## COLLAPSE OF STEAM BUBBLES

## IN SUBCOOLED WATER

by

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#### Abstract

Condensation, of steam bubbles generated at an orifice and rising freely through water, subcooled from 5 $K$ to 36.6 K at pressures of 1 bar and 2 bar, has been analysed theoretically and experimentally. Orifice diameters were 1 mm and 2 mm , and steam flow rates of 0.5 , 1 and $1.5 \mathrm{~g} / \mathrm{min}$ were used.

The data indicate a decrease in collapse fourier number with increase in either Jakob number or steam flow rate, or with a decrease in pressure, while change in orifice diameter does not have a significant effect on collapse Fourier number.

Average values of heat transfer coefficient around the collapsing bubbles have been determined to be between $0.15 .10^{5}-0.35 .10^{5} \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$.

The effect of bubble distortion and of local heating of the liquid, close to the orifice, due to condensation of the bubbles, have both been included in the quasi-steady state theory which has been presented. The experimental data is compared with the theoretical predictions.

A semi-empirical correlation for bubble rise height has been proposed, which is also based on the quasi-steady theory combined with a correlation for the velocity of steam bubbles condensing in subcooled water.


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F1g.A 5.5. Fig.A 5.6, Fig.A 6.2

## NOMENCLATURE

Ar Archimedes number $=\frac{g R Q^{2}}{v^{2}}$

Ar the surface area at the rear of the bubble, $m^{2}$
$B \quad$ parameter defined as $B=J a^{2} \cdot \frac{\alpha}{R_{0}} \cdot \sqrt{\frac{\rho_{f}}{\Delta P}}$

Beff parameter defined by equation (2.10)

C1 variable defined by equation (A 6.19)
$C_{b}$ constant defined by equation (A 6:16)
$C_{1}$ the centre line of the inlet orifice

CX horizontal distance of the centroid of the bubble from the orifice centre line, mm

CY vertical distance of the centroid of the bubble above the orifice, mm
$c_{p}$ specific heat, J/kg.K

D diffusion coefficient, $\mathrm{m}^{2} / \mathrm{s}$
d orifice diameter, mm

Ec Eckert number $=\frac{\dot{m}^{2}}{d^{4} \cdot c_{p} \cdot P_{o} \cdot\left(T_{0}-T_{W}\right)}$
Eu Euler number $=\frac{P_{\cdot} \cdot \rho_{0} \cdot d^{4}}{\dot{m}^{2}}$
F number of frames in cine film

Fo Fourler number for the collapse
region $-\frac{\alpha_{f} \cdot t}{4 R_{0}{ }^{2}}$
Fo2 Fourier number for the growth and

$$
\text { collapse regions }=\frac{\alpha_{f} \cdot t}{4 R_{m}{ }^{2}}
$$

Foc collapse fourier number $=\frac{\alpha_{f} \cdot t_{c}}{4 R_{o}{ }^{2}}$
Fr Froude number $=\frac{\dot{m}^{2}}{\mathrm{gd}^{5} \rho_{O^{2}}}$
g gravitational acceleration, $\mathrm{m} / \mathrm{s}^{2}$
$n$ Instantaneous heat transfer coefficient
; heat transfer coefficient, $W / \mathrm{m}^{2} K$
$h_{f g}$ specific enthalpy of evaporation, J/kg

Ja Jakob number $=\frac{\rho_{f} \cdot{ }_{c_{p f}} \cdot \Delta T}{\rho_{g} \cdot h_{f}}$
K thermal conductivity, W/mK
$k_{m}$ mass transfer coefficient, $m / s$

LE distance of leading edge (1.e. top most point) of the bubble above the orifice, mm

LP distance of lowest point of the bubble above the orifice, mm
$\dot{m}_{\mathrm{m}} \dot{m}_{s}$ steam mass flow rate, g/min

Nu Instantaneous Nusselt number $=\frac{2 h R}{k_{f}}$
$P$ prejsiupé in the bubble chamber, bar ; pressure, $N / m^{2}$

Pe Peclet number $-\frac{2 R U}{\alpha}$
Peo Peclet number $=\frac{2 R_{0} U}{\alpha}$
Pr Prandtl number $=\frac{v}{\alpha}$
Q rate of heat transfer from the bubble at time $t$, $W$
$q_{0}$ heat flux at angular position $\theta$ on the surface of the bubble, $W / m^{2}$
$R$ equivalent bubble radius, mm; bubble radius, m
$\dot{R} \quad$ bubble wall radial velocity $=\frac{d R}{d t}, \mathrm{~m} / \mathrm{s}$
Re Reynolds number $=\frac{2 R U}{v}=\frac{2 R U_{\rho}}{\mu}$
Reo Reynolds number $-\frac{2 R_{0} U}{V}$
Ro bubble radius at detachment, m
$r$ radial coordinate, $m$
s surface renewal rate, $1 / \mathrm{s}$

T temperature, $K$
$\Delta T$ aubcooling or superheating of water relative
to steam, $K$
$t$ time from the beginning of bubble growth or bubble detachment, ms; time, s
$\bar{t} \quad$ time from detachment to a mean point between successive frames, ms
at time interval between alternate frames in cine fllm. ms
$U$ average vertical velocity of bubble during condensation (after detachment), mm/s; bubble rise velocity, m/s
$u_{r}$ velocity in the radial direction; $\mathrm{m} / \mathrm{s}$
$u_{0} \quad$ velocity in the tangential direction, $\mathrm{m} / \mathrm{s}$
v bubble volume, ma
$V_{s}$ steam volume flow rate, $\mathrm{mm}^{3} / \mathrm{s}$
We Weber number - $\frac{\dot{m}^{2}}{d^{3} \cdot \sigma \cdot \rho_{0}}$
$w \quad$ velocity of subcooled ilquid in bubble chamber or flow channel, mm/s
$y$ radial distance from the bubble surface $=r-R$, $m$
$z$ dimensionless bubble volume $=\frac{V}{\frac{4}{3} \cdot \pi R_{0}^{3}}$
$\alpha$

B
dimensionless radius based on maximum
$\operatorname{radius}=\frac{R}{R_{m}}$
Y polar angle from vertical defining the base of the spherical-cap bubble, radians
$\delta$ thermal boundary layer thickness, m
$\varepsilon \quad$ dimensionless variable $=\left(\frac{\delta}{R}\right)^{2}$; rate of
energy dissipation by turbulence per unit of mass, $W / k g$ or $\mathrm{m}^{2} / \mathrm{s}^{3}$

ET total rate of energy dissipation per unit of mass. $W / k g$ or $\mathrm{m}^{2} / \mathrm{s}^{8}$

O polar angle from vertical, radians
$\mu \quad$ dynamic viscosity, Ns/m²
$v \quad$ kinematic viscosity $\quad \mu / \rho, m^{2} / s$
$p$ density, $k g / m^{3}$
$\sigma$
surface tension, $N / m$
dimensionless temperature ratio $=\frac{T-T_{\infty}}{T_{1}-T_{\infty}}$
$\Psi$ bubble shape factor defined by equation (5.50)

## Subseripts

a average
b bubble; back
c condensation or collapse region
d detachment
f sluid : front
g growth region

1 vapour water interface

1 11quid
m maximum ; mean
o value at detachment ; initial value
$r$ rear
s steam ; separation
sat saturation

T,t total
v vapour
w water : wake

- bulk liquid value
$\gamma \quad$ to define heat transfer from spherical-cap bubbles

Note: The nomenclature for experimental parameters and the experimental results on the test data sheets are given in section 4.2.

### 1.1 General aspects of direct contact condensation

Direct contact condensation is important in the design of equipment such as condensers, cooling towers, feed water heaters and deaerators. It is of current lmportance 1 n the development of economic water desalination units, utilisation of geothermal brine for energy production and emergency cooling of nuclear reactors.

The advantages of direct contact condensation over conventional processes using metallic transfer surfaces, are due to relative simplicity of design, reduced corrosion and scallng problems, lower malntenance costs, greater heat transfer area for a given volume, higher heat transfer rates and iower temperature driving forces.

The limitations of direct contact condensation are due to the physical properties of the fluids in contact; low solublilty, low viscosity and surface tension, differences 1 n the specific gravities for ease of separation with no affinity for stable emulsions, chemical inertness and stablifty belng required for economical and trouble free operation.

While condensation on a solid surface is limited only by the extent of the surface and the rate of cooling of the surface, direct contact condensation is naturally limited by the balance between the latent heat of condensation and the sensible heat that the ilquid can absorb until saturation is attained.

### 1.2 Emergency cooling of nuclear reactors

When a loss of coolant accident (LOCA) occurs in a nuclear reactor, due for example to the fracture of a main pipe $1 n$ a Pressurised Water Reactor (PWR), various types of direct contact condensation may occur, dependent on the methods adopted to counteract the effects of the fracture (seefig. 1. for a typical PWR loop). The resultant direct contact condensation between vapour and water may be :

1) Condensation of steam bubbles or jets in a pool of water : e.g.,
2) In the upper plenum and/or reactor core when hot leg emergency core cooling (ECC) injection is employed,
3) In parts of the downcomer,
4) In the contalnment vessel.
5) Steam condensed by a spray of water : e.g., in the upper plenum with not leg ECC injection.
6) Steam condensed in a parallel stream with water: e.8..
7) As a falling film from the upper plenum.
8) In the downcomer,
9) In the lower plenum.

When the emergency core coolant is injected through a hot leg during a LOCA in a PWR, water enters the upper plenum of the reactor vessel and falls on to an upper core plate (adapter plate or tie plate) and tries to penetrate to the reactor core against a flow of rising steam. In the safety analysis and prediction, information is required on the effectiveness with which the water penetrates to the core and on the steam water interactions in the upper plenum and at the core plate.


### 1.3 Project description

This work is a study of the behaviour of steam bubbles generated at an orlfice and passing upward through a water pool in a bubble chamber. The experiments were carried out to find out whether the bubbles condensed completely within a prescribed liquid pool helght or whether they escaped through the pool.

The main variables in the tests are, the helght of the liquid pool, the water subcooling relative to the steam, the steam mass flow rate, the orifice diameter and the system pressure. The experimental results are compared with predicted bubble collapse rates obtalned through a theoretical analysis.

## CHAPTER 2

## LITERATURE REVIEW

### 2.1 Introduction

The processes governing the growth or collapse of vapour bubbles in a liquid pool are determined mainly by the temperature of the $11 q u i d$ and by interactions resulting from the hydrodynamic behaviour of the bubbles In the liquid pool. The precise effects of these parameters can involve both heat transfer between bubble and liquid, and liquid inertia. In some cases liquid inertia may be the dominant feature while in others heat transfer may be the main factor controliling growth or collapse. Many research workers, using an extended form of the Rayleigh equation, originally developed for the collapse of a cavity in a liquid, have demonstrated the close relationship between the phenomena of bubble growth and those of bubble collapse.

This, of course, implies that many of the features noted by these workers in bubble growth may also be important factors in the fleld of bubble collapse.
2.2 Growth of vapour bubbles

An extensive amount of work has been carried out into the growth of vapour bubbles in a superheated liquid and much of this has been the subject of a comprehensive review in a book on bolling phenomena by Stralen and Cole [1]. In addition, Bankoff [7] has given an extensive review of heat transfer controlled bubble growth.

As mentioned earlier much of this work owes a great deal to Raylelgh [3]. In his work on the collapse of a spherical cavity in a liquid under a constant pressure difference, he derived an equation giving the time for the Inertia controlled collapse of a bubble as

$$
\begin{equation*}
t=R_{0} \sqrt{\frac{3 \rho_{l}}{2 \dot{\Delta} P}} \int_{\beta}^{1} \frac{\beta^{3 / 2} d \beta}{\left(1-\beta^{3}\right)^{2 / 2}} \tag{2.1}
\end{equation*}
$$

Further work by Forster and Zuber [4], Plesset and Zwich [5] and others led to an extended form for the equation of motion as

$$
\begin{equation*}
\ddot{R} \ddot{R}+\frac{3}{2} \dot{R}^{2}=\frac{\dot{H} P(t)}{\rho_{l}}-\frac{2 \sigma}{\rho_{l} R}-4 v \frac{\dot{R}}{R} \tag{2.2}
\end{equation*}
$$

the last two terms being included to account for surface tension and viscous effects.

In boiling, the initial stage of bubble growth, when the vapour temperature equals the temperature of the superheated liquid around it, is controlled mainly by liquid inertia effects and a solution can be obtained corresponding to that given by equation (2.1). As growth continues, heat transfer effects become more significant and, towards the later stages of growth, when the vapour pressure inside the bubble is approximately the same as the system pressure outside the bubble, growth is controlled by heat transfer. In general after a few microseconds of inertia controlled growth the remaining bubble growth takes place in a few milliseconds and in most of the work reported in the next paragraph heat transfer effectively controls the growth.

Stralen and Cole [1] reported that Bosnjakovic [38], by assuming a conduction layer surrounding a spherical bubble and by equating the heat conducted through this layer to the heat absorbed by evaporation, showed that the bubble radius was given by

$$
\begin{equation*}
R(t)=C J a(\alpha t)^{1 / 2} \tag{2.3}
\end{equation*}
$$

where the value of $C$ depended on the time dependent thickness assumed for the conduction layer, and Stralen suggested

$$
C=\frac{2}{\pi^{2 / 2}} .
$$

The same result was obtained by Forster and Zuber [4] who considered the bubble boundary as a moving sink. A different value,

$$
c=\left(\frac{12}{\pi}\right)^{1 / 2}
$$

was obtained by Plesset and Zwick [5] who assumed a thin boundary layer between the vapour bubble and the liquid and applied a perturbation technique to a stationary growing bubble. This value of

$$
C=\left(\frac{12}{\pi}\right)^{1 / 2}
$$

was also obtained, by Birkhoff, Margulies and Horning [39], in a detailed solution for a growing bubble and by Scriven [6] for large superheats. Scriven [6] also gave an equation which he suggested was more accurate for small superheats as

$$
\begin{equation*}
R(t) \cong(2 \mathrm{Ja})^{2 / 2}(\alpha t)^{1 / 2} \tag{2.4}
\end{equation*}
$$

Ruckenstein [8] investigated the effect of a constant translational velocity on the bubble growth rate. He reduced the energy equation in the liquid to the diffusion problem solved by Levich [9] giving the temperature distribution around the bubble as

$$
\frac{T-T_{1}}{T_{\infty}-T_{i}}=\frac{2}{\sqrt{\pi}} \operatorname{erf} \frac{\frac{3}{2} U R y \sin ^{2} \theta}{2 \sqrt{\frac{3}{2} U R^{3} \alpha\left(\frac{2}{3}-\cos \theta+\frac{1}{3} \cos ^{3} \theta\right)}} \quad \text { (2.5) }
$$

and gave a correlation for the heat transfer rate as

$$
\begin{equation*}
\mathrm{Nu}=\left(\frac{4}{\pi}\right)^{9 / 2} \mathrm{Pe}^{x / 2} \tag{2.6}
\end{equation*}
$$

A further theoretical analysis by Ruckenstein and Davis [31] assumed potential flow around a spherical bubble and showed that for zero translational velocity, apart from the effect of an initial radius, the growth rates conformed closely to the values given by Plesset and Zwick [5], (equation (2.3) with $\left.C=(12 / \pi)^{1 / 2}\right)$, but noted that, at lower values of Jakob number the growth rates increased with increasing velocity although this effect reduced with increase in Jakob number as radial convection increased, being of no significance for Ja > 50. They also pointed out that caution should be exercised in applying this analysis to low Jakob numbers, especially at low translational velocities because, under these conditions, the assumption of a thin boundary layer became questionable. The theory was in good agreement with Florschuetz et al's [40] data on growing and translating vapour bubbles at uniform superheats in the low Jakob number range ( $3<\mathrm{Ja}<10$ ).

Growth and collapse of vapour bubbles in liquid nitrogen was studied by Hewitt and Parker [41]. Their experimental data for bubble growth in boiling was in good agreement with the Plesset and Zwick [5] theory (equation (2.3) with $\left.C=(12 / \pi)^{1 / 2}\right)$ 。

### 2.3 Condensation of vapour bubbles

Although it was implied in the previous section that, for bubble growth in a liquid of small superheat, liquid inertia effects soon became negligible and thereafter growth was controlled by heat transfer, the same conditions do not necessarily apply throughout bubble collapse. Thus, in highly subcooled boiling, where the temperature driving force remains relatively large, bubble collapse appears to be dominated by liquid inertia.

Florschuetz and Chao [10] investigated the relative importance of liquid inertia and heat transfer on the spherically symmetrical collapse rate of vapour bubbles (see table (2.1) in the attached portfolio for experimental values).

Bubble collapse was classified into 3 regions :
i) Where the Jakob number was high, so that the wall temperature was approximately equal to the bulk liquid temperature so that the vapour pressure remained constant at the initial value throughout collapse, this implying liquid" inertia controlled collapse and conforming to the simple Rayleigh solution (see equ. (2.1)), the solution for the dimensionless collapse time being given as

$$
\begin{equation*}
\tau_{I}=\frac{t}{R_{O}} \sqrt{\frac{2 \Delta P}{3 p_{L}}} \tag{2.7}
\end{equation*}
$$

1i) Where the vapour pressure in the bubble remained equal to the system pressure, so that liquid inertia effects were negligible and collapse was dominated by heat transfer. Neglecting the effects of surface curvature, a plane interface solution was derived for heat transfer
controlled collapse as

$$
\begin{equation*}
B=1-\tau_{H} \frac{y}{z} \tag{2.8}
\end{equation*}
$$

where $\tau_{H}=\frac{4}{\pi} \mathrm{Ja}^{2} \frac{\alpha t}{R_{0}{ }^{2}}$
The Plesset and Zwick temperature integral was solved leading to a solution for the dimensionless time, $\tau$, in heat transfer controlled collapse as

$$
\begin{equation*}
\tau_{H}=\frac{1}{3}\left(\frac{2}{\beta}+\beta^{2}-3\right) \tag{2.9}
\end{equation*}
$$

Although both equations (2.8) and (2.9) were approximate results, nevertheless it was concluded that these two equations should represent lower and upper limits to the collapse time for heat transfer controlled collapse and that in each case the nature of the collapse was quite different from the inertia controlled collapse of equation (2.1).
1ii) In general cases, where both liquid inertia and heat transfer affected the collapse, the relative importance of the two effects was considered to depend on the value of a parameter,

$$
B=J a^{2} \frac{\alpha}{R_{0}} \sqrt{\frac{\rho_{f}}{\Delta P}} \text {, heat transfer effects }
$$

becoming more important as $B$ decreased and liquid.inertia effects more important as $B$ increased. They also included an interesting indication that, for certain intermediate values of $\mathrm{B}(0.05<\mathrm{B}<10.0$ ) the combined effects of liquid inertia and heat transfer could produce a range of alternate collapse and regrowth periods before collapse finally continued.

An effective value of $B$ was defined as

$$
\begin{equation*}
B_{\text {eff }}=\psi^{2} B\left(\frac{\rho_{v s a t}}{\bar{\rho}_{V}}\right)^{2} \tag{2.10}
\end{equation*}
$$

where, $\psi$ was a factor introduced to account for the non-linearity of the vapour pressure and temperature relation and used an average mean value of reference density, $\bar{\rho}_{V}$ for the temperature range $T_{\infty}$ to $T_{s a t}$.

They suggested that collapse would be heat transfer controlled for Beff < 0.05 , and liquid inertia controlled for $B_{e f f}>10.0$ with both effects present in varying amounts between these values. However it may be noted that, in the example given for heat transfer controlled collapse of water vapour bubbles at atmospheric pressure, for $40 \leq J a \leq 100$ the value of Beff was always greater than 0.05 while for inertia controlled collapse with $150 . \leq$ $\mathrm{Ja}=250$ Beff was always less than 10 .

For heat transfer controlled collapse, the collapse rates were, in general, faster than given by equation (2.9) although agreement was reasonably good with theoretical predictions for low translational velocities, but the collapse rates increased if translational velocities were significant, in some cases collapse being more rapid than that given by the plane interface solution of equation (2.8). It was suggested that deviation from the slow collapse rate of equation (2.9) would be greater for small values of Jakob number, due to the relatively earlier breakdown of the thin thermal boundary layer approximation for slowly collapsing bubbles. They noted that results published by Levenspiel [21] gave a more rapid collapse rate than any of the above and showed a markedly faster collapse even than that indicated by the lower bound equation (2.8). However these collapse rates
were still very much slower than for purely inertia controlled collapse and it was suggested that the rapidity of collapse in this case was due to the high translational velocities of the bubble and that liquid inertia effects were insignificant.

Wittke and Chao [11] used the same apparatus as Florschuetz and Chao [10], modified to study the effects of translational velocity and the presence of noncondensables in the collapsing bubbles (see table (2.1)). As it was considered that translational velocity would have little effect in liquid inertia controlled collapse, attention was directed solely at heat transfer controlled collapse, but they rejected the thin boundary layer assumption of Plesset and Zwick as unsatisfactory and obtained a numerical solution of the governing equations. Their experimental results showed reasonably good agreement with their theory. It was shown that, at any value of Jakob number, an increase in translational velocity not only increased the collapse rate, but in the absence of noncondensables caused it to increase with time. However, the presence' of noncondensables, except for stationary bubbles at very low Jakob numbers, caused a reduction in the collapse rate, with the collapse rate decreasing as collapse continued. They found that the Florschuetz and Chao upper bound solution was theoretically valid for large Jakob numbers but the applicability was restricted as heat transfer controlled collapse would only occur at low values of Jakob number.

Brucker and Sparrow [12] investigated the collapse of water vapour bubbles of about 3 mm initial diameter, at pressures in the range 10.3 to 62.1 bar, subcoolings from 15 to 100 K (see table (2.1)). They found the bubble translational velocities approximately constant, with a range from 150 to $220 \mathrm{~mm} / \mathrm{s}$, and also that the bubbles
followed a pattern of spherical at detachment, changing to hemispherical, then to ellipsoidal at which condition the shorter lived bubbles collapsed, while longer lived bubbles changed again to spherical shape before final collapse. It was noted that, irrespective of the total bubble collapse time, the times of change to hemispherical and then to ellipsoidal were essentially constant at about 0.010 and 0.020 seconds respectively. The average heat transfer coefficients obtained were about $10^{4} \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$ and both collapse time and height to collapse increased with increasing pressure and decreasing temperature difference.

Empirical relations were given for both the collapse time and the height to collapse in the form

$$
\begin{equation*}
F O_{C}=13.9 / \mathrm{Ja}^{3 / 4} \cdot \mathrm{Ra}^{1 / 2} \tag{2.11}
\end{equation*}
$$

and $\frac{Z_{c}}{R_{0}}=1536 / \mathrm{Ja}^{3 / \mathrm{s}} \cdot \mathrm{Ra}^{\mathrm{r} / 4}$
where $R=\left[8 g\left(\rho_{l}-\rho_{V}\right) R_{o} 3 / \rho_{f} \nu_{f}^{2}\right] . \operatorname{Pr}_{f}$.
Nordmann [13] studied the heat transfer around bubbles supplied through an orifice into a slightly downwards flow of water (see table (2.1)). Using sensitive pressure transducers, coupled with high speed film of the emerging bubbles, pressure fluctuations were recorded near the surface of the condensing bubbles. These measurements, showed that the maximum pressure fluctuations occurred at detachment, with a further slightly lower amplitude fluctuation towards the end of collapse. It was noted that the magnitude of the fluctuation depended on the Jakob number, the effect being much greater for Ja > 100 than for lower values of Ja.

The amplitude of the fluctuation against condensation time was plotted as shown in Fig. 2.1, which clearly
indicated, that for $J a>100$, the amplitude increased rapidly as the condensation time was reduced, while for Ja < 100 there was comparatively little change of amplitude with change in condensation time.


Fig. 2.1 Maximum pressure amplitude - $\tilde{P}_{k}$ at the end of condensation as a function of condensation time, $t_{k}$ [13].

Holographic interferometry was also used, this being only feasible while a stable smooth boundary layer prevailed at the phase boundary of the condensing bubbles. This was the case for $J a<30$, the irregularity of the surface increasing as the Jakob number increased, eliminating the regularity of the fringe pattern around the bubbles. It was concluded that the presence of the thermal boundary layer indicated that the bubble collapse was dominated by heat transfer for $J a<30$, while the pressure fluctuation pattern indicated that for $J a>100$, the collapse was dominated by liquid inertia effects, and for $30<J a<100$ there was a transition region in which both effects were present.

Values of Beff, as defined by Florschuetz and Chao [10], were obtained as Beff < 0.025 for heat transfer
controlled collapse and Beff $>0.44$ for liquid inertia controlled collapse compared with the values of Beff $\left\langle 0.05\right.$ and $\left.B_{e f f}\right\rangle 10$ given by Florschuetz and Chao.

In the heat transfer controlled region, it was observed that the bubble formed a long neck before detaching as hemispherical, changing to pear-shaped and back to hemispherical before collapsing as horizontal ellipsoids. Also, after detachment the bottom of the bubble rose rapidly, passing through the bubble volume, penetrating and deforming the top of the bubble. This sometimes caused entrainment of water into the bubble and the emergence of tiny water droplets or steam bubbles at the top of the bubble.

In the transition region, the turbulence at the phase boundary increased with increase in Jakob number and the detached bubble shape alternated between hemispherical and ellipsoidal. The neck at detachment became shorter, condensation occurred more around the bubble, the translational movement of the centre of gravity was greatly reduced. The top of a bubble frequently penetrated the preceding bubble producing a toroidal volume with rapid collapse.

In the inertia controlled region, the turbulent bubble surface continually changed shape during growth. Detachment occurred with a short neck and after detachment condensation took place all around the bubble, with no apparent translational movement of the centre of gravity.

Nordmann [13] applied the Nusselt equation for a solid sphere, $N u=c R e^{m} \cdot \operatorname{Pr} n[14]$ and, using the bubble wall velocity as the characteristic velocity, obtained for the heat transfer controlled and transitional regions the equation

$$
\begin{equation*}
B=\left(1-2.95 .10^{2} \operatorname{Pr}^{0 .}{ }^{26} \mathrm{Ja}^{0.87} \mathrm{FO}^{3}\right. \tag{2.13}
\end{equation*}
$$

wheres $s=1-0.005 . J a($ for $0<J a<100)$.

For the heat transfer controlled region, heat transfer coefficients were determined from the fringe patterns around the bubbles, and the disturbance in these fringes meant that in only a few cases was it possible to calculate the coefficients after detachment and even then only for one or two frames. This meant that in most cases the heat transfer coefficients were determined just before detachment.

The heat transfer around the top half of the bubble was related to the equation $N u=c e^{m_{P}}{ }^{n}$, modified by inclusion of the Jakob number and the density ratio, $\mathcal{R}^{\prime} \rho_{l}$ with the downward velocity of water as the characteristic velocity. Two values of Nusselt number were determined; one at the top of the bubble $\left(\theta=0^{\circ}\right) N u_{t}$ and one at the side of the bubble $\left(\theta=90^{\circ}\right) \mathrm{Nu}_{\mathrm{s}}$, and these were expressed as

$$
\begin{align*}
& N u_{t}=4000 . R^{0.25} \mathrm{Pr}^{0.65} . \mathrm{Ja}^{0.27}\left(\frac{P_{V}}{P_{L}}\right)^{0.75}  \tag{2.14}\\
& N u_{S}=253 . \text { Re }^{0}: 41 \cdot \operatorname{Pro.66} \cdot \mathrm{Ja}^{0.36} \cdot\left(\frac{\rho_{v}}{\rho_{L}}\right)^{0.75} \tag{2.15}
\end{align*}
$$

where Re $=\frac{2 R W}{\nu_{f}}$
A mean Nusselt number defined as $N u_{m}=\sqrt{N u_{t} \cdot N u_{s}}$ was thus given as

$$
\begin{equation*}
N u_{m}=1000 . R^{0.33} \mathrm{Pr}^{0.66} . \mathrm{Ja}^{0.27} \cdot\left(\frac{P_{V}}{\rho_{L}}\right)^{0.79} \tag{2.16}
\end{equation*}
$$

The values of heat transfer coefficient calculated were up to $65000 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$, with a maximum value at the top of
the bubble gradually decreasing down towards the $90^{\circ}$ position. The mean value around the bubble surface increased with increase in pressure. Thick thermal boundary layers formed around the neck of the attached bubble reducing the heat transfer in that region.

Mayinger and Chen [15], using the same apparatus as Nordmann, carried out experiments on the collapse of bubbles of R113 vapour having initial diameters about 2 mm (see table (2.1)). The tests were carried out over a range of pressures from 0.4 to 4 bar with subcoolings from 5 to 60 K . The experimental data were correlated, together with those of Nordmann [13] and Brucker and Sparrow [12], to show that the heat transfer coefficient was apparently unaffected by the value of Jakob number in the range $1.6 \leq J a \leq 60$, and agreement, within the range of the experimental data, was obtained, to within $\pm 50 \%$, with the Nusselt equations for a solid sphere

$$
\begin{equation*}
N u=0.37 \operatorname{Re}^{0.6} \operatorname{Pr}^{8 / 3} \tag{2.17}
\end{equation*}
$$

and

$$
\begin{equation*}
N u=2.0+0.6 \mathrm{Re}^{0.5} \mathrm{Pr}^{\mathrm{s} / 3} \tag{2.18}
\end{equation*}
$$

where $\quad \operatorname{Re}=\frac{2 R_{\text {ow }}}{v_{f}}$
Denekamp et al [16] carried out a study into the behaviour of water vapour bubbles, injected under sub-atmospheric conditions into a co-current flow of water subcooled by $1-5 \mathrm{~K}$ (see table( 2.1)). Their main interest lay in the behaviour of the bubble prior to detachment. It was observed that the bubbles left the orifice in pairs, and that after detachment, the second and smaller bubble was sucked into the leading bubble so
that the combined bubble formed a mushroom shape. Values of condensed mass, detachment diameter and bubble frequency were presented related to subcooling, with water temperature and water level above the orifice as parameters. Approximately half of the vapour was condensed before detachment. The governing conservation equations were solved numerically, and attention was directed at the leading bubble of each pair to derive a theoretical prediction for detachment radius and amount of condensation before detachment.

Delmas et al [17] investigated the influence of noncondensable gas on the collapse of vapour bubbles (see table (2.1)). The bubbles were obtained by boiling under vacuum and collapse initiated by increasing the pressure to atmospheric. Collapse rates were compared with the inertia controlled collapse of Rayleigh modified by assuming an isentropic pressure change within the bubble. This resulted in a much slower collapse, together with a significant oscillation in bubble radius, these oscillations also appearing in the experimental measurements, with a time lag of 0.4 to 0.8 ms . The collapse rates were, in general, between the values given by Florschuetz and Chao [10] (equations (2.8) and (2.9)) although as the nitrogen content increased, the collapse rates were much slower than those indicated by equation (2.9). The effect of the noncondensable gas on bubble stability was also noted and four regions indicated depending on bubble initial radius and on pressure difference :

1) Stable region at low initial radius and low pressure difference,
2) At high initial radius and high pressure difference, considerable oscillation occurred during collapse with a sudden explosive rupture
```
when the bubble attained a minimum size,
iii) At high initial radius but low pressure difference, bubble rupture by progressive division due to condensation in preferential sites,
iv) A mixed region, between (ii) and (iii) at medium pressure difference and high initial radius.
```

Bankoff and Mason [18] measured turbulent heat transfer coefficients at the surface of bubbles collapsing in a counter current stream of subcooled water (see table (2.1)) and observed three different types of bubble depending on steam flow rate and water temperature :
i) At low steam flow rates and high subcoolings (50-73 K): essentially ellipsoidal bubbles of smooth surface with initial diameter of about 0.5 mm and a frequency of about 2500 Hz corresponding to the bubbles observed by Gunther [43] in subcooled nucleate boiling. These bubbles were correlated to within $\pm 20 \%$ by the equation,
$\mathrm{Nu}=0.000374 \mathrm{Pe}^{2.74} \mathrm{St}^{\mathrm{I} .5}$
where the Strouhal number, $S t$, is the ratio of mean bubble wall velocity and liquid jet velocity
i1) At increased steam flow rates and decreased subcoolings (about 50 K ) the bubbles had a frequency of about 720 Hz and collapsed as irregular ellipsoids conforming, to within $\pm$ $20 \%$, to the equation
$\mathrm{Nu}=21.1 \mathrm{Pe}{ }^{0.404} \mathrm{St}^{0.2}$
iii) At even lower subcoolings (18-30 K) the bubbles were of irregular shape, of initial diameter from about 1.2 to 4.5 mm , with large volume oscillations, had a frequency range of 200 to 1000 Hz , did not collapse completely and conformed, to within $\pm 50 \%$, to the equation

$$
\begin{equation*}
\mathrm{Nu}=63.7 \mathrm{Pe}^{0.265} \mathrm{St}^{-0.1} \tag{2.21}
\end{equation*}
$$

The authors also concluded that a much larger proportion of the heat transfer was due to latent heat transport than had been previously estimated, this fraction decreasing with increase in subcooling and increase in liquid jet velocity but increasing with heat flux and dominating near burn-out (max. heat flux).

Dimic [19] carried out a theoretical investigation into the collapse of one component vapour bubbles with translatory motion in subcooled liquid. The differential energy equation was solved to obtain an expression for the thickness of the thermal boundary layer around the bubble and hence a general solution was derived for the variation in bubble radius with time, qualitatively similar to that of Ruckenstein's [31] for bubble growth. A solution was then derived for the collapse of a stationary bubble as
$\beta=2\left(1+\frac{16}{\pi} \mathrm{Ja}{ }^{2} \mathrm{FO}\right)^{1 / 2} \cdot \cos \left[\frac{1}{3}\left[\pi+\arccos \left(1+\frac{16}{\pi} \mathrm{Ja}^{2} \mathrm{FO}\right)^{-3 / 2}\right]\right]$
which agreed, to within $\pm 6 \%$, with equation (2.9) of Florschuetz and Chao and equation (2.24) of Voloshko and Vurgaft [20], and was in qualitative agreement with the bubble growth solution of Skinner and Bankoff [44].

By assuming quasi steady state conditions throughout collapse and assuming a range of functional relationships between translational velocity and bubble radius, a range of solutions was derived for bubble radius versus time.

It is interesting to note that the solution for constant translational velocity, i.e.
$\beta=\left(1-\frac{6}{\sqrt{\pi}} \mathrm{JaPe}^{0.5} \mathrm{Fo}\right)^{2 / 3}$
was also obtained by Isenberg, Moalem and Sideman [23].

Four other solutions were given for different radius dependent relationships in the form

$$
\begin{equation*}
B=\left(1-C A r^{a} \operatorname{Pr} b J a F o\right)^{e} \tag{2.23}
\end{equation*}
$$

Where Ar is the Archimedes number.
Voloshko and Vurgaft [20] presented an empirical correlation for the collapse of steam bubbles in subcooled water for $40 \leq \mathrm{Ja} \leqq 75$ (see table (2.2) in the attached portfolio),

$$
\begin{equation*}
B=1-6.776 .10^{4 .} \mathrm{FO} \tag{2.24}
\end{equation*}
$$

which agreed with the experimental data to within $\pm 30 \%$. When compared with the Florschuetz and Chao [10] equation (2.8), the collapse rate was slower for the early part (down to $\beta=0.8$ ) of the collapse time, but accelerated during the later part to give a more rapid collapse, this being possibly associated with the effect of translational velocity.

Using equation (2.24) they obtained the equation for heat transfer coefficient as

$$
\begin{equation*}
\mathrm{h}=1.694 .10^{4} \frac{\mathrm{k}_{\mathrm{f}}}{\mathrm{JaR}_{0}} \tag{2.25}
\end{equation*}
$$

Levenspiel [21] studied the collapse of steam bubbles in water by sudden pressurisation while boiling under vacuum conditions (see table (2.2)). Considering a differential heat balance for the condensing bubble, he derived a correlation for the instantaneous heat transfer coefficient for his experimental data :

$$
\begin{equation*}
h=14163 \cdot R \cdot h_{f g} \cdot \rho_{v} \tag{2.26}
\end{equation*}
$$

He then deduced the mean heat transfer coefficient over the collapse period as

where $Q$ was the total heat transferred during the collapse, and concluded that the mean value was 0.75 times the instantaneous initial value, $h_{0}$.

