0.679	0.707	0.669
	normalised ratings	0.269	0.483	0.477	0.496	0.470
	weighted normalised ratings	0.028	0.050	0.050	0.052	0.049
A_4	defuzzified aggregated values	0.393	0.586	0.496	0.487	0.445
	normalised ratings	0.362	0.540	0.457	0.449	0.410
	weighted normalised ratings	0.031	0.046	0.039	0.038	0.035
A_5	defuzzified aggregated values	0.452	0.550	0.837	0.800	0.872
	normalised ratings	0.280	0.340	0.519	0.495	0.540
	weighted normalised ratings	0.028	0.033	0.051	0.049	0.053
A_6	defuzzified aggregated values	0.629	0.453	0.217	0.162	0.133
	normalised ratings	0.756	0.545	0.261	0.195	0.159
	weighted normalised ratings	0.068	0.049	0.024	0.018	0.014
A_7	defuzzified aggregated values	12.439	14.402	18.366	24.817	46.313
	normalised ratings	0.211	0.245	0.312	0.422	0.787
	weighted normalised ratings	0.016	0.019	0.024	0.033	0.061
A_8	defuzzified aggregated values	11.572	16.667	24.910	37.742	54.809
	normalised ratings	0.157	0.226	0.337	0.511	0.742
	weighted normalised ratings	0.010	0.014	0.021	0.032	0.047
	defuzzified aggregated values	3.931	4.667	5.538	6.753	8.130
	normalised ratings	0.293	0.348	0.413	0.504	0.607

A_9	weighted normalised ratings	0.019	0.022	0.026	0.032	0.039
A_{10}	defuzzified aggregated values	6.619	8.000	12.756	16.346	19.583
	normalised ratings	0.218	0.264	0.420	0.539	0.645
	weighted normalised ratings	0.013	0.016	0.026	0.033	0.040
A_{11}	defuzzified aggregated values	1.156	1.250	1.318	1.345	1.350
	normalised ratings	0.402	0.435	0.458	0.468	0.470
	weighted normalised ratings	0.025	0.027	0.028	0.029	0.029
A_{12}	defuzzified aggregated values	9.402	13.000	13.674	18.383	19.813
	normalised ratings	0.274	0.379	0.399	0.536	0.578
	weighted normalised ratings	0.017	0.023	0.024	0.032	0.035
A_{13}	defuzzified aggregated values	8.269	11.345	11.797	16.001	25.220
	normalised ratings	0.236	0.324	0.337	0.457	0.720
	weighted normalised ratings	0.014	0.020	0.020	0.028	0.044

Table 9. Positive and negative ideal solutions with regard to attributes for the alternatives

Attributes	Positive ideal solution	Negative ideal solution
A_1	0.0317	0.0553
A_2	0.0486	0.0128
A_3	0.0518	0.0281
A_4	0.0457	0.0306
A_5	0.0275	0.0530
A_6	0.0144	0.0682
A_7	0.0608	0.0163
A_8	0.0472	0.0100
A_9	0.0386	0.0187
A_{10}	0.0399	0.0135
A_{11}	0.0290	0.0248
A_{12}	0.0350	0.0166
A_{13}	0.0436	0.0143

According to the final ranking that is acquired from the C_i^+ values as shown in *Table 10*, alternative X_4 (condition based maintenance approach) is the most proper one since it has the closest distance from the positive ideal solution and at the same time, the farthest to the negative ideal solution. As a result, the ranking C_i^+ of this is the highest among of all the proposed alternatives. On the other hand, the X_5 (computerised maintenance management system) is assessed as second preferred maintenance despite the minuscule value with the first alternative X_4. In the following section, it described that this slight difference with them can be changed regarding to the variance of the beta coefficient (β). The similarity of the value of C_i^+ is also discovered between X_2 (preventive maintenance approach) and X_3 (reliability centred maintenance). The priority of them is $X_3 > X_2$, where the third alternative is the leader and the second alternative ranked below. On the contrary, X_1 (corrective maintenance) has the smallest C_i^+ value with the largest value of S_i^+ and the smallest value of S_i^-. It proves that experts from various area (crew of the vessel, manager of the shipping company, constitutor of IMO) are not prefer to select the corrective maintenance (X_1) in the problem of ship maintenance such as a breakdown of machinery in a vessel.

Table 10. Final ranking for solutions (β =0.5)

	X_1	X_2	X_3	X_4	X_5
S_i^+	0.1058	0.0762	0.0615	0.0445	0.0287
S_i^-	0.0255	0.0515	0.0649	0.0777	0.1029
C_i^+	0.19421	0.40299	0.51362	0.63564	0.78196
Final ranking	5	4	3	2	1

5.5. Sensitivity analysis

Sensitivity analysis of β-factor is carried out to monitor the influence of facilitator towards the result of C_i^+ value. In this regard, the effect of the coordinator is reflected on the overall decision making procedure. As *Table 11* shows, β has a value in the scope between 0 and 1. The closer the β value is to 0, the less the effect on the FMAGDM procedure. Conversely, the closer to 1 the β value becomes, the more important the selection of facilitator towards initial weighting by the expertise. Therefore, the result of rating for the five alternatives is presented as *Figure 17*. According to the Figure, even if β changes from 0 to 1, in total ranking of the procedure, is no different. In other words, this case is insensitive with the beta coefficient.

However, the impact of facilitator on each alternative can be confirmed. For example, CMMS increases its value as β ascends, but X_2, X_3, and X_4 decline while X_1 is not affected by the facilitator. This means that as the beta coefficient approaches 1, the gap between the first alternative and the remaining alternatives becomes larger. Consequently, according to the sensitivity analysis, the fifth alternative (CMMS) is evaluated as the first of all other alternatives.

