ESSAYS ON CONDITIONAL VOLATILITY IN ASSET RETURNS

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ABSTRACT

This dissertation consists of four papers that examine various aspects of the temporal patterns in the volatility of asset returns.

The first paper compares the predictive performance of various parametric ARCH models. We find that ARCH models are generally good descriptions of the time-varying volatility of UK stock returns. There appears to be asymmetry in the conditional volatility, although no single model outperforms the rest in all instances.

In the second paper, we uncover evidence of asymmetric predictability in the conditional variance of firms of different size. Large firms shocks affect the future volatility of small firms, but not vice versa. We also find that trading period shocks have a significant impact on future volatility, but not nontrading period shocks.

In the third paper, we document a contemporaneous volatility-volume relationship. We find that volatility is related to change in trading volume, and we propose a conditional volatility model that incorporate this contemporaneous volatility-volume relationship.

In the final paper, we examine the various method of adjusting for nontrading effects in ARCH models, and we propose a new diagnostic test to detect the validity of such adjustments. We also uncover evidence that conditional volatility increases prior to market closure, but declines after market opening.

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CHAPTER 1

INTRODUCTION

1. BACKGROUND

In the last 30 years or so, the financial markets of the world have undergone a period of tremendous growth. Following this increase in financial activities, the behaviour of asset prices has come under the scrutiny of academics and practitioners alike. However, interest in speculative price behaviour can be traced back to the beginning of this century, when the French mathematician, Louis Bachelier (1900) developed a mathematical theory of asset prices. Unfortunately, much of his work has largely been ignored until some decades later, only to be 'rediscovered' by others (see Mandelbrot, 1989, for an account of Bachelier's work). On the other hand, empirical research into asset prices did not begin until 1930s, and the resulting research papers were far and few. Much of these early works (e.g. Cowles, 1933, and Working, 1934) focused on the predictability of price changes.

In the 1960s, several important empirical works appeared, the most influential being Fama (1965) and Mandelbrot (1963). Since then, various researchers have documented empirical regularities in asset returns. These include returns exhibiting fat-tailness (i.e. leptokurtosis), large (small) changes followed by large (small) changes, of either sign (i.e. volatility clustering), changes in stock prices being negatively related to changes in stock volatility (i.e. leverage effects[1]), and differences in volatility between open and closed markets (i.e. nontrading effects). Mandelbrot used the term "stylised facts" to describe some of these empirical features.

The use of time series models in empirical research gained considerable momentum in the 1970s, following the work of Box and Jenkins (1976). Part of the success of time series models can be attributed to their use in forecasting applications. By focusing on the *conditional* mean rather than the *unconditional* mean, time series models were able to give more accurate forecasts of future prices/returns. However, most conventional models (such as the ARIMA/Box-Jenkins models) assumed that the covariance matrix remains constant over time. Given the empirical results of Mandelbrot (1963) that volatility of asset returns changes over time, this assumption is clearly violated.

The problem of heteroskedasticity in time series data is well known among statisticians and econometricians. Though there have been earlier attempts to model time-varying variances (see for example Khan, 1977, and Klien, 1977), none of the methods proved to be popular. One common approach is to express the time-varying variances as a function of some exogenously determined time-varying variables. Alternatively, researchers apply some sort of variance-stabilising transformation such

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[1] Black (1976) suggests that a decrease in stock prices increases the leverage of the firm, thereby increasing its risk. Hence, the increase in stock volatility is brought about by an increase in the leverage of the firm.

as the Box-Cox (1964) transformation on the data to get rid of the heteroscedasticity. Neither method proves to be satisfactory, especially in forecasting applications.

The breakthrough came when Engle (1982) introduced the class of AutoRegressive Conditional Heteroskedastic (i.e. ARCH) models. Engle's insight was to express the *conditional* variance at time *t* as a (linear) function of variables known only at time *t*-1. Hence, the conditioning information set at time *t*-1 would include past errors and past variables. This approach to modelling the conditional variance is analogous to the Box-Jenkins methodology of modelling the conditional mean. Further refinements came from Bollerslev (1986), who introduced a generalized version of Engle's ARCH model (i.e. GARCH)[2].

The introduction of ARCH models[3] has opened up a whole range of different research. With modern economic theory showing increasing emphasis on risk and uncertainty, it is now possible to model the time-variation in second moments using a dynamic (ARCH) framework. It is therefore hardly surprising to find an explosion in the ARCH literature. Several other reasons also contribute to ARCH models' success. First, ARCH models are consistent with some of the previously mentioned empirical regularities of financial data. Second, the conditional mean and conditional variance parameters can be jointly estimated using conventional econometric methods.

[2] Taylor (1986) independently proposed a class of ARMACH (AutoRegressive Moving-Average Conditional Heteroskedastic) models, which is identical to the GARCH model of Bollerslev (1986). However, the ARMACH acronym never caught on.

[3] Henceforth, ARCH models, as used in the plural, shall denote the class of models that exhibit conditional heteroskedastic behaviour, to distinguish from Engle's original ARCH model.

Finally, in the absence of theoretical economic models in explaining temporal dependence in higher moments, ARCH models provide a useful and simple approach.

Research on ARCH models have proceeded in many different directions. On a theoretical level, there has been some attempts to incorporate ARCH effects into economic basis of asset pricing models. Gallant, Hsieh and Tauchen (1991) provide a rationale to explain the presence of dependency in higher moments. Nelson (1990) shows that ARCH models are not necessarily incompatible with diffusion models commonly found in asset pricing literature. Campbell and Hentschel (1992) develop a model to explain asymmetry in the volatility of stock returns. Unfortunately, such works on the economic foundations of ARCH effects are far and few.

On the other hand, ARCH models have found an overwhelmingly large number of empirical applications. Many economic and financial time series are found to exhibit ARCH effects, including stock returns (Akigray, 1989, Baillie and DeGennaro, 1990, Corhay and Rad, 1991, Engle and Ng, 1993, Kearns and Pagan, 1993, Pagan and Schwert, 1991, and Poon and Taylor, 1992), interest rates (Engle, Lilien and Robbins, 1987, Engle, Ng and Rothschild, 1990, Lee and Tse, 1991, and Weiss, 1984), exchange rates (Baillie and Bollerslev, 1989, Bollerslev, 1990, Gallant, Hsieh and Tauchen, 1991, and Hsieh, 1989), and futures prices (Antoniou and Foster, 1992, Cheung and Ng, 1990, Gagnon, Morgan and Neave, 1993, and McCurdy and Morgan, 1987). For a more complete citation on the huge amount of literature on

ARCH models, see Bera and Higgins (1993), Bollerslev, Chou and Kroner (1992) Bollerslev, Engle and Nelson (1993), Engle (1993) and Nijman and Palm (1991).

The analysis of financial market volatility is of significant importance, from both an academic and practitioner viewpoint. Following the stock market crash of October 1987, there has been considerable interest in the subject. In the words of Becketti and Sellon (1989):

> *"Some volatility in the prices of financial assets is a normal part of the process of allocating investable funds among competing uses. Excessive or extreme volatility of stock prices, interest rates and exchange rates may be detrimental, however, because such volatility may impair the smooth functioning of the financial system and adversely affect economic performance."*

Academic interest in volatility stems mainly from the efficient market hypothesis. In an efficient market, all available information in the market is impounded in current prices. Any unexpected news will cause the investor to revise his expectations of the future. This revision in expectations will induce a price change, since the new price will reflect the piece of unexpected information[4]. Implicitly, the efficient market hypothesis suggests that information is exogenously determined, and news arrival is

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[4] See Ross (1989) for a formal analysis of the relationship between variance of price changes and rate of information flow.

random. On the other hand, others suggest that information is generated endogenously by the mechanism of trading. Uninformed investors infer information from the actions of informed traders through the trading activities of the latter. The inferred information causes the uninformed traders to revise their expectations, which induces a change in price.

Regardless of whether information is generated endogenously or exogenously, it is clear that volatility is determined by the rate of information flow. ARCH models allow one to analyse volatility in a dynamic framework. For example, Masulis and Ng (1991) directly examine the hypothesis that stock volatility is a function of trading activities of investors, Susmel and Engle (1994) and Engle, Ito and Lin (1990) look at the spillover of volatility across national stock markets and foreign exchange markets respectively, Cheung and Ng (1990) explore the relationship between the volatilities of cash and future markets, and Conrad, Gultekin and Kaul (1991) investigate the relationship between the volatilities of firms of different sizes.

For practitioners such as fund managers, an understanding of volatility is important for hedging and portfolio selection. The ability to model and forecast volatility has consequential implications in asset pricing and tactical asset allocation decisions. For example, in the pricing of an option, one key variable is the volatility of the underlying asset, and accurate option prices can only be calculated if the volatility is accurately estimated. Day and Lewis (1992), Engle and Mustafa (1992), and Engle, Hong, Kane and Noh (1993) show that conditional volatility of stock returns as obtained from ARCH models have significant information content. In hedging decisions involving futures contract, again the ability to estimate the hedge ratios is of crucial importance. Baillie and Myers (1991) and Kroner and Sultan (1991) show that time varying hedge ratios estimated using ARCH models are superior to those estimated with linear regressions.

For public policy makers, the need for stability in financial markets requires an understanding of volatility and its impact on the economy. With the phenomenal growth in financial market activity, especially in the markets for derivative instruments, there is an emphasis on curtailing any 'excess' volatility. ARCH models allow for a dynamic analysis of volatility. For example, Antoniou and Foster (1992) use a GARCH model to examine the impact of futures trading on the spot price volatility of Brent Crude Oil, while Kupiec (1990) and Seguin (1990) analyse the effects of margins on volatility.

It must be said however, that ARCH models are not the only models used in empirical analysis. There are several competing models commonly in used in the finance and economic literature. These models possess nonlinear characteristics, unlike ARIMA models (see Granger and Teräsvirta, 1993, and Tong, 1990). These include the bilinear model (Granger and Anderson, 1978), the threshold autoregressive model (Tong and Lim, 1980), the exponential autoregressive model (Haggan and Ozaki, 1981), and Hamilton two-state switching-regime model (Hamilton, 1989). For details, see Granger and Teräsvirta (1993), Mills (1990 and

1993), and Tong (1990). Strictly speaking, most of these nonlinear techniques do not model volatility explicitly, although one common feature is that forecasts from such models are usually nonlinear functions of past observations.

As mentioned earlier, the simplicity of ARCH models and their ability to account for the observed "stylized facts" of financial data such as leptokurtosis and volatility clustering explain why ARCH models are more popular than the competing nonlinear models. As pointed out by Bera and Higgins (1993), ARCH models can also be subjected to different interpretations. For instance, an ARCH model can be re-interpreted as either a random coefficient model, or a bilinear model (see Bera and Higgins, 1993, Section 3 for details). This flexibility in interpretation also contribute to the popularity of ARCH models in research work.

2. MOTIVATION

This dissertation seeks to extend the literature on ARCH models. It consists of four papers:

2.1 An Empirical Analysis of Alternative Parametric ARCH Models

Since the introduction of the GARCH model by Bollerslev (1986), many newer parameterisations have been suggested. These include the Exponential GARCH of Nelson (1991), the Quadratic GARCH of Sentana (1991), the Threshold GARCH of

Glosten, Jagannathan and Runkle (1993) and Zakoian (1991), and the Nonlinear GARCH of Higgins and Bera (1992). In proposing these newer formulations, the practice is almost always to compare with the standard GARCH.

Any new formulation however, should meet the following two criteria. First, it should describe empirical facts that are not explained by the standard GARCH model. Second, the parameterisation must not only model volatility well on an in-sample basis, but also perform well out-of-sample.

One documented empirical regularity not accounted for in the GARCH model is the leverage effect. As noted by Black (1976) and Christie (1982), there appears to be an inverse relationship between current stock returns and future volatility. This phenomenon is attributed to the fact that a reduction in the value of equity increases the debt ratio of the company, and this increases the risk of the firm in the future. In both Engle's ARCH model and Bollerslev's GARCH model, the conditional volatility response is symmetric to current stock returns i.e. future volatility is a function of current stock returns, regardless of the signs of the returns (hence, they are usually called linear ARCH models). Several newer 'nonlinear' models like those mentioned above have been proposed to account for this leverage effect in the variance equation. However, each model accounts for this asymmetry in different ways. It would therefore be interesting to compare the empirical performance of each of this model using a consistent dataset.

A more important consideration is that these models may have different out-of-sample performance. For a volatility model to be useful, one must test the *ex ante* performance. As noted by Pagan and Schwert (1990), nonparametric volatility models perform very well in-sample, but fare badly on an out-of-sample basis. This is attributed to 'over-fitting', which is also a problem that plagues forecasting of the mean using nonparametric methods.

This paper seeks to compare the empirical performance of various parametric 'post-GARCH' models (i.e. models that are introduced after the GARCH model). Performance is examined on an in-sample basis, as well as in their out-of-sample predictive ability, using a variety of measures. A number of diagnostics proposed in existing literature is also examined. Results are evaluated over different time periods and across portfolios of different market sizes.

2.2 Asymmetry in Volatilities and the Private Information Hypothesis

Another well documented empirical regularity relates to the differences in the volatility of stock returns between trading and nontrading periods. It has been hypothesised that the volatility of a stock price is driven solely by the random arrival of new information (see for example Clark, 1973 and Tauchen and Pitts, 1983). If this is true, the variance of the stock returns between the close of trading on a Friday and the close of trading a Monday (i.e. over the weekend) is likely to be three times that of the variance between the close of one trading day and the close of the following trading day, without any intervening nontrading day.

Fama (1965) and French (1980) both find that the variance over the weekend period is only about 20 percent higher. French and Roll (1986) suggest three possible explanations:

(1) public information is released more frequently during trading hours

(2) private information is inferred from the trading activities of informed traders

(3) presence of irrational noise traders

An understanding of the mechanics of volatility is necessary because these hypotheses have different implications for both corporate treasurers and public policy makers. For example, if the public information hypothesis is true, it may be desirable to 'spread' the release of company announcements evenly over trading and nontrading periods so as to stabilise stock volatility. If the private information hypothesis is true, a more logical approach would be to extend the number of trading hours so that there will not be alternating periods of high volatility (during trading periods) and low volatility (during nontrading periods). If however, volatility is driven by irrational traders' activities, steps should be taken to reduce the amount of noise.

Two previous studies have attempted to examine the above hypotheses. Barclay, Litzenberger and Warner (1990) use a static framework to examine the differences in variances between trading and nontrading Saturdays in the Japanese stock market. Masulis and Ng (1991) employ a dynamic ARCH model to examine the differences in trading and nontrading shocks on future volatility using UK data. Both studies appear to support the private information hypothesis.

Along a different line, there is also empirical evidence to indicate that predictive (i.e. conditional) volatility is related to the size of the company. Conrad, Gultekin and Kaul (1991) provide evidence that in the US, the volatility of large size firms can predict the volatility of small size firms, but not vice versa. This finding mirrors that of Lo and MacKinlay (1990a)'s, who uncover evidence of asymmetric predictability in the (mean) returns of US firms of different sizes.

This paper looks at the volatility of UK stock returns in two contexts. First, it analyse the predictive ability in the conditional volatility of different size firms, using a GARCH model. A distinction is then made between trading and nontrading period shocks of large and small firms, and an examination is made on the effects of such shocks on future volatility of small and large firms respectively.

2.3 An Empirical Examination of the Volatility-Volume Relationship

Though the ARCH class of models has a rather short history, it has enjoyed great success as a modelling technique. Its widespread use lies in its ability to capture the "stylized" facts associated with financial time series. However, with few exceptions, there has been little emphasis on the sources of ARCH effects, especially for stock returns. Nevertheless, there have been some attempts to investigate the nature of temporal behaviour of stock market volatility. Such research falls into two categories: studies that link ARCH effects to macroeconomic variables, and studies that relate ARCH behaviour to microeconomic events. Among the studies in the first category include Attanasio (1991), Glosten, Jagannathan and Runkle (1993), Kupiec (1990), Schwert (1989), and Seguin (1990).

Studies in the second category attempt to explain GARCH effects in the data generation process. As noted by Diebold (1986), and Gallant, Hsieh and Tauchen (1991), ARCH effects may be a result of the sequential correlation of information arrival. This explanation stems from the works of Clark (1973) and Tauchen and Pitts (1983), who suggest that returns can be characterised as a subordinated process. In these models, information is explicitly assumed to be derived exogenously. On the other hand, the "price formation" models of Admati and Pfleiderer (1988), Foster and Viswanathan (1990), and Kyle (1985) assume that information is only partially exogenous, and some information is endogenously generated by the trading process. Nevertheless, regardless of whether information is exogenously or endogenously produced, these models would suggest that volatility is a function of news. In empirical tests, explicit modelling of the rate of information would be impossible, since by definition, news is unpredictable. As such, suitable proxies are required. Common proxies used in current literature include volatility shocks (Ahbyanker, 1993, Cheung and Ng, 1990, and Engle, Ito and Lin, 1990), number of price changes per period (Laux and Ng, 1993), and volume. The use of volume as a proxy for news arrival is especially popular because of two reasons. First, trading volume is frequently reported with price changes and is therefore easily available. But more importantly, the use of volume is heuristically appealing, since there is a common perception that "*It takes volume to make prices move*" (Karpoff, 1987, pg 112).

The last statement would suggest that price changes and volume are jointly determined. Indeed, Karpoff (1987) cites various studies that find a contemporaneous relationship between price volatility and trading volume across different markets. Interestingly, a number of these studies also document an asymmetric relationship between volatility and volume.

Karpoff (1987) suggests four reasons why an understanding of the relationship between market volatility and trading activity is important. First, it gives an insight into the structure of financial markets. Second, it has implications for event studies that draw inferences from volume behaviour. Third, it sheds light on the stochastic process of asset prices. Fourth, it is of significance to research into futures market.

The motivation of this paper is along these lines. Additionally, we seek to synthesise recent works on conditional volatility models with existing literature on the volatility-volume relationship. Our framework of analysis will be on the empirical relationship between stock index volatility and trading volume in the UK. We believe that this volatility-volume behaviour is of significance in modelling conditional volatility, and we suggest a means of incorporating this relationship to improve the performance of a conventional GARCH model.

2.4 A Test for Omitted Deterministic Dummy Variables in ARCH Models

The use of ARCH models has become very popular in empirical research, especially in modelling time varying conditional variances. Maximum likelihood estimation of ARCH models is rather straightforward, and some econometric software even provide "canned" instructions for estimating the parameters of a standard GARCH model.

However, as pointed out by Nelson (1992a), ARCH models are merely statistical models that approximate reality. Hence in empirical work, there is a possibility of choosing a misspecified model. Nelson proceeds to demonstrate that under fairly general conditions, conditional variances as estimated using misspecified ARCH models converge to the true conditional variances. He also suggests that the success of ARCH models in short term forecasting using high frequency data may be a direct result of this property, since misspecified ARCH models can still provide relatively accurate estimates of volatility.

Several other papers have also examined the effects of misspecified ARCH models, though in a slightly different context. Nelson and Cao (1992) look at the effect of constraining the parameters of the GARCH model as first suggested by Bollerslev (1986). They find that the Bollerslev's suggestions are too restrictive, and provide more general conditions[5]. Baillie and DeGennaro (1990) examine the issue from the point of a misspecified conditional distribution. They find that the use of a conditional student-*t* distribution instead of a conditional normal distribution changes the significance of the parameter for the conditional variance entering into the mean equation (i.e. the 'risk-premium' parameter)[6]. Finally, Lamoureux and Lastrapes (1990a) consider the impact of failing to account for deterministic structural changes on persistence in variance. They report that a misspecified GARCH model (i.e. one that did not account for structural shifts) can lead to "*a misinterpretation of estimates of volatility persistence*".

Without a theoretical model, it is therefore not surprising to find that ARCH models tend to be formulated in rather ad hoc ways. This can lead to misspecified ARCH models. While this may not be much of a problem in forecasting volatility (c.f. Nelson, 1992a), a wrong inference may be made from misspecified ARCH models.

Many empirical studies have documented anomalies in financial time series data. For example, returns are found to exhibit hour-of-the-day effect, day-of-the-week effect, week-of-the-month effect, and month-the-year effect. Similar effects are also found

[5] In the results of French, Schwert and Stambaugh (1987), the GARCH model has been constrained as per Bollerslev's suggestion. Nelson and Cao (1992) re-estimated the GARCH model using the less restrictive constraints and find some differences in the parameters of the model.

[6] Similar results are reported in Bollerslev and Wooldridge (1992) and Susmel and Engle (1994). This leads Bollerslev and Wooldridge to suggest the use of quasi-maximum likelihood estimation. Alternatively, one could use nonparametric techniques, as in Pagan and Schwert (1990), or semi-nonparametric methods, as in Gallant, Hsieh and Tauchen (1991).

in the *unconditional* volatility of asset returns. However, there has been little emphasis in trying to account for such effects in conditional volatility models.

Consider the problem associated with nontrading. It is well documented that the volatility over a nontrading period differs from the volatility over a trading period. It is therefore difficult to envisage that the data generating process over both periods is identical. In a dynamic framework like that of ARCH models, current volatility is a function of past volatility. Failure to distinguish between nontrading and trading volatility results in an incomplete or incorrect model. This problem will be especially severe if the researcher is working with high frequency data (for example, hourly data), since the ratio of the length of nontrading to trading period increases.

Focusing on the issue of nontrading, the paper analyse the approaches commonly used in existing empirical work to account for such effects. A diagnostic test is proposed to determine the specific form of accounting for nontrading effects. Using the test, the *conditional* volatility surrounding a nontrading period is then examined.

3. ORGANIZATION OF THE DISSERTATION

This dissertation is organised as follows:

(a) Chapter 2 provides an empirical comparison of various parametric ARCH models using a variety of benchmarks and diagnostics. It is based on UK stock data, spanning from January 1971 to December 1990[7].

(b) Chapter 3 examines the asymmetric predictability of conditional volatilities between small and large companies. It looks at the differences in shocks of small and large companies on future volatilities of large and small companies respectively. The analysis also addresses the issue of whether volatility is driven by the private information of informed investors. The study uses daily UK stock data from January 1990 to June 1993.

(c) Chapter 4 looks at the empirical relationship between the conditional volatility and volume. Daily UK stock index data between January 1990 and December 1993 is used in the analysis.

(d) Chapter 5 proposes a diagnostic test for omitted dummy variables in ARCH models. This test aids in the identification of a more "complete" conditional

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[7] First draft, 9 June 1992, revised 15 November 1993. This paper has been presented to *BAA Annual Conference* 1993, *French Finance Association Conference* 1993, and *INQUIRE-Europe Conference* 1993. The paper will be presented at the forthcoming *Western Finance Association Conference* 1994.

volatility model. The conditional volatility surrounding a nontrading period is also examined. Empirical evidence is based on hourly US stock index futures data from June 1983 to June 1987. The dataset is obtained from Professor Pradeep K Yadav, courtesy of Craig MacKinlay (Wharton).

(e) Chapter 6 summarises the main conclusions and suggests some directions for future research.

CHAPTER 2

AN EMPIRICAL ANALYSIS OF ALTERNATIVE PARAMETRIC ARCH MODELS[8]

Abstract

This paper compares the performance of various parametric ARCH models. Using data based on 20 years of UK daily stock returns on a value weighted stock index and ten size sorted equally weighted stock portfolios for the period 1971-90, it is found that ARCH models are good descriptions of the time-varying volatility. The conditional variance is an unbiased predictor of the actual variance in all cases, but the assumption of normal conditional densities is inadequate. None of the three Engle and Ng (1993)'s diagnostic tests based on the news impact curve are able to discriminate between the different models. The relative performance of the models appears to differ across different periods and across different portfolios. Nevertheless, we find that the parameters proxying for asymmetry in the post-GARCH models are usually statistically significant, with the TGARCH and EGARCH models significantly outperforming the GARCH model over each of the more recent sub-periods 1981-85 and 1986-90. The superior performance disappears when a benchmark that gives relatively greater weight to errors in predicting small variances is used. In addition, there do not appear to be any systematic firm size related differences in the relative predictive ability of the different parametric ARCH models.

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AN EMPIRICAL ANALYSIS OF ALTERNATIVE PARAMETRIC ARCH MODELS[9]

1. INTRODUCTION

There is extensive empirical evidence that stock market volatility varies systematically with time. The evidence dates back to the pioneering studies of Mandelbrot (1963) and Fama (1965) who found that large price changes tend to be followed by large price changes and small price changes by small price changes. More recent evidence is provided by Poterba and Summers (1986), French, Schwert and Stambaugh (1987), Chou (1988) and Schwert (1990).

There is also strong evidence that ARCH models are good descriptions of this time-varying volatility in stock returns. Significant ARCH effects are documented *inter-alia* by Engle and Mustafa (1992) for individual US stocks, Akgiray (1989) for US stock indices, Poon and Taylor (1992) for a UK stock index, Corhay and Rad (1991) for a selection of international stock indices and Frennberg and Kansson (1992) for the Swedish stock market.

Systematic temporal variation in volatility of asset returns implies that the variance at time t can be broken up into predictable and unpredictable components. The

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predictable component is called the conditional variance and is a function of information available at time t-1. This information can include relevant firm specific and economy wide variables, and also the past history of the returns' series. Clearly, modelling conditional variance of stock returns is important because expected volatility is a fundamental input in portfolio selection decisions and in models of asset and option pricing.

In its most general form (see Engle, 1982, eqs 1-5) a univariate ARCH model makes conditional variance σ_t^2 at time t a function of exogenous and lagged endogenous variables, time, the vector of parameters, and past residuals. A variety of different parameterisations for the functional dependence of σ_t^2 on the above variables have been proposed by econometricians. These started with the original linear ARCH(q) specification of Engle (1982) and the linear GARCH(p,q) model of Bollerslev (1986), but now include for example the exponential GARCH of Nelson (1991), the threshold GARCH of Glosten *et. al.* (1991) and Zakoian (1991), the quadratic GARCH of Sentana (1991), the generalised augmented ARCH of Bera *et. al.* (1990), the asymmetric GARCH of Engle (1990), the non-linear GARCH of Higgins and Bera (1992), and the log-GARCH models of Geweke (1986) and Pantula (1986).

All of the above parameterisations of conditional heteroskedasticity have been motivated either by the need to more effectively model some specific empirical features of the underlying data (e.g. the negative correlation between current returns and future volatility documented by Black, 1976, and Christie, 1982) or by considerations of computational simplicity (e.g. EGARCH avoids the need for non-negativity constraints in estimation). There has been relatively little work on the economic foundations of ARCH models[10], or the formulation, from first principles, of equilibrium asset pricing models with ARCH features. Since the different parameterisations of ARCH models have been motivated essentially on the basis of data specific empirical features, their relative usefulness can only be judged by the extent to which they are able to explain time variation in conditional volatility in actual data. The more recent ARCH models include more parameters than in the linear ARCH(q) of Engle (1982) and the linear GARCH(p,q) of Bollerslev (1986). Hence, clearly these recent models could be potentially better *ex post* descriptions of the return generating process. But the *ex ante* usefulness of these models for portfolio selection and asset pricing decisions depends on the out of sample predictive ability of these models. It also depends on whether the outperformance is consistent across different sub-periods and across different categories of stocks. Furthermore, it is important to note that the results of Bollerslev and Domowitz (1991) show that the trade execution process can significantly alter the intertemporal dependence in conditional volatility of high frequency returns because of the differences in serial correlation in market spreads across different trading systems.

Several studies have examined the effectiveness of different individual ARCH models in modelling conditional stock market volatility relative to the linear ARCH and the

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[10] This has been pointed out eg by Bollerslev *et. al.* (1992, section 2.9).

linear GARCH model. Comparisions of EGARCH with GARCH are reported *inter-alia* by Nelson (1990) using daily CRSP index returns from July 1962 to December 1987; Kearns and Pagan (1991) using 88 years monthly Australian stock index data; Poon and Taylor (1992) using daily, weekly, fortnightly and monthly UK stock index data over a period exceeding 20 years; and Zakoian (1991) using 2 years of daily French CAC40 index data. Sentana (1991) compares ARCH models with the linear GARCH using a century of US stock index data. Rabemanjara and Zakoian (1993) compare a threshold GARCH model with the linear GARCH using about two year of daily data on the French CAC40 index and some individual stocks comprising the index. Pagan and Schwert (1990) analyse several alternative conditional volatility models using monthly data from 1835 to 1925, but most of these are non-parametric models, the only parametric ARCH models being the EGARCH and the linear GARCH. To the best of our knowledge, Engle and Ng (1993) is the only major study to date which evaluates a wide spectrum of parametric ARCH models. They examine six parametric ARCH models using eight years daily returns on the Japanese Topix Index. However, the main focus of their study is the development of new diagnostic tests and the asymmetry of the volatility response to news, and though they do a sub-sample robustness check, their empirical analysis does not include an evaluation of the out-of-sample predictive ability of the models.

This paper provides comprehensive empirical evidence on different parametric ARCH models using daily data on a value weighted stock index and ten size based portfolios of stocks, covering the 20 year period 1971-1990. It seeks to make a contribution in several directions. First, it documents for the same data, the relative effectiveness of most of the major parametric ARCH models that have been proposed in the literature. Second, the *ex ante* usefulness of these models for portfolio selection and asset pricing decisions is evaluated by quantifying their out-of-sample predictive ability. Third, it examines the relative performance not only across different time periods but also across portfolios corresponding to different market value deciles. Finally, it employs data from the UK market where reported prices represent firm dealer quotes rather than last transaction prices as in the US or Japan on which most of the earlier work has been based.

This paper is organised as follows: Section 2 provides a specification of the parametric ARCH models examined; Section 3 describes the data; Section 4 outlines the methodology used; Section 5 documents the results; and Section 6 summarises the conclusions.

2. PARAMETRIC ARCH MODELS

Consider a time series of stock returns y_t which can be modelled as

$$y_t = f(x_t^{/}) + u_t$$

where $x_t^{/}$ denotes a vector of variables (either lagged dependent or exogenous) that affect the conditional mean of the series y_t, and u_t is an independent and identically distributed error term with zero mean, a variance σ_t^2 conditional on Ω_{t-1}, the information set available at time t-1, and a density function D(u_t). A general conditional volatility model can be written as

$$E\left[\sigma_t^2 \mid \Omega_{t-1}\right] = f(\Omega_{t-1})$$

Clearly, there are two major issues that need to be addressed: first, what variables should be in Ω_{t-1}; and second what is the nature of the function $f(\cdot)$? Ideally, Ω_{t-1} should contain all (relevant) variables that are observable at time t-1. Given the complexity of financial markets, these are virtually impossible to specify completely or precisely. If markets are efficient, volatility (like price) at time t should reflect all information at time t. Hence natural proxies for Ω_{t-1} are lagged values of σ_t^2 and u_t^2. The nature of the parametric ARCH model is determined by the specific functional form of $f(\cdot)$. In the linear ARCH(q) model of Engle (1982),

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_j u_{t-j}^2$$

with $\alpha_0 > 0$, $\alpha_j \geq 0, j = 1, \ldots\ldots, q$ to ensure a positive conditional (and unconditional) variance.

Bollerslev (1986) generalised the ARCH model to the linear GARCH(p,q) model where:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j u_{t-j}^2$$

with $\alpha_0 > 0$, $\beta_i \geq 0$, $i = 1, \ldots\ldots, p$, $\alpha_j \geq 0, j = 1, \ldots\ldots, q$. The simple GARCH(1,1) model has often been the most popular for modelling financial time series. Engle and Bollerlev (1986) extended GARCH to the integrated GARCH in which the coefficients sum to unity implying infinite unconditional variance as in the stable paretian distributions suggested by Fama (1965) and Mandelbrot (1963).