Prisnyakov [22] applied the first law of
thermodynamics, for stationary condensing bubbles assuming the vapour obeyed the perfect gas laws, and obtained the following equation,

$$
\begin{equation*}
\beta=1-2 \varepsilon\left(\frac{16}{\pi} \mathrm{Ja}^{2} \mathrm{FO}\right)^{1 / 2} \tag{2.27}
\end{equation*}
$$

where $\varepsilon$ is a physical constant to be calculated for the experimental condition, having values 1 to 0.96 for water subcoolings 0 to 22 K .

Isenberg et al [23], assuming potential flow and quasi-steady state conditions around condensing bubbles derived an approximate analytical solution, including the effect of noncondensables. As the solution related to low Jakob numbers, the effect of bubble wall velocity was
neglected compared with that of translational velocity. Using Ruckenstein's [8] heat transfer equation for spherical bubbles rising independently in boiling liquid they obtained the solution
$\mathrm{JaPe}^{1 / 2} \mathrm{FO}=\frac{\pi^{1 / 2}}{4}\left[\frac{2}{3}\left(1-\beta^{3 / 2}\right)+\frac{\beta_{f}}{3} \mathrm{~B}^{3 / 2} \ln \left[\frac{\left(1-\beta_{f}\right)^{3 / 2}\left(\beta^{3 / 2}+\beta_{f}^{3 / 2}\right)}{\left(1+\beta_{f}^{3 / 2}\right)\left(\beta^{3 / 2}-\beta_{f}^{3 / 2}\right)}\right]\right.$

For a pure vapour $\beta_{f}=0$, and the equation reduces to

$$
\begin{equation*}
B=\left(1-\frac{6}{\sqrt{\pi}} \mathrm{Ja} \cdot \mathrm{Pe}^{1 / 2} \mathrm{FO}\right)^{2 / 3} \tag{2.29}
\end{equation*}
$$

Isenberg and Sideman [24] also presented a finite difference numerical solution for bubbles with noncondensables, condensing either in their own liquid or in another immiscible liquid. Their experimental data with pentane and steam bubbles condensing in their own liquids agreed well with equation (2.28) especially at high values of Peclet number. The agreement between the numerical and analytical solutions was better at high Peclet numbers and at low Jakob numbers but correlation of the experimental data was better with the numerical solution for Ja>30.

In the above analysis, involving noncondensables, it has been assumed that the noncondensables have been homogeneously mixed with the vapour. Moalem and Sideman [25] extended the analysis to include the build up of a concentration of noncondensables near the phase boundary, which is likely to occur in two component systems where internal circulation is restricted, indicating that this would slow down the collapse process. Moalem and Sideman [26] also included the effects of translational velocity, taken as constant for $2 \mathrm{~mm}<$ Ro $<4 \mathrm{~mm}$, the solution
corresponding to equation (2.28), and for Ro < 1 mm assuming $U \propto \sqrt{R}$, obtained the relation

$$
\begin{equation*}
\mathrm{JaPe}_{0}^{1 / 2} \cdot d F o=-\frac{\sqrt{\pi}}{4}\left(\frac{\beta^{13 / 4}}{\left.\beta^{3}-\beta_{f}^{3}\right)} d \beta\right. \tag{2.30}
\end{equation*}
$$

which in the case of noncondensables reduced to

$$
\begin{equation*}
B=\left(1-\frac{5}{\sqrt{\pi}} \mathrm{JaPe}_{0}^{1 / 2} \cdot \mathrm{FO}_{\mathrm{O}}\right)^{4 / 5} \tag{2.31}
\end{equation*}
$$

where $\mathrm{Pe}_{0}=\frac{2 \mathrm{R}_{0} \mathrm{U}_{0}}{\alpha}$.
It is clear that, with increase in noncondensables the rate of collapse would decrease, and in such cases the radius dependent velocity changes would be slower, so that the effect of the velocity would decrease with increase in noncondensable content. This is borne out by equations (2.28) and (2.30). When the analysis was extended to forced flow boiling, where the bubbles detaching from the surface were subject to a transverse flow of velocity $U_{L}$, while rising with the velocity $U_{b}=c \sqrt{R}$, the equation resulted as
$\mathrm{JaPe}_{\mathrm{O}}{ }^{2 / 2} \mathrm{dFO}=-\frac{\sqrt{\pi}}{4\left(\beta+\mathrm{U}^{2}\right)^{1 / 4}} \cdot \frac{\beta^{7 / 2}}{\beta^{3}-\beta_{f}^{3}} \mathrm{~d} \beta$
where $U_{R}=U_{L} / U_{m}$ and $U_{m}$ was the rise velocity corresponding to the maximum radius at detachment, which for the case of zero translational velocity reduced to. equation (2.30).

Equation (2.32) gave good agreement with the experimental data of Abdelmessin et al [27] for a noncondensable content corresponding to $\beta_{f}=0.4$.

Moalem et al [29] presented an approximate analytical solution, related to the changes in temperature field and flow velocity in bubble trains, for pentane bubbles, with
or without noncondensables, condensing in liquid pentane or in water. They showed the effect of bubble frequency and the number of bubbles in a train on both the temperature field and the flow velocity. The results predicted the amount of liquid subcooling to maintain a bubble train and the liquid height for complete condensation, for different numbers of bubbles in a train at given frequencies.

For a given number of bubbles in a train, increase in frequency necessitated higher bulk liquid subcooling and this effect was increased by the presence of noncondensables for a given frequency. For a given bulk liquid subcooling, frequency did not have any significant influence on condensation height but the presence of noncondensables increased the condensation height. Moalem et al [30] also presented an exact numerical analysis, solving the potential flow field and the energy equation simultaneously for a bubble train. The solution in the case of one bubble was in good agreement with the previous analysis of Isenberg and Sideman [24] confirming the reliability of the theory.

The agreement between the approximate analytical [29] and the exact numerical [30] solutions was good, especially at frequencies above 10 bubbles per second. The assumption of the energy balance in the analytical solution, that all the heat accumulated within the liquid envelope between the condensing bubble and the bubble following it, was not completely fulfilled at low frequencies when bubbles were far apart.

Moalem et al [30] showed that the bubble frequency affected the condensation rate in two ways : (1) by reducing the effective subcooling (due to condensation of the preceding bubbles) leading to a reduction in the
condensation rate, and (2) by increasing the bubble rise velocity leading to an increase in the condensation rate. At low frequencies (up to 12-14 bubbles per second), increase in frequency reduced the effective subcooling and had little effect on the rise velocity, and thus decreased the condensation rate. However, at high frequencies, each bubble entered the wake region of the preceding bubble, increasing the rise velocity and hence increased the convection effects and the collapse rate which, for 26 bubbles per second, approached that of a single bubble. If noncondensables were present, they had little effect on the rise velocity but affected the temperature field tending to reduce the condensation rate even more.

The experimental data for pentane-pentane and pentane-water systems were in good agreement with the analytical solution except at high frequencies and/or high noncondensable content. The bubbles deviated from the assumed axial symmetry and, at high frequencies, different interaction effects occurred as the bubbles approached each other. The assumption of homogeneous distribution of noncondensables was considered inappropriate when the noncondensable content was high.

Schmidt [35] carried out a range of tests in which highly superheated steam (100-200 K) was injected into saturated water at high system pressures (see table (2.2)). In such conditions, in contrast to those where saturated steam is injected into subcooled water, the main thermal resistance is inside the bubble, where a thermal boundary layer forms on the steam side of the steam-water interface. He defined an average heat transfer coefficient, in the range of $0.5-7.5 \mathrm{~kW} / \mathrm{m}^{2} \mathrm{~K}$, as

$$
\begin{equation*}
h=\frac{\dot{m} c_{p}}{S} \ln \frac{T_{0}-T_{W}}{T_{S}-T_{W}} \tag{2.33}
\end{equation*}
$$

where $S$ is average bubble surface area during growth, $\dot{m}$ is steam mass flow rate. $T_{s}$ and $T_{o}$ are the respective steam temperatures at orifice outlet and inside the bubble at detachment. The values obtained were about one order of magnitude lower than values reported for the collapse of saturated steam bubbles in subcooled water (see tables (2.1) and (2.2) for values reported in the literature), this indicating a better heat transfer when the main thermal resistance was on the water side of the interface rather than on the steam side.

Schmidt [35] reported an increase in heat transfer coefficient with increased pressure (due to changes in enthalpy of evaporation and in steam viscosity and thermal conductivity). The coefficient also increased with increase in orifice diameter and mass flow rate, the rate of increase being greater for $\operatorname{Re}<1300$, at which value there was a transition from steady state to unsteady state bubble generation. There was no significant effect on the heat transfer rate due to increase in the amount of superheat in the bubble. The heat transfer coefficients were presented as Stanton number against Reynolds number, to within $\pm 20 \%$, in the form

$$
\begin{equation*}
\text { St }=\frac{h d}{\dot{\dot{m} c} c_{p}} \sqrt{\frac{\sigma}{g\left(\rho_{W}-\rho_{S}\right)}}=\text { constant. Re }{ }^{0.766} \tag{2.34}
\end{equation*}
$$

where the constant depends on orifice diameter (ranging from 8 at $d=3 \mathrm{~mm}$ to 12 at $d=1.5 \mathrm{~mm}$ ).

Schmidt [34] also noted that the bubble volume at detachment and the bubble frequency both increased with steam mass flow rate, the volume also increasing with decrease in system pressure and increase in orifice diameter, while, for a given mass flow rate, the frequency increased with decrease in orifice diameter.

The bubble surface area at detachment was given, to within $\pm 30 \%$, as

$$
\begin{equation*}
\mathrm{S}=1.913 \cdot 10^{-3} \cdot \mathrm{~d}^{2}\left[\frac{\mathrm{Fr}}{\mathrm{Re}^{2} \mathrm{We}^{2}}\left(\frac{(\mathrm{Eu} \cdot E \mathrm{EC})^{1 / 2}}{\mathrm{Re}}\right)^{5.35}\right]^{1 / 3} \tag{2.35}
\end{equation*}
$$

and the frequency of formation, to within $\pm 10 \%$, as

$$
\begin{equation*}
f=3.08 \cdot 10^{-2}\left(\frac{g}{d}\right)^{1 / 2} \cdot \mathrm{Re}^{0.426} \tag{2.36}
\end{equation*}
$$

where $\operatorname{Re}=\dot{m} / d \mu_{0}$.

Akiyama [36] investigated growth and collapse of vapour bubbles in subcooled boiling (see table (2.2)). Employing the heat transfer correlation, given by Grigull [37], for solid spheres in laminar flow

$$
\begin{equation*}
N u=0.37 \cdot \operatorname{Re}^{0.6} \cdot \mathrm{Pr}^{2 / 3} \tag{2.37}
\end{equation*}
$$

and from a heat balance for a collapsing bubble he derived the equation,

$$
\begin{equation*}
B=\left(1-1.036 . \mathrm{JaPe}^{0.6} \mathrm{Pr}^{-0.27 \mathrm{FO}^{5 / 7}}\right. \tag{2.38}
\end{equation*}
$$

This correlation overpredicted bubble collapse rates when compared with the experimental data.
2.4 Review findings

The following conclusions may be drawn from the preceding literature review :

There is a limited amount of experimental data and theoretical study on vapour bubbles condensing in subcooled liquids. When such experiments were performed, no extensive or accurate analysis of the cine-films was carried out because the necessary electronic equipment was not in general use.

None of the existing quasi-steady state theories take account of radial velocities, bubble distortion or local temperature differences produced around the condensing bubbles. Each of these may have some effect on the collapse rate of the bubble although the relative magnitude of the different effects should depend on the parameters controlling the collapse. For example, radial velocities may be comparable with the bubble rise velocity especially at high subcoolings, there may be bubble distortion especially early in the collapse and the condensation of the bubble mustlead to local heating of the water around the bubble.

Ruckenstein and Davis [31] presented a general solution for a growing bubble, with allowance made for radial velocity effects but with no allowance made for either distortion or local temperature changes. This gave the bubble radius $R$ at any time $t$ in terms of a complicated integral equation, which was solved iteratively for certain cases of the growth of vapour bubbles, but the result is found to be too complicated for ready engineering use.

It would therefore be desirable to obtain a more complete solution, easier to use, and including the effects of the three variables mentioned.

## CHAPTER 3

3.1 Test rig

A schematic layout of the bubble chamber test rig is shown in fig. 3.1. It consisted of three main parts; a steam boiler, a steam flow measuring section, and a bubble test chamber, including an injection orifice.
3.1.1 The steam boiler

The stainless steel boiler was a vertical cylinder of 230 mm inside diameter, 380 mm height and 12.7 mm thickness. It was enclosed in another cylindrical steel container insulated by two layers of bricks at the bottom. The boiler was placed on top of a 3 kW external ring heater embedded, in a layer of Kaowool ceramic fibre ( $k=0.08 \mathrm{~W} / \mathrm{mK}$ ), on the bricks. The sides and top of the boiler were insulated by fibre glass ( $k=0.03 \mathrm{~W} / \mathrm{mK}$ ). An additional 325 W immersion heater, controlled by a variac transformer, was used during the experiments, to maintain the pressure in the boiler constant for different steam flow rates. A sight glass, to observe the water level inside the boiler, and a safety valve, to prevent unwanted high pressures, were fitted to the boiler. Two pressure gauges were fitted, one for subatmospheric and the other for elevated pressures.

### 3.1.2 Flow measuring section and calibration

Steam, generated from distilled water in the boiler, was passed through a 6 mm diameter copper pipe to the flow measuring section. This part consisted of a 1.5 mm diameter, 200 mm long stainless steel pipe. Another 6 mm diameter copper pipe carried steam from the flow measuring section to the bubble chamber. The two copper pipes and

Fig. 3.1 Layout of steam bubble condensation rig
the stainless steel pipe were each wrapped by a 260 watt electric tape heater controlled by variac transformers to superheat the steam before it reached the bubble chamber.

Two pressure tappings at each end of the measuring section were connected to a $30^{\circ}$ inclined manometer. Carbon tetrachloride $\left(\mathrm{CCl}_{4}\right)\left(\rho=1594 \mathrm{~kg} / \mathrm{m}^{3}\right)$, dyed red to differentiate it from water, was used as the manometric fluid. Two copper constantan thermocouples, one before and one after the measuring section, were installed on the copper pipes to indicate steam temperatures. The tape heater was adjusted to maintain the steam temperature constant and ensure single phase superheated steam throughout the measuring pipe.

During the calibration the temperatures at both ends of the copper pipes were kept at $160^{\circ} \mathrm{C}$. In order to establish the relationship between the temperature on the pipe and the steam pressure inside the pipe, tests were carried out, at different pressures, using saturated steam and it was found that the steam temperature was about 4 K above the temperature of the pipe surface. Therefore the steam temperature inside the pipe during calibration was assumed to be $164^{\circ} \mathrm{C}$. Apart from the manometric liquid $\mathrm{CCl}_{4}$, the arms of the inclined manometer and the copper pipes connecting them to the pressure tappings were filled with water. The valve to the test chamber was closed and differences in steam flow rate were obtained by adjustment of the valve leading to the condenser.


Fig. 3.2 Pressure drop in inclined manometer

As illustrated in fig. 3.2, pressures on both arms of the manometer should be equal at plane $A-A$.

$$
\begin{align*}
& P_{2}+\rho_{W} g \operatorname{Sin} \alpha=P_{2}+\rho_{C} g z \operatorname{Sin} \alpha \\
\therefore \quad & \quad \dot{H}=P_{z}-P_{2}=\operatorname{gzSin} \alpha\left(\rho_{C}-\rho_{W}\right) \tag{3.1}
\end{align*}
$$

with the density of water, $\rho_{W}=998 \mathrm{~kg} / \mathrm{m}^{3}$ and of $\mathrm{CCl} \mathrm{I}_{4}$, $\rho_{C}=1594 \mathrm{~kg} / \mathrm{m}^{3}$ (both at $16^{\circ} \mathrm{C}$ ). $\quad \Delta P$ is the pressure drop in the pipe, which for a manometer inclination of $30^{\circ}$ is given as

$$
\begin{equation*}
\Delta P=2.922 . z \quad N / m^{2} \tag{3.2}
\end{equation*}
$$

where $z(\mathrm{~mm})$ is the difference in CCly levels in the limbs of the manometer.

While carrying out the calibration, the steam flow rate $(\dot{m})$ was measured in $g / m i n$ by collection of condensate
while, at the same time, recording the corresponding pressure difference ( $z$ ) on the manometer.

The pressure drop in a pipe having a diameter d and length 1 is given by

$$
\begin{equation*}
\Delta P=\lambda \frac{1}{d} \frac{\rho U^{2}}{2} \tag{3.3}
\end{equation*}
$$

where $\lambda=\lambda(R e)$ is the flow resistance coefficient and $U$ the mean velocity of flow.

Multiplying both sides by $\frac{\rho^{2}}{\mu^{2}}$ in eqn. (3.3),
$\frac{\rho d^{2} \dot{L} P}{\mu^{2}}=\frac{\lambda 1}{2 d} \cdot \frac{U^{2} \rho^{2} d^{2}}{\mu^{2}}=\frac{\lambda 1}{2 d} \cdot R e^{2}=\frac{1}{2 d} f(R e)$
This indicates that for a given pipe length and diameter, the dimensionless group
$B=\frac{\rho d^{2} \dot{L} P}{\mu^{2}}$ is a function of Reynolds number (Re) only.
This is applicable for all fluids and conditions (temperatures and pressures).

From the measurement of condensate and manometric pressure difference, the values of
$B$ and $\operatorname{Re}\left(=\frac{4 \dot{m}}{\pi d \mu}\right)$ were determined and the calibration curve was plotted as shown in Fig. 3.3. As a check on the calibration curve, measurements were also taken using air, with the mass flow rate determined by a rotameter.

Fig. 3.3 The calibration curve

After passing through the flow measuring section, steam passed to the injection orifice situated at the base of the bubble chamber, where bubbles were formed which then rose into the subcooled distilled water. The steel chamber was 180 mm square x 240 mm high with borosilicate glass windows installed in three of the sides to permit observation and photography of the condensation process. A layer of low conductivity polycarbonate was fitted to the base of the test chamber to house the injection orifice, the intention being to minimise heat losses from the steam in the orifice before bubble formation and to reduce convection currents in the region surrounding the issuing steam.

Apart from the windows, the sides of the chamber were insulated by a layer of low conductivity cement. The top and the base were also insulated using fibre glass. A 325 W immersion heater was installed at the wall, near the base, to heat up the water in the chamber to near boiling temperature and hence to obtain different degrees of subcooling. A valve situated near the base was used to drain water from the chamber.

The test chamber could be filled to any required level with distilled water and could be pressurised up to 4 bar by means of a pressure regulator, using compressed air from the main supply. A pressure gauge and a valve, to release pressure if required, were situated at the top of the chamber. A thermocouple probe was installed for measuring the water temperature in the chamber at six different levels in 38 mm intervals, the first one being 6.3 mm away from the base. A valve, situated before the orifice on the copper pipe, was used to adjust the steam flow rate.

The injection orifice was initially manufactured from brass with the tip just emerging above the layer of polycarbonate and with two thermocouples embedded into the brass wall. However, due to the relatively high thermal conductivity of the brass, excessive convection currents were set up, particularly at low steam flow rates, in the region surrounding the emerging bubble. In addition, and especially at high water subcoolings, significant
pre-detachment condensation occurred. To minimise these effects, the orifice material was changed from brass to PTFE which has a thermal conductivity of $0.25 \mathrm{~W} / \mathrm{mK}$ and could be used at temperatures up to $250^{\circ} \mathrm{C}$. A sketch of the 2 mm diameter orifice is shown in Fig. 3.4.

Due to the low conductivity of PTFE it was not practicable to embed thermocouples in the orifice walls. Hence a thermocouple was fixed to the wall of the copper supply pipe just before entry to the orifice. In order to relate the temperature indicated by this thermocouple to the actual temperature of the emerging steam bubbles, provision was made for a thermocouple probe to be inserted through the roof of the bubble chamber, capable of being lowered into the steam bubbles at the orifice mouth. After some tests, at different steam flow rates and water subcoolings, it was found that a temperature of $165^{\circ} \mathrm{C}$ on the copper wall before the orifice would provide steam, either slightly superheated or saturated, at the orifice mouth. Therefore the temperature on the copper wall was maintained at $165^{\circ} \mathrm{C}$ during the experiments.
3.2 Photographic technique

A shadowgraph (direct shadow) optical system, illustrated in Fig. 3.5, was used to obtain photographic

Fig. 3.4 Axial cross-section of injection orifice

recordings of growing and collapsing bubbles. The theory of the direct-shadow method to detect density gradients in colourless fluids is well documented and will not be presented here. A thorough discussion and review is given by Holder and North [33].

The light source used was a 250 .watt mercury vapour discharge lamp having a 250 volt D.C. supply. The condensing lens was positioned to give a sharp image of the arc at a slit located at the focus of a spherical concave mirror, having 8 inches diameter and a focal. length of 6 feet, this reflecting a parallel beam of light through the test chamber. All the light passing outside the lens was cut off by a 4-sided covering fitted around the lamp and the lens. To minimise aberrations the reflection angle, $\theta$ at the mirror was kept as small as possible ( $\theta<10$ degrees).

A Hitachi 16 HM high speed camera with a speed range of up to 20,000 frames per second was located on a horizontal rail which had a vertical adjustment over a range of 60 mm . The camera could be moved horizontally along the rail to give either distant or close up images of the emerging bubbles and the associated convection currents. Close up views gave useful detail of bubble growth, detachment and collapse at high and medium subcoolings; distant views were useful, particularly, for studying bubble train effects, when condensation effects were small, or for more extensive pictorial records of bubble movement. During filming, the camera was focussed at the plane passing through the tip of the orifice to get a sharp image of the bubble and the tip, the diameter of the tip being used as a reference measure to calculate bubble dimensions during analysis of the cine films.

A camera speed of 500 frames per second was used for zero and low water subcoolings, this being increased up to 2500 frames per second as subcooling increased and bubbles condensed more quickly. 16 mm Kodak Tri-X reversal film of 30.5 m length (Type No. 7278) was used. The films were processed to the negative and used in that form during the analysis. A timing light generator connected to the camera produced marks on the film in time intervals of 10 milliseconds.

### 3.3 Test procedure and programme

A considerable effort was spent to obtain effective evacuation of air from the apparatus and finally the following procedure was adopted :

The boiler was filled with distilled water to the prescribed level and then a vacuum pump was connected to the boiler and run for about three hours without any heating. The external and immersion heaters were then switched on and, when steam formation started (still under vacuum), the valve connected to the vacuum pump was turned down to a minimum level to prevent steam going to the pump and damaging it. When atmospheric pressure inside the boiler was reached, the vacuum pump was switched off and the pressure allowed to rise. During the pressure rise, a purge valve on the boiler was opened to remove any residual air.

The vacuum pump was then used to evacuate the pipes leading to the test chamber and, after several hours, the supply valve from the boiler was opened and the pipe system allowed to fill with steam. The vacuum pump was once more disconnected and the steam again purged to atmosphere. The boiler and the pipe systems were
thereafter maintained at a pressure higher than
atmospheric (about 1.5 bar), by suitable regulation of the immersion heater in the boiler, to compensate for heat losses from the system.

In the tests, a water level of 40 mm above the orifice was maintained in the test chamber, with the water being heated up to boiling point by the immersion heater and by steam injected via the orifice plate, the former being switched off when the required conditions were attained. After convection currents around the heater disappeared and while the water temperature was still near boiling, suitable cine films were recorded while all the other relevant measurements were taken.

The water in the chamber was then allowed to cool progressively, to give where possible Jakob numbers up to 75, in increments of 15, with cine film and measurements recorded at each Jakob number. In each case, whenever bubbles formed, cine film recording was continued throughout bubble collapse. However, at high subcoolings the intensity of pre-detachment condensation meant that, in some cases, no bubbles detached from the orifice.

The range of test conditions covered is as follows:


### 3.4 Analysis of films

A DT-11A HIPAD digitiser, interfaced to an Apple-II micro-computer, was used to obtain, from the cine films, measured values of bubble volume, bubble surface area and bubble position (all related to time), throughout both growth and collapse periods. The speed at a particular section of film was determined from timing marks on the film.

To carry out the measurements, the cine film was projected on to the tablet of the digitiser via a mirror (silverised on the front face to avoid image distortion) inclined at $45^{\circ}$ to the horizontal. The cursor of the digitiser was then traversed around the bubble outline and the resulting $X$ and $Y$ coordinates fed to the computer for transformation to the required bubble volume, surface area, centroid position, etc., using a computer program written specially for the purpose. To facilitate the coordinate measurements, the tablet was covered with paper printed with a 1 mm square grid. The arrangement of digitiser and computer together with projector and mirror is illustrated in Fig. 3.6.

The bubble projected on the graph paper was assumed to be comprised of frustums of cones, each having a height of either 10 mm or in some cases 5 mm . The computer program took 4 coordinates of a frustum and calculated volume, surface area, and position of centroid. An illustration of bubble projection on graph paper is given in Fig. 3.7.

The calculations were carried out as follows :
Projector . Silvered [front face]



Fig. 3.7 Bubble projection on graph paper
i) The volume of the $i$ th frustum (hatched in fig. 3.7) was calculated from the coordinate measurements, $X$ and $Y$ as
$V_{i}=\frac{\pi}{3} h_{i}\left(R_{t i}{ }^{2}+R_{t i} R_{b i}+R_{b i}{ }^{2}\right)$
where $R_{b}$ and $R_{t}$ are respectively the radii of the bottom and top surfaces, and $h$ is the height of the frustum. Hence the total volume of the bubble :
$V_{B}=\sum_{i=1}^{n} V_{i}$
(1i) The lateral area of the $i$ th frustum was
$A_{i}=\pi\left(R_{b i}+R_{t i}\right) \sqrt{h_{i}^{2}+\left(R_{t i}-R_{b i}\right)^{2}}(3.7)$

Hence the total surface area of the bubble was
n
$S_{B}=\sum_{i=1} A_{i}+S_{b}+S_{t}$
where $S_{b}$ \& $S_{t}$ are the respective areas at the bottom of the first frustum and at the top of the last frustum.
iii) The equivalent radius of the bubble,
$R=\left(\frac{3 V_{B}}{4 \pi}\right)^{1 / 3}$
iv) Location of the centroid of each frustum was calculated according to the formula,
$c=\frac{1}{4} d\left(\frac{1+2 k+3 k^{2}}{1+k+k^{2}}\right)$
where $d$ is the distance between the centroids of the bases of frustum, and

```
k=\frac{R}{\mp@subsup{R}{b}{}}}\mathrm{ is the ratio of the radius of the top
surface to the radius of the base.
```

The centroid of each frustum is at a distance $c$ from the centroid of the base along the line of length joining the centroids of the base and the top surface as illustrated in Fig. 3.8.


Fig. 3.8 The centroid of the frustum of a cone

Hence the bubble centroid position ( $X_{C B}, Y_{C B}$ ) was calculated as

$$
\begin{equation*}
X_{C B}=\frac{\sum_{i=1}^{n} V_{i} X_{C i}}{\sum_{i=1}^{n} V_{i}}, Y_{C B}=\frac{\sum_{i=1}^{n} V_{i} Y_{C i}}{\sum_{i=1}^{n} V_{i}} \tag{3.11}
\end{equation*}
$$

## CHAPTER 4

EXPERIMENTAL RESULTS AND DATA ANALYSIS

### 4.1 Introduction

The test results are typified by the set shown in Figures 4.1 and 4.2 , these being printed by the plotter. These results relate to a particular steam flow rate ( $0.45 \mathrm{~g} / \mathrm{min}$ ) and water subcooling ( 9.3 K ) at 2 bar pressure with a 2 mm diameter orifice and include : (1) An artistic impression of the bubble shape over a range of frames in the cine film
(ii) A table of experimental parameters
(iii) A table of experimental results
(iv) 10 plots of experimental or derived data
4.2 Format of Test Results
4.2.1 Impressions of Bubble Shape

For any particular test condition, several sequences of bubble growth and collapse were photographed. The impressions illustrated are typical of the events at that particular test condition.

### 4.2.2 Experimental Parameters

The identification of parameters and other details relevant to Fig.4.1 is as follows:
d : orifice diameter, mm
$\dot{m}_{S}$ : steam mass flow rate, g/min
$V_{s}$ : steam volume flow rate, $\mathrm{mm}^{3} / \mathrm{s}$
AT : water subcooling, $K$
$Z$ : water level above orifice in chamber ( $=40 \mathrm{~mm}$ in all tests)
$P$ : pressure in bubble chamber steam space, bar

EXPERIMENTAL PARAMETERS :

| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | .45 |
| 1 | $V_{s}$ |
| $\Delta T$ | 6643 |
|  | 9.3 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.005 |
| $\Delta t$ | 2.37 |
| 1 | $C S$ |


| $F$ | 24 |
| :--- | :--- |
| $J_{Q}$ | 15 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| $t_{g}$ | 28.46 |
| :--- | :--- |
| $t_{c}$ | 24.7 |
| $t_{t}$ | 53.16 |
| $t_{s}$ | 38 |
| $R_{m}$ | 2.51 |
| $R_{0}$ | 2.45 |

TEST NO: (4.1). 2

| $h_{c}$ | 17086 |
| :--- | :--- |
| $N u_{c}$ | 63 |
| $Z_{d}$ | 4.22 |
| $Z_{c}$ | 11.36 |
| $U$ | 250 |
| $F_{o_{c}}$ | $1.749 \mathrm{E}-04$ |


| $P_{\theta_{0}}$ | 7210 |
| :--- | :--- |




Fig. 4.1 Experimental parameters and results

|  |  |  |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |



The identification of the items in Fig. 4.1 is as follows:
ty : time from bubble initiation to detachment from orifice, ms
$t_{c}$ : time from bubble detachment to complete collapse due to condensation, ms
$t_{t}$ : total time from initiation to complete collapse $=t g+t_{c}, m s$
$f_{S}$ : bubble frequency, $1 / s$
$R_{m}$ : maximum radius of bubble, mm
$R_{0}$ : bubble radius at detachment, mm
$h_{c}$ : average heat transfer coefficient during condensation (after detachment), W/m $\mathbf{m}^{\mathbf{K}}$

Nu ce average Nusselt number during condensation
$Z_{d}$ : bubble centroid height at detachment (above orifice), mm
$Z_{c}$ : bubble centroid height at collapse, mm
$U$ : average vertical velocity of bubble during condensation (after detachment), mmes

For : collapse Fourier number $=\frac{\alpha_{w} t_{c}}{4 R_{o}{ }^{2}}$
4.2.4 Data Plots

There are 10 separate data plots (identified as 'a' to ' $k$ ' over Figs. 4.1 and 4.2), the data points on which relate to computed values. The vertical chaindotted line on plot (d) indicates the centre line of the orifice while the vertical chaindotted line on plots (a), (b), (c), (e) and (g) indicates the point of detachment and thereby separates the bubble growth and bubble collapse (condensation) regions.
4.3 Comments on individual data plots

Plot (a) : This is a plot of bubble volume versus time, as obtained from the cine film analysis. The data clearly show the bubble growth period, the bubble volume reaching a maximum value just before detachment, i.e. just before detachment the reduction in volume due to condensation is apparently beginning to overcome the growth due to steam flow. For this subcooling value of 9.3 K, the data indicate that, after detachment, bubble collapse due to condensation is fairly rapid.

Plot (b) : This shows bubble radius $R$ to a base of time and is an alternate form of plot (a) since $R$ was evaluated from $V=\frac{4}{3} \pi R^{3}$.

Plot (c) : This shows the position of the $Y$ coordinate of the centroid of the bubble with respect to time. It shows the bubble centroid rising fairly uniformly throughout bubble growth, with a slight acceleration at detachment and with a more rapid centroid movement after detachment than during growth.

* It was not possible to sustain a constant steady steam flow rate, unaffected by fluctuations, throughout the experiment .

Plot (d) : This is a spatial plot of the $X$ and $Y$ coordinates of the centroid of the bubble during growth and collapse. In this plot, the vertical chaindotted line represents the centre line of the inlet orifice and the data indicate the bubble movement relative to this from initiation to collapse. The plot clearly shows that, during bubble growth, the bubble centroid position deviates only slightly from the orifice centreline. After detachment, however, the bubble does not travel stralght upwards, but deviates from the centre line by up to about 0.6 mm , (or perhaps more since these are $X$-projected values and the radial deviation may be greater), which is much less than the bubble diameter at detachment. The height above the inlet orifice at which complete collapse occurred is about 12 mm , which means the bubble rose about 8 mm between detachment and complete collapse.

Plot (e) : This is a plot of the position of the "leading edge" (LE) of the bubble (i.e. top most point) against time and, as such, is an alternative to plot (c).

Plot (f) : This is a similar plot to (e) but with the "lowest point" (LP) of the bubble being plotted against time, for the collapse region only. A comparison of plots (c), (e) and (f) gives some indication of the distortion in bubble shape from the spherical and also suggests that, after detachment, condensation occurs mainly on the upper part of the bubble.

Plot (g) : This is a dimensionless plot which shows $\beta_{2}=\frac{R}{R_{m}}$ against Fourier number $\mathrm{FO}_{2}=\frac{\alpha_{r} \cdot t}{4 R_{m}^{2}}$ and is an attempt to physically relate the bubble growth and collapse to the heat conduction through the bubble boundary layer.

Plot (h): This is another dimensionless plot of the data, similar to plot (g), but using the bubble radius at detachment $R_{o}$, instead of the maximum bubble radius $R_{m}$, in the expressions for $\beta$ and $F_{o}$, and with time $t$ being measured from the point of detachment. Here
$\beta=\frac{R}{R_{0}}$ and $F_{0}=\frac{\alpha_{w} \cdot t}{4 R_{0}{ }^{2}}$
and the data are plotted only for the collapse region.

Plot (j): This is a plot of instantaneous heat transfer coefficient $h$, between the steam bubble and the water, versus time for the collapse region. The values of $h$ were evaluated as

$$
\begin{equation*}
h=\frac{\frac{\Delta V}{\Delta t} \cdot \rho_{s} \cdot h_{f g}}{A \cdot \Delta T} \tag{4.2}
\end{equation*}
$$

where $\Delta V=V_{1}-V_{i+1}$, the difference in bubble volume between two successive frames
$A=\frac{A_{i}+A_{i+1}}{2}$, the average bubble surface area between two successive frames.

In the plot, $\bar{t}=\frac{t_{1}+t_{1}+1}{2}$ indicates the time to a point between two successive frames. The data show condensation heat transfer coefficients ranging up to about $35 \mathrm{~kW} / \mathrm{m}^{2} \mathrm{~K}$.

Plot (k): This shows the instantaneous Nusselt
number plotted against time for the collapse region. The Nusselt number was calculated as

$$
\begin{equation*}
\mathrm{Nu}=\frac{2 \mathrm{~h} \cdot \mathrm{R}}{\mathrm{k}_{\mathrm{W}}} \tag{4.3}
\end{equation*}
$$

$\Delta T$ is taken as the nominal temperature difference between steam and water , $T_{s}-T_{\infty}$.
where $R=\frac{R_{i}+R_{i+1}}{2}$, the average bubble equivalent radius between two successive frames.

### 4.4 Experimental Results

The grid of the test conditions together with the corresponding test numbers is given in Table 4.1. Water subcoolings, corresponding to Jakob numbers at different pressures, are also presented in Table 4.1. For example, Test ${ }^{\prime}$. $^{\prime}$ indicates a 2 mm diameter orifice being used, at 2 bar pressure with a steam mass flow rate of 0.92 $\mathrm{g} / \mathrm{min}$ and a water subcooling of 18.5 K (corresponding to a Jakob number value of 30 ). Three bubbles were analysed for each of the 38 conditions. Experimental results, for one bubble in each condition, are given in the following pages. The numerical values of some of the variables are given in Appendix 4.