Table 11. values of C_i^+ in respect of β values

β	X_1	X_2	X_3	X_4	X_5
0	0.1954	0.4156	0.5195	0.6403	0.7781
0.1	0.1952	0.4131	0.5184	0.6394	0.7789
0.2	0.1949	0.4106	0.5173	0.6386	0.7797
0.3	0.1947	0.4080	0.5161	0.6376	0.7804
0.4	0.1944	0.4055	0.5149	0.6367	0.7812
0.5	0.1942	0.4030	0.5136	0.6356	0.7820
0.6	0.1940	0.4005	0.5124	0.6346	0.7827
0.7	0.1938	0.3979	0.5111	0.6334	0.7834
0.8	0.1936	0.3954	0.5097	0.6322	0.7841
0.9	0.1934	0.3929	0.5084	0.6310	0.7848
1	0.1932	0.3904	0.5070	0.6297	0.7855

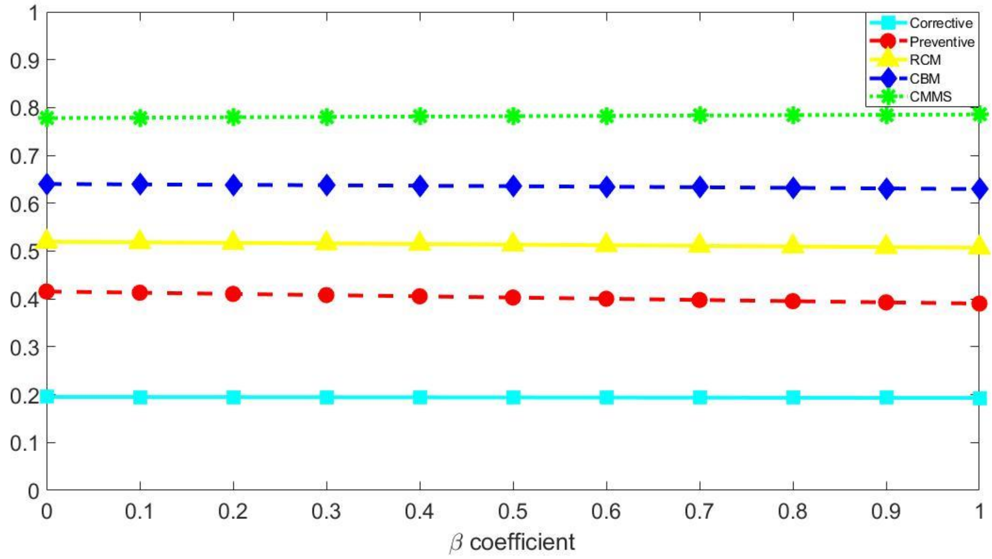


Figure 17. Sensitivity analysis for the five alternatives attributed to the beta coefficient (β)

Meantime, to investigate the effect of performance attributes on the overall decision-making technique, sensitivity analysis only on maintenance attributes is performed as *Figure 18* below. In this instance, CBM is interpreted as the best maintenance solution. Even the difference between CMMS, which was the best alternative to all the attributes considered, is a slight but widening gap as β increases. Nevertheless, the three solutions categorized as predictive maintenance do not show large disparities. This is caused by the absence of the performance attributes that make a clear numerical difference between the predictive maintenance approaches.

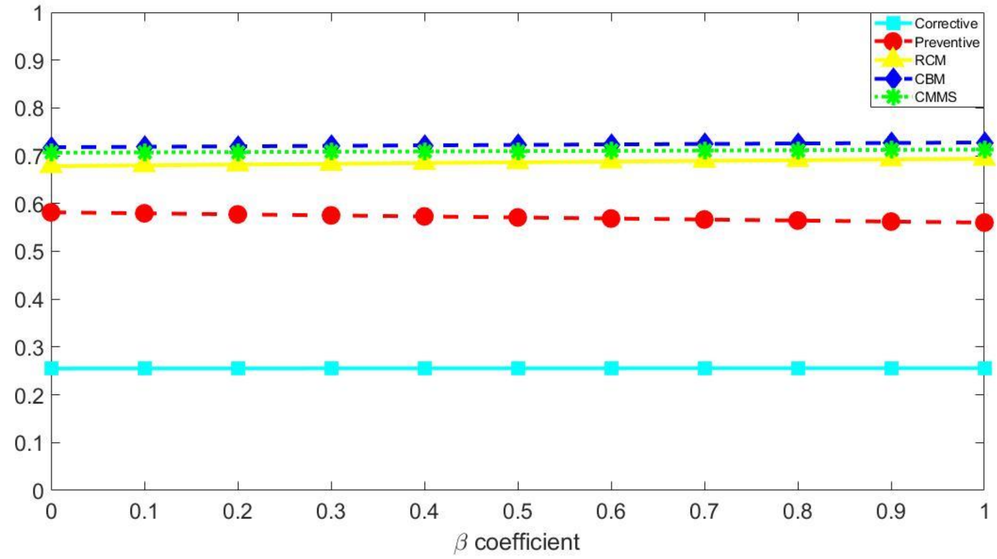


Figure 18. Sensitivity analysis with only maintenance attributes (A_1~A_6)

To the next, the results of each expert group are examined and analyse their characteristics.

In the beginning, sensitivity analysis with respect to the crew group is executed as shown in Figure 19. Compared to the results of other expert groups, the gap between CMMS which is the best maintenance and other methods is bigger. This is because the CMMS received higher scores for all performance attributes than the other methods and the crews of this group all gave an outstanding weight to the performance data.

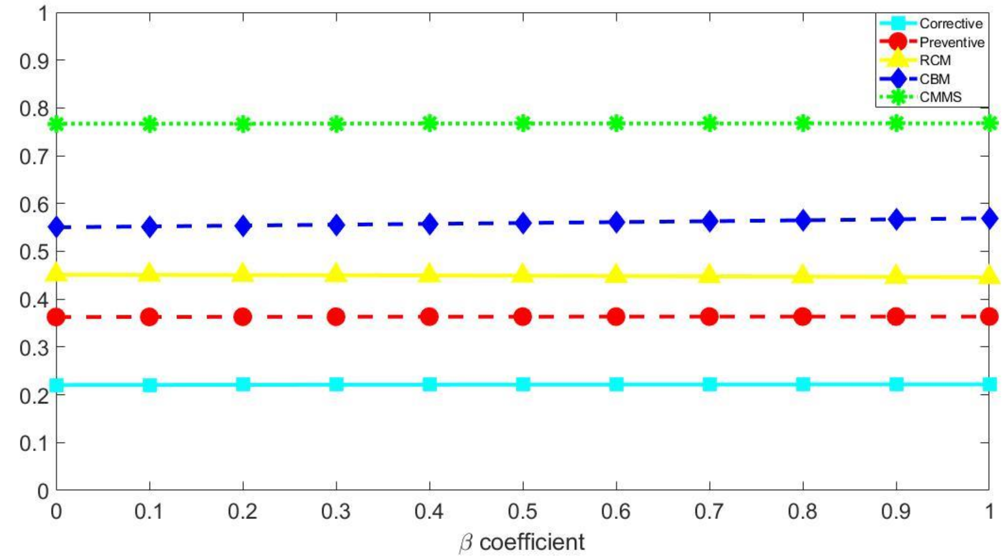


Figure 19. Sensitivity analysis with respect to the crew group ($E_1 \sim E_3$)