However, there are several difficulties with the linear GARCH model. First, there are features of the data which the model is not capable of describing. In particular, Black (1976) and Christie (1982) document a negative correlation between current returns and future volatility. There is an asymmetry in the impact of news on volatility. Negative news surprises increase predictable volatility more than positive news comprises. The GARCH model fails to capture this "leverage" effect because it is a symmetric (i.e. quadratic) function of u_t. Second, GARCH estimation must neccessarily restrict α_j and β_i to positive values to ensure non-negativity of the conditional variances. This makes computations more difficult. Furthermore, as highlighted by Rabemanjara and Zakoian (1993), the impact of past volatility shocks, irrespective of sign, always increases with the magnitude of the shock in the GARCH model making it incapable of describing cyclical or non-linear behaviour in the volatility. These limitations have motivated most of the post-GARCH models.

Nelson (1990) proposes the Exponential GARCH (EGARCH) model:

$$\log\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}\beta_i \log\sigma_{t-i}^2 + \sum_{j=1}^{q}\alpha_j\left[\theta_{t-j}\frac{u_{t-j}}{\sigma_{t-j}} + \gamma\left(\left|\frac{u_{t-j}}{\sigma_{t-j}}\right| - \sqrt{\frac{2}{\pi}}\right)\right]$$

The EGARCH accounts for the asymmetric relationship between returns and volatility, and avoids the need to restrict α_j and β_i to positive values to ensure non-negativity of the conditional variances.

Glosten *et. al.* (1993) and Zakoian (1991) model the "leverage" effect by addition of terms based on threshold values of u_t to the GARCH model. The Threshold GARCH (TGARCH) can be expressed as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}(\beta_i\sigma_{t-i}^2 + \gamma_i S_{t-i}^-\sigma_{t-i}^2) + \sum_{j=1}^{q}(\alpha_j u_{t-j}^2 + \delta_j S_{t-j}^- u_{t-j}^2)$$

where $S_t^- = 1$ if $u_t < 0$, $S_t^- = 0$ otherwise.

A general parameterisation of the conditional variance equation is the Quadratic ARCH model, put forward by Sentana (1991). This model can be viewed as a second-order Taylor expansion of the (unknown) conditional variance function. In its most general form, the Quadratic GARCH (QGARCH) model is given by:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}\beta_i\sigma_{t-i}^2 + \sum_{j=1}^{q}\gamma_j u_{t-j} + \sum_{j=1}^{q}\sum_{k=1}^{q}\alpha_{jk}u_{t-j}u_{t-k}$$

The QGARCH model encompasses several models, including the standard linear GARCH, the Generalised Augmented ARCH (GAARCH) of Bera, Lee and Higgins (1990), the Asymmetric GARCH (AGARCH) of Engle (1990) and Robinson's (1991) linear standard deviation model. For example, Engle's AGARCH is given by

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} (\alpha_j |u_{t-j}|^{\theta} + \gamma_j u_{t-j})$$

Suitably formulated, the QGARCH model, and its variants, can capture the asymmetric relationship between returns and volatility.

Another approach to reparameterising the conditional variance equation has been recently suggested by Higgins and Bera (1992). By applying a Box-Cox (1964) transformation to the variance equation, a nonlinear GARCH (NGARCH) model can be obtained.

$$\sigma_t^2 = \left[(\alpha_0)^{\delta} + \sum_{i=1}^{p} \beta_i (\sigma_{t-i}^2)^{\delta} + \sum_{j=1}^{q} \alpha_j (u_{t-j}^2)^{\delta} \right]^{\frac{1}{\delta}}$$

Like the QGARCH model, the NGARCH model encompasses several other models, such as the standard linear GARCH, and the log-GARCH form of Geweke (1986) and Pantula (1986).

In this study, the information set Ω_{t-1} governing conditional volatility is assumed to contain only one time period lagged variables. In particular, it is assumed that[11]:

[11] See Pagan and Schwert (1990) on using e_t as conditioning variables instead of u_t.

$$E\left[\sigma_t^2 \mid \Omega_{t-1}\right] = f(\sigma_{t-1}, e_{t-1})$$

Longer lags of either σ_t or e_t are not reported for three reasons. First, earlier studies have found that lags of one are not only sufficient but often provide the best fit (see eg Corhay and Rad, 1991, Chou, 1988, Baillie and DeGennaro, 1990, and Poon and Taylor, 1992). Second, as Engle and Ng (1993) point out, by holding the information before t-1 constant, the impact of new information on volatility can be examined. Third, for several of our models, we did examine a longer lag structure of up to 3 lags. Our results were not affected either quantitatively or qualitatively by including the additional lags.

More importantly, it is neccessary to include the effect of holidays and weekends on the volatility equation. There is clear evidence that the volatility over holidays and weekends differs from the volatility over trading days (see for example French and Roll, 1986, and Lockwood and Linn, 1991). To account for this, we modify the conditional volatility equation to

$$E\left[\sigma_t^2 \mid \Omega_{t-1}\right] = f([\sigma_{t-1} - \omega X_{t-1} - \psi Y_{t-1}], e_{t-1}) + \omega X_t + \psi Y_t$$

where X_t takes the value of one if period t follows a holiday and zero otherwise, and Y_t takes the value of one if period t follows a weekend and zero otherwise. The aim is to subtract volatility due to the holiday and weekend effect from the 'normal' trading day volatility[12]. In this context, the models described in this section so far are estimated in the following form:

GARCH(1,1)

$$\sigma_t^2 = A_0 + A_1(\sigma_{t-1}^2 - \omega X_{t-1} - \psi Y_{t-1}) + A_2 e_{t-1}^2 + \omega X_t + \psi Y_t$$

EGARCH(1,1)

$$\sigma_t^2 = \exp\left[A_0 + A_1\, \varsigma_{t-1}^2 + A_3\left(A_2 \frac{e_{t-1}}{\varsigma_{t-1}} + \left|\frac{e_{t-1}}{\varsigma_{t-1}}\right| - \sqrt{\frac{2}{\pi}}\right)\right] + \omega X_t + \psi Y_t$$

where $\varsigma_t^2 = \sigma_t^2 - \omega X_t - \psi Y_t$

TGARCH(1,1)

$$\sigma_t^2 = A_0 + A_1(\sigma_{t-1}^2 - \omega X_{t-1} - \psi Y_{t-1}) + A_2 e_{t-1}^2 + A_3 S^- e_{t-1}^2 + \omega X_t + \psi Y_t$$

where $S^- = 1$ if $e_{t-1} < 0$, and $S^- = 0$ otherwise.

QGARCH(1,1)

$$\sigma_t^2 = A_0 + A_1(\sigma_{t-1}^2 - \omega X_{t-1} - \psi Y_{t-1}) + A_2 e_{t-1}^2 + A_3 e_{t-1} + \omega X_t + \psi Y_t$$

NGARCH(1,1)

$$\sigma_t^2 = \left[A_0 + A_1\left(\sigma_{t-1}^2 - \omega X_{t-1} - \psi Y_{t-1}\right)^{A_3} + A_2\left(e_{t-1}^2\right)^{A_3}\right]^{1/A_3} + \omega X_t + \psi Y_t$$

AGARCH(1,1)

$$\sigma_t^2 = A_0 + A_1(\sigma_{t-1}^2 - \omega X_{t-1} - \psi Y_{t-1}) + A_2\left|e_{t-1}\right|^{A_3} + A_4 e_{t-1} + \omega X_t + \psi Y_t$$

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[12] Such a formulation is employed for example, by Cheung and Ng (1990).

Engle and Ng (1993) introduce the *News Impact Curve* to examine how new information affects the next period variance. The curve represents the relationship between innovations in returns e_{t-1} and the volatility σ_t^2 implied by a conditional variance model estimated at the sample mean and holding constant information dated *t*-2 and earlier. For the GARCH and NGARCH models, a quadratic curve is obtained, symmetric at $e_{t-1} = 0$. The curves for EGARCH, TGARCH, QGARCH and the AGARCH models have their minimum also at $e_{t-1} = 0$, but they are asymmetric. Engle and Ng (1993) also propose a Nonlinear Asymmetric GARCH (NAGARCH), which not only captures the asymmetric relationship between e_{t-1} and σ_t^2, but also allows for the asymmetric impact response curve to be centred at a non-zero e_{t-1}:

NAGARCH(1,1)

$$\sigma_t^2 = A_0 + A_1 \varsigma_{t-1}^2 + A_2 [e_{t-1} + A_3 \varsigma_{t-1}]^2 + \omega X_t + \psi Y_t$$

$$\text{where} \qquad \varsigma_t^2 = \sigma_t^2 - \omega X_t - \psi Y_t$$

Additionally, other models based on some other nonlinear functions of the conditional variance equation that can also be proposed. First, one can allow for interactions between e_{t-1} and σ_{t-1}^2 (the NAGARCH model can be viewed as a standard GARCH with an additional $e_{t-1}\sigma_{t-1}$ term, or a modified form of the QGARCH). This leads to a GARCH model with an interaction term (INGARCH):

INGARCH(1,1)

$$\sigma_t^2 = A_0 + A_1 \varsigma_{t-1}^2 + A_2 e_{t-1}^2 + A_3 e_{t-1} \varsigma_{t-1}^2 + \omega X_t + \psi Y_t$$

where $\quad \varsigma_t^2 = \sigma_t^2 - \omega X_t - \psi Y_t$

One can combine several of the above models to obtain new parametric formulations. For example, by combining the TGARCH with the QGARCH, one gets a TQGARCH model:

TQGARCH(1,1)

$$\sigma_t^2 = A_0 + A_1(\sigma_{t-1}^2 - \omega X_{t-1} - \psi Y_{t-1}) + A_2(e_{t-1} - A_3)^2 + A_4 S^-(e_{t-1} - A_3)^2 + \omega X_t + \psi Y_t$$

Likewise, the Box-Cox transformation technique of NGARCH can be applied on QGARCH to yield the Nonlinear Quadratic GARCH (NQGARCH):

NQGARCH1(1,1)

$$\sigma_t^2 = \left[A_0 + A_1(\sigma_{t-1}^2 - \omega X_{t-1} - \psi Y_{t-1})^{A_4} + [A_2(e_{t-1} - A_3)^2]^{A_4}\right]^{1/A_4} + \omega X_t + \psi Y_t$$

NQGARCH1 nests the following models

a) GARCH ($A_3 = 0$ and $A_4 = 1$)

b) QGARCH ($A_4 = 1$)

c) NGARCH ($A_3 = 0$)

A variant to the above model is to apply the Box-Cox transformation only to σ_t^2 and σ_{t-1}^2 terms i.e.

NQGARCH2(1,1)

$$\sigma_t^2 = \left[A_0 + A_1(\sigma_{t-1}^2 - \omega X_{t-1} - \psi Y_{t-1})^{A_4} + A_2(e_{t-1} - A_3)^2\right]^{1/A_4} + \omega X_t + \psi Y_t$$

Note that the NQGARCH2 is almost identical to the EGARCH when $A_4 \rightarrow 0$.

Finally, one could combine the NGARCH with the TGARCH to get

NTGARCH1(1,1)

$$\sigma_t^2 = \left[A_0 + A_1(\sigma_{t-1}^2 - \omega X_{t-1} - \psi Y_{t-1})^{A_4} + A_2(e_{t-1}^2)^{A_4} + A_3 S^-(e_{t-1}^2)^{A_4}\right]^{1/A_4} + \omega X_t + \psi Y_t$$

NTGARCH2(1,1)

$$\sigma_t^2 = \left[A_0 + A_1(\sigma_{t-1}^2 - \omega X_{t-1} - \psi Y_{t-1})^{A_4} + A_2 e_{t-1}^2 + A_3 S^- e_{t-1}^2\right]^{1/A_4} + \omega X_t + \psi Y_t$$

Like the NAGARCH, the INGARCH, TQGARCH and both variants of NQGARCH and NTGARCH are formulated such that the models' *News Impact Curve* depends on various (nonlinear) terms. As such, these models also attempt to capture observed asymmetric leverage effects.

3. DATA

The empirical analysis is based on 20 years daily data of the London Stock Exchange from January 1971 to December 1990. 11 returns' series are analysed corresponding to the FT All Share Index and ten size sorted portfolios. Daily values of the FT All Share Index for the sample period were collected from Datastream and used to calculate daily log returns. To construct the ten size sorted portfolios, all stocks which satisfied the following criteria were selected:

(a) Daily "adjusted"[13] and unadjusted prices for the stock were available on Datastream for the entire sample period,

(b) Dividends and ex dividend dates for the stock were available from the London Business School Share Price Database for the entire sample period, and

(c) The stock had traded at least once during every calendar month in the sample period.

Criteria (a) and (b) above clearly introduce a selection bias but this does not appear important in the context of the study. Criteria (c) above was included to mitigate potential distortions due to infrequent trading[14].

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[13] "Adjusted" prices control for changes in capitalisation of the stock.

[14] Infrequent trading related effects are also filtered out before estimating conditional volatility. See Section 4 later.

176 stocks satisfied the above criteria. Daily adjusted and unadjusted prices for these stocks were collected from Datastream and dividend data was collected from the LBS Share Price Database and these were used to calculate daily returns for each day in the sample period. Using the market value at the start of the sample period as the basis of sorting, ten equally weighted size based portfolio returns' series were constructed.

4. METHODOLOGY

4.1 Empirical Research Design

The empirical research design adopted in this study is essentially similar to that of Pagan and Schwert (1990), though a wider spectrum of diagnostic tests are employed including in particular the tests based on the *News Impact Curve* recently proposed by Engle and Ng (1993). The 20 year sample period January 1971 to December 1990 is divided into two equal 10 year periods - January 1971 to December 1980 and January 1981 to December 1990. The second half of the sample is itself split into two sub-periods - January 1981 to December 1985 and January 1986 to December 1990 - because it includes the major market crash on Black Monday October 19, 1987 and the mini crash on Black Friday October 16, 1989, both of which involved high volatility, localised in time, following high negative returns.

An important issue is the choice of the conditional density function $D(u_t)$ for the error term. The obvious first possible choice is a normal distribution. However, unconditional stock return distributions tend to have fatter tails than a normal distribution[15], and the standardised residuals from estimated parametric ARCH models are often leptokurtic for financial time series (Bollerslev *et al*, 1992, Section 2.3). Several other forms of the conditional density function $D(u_t)$ have been suggested in the literature, e.g. the Student-*t* distribution in Bollerslev (1987), the normal-Poisson mixture distribution in Jorion (1988), the power exponential distribution in Baillie and Bollerslev (1989), the normal-lognormal mixture distribution in Hsieh (1989), and the generalised exponential distribution in Nelson (1990). To test whether our rankings of the different parametric ARCH models are robust to the choice of the conditional density function, we run all computations for the FT All Share Index in two ways. First, we base our estimations on log-likelihood functions that correspond to a normal distribution. Second, we assume that the error term is Student-*t* distributed, as in Bollerslev (1987). We find that our relative rankings of the different parametric ARCH models remain exactly the same for the FT All Share Index. Hence, all further estimations are based on assuming conditional normality, and all the results reported in the tables in this paper are for a conditional normal density function[16].

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[15] See eg Mandelbrot (1965) and Fama (1965) for early evidence.

[16] Results based on the use of Student-*t* as the conditional distribution are available from the author on request.

The parameters of each of the variance equations (corresponding to the different parametric ARCH models) are estimated separately for each of the 4 periods (1971-80, 1981-90, 1981-85 and 1986-90), and for each of 11 returns' series (the FT All Share Index and the 10 size based portfolios). These parameters are used to estimate the daily conditional volatility series for each of the 44 cases (11 returns' series and 4 periods), and together with the diagnostics constitute the in-sample set of results. To estimate the *ex-ante* out-of-sample predictive power of the different models, the parameters of the relevant variance equation estimated from an earlier sub-period, are used to compute the conditional volatility in the following period. Therefore, parameters estimated from the 1971-80 data are used to predict conditional volatility for the 1981-90 and the 1981-85 periods and the parameters estimated from the 1981-85 data are used to predict conditional volatility for the 1986-90 period.

Since the focus of this study is on conditional volatility it is necessary to remove possible predictability in the conditional mean. Such predictability can potentially arise from two sources - calender based seasonalities and infrequent trading of portfolio stocks. The method followed to filter out such predictability is similar to that used by Pagan and Schwert (1990), Engle and Ng (1991) and Kearn and Pagan (1990). For each return series, we first regress the returns y_t on five day-of-the-week dummies - MON, TUE, WED, THU, FRI - and a dummy variable X_t for the day following a holiday:

$$y_t = \lambda_1 MON_t + \lambda_2 TUE_t + \lambda_3 WED_t + \lambda_4 THU_t + \lambda_5 FRI_t + \lambda_6 X_t + u_t$$

u_t is then regressed against a constant and ten autoregressive lags to give the residuals, ϵ_t.

$$u_t = \upsilon_0 + \sum_{l=1}^{10} \upsilon_{t-l} u_{t-l} + \epsilon_t$$

From the series ϵ_t, we then estimate by means of maximum likelihood the following equation using the algorithm of Berndt, Hall, Hall and Hausman (1974)

$$\epsilon_t = B_0 + e_t$$

where e_t is distributed as conditionally normal, with zero mean and a time-varying variance σ_t^2 based on various parametric formulations described in Section 2.

4.2 Benchmarks and Diagnostics

Several benchmarks and diagnostics are examined. Since all the models are estimated using maximum likelihood, the first benchmark is the log-likelihood. In this context, it is relevant to note that except for the EGARCH model, the GARCH model is nested within all the other parametric ARCH models examined. Therefore, a likelihood ratio test can be performed. For models with one additional parameter i.e. the TGARCH, QGARCH, NGARCH, NAGARCH and INGARCH models, the log-likelihood must improve by at least 1.92 before we can reject the null hypothesis that the additional parameters are not significantly different from zero at the 5% level. Similarly, for models with two additional parameters, i.e. the AGARCH, TQGARCH1, TQGARCH2, NQGARCH1, NQGARCH2, NTGARCH1 and NTGARCH2, the log-likelihood must improve by at least 3.00 for the additional parameters to be significantly different from zero at the 5% level.

Second, the sample skewness and excess kurtosis of the standardised residuals from each of the models is calculated and the hypothesis that they are equal to zero is tested. In this paper, all directly reported results assume conditionally normally distributed errors. However, results (on other financial assets) of McCurdy and Morgan (1987), Milhøj (1987), Hsieh (1989) and Baillie and Bollerslev (1989) suggest that such an assumption does not capture all the observed skewness and excess kurtosis. This test will provide empirical evidence in this regard on UK stock returns.

Third, we also test for serial correlation in the standardised residuals and the square of the standardised residuals. These are standard tests employed in the ARCH literature to check if the models fully capture heteroskedastic behaviour.

Fourth, we run the following regression:

$$e_t^2 = \alpha + \beta \sigma_t^2 + v_t$$

and look at the R^2 (the coefficient of determination)[17]. The higher the R^2, the better will be the prediction of the actual variance e_t^2 with the estimated variance σ_t^2. We also run the logarithmic version of the above regression ie

$$\log e_t^2 = \alpha + \beta \log \sigma_t^2 + \nu_t$$

and look at the R^2 (which we shall term as R^2-for-logs). This assumes a proportional loss function (as opposed to a quadratic one implied in the linear regression model) between e_t^2 and σ_t^2 (see Pagan and Schwert, 1990). The regression for logs implies that mistakes in predicting small variances are given more weight than in the linear regression.

In the context of the use of R^2 or R^2-for-logs as benchmarks of performance, it is important to emphasise that the values of R^2 or R^2-for-logs do not have "standard error" estimates around them, and so it very difficult to come to any reasonably firm conclusion about whether the differences in R^2, or R^2-for logs, between different models are statistically significant or not, except through consistency across time or consistency across portfolios.

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[17] The R^2 measures the explanation of the squared innovations e_t^2 by the conditional variance σ_t^2, while the log-likelihood pertains to the innovations e_{t-1}. As noted by Pagan and Schwert (1990), while the two measures point in the same direction, they are not comparable.

The regression of the actual variance e_t^2 on the estimated variance σ_t^2 also tests for model adequacy (Pagan and Sabau, 1987). If the model is well specified, the intercept α should be zero and the slope β should be unity. Furthermore, if persistence in volatility is fully captured by the models, the residuals v_t should be serially uncorrelated.

Our final set of diagnostics come from Engle and Ng (1993). The tests are based on the *News Impact Curve* which represents the relationship between innovations in returns e_{t-1} on the volatility σ_t^2 implied by a conditional variance model estimated at the sample mean and holding constant information dated *t*-2 and earlier. The tests are designed to examine if the volatility models represent the data adequately. There are three tests. The sign-bias test indicates whether *positive* and *negative* return innovations have different impact on the volatility not predicted by the null volatility model. The positive-size-bias test indicates the difference in impact between *large* and *small* positive innovations on volatility not explained by the null volatility model. Finally, the negative size bias test indicates the difference in impact between *large* and *small* negative innovations on volatility not predicted by the null volatility model. To conduct the tests, the following regression are estimated:

$$z_t^2 = a + b_1 S_t^- + b_2 S_t^- e_{t-1} + b_3 S_t^+ e_{t-1} + \upsilon_t$$

where $z_t = \dfrac{e_t}{\sigma_t}$; $S_t^- = 1$ if $e_{t-1} < 0$ and $S_t^- = 0$ otherwise; and $S_t^+ = 1 - S_t^-$.

The t-ratios for b_1, b_2 and b_3 are the test statistics for the sign-bias test, the negative-size-bias test, and the positive size-bias test respectively. We also do a joint F-test for any of the three statistics to be different from zero.

5. RESULTS

5.1 Unconditional Estimates

Table 1 shows the summary statistics for the residuals ϵ_t of each of the 11 portfolios over the 4 periods. The unconditional volatility is highest for the 1971-80 period for all portfolios. The recursive variance plots[18] show that this is driven largely by the high volatility in the 1973-75 period. The unconditional volatility during the 1986-90 sub-period is also higher than the 1981-85 sub-period, once again driven by the localised high volatility around Black Monday October 19, 1987. The unconditional volatility also appears to be related to firm size: portfolios of large-sized companies generally have a higher unconditional variance than that of small-sized companies.

In all cases, the residuals display skewness and excess kurtosis that are significantly different from that of a normal distribution. Portfolios of small-sized companies tend to be more negatively skewed than portfolios of large-sized companies. Significant excess kurtosis indicates fat-tailed distributions for all portfolios. Kurtosis appears

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[18] These are available from the author on request.

to be size related, with portfolios of small-sized companies displaying greater kurtosis. Kurtosis is greatest during the 1986-90 period, presumably arising from the relatively higher localised variations in distributional parameters over this period due to the stock market crash on Black Monday.

Table 1 also shows the results of the Ljung-Box test for serial correlation of up to 10 lags for both the residuals ϵ_t and squared residuals ϵ_t^2. While the residuals are not serially correlated (since all predictability has been removed from the mean), serial correlation is present in the squared residuals for all portfolios across all periods, suggesting time-varying volatility.

5.2 The FT All Share Index

5.2.1 In-sample estimation

Tables 2A, 2B, 2C and 2D summarise the benchmarks and diagnostics related to the in-sample performance of the different parametric ARCH models for the FT All Share Index for the four periods 1971-80, 1981-90, 1981-85 and 1986-90 respectively. The tables have several interesting features. First, except for the 1981-85 period, the standardised residuals from the parametric volatility models continue to display significant skewness and excess kurtosis, even though the extent of skewness and excess kurtosis is markedly lower than the unconditional estimates reported in Table 1. However, for all models and all periods, the 10-lags Ljung Box statistic for the squared standardised residuals ie $Q^2(10)$ is not significant even at the

10% level when it was highly significant for the unconditional estimates in Table 1. This shows that all the ARCH models estimated are generally good descriptions of the time-varying volatility in the index.

Second, examination of the log-likelihood reveals that, in the majority of cases, the log-likelihood of the additional parameter models (which nest the GARCH model as a special case) is significantly greater (at the 5% level) than that of the GARCH model. However, it is only the NAGARCH and the TQGARCH which outperform the GARCH on this criteria in all periods. Nevertheless, this does show that the parameters proxying for asymmetry in the post-GARCH models are usually statistically significant.

With R^2 as the benchmark of performance, the results vary significantly from the first to the second half of the sample period. For 1971-80, the R^2 of the GARCH model is *higher* than the R^2 of each of the post-GARCH models, while for 1981-90, and the two sub periods 1981-85 and 1986-90, the R^2 of the GARCH model is *lower* than the R^2 of each of the post-GARCH models. It is also relevant to note that in the 1981-90 period and the 1986-90 sub-period, the performance of EGARCH on the basis of R^2 is markedly better than the performance of each of the other models, while it is largely similar to the other models in remaining 1971-80 period and 1981-85 sub-period.

On the basis of R^2-for-logs, the EGARCH in-sample prediction is largely similar to the other models in all periods. On the other hand, the R^2-for-logs is, in almost all cases, both across periods and across models, lower for the GARCH model than for the post-GARCH models. This could suggest that all the alternative parametric ARCH models consistently perform better than the GARCH in predicting e_t^2 for small values. However, anticipating later results, such a conclusion is not valid since it is not robust across different size based portfolios, and does not carry over to out-of-sample estimation.

The intercept and slope coefficients in the Pagan and Sabau (1987) regression of e_t^2 on σ_t^2 are not significantly different from zero (at even the 10% significance level) in any period for any model. This shows that none of the volatility models are misspecified. The residuals from the regression do not also show any evidence of serial correlation and the heteroskedasticity-corrected Box-Pierce (1970) statistic cannot reject the null of serially uncorrelated residuals at 10 lags. This suggests that there is no persistence in volatility in addition to that captured by these models.

Finally, none of three Engle and Ng (1993) diagnostic tests are able to distinguish significantly between the different models in any period. The test statistics for the sign-bias test as well as for the positive-size-bias test are not significantly different from zero for any period for any model. The test statistics for the negative-size-bias test are also not significantly different from zero for any model for the periods 1971-80 and 1981-90 and the sub-period 1986-90. The test statistics for the negative-size-bias test are significantly different from zero for some models only for the sub-period 1981-85. However, the *t*-statistics vary from -2.18 for GARCH to -1.25 for INGARCH, and the *differences* across models do not appear statistically significant. The *F*-statistic for the joint test is also not significant in any case except for the GARCH model during the 1981-85 sub-period.

5.2.2 Out-of-Sample Estimation

Tables 3A, 3B and 3C summarise the benchmarks and diagnostics related to the out-of-sample performance of the different parametric ARCH models for the period 1981-90, and the sub-periods 1981-85 and 1986-90. The basic results are largely similar to the in-sample case. First, as expected, the standardised residuals from the different volatility models display skewness and excess kurtosis which is not as significant as the unconditional estimates in Table 1, but are nevertheless more significant than the in-sample estimates of Tables 2B, 2C and 2D. The 10-lags Ljung-Box statistic for the squared standardised residuals continues to be statistically insignificant for the 1981-85 and the 1986-90 sub-periods, but is significant for the overall 1981-90 period. This means that all the ARCH models are good *ex-ante* descriptions of time varying volatility provided the parameter estimates do not span too long an interval. This suggests some non-stationarity in the estimated parameters of the ARCH models. However, there are no systematic and significant differences between models in this regard.

With R^2 as the benchmark of performance, the EGARCH, TGARCH, AGARCH, TQGARCH, NQGARCH1, NTGARCH1 and NTGARCH2 outperform the GARCH in both the sub-periods 1981-85 and 1986-90. Similar to the in-sample results, the EGARCH out-of-sample prediction is markedly better than all other models for 1986-90 and largely similar to the other models for 1981-85. The EGARCH, TGARCH and NTGARCH1 rank in the top quartile of all models in *both* sub-periods 1981-85 and 1986-90.

However, with R^2-for-logs as the measure of performance, the GARCH outperforms all these models in the sub-period 1981-85 and underperforms all other models in the sub-period 1986-90. The results for 1981-85 are clearly not consistent with the earlier results for in-sample estimation. The EGARCH outperforms most of the other models for the 1981-85 period and is largely similar to the other models for the 1986-90 period.

The Pagan and Sabau (1987) test shows that none of the volatility models are mis-specified even for *ex-ante* out-of-sample conditional volatility prediction. The intercept and slope are not significantly different from zero and one respectively for any period and any model. The residuals from the Pagan-Sabau regression do not show any evidence of serial correlation and the heteroskedasticity-corrected Box-Pierce (1970) statistic cannot again reject in any case the null hypothesis of serially uncorrelated residuals at ten lags.

Finally, the Engle and Ng (1993) sign-bias test, positive-size-bias test and negative-size-bias test again fail to distinguish between the different models. None of the test statistics for the sign-bias test and the positive-size-bias test are significantly different from zero for any period or any model. The test statistics for the negative-size-bias test are also not significantly different from zero for any model for the sub-period 1986-90. The test statistics for the negative-size-bias test are significantly different from zero for *all* models for the sub-period 1981-85 and the overall period 1981-90. The *t*-statistics vary from -3.47 for EGARCH to -1.93 for NGARCH during the overall 1981-90 period, and from -2.96 for NGARCH to -2.29 for EGARCH during the 1981-85 sub-period. The *F*-statistics for the joint test are also not significant for any model for the 1986-90 sub-period, and significant for all models for the overall period 1981-90 (p-values < 0.09) and for the sub-period 1981-85 (p-values < 0.06). Clearly, inferences regarding the *differences* across models are not consistent, and also do not appear to be statistically significant.

5.3 Size based Portfolios

Analysis similar to that discussed for the FT All Share Index is conducted for each of the 10 size based portfolios and tables corresponding to Tables 2A, 2B, 2C, 2D, 3A, 3B and 3C are prepared in each case. These 70 tables are not being appended to the paper because of their sheer volume, but are available from the author on request. Qualitatively, the results relating to the *relative* performance of the different parametric ARCH models are largely similar to those obtained for the FT All Share

Index with no apparent sized based differences across portfolios. It is found that in all cases:

(a) the standardised residuals from the different volatility models continue to display significant skewness and excess kurtosis even though the magnitudes are much lower than the corresponding unconditional estimates;

(b) the squared standardised residuals are not serially correlated suggesting that ARCH models are generally good descriptions of the time variation in volatility;

(c) the intercept and slope coefficients in the Pagan and Sabau (1987) regression of e_t^2 on σ_t^2 are not significantly different from zero and one respectively (at even the 10% significance level) in any period for any model, and the regression residuals are serially uncorrelated, indicating that none of the volatility models are misspecified;

(d) for all portfolios, in the majority of cases, the log-likelihood of the additional parameter models is significantly greater than that of the GARCH model, at least in some periods, suggesting that the parameters proxying for asymmetry in the post-GARCH models are usually statistically significant; and

(e) none of the three Engle and Ng (1993) sign bias tests are able to *distinguish* between the different parametric ARCH models.

However, the inferences based on R^2 and R^2-for-logs as the benchmark of performance are not completely consistent across portfolios. To analyse the differences, the performance of the different models based on R^2, and R^2-for-logs (for different portfolios and different periods) was ranked across models from 1 to 13, and summed across different portfolios to infer the mean rank of different models for each period separately. The statistical significance of this mean rank was calculated as in Walmsley *et. al.* (1992). Tables 4A, 4B, 4C and 4D (based on in-sample R^2, out-of-sample R^2, in-sample R^2-for-logs and out-of-sample R^2-for-logs respectively) report the mean rank of different models for each of time periods 1971-80, 1981-90, 1981-85 and 1986-90, as well as the z-statistics for the null hypotheses that the mean rank is not significantly different from the expected value of 7 if the performance of all the models is the same.

