## UNIVERSITY OF STRATHCLYDE

# Length-structured approach to Fisheries stock assessment 

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If I have seen further, it is by standing on the shoulders of giants


#### Abstract

Modern fishing fleets have the capacity to over-exploit fish stocks. Inaccurate assessments could overestimate the stock size and as a result Total Allowable Catches (TAC) are set too high for sustainable stock conservation. Fisheries management need robust and reliable stock assessments to ensure that the species and environmental effect of fishing is sustainable. Since the demand for ecosystembased approaches to management has increased, the needs for improved estimates of un-assessed abundance have risen. Managers simply need to know how many fish left in the see and how much to limit the fishermen to fish to have sustainable fisheries. Therefore, accurate assessment of the market as well as by-catch stocks and records of true landings and discards are critical aspects of the scientific advice to the fisheries managers to accurately set TACs.

Here, we consider the marine species that are left un-assessed. That is because they cannot be assessed by the existing methods. We therefore sought to fill the key gap with this matter. This thesis has five key elements. First we reviewed the stock assessment method with the emphasis on the length-structured models. Second, we produced a population model (so called survey-landings model) to make the use of survey frequency data extracted from International Bottom Trawl Survey and total annual landed biomass from commercial reports. Third, within a twin-experiment context and sensitivity analysis the model was assessed for accuracy and robustness in variability in initial parameter values and observational noise. Forth, applying the survey-landings model the population dynamics of the North Sea haddock was assessed and the results were compared with the International Council for Exploitation of the Sea assessment. Fifth, after the model proved to be reliable it is used as an alternative for age- or catch-at-length model, the population of the North Sea grey gurnards were modelled with confidence. This model enabled un-assessed species such as grey gurnards to be modelled and assessed for the first time.


## Preface

This research involved the critical use and interpretation of data from research vessels. It was widely thought that the quickest and easiest, if not most comfortable, way to gain this understanding is to join a research vessel for a cruise.

I joined the ground fish survey trip on the FRV Scotia 0912S for a 10 day cruise of the North Sea, travelling as far north as the Norwegian borders. The aim was to observe and take part in fish sampling and collecting data. In the cruise I attended, fish were sampled from five different stations every day; and each sample held around half a tone of fish. Depending on the type of species, different data were collected. Some species such as flat fish, grey gurnard, and shellfish were measured for their length only, which was quite a fast and straightforward process. However, for each length class of haddock and whiting one or two otoliths were taken as well and the information about sex and weight was recorded. The survey data was collected with more details for pelagic fish such as mackerel and herrings.

The vessel left Aberdeen at 7AM on the morning of $22^{\text {nd }}$ July 2012. The first morning was spent on musters, emergency trainings and testing the sampling equipment prior to moving into deeper waters. By afternoon, the first sample was in the fish lab. The chief scientists Finlay Burns and Jim Drewery gave me an overview of the equipment used and the processing methodology. I also appreciate Dr Toyonobu Fujii (University of Aberdeen), who kindly shared his knowledge and experience throughout the trip. I am very grateful that the crew and scientists on Scotia were friendly and supportive. I joined the scientists group and took part in every survey activities in the fish laboratory including sorting the fish, weighing, measuring, gutting, data entry, taking out the otoliths and preparing them to be read. Scotia berthed in Aberdeen on the evening of $31^{\text {st }}$ July.

To meet the legal requirements for boarding a research vessel, I was required to attend a MCA approved Sea Survival Course, and take an ENG1 Medical Test. I attended a Sea Survival Course in Glasgow Nautical College prior to boarding. This was split into two sections. The first, much easier, section was a three-hour taught session on survival techniques and the correct usage of survival equipment. This was then followed by a wet session in a specialist pool. The aim of this was to simulate a
number of different survival situations. They included group swimming, boarding a life raft, abandoning a ship and being lifted to safety. The ENG1 medical was a standard medical test to ensure I did not have any medical conditions that would cause significant risk on board.

My motion sickness was almost over when it was time to go back to land! By the end of the trip I had a better and clear view on how the IBTS data is collected and what sort of information could be recorded for a desired assessment model. I was, therefore, more aware of the hard sampling process as well as the issues that scientist come across in fish sampling. A valuable lesson was that marine biology could be more challenging than what I previously had thought.

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## Glossary

| CFP | Common Fisheries Policy |
| :---: | :---: |
| CPUE | Catch Per Unit of Effort |
| CWP | Coordination Working Party |
| Eurostat | EC's body for European statistics |
| EEZ | Exclusive Economic Zone |
| FAO | Food and Agriculture Organization of the United Nations |
| Giga | 1000,000,000 |
| Gt | Giga tonnes |
| IBTS | International Bottom Trawl Survey |
| IBTSWG | The International Bottom Trawl Survey Working Group |
| ICES | International Council for Exploration of the Sea |
| IUU | Illegal Unregulated and Unreported |
| IYFS | International Young Fish Survey |
| kt | 1000 tonnes |
| MPA | Marine Protected Area |
| MSC | Marine Stewardship Council |
| Mega | 1000,000 |
| Mt | Mega tonnes |
| MASTS | Marine Alliance for Science and Technology Scotland |
| MPMG | Marine Population Modeling Group |
| MSC | Marine Stewardship Council |
| PICTs | Pacific Island Countries and Territories |
| PSM | Port State Measure |
| SSF | Small-scale fisheries |

STECF Scientific, Technical and Economic Committee for Fisheries
TACs Total Allowable Catches
VPA Virtual Population Analysis

## Chapter 1

## Introduction and thesis outline

### 1.1 Fisheries management

The European commission of fisheries management has set of rules (CFP) since 1970s to manage European fishing fleets and to conserve fish stocks. The Common Fisheries Policy is to ensure the fishing industry is environmentally, economically and socially sustainable. Fishing is also controlled to protect fish population size and productivity from being at danger in long term. Some principal aims of the fisheries management are to guarantee a high long-term maximum sustainable yield for all stocks by 2020 and to minimise unwanted catches through landing obligations. Maximum sustainable yield is the level at which fish source can be routinely exploited without long term depletion (European Commission).

Fisheries management takes control by setting the rules for vessels to access waters, fishing capacity, gear usage and time and area that fisherman can fish. The amount of fish from fisheries is also regulated through total allowable catches (TACs) by the Council of fisheries ministers based on advisory bodies of the International Council of Exploitation of the Sea (ICES) and Scientific, Technical and Economic Committee for Fisheries (STECF). TACs are specified for each stock and shared as a percentage between EU countries in the form of national quotas. The fishery has to be closed when all the available quota of a species is fished (European Commission).

Although stocks may be renewable, they are finite. Modern fishing fleets have the capacity to over-exploit fish stocks, which have caused environmental and socioeconomic problems in recent years (Worm, et al. 2009). One example is the fall of Canadian population of Atlantic cod, which was argued to be due to poor recruitment of cod to the fisheries as the result of young fish being discarded while the fishing increased (Myers et al., 1997). This is probably because the trend in stock was not correctly understood (Hilborn et al., 2003). Other examples can be found in the three Asian countries, Malaysia, Philippines and Thailand. Over 12 to 49 years (depend on the species) their total demersal biomass (i.e. weight of fish) substantially declined up to $64 \%$ (Stobutzki et al, 2006). These results raise concern for fisheries resources and require necessary management actions to maintain ecosystem integrity.

The result of a study on 31 ecosystems shows that with increase in exploitation rate, the fish catch increases (five-folded since 1950) to reach the maximum sustainable yield and decrease thereafter, when is defined as overfishing (Worm et al. 2009). One third of the studied fisheries are overfished (Worm, et al., 2009, FAO, 2010; Branch, et al., 2011) and a majority of world's stocks are intensively exploited (Hilborn et al., 2003). The increase in exploitation rate also causes the reduction in overall biomass, collapse in species and decrease in average body size. Looking from a different angle, setting the exploitation rate below the maximum sustainable yields reduces the cost of fishing and contributes to rebuilding some stocks (Worm et al. 2009). In the assessment of 166 stocks, of which the majority come from fisheries of developed countries, $28 \%$ have continuing overfishing and fisheries are exploited in another $28 \%$ but would allow for rebuilding (Worm et al., 2009). Since 1977 there was 11\% decline observed in total biomass, mainly in pelagic species. Analyses of research trawl surveys revealed $32 \%$ decline in overall biomass, while the big demersal species had $56 \%$ decline. Additionally, the analyses show an overall decline in average maximum size (Worm et al., 2009).

Fishing has both direct and indirect effects on marine ecosystem function (Hilborn et al., 2003; Gutlerrez et al., 2012). The damage due to fishing, although not easy to measure, has been explored by Hilborn (2011). The successful fisheries, which are biologically sustainable and economically profitable, as well as unsuccessful ones, in
which the stock has been reduced to very low level or in the case of economically failure, are investigated in Hilborn et al (2005).

Discarding fish and illegal fishing are another huge issue for fisheries management, which are caused by economic and regulatory constraints (Hilborn et al., 2003). Discards refer to the part of unwanted catch that is returned to the sea during fishing operation. By-catch (non-target catch) could also influence fisheries management by increasing the fishing mortality level or may contribute to overfishing. Fish are discarded due to either being too small, unmarketable or the fisherman has no quota or due to some catch competition rules. Addressing these issues largely depend on the governance enforcements. Fishermen in Norway, for example, are obliged by law to land all their by-catch (Isaksen, 1997). With regards to illegal fishing, the Chilean government, addressed the issue by granting ownership of the fishing beds to local cooperatives to police to protect and police their fishing areas (Hilborn et al., 2003). The reformed CFP is releasing some landing obligation rules for all commercial fisheries to prevent wasteful practice of discarding. Under the new rule, all the catches should be kept on-board and landed and counted against quotas. For example from 2016, North Sea vessels using gear 100 mm need to land all catches of plaice, haddock, cod, whiting, northern prawn, sole and nephrops.

Fisheries resources and products are also fundamental for human feeding. The global demand for Seafood emphasizes its important role in diet and its contribution to a healthy nutritional status (FAO 2010). It is rich in a number of important nutrients that are not easily found in other food. It is also a very high source of proteins, omega-3 fatty acids (EPA and DHA) and vitamin D (Fisheries.no). The outstandingly high consumption of fish by Pacific island countries and territories (PICTs) shows the importance of contribution of the seafood security of the Pacific. Providing fish for food security will need improvement in most of fisheries management in the region and assistance from fisheries organizations (Bell et al., 2009). With regards to the increasing demand of seafood, the Marine Stewardship Council (MSC) certifies fisheries that adopt environmentally sound activities (Parkes et al., 2010), among which is the Scottish fisheries. Of the studied fisheries, 45 fisheries were certified and 179 uncertified. Among the certified fisheries, $74 \%$ are above biomass level, while it is the case for only $44 \%$ of the uncertified fisheries. Also, the increase rate in
biomass in certified stocks was on average $46 \%$ over 10 years, while it was only $9 \%$ for uncertified stocks (Gutlerrez et al., 2012).

In addition to human feeding, fishing industry and fisheries resources are important for employment, in Scotland in particular compared to the rest of UK. Over $60 \%$ of the total UK catch of fish is landed in Scotland, although it has just less than $9 \%$ of the UK population (The Royal Society of Edinburgh, 2004). In the worldwide scale, Asia contributes over half of the global fisheries production and has $85 \%$ of the world's fishers (FAO, 2002). Mahon (1997) discusses that the majority of fish stocks are small-scale and predominantly located in developing countries. Nevertheless, there have not been complementary projects and assistance for low-budget management situations. Instead, international fisheries development and management institutions have focused on transferring methods and technology from developed to developing countries. Social and economic health of fisheries are another important aspects. Fisheries export is very variable worldwide, from $1 \%$ in Korea and Netherlands to $64 \%$ in Iceland (Hilborn et al., 2003). They are heavily subsidised to maintain the fragile fisheries industry.

The extinction of marine species, although it seems very unlikely, could be caused by overfishing. Not only the fisheries exploitation changes the abundance of stock (number of fish in the sea) but also they contribute to change in the size structure of the population (Hilborn et al., 2003). Depending on local areas and governance system, fisheries objectives can be achieved by management actions such as catch controls and reduction in Total Allowable Catches (TAC) for target species (Worm et al., 2009), gear modifications, precautionary approach and marine reserves. One example is Logan Bay in UK where fishing herring was not permitted due to collapse of Northern Irish Sea herring in 1980 (Rogers 1997). It has been argued that the path to successful fisheries management is to establish appropriate institutions, which include reward to maximise the welfare of managers, fishermen and scientists (Hilborn et al., 2005). In their review, Hilborn et al (2004) suggest that the marine reserves (areas that are legally protected against fishing), together with other fisheries management tools, can be successful for long-term fisheries management and conservation of biodiversity, although it has its downsides depends on the species and characteristics of the ecosystem. Their approach is that fisheries reserves can
potentially increase yield if heavy fishing mortality has reduced recruitment (number of new fish added to the fisheries). Also with regards to sedentary organisms, spatial management such as marine reserves is more effective than catch regulations; whereas for highly mobile species marine reserves need to be very big to protect breeding population. For fisheries that target many species, marine reserves could be more cost-effective than increasing quota or even setting catch limit from Hilborn et al (2004)'s review. The Scottish marine protected area (MPA) network covers about $20 \%$ of Scottish waters in the Sea (The Scottish Government).

All of the above concerns need robust and reliable stock assessments to ensure that the species and environmental effect of fishing is sustainable. Since the demand for ecosystem-based approaches to management has increased, the needs for improved estimates of un-assessed abundance have risen. Managers simply need to know how many fish are left in the sea and how much to limit the fishermen in order to have sustainable fisheries. As a result, accurate assessment of the target as well as by-catch stocks and records of true landings and discards are critical aspects of the scientific advice, which enable the fisheries managers to accurately set TACs.

### 1.2 Traditional population model assumptions

Fisheries science has learned a great deal from the past. This includes methods, data usage and improvements in computing power, which enabled development of those used in previous assessments. . For example the data are no longer assumes to come from a population at equilibrium; neither do assessments rely more on the catch per unit of effort (CPUE) than survey data (Hilborn and Liermann, 1998). Researchers used to assume that the dynamics of fish population are deterministic and the length distributions were studied in steady state over time (Fry, 1949; Pope, 1972; Lai and Gallucci, 1988) and therefore it was not possible to analyse the uncertainties and estimate the parameters. The primary assumptions were based on the recruitment and mortality being constant over time. It was also assumed that there was a fixed age at which fish are recruited (knife-edged recruitment) and before that age the fish are
invulnerable to fishery (Deriso, 1980; Schnute, 1985). Hence, the growth was deterministic and the length at age could be well estimated.

These assumptions significantly limited the effectiveness of population assessment methods (Sullivan et al 1990). Throughout the improvement in population stock assessment, it became obvious that the population parameters hardly remain constant. Moreover, population dynamics follow stochastic processes and spawning occurs during several months. In fact, not only do they not recruit simultaneously (Sullivan et al., 1990) but also they are influenced by other factors (Beverton and Holt, 1957) such as water temperature and population density.

### 1.3 Stock assessment methods

The purpose of stock assessment is to provide the basis for fisheries management decision-making. Over the past twenty years the methods used in stock assessment have changed from using only one source of data (catch, catch-at-age, survey or CPUE) to using all available data simultaneously, often called integrated analysis, to capture all knowledge about stock size and productivity (Hilborn, 2003). Further, the modern methods got more complex and as a result more capable of expressing the uncertainties through sensitivity analysis, bootstrapping and Bayesian approaches (Punt and Hilborn, 1997; Liermann and Hilborn, 1997; McAllister and Ianelli, 1997; Cook, 2013).

Estimates of fish stock biomass and mortality for assessed commercially exploited species in EU waters are currently based on fishery landing statistics from the different EU nations, which are collated by the International Council for the Exploration of the Sea (ICES). The standard stock assessment approaches are agebased methods, which have been developed from cohort analysis and Virtual Population Analysis (VPA). The methods have been in use for over 30 years and rely on catch-at-age data as the model input. They can be subject to error if misreporting, illegal landings and unrecorded discards calculating stock size are high. To obtain reliable catch-at-age data (for those that can be aged), a great number of fish need to
be aged, which is costly and time consuming, and this puts practical limits on the number of species that can be assessed in this way.

In contrast to age data, information on length distributions of fish ${ }^{1}$ is more easily available from commercial landings, market samplings and scientific surveys such as the International Bottom Trawl Survey (IBTS), which covers the North East Atlantic. To get the length distribution of a fish population, a large random (and unbiased) sample of the stock should be taken and the length of each fish in it measured. In addition to the advantage of the ease of the measure, population parameters such as growth, mortality and maturity may be better related to size rather than to age. This has led to the development of a variety of length-based assessment methods.

Early length-based approaches, such as age-length analysis, focussed on using fish growth models (mainly von Bertalanffy growth function ${ }^{2}$ ) to decompose the length distributions of catches into age classes, which are then analysed in the standard agedbased method (Froysa et al. 2001).

Increasingly, however, there has been interest in entirely length-based approaches such as catch-length analysis. This approach, which was originally developed by Sullivan et al. (1990), is a somewhat more advanced method that incorporates variability in growth to account for individual differences and generates a modelled population and catch length distribution that can be fitted to observations. , It uses a forward-running length-structured matrix model in which growth increments are modelled by a gamma distribution, and where the model parameters are estimated by fitting to catch-at-length observation data from commercially reported landings. Although the catch-length analysis removes the need for age data, the length distribution of commercial catches are not always routinely recorded, especially in regions such as the North Sea where the fishery involves fleets from different EU nations.

[^0]For some of those species that the length distribution of commercial catches is not known, total landed biomass (total weight of landed fish) is available (i.e, gurnard, dogfish, lemon sole). Length distribution of survey from the scientific survey data such as the IBTS is also routinely recorded for all species.

Marine sampled species/stock can be divided into three main categories where information is available:

1. Well-sampled species/stock

- Total weight landed, number at length and age in the landing
- Total weight discarded, number at length and age in discards
- Survey data on numbers at length and age in the sea
- Cod, haddock, whiting, plaice, hake, herring, mackerel, scallops ${ }^{3}$, ...

2. Moderately sampled species/stock

- Total weight landed, number at length and age in the landings
- Possibly some information on total weight discarded
- Survey data on numbers at length and age in the sea
- Anglerfish, lemon sole, turbot, ...

3. Poorly sampled species/stock

- Total weight landed
- Possibly the average minimum landing size
- Survey data on numbers at length and age in the sea
- Dab, flounder, dogfish, gurnard ${ }^{4}$, wolfish, ...

The catch-at-length model could only be applied to the well-sampled species and some moderately sampled species where the number of catch-at-length is recorded.

In this research work, which aims to improve the description of individual growth, stock biomass and fishing mortality, a new population model based on the catch-atlength approach was developed. This work enables the length-structured model to assess those stocks for which neither the age nor the commercial catch-at-length is available. The new model, which is so-called survey-landings model, adopted the

[^1]catch-at-length approach and modified it to make the use of the total landed biomass (instead of catch-at-length) data from commercially reported landings and the length frequency distributions of fish from the North Sea IBTS to estimate the model parameters.

In the following sections, the history of assumptions of stock assessment and the ageas well as age-length-structured assessment methods are briefly discussed. The next chapter provides a review of the length-structured stock assessment in more mathematical and methodological depth.

### 1.3.1 Age structured models

Most stock estimation methods are based on age data, in which the population is composed of groups of fish of equal age. It is by far the most widely used assessment approach in the International Council for Exploration of the Sea (ICES). Of these, Virtual Population Analysis (VPA) (Beverton and Holt, 1957; Gulland, 1965) is the most widely used and accepted stock assessment method when the historical catch-atage data are available. This runs backwards in time to solve for the fishing mortality and the stock size in each cohort, for which the catch and natural mortality must be known (Sims, $1982 \&$ 1984). In the actual application of both Single Species VPA (SSVPA) and Multi Species VPA (MSVPA) values of fishing mortality for all age groups in the last year and for the older age group in other years is used as input into the model. It then adds all the catches and deaths throughout the cohort to the population number in the oldest age class to estimate the population numbers at younger ages within the same cohort (Vinther, 2001; Fry, 1949; Beverton and Holt, 1957; Murphy, 1965). The problem here is that the fishing mortality of the final year is not known. However, in practice, the backward process is independent of the value of the terminal fishing mortality. VPA is not a statistical model and treats input catch-at-age data as being exact; also the variables of each cohort are calculated independently from other cohorts. Cohort analysis or follow up age analysis in VPA can be applied over several years by keeping the fishing mortality fixed in each year class. To have a better understanding of how the age-structured assessment works,
some effort was put to write a program to run a population assessment for haddock data using a simple VPA method. The step-by-step notes and the related R codes are presented in Appendix 1.1.

Statistical catch-at-age analysis (CAGEAN), in which numbers of catch-at-age are not assumed to be exact inputs into the model (Deriso et al., 1985; Fournier and Archibald, 1982). The advantage of the statistical catch-at-age analysis compared to VPA is that the confidence intervals for the model parameters can be estimated, which is beneficial for management-based decisions. Bayesian statistical approaches in catch-at-age analysis have also been applied to incorporate uncertainties of the natural mortality, recruitment, fishing selectivity and catch (e.g., McAllister and Ianelli, 1997; Nielsen and Lewy, 2002; Cook, 2013), which assess the likelihood of the variety of outcomes of the population dynamics. Punt and Hilborn (1997) used all the available data within age-structured stock assessment in a Bayesian context.

The VPA technique is particularly sensitive to the observation error in final age class, because it needs a reliable assumption about the fishing mortality for the oldest cohort. Various errors in VPA as a result of the errors in assumed natural mortality are discussed in Ulltang (1977). If the natural mortality is assumed fixed, the fluctuations of the total mortality, caused by random variations in natural mortality, will disappear. However, Ulltang (1977) discusses that errors in VPA caused by uneven natural or fishing mortality are generally small and that using fixed natural mortality has a very little influence on the total catch in short term (Pope, 1991). The study in Sims (1984) concludes that an overestimate of the natural mortality rate leads to considerably higher percentage errors in stock-size estimates than does an underestimate. Pope and Shepherd (1982) applied a least-square method to tune the VPA and gain a consistent and more reliable fishing mortality calculation. Different methods were also tested in Pope et al. (1985) based on constant or varying catchability for better determination of fishing mortality. Shepherd (1999) developed the VPA method that was first introduced by Doubleday (1981) and that uses all the available data of the population of survivors of each cohort to estimate the fishing mortality in the final year. He discussed that in the presence of a well-sampled catch, tuned VPA is the most accurate estimate for the dynamics of population. Anderson
(1976) has used commercial catch per unit effort and research vessel survey catch per tow to apply VPA in documenting the changes in species abundance.

The age-based model relies mostly on commercial catch-at-age data, with survey data used for tuning the final year parameter estimates (Dobby, 2004). Survey-based assessment models were developed by Cook (1997) and later implemented by Needle (2002).

The primary component of age-based stock assessment is the number of caught fish in each age group. Aging is usually determined by counting the number of growth zones on the otoliths, which are usually located to the back of fish ears ${ }^{5}$. Hence, a great number of fish should be aged which is a very difficult, costly and time-consuming process. In marine world, nevertheless, many animals such as Blacklip abalone (Haddon et al., 2008) and lobsters (e.g., Nephrops) due to moulting, (Zheng et al., 1996; Wang and Ellis, 2005) are extremely difficult to age. Aging does not work for shrimps because no calcium-based structure survives the periodic shedding of the exoskeleton. Even tagging may affect the growth process (Etim and Sankare, 1998). Uncertainties of age reading of anglerfish make the age-based assessment of this stock very challenging (Dobby, 2000, 2004). The quality of the stock assessment, which is affected by the discrepancies in age reading between the readers (e.g., cod in the Baltic, pelagic fish species in Northeast Central Atlantic) from different countries (ICES, 2003), adds to the issues of age-structured stock assessment methods. Consequently, all of the issues mentioned above limit the number of species that can be assessed (Drouineau et al., 2007. Additionally, lack of valid (Parrack, 1992) or direct aging methods in some situations (Sullivan et al., 1990), plus the fact that some animals such as Skipjack tuna fish cannot be aged due to lack of clearly defined annual rings in the otoliths (Hillary 2011), leads to the need to search for sensible alternatives.

[^2]
### 1.3.2 Age-length-structured models

Age-length-structured models, which are basically age-based models using length data, rely on the age composition data derived from the age-length relationship. The length of the fish at each age class is assumed to be normally distributed with the mean lengths-at-age lying on or near the von Bertalanffy growth curve and the standard deviation of the lengths are linear functions of the mean (Fournier et al., 1990; Fournier et al., 1998). To reduce the error caused by the length-at-age variation, Fournier et al. (1998) suggest estimating the parameters of the age composition function and the parameters of the age-structured model simultaneously.

Age-length-structured models are likely to be more realistic than the models based on age only. They can better describe biological and fishery selectivity processes. However, the level of complexity is higher because of demanding more data and estimation more parameters. Hence, using these models may have more uncertainties in comparison to the models based on age or length only and therefore their advantages may be weakened.

Froysa et al. (2002) apply a self-contained model to North-East Arctic cod, using age and length data to deal with the population stock assessment. The Age-length structured method, Fleksibest, which is the length-structured version of CAGEAN, uses a function of length as a transformed version of catch-at-age analysis. The population size is, therefore, structured by a matrix of numbers of fish at length at age. Parametric models estimate growth, mortality and maturation where the model parameters are subject to estimation.

Converting the catch-at-length into catch-at-age, often carried out using the agelength key, raises some limitations. The inaccuracy of determining age at length, for some species, results in uncertainty in the population assessment (Drouineau et al., 2007). Significant variations among individual length within an age class could result in inaccurate estimate of fish stock assessment by age (Froysa et al., 2002; Drouineau et al., 2007). The variation could be more significant when the growth speed is different in cohorts due to density variations in cohort's population. As a result, agelength relationships are feasible only if the relationship is accurately known and the
age classes do not overlap too much (Fournier and Doonan, 1987). The procedure suggested in Schnute and Fournier (1980) applies the length-frequency data to derive the percentage of fish at each age class, which reduces the overlaps between breaking points in age groups.

### 1.4 Thesis outline

In Chapter 2 a methodological and mathematical review of length-structured stock assessment methods in marine world is presented. It takes the reader on a journey through length-cohort analysis and then basic length-based models to stage-structured models and parameterisation of stochastic models. The aim is to provide a general mathematical view of available length-structured models. The underlying assumptions, data requirements, growth modelling, mortality and recruitment parameter estimation are also discussed. The development and pros and cons of each approach are outlined.

Chapter 3 aims to describe the International Bottom Trawl Survey (IBTS) Working Group and their responsibilities in bringing together the survey. The source of fisheries catch data is also given. The standards and the process of data collection and the challenges and risk factors that may reduce the accuracy of the samples and fisheries statistics are also discussed here.

In Chapter 4, the modified and developed length-structured model, for the situation when age and the catch-at-length is unavailable, is presented. The methodology of the model, so called survey-landings model, is discussed. All the components and structure of the model are explored separately and combined to make a modelled dynamics of the population. The survey-landings model is also taken through basic checks for consistency of the model with the characteristics and dynamics of the population in the sea

In Chapter 5, the survey-landings model is used to simulate and generate observations of stock and mortality. Investigation of the model is conducted within a twinexperiment context to check the accuracy and robustness of parameters that are subject to estimation. Different assumptions regarding parameter values are run and
results are compared to identify which factor is more influential and therefore which parameters are more sensitive to variations. This is a useful practice to understand the behaviour of the model. The strengths and limitations of the model are discussed with regards to parameters that are estimated within the model and those which can be estimated externally and independently of the model.

In Chapter 6 the abundance of the North Sea haddock is assessed by applying the length-structured survey-landings model and the assessment outputs are compared with the standard age-structured ICES assessment. The aim is to investigate how close our model output is to the standard method. The growth parameters are first considered constant for the whole study years from 1969 to 2012. In reality, however, growth trend is not constant and varies over time. In order to put the variability of growth trend into consideration, the years are broken into 4 separate time lines and growth parameters are estimated separately for the periods from 1969-1979, 19801989, 1990-1999 and 2000-2012. The results indicate that the growth trend is not fixed but varies over time. Therefore, the assessment output is more reasonable and promising when the model is applied on separated time lines of haddock.

Chapter 7 is where the survey-landings model takes a further step to assess the stock of the un-assessed population of the North Sea grey gurnard. Since there is not standard assessment for grey gurnard is available, there would be no standard source to compare the model assessment result with. The reliability of the survey-landing model is decided by the accuracy of estimating the observation data as well as growth and mortality parameters. The model initial parameters are taken from published literature and FishBase where possible to model the population.

In Chapter 8, the rationale and the journey of this research work is briefly discussed. The aims and objectives of this research work as well as the summary of results and limitations along with the strengths and practical usage of the model are highlighted.

### 1.5 Appendix 1.1: Virtual Population Analysis:

## The step by step approach and $R$ codes

In the actual application of Virtual Population Analysis (VPA), one starts with the value of fishing mortality in the final year or for the oldest age class. The problem is that the fishing mortality of the final year and the fish population in the final year is not known. However, in practice, the backward process is independent of the value of the terminal fishing mortality. The VPA adds all the catches and deaths throughout the cohort to the population number in the oldest age class.

In this appendix a simple VPA step-by-step for catch-at-age haddock is presented. It is then accompanied by a piece of R code used to run the model when the catch-at-age data is used as the model observation.

The VPA age-based method estimates the mortality and abundance in a backward approach. The observation data is the number of catch-by-age through 46 years, which makes a 9 by 46 matrix. The rows represent the age and the columns are time steps (years):

| Catch-at-age | Y 1 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | Y 46 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A 1 | $C_{1,1}$ | . | ... | . | . | $C_{1,46}$ |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| A 9 | $C_{9,1}$ | . | . | . | . | $C_{9,46}$ |

Fishing mortality, $F$, is fixed and assumed as known for the oldest age throughout the time steps:

| Fishing <br> mortality | Y 1 | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | Y 46 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A 1 |  |  |  |  |  |  |
| . |  |  |  |  |  |  |
| . |  |  |  |  |  |  |
| . |  |  |  |  |  |  |
| A 9 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |

The abundance, $N$, for the bottom row is estimated, while natural mortality, $M$, is set equal to 0.2 :

$$
\begin{equation*}
N_{9, Y}=\frac{C_{9, Y}\left(F_{9, Y}+M\right)}{F_{9, Y}\left(1-e^{\left(-F_{9, Y}-M\right)}\right)} \tag{1}
\end{equation*}
$$

| Abundance | Y 1 | $\ldots$ | $\ldots$ | $\ldots$ | Y 45 | Y 46 |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| A 1 |  |  |  |  |  |  |
| . |  |  |  |  |  |  |
| . |  |  |  |  |  |  |
| . |  |  |  |  |  |  |
| A 9 | $N_{9,1}$ | $N_{9, .}$ | $N_{9, .}$ | $N_{9, .}$ | $N_{9,45}$ | $N_{9,46}$ |

The abundance is estimated diagonally in a backward approach. The estimation actually starts from age 8 at year 45 . For the start $N_{8,45}=N_{A, Y}$ and $N_{9,46}=N_{A+1, Y+1}$ :

$$
\begin{equation*}
C_{A, Y}=\left(1-\frac{M}{\ln \left(\frac{N_{A, Y}}{N_{A+1, Y+1}}\right)}\right)\left(N_{A, Y}-N_{A+1, Y+1}\right) \tag{2}
\end{equation*}
$$

| Abundance | Y 1 | ... | ... | ... | Y 45 | Y 46 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A 1 | $N_{1,1}$ | $N_{1,}$. |  |  |  |  |
| . | $N_{., 1}$ | N, | $N_{\text {., }}$ |  |  |  |
| $\cdot$ | $N_{\text {, }, 1}$ | $N_{\text {,., }}$ | $N_{\text {,., }}$ | $N_{\text {,., }}$ |  |  |
| A 8 | $N_{8,1}$ | $N_{8,}$, | $N_{8,}$ | $N_{8,}$ | $N_{8,45}$ |  |
| A 9 | $N_{9,1}$ | $N_{9,}$. | $N_{9}$, | $N_{9,}$. | $N_{9,45}$ | $N_{9,46}$ |

The next step is to estimate the left bottom triangle in the fishing mortality matrix. It is calculated by applying the exponential relationship between the abundance in one year and the abundance in the year before:

$$
\begin{align*}
& N_{A+1, Y+1}=N_{A, Y} e^{\left(-F_{A, Y}-M\right)} \Rightarrow \\
& \quad F_{A, Y}=-M-\operatorname{Ln}\left(\frac{N_{A+1, Y+1}}{N_{A, Y}}\right) \tag{3}
\end{align*}
$$

| Fishing <br> mortality | Y 1 | $\cdots$ | $\cdots$ | $\cdots$ | Y 45 | Y 46 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A 1 | $F_{1,1}$ | $F_{1, Y}$ |  |  |  |  |
| $\cdot$ | $F_{A .1}$ | $F_{A, Y}$ | $F_{A, Y}$ |  |  |  |
| $\cdot$ | $F_{A, 1}$ | $F_{A, Y}$ | $F_{A, Y}$ | $F_{A, Y}$ |  |  |


| A 8 | $F_{8,1}$ | $F_{8, Y}$ | $F_{8, Y}$ | $F_{8, Y}$ | $F_{8,45}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A 9 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |

The top right triangles of the fishing mortality and abundance matrices are estimated.
In a loop:
I. The fishing mortality of the last year is set equal to the fishing mortality of the year before for every age group $\left(F_{A, 46}=F_{a, 45}\right)$.

| Fishing <br> mortality | Y 1 |  | $\cdots$ | $\cdots$ | Y 45 | Y 46 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A 1 | $F_{1,1}$ | $F_{1, Y}$ |  |  |  |  |
| . | $F_{A .1}$ | $F_{A, Y}$ | $F_{A, Y}$ |  |  |  |
| . | $F_{A, 1}$ | $F_{A, Y}$ | $F_{A, Y}$ | $F_{A, Y}$ |  |  |
| A 8 | $F_{8,1}$ | $F_{8, Y}$ | $F_{8, Y}$ | $F_{8, Y}$ | $F_{8,45}$ | $F_{8,46}=F_{8,45}$ |
| A 9 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |

II. Hence, the abundance of the last year (year 46), $N_{A, 46}$, is calculated using Equation (1):

$$
N_{A, 46}=\frac{C_{A, 46}\left(F_{A, 46}+M\right)}{F_{A, 46}\left(1-e^{\left(-F_{A, 46}-M\right)}\right)}
$$

| Abundance | Y 1 | $\ldots$ | $\ldots$ | $\ldots$ | Y 45 | Y 46 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A 1 | $N_{1,1}$ | $N_{1, Y}$ |  |  |  |  |
| . | $N_{A, 1}$ | $N_{A, Y}$ | $N_{A, Y}$ |  |  |  |
| $\cdot$ | $N_{A, 1}$ | $N_{A, Y}$ | $N_{A, Y}$ | $N_{A, Y}$ |  |  |
| A 8 | $N_{8,1}$ | $N_{8, .}$ | $N_{8, .}$ | $N_{8, Y}$ | $N_{8,45}$ | $N_{8,46}$ |
| A 9 | $N_{9,1}$ | $N_{9, Y}$ | $N_{9, Y}$ | $N_{9, Y}$ | $N_{9,45}$ | $N_{9,46}$ |

III. The population values in backward diagonals are estimated from Equation (2)
IV. The fishing mortality is derived from Equation (3)

The end of this loop completes the top right triangles of both fishing mortality and abundance matrices. There is one cell in each matrix left to the top right one, which belongs to the age 1 haddock at the last year.

| Abundance | Y 1 | $\ldots$ | $\ldots$ | $\ldots$ | Y 45 | Y 46 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A 1 | $N_{1,1}$ |  | $N_{1, Y}$ |  | $N_{1, Y}$ |  |
| $N_{1, Y}$ | $N_{1,45}$ |  |  |  |  |  |


| $\cdot$ | $N_{A, 1}$ | $N_{A, Y}$ | $N_{A, Y}$ | $N_{A, Y}$ | $N_{A, 45}$ | $N_{A, 46}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\cdot$ | $N_{A, 1}$ | $N_{A, Y}$ | $N_{A, Y}$ | $N_{A, Y}$ | $N_{A, 45}$ | $N_{A, 46}$ |
| A 8 | $N_{8,1}$ | $N_{8, .}$ | $N_{8, r}$ | $N_{8, Y}$ | $N_{8,45}$ | $N_{8,46}$ |
| A 9 | $N_{9,1}$ | $N_{9, Y}$ | $N_{9, Y}$ | $N_{9, Y}$ | $N_{9,45}$ | $N_{9,46}$ |


| Fishing mortality | Y 1 | ... | ... | ... | Y 45 | Y 46 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A 1 | $F_{1,1}$ | $F_{1, Y}$ | $F_{1, Y}$ | $F_{1, Y}$ | $F_{1,45}$ |  |
|  | $F_{\text {A. } 1}$ | $F_{A, Y}$ | $F_{A, Y}$ | $F_{A, Y}$ | $F_{A, 45}$ | $F_{A, 46}=F_{A, 45}$ |
| . | $F_{A, 1}$ | $F_{A, Y}$ | $F_{A, Y}$ | $F_{A, Y}$ | $F_{A, 45}$ | $F_{A, 46}=F_{A, 45}$ |
| A 8 | $F_{8,1}$ | $F_{8, Y}$ | $F_{8, Y}$ | $F_{8, Y}$ | $F_{8,45}$ | $F_{8,46}=F_{8,45}$ |
| A 9 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |

By setting the fishing mortality value of the 1 -year-old fish at the last year, $F_{1,46}$, equal to the fishing mortality of the year before, $F_{1,45}$ (similar part I of the loop), the abundance of the first age class in the last year is calculated from Equation (1).

$$
N_{1,46}=\frac{C_{1,46}\left(F_{1,46}+M\right)}{F_{1,46}\left(1-e^{\left(-F_{1,46}-M\right)}\right)}
$$

## Author: Andisheh Bakhshi

\# Department of Mathematics and statistics
\# University of Strathclyde
\# Description:
\# Virtual Population Analysis
\# Codes are written in package R as part of my practice of how the VPA works in
\# population assessment
\# a: number of age classes (year)
\# $\quad \mathrm{t}$ : number of time steps (year)
\# M: Natural mortality rate 1/year
\# lastA: number of age category
\# ages: Vector of age
\# lastY: number of year category
\# years: vector of year
\# population: a*t matrix of population numbers
\# FishM: a*t matrix of fishing mortality rate

```
rm (list=ls(all=T))
\(\mathrm{a}<-9\)
\(\mathrm{t}<-46\)
catch <- matrix(scan(paste("/haddock.txt",sep=""), n=a*t), a,t, byrow=TRUE)
    M <- 0.2
lastA \(<-\operatorname{dim}\) (catch)[1]
ages \(<-\operatorname{seq}(\operatorname{dim}(\) catch \()[1])\)
lastY \(<-\operatorname{dim}\) (catch)[2]
years \(<-\operatorname{seq}(\operatorname{dim}(\) catch \()[2])\)
population <- matrix(nrow=lastA, ncol=lastY)
FishM \(<-\) matrix (nrow=lastA, ncol=lastY)
```


# 1)

```
# 1)
#Fixing the Fishing Mortality values for the last year
#Fixing the Fishing Mortality values for the last year
FishM[lastA , 1:ncol(FishM)]<- 0.5
FishM[lastA , 1:ncol(FishM)]<- 0.5
#Estimating the population of the last row
#Estimating the population of the last row
population[lastA, 1:ncol(population)] <-
population[lastA, 1:ncol(population)] <-
    (catch[lastA,1:ncol(population)]*(FishM[lastA,1:ncol(population)]+M))/
    (catch[lastA,1:ncol(population)]*(FishM[lastA,1:ncol(population)]+M))/
    (FishM[lastA,1:ncol(population)]*(1-exp(- FishM[lastA,1:ncol(population)]-M)))
    (FishM[lastA,1:ncol(population)]*(1-exp(- FishM[lastA,1:ncol(population)]-M)))
# 2)
#Estimating the population and fishing mortality in diagonals
vpa<- function(X, Next, C, M) {
    U <- (1-M/log(X/Next))*(X-Next)
    Error <-(U-C)^2
    return(Error) }
Total.Ages<- max(ages)
Total.Years<- max(years)
Seq.Ages<- seq(Total.Ages - 1, 1, by=-1)
for (i in Seq.Ages) {
    Seq.Years <- seq(Total.Years-Total.Ages+i, 1, by=-1)
    for (j in Seq.Years) {
        optimum <- optimise (vpa, lower=0, upper=10^15, Next=population[i+1,j+1],
        C=catch[i,j],M=M)
        population[i,j] <- optimum$minimum
        #Estimating the Fishing mortality in diagonals
```

```
        FishM[i,j] <- -M-log(population[i+1,j+1]/population[i,j])
    } }
#3)
#Estimating the Fishing mortality and population for the top right triangle
vpa <- function(X, Next, C, M){
    U <- (1-M/log(X/Next))*(X-Next)
    Error <-(U-C)^2
    return(Error) }
Total.Ages<- max(ages)
Total.Years<- max(years)
Seq.AgesT<- seq(Total.Ages - 2, 1, by=-1)
for (i in Seq.AgesT) {
    FishM[i+1,Total.Years]<- FishM[i+1, Total.Years-1]
    population[i+1,Total.Years] <- (catch[i+1,Total.Years]*(FishM[i+1,Total.Years]+M))/
    (FishM[i+1, Total.Years]*(1-exp(-FishM[i+1,Total.Years]-M)))
    Seq.YearsT <- seq(Total.Years ,Total.Years-Total.Ages+i+2, by=-1)
    for (j in Seq.YearsT)
        {
        optimum <- optimise (vpa, lower=0, upper=10^15, Next=population[i+1,j],
        C=catch[i,j-1],M=M)
        population[i,j-1]<- optimum$minimum
        FishM[i,j-1] <- -M-log(population[i+1,j]/population[i,j-1])
    }}
```

FishM[1,Total.Years]<- FishM[1, Total.Years-1]
population[1, Total.Years] <- (catch[1,Total.Years]*(FishM[1,Total.Years]+M))/(FishM[1, Total.Years]*(1-exp(-FishM[1,Total.Years]-M)))

## Chapter 2

## A review of length-structured stock assessment methods

### 2.1 Introduction

Many research works have established methods to assess marine abundance. The available models are mainly based on age, age and length or length of fish usually from commercial fisheries and survey. They range from relatively simple population dynamic models to more complex biomass analysis. Very complex theoretical models seem to be accurate and less biased but would increase the variance of the assessment due to estimating many parameters, which would potentially reduce the reliability of the whole assessment (Hilborn and Liermann, 1998; Fu and Quinn, 2000). In parallel, a simple model would be criticised due to lack of power to analyse such a complex biological process. Many assumptions (i.e. fixed natural mortality) in stock assessment models are set to keep the model as simple as possible, although a great number of which are likely to be false and, as such, the researcher is aware of the uncertainty of the assessment (Hilborn and Liermann, 1998). The best justification of simplifying the models is Monte-Carlo simulation trials that show simple models, with so many fixed assumptions, still work well (Hilborn, 1979; Punt, 1993).. The biological uncertainties also affect stock assessment and predictions for future management (Drouineau et al., 2010). The population dynamics model should be able to produce the outputs needed for management. Since availability of data is different for each species and stock, the model should also be able to utilise data to assess the stocks (Punt et al., 2013). Here the experiences
from the past are to be learned and applied to let the researchers admit the uncertainty of analysis and provide constraints on them to contribute to improvement in model assumptions.

The age- and age-length methods were briefly discussed in Chapter 1. This chapter reviews approaches to length-based stock assessment. It starts from general mathematical aspects of basic length-based models and finishes off with stage-structured models and estimation of stochastic model parameters. The underlying assumptions, data requirements, growth modelling, mortality and recruitment parameter estimation are discussed. The development and pros and cons of each approach are also highlighted.

### 2.2 Length-structured population models

Length-structured models do not perform the conversion from length frequency to age frequency. One of the main reasons of applying these models is that they deal with the data that cannot be studied by age cohort. In addition many biological and fisheries-related processes such as population, exploitation, maturity, growth, natural mortality (through predation) and fishing selectivity are rapidly responded and better described by length compared to age (Drouineau et al., 2007; Morales-Bojorquez and Nevarez-Martinez, 2010; Hillary, 2011).

The issues related to age frequency can be addressed by moving the population modelling applying size and time rather than age and years. The idea of length cohort analysis (Jones, 1981) is to use the length frequency data to assess the stock by applying the same backcalculation as in VPA methods. The catch-at-size analysis (CASA), however, relates the length distribution of individuals to the abundance (when age data is unavailable) and models the transition of individuals from one length group to another. Number of individuals in each length class at each time is related to the number of fish that grew and survived at some later time $\left(N_{t+1} \propto N_{t}\right)$. In length-based stock synthesis (Quinn and Deriso, 1999) all catch-at-age and catch-at-length methods are considered as a family, in which a form of length-based model is applied on the catch-at-age framework. The last method (length-based stock synthesis) is not discussed here since it heavily depends on age data and is not the purpose of this research work.

### 2.2.1 Length Cohort Analysis

The extension of cohort and Virtual Population Analysis (VPA), for length data, was first developed by Jones (1981) and then explored further by Lai and Gallucci (1988). The aim is to use the length frequency of catches to assess the stock and mortality with the same backcalculation method as in VPA. In addition to the assumption of the age cohort analysis, it is also assumed that catch-at-length is available without error and growth is modelled using the deterministic von Bertalanffy (LVB) model.

Using the backward equation for cohort analysis with constant natural mortality (M) in Quinn and Deriso (1999, Chapter 8):

$$
\begin{equation*}
N_{a}=N_{a+1} e^{M}+C_{a} e^{M / 2} \tag{2.1}
\end{equation*}
$$

where $N_{a}$ is abundance at age $a$ and $C_{a}$ is catch numbers at age $a$ and can be generalised for any age $a+\tau$ :

$$
\begin{equation*}
N_{a}=N_{a+\tau} e^{M \tau}+C_{a} e^{M \tau / 2} \tag{2.2}
\end{equation*}
$$

Now the two ages $a$ and $a+\tau$ can be considered as two ends of a length class $\left(l_{l o w}, l_{u p}\right)$, in which $l_{u p}$ is the start of the next length class $(l+1)$.

## 6

$N_{a}=N_{a-1} e^{-Z_{a-1}}=N_{a-1} S_{a-1}$
$N_{a+1} e^{M}=N_{a} e^{-F_{a}}=N_{a}-N_{a}\left(1-e^{-F_{a}}\right)=N_{a}-C_{a} \frac{Z_{a}\left(1-e^{-F_{a}}\right)}{F_{a}\left(1-e^{-Z_{a}}\right)}$
Where $C_{a}=\frac{F_{a}}{Z_{a}}\left(1-e^{-Z}\right) N_{a}$
Since $\frac{1-e^{-x}}{x} \approx e^{\frac{-x}{2}}$ therefore $N_{a+1} e^{M} \approx N_{a}-C_{a} \frac{e^{-F_{a / 2}}}{e^{-z_{a / 2}}}=N_{a}-C_{a} e^{\frac{-M_{a}}{2}}$
$\rightarrow N_{a+1}=N_{a} e^{-M_{a}}-C_{a} e^{\frac{-M_{a}}{2}}=\left(N_{a} e^{\frac{-M_{a}}{2}}-C_{a}\right) e^{\frac{-M_{a}}{2}}$
$\rightarrow N_{a}=N_{a+1} e^{M_{a}}+C_{a} e^{\frac{M}{2}}$

As the result, the increment of growth from time $t$ to time $t+1$ (Gulland, 1983) is calculated as:

$$
\begin{equation*}
\Delta L(t) \equiv L(t+\tau)-L(t)=\left[L_{\infty}-L(t)\right]\left(1-e^{-k \tau}\right) \tag{2.3}
\end{equation*}
$$

Where $L_{\infty}$ is the maximum asymptotic length and $L(t)$ is the average length at time $t$. Growth increment is a linear function of $L(t)$ with slope $-\left(1-e^{-k \tau}\right)$ and intercept $L_{\infty}$.

Quinn and Deriso (1999, Chapter 9) show the time increment $\tau$ as:

$$
\begin{equation*}
\tau \equiv \tau_{l}=\frac{1}{k} \ln \left(\frac{L_{\infty}-l_{l o w}}{L_{\infty}-l_{\text {up }}}\right) \tag{2.4}
\end{equation*}
$$

By substituting (2.4) into (2.2) and changing age subscripts to length subscripts, the backward model; for length cohort analysis is:

$$
\begin{equation*}
N_{l}=\left(N_{l+1}\left(\left(\frac{L_{\infty}-l_{\text {low }}}{L_{\infty}-l_{u p}}\right)^{M /(2 k)}\right)_{l}+C_{l}\right)\left(\frac{L_{\infty}-l_{\text {low }}}{L_{\infty}-l_{u p}}\right)^{M /(2 k)} \tag{2.5}
\end{equation*}
$$

where $N_{l}$ is the abundance at the start of length class $l$ and $N_{l+1}$ is the abundance at the start of length class $l+l$ and $C_{l}=\frac{F_{l}}{Z_{l}}\left(1-e^{-Z}\right) N_{l}$.

The back calculation starts with the last length class L or the class with larger fish. The Baranov equation (Eq. 2.27) is used to calculate the abundance at length class L, in which a fixed value for $\frac{F_{l}}{Z_{l}}$ is assumed, where $F_{l}$ is the fishing mortality rate for the fish with length $l$ and $Z_{l}$ is the total mortality rate for the fish at with length $l$ :

$$
\begin{equation*}
N_{L}=\frac{C_{L}}{F_{l} / Z_{l}} \tag{2.6}
\end{equation*}
$$

Then (2.5) is used to calculate the abundance at the start of the other length classes. The fraction $F / Z$ for the other length classes is derived from:

$$
\frac{F_{l}}{Z_{l}}=\frac{C_{l}}{N_{l}-N_{l+1}}
$$

where the denominator is the difference between the abundance at two neighbouring length classes, which is in fact the number of deaths at length class $l$.

In the presence of a precise value for M , both total mortality and fishing mortality are:

$$
\begin{equation*}
Z_{l}=\frac{M}{\left(1-F_{l} / Z_{l}\right)} \tag{2.9}
\end{equation*}
$$

$$
F_{l}=\frac{F_{l}}{Z_{l}} Z_{l}
$$

and the values $Z_{l} \tau_{l}$ and $F_{l} \tau_{l}$ are calculated as:

$$
Z_{l} \tau_{l}=-\operatorname{Ln}\left(\frac{N_{l+1}}{N_{l}}\right)
$$

$$
F_{l} \tau_{l}=\frac{F_{l}}{Z_{l}}\left(Z_{l} \tau_{l}\right)
$$

And finally the average abundance in length class $l$ is calculated as:

$$
\bar{N}_{l}=\frac{N_{l}-N_{l+1}}{Z_{l} \tau_{l}}
$$

The length cohort analysis is ideally applied on a catch-length data from a year-class over its life cohort (usually is not possible though), which requires age-length relationship. It therefore has the assumption that the length frequency distribution of a year represents the catch of a year-class throughout its lifetime. This is possible only if the recruitment and growth are constant over time.

As in cohort analysis, incorrect assumptions about $M$ in the last length class lead to large errors in abundance of fish (Lai and Gallucci, 1988). It was also concluded in Lai and Gallucci (1988) that the greater dependence on steady state assumption in the length cohort analysis makes it less reliable than cohort analysis.

### 2.2.2 Basic length-structured models

The basic age-based cohort model of abundance (Quinn and Deriso, 1999: Chapter 1) can be transformed into a length-based population, $N$, model by changing age, $a$, to length, $l$, and time (year) is described by $t$. Hence, the instantaneous change of population at time is simply described as:

$$
\begin{equation*}
\frac{d N}{d t}=-Z_{l, t} N \tag{2.13}
\end{equation*}
$$

where $Z_{l, t}$ is the total instantaneous mortality at size $l$ at time $t$. With the absence of growth, the numbers at each length class at next time step are reduced by mortality only.

Now, if the instantaneous change of size at time (growth) is explained by a length-based growth function (i.e. from von Bertalanffy (LVB) growth function; Quinn and Deriso, 1999: Chapter 4):

$$
\frac{d L}{d t}=G_{l}=k\left(L_{\infty}-L\right), \quad k>0, L_{\infty}>0
$$

where $k$ is growth rate with units of $t^{-1}$ and $L_{\infty}$ is the maximum asymptotic length then the instantaneous change of abundance as a function of length is the negative of the ratio of mortality and growth (Beyer, 1989):

$$
\begin{equation*}
\frac{d N}{d L}=\frac{d N / d t}{d L / d t}=-\frac{Z_{l, t}}{G_{l}} N \tag{2.15}
\end{equation*}
$$

Thus, by separation of variables and using the initial condition the solution for Equation (2.15) is:

$$
N_{l, t}=N_{l, 0} \exp \left[-\int_{l_{0}}^{l_{t}} \frac{Z_{l, t}}{G_{l}} d l\right]
$$

where $N_{l, 0}$ is the population size when the individuals are at recruitment length (at time zero) (Quinn and Deriso, 1999: Chapter 6). Similarly, catch-at-length function can be formulated as:

$$
C=\int_{l 1}^{l 2} \frac{F_{l, t}}{G_{l}} N_{l, t} d l
$$

where $l 1$ and $l 2$ are lengths of individuals and $F_{l, t}$ is the fishing mortality at length $l$ and time $t$.

With regards to LVB growth function, if the initial length at time zero is $L_{0}$, the solution for Equation (2.14) for length at time $t$ (Gallucci and Quinn, 1979) is:

$$
L_{t}=L_{\infty}-\left(L_{\infty}-L_{0}\right)\left(\exp \left[-k\left(t-t_{0}\right)\right]\right)
$$

In size-structured models, it is very important to generate anunbiased mathematical and statistical description of growth. Otherwise, it would negatively affect the quality of the stock assessment (Haddon et al., 2008). In age-based assessments individuals are one year older in the next year cohort, which means that age increases linearly with time; while in length-based models individuals are not necessarily growing at a fixed rate in time. Hence, length-based stock assessments methods need estimates of growth parameters to relate length to time. One approach to determine the growth is to apply the tagging technique (Hillary, 2011 and Haddon et al., 2008), which is very expensive as a great number of fish need to be tagged and need to be recovered for an accurate assessment. In this method, the growth increment (the difference between the length at the time of tagging and the length at recaptured) is assumed to follow a normal or t - distribution around the model predicted length increment.

Some studies have been conducted by Basson et al. (1988) to test the accuracy of the estimation of growth parameters within the assessment model. Variability of other parameters in the length-based model can also affect the length-based analysis. For example Lai and Gallucci (1998) discuss that the choice of maximum asymptotic length and growth rate (in a von Bertalanffy growth function) is fundamental and that the fishing mortality is underestimated as the width of length classes increases. A unit change in the asymptotic length leads to a big change in the abundance. The choice of growth rate is more complicated though because it cannot be guessed from observations.

Sullivan et al. (1990) estimate the asymptotic length in the von Bertalanffy growth function within the population assessment model while Meise et al. (1999) use a fixed value. Wang and Ellis (2005) in one example assume normally distributed individual variability for the asymptotic length, truncated at the recruitment length. In another attempt, they use known values for maximum asymptotic length and growth rate, which reduces the complexity of estimating the remaining parameters. The asymptotic length is an input parameter with an assigned value to it in the modified length-based model applied in DeLong et al. (2001), but the model is relatively insensitive to some reasonable assigned values.

In a case where mortality $Z$ is constant and growth follows the LVB growth function, the population size at time $t$ is given by:

$$
N_{l, t}=N_{l, 0} \exp \left[-Z\left(t-t_{0}\right)\right]
$$

Then using Equation 2.18 the length-based population is defined as:

$$
N_{l, t}=N_{l, 0}\left(\frac{L_{\infty}-L}{L_{\infty}-L_{0}}\right)^{Z_{l, t} / k}
$$

A length-based population dynamic model for marine animals needs a transition matrix to model the growth of marine animals in terms of their length and to relate them to changes in frequencies in time steps (Sullivan et al., 1990; Morales-Bojorquez and Nevarez-Martinez, 2010). Therefore, in order to represent the characteristics of population, the generalisation of the Leslie matrix is applied to enable the model to utilise the size classes. The generalisation of Leslie matrix has been covered in a number of publications including Sainsbury (1982) and Sullivan et al. (1990). It is a matrix of probabilities of transitions between length classes (Morales-Bojorquez and Nevarez-Martinez, 2010; Hillary, 2011). Instead of moving from one age to the next one, in length-based methods the process of aging is defined by the moves through different lengths over time. Large individual variability and variations in growth rate are taken into account in transition matrix models (i.e. it allows individuals of the same size to have different growth rates). Therefore, if the growth dynamics are presented in the transition matrices, the length distribution of the population and catch can be derived in addition to modelling the mean growth and growth variability (DeLong et al., 2001; Wang and Ellis, 2005). In the one-year gap transition matrix, individuals may stay in their length classes or grow to bigger length classes. The assumption is that they do not shrink.

Hence, the population abundance for length classes (apart from the first length class) is sum of all the survivors of size classes that grow into the given size or stayed in that length class:

$$
N_{L, t+1}=\sum_{l=1}^{i} S_{l} P_{l, i} N_{l, t}
$$

where $N_{l, t}$ is population size at length class $l$ and time $t ; S_{l}$ is survival rate at length class $l$ and $P_{l, \tilde{l}}$, which defines the transition matrix, is the proportion of individuals moving from length class $l$ to length class $l$.

Population size in the first length class $(l=1)$, however, depends on the surviving egg production as well as individuals in the first year class that do not grow to the bigger length class:

$$
N_{1, t+1}=S_{0} \sum_{l=1}^{L} f_{l} N_{l, t}+S_{1} P_{1,1} N_{1, t}
$$

where $f_{l}$ is net fecundity.

Hillary (2011) presents a growth transition matrix based on a Bayesian approach to estimate the probability of individuals moving from one length interval to another length interval. In the length-based approach, the transition matrices, which are the proportion of individuals growing from one length class to another, can also be calculated by integrating over the length classes from gamma distribution for a given time step. The length classes mostly have equal width and the length of individuals in each class is assumed to equal the midpoint (Sullivan et al., 1990; Quinn and Deriso, 1999; Morales-Bojorquez and Nevarez-Martinez, 2010).

Equations (2.21) and (2.22) can be written in matrix form as:

$$
N_{(t+1)}=A N_{(t)}
$$

$$
\left[\begin{array}{cccccc}
S_{1} P_{1,1}+S_{0} f_{1} & S_{0} f_{2} & S_{0} f_{3} & \cdots & S_{0} f_{L-1} & S_{0} f_{L} \\
S_{1} P_{1,2} & S_{2} P_{2,2} & 0 & \cdots & 0 & 0 \\
S_{1} P_{1,3} & S_{2} P_{2,3} & S_{3} P_{3,3} & \ddots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & 0 & \vdots \\
\vdots & \vdots & \vdots & \ddots & S_{L-1} P_{L-1, L-1} & 0 \\
S_{1} P_{1, L} & S_{2} P_{2, L} & \cdots & \cdots & S_{L-1} P_{L-1, L} & S_{L} P_{L, L}
\end{array}\right]
$$

where $P_{L, L}=1$ and $\boldsymbol{N}_{(t)}$ is vector of number of individuals at length.
If an initial condition $N_{0}$ (which is the total abundance at time zero $\left(\sum_{l} N_{l, 0}\right)$ ) is available, the population can be projected by repeated application of Equation (2.23). The matrix and Equation (2.23) can be re-formatted as:

$$
N_{(t+1)}=(P S+R) N_{(t)}
$$

where $\mathbf{P}$ is the growth transition matrix, $\mathbf{S}$ is the survival matrix and $\mathbf{R}$ is the recruitment matrix and represents the total annual number of recruitments:

$$
\begin{gathered}
P=\left[\begin{array}{ccc}
P_{1,1} & 0 & 0 \\
\vdots & \ddots & 0 \\
P_{1, L} & \ldots & P_{L, L}
\end{array}\right] \\
S=\left[\begin{array}{ccc}
S_{1} & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & S_{L}
\end{array}\right] \\
R=\left[\begin{array}{ccc}
S_{0} f_{1} & \ldots & S_{0} f_{l} \\
0 & \ldots & 0 \\
0 & \ldots & 0
\end{array}\right]
\end{gathered}
$$

Alternatively, Equation (2.24) can be written as

$$
N_{(t+1)}=P S N_{(t)}+R_{(t)}
$$

where $R_{(t)}$ is the vector:

$$
R_{t}=\left[\begin{array}{c}
S_{0} N_{0, t} \\
0 \\
\vdots \\
0
\end{array}\right]
$$

and $N_{0, t}$ is the egg production given in Equation (2.22), $\sum_{l=1}^{L} f_{l} N_{l, t}$.

The main benefit of the model structure (2.25) is that any model of recruitment can be implemented.

### 2.2.3 Catch-at-length Analysis

Stock assessment methods based on length or catch-at-size analysis (CASA) have become increasingly important in the past decade. They have been developed to avoid the assumptions of recruitment stationary and deterministic growth in length-cohort analysis. Schnute et al (1989) wrote one of the first series of papers that introduces a model based on size (or weight) only growth assessment frequency data. The size-based growth assessment approach, which is based on length category of fish, is extended later with more flexible assumptions (than age-structured models) around the distribution of recruitment in particular (Sullivan et al., 1990; Parrack, 1992; Zheng et al., 1996; Meise et al., 1999; DeLong et al., 2001; Wang and Ellis, 2005; Drouineau et al., 2010; Morales-Bojorquez and NevarezMartinez, 2010; Hillary, 2011).

Parrack (1992) applies Monte Carlo methods to assess the size-structured stock assessment models where age of individuals is not available. With the help of simulation approaches, he concludes that there is no age data or assumptions of VPA methods is needed for stock assessment and that the size data can provide precise and accurate estimations for any fish stock assessment even with no prior knowledge of the unobserved change data (such as migration, predation and disease).

In this section, the description of catch-at-length analysis, which is taken from Sullivan et al (1990), is explored. The population model is an extension of Equation (2.25), which allows
for additive log normal errors. The terms $P, S, N$ and $R$ are matrices in the model. Survival, $S_{l, t}$, is allowed to vary overtime and recruitment is added in the start of time $t+l$ rather than $t$.

$$
N_{l, t+1}=P_{l, i} S_{l, t} N_{l, t}+R_{l, t+1}+\omega_{l, t}
$$

The exploitation fraction of Baranov equation is used as:

$$
\mu_{l, t}=\frac{F_{l, t}}{Z_{l, t}}\left(1-S_{l, t}\right)
$$

where total mortality, $Z$, is sum of the natural mortality, M and fishing mortality, $F$, and survival is modelled as:

$$
S_{l, t}=e^{-Z_{l, t}}
$$

Natural mortality is an essential parameter in marine stock assessment. The accuracy of abundance estimates relies on the knowledge of natural mortality and how it is estimated (Zheng et al., 1996). As an alternative to being kept constant, natural mortality can be described well as functions of length (Wang and Ellis, 2005; Haddon et al., 2008; Meise et al., 1999). It is the most remarkable factor in controlling population dynamics (Fu and Quinn, 2000) and has a considerable effect on absolute abundance. When the natural mortality is high, the estimate of abundance is also higher. Without an estimate of natural mortality, fishing mortality cannot be obtained (Gislason et al., 2010). Due to the density-mortality and water temperature-mortality relationship, changes in density and water temperature could cause variation of natural mortality (DeLong et al., 2001).

The fishing mortality, $F_{l, t}$, is based on gear selectivity and effort. Hence, it is separable as a function of time-dependent full-recruitment fishing mortality, $f_{t}$, and a length-dependent function of selectivity, $s_{l}$

$$
F_{l, t}=s_{l} f_{t}
$$

Catch-at-length analysis could provide accurate estimates of the population if the natural mortality can be approximated (Zheng et al., 1996). However, due to natural mortality being confounded with fishing mortality (Sullivan at al, 1990; Fu and Quinn II, 2000; Cook, 2013), the estimates of them become very difficult and unreliable when both are unknown parameters in the model. For that reason, natural mortality should be estimated separately from the abundance analysis. Pet et al. (1997) applies Pauly's formula (Pauly, 1980) to estimate the natural mortality in analysing the growth of sardine while the fishing mortality estimation follows after. In a great number of length-based stock assessment research works, natural mortality is not estimated but a fixed value is taken from previous research (Sullivan et al., 1990; Drouineau et al., 2010; Morales-Bojorquez and Nevarez-Matines, 2010). In another case, several fixed values are tested and the one with the best fit to the model is selected (Zheng et al., 1996). Although the natural mortality in Meise at al. (1999) is modelled as a two-parameter allometric function, repeating the model runs with fixed values enables the choice of the first coefficient parameter. Wang and Ellis (2005) conclude that in the model with known asymptotic length and growth rate, natural mortality and fishing mortality can be estimated more reliably when there is substantial contrast in the effort pattern. To test the robustness of the model, they once estimate the natural mortality. Then other parameters were estimated for a range of fixed values of natural mortality. It is similar to what Sullivan et al (1990) did with the growth increment parameter $(\beta)$, in a gamma distribution.

On the other hand, Zheng et al. (1996) suggest relative abundance is fairly robust to changes in natural mortality. Therefore, if the population or recruitment trend is of interest, then uncertainty in natural mortality is not a big issue. Sullivan et al. (1990), Zheng et al. (1996) and Morales-Bojorquez and Nevarez-Martinez (2010) assume the natural mortality is constant over time, while Zheng et al. (1995) consider it as a time-dependent factor but fixed for all the length classes. That is because, in their study, the natural mortality of the red king crab was found to be much higher in early 1980's than other time periods. DeLong et al. (2001) model the instantaneous monthly natural mortality as a decreasing allometric function of length. Gislason et al. (2010) suggest that natural mortality is significantly related to body size as well as the growth parameters (asymptotic length and growth rate); and recommend
using an empirical formula to include all the three factors of length asymptotic length and growth rate, to estimate the natural mortality of marine and brackish water fish.

In the stock assessment in which survey numbers are input data, changing of the selectivity of survey gear over time could affect the accuracy of the catchability estimates. Survey biomass and population biomass are related by the catchability of the survey gear, which is closely correlated with natural mortality (Fu and Quinn II, 2000).

Changes in fishing gear selectivity, while the selectivity parameters in the model are fixed over time, could affect the estimation of natural mortality. Through their simulationestimation experiment to assess the robustness of the length-based model, Fu and Quinn II (2000) recommend that natural mortality and the selectivity process should be modelled correctly as time-varying factors, while catchability coefficient is kept as a constant value (even if it varies over time). Wang and Ellis (2005) estimate the catchability coefficient with setting it equal to the fishing mortality. In this case, the fishing mortality and natural mortality is estimated simultaneously.

The predicted catch by length at each time (year) is basically the product of the exploitation fraction and abundance:

$$
C_{l, t}=\mu_{l, t} N_{l, t}
$$

Since most individuals spawn over several months or throughout the entire year, the growth process of young fish and recruitment to fisheries is continuous and as a result, discrete monthly or annual recruitment does not usually occur. Therefore, the recruitment $R_{l, t}$ is not the Leslie matrix progression in Sullivan et al. (1990) and Zheng et al. (1996); but it is assessed with the gradual recruitment patterns as functions of length and time. It is assumed that the recruitment can be added at any length throughout the year and that it is separable into time-based component, $r_{t}$, and length-based component, $p_{l}$, as:

$$
R_{l, t}=r_{t} p_{l}
$$

The patterns of recruitment in the length classes (at each time step) are assumed the continuous gamma distribution. The gamma distribution shows the distribution of recruits over the length classes. In other cases, however, it is truncated (gamma distribution) for the small length classes of the red king crabs (Zheng et al.,1995) or the recruitment pattern is not modelled and considered as known (Wang and Ellis, 2005). Parrack (1992) considers uniform as well as seasonal pattern for recruits. In the seasonal pattern, the date each fish grow to minimum size class is taken from normal distribution. The recruitment for fresh water shrimp, based on abundance of juvenile, is considered as one pulse in a year rather than year around in Etim and Sankare (1998). Also, Wang and Ellis (2005) assume a known length for tiger prawn recruits with a an annually fixed pattern.

A LVB model of individual growth by length is applied to obtain the lower triangular growth transition matrix, $P_{l, i}$, in (2.26). It is used to find the proportion of fish in length class $l$ at time $t$ that survive and are in length class $l$ at time $t+1$. If $l_{l o w}$ and $l_{u p}$ are the lower and upper limit of length class $l$, then $l^{*}$, the mid-point, is:

$$
l^{*}=\frac{\left(l_{\text {low }}+l_{u p}\right)}{2}
$$

Haddon et al. (2008) propose an inverse logistic model to estimate the average growth increment. The advantage of this model is that it removes the assumption of rapid or slow growth in the early stages. The variability of individual growth increment around the mean for blacklip abalone population is determined with either a second inverse-logistic relationship (standard deviation vs. initial length) or by a power relationship (standard deviation vs. predicted growth increment).

In a very few cases, the expected value of growth increment is parameterised as a linear function of length (Zheng et al., 1996). The most common way, however, to determine the growth is to use the average growth increment at each length class by using a growth equation that is usually the transformed version of von Bertalanffy (1939) growth function, where the variance is related to the mean (Sullivan et al., 1990; DeLong et al., 2001; Drouineau et al., 2007; Drouineau et al., 2010, Pet et al., 1997, Etim and Sankare, 1998). In a transformed function, in order to calculate the mean growth increment in each time step, the
age parameter has been replaced by time differences (Sullivan et al., 1990; Meise et al., 1999; Morales-Bojorquez and Nevarez-Martinez, 2010; Hillary, 2011).

Although the Von Bertalanffy growth function is considerably widely used, it is mainly adapted to the fast growing assumptions. By contrast, the Gompertz growth model, which has been less popular, implies the slow growing assumption. The growth increment in Von Bertalanffy growth model is linearly decreasing for smaller size fish but it is exponentially decreasing in Gompertz. The popularity of the von Bertalanffy function could be due to the ease of fitting to the growth data (Haddon et al., 2008). The main parameters to be estimated in the Von Bertalanffy growth equation are the asymptotic maximum length and the growth rate coefficient. In Wang and Ellis (2005) the asymptotic maximum length is a random variable from a normal distribution.

Using the LVB model, the expected change in length or average growth increment over one time span for an individual at mid length $l^{*}$ is:

$$
\bar{\Delta}_{l}=\left(L_{\infty}-l^{*}\right)\left(1-e^{-k}\right)
$$

Therefore, the expected length for an individual of mid-length $l^{*}$ in the next time span is:

$$
E(x)=l^{*}+\bar{\Delta}_{l}
$$

In a significant number of the stock assessments that are based on length frequency data, a stochastic model is used. The variability of growth increment of individuals in the population is then statistically modelled. Here it assumes that the length increment of fish follows the two-parameter gamma distribution, whose mean is derived from the von Bertalanffy growth model (Sullivan et al., 1990; Zheng et al., 1996; Drouineau et al., 2010; Morales-Bojorquez and Nevarez-Martinez, 2010):

$$
g\left(x \mid \alpha_{l}, \beta\right)=\frac{1}{\beta^{\alpha_{l}} \Gamma\left(\alpha_{l}\right)} x^{\alpha_{l}-1} e^{-x / \beta}
$$

for which the mean and variance of gamma distribution are:

$$
E(x)=\alpha_{l} \beta, \operatorname{Var}(x)=\alpha_{l} \beta^{2}=\beta E(x)
$$

The gamma distribution has been fitted to the length frequency data of various marine species such as pacific cod, Pseudotolithustypus and Decapterusrussellii (Sullivan et al., 1990), red king crab (Zheng et al., 1996), jumbo squid in Mexico (Morales-Bojorquez and NevarezMartinez, 2010), skipjack tuna in Indian Ocean (Hillary, 2011), pink shrimp in Alaska (Fu and Quinn, 2000), tiger crab (Wang and Ellis, 2005), European hake (Drouineau et al. 2010) and juvenile winter flounder in Rhode Island (DeLong et al. 2001), which means that the parameters of gamma distribution estimated as part of stock assessment. Both parameters in gamma are estimated in the Sullivan's model but Zheng et al. (1996) assume a fix value for the parameter $\beta$.

From (2.34) and (2.36) the parameter $\alpha_{l}$ is calculated as:

$$
\alpha_{l}=\frac{l^{*}+\bar{\Delta}_{l}}{\beta}
$$

and that parameters $\alpha_{l}$ are functions of $L_{\infty}$ and $k$ if $l^{*}$ and $\beta$ are known.

Finally the growth increment transition probabilities, which make matrix $\mathbf{P}$, from length class $l$ to length class $l$ can be calculated as:

$$
P_{l, i}=\int_{l}^{i} g\left(x \mid \alpha_{l}, \beta\right) d x
$$

There are $L \mathrm{x} T$ observations of catch at length $l$ and time $t$. Equation (2.30) is the observed catch, which is different from predicted catch with additive normally distributed error $v_{l, t} \sim N\left(0, \sigma^{2}\right):$

$$
\dot{C}_{l, t}=C_{l, t}+v_{l, t}
$$

If, however, fishing effort $E_{t}$ is available then it can be assumed that the full-recruitment fishing mortality deviates from the constant catchability (Quinn and Deriso, 1999: Chapter 8):

$$
f_{t}=q E_{t} e^{\epsilon_{q}}
$$

where $e^{\epsilon_{q}}$ is a lognormal error, $\epsilon_{q} \sim N\left(0, \sigma^{2}\right)$

The classical approach to estimate the main population model parameters, in the catch-atlength stock assessment method, is the least square method due to being robust to error structure (Sullivan et al., 1990; Zheng et al., 1995 and 1996; Fu and Quinn, 2000). If the process error in (2.26) is negligible ( $\omega_{l, t}=0$ ), from (2.26), (2.40) the least square can be used to minimise the residual sums of squares:

$$
R S S_{L}=\sum_{l} \sum_{t}\left(\dot{C}_{l, t}-C_{l, t}\right)^{2}+R S S_{a u x}
$$

In application to catch-at-length model to a simulated Pacific cod (gadus macrocephalus), commercial pseudotolithus typus catch-at-length and market sampling data from commercial catch of decapterus russellii, Sullivan et al. (1990) found that the measurement-error model works well if the distribution of recruitment is modelled with care and the gamma parameter $\beta$ is kept fixed.

DeLong et al. (2001) adopted the length-based model proposed by Sullivan et al. (1990) and modified it to study the effect of density and environmental factors on the growth and mortality of winter flounders. The model construct is an analysis based on catch-at-length in which growth is stochastic and recruitment is separated into time and length factors. The mortality and growth is calculated directly from the data and tested for their correlation with average length, density and temperature, $Z_{t+1}=-\log \left(\frac{N_{t+1}}{N_{t}}\right), G r_{t+1}=\bar{L}_{t+1}-\bar{L}_{t}$. The results are interesting and show significant correlations among monthly growth and mean length, density, mortality and monthly water temperature. The study implies that the environmental factors and density should also be considered in modelling the stock assessment process. Meise et al. (1999) too developed a length-based population model for winter flounder based on the model that Sullivan et al. (1990) proposed with a lengthdependant natural mortality; but the length-based selectivity function which is an important factor in catch-at-length model is not included. The only mortality factor is modelled as a power function of length $\left(Z=\alpha l^{-\beta}\right)$, which is not separated into fishing and natural mortality.

Using the survey data only, Dobby (2004) applies the Sullivan et al. (1990)'s size-structure approach to assess the trends in West of Scotland haddock. By comparing the results with the ICES assessment, which relies heavily on commercial catch-at-age data, she aimed at exploring whether the model can be extended to anglerfish, for which the official report of landing data is not reliable. Survey length frequency $\left(I_{l, y}\right)$ is modelled as a product of survey catchability $\left(q_{l}\right)$, modelled population numbers and survival rate, $I_{l, y}=q_{l} \widehat{N}_{l, y} e^{-p z_{l, y}}$. This paper, however, states that the mortality is known (i.e. not estimated by the model) and taken from external length at age data.

Wang and Ellis (2005) use the length-frequency data of tiger prawn to estimate the natural and fishing mortality. They have also considered individual variability in growth. The lengthbased models along with the modified versions have also been applied to red king crab (Zheng et al., 1995) and shrimp (Etim and Sankare, 1998).

Nevarez-Martinez et al. (2010) propose a catch-at-size model for jumbo squid, which does not depend on fishing effort or gear selectivity. The probability of catch is modelled as a logistic function and does not rely on the abundance.

The weak point of the least square method, nevertheless, is that it could provide biased estimation due to ignoring the individual variation. The maximum likelihood estimation (MLE) method is an alternative method for estimation of the confidence intervals for the model parameters (Meise et al., 1999; DeLong et al., 2001; Wang and Ellis, 2005; Shackell et al., 1997; Haddon et al., 2008; Drouineau et al., 2010). The second derivative of the loglikelihood of the probability distribution function, which is fitted to the length frequency data, is set equal to zero to estimate the values that may maximise the function. Although the maximum likelihood estimation method is generally slower than the least square method in finding the best value, it is more structured due to the fact that it applies a statistical distribution as a base function. Nevertheless, the support for the assumptions of the distribution is not always accurate. Hence, the estimator is not always precise for the stock assessment.

The Bayesian framework (Fournier and Doonan, 1987; Fournier et al., 1998; Hillary, 2011; Cook, 2013) is an alternative method for to both least square and maximum likelihood. The Bayesian framework allows for more natural interpretation of the parameter confidence interval, which is easier to assess as a parameter density (Congdon, 2004). The main advantage is that the estimated confidence interval can be updated as new information is added.

### 2.3 Discussion and conclusion

Selection of the model very much depends on the availability of the data as well as the purpose of the assessment model. If the actual age data and the natural mortality are known, the age-structured stock assessment method could be a powerful approach. That is because aging is a natural process as individuals move over time and therefore it is easier to observe the cohort. The size-based model is usually applied when there are some growth and catch-atsize information available but not age data. The length-based model, which takes catch-atlength as input, shows to be promising in modelling the marine population abundance and in some situations fits the data very well. Nevertheless, it may underestimate the growth of individuals with younger age (DeLong et al., 2001) or at some time steps. It may not be suitable to model the species that are either very slow growing or have high variability in
growth rate. For that, growth should be modelled with more care to avoid bias and inaccuracy.

The performance of the stock assessment method also relies on the structure of the data. The data for the study of population dynamics are usually taken from commercial landings, survey numbers, fishing effort and tagging. Mark and recapture data, for instance, are more useful for estimating the growth transition matrices. The data that are collected constantly throughout the year enable researchers to produce a time series of the fluctuations in movement as well as growth, while periodically data collection approach fixes the variability of seasonal effect.

The modelling attitude towards the factors and parameters make differences to the accuracy of the stock assessment. Depending on the model structure, factors such as survey and fishing selectivity, density, water temperature, size of the fish, choice of asymptotic maximum length, growth rate and time of the survey could affect the variability of the natural mortality. Mortality and the maximum asymptotic length could improve assessment methods if they are correctly modelled and estimated.

Since length of fish and time are continuous variables, a continuous approach for population abundance is more suitable for the biological process. The continuous equations, however, are extremely complicated to apply. The choice of time steps and the width of length classes are very important in using the discrete models of continuous variable. The discretisation needs to be done with a special care, so that it has the least negative effect on the accuracy of the estimates. The assessment of results of the impact of discretisation of time and fish length on the stock assessment models allows the proposal of approaches to improve the model in case the discrete substitutions are applied (Drouineau et al. 2007). Some of these approaches include reducing the width of the length class as well as shortening the time spans.

Drouineau et al. (2007) assess the impact of discretisation of length and time on lengthstructured population growth model by fitting three continuous statistical distributions to the length frequency simulated data of European hake and estimating the growth rate parameter, while the asymptotic length is assumed as known. The fitted models are Normal, Lognormal and Gamma. The overall best fits were obtained with the smaller length class interval and the shortest time steps. Also weak inter-individual variability was concluded to improve the estimation of the coefficient parameter. Normal distribution for the growth increment
produces a very weak and biased estimation growth rate parameter. The Gamma process was found to be most appropriate in describing the properties of length frequency data (Froysa et al., 2002; Drouineau et al., 2008). In fact, the flexibility of the gamma distribution could be the reason for its suitability for practically describing the variability of the growth increment.

Catch-at-length data that is used in length-structured models could only be applied to the well-sampled species and some moderately sampled species (section 1.3) for which the number of catch-at-length from commercial landings is recorded. This, however, is not available for all the sampled species. There are some very poorly sampled species (see Chapter 1) for which the only reported commercial data is total landed biomass. For those species, length distributions from the scientific survey data such as International Bottom Trawl Survey (IBTS) are routinely available. A population dynamic model is, therefore, required to make the use of available data and assess the very poorly sampled species stocks. This is the aim of this study and will be discussed in the next few chapters. In this piece of work, a new population dynamic model, so called survey-landings model, was developed. It adopted the underlying catch-at-length approach and developed the survey-landing model to enable it to be fitted to the landed biomass from commercial data and the length distributions of fish from the NS IBTS (North Sea International Bottom Trawl Survey). Hence, under the new model the poor sampled species could be assessed as well.


### 2.4 Appendix 2.1: Some examples of the stock assessment methods, general benefits and disadvantages

| Method | Advantages | Disadvantages | Reference |
| :---: | :---: | :---: | :---: |
| Age-structured (VPA) | Very powerful if the actual age data and the natural mortality are known. | Input catch-at-age is treated as exact. <br> Aging is costly, very difficult or impossible. <br> The model is sensitive to the observation error in mortality rate of the final age class, but it is not always known. <br> Size-dependant selectivity function cannot be taken into account. <br> Recruitment is assumed to be constant throughout the year. | Anderson 1976 <br> Beverton and Holt, 1957 <br> Doubleday, 1981 <br> Fry 1949; <br> Gulland, 1965 <br> Murphy 1965 <br> Pope, 1991 <br> Pope et al., 1982 <br> Pope et al., 1985 <br> Shepherd, 1999 <br> Sims, 1982 \& 1984 <br> Vinther, 2001 |
| Age-length-structured | Age data is not required. <br> Age is derived by the agelength transformation function. <br> Age-based method can still be applied by using catch-at-length data. <br> The biological and fishery selectivity factors are taken into account. | Significant variations in length-at-age are ignored and can cause inaccuracy of estimates of stock. <br> MCMC simulations show weakness in the transformation method. | Basson et al., 1988 <br> Fournier et al., 1990 <br> Fournier et al., 1998 <br> Froysa et al, 2002 <br> (Schnute and Fournier 1980) |
| Length-structured | No age data is required. <br> No steady state assumptions. <br> Size data is easier and cheaper to collect. <br> Biological and fisheriesrelated process is related to size. <br> Models the length specific fishing selectivity and allows fishing effort to vary over time. <br> Growth is modelled by length of individuals. <br> Recruitment is modelled at time over length classes. | Sensitive to the choice of growth parameters in VBL curve and class length width. <br> Natural mortality is affected by changes in fishing and survey gear selectivity. <br> Discretisation can affect the accuracy of the estimates if not done with care. | DeLong et al., 2001 <br> Drouineau et al., 2007 <br> Drouineau et al., 2010 <br> Fu and Quinn, 2000 <br> Hillary, 2011 <br> Meise et al., 1999 <br> Morales-Bojorquez and Nevarez- <br> Martinez, 2010 <br> Parrack, 1992 <br> Schnute et al., 1989 <br> Sullivan et al., 1990 <br> Wang and Ellis, 2005 <br> Zheng et al., 1996 |

## Chapter 3

## Data collection

### 3.1 Introduction

Fisheries data are used in scientific surveys, fisheries management and strategic planning for industry in the communities that rely on fisheries. Fishermen themselves may use fisheries data for their future plan or change of strategy in fishing. However, the primary use of fisheries data is to conduct stock assessment and to evaluate the exploitation of the sea for long term fishing sustainability purposes.

Due to providing information on stock trends, survey data has become an essential part of the annual fish stock assessment. They are used in this research as the standard and reliable observations against which to develop and model the dynamics of the population.

This chapter aims to describe the International Bottom Trawl Survey (IBTS) Working Group and their responsibilities in bringing together the survey. The source of fisheries catch data is also given in this chapter. The standards and the process of data collection and the challenges and risk factors that may reduce the accuracy of the samples and fisheries statistics are also discussed here.

### 3.2 International Bottom Trawl Survey

The International Bottom Trawl Survey Working Group (IBTSWG) is responsible for bringing together multi-species surveys that are conducted by research vessels within ICES areas IIIa or IV-IX. The surveys are otter trawl surveys and follow IBTSWG standardised sampling methods as well as haul duration and vessel speed to ensure that the result is fairly
reliable to use for annual evaluations. Data from all participating nations are stored in DATRAS database and presented at the regular meetings of IBTSWG (ICES, 2012).