In each case the bubble is represented by one of the following symbols ( $\mathbb{X},+$ and $*$ ), each of which relates to the same bubble in all plots. In the following chapters all 3 bubbles in each condition are plotted in dimensionless form as $\beta$ vs Fo.
4.5 Determination of bubble rise velocity, peclet number and collapse fourier number

The bubble rise velocities, $U$ were determined "for each bubble from the slopes of the 'Cy versus $t$ ' curves. The detachment radius of the bubble, $R_{0}$ and the collapse time after detachment, $t_{c}$ were determined, in most cases, from the 'V versus $t$ ' curves as they showed more clearly the effect of noncondensables towards the end of collapse

| $\begin{gathered} \mathrm{d} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{P} \\ (\mathrm{bar}) \end{gathered}$ |  | 15 | 30 | 45 | 60 | 75 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1 | 0.5 | Test number |  |  |  |  |
|  |  |  | 1.1 | 1.2 | - | - | - |
|  |  | 1.0 | 2.1 | 2.2 | 2.3 | - | - |
|  |  | 1.5 | 3.1 | 3.2 | 3.3 | 3.4 | - |
|  | 2 | 0.45 | 4.1 | 4.2 | - | - | - |
|  |  | 0.92 | 5.1 | 5.2 | 5.3 | - | - |
|  |  | 1.42 | 6.1 | 6.2 | 6.3 | 6.4 | - |
| 1 | 1 | 0.42 | 7.1 | 7.2 | 7.3 | - | - |
|  |  | 1.0 | 8.1 | 8.2 | 8.3 | 8.4 | - |
|  |  | 1.42 | - | 9.2 | 9.3 | 9.4 | 9.5 |
|  | 2 | 0.45 | 10.1 | 10.2 | - | - | - |
|  |  | 0.82 | 11.1 | 11.2 | 11.3 | - | - |
|  |  | 1.39 | 12.1 | 12.2 | 12.3 | 12.4 | - |


| Ja | 15 | 30 | 45 | 60 | 75 | P <br> (bar) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (T) <br> $(K)$ | 5 | 10 | 14.9 | 19.8 | 24.7 | 1 |

Table 4.1 The grid of the test conditions, and water subcoolings corresponding to Jakob numbers


FRAME
NUMBERS :
EXPERIMENTAL PARAMETERS :

| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | .5 |
| $\left(V_{s}\right.$ | 13935 |
| $\Delta T$ | 5 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | .991 |
| $\Delta t$ | 2.29 |
| 1 | $C S$ |


| $F$ | 34 |
| :--- | :--- |
| $J_{a}$ | 15 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| $t_{g}$ | 50.46 |
| :--- | :--- |
| $t_{c}$ | 26 |
| $t_{t}$ | 76.46 |
| $f_{s}$ | 20 |
| $R_{m}$ | 3.84 |
| $R_{0}$ | 3.46 |


| $h_{c}$ | 21285 |
| :--- | :--- |
| $N u_{c}$ | 121 |
| $Z_{d}$ | 7.76 |
| $Z_{c}$ | 18.12 |
| $U$ | 480 |
| $\mathrm{Fo}_{c}$ | $9.09 \mathrm{E}-05$ |


| $\mathrm{Pe}_{\mathrm{o}}$ | 19840 |
| :--- | :--- |



TEST NO : (1.1). 3
FILM NO : 34-2/5

| 25 CrimM) 20 15 10 10 5 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \beta_{2}^{1} \\ .8 \\ .6 \\ .4 \\ .2 \\ 0 \end{gathered}$ |  |  |  |  |  |  |  |
| TEST NO : (1.1).3 FILM NO : 34-2/5 |  | (1.1).3 FILM NO : 34-2/5 |  |  |  |  |  |



NUMBERS :
EXPERIMENTAL PARAMETERS:

| $d$ | 2 |
| :---: | :--- |
| $\dot{m}_{s}$ | .5 |
| $l$ | $V_{s}$ |
| $\Delta T$ | 13935 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | .991 |
| $\Delta t$ | 2.31 |
| $l$ | $C S$ | 433,


| $F$ | 19 |
| :--- | :--- |
| $J_{a}$ | 30 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:

| $t_{g}$ | 32.33 |
| :--- | :--- |
| $t_{c}$ | 8.25 |
| $t_{t}$ | 40.58 |
| $t_{s}$ | 33 |
| $R_{m}$ | 2.82 |
| $R_{o}$ | 2.3 |


| $h_{c}$ | 17905 |
| :--- | :--- |
| $N u_{c}$ | 65 |
| $Z_{d}$ | 4.92 |
| $Z_{c}$ | 9.24 |
| $U$ | 437 |
| $F_{o_{c}}$ | $6.49 \mathrm{E}-05$ |


| $\mathrm{Pe}_{0}$ | 12080 |
| ---: | ---: |



$$
\text { TEST NO : (1.2).1 FILM NO: } 34-3 / 6
$$




FRAME
NUMBERS:
EXPERIMENTAL PARAMETERS :

| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1 |
| 1 | $V_{s}$ |
| $\Delta T$ | 27871 |
|  | 5 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | .995 |
| $\Delta t$ | 2.37 |
| $l$ | $C S$ |


| $F$ | 30 |
| :--- | :--- |
| $J_{a}$ | 15 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| $t_{g}$ | 37.92 |
| :--- | :--- |
| $t_{c}$ | 35 |
| $t_{t}$ | 72.92 |
| $f_{s}$ | 28 |
| $R_{m}$ | 4.68 |
| $R_{0}$ | 4.49 |


| $h_{c}$ | 26596 |
| :--- | :--- |
| $N u_{c}$ | 195 |
| $Z_{d}$ | 13.71 |
| $Z_{c}$ | 30.54 |
| $U$ | 524 |
| $F o_{c}$ | $7.26 E-05$ |


| $\mathrm{Pe}_{0}$ | 28110 |
| :--- | :--- |


|  |  |
| :---: | :---: |
| TEST NO: $(2.1) .2$ | FILM NO: $38-1 / 2$ |


ERAME $: 12$

| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1 |
| $\left(V_{s}\right.$ | 27871 |
| $\Delta T$ | 10 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | .995 |
| $\Delta t$ | 1.14 |
| $l$ | $C S$ |


| $F$ | 36 |
| :--- | :--- |
| $\mathrm{~J}_{\mathrm{a}}$ | 30 |
| $\mathrm{~T}_{\mathrm{p}}$ | 165 |

## EXPERIMENTAL RESULTS:

| $t_{g}$ | 2.6 .39 |
| :--- | :--- |
| $t_{c}$ | 15 |
| $t_{t}$ | 41.39 |
| $f_{s}$ | 40 |
| $R_{m}$ | 3.9 |
| $R_{o}$ | 3.8 |


| $h_{c}$ | 26206 |
| :--- | :--- |
| $N u_{c}$ | 159 |
| $Z_{d}$ | 9.27 |
| $Z_{c}$ | 18.42 |
| $U$ | 520 |
| $F_{O_{c}}$ | $4.32 \mathrm{E}-05$ |


| $\mathrm{Pe}_{0}$ | 23750 |
| :--- | :--- |


|  |  |
| :---: | :---: |
| TEST NO: 2.21 .3 | FILM NO: $38-2 / 3$ |



NUMBERS :
EXPERIMENTAL PARAMETERS:

| $d$ | 2 |
| :---: | :--- |
| $\dot{m}_{s}$ | 1 |
| $1 \mathrm{~V}_{s}$ | 27871 |
| $\Delta T$ | 15 |


| Z | 40 |
| :---: | :---: |
| P | . 995 |
| $\Delta t$ | 1.13 |
| 1 CS | 885 |


| $F$ | 15 |
| :--- | :--- |
| $J_{a}$ | 45 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:

| $t_{g}$ | 12.43 |
| :--- | :--- |
| $t_{c}$ | 4.5 |
| $t_{t}$ | 16.93 |
| $f_{s}$ | 88 |
| $R_{m}$ | 2.43 |
| $R_{0}$ | 2.15 |


| $h_{c}$ | 26367 |
| :--- | :--- |
| $N_{c}$ | 91 |
| $Z_{d}$ | 4.85 |
| $Z_{c}$ | 8.14 |
| $U$ | 775 |
| $F o_{c}$ | $4.02 \mathrm{E}-05$ |


| $\mathrm{Pa}_{0}$ | 20170 |
| ---: | ---: |




|  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| d | 2 | Z | 40 | F | 31 |
| $\dot{m}_{s}$ | 1.5 | P | . 995 | Ja | 15 |
| $1 \mathrm{~V}_{\mathrm{s}}$ | 41806 ) | $\Delta t$ | 2.18 | $T_{p}$ | 165 |
| $\Delta T$ | 5 | 1 CS | 458 |  |  |

EXPERIMENTAL RESULTS:

| $t_{g}$ | 37.11 |
| :--- | :--- |
| $t_{c}$ | 35.6 |
| $t_{t}$ | 72.71 |
| $f_{s}$ | 29 |
| $R_{m}$ | 5.4 |
| $R_{0}$ | 5.25 |


| $h_{c}$ | 33643 |
| :--- | :--- |
| $\mathrm{Nu}_{c}$ | 281 |
| $Z_{d}$ | 13.86 |
| $Z_{c}$ | 30.25 |
| $U$ | 643 |
| $\mathrm{Fo}_{c}$ | $5.41 \mathrm{E}-05$ |


| $P \theta_{0}$ | 40330 |
| :--- | :--- |



TEST NO: (3.1).2 FILM NO: 35-3/2



NUMBERS:
EXPERIMENTAL PARAMETERS:

| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1.5 |
| $L$ | $V_{s}$ |
| $\Delta T$ | 41806 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | .995 |
| $\Delta t$ | 1.37 |
| $C$ | $C S$ |


| $F$ | 38 |
| :---: | :---: |
| $\cdots J_{Q}$ | 30 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:

| $t_{g}$ | 31.51 |
| :--- | :--- |
| $t_{c}$ | 19 |
| $t_{t}$ | 50.51 |
| $f_{s}$ | 33 |
| $R_{m}$ | 4.87 |
| $R_{0}$ | 4.63 |


| $h_{c}$ | 24055 |
| :--- | :--- |
| $N_{c}$ | 179 |
| $Z_{d}$ | 10.63 |
| $Z_{c}$ | 21.88 |
| $U$ | 590 |
| $F_{o_{c}}$ | $3.69 E-05$ |


| $\mathrm{Pe}_{0}$ | 32830 |
| :--- | :--- |





EXPERIMENTAL RESULTS:

| $t_{g}$ | 18.79 |
| :--- | :--- |
| $t_{c}$ | 8.3 |
| $t_{t}$ | 27.09 |
| $f_{s}$ | 57 |
| $R_{m}$ | 4.01 |
| $R_{0}$ | 3.55 |


| $h_{c}$ | 24719 |
| :--- | :--- |
| $N u_{c}$ | 141 |
| $Z_{d}$ | 7.26 |
| $Z_{c}$ | 16.34 |
| $U$ | 800 |
| $F_{O_{c}}$ | $2.72 \mathrm{E}-05$ |


| $\mathrm{Pe}_{0}$ | 34380 |
| :--- | :--- |



| 25 crimM) 20 15 10 5 0 |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \beta_{2} \\ .8 \\ .6 \\ .4 \\ .2 \\ 0 \end{gathered}$ |  |  |  |  |  |  |  |  |
| TEST NO : 13.31 .1 |  |  |  | FILM NO : 36-2/1 |  |  |  |  |



EXPERIMENTAL RESULTS:

| $t_{g}$ | 16.72 |
| :--- | :--- |
| $t_{c}$ | 5.5 |
| $t_{t}$ | 22.22 |
| $f_{s}$ | 63 |
| $R_{m}$ | 3.33 |
| $R_{0}$ | 3.08 |


| $h_{c}$ | 24967 |
| :--- | :--- |
| $N u_{c}$ | 125 |
| $Z_{d}$ | 6.27 |
| $Z_{c}$ | 10.43 |
| $U$ | 810 |
| $F_{o_{c}}$ | $2.38 \mathrm{E}-05$ |


| $\mathrm{Pe}_{0}$ | 30390 |
| :--- | :--- |


|  |  |
| :---: | :---: |
| TEST NO: (3.4).1 | FILM NO: $37-1 / 2$ |



FRAME
$1-14$
15-16
17-19
20-22
23-24
25
NUMBERS:

EXPERIMENTAL PARAMETERS:

| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | .45 |
| $l$ | $V_{s}$ |
| $\Delta T$ | 6643 |
|  | 18.5 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.005 |
| $\Delta t$ | 1.51 |
| 1 | $C S$ |


| $F$ | 25 |
| :--- | :--- |
| $J_{a}$ | 30 |
| $T_{P}$ | 165 |

EXPERIMENTAL RESULTS :


| カ/I-St : DN W7İ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (SW) $\frac{1}{2 t}$ | 0 02 00 09 08 |  |  |  |  |  |  |
| (SW) 7 |  |  |  |  | 0 <br> 2 <br> $\square$ <br> 9 <br> $\stackrel{8}{\text { (wh) }}$ <br> 01 |  | 0 2 0 9 8 8 LWH 0 01 |



FRAME
$1-8$
9-10
11-12
13-16
17-20
21-27
NUMBERS:
EXPERIMENTAL PARAMETERS:

| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | .92 |
| $1 V_{s}$ | 13560 |
| $\Delta T$ | 9.3 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | 2.23 |
| 1 | $C S$ | 449,$\quad$|  |
| :--- |


| $F$ | 27 |
| :--- | :--- |
| $J_{a}$ | 15 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:

| $t_{g}$ | 22.26 |
| :--- | :--- |
| $t_{c}$ | 30 |
| $t_{t}$ | 52.26 |
| $f_{s}$ | 50 |
| $R_{m}$ | 3.5 |
| $R_{0}$ | 3.42 |


| $h_{c}$ | 20042 |
| :--- | :--- |
| $N u_{c}$ | 104 |
| $Z_{d}$ | 6.25 |
| $Z_{c}$ | 17.78 |
| $U$ | 338 |
| $\mathrm{Fo}_{c}$ | $1.09 E-04$ |


| $\mathrm{Pe}_{0}$ | 13600 |
| :---: | :---: |


|  |  |
| :---: | :---: |
| TEST NO: (5.1). 2 | FILM NO: 51-1/6 |


|  |  |  |
| :---: | :---: | :---: |
|  | 品洜品 品 只 品 | $\stackrel{0}{-1}$ |
|  |  | $\square$ <br> 1 <br>  <br> 0 <br> 2 <br> $\Sigma$ <br>  |
|  |  |  |
|  |  | $\sim$ $\underset{\sim}{1}$ $\stackrel{y}{*}$ |
|  |  |  |

FRAME
NUMBERS :
EXPERIMENTAL PARAMETERS :

| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | .92 |
| $1 V_{s}$ | 13560 |
| $\Delta T$ | 18.5 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.008 |
| $\Delta t$ | 1.58 |
| $C C S$ | 633 |


| $F$ | 26 |
| :--- | :--- |
| $J_{a}$ | 30 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:

| $t_{g}$ | 25.25 |
| :--- | :--- |
| $t_{c}$ | 14.2 |
| $t_{t}$ | 39.48 |
| $f_{s}$ | 42 |
| $R_{m}$ | 2.58 |
| $R_{0}$ | 2.74 |


| $h_{c}$ | 15874 |
| :--- | :--- |
| $N u_{c}$ | 70 |
| $Z_{d}$ | 4.06 |
| $Z_{c}$ | 7.32 |
| $U$ | 320 |
| $F o_{c}$ | $7.99 E-05$ |


| $\mathrm{Pe}_{0}$ | 10380 |
| :---: | :---: |





EXPERIMENTAL PARAMETERS :

| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | .92 |
| 1 | $V_{s}$ |
| $\Delta T$ | 13560 |
|  | 27.6 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.008 |
| $\Delta t$ | .77 |
| $l$ | $C S$ |


| $F$ | 40 |
| :--- | :--- |
| $J_{Q}$ | 45 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| $t_{g}$ | 21.56 |
| :--- | :--- |
| $t_{c}$ | 8.7 |
| $t_{t}$ | 30.26 |
| $f_{s}$ | 48 |
| $R_{m}$ | 2.49 |
| $R_{0}$ | 2.29 |


| $h_{c}$ | 16664 |
| :--- | :--- |
| $N u_{c}$ | 61 |
| $Z_{d}$ | 3.91 |
| $Z_{c}$ | 6.36 |
| $U$ | 247 |
| $F_{o_{c}}$ | $6.93 E-05$ |


| $\mathrm{Pe}_{0}$ | 6770 |
| :--- | :--- |


|  |  |
| :---: | :---: |
| TEST NO: (5.3).3 | FILM NO: 43-1/3 |




| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1.42 |
| 1 | $V_{s}$ |
| $\Delta T$ | 20963 |
|  | 9.3 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | 1.12 |
| 1 | $C S$ |


| $F$ | 41 |
| :--- | :--- |
| $J_{Q}$ | 15 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| $t_{g}$ | 21.21 |
| :--- | :--- |
| $t_{c}$ | 23 |
| $t_{t}$ | 44.21 |
| $f_{s}$ | 50 |
| $R_{m}$ | 3.7 |
| $R_{0}$ | 3.37 |


| $h_{c}$ | 28956 |
| :--- | :--- |
| $N u_{c}$ | 148 |
| $Z_{d}$ | 9.34 |
| $Z_{c}$ | 22.58 |
| $U$ | 589 |
| $F_{o_{c}}$ | $8.61 \mathrm{E}-05$ |


| $\mathrm{Pe}_{0}$ | 23350 |
| :---: | :---: |





FRAME
EXPERIMENTAL PARAMETERS:

| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1.42 |
| $1 \mathrm{~V}_{\mathrm{s}}$ | 20963 |
| $\Delta \mathrm{~T}$ | 18.5 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | .81 |
| 1 | $C S$ |


| $F$ | 30 |
| :--- | :--- |
| $J_{Q}$ | 30 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| $t_{g}$ | 11.39 | $h_{c}$ | 22580 | $P \theta_{0}$ | 17810 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $t_{c}$ | 12.5 | $\mathrm{Nu}_{c}$ | 109 |  |  |
| $t_{t}$ | 23.89 | $z_{d}$ | 5.06 |  |  |
| $f_{s}$ | 95 | $z_{c}$ | 11.54 |  |  |
| $\mathrm{R}_{\mathrm{m}}$ | 3.13 | $U$ | 495 |  |  |
| $\mathrm{R}_{0}$ | 3.04 | $\mathrm{Fo}_{\mathrm{c}}$ | 5.71E-05 |  |  |
|  |  |  |  |  |  |
| TEST | NO: | . 2 | FILM | NO : 4 | $-2 / 3$ |




EXPERIMENTAL PARAMETERS :

| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1.42 |
| $1 V_{s}$ | 209631 |
| $\Delta T$ | 27.6 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | .81 |
| $l$ | $C S$ |


| $F$ | 27 |
| :--- | :--- |
| $J_{Q}$ | 45 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:

| $t_{g}$ | 14.6 |
| :--- | :--- |
| $t_{c}$ | 7.5 |
| $t_{t}$ | 22.1 |
| $f_{s}$ | 73 |
| $R_{m}$ | 2.9 |
| $R_{0}$ | 2.73 |


| $h_{c}$ | 23346 |
| :--- | :--- |
| $N u_{c}$ | 107 |
| $Z_{d}$ | 4.07 |
| $Z_{c}$ | 6.85 |
| $U$ | 447 |
| $F 0_{c}$ | $4.2 E-05$ |


| $\mathrm{Pe}_{0}$ | 14610 |
| :---: | :---: |



TEST NO: (6.3). 3
FILM NO : 49-1/5


FRAME
EXPERIMENTAL PARAMETERS :

| $d$ | 2 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1.42 |
| $\left(\mathrm{~V}_{s}\right.$ | 20963 l |
| $\Delta T$ | 36.6 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | .81 |
| $l$ | $C S$ |


| $F$ | 27 |
| :--- | :--- |
| $J_{Q}$ | 60 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| $t_{g}$ | 17.03 |
| :--- | :--- |
| $t_{c}$ | 5 |
| $t_{t}$ | 22.03 |
| $f_{s}$ | 62 |
| $R_{m}$ | 2.47 |
| $R_{0}$ | 2.27 |


| $h_{c}$ | 21777 |
| :--- | :--- |
| $N u_{c}$ | 79 |
| $Z_{d}$ | 3.65 |
| $Z_{c}$ | 6.41 |
| $U$ | 447 |
| $F_{c}$ | $4 \mathrm{E}-05$ |


| $\mathrm{Pe}_{0}$ | 12300 |
| :--- | :--- |



TEST NO : (6.4). 1
FILM NO : $50-1 / 2$



| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | .42 |
| $\left(V_{s}\right.$ | 11706 |
| $\Delta T$ | 5 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 1.02 |
| $\Delta t$ | 2.15 |
| $(C S$ | 466 |


| $F$ | 27 |
| :--- | :--- |
| $J_{a}$ | 15 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :




EXPERIMENTAL PARAMETERS :

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | .42 |
| $\left(V_{s}\right.$ | 11706 |
| $\Delta T$ | 10 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 1.02 |
| $\Delta t$ | 1.03 |
| $1 C S$ | 967 |


| $F$ | 33 |
| :--- | :--- |
| $J_{a}$ | 30 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:

| $t_{g}$ | 22.74 |
| :--- | :--- |
| $t_{c}$ | 10.5 |
| $t_{t}$ | 33.24 |
| $f_{s}$ | 46 |
| $R_{m}$ | 2.96 |
| $R_{0}$ | 2.63 |


| $h_{c}$ | 23402 |
| :--- | :--- |
| $N u_{c}$ | 97 |
| $Z_{d}$ | 7.89 |
| $Z_{c}$ | 12.21 |
| $U$ | 515 |
| $F_{o_{c}}$ | $6.3 E-05$ |


| $P e_{0}$ | 16320 |
| :--- | :--- |




[^0]|  |  |  |
| :---: | :---: | :---: |
|  |  |  |
|  |  | m $\stackrel{y}{n}$ $\vdots$ |
|  |  | $\cdot$ <br> 0 <br> 2 <br> $\sim$ <br> $\sim$ |


FRAME
NUMBERS:
$1-11$
12
13-14
15-16
17

EXPERIMENTAL PARAMETERS:

| $d$ | 1 |
| :---: | :--- |
| $\dot{m}_{s}$ | .42 |
| 1 | $V_{s}$ |
| $\Delta T$ | 11706 |
| $\Delta T$ |  |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 1.02 |
| $\Delta t$ | $.78^{\circ}$ |
| 1 | $C S$ |


| $F$ | 17 |
| :--- | :--- |
| $J_{a}$ | 45 |
| $T_{p}$ | 165 |

## EXPERIMENTAL RESULTS :

| $t_{g}$ | 9.32 |
| :--- | :--- |
| $t_{c}$ | 4 |
| $t_{t}$ | 13.32 |
| $f_{s}$ | 117 |
| $R_{m}$ | 2.13 |
| $R_{0}$ | 1.85 |


| $h_{c}$ | 27982 |
| :--- | :--- |
| $N u_{c}$ | 84 |
| $Z_{d}$ | 7.33 |
| $Z_{c}$ | 9.59 |
| $U$ | 533 |
| $F_{o_{c}}$ | $4.77 E-05$ |


| $\mathrm{Pe}_{0}$ | 11950 |
| :---: | :---: |



TEST NO: (7.3).1
FILM NO : 54-2/2


FRAME
NUMBERS :

EXPERIMENTAL PARAMETERS :

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1 |
| 1 | $V_{s}$ |
| $\Delta T$ | 27871 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 1.017 |
| $\Delta t$ | 2.25 |
| 1 | $C S$ |


| $F$ | 10 |
| :--- | :--- |
| $J_{Q}$ | 15 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:

| $t_{g}$ | - |
| :--- | :--- |
| $t_{c}$ | 18.8 |
| $t_{t}$ | - |
| $f_{s}$ | - |
| $R_{m}$ | 3.44 |
| $R_{0}$ | 3.44 |


| $h_{c}$ | 33435 |
| :--- | :--- |
| $N u_{c}$ | 177 |
| $Z_{d}$ | 12.57 |
| $Z_{c}$ | 24.4 |
| $U$ | 680 |
| $F_{o_{c}}$ | $6.63 E-05$ |


| $\mathrm{Pe}_{0}$ | 28010 |
| :--- | :--- |



TEST NO : (8.1). 3
FILM NO : 55-2/5



$\Omega$
$\infty$
9
10-11
4-6
$7-8$

FRAME $1-3$

| $F$ | 11 |
| :--- | :--- |
| $J_{Q}$ | 30 |
| $T_{p}$ | 165 |

EXPERIMENTAL PARAMETERS :

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1 |
| $l$ | $V_{s}$ |
| $\Delta T$ | 27871 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 1.017 |
| $\Delta t$ | 1.07 |
| $l$ | $C S$ |

EXPERIMENTAL RESULTS :

| $t_{g}$ | - | $h_{c}$ | 29081 | $P e_{0}$ | 22360 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{t}$ | 10.9 | $\mathrm{Nu}_{\mathrm{c}}$ | 149 |  |  |
| $t_{t}$ | - | $z_{d}$ | 8.25 |  |  |
| ${ }_{\text {f }}$ | - | $z_{c}$ | 16.26 |  |  |
| $\mathrm{R}_{\mathrm{m}}$ | 3.17 | U | 593 |  |  |
| $\mathrm{R}_{0}$ | 3.13 | $\mathrm{Fo}_{\mathrm{C}}$ | 4.62E-05 |  |  |
|  |  |  |  |  |  |
| TEST | O: | . 3 | FILM | NO: 5 | 1/3 |




FRAME NUMBERS:
EXPERIMENTAL PARAMETERS:

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1 |
| 1 | $\mathrm{~V}_{s}$ |
| $\Delta \mathrm{~T}$ | 27871 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 1.017 |
| $\Delta t$ | .79 |
| $l$ | $C S$ |


| $F$ | 27 |
| :--- | :--- |
| $J_{a}$ | 45 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| $t_{g}$ | 16.51 |
| :--- | :--- |
| $t_{c}$ | 5.5 |
| $t_{t}$ | 22.01 |
| $f_{s}$ | 64 |
| $R_{m}$ | 2.84 |
| $R_{0}$ | 2.73 |


| $h_{c}$ | 34227 |
| :--- | :--- |
| $N u_{c}$ | 155 |
| $Z_{d}$ | 6.58 |
| $Z_{c}$ | 11.51 |
| $U$ | 650 |
| $F_{o_{c}}$ | $3.05 \mathrm{E}-05$ |


| $\mathrm{Pe}_{0}$ | 21510 |
| :---: | :---: |



TEST NO: (8.3). 3
FILM NO : 56-2/4


FRAME
1
$2-3$
4-5
$6-8$
NUMBERS:

EXPERIMENTAL PARAMETERS :

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1 |
| 1 | $V_{s}$ |
| $\Delta T$ | 27871 |
|  | 20 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 1.017 |
| $\Delta t$ | .58 |
| $l$ | $C S$ |


| $F$ | 8 |
| :--- | :--- |
| $J_{a}$ | 60 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:

| $t_{g}$ | - |
| :---: | :--- |
| $t_{c}$ | 3.6 |
| $t_{t}$ | - |
| $f_{s}$ | - |
| $R_{m}$ | 2.06 |
| $R_{o}$ | 2.06 |


| $h_{c}$ | 25525 |
| :--- | :--- |
| $N u_{c}$ | 84 |
| $Z_{d}$ | 6.18 |
| $Z_{c}$ | 9.29 |
| $U$ | 547 |
| $\mathrm{Fo}_{c}$ | $3.47 E-05$ |


| $\mathrm{Pa}_{0}$ | 13740 |
| :---: | :---: |





| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1.42 |
| 1 | $V_{s}$ |
| $\Delta T$ | 39576 |
|  | 10 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 1.003 |
| $\Delta t$ | 1.02 |
| 1 | $C S$ |


| $F$ | 13 |
| :--- | :--- |
| $J_{a}$ | 30 |
| $T_{p}$ | 165 |

EXPERIMENTAL" RESULTS:

| $\mathrm{t}_{g}$ | - | $h_{c}$ | 29663 | $\mathrm{Pe}{ }_{0}$ | 26520 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{t}$ | 12 | $\mathrm{Nu}_{\mathrm{c}}$ | 167 |  |  |
| $t_{t}$ | - | $z_{d}$ | 10.95 |  |  |
| ${ }_{\text {f }}$ | - | $z_{c}$ | 18.26 |  |  |
| $\mathrm{R}_{\mathrm{m}}$ | 3.51 | U | 627 |  |  |
| $\mathrm{R}_{0}$ | 3.51 | $\mathrm{Fo}_{\mathrm{c}}$ | 4.04E-05 |  |  |
|  |  |  |  |  |  |
| TEST | 0 : |  | FILM | 5 | $2 / 1$ |


FILM NO : 58-2/1

FRAME
$1-15$
16-19
20-21
22-25
26
27-28

NUMBERS:
EXPERIMENTAL PARAMETERS :

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1.42 |
| 1 | $V_{s}$ |
| $\Delta T$ | 39576 |
|  | 15 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 1.003 |
| $\Delta t$ | .79 |
| $l$ | $C S$ |


| $F$ | 28 |
| :--- | :--- |
| $J_{Q}$ | 45 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| $t_{g}$ | 15.72 |
| :--- | :--- |
| $t_{c}$ | 6.6 |
| $t_{t}$ | 22.32 |
| $f_{s}$ | 67 |
| $R_{m}$ | 3.02 |
| $R_{o}$ | 2.8 |


| $h_{c}$ | 29261 |
| :--- | :--- |
| $N u_{c}$ | 132 |
| $Z_{d}$ | 7.2 |
| $Z_{c}$ | 11.91 |
| $U$ | 600 |
| $F_{o_{c}}$ | $3.47 E-05$ |


| $\mathrm{Pe}_{0}$ | 20360 |
| :---: | :---: |


|  |  |
| :---: | :---: |
| TEST NO: 19.3$) .2$ | FILM NO: 59-1/2 |



EXPERIMENTAL PARAMETERS :

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1.42 |
| $1 V_{s}$ | 39576 |
| $\Delta T$ | 20 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 1.003 |
| $\Delta t$ | .58 |
| 1 | $C S$ |


| $F$ | 7 |
| :--- | :--- |
| $J_{a}$ | 60 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:-

| $t_{g}$ | - |
| :--- | :--- |
| $t_{c}$ | 3.5 |
| $t_{t}$ | - |
| $f_{s}$ | - |
| $R_{m}$ | 2.09 |
| $R_{0}$ | 2.09 |


| $h_{c}$ | 29328 |
| :--- | :--- |
| $N u_{c}$ | 99 |
| $Z_{d}$ | 6.72 |
| $Z_{c}$ | 9.03 |
| $U$ | 587 |
| $F_{o_{c}}$ | $3.29 E-05$ |


| $\mathrm{Pe}_{0}$ | 14960 |
| :--- | :--- |




TEST NO : (9.4).3
FILM NO : 60-1/4

|  |  |  |
| :---: | :---: | :---: |
|  |  | 2 <br> 1 <br> 0 <br> 0 <br> 1 <br> $\cdots$ <br> 0 <br> 2 <br> $\Sigma$ |
|  |  |  |
|  |  | $M$ <br>  <br> $\square$ <br> $\square$ |
|  |  | $\cdots$ 0 $z$ $\leftarrow$ $\sim$ $\sim$ |



| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1.42 |
| $\left(V_{s}\right.$ | 39576 |
| $\Delta T$ | 25 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 1.003 |
| $\Delta t$ | .46 |
| 1 | $C S$ |


| $F$ | 5 |
| :--- | :--- |
| $J_{a}$ | 75 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| ${ }^{\text {tg }}$ | - | $h_{c}$ | 29127 |  | Pe 。 | 13270 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{t}$ | 2.1 | $\mathrm{Nu}_{c}$ | 82 |  |  |  |
| $t_{t}$ | - | $z_{d}$ | 5.62 |  |  |  |
| ${ }_{\text {f }}$ | - | $z_{c}$ | 6.96 |  |  |  |
| $\mathrm{R}_{\mathrm{m}}$ | 1.65 | U | 640 |  |  |  |
| $\mathrm{R}_{0}$ | 1.69 | $\mathrm{FO}_{\mathrm{c}}$ | 3E-05 |  |  |  |
|  |  |  |  |  | [b] | $\begin{array}{rl} 1 & 1 \\ 4 & 5 \\ 1 & (M S) \end{array}$ |
| TEST | 0 : |  | FILM | NO | : 61 | $1 / 6$ |



0
$\infty$
0


9-10
11-17
$1-2$
3-5
$6-8$

| $F$ | 17 |
| :--- | :--- |
| $\mathrm{Ja}_{\mathrm{a}}$ | 15 |
| $\mathrm{~T}_{\mathrm{P}}$ | 165 |

EXPERIMENTAL RESULTS:

| ${ }^{t_{g}}$ | - |
| :--- | :--- |
| $t_{c}$ | 29.3 |
| $t_{t}$ | - |
| $f_{s}$ | - |
| $R_{m}$ | 2.63 |
| $R_{0}$ | 2.63 |


| $h_{c}$ | 16913 |
| :--- | :--- |
| $N u_{c}$ | 68 |
| $Z_{d}$ | 4.82 |
| $Z_{c}$ | 12.96 |
| $U$ | 258 |
| $F o_{c}$ | $1.8 \mathrm{E}-04$ |


| $\mathrm{Pe}_{0}$ | 7980 |
| :--- | :--- |





EXPERIMENTAL PARAMETERS:

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | .45 |
| 1 | $V_{s}$ |
| $\Delta T$ | 6643 |
|  | 18.5 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | .82 |
| 1 | $C S$ |


| $F$ | 34 |
| :--- | :--- |
| $J_{a}$ | 30 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| $t_{g}$ | 18.04 |
| :--- | :--- |
| $t_{c}$ | 9.3 |
| $t_{t}$ | 27.34 |
| $f_{s}$ | 58 |
| $R_{m}$ | 1.94 |
| $R_{o}$ | 1.93 |


| $h_{c}$ | 18618 |
| :--- | :--- |
| $N u_{c}$ | 58 |
| $Z_{d}$ | 3.06 |
| $Z_{c}$ | 5.98 |
| $U$ | 285 |
| $F_{o_{c}}$ | $1.055 \mathrm{E}-04$ |


| $\mathrm{Pe}_{0}$ | 6510 |
| :---: | :---: |




TEST NO : (10.2).1
FILM NO : 65-2/1



NUMBERS:
EXPERIMENTAL PARAMETERS :

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | .82 |
| 1 | $V_{s}$ |
| $\Delta T$ | 12105 |
| $\Delta T$ |  |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | 1.74 |
| $(C S$ | 575 |


| $F$ | 16 |
| :--- | :--- |
| $J_{Q}$ | 15 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| $t_{g}$ | 10.44 | $h_{c}$ | 26177 |  | $P e_{0}$ | 12140 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{\circ} \mathrm{c}$ | 18 | $\mathrm{Nu}_{c}$ | 94 |  |  |  |
| ${ }^{t}$ t | 28.44 | $z_{d}$ | 5.69 |  |  |  |
| ${ }_{\text {f }}$ | 115 | $z_{c}$ | 13.99 |  |  |  |
| $\mathrm{R}_{\mathrm{m}}$ | 2.38 | U | 443 |  |  |  |
| $\mathrm{R}_{0}$ | 2.33 | $\mathrm{Fo}_{\mathrm{C}}$ | $1.409 \mathrm{E}-04$ |  |  |  |
|  |  |  |  |  |  |  |
| TEST | NO : | 1.1 | FILM | NO | : 6 | 2/2 |


|  |  |  |
| :---: | :---: | :---: |
|  |  | $N$ <br> $N$ <br> 1 <br> $\sim$ <br> 0 |
|  |  | $\stackrel{-}{\square}$ |
|  |  |  |



FRAME 1 -16
NUMBERS:
EXPERIMENTAL PARAMETERS :

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | .82 |
| $\left(V_{s}\right.$ | 12105 |
| $\Delta T$ | 18.5 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | .95 |
| 1 | $C S$ |


| $F$ | 26 |
| :--- | :--- |
| $J_{a}$ | 30 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:




20-21
FRAME $\quad 1$-14
15-17
18-19
NUMBERS:
EXPERIMENTAL PARAMETERS :

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | .82 |
| $V_{s}$ | 12105 |
| $\Delta T$ | 27.6 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | .74 |
| 1 | $C S$ |


| $F$ | 21 |
| :--- | :--- |
| $J_{a}$ | 45 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| ${ }^{\text {t }} \mathrm{g}$ | 11.1 | $h_{c}$ | 22474 | Pe |  | 8500 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{t}$ | 4.6 | $\mathrm{Nu}_{\mathrm{c}}$ | 60 |  |  |  |
| $t_{t}$ | 15.7 | $z_{d}$ | 3.25 |  |  |  |
| ${ }_{\text {f }}$ | 97 | $z_{c}$ | 5.9 |  |  |  |
| $\mathrm{R}_{\mathrm{m}}$ | 1.83 | U | 420 |  |  |  |
| $\mathrm{R}_{0}$ | 1.69 | $\mathrm{Fo}_{\mathrm{C}}$ | 6.72E-05 |  |  |  |
|  |  |  |  |  |  |  |
| TEST | NO : | 3). 1 | FILM | NO : 6 | $63-$ | 2/1 |

EXPERIMENTAL PARAMETERS :

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1.39 |
| 1 | $V_{s}$ |
| $\Delta T$ | 20520 |
|  | 9.3 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | 2.13 |
| $l$ | $C S$ |


| $F$ | 28 |
| :--- | :--- |
| $J_{Q}$ | 15 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:




| $d$ | 1 |
| :---: | :---: |
| $m_{s}$ | 1.39 |
| $\left(V_{s}\right.$ | 20520 |
| $\Delta T$ | 18.5 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | 1.17 |
| $C$ | $C S$ |


| $F$ | 22 |
| :--- | :--- |
| $J_{a}$ | 30 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:

| $t_{g}$ | 15.17 |
| :--- | :--- |
| $t_{c}$ | 9.5 |
| $t_{t}$ | 24.67 |
| $f_{s}$ | 71 |
| $R_{m}$ | 2.55 |
| $R_{0}$ | 2.41 |


| $h_{c}$ | 22186 |
| :---: | :--- |
| $N_{c}$ | 85 |
| $Z_{d}$ | 6.08 |
| $Z_{c}$ | 10.36 |
| $U$ | 447 |
| $F_{c}$ | $6.91 E-05$ |


| $P e_{0}$ | 12750 |
| :--- | :--- |



TEST NO : (12.2).2
FILM NO : 67-1/2



FRAME
16
17
NUMBERS :
EXPERIMENTAL PARAMETERS :

| $d$ | 1 |
| :---: | :--- |
| $\dot{m}_{s}$ | 1.39 |
| $1 \mathrm{~V}_{s}$ | 20520 |
| $\Delta \mathrm{~T}$ | 27.6 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | .77 |
| 1 | $C S$ |


| $F$ | 17 |
| :--- | :--- |
| $J_{a}$ | 45 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS :

| ${ }^{\text {t }}$ | 8.49 | $h_{c}$ | 27144 |  | Pe | 11310 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{t}$ | 4.8 | $\mathrm{Nu}_{c}$ | 89 |  |  |  |
| $t_{t}$ | 13.29 | $z_{d}$ | 4.32 |  |  |  |
| ${ }_{\text {f }}$ | 130 | $z_{c}$ | 7.41 |  |  |  |
| $\mathrm{R}_{\mathrm{m}}$ | 2.16 | U | 463 |  |  |  |
| $\mathrm{R}_{0}$ | 2.04 | $\mathrm{FO}_{\mathrm{c}}$ | 4.82E-05 |  |  |  |
|  |  |  |  |  |  |  |
| TEST | $\bigcirc$ : | ) 3 | FILM | NO | : 6 | $2 / 5$ |



$1-11$
12-13
14-15
16-18
FRAME
EXPERIMENTAL PARAMETERS:

| $d$ | 1 |
| :--- | :--- |
| $\dot{m}_{s}$ | 1.39 |
| 1 | $V_{s}$ |
| $\Delta T$ | 20520 |
|  | 36.6 |


| $Z$ | 40 |
| :--- | :--- |
| $P$ | 2.011 |
| $\Delta t$ | .56 |
| $l$ | $C S$ |


| $F$ | 18 |
| :--- | :--- |
| $J_{a}$ | 60 |
| $T_{p}$ | 165 |

EXPERIMENTAL RESULTS:


FILM NO : 68-1/3
especially at low subcoolings. Values of $R_{0}$, $t_{c}$ and $U$ for each bubble are given in Table 4.2. These values, together with the thermal diffusivity of water as given in Appendix 1, were used to determine the collapse Fourier
numbers $\left(F O_{c}=\frac{\alpha t}{4 R_{o}^{2}}\right)^{2}$ and the Peclet numbers
$\left(P e_{0}=\frac{2 U R}{\alpha}\right)$, and these are also shown in Table 4.2.