Based on R^2 and in-sample estimation, consistent with earlier results, the GARCH model significantly outperforms the other models in the 1971-80 period and significantly underperforms in each of the periods 1981-90, 1981-85 and 1986-90. The EGARCH model significantly outperforms all other models for the sub-period 1986-90 and for the overall period 1981-90, and is not very different from the average model in the period 1971-80 and the sub-period 1981-85. While no model *significantly* outperforms the median model in each of the 4 periods analysed, the

EGARCH, TGARCH and NTGARCH1 rank above the median in *both* sub-periods 1981-85 and 1986-90. Based on R^2 and out-of-sample estimation, no model *significantly* outperforms others in both sub-periods 1981-85 and 1986-90, but once again the EGARCH, TGARCH and NTGARCH1 rank above the median in *both* sub-periods.

Based on R^2-for-logs, no model exhibits a performance that is significantly different from the average performance, both for in-sample and out-of-sample estimation. While individual portfolio results may have signalled superior performance for individual models in some periods, there is no clear bottomline for a particular portfolio in all periods, or for all portfolios in a particular period. This suggest that the superior performance of some of the parametric models highlighted earlier could be driven by the ability to predict large variances accurately, since the superior performance disappears when a benchmark which gives relatively greater weight to errors in predicting small variances is used. Alternatively, as highlighted earlier, the distribution of R^2 values are unknown, and so the only way to come to any reasonably firm conclusion about whether the differences in R^2, or R^2-for-logs, between different models are statistically significant or not is to analyse consistency across time or consistency across portfolios. The results could also suggest that a benchmark based on R^2-for-logs is not statistically powerful enough to distinguish between the different models examined.

Finally, in results not reported in this paper but available from the author on request, we repeat the analysis by splitting our 10 size-sorted portfolios into two sub-groups. The first sub-group consists of the first 5 portfolios and hence represents large-sized companies and the second sub-group consists of the remaining 5 portfolios and hence represents the small-sized companies. A similar pattern of predictive performance is observed for both sub-groups. This suggests that the performance of a particular model in predicting volatility is largely unrelated to firm size.

6. SUMMARY AND CONCLUSIONS

This paper presents empirical evidence on the effectiveness of different parametric ARCH models in describing daily stock returns. 20 years UK daily data on a value weighted stock index and ten size sorted portfolios are investigated for the period 1971-90. Several interesting results are documented. First, the squared standardised residuals from all models are serially uncorrelated suggesting that ARCH models are generally good descriptions of the time variation in volatility. Second, none of the volatility models are misspecified since the conditional variance is an unbiased predictor of the actual variance in all cases. Third, the assumption of normal conditional densities is inadequate for all models since the resulting standardised residuals continue to display significant skewness and excess kurtosis. Fourth, in most of the cases, the log-likelihood of the additional parameter models is significantly greater than that of the GARCH model, suggesting that the parameter(s) proxying for asymmetry in the post-GARCH models are usually statistically significant. Fifth, none of the three Engle and Ng (1993) sign bias tests, based on the *News Impact Curve*, are able to *distinguish* between the different parametric ARCH models. Sixth, the relative performance of different parametric ARCH models has not been totally consistent across different periods and across different portfolios, but, among the models currently proposed in the literature, the TGARCH and EGARCH models significantly outperform the GARCH model[19] both on an in-sample and out-of-sample basis over each of the more recent sub-periods 1981-85 and 1986-90. However, all superior performance disappears when a benchmark which gives relatively greater weight to errors in predicting small variances is used. Finally, there do not appear to be any systematic firm size related differences in the relative predictive ability of the different parametric ARCH models.

[19] Engle and Ng (1993) arrive at a similar conclusion using the diagnostics based on the *New Impact Curve*.

TABLE 1

Sample statistics for different periods. **Q(10)** and **Q^2(10)** denotes Ljung-Box statistics of 10 lags for residuals and squared residuals respectively. Standard errors in brackets [ ], p-values in parenthesis ( ).

	Port.1	Port.2	Port.3	Port.4	Port.5	Port.6	Port.7	Port.8	Port.9	Port.10	FTA
1971 to 1980											
Variance	1.9115	2.0403	1.7363	1.4782	1.7312	1.0640	1.3143	0.9821	1.0738	0.9180	1.4078
Skewness*	0.2896 (0.0000)	0.2827 (0.0000)	0.1493 (0.0023)	0.0680 (0.1644)	0.1853 (0.0002)	0.0964 (0.0486)	0.1600 (0.0011)	-0.1418 (0.0037)	-0.1011 (0.0387)	-0.2422 (0.0000)	-0.0060 (0.9026)
Excess Kurtosis**	3.0132 (0.0000)	4.0372 (0.0000)	4.5912 (0.0000)	5.1069 (0.0000)	5.3314 (0.0000)	5.7186 (0.0000)	4.9659 (0.0000)	4.9923 (0.0000)	7.4747 (0.0000)	8.5402 (0.0000)	4.9186 (0.0000)
Q(10)	0.5285 (1.0000)	0.3398 (1.0000)	0.2350 (1.0000)	0.1383 (1.0000)	0.3696 (1.0000)	0.2684 (1.0000)	0.5274 (1.0000)	0.4574 (1.0000)	0.3488 (1.0000)	0.0991 (1.0000)	0.3902 (1.0000)
Q^2(10)	1195.81 (0.0000)	1418.98 (0.0000)	1850.07 (0.0000)	2190.93 (0.0000)	1230.33 (0.0000)	1933.47 (0.0000)	1688.98 (0.0000)	1828.79 (0.0000)	1228.91 (0.0000)	1086.31 (0.0000)	1783.13 (0.0000)
1981 to 1990											
Variance	1.1667	1.1031	1.0423	0.8957	0.8829	0.8832	0.8581	0.6112	0.8187	0.7571	0.8341
Skewness*	-1.0398 (0.0000)	-0.8087 (0.0000)	-1.2064 (0.0000)	-1.0198 (0.0000)	-1.8847 (0.0000)	-1.8658 (0.0000)	-1.1086 (0.0000)	-1.2001 (0.0000)	-1.6976 (0.0000)	-2.5193 (0.0000)	-1.1968 (0.0000)
Excess Kurtosis**	12.7828 (0.0000)	10.6522 (0.0000)	17.1224 (0.0000)	11.6012 (0.0000)	24.9309 (0.0000)	31.6424 (0.0000)	14.5411 (0.0000)	16.1254 (0.0000)	27.9055 (0.0000)	51.8452 (0.0000)	16.4471 (0.0000)
Q(10)	0.0310 (1.0000)	0.1478 (1.0000)	0.0828 (1.0000)	0.2939 (1.0000)	0.6107 (1.0000)	0.3479 (1.0000)	0.4649 (1.0000)	0.1639 (1.0000)	0.6972 (1.0000)	0.1756 (1.0000)	0.1558 (1.0000)
Q^2(10)	1222.59 (0.0000)	1531.74 (0.0000)	928.59 (0.0000)	1529.67 (0.0000)	1088.90 (0.0000)	762.15 (0.0000)	1671.19 (0.0000)	888.81 (0.0000)	1213.22 (0.0000)	933.79 (0.0000)	1548.80 (0.0000)

TABLE 1 *(continue)*

	Port.1	Port.2	Port.3	Port.4	Port.5	Port.6	Port.7	Port.8	Port.9	Port.10	FTA
1981 to 1985											
Variance	1.0886	0.9544	0.9451	0.7377	0.6723	0.6804	0.6685	0.5410	0.5851	0.5014	0.6807
Skewness*	0.0083 (0.9054)	0.0422 (0.5446)	-0.0121 (0.8623)	0.0107 (0.8780)	-0.5704 (0.0000)	0.0338 (0.6277)	-0.1174 (0.0917)	-0.1402 (0.0439)	-0.2463 (0.0004)	-1.6519 (0.0000)	0.0102 (0.8840)
Excess Kurtosis**	0.5580 (0.0001)	0.5878 (0.0000)	1.4303 (0.0000)	1.7776 (0.0000)	6.6576 (0.0000)	3.7132 (0.0000)	1.8233 (0.0000)	5.3331 (0.0000)	9.1339 (0.0000)	23.8377 (0.0000)	2.0076 (0.0000)
Q(10)	0.0446 (1.0000)	0.0343 (1.0000)	0.1256 (1.0000)	0.0486 (1.0000)	0.0522 (1.0000)	0.0263 (1.0000)	0.0805 (1.0000)	0.0194 (1.0000)	0.0667 (1.0000)	0.0710 (1.0000)	0.0517 (1.0000)
$Q^2(10)$	188.94 (0.0000)	154.80 (0.0000)	230.70 (0.0000)	235.07 (0.0000)	190.65 (0.0000)	391.05 (0.0000)	263.33 (0.0000)	453.85 (0.0000)	418.94 (0.0000)	310.02 (0.0000)	361.24 (0.0000)
1986 to 1990											
Variance	1.2108	1.2206	1.1101	1.0411	1.0627	1.0741	1.0255	0.6662	1.0263	0.9875	0.9679
Skewness*	-1.6306 (0.0000)	-1.2207 (0.0000)	-2.0164 (0.0000)	-1.5137 (0.0000)	-2.3902 (0.0000)	-2.7730 (0.0000)	-1.5617 (0.0000)	-1.9083 (0.0000)	-2.0301 (0.0000)	-2.6999 (0.0000)	-1.8234 (0.0000)
Excess Kurtosis**	20.9915 (0.0000)	15.7974 (0.0000)	27.1818 (0.0000)	14.8998 (0.0000)	30.1388 (0.0000)	41.1876 (0.0000)	18.5255 (0.0000)	22.8052 (0.0000)	27.8735 (0.0000)	52.6432 (0.0000)	22.3505 (0.0000)
Q(10)	0.2493 (1.0000)	0.5949 (1.0000)	0.3952 (1.0000)	0.4976 (1.0000)	1.2321 (0.9996)	0.4944 (1.0000)	0.8757 (0.9999)	0.7257 (1.0000)	1.6794 (0.9983)	0.5403 (1.0000)	0.4134 (1.0000)
$Q^2(10)$	769.19 (0.0000)	860.74 (0.0000)	506.55 (0.0000)	868.06 (0.0000)	504.40 (0.0000)	342.13 (0.0000)	854.59 (0.0000)	414.21 (0.0000)	633.83 (0.0000)	460.38 (0.0000)	809.18 (0.0000)

* p-values of two-tailed test that skewness equals zero (i.e. normal skewness).
** p-values of two-tailed test that excess kurtosis equals 0 (i.e. normal kurtosis).

LEGEND TABLE FOR TABLES 2A-2D AND 3A-3D

Description of column headings appearing on Tables 2A-2D and Tables 3A-3D.

Log-L : Denotes log-likelihood value. Asterisk denotes significance of likelihood ratio test over the GARCH model.

Skewness : Skewness computed for the standardised residuals. p-values (shown in parenthesis) denote test for normal skewness.

Kurt. : Excess kurtosis computed for the standardised residuals. p-values (shown in parenthesis) denote test for zero excess (i.e. normal) kurtosis.

CONST. and **SLOPE** : Denote the Pagan and Sabau (1987) test for misspecification of the conditional variance equation i.e.

$$e_t^2 = \text{CONST} + \text{SLOPE}\sigma_t^2 + v_t$$

If conditional variance is correctly specified, CONST. should equal zero and SLOPE equal one.

R^2 : Denotes the R-square of the above regression.

R^2-logs : Denotes rhe R-square for the log version of the above regression.

Qv(10) : Heteroskedasticity-adjusted Box-Pierce statistic for the residuals from the above regression i.e. v_t.

Qx(10) : Ljung-Box statistic for the standardised residuals.

$Qx^2(10)$: Ljung-Box statistic for the square of standardised residuals.

S-, S-e, S+e, F-test : Denote Engle and Ng (1993) diagnostic tests.

TABLE 2A

In-sample statistics for FT All Share Index (1971-80). p-values in parenthesis ( ) and standard errors in brackets [ ].
For description of the column headings, see Legend Table appearing before TABLE 2A.

	Log-L	Skewness	Kurt.	CONST.	SLOPE	R^2	R^2-logs	Qv(10)	Qx(10)	Qx^2(10)	S-	S-e	S+e	F-test
GARCH	-1266.34	-0.2651*	1.3080*	0.0332	0.9747	0.2019	0.0922	8.2442	16.5787	7.2485	-0.0546	-0.0952	0.0220	0.8350
		(0.0000)	(0.0000)	[0.1494]	[0.1305]			(0.6050)	(0.0842)	(0.7018)	[0.1038]	[0.0618]	[0.0619]	(0.4745)
EGARCH	-1270.24	-0.2648*	1.3040*	-0.0325	1.0313	0.1898	0.0938	9.9903	15.7425	8.6137	-0.0373	-0.0694	0.0482	0.6480
		(0.0000)	(0.0000)	[0.1572]	[0.1384]			(0.4413)	(0.1072)	(0.5691)	[0.1036]	[0.0620]	[0.0615]	(0.5842)
TGARCH	-1264.49	-0.2561*	1.2984*	0.0147	0.9921	0.1939	0.0953	8.7808	15.8105	6.9590	-0.0283	-0.0737	0.0514	0.7127
		(0.0000)	(0.0000)	[0.1466]	[0.1302]			(0.5530)	(0.1052)	(0.7293)	[0.1035]	[0.0619]	[0.0614]	(0.5443)
QGARCH	-1263.37*	-0.2519*	1.3177*	0.0171	0.9887	0.1986	0.0951	8.7999	15.7349	6.8131	-0.0555	-0.0799	0.0427	0.7522
		(0.0000)	(0.0000)	[0.1480]	[0.1305]			(0.5512)	(0.1075)	(0.7430)	[0.1037]	[0.0621]	[0.0615]	(0.5210)
NGARCH	-1265.94	-0.2586*	1.2883*	0.0340	0.9713	0.2004	0.0915	8.0350	16.3762	7.3117	-0.0464	-0.0948	0.0302	0.8715
		(0.0000)	(0.0000)	[0.1447]	[0.1272]			(0.6254)	(0.0894)	(0.6957)	[0.1034]	[0.0617]	[0.0616]	(0.4551)
NAGARCH	-1263.95*	-0.2574*	1.3390*	0.0065	0.9999	0.1933	0.0963	9.0067	15.4805	6.9606	-0.0437	-0.0765	0.0551	0.8133
		(0.0000)	(0.0000)	[0.1465]	[0.1304]			(0.5315)	(0.1155)	(0.7292)	[0.1041]	[0.0623]	[0.0618]	(0.4864)
INGARCH	-1265.59	-0.2639*	1.3365*	0.0023	1.0060	0.1914	0.0945	8.7129	15.7436	7.0791	-0.0319	-0.0805	0.0542	0.8218
		(0.0000)	(0.0000)	[0.1466]	[0.1308]			(0.5596)	(0.1072)	(0.7180)	[0.1041]	[0.0622]	[0.0619]	(0.4817)
AGARCH	-1263.18*	-0.2508*	1.3089*	0.0780	0.9324	0.1977	0.0958	8.8573	15.9411	6.9048	-0.0618	-0.0786	0.0332	0.6711
		(0.0000)	(0.0000)	[0.1431]	[0.1247]			(0.5457)	(0.1013)	(0.7344)	[0.1036]	[0.0620]	[0.0615]	(0.5697)
TQGARCH	-1263.18*	-0.2503*	1.3212*	0.0361	0.9704	0.2016	0.0934	8.8217	15.8666	6.9335	-0.0709	-0.0871	0.0332	0.7970
		(0.0000)	(0.0000)	[0.1477]	[0.1291]			(0.5491)	(0.1035)	(0.7317)	[0.1038]	[0.0621]	[0.0616]	(0.4954)
NQGARCH1	-1263.33*	-0.2502*	1.3090*	0.0168	0.9882	0.1983	0.0953	8.6982	15.7183	6.8464	-0.0557	-0.0809	0.0435	0.7749
		(0.0000)	(0.0000)	[0.1468]	[0.1297]			(0.5610)	(0.1080)	(0.7399)	[0.1036]	[0.0620]	[0.0615]	(0.5080)
NQGARCH2	-1263.26*	-0.2528*	1.3180*	0.0715	0.9392	0.1976	0.0951	8.9495	15.9287	6.8646	-0.0609	-0.0779	0.0339	0.6649
		(0.0000)	(0.0000)	[0.1446]	[0.1260]			(0.5369)	(0.1017)	(0.7382)	[0.1037]	[0.0621]	[0.0616]	(0.5736)
NTGARCH1	-1264.44	-0.2545*	1.2910*	0.0175	0.9888	0.1939	0.0952	8.6459	15.8059	6.9484	-0.0308	-0.0755	0.0508	0.7328
		(0.0000)	(0.0000)	[0.1451]	[0.1290]			(0.5660)	(0.1053)	(0.7303)	[0.1034]	[0.0618]	[0.0614]	(0.5324)
NTGARCH2	-1264.34	-0.2570*	1.2968*	0.0769	0.9356	0.1927	0.0950	8.9471	16.0320	7.0157	-0.0312	-0.0704	0.0423	0.5928
		(0.0000)	(0.0000)	[0.1427]	[0.1252]			(0.5371)	(0.0987)	(0.7240)	[0.1035]	[0.0619]	[0.0614]	(0.6197)

* Significant at five percent level.

TABLE 2B

In-sample statistics for FT All Share Index (1981-90). p-values in parenthesis ( ) and standard errors in brackets [ ].
For description of the column headings, see Legend Table appearing before TABLE 2A.

	Log-L	Skewness	Kurt.	CONST.	SLOPE	R^2	R^2-logs	Qv(10)	Qx(10)	Qx^2(10)	S-	S-e	S+e	F-test
GARCH	-729.69	-0.6699* (0.0000)	4.4327* (0.0000)	-0.2022 [0.2688]	1.2976 [0.3898]	0.1832	0.0439	8.1760 (0.6116)	11.9320 (0.2896)	14.6799 (0.1442)	-0.0235 [0.1473]	-0.1801 [0.1020]	-0.0983 [0.1289]	2.0895 (0.0995)
EGARCH	-723.61*	-0.5608* (0.0000)	3.4938* (0.0000)	-0.7229 [0.4514]	2.0259 [0.6413]	0.2808	0.0464	6.6907 (0.7543)	12.1509 (0.2751)	8.4672 (0.5833)	-0.0218 [0.1359]	-0.0966 [0.0945]	-0.0330 [0.1180]	0.5373 (0.6568)
TGARCH	-724.26*	-0.6263* (0.0000)	4.1626* (0.0000)	-0.1301 [0.2516]	1.1958 [0.3644]	0.1987	0.0468	7.6475 (0.6632)	11.9615 (0.2876)	9.7175 (0.4656)	0.0207 [0.1441]	-0.1065 [0.1000]	-0.0345 [0.1254]	0.8582 (0.4621)
QGARCH	-723.28*	-0.6045* (0.0000)	3.9163* (0.0000)	-0.2170 [0.2746]	1.3239 [0.3997]	0.1913	0.0475	8.0185 (0.6270)	11.7914 (0.2993)	13.5950 (0.1923)	-0.0617 [0.1411]	-0.1517 [0.0981]	-0.0576 [0.1226]	1.0937 (0.3505)
NGARCH	-728.93	-0.6723* (0.0000)	4.5352* (0.0000)	-0.1667 [0.2615]	1.2360 [0.3756]	0.2030	0.0436	7.9733 (0.6314)	11.6478 (0.3093)	9.0868 (0.5239)	0.0146 [0.1485]	-0.1385 [0.1028]	-0.0876 [0.1299]	1.6256 (0.1813)
NAGARCH	-722.15*	-0.5953* (0.0000)	3.7842* (0.0000)	-0.1392 [0.2576]	1.2174 [0.3749]	0.1906	0.0482	7.4684 (0.6806)	11.7090 (0.3050)	11.6788 (0.3071)	-0.0548 [0.1395]	-0.1208 [0.0970]	-0.0462 [0.1212]	0.6854 (0.5609)
INGARCH	-721.01*	-0.6093* (0.0000)	3.8380* (0.0000)	-0.1562 [0.2658]	1.2528 [0.3899]	0.1907	0.0481	5.7841 (0.8331)	11.7028 (0.3054)	12.1460 (0.2754)	-0.0041 [0.1403]	-0.0997 [0.0975]	0.0063 [0.1219]	0.4819 (0.6949)
AGARCH	-722.50*	-0.6049* (0.0000)	3.9800* (0.0000)	0.0049 [0.2133]	1.0060 [0.3085]	0.1990	0.0477	8.0729 (0.6217)	11.7021 (0.3055)	7.0796 (0.7179)	-0.0231 [0.1419]	-0.0916 [0.0986]	-0.0607 [0.1233]	0.5818 (0.6269)
TQGARCH	-723.22*	-0.6061* (0.0000)	3.9415* (0.0000)	-0.2066 [0.2722]	1.3085 [0.3960]	0.1938	0.0476	7.9092 (0.6377)	11.8352 (0.2962)	12.7358 (0.2388)	-0.0474 [0.1414]	-0.1419 [0.0983]	-0.0515 [0.1228]	0.9953 (0.3940)
NQGARCH1	-723.04*	-0.6090* (0.0000)	4.0066* (0.0000)	-0.1969 [0.2697]	1.2898 [0.3910]	0.2001	0.0477	7.9321 (0.6355)	11.8025 (0.2985)	10.7295 (0.3790)	-0.0333 [0.1422]	-0.1294 [0.0988]	-0.0508 [0.1235]	0.8973 (0.4418)
NQGARCH2	-722.58*	-0.6018* (0.0000)	3.9187* (0.0000)	0.0672 [0.1970]	0.9252 [0.2856]	0.1877	0.0475	8.1837 (0.6109)	11.7931 (0.2991)	7.7220 (0.6560)	-0.0260 [0.1411]	-0.0904 [0.0981]	-0.0620 [0.1226]	0.5674 (0.6365)
NTGARCH1	-724.21*	-0.6217* (0.0000)	4.0998* (0.0000)	-0.1424 [0.2552]	1.2163 [0.3705]	0.1946	0.0468	7.6556 (0.6624)	12.0109 (0.2843)	10.9815 (0.3590)	0.0110 [0.1433]	-0.1144 [0.0995]	-0.0336 [0.1247]	0.8867 (0.4472)
NTGARCH2	-724.10*	-0.6275* (0.0000)	4.1820* (0.0000)	0.0041 [0.2140]	1.0076 [0.3095]	0.1969	0.0467	7.6965 (0.6585)	11.9354 (0.2894)	7.4002 (0.6872)	0.0360 [0.1443]	-0.0804 [0.1002]	-0.0396 [0.1256]	0.7045 (0.5493)

* Significant at five percent level.

TABLE 2C

In-sample statistics for FT All Share Index (1981-85). p-values in parenthesis ( ) and standard errors in brackets [ ].
For description of the column headings, see Legend Table appearing before TABLE 2A.

	Log-L	Skewness	Kurt.	CONST.	SLOPE	R^2	R^2-logs	Qv(10)	Qx(10)	Qx^2(10)	S-	S-e	S+e	F-test
GARCH	-314.85	-0.0810 (0.2445)	0.1784 (0.2006)	-0.0178 [0.2430]	1.0258 [0.3903]	0.1044	0.0179	5.3662 (0.8654)	4.3143 (0.9321)	6.8809 (0.7366)	-0.1829 [0.1301]	-0.2431* [0.1114]	-0.1835 [0.1114]	2.8666* (0.0355)
EGARCH	-314.33	-0.0503 (0.4701)	0.1321 (0.3433)	-0.3208 [0.3429]	1.4972 [0.5462]	0.1422	0.0234	5.1987 (0.8775)	2.9388 (0.9828)	7.1852 (0.7079)	-0.1886 [0.1289]	-0.2272* [0.1115]	-0.1152 [0.1092]	1.8016 (0.1450)
TGARCH	-311.58*	-0.0533 (0.4440)	0.1219 (0.3820)	-0.1226 [0.2808]	1.1855 [0.4472]	0.1424	0.0228	5.6387 (0.8447)	3.3089 (0.9732)	4.9565 (0.8941)	-0.1390 [0.1286]	-0.1685 [0.1110]	-0.1267 [0.1092]	1.3332 (0.2620)
QGARCH	-313.33	-0.0635 (0.3615)	0.1503 (0.2809)	-0.0637 [0.2585]	1.0980 [0.4150]	0.1153	0.0219	5.4901 (0.8561)	3.5133 (0.9666)	5.8840 (0.8249)	-0.2072 [0.1293]	-0.2318* [0.1114]	-0.1564 [0.1101]	2.1946 (0.0870)
NGARCH	-314.66	-0.0809 (0.2449)	0.1761 (0.2064)	0.0128 [0.2331]	0.9772 [0.3742]	0.1045	0.0174	5.4376 (0.8601)	4.3155 (0.9320)	6.4333 (0.7776)	-0.1773 [0.1300]	-0.2338* [0.1112]	-0.1847 [0.1115]	2.7628* (0.0409)
NAGARCH	-312.59*	-0.0614 (0.3774)	0.1384 (0.3207)	-0.1005 [0.2758]	1.1528 [0.4402]	0.1305	0.0229	5.5793 (0.8493)	3.3897 (0.9707)	5.2714 (0.8723)	-0.1911 [0.1291]	-0.2050 [0.1114]	-0.1449 [0.1096]	1.7594 (0.1531)
INGARCH	-310.91*	-0.0569 (0.4134)	0.1184 (0.3958)	-0.2787 [0.3253]	1.4199 [0.5107]	0.2036	0.0237	5.6321 (0.8452)	3.1609 (0.9774)	4.1974 (0.9380)	-0.1636 [0.1286]	-0.1396 [0.1110]	-0.1305 [0.1092]	1.0057 (0.3894)
AGARCH	-313.32	-0.0639 (0.3585)	0.1497 (0.2830)	-0.0395 [0.2535]	1.0600 [0.4066]	0.1146	0.0218	5.5042 (0.8551)	3.5450 (0.9655)	5.7336 (0.8371)	-0.2024 [0.1293]	-0.2246* [0.1114]	-0.1570 [0.1101]	2.1130 (0.0968)
TQGARCH	-311.17*	-0.0544 (0.4343)	0.1186 (0.3950)	-0.1078 [0.2781]	1.1596 [0.4411]	0.1515	0.0212	5.6691 (0.8423)	3.4837 (0.9676)	4.6529 (0.9131)	-0.1106 [0.1285]	-0.1442 [0.1107]	-0.1306 [0.1094]	1.2393 (0.2941)
NQGARCH1	-312.78	-0.0586 (0.3995)	0.1432 (0.3044)	-0.0222 [0.2438]	1.0310 [0.3909]	0.1174	0.0211	5.5853 (0.8488)	3.4954 (0.9673)	5.3451 (0.8670)	-0.1707 [0.1292]	-0.1995 [0.1114]	-0.1441 [0.1099]	1.7554 (0.1539)
NQGARCH2	-313.31	-0.0634 (0.3626)	0.1470 (0.2917)	-0.0261 [0.2533]	1.0391 [0.4058]	0.1154	0.0214	5.5027 (0.8552)	3.5359 (0.9659)	5.6271 (0.8456)	-0.1986 [0.1293]	-0.2172 [0.1113]	-0.1556 [0.1101]	2.0084 (0.1110)
NTGARCH1	-311.49*	-0.0537 (0.4407)	0.1231 (0.3771)	-0.1007 [0.2730]	1.1503 [0.4345]	0.1420	0.0223	5.6782 (0.8415)	3.3596 (0.9717)	4.8227 (0.9027)	-0.1301 [0.1287]	-0.1614 [0.1110]	-0.1273 [0.1093]	1.2964 (0.2742)
NTGARCH2	-311.58*	-0.0533 (0.4438)	0.1217 (0.3827)	-0.1160 [0.2798]	1.1750 [0.4454]	0.1427	0.0228	5.6430 (0.8443)	3.3139 (0.9730)	4.9331 (0.8956)	-0.1374 [0.1286]	-0.1660 [0.1110]	-0.1267 [0.1092]	1.3099 (0.2697)

* Significant at five percent level.

TABLE 2D

In-sample statistics for FT All Share Index (1986-90). p-values in parenthesis ( ) and standard errors in brackets [ ].
For description of the column headings, see Legend Table appearing before TABLE 2A.

	Log-L	Skewness	Kurt.	CONST.	SLOPE	R^2	R^2-logs	Qv(10)	Qx(10)	$Qx^2(10)$	S-	S-e	S+e	F-test
GARCH	-393.74	-0.9700* (0.0000)	5.4523* (0.0000)	-0.4047 [0.3714]	1.5722 [0.5173]	0.1937	0.0493	9.4977 (0.4856)	12.5990 (0.2470)	14.0334 (0.1715)	0.0772 [0.2173]	-0.1876 [0.1323]	0.0178 [0.1884]	1.2345 (0.2958)
EGARCH	-392.36	-0.8722* (0.0000)	4.9303* (0.0000)	-0.9188 [0.5547]	2.2197 [0.7368]	0.3394	0.0436	7.6883 (0.6593)	13.3599 (0.2042)	5.3943 (0.8633)	0.1493 [0.2091]	-0.0209 [0.1283]	0.0956 [0.1797]	0.2665 (0.8496)
TGARCH	-391.88	-0.9376* (0.0000)	5.3379* (0.0000)	-0.3318 [0.3530]	1.4694 [0.4886]	0.2121	0.0497	8.9585 (0.5360)	13.1287 (0.2166)	9.2934 (0.5045)	0.1292 [0.2151]	-0.1100 [0.1320]	0.0723 [0.1848]	0.6612 (0.5760)
QGARCH	-389.79*	-0.8862* (0.0000)	5.0611* (0.0000)	-0.3969 [0.3762]	1.5666 [0.5233]	0.2138	0.0510	9.0422 (0.5281)	13.2561 (0.2097)	10.1963 (0.4234)	0.0388 [0.2109]	-0.1273 [0.1295]	0.0570 [0.1808]	0.4887 (0.6902)
NGARCH	-388.63*	-0.9420* (0.0000)	5.3553* (0.0000)	-0.4096 [0.3806]	1.5167 [0.5048]	0.2666	0.0530	9.1695 (0.5161)	12.5779 (0.2482)	4.7609 (0.9066)	0.1665 [0.2150]	-0.0641 [0.1320]	0.0388 [0.1841]	0.5942 (0.6188)
NAGARCH	-389.58*	-0.8748* (0.0000)	4.9601* (0.0000)	-0.2466 [0.3377]	1.3701 [0.4688]	0.2047	0.0535	8.4834 (0.5817)	13.6756 (0.1883)	9.1217 (0.5206)	0.0497 [0.2095]	-0.1016 [0.1287]	0.0760 [0.1794]	0.3528 (0.7871)
INGARCH	-391.18*	-0.9277* (0.0000)	5.2289* (0.0000)	-0.2612 [0.3438]	1.4037 [0.4824]	0.1920	0.0490	8.2378 (0.6056)	13.4857 (0.1978)	11.0885 (0.3507)	0.1228 [0.2137]	-0.1192 [0.1311]	0.1212 [0.1836]	0.6760 (0.5668)
AGARCH	-386.44*	-0.8678* (0.0000)	5.0092* (0.0000)	0.2031 [0.1860]	0.7541 [0.2490]	0.2332	0.0542	9.0520 (0.5272)	13.8936 (0.1779)	2.7837 (0.9861)	0.1316 [0.2101]	0.0242 [0.1290]	0.0384 [0.1799]	0.1462 (0.9321)
TQGARCH	-389.28*	-0.8749* (0.0000)	4.9517* (0.0000)	-0.3713 [0.3676]	1.5358 [0.5130]	0.2008	0.0456	9.4979 (0.4856)	13.1503 (0.2154)	12.0363 (0.2826)	-0.0076 [0.2090]	-0.1578 [0.1283]	0.0323 [0.1791]	0.6273 (0.5974)
NQGARCH1	-387.70*	-0.9509* (0.0000)	5.3287* (0.0000)	-0.4474 [0.3965]	1.5784 [0.5289]	0.2734	0.0481	9.2431 (0.5092)	12.4125 (0.2584)	5.0983 (0.8845)	0.1607 [0.2163]	-0.0805 [0.1328]	0.0288 [0.1855]	0.6938 (0.5559)
NQGARCH2	-388.16*	-0.8784* (0.0000)	5.0305* (0.0000)	0.1907 [0.1986]	0.7966 [0.2698]	0.2002	0.0521	9.0420 (0.5281)	13.9027 (0.1775)	4.3905 (0.9280)	0.1144 [0.2106]	-0.0124 [0.1293]	0.0390 [0.1805]	0.1623 (0.9217)
NTGARCH1	-387.40*	-0.9462* (0.0000)	5.2700* (0.0000)	-0.5358 [0.4248]	1.6879 [0.5656]	0.2680	0.0499	9.4070 (0.4940)	12.3279 (0.2637)	5.8708 (0.8260)	0.1423 [0.2138]	-0.0990 [0.1314]	0.0245 [0.1831]	0.7583 (0.5176)
NTGARCH2	-390.56*	-0.9308* (0.0000)	5.2803* (0.0000)	0.1845 [0.1970]	0.7977 [0.2666]	0.1965	0.0454	9.0308 (0.5292)	13.5264 (0.1957)	4.6942 (0.9106)	0.1707 [0.2144]	-0.0252 [0.1314]	0.0333 [0.1843]	0.4236 (0.7361)

* Significant at five percent level

TABLE 3A

Out-of-sample statistics for FT All Share Index (1981-90). p-values in parenthesis ( ) and standard errors in brackets [ ].
For description of column headings, see Legend Table appearing before TABLE 2A.