Secondly, in the shipping company experts' case, as with the sensitivity results for all experts, CMMS is the best alternative. At the same time, the corrective maintenance value decreases drastically as the β value increases since maintenance cost (A_1) is the important attribution for the marine corporation. Most experts in this study determine that the expense of maintenance for the first alternative (corrective maintenance) is high. In addition, CBM, RCM, and preventive maintenance slightly escalate as the beta value changes from 0 to 1, while CMMS shows a relatively big increase. This result is mainly due to the weight of the first, third and fifth attributes. The shipping company's experts gave a high weight to the maintenance cost (A_1), with CMMS having a much lower linguistic variable than the other alternatives, and experts weighted lower importance to crew training (A_5), but the fifth alternative (X_5) requires more training than other alternatives. In terms of the third attribute, it is given a great weight and also has a more significant linguistic element than the other solutions.

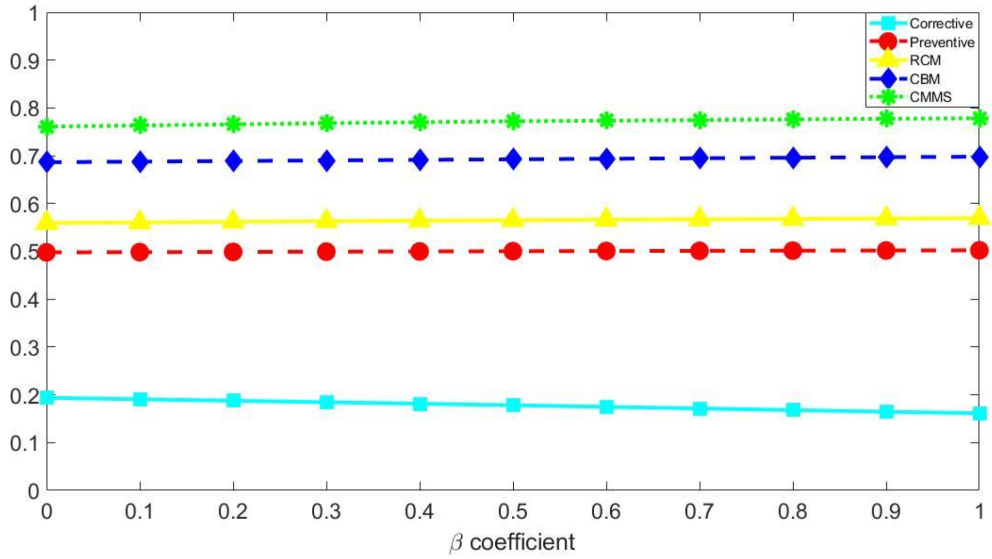


Figure 20. Sensitivity analysis with respect to the shipping company group ($E_4 \sim E_6$)

Finally, decision maker group for the regulator rank the same alternatives as other expert groups and show little difference between the top alternative and the remaining alternatives compared to the final result which is considered all experts. In particular, the best maintenance method is undervalued because regulators have very little importance on performance attributes. Likewise, corrective maintenance is relatively high as engine parameters are not considered significant.

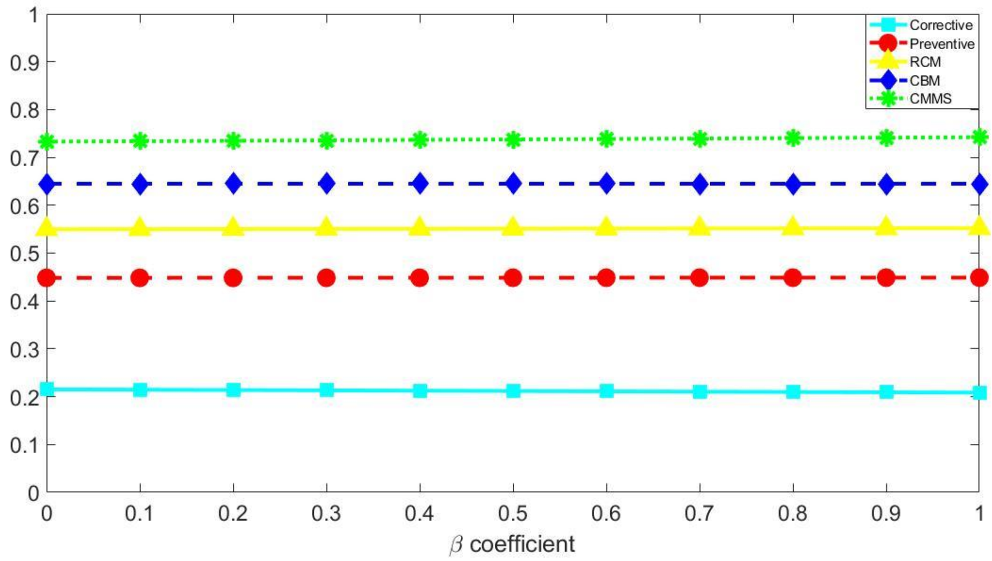


Figure 21. Sensitivity analysis with respect to the regulator group ($E_7 \sim E_9$)

6. Discussion

The experts included in this paper are divided into three groups: crew, shipping company, and regulator. Among them, crew group obtained questionnaire results from three professionals with boarding experience and the data from the shipping company's experts were obtained from the study of I. Lazakis (2014). However, the evaluation of the regulator is a hypothetical data based on the authors' conjecture, and this section expects more reliable results based on the evaluation of real experts in future studies.

FMAGDM using the TOPSIS method is applied where fuzzy and non-fuzzy evaluation is required. The MADGM problem can have data in the form of linguistic terms, fuzzy numbers and/or crisp numbers by the proposed approach. It makes decision models that are more realistic, accurate, and reliable than the existent technique. The proposed method provides a method to systematically compile expert opinions according to the preference of mediators using triangular fuzzy numbers.

In this study, it was confirmed that a large number of crisp attributes, which were not covered much in previous studies, could be considered to be reasonable even if they were considered together with the attributes of linguistic variables.