The survey data are used in ICES for various assessment purposes including examining changes in the relative population distribution and to calculate the biological parameters of commercial fish species. The first of these surveys was under the former name, International Young Fish Survey (IYFS), for herring in the North Sea and Skagerrak/Kattegat in late 1960s in quarter 1 only, accompanied by a manual for scientists. Having evolved from only herring to roundfish species such as cod, haddock and whiting in 1991, the International Bottom Trawl Survey (IBTS) data set now includes all finfish species. The survey was also expanded to include the other three quarters of the year in 1991. The manual was revised up to eight times by 2012 and other ICES areas were added to the survey coverage area (ICES, 2012).

Until 1983, due to lack of a unified gear, different gears were being used by different nations. Since then the unified multi-purpose gear GOV (Grande Ouverture Verticale) 36/47 were recommended as the standard gear for all the nations participating in the quarter 1 survey; and by 1992 the GOV trawl was used in all quarters of the IBTS. A series of checks are carried out regularly to ensure that the GOV is equipped correctly and securely on the vessel (ICES, 2012).

Since the latest allocation of sampling stations in 1991, the aim is to keep at least one vessel in each ICES subarea ( $30 * 30$ nautical miles). Additionally, three different grids were introduced and the idea was that in every quarter at least four vessels would participate. The initial haul was one hour but changed to the current 30 -minute tow a few times a day.

With regards to fishing method, it is suggested that all nations carry out additional hauls at the start of survey to make sure all the equipment are working correctly. The standard fishing speed is set to be on average 4 knots, with the maximum depth of 200 m in the North Sea and 250 m in Division IIII. The tow, from the time the door spread is vertically stable to when the net goes back in, should take 30 minutes; any time under 15 minutes is considered invalid. The vessels are free to chose anywhere within the statistical allocated rectangle. The study on haddock, dab, Norway pout, whiting and grey gurnards, shows that the departure from the target speed influences their catch rates. For small haddock and whiting, grey gurnard and dab the catch rates increase by speed over ground and the catch rates for Norway pout and large whiting increase by speed through water (Adlerstein and Siegfried, 2002).

Length distributions of all the caught fish species are obtained by measuring to 0.1 cm below for shellfish, to 0.5 cm below for herring and sprat and to 1 cm below for all other species. If the number of caught fish is too large to measure, a representative sample of at least 75 fish is selected instead and raised up (methods for exception occasion are applied). All catches are also sorted to the lowest possible species level and submitted to DATRAS (ICES, 2012).

In some specified sampling areas otolith samples are also collected, but only after their lengths are measured so that the deformation due to extracting otoliths does not affect the length measurement. For haddock, 8 otoliths are taken for every 1 cm class. Sex, maturity and weight of individuals are also recorded for those individuals whose age data are collected. IBTS also collects the marine litter from the GOV trawl (ICES, 2012).

Along with each sampling, some environmental data such as temperature and salinity for surface and bottom are recorded. Surface and bottom current, wind and swell directions and speed are also measured (ICES, 2012). This extra information recorded at the time of survey is particularly useful to investigate any differences between the spatial distributions derived from bottom trawl surveys that could be due to natural and environmental factors. A generalised linear model used in Wieland et al. (2011) revealed that the wind speed prior and during the survey had the significant effect on the cod catch abundance in deeper areas for the trawlers.

Apart from wind speed that could influence the catch rate, bottom trawl surveys face other challenges as well. Engas et al. (1992) show that the trawl catches have different catch rates for cod and haddock at different time of the day; and concluded that bottom trawl surveys struggle to provide representative sample of small fish in the allocated area. The variations in catch rates for all the = species (with the exception of large whiting), between day and night, were also confirmed in the investigation by Adlerstein and Siegfried (2002). Some physical and biological factors such as depth location, water temperature, light and fish density may cause variation between hauls in the day-to-day surveys of cod and haddock. This also could reduce the precision of the annual evaluation of the stock, for which the bottom trawl surveys are used as base sources (Hjellvlk et al., 2004). The type of bottom of the sea also affects the catch rates of the North Sea cod, as they were significantly higher in the stone bottom than the sandy bottom in Wieland et al. (2009)'s study. The variation in the small fish survey for haddock and cod increased linearly with the depth of the location and the density was the important biological factor for small cod samples (Hjellvlk et al., 2004).

The indices in the North Sea IBTS are calculated as mean number at length per haul then per statistical rectangle over the index area by ICES (ICES DATRAS Report 2006); while this work applied slightly a different approach (chapter 6). Since the model output in this research work have been compared with the ICES assessment results, the survey data that are extracted from NS-IBTS from both ICES approach are graphically compared and statistically tested to make sure they are not significantly different from each other.

The ICES calculation method has been duplicated (ICES DATRAS Report 2006) for three blocks of samples of 5 years each. One from the beginning of time line, one from middle and one block from the end. The extracted length distribution of observation from ICES approach are plotted against the survey data extracted in this work to be used in the survey-landings model. Then, the non-parametric independent two-group Mann-Whitney $U$ test is applied for to compare the magnetite and two-sample Kolmogorov-Smirnoff test was applied for the distribution comparison.

Graphs are illustrated in Figure 3.1, which shows no significant distance between the two distributions at each timeline. It is also supported by the statistical tests given in Table 3.1.


Figure 3.1: Extracted NS haddock survey frequency length distribution;
ICES approach: blue circles) survey-landings model approach: red dashed

Table 3.1: Statistical test results for comparison between the ICES method indices survey calculation and survey-landings model

|  | p-value | p-value |  |
| :--- | :--- | :--- | :---: |
| Year | Mann-Whitney U test <br> (ICES Method vs Survey-landings) <br> Kolmogorov-Smirnoff test <br> (ICES <br> model) |  |  |
| 1969 | 0.715 | 0.935 |  |
| 1970 | 0.758 | 0.727 |  |
| 1971 | 0.81 | 0.998 |  |
| 1972 | 0.858 | 0.845 |  |
| 1973 | 0.891 | 1 |  |
| 1990 | 0.788 | 0.935 |  |
| 1991 | 0.785 | 0.935 |  |
| 1992 | 0.952 | 1 |  |
| 1993 | 0.882 | 0.999 |  |
| 1994 | 0.83 | 0.984 |  |
| 2008 | 0.754 | 0.935 |  |
| 2009 | 0.845 | 0.983 |  |
| 2010 | 0.837 | 0.998 |  |
| 2011 | 0.793 | 0.984 |  |
| 2012 | 0.793 | 0.935 |  |

Frequency of haul numbers in stats rectangles are also illustrated in Figure 3.2



Figure 3.2: Spatial distribution of the North Sea hauls at stat-rectangle at year

### 3.3 Fisheries landing data

### 3.3.1 Data collection

Twenty ICES member countries submit fishing effort and fisheries landing data of more than 200 fish and shellfish species in the Northeast Atlantic. While this has been practised since

1904 beginning for data from the year 1903, currently the data is collected in collaboration with Statistical Office of the European Communities (EUROSTAT) covering from 1973. Catches of fish, crustaceans and other aquatic organisms are reported in whole tonnes live weight (TLW) to the nearest 1000 tonnes by species, fishing area, country and year (Lassen et al., 2012).

Vessels with length 10 meters and under are not legally required to report their catches; instead all registered buyers are required to report commercially sold fish (GOV.UK: Fishing data collection, coverage, processing and revisions).

Having been set out by EU legislation, for the vessels over 10 metres in length, the details of catch by species are collected in a fishing logbook within 48 hours of sale for every trip and each activity within the trip. Following the standards defined by ICES, the fishing gear and the geographical fishing areas are also reported. Currently all the vessels 12 metres and longer also have to report the landing declaration electronically. Fisheries administration conduct checking, accordingly for accuracy and validation (GOV.UK: Fishing data collection, coverage, processing and revisions).

The aim of collecting catch data is to use it for management of marine resources, for which availability of high quality data is essential (Lassen et al., 2012). This data provides a platform for researchers as well as management and policy makers for structure and economic development.

The official catch and landing data are documented by the national statistical offices and kept in Excel spreadsheets as well as in csv format, which are not corrected for non-reported landings. Therefore, in some cases, the data might be different from those presented in ICES fish stock assessment working group reports (Lassen et al., 2012; Catch Statistics 20062013).

The electronic data set was originally only available from 1973 onward. However, due to increase of demand from fishery managers, scientists and environmentalists to extend the database to include earlier years; the data has now been made available on CDs and ICES website. The data for 1903 to 1949 are provided in Excel Workbooks for each country and are available only on the ICES website. The main species that were reported in this period are herring, cod, haddock and plaice and very few data on shellfish. The catch data since 1950 is available from the Eurostat and ICES websites. The data from 1950 up to 2008 is in the form
of annual time series for each species in the ICES geographical area (Lassrn et al., 2012) (Figure 3.3 and Figure 3.4).


Figure 3.3: ICES subarea and divisions. Source: CWP Handbook of Fishery Statistical Standards (FAO 2003)


Figure 3.4: ICES subdivision around the British Isles. Source: CWP Handbook of Fishery Statistical Standards (FAO 2003)

### 3.3.2 Catch report accuracy and discrepancies

The breakdown of the geographical areas and the scope of statistical reports changed over years. In the latest update, all the reports from outside the FAO fishing area, Northeast Atlantic Ocean, have been removed. That removal process affected the data prior to 1964. The data from FAO should, in principle, be identical to the Euorostats/ICES data when all the geographical areas were added together. This, however, was not the case for all the data. Although there were some discrepancies in the 2000s data, some of the discrepancies for older data were due to the geographical area definitions. To reduce the scale of discrepancies, FAO took the lead through establishment of CWP for Atlantic Fishery Statistics to develop a standardisation of definitions, classifications, collection and compilation of fishery statistics. ICES and Eurostat also collaborated to validate the submitted data by introducing automatic data-check at the time of data submission to reduce the discrepancies and to provide an efficient source for use. FAO catch data was also used for validation purposes. The
discrepancies between ICES and Eurostat were mainly due to inconsistent reporting of data by national authorities or failure of correction by national authorities to be recorded in the database. The validation was completed in 2009 (Lassrn et al., 2012).

The methodology of data collection and compilation changed several times during the years 1903 to 2008. Because these changes are not documented, it is difficult to assess the consistency of the data set throughout these years. Events such as World War I and II had obvious effects on the data collections. Also after introducing total allowable catches in the 1970s over reporting and under-reporting influenced the accuracy of the catch report.

The accuracy and reliability of collected data very much depend on the information provided by fishermen. Also for a full evaluation of the impact of fishing on the marine ecosystem, the total exploitation of the sea by fishing must be known. In the UK, although legal obligations are set for fishermen to report catches, unreported catch or illegal fishing are estimated to be between $5 \%$ and $13 \%$ of all catches (Agnew et al., 2009). The root of Illegal unreported and unregulated (IUU) is mainly lack of effective control (FAO, Fisheries and Aquaculture Department). The international communities took action in 1992 to emphasise on the use of port State measures for fisheries management and long term sustainability (Doulman and Swan, 2012; FAO, Fisheries and Aquaculture Department). IUU activities are widespread though (Sumaila et al., 2006), however, and this rate is even higher in developing countries due to being more vulnerable to illegal activities (Agnew et al., 2009). Since countries fail to plan any recovery for any decline of stock, unreported catch could significantly influence on stock size and the balance of ecosystem models. The cooperation between regional management authorities and developing tested harvest strategies, as part of controlling activities, would contribute to the reduction of bias reports (Agnew et al., 2009). Also the statistical methods that are used to estimate confidence intervals for IUU (Pitcher et al., 2002; Tesfamichael et al., 2007) provide an overview of the actual catch rate for sustainability evaluation and help the assessment of actual resources.

### 3.4 Discussion

Survey data, which is brought together by IBTS, is assumed to reflect the relative distribution of the fish in the sea. It is also important to realise that IBTS describes a snapshot of the distribution of species in a region at a particular time, while population is a dynamic pattern.

Although it comes across many physical and environmental factors, it is regarded as a reliable source for assessing the distribution of marine species. In our study, it is assumed that the IBTS survey data is proportional to the actual abundance and unbiased representation of the population numbers at length in the sea. It is therefore used in the survey-landings model as one part of the model's observation. In this model the numbers caught over all hauls are summed while the spatial distribution of hauls are given for the variability of potential sampling effort (Figure 3.2).

In order to make sure that the haddock assessment from the survey-landings model is comparable with the ICES assessment (chapter 6), their extracted survey data that are used as observation are compared. The result show that the they are not significantly different.

Fisheries catch and landing data are collected from commercial reports; of which the accuracy heavily depends on the individual reports and the communities control and management to combat IUU. Although it is the main source of official landing statistics of the major species for all the ICES members, it does not provide a description of international effort. This makes the comparison between the stocks of other countries unreliable. However, with regards to the North Atlantic fisheries, it is the most accurate source for fisheries effort and stock assessment as well as fisheries management for policymaking and long-term sustainability in exploitation of the sea. To develop the survey-landings model, the total landed biomass extracted from ICES Working Group report is one of the observation data sets in this study.

There is also another source of fisheries data (which is not used in this study) collected by the UK Sea Fisheries Inspectorate through the flights over UK EEZ to monitor commercial fishing activities. The survey is conducted at each ICES rectangle, approximately once a week throughout the year (Jennings et al., 2000). For every vessel that is observed, information about the nationality, activity, type of gear and date and the time and the altitude of the observation are recorded. The average unit effort data are calculated for the trawlers.

## Chapter 4

# Survey-landings length-structured model for marine population dynamics: Model development 

### 4.1 Introduction

A new length-structured stock assessment model is developed to estimate the unassessed and poorly assessed stocks, (Chapter 1). The survey-landings population dynamic model adopted the catch-at-length approach (Chapter 2) and developed it to make the use of the total landed biomass from commercial reports and the frequency length distributions of fish from the International Bottom Trawl Survey (IBTS).

The survey-landings model aims to improve the description of variability in growth, stock biomass and fishing mortality. In this chapter, the methodology and the development of the survey-landings model are described. All of the components including growth, natural and fishing mortality and recruitment are explored and modelled separately then combined to make a modelled population dynamic.

Total landed biomass and survey numbers as well as population, growth, natural and fishing mortality and recruitment are simulated in the model. At the end of this
chapter, the survey-landings model is taken through basic checks before it is applied to the simulated data and assessed in the next chapter.

Sullivan et al. (1990) used a 'Levenberg-Marquardt' optimisation method (Lourakis, 2005) to find the least square estimates of the model parameters in the catch-at-size analysis. The program was written in FORTRAN 77 in MS-DOS platform. However, all the simulations and analysis programs in this research work are written in the statistical software R version 3.0.3 ${ }^{7}$ (RStudio Version 0.98 .953 - © 2009-2013), which is an open-source environment for statistical computing and graphics. Some algorithms from the packages 'stats', part of the base R built, are applied for the parameter optimisation while exploring the original catch-at-length model as well as in the simulation process. Other methods such as Levenberg Marquardt and optimisation by differential equation are also explored, tested and applied for estimation accuracy and comparison in the model assessment and the application of the model on the real observations.

### 4.2 Method and Model Development

The survey-landings model describes the dynamics of the population in terms of numbers of fish at length over a period of time. Each component of the model, including growth, fishing and natural mortality and recruitment is explored and modelled separately before all are combined to form an improved model of abundance. The input is a time series of total landed biomass (total weight of all the landed fish) from commercial reports and survey numbers in the form of time series of frequency length distributions (Chapter 3). Total discarded biomass is modelled and added to the observations if reliable data are available.

The model is based on three main relationships:

- The relationship between catch and abundance
- The relationship between the abundance at two time steps

[^3]- The relationship between survey and abundance


### 4.2.1 Relationship between catch and abundance

The number of catch $\left(C_{l, t}\right)$ of length $l$ at time $t$ taken by fisheries is related to the population of individuals of length $l$ in the sea $\left(N_{l, t}\right)$ at time $t$ by an exploitation rate $\mu_{l, t}$ and modelled using Baranov catch equation (Ricker, 1975) with log normal process error $e^{\omega_{l, t}}$ :

$$
\begin{equation*}
C_{l, t}=\mu_{l, t} \cdot N_{l, t}+e^{\omega_{l, t}} \tag{4.1}
\end{equation*}
$$

The Baranov exploitation rate $\left(\mu_{l, t}\right)$ represents the proportion of individuals' death that is caused by fishing given that they were at length $l$ at time $t$ at the time of catch:

$$
\begin{equation*}
\mu_{l, t}=\frac{F_{l, t}}{Z_{l, t}} \cdot\left(1-e^{-Z_{l, t}}\right) \tag{4.2}
\end{equation*}
$$

The exploitation rate is related to the fishing mortality $\left(F_{l, t}\right)$ and total mortality $\left(Z_{l, t}\right)$ of individuals at length $l$ at time $t$. Total mortality is the overall mortality combining the fishing and natural mortality.

Fishing mortality rate $\left(F_{l, t}\right)$ at length class $l$ at time $t$ is the mortality rate that is caused by fishing with the units of $\Delta t^{-1}$ ( $\Delta t$ is the time interval). Fishing mortality rate is different for different size of individuals. Selective fishing aims to target and capture the individuals by size (and species) and allowing the others to be avoided or scape unharmed (Marchal et al., 2002). Hence, the fishing gears are adjusted to catch the fish with the desirable or marketable size and tend to avoid very small or
immature fish. Additionally, fishing mortality rate varies by time. Hence, it is affected by the amount of time and fishing gear of a specific type spent fishing as well as the width and the mesh size of fishing trawl gear. Fishing mortality is therefore a product of length-dependant gear selectivity, $s_{l}$, and time-dependent factor, $f_{t}$ :

$$
\begin{equation*}
F_{l, t}=s_{l} f_{t} \tag{4.3}
\end{equation*}
$$

The length-dependent fishing gear selectivity, $s_{l}$, is the probability of fish with length $l$ entering and being retained by the fishing gear. Two assumptions are considered in modelling the fishing gear selectivity. First is similar to the model proposed by Sullivan et al (1990). It is represented by a two-parameter logistic function, which describes that a fish with a bigger size is more likely to be caught and retained in the fishing net than a smaller fish; and all the fish bigger than a particular size are caught (Figure 4.1a).

$$
\begin{equation*}
s_{l}=\frac{1}{1+\alpha_{s} \exp \left(-\beta_{s} l\right)} \tag{4.3a}
\end{equation*}
$$

Where $\alpha_{s}$ is the shape and $\beta_{s}$ is the scale parameter (Figure 4.1a)
The alternative assumption is that not all the big fish are necessarily caught in the fishing net, but the fish behaviour and its experience could change the likelihood. The big fish, which are generally older, would avoid the net due to behavioural differences or greater swimming speed to enable them to void being caught (Figure 4.1b). This is described by the concept of catchability, which may reduce when the fish gets older. The alternative fishing selectivity is therefore assumed to have an increasing logistic trend to reach its peak (i.e. $s_{l}=1$ ) before declining to make a
dome curve. A double logistic function would make the dome curve. As a result, it is modelled as a product of two logistic functions. First is catchability, which is a twoparameter $\left(\alpha_{c}, \beta_{s 2}\right)$ decreasing function by length, and the other one is similar to Equation (4.3a) and represents that fishing trawl is length selective and targeting fish with bigger body size.

$$
\begin{gather*}
s_{l}=\frac{1}{1+\alpha_{c} \exp \left(\beta_{s 2} l\right)} \cdot \frac{1}{1+\alpha_{s} \exp \left(-\beta_{s 1} l\right)} \underset{\alpha_{c}=\alpha_{s}}{\Longrightarrow} \\
s_{l}=\frac{1}{1+\alpha_{s} \exp \left(-\beta_{s 1} l\right)+\alpha_{s} \exp \left(\beta_{s 2} \cdot l\right)+\alpha_{s}^{2} \exp \left(l\left(\beta_{s 2}-\beta_{s 1}\right)\right)} \tag{4.3b}
\end{gather*}
$$

If the two parameters $\alpha_{c}$ and $\alpha_{s}$ are set equal to avoid extra parameter, the lengthdependent fishing gear selectivity would be formulated as Equation 4.3b (Figure 4.1b).

The time-dependent factor, $f_{t}$, in fishing mortality is fishing mortality scalar, which is in fact a measure of overall fishing mortality at the full effect of gear selectivity (i.e. when $s_{l}=1$ ). It is the time-varying parameter to be estimated in the stock assessment model.

Natural mortality, $M$, plays an important role in simulating the population length distribution and is a remarkable factor in controlling the dynamic of the population. It describes the mortality rate caused by any other factors rather than fishing. It is strongly discussed that natural mortality not only depends on the size of the species, but it also changes over time and should not be considered to be a fixed value. One approach to estimate $M$ is first to fix it in the model as a power function of weight (Lorenzen, 1996), and then to use the weight-length relationship (Coull et al. 1989). In this research work, survey-landings model, natural mortality is formulated as a decreasing function of length, but unchanged over time (Figure 4.1e). Due to natural mortality being confounded with fishing mortality, the estimates of these parameters
become very difficult and unreliable when both are unknown parameters in the model. In order to reduce inaccuracy in estimating fishing mortality parameters, the natural mortality is entered as an input vector into the model, while its parameters are estimated outside the model.

$$
\begin{equation*}
M_{l}=\alpha_{M} l^{\beta_{M}} \tag{4.4}
\end{equation*}
$$

The total instantaneous mortality at length class $l$ at time $t$ is the sum of natural and fishing mortality.

$$
\begin{equation*}
Z_{l, t}=M_{l}+F_{l, t} \tag{4.5}
\end{equation*}
$$

Hence, the survival rate is derived as:

$$
\begin{equation*}
S_{l, t}=e^{-Z_{l, t}} \tag{4.6}
\end{equation*}
$$

What is described in the Equation 4.1 is the relationship between the frequency length distribution of catch (catch-at-length) and the frequency length distribution of abundance. However, the catch-at-length is not available in survey-landing model. To make the use of total landed biomass, catch numbers at length are first transformed into catch numbers at weight using the length-weight relationship, which is described by a two-parameter allometric function (Quinn, T. J. and Deriso, R. B., 1999; Coull et al. 1989) (Figure 4.1c):

$$
\begin{equation*}
W_{l}=\alpha_{w} \cdot l^{\beta_{w}} \tag{4.7}
\end{equation*}
$$

Weight distribution of catch (catch biomass for individuals of length $l$ at time $t$ ), $C B_{l . t}$, is the product of the catch frequency at each length class at time $t$ and the weight equivalent of fish at that length:

$$
\begin{equation*}
C B_{l, t}=C_{l, t} \cdot W_{l} \tag{4.8}
\end{equation*}
$$

Using (4.8), the total catch biomass at time $t$ is derived by summing all the individuals' biomass over the length classes:

$$
\begin{equation*}
T C B_{t}=\sum_{l} C B_{l, t} \tag{4.9}
\end{equation*}
$$

Among all the fish caught, in the current regulation, only a fraction is landed and reported. The rest is discarded due to being unmarketable, smaller than the minimum landing size lack of quota or high grading to include larger individuals. The total landed biomass at time $t, T L B_{t}$, which is the total weight of landed fish at time $t$, is then described by:

$$
\begin{equation*}
T L B_{t}=\sum_{l} C B_{l, t} \cdot \text { Plnd }_{l} \tag{4.10}
\end{equation*}
$$

The probability of landing or retention ogive $\left(P \ln d_{l}\right)$ is represented by a twoparameter logistic function to describe the probability of landings for individuals with length $l$. Based on the regulations for minimum landing size, no fish should be landed if they are smaller than a certain size $^{8}$; so the probability of landing should be 0 for the fish smaller than the minimum landing size and 1 for bigger fish. Since it is not always the case in reality (due to human error), it is modelled as a continuous

[^4]function to represent very low landing probability for small fish with a logistic increase with length. To be able to model the probability of landing, it is assumed that around $50 \%$ of fish with minimum landing size be commercially reported as landed (Figure 4.1d). This assumption could change in different species if size is not the only factor for landing but quota and market demand can influence the probability of landing.
\[

$$
\begin{equation*}
P_{-} \ln d_{l}=\frac{1}{1+\alpha_{D} \exp \left(-\beta_{D} l\right)} \tag{4.11}
\end{equation*}
$$

\]

The value 1 in the numerator shows the maximum probability of landing for a fish bigger than a particular size.


Figure 4.1: Example of a) Fishing gear selectivity, logistic curve, b) Fishing gear selectivity, double logistic curve, c) An example of length-weight relationship using Equation 4.7, d) Logistic curve of landing probability, e) Natural mortality decreasing with length

Discard biomass is calculated by subtracting the total landed biomass from total catch biomass. If the discard biomass, $T D B_{t}$, is available, it is added to the model and used to fit the landing parameters:

$$
\begin{equation*}
T D B_{t}=T C B_{t}-T L B_{t} \tag{4.12}
\end{equation*}
$$

### 4.2.2 Relationship between abundance at two time steps

The survey-landings model is a forward running model, in which the abundance (population numbers) at each time step is related to the abundance at the previous time step.

$$
\begin{equation*}
N_{t+1} \propto N_{t} \tag{4.13}
\end{equation*}
$$

Abundance is affected by the growth of individuals at each length class as well as the survival rate. There are also some new fish (recruitment) added to the population to change the abundance distribution. All are considered to model the relationship between the length distributions of population at two time steps.

Since individuals are not necessarily growing at a fixed rate in time, lengthstructured stock assessment methods need estimates of growth parameters to relate length to time. The most common method to estimate the average growth is to use the von Bertalanffy growth function, which relates the average length to age. Mean length at age typically follows a distribution similar to normal distribution with a specific standard deviation. It is because the fish don't spawn at exactly the same time. Those spawned at the beginning of the year are expected to be bigger than those spawned later. The average growth is calculated from the differences between each length or mean length at consecutive ages. The original von Bertalanffy growth function shows that the growth rate of a marine species decline with size, so the rate
of change in length ( $l$ ) can be described by $\frac{d l}{d t}=K\left(L_{\infty}-l\right)$ where $t$ is age, $L_{\infty}$ represents the asymptotic maximum length and $K$ is the growth rate coefficient (curvature) with units of $t^{-1}$. Asymptotic length is the maximum length the species in a stock would attain if they were to grow for a long period. It is not the largest observed size of a species. By integrating the growth rate, the average length at age $t$ is described by Equation 4.14a, where $t_{0}$ is the age of individuals when the fish has zero size (Figure 4.2).

$$
\begin{equation*}
E\left(l_{t}\right)=L_{\infty}\left(1-e^{-K\left(t-t_{0}\right)}\right) \tag{4.14a}
\end{equation*}
$$



Figure 4.2: An example of von Bertalanffy growth curve for 20 age cohorts when $L_{\infty}=80, K=$ 0.2 and $t_{0}=0$. The $y$-axis shows the average length of individuals at time or age. The curves around points represent the possible variability around the average lengths. The size of fish at age 2 , $\left(l_{2}\right)$, is $\mathbf{1 7 . 0 7} \mathbf{~ c m}$ and $l_{3}$ is $\mathbf{3 0 . 5 0} \mathbf{~ c m}$. Hence, the growth increment for a fish with the original length of 17.07 cm is $13.43 \mathrm{~cm}\left(\Delta l=\Delta 2=l_{3}-l_{2}\right)$. The growth increment declines linearly with length.

In Equation 4.14a the length of fish at birth is assumed zero. For many individuals the growth rate in the first year is faster than flowing years. Therefore, the average length $E\left(l_{t}\right)$ at age $t$ is modelled as Equation 4.14b, in which the average length at age zero $\left(L_{0}\right)$ is reformulated from VBL and calculated below zero.

$$
\begin{equation*}
E\left(l_{t}\right)=L_{\infty}-\left(L_{\infty}-L_{0}\right) e^{-K(t)} \tag{4.14b}
\end{equation*}
$$

The age data is not used in survey-landing model. Therefore, the average growth increment $E(\Delta l)$ of a fish that was originally of length $l$ is determined by using the transformed version of the von Bertalanffy (1939) growth function:

$$
\begin{equation*}
E(\Delta l)=\left(L_{\infty}-l^{*}\right)\left(1-e^{-K \Delta t}\right) \tag{4.15}
\end{equation*}
$$

In the transformed function (4.15) that shows the changes of size between two time steps (Figure 4.3), the age parameter has been replaced by time differences $(\Delta t)$. The unit of the time differences $(\Delta t)$ is the time increment over which the data are collected, which is year in this study. The parameter $l$. is the mid length in the length class ( $l$ ). If the length classes interval is short (e.g. 1 cm for some species), the mid length $l^{*}$ and length $l$ are assumed identical.


Figure 4.3: Transformed von Bertalanffy growth curve (Equation 4.15); The $x$-axis is the mid length of individuals and $y$-axis represents the average growth increment of individuals with that length.

The variability of growth increment, $\Delta l$, which accounts for changes and variation in growth between individuals of length $l$, is statistically modelled as a gamma distribution with a shape parameter $\alpha_{l}$, which varies with length of individuals, and a scale parameter $\beta$ (Figure 4.4) and Equation 4.16:

$$
\begin{gathered}
x=\Delta l \sim \operatorname{gamma}\left(\alpha_{l}, \beta\right) \text { for } \Delta l>0, \alpha_{l}>0 \text { and } \beta>0 \\
\rightarrow f\left(\Delta l \mid \alpha_{l}, \beta\right)=\frac{(\Delta l)^{\alpha_{l}-1} \cdot e^{\left(\frac{-\Delta l}{\beta}\right)}}{\beta^{\alpha_{l}} \cdot \Gamma\left(\alpha_{l}\right)} \\
\mu=E(\Delta l)=\alpha_{l} \beta \quad, \quad \sigma_{\Delta l}^{2}=\alpha_{l} \beta^{2}=\beta \mu
\end{gathered}
$$



Figure 4.4: Probability density curve from gamma distribution with different shape and scale values

The flexibility of the gamma distribution in describing different patterns of growth (small or big fish) is the reason that it is suitable for practically describing the variability of the growth increment. Parameters of the gamma distribution are estimated using the transformed von Bertalanffy growth function, because the average change in length $(E(\Delta l))$ is a function of the von Bertalanffy parameters.

Probability of growth is then described by the proportion $P_{l, i}$ of surviving individuals that grow from length class $l$ to length class $l$ during a fixed time interval derived by integrating over the gamma distribution.

$$
\begin{equation*}
P_{l, i}=\int_{l}^{\imath} f(\Delta l) d \Delta l \tag{4.17}
\end{equation*}
$$

The outcome is a probability transition matrix, where columns show the original length of fish at time $t$ and rows represent the length at time $t+1$. With the assumption that fish either grow or stay in the same length and do not shrink; the top right triangle of the matrix is zero (Table 4.1). The fish with the original body length bigger than $L_{\infty}$ is assumed to remain in the length class (Figure 4.3), while the fish with original body length smaller than $L_{\infty}$ can grow beyond $L_{\infty}$ (i.e. if the $L_{\infty}$ is at length 4, there is a probability, $P_{3,5}$, estimated for fish with size 3 at time $t$ to grow to length 5 at time $t+1$ ).

Table 4.1: Example of a probability transition matrix with 5 length classes. The columns are the length at time $t$ and the rows are the length at $t+1$. Cells represent the proportion of individuals with length $l$ at time $t$ growing to another length class at time $t+1\left(P_{i, j}\right.$ is the probability of growing from length class $i$ to length class $j$ ).

|  | $\boldsymbol{l}_{\boldsymbol{t}}=\mathbf{1}$ | $\boldsymbol{l}_{\boldsymbol{t}}=\mathbf{2}$ | $\boldsymbol{l}_{\boldsymbol{t}}=\mathbf{3}$ | $\boldsymbol{l}_{\boldsymbol{t}}=\mathbf{4}=\boldsymbol{L}_{\infty}$ | $\boldsymbol{l}_{\boldsymbol{t}}=\mathbf{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{l}_{\boldsymbol{t}+\mathbf{1}}=\mathbf{1}$ | $P_{1,1}$ | 0 | 0 | 0 | 0 |
| $\boldsymbol{l}_{\boldsymbol{t} \mathbf{1}}=\mathbf{2}$ | $P_{1,2}$ | $P_{2,2}$ | 0 | 0 | 0 |
| $\boldsymbol{l}_{\boldsymbol{t}+\mathbf{1}}=\mathbf{3}$ | $P_{1,3}$ | $P_{2,3}$ | $P_{3,3}$ | 0 | 0 |
| $\boldsymbol{l}_{\boldsymbol{t} \boldsymbol{+}}=\mathbf{4}=\boldsymbol{L}_{\infty}$ | $P_{1, \boldsymbol{L}_{\infty}}$ | $P_{2, L_{\infty}}$ | $P_{3, L_{\infty}}$ | $P_{L_{\infty}, L_{\infty}}=1$ | 0 |
| $\boldsymbol{l}_{\boldsymbol{t + \boldsymbol { 1 }}}=\mathbf{5}$ | $P_{1,5}$ | $P_{2,5}$ | $P_{3,5}$ | $P_{4,5}=0$ | 1 |

Recruits to fishery $\left(R_{l, t}\right)$ are the number of individuals of length $l$ that are added to the fisheries population at each time step (i.e. every year). In fact, new recruits are the individuals that survived over the past time step(s) and are big enough to be caught in fishing nets. Fish that are hatched at year $t$ are too small to be caught in fishing nets. They appear in the fisheries population in the next year when they are about one year old. Therefore, the recruitment term $\left(R_{l, t+1}\right)$ that is added to the population at year $t$ to make the population at year $t+1$ is in fact the recruits that are born in year $t$ and are 1 year old when they appear in the fisheries population. Recruitment may occur over the range of length classes in a time period and throughout the year. It is, therefore, separable into time-dependent, $r_{t}$, and lengthdependent probability, $P_{l}$, components. Here, $r_{t}$ is the total number of recruits at the
beginning of each time step. Then, they are distributed over length classes by a proportion $p_{l}$, derived from another gamma distribution similar to Equation (4.16) with parameters $\left(\alpha_{R}, \beta_{R}\right)$. This is a vector of values ranging from 0 to 1 , representing the proportion of recruits going to each length class:

$$
\begin{equation*}
p_{l}=\int_{1}^{l} f(l) d l \tag{4.18}
\end{equation*}
$$

The expected value of the distribution ( $\mu_{R}=\alpha_{R} \cdot \beta_{R}$ ) is the expected length of fish at the time of recruitment. The length-dependent probability $p_{l}$ calculates the probability of fish growing to length class $l$ at time $t+1$ when it is at the first length class at time $t$. The number of recruits at time $t$ when the fish are at length $l$ is formulated as the product of the total number of recruits at the beginning of time $t$ and the probability of occurrence for each length class.

$$
\begin{equation*}
R_{l, t}=p_{l} \cdot r_{t} \tag{4.19}
\end{equation*}
$$

If the effect of growth probability (Equation 4.17) and the survival rate (Equation 4.6) are combined, plus the new yearly recruits (Equation 4.19), the population dynamic function is then modelled as a form of matrix with log normal error term as it is in the population:

$$
\begin{equation*}
N_{l, t+1}=P_{l, i} \cdot S_{l, t} \cdot N_{l, t}+R_{l, t+1}+e^{\theta_{l, t}} \tag{4.20}
\end{equation*}
$$

### 4.2.3 Relationship between survey and abundance

In order to incorporate survey, it is assumed that the frequency length distribution of survey, $N s_{l, t}$, is directly proportional to the length distribution of actual population in the sea.

$$
\begin{equation*}
N s_{l, t} \propto N_{l, t} \tag{4.21}
\end{equation*}
$$

The proportionality depends on the trawl selectivity of the survey vessel, which is a length-dependent factor, as well as the survey sampling area of the sea.

The length-dependent survey gear selectivity is the probability of fish with length $l$ being caught in survey trawls. Similar to fishing gear selectivity, there are two assumptions in modelling the survey selectivity. First is a three-parameter logistic function, which assumes that the probability of a fish caught in the survey trawl increases logistically by length of fish. Therefore, the bigger fish are more likely to be caught.

$$
\begin{equation*}
q_{l}=\frac{q_{\max }}{1+\alpha_{v} \exp \left(-\beta_{v} l\right)} \tag{4.22a}
\end{equation*}
$$

The numerator, $q_{\max }$, represents the maximum catchability in the sampling area, which is usually set equal to 1 based on the assumption that the fish are evenly spread. However, it is not always the case. Maximum catchability value can be more than 1 if a higher proportion of fish gather in the sampling area (i.e. due to fishing gear which can herd the fish in the area) and can go below 1 otherwise.

In the second assumption, as it was discussed earlier, the selectivity follows a dome shape modelled as a double logistic function.

$$
\begin{equation*}
q_{l}=\frac{q_{\max }}{1+\alpha_{v} \exp \left(-\beta_{v 1} l\right)+\frac{1}{\alpha_{v}} \exp \left(\beta_{v 2} \cdot l\right)+\exp \left(l\left(\beta_{v 2}-\beta_{v 1}\right)\right)} \tag{4.22b}
\end{equation*}
$$

The parameters of the fishing gear selectivity are different from those of the survey gear selectivity function. That is because the fishing selectivity is adjusted to catch the fish with the desirable or marketable size. Survey, however, are conducted with a small mesh so that all length classes are retained and the samples represent the true proportional length distribution of the actual numbers of individuals in the sea.

The proportion of the sampling area $\left(P a_{t}\right)$ is the fraction of the sea that is covered by the survey. It varies over time and is calculated by dividing the total survey swept area for each time step by the whole area of the sea.

$$
\begin{equation*}
P a_{t}=\frac{\sum_{a}(\text { Sweeping area })_{t}}{\text { Area of the Sea }} \tag{4.23}
\end{equation*}
$$

Where $a$ represent the survey area

The sweeping area is derived from the product of the trawl wingspread and the distance that the survey vessel travels during the sampling time. They are extracted from IBTS data:

$$
\begin{equation*}
\text { Sweeping area }\left(m^{2}\right)=\operatorname{wingspread}(m) * \operatorname{distance}(m) \tag{4.24}
\end{equation*}
$$

The relationship between survey and abundance is modelled by combining the two main factors, survey length specific gear selectivity, $q_{l}$, and the proportion of the sampling area $P a_{t}$, with the log normal error term of the population:

$$
\begin{equation*}
N s_{l, t}=q_{l} \cdot P a_{t} \cdot N_{l, t}+e^{\delta_{l, t}} \tag{4.25}
\end{equation*}
$$

### 4.3 Initial Values and basic checks

In this section, the survey-landing model is taken through some basic checks prior to the sensitivity analysis in Chapter 5. It aims to make sure that the model is consistent with the characteristics and dynamics of the population in the sea.

The population dynamics are simulated for 9 years with the assumption of constant natural mortality to make the checking process more straightforward. The 9 years was chosen since it was enough to show the changes and checks. The survey model is not included in the initial checks, therefore survey selectivity parameters and the proportion of sampling area are not used either. The effect of growth, recruitment and mortality are investigated on the population stock. A list of the parameter values is given in Table 4.2. Selection methods for parameter values are not given here because they are discussed in details in the next chapter.

Table 4.2: List of the parameters along with the values used for simulating the dynamics of the population in survey-landing population model

| Parameter | Definition | Parameter value | Parameter | Parameter value |
| :--- | :--- | :--- | :--- | :--- |
| $L_{\infty}$ |  | $8.30 \mathrm{E}+01$ | $p A_{t(t=1, \ldots, 9)}$ | NULL |
| $K$ |  | $2.40 \mathrm{E}-01$ | $f_{t(t=1, \ldots, 9)}$ | $f_{1}=0.4 ; f_{t+1}=f_{t}+$ Uniform $[-0.07,0.07]$ |
| $\beta$ | Gamma <br> distribution for 1.00E-00 | $r_{t(t=1, \ldots, 9)}$ | $3 e 11 * E X P(n=9, \lambda=60)^{9}$ |  |
|  | growth |  |  |  |
| $\alpha_{s}$ | Fishing gear | $5.00 \mathrm{E}+06$ | rhist $t_{t(t=1, \ldots, 9)}$ | $3 e 11 * E X P(n=1, \lambda=60)$ |
| $\beta_{s 1}$ | selectivity | $2.00 \mathrm{E}-01$ | $\Delta t$ | 1 |
| $\beta_{s 2}$ |  | $\Delta L$ | $0.5,1,2$ |  |
| $\alpha_{R}$ | Recruitment | $3.70 \mathrm{E}+01$ | $W_{l}$ | $\left(0.002 l^{1.514}\right) / 1 E+3$ |

[^5]| $\beta_{R}$ | gamma <br> distribution | $5.00 \mathrm{E}-01$ |  |
| :--- | :--- | :--- | :--- |
| $q_{\max }$ |  | NULL |  |
| $\alpha_{v}$ | Survey gear <br> selectivity | NULL | NULL |
| $\beta_{v 1}$ |  | NULL |  |
| $\beta_{v 2}$ |  | $2.00 \mathrm{E}-01$ |  |
| $M$ |  | NULL |  |
| $\alpha_{D}$ | Landing |  |  |
| $\beta_{D}$ | probability | NULL |  |

## Check 1: Constant total abundance

The first check is of total population size (total stock numbers) in an environment where no fish is caught or die due to fishing or natural reasons (Eq. 4.5) and no new fish are born either (Eq. 4.19). As the result, the dynamics of the population are modelled on a single cohort and depend on growth only. Therefore, the population model changes from the original structure in Equation 4.20, $N_{l, t+1}=P_{l, i} \cdot S_{l, t} \cdot N_{l, t}+$ $R_{l, t+1}+e^{\theta_{l, t}}$, to Equation 4.26.

$$
\left\{\begin{array}{c}
Z=0 \Rightarrow S=1  \tag{4.26}\\
R=0
\end{array}\right\} \Rightarrow N_{l, t+1}=P_{l, \dot{c}} \cdot N_{l, t}
$$

Since the population is influenced by growth only, total abundance is expected to be constant and equal to the initial stock number which can be seen to be true in Figure 4.5.


Figure 4.5: Check 1; with the absence of mortality and recruits, total stock size at each time step is equal to the initial total size year 1

## Check 2: Average length at cohort

In the population model based on growth only (Eq. 4.26), the simulated average length of individuals in the cohort $\left(\mu L_{t}\right)$ should follow the von Bertalanffy growth curve $\left(E(\Delta l)_{\text {age }}\right)$, which represents the average length of fish at each age class (Figure 4.6).

$$
\begin{gather*}
\mu L_{t}=\sum_{l} \frac{l . N_{l, t}}{\sum_{l} N_{l, t}}  \tag{4.27}\\
E(\Delta l)_{a g e}=L_{\infty}(1-\exp (-k . \text { age })) \tag{4.28}
\end{gather*}
$$



Figure 4.6: Check 2; simulated mean length of cohort in the absence of mortality and recruitment. The average lengths are closer to the von Bertalanffy curve (Black solid line) when the length class width is smaller; 2 cm length-class (red circles), 1 cm length-class (blue stars) and 0.5 cm length-class (green triangles). The asymptotic maximum length is shown by the purple dotted line.

Results suggest a departure between the simulation and the underlying parametric VB model when the length class width is 2 cm (Figure 4.6, red points). However, the gap is reduced by re-running the simulation with 1 cm length class width (Figure 4.6; blue star points). Further, it is even much closer to the target when the model is run with 0.5 cm length-class (Figure 4.6; green triangles). Reassuringly the model approaches the parametric von Bertalanffy growth curve as the discrete size class gets smaller. In the catch-at-length model, Sullivan et al. (1990) used 2 cm length class with length rounded down. In this research work, nevertheless, 1 cm lengthclasses are used.

The distance from the VBL line is due to the gap between the calculated mean length in the data and the VBL model. It gets bigger at older age because the frequencies are lower and the gap is clearer.

## Check 3: Constant natural mortality

In a population modelled by Equation 4.26, where the dynamics of population rely on growth only, natural mortality $(\mathrm{M})$ is added as the new component. When the only cause of death is natural mortality, the total mortality $(Z)$ equals to natural mortality (Eq. 4.5). For simplicity a constant value of mortality, $\mathrm{M}=0.2$, is considered. The results show that despite the distribution changes due to growth (Figure 4.7), the total population size declines exponentially at a correct rate, which is natural mortality rate in this case (Figure 4.8).


Figure 4.7: Check 3; simulated length distribution of population when the dynamics depend on growth and natural mortality only. Each curve represents a time step (cohort) from 1 to 9.


Figure 4.8: Check 3; Total stock size at time when the dynamics of population rely on growth and natural mortality only. The decline is exponential at natural mortality rate

### 4.4 Summary

The survey-landings stock assessment model was developed to use total landed biomass from commercial landing annual reports and length distribution of scientific samples from the IBTS as the model observations. In the case of availability, the discard data from landing reports is also added to the model observations.

Equations (4.1), (4.20) and (4.25) are the main constructions of the linear dynamic system in the survey-landing model:

$$
\begin{gathered}
C_{l, t}=\mu_{l, t} \cdot N_{l, t}+e^{\omega_{l, t}} \\
N_{l, t+1}=P_{l, i} \cdot S_{l, t} \cdot N_{l, t}+R_{l, t+1}+e^{\theta_{l, t}} \\
\text { Survey }_{l, t}=q_{l} \cdot P a_{t} \cdot N_{l, t}+e^{\delta_{l, t}}
\end{gathered}
$$

The model is presented as a modified version of the catch-at-length model. It differs from the original catch-at-length model (Sullivan et al., 1990) in its input and aims to improve the assessment procedure for the species for which very little data about age and/or catch-at-length is available. The model is constructed to use the total landed biomass and length frequency of survey instead of catch numbers at length.

The survey-landings model has also a unique assumption about the fishing mortality model structure. In this model, fishing gear selectivity is assumed to have a dome shape rather than following an increasing logistic curve. Although this assumption is not necessarily feasible for all species, the application of the model in haddock is considered in Chapter 6 .

Error structures $\left(e^{\omega_{l, t}}, e^{\theta_{l, t}}, e^{\delta_{l, t}}\right)$ are differently presented from previously published models assumed to be multiplicative with lognormal distribution for which $\omega_{l, t}, \delta_{l, t}, \theta_{l, t}$ are independently and identically normally distributed with the $\mu=0$. The assumption of the multiplicative error terms is made to make it closer to the noise in real population in the Sea. The assumptions of the noise structures are discussed in Chapter 5 where the sensitivity analysis is performed.

There are two main sets of parameters that are subject to estimation. First, the 15 constant parameters including the parameters describing growth $\left(L_{\infty}, k, \beta\right)$ in Equations (4.15) and (4.16), parameters in the fishing selectivity function $\left(\alpha_{s}, \beta_{s 1}\right.$, $\beta_{s 2}$ ) in Equation (4.3b), parameters that distribute the recruits over length classes ( $\alpha_{R}, \beta_{R}$ ) in Equation (4.18), the parameters describing the shape of survey selectivity ( $q_{\max }, \alpha_{v}, \beta_{v 1}, \beta_{v 2}$ ) in Equation (4.22b) and the parameters of the probability of landing $\left(\alpha_{D}, \beta_{D}\right)$. The assessment algorithm needs a supply of initial abundance value $\left(N_{0}\right)$ for time zero before the start of the estimation process. Sullivan et al. (1990) have estimated it using the proportion of catch at each length class. In the survey-landings model, in the absence of catch-at-length data, it is estimated from the historical recruits and natural mortality inside the model. However, if it can be extracted from other previously assessed sources, $N_{0}$ would be exempt from estimation. The model is also estimating the time-varying parameters $f_{t}$ and $r_{t}$
( $t=1, \ldots$, max year) representing fishing mortality scalar and total number of recruits, respectively. The number of time-varying parameters depends on the number of time steps (years) in the model. To estimate $N_{0}$, some extra $f_{t}$ and $r_{t}$ need to be estimated for the years before the start of the data. The model parameters are defined in Appendix 4.1.

In practice, any parameters that happen to be known are not estimated. This is mainly to remove the confounding between parameters as well as reduce the inaccuracy and complexity of estimation due to the large number of parameters. The two parameters of natural mortality $\left(\alpha_{M}, \beta_{M}\right)$, which is modelled as a decreasing function of length (Equation 4.4), are estimated externally. The natural mortality is then called as an input vector into the model. The parameters ( $\alpha_{w}, \beta_{w}$ ) of weight-length relationship (Equation 4.7) have been estimated for a number of marine species and are added into the model as known parameters for those species (Coull et al., 1989). The proportion of sampling area at each survey year is either available or can be extracted and calculated from IBTS data. Therefore it is not a parameter to be estimated.

The least square estimation method is applied to fit the model onto observations and minimise the residuals sums of squares between the modelled and observed data:

$$
\begin{aligned}
& R S S=\lambda_{1} \sum_{t}\left(T L B_{M}-T L B_{O}\right)^{2} \\
&+\lambda_{2} \sum_{l, t}\left(N s_{M}-N s_{O}\right)^{2}+\lambda_{3} \sum_{t}\left(T D B_{M}-T D B_{O}\right)^{2}
\end{aligned}
$$

Model observations have different scales and units. The landed and discarded biomasses represent weight and have one value for each time step. The survey, however, are in numbers and presented in a matrix of rows, which represent length classes, and columns of time steps. The parameter $\lambda$ is used for degree of confidence in the commercial reports and to make a balance in the scale gap between the survey
and official landings. The parameter $\lambda$, nevertheless, is valued 1 in this work. That is because (as will be discussed in chapter 5) weighting it did not improve the parameter estimation.

All analysis is conducted in the statistical programing environment R version 3.0.3 using RStudio version 0.98 .953 - © 2009-2013. Three optimization methods are applied to find the least square estimates of the model parameters and tested for accuracy and robustness of estimation and convergence. The Levenberg-Marquardt optimisation algorithm (Lourakis, 2005) in package 'minpack.lm’ (Elzhov et al, 2015) is initially used in the simulation process. The 'optim' in 'stats' package (part of base R), using Nelder and Mead (1965) optimisation method, which is reliable for general optimisation, is also applied. Later, package 'DEoptim' (Ardia et al, 2015) for global optimisation, using differential equations, is also applied. It is very much slower that the previous methods, but converges better for some functions.

In Chapter 5 the survey-landing model is used to simulate observations including the length frequency of survey, total landed and discard biomass. Abundance, catch, mortality and recruitment are also generated inside the model. The Twin-experiment method is applied to check the accuracy, sensitivity and robustness of parameters that are subject to estimation. The limitations of the model are investigated and the parameters that can be estimated externally and independently of the model, for its accuracy and robustness in a twin-experiment context, are also discussed.

### 4.5 Appendix 4.1: Model parameters and definitions

Model parameters and definitions of the length-structured survey-landings model

| Parameter <br> codes | Parameter <br> codes in $\mathbf{R}$ | Definitions |
| :--- | :--- | :--- |
| $L_{\infty}$ | Linf | Asymptotic length in von Bertalanffy growth function |
| $k$ | K | Growth rate or curvature in von Bertalanffy growth function |
| $\beta$ | beta | Variability of growth; scale parameter in the gamma distribution for the <br> calculation probability transition matrix |
| $\alpha_{s}$ | alphaS | Shape parameter in the double logistic function of fishing gear selectivity; it <br> controls the position of the curve |
| $\beta_{s 1}$ | betaS1 | Scale parameter in the double logistic function of the fishing gear selectivity; it <br> controls the curvature before the curve reaches its peak point |
| $\beta_{s 2}$ | betaS2 | Scale parameter in the double logistic function of the fishing gear selectivity; it <br> controls the negative slope of the curve after it reaches its peak point |
| $\alpha_{R}$ | alphaR | Shape parameter in the gamma distribution to distribute recruits over length |
| $\beta_{R}$ | betaR | Scale parameter in the gamma distribution to distribute recruits over length |
| $\mu_{R}$ | MuR | Mean parameter in gamma distribution; average length at recruitment |
| $q_{m a x}$ | qmax | Maximum survey catchability in the sampling area |
| $\alpha_{v}$ | alphaV | Shape parameter in the double logistic function of survey selectivity; it controls <br> the position of the curve |
| $\beta_{v 1}$ | betaV1 | Scale parameter in the double logistic function of the survey gear selectivity; it <br> controls the curvature before the curve reaches its peak point |
| $\beta_{v 2}$ | betaV2 | Scale parameter in the double logistic function of the survey gear selectivity; it <br> controls the negative slope of the curve after it reaches its peak point |
| $\alpha_{D}$ | alphaD | Shape parameter in the logistic function of the probability of landing |
| $\beta_{D}$ | betaD | Scale parameter in the logistic function of the probability of landing |
| $\alpha_{M}$ | alphaM | Parameter of allometric function of natural mortality |
| $\beta_{M}$ | betaM | Parameter of allometric function of natural mortality |
| $f_{t}$ | Ft | Numhing effort scalar at time $t$ |
| $r_{t}$ | Number of Recruits at year $t$ |  |
| $r h i s t_{t}$ | Number of historical recruits before the start of simulation |  |
| $\mathbf{R S S}$ | Residual sums of squares |  |

## Chapter 5

## Simulation and testing the survey-landings model

### 5.1 Abstract

The survey-landings model is a new length-structured approach, which relies on the scientific survey and total landed biomass from commercial landings reports to study the dynamics of fish population. For such a complex model it is vital to focus on the most influencing parameters and factors (Arhonditsis and Brett, 2004).

In this chapter the survey-landings model is used to simulate and generate observations (Mesnil, 2003) to estimate stock and mortality. The twin-experiment method is applied to check the accuracy and robustness of parameters that are subject to estimation (Friedrichs, 2001). The model sensitivity analysis employs the Morris one-at-a-time (OAT) method (Morris et al., 2014) with respect to existence and variation of the model parameters. Sensitivity analysis is used to identify which factor is most influential and therefore which parameter is more sensitive to variation. The strength and limitations of the model are discussed with regards to parameters that are estimated within the model and those which can be estimated externally and independently of the model. In order to simplify the model, any parameters that can be estimated externally are excluded from estimation.

The sensitivity analysis indicates that the survey-landings model is capable of providing reliable assessment of the stock. The results of the twin-experiment are promising and confirm the robustness of the model to the noise in the initial parameter values as well as the noise in the observations.

### 5.2 Introduction

Testing and model assessment form an essential part of modelling process to confirm the model is feasible to apply (Morris et al., 2014). The aim of validation of this survey-landings model is to increase reliability in making predictions of the dynamics of fish population in the sea. If the model can obtain robust estimates of parameters, the survey-landings model will then be a promising approach to assess the stock of poorly sampled species, for which neither age nor catch-at-length is available (Mesnil, 2003).

Investigation of the model is conducted within a twin-experiment context (Friedrichs, 2001). The estimability of parameters is examined by perturbing the observations as well as initial parameter values. A similar method was proposed for developing catch-at-length model (Sullivan et al., 1990) but the main difference is that the model was assessed by adding noise to the initial parameters values only.

The survey-landings model is a new approach to modelling the length-structured fish stock assessment to produce a model platform for when there is neither age nor catch-at-length data is available. The double logistic function for fishing selectivity $\left(S_{l}\right)$ makes a dome shape rather than an increasing trend. It also provides a new assumption about fishing mortality $\left(F_{l, t}\right)$ curve (Chapter 4).

The average length at which most recruitment takes place is fixed over years in surveylandings model, although the total recruitment $\left(r_{t}\right)$ is allowed to vary over time. Natural mortality $\left(M_{l}\right)$ is modelled as a decreasing function of length while the fishing mortality $\left(F_{l, t}\right)$ depends on the length of individuals and changes over time too. The sum of natural and fishing mortality is the total mortality $\left(Z_{l, t}\right)$.

The survey-landings stock assessment model is applied to simulate observations including length frequency of survey $\left(N s_{l, t}\right)$ and total landed $\left(T L B_{t}\right)$ and discards $\left(T D B_{t}\right)$ biomass. Other important factors such as total annual recruits $\left(r_{t}\right)$, fishing mortality $\left(F_{l, t}\right)$ and catch-atlength $\left(C_{l, t}\right)$ are estimated in the model. The components of the survey-landings model are gradually added into the model and simple feasibility checks are conducted too. The estimated abundance ( $N_{l, t}$ ) and stock biomass $\left(T S B_{t}\right)$ are then estimated.

The feasibility of the model is tested in three main steps within the twin-experiment context. First the model is applied to the simulated observations to examine if the identical parameters are recovered through estimation. Next, the initial values are muddled to examine the sensitivity of the model in estimating the parameters. At this stage, the parameters are moved one by one into the model from fixed position to fitting position. Finally, the model is applied to the perturbed simulated observations to test the robustness of the model against the variation of observations.

### 5.3 Generating Data

For testing of estimation process, the survey-landings model, which was discussed in Chapter 4, is applied to create observation (Mesnil, 2003) and modelled data for 9 time steps (years).

$$
\begin{align*}
& C_{l, t}=\mu_{l, t} \cdot N_{l, t}+e^{\omega_{l, t}}  \tag{5.1}\\
& N_{l, t+1}=P_{l, i} \cdot S_{l, t} \cdot N_{l, t}+R_{l, t+1}+e^{\theta_{l, t}}  \tag{5.2}\\
& N s_{l, t}=q_{l} \cdot P a_{t} \cdot N_{l, t}+e^{\delta_{l, t}} \tag{5.3}
\end{align*}
$$

The catch $\left(\boldsymbol{C}_{l, t}\right)$ for given length class and time is related to the number of individuals in the sea as well as the exploitation rate $\left(\mu_{l, t}\right)$. The relationship between the population at time $t$ and $t+1$ is described in terms of number of fish surviving and growing to next length class $\left(\boldsymbol{P}_{l, \boldsymbol{l}}\right)$ as well as number of new fish $\left(\boldsymbol{R}_{\boldsymbol{l}, \boldsymbol{+ 1}}\right)$ added to the population at year $t+1$. Recruitment at year $t+1$ in the survey-landings model is the individuals that are born at year $t$ when they are too small to be captured in survey vessels. They are added to the fisheries in the next year $(t+1)$ when they are about one year old. The average length at recruitment is set to 14.50 cm (first peak in length frequency of stock) in the simulation, which is in fact the average length at which the individuals are one year old in von Bertalanffy growth curve (Figure 5.2b; Figure 5.6).

The number of individuals in survey $\left(\boldsymbol{N} \boldsymbol{s}_{\boldsymbol{l}, t}\right)$ is assumed to be proportional to the actual population size $\left(\boldsymbol{N}_{\boldsymbol{l}, \boldsymbol{t}}\right)$ (Chapter 4 and Section 5.2.3). This relationship depends on the survey capture selectivity and the proportion of the sea area used for the survey (Eq. 5.3). The first peak in each graph in Figure 5.6 shows the average length at which the fish entered the fisheries, which are usually one year-old fish.

Landing (Figure 5.1a; Figure 5.7) and discard (Figure 5.1c) biomass as well as survey numbers (Figure 5.1d; Figure 5.6) are simulated as the observation data in the survey-landings model. Length frequency of catch (Figure 5.7) and stock (Figure 5.5) and also mortality rates (Figure 5.3; Figure 5.5) are estimated in the model using the generated observations.

The data was generated from an artificial population and assumed that the population's characteristics are known. Having used information available for haddock, parameter values representing growth, mortality, recruitment, population size and survey are selected either from published literature for haddock or individual trials. Using the haddock example is mainly due to availability of information and to ensure the model dynamics represents the observation from natural system as much as possible.

The initial population $\left(N 0_{l, 0}\right)$, representing the population size at time zero (before data started) for individuals at length $l$ is estimated inside the model using 6-year historical recruits. It is an assumption about stock size based on 6 years historical recruits. Biomass values are calculated using a weight-length relationship, $W_{l}$ (Coull et al. 1989) (Figure 5.1e, f $\& \mathrm{~h})$. Smallest length class is set to 2 centimetres with one-centimetre-wide increments until the maximum of 100 centimetres.

The time-varying parameters such as fishing mortality scalar $\left(f_{t}\right)$ (Figure 5.2 d ) and total annual recruitment $\left(R_{t}\right)$ (Figure 5.1i) are allowed to differ from one year to the next and are generated randomly from a uniform and exponential distribution, respectively. Fishing selectivity $\left(S_{l}\right)$ and survey selectivity $\left(q_{l}\right)$ are assumed to be a double logistic function (Eq. $5.3 \mathrm{~b} \& 5.22 \mathrm{~b}$ ), where the catchability goes up to the maximum at a length then reduces for bigger fish. Fishing boats target big and marketable fish, while survey vessels need to take a representative sample of all the fish in the sea (even very small fish) so that the length distribution of captured fish reflects the distribution of the whole population of the fish. Survey selectivity, compared to fishing selectivity, is higher for smaller fish and lower for bigger fish. Therefore, the fishing and survey selectivity parameters are set to reflect fishing
and survey selectivity as it happens in reality (Figure 5.2c). Maximum survey catchability $\left(q_{\max }\right)$ is set to 1 in simulation. In reality $q_{\max }$ is 1 when fish population spread evenly in the sea; and can be bigger if fish are distributed more in the sampling area compared to the rest of the sea area. The proportion of the sampling area $P a_{t}$ (Eq. 4.23) is simulated from a normal distribution with a mean close to the actual survey sampling proportion area that is recorded in IBTS FishBase.

The average length at recruitment was set equal to the average length at age 1 , extracted from von Bertalanffy growth function.

Fishing mortality scalar $\left(f_{t}\right)$, representing the fishing mortality at the maximum selectivity (e.g. when selectivity is 1 ), is then multiplied by the selectivity function to mark the fishing mortality rate at length at time (Figure 5.3). Figure 5.2 f shows the average fishing mortality rate for individuals of length 25 to 42 cm , with the assumption that they have the highest catchability.

Natural mortality $\left(M_{l}\right)$ is modelled to have decreasing trend with length (Figure 5.2e) (DeLong et al., 2001). For specific species, the weight-related natural mortality is calculated first (Lorenzen, 1996) then the rate at length is estimated applying the weight-to-length transformation (Coull et al. 1989). When $M_{l}$ is added to the fishing mortality $\left(F_{l, t}\right)$, it makes the total mortality as shown in (Figure 5.4). High total mortality for small fish is affected by the high natural mortality and increases when the fishing mortality reaches its peak.

Growth increment $(E(\Delta l))$ and then the probability transition matrix $\left(P_{l, i}\right)$ are calculated for each length class using von Bertalanffy function, while the variance in growth $\left(\alpha \beta^{2}\right)$ is set to be half of the mean of the distribution (Section 4.2.2).

Not all the caught fish, in the survey-landings model are landed; but only the fish with the marketable size is allowed landing. In this simulation the minimum landing size is assumed to be at length 27 cm , which means that the fish smaller than 27 cm are to be discarded. Instead of using binomial probability of 0 and 1 , the probability of landing is assumed to be a logistic function (Eq. 4.11), in which the probability of landing for the fish size 27 cm is 0.50 (Figure 5.2; Figure 5.7).

All the results in the next section are based on true values that were obtained from data generation.

Table 5.1: Table of all the parameter values that were used to generate observations

| Parameter | Parameter value | Parameter | Parameter value |
| :--- | ---: | :---: | ---: |
| $L_{\infty}$ | $7.00 \mathrm{E}+01$ | $f_{8}$ | $4.16 \mathrm{E}-01$ |
| $K$ | $2.40 \mathrm{E}-01$ | $f_{9}$ | $3.73 \mathrm{E}-01$ |
| $\beta$ | $5.00 \mathrm{E}-01$ | $r_{1}$ | $7.71 \mathrm{E}+09$ |
| $\alpha_{s}$ | $5.00 \mathrm{E}+06$ | $r_{2}$ | $1.98 \mathrm{E}+09$ |
| $\beta_{s 1}$ | $6.00 \mathrm{E}-01$ | $r_{3}$ | $5.44 \mathrm{E}+09$ |
| $\beta_{s 2}$ | $2.00 \mathrm{E}-05$ | $r_{4}$ | $2.69 \mathrm{E}+09$ |
| $\beta_{R}$ | $3.00 \mathrm{E}-01$ | $r_{5}$ | $3.72 \mathrm{E}+09$ |
| $q_{\text {max }}$ | $1.00 \mathrm{E}+00$ | $r_{6}$ | $1.85 \mathrm{E}+09$ |
| $\alpha_{v}$ | $4.79 \mathrm{E}+06$ | $r_{7}$ | $1.53 \mathrm{E}+09$ |
| $\beta_{v 1}$ | $1.28 \mathrm{E}-01$ | $r_{8}$ | $8.18 \mathrm{E}+08$ |
| $\beta_{v 2}$ | $4.73 \mathrm{E}-01$ | $r_{9}$ | $5.51 \mathrm{E}+09$ |
| $f_{1}$ | $4.00 \mathrm{E}-01$ | $r h i s t_{1}$ | $1.39 \mathrm{E}+09$ |
| $f_{2}$ | $3.60 \mathrm{E}-01$ | $r h i s t_{2}$ | $6.08 \mathrm{E}+09$ |
| $f_{3}$ | $3.94 \mathrm{E}-01$ | $r h i s t_{3}$ | $2.27 \mathrm{E}+10$ |
| $f_{4}$ | $3.43 \mathrm{E}-01$ | $r{ }_{2}$ | $r_{i s t_{4}}$ |
| $f_{5}$ | $3.94 \mathrm{E}-01$ | rhist $_{5}$ | $5.58 \mathrm{E}+08$ |
| $f_{6}$ | $3.39 \mathrm{E}-01$ | rhist $_{6}$ | $1.32 \mathrm{E}+09$ |
| $f_{7}$ | $4.00 \mathrm{E}-01$ |  | $5.68 \mathrm{E}+08$ |



Figure 5.1: Simulated observations and population dynamics components using the survey-landings stock assessment model; a) Total landed biomass; b) Total catch biomass; c) Total discards biomass; d) Total survey numbers; e) Total survey biomass; f) Total landing numbers; g) Total stock numbers; h) Total stock biomass; i) Total recruits.


Figure 5.2: Components and parameters used for simulation; a) Logistic curve of probability of landing for individuals at each length class; Due to the minimum landing size being set at 27 cm , the probability of landing for a $27-\mathrm{cm}$ fish is 0.50 ; b) Average length at age in von Bertalanffy growth curve; c) Double logistics fishing selectivity (solid purple) and survey selectivity (dashed orange) curve; d) Fishing scalars (Ft) represents the fishing mortality at the maximum fishing selectivity (i.e. $S_{l}=1$ ); e) Natural mortality decreasing by length of fish; f) Average fishing mortality rate for age 2-4 or 25-42 cm individuals after the influence of fishing selectivity.


Figure 5.3: Simulated fishing mortality rate $\left(F_{l, t}\right)$ at length for 9 time steps


Figure 5.4: Simulated total mortality rate $\left(Z_{l, t}\right)$ at length for 9 time steps


Figure 5.5: Simulated length frequency of stock ( $N_{l, t}$ ) for 9 time steps (year 1 to year 9). The graph shows the stock numbers at each length class at time steps (years)


Figure 5.6: Simulated length frequency of survey ( $N s_{l, t}$ ) for 9 time steps. The graphs illustrate the number of fish at each length class that is captured in survey vessels at each time step (years)


Figure 5.7: Simulated catch-at-length $\left(C_{l, t}\right)$ at time (solid purple) and simulated length frequency of landings ( $L n_{l, t}$ ) (dotted green); difference between catch and landing is mainly for small fish where the probability of landing is very small

### 5.4 Accuracy of the survey-landings model

In this section three different approaches are applied to test the accuracy, limitations and robustness of the survey-landings stock assessment model.

First the model is applied on the simulated observations, while the initial values of all the parameters are set to true values. True values are the parameter values that were used for simulation. Since neither the simulated observations nor the parameter initial values change, a feasible model is expected to recover the true parameters values with very small errors. Next, the parameters are moved one by one from fixed position to fitting position. At each step the initial values are perturbed to examine the sensitivity and estimability of the model parameters. Finally, the model is applied to the noisy simulated observations to test the robustness of the model against the variation of observations.

The least square method (Nelder and Mead, 1965) is used to estimate the parameters by minimising the residual sums of squares. In order to make the assessment process easier to follow, only the graphs of total numbers and biomass along with the estimated parameters are presented here. For each step, the graphs of the length distribution of stock, survey, catch; landing and mortality are presented in Appendix 5.1.

Table 5.2 lists the input data set (observations) and the data that are estimated as well as fixed and fitting parameters and the input vectors that are calculated or estimated externally. In the following sections the fitting parameters move between fitting and fixed position depends on the characteristics of the validation.

## Table 5.2: Survey-landings model components

| Observation data | Estimated data | Fixed Parameters | Fixed vectors | Fitting Parameters |
| :---: | :---: | :---: | :---: | :---: |
| - Survey numbers at length at time <br> - Total landed biomass <br> - Total discards biomass | - Length distribution of fishing mortality <br> - Length distribution of total mortality <br> - Length distribution of stock numbers <br> - Length distribution of catch <br> - Length distribution of landings <br> - Total catch biomass <br> - Total landed biomass <br> - Total discards biomass <br> - Total stock numbers <br> - Total stock biomass | dt dL $\alpha_{D}$ $\beta_{D}$ $\mu_{R}$ | $\begin{aligned} & W_{l} \\ & M_{l} \\ & P a_{t} \end{aligned}$ | $\begin{gathered} L_{\infty} \\ K \\ \beta \\ \alpha_{s} \\ \beta_{s 1} \\ \beta_{s 2} \\ \beta_{R} \\ q_{\max } \\ \alpha_{v} \\ \beta_{v 1} \\ \beta_{v 2} \\ f_{t} \\ r_{t} \end{gathered}$ |

### 5.4.1 Applying survey-landings model on simulated

## observation; initial parameter set equal to true values

The survey-landings model is applied to the simulated observations. For the first estimation with the simulated observations, all the parameter values are kept equal to the true values that were used to generate observations.

Table 5.3 shows the list of known and fitting parameters. The gap (dt) between two time steps is set to 1 year and length classes are set to be 1 centimetre wide. Parameters $W_{l}$ and $M_{l}$ are vectors of weight (tonnes) and natural mortality ( year $^{-1}$ ) of fish at length ( cm ), respectively. Weight at length can be calculated externally (Coull et al., 1989) for different species and used as input into the model. Natural mortality needs to be estimated externally and used as a known vector in the model. This is because fishing selectivity and natural mortality may be confounded and therefore assumptions of one are required to estimate the other (Cook, 2013). The probability of the survey area $\left(P a_{t}\right)$, if not known as discussed in Section 4.2.3, could be calculated from IBTS data at each survey year. If the minimum landing size is known, the parameters of the landing probability ( $\alpha_{D}, \beta_{D}$ ) function (Equation 4.22) are estimated to mark very small probabilities for fish smaller than minimum landing size; while the curve reaches
around 0.50 for the individuals at the minimum landing size ( 27 cm in this simulation). Then it goes up to the maximum probability of 1 for bigger fish. Exempting the two landing parameters from estimation reduces the complication of the model and boosts the accuracy of other parameter estimation. In the simulation process the gamma distribution parameters for recruitment $\left(\alpha_{R}, \beta_{R}\right)$ was set to mark the average length at recruit, $\left(\mu_{R}=\alpha_{R} . \beta_{R}\right)$, equal to the average length at age $1(13.5 \mathrm{~cm})$ in von Bertalanffy growth function. The first peak in the frequency length distribution of survey is usually when the new fish are added to fisheries. In order to keep the average length at recruitment unchanged, $\mu_{R}$ is kept fixed to the average length at age 1 , which is 13.5 cm in this simulation. The scale parameter $\beta_{R}$, which affects the variance of the distribution, is allowed to vary.

The survey-landings model is able to recover the exact parameters with almost no errors (Figure 5.8). A list of initial and estimated parameter values are given in Table 5.3. This result confirms that the model has one way only to calculate the outputs with the same given input.

Table 5.3: Initial and estimated parameter values when the survey-landings model is applied on the simulated observation, while the initial values are set equal to the true values

| Initial (=true) |  |  | ParameterInitial (=true) value |  | Estimated |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{\infty}$ | $7.00 \mathrm{E}+01$ | $7.00 \mathrm{E}+01$ | $f_{8}$ | $4.16 \mathrm{E}-01$ | $4.16 \mathrm{E}-01$ |
| K | $2.40 \mathrm{E}-01$ | $2.40 \mathrm{E}-01$ | $f_{9}$ | $3.73 \mathrm{E}-01$ | $3.73 \mathrm{E}-01$ |
| $\beta$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $r_{1}$ | $7.71 \mathrm{E}+09$ | $7.71 \mathrm{E}+09$ |
| $\alpha_{s}$ | $5.00 \mathrm{E}+06$ | $5.00 \mathrm{E}+06$ | $r_{2}$ | $1.98 \mathrm{E}+09$ | $1.98 \mathrm{E}+09$ |
| $\beta_{s 1}$ | $6.00 \mathrm{E}-01$ | $6.00 \mathrm{E}-01$ | $r_{3}$ | $5.44 \mathrm{E}+09$ | $5.44 \mathrm{E}+09$ |
| $\beta_{s 2}$ | $2.00 \mathrm{E}-05$ | $2.00 \mathrm{E}-05$ | $r_{4}$ | $2.69 \mathrm{E}+09$ | $2.69 \mathrm{E}+09$ |
| $\beta_{R}$ | $3.00 \mathrm{E}-01$ | $3.00 \mathrm{E}-01$ | $r_{5}$ | $3.72 \mathrm{E}+09$ | $3.72 \mathrm{E}+09$ |
| $q_{\text {max }}$ | $1.00 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ | $r_{6}$ | $1.85 \mathrm{E}+09$ | $1.85 \mathrm{E}+09$ |
| $\alpha_{v}$ | $4.79 \mathrm{E}+06$ | $4.79 \mathrm{E}+06$ | $r_{7}$ | $1.53 \mathrm{E}+09$ | $1.53 \mathrm{E}+09$ |
| $\beta_{v 1}$ | $1.28 \mathrm{E}-01$ | $1.28 \mathrm{E}-01$ | $r_{8}$ | 8.18E+08 | $8.18 \mathrm{E}+08$ |
| $\beta_{v 2}$ | $4.73 \mathrm{E}-01$ | $4.73 \mathrm{E}-01$ | $r_{9}$ | $5.51 \mathrm{E}+09$ | $5.51 \mathrm{E}+09$ |
| $f_{1}$ | $4.00 \mathrm{E}-01$ | $4.00 \mathrm{E}-01$ | rhist ${ }_{1}$ | $1.39 \mathrm{E}+09$ | $1.39 \mathrm{E}+09$ |
| $f_{2}$ | $3.60 \mathrm{E}-01$ | $3.60 \mathrm{E}-01$ | rhist $_{2}$ | $6.08 \mathrm{E}+09$ | $6.08 \mathrm{E}+09$ |
| $f_{3}$ | $3.94 \mathrm{E}-01$ | $3.94 \mathrm{E}-01$ | rhist $_{3}$ | $2.27 \mathrm{E}+10$ | $2.27 \mathrm{E}+10$ |
| $f_{4}$ | $3.43 \mathrm{E}-01$ | $3.43 \mathrm{E}-01$ | rhist $_{4}$ | $5.58 \mathrm{E}+08$ | $5.58 \mathrm{E}+08$ |
| $f_{5}$ | $3.94 \mathrm{E}-01$ | $3.94 \mathrm{E}-01$ | rhist $_{5}$ | $1.32 \mathrm{E}+09$ | $1.32 \mathrm{E}+09$ |
| $f_{6}$ | $3.39 \mathrm{E}-01$ | $3.39 \mathrm{E}-01$ | rhist ${ }_{6}$ | $5.68 \mathrm{E}+08$ | $5.68 \mathrm{E}+08$ |
| $f_{7}$ | $4.00 \mathrm{E}-01$ | $4.00 \mathrm{E}-01$ | Error V |  | 1E-18 |



Figure 5.8: Survey-landings model is applied on the simulated observations, when the initial parameter values set equal to the true values. Simulated observation (solid blue lines) and estimated (red points); a) Total landed biomass; b) Total survey numbers; c) Total catch biomass; d) Total survey biomass; e) Total discards biomass; f) Total landing numbers.

### 5.4.2 Applying survey-landings model on simulated

## observations; Initial parameters values are perturbed

This section aims to test the limitation and the estimability of the survey-landings model parameters. To reduce the complexity of the model, this section explores the parameters that should and can be estimated externally before being added to the model.

The survey-landings model is applied on the simulated observations, while the initial parameter values are perturbed from their true values. The noise factor, $\tau$, added to the initial parameter values, is assumed to follow a normal distribution with mean 0 and the coefficient of variation $\left(c v=\frac{\sigma}{\mu}\right)$ varying up to 0.3 .

The model is assessed for the robustness of recovering the parameters while they are exposed to noise. The coefficient of variation starts from its maximum level of 0.3 , but is reduced if the model cannot recover the parameter at this level of noise.

All of the fitting parameters are moved to the fixed position but taken back one by one to the fitting list; then perturbed using the same process as above before the model is applied to the simulated observations.

### 5.4.2.1 Step1: Estimating $f_{t}, r_{t}$

In the second phase of estimation with simulated observation, all the fitting parameters apart from fishing scalars $\left(f_{t}\right)$ and recruitments $\left(r_{t}\right)$ are considered as known and kept fixed (Table 5.4). Fishing scalar and recruitment are fundamental parameters in the population dynamics and should be estimated inside the model. The two sets of time-varying parameters are perturbed by $30 \%(\mathrm{cv}=0.30)$ of their true values and are subject to estimation. The model is expected to recover the fishing scalar and recruit parameters with any high noise level added to the initial values. That is because all the other parameters that are linked to fishing scalar and recruitment are kept as known. A list of parameter initial values and estimated values is given in Table 5.4.

Observations are recovered and total catch and stock are estimated identical to the simulation (Figure 5.9). The model recovers the recruitment numbers although they are perturbed. The level of the noise added to $f_{t}$ is very high; still fishing scalar parameter and, as the result, fishing mortality are estimated very close to the true values. Although they are slightly underestimated in the final year (Figure 5.10), the result does not affect the survey and stock distribution (Appendix 5.1, Figure 2 \& 3).

Table 5.4: Initial and estimated parameter values; step 1

| Parameter | True value | Initial ( $\pm 30 \%$ ) | Estimated | Parameter | True value | Initial ( $\pm 30 \%$ ) | Estimated |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{1}$ | $4.00 \mathrm{E}-01$ | 3.81E-01 | $4.08 \mathrm{E}-01$ | $r_{1}$ | $7.71 \mathrm{E}+09$ | $7.82 \mathrm{E}+09$ | $7.74 \mathrm{E}+09$ |
| $f_{2}$ | $3.60 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $3.62 \mathrm{E}-01$ | $r_{2}$ | $1.98 \mathrm{E}+09$ | $2.26 \mathrm{E}+09$ | $1.99 \mathrm{E}+09$ |
| $f_{3}$ | $3.94 \mathrm{E}-01$ | $3.54 \mathrm{E}-01$ | $3.93 \mathrm{E}-01$ | $r_{3}$ | $5.44 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $5.46 \mathrm{E}+09$ |
| $f_{4}$ | $3.43 \mathrm{E}-01$ | $3.97 \mathrm{E}-01$ | $3.45 \mathrm{E}-01$ | $r_{4}$ | $2.69 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $2.64 \mathrm{E}+09$ |
| $f_{5}$ | $3.94 \mathrm{E}-01$ | $5.19 \mathrm{E}-01$ | $3.95 \mathrm{E}-01$ | $r_{5}$ | $3.72 \mathrm{E}+09$ | $3.27 \mathrm{E}+09$ | $3.66 \mathrm{E}+09$ |
| $f_{6}$ | $3.39 \mathrm{E}-01$ | $3.04 \mathrm{E}-01$ | $3.40 \mathrm{E}-01$ | $r_{6}$ | $1.85 \mathrm{E}+09$ | $1.49 \mathrm{E}+09$ | $1.83 \mathrm{E}+09$ |
| $f_{7}$ | $4.00 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.03 \mathrm{E}-01$ | $r_{7}$ | $1.53 \mathrm{E}+09$ | $1.47 \mathrm{E}+09$ | $1.70 \mathrm{E}+09$ |
| $f_{8}$ | $4.16 \mathrm{E}-01$ | $2.31 \mathrm{E}-01$ | $4.18 \mathrm{E}-01$ | $r_{8}$ | $8.18 \mathrm{E}+08$ | $5.89 \mathrm{E}+08$ | $1.57 \mathrm{E}+09$ |
| $f_{9}$ | $3.73 \mathrm{E}-01$ | $3.15 \mathrm{E}-01$ | $3.43 \mathrm{E}-01$ | $r_{9}$ | $5.51 \mathrm{E}+09$ | $4.84 \mathrm{E}+09$ | $5.52 \mathrm{E}+09$ |
|  |  |  |  | rhist ${ }_{1}$ | $1.39 \mathrm{E}+09$ | $2.00 \mathrm{E}+09$ | $4.82 \mathrm{E}+09$ |
|  |  |  |  | rhist $_{2}$ | $6.08 \mathrm{E}+09$ | $7.42 \mathrm{E}+09$ | $1.79 \mathrm{E}+10$ |
|  |  |  |  | rhist $_{3}$ | $2.27 \mathrm{E}+10$ | $2.14 \mathrm{E}+10$ | $1.61 \mathrm{E}+10$ |
|  |  |  |  | rhist ${ }_{4}$ | $5.58 \mathrm{E}+08$ | $4.69 \mathrm{E}+08$ | $3.94 \mathrm{E}+07$ |
|  |  |  |  | rhist $_{5}$ | $1.32 \mathrm{E}+09$ | $1.91 \mathrm{E}+09$ | $1.59 \mathrm{E}+09$ |
|  |  |  |  | rhist ${ }_{6}$ | $5.68 \mathrm{E}+08$ | $6.86 \mathrm{E}+08$ | $5.47 \mathrm{E}+08$ |
|  |  |  |  | Error value |  | 1E10 | 2.3 E 7 |



Figure 5.9: Step 1; Simulated (solid blue), model estimation (dotted red) and pre-estimation (dashed black); In (i) the dashed black line is the perturbed initial recruits

b)


Figure 5.10: Step 1; Simulated (solid blue) and estimated (dashed red); a) Fishing mortality scalar ( $f_{t}$ ); b) Average fishing mortality for fish with length 25 to $\mathbf{4 2} \mathbf{~ c m}$; Black dotted dashed line in (a) is the perturbed fishing scalar and in (b) is the pre-estimation average fishing mortality

### 5.4.2.2 Step 2: Estimating $f_{t}, r_{t}$ and $\beta_{R}$

The parameters of fishing mortality scalar $\left(f_{t}\right)$, time dependent recruitment and recruitment scale parameter $\left(r_{t}\right)$ are fitted in the model. The rest of parameters in table $\left(\beta_{R}\right)$ are moved to fixed position. The recruitment gamma parameters in the simulation are set to make the average length at recruitment at $13.5 \mathrm{~cm}\left(\mu=\alpha_{r} . \beta_{r}\right)$. This is the length at which fish are 1 year old in von Bertalanffy growth curve.

In order to keep the average length at recruitment unchanged, the mean of the gamma distribution is set fixed at 13.5 cm . That is because we know that the average length at recruit is the first peak in the length distribution of survey and it can be estimated from observations. The scale parameter $\beta_{r}$, which affects the variance of the distribution is, however, allowed to vary and added to the fitting parameters. Along with the time-varying fishing scalar and recruits, the gamma scale parameter is perturbed by $30 \%$ of its true value.

Survey numbers (Figure 5.13) and yearly recruits (Figure 5.11i) are estimated more accurately when both recruits and the gamma parameter are fitting in the model compared to when $\beta_{r}$ was a fixed parameter. The other two observations $T L B_{t}$ and $T D B_{t}$ are not affected by the
extra fitting parameter. It is mainly the survey number that is more sensitive to the number of annual recruits (Appendix 5.1 Figure 4). Additionally, because the average length at recruit is kept fixed, the survey numbers are expected to recover securely. Fishing scalar, $f_{t}$, however is slightly affected by the variability of recruitment gamma parameter $\beta_{r}$.