	Skewness	Kurt.	CONST.	SLOPE	R^2	R^2-logs	Qe(10)	Qx(10)	$Qx^2(10)$	S-	S-e	S+e	F-test
GARCH	-0.8968*	7.8231*	0.0366	0.8927	0.1172	0.0448	8.9529	13.2262	27.8083*	-0.0482	-0.2966*	-0.0755	3.0184*
	(0.0000)	(0.0000)	[0.1811]	[0.2483]			(0.5366)	(0.2113)	(0.0019)	[0.1747]	[0.1215]	[0.1516]	(0.0288)
EGARCH	-0.7597*	5.9925*	-0.0165	0.9798	0.1012	0.0404	8.9455	14.4458	58.8944*	-0.1393	-0.3849*	-0.0588	4.8006*
	(0.0000)	(0.0000)	[0.1911]	[0.2675]			(0.5373)	(0.1536)	(0.0000)	[0.1590]	[0.1108]	[0.1372]	(0.0025)
TGARCH	-0.8492*	7.3415*	0.0686	0.8445	0.1202	0.0410	8.8619	13.2443	23.9520*	-0.0162	-0.2511*	-0.0505	2.4038
	(0.0000)	(0.0000)	[0.1737]	[0.2367]			(0.5453)	(0.2103)	(0.0077)	[0.1696]	[0.1181]	[0.1466]	(0.0657)
QGARCH	-0.8091*	6.8238*	0.0476	0.8838	0.1145	0.0420	9.0312	13.3461	31.8271*	-0.0629	-0.2931*	-0.0496	2.8819*
	(0.0000)	(0.0000)	[0.1760]	[0.2437]			(0.5291)	(0.2050)	(0.0004)	[0.1666]	[0.1162]	[0.1435]	(0.0346)
NGARCH	-0.9112*	8.1510*	0.0565	0.8523	0.1282	0.0443	8.8507	12.7783	17.5796	-0.0037	-0.2378	-0.0722	2.2530
	(0.0000)	(0.0000)	[0.1791]	[0.2421]			(0.5463)	(0.2363)	(0.0625)	[0.1774]	[0.1233]	[0.1539]	(0.0803)
NAGARCH	-0.8376*	7.1228*	0.0756	0.8477	0.1118	0.0422	8.9286	13.2690	31.2734*	-0.0543	-0.2935*	-0.0508	2.8881*
	(0.0000)	(0.0000)	[0.1693]	[0.2341]			(0.5389)	(0.2090)	(0.0005)	[0.1691]	[0.1179]	[0.1458]	(0.0343)
INGARCH	-0.8797*	7.6108*	0.1021	0.8110	0.1101	0.0431	8.7674	13.2804	28.9106*	-0.0410	-0.2889*	-0.0619	2.8771*
	(0.0000)	(0.0000)	[0.1650]	[0.2266]			(0.5543)	(0.2084)	(0.0013)	[0.1730]	[0.1204]	[0.1497]	(0.0348)
AGARCH	-0.8130*	6.9187*	0.1008	0.8113	0.1162	0.0418	9.0294	13.1059	26.2845*	-0.0401	-0.2661*	-0.0468	2.5034
	(0.0000)	(0.0000)	[0.1636]	[0.2254]			(0.5293)	(0.2178)	(0.0034)	[0.1673]	[0.1168]	[0.1442]	(0.0575)
TQGARCH	-0.8028*	6.7222*	0.0412	0.9000	0.1100	0.0417	9.1521	13.3006	37.6867*	-0.0894	-0.3269*	-0.0504	3.3724*
	(0.0000)	(0.0000)	[0.1754]	[0.2452]			(0.5177)	(0.2073)	(0.0000)	[0.1666]	[0.1163]	[0.1436]	(0.0177)
NQGARCH1	-0.8137*	6.9243*	0.0522	0.8720	0.1181	0.0420	8.9977	13.2482	27.4332*	-0.0450	-0.2712*	-0.0487	2.5775
	(0.0000)	(0.0000)	[0.1759]	[0.2422]			(0.5323)	(0.2101)	(0.0022)	[0.1671]	[0.1167]	[0.1440]	(0.0521)
NQGARCH2	-0.8109*	6.8535*	0.0919	0.8269	0.1132	0.0419	9.0501	13.2250	30.1766*	-0.0536	-0.2852*	-0.0467	2.7746*
	(0.0000)	(0.0000)	[0.1651]	[0.2284]			(0.5274)	(0.2114)	(0.0008)	[0.1668]	[0.1164]	[0.1437]	(0.0400)
NTGARCH1	-0.8568*	7.4816*	0.0742	0.8320	0.1243	0.0424	8.8314	13.0904	20.2518*	-0.0007	-0.2311	-0.0508	2.1639
	(0.0000)	(0.0000)	[0.1734]	[0.2349]			(0.5482)	(0.2187)	(0.0270)	[0.1707]	[0.1189]	[0.1475]	(0.0903)
NTGARCH2	-0.8499*	7.3645*	0.1218	0.7765	0.1187	0.0419	8.8801	13.1028	22.1976*	-0.0061	-0.2411*	-0.0477	2.2894
	(0.0000)	(0.0000)	[0.1604]	[0.2181]			(0.5435)	(0.2180)	(0.0141)	[0.1699]	[0.1183]	[0.1468]	(0.0765)

* Significant at five percent level.

TABLE 3B

Out-of-sample statistics for FT All Share Index (1981-85). p-values in parenthesis ( ) and standard errors in brackets [ ].
For description of column headings, see Legend Table appearing before TABLE 2A.

	Skewness	Kurt.	CONST.	SLOPE	R^2	R^2-logs	Qe(10)	Qx(10)	Qx^2(10)	S-	S-e	S+e	F-test
GARCH	-0.0920 (0.1861)	0.2370 (0.0891)	0.0477 [0.1957]	0.8374 [0.2897]	0.0849	0.0186	5.1934 (0.8779)	4.8066 (0.9037)	14.1301 (0.1671)	-0.2030 [0.1205]	-0.3039* [0.1042]	-0.1431 [0.1021]	3.8762* (0.0090)
EGARCH	-0.0712 (0.3066)	0.2208 (0.1132)	-0.0110 [0.2191]	0.9184 [0.3220]	0.0965	0.0184	5.1047 (0.8841)	4.7779 (0.9055)	12.3231 (0.2640)	-0.2048 [0.1226]	-0.2446* [0.1066]	-0.1421 [0.1032]	2.4826 (0.0594)
TGARCH	-0.0772 (0.2670)	0.1929 (0.1664)	0.0080 [0.2141]	0.8934 [0.3152]	0.0978	0.0167	5.1726 (0.8794)	4.4095 (0.9270)	10.9701 (0.3599)	-0.1688 [0.1196]	-0.2602* [0.1040]	-0.1184 [0.1007]	2.8538* (0.0362)
QGARCH	-0.0745 (0.2841)	0.2005 (0.1503)	0.0330 [0.2008]	0.8609 [0.2980]	0.0916	0.0184	5.1525 (0.8808)	4.2150 (0.9371)	11.6171 (0.3115)	-0.1901 [0.1206]	-0.2696* [0.1050]	-0.1172 [0.1015]	2.8248* (0.0376)
NGARCH	-0.0916 (0.1882)	0.2403 (0.0847)	0.0668 [0.1892]	0.8113 [0.2806]	0.0838	0.0176	5.2329 (0.8751)	4.6966 (0.9105)	14.8411 (0.1380)	-0.1942 [0.1205]	-0.3080* [0.1042]	-0.1340 [0.1021]	3.9288* (0.0083)
NAGARCH	-0.0792 (0.2552)	0.1992 (0.1531)	0.0205 [0.2087]	0.8771 [0.3083]	0.0947	0.0181	5.1582 (0.8804)	4.3498 (0.9302)	11.5545 (0.3160)	-0.1819 [0.1202]	-0.2679* [0.1046]	-0.1181 [0.1011]	2.8805* (0.0349)
INGARCH	-0.0876 (0.2082)	0.2168 (0.1199)	0.0223 [0.2089]	0.8725 [0.3076]	0.0924	0.0174	5.1718 (0.8794)	4.6531 (0.9131)	12.5115 (0.2523)	-0.1902 [0.1201]	-0.2824* [0.1042]	-0.1328 [0.1014]	3.3386* (0.0187)
AGARCH	-0.0758 (0.2760)	0.1991 (0.1532)	0.0477 [0.1986]	0.8436 [0.2957]	0.0900	0.0180	5.1848 (0.8785)	4.0880 (0.9433)	12.7993 (0.2351)	-0.1845 [0.1206]	-0.2783* [0.1050]	-0.1081 [0.1014]	2.9421* (0.0321)
TQGARCH	-0.0767 (0.2704)	0.2132 (0.1262)	0.0558 [0.1898]	0.8308 [0.2834]	0.0858	0.0182	5.1547 (0.8806)	4.1486 (0.9404)	13.0248 (0.2223)	-0.2064 [0.1213]	-0.2876* [0.1056]	-0.1184 [0.1020]	3.0802* (0.0266)
NQGARCH1	-0.0742 (0.2862)	0.1996 (0.1522)	0.0369 [0.1995]	0.8550 [0.2959]	0.0914	0.0182	5.1612 (0.8802)	4.2048 (0.9376)	11.7422 (0.3027)	-0.1840 [0.1204]	-0.2693* [0.1048]	-0.1140 [0.1013]	2.8344* (0.0371)
NQGARCH2	-0.0755 (0.2778)	0.1972 (0.1571)	0.0402 [0.2013]	0.8538 [0.2993]	0.0910	0.0182	5.1745 (0.8792)	4.1165 (0.9419)	12.4298 (0.2573)	-0.1860 [0.1206]	-0.2752* [0.1050]	-0.1104 [0.1014]	2.8953* (0.0342)
NTGARCH1	-0.0778 (0.2633)	0.1949 (0.1620)	0.0142 [0.2119]	0.8849 [0.3121]	0.0971	0.0159	5.1837 (0.8786)	4.3955 (0.9277)	11.1812 (0.3436)	-0.1660 [0.1196]	-0.2626* [0.1040]	-0.1165 [0.1007]	2.9072* (0.0336)
NTGARCH2	-0.0784 (0.2599)	0.1924 (0.1675)	0.0162 [0.2149]	0.8855 [0.3172]	0.0972	0.0163	5.1992 (0.8775)	4.2979 (0.9329)	11.9544 (0.2881)	-0.1654 [0.1197]	-0.2666* [0.1041]	-0.1114 [0.1008]	2.9274* (0.0327)

* Significant at five percent level.

TABLE 3C

Out-of-sample statistics for FT All Share Index (1986-90). p-values in parenthesis ( ) and standard errors in brackets [ ]. For description of column headings, see Legend Table appearing before TABLE 2A.

	Skewness	Kurt.	CONST.	SLOPE	R^2	R^2-logs	Qe(10)	Qx(10)	$Qx^2(10)$	S-	S-e	S+e	F-test
GARCH	-1.5180* (0.0000)	12.8058* (0.0000)	-0.0166 [0.2467]	1.1147 [0.3624]	0.1486	0.0453	10.1915 (0.4239)	12.9232 (0.2280)	7.8332 (0.6451)	0.1836 [0.3389]	-0.1624 [0.2071]	-0.0374 [0.2923]	0.8086 (0.4891)
EGARCH	-1.2390* (0.0000)	8.9733* (0.0000)	-0.5792 [0.4484]	1.9760 [0.6776]	0.1722	0.0520	9.4496 (0.4900)	13.3156 (0.2066)	18.9971* (0.0403)	0.0674 [0.2890]	-0.3041 [0.1781]	0.1139 [0.2460]	1.4078 (0.2389)
TGARCH	-1.3718* (0.0000)	11.0167* (0.0000)	0.0314 [0.2356]	1.0189 [0.3362]	0.1596	0.0519	9.8798 (0.4511)	13.2651 (0.2092)	5.9923 (0.8159)	0.1733 [0.3126]	-0.0961 [0.1922]	0.0117 [0.2671]	0.4541 (0.7145)
QGARCH	-1.3622* (0.0000)	10.7790* (0.0000)	-0.0353 [0.2518]	1.1511 [0.3744]	0.1482	0.0526	10.2089 (0.4224)	13.2359 (0.2108)	9.5988 (0.4764)	0.1121 [0.3138]	-0.1816 [0.1927]	-0.0078 [0.2687]	0.7054 (0.5489)
NGARCH	-1.5451* (0.0000)	13.3016* (0.0000)	0.0121 [0.2390]	1.0486 [0.3438]	0.1643	0.0472	10.0719 (0.4342)	12.7664 (0.2370)	4.6086 (0.9157)	0.2222 [0.3444]	-0.0887 [0.2103]	-0.0534 [0.2972]	0.6333 (0.5936)
NAGARCH	-1.3434* (0.0000)	10.4547* (0.0000)	0.0155 [0.2407]	1.0853 [0.3571]	0.1435	0.0522	9.9341 (0.4463)	13.3887 (0.2027)	9.7865 (0.4594)	0.0938 [0.3107]	-0.1811 [0.1911]	-0.0004 [0.2655]	0.6417 (0.5882)
INGARCH	-1.3103* (0.0000)	9.8002* (0.0000)	0.0742 [0.2576]	1.0382 [0.3803]	0.1381	0.0551	8.3664 (0.5931)	13.9748 (0.1741)	9.6822 (0.4688)	0.1280 [0.3059]	-0.1547 [0.1885]	0.0783 [0.2607]	0.5085 (0.6765)
AGARCH	-1.3687* (0.0000)	10.9065* (0.0000)	0.0209 [0.2358]	1.0629 [0.3467]	0.1511	0.0529	10.1648 (0.4262)	13.2344 (0.2109)	7.6818 (0.6599)	0.1304 [0.3150]	-0.1443 [0.1935]	-0.0196 [0.2697]	0.5889 (0.6223)
TQGARCH	-1.4429* (0.0000)	12.0081* (0.0000)	0.0993 [0.2166]	0.9030 [0.2997]	0.1651	0.0532	9.7331 (0.4642)	13.0478 (0.2210)	4.0145 (0.9467)	0.2404 [0.3231]	-0.0397 [0.1984]	0.0079 [0.2766]	0.4490 (0.7180)
NQGARCH1	-1.3804* (0.0000)	11.2320* (0.0000)	0.0066 [0.2409]	1.0421 [0.3433]	0.1723	0.0529	10.0621 (0.4351)	12.9910 (0.2242)	4.3504 (0.9302)	0.1847 [0.3171]	-0.0635 [0.1948]	-0.0222 [0.2715]	0.4255 (0.7348)
NQGARCH2	-1.3652* (0.0000)	10.8610* (0.0000)	0.0803 [0.2195]	0.9797 [0.3200]	0.1476	0.0527	10.1496 (0.4275)	13.3075 (0.2070)	7.3433 (0.6927)	0.1318 [0.3142]	-0.1341 [0.1929]	-0.0244 [0.2690]	0.5579 (0.6429)
NTGARCH1	-1.3945* (0.0000)	11.4067* (0.0000)	0.0544 [0.2282]	0.9650 [0.3191]	0.1695	0.0526	9.8416 (0.4545)	13.1094 (0.2176)	4.2182 (0.9370)	0.2017 [0.3163]	-0.0517 [0.1945]	-0.0007 [0.2703]	0.3972 (0.7550)
NTGARCH2	-1.3732* (0.0000)	11.0428* (0.0000)	0.0522 [0.2295]	0.9885 [0.3264]	0.1596	0.0519	9.8656 (0.4524)	13.2712 (0.2089)	5.6827 (0.8412)	0.1768 [0.3128]	-0.0881 [0.1924]	0.0087 [0.2673]	0.4387 (0.7254)

* Significant at five percent level.

TABLE 4A

Estimated Median and Mean Rankings of in-sample R^2 for the various parametric ARCH Models.
p-values denote Friedman test that the ranks of in-sample R^2 for all models are equal.

	1971-80			1981-90			1981-85			1986-90		
	Est. Median	Mean Ranking	Z-stat.	Est. Median	Mean Ranking	Z-stat.	Est. Median	Mean Ranking	Z-stat.	Est. Median	Mean Ranking	Z-stat.
GARCH	0.1811	12.00	4.2258*	0.1663	2.82	-3.1489*	0.0798	2.00	-4.2258*	0.1665	3.09	-3.0062*
EGARCH	0.1712	3.82	-1.0511	0.2378	12.73	6.3317*	0.1114	8.91	0.4820	0.2784	12.73	6.3317*
TGARCH	0.1738	5.91	-0.4573	0.1784	8.36	0.9082	0.1056	8.73	0.8617	0.1807	8.64	0.9363
QGARCH	0.1770	9.45	2.9926*	0.1732	5.91	-0.7933	0.0883	4.64	-2.5569*	0.1767	7.18	0.1298
NGARCH	0.1803	10.82	1.3203	0.1833	9.45	0.7585	0.0802	3.64	-1.1178	0.1969	8.82	0.5442
NAGARCH	0.1723	4.55	-1.7049	0.1718	5.18	-1.0222	0.1037	8.09	0.5839	0.1731	5.36	-1.1416
INGARCH	0.1695	2.36	-2.2478*	0.1668	4.27	-0.6390	0.1582	12.45	3.0151*	0.1646	4.45	-0.5768
AGARCH	0.1762	8.36	0.6321	0.1758	7.27	0.1396	0.0884	4.36	-1.4184	0.1781	7.45	0.2199
TQGARCH	0.1800	11.09	2.4939*	0.1766	7.55	0.2023	0.1101	10.27	1.5229	0.1762	7.27	0.0898
NQGARCH1	0.1762	8.00	0.3333	0.1841	9.91	1.0925	0.0925	6.18	-0.2936	0.1999	9.73	0.8001
NQGARCH2	0.1755	6.64	-0.2081	0.1633	1.73	-8.1537*	0.0850	3.27	-1.9075	0.1603	2.09	-3.7752*
NTGARCH1	0.1731	4.36	-1.2491	0.1847	10.64	2.1523*	0.1111	10.36	1.9909*	0.1991	10.18	1.4002
NTGARCH2	0.1716	3.64	-1.0608	0.1719	5.18	-0.8511	0.1027	8.09	0.6014	0.1675	4.00	-1.9365
p-value of Friedman Test	0.000*			0.000*			0.000*			0.000*		

* Significant at the five percent level.

TABLE 4B

Estimated Median and Mean Rankings of in-sample R^2-for-logs for the various parametric ARCH Models.
p-values denote Friedman test that the ranks of in-sample log-R^2 for all models are equal.

	1971-80			1981-90			1981-85			1986-90		
	Est. Median	Mean Ranking	Z-stat.	Est. Median	Mean Ranking	Z-stat.	Est. Median	Mean Ranking	Z-stat.	Est. Median	Mean Ranking	Z-stat.
GARCH	0.0784	4.09	-0.9443	0.0494	6.27	-0.1509	0.0356	7.27	0.0523	0.0729	6.73	-0.0571
EGARCH	0.0784	4.64	-0.6752	0.0484	3.73	-0.9141	0.0331	3.36	-1.0304	0.0713	4.45	-0.5922
TGARCH	0.0789	5.91	-0.3164	0.0495	6.09	-0.2513	0.0358	7.36	0.1114	0.0733	7.00	0.0000
QGARCH	0.0795	8.09	0.3701	0.0499	7.45	0.1238	0.0352	6.64	-0.1686	0.0727	6.05	-0.3738
NGARCH	0.0783	4.18	-0.8288	0.0488	5.00	-0.4344	0.0356	7.27	0.0515	0.0742	8.64	0.3469
NAGARCH	0.0796	10.36	1.3240	0.0501	8.45	0.4583	0.0350	6.36	-0.1775	0.0740	8.55	0.4412
INGARCH	0.0792	7.36	0.0919	0.0504	9.00	0.5384	0.0360	8.91	0.4347	0.0726	6.27	-0.2232
AGARCH	0.0794	8.55	0.4007	0.0501	8.00	0.2599	0.0357	8.45	0.6589	0.0738	8.50	0.5254
TQGARCH	0.0793	7.91	0.2552	0.0497	7.18	0.0413	0.0353	6.36	-0.1608	0.0718	4.91	-0.5058
NQGARCH1	0.0794	7.73	0.1773	0.0499	8.36	0.6772	0.0358	8.64	0.5438	0.0741	8.18	0.3076
NQGARCH2	0.0796	9.55	0.9710	0.0498	7.00	0.0000	0.0351	6.27	-0.3313	0.0730	6.82	-0.0679
NTGARCH1	0.0790	6.09	-0.2951	0.0499	8.27	0.4804	0.0354	7.55	0.1754	0.0742	8.55	0.4307
NTGARCH2	0.0790	6.55	-0.1330	0.0495	6.18	-0.2703	0.0351	6.55	-0.1187	0.0730	6.36	-0.1932
p-value of Friedman Test	0.001*			0.069			0.139			0.139		

* Significant at the five percent level.

TABLE 4C

Estimated Median and Mean Rankings of out-of-sample R^2 *for the various parametric* ARCH Models.
p-values denote Friedman test that the ranks of out-of-sample R^2 for all models are equal.

	1981-90			1981-85			1986-90		
	Est. Median	Mean Ranking	Z-stat.	Est. Median	Mean Ranking	Z-stat.	Est. Median	Mean Ranking	Z-stat.
GARCH	0.1155	8.73	1.5650	0.0694	2.64	-2.2226*	0.1332	5.09	-1.1228
EGARCH	0.1001	2.91	-1.1307	0.0764	7.73	0.1572	0.1519	9.82	0.6185
TGARCH	0.1182	11.09	2.2551*	0.0783	10.73	1.8148	0.1430	9.00	0.9759
QGARCH	0.1135	7.18	0.0937	0.0736	6.36	-0.4062	0.1341	5.91	-0.7544
NGARCH	0.1169	9.00	0.4588	0.0694	2.18	-6.4178*	0.1426	7.09	0.0216
NAGARCH	0.1124	6.18	-0.3350	0.0777	9.82	1.7601	0.1330	4.82	-1.0959
INGARCH	0.1103	4.36	-0.8485	0.0798	10.36	1.5591	0.1221	2.82	-1.2298
AGARCH	0.1145	7.82	0.5562	0.0729	5.55	-0.7208	0.1358	5.64	-0.6189
TQGARCH	0.1087	4.27	-0.7445	0.0697	3.73	-1.0664	0.1479	10.45	1.9707
NQGARCH1	0.1100	5.64	-0.3092	0.0734	6.00	-0.4767	0.1475	8.18	0.3035
NQGARCH2	0.1121	5.27	-0.8038	0.0732	5.55	-0.6729	0.1305	4.55	-0.7513
NTGARCH1	0.1166	8.91	0.3922	0.0782	10.27	1.0664	0.1523	9.91	0.8508
NTGARCH2	0.1164	9.64	1.8393	0.0780	10.09	1.4271	0.1400	7.73	0.1852
p-value of Friedman Test		0.000			0.000			0.000	

* Significant at five percent level.

TABLE 4D

Estimated Median and Mean Rankings of out-of-sample R^2-for-logs for the various parametric ARCH Models.
p-values denote Friedman test that the ranks of out-of-sample log-R^2 for all models are equal.

	1981-90			1981-85			1986-90		
	Est. Median	Mean Ranking	Z-stat.	Est. Median	Mean Ranking	Z-stat.	Est. Median	Mean Ranking	Z-stat.
GARCH	0.0488	6.82	-0.0409	0.0277	10.00	0.9129	0.0651	7.45	0.0946
EGARCH	0.0454	2.82	-1.0533	0.0259	4.27	-0.6814	0.0622	3.73	-0.9002
TGARCH	0.0491	8.64	0.4793	0.0276	9.36	0.8035	0.0651	7.55	0.1625
QGARCH	0.0489	7.14	0.0587	0.0268	6.55	-0.1405	0.0645	6.55	-0.1418
NGARCH	0.0486	5.91	-0.2484	0.0277	8.64	0.4059	0.0650	7.09	0.0193
NAGARCH	0.0491	9.27	0.6668	0.0273	7.00	0.0000	0.0657	9.18	0.8516
INGARCH	0.0492	9.27	0.7041	0.0279	9.00	0.4688	0.0658	8.82	0.3934
AGARCH	0.0487	6.55	-0.1257	0.0268	5.36	-0.5628	0.0648	7.09	0.0375
TQGARCH	0.0488	6.68	-0.0750	0.0264	3.64	-1.2848	0.0644	7.00	0.0000
NQGARCH1	0.0488	7.09	0.0286	0.0267	5.18	-0.7445	0.0644	6.64	-0.1114
NQGARCH2	0.0488	6.64	-0.1686	0.0268	5.73	-0.4947	0.0638	4.73	-0.8234
NTGARCH1	0.0488	7.45	0.1058	0.0276	9.09	0.6456	0.0654	9.09	0.7176
NTGARCH2	0.0489	6.73	-0.1094	0.0274	7.18	0.0570	0.0642	6.09	-0.2834
p-value of Friedman Test		0.025			0.000			0.042	

* Significant at five percent level.

CHAPTER 3

ASYMMETRY IN VOLATILITIES AND THE PRIVATE INFORMATION HYPOTHESIS

Abstract

In a recent paper, Conrad, Gultekin and Kaul (1991) report an asymmetric effect in the conditional volatilities of large and small market value firms using weekly stock returns data from the US. This paper documents a similar effect for UK daily stock returns. Shocks from a portfolio of large firms affect the future volatility of a portfolio of small firms, but shocks from the portfolio of small firms have no effect on the future volatility of the portfolio of large firms. We also examine the differences in nontrading and trading shocks on future volatility. We find that trading period shocks from one portfolio tend to affect subsequent volatility of the other portfolio, but not nontrading period shocks. Volatility of small firms appears to be affected more by the trading shocks of large firms than volatility of large firms by trading shocks of small firms. The evidence in this paper therefore support the hypothesis that higher trading period volatility is due to the release of private information through the trading activities of informed investors.

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1. INTRODUCTION

There is considerable evidence on the time-series predictability of stock returns. Fama (1965), Fisher (1966), French and Roll (1986), Lo and MacKinlay (1990a), and Scholes and Williams (1977) have documented serial dependence in stock returns, and Lo and MacKinlay (1990a) find that lagged returns of large size firms can predict returns of small size firms, but not vice versa. Cohen *et. al.* (1980) and Scholes and Williams (1977) have ascribed the observed serial dependence in returns to problems of nonsynchronous trading. Given that stocks of different market capitalization exhibit different degrees of nonsynchronous trading, this may also explain the asymmetric predictability in the returns of different size stocks, as documented by Lo and MacKinlay (1990a). However, Lo and MacKinlay (1990b) have argued that attributing all observed serial dependence and cross predictability to nonsynchronous trading would suggest an absurdly thin market.

The introduction of Autoregressive Conditional Heteroskedasticity (ARCH) models by Engle (1982) and others (e.g. Bollerslev, 1986, Nelson, 1991) have allowed analysis to be extended to higher moments of stock returns, namely volatility. Many researchers have found that there is considerable dependence in the variance of stock returns, suggesting predictability in stock volatility (see Chou, 1988, French *et. al.*, 1987, and Watt and Yadav, 1993[1]). Conrad *et. al.* (1991) find that the asymmetric predictability extends to the conditional variance as well, i.e. the conditional variance of large firms can predict the conditional variance of small firms, but not the other way round. Using the Lo and MacKinlay (1990b) nontrading model, they proceed to show that the observed asymmetric predictability of conditional variance cannot be caused by nonsynchronous trading[2].

Such findings in the (asymmetric) predictability in the mean and variance of stock returns add to the conundrum of empirical evidence on asset prices. For example, autocorrelations of individual stock returns are only weakly positive or negative (Fama, 1965, French and Roll, 1986, and Lo and MacKinlay, 1990a), but portfolio returns appear to be positively autocorrelated (Cohen *et. al.*, 1980, and Scholes and Williams, 1977). Fama (1965), and French and Roll (1986) also find that daily stock return variances in the US are higher when the markets are open than when they are closed, while Barclay *et. al.* (1990) provide similar evidence for the Japanese stock market.

It is generally agreed the such empirical behaviour of stock returns reflects the trading behaviour of investors and the microstructure of the trading system. For example, French and Roll (1986) suggest three explanations on why trading period volatility is higher than nontrading period volatility: 1) more public information is

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[1] This paper is based on Chapter 2 of this dissertation.

[2] The simulations were done under assumptions of both homoskedasticity and heteroskedasticity in the distribution of the returns. In both cases, the cross-effects are not statistically different from zero.

released during trading hours; 2) private information is released through the trading activities of informed investors; 3) the effects of noise traders. There have also been attempts to model and predict the empirical patterns, first by Kyle (1985), and more recently by Admati and Pfleiderer (1988), and Foster and Viswanathan (1990, 1993). In these models, the market maker does not know if the trade is executed by an informed trader or a liquidity trader. As such the market maker has to infer the informed trader's beliefs from the order flow and set prices accordingly. This would support the private information hypothesis.

Chan (1993) extends the models of Kyle (1985) and Admati and Pfleiderer (1988) to a setting in which, in each period, the market maker derives a signal about the value of his stock. If the signal contains both market-wide information and uncorrelated noise, obtaining more signals will allow the market maker to better extract market-wide information. While the market maker can readily get the signal from his own stock, signals about the value of other stocks are however not instantaneously obtainable. Nevertheless, lag values of other stocks are available, and this induces cross-autocorrelation patterns in the stock prices. If the signal quality of large firms is better than small firms, this will also explain the observed asymmetry of returns of companies of different market values[3]. Following Ross

[3] US market makers (i.e. specialists) typically deal in only a handful of stocks, but have a preferential position with respect to the order flow information on these stocks. UK market makers on the other hand, typically make a market in 100+ stocks, but only see a fraction of the order flow in each (there are more than one market maker for each stock). Despite this difference, Chan (1993)'s results may still be applicable under the UK setting. Each stock in the inventory of the UK market maker contains less market-wide information relative to the stock under US specialist's inventory. However, the UK market maker has more stocks to derive market-wide information than the US specialist. Unless the UK market maker can obtain a better overall signal from *all* the stocks under his inventory than the corresponding US specialist from his inventory, a cross correlation pattern can still exist.