The experimental values of $\mathrm{FO}_{\mathrm{c}}$ and $\mathrm{Pe}_{\mathrm{o}}$ are plotted as shown in Figs. 4.3 to 4.10 (as functions of Jakob number and steam mass flow rate), and the resulting smoothed values for each experimental condition, as shown in Table 4.3, are used in the comparison between the experimental data and the theoretical analysis.
4.6 The bubble rise velocity

The curves of 'CY and $t$ ' shown in section 4.4 suggested that the velocity of bubble rise after detachment was generally constant, despite the effects of bubble collapse, oscillations, and distortion from the spherical shape. Thus a comparison was made of the measured rise velocities with various correlations in the literature for large bubbles. A popular one is the combination of the Peebles and Garber [46] expression
expression

$$
U=\left(\frac{\sigma}{R \rho_{f}}\right)^{\frac{1}{2}} \text { with the Davies and Taylor [47] }
$$

| Test No. | Bubble symbol | $\begin{gathered} \mathrm{R}_{\mathrm{O}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{t}_{\mathrm{c}} \\ (\mathrm{~ms}) \end{gathered}$ | $\begin{gathered} \mathrm{U} \\ (\mathrm{~mm} / \mathrm{s}) \end{gathered}$ | Foc. $\mathrm{l}^{10^{4}}$ | $\begin{aligned} & 136 \\ & \mathrm{Pe}_{0} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.1 | $\pm$ | 3.54 | 27.0 | 476 | 0.902 | 20130 |
|  | + | 3.57 | 27.5 | 465 | 0.903 | 19830 |
|  | * | 3.46 | 26.0 | 480 | 0.909 | 19840 |
| 1.2 | I | 2.30 | 8.25 | 437 | 0.649 | 12080 |
|  | + | 2.31 | 8.0 | 580 | 0.624 | 16100 |
|  | * | 2.01 | 7.0 | 560 | 0.721 | 13530 |
| 2.1 | I | 4.70 | 34.7 | 522 | 0.657 | 29310 |
|  | + | 4.49 | 35.0 | 524 | 0.726 | 28110 |
|  | * | 4.39 | 32.0 | 514 | 0.695 | 26960 |
| 2.2 | $\pm$ | 3.34 | 13.0 | 605 | 0.485 | 24290 |
|  | + | 3.58 | 13.5 | 502 | 0.438 | 21600 |
|  | * | 3.80 | 15.0 | 520 | 0.432 | 23750 |
| $2 \cdot 3$ | $\pm$ | 2.15 | 4.5 | 775 | 0.402 | 20170 |
|  | + | 2.29 | 4.5 | 533 | 0.354 | 14780 |
|  | * | 2.27 | 4.0 | 533 | 0.321 | 14650 |
| 3.1 | x | 4.92 | 31.0 | 714 | 0.536 | 41970 |
|  | + | 5.25 | 35.6 | 643 | 0.541 | 40330 |
|  | * | 3.87 | 20.0 | 802 | 0.559 | 37080 |
| 3.2 | I | 4.63 | 19.0 | 590 | 0.369 | 32830 |
|  | + | 4.39 | 17.0 | 600 | 0.367 | 31660 |
|  | * | 4.58 | 18.0 | 590 | 0.357 | 32480 |
| $3 \cdot 3$ | $\pm$ | 3.55 | 8.3 | 800 | 0.272 | 34380 |
|  | + | 3.13 | 6.1 | 830 | 0.257 | 31450 |
|  | * | 3.39 | 6.7 | 800 | 0.241 | 32830 |

Table 4.2 Values of $R_{o}, t_{c}, U, F O_{c}$ and $P e_{o}$ for each bubble


| Test No. | Bubble symbol | $\begin{gathered} R_{0} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{t}_{\mathrm{c}} \\ (\mathrm{~ms}) \end{gathered}$ | $\begin{gathered} \mathrm{U} \\ (\mathrm{~mm} / \mathrm{s}) \end{gathered}$ | $\mathrm{FOC.O}^{104}$ | $\mathrm{Pe}{ }_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6.3 | 8 | 2.56 | 7.5 | 493 | 0.478 | 15115 |
|  | + | 2.71 | 8.0 | 460 | 0.455 | 14930 |
|  | * | 2.73 | 7.5 | 447 | 0.420 | 14610 |
| 6.4 | z | 2.27 | 5.0 | 447 | 0.400 | 12300 |
|  | + | 2.27 | 5.2 | 423 | 0.416 | 11640 |
|  | * | 2.40 | 5.9 | 427 | 0.422 | 12420 |
| 7.1 | I | 3.73 | 29.0 | 500 | 0.870 | 22340 |
|  | + | 3.62 | 27.5 | 465 | 0.877 | 20160 |
|  | * | 3.65 | 27.0 | 495 | 0.846 | 21640 |
| 7.2 | z | 3.00 | 11.5 | 450 | 0.530 | 16270 |
|  | + | 2.62 | 10.5 | 515 | 0.635 | 16260 |
|  | * | 2.63 | 10.5 | 515 | 0.630 | 16320 |
| $7 \cdot 3$ | $\pm$ | 1.85 | 4.0 | 533 | 0.477 | 11950 |
|  | + | 1.91 | 4.9 | 533 | 0.554 | 12340 |
|  | * | 2.00 | 3.9 | 733 | 0.402 | 17770 |
| 8.1 | x | 4.03 | 22.5 | 600 | 0.578 | 28960 |
|  | + | 3.90 | 25.0 | 570 | 0.686 | 26620 |
|  | * | 3.44 | 18.8 | 680 | 0.663 | 28010 |
| 8.2 | $\pm$ | 2.99 | 9.6 | 613 | 0.446 | 22080 |
|  | + | 3.09 | 10.2 | 593 | 0.443 | 22080 |
|  | * | 3.13 | 10.9 | 593 | 0.462 | 22360 |
| 8.3 | $\pm$ | 2.34 | 6.2 | 600 | 0.467 | 17020 |
|  | + | 2.71 | 6.4 | 510 | 0.359 | 16750 |
|  | * | 2.73 | 5.5 | 650 | 0.305 | 21510 |

Table 4.2 (continued)

| Test No. | Bubble symbol | $\begin{gathered} \mathrm{R}_{\mathrm{O}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{t}_{\mathrm{c}} \\ (\mathrm{~ms}) \end{gathered}$ | $\begin{gathered} \mathrm{U} \\ (\mathrm{~mm} / \mathrm{s}) \end{gathered}$ | $\mathrm{FOCO}^{1} 0^{4}$ | Pe 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8.4 | $\pm$ | 2.02 | 3.3 | 740 | 0.332 | 18230 |
|  | + | 2.41 | 4.1 | 560 | 0.289 | 16460 |
|  | * | 2.06 | 3.6 | 547 | 0.347 | 13740 |
| 9.2 | I | 3.51 | 12.0 | 627 | 0.404 | 26520 |
|  | + | 3.52 | 11.0 | 627 | 0.368 | 26590 |
|  | * | 3.42 | 11.4 | 700 | 0.404 | 28840 |
| 9.3 | $\pm$ | 2.73 | 6.3 | 650 | 0.349 | 21510 |
|  | + | 2.80 | 6.6 | 600 | 0.347 | 20360 |
|  | * | 3.33 | 8.8 | 540 | 0.327 | 21800 |
| 9.4 | $\pm$ | 2.06 | 3.7 | 587 | 0.357 | 14750 |
|  | + | 2.07 | 3.7 | 587 | 0.354 | 14820 |
|  | * | 2.09 | 3.5 | 587 | 0.329 | 14960 |
| 9.5 | I | 1.60 | 1.87 | 840 | 0.298 | 16490 |
|  | + | 1.74 | 1.90 | 764 | 0.256 | 16310 |
|  | * | 1.69 | 2.10 | 640 | 0.300 | 13270 |
| 10.1 | I | 2.66 | 30.0 | 258 | 1.802 | 8070 |
|  | + | 2.63 | 29.3 | 258 | 1.800 | 7980 |
|  | * | 2.55 | 28.0 | 258 | 1.830 | 7740 |
| 10.2 | $\pm$ | 1.93 | 9.3 | 285 | 1.055 | 6510 |
|  | + | 1.96 | 8.8 | 284 | 0.968 | 6590 |
|  | * | 2.01 | 10.0 | 284 | 1.046 | 6760 |
| 11.1 | $\pm$ | 2.33 | 18.0 | 443 | 1.409 | 12140 |
|  | + | 2.65 | 24.7 | 305 | 1.495 | 9510 |
|  | * | 2.53 | 23.3 | 329 | 1.547 | 9790 |

Table 4.2 (continued)

| Test No. | Bubble <br> symbol | $\begin{gathered} \mathrm{R}_{\mathrm{O}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{t}_{\mathrm{c}} \\ (\mathrm{~ms}) \end{gathered}$ | $\begin{gathered} \mathrm{U} \\ (\mathrm{~mm} / \mathrm{s}) \end{gathered}$ | $\mathrm{Foc}_{\mathrm{c}} \cdot 10^{4}$ | $\mathrm{Pe}_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11.2 | \% | 2.05 | 8.0 | 727 | 0.804 | 17640 |
|  | + | 2.00 | 7.5 | 633 | 0.792 | 14980 |
|  | * | 1.88 | 8.0 | 413 | 0.956 | 9190 |
| 11.3 | I | 1.69 | 4.6 | 420 | 0.672 | 8500 |
|  | + | 1.60 | 4.5 | 560 | 0.734 | 10730 |
|  | * | 1.74 | 4.7 | 520 | 0.648 | 10840 |
| 12.1 | I | 3.54 | 30.0 | 412 | 1.017 | 17160 |
|  | + | 3.15 | 28.0 | 471 | 1.199 | 17450 |
|  | * | 3.29 | 28.0 | 471 | 1.099 | 18230 |
| 12.2 | $\pm$ | 2.41 | 9.5 | 433 | 0.691 | 12350 |
|  | + | 2.41 | 9.5 | 447 | 0.691 | 12750 |
|  | * | 2.52 | 9.5 | 420 | 0.632 | 12530 |
| 12.3 | $\pm$ | 1.92 | 5.4 | 480 | 0.612 | 11040 |
|  | + | 2.01 | 4.5 | 540 | 0.465 | 13000 |
|  | * | 2.04 | 4.8 | 463 | 0.482 | 11310 |
| 12.4 | I | 1.56 | 2.70 | 533 | 0.458 | 10080 |
|  | + | 1.71 | 2.87 | 760 | 0.405 | 15750 |
|  | * | 1.78 | 3.00 | 380 | 0.391 | 8200 |

Table 4.2 (continued)









| Test No. | Foc.104 | Peo |
| :--- | ---: | ---: |
| 1.1 | 0.90 | 19800 |
| 1.2 | 0.66 | 15000 |
| 2.1 | 0.70 | 29000 |
| 2.2 | 0.46 | 22300 |
| 2.3 | 0.35 | 18400 |
| 3.1 | 0.55 | 41000 |
| 3.2 | 0.37 | 34000 |
| 3.3 | 0.27 | 31500 |
| 3.4 | 0.23 | 30200 |
|  |  |  |
|  | 1.72 | 7400 |
| 4.1 | 1.10 | 5300 |
| 4.2 | 1.10 | 13800 |
| 5.1 | 0.79 | 10000 |
| 5.2 | 0.70 | 7400 |
| 5.3 | 0.86 | 22400 |
| 6.1 | 0.55 | 18300 |
| 6.2 | 0.43 | 14800 |
| 6.3 | 0.40 | 12100 |
| 6.4 |  |  |

Table 4.3 The smoothed values of Foc and $\mathrm{Pe}_{\mathrm{o}}$ for each experimental condition

| Test No. | Fo. $10^{4}$ | Peo |
| :--- | :---: | :---: |
| 7.1 | 0.87 | 22000 |
| 7.2 | 0.55 | 16500 |
| 7.3 | 0.46 | 12500 |
| 8.1 | 0.67 | 28000 |
| 8.2 | 0.45 | 22000 |
| 8.3 | 0.36 | 16600 |
| 8.4 | 0.33 | 13300 |
| 9.2 | 0.41 | 28000 |
| 9.3 | 0.34 | 20500 |
| 9.4 | 0.31 | 15000 |
| 9.5 | 0.30 | 13000 |
|  |  |  |
|  | 1.80 | 8000 |
| 10.1 | 1.06 | 6500 |
| 10.2 | 1.40 | 12000 |
| 11.1 | 0.82 | 9200 |
| 11.2 | 0.66 | 1.05 |
| 11.3 | 0.68 | 18200 |
| 12.1 | 0.49 | 13500 |
| 12.2 | 0.43 | 11000 |
| 12.3 |  | 9500 |
| 12.4 |  |  |

Table 4.3 (Continued)
$U=\left(\frac{\rho_{f}-\rho_{g}}{\rho_{f}} . g R\right)^{\frac{j}{2}}$ in the form,
$U_{i}=\left(\frac{\sigma}{R \rho_{f}}+\frac{\rho_{f}-\rho_{g}}{\rho_{f}} . g R\right)^{n / 2}$
A justification for this type of expression was given by Mendelson [48]. The average rise velocities, U and the initial radii, $R_{o}$ for the different Jakob numbers and system pressures are given in table 4.4. Bubble rise velocities, $U_{z}$ determined from equation (4.4) using the initial bubble radius are also presented in table 4.4 together with the ratio of the velocities.

A comparison of the velocities according to equation (4.4) with the observed velocities, as shown in Fig. 4.11, indicated that equation (4.4) underpredicted the experimental results by a factor between 1.67 and 3.18, and that in addition $U$ appeared to vary with the collapse rate through the Jakob number and with the pressure relative to a reference pressure, $P_{0}=1$ bar. Consequently, we employed an empirical correlation of the form

$$
\begin{equation*}
U=2.148\left(\frac{P_{0}}{P}\right)^{1 / 2}\left(1+6.52 .10^{-3} \mathrm{Ja}\right)\left(\frac{\sigma}{R_{0} P_{f}}+\frac{\rho_{f}-\rho_{g}}{\rho_{f}} . g . R_{o}\right)^{1 / 2} \tag{4.5}
\end{equation*}
$$

> and this was found to describe the
experimental data with a percentage deviation of about $2 \%$, as shown in Fig. 4.12.

| Ja | $\underset{(\mathrm{mm} / \mathrm{s})}{\mathrm{U}}$ | $\begin{gathered} \mathrm{R}_{\mathrm{o}} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \mathrm{U}_{1} \\ (\mathrm{~mm} / \mathrm{s}) \end{gathered}$ | $\frac{U}{U_{1}} \cdot\left(\frac{P}{P_{0}}\right)^{\frac{1}{2}}$ |
| :---: | :---: | :---: | :---: | :---: |
| 15 | 563 | 4.04 | 234 | 2.41 |
| 30 | 568 | 3.27 | 226 | 2.51 |
| 45 | 641 | 2.61 | 223 | 2.87 |
| 60 | 678 | 2.41 | 224 | 3.03 |
| 75 | 748 | 1.68 | 235 | 3.18 |


| $P=2 \mathrm{bar}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ja | $U$ <br> $(\mathrm{~mm} / \mathrm{s})$ | $R_{0}$ <br> $(\mathrm{~mm})$ | $U_{1}$ <br> $(\mathrm{~mm} / \mathrm{s})$ | $\frac{U}{U_{1}}$ | $\frac{U}{U_{1}} \cdot\left(\frac{P}{P_{0}}\right)^{\frac{2}{2}}$ |  |
| 15 | 371 | 2.94 | 222 | 1.67 | 2.36 |  |
| 30 | 398 | 2.39 | 221 | 1.80 | 2.55 |  |
| 45 | 434 | 2.13 | 224 | 1.94 | 2.74 |  |
| 60 | 495 | 2.00 | 227 | 2.18 | 3.08 |  |

Table 4.4 Measured and calculated bubble rise velocities and the velocity ratio. ( $P_{0}=1$ bar)


| Ja | 1 bar | 2 bar |
| :---: | :---: | :---: |
| 15 | + | ( + |
| 30 | x | (8) |
| 45 | $\bigcirc$ | © |
| 60 | * | $\circledast$ |
| 75 | $\Delta$ |  |

## CHAPTER 5

THEORETICAL ANALYSIS

It has already been pointed out that very little research has been carried out into the condensation of vapour bubbles in subcooled liquids and that the existing quasi-steady state theories did not, in general, take account of radial velocities, despite the fact that, especially at high subcoolings, the radial velocities were comparable with the rise velocity of the bubble.

Ruckenstein and Davis [31] presented a general solution for a growing bubble with allowance made for radial velocity effects, but this solution is not considered to be suitable for ready engineering use.

A simplified approximate solution is presented below, allowing for radial velocities in addition to the velocities due to the vertical motion of the bubbles.
5.2 The Model

The collapsing bubble (illustrated in Fig. 5.1) is assumed to be spherical and rising freely in a vertical path with a constant velocity, U. Alternatively, it can be visualised as being at rest with the continuous medium moving against it with an approach velocity, $U$.
5.2.1 The energy equation

The convection of heat in the water flowing around a spherical bubble is assumed to be given, in spherical coordinates, by the energy equation as


Fig.5.1 Theoretical bubble collapse model
$\frac{\partial T}{\partial t}+u_{r} \frac{\partial T}{\partial r}+\frac{u}{r} \theta \frac{\partial T}{\partial \theta}=\alpha\left(\frac{\partial^{2} T}{\partial r^{2}}+\frac{2}{r} \frac{\partial T}{\partial r}\right)$
where conduction in the tangential direction is assumed to be much smaller than in the radial direction, and hence is neglected. Viscous dissipation is also neglected because flow around the bubble is assumed to be inviscid.

Assuming simplified potential flow around the bubble and that the thickness of the thermal boundary layer is small, the velocities in the radial direction, $u_{r}$ and in the tangential direction $u_{\theta}$ are given as
$u_{r}=-U\left[1-\left(\frac{R}{r}\right)^{3}\right] \cos \theta+\dot{R} \frac{R^{2}}{r^{2}}$
$u_{\theta}=U\left[1+\frac{1}{2}\left(\frac{R}{r}\right)^{3}\right] \sin \theta$
Changing the coordinate system to a moving coordinate, $y=r-R$ and neglecting second or higher order terms since

$$
\frac{y}{R} \ll 1
$$

$$
u_{r}=-U\left[1-\left(1+\frac{y}{R}\right)^{-3}\right] \cos \theta+\dot{R}\left(1+\frac{y}{R}\right)^{-2}
$$

$$
\cong-U\left[1-\left(1-3 \frac{y}{R}\right)\right] \cos \theta+\dot{R}\left(1-2 \frac{y}{R}\right)
$$

$$
\begin{equation*}
\therefore u_{r} \cong-3 U \frac{y}{R} \cos \theta+\dot{R}\left(1-2 \frac{y}{R}\right) \tag{5.4}
\end{equation*}
$$

$$
u_{\theta}=U\left[1+\frac{1}{2}\left(1-3 \frac{y}{R}\right)\right] \sin \theta
$$

neglecting $y / R$ term,

$$
\begin{equation*}
u_{\theta} \cong \frac{3}{2} u \sin \theta \tag{5.5}
\end{equation*}
$$

Returning to the energy equation (5.1),
$\frac{\partial^{2} T}{\partial r^{2}} \sim \frac{T}{y^{2}}$ and $\frac{2}{r} \frac{\partial T}{\partial r} \sim \frac{2}{r} \frac{T}{y}$
Hence $\frac{\partial^{2} T}{\partial r^{2}} \gg \frac{2}{r} \frac{\partial T}{\partial r}$, and the term $\frac{2}{r} \frac{\partial T}{\partial r}$
can be neglected.
In the moving coordinate system,
$\left(\frac{\partial T}{\partial t}\right)_{r, \theta}=\left(\frac{\partial T}{\partial t}\right)_{y, \theta}-\dot{R}\left(\frac{\partial T}{\partial y}\right)$ and $\frac{\partial T}{\partial r}=\frac{\partial T}{\partial y}$
Thus equation (5.1) becomes,
$\frac{\partial T}{\partial t}-\frac{y}{R}[3 U \cos \theta+2 \dot{R}] \frac{\partial T}{\partial y}+\frac{3}{2} \frac{U}{R} \sin \theta \frac{\partial T}{\partial \theta}=\alpha \frac{\partial^{2} T}{\partial y^{2}}$
which is equation (9) of Ruckenstein and Davis [31]. If it is now further assumed that *
$\frac{\partial T}{\partial t} \ll \frac{y}{R} \dot{R} \frac{\partial T}{\partial y}$ and if equation (5.6) is integrated across the thermal boundary layer thickness $\delta(\theta)$,
$\alpha \int_{0}^{\delta} \frac{\partial^{2} \phi}{\partial y^{2}} d y=-\left(3 \frac{U}{R} \cos \theta+2 \frac{\dot{R}}{R}\right) \int_{0}^{\delta} y \frac{\partial \phi}{\partial y} d y+\frac{3}{2} \frac{U}{R} \sin \theta \frac{\partial}{\partial \theta} \int_{0}^{\delta} \phi d y(5.7)$
where $\phi$ is the dimensionless temperature ratio,
$\frac{T-T_{\infty}}{T_{1}-T_{\infty}}$
Assuming the boundary condition,
$\frac{\partial \phi}{\partial y}=0$ at $y=\delta$
the left hand side of equation (5.7) becomes
$\alpha \int_{0}^{\delta} \frac{\partial^{2} \phi}{\partial y^{2}} d y=\left.\alpha \frac{\partial \phi}{\partial y}\right|_{0} ^{\delta}=-\alpha\left(\frac{\partial \phi}{\partial y}\right)_{y=0}$
and equation (5.7) is expressed as
$-\alpha\left(\frac{\partial \phi}{\partial y}\right)_{y=0}=-\left(3 \frac{U}{R} \cos \theta+2 \frac{\dot{R}}{R}\right) \int_{0}^{\delta} y \frac{\partial \phi}{\partial y} d y+\frac{3}{2} \frac{U}{R} \sin \theta \frac{\partial}{\partial \theta} \int_{0}^{\delta} \phi d y$

As in boundary layer theory, if $\phi$ is taken to be a simple power series in $y$, equation (5.9) can be simplified further. Here a cubic of the form,

$$
\begin{equation*}
\phi=\frac{3}{2}\left(1-\frac{y}{\delta}\right)^{2}-\frac{1}{2}\left(1-\frac{y}{\delta}\right)^{3} \tag{5.10}
\end{equation*}
$$

is assumed, with boundary conditions
$\phi=1, \frac{\partial^{2} \phi}{\partial y^{2}}=0$ at $y=0$
and $\phi=\frac{\partial \phi}{\partial y}=0 \quad$ at $y=\delta$
As the temperature at the vapour-ilquid interface,
$T_{1}$ is constant
all points on the bubble surface,

$$
\begin{equation*}
\frac{\partial \phi}{\partial \theta}=0 \tag{5.11}
\end{equation*}
$$

Calculating the terms in equation (5.9) by
substituting the value of $\phi$, given by equation (5.10),

$$
\begin{equation*}
\left(\frac{\partial \phi}{\partial y}\right)_{y=0}=\left(-\frac{3}{\delta}\left(1-\frac{y}{\delta}\right)+\frac{3}{2 \delta}\left(1-\frac{y}{\delta}\right)^{2}\right)_{y=0}=-\frac{3}{2 \delta} \tag{5.12}
\end{equation*}
$$

$\int_{0}^{\delta} y \frac{\partial \phi}{\partial y} d y=\frac{1}{\delta} \int_{0}^{\delta} y\left[-3\left(1-\frac{y}{\delta}\right)+\frac{3}{2}\left(1-\frac{y}{\delta}\right)^{2}\right] d y$
$=\frac{1}{\delta} \int_{0}^{\delta} y\left[-3+\frac{3 y}{\delta}+\frac{3}{2}\left(1-\frac{2 y}{\delta}+\frac{y^{2}}{\delta^{2}}\right)\right] d y$
$=\frac{1}{\delta} \int_{0}^{\delta} y\left[-3+\frac{3}{2}+\frac{3}{2} \frac{y^{2}}{\delta^{2}}\right] d y$
$=\frac{3}{2 \delta} \int_{0}^{\delta} y\left(\frac{y^{2}}{\delta^{2}}-1\right) d y=\frac{3}{2 \delta}\left|\frac{y^{4}}{4 \delta^{2}}-\frac{y^{2}}{2}\right|_{0}^{\delta}$
$=\frac{3}{2 \delta}\left(\frac{\delta^{2}}{4}-\frac{\delta^{2}}{2}\right)$
$\therefore \int_{0}^{\delta} y \frac{\partial \phi}{\partial y} d y=-\frac{3}{8} \delta$
$\int_{0}^{\delta} \phi d y=\frac{3}{2} \int_{0}^{\delta}\left(1-\frac{y}{\delta}\right)^{2} d y-\frac{1}{2} \int_{0}^{\delta}\left(1-\frac{y}{\delta}\right)^{3} d y$
$=-\left|\frac{\delta}{2}\left(1-\frac{y}{\delta}\right)^{3}\right|_{0}^{\delta}+\left|\frac{\delta}{8}\left(1-\frac{y}{\delta}\right)^{4}\right|_{0}^{\delta}=\delta\left(\frac{1}{2}-\frac{1}{8}\right)=\frac{3}{8} \delta$
$\therefore \frac{\partial}{\partial \theta} \int_{0}^{\delta} \phi d y=\frac{3}{8} \frac{\partial \delta}{\partial \theta}$
Substitution of equations (5.12), (5.13) and (5.14) in equation (5.9) gives.
$\frac{3}{2} \frac{\ddot{\alpha}}{\delta}=\left(2 \frac{\dot{R}}{R}+3 \frac{U}{R} \cos \theta\right) \frac{3}{8} \delta+\frac{9}{16} \frac{U}{R} \sin \theta \frac{\partial \delta}{\partial \theta}$
$\therefore \frac{3}{8} \frac{U}{R} \sin \theta \frac{\partial \delta}{\partial \theta}=\frac{\alpha}{\delta}-\frac{1}{4}\left(2 \frac{\dot{R}}{R}+\frac{3 U}{R} \cos \theta\right) \delta$
$\therefore \frac{\partial \delta}{\partial \theta}=\frac{\alpha}{\delta} \frac{8 R}{3 U \sin \theta}-\frac{2 R}{3 U \sin \theta}\left(2 \frac{\dot{R}}{R}+\frac{3 U}{R} \cos \theta\right) \delta$
$\therefore \frac{1}{2} \frac{\partial \delta^{2}}{\partial \theta}=\frac{8}{3} \frac{\alpha R}{U \sin \theta}-\frac{2}{3 \sin \theta}\left(\frac{2 \dot{R}}{U}+3 \cos \theta\right) \delta^{2}$
Dividing by $R^{2}$ and letting $\varepsilon=\left(\frac{\delta}{R}\right)^{2}$ and hence $\partial \delta^{2}=R^{2} \partial \varepsilon$,
$\frac{\partial \varepsilon}{\partial \theta}=\frac{16}{3} \frac{\alpha}{U R \sin \theta}-\frac{4}{3 \sin \theta}\left(\frac{2 \dot{R}}{U}+3 \cos \theta\right) \varepsilon$
$\therefore \frac{\partial \varepsilon}{\partial \theta}=-\frac{4}{3}\left(\frac{2 \dot{R}}{U}+3 \cos \theta\right) \frac{\varepsilon}{\sin \theta}+\frac{16}{3}\left(\frac{\alpha}{U R}\right) \frac{1}{\sin \theta}$
This is a first order, first degree, linear differential equation in $\varepsilon$, which can be solved as follows:
$\frac{\partial \varepsilon}{\partial \theta}+P \varepsilon=Q$
where $P=\frac{4}{3 \sin \theta}\left(\frac{2 \dot{R}}{U}+3 \cos \theta\right)$ and

$$
Q=\frac{16}{3}\left(\frac{\alpha}{U R}\right) \frac{1}{\sin \theta}
$$

The solution for the differential equation (5.16)
is given as
$\varepsilon=e^{-\int P d \theta}\left[\int e^{\int P d \theta} \cdot Q d \theta+c\right]$
Assuming $\varepsilon=0$ when $\theta=0$, gives $c=0$.
Substituting the values of $P$ and $Q$,
$\int P d \theta=\frac{4}{3} \int\left(\frac{2 \dot{R}}{U \sin \theta}+\frac{3 \cos \theta}{\sin \theta}\right) d \theta$
$=\frac{4}{3}\left[\frac{2 \dot{R}}{U} \ln \left(\tan \frac{\theta}{2}\right)+3 \ln (\sin \theta)\right]$
$=\ln \left[\sin ^{4} \theta \cdot\left(\tan \frac{\theta}{2}\right)^{\frac{8}{3} \frac{\dot{R}}{U}}\right]$
$\therefore e^{\int P d \theta}=\sin ^{4} \theta \cdot\left(\tan \frac{\theta}{2}\right)^{\frac{8}{3} \frac{\dot{R}}{U}}$

Hence, equation (5.17) becomes,

$\therefore \frac{1}{R}=\frac{1}{\sin ^{2} \theta \cdot\left(\tan \frac{\theta}{2}\right)^{\frac{4}{3} \frac{R}{U}} \cdot \frac{4}{\sqrt{3}}\left(\frac{\alpha}{U R}\right)^{1 / 2}\left[\int_{0}^{\theta} \sin ^{3} \theta \cdot\left(\tan \frac{\theta}{2}\right)^{\frac{8}{3} \frac{\dot{R}}{U}} d \theta\right]^{\frac{1 / 2}{2}}} \quad \therefore$ (5.19)
$\therefore \frac{\delta}{R}=\frac{4}{\sqrt{3}}\left(\frac{\alpha}{U R}\right)^{1 / 2} \frac{1}{\sqrt{\sin \theta \cdot F\left(\theta, \frac{R}{U}\right)}}\left[\int_{0}^{\theta} F\left(\theta, \frac{\dot{R}}{U}\right) d \theta\right]^{1 / 2}$
where $F\left(\theta, \frac{\dot{R}}{U}\right)=\left[\sin ^{3} \theta \cdot\left(\tan \frac{\theta}{2}\right)^{\frac{8}{3} \frac{\dot{R}}{U}}\right]$

The integral can be evaluated further in terms of incomplete beta functions, as follows,

Let $\zeta=\sin ^{2} \frac{\theta}{2}, \quad \alpha=-\frac{4}{3} \frac{\dot{R}}{U}$

$$
d \zeta=\sin \frac{\theta}{2} \cos \frac{\theta}{2} d \theta
$$

$\therefore \frac{\sin ^{2} \theta d \theta}{\left(\tan ^{2} \frac{\theta}{2}\right)^{\alpha}}=\frac{8 \sin ^{3} \frac{\theta}{2} \cdot \cos ^{3} \frac{\theta}{2} \cdot\left(\cos ^{2} \frac{\theta}{2}\right)^{\alpha}}{\left(\sin ^{2} \frac{\theta}{2}\right)^{\alpha}} d \theta$
$=\frac{8 \sin ^{2} \frac{\theta}{2} \cos ^{2} \frac{\theta}{2}\left(\cos ^{2} \frac{\theta}{2}\right)^{\alpha}}{\left(\sin ^{2} \frac{\theta}{2}\right)^{\alpha}} \mathrm{d} \zeta=8 \zeta^{1-\alpha}(1-\zeta)^{1+\alpha} \mathrm{d} \zeta$
Hence,
$\int_{0}^{\theta} \frac{\sin ^{3} \theta}{\left(\tan ^{2} \frac{\theta}{2}\right)^{\alpha}} d \theta=8 \int_{0}^{\zeta} \zeta^{1-\alpha}(1-\zeta)^{1+\alpha} d \zeta$
$=8 \int_{\zeta} \zeta^{(2-\alpha)-1}(1-\zeta)^{(2+\alpha)-1} d \zeta=8 B_{\zeta}((2-\alpha),(2+\alpha))$
where $0 \leq \zeta \leq 1$.

Incomplete beta functions are tabulated in mathematical tables, e.g. Pearson [49].

Substitution of equation (5.20) in (5.10) gives the temperature distribution $\phi$.
5.2.2 Total heat transfer from the bubble

The heat flux at an angular position $\theta$ on the surface of the bubble is given as
$q_{\theta}=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}=-k\left(\frac{\partial \phi}{\partial y}\right)_{y=0} \Delta T=\frac{3}{2} \frac{k \Delta T}{R}\left(\frac{R}{\delta}\right)$
where $(\delta / R)$ is given by equation (5.20) and $\Delta T=T_{1}-T_{\infty}$ is the liquid subcooling.