Table 5.5: Step 2; Initial and estimated parameter values

| Parameter | True value | Initial ( $\pm 30 \%$ ) | Estimated | Parameter | True value | Initial ( $\pm 30 \%$ ) | Estimated |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\beta_{R}$ | $3.00 \mathrm{E}-01$ | $3.28 \mathrm{E}-01$ | $3.01 \mathrm{E}-01$ | $r_{1}$ | $7.71 \mathrm{E}+09$ | $7.82 \mathrm{E}+09$ | $6.65 \mathrm{E}+09$ |
| $f_{1}$ | $4.00 \mathrm{E}-01$ | 3.81E-01 | $4.16 \mathrm{E}-01$ | $r_{2}$ | $1.98 \mathrm{E}+09$ | $2.26 \mathrm{E}+09$ | $1.96 \mathrm{E}+09$ |
| $f_{2}$ | $3.60 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $3.74 \mathrm{E}-01$ | $r_{3}$ | $5.44 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $5.31 \mathrm{E}+09$ |
| $f_{3}$ | $3.94 \mathrm{E}-01$ | $3.54 \mathrm{E}-01$ | $4.11 \mathrm{E}-01$ | $r_{4}$ | $2.69 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $2.77 \mathrm{E}+09$ |
| $f_{4}$ | $3.43 \mathrm{E}-01$ | $3.97 \mathrm{E}-01$ | $3.60 \mathrm{E}-01$ | $r_{5}$ | $3.72 \mathrm{E}+09$ | $3.27 \mathrm{E}+09$ | $3.80 \mathrm{E}+09$ |
| $f_{5}$ | $3.94 \mathrm{E}-01$ | $5.19 \mathrm{E}-01$ | $4.14 \mathrm{E}-01$ | $r_{6}$ | $1.85 \mathrm{E}+09$ | $1.49 \mathrm{E}+09$ | $1.94 \mathrm{E}+09$ |
| $f_{6}$ | $3.39 \mathrm{E}-01$ | $3.04 \mathrm{E}-01$ | $3.51 \mathrm{E}-01$ | $r_{7}$ | $1.53 \mathrm{E}+09$ | $1.47 \mathrm{E}+09$ | $1.56 \mathrm{E}+09$ |
| $f_{7}$ | $4.00 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.07 \mathrm{E}-01$ | $r_{8}$ | $8.18 \mathrm{E}+08$ | $5.89 \mathrm{E}+08$ | $8.85 \mathrm{E}+08$ |
| $f_{8}$ | $4.16 \mathrm{E}-01$ | $2.31 \mathrm{E}-01$ | $4.19 \mathrm{E}-01$ | $r_{9}$ | $5.51 \mathrm{E}+09$ | $4.84 \mathrm{E}+09$ | $5.49 \mathrm{E}+09$ |
| $f_{9}$ | $3.73 \mathrm{E}-01$ | $3.15 \mathrm{E}-01$ | $3.71 \mathrm{E}-01$ | rhist $_{1}$ | $1.39 \mathrm{E}+09$ | $2.00 \mathrm{E}+09$ | $2.82 \mathrm{E}+10$ |
|  |  |  |  | rhist $_{2}$ | $6.08 \mathrm{E}+09$ | $7.42 \mathrm{E}+09$ | $4.92 \mathrm{E}+09$ |
|  |  |  |  | rhist ${ }_{3}$ | $2.27 \mathrm{E}+10$ | $2.14 \mathrm{E}+10$ | $1.64 \mathrm{E}+10$ |
|  |  |  |  | rhist $_{4}$ | $5.58 \mathrm{E}+08$ | $4.69 \mathrm{E}+08$ | $2.24 \mathrm{E}+08$ |
|  |  |  |  | rhist $_{5}$ | $1.32 \mathrm{E}+09$ | $1.91 \mathrm{E}+09$ | $1.44 \mathrm{E}+09$ |
|  |  |  |  | rhist $_{6}$ | $5.68 \mathrm{E}+08$ | $6.86 \mathrm{E}+08$ | $1.52 \mathrm{E}+09$ |
|  |  |  |  | Error value |  | 1E10 | 5.9E6 |



Figure 5.11: Step 2; a) Total landed biomass, b) Total catch biomass, c) Total discards biomass, d) Total survey numbers, e) total survey biomass, f) Total landing numbers, g) total stock numbers, h) Total stock biomass, i) Recruit numbers; Simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)
a)

b)

Fishing Mortality
(Age: 2-4 yrs) (L: 25-42 cm)

$\begin{array}{llllllll}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \text { Year }\end{array}$

Figure 5.12: Step 2; a) Fishing mortality scalar (Ft); b) Average fishing mortality for length classes 25-42 cm. Simulated (solid blue) and estimated (dashed red); Black dotted-dashed line in (a) is the perturbed fishing scalar and in (b) is the pre-estimation average fishing mortality


Figure 5.13: Step 2; frequency length distribution of survey for 9 years; simulated (solid blue), estimated (red points), pre-estimated (black dotted-dashed line)

### 5.4.2.3 Step 3: Estimating $f_{t}, r_{t}, \beta_{R}, \alpha_{s}, \beta_{s 1}, \beta_{s 2}$

Fishing selectivity follows a double logistic function with three parameters (Equation 3.3b).

$$
\begin{equation*}
s_{l}=\frac{1}{1+\alpha_{s} \exp \left(-\beta_{s 1} l\right)+\frac{1}{\alpha_{s}} \exp \left(\beta_{s 2} \cdot l\right)+\exp \left(l\left(\beta_{s 2}-\beta_{s 1}\right)\right)} \tag{5.4}
\end{equation*}
$$

The survey-landings model is a length-structured stock assessment model and relies on the information on length of fish to estimate the dynamics of the population. Also, fishing selectivity function is formed based on the length of fish to estimate the probability of fish being caught in fishing vessels. Therefore, fishing selectivity parameters need to be estimated inside the model. There is hardly any information about length-based selectivity of marine population available to construct the selectivity without estimation. The additional parameters that are perturbed in this phase (Table 5.6) include shape parameter $\alpha_{s}$, which decides about
the position of the curve, $\beta_{s 1}$, showing the slope, and $\beta_{s 2}$, which is responsible for the reversing slope when the selectivity reaches its maximum.

All three parameters of fishing selectivity have been added noise ( $\mathrm{cv}=0.3$ ) before moved to the fitting position. The survey-landings model is then applied to the simulated observations, while the parameters that are subject to estimation are all perturbed. A list of observations and estimated data and parameters is given in Table 5.6.

As the result of perturbed selectivity parameters, fishing vessels hardly catch any fish smaller than 40 cm (Figure 5.15a). The pre-estimation fishing mortality is therefore very low (Figure 5.15 c ), which affects catch and landings. The pre-estimation landed biomass (Figure 5.14a) and catch biomass (Figure 5.14b) observations are significantly different from the simulated data. Pre-estimation length distribution of catch and landings (Appendix 5.1 step 3) show that they are heavily pushed towards the bigger fish. The noise to fishing selectivity parameters has less effect on the survey and stock numbers (Figure $5.14 \mathrm{~d} \& \mathrm{~g}$ ).

Although the estimated selectivity parameters are not very close to the true parameter values, they are close enough to keep the selectivity at length unaffected. Furthermore, the model is able to estimate the observation data sets very accurately. Time-varying fishing scalar and recruitments parameters recovered well, though not accurately. Considering the huge differences between pre-estimated and true observations, the model did very well in recovering the observations. This result shows that the model is robust to the variability in fishing selectivity parameters (Appendix 5.1, Figures 10-15). That is a promising outcome, because in real data there is little information about the selectivity and that the estimation process starts from initial parameters that might be far from true values. However, $f_{t}$ becomes slightly unsettled with the variation of selectivity parameters and that means that fishing scalar should be dealt with care in perturbing. The list of initial parameter values and estimated values is given in Table 5.6.

Table 5.6: Step 3; True, perturbed and estimated parameter values

| Parameter | True value | $\begin{gathered} \text { Initial } \\ ( \pm 30 \%) \end{gathered}$ | Estimated | Parameter | True value | $\begin{array}{r} \text { Initial } \\ ( \pm 30 \%) \end{array}$ | Estimated |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha_{s}$ | $5.00 \mathrm{E}+06$ | $4.84 \mathrm{E}+06$ | $3.28 \mathrm{E}+06$ | $r_{1}$ | $7.71 \mathrm{E}+09$ | $7.82 \mathrm{E}+09$ | $7.69 \mathrm{E}+09$ |
| $\beta_{s 1}$ | $6.00 \mathrm{E}-01$ | $3.01 \mathrm{E}-01$ | $5.75 \mathrm{E}-01$ | $r_{2}$ | $1.98 \mathrm{E}+09$ | $2.26 \mathrm{E}+09$ | $1.94 \mathrm{E}+09$ |
| $\beta_{s 2}$ | $2.00 \mathrm{E}-05$ | $1.92 \mathrm{E}-05$ | $1.16 \mathrm{E}-05$ | $r_{3}$ | $5.44 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $5.48 \mathrm{E}+09$ |
| $\beta_{R}$ | $4.50 \mathrm{E}+01$ | $2.99 \mathrm{E}-01$ | $2.97 \mathrm{E}-01$ | $r_{4}$ | $2.69 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $2.81 \mathrm{E}+09$ |
| $f_{1}$ | $4.00 \mathrm{E}-01$ | $3.81 \mathrm{E}-01$ | $4.42 \mathrm{E}-01$ | $r_{5}$ | $3.72 \mathrm{E}+09$ | $3.27 \mathrm{E}+09$ | $3.95 \mathrm{E}+09$ |
| $f_{2}$ | $3.60 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.07 \mathrm{E}-01$ | $r_{6}$ | $1.85 \mathrm{E}+09$ | $1.49 \mathrm{E}+09$ | $2.04 \mathrm{E}+09$ |
| $f_{3}$ | $3.94 \mathrm{E}-01$ | $3.54 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $r_{7}$ | $1.53 \mathrm{E}+09$ | $1.47 \mathrm{E}+09$ | $1.65 \mathrm{E}+09$ |
| $f_{4}$ | $3.43 \mathrm{E}-01$ | $3.97 \mathrm{E}-01$ | $3.80 \mathrm{E}-01$ | $r_{8}$ | $8.18 \mathrm{E}+08$ | $5.89 \mathrm{E}+08$ | $1.01 \mathrm{E}+09$ |
| $f_{5}$ | $3.94 \mathrm{E}-01$ | $5.19 \mathrm{E}-01$ | $4.19 \mathrm{E}-01$ | $r_{9}$ | $5.51 \mathrm{E}+09$ | $4.84 \mathrm{E}+09$ | $4.85 \mathrm{E}+09$ |
| $f_{6}$ | $3.39 \mathrm{E}-01$ | $3.04 \mathrm{E}-01$ | $3.51 \mathrm{E}-01$ | rhist ${ }_{1}$ | $1.39 \mathrm{E}+09$ | $2.00 \mathrm{E}+09$ | $9.07 \mathrm{E}+09$ |
| $f_{7}$ | $4.00 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $3.94 \mathrm{E}-01$ | rhist $_{2}$ | $6.08 \mathrm{E}+09$ | $7.42 \mathrm{E}+09$ | $7.51 \mathrm{E}+09$ |
| $f_{8}$ | $4.16 \mathrm{E}-01$ | $2.31 \mathrm{E}-01$ | $3.97 \mathrm{E}-01$ | rhist ${ }_{3}$ | $2.27 \mathrm{E}+10$ | $2.14 \mathrm{E}+10$ | $1.96 \mathrm{E}+10$ |
| $f_{9}$ | $3.73 \mathrm{E}-01$ | $3.15 \mathrm{E}-01$ | $3.36 \mathrm{E}-01$ | rhist ${ }_{4}$ | $5.58 \mathrm{E}+08$ | $4.69 \mathrm{E}+08$ | $2.81 \mathrm{E}+08$ |
|  |  |  |  | $\text { rhist }_{5}$ | $1.32 \mathrm{E}+09$ | $1.91 \mathrm{E}+09$ | $1.30 \mathrm{E}+09$ |
|  |  |  |  | $\text { rhist }_{6}$ | $5.68 \mathrm{E}+08$ | $6.86 \mathrm{E}+08$ | $4.69 \mathrm{E}+08$ |
|  |  |  |  | Error value |  | 7E10 | 1.2 E 7 |



Figure 5.14: Step 3; a) Total landed biomass, b) Total catch biomass, c) Total discards biomass, d) Total survey numbers, e) total survey biomass, f) Total landing numbers, g) total stock numbers, h) Total stock biomass, i) Recruit numbers; Simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.15: Step 3; a) Fishing selectivity curve, b) Fishing scalar and c)Average Fishing mortality rate for length class $\mathbf{2 5 - 4 2} \mathbf{~ c m}$; Simulated or true value (solid blue), estimated (dashed red) and pre-estimation or perturbed (dashed black)

### 5.4.2.4 Step 4: Estimating $\boldsymbol{f}_{t}, \boldsymbol{r}_{t}, \boldsymbol{\beta}_{R}, \boldsymbol{\alpha}_{s}, \boldsymbol{\beta}_{s 1}, \boldsymbol{\beta}_{s 2}, \boldsymbol{\alpha}_{v}, \boldsymbol{\beta}_{v 1}, \boldsymbol{\beta}_{v 2}$

Survey vessels aim to take samples to represent the distribution of the actual population in the sea. The survey selectivity function in the model decides about the probability of an individual in length class $l$ to be captured in survey vessels. Similar to fishing selectivity, the survey selectivity in the survey-landings model follows a double logistic function (Equation $3.22 b)$.

$$
\begin{equation*}
q_{l}=\frac{q_{\max }}{1+\alpha_{v} \exp \left(-\beta_{v 1} l\right)+\frac{1}{\alpha_{v}} \exp \left(\beta_{v 2} . l\right)+\exp \left(l\left(\beta_{v 2}-\beta_{v 1}\right)\right)} \tag{5.5}
\end{equation*}
$$

The maximum catchability, $q_{\max }$, is set to 1 in the simulation. All of the parameters of survey selectivity, apart from catchability, along with the fitting parameters from the previous phase are perturbed by $30 \%$ of their true values and moved to fitting position (Table 5.7). Maximum survey catchability, $q_{\max }$, varies by the distribution of fish in the sampling area. It is not, therefore, an influencing length-based parameter. Because of that, it is kept fixed to assess the sensitivity of the selectivity parameters only.

The noise to the survey selectivity parameters moved the survey selectivity function towards bigger fish. It means that survey vessels catch fewer fish from smaller length classes (Figure $5.17 \mathrm{c})$. Nevertheless, the model is robust to overcome the noisy selectivity parameters and recover the observations and parameters. Fishing scalars, $f_{t}$, are estimated more accurately this time. Not only the total survey numbers and stock biomass are recovered, the model captured all the curves and peaks in the length distribution too (Appendix 5.1; Figures 16-21). This is a particularly important result since the survey selectivity parameters cannot be calculated externally and need to be estimated inside the model.

Table 5.7: Step 4; True, perturbed and estimated parameter values when the survey-landings model is applied on the simulated observations.

| Parameter | True value | Initial $( \pm 30 \%)$ | Estimated | Parameter | True value | $\begin{aligned} & \hline \text { Initial } \\ & ( \pm 30 \%) \end{aligned}$ | Estimated |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha_{v}$ | $4.79 \mathrm{E}+06$ | $5.00 \mathrm{E}+06$ | $5.28 \mathrm{E}+06$ | $r_{1}$ | $7.71 \mathrm{E}+09$ | 7.82E+09 | $8.19 \mathrm{E}+09$ |
| $\beta_{v 1}$ | $1.28 \mathrm{E}-01$ | $1.60 \mathrm{E}-01$ | $1.25 \mathrm{E}-01$ | $r_{2}$ | $1.98 \mathrm{E}+09$ | $2.26 \mathrm{E}+09$ | $2.08 \mathrm{E}+09$ |
| $\beta_{v 2}$ | $4.73 \mathrm{E}-01$ | $3.85 \mathrm{E}-01$ | $4.75 \mathrm{E}-01$ | $r_{3}$ | $5.44 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $5.42 \mathrm{E}+09$ |
| $\alpha_{s}$ | $5.00 \mathrm{E}+06$ | $4.84 \mathrm{E}+06$ | $6.19 \mathrm{E}+06$ | $r_{4}$ | $2.69 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $2.93 \mathrm{E}+09$ |
| $\beta_{s 1}$ | $6.00 \mathrm{E}-01$ | $3.01 \mathrm{E}-01$ | 6.11E-01 | $r_{5}$ | $3.72 \mathrm{E}+09$ | $3.27 \mathrm{E}+09$ | $3.55 \mathrm{E}+09$ |
| $\beta_{s 2}$ | $2.00 \mathrm{E}-05$ | $1.92 \mathrm{E}-05$ | $1.17 \mathrm{E}-05$ | $r_{6}$ | $1.85 \mathrm{E}+09$ | $1.49 \mathrm{E}+09$ | $1.85 \mathrm{E}+09$ |
| $\beta_{R}$ | $4.50 \mathrm{E}+01$ | $2.99 \mathrm{E}-01$ | $3.17 \mathrm{E}-01$ | $r_{7}$ | $1.53 \mathrm{E}+09$ | $1.47 \mathrm{E}+09$ | $1.62 \mathrm{E}+09$ |
| $f_{1}$ | $4.00 \mathrm{E}-01$ | $3.81 \mathrm{E}-01$ | $4.26 \mathrm{E}-01$ | $r_{8}$ | $8.18 \mathrm{E}+08$ | $5.89 \mathrm{E}+08$ | $7.55 \mathrm{E}+08$ |
| $f_{2}$ | $3.60 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $3.88 \mathrm{E}-01$ | $r_{9}$ | $5.51 \mathrm{E}+09$ | $4.84 \mathrm{E}+09$ | $5.64 \mathrm{E}+09$ |
| $f_{3}$ | $3.94 \mathrm{E}-01$ | $3.54 \mathrm{E}-01$ | 4.11E-01 | rhist ${ }_{1}$ | $1.39 \mathrm{E}+09$ | $2.00 \mathrm{E}+09$ | 5.87E+09 |
| $f_{4}$ | $3.43 \mathrm{E}-01$ | $3.97 \mathrm{E}-01$ | $3.57 \mathrm{E}-01$ | rhist ${ }_{2}$ | $6.08 \mathrm{E}+09$ | 7.42E+09 | $1.30 \mathrm{E}+10$ |
| $f_{5}$ | $3.94 \mathrm{E}-01$ | $5.19 \mathrm{E}-01$ | $4.01 \mathrm{E}-01$ | rhist ${ }_{3}$ | $2.27 \mathrm{E}+10$ | $2.14 \mathrm{E}+10$ | $1.68 \mathrm{E}+10$ |
| $f_{6}$ | $3.39 \mathrm{E}-01$ | $3.04 \mathrm{E}-01$ | $3.41 \mathrm{E}-01$ | rhist ${ }_{4}$ | $5.58 \mathrm{E}+08$ | $4.69 \mathrm{E}+08$ | $8.74 \mathrm{E}+08$ |
| $f_{7}$ | $4.00 \mathrm{E}-01$ | 4.85E-01 | $4.05 \mathrm{E}-01$ | rhist ${ }_{5}$ | $1.32 \mathrm{E}+09$ | $1.91 \mathrm{E}+09$ | $1.25 \mathrm{E}+09$ |
| $f_{8}$ | 4.16E-01 | $2.31 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | rhist ${ }_{6}$ | $5.68 \mathrm{E}+08$ | $6.86 \mathrm{E}+08$ | $1.94 \mathrm{E}+08$ |
| $f_{9}$ | $3.73 \mathrm{E}-01$ | $3.15 \mathrm{E}-01$ | $3.71 \mathrm{E}-01$ | Error value |  | 5.2 E 10 | 7.1E6 |



Figure 5.16: Step 4; a) Total landed biomass, b) Total catch biomass, c) Total discards biomass, d) Total survey numbers, e) total survey biomass, f) Total landing numbers, g) total stock numbers, h) Total stock biomass, i) Recruit numbers. Simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.17: Step 4; a) Fishing selectivity, b) Fishing scalar and c) survey selectivity, d) average Fishing mortality for length classes 25 to $\mathbf{4 2} \mathrm{cm}$; Simulated or true value (solid blue), estimated (dashed red) and pre-estimation or perturbed (dashed black)

### 5.4.2.5 Step 5: Estimating $\boldsymbol{f}_{t}, \boldsymbol{r}_{t}, \boldsymbol{\beta}_{R}, \boldsymbol{\alpha}_{s}, \boldsymbol{\beta}_{s 1}, \boldsymbol{\beta}_{s 2}, \boldsymbol{\alpha}_{v}, \boldsymbol{\beta}_{v 1}, \boldsymbol{\beta}_{v 2}, \boldsymbol{q}_{\max }$

At this stage, the maximum catchability parameter, $q_{\max }$, in survey selectivity is under assessment. Although it is usually set to 1 , it can be bigger or smaller than 1 if the fish is not evenly spread in the sampling area. If $q_{\max }=1$ the survey selectivity has the characteristics of probability. That's because the survey selectivity function ranges from 0 to 1 . However, if $q_{\max } \neq 1$, then the survey selectivity can have a maximum value which is equal to $q_{\max }$.

The parameter $q_{\max }$ along with other fitting parameters is perturbed by $30 \%$ of its true value. The only difference between this step and step 4 is that $q_{\max }$ is added noise and subject to estimation. The peak pre-estimation survey selectivity is no longer 1 and is affected by the survey numbers too (Appendix 5.1, Figures 22-27; Figure 5.18d).

The observations and yearly recruits are all recovered (Figure 5.18). The fishing and selectivity curves are estimated with accuracy, in spite of the pre-estimated curves being pulled towards bigger individuals. Although $q_{\max }$ is estimated very close to the true value, the fishing scalar is affected (Figure 5.19). This is quite expected, since $q_{\max }$ decides about the peak point in the selectivity function. When it changes, all the parameters in the selectivity function move around and estimate more or fewer stock, which can change the exploitation rate and fishing mortality. This is a sign of sensitivity in estimating $q_{\max }$ that with a very insignificant variability, the estimation of other parameters is influenced. For that reason, $q_{\max }$ is moved back to known position so that the other parameters are assessed without its interaction.

The consequence of keeping $q_{\max }$ known in the model is that it has to be calculated or estimated externally before added into the model as an input. Similar to the other three parameters in the survey selectivity function, the maximum catchability cannot be estimated externally. One alternative solution is to adjust it manually in different trials prior to fitting then select the value that suits best in the model with regards to estimating the survey data.

Table 5.8: Step 5; True, perturbed and estimated parameter values

|  |  | Initial |  |  | Initial |  |  |
| :---: | :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| Parameter | True value | $( \pm 30 \%)$ | Estimated | Parameter | True value | $( \pm 30 \%)$ | Estimated |
| $q_{m a x}$ | 1 | $8.10 \mathrm{E}-01$ | $1.04 \mathrm{E}+00$ | $r_{1}$ | $7.71 \mathrm{E}+09$ | $7.82 \mathrm{E}+09$ | $6.03 \mathrm{E}+09$ |
| $\alpha_{v}$ | $4.79 \mathrm{E}+06$ | $5.00 \mathrm{E}+06$ | $2.51 \mathrm{E}+06$ | $r_{2}$ | $1.98 \mathrm{E}+09$ | $2.26 \mathrm{E}+09$ | $1.69 \mathrm{E}+09$ |
| $\beta_{v 1}$ | $1.28 \mathrm{E}-01$ | $1.60 \mathrm{E}-01$ | $1.28 \mathrm{E}-01$ | $r_{3}$ | $5.44 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $5.36 \mathrm{E}+09$ |
| $\beta_{v 2}$ | $4.73 \mathrm{E}-01$ | $3.85 \mathrm{E}-01$ | $4.51 \mathrm{E}-01$ | $r_{4}$ | $2.69 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $2.59 \mathrm{E}+09$ |
| $\alpha_{s}$ | $5.00 \mathrm{E}+06$ | $5.81 \mathrm{E}+06$ | $9.43 \mathrm{E}+06$ | $r_{5}$ | $3.72 \mathrm{E}+09$ | $3.27 \mathrm{E}+09$ | $3.50 \mathrm{E}+09$ |
| $\beta_{s 1}$ | $6.00 \mathrm{E}-01$ | $3.55 \mathrm{E}-01$ | $6.20 \mathrm{E}-01$ | $r_{6}$ | $1.85 \mathrm{E}+09$ | $1.49 \mathrm{E}+09$ | $1.65 \mathrm{E}+09$ |
| $\beta_{s 2}$ | $2.00 \mathrm{E}-05$ | $8.09 \mathrm{E}-06$ | $7.97 \mathrm{E}-06$ | $r_{7}$ | $1.53 \mathrm{E}+09$ | $1.47 \mathrm{E}+09$ | $1.74 \mathrm{E}+09$ |
| $\beta_{R}$ | $4.50 \mathrm{E}+01$ | $4.03 \mathrm{E}-01$ | $2.97 \mathrm{E}-01$ | $r_{8}$ | $8.18 \mathrm{E}+08$ | $5.89 \mathrm{E}+08$ | $7.79 \mathrm{E}+08$ |
| $f_{1}$ | $4.00 \mathrm{E}-01$ | $3.81 \mathrm{E}-01$ | $4.24 \mathrm{E}-01$ | $r_{9}$ | $5.51 \mathrm{E}+09$ | $4.84 \mathrm{E}+09$ | $5.16 \mathrm{E}+09$ |
| $f_{2}$ | $3.60 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ | $r h i s t_{1}$ | $1.39 \mathrm{E}+09$ | $2.00 \mathrm{E}+09$ | $2.97 \mathrm{E}+10$ |
| $f_{3}$ | $3.94 \mathrm{E}-01$ | $3.54 \mathrm{E}-01$ | $4.27 \mathrm{E}-01$ | $r h i s t_{2}$ | $6.08 \mathrm{E}+09$ | $7.42 \mathrm{E}+09$ | $6.01 \mathrm{E}+08$ |
| $f_{4}$ | $3.43 \mathrm{E}-01$ | $3.97 \mathrm{E}-01$ | $3.83 \mathrm{E}-01$ | $r h i s t_{3}$ | $2.27 \mathrm{E}+10$ | $2.14 \mathrm{E}+10$ | $1.74 \mathrm{E}+10$ |
| $f_{5}$ | $3.94 \mathrm{E}-01$ | $5.19 \mathrm{E}-01$ | $4.35 \mathrm{E}-01$ | $r h i s t_{4}$ | $5.58 \mathrm{E}+08$ | $4.69 \mathrm{E}+08$ | $6.62 \mathrm{E}+08$ |
| $f_{6}$ | $3.39 \mathrm{E}-01$ | $3.04 \mathrm{E}-01$ | $3.78 \mathrm{E}-01$ | $r h i s t_{5}$ | $1.32 \mathrm{E}+09$ | $1.91 \mathrm{E}+09$ | $1.26 \mathrm{E}+09$ |
| $f_{8}$ | $4.00 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.59 \mathrm{E}-01$ | $r h i s t_{6}$ | $5.68 \mathrm{E}+08$ | $6.86 \mathrm{E}+08$ | $2.02 \mathrm{E}+09$ |
|  | $3.73 \mathrm{E}-01$ | $3.15 \mathrm{E}-01$ | $4.19 \mathrm{E}-01$ |  |  |  | 5.2 E 10 |



Figure 5.18: Step 5; a) Total landed biomass, b) Total catch biomass, c) Total discards biomass, d) Total survey numbers, e) total survey biomass, f) Total landing numbers, g) total stock numbers, h) Total stock biomass, i) Recruit numbers. Simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.19: Step 5; a) Fishing selectivity, b) Fishing scalar and c) survey selectivity, d) average Fishing mortality for length classes 25 to $\mathbf{4 2} \mathbf{~ c m}$; Simulated or true value (solid blue), estimated (dashed red) and pre-estimation or perturbed (dashed black)

### 5.4.2.6 Step 6: Estimating $f_{t}, r_{t}, \beta_{R}, \alpha_{s}, \beta_{s 1}, \beta_{s 2}, \alpha_{v}, \beta_{v 1}, \beta_{v 2}, L_{\infty}$

In this step, the asymptotic length, $L_{\infty}$, in von Bertalanffy growth function is perturbed and added to the fitting parameters (Table 5.9). Parameter $L_{\infty}$ is particularly important in estimating the probability transition matrix. The probability of moving from one length class to another is zero for individuals that reach their asymptotic length. Therefore, an inaccurate $L_{\infty}$ (either too small or to too big) affects the distribution of population and as a result the mortality and survival are influenced.

In this trial, although $L_{\infty}$ is perturbed by $30 \%$ of its true value, the model is able to estimate it very close to its true value. Model observations are estimated very well with insignificant differences. Recruitment is also unaffected (Figure 5.20). However, the result shows that it is confounded with $f_{t}$ and brings instability in the time-varying fishing scalars by estimating them systematically higher than their true values, although fishing and survey selectivity are unaffected (Figure 5.21).

In a different trial, the model is more robust and $f_{t}$ is less affected when the variation of $L_{\infty}$ is only by $20 \%$ of its true value (Figure 5.23).

The asymptotic length $L_{\infty}$ can be estimated externally from the model and used as a known input to avoid confounding with fishing scalar. Survey data is a good source of estimating $L_{\infty}$ by using the length at two clear peaks as the average length in two year-classes in von Bertalanffy function. Even if the external estimation of $L_{\infty}$ is used as a fitting parameter, it is close enough to the true value.

Table 5.9: Step 6; True, perturbed and estimated parameter values

| Parameter | True value | Initial $( \pm 30 \%)$ | Estimated | Parameter | True value | Initial $( \pm 30 \%)$ | Estimated |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{\infty}$ | $7.00 \mathrm{E}+01$ | $8.74 \mathrm{E}+01$ | $7.05 \mathrm{E}+01$ | $r_{1}$ | $7.71 \mathrm{E}+09$ | $7.82 \mathrm{E}+09$ | $6.38 \mathrm{E}+09$ |
| $\alpha_{v}$ | $4.79 \mathrm{E}+06$ | $5.00 \mathrm{E}+06$ | $9.21 \mathrm{E}+06$ | $r_{2}$ | $1.98 \mathrm{E}+09$ | $2.26 \mathrm{E}+09$ | $1.84 \mathrm{E}+09$ |
| $\beta_{v 1}$ | $1.28 \mathrm{E}-01$ | $1.60 \mathrm{E}-01$ | $1.17 \mathrm{E}-01$ | $r_{3}$ | $5.44 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $4.94 \mathrm{E}+09$ |
| $\beta_{v 2}$ | $4.73 \mathrm{E}-01$ | $3.85 \mathrm{E}-01$ | $4.88 \mathrm{E}-01$ | $r_{4}$ | $2.69 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $2.36 \mathrm{E}+09$ |
| $\alpha_{s}$ | $5.00 \mathrm{E}+06$ | $4.84 \mathrm{E}+06$ | $6.48 \mathrm{E}+06$ | $r_{5}$ | $3.72 \mathrm{E}+09$ | $3.27 \mathrm{E}+09$ | $3.58 \mathrm{E}+09$ |
| $\beta_{s 1}$ | $6.00 \mathrm{E}-01$ | $3.01 \mathrm{E}-01$ | 5.92E-01 | $r_{6}$ | $1.85 \mathrm{E}+09$ | $1.49 \mathrm{E}+09$ | $1.46 \mathrm{E}+09$ |
| $\beta_{s 2}$ | $2.00 \mathrm{E}-05$ | $1.92 \mathrm{E}-05$ | $3.24 \mathrm{E}-05$ | $r_{7}$ | $1.53 \mathrm{E}+09$ | $1.47 \mathrm{E}+09$ | $1.43 \mathrm{E}+09$ |
| $\beta_{R}$ | $4.50 \mathrm{E}+01$ | $2.99 \mathrm{E}-01$ | $3.08 \mathrm{E}-01$ | $r_{8}$ | $8.18 \mathrm{E}+08$ | $5.89 \mathrm{E}+08$ | $5.19 \mathrm{E}+08$ |
| $f_{1}$ | $4.00 \mathrm{E}-01$ | $3.81 \mathrm{E}-01$ | 5.52E-01 | $r_{9}$ | $5.51 \mathrm{E}+09$ | $4.84 \mathrm{E}+09$ | $3.97 \mathrm{E}+09$ |
| $f_{2}$ | $3.60 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $5.05 \mathrm{E}-01$ | rhist ${ }_{1}$ | $1.39 \mathrm{E}+09$ | $2.00 \mathrm{E}+09$ | $1.48 \mathrm{E}+09$ |
| $f_{3}$ | $3.94 \mathrm{E}-01$ | $3.54 \mathrm{E}-01$ | $5.28 \mathrm{E}-01$ | rhist ${ }_{2}$ | $6.08 \mathrm{E}+09$ | $7.42 \mathrm{E}+09$ | $1.90 \mathrm{E}+08$ |
| $f_{4}$ | $3.43 \mathrm{E}-01$ | $3.97 \mathrm{E}-01$ | $4.74 \mathrm{E}-01$ | rhist $_{3}$ | $2.27 \mathrm{E}+10$ | $2.14 \mathrm{E}+10$ | $2.24 \mathrm{E}+10$ |
| $f_{5}$ | $3.94 \mathrm{E}-01$ | 5.19E-01 | $5.35 \mathrm{E}-01$ | rhist ${ }_{4}$ | $5.58 \mathrm{E}+08$ | $4.69 \mathrm{E}+08$ | $7.99 \mathrm{E}+08$ |
| $f_{6}$ | $3.39 \mathrm{E}-01$ | $3.04 \mathrm{E}-01$ | $4.67 \mathrm{E}-01$ | rhist $_{5}$ | $1.32 \mathrm{E}+09$ | $1.91 \mathrm{E}+09$ | $1.49 \mathrm{E}+09$ |
| $f_{7}$ | $4.00 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $5.44 \mathrm{E}-01$ | rhist $_{6}$ | $5.68 \mathrm{E}+08$ | $6.86 \mathrm{E}+08$ | $1.06 \mathrm{E}+09$ |
| $f_{8}$ | $4.16 \mathrm{E}-01$ | $2.31 \mathrm{E}-01$ | $6.15 \mathrm{E}-01$ | Error value |  | 5E10 | 2.7 E 7 |
| $f_{9}$ | $3.73 \mathrm{E}-01$ | $3.15 \mathrm{E}-01$ | $6.05 \mathrm{E}-01$ |  |  |  |  |



Figure 5.20: Step 6; a) Total landed biomass, b) Total catch biomass, c) Total discards biomass, d) Total survey numbers, e) total survey biomass, f) Total landing numbers, g) total stock numbers, h) Total stock biomass, i) Annual recruit numbers. Simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black); All parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.


Figure 5.21: Step 6; a) Fishing selectivity, b) Fishing scalar and c) survey selectivity, d) average Fishing mortality for length classes $\mathbf{2 5}$ to $\mathbf{4 2} \mathbf{~ c m}$, d) von Bertalanffy growth curve; Simulated or true value (solid blue), estimated (dashed red) and pre-estimation or perturbed (dashed black); All parameters are perturbed by $\mathbf{3 0 \%}$


Figure 5.22: Step 6; a) Total landed biomass, b) Total catch biomass, c) Total discards biomass, d) Total survey numbers, e) total survey biomass, f) Total landing numbers, g) total stock numbers, h) Total stock biomass, i) Recruit numbers. Simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black). All fitting parameters are perturbed by $\mathbf{3 0 \%}$ apart from $L_{\infty}$, which is added $\mathbf{2 0 \%}$ noise.


Figure 5.23: Step 6; a) Fishing selectivity, b) Fishing scalar and c) survey selectivity, d) average Fishing mortality for length classes 25 to $\mathbf{4 2} \mathrm{cm}$, d) von Bertalanffy growth curve; Simulated or true value (solid blue), estimated (dashed red) and pre-estimation or perturbed (dashed black); All fitting parameters are perturbed by $30 \%$ apart from $L_{\infty}$, which is added $20 \%$ noise.

### 5.4.2.7 Step 7: Estimating $\boldsymbol{f}_{t}, \boldsymbol{r}_{t}, \boldsymbol{\beta}_{R}, \boldsymbol{\alpha}_{s}, \boldsymbol{\beta}_{s 1}, \boldsymbol{\beta}_{s 2}, \boldsymbol{\alpha}_{v}, \boldsymbol{\beta}_{v 1}, \boldsymbol{\beta}_{v 2}, \boldsymbol{K}$

As a parameter in von Bertalanffy growth function, growth rate coefficient (K) shows how fast individuals reach their maximum length. Similar to $L_{\infty}$, it plays a significantly important
role in the growth function and the estimation of the probability transition matrix. In this step $K$ is perturbed by $30 \%$ of its true value and is subject to estimation. Parameter $L_{\infty}$ is, however, moved back to fixed position to make sure the estimation of $K$ is not affected by the variability of $L_{\infty}$.

The results show that the model is robust in estimating observations and population matrices when $K$ is noisy (Figure 5.24). The length distribution of survey and stock as well as catch and landings are also estimated close to simulation (Appendix 5.1 Figures 34-39). However, fishing scalar is again affected by the variation of von Bertalanffy parameter. Although not hugely, the time-varying fishing scalar parameters are systematically over estimated.

A method to avoid the confounding between $K$ and $f_{t}$ is that the parameters of the VBL growth function are estimated externally before importing into model.

Table 5.10: Step 7; True, perturbed and estimated parameter values

| Parameter | True value | Initial $( \pm 30 \%)$ | Estimated | Parameter | True value | Initial $( \pm 30 \%)$ | Estimated |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $2.40 \mathrm{E}-01$ | $2.31 \mathrm{E}-01$ | $2.41 \mathrm{E}-01$ | $r_{1}$ | $7.71 \mathrm{E}+09$ | $7.82 \mathrm{E}+09$ | $4.41 \mathrm{E}+09$ |
| $\alpha_{v}$ | $4.79 \mathrm{E}+06$ | $5.00 \mathrm{E}+06$ | $1.56 \mathrm{E}+07$ | $r_{2}$ | $1.98 \mathrm{E}+09$ | $2.26 \mathrm{E}+09$ | $1.90 \mathrm{E}+09$ |
| $\beta_{v 1}$ | $1.28 \mathrm{E}-01$ | $1.60 \mathrm{E}-01$ | $1.23 \mathrm{E}-01$ | $r_{3}$ | $5.44 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $5.21 \mathrm{E}+09$ |
| $\beta_{v 2}$ | $4.73 \mathrm{E}-01$ | $3.85 \mathrm{E}-01$ | $5.06 \mathrm{E}-01$ | $r_{4}$ | $2.69 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $2.51 \mathrm{E}+09$ |
| $\alpha_{s}$ | $5.00 \mathrm{E}+06$ | $4.84 \mathrm{E}+06$ | $1.41 \mathrm{E}+05$ | $r_{5}$ | $3.72 \mathrm{E}+09$ | $3.27 \mathrm{E}+09$ | $3.53 \mathrm{E}+09$ |
| $\beta_{s 1}$ | $6.00 \mathrm{E}-01$ | $3.01 \mathrm{E}-01$ | $4.43 \mathrm{E}-01$ | $r_{6}$ | $1.85 \mathrm{E}+09$ | $1.49 \mathrm{E}+09$ | $1.59 \mathrm{E}+09$ |
| $\beta_{s 2}$ | $2.00 \mathrm{E}-05$ | $1.92 \mathrm{E}-05$ | $1.20 \mathrm{E}-05$ | $r_{7}$ | $1.53 \mathrm{E}+09$ | $1.47 \mathrm{E}+09$ | $1.63 \mathrm{E}+09$ |
| $\beta_{R}$ | $4.50 \mathrm{E}+01$ | $2.99 \mathrm{E}-01$ | $2.80 \mathrm{E}-01$ | $r_{8}$ | $8.18 \mathrm{E}+08$ | $5.89 \mathrm{E}+08$ | $9.32 \mathrm{E}+08$ |
| $f_{1}$ | $4.00 \mathrm{E}-01$ | $3.81 \mathrm{E}-01$ | $4.69 \mathrm{E}-01$ | $r_{9}$ | $5.51 \mathrm{E}+09$ | $4.84 \mathrm{E}+09$ | $4.79 \mathrm{E}+09$ |
| $f_{2}$ | $3.60 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.47 \mathrm{E}-01$ | rhist ${ }_{1}$ | $1.39 \mathrm{E}+09$ | $2.00 \mathrm{E}+09$ | $7.41 \mathrm{E}+09$ |
| $f_{3}$ | $3.94 \mathrm{E}-01$ | $3.54 \mathrm{E}-01$ | $4.65 \mathrm{E}-01$ | rhist $_{2}$ | $6.08 \mathrm{E}+09$ | $7.42 \mathrm{E}+09$ | $1.56 \mathrm{E}+10$ |
| $f_{4}$ | $3.43 \mathrm{E}-01$ | $3.97 \mathrm{E}-01$ | 4.13E-01 | rhist $_{3}$ | $2.27 \mathrm{E}+10$ | $2.14 \mathrm{E}+10$ | $1.51 \mathrm{E}+10$ |
| $f_{5}$ | $3.94 \mathrm{E}-01$ | 5.19E-01 | $4.68 \mathrm{E}-01$ | rhist ${ }_{4}$ | $5.58 \mathrm{E}+08$ | $4.69 \mathrm{E}+08$ | $1.31 \mathrm{E}+08$ |
| $f_{6}$ | $3.39 \mathrm{E}-01$ | $3.04 \mathrm{E}-01$ | $4.07 \mathrm{E}-01$ | rhist $_{5}$ | $1.32 \mathrm{E}+09$ | $1.91 \mathrm{E}+09$ | $1.31 \mathrm{E}+09$ |
| $f_{7}$ | $4.00 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.84 \mathrm{E}-01$ | rhist $_{6}$ | $5.68 \mathrm{E}+08$ | $6.86 \mathrm{E}+08$ | $3.62 \mathrm{E}+09$ |
| $f_{8}$ | $4.16 \mathrm{E}-01$ | $2.31 \mathrm{E}-01$ | $5.21 \mathrm{E}-01$ | Error value |  | 6.2 E 10 | 1.6 E 7 |
| $f_{9}$ | $3.73 \mathrm{E}-01$ | $3.15 \mathrm{E}-01$ | $4.53 \mathrm{E}-01$ |  |  |  |  |



Figure 5.24: Step 7; a) Total landed biomass, b) Total catch biomass, c) Total discards biomass, d) Total survey numbers, e) total survey biomass, f) Total landing numbers, g) total stock numbers, h) Total stock biomass, i) Recruit numbers. Simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.25: Step 7; a) Fishing selectivity, b) Fishing scalar and c) survey selectivity, d) average Fishing mortality for length classes 25 to 42 cm , d) von Bertalanffy growth curve; Simulated or true value (solid blue), estimated (dashed red) and pre-estimation or perturbed (dashed black)

### 5.4.2.8 Step 8: Estimating $f_{t}, r_{t}, \beta_{R}, \alpha_{s}, \beta_{s 1}, \beta_{s 2}, \alpha_{v}, \boldsymbol{\beta}_{v 1}, L_{\infty}, \boldsymbol{\beta}_{v 2}, K$

In this trial both von Bertalanffy parameters, asymptotic length $L_{\infty}$ and $K$, are perturbed before being added to the list of fitting variables. Although the pre-estimated landing is very different from the simulation, the estimation of landed biomass is not affected by the noise in growth parameters. What is most influenced, however, is the survey (Appendix 5.1 Figures 40). It is also clear in the total survey numbers (Figure 5.26 d ). The survey selectivity also moved away from simulation (Figure 5.27c). When only one of the two growth parameters were perturbed, the model was able to manage the stock and survey well; but it struggles to cope when both are noisy simultaneously (Appendix 5.1 Figures 40-45).

In a different trial, when the growth parameters are perturbed by only $20 \%$ of their true values, even if all the other fitting parameters are added noise by $30 \%$, the model is capable of recovering the observations and parameters including the fishing scalars (Figure 5.28 \& Figure 5.29). The estimation improves a lot when the variation of growth parameter is reduced (Appendix 5.1, Figures 46-51).

Table 5.11: Step 8; true, perturbed and estimated parameter values

| Parameter | True value | Initial $( \pm 30 \%)$ | Estimated | Parameter | True value | Initial <br> ( $\pm 30 \%$ ) | Estimated |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{\infty}$ | $7.00 \mathrm{E}+1$ | $8.74 \mathrm{E}+01$ | $8.10 \mathrm{E}+01$ | $r_{1}$ | $7.71 \mathrm{E}+09$ | $7.82 \mathrm{E}+09$ | $4.46 \mathrm{E}+09$ |
| $K$ | $2.40 \mathrm{E}-01$ | $2.31 \mathrm{E}-01$ | $2.53 \mathrm{E}-01$ | $r_{2}$ | $1.98 \mathrm{E}+09$ | $2.26 \mathrm{E}+09$ | $1.59 \mathrm{E}+09$ |
| $\alpha_{v}$ | $4.79 \mathrm{E}+06$ | $5.00 \mathrm{E}+06$ | $1.66 \mathrm{E}+06$ | $r_{3}$ | $5.44 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $3.07 \mathrm{E}+09$ |
| $\beta_{v 1}$ | $1.28 \mathrm{E}-01$ | $1.60 \mathrm{E}-01$ | $1.70 \mathrm{E}-01$ | $r_{4}$ | $2.69 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $2.52 \mathrm{E}+09$ |
| $\beta_{v 2}$ | 4.73E-01 | $3.85 \mathrm{E}-01$ | $5.24 \mathrm{E}-01$ | $r_{5}$ | $3.72 \mathrm{E}+09$ | $3.27 \mathrm{E}+09$ | $2.75 \mathrm{E}+09$ |
| $\alpha_{s}$ | $5.00 \mathrm{E}+06$ | $4.84 \mathrm{E}+06$ | $1.98 \mathrm{E}+06$ | $r_{6}$ | $1.85 \mathrm{E}+09$ | $1.49 \mathrm{E}+09$ | $1.35 \mathrm{E}+09$ |
| $\beta_{s 1}$ | $6.00 \mathrm{E}-01$ | $3.01 \mathrm{E}-01$ | 5.91E-01 | $r_{7}$ | $1.53 \mathrm{E}+09$ | $1.47 \mathrm{E}+09$ | $1.14 \mathrm{E}+09$ |
| $\beta_{s 2}$ | $2.00 \mathrm{E}-05$ | $1.92 \mathrm{E}-05$ | $3.65 \mathrm{E}-05$ | $r_{8}$ | $8.18 \mathrm{E}+08$ | $5.89 \mathrm{E}+08$ | $6.72 \mathrm{E}+08$ |
| $\beta_{R}$ | $4.50 \mathrm{E}+01$ | $2.99 \mathrm{E}-01$ | $1.71 \mathrm{E}-01$ | $r_{9}$ | $5.51 \mathrm{E}+09$ | $4.84 \mathrm{E}+09$ | $6.14 \mathrm{E}+09$ |
| $f_{1}$ | $4.00 \mathrm{E}-01$ | $3.81 \mathrm{E}-01$ | $6.31 \mathrm{E}-01$ | rhist $_{1}$ | $1.39 \mathrm{E}+09$ | $2.00 \mathrm{E}+09$ | $3.73 \mathrm{E}+09$ |
| $f_{2}$ | $3.60 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | 4.33E-01 | rhist $_{2}$ | $6.08 \mathrm{E}+09$ | $7.42 \mathrm{E}+09$ | $1.43 \mathrm{E}+10$ |
| $f_{3}$ | $3.94 \mathrm{E}-01$ | $3.54 \mathrm{E}-01$ | $5.51 \mathrm{E}-01$ | rhist $_{3}$ | $2.27 \mathrm{E}+10$ | $2.14 \mathrm{E}+10$ | $8.48 \mathrm{E}+09$ |
| $f_{4}$ | $3.43 \mathrm{E}-01$ | $3.97 \mathrm{E}-01$ | $4.47 \mathrm{E}-01$ | rhist $_{4}$ | $5.58 \mathrm{E}+08$ | $4.69 \mathrm{E}+08$ | $2.97 \mathrm{E}+09$ |
| $f_{5}$ | $3.94 \mathrm{E}-01$ | 5.19E-01 | 4.97E-01 | rhist $_{5}$ | $1.32 \mathrm{E}+09$ | $1.91 \mathrm{E}+09$ | $7.91 \mathrm{E}+08$ |
| $f_{6}$ | $3.39 \mathrm{E}-01$ | $3.04 \mathrm{E}-01$ | $3.72 \mathrm{E}-01$ | rhist 6 | $5.68 \mathrm{E}+08$ | $6.86 \mathrm{E}+08$ | $9.13 \mathrm{E}+08$ |
| $f_{7}$ | $4.00 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.54 \mathrm{E}-01$ | Error value |  | 1.2E11 | 7E8 |
| $f_{8}$ | 4.16E-01 | $2.31 \mathrm{E}-01$ | 4.65E-01 |  |  |  |  |
| $f_{9}$ | $3.73 \mathrm{E}-01$ | $3.15 \mathrm{E}-01$ | $4.23 \mathrm{E}-01$ |  |  |  |  |



Figure 5.26: Step 8; a) Total landed biomass, b) Total catch biomass, c) Total discards biomass, d) Total survey numbers, e) total survey biomass, f) Total landing numbers, g) total stock numbers, h) Total stock biomass, i) Recruit numbers. Simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black); all fitting parameters are perturbed by $\mathbf{3 0 \%}$ ( $\mathbf{c v}=\mathbf{0 . 3 0}$ ).


Figure 5.27: Step 8; a) Fishing selectivity, b) Fishing scalar and c) survey selectivity, d) average Fishing mortality for length classes 25 to $\mathbf{4 2} \mathbf{~ c m}$, d) von Bertalanffy growth curve; Simulated or true value (solid blue), estimated (dashed red) and pre-estimation or perturbed (dashed black); all fitting parameters are perturbed by $\mathbf{3 0 \%}$ (cv=0.30)


Figure 5.28: Step 8; a) Total landed biomass, b) Total catch biomass, c) Total discards biomass, d) Total survey numbers, e) total survey biomass, f) Total landing numbers, g) total stock numbers, h) Total stock biomass, i) Recruit numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black); all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $20 \%$ (cv=0.20), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.


Figure 5.29: Step 8; a) Fishing selectivity, b) Fishing scalar and c) survey selectivity, d) average Fishing mortality for length classes 25 to $\mathbf{4 2} \mathbf{~ c m}$, d) von Bertalanffy growth curve; Simulated or true value (solid blue), estimated (dashed red) and pre-estimation or perturbed (dashed black); apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}$ ( $\mathbf{c v}=0.20$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.

### 5.4.2.9 Step 9: Estimating $f_{t}, r_{t}, \beta_{R}, \alpha_{s}, \beta_{s 1}, \beta_{s 2}, \alpha_{v}, \beta_{v 1}, \beta_{v 2}, L_{\infty}, K, \beta$

In this trial, the gamma parameter growth variance $\beta$ is added to the fitting parameters. All of the fitting parameters, apart from growth parameters, are perturbed by $30 \%$. The growth parameters $\left(L_{\infty}, K\right)$ are however perturbed by $20 \%$ ( $\mathrm{cv}=0.20$ ); this is due to the significant influence they have on the estimation of model parameters. Observations are estimated very close to the original data (Figure 5.30) and the model is capable of recovering the parameters close to the true values (Table 5.12). Fishing scalars (Figure 5.31b) are again overestimated with the variability of growth variance parameter.

Growth parameter $\beta$ can affect the accuracy of the model estimation, fishing scalars in particular. If any information is available, it is more feasible to keep it as known in the model. It is similar to the approach in Sullivan et al. (1990), in which three different fixed values were selected and tested for $\beta$.

Table 5.12: Step 9; True, perturbed and estimated parameter values (growth parameters $\boldsymbol{L}_{\infty}, \boldsymbol{K}, \boldsymbol{\beta}$ are perturbed by $\mathbf{2 0 \%}$ while all the other fitting parameters are added $\mathbf{3 0 \%}$ noise)

| Parameter | True value | Initial $( \pm 30 \% \text { or } \pm 20 \%)$ | Estimated | Parameter | True value | Initial $( \pm 30 \%)$ | Estimated |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{\infty}$ | $7.00 \mathrm{E}+1$ | 4.97E+01 | $7.24 \mathrm{E}+01$ | $r_{1}$ | $7.71 \mathrm{E}+09$ | $7.82 \mathrm{E}+09$ | 7.37E +09 |
| $K$ | $2.40 \mathrm{E}-01$ | $2.09 \mathrm{E}-01$ | $2.31 \mathrm{E}-01$ | $r_{2}$ | $1.98 \mathrm{E}+09$ | $2.26 \mathrm{E}+09$ | $1.62 \mathrm{E}+09$ |
| $\beta$ | $5.00 \mathrm{E}-01$ | $4.79 \mathrm{E}-01$ | 4.96E-01 | $r_{3}$ | $5.44 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $5.27 \mathrm{E}+09$ |
| $\alpha_{v}$ | $4.79 \mathrm{E}+06$ | $5.00 \mathrm{E}+06$ | $1.09 \mathrm{E}+06$ | $r_{4}$ | $2.69 \mathrm{E}+09$ | $2.05 \mathrm{E}+09$ | $2.84 \mathrm{E}+09$ |
| $\beta_{v 1}$ | $1.28 \mathrm{E}-01$ | $1.60 \mathrm{E}-01$ | $1.28 \mathrm{E}-01$ | $r_{5}$ | $3.72 \mathrm{E}+09$ | $3.27 \mathrm{E}+09$ | $3.61 \mathrm{E}+09$ |
| $\beta_{v 2}$ | $4.73 \mathrm{E}-01$ | $3.85 \mathrm{E}-01$ | $4.29 \mathrm{E}-01$ | $r_{6}$ | $1.85 \mathrm{E}+09$ | $1.49 \mathrm{E}+09$ | $1.84 \mathrm{E}+09$ |
| $\alpha_{s}$ | $5.00 \mathrm{E}+06$ | $4.84 \mathrm{E}+06$ | $4.13 \mathrm{E}+07$ | $r_{7}$ | $1.53 \mathrm{E}+09$ | $1.47 \mathrm{E}+09$ | $2.11 \mathrm{E}+09$ |
| $\beta_{s 1}$ | $6.00 \mathrm{E}-01$ | $3.01 \mathrm{E}-01$ | $6.73 \mathrm{E}-01$ | $r_{8}$ | $8.18 \mathrm{E}+08$ | $5.89 \mathrm{E}+08$ | $1.19 \mathrm{E}+09$ |
| $\beta_{s 2}$ | $2.00 \mathrm{E}-05$ | $1.92 \mathrm{E}-05$ | 4.06E-06 | $r_{9}$ | $5.51 \mathrm{E}+09$ | $4.84 \mathrm{E}+09$ | $4.94 \mathrm{E}+09$ |
| $\beta_{R}$ | $4.50 \mathrm{E}+01$ | $2.99 \mathrm{E}-01$ | $3.03 \mathrm{E}-01$ | rhist ${ }_{1}$ | $1.39 \mathrm{E}+09$ | $2.00 \mathrm{E}+09$ | $3.43 \mathrm{E}+09$ |
| $f_{1}$ | $4.00 \mathrm{E}-01$ | $3.81 \mathrm{E}-01$ | 4.63E-01 | rhist ${ }_{2}$ | $6.08 \mathrm{E}+09$ | $7.42 \mathrm{E}+09$ | $3.59 \mathrm{E}+09$ |
| $f_{2}$ | $3.60 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.29 \mathrm{E}-01$ | rhist $_{3}$ | $2.27 \mathrm{E}+10$ | $2.14 \mathrm{E}+10$ | $2.19 \mathrm{E}+10$ |
| $f_{3}$ | $3.94 \mathrm{E}-01$ | $3.54 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | rhist $_{4}$ | $5.58 \mathrm{E}+08$ | $4.69 \mathrm{E}+08$ | $6.28 \mathrm{E}+08$ |
| $f_{4}$ | $3.43 \mathrm{E}-01$ | $3.97 \mathrm{E}-01$ | $4.11 \mathrm{E}-01$ | rhist ${ }_{5}$ | $1.32 \mathrm{E}+09$ | $1.91 \mathrm{E}+09$ | $9.60 \mathrm{E}+08$ |
| $f_{5}$ | $3.94 \mathrm{E}-01$ | $5.19 \mathrm{E}-01$ | $4.50 \mathrm{E}-01$ | rhist ${ }_{6}$ | $5.68 \mathrm{E}+08$ | $6.86 \mathrm{E}+08$ | $5.78 \mathrm{E}+08$ |
| $f_{6}$ | $3.39 \mathrm{E}-01$ | $3.04 \mathrm{E}-01$ | $3.75 \mathrm{E}-01$ | Error value |  | 2E11 | 3E7 |


| $f_{7}$ | $4.00 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.36 \mathrm{E}-01$ |
| :--- | :--- | :--- | :--- |
| $f_{8}$ | $4.16 \mathrm{E}-01$ | $2.31 \mathrm{E}-01$ | $4.28 \mathrm{E}-01$ |
| $f_{9}$ | $3.73 \mathrm{E}-01$ | $3.15 \mathrm{E}-01$ | $4.23 \mathrm{E}-01$ |











Figure 5.30: Step 9; a) Total landed biomass, b) Total catch biomass, c) Total discards biomass, d) Total survey numbers, e) total survey biomass, f) Total landing numbers, g) total stock numbers, h) Total stock biomass, i) Recruit numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)

fgure 5.31: Step 9; a) Fishing selectivity, b) Fishing scalar and c) survey selectivity, d) average fishing mortality rate for length classes 25 to $\mathbf{4 2} \mathbf{~ c m}, ~ d$ ) von Bertalanffy growth curve; Simulated or true value (solid blue), estimated (dashed red) and pre-estimation or perturbed (dashed black)

### 5.4.3 Applying survey-landings model on perturbed

## observation

In this section, the limitation of the survey-landings model is assessed from a different angle. The model is applied to noisy observations. Since the real noise in the sea is closer to multiplicative noise rather than additive noise, the three observations including total landed biomass, total discards biomass and survey numbers are exposed to multiplicative noise simultaneously. The noise, which is added exponentially to every value of the observations, follows a normal distribution with coefficient of variation (cv) set equal to 0.3 :

$$
\begin{align*}
& \text { Noisy obs }=(\text { sim obs })\left(e^{c v . \tau}\right)  \tag{5.6}\\
& \quad \tau \sim N(0,1)
\end{align*}
$$

The survey-landings model is applied to noisy observations while the initial parameters are not perturbed and set equal to true values. All the parameters that represent growth, fishing and survey selectivity, fishing mortality and recruitment are subject to estimation (Table 5.13).

This process is designed to assess whether the survey-landings model is capable of recovering the parameters in a noisy environment of observations. Aditionally, it highlights the parameters that are more sensitive to variability of input observations.

Perturbed observations are randomly scattered around the original simulated observations. Therefore, the ideal result is that the model estimates the perturbed observations with parameters estimated reasonably close to the true values.

The results show that all three noisy observation sets are recovered (Figure 5.32 \& Figure 5.33). The model parameters (Table 5.13) also show that they are robust to noisy observations. The exception, however, is $f_{t}$ (Figure 5.34). It is quite expected for fishing scalar to be sensitive to variability of observations. Keeping other factors fixed, with more or less landed biomass, the fishing scalar goes up or down to make the model robust in recovering the observation. Nevertheless, the estimation of fishing scalars is fluctuating randomly around the true values. It makes the model more reliable compared to the situation when the parameters are all over or under estimated.

Table 5.13: True (initial) parameter values and estimated parameter values; the survey-landings model is applied on perturbed simulated observations.

| Parameter | Initial = True value | Estimated | Parameter | Initial $=$ True value | Estimated |
| :---: | :--- | :--- | :---: | :--- | :--- |
| $L_{\infty}$ | $7.00 \mathrm{E}+1$ | $7.26 \mathrm{E}+01$ |  | $7.71 \mathrm{E}+09$ | $7.04 \mathrm{E}+09$ |
| $K$ | $2.40 \mathrm{E}-01$ | $2.28 \mathrm{E}-01$ | $r_{2}$ | $1.98 \mathrm{E}+09$ | $2.59 \mathrm{E}+09$ |
| $\beta$ | $5.00 \mathrm{E}-01$ | $4.75 \mathrm{E}-01$ | $r_{3}$ | $5.44 \mathrm{E}+09$ | $6.13 \mathrm{E}+09$ |
| $q_{m a x}$ | $1.00 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $r_{4}$ | $2.69 \mathrm{E}+09$ | $2.09 \mathrm{E}+09$ |
| $\alpha_{v}$ | $4.79 \mathrm{E}+06$ | $8.66 \mathrm{E}+06$ | $r_{5}$ | $3.72 \mathrm{E}+09$ | $3.65 \mathrm{E}+09$ |
| $\beta_{v 1}$ | $1.28 \mathrm{E}-01$ | $1.36 \mathrm{E}-01$ | $r_{6}$ | $1.85 \mathrm{E}+09$ | $2.04 \mathrm{E}+09$ |
| $\beta_{v 2}$ | $4.73 \mathrm{E}-01$ | $4.96 \mathrm{E}-01$ | $r_{7}$ | $1.53 \mathrm{E}+09$ | $1.94 \mathrm{E}+09$ |
| $\alpha_{s}$ | $5.00 \mathrm{E}+06$ | $9.69 \mathrm{E}+06$ | $r_{8}$ | $8.18 \mathrm{E}+08$ | $1.07 \mathrm{E}+09$ |
| $\beta_{s 1}$ | $6.00 \mathrm{E}-01$ | $6.05 \mathrm{E}-01$ | $r_{9}$ | $5.51 \mathrm{E}+09$ | $6.05 \mathrm{E}+09$ |
| $\beta_{s 2}$ | $2.00 \mathrm{E}-05$ | $1.56 \mathrm{E}-05$ | $r h i s t_{1}$ | $1.39 \mathrm{E}+09$ | $6.64 \mathrm{E}+08$ |
| $\beta_{R}$ | $4.50 \mathrm{E}+01$ | $2.68 \mathrm{E}-01$ | $r h i s t_{2}$ | $6.08 \mathrm{E}+09$ | $5.13 \mathrm{E}+08$ |


| $f_{1}$ | $4.00 \mathrm{E}-01$ | $5.58 \mathrm{E}-01$ | rhist $_{3}$ | $2.27 \mathrm{E}+10$ | $2.79 \mathrm{E}+10$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $f_{2}$ | $3.60 \mathrm{E}-01$ | $3.35 \mathrm{E}-01$ | rhist $_{4}$ | $5.58 \mathrm{E}+08$ | $1.86 \mathrm{E}+09$ |
| $f_{3}$ | $3.94 \mathrm{E}-01$ | $2.55 \mathrm{E}-01$ | rhist $_{5}$ | $1.32 \mathrm{E}+09$ | $8.63 \mathrm{E}+08$ |
| $f_{4}$ | $3.43 \mathrm{E}-01$ | $3.63 \mathrm{E}-01$ | rhist $_{6}$ | $5.68 \mathrm{E}+08$ | $1.72 \mathrm{E}+09$ |
| $f_{5}$ | $3.94 \mathrm{E}-01$ | $5.51 \mathrm{E}-01$ | Error value | 2.1 E 10 | 3.5 E 8 |
| $f_{6}$ | $3.39 \mathrm{E}-01$ | $4.04 \mathrm{E}-01$ |  |  |  |
| $f_{7}$ | $4.00 \mathrm{E}-01$ | $2.67 \mathrm{E}-01$ |  |  |  |
| $f_{8}$ | $4.16 \mathrm{E}-01$ | $3.46 \mathrm{E}-01$ |  |  |  |
| $f_{9}$ | $3.73 \mathrm{E}-01$ | $5.89 \mathrm{E}-01$ |  |  |  |



Figure 5.32: a) Total landed biomass; b) Total discards biomass; c) Total survey numbers; the surveylandings model is applied on perturbed observations; perturbed observation (solid blue line), Simulated observation (dotted green) and estimated observations (red circles)


Figure 5.33: Frequency length distribution of survey; perturbed (solid blue line), original simulated (dotted green) and estimated (red circles)


Figure 5.34: True and estimated parameters when the model is applied on perturbed observations; a) Fishing selectivity, b) Fishing scalar; c) survey selectivity; d) average Fishing mortality for length classes $\mathbf{2 5}$ to $\mathbf{4 2} \mathbf{~ c m}$; e) von Bertalanffy growth curve; f) total yearly recruits; True value (solid blue), estimated (dashed red) and pre-estimation or perturbed (dashed black)

### 5.5 Discussion

The length-structured survey-landings model was developed to assess the stock of poorly sampled marine species, for which neither age nor length distribution of catch is available. In the first step, it applies and develops the length-based stock assessment that Sullivan et al. (1990) originally proposed. The survey-landings model uses total landed biomass (taken from commercial reports) and frequency length distribution of survey (extracted from IBTS) instead of catch-at-length in Sullivan et al. (1990). The model then proposes a new assumption about fishing and survey selectivity curves. The selectivity curves follow double logistic functions instead of increasing logistic trend. All the main components of growth, mortality and recruitments are modelled separately then combined to make a modelled population dynamics (see Chapter 4).

Prior to applying to real data, the twin-experiment method was used to assess the robustness and limitations of the model when the parameters or observations are perturbed.

The three main observations ( $T L B_{t}, T D B_{t}, N s_{l, t}$ ) were generated first, then the model was applied to the simulated observations while the initial parameter values were kept equal to the true values that were used for simulation. The result ensures that with the same given input the model produces identical estimation and there is no second way or any minimum value elsewhere, otherwise the model estimation is not accurate..

Next, all the parameters were considered known but were perturbed by up to $30 \%$ and added to the fitting position one by one. The magnitude of noise is high enough to highlight any estimation instability and parameter sensitivity. The noise is also comparable to the validation approach in Sullivan et al. (1990). Stepwise procedures in moving parameters from fixed position to being subject to estimation makes it easier to assess the limitation of the model. The process also spots the parameters that are more sensitive to variations and the parameters that should be estimated externally to bring more accuracy to the estimation process.

Stepwise sensitivity analysis and validation show that the variability of $q_{\max }$ brings instability to $f_{t}$. The maximum survey catchability, $q_{\max }$, is set to 1 in simulation; but in reality it can be bigger or smaller than 1 depending on the distribution of fish in the sampling area. It is an unknown parameter and is not possible to enter into the model as a fixed input.

However, a sensible approach, in the application with real data, is to adjust it manually for the best fit before fitted in the model.

The model struggles to estimate survey observation data accurately when $30 \%$ noise is exposed to the growth functions $L_{\infty}$ and $K$. Variation of the growth parameters also reduces the accuracy of estimation $f_{t}$. It is not the case though when the growth parameters are perturbed by only $20 \%$ of their true values. Both $L_{\infty}$ and $K$ are possible to be estimated externally and used as either fixed input into the model or as initial values close enough to the true value ( $20 \%$ of the true values). The frequency length distribution of survey is a practical platform for this purpose. In this simulation, the first two peaks of the distribution at each time step are the number of one- and two-year old fish. Extracting the length classes related to the peaks and using VBL growth function estimate feasible values for the growth parameters. It can also help to set initial values close to the true values.

Variation of growth parameter $\beta$ affects the estimation of fishing scalar. The approach to estimate $\beta$ is not as practical as the other two growth parameters. However, it too can be adjusted manually and by applying trials to set a fixed value. This approach has been applied before in the length-based method that Sullivan et al. (1990) proposed.

All the other parameters in the model proved to be robust to the noise in initial parameters. The fishing and survey selectivity functions are estimated very well even though the parameters are estimated quite far away from the initial values.

In the next validation phase, when the observations are perturbed, the model is very capable of estimating the observations. The variability in survey and landing appeared to influence the estimation of fishing scalar $f_{t}$.

In the next chapter, the survey-landings model is applied to the North Sea haddock. The results are compared to the assessment from ICES working group (ICES WGNSSK REPORT 2013) to investigate if both provide same output with regards to popualation.

## Appendix 5.1

The length distribution of survey, stock numbers, fishing mortality, total mortality, catch and landings are given here. Figures are coded with their matching trial step in section 5.4.2.


Figure 5.35: Frequency length distribution of survey; survey-landing model was applied on simulated observation; all initial parameter values set equal to true values; Simulated (solid blue lines) and estimated (red points)



## ear 4



Year 5


Year 8



Figure 5.36: Step 1; Frequency length distribution of survey numbers; simulated observation (solid blue), model estimation (dotted red) and pre-estimation (dashed black)









Figure 5.37: Step 1; Frequency length distribution of stock numbers; simulated observation (solid blue), model estimation (dotted red) and pre-estimation (dashed black)






Year 8


Year 9


Figure 5.38: Step 2; Frequency length distribution of survey numbers; simulated observation (solid blue), model estimation (dotted red) and pre-estimation (dashed black)


Figure 5.39: Step 2; Frequency length distribution of stock numbers; simulated observation (solid blue), model estimation (dotted red) and pre-estimation (dashed black)


Figure 5.40: Step2; Frequency length distribution of fishing mortality; simulated observation (solid blue), model estimation (dotted red) and pre-estimation (dashed black)


Figure 5.41: Step 2; Frequency length distribution of total mortality; simulated observation (solid blue), model estimation (dotted red) and pre-estimation (dashed black)


Figure 5.42: Step 2; Frequency length distribution of catch numbers; simulated observation (solid blue), model estimation (dotted red) and pre-estimation (dashed black)


Figure 5.43: Step 2; Frequency length distribution of landing numbers; simulated observation (solid blue), model estimation (dotted red) and pre-estimation (dashed black)




Year 2


Year 5


Year 8



Figure 5.44: Step 3; length distribution of survey numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Year 1

Year 4


Year 7


Year 2


Year 5


Year 8



Figure 5.45: Step 3; length distribution of stock number; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.46: Step 3; length distribution of fishing mortality; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.47: Step 3; length distribution of total mortality; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.48: Step 3; length distribution of catch numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black).


Figure 5.49: Step 3; length distribution of landings; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)




Year 2

## 

Year 5


Year 8



Year 9


Figure 5.50: Step 4; length distribution of survey numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.51: Step 4; length distribution of stock numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.52: Step 4; length distribution of fishing mortality; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.53: Step4; length distribution of total mortality; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.54: Step 4; length distribution of catch numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.55: Step 4; length distribution of landings; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)



Year 2

Year 5

Year 8




Figure 5.56: Step 5; length distribution of survey numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.57: Step 5; length distribution of stock numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.58: Step 5; length distribution of fishing mortality rate; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.59: Step 5; length distribution of total mortality rate; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.60: Step 5; length distribution of catch numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.61: Step 5; length distribution of landing numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)










Figure 5.62: Step 6; length distribution of survey numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.63: Step 6; length distribution of stock numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.64: Step 6; length distribution of fishing mortality rate; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.65: Step 6; length distribution of total mortality rate; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.66: Step 6; length distribution of catch numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.67: Step 6; length distribution of landing numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)




Year 2



Year 8



Figure 5.68: Step 7; length distribution of survey numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)

Year 2
Year 3



Year 8
Year 9




Figure 5.69: Step 7; length distribution of stock numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.70: Step 7; length distribution of fishing mortality rate; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.71: Step 7; length distribution of total mortality rate; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.72: Step 7; length distribution of catch numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)


Figure 5.73: Step 7; length distribution of landing numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black)




Year 2


Year 8




Figure 5.74: Step 8; length distribution of survey numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), when all the fitting parameters are perturbed by30\%


Figure 5.75: Step 8; length distribution of stock numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), when all the fitting parameters are perturbed by30\%


Figure 5.76: Step 8; length distribution of fishing mortality rate; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), when all the fitting parameters are perturbed by30\%


Figure 5.77: Step 8; length distribution of total mortality rate; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), when all the fitting parameters are perturbed by30\%


Figure 5.78: Step 8; length distribution of catch numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), when all the fitting parameters are perturbed by30\%


Figure 5.79: Step 8; length distribution of landing numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), when all the fitting parameters are perturbed by30\%




Year 2
$\qquad$

Year 5


Year 8



Year 9


Figure 5.80: Step 8; length distribution of survey numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}$ ( $\mathbf{c v}=\mathbf{0 . 2 0}$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.




Year 8





Figure 5.81: Step 8; length distribution of stock numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}(\mathbf{c v}=\mathbf{0 . 2 0}$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.


Figure 5.82: Step 8; length distribution of fishing mortality rate; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}$ ( $\mathbf{c v = 0 . 2 0}$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.


Figure 5.83: Step 8; length distribution of total mortality rate; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}(\mathbf{c v}=\mathbf{0 . 2 0}$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.


Figure 5.84: Step 8; length distribution of catch numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}$ ( $\mathbf{c v}=\mathbf{0 . 2 0}$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.


Figure 5.85: Step 8; length distribution of landing numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}(\mathbf{c v}=\mathbf{0 . 2 0}$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.








Figure 5.86: Step 9; length distribution of survey numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}$ ( $\mathbf{c v = 0 . 2 0}$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values


Figure 5.87: Step 9; length distribution of stock numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}(\mathbf{c v}=\mathbf{0 . 2 0}$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.


Figure 5.88: Step 9; length distribution of fishing mortality rate; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}$ ( $\mathbf{c v = 0 . 2 0}$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.


Figure 5.89: Step 9; length distribution of total mortality rate; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}(\mathbf{c v}=\mathbf{0 . 2 0}$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.


Figure 5.90: Step 9; length distribution of catch numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}$ ( $\mathbf{c v = 0 . 2 0}$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.


Figure 5.91: Step 9; length distribution of landing numbers; simulated (solid blue), estimated (dashed red) and pre-estimation (dashed black), all fitting parameters, apart from $L_{\infty}$ and $K$ that are perturbed by $\mathbf{2 0 \%}$ ( $\mathbf{c v = 0 . 2 0}$ ), all fitting parameters are perturbed by $\mathbf{3 0 \%}$ of their true values.

## Chapter 6

## North Sea haddock stock assessment: Application of length-structured surveylandings model

### 6.1 Abstract

The length-structured survey-landings model is used to assess the stock size and the dynamics of the North Sea haddock population. The assessment outputs from lengthstructured survey-landings model are compared with the standard age-structured method from ICES assessment results. The aim is to investigate how close this model result is to the ICES assessments. The outcome is promising and shows that both models provide similar assessments.

### 6.2 Introduction

### 6.2.1 Biology, ecology and spatial distribution

Haddock (Melanogrammus eaglefinus) is a salt-water fish from the cod family found in the Atlantic Ocean but more common on the European side. In the UK haddock is more located to the north of the British Isles (British Sea Fishing) and is not found in great quantity in south of the English Channel (FAO: The haddock). Aside from the UK, haddock is reported in other 20 countries ${ }^{10}$.

The black lateral line along its white side and the spot make haddock recognisable from other fish (British Sea Fishing). The head, back and sides are dark purplish grey, while the belly and lower sides of the head are white. It has small mouth and small teeth with the lower jaw smaller than the upper one. Three fins are along the back and two on the underside; the rear edge of the tail is almost straight (FAO: The haddock). Haddock has been called different names in different part of Britain, most of which associated with the size of the fish, such as 'jumbo' for very large haddock, 'gibber' for large, 'kit' for medium, 'calfie', 'chat', 'danny', 'pinger', 'powie', and 'tiddley' for small and 'ping pong' and 'seed' for very small haddock (FAO: The haddock).

Haddock is generally a bottom-living species with adults mostly in deeper part of the North Sea in the depth of 75 to 125 metres $^{11}$ (ICES FishMap; Daan et al. 1990; Hedger et al., 2004) usually at colder temperatures around $4^{\circ} c$ to $10^{\circ} c$, with the peak abundance at 1980s being at around 6.5 c and shifted to around 8c at 1990s (Hedger et al., 2004). The biggest abundance reported is to the east of Scotland, north of Outer Hebrides, around the Orkneys and west of Norwegian Trench (Daan et al., 1990; Knijn et al., 1993; Hedger et al., 2004). The result of generalised additive modelling on spatial distribution of haddock shows that the abundance increases with

[^6]temperature and salinity (Hedger et al., 2004). Haddock in Norwegian Deep migrates to shallow area in summer and deeper area in winter (Albert, 1994). Unlike other species in the North Sea, juvenile haddock do not have nursery ground (Daan et al., 1990). Although the distributions of adult and juvenile haddock overlap, the juvenile are more abundant in ICES area IIIa, Skagerrak, (ICES FishMap) and prefer shallower water (Figure 6.1). Haddock feeds on small bottom-living organisms, worms and shellfish, with bigger haddock hunting other fish (British Sea Fishing). It also goes to mid-water (Albert, 1994), to feed on sandeel, Norway pout long rough dab, sprat and herring.


Figure 6.1: Average annual catch rate for juvenile ( $<\mathbf{3 0} \mathrm{cm}$, left) and adult ( $>30 \mathrm{~cm}$, right) haddock in the quarter 1 IBTS survey, 1977-2005 (picture from ICES FishMap)

Haddock move to the northern water around Norway, between the Shetland Islands and Norwegian Deep, in springtime (Daan et al., 1990) and remain in deep water, 100-150m (ICES FishMap), to spawn. Although the peak spawning time is in March and April, it can vary from January to May (Daan et al., 1990). After spawning adult haddock migrate to west toward the Orkney and Shetland Islands and the central part of North Sea to feed (Bjorke and Saetre, 1994). Millions of released eggs, with
roughly 1-1.6 mm (FAO: The haddock), float on the surface of water for 10 to 14 days ${ }^{12}$ until they hatch. The number of eggs a female haddock lays depends on her body size. An average sized female haddock produces about 850,000 eggs, while larger females can produce up to 3 million eggs every year. The black 3-4-mm-larvae (FAO: The haddock) stay for another few weeks on the upper surface until they reach around 7 cm (ICES FishMap) to feed, mainly on immature copepods (Russell, 1976), and then move down to the seabed (British Sea Fishing) to feed on decapod larvae, copepods and fish (Robb and Hislop, 1980). Summer is the time that juvenile haddock are at their highest density off the northeast coast of Scotland (Albert, 1994).

Haddock can grow up to 100 metres and very few live over 5 years (Albert, 1994) but the maximum reported age was 10 years (Wright and Gibb, 2005). In the study Albert (1994) conducted of the haddock in Norwegian Deep, the mean length at age for juveniles was larger in Skagerrak compared to other area, which may indicate that Skagerrak has better growth conditions. Haddock matures at around 2 to 3 years of age when it is about $25-30 \mathrm{~cm}$, but the eggs produced by 2 -year-old haddock are smaller than those produced by older age classes (Wright and Gibb, 2005). Growth rates of haddock have changed over the past 30 to 40 years. A comparison between maturity rates of 2-year-old haddock over two time periods shows that the maturity rate increased among female and decreased for male haddock (Hislop and Shanks 1981).

### 6.2.2 Commercial importance

The North Sea is one of the world's most important fishing ground, which targets both pelagic and demersal fish stocks. The otter trawl net vessels in the mixed demersal fishery, which is a very important fishery in the North Sea, target cod, haddock and whiting in the central and northern part. Along with cod, salmon, tuna

[^7]and prawns, haddock makes up $60 \%$ of seafood consumption in Britain (British Sea Fishing).

The haddock fisheries were highly affected by the limited cod and whiting quotas, which pulled fishermen's concentration particularly in years 2006 and 2007 towards fishing more. In the 44 year period between 1969 and 2012 inclusive, the highest record of 524,000 tonnes of North Sea haddock landed in year 1970 and minimum of just under 30,000 tonnes in 2010 (ICES Advice Book, 2013). The North Sea haddock population is characterised by the highly fluctuating numbers of young fish or recruits entering the population every year. 1999 was the strongest year with ICES estimated of $1.4 \mathrm{e}^{11}$ recruits while 2011 was estimated to have had the lowest with $6.8 \mathrm{e}^{8}$ recruits.

Haddock was once less popular than cod, but (depending on the location) the expansion of its trade and the good keeping quality and size increased the acceptance of haddock (GMA). Haddock is sold at British port markets as usually a whole gutted fish or fillets chilled or frozen. As a food, haddock is very popular in Britain, often served with chips. Fresh haddock has a white flesh and is prepared in the same way as cod. One popular form of haddock in Scotland is called 'Finnan haddie', which is served poached in milk for breakfast. It is also served as hot-smoked in town of Arbroath on the east coast of Scotland or even in the soup-like dish named as 'Cullen Skink'. The flesh contains around $80 \%$ water, $15-20 \%$ protein, less than $2 \%$ fat and carbohydrate and 1-2\% minerals (FAO: The haddock).

### 6.2.3 Model overview

The length-structured survey-landings model, was introduced and discussed in Chapter 4 and assessed on simulated observations in Chapter 5. here it is used to assess the stock size and the dynamics of the North Sea haddock population. The model components and functions from Equations 4.1, 4.20 and 4.25 are applied to predict and estimate the dynamics of the population of the North Sea haddock in the ICES area IVa, IVb, IVc and IIIa (Figure 6.2).


Figure 6.2: The North Sea area in ICES; the survey-landings model is applied on haddock in area IVa, IVb, IVe, and IIIa.

$$
\begin{align*}
& C_{l, t}=\mu_{l, t} \cdot N_{l, t}+e^{\omega_{l, t}}  \tag{6.1}\\
& N_{l, t+1}=P_{l, i} \cdot S_{l, t} \cdot N_{l, t}+R_{l, t+1}+e^{\theta_{l, t}}  \tag{6.2}\\
& \text { Survey }_{l, t}=q_{l} \cdot P a_{t} \cdot N_{l, t}+e^{\delta_{l, t}} \tag{6.3}
\end{align*}
$$

Haddock was selected due to its commercial importance in fisheries and the fact that the result of ICES annual assessments is available from the ICES advice book. Also, the availability of length data makes the comparison between age- and lengthstructured methods possible. The assessment outputs from the length-structured
survey-landings model are compared with age-structured ICES assessment results. The survey-landings model uses survey numbers at length extracted from IBTS and total landed biomass from commercial landings reports (Chapter 3), while ICES applies the age-structured assessment method using catch-at-age data from commercial landings reports. Although the haddock stock in the North Sea is linked to its stock in the west coast due to migration (ICES, 2014), and the newly updated approach is the combined assessment, the North Sea haddock is considered to be a single stock in this research work and survey-landings model is applied on the North Sea haddock only. One reason is the availability and reliability of data throughout the studied cohort for the North Sea data in comparison to the haddock data in the West of Scotland (ICES, 2014). The possibility of the significant differences between the natural mortalities of the two areas was another reason. The aim is initially to check the model and see whether our model is capable of producing the same result as in ICES, in which case North Sea meet the technical expectations.

In this chapter, the growth parameters are first considered constant for the 44 study years from 1969 to 2012. In reality, however, growth trend is not constant and varies over time, which is clear in the length distribution of survey. In order to put the variability of growth trend into consideration, the study period is broken into 4 separate timelines, and growth parameters are estimated separately for periods 19691979, 1980-1989, 1990-1999 and 2000-2012.

### 6.3 Model Observations

The length-structured survey-landings model takes three observation data sets to assess the stock of the North Sea haddock. Observations include length frequency of sampled fish from the scientific IBTS, total landed biomass and total discards biomass from commercial reports extracted from ICES Working Group Report. The details of the source of data sets are given in Chapter 3.

The $1^{\text {st }}$ quarter haddock survey numbers at length for 44 years from 1969 to 2012 at 1 -year time-steps $(d t=1)$ is extracted from IBTS. The survey data are assumed to
be the proportional sample of the haddock in the North Sea and to have the same distribution as the actual population in the sea. Quarter 1 of the year is selected since it is the time when most of sampling takes place and the most completed data is available. Data are recorded in a Microsoft Excel spreadsheet and read into R in $c s v$ format. The time series of the total survey numbers for the North Sea haddock over the study years is given in Figure 6.3.


Figure 6.3: The North Sea haddock's $1^{\text {st }}$ quarter total survey numbers from 1969 to 2012 extracted from IBTS

The frequency length distribution of survey for the North Sea haddock is given in Figure 6.4. Graphs are plotted on the same scale throughout the period to reflect the absolute changes in frequencies over years. Figure 6.5, in which the length frequency of survey is normalised by the total number at each time, illustrates the structure and shape of the population. A complete table of survey numbers at length for the 44 years as well as the separated distribution plots is given in Appendix 6.5.



Figure 6.4: Survey frequency length distribution (survey numbers at length at year) taken from IBTS data from year 1969 to 2012; used as observation is survey-landing model

Three variables from the original IBTS length data file are extracted to make a matrix of survey numbers at length. They include the columns that represent year of sampling, length of captured haddock and number of captured haddock at length at year. Programing codes in $R$ (Appendix 6.1) are used to sum the number of fish at each length at each year to make a matrix of input observations, where rows represent the length classes and columns are the years.

| 0.20 |
| :--- |
| 0.15 |
| 0.10 |
| 0.05 |
| 0.00 |$-1969$





1978


1982

$\qquad$


1975


1979

1983


1972
$\qquad$

1976


1980


1984
$\int_{0}^{1}$
1986
1987


1990


1994


1998


1991


1995


1999


Length (cm)


Figure 6.5: Normalised survey frequency length distribution (normalised by total frequency at year) taken from IBTS data from year 1969 to 2012

The first available North Sea haddock data in IBTS is 1965. Length classes are from 3 to 87 cm , which are the smallest and largest haddock captured in the survey in the 44 study years. Length classes are set to 1 cm width $(d L=1)$ to match the length class in the IBTS data. The length for some minority of fish that are recorded in decimals is rounded down to fit into the length classes (i.e. a 3.5 cm haddock is in 3 cm length class).