(1989), who shows that the variance of price changes is directly related to the rate of information flow, Chan (1993)'s model appears to be consistent with the findings of asymmetric predictability in volatilities. The model also lends support to the private information hypothesis.

Various empirical research papers tend to support the private information hypothesis. French and Roll (1986) compare multi-day return variances over weekends and exchange holidays to single trading day variances and conclude that the evidence is consistent with the private information hypothesis. A similar conclusion is reached by Barclays *et. al.* (1990) when they compare variances of trading and nontrading Saturdays in the Japanese stock market. Houston and Ryngaert (1992) examine the effect of reductions in trading hours on intra-week pattern of trading volume and volatility in the US stock market.. They find evidence that lower volume and volatility caused by a decrease in trading hours is more often made up after a closing, rather than before. They suggest that this is consistent with the private information hypothesis. Masulis and Ng (1991) analyse the UK stock index returns with a dynamic ARCH model that allows for temporal variation in volatility. They find evidence of differences in the impact and persistence of shocks arising from trading and nontrading periods on future volatility. Shocks originating from nontrading periods are less persistent than those originating from trading periods. This suggests that stock volatility is related to the trading activities of informed investors, and that private information held by them are released through trading activities.

This paper seeks to address the following issues. First, the results of Conrad *et. al.* (1991) on the asymmetric predictability in conditional volatilities may be a direct result of the use of transaction prices. Nonsynchronous trading of stocks of different market values may result in stale transaction prices, which may induce the observed asymmetry in the predictability of the conditional variances. Second, in a reasonably efficient market, the asymmetric predictability is likely to occur over a shorter return interval[4]. Third, the differences in market making arrangement between the US and UK may lead to different degrees of asymmetric predictability. Finally, we seek to address the issue of information and predictability of conditional volatilities. Assuming that there is more information in trading periods than nontrading periods (i.e. private information is released through trading), the degree of predictability should therefore differ as well.

Our approach is essentially similar to that of Conrad *et. al.* (1991). Using daily returns data from the London stock exchange, we begin our analysis by first applying a univariate GARCH specification to model the conditional volatilities of two size-sorted portfolios, and to determine the degree of (asymmetric) predictability between the two portfolios[5]. Since such a technique may be inefficient, we then proceed to re-estimate the above using a multivariate GARCH specification of Baba *et. al.* (1991). In both cases, our results show that the daily shocks of the portfolio of large-

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[4] It should be noted that the choice of using weekly returns in Conrad *et. al.* (1991) is largely an arbitrary one. It represents a "compromise between the relatively few monthly observations and the potential biases associated with nontrading, the bid-ask effect, etc." (pg 603).

[5] Since the UK data is based on the mid-market quote of dealer prices (i.e. average of the dealer's bid and ask prices), it does not suffer from problems associated with nontrading and bid-ask spreads.

size companies can predict the conditional volatility of the portfolio of smaller companies, but daily shocks of the portfolio of small-size companies have no impact on the conditional volatility of the portfolio of large-size companies.

We also extend our analysis further by looking at the differences in asymmetric response of conditional volatilities of the two size-sorted portfolio between trading and nontrading periods. Given the similarity in the results of both univariate and multivariate specifications, we limit our analysis to the univariate specification only[6]. This specification is based on that of Masulis and Ng (1991), adapted in our case to examine the interaction in the conditional volatilities between the two size-sorted portfolios. Our findings show that trading period shocks of one portfolio affect the future conditional volatility of the other portfolio, though trading period shocks from the large firms portfolio have a more significant impact on the small firms portfolio than the other way round. On the other hand, nontrading period shocks of large (small) companies have relatively little impact on small (large) companies.

The plan of this paper is as follows. Section 2 describes the data used in this study. Section 3 outlines the methodology we used. Section 4 presents the results of our univariate and multivariate models using daily stock returns data, as well as the trading/nontrading periods results. Our summary and conclusion is presented in Section 5.

[6] One problem of multivariate ARCH models is the number of parameters that are to be estimated (see Baba *et. al.,* 1991, Bollerslev, Engle and Wooldridge, 1988, and Bollerslev, 1990).

2. DATA

Daily closing returns of 200 of the largest companies listed on the London Stock Exchange are obtained from Datastream for the period from January 1990 to June 1993. The companies are then sorted into two equally weighted portfolios based on their market capitalization as at the beginning of the sample period. The daily returns are also divided into trading period returns i.e. returns computed from the opening price and closing price of the same day, and nontrading period returns i.e. returns computed from the closing price and the opening price of the following day.

3. METHODOLOGY

3.1 Univariate Specification - Daily Returns

We first model the returns of the two size-sorted portfolios as an autoregressive process with GARCH distributed errors. The GARCH model was introduced by Bollerslev (1986) and is based on the generalization of the ARCH model of Engle (1982). Essentially, the GARCH model expressed volatility as a function of previous volatility and past shocks.

Three problems exist with the use of UK daily data. First, initial data analysis reveals that returns on the two portfolios exhibit a rather high degree of serial correlation. Second, weekends and holidays tend to induce dependence in the returns.

Third, in the UK, stock transactions are settled according to an account period of approximately two weeks. As pointed out by Crouhy and Galai (1992) and Yadav and Pope (1992), this microstructure settlement procedure has implications on stock returns and volatility. After experimenting with various formulations, we use the following specification[7]

$$\begin{aligned} R_{it} = \beta_{i0} + \beta_{i1}R_{i,t-1} + \beta_{i2}R_{i,t-2} + \beta_{i3}R_{i,t-3} + \beta_{i4}R_{i,t-4} + \\ \delta_{i0}D_t + \delta_{i1}D_{t-1} + \phi_{i0}S_t + \phi_{i1}S_{t-1} + e_{it} \end{aligned} \quad . \qquad \textbf{(1.1)}$$

with $e_{it} \mid \Omega_{i,t-1} \sim N\left(0, h_{it}\right)$,

$$h_{it} = c_i + a_i e^2_{i,t-1} + b_i h_{i,t-1} \qquad \textbf{(1.2)}$$

where R_{it} is the return of security i in period t;

D_t = 1 if period t follows either a holiday or a weekend, and zero otherwise;

S_t = 1 if period t marks the start of an account period, and zero otherwise;

Ω_{t-1} is the set of all information available at time t - 1.

To determine the degree of spillover from one portfolio to another, we then perform a "second-pass" estimation using the residuals obtained from the above. This is given by

$$\begin{aligned} R_{it} = \beta_{i0} + \beta_{i1}R_{i,t-1} + \beta_{i2}R_{i,t-2} + \beta_{i3}R_{i,t-3} + \beta_{i4}R_{i,t-4} + \\ \delta_{i0}D_t + \delta_{i1}D_{t-1} + \phi_{i0}S_t + \phi_{i1}S_{t-1} + e_{it} + \theta_i e_{j,t-1} \end{aligned} \qquad \textbf{(2.1)}$$

[7]None of the dummy variables used in the mean equation appears to be significant in the conditional variance equation and are thus omitted in the latter.

with $e_{it} \mid \Omega_{i,t-1} \sim N(0, h_{it})$,

$$h_{it} = c_i + a_i e^2_{i,t-1} + b_i h_{i,t-1} + k_i e^2_{j,t-1} \qquad i,j = 1,2 \; ; \; i \neq j \qquad (2.2)$$

where θ_i and k_i measure the degree of spillover from portfolio j to portfolio i in the mean and variance respectively.

As pointed out by Conrad *et. al.* (1991), this univariate approach assumes that lagged values from portfolio j are treated as exogenous variables, and care should be taken when interpreting the results of the spillover effects. As such, we employ a multivariate approach that estimates the parameters for both portfolios simultaneously.

3.1 Multivariate Specification - Daily Returns

For the multivariate approach, we use the positive definite formulation of Baba *et. al.* (1991), which is also known as the BEKK model after the authors. The BEKK model essentially reparameterises the conditional variance equations (two in our case) into a matrix of the following form

$$\mathbf{H}_t = \mathbf{C}'\mathbf{C} + \mathbf{A}' e_{t-1} e'_{t-1} \mathbf{A} + \mathbf{B}' \mathbf{H}_{t-1} \mathbf{B} \qquad (3.1)$$

where

$$\mathbf{H}_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix}; \; \mathbf{C}'\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}; \; \mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}; \; \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}; \; e_{t-1} = \begin{bmatrix} e_{1,t-1} \\ e_{2,t-1} \end{bmatrix}$$

Note that a_{21} and a_{12} measures the shocks of Portfolio 2 (small size firms) on the conditional volatility of Portfolio 1 (large size firms), and vice versa respectively.

The BEKK representation is shown to be superior to several other formulations (e.g. the diagonal model of Bollerslev *et. al.*, 1988, or the constant correlation model of Bollerslev, 1990) in that it achieves parsimony without having to impose specific restrictions on the parameters of the model (see Bera and Higgins, 1993, Section 6). This multivariate approach is also more efficient than the univariate specification because the parameters for both portfolios are estimated simultaneously.

3.3 Univariate Specification - Open-to-Close and Close-to-Open Returns

To model the different impact of shocks arising from trading and nontrading periods on future volatility, we follow the method of Masulis and Ng (1991). We first divide the daily returns for each of the two portfolios into close-to-open and open-to-close returns. The close-to-open and open-to-close returns for portfolio i are specified respectively as

$$\begin{aligned} R_{it}^{C} = \beta_{i0}^{C} + \sum_{m=1,3,5,7} \beta_{im}^{O} R_{i,t-m}^{O} + \sum_{n=2,4,6,8} \beta_{in}^{C} R_{i,t-n}^{C} \\ + \delta_{i0} D_t + \phi_{i0} S_t + e_{it}^{C} \end{aligned} \quad \textbf{(4.1a)}$$

$$\begin{aligned} R_{it}^{O} = \beta_{i0}^{O} + \sum_{m=1,3,5,7} \beta_{im}^{C} R_{i,t-m}^{C} + \sum_{n=2,4,6,8} \beta_{in}^{O} R_{i,t-n}^{O} \\ + \delta_{i1} D_{t-1} + \delta_{i2} D_{t+1} + \phi_{i1} S_{t-1} + \phi_{i2} S_{t+1} + e_{it}^{O} \end{aligned} \quad \textbf{(4.1b)}$$

with $e_{it}^C \mid \Omega_{i,t-1} \sim N\left(0, h_{it}^C\right)$ and $e_{it}^O \mid \Omega_{i,t-1} \sim N\left(0, h_{it}^O\right)$,

$$\begin{aligned} h_{it}^C &= w_i^C + (b^C + g^C) h_{i,t-2}^C - b^C g^C h_{i,t-4}^C \\ &+ a^C (e_{i,t-2}^C)^2 - a^C g^C (e_{i,t-4}^C)^2 \\ &+ d^C (e_{i,t-1}^O)^2 - d^C b^C (e_{i,t-3}^O)^2 \end{aligned} \qquad \textbf{(4.2a)}$$

$$\begin{aligned} h_{it}^O &= w_i^O + (b^O + g^O) h_{i,t-2}^O - b^O g^O h_{i,t-4}^O \\ &+ a^O (e_{i,t-2}^O)^2 - a^O g^O (e_{i,t-4}^O)^2 \\ &+ d^O (e_{i,t-1}^C)^2 - d^O b^O (e_{i,t-3}^C)^2 \end{aligned} \qquad \textbf{(4.2b)}$$

where R_{it}^C and R_{it}^O denote the close-to-open and open-to-close return for portfolio i in period t respectively; D_t equals one if t spans a weekend/holiday, and zero otherwise; and S_t equals one if the immediate (trading) period following t marks the start of a new account period, and zero otherwise. Since we have split the daily returns into close-to-open and open-to-close returns, period t refers either to a nontrading period (from which we get the close-to-open returns, R_{it}^C) or a trading period (from which we get the open-to-close returns, R_{it}^O). Note also that if t is a trading period, t-1 and t+1 are nontrading periods, and vice versa. As per Masulis and Ng (1991), for each portfolio, we combine the Equations (4.1a) and (4.2a) with (4.1b) and (4.2b) respectively by introducing two dummy variables that orthogonalize the pair of intraday returns.

We then perform a "second pass" estimation using a similar specification. However, for the "second pass" estimation, we allow for interaction effects between the two portfolios in both the mean and variance equation. Specifically, our set-up is

$$
\begin{aligned}
R_{it}^{C} = \beta_{i0}^{C} &+ \sum_{m - 1,3,5,7} \beta_{im}^{O} R_{i,t-m}^{O} + \sum_{n=2,4,6,8} \beta_{in}^{C} R_{i,t-n}^{C} \\
&+ \delta_{i0} D_{t} + \phi_{i0} S_{t} + e_{it}^{C} + \theta_{i1}^{C} e_{j,t-1}^{O} + \theta_{i2}^{C} e_{j,t-2}^{C}
\end{aligned}
\qquad \textbf{(5.1a)}
$$

$$
\begin{aligned}
R_{it}^{O} = \beta_{i0}^{O} &+ \sum_{m=1,3,5,7} \beta_{im}^{C} R_{i,t-m}^{C} + \sum_{n=2,4,6,8} \beta_{in}^{O} R_{i,t-n}^{O} \\
&+ \delta_{i1} D_{t\ 1} + \delta_{i2} D_{t+1} + \phi_{i1} S_{t-1} + \phi_{i2} S_{t+1} \\
&+ e_{it}^{O} + \theta_{i1}^{O} e_{j,t-1}^{C} + \theta_{i2}^{O} e_{j,t-2}^{O}
\end{aligned}
\qquad \textbf{(5.1b)}
$$

with $e_{it}^{C} \mid \Omega_{i,t\ 1} \sim N\left(0, h_{it}^{C}\right)$ and $e_{it}^{O} \mid \Omega_{i,t-1} \sim N\left(0, h_{it}^{O}\right)$,

$$
\begin{aligned}
h_{it}^{C} = w_{i}^{C} &+ (b^{C} + g^{C}) h_{i,t-2}^{C} - b^{C} g^{C} h_{i,t-4}^{C} \\
&+ a^{C} (e_{i,t-2}^{C})^{2} - a^{C} g^{C} (e_{i,t-4}^{C})^{2} \\
&+ d^{C} (e_{i,t-1}^{O})^{2} - d^{C} b^{C} (e_{i,t-3}^{O})^{2} \\
&+ k_{i1}^{C} (e_{j,t-1}^{O})^{2} + k_{i2}^{C} (e_{j,t-2}^{C})^{2}
\end{aligned}
\qquad \textbf{(5.2a)}
$$

$$
\begin{aligned}
h_{it}^{O} = w_{i}^{O} &+ (b^{O} + g^{O}) h_{i,t-2}^{O} - b^{O} g^{O} h_{i,t-4}^{O} \\
&+ a^{O} (e_{i,t-2}^{O})^{2} - a^{O} g^{O} (e_{i,t-4}^{O})^{2} \\
&+ d^{O} (e_{i,t-1}^{C})^{2} - d^{O} b^{O} (e_{i,t-3}^{C})^{2} \\
&+ k_{i1}^{O} (e_{j,t\ 1}^{C})^{2} + k_{i2}^{O} (e_{j,t-2}^{O})^{2}
\end{aligned}
\qquad \textbf{(5.2b)}
$$

As before, we combine Equations (5.1a) and (5.2a) with (5.1b) and (5.2b) respectively using a pair of indicator variables. Note that this specification allows for spillover effects arising from trading and nontrading periods to have different impacts on subsequent returns and volatility. For example θ_{i1}^{C} and θ_{i2}^{C} measure the immediately preceding open-to-close and close-to-open spillover effects of portfolio j on the close-to-open returns of portfolio i respectively, while θ_{i1}^{O} and θ_{i2}^{O} measures the close-to-open and open-to-close spillover effects of portfolio j on the open-to-close returns of portfolio i. A similar interpretation is given to k_{i1}^{C}, k_{i2}^{C}, k_{i1}^{O} and k_{i2}^{O} for spillover effects in the variance.

4. RESULTS

4.1 Univariate Specification - Daily Returns

Table 1 shows the results of estimating Equations (1.1) and (1.2) for the two portfolios. Several points are worth noting. First, the returns of the two portfolios exhibit a "settlement" effect and (to a certain extent) a holiday/weekend effect. Second, the returns display a time-varying volatility, as evidenced by the statistical significance of the parameters in the conditional variance equation (1.2). Third, the AR(4)-GARCH(1,1) with the settlement period and holiday/weekend dummy variables appear to be well specified, since neither the standardised residuals nor the standardised squared residuals exhibit serial correlation up to nine lags.

To determine the spillover effects from one portfolio to another, we perform a "second-pass" estimation as per Equations (2.1) and (2.2). The results are presented in Table 2. For brevity, we only show the parameters that are of interest, namely those that indicate the spillover effects in the mean and volatility. It can be seen that neither the returns nor the volatility of Portfolio 1 (i.e. the portfolio consisting of larger companies) appears to be influenced by the returns and volatility of Portfolio 2 (i.e. the portfolio consisting of smaller companies) respectively. However, the converse is not true. The returns of Portfolio 2 appears to be influenced by Portfolio 1's previous returns, although it is not very significant. On the other hand, Portfolio 2's variance is strongly influenced by Portfolio 1's volatility shock, indicating that Portfolio 2's future (i.e. conditional) volatility is predictable from the surprises of Portfolio 1. A likelihood ratio test also shows that the two additional parameters associated with the previous return and volatility of Portfolio 2 are not jointly significant on the returns and volatility of Portfolio 1, but the previous return and volatility of Portfolio 1 are jointly significant on the returns and volatility of Portfolio 2. This evidence is consistent with the findings of Conrad *et. al.* (1991).

One potential problem concerning the significance of the parameters to Equations (2.1) and (2.2) relate to the issue of a misspecified conditional distribution. Bollerslev and Wooldridge (1992) and Susmel and Engle (1994) show that if the assumption of conditional normality of the error term is violated, the resulting parameters in both the mean and variance equation are inconsistent. As such, we compute the robust *t*-statistics using quasi-maximum likelihood estimation, as suggested by Bollerslev and Wooldridge (1992), and these are shown in brackets in Table 2. Again this points to an asymmetric effect in both the mean and variance of the two portfolios.

4.2 Multivariate Specification - Daily Returns

The results of using the multivariate specification are presented in Table 3. Since the main difference between the univariate and multivariate specification is in the parameterisation of the conditional variance equation, we show only the spillover effects in volatility. As in the univariate case, there is a distinct asymmetry in the predictability of the conditional variance. Shocks arising from Portfolio 1 affects the future volatility of Portfolio 2, but shocks from Portfolio 2 do not have a significant influence on the future volatility of Portfolio 1. Again the findings are similar to that of Conrad *et. al.* (1991).

4.3 Univariate Specification - Close-to-Open and Open-to-Close Returns

Given the results of Masulis and Ng (1991), it would be interesting to analyse the asymmetric predictability of the two portfolios in the context of close-to-open and open-to-close returns. Since both our earlier univariate and multivariate analysis yield essentially the same results, we decide to stick to the simpler univariate approach. As before, this involve performing a "two-pass" estimation, the first using Equations (4.1a), (4.1b), (4.2a) and (4.2b), and the second using Equations (5.1a), (5.1b), (5.2a) and (5.2b). The results are summarised in Table 4. Again for brevity, we only show the spillover effects in the mean and variance equation between the two portfolios. Unlike the earlier univariate results using daily returns, we only show the robust *t*-statistics computed using quasi-maximum likelihood techniques, since such an estimation is shown to be more robust to misspecification in the conditional distribution.

Once again, we find a distinct asymmetry in the predictability of the returns and volatility of the portfolio of small companies with the returns and volatility of the portfolio of large companies. The previous open-to-close returns and volatility of Portfolio 2 has a weakly significant impact on the close-to-open returns and volatility of Portfolio 1, but the previous close-to-open returns and volatility of Portfolio 2 has no significant impact on the close-to-open returns and volatility of Portfolio 1. However, neither the previous close-to-open nor the previous open-to-close returns and volatilities of Portfolio 2 have any impact on the open-to-close returns and volatilities of Portfolio 1. On the other hand, both the previous close-to-open and open-to-close returns of Portfolio 1 affect the open-to-close and close-to-open returns of Portfolio 2. The effect is more significant in the immediately preceding period. For example, nontrading period returns have a more significant impact than previous trading period returns on subsequent trading period returns, and trading period returns have a more significant impact than previous nontrading returns on subsequent nontrading period returns. As for volatility, shocks of Portfolio 1 arising from the immediately preceding trading period exhibit a very significant impact on the nontrading period volatility of Portfolio 2. All other shocks of Portfolio 1 have no significant effects on Portfolio 2's volatility (for both trading and nontrading periods).

The above indicates that the degree of asymmetric predictability in the mean and variance of large and small firms is different between trading and nontrading periods. The evidence suggests that trading periods contain a richer information set which is impounded in the returns and volatility of the subsequent nontrading period. This is consistent with earlier studies on differences between trading and nontrading period volatilities, most notably that of French and Roll (1986). Like the results of Masulis and Ng (1991), the evidence here appears to support the hypothesis that stock price volatility is affected by the trading activities of informed investors.

The evidence also shows that the information set implied in the returns and volatility of the portfolio of larger companies contains valuable information which can predict the returns and volatilities of the portfolio of smaller companies. This is consistent with the empirical findings of cross serial dependence in returns, and is also compatible with the imperfect information model of Chan (1993).

5. CONCLUSION

In this paper, we document the asymmetric predictability in the conditional volatility of firms of different market values. Using daily data from the UK market, we find that the shocks to larger firms affect the future returns and volatility of smaller size firms, but not vice versa. As stated by Conrad *et. al.* (1991), while one possible explanation is that there are timing differences in the way information is incorporated into the prices of large and small companies, other explanations not related to timing effects are also plausible, such as the conditional volatility being driven by some factors that are more closely associated with the shocks to large firms.

Our intraday analysis shows that the degree of asymmetric shocks differ between trading and nontrading periods. Open-to-close returns and volatility of large firms tend to have a much more significant impact on subsequent close-to-open returns and volatility of small firms. This suggests that information impounded in trading period returns and volatility is richer. This supports the hypothesis that stock volatility is related to the trading activities of informed investors. It also suggests that the asymmetric predictability may be a result of timing rather than non-timing effects.

TABLE 1

Univariate AR(4)-GARCH(1,1) model of daily returns of two equally weighted portfolios for the period from 2 January 1990 to 30 June 1993. Portfolio 1 consists of the 100 largest stocks listed on the London Stock Exchange, with Portfolio 2 consisting of the next 100 largest stocks (based on their capitalization as at the start of the sample period). The model is given by

$$R_{it} = \beta_{i0} + \beta_{i1}R_{i,t-1} + \beta_{i2}R_{i,t-2} + \beta_{i3}R_{i,t-4} + \beta_{i4}R_{i,t-4} + \delta_{i0}D_t + \delta_{i1}D_{t-1} + \phi_{i0}S_t + \phi_{i1}S_{t-1} + e_{it} \tag{1.1}$$

with $e_{it} \mid \Omega_{i,t-1} \sim N\left(0, h_{it}\right)$,

$$h_{it} = c_i + a_i e^2_{i,t-1} + b_i h_{i,t-1} \tag{1.2}$$

Parameters	Portfolio 1	Portfolio 2
β_{i0}	-0.0043 (-0.1032)	-0.0099 (-0.2809)
β_{i1}	0.0805 (2.1829)	0.2240 (6.5362)
β_{i2}	-0.0032 (-0.0877)	0.0635 (1.5715)
β_{i3}	0.0669 (1.9101)	0.1263 (3.3896)
β_{i4}	0.0952 (2.7177)	0.1485 (4.9220)
δ_{i0}	0.2124 (2.4549)	0.0411 (0.6771)
δ_{i1}	-0.1295 (-1.3384)	-0.1065 (-1.6248)
ϕ_{i0}	-0.5712 (-4.9324)	-0.3033 (-3.4477)
ϕ_{i1}	0.2922 (2.3025)	0.1794 (1.7850)
c_i	0.0438 (3.5116)	0.0286 (3.9040)
a_i	0.1165 (5.7773)	0.2218 (9.4003)
b_i	0.8365 (29.7817)	0.7606 (32.5040)
Log-likelihood	-328.5537	-134.7977
Q(9)	3.2518 [0.6612]	7.0267 [0.2187]
$Q^2(9)$	3.9846 [0.7917]	2.2524 [0.8132]

Asymptotic *t*-statistics denoted in parentheses, p-values in brackets. Q(9) and $Q^2(9)$ refer to Ljung-Box statistics of 9 lags for the standardised residuals and squared standardised residuals respectively.

TABLE 2

Univariate AR(4)-GARCH(1,1) model with spillover effects of daily returns of two equally weighted portfolios for the period 2 January 1990 to 30 June 1993. Portfolio 1 consists of the 100 largest stocks listed on the London Stock Exchange, with Portfolio 2 consisting of the next 100 largest stocks (based on their capitalization as at the start of the sample period). The model is given by

$$R_{it} = \beta_{i0} + \beta_{i1}R_{i,t-1} + \beta_{i2}R_{i,t-2} + \beta_{i3}R_{i,t-4} + \beta_{i4}R_{i,t-4} + \delta_{i0}D_t + \delta_{i1}D_{t-1} + \phi_{i0}S_t + \phi_{i1}S_{t-1} + e_{it} + \theta_i e_{j,t-1} \tag{2.1}$$

with $e_{it} \mid \Omega_{i,t-1} \sim N\left(0, h_{it}\right)$,

$$h_{it} = c_i + a_i e^2_{i,t-1} + b_i h_{i,t-1} + k_i e^2_{j,t-1} \qquad i,j = 1,2 \; ; \; i \neq j \tag{2.2}$$

Parameters	Portfolio 1	Portfolio 2
θ_i	-0.0836 (-0.8434) [-0.8767]	0.0899 (1.3061) [1.9886]
k_i	0.0119 (0.2796) [0.1722]	0.0914 (3.6848) [2.0554]
Log-likelihood	-328.1837	-128.5872

Asymptotic t-statistics denoted in parentheses. Robust t-statistics (see Bollerslev and Wooldridge, 1992) denoted in brackets.

TABLE 3

Multivariate AR(4)-GARCH(1,1) model with spillover effects of daily returns of two equally weighted portfolios for the period 2 January 1990 to 30 June 1993. Portfolio 1 consists of the 100 largest stocks listed on the London Stock Exchange, with Portfolio 2 consisting of the next 100 largest stocks (based on their capitalization as at the start of the sample period). The model is given by

$$R_{it} = \beta_{i0} + \beta_{i1}R_{i,t-1} + \beta_{i2}R_{i,t-2} + \beta_{i3}R_{i,t-4} + \beta_{i4}R_{i,t-4} + \delta_{i0}D_t + \delta_{i1}D_{t-1} + \phi_{i0}S_t + \phi_{i1}S_{t-1} + e_{it} + \theta_i e_{j,t-1} \quad (2.1)$$

with $e_t \mid \Omega_{t-1} \sim MVN\left(0, \mathbf{H}_t\right)$,

$$\mathbf{H}_t = \mathbf{C}'\mathbf{C} + \mathbf{A}' e_{t-1} e_{t-1}' \mathbf{A} + \mathbf{B}'\mathbf{H}_{t-1}\mathbf{B} \quad (3.1)$$

Parameters	Portfolio 1	Portfolio 2
a_{21} / a_{12}	0.1530 (1.5893)	0.2276 (4.1954)

Asymptotic *t*-statistics shown in parentheses.

TABLE 4

Univariate AR(4)-GARCH(1,1) model with spillover effects of daily returns of two equally weighted portfolios for the period 2 January 1990 to 30 June 1993. Portfolio 1 consists of the 100 largest stocks listed on the London Stock Exchange, with Portfolio 2 consisting of the next 100 largest stocks (based on their capitalization as at the start of the sample period). The model is given by

$$R_{it}^{C} = \beta_{i0}^{C} + \sum_{m=1,3,5,7} \beta_{im}^{O} R_{i,t-m}^{O} + \sum_{n=2,4,6,8} \beta_{in}^{C} R_{i,t-n}^{C} + \delta_{i0} D_t + \phi_{i0} S_t + e_{it}^{C} + \theta_{i1}^{C} e_{j,t-1}^{O} + \theta_{i2}^{C} e_{j,t-2}^{C} \qquad (5.1a)$$

$$R_{it}^{O} = \beta_{i0}^{O} + \sum_{m=1,3,5,7} \beta_{im}^{C} R_{i,t-m}^{C} + \sum_{n=2,4,6,8} \beta_{in}^{O} R_{i,t-n}^{O} + \delta_{i1} D_{t-1} + \delta_{i2} D_{t+1} + \phi_{i1} S_{t-1} + \phi_{i2} S_{t+1} + e_{it}^{O} + \theta_{i1}^{O} e_{j,t-1}^{C} + \theta_{i2}^{O} e_{j,t-2}^{O} \qquad (5.1b)$$

with $e_{it}^{C} \mid \Omega_{i,t-1} \sim N\left(0, h_{it}^{C}\right)$ and $e_{it}^{O} \mid \Omega_{i,t-1} \sim N\left(0, h_{it}^{O}\right)$,

$$h_{it}^{C} = w_i^{C} + (b^{C} + g^{C}) h_{i,t-2}^{C} - b^{C} g^{C} h_{i,t-4}^{C} + a^{C} (e_{i,t-2}^{C})^2 - a^{C} g^{C} (e_{i,t-4}^{C})^2 + d^{C} (e_{i,t-1}^{O})^2 - d^{C} b^{C} (e_{i,t-3}^{O})^2 + k_{i1}^{C} (e_{j,t-1}^{O})^2 + k_{i2}^{C} (e_{j,t-2}^{C})^2 \qquad (5.2a)$$

$$h_{it}^{O} = w_i^{O} + (b^{O} + g^{O}) h_{i,t-2}^{O} - b^{O} g^{O} h_{i,t-4}^{O} + a^{O} (e_{i,t-2}^{O})^2 - a^{O} g^{O} (e_{i,t-4}^{O})^2 + d^{O} (e_{i,t-1}^{C})^2 - d^{O} b^{O} (e_{i,t-3}^{C})^2 + k_{i1}^{O} (e_{j,t-1}^{C})^2 + k_{i2}^{O} (e_{j,t-2}^{O})^2 \qquad (5.2b)$$

Parameters	Portfolio 1	Portfolio 2
θ_{i1}^{C}	-0.0769 [-2.0235]	0.1174 [6.4158]
θ_{i2}^{C}	0.0798 [0.8168]	-0.2054 [-3.8868]
θ_{i1}^{O}	-0.1416 [-0.4690]	0.5956 [6.7247]
θ_{i2}^{O}	0.0601 [0.3727]	-0.0861 [-2.4896]
k_{i1}^{C}	0.1316 [2.1738]	0.0964 [4.3117]
k_{i2}^{C}	0.4794 [0.6619]	0.0028 [0.6398]
k_{i1}^{O}	0.0000 [0.0000]	0.0000 [0.0044]
k_{i2}^{O}	0.0247 [1.3861]	0.0000 [0.0013]

Robust t-statistics (see Bollerslev and Wooldridge, 1992) denoted in brackets.