For the simple case of $\frac{\dot{R}}{U}=0$,
equation (5.22) can be calculated as follows:

$$
F(\theta, 0)=\sin ^{3} \theta
$$

$\left[\int_{0}^{\theta} F(\theta, 0) d \theta\right]^{1 / 2}=\left[\int_{0}^{\theta} \sin ^{3} \theta d \theta\right]^{1 / 2}$

$$
\begin{equation*}
=\left[\frac{\cos ^{3} \theta}{3}-\cos \theta+\frac{2}{3}\right]^{1 / 2} \tag{5.23}
\end{equation*}
$$

Substituting the value of $\frac{\delta}{R}$ (with $\left.\frac{\dot{R}}{U}=0\right)$
from equation (5.20) in equation (5.22) gives,
$q_{\theta}=\frac{3}{2} \cdot \frac{k \cdot \Delta T}{R} \cdot \frac{\sin ^{2} \theta}{\sqrt{\frac{16}{3}\left(\frac{\alpha}{U R}\right)\left(\frac{2}{3}-\cos \theta+\frac{1}{3} \cos ^{3} \theta\right)}}$
$\therefore q_{\theta}=\frac{\rho_{\ell} \cdot c_{p} \cdot \Delta T \cdot\left(\frac{3}{2}\right) \alpha U \cdot R \sin ^{2} \theta}{\sqrt{\frac{16}{3} \alpha U R^{3}\left(\frac{2}{3}-\cos \theta+\frac{1}{3} \cos ^{3} \theta\right)}}$
which is of the same form as the exact solution of Rückenstein [8] but gives a value $6 \%$ lower than his solution.

An energy balance on the steam bubble states that the total heat transferred from the bubble at time t is given as
$Q=\int_{0}^{\pi} 2 \pi R^{2} \sin \theta q_{\theta} d \theta=-h_{f} \rho_{V} \frac{d V}{d t}$
where $\frac{d V}{d t}$ is the rate of change of volume of the bubble. 5.2.3 Bubble collapse rate

$$
\text { Substitution of the value of }\left(\frac{R}{\delta}\right)\left(\text { wher } e \frac{\dot{R}}{U} \neq 0\right)
$$

from equation (5.20) in (5.22) gives,
$q_{\theta}=\frac{3 \sqrt{3}}{8} \cdot \frac{K \cdot \Delta T}{R}\left(\frac{U R}{\alpha}\right)^{1 / 2} \frac{\left[\sin \theta \cdot F\left(\theta, \frac{\dot{R}}{U}\right)\right]^{1 / 2}}{\left[\int_{0}^{\theta} F\left(\theta, \frac{\dot{R}}{U}\right) d \theta\right]^{1 / 2}}$
This value is then introduced into equation (5.25) to give,
$-h_{f g} \cdot \rho_{V} \cdot \frac{d V}{d t}=\frac{3 \sqrt{3}}{8} \cdot k \cdot \Delta T \cdot\left(\frac{U R}{a}\right)^{1 / 2} \cdot 2 \pi R \int_{0}^{\pi} \frac{\left[\sin ^{3} \theta \cdot F\left(\theta, \frac{\dot{R}}{U}\right)\right]^{1 / 2}}{\left[\int_{0}^{\theta} F\left(\theta, \frac{\dot{R}}{U}\right) d \theta\right]^{1 / 2}} d \theta$
$\therefore-\frac{d V}{d t}=\frac{3 \sqrt{6}}{8} \cdot\left(\frac{c_{p} \cdot \Delta T \cdot \rho_{1}}{h_{f g} \cdot P_{V}}\right) \cdot\left(\frac{2 U R}{\alpha}\right)^{1 / 2} \pi R \alpha \int_{0}^{\pi} \frac{\left[\sin ^{3} \theta \cdot F\left(\theta, \frac{\dot{R}}{U}\right)\right]^{1 / 2}}{\left[\int_{0}^{\theta} F\left(\theta, \frac{\dot{R}}{U}\right) d \theta\right]^{1 / 2}} d \theta(5.27)$
The rate of change of bubble volume is expressed as
$-\frac{d V}{d t}=-\frac{d}{d t}\left(\frac{4}{3} \pi R^{3}\right)=-4 \pi R^{2} \dot{R}=-4 \pi R_{0}{ }^{3} \dot{\beta} B^{2}$
where the dimensionless radius, $\beta=\frac{R}{R_{0}}$.
Introducing the Fourier number, Fo $=\frac{\alpha t}{4 R_{0}{ }^{2}}$ and
substituting $\frac{d \beta}{d t}=\frac{d \beta}{d F o} \cdot \frac{\alpha}{4 R_{0}{ }^{2}}$ in equation (5.28) gives,
$-\frac{d V}{d t}=-\pi R_{O} \beta^{2} \alpha \frac{d \beta}{d F o}$
Thus equation (5.27) can be rearranged to give,
$-\pi R_{O} B^{2} \alpha \cdot \frac{d B}{d F O}=\frac{3 \sqrt{6}}{8} \cdot J a\left(\frac{2 U R}{\alpha}\right)^{1 / 2} \pi R \alpha \cdot \int_{0}^{\pi} \frac{\left[\sin ^{3} \theta \cdot F\left(\theta, \frac{\dot{R}}{U}\right)\right]^{1 / 2}}{\left[\int_{0}^{\theta} F\left(\theta \cdot \frac{\dot{R}}{U}\right) d \theta\right]^{1 / 2}} d \theta$
$\therefore \beta \frac{d B}{d F O}=-\frac{9}{4 \sqrt{6}} \cdot J a\left(\frac{2 U R_{0}}{\alpha}\right)^{1 / 2} \beta^{1 / 2} \int_{0}^{\pi} \frac{\left[\sin ^{3} \theta \cdot F\left(\theta, \frac{\dot{R}}{U}\right)\right]^{1 / 2}}{\left[\int_{0}^{\theta} F\left(\theta, \frac{\dot{R}}{U}\right) d \theta\right]^{1 / 2}} d \theta$
Thus the equation for the rate of collapse of the bubble is obtalned as
$\beta_{B}^{1 / 2} \frac{d B}{d F O}=-\frac{9}{4 \sqrt{6}} J a P e_{0}^{1 / 2} \int_{0}^{\pi} \frac{\left[\sin ^{3} \theta \cdot F\left(\theta \cdot \frac{\dot{R}}{U}\right)\right]^{1 / 2}}{\left[\int_{0}^{\theta} F\left(\theta, \frac{\dot{R}}{U}\right) d \theta\right]^{1 / 2}} d \theta$

Also,
$\frac{\dot{R}}{U}=\frac{R_{0}}{U} \cdot \frac{d B}{d t}=\frac{R_{0}}{U} \frac{\alpha}{4 R_{0}{ }^{2}} \frac{d \beta}{d F O}=\frac{1}{2}\left(\frac{\alpha}{2 U R_{0}}\right) \cdot \frac{d \beta}{d F_{0}}$
$\therefore \frac{\dot{R}}{U}=\frac{1}{2 \mathrm{Pe}_{\mathrm{O}}} \frac{\mathrm{d} \beta}{\mathrm{dFO}}$

Equations (5.30) and (5.31) can be combined to give $\beta$ in terms of $\frac{\dot{R}}{U}$ with $J a$ and $P e_{o}$ as parameters, viz.,

$$
\begin{equation*}
\beta=\left[-\frac{9}{8 \sqrt{6}}\left(\frac{\mathrm{Ja}}{\mathrm{Pe}_{0}^{1 / 2}}\right)\left(\frac{1}{K / U}\right) \int_{0}^{\pi} \frac{\left[\sin ^{3} \theta \cdot F\left(\theta, \frac{\dot{R}}{U}\right)\right]^{1 / 2}}{\left[\int_{0} F\left(\theta, \frac{\dot{R}}{U}\right) d \theta\right]^{1 / 2}} d \theta\right]^{2} \tag{5.32}
\end{equation*}
$$

A typical plot of $\left(-\frac{U}{\dot{R}}\right)$ versus $\beta$ is shown in figure (5.2) for $\mathrm{Ja}=15$ and $\mathrm{Pe}_{\mathrm{O}}=10^{4}$.
Equation (5.31) can be rewritten as
$F O_{i}=\frac{1}{2 P e_{0}} \int_{\beta_{i}}^{1}\left(-\frac{U}{\dot{R}}\right) d \beta$
Thus, when the curve in Figure 5.2 is integrated the corresponding values of $\mathrm{FO}_{1}$ and $\beta_{1}$ can be found, and plotted as shown in figure 5.3. Details of the calculation of these curves by a simulation program, TUTSIM are given in Appendix 3.


Fig.5.2 A typical plot of $\left(-\frac{U}{\dot{R}}\right)$ versus $\beta$ according to equation(5.32).


Fig.5.3 $\beta$ versus $F$ a according to equation(5.33)
5.2.4 Negligible bubble wall velocity $(\dot{R}=0)$

When $\frac{\dot{R}}{U}=0, F\left(\theta, \frac{\dot{R}}{U}\right)_{\dot{R}=0}=\sin ^{3} \theta$, and equation (5.30) becomes,
$\beta^{1 / 2 d \beta} \frac{d F 0}{d F}-\frac{9}{4 \sqrt{6}} \cdot \mathrm{JaPe}_{0}^{1 / 2} \int_{0}^{\pi} \frac{\left[\sin ^{6} \theta\right]^{1 / 2}}{\left[\int_{0}^{\theta} \sin ^{3} \theta d \theta\right]^{1 / 2}} d \theta$
Substituting $Z=\int_{0}^{\theta} \sin ^{3} \theta d \theta=\left(\frac{\cos ^{3} \theta}{3}-\cos \theta+\frac{2}{3}\right)$,
$d Z=\sin ^{3} \theta d \theta$, the integral can be calculated as
$\int_{0}^{\pi} \frac{\sin ^{3} \theta d \theta}{\left[\int_{0}^{\theta} \sin ^{3} \theta d \theta\right]^{1 / 2}}=\int_{0}^{4 / 3} \frac{d z}{z^{1 / 2}}=\left.2 z^{1 / 2}\right|_{0} ^{4 / 3}=\frac{4}{\sqrt{3}}$
Therefore equation (5.34) becomes
$\beta^{1 / 2} \frac{d \beta}{d F 0}=-\frac{3}{\sqrt{2}}$ Ja $P e_{0}^{1 / 2}$
Hence
$\frac{2}{3} \frac{d \beta^{3 / 2}}{d F O}=-\frac{3}{\sqrt{2}} \mathrm{Ja} \mathrm{Pe}{ }_{0}^{1 / 2}$
Integrating,

$$
\begin{equation*}
\left.{ }_{\beta}^{3 / 2}\right|_{1} ^{\beta}=-\left.\frac{9}{2 \sqrt{2}} \mathrm{Ja} \mathrm{Pe} \mathrm{O}_{\mathrm{o}}^{1 / 2} \mathrm{Fo}\right|_{0} ^{\mathrm{FO}} \tag{5.38}
\end{equation*}
$$

or
$\beta^{3 / 2}-1=-\frac{9}{2 \sqrt{2}} \mathrm{Ja} \mathrm{Pe}{ }_{o}^{1 / 2} \mathrm{FO}$
$\therefore \beta=\left(1-3.182 \mathrm{JaPe} \mathrm{O}_{0}^{1 / 2} \mathrm{FO}\right)^{2 / 3}$

It should be noted that Isenberg et al.[23]
obtained a very similar equation by employing Ruckenstein's [8] exact heat transfer solution for bubbles rising with negligible radial velocity as

$$
\begin{equation*}
\beta=\left(1-3 \cdot 385 \mathrm{Ja} \cdot \mathrm{Pe}_{0}^{0.5} \cdot \mathrm{FO}\right)^{2 / 3} \tag{5.40}
\end{equation*}
$$

5.3 Simplification of the theory

Equation (5.30) can be written as
$\beta^{1 / 2} \frac{d B}{d F O}=-\frac{9}{4 \sqrt{6}}$ Ja Pe $0_{0}^{1 / 2} F_{1}\left(\frac{\dot{R}}{U}\right)$
where
$F_{1}\left(\frac{\dot{R}}{U}\right)=\int_{0}^{\pi} \frac{\left[\sin ^{3} \theta F\left(\theta, \frac{\dot{R}}{U}\right)\right]^{1 / 2}}{\left[\int_{0}^{\theta} F\left(\theta, \frac{\dot{R}}{U}\right) d \theta\right]^{1 / 2}} d \theta$

An evaluation of $F_{1}\left(\frac{\dot{R}}{U}\right)$ versus $\left(-\frac{\dot{R}}{U}\right)$ presented in Fig. 5.4, gives the relationship as

$$
\begin{equation*}
F_{1}=2.31-1.33\left(-\frac{\dot{R}}{U}\right) \tag{5.43}
\end{equation*}
$$

for $0 \leq-\frac{\dot{R}}{U} \leq 1.4$
Hence, equation (5.41) becomes
$\frac{2}{3} \frac{d \beta^{3 / 2}}{d F O}=-\frac{9}{4 \sqrt{6}} \mathrm{JaPe}_{0}^{1 / 2}\left(2.31+1.33\left(\frac{\dot{R}}{\mathrm{U}}\right)\right)$


Substituting $\frac{\dot{R}}{U}=\frac{1}{2 \mathrm{Pe}_{0}} \frac{\mathrm{~d} \beta}{\mathrm{dFO}}$ from equation (5.31)
$\frac{2}{3} \frac{d \beta^{3 / 2}}{d F O}=-\frac{9}{4 \sqrt{6}} \mathrm{Ja} \mathrm{Pe}{ }_{0}^{1 / 2}\left(2.31+\frac{1.33}{2 P e_{0}} \frac{d \beta}{d F O}\right)$
and integrating,
$\left.\beta^{3 / 2}\right|_{1} ^{\beta}=-\frac{27}{8 \sqrt{6}} \mathrm{Ja} \mathrm{Pe} 0^{1 / 2}\left[\left.2.31 \mathrm{Fo}\right|_{0} ^{F O}+\left.\frac{1.33}{2 \mathrm{Pe}_{0}} \cdot \beta\right|_{1} ^{\beta}\right] \quad(5.46)$
$\therefore\left(1-\beta^{3 / 2}\right)=3.182 \mathrm{Ja} \mathrm{Pe} o^{1 / 2}\left[F o-\frac{0.288}{P e_{0}}(1-\beta)\right]$

Hence, the simplified form of equation (5.32) can be expressed as
$B=\left[1-3.182 \mathrm{Ja} \mathrm{Pe} 0^{1 / 2}\left[F 0-\frac{0.288}{P e_{0}}(1-\beta)\right]\right]^{2 / 3}$
A comparison of the curve according to the complete equations (5.32) and (5.33) calculated by a computer simulation program, TUTSIM with the curve according to the simplified equation (5.48) is given in Fig. 5.5 for Ja $=$ 15 and $\mathrm{Pe}_{\mathrm{o}}=10^{4}$. The collapse Fourier number according to the simplified equation is about $2.5 \%$ higher than that from equations (5.32) and (5.33).


The collapse Fourier number for the simplified theory in equation (5.48) is given by

$$
\begin{equation*}
E O_{c}=\frac{1}{3.182 \mathrm{Ja} \mathrm{Pe}{ }_{0}^{1 / 2}}+\frac{0.288}{\mathrm{Pe}_{\mathrm{o}}} \tag{5.49}
\end{equation*}
$$

When $\dot{R}=0$ the term $\frac{0.288}{\mathrm{Pe}_{\mathrm{o}}}$ vanishes and equation (5.48) reduces to equation (5.39).
5.4 Effect of bubble distortion and heating of liquid near the injection orifice

As can be seen from the impressions of bubble shapes (section 4.2.1), the bubbles were generally far prom spherical and indeed changed in shape, sometimes in an oscillating manner, after detachment.

In section 3.4 , the equivalent bubble radius, $R$ was calculated as the radius of a sphere of volume equal to the measured bubble volume $V_{B}$. This radius was used in the heat transfer calculations in equation (5.25).

To account for the non-spherical shape of the bubble a shape factor, $\psi$ defined as

$$
\begin{equation*}
\psi=\frac{\text { measured bubble surface area }}{4 \cdot \pi \cdot R^{2}} \tag{5.50}
\end{equation*}
$$

was included in equation (5.25) to give

$$
Q=\left[\int_{0}^{\pi} 2 \pi R^{2} \sin \theta q_{\theta}\right] \cdot \psi \cdot d \theta
$$

Assuming $\psi=\psi(B)$, the simplified equation for the bubble collapse rate is now obtained from equation (5.41) as
$\beta^{\frac{3}{2} \frac{d B}{d F O}}=-\frac{9}{4 \sqrt{6}} \mathrm{Ja} \mathrm{Pe}_{0}^{1 / 2} \psi(\beta) . \mathrm{F}_{1}\left(\frac{\ddot{R}}{U}\right)$
Substituting the approximate expression, given in equation (5.43), for
$F_{1}\left(\frac{\dot{R}}{U}\right)$ together with the value of $\left(\frac{\dot{R}}{U}\right)$ in equation (5.31), equation (5.52) can be written as

$$
\begin{equation*}
\beta^{\frac{1}{2}} \frac{d \beta}{d F O}=-\frac{9}{4 \sqrt{6}} \mathrm{Ja} \mathrm{Pe}{ }_{0}^{1 / 2} \psi(\beta)\left[2.31+\frac{1.33}{2 \mathrm{Pe}_{0}} \frac{\mathrm{~d} \beta}{\mathrm{dFo}}\right] \tag{5.53}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
\int_{1}^{\beta} \frac{\beta^{1 / 2} \mathrm{~d} \beta}{\psi(\beta)}=-2.122 \mathrm{Ja} \mathrm{Pe} 0^{1 / 2}\left[\left.F O\right|_{0} ^{F O}+\left.\frac{0.288}{P e_{0}} \beta\right|_{1} ^{\beta}\right] \tag{5.54}
\end{equation*}
$$

$\therefore \int_{1}^{\beta} \frac{\beta^{2 / 2}}{\psi(\beta)} d \beta=-2.122 \mathrm{Ja} \mathrm{Pe} O^{3 / 2}\left[F O-\frac{0.288}{P e_{0}}(1-\beta)\right]$

$$
\text { In the case of } \psi(\beta)=\text { constant, equation (5.55) }
$$

gives
$\beta=\left[1-3.182 \mathrm{Ja} \mathrm{Pe} \mathrm{P}^{1 / 2} \psi\left[F O-\frac{0.288}{P e_{0}}(1-\beta)\right]^{2 / 2}\right.$
Examination of the bubble volume and equivalent radius versus time curves (section 4.4) showed that there was significant condensation of the bubble before detachment from the orifice. Therefore, it was assumed that the condensation of the predetached bubble heated the liquid close to the orifice, so that after detachment the bubble rose through liquid where the temperature difference, $\Delta T$ ' between the steam and the surrounding water rose from zero to $\Delta T$, the nominal value, according to the simple expression,
$\frac{\Delta T^{\prime}}{\Delta T}=\frac{J a^{\prime}}{J a}=\left(\frac{Z}{Z_{c}}\right)^{n}=\left(\frac{F O}{F O_{c}}\right)^{n}$
where $Z_{c}$ and $F O_{c}$ are the collapse height and collapse Fourier number respectively. It is expected that the
index $n$ will be between zero (no heating) and 1 (plane heating) and can be used as an adjustable parameter.

In the case of $\psi=$ const. and negligible bubble wall velocity $(\dot{R}=0)$, the simplified equation (5.53) for the bubble collapse rate can now be written as
$\frac{2}{3} \frac{d \beta^{3 / 2}}{d F O}=-2.122 \psi \mathrm{Ja} \cdot \mathrm{Peo}^{1 / 2}$
Substituting Ja' $=\mathrm{Ja}\left(\frac{\mathrm{F}_{0}}{\mathrm{FO}_{\mathrm{C}}}\right)^{\mathrm{n}}$ from equation (5.57) and integrating,
$\left.\beta^{3 / 2}\right|_{1} ^{\beta}=\left.\frac{-3.182 \psi \mathrm{Ja} \mathrm{Pe}_{\rho_{2}}^{2 / 2}}{\mathrm{FO}_{C}{ }^{n}} \frac{\mathrm{Fo}}{\mathrm{n}+1}\right|_{0} ^{\mathrm{Fo}}$
$\therefore 1-\beta^{3 / 2}=\frac{3.182 \psi \mathrm{JaPP}_{0}{ }^{1 / 2}}{\mathrm{n}+1} \frac{F 0^{n+1}}{\mathrm{FO}_{\mathrm{C}}{ }^{n}}$
The collapse Fourier number, $F o_{c}=F o$ at $\beta=0$, is

$$
\begin{equation*}
F o_{C}=\frac{n+1}{3.182 \psi \mathrm{Ja} \mathrm{Pe} \mathrm{O}_{\mathrm{o}}^{5 / 2}} \tag{5.61}
\end{equation*}
$$

Thus, equation (5.60) can be written as
$\left(1-\beta^{3 / 2}\right)=\left(\frac{3.182 \psi}{n+1}\right)^{n+1} \mathrm{Ja} a^{\mathrm{n}+1} \mathrm{Pe} 0^{\frac{\mathrm{n}+1}{2}} \mathrm{Fo}$
where a value of $n$ can be chosen to suit the experimental conditions.

In our case, as can be seen later in chapter 6, the experimental conditions suggest a value of $n=0.1$
5.5 Collapse of spherical-cap bubbles

In some cases it was observed that when a near spherical bubble detached from the orifice, it became slightly flattened at the rear of the bubble, passing through hemispherical to spherical-cap shape as represented in the impressions of bubble shape given for tests $4.1,4.2,3.2$ and 7.1 in section 4.4 and illustrated in the collapse pattern shown in Fig. 5.6.


Fig.5.6 Impressions of collapse pattern of spherical-cap bubbles

The effect of conduction into the wake at the rear of such bubbles may be a significant factor in the total amount of heat transferred from the bubble and hence on the rate of collapse of the bubble.

In Appendix 6, the heat transfer into the wake is derived by applying a heat transfer analogy to a mass transfer approach suggested by Coypus [50]. When this is combined with the heat transfer over the front spherical surface, the bubble collapse is expressed by the equations
$F O=\frac{-3}{16 J a P e_{0}{ }^{1 / 2}} \cdot \int_{\pi}^{\gamma} \frac{\sin ^{3} \gamma \cdot d \gamma}{\frac{3}{2} \cdot \frac{\sqrt{3}}{\pi} \cdot C_{1}^{1 / 2}+\left(\frac{3}{4} \cdot C_{b} \cdot \operatorname{Re}_{0}^{1 / 4}\right) \cdot\left(\frac{3}{4}\right)^{1 / 4}} \frac{\sin ^{2} \gamma \cdot C_{1} / /^{4}}{\left(1-\frac{3}{4} C_{1}\right)^{1 / 4}}$
and

$$
\begin{equation*}
B=\left(\frac{3}{4} \dot{C}_{1}\right)^{1 / 3} \tag{5.64}
\end{equation*}
$$

where $C_{1}=\frac{2}{3}-\cos \gamma+\frac{\cos ^{3} \gamma}{3}$ and
$C_{b}=0.30\left(\frac{g R}{8 U^{2}}\right)^{0.25}$

The experimental data is compared with these equations in chapter 6 .

# CHAPTER 6 

DISCUSSION
6.1 Comparison of theory with experimental data

Preliminary study of the radius ratio versus
Fourier number curves in Section 4.4 suggested that the shape curve could well be followed if $n=0.1$ in equation (5.62), thus reducing that equation to the form,

$$
\begin{equation*}
\left(1-\beta^{3 / 2}\right)=\left(\frac{3.182 . \psi}{1.1}\right)^{1.1} \mathrm{Ja}^{2.2} \cdot \mathrm{Pe}_{0}^{0.53} \cdot \mathrm{Fo}^{\mathrm{I} \cdot 1} \tag{6.1}
\end{equation*}
$$

A plot of all the experimental data in the form of $\beta^{3 / 2}$ versus Ja'. ${ }^{2} \mathrm{Pe}_{\mathrm{o}}{ }^{0.5{ }^{5} \mathrm{FO}^{\prime \prime} \text { : is shown in Fig. 6.1. The }}$ equation,
$\left(1-\beta^{3 / 2}\right)=4.35 \mathrm{Ja}^{1.2} \cdot \mathrm{Pe}^{0.55} \cdot \mathrm{FO}^{1.10}$
fits the 1134 experimental points with a percentage standard deviation, $S_{1}$ of $27.5 \%$ of the $X$ coordinate calculated as
$S_{1}=\sqrt{\frac{\sum_{i=1}^{N}[(F-X) / F]^{2}}{N}} \times 100 \%$
where X is the data point given by (Jai.' Peo.ss fo'.') for any particular value of $\beta^{3 / 2}$ and $F$ is the corresponding parameter determined by equation (6.2).

The standard deviation defined as
$S_{2}=\sqrt{\frac{\sum(F-X)^{2}}{N}}$
is calculated to be 0.0285 .


Fig.6.1 Plot of all the experimental data

From equation (6.2), the value of $X$ at the end of bubble collapse is given as

$$
\begin{equation*}
X_{0}=J a^{x .2} \mathrm{Pe}^{0.35} \mathrm{Fo}_{\mathrm{c}}^{2.2}=\frac{1}{4.35} \cong 0.23 \tag{6.5}
\end{equation*}
$$

Thus, a percentage standard deviation related to the collapse Fourier number is found as

$$
\begin{equation*}
\frac{S_{2}}{X_{0}} \cdot 100=12.4 \% \tag{6.6}
\end{equation*}
$$

Since $4.35=\left(\frac{3.182 \Psi}{1.1}\right)^{1}$, the coefficient
4.35 implies a shape factor, $\psi$ of 1.316 , about $16 \%$ higher than the average of the shape factors $\left(\psi_{a}=1.134\right)$ presented in Table A 5.1 in Appendix 5.

In general, as is implied by equation (6.2) the bubbles condensed more slowly at low values of Jakob number and even in a few cases, at a pressure of 1 bar, reached the surface of the water without collapsing completely.

In Figures 6.2, the experimental data plotted individually for each test as radius ratio ( $\beta$ ) versus Fourier number (Fo) are compared with equation (6.2). The plots indicate that in most cases equation (6.2) gives a fairly accurate prediction of the collapse time. However, the form of the collapse curve for tests carried out at the lowest value of Jakob number (Ja = 15) differs from the form at higher Jakob numbers, in that the curve $1 s$ very nearly linear with a collapse time shorter than that predicted by equation (6.2). For these low Jakob number tests the collapse time as predicted by equation (6.2) is up to $70 \%$ greater than the experimental collapse time. Exceptions to this early collapse time are noted for tests $10.1,11.1$ and 12.1 , which are all at 2 bar with


Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number (Fo)


Fo $* 1014$

Fo *1014
Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number ( Fo )


Fo *1014


Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number (Fo)


Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number ( $F 0$ )



Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number ( $F$ )


Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number (Fo)



Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number (Fo)



Fo *1014
Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number (Fo)


Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number (Fo)



Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number (Fo)



Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number ( Fo )


Fo* 1014


Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number ( $F$ )



Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number ( $F$ ( )



Fig.6.2 Radius ratio ( $\beta$ ) versus Fourier number (Fo)
the smaller orifice of 1 mm , and show reasonable agreement with equation (6.2). In addition the form of the collapse curve for test 4.1 is much more curved and closer to the form indicated for higher values of Jakob number.

It was noted during both testing and the subsequent analysis of the cine films that, at 1 bar and $J a=15$, the bubbles were larger, elongated and more distorted and the water in the chamber was more agitated than for the other tests. This may have been because there was less pre-detachment condensation at the lowest value of Jakob number. This is illustrated in Fig. 6.3, in a typical set of photographs, taken from test 2.1, showing the bubble growth at the orifice and the collapse after detachment. At 2 bar and $J a=15$, because of the greater steam density, the bubbles were smaller for the same steam flow rate. This meant that both water agitation and bubble distortion were lower than for a pressure of 1 bar. Thus it would appear that the faster collapse of the bubbles for $J a=15$ and 1 bar could be attributed to the effects of large bubble size giving oscillation, greater distortion and increased water agitation.

In many of the tests, especially at higher Jakob numbers, the collapse rate is slower, during the early stages of collapse, than the predicted collapse rate, but later on the rate of collapse increases so that the final collapse time shows reasonable agreement with that predicted, and in a few cases gives a faster collapse than the predicted rate. This type of collapse is illustrated in Fig. 6.4, in a typical set of photographs taken from test 5.2.

This collapse pattern may be due to the effect of the bubble condensation giving some heating of the water near the orifice and so reducing the effective driving


Fig.6.3 Photographs of bubble growth and collapse Test 2.1 , $\Delta t /$ Frame $=2.37 \mathrm{~ms}$

* Detachment point


Fig.6.4 Photographs of bubble growth and collapse Test 5.2 , $\Delta t /$ Frame $=0.79 \mathrm{~ms}$

* Detachment point
force for condensation, while increased bubble distortion leads to a more rapid.increase in the collapse rate towards the later stages of collapse.

In a number of "the tests, the bubble, while near spherical at detachment, passes through a short spherical cap phase before reverting to spherical before the final collapse. However, this does not appear to affect the collapse pattern which follows that outlined above for the tests at higher Jakob numbers. This type of collapse is illustrated in the photographs, taken from test 4.1 and shown in Fig. 6.5.

Considering the accuracy in the determination of bubble rise velocities for each bubble and later in deciding the smoothed values of Peclet numbers for the different conditions, the agreements between the data and the curves are good.

Although considerable effort was spent in removing alr from the apparatus and hence from the bubbles, the data still show signs of the presence of noncondensables especially in the latest stages of collapse for tests 1.2 , 4.1, 5.1. 5.3. 10.1. 12.1. In these tests, the collapse rate tends to slow down late in the collapse with very iftle change in radius ratio with increase in time.

In Fig. 6.6 (a to d), experimental data are
presented for the larger orifice diameter of 2 mm , for two different pressures and four different Jakob numbers, with a nominal mass flow rate of $1.5 \mathrm{~g} / \mathrm{min}$. Equation (6.2) is alsif shown for each condition. The effect of increased Jak\&b number leading to a faster collapse is clear from the plots. For the same Jakob number, the bubbles have a smaller value of Peclet number at a pressure of 2 bar and
$\frac{\text { Frame }}{\text { No. }}$
$\frac{\text { Frame }}{\text { No. }}$

42

40

38


Fig.6.5 Photographs of bubble growth and collapse
Test 4.1 , $\Delta t /$ Frame $=1.186 \mathrm{~ms}$
Detachment point
$\beta \quad x($ Test 3.1$)\left[\begin{array}{l}1 \text { bar, Pe }=41000 \\ \dot{m}=1.5 \mathrm{~g} / \mathrm{min}\end{array}\right]$ $+\left(\right.$ Test 6.1) $\left[\begin{array}{l}2 \text { bar } \\ \mathrm{Pe}_{0}=22400 \\ \dot{\mathrm{~m}}=1.42 \mathrm{~g} / \mathrm{min}\end{array}\right]$



.4
.2
0
$+$
Fig.6.6 Radius ratio versus Fourier number with pressure and Jakob number as parameters


Fig 6.6 Radius ratio versus Fourier number with pressure and Jakob number as parameters
consequently collapse at a slower rate than those at a pressure of 1 bar.

In Fig. 6.7 (a to c), the data are presented, for a Jakob number of 30 and a pressure of 2 bar, for different steam flow rates and different orifice diameters. With increase in steam flow rate the Peclet number increases, and consequently the collapse rate also increases, but there is little indication of any effect from change in orifice diameter.

The effect of Jakob number is explicitly allowed for in the theory but the effects of orifice diameter, steam flow rate and pressure are only taken into account indirectly through the properties of steam and water, the initial bubble radius and the rise velocity of the bubble.

The collapse Fourier number ( $\mathrm{FO}_{\mathrm{C}}$ ) according to equation (6.2) is given, when $\beta=0$, as

$$
\begin{equation*}
\frac{1}{1: 10} \tag{6.7}
\end{equation*}
$$

$F O_{c}=\left(\frac{1}{4.35}\right) \cdot \frac{1}{\mathrm{Ja.Pe}}=\frac{0.263}{\mathrm{Ja.Pe}}$

The collapse height, $Z_{c}$ is given as
$Z_{c}=t_{c} \cdot U=F O_{C} \cdot \frac{4 R_{O}{ }^{2}}{\alpha} \cdot U$
Substituting the value of bubble rise velocity, $U$ from equation (4.5), in this equation, gives

(a)

(b) Fo*1014

Fig.6.7 Radius ratio versus Fourier number with orifice diameter and steam flow rate as parameters. $\mathrm{Ja}=30$, $\mathrm{P}=2$ bar

(c)

Fig.6.7 Radius ratio versus Fourier number with orifice diameter and steam flow rate as parameters. $J a=30, P=2$ bar


Fig. 6.11 Theoretical distance to collapse $Z_{c}$ versus experimental distance to collapse $Z_{c}^{\prime}$.

The theoretical distance to collapse $Z_{c}$ is calculated according to equ.(6.9) .
The experimental distance to collapse $Z_{c}^{\prime}$ is the experimental height at collapse minus the experimental height at detachment. As can be seen from Fig. 6.11, approximately 80 percent of the experimental data indicate experimental distances to collapse within $\pm 30 \%$ of the predicted values

$$
\begin{align*}
Z_{C} & =\frac{\frac{0.263}{\mathrm{Ja} P e^{0.5}} \cdot \frac{4 R_{O}^{2}}{\alpha} 2.148\left(\frac{P_{O}}{P}\right)^{\frac{2}{2}}\left(1+6.52 .10^{-3} \mathrm{Ja}\right)}{\left(\frac{\sigma}{R_{O} \rho_{f}}+\frac{\rho_{f}-\rho_{g}}{\rho_{f}} g R_{O}\right)^{-\frac{2}{2}}} \\
\therefore \frac{Z_{C}}{R_{O}} & =\frac{2.26 \cdot\left(1+6.52 \cdot 10^{-3} \mathrm{Ja}\right)\left(\frac{\sigma}{R_{O} \rho_{F}}+\left(\frac{\rho_{F}-\rho_{g}}{\rho_{F}}\right) \mathrm{g} \cdot \mathrm{R}_{O}\right)^{1 / 2}\left(\frac{R_{O}}{\alpha}\right)}{\left(\frac{P}{P_{O}}\right)^{\frac{q}{2}} \cdot \mathrm{Ja} \cdot P e^{0.5}}
\end{align*}
$$

If it is assumed that there is unlikely to be entrainment of water by the rising steam bubble, provided the liquid height is greater than that given by equation (6.9), then this equation should prove useful in determining the likelihood of entrainment for the range of Jakob number and Peclet number considered in this work.*

The complete solutions (equations (5.32) and (5.33)) or the simplified solution (equation (5.48)), which does not consider bubble distortion or heating of the water around the bubbles, both overpredict the collapse time as shown for test 4.2 in Fig. 6.8.

The theoretical model for a spherical-cap bubble derived in Appendix 6 leads to the bubble collapse being expressed in equations (5.63) and (5.64). This theory is compared with the data for tests 4.1 and 3.2 in Fig. 6.9 (a and b) which indicates that the theory greatly underpredicts the collapse time.

There may be several reasons to account for the underprediction of the collapse time by this theory.

Firstly, Coppus [50] stated that the value of the proportionality constant used for the average mass transfer coefficient would vary between 0.2 and 0.4 for
Fig.6.8 Comparison of collapse data (Test 4.2) with both complete and simplified solutions


Fig.6.9 Comparison of collapse data (Tests 4.1 \& 3.2) with solution for spherical-cap bubble
open channel flow or film flow. As there is no information available about the value of this constant in the wake of a bubble we have chosen a value of 0.3 which led to the relationship

$$
C_{b}=0.30\left(\frac{g R}{8 U}\right)^{0.25} .
$$

A lower value of constant would result in a lower value of $C_{b}$ in equation (5.63), this leading to an increase in the predicted collapse time.

Secondly, the calculation of wake volume in equation (A 6.17) assumes that the closed wake occupies the remaining portion of the sphere behind the bubble. This assumption may not be valid, especially when $\gamma$ is smaller than 90 degrees.

Thirdly, we have assumed turbulent flow in the wake, which implies a relatively quick collapse. However, as we have no real information on whether the wake is fully turbulent or not it is not possible to evaluate the collapse rate with any degree of certainty.

Because of the uncertainties associated with these effects, the theoretical model for the spherical-cap bubble collapse has not been pursued further.
6.2 Comparison with other theories.

The experimental data for test 11.1 is compared in Fig. 6.10 with equation (6.2) and also with five other theoretical relationships. It is clear from the figure that equation (6.2) gives a much closer agreement with the experimental data than any of the other theories and should be much more useful in predicting the collapse

Fourier number. Of the other theories the heat transfer controlled relationships developed by Florschuetz and Chao [10] (i.e. equations (2.8) and (2.9)) clearly indicate a different collapse pattern from all other theories. These two relationships show initially a rapid rate of collapse, which decreases rapidly as collapse proceeds. With equation (2.9) complete collapse cannot occur while with equation (2.8) the collapse time will be very large (e.g. for the condition in Fig. 6.10, $t_{c}=0.128$ seconds). Thus, neither of these relationships appears to be much value in establishing the collapse rate of bubbles under the specified conditions.