Total landed biomass is the total weight (tonnes) of landed North Sea haddock in area IV and IIIa at each study year extracted from ICES Working Group Report 2013 (Figure 6.6, solid blue line with filled circle points). The earliest available data in ICES was in year 1963. The first six years are used as historical to estimate the initial population number.

Total annual discard biomass is available for North Sea haddock and is used as a model observation to gain more accuracy in estimation. Haddock discard data are not a recorded observation data like landings biomass but it is regularly sampled and estimated. For that, it is considered as a reliable observation in the survey-landing model (Figure 6.6, dashed purple line with filled triangles).


Figure 6.6: Total landed biomass (blue solid line with filled circles) and total discard biomass (purple dashed line with filled triangles) extracted from ICES Working Group Report 2013; used as vectors of observation in survey-landing model

### 6.4 Input data

In order to make the model as simple as possible, all the parameters that happen to be known or can be estimated externally are added to the model as fixed input. Since the result is compared to ICES assessment, basic assumptions are taken from ICES report where possible.

Following the sensitivity results in Chapter 5 , growth parameter $\beta$ is kept as a fixed constant in the model to improve the accuracy of the von Bertalanffy parameters $\left(L_{\infty}, K\right)$. Weight of haddock at length, $W_{l}$, is calculated externally (Coull et al., 1989) and is added as a fixed length-dependent vector into the model. Annual proportion of the North Sea $\left(P a_{t}\right)$ that is used for sampling haddock is calculated externally (Chapter 4, Equation 4.23) from the IBTS data and is imported as a fixed time-dependent vector. Natural mortality rate at age is extracted from ICES report; it
is then transformed to natural mortality at length $\left(M_{l}\right)$ using the model estimated growth parameters (Appendix 6.2) and approximated over length classes. Parameters of the landing probability function $\left(\alpha_{D}, \beta_{D}\right)$ are also exempt from estimation. That is because they can be adjusted using the minimum landing size (Appendix 6.6). In the following subsections, input data are discussed in detail.

### 6.4.1 Growth variation

Due to being sensitive to noise and variability (see Chapter 5), parameter $\beta$ is added into the survey-landing model as a fixed parameter and set equal to 1.36 when the model is applied to the whole period. Throughout the model application on the years, $\beta$ is adjusted manually through different model runs for the best estimation result and closest variability to the survey distribution. A similar approach has been applied previously in the literature (Sullivan et al., 1990), in which different values of $\beta$ are manually tested. A fixed value of $\beta$, where possible, provides more accurate estimation of $L_{\infty}$ and $K$.

### 6.4.2 Weight at length

Weigh-length relationship, which is used to determine the weight of haddock at length, can be calculated externally and put as a vector into the survey-landing model. A sensible approach is to extract the weight-length parameters from marine lab report for haddock (Coull et al., 1989) in Equation 4.7. The weight-length function estimates the gutted weight and consequently the result is multiplied by 1.16 (Coull et al., 1989) to provide full weight at length. Since the measure of length $l$ is in centimetres, the outcome weight is in grams; the denominator in Equation (6.4) is to get the weight in tonnes for model application.

$$
\begin{equation*}
W_{l}=\left(0.0157 . l^{2.8268} * 1.16\right) /\left(1^{6}\right) \tag{6.4}
\end{equation*}
$$

An alternative method is to use average weight at age recorded in the 2013 ICES Working Group Report for haddock in area IV and IIIa. ICES use average weight at age in the total catch in calculation of stock biomass. Since the length at age are not available in ICES report, the average length at age that was estimated from IBTS (Appendix 6.2), is used to estimate the parameters for the allometric weight-length function in Equation 4.7. The unit of length is centimetres and final weight is in tonnes (Eq. 6.4). The methods and R codes are given in Appendix 6.3.

$$
\begin{equation*}
W_{l}=\left(0.002 . l^{1.514}\right) /\left(1^{3}\right) \tag{6.5}
\end{equation*}
$$

The two weight-length functions give different results, in particular for bigger individuals. It should, however, not make a huge difference in the model because there is not a big stock for very large fish. Although the weight-at-length from Marine Lab is more realistic (Figure 6.7) due to be calculated regularly from samples, the latter method resulted in better estimation in stock biomass and fishing mortality when compared with ICES assessment.


Figure 6.7: Weight-at-length for North Sea haddock extracted from Marine Lab report is compared with the weight at length estimated from average weight at age in ICES report

### 6.4.3 Proportion of the sampling area

Haddock survey number at length is assumed to be proportional to the real abundance (number) of haddock in the North Sea by the survey selectivity $q_{l}$ and proportion of the sampling area $P a_{t}$ (Eq. 4.25):

$$
\begin{equation*}
N s_{l, t}=q_{l} \cdot P a_{t} \cdot N_{l, t}+e^{\delta_{l, t}} \tag{6.6}
\end{equation*}
$$

Trawl wingspread and distance that survey vessel travels during sampling time is estimated for every survey station and is available from IBTS data. Sweeping area, which is the area, in which the sampling takes place, is calculated from Equation 4.24:

$$
\begin{equation*}
\text { Sweeping area }\left(m^{2}\right)=\operatorname{wingspread}(m) * \operatorname{distance}(m) \tag{6.7}
\end{equation*}
$$

The sum of all the sweeping area in a year becomes the total area in the North Sea that is used to take haddock samples in that year. The proportion of the sampling area at year $t, P a_{t}$, is calculated from Equation 4.23 for the years from 1969 to 2012 (Table 6.1):

$$
\begin{equation*}
P a_{t}=\frac{\sum_{a}(\text { Sweeping area })_{t}}{\text { Area of the Sea }} \tag{6.8}
\end{equation*}
$$

Proportions, as expected, are very small and values and change slightly every year (Figure 6.8). It is, therefore, calculated externally and added onto the model as a factor of a time series of Pa .

Table 6.1: Proportion of haddock sampling area in the North Sea area IV and IIIa from year 1969 to 2012 calculated from IBTS data.

| $t$ | $P a_{t}$ | $t$ | $P a_{t}$ | $t$ | $P a_{t}$ | $t$ | $P a_{t}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1969 | $2.01 \mathrm{E}-05$ | 1980 | $4.82 \mathrm{E}-05$ | 1991 | $5.40 \mathrm{E}-05$ | 2002 | $4.28 \mathrm{E}-05$ |
| 1970 | $2.46 \mathrm{E}-05$ | 1981 | $3.87 \mathrm{E}-05$ | 1992 | $4.62 \mathrm{E}-05$ | 2003 | $4.16 \mathrm{E}-05$ |
| 1971 | $3.12 \mathrm{E}-05$ | 1982 | $4.27 \mathrm{E}-05$ | 1993 | $4.57 \mathrm{E}-05$ | 2004 | $3.88 \mathrm{E}-05$ |
| 1972 | $3.45 \mathrm{E}-05$ | 1983 | $5.13 \mathrm{E}-05$ | 1994 | $4.54 \mathrm{E}-05$ | 2005 | $4.00 \mathrm{E}-05$ |
| 1973 | $3.36 \mathrm{E}-05$ | 1984 | $5.38 \mathrm{E}-05$ | 1995 | $4.09 \mathrm{E}-05$ | 2006 | $3.93 \mathrm{E}-05$ |
| 1974 | $3.72 \mathrm{E}-05$ | 1985 | $6.08 \mathrm{E}-05$ | 1996 | $3.92 \mathrm{E}-05$ | 2007 | $3.64 \mathrm{E}-05$ |
| 1975 | $5.58 \mathrm{E}-05$ | 1986 | $6.17 \mathrm{E}-05$ | 1997 | $4.29 \mathrm{E}-05$ | 2008 | $3.82 \mathrm{E}-05$ |
| 1976 | $5.59 \mathrm{E}-05$ | 1987 | $6.58 \mathrm{E}-05$ | 1998 | $4.64 \mathrm{E}-05$ | 2009 | $3.86 \mathrm{E}-05$ |
| 1977 | $6.11 \mathrm{E}-05$ | 1988 | $4.89 \mathrm{E}-05$ | 1999 | $3.78 \mathrm{E}-05$ | 2010 | $4.00 \mathrm{E}-05$ |
| 1978 | $5.00 \mathrm{E}-05$ | 1989 | $5.18 \mathrm{E}-05$ | 2000 | $4.01 \mathrm{E}-05$ | 2011 | $3.84 \mathrm{E}-05$ |
| 1979 | $5.69 \mathrm{E}-05$ | 1990 | $4.71 \mathrm{E}-05$ | 2001 | $4.14 \mathrm{E}-05$ | 2012 | $3.77 \mathrm{E}-05$ |



Figure 6.8: Proportion of the sweeping area by survey vessels, calculated from IBTS data for the North Sea haddock

### 6.4.4 Natural mortality

Natural mortality, $M$, describes the mortality rate caused by any factors other than fishing. Due to natural mortality being confounded with fishing mortality (Cook, 2013), the estimates of them become very difficult and unreliable when both are unknown parameters in the model (Eq. 4.5). Following the discussion in literature review as well as in Section 4.2.1, it is unrealistic to assume that natural mortality is constant, although this approach was accepted in Sullivan et al (1990). ICES Working Group Report 2013 has estimated variable values for $M$ at age groups, which is highest for the zero age group and decreases exponentially by age (Table 6.2, Figure 6.9). It has not been explained though how the natural mortality was estimated in ICES report.

Table 6.2: Natural mortality rate (year ${ }^{-1}$ ) at age for North Sea haddock extracted from ICES Working Group Report 2013

| Age (year) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Natural mortality (M) | 2.05 | 1.65 | 0.40 | 0.25 | 0.25 | 0.2 | 0.2 | 0.2 | 0.2 |



Figure 6.9: Natural mortality rate at age for the North Sea haddock extracted from ICES Working Group Report 2013

In order to compare the model result with ICES assessment also to reduce inaccuracy in estimating fishing mortality parameters, the ICES's basic assumptions and estimations of natural mortality at age are used in the survey-landing model. Natural mortality at age $\left(M_{a}\right)$, which is extracted from ICES Report, is transformed to natural mortality at the equivalent length $\left(M_{l}\right)$ inside the survey-landing model. Then it is smoothly approximated over all the length classes in the R programing codes The length-at-age is calculated from the estimated growth parameters in the model (Appendix 6.4).

The advantage of the above method is that the natural mortality is a vector of a fixed input into the model with the values matching the assumption of natural mortality in ICES assessment. It would make the estimated stock size and fishing mortality in survey-landings model more directly comparable with ICES assessment.

### 6.4.5 Probability of landing

The logistic model in Equation 4.11 is used to model the probability of landing (see Section 4.2.1) for the North Sea haddock:

$$
\begin{equation*}
P_{-} \ln d_{l}=\frac{1}{1+\alpha_{D} \exp \left(-\beta_{D} l\right)} \tag{6.9}
\end{equation*}
$$

Minimum landing size for the North Sea haddock is 30 cm in ICES area IV and 27 cm in area $\mathrm{IIIa}^{13}$. Availability of haddock of minimum landing size made it possible to estimate the function parameters externally and to reflect the logistic pattern of the permitted landing. Parameters $\left(\alpha_{D}, \beta_{D}\right)$ were estimated to get the probability of landing for a 30 cm haddock at around 0.5 (Figure 6.10). For application to different years, it was then adjusted manually to achieve the most possible accurate estimation of total discards biomass as well as fishing mortality.

$$
\begin{equation*}
P_{-} \ln d_{l}=\frac{1}{1+36554 \exp (-0.35 l)} \tag{6.10}
\end{equation*}
$$

[^8]

Figure 6.10: Probability of landing for the North Sea haddock. The probability has a logistic increasing trend from 0 to 1 . The probability that a 30 cm haddock is landed is 0.50 (red dotted line).

### 6.5 Initial parameter values

In this section, the parameters that are subject to estimation in survey-landing model are discussed. Preferable approaches to calculate or set values as the initials for estimation are also highlighted. A list of initial parameter values for estimation along with the estimated values is given at the end of the section. The R codes are provided in appendix 6.4.

### 6.5.1 Growth

As discussed in Chapter 5, the model is sensitive in large variability of growth parameters. In order to use initial values of growth parameters $\left(L_{\infty}, K\right)$ as close as possible to the true values, they are estimated (Eq. 4.15) and extracted from external sources before adding them as initial values into the survey-landing model. The
maximum asymptotic length $\left(L_{\infty}\right)$ for North Sea haddock with UK fisheries is estimated within a range of 53 to 74 cm and the curvature parameter $K$ ranges from 0.18 to 0.36 in FishBase (Froese, R. and Pauly, D., 2013) ( ). It is, however, not clear at which years the values have been estimated.

Table 6.3: Growth parameters estimated in FishBase by UK fisheries for North Sea haddock

| $L_{\infty}$ | 53 | 53.5 | 58 | 61.4 | 63.5 | 63.5 | 63.5 | 64 | 66.7 | 68 | 73 | 74 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $K$ | 0.20 | 0.36 | 0.29 | 0.25 | 0.24 | 0.25 | 0.26 | 0.23 | 0.18 | 0.19 | 0.18 | 0.18 |

There is also another method to estimate the growth parameters. It involves using the age-length data from IBTS. This is different from the data, from which survey numbers at length are extracted. Since a limited number of caught fish are aged, conditional probability is applied to estimate the average length-at-age and then to estimate the von Bertalanffy parameters (Eq. 4.14a). This method resulted in 95 for $L_{\infty}$ and 0.17 for $K$. Estimation method along with R code is given in Appendix 6.2. This approach is used to estimate the growth parameters to be used as initial values. That is because growth parameters have a great influence on survey and that the estimation is more accurate if the growth parameters calculated from survey data. In addition to that, the initial parameter values are adjusted manually for the best and closest estimation to ICES assessment.

With regards to estimation of von Bertalanffy curve, average length at age 1 is fixed to $\mu_{R}=17 \mathrm{~cm}$ (Section 6.5.3). This is the most common length at the first peak in the length distribution of survey. Since haddock's growth rate in the first year of birth is different (usually faster) from the later stages in their lives, the average length at age zero is also calculated separately using Equation 4.14b:

$$
\begin{equation*}
E\left(l_{0}\right)=\frac{\left(\mu_{R}-L_{\infty}\left(1-e^{-K}\right)\right)}{e^{-K}}, E\left(l_{1}\right)=\mu_{R} \tag{6.11}
\end{equation*}
$$

Average length at age $2+$ is then estimated inside the model using VBL growth function:

$$
\begin{equation*}
E\left(l_{a \geq 2}\right)=L_{\infty}-\left(L_{\infty}-E\left(l_{0}\right)\right) e^{-(K)(a)} \tag{6.12}
\end{equation*}
$$

### 6.5.2 Gear selectivity

Survey selectivity parameters $\left(q_{\max }, \alpha_{v}, \beta_{v 1}, \beta_{v 2}\right)$ in Equation 4.22 b are all subject to estimation:

$$
\begin{equation*}
q_{l}=\frac{q_{\max }}{1+\alpha_{v} \exp \left(-\beta_{v 1} l\right)+\frac{1}{\alpha_{v}} \exp \left(\beta_{v 2} . l\right)+\exp \left(l\left(\beta_{v 2}-\beta_{v 1}\right)\right)} \tag{6.13}
\end{equation*}
$$

Although the result of sensitivity analysis suggests keeping $q_{\max }$ as a fixed parameter, it is estimated in the model due to lack of any reliable information.

Fishing selectivity function (Eq. 4.3b) also has three estimating parameters $\left(\alpha_{s}, \beta_{s 1}, \beta_{s 2}\right)$ :

$$
\begin{equation*}
s_{l}=\frac{1}{1+\alpha_{s} \exp \left(-\beta_{s 1} l\right)+\frac{1}{\alpha_{s}} \exp \left(\beta_{s 2} \cdot l\right)+\exp \left(l\left(\beta_{s 2}-\beta_{s 1}\right)\right)} \tag{6.14}
\end{equation*}
$$

Both fishing and survey gear selectivity functions are modelled to follow double logistic curves. Through a number of trials and simulations, the initial parameters are set to reflect both of the selectivity patterns as what happen in reality. For fishing gear selectivity the shape of fishing mortality curve (at age) was the reference. In

Chapter 5, when simulation is discussed, the main differences between survey and fishing selectivity are highlighted. Fishing boats target big and marketable fish, while survey vessels need to take a representative sample of all the fish in the sea (even very small fish) so that the length distribution of captured fish reflects the distribution of the population of the fish in the sea. Survey selectivity, compared to fishing selectivity, is higher for smaller fish and lower for bigger fish.

### 6.5.3 Annual recruitment

Recruits are defined as the new fish that are added to the population and had survived over (usually) a year, when they could be captured in fisheries. In survey the sampling takes place in the first quarter of the year when the haddock have not hatched yet. The recruits of the year, therefore, are added to the population in the following year, or even the year following that, depending on their size. Hence, the recruits that appeared in year $t+1$ in the survey-landings model are in fact the individuals that were born in year $t$ and are 1 year old at year $t+1$.

In ICES assessment all the year cohorts are estimated at January each year with the exception of the age zero group, which are considered at July (with 6 months delay). This means that ICES abundance assessment at year $t$ includes the newborn haddock (0-group) at that year, while the model abundance assessment at year $t$ does not include newborn individuals but the abundance starts from the 1 -year-old haddock. Therefore, the recruits at year $t$ in the survey-landings model are in fact the 1-yearold haddock in ICES assessment, which are called 1-group.

Annual recruit numbers vary from one year to another and therefore they are considered to be time-varying parameters and are subject to estimation for the 44 study years from 1969 to 2012. The closest guess for the recruits' initial values is, therefore, the 1 -group (1 year old group) values extracted from ICES stock assessment. The ICES 1-group population size from 1963 to 1968 is used as initial values for the 6 -year historical recruits, to calculate the initial condition $N 0$ (population size at time zero).

In the estimation process, the gamma distribution parameters for distributing recruits over length classes $\left(\alpha_{R}, \beta_{R}\right)$ were set to mark the average length at recruit, ( $\mu_{R}=\alpha_{R} \cdot \beta_{R}$ ), equal to the average length at age $1(17 \mathrm{~cm})$ in von Bertalanffy growth function. The set value for $\mu_{R}$ changes when the model is applied to a different time period. They are basically taken from the length distribution of survey in the target years. The first peak in the survey length distribution is a reliable guide for the average length of individuals at age 1 . In order to keep the average length at recruitment unchanged, $\mu_{R}$ is kept fixed to the average length at age $1(17 \mathrm{~cm})$. The scale parameter $\beta_{R}$, which affects the variance of the distribution, is allowed to vary.

### 6.5.4 Annual fishing mortality scalar

Annual fishing mortality scalar $f_{t}$ is a measure of overall fishing mortality at the full effect of gear selectivity (i.e. when $s_{l}=1$ ). Since it changes over time, $f_{t}$ is estimated for every time step from 1969 to 2012.

Due to differences in stock assessment methods, $f_{t}$ is not estimated or recorded in ICES assessment report. Instead, fishing mortality at age and therefore average fishing mortality for age 2 to 4 ( $F_{a=2 \text { to } 4 y r s}$ ) is available for haddock. Fishing mortality is at its highest rate (i.e. when $s_{l}=1$ ) for 2-4 year-old haddock, which concludes that fishing mortality and fishing scalar are identical for the full effect of gear selectivity (Eq. 4.3):

$$
F_{l, t}=s_{l} f_{t} \rightarrow F_{l, t}=1 f_{t}
$$

Initially, extracted annual average fishing mortality rates from ICES report are sensible starting values for fishing scalar $f_{t}$. Nevertheless, the gear selectivity is not 1 for all the length classes, but has a double logistic increasing trend (Eq. 4.3b and Figure 5.2). Fishing mortality $F_{l, t}$ is in fact a fraction of fishing scalar $f_{t}$. If the
average of the proportion overall the length classes is $p(0<p<1)$, then the initial values can be set equal to $\frac{1}{p} F$ which is slightly bigger than $F_{a=2}$ to $4 y r s$. In the application of the survey-landings model to NS haddock, the ICES assessment average annual fishing mortality is used for fishing scalars' initial values. The initial values are adjusted when applied to time spans to get the closest estimation to ICES assessment where possible.

### 6.6 Survey-landing model: Application on the North Sea haddock over the total 44 years

Observations and fixed parameters are put into the survey-landing model. Least square parameter estimation method within 'optim' function in R is used to minimise the residual sums of square (Appendix 6.4). Since the observations are in different scales and units, it is argued that the model could estimate one set of observations better than the other, which can be avoided by weighting the data (Francis, 2011); and can substantially change the result of assessment. Therefore, model parameters were estimated once when equal weights are given to the least square function and once with different weights. It was to explore if the least square function with unequal weights could improve the accuracy of the parameter estimation. The estimation was overall better when the least square components were given equal weight. Therefore, to avoid cnfusion, the details of the unequal weight is not given here.

### 6.6.1 Least square functions with equal weights

At this point all the least square components in Equation 4.29 are given equal weights $\left(\lambda_{i(i=1,2,3)}=1\right)$ :

$$
\begin{aligned}
& R S S=\lambda_{1} \sum_{t}\left(T L B_{M}-T L B_{O}\right)^{2} \\
&+\lambda_{2} \sum_{l, t}\left(N s_{M}-N s_{O}\right)^{2}+\lambda_{3} \sum_{t}\left(T D B_{M}-T D B_{O}\right)^{2}
\end{aligned}
$$

Figure 6.11 (a) and (e) show that survey-landings model estimated the biomass observations very accurately. Apart from the first year, which is due to the estimation of N0 from historical recruits, catch biomass is estimated identical to the ICES assessment. As the result, total annual landing numbers is well estimated too, which confirms the feasible function being used for weight-length relationship and landing probability. The landing biomass and discards biomass for year 1969 is well estimated, therefore it is expected the same for catch biomass, while it is not the case. That is because catch is estimated from Baranov equation (Equation 4.2) and then multiplied by weight equivalent of fish at length (Equation 4.7 and 4.8) before it is summed over length classes. It is calculated seperate from landed biomass, which is modelled as an increasing logistic function.

b)

d)

f)


진

Figure 6.11: Model estimated (red dashed line) and observed (blue solid line) of a) total landed biomass, b) total survey numbers, c) total catch biomass, d) total survey biomass, e) total discards biomass; and $f$ ) total landing numbers over the 44 years. This is the result when equal weight is given to the components of the least square estimation function.

The model captured total and length distribution of survey numbers at some years better than others (Figure 6.11b and Figure 6.12)



Figure 6.12: Estimated (red dotted lines) and observed (blues vertical bars) survey numbers at length at year. This is the result when equal weight is given to the components of the least square estimation function.

If the 44 study years are considered as three time spans, total survey is overestimated in the first 11 years from 1969 to 1979. In the next 11 years (1980-90) they are estimated close to the observations, while the observations are underestimated in the last 22 years from 1991 to 2012. It is clearer in Figure 6.12 where the survey distributions are plotted separately at each year. In most of the estimated distributions the main peaks, which are 1- and 2 -year olds, are not captured well. That is due to the very high estimation of $L_{\infty}$ or large variation of growth. The biggest influence of the growth parameters is on the survey distribution rather than landed biomass. The result also implies that the growth is not constant but changes over time.

Although the parameters of the landing probability function are fixed inputs, the curve is plotted (Figure 6.13a) to illustrate the landing threshold in comparison with estimated selectivity curves (Figure 6.13c). Estimated gear selectivity curves are reasonable and reflect what is actually happening in the fishing and sampling. Survey vessels are designed to capture more of small haddock compared to fishing boats but fewer big haddock. The peak in survey selectivity is higher than 1 ; that is due to the high estimated value of $q_{\max }$ (Section 4.2.3).


Figure 6.13: a) landing probability, b) estimated von Bertalanffy growth curve, c) estimated fishing and survey gear selectivity curve, d) time-dependent fishing mortality scalar, e) approximated natural mortality over length classes and f) estimated average fishing mortality at length 24 to 42 cm with ICES assessment of average fishing mortality at age 2 to 4 . This is the result when equal weight is given to the components of the least square estimation function.

Growth parameters are estimated in the model to get the average length at age and to shape the von Bertalanffy growth curve (Figure 6.13b, Table 6.4).

Table 6.4: Model estimated average length at age $\left(E\left(l_{t}\right)\right.$ ) using von Bertalanffy function (Figure 6.13b), when equal weight is given to the components of the least square estimation function

| Age (year) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Estimated average length | 1.2 | 13.6 | 24.3 | 33.6 | 41.7 | 48.6 | 54.7 | 59.9 | 64.4 |

Since the growth speed in the first year is different from the rest of haddock's life, the average length at age zero is calculated separately. This is why the average length does not start at zero. The estimated VBL curve has a very high $L_{\infty}$ and a very low $K$, which explains why the length distribution of survey is not accurately estimated. This is the best result after trying variety of initial growth parameter values. With
smaller initial $L_{\infty}$, the trend in modelled estimated fishing mortality is completely off the trend from ICES assessment.

Average fishing selectivity at length 24 to 42 cm , which is the estimated average length at age 2 to 4 years from VBL function, is multiplied by the estimated fishing scalar (Figure 6.13d; Eq. 3.3) to give the average fishing mortality (Figure 6.13f) comparable to fishing mortality estimated in ICES assessment report. The estimated fishing mortality curve matches well with ICES assessment in the period from 1977 to 1991 and 2000 to 2012. Fishing mortality estimation is, however, distanced from ICES mainly in the years 1973-75 and 1990s. Since the fishing mortality in ICES report is estimated too, it is impossible to know which one, if either, is reflecting the true fishing mortality rate. However, the aim is to assess the stock as close as possible to ICES assessment.

Natural mortality at age is an input vector extracted from ICES report. Then it is approximated over length classes using the model estimated growth functions. The approximated natural mortality at length (Figure 6.13e) follows similar pattern as natural mortality at age in ICES assessment.

A very important part of this work is to determine whether the model stock assessment and the ICES assessment differ in results. The result is very promising and shows that the model has similar assessment in dynamics of the North Sea haddock population to ICES. Total stock biomass is estimated quite close too. The consistency in the dynamics of the population between model estimation and ICES assessment means that the model has achieved its overall aim. It is the same case with the annual recruits (Figure 6.14). Model recruitments are in fact the 1 -year-old group in ICES Report, which survived during the previous year and added to fisheries. A list of the initial and estimated parameter values in the model is given in


Figure 6.14: comparison between model estimated (red dashed lines) and ICES assessment (blue solid lines) of a) total stock size (abundance), b) total stock biomass and c) annual recruitment. This is the result when equal weight is given to the components of the least square estimation function.

Table 6.5: List of parameter initial and estimated values when the survey-landing model is applied on the North Sea haddock (All the components of the least square function have equal weights)

| Model <br> parameters | Initial values | Estimated <br> values | Model <br> parameters | Initial values | Estimated <br> values |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{L}_{\infty}$ | $9.25 \mathrm{E}+01$ | $9.36 \mathrm{E}+01$ | $\boldsymbol{r}_{\mathbf{1}}$ | $2.20 \mathrm{E}+09$ | $1.03 \mathrm{E}+09$ |
| $\boldsymbol{K}$ | $1.60 \mathrm{E}-01$ | $1.44 \mathrm{E}-01$ | $\boldsymbol{r}_{\mathbf{2}}$ | $1.54 \mathrm{E}+09$ | $6.79 \mathrm{E}+09$ |
| $\boldsymbol{q}_{\boldsymbol{m a x}}$ | $1.20 \mathrm{E}+00$ | $1.79 \mathrm{E}+00$ | $\boldsymbol{r}_{\mathbf{3}}$ | $1.09 \mathrm{E}+10$ | $8.93 \mathrm{E}+09$ |
| $\boldsymbol{\alpha}_{\boldsymbol{v}}$ | $3.02 \mathrm{E}+04$ | $4.20 \mathrm{E}+04$ | $\boldsymbol{r}_{\boldsymbol{4}}$ | $9.95 \mathrm{E}+09$ | $6.31 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{\boldsymbol{v 1}}$ | $5.90 \mathrm{E}-01$ | $6.57 \mathrm{E}-01$ | $\boldsymbol{r}_{\mathbf{5}}$ | $2.67 \mathrm{E}+09$ | $4.01 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{\boldsymbol{v} \mathbf{2}}$ | $3.00 \mathrm{E}-01$ | $2.70 \mathrm{E}-01$ | $\boldsymbol{r}_{\mathbf{6}}$ | $9.37 \mathrm{E}+09$ | $9.29 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{\boldsymbol{R}}$ | $1.30 \mathrm{E}-01$ | $3.41 \mathrm{E}-01$ | $\boldsymbol{r}_{\mathbf{7}}$ | $1.69 \mathrm{E}+10$ | $1.21 \mathrm{E}+10$ |
| $\boldsymbol{\alpha}_{\boldsymbol{s}}$ | $3.06 \mathrm{E}+02$ | $3.66 \mathrm{E}+02$ | $\boldsymbol{r}_{\mathbf{8}}$ | $1.45 \mathrm{E}+09$ | $1.97 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{\boldsymbol{s} \mathbf{1}}$ | $2.90 \mathrm{E}-01$ | $2.63 \mathrm{E}-01$ | $\boldsymbol{r}_{\mathbf{9}}$ | $2.05 \mathrm{E}+09$ | $3.28 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{\boldsymbol{s} \mathbf{2}}$ | $5.00 \mathrm{E}-04$ | $6.93 \mathrm{E}-05$ | $\boldsymbol{r}_{\mathbf{1 0}}$ | $3.33 \mathrm{E}+09$ | $5.29 \mathrm{E}+09$ |
| $\boldsymbol{f}_{\mathbf{1}}$ | $1.40 \mathrm{E}+00$ | $9.23 \mathrm{E}-01$ | $\boldsymbol{r}_{\mathbf{1 1}}$ | $5.02 \mathrm{E}+09$ | $3.63 \mathrm{E}+09$ |
| $\boldsymbol{f}_{\mathbf{2}}$ | $1.44 \mathrm{E}+00$ | $1.44 \mathrm{E}+00$ | $\boldsymbol{r}_{\mathbf{1 2}}$ | $9.05 \mathrm{E}+09$ | $1.02 \mathrm{E}+10$ |
| $\boldsymbol{f}_{\mathbf{3}}$ | $9.66 \mathrm{E}-01$ | $1.37 \mathrm{E}+00$ | $\boldsymbol{r}_{\mathbf{1 3}}$ | $1.90 \mathrm{E}+09$ | $2.73 \mathrm{E}+09$ |
| $\boldsymbol{f}_{\mathbf{4}}$ | $1.40 \mathrm{E}+00$ | $1.34 \mathrm{E}+00$ | $\boldsymbol{r}_{\mathbf{1 4}}$ | $3.97 \mathrm{E}+09$ | $3.98 \mathrm{E}+09$ |
| $\boldsymbol{f}_{\mathbf{5}}$ | $1.08 \mathrm{E}+00$ | $1.50 \mathrm{E}+00$ | $\boldsymbol{r}_{\mathbf{1 5}}$ | $2.54 \mathrm{E}+09$ | $4.08 \mathrm{E}+09$ |
| $\boldsymbol{f}_{\mathbf{6}}$ | $1.20 \mathrm{E}+00$ | $2.06 \mathrm{E}+00$ | $\boldsymbol{r}_{\mathbf{1 6}}$ | $8.39 \mathrm{E}+09$ | $8.24 \mathrm{E}+09$ |
| $\boldsymbol{f}_{\boldsymbol{7}}$ | $1.38 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $\boldsymbol{r}_{\mathbf{1 7}}$ | $2.18 \mathrm{E}+09$ | $3.19 \mathrm{E}+09$ |


| $f_{8}$ | $1.22 \mathrm{E}+00$ | $1.60 \mathrm{E}+00$ | $r_{18}$ | $3.03 \mathrm{E}+09$ | $2.10 \mathrm{E}+09$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{9}$ | $1.29 \mathrm{E}+00$ | $1.65 \mathrm{E}+00$ | $r_{19}$ | $6.29 \mathrm{E}+09$ | $5.94 \mathrm{E}+09$ |
| $\boldsymbol{f}_{10}$ | $1.33 \mathrm{E}+00$ | $1.57 \mathrm{E}+00$ | $r_{20}$ | $5.31 \mathrm{E}+08$ | $7.90 \mathrm{E}+08$ |
| $f_{11}$ | $1.23 \mathrm{E}+00$ | $8.46 \mathrm{E}-01$ | $r_{21}$ | $1.07 \mathrm{E}+09$ | $1.68 \mathrm{E}+09$ |
| $f_{12}$ | $1.12 \mathrm{E}+00$ | $9.41 \mathrm{E}-01$ | $r_{22}$ | $1.10 \mathrm{E}+09$ | $3.02 \mathrm{E}+09$ |
| $f_{13}$ | $8.24 \mathrm{E}-01$ | $6.93 \mathrm{E}-01$ | $r_{23}$ | $3.63 \mathrm{E}+09$ | $2.25 \mathrm{E}+09$ |
| $f_{14}$ | $8.24 \mathrm{E}-01$ | $7.65 \mathrm{E}-01$ | $r_{24}$ | $3.49 \mathrm{E}+09$ | $4.81 \mathrm{E}+09$ |
| $f_{15}$ | $1.11 \mathrm{E}+00$ | $9.65 \mathrm{E}-01$ | $r_{25}$ | $5.30 \mathrm{E}+09$ | $3.94 \mathrm{E}+09$ |
| $f_{16}$ | $1.09 \mathrm{E}+00$ | $9.71 \mathrm{E}-01$ | $r_{26}$ | $1.64 \mathrm{E}+09$ | $2.12 \mathrm{E}+09$ |
| $\boldsymbol{f}_{17}$ | $1.09 \mathrm{E}+00$ | $1.12 \mathrm{E}+00$ | $r_{27}$ | $7.18 \mathrm{E}+09$ | $5.67 \mathrm{E}+09$ |
| $\mathrm{f}_{18}$ | $1.50 \mathrm{E}+00$ | $1.17 \mathrm{E}+00$ | $r_{28}$ | $1.77 \mathrm{E}+09$ | $1.75 \mathrm{E}+09$ |
| $f_{19}$ | $1.28 \mathrm{E}+00$ | $1.35 \mathrm{E}+00$ | $r_{29}$ | $2.71 \mathrm{E}+09$ | $4.16 \mathrm{E}+09$ |
| $f_{20}$ | $1.39 \mathrm{E}+00$ | $1.35 \mathrm{E}+00$ | $r_{30}$ | $1.63 \mathrm{E}+09$ | $1.38 \mathrm{E}+09$ |
| $f_{21}$ | $1.19 \mathrm{E}+00$ | $1.20 \mathrm{E}+00$ | $r_{31}$ | $1.27 \mathrm{E}+09$ | $9.79 \mathrm{E}+08$ |
| $\boldsymbol{f}_{22}$ | $1.39 \mathrm{E}+00$ | $1.29 \mathrm{E}+00$ | $r_{32}$ | $1.78 \mathrm{E}+10$ | $1.30 \mathrm{E}+10$ |
| $f_{23}$ | $1.11 \mathrm{E}+00$ | $1.10 \mathrm{E}+00$ | $r_{33}$ | $3.41 \mathrm{E}+09$ | $3.41 \mathrm{E}+09$ |
| $\boldsymbol{f}_{24}$ | $1.23 \mathrm{E}+00$ | $1.36 \mathrm{E}+00$ | $r_{34}$ | $3.65 \mathrm{E}+08$ | $1.22 \mathrm{E}+08$ |
| $f_{25}$ | $1.12 \mathrm{E}+00$ | $1.54 \mathrm{E}+00$ | $r_{35}$ | $4.62 \mathrm{E}+08$ | $1.82 \mathrm{E}+08$ |
| $f_{26}$ | $1.04 \mathrm{E}+00$ | $1.37 \mathrm{E}+00$ | $r_{36}$ | $4.99 \mathrm{E}+08$ | $3.70 \mathrm{E}+08$ |
| $\boldsymbol{f}_{27}$ | $9.16 \mathrm{E}-01$ | $1.42 \mathrm{E}+00$ | $r_{37}$ | $4.78 \mathrm{E}+08$ | $5.14 \mathrm{E}+08$ |
| $\boldsymbol{f}_{28}$ | $8.60 \mathrm{E}-01$ | $1.27 \mathrm{E}+00$ | $r_{38}$ | $5.45 \mathrm{E}+09$ | $2.54 \mathrm{E}+09$ |
| $f_{29}$ | $6.71 \mathrm{E}-01$ | $1.50 \mathrm{E}+00$ | $r_{39}$ | $1.16 \mathrm{E}+09$ | $1.09 \mathrm{E}+08$ |
| $f_{30}$ | $7.55 \mathrm{E}-01$ | $1.61 \mathrm{E}+00$ | $r_{40}$ | $6.80 \mathrm{E}+08$ | $2.54 \mathrm{E}+09$ |
| $f_{31}$ | $8.93 \mathrm{E}-01$ | $1.93 \mathrm{E}+00$ | $r_{41}$ | $5.52 \mathrm{E}+08$ | $8.11 \mathrm{E}+07$ |
| $f_{32}$ | $9.56 \mathrm{E}-01$ | $8.44 \mathrm{E}-01$ | $r_{42}$ | $4.26 \mathrm{E}+09$ | $2.82 \mathrm{E}+09$ |
| $f_{33}$ | $6.15 \mathrm{E}-01$ | $6.96 \mathrm{E}-01$ | $r_{43}$ | $2.44 \mathrm{E}+08$ | $9.22 \mathrm{E}+08$ |
| $f_{34}$ | $2.86 \mathrm{E}-01$ | $2.93 \mathrm{E}-01$ | $r_{44}$ | $9.82 \mathrm{E}+07$ | $5.83 \mathrm{E}+08$ |
| $\boldsymbol{f}_{35}$ | $2.51 \mathrm{E}-01$ | $1.75 \mathrm{E}-01$ | rhist $_{1}$ | $2.55 \mathrm{E}+10$ | $1.98 \mathrm{E}+09$ |
| $f_{36}$ | $3.29 \mathrm{E}-01$ | $2.04 \mathrm{E}-01$ | rhist $_{2}$ | $2.98 \mathrm{E}+08$ | $9.78 \mathrm{E}+09$ |
| $f_{37}$ | $3.88 \mathrm{E}-01$ | $2.64 \mathrm{E}-01$ | rhist $_{3}$ | $1.13 \mathrm{E}+09$ | $2.42 \mathrm{E}+08$ |
| $f_{38}$ | $6.39 \mathrm{E}-01$ | $3.27 \mathrm{E}-01$ | rhist $_{4}$ | $3.15 \mathrm{E}+09$ | $8.57 \mathrm{E}+09$ |
| $f_{39}$ | $4.98 \mathrm{E}-01$ | $2.77 \mathrm{E}-01$ | rhist $_{5}$ | $8.27 \mathrm{E}+09$ | $2.90 \mathrm{E}+10$ |
| $f_{40}$ | $2.84 \mathrm{E}-01$ | $2.45 \mathrm{E}-01$ | $\text { rhist }_{6}$ | $4.99 \mathrm{E}+10$ | $2.80 \mathrm{E}+09$ |
| $f_{41}$ | $2.61 \mathrm{E}-01$ | $2.44 \mathrm{E}-01$ | RSS |  | $4.80 \mathrm{E}+10$ |
| $f_{42}$ | $2.91 \mathrm{E}-01$ | $1.75 \mathrm{E}-01$ |  |  |  |
| $f_{43}$ | $3.73 \mathrm{E}-01$ | $2.35 \mathrm{E}-01$ |  |  |  |
| $f_{44}$ | $2.20 \mathrm{E}-01$ | $1.73 \mathrm{E}-01$ |  |  |  |

Survey-landing model did well in recovering the landing and discards. The result also was promising in assessment the stock size and biomass. The annual recruitment too was estimated closely to the ICES assessment. However, the survey numbers are not quite what is expected from the model. Moreover, if it is assumed that the fishing mortality estimated in ICES is the reference for comparison, the two gaps in fishing mortality is something to be addressed to improve the survey-landing model.

### 6.7 Survey-landings model: Application on the North Sea

## haddock over shorter time lines

In real world, for many reasons, growth parameters including the maximum asymptotic length $L_{\infty}$ and growth rate $K$ are not fixed but change over time. In the previous section (Section 6.6) the model seemed to be under constraint to manage between the different growth rates and maximum lengths throughout the long 44 years.

One approach to potentially address that is to change growth parameters from constant to time-varying parameters in the model. This approach makes the model very complicated and adds more uncertainties to the model due to the number of estimating parameters. In order to manage the effect of the variability of growth over time, the 44-year study time is broken down into four time lines 1969-1979, 19801989, 19990-1999 and 2000-2012. The survey-landings model is then applied to each period separately. This allows the model to assess the dynamics of the populations independently of each other, which could hold very different growth models. The following subsections discuss the details of the results starting from the oldest time 1969-1979 and ends with the most recent years 2000-2012.

Input vectors, including the weight-length relationship and natural mortality-at-age, are similar in all periods. Proportion of the sea area swept by sampling is calculated and extracted from IBTS for the equivalents years. Six historical years are used for the estimation of initial abundance at the start of each time line. The average length
at age 1 is manually adjusted and fixed in the model based on the survey distribution at each time line. Length-dependent probability of landing is adjusted externally to get the best estimation of landing numbers before adding them to the model as fixed parameters. Also the parameter $\beta$, representing the variation of growth, is kept fixed but adjusted for the best estimation output.

### 6.7.1 Model application on the North Sea haddock over the period 1969-1979

The survey-landings model is attempting to assess the North Sea haddock in the oldest study period from 1969 to 1979. The survey observations from IBTS and the landings from commercial reports are obviously not as accurate as the recent decade.

The length-dependent probability of landing (EQ. 4.11) is manually adjusted (Figure 6.17.a) for the better estimation of landing numbers. The probability of landing is set at 0.50 for haddock at length 27 cm :

$$
\begin{equation*}
P_{-} \ln d_{l}=\frac{1}{1+549303 \exp (-0.5 l)} \tag{6.21}
\end{equation*}
$$

A list of initial and estimated parameters values is given in .

Two observations from commercial reports including total landed biomass and total discards biomass (Figure 6.15.a and e) are captured very well in the survey-landing model. Length distribution of survey is, however, underestimated in years 1973, 1974 and 1979 (Figure 6.15.b, Figure 6.16). Although the survey data is not as accurate as expected in this decade, total survey numbers are captured well. The estimated peaks are clear but the distribution is still better in some years than the others.
a)


e)

b)

d)

f)

(puesnouı)
sıəqunu Кəəıns o

Figure 6.15: Model estimated (red dashed line) and observed (blue solid line) over the 11 years 1969-1979; a) Total landed biomass, b) total survey numbers, c) total catch biomass, d) total survey biomass, e) total discards biomass and f) total landing numbers


Figure 6.16: Estimated (red dotted lines) and observed (blues vertical bars) survey numbers at length at year. This is the result when survey-landing model is applied on the 11 years 19691979

The growth parameter $L_{\infty}$ that the model estimated is 100 , which is much bigger than the estimated $L_{\infty}$ in the other three periods that comes in later sections. Estimated growth rate is also lower (Figure 6.17.b). Since the speed of growth is different during the first year, the average length at age zero is estimated separately. The estimated average length of -0.5 cm at age 0 may not seem to make sense but it is just to adjust the growth rate of the first year with the rest of fish's life ( ).

Table 6.6: Model estimated average length at age $\left(E\left(l_{t}\right)\right)$ using von Bertalanffy function when survey-landing model is applied on the years 1969-1979

| Age (year) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Estimated average length (cm) | -0.5 | 14 | 26.4 | 37.1 | 46.3 | 54.1 | 60.8 | 66.5 | 71.4 |

Fishing selectivity is reaching its highest at length $26-46 \mathrm{~cm}$, which is equivalent to age 2-4 years in the model (Figure 6.17.b and c). Fishing mortality, as a product of fishing scalar in Figure 6.17 d and average fishing selectivity at length $26-46 \mathrm{~cm}$, is slightly distanced from the fishing mortality at age 2-4 in ICES assessment. However, it may arguably be accepted due the result of stock size and stock biomass estimation. Model estimated stock size and stock biomass follow the trend in the ICES assessment result (Figure 6.18.a and b).

The overall structure of estimated survey selectivity curve (Figure 6.17.c) is realistic with the increasing selectivity at length as well as in comparison with the structure of fishing selectivity curve. Although survey selectivity is very low at length 46 (equivalent to age 4), it does not affect total estimation of survey. This is because there is not very many big fish captured in survey (Figure 6.16).

The model estimated recruits could also be argued that they are distanced from ICES assessment (Figure 6.18.c).

Estimated population numbers at length are plotted in Figure 6.19 with identical y-axis to show the changes in distribution and stock size over time more obviously. The combined effect of fishing mortality (Figure 6.20) and natural mortality (Figure 6.17.e) is clearly in the total mortality (Figure 6.21). Also the variations in annual recruits and mortality explain changes in the dynamics of the population at time.


Figure 6.17: a) landing probability, b) estimated von Bertalanffy growth curve, c) estimated fishing and survey gear selectivity curve, d) time-dependent fishing mortality scalar, e) approximated natural mortality over length classes and f) model estimated average fishing mortality at length 26 to 46 cm (dashed red) with ICES assessment of average fishing mortality at age 2 to 4 (solid blue).
This is the result when survey-landings model is applied on 11 years 1969-1979


Figure 6.18: comparison between model estimated (red dashed lines) and ICES assessment (blue solid lines) of a) total stock size (abundance), b) total stock biomass and c) annual recruitment. This is the result when survey-landing model is applied on 11 years 1969-1979


Figure 6.19: Model estimated stock size (population) at length at year. This is the result when survey-landing model is applied on 11 years 1969-1979


Figure 6.20: Model estimated fishing mortality curve at length at year. This is the result when survey-landing model is applied on 11 years1969-1979


Figure 6.21: Model estimated total mortality curve at length at year. This is the result when survey-landing model is applied on 11 years 1969-1979

Table 6.7: List of parameter initial and estimated values when the survey-landing model is applied on the North Sea haddock over 11 years (1969-1979)

| $\begin{gathered} \text { Model } \\ \text { parameters } \end{gathered}$ | Initial values | Estimated values | Model parameters | Initial values | Estimated values |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{\infty}$ | $9.50 \mathrm{E}+01$ | $1.00 \mathrm{E}+02$ | $r_{1}$ | $2.20 \mathrm{E}+09$ | $2.40 \mathrm{E}+08$ |
| $K$ | $1.40 \mathrm{E}-01$ | $1.56 \mathrm{E}-01$ | $r_{2}$ | $1.54 \mathrm{E}+09$ | $2.91 \mathrm{E}+09$ |
| $q_{\text {max }}$ | $1.79 \mathrm{E}+00$ | $1.13 \mathrm{E}+00$ | $r_{3}$ | $1.09 \mathrm{E}+10$ | $1.07 \mathrm{E}+10$ |
| $\alpha_{v}$ | $4.20 \mathrm{E}+06$ | $8.45 \mathrm{E}+06$ | $r_{4}$ | $9.95 \mathrm{E}+09$ | $3.32 \mathrm{E}+09$ |
| $\beta_{v 1}$ | $6.00 \mathrm{E}-01$ | $9.99 \mathrm{E}-01$ | $r_{5}$ | $2.67 \mathrm{E}+09$ | $5.27 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{v 2}$ | $2.70 \mathrm{E}-01$ | $4.63 \mathrm{E}-01$ | $r_{6}$ | $9.37 \mathrm{E}+09$ | $6.85 \mathrm{E}+09$ |
| $\beta_{R}$ | $3.10 \mathrm{E}-01$ | $4.13 \mathrm{E}-01$ | $r_{7}$ | $1.69 \mathrm{E}+10$ | $1.15 \mathrm{E}+10$ |
| $\alpha_{s}$ | $3.66 \mathrm{E}+02$ | $5.40 \mathrm{E}+02$ | $r_{8}$ | $1.45 \mathrm{E}+09$ | $2.57 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{s 1}$ | $2.60 \mathrm{E}-01$ | $3.38 \mathrm{E}-01$ | $r_{9}$ | $2.05 \mathrm{E}+09$ | $1.69 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{s 2}$ | $6.90 \mathrm{E}-05$ | 5.13E-05 | $r_{10}$ | $3.33 \mathrm{E}+09$ | $4.72 \mathrm{E}+09$ |
| $\boldsymbol{f}_{1}$ | $9.75 \mathrm{E}-01$ | $7.15 \mathrm{E}-01$ | $r_{11}$ | $5.02 \mathrm{E}+09$ | $4.73 \mathrm{E}+08$ |
| $f_{2}$ | $9.38 \mathrm{E}-01$ | $1.22 \mathrm{E}+00$ | rhist $_{1}$ | $2.55 \mathrm{E}+10$ | $1.03 \mathrm{E}+10$ |
| $f_{3}$ | $6.94 \mathrm{E}-01$ | $1.30 \mathrm{E}+00$ | $r_{\text {hist }}^{2}$ | $2.98 \mathrm{E}+08$ | $3.70 \mathrm{E}+08$ |
| $f_{4}$ | $8.56 \mathrm{E}-01$ | $1.02 \mathrm{E}+00$ | rhist $_{3}$ | $1.13 \mathrm{E}+09$ | $3.19 \mathrm{E}+09$ |
| $f_{5}$ | $7.50 \mathrm{E}-01$ | $1.26 \mathrm{E}+00$ | rhist $_{4}$ | $3.15 \mathrm{E}+09$ | $5.62 \mathrm{E}+09$ |
| $\boldsymbol{f}_{6}$ | $7.23 \mathrm{E}-01$ | $1.49 \mathrm{E}+00$ | rhist $_{5}$ | $8.27 \mathrm{E}+09$ | $2.48 \mathrm{E}+10$ |
| $\boldsymbol{f}_{7}$ | $9.10 \mathrm{E}-01$ | $1.59 \mathrm{E}+00$ | rhist $_{6}$ | $4.99 \mathrm{E}+10$ | $4.82 \mathrm{E}+09$ |
| $\boldsymbol{f}_{8}$ | $8.54 \mathrm{E}-01$ | $1.12 \mathrm{E}+00$ | RSS |  | $4.93 \mathrm{E}+09$ |
| $f_{9}$ | $8.29 \mathrm{E}-01$ | $1.05 \mathrm{E}+00$ |  |  |  |
| $f_{10}$ | $8.05 \mathrm{E}-01$ | $1.15 \mathrm{E}+00$ |  |  |  |
| $f_{11}$ | $7.02 \mathrm{E}-01$ | 8.59E-01 |  |  |  |

Survey-landings model application on the North Sea haddock over the years 19691979 was overall successful since the model managed to estimated landing and discards accurately. Estimated total survey numbers is close to the sample numbers and the survey distribution is well captured, in particular at the first peak when the recruits are adding to the population. Although the fishing mortality is slightly distanced from ICES assessment, the model estimated similar population dynamic approaches to ICES assessment.

### 6.7.2 Model application on the North Sea haddock over the period 1980-1989

The survey-landings model is applied to the North Sea haddock over the 10 -year period from 1980 to 1989 at 1-year time steps. The length classes are unchanged from 3 cm to 87 cm at 1 cm width.

Average length at recruitment $\left(\mu_{R}\right)$ is kept fixed at 14 cm . That is because the first peak in length distribution of survey, which represents the approximate average length of individuals at age 1 , is at length class 14 cm . For the input vector of landing probability $\left(P_{-} \ln d_{l}\right)$ Equation 6.3 is applied, in which the threshold length for landing is set at 30 cm . Proportion of the sampling area $\left(P a_{t}\right)$ is extracted and calculated from IBTS data and added as a vector into the model.




d)

f)


Figure 6.22: Model estimated (red dashed line) and observed (blue solid line) over 10 years 1980-1989; a) Total landed biomass, b) total survey numbers, c) total catch biomass, d) total survey biomass, e) total discards biomass and f) total landing numbers

All three sets of observation are well estimated in survey-landing model (Figure 6.22.a,b,e). A promising result is that the model estimated survey numbers match very well with the length distribution of observed survey. Both first and second peaks are captured closely to the peaks in the observations and that they are at the right length (Figure 6.23).


Figure 6.23: Estimated (red dotted lines) and observed (blues vertical bars) survey numbers at length at year. This is the result when survey-landing model is applied on 10 years 1980-1989

Growth rate $K$ is estimate higher and $L_{\infty}$ is lower (Figure 6.24.b) than the previous period. Fishing selectivity reaches its highest value at length 26 cm , which is equivalent to age 2 in the ICES assessment. It has its highest selectivity at length 44 cm (4-years old haddock) (Figure 6.24.c). The model estimated the average fishing mortality at the equivalent length of age 2 to $4(26-44 \mathrm{~cm})$ estimated from the VBL growth function inside the model. Apart from the last two years, in which it is underestimated, the model estimated fishing mortality is accurately close to the ICES assessment (Figure 6.24.f).


Figure 6.24: a) landing probability, b) estimated von Bertalanffy growth curve, c) estimated fishing and survey gear selectivity curve, d) time-dependent fishing mortality scalar, e) approximated natural mortality over length classes and f) model estimated average fishing mortality at length 26 to 44 cm (dashed red) with ICES assessment of average fishing mortality at age 2 to 4 (solid blue).
This is the result when survey-landing model is applied on the 10-year period 1980-1989

Table 6.8: Model estimated average length at age $\left(E\left(l_{t}\right)\right)$ using von Bertalanffy function when survey-landing model is applied on North Sea haddock over the 10-year period 1980-1989

| Age (year) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Estimated average length (cm) | -1.7 | 14 | 26.3 | 36.0 | 43.7 | 49.6 | 54.3 | 58.0 | 60.9 |

There is another promising model result and that is the model estimation of stock size, stock biomass and recruitment follow the overall trend and that they are very close to the ICES assessment (Figure 6.25).


Figure 6.25: comparison between model estimated (red dashed lines) and ICES assessment (blue solid lines) of a) total stock size (abundance), b) total stock biomass and $c$ ) annual recruitment. This is the result when survey-landing model is applied on the 10-year period 1980-1989

Model estimated length distribution of population is plotted with identical scales to illustrate the changes in total size and distribution of population over time (Figure 6.26). The ICES assessment uses age-structured methods, for which length frequency distribution is not available to compare it with the result from survey-landings model. Fishing mortality and total mortality share the same comparison issue. Until at length 20 cm , total mortality (Figure 6.28) is highly influenced by the length-dependent trend in natural mortality (Figure 6.24.e). Fishing mortality (Figure 6.27) is the main source of variability when fish are bigger and exposed to fishing mortality. This is clear when the fishing mortality and total mortality plots are compared.


Figure 6.26: Model estimated stock size (population) at length at year. This is the result when survey-landing model is applied on the 10-year period 1980-1989


Figure 6.27: Model estimated fishing mortality curve at length at year. This is the result when survey-landing model is applied on the 10-year period 1980-1989


Figure 6.28: Model estimated total mortality curve at length at year. This is the result when survey-landing model is applied on the 10-year period 1980-1989

Table 6.9: List of parameter initial and estimated values when the survey-landing model is applied on the North Sea haddock over the 10-year period (1980-1989)

| Model parameters | Initial values | Estimated values | Model parameters | Initial values | Estimated values |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{\infty}$ | $7.00 \mathrm{E}+01$ | $7.13 \mathrm{E}+01$ | $r_{1}$ | $9.05 \mathrm{E}+09$ | $5.21 \mathrm{E}+09$ |
| K | $2.40 \mathrm{E}-01$ | $2.43 \mathrm{E}-01$ | $r_{2}$ | $1.90 \mathrm{E}+09$ | $1.89 \mathrm{E}+09$ |
| $\boldsymbol{q}_{\max }$ | $1.50 \mathrm{E}+00$ | $1.51 \mathrm{E}+00$ | $r_{3}$ | $3.97 \mathrm{E}+09$ | $4.33 \mathrm{E}+09$ |
| $\alpha_{v}$ | $7.00 \mathrm{E}+05$ | $1.08 \mathrm{E}+06$ | $r_{4}$ | $2.54 \mathrm{E}+09$ | $4.78 \mathrm{E}+09$ |
| $\beta_{v 1}$ | $9.00 \mathrm{E}-01$ | $8.80 \mathrm{E}-01$ | $r_{5}$ | $8.39 \mathrm{E}+09$ | $7.01 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{v 2}$ | $3.00 \mathrm{E}-01$ | $3.77 \mathrm{E}-01$ | $r_{6}$ | $2.18 \mathrm{E}+09$ | $2.09 \mathrm{E}+09$ |
| $\beta_{R}$ | $5.00 \mathrm{E}-01$ | $2.70 \mathrm{E}-01$ | $r_{7}$ | $3.03 \mathrm{E}+09$ | $4.02 \mathrm{E}+09$ |
| $\alpha_{s}$ | $5.00 \mathrm{E}+05$ | $3.56 \mathrm{E}+06$ | $r_{8}$ | $6.29 \mathrm{E}+09$ | $6.37 \mathrm{E}+09$ |
| $\beta_{s 1}$ | $7.00 \mathrm{E}-01$ | $5.88 \mathrm{E}-01$ | $r_{9}$ | $5.31 \mathrm{E}+08$ | $2.19 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{s 2}$ | $2.00 \mathrm{E}-04$ | $3.04 \mathrm{E}-02$ | $r_{10}$ | $1.07 \mathrm{E}+09$ | $1.27 \mathrm{E}+09$ |
| $f_{1}$ | $6.93 \mathrm{E}-01$ | $9.48 \mathrm{E}-01$ | $\text { rhist }_{1}$ | $9.37 \mathrm{E}+09$ | $2.50 \mathrm{E}+09$ |
| $f_{2}$ | $6.02 \mathrm{E}-01$ | $6.52 \mathrm{E}-01$ | $r_{\text {hist }}^{2}$ | $1.69 \mathrm{E}+10$ | $4.99 \mathrm{E}+09$ |
| $f_{3}$ | $6.37 \mathrm{E}-01$ | $7.93 \mathrm{E}-01$ | rhist $_{3}$ | $1.45 \mathrm{E}+09$ | $1.71 \mathrm{E}+09$ |
| $f_{4}$ | $6.18 \mathrm{E}-01$ | $1.12 \mathrm{E}+00$ | $\text { rhist }_{4}$ | $2.05 \mathrm{E}+09$ | $2.28 \mathrm{E}+09$ |
| $\boldsymbol{f}_{5}$ | $6.09 \mathrm{E}-01$ | $1.00 \mathrm{E}+00$ | rhist $_{5}$ | $3.33 \mathrm{E}+09$ | $7.54 \mathrm{E}+09$ |
| $f_{6}$ | $6.23 \mathrm{E}-01$ | $1.02 \mathrm{E}+00$ | rhist $_{6}$ | $5.02 \mathrm{E}+09$ | $1.29 \mathrm{E}+09$ |
| $f_{7}$ | $7.48 \mathrm{E}-01$ | $1.21 \mathrm{E}+00$ | RSS |  |  |
| $\boldsymbol{f}_{8}$ | $7.40 \mathrm{E}-01$ | $1.14 \mathrm{E}+00$ |  |  |  |
| $f_{9}$ | $8.70 \mathrm{E}-01$ | $7.80 \mathrm{E}-01$ |  |  |  |
| $f_{10}$ | $9.44 \mathrm{E}-01$ | $4.18 \mathrm{E}-01$ |  |  |  |

Model application on the NS haddock over years 1980-1989 proved to be much more accurate than the previous period. It is more obvious in survey numbers in particular. This estimation difference could be due to the better availability of data. The surveylandings model has achieved its aim estimating the dynamics of population close to ICES results by using different source of observation.

### 6.7.3 Model application on the North Sea haddock over the period 1990-1999

The survey-landings model is now applied on the NS haddock in the third time period (1990 to 1999). The weight-length function and natural mortality at age is unchanged. Parameters of the landing probability are estimated to set the landing probability of a 30 cm haddock equal to 0.50 . The variability of growth $\beta$ is fixed at 1 and the average length at age 1 is set equal to $15 \mathrm{~cm}\left(\mu_{R}=15\right)$.

The two commercial observation data sets, landed and discards biomass, are recovered accurately (Figure 6.29.a and e). Total survey numbers are fairly close to the true values (Figure 6.29.b). Estimated survey distribution has done well in capturing the distribution curves (Figure 6.30). The first peak is slightly over estimated at 1990 and 1994 and second peak is underestimated at 1997, which is also reflected in the total survey numbers.
a)

c)

e)

b)

d)

(puesnoчı)
sıəqunu Кəлıns


Figure 6.29: Model estimated (red dashed line) and observed (blue solid line) over the 10 years 1990-1999; a) Total landed biomass, b) total survey numbers, c) total catch biomass, d) total survey biomass, e) total discards biomass and f) total landing numbers


Figure 6.30: Estimated (red dotted lines) and observed (blues vertical bars) survey numbers at length at year. This is the result when survey-landing model is applied on the 10 -year period 1990-1999

The asymptotic maximum length is still lower than the previous study period, while the growth rate is unexpectedly large (Figure 6.31.b). Fishing selectivity curve reaches its highest point at length 30 cm to match the high fishing mortality of age 2-4 in ICES assessment (Figure 6.31.c). Fishing mortality for the length $26-39 \mathrm{~cm}$ (equivalent the age 2-4) is slightly distanced from fishing mortality extracted from ICES Report for almost all of the time line (Figure 6.31.f).

The survey selectivity curve is capturing smaller haddock compared to fishing selectivity. The survey selectivity curve domed at length 40 cm when there is hardly any fish to capture at bigger size (Figure 6.31.c). The high peak at survey selectivity is due to parameter $q_{\max }$.


Figure 6.31: a) landing probability, b) estimated von Bertalanffy growth curve, c) estimated fishing and survey gear selectivity curve, d) time-dependent fishing mortality scalar, e) approximated natural mortality over length classes and f) model estimated average fishing mortality at length 26 to 44 cm (dashed red) with ICES assessment of average fishing mortality at age 2 to 4 (solid blue).
This is the result when survey-landing model is applied on the 10-year period 1990-1999

Table 6.10: Model estimated average length at age $\left(E\left(l_{t}\right)\right)$ using von Bertalanffy function when survey-landing model is applied over years 1990-1999

| Age (year) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Estimated average length (cm) | -2.2 | 15 | 26.3 | 33.8 | 38.8 | 42.0 | 44.2 | 45.6 | 46.5 |

In spite of the gap between the model-estimated and ICES-estimated fishing mortality, stock size and biomass have similar trends and are close together (Figure 6.32.a and b). That is very promising to see the standard assessment method applied by ICES shares similar result with survey-landing model in estimating the dynamics of the population. The recruitment numbers are encouraging too (Figure 6.32.c).


Figure 6.32: comparison between model estimated (red dashed lines) and ICES assessment (blue solid lines) of a) total stock size (abundance), b) total stock biomass and $c$ ) annual recruitment. This is the result when survey-landing model is applied on the 10-year period 1990-1999

Model estimated population numbers at length is given in Figure 6.33 for all the period. ICES do not provide any length distribution of abundance to compare the model results with.


Figure 6.33: Model estimated stock size (population) at length at year. This is the result when survey-landing model is applied on the 10-year period 1990-1999


Figure 6.34: Model estimated fishing mortality curve at length at year. This is the result when survey-landing model is applied on the 10-year period 1990-1999


Figure 6.35: Model estimated total mortality curve at length at year. This is the result when survey-landing model is applied on the 10-year period 1990-1999

Table 6.11: List of parameter initial and estimated values when the survey-landing model is applied on the North Sea haddock over the 10-year period (1990-1999)

| Model parameters | Initial values | Estimated values | Model parameters | Initial values | Estimated values |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{\infty}$ | $6.50 \mathrm{E}+01$ | $4.83 \mathrm{E}+01$ | $r_{1}$ | $1.10 \mathrm{E}+09$ | $3.95 \mathrm{E}+08$ |
| $K$ | $3.00 \mathrm{E}-01$ | $4.16 \mathrm{E}-01$ | $r_{2}$ | $3.63 \mathrm{E}+09$ | $2.56 \mathrm{E}+09$ |
| $\boldsymbol{q}_{\text {max }}$ | $1.20 \mathrm{E}+00$ | $2.56 \mathrm{E}+00$ | $r_{3}$ | $3.49 \mathrm{E}+09$ | $4.28 \mathrm{E}+09$ |
| $\alpha_{v}$ | $3.02 \mathrm{E}+04$ | $1.88 \mathrm{E}+05$ | $r_{4}$ | $5.30 \mathrm{E}+09$ | $4.38 \mathrm{E}+09$ |
| $\beta_{v 1}$ | $5.90 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $r_{5}$ | $1.64 \mathrm{E}+09$ | $2.17 \mathrm{E}+09$ |
| $\beta_{v 2}$ | $3.00 \mathrm{E}-01$ | $3.67 \mathrm{E}-01$ | $r_{6}$ | $7.18 \mathrm{E}+09$ | $5.53 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{R}$ | $5.00 \mathrm{E}-01$ | $2.82 \mathrm{E}-01$ | $\boldsymbol{r}_{7}$ | $1.77 \mathrm{E}+09$ | $2.46 \mathrm{E}+09$ |
| $\alpha_{s}$ | $5.00 \mathrm{E}+06$ | $4.23 \mathrm{E}+07$ | $r_{8}$ | $2.71 \mathrm{E}+09$ | $3.14 \mathrm{E}+09$ |
| $\beta_{s 1}$ | $8.00 \mathrm{E}-01$ | $9.00 \mathrm{E}-01$ | $r_{9}$ | $1.63 \mathrm{E}+09$ | $1.97 \mathrm{E}+09$ |
| $\beta_{s 2}$ | $5.00 \mathrm{E}-04$ | $2.94 \mathrm{E}-02$ | $r_{10}$ | $1.27 \mathrm{E}+09$ | $1.48 \mathrm{E}+09$ |
| $\boldsymbol{f}_{1}$ | $9.22 \mathrm{E}-01$ | $4.43 \mathrm{E}-01$ | rhist ${ }_{1}$ | $8.39 \mathrm{E}+09$ | $2.03 \mathrm{E}+09$ |
| $\boldsymbol{f}_{2}$ | $1.02 \mathrm{E}+00$ | $4.37 \mathrm{E}-01$ | $\boldsymbol{r h i s t}_{2}$ | $2.18 \mathrm{E}+09$ | $1.87 \mathrm{E}+09$ |
| $f_{3}$ | $9.19 \mathrm{E}-01$ | $6.10 \mathrm{E}-01$ | $r_{\text {hist }}^{3}$ | $3.03 \mathrm{E}+09$ | $6.25 \mathrm{E}+08$ |
| $\boldsymbol{f}_{4}$ | $7.64 \mathrm{E}-01$ | $6.89 \mathrm{E}-01$ | $\boldsymbol{r h i s t}_{4}$ | $6.29 \mathrm{E}+09$ | $5.84 \mathrm{E}+09$ |
| $\boldsymbol{f}_{5}$ | $8.50 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | rhist $_{5}$ | $5.31 \mathrm{E}+08$ | $1.01 \mathrm{E}+09$ |
| $f_{6}$ | $7.73 \mathrm{E}-01$ | $6.02 \mathrm{E}-01$ | $r_{\text {hist }}^{6}$ | $1.07 \mathrm{E}+09$ | $1.71 \mathrm{E}+09$ |
| $\boldsymbol{f}_{7}$ | $9.74 \mathrm{E}-01$ | $5.53 \mathrm{E}-01$ | RSS |  | $3.02 \mathrm{E}+09$ |
| $f_{8}$ | $4.50 \mathrm{E}-01$ | $5.80 \mathrm{E}-01$ |  |  |  |
| $f_{9}$ | 4.19E-01 | $5.59 \mathrm{E}-01$ |  |  |  |
| $f_{10}$ | $5.15 \mathrm{E}-01$ | $6.00 \mathrm{E}-01$ |  |  |  |

The model result is promising for estimation the time vector observations as well as the length frequency of survey. Gear selectivity curves were in expected shape and match well with the estimated length at age as well as the landing probability threshold. The main point of concern is the gap between model and ICES fishing mortality assessment, and mainly because the gap is more systematic than random. Nevertheless, the stock size and biomass is the promising point of the result. Also the recruits, as a reliable source of comparison, are estimated well.

### 6.7.4 Model application on the North Sea haddock over the period 2000-2012

The survey-landings model is applied to survey numbers, landed biomass and discards biomass of the North Sea haddock from the year 2000 to the year 2012. Year 2000 has been chosen as the start of the time line because it was a very strong year for recruitment. The previous six years (1994-1999) are used as historical data to estimate initial population size. Input values and vectors are unchanged but the initial parameter values are manually adjusted after every model run to get the best estimation. The components of the least square function are given equal weights.

It is clear that there has been a great success in estimation of total survey numbers (Figure 6.36b). The model well captured the first year and second year peaks (Figure 6.39). Landed biomass and discards biomass are also well estimated (Figure 6.36.a and e). The over-estimated landed biomass in 2001 is due the year 2000 being a much stronger year compared to all the other years. The same effect applies on landing numbers (Figure 6.36f).

c)

e)

b)
d)


f)

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Figure 6.36: Model estimated (red dashed line) and observed (blue solid line) over the 13 years 2000-2012; a) total landed biomass, b) total survey numbers, c) total catch biomass, d) total survey biomass, e) total discards biomass and f) total landing numbers

Over the time line between 2000 and 2001, the model estimated quite a high growth rate of 0.32 and low $L_{\infty}$ of 41 cm for the North Sea haddock (Figure 6.37b). Although the estimated growth parameters seem to be distanced from previous period, the literature supports the increasing trend of growth rate and that the maximum asymptotic length has become smaller in the past decades (Baudron and Needle et al., 2011; Bolle et al., 2004). Additionally, with the model estimated growth parameters, frequency length distribution of survey is better estimated.

Parameter $q_{\max }$ in the survey selectivity function is estimated to be 3.36 , which means that fish in the survey area not only are not distributed evenly but also the sampling area has a higher density of fish compared to the other part of the sea. It was decided based on manual adjustment. As expected, in the model-estimated selectivity curves, survey vessels capture more small fish compared to fishing boats. The higher peak in survey is influenced by the maximum catchability $q_{\max }$ in survey selectivity function. Survey vessels are then less likely to capture very big fish, while commercial boats' main target are those big fish in the sea. That is why the tail in
survey selectivity curve is lower than the estimated fishing selectivity curve (Figure $6.37 \mathrm{c})$.

Using the model estimated von Bertalanffy growth curve, equivalent average lengths for haddock at age 2 and 4 are 24 and 32 cm , respectively. The product of estimated average fishing selectivity at length $24-32 \mathrm{~cm}$ and fishing scalar $f_{t}$ at time estimates fishing mortality rate at lengths equivalent to age 2 to 4 cm . The model result is reassuringly close to the ICES assessment (Figure 6.37f). Natural mortality at age, extracted from ICES report, is approximated over the length classes by applying the estimated VBL average length age (Figure 6.37e).


Figure 6.37: a) landing probability, b) estimated von Bertalanffy growth curve, c) estimated fishing and survey gear selectivity curve, d) time-dependent fishing mortality scalar, e) approximated natural mortality over length classes and f) Model estimated average fishing mortality at length 24 to 32 cm (dashed red) with ICES assessment of average fishing mortality at age 2 to 4 (solid blue).
This is the result when survey-landing model is applied on the 13-year period 2000-2012

Model estimated abundance and stock biomass is comparably close to ICES assessment (Figure 6.38). Thorough the study years, 2000 is a strong year with high recruitment and high fishing mortality. Having been the starting year of the period, year 2000 was underestimated in its stock size, biomass and the recruit numbers. In
addition to that, the first year's estimation depends on the historical recruits, which is not as strong as the year 2000.


Figure 6.38: comparison between model estimated (red dashed lines) and ICES assessment (blue solid lines) of a) total stock size (abundance), b) total stock biomass and c) annual recruitment. This is the result when survey-landing model is applied on the 13-year period 2000-2012


Figure 6.39: Estimated (red dotted lines) and observed (blues vertical bars) survey numbers at length at year. This is the result when survey-landing model is applied on the 13 years 20002012

The length distribution of population size is given in Figure 6.40. The scale is very different from one year to another. Therefore, the distributions are plotted with individual scales to make sure the curves and peaks are clearly distinguished. It can be misleading because changes in stock size from one year to another is not clear if the scales in y-axis is overlooked. Log transformation is a sensible alternative, but the age-cohorts within each year are not as clear as the distribution plot of actual numbers. Figure 6.41 also illustrates the abundance similar to Figure 6.40 but with unified $y$-axis.


Figure 6.40: Model estimated stock size (population) at length at year. This is the result when survey-landing model is applied on the 13-year period 2000-2012


Figure 6.41: Model estimated stock size (population) at length at year with unified scales. This is the result when survey-landing model is applied on the 13-year period 2000-2012

Fishing mortality curves at length throughout the 13 timelines is given in Figure 6.42. A quick glance at the estimated fishing mortality at each year justifies the change of abundance distribution of the following year.


Figure 6.42: Model estimated fishing mortality curve at length at year. This is the result when survey-landing model is applied on the 13-year period 2000-2012

Table 6.12 : List of parameter initial and estimated values when the survey-landing model is applied on the North Sea haddock over the 13-year period (2000-2012)

| Model parameters | Initial values | Estimated values | Model parameters | Initial values | Estimated values |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{\infty}$ | $4.35 \mathrm{E}+01$ | $4.14 \mathrm{E}+01$ | $r_{1}$ | $1.78 \mathrm{E}+10$ | $9.05 \mathrm{E}+09$ |
| K | $3.32 \mathrm{E}-01$ | $3.22 \mathrm{E}-01$ | $r_{2}$ | $3.41 \mathrm{E}+09$ | $3.29 \mathrm{E}+09$ |
| $\boldsymbol{q}_{\text {max }}$ | $3.06 \mathrm{E}+00$ | $3.36 \mathrm{E}+00$ | $r_{3}$ | $3.65 \mathrm{E}+08$ | $4.76 \mathrm{E}+08$ |
| $\alpha_{v}$ | $2.06 \mathrm{E}+06$ | $4.79 \mathrm{E}+06$ | $r_{4}$ | $4.62 \mathrm{E}+08$ | $6.97 \mathrm{E}+08$ |
| $\boldsymbol{\beta}_{\boldsymbol{v 1}}$ | $2.07 \mathrm{E}-01$ | $1.28 \mathrm{E}-01$ | $r_{5}$ | $4.99 \mathrm{E}+08$ | $6.54 \mathrm{E}+08$ |
| $\beta_{v 2}$ | $5.07 \mathrm{E}-01$ | $4.73 \mathrm{E}-01$ | $r_{6}$ | $4.78 \mathrm{E}+08$ | $3.19 \mathrm{E}+07$ |
| $\boldsymbol{\beta}_{\boldsymbol{R}}$ | $1.30 \mathrm{E}-01$ | $3.26 \mathrm{E}-01$ | $r_{7}$ | $5.45 \mathrm{E}+09$ | $5.38 \mathrm{E}+09$ |
| $\alpha_{s}$ | $6.92 \mathrm{E}+06$ | $5.44 \mathrm{E}+06$ | $r_{8}$ | $1.16 \mathrm{E}+09$ | $7.55 \mathrm{E}+07$ |
| $\beta_{s 1}$ | $7.38 \mathrm{E}-01$ | $6.96 \mathrm{E}-01$ | $r_{9}$ | $6.80 \mathrm{E}+08$ | $9.68 \mathrm{E}+08$ |
| $\beta_{s 2}$ | $1.84 \mathrm{E}-04$ | $2.69 \mathrm{E}-05$ | $r_{10}$ | $5.52 \mathrm{E}+08$ | $3.60 \mathrm{E}+08$ |
| $\boldsymbol{f}_{1}$ | $3.59 \mathrm{E}-01$ | $8.29 \mathrm{E}-01$ | $r_{11}$ | $4.26 \mathrm{E}+09$ | $3.11 \mathrm{E}+09$ |
| $\boldsymbol{f}_{2}$ | $2.22 \mathrm{E}-01$ | $5.86 \mathrm{E}-01$ | $r_{12}$ | $2.44 \mathrm{E}+08$ | $4.19 \mathrm{E}+07$ |
| $\boldsymbol{f}_{3}$ | $1.31 \mathrm{E}-01$ | $3.50 \mathrm{E}-01$ | $r_{13}$ | $9.82 \mathrm{E}+07$ | $2.11 \mathrm{E}+07$ |
| $\boldsymbol{f}_{4}$ | $1.13 \mathrm{E}-01$ | $2.39 \mathrm{E}-01$ | $r_{\text {hist }}^{1}$ | $1.64 \mathrm{E}+09$ | $1.54 \mathrm{E}+09$ |
| $f_{5}$ | $1.24 \mathrm{E}-01$ | $3.16 \mathrm{E}-01$ | $r_{\text {hist }}^{2}$ | $7.18 \mathrm{E}+09$ | $8.95 \mathrm{E}+09$ |
| $f_{6}$ | $1.57 \mathrm{E}-01$ | $4.11 \mathrm{E}-01$ | rhist $_{3}$ | $1.77 \mathrm{E}+09$ | $4.80 \mathrm{E}+09$ |
| $\boldsymbol{f}_{7}$ | $2.54 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $\mathrm{rhist}_{4}$ | $2.71 \mathrm{E}+09$ | $9.05 \mathrm{E}+07$ |
| $\boldsymbol{f}_{8}$ | $2.28 \mathrm{E}-01$ | $3.44 \mathrm{E}-01$ | rhist $_{5}$ | $1.63 \mathrm{E}+09$ | $2.89 \mathrm{E}+07$ |
| $f_{9}$ | $1.36 \mathrm{E}-01$ | $2.57 \mathrm{E}-01$ | rhist $_{6}$ | $1.27 \mathrm{E}+09$ | $3.14 \mathrm{E}+09$ |
| $f_{10}$ | $1.22 \mathrm{E}-01$ | $2.95 \mathrm{E}-01$ | RSS |  | $9.77 \mathrm{E}+9$ |
| $f_{11}$ | $1.27 \mathrm{E}-01$ | $3.20 \mathrm{E}-01$ |  |  |  |
| $f_{12}$ | $1.98 \mathrm{E}-01$ | $3.38 \mathrm{E}-01$ |  |  |  |
| $f_{13}$ | $1.06 \mathrm{E}-01$ | $3.78 \mathrm{E}-01$ |  |  |  |

The survey-model application on NS haddock in the most recent years proved to be promising in estimating both the observations and the stock assessment in comparison with ICES assessments. The fishing mortality estimation and as the result stock size and stock biomass, were by far the closest estimation to ICES assessment.

### 6.8 Model application on the North Sea haddock with random initial recruitment and random fishing scalar

In this section, the survey-landings model is applied to survey numbers, landed biomass and discards biomass of the North Sea haddock from the final period. The difference between this section and section 6.7.4 is in the recruitment and fishing scalar initial values. This time, the initial values are not taken from age-based ICES assessment, but they are randomly generated and used in the estimating process. It is assumed that no information about fishing mortality and recruitments is available from ICES. The aim is to validate the model without the reference to the one-year group from ICES assessment and without the assumption about the fishing mortality at age 2-4. Also, year 2000 was removed from the period due to being an outlier in comparison with the years 2001 to 2012. Being a very strong year compare to other years could influence the estimation result. For initial recruitment values, 12 random numbers were generated from normal distribution with mean and standard deviation set equal to $2^{*} 10^{\wedge} 9$ and $10^{\wedge} 9$, respectively. The six years (1994-1999) historical recruits are also calculated similarly. Random initial values for fishing scalar are generated by uniform distribution between 0.3 and 0.5 . Input values, vectors and other initial values are same as section 6.7.4. The components of the least square function are also given equal weights.

The result is promising as the model recovered the observation although the initial recruits and fishing scalar were randomly calculated (


Figure 6.43). Landing numbers in both methods is over estimated at early years of the periods.

c)

b)

d)

f)


Figure 6.43: Model estimated (red dashed line) and observed (blue solid line) over the $\mathbf{1 2}$ years 2001-2012; a) total landed biomass, b) total survey numbers, c) total catch biomass, d) total survey biomass, e) total discards biomass and f) total landing numbers

Over the study period, the model estimated growth rate $L_{\infty}$ are similar to section 6.7.4 (Figure 6.44).

Parameter $q_{\max }$ in the survey selectivity function is estimated slightly higher. Both gear selectivity and survey selectivity are estimated as double logistic curves. As expected in the model-estimated selectivity curves, survey vessels capture more small fish compared to fishing boats.

Similar to section 6.7.4, the equivalent average lengths for haddock at age 2 and 4 are 24 and 32 cm , respectively. The product of estimated average fishing selectivity at length $24-32 \mathrm{~cm}$ and fishing scalar $f_{t}$ at time estimates fishing mortality rate at lengths equivalent to age 2 to 4 cm . Considering that the initial values for $f_{t}$ were generated randomly, comparing the estimated equivalent fishing mortality for age 2 to 4 with fishing mortality in ICES assessment for the same age group is promising (Figure 6.44f). Natural mortality at age, extracted from ICES report, is approximated over the length classes by applying the estimated VBL average length age (Figure $6.44 b)$.


Figure 6.44: a) landing probability, b) estimated von Bertalanffy growth curve, c) estimated fishing and survey gear selectivity curve, $d$ ) time-dependent fishing mortality scalar, e) approximated natural mortality over length classes and f) Model estimated average fishing mortality at length 24 to 32 cm (dashed red) with ICES assessment of average fishing mortality at age 2 to 4 (solid blue). This is the result when survey-landing model is applied on the 12-year period 2001-2012

Although the initial recruits and fishing mortality scalars are randomly generated, the model result is reassuringly close to the ICES assessment (Figure 6.45). Regardless of the year 2000, which was the excluded in this section, the estimation results in section 6.7.4 is closer to ICES assessment but it is due to the fact that the initial values are taken from ICES source. Model estimated abundance and stock biomass are also comparably close to ICES assessment. The black dotted line in the recruitment graph () shows the initial random values that were used to fit the model. Nevertheless, th estimated recruits is close to the ICES assessment.


Figure 6.45: Comparison between model estimated (red dashed lines) and ICES assessment (blue solid lines) of a) total stock size (abundance), b) total stock biomass and c) annual recruitment; the black solid line is the random initial recruitment values. This is the result when survey-landing model is applied on the 12-year period 2001-2012.


Figure 6.46 :Estimated (red dotted lines) and observed (blues vertical bars) survey numbers at length at year. This is the result when survey-landing model is applied on the 12 years 20012012


Figure 6.47: Model estimated stock size (population) at length at year. This is the result when survey-landing model is applied on the 12-year period 2001-2012

The length distribution of population size is given in Figure 6.47. Due to differences in scales, the distributions are plotted with individual scales to make sure the curves and peaks are clearly distinguished. All the initial values for parameters and estimated parameters are listed in table 6.12.