CHAPTER 4

AN EMPIRICAL EXAMINATION OF THE VOLATILITY-VOLUME RELATIONSHIP

Abstract

This paper examines the empirical relationship between stock index volatility and trading volume in the UK. There is some evidence of contemporaneous relationship between volatility and volume levels, although the volatility appears to be affected more strongly by *changes* in volume levels instead. However, when a GARCH framework is used, we find that neither contemporaneous nor lag daily trading volume has any explanatory power on conditional volatility. The evidence appears to be inconsistent with the results of Lamoureux and Lastrapes (1990), and Najand and Yung (1991) in so far as volume levels are concerned. On the hand, contemporaneous *changes* in volume levels has significant explanatory power on the conditional variance, but not lag changes in volume, and this appears to be robust to the choice of the conditional distribution. Nevertheless, ARCH effects remain strongly significant. Inclusion of contemporaneous changes in volume also reduces the kurtosis of the conditional distribution i.e. the standardised residuals. These results therefore suggest that volume and price changes are jointly determined.

AN EMPIRICAL EXAMINATION OF THE VOLATILITY-VOLUME RELATIONSHIP

1. INTRODUCTION

There has been considerable interest in the relationship between volatility of asset returns and trading volume. Much of the interest stems from the belief that volume is needed to move prices. This belief is further strengthened by substantial anecdotal evidence, such as the 'Black Monday' of October 1987, in which stock markets of the world experienced a sharp fall in prices, accompanied by unprecedented trading volume. Hence, it is not surprising to find early academic research focusing on the price-volatility-volume relationship *per se*. For example, Osborne (1959) models stock price changes as a diffusion process with variance being a function of the number of transactions, while empirical research such as Crouch (1970), Godfrey, Granger and Morgenstern (1964), and Ying (1966) tends to examine the price-volatility-volume behaviour without considering the fundamental nature of trading volume[1].

A related development on the price-volatility-volume relationship comes partly from the work on *unconditional* distributions of asset returns. Fama (1965) and Mandebrot

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[1] In this paper, we make a distinction between price-volume behaviour and volatility-volume behaviour. This is of course a matter of convenience only. The former shall refer to what Karpoff (1987) termed "price change *per se*", while the latter is the "absolute value of price change". This distinction is also helpful in distinguishing the literature on returns and volume relationship (such as Antoniewicz, 1992, Campbell, Grossman and Wang, 1994, and Duffee, 1992, etc.) from studies on volatility-volume relationship.

(1963) first suggest that asset prices/returns are characterised by stable Paretian distributions because of their kurtotic nature. On the other hand, Clark (1973) and Tauchen and Pitts (1983) propose that returns behave as subordinated stochastic processes. Given that price movements are a result of news, observed returns are therefore drawn from a mixture of distributions, with the directing/mixing variable being the number of news arrivals to the market.

Empirical tests of which hypothesis best characterised asset returns distributions tend to support the mixture of distribution hypothesis (MDH) over the stable Paretian. Such tests can be divided into two groups: those that test the stable Paretian against alternative distributions, and those that test the MDH directly. Among the studies in the first group are Akgiray and Booth (1988), Blattberg and Gonedes (1974), Lau, Lau and Wingender (1990), Officer (1972), and Tucker (1992). Despite using different methods, these studies tend to reject the stable Paretian distribution over some alternative distributions in describing asset returns.

Direct test of the MDH requires a mixing variable, which is the rate of news arrival to the market, and this is either unobservable or unmeasurable. Studies in the second group therefore require a proxy for the mixing variable, and trading volume is frequently used. Examples include Clark (1973), Harris (1987), and Tauchen and Pitts (1983), who all document evidence supporting the MDH. However, volume is not the only available proxy for information arrival, though it is by far the most common. Volatility shocks (Engle, Ito and Lin, 1990), the number of price changes within a period (Laux and Ng, 1993), the number of quote arrivals per period, the bid-ask spread, and the average duration between trades (Bollerslev and Domowitz, 1993) have been used as proxies in other studies.

The essence of the models of Clark (1973), and Tauchen and Pitts (1983) is that the rate of news arrival varies across calendar time. Information arrival may be constant across economic time, but economic time differs from calendar time. Since we are measuring returns across calendar time, the returns therefore appear to be drawn from a mixture of distributions (see Stock 1987, 1988, for more information on this concept of time deformation). Explicitly, these models suggest that information is derived exogenously (hence the term arrival), and is in a sense, compatible to the "sequential information arrival" model of Copeland (1976). A feature of exogenous information models is that there is a joint impact of both price changes and volume to information shocks. For example, in the Tauchen and Pitts model, an exogenous revision of traders' expectations results in a change in prices and volume. Depending on whether the new information is trader specific or common knowledge to all investors, a large (small) change in volume is accompanied by a small (large) change in price. Another model by Epps and Epps (1976), suggests that price volatility and volume is directly related to the extent in which investors disagree about the value of new information.

On the other hand, endogenous information models do not assume information to be fully exogenous. In the "price formation" models of Admati and Pfleiderer (1988), Foster and Viswanathan (1990), and Kyle (1985), information is only partially exogenous. Some information is generated endogenously by the trading mechanism. The framework of these models typically requires a market maker, informed traders and liquidity traders (both discretionary and non-discretionary), with the market maker setting prices at which trade occurs. To distinguish between informed and unformed trades, the market maker has to infer the informed trader's beliefs from the market order flow. These models predict several patterns related to trading activity, one of which is a positive correlation between price changes and trading volume.

From an empirical viewpoint, both exogenous and endogenous information models show a positive relationship between volatility and volume. Indeed, many previous studies provide evidence to support this behaviour. Karpoff (1987) cites some 17 previous studies that document a positive contemporaneous volatility-volume relationship. However, most of these studies are examined in the context of exogenously generated information.

The development of ARCH models by Engle (1982) and Bolleslev (1986) has allowed the issue to be re-examined. A feature of ARCH models is that it captures some of the empirical regularities of financial data, such as temporal patterns in volatility and excessive kurtosis in the unconditional distribution. Unfortunately, economic theory does not fully explain why variances are autocorrelated. As noted by Diebold (1986), and Gallant, Hsieh and Tauchen (1991), the time varying dependencies in the conditional variances as fitted by ARCH models may be a manifestation of the (serially correlated) information arrival process. In that respect, ARCH processes can therefore be interpreted as evidence supporting MDH.

Lamoureux and Lastrapes (1990b) provide a framework linking MDH to ARCH effects. Suppose δ_{it} is the ith intraday increment of the equilibrium price in day t. Suppose further that δ_{it} is independent and identically distributed, with a zero mean and variance corresponding to σ^2. The equilibrium price change across fixed intervals, ϵ_t, is given by

$$\epsilon_t = \sum_{i=1}^{n_t} \delta_{it} \tag{1}$$

where n_t is a random variable representing the stochastic rate of information arrival. Two important properties become apparent. First, ϵ_t is subordinate to δ_{it}, with n_t being the directing process. Second, if n_t becomes very large, then Central Limit Theorem would suggest that

$$\epsilon_t \mid n_t \sim N\left(0, \sigma^2 n_t\right) \tag{2}$$

If the rate of information arrival is sequentially related i.e.

$$n_t = k + b(L) n_{t-1} + u_t \tag{3}$$

where k is a constant $b(L)$ is a lag operator of order q, and u_t is white noise, shocks to the mixing n_t will persist according to the pattern of $b(L)$. From this, it can be shown that

$$\begin{aligned} E[\epsilon_t^2 \mid n_t] &= \sigma^2 n_t \\ &= \sigma^2 k + b(L) E[\epsilon_{t-1}^2 \mid n_{t-1}] + \sigma^2 u_t \end{aligned} \tag{4}$$

which is similar in structure to the conditional variance parameterisation of an ARCH(q) model.

Lamoureux and Lastrapes (1990b) proceed to estimate a GARCH model with an additional volume parameter in the conditional variance equation i.e.

$$h_t = \alpha_0 + \alpha_1 \epsilon_t^2 + \alpha_2 h_{t-1} + \alpha_3 V_t \tag{5}$$

where V_t is the contemporaneous trading volume. They argue that if the MDH is correct, and volume is an accurate proxy for the mixing variable, then α_3 would be significantly greater than zero and that the persistance in variance as measured by ($\alpha_1 + \alpha_2$) will become negligble, with both α_1 and α_2 becoming insignificant i.e. ARCH effects will disappear. This conclusion appears to be supported by their empirical results, which incidentally also show that lagged volume has little explanatory power in the variance equation.

There are several problems associated with Lamoureux and Lastrape's specification of the conditional variance i.e. Equation (5). First, volume (or log-volume) may be a nonstationary variable, and introducing it into the conditional variance equation may not be appropriate. Second, the equation suggests that the volatility-volume relationship is linear. Karpoff (1987) gives an example of a nonlinear relationship that is still consistent with empirical observations. Finally, and perhaps most importantly, if price and volume are jointly determined, or if information is not strictly exogenous, then Equation (5) may have some simultaneity bias[2].

Several authors have addressed these problems in different ways. Connolly (1990) uses a nonlinear specification of volume in the variance equation i.e.

$$h_t = \alpha_0 + \alpha_1 \epsilon_t^2 + \alpha_2 h_{t-1} + \alpha_3 V_t + \alpha_4 V_t^2 \tag{6}$$

and finds that contemporaneous volume has explanatory power regarding the variance in foreign exchange spot markets. It reduces the ARCH effects present in the data, but does not completely eliminates them. On the other hand, contemporaneous volume completely eliminates the ARCH effects in foreign exchange futures market.

Najand and Yung (1991) present some results that differ from the Lamoureux and Lastrapes (1990b) study. Using Equation (5), they find that there is no relationship between volatility and contemporaneous volume in the Treasury-bond futures market i.e. α_3 is not significantly different from zero, and that ARCH effects are still present. However, they find that lagged volume is significant in explaining some of the volatility of the market, although it neither reduces the ARCH effects nor reduces the level of persistance of past shocks. They argue that their results support the hypothesis that information arrival is probably not exogenous, with price and volume likely to be jointly determined. Equation (5) is therefore misspecified because of the

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[2] If volume is not exogenous but forms part of a system of simultaneous equations, using maximum likelihood techniques on Equation (5) would not be appropriate. See Harvey (1990, Section 9.4) for a brief discussion on simultaneous equation bias.

simultaneity bias. Nevertheless, the signficance of lagged volume is evidence in support of MDH.

Laux and Ng (1993) provide further evidence supporting the MDH. They suggest that the use of intraday data mitigates the simultaneity bias as investors are likely to react on any new information with a small time lag. They also argue that a more inclusive proxy for information arrival is the number of price changes per period, since it may contain information not reflected in volume data. Using a multivariate GARCH specification, they decompose the volatility of foreign exchange futures contracts into systematic and unique components. Their results show that the MDH explains some of the GARCH behaviour of unique risks, but the GARCH effects in systematic risks are still present[3].

Using a different proxy for information arrival, Bollerslev and Domowitz (1993) find that the (lagged) number of quote arrivals per period is neither economically or statistically significant with regards to the conditional volatility of the deutschemark-dollar intraday exchange rates. Another proxy, the average duration per trade over a lagged interval, is also statistically insignificant. A third proxy, the bid-ask spread in the previous period, however shows a statistically significant relationship with conditional volatility. The last finding is not surprising, since Bollerslev and

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[3] The study used contemporaneous number of price changes. Bollerslev and Domowitz (1993) argue that any contemporaneous activity variable is not only subjected to simultaneity bias, but also inappropriate because it is not in the traders' information set at the time the decisions are made. Laux and Ng (1993) also used the predicted number of price changes, but this does not change their results.

Domowitz (1991) show that the market microstructure mechanism has a potential impact on conditional volatility.

Several other studies also examine the volatility-volume relationship, though from slightly different perspectives. Abhyankar (1993) examines the Eurodollar futures market traded on both the Chicago Mercantile Exchange and Singapore International Monetary Exchange and finds evidence that lagged volume affects conditional volatility, but has no impact on the GARCH parameters. In addition, the volume in the previous market (in terms of trading time) is found to have a significant impact on the conditional volatility of the subsequent market. Bessembinder and Seguin (1992, 1993) find that for a number of assets, volatility is affected by both expected and unexpected components of trading activy. Moser (1992) documents evidence that volatility is related to program and nonprogram trading activity.

Given the above empirical evidence, clearly the question of the volatility-volume behaviour is a complex issue. Without an economic model to guide us, some of the issues can only be addressed empirically. For a market trader however, the more important question is whether this relationship can be used for forecasting purposes.

In this paper, we first examine the empirical relationship between volatility and trading volume. Our findings show that there is a relationship between trading activity, as measured by market volume, and returns volatility. We find that volatility is a function of the change in trading volume. We also uncover evidence that there is a strong contemporaneous volume-volatility relationship. We then suggest a simple modification to the GARCH model to incorporate this effect.

The organisation of this paper is as follows. In Section 2, we describe the data used in our study, and we present some preliminary statistics. In Section 3, we model the volatility of the data using different GARCH specifications. We examine the volume-volatility relationship under a GARCH framework in Section 4. In Section 5 we then propose a modification to the GARCH model that incorporates the volume-volatility relationship. Section 6 concludes.

2. DATA SOURCES AND SUMMARY STATISTICS

The data analysed consist of daily closing mid-market quotes of the Financial Times-Stock Exchange 100 (FT-SE 100) from January 1989 to December 1993, obtained from Datastream. This is a value-weighted index and includes the 100 largest UK companies in terms of market capitalisation. The index was first started in 1984, and its constituents are amended quarterly. For the same period, we obtain from Datastream the total trading volume of the constituent stocks. Returns are calculated as the natural logarithm of the price relatives. For trading volume, because of the magnitude, we decide to transform it by taking its natural logs. This transformation, which is commonly used in empirical research, also removes heteroskedasticity and low frequency variation in the volume.

Table 1 shows the summary statistics. The returns series exhibit the typical leptokurtosis, with some degree of skewness. There is mild serial correlation in the levels, as evidenced by Q(10), the 10-lag Ljung-Box statistic, with the largest autocorrelation coefficient coming from lag one. On the other hand, the squares are strongly autocorrelated, with the 10-lag Ljung-Box statistic at about 50.3. The volume series is symmetric, slightly kurtotic, and strongly autocorrelated. A Dickey-Fuller (1979) test indicates that the volume series is stationary.

As a framework for subsequent analysis, we want to determine the contemporaneous volume-volatility relationship. Following Jain and Joh (1988), we run the following regression:

$$V_t = b_0 + b_1 |r_t| + b_2 D_t |r_t| \tag{7}$$

where V_t is the day t trading volume, r_t is the returns on day t, and D_t is a dummy variable that equals one if $r_t < 0$, and one otherwise. We also want to find the relationship betwen contemporaneous *change* in trading volume and volatility. For this we run the following regression:

$$(V_t - V_{t-1}) = b_0 + b_1 |r_t| + b_2 D_t |r_t| \tag{8}$$

Table 2 summarises the results of the above regressions. It appears that volatility is related to both contemporaneous trading volume levels and changes in volume, since b_1 is statistically significant for both equations. There is also a statistically significant relationship between price change *per se* and trading volume. A negative price change is assoicated with lower volume and smaller (i.e. less positive) changes in volume. R^2 for changes in volume is higher than that for volume levels, indicating that volatility is more closely related to changes in trading activities.

The observation that volatility is more strongly affected by changes in volume levels rather than levels themselves is not inconsistent with some of the "price formation" models discussed earlier. In the absence of new information to the market, there will be an equilibrium volume level that clears the market. Suppose new information arrives, and this increases the volatility of the stock. If volume is an accurate proxy for information, then changes in volume will reflect the amount of change in information i.e. the amount of new (incremental) information. Under this scenario, volatility shocks will be better explained by changes in volume.

3. GARCH SPECIFICATIONS

A number of studies have successfully used low order GARCH models to describe asset returns. Chou (1987), Corhay and Rad (1991), Poon and Taylor (1992), and Watt and Yadav (1993) all find that the GARCH(1,1) model is sufficient. As such, we model the returns process as:

$$r_t = a_0 + a_1 \epsilon_{t-1} + \epsilon_t \tag{9a}$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} \tag{9b}$$

$$E[\epsilon_t \mid \Omega_{t-1}] \sim N(0, h_t) \tag{9c}$$

Bollerslev (1987), Hsieh (1989) and Nelson (1991) have shown that a conditional normal distribution is insufficient to account for the excess unconditional kurtosis.

Following Bollerslev (1987) suggestion, we also estimate another model by replacing the conditional normal distribution of Equation (9c) with a student-*t* distribution with ν degrees of freedom i.e.

$$E[\epsilon_t \mid \Omega_{t-1}] \sim t(0, h_t, \nu) \qquad \textbf{(9d)}$$

A problem with financial data is that there are often deterministic effects in the returns. For our returns series, this include weekends, exchange holidays and account periods[4]. To control for this, we introduce a number of indicator variables into the mean and variance equations. We also estimate the parameters using both the conditional normal and student-*t* distribution i.e.

$$r_t = \sum_{i=1}^{5} c_i \mathrm{DAY}_{it} + \sum_{j=0}^{2} d_j \mathrm{HOL}_{j,t+j-1} + \sum_{k=0}^{2} e_j \mathrm{SET}_{k,t+k-1} + a_1 \epsilon_{t-1} + \epsilon_t \qquad \textbf{(10a)}$$

$$h_t = \sum_{i=1}^{5} \beta_i \mathrm{DAY}_{it} + \sum_{j=0}^{2} \gamma_j \mathrm{HOL}_{j,t+j-1} + \sum_{k=0}^{2} \eta_k \mathrm{SET}_{k,t+k-1} + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} \qquad \textbf{(10b)}$$

$$E[\epsilon_t \mid \Omega_{t-1}] \sim N(0, h_t) \qquad \textbf{(10c)}$$

$$E[\epsilon_t \mid \Omega_{t-1}] \sim t(0, h_t, \nu) \qquad \textbf{(10d)}$$

where DAY_{it} is the day-of-the-week dummy variables; HOL_{jt} equals one if *t* spans a holiday, and zero otherwise; and SET_{kt} equals one if *t* is the last day of the account

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[4] In the UK, trades are settled according to a system of account periods of appoximately 14 calendar days. Trade done within an account period is usually settled at the end of an account period. As documented by Yadav and Pope (1992), who examined the UK settlement system, this has an impact on the observed prices. Crouhy and Galai (1992) provide similar evidence on the French settlement system, which has a 30-day account period.

period, and zero otherwise. The inclusion of more than one dummy variables for both holiday and settlement effects is to account for changes *around* these days.

Instead of reporting the results of all the parameters, in Table 3 we only report those parameters that are of interest. We also report the log-likelihood values for each of the specifications, and some summary statistics for the standardised residuals from these specifications. To check for in-sample prediction performance, we run the following two regressions

$$\epsilon_t^2 = A_0 + A_1 h_t \tag{11}$$

$$\log \epsilon_t^2 = B_0 + B_1 \log h_t \tag{12}$$

and report their R-squares (the latter shall be termed R^2-for-logs).

Several conclusions can be drawn. First, the returns exhibit significant ARCH effects, and there is a fair degree of persistence in the variance ($\alpha_1 + \alpha_2$ being greater than 0.94 in all cases). Second, a conditional normal distribution is inadequate, with the standardised residuals still exhibiting greater than normal kurtosis (Columns 2 and 4). Third, the student-*t* distribution is more suitable, since it gives a significantly higher log-likelihood (Columns 2 and 3, and Columns 4 and 5), and the parameter 1/v is significantly greater than zero[5] (Columns 3 and 5). Fourth, the addition of

[5] See Bollerslev (1987) on why 1/v is used instead of v. If the distribution is normal, then 1/v would not be significantly different from zero, since the student-*t* approaches the normal when v tends to infinity.

indicator variables in the mean and variance equations increases the in-sample predictive power and reduces the excess kurtosis of the standardised residuals[6].

4. CONDITIONAL VOLATILITY WITH TRADING VOLUME

In this section, we examine the significance of trading volume on conditional volatility estimates. Given the initial results in Section 2, our analysis will focus on both trading volume levels as well as changes in volume.

4.1 GARCH and Trading Volume Levels

Following Lamoureux and Lastrapes (1990b) and Najand and Yung (1991), we incorporate the contemporaneous trading volume level into the conditional variance equation. Specifically, the conditional variance of Equations (9b) and (10b) becomes

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 V_t \quad \textbf{(13a)}$$

$$\begin{aligned} h_t = {} & \sum_{i=1}^{5} \beta_i \mathrm{DAY}_{it} + \sum_{j=0}^{2} \gamma_j \mathrm{HOL}_{j,t+j-1} + \\ & \sum_{k=0}^{2} \eta_k \mathrm{SET}_{k,t+k-1} + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 V_t \end{aligned} \quad \textbf{(13b)}$$

respectively, where V_t is the contemporaneous trading volume level at day t.

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[6] The reduction in the excess kurtosis would suggest that the fat-tails of the distribution are generated in part by omitted variables. In a sense, if the objective of using conditional volatility models is to undo the leptokurtosis as much as possible (see Nelson, 1992b), a more appropriate specification would be one that includes the omitted variables.

Table 4A reports the parameters of interest. It appears that ARCH effects and persistence in volatility remains despite the addition of the volume variable in the volatility equation. The volume parameter is also not statistically significant. This result is inconsistent with Lamoureux and Lastrapes (1990b), but is consistent with the findings of Najand and Yung (1991).

We repeat the analysis next with lagged volume. The conditional variance equation is given by

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 V_{t-1} \tag{14a}$$

$$h_t = \sum_{i=1}^{5} \beta_i \text{DAY}_{it} + \sum_{j=0}^{2} \gamma_j \text{HOL}_{j,t+j-1} + \sum_{k=0}^{2} \eta_k \text{SET}_{k,t+k-1} + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 V_{t-1} \tag{14b}$$

The results are reported in Table 4B. Consistent with Najand and Yung (1991), lagged volume has significant explanatory power on conditional volatility. However looking at α_3, it would appear that lagged volume is negatively related to volatility i.e. an increase in current trading volume levels will reduce future volatility! This appears to be inconsistent with the positive lagged volume coefficient in Najand and Yung (1991). It is also inconsistent with current literature documenting a positive volume-volatility relationship (e.g. Gallant, Rossi and Tauchen, 1992, Karpoff, 1987, and Najand and Yung, 1991).

One explanation for our results lies in the trading volume variable that we used. As mentioned earlier, we perform a log transformation to reduce heteroskedasticity and remove low frequency variation. We can think of two other problems plaguing this series. First, there may be a trend, even in the log-volume series. Second, we have not accounted for intraweekly patterns in the volume, and this may give rise to the problem of spurious correlation (see for example, Harvey, 1990).

We solve this problem by forming a detrended volume series, $\tilde{V}_t$. This is done by first forming a 100-day backward moving average of the actual volume series before performing a log transformation[7] i.e.

$$\tilde{V}_t = \log\left(\frac{\text{VOL}_t}{\frac{1}{100}\sum_{i=1}^{100}\text{VOL}_{t-i}}\right) \tag{15}$$

where VOL_t is the actual volume series on day t. Using $\tilde{V}_t$ instead of V_t, we reestimate the GARCH model with the following conditional variance specification using both a normal and student-t distribution:

$$h_t = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3\tilde{V}_t \tag{16a}$$

$$h_t = \sum_{i=1}^{5}\beta_i\text{DAY}_{it} + \sum_{j=0}^{2}\gamma_j\text{HOL}_{j,t+j-1} + \sum_{k=0}^{2}\eta_k\text{SET}_{k,t+k-1} + \alpha_1\epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3\tilde{V}_t \tag{16b}$$

$$h_t = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3\tilde{V}_{t-1} \tag{17a}$$

$$h_t = \sum_{i=1}^{5}\beta_i\text{DAY}_{it} + \sum_{j=0}^{2}\gamma_j\text{HOL}_{j,t+j-1} + \sum_{k=0}^{2}\eta_k\text{SET}_{k,t+k-1} + \alpha_1\epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3\tilde{V}_{t-1} \tag{17b}$$

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[7] This and similar techniques are commonly used in previous studies, such as Antoniewicz (1992), Campbell, Grossman and Wang (1994), and Schwert (1989).

The results for contemporaneous detrended volume and lagged detrended volume are reported in Tables 5A and 5B respectively. Together with Tables 4A and 4B, the following observations and conclusions can be drawn:

1) The coefficient for contemporaneous detrended volume is strongly significant for a conditional normal distribution, but contemporaneous log-volume is negative and insignificant. This is consistent with the findings of Lamoureux and Lastrapes (1990b), except that in our case, ARCH effects and volatility persistence are not reduced. On the other hand, the coefficient for lagged detrended volume is negative and insignificant, but negative and significant for lagged log-volume. Taken together, this suggests that the significance of both contemporaneous and lagged volume on conditional volatility may be driven by some other factors, such as calendar effects. This explanation can also reconcile the different findings of Lamoureux and Lastrapes (1990b) and Najand and Yung (1991). Our findings of a negative but insignificant lagged detrended volume coefficient is consistent with Bollerslev and Domowitz (1993), who also find a negative but economically and statistically insignificant relationship between market activity (proxied by the number of quote arrivals per period) and conditional volatility in the deutschemark-dollar currency market[8].

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[8] A negative relationship between volume and volatility is not implausible. If volume is an indication of liquidity, and if higher liquidity implies lower volatility, then higher trading volume will be associated with lower volatility.

2) The significance of the volume parameter appears to be affected by the choice of the conditional distribution. Baillie and DeGennaro (1990) and Pagan and Ullah (1988) have documented that the choice of the conditional distribution affects the significance of the parameters in the mean equation. It would therefore appears that the choice of the conditional distribution also affects the variables in the variance equation.

3) The omission of the indicator variables also appears to be affecting the significance of the volume parameter. Given that patterns associated with calendar effects can be seen for both volatility and volume, the significance of α_3 in specifications without incorporating the calendar effects is likely a result of spurious correlations. This can also explain why log-volume is showing a higher significance than detrended volume.

4) Another indication why detrended volume is better than log-volume is that the excess kurtosis of the standardised residuals from models incorporating log-volume (both contemporaneous and lagged) is higher than those incorporating detrended volume. This follows from the theoretical results of Nelson (1992b), who shows that adding noise to the information set of a conditional volatility model thickens the tails of the standardised residuals.

In summary, the results in this subsection suggest that volume is (at best) weakly related to conditional volatility. If information is exogenous, and volume is a proxy to rate of information arrival, this would suggest that the evidence here does not support the MDH.

4.2 GARCH and Changes in Trading Volume

Our initial results in Section 2 suggest that volatility is more strongly related to changes in trading volume, and we have suggested a scenario in which such a relationship is likely. To explore this hypothesis, we use both contemporaneous and lagged changes in detrended volume i.e.

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \left(\tilde{V}_t - \tilde{V}_{t-1} \right) \quad \textbf{(18a)}$$

$$\begin{aligned} h_t = & \sum_{i=1}^{5} \beta_i \mathrm{DAY}_{it} + \sum_{j=0}^{2} \gamma_j \mathrm{HOL}_{j,t+j-1} + \sum_{k=0}^{2} \eta_k \mathrm{SET}_{k,t+k-1} \\ & + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \left(\tilde{V}_t - \tilde{V}_{t-1} \right) \end{aligned} \quad \textbf{(18b)}$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \left(\tilde{V}_{t-1} - \tilde{V}_{t-2} \right) \quad \textbf{(19a)}$$

$$\begin{aligned} h_t = & \sum_{i=1}^{5} \beta_i \mathrm{DAY}_{it} + \sum_{j=0}^{2} \gamma_j \mathrm{HOL}_{j,t+j-1} + \sum_{k=0}^{2} \eta_k \mathrm{SET}_{k,t+k-1} \\ & + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \left(\tilde{V}_{t-1} - \tilde{V}_{t-2} \right) \end{aligned} \quad \textbf{(19b)}$$

The GARCH estimates are reproduced in Tables 6A and 6B. Contemporaneous changes in detrended volume is strongly significant in explaining conditional volatility[9], but lagged changes in detrended volume has no explanatory power. The

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[9] We also used a variety of other change in volume measures, such as change in log-volume and residuals from a linearly filtered log-volume series, as well as changing the moving average window. The results do not appear to differ either qualitatively or quantitatively. Note that the linearly filtered log-volume series is analogous to Bessembinder and Seguin (1992, 1993)'s technique of partitioning the volume into expected and unexpected components. In a sense, our definition of change in volume is similar to their definition of surprise volume.

kurtosis of the standardised residuals from the GARCH model with contemporaneous volume is not significantly different from normal, and the $1/v$ parameter is not statistically different from zero (Table 6A, Columns 4 and 5), suggesting that *changes* in trading volume is useful information. Incorporating contemporaneous changes in volume in the volatility equation also increases the in-sample predictive performance of the model, in some cases by almost three times (Table 6A, columns 4 and 5).

If information arrival is an exogenous process, the results above show that changes in volume contains valuable information about the pricing dynamics of assets. This information is not fully captured in GARCH models, which also appear to capture some other information not reflected in the volume. On the other hand, if information is endogenously generated, the above analysis may be subject to some specification bias. We recognise that the results of using contemporaenous volume (levels and changes) may be subjected to simultaneity bias. Indeed if anything, they indicate that the volatility-volume relationship is much more complex than is often thought, and that price volatility and trading volume is jointly determined. Nevertheless, we believe that the above econometric specifications help to uncover the partial relations between trading volume and volatility.

5. EXTENSION OF GARCH - THE VOLUME-GARCH MODEL

A major success of ARCH models is its ability to forecast future volatility. To forecast the volatility of period t, information is only required of those variables that are available or measurable at time t-1. This differs from some models which need to use variables measured at period t, which in themselves may not be available. This would therefore suggest that despite good (in-sample) prediction, the conditional variance specifications of Equations (18a) and (18b) are infeasible, since the variable $\tilde{V}_t$ is not in the information set of the investor at time t-1.

To make the model useful, Equations (18a) and (18b) must be modified in such a way that contains only variables measurable at time t-1. Simply substituting $\tilde{V}_{t-1}$ for $\tilde{V}_t$ is not useful, since we have already shown the lack of predictive power of lagged volume (Table 6B). Instead, our modified conditional variance should be

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \left(\tilde{V}_{t-1} - \tilde{V}_{t-2}\right) \epsilon_{t-1}^2 \tag{20a}$$

and

$$\begin{aligned} h_t = & \sum_{i=1}^{5} \beta_i \mathrm{DAY}_{it} + \sum_{j=0}^{2} \gamma_j \mathrm{HOL}_{j,t+j-1} + \sum_{k=0}^{2} \eta_k \mathrm{SET}_{k,t+k-1} \\ & + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \left(\tilde{V}_{t-1} - \tilde{V}_{t-2}\right) \epsilon_{t-1}^2 \end{aligned} \tag{20b}$$

The last term in Equations (20a) and (20b) attempt to capture the joint volatility volume relationship at time t-1. We have suggested earlier that the simple GARCH model fails to capture the complex volatility-volume relationship, and this is one way of incorporating the contemporaneous (at time t-1) behaviour. This specification is in a sense, supported by the results of Gallant, Rossi and Tauchen (1993), who document joint price and volume shocks.

In Table 7A, we report the results of our estimations using the conditional variance specifications of Equations (20a) and (20b). The results are encouraging. The interaction between changes in volume and volatility shocks is statistically significant. In-sample predictive performance (both R^2 and R^2-for-logs) show some improvements with the additional parameter. The kurtosis of the standardised residuals is also reduced, indicating that the additional term contains useful information.