The evaluations of the alternatives in each expert group are almost identical to the aggregate results (with nine experts). In addition, there is no significant difference in the comparison between the expert groups. The consistently large gaps in the range of performance data for each alternative had a significant impact on the results. Furthermore, one expert group does not have significantly lower weights for other specialist groups for certain attributes with high weights. As the maintenance technique becomes more sophisticated and the awareness of maintenance becomes higher, a similar evaluation is made except for some characteristic features of each expert group. This problem can be improved by further subdividing the attributes or by adding attributes that represent the characteristics of each expert group.

In addition to the existing research that finds appropriate alternatives for a particular group, it also considers multiple groups at the same time to derive optimal solutions. It is expected to present a compromise plan for various organizations that have conflicts of interest in any future issues. From the perspective of ship maintenance and repair, this FMAGDM technique is applicable to various realistic issues related to ship maintenance as a generalized model.

7. Conclusion and Future work suggestions

This paper presents a practical FMAGDM technique that is effective in dealing with the multi-attribute decision problem of subjective and inaccurate information. This classical FMAGDM method is based on fuzzy set theory and AHP as reviewed in Chapter 4. Analytical hierarchy process is implemented to support the weight of attributes in multiple attribute decision processes.

A detailed review of various maintenance methodologies is conducted to identify and analyse the advantages and disadvantages identified in the existing maintenance system. Besides, the numerical analysis of ambiguity and uncertainty is considered by fuzzy numbers. It provides an approach to systematically compile subjective opinion of experts using triangular fuzzy numbers. In terms of employing of attribute, not only maintenance attributes such as maintenance cost, reliability, and crew training, but also performance data that directly affects engine performance and failure are considered. All these attributes are applied as criteria for the five maintenance approaches.

The proposed method allows to integrate and aggregate subjective views of expert groups. In this respect, alternatives are estimated by collecting the evaluations of various expert groups with diverse interests. This helps rational decision-making in complex environments where the goals and objectives of other groups are different. The suggested method is uncomplicated to use, easy to understand, and can import data in the linguistic variables, fuzzy numbers, or crisp numbers format. This provides a more reasonable and dependable model than existent decision problems.

According to a case study of the maintenance problem of a marine diesel engine, it appears that CMMS is the best maintenance approach. Unlike the general decision making of maintenance method considering only the management attribute, the performance attribute is considered together to produce a different result. Additionally, although final ranking is not changed by the beta coefficient, a significant result can be found that the difference between the best method and some other maintenance approaches increases as the beta value increases.

For future work, it is possible to obtain credible results by collecting actual opinions of experts in each field rather than virtual data. The proposed methodology will be extended to specific and practical the problem of ship maintenance. Regarding that, alternatives for a case can be not only maintenance approaches but also ship’s technical system. In addition, the FMAGDM method can be employed in any area where decision-making is required. By combining the evaluation of different decision-makers groups in the complex decision-making problem of multiple variables, this approach can be widely applied as a useful tool for presenting a clear solution.