Equation (2.13) developed by Nordmann [13] for $0<J a<100$, which agreed with his data, also indicated a much slower collapse rate than the other theories,apart from the above two heat transfer controlled relationships. It should be noted that he used a stainless steel orifice, which, because of its relatively high thermal conductivity, probably heated the water around the orifice so producing a slower collapse rate. In contrast, in this work, the heating around the orifice was minimized by the use of a low thermal conductivity polycarbonate fitted to the base of the chamber, and a PTFE orifice was used. Although it is clear that the peclet number would have a significant effect on the collapse rate, no peclet number term appears in Nordmann's relationship.

Equation (2.31) by Moalem and Sideman [26], for small bubbles ( $R_{0}<1 \mathrm{~mm}$ ) with the translational velocity $U \propto \sqrt{R}$, is closer to the present data but still predicts a significantly slower collapse rate than equation (6.2).

The relationship leading to equation (2.29) by Isenberg et al [23], derived for low Jakob number conditions, is closer to equation (6.2) than any of the
others but, in this case,still predicts a final collapse time which is about $12 \%$ slower than that predicted by equation (6.2).

A number of observations have been made during this study, and these can be summarised as follows:

The condensation process of steam bubbles injected through an orifice is quite complicated, since all bubbles are distorted to some extent, although the distortion is greater for the larger bubbles corresponding to the lowest value of Jakob number (Ja = 15). Also the heating of the water, close to the orifice, caused by the condensing bubbles, introduces changes in the experimental conditions around the emerging and condensing bubbles.

As has been observed in a few cases, that the presence of air in the bubble has no significant effect on the collapse except towards the latest stages of collapse.

The experimental results show that bubbles generally rise at more or less constant velocity, but at higher values than the velocities predicted by existing correlations for gas bubbles. A correlation, for the velocity of steam bubbles condensing in subcooled water is proposed in this work, to describe the experimentally obtained values of the velocity.

It has been observed that, for the limited range considered, the orifice diameter has no significant influence on the collapse rate, but an increase in the steam flow rate increases the peclet number and hence the bubble collapse rate. However, an increase in pressure decreases the peclet number and also the collapse rate.

Within the range of the experimental parameters considered in this work, average values of heat transfer coefficient around the collapsing bubbles have been determined to be between $0.15 .10^{5}-0.35 .10^{5} \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$.

The semi-empirical equation (6.2), which considers both bubble distortion and the heating of the liquid around the condensing bubble, correlates the experimental data reasonably well. Within the range of Jakob numbers and Peclet numbers considered, equation (6.9) gives a method for determining the bubble height to collapse. This should prove useful in determining whether entrainment of water by steam bubbles is likely to occur or not.

## APPENDIX 1.

## Physical Properties

The properties of steam and water, except for the surface tension, are given by Mayhew and Rogers [45]. The values of surface tension of saturated water are given by Parker et al.[52]. These, together with the corresponding Jakob numbers at the two pressures used, are as follows:

| $T\left({ }^{\circ} \mathrm{C}\right)$ | $\alpha_{f} \cdot 10^{7}$ <br> $\left(\mathrm{~m}^{2} / \mathrm{s}\right)$ | $\mathrm{k}_{\mathrm{f}}$ <br> $(\mathrm{W} / \mathrm{mK})$ | $\mathrm{C}_{\mathrm{pf}}$ <br> $(\mathrm{kJ} / \mathrm{kgK})$ | $\sigma_{f} \cdot 10^{4}$ <br> $(\mathrm{~N} / \mathrm{m})$ | Ja (at <br> (abr) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 110.9 | 1.70 | 0.684 | 4.234 | 567 | 15 |
| 101.7 | 1.69 | 0.682 | 4.221 | 585 | 30 |
| 92.6 | 1.67 | 0.677 | 4.221 | 602 | 45 |
| 83.6 | 1.65 | 0.672 | 4.202 | 619 | 60 |


| $T\left({ }^{\circ} \mathrm{C}\right)$ | $\rho_{f}$ <br> $\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$ | $\mu_{f} \cdot 10^{6}$ <br> $(\mathrm{~kg} / \mathrm{ms})$ | $\nu_{f} \cdot 10^{7}$ <br> $\left(\mathrm{~m}^{2} / \mathrm{s}\right)$ | $\mathrm{Pr}_{f}$ |
| :---: | :---: | :---: | :---: | :---: |
| 110.9 | 950 | 250 | 2.63 | 1.55 |
| 101.7 | 957 | 274 | 2.86 | 1.70 |
| 92.6 | 963 | 302 | 3.14 | 1.88 |
| 83.6 | 970 | 336 | 3.46 | 2.10 |


| $\mathrm{T}\left({ }^{\circ} \mathrm{C}\right)$ | $\begin{aligned} & \alpha_{f} \cdot 10^{7} \\ & \left(\mathrm{~m}^{2} / \mathrm{s}\right) \end{aligned}$ | $\begin{gathered} \mathrm{k}_{\mathrm{f}} \\ (\mathrm{~W} / \mathrm{mK}) \end{gathered}$ | $\begin{gathered} C_{\mathrm{pf}} \\ (\mathrm{~kJ} / \mathrm{kgK}) \end{gathered}$ | $\begin{aligned} & \sigma_{f} \cdot 10^{4} \\ & (\mathrm{~N} / \mathrm{m}) \end{aligned}$ | Ja (at 1 bar) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 95 | 1.67 | 0.678 | 4.213 | 597 | 15 |
| 90 | 1.66 | 0.676 | 4.208 | 607 | 30 |
| 85.1 | 1.65 | 0.673 | 4.203 | 616 | 45 |
| 80.2 | 1.64 | 0.670 | 4.198 | 625 | 60 |
| 75.3 | 1.63 | 0.666 | 4.194 | 634 | 75 |
| $\mathrm{T}\left({ }^{\circ} \mathrm{C}\right)$ | $\stackrel{\rho_{f}}{\left(k g / m^{3}\right)}$ | $\begin{aligned} & \mu_{f} \cdot 10^{6} \\ & (\mathrm{~kg} / \mathrm{ms}) \end{aligned}$ | $\begin{aligned} & v_{f} \cdot 10^{7} \\ & \left(\mathrm{~m}^{2} / \mathrm{s}\right) \end{aligned}$ | - $\mathrm{Pr}_{f}$ |  |
| 95 | 962 | 294 | 3.06 | 1.83 |  |
| 90 | 965 | 311 | 3.22 | 1.94 |  |
| 85.1 | 969 | 330 | 3.41 | 2.06 |  |
| 80.2 | 972 | 351 | 3.61 | 2.20 |  |
| 75.3 | 975 | 374 | 3.84 | 2.36 |  |

Density of steam, saturation temperature and specific enthalpy of evaporation at the two pressures are:

| $\rho_{S}$ <br> $\left(k g / m^{3}\right)$ | $T_{S}$ <br> $\left({ }^{\circ} \mathrm{C}\right)$ | $\mathrm{h}_{\mathrm{fg}}$ <br> $(\mathrm{kJ} / \mathrm{kg})$ | P <br> $(\mathrm{bar})$ |
| :---: | :---: | :---: | :---: |
| 0.590 | 99.6 | 2258 | 1 |
| 1.129 | 120.2 | 2202 | 2 |

A 2.1 Introduction

The computer program, written in APPLESOFT II BASIC language, requires the following input data : test number (TITLE), date, $F, \Delta t$ (DT in the program), $\alpha_{W}(A L F A F), \rho_{S}(D E S), h_{f}(H F G), \Delta T(T E M D), \dot{m}_{S}(M S), V_{S}$ (VS), $\mathrm{k}_{\mathrm{w}}(\mathrm{KF}), \mathrm{Ja}(\mathrm{JA})$.

Then, $N$ - the number of pairs of points whose coordinates are required to be recorded; the first pair of points for the reference length taken as the diameter of the orifice tip, the rest for frustums of cones. For example, in Fig. 3.7, the number of frustums of cones forming the bubble is $n=16$, therefore $N=1+16+1=18$.

Afterwards, the computer is connected to the digitiser and the coordinates of the $N$ pair of points are sent to the computer by placing the cursor of the digitiser on each point in turn and pushing the cursor button in.

The program then calculates bubble volume, bubble surface area and bubble position as described in Chapter 3 (section 3.4), and some other parameters, such as dimensionless radius, Fourier number, as described in Chapter 4 (section 4.2.4). The results are later printed, and stored in a diskette for future use.

## A.2.2 Sample input

Below is the input data for the bubble for which data plots and other information are presentedinfig. 4.1 and Fig. 4.2.

```
JFUNH
HHAT TITLES (4.1).2
CHTE? 12 HF'% 1584
```

T'r'PE F:HAHEER OF FRAHES TO EE AHALYSED 24
TYPE DT: TIHE EETHEEN THE FFIHHES HS EEC.
$2.37 \mathrm{EE}-3$
TYFE ALFAF: THEFHFAL DIFFUSIUITY OF HATER RS HへでCEC.
1. 7 EE 7
TVPE DES: OEHEITY DF STEAH RS KGノHへ3 1.123
T'PE HFG: LATENT HEAT OF EUAFOFiATIO\& AS K゙いハKG ことGこ
TYPE TEHD: SUECODLIHG DF HATER HS DEGREE CEHTIGRADE 3.3
T'PE HE: (HASS) STEFit FLUH FHTE AS B, GItt. 0.45


T'YE JA: JRCOE NUHEER 15
$F=24 \quad \quad \mathrm{DT}=2.372 \mathrm{EE}-93$
HLFAF=1.TE-B7 TES=1.123
$H F G=2202 \quad T E H[=9.3$
$V 5=6 E 43 \quad K F=.504 \quad J A=15$
PFEES FETUFH TO COTHTINUE
RTTEHTIOH! THIS PFIOGRAHA IS SET FOR FRAHE FUHEERS SHALLER THAH TB
PRESS FETURH TO COHITIHUE
HIHH TYPE H
$\because 15$
A 2.3 Program Listing.
A listing of the computer program is given in
Fig. A 2.1 .


US：PRINT
SF：PRINT！
 IHFUT＂FRESS RETURH TO COHTIHUE＂；Q $\$$ IS
HOHE ：PRIHT＂ATTENTIOH ！THIS PROGRAH IS SET FOR FRAHE HUHBERS＇SHALL＇


UR（76）
HTN

 K：JACOB HUHBER＂；JH：FRIHT ＂：TEHD：PRTINT
KF：THERHAL COHDUCTIUITY OF HATER AS H／HK＊K
JR：JACOB +HUHBER

> "JH="JH
山鿊
$0=" T E H D$ ．
；ALFAF：PRIHT
宸比品
PGO F

B），HTIMF（70），मiC（70），LP（70）

Fig．A2． 1 The listing of the computer program for the analysis of cine films


Fig.A2.1 (continued)

$$
" ; G T
$$

$$
\begin{aligned}
& =\operatorname{cc}(J) *(7)=L F(J)=L F(J\rangle) \\
& 11): L P)
\end{aligned}
$$

Fig.A2. 1 (continued)



Fig.A2. 1 (continued)
Fig.A2. 1 (continued)
Fig.A2. 1 (continued)
SUB．FOR DRAWING LINE ACROSS THE PRGE

For drahinig Line
RETURH
FOR DR
hCROSE THE PRGE
ヴッ
$\stackrel{H}{7}$
孚要

票垷

Fig．A2． 1 （continued）

Computer program for calculation of bubble
collapse history
A.3.1 Introduction

The computer package used for calculation of bubble collapse history was "TUTSIM V8201" (copyright Twente University, The Netherlands). This is a simulation program in BASIC language to be used in an APPLE II computer, to calculate models of continuous dynamic systems defined as block diagrams or block graphs. When simulation data are put into the computer, the results are shown directly in a graphical way on a display screen. Simulation model data can be stored on floppy disk, and model listings and numerical or graphical results can be printed.

A simulation with TUTSIM is performed by means of calculations by the computer. A block diagram of the model is drawn for solving differential equations or calculating integrals in an equation.


For a given experimental condition both Jakob
number and peclet number are known and the value of $\beta=1$, is obtained for a particular value of
$\left(-\frac{U}{R}\right)$ (positive) according to equation (5.32).
This $\left(-\frac{U}{\dot{R}}\right)$ value is found by trial and error and fed into
the program as an input. The program changes
$\left(-\frac{U}{\dot{R}}\right)$ values between this maximum value and a value very near to zero in 80 steps and, as shown in Fig. 5.2, calculates corresponding $\beta$ values according to equation (5.32). At the same time the integral in equation (5.33) is calculated and the. $\beta$ versus Fourier number relationship is drawn as shown in Fig. 5.3.

The block diagram for these calculations is presented in Fig. A 3.1 ( $a$ and $b$ ) in the attached portfolio. The Listing of the model is given in table A 3.1.

```
0.30000E-01
0.13000E+02
```

outputelocks And ranges

| $X 1:$ | 78 | $-0.00000 \mathrm{E}+00$ | $0.10000 \mathrm{E}+01$ |
| ---: | :---: | ---: | ---: |
| $Y 1:$ | 85 | $0.00000 \mathrm{E}+00$ | $0.15000 \mathrm{E}-0 \mathrm{~S}$ |
| $\mathrm{YZ}:$ | 87 | $0.00000 \mathrm{E}+00$ | $0.15000 \mathrm{E}-0 \mathrm{~S}$ |
| YS: | 1 | $0.00000 \mathrm{E}+00$ | $0.10000 \mathrm{E}-04$ |
| Y4: | 1 | $0.00000 \mathrm{E}+00$ | $0.10000 \mathrm{E}-04$ |

MODEL


Table A3.1 The listing of the model for the complete solution



Experimental Data

| NO : (1.1).3 |  |  | FILM NO : 34-2/5 |  |
| :---: | :---: | :---: | :---: | :---: |
| FR | TIME | VOLUME | AREA | RADIUS |
| 1 | 2.29 | 3.69 | 15.07 | . 96 |
| 2 | 4.59 | 40.72 | 64.08 | 2.13 |
| 3 | 6.88 | 98.52 | 115.22 | 2.87 |
| 4 | 9.18 | 127.6 | 135.28 | 3.12 |
| 5 | 11.47 | 103.02 | 118.03 | 2.91 |
| 6 | 13.76 | 110.15 | 122.63 | 2.97 |
| 7 | 16.06 | 140.72 | 141.86 | 3.23 |
| 8 | 18.35 | 146.65 | 146.54 | 3.27 |
| 9 | 20.65 | 133.21 | 139.83 | 3.17 |
| 10 | 22.94 | 155.14 | 156.29 | 3.33 |
| 11 | 25.23 | 180.46 | 169.28 | 3.51 |
| 12 | 27.53 | 195.21 | 171.03 | 3.6 |
| 13 | 29.82 | 164.67 | 162.71 | 3.4 |
| 14 | 32.12 | 177.93 | 203.79 | 3.49 |
| 15 | 34.41 | 237.92 | 244.73 | 3.84 |
| 16 | 36.7 | 217.23 | 232.12 | 3.73 |
| 17 | 39 | 204.32 | 228.03 | 3.65 |
| 18 | 41.29 | 205.27 | 231.73 | 3.66 |
| 19 | 43.59 | 219.29 | 238.14 | 3.74 |
| 20 | 45.88 | 211.45 | 233.53 | 3.7 |
| 21 | 48.17 | 202.94 | 231.09 | 3.65 |
| 22 | 50.47 | 173.01 | 215.26 | 3.46 |
| 23 | 52.76 | 135.06 | 207.6 | 3.18 |
| 24 | 55.06 | 92.11 | 112.65 | 2.8 |
| 25 | 57.35 | 91.94 | 111.45 | 2.8 |
| 26 | 59.64 | 86.09 | 103.11 | 2.74 |
| 27 | 61.94 | 68.75 | 84.94 | 2.54 |
| 28 | 64.23 | 52.2 | 69.42 | 2.32 |
| 29 | 66.53 | 35.12 | 54.92 | 2.03 |
| 30 | 68.82 | 27.86 | 51.38 | 1.88 |
| 31 | 71.11 | 21.88 | 46.2 | 1.74 |
| 32 | 73.41 | 8.3 | 33.41 | 1.26 |
| 33 | 75.7 | 1.2 | 13.38 | . 66 |
| 34 | 78 | 1.19 | 8.8 | . 66 |
| FR | CY | CX | LE | LP |
| 1 | . 39 | . 03 | . 91 | - |
| 2 | 1.12 | . 13 | 2.98 | - |
| 3 | 1.7 | . 12 | 4.74 | - |
| 4 | 1.99 | . 1 | 5.72 | - |
|  |  |  |  | CONT.- |



| TEST NO: (1.2).1 FILM NO : 34-3/6 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FR | TIME | VOLUME | AREA | RADIUS |
| 1 | 2.31 | . 14 | 1.97 | . 32 |
| 2 | 4.62 | . 33 | 5.13 | . 43 |
| 3 | 6.93 | 19.76 | 41.4 | 1.68 |
| 4 | 9.24 | 71.71 | 93.29 | 2.58 |
| 5 | 11.54 | 94.36 | 110.61 | 2.82 |
| 6 | 13.85 | 73.05 | 88.45 | 2.59 |
| 7 | 16.16 | 66.38 | 83.22 | 2.51 |
| 8 | 18.47 | 83.98 | 101.21 | 2.72 |
| 9 | 20.78 | 79.49 | 102.9 | 2.67 |
| 10 | 23.09 | 82.61 | 108.08 | 2.7 |
| 11 | 25.4 | 64.99 | 83.84 | 2.49 |
| 12 | 27.71 | 58.53 | 80.12 | 2.41 |
| 13 | 30.02 | 61.85 | 85.25 | 2.45 |
| - 14 | 32.33 | 50.94 | 70.39 | 2.3 |
| 15 | 34.63 | 32.73 | 52.53 | 1.98 |
| 16 | 36.94 | 17.07 | 34.24 | 1.6 |
| 17 | 39.25 | 8.78 | 24.73 | 1.28 |
| 18 | 41.56 | . 51 | 4.06 | . 5 |
| 19 | 43.87 | . 06 | . 84 | . 24 |
| $\overline{F R}-\overline{C Y}-\overline{C X}-\overline{L E}-\overline{L P}-\overline{\underline{L P}}-$ |  |  |  |  |
| 1 | . 11 | . 46 | . 26 | - |
| 2 | . 11 | . 48 | . 28 | - |
| 3 | . 78 | . 08 | 2.02 | - |
| 4 | 1.4 | . 12 | 3.97 | - |
| 5 | 1.78 | . 21 | 5.32 | - |
| 6 | 1.99 | . 28 | 4.69 | - |
| 7 | 2.4 | . 21 | 4.57 | - |
| 8 | 2.84 | . 23 | 5.29 | - |
| 9 | 2.94 | . 22 | 6.11 | - |
| 10 | 3.23 | . 21 | 7.38 | - |
| 11 | 3.39 | . 13 | 6.84 | - |
| 12 | 3.63 | . 14 | 7.11 | - |
| 13 | 4.16 | . 18 | 8.03 | - |
| - |  |  |  |  |
| 14 | 4.92 | .15 | 8.55 | - |
| 15 | 5.66 | . 11 | 8.88 | - |
| 16 | 6.77 | . 2 | 8.56 | - |
| 17 | 7.97 | . 3 | 9.18 | - |
| 18 | 8.36 | -. 21 | 8.72 | - |
|  |  |  |  | CONT |


| $\frac{F R}{19}$ | $\frac{C Y}{9.24}$ | $\underline{C X}$ | LE | LP |
| :--- | :--- | :--- | :--- | :--- |
| 19 | 9.24 | -.69 | 9.45 | - |
|  | - | - | - | - |

## EXPERIMENTAL PARRMETERS :

| $d=2$ | $\dot{m}_{s}=.5$ | $V_{s}=13935$ | $P=.991$ |
| :--- | :--- | :--- | :--- |
| $J_{a}=30$ | $\Delta T=10$ | $\Delta t=2.31$ | $C S=433$ |
| $F=19$ | $Z=40$ | $T_{p}=165$ |  |

EXPERIMENTAL RESULTS :
$t_{g}=32.33$
$t_{c}=8.25$
$U^{\prime}=437$
$P_{\theta_{0}}=12080$
$Z_{c}=9.24$
$t_{t}=40.58$
$h_{c}=1790$
$N u_{c}=65$
$R_{m}=2.82$
$Z_{d}=4.92$
$\mathrm{Fo}_{\mathrm{c}}=6.49 \mathrm{E}-05$




| FR | CY | CX | LE | LP |
| :---: | :---: | :---: | :---: | :---: |
| 3 | 2.86 | . 18 | 5.7 | - |
| 4 | 3.04 | . 28 | 6.34 | - |
| 5 | 3.18 | . 22 | 7.28 | - |
| 6 | 3.19 | . 18 | 7.72 | - |
| 7 | 3.31 | . 16 | 8.32 | - |
| 8 | 3.42 | . 18 | 8.35 | - |
| 9 | 3.48 | . 15 | 8.45 | - |
| 10 | 3.76 | . 17 | 8.9 | - |
| 11 | 4.19 | . 24 | 9.73 | - |
| 12 | 4.65 | . 31 | 10.59 | - |
| 13 | 4.96 | . 19 | 11.31 | - |
| 14 | 5.15 | . 19 | 11.55 | - |
| 15 | 5.36 | . 21 | 11.85 | - |
| 16 | 5.6 | . 24 | 12.33 | - |
| 17 | 6.06 | . 22 | 12.78 | - |
| 18 | 6.51 | . 2 | 13.32 | - |
| 19 | 6.83 | . 26 | 13.72 | - |
| 20 | 7.45 | . 19 | 14.34 | - |
| 21 | 8.04 | . 17 | 14.87 | - |
| 22 | 8.59 | . 19 | 15.51 | - |
| - | - | - | - | - - |
| 23 | 9.27 | . 21 | 15.86 | - |
| 24 | 9.91 | . 15 | 16.01 | - |
| 25 | 10.35 | . 06 | 16.26 | - |
| 26 | 10.96 | . 02 | 16.43 | - |
| 27 | 11.72 | . 05 | 16.77 | - |
| 28 | 12.5 | . 06 | 17.2 | - |
| 29 | 12.98 | -. 02 | 17.41 | - |
| 30 | 13.42 | -. 09 | 17.46 | - |
| 31 | 14.19 | -. 07 | 17.88 | - |
| 32 | 14.65 | -. 2 | 17.69 | - |
| 33 | 15.29 | -. 15 | 17.71 | - |
| 34 | 16.6 | . 01 | 17.9 | - |
| 35 | 17.54 | . 02 | 18.7 | - |
| 36 | 18.42 | -. 15 | 19.19 | - |
| EXPERIMENTAL PARAMETERS |  |  |  |  |
| $d=$ | $\dot{m}_{s}=1 \quad V_{s}=27871$ |  |  | $\mathrm{P}=.995$ |
| Ja | $\Delta T=10$ |  | $\Delta t=1.14$ | $C S=875$ |
| F= | $Z=40 \quad T_{p}=165$ |  |  |  |
| EXPERIMENTAL RESULTS : |  |  |  |  |
| $t_{g}=26.39 \quad t_{c}=15$ | $39 \quad{ }^{t_{c}}=15$ |  | $t_{t}=41.39$ | $f_{s}=4 Q 0 n t .-$ |



## EXPERIMENTAL PARAMETERS :

| $\dot{d}=2$ | $\dot{m}_{s}=1$ | $V_{s}=27871$ | $P=.995$ |
| :--- | :--- | :--- | :--- |
| $J_{a}=45$ | $\Delta T=15$ | $\Delta t=1.13$ | $C S=885$ |
| $F=15$ | $Z=40$ | $T_{p}=165$ |  |

EXPERIMENTAL RESULTS :

| $t_{g}=12.43$ | $t_{c}=4.5$ | $t_{t}=16.93 \quad f_{s}=88$ |
| :--- | :--- | :--- |
| $R_{o}=2.15$ | $U=775$ | $h_{c}=26367$ |
| $R_{m}=2.43$ | $P_{0}=20170$ | $N u_{c}=91$ |
| $Z_{d}=4.85$ | $Z_{c}=8.14$ | $F o_{c}=4.02 E-05$ |





| FR | CY | CX | LE | LP |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 2.84 | . 1 | 6.91 | - |
| 3 | 3.15 | . 1 | 7.51 | - |
| 4 | 3.34 | . 15 | 8.03 | - |
| 5 | 3.45 | . 12 | 8.01 | - |
| 6 | 3.42 | . 15 | 8.52 | - |
| 7 | 3.34 | . 17 | 8.68 | - |
| 8 | 3.56 | . 27 | 9.17 | - |
| 9 | 3.79 | . 22 | 9.71 | - |
| 10 | 4.16 | . 26 | 10.56 | - |
| 11 | 4.37 | . 34 | 11.06 | - |
| 12 | 4.74 | . 33 | 11.79 | - |
| 13 | 5.16 | . 36 | 12.32 | - |
| 14 | 5.66 | . 35 | 12.93 | - |
| 15 | 6.06 | . 4 | 13.64 | - |
| 16 | 6.5 | . 35 | 14.45 | - |
| 17 | 7.1 | . 28 | 15.05 | - |
| 18 | 7.62 | . 29 | 15.53 | - |
| 19 | 8.04 | . 3 | 15.95 | - |
| 20 | 8.6 | . 29 | 16.7 | - |
| 21 | 9.11 | . 24 | 17.03 | - |
| 22 | 9.82 | . 22 | 17.54 | - |
| - | - | - | - | - - |
| 23 | 10.63 | . 29 | 17.84 | - |
| 24 | 11.01 | . 33 | 17.78 | - |
| 25 | 11.94 | . 28 | 18.16 | - |
| 26 | 12.68 | . 24 | 18.36 | - |
| 27 | 13.5 | . 24 | 18.87 | - |
| 28 | 14.14 | . 3 | 19.11 | - |
| 29 | 14.61 | . 25 | 19.24 | - |
| 30 | 15.01 | . 24 | 19.15 | - |
| 31 | 15.51 | . 27 | 19.32 | - |
| 32 | 15.87 | . 21 | 19.05 | - |
| 33 | 16.56 | . 26 | 19.63 | - |
| 34 | 17.45 | . 35 | 20.61 | - |
| 35 | 19.23 | . 57 | 21.36 | - |
| 36 | 20.69 | 2.72 | 22.07 | - |
| 37 | 21.49 | . 25 | 22.5 | - |
| 38 | 21.88 | -. 03 | 22.78 | - |
|  |  |  |  |  |

EXPERIMENTAL PARAMETERS:

| $d=2$ | $\dot{m}_{s}=1.5$ | $V_{s}=41806$ | $P=.995$ |
| :--- | :--- | :--- | :--- |
| $J_{a}=30$ | $\Delta T=10$ | $\Delta t=1.37$ | $C S=730$ |
| $\dot{F}=38$ | $Z=40$ | $T_{p}=165$ |  |

EXPERIMENTAL RESULTS :
$\begin{array}{ll}t_{g}=31.51 . & t_{c}=19 \\ R_{0}=4.63 & U=590\end{array}$
$f_{s}=33$
$\mathrm{R}_{\mathrm{m}}=4.87 \quad \mathrm{Pe}_{0}=32830$
$N u_{c}=179$
$z_{d}=10.63$
$Z_{c}=21.88$
$\mathrm{Fo}_{\mathrm{c}}=3.69 \mathrm{E}-05$



| TEST NO: 3.4 . 13 FILM NO: 37-1/2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FR | TIME | VOLUME | AREA | RADIUS |
| 1. | . 8 | 39.79 | 61.6 | 2.12 |
| 2 | 1.59 | 60.6 | 81.56 | 2.44 |
| 3 | 2.39 | 81.64 | 99.51 | 2.69 |
| 4 | 3.18 | 104.86 | 117.86 | 2.93 |
| 5 | 3.98 | 122.54 | 130.8 | 3.08 |
| 6 | 4.78 | 135.85 | 143.93 | 3.19 |
| 7 | 5.57 | 135.46 | 144.19 | 3.19 |
| 8 | 6.37 | 139.56 | 146.45 | 3.22 |
| - 9 | 7.16 | 135.75 | 144.58 | 3.19 |
| 10 | 7.96 | 135.95 | 147.32 | 3.19 |
| 11 | 8.76 | 121.09 | 134.82 | 3.07 |
| 12 | 9.55 | 120.15 | 138.66 | 3.06 |
| 13 | 10.35 | 135.14 | 145.7 | 3.18 |
| 14 | 11.14 | 146.8 | 154.08 | 3.27 |
| 15 | 11.94 | 142.91 | 151.78 | 3.24 |
| 16 | 12.74 | 149.54 | 154.56 | 3.29 |
| 17 | 13.53 | 155.06 | 158.1 | 3.33 |
| 18 | 14.33 | 144.55 | 149.79 | 3.26 |
| 19 | 15.12 | 143.73 | 152.1 | 3.25 |
| 20 | 15.92 | 138.52 | 149.32 | 3.21 |
| 21 | 16.72 | 121.84 | $\overline{133}$ | $3.08$ |
| 22 | 17.51 | 99.85 | 117.18 | 2.88 |
| 23 | 18.31 | 83.02 | 100.7 | 2.71 |
| 24 | 19.1 | 67.04 | 87.35 | 2.52 |
| 25 | 19.9 | 51.89 | 72.28 | 2.31 |
| 26 | 20.7 | 35.71 | 57.33 | 2.04 |
| 27 | 21.49 | 16.07 | 38.09 | 1.57 |
| 28 | 22.29 | . 27 | 2.42 | . 4 |
| FR | CY | CX | LE | $\underline{\underline{P}}$ |
| 1 | 1.49 | . 14 | 4.53 | - |
| 2 | 1.61 | . 19 | 5.1 | - |
| 3 | 1.73 | . 25 | 5.01 | - |
| 4 | 1.75 | . 31 | 5.15 | - |
| 5 | 1.96 | . 3 | 5.47 | - |
| 6 | 2.08 | . 33 | 5.98 | - |
| 7 | 2.13 | . 29 | 6.15 | - |
| 8 | 2.23 | . 28 | 6.79 | - |
| 9 | 2.33 | . 25 | 7.18 | - |
| 10 | 2.5 | . 23 | 7.67 | - |
|  |  |  |  | CONT |


| FR | CY | CX | LE | LP |
| :---: | :---: | :---: | :---: | :---: |
| 11 | 2.63 | . 07 | 7.75 | - |
| 12 | 2.91 | . 14 | 8.15 | - |
| 13 | 3.15 | . 14 | 8.8 | - |
| 14 | 3.46 | . 12 | 9.25 | - |
| 15 | 3.77 | . 14 | 9.56 | - |
| 16 | 4.05 | . 13 | 9.91 | - |
| 17 | 4.4 | . 13 | 10.39 | - |
| 18 | 4.92 | . 1 | 10.57 | - |
| 19 | 5.43 | . 04 | 10.93 | - |
| 20 | 5.88 | . 01 | 11.16 | - |
| - | - | - | - | - - |
| 21 | 6.27 | . 05 | 11.12 | - |
| 22 | 6.89 | -. 05 | 11.27 | - |
| 23 | 7.38 | -. 13 | 11.42 | - |
| 24 | 8 | -. 08 | 11.85 | - |
| 25 | 8.83 | -. 08 | 12.66 | - |
| 26 | 9.65 | -. 14 | 13.5 | - |
| 27 | 10.43 | -. 12 | 13.72 | - |
| 28 | 10.43 | -. 25 | 10.69 | - |
|  | - | - | - - | - |
| EXPERIMENTAL PARAMETERS |  |  |  |  |
| $d=$ |  |  | 41806 | $P=.995$ |
|  |  |  | . 8 | $C S=1256$ |
| $F=$ |  |  |  |  |
| EXPERIMENTAL RESULTS : |  |  |  |  |
| $\mathrm{t}_{\mathrm{g}}=16.72$ |  | ${ }^{t}{ }_{c}=5.5$ | $t_{t}=22.22$ | $f_{s}=63$ |
| $R_{0}=3.08$ |  | $U=810$ | $h_{c}=24967$ |  |
| $\mathrm{R}_{\mathrm{m}}=3.33$ |  | $\mathrm{Pe}_{0}=30390$ | $\mathrm{Nu}_{\mathrm{c}}=125$ |  |
| $z_{d}=6.27$ |  | $Z_{c}=10.43$ | $\mathrm{Fo}_{\mathrm{C}}=2.38 \mathrm{E}-05$ |  |



| FR | CY | CX | LE | LP |
| :---: | :---: | :---: | :---: | :---: |
| 14. | 5.66 | . 3 | 8.07 | 3.9 |
| 15 | 6.08 | . 18 | 8.3 | 4.62 |
| 16 | 6.42 | . 29 | 8.49 | 5.11 |
| 17 | 7.12 | . 22 | 8.99 | 6.06 |
| 18 | 7.62 | . 24 | 9.14 | 6.77 |
| 19 | 8.18 | . 41 | 9.78 | 7.44 |
| 20 | 8.89 | . 27 | 9.56 | 8.31 |
| 21 | 9.5 | . 44 | 10.27 | 8.45 |
| 22 | 9.68 | . 57 | 10.93 | 8.61 |
| 23 | 10.57 | . 5 | 11.35 | 9.76 |
| 24 | 11.36 | . 58 | 11.76 | 10.85 |
| EXPERIMENTAL PARAMETERS |  |  |  |  |
|  |  |  |  |  |
| $d=$ |  |  | 6643 | $P=2.005$ |
| Ja |  |  | 2.37 | $C S=422$ |
| F $=$ |  |  | 165 |  |
| EXPERIMENTAL RESULTS : |  |  |  |  |
| $t_{g}=28.46$ |  | $t_{c}=24.7$ | 53.16 | $f_{s}=38$ |
| $\mathrm{R}_{0}=2.45$ |  | $U=250$ | $h_{c}=17086$ |  |
| $\mathrm{R}_{\mathrm{m}}=2.51$ |  | $\mathrm{Pe}_{0}=7210$ | $N u_{c}=63$ |  |
| $z_{d}=4.22$ |  | $Z_{c}=11.36$ | $\mathrm{Fo}_{\mathrm{c}}=1.749 \mathrm{E}-04$ |  |



| FR | CY | cx | LE | LP |
| :---: | :---: | :---: | :---: | :---: |
| 14 | 2.67 | . 1 | 4.83 | 0 |
| 15 | 3.03 | . 12 | 5.16 | 0 |
| 16 | 3.28 | . 07 | 5.52 | 0 |
| - - - |  |  |  |  |
| 17 | 3.58 | . 1 | 5.97 | . 85 |
| 18 | 3.91 | . 07 | 6.18 | 1.75 |
| 19 | 4.26 | . 1 | 6.27 | 2.28 |
| 20 | 4.86 | . 09 | 6.54 | 3.46 |
| 21 | 5.31 | . 09 | 6.71 | 4.15 |
| 22 | 5.66 | . 13 | 7.04 | 4.79 |
| 23 | 6.11 | . 12 | 6.97 | 5.57 |
| 24 | 6.46 | . 01 | 6.85 | 6.07 |
| 25 | 6.67 | 1.25 | 7.28 | 6.4 |
| EXPERIMENTAL PARAMETERS : - - - - |  |  |  |  |
| d $=$ |  |  | 6643 | $P=2.005$ |
|  |  | 8.5 | 1. 51 | $C S=663$ |
| $\mathrm{F}=$ |  |  | $=165$ |  |
| EXPERIMENTAL RESULTS: |  |  |  |  |
| $t^{\text {g }}=25.64$$R_{0}=2.16$ |  | $t_{c}=12$ | '37.64 | $f_{s}=41$ |
|  |  | $U=249$ | $h_{c}=16723$ |  |
| $\mathrm{R}_{\mathrm{m}}=2.16$ |  | $\mathrm{Pe}_{0}=6360$ | $N u_{c}=58$ |  |
| $R_{m}=2.16$$Z_{d}=3.58$ |  | $Z_{c}=6.87$ | $\mathrm{Fo}_{\mathrm{c}} \mathrm{C}=1.087 \mathrm{E}-04$ |  |