There is also empirical evidence showing some relationship between price change *per se* and volume. We capture this with the following specifications:

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \left(\tilde{V}_{t-1} - \tilde{V}_{t-2}\right) \epsilon_{t-1} \qquad \textbf{(21a)}$$

$$\begin{aligned} h_t = {} & \sum_{i=1}^{5} \beta_i \mathrm{DAY}_{it} + \sum_{j=0}^{2} \gamma_j \mathrm{HOL}_{j,t+j-1} + \sum_{k=0}^{2} \eta_k \mathrm{SET}_{k,t+k-1} \\ & + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \left(\tilde{V}_{t-1} - \tilde{V}_{t-2}\right) \epsilon_{t-1} \end{aligned} \qquad \textbf{(21b)}$$

Table 7B reports the results of the specifications of Equations (21a) and (21b). Again the interaction between volume and error term is statistically significant, and R^2 and R^2-for-logs is also higher than without the interaction term (i.e. Table 3).

Finally, we combine both types of interaction terms to the GARCH model to give the following conditional variance specifications:

$$\begin{aligned} h_t = {} & \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \left(\tilde{V}_{t-1} - \tilde{V}_{t-2}\right) \epsilon_{t-1}^2 + \\ & \alpha_4 \left(\tilde{V}_{t-1} - \tilde{V}_{t-2}\right) \epsilon_{t-1} \end{aligned} \qquad \textbf{(22a)}$$

$$h_t = \sum_{i=1}^{5} \beta_i \text{DAY}_{it} + \sum_{j=0}^{2} \gamma_j \text{HOL}_{j,t+j-1} + \sum_{k=0}^{2} \eta_k \text{SET}_{k,t+k-1} + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \left(\tilde{V}_{t-1} - \tilde{V}_{t-2}\right) \epsilon_{t-1}^2 + \alpha_4 \left(\tilde{V}_{t-1} - \tilde{V}_{t-2}\right) \epsilon_{t-1} \quad \textbf{(22b)}$$

Comparing Table 7C with Table 3, we find that the additional terms are significant (both in terms of log-likelihood values and in terms of *t*-statistics). R^2 and R^2-for-logs is much higher, and the standardised residuals is less fat-tailed. This suggests that the interaction terms have valuable information on the conditional volatility of future returns, and is further evidence of a joint volatility-volume effect.

6. SUMMARY AND CONCLUSION

In this paper, we investigate the empirical relationship between daily trading volume and stock returns volatility of the FTSE100 index. Using a dynamic volatility model, we find that change in volume is more strongly related to conditional volatility than volume itself. The use of contemporaneous change in trading volume in the GARCH model effectively reduces the excess kurtosis of the standardised residuals, but it does not affect either the ARCH effects or volatility persistence. This suggests that changes in trading volume possess valuable information about the distribution of the asset returns, and this information is not fully reflected in ARCH models. A further implication is that volume and volatility is jointly determined. We suggest the incorporation of the lagged joint volatility-volume effects into the standard GARCH model using simple interaction terms. We find that the interaction terms possess explanatory power regarding the conditional variance, further confirming that there is a contemporaneous relationship between volatility and volume.

TABLE 1

Sample statistics of FTSE100 returns and log-volume for period January 1989 to December 1993.
t-statistics given in brackets [ ], p-values given in parenthesis ( ).

	Returns	Volume
Mean	0.0522 [2.1792]	12.0966 [11187.1350]
Variance	0.7229	0.1309
Skewness	0.2423 (0.0004)	0.0038 (0.9557)
Excess Kurtosis	2.7004 (0.0000)	0.7870 (0.0000)
ρ_1	0.0748	0.6353
ρ_2	-0.0010	0.5265
ρ_3	0.0177	0.4851
ρ_4	0.0660	0.4931
ρ_5	0.0124	0.5290
Q(10)	22.0313 (0.0149)	3119.0484 (0.0000)
$Q^2(10)$	50.2977 (0.0000)	3142.7971 (0.0000)

Note: ρ_k denote the *k*-lag autocorrelation. Q(10) and $Q^2(10)$ denote the 10-lag Ljung-Box statistics for the returns and squared returns respectively.

TABLE 2

Results of regressing log-volume against absolute returns
and changes in log-volume against absolute returns of FTSE100
for the period January 1989 to December 1990.

$$V_t = b_0 + b_1 |r_t| + b_2 D_t |r_t| \quad (1)$$

$$(V_t - V_{t-1}) = b_0 + b_1 |r_t| + b_2 D_t |r_t| \quad (2)$$

Asymptotic t-statistics given in brackets [ ],

White (1980) t-statistics given in parenthesis ( ),

Newey-West (1987) t-statistics given in braces { }.

Dependent Variable	b_0	b_1	b_2	R^2
V_t	12.0341 (769.5893) [688.7170] {461.1501}	0.1379 (6.6808) [5.0974] {4.2964}	-0.0914 (-3.8420) [-3.4607] {-3.0257}	0.0348
$(V_t - V_{t-1})$	-0.0814 (-6.1921) [-6.0271] {-7.0114}	0.1612 (9.2849) [8.8555] {9.1288}	-0.7817 (-3.9089) [-3.5858] {-3.7104}	0.0642

Note: V_t denotes the log trading volume of the FTSE100 on day t.
r_t denote the (log) returns of FTSE100 on day t.
$D_t = 1$ if $r_t < 0$, and zero otherwise.

TABLE 3

GARCH(1,1) estimates of FTSE100 returns for period January 1989 to December 1993.
Asymptotic *t*-statistics given in brackets [ ], p-values given in parenthesis ( ).
See Section 3 for the different specifications estimated.

Parameters	Equations 9a, 9b, 9c	Equations 9a, 9b, 9d	Equations 10a, 10b, 10c	Equations 10a, 10b, 10d
a_1	0.0707 [2.2618]	0.0581 [1.9420]	0.0638 [1.9286]	0.0526 [1.7446]
α_1	0.0776 [5.1630]	0.0588 [3.0871]	0.0722 [4.4447]	0.0554 [3.1911]
α_2	0.8657 [32.2094]	0.8933 [23.9218]	0.8806 [30.7846]	0.9020 [27.8027]
$1/\nu$		0.0950 [4.8406]		0.0894 [4.1397]
Log-likelihood	-1546.25	-1530.29	-1523.93	-1511.88
R^2	0.0279	0.0275	0.0369	0.0356
R^2-for-logs	0.0144	0.0148	0.0242	0.0234
Skewness	0.0927 (0.1797)	0.1216 (0.0783)	0.1600 (0.0205)	0.1964 (0.0045)
Excess Kurtosis	1.5488 (0.0000)	1.6846 (0.0000)	1.0956 (0.0000)	1.4308 (0.0000)
Q(10)	13.4689 (0.1425)	13.5911 (0.1376)	10.4234 (0.3173)	11.1265 (0.2671)
$Q^2(10)$	4.2223 (0.9368)	3.8708 (0.9530)	4.9796 (0.8925)	4.2535 (0.9352)

Notes: R^2 denotes the R-square for the regression of Equation (11).
R^2-for-logs dentes the R-square for the regression of Equation (12).
Q(10) and $Q^2(10)$ denote the 10-lag Ljung-Box statistics for the standardised residuals and squared standardised residuals respectively.

TABLE 4A

GARCH(1,1) with contemporaneous log-volume estimates of FTSE100 returns for period January 1989 to December 1993.
Asymptotic *t*-statistics given in brackets [ ], p-values given in parenthesis ( ).
See Section 4 for the different specifications estimated.

Parameters	Equations 9a, 13a, 9c	Equations 9a, 13a, 9d	Equations 10a, 13b, 10c	Equations 10a, 13b, 10d
a_1	0.0732 [2.2949]	0.0595 [2.0240]	0.0665 [2.0284]	0.0534 [1.8123]
α_1	0.0702 [4.9166]	0.0456 [2.8044]	0.0617 [4.2359]	0.0426 [2.9517]
α_2	0.8776 [32.6581]	0.9119 [26.0672]	0.8951 [33.1404]	0.9186 [31.0427]
α_3	-0.0129 [-1.1557]	-0.0230 [-1.5783]	-0.0179 [-1.5846]	-0.0214 [-1.6374]
$1/\nu$		0.0950 [4.8406]		0.0939 [4.4093]
Log-likelihood	-1545.73	-1527.70	-1522.63	-1509.26
R^2	0.0277	0.0258	0.0355	0.0323
R^2-for-logs	0.0184	0.0210	0.0256	0.0265
Skewness	0.1179 (0.0877)	0.1841 (0.0077)	0.1937 (0.0050)	0.2441 (0.0004)
Excess Kurtosis	1.6696 (0.0000)	1.9878 (0.0000)	1.1689 (0.0000)	1.6282 (0.0000)
Q(10)	13.0033 (0.1625)	12.7920 (0.1722)	9.9397 (0.3554)	10.8103 (0.2889)
$Q^2(10)$	4.1052 (0.9425)	4.2041 (0.9377)	5.0504 (0.8878)	5.2355 (0.8749)

Notes: R^2 denotes the R-square for the regression of Equation (11).
R^2-for-logs dentes the R-square for the regression of Equation (12).
Q(10) and $Q^2(10)$ denote the 10-lag Ljung-Box statistics for the standardised residuals and squared standardised residuals respectively.

TABLE 4B

GARCH(1,1) with lagged log-volume estimates of FTSE100 returns
for period January 1989 to December 1993.
Asymptotic *t*-statistics given in brackets [ ], p-values given in parenthesis ( ).
See Section 4 for the different specifications estimated.

Parameters	Equations 9a, 14a, 9c	Equations 9a, 14a, 9d	Equations 10a, 14b, 10c	Equations 10a, 14b, 10d
a_1	0.0752 [2.3819]	0.0592 [1.9948]	0.0668 [2.0401]	0.0530 [1.7726]
α_1	0.0689 [4.9210]	0.0516 [2.7801]	0.0646 [4.1538]	0.0505 [2.8603]
α_2	0.8704 [33.2454]	0.8910 [22.0882]	0.8951 [30.2601]	0.8945 [24.4455]
α_3	-0.0340 [-2.7736]	-0.0377 [-2.0145]	-0.0364 [-2.6817]	-0.0370 [-2.0900]
$1/v$		0.0979 [5.0327]		0.0939 [4.4093]
Log-likelihood	-1542.61	-1525.29	-1519.13	-1506.69
R^2	0.0293	0.0284	0.0373	0.0354
R^2-for-logs	0.0227	0.0234	0.0276	0.0295
Skewness	0.1326 (0.0549)	0.1721 (0.0127)	0.2060 (0.0028)	0.2384 (0.0006)
Excess Kurtosis	1.6639 (0.0000)	1.8387 (0.0000)	1.1157 (0.0000)	1.4806 (0.0000)
Q(10)	12.4709 (0.1880)	12.6533 (0.1779)	9.7350 (0.3724)	10.7080 (0.2963)
$Q^2(10)$	4.3591 (0.9297)	4.4033 (0.9273)	5.6010 (0.8476)	5.6471 (0.8440)

Notes: R^2 denotes the R-square for the regression of Equation (11).
R^2-for-logs dentes the R-square for the regression of Equation (12).
Q(10) and $Q^2(10)$ denote the 10-lag Ljung-Box statistics for the standardised residuals and squared standardised residuals respectively.

TABLE 5A

GARCH(1,1) with contemporaneous detrended volume estimates of FTSE100 returns for period January 1989 to December 1993.
Asymptotic t-statistics given in brackets [ ], p-values given in parenthesis ( ).
See Section 4 for the different specifications estimated.

Parameters	Equations 9a, 16a, 9c	Equations 9a, 16a, 9d	Equations 10a, 16b, 10c	Equations 10a, 16b, 10d
a_1	0.0654 [2.0511]	0.0570 [1.8857]	0.0332 [1.5205]	0.0500 [1.6337]
α_1	0.0939 [4.8425]	0.0636 [3.0956]	0.0936 [4.1584]	0.0634 [3.1460]
α_2	0.7850 [23.5139]	0.8766 [21.4511]	0.7777 [17.5141]	0.8774 [22.8590]
α_3	0.1018 [3.7740]	0.0206 [0.7072]	0.1445 [3.7220]	0.0332 [1.0312]
$1/v$		0.0914 [4.4062]		0.0837 [3.6839]
Log-likelihood	-1542.38	-1530.06	-1518.96	-1511.53
R^2	0.0334	0.0291	0.0446	0.0383
R^2-for-logs	0.0072	0.0122	0.0221	0.0216
Skewness	0.0404 (0.5589)	0.1056 (0.1264)	0.0842 (0.2225)	0.1653 (0.0167)
Excess Kurtosis	1.1013 (0.0000)	1.5189 (0.0000)	0.7503 (0.0000)	1.2143 (0.0000)
Q(10)	14.1928 (0.1156)	13.8478 (0.1278)	11.2414 (0.2595)	11.3180 (0.2545)
$Q^2(10)$	6.3646 (0.7838)	4.1445 (0.9406)	7.1646 (0.7098)	4.5899 (0.9168)

Notes: R^2 denotes the R-square for the regression of Equation (11).
R^2-for-logs dentes the R-square for the regression of Equation (12).
Q(10) and $Q^2(10)$ denote the 10-lag Ljung-Box statistics for the standardised residuals and squared standardised residuals respectively.

TABLE 5B

GARCH(1,1) with lagged detrended volume estimates of FTSE100 returns for period January 1989 to December 1993.
Asymptotic *t*-statistics given in brackets [ ], p-values given in parenthesis ( ).
See Section 4 for the different specifications estimated.

Parameters	Equations 9a, 17a, 9c	Equations 9a, 17a, 9d	Equations 10a, 17b, 10c	Equations 10a, 17b, 10d
a_1	0.0717 [2.2668]	0.0587 [1.9702]	0.0653 [1.9729]	0.0534 [1.7925]
α_1	0.0771 [5.1879]	0.0586 [3.1336]	0.0716 [4.4576]	0.0528 [3.2868]
α_2	0.8689 [33.2526]	0.8979 [25.3915]	0.8857 [32.1355]	0.9128 [31.5512]
α_3	-0.0200 [-0.8032]	-0.0316 [-1.1563]	-0.0336 [-1.2777]	-0.0441 [-1.6352]
$1/v$		0.0962 [4.9363]		0.0917 [4.3174]
Log-likelihood	-1545.94	-1529.38	-1523.14	-1510.25
R^2	0.0282	0.0278	0.0378	0.0359
R^2-for-logs	0.0157	0.0166	0.0265	0.0268
Skewness	0.0962 (0.1636)	0.1281 (0.0636)	0.1751 (0.0112)	0.2227 (0.0013)
Excess Kurtosis	1.5886 (0.0000)	1.7654 (0.0000)	1.1403 (0.0000)	1.5507 (0.0000)
Q(10)	13.2892 (0.1500)	13.3010 (0.1495)	10.2702 (0.3291)	10.9850 (0.2767)
$Q^2(10)$	4.1239 (0.9416)	3.8398 (0.9543)	4.9659 (0.8934)	4.4477 (0.9249)

Notes: R^2 denotes the R-square for the regression of Equation (11).
R^2-for-logs dentes the R-square for the regression of Equation (12).
Q(10) and $Q^2(10)$ denote the 10-lag Ljung-Box statistics for the standardised residuals and squared standardised residuals respectively.

TABLE 6A

GARCH(1,1) with contemporaneous change in detrended volume estimates of FTSE100 returns for period January 1989 to December 1993.
Asymptotic *t*-statistics given in brackets [ ], p-values given in parenthesis ( ).
See Section 4 for the different specifications estimated.

Parameters	Equations 9a, 18a, 9c	Equations 9a, 18a, 9d	Equations 10a, 18b, 10c	Equations 10a, 18b, 10d
a_1	0.0645 [2.1882]	0.0559 [1.9136]	0.0323 [1.0464]	0.0279 [0.8835]
α_1	0.0325 [4.5036]	0.0306 [3.5793]	0.0513 [3.3548]	0.0607 [3.2363]
α_2	0.9511 [86.1252]	0.9520 [69.7673]	0.9014 [34.4485]	0.8649 [25.5976]
α_3	0.3153 [11.0705]	0.3098 [8.0850]	0.8356 [11.0253]	0.8703 [9.7665]
$1/v$		0.0632 [3.2825]		0.0069 [0.3636]
Log-likelihood	-1518.59	-1512.26	-1465.02	-1464.25
R^2	0.0573	0.0581	0.1076	0.1134
R^2-for-logs	0.0276	0.0284	0.0486	0.0479
Skewness	0.0504 (0.4654)	0.0567 (0.4112)	-0.0574 (0.4062)	-0.0571 (0.4085)
Excess Kurtosis	0.8102 (0.0000)	0.8387 (0.0000)	0.0802 (0.5623)	0.0782 (0.5721)
Q(10)	12.3278 (0.1955)	12.4911 (0.1870)	10.1154 (0.3412)	10.2527 (0.3304)
$Q^2(10)$	6.7549 (0.7484)	7.1958 (0.7068)	3.9363 (0.9502)	4.1634 (0.9397)

Notes: R^2 denotes the R-square for the regression of Equation (11).
R^2-for-logs dentes the R-square for the regression of Equation (12).
Q(10) and $Q^2(10)$ denote the 10-lag Ljung-Box statistics for the standardised residuals and squared standardised residuals respectively.

TABLE 6B

GARCH(1,1) with lagged change in detrended volume estimates of FTSE100 returns for period January 1989 to December 1993.
Asymptotic *t*-statistics given in brackets [ ], p-values given in parenthesis ( ).
See Section 4 for the different specifications estimated.

Parameters	Equations 9a, 19a, 9c	Equations 9a, 19a, 9d	Equations 10a, 19b, 10c	Equations 10a, 19b, 10d
a_1	0.0702 [2.2375]	0.0579 [1.9335]	0.0630 [1.8913]	0.0524 [1.7292]
α_1	0.0759 [4.9374]	0.0585 [3.0409]	0.0694 [4.0015]	0.0546 [3.0151]
α_2	0.8693 [32.1190]	0.8939 [23.7971]	0.8847 [29.5510]	0.9032 [31.5512]
α_3	0.0840 [0.9016]	0.0278 [0.2307]	0.0873 [0.6930]	0.0505 [0.3544]
1/v		0.0944 [4.8159]		0.0889 [4.1336]
Log-likelihood	-1545.88	-1530.26	-1523.75	-1511.81
R^2	0.0285	0.0278	0.0376	0.0361
R^2-for-logs	0.0141	0.0144	0.0227	0.0228
Skewness	0.0966 (0.1617)	0.1220 (0.0773)	0.1601 (0.0204)	0.1943 (0.0049)
Excess Kurtosis	1.5244 (0.0000)	1.6733 (0.0000)	1.0947 (0.0000)	1.4290 (0.0000)
Q(10)	13.2350 (0.1523)	13.5040 (0.1411)	10.3046 (0.3264)	11.0442 (0.2727)
$Q^2(10)$	4.1676 (0.9395)	3.8251 (0.9549)	4.9138 (0.8969)	4.0984 (0.9428)

Notes: R^2 denotes the R-square for the regression of Equation (11).
R^2-for-logs dentes the R-square for the regression of Equation (12).
Q(10) and $Q^2(10)$ denote the 10-lag Ljung-Box statistics for the standardised residuals and squared standardised residuals respectively.

TABLE 7A

Specification 1 of Volume-GARCH(1,1) estimates of FTSE100 returns
for period January 1989 to December 1993.
Asymptotic *t*-statistics given in brackets [ ], p-values given in parenthesis ( ).
See Section 4 for the different specifications estimated.

Parameters	Equations 9a, 20a, 9c	Equations 9a, 20a, 9d	Equations 10a, 20b, 10c	Equations 10a, 20b, 10d
a_1	0.0671 [2.1994]	0.0567 [1.9419]	0.0598 [1.9074]	0.0518 [1.7664]
α_1	0.0894 [5.3454]	0.0685 [3.1720]	0.0843 [4.7363]	0.0661 [3.3141]
α_2	0.8836 [34.0278]	0.9115 [28.6676]	0.8946 [34.5002]	0.9140 [32.1459]
α_3	-0.0883 [-4.1251]	-0.0665 [-2.5108]	-0.0810 [-3.6248]	-0.0608 [-2.3879]
$1/\nu$		0.0915 [4.5953]		0.0841 [3.7803]
Log-likelihood	-1540.84	-1526.56	-1518.65	-1508.56
R^2	0.0345	0.0330	0.0412	0.0392
R^2-for-logs	0.0187	0.0215	0.0273	0.0242
Skewness	0.0828 (0.2306)	0.1103 (0.1102)	0.1383 (0.0452)	0.1679 (0.0150)
Excess Kurtosis	1.4181 (0.0000)	1.5585 (0.0000)	0.9578 (0.0000)	1.2679 (0.0000)
Q(10)	12.6204 (0.1805)	12.8832 (0.1680)	8.9936 (0.4379)	10.0731 (0.3446)
$Q^2(10)$	3.5832 (0.9642)	3.6723 (0.9609)	4.5323 (0.9202)	3.9993 (0.9474)

Notes: R^2 denotes the R-square for the regression of Equation (11).
R^2-for-logs dentes the R-square for the regression of Equation (12).
Q(10) and $Q^2(10)$ denote the 10-lag Ljung-Box statistics for the standardised residuals and squared standardised residuals respectively.

TABLE 7B

Specification 2 of Volume-GARCH(1,1) estimates of FTSE100 returns
for period January 1989 to December 1993.
Asymptotic t-statistics given in brackets [ ], p-values given in parenthesis ( ).
See Section 4 for the different specifications estimated.

Parameters	Equations 9a, 21a, 9c	Equations 9a, 21a, 9d	Equations 10a, 21b, 10c	Equations 10a, 21b, 10d
a_1	0.0682 [2.2032]	0.0568 [1.9020]	0.0540 [1.6817]	0.0484 [1.6124]
α_1	0.0797 [4.8434]	0.0651 [3.2022]	0.0699 [4.2638]	0.0607 [3.2234]
α_2	0.8604 [30.9125]	0.8776 [22.6372]	0.8823 [29.7846]	0.8907 [25.3616]
α_3	-0.1663 [-3.7289]	-0.1387 [-2.2947]	-0.1545 [-3.4960]	-0.1292 [-2.3929]
$1/v$		0.0917 [4.6467]		0.0842 [3.8644]
Log-likelihood	-1542.05	-1527.38	-1518.57	-1508.86
R^2	0.0322	0.0321	0.0406	0.0397
R^2-for-logs	0.0188	0.0182	0.0298	0.0236
Skewness	0.0734 (0.2881)	0.0951 (0.1683)	0.1659 (0.0163)	0.1830 (0.0081)
Excess Kurtosis	1.4297 (0.0000)	1.5086 (0.0000)	0.9771 (0.0000)	1.2247 (0.0000)
Q(10)	13.4438 (0.1435)	13.6677 (0.1346)	10.1328 (0.3398)	11.0216 (0.2742)
$Q^2(10)$	4.3547 (0.9299)	4.0117 (0.9468)	5.5931 (0.8482)	4.7772 (0.9056)

Notes: R^2 denotes the R-square for the regression of Equation (11).
R^2-for-logs dentes the R-square for the regression of Equation (12).
Q(10) and $Q^2(10)$ denote the 10-lag Ljung-Box statistics for the standardised residuals and squared standardised residuals respectively.

TABLE 7C

Specification 3 of Volume-GARCH(1,1) estimates of FTSE100 returns
for period January 1989 to December 1993.
Asymptotic *t*-statistics given in brackets [ ], p-values given in parenthesis ( ).
See Section 4 for the different specifications estimated.

Parameters	Equations 9a, 22a, 9c	Equations 9a, 22a, 9d	Equations 10a, 22b, 10c	Equations 10a, 22b, 10d
a_1	0.0660 [2.1246]	0.0561 [1.8955]	0.0562 [1.7713]	0.0488 [1.6518]
α_1	0.0925 [4.7723]	0.0730 [3.1032]	0.0822 [4.1973]	0.0692 [3.2028]
α_2	0.8710 [30.4007]	0.8979 [25.2740]	0.8906 [30.7029]	0.9049 [28.6012]
α_3	-0.0781 [-3.0982]	-0.0612 [-2.1957]	-0.0676 [-2.6161]	-0.0560 [-2.0464]
α_4	-0.1039 [-2.3854]	-0.0846 [-1.7041]	-0.1038 [-2.4195]	-0.0920 [-1.9597]
$1/v$		0.0895 [4.5140]		0.0804 [3.6687]
Log-likelihood	-1538.31	-1524.68	-1515.36	-1506.04
R^2	0.0368	0.0361	0.0437	0.0423
R^2-for-logs	0.0215	0.0232	0.0276	0.0254
Skewness	0.0801 (0.2435)	0.1034 (0.1342)	0.1549 (0.0249)	0.1719 (0.0128)
Excess Kurtosis	1.3560 (0.0000)	1.4553 (0.0000)	0.9256 (0.0000)	1.1480 (0.0000)
Q(10)	12.9869 (0.1632)	13.1851 (0.1544)	8.9039 (0.4462)	9.9173 (0.3572)
$Q^2(10)$	3.8363 (0.9544)	3.6744 (0.9608)	4.9266 (0.8960)	4.2786 (0.9339)

Notes: R^2 denotes the R-square for the regression of Equation (11).
R^2-for-logs dentes the R-square for the regression of Equation (12).
Q(10) and $Q^2(10)$ denote the 10-lag Ljung-Box statistics for the standardised residuals and squared standardised residuals respectively.

CHAPTER 5

A TEST FOR OMITTED DETERMINISTIC DUMMY VARIABLES IN ARCH MODELS

Abstract

This paper proposes a simple diagnostic test for deterministic dummy variables in ARCH models. Deterministic dummy variables are indicator variables that are included in either the conditional mean or conditional variance equation to capture some of the *a priori* observed anomalies/effects of financial data. Notable examples are day-of-the-week effects in returns, and observed increased in volatility at certain calendar periods. In the current literature, such effects are either ignored, or captured with ad-hoc adjustments. Several commonly used methods in current empirical work are applied to model intraday returns and volatility of the S&P500 index futures. We find that the traditional diagnostic tests fail to detect omitted determinstic effects associated with nontrading, even when such effects are ignored. We also find that none of the currently employed methods of adjustments are sufficient to account for the pattern surrounding a nontrading period. Using dummy variables in the GARCH, we find that conditional volatility tends to increase prior to market closure, and across the nontrading period. However when the market reopens, the conditional volatility declines. This suggest that the volatility structure surrounding a nontrading period is far more complicated than what current empirical adjustments allow for.

A TEST FOR OMITTED DETERMINISTIC DUMMY VARIABLES IN ARCH MODELS

1. INTRODUCTION

As early as the sixties, researchers have documented that the variances and covariances of returns on speculative assets change over time (see Fama, 1965 and Mandelbrot, 1963). The observed phenomena of fat-tailed (unconditional) distribution (leptokurtosis), non-stationary variances (time-varying volatility) and large (small) changes following large (small) changes (volatility clustering) represent "stylised facts" (coined by Mandelbrot) for many financial and economic time series. Initial attempts to model such time series focus on the unconditional distribution. It wasn't until the seminal paper by Engle (1982) that the focus shifts to modelling the conditional distribution. Since then, there has been an explosion of literature on Autoregressive Conditional Heteroskedasticity (ARCH) models (see survey paper by Bollerslev *et. al.*, 1992).

Despite their widespread growth, ARCH models are essentially developed more as econometric and modelling tools, than being motivated by any economic and/or asset pricing theories. Only recently have there been some serious theoretical attempts to explain the observed ARCH effects in financial data.

One major problem of applying ARCH models in empirical modelling of financial data is that the data is often 'contaminated'. Two well-known anomalies are calender effects and nontrading periods. The latter becomes especially important when the frequency of the data set increases, since the ratio of the length of nontrading period to trading period will increase. In a dynamic model such as that of the ARCH, these effects may require some form of adjustment to the basic model. Unfortunately, many researchers either do not take into account such factors, or if they do, the adjustments are either ad-hoc or based on intuition. Making the wrong adjustments or no adjustments at all may result in an incorrect or incomplete model. This in turn may lead to wrong inferences. While we have no solution as to what sort of adjustments we should make to account for the deterministic effects, we propose that any adjustments should be subjected to the simple diagnostic test proposed here.

The plan of the paper is as follows. The basic ARCH models are laid out in the following section. In Section 3, we propose a very simple diagnostic test to determine the adequacy of the adjustment (if any) in the mean and conditional variance equation. In Section 4, we apply a GARCH(1,1) model (with and without nontrading period adjustments) to the high frequency data set of MacKinlay and Ramaswamy (1989) and show that deterministic patterns still exist in the residuals if we fail to properly account for the nontrading effect. Section 5 concludes.

2. ARCH MODELS IN EMPIRICAL RESEARCH

Consider a time series of observations y_t. According to Engle (1982), the ARCH(q) model is given by:

$$y_t = \phi x_t + \epsilon_t \tag{1}$$

$$\epsilon_t \mid \Omega_{t-1} \sim N(0, h_t) \tag{2}$$

$$h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2 \tag{3}$$

Equation (1) is commonly known as the mean equation. The observations y_t depend on variables x_t (which are either exogenous, lagged endogenous or deterministic). Equation (2) states that conditioned on the information set at time t-1 (denoted by Ω_{t-1}), the errors of the mean equation ϵ_t are normally distributed with a zero mean[1] and a time-varying variance h_t. The conditional variance is given by Equation (3), which is simply the (weighted) sum of q previous lags of the square of the error term. The coefficients in the conditional variance equation must be positive to ensure that the series have a positive unconditional and conditional variance.

Bollerslev (1986) extends the ARCH(q) model to a generalised form. Specifically, the GARCH(p,q) model has a conditional variance given by

$$h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \tag{4}$$

[1] The errors have a zero mean because we have accounted for all deterministic components in the mean equation.

Again typically, the coefficients in the conditional variance equation are constrained to take on positive values only to ensure a non-negative (and hence defined) second moment[2].

Several other models have also been proposed to parameterise the conditional variance equation. This include the Exponential GARCH of Nelson (1991), the Quadratic GARCH of Sentana (1992), and the Nonlinear GARCH of Higgins and Bera (1993). This list is by no means exhaustive (see Chapter 2 for a discussion and empirical analysis of these and other parametric ARCH models).

Applying these models to empirical data, especially financial data, poses several problems. Asset returns are typically 'contaminated', being plagued by well documented anomalies such as calendar effects and nontrading periods. Accounting for such behaviour in the mean equation is relatively easy, and this is usually accomplished by the use of dummy variables. On the other hand, the dynamic structure of the conditional variance equation makes any similar adjustments difficult, since the recursive nature of ARCH models means that incorrect adjustments will tend to persist. Analysis using these models to infer relationships such as lead-lag behaviour of two equivalent markets (Cheung and Ng, 1990) and asymmetric volatility effects (Conrad *et. al.*, 1991) may therefore become invalid.

[2] See Nelson and Cao (1992) for the conditions in which some of the coefficients can take on negative values.

Most of the current empirical work in this area either ignores any deterministic effects, or makes an ad hoc adjustment to the conditional variance equation which may potentially be incorrect. For the former, we cite works by Akgiray (1989), Bollerslev (1987) and Taylor (1986), which ignore deterministic anomalies like calendar effects. For the latter, several methods are used. The most obvious is to simply include one or more indicator variables in the variance equation to indicate the deterministic event such as January effects, post-holiday effects etc. Gagnon, Morgan and Neave (1993), Conrad *et. al.* (1991), and Cheung and Ng (1990) use this adjustment method.

In the absence of any formal theoretical model of volatility, it is difficult to determine the correct form of adjustments required to account for calendar and non-trading effects. Nevertheless, we suggest in the following section a very simple diagnostic test which should be applied to determine the validity (if any) of the adjustments.