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Appendix

Matlab Code

```
% Experts's decision

E1 = ...
  [0    0    0.2 0.3 0.5 0.7 0.5 0.65    0.8 0.5 0.65    0.8 0.6 0.9 1
0    0.1 0.3 0.3 0.5 0.7 0.5 0.65    0.8 0.5 0.65    0.8 0.6 0.8 1
0    0.1 0.3 0.3 0.5 0.7 0.5 0.65    0.8 0.5 0.65    0.8 0.6 0.9 1
0    0.2 0.4 0.3 0.5 0.7 0.3 0.5 0.7 0.5 0.65    0.8 0.6 0.9 1
0.2 0.35    0.5 0.2 0.35    0.5 0.6 0.8 1    0.6 0.8 1    0.7 0.9 1
0.5 0.65    0.8 0.3 0.5 0.7 0    0.1 0.3 0    0.1 0.3 0    0.1 0.3
65.5    70   75.2    66   70   74   67.6    70   73   68   70   72.4    69.2    70
70.4
315 340 365 320 340 360 325 340 355 330 340 350 335 340 345
10.5    13   15.5    11   13   15   11.5    13   14.5    12   13   14   12.5    13
13.5
33   38   43   34   38   42   35   38   41   36   38   40   37   38   39
1.15    1.4 1.65    1.2 1.4 1.6 1.25    1.4 1.55    1.3 1.4 1.5 1.35    1.4
1.45
35.5    38   40.5    36   38   40   36.5    38   40.5    37   38   41   37.5    38
40.5
40   45   50   41   45   49   42   45   48   43   45   47   44   45   46
];

E2 = ...
  [0.8   1    1    0    0.2 0.4 0.3 0.5 0.7 0.7 0.9 1    0.7 0.9 1
0    0.2 0.4 0.3 0.5 0.7 0.3 0.5 0.7 0.7 0.9 1    0.7 0.9 1
0    0.1 0.3 0.5 0.65    0.8 0.5 0.65    0.8 0.3 0.5 0.7 0.3 0.5 0.7
0    0.2 0.4 0.6 0.9 1    0.2 0.35    0.5 0    0.2 0.4 0    0.2 0.4
0.2 0.35    0.5 0.2 0.35    0.5 0.6 0.8 1    0.8 1    1    0.8 1    1
0.6 0.8 1    0.5 0.65    0.8 0    0.2 0.4 0    0.1 0.3 0    0.1 0.3
65.2    70   75.3    66   70   74   67.5    70   73.1    68.4    70   72.7
69.9    70   70.3
319 341.5    364 320 340 360 326 340.5    355 334 340 351 336.4    340 349.2
10.5    13   16.2    11   13   15   11.4    13   14.5    12.1    13   13.6
12.3    13   13.4
31.9    38   44   34   38   42   35.5    38   40   36.8    38   39.2    37   38
38.9
1.1 1.4 1.9 1.2 1.4 1.6 1.33    1.4 1.52    1.34    1.4 1.46    1.34    1.4
1.46
35   38   42   36   38   40   36.5    38   40.2    37   38   39   37.2    38   39
40   45   50   42   45   48   42.5    45   47.5    43.5    45   46.7    44.3    45
45.9
];

E3 = ...
  [0.3   0.5 0.7 0.6 0.9 1    0.3 0.5 0.7 0.7 0.9 1    0.7 0.9 1
0    0.2 0.4 0.6 0.8 1    0.3 0.5 0.7 0.3 0.5 0.7 0.3 0.5 0.7
0.3 0.5 0.7 0.3 0.5 0.7 0.3 0.5 0.7 0.3 0.5 0.7 0.3 0.5 0.7
0.2 0.35    0.5 0.5 0.65    0.8 0.3 0.5 0.7 0.5 0.65    0.8 0.2 0.35    0.5
0.2 0.35    0.5 0.2 0.35    0.5 0.8 1    1    0.8 1    1    0.8 1    1
0.5 0.65    0.8 0.5 0.65    0.8 0    0.2 0.4 0    0.1 0.3 0    0    0.2
65.3    70   75.1    66.5    70   74.3    67   70   73.3    68.2    70   72.5
69.7    70   70.2
316 342 368 320 340 360 328 341 354 333.5    340 350.2    336.6    340 349.6
```

```
10.4	13	16.5	11	13	15	11.2	13	14.4	11.8	13	13.9
12.2	13	13.7
32.1	38	45	34	38	42	35.8	38	38.9	36.3	38	39.4	36.8
38	38.8
1.05	1.4	2	1.2	1.4	1.6	1.32	1.4	1.54	1.34	1.4	1.46	1.34
1.4	1.46
34.5	38	42	36	38	40	36.5	38	40.6	37	38	39	37.3	38	39
40.5	45	49.9	42	45	48	42	45	48	43	45	47	44.5	45	46
];

E4 = ...
  [0.6	0.9	1	0	0.2	0.4	0.3	0.5	0.7	0.5	0.65	0.8	0.5	0.65	0.8
0	0	0.2	0.5	0.65	0.8	0.5	0.65	0.8	0.5	0.65	0.8	0.8	1	1
0	0	0.2	0.8	1	1	0	0.2	0.4	0.5	0.65	0.8	0.2	0.35	0.5
0.8	1	1	0.6	0.8	1	0.2	0.35	0.5	0.2	0.35	0.5	0	0	0.2
0	0	0.2	0.5	0.65	0.8	0.8	1	1	0.3	0.5	0.7	0.6	0.8	1
0.6	0.8	1	0	0.1	0.3	0	0.2	0.4	0	0.1	0.3	0	0.1	0.3
65.5	70	75	66.2	70	74.4	67	70	73.1	68.6	70	72.5
69.4	70	71
318	341.5	365	320	340	360	328	340.5	353	333.6	340	350.9	336.8	340
349.2
10.8	13	16.1	11	13	15	11.2	13	14.3	12.2	13	13.95
12.4	13	13.65
32.5	38	43.5	34	38	42	35.3	38	39.4	36.6	38	39	36.9
38	38.8
1	1.4	1.9	1.2	1.4	1.6	1.3	1.4	1.5	1.34	1.4	1.46	1.34	1.4	1.46
35	38	41	36	38	40	36.5	38	40.3	37	38	39	37.3	38	39
40	45	51	42	45	48	42	45	48	43.2	45	46.6	44.2	45	45.2
];

E5 = ...
  [0.8	1	1	0.3	0.5	0.7	0	0.2	0.4	0	0	0.2	0	0.2	0.4
0	0	0.2	0.3	0.5	0.7	0.7	0.9	1	0.7	0.9	1	0.7	0.9	1
0	0	0.2	0	0.2	0.4	0.8	1	1	0.6	0.8	1	0.3	0.5	0.7
0	0	0.2	0.3	0.5	0.7	0.8	1	1	0.5	0.65	0.8	0.6	0.8	1
0.6	0.8	1	0.3	0.5	0.7	0.8	1	1	0.3	0.5	0.7	0.7	0.9	1
0.5	0.65	0.8	0.2	0.35	0.5	0	0.1	0.3	0	0	0.2	0	0	0.2
66.1	70	74.6	66	70	74	66.9	70	73.5	68.9	70	71.5
69.6	70	70.4
318	341	364	320	340	360	327	340.5	354	334.2	340	350.3	336.2	340
349.8
10.7	13	16	11	13	15	11.3	13	14.1	12	13	14.2	12.45	13
13.6
32.5	38	43	34	38	42	35.2	38	39	36.5	38	39.1	37	38
38.6
1	1.4	1.85	1.2	1.4	1.6	1.3	1.4	1.5	1.34	1.4	1.46	1.34	1.4
1.46
35	38	41.5	36	38	40	36.5	38	40.1	37	38	39	37.5	38	39
41	45	49	42	45	48	42	45	48	43	45	46.4	44.6	45	45.5
];

E6 = ...
  [0.6	0.8	1	0.3	0.5	0.7	0	0	0.2	0.2	0.35	0.5	0	0.2	0.4
0	0	0.2	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.8	1	1
0	0	0.2	0.6	0.8	1	0.3	0.5	0.7	0.5	0.65	0.8	0.5	0.65	0.8
0.3	0.5	0.7	0.3	0.5	0.7	0.8	1	1	0.5	0.65	0.8	0.5	0.65	0.8
0	0.1	0.3	0.3	0.5	0.7	0.5	0.65	0.8	0.6	0.8	1	0.7	0.9	1
0.5	0.65	0.8	0.2	0.35	0.5	0	0.2	0.4	0	0.1	0.3	0	0	0.2
65.3	70	75.2	66	70	74	67.4	70	72.9	68.3	70	71.8
69.6	70	70.8
```

```
317.5   341 364.5   320 340 360 326 340.5   355 333.4   340 350.5   336.5
340 349.4
10.7    13  16.1    11  13  15  11.2    13  14  11.9    13  13.9    12.4
13  13.6
33  38  43  34  38  42  35  38  39.9    36  38  38.7    37.2    38  39.2
1.1 1.4 1.95    1.2 1.4 1.6 1.31    1.4 1.51    1.34    1.4 1.46    1.34
1.4 1.46
35  38  42  36  38  40  36.5    38  39.7    37  38  39  37.1    38  39
40  45  50  42  45  48  42  45  48  43.2    45  47.1    44.2    45  45.9
];

E7 = ...
  [0.3  0.5 0.7 0   0.2 0.4 0.3 0.5 0.7 0.3 0.5 0.7 0.2 0.35    0.5
0   0.2 0.4 0.3 0.5 0.7 0.3 0.5 0.7 0.3 0.5 0.7 0.3 0.5 0.7
0.7 0.9 1   0.8 1   1   0.8 1   1   0.8 1   1   0.8 1   1
0.3 0.5 0.7 0.5 0.65    0.8 0.2 0.35    0.5 0.3 0.5 0.7 0.3 0.5 0.7