## EXPERIMENTAL PARAMETERS:

| $\dot{d}=2$ | $\dot{m}_{s}=.92$ | $V_{s}=13560$ | $P=2.008$ |
| :--- | :--- | :--- | :--- |
| $J_{a}=45$ | $\Delta T=27.6$ | $\Delta_{t}=.77$ | $C S=1298$ |
| $F=40$ | $Z=40$ | $T_{p}=165$ |  |
| EXPERIMENTAL RESULTS $:$ |  |  |  |

$t_{g}=21.56$
$R_{0}=2.29$
$t_{t}=30.26$
$f_{s}=48$
$R_{m}=2.49$
$h_{c}=16664$
$z_{d}^{m}=3.91$
$t_{c}=8.7$
$N u_{c}=61$
$\mathrm{Fo}_{\mathrm{c}}=6.93 \mathrm{E}-05$



## EXPERIMENTAL PARAMETERS:

| $d=2$ | $\dot{m}_{s}=1.42$ | $V_{s}=20963$ | $P=2.011$ |
| :--- | :--- | :--- | :--- |
| $J_{a}=15$ | $\Delta T=9.3$ | $\Delta t=1.12$ | $C S=896$ |
| $F=41$ | $Z=40$ | $T_{p}=165$ |  |

EXPERIMENTAL RESULTS:
$t_{g}=21.21$
$R_{0}=3.37$
$t_{c}^{c}=23$
$U^{c}=589$
$t_{t}=44.21$
$f_{s}=50$
$R_{\text {en }}=3.7$
$P e_{0}=23350$
$h_{c}=28956$
$z_{d}=9.34$
$Z_{c}=22.58$
$N u_{c}=148$

| TEST | NO : (6.2).2 |  | FILM NO: | 48-2/3 |
| :---: | :---: | :---: | :---: | :---: |
| FR | TIME | VOLUME | AREA | RADIUS |
| 1 | . 81 | 83.33 | 101.13 | 2.71 |
| 2 | 1.63 | 87.08 | 104.41 | 2.75 |
| 3 | 2.44 | 88.52 | 105.11 | 2.76 |
| 4 | 3.26 | 86.72 | 103.37 | 2.75 |
| 5 | 4.07 | 89.97 | 106.38 | 2.78 |
| 6 | 4.88 | 98.28 | 112.21 | 2.86 |
| 7 | 5.7 | 109.29 | 119.86 | 2.97 |
| 8 | 6.51 | 118.75 | 127.13 | 3.05 |
| 9 | 7.33 | 124.83 | 131.28 | 3.1 |
| 10 | 8.14 | 128.93 | 134.54 | 3.13 |
| 11 | 8.95 | 125.52 | 133.47 | 3.11 |
| 12 | 9.77 | 126.01 | 134.69 | 3.11 |
| 13 | 10.58 | 124.18 | 134.15 | 3.09 |
|  | - | 11 | 12 | -3.03 |
|  |  |  |  | 3.03 |
| 15 | 12.21 | 111.38 | 124.02 | 2.98 |
| 16 | 13.02 | 98.24 | 114.53 | 2.86 |
| 17 | 13.84 | 87.86 | 108.43 | 2.76 |
| 18 | 14.65 | 76.86 | 100.36 | 2.64 |
| 19 | 15.47 | 65.26 | 93.09 | 2.5 |
| 20 | 16.28 | 59.85 | 87.64 | 2.43 |
| 21 | 17.09 | 54.54 | 81.71 | 2.35 |
| 22 | 17.91 | 49.33 | 76.3 | 2.28 |
| 23 | 18.72 | 42.72 | 67.74 | 2.17 |
| 24 | 19.54 | 35.56 | 59.76 | 2.04 |
| 25 | 20.35 | 23.05 | 44.97 | 1.77 |
| 26 | 21.16 | 13.33 | 36.09 | 1.47 |
| 27 | 21.98 | 10.22 | 28.74 | 1.35 |
| 28 | 22.79 | 2.39 | 12.29 | . 83 |
| 29 | 23.61 | . 7 | 5.16 | . 55 |
| 30 | 24.42 | . 29 | 2.8 | . 41 |
| - - | - - | - - - | - | - - - |
| FR | CY | CX | LE | LP |
| 1 | 2.68 | . 15 | 7.38 |  |
| 2 | 2.84 | . 11 | 7.47 | 0 |
| 3 | 2.93 | . 11 | 7.47 | 0 |
| 4 | 3.01 | . 12 | 7.56 |  |
| 5 | 3.21 | . 13 | 7.91 | 0 |
| 6 | 3.4 | . 07 | 8.23 | 0 |
| 7 | 3.57 | . 11 | 8.55 | 0 |
| 8 | 3.76 | . 13 | 9.01 | 0 |
|  |  |  |  | CONT.- |


| FR | CY | CX | LE | LP |
| :---: | :---: | :---: | :---: | :---: |
| 9 | 4.03 | . 14 | 9.38 | 0 |
| 10 | 4.19 | . 15 | 9.64 | 0 |
| 11 | 4.34 | . 14 | 9.88 | 0 |
| 12 | 4.65 | . 18 | 10.13 | 0 |
| 13 | 4.74 | . 17 | 10.33 | 0 |
|  |  |  |  |  |
|  |  |  |  |  |
| 15 | 5.48 | . 2 | 10.74 | 1.82 |
| 16 | 5.82 | . 25 | 10.93 | 2.54 |
| 17 | 6.51 | . 29 | 11.24 | 3.71 |
| 18 | 6.99 | . 29 | 11.46 | 4.21 |
| 19 | 7.6 | . 3 | 12.34 | 5 |
| 20 | 7.95 | . 33 | 12.75 | 5.31 |
| 21 | 8.14 | . 36 | 12.67 | 5.51 |
| 22 | 8.36 | . 35 | 12.55 | 5.81 |
| 23 | 8.43 | . 36 | 11.83 | 6.08 |
| 24 | 8.67 | . 4 | 11.66 | 6.5 |
| 25 | 9.33 | . 47 | 11.69 | 6.63 |
| 26 | 9.98 | . 47 | 13.83 | 8.13 |
| 27 | 10.67 | . 41 | 11.78 | 9.15 |
| 28 | 10.95 | . 49 | 12.26 | 10.66 |
| 29 | 11.53 | . 76 | 12.57 | 11.74 |
| 30 | 11.54 | 1.15 | 13 | 12.49 |
| EXPERIMENTAL PARAMETERS |  |  |  |  |
| $d=$ |  | . 42 | 20963 | $\mathrm{P}=2.011$ |
| Ja |  | 8.5 | . 81 | $C S=1228$ |
| $\mathrm{F}=$ |  |  |  |  |
| EXPERIMENTAL RESULTS : |  |  |  |  |
|  |  | 2.5 |  | $f_{s}=95$ |
| $\mathrm{R}_{0}$ |  |  | 22580 |  |
|  |  |  | $=109$ |  |
| $Z_{d}^{m}=5.06$ |  | $Z_{C}{ }^{0}=11.54 \quad \mathrm{Fo}_{\mathrm{c}}=1.71 \mathrm{E}-05$ |  |  |




| TEST NO : (6.4). 1 |  |  | FILM NO : 50-1/2 |  |
| :---: | :---: | :---: | :---: | :---: |
| FR | TIME | VOLUME | AREA | RADIUS |
| 1 | . 81 | 5.01 | 19.14 | 1.06 |
| 2 | 1.62 | 10.94 | 28.59 | 1.38 |
| 3 | 2.43 | 22.75 | 44.04 | 1.76 |
| 4 | 3.24 | 31.3 | 53.93 | 1.96 |
| 5 | 4.06 | 39.94 | 63.01 | 2.12 |
| 6 | 4.87 | 44.79 | 69.63 | 2.2 |
| 7 | 5.68 | 49.4 | 74.04 | 2.28 |
| 8 | 6.49 | 50.63 | 75.14 | 2.29 |
| 9 | 7.3 | 49.4 | 73.17 | 2.28 |
| 10 | 8.11 | 47.72 | 69.84 | 2.25 |
| 11 | 8.92 | 50.9 | 72.56 | 2.3 |
| 12 | 9.73 | 53.96 | 74.45 | 2.34 |
| 13 | 10.54 | 59.5 | 78.57 | 2.42 |
| 14 | 11.35 | 63.16 | 81.28 | 2.47 |
| 15 | 12.17 | 63 | 80.34 | 2.47 |
| 16 | 12.98 | 62.71 | 79.71 | 2.46 |
| 17 | 13.79 | 61.19 | 78.42 | 2.44 |
| 18 | 14.6 | 56.29 | 74.69 | 2.38 |
| 19 | 15.41 | 54. | 73.44 | 2.34 |
| 20 | 16.22 | 55.54 | 74.67 | 2.37 |
| - | - | - - | - - | - - |
| 21 | 17.03 | 48.76 | 66.31 | 2.27 |
| 22 | 17.84 | 37.14 | 55.84 | 2.07 |
| 23 | 18.65 | 30.07 | 49.48 | 1.93 |
| 24 | 19.46 | 22.56 | 41.71 | 1.75 |
| 25 | 20.28 | 15.93 | 34.44 | 1.56 |
| 26 | 21.09 | 2.61 | 11.65 | . 85 |
| 27 | 21.9 | . 37 | 2.78 | . 45 |
| FR | CY | CX | LE | LP |
| 1 | . 56 | . 03 | 1.36 | 0 |
| 2 | . 68 | -. 01 | 1.68 | 0 |
| 3 | . 97 | . 02 | 2.39 | 0 |
| 4 | 1.12 | . 05 | 2.94 | 0 |
| 5 | 1.16 | . 06 | 3.29 | 0 |
| 6 | 1.19 | . 01 | 3.44 | 0 |
| 7 | 1.27 | -. 02 | 3.64 | 0 |
| 8 | 1.31 | . 02 | 3.9 | 0 |
| 9 | 1.37 | . 02 | 4.15 | 0 |
| 10 | 1.49 | . 02 | 4.44 | 0 |
| 11 | 1.64 | . 03 | 4.57 | 0 |
|  |  |  |  | CONT. - |




| FR | CY | CX | LE | LP |
| :---: | :---: | :---: | :---: | :---: |
| 12 | 7.86 | -. 16 | 14.59 | 0 |
|  |  | - | - | - - - |
| 13 | 9.13 | -. 24 | 15.12 | 2.54 |
| 14 | 10.54 | -. 32 | 15.8 | 6.09 |
| 15 | 12.28 | -. 38 | 16.3 | 8.91 |
| 16 | 13.13 | -. 46 | 16.54 | 10.52 |
| 17 | 13.95 | -. 61 | 17.4 | 11.77 |
| 18 | 14.6 | -. 75 | 17.76 | 12.8 |
| 19 | 15.19 | -. 8 | 17.86 | 13.31 |
| 20 | 15.54 | -. 78 | 18.05 | 14.06 |
| 21 | 15.82 | -1.13 | 17.7 | 14.51 |
| 22 | 16.67 | -1.31 | 18.31 | 15.26 |
| 23 | 18.6 | -. 9 | 19.79 | 17.7 |
| 24 | 19.86 | -. 83 | 21.1 | 18.78 |
| 25 | 20.86 | -. 98 | 21.83 | 19.74 |
| 26 | 21.73 | -. 98 | 22.24 | 21.19 |
| 27 | 22.55 | -1.02 | 22.93 | 22.2 |
| EXPERIMENTAL PARAMETERS |  |  |  |  |
|  |  |  |  |  |
| $d=$ |  |  | 11706 | $P=1.02$ |
| Ja |  |  | 2.15 | $C S=466$ |
| $F=$ |  |  |  |  |
| EXPERIMENTAL RESULTS : |  |  |  |  |
| $t_{g}=27.98$ |  | $\mathrm{t}_{\mathrm{c}}=29$ | 56.93 | $f_{s}=39$ |
| $\mathrm{R}_{0}=3.73$ |  | $U=500$ | 26777 |  |
| $\mathrm{R}_{\mathrm{m}}=3.73$ |  | $\mathrm{Pe}_{0}=22340$ | $=153$ |  |
| $z_{\text {m }}=9.13$ |  | $z_{c}=22.55$ | $\mathrm{Fo}_{\mathrm{c}} \mathrm{c}=8.7 \mathrm{E}-05$ |  |



| FR | CY | CX | LE | LP |
| :---: | :---: | :---: | :---: | :---: |
| 6 | 1.62 | -. 08 | 5.14 | 0 |
| 7 | 1.91 | -. 09 | 5.9 | 0 |
| 8 | 2.19 | -. 09 | 6.41 |  |
| 9 | 2.5 | -. 04 | 6.87 | 0 |
| 10 | 2.76 | -. 05 | 7.49 | 0 |
| 11 | 3.07 | -. 06 | 8.02 | 0 |
| 12 | 3.35 | -. 06 | 8.6 | 0 |
| 13 | 3.75 | -. 04 | 9.13 | 0 |
| 14 | 4.27 | -. 03 | 9.66 | 0 |
| . 15 | 4.68 | -. 06 | 10.17 | 0 |
| 16 | 5.05 | -. 08 | 10.52 | 0 |
| 17 | 5.37 | -. 1 | 10.71 | 0 |
| 18 | 5.73 | -. 12 | 11.04 | 0 |
| 19 | 6.15 | -. 13 | 11.28 | 0 |
| 20 | 6.7 | -. 16 | 11.64 | 0 |
| 21 | 7.18 | -. 13 | 11.81 | 0 |
| - | - |  | - | - 3 |
| 22 | 7.89 | -. 14 | 11.91 | 3.87 |
| 23 | 8.64 | -. 18 | 12.56 | 5.15 |
| 24 | 9.73 | -. 28 | 12.57 | 7.1 |
| 25. | 10.32 | -. 3 | 12.51 | 8.37 |
| 26 | 10.72 | -. 3 | 12.54 | 8.9 |
| 27 | 11.18 | -. 32 | 12.77 | 9.27 |
| 28 | 11.56 | -. 41 | 12.75 | 9.97 |
| 29 | 11.75 | -. 29 | 12.87 | 10.33 |
| 30 | 12.09 | -. 56 | 13.09 | 11.06 |
| 31 | 11.84 | -1.19 | 12.52 | 11.06 |
| 32 | 11.9 | -1.64 | 12.37 | 11.42 |
| 33 | 12.21 | -1.45 | 12.33 | 12.1 |
| EXPERIMENTAL PARAMETERS |  |  |  |  |
| $d=$ |  |  | 11706 | $P=1.02$ |
| Ja |  |  | 1.03 | $C S=967$ |
| $\mathrm{F}=$ |  |  | 165 |  |
| EXPERIMENTAL RESULTS: |  |  |  |  |
| $\begin{aligned} t^{\text {g }} & =22.74 \\ R_{0} & =2.63\end{aligned}$ |  | ${ }^{t}{ }_{c}=10.5$ | $t_{t}=33.24$$h_{c}=23402$ | $f_{s}=46$ |
|  |  | $U=515$ |  |  |
| $\mathrm{R}_{\mathrm{m}}=2.96$ |  | $\mathrm{Pe}_{0}=16320$ | $\mathrm{Nu}_{c}=97$ |  |
| $z_{d}=7.89$ |  | $Z_{c}=12.21$ | $\mathrm{Fo}_{\mathrm{c}}=6.3 \mathrm{E}-05$ |  |



## EXPERIMENTAL PARAMETERS :

| $d=1$ | $\dot{m}_{s}=.42$ | $V_{s}=11706$ | $P=1.02$ |
| :--- | :--- | :--- | :--- |
| $J_{a}=45$ | $\Delta T=15$ | $\Delta t=.78$ | $C S=1289$ |
| $F=17$ | $Z=40$ | $T_{p}=165$ |  |

EXPERIMENTAL RESULTS :

$$
\begin{array}{llll}
t_{g}=9.32 & t_{c}=4 & t_{t}=13.32 & f_{s}=117 \\
R_{o}=1.85 & U=533 & h_{c}=27982 & \\
R_{m}=2.13 & P_{e}=11950 & \mathrm{Nu}_{c}=84 \\
Z_{d}=7.33 & Z_{c}=9.59 & F_{o}=4.77 E-05
\end{array}
$$





| FR | CY | CX | LE | LP |
| :---: | :---: | :---: | :---: | :---: |
| 12 | 2.49 | . 11 | 7.03 | 0 |
| 13 | 2.71 | . 09 | 7.54 | 0 |
| 14 | 3.09 | . 06 | 8.03 | 0 |
| 15 | 3.59 | . 03 | 8.7 | 0 |
| 16 | 3.82 | -. 01 | 8.97 | 0 |
| 17 | 4.26 | -. 07 | 9.44 | 0 |
| 18 | 4.8 | -. 08 | 9.88 | 0 |
| 19 | 5.31 | -. 13 | 10.29 | 0 |
| 20 | 5.95 | -. 2 | 10.73 | 0 |
| - | - |  | - | - |
| 21 | 6.58 | -. 25 | 10.99 | 2.23 |
| 22 | 7.14 | -. 26 | 11.12 | 3.38 |
| 23 | 7.86 | -. 32 | 11.09 | 4.5 |
| 24 | 8.35 | -. 42 | 11.13 | 5.66 |
| 25 | 8.97 | -. 65 | 11.09 | 6.57 |
| 26 | 9.51 | -. 87 | 12.11 | 7.67 |
| 27 | 11.51 | -1.4 | 13.09 | 10.29 |
| EXPERIMENTAL PARAMETERS |  |  |  |  |
| $d=$ |  |  | 27871 | 1.017 |
| Ja |  |  | . 79 | $=1272$ |
| F $=$ |  |  | 165 |  |
| EXPERIMENTAL RESULTS : ${ }^{p}$ |  |  |  |  |
|  | $t_{c}=5.5$$U=650$ |  | $22.01$ | $s=64$ |
| $\mathrm{R}_{0}$ |  |  | 34227 |  |
|  | $\mathrm{Pe}_{0}=21510$ |  | $\mathrm{Nu}_{c}=155$ |  |
| $z_{d}=6.58$ | $z_{c}=11.51$ |  | $\mathrm{Fo}_{\mathrm{c}}=3.05 \mathrm{E}-05$ |  |


| TES | O: 1 | FILM NO : 57-1/4 |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FR | TIME | VOLUME | AREA | RADIUS |
| 1 | . 58 | 36.8 | 62.58 | 2.06 |
| 2 | 1.16 | 25.35 | 44.92 | 1.82 |
| 3 | 1.75 | 18.13 | 34.16 | 1.63 |
| 4 | 2.33 | 12.86 | 27.13 | 1.45 |
| 5 | 2.91 | 8.53 | 20.99 | 1.27 |
| 6 | 3.49 | 4.72 | 14.17 | 1.04 |
| 7 | 4.07 | . 56 | 4.02 | . 51 |
| 8 | 4.66 | . 03 | . 5 | . 18 |
| FR | CY | CX | LE | LP |
| 1 | 6.18 | -. 02 | 9.36 | 1.7 |
| 2 | 7 | . 03 | 9.23 | 3.44 |
| 3 | 7.49 | . 08 | 9.28 | 5.57 |
| 4 | 7.8 | . 14 | 9.22 | 6.45 |
| 5 | 8.12 | . 18 | 9.29 | 7.17 |
| 6 | 8.25 | . 12 | 9.24 | 7.51 |
| 7 | 8.7 | -. 04 | 9.51 | 8.33 |
| 8 | 9.29 | . 33 | 9.49 | 9.05 |
| EXPERIMENTAL PARAMETERS : |  |  |  |  |
| $d=1$ |  | $\dot{m}_{s}=1$ | $V_{s}=27871$ | $P=1.017$ |
| $\mathrm{J}_{\mathrm{a}}=$ |  | $\Delta T=20$ | $\Delta t=.58$ | CS: $=1718$ |
| $F=8$ |  | $Z=40$ | $T_{p}=165$ |  |
| EXPERIMENTAL RESULTS: |  |  |  |  |
|  |  | $t_{c}=3.6$ | $t_{t}=-\quad f_{s}=-$ |  |
| $\mathrm{R}_{0}=$ |  | $U=547$ | $h_{c}=25525$ |  |
| $\mathrm{R}_{\mathrm{m}}=$ |  | $\mathrm{Pe}_{0}=13740$ | $N u_{c}=84$ |  |
| $z_{d}=$ | $18$ | $z_{c}=9.29$ | $\mathrm{FO}_{\mathrm{c}}=3.47 \mathrm{E}-05$ |  |



| TEST NO: (9.3).2 FILM NO: 59-1/2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FR | TIME | VOLUME | AREA | RADIUS |
| 1 | .79 | 9.8 | 24.24 | 1.33 |
| 2 | 1.57 | 19.66 | 38.22 | 1.67 |
| 3 | 2.36 | 33.22 | 54.64 | 1.99 |
| 4 \% | 3.14 | 45.5 | 69.67 | 2.21 |
| 5 | 3.93 | 53.48 | 79.49 | 2.34 |
| 6 | 4.72 | 63.89 | 90.67 | 2.48 |
| 7 | 5.5 | 67.84 | 96.29 | 2.53 |
| 8 | 6.29 | 76.6 | 102.98 | 2.63 |
| 9 | 7.07 | 80.28 | 105.37 | 2.68 |
| 10 | 7.86 | 86.37 | 109.9 | 2.74 |
| 11 | 8.65 | 93.12 | 114.29 | 2.81 |
| 12 | 9.43 | 96.87 | 117.09 | 2.85 |
| 13 | 10.22 | 100.12 | 119.17 | 2.88 |
| 14 | 11 | 106.51 | 123.46 | 2.94 |
| 15 | 11.79 | 106.04 | 123.86 | 2.94 |
| 16 | 12.58 | 109.93 | 127.79 | 2.97 |
| 17 | 13.36 | 109.53 | 129.22 | 2.97 |
| 18 | 14.15 | 113.44 | 134.07 | 3 |
| 19 | 14.93 | 115.6 | 135.49 | 3.02 |
|  | - | - - | - - | - - |
| 20 | 15.72 | 92.42 | 112.13 | 2.8 |
| 21 | 16.51 | 75.93 | 96.27 | 2.63 |
| 22 | 17.29 | 61.64 | 79.54 | 2.45 |
| 23 | 18.08 | 48.86 | 66.81 | 227 |
| 24 | 18.86 | 39.98 | 57.88 | 2.12 |
| 25 | 19.65 | 29.51 | 48.7 | 1.92 |
| 26 | 20.44 | 18.08 | 37.47 | 1.63 |
| 27 | 21.22 | 4.4 | 15.94 | 1.02 |
| 28 | 22.01 | . 67 | 4.21 | . 54 |
| FR | CY | CX | LE | LP |
| 1 | 1.09 | -. 15 | 2.46 | 0 |
| 2 | 1.17 | -. 18 | 2.96 | 0 |
| 3 | 1.34 | -. 14 | 3.85 | 0 |
| 4 | 1.62 | -. 17 | 4.59 | 0 |
| 5 | 1.8 | -. 2 | 5.39 | 0 |
| 6 | 2.04 | -. 16 | 5.96 | 0 |
| 7 | 2.34 | -. 22 | 6.48 | 0 |
| 8 | 2.5 | -. 2 | 6.81 | 0 |
| 9 | 2.74 | -. 25 | 7.12 | 0 |
| 10 | 3 | -. 21 | 7.48 | 0 |
|  |  |  |  | CONT. |




| TES | O: |  | M NO | 61-1/6 |
| :---: | :---: | :---: | :---: | :---: |
| FR | TIME | VOLUME | AREA | RADIUS |
| 1. | . 46 | 18.75 | 35.78 | 1.65 |
| 2 | . 92 | 13.8 | 29.07 | 1.49 |
| 3 | 1.38 | 8.06 | 20.38 | 1.24 |
| 4 | 1.84 | 4.2 | 13.7 | 1 |
| 5 | 2.3 | 1.24 | 7.2 | . 67 |
| FR | CY | CX | LE | $L P$ |
| 1 | 5.62 | . 03 | 7.79 | 3.28 |
| 2 | 5.94 | . 03 | 7.85 | 4.44 |
| 3 | 6.35 | 0 | 7.83 | 5.24 |
| 4 | 6.82 | . 01 | 7.8 | 6.18 |
| 5 | 6.96 | 0 | 7.62 | 6.61 |
| EXP | MENTA | AMETERS |  |  |
| $d=$ |  | . 42 | 39576 | $P=1.003$ |
| Ja |  |  | . 46 | $C S=2177$ |
| $F=$ |  |  | 165 |  |
| EXPERIMENTAL RESULTS: ${ }^{p}$ |  |  |  |  |
| $\begin{array}{llll} t_{g}=- & t_{c}=2.1 & t_{t}=- & t_{s}=- \\ R_{0}=1.69 & U=640 & h_{c}=29127 & \end{array}$ |  |  |  |  |
|  |  |  |  |  |
| $R_{m}=1.65$ |  | $P_{\theta_{0}}=13270$ | $\mathrm{Nu}_{\mathrm{c}}=82$ |  |
| $z_{d}^{m}=5.62$ |  | . 96 | $\mathrm{Fo}_{\mathrm{c}}=3 \mathrm{E}-05$ |  |


| TEST | NO: (10.1).2 |  | FILM NO : 65-1/7 |  |
| :---: | :---: | :---: | :---: | :---: |
| FR | TIME | VOLUME | AREA | RADIUS |
| 1 | 1.99 | 76.09 | 91.94 | 2.63 |
| 2 | 3.98 | 68.65 | 85.23 | 2.54 |
| 3 | 5.97 | 51.63 | 69.97 | 2.31 |
| $4 \ldots$ | 7.96 | 43.51 | 63.58 | 2.18 |
| 5 | 9.95 | 39.66 | 59.86 | 2.12 |
| 6 | 11.93 | 36.78 | 57.23 | 2.06 |
| 7 | 13.92 | 33.21 | 55.31 | 1.99 |
| 8 | 15.91 | 28.87 | 54.38 | 1.9 |
| 9 | 17.9 | 12 | 37.4 | 1.42 |
| 10 | 19.89 | 10.37 | 34.11 | 1.35 |
| 11 | 21.88 | 4.78 | 15.11 | 1.04 |
| 12 | 23.87 | 4.7 | 14.81 | 1.04 |
| 13 | 25.86 | 2.16 | 8.51 | . 8 |
| 14 | 27.85 | 1.53 | 6.89 | . 71 |
| 15 | 29.84 | . 77 | 4.64 | . 57 |
| 16 | 31.82 | . 34 | 2.68 | . 43 |
| 17 | 33.81 | . 2 | 1.94 | . 36 |
| FR | CY | CX | LE | LP |
| 1 | 4.82 | . 13 | 8.87 | . 96 |
| 2 | 5.54 | . 15 | 9.05 | 2.56 |
| 3 | 6.27 | . 09 | 9.12 | 3.84 |
| 4 | 7.07 | . 09 | 9.3 | 5.33 |
| 5 | 7.6 | . 12 | 9.72 | 6.06 |
| 6 | 7.99 | . 14 | 9.97 | 6.6 |
| 7 | 8.3 | . 14 | 9.99 | 7.15 |
| 8 | 8.58 | . 16 | 9.99 | 7.65 |
| 9 | 9.15 | . 16 | 10.23 | 8.03 |
| 10 | 9.48 | -. 02 | 10.61 | 8.94 |
| 11 | 9.97 | -. 15 | 10.9 | 9.09 |
| 12 | 10.37 | -. 08 | 11.08 | 9.36 |
| 13 | 10.94 | -. 02 | 11.7 | 10.21 |
| 14 | 11.44 | -. 06 | 12.08 | 10.74 |
| 15 | 11.83 | -. 16 | 12.35 | 11.35 |
| 16 | 12.43 | -. 11 | 12.77 | 12.1 |
| 17 | 12.96 | -. 08 | 13.2 | 12.73 |
| - - |  | - - | - | - - - |
| - |  |  |  | CONT.- |

## EXPERIMENTAL PARAMETERS :

| $\dot{\alpha}^{2}=1$ | $\dot{m}_{s}=.45$ | $V_{s}=6643$ | $P=2.011$ |
| :--- | :--- | :--- | :--- |
| $J_{a}=15$ | $\Delta T=9.3$ | $\Delta t=1.99$ | $C S=503$ |
| $F=17$ | $Z=40$ | $T_{P}=165$ |  |

EXPERIMENTAL RESULTS :

| $t_{g}=-$ | $t_{c}=29.3$ | $t_{t}=-$ | $f_{s}=-$ |
| :--- | :--- | :--- | :--- |
| $R_{0}=2.63$ | $U=258$ | $h_{c}=16913$ |  |
| $R_{m}=2.63$ | $P e_{o}=7980$ | $N u_{c}=68$ |  |
| $Z_{d}=4.82$ | $Z_{c}=12.96$ | $F o_{c}=1.8 E-04$ |  |





## EXPERIMENTAL PARAMETERS :

| $d=1$ | $\dot{m}_{s}=.82$ | $V_{s}=12105$ | $P=2.011$ |
| :--- | :--- | :--- | :--- |
| $J a=15$ | $\Delta T=9.3$ | $\Delta t=1.74$ | $C S=575$ |
| $F=16$ | $Z=40$ | $T_{p}=165$ |  |

EXPERIMENLAL RESULTS:
$\mathrm{t}_{\mathrm{g}}=10.44$
$t_{c}=18$
$t_{t}=28.44$
$h_{c}=26177$
$f_{s}=115$
$R_{o}=2.33$
$R_{m}=2.38$
$Z_{d}=5.69$
$U=443$
$\mathrm{Nu}_{\mathrm{c}}=94$
$\mathrm{Pe}_{\mathrm{o}}=12140$
$\mathrm{FO}_{\mathrm{c}}^{\mathrm{c}}=1.409 \mathrm{E}-04$




| FR | CY | CX | LE | LP |
| :---: | :---: | :---: | :---: | :---: |
| 17 | 3.92 | . 1 | 6.36 | 2.11 |
| 18 | 4.33 | . 1 | 6.72 | 2.72 |
| 19 | 4.63 | . 06 | 6.72 | 3.3 |
| 20 | 4.76 | . 06 | 6.16 | 3.64 |
| 21 | 5.9 | . 32 | 6.27 | 5.54 |
| EXPERIMENTAL PARAMETERS |  |  |  |  |
| $d=$ |  |  | 12105 | $P=2.011$ |
| Ja |  | 7.6 | . 74 | $C S=1351$ |
| $\mathrm{F}=$ |  |  | 165 |  |
| EXPERIMENTAL RESULTS: |  |  |  |  |
| $t_{g}=11.1 \quad t_{c}=4.6 \quad t_{t}=15.7 \quad f_{s}=97$ |  |  |  |  |
| $R_{0}=1.69^{-} \quad U=420 \quad h_{c}=22474$ |  |  |  |  |
| $R_{m}=1.83 \quad \mathrm{Pe}=8500 \quad N u_{0}=60$ |  |  |  |  |
| $z_{d}=3.25 \quad z_{0}=5.9 \quad F_{0}=6.72 \mathrm{E}$ |  |  | 6.72 E |  |


| TEST NO: 112.1$) .3$ FILM NO: 66-2/3 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FR | TIME | VOLUME | AREA | RADIUS |
| 1 | 2.13 | 1.35 | 6.93 | . 69 |
| 2 | 4.26 | 24.37 | 46.4 | 1.8 |
| 3 | 6.39 | 63.66 | 93.13 | 2.48 |
| 4 | 8.52 | 79.62 | 111.99 | 2.07 |
| 5 | 10.65 | 95.13 | 136.65 | 2.83 |
| 6 | 12.78 | 116.8 | 144.54 | 3.03 |
| 7 | 14.91 | 132.03 | 156.71 | 3.16 |
| 8 | 17.04 | 136.16 | 157.17 | 3.19 |
| 9 | 19.17 | 137.18 | 159.67 | 3.2 |
| 10 | 21.3 | 15E. 81 | 177.3 | 3.35 |
| 11 | 23.43 | 162.03 | 183.24 | 3.38 |
| 12 | 25.56 | 171.3 | 186.55 | 3.45 |
| 13 | 27.69 | - $\overline{48}$ | - 53.7 | $\overline{320}$ |
| 14 | 29.82 | 119.68 | 131.59 | 3.06 |
| 15 | 31.95 | 104.57 | 115.75 | 2.92 |
| 16 | 34.08 | 97.31 | 112.94 | 2.85 |
| 17 | 36.21 | 86.07 | 98.69 | 2.74 |
| 18 | 38.34 | 63.83 | 81.27 | 2.48 |
| 19 | 40.47 | 49.17 | 72.82 | 2.27 |
| 20 | 42.6 | 33.38 | 57.37 | 2 |
| 21 | 44.73 | 26.06 | 52.01 | 1.84 |
| 22 | 46.86 | 15.55 | 41.97 | 1.55 |
| 23 | 48.99 | 5.47 | 19.16 | 1.09 |
| 24 | 51.12 | 4.27 | 13.92 | 1.01 |
| 25 | 53.25 | 2.28 | 9.02 | . 82 |
| 26 | 55.38 | 1.17 | 6.24 | . 65 |
| 27 | 57.51 | . 6 | 4 | . 52 |
| 28 | 59.64 | . 26 | 2.64 | . 39 |
| $\overline{F R}-\overline{C Y}-\overline{C X}-\overline{L E}-\overline{L P}-\overline{-}-\overline{-}$ |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| $\begin{array}{lllll}2 & 1.08 & -.03 & 2.67 & 0\end{array}$ |  |  |  |  |
| $\begin{array}{lllll}3 & 1.6 & -.1 & 5.19 & 0\end{array}$ |  |  |  |  |
| $\begin{array}{lllll}4 & 2.11 & -.12 & 7.05 & 0\end{array}$ |  |  |  |  |
| $\begin{array}{lllll}5 & 2.67 & -.26 & 8.29 & 0\end{array}$ |  |  |  |  |
| $\begin{array}{lllll}6 & 3.35 & -.24 & 9.37 & 0\end{array}$ |  |  |  |  |
| $\begin{array}{lllll}7 & 4.21 & -.19 & 10.73 & 0\end{array}$ |  |  |  |  |
| $8 \quad 5.17$ - 6 - 20 |  | -. 2 | 11.88 | 0 |
| $\begin{array}{lllll}9 & 6.2 & -.21 & 12.78 & 0\end{array}$ |  |  |  |  |
| 10 | 7.04 | $-.2$ | 13.81 | 0 |
|  |  |  |  | CONT.- |




| FR | CY | CX | LE | LP |
| :---: | :---: | :---: | :---: | :---: |
| 16 | 8.68 | . 16 | 10.77 | 6.89 |
| 17 | 9.11 | . 19 | 10.77 | 7.76 |
| 18 | 9.4 | . 24 | 10.65 | 8.4 |
| 19 | 9.69 | . 25 | 10.63 | 9.05 |
| 20 | 9.95 | . 47 | 10.37 | 9.56 |
| 21 | 10.15 | . 8 | 10.34 | 9.78 |
| 22 | 10.36 | 1.55 | 10.56 | 10.17 |
| EXPERIMENTAL PARAMETERS |  |  |  |  |
| $d=$ |  | . 39 | 20520 | $P=2.011$ |
| Ja |  | 8.5 | 1.17 | $C S=857$ |
| $\mathrm{F}=$ |  |  | 165 |  |
| EXPERIMENTAL RESULTS: |  |  |  |  |
| $t_{g}=15.17$ |  | $t_{c}=9.5$ | 24.67 | $f_{s}=71$ |
| $R_{0}=2.41$ |  | $U=447$ | $h_{c}=22186$ |  |
| $\mathrm{R}_{\mathrm{m}}=2.55$ |  | $P_{e_{0}}=12750$ | $N u_{c}=85$ |  |
| $z_{d}=6.08$ |  | $z_{c}=10.36$ | $\mathrm{Fo}_{\mathrm{c}}=6.91 \mathrm{E}-05$ |  |


| TEST NO : (12.3).3 |  |  | FILM NO : 67-2/5 |  |
| :---: | :---: | :---: | :---: | :---: |
| FR | TIME | VOLUME | AREA | RADIUS |
| 1 | . 77 | 17.87 | 36.42 | 1.62 |
| 2 | 1.54 | 22.87 | 43.31 | 1.76 |
| 3 | 2.32 | 28.45 | 50.02 | 1.89 |
| 4 | 3.09 | 32.42 | 55.5 | 1.98 |
| 5 | 3.86 | 35.12 | 59 | 2.03 |
| 6 | 4.63 | 37.05 | 60.98 | 2.07 |
| 7 | 5.4 | 40.24 | 64 | 2.13 |
| 8 | 6.18 | 40.9 | 64.67 | 2.14 |
| 9 | 6.95 | 42.25 | 66.11 | 2.16 |
| 10 | 7.72 | 40.85 | 64.87 | 2.14 |
| - |  |  |  |  |
| 11 | 8.49 | 35.8 | 57.73 | 2.04 |
| 12 | 9.26 | 29.45 | 50.07 | 1.92 |
| 13 | 10.04 | 21.72 | 40.51 | 1.73 |
| 14 | 10.81 | 16.99 | 34.01 | 1.59 |
| 15 | 11.58 | 10.7 | 24.7 | 1.37 |
| 16 | 12.35 | 4.84 | 17.74 | 1.05 |
| 17 | 13.12 | . 41 | 2.82 | . 46 |
| FR | CY | CX | LE | LP |
| 1 | 1.4 | -. 16 | 3.87 | 0 |
| 2 | 1.55 | -. 2 | 4.23 | 0 |
| 3 | 1.68 | -. 24 | 4.61 | 0 |
| 4 | 1.9 | -. 23 | 5.09 | 0 |
| 5 | 2.13 | -. 22 | 5.45 | 0 |
| 6 | 2.42 | -. 25 | 5.78 | 0 |
| 7 | 2.64 | -. 24 | 6.22 | 0 |
| 8 | 2.95 | -. 24 | 6.57 | 0 |
| 9 | 3.37 | -. 2 | 6.98 | 0 |
| 10 | 3.89 | -. 19 | 7.45 | 0 |
| -11 | - | - | - | - - |
|  | 4.32 | -. 16 | 7.54 | 1.11 |
| 12 | 4.74 | -. 2 | 7.64 | 1.68 |
| 13 | 5.22 | -. 13 | 7.7 | 2.75 |
| 14 | 5.55 | -. 12 | 7.65 | 3.35 |
| 15 | 5.95 | -. 17 | 7.54 | 4.55 |
| 16 | 6.26 | -. 23 | 7.45 | 5.58 |
| 17 | 7.41 | -. 3 | 7.82 | 6.93 |
|  | - | - - | - - - | - - - |

## EXPERIMENTAL PARAMETERS :

| $d=1$ | $\dot{m}_{s}=1.39$ | $V_{s}=20520$ | $P=2.011$ |
| :--- | :--- | :--- | :--- |
| $J a=45$ | $\Delta T=27.6$ | $\Delta t=.77$ | $C S=1295$ |
| $F=17$ | $Z=40$ | $T_{p}=165$ |  |
| EXPERIMENTAL RESULTS $:$ |  |  |  |
| $t_{t}=8.49$ | $t_{c}=4.8$ | $t_{t}=13.29$ | $f_{s}=130$ |
| $R_{0}=2.04$ | $U=463$ | $h_{c}=27144$ |  |
| $R_{m}=2.16$ | $P e_{0}=11310$ | $N u_{c}=89$ |  |
| $Z_{d}=4.32$ | $Z_{c}=7.41$ | $F_{c}=4.82 E-05$ |  |



## EXPERIMENTAL PARAMETERS :

| $d=1$ | $\dot{m}_{s}=1.39$ | $V_{s}=20520$ | $P=2.011$ |
| :--- | :--- | :--- | :--- |
| $J_{a}=60$ | $\Delta T=36.6$ | $\Delta t=.56$ | $C S=1800$ |
| $F=18$ | $Z=40$ | $T_{p}=165$ |  |

EXPERIMENTAL RESULTS :
$t_{g}=6.68$
$R_{o}=1.78$
$R_{m}=1.91$
$Z_{d}=4.16$
$t_{c}=3$
$U=380$
$t_{t}=9.68$
$h_{c}=25544$
$f_{s}=164$
$\mathrm{Pe}_{0}=8200$
$Z_{c}=6.14$
$\mathrm{Nu}_{\mathrm{c}}=72$
$\mathrm{Fo}_{\mathrm{c}}=3.91 \mathrm{E}-05$

APPENDIX 5.