Table 6.13: List of parameter initial and estimated values when the survey-landing model is applied on the North Sea haddock over the 12-year period (2001-2012)

| Model <br> parameters | Initial values | Estimated <br> values | Model <br> parameters | Initial values | Estimated <br> values |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{L}_{\infty}$ | $4.35 \mathrm{E}+01$ | $4.20 \mathrm{E}+01$ | $\boldsymbol{r}_{\mathbf{1}}$ | $1.80 \mathrm{E}+09$ | $2.51 \mathrm{E}+09$ |
| $\boldsymbol{K}$ | $3.32 \mathrm{E}-01$ | $3.18 \mathrm{E}-01$ | $\boldsymbol{r}_{\boldsymbol{2}}$ | $3.23 \mathrm{E}+09$ | $1.34 \mathrm{E}+09$ |
| $\boldsymbol{q}_{\boldsymbol{\operatorname { m a x }}}$ | $3.06 \mathrm{E}+00$ | $3.38 \mathrm{E}+00$ | $\boldsymbol{r}_{\boldsymbol{3}}$ | $1.52 \mathrm{E}+09$ | $1.26 \mathrm{E}+09$ |
| $\boldsymbol{\alpha}_{\boldsymbol{v}}$ | $2.06 \mathrm{E}+06$ | $8.76 \mathrm{E}+06$ | $\boldsymbol{r}_{\mathbf{4}}$ | $8.88 \mathrm{E}+08$ | $5.92 \mathrm{E}+07$ |
| $\boldsymbol{\beta}_{\boldsymbol{v} \mathbf{1}}$ | $2.07 \mathrm{E}-01$ | $2.07 \mathrm{E}-01$ | $\boldsymbol{r}_{\mathbf{5}}$ | $6.24 \mathrm{E}+08$ | $9.07 \mathrm{E}+08$ |
| $\boldsymbol{\beta}_{\boldsymbol{v} \mathbf{2}}$ | $5.07 \mathrm{E}-01$ | $5.49 \mathrm{E}-01$ | $\boldsymbol{r}_{\boldsymbol{6}}$ | $2.48 \mathrm{E}+08$ | $5.90 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{\boldsymbol{R}}$ | $1.30 \mathrm{E}-01$ | $1.75 \mathrm{E}-01$ | $\boldsymbol{r}_{\boldsymbol{7}}$ | $3.16 \mathrm{E}+09$ | $1.22 \mathrm{E}+09$ |
| $\boldsymbol{\alpha}_{\boldsymbol{s}}$ | $6.92 \mathrm{E}+06$ | $2.63 \mathrm{E}+07$ | $\boldsymbol{r}_{\mathbf{8}}$ | $1.72 \mathrm{E}+09$ | $1.92 \mathrm{E}+09$ |
| $\boldsymbol{\beta}_{\boldsymbol{s} \mathbf{1}}$ | $7.38 \mathrm{E}-01$ | $7.66 \mathrm{E}-01$ | $\boldsymbol{r}_{\mathbf{9}}$ | $4.24 \mathrm{E}+08$ | $8.45 \mathrm{E}+08$ |
| $\boldsymbol{\beta}_{\boldsymbol{s} \mathbf{2}}$ | $1.84 \mathrm{E}-04$ | $4.84 \mathrm{E}-01$ | $\boldsymbol{r}_{\mathbf{1 0}}$ | $2.47 \mathrm{E}+09$ | $4.14 \mathrm{E}+09$ |
| $\boldsymbol{f}_{\boldsymbol{1}}$ | $4.22 \mathrm{E}-01$ | $5.29 \mathrm{E}-01$ | $\boldsymbol{r}_{\mathbf{1 1}}$ | $2.73 \mathrm{E}+09$ | $1.70 \mathrm{E}+09$ |


| $\boldsymbol{f}_{\boldsymbol{2}}$ | $4.03 \mathrm{E}-01$ | $3.33 \mathrm{E}-01$ | $\boldsymbol{r}_{\boldsymbol{1} \boldsymbol{2}}$ | $1.03 \mathrm{E}+09$ | $6.32 \mathrm{E}+08$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{f}_{\boldsymbol{3}}$ | $4.05 \mathrm{E}-01$ | $2.57 \mathrm{E}-01$ | $\boldsymbol{r h i s t}_{\boldsymbol{1}}$ | $4.65 \mathrm{E}+09$ | $5.62 \mathrm{E}+08$ |
| $\boldsymbol{f}_{\boldsymbol{4}}$ | $4.03 \mathrm{E}-01$ | $3.44 \mathrm{E}-01$ | $\boldsymbol{r h i s t}_{\boldsymbol{2}}$ | $5.04 \mathrm{E}+08$ | $1.08 \mathrm{E}+09$ |
| $\boldsymbol{f}_{\mathbf{5}}$ | $3.15 \mathrm{E}-01$ | $5.94 \mathrm{E}-01$ | $\boldsymbol{r h i s t}_{\boldsymbol{3}}$ | $9.57 \mathrm{E}+08$ | $3.19 \mathrm{E}+08$ |
| $\boldsymbol{f}_{\mathbf{6}}$ | $3.66 \mathrm{E}-01$ | $7.68 \mathrm{E}-01$ | $\boldsymbol{r h i s t}_{\boldsymbol{4}}$ | $1.85 \mathrm{E}+09$ | $5.24 \mathrm{E}+08$ |
| $\boldsymbol{f}_{\boldsymbol{7}}$ | $3.59 \mathrm{E}-01$ | $3.97 \mathrm{E}-01$ | $\boldsymbol{r h i s t}_{\mathbf{5}}$ | $2.47 \mathrm{E}+09$ | $1.41 \mathrm{E}+10$ |
| $\boldsymbol{f}_{\mathbf{8}}$ | $4.42 \mathrm{E}-01$ | $2.64 \mathrm{E}-01$ | $\boldsymbol{r h i s t}_{\boldsymbol{6}}$ | $1.46 \mathrm{E}+09$ | $4.79 \mathrm{E}+08$ |
| $\boldsymbol{f}_{\boldsymbol{9}}$ | $4.21 \mathrm{E}-01$ | $3.09 \mathrm{E}-01$ | $\boldsymbol{R S S}$ |  | $3.58 \mathrm{E}+09$ |
| $\boldsymbol{f}_{\mathbf{1 0}}$ | $3.30 \mathrm{E}-01$ | $2.40 \mathrm{E}-01$ |  |  |  |
| $\boldsymbol{f}_{\mathbf{1 1}}$ | $3.44 \mathrm{E}-01$ | $2.61 \mathrm{E}-01$ |  |  |  |
| $\boldsymbol{f}_{\mathbf{1 2}}$ | $4.91 \mathrm{E}-01$ | $2.33 \mathrm{E}-01$ |  |  |  |

### 6.9 Discussion and conclusion

This research work aims to develop a marine length-structured stock assessment method, from which the very poorly sampled species can be assessed. For these poorly sampled neither age nor catch-at-length are available. Therefore, the standard age-structured methods and length-structured methods in which catch-at-length is used as model input are not of use. The survey-landings model that has been developed in this piece of research work is a length-structured stock assessment model and is a feasible as well as a practical alternative.

The survey-landings model is first applied on the North Sea haddock for the whole available years from 1969 to 2012. The result is overall encouraging but the timevarying growth is neglected in the 44 years as a whole period. To allow the growth parameters to vary over time, the model is applied on four smaller timelines.

The survey-landings model is applicable on all four periods. In all the cases, landing and discards biomass are accurately estimated. The result for length distribution of survey is by far more accurate and promising in the most resent years (2000-2012). That could be due to the better and more reliable commercial and survey data compared to the previous years.

Since the survey-landings model is applied on four timelines, four separate sets of growth parameters were estimated. The results show that not only they are not constant but also follow a trend over time. The asymptotic maximum length is decreasing over time, while the growth rate (the curvature in VBL curve) is going up apart from the exception in the third study period (Table 6.14). Also the estimated value of $q_{\max }$ is going up over years.

Table 6.14: Model estimated growth parameters at study periods

| Growth parameters | $1969-1979$ | $1980-1989$ | $1990-1999$ | $2000-2012$ |
| :---: | :---: | :---: | :---: | :---: |
| $L_{\infty}$ | 100 | 71 | 48 | 41.4 |
| $K$ | 0.16 | 0.24 | 0.42 | 0.32 |

In addition to survey and growth parameters, the parameters that shape fishing mortality curve are changing over time to adjust the model.

A different approach was applied in modelling and that was to use a random initial values for recruits and fishing mortality scalars. The purpose was to investigate if the model still can recover the observations and if the results are close to ICES assessment even if the assumption of the ICES are not used. The approach was applied on the last period only, and the result was not distanced from the original approach.

Although the age-structured stock assessment method applied by ICES is well accepted over years and considered as the standard approach, ICES estimated fishing mortality is an estimation itself. Therefore, in the case of discrepancies it is not straightforward to suggest which one is more accurate. However, the model result is compared with ICES assessments to see if both give similar abundance and mortality prediction for the same marine species. If this is the case, the survey-landing model is considered as a reliable substitute to assess the stock of moderately and poorly sampled species.

A great deal of effort was put into testing the model in estimating the length distribution of survey numbers over the time steps. Survey numbers are recorded regularly by survey vessels (see Chapter 3) and are assumed to be a consistently
proportional to the true population numbers in the sea. Therefore, they are considered as a reliable source of data for assessing the dynamics of population with regards to numbers at length. The outputs show that the survey-landings model is capable of estimating the survey numbers at length and more importantly capturing the first and second peaks.

Since the assessed stock in the model is very close to the ICES assessment in all four timelines and the most recent one in particular, it can be argued that in the absence of age or even catch-at-length data, the survey-landings model is capable of modelling the dynamic of the population with a reliable precision.

### 6.10Appendix 6.1: Adding up the number of North Sea haddock at length

This R code adds up the number of North Sea haddock at length captured in quarter 1 at each year in the area IV and III from NS IBTS
rm(list=ls(all=T))
\# Read the csv data file
\# Make a matrix of the survey numbers at length at year
\# Length of haddock from 3 cm to $87 \mathrm{~cm}, 1 \mathrm{~cm}$ length class
\# Year from 1969 to 2012
\# HAD_NS_IBTS : Name of csv file
\# length.CatchHLNoAtLngt: frequency of captured fish
\# haul.Year: year
\# length.EquivLength: Length of captured fish
\# All the NA values set equal to 0

HAD_NS_IBTS <- read.csv ('mpm_NS-IBTS_164744_All_All_Q1_length_edit.csv', header=T)

HAD_NS_IBTS_LenYear <- t (tapply (HAD_NS_IBTS\$length.CatchHLNoAtLngt, list (factor HAD_NS_IBTS\$haul.Year), factor(HAD_NS_IBTS\$length.EquivLength)), sum, na.rm=T))

HAD_NS_IBTS_LenYear [is.na (HAD_NS_IBTS_LenYear)] <-0

### 6.11 Appendix 6.2: Estimating growth parameters from IBTS age data

Age data from the $1^{\text {st }}$ quarter sampling for North Sea haddock (1981-2012) for area IV $\mathrm{a}, \mathrm{b}$ and c and IIIa from IBTS are used to estimate average length at age and then the VBL growth parameters. Since a limited number of caught fish is aged, the conditional probability is used to calculate the probability of length given age. Then the mean length at age is calculated and VBL is applied to estimate the growth parameters $L_{\infty}$ and $K$.

| \#setwd('...') |  |
| :--- | :--- |
| \# Creating a data base matrix of length at age and all NA values set equal to 0  <br> \# nl: Numbers at each length class <br> \# Tnl: Total sampled numbers, total numbers for all length classes <br> \# Pl: probability (fraction) of fish at each length class <br> \# P_al: probability of fish being at age a given being at length 1 <br> \# P_la_1: Numerator of the Bayes theorem == P_al*pl <br> \# P_la_2: Denominator of the Bayes theorem == Total probability of length at <br> given age $===$ sum (P_la_1) over length classes <br> \# P_la: Probability of length at age <br> \# Mean_A_at_L: Average age at length <br> \# Mean_L_at_Age: Average length at age |  |

L_Age_IBTS <- read.csv ('L_at_age_IBTS_1981_2012.csv',header=T)
L_Age_81_12 <- t (tapply (L_Age_IBTS\$CatchNo, List (factor (L_Age_IBTS\$Age), factor
(L_Age_IBTS\$Length)), sum, na.rm=T))
L_Age_81_12 [is.na (L_Age_81_12)] <- 0
LAGE <- L_Age_81_12
L_class <- scan ('Len_IBTS.csv') \#length classes (cm)
Age_class <- scan ('Age_IBTS.csv') \#age classes(year)
$\mathrm{L}<-\operatorname{dim}(\mathrm{LAGE})[1]$
$\mathrm{AG}<-\operatorname{dim}$ (LAGE)[2]
Empty <- matrix ( 0 ,nrow=L,ncol=AG)
Nl <- apply (L_Age_81_12,1,sum)

```
Tnl <- sum(nl)
Pl <- nl/Tnl
P_al <- empty
P_la_1<- empty
P_la <- empty
for( J in 1:AG) {
    for (I in 1:L) {
        P_al[I,J]<-LAGE[I,J]/nl[I]
        P_la_1[I,J]<- P_al[I,J]*Pl[I]
    }}
P_la_2 <- apply (P_la_1,1,sum)
for ( J in 1:AG) {
    for(I in 1:L){
    P_la[I,J] <- P_la_1[I,J]/P_la_2[I]
    }}
write.table (P_la,'P_la.csv', sep=',', row.names=F, col.names=F )
Mean_A_at_L<- rep(1:L)
for (I in 1:L) {
    Mean_A_at_L[I] <- floor (sum (P_la[I,]*Age_class))
    }
Mean_L_at_Age <- t ( tapply (L_class, list (factor (Mean_A_at_L)), mean, na.rm=T))
SD_L_at_Age <- t ( tapply ( L_class, list ( factor (Mean_A_at_L)), sd, na.rm=T))
A_factor <- c(1, 2, 3, 4, 5, 6, 7, 8)
Mean_L <- c(14.3, 28, 34, 40.14286, 55.85714, 61.3, 69.5, 81.33333)
SD_L <- c(6.275265, 2.160247, 1.581139, 11.32633, 9.044862, 4.854551, 4.105745,
6.027714)
func <- function(pars) {
    Lt2<- pars[1]*(1-exp(-pars[2]*A_factor))
    return(sum((Lt2-Mean_L)^2))
    }
library (DEoptim)
fittt <- DEoptim (func, lower= c(20,0.05), upper = c(190, 0.3), DEoptim.control (VTR=0,
NP=20, itermax=12000))
```


### 6.12 Appendix 6.3: Estimating parameters of weight-atlength function for North Sea haddock from ICES report

Average weight at age for total catch for North Sea haddock in area 4 and 3a is extracted from ICES Working Group Report 2013. Since the length at age is not available in ICES report, the mean length at age that was estimated from IBTS data (appendix 6.2) is applies.

```
# W(kg)= alpha * L(cm)^ beta
# W_at_A: Average weight at age for total catch from 1969 to 2012 extracted
    from ICES working group report }201
# Age: vector of age
# L_at_A: Mean length at age estimated from IBTS data (1981-2012)
W_at_A <- c (0.13, 0.27, 0.40, 0.57, 0.75, 0.95, 1.15, 1.49)
Age <- c(1, 2, 3, 4, 5, 6, 7, 8)
L_at_A <- c (14.3, 28, 34, 40.14, 55.86, 61.30, 69.50, 81.33)
W_at_L <- nls (W_L~alpha*L^beta,
    data=list (W_L=W_at_A, L=L_at_A), start=list (alpha=.01, beta=2.8),
    nls.control (maxiter=10000, tol=1e-3, printEval=F), trace=T, model=F)
```


### 6.13Appendix 6.4: Survey-landings model: R codes; Source

 and assessment```
# Author: Dr Douglas C. Speirs
# Modification and application: Andisheh Bakhshi
# Department of Mathematics & Statistics
# University of Strathclyde
rm (list= ls (all=T) )
# Utlity function to truncate a vector at zero
non.negative <- function (x) {
                                    x [ }\textrm{x}<0]<-
                                    return (x ) }
# Function returning the probability over interval (x1,x2) from a Gamma
# Distribution with positive parameters " }a\mathrm{ " and " " "
# mean =a*b, variance =a*b^2
pgam<- function (x1, x2, a, b ) {
    pgamma ( x2, shape =a, scale=b) - pgamma ( x1, shape =a, scale =b) }
# Probability of growing from length L0 to L over time-steps
pdL <- function (L, L0, Linf, k, beta, dL, dt) {
    MeanDeltaL <- non.negative (( Linf-L0 ) * ( 1-exp ( -k *dt )))
    alphaL <- MeanDeltaL/beta
    DeltaL <- L-L0
    return ( pgam ( DeltaL, DeltaL+dL, alphaL, beta )) }
# Logistic function of length used in landing fraction
logistic <- function (L, alpha, beta ) {
    return ( 1/( +alpha*exp (-beta*L ))) }
# Logistic function of length used in gear selectivity
logistic2 <- function (L, alpha, beta1, beta2 ) {
    logit2<-1/ ( 1+alpha*exp ( -beta1*L ) + ( 1/alpha ) * exp ( beta2*L ) + exp ( L* (
    beta2-beta1 )))
    return( logit2/max ( logit2 )) }
```

```
# Description of survey:
# Samples a population matrix of number-at-length by year given the survey swept
# area each year, and double logistic survey gear selectivity to yield a matrix of survey
# number-at-length by year.
# Usage:
# survey (N, L, Pa, qmax, alpha, beta)
# Arguments:
#N: an n by maxiter array where N[,j] is the population length distribution in
# timestep j; n is the number of length classes, and maxiter is the number of
# timesteps
# pA: a vector of length maxiter containing the proportion of area swept by the
# survey each at timestep (dimensionless)
#L: a vector of length n containing the lengths of each length class (cm)
# qmax: maximum survey selectivity (dimensionless)
# alpha: survey selectivity parameter
# betal: survey selectivity parameter
# beta2: survey selectivity parameter
# Value:
# An n by maxiter array of surveyed number-at-length for each year
survey <- function (N, L, pA = NULL, alpha, beta1, beta2, qmax ) {
    if (is.null (pA )) {pA<- rep (1, dim (N)[2])}
    q <- qmax*logistic2 (L, alpha, beta1, beta2 )
    Survey.N <- q %o% pA*N
    return ( Survey.N ) }
# Function to estimate initial stock size prior to the main model run
# Consistent with the growth model and a 'historical' recruitment
# The historical recruitment values are required to achieve the initial condition
# The historical recruitments are fitting parameters.
# Returns a vector of number at length
InitLenDist <- function (P, Z, p, s, n, dt, Rhist ) {
    nhist <- length( Rhist)
    N <- rep (0, n)
    S<- diag( exp (-Z*dt))
    for (i in 1:nhist ){ N<- P %*% S %*% N + p * Rhist [ i ] }
    return (N ) }
```

```
MakeParList <- function (fitting.pars, fixed.pars ){
    attach (fixed.pars)
    n1<- length (fitting.pars ) - 2* maxiter - nhist
    n2<- n1 + maxiter
    n3 <- n2 + maxiter
    n4<- n3 + nhist
    detach (fixed.pars)
    scalar.pars <- as.list (fitting.pars [1:n1])
    vector.pars <- list ( Ft = unname ( fitting.pars [ (n1 + 1) : n2 ]),
    Rt = unname ( fitting.pars [( n2 + 1 ) : n3 ]),
    Rhist = unname ( fitting.pars [( n3 + 1) : n4 ]))
    pars <- c( scalar.pars, vector.pars, fixed.pars )
    return ( as.list (pars )) }
# Description:
# Simulates a length-structured fish population using the survey-landing model
# Usage:
# len.sim (par.list)
# Arguments:
# par.list: a list containing the following elements:
# Linf: von Bertalanffy asymptotic length (cm)
#k: von Bertalanffy growth rate (1/year)
# beta: gamma distribution scale parameter for growth increments
#M: natural mortality rate (1/year)
# alphaS: fishing mortality selectivity parameter
# betaS1: fishing mortality selectivity parameter
# betaS2: fishing mortality selectivity parameter
# alphaR: gamma distribution shape parameter for recruitment length
# distribution
# betaR: gamma distribution scale parameter for recruitment length distribution
# qmax: survey maximum selectivity (dimensionless, only used if Survey=T)
# alphaV: survey selectivity parameter (only used if Survey=T)
# betaV1: survey selectivity parameter (only used if Survey=T)
# betaV2: survey selectivity parameter (only used if Survey=T)
# Ft: vector of time-dependent component to the fishing mortality (must be
```

```
# of length maxiter) (1/year)
# Rt: vector of time-dependent recruitment
# Rhist: vector of time-dependent recruitment used by function InitLenDist to
# generate the initial condition if that is not provided separately
#dt: timestep (years)
# maxiter: number of timesteps
#dL: lengh-class width (cm)
# L: a vector of length class sizes (cm)
# W: a vector of individual weight at length (tonnes), must have the same
# length as 'L'
# alphaD: discarding selectivity parameter (only used if Discard=T)
# betaD: discarding selectivity parameter (only used if Discard=T)
# pA: vector of length maxiter giving the proportion of population area swept
#
# Value:
# A list with the following elements:
# N: an n by maxiter matrix where N[,j] is the population number-at-length
# in timestep j.
#C: an n by maxiter matrix where C[,j] is the catch number-at-length
# in timestep j.
# Fm: an n by maxiter matrix where F[,j] is the fishing mortality rate at length
# in timestep j.
# Ln: an n by maxiter matrix where Ln[,j] is the landed numbers at length
# in timestep j.
# TLb: a vector of length maxiter containing the landed biomass (tonnes) by
# timestep
# Ns: an n by maxiter matrix where Ns[j] is the survey numbers-at-length
# in timestep j.
len.sim <- function ( par.list, N0=NULL ) {
attach ( par.list)
n<- length (L )
empty <- matrix ( 0, nrow=n, ncol=maxiter )
P <- outer ( L, L, pdL, Linf, k, beta, dL, dt)
P [ n, ]<-1 - apply ( P [ - n , ], 2, sum )
Lzero <- ( MuR-Linf * (1-exp ( -k )))/ exp( -k )
```

```
L1 <- Linf-(Linf-Lzero )* exp(-k * Age )
L2 <- c( Lzero,MuR,L1)
APPR_M<- approx. (x=L2, y=MAge, xout=L, method='linear', rule=2)
M<- APPR_M$y
alphaR <- MuR/betaR
p <- pgam ( L, L+dL, alphaR, betaR )
s <- logistic2 (L, alphaS, betaS1, betaS2)
Fm <- s %o% Ft
Z<- Fm + M
N <- empty
N1Plus <- empty
N[,1] <- if ( is.null (N0)) InitLenDist (P,Z[,1], p, s, n, dt, Rhist) + p * Rt[1] else N0
for (i in 1:(maxiter-1)) {
S <- diag ( exp (-Z[, i] * dt )
N [,i+1]<- P %*% S %*% N [, i] + p * Rt[i+1 ]
N1Plus [, i] <- P %*% S %*% N[, i]}
S<- diag (exp (-Z [,maxiter] * dt))
N1Plus [, maxiter] <- P %*% S %*% N[ , maxiter]
C <- Fm / Z * (1- exp (-Z*dt )) * N
Ln<- if (Discard==T) logistic (L, alphaD, betaD ) * C else C
Ns <- if (Survey==T) survey (N, L, pA, alphaV, betaV1, betaV2, qmax) else empty
TLb <- apply ( W * Ln, 2, sum)
TNs <- apply (Ns, 2, sum)
Bs <- W * Ns
TBs <- apply (Bs, 2, sum)
TN <- apply (N, 2, sum)
TSb <- apply (W * N, 2, sum)
TSb1Plus <- apply ( W * N1Plus, 2, sum)
TN1Plus <- apply (N1Plus, 2, sum)
TC <- apply ( C, 2, sum)
TCb <- apply (W * C, 2, sum)
TDb<- TCb - TLb
TLn <- apply ( Ln, 2, sum)
```

$\mathrm{q}<-\mathrm{qmax} *$ logistic2 (L, alphaV, betaV1, betaV2)
detach ( par.list )
return ( $\operatorname{list}(\mathrm{TLb}=\mathrm{TLb}, \mathrm{Ns}=\mathrm{Ns}, \mathrm{N}=\mathrm{N}, \mathrm{C}=\mathrm{C}, \mathrm{Ln}=\mathrm{Ln}, ~ N m a t=\mathrm{Nmat}, \mathrm{TNs}=\mathrm{TNs}$, $\mathrm{TN}=\mathrm{TN}, \quad \mathrm{TC}=\mathrm{TC}, \quad \mathrm{TL}=\mathrm{TL}, \quad \mathrm{N} 1$ Plus=N1Plus, TN1Plus=TN1Plus, $\mathrm{TCb}=\mathrm{TCb}, \mathrm{TSb}=\mathrm{TSb}, \mathrm{TDb}=\mathrm{TDb}, \mathrm{TBs}=\mathrm{TBs}, \mathrm{TSb} 1 \mathrm{Plus}=\mathrm{TSb} 1 \mathrm{Plus}, \mathrm{Fm}=\mathrm{Fm}$, $\mathrm{Z}=\mathrm{Z}, \mathrm{Bs}=\mathrm{Bs}, \mathrm{q}=\mathrm{q}, \mathrm{M}=\mathrm{M}, \mathrm{s}=\mathrm{s})$ ) $\}$

## \# Estimation and assessment

\# Fitting parameters
fitting.pars $<-\mathrm{c}($
Linf $=92.5$,
$\mathrm{k} \quad=0.16$,
qmax $\quad=1.2$,
alphaV $=30169$,
betaV1 $=0.59$,
betaV2 $=0.30$
betaR $=0.3$,
\#alphaM $=45.64$,
\#betaM $\quad=1.2$,
alphaS $=306$,
betaS $1=0.29$,
betaS2 $=.0005$,
Ft $\quad=1.25 * \operatorname{scan}($ "Fm_ICES_2_4.csv"),
Rt $\quad=\operatorname{scan}($ "N1group_ICES.csv"),
Rhist = scan ("N1Hist_63_68.csv")
)
\# Input parameters and vectors
fixed.pars $<-$ list (
$\mathrm{dt} \quad=1$,
maxiter $=44$
nhist $=6$,
$\mathrm{dL} \quad=1$,
$\mathrm{L} \quad=\operatorname{seq}(3,87,1)$,
$\mathrm{W} \quad=\left(0.002 * \operatorname{seq}(3,87,1)^{\wedge} 1.514\right) / 1 \mathrm{e} 3$,
Discard = TRUE,
Survey = TRUE,
pA $=\operatorname{scan}\left(\right.$ ( $P_{-}$sampl_69_12.csv"),
\#M $\quad=\operatorname{scan}($ "M_pred_test2.csv"),
Age $\quad=\operatorname{seq}(0,8,1)$,
MAge $\quad=c(2.05,1.65,0.40,0.25,0.25,0.2,0.2,0.2,0.2)$,
alphaD $=549303$,
betaD $\quad=0.5$,