0.5 0.65    0.8 0.6 0.8 1   0.8 1   1   0.8 1   1   0.8 1   1
0.5 0.65    0.8 0.3 0.5 0.7 0   0.2 0.4 0   0.1 0.3 0   0.1 0.3
65.6    70  75.2    66  70  74  67.8    70  73.9    68.2    70  71.6    69
70  70.7
318 341.5   365 320 340 360 328 340.5   353 333.2   340 350 336.5   340
349.4
11  13  16  11  13  15  11.2    13  14.1    11.85   13  14.1    12.3    13
13.7
34  38  42  34  38  42  35  38  39  36.5    38  39  37  38  39
1   1.4 1.9 1.2 1.4 1.6 1.32    1.4 1.52    1.34    1.4 1.46    1.34    1.4
1.46
36  38  42  36  38  40  36.5    38  40.6    37  38  39  37  38  39
41  45  49  42  45  48  42  45  48  43  45  47  44  45  46
];

E8 = ...
  [0.5  0.65    0.8 0   0   0.2 0   0.2 0.4 0   0.2 0.4 0   0.2 0.4
0   0.1 0.3 0   0.1 0.3 0.2 0.35    0.5 0.2 0.35    0.5 0   0.2 0.4
0.7 0.9 1   0.7 0.9 1   0.8 1   1   0.8 1   1   0.8 1   1
0   0.2 0.4 0.2 0.35    0.5 0.2 0.35    0.5 0.3 0.5 0.7 0.2 0.35    0.5
0.6 0.8 1   0.7 0.9 1   0.8 1   1   0.8 1   1   0.8 1   1
0.3 0.5 0.7 0.3 0.5 0.7 0   0.2 0.4 0   0.2 0.4 0   0.1 0.3
65.3    70  75  66  70  74  67.3    70  73  68.5    70  72  69.3    70
70.3
318 341.5   365 320 340 360 328 340.5   353 334 340 350 336.5   340 349.4
11  13  16.2    11  13  15  11.2    13  14.1    11.7    13  14  12.1    13
13.7
34  38  42  34  38  42  35  38  39.5    36.5    38  39  37  38  39
1   1.4 1.8 1.2 1.4 1.6 1.32    1.4 1.52    1.34    1.4 1.46    1.34    1.4
1.46
35  38  42  36  38  40  36.5    38  40.5    37  38  39  37  38  39
41  45  49  42  45  48  42  45  48  43  45  47  44  45  46
];

E9 = ...
  [0.6  0.8 1   0.2 0.35    0.5 0.2 0.35    0.5 0   0   0.2 0   0   0.2
0.2 0.35    0.5 0   0.2 0.4 0.3 0.5 0.7 0   0.1 0.3 0.2 0.35    0.5
0.8 1   1   0.7 0.9 1   0.7 0.9 1   0.8 1   1   0.8 1   1
0.3 0.5 0.7 0.5 0.65    0.8 0   0.2 0.4 0   0.1 0.3 0   0.2 0.4
0.3 0.5 0.7 0.5 0.65    0.8 0.7 0.9 1   0.8 1   1   0.8 1   1
0.3 0.5 0.7 0.2 0.35    0.5 0   0   0.2 0   0   0.2 0   0   0.2
65.2    70  75.2    66  70  74  67.7    70  72.8    68.3    70  71.9    69
70  70.3
318 341.5   365 320 340 360 328 340.5   353 332 340 350 336.5   340 349.4
```

```
10.9     13  16.2     11  13  15  11.2     13  14.1     11.85    13  14.1
12.2     13  13.8
34  38  42  34  38  42  35  38  39  36.5     38  39  37  38  39
1   1.4 1.8 1.2 1.4 1.6 1.32     1.4 1.52     1.34     1.4 1.46     1.34     1.4
1.46
36  38  42  36  38  40  36.5     38  40.5     37  38  39  37  38  39
41  45  50  42  45  48  42  45  48  43  45  47  44  45  46
];


% Beta deifinition
b=0.9;
%%b=0:0.1:1;

% AA definition of Attributes
a1AA=[0.456                                 0.773
0.519                                 0.688
0.575
0.719                                 0.773
0.810                                 0.552
0.554
0.690                                 0.481
0.810                                 0.552
0.554
0.781                                 0.773
0.810                                 0.688
0.644
0.719                                 0.773
0.735                                 0.548
0.663
0.794                                 0.773
0.619                                 0.688
0.663
0.738                                 0.775
0.792                                 0.723
0.698
0.746                                 0.656
0.735                                 0.631
0.663
0.813                                 0.794
0.773                                 0.565
0.563
];

a2AA=[0.906                                 0.860
0.842                                 0.788
0.744
0.906                                 0.860
0.898                                 0.652
0.740
0.898                                 0.635
0.898                                 0.806
0.702
0.906                                 0.767
0.842                                 0.788
0.698
0.906                                 0.860
0.652                                 0.652
0.740
```

```
0.906
0.898
0.698
0.917
0.879
0.683
0.906
0.767
0.477
0.785
0.879
0.590
];

a3AA=[0.646
0.773
0.754
0.646
0.773
0.758
0.533
0.717
0.758
0.621
0.454
0.627
0.621
0.721
0.758
0.621
0.717
0.777
0.471
0.729
0.717
0.396
0.721
0.717
0.413
0.754
0.717
];

a4AA=[0.760
0.779
0.556
0.760
0.798
0.698
0.750
0.779
0.754
0.381
0.798
0.581
0.644
0.508
0.585
0.760
0.508
0.679
```

```
0.860
0.806

0.852
0.833

0.615
0.713

0.665
0.550

   0.706
0.838

0.763
0.744

0.706
0.744

0.731
0.838

0.444
0.819

0.781
0.838

0.731
0.769

0.773
0.769

0.773
0.769

   0.846
0.804

0.742
0.673

0.865
0.804

0.771
0.767

0.846
0.804

0.846
0.804
```

```
0.723
0.817
0.754
0.731
0.798
0.754
0.723
0.685
0.717
];


a5AA=[0.769
0.890
0.950
0.769
0.890
0.958
0.698
0.923
0.958
0.575
0.923
0.892
0.600
0.923
0.950
0.633
0.758
0.950
0.694
0.931
0.958
0.529
0.923
0.958
0.750
0.923
0.958
];


a6AA=[0.925
0.942
0.967
0.831
0.967
0.967
0.896
0.967
0.958
0.831
0.967
0.967
0.925
0.942
0.958
0.925
0.967
0.958
```

```
0.827
0.869

0.715
0.823

0.827
0.660


   0.767
0.842

0.767
0.858

0.767
0.858

0.804
0.654

0.823
0.654

0.823
0.842

0.721
0.858

0.652
0.858

0.815
0.858


   0.860
0.975

0.767
0.975

0.767
0.975

0.652
0.975

0.842
0.933

0.842
0.975
```

```
0.925                                   0.879
0.983                                   0.975
0.975
0.802                                   0.860
0.967                                   0.917
0.967
0.831                                   0.860
0.900                                   0.933
0.967
];

a7AA=[0.871                                 0.942
0.792                                   0.704
0.813