## Bubble Shape Factor

The bubble shape factor, as defined in section 5.4 is

$$
\begin{equation*}
\psi=\frac{\text { Measured bubble surface area }}{4 \cdot \pi \cdot R^{2}} \tag{A1.1}
\end{equation*}
$$

where $R$ is the radius of a sphere of volume equal to the measured bubble volume. This means that $\psi$ will have a value greater than or equal to unity, the higher the value is, giving an indication of the level of bubble distortion.

Some examples of bubble shape factor are given as a function of radius ratio ( $\beta$ ) in Figs. A 5.1 and $A 5.2$.

The values of $\psi(\beta)$ generally oscillated with a value always greater than unity, as shown, for test 11.2 , in Fig. A 5.1. In a few cases, a near spherical bubble detached from the orifice and while condensing, became hemispherical then spherical-cap shaped, and finally collapsed as spherical. In such cases as illustrated, for test 4.2, in Fig. A 5.1, the shape factor increased until condensation was about $87 \%$ complete and then decreased towards unity in the later stages of collapse. In two or three cases, the shape factor continued to increase throughout collapse, indicating that, in these cases, bubble distortion continued throughout the entire collapse process. An example of this type of collapse occurs in test 6.3 as shown in Fig. A 5.2. In a few cases, the bubble, which was distorted at detachment, became near spherical, before again becoming slightly distorted as collapse continued. An example of this type of collapse occurs in test 11.1 as shown in Fig. A 5.2.



Fig.A5.1 Bubble shape factor as function of radius ratio


Fig A5.2 Bubble shape factor as function of radius ratio

A simple average of $\psi$ values calculated for all the data points (N) in any test is given by

$$
\begin{equation*}
\psi_{\mathrm{m}}=\frac{\sum_{1}^{N} \psi}{\mathrm{~N}} \tag{A5.2}
\end{equation*}
$$

The values obtained for each experimental condition are listed in table A 5.1. A mean average value for all the data was calculated to be 1.134.

In section 5.4 the effect of bubble distortion has been analysed and equation (5.55) was derived using the simplified bubble collapse equation.

Using a simulation program, TUTSIM, the values of $\psi$ corresponding to four different functions have been plotted in Figs. A 5.3 and A 5.4 for test conditions corresponding to those shown in Figs. A 5.1 and $A 5.2$. In each case the program also gives a plot of equation (5.55) for the corresponding $\psi$ function. In addition, curves for equation (5.48) corresponding to $\psi=1$ are also plotted.

In all cases the bubbles collapse faster when $\psi$ has values greater than unity.

Information about TUTSIM is given in Appendix 3. The block diagram and the listing of the model for the bubble shape factor function being $\psi=1+a(1-\beta)^{b}$, are given in Fig. A 5.5 in the attached portfolio and in table A 5.2, respectively. For $\psi=a+b \cos (2 \pi \beta)$, the block diagram and the listing are presented in Fig. A 5.6 in the attached portfolio and in table A 5.3.

| Test No. | $\psi_{m}$ | Test No. | $\Psi_{\text {m }}$ |
| :---: | :---: | :---: | :---: |
| 1.1 | 1.312 | 7.1 | 1.127 |
| 1.2 | 1.165 | 7.2 | 1.121 |
| 2.1 | 1.187 | 7.3 | 1.139 |
| 2.2 | 1.142 | 8.1 | 1.136 |
| 2.3 | 1.070 | 8.2 | 1.135 |
| 3.1 | 1.146 | 8.3 | 1.110 |
| 3.2 | 1.137 | 8.4 | 1.080 |
| 3.3 | 1.120 | 9.2 | 1.103 |
| 3.4 | 1.135 | 9.3 | 1.097 |
|  |  | 9.4 | 1.065 |
|  |  | 9.5 | 1.080 |
| 4.1 | 1.133 | 10.1 | 1.138 |
| 4.2 | 1.100 | 10.2 | 1.118 |
| 5.1 | 1.143 | 11.1 | 1.139 |
| 5.2 | 1.190 | 11.2 | 1.150 |
| 5.3 | 1.120 | 11.3 | 1.177 |
| 6.1 | 1.162 | 12.1 | 1.143 |
| 6.2 | 1.168 | 12.2 | 1.151 |
| 6.3 | 1.111 | 12.3 | 1.101 |
| 6.4 | 1.091 | 12.4 | 1.144 |

Table A 5.1 Average values of shape factors for each experimental condition


Fig.A5. 3 Shape factor and Fourier number versus radius ratio for different values of shape factor function


TIMING
$0.10000 E-01$ O. $10000 E+01$
DUTFUTELOCKS AND RANGES

| $\mathrm{X1:}$ | 21 | $0.00000 \mathrm{E}+00$ | $0.10000 \mathrm{E}+01$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{Y} 1:$ | 27 | $0.00000 \mathrm{E}+00$ | $0.25000 \mathrm{E}-03$ |
| $\mathrm{Y}:$ | 23 | $0.00000 \mathrm{E}+00$ | $0.20000 \mathrm{E}+01$ |

MODEL


Table A5.2 The listing of the model for the bubble collapse equation (5.55) .with

$$
\psi=1+a(1-\beta)^{b}
$$

## timine

$$
0.10000 \mathrm{E}-01 \quad 0.10000 \mathrm{E}+01
$$

## dutfutblocks and finiges

| $\mathrm{Y} 1:$ | 21 | $0.00000 \mathrm{E}+00$ | $0.10000 \mathrm{E}+01$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{Y} 1:$ | 27 | $0.00000 \mathrm{E}+00$ | $0.25000 \mathrm{E}-03$ |
| $Y \mathrm{Y}:$ | 23 | $0.00000 \mathrm{E}+00$ | $0.20000 \mathrm{E}+01$ |

MODEL

|  | 2 | $\begin{aligned} & \text { TIM } \\ & \text { SUM } \end{aligned}$ | 1 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $0.62832 E+01$ | 3 | CON |  |  |  |
|  | 4 | MUL | 21 | 3 |  |
|  | 5 | cos | 4 |  |  |
| $0.250005+00$ | $\epsilon$ | Can |  |  |  |
| $0.10000 E+01$ | 7 | COH |  |  |  |
|  | 8 | MUL | 5 | 6 |  |
| $0.00000 E+00$ | \% | CON |  |  |  |
|  | 10 | Sum | 7 | 6 | 8 |
|  | 11 | DIU | 22 | 23 |  |
| $0.00000 E+00$ | 12 | INT | 11 |  |  |
|  | 13 | DI! | 12 | 20 |  |
| $0.10000 E+05$ | 14 | COH |  |  |  |
|  | 15 | SOT | 14 |  |  |
| $0.15000 E+02$ | 16 | cond |  |  |  |
|  | 17 | TIM |  |  |  |
|  | 18 | TIM |  |  |  |
| $0.21220 E+01$ | 19 | CON |  |  |  |
|  | 20 | MUL | 15 | 16 | 19 |
|  | 21 | SUM | 7 | -1 |  |
|  | 22 | SOT | 21 |  |  |
| 0.10000E+01 | 23 | GAI | 10 |  |  |
| $0.28800 E+00$ | 24 | CON |  |  |  |
|  | 25 | DIV | 24 | 14 |  |
|  | 26 | MUL | 1 | 25 |  |
|  | 27 | Sum | 13 | 26 |  |

Table A5.3 The listing of the model for calculation of the bubble collapse equation(5.55) with $\psi=a+b \operatorname{Cos}(2 \pi \beta)$

## APPENDIX 6

## Collapse of spherical-cap bubbles

When a spherical-cap bubble rises, with a constant velocity $U$, through a subcooled liquid, heat transfer will take place from the top of the bubble and in addition some heat transfer may take place by conduction into the wake.

If we consider that the spherical-cap bubble shown in Fig. A 6.1 has a constant radius, $R_{o}$ and an angle $\gamma$, which decreases as collapse continues, we can, by assuming simplified potential flow over the spherical surface, determine the heat transfer from the top surface of the bubble. If we neglect bubble radial velocity (i.e. $\dot{R}=0$ ), the velocities $u_{r}$ and $u_{\theta}$ can be written from equations (5.4) and (5.5) as

$$
\begin{align*}
& u_{r} \cong-3 U \frac{y}{R_{0}} \cos \theta  \tag{A6.1}\\
& u_{\theta} \cong \frac{3}{2} U \cdot \sin \theta \tag{A.6.2}
\end{align*}
$$

For these conditions, Ruckenstein [8] gave the heat flux through the liquid boundary layer at angle $\theta$ as

$$
\begin{equation*}
q_{\theta}=c_{p} \cdot p_{l} \cdot \Delta T \cdot \frac{\frac{3}{2} \cdot U \cdot R_{0} \alpha \cdot \sin ^{2} \theta}{\sqrt{\frac{3}{2} \cdot \pi \cdot \alpha U \cdot R_{0}{ }^{3}\left(\frac{2}{3}-\cos \theta+\frac{\cos ^{3} \theta}{3}\right)}} \tag{A6.3}
\end{equation*}
$$

Thus, the heat transferred at the upper (front) surface, between $\theta=0$ and $\theta=\gamma$, is given as

$$
110
$$


$R=R_{0}=$ constant

Fig.A6.1 Theoretical model for collapse of sphericalcap bubbles

$$
Q_{\gamma f}=\int_{0}^{Y} 2 \pi R_{o}^{2} \sin \theta \cdot q_{\theta} \cdot d \theta
$$

$=\frac{2 \pi R_{0}{ }^{2} c_{p} \rho_{1} \cdot \Delta T \cdot \frac{3}{2} \cdot U R_{0} \alpha}{\sqrt{\frac{3}{2} \pi \cdot \alpha U R_{0}{ }^{3}}} \cdot \int_{0}^{\gamma} \frac{\sin ^{3} \theta}{\left(\frac{2}{3}-\cos \theta+\frac{1}{3} \cos ^{3} \theta\right)^{1 / 2}} d \theta$
$=3 \sqrt{\frac{2 \pi}{3}} \cdot \frac{R_{0}{ }^{3} c_{p} \cdot \rho_{1} \cdot \Delta T \cdot U \cdot \alpha}{\sqrt{\alpha \cdot U \cdot R_{0}{ }^{3}}} \int_{0}^{\gamma} \frac{\sin ^{3} \theta}{\left(\frac{2}{3}-\cos \theta+\frac{1}{3} \cos ^{3} \theta\right)^{\frac{3}{2}}} d \theta$

Let $y=\frac{2}{3}-\cos \theta+\frac{1}{3} \cos ^{3} \theta$
$\therefore d y=\left[\sin \theta-\cos ^{2} \theta \sin \theta\right] d \theta=\sin ^{3} \theta d \theta$
$\therefore \int_{0}^{\gamma} \frac{\sin ^{3} \theta}{\left(\frac{2}{3}-\cos \theta+\frac{1}{3} \cos ^{3} \theta\right)^{\frac{1}{2}}} d \theta=\int_{0}^{y_{\gamma}} \frac{d y}{y^{\frac{y}{2}}}=\left.2 y^{\frac{2}{2}}\right|_{0} ^{y_{\gamma}}$
$=\left.2\left(\frac{2}{3}-\cos \theta+\frac{1}{3} \cos ^{3} \theta\right)^{\frac{1}{2}}\right|_{0} ^{\gamma}=2\left(\frac{2}{3}-\cos \gamma+\frac{1}{3} \cos ^{3} \gamma\right)^{\frac{2}{2}}$
(A 6.5)
Therefore,
$Q_{Y f}=3 \sqrt{\frac{2 \pi}{3}} \cdot \frac{R_{0}{ }^{3} c_{p} \cdot \rho_{1} \cdot \Delta T \cdot U \cdot \alpha}{\sqrt{\alpha \cdot U \cdot R_{O}{ }^{3}}} \cdot 2 \cdot\left[\frac{2}{3}-\cos \gamma+\frac{1}{3} \cos ^{3} \gamma\right]^{\frac{1}{2}}$
$=3 \sqrt{\frac{2 \pi}{3}} \cdot\left(c_{p} \cdot \rho_{1} \cdot \Delta T\right) \cdot\left(U \cdot \alpha R_{0}{ }^{3}\right)^{3 / 2} \cdot 2\left[\frac{2}{3}-\cos \gamma+\frac{1}{3} \cos ^{3} \gamma\right]^{\frac{1}{2}}$

$$
\begin{array}{r}
=3 \sqrt{\frac{2 \pi}{3}} \cdot\left(c_{p} \cdot \rho_{l} \cdot \Delta T\right) \cdot\left(\frac{2 U R_{0}}{\alpha}\right)^{\frac{1}{2}} \cdot \sqrt{2} \cdot \alpha \cdot R_{0}\left[\frac{2}{3}-\cos \gamma+\frac{1}{3} \cos ^{3} \gamma\right]^{\frac{1}{2}} \\
(A 6 \cdot 6)
\end{array}
$$

$$
\begin{equation*}
Q_{\gamma f}=6 \sqrt{\frac{\pi}{3}} \cdot\left(c_{p} \cdot \rho_{l} \cdot \Delta T\right) P e_{o}{ }^{\frac{1}{2}} \cdot \alpha R_{0}\left[\frac{2}{3}-\cos \gamma+\frac{1}{3} \cos ^{3} \gamma\right]^{\frac{1}{2}} \tag{A6.7}
\end{equation*}
$$

In addition to this heat transferred at the top surface of the bubble, heat may also be conducted into the water in the wake, and this can be determined by using a method similar to that of Coppus [50], who employed Danckwert's [51] surface renewal model for mass transfer at the wake of a spherical-cap bubble in the form

$$
\begin{equation*}
k_{m} \sim \sqrt{D . s} \tag{A6.8}
\end{equation*}
$$

where $D$ is the diffusion coefficient and $s$ is the surface renewal rate. Lamont and Scott [53], assuming that mass transfer is controlled by small eddies, obtained a value of average mass transfer coefficient as

$$
\begin{equation*}
k_{m} \sim\left(\frac{D}{v}\right)^{0.5} \cdot(\varepsilon v)^{0.25} \tag{A6.9}
\end{equation*}
$$

where $\varepsilon$ is the rate of energy dissipation by turbulence per unit mass.

Coppus [50] estimated $\varepsilon$ from the total energy dissipation rate of the bubble, $\Gamma$ as follows :

$$
\begin{equation*}
\Gamma=\operatorname{Drag} \cdot \mathrm{U}=\left(\rho_{l} \cdot \mathrm{~g} \cdot \mathrm{~V}_{\mathrm{b}}\right) \cdot \mathrm{U} \tag{A6.10}
\end{equation*}
$$

He assumed that all the energy is dissipated in the closed wake behind the bubble and determined the total energy dissipation rate per unit mass as

$$
\begin{equation*}
E_{T}=\frac{\Gamma}{\rho_{l} V_{W}}=\frac{g V_{D} \cdot U}{V_{W}}=\frac{g U}{V_{W} / V_{b}} \tag{A6.11}
\end{equation*}
$$

where $V_{b}$ is the volume of the bubble and $V_{W}$ is the volume of the wake.

Coppus suggested that the energy dissipated by turbulence was equal to the total energy dissipation rate times a function of the Reynolds number or

$$
\begin{equation*}
\varepsilon=\varepsilon_{T} \cdot f(R e)=\frac{g U}{V_{W} / V_{b}} \cdot f(R e) \tag{A6.12}
\end{equation*}
$$

where for a low value of Reynolds number (laminar wake flow) $f(R e)=0$ and for a high value of Reynolds number (turbulent wake flow) $f(R e)=1$.

Since we have turbulent flow,

$$
\begin{equation*}
\varepsilon=\frac{g U}{V_{W} / V_{b}} \tag{A6.13}
\end{equation*}
$$

Substituting this in equation (A 6.9) and inserting a constant of proportionality of 0.30 (Coppus suggested a value between 0.20 and 0.40 ) gives,

$$
k_{m}=0.30\left(\frac{D}{v}\right)^{0.5} \cdot\left(\frac{g U v}{V_{W} / V_{b}}\right)^{0.25}=0.30\left(\frac{D}{v}\right)^{0.5} \cdot\left(\frac{V_{b}}{V_{W}}\right)^{0.25}(g U v)^{0.25}
$$

$$
0.5 \quad 0.25 \quad 0.25
$$

$\therefore \frac{k_{m} \cdot R}{D}=0.30\left(\frac{V}{D}\right) \quad \cdot\left(\frac{g R}{U^{2}} \cdot \frac{U^{3} R^{3}}{v^{3}}\right) \quad \cdot\left(\frac{V_{b}}{V_{W}}\right)$

$$
=0.30\left(\frac{V}{D}\right)^{0.5} \cdot\left(\frac{g R}{8 U^{2}}\right)^{0.25} \cdot\left(\frac{2 U R}{V}\right)^{0.75} \cdot\left(\frac{V_{b}}{V_{W}}\right)^{0.25}
$$

By using an analogous heat transfer relationship
which substitutes $\frac{h}{\rho_{l}{ }^{c_{p}}}$ for the mass transfer coefficient $\mathrm{k}_{\mathrm{m}}$ and the thermal diffusivity $\alpha$ for the diffusion coefficient $D$, we can write

$$
\frac{\frac{h}{p-c_{p}} \cdot R}{-1}=0.30\left(\frac{v}{\alpha}\right)^{0.5} \cdot\left(\frac{g R}{8 U^{2}}\right)^{0.25} \cdot R e^{0.75} \cdot\left(\frac{V_{b}}{V_{W}}\right)^{0.25}
$$

or


$$
=C_{b} \cdot P e^{0.50} \cdot \mathrm{Re}^{0.25} \cdot\left(\frac{V_{b}}{V_{W}}\right)^{0.25}
$$

where $C_{b}=0.30\left(\frac{\mathrm{gR}}{8 \mathrm{U}^{2}}\right)^{0.25}$
Assuming a closed spherical wake,
$\frac{V_{b}}{V_{W}}=\frac{\pi R^{3}\left[\frac{2}{3}-\cos \gamma+\frac{\cos ^{3} \gamma}{3}\right]}{\frac{4}{3} \pi R^{3}-\pi R^{3}\left[\frac{2}{3}-\cos \gamma+\frac{\cos ^{3} \gamma}{3}\right]}$
$\therefore \frac{V_{b}}{V_{W}}=\frac{\left[\frac{2}{3}-\cos \gamma+\frac{\cos ^{3} \gamma}{3}\right]}{\frac{4}{3}-\left[\frac{2}{3}-\cos \gamma+\frac{\cos ^{3} \gamma}{3}\right]}=\frac{c_{1}}{\frac{4}{3}-c_{1}}$
where $C_{1}=\frac{2}{3}-\cos \gamma+\frac{\cos ^{3} \gamma}{3}$

Hence, equation (A 6.15) can be written as
$\frac{\frac{h}{\rho_{L} c_{p}} \cdot R}{\alpha}=C_{b} P e^{\frac{z}{2}} \cdot R e^{2 / 4} \cdot\left[\frac{C_{1}}{\frac{4}{3}-C_{1}}\right]^{1 / 4}$

Thus, heat transferred to the wake at the rear of the bubble is expressed as
$Q_{\gamma r}=h \cdot A_{r} \cdot \Delta T=h \cdot \Delta T \cdot \pi(R \sin \gamma)^{2}$
where $A_{r}$ is the surface area at the rear of the bubble.
$\therefore Q_{\gamma r}=C_{b} \rho_{L} c_{p} \cdot \pi \cdot \Delta T \cdot R^{2} \cdot \sin ^{2} \gamma \cdot \frac{\alpha}{R} \cdot P e^{1 / 2} \cdot R e^{1 / 4} \cdot\left[\frac{C_{1}}{\frac{4}{3}-C_{1}}\right]^{1 / 4}$

Since $R=R_{0}=$ constant, $P e=P e_{0}, R e \equiv e_{0}$,
$Q_{\gamma r}=\pi C_{b} \cdot\left(c_{p} \rho_{L} \Delta T\right) \cdot R_{0} \cdot \alpha \cdot \operatorname{Pe}_{o}^{\frac{3 / 2}{2}} \cdot \operatorname{Re}_{0}^{1 / 4} \cdot \sin ^{2} \gamma\left[\frac{C_{1}}{\frac{4}{3}-C_{1}}\right]^{1 / 4}$

Combining equations (A 6.7) and (A 6.20) to obtain the total heat transfer from the bubble,

$$
\begin{aligned}
& Q_{\gamma}=Q_{\gamma f}+Q_{\gamma r}=6 \sqrt{\frac{\pi}{3}} \cdot\left(c_{p} \cdot \rho_{L} \cdot \Delta T\right) \cdot P e_{o}^{1 / 2} \cdot \alpha \cdot R_{o} \cdot\left[\frac{2}{3}-\cos \gamma+\frac{\cos ^{3} \gamma}{3}\right]^{\frac{2}{2}}+ \\
& \pi C_{b} \cdot\left(c_{p} \rho_{L} \Delta T\right) \operatorname{Pe}_{o}^{1 / 2} R_{o} \quad \cdot R_{0} \cdot \alpha \cdot \sin ^{2} \gamma \cdot\left[\frac{C_{1}}{\frac{4}{3}-C_{1}}\right]^{1 / 4} \\
& \therefore Q_{\gamma}=\left(c_{p} \rho_{l} \Delta T\right) P e_{o}^{\frac{1}{2}} \cdot R_{o \alpha}\left[6 \sqrt{\frac{\pi}{3}} C_{1}^{\frac{1 / 2}{}}+\pi C_{b} \cdot R e_{o}^{1 / 4} \cdot \sin ^{2} \gamma \cdot \frac{C C_{1}^{1 / 4}}{\left(\frac{4}{3}-C_{1}\right)^{1 / 4}}\right]
\end{aligned}
$$

Using a similar method to that adopted in section 5.2.2, to determine the collapse rate of the bubble, equation (5.25) can now be written as

$$
\begin{aligned}
& Q_{\gamma}=-\rho_{V} \cdot h_{f g} \cdot \frac{d V}{d t}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{C_{1}^{I / 4}}{\left.\left(\frac{4}{3}-C_{1}\right)^{1 / 4}\right]} \\
& \therefore \frac{d V}{d t}=-4 J a \cdot \operatorname{Pe}_{0}^{2 / 2} \cdot \frac{\alpha}{4 R_{0}{ }^{2}}\left[6 \sqrt{\frac{\pi}{3}} R_{0}{ }^{3} C_{1}^{2 / 2}+\pi C_{b} \cdot \operatorname{Re}_{0} \quad 1 / 4 \quad \cdot R_{0}{ }^{3} \cdot \sin ^{2} \gamma \cdot \frac{C_{1}{ }^{1 / 4}}{\left(\frac{4}{3}-C_{1}\right)^{1 / 4}}\right]
\end{aligned}
$$

$\therefore \frac{d V}{d F o}=-4 \mathrm{JaPe}_{0}{ }^{1 / 2}\left[6 \sqrt{\frac{\pi}{3}} \cdot \mathrm{RO}_{0}{ }^{3} \mathrm{C}_{1}{ }^{1 / 2}+\pi \cdot \mathrm{C}_{\mathrm{b}} \cdot \mathrm{Re}_{0}^{1 / 4} \quad \cdot R_{0}{ }^{3} \cdot \sin ^{2} \gamma \cdot \frac{C_{1} 1 / 4}{\left(\frac{4}{3}-C_{1}\right)^{1 / 4}}\right]$
(A 6.21)

Let $Z=\frac{V}{V_{O}}=\frac{V}{\frac{4}{3} \pi R_{O}{ }^{3}}=\frac{\pi R_{0}{ }^{3}\left[\frac{2}{3}-\cos \gamma+\frac{\cos ^{3} \gamma}{3}\right]}{\frac{4}{3} \cdot \pi \cdot R_{0}{ }^{3}}$
$\therefore z=\frac{3}{4}\left[\frac{2}{3}-\cos \gamma+\frac{\cos ^{3} \gamma}{3}\right]=\frac{3}{4} c_{1}$
and $d Z=\frac{1}{\frac{4}{3} \pi R_{0}{ }^{3}} \cdot d V$
Hence, equation (A 6.21) can be written as
$\frac{d Z}{d F o}=-4 \mathrm{JaPe}_{0}{ }^{\frac{1}{2}}\left[6 \sqrt{\frac{\pi}{3} \cdot \frac{C_{1}}{4} \cdot \frac{1 / 2}{3} \cdot \pi}+\pi \mathrm{C}_{\mathrm{b} R \mathrm{Re}_{0}}^{1 / 4} \cdot \frac{1}{\frac{4}{3} \cdot \pi} \cdot \sin ^{2} \gamma \cdot \frac{C_{1}{ }^{1 / 4}}{\left(\frac{4}{3}-C_{1}\right)^{1 / 4}}\right]$
$=-4 \mathrm{JaPe}_{o}{ }^{1 / 2}\left[6 \sqrt{\frac{\pi}{3}} \cdot \frac{\left(\frac{4}{3} z\right)^{1 / 2}}{\frac{4}{3} \cdot \pi}+\mathrm{C}_{\mathrm{b}} \cdot \mathrm{Re}_{0}^{1 / 4} \cdot \frac{1}{\frac{4}{3}} \sin ^{2} \gamma \frac{\left(\frac{4}{3} z\right)^{1 / 4}}{\left(\frac{4}{3}-\frac{4}{3} z\right)^{1 / 4}}\right]$
$=-4 \mathrm{JaPe}_{0}{ }^{\frac{1}{2}}\left[6 \cdot \sqrt{\frac{\pi}{3}} \cdot \sqrt{\frac{3}{4}} \cdot \frac{z^{2 / 2}}{\pi}+\frac{3}{4} \cdot \mathrm{C}_{\mathrm{b}} \cdot \operatorname{Re}_{0}^{1 / 4} \cdot \sin ^{2} \gamma \cdot \frac{z^{8 / 4}}{(1-z)^{3 / 4}}\right]$

Hence,

$$
\frac{d Z}{d F o}=-4 \mathrm{Ja} \cdot \mathrm{Pe}_{0} \mathrm{o}^{\frac{1}{2}} \cdot\left[\frac{3}{\sqrt{\pi}} \cdot z^{\frac{3}{2}}+\left(\frac{3}{4} \mathrm{C}_{\mathrm{b}} R e^{8 / 4}\right) \cdot \sin ^{2} \gamma \cdot \frac{z^{8 / 4}}{(1-z)^{1 / 4}}\right]
$$

(A 6.24)
Integrating gives,

$$
F O=-\frac{1}{4 \cdot J a \cdot P_{0}{ }^{1 / 2}} \cdot \int_{Z_{0}}^{Z} \frac{d Z}{\frac{3}{\sqrt{\pi} \cdot Z^{\frac{1}{2}}+\left(\frac{3}{4} C_{b} \cdot \operatorname{Re}_{0}^{1 / 4}\right) \sin ^{2} \gamma \cdot \frac{Z^{1 / 4}}{(1-Z)^{1 / 4}}}}
$$

where $Z=\frac{3}{4}\left[\frac{2}{3}-\cos \gamma+\frac{\cos ^{3} \gamma}{3}\right]$
From equation (A 6.23),

$$
\begin{align*}
\frac{d Z}{d \gamma} & =\frac{3}{4} \frac{d}{d \gamma}\left(\frac{2}{3}-\cos \gamma+\frac{\cos ^{3} \gamma}{3}\right) \\
\therefore d Z & =\frac{3}{4}\left(\sin \gamma-\sin \gamma \cdot \cos ^{2} \gamma\right) d \gamma=\frac{3}{4} \sin ^{3} \gamma \cdot d \gamma \tag{A6.26}
\end{align*}
$$

Therefore, equation (A 6.25) can be written in terms of $\gamma$ as

$$
F O=-\frac{1}{4 J a P e_{0}^{1 / 2}} \cdot \int_{\pi}^{\gamma} \frac{\frac{3}{4} \cdot \sin ^{3} \gamma \cdot d \gamma}{\frac{3}{\sqrt{\pi}} \cdot \frac{\sqrt{3}}{2} \cdot C_{1}^{1 / 2}+\left(\frac{3}{4} C_{b} \operatorname{Re}_{0}^{1 / 4}\right) \cdot \sin ^{2} \gamma \cdot \frac{\left(\frac{3}{4}\right)^{1 / 4} \cdot C_{1}^{1 / 4}}{\left(1-\frac{3}{4} C_{1}\right)^{1 / 4}}}
$$

$$
\begin{equation*}
\therefore F O=-\frac{3}{16 J a P e_{o}^{1 / 2}} \cdot \int_{\pi}^{\gamma} \frac{\sin ^{3} \gamma d \gamma}{\frac{3}{2} \cdot \frac{\sqrt{3}}{\pi} \cdot C_{1}^{2 / 2}+\left(\frac{3}{4} \cdot C_{b} \cdot \operatorname{Re}_{o}^{1 / 4}\right) \cdot\left(\frac{3}{4}\right)^{1 / 4} \cdot \frac{\sin ^{2} \gamma \cdot C_{1}^{1 / 4}}{\left(1-\frac{3}{4} C_{1}\right)^{1 / 4}}} \tag{A6.27}
\end{equation*}
$$

From the definition of $Z$ in equation ( $A$ 6.22), the dimensionless radius, $\beta$ can be calculated as

$$
\begin{align*}
\beta & =\left(\frac{V}{V_{O}}\right)^{1 / 3}=Z^{1 / 3}  \tag{A6.28}\\
\therefore \beta & =\left[\frac{3}{4}\left[\frac{2}{3}-\cos \gamma+\frac{\cos ^{3} \gamma}{3}\right]\right]^{1 / 3} \tag{A6.29}
\end{align*}
$$

For a given experimental condition the Jakob number, Peclet number, Reynolds number and $C_{b}$ are known. For a given angle, $\gamma$, equation ( $A$ 6.27) gives the value of Fourier number ( $F O$ ) and equation ( $A$ 6.29) determines the value of $\beta$.

A simulation program, TUTSIM was used to calculate and draw $\beta$ versus fo curve according to the above equations. The block diagram and the listing of the model are given in Fig. A 6.2 in the attached portfolio and table A 6.1, respectively. Information about TUTSIM is given in Appendix 3. The comparison of this theoretical analysis with the experimental data is given in Chapter 6.

Equation (A 6.24) can be rewritten as
$\frac{d Z}{d F o}=-4 \mathrm{JaPe}_{0}{ }^{\frac{1}{2}}\left[\frac{3}{\sqrt{\pi}} \cdot Z^{1 / 2}+A \cdot \sin { }^{2} \gamma \cdot\left(\frac{Z}{1-Z}\right)^{1 / 4}\right]$
where $A=\frac{3}{4} C_{b} \operatorname{Re}_{0}^{1 / 4 .} \quad$.

When heat transfer at the rear of the bubble is neglected, $A=0$ and equation (A 6.30) reduces to

## TIMING <br> $0.50000 \mathrm{E}-01 \quad 0.31416 \mathrm{E}+01$

## OUTFUTELOCKS AND RANGES

| $X_{1}:$ | $2 S$ | $0.00000 E+00$ | $0.10000 E-03$ |
| :--- | :--- | :--- | :--- |
| $Y 1:$ | 24 | $0.00000 E+00$ | $0.10000 E+01$ |

MODEL


Table A6.1 The listing of the model for the collapse of spherical-cap bubbles
$\frac{d z}{d F O}=-\frac{12}{\sqrt{\pi}} \mathrm{JaPe}_{0}{ }^{2 / 2} z^{1 / 2}$

Hence,
$\frac{d Z}{Z^{2 / 2}}=-\frac{12}{\sqrt{\pi}} \cdot \mathrm{Ja} \mathrm{Pe}_{\mathrm{o}}^{1 / 2} \mathrm{dFo}$
Integrating gives,
$2 Z^{\frac{1}{2}}=-\frac{12}{\sqrt{\pi}} \cdot \mathrm{JaPe}_{0}^{\frac{y}{2}} \mathrm{FO}+\mathrm{c}$
at $\gamma=\pi, Z=\frac{3}{4}\left(\frac{2}{3}+1-\frac{1}{3}\right)=1$ and $F O=0$
$\therefore c=2$

Therefore,

$$
\begin{aligned}
& 2 Z^{\frac{1}{2}}=-\frac{12}{\sqrt{\pi}} \cdot \mathrm{JaPe}_{0} \mathrm{~K} / \mathrm{FO}+2 \\
\therefore & z=\left(1-\frac{6}{\sqrt{\pi}} \mathrm{Ja} \mathrm{Pe}_{0} \mathrm{~F} / 2\right)^{2}
\end{aligned}
$$

or

$$
\begin{equation*}
B=\left(1-\frac{6}{\sqrt{\pi}} \text { Ja Pe } o_{o}^{\frac{y}{2}} \mathrm{FO}\right)^{2 / 3} \tag{A6.32}
\end{equation*}
$$

which is equation (2.29) given by Isenberget al. [23] for the collapse of spherical bubbles rising freely with negligible bubble radial velocity.

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[^0]:    TEST NO :
    $: 17.21 .3$
    FILM NO: 54-1/3