```
    beta
        =1.36,
)
```

\# Observation data
obs $<-$ list $($
TLb = scan("TLb_ICES_1969_2012.csv"),
Ns = read.csv("HAD_NS_IBTS_LenYear_1969_2012.csv",header=F),
TDb = scan("TDB_ICES_69_12.csv")
)
fun $<$ - function ( ln.fitting.pars, fixed.pars, obs ) \{
pars $<-\exp$ ( ln.fitting.pars )
mod <- len.sim(MakeParList(pars,fixed.pars))
return (
sum (( mod\$TLb-obs\$TLb ) $\left.{ }^{\wedge} 2\right)$
$+\operatorname{sum}\left((\bmod \$ \mathrm{Ns}-\mathrm{obs} \$ \mathrm{Ns})^{\wedge} 2\right)$
$\left.+\operatorname{sum}((\bmod \$ T D b-o b s \$ T D b))^{\wedge} 2\right)$
) $\}$
$\ln . f i t t i n g . p a r s<-\log$ ( fitting.pars $)$
\# Parameter estimation
fit $<-$ optim ( ln.fitting.pars, fun, gr $=$ NULL, fixed.pars, obs, control $=\operatorname{list}(\operatorname{maxit}=20000$, abstol $=1 \mathrm{e}-15)$ )
pred $<-$ len.sim ( MakeParList ( $\exp$ ( fit\$par ), fixed.pars ))

### 6.14Appendix 6.5: North Sea haddock Survey numbers

In this appendix table of survey numbers at length for the 44 years from 1969 to 2012 is given. Rows represent length classes with 1 cm width start from 3 cm and end with 87 cm , which are the smallest and biggest haddock captured in the North Sea survey in area VI and IIIa over years 1969 to 2012. The frequency distribution of survey at length is also plotted. These plots show the same data Figure 6.4 but the scales are not unified so that the distribution of survey numbers is clearer at each time step.

|  | 1969 | 1970 | 1971 | 1972 | 1973 | 1974 | 1975 | 1976 | 1977 | 1978 | 1979 | 1980 | 1981 | 1982 | 1983 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 4 | 8 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 22 | 0 | 0 | 1 | 0 | 197 | 31 | 33 | 1 |
| 10 | 0 | 0 | 0 | 9 | 0 | 137 | 276 | 1 | 44 | 10 | 121 | 1272 | 585 | 862 | 40 |
| 11 | 0 | 1 | 1 | 188 | 4 | 390 | 1058 | 37 | 408 | 135 | 773 | 5047 | 1950 | 4104 | 391 |
| 12 | 0 | 1 | 12 | 899 | 125 | 2282 | 3579 | 218 | 876 | 1009 | 3173 | 9554 | 4107 | 8940 | 1213 |
| 13 | 39 | 17 | 226 | 2948 | 854 | 6828 | 11340 | 1128 | 1683 | 4106 | 5850 | 14152 | 4411 | 8349 | 2890 |
| 14 | 322 | 42 | 924 | 5394 | 2686 | 15936 | 20917 | 1378 | 2434 | 8019 | 8618 | 22302 | 3785 | 7062 | 4737 |
| 15 | 585 | 138 | 3327 | 6396 | 3862 | 21948 | 32405 | 1464 | 2749 | 9302 | 11845 | 29154 | 3220 | 5534 | 6599 |
| 16 | 836 | 633 | 7055 | 5520 | 4672 | 21895 | 30372 | 1532 | 1931 | 9471 | 13170 | 23686 | 2091 | 5574 | 7737 |
| 17 | 782 | 474 | 10202 | 6038 | 3042 | 15827 | 24528 | 1520 | 1691 | 7372 | 12127 | 15612 | 1316 | 4464 | 7710 |
| 18 | 812 | 389 | 9594 | 5570 | 2739 | 9008 | 16206 | 2492 | 1224 | 5003 | 7522 | 8171 | 994 | 3494 | 6710 |
| 19 | 1065 | 204 | 8103 | 3758 | 3735 | 7033 | 11125 | 3942 | 1152 | 2683 | 4053 | 4036 | 1718 | 2504 | 4985 |
| 20 | 4786 | 172 | 5132 | 1818 | 5933 | 4962 | 8250 | 5381 | 1134 | 1557 | 1707 | 2759 | 3123 | 2267 | 4073 |
| 21 | 15582 | 293 | 3051 | 1058 | 8430 | 4215 | 7105 | 7670 | 1220 | 1396 | 1012 | 3186 | 5131 | 2313 | 3233 |
| 22 | 26705 | 467 | 1376 | 1005 | 10562 | 3830 | 5938 | 8961 | 1187 | 1691 | 1354 | 5565 | 8205 | 2993 | 3899 |
| 23 | 30537 | 563 | 506 | 1369 | 12282 | 3743 | 5908 | 10669 | 1293 | 2052 | 2080 | 7824 | 10320 | 3892 | 4541 |
| 24 | 26659 | 907 | 292 | 1851 | 14660 | 4159 | 5644 | 12458 | 1608 | 2424 | 2989 | 9305 | 10769 | 4604 | 5522 |
| 25 | 22548 | 1870 | 316 | 2607 | 13446 | 3947 | 4574 | 10998 | 2031 | 2201 | 3909 | 7695 | 9805 | 4100 | 5981 |
| 26 | 12682 | 2314 | 429 | 3000 | 12612 | 3453 | 4419 | 9660 | 2439 | 1758 | 4159 | 6828 | 9281 | 3923 | 6269 |
| 27 | 6815 | 4233 | 567 | 3787 | 10094 | 3474 | 3942 | 8237 | 2724 | 1664 | 4353 | 5602 | 7168 | 4544 | 7393 |
| 28 | 2549 | 4507 | 652 | 3444 | 7767 | 2876 | 3473 | 6472 | 2734 | 1680 | 3662 | 4669 | 5336 | 4490 | 7238 |
| 29 | 1813 | 4460 | 779 | 2975 | 5967 | 2742 | 3326 | 5120 | 2740 | 1587 | 3291 | 3797 | 4525 | 4423 | 6817 |
| 30 | 1163 | 5289 | 729 | 1490 | 3581 | 2696 | 2450 | 4607 | 2989 | 1625 | 2820 | 2912 | 3931 | 5365 | 6150 |
| 31 | 730 | 3403 | 890 | 1075 | 3091 | 2245 | 1888 | 3866 | 2611 | 1779 | 2371 | 2343 | 2847 | 4454 | 5285 |
| 32 | 657 | 2637 | 884 | 605 | 2101 | 2036 | 1465 | 3422 | 2028 | 1451 | 1788 | 1920 | 2797 | 4803 | 4225 |
| 33 | 493 | 1577 | 1158 | 531 | 2927 | 1744 | 1288 | 2637 | 1733 | 1453 | 1505 | 1506 | 2136 | 4687 | 3252 |
| 34 | 212 | 836 | 1262 | 522 | 2579 | 1414 | 989 | 2476 | 1456 | 1454 | 1316 | 1363 | 1569 | 4473 | 2947 |
| 35 | 232 | 581 | 1148 | 317 | 1932 | 1216 | 918 | 1959 | 1256 | 1325 | 994 | 1318 | 1599 | 4681 | 2857 |
| 36 | 120 | 750 | 1274 | 271 | 1618 | 1110 | 828 | 1573 | 936 | 1190 | 795 | 1234 | 1341 | 3448 | 2899 |
| 37 | 145 | 464 | 1018 | 387 | 1224 | 705 | 787 | 1107 | 683 | 1137 | 780 | 954 | 1122 | 3126 | 2285 |
| 38 | 145 | 303 | 967 | 306 | 952 | 533 | 636 | 1097 | 436 | 850 | 638 | 889 | 944 | 2256 | 2373 |
| 39 | 114 | 180 | 809 | 252 | 754 | 625 | 384 | 764 | 392 | 761 | 461 | 634 | 652 | 1793 | 2280 |
| 40 | 116 | 177 | 509 | 225 | 874 | 417 | 365 | 720 | 322 | 624 | 474 | 626 | 570 | 1649 | 2172 |
| 41 | 65 | 145 | 538 | 235 | 642 | 364 | 305 | 508 | 235 | 463 | 352 | 480 | 407 | 863 | 1754 |
| 42 | 68 | 138 | 372 | 96 | 359 | 338 | 276 | 338 | 247 | 381 | 249 | 373 | 322 | 839 | 1599 |
| 43 | 51 | 74 | 376 | 168 | 245 | 246 | 195 | 275 | 135 | 284 | 259 | 276 | 265 | 621 | 1182 |
| 44 | 56 | 35 | 281 | 98 | 430 | 244 | 179 | 279 | 130 | 204 | 210 | 225 | 241 | 505 | 1205 |
| 45 | 23 | 21 | 201 | 142 | 144 | 223 | 132 | 169 | 109 | 148 | 192 | 202 | 199 | 505 | 684 |
| 46 | 31 | 20 | 182 | 110 | 102 | 150 | 97 | 178 | 94 | 87 | 158 | 158 | 130 | 289 | 484 |
| 47 | 24 | 6 | 123 | 107 | 69 | 193 | 89 | 92 | 75 | 61 | 160 | 149 | 104 | 220 | 398 |
| 48 | 21 | 15 | 75 | 82 | 65 | 150 | 63 | 77 | 87 | 70 | 59 | 107 | 83 | 197 | 403 |
| 49 | 22 | 29 | 48 | 25 | 45 | 104 | 83 | 71 | 38 | 61 | 60 | 110 | 68 | 185 | 200 |
| 50 | 21 | 15 | 77 | 48 | 71 | 99 | 27 | 41 | 32 | 56 | 40 | 108 | 90 | 167 | 187 |
| 51 | 17 | 2 | 13 | 65 | 91 | 92 | 42 | 34 | 29 | 23 | 34 | 50 | 157 | 83 | 81 |
| 52 | 28 | 34 | 51 | 25 | 71 | 52 | 45 | 35 | 19 | 20 | 45 | 52 | 59 | 72 | 100 |
| 53 | 14 | 6 | 10 | 55 | 28 | 51 | 45 | 15 | 17 | 15 | 27 | 51 | 39 | 79 | 71 |
| 54 | 4 | 2 | 10 | 22 | 35 | 43 | 23 | 17 | 9 | 20 | 16 | 30 | 44 | 24 | 43 |
| 55 | 23 | 5 | 10 | 8 | 27 | 35 | 7 | 49 | 14 | 25 | 22 | 26 | 23 | 36 | 58 |
| 56 | 17 | 3 | 23 | 21 | 27 | 44 | 15 | 15 | 5 | 14 | 16 | 27 | 24 | 31 | 26 |
| 57 | 10 | 2 | 20 | 9 | 15 | 25 | 7 | 29 | 2 | 9 | 23 | 33 | 10 | 28 | 42 |
| 58 | 13 | 2 | 22 | 18 | 15 | 17 | 9 | 4 | 3 | 9 | 12 | 10 | 13 | 7 | 18 |
| 59 | 5 | 16 | 12 | 4 | 11 | 22 | 9 | 3 | 4 | 7 | 8 | 11 | 10 | 25 | 9 |
| 60 | 37 | 16 | 5 | 11 | 7 | 30 | 30 | 15 | 7 | 8 | 21 | 18 | 8 | 7 | 8 |



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| 0 | 1 | 0 | 17 | 5 | 9 |
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| 0 | 0 | 0 | 6 | 0 | 2 |
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2 $\begin{array}{rl}1 & 2 \\ 12 & 3 \\ 7 & 4\end{array}$ $\begin{array}{ll}1 & 3 \\ 0 & 3\end{array}$ $\left[\begin{array}{ll}0 & 4 \\ 1 & 2 \\ 0\end{array}\right.$
 $\square$ $\begin{array}{ll}0 & 1 \\ 2 & 1 \\ 0 & 0\end{array}$ $\begin{array}{ll}0 & 1 \\ 0 & 1 \\ 2 & 0 \\ 0 & 0\end{array}$ $\begin{array}{ll}1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0\end{array}$ 0

|  | 1984 | 1985 | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 |
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| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 6 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 |
| 8 | 12 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 0 | 0 | 0 | 0 |
| 9 | 33 | 1 | 28 | 17 | 3 | 0 | 0 | 0 | 24 | 69 | 26 | 50 | 11 | 56 | 0 |
| 10 | 487 | 202 | 595 | 287 | 55 | 0 | 8 | 43 | 394 | 712 | 159 | 1087 | 180 | 381 | 62 |
| 11 | 2664 | 911 | 3451 | 3453 | 462 | 70 | 59 | 205 | 1956 | 4555 | 712 | 3762 | 901 | 1447 | 325 |
| 12 | 7001 | 2186 | 8418 | 10167 | 1476 | 172 | 275 | 1060 | 7080 | 10746 | 1972 | 7660 | 2131 | 5287 | 1436 |
| 13 | 13626 | 3412 | 12098 | 19704 | 2231 | 529 | 842 | 3680 | 13032 | 16809 | 3720 | 13535 | 3537 | 11175 | 4479 |
| 14 | 20787 | 4890 | 16079 | 27843 | 2469 | 976 | 1779 | 6840 | 21192 | 20828 | 4143 | 17180 | 4141 | 16554 | 7964 |
| 15 | 23260 | 6403 | 17835 | 30864 | 2113 | 1980 | 3091 | 12207 | 26624 | 24798 | 4821 | 22623 | 5364 | 21435 | 10902 |
| 16 | 25022 | 6002 | 13011 | 27345 | 1837 | 2898 | 4905 | 17949 | 26184 | 23238 | 5877 | 24373 | 5550 | 19368 | 10682 |
| 17 | 23143 | 5436 | 7979 | 18785 | 1769 | 4764 | 6270 | 19811 | 21775 | 21383 | 6025 | 23789 | 6565 | 15148 | 7623 |
| 18 | 18801 | 4034 | 4225 | 11228 | 2390 | 5491 | 5531 | 18602 | 17018 | 18083 | 4729 | 17421 | 6073 | 10457 | 4287 |
| 19 | 13365 | 2960 | 2495 | 7085 | 4710 | 5235 | 3934 | 13846 | 11638 | 15255 | 3869 | 9671 | 7169 | 6898 | 3075 |
| 20 | 9116 | 4544 | 2564 | 5125 | 8472 | 4159 | 2524 | 9097 | 6482 | 11511 | 5340 | 5045 | 9733 | 5586 | 2601 |
| 21 | 6057 | 8144 | 4212 | 4605 | 11718 | 2927 | 2085 | 5431 | 3411 | 8108 | 7101 | 3108 | 14241 | 4165 | 3864 |
| 22 | 4230 | 14657 | 6115 | 5454 | 13059 | 2499 | 2073 | 2931 | 2216 | 6052 | 9990 | 2758 | 17497 | 4356 | 7001 |
| 23 | 3081 | 20692 | 6789 | 6922 | 12815 | 2529 | 2185 | 1978 | 1946 | 7072 | 12259 | 2697 | 18517 | 4988 | 9685 |
| 24 | 2964 | 22814 | 6430 | 7552 | 12486 | 3395 | 2493 | 1862 | 2333 | 8225 | 11509 | 3090 | 15765 | 6165 | 10715 |
| 25 | 3592 | 21047 | 6597 | 7258 | 10406 | 4520 | 3008 | 2242 | 3340 | 9035 | 10449 | 3544 | 12729 | 8536 | 10214 |
| 26 | 4207 | 17335 | 5784 | 7204 | 8699 | 4977 | 3003 | 2633 | 4554 | 8855 | 7866 | 3850 | 9733 | 11303 | 7827 |
| 27 | 4857 | 13200 | 5906 | 8082 | 6644 | 5412 | 3536 | 3105 | 6729 | 9188 | 6088 | 5062 | 7694 | 13691 | 5872 |
| 28 | 5092 | 9836 | 6225 | 7122 | 4547 | 4756 | 4097 | 2983 | 7183 | 9440 | 4206 | 4741 | 5345 | 13203 | 4629 |
| 29 | 4865 | 7392 | 7046 | 6053 | 3670 | 5122 | 3532 | 2774 | 6745 | 7576 | 3008 | 4451 | 4179 | 10750 | 4168 |
| 30 | 4575 | 5389 | 7041 | 3840 | 2531 | 4616 | 3108 | 2120 | 4952 | 6649 | 2301 | 3650 | 2795 | 10248 | 4441 |
| 31 | 3565 | 3726 | 7050 | 2787 | 2220 | 3902 | 2388 | 1489 | 3360 | 5090 | 1918 | 2652 | 2476 | 7153 | 4415 |
| 32 | 2883 | 2916 | 6792 | 2228 | 1965 | 3387 | 1851 | 1196 | 2271 | 4412 | 1400 | 2181 | 2044 | 5237 | 4434 |
| 33 | 2629 | 2536 | 6236 | 1798 | 1743 | 2377 | 1531 | 926 | 1322 | 3391 | 1340 | 1718 | 1663 | 3476 | 3441 |
| 34 | 2420 | 2007 | 4346 | 1703 | 1491 | 1689 | 1281 | 745 | 895 | 2588 | 1230 | 1132 | 1306 | 2535 | 2967 |
| 35 | 2288 | 1983 | 3388 | 1545 | 1398 | 1091 | 1084 | 648 | 530 | 2342 | 1081 | 811 | 1316 | 1917 | 2454 |
| 36 | 2069 | 1819 | 2053 | 1611 | 1067 | 791 | 830 | 491 | 352 | 1728 | 820 | 751 | 1105 | 1050 | 1571 |
| 37 | 2043 | 1300 | 1477 | 1362 | 999 | 591 | 632 | 405 | 254 | 1544 | 581 | 470 | 733 | 667 | 1244 |
| 38 | 1675 | 1110 | 950 | 1301 | 660 | 364 | 537 | 332 | 197 | 1027 | 465 | 400 | 510 | 479 | 751 |
| 39 | 1598 | 877 | 815 | 1002 | 538 | 299 | 442 | 266 | 140 | 759 | 343 | 284 | 394 | 527 | 573 |
| 40 | 1080 | 759 | 743 | 668 | 424 | 217 | 329 | 232 | 112 | 643 | 242 | 282 | 308 | 326 | 369 |
| 41 | 829 | 620 | 485 | 549 | 346 | 144 | 204 | 173 | 103 | 399 | 177 | 207 | 232 | 222 | 240 |
| 42 | 670 | 508 | 366 | 441 | 236 | 136 | 169 | 170 | 69 | 273 | 121 | 134 | 160 | 147 | 165 |
| 43 | 527 | 321 | 287 | 333 | 226 | 125 | 141 | 138 | 83 | 151 | 97 | 141 | 151 | 134 | 102 |
| 44 | 495 | 284 | 261 | 193 | 137 | 81 | 85 | 80 | 49 | 153 | 54 | 72 | 74 | 70 | 50 |
| 45 | 468 | 195 | 288 | 209 | 144 | 77 | 75 | 80 | 33 | 84 | 53 | 30 | 77 | 143 | 49 |
| 46 | 415 | 176 | 186 | 143 | 121 | 77 | 48 | 61 | 30 | 68 | 29 | 44 | 47 | 207 | 32 |
| 47 | 354 | 193 | 144 | 101 | 140 | 45 | 39 | 74 | 23 | 32 | 23 | 30 | 46 | 27 | 27 |
| 48 | 274 | 143 | 115 | 73 | 98 | 35 | 49 | 54 | 21 | 33 | 29 | 21 | 34 | 17 | 23 |
| 49 | 255 | 85 | 79 | 64 | 64 | 32 | 20 | 20 | 15 | 11 | 25 | 46 | 25 | 14 | 4 |
| 50 | 192 | 59 | 128 | 53 | 75 | 40 | 15 | 50 | 7 | 35 | 20 | 27 | 20 | 12 | 11 |
| 51 | 115 | 70 | 38 | 49 | 31 | 30 | 10 | 13 | 9 | 6 | 7 | 20 | 18 | 3 | 8 |
| 52 | 76 | 47 | 47 | 51 | 34 | 22 | 15 | 14 | 7 | 9 | 9 | 21 | 12 | 0 | 12 |
| 53 | 110 | 42 | 41 | 19 | 35 | 27 | 9 | 11 | 6 | 5 | 5 | 7 | 13 | 6 | 2 |
| 54 | 46 | 54 | 43 | 23 | 20 | 27 | 7 | 12 | 6 | 2 | 6 | 3 | 3 | 5 | 4 |
| 55 | 46 | 67 | 22 | 26 | 21 | 20 | 6 | 16 | 2 | 13 | 5 | 0 | 13 | 2 | 10 |
| 56 | 25 | 26 | 16 | 19 | 14 | 5 | 5 | 19 | 5 | 2 | 2 | 6 | 2 | 1 | 3 |
| 57 | 46 | 15 | 46 | 14 | 15 | 7 | 4 | 12 | 5 | 7 | 3 | 2 | 0 | 2 | 10 |
| 58 | 33 | 14 | 13 | 8 | 3 | 4 | 2 | 13 | 5 | 5 | 2 | 0 | 0 | 4 | 2 |
| 59 | 8 | 15 | 13 | 7 | 5 | 5 | 3 | 12 | 3 | 1 | 6 | 4 | 1 | 1 | 8 |
| 60 | 12 | 11 | 9 | 16 | 13 | 14 | 3 | 2 | 6 | 5 | 4 | 10 | 2 | 0 | 1 |




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| 13 | 1161 | 50408 | 16981 | 229 | 161 | 42 | 38 | 1444 | 867 | 235 | 319 | 1061 | 16 | 3 |
| 14 | 2233 | 54219 | 14488 | 378 | 452 | 108 | 202 | 4548 | 1097 | 472 | 787 | 2765 | 92 | 15 |
| 15 | 3476 | 62002 | 14711 | 588 | 1127 | 313 | 617 | 13352 | 1424 | 664 | 1303 | 7143 | 191 | 95 |
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| 17 | 4078 | 59439 | 12299 | 1078 | 2506 | 1442 | 1671 | 32389 | 2060 | 1235 | 1620 | 15761 | 465 | 255 |
| 18 | 3293 | 33645 | 14354 | 1530 | 2532 | 1929 | 1710 | 25112 | 1715 | 1190 | 1230 | 17429 | 547 | 426 |
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| 39 | 442 | 343 | 276 | 276 | 373 | 453 | 593 | 550 | 297 | 205 | 238 | 305 | 668 | 683 |
| 40 | 253 | 288 | 161 | 161 | 182 | 445 | 422 | 427 | 217 | 155 | 130 | 270 | 801 | 348 |
| 41 | 220 | 180 | 129 | 152 | 149 | 193 | 190 | 288 | 235 | 115 | 134 | 131 | 560 | 262 |
| 42 | 87 | 123 | 109 | 124 | 105 | 81 | 131 | 166 | 137 | 69 | 75 | 95 | 372 | 179 |
| 43 | 66 | 83 | 43 | 40 | 58 | 66 | 68 | 133 | 90 | 59 | 73 | 90 | 114 | 137 |
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### 6.15Appendix 6.6: Minimum fish landing size

## MINIMUM FISH SIZES

## MINIMUM FISH SIZES

## General

36. The EU minimum sizes are set out in Annex XII of Regulation 850/98 (Table 5) For both finfish and shellfish undersized animals are not to be retained on board, transhipped, landed, transported, stored, sold, displayed or offered for sale. Undersized animals must be returned immediately to the sea.

## Shellfish

## Lobster \& Crawfish

37. It should be noted that:

- carapace length is now the only measurement to decide minimum size
- the minimum size for lobster from 1 danuary 2002 has been increased to 87 mm on an EC wide basis.


## Edible Crab

38. Edible crab landing sizes were introduced at EC level for the first time on 1 lanuary 2000. The sizes are

- 140 mm north of $56^{\circ} \mathrm{N}$ both to West of Scotland and in the North Sea;
- 130 mm in the remainder of the North Sea except the Eastern Sea Fisheries District;
- 115 mm in the Eastern Sea Fisheries District;
- 140 mm in the Channel and around the Southwest Peninsula i.e. areas VIId, e and f;
- 130 mm elsewhere.

39. Please remember that UK national legislation already applies higher crab landing sizes in the UK. Off Devon, Cornwall and the Scilly Isles the UK size for male edible crab is set at 160 mm rather than 140 mm established by the EC. The size for female edible crabs in this area is 140 mm . For male Spider crabs the UK has set a size of 130 mm . Females may be
landed at 120 mm in accordance with the EC requirements
40. For catches of edible crabs made by pots or creels a maximum of $1 \%$ by weight of the total of edible crabs or parts of edible crabs may consist of detached crab claws. For catches of edible crabs made by any other fishing gear a maximum of 75 kgs of detached crab claws may be retained on board or landed at the end of any fishing voyage. Please note, however, that the Marketing Regulation restricts crab claws to a small final quantity necessary for local processing

## Scallops

41. For scallops the size is 110 mm for the Irish Sea from its northern boundary down as far as $52^{\circ} 30^{\prime} \mathrm{N} .110 \mathrm{~mm}$ is also the size set in area VIId. Bsewhere the size remains 100mm.

## Whelks

42. The minimum landing size for whelk is 45 mm shell length.



Table 5
Council Regulation 850/98 ANNEX XII
MINIM UM SIZES

| Species | Minimum size |  |
| :---: | :---: | :---: |
|  | Regions 1 to 5 except Skagerrak/ Kattegat | Skagerrak/ Kattegat |
| Cod (Gadus Morhua) | 35 cm | 30 cm |
| Haddock (Melanogrammus aeglefinus) | 30 cm | 27 cm |
| Saithe (Pollachius virens) | 35 cm | 30 cm |
| Pollack (Pollachius pollachius) | 30 cm | - |
| Hake (Merluccius merluccius) | 27 cm | 30 cm |
| Megrim (Lepidorhombus spp.) | 20 cm | 25 cm |
| Sole (Solea spp.) | 24 cm | 24 cm |
| Plaice (Pleuronectes platessa) | 27 cm | 27 cm |
| Whiting (Merlangius merlangus) | 27 cm | 23 cm |
| Ling (Molva molva) | 63 cm | - |
| Blue ling (Molva dipterygia) | 70 cm | - |
| Bass (Dicentrarchus labrax) | 36 cm | $\stackrel{-}{ }$ |
| Norway lobster (Nephrops norvegicus) $^{1}$ Norway lobster tails |  | 130 (40) mm ${ }^{1}$ |
| Mackerel (Scomber spp.) | 20 cm (30cm in North Sea) | $20 \mathrm{~cm}{ }^{2}$ |
| Herring (Clupea harengus) | 20 cm | 18 cm |
| Horse mackerel (Trachurus spp.) ( ${ }^{3}$ ) | 15 cm | 15 cm |
| Sardine (Sardina pilchardus) | 11 cm | ${ }^{-}$ |
| Lobster (Homarus gammarus) | 87 mm | 220 (78) mm ${ }^{1}$ |
| Spinous spider crab (Maia squinado) | 120 mm | - |
| Queen scallop (Chlamys spp.) | 40 mm | - |
| Grooved carpetshell (Ruditapes decussatus) | 40 mm | - |
| Carpetshell ( Venerupis pullastra) | 38 mm | - |
| Short-necked clam (Ruditapes phillippinarum) | 40 mm | - |
| Clam (Venus verrucosa) | 40 mm | - |
| Hard clam (Callista chione) | 6 cm |  |
| Razor clam (Ensis spp.) | 10 cm |  |
| Surf clams (Spisula Solida) | 25 mm |  |
| Donax clams (Donax spp.) | 25 mm |  |
| Bean solen (Pharus legumen) | 65 mm |  |
| Whelk (Buccinum undatum) | 45 mm | - |
| Octopus (Octopus vulgaris) | 750 grammes |  |
| Swordfish (Xiphias gladius) ( ${ }^{4}$ ) | 25 kg or 125 cm (lower mandible) |  |
| Bluefin tuna (Thunnus thynnus) ( ${ }^{5}$ ) | 6.4 kg (or 70 cm ) |  |
| Crawfish (Palinurus spp.) | 95 mm |  |
| Deepwater rose shrimp (Parapenaeus longirostirs) | 22 mm (carapace length) |  |

Total carapace length
${ }^{2} 30 \mathrm{~cm}$ for industrial purposes only
${ }^{3}$ No minimum size will apply to horse mackerel caught in waters adjacent to the Azores islands and under the sovereignty or jurisdiction of Portugal
${ }^{4}$ It is prohibited to land more than $15 \%$ in number of swordfish below 25 kg or 125 cm
${ }^{5}$ It is prohibited to land more than $15 \%$ in number of bluefin tuna below 6.4 kg or 70 cm . In addition it is prohibited to land any individual tuna below 1.8 kg .

| Species | Minimum size, Regions 1-5 except Skagerrak/Kattegat |
| :---: | :---: |
| Norway lobster (Nephrops norvegicus) | Whole area, except Region 3 and ICES VIa, VIla: total length 85 mm , carapace length 25 mm |
|  | ICES Vla, VIla; Region 3: total length 70 mm , carapace length 20 mm |
| Norway lobster tails | Whole area, except Region 3 and ICES Vla, VIla 46 mm |
|  | ICES Vla, VIla; Region 3: 37 mm |
| M ackerel (Scomber spp.) | Whole area, except North Sea: 20 cm |
|  | North Sea: 30 cm |
| Anchovy (Engraulis encrasicolus) | Whole area, except ICES IXa east of longitude $7^{\circ} 23^{\prime} 48^{\prime \prime}$ W: 12 cm |
|  | ICES IXa east of longitude $7^{\circ} 23^{\prime} 48^{\prime \prime} \mathrm{W}$ : 10 cm |
| Edible Crab (Cancer pagarus) | Regions 1 and 2 north of $56^{\circ} \mathrm{N}: 140 \mathrm{~mm}$ |
|  | Region 2 south of $56^{\circ} \mathrm{N}$ except ICES Divisions VIId, e, f and ICES Divisions IVb , c south of $56^{\circ} \mathrm{N}$ : 130 mm |
|  | ICES Divisions $\mathrm{IVb}, \mathrm{c}$ south of $56^{\circ} \mathrm{N}$ : 130 mm (except for an area limited by a point at $53^{\circ} 28^{\prime} 22^{\prime \prime} \mathrm{N} 0^{\circ} 09^{\prime} 24^{\prime \prime}$ Eon the coast of England, a straight line joining this point with $53^{\circ} 28^{\prime} 22^{\prime \prime} \mathrm{N} 0^{\circ} 22^{\prime} 24^{\prime \prime} \mathrm{E}$ the 6 mile boundary of the United Kingdom and a straight line connecting a point at $51^{\circ} 54^{\prime} 06^{\prime \prime} \mathrm{N} 1^{\circ} 30^{\prime} 30^{\prime \prime}$ E with a point on the coast of England at $51^{\circ} 55^{\prime} 48^{\prime \prime} \mathrm{N} 1^{\circ} 17^{\prime} 00$ " - the Eastern sea fisheries district - where the landing size shall be 115 mm ). |
|  | ICES Divisions VIId, e, f: 140 mm |
|  | Region 3: 130 mm |
| Scallop (Pecten maximus) | Whole area, except ICES VIIa, north of $52^{\circ} 30^{\prime} \mathrm{N}$ VIId: 100 mm |
|  | ICES VIla, north of $52^{\circ} 30^{\prime} \mathrm{N}$ VIId: 110 mm |

## Chapter 7

## North Sea grey gurnard stock assessment: <br> Applying length-structured survey-landings model


#### Abstract

7.1 Abstract

The output of the survey-landings model application on the North Sea haddock was reasonably close to the ICES assessment suggesting that the model could be an alternative method to assess the stock of marine species for which no age neither catch-at-length is available. In this Chapter the length-structured survey-landings model is applied to assess the stock of one of the poorly sampled species, the North Sea grey gurnard. Although there is no standard stock assessment for the grey gurnard (ICES WGNEW, 2014; ICES stock annex for grey gurnard, 2014) to compare the result with, the output results show that it is a reliable model for population dynamics modelling.


### 7.2 Biology and ecology

The grey gurnard (Eutrigla gurnardus) is a small predatory demersal fish and a large member of the sea robin family. It has a large head (compared to its body size) that can spin around the body with a sloping forehead and a body that tapers towards the tail. It walks along the seabed using its feelers and also it has pectoral fins to fly through the sea. This is why it is
also called sea robin (British Sea Fishing.co.uk). It is usually greyish-brown with a red tinge on the back and white spots on the sides (The marine Life Information Network).

The grey gurnard is found on sandy bottoms down to 140 m depth as well as on rocky or muddy seabed. Geographically, it is found off the coasts of western Scotland, southern and western England, Wales, eastern and southern Ireland, eastern England and Scotland (The marine Life Information Network). The spatial distribution of the grey gurnard changes seasonally. In winter, the population concentrates in the western part of the central North Sea, whereas they scatter in spring and summer (Knijn et al. 1993; ICES_Denmark) (Figure 7.1). Their abundance is very high in 50-metre-depth in both spring and summer (Heessen and Daan, 1994). Climate change, however, in relation to warming, has also affected the spatial distribution of a few species including grey gurnard and shifted it northward and in depth as well towards colder water (Perry et al., 2005).


Figure 7.1: Average annual catch (number per fishing hour for all length classes combined) for grey gurnard in the quarter 1 IBTS survey, 1977-2005 (the picture is taken from ICES-FishMap)

The grey gurnard has muscles that can drum against its swim bladder to make sounds (FishBase), which are believed to help them stay in contact with schools (ICES_Denmark). It emits frequent knocks and grunts and growls sounds during its competitive feeding (Amorim et al., 2004). Knocking is less aggressive and is a single sound composed of 1-2 pulses, while grunts and growls are longer with 4-8 and over 10 pulses, respectively (Amorim et al., 2004). The result of the study in Clara et al., (2005) indicates that both feeding behaviour and sound production change with the gurnard's size. Sound production rate is lower for a larger grey gurnard, which may imply that the larger body size gives advantage of locating and capturing the prey and that making sounds is less needed.

The range of grey gurnard's maximum reported length varies according to different sources. The marine Life Information Network reported that grey gurnard's maximum length is 30 cm (rarely 50 cm ), while in ICES maximum length of grey gurnard is said to be 45 cm and that they live for maximum of 9 years. FishBase, however, recorded 60 cm for its maximum length and 30 cm for its common length with maximum published weight of 956 grams. Females grow faster and live longer; on average a 1-year-old grey gurnard was $13-14 \mathrm{~cm}$ and a 2-year-old was $19-20 \mathrm{~cm}$ in the sample collected in 1978 (ICES). It spawns in the period between April and August. A male gurnard matures at 18 cm while maturity is at length 24 cm for a female gurnard (at the age of 1 or 2 years)(FishBase).

Grey gurnard has mixed diet according to the findings in its stomach. The small fish feed on crustaceans, mostly shrimp and shore crabs (FishBase), while the percentage of fish significantly increases in the diet of an adult grey gurnard (Sobecka et al., 2014; MorenoAmich, 1995; de Gee and Kikkert, 1993) at length around 20 cm (de Gee and Kikkert, 1993). Based on Weinert et al. (2010), the most abundant species found in the North Sea gurnard's stomach was the amphipod and shrimp and that the dominant vertebrates was the sandeel family; while in Sobecka et al. (2014)'s study poor cod is the most dominated vertebrates.

Between $18 \%$ and $32 \%$ of stomach contents of grey gurnards larger than 30 cm is Atlantic cod (Gadus morhua) and whiting (Merlangius merlangus). Also $90 \%$ of cod and $60 \%$ of whiting in grey gurnards' stomach are smaller than 10 cm (de Gee and Kikkert, 1993). The habit of grey gurnard of feeding on juveniles of commercially exploited fish may have played an important role in the slow of recovery of North Sea cod and whiting, Frank et al. (2007) argue. Also the result of MSVPA in Floeter et al. 2005 suspects that grey gurnard might be responsible for about $60 \%$ of the total predation mortality on 0 -group Atlantic cod, while the

Holling type II functional response was not responsible for the extinction of cod in their model. In addition to that, the implementation of grey gurnard in the North Sea MSVPA model did not conclude any quantitative influence on of grey gurnard on cod (ICES, 2003a). The size and speed of food eaten changes through seasons. In winter, grey gurnard eat slower but larger prey (Moreno-Amich, 1995)

The grey gurnard, historically, is considered as bycatch species and landed mainly for human consumption (ICES WGNEW, 2014). However, due to market limitation, a great proportion is discarded. Since 2014 the grey gurnard in area IIIa, IVand VIId was included in the intercatch data with Denmark (454t) and England (265t) reported the largest landings and the largest discards reported for the Dutch (1152t). The highest catches belong to the Netherlands, followed by England and Scotland. However, the data still remains incomplete since not all countries manage to provide data in time (ICES WGNEW, 2014). The grey gurnard is included as 'other predators' in the North Sea MSVPA since the significant consumption of commercial fish by grey gurnard was revealed in 1991 (ICES, 1997); also when its catch rates has increased since the late 1980s (Floeter et al. 2005). This is while Heessen and Daan (1994) had already concluded that grey gurnard might play an important role in the North Sea ecosystem (de Gee and Kikkert, 1993). Due to having little economic value as a non-target species, it is very difficult to understand the effect of fishing on grey gurnard. Nevertheless, Pope et al., (2000) discus the importance of developing a model which can help understand the changes of a community as the result of exploitation. Their works suggest that grey gurnard is not vulnerable to fishing mortality levels. That is because the size of first maturity (around 19 cm ) is lower than the size at which it is fully recruited to the fishery ( $20-25 \mathrm{~cm}$ ). Consequently, females have an opportunity to spawn before they are much fished and hence their spawning potential is hardly affected by existing levels of fishing mortality (Pope et al. 2000). Also being non-target species, the assumption is that fishing effort is not aimed at grey gurnards and therefore it is safe to use a random term for fishing this species (Pope et al. 2000).

Due to being an abundant demersal fish in the North Sea as well as being considered as sustainable stock, grey gurnard is increasingly deployed to the food source. Karl and Levsen (2011) studied the North Sea grey gurnard with regards to safety as a food source. The result of the study shows that the North Sea grey gurnard can be heavily infected with anisakid nematodes, with the infection rate in west of east coast of British main land being
significantly higher than the other parts of the North Sea (Levsen and Karl, 2014). However, it was further shown that most of the larvae infecting the fish are located in the belly flaps and that the abundance of infecting larvae could reduce by removing the belly flaps (Levsen and Karl, 2014). More study should be conducted to clarify if the situation exists in the final product for human consumption.

### 7.3 Population structure based on survey frequency distribution

The stock size for grey gurnard is unknown (ICES WGNEW, 2014) but the survey catches shows an increase from late 1980s to late 1990s (Figure 7.2).


Figure 7.2: The North Sea grey gurnard's total survey numbers from 1979 to 2010 extracted from IBTS

The length distributions of grey gurnard survey from IBTS during years 1979 to 2010 in the North Sea and Skagerrak are very similar from year to year but the scale is very different. The absence of small fish in the North Sea suggests that the IBTS survey does not completely
cover the nursery population. That could conclude that the juvenile fish stay on the rough seabed, which is avoided by the fishing trawls.

The length distribution of survey numbers are normalised to illustrate the structure of the sample population (Figure 7.3). Since the scale of survey numbers was clearly different from year to year, the absolute frequency graph was avoided.



Figure 7.3: Normalised frequency length distribution of survey for the North Sea grey gurnard from 1979 to 2010 extracted from IBTS

### 7.4 Changing in landing biomass

Due to the grey gurnards being non-target species, its fishing mortality is unknown (ICES WGNEW, 2014), while the total weight of commercially landed fish is recorded if reported. Landing statistics is very poorly available for grey gurnard. It is either unreported or not reported especially for grey gurnard (i.e. gurnard). ICES-FAO (2011 and 2014) shows an exceptionally large increase between 1986 and 1987 (46598 tonnes in 1987 due to Danish landing report) before it declines to its usual level in 1993. Heath and Cook (2015), however, consider the scale of increase to be implausible and that it may reflect the misreporting of another species. Therefore, in a supplementary table, they disregard the official data between 1987 and 1992 and replace it by linear interpretation the mean for 1985 and 1986 and the mean for 1993 and 1994. Heath and Cook's assumption and landing estimate for landing is used in the survey-landings model for grey gurnard's stock assessment. There are two particular reason to use this estimate in the population dynamic model. First, Heath and Cook (2015) estimated the discards biomass too based on that assumption, which is used as an
input in the survey-landings stock assessment model. Second, the exceptionally high landing report could influence the model and pull the estimation towards this point and make unrealistic estimation by ignoring other landing records. Therefore, the spike belonging to the Danish landing is moderated (Figure 7.4). The possible issue in using this landing estimate, however, is that they belong to all gurnard family (grey gurnard, red gurnard, tub gurnard, gurnards and sea robins) and not specifically for grey gurnard. Alternatively the landing data from WGNSSK (2014), which was reported specifically for grey gurnard could have been a valid source for commercial fishery input data. But the discards reports are missing.

Based on the Heath and Cook (2015) landing and discards records, the annual reported landed biomass, since 1979, has its peak at over 4500 tonnes in 1984 when it dramatically dropped to one of its lowest rates of around 650 tonnes in 1986. After recovery to less than 2000 tonnes in 1987, it has its slow decrease again until 1999. The 2000s is when landed biomass recovers again to level off around 2500 tonnes. Year 2010 is the exception, though, with less than 100 tonnes (Figure 7.4).

Throughout the study years, biomass of discarded fish is considerably higher than landings. The gap between landings and discards increases by time, which implies that a larger amount of catch is discarded every year. With regards to [grey] gurnard, discarding is not just about size but include many fish that are of good size but not landed due to low value (Figure 7.4).


Figure 7.4: Total landed biomass and total discards biomass for the North Sea [grey] gurnard extracted from IBTS.

### 7.5 Model observations

The length-structured survey-landings model takes its observations from three data sets (see Chapter 3 and 4) to assess the stock of the North Sea grey gurnard. Observations include length frequency of sampled fish from the scientific IBTS, total landed biomass and total discards biomass from commercial reports extracted and estimated (supplimentary tables in Heath \& Cook, 2015) from ICES (2011).

The study period is 32 years from the year 1979 to 2010 in 1-year time steps $(d t=1)$. The IBTS has recorded the North Sea grey gurnard's survey numbers at length since 1966, of which the $1^{\text {st }}$ quarter of year is selected since it is the time when most of sampling takes place and most complete data are available.

Total landed and discards biomass, which are the total weight (tonnes) of landed and discarded North Sea [grey] gurnard in area IV and IIIa and VIId at each year, were extracted from ICES-FAO with being moderated by Heath and Cook (2015) for years 1978 to 2010. The estimation of the first year (1978) was kept to use as historical data to estimate the initial condition (Chapter 4). Total annual discarded biomass was used as model observation to gain more accuracy in estimation. Gurnard's discard moderated data is not a recorded observation data like landings but it is regularly investigated and estimated. For that, it is considered as a reliable observation in survey-landings model.

Survey data is recorded in a Microsoft Excel spread sheet and read in R in csv format. In order to make a matrix of survey numbers at length, three variables of the original IBTS length data file are extracted. They include the columns that represent year of sampling, length of captured grey gurnards and number of captured grey gurnards at length at year. The same codes as in Appendix 6.1 in the programming package R are used to sum the number of fish at each length at each year to make a matrix of input observations, where rows represent the length classes and columns are the years. Length classes are from 3 to 49 cm , which are the smallest and biggest gurnards captured in the survey in the 32 years period from 1979 to 2010. Length classes are set to 1 cm width $(d L=1)$ to match the length class in the IBTS data.

### 7.6 Input data

In order to make the assessment model as simple as possible, all the parameters that happen to be known or can be estimated externally are added to the model as fixed input. Unlike the haddock (Chapter 6), the stock of the North Sea grey gurnard has never been assessed by any of the stock assessment methods. This makes the start value of the estimating parameters is more challenging.

### 7.6.1 Growth variations

Due to being sensitive to noise and variability (Chapter 5), growth parameter $\beta$ is kept fixed and constant in the survey-landings model to improve the accuracy of the von Bertalanffy parameters $\left(L_{\infty}, K\right)$. Throughout the model applications, $\beta$ is adjusted manually for the best estimation result and closest variability to the survey distribution. A similar approach has been applied previously (Sullivan et al., 1990), in which different values of $\beta$ are manually tested. Finally, it was set equal to 0.5 , which is closest to the growth variation in the survey distribution.

### 7.6.2 Weight at length

Weight of grey gurnard at length, $W_{l}$, is extracted externally from Marine Lab Report (FishBase, Trawl record data base, Simon Greenstreet) and is added as a fixed vector into the model (Figure 7.5).

$$
W_{l}=0.0082 . l^{3.015}
$$



Figure 7.5: Weight-length curve for the North Sea grey gurnard
The weight-length parameters from WGNSSK gurnard (2014) was also available ( $W_{l}=$ $\left.0.006 . l^{3.094}\right)$. Although there was little difference in the overall relationship, the model resulted better in if using the FishBase parameters.

### 7.6.3 Proportion of the sampling area

The survey number of grey gurnards is assumed to be proportional to the real abundance (number) of grey gurnards in the North Sea by the survey selectivity $q_{l}$ and proportion of the sampling area $P a_{t}$ (Eq. 4.25 and 7.2). The calculation method is avoided here because it is explained in detail in section 6.4.3.

Table 7.1: Proportion of the grey gurnard sampling area in the North Sea area IV and IIIa from year 1979 to 2010 calculated from IBTS data

| $t$ | $P a_{t}$ | $t$ | $P a_{t}$ | $t$ | $P a_{t}$ | $t$ | $P a_{t}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1979 | $5.69 \mathrm{E}-05$ | 1987 | $6.58 \mathrm{E}-05$ | 1995 | $4.09 \mathrm{E}-$ | 2003 | $4.28 \mathrm{E}-05$ |
| 1980 | $4.82 \mathrm{E}-05$ | 1988 | $4.89 \mathrm{E}-05$ | 1996 | $3.92 \mathrm{E}-$ | 2004 | $4.16 \mathrm{E}-05$ |
| 1981 | $3.87 \mathrm{E}-05$ | 1989 | $5.18 \mathrm{E}-05$ | 1997 | $4.29 \mathrm{E}-$ | 2005 | $3.88 \mathrm{E}-05$ |
| 1982 | $4.27 \mathrm{E}-05$ | 1990 | $4.71 \mathrm{E}-05$ | 1998 | $4.64 \mathrm{E}-$ | 2006 | $4.00 \mathrm{E}-05$ |
| 1983 | $5.13 \mathrm{E}-05$ | 1991 | $5.40 \mathrm{E}-05$ | 1999 | $3.78 \mathrm{E}-$ | 2007 | $3.93 \mathrm{E}-05$ |
| 1984 | $5.38 \mathrm{E}-05$ | 1992 | $4.62 \mathrm{E}-05$ | 2000 | $4.71 \mathrm{E}-$ | 2008 | $3.64 \mathrm{E}-05$ |
| 1985 | $6.08 \mathrm{E}-05$ | 1993 | $4.57 \mathrm{E}-05$ | 2001 | $4.01 \mathrm{E}-$ | 2009 | $3.82 \mathrm{E}-05$ |
| 1986 | $6.17 \mathrm{E}-05$ | 1994 | $4.54 \mathrm{E}-05$ | 2002 | $4.14 \mathrm{E}-$ | 2010 | $3.86 \mathrm{E}-05$ |



Figure 7.6: Proportion of the sweeping area by survey vessels, calculated from IBTS data for the North Sea grey gurnard

### 7.6.4 Natural mortality

The Natural mortality rate, $M$, for grey gurnards has not previously been recorded or estimated. Therefore, in order to get the best result, different assumptions were tested. Different known mortality rates of $0.3,0.40,0.50$ and 0.60 were tested before the model was applied based on the assumption of variability of natural mortality by length. Since no information was available for the mortality at length either, it was externally calculated using the relationship between body weight and natural mortality (Lorenzen, 1996; ICES 1988b). The natural mortality at weight was then transformed to natural mortality at length $\left(M_{l}\right)$ (Table 7.2 and Figure 7.7).

Table 7.2: Natural mortality rate (year ${ }^{-1}$ ) at length for North Sea grey gurnard calculated from mortality at weight

| Length (cm) | Natural <br> mortality <br> (1/year) | Length (cm) | Natural <br> mortality <br> $(\mathbf{1} / \mathbf{y e a r})$ | Length (cm) | Natural <br> mortality <br> (1/year) | Length (cm) | Natural <br> mortality <br> (1/year) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 9.30 | 15 | 1.43 | 27 | 0.72 | 39 | 0.47 |
| 4 | 6.65 | 16 | 1.33 | 28 | 0.69 | 40 | 0.46 |
| 5 | 5.13 | 17 | 1.24 | 29 | 0.66 | 41 | 0.44 |
| 6 | 4.15 | 18 | 1.16 | 30 | 0.64 | 42 | 0.43 |
| 7 | 3.47 | 19 | 1.09 | 31 | 0.61 | 43 | 0.42 |
| 8 | 2.97 | 20 | 1.02 | 32 | 0.59 | 44 | 0.41 |
| 9 | 2.59 | 21 | 0.97 | 33 | 0.57 | 45 | 0.40 |
| 10 | 2.29 | 22 | 0.91 | 34 | 0.55 | 46 | 0.39 |
| 11 | 2.05 | 23 | 0.87 | 35 | 0.53 | 47 | 0.38 |
| 12 | 1.85 | 24 | 0.83 | 36 | 0.52 | 48 | 0.37 |
| 13 | 1.69 | 25 | 0.79 | 37 | 0.50 | 49 | 0.36 |
| 14 | 1.55 | 26 | 0.75 | 38 | 0.48 |  |  |



Figure 7.7: Natural mortality rate at length for the North Sea grey gurnard calculated from mortality at weight

### 7.6.5 Probability of landing

Parameters of the landing probability function $\left(\alpha_{D}, \beta_{D}\right)$ are also exempt from estimation. That is because they were adjusted using the minimum landing size ( 30 cm ) available in Heath and Cook (2015), no minimum landing size is set for grey gurnard by the EU legislation (ICES, Stock annex for grey gurnard, 2014) though. Also they are confounded with fishing selectivity parameters (Chapter 4 and 5) and better to be fixed in the model. The logistic model in Equation 4.11 is adapted to model the probability of landing (Section 4.2.1) for the North Sea grey gurnard:

$$
P_{-} \ln d_{l}=\frac{0.5}{1+\alpha_{D} \exp \left(-\beta_{D} l\right)}
$$

Landing grey gurnard is not just about the size but value of fish as well. Many fish are in perfect size but are not landed due to very low value. For that, the maximum landing probability is much smaller than 1.

### 7.6.6 Average length at recruit

The average length at age 1 , which is in fact the average length at recruit, $E\left(l_{1}\right)=\mu_{R}$, is set as a fixed parameter in the model. It is the most common length at the first peak in the length frequency of survey. Having this parameter fixed helps the growth parameters to be estimated more accurately.

### 7.6.7 Maximum survey catchability

The result of sensitivity analysis in Chapter 5 suggests that it is sensible to keep the maximum catchability, in the numerator of Equation 4.22, as a fixed parameter.

$$
q_{l}=\frac{q_{\max }}{1+\alpha_{v} \exp \left(-\beta_{v 1} l\right)+\frac{1}{\alpha_{v}} \exp \left(\beta_{v 2} \cdot l\right)+\exp \left(l\left(\beta_{v 2}-\beta_{v 1}\right)\right)}
$$

Due to lack of any information, this parameter was taken from haddock stock assessment (Chapter 6, Section 6.5.2). Then different values of $q_{\max }$ around the estimated value were tested and finally it was set as fixed at 2.7 for the final model application. The fixed maximum catchability, being bigger than 1 , implies that fish in the survey area not only are not distributed evenly but also the sampling area has a higher density of fish compared to the other part of the sea.

### 7.7 Initial parameter values

In this section, the approaches to calculate the initial values for estimating parameters are discussed. A list of all initial parameter values along with the estimates values is given at the end of this section (Table 7.5). The initial parameters values were estimated, calculated or even guessed to start with. In order to gain the best possible assessment, the output of the assessment was used as initials to run the models. This process was repeated until the result does not change or start distancing from being reasonable.

With some minor changes, the codes are similar to Appendix 6.4 for haddock. Unlike haddock assessment, for which the fishing selectivity had a double logistic curve (Chapter 4, Equation 4.3b) with the maximum selectivity of 1 , the fishing selectivity for grey gurnards follows an increasing logistic function (Chapter 4, Equation 4.3a) with the maximum selectivity of 0.9 . That is because it provided a more meaningful result:

$$
\begin{equation*}
s_{l}=\frac{0.9}{1+\alpha_{s} \exp \left(-\beta_{s} l\right)} \tag{7.7}
\end{equation*}
$$

The maximum probability of landing is 0.5 for grey gurnard. That is due to the fact that the fish size is not the only factor for discarding but the value too. The fish with the acceptable size but with low value is not landed. Therefore the maximum landing probability is much smaller than 1 :

$$
P_{-} \operatorname{lnd} d_{l}=\frac{0.5}{1+\alpha_{D} \exp \left(-\beta_{D} l\right)}
$$

### 7.7.1 Growth parameters

As discussed in Chapter 5, the model is sensitive to large variability of growth parameters. In order to use initial values of growth parameters $\left(L_{\infty}, K\right)$ as close as possible to the true values, they are estimated (Eq. 4.15) and extracted from external sources before adding them as initial values into the survey-landings model. The maximum asymptotic length $\left(L_{\infty}\right)$ for the North Sea grey gurnards with UK fisheries is estimated within a range of 35 to 46 cm and the curvature parameter $K$ ranges from 0.15 to 0.50 in FishBase and Trawl Record Management Tool (Table 7.3). In addition to that, the average length of gurnard at age from ICES-Denmark was used to estimate the growth parameters. Each pair was tried and tested in the model.

Table 7.3: Growth parameters estimated for the North Sea grey gurnards in FishBase, Trawl Record Management Tool and ICES-Denmark

| $L_{\infty}$ | 35 | 35 | 37 | 43 | 46 | 63.5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $K$ | 0.50 | 0.48 | 0.32 | 0.15 | 0.25 | 0.16 |

From the growth curve in ICES-Denmark, the growth in the first year of birth is faster than the later stages in gurnards' lives. Therefore, the average length at age zero is calculated separately from the Equation 4.14b:

$$
E\left(l_{0}\right)=\frac{\left(\mu_{R}-L_{\infty}\left(1-e^{-K}\right)\right)}{e^{-K}}
$$

where $\mu_{R}=E\left(l_{1}\right)$ is the average length at age 1 , which is fixed and set equal to 15 cm . It is the most common length at the first peak in the length distribution of survey. Average length at age $2+$ is then estimated inside the model using VBL growth function:

$$
E\left(l_{a \geq 2}\right)=L_{\infty}-\left(L_{\infty}-E\left(l_{0}\right)\right) e^{-(K)(a)}
$$

### 7.7.2 Gear selectivity

The three remaining parameters ( $\alpha_{v}, \beta_{v 1}, \beta_{v 2}$ ) in the double logistic survey selectivity (Equation 4.22b) are subject to estimation:

$$
q_{l}=\frac{q_{\max }}{1+\alpha_{v} \exp \left(-\beta_{v 1} l\right)+\frac{1}{\alpha_{v}} \exp \left(\beta_{v 2} \cdot l\right)+\exp \left(l\left(\beta_{v 2}-\beta_{v 1}\right)\right)}
$$

The initial values were taken from haddock's but adjusted for the best result. Fishing selectivity follows the increasing logistic function (Eq. 4.3a) and has two estimating parameters.

$$
s_{l}=\frac{0.9}{1+\alpha_{s} \exp \left(-\beta_{s} l\right)}
$$

The first clue for the fishing selectivity initial parameters values was taken from Pope et al. (2000), in which a graph of gurnard's fishing mortality was plotted. The fishing scalar, $f_{t}$, was assumed constant at 0.44 and initial values for $s_{l}$ parameters were estimated. Throughout the runs of the model, the initial values were adjusted for the best result.

### 7.7.3 Fishing Mortality Scalar

Annual fishing mortality scalar $f_{t}$ is a measure of overall fishing mortality at the full effect of gear selectivity (i.e. when $s_{l}=1$ ). Since it changes over time, $f_{t}$ is estimated for each of the 32 time steps from 1979 to 2010.

Apart from Pope et al. (2000) who assume a constant $f_{t}$ of 0.44 , there is no record of $f_{t}$ in any publication for the North Sea grey gurnards. That is because the grey gurnard has never been assessed before. The initial value, therefore, started from the constant value of 0.44 , then was adjusted in the later runs of the model.

### 7.7.4 Recruitment

Recruits are defined as the new fish that are added to the population and had survived over (usually) a year, when they could be captured in fisheries. In survey the sampling takes place in the first quarter of the year when the fish have not hatched yet. The recruits of the year, therefore, are added to the population in the year after or even the following year (depends on their sizes). Hence, the recruits that appeared in year $t+1$ in survey-landings model are in fact the individuals that were born in year $t$ and are 1 year old at year $t+1$.

This assumption was used to calculate the initial values for recruitment parameters. First, the length range of a 1 -year-old grey gurnard was assumed to be between 9 cm to 15 cm . Then stock size was roughly calculated from Equation 4.25, 4.22a and 4.23:

$$
N_{l, t}=\frac{N s_{l, t}}{\left(P a_{t}\right) \cdot\left(q_{\max }\right) \cdot(0.5) /(1+\alpha \cdot \exp (-\beta l))}
$$

where the survey numbers in the numerator and proportion of sampling area are known; and the parameters of the survey selectivity was selected to provide a sensible catchability curve ( $q_{m q x}=2.7, \alpha=16.024, \beta=0.113$ ). The initial idea of the shape of the curve was taken from fishing mortality in Pope et al. (2000) then adjusted for survey selectivity. Next the sum of the abundance of fish at size 9 cm to 15 cm at each year was set as the initial value for the annual recruitment.

As discussed earlier in the estimation process, the gamma distribution parameters for distributing recruits over length classes $\left(\alpha_{R}, \beta_{R}\right)$ were set to mark the average length at recruit, ( $\mu_{R}=\alpha_{R} \cdot \beta_{R}$ ); and set equal to the average length at age $1(15 \mathrm{~cm})$ in von Bertalanffy growth function. They are therefore taken from the length distribution of survey in the target years. The first peak in the survey length distribution is a reliable guide for the average length of individuals at age 1 . In order to keep the average length at recruitment unchanged, $\mu_{R}$ is kept fixed to the average length at age $1(15 \mathrm{~cm})$. The scale parameter $\beta_{R}$, which affects the variance of the distribution, is allowed to vary.

### 7.7.5 Initial Condition

The survey numbers as well as the landed ad discards biomass of year 1978 were used to estimate the initial abundance $\left(N_{0}\right)$ before the start of estimation. The initial abundance was estimated inside the model and the same method used for the initial parameter values at time zero.

### 7.8 Survey-landings model: Result of model application on the North Sea grey gurnard (1979-2010)

Observations, including the survey numbers at length and total landed and discards biomass for the 32 years from 1979 to 2010, and fixed parameters are put into the survey-landings model. Least square parameter estimation method within 'optim' function in R is used to minimise the residual sums of square (in a similar approach to Appendix 6.4).

All the least square components are given equal weights $\left(\lambda_{i(i=1,2,3)}=1\right)$ :

$$
R S S=\lambda_{1} \sum_{t}\left(T L B_{M}-T L B_{O}\right)^{2}+\lambda_{2} \sum_{l, t}\left(N s_{M}-N s_{O}\right)^{2}+\lambda_{3} \sum_{t}\left(T D B_{M}-T D B_{O}\right)^{2}
$$

The model managed to estimate the total survey numbers fairly close to the actual values (Figure 7.8b). It also captured the peaks and the increasing trend of the total survey numbers.

The variation of the grey gurnard's landed biomass is very high from the maximum of 4541 tonnes in 1984 to the minimum of 463 tonnes in 1999. Nevertheless, the model was capable of estimating them with reasonable accuracy of capturing the increasing and decreasing trend throughout the period. In the peak period of the early 1980s the model slightly underestimated landed biomass (Figure 7.8a), and overestimated the landings in late 1990s.
a)

b)

d)

f)


Time (Year)

Figure 7.8: Model estimated (red dashed line) and observed (blue solid line) of a) total landed biomass, b) total survey numbers, c) total catch biomass, d) total discards biomass, e) Total stock size, f) total stock biomass and $g$ ) total annual recruit numbers over the 32 years. Figures ' $e$ ', ' $f$ ' and ' $g$ ' are the estimated out put, for which there is no observation or other standard assessment to compare with.

The model accurately estimated the discards biomass (Figure 7.8.d). It confirming that a feasibl function and minimum landing size was used for the probability of landing curve. Although there was no total catch observation available, the sum of the landings and discards could be considered as total observed catch. The model's output for total catch biomass is well matched with the observations (Figure 7.8.c).

Since the dynamics of the North Sea grey gurnard's population has never been assessed, there is no source to compare the assessment results with. In fact, that is the main purpose of this study, which is to design a population model capable of assessing the stock of un-assessed fish species. Both stock size (Figure 7.8.e) and stock biomass (Figure 7.8f) reflect the increasing pattern of survey numbers (Figure 7.8.g).

The scale of survey numbers changes year by year, for that the length distribution of survey was normalised to the total number at each year (Figure 7.9). With this method, the shape and the structure of the graphs are more visible. The actual numbers are presented in the graph that illustrates the total survey numbers (Figure 7.8.b).


Figure 7.9: Estimated (red dotted lines) and observed (blues vertical bars) survey normalised numbers at length at year.

The logistic function of landing probability was adjusted to be $50 \%$ for the grey gurnard at its minimum landing size $(30 \mathrm{~cm})$. However, since the size is not the only factor of discarding the fish and that they are discarded due to low values as well, the maximum probability is set to 0.50 instead of 1 (Figure 7.10.a).

The fishing selectivity was initially modelled as a double (dome shaped) logistic function. While the estimated curve follows an increasing pattern with very low selectivity (Figure 7.10.c). It reaches the maximum of 0.02 for the gurnards of size 50 cm . That is because grey gurnard is not a target species and basically is caught randomly through by fishing trawls (Pope at al., 2000).

The time-dependent fishing mortality scalar is estimated to be very high in the first decade, which reflects the high landing biomass in the same period. It then goes down and level off in 2000s (Figure 7.10d). Fishing mortality scalar in grey gurnard is affected by the scalar for the target species in the North Sea. This is very clear when the fishing mortality scalar for grey gurnard is compared with the output for haddock in Figure 7.10.d.

Survey selectivity is a double logistic curve with the peak catchability at around 24 cm (Figure 7.10.e). Since there are hardly any small fish in the survey, the selectivity is also zero for very small gurnards. The survey selectivity curve well represents the true sampling pattern, in which a representative sample of the fish is of interest rather than the very large and more valuable fish. Because of that the selectivity reduces for the top biggest fish.

The natural mortality is modelled as a decreasing function of length and calculated based on mortality at weight function, which is then transformed to length using the weight length relationship (Figure 7.10.f). It is a fixed vector in the model.

Growth parameters are estimated in the model to get the average length at age and to shape the von Bertalanffy growth curve. They are reasonably estimated (Table 7.4). The result is within the range of growth parameters that were estimated in literature. Since the growth speed in the first year is different from the rest of haddock's life, the average length at age zero is calculated separately. That is why the average length does not start from zero.


Figure 7.10: a) landing probability, b) estimated von Bertalanffy growth curve, c) estimated fishing gear selectivity curve, d) time-dependent fishing mortality scalar, e) estimated survey gear selectivity curve and f) natural mortality

Table 7.4: Model estimated average length at age $\left(E\left(l_{t}\right)\right)$ using von Bertalanffy function when surveylandings model is applied on the North Sea grey gurnards over the 32 years (1979-2010)

| Age (year) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Estimated average length (cm) | 10.4 | 15 | 18.9 | 22.2 | 25.1 | 27.5 | 29.6 | 31.4 | 32.9 |

The fishing mortality rate at length at time is the product of the estimated fishing selectivity and fishing scalar. The curve shows a logistically increasing pattern by length, although the scale of the estimated fishing mortality is very low. That could be due to grey gurnard being a non-target species (Figure 7.11).


Figure 7.11: Estimated fishing mortality rate (1/year) for the North Sea grey gurnard

Since the fishing mortality rate is low (Figure 7.11) compared to the gurnard's natural mortality (Figure 7.10.f), the total mortality rate (Figure 7.12) is mostly influenced by natural mortality.


Figure 7.12: Estimated total mortality rate (1/year) for the north Sea grey gurnard

The estimated length frequency of gurnard's population is illustrated in Figure 7.13. The scales are unified for all the distributions to reflect the change of stock size over years. Apart from 1981, the gurnard's stock size in the first 12 years is very low, while, it starts to recover from 1989. The estimated stock size is given in a table in Appendix 7.1.



Figure 7.13: Estimated abundance at length at year for grey gurnard

As discussed earlier in this chapter, a very small proportion of caught grey gurnard is landed. The deciding factor is size as well as the value of the fish. The minimum landing size is fixed at 30 cm , at which the probability of landing is 0.5 . However, not all the fish bigger than the minimum threshold are landed (Figure 7.14). As a non-target species, the number of caught fish is significantly higher than the amount of landings. This can be explained by saying that although the small gurnards are not targeted, they are stuck in the trawl and that they have to be discarded for the size and low value reason.


Figure 7.14: Estimated catch-at-length (light blue vertical bars) and estimated landing numbers at length (green vertical bars) for the north Sea grey gurnards.

Table 7.5: List of parameter initial and estimated values when the survey-landings model is applied on the North Sea grey gurnard over years 1979 to 210

| Model parameters | Initial values | Estimated values | Model parameters | Initial values | Estimated values |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{\infty}$ | $4.40 \mathrm{E}+01$ | $4.18 \mathrm{E}+01$ | $r_{1}$ | $1.71 \mathrm{E}+08$ | $1.68 \mathrm{E}+08$ |
| K | $1.60 \mathrm{E}-01$ | $1.58 \mathrm{E}-01$ | $r_{2}$ | $5.39 \mathrm{E}+07$ | $9.92 \mathrm{E}+07$ |
| $\alpha_{v}$ | $2.22 \mathrm{E}+06$ | $2.38 \mathrm{E}+06$ | $r_{3}$ | $7.67 \mathrm{E}+08$ | $7.17 \mathrm{E}+08$ |
| $\beta_{v 1}$ | $3.55 \mathrm{E}-01$ | $3.62 \mathrm{E}-01$ | $r_{4}$ | $2.79 \mathrm{E}+05$ | $6.45 \mathrm{E}+04$ |
| $\beta_{v 2}$ | $6.69 \mathrm{E}-01$ | $6.73 \mathrm{E}-01$ | $r_{5}$ | $3.83 \mathrm{E}+08$ | $3.47 \mathrm{E}+08$ |
| $\boldsymbol{\beta}_{R}$ | $2.38 \mathrm{E}+00$ | $2.37 \mathrm{E}+00$ | $r_{6}$ | $3.17 \mathrm{E}+06$ | $2.84 \mathrm{E}+08$ |
| $\alpha_{s}$ | $1.80 \mathrm{E}+03$ | $1.80 \mathrm{E}+03$ | $r_{7}$ | $2.75 \mathrm{E}+07$ | $4.73 \mathrm{E}+04$ |
| $\boldsymbol{\beta}_{s}$ | $7.54 \mathrm{E}-02$ | 7.53E-02 | $r_{8}$ | $3.45 \mathrm{E}+08$ | $2.90 \mathrm{E}+08$ |
| $\boldsymbol{f}_{1}$ | $2.08 \mathrm{E}-01$ | $2.31 \mathrm{E}-01$ | $r_{9}$ | $1.90 \mathrm{E}+07$ | $2.90 \mathrm{E}+07$ |
| $f_{2}$ | $3.29 \mathrm{E}-01$ | $3.19 \mathrm{E}-01$ | $r_{10}$ | $5.59 \mathrm{E}+06$ | $5.47 \mathrm{E}+05$ |
| $\boldsymbol{f}_{3}$ | $1.13 \mathrm{E}-01$ | $1.24 \mathrm{E}-01$ | $r_{11}$ | $2.74 \mathrm{E}+08$ | $2.60 \mathrm{E}+08$ |
| $\boldsymbol{f}_{4}$ | $1.71 \mathrm{E}-01$ | $1.95 \mathrm{E}-01$ | $r_{12}$ | $3.65 \mathrm{E}+08$ | $4.09 \mathrm{E}+08$ |
| $\boldsymbol{f}_{5}$ | $1.25 \mathrm{E}-01$ | $1.52 \mathrm{E}-01$ | $r_{13}$ | $3.70 \mathrm{E}+08$ | $4.00 \mathrm{E}+08$ |
| $\boldsymbol{f}_{6}$ | $1.97 \mathrm{E}-01$ | $1.48 \mathrm{E}-01$ | $r_{14}$ | $5.84 \mathrm{E}+08$ | $5.44 \mathrm{E}+08$ |
| $\boldsymbol{f}_{7}$ | $2.09 \mathrm{E}-01$ | $1.77 \mathrm{E}-01$ | $r_{15}$ | $6.99 \mathrm{E}+05$ | $2.34 \mathrm{E}+08$ |
| $\boldsymbol{f}_{8}$ | $1.43 \mathrm{E}-01$ | $1.40 \mathrm{E}-01$ | $r_{16}$ | $4.96 \mathrm{E}+08$ | $3.60 \mathrm{E}+08$ |
| $f_{9}$ | $1.47 \mathrm{E}-01$ | $1.50 \mathrm{E}-01$ | $r_{17}$ | $3.47 \mathrm{E}+08$ | $4.27 \mathrm{E}+08$ |
| $\boldsymbol{f}_{10}$ | $1.64 \mathrm{E}-01$ | $2.11 \mathrm{E}-01$ | $r_{18}$ | $3.22 \mathrm{E}+08$ | $3.64 \mathrm{E}+08$ |
| $\boldsymbol{f}_{11}$ | $1.29 \mathrm{E}-01$ | $1.47 \mathrm{E}-01$ | $r_{19}$ | $7.91 \mathrm{E}+08$ | $7.44 \mathrm{E}+08$ |
| $\mathrm{f}_{12}$ | $9.62 \mathrm{E}-02$ | $1.00 \mathrm{E}-01$ | $r_{20}$ | $5.74 \mathrm{E}+08$ | $5.68 \mathrm{E}+08$ |
| $f_{13}$ | $8.55 \mathrm{E}-02$ | $9.84 \mathrm{E}-02$ | $r_{21}$ | $1.12 \mathrm{E}+09$ | $1.14 \mathrm{E}+09$ |
| $\boldsymbol{f}_{14}$ | $5.86 \mathrm{E}-02$ | $6.06 \mathrm{E}-02$ | $r_{22}$ | $8.45 \mathrm{E}+08$ | $8.14 \mathrm{E}+08$ |
| $f_{15}$ | $9.19 \mathrm{E}-02$ | $7.55 \mathrm{E}-02$ | $r_{23}$ | $1.02 \mathrm{E}+09$ | $9.17 \mathrm{E}+08$ |
| $\boldsymbol{f}_{16}$ | $6.46 \mathrm{E}-02$ | $7.05 \mathrm{E}-02$ | $r_{24}$ | $2.15 \mathrm{E}+05$ | $6.57 \mathrm{E}+08$ |
| $\boldsymbol{f}_{17}$ | $7.01 \mathrm{E}-02$ | $6.78 \mathrm{E}-02$ | $r_{25}$ | $1.28 \mathrm{E}+09$ | $1.12 \mathrm{E}+09$ |
| $f_{18}$ | $6.41 \mathrm{E}-02$ | $7.28 \mathrm{E}-02$ | $r_{26}$ | $7.46 \mathrm{E}+08$ | $7.61 \mathrm{E}+08$ |
| $f_{19}$ | $6.10 \mathrm{E}-02$ | $6.36 \mathrm{E}-02$ | $r_{27}$ | $1.23 \mathrm{E}+09$ | $1.14 \mathrm{E}+09$ |
| $\boldsymbol{f}_{20}$ | $5.57 \mathrm{E}-02$ | 5.99E-02 | $r_{28}$ | $1.40 \mathrm{E}+09$ | $1.31 \mathrm{E}+09$ |
| $\boldsymbol{f}_{21}$ | $5.79 \mathrm{E}-02$ | $6.33 \mathrm{E}-02$ | $r_{29}$ | $1.50 \mathrm{E}+09$ | $1.58 \mathrm{E}+09$ |
| $\boldsymbol{f}_{22}$ | $4.44 \mathrm{E}-02$ | $4.92 \mathrm{E}-02$ | $r_{30}$ | $9.47 \mathrm{E}+08$ | $9.62 \mathrm{E}+08$ |
| $\boldsymbol{f}_{23}$ | $3.35 \mathrm{E}-02$ | $3.47 \mathrm{E}-02$ | $r_{31}$ | $6.23 \mathrm{E}+08$ | $6.06 \mathrm{E}+08$ |
| $f_{24}$ | $4.94 \mathrm{E}-02$ | $4.87 \mathrm{E}-02$ | $r_{32}$ | $1.02 \mathrm{E}+09$ | $1.01 \mathrm{E}+09$ |
| $f_{25}$ | $3.81 \mathrm{E}-02$ | $3.95 \mathrm{E}-02$ | rhist $_{1}$ | $8.52 \mathrm{E}+06$ | $2.72 \mathrm{E}+05$ |
| $f_{26}$ | $4.23 \mathrm{E}-02$ | $4.70 \mathrm{E}-02$ | RSS |  | $3.30 \mathrm{E}+8$ |
| $\boldsymbol{f}_{27}$ | 3.99E-02 | $4.44 \mathrm{E}-02$ |  |  |  |
| $\boldsymbol{f}_{28}$ | 3.32E-02 | $3.64 \mathrm{E}-02$ |  |  |  |
| $\boldsymbol{f}_{29}$ | $3.19 \mathrm{E}-02$ | $3.67 \mathrm{E}-02$ |  |  |  |
| $f_{30}$ | $2.84 \mathrm{E}-02$ | $3.19 \mathrm{E}-02$ |  |  |  |
| $f_{31}$ | $3.28 \mathrm{E}-02$ | $3.61 \mathrm{E}-02$ |  |  |  |
| $f_{32}$ | $3.43 \mathrm{E}-02$ | $3.41 \mathrm{E}-02$ |  |  |  |

### 7.9 Discussion

In Chapter 6, in the comparison between the model results and ICES assessment for haddock, the model successfully passed validation the test as a reliable stock assessment method. Applying the survey-landings model on the grey gurnard is one step further to establish this model as an alternative stock assessment method for where there is no age or catch-at-length data available. The survey-landings model proved to be capable of estimating the observation to great level of accuracy. It also managed to estimate the recruits and population over the 32 years.

The North Sea grey gurnard is an environmentally important marine species for its role as a predator. It also can be a sustainable alternative seafood to overfished species (The Independent, August 2008). Nonetheless, the North Sea grey gurnard is among the very poorly sampled species, for which only the total landed is weighted and reported and only from 2014 has been included in inter catch for which specific commercial landing is reported. Also due to have been a by-catch species, its stock has never been assessed. The main information available about the grey gurnard are the growth parameters and weight-length relationship.

The survey-landings model was designed to make the stock assessment happen and that the dynamics of gurnard's population is studied. The model does not need any information about age or catch-at-length but takes the total landed biomass and survey numbers as input.

To feed into the survey-landing model, the length frequency distribution of grey gurnards from survey was used as one of the three observation data. The main weakness of the model is that the landing data are the one that was reported for gurnard species and not specifically for grey gurnard. Also, within this source the discards data was recorded and estimated to feed into the model as the third observation component. If the discards data for grey gurnard (not gurnard species) were available the commercial report from WGNSSK gurnard (2014) would have been a sensible source.

### 7.10 Appendix 7.1

In this section, the output values of the application of survey-landings model on the North Sea grey gurnard is given in the form of tables:

- Estimated total mortality at length at time (Table 7.6)

Estimated fishing mortality at length at time (

- Table 7.7)
- Estimated catch numbers at length at time (Table 7.8)
- Estimated stock size at length at time (Table 7.9)
- Estimated total annual stock biomass, catch biomass, landed biomass, discards biomass, survey biomass (Table 7.10)
- The probability transition matrix; the probability transition matrix for gurnard's growing from one length class to the other length class comes next. The top triangle of the matrix is 0 due to the assumption of fish do not shrink. It is also assumed that the fish d not grow more that the asymptotic length, which means that the probability of fish at its maximum asymptotic length staying in that length class is 1 (Table 7.11)

Table 7.6: Estimated total mortality rate (1/year) at length at year for North Sea grey gurnard

| Length/year | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ |
| 4 | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ |
| 5 | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ |
| 6 | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ |
| 7 | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ |
| 8 | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ |
| 9 | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ |
| 10 | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ |
| 11 | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ |
| 12 | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ |
| 13 | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ |
| 14 | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ |
| 15 | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ |
| 16 | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ |
| 17 | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ |
| 18 | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ |
| 19 | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ |
| 20 | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ |
| 21 | $9.66 \mathrm{E}-01$ | $9.67 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ |
| 22 | $9.16 \mathrm{E}-01$ | $9.16 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.16 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ |
| 23 | $8.70 \mathrm{E}-01$ | $8.70 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ |
| 24 | 8.28E-01 | $8.28 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ |
| 25 | $7.89 \mathrm{E}-01$ | $7.90 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ |
| 26 | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ |
| 27 | $7.22 \mathrm{E}-01$ | $7.22 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.22 \mathrm{E}-01$ | $7.22 \mathrm{E}-01$ | $7.22 \mathrm{E}-01$ | $7.22 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ |
| 28 | $6.92 \mathrm{E}-01$ | $6.92 \mathrm{E}-01$ | $6.92 \mathrm{E}-01$ | $6.92 \mathrm{E}-01$ | $6.92 \mathrm{E}-01$ | $6.92 \mathrm{E}-01$ | $6.92 \mathrm{E}-01$ | $6.92 \mathrm{E}-01$ |
| 29 | $6.64 \mathrm{E}-01$ | $6.65 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ |
| 30 | $6.39 \mathrm{E}-01$ | $6.39 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.39 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.39 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ |
| 31 | $6.15 \mathrm{E}-01$ | $6.16 \mathrm{E}-01$ | $6.15 \mathrm{E}-01$ | $6.15 \mathrm{E}-01$ | $6.15 \mathrm{E}-01$ | $6.15 \mathrm{E}-01$ | $6.15 \mathrm{E}-01$ | $6.15 \mathrm{E}-01$ |
| 32 | $5.93 \mathrm{E}-01$ | $5.93 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.93 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.93 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ |
| 33 | $5.72 \mathrm{E}-01$ | $5.73 \mathrm{E}-01$ | $5.72 \mathrm{E}-01$ | $5.72 \mathrm{E}-01$ | $5.72 \mathrm{E}-01$ | $5.72 \mathrm{E}-01$ | $5.72 \mathrm{E}-01$ | $5.72 \mathrm{E}-01$ |
| 34 | $5.53 \mathrm{E}-01$ | $5.53 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.53 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ |
| 35 | $5.35 \mathrm{E}-01$ | $5.35 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ |
| 36 | $5.18 \mathrm{E}-01$ | $5.18 \mathrm{E}-01$ | $5.17 \mathrm{E}-01$ | $5.17 \mathrm{E}-01$ | $5.17 \mathrm{E}-01$ | $5.17 \mathrm{E}-01$ | $5.17 \mathrm{E}-01$ | $5.17 \mathrm{E}-01$ |
| 37 | $5.01 \mathrm{E}-01$ | $5.02 \mathrm{E}-01$ | $5.01 \mathrm{E}-01$ | $5.01 \mathrm{E}-01$ | $5.01 \mathrm{E}-01$ | $5.01 \mathrm{E}-01$ | $5.01 \mathrm{E}-01$ | $5.01 \mathrm{E}-01$ |
| 38 | $4.86 \mathrm{E}-01$ | $4.87 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.86 \mathrm{E}-01$ | $4.86 \mathrm{E}-01$ | $4.86 \mathrm{E}-01$ | $4.86 \mathrm{E}-01$ | $4.86 \mathrm{E}-01$ |
| 39 | $4.72 \mathrm{E}-01$ | $4.73 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.72 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.72 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ |
| 40 | $4.59 \mathrm{E}-01$ | $4.60 \mathrm{E}-01$ | $4.58 \mathrm{E}-01$ | $4.58 \mathrm{E}-01$ | $4.58 \mathrm{E}-01$ | $4.58 \mathrm{E}-01$ | $4.58 \mathrm{E}-01$ | $4.58 \mathrm{E}-01$ |
| 41 | $4.46 \mathrm{E}-01$ | $4.47 \mathrm{E}-01$ | $4.45 \mathrm{E}-01$ | $4.45 \mathrm{E}-01$ | $4.45 \mathrm{E}-01$ | $4.45 \mathrm{E}-01$ | $4.45 \mathrm{E}-01$ | $4.45 \mathrm{E}-01$ |
| 42 | $4.34 \mathrm{E}-01$ | $4.35 \mathrm{E}-01$ | $4.33 \mathrm{E}-01$ | $4.33 \mathrm{E}-01$ | $4.33 \mathrm{E}-01$ | $4.33 \mathrm{E}-01$ | 4.33E-01 | $4.33 \mathrm{E}-01$ |
| 43 | $4.22 \mathrm{E}-01$ | $4.23 \mathrm{E}-01$ | $4.21 \mathrm{E}-01$ | $4.22 \mathrm{E}-01$ | $4.21 \mathrm{E}-01$ | $4.21 \mathrm{E}-01$ | $4.22 \mathrm{E}-01$ | $4.21 \mathrm{E}-01$ |
| 44 | $4.12 \mathrm{E}-01$ | $4.13 \mathrm{E}-01$ | $4.10 \mathrm{E}-01$ | $4.11 \mathrm{E}-01$ | $4.10 \mathrm{E}-01$ | $4.10 \mathrm{E}-01$ | $4.11 \mathrm{E}-01$ | $4.10 \mathrm{E}-01$ |
| 45 | $4.01 \mathrm{E}-01$ | $4.02 \mathrm{E}-01$ | $4.00 \mathrm{E}-01$ | $4.01 \mathrm{E}-01$ | $4.00 \mathrm{E}-01$ | $4.00 \mathrm{E}-01$ | $4.00 \mathrm{E}-01$ | $4.00 \mathrm{E}-01$ |
| 46 | $3.91 \mathrm{E}-01$ | $3.93 \mathrm{E}-01$ | $3.90 \mathrm{E}-01$ | $3.91 \mathrm{E}-01$ | $3.90 \mathrm{E}-01$ | $3.90 \mathrm{E}-01$ | $3.91 \mathrm{E}-01$ | $3.90 \mathrm{E}-01$ |
| 47 | $3.82 \mathrm{E}-01$ | $3.84 \mathrm{E}-01$ | $3.80 \mathrm{E}-01$ | $3.82 \mathrm{E}-01$ | $3.81 \mathrm{E}-01$ | $3.81 \mathrm{E}-01$ | $3.81 \mathrm{E}-01$ | $3.81 \mathrm{E}-01$ |
| 48 | $3.73 \mathrm{E}-01$ | $3.75 \mathrm{E}-01$ | $3.71 \mathrm{E}-01$ | $3.73 \mathrm{E}-01$ | $3.72 \mathrm{E}-01$ | $3.72 \mathrm{E}-01$ | $3.72 \mathrm{E}-01$ | $3.72 \mathrm{E}-01$ |
| 49 | $3.65 \mathrm{E}-01$ | $3.67 \mathrm{E}-01$ | $3.63 \mathrm{E}-01$ | $3.64 \mathrm{E}-01$ | $3.63 \mathrm{E}-01$ | $3.63 \mathrm{E}-01$ | $3.64 \mathrm{E}-01$ | $3.63 \mathrm{E}-01$ |

(Continued Table 7.6) Estimated total mortality rate (1/year) at length at year for North Sea grey gurnard

| Length/year | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ |
| 4 | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ |
| 5 | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ |
| 6 | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ |
| 7 | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ |
| 8 | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ |
| 9 | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ |
| 10 | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ |
| 11 | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ |
| 12 | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ |
| 13 | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ |
| 14 | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ |
| 15 | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ |
| 16 | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ |
| 17 | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ |
| 18 | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ |
| 19 | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ |
| 20 | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ |
| 21 | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ |
| 22 | $9.15 \mathrm{E}-01$ | $9.16 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ |
| 23 | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ |
| 24 | $8.27 \mathrm{E}-01$ | $8.28 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ |
| 25 | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ |
| 26 | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ |
| 27 | $7.22 \mathrm{E}-01$ | $7.22 \mathrm{E}-01$ | $7.22 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ |
| 28 | $6.92 \mathrm{E}-01$ | $6.92 \mathrm{E}-01$ | $6.92 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ |
| 29 | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ |
| 30 | $6.38 \mathrm{E}-01$ | $6.39 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ |
| 31 | $6.15 \mathrm{E}-01$ | $6.15 \mathrm{E}-01$ | $6.15 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ |
| 32 | $5.92 \mathrm{E}-01$ | $5.93 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ |
| 33 | $5.72 \mathrm{E}-01$ | $5.72 \mathrm{E}-01$ | $5.72 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ |
| 34 | $5.52 \mathrm{E}-01$ | $5.53 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ |
| 35 | $5.34 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ |
| 36 | $5.17 \mathrm{E}-01$ | $5.17 \mathrm{E}-01$ | $5.17 \mathrm{E}-01$ | $5.17 \mathrm{E}-01$ | $5.17 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ |
| 37 | $5.01 \mathrm{E}-01$ | $5.01 \mathrm{E}-01$ | $5.01 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ |
| 38 | $4.86 \mathrm{E}-01$ | $4.86 \mathrm{E}-01$ | $4.86 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ |
| 39 | $4.71 \mathrm{E}-01$ | $4.72 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ |
| 40 | $4.58 \mathrm{E}-01$ | $4.58 \mathrm{E}-01$ | $4.58 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ |
| 41 | $4.45 \mathrm{E}-01$ | $4.46 \mathrm{E}-01$ | $4.45 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ |
| 42 | $4.33 \mathrm{E}-01$ | $4.34 \mathrm{E}-01$ | $4.33 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ |
| 43 | $4.21 \mathrm{E}-01$ | $4.22 \mathrm{E}-01$ | $4.21 \mathrm{E}-01$ | $4.21 \mathrm{E}-01$ | $4.21 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ |
| 44 | $4.10 \mathrm{E}-01$ | $4.11 \mathrm{E}-01$ | $4.10 \mathrm{E}-01$ | $4.10 \mathrm{E}-01$ | $4.10 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ |
| 45 | $4.00 \mathrm{E}-01$ | $4.01 \mathrm{E}-01$ | $4.00 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ |
| 46 | $3.90 \mathrm{E}-01$ | $3.91 \mathrm{E}-01$ | $3.90 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ |
| 47 | $3.81 \mathrm{E}-01$ | $3.82 \mathrm{E}-01$ | $3.81 \mathrm{E}-01$ | $3.80 \mathrm{E}-01$ | $3.80 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ |
| 48 | $3.72 \mathrm{E}-01$ | $3.73 \mathrm{E}-01$ | $3.72 \mathrm{E}-01$ | $3.71 \mathrm{E}-01$ | $3.71 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ |
| 49 | $3.63 \mathrm{E}-01$ | $3.64 \mathrm{E}-01$ | $3.63 \mathrm{E}-01$ | $3.62 \mathrm{E}-01$ | $3.62 \mathrm{E}-01$ | $3.62 \mathrm{E}-01$ | $3.62 \mathrm{E}-01$ | $3.62 \mathrm{E}-01$ |


| Length/year | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ |
| 4 | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ |
| 5 | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ |
| 6 | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ |
| 7 | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ |
| 8 | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ |
| 9 | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ |
| 10 | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ |
| 11 | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ |
| 12 | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ |
| 13 | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ |
| 14 | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ |
| 15 | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ |
| 16 | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ |
| 17 | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ |
| 18 | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ |
| 19 | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ |
| 20 | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ |
| 21 | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ |
| 22 | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ |
| 23 | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ |
| 24 | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ |
| 25 | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ |
| 26 | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ | $7.53 \mathrm{E}-01$ | $7.54 \mathrm{E}-01$ |
| 27 | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ |
| 28 | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ |
| 29 | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ |
| 30 | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ |
| 31 | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ |
| 32 | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ |
| 33 | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ |
| 34 | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ |
| 35 | $5.33 \mathrm{E}-01$ | $5.34 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ |
| 36 | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ |
| 37 | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ |
| 38 | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ |
| 39 | $4.71 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.71 \mathrm{E}-01$ | $4.70 \mathrm{E}-01$ | $4.70 \mathrm{E}-01$ | $4.70 \mathrm{E}-01$ |
| 40 | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ |
| 41 | $4.44 \mathrm{E}-01$ | 4.44E-01 | $4.44 \mathrm{E}-01$ | 4.44E-01 | $4.44 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ |
| 42 | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ |
| 43 | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ |
| 44 | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ |
| 45 | $3.99 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ | $3.98 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ |
| 46 | $3.89 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ | $3.88 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ |
| 47 | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ |
| 48 | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ |
| 49 | $3.62 \mathrm{E}-01$ | $3.62 \mathrm{E}-01$ | $3.62 \mathrm{E}-01$ | $3.61 \mathrm{E}-01$ | $3.62 \mathrm{E}-01$ | $3.61 \mathrm{E}-01$ | $3.61 \mathrm{E}-01$ | $3.61 \mathrm{E}-01$ |


| Length/year | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ | $9.30 \mathrm{E}+00$ |
| 4 | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ | $6.65 \mathrm{E}+00$ |
| 5 | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ | $5.13 \mathrm{E}+00$ |
| 6 | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ | $4.15 \mathrm{E}+00$ |
| 7 | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ | $3.47 \mathrm{E}+00$ |
| 8 | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ | $2.97 \mathrm{E}+00$ |
| 9 | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ | $2.59 \mathrm{E}+00$ |
| 10 | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ | $2.29 \mathrm{E}+00$ |
| 11 | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ | $2.05 \mathrm{E}+00$ |
| 12 | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ | $1.85 \mathrm{E}+00$ |
| 13 | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ |
| 14 | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ | $1.55 \mathrm{E}+00$ |
| 15 | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ |
| 16 | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ | $1.33 \mathrm{E}+00$ |
| 17 | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ | $1.24 \mathrm{E}+00$ |
| 18 | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ |
| 19 | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ | $1.09 \mathrm{E}+00$ |
| 20 | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ | $1.02 \mathrm{E}+00$ |
| 21 | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ | $9.66 \mathrm{E}-01$ |
| 22 | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ | $9.15 \mathrm{E}-01$ |
| 23 | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ | $8.69 \mathrm{E}-01$ |
| 24 | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ | $8.27 \mathrm{E}-01$ |
| 25 | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ | $7.89 \mathrm{E}-01$ |
| 26 | $7.53 \mathrm{E}-01$ | $7.53 \mathrm{E}-01$ | $7.53 \mathrm{E}-01$ | $7.53 \mathrm{E}-01$ | $7.53 \mathrm{E}-01$ | $7.53 \mathrm{E}-01$ | $7.53 \mathrm{E}-01$ | $7.53 \mathrm{E}-01$ |
| 27 | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ | $7.21 \mathrm{E}-01$ |
| 28 | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ | $6.91 \mathrm{E}-01$ |
| 29 | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ | $6.64 \mathrm{E}-01$ |
| 30 | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ | $6.38 \mathrm{E}-01$ |
| 31 | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ | $6.14 \mathrm{E}-01$ |
| 32 | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ | $5.92 \mathrm{E}-01$ |
| 33 | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ | $5.71 \mathrm{E}-01$ |
| 34 | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ | $5.52 \mathrm{E}-01$ |
| 35 | $5.33 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ | $5.33 \mathrm{E}-01$ |
| 36 | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ |
| 37 | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ |
| 38 | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ | $4.85 \mathrm{E}-01$ |
| 39 | $4.70 \mathrm{E}-01$ | $4.70 \mathrm{E}-01$ | $4.70 \mathrm{E}-01$ | $4.70 \mathrm{E}-01$ | $4.70 \mathrm{E}-01$ | $4.70 \mathrm{E}-01$ | $4.70 \mathrm{E}-01$ | $4.70 \mathrm{E}-01$ |
| 40 | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ | $4.57 \mathrm{E}-01$ |
| 41 | $4.44 \mathrm{E}-01$ | 4.44E-01 | $4.44 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ | $4.44 \mathrm{E}-01$ |
| 42 | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.31 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ | $4.32 \mathrm{E}-01$ |
| 43 | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ | $4.20 \mathrm{E}-01$ |
| 44 | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ | $4.09 \mathrm{E}-01$ |
| 45 | $3.98 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ | $3.99 \mathrm{E}-01$ | $3.98 \mathrm{E}-01$ | $3.98 \mathrm{E}-01$ | $3.98 \mathrm{E}-01$ | $3.98 \mathrm{E}-01$ | $3.98 \mathrm{E}-01$ |
| 46 | $3.88 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ | $3.89 \mathrm{E}-01$ | $3.88 \mathrm{E}-01$ | $3.88 \mathrm{E}-01$ | $3.88 \mathrm{E}-01$ | $3.88 \mathrm{E}-01$ | $3.88 \mathrm{E}-01$ |
| 47 | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ | $3.79 \mathrm{E}-01$ |
| 48 | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ |
| 49 | $3.61 \mathrm{E}-01$ | $3.61 \mathrm{E}-01$ | $3.61 \mathrm{E}-01$ | $3.61 \mathrm{E}-01$ | $3.61 \mathrm{E}-01$ | $3.61 \mathrm{E}-01$ | $3.61 \mathrm{E}-01$ | $3.61 \mathrm{E}-01$ |

Table 7.7: Estimated fishing mortality rate (1/year) at length at year for North Sea grey gurnard

| Length/year | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $1.44 \mathrm{E}-04$ | $1.99 \mathrm{E}-04$ | $7.73 \mathrm{E}-05$ | $1.22 \mathrm{E}-04$ | $9.54 \mathrm{E}-05$ | $9.26 \mathrm{E}-05$ | $1.11 \mathrm{E}-04$ | $8.74 \mathrm{E}-05$ |
| 4 | $1.56 \mathrm{E}-04$ | $2.15 \mathrm{E}-04$ | $8.33 \mathrm{E}-05$ | $1.32 \mathrm{E}-04$ | $1.03 \mathrm{E}-04$ | $9.98 \mathrm{E}-05$ | $1.19 \mathrm{E}-04$ | $9.42 \mathrm{E}-05$ |
| 5 | $1.68 \mathrm{E}-04$ | $2.32 \mathrm{E}-04$ | $8.98 \mathrm{E}-05$ | $1.42 \mathrm{E}-04$ | $1.11 \mathrm{E}-04$ | $1.08 \mathrm{E}-04$ | $1.29 \mathrm{E}-04$ | $1.02 \mathrm{E}-04$ |
| 6 | $1.81 \mathrm{E}-04$ | $2.50 \mathrm{E}-04$ | $9.68 \mathrm{E}-05$ | $1.53 \mathrm{E}-04$ | $1.20 \mathrm{E}-04$ | $1.16 \mathrm{E}-04$ | $1.39 \mathrm{E}-04$ | $1.10 \mathrm{E}-04$ |
| 7 | $1.95 \mathrm{E}-04$ | $2.70 \mathrm{E}-04$ | $1.04 \mathrm{E}-04$ | $1.65 \mathrm{E}-04$ | $1.29 \mathrm{E}-04$ | $1.25 \mathrm{E}-04$ | $1.50 \mathrm{E}-04$ | $1.18 \mathrm{E}-04$ |
| 8 | $2.10 \mathrm{E}-04$ | $2.91 \mathrm{E}-04$ | $1.13 \mathrm{E}-04$ | $1.78 \mathrm{E}-04$ | $1.39 \mathrm{E}-04$ | $1.35 \mathrm{E}-04$ | $1.61 \mathrm{E}-04$ | $1.27 \mathrm{E}-04$ |
| 9 | $2.27 \mathrm{E}-04$ | $3.13 \mathrm{E}-04$ | $1.21 \mathrm{E}-04$ | $1.92 \mathrm{E}-04$ | $1.50 \mathrm{E}-04$ | $1.45 \mathrm{E}-04$ | $1.74 \mathrm{E}-04$ | $1.37 \mathrm{E}-04$ |
| 10 | $2.45 \mathrm{E}-04$ | $3.38 \mathrm{E}-04$ | $1.31 \mathrm{E}-04$ | $2.07 \mathrm{E}-04$ | $1.61 \mathrm{E}-04$ | $1.57 \mathrm{E}-04$ | $1.87 \mathrm{E}-04$ | $1.48 \mathrm{E}-04$ |
| 11 | $2.64 \mathrm{E}-04$ | $3.64 \mathrm{E}-04$ | $1.41 \mathrm{E}-04$ | $2.23 \mathrm{E}-04$ | $1.74 \mathrm{E}-04$ | $1.69 \mathrm{E}-04$ | $2.02 \mathrm{E}-04$ | $1.60 \mathrm{E}-04$ |
| 12 | $2.84 \mathrm{E}-04$ | $3.93 \mathrm{E}-04$ | $1.52 \mathrm{E}-04$ | $2.40 \mathrm{E}-04$ | $1.88 \mathrm{E}-04$ | $1.82 \mathrm{E}-04$ | $2.18 \mathrm{E}-04$ | $1.72 \mathrm{E}-04$ |
| 13 | $3.06 \mathrm{E}-04$ | $4.23 \mathrm{E}-04$ | $1.64 \mathrm{E}-04$ | $2.59 \mathrm{E}-04$ | $2.02 \mathrm{E}-04$ | $1.96 \mathrm{E}-04$ | $2.35 \mathrm{E}-04$ | $1.85 \mathrm{E}-04$ |
| 14 | $3.30 \mathrm{E}-04$ | $4.56 \mathrm{E}-04$ | $1.77 \mathrm{E}-04$ | $2.79 \mathrm{E}-04$ | $2.18 \mathrm{E}-04$ | $2.12 \mathrm{E}-04$ | $2.53 \mathrm{E}-04$ | $2.00 \mathrm{E}-04$ |
| 15 | $3.56 \mathrm{E}-04$ | $4.92 \mathrm{E}-04$ | $1.91 \mathrm{E}-04$ | $3.01 \mathrm{E}-04$ | $2.35 \mathrm{E}-04$ | $2.28 \mathrm{E}-04$ | $2.73 \mathrm{E}-04$ | $2.15 \mathrm{E}-04$ |
| 16 | $3.84 \mathrm{E}-04$ | $5.30 \mathrm{E}-04$ | $2.05 \mathrm{E}-04$ | $3.24 \mathrm{E}-04$ | $2.54 \mathrm{E}-04$ | $2.46 \mathrm{E}-04$ | $2.94 \mathrm{E}-04$ | $2.32 \mathrm{E}-04$ |
| 17 | $4.14 \mathrm{E}-04$ | $5.72 \mathrm{E}-04$ | $2.21 \mathrm{E}-04$ | $3.50 \mathrm{E}-04$ | $2.73 \mathrm{E}-04$ | $2.65 \mathrm{E}-04$ | $3.17 \mathrm{E}-04$ | $2.50 \mathrm{E}-04$ |
| 18 | $4.46 \mathrm{E}-04$ | $6.16 \mathrm{E}-04$ | $2.39 \mathrm{E}-04$ | $3.77 \mathrm{E}-04$ | $2.95 \mathrm{E}-04$ | $2.86 \mathrm{E}-04$ | $3.42 \mathrm{E}-04$ | $2.70 \mathrm{E}-04$ |
| 19 | $4.81 \mathrm{E}-04$ | $6.64 \mathrm{E}-04$ | $2.57 \mathrm{E}-04$ | $4.06 \mathrm{E}-04$ | $3.18 \mathrm{E}-04$ | $3.08 \mathrm{E}-04$ | $3.69 \mathrm{E}-04$ | $2.91 \mathrm{E}-04$ |
| 20 | $5.19 \mathrm{E}-04$ | $7.16 \mathrm{E}-04$ | $2.77 \mathrm{E}-04$ | $4.38 \mathrm{E}-04$ | $3.42 \mathrm{E}-04$ | $3.33 \mathrm{E}-04$ | $3.97 \mathrm{E}-04$ | $3.14 \mathrm{E}-04$ |
| 21 | $5.59 \mathrm{E}-04$ | $7.72 \mathrm{E}-04$ | $2.99 \mathrm{E}-04$ | $4.72 \mathrm{E}-04$ | $3.69 \mathrm{E}-04$ | $3.58 \mathrm{E}-04$ | $4.28 \mathrm{E}-04$ | $3.38 \mathrm{E}-04$ |
| 22 | $6.03 \mathrm{E}-04$ | $8.32 \mathrm{E}-04$ | $3.22 \mathrm{E}-04$ | $5.09 \mathrm{E}-04$ | $3.98 \mathrm{E}-04$ | $3.86 \mathrm{E}-04$ | $4.62 \mathrm{E}-04$ | $3.65 \mathrm{E}-04$ |
| 23 | $6.50 \mathrm{E}-04$ | $8.97 \mathrm{E}-04$ | $3.48 \mathrm{E}-04$ | $5.49 \mathrm{E}-04$ | $4.29 \mathrm{E}-04$ | $4.17 \mathrm{E}-04$ | $4.98 \mathrm{E}-04$ | $3.93 \mathrm{E}-04$ |
| 24 | $7.00 \mathrm{E}-04$ | $9.67 \mathrm{E}-04$ | $3.75 \mathrm{E}-04$ | $5.92 \mathrm{E}-04$ | $4.62 \mathrm{E}-04$ | $4.49 \mathrm{E}-04$ | $5.37 \mathrm{E}-04$ | $4.24 \mathrm{E}-04$ |
| 25 | $7.55 \mathrm{E}-04$ | $1.04 \mathrm{E}-03$ | $4.04 \mathrm{E}-04$ | $6.38 \mathrm{E}-04$ | $4.98 \mathrm{E}-04$ | $4.84 \mathrm{E}-04$ | $5.79 \mathrm{E}-04$ | $4.57 \mathrm{E}-04$ |
| 26 | 8.14E-04 | $1.12 \mathrm{E}-03$ | $4.35 \mathrm{E}-04$ | $6.87 \mathrm{E}-04$ | $5.37 \mathrm{E}-04$ | $5.22 \mathrm{E}-04$ | $6.24 \mathrm{E}-04$ | $4.92 \mathrm{E}-04$ |
| 27 | $8.77 \mathrm{E}-04$ | $1.21 \mathrm{E}-03$ | $4.69 \mathrm{E}-04$ | $7.41 \mathrm{E}-04$ | $5.79 \mathrm{E}-04$ | $5.62 \mathrm{E}-04$ | $6.72 \mathrm{E}-04$ | $5.31 \mathrm{E}-04$ |
| 28 | $9.45 \mathrm{E}-04$ | $1.31 \mathrm{E}-03$ | $5.06 \mathrm{E}-04$ | $7.99 \mathrm{E}-04$ | $6.24 \mathrm{E}-04$ | $6.06 \mathrm{E}-04$ | $7.24 \mathrm{E}-04$ | $5.72 \mathrm{E}-04$ |
| 29 | $1.02 \mathrm{E}-03$ | $1.41 \mathrm{E}-03$ | $5.45 \mathrm{E}-04$ | $8.61 \mathrm{E}-04$ | $6.73 \mathrm{E}-04$ | $6.53 \mathrm{E}-04$ | $7.81 \mathrm{E}-04$ | $6.16 \mathrm{E}-04$ |
| 30 | $1.10 \mathrm{E}-03$ | $1.52 \mathrm{E}-03$ | $5.87 \mathrm{E}-04$ | $9.28 \mathrm{E}-04$ | $7.25 \mathrm{E}-04$ | $7.04 \mathrm{E}-04$ | $8.42 \mathrm{E}-04$ | $6.64 \mathrm{E}-04$ |
| 31 | $1.18 \mathrm{E}-03$ | $1.63 \mathrm{E}-03$ | $6.33 \mathrm{E}-04$ | $1.00 \mathrm{E}-03$ | $7.82 \mathrm{E}-04$ | $7.59 \mathrm{E}-04$ | $9.07 \mathrm{E}-04$ | $7.16 \mathrm{E}-04$ |
| 32 | $1.28 \mathrm{E}-03$ | $1.76 \mathrm{E}-03$ | $6.82 \mathrm{E}-04$ | $1.08 \mathrm{E}-03$ | $8.42 \mathrm{E}-04$ | 8.18E-04 | $9.78 \mathrm{E}-04$ | $7.72 \mathrm{E}-04$ |
| 33 | $1.37 \mathrm{E}-03$ | $1.90 \mathrm{E}-03$ | $7.35 \mathrm{E}-04$ | $1.16 \mathrm{E}-03$ | $9.08 \mathrm{E}-04$ | $8.81 \mathrm{E}-04$ | $1.05 \mathrm{E}-03$ | $8.32 \mathrm{E}-04$ |
| 34 | $1.48 \mathrm{E}-03$ | $2.05 \mathrm{E}-03$ | $7.93 \mathrm{E}-04$ | $1.25 \mathrm{E}-03$ | $9.78 \mathrm{E}-04$ | $9.50 \mathrm{E}-04$ | $1.14 \mathrm{E}-03$ | $8.96 \mathrm{E}-04$ |
| 35 | $1.60 \mathrm{E}-03$ | $2.20 \mathrm{E}-03$ | $8.54 \mathrm{E}-04$ | $1.35 \mathrm{E}-03$ | $1.05 \mathrm{E}-03$ | $1.02 \mathrm{E}-03$ | $1.22 \mathrm{E}-03$ | $9.66 \mathrm{E}-04$ |
| 36 | $1.72 \mathrm{E}-03$ | $2.38 \mathrm{E}-03$ | $9.20 \mathrm{E}-04$ | $1.45 \mathrm{E}-03$ | $1.14 \mathrm{E}-03$ | $1.10 \mathrm{E}-03$ | $1.32 \mathrm{E}-03$ | $1.04 \mathrm{E}-03$ |
| 37 | $1.85 \mathrm{E}-03$ | $2.56 \mathrm{E}-03$ | $9.92 \mathrm{E}-04$ | $1.57 \mathrm{E}-03$ | $1.22 \mathrm{E}-03$ | $1.19 \mathrm{E}-03$ | $1.42 \mathrm{E}-03$ | $1.12 \mathrm{E}-03$ |
| 38 | $2.00 \mathrm{E}-03$ | $2.76 \mathrm{E}-03$ | $1.07 \mathrm{E}-03$ | $1.69 \mathrm{E}-03$ | $1.32 \mathrm{E}-03$ | $1.28 \mathrm{E}-03$ | $1.53 \mathrm{E}-03$ | $1.21 \mathrm{E}-03$ |
| 39 | $2.15 \mathrm{E}-03$ | $2.97 \mathrm{E}-03$ | $1.15 \mathrm{E}-03$ | $1.82 \mathrm{E}-03$ | $1.42 \mathrm{E}-03$ | $1.38 \mathrm{E}-03$ | $1.65 \mathrm{E}-03$ | $1.30 \mathrm{E}-03$ |
| 40 | $2.32 \mathrm{E}-03$ | $3.20 \mathrm{E}-03$ | $1.24 \mathrm{E}-03$ | $1.96 \mathrm{E}-03$ | $1.53 \mathrm{E}-03$ | $1.49 \mathrm{E}-03$ | $1.78 \mathrm{E}-03$ | $1.40 \mathrm{E}-03$ |
| 41 | $2.50 \mathrm{E}-03$ | $3.45 \mathrm{E}-03$ | $1.34 \mathrm{E}-03$ | $2.11 \mathrm{E}-03$ | $1.65 \mathrm{E}-03$ | $1.60 \mathrm{E}-03$ | $1.91 \mathrm{E}-03$ | $1.51 \mathrm{E}-03$ |
| 42 | $2.69 \mathrm{E}-03$ | $3.72 \mathrm{E}-03$ | $1.44 \mathrm{E}-03$ | $2.27 \mathrm{E}-03$ | $1.78 \mathrm{E}-03$ | $1.72 \mathrm{E}-03$ | $2.06 \mathrm{E}-03$ | $1.63 \mathrm{E}-03$ |
| 43 | $2.90 \mathrm{E}-03$ | $4.00 \mathrm{E}-03$ | $1.55 \mathrm{E}-03$ | $2.45 \mathrm{E}-03$ | $1.91 \mathrm{E}-03$ | $1.86 \mathrm{E}-03$ | $2.22 \mathrm{E}-03$ | $1.75 \mathrm{E}-03$ |
| 44 | $3.12 \mathrm{E}-03$ | $4.31 \mathrm{E}-03$ | $1.67 \mathrm{E}-03$ | $2.64 \mathrm{E}-03$ | $2.06 \mathrm{E}-03$ | $2.00 \mathrm{E}-03$ | $2.39 \mathrm{E}-03$ | $1.89 \mathrm{E}-03$ |
| 45 | $3.36 \mathrm{E}-03$ | $4.64 \mathrm{E}-03$ | $1.80 \mathrm{E}-03$ | $2.84 \mathrm{E}-03$ | $2.22 \mathrm{E}-03$ | $2.15 \mathrm{E}-03$ | $2.58 \mathrm{E}-03$ | $2.03 \mathrm{E}-03$ |
| 46 | $3.62 \mathrm{E}-03$ | $5.00 \mathrm{E}-03$ | $1.94 \mathrm{E}-03$ | $3.06 \mathrm{E}-03$ | $2.39 \mathrm{E}-03$ | $2.32 \mathrm{E}-03$ | $2.77 \mathrm{E}-03$ | $2.19 \mathrm{E}-03$ |
| 47 | $3.90 \mathrm{E}-03$ | $5.38 \mathrm{E}-03$ | $2.08 \mathrm{E}-03$ | $3.29 \mathrm{E}-03$ | $2.57 \mathrm{E}-03$ | $2.50 \mathrm{E}-03$ | $2.99 \mathrm{E}-03$ | $2.36 \mathrm{E}-03$ |
| 48 | $4.20 \mathrm{E}-03$ | $5.79 \mathrm{E}-03$ | $2.24 \mathrm{E}-03$ | $3.54 \mathrm{E}-03$ | $2.77 \mathrm{E}-03$ | $2.69 \mathrm{E}-03$ | $3.22 \mathrm{E}-03$ | $2.54 \mathrm{E}-03$ |
| 49 | $4.52 \mathrm{E}-03$ | $6.24 \mathrm{E}-03$ | $2.42 \mathrm{E}-03$ | $3.82 \mathrm{E}-03$ | $2.98 \mathrm{E}-03$ | $2.90 \mathrm{E}-03$ | $3.46 \mathrm{E}-03$ | $2.73 \mathrm{E}-03$ |

Table 7.7) Estimated fishing mortality rate (1/year) at length at year for North Sea grey gurnard

| Length/year | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $9.38 \mathrm{E}-05$ | $1.32 \mathrm{E}-04$ | $9.21 \mathrm{E}-05$ | $6.26 \mathrm{E}-05$ | $6.15 \mathrm{E}-05$ | $3.79 \mathrm{E}-05$ | $4.72 \mathrm{E}-05$ | $4.41 \mathrm{E}-05$ |
| 4 | $1.01 \mathrm{E}-04$ | $1.42 \mathrm{E}-04$ | $9.93 \mathrm{E}-05$ | $6.75 \mathrm{E}-05$ | $6.63 \mathrm{E}-05$ | $4.09 \mathrm{E}-05$ | $5.09 \mathrm{E}-05$ | $4.75 \mathrm{E}-05$ |
| 5 | $1.09 \mathrm{E}-04$ | $1.54 \mathrm{E}-04$ | $1.07 \mathrm{E}-04$ | $7.28 \mathrm{E}-05$ | $7.15 \mathrm{E}-05$ | $4.41 \mathrm{E}-05$ | $5.49 \mathrm{E}-05$ | $5.12 \mathrm{E}-05$ |
| 6 | $1.17 \mathrm{E}-04$ | $1.66 \mathrm{E}-04$ | $1.15 \mathrm{E}-04$ | $7.85 \mathrm{E}-05$ | $7.71 \mathrm{E}-05$ | $4.75 \mathrm{E}-05$ | $5.92 \mathrm{E}-05$ | $5.52 \mathrm{E}-05$ |
| 7 | $1.27 \mathrm{E}-04$ | $1.79 \mathrm{E}-04$ | $1.24 \mathrm{E}-04$ | $8.46 \mathrm{E}-05$ | $8.31 \mathrm{E}-05$ | $5.12 \mathrm{E}-05$ | $6.38 \mathrm{E}-05$ | $5.96 \mathrm{E}-05$ |
| 8 | $1.37 \mathrm{E}-04$ | $1.92 \mathrm{E}-04$ | $1.34 \mathrm{E}-04$ | $9.12 \mathrm{E}-05$ | $8.96 \mathrm{E}-05$ | $5.52 \mathrm{E}-05$ | $6.88 \mathrm{E}-05$ | $6.42 \mathrm{E}-05$ |
| 9 | $1.47 \mathrm{E}-04$ | $2.07 \mathrm{E}-04$ | $1.45 \mathrm{E}-04$ | $9.83 \mathrm{E}-05$ | $9.66 \mathrm{E}-05$ | $5.95 \mathrm{E}-05$ | $7.41 \mathrm{E}-05$ | $6.92 \mathrm{E}-05$ |
| 10 | $1.59 \mathrm{E}-04$ | $2.24 \mathrm{E}-04$ | $1.56 \mathrm{E}-04$ | $1.06 \mathrm{E}-04$ | $1.04 \mathrm{E}-04$ | $6.42 \mathrm{E}-05$ | $7.99 \mathrm{E}-05$ | $7.46 \mathrm{E}-05$ |
| 11 | $1.71 \mathrm{E}-04$ | $2.41 \mathrm{E}-04$ | $1.68 \mathrm{E}-04$ | $1.14 \mathrm{E}-04$ | $1.12 \mathrm{E}-04$ | $6.92 \mathrm{E}-05$ | $8.62 \mathrm{E}-05$ | $8.05 \mathrm{E}-05$ |
| 12 | $1.85 \mathrm{E}-04$ | $2.60 \mathrm{E}-04$ | $1.81 \mathrm{E}-04$ | $1.23 \mathrm{E}-04$ | $1.21 \mathrm{E}-04$ | $7.46 \mathrm{E}-05$ | $9.29 \mathrm{E}-05$ | $8.68 \mathrm{E}-05$ |
| 13 | $1.99 \mathrm{E}-04$ | $2.80 \mathrm{E}-04$ | $1.95 \mathrm{E}-04$ | $1.33 \mathrm{E}-04$ | $1.31 \mathrm{E}-04$ | $8.04 \mathrm{E}-05$ | $1.00 \mathrm{E}-04$ | $9.35 \mathrm{E}-05$ |
| 14 | $2.14 \mathrm{E}-04$ | $3.02 \mathrm{E}-04$ | $2.11 \mathrm{E}-04$ | $1.43 \mathrm{E}-04$ | $1.41 \mathrm{E}-04$ | $8.67 \mathrm{E}-05$ | $1.08 \mathrm{E}-04$ | $1.01 \mathrm{E}-04$ |
| 15 | $2.31 \mathrm{E}-04$ | $3.26 \mathrm{E}-04$ | $2.27 \mathrm{E}-04$ | $1.54 \mathrm{E}-04$ | $1.52 \mathrm{E}-04$ | $9.35 \mathrm{E}-05$ | $1.16 \mathrm{E}-04$ | $1.09 \mathrm{E}-04$ |
| 16 | $2.49 \mathrm{E}-04$ | $3.51 \mathrm{E}-04$ | $2.45 \mathrm{E}-04$ | $1.66 \mathrm{E}-04$ | $1.64 \mathrm{E}-04$ | $1.01 \mathrm{E}-04$ | $1.25 \mathrm{E}-04$ | $1.17 \mathrm{E}-04$ |
| 17 | $2.69 \mathrm{E}-04$ | $3.79 \mathrm{E}-04$ | $2.64 \mathrm{E}-04$ | $1.79 \mathrm{E}-04$ | $1.76 \mathrm{E}-04$ | $1.09 \mathrm{E}-04$ | $1.35 \mathrm{E}-04$ | $1.26 \mathrm{E}-04$ |
| 18 | $2.90 \mathrm{E}-04$ | $4.08 \mathrm{E}-04$ | $2.85 \mathrm{E}-04$ | $1.93 \mathrm{E}-04$ | $1.90 \mathrm{E}-04$ | $1.17 \mathrm{E}-04$ | $1.46 \mathrm{E}-04$ | $1.36 \mathrm{E}-04$ |
| 19 | $3.12 \mathrm{E}-04$ | $4.40 \mathrm{E}-04$ | $3.07 \mathrm{E}-04$ | $2.09 \mathrm{E}-04$ | $2.05 \mathrm{E}-04$ | $1.26 \mathrm{E}-04$ | $1.57 \mathrm{E}-04$ | $1.47 \mathrm{E}-04$ |
| 20 | $3.37 \mathrm{E}-04$ | $4.74 \mathrm{E}-04$ | $3.31 \mathrm{E}-04$ | $2.25 \mathrm{E}-04$ | $2.21 \mathrm{E}-04$ | $1.36 \mathrm{E}-04$ | $1.69 \mathrm{E}-04$ | $1.58 \mathrm{E}-04$ |
| 21 | $3.63 \mathrm{E}-04$ | $5.11 \mathrm{E}-04$ | $3.57 \mathrm{E}-04$ | $2.42 \mathrm{E}-04$ | $2.38 \mathrm{E}-04$ | $1.47 \mathrm{E}-04$ | $1.83 \mathrm{E}-04$ | $1.71 \mathrm{E}-04$ |
| 22 | $3.91 \mathrm{E}-04$ | $5.51 \mathrm{E}-04$ | $3.84 \mathrm{E}-04$ | $2.61 \mathrm{E}-04$ | $2.57 \mathrm{E}-04$ | $1.58 \mathrm{E}-04$ | $1.97 \mathrm{E}-04$ | $1.84 \mathrm{E}-04$ |
| 23 | $4.22 \mathrm{E}-04$ | $5.94 \mathrm{E}-04$ | $4.14 \mathrm{E}-04$ | $2.82 \mathrm{E}-04$ | $2.77 \mathrm{E}-04$ | $1.71 \mathrm{E}-04$ | $2.12 \mathrm{E}-04$ | $1.98 \mathrm{E}-04$ |
| 24 | $4.55 \mathrm{E}-04$ | $6.41 \mathrm{E}-04$ | $4.47 \mathrm{E}-04$ | $3.04 \mathrm{E}-04$ | $2.98 \mathrm{E}-04$ | $1.84 \mathrm{E}-04$ | $2.29 \mathrm{E}-04$ | $2.14 \mathrm{E}-04$ |
| 25 | $4.90 \mathrm{E}-04$ | $6.90 \mathrm{E}-04$ | $4.81 \mathrm{E}-04$ | $3.27 \mathrm{E}-04$ | $3.22 \mathrm{E}-04$ | $1.98 \mathrm{E}-04$ | $2.47 \mathrm{E}-04$ | $2.30 \mathrm{E}-04$ |
| 26 | $5.28 \mathrm{E}-04$ | $7.44 \mathrm{E}-04$ | $5.19 \mathrm{E}-04$ | $3.53 \mathrm{E}-04$ | $3.47 \mathrm{E}-04$ | $2.14 \mathrm{E}-04$ | $2.66 \mathrm{E}-04$ | $2.48 \mathrm{E}-04$ |
| 27 | $5.69 \mathrm{E}-04$ | $8.02 \mathrm{E}-04$ | $5.59 \mathrm{E}-04$ | $3.80 \mathrm{E}-04$ | $3.74 \mathrm{E}-04$ | $2.30 \mathrm{E}-04$ | $2.87 \mathrm{E}-04$ | $2.68 \mathrm{E}-04$ |
| 28 | $6.14 \mathrm{E}-04$ | $8.65 \mathrm{E}-04$ | $6.03 \mathrm{E}-04$ | $4.10 \mathrm{E}-04$ | $4.03 \mathrm{E}-04$ | $2.48 \mathrm{E}-04$ | $3.09 \mathrm{E}-04$ | $2.89 \mathrm{E}-04$ |
| 29 | $6.61 \mathrm{E}-04$ | $9.32 \mathrm{E}-04$ | $6.50 \mathrm{E}-04$ | $4.42 \mathrm{E}-04$ | $4.34 \mathrm{E}-04$ | $2.67 \mathrm{E}-04$ | $3.33 \mathrm{E}-04$ | $3.11 \mathrm{E}-04$ |
| 30 | $7.13 \mathrm{E}-04$ | $1.00 \mathrm{E}-03$ | $7.00 \mathrm{E}-04$ | $4.76 \mathrm{E}-04$ | $4.68 \mathrm{E}-04$ | $2.88 \mathrm{E}-04$ | $3.59 \mathrm{E}-04$ | $3.35 \mathrm{E}-04$ |
| 31 | $7.68 \mathrm{E}-04$ | $1.08 \mathrm{E}-03$ | $7.55 \mathrm{E}-04$ | $5.13 \mathrm{E}-04$ | $5.04 \mathrm{E}-04$ | $3.11 \mathrm{E}-04$ | $3.87 \mathrm{E}-04$ | $3.61 \mathrm{E}-04$ |
| 32 | 8.28E-04 | $1.17 \mathrm{E}-03$ | $8.14 \mathrm{E}-04$ | $5.53 \mathrm{E}-04$ | $5.43 \mathrm{E}-04$ | $3.35 \mathrm{E}-04$ | $4.17 \mathrm{E}-04$ | $3.89 \mathrm{E}-04$ |
| 33 | $8.92 \mathrm{E}-04$ | $1.26 \mathrm{E}-03$ | $8.77 \mathrm{E}-04$ | $5.96 \mathrm{E}-04$ | $5.86 \mathrm{E}-04$ | $3.61 \mathrm{E}-04$ | $4.49 \mathrm{E}-04$ | $4.20 \mathrm{E}-04$ |
| 34 | $9.62 \mathrm{E}-04$ | $1.36 \mathrm{E}-03$ | $9.45 \mathrm{E}-04$ | $6.42 \mathrm{E}-04$ | $6.31 \mathrm{E}-04$ | $3.89 \mathrm{E}-04$ | $4.84 \mathrm{E}-04$ | $4.52 \mathrm{E}-04$ |
| 35 | $1.04 \mathrm{E}-03$ | $1.46 \mathrm{E}-03$ | $1.02 \mathrm{E}-03$ | $6.92 \mathrm{E}-04$ | $6.80 \mathrm{E}-04$ | $4.19 \mathrm{E}-04$ | $5.22 \mathrm{E}-04$ | $4.87 \mathrm{E}-04$ |
| 36 | $1.12 \mathrm{E}-03$ | $1.57 \mathrm{E}-03$ | $1.10 \mathrm{E}-03$ | $7.46 \mathrm{E}-04$ | $7.33 \mathrm{E}-04$ | $4.52 \mathrm{E}-04$ | $5.62 \mathrm{E}-04$ | $5.25 \mathrm{E}-04$ |
| 37 | $1.20 \mathrm{E}-03$ | $1.70 \mathrm{E}-03$ | $1.18 \mathrm{E}-03$ | $8.03 \mathrm{E}-04$ | $7.90 \mathrm{E}-04$ | $4.87 \mathrm{E}-04$ | $6.06 \mathrm{E}-04$ | $5.66 \mathrm{E}-04$ |
| 38 | $1.30 \mathrm{E}-03$ | $1.83 \mathrm{E}-03$ | $1.27 \mathrm{E}-03$ | $8.66 \mathrm{E}-04$ | $8.51 \mathrm{E}-04$ | $5.24 \mathrm{E}-04$ | $6.53 \mathrm{E}-04$ | $6.10 \mathrm{E}-04$ |
| 39 | $1.40 \mathrm{E}-03$ | $1.97 \mathrm{E}-03$ | $1.37 \mathrm{E}-03$ | $9.33 \mathrm{E}-04$ | $9.17 \mathrm{E}-04$ | $5.65 \mathrm{E}-04$ | $7.03 \mathrm{E}-04$ | $6.57 \mathrm{E}-04$ |
| 40 | $1.50 \mathrm{E}-03$ | $2.12 \mathrm{E}-03$ | $1.48 \mathrm{E}-03$ | $1.00 \mathrm{E}-03$ | $9.87 \mathrm{E}-04$ | $6.09 \mathrm{E}-04$ | $7.58 \mathrm{E}-04$ | $7.08 \mathrm{E}-04$ |
| 41 | $1.62 \mathrm{E}-03$ | $2.28 \mathrm{E}-03$ | $1.59 \mathrm{E}-03$ | $1.08 \mathrm{E}-03$ | $1.06 \mathrm{E}-03$ | $6.56 \mathrm{E}-04$ | $8.16 \mathrm{E}-04$ | $7.62 \mathrm{E}-04$ |
| 42 | $1.75 \mathrm{E}-03$ | $2.46 \mathrm{E}-03$ | $1.72 \mathrm{E}-03$ | $1.17 \mathrm{E}-03$ | $1.15 \mathrm{E}-03$ | $7.06 \mathrm{E}-04$ | $8.79 \mathrm{E}-04$ | $8.21 \mathrm{E}-04$ |
| 43 | $1.88 \mathrm{E}-03$ | $2.65 \mathrm{E}-03$ | $1.85 \mathrm{E}-03$ | $1.26 \mathrm{E}-03$ | $1.23 \mathrm{E}-03$ | $7.61 \mathrm{E}-04$ | $9.47 \mathrm{E}-04$ | $8.84 \mathrm{E}-04$ |
| 44 | $2.03 \mathrm{E}-03$ | $2.85 \mathrm{E}-03$ | $1.99 \mathrm{E}-03$ | $1.35 \mathrm{E}-03$ | $1.33 \mathrm{E}-03$ | 8.19E-04 | $1.02 \mathrm{E}-03$ | $9.53 \mathrm{E}-04$ |
| 45 | $2.18 \mathrm{E}-03$ | $3.07 \mathrm{E}-03$ | $2.14 \mathrm{E}-03$ | $1.46 \mathrm{E}-03$ | $1.43 \mathrm{E}-03$ | $8.82 \mathrm{E}-04$ | $1.10 \mathrm{E}-03$ | $1.03 \mathrm{E}-03$ |
| 46 | $2.35 \mathrm{E}-03$ | $3.31 \mathrm{E}-03$ | $2.31 \mathrm{E}-03$ | $1.57 \mathrm{E}-03$ | $1.54 \mathrm{E}-03$ | $9.50 \mathrm{E}-04$ | $1.18 \mathrm{E}-03$ | $1.10 \mathrm{E}-03$ |
| 47 | $2.53 \mathrm{E}-03$ | $3.56 \mathrm{E}-03$ | $2.49 \mathrm{E}-03$ | $1.69 \mathrm{E}-03$ | $1.66 \mathrm{E}-03$ | $1.02 \mathrm{E}-03$ | $1.27 \mathrm{E}-03$ | $1.19 \mathrm{E}-03$ |
| 48 | $2.72 \mathrm{E}-03$ | $3.84 \mathrm{E}-03$ | $2.68 \mathrm{E}-03$ | $1.82 \mathrm{E}-03$ | $1.79 \mathrm{E}-03$ | $1.10 \mathrm{E}-03$ | $1.37 \mathrm{E}-03$ | $1.28 \mathrm{E}-03$ |
| 49 | $2.93 \mathrm{E}-03$ | $4.13 \mathrm{E}-03$ | $2.88 \mathrm{E}-03$ | $1.96 \mathrm{E}-03$ | $1.92 \mathrm{E}-03$ | $1.19 \mathrm{E}-03$ | $1.48 \mathrm{E}-03$ | $1.38 \mathrm{E}-03$ |

(Continued Table 7.7) Estimated fishing mortality rate (1/year) at length at year for North Sea grey gurnard

| Length/year | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $4.24 \mathrm{E}-05$ | $4.55 \mathrm{E}-05$ | $3.98 \mathrm{E}-05$ | $3.75 \mathrm{E}-05$ | $3.96 \mathrm{E}-05$ | $3.08 \mathrm{E}-05$ | $2.17 \mathrm{E}-05$ | $3.04 \mathrm{E}-05$ |
| 4 | $4.57 \mathrm{E}-05$ | $4.91 \mathrm{E}-05$ | $4.29 \mathrm{E}-05$ | $4.04 \mathrm{E}-05$ | $4.27 \mathrm{E}-05$ | $3.32 \mathrm{E}-05$ | $2.34 \mathrm{E}-05$ | $3.28 \mathrm{E}-05$ |
| 5 | $4.93 \mathrm{E}-05$ | $5.29 \mathrm{E}-05$ | $4.63 \mathrm{E}-05$ | $4.36 \mathrm{E}-05$ | $4.60 \mathrm{E}-05$ | $3.58 \mathrm{E}-05$ | $2.52 \mathrm{E}-05$ | $3.54 \mathrm{E}-05$ |
| 6 | $5.32 \mathrm{E}-05$ | $5.71 \mathrm{E}-05$ | $4.99 \mathrm{E}-05$ | $4.70 \mathrm{E}-05$ | $4.96 \mathrm{E}-05$ | $3.85 \mathrm{E}-05$ | $2.72 \mathrm{E}-05$ | $3.81 \mathrm{E}-05$ |
| 7 | $5.73 \mathrm{E}-05$ | $6.15 \mathrm{E}-05$ | $5.38 \mathrm{E}-05$ | $5.06 \mathrm{E}-05$ | $5.35 \mathrm{E}-05$ | $4.16 \mathrm{E}-05$ | $2.93 \mathrm{E}-05$ | $4.11 \mathrm{E}-05$ |
| 8 | $6.18 \mathrm{E}-05$ | $6.63 \mathrm{E}-05$ | $5.80 \mathrm{E}-05$ | $5.46 \mathrm{E}-05$ | $5.77 \mathrm{E}-05$ | $4.48 \mathrm{E}-05$ | $3.16 \mathrm{E}-05$ | $4.43 \mathrm{E}-05$ |
| 9 | $6.66 \mathrm{E}-05$ | $7.15 \mathrm{E}-05$ | $6.25 \mathrm{E}-05$ | $5.89 \mathrm{E}-05$ | $6.22 \mathrm{E}-05$ | $4.83 \mathrm{E}-05$ | $3.41 \mathrm{E}-05$ | $4.78 \mathrm{E}-05$ |
| 10 | $7.18 \mathrm{E}-05$ | $7.71 \mathrm{E}-05$ | $6.74 \mathrm{E}-05$ | $6.35 \mathrm{E}-05$ | $6.70 \mathrm{E}-05$ | $5.21 \mathrm{E}-05$ | $3.68 \mathrm{E}-05$ | $5.15 \mathrm{E}-05$ |
| 11 | $7.75 \mathrm{E}-05$ | $8.31 \mathrm{E}-05$ | $7.26 \mathrm{E}-05$ | $6.84 \mathrm{E}-05$ | $7.23 \mathrm{E}-05$ | $5.61 \mathrm{E}-05$ | $3.96 \mathrm{E}-05$ | $5.56 \mathrm{E}-05$ |
| 12 | $8.35 \mathrm{E}-05$ | $8.96 \mathrm{E}-05$ | $7.83 \mathrm{E}-05$ | $7.38 \mathrm{E}-05$ | $7.79 \mathrm{E}-05$ | $6.05 \mathrm{E}-05$ | $4.27 \mathrm{E}-05$ | $5.99 \mathrm{E}-05$ |
| 13 | $9.00 \mathrm{E}-05$ | $9.66 \mathrm{E}-05$ | $8.44 \mathrm{E}-05$ | $7.95 \mathrm{E}-05$ | $8.40 \mathrm{E}-05$ | $6.53 \mathrm{E}-05$ | $4.61 \mathrm{E}-05$ | $6.46 \mathrm{E}-05$ |
| 14 | $9.71 \mathrm{E}-05$ | $1.04 \mathrm{E}-04$ | $9.10 \mathrm{E}-05$ | $8.57 \mathrm{E}-05$ | $9.06 \mathrm{E}-05$ | $7.04 \mathrm{E}-05$ | $4.97 \mathrm{E}-05$ | $6.96 \mathrm{E}-05$ |
| 15 | $1.05 \mathrm{E}-04$ | $1.12 \mathrm{E}-04$ | $9.81 \mathrm{E}-05$ | $9.24 \mathrm{E}-05$ | $9.76 \mathrm{E}-05$ | $7.58 \mathrm{E}-05$ | $5.35 \mathrm{E}-05$ | $7.51 \mathrm{E}-05$ |
| 16 | $1.13 \mathrm{E}-04$ | $1.21 \mathrm{E}-04$ | $1.06 \mathrm{E}-04$ | $9.96 \mathrm{E}-05$ | $1.05 \mathrm{E}-04$ | $8.18 \mathrm{E}-05$ | $5.77 \mathrm{E}-05$ | $8.09 \mathrm{E}-05$ |
| 17 | $1.22 \mathrm{E}-04$ | $1.31 \mathrm{E}-04$ | $1.14 \mathrm{E}-04$ | $1.07 \mathrm{E}-04$ | $1.13 \mathrm{E}-04$ | $8.82 \mathrm{E}-05$ | $6.22 \mathrm{E}-05$ | $8.72 \mathrm{E}-05$ |
| 18 | $1.31 \mathrm{E}-04$ | $1.41 \mathrm{E}-04$ | $1.23 \mathrm{E}-04$ | $1.16 \mathrm{E}-04$ | $1.22 \mathrm{E}-04$ | $9.50 \mathrm{E}-05$ | $6.71 \mathrm{E}-05$ | $9.41 \mathrm{E}-05$ |
| 19 | $1.41 \mathrm{E}-04$ | $1.52 \mathrm{E}-04$ | $1.33 \mathrm{E}-04$ | $1.25 \mathrm{E}-04$ | $1.32 \mathrm{E}-04$ | $1.02 \mathrm{E}-04$ | $7.23 \mathrm{E}-05$ | $1.01 \mathrm{E}-04$ |
| 20 | $1.52 \mathrm{E}-04$ | $1.64 \mathrm{E}-04$ | $1.43 \mathrm{E}-04$ | $1.35 \mathrm{E}-04$ | $1.42 \mathrm{E}-04$ | $1.10 \mathrm{E}-04$ | $7.80 \mathrm{E}-05$ | $1.09 \mathrm{E}-04$ |
| 21 | $1.64 \mathrm{E}-04$ | $1.76 \mathrm{E}-04$ | $1.54 \mathrm{E}-04$ | $1.45 \mathrm{E}-04$ | $1.53 \mathrm{E}-04$ | $1.19 \mathrm{E}-04$ | $8.40 \mathrm{E}-05$ | $1.18 \mathrm{E}-04$ |
| 22 | $1.77 \mathrm{E}-04$ | $1.90 \mathrm{E}-04$ | $1.66 \mathrm{E}-04$ | $1.56 \mathrm{E}-04$ | $1.65 \mathrm{E}-04$ | $1.28 \mathrm{E}-04$ | $9.06 \mathrm{E}-05$ | $1.27 \mathrm{E}-04$ |
| 23 | $1.91 \mathrm{E}-04$ | $2.05 \mathrm{E}-04$ | $1.79 \mathrm{E}-04$ | $1.69 \mathrm{E}-04$ | $1.78 \mathrm{E}-04$ | $1.38 \mathrm{E}-04$ | $9.76 \mathrm{E}-05$ | $1.37 \mathrm{E}-04$ |
| 24 | $2.06 \mathrm{E}-04$ | $2.21 \mathrm{E}-04$ | $1.93 \mathrm{E}-04$ | $1.82 \mathrm{E}-04$ | $1.92 \mathrm{E}-04$ | $1.49 \mathrm{E}-04$ | $1.05 \mathrm{E}-04$ | $1.48 \mathrm{E}-04$ |
| 25 | $2.22 \mathrm{E}-04$ | $2.38 \mathrm{E}-04$ | $2.08 \mathrm{E}-04$ | $1.96 \mathrm{E}-04$ | $2.07 \mathrm{E}-04$ | $1.61 \mathrm{E}-04$ | $1.13 \mathrm{E}-04$ | $1.59 \mathrm{E}-04$ |
| 26 | $2.39 \mathrm{E}-04$ | $2.57 \mathrm{E}-04$ | $2.24 \mathrm{E}-04$ | $2.11 \mathrm{E}-04$ | $2.23 \mathrm{E}-04$ | $1.73 \mathrm{E}-04$ | $1.22 \mathrm{E}-04$ | $1.71 \mathrm{E}-04$ |
| 27 | $2.58 \mathrm{E}-04$ | $2.77 \mathrm{E}-04$ | $2.42 \mathrm{E}-04$ | $2.28 \mathrm{E}-04$ | $2.40 \mathrm{E}-04$ | $1.87 \mathrm{E}-04$ | $1.32 \mathrm{E}-04$ | $1.85 \mathrm{E}-04$ |
| 28 | $2.78 \mathrm{E}-04$ | $2.98 \mathrm{E}-04$ | $2.60 \mathrm{E}-04$ | $2.45 \mathrm{E}-04$ | $2.59 \mathrm{E}-04$ | $2.01 \mathrm{E}-04$ | $1.42 \mathrm{E}-04$ | $1.99 \mathrm{E}-04$ |
| 29 | $2.99 \mathrm{E}-04$ | $3.21 \mathrm{E}-04$ | $2.81 \mathrm{E}-04$ | $2.64 \mathrm{E}-04$ | $2.79 \mathrm{E}-04$ | $2.17 \mathrm{E}-04$ | $1.53 \mathrm{E}-04$ | $2.15 \mathrm{E}-04$ |
| 30 | $3.23 \mathrm{E}-04$ | $3.46 \mathrm{E}-04$ | $3.03 \mathrm{E}-04$ | $2.85 \mathrm{E}-04$ | $3.01 \mathrm{E}-04$ | $2.34 \mathrm{E}-04$ | $1.65 \mathrm{E}-04$ | $2.31 \mathrm{E}-04$ |
| 31 | $3.48 \mathrm{E}-04$ | $3.73 \mathrm{E}-04$ | $3.26 \mathrm{E}-04$ | $3.07 \mathrm{E}-04$ | $3.24 \mathrm{E}-04$ | $2.52 \mathrm{E}-04$ | $1.78 \mathrm{E}-04$ | $2.49 \mathrm{E}-04$ |
| 32 | $3.75 \mathrm{E}-04$ | $4.02 \mathrm{E}-04$ | $3.51 \mathrm{E}-04$ | $3.31 \mathrm{E}-04$ | $3.50 \mathrm{E}-04$ | $2.72 \mathrm{E}-04$ | $1.92 \mathrm{E}-04$ | $2.69 \mathrm{E}-04$ |
| 33 | $4.04 \mathrm{E}-04$ | $4.33 \mathrm{E}-04$ | $3.79 \mathrm{E}-04$ | $3.57 \mathrm{E}-04$ | $3.77 \mathrm{E}-04$ | $2.93 \mathrm{E}-04$ | $2.07 \mathrm{E}-04$ | $2.90 \mathrm{E}-04$ |
| 34 | $4.35 \mathrm{E}-04$ | $4.67 \mathrm{E}-04$ | $4.08 \mathrm{E}-04$ | $3.84 \mathrm{E}-04$ | $4.06 \mathrm{E}-04$ | $3.15 \mathrm{E}-04$ | $2.23 \mathrm{E}-04$ | $3.12 \mathrm{E}-04$ |
| 35 | $4.69 \mathrm{E}-04$ | $5.03 \mathrm{E}-04$ | $4.40 \mathrm{E}-04$ | $4.14 \mathrm{E}-04$ | $4.38 \mathrm{E}-04$ | $3.40 \mathrm{E}-04$ | $2.40 \mathrm{E}-04$ | $3.36 \mathrm{E}-04$ |
| 36 | $5.05 \mathrm{E}-04$ | $5.42 \mathrm{E}-04$ | $4.74 \mathrm{E}-04$ | $4.46 \mathrm{E}-04$ | $4.72 \mathrm{E}-04$ | $3.66 \mathrm{E}-04$ | $2.59 \mathrm{E}-04$ | $3.63 \mathrm{E}-04$ |
| 37 | $5.45 \mathrm{E}-04$ | $5.84 \mathrm{E}-04$ | $5.11 \mathrm{E}-04$ | $4.81 \mathrm{E}-04$ | $5.08 \mathrm{E}-04$ | $3.95 \mathrm{E}-04$ | $2.79 \mathrm{E}-04$ | $3.91 \mathrm{E}-04$ |
| 38 | $5.87 \mathrm{E}-04$ | $6.30 \mathrm{E}-04$ | $5.50 \mathrm{E}-04$ | $5.18 \mathrm{E}-04$ | $5.47 \mathrm{E}-04$ | $4.25 \mathrm{E}-04$ | $3.00 \mathrm{E}-04$ | $4.21 \mathrm{E}-04$ |
| 39 | $6.32 \mathrm{E}-04$ | $6.78 \mathrm{E}-04$ | $5.93 \mathrm{E}-04$ | $5.58 \mathrm{E}-04$ | $5.90 \mathrm{E}-04$ | $4.58 \mathrm{E}-04$ | $3.23 \mathrm{E}-04$ | $4.53 \mathrm{E}-04$ |
| 40 | $6.81 \mathrm{E}-04$ | $7.31 \mathrm{E}-04$ | $6.39 \mathrm{E}-04$ | $6.01 \mathrm{E}-04$ | $6.35 \mathrm{E}-04$ | $4.94 \mathrm{E}-04$ | $3.48 \mathrm{E}-04$ | $4.89 \mathrm{E}-04$ |
| 41 | $7.34 \mathrm{E}-04$ | $7.87 \mathrm{E}-04$ | $6.88 \mathrm{E}-04$ | $6.48 \mathrm{E}-04$ | $6.85 \mathrm{E}-04$ | $5.32 \mathrm{E}-04$ | $3.75 \mathrm{E}-04$ | $5.26 \mathrm{E}-04$ |
| 42 | $7.90 \mathrm{E}-04$ | $8.48 \mathrm{E}-04$ | $7.41 \mathrm{E}-04$ | $6.98 \mathrm{E}-04$ | $7.37 \mathrm{E}-04$ | $5.73 \mathrm{E}-04$ | $4.04 \mathrm{E}-04$ | $5.67 \mathrm{E}-04$ |
| 43 | $8.51 \mathrm{E}-04$ | $9.14 \mathrm{E}-04$ | $7.98 \mathrm{E}-04$ | $7.52 \mathrm{E}-04$ | $7.94 \mathrm{E}-04$ | $6.17 \mathrm{E}-04$ | $4.36 \mathrm{E}-04$ | $6.11 \mathrm{E}-04$ |
| 44 | $9.17 \mathrm{E}-04$ | $9.84 \mathrm{E}-04$ | $8.60 \mathrm{E}-04$ | $8.10 \mathrm{E}-04$ | $8.55 \mathrm{E}-04$ | $6.65 \mathrm{E}-04$ | $4.69 \mathrm{E}-04$ | $6.58 \mathrm{E}-04$ |
| 45 | $9.87 \mathrm{E}-04$ | $1.06 \mathrm{E}-03$ | $9.26 \mathrm{E}-04$ | $8.72 \mathrm{E}-04$ | $9.21 \mathrm{E}-04$ | $7.16 \mathrm{E}-04$ | $5.05 \mathrm{E}-04$ | $7.08 \mathrm{E}-04$ |
| 46 | $1.06 \mathrm{E}-03$ | $1.14 \mathrm{E}-03$ | $9.97 \mathrm{E}-04$ | $9.39 \mathrm{E}-04$ | $9.92 \mathrm{E}-04$ | $7.71 \mathrm{E}-04$ | $5.44 \mathrm{E}-04$ | $7.63 \mathrm{E}-04$ |
| 47 | $1.14 \mathrm{E}-03$ | $1.23 \mathrm{E}-03$ | $1.07 \mathrm{E}-03$ | $1.01 \mathrm{E}-03$ | $1.07 \mathrm{E}-03$ | $8.30 \mathrm{E}-04$ | $5.86 \mathrm{E}-04$ | $8.21 \mathrm{E}-04$ |
| 48 | $1.23 \mathrm{E}-03$ | $1.32 \mathrm{E}-03$ | $1.16 \mathrm{E}-03$ | $1.09 \mathrm{E}-03$ | $1.15 \mathrm{E}-03$ | $8.93 \mathrm{E}-04$ | $6.31 \mathrm{E}-04$ | $8.84 \mathrm{E}-04$ |
| 49 | $1.33 \mathrm{E}-03$ | $1.42 \mathrm{E}-03$ | $1.24 \mathrm{E}-03$ | $1.17 \mathrm{E}-03$ | $1.24 \mathrm{E}-03$ | $9.62 \mathrm{E}-04$ | $6.79 \mathrm{E}-04$ | $9.52 \mathrm{E}-04$ |

(Continued Table 7.7) Estimated fishing mortality rate (1/year) at length at year for North Sea grey gurnard

| Length/year | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $2.47 \mathrm{E}-05$ | $2.94 \mathrm{E}-05$ | $2.78 \mathrm{E}-05$ | $2.28 \mathrm{E}-05$ | $2.30 \mathrm{E}-05$ | $2.00 \mathrm{E}-05$ | $2.26 \mathrm{E}-05$ | $2.13 \mathrm{E}-05$ |
| 4 | $2.67 \mathrm{E}-05$ | $3.17 \mathrm{E}-05$ | $3.00 \mathrm{E}-05$ | $2.45 \mathrm{E}-05$ | $2.48 \mathrm{E}-05$ | $2.15 \mathrm{E}-05$ | $2.43 \mathrm{E}-05$ | $2.30 \mathrm{E}-05$ |
| 5 | $2.87 \mathrm{E}-05$ | $3.42 \mathrm{E}-05$ | $3.23 \mathrm{E}-05$ | $2.65 \mathrm{E}-05$ | $2.67 \mathrm{E}-05$ | $2.32 \mathrm{E}-05$ | $2.62 \mathrm{E}-05$ | $2.48 \mathrm{E}-05$ |
| 6 | $3.10 \mathrm{E}-05$ | $3.68 \mathrm{E}-05$ | $3.48 \mathrm{E}-05$ | $2.85 \mathrm{E}-05$ | $2.88 \mathrm{E}-05$ | $2.50 \mathrm{E}-05$ | $2.83 \mathrm{E}-05$ | $2.68 \mathrm{E}-05$ |
| 7 | $3.34 \mathrm{E}-05$ | $3.97 \mathrm{E}-05$ | $3.76 \mathrm{E}-05$ | $3.07 \mathrm{E}-05$ | $3.10 \mathrm{E}-05$ | $2.70 \mathrm{E}-05$ | $3.05 \mathrm{E}-05$ | $2.88 \mathrm{E}-05$ |
| 8 | $3.60 \mathrm{E}-05$ | $4.28 \mathrm{E}-05$ | $4.05 \mathrm{E}-05$ | $3.32 \mathrm{E}-05$ | $3.35 \mathrm{E}-05$ | $2.91 \mathrm{E}-05$ | $3.29 \mathrm{E}-05$ | $3.11 \mathrm{E}-05$ |
| 9 | $3.88 \mathrm{E}-05$ | $4.62 \mathrm{E}-05$ | $4.37 \mathrm{E}-05$ | $3.57 \mathrm{E}-05$ | $3.61 \mathrm{E}-05$ | $3.13 \mathrm{E}-05$ | $3.54 \mathrm{E}-05$ | $3.35 \mathrm{E}-05$ |
| 10 | $4.19 \mathrm{E}-05$ | $4.98 \mathrm{E}-05$ | $4.71 \mathrm{E}-05$ | $3.85 \mathrm{E}-05$ | $3.89 \mathrm{E}-05$ | $3.38 \mathrm{E}-05$ | $3.82 \mathrm{E}-05$ | $3.62 \mathrm{E}-05$ |
| 11 | $4.51 \mathrm{E}-05$ | $5.37 \mathrm{E}-05$ | $5.07 \mathrm{E}-05$ | $4.15 \mathrm{E}-05$ | $4.19 \mathrm{E}-05$ | $3.64 \mathrm{E}-05$ | $4.12 \mathrm{E}-05$ | $3.90 \mathrm{E}-05$ |
| 12 | $4.87 \mathrm{E}-05$ | $5.79 \mathrm{E}-05$ | $5.47 \mathrm{E}-05$ | $4.48 \mathrm{E}-05$ | $4.52 \mathrm{E}-05$ | $3.93 \mathrm{E}-05$ | $4.44 \mathrm{E}-05$ | $4.20 \mathrm{E}-05$ |
| 13 | $5.25 \mathrm{E}-05$ | $6.24 \mathrm{E}-05$ | $5.90 \mathrm{E}-05$ | $4.83 \mathrm{E}-05$ | $4.87 \mathrm{E}-05$ | $4.23 \mathrm{E}-05$ | $4.79 \mathrm{E}-05$ | $4.53 \mathrm{E}-05$ |
| 14 | $5.66 \mathrm{E}-05$ | $6.72 \mathrm{E}-05$ | $6.36 \mathrm{E}-05$ | $5.21 \mathrm{E}-05$ | $5.25 \mathrm{E}-05$ | $4.56 \mathrm{E}-05$ | 5.16E-05 | $4.88 \mathrm{E}-05$ |
| 15 | $6.10 \mathrm{E}-05$ | $7.25 \mathrm{E}-05$ | $6.86 \mathrm{E}-05$ | $5.61 \mathrm{E}-05$ | $5.66 \mathrm{E}-05$ | $4.92 \mathrm{E}-05$ | $5.56 \mathrm{E}-05$ | $5.27 \mathrm{E}-05$ |
| 16 | $6.57 \mathrm{E}-05$ | $7.82 \mathrm{E}-05$ | $7.39 \mathrm{E}-05$ | $6.05 \mathrm{E}-05$ | $6.11 \mathrm{E}-05$ | $5.30 \mathrm{E}-05$ | $6.00 \mathrm{E}-05$ | $5.68 \mathrm{E}-05$ |
| 17 | $7.09 \mathrm{E}-05$ | $8.43 \mathrm{E}-05$ | $7.97 \mathrm{E}-05$ | $6.52 \mathrm{E}-05$ | $6.58 \mathrm{E}-05$ | $5.72 \mathrm{E}-05$ | $6.47 \mathrm{E}-05$ | $6.12 \mathrm{E}-05$ |
| 18 | $7.64 \mathrm{E}-05$ | $9.08 \mathrm{E}-05$ | $8.59 \mathrm{E}-05$ | $7.03 \mathrm{E}-05$ | $7.10 \mathrm{E}-05$ | $6.16 \mathrm{E}-05$ | $6.97 \mathrm{E}-05$ | $6.60 \mathrm{E}-05$ |
| 19 | $8.24 \mathrm{E}-05$ | $9.79 \mathrm{E}-05$ | $9.26 \mathrm{E}-05$ | $7.58 \mathrm{E}-05$ | $7.65 \mathrm{E}-05$ | $6.65 \mathrm{E}-05$ | 7.51E-05 | $7.11 \mathrm{E}-05$ |
| 20 | $8.88 \mathrm{E}-05$ | $1.06 \mathrm{E}-04$ | $9.98 \mathrm{E}-05$ | $8.17 \mathrm{E}-05$ | $8.25 \mathrm{E}-05$ | $7.16 \mathrm{E}-05$ | $8.10 \mathrm{E}-05$ | $7.67 \mathrm{E}-05$ |
| 21 | $9.57 \mathrm{E}-05$ | $1.14 \mathrm{E}-04$ | $1.08 \mathrm{E}-04$ | 8.81E-05 | $8.89 \mathrm{E}-05$ | $7.72 \mathrm{E}-05$ | $8.73 \mathrm{E}-05$ | $8.26 \mathrm{E}-05$ |
| 22 | $1.03 \mathrm{E}-04$ | $1.23 \mathrm{E}-04$ | $1.16 \mathrm{E}-04$ | $9.50 \mathrm{E}-05$ | $9.58 \mathrm{E}-05$ | $8.32 \mathrm{E}-05$ | $9.41 \mathrm{E}-05$ | $8.91 \mathrm{E}-05$ |
| 23 | $1.11 \mathrm{E}-04$ | $1.32 \mathrm{E}-04$ | $1.25 \mathrm{E}-04$ | $1.02 \mathrm{E}-04$ | $1.03 \mathrm{E}-04$ | $8.97 \mathrm{E}-05$ | $1.01 \mathrm{E}-04$ | $9.60 \mathrm{E}-05$ |
| 24 | $1.20 \mathrm{E}-04$ | $1.43 \mathrm{E}-04$ | $1.35 \mathrm{E}-04$ | $1.10 \mathrm{E}-04$ | $1.11 \mathrm{E}-04$ | $9.67 \mathrm{E}-05$ | $1.09 \mathrm{E}-04$ | $1.04 \mathrm{E}-04$ |
| 25 | $1.29 \mathrm{E}-04$ | $1.54 \mathrm{E}-04$ | $1.45 \mathrm{E}-04$ | $1.19 \mathrm{E}-04$ | $1.20 \mathrm{E}-04$ | $1.04 \mathrm{E}-04$ | $1.18 \mathrm{E}-04$ | $1.12 \mathrm{E}-04$ |
| 26 | $1.39 \mathrm{E}-04$ | $1.66 \mathrm{E}-04$ | $1.57 \mathrm{E}-04$ | $1.28 \mathrm{E}-04$ | $1.29 \mathrm{E}-04$ | $1.12 \mathrm{E}-04$ | $1.27 \mathrm{E}-04$ | $1.20 \mathrm{E}-04$ |
| 27 | $1.50 \mathrm{E}-04$ | $1.79 \mathrm{E}-04$ | $1.69 \mathrm{E}-04$ | $1.38 \mathrm{E}-04$ | $1.39 \mathrm{E}-04$ | $1.21 \mathrm{E}-04$ | $1.37 \mathrm{E}-04$ | $1.30 \mathrm{E}-04$ |
| 28 | $1.62 \mathrm{E}-04$ | $1.92 \mathrm{E}-04$ | $1.82 \mathrm{E}-04$ | $1.49 \mathrm{E}-04$ | $1.50 \mathrm{E}-04$ | $1.31 \mathrm{E}-04$ | $1.48 \mathrm{E}-04$ | $1.40 \mathrm{E}-04$ |
| 29 | $1.74 \mathrm{E}-04$ | $2.07 \mathrm{E}-04$ | $1.96 \mathrm{E}-04$ | $1.61 \mathrm{E}-04$ | $1.62 \mathrm{E}-04$ | $1.41 \mathrm{E}-04$ | $1.59 \mathrm{E}-04$ | $1.51 \mathrm{E}-04$ |
| 30 | $1.88 \mathrm{E}-04$ | $2.24 \mathrm{E}-04$ | $2.11 \mathrm{E}-04$ | $1.73 \mathrm{E}-04$ | $1.75 \mathrm{E}-04$ | $1.52 \mathrm{E}-04$ | $1.72 \mathrm{E}-04$ | $1.62 \mathrm{E}-04$ |
| 31 | $2.03 \mathrm{E}-04$ | $2.41 \mathrm{E}-04$ | $2.28 \mathrm{E}-04$ | $1.86 \mathrm{E}-04$ | $1.88 \mathrm{E}-04$ | $1.63 \mathrm{E}-04$ | $1.85 \mathrm{E}-04$ | $1.75 \mathrm{E}-04$ |
| 32 | $2.18 \mathrm{E}-04$ | $2.60 \mathrm{E}-04$ | $2.45 \mathrm{E}-04$ | $2.01 \mathrm{E}-04$ | $2.03 \mathrm{E}-04$ | $1.76 \mathrm{E}-04$ | $1.99 \mathrm{E}-04$ | $1.89 \mathrm{E}-04$ |
| 33 | $2.35 \mathrm{E}-04$ | $2.80 \mathrm{E}-04$ | $2.65 \mathrm{E}-04$ | $2.17 \mathrm{E}-04$ | $2.19 \mathrm{E}-04$ | $1.90 \mathrm{E}-04$ | $2.15 \mathrm{E}-04$ | $2.03 \mathrm{E}-04$ |
| 34 | $2.54 \mathrm{E}-04$ | $3.02 \mathrm{E}-04$ | $2.85 \mathrm{E}-04$ | $2.33 \mathrm{E}-04$ | $2.36 \mathrm{E}-04$ | $2.05 \mathrm{E}-04$ | $2.31 \mathrm{E}-04$ | $2.19 \mathrm{E}-04$ |
| 35 | $2.73 \mathrm{E}-04$ | $3.25 \mathrm{E}-04$ | $3.07 \mathrm{E}-04$ | $2.52 \mathrm{E}-04$ | $2.54 \mathrm{E}-04$ | $2.21 \mathrm{E}-04$ | $2.49 \mathrm{E}-04$ | $2.36 \mathrm{E}-04$ |
| 36 | $2.95 \mathrm{E}-04$ | $3.50 \mathrm{E}-04$ | $3.31 \mathrm{E}-04$ | $2.71 \mathrm{E}-04$ | $2.74 \mathrm{E}-04$ | $2.38 \mathrm{E}-04$ | $2.69 \mathrm{E}-04$ | $2.54 \mathrm{E}-04$ |
| 37 | $3.17 \mathrm{E}-04$ | $3.77 \mathrm{E}-04$ | $3.57 \mathrm{E}-04$ | $2.92 \mathrm{E}-04$ | $2.95 \mathrm{E}-04$ | $2.56 \mathrm{E}-04$ | $2.90 \mathrm{E}-04$ | $2.74 \mathrm{E}-04$ |
| 38 | $3.42 \mathrm{E}-04$ | $4.06 \mathrm{E}-04$ | $3.84 \mathrm{E}-04$ | $3.15 \mathrm{E}-04$ | $3.18 \mathrm{E}-04$ | $2.76 \mathrm{E}-04$ | $3.12 \mathrm{E}-04$ | $2.95 \mathrm{E}-04$ |
| 39 | $3.68 \mathrm{E}-04$ | $4.38 \mathrm{E}-04$ | $4.14 \mathrm{E}-04$ | $3.39 \mathrm{E}-04$ | $3.42 \mathrm{E}-04$ | $2.97 \mathrm{E}-04$ | $3.36 \mathrm{E}-04$ | $3.18 \mathrm{E}-04$ |
| 40 | $3.97 \mathrm{E}-04$ | $4.72 \mathrm{E}-04$ | $4.46 \mathrm{E}-04$ | $3.65 \mathrm{E}-04$ | $3.69 \mathrm{E}-04$ | $3.20 \mathrm{E}-04$ | $3.62 \mathrm{E}-04$ | $3.43 \mathrm{E}-04$ |
| 41 | $4.28 \mathrm{E}-04$ | $5.08 \mathrm{E}-04$ | $4.81 \mathrm{E}-04$ | $3.93 \mathrm{E}-04$ | $3.97 \mathrm{E}-04$ | $3.45 \mathrm{E}-04$ | $3.90 \mathrm{E}-04$ | $3.69 \mathrm{E}-04$ |
| 42 | $4.61 \mathrm{E}-04$ | $5.48 \mathrm{E}-04$ | $5.18 \mathrm{E}-04$ | $4.24 \mathrm{E}-04$ | $4.28 \mathrm{E}-04$ | $3.72 \mathrm{E}-04$ | $4.20 \mathrm{E}-04$ | $3.98 \mathrm{E}-04$ |
| 43 | $4.96 \mathrm{E}-04$ | $5.90 \mathrm{E}-04$ | $5.58 \mathrm{E}-04$ | $4.57 \mathrm{E}-04$ | $4.61 \mathrm{E}-04$ | $4.00 \mathrm{E}-04$ | $4.53 \mathrm{E}-04$ | $4.28 \mathrm{E}-04$ |
| 44 | $5.34 \mathrm{E}-04$ | $6.35 \mathrm{E}-04$ | $6.01 \mathrm{E}-04$ | $4.92 \mathrm{E}-04$ | $4.96 \mathrm{E}-04$ | $4.31 \mathrm{E}-04$ | $4.87 \mathrm{E}-04$ | $4.61 \mathrm{E}-04$ |
| 45 | $5.75 \mathrm{E}-04$ | $6.84 \mathrm{E}-04$ | $6.47 \mathrm{E}-04$ | $5.30 \mathrm{E}-04$ | $5.34 \mathrm{E}-04$ | $4.64 \mathrm{E}-04$ | $5.25 \mathrm{E}-04$ | $4.97 \mathrm{E}-04$ |
| 46 | $6.20 \mathrm{E}-04$ | $7.37 \mathrm{E}-04$ | $6.97 \mathrm{E}-04$ | $5.70 \mathrm{E}-04$ | $5.75 \mathrm{E}-04$ | $5.00 \mathrm{E}-04$ | $5.65 \mathrm{E}-04$ | $5.35 \mathrm{E}-04$ |
| 47 | $6.67 \mathrm{E}-04$ | $7.93 \mathrm{E}-04$ | $7.50 \mathrm{E}-04$ | $6.14 \mathrm{E}-04$ | $6.20 \mathrm{E}-04$ | $5.38 \mathrm{E}-04$ | $6.09 \mathrm{E}-04$ | $5.76 \mathrm{E}-04$ |
| 48 | $7.18 \mathrm{E}-04$ | $8.54 \mathrm{E}-04$ | $8.07 \mathrm{E}-04$ | $6.61 \mathrm{E}-04$ | $6.67 \mathrm{E}-04$ | $5.80 \mathrm{E}-04$ | $6.55 \mathrm{E}-04$ | $6.20 \mathrm{E}-04$ |
| 49 | $7.73 \mathrm{E}-04$ | $9.19 \mathrm{E}-04$ | $8.69 \mathrm{E}-04$ | 7.12E-04 | 7.18E-04 | $6.24 \mathrm{E}-04$ | $7.05 \mathrm{E}-04$ | $6.68 \mathrm{E}-04$ |

Table 7.8: Estimated catch numbers at length at time for the North Sea grey gurnard

| Length/year | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 10 | 8 | 22 | 0 | 13 | 11 | 0 | 10 |
| 4 | 36 | 30 | 83 | 0 | 49 | 39 | 0 | 38 |
| 5 | 96 | 78 | 218 | 0 | 130 | 103 | 0 | 100 |
| 6 | 201 | 163 | 457 | 0 | 273 | 217 | 0 | 209 |
| 7 | 357 | 291 | 813 | 0 | 486 | 387 | 0 | 373 |
| 8 | 562 | 458 | 1280 | 2 | 765 | 609 | 1 | 586 |
| 9 | 804 | 658 | 1830 | 11 | 1093 | 873 | 4 | 838 |
| 10 | 1063 | 879 | 2424 | 37 | 1447 | 1164 | 13 | 1109 |
| 11 | 1322 | 1114 | 3018 | 102 | 1799 | 1467 | 37 | 1378 |
| 12 | 1561 | 1358 | 3574 | 231 | 2125 | 1773 | 84 | 1628 |
| 13 | 1765 | 1608 | 4058 | 450 | 2406 | 2074 | 164 | 1842 |
| 14 | 1924 | 1859 | 4448 | 770 | 2631 | 2362 | 284 | 2010 |
| 15 | 2031 | 2105 | 4733 | 1187 | 2802 | 2630 | 446 | 2131 |
| 16 | 2086 | 2333 | 4911 | 1674 | 2928 | 2867 | 646 | 2208 |
| 17 | 2092 | 2532 | 4986 | 2194 | 3026 | 3063 | 878 | 2249 |
| 18 | 2054 | 2686 | 4968 | 2704 | 3115 | 3210 | 1129 | 2265 |
| 19 | 1979 | 2785 | 4870 | 3163 | 3206 | 3308 | 1387 | 2267 |
| 20 | 1875 | 2824 | 4705 | 3540 | 3301 | 3363 | 1635 | 2264 |
| 21 | 1750 | 2801 | 4485 | 3813 | 3392 | 3385 | 1860 | 2261 |
| 22 | 1611 | 2720 | 4223 | 3976 | 3466 | 3384 | 2051 | 2258 |
| 23 | 1465 | 2590 | 3927 | 4028 | 3505 | 3368 | 2206 | 2252 |
| 24 | 1317 | 2421 | 3608 | 3978 | 3495 | 3337 | 2326 | 2240 |
| 25 | 1172 | 2224 | 3276 | 3837 | 3427 | 3284 | 2415 | 2218 |
| 26 | 1033 | 2010 | 2939 | 3621 | 3300 | 3200 | 2472 | 2184 |
| 27 | 903 | 1791 | 2607 | 3346 | 3115 | 3076 | 2495 | 2137 |
| 28 | 783 | 1573 | 2287 | 3030 | 2881 | 2906 | 2474 | 2076 |
| 29 | 673 | 1365 | 1984 | 2692 | 2610 | 2692 | 2401 | 1997 |
| 30 | 575 | 1171 | 1704 | 2347 | 2315 | 2438 | 2271 | 1893 |
| 31 | 488 | 994 | 1450 | 2010 | 2010 | 2157 | 2084 | 1757 |
| 32 | 412 | 835 | 1222 | 1693 | 1710 | 1861 | 1851 | 1588 |
| 33 | 345 | 695 | 1021 | 1403 | 1426 | 1567 | 1588 | 1389 |
| 34 | 288 | 573 | 847 | 1145 | 1168 | 1288 | 1314 | 1171 |
| 35 | 239 | 469 | 698 | 923 | 940 | 1035 | 1051 | 952 |
| 36 | 198 | 381 | 571 | 734 | 746 | 816 | 813 | 746 |
| 37 | 163 | 308 | 464 | 578 | 584 | 633 | 613 | 568 |
| 38 | 133 | 247 | 376 | 452 | 454 | 486 | 453 | 423 |
| 39 | 109 | 198 | 303 | 351 | 352 | 371 | 332 | 312 |
| 40 | 88 | 158 | 244 | 273 | 272 | 284 | 243 | 231 |
| 41 | 71 | 126 | 196 | 213 | 212 | 219 | 181 | 174 |
| 42 | 58 | 100 | 157 | 166 | 166 | 170 | 136 | 133 |
| 43 | 46 | 80 | 126 | 131 | 131 | 134 | 105 | 103 |
| 44 | 37 | 64 | 101 | 106 | 106 | 108 | 86 | 84 |
| 45 | 30 | 52 | 81 | 86 | 85 | 88 | 70 | 68 |
| 46 | 24 | 41 | 65 | 69 | 69 | 71 | 57 | 55 |
| 47 | 19 | 33 | 52 | 56 | 55 | 57 | 47 | 45 |
| 48 | 15 | 26 | 41 | 45 | 44 | 46 | 38 | 36 |
| 49 | 12 | 21 | 32 | 36 | 35 | 37 | 30 | 29 |

(Continued Table 7.8) Estimated catch numbers at length at time for the North Sea grey gurnard

| Length/year | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 1 | 0 | 10 | 10 | 10 | 8 | 4 | 6 |
| 4 | 4 | 0 | 36 | 38 | 37 | 31 | 16 | 24 |
| 5 | 11 | 0 | 94 | 101 | 97 | 81 | 43 | 62 |
| 6 | 22 | 1 | 198 | 211 | 203 | 170 | 91 | 131 |
| 7 | 40 | 1 | 352 | 376 | 361 | 303 | 162 | 233 |
| 8 | 64 | 2 | 553 | 592 | 569 | 477 | 256 | 367 |
| 9 | 93 | 3 | 791 | 847 | 815 | 683 | 368 | 525 |
| 10 | 130 | 5 | 1047 | 1125 | 1086 | 908 | 494 | 698 |
| 11 | 179 | 8 | 1301 | 1409 | 1366 | 1138 | 630 | 874 |
| 12 | 246 | 15 | 1537 | 1684 | 1645 | 1363 | 777 | 1046 |
| 13 | 337 | 27 | 1738 | 1939 | 1915 | 1575 | 935 | 1206 |
| 14 | 454 | 47 | 1894 | 2165 | 2169 | 1766 | 1103 | 1350 |
| 15 | 594 | 79 | 2001 | 2355 | 2401 | 1934 | 1279 | 1477 |
| 16 | 750 | 129 | 2059 | 2502 | 2607 | 2076 | 1459 | 1588 |
| 17 | 909 | 201 | 2070 | 2602 | 2778 | 2189 | 1636 | 1684 |
| 18 | 1059 | 300 | 2043 | 2652 | 2911 | 2274 | 1802 | 1768 |
| 19 | 1192 | 425 | 1987 | 2652 | 3000 | 2329 | 1950 | 1842 |
| 20 | 1301 | 569 | 1913 | 2605 | 3040 | 2356 | 2075 | 1906 |
| 21 | 1388 | 722 | 1833 | 2517 | 3031 | 2353 | 2172 | 1959 |
| 22 | 1455 | 874 | 1753 | 2396 | 2973 | 2322 | 2237 | 2000 |
| 23 | 1509 | 1015 | 1678 | 2252 | 2867 | 2262 | 2270 | 2025 |
| 24 | 1551 | 1140 | 1610 | 2095 | 2722 | 2174 | 2267 | 2030 |
| 25 | 1585 | 1248 | 1547 | 1935 | 2546 | 2059 | 2228 | 2013 |
| 26 | 1607 | 1340 | 1488 | 1778 | 2349 | 1923 | 2153 | 1971 |
| 27 | 1617 | 1416 | 1431 | 1627 | 2142 | 1768 | 2044 | 1902 |
| 28 | 1612 | 1474 | 1376 | 1485 | 1935 | 1603 | 1905 | 1805 |
| 29 | 1590 | 1512 | 1321 | 1353 | 1735 | 1435 | 1741 | 1682 |
| 30 | 1547 | 1526 | 1265 | 1231 | 1546 | 1268 | 1562 | 1535 |
| 31 | 1478 | 1510 | 1203 | 1117 | 1370 | 1110 | 1377 | 1371 |
| 32 | 1375 | 1455 | 1132 | 1010 | 1208 | 963 | 1194 | 1198 |
| 33 | 1237 | 1355 | 1044 | 904 | 1057 | 827 | 1020 | 1025 |
| 34 | 1066 | 1205 | 934 | 796 | 915 | 704 | 859 | 859 |
| 35 | 877 | 1013 | 800 | 682 | 778 | 590 | 711 | 707 |
| 36 | 688 | 802 | 651 | 563 | 642 | 485 | 577 | 569 |
| 37 | 515 | 596 | 501 | 444 | 511 | 387 | 456 | 446 |
| 38 | 373 | 420 | 367 | 337 | 393 | 301 | 350 | 341 |
| 39 | 265 | 287 | 262 | 250 | 295 | 228 | 262 | 254 |
| 40 | 189 | 194 | 187 | 185 | 221 | 173 | 194 | 189 |
| 41 | 137 | 135 | 136 | 140 | 167 | 132 | 146 | 141 |
| 42 | 102 | 97 | 102 | 107 | 129 | 102 | 112 | 108 |
| 43 | 78 | 73 | 79 | 84 | 102 | 81 | 87 | 84 |
| 44 | 64 | 60 | 64 | 68 | 82 | 65 | 71 | 68 |
| 45 | 52 | 50 | 52 | 55 | 66 | 53 | 57 | 55 |
| 46 | 43 | 41 | 43 | 45 | 54 | 42 | 47 | 45 |
| 47 | 35 | 34 | 35 | 36 | 43 | 34 | 38 | 36 |
| 48 | 28 | 28 | 28 | 29 | 35 | 27 | 30 | 29 |
| 49 | 23 | 23 | 23 | 23 | 28 | 22 | 24 | 24 |

(Continued Table 7.8) Estimated catch numbers at length at time for the North Sea grey gurnard

| Length/year | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 7 | 7 | 12 | 9 | 18 | 10 | 8 | 8 |
| 4 | 27 | 25 | 44 | 32 | 67 | 37 | 30 | 30 |
| 5 | 71 | 65 | 116 | 84 | 177 | 98 | 78 | 79 |
| 6 | 149 | 137 | 244 | 176 | 372 | 207 | 164 | 165 |
| 7 | 266 | 243 | 435 | 313 | 662 | 368 | 293 | 294 |
| 8 | 419 | 383 | 685 | 493 | 1042 | 580 | 461 | 463 |
| 9 | 599 | 549 | 980 | 706 | 1491 | 831 | 660 | 664 |
| 10 | 797 | 732 | 1301 | 942 | 1980 | 1109 | 878 | 886 |
| 11 | 1001 | 922 | 1626 | 1189 | 2476 | 1401 | 1103 | 1120 |
| 12 | 1201 | 1114 | 1939 | 1440 | 2952 | 1699 | 1325 | 1358 |
| 13 | 1392 | 1302 | 2224 | 1688 | 3387 | 1997 | 1537 | 1596 |
| 14 | 1567 | 1483 | 2472 | 1930 | 3767 | 2291 | 1734 | 1832 |
| 15 | 1722 | 1653 | 2677 | 2161 | 4082 | 2572 | 1914 | 2061 |
| 16 | 1854 | 1810 | 2836 | 2372 | 4330 | 2834 | 2075 | 2278 |
| 17 | 1961 | 1948 | 2950 | 2557 | 4510 | 3066 | 2215 | 2478 |
| 18 | 2041 | 2064 | 3020 | 2708 | 4626 | 3260 | 2333 | 2657 |
| 19 | 2095 | 2156 | 3051 | 2819 | 4682 | 3407 | 2430 | 2811 |
| 20 | 2125 | 2220 | 3045 | 2889 | 4682 | 3504 | 2502 | 2937 |
| 21 | 2134 | 2257 | 3007 | 2916 | 4630 | 3551 | 2548 | 3034 |
| 22 | 2127 | 2266 | 2941 | 2901 | 4529 | 3547 | 2566 | 3101 |
| 23 | 2106 | 2249 | 2848 | 2850 | 4383 | 3496 | 2553 | 3136 |
| 24 | 2073 | 2211 | 2734 | 2765 | 4197 | 3402 | 2509 | 3135 |
| 25 | 2028 | 2156 | 2602 | 2651 | 3977 | 3269 | 2436 | 3099 |
| 26 | 1971 | 2085 | 2455 | 2514 | 3728 | 3101 | 2334 | 3024 |
| 27 | 1898 | 2002 | 2299 | 2358 | 3458 | 2905 | 2207 | 2911 |
| 28 | 1807 | 1906 | 2137 | 2189 | 3173 | 2687 | 2058 | 2760 |
| 29 | 1697 | 1797 | 1971 | 2012 | 2880 | 2452 | 1892 | 2577 |
| 30 | 1567 | 1671 | 1803 | 1830 | 2586 | 2207 | 1712 | 2364 |
| 31 | 1416 | 1526 | 1629 | 1646 | 2296 | 1959 | 1525 | 2128 |
| 32 | 1251 | 1364 | 1450 | 1461 | 2013 | 1714 | 1336 | 1878 |
| 33 | 1078 | 1188 | 1264 | 1274 | 1740 | 1476 | 1148 | 1622 |
| 34 | 907 | 1006 | 1076 | 1085 | 1477 | 1247 | 968 | 1368 |
| 35 | 744 | 827 | 890 | 900 | 1226 | 1032 | 798 | 1125 |
| 36 | 596 | 660 | 715 | 724 | 991 | 832 | 642 | 900 |
| 37 | 466 | 513 | 559 | 565 | 781 | 653 | 502 | 700 |
| 38 | 355 | 388 | 428 | 430 | 602 | 500 | 384 | 529 |
| 39 | 265 | 288 | 322 | 322 | 457 | 378 | 289 | 393 |
| 40 | 197 | 212 | 242 | 241 | 347 | 284 | 217 | 291 |
| 41 | 148 | 159 | 183 | 182 | 266 | 216 | 164 | 218 |
| 42 | 113 | 121 | 141 | 140 | 206 | 167 | 126 | 167 |
| 43 | 88 | 94 | 111 | 109 | 162 | 131 | 99 | 130 |
| 44 | 71 | 76 | 89 | 89 | 131 | 106 | 80 | 105 |
| 45 | 58 | 62 | 73 | 72 | 106 | 86 | 65 | 86 |
| 46 | 47 | 50 | 59 | 58 | 86 | 69 | 53 | 70 |
| 47 | 38 | 41 | 47 | 47 | 69 | 56 | 42 | 56 |
| 48 | 31 | 33 | 38 | 38 | 55 | 45 | 34 | 45 |
| 49 | 25 | 26 | 30 | 30 | 44 | 36 | 27 | 36 |

(Continued Table 7.8) Estimated catch numbers at length at time for the North Sea grey gurnard

| Length/year | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 11 | 9 | 13 | 12 | 15 | 8 | 6 | 9 |
| 4 | 41 | 33 | 47 | 45 | 54 | 29 | 20 | 32 |
| 5 | 109 | 88 | 124 | 117 | 143 | 75 | 54 | 85 |
| 6 | 228 | 185 | 261 | 246 | 299 | 158 | 113 | 178 |
| 7 | 407 | 329 | 464 | 438 | 533 | 282 | 201 | 317 |
| 8 | 640 | 518 | 731 | 690 | 840 | 444 | 316 | 499 |
| 9 | 916 | 743 | 1047 | 988 | 1203 | 637 | 454 | 714 |
| 10 | 1217 | 992 | 1391 | 1315 | 1600 | 852 | 606 | 949 |
| 11 | 1524 | 1254 | 1743 | 1651 | 2008 | 1079 | 768 | 1188 |
| 12 | 1820 | 1523 | 2085 | 1983 | 2410 | 1315 | 934 | 1419 |
| 13 | 2094 | 1793 | 2404 | 2298 | 2790 | 1556 | 1105 | 1633 |
| 14 | 2337 | 2062 | 2692 | 2589 | 3140 | 1799 | 1278 | 1824 |
| 15 | 2543 | 2322 | 2942 | 2849 | 3452 | 2042 | 1453 | 1987 |
| 16 | 2713 | 2568 | 3153 | 3072 | 3718 | 2278 | 1630 | 2123 |
| 17 | 2844 | 2790 | 3325 | 3253 | 3936 | 2500 | 1807 | 2232 |
| 18 | 2940 | 2981 | 3458 | 3389 | 4102 | 2701 | 1983 | 2317 |
| 19 | 3003 | 3133 | 3555 | 3478 | 4213 | 2874 | 2155 | 2383 |
| 20 | 3037 | 3244 | 3617 | 3523 | 4270 | 3013 | 2318 | 2433 |
| 21 | 3043 | 3313 | 3643 | 3527 | 4274 | 3113 | 2468 | 2472 |
| 22 | 3026 | 3341 | 3633 | 3492 | 4227 | 3171 | 2597 | 2502 |
| 23 | 2985 | 3330 | 3589 | 3424 | 4133 | 3185 | 2698 | 2524 |
| 24 | 2924 | 3286 | 3510 | 3325 | 3999 | 3156 | 2765 | 2535 |
| 25 | 2843 | 3212 | 3401 | 3200 | 3829 | 3086 | 2792 | 2533 |
| 26 | 2743 | 3111 | 3264 | 3051 | 3630 | 2977 | 2777 | 2511 |
| 27 | 2621 | 2986 | 3105 | 2881 | 3408 | 2835 | 2718 | 2465 |
| 28 | 2479 | 2837 | 2926 | 2695 | 3167 | 2665 | 2616 | 2389 |
| 29 | 2314 | 2666 | 2731 | 2496 | 2913 | 2473 | 2477 | 2282 |
| 30 | 2128 | 2471 | 2520 | 2287 | 2650 | 2263 | 2304 | 2143 |
| 31 | 1924 | 2252 | 2294 | 2070 | 2382 | 2041 | 2105 | 1974 |
| 32 | 1705 | 2013 | 2053 | 1846 | 2112 | 1813 | 1885 | 1782 |
| 33 | 1479 | 1758 | 1799 | 1617 | 1841 | 1581 | 1653 | 1571 |
| 34 | 1251 | 1495 | 1536 | 1383 | 1572 | 1350 | 1415 | 1350 |
| 35 | 1031 | 1235 | 1274 | 1151 | 1309 | 1122 | 1177 | 1125 |
| 36 | 826 | 988 | 1023 | 927 | 1058 | 905 | 946 | 907 |
| 37 | 643 | 765 | 795 | 724 | 829 | 706 | 732 | 703 |
| 38 | 488 | 577 | 601 | 549 | 632 | 533 | 547 | 525 |
| 39 | 365 | 426 | 445 | 408 | 473 | 394 | 398 | 382 |
| 40 | 271 | 314 | 329 | 303 | 353 | 290 | 287 | 276 |
| 41 | 205 | 234 | 247 | 228 | 266 | 217 | 210 | 202 |
| 42 | 157 | 178 | 188 | 175 | 205 | 165 | 158 | 152 |
| 43 | 122 | 139 | 147 | 136 | 160 | 128 | 122 | 117 |
| 44 | 99 | 113 | 119 | 110 | 130 | 104 | 99 | 95 |
| 45 | 81 | 92 | 97 | 90 | 105 | 85 | 81 | 78 |
| 46 | 65 | 74 | 78 | 73 | 85 | 69 | 66 | 63 |
| 47 | 53 | 60 | 63 | 59 | 69 | 56 | 54 | 51 |
| 48 | 42 | 49 | 51 | 47 | 55 | 45 | 43 | 42 |
| 49 | 34 | 39 | 41 | 38 | 44 | 36 | 35 | 34 |

Table 7.9: Estimated stock size at length at time for the North Sea grey gurnard

| Length/year | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $6.3 \mathrm{E}+05$ | $3.7 \mathrm{E}+05$ | $2.7 \mathrm{E}+06$ | $2.4 \mathrm{E}+02$ | $1.3 \mathrm{E}+06$ | $1.1 \mathrm{E}+06$ | $1.8 \mathrm{E}+02$ | $1.1 \mathrm{E}+06$ |
| 4 | $1.6 \mathrm{E}+06$ | $9.1 \mathrm{E}+05$ | $6.6 \mathrm{E}+06$ | $6.0 \mathrm{E}+02$ | $3.2 \mathrm{E}+06$ | $2.6 \mathrm{E}+06$ | $4.4 \mathrm{E}+02$ | $2.7 \mathrm{E}+06$ |
| 5 | $2.9 \mathrm{E}+06$ | $1.7 \mathrm{E}+06$ | $1.3 \mathrm{E}+07$ | $1.2 \mathrm{E}+03$ | $6.1 \mathrm{E}+06$ | $5.0 \mathrm{E}+06$ | $8.4 \mathrm{E}+02$ | $5.1 \mathrm{E}+06$ |
| 6 | $4.7 \mathrm{E}+06$ | $2.8 \mathrm{E}+06$ | $2.0 \mathrm{E}+07$ | $2.4 \mathrm{E}+03$ | $9.6 \mathrm{E}+06$ | $7.9 \mathrm{E}+06$ | $1.6 \mathrm{E}+03$ | $8.1 \mathrm{E}+06$ |
| 7 | $6.6 \mathrm{E}+06$ | $3.9 \mathrm{E}+06$ | $2.8 \mathrm{E}+07$ | $8.8 \mathrm{E}+03$ | $1.4 \mathrm{E}+07$ | $1.1 \mathrm{E}+07$ | $4.3 \mathrm{E}+03$ | $1.1 \mathrm{E}+07$ |
| 8 | $8.4 \mathrm{E}+06$ | $4.9 \mathrm{E}+06$ | $3.6 \mathrm{E}+07$ | $4.1 \mathrm{E}+04$ | $1.7 \mathrm{E}+07$ | $1.4 \mathrm{E}+07$ | $1.7 \mathrm{E}+04$ | $1.4 \mathrm{E}+07$ |
| 9 | $9.9 \mathrm{E}+06$ | $5.9 \mathrm{E}+06$ | $4.2 \mathrm{E}+07$ | $1.5 \mathrm{E}+05$ | $2.0 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ | $6.2 \mathrm{E}+04$ | $1.7 \mathrm{E}+07$ |
| 10 | $1.1 \mathrm{E}+07$ | $6.6 \mathrm{E}+06$ | $4.7 \mathrm{E}+07$ | $4.5 \mathrm{E}+05$ | $2.3 \mathrm{E}+07$ | $1.9 \mathrm{E}+07$ | $1.8 \mathrm{E}+05$ | $1.9 \mathrm{E}+07$ |
| 11 | $1.2 \mathrm{E}+07$ | $7.2 \mathrm{E}+06$ | $5.0 \mathrm{E}+07$ | $1.1 \mathrm{E}+06$ | $2.4 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $4.3 \mathrm{E}+05$ | $2.0 \mathrm{E}+07$ |
| 12 | $1.2 \mathrm{E}+07$ | $7.6 \mathrm{E}+06$ | $5.2 \mathrm{E}+07$ | $2.1 \mathrm{E}+06$ | $2.5 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $8.5 \mathrm{E}+05$ | $2.1 \mathrm{E}+07$ |
| 13 | $1.2 \mathrm{E}+07$ | $7.9 \mathrm{E}+06$ | $5.1 \mathrm{E}+07$ | $3.6 \mathrm{E}+06$ | $2.5 \mathrm{E}+07$ | $2.2 \mathrm{E}+07$ | $1.4 \mathrm{E}+06$ | $2.1 \mathrm{E}+07$ |
| 14 | $1.1 \mathrm{E}+07$ | $8.0 \mathrm{E}+06$ | $4.9 \mathrm{E}+07$ | $5.4 \mathrm{E}+06$ | $2.4 \mathrm{E}+07$ | $2.2 \mathrm{E}+07$ | $2.2 \mathrm{E}+06$ | $2.0 \mathrm{E}+07$ |
| 15 | $1.1 \mathrm{E}+07$ | $8.0 \mathrm{E}+06$ | $4.7 \mathrm{E}+07$ | $7.4 \mathrm{E}+06$ | $2.2 \mathrm{E}+07$ | $2.2 \mathrm{E}+07$ | $3.1 \mathrm{E}+06$ | $1.9 \mathrm{E}+07$ |
| 16 | $9.8 \mathrm{E}+06$ | $7.9 \mathrm{E}+06$ | $4.3 \mathrm{E}+07$ | $9.3 \mathrm{E}+06$ | $2.1 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $4.0 \mathrm{E}+06$ | $1.7 \mathrm{E}+07$ |
| 17 | $8.8 \mathrm{E}+06$ | $7.7 \mathrm{E}+06$ | $3.9 \mathrm{E}+07$ | $1.1 \mathrm{E}+07$ | $1.9 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $4.8 \mathrm{E}+06$ | $1.6 \mathrm{E}+07$ |
| 18 | $7.8 \mathrm{E}+06$ | $7.4 \mathrm{E}+06$ | $3.5 \mathrm{E}+07$ | $1.2 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ | $1.9 \mathrm{E}+07$ | $5.6 \mathrm{E}+06$ | $1.4 \mathrm{E}+07$ |
| 19 | $6.7 \mathrm{E}+06$ | $6.9 \mathrm{E}+06$ | $3.1 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ | $6.2 \mathrm{E}+06$ | $1.3 \mathrm{E}+07$ |
| 20 | $5.8 \mathrm{E}+06$ | $6.3 \mathrm{E}+06$ | $2.7 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ | $1.6 \mathrm{E}+07$ | $6.6 \mathrm{E}+06$ | $1.2 \mathrm{E}+07$ |
| 21 | $4.9 \mathrm{E}+06$ | $5.7 \mathrm{E}+06$ | $2.3 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $1.4 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ | $6.8 \mathrm{E}+06$ | $1.0 \mathrm{E}+07$ |
| 22 | $4.1 \mathrm{E}+06$ | $5.0 \mathrm{E}+06$ | $2.0 \mathrm{E}+07$ | $1.2 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $6.8 \mathrm{E}+06$ | $9.5 \mathrm{E}+06$ |
| 23 | $3.4 \mathrm{E}+06$ | $4.3 \mathrm{E}+06$ | $1.7 \mathrm{E}+07$ | $1.1 \mathrm{E}+07$ | $1.2 \mathrm{E}+07$ | $1.2 \mathrm{E}+07$ | $6.6 \mathrm{E}+06$ | $8.6 \mathrm{E}+06$ |
| 24 | $2.8 \mathrm{E}+06$ | $3.7 \mathrm{E}+06$ | $1.4 \mathrm{E}+07$ | $9.9 \mathrm{E}+06$ | $1.1 \mathrm{E}+07$ | $1.1 \mathrm{E}+07$ | $6.4 \mathrm{E}+06$ | $7.8 \mathrm{E}+06$ |
| 25 | $2.2 \mathrm{E}+06$ | $3.1 \mathrm{E}+06$ | $1.2 \mathrm{E}+07$ | $8.7 \mathrm{E}+06$ | $9.9 \mathrm{E}+06$ | $9.8 \mathrm{E}+06$ | $6.0 \mathrm{E}+06$ | $7.0 \mathrm{E}+06$ |
| 26 | $1.8 \mathrm{E}+06$ | $2.5 \mathrm{E}+06$ | $9.6 \mathrm{E}+06$ | $7.5 \mathrm{E}+06$ | $8.7 \mathrm{E}+06$ | $8.7 \mathrm{E}+06$ | $5.6 \mathrm{E}+06$ | $6.3 \mathrm{E}+06$ |
| 27 | $1.4 \mathrm{E}+06$ | $2.1 \mathrm{E}+06$ | $7.8 \mathrm{E}+06$ | $6.3 \mathrm{E}+06$ | $7.6 \mathrm{E}+06$ | $7.7 \mathrm{E}+06$ | $5.2 \mathrm{E}+06$ | $5.7 \mathrm{E}+06$ |
| 28 | $1.1 \mathrm{E}+06$ | $1.7 \mathrm{E}+06$ | $6.3 \mathrm{E}+06$ | $5.3 \mathrm{E}+06$ | $6.4 \mathrm{E}+06$ | $6.6 \mathrm{E}+06$ | $4.7 \mathrm{E}+06$ | $5.0 \mathrm{E}+06$ |
| 29 | $9.0 \mathrm{E}+05$ | $1.3 \mathrm{E}+06$ | $5.0 \mathrm{E}+06$ | $4.3 \mathrm{E}+06$ | $5.3 \mathrm{E}+06$ | $5.6 \mathrm{E}+06$ | $4.2 \mathrm{E}+06$ | $4.4 \mathrm{E}+06$ |
| 30 | $7.1 \mathrm{E}+05$ | $1.0 \mathrm{E}+06$ | $3.9 \mathrm{E}+06$ | $3.4 \mathrm{E}+06$ | $4.3 \mathrm{E}+06$ | $4.7 \mathrm{E}+06$ | $3.7 \mathrm{E}+06$ | $3.9 \mathrm{E}+06$ |
| 31 | $5.5 \mathrm{E}+05$ | $8.1 \mathrm{E}+05$ | $3.1 \mathrm{E}+06$ | $2.7 \mathrm{E}+06$ | $3.4 \mathrm{E}+06$ | $3.8 \mathrm{E}+06$ | $3.1 \mathrm{E}+06$ | $3.3 \mathrm{E}+06$ |
| 32 | $4.3 \mathrm{E}+05$ | $6.3 \mathrm{E}+05$ | $2.4 \mathrm{E}+06$ | $2.1 \mathrm{E}+06$ | $2.7 \mathrm{E}+06$ | $3.0 \mathrm{E}+06$ | $2.5 \mathrm{E}+06$ | $2.7 \mathrm{E}+06$ |
| 33 | $3.3 \mathrm{E}+05$ | $4.8 \mathrm{E}+05$ | $1.8 \mathrm{E}+06$ | $1.6 \mathrm{E}+06$ | $2.1 \mathrm{E}+06$ | $2.3 \mathrm{E}+06$ | $2.0 \mathrm{E}+06$ | $2.2 \mathrm{E}+06$ |
| 34 | $2.5 \mathrm{E}+05$ | $3.6 \mathrm{E}+05$ | $1.4 \mathrm{E}+06$ | $1.2 \mathrm{E}+06$ | $1.6 \mathrm{E}+06$ | $1.8 \mathrm{E}+06$ | $1.5 \mathrm{E}+06$ | $1.7 \mathrm{E}+06$ |
| 35 | $1.9 \mathrm{E}+05$ | $2.7 \mathrm{E}+05$ | $1.1 \mathrm{E}+06$ | $8.8 \mathrm{E}+05$ | $1.2 \mathrm{E}+06$ | $1.3 \mathrm{E}+06$ | $1.1 \mathrm{E}+06$ | $1.3 \mathrm{E}+06$ |
| 36 | $1.5 \mathrm{E}+05$ | $2.1 \mathrm{E}+05$ | $7.9 \mathrm{E}+05$ | $6.5 \mathrm{E}+05$ | $8.4 \mathrm{E}+05$ | $9.5 \mathrm{E}+05$ | $7.9 \mathrm{E}+05$ | $9.2 \mathrm{E}+05$ |
| 37 | $1.1 \mathrm{E}+05$ | $1.5 \mathrm{E}+05$ | $6.0 \mathrm{E}+05$ | $4.7 \mathrm{E}+05$ | $6.1 \mathrm{E}+05$ | $6.8 \mathrm{E}+05$ | $5.5 \mathrm{E}+05$ | $6.4 \mathrm{E}+05$ |
| 38 | $8.4 \mathrm{E}+04$ | $1.1 \mathrm{E}+05$ | $4.4 \mathrm{E}+05$ | $3.4 \mathrm{E}+05$ | $4.3 \mathrm{E}+05$ | $4.8 \mathrm{E}+05$ | $3.7 \mathrm{E}+05$ | $4.4 \mathrm{E}+05$ |
| 39 | $6.3 \mathrm{E}+04$ | $8.3 \mathrm{E}+04$ | $3.3 \mathrm{E}+05$ | $2.4 \mathrm{E}+05$ | $3.1 \mathrm{E}+05$ | $3.4 \mathrm{E}+05$ | $2.5 \mathrm{E}+05$ | $3.0 \mathrm{E}+05$ |
| 40 | $4.7 \mathrm{E}+04$ | $6.1 \mathrm{E}+04$ | $2.5 \mathrm{E}+05$ | $1.7 \mathrm{E}+05$ | $2.2 \mathrm{E}+05$ | $2.4 \mathrm{E}+05$ | $1.7 \mathrm{E}+05$ | $2.1 \mathrm{E}+05$ |
| 41 | $3.5 \mathrm{E}+04$ | $4.5 \mathrm{E}+04$ | $1.8 \mathrm{E}+05$ | $1.2 \mathrm{E}+05$ | $1.6 \mathrm{E}+05$ | $1.7 \mathrm{E}+05$ | $1.2 \mathrm{E}+05$ | $1.4 \mathrm{E}+05$ |
| 42 | $2.6 \mathrm{E}+04$ | $3.3 \mathrm{E}+04$ | $1.3 \mathrm{E}+05$ | $9.0 \mathrm{E}+04$ | $1.1 \mathrm{E}+05$ | $1.2 \mathrm{E}+05$ | $8.2 \mathrm{E}+04$ | $1.0 \mathrm{E}+05$ |
| 43 | $2.0 \mathrm{E}+04$ | $2.4 \mathrm{E}+04$ | $1.0 \mathrm{E}+05$ | $6.6 \mathrm{E}+04$ | $8.4 \mathrm{E}+04$ | $8.8 \mathrm{E}+04$ | $5.8 \mathrm{E}+04$ | $7.2 \mathrm{E}+04$ |
| 44 | $1.5 \mathrm{E}+04$ | $1.8 \mathrm{E}+04$ | $7.4 \mathrm{E}+04$ | $4.9 \mathrm{E}+04$ | $6.2 \mathrm{E}+04$ | $6.6 \mathrm{E}+04$ | $4.4 \mathrm{E}+04$ | $5.4 \mathrm{E}+04$ |
| 45 | $1.1 \mathrm{E}+04$ | $1.3 \mathrm{E}+04$ | $5.5 \mathrm{E}+04$ | $3.7 \mathrm{E}+04$ | $4.7 \mathrm{E}+04$ | $4.9 \mathrm{E}+04$ | $3.3 \mathrm{E}+04$ | $4.1 \mathrm{E}+04$ |
| 46 | $7.9 \mathrm{E}+03$ | $1.0 \mathrm{E}+04$ | $4.0 \mathrm{E}+04$ | $2.7 \mathrm{E}+04$ | $3.5 \mathrm{E}+04$ | $3.7 \mathrm{E}+04$ | $2.5 \mathrm{E}+04$ | $3.1 \mathrm{E}+04$ |
| 47 | $5.8 \mathrm{E}+03$ | $7.4 \mathrm{E}+03$ | $3.0 \mathrm{E}+04$ | $2.0 \mathrm{E}+04$ | $2.6 \mathrm{E}+04$ | $2.7 \mathrm{E}+04$ | $1.9 \mathrm{E}+04$ | $2.3 \mathrm{E}+04$ |
| 48 | $4.3 \mathrm{E}+03$ | $5.4 \mathrm{E}+03$ | $2.2 \mathrm{E}+04$ | $1.5 \mathrm{E}+04$ | $1.9 \mathrm{E}+04$ | $2.0 \mathrm{E}+04$ | $1.4 \mathrm{E}+04$ | $1.7 \mathrm{E}+04$ |
| 49 | $3.1 \mathrm{E}+03$ | $4.0 \mathrm{E}+03$ | $1.6 \mathrm{E}+04$ | $1.1 \mathrm{E}+04$ | $1.4 \mathrm{E}+04$ | $1.5 \mathrm{E}+04$ | $1.1 \mathrm{E}+04$ | $1.3 \mathrm{E}+04$ |

(Continued Table 7.9) Estimated stock size at length at time for the North Sea grey gurnard

| Length/year | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $1.1 \mathrm{E}+05$ | $2.1 \mathrm{E}+03$ | $9.8 \mathrm{E}+05$ | $1.5 \mathrm{E}+06$ | $1.5 \mathrm{E}+06$ | $2.0 \mathrm{E}+06$ | $8.8 \mathrm{E}+05$ | $1.4 \mathrm{E}+06$ |
| 4 | $2.7 \mathrm{E}+05$ | $5.0 \mathrm{E}+03$ | $2.4 \mathrm{E}+06$ | $3.8 \mathrm{E}+06$ | $3.7 \mathrm{E}+06$ | $5.0 \mathrm{E}+06$ | $2.2 \mathrm{E}+06$ | $3.3 \mathrm{E}+06$ |
| 5 | $5.1 \mathrm{E}+05$ | $9.6 \mathrm{E}+03$ | $4.5 \mathrm{E}+06$ | $7.1 \mathrm{E}+06$ | $7.0 \mathrm{E}+06$ | $9.5 \mathrm{E}+06$ | 4.1E+06 | $6.3 \mathrm{E}+06$ |
| 6 | $8.1 \mathrm{E}+05$ | $1.5 \mathrm{E}+04$ | $7.2 \mathrm{E}+06$ | $1.1 \mathrm{E}+07$ | $1.1 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ | $6.5 \mathrm{E}+06$ | $1.0 \mathrm{E}+07$ |
| 7 | $1.1 \mathrm{E}+06$ | $2.2 \mathrm{E}+04$ | $1.0 \mathrm{E}+07$ | $1.6 \mathrm{E}+07$ | $1.6 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $9.1 \mathrm{E}+06$ | $1.4 \mathrm{E}+07$ |
| 8 | $1.5 \mathrm{E}+06$ | $2.9 \mathrm{E}+04$ | $1.3 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ | $1.2 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ |
| 9 | $1.8 \mathrm{E}+06$ | $3.8 \mathrm{E}+04$ | $1.5 \mathrm{E}+07$ | $2.4 \mathrm{E}+07$ | $2.4 \mathrm{E}+07$ | $3.2 \mathrm{E}+07$ | $1.4 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ |
| 10 | $2.1 \mathrm{E}+06$ | $5.4 \mathrm{E}+04$ | $1.7 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ | $3.6 \mathrm{E}+07$ | $1.6 \mathrm{E}+07$ | $2.4 \mathrm{E}+07$ |
| 11 | $2.5 \mathrm{E}+06$ | $8.3 \mathrm{E}+04$ | $1.8 \mathrm{E}+07$ | $2.9 \mathrm{E}+07$ | $2.9 \mathrm{E}+07$ | $3.9 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ | $2.6 \mathrm{E}+07$ |
| 12 | $2.9 \mathrm{E}+06$ | $1.3 \mathrm{E}+05$ | $1.9 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $4.0 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ | $2.6 \mathrm{E}+07$ |
| 13 | $3.5 \mathrm{E}+06$ | $2.0 \mathrm{E}+05$ | $1.8 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $4.1 \mathrm{E}+07$ | $1.9 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ |
| 14 | $4.2 \mathrm{E}+06$ | $3.1 \mathrm{E}+05$ | $1.8 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $4.0 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $2.6 \mathrm{E}+07$ |
| 15 | $4.8 \mathrm{E}+06$ | $4.6 \mathrm{E}+05$ | $1.7 \mathrm{E}+07$ | $2.9 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $3.9 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $2.6 \mathrm{E}+07$ |
| 16 | $5.4 \mathrm{E}+06$ | $6.6 \mathrm{E}+05$ | $1.5 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ | $2.9 \mathrm{E}+07$ | $3.7 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $2.4 \mathrm{E}+07$ |
| 17 | $5.9 \mathrm{E}+06$ | $9.2 \mathrm{E}+05$ | $1.4 \mathrm{E}+07$ | $2.5 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ | $3.5 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $2.3 \mathrm{E}+07$ |
| 18 | $6.2 \mathrm{E}+06$ | $1.2 \mathrm{E}+06$ | $1.2 \mathrm{E}+07$ | $2.3 \mathrm{E}+07$ | $2.6 \mathrm{E}+07$ | $3.3 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $2.2 \mathrm{E}+07$ |
| 19 | $6.3 \mathrm{E}+06$ | $1.6 \mathrm{E}+06$ | $1.1 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $2.4 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ |
| 20 | $6.2 \mathrm{E}+06$ | $1.9 \mathrm{E}+06$ | $9.2 \mathrm{E}+06$ | $1.9 \mathrm{E}+07$ | $2.2 \mathrm{E}+07$ | $2.8 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $1.9 \mathrm{E}+07$ |
| 21 | $6.0 \mathrm{E}+06$ | $2.2 \mathrm{E}+06$ | $8.0 \mathrm{E}+06$ | $1.6 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $2.5 \mathrm{E}+07$ | $1.9 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ |
| 22 | $5.7 \mathrm{E}+06$ | $2.4 \mathrm{E}+06$ | $7.0 \mathrm{E}+06$ | $1.4 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ | $2.2 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ |
| 23 | $5.4 \mathrm{E}+06$ | $2.6 \mathrm{E}+06$ | $6.1 \mathrm{E}+06$ | $1.2 \mathrm{E}+07$ | $1.6 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $1.6 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ |
| 24 | $5.0 \mathrm{E}+06$ | $2.6 \mathrm{E}+06$ | $5.3 \mathrm{E}+06$ | $1.0 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ | $1.4 \mathrm{E}+07$ |
| 25 | $4.7 \mathrm{E}+06$ | $2.6 \mathrm{E}+06$ | $4.6 \mathrm{E}+06$ | $8.6 \mathrm{E}+06$ | $1.1 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ |
| 26 | $4.3 \mathrm{E}+06$ | $2.6 \mathrm{E}+06$ | $4.1 \mathrm{E}+06$ | $7.2 \mathrm{E}+06$ | $9.6 \mathrm{E}+06$ | $1.3 \mathrm{E}+07$ | $1.2 \mathrm{E}+07$ | $1.1 \mathrm{E}+07$ |
| 27 | $4.0 \mathrm{E}+06$ | $2.5 \mathrm{E}+06$ | $3.6 \mathrm{E}+06$ | $6.0 \mathrm{E}+06$ | $8.0 \mathrm{E}+06$ | $1.1 \mathrm{E}+07$ | $1.0 \mathrm{E}+07$ | $1.0 \mathrm{E}+07$ |
| 28 | $3.6 \mathrm{E}+06$ | $2.4 \mathrm{E}+06$ | $3.2 \mathrm{E}+06$ | $5.0 \mathrm{E}+06$ | $6.7 \mathrm{E}+06$ | $8.9 \mathrm{E}+06$ | $8.5 \mathrm{E}+06$ | $8.7 \mathrm{E}+06$ |
| 29 | $3.3 \mathrm{E}+06$ | $2.2 \mathrm{E}+06$ | $2.8 \mathrm{E}+06$ | $4.2 \mathrm{E}+06$ | $5.5 \mathrm{E}+06$ | $7.3 \mathrm{E}+06$ | $7.2 \mathrm{E}+06$ | $7.4 \mathrm{E}+06$ |
| 30 | $2.9 \mathrm{E}+06$ | $2.1 \mathrm{E}+06$ | $2.4 \mathrm{E}+06$ | $3.5 \mathrm{E}+06$ | $4.5 \mathrm{E}+06$ | $6.0 \mathrm{E}+06$ | $5.9 \mathrm{E}+06$ | $6.2 \mathrm{E}+06$ |
| 31 | $2.6 \mathrm{E}+06$ | $1.9 \mathrm{E}+06$ | $2.1 \mathrm{E}+06$ | $2.9 \mathrm{E}+06$ | $3.6 \mathrm{E}+06$ | $4.8 \mathrm{E}+06$ | $4.8 \mathrm{E}+06$ | $5.1 \mathrm{E}+06$ |
| 32 | $2.2 \mathrm{E}+06$ | $1.7 \mathrm{E}+06$ | $1.8 \mathrm{E}+06$ | $2.4 \mathrm{E}+06$ | $2.9 \mathrm{E}+06$ | $3.8 \mathrm{E}+06$ | $3.8 \mathrm{E}+06$ | $4.1 \mathrm{E}+06$ |
| 33 | $1.8 \mathrm{E}+06$ | $1.4 \mathrm{E}+06$ | $1.6 \mathrm{E}+06$ | $2.0 \mathrm{E}+06$ | $2.4 \mathrm{E}+06$ | $3.0 \mathrm{E}+06$ | $3.0 \mathrm{E}+06$ | $3.2 \mathrm{E}+06$ |
| 34 | $1.4 \mathrm{E}+06$ | $1.2 \mathrm{E}+06$ | $1.3 \mathrm{E}+06$ | $1.6 \mathrm{E}+06$ | $1.9 \mathrm{E}+06$ | $2.4 \mathrm{E}+06$ | $2.3 \mathrm{E}+06$ | $2.5 \mathrm{E}+06$ |
| 35 | $1.1 \mathrm{E}+06$ | $9.0 \mathrm{E}+05$ | $1.0 \mathrm{E}+06$ | $1.3 \mathrm{E}+06$ | $1.5 \mathrm{E}+06$ | $1.8 \mathrm{E}+06$ | $1.8 \mathrm{E}+06$ | $1.9 \mathrm{E}+06$ |
| 36 | $7.9 \mathrm{E}+05$ | $6.5 \mathrm{E}+05$ | $7.6 \mathrm{E}+05$ | $9.7 \mathrm{E}+05$ | $1.1 \mathrm{E}+06$ | $1.4 \mathrm{E}+06$ | $1.3 \mathrm{E}+06$ | $1.4 \mathrm{E}+06$ |
| 37 | $5.4 \mathrm{E}+05$ | $4.5 \mathrm{E}+05$ | $5.4 \mathrm{E}+05$ | $7.0 \mathrm{E}+05$ | $8.2 \mathrm{E}+05$ | $1.0 \mathrm{E}+06$ | $9.6 \mathrm{E}+05$ | $1.0 \mathrm{E}+06$ |
| 38 | $3.6 \mathrm{E}+05$ | $2.9 \mathrm{E}+05$ | $3.6 \mathrm{E}+05$ | $4.9 \mathrm{E}+05$ | $5.8 \mathrm{E}+05$ | $7.2 \mathrm{E}+05$ | $6.8 \mathrm{E}+05$ | $7.1 \mathrm{E}+05$ |
| 39 | $2.4 \mathrm{E}+05$ | $1.8 \mathrm{E}+05$ | $2.4 \mathrm{E}+05$ | $3.4 \mathrm{E}+05$ | $4.0 \mathrm{E}+05$ | $5.1 \mathrm{E}+05$ | $4.7 \mathrm{E}+05$ | $4.9 \mathrm{E}+05$ |
| 40 | $1.6 \mathrm{E}+05$ | $1.1 \mathrm{E}+05$ | $1.6 \mathrm{E}+05$ | $2.3 \mathrm{E}+05$ | $2.8 \mathrm{E}+05$ | $3.5 \mathrm{E}+05$ | $3.2 \mathrm{E}+05$ | $3.3 \mathrm{E}+05$ |
| 41 | $1.0 \mathrm{E}+05$ | $7.3 \mathrm{E}+04$ | $1.1 \mathrm{E}+05$ | $1.6 \mathrm{E}+05$ | $1.9 \mathrm{E}+05$ | $2.5 \mathrm{E}+05$ | $2.2 \mathrm{E}+05$ | $2.3 \mathrm{E}+05$ |
| 42 | $7.2 \mathrm{E}+04$ | $4.9 \mathrm{E}+04$ | $7.3 \mathrm{E}+04$ | $1.1 \mathrm{E}+05$ | $1.4 \mathrm{E}+05$ | $1.8 \mathrm{E}+05$ | $1.6 \mathrm{E}+05$ | $1.6 \mathrm{E}+05$ |
| 43 | $5.1 \mathrm{E}+04$ | $3.4 \mathrm{E}+04$ | $5.2 \mathrm{E}+04$ | $8.2 \mathrm{E}+04$ | $1.0 \mathrm{E}+05$ | $1.3 \mathrm{E}+05$ | $1.1 \mathrm{E}+05$ | $1.2 \mathrm{E}+05$ |
| 44 | $3.8 \mathrm{E}+04$ | $2.6 \mathrm{E}+04$ | $3.9 \mathrm{E}+04$ | $6.1 \mathrm{E}+04$ | $7.5 \mathrm{E}+04$ | $9.7 \mathrm{E}+04$ | $8.5 \mathrm{E}+04$ | $8.7 \mathrm{E}+04$ |
| 45 | $2.9 \mathrm{E}+04$ | $2.0 \mathrm{E}+04$ | $3.0 \mathrm{E}+04$ | $4.6 \mathrm{E}+04$ | $5.6 \mathrm{E}+04$ | $7.2 \mathrm{E}+04$ | $6.3 \mathrm{E}+04$ | $6.6 \mathrm{E}+04$ |
| 46 | $2.2 \mathrm{E}+04$ | $1.5 \mathrm{E}+04$ | $2.2 \mathrm{E}+04$ | $3.4 \mathrm{E}+04$ | $4.2 \mathrm{E}+04$ | $5.4 \mathrm{E}+04$ | $4.8 \mathrm{E}+04$ | $4.9 \mathrm{E}+04$ |
| 47 | $1.7 \mathrm{E}+04$ | $1.1 \mathrm{E}+04$ | $1.7 \mathrm{E}+04$ | $2.6 \mathrm{E}+04$ | $3.1 \mathrm{E}+04$ | $4.0 \mathrm{E}+04$ | $3.6 \mathrm{E}+04$ | $3.7 \mathrm{E}+04$ |
| 48 | $1.2 \mathrm{E}+04$ | $8.6 \mathrm{E}+03$ | $1.3 \mathrm{E}+04$ | $1.9 \mathrm{E}+04$ | $2.3 \mathrm{E}+04$ | $3.0 \mathrm{E}+04$ | $2.6 \mathrm{E}+04$ | $2.7 \mathrm{E}+04$ |
| 49 | $9.3 \mathrm{E}+03$ | $6.5 \mathrm{E}+03$ | $9.3 \mathrm{E}+03$ | $1.4 \mathrm{E}+04$ | $1.7 \mathrm{E}+04$ | $2.2 \mathrm{E}+04$ | $2.0 \mathrm{E}+04$ | $2.0 \mathrm{E}+04$ |

(Continued Table 7.9) Estimated stock size at length at time for the North Sea grey gurnard

| Length/year | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $1.6 \mathrm{E}+06$ | $1.4 \mathrm{E}+06$ | $2.8 \mathrm{E}+06$ | $2.1 \mathrm{E}+06$ | $4.3 \mathrm{E}+06$ | $3.1 \mathrm{E}+06$ | $3.4 \mathrm{E}+06$ | $2.5 \mathrm{E}+06$ |
| 4 | $3.9 \mathrm{E}+06$ | $3.4 \mathrm{E}+06$ | $6.9 \mathrm{E}+06$ | $5.2 \mathrm{E}+06$ | $1.0 \mathrm{E}+07$ | $7.5 \mathrm{E}+06$ | $8.5 \mathrm{E}+06$ | $6.1 \mathrm{E}+06$ |
| 5 | $7.4 \mathrm{E}+06$ | $6.3 \mathrm{E}+06$ | $1.3 \mathrm{E}+07$ | $9.9 \mathrm{E}+06$ | $2.0 \mathrm{E}+07$ | $1.4 \mathrm{E}+07$ | $1.6 \mathrm{E}+07$ | $1.1 \mathrm{E}+07$ |
| 6 | $1.2 \mathrm{E}+07$ | $1.0 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $1.6 \mathrm{E}+07$ | $3.2 \mathrm{E}+07$ | $2.3 \mathrm{E}+07$ | $2.5 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ |
| 7 | $1.7 \mathrm{E}+07$ | $1.4 \mathrm{E}+07$ | $2.9 \mathrm{E}+07$ | $2.2 \mathrm{E}+07$ | $4.4 \mathrm{E}+07$ | $3.2 \mathrm{E}+07$ | $3.6 \mathrm{E}+07$ | $2.6 \mathrm{E}+07$ |
| 8 | $2.1 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ | $3.7 \mathrm{E}+07$ | $2.8 \mathrm{E}+07$ | $5.7 \mathrm{E}+07$ | $4.0 \mathrm{E}+07$ | $4.6 \mathrm{E}+07$ | $3.3 \mathrm{E}+07$ |
| 9 | $2.5 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $4.4 \mathrm{E}+07$ | $3.4 \mathrm{E}+07$ | $6.7 \mathrm{E}+07$ | $4.8 \mathrm{E}+07$ | $5.4 \mathrm{E}+07$ | $3.9 \mathrm{E}+07$ |
| 10 | $2.8 \mathrm{E}+07$ | $2.4 \mathrm{E}+07$ | $4.9 \mathrm{E}+07$ | $3.8 \mathrm{E}+07$ | $7.5 \mathrm{E}+07$ | $5.4 \mathrm{E}+07$ | $6.1 \mathrm{E}+07$ | $4.4 \mathrm{E}+07$ |
| 11 | $3.0 \mathrm{E}+07$ | $2.6 \mathrm{E}+07$ | $5.3 \mathrm{E}+07$ | $4.1 \mathrm{E}+07$ | $8.1 \mathrm{E}+07$ | $5.9 \mathrm{E}+07$ | $6.5 \mathrm{E}+07$ | $4.7 \mathrm{E}+07$ |
| 12 | $3.2 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ | $5.4 \mathrm{E}+07$ | $4.3 \mathrm{E}+07$ | $8.3 \mathrm{E}+07$ | $6.2 \mathrm{E}+07$ | $6.8 \mathrm{E}+07$ | $5.0 \mathrm{E}+07$ |
| 13 | $3.2 \mathrm{E}+07$ | $2.8 \mathrm{E}+07$ | $5.5 \mathrm{E}+07$ | $4.4 \mathrm{E}+07$ | $8.4 \mathrm{E}+07$ | $6.3 \mathrm{E}+07$ | $6.9 \mathrm{E}+07$ | $5.1 \mathrm{E}+07$ |
| 14 | $3.2 \mathrm{E}+07$ | $2.8 \mathrm{E}+07$ | $5.3 \mathrm{E}+07$ | $4.4 \mathrm{E}+07$ | $8.2 \mathrm{E}+07$ | $6.4 \mathrm{E}+07$ | $6.9 \mathrm{E}+07$ | $5.2 \mathrm{E}+07$ |
| 15 | $3.1 \mathrm{E}+07$ | $2.8 \mathrm{E}+07$ | $5.1 \mathrm{E}+07$ | $4.4 \mathrm{E}+07$ | $7.9 \mathrm{E}+07$ | $6.4 \mathrm{E}+07$ | $6.7 \mathrm{E}+07$ | $5.2 \mathrm{E}+07$ |