0.829                                   0.942
0.808                                   0.700
0.737
0.875                                   0.750
0.746                                   0.750
0.767
0.854                                   0.808
0.771                                   0.725
0.713
0.571                                   0.942
0.675                                   0.579
0.821
0.879                                   0.942
0.788                                   0.775
0.763
0.846                                   0.942
0.554                                   0.708
0.721
0.863                                   0.942
0.804                                   0.775
0.821
0.854                                   0.942
0.712                                   0.775
0.754
];

a8AA=[-0.646                                        1.000
-0.396                                  -0.283
-0.975
-0.083                                  1.000
0.042                                   0.392
0.642
-1.229                                  1.000
0.188                                   0.587
0.617
0.375                                   1.000
0.292                                   0.471
0.575
0.146                                   1.000
0.292                                   0.471
0.525
0.188                                   1.000
0.042                                   0.554
0.679
0.375                                   1.000
0.292                                   0.567
0.679
```

```
0.167                                       1.000
0.292                                       0.525
0.679
0.375                                       1.000
0.292                                       0.317
0.679
];

a9AA=[0.688                                     1.000
0.813                                       0.906
0.875
0.842                                       1.000
0.842                                       0.790
0.875
0.675                                       1.000
0.896                                       0.890
0.913
0.867                                       1.000
0.908                                       0.854
0.923
0.858                                       1.000
0.904                                       0.865
0.910
0.871                                       1.000
0.888                                       0.902
0.921
0.825                                       1.000
0.917                                       0.885
0.925
0.779                                       1.000
0.917                                       0.869
0.883
0.854                                       1.000
0.917                                       0.885
0.883
];

a10AA=[0.479                                        1.000
0.371                                       0.529
0.933
0.150                                       1.000
0.600                                       0.754
0.929
-0.242                                      1.000
0.542                                       0.767
0.867
0.396                                       1.000
0.717                                       0.829
0.896
0.458                                       1.000
0.713                                       0.846
0.858
0.479                                       1.000
0.683                                       0.675
0.817
0.229                                       1.000
0.700                                       0.850
0.933
0.071                                       1.000
0.733                                       0.850
0.933
```

```
0.229                                   1.000
0.700                                   0.850
0.933
];

a11AA=[0.881                                    1.000
0.967                                   0.973
0.993
0.948                                   1.000
0.988                                   0.997
0.999
0.925                                   1.000
0.986                                   0.997
0.999
0.956                                   1.000
0.985                                   0.997
0.999
0.954                                   1.000
0.985                                   0.997
0.999
0.938                                   1.000
0.989                                   0.997
0.999
0.956                                   1.000
0.990                                   0.997
0.999
0.944                                   1.000
0.990                                   0.997
0.999
0.948                                   1.000
0.990                                   0.997
0.999
];

a12AA=[0.375                                    1.000
0.921                                   0.333
0.392
0.750                                   1.000
0.900                                   0.917
0.875
0.563                                   1.000
0.900                                   0.917
0.871
0.583                                   1.000
0.913                                   0.917
0.871
0.688                                   1.000
0.879                                   0.917
0.829
0.750                                   1.000
0.763                                   0.917
0.871
0.542                                   1.000
0.900                                   0.917
0.858
0.708                                   1.000
0.921                                   0.917
0.858
0.542                                   1.000
0.921                                   0.917
0.858
```

```
];

a13AA=[0.642                              0.333
0.958                           0.904
0.863
0.642                           0.917
0.667                           0.754
0.854
0.617                           0.917
0.958                           0.904
0.817
0.350                           0.917
0.958                           0.821
0.696
0.525                           0.917
0.958                           0.787
0.700
0.642                           0.917
0.958                           0.850
0.867
0.567                           0.917
0.958                           0.904
0.863
0.483                           0.917
0.958                           0.904
0.863
0.683                           0.917
0.958                           0.904
0.863
];

weight=[0.123287671 0.102739726 0.0959  0.136986301 0.136986301 0.136986301
0.082191781 0.095890411 0.089041096
0.148148148 0.133333333 0.1185  0.118518519 0.103703704 0.111111111
0.088888889 0.096296296 0.081481481
0.112426036 0.112426036 0.1006  0.112426036 0.106508876 0.112426036
0.112426036 0.112426036 0.118343195
0.087591241 0.087591241 0.0876  0.131386861 0.138686131 0.131386861
0.109489051 0.124087591 0.102189781
0.100628931 0.125786164 0.1195  0.081761006 0.094339623 0.106918239
0.125786164 0.125786164 0.119496855
0.116438356 0.109589041 0.1370  0.116438356 0.089041096 0.116438356
0.095890411 0.109589041 0.109589041
0.134453782 0.159663866 0.1176  0.134453782 0.117647059 0.134453782
0.033613445 0.100840336 0.067226891
0.174757282 0.174757282 0.1553  0.145631068 0.097087379 0.116504854
0.058252427 0.038834951 0.038834951
0.174757282 0.174757282 0.1553  0.145631068 0.097087379 0.116504854
0.058252427 0.038834951 0.038834951
0.16    0.19    0.1600  0.17    0.09    0.13    0.04    0.03    0.03
0.16    0.19    0.1600  0.17    0.09    0.13    0.04    0.03    0.03
0.163265306 0.163265306 0.1633  0.163265306 0.102040816 0.132653061
0.040816327 0.030612245 0.040816327
0.163265306 0.163265306 0.1633  0.163265306 0.102040816 0.132653061
0.040816327 0.030612245 0.040816327
];

w=[0.090179123
0.083384805
0.104385423
0.084620136
```

```
0.098208771
0.090179123
0.077208153
0.063619518
0.063619518
0.061766523
0.061766523
0.060531192
0.060531192
];

AA=zeros(9,5,13)
AA(:,:,1)=a1AA;
AA(:,:,2)=a2AA;
AA(:,:,3)=a3AA;
AA(:,:,4)=a4AA;
AA(:,:,5)=a5AA;
AA(:,:,6)=a6AA;
AA(:,:,7)=a7AA;
AA(:,:,8)=a8AA;
AA(:,:,9)=a9AA;
AA(:,:,10)=a10AA;
AA(:,:,11)=a11AA;
AA(:,:,12)=a12AA;
AA(:,:,13)=a13AA;

% RA definition of AA
RA=zeros(9,5,13)
for i=1:9
    for k=1:5
        for m=1:13
        RA(i,k,m)=AA(i,k,m)/sum(AA(:,k,m));
        end
    end
end

% CC definition according to RA and Weight and Beta
CC=zeros(9,5,13);
for i=1:9
    for k=1:5
    for m=1:13
    CC(i,k,m)=(b*weight(m,i)+(1-b)*RA(i,k,m));
    end
    end
end

% R aggregation result
R=zeros(13,15)
for k=1:13
    for i=1:15
        R(k,i)=E1(k,i)*CC(1,floor(1+(i-1)/3))+E2(k,i)*CC(2,floor(1+(i-
1)/3))+E3(k,i)*CC(3,floor(1+(i-1)/3))+E4(k,i)*CC(4,floor(1+(i-
1)/3))+E5(k,i)*CC(5,floor(1+(i-1)/3))+E6(k,i)*CC(6,floor(1+(i-
1)/3))+E7(k,i)*CC(7,floor(1+(i-1)/3))+E8(k,i)*CC(8,floor(1+(i-
1)/3))+E9(k,i)*CC(9,floor(1+(i-1)/3));
    end
end

% Right & Left & Total score
RS=zeros(13,5);
```

```
for i=1:13;
    for k=1:5
RS(i,k)=R(i,3*k)./(1+R(i,3*k)-R(i,3*k-1))
    end
end

LS=zeros(13,5);
for i=1:13
    for k=1:5
        LS(i,k)=1-R(i,3*k-1)./(1+R(i,3*k-1)-R(i,3*k-2))
    end
end

TS=(RS+1-LS)./2

sum2=zeros(13,1)
for i=1:13
    sum2(i,1)=sqrt(TS(i,1).^2+TS(i,2).^2+TS(i,3).^2+TS(i,4).^2+TS(i,5).^2)
end

% definition of rij
rij=zeros(13,1,5)
for i=1:13
    for k=1:5
    rij(i,k)=TS(i,k)./sum2(i,1)
    end
end

% definition of vij
vij=zeros(13,1,5)
for i=1:13
    for k=1:5
        vij(i,k)=rij(i,k).*w(i,1)
    end
end

% v+ and v- definition
vplus=zeros(13,1)
vminus=zeros(13,1)
for i=1
vplus(i,1)=min(vij(i,1:5))
vminus(i,1)=max(vij(i,1:5))
end
for i=5
vplus(i,1)=min(vij(i,1:5))
vminus(i,1)=max(vij(i,1:5))
end
for i=6
vplus(i,1)=min(vij(i,1:5))
vminus(i,1)=max(vij(i,1:5))
end
for i=7
vplus(i,1)=max(vij(i,1:5))
vminus(i,1)=min(vij(i,1:5))
end
for i=2
vplus(i,1)=max(vij(i,1:5))
vminus(i,1)=min(vij(i,1:5))
end
for i=3
```

```
vplus(i,1)=max(vij(i,1:5))
vminus(i,1)=min(vij(i,1:5))
end
for i=4
vplus(i,1)=max(vij(i,1:5))
vminus(i,1)=min(vij(i,1:5))
end
for i=8
vplus(i,1)=max(vij(i,1:5))
vminus(i,1)=min(vij(i,1:5))
end
for i=9
vplus(i,1)=max(vij(i,1:5))
vminus(i,1)=min(vij(i,1:5))
end
for i=10
vplus(i,1)=max(vij(i,1:5))
vminus(i,1)=min(vij(i,1:5))
end
for i=11
vplus(i,1)=max(vij(i,1:5))
vminus(i,1)=min(vij(i,1:5))
end
for i=12
vplus(i,1)=max(vij(i,1:5))
vminus(i,1)=min(vij(i,1:5))
end
for i=13
vplus(i,1)=max(vij(i,1:5))
vminus(i,1)=min(vij(i,1:5))
end

% positive and negative ideal solution
posid=zeros(13,5)
for i=1:13
    for k=1:5
    posid(i,k)=(vij(i,k)-vplus(i,1)).^2
    end
end

negid=zeros(13,5)
for i=1:13
    for k=1:5
    negid(i,k)=(vij(i,k)-vminus(i,1)).^2
    end
end

% Splus & Sminus definition
Splus=zeros(1,5)
for k=1:5
    Splus(1,k)=sqrt(sum(posid(:,k)))
end

Sminus=zeros(1,5)
for k=1:5
    Sminus(1,k)=sqrt(sum(negid(:,k)))
end

% C definition
C=zeros(1,5)
```

```
for k=1:5
    C(1,k)=Sminus(1,k)./(Splus(1,k)+Sminus(1,k))
End


% Create figure

f1=xlsread('FuzzyFinal.xlsx','Sheet1','I3:M13');

figure;
plot(f1(:,1),'-ks','markerfacecolor','k');
hold on;
plot(f1(:,2),'--ko','markerfacecolor',[0.3 0.3 0.3]);
plot(f1(:,3),'-k^','markerfacecolor',[0.5 0.5 0.5]);
plot(f1(:,4),'--kd','markerfacecolor',[0.7 0.7 0.7]);
plot(f1(:,5),':k*');



ylim([0 1]);
legend('Corrective','Preventive','RCM','CBM','CMMS');
xticklabels({'0','0.1','0.2','0.3','0.4','0.5','0.6','0.7','0.8','0.9','1'}
)
xlabel('\beta coefficient')

l7.FontSize=10;
l7.Location='east';
ax=gca;
ax.FontSize =10;
```