| 16 | $3.0 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ | $4.8 \mathrm{E}+07$ | $4.3 \mathrm{E}+07$ | $7.4 \mathrm{E}+07$ | $6.3 \mathrm{E}+07$ | $6.5 \mathrm{E}+07$ | $5.1 \mathrm{E}+07$ |
| 17 | $2.8 \mathrm{E}+07$ | $2.6 \mathrm{E}+07$ | $4.5 \mathrm{E}+07$ | $4.1 \mathrm{E}+07$ | $6.9 \mathrm{E}+07$ | $6.1 \mathrm{E}+07$ | $6.2 \mathrm{E}+07$ | $4.9 \mathrm{E}+07$ |
| 18 | $2.6 \mathrm{E}+07$ | $2.5 \mathrm{E}+07$ | $4.1 \mathrm{E}+07$ | $3.9 \mathrm{E}+07$ | $6.4 \mathrm{E}+07$ | $5.8 \mathrm{E}+07$ | $5.9 \mathrm{E}+07$ | $4.8 \mathrm{E}+07$ |
| 19 | $2.4 \mathrm{E}+07$ | $2.3 \mathrm{E}+07$ | $3.8 \mathrm{E}+07$ | $3.7 \mathrm{E}+07$ | $5.8 \mathrm{E}+07$ | $5.5 \mathrm{E}+07$ | $5.5 \mathrm{E}+07$ | $4.5 \mathrm{E}+07$ |
| 20 | $2.2 \mathrm{E}+07$ | $2.2 \mathrm{E}+07$ | $3.4 \mathrm{E}+07$ | $3.4 \mathrm{E}+07$ | $5.3 \mathrm{E}+07$ | $5.1 \mathrm{E}+07$ | $5.1 \mathrm{E}+07$ | $4.3 \mathrm{E}+07$ |
| 21 | $2.0 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $3.1 \mathrm{E}+07$ | $4.7 \mathrm{E}+07$ | $4.7 \mathrm{E}+07$ | $4.7 \mathrm{E}+07$ | $4.0 \mathrm{E}+07$ |
| 22 | $1.8 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ | $2.8 \mathrm{E}+07$ | $4.2 \mathrm{E}+07$ | $4.2 \mathrm{E}+07$ | $4.3 \mathrm{E}+07$ | $3.7 \mathrm{E}+07$ |
| 23 | $1.7 \mathrm{E}+07$ | $1.6 \mathrm{E}+07$ | $2.4 \mathrm{E}+07$ | $2.5 \mathrm{E}+07$ | $3.7 \mathrm{E}+07$ | $3.8 \mathrm{E}+07$ | $3.9 \mathrm{E}+07$ | $3.4 \mathrm{E}+07$ |
| 24 | $1.5 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $2.2 \mathrm{E}+07$ | $3.2 \mathrm{E}+07$ | $3.4 \mathrm{E}+07$ | $3.5 \mathrm{E}+07$ | $3.1 \mathrm{E}+07$ |
| 25 | $1.3 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $2.8 \mathrm{E}+07$ | $2.9 \mathrm{E}+07$ | $3.1 \mathrm{E}+07$ | $2.8 \mathrm{E}+07$ |
| 26 | $1.2 \mathrm{E}+07$ | $1.2 \mathrm{E}+07$ | $1.6 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ | $2.4 \mathrm{E}+07$ | $2.5 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ | $2.5 \mathrm{E}+07$ |
| 27 | $1.0 \mathrm{E}+07$ | $1.0 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $2.2 \mathrm{E}+07$ | $2.3 \mathrm{E}+07$ | $2.2 \mathrm{E}+07$ |
| 28 | $9.0 \mathrm{E}+06$ | $8.9 \mathrm{E}+06$ | $1.1 \mathrm{E}+07$ | $1.2 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $1.9 \mathrm{E}+07$ |
| 29 | $7.8 \mathrm{E}+06$ | $7.7 \mathrm{E}+06$ | $9.6 \mathrm{E}+06$ | $1.0 \mathrm{E}+07$ | $1.4 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ | $1.6 \mathrm{E}+07$ |
| 30 | $6.6 \mathrm{E}+06$ | $6.5 \mathrm{E}+06$ | $8.1 \mathrm{E}+06$ | $8.7 \mathrm{E}+06$ | $1.2 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $1.4 \mathrm{E}+07$ | $1.4 \mathrm{E}+07$ |
| 31 | $5.5 \mathrm{E}+06$ | $5.5 \mathrm{E}+06$ | $6.7 \mathrm{E}+06$ | $7.2 \mathrm{E}+06$ | $9.5 \mathrm{E}+06$ | $1.0 \mathrm{E}+07$ | $1.1 \mathrm{E}+07$ | $1.1 \mathrm{E}+07$ |
| 32 | $4.4 \mathrm{E}+06$ | $4.5 \mathrm{E}+06$ | $5.5 \mathrm{E}+06$ | $5.8 \mathrm{E}+06$ | $7.6 \mathrm{E}+06$ | $8.4 \mathrm{E}+06$ | $9.2 \mathrm{E}+06$ | $9.3 \mathrm{E}+06$ |
| 33 | $3.5 \mathrm{E}+06$ | $3.6 \mathrm{E}+06$ | $4.4 \mathrm{E}+06$ | $4.7 \mathrm{E}+06$ | $6.1 \mathrm{E}+06$ | $6.6 \mathrm{E}+06$ | $7.3 \mathrm{E}+06$ | $7.3 \mathrm{E}+06$ |
| 34 | $2.7 \mathrm{E}+06$ | $2.8 \mathrm{E}+06$ | $3.4 \mathrm{E}+06$ | $3.7 \mathrm{E}+06$ | $4.7 \mathrm{E}+06$ | $5.1 \mathrm{E}+06$ | $5.7 \mathrm{E}+06$ | $5.7 \mathrm{E}+06$ |
| 35 | $2.0 \mathrm{E}+06$ | $2.1 \mathrm{E}+06$ | $2.6 \mathrm{E}+06$ | $2.8 \mathrm{E}+06$ | $3.6 \mathrm{E}+06$ | $3.9 \mathrm{E}+06$ | $4.3 \mathrm{E}+06$ | $4.3 \mathrm{E}+06$ |
| 36 | $1.5 \mathrm{E}+06$ | $1.6 \mathrm{E}+06$ | $1.9 \mathrm{E}+06$ | $2.1 \mathrm{E}+06$ | $2.7 \mathrm{E}+06$ | $2.9 \mathrm{E}+06$ | $3.2 \mathrm{E}+06$ | $3.2 \mathrm{E}+06$ |
| 37 | $1.1 \mathrm{E}+06$ | $1.1 \mathrm{E}+06$ | $1.4 \mathrm{E}+06$ | $1.5 \mathrm{E}+06$ | $2.0 \mathrm{E}+06$ | $2.1 \mathrm{E}+06$ | $2.3 \mathrm{E}+06$ | $2.3 \mathrm{E}+06$ |
| 38 | $7.6 \mathrm{E}+05$ | $7.8 \mathrm{E}+05$ | $9.8 \mathrm{E}+05$ | $1.0 \mathrm{E}+06$ | $1.4 \mathrm{E}+06$ | $1.5 \mathrm{E}+06$ | $1.6 \mathrm{E}+06$ | $1.6 \mathrm{E}+06$ |
| 39 | $5.3 \mathrm{E}+05$ | $5.3 \mathrm{E}+05$ | $6.8 \mathrm{E}+05$ | $7.2 \mathrm{E}+05$ | $9.7 \mathrm{E}+05$ | $1.0 \mathrm{E}+06$ | $1.1 \mathrm{E}+06$ | $1.1 \mathrm{E}+06$ |
| 40 | $3.6 \mathrm{E}+05$ | $3.6 \mathrm{E}+05$ | $4.7 \mathrm{E}+05$ | $5.0 \mathrm{E}+05$ | $6.8 \mathrm{E}+05$ | $7.2 \mathrm{E}+05$ | $7.7 \mathrm{E}+05$ | $7.4 \mathrm{E}+05$ |
| 41 | $2.5 \mathrm{E}+05$ | $2.5 \mathrm{E}+05$ | $3.3 \mathrm{E}+05$ | $3.5 \mathrm{E}+05$ | $4.8 \mathrm{E}+05$ | $5.0 \mathrm{E}+05$ | $5.4 \mathrm{E}+05$ | $5.1 \mathrm{E}+05$ |
| 42 | $1.8 \mathrm{E}+05$ | $1.8 \mathrm{E}+05$ | $2.3 \mathrm{E}+05$ | $2.5 \mathrm{E}+05$ | $3.4 \mathrm{E}+05$ | $3.6 \mathrm{E}+05$ | $3.8 \mathrm{E}+05$ | $3.6 \mathrm{E}+05$ |
| 43 | $1.3 \mathrm{E}+05$ | $1.3 \mathrm{E}+05$ | $1.7 \mathrm{E}+05$ | $1.8 \mathrm{E}+05$ | $2.5 \mathrm{E}+05$ | $2.6 \mathrm{E}+05$ | $2.8 \mathrm{E}+05$ | $2.6 \mathrm{E}+05$ |
| 44 | $9.5 \mathrm{E}+04$ | $9.4 \mathrm{E}+04$ | $1.3 \mathrm{E}+05$ | $1.3 \mathrm{E}+05$ | $1.9 \mathrm{E}+05$ | $1.9 \mathrm{E}+05$ | $2.1 \mathrm{E}+05$ | $2.0 \mathrm{E}+05$ |
| 45 | $7.1 \mathrm{E}+04$ | $7.1 \mathrm{E}+04$ | $9.5 \mathrm{E}+04$ | $1.0 \mathrm{E}+05$ | $1.4 \mathrm{E}+05$ | $1.5 \mathrm{E}+05$ | $1.6 \mathrm{E}+05$ | $1.5 \mathrm{E}+05$ |
| 46 | $5.3 \mathrm{E}+04$ | $5.3 \mathrm{E}+04$ | $7.1 \mathrm{E}+04$ | $7.5 \mathrm{E}+04$ | $1.0 \mathrm{E}+05$ | $1.1 \mathrm{E}+05$ | $1.2 \mathrm{E}+05$ | $1.1 \mathrm{E}+05$ |
| 47 | $4.0 \mathrm{E}+04$ | $4.0 \mathrm{E}+04$ | $5.3 \mathrm{E}+04$ | $5.6 \mathrm{E}+04$ | $7.7 \mathrm{E}+04$ | $8.1 \mathrm{E}+04$ | $8.7 \mathrm{E}+04$ | $8.2 \mathrm{E}+04$ |
| 48 | $3.0 \mathrm{E}+04$ | $3.0 \mathrm{E}+04$ | $3.9 \mathrm{E}+04$ | $4.1 \mathrm{E}+04$ | $5.7 \mathrm{E}+04$ | $6.0 \mathrm{E}+04$ | $6.5 \mathrm{E}+04$ | $6.1 \mathrm{E}+04$ |
| 49 | $2.2 \mathrm{E}+04$ | $2.2 \mathrm{E}+04$ | $2.9 \mathrm{E}+04$ | $3.1 \mathrm{E}+04$ | $4.2 \mathrm{E}+04$ | $4.5 \mathrm{E}+04$ | $4.8 \mathrm{E}+04$ | $4.6 \mathrm{E}+04$ |

(Continued Table 7.9) Estimated stock size at length at time for the North Sea grey gurnard

| Length/year | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $4.2 \mathrm{E}+06$ | $2.9 \mathrm{E}+06$ | $4.3 \mathrm{E}+06$ | $4.9 \mathrm{E}+06$ | $5.9 \mathrm{E}+06$ | $3.6 \mathrm{E}+06$ | $2.3 \mathrm{E}+06$ | $3.8 \mathrm{E}+06$ |
| 4 | $1.0 \mathrm{E}+07$ | $7.0 \mathrm{E}+06$ | $1.0 \mathrm{E}+07$ | $1.2 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ | $8.9 \mathrm{E}+06$ | $5.6 \mathrm{E}+06$ | $9.3 \mathrm{E}+06$ |
| 5 | $2.0 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $2.3 \mathrm{E}+07$ | $2.8 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ | $1.1 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ |
| 6 | $3.1 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $3.2 \mathrm{E}+07$ | $3.6 \mathrm{E}+07$ | $4.4 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ | $2.8 \mathrm{E}+07$ |
| 7 | $4.4 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $4.4 \mathrm{E}+07$ | $5.1 \mathrm{E}+07$ | $6.2 \mathrm{E}+07$ | $3.7 \mathrm{E}+07$ | $2.4 \mathrm{E}+07$ | $3.9 \mathrm{E}+07$ |
| 8 | $5.6 \mathrm{E}+07$ | $3.8 \mathrm{E}+07$ | $5.7 \mathrm{E}+07$ | $6.5 \mathrm{E}+07$ | $7.9 \mathrm{E}+07$ | $4.8 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $5.0 \mathrm{E}+07$ |
| 9 | $6.6 \mathrm{E}+07$ | $4.5 \mathrm{E}+07$ | $6.7 \mathrm{E}+07$ | $7.7 \mathrm{E}+07$ | $9.3 \mathrm{E}+07$ | $5.7 \mathrm{E}+07$ | $3.6 \mathrm{E}+07$ | $6.0 \mathrm{E}+07$ |
| 10 | $7.4 \mathrm{E}+07$ | $5.1 \mathrm{E}+07$ | $7.5 \mathrm{E}+07$ | $8.7 \mathrm{E}+07$ | $1.0 \mathrm{E}+08$ | $6.4 \mathrm{E}+07$ | $4.0 \mathrm{E}+07$ | $6.7 \mathrm{E}+07$ |
| 11 | $7.9 \mathrm{E}+07$ | $5.5 \mathrm{E}+07$ | $8.1 \mathrm{E}+07$ | $9.4 \mathrm{E}+07$ | $1.1 \mathrm{E}+08$ | $7.0 \mathrm{E}+07$ | $4.4 \mathrm{E}+07$ | $7.2 \mathrm{E}+07$ |
| 12 | $8.2 \mathrm{E}+07$ | $5.8 \mathrm{E}+07$ | $8.4 \mathrm{E}+07$ | $9.7 \mathrm{E}+07$ | $1.2 \mathrm{E}+08$ | $7.4 \mathrm{E}+07$ | $4.6 \mathrm{E}+07$ | $7.4 \mathrm{E}+07$ |
| 13 | $8.3 \mathrm{E}+07$ | $6.0 \mathrm{E}+07$ | $8.4 \mathrm{E}+07$ | $9.9 \mathrm{E}+07$ | $1.2 \mathrm{E}+08$ | $7.6 \mathrm{E}+07$ | $4.8 \mathrm{E}+07$ | $7.5 \mathrm{E}+07$ |
| 14 | $8.1 \mathrm{E}+07$ | $6.0 \mathrm{E}+07$ | $8.3 \mathrm{E}+07$ | $9.8 \mathrm{E}+07$ | $1.2 \mathrm{E}+08$ | $7.8 \mathrm{E}+07$ | $4.9 \mathrm{E}+07$ | $7.3 \mathrm{E}+07$ |
| 15 | $7.8 \mathrm{E}+07$ | $6.0 \mathrm{E}+07$ | $8.1 \mathrm{E}+07$ | $9.5 \mathrm{E}+07$ | $1.1 \mathrm{E}+08$ | $7.8 \mathrm{E}+07$ | $4.9 \mathrm{E}+07$ | $7.1 \mathrm{E}+07$ |
| 16 | $7.4 \mathrm{E}+07$ | $5.9 \mathrm{E}+07$ | $7.7 \mathrm{E}+07$ | $9.2 \mathrm{E}+07$ | $1.1 \mathrm{E}+08$ | $7.8 \mathrm{E}+07$ | $4.9 \mathrm{E}+07$ | $6.8 \mathrm{E}+07$ |
| 17 | $7.0 \mathrm{E}+07$ | $5.8 \mathrm{E}+07$ | $7.3 \mathrm{E}+07$ | $8.7 \mathrm{E}+07$ | $1.0 \mathrm{E}+08$ | $7.6 \mathrm{E}+07$ | $4.9 \mathrm{E}+07$ | $6.4 \mathrm{E}+07$ |
| 18 | $6.5 \mathrm{E}+07$ | $5.5 \mathrm{E}+07$ | $6.8 \mathrm{E}+07$ | $8.1 \mathrm{E}+07$ | $9.8 \mathrm{E}+07$ | $7.4 \mathrm{E}+07$ | $4.8 \mathrm{E}+07$ | $5.9 \mathrm{E}+07$ |
| 19 | $6.0 \mathrm{E}+07$ | $5.2 \mathrm{E}+07$ | $6.3 \mathrm{E}+07$ | $7.5 \mathrm{E}+07$ | $9.0 \mathrm{E}+07$ | $7.1 \mathrm{E}+07$ | $4.7 \mathrm{E}+07$ | $5.5 \mathrm{E}+07$ |
| 20 | $5.5 \mathrm{E}+07$ | $4.9 \mathrm{E}+07$ | $5.8 \mathrm{E}+07$ | $6.9 \mathrm{E}+07$ | $8.3 \mathrm{E}+07$ | $6.7 \mathrm{E}+07$ | $4.6 \mathrm{E}+07$ | $5.1 \mathrm{E}+07$ |
| 21 | $5.0 \mathrm{E}+07$ | $4.5 \mathrm{E}+07$ | $5.3 \mathrm{E}+07$ | $6.2 \mathrm{E}+07$ | $7.5 \mathrm{E}+07$ | $6.3 \mathrm{E}+07$ | $4.4 \mathrm{E}+07$ | $4.7 \mathrm{E}+07$ |
| 22 | $4.5 \mathrm{E}+07$ | $4.2 \mathrm{E}+07$ | $4.8 \mathrm{E}+07$ | $5.6 \mathrm{E}+07$ | $6.7 \mathrm{E}+07$ | $5.8 \mathrm{E}+07$ | $4.2 \mathrm{E}+07$ | $4.3 \mathrm{E}+07$ |
| 23 | $4.0 \mathrm{E}+07$ | $3.8 \mathrm{E}+07$ | $4.3 \mathrm{E}+07$ | $5.0 \mathrm{E}+07$ | $6.0 \mathrm{E}+07$ | $5.3 \mathrm{E}+07$ | $4.0 \mathrm{E}+07$ | $3.9 \mathrm{E}+07$ |
| 24 | $3.6 \mathrm{E}+07$ | $3.4 \mathrm{E}+07$ | $3.8 \mathrm{E}+07$ | $4.4 \mathrm{E}+07$ | $5.3 \mathrm{E}+07$ | $4.8 \mathrm{E}+07$ | $3.7 \mathrm{E}+07$ | $3.6 \mathrm{E}+07$ |
| 25 | $3.2 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $3.4 \mathrm{E}+07$ | $3.9 \mathrm{E}+07$ | $4.6 \mathrm{E}+07$ | $4.3 \mathrm{E}+07$ | $3.4 \mathrm{E}+07$ | $3.3 \mathrm{E}+07$ |
| 26 | $2.8 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ | $3.4 \mathrm{E}+07$ | $4.0 \mathrm{E}+07$ | $3.8 \mathrm{E}+07$ | $3.1 \mathrm{E}+07$ | $3.0 \mathrm{E}+07$ |
| 27 | $2.5 \mathrm{E}+07$ | $2.3 \mathrm{E}+07$ | $2.6 \mathrm{E}+07$ | $2.9 \mathrm{E}+07$ | $3.4 \mathrm{E}+07$ | $3.3 \mathrm{E}+07$ | $2.8 \mathrm{E}+07$ | $2.7 \mathrm{E}+07$ |
| 28 | $2.1 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $2.2 \mathrm{E}+07$ | $2.5 \mathrm{E}+07$ | $2.9 \mathrm{E}+07$ | $2.8 \mathrm{E}+07$ | $2.5 \mathrm{E}+07$ | $2.4 \mathrm{E}+07$ |
| 29 | $1.8 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ | $1.9 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $2.5 \mathrm{E}+07$ | $2.4 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ |
| 30 | $1.5 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ | $1.6 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ | $2.1 \mathrm{E}+07$ | $2.0 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ | $1.8 \mathrm{E}+07$ |
| 31 | $1.3 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ | $1.7 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ | $1.5 \mathrm{E}+07$ |
| 32 | $1.0 \mathrm{E}+07$ | $1.0 \mathrm{E}+07$ | $1.1 \mathrm{E}+07$ | $1.2 \mathrm{E}+07$ | $1.4 \mathrm{E}+07$ | $1.4 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ | $1.3 \mathrm{E}+07$ |
| 33 | $8.2 \mathrm{E}+06$ | $8.2 \mathrm{E}+06$ | $8.9 \mathrm{E}+06$ | $9.8 \mathrm{E}+06$ | $1.1 \mathrm{E}+07$ | $1.1 \mathrm{E}+07$ | $1.0 \mathrm{E}+07$ | $1.0 \mathrm{E}+07$ |
| 34 | $6.4 \mathrm{E}+06$ | $6.5 \mathrm{E}+06$ | $7.0 \mathrm{E}+06$ | $7.7 \mathrm{E}+06$ | $8.7 \mathrm{E}+06$ | $8.6 \mathrm{E}+06$ | $8.0 \mathrm{E}+06$ | $8.0 \mathrm{E}+06$ |
| 35 | $4.9 \mathrm{E}+06$ | $4.9 \mathrm{E}+06$ | $5.3 \mathrm{E}+06$ | $5.9 \mathrm{E}+06$ | $6.7 \mathrm{E}+06$ | $6.6 \mathrm{E}+06$ | $6.1 \mathrm{E}+06$ | $6.2 \mathrm{E}+06$ |
| 36 | $3.6 \mathrm{E}+06$ | $3.6 \mathrm{E}+06$ | $4.0 \mathrm{E}+06$ | $4.4 \mathrm{E}+06$ | $5.0 \mathrm{E}+06$ | $4.9 \mathrm{E}+06$ | $4.5 \mathrm{E}+06$ | $4.6 \mathrm{E}+06$ |
| 37 | $2.6 \mathrm{E}+06$ | $2.6 \mathrm{E}+06$ | $2.8 \mathrm{E}+06$ | $3.1 \mathrm{E}+06$ | $3.6 \mathrm{E}+06$ | $3.5 \mathrm{E}+06$ | $3.2 \mathrm{E}+06$ | $3.3 \mathrm{E}+06$ |
| 38 | $1.8 \mathrm{E}+06$ | $1.8 \mathrm{E}+06$ | $2.0 \mathrm{E}+06$ | $2.2 \mathrm{E}+06$ | $2.5 \mathrm{E}+06$ | $2.4 \mathrm{E}+06$ | $2.2 \mathrm{E}+06$ | $2.2 \mathrm{E}+06$ |
| 39 | $1.2 \mathrm{E}+06$ | $1.2 \mathrm{E}+06$ | $1.3 \mathrm{E}+06$ | $1.5 \mathrm{E}+06$ | $1.7 \mathrm{E}+06$ | $1.7 \mathrm{E}+06$ | $1.5 \mathrm{E}+06$ | $1.5 \mathrm{E}+06$ |
| 40 | $8.5 \mathrm{E}+05$ | $8.3 \mathrm{E}+05$ | $9.2 \mathrm{E}+05$ | $1.0 \mathrm{E}+06$ | $1.2 \mathrm{E}+06$ | $1.1 \mathrm{E}+06$ | $9.9 \mathrm{E}+05$ | $1.0 \mathrm{E}+06$ |
| 41 | $5.9 \mathrm{E}+05$ | $5.7 \mathrm{E}+05$ | $6.4 \mathrm{E}+05$ | $7.2 \mathrm{E}+05$ | $8.3 \mathrm{E}+05$ | $7.8 \mathrm{E}+05$ | $6.7 \mathrm{E}+05$ | $6.8 \mathrm{E}+05$ |
| 42 | $4.2 \mathrm{E}+05$ | $4.0 \mathrm{E}+05$ | $4.5 \mathrm{E}+05$ | $5.1 \mathrm{E}+05$ | $5.9 \mathrm{E}+05$ | $5.5 \mathrm{E}+05$ | $4.6 \mathrm{E}+05$ | $4.7 \mathrm{E}+05$ |
| 43 | $3.0 \mathrm{E}+05$ | $2.9 \mathrm{E}+05$ | $3.2 \mathrm{E}+05$ | $3.7 \mathrm{E}+05$ | $4.3 \mathrm{E}+05$ | $3.9 \mathrm{E}+05$ | $3.3 \mathrm{E}+05$ | $3.4 \mathrm{E}+05$ |
| 44 | $2.3 \mathrm{E}+05$ | $2.2 \mathrm{E}+05$ | $2.4 \mathrm{E}+05$ | $2.7 \mathrm{E}+05$ | $3.2 \mathrm{E}+05$ | $2.9 \mathrm{E}+05$ | $2.5 \mathrm{E}+05$ | $2.5 \mathrm{E}+05$ |
| 45 | $1.7 \mathrm{E}+05$ | $1.6 \mathrm{E}+05$ | $1.8 \mathrm{E}+05$ | $2.1 \mathrm{E}+05$ | $2.4 \mathrm{E}+05$ | $2.2 \mathrm{E}+05$ | $1.9 \mathrm{E}+05$ | $1.9 \mathrm{E}+05$ |
| 46 | $1.3 \mathrm{E}+05$ | $1.2 \mathrm{E}+05$ | $1.4 \mathrm{E}+05$ | $1.5 \mathrm{E}+05$ | $1.8 \mathrm{E}+05$ | $1.7 \mathrm{E}+05$ | $1.4 \mathrm{E}+05$ | $1.4 \mathrm{E}+05$ |
| 47 | $9.5 \mathrm{E}+04$ | $9.1 \mathrm{E}+04$ | $1.0 \mathrm{E}+05$ | $1.1 \mathrm{E}+05$ | $1.3 \mathrm{E}+05$ | $1.2 \mathrm{E}+05$ | $1.1 \mathrm{E}+05$ | $1.1 \mathrm{E}+05$ |
| 48 | $7.1 \mathrm{E}+04$ | $6.8 \mathrm{E}+04$ | $7.6 \mathrm{E}+04$ | $8.5 \mathrm{E}+04$ | $9.9 \mathrm{E}+04$ | $9.3 \mathrm{E}+04$ | $7.9 \mathrm{E}+04$ | $8.0 \mathrm{E}+04$ |
| 49 | $5.3 \mathrm{E}+04$ | $5.1 \mathrm{E}+04$ | $5.6 \mathrm{E}+04$ | $6.3 \mathrm{E}+04$ | $7.3 \mathrm{E}+04$ | $6.9 \mathrm{E}+04$ | $5.9 \mathrm{E}+04$ | $6.0 \mathrm{E}+04$ |

Table 7.10: Estimated total stock biomass (TSB), total catch biomass (TCB), total landed biomass (TLB), total discards biomass (TDB) and total survey biomass (TSurB) in tonnes for the North Sea grey gurnard

| Year | TSB | TCB | TLB | TDB | TSurB |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1979 | $6.9 \mathrm{E}+06$ | $3.3 \mathrm{E}+03$ | $6.1 \mathrm{E}+02$ | $2.7 \mathrm{E}+03$ | $4.6 \mathrm{E}+02$ |
| 1980 | $7.6 \mathrm{E}+06$ | $5.7 \mathrm{E}+03$ | $1.2 \mathrm{E}+03$ | $4.6 \mathrm{E}+03$ | $4.6 \mathrm{E}+02$ |
| 1981 | $3.3 \mathrm{E}+07$ | $8.9 \mathrm{E}+03$ | $1.8 \mathrm{E}+03$ | $7.2 \mathrm{E}+03$ | $1.5 \mathrm{E}+03$ |
| 1982 | $1.8 \mathrm{E}+07$ | $9.6 \mathrm{E}+03$ | $2.2 \mathrm{E}+03$ | $7.4 \mathrm{E}+03$ | $9.7 \mathrm{E}+02$ |
| 1983 | $2.4 \mathrm{E}+07$ | $9.5 \mathrm{E}+03$ | $2.2 \mathrm{E}+03$ | $7.3 \mathrm{E}+03$ | $1.4 \mathrm{E}+03$ |
| 1984 | $2.5 \mathrm{E}+07$ | $9.8 \mathrm{E}+03$ | $2.3 \mathrm{E}+03$ | $7.5 \mathrm{E}+03$ | $1.5 \mathrm{E}+03$ |
| 1985 | $1.4 \mathrm{E}+07$ | $7.9 \mathrm{E}+03$ | $2.1 \mathrm{E}+03$ | $5.7 \mathrm{E}+03$ | $8.8 \mathrm{E}+02$ |
| 1986 | $2.0 \mathrm{E}+07$ | $7.6 \mathrm{E}+03$ | $1.9 \mathrm{E}+03$ | $5.7 \mathrm{E}+03$ | $1.3 \mathrm{E}+03$ |
| 1987 | $1.2 \mathrm{E}+07$ | $5.8 \mathrm{E}+03$ | $1.6 \mathrm{E}+03$ | $4.2 \mathrm{E}+03$ | $8.0 \mathrm{E}+02$ |
| 1988 | $6.9 \mathrm{E}+06$ | $5.5 \mathrm{E}+03$ | $1.7 \mathrm{E}+03$ | $3.8 \mathrm{E}+03$ | $2.8 \mathrm{E}+02$ |
| 1989 | $1.5 \mathrm{E}+07$ | $5.8 \mathrm{E}+03$ | $1.5 \mathrm{E}+03$ | $4.3 \mathrm{E}+03$ | $7.8 \mathrm{E}+02$ |
| 1990 | $2.5 \mathrm{E}+07$ | $6.1 \mathrm{E}+03$ | $1.4 \mathrm{E}+03$ | $4.7 \mathrm{E}+03$ | $1.3 \mathrm{E}+03$ |
| 1991 | $3.0 \mathrm{E}+07$ | $7.3 \mathrm{E}+03$ | $1.7 \mathrm{E}+03$ | $5.7 \mathrm{E}+03$ | $1.9 \mathrm{E}+03$ |
| 1992 | $3.8 \mathrm{E}+07$ | $5.8 \mathrm{E}+03$ | $1.3 \mathrm{E}+03$ | $4.5 \mathrm{E}+03$ | $2.1 \mathrm{E}+03$ |
| 1993 | $3.0 \mathrm{E}+07$ | $6.4 \mathrm{E}+03$ | $1.5 \mathrm{E}+03$ | $4.8 \mathrm{E}+03$ | $1.6 \mathrm{E}+03$ |
| 1994 | $3.2 \mathrm{E}+07$ | $6.1 \mathrm{E}+03$ | $1.5 \mathrm{E}+03$ | $4.6 \mathrm{E}+03$ | $1.6 \mathrm{E}+03$ |
| 1995 | $3.5 \mathrm{E}+07$ | $6.4 \mathrm{E}+03$ | $1.6 \mathrm{E}+03$ | $4.8 \mathrm{E}+03$ | $1.6 \mathrm{E}+03$ |
| 1996 | $3.4 \mathrm{E}+07$ | $6.8 \mathrm{E}+03$ | $1.7 \mathrm{E}+03$ | $5.2 \mathrm{E}+03$ | $1.5 \mathrm{E}+03$ |
| 1997 | $4.9 \mathrm{E}+07$ | $8.0 \mathrm{E}+03$ | $1.9 \mathrm{E}+03$ | $6.1 \mathrm{E}+03$ | $2.4 \mathrm{E}+03$ |
| 1998 | $5.0 \mathrm{E}+07$ | $7.9 \mathrm{E}+03$ | $1.9 \mathrm{E}+03$ | $6.1 \mathrm{E}+03$ | $2.7 \mathrm{E}+03$ |
| 1999 | $7.4 \mathrm{E}+07$ | $1.2 \mathrm{E}+04$ | $2.7 \mathrm{E}+03$ | $9.0 \mathrm{E}+03$ | $3.2 \mathrm{E}+03$ |
| 2000 | $7.3 \mathrm{E}+07$ | $9.5 \mathrm{E}+03$ | $2.2 \mathrm{E}+03$ | $7.3 \mathrm{E}+03$ | $3.4 \mathrm{E}+03$ |
| 2001 | $7.8 \mathrm{E}+07$ | $7.2 \mathrm{E}+03$ | $1.7 \mathrm{E}+03$ | $5.5 \mathrm{E}+03$ | $3.7 \mathrm{E}+03$ |
| 2002 | $7.0 \mathrm{E}+07$ | $9.5 \mathrm{E}+03$ | $2.3 \mathrm{E}+03$ | $7.1 \mathrm{E}+03$ | $3.3 \mathrm{E}+03$ |
| 2003 | $8.5 \mathrm{E}+07$ | $8.9 \mathrm{E}+03$ | $2.1 \mathrm{E}+03$ | $6.8 \mathrm{E}+03$ | $3.9 \mathrm{E}+03$ |
| 2004 | $7.8 \mathrm{E}+07$ | $1.0 \mathrm{E}+04$ | $2.5 \mathrm{E}+03$ | $7.6 \mathrm{E}+03$ | $3.3 \mathrm{E}+03$ |
| 2005 | $9.0 \mathrm{E}+07$ | $1.1 \mathrm{E}+04$ | $2.6 \mathrm{E}+03$ | $8.1 \mathrm{E}+03$ | $3.9 \mathrm{E}+03$ |
| 2006 | $1.0 \mathrm{E}+08$ | $9.9 \mathrm{E}+03$ | $2.4 \mathrm{E}+03$ | $7.5 \mathrm{E}+03$ | $4.5 \mathrm{E}+03$ |
| 2007 | $1.2 \mathrm{E}+08$ | $1.2 \mathrm{E}+04$ | $2.7 \mathrm{E}+03$ | $8.9 \mathrm{E}+03$ | $5.0 \mathrm{E}+03$ |
| 2008 | $1.1 \mathrm{E}+08$ | $9.4 \mathrm{E}+03$ | $2.3 \mathrm{E}+03$ | $7.1 \mathrm{E}+03$ | $4.5 \mathrm{E}+03$ |
| 2009 | $8.4 \mathrm{E}+07$ | $9.0 \mathrm{E}+03$ | $2.3 \mathrm{E}+03$ | $6.6 \mathrm{E}+03$ | $3.4 \mathrm{E}+03$ |
| 2010 | $8.8 \mathrm{E}+07$ | $8.6 \mathrm{E}+03$ | $2.2 \mathrm{E}+03$ | $6.4 \mathrm{E}+03$ | $3.7 \mathrm{E}+03$ |

Table 7.11: Probability transition matrix; Probability of a gurnard at size (column) growing to size (row)

| Growing to length (cm) | From Length (cm) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 3 | 1.1E-06 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 4 | 7.8E-04 | $1.8 \mathrm{E}-06$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 5 | $1.7 \mathrm{E}-02$ | $1.1 \mathrm{E}-03$ | 3.1E-06 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 6 | 8.6E-02 | 2.2E-02 | $1.5 \mathrm{E}-03$ | 5.3E-06 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 7 | 1.9E-01 | $9.9 \mathrm{E}-02$ | $2.7 \mathrm{E}-02$ | $2.1 \mathrm{E}-03$ | $8.9 \mathrm{E}-06$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 8 | 2.3E-01 | $2.0 \mathrm{E}-01$ | $1.1 \mathrm{E}-01$ | $3.3 \mathrm{E}-02$ | $3.0 \mathrm{E}-03$ | $1.5 \mathrm{E}-05$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 9 | $2.1 \mathrm{E}-01$ | $2.4 \mathrm{E}-01$ | $2.1 \mathrm{E}-01$ | $1.3 \mathrm{E}-01$ | $4.1 \mathrm{E}-02$ | $4.1 \mathrm{E}-03$ | $2.5 \mathrm{E}-05$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 10 | $1.4 \mathrm{E}-01$ | $2.0 \mathrm{E}-01$ | $2.4 \mathrm{E}-01$ | $2.2 \mathrm{E}-01$ | $1.4 \mathrm{E}-01$ | $5.0 \mathrm{E}-02$ | $5.5 \mathrm{E}-03$ | $4.0 \mathrm{E}-05$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 11 | $7.6 \mathrm{E}-02$ | $1.3 \mathrm{E}-01$ | $1.9 \mathrm{E}-01$ | $2.4 \mathrm{E}-01$ | $2.3 \mathrm{E}-01$ | $1.6 \mathrm{E}-01$ | $6.0 \mathrm{E}-02$ | $7.4 \mathrm{E}-03$ | $6.6 \mathrm{E}-05$ | $0.0 \mathrm{E}+00$ |
| 12 | 3.6E-02 | $6.8 \mathrm{E}-02$ | $1.2 \mathrm{E}-01$ | $1.8 \mathrm{E}-01$ | $2.4 \mathrm{E}-01$ | $2.4 \mathrm{E}-01$ | $1.8 \mathrm{E}-01$ | $7.2 \mathrm{E}-02$ | $1.0 \mathrm{E}-02$ | $1.1 \mathrm{E}-04$ |
| 13 | $1.5 \mathrm{E}-02$ | $3.1 \mathrm{E}-02$ | $6.0 \mathrm{E}-02$ | $1.1 \mathrm{E}-01$ | $1.7 \mathrm{E}-01$ | $2.3 \mathrm{E}-01$ | $2.5 \mathrm{E}-01$ | $1.9 \mathrm{E}-01$ | 8.6E-02 | $1.3 \mathrm{E}-02$ |
| 14 | 5.6E-03 | $1.3 \mathrm{E}-02$ | $2.7 \mathrm{E}-02$ | $5.3 \mathrm{E}-02$ | $9.7 \mathrm{E}-02$ | $1.6 \mathrm{E}-01$ | $2.3 \mathrm{E}-01$ | $2.6 \mathrm{E}-01$ | $2.1 \mathrm{E}-01$ | $1.0 \mathrm{E}-01$ |
| 15 | $1.9 \mathrm{E}-03$ | $4.6 \mathrm{E}-03$ | $1.0 \mathrm{E}-02$ | $2.3 \mathrm{E}-02$ | $4.6 \mathrm{E}-02$ | $8.7 \mathrm{E}-02$ | $1.5 \mathrm{E}-01$ | $2.2 \mathrm{E}-01$ | 2.6E-01 | $2.3 \mathrm{E}-01$ |
| 16 | 6.2E-04 | $1.5 \mathrm{E}-03$ | $3.7 \mathrm{E}-03$ | $8.6 \mathrm{E}-03$ | $1.9 \mathrm{E}-02$ | $4.0 \mathrm{E}-02$ | $7.8 \mathrm{E}-02$ | $1.4 \mathrm{E}-01$ | $2.1 \mathrm{E}-01$ | $2.6 \mathrm{E}-01$ |
| 17 | $1.9 \mathrm{E}-04$ | $4.8 \mathrm{E}-04$ | $1.2 \mathrm{E}-03$ | $3.0 \mathrm{E}-03$ | $7.1 \mathrm{E}-03$ | $1.6 \mathrm{E}-02$ | $3.4 \mathrm{E}-02$ | $6.9 \mathrm{E}-02$ | $1.3 \mathrm{E}-01$ | $2.0 \mathrm{E}-01$ |
| 18 | 5.3E-05 | $1.4 \mathrm{E}-04$ | $3.7 \mathrm{E}-04$ | $9.6 \mathrm{E}-04$ | $2.4 \mathrm{E}-03$ | 5.8E-03 | $1.3 \mathrm{E}-02$ | $2.9 \mathrm{E}-02$ | $6.0 \mathrm{E}-02$ | $1.1 \mathrm{E}-01$ |
| 19 | $1.5 \mathrm{E}-05$ | $4.0 \mathrm{E}-05$ | $1.1 \mathrm{E}-04$ | $2.9 \mathrm{E}-04$ | $7.5 \mathrm{E}-04$ | $1.9 \mathrm{E}-03$ | $4.7 \mathrm{E}-03$ | $1.1 \mathrm{E}-02$ | $2.5 \mathrm{E}-02$ | $5.2 \mathrm{E}-02$ |
| 20 | 3.8E-06 | $1.1 \mathrm{E}-05$ | $3.0 \mathrm{E}-05$ | $8.1 \mathrm{E}-05$ | $2.2 \mathrm{E}-04$ | $5.8 \mathrm{E}-04$ | $1.5 \mathrm{E}-03$ | $3.7 \mathrm{E}-03$ | $9.0 \mathrm{E}-03$ | $2.1 \mathrm{E}-02$ |
| 21 | 9.6E-07 | 2.8E-06 | $7.8 \mathrm{E}-06$ | $2.2 \mathrm{E}-05$ | $6.1 \mathrm{E}-05$ | $1.7 \mathrm{E}-04$ | $4.4 \mathrm{E}-04$ | $1.2 \mathrm{E}-03$ | $3.0 \mathrm{E}-03$ | $7.3 \mathrm{E}-03$ |
| 22 | 2.3E-07 | $6.8 \mathrm{E}-07$ | $2.0 \mathrm{E}-06$ | $5.7 \mathrm{E}-06$ | $1.6 \mathrm{E}-05$ | $4.5 \mathrm{E}-05$ | $1.2 \mathrm{E}-04$ | $3.4 \mathrm{E}-04$ | $9.0 \mathrm{E}-04$ | $2.3 \mathrm{E}-03$ |
| 23 | 5.5E-08 | $1.6 \mathrm{E}-07$ | $4.8 \mathrm{E}-07$ | $1.4 \mathrm{E}-06$ | $4.1 \mathrm{E}-06$ | $1.2 \mathrm{E}-05$ | $3.3 \mathrm{E}-05$ | $9.3 \mathrm{E}-05$ | $2.6 \mathrm{E}-04$ | $6.9 \mathrm{E}-04$ |
| 24 | $1.3 \mathrm{E}-08$ | $3.8 \mathrm{E}-08$ | $1.1 \mathrm{E}-07$ | $3.4 \mathrm{E}-07$ | $9.9 \mathrm{E}-07$ | 2.9E-06 | 8.4E-06 | $2.4 \mathrm{E}-05$ | $6.8 \mathrm{E}-05$ | $1.9 \mathrm{E}-04$ |
| 25 | 2.9E-09 | $8.7 \mathrm{E}-09$ | $2.6 \mathrm{E}-08$ | $7.9 \mathrm{E}-08$ | $2.4 \mathrm{E}-07$ | $7.0 \mathrm{E}-07$ | $2.1 \mathrm{E}-06$ | $6.0 \mathrm{E}-06$ | $1.7 \mathrm{E}-05$ | $5.0 \mathrm{E}-05$ |
| 26 | $6.3 \mathrm{E}-10$ | $1.9 \mathrm{E}-09$ | $5.9 \mathrm{E}-09$ | $1.8 \mathrm{E}-08$ | $5.4 \mathrm{E}-08$ | $1.6 \mathrm{E}-07$ | $4.8 \mathrm{E}-07$ | $1.4 \mathrm{E}-06$ | $4.3 \mathrm{E}-06$ | $1.2 \mathrm{E}-05$ |
| 27 | $1.4 \mathrm{E}-10$ | $4.2 \mathrm{E}-10$ | $1.3 \mathrm{E}-09$ | $3.9 \mathrm{E}-09$ | $1.2 \mathrm{E}-08$ | $3.7 \mathrm{E}-08$ | $1.1 \mathrm{E}-07$ | 3.3E-07 | $1.0 \mathrm{E}-06$ | $3.0 \mathrm{E}-06$ |
| 28 | $2.9 \mathrm{E}-11$ | $9.0 \mathrm{E}-11$ | $2.8 \mathrm{E}-10$ | $8.5 \mathrm{E}-10$ | $2.6 \mathrm{E}-09$ | 8.1E-09 | $2.5 \mathrm{E}-08$ | $7.5 \mathrm{E}-08$ | $2.3 \mathrm{E}-07$ | $6.9 \mathrm{E}-07$ |
| 29 | $6.0 \mathrm{E}-12$ | $1.9 \mathrm{E}-11$ | $5.8 \mathrm{E}-11$ | $1.8 \mathrm{E}-10$ | $5.6 \mathrm{E}-10$ | $1.7 \mathrm{E}-09$ | $5.4 \mathrm{E}-09$ | $1.7 \mathrm{E}-08$ | $5.1 \mathrm{E}-08$ | $1.5 \mathrm{E}-07$ |
| 30 | $1.2 \mathrm{E}-12$ | $3.9 \mathrm{E}-12$ | $1.2 \mathrm{E}-11$ | $3.8 \mathrm{E}-11$ | $1.2 \mathrm{E}-10$ | $3.7 \mathrm{E}-10$ | $1.1 \mathrm{E}-09$ | $3.5 \mathrm{E}-09$ | $1.1 \mathrm{E}-08$ | $3.4 \mathrm{E}-08$ |
| 31 | $2.5 \mathrm{E}-13$ | $7.8 \mathrm{E}-13$ | $2.5 \mathrm{E}-12$ | $7.7 \mathrm{E}-12$ | $2.4 \mathrm{E}-11$ | $7.5 \mathrm{E}-11$ | $2.4 \mathrm{E}-10$ | $7.4 \mathrm{E}-10$ | $2.3 \mathrm{E}-09$ | $7.1 \mathrm{E}-09$ |
| 32 | $4.9 \mathrm{E}-14$ | $1.6 \mathrm{E}-13$ | $4.9 \mathrm{E}-13$ | $1.6 \mathrm{E}-12$ | $4.9 \mathrm{E}-12$ | $1.5 \mathrm{E}-11$ | $4.8 \mathrm{E}-11$ | $1.5 \mathrm{E}-10$ | $4.7 \mathrm{E}-10$ | $1.5 \mathrm{E}-09$ |
| 33 | $9.8 \mathrm{E}-15$ | $3.1 \mathrm{E}-14$ | $9.7 \mathrm{E}-14$ | $3.1 \mathrm{E}-13$ | $9.7 \mathrm{E}-13$ | $3.1 \mathrm{E}-12$ | $9.6 \mathrm{E}-12$ | $3.0 \mathrm{E}-11$ | $9.6 \mathrm{E}-11$ | $3.0 \mathrm{E}-10$ |
| 34 | $1.9 \mathrm{E}-15$ | $6.0 \mathrm{E}-15$ | $1.9 \mathrm{E}-14$ | $6.0 \mathrm{E}-14$ | $1.9 \mathrm{E}-13$ | $6.0 \mathrm{E}-13$ | $1.9 \mathrm{E}-12$ | $6.0 \mathrm{E}-12$ | $1.9 \mathrm{E}-11$ | $6.0 \mathrm{E}-11$ |
| 35 | $3.3 \mathrm{E}-16$ | $1.2 \mathrm{E}-15$ | $3.7 \mathrm{E}-15$ | $1.2 \mathrm{E}-14$ | $3.7 \mathrm{E}-14$ | $1.2 \mathrm{E}-13$ | $3.7 \mathrm{E}-13$ | $1.2 \mathrm{E}-12$ | $3.7 \mathrm{E}-12$ | $1.2 \mathrm{E}-11$ |
| 36 | $1.1 \mathrm{E}-16$ | $2.2 \mathrm{E}-16$ | $7.8 \mathrm{E}-16$ | $2.1 \mathrm{E}-15$ | $7.0 \mathrm{E}-15$ | $2.2 \mathrm{E}-14$ | $7.1 \mathrm{E}-14$ | $2.3 \mathrm{E}-13$ | $7.2 \mathrm{E}-13$ | $2.3 \mathrm{E}-12$ |
| 37 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $1.1 \mathrm{E}-16$ | $4.4 \mathrm{E}-16$ | $1.3 \mathrm{E}-15$ | $4.2 \mathrm{E}-15$ | $1.3 \mathrm{E}-14$ | $4.3 \mathrm{E}-14$ | $1.4 \mathrm{E}-13$ | $4.4 \mathrm{E}-13$ |
| 38 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $1.1 \mathrm{E}-16$ | $2.2 \mathrm{E}-16$ | $7.8 \mathrm{E}-16$ | $2.6 \mathrm{E}-15$ | $8.1 \mathrm{E}-15$ | $2.6 \mathrm{E}-14$ | $8.2 \mathrm{E}-14$ |
| 39 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $1.1 \mathrm{E}-16$ | $2.2 \mathrm{E}-16$ | $4.4 \mathrm{E}-16$ | $1.4 \mathrm{E}-15$ | $4.7 \mathrm{E}-15$ | $1.5 \mathrm{E}-14$ |
| 40 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $1.1 \mathrm{E}-16$ | $2.2 \mathrm{E}-16$ | $8.9 \mathrm{E}-16$ | $2.8 \mathrm{E}-15$ |
| 41 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $1.1 \mathrm{E}-16$ | $2.2 \mathrm{E}-16$ | $5.6 \mathrm{E}-16$ |
| 42 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $1.1 \mathrm{E}-16$ |
| 43 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 44 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 45 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 46 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 47 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 48 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |
| 49 | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ | $0.0 \mathrm{E}+00$ |

(Continued Table 7.11) Probability transition matrix; Probability of a gurnard at size (column) grow to size (row)

| Growing | From Length (cm) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| length (cm) | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 11 | 21 |
| 3 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 4 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 5 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 6 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 7 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 8 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 9 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 10 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 11 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 12 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 13 | $1.71 \mathrm{E}-04$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.71 \mathrm{E}-04$ |
| 14 | $1.74 \mathrm{E}-02$ | $2.71 \mathrm{E}-04$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.74 \mathrm{E}-02$ |
| 15 | $1.18 \mathrm{E}-01$ | $2.28 \mathrm{E}-02$ | $4.28 \mathrm{E}-04$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.18 \mathrm{E}-01$ |
| 16 | $2.44 \mathrm{E}-01$ | $1.37 \mathrm{E}-01$ | $2.96 \mathrm{E}-02$ | $6.69 \mathrm{E}-04$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $2.44 \mathrm{E}-01$ |
| 17 | $2.62 \mathrm{E}-01$ | $2.59 \mathrm{E}-01$ | $1.57 \mathrm{E}-01$ | $3.81 \mathrm{E}-02$ | $1.04 \mathrm{E}-03$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $2.62 \mathrm{E}-01$ |
| 18 | $1.87 \mathrm{E}-01$ | $2.59 \mathrm{E}-01$ | $2.72 \mathrm{E}-01$ | $1.79 \mathrm{E}-01$ | $4.85 \mathrm{E}-02$ | $1.59 \mathrm{E}-03$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.87 \mathrm{E}-01$ |
| 19 | $1.01 \mathrm{E}-01$ | $1.74 \mathrm{E}-01$ | $2.53 \mathrm{E}-01$ | $2.82 \mathrm{E}-01$ | $2.02 \mathrm{E}-01$ | $6.12 \mathrm{E}-02$ | $2.42 \mathrm{E}-03$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.01 \mathrm{E}-01$ |
| 20 | $4.49 \mathrm{E}-02$ | $8.97 \mathrm{E}-02$ | $1.61 \mathrm{E}-01$ | $2.45 \mathrm{E}-01$ | $2.90 \mathrm{E}-01$ | $2.25 \mathrm{E}-01$ | $7.66 \mathrm{E}-02$ | $3.63 \mathrm{E}-03$ | $0.00 \mathrm{E}+00$ | $4.49 \mathrm{E}-02$ |
| 21 | $1.72 \mathrm{E}-02$ | $3.82 \mathrm{E}-02$ | $7.89 \mathrm{E}-02$ | $1.47 \mathrm{E}-01$ | $2.34 \mathrm{E}-01$ | $2.95 \mathrm{E}-01$ | $2.48 \mathrm{E}-01$ | $9.48 \mathrm{E}-02$ | $5.41 \mathrm{E}-03$ | $1.72 \mathrm{E}-02$ |
| 22 | $5.84 \mathrm{E}-03$ | $1.41 \mathrm{E}-02$ | $3.22 \mathrm{E}-02$ | $6.86 \mathrm{E}-02$ | $1.33 \mathrm{E}-01$ | $2.22 \mathrm{E}-01$ | $2.97 \mathrm{E}-01$ | $2.71 \mathrm{E}-01$ | 1 | 03 |
| 23 | $1.81 \mathrm{E}-03$ | $4.64 \mathrm{E}-03$ | $1.14 \mathrm{E}-02$ | $2.69 \mathrm{E}-02$ | $5.91 \mathrm{E}-02$ | $1.18 \mathrm{E}-01$ | $2.08 \mathrm{E}-01$ | $2.95 \mathrm{E}-01$ | $2.92 \mathrm{E}-01$ | $1.81 \mathrm{E}-03$ |
| 24 | $5.22 \mathrm{E}-04$ | $1.40 \mathrm{E}-03$ | $3.65 \mathrm{E}-03$ | $9.21 \mathrm{E}-03$ | $2.22 \mathrm{E}-02$ | $5.03 \mathrm{E}-02$ | $1.05 \mathrm{E}-01$ | $1.92 \mathrm{E}-01$ | $2.90 \mathrm{E}-01$ | $5.22 \mathrm{E}-04$ |
| 25 | $1.41 \mathrm{E}-04$ | $3.92 \mathrm{E}-04$ | $1.07 \mathrm{E}-03$ | $2.84 \mathrm{E}-03$ | $7.33 \mathrm{E}-03$ | $1.81 \mathrm{E}-02$ | $4.23 \mathrm{E}-02$ | $9.14 \mathrm{E}-02$ | $1.76 \mathrm{E}-01$ | $1.41 \mathrm{E}-04$ |
| 26 | $3.61 \mathrm{E}-05$ | $1.03 \mathrm{E}-04$ | $2.92 \mathrm{E}-04$ | 8.08E-04 | $2.19 \mathrm{E}-03$ | $5.77 \mathrm{E}-03$ | $1.46 \mathrm{E}-02$ | $3.52 \mathrm{E}-02$ | $7.88 \mathrm{E}-02$ | $3.61 \mathrm{E}-05$ |
| 27 | $8.82 \mathrm{E}-06$ | $2.58 \mathrm{E}-05$ | $7.50 \mathrm{E}-05$ | $2.15 \mathrm{E}-04$ | $6.04 \mathrm{E}-04$ | $1.67 \mathrm{E}-03$ | $4.48 \mathrm{E}-03$ | $1.16 \mathrm{E}-02$ | $2.89 \mathrm{E}-02$ | 8.82E-06 |
| 28 | $2.07 \mathrm{E}-06$ | $6.19 \mathrm{E}-06$ | $1.83 \mathrm{E}-05$ | $5.38 \mathrm{E}-05$ | $1.56 \mathrm{E}-04$ | $4.47 \mathrm{E}-04$ | $1.26 \mathrm{E}-03$ | $3.45 \mathrm{E}-03$ | $9.17 \mathrm{E}-03$ | $2.07 \mathrm{E}-06$ |
| 29 | $4.70 \mathrm{E}-07$ | $1.43 \mathrm{E}-06$ | $4.30 \mathrm{E}-06$ | $1.29 \mathrm{E}-05$ | $3.83 \mathrm{E}-05$ | $1.12 \mathrm{E}-04$ | $3.27 \mathrm{E}-04$ | $9.33 \mathrm{E}-04$ | $2.61 \mathrm{E}-03$ | $4.70 \mathrm{E}-07$ |
| 30 | $1.04 \mathrm{E}-07$ | $3.18 \mathrm{E}-07$ | $9.72 \mathrm{E}-07$ | $2.96 \mathrm{E}-06$ | $8.94 \mathrm{E}-06$ | $2.69 \mathrm{E}-05$ | $8.00 \mathrm{E}-05$ | $2.36 \mathrm{E}-04$ | $6.85 \mathrm{E}-04$ | $1.04 \mathrm{E}-07$ |
| 31 | $2.22 \mathrm{E}-08$ | $6.88 \mathrm{E}-08$ | $2.13 \mathrm{E}-07$ | $6.55 \mathrm{E}-07$ | $2.01 \mathrm{E}-06$ | $6.14 \mathrm{E}-06$ | $1.87 \mathrm{E}-05$ | $5.63 \mathrm{E}-05$ | $1.68 \mathrm{E}-04$ | $2.22 \mathrm{E}-08$ |
| 32 | $4.64 \mathrm{E}-09$ | $1.45 \mathrm{E}-08$ | $4.52 \mathrm{E}-08$ | $1.41 \mathrm{E}-07$ | $4.37 \mathrm{E}-07$ | $1.35 \mathrm{E}-06$ | $4.17 \mathrm{E}-06$ | $1.28 \mathrm{E}-05$ | $3.90 \mathrm{E}-05$ | $4.64 \mathrm{E}-09$ |
| 33 | $9.49 \mathrm{E}-10$ | $2.98 \mathrm{E}-09$ | $9.38 \mathrm{E}-09$ | $2.94 \mathrm{E}-08$ | $9.21 \mathrm{E}-08$ | $2.88 \mathrm{E}-07$ | 8.99E-07 | $2.80 \mathrm{E}-06$ | $8.66 \mathrm{E}-06$ | $9.49 \mathrm{E}-10$ |
| 34 | $1.90 \mathrm{E}-10$ | $6.02 \mathrm{E}-10$ | $1.90 \mathrm{E}-09$ | $6.00 \mathrm{E}-09$ | $1.89 \mathrm{E}-08$ | $5.96 \mathrm{E}-08$ | $1.88 \mathrm{E}-07$ | $5.90 \mathrm{E}-07$ | $1.85 \mathrm{E}-06$ | $1.90 \mathrm{E}-10$ |
| 35 | $3.75 \mathrm{E}-11$ | $1.19 \mathrm{E}-10$ | $3.78 \mathrm{E}-10$ | $1.20 \mathrm{E}-09$ | $3.80 \mathrm{E}-09$ | $1.20 \mathrm{E}-08$ | $3.82 \mathrm{E}-08$ | $1.21 \mathrm{E}-07$ | $3.82 \mathrm{E}-07$ | $3.75 \mathrm{E}-11$ |
| 36 | $7.27 \mathrm{E}-12$ | $2.31 \mathrm{E}-11$ | $7.36 \mathrm{E}-11$ | $2.34 \mathrm{E}-10$ | $7.47 \mathrm{E}-10$ | $2.38 \mathrm{E}-09$ | $7.57 \mathrm{E}-09$ | $2.41 \mathrm{E}-08$ | $7.68 \mathrm{E}-08$ | $7.27 \mathrm{E}-12$ |
| 37 | $1.39 \mathrm{E}-12$ | $4.43 \mathrm{E}-12$ | $1.41 \mathrm{E}-11$ | $4.51 \mathrm{E}-11$ | $1.44 \mathrm{E}-10$ | $4.60 \mathrm{E}-10$ | $1.47 \mathrm{E}-09$ | $4.70 \mathrm{E}-09$ | $1.51 \mathrm{E}-08$ | $1.39 \mathrm{E}-12$ |
| 38 | $2.61 \mathrm{E}-13$ | $8.34 \mathrm{E}-13$ | $2.67 \mathrm{E}-12$ | $8.53 \mathrm{E}-12$ | $2.73 \mathrm{E}-11$ | $8.75 \mathrm{E}-11$ | $2.80 \mathrm{E}-10$ | $8.99 \mathrm{E}-10$ | $2.89 \mathrm{E}-09$ | $2.61 \mathrm{E}-13$ |
| 39 | $4.85 \mathrm{E}-14$ | $1.55 \mathrm{E}-13$ | $4.97 \mathrm{E}-13$ | $1.59 \mathrm{E}-12$ | $5.10 \mathrm{E}-12$ | $1.64 \mathrm{E}-11$ | $5.25 \mathrm{E}-11$ | $1.69 \mathrm{E}-10$ | $5.42 \mathrm{E}-10$ | $4.85 \mathrm{E}-14$ |
| 40 | $8.88 \mathrm{E}-15$ | $2.85 \mathrm{E}-14$ | $9.14 \mathrm{E}-14$ | $2.93 \mathrm{E}-13$ | $9.39 \mathrm{E}-13$ | $3.01 \mathrm{E}-12$ | $9.68 \mathrm{E}-12$ | $3.11 \mathrm{E}-11$ | $1.00 \mathrm{E}-10$ | $8.88 \mathrm{E}-15$ |
| 41 | $1.67 \mathrm{E}-15$ | $5.22 \mathrm{E}-15$ | $1.65 \mathrm{E}-14$ | $5.32 \mathrm{E}-14$ | $1.71 \mathrm{E}-13$ | $5.48 \mathrm{E}-13$ | $1.76 \mathrm{E}-12$ | $5.66 \mathrm{E}-12$ | $1.82 \mathrm{E}-11$ | $1.67 \mathrm{E}-15$ |
| 42 | $2.22 \mathrm{E}-16$ | $8.88 \mathrm{E}-16$ | $3.00 \mathrm{E}-15$ | $9.55 \mathrm{E}-15$ | $3.06 \mathrm{E}-14$ | $9.84 \mathrm{E}-14$ | $3.16 \mathrm{E}-13$ | $1.02 \mathrm{E}-12$ | $3.27 \mathrm{E}-12$ | $2.22 \mathrm{E}-16$ |
| 43 | $1.11 \mathrm{E}-16$ | $2.22 \mathrm{E}-16$ | $5.55 \mathrm{E}-16$ | $1.78 \mathrm{E}-15$ | $5.55 \mathrm{E}-15$ | $1.75 \mathrm{E}-14$ | $5.61 \mathrm{E}-14$ | $1.80 \mathrm{E}-13$ | $5.79 \mathrm{E}-13$ | $1.11 \mathrm{E}-16$ |
| 44 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.11 \mathrm{E}-16$ | $2.22 \mathrm{E}-16$ | $8.88 \mathrm{E}-16$ | $3.00 \mathrm{E}-15$ | $9.77 \mathrm{E}-15$ | $3.16 \mathrm{E}-14$ | $1.01 \mathrm{E}-13$ | $0.00 \mathrm{E}+00$ |
| 45 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.11 \mathrm{E}-16$ | $2.22 \mathrm{E}-16$ | $5.55 \mathrm{E}-16$ | $1.78 \mathrm{E}-15$ | $5.44 \mathrm{E}-15$ | $1.74 \mathrm{E}-14$ | $0.00 \mathrm{E}+00$ |
| 46 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.11 \mathrm{E}-16$ | $2.22 \mathrm{E}-16$ | $8.88 \mathrm{E}-16$ | $3.00 \mathrm{E}-15$ | $0.00 \mathrm{E}+00$ |
| 47 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.11 \mathrm{E}-16$ | $2.22 \mathrm{E}-16$ | $5.55 \mathrm{E}-16$ | $0.00 \mathrm{E}+00$ |
| 48 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.11 \mathrm{E}-16$ | $0.00 \mathrm{E}+00$ |
| 49 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |

(Continued Table 7.11) Probability transition matrix; Probability of a gurnard at size (column) grow to size (row)

| Growing |  |  |  |  | Len |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| length (cm) | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |
| 3 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 4 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 5 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 6 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 7 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 8 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 9 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 10 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 11 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 12 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 13 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 14 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 15 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 16 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 17 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 18 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 19 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 20 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 21 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 22 | $7.97 \mathrm{E}-03$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 23 | $1.41 \mathrm{E}-01$ | $1.16 \mathrm{E}-02$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 24 | $3.11 \mathrm{E}-01$ | $1.68 \mathrm{E}-01$ | $1.67 \mathrm{E}-02$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 25 | $2.81 \mathrm{E}-01$ | $3.27 \mathrm{E}-01$ | $1.99 \mathrm{E}-01$ | $2.38 \mathrm{E}-02$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 26 | $1.58 \mathrm{E}-01$ | $2.69 \mathrm{E}-01$ | $3.40 \mathrm{E}-01$ | $2.33 \mathrm{E}-01$ | $3.35 \mathrm{E}-02$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 27 | $6.70 \mathrm{E}-02$ | $1.41 \mathrm{E}-01$ | $2.54 \mathrm{E}-01$ | $3.47 \mathrm{E}-01$ | $2.68 \mathrm{E}-01$ | $4.64 \mathrm{E}-02$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 28 | $2.34 \mathrm{E}-02$ | $5.62 \mathrm{E}-02$ | $1.24 \mathrm{E}-01$ | $2.36 \mathrm{E}-01$ | $3.50 \mathrm{E}-01$ | $3.04 \mathrm{E}-01$ | $6.36 \mathrm{E}-02$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 29 | $7.12 \mathrm{E}-03$ | $1.87 \mathrm{E}-02$ | $4.65 \mathrm{E}-02$ | $1.07 \mathrm{E}-01$ | $2.16 \mathrm{E}-01$ | $3.47 \mathrm{E}-01$ | $3.40 \mathrm{E}-01$ | $8.59 \mathrm{E}-02$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 30 | $1.96 \mathrm{E}-03$ | $5.45 \mathrm{E}-03$ | $1.47 \mathrm{E}-02$ | $3.79 \mathrm{E}-02$ | $9.08 \mathrm{E}-02$ | $1.94 \mathrm{E}-01$ | 3.38E-01 | $3.74 \mathrm{E}-01$ | $1.14 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ |
| 31 | $4.96 \mathrm{E}-04$ | $1.44 \mathrm{E}-03$ | $4.12 \mathrm{E}-03$ | $1.14 \mathrm{E}-02$ | $3.03 \mathrm{E}-02$ | $7.59 \mathrm{E}-02$ | $1.72 \mathrm{E}-01$ | $3.23 \mathrm{E}-01$ | $4.04 \mathrm{E}-01$ | $1.50 \mathrm{E}-01$ |
| 32 | $1.18 \mathrm{E}-04$ | $3.55 \mathrm{E}-04$ | $1.05 \mathrm{E}-03$ | $3.06 \mathrm{E}-03$ | $8.70 \mathrm{E}-03$ | $2.39 \mathrm{E}-02$ | $6.23 \mathrm{E}-02$ | $1.49 \mathrm{E}-01$ | $3.02 \mathrm{E}-01$ | $4.27 \mathrm{E}-01$ |
| 33 | $2.67 \mathrm{E}-05$ | $8.20 \mathrm{E}-05$ | $2.50 \mathrm{E}-04$ | $7.53 \mathrm{E}-04$ | $2.24 \mathrm{E}-03$ | $6.52 \mathrm{E}-03$ | $1.85 \mathrm{E}-02$ | $5.02 \mathrm{E}-02$ | $1.26 \mathrm{E}-01$ | $2.77 \mathrm{E}-01$ |
| 34 | $5.79 \mathrm{E}-06$ | $1.81 \mathrm{E}-05$ | $5.61 \mathrm{E}-05$ | $1.73 \mathrm{E}-04$ | $5.30 \mathrm{E}-04$ | $1.61 \mathrm{E}-03$ | $4.80 \mathrm{E}-03$ | $1.40 \mathrm{E}-02$ | $3.95 \mathrm{E}-02$ | $1.05 \mathrm{E}-01$ |
| 35 | $1.21 \mathrm{E}-06$ | $3.81 \mathrm{E}-06$ | $1.20 \mathrm{E}-05$ | $3.77 \mathrm{E}-05$ | $1.18 \mathrm{E}-04$ | $3.67 \mathrm{E}-04$ | $1.13 \mathrm{E}-03$ | $3.47 \mathrm{E}-03$ | $1.04 \mathrm{E}-02$ | $3.05 \mathrm{E}-02$ |
| 36 | $2.45 \mathrm{E}-07$ | $7.78 \mathrm{E}-07$ | $2.48 \mathrm{E}-06$ | $7.87 \mathrm{E}-06$ | $2.50 \mathrm{E}-05$ | $7.91 \mathrm{E}-05$ | $2.50 \mathrm{E}-04$ | $7.85 \mathrm{E}-04$ | $2.45 \mathrm{E}-03$ | $7.57 \mathrm{E}-03$ |
| 37 | $4.82 \mathrm{E}-08$ | $1.54 \mathrm{E}-07$ | $4.94 \mathrm{E}-07$ | $1.58 \mathrm{E}-06$ | $5.07 \mathrm{E}-06$ | $1.63 \mathrm{E}-05$ | $5.21 \mathrm{E}-05$ | $1.67 \mathrm{E}-04$ | $5.32 \mathrm{E}-04$ | $1.69 \mathrm{E}-03$ |
| 38 | $9.27 \mathrm{E}-09$ | $2.98 \mathrm{E}-08$ | $9.59 \mathrm{E}-08$ | $3.09 \mathrm{E}-07$ | $9.96 \mathrm{E}-07$ | $3.21 \mathrm{E}-06$ | $1.04 \mathrm{E}-05$ | $3.36 \mathrm{E}-05$ | $1.09 \mathrm{E}-04$ | $3.53 \mathrm{E}-04$ |
| 39 | $1.75 \mathrm{E}-09$ | $5.63 \mathrm{E}-09$ | $1.82 \mathrm{E}-08$ | $5.87 \mathrm{E}-08$ | $1.90 \mathrm{E}-07$ | $6.15 \mathrm{E}-07$ | $2.00 \mathrm{E}-06$ | $6.51 \mathrm{E}-06$ | $2.12 \mathrm{E}-05$ | $6.94 \mathrm{E}-05$ |
| 40 | $3.23 \mathrm{E}-10$ | $1.04 \mathrm{E}-09$ | $3.37 \mathrm{E}-09$ | $1.09 \mathrm{E}-08$ | $3.53 \mathrm{E}-08$ | $1.15 \mathrm{E}-07$ | $3.73 \mathrm{E}-07$ | $1.22 \mathrm{E}-06$ | $3.99 \mathrm{E}-06$ | $1.31 \mathrm{E}-05$ |
| 41 | $5.87 \mathrm{E}-11$ | $1.90 \mathrm{E}-10$ | $6.13 \mathrm{E}-10$ | $1.98 \mathrm{E}-09$ | $6.43 \mathrm{E}-09$ | $2.09 \mathrm{E}-08$ | $6.81 \mathrm{E}-08$ | $2.22 \mathrm{E}-07$ | $7.28 \mathrm{E}-07$ | $2.39 \mathrm{E}-06$ |
| 42 | $1.05 \mathrm{E}-11$ | $3.40 \mathrm{E}-11$ | $1.10 \mathrm{E}-10$ | $3.55 \mathrm{E}-10$ | $1.15 \mathrm{E}-09$ | $3.73 \mathrm{E}-09$ | $1.21 \mathrm{E}-08$ | $3.96 \mathrm{E}-08$ | $1.29 \mathrm{E}-07$ | $4.25 \mathrm{E}-07$ |
| 43 | $1.86 \mathrm{E}-12$ | $6.00 \mathrm{E}-12$ | $1.93 \mathrm{E}-11$ | $6.25 \mathrm{E}-11$ | $2.02 \mathrm{E}-10$ | $6.55 \mathrm{E}-10$ | $2.12 \mathrm{E}-09$ | $6.91 \mathrm{E}-09$ | $2.25 \mathrm{E}-08$ | $7.37 \mathrm{E}-08$ |
| 44 | $3.25 \mathrm{E}-13$ | $1.05 \mathrm{E}-12$ | $3.37 \mathrm{E}-12$ | $1.08 \mathrm{E}-11$ | $3.50 \mathrm{E}-11$ | $1.13 \mathrm{E}-10$ | $3.66 \mathrm{E}-10$ | 1.19E-09 | $3.86 \mathrm{E}-09$ | $1.26 \mathrm{E}-08$ |
| 45 | $5.62 \mathrm{E}-14$ | $1.80 \mathrm{E}-13$ | $5.79 \mathrm{E}-13$ | $1.86 \mathrm{E}-12$ | $5.98 \mathrm{E}-12$ | $1.93 \mathrm{E}-11$ | $6.21 \mathrm{E}-11$ | $2.01 \mathrm{E}-10$ | $6.49 \mathrm{E}-10$ | $2.10 \mathrm{E}-09$ |
| 46 | $9.55 \mathrm{E}-15$ | $3.08 \mathrm{E}-14$ | $9.85 \mathrm{E}-14$ | $3.15 \mathrm{E}-13$ | $1.01 \mathrm{E}-12$ | $3.24 \mathrm{E}-12$ | $1.04 \mathrm{E}-11$ | $3.35 \mathrm{E}-11$ | $1.08 \mathrm{E}-10$ | $3.47 \mathrm{E}-10$ |
| 47 | $1.67 \mathrm{E}-15$ | $5.22 \mathrm{E}-15$ | $1.65 \mathrm{E}-14$ | $5.30 \mathrm{E}-14$ | $1.69 \mathrm{E}-13$ | $5.40 \mathrm{E}-13$ | $1.73 \mathrm{E}-12$ | $5.52 \mathrm{E}-12$ | $1.77 \mathrm{E}-11$ | $5.65 \mathrm{E}-11$ |
| 48 | $3.33 \mathrm{E}-16$ | $7.77 \mathrm{E}-16$ | $2.78 \mathrm{E}-15$ | $8.77 \mathrm{E}-15$ | $2.81 \mathrm{E}-14$ | 8.92E-14 | $2.83 \mathrm{E}-13$ | $9.02 \mathrm{E}-13$ | $2.87 \mathrm{E}-12$ | $9.11 \mathrm{E}-12$ |
| 49 | $0.00 \mathrm{E}+00$ | $2.22 \mathrm{E}-16$ | $5.55 \mathrm{E}-16$ | $1.78 \mathrm{E}-15$ | $5.44 \mathrm{E}-15$ | 1.73E-14 | $5.50 \mathrm{E}-14$ | $1.74 \mathrm{E}-13$ | $5.48 \mathrm{E}-13$ | $1.73 \mathrm{E}-12$ |

(Continued Table 7.11) Probability transition matrix; Probability of a gurnard at size (column) grow to size (row)

| Growing |  |  |  |  | rom Le |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| length <br> (cm) | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 |
| 3 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 4 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 5 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 6 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 7 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 8 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 9 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 10 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 11 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 12 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 13 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 14 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 15 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 16 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 17 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 18 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 19 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 20 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 21 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 22 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 23 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 24 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 25 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 26 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 27 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 28 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 29 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 30 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 31 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 32 | $1.94 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 33 | $4.43 \mathrm{E}-01$ | $2.46 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 34 | $2.48 \mathrm{E}-01$ | $4.48 \mathrm{E}-01$ | $3.08 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 35 | $8.51 \mathrm{E}-02$ | $2.17 \mathrm{E}-01$ | $4.42 \mathrm{E}-01$ | $3.77 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 36 | $2.29 \mathrm{E}-02$ | $6.72 \mathrm{E}-02$ | $1.84 \mathrm{E}-01$ | $4.23 \mathrm{E}-01$ | $4.55 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 37 | $5.37 \mathrm{E}-03$ | $1.68 \mathrm{E}-02$ | $5.16 \mathrm{E}-02$ | $1.51 \mathrm{E}-01$ | $3.91 \mathrm{E}-01$ | $5.38 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 38 | $1.14 \mathrm{E}-03$ | $3.70 \mathrm{E}-03$ | $1.20 \mathrm{E}-02$ | $3.83 \mathrm{E}-02$ | $1.20 \mathrm{E}-01$ | $3.48 \mathrm{E}-01$ | $6.23 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 39 | $2.28 \mathrm{E}-04$ | $7.51 \mathrm{E}-04$ | $2.48 \mathrm{E}-03$ | $8.24 \mathrm{E}-03$ | $2.74 \mathrm{E}-02$ | $9.12 \mathrm{E}-02$ | $2.96 \mathrm{E}-01$ | $7.08 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 40 | 4.33E-05 | $1.43 \mathrm{E}-04$ | $4.79 \mathrm{E}-04$ | $1.61 \mathrm{E}-03$ | $5.47 \mathrm{E}-03$ | $1.88 \mathrm{E}-02$ | $6.61 \mathrm{E}-02$ | $2.38 \mathrm{E}-01$ | $7.88 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ |
| 41 | $7.90 \mathrm{E}-06$ | $2.62 \mathrm{E}-05$ | $8.76 \mathrm{E}-05$ | $2.95 \mathrm{E}-04$ | $1.01 \mathrm{E}-03$ | $3.48 \mathrm{E}-03$ | $1.23 \mathrm{E}-02$ | $4.52 \mathrm{E}-02$ | $1.78 \mathrm{E}-01$ | $8.60 \mathrm{E}-01$ |
| 42 | $1.40 \mathrm{E}-06$ | $4.64 \mathrm{E}-06$ | $1.54 \mathrm{E}-05$ | $5.18 \mathrm{E}-05$ | $1.76 \mathrm{E}-04$ | $6.03 \mathrm{E}-04$ | $2.11 \mathrm{E}-03$ | $7.60 \mathrm{E}-03$ | $2.88 \mathrm{E}-02$ | $1.21 \mathrm{E}-01$ |
| 43 | $2.42 \mathrm{E}-07$ | $7.98 \mathrm{E}-07$ | $2.64 \mathrm{E}-06$ | $8.80 \mathrm{E}-06$ | $2.95 \mathrm{E}-05$ | $1.00 \mathrm{E}-04$ | $3.44 \mathrm{E}-04$ | $1.21 \mathrm{E}-03$ | $4.36 \mathrm{E}-03$ | $1.66 \mathrm{E}-02$ |
| 44 | $4.10 \mathrm{E}-08$ | $1.34 \mathrm{E}-07$ | $4.41 \mathrm{E}-07$ | $1.46 \mathrm{E}-06$ | $4.83 \mathrm{E}-06$ | $1.61 \mathrm{E}-05$ | $5.43 \mathrm{E}-05$ | $1.85 \mathrm{E}-04$ | $6.40 \mathrm{E}-04$ | $2.27 \mathrm{E}-03$ |
| 45 | $6.82 \mathrm{E}-09$ | $2.22 \mathrm{E}-08$ | $7.22 \mathrm{E}-08$ | $2.36 \mathrm{E}-07$ | $7.73 \mathrm{E}-07$ | $2.54 \mathrm{E}-06$ | $8.39 \mathrm{E}-06$ | 2.78E-05 | $9.25 \mathrm{E}-05$ | $3.09 \mathrm{E}-04$ |
| 46 | $1.12 \mathrm{E}-09$ | $3.61 \mathrm{E}-09$ | $1.16 \mathrm{E}-08$ | $3.76 \mathrm{E}-08$ | $1.22 \mathrm{E}-07$ | $3.94 \mathrm{E}-07$ | $1.27 \mathrm{E}-06$ | $4.11 \mathrm{E}-06$ | $1.32 \mathrm{E}-05$ | $4.20 \mathrm{E}-05$ |
| 47 | $1.81 \mathrm{E}-10$ | $5.79 \mathrm{E}-10$ | $1.85 \mathrm{E}-09$ | $5.92 \mathrm{E}-09$ | $1.89 \mathrm{E}-08$ | $6.02 \mathrm{E}-08$ | $1.91 \mathrm{E}-07$ | $6.02 \mathrm{E}-07$ | $1.87 \mathrm{E}-06$ | $5.71 \mathrm{E}-06$ |
| 48 | $2.89 \mathrm{E}-11$ | $9.18 \mathrm{E}-11$ | $2.91 \mathrm{E}-10$ | $9.20 \mathrm{E}-10$ | $2.90 \mathrm{E}-09$ | $9.09 \mathrm{E}-09$ | $2.83 \mathrm{E}-08$ | $8.72 \mathrm{E}-08$ | $2.64 \mathrm{E}-07$ | $7.75 \mathrm{E}-07$ |
| 49 | $5.44 \mathrm{E}-12$ | $1.71 \mathrm{E}-11$ | $5.35 \mathrm{E}-11$ | $1.67 \mathrm{E}-10$ | $5.18 \mathrm{E}-10$ | $1.60 \mathrm{E}-09$ | $4.87 \mathrm{E}-09$ | $1.47 \mathrm{E}-08$ | $4.30 \mathrm{E}-08$ | $1.22 \mathrm{E}-07$ |

(Continued Table 7.11) Probability transition matrix; Probability of a gurnard at a size (column) grow to size (row)

| Growing to | From Length (cm) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| length (cm) | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 3 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 4 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 5 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 6 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 7 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 8 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 9 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 10 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 11 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 12 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 13 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 14 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 15 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 16 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 17 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 18 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 19 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 20 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 21 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 22 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 23 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 24 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 25 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 26 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 27 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 28 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 29 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 30 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 31 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 32 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 33 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 34 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 35 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 36 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 37 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 38 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 39 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 40 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 41 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 42 | $9.18 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 43 | $7.21 \mathrm{E}-02$ | $9.63 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 44 | $8.36 \mathrm{E}-03$ | $3.38 \mathrm{E}-02$ | $9.91 \mathrm{E}-01$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 45 | $1.03 \mathrm{E}-03$ | $3.31 \mathrm{E}-03$ | $8.10 \mathrm{E}-03$ | $1.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 46 | $1.30 \mathrm{E}-04$ | $3.66 \mathrm{E}-04$ | $6.69 \mathrm{E}-04$ | $0.00 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 47 | $1.66 \mathrm{E}-05$ | $4.27 \mathrm{E}-05$ | $6.65 \mathrm{E}-05$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 48 | $2.14 \mathrm{E}-06$ | $5.14 \mathrm{E}-06$ | $7.18 \mathrm{E}-06$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ |
| 49 | $3.21 \mathrm{E}-07$ | $7.22 \mathrm{E}-07$ | $9.22 \mathrm{E}-07$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |

## Chapter 8

## Discussion and conclusion

Many research works have established methods to assess marine abundance. The available models are mainly based on age, age and length or length of fish usually from commercial fisheries and survey. They range from relatively simple population dynamic models to more complex biomass analysis. Very complex theoretical models seem to be accurate and less biased but would increase the uncertainty in the assessment process due to adding too many parameters. In parallel, a simple model would be criticised due to lack of power to analyse such a complex biological process. The feasibility of the model is another important factor. The biological uncertainties also affect stock assessment and predictions for future management. The population dynamics model that is the core of this piece of research work should be able to produce the outputs needed for mangement decision-making. Since data availability and data collection is different for different species, the model should also be able to utilise the available data to assess the stocks.

### 8.1 Research Problems

Estimates of fish stock biomass and mortality for assessed commercially exploited species in EU waters are currently based on fishery landing statistics from the different EU nations, which are obtained by the ICES. The standard approaches are age-based methods, which are the descendant cohort analysis and VPA. The methods have been in use for over 30 years and rely on catch-at-age data as the model input. They are subject to error if misreporting, illegal landings and unrecorded discards calculating stock size are high. To obtain reliable catch-at-
age data, a great number of fish need to be aged, which is costly and time consuming and this puts practical limits to the number of species that can be assessed.

For those species, for which ageing is not feasible, the catch-at-length analysis removes the need for age data. It uses a forward running length-structured matrix model, in which the model parameters are estimated by fitting to catch-at-length observation data from commercially reported landings.

The problem is that the length distributions of commercial catches are not routinely recorded for all species either. It is more of the issue for non-target species (i.e. da, flounder, dogfish, gurnard, wolfish and megrim, ...) in regions such as the North Sea where the fisheries involves fleets from different EU nations. Hence, neither age-structured nor catch-at-length analysis is capable of modelling the dynamics of the population if age and length distribution of catch is not available.

Hence, the gap here is a feasible method to enable the species for which there is no age or catch-at-length data available to be assessed.

### 8.2 Objectives

The main objectives of this piece of this research are:

- To improve the description of individual growth, stock biomass, and fishing mortality for poorly sampled marine species
- To create a length-structured stock assessment method (so called survey-landings model) as an alternative to age-structured and catch-at-length analysis models for marine species with no age or catch-at-length data available. This includes the modification of the existing catch-at-length analysis model and making the use of the available data as input observations
- To model the fishing and natural mortality for the species of interest
- To estimate the number of the new fish that survived and added to fisheries (recruitments) as part of the model development
- To assess and validate the model with the help of simulation and twin-experiment method
- To compare the new result of the new model with ICES assessment to assess the accuracy of the output
- To apply the tested model on a species for which there is no formal assessment has been conducted (grey gurnard in this piece of research work)


### 8.3 Methodology and novel contributions

The survey-landings model is a modification and transformation of the existing catch-atlength analysis that was originally introduced by Sullivan et al. (1990). The survey-landings model made the use of survey frequency and landed biomass (and discards biomass where available) simultaneously in the length-structured stock assessment model. In this new model no age data is needed and the catch-at-length data is substituted by the length frequency of survey as well as total landed biomass and total discards biomass from commercial reports. The survey-landings model describes the dynamics of the population in terms of numbers of fish at length over a period of time. It is a modelled combination of natural mortality, fishing mortality, growth and recruitment functions. The model was built and assessed based on three main equations including the relationship between catch and abundance, relationship between abundance at two time steps and the relationship between survey and abundance.

It has enabled the non-target fish and poorly sampled species to be assessed reliably. The poorly sampled species, although they have been largely neglected, are very important environmentally and could play significant roles in the mortality of other species as predators. By assessing the stock of these species their unknown characteristics, growth and mortality is discovered.

The survey-landings model data was first simulated before the model was examined within the twin-experiment context for its accuracy and robustness. The sensitivity of the estimating parameters was also tested to identify the most influencing factor and the most sensitive parameters to variations and noise. Strengths and limitations of the model are also discussed with regards to the parameters that can be estimated externally and independently of the model. Feasibility of the model was tested in three main steps within the twin-experiment context. First the model was applied onto the simulated observations to examine if the identical parameters were recovered through estimation. Next, the initial parameter values
were perturbed to examine the sensitivity of the model in estimating the parameters. At this stage, the parameters were moved one by one into the model from fixed position to fitting position. Finally the model was applied to the perturbed simulated observations to test the robustness of the model against the variation of observations.

This piece of research work has the potential to transform the stock assessment of marine population by exploring the dynamics of population of the species that have never been assessed before.

The catch-abundance function is modelled using the exploitation rate that includes fishing and total mortality rates. Being a length-structured model, length-dependent fishing selectivity in the survey-landings model is modelled within the fishing mortality and multiplied by the time-dependent fishing scalar. In theory, fishing selectivity has an increasing logistic function; in practice, however, it very much depends on the species. In this work, the challenge was to ensure the fishing curve represents a relatively close pattern to what it actually is in real world. In haddock's case, the age-based fishing mortality estimated by ICES had a dome shape with the peak belonging to the 2-4-year-old age group. In order to extend the structure of the age-based fishing selectivity to the length-based platform, the age was transformed to length and extended over the length classes. As the result, the lengthdependent fishing selectivity followed the dome shape that was finally modelled by a double logistic function. This transformation played a massive role in fitting the survey-landings model on the haddock data. The grey gurnard had a different pattern, though. Since it is not a target species, fishing selectivity does not significantly affect its mortality; and it is not expected to have a peak at a particular marketable size. Therefore, the curve and the initial parameters are set to have an increasing function by length. Experience from the current work showed that fishing selectivity is extremely important in forming the dynamics of the population. This emphasises that extra care should be taken in assumptions of the shape of fishing selectivity.

Although the population model would have been less complicated if a fixed constant value for natural mortality had been assumed, it was modelled as a decreasing function of length to incorporate the effect of size. In applying the survey-landings model on haddock, both constant and length-dependent natural mortality were tested. Although in the existing catch-at-length models natural mortality is a known constant, the fitting process in survey-landings model improved when natural mortality was considered as a decreasing function of length.

Therefore, the same assumption was applied for grey gurnard. The conclusion is that natural mortality is a remarkable factor in controlling the dynamics of population and needs to be varying by length classes in the survey-landings model.

Only a fraction of caught fish is commercially permitted to land. To gain the best estimation of fishing mortality, the parameters of the logistic function of landing proportion were added to the model as fixed and known values. The curve was formed using the minimum landing size, at which the probability of landing was set at 0.50 . The advantage of this approach was that it provided a logical assumption for the landing pattern. It reduced the unnecessary complexity of the model as well as removing the confounding effect between the probability of landing and fishing mortality.

Having been designed as a forward running model, the abundance in the survey-landings at each time step is related to the abundance at the previous time step. Since individuals are not necessarily growing at a fixed rate in time, length-structured stock assessment methods estimated the growth parameters to relate length to time. A distribution like the normal distribution is assumed to be around the mean length with some standard deviations. It is because the fish don't spawn at exactly the same time. The gamma distribution was chosen to model the variability of growth increment. The probability of growth from one length class to another is the direct outcome of the distribution. The assumption is that fish do not shrink and do not grow when they reach the maximum asymptotic length. Recruits were modelled as a product of the two time-varying and length-dependent component.

The length distribution of survey is assumed to be proportional to the actual distribution of fish in the sea by the survey selectivity and the proportion of the sampling area of the sea. The structure of the survey selectivity plays a fundamental role in forming this proportion. The initial thought was that survey should follow the same curve as the fishing selectivity but with a slight shift to the left to make sure the survey vessels take sample of very small fish and do not concentrate on commercially preferred fish. For both haddock and grey gurnard, the survey selectivity follows a dome-shaped double logistic curve to take the sample of small fish as much as possible and avoid the very big fish.

Model parameters are estimated using the least square estimation method, programmed in R, by fitting to annual landed and discard biomass as well as the annual length frequency of survey.

Model sensitivity to parameter variations and fluctuations in observations and model robustness were assessed in the twin-experiment context. However, in order to test the reliability and feasibility of the model, the assessment result of the model application on the North Sea haddock was compared with the ICES assessment. Haddock was selected because both age and length information is recorded and that the ICES undertake routine assessment for haddock. Collation of information is small in haddock, while fluctuation rate is huge. Also all the products that is required for assessment is complete. These make the assessment works better. When the survey-landings model proved to be robust and reliable, it was taken further to undertake the population assessment of one of the un-assessed and poorly sampled species (grey gurnard) in the North Sea.

### 8.4 Results

Testing and model assessment is an essential part of the modelling process to confirm the model is feasible to apply. It was carried out within the twin-experiment context. In the first phase of assessment, where the model was applied on simulated observations, the surveylandings model was able to recover the exact parameter value.

In the second phase, where the initial parameters were perturbed by $30 \%$ ( $\mathrm{cv}=0.30$ ), the timedependent fishing scalar and annual recruits proved to be robust too. Although the fishing scalar is slightly affected by the variability of the recruitment gamma parameter, the survey numbers is recovered better if both annual recruits and the gamma parameter for recruitment are simultaneously estimated in the model. If the initial assumptions about the shapes of the fishing and survey selectivity curves (i.e. increasing logistic curve or dome shape) are close enough to the actual shape, the selectivity parameters are robust to the variations. That is important, because these parameters have to be estimated in the model and cannot be externally dealt with. The exception, however, is the catchability parameter in the survey selectivity. Being the estimating parameter, the catchability affects the estimation of fishing scalar and as a result, the distribution of mortality is also influenced. In applying the surveylandings model to haddock data, the survey catchability was considered to be the estimating parameter, rather than being a known constant. That was due to the lack of any sources to guess a feasible value. Later, in applying the model to grey gurnard, the estimated result from haddock was used to manually adjust the catchability before keeping it fixed in the model.

The growth parameters can be confounded with the fishing scalar and cause instability if the initial values are set too far from the actual values. The feasible approach is to estimate growth parameters externally if possible before adding them into the model, rather than starting with a very random guess. The variation of the growth can affect the accuracy of the model estimation, fishing scalar in particular. If any information is available, it is more feasible to keep it as a known constant, or manually adjusted.

In the third phase, where the model is tested on the multiplicative noisy observations, the model proved to be unaffected. The exception is the fishing scalar but it does not affect the accuracy of the population dynamics as a whole model. That is because the affect on the fishing scalar is not systematic.

The ICES assessment method has been widely accepted as a standard method to assess the stock of marine species. It is an assessment on its own, but it has been used for long enough to be used as the source of judgement to test the reliability of the survey-landings model. Although the result from applying the survey-landings model on 44 years of haddock observations was reasonably close to the ICES assessment on the same time line, the survey numbers were better at some years than others. That was because the growth parameters were constant in the model, while in reality they are time-varying parameters. The issue was addressed by splitting the study years into 4 different time periods. The purpose was to allow the growth to vary over timelines so that the model could estimate the growth parameters in each time period independently from another. The consistency in the dynamics of the population and recruits between model estimation and ICES assessment means that the model has achieved its overall aim. The most recent observations resulted in a better fit, which could be due to the more accurate sampling and data collection. It was also discovered that the maximum asymptotic length is getting smaller over years from 100 in the oldest study years to 41 years in the most recent ones. This is while the growth rate (curvature) has increased from 0.16 in the first period to 0.42 in 1990s and then down to 0.32 in 2000s. The variation of growth was manually adjusted before it was added as a known value into the model. Estimated fishing mortality at its highest rate was consistent with the assessed fishing mortality at ICES for haddock throughout the study years. The fishing and survey selectivity curves reflected the reality of the how the fishing boats and survey vessel would work in catching and sampling the fish in the sea. The first and second peaks in the population distribution match with the 1 -year and 2-year group at length in the ICES assessment. The
overall promising result allows the conclusion that the survey-landings model could be used as an alternative method to assess the population size when the age and catch-at-length data are not available.

The survey-landings model was taken further and was applied onto the North Sea grey gurnards, of which the stock size and other aspects of its dynamics are unknown. Although the population of the North Sea grey gurnard has never been assessed and therefore there is no standard platform to judge the accuracy of the results, the estimation of growth and the overall population output would determine the model's reliability in assessing the stock size. It is obvious that there is always room for improvement to make the model more accurate, but this model can revolutionarily fill the knowledge gap about the non-target species such as grey gurnard. The discards biomass was recovered very accurately, which approved the correct function of the landing probability. The estimated fishing scalar is comparable with the one for haddock. It is not unrealistic to compare these two results, because these two species could be caught with the same trawl in the fishing season. Since grey gurnard is a bycatch species in the North Sea, the fishing mortality does not affect it strongly and therefore it is not expected to have a peak or a large value at any length classes, which is again reflected in the fishing selectivity estimation. The growth parameters are estimated within the range of what was found in other publications, which is very promising and reassuring in the sense that the model moving on the right path.

### 8.5 Future Work

The main objectives of this research work, which was to make a population model for poorly sampled specie and improve the description of growth and mortality, were achieved. There are a few parameters that if known would increase the reliability and accuracy of the model in estimating the parameters. The time-dependent fishing scalar is affected by variations of growth parameters. As the result if fishing scalar can somehow be dealt with externally, less effort would be needed to improve the fitting process. In the survey-landings model, the model worked better when the growth parameters were estimated in the model. Nevertheless, if they can be accurately estimated externally, two parameters would be removed from the model and the fishing mortality parameters are more reliably estimated.

The output of the haddock's stock assessment indicated that growth is not constant but changes over time. In the current work, this matter was considered by separating the study years into four time periods. Growth was assumed constant at each period while it was allowed to vary from one time line to another. This approach fulfilled the aim of the study and provided a promising result, without adding extra parameters to the model. For future work, an alternative is that both asymptotic length and growth rate are considered as timedependent estimating parameters. This action enables the model to observe the changes of growth as a time trend at every time step. It, on the other hand, adds more complexity to the model, especially if a long study time is of interest.

The model's components include growth, mortality and recruitment. Spatial analysis was avoided in the model structure, due to lack of sufficient information. Spatial analysis could potentially improve the knowledge of the population dynamics in various stations of the North Sea. Global warming and migration are some of the environmental factors that could significantly influence recruitment and abundance distribution. In order to avoid making the model more complicated, none of the environmental and spatial factors were considered in the length-structured survey-landings model. However, the model could be extended in future work to incorporate such factors.

Using the survey data should ensure that it is an unbiased representative of the numbers at length in the real population. Looking at the general trend of the distribution can provide that assumption. However, different sampling effort in different areas could make selection bias. Nevertheless, the aim of this research work is to make the use of the only available survey data to assess the stock.

### 8.6 Conclusion

The survey-landings model has been tested for robustness, sensitivity and reliability. The output for NS haddock correspond to the assessment results fron the ICES. It, therefore, proved to be a reliable method to contribute to development of fish stock assessment, which removes the need for detailed catch-at-length and age composition. It is not expected to take over the age-based or catch-at-length assessment methods, but
makes the assessment of currently unassessed and limited species potentially possible. The outstanding example is the assessment of the grey gurnard's stock.

From modelling and technical point of view, the survey-landings model is robust to variation of the initial parameters. However, the shapes assumptions of the fishing and selectivity curves need be close to the actual curves to give robust selectivity estimates. One of the main challenges is catchability though, which could be a huge development in the assessment model if it is known or can be estimated from elsewhere, as it is influencing the mortality curve too.

To count for the time varying growth parameters, the model was applied onto four different time blocks. The assessment result would be more reliable if the model is capable of estimating the time varying growth parameters inside the model simultaneously with other parameters.

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[^0]:    ${ }^{1}$ The most common way to determine the length is the total length. It is measured by pushing the fish's snout up against a vertical surface with the mouth closed and the tail fin pinched together while the fish laying along a tape measure.
    ${ }^{2}$ The von Bertalanffy growth function (VBGF), published by von Bertalanffy in 1934, and is based on a bio-energetic expression of fish growth. It is widely used in biological models and exists in a number of permutations. In its simplest version the equation is expressed as a differential equation of length ( L ) over time ( t )

[^1]:    ${ }^{3}$ It depends more on stock than species; i.e. Scottish scallops are well samples but English are not.
    ${ }^{4}$ Dogfish and gurnard include a number of species

[^2]:    ${ }^{5}$ http://www.nefsc.noaa.gov/fbp/basics.htm

[^3]:    ${ }^{7}$ http://www.r-project.org

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[^5]:    ${ }^{9}$ Exponential distribution

[^6]:    ${ }^{10}$ Belgium, Denmark, Faeroe Island, France, Germany, Iceland, Ireland, Isle of Man, Latvia, Lithuania, Netherlands, Norway, Poland, Russia, Spain, Svalbard, Sweden, Canada, Greenland and USA (FishBase)
    ${ }^{11}$ The depth is 40-300 m in FAO: http://www.fao.org/wairdocs/tan/x5939e/x5939e01.htm

[^7]:    1213 to 15 days in FAO: http://www.fao.org/wairdocs/tan/x5939e/x5939e01.htm and 7-21 days in ICES FishMap

[^8]:    ${ }^{13}$ http://www.ne-ifca.gov.uk/minimum-landing-sizes/ and appendix 5.5

[^9]:    4