3. A SIMPLE DIAGNOSTIC TEST FOR DETERMINISTIC EVENTS

3.1 A Heuristic Approach

We can rewrite the error term of Equation (1) as follows

$$\epsilon_t = z_t \sqrt{h_t}$$

where h_t follows a specific conditional variance parameterisation e.g. GARCH(1,1) and z_t is a standard normal. Rewriting the above, we get

$$z_t = \frac{\epsilon_t}{\sqrt{h_t}}$$

In other words, the standardised residuals must be a white noise process. Relying on this property, one of the standard diagnostic tests for correct specification of the conditional variance (and mean) equation is that the standardised residuals are not autocorrelated. The portmanteau test of Ljung and Box (1978) for serial correlation is almost universally employed as a diagnostic test in the GARCH literature. Along the same lines, Bollerslev (1986) suggests using the Ljung-Box test on the squares of the standardised residuals. This follows from the work of McLeod and Li (1983).

Our proposed diagnostic test extends the above. Suppose we have a deterministic event for which we use a dummy variable (either in the mean or variance equation or both). The resulting standardised residuals must therefore not contain any deterministic components i.e. it must be a white noise process. In that sense, if we regress the standardised residuals against this dummy variable, the coefficient should not be significant. Likewise, by regressing the squares of the standardised residuals against the dummy variable, we would be able to test if we have sufficiently accounted for the deterministic event in the conditional variance equation. We could extend this further by including additional dummy variables surrounding the deterministic event to test if the adjustments we make to the conditional mean and variance equation are adequate.

3.2 A Formal Approach

The diagnostic test for omitted (dummy) variables in a regression model (i.e. the mean equation) is straightforward, and can be found in many econometric textbooks (e.g. Davidson and MacKinnon, 1993, Greene, 1993, and Harvey, 1990). Detailed proofs can be found in Pagan and Hall (1983) and will not be given here. Essentially, the test is based on the Lagrange Multiplier (LM) test. The procedure involves regressing the standardised residuals, z_t, on the set of dummy variables. If the dummy variables are significant in the regression, this implies that the mean equation has omitted variables[3].

The diagnostic test for omitted (dummy) variables in the variance equation is slightly more complicated, although a similar principle applies. Let $z_t = e_t h^{-1/2}$, Z^*_{t-1} be a vector of measurable variables used as a proxy for the information set Ω_{t-1}, and Z_{t-1} be the vector of (possibly) missing variables. Engle and Ng (1993) show that if the conditional volatility model is well specified, running the following regression

[3] As pointed out in Maddala (1992), if we simply regress ϵ_t against the dummy variables (as suggested in the earlier section), the test has low power.

$$z_t^2 = a + b Z_{t-1} + \Phi Z^*_{t-1} + v_t$$

should result in $b = 0$ and v_t being a white noise. By selecting different variables for Z_{t-1}, different tests can be constructed against a specific alternative. Engle and Ng (1993) suggest three different diagnostic tests based on ϵ_t, which they termed *the sign bias test*, *the negative size bias test*, and *the positive size bias test*. In our diagnostic test, we propose the use of indicator variables to denote a nontrading period. Likewise, by introducing additional indicator variables *around* a nontrading period, we can determine if we need to account for temporal patterns in the conditional volatility surrounding the nontrading period.

4. VOLATILITY AROUND NONTRADING PERIODS : THE CASE OF S&P500 INDEX FUTURES

As a practical example, we compare the various methods of accounting for the non-trading effect in asset returns by estimating various GARCH(1,1) models (with and without non-trading period adjustments) on the futures data of MacKinlay and Ramaswamy (1988)'s dataset. The original dataset consist of 15 minute interval prices of the S&P 500 index futures supplied by Chicago Mercantile Exchange. The data relates only to the near contract from the expiration of the previous contract until its own expiration, beginning with the September 1983 contract and ending with the June 1987 contract (16 contracts in all). However, we use prices sampled at a 30 minute interval. Observations per contract range from 780 for the June 1984 contract to 896 for the December 1986 contract[4].

A GARCH(1,1) model is estimated, using the following methods of adjustments for non-trading (i.e. turn-of-the-day/turn-of-the-week) period:

Method 1: No dummy variables in either both the mean and conditional variance equations i.e.

$$\begin{aligned} r_t &= \mu + \epsilon_t \\ \epsilon_t &\sim N(0,h_t) \\ h_t &= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} \end{aligned} \qquad (5)$$

Method 2: One dummy variable in the mean equation and one dummy variable in the conditional variance equation i.e.

$$\begin{aligned} r_t &= \mu + \eta_0 D_t + \epsilon_t \\ \epsilon_t &\sim N(0,h_t) \\ h_t &= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + \zeta_0 D_t \end{aligned} \qquad (6)$$

where D_t equals one if t spans a nontrading period, and zero otherwise. Note that the recursive nature of the GARCH model means that the increase in conditional variance over the nontrading period will persist for some time.

[4] Futures trading hours were 9am Central Time to 3pm Central Time up to September 27, 1985; and 8.30am Central Time to 3am Central Time thereafter.

Method 3: Cheung and Ng (1990) adjustment. This consists of one dummy variable in the mean equation and one dummy variable in the conditional variance equation. However, the same dummy variable is subtracted from the previous conditional volatility only if the previous return spans a non-trading period i.e.

$$\begin{aligned} r_t &= \mu + \eta_0 D_t + \epsilon_t \\ \epsilon_t &\sim N(0, h_t) \\ h_t &= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \left(h_{t-1} - \zeta_0 D_{t-1} \right) + \zeta_0 D_t \end{aligned} \tag{7}$$

where D_t equals one if t spans a nontrading period, and zero otherwise. The idea is to remove any increase in volatility due to the nontrading period from subsequent period volatility. In other words, the increased in volatility due to the nontrading period is not allowed to persist into future periods.

Method 4: One dummy variable in the mean equation, and two dummy variables in the conditional variance equation i.e.

$$\begin{aligned} r_t &= \mu + \eta_0 D_t + \epsilon_t \\ \epsilon_t &\sim N(0, h_t) \\ h_t &= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \left(h_{t-1} - \zeta_1 D_{t-1} \right) + \zeta_0 D_t \end{aligned} \tag{8}$$

where D_t equals one if t spans a nontrading period, and zero otherwise. This adjustment nests the Cheung and Ng (1991) adjustment if $\zeta_1 = \zeta_0$, and is based on the argument that the increased in volatility associated with the nontrading period has temporary and permanent components. The temporary component has no impact on future volatility, while the permanent component can persist into the future.

For each of the methods, we then perform the usual Ljung-Box (1978) portmanteau test on the standardised residuals and squares of standardised residuals respectively. The results are summarised in Table 1, together with their log-likelihood values. With one or two exceptions, it appears that all 4 methods of adjustments are well-specified for all 16 contracts, as indicated by the lack of significance in the Ljung-Box statistics for both the levels and their squares. Now, clearly the GARCH(1,1) model without any nontrading period adjustments is not correctly specified. Yet, the Ljung-Box test does not show any evidence to indicate model inadequacy. Note that the addition of dummy variables (regardless of form) significantly increases the log-likelihood values, indicating that non-trading period adjustments are necessary[5].

To examine the behaviour of returns and conditional volatility spanning the non-trading period, we examine a window of three 30-minute intervals prior to market close and three 30-minute intervals after market open[6]. We regress (using OLS) the levels and squares of the standardised residuals against 7 dummy variables i.e.

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[5] A likelihood ratio test can be performed to check for the significance of the additional parameter(s). For two (three) additional parameters (one in the mean equation and one (two) in the conditional variance equation), the likelihood value must increase by 2.996 (3.907) at the 5 percent significance level.

[6] Initially, our window consist of five 30-minute interval periods before market closure and five 30-minute interval periods after market has opened. However, since the two 30-minute interval furthest away from market closure and market opening does not appear to have a deterministic time component, we reduce the window to only three 30-minute interval periods on either side of the non-trading period.

$$\frac{\epsilon_t}{\sqrt{h_t}} = \kappa + \sum_{k=+3}^{-3} \theta_k D_{t+k} + \phi x_t' \qquad (9)$$

and

$$\frac{\epsilon_t^2}{h_t} = \lambda + \sum_{l=+3}^{-3} \Theta_l D_{t+l} + \Phi Z_{t-1}^* \qquad (10)$$

where as before D_t equals one if t spans a nontrading period, and zero otherwise. By examining the significance of the coefficients in both regressions (i.e. θ_k and Θ_l), we can determine if the particular method of adjustment has fully accounted for deterministic effects of the nontrading period. The results are reported in Tables 2A to 2D (for the levels) and 3A to 3D (for the squares) for each of the 4 methods of adjustments respectively.

From Tables 2A to 2D, it appears that nontrading periods have little or no impact in the levels (i.e. mean equation). This suggests that nontrading periods do not affect returns, since the exclusion of the dummy variable (Method 1 adjustment) does not exhibit a deterministic component (with respect to the nontrading period) in the standardised residuals[7]. It is also worthwhile to note that there is no pattern in the levels within our window of three 30-minute interval periods before and three 30-minute interval periods after a nontrading period.

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[7] Detailed results of the GARCH(1,1) parameters using maximum likelihood estimation are available separately. Regardless of which adjustment method is used, the coefficient of the dummy variable in the mean equation (i.e. η_0) is not significant for more than half of the 16 contracts. The results remain unchanged when quasi-maximum likelihood estimation (see Bollerslev and Wooldridge, 1992) is used.

On the other hand, not adjusting for the nontrading effect in the conditional volatility equation has some serious consequences. Table 3A shows the results of regressing the squares of the standardised residuals without adjusting for the non-trading effect (Method 1) against dummy variables in our window. Clearly, the squared residuals are no longer white noise, since the coefficients are highly significant, especially when the period spans a non-trading period. Indeed, this suggests that the GARCH(1,1) model without the nontrading period adjustment is incorrectly specified because such a model has not been fully conditioned on the information set Ω_t[8].

Tables 3B to 3D show that while the other 3 methods of adjustment reduce the deterministic component of the conditional volatility across the nontrading period, there are still uncaptured deterministic effects. In fact, this suggests that the (conditional) volatility structure surrounding the nontrading period cannot be fully captured by adding one (or two) dummy variables.

Given the above results, we re-estimate a GARCH(1,1) model for all 16 futures contracts. However, we use 7 dummy variables in both the mean and conditional variance equation i.e.

$$\begin{gathered} r_t = \mu + \sum_{i=+3}^{-3} \eta_i D_{t+i} + \epsilon_t \\ \epsilon_t \sim N(0, h_t) \\ h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + \sum_{j=+3}^{-3} \zeta_j D_{t+j} \end{gathered} \tag{11}$$

[8] For example, we know beforehand, when the market will be closed (for the night or for the weekend). This knowledge will form part of our information set, which should condition our (conditional) volatility estimate for the next (trading) period.

Table 4 shows log-likelihood values and Ljung-Box statistics for the levels and squares. Comparing with Table 1, the log-likelihood values for all 16 contracts are larger than their corresponding values, while the Ljung-Box statistics are essentially similar. This further suggests that the portmanteau tests have little power against omitted deterministic variables in either the mean or variance equation.

The suggested diagnostic tests of regressing the standardised residuals and squares of standardised residuals against deterministic dummies are performed, and the results are summarised in Tables 5A and 5B respectively. Since we have accounted for the deterministic dummies in our mean and conditional variance equation, the coefficients of the diagnostic regression for almost all contracts are not significantly different from zero.

As a matter of comparison, the quasi-maximum likelihood[9] estimates of the dummy variables for the mean and conditional variance equation are reported in Table 6A and 6B. It can be seen that the *t*-statistics of the coefficients of our diagnostic regressions tend to be larger for those dummy variables that are significant. This indicates that our proposed diagnostic regressions can identify omitted deterministic (dummy) variables in either the mean or variance equation.

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[9] Bollerslev and Wooldridge (1992) and Susmel and Engle (1994) show that quasi-maximum likelihood estimation is robust to violation of the assumption of conditional normality of the error term.

An interesting pattern that emerges from Table 6B is that conditional variance tends to *increase* prior to market closure. Of the 16 futures contract we examine, more than half of them experience a significant increase in conditional volatility within the last one and a half hours prior to market closure, while only two show a significant decrease. For the remaining contracts, the sign of the dummy variables tends to be positive, indicating a tendency for conditional volatility to rise. On the other hand, conditional volatility over the nontrading period (turn-of-the-day/turn-of-the-week) is higher for 15 of the 16 contracts, 8 of which are significant. However, once the market opens, the conditional volatility tends to decline. The decline usually occurs within the first 30 minutes of trading, but can last as long as one and a half hours[10].

5. CONCLUSION

ARCH models have been successfully applied to a wide range of financial time series. Unfortunately, financial time series exhibit deterministic patterns and anomalies which have little theoretical explanations. Examples include day-of-the-week effects, turn-of-the-year effects, and differences between nontrading and trading period's volatility. Current empirical work either ignores such effects, or makes ad hoc adjustments based on intuition or observed phenomenon. Unfortunately, the dynamic nature of ARCH models make such adjustments difficult. In this paper we propose a simple diagnostic to determine the adequacy of any adjustment that

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[10] Note that this pattern in conditional volatility mirrors the U-shaped pattern in *unconditional* volatility of the cash market e.g. Lockwood and Linn (1990).

incorporates deterministic variables in the mean and variance equation. As a practical example, we apply a GARCH(1,1) model to 30-minute returns of the S&P500 index futures, using several methods to account for the changes in volatility surrounding a nontrading period. We find that the traditional Ljung-Box (1978) diagnostics could neither detect omitted deterministic variables nor discriminate among the various ad-hoc adjustment methods. On the other hand, our proposed diagnostic regressions indicate that all 4 methods are inadequate, suggesting that the conditional volatility structure surrounding the nontrading period is more complicated. The conditional volatility of the index futures appears to rise prior to market closure, but declines once the market opens, with the duration being as long as one and a half hours. Any adjustment methods that do not allow for this pattern (e.g. Cheung and Ng, 1990) may lead to incorrect estimates of the conditional variance.

TABLE 1

Log-likelihood values and diagnostic tests.

Contract	Method 1			Method 2			Method 3			Method 4		
	Log-likelihood	$Q(10)$	$Q^2(10)$	Log-likelihood	$Q(10)$	$Q^2(10)$	Log-likelihood	$Q(10)$	$Q^2(10)$	Log-likelihood	$Q(10)$	$Q^2(10)$
Sep 1983	715.884	11.955	2.851	749.744	12.902	7.023	750.492	11.989	5.062	771.817	11.881	7.863
Dec 1983	993.246	10.598	12.478	994.525	10.968	14.281	995.998	11.297	14.038	1007.064	12.132	12.103
Mar 1984	872.593	9.001	6.587	877.801	9.871	6.852	885.019	9.311	5.231	887.630	9.071	4.369
Jun 1984	822.215	9.027	18.149	842.534	9.359	22.409	842.589	9.059	19.537	842.606	8.999	19.831
Sep 1984	800.722	17.032	7.081	814.204	16.999	10.384	824.870	17.233	6.270	833.022	17.448	4.842
Dec 1984	875.509	5.753	3.254	880.692	6.495	3.642	881.243	6.665	2.783	885.424	7.202	8.669
Mar 1985	803.698	17.830	1.738	806.520	17.630	1.334	806.602	17.496	1.355	817.240	17.781	1.929
Jun 1985	1156.131	8.884	3.152	1170.640	9.214	3.158	1170.439	9.095	3.035	1181.649	11.006	10.826
Sep 1985	1137.590	10.974	2.585	1143.245	10.557	2.760	1142.240	10.389	2.737	1144.100	12.057	4.415
Dec 1985	1098.242	9.921	8.699	1099.804	10.585	9.133	1101.422	10.411	7.547	1102.165	9.938	7.638
Mar 1986	818.231	8.695	9.304	822.441	8.956	8.560	822.134	8.806	7.857	822.448	8.946	8.439
Jun 1986	796.881	22.418	9.480	800.102	23.763	9.844	803.260	22.033	10.759	810.973	23.752	12.662
Sep 1986	751.336	13.178	5.373	766.574	13.486	3.488	757.256	13.404	4.642	768.569	13.792	3.841
Dec 1986	785.771	7.140	3.238	818.281	8.462	5.473	817.011	8.495	5.863	818.387	8.436	5.289
Mar 1987	786.107	5.106	6.658	797.977	4.474	10.793	794.420	4.640	10.183	798.548	4.580	10.487
Jun 1987	509.799	3.982	5.143	551.667	5.061	8.598	555.346	5.694	4.389	555.518	5.578	4.412

Note: $Q(10)$ and $Q^2(10)$ denote the Ljung-Box (1978) test for up to 10th-order serial correlation in the standardised residuals and squares of standardised residuals respectively. Under the null hypothesis of no serial correlation, the test statistic is distributed as a chi-square with 10 degrees of freedom. Critical values at the 1 and 5 percent significance level are 23.209 and 18.307 respectively.

TABLE 2A

Coefficients and t-statistics of regression of standardised residuals (Method 1 adjustment) against various dummy variables, as per Equation (9).

Contract	θ_{t+3}	θ_{t+2}	θ_{t+1}	θ_t	θ_{t-1}	θ_{t-2}	θ_{t-3}
Sep 1983	0.1331 0.8548	0.1321 0.8482	-0.1768 -1.1352	0.2793 1.7934	-0.0944 -0.6062	-0.0840 -0.5396	-0.0571 -0.3665
Dec 1983	0.1107 0.7091	0.1646 1.0543	0.0300 0.1925	0.0651 0.4169	0.0081 0.0519	-0.0147 -0.0941	0.0882 0.5653
Mar 1984	-0.2547 -1.6323	0.0729 0.4670	-0.2200 -1.4100	0.2906 1.8619	0.2191 1.4039	-0.1104 -0.7076	-0.0421 -0.2698
Jun 1984	0.1973 1.2343	-0.0177 -0.1110	-0.0328 -0.2052	0.0173 0.1082	0.0057 0.0355	-0.3644 -2.2801*	-0.0189 -0.1186
Sep 1984	0.0521 0.3516	0.0018 0.0122	-0.3983 -2.6871*	0.0247 0.1668	0.0717 0.4835	-0.0625 -0.4219	-0.2972 -2.0054*
Dec 1984	-0.2112 -1.3716	-0.2931 -1.9033	0.1593 1.0346	0.2446 1.5879	-0.0050 -0.0325	-0.0613 -0.3980	-0.0337 -0.2185
Mar 1985	0.0960 0.5853	0.1027 0.6260	-0.1464 -0.8921	-0.0054 -0.0327	0.1478 0.9006	-0.0731 -0.4455	0.2732 1.6648
Jun 1985	-0.0704 -0.4745	0.1360 0.9172	-0.0179 -0.1205	0.3539 2.3863*	-0.0461 -0.3107	-0.0207 -0.1393	-0.0634 -0.4278
Sep 1985	0.1016 0.6531	-0.3330 -2.1395*	0.0685 0.4402	0.1888 1.2128	-0.0061 -0.0392	-0.1017 -0.6534	-0.0355 -0.2278
Dec 1985	-0.1348 -0.9214	0.0776 0.5307	-0.0381 -0.2603	0.1169 0.7991	0.0068 0.0462	-0.1107 -0.7569	-0.0615 -0.4204
Mar 1986	0.0386 0.2628	0.1879 1.2791	0.1225 0.8341	0.1798 1.2242	0.0810 0.5517	-0.0110 -0.0748	0.1734 1.1808
Jun 1986	0.2245 1.5364	0.1377 0.9421	0.0326 0.2229	0.2669 1.8269	0.0416 0.2848	0.0318 0.2174	-0.1652 -1.1309
Sep 1986	-0.1132 -0.7824	0.3487 2.4093*	-0.2181 -1.5065	0.0757 0.5230	0.0206 0.1426	-0.0213 -0.1474	0.3437 2.3747*
Dec 1986	-0.0525 -0.3622	-0.0256 -0.1762	0.0643 0.4431	-0.2805 -1.9331	0.2724 1.8777	-0.1089 -0.7507	-0.2120 -1.4610
Mar 1987	-0.0927 -0.6264	0.0433 0.2927	-0.1653 -1.1175	0.1833 1.2389	0.0349 0.2357	-0.1146 -0.7748	-0.0746 -0.5044
Jun 1987	0.0577 0.3895	-0.1977 -1.3359	-0.0533 -0.3599	0.0173 0.1168	0.0507 0.3427	-0.0895 -0.6050	0.1950 1.3176

* Significant at the five percent level.

TABLE 2B

Coefficients and *t*-statistics of regression of standardised residuals (Method 2 adjustment) against various dummy variables, as per Equation (9).

Contract	θ_{t+3}	θ_{t+2}	θ_{t+1}	θ_{t}	θ_{t-1}	θ_{t-2}	θ_{t-3}
Sep 1983	0.1513 0.9693	0.1317 0.8441	-0.1773 -1.1357	-0.1012 -0.6485	-0.1015 -0.6503	-0.0930 -0.5956	-0.0660 -0.4227
Dec 1983	0.1125 0.7210	0.1633 1.0461	0.0317 0.2031	0.0440 0.2816	0.0075 0.0479	-0.0133 -0.0853	0.0921 0.5903
Mar 1984	-0.2547 -1.6267	0.0733 0.4684	-0.2213 -1.4134	-0.1025 -0.6543	0.2134 1.3628	-0.1130 -0.7214	-0.0421 -0.2689
Jun 1984	0.1949 1.2208	-0.0219 -0.1371	-0.0574 -0.3598	-0.0798 -0.5003	-0.0021 -0.0129	-0.4096 -2.5661*	-0.0152 -0.0951
Sep 1984	0.0677 0.4597	0.0144 0.0981	-0.4244 -2.8832*	-0.1524 -1.0354	0.0408 0.2769	-0.0657 -0.4464	-0.3298 -2.2403*
Dec 1984	-0.2189 -1.4168	-0.2986 -1.9328	0.1613 1.0443	-0.0863 -0.5590	-0.0230 -0.1489	-0.0689 -0.4458	-0.0328 -0.2121
Mar 1985	0.0976 0.5968	0.1052 0.6432	-0.1509 -0.9229	0.0217 0.1326	0.1477 0.9032	-0.0755 -0.4618	0.2774 1.6967
Jun 1985	-0.0742 -0.4985	0.1415 0.9509	-0.0201 -0.1353	-0.0129 -0.0868	-0.0497 -0.3340	-0.0282 -0.1898	-0.0667 -0.4485
Sep 1985	0.1063 0.6825	-0.3431 -2.2023*	0.0699 0.4487	-0.0467 -0.2998	-0.0060 -0.0387	-0.1045 -0.6709	-0.0366 -0.2347
Dec 1985	-0.1402 -0.9575	0.0773 0.5279	-0.0412 -0.2812	-0.0483 -0.3298	0.0018 0.0123	-0.1161 -0.7928	-0.0637 -0.4354
Mar 1986	0.0396 0.2700	0.1929 1.3140	0.1247 0.8491	0.0422 0.2876	0.0817 0.5562	-0.0114 -0.0778	0.1768 1.2043
Jun 1986	0.2153 1.4713	0.1308 0.8940	0.0256 0.1752	0.0047 0.0323	0.0403 0.2754	0.0309 0.2115	-0.1700 -1.1616
Sep 1986	-0.1236 -0.8566	0.3700 2.5650*	-0.2404 -1.6670	0.0649 0.4500	0.0364 0.2522	-0.0119 -0.0825	0.3527 2.4450*
Dec 1986	-0.0695 -0.4792	-0.0251 -0.1733	0.0738 0.5084	-0.0514 -0.3540	0.2594 1.7877	-0.1328 -0.9154	-0.2305 -1.5886
Mar 1987	-0.0926 -0.6261	0.0506 0.3419	-0.1763 -1.1914	0.0486 0.3286	0.0326 0.2206	-0.1065 -0.7196	-0.0771 -0.5213
Jun 1987	0.0638 0.4324	-0.2101 -1.4234	-0.0683 -0.4630	-0.0105 -0.0709	0.0549 0.3716	-0.0969 -0.6562	0.2233 1.5127

* Significant at the five percent level.

TABLE 2C

Coefficients and t-statistics of regression of standardised residuals (Method 3 adjustment) against various dummy variables, as per Equation (9).

Contract	θ_{t+3}	θ_{t+2}	θ_{t+1}	θ_{t}	θ_{t-1}	θ_{t-2}	θ_{t-3}
Sep 1983	0.1494 0.9571	0.1279 0.8196	-0.1894 -1.2131	-0.1141 -0.7308	-0.1109 -0.7105	-0.1046 -0.6700	-0.0672 -0.4303
Dec 1983	0.1123 0.7195	0.1674 1.0729	0.0302 0.1938	0.0508 0.3256	0.0078 0.0502	-0.0148 -0.0946	0.0899 0.5762
Mar 1984	-0.2610 -1.6690	0.0755 0.4830	-0.2263 -1.4472	-0.0984 -0.6295	0.2249 1.4385	-0.1155 -0.7385	-0.0439 -0.2807
Jun 1984	0.2017 1.2635	-0.0223 -0.1399	-0.0493 -0.3087	-0.0646 -0.4045	-0.0009 -0.0058	-0.3965 -2.4836*	-0.0178 -0.1112
Sep 1984	0.0587 0.3970	-0.0065 -0.0442	-0.4317 -2.9215*	-0.0265 -0.1796	0.0673 0.4555	-0.0724 -0.4900	-0.3212 -2.1734*
Dec 1984	-0.2179 -1.4108	-0.3003 -1.9447	0.1609 1.0419	-0.0859 -0.5559	-0.0208 -0.1346	-0.0643 -0.4166	-0.0337 -0.2183
Mar 1985	0.0977 0.5959	0.1050 0.6400	-0.1495 -0.9116	0.0319 0.1943	0.1502 0.9156	-0.0755 -0.4602	0.2788 1.6999
Jun 1985	-0.0740 -0.4977	0.1414 0.9510	-0.0198 -0.1333	-0.0149 -0.1003	-0.0500 -0.3359	-0.0303 -0.2034	-0.0656 -0.4408
Sep 1985	0.1050 0.6738	-0.3399 -2.1813*	0.0692 0.4441	-0.0516 -0.3314	-0.0069 -0.0444	-0.1052 -0.6752	-0.0365 -0.2344
Dec 1985	-0.1361 -0.9303	0.0779 0.5324	-0.0372 -0.2542	-0.0385 -0.2630	0.0064 0.0437	-0.1154 -0.7884	-0.0655 -0.4475
Mar 1986	0.0399 0.2719	0.1909 1.3001	0.1232 0.8390	0.0441 0.3005	0.0798 0.5436	-0.0128 -0.0874	0.1763 1.2006
Jun 1986	0.2333 1.5931	0.1441 0.9842	0.0388 0.2650	0.0190 0.1297	0.0423 0.2890	0.0349 0.2382	-0.1685 -1.1507
Sep 1986	-0.1161 -0.8034	0.3598 2.4887*	-0.2232 -1.5440	0.0621 0.4295	0.0233 0.1614	-0.0205 -0.1417	0.3542 2.4501*
Dec 1986	-0.0664 -0.4580	-0.0261 -0.1799	0.0737 0.5082	-0.0253 -0.1747	0.2990 2.0622*	-0.1364 -0.9409	-0.2300 -1.5865
Mar 1987	-0.0934 -0.6310	0.0473 0.3193	-0.1707 -1.1535	0.0767 0.5179	0.0357 0.2413	-0.1164 -0.7863	-0.0761 -0.5141
Jun 1987	0.0616 0.4165	-0.2151 -1.4545	-0.0444 -0.3002	-0.0331 -0.2240	0.0550 0.3717	-0.1140 -0.7710	0.2004 1.3555

* Significant at the five percent level.

TABLE 2D

Coefficients and t-statistics of regression of standardised residuals (Method 4 adjustment) against various dummy variables, as per Equation (9).

Contract	θ_{t+3}	θ_{t+2}	θ_{t+1}	θ_{t}	θ_{t-1}	θ_{t-2}	θ_{t-3}
Sep 1983	0.1247 0.7987	0.1391 0.8911	-0.1899 -1.2164	-0.0953 -0.6108	-0.1406 -0.9011	-0.0744 -0.4769	-0.0473 -0.3030
Dec 1983	0.0909 0.5827	0.1416 0.9085	0.0125 0.0799	0.0305 0.1959	-0.0127 -0.0812	-0.0399 -0.2561	0.0906 0.5810
Mar 1984	-0.2509 -1.6042	0.0731 0.4673	-0.2169 -1.3872	-0.0791 -0.5055	0.2483 1.5876	-0.1263 -0.8077	-0.0501 -0.3204
Jun 1984	0.2012 1.2604	-0.0222 -0.1390	-0.0495 -0.3104	-0.0667 -0.4180	-0.0004 -0.0026	-0.3972 -2.4885*	-0.0176 -0.1104
Sep 1984	0.0471 0.3187	-0.0123 -0.0830	-0.3993 -2.7022*	-0.0019 -0.0130	0.0830 0.5616	-0.0793 -0.5365	-0.3468 -2.3466*
Dec 1984	-0.2083 -1.3497	-0.3172 -2.0551*	0.1509 0.9777	-0.1463 -0.9478	-0.0095 -0.0616	-0.0909 -0.5891	-0.0474 -0.3070
Mar 1985	0.0919 0.5641	0.0976 0.5992	-0.1374 -0.8431	0.0260 0.1597	0.1703 1.0450	-0.0923 -0.5666	0.3092 1.8973
Jun 1985	-0.0359 -0.2388	0.1331 0.8846	-0.0561 -0.3730	-0.0223 -0.1482	-0.0737 -0.4898	-0.0415 -0.2758	-0.0803 -0.5340
Sep 1985	0.1044 0.6847	-0.3480 -2.2816*	0.0727 0.4766	-0.0413 -0.2709	-0.0080 -0.0525	-0.1061 -0.6959	-0.0374 -0.2451
Dec 1985	-0.1279 -0.8742	0.0755 0.5157	-0.0367 -0.2510	-0.0329 -0.2249	0.0140 0.0960	-0.1173 -0.8017	-0.0661 -0.4516
Mar 1986	0.0398 0.2712	0.1927 1.3126	0.1245 0.8482	0.0425 0.2892	0.0815 0.5549	-0.0115 -0.0786	0.1768 1.2041
Jun 1986	0.2133 1.4563	0.1295 0.8846	0.0269 0.1840	0.0089 0.0610	0.0469 0.3201	0.0389 0.2655	-0.1894 -1.2936
Sep 1986	-0.1242 -0.8615	0.3750 2.6003*	-0.2381 -1.6514	0.0825 0.5720	0.0516 0.3580	-0.0017 -0.0115	0.3557 2.4669*
Dec 1986	-0.0689 -0.4749	-0.0262 -0.1809	0.0737 0.5080	-0.0549 -0.3784	0.2483 1.7108	-0.1350 -0.9298	-0.2325 -1.6020
Mar 1987	-0.0956 -0.6464	0.0506 0.3421	-0.1767 -1.1941	0.0410 0.2771	0.0288 0.1949	-0.1061 -0.7172	-0.0776 -0.5247
Jun 1987	0.0612 0.4140	-0.2148 -1.4526	-0.0444 -0.3005	-0.0268 -0.1810	0.0603 0.4075	-0.1158 -0.7833	0.2006 1.3569

* Significant at the five percent level.

TABLE 3A

Coefficients and t-statistics of regression of squares of standardised residuals (Method 1 adjustment) against various dummy variables, as per Equation (10).

Contract	Θ_{t+3}	Θ_{t+2}	Θ_{t+1}	Θ_{t}	Θ_{t-1}	Θ_{t-2}	Θ_{t-3}
Sep 1983	0.4717 1.2939	1.1619 3.1869*	0.5201 1.4265	2.2206 6.0908*	0.0235 0.0644	-0.0660 -0.1809	0.0066 0.0181
Dec 1983	0.5474 2.2142*	0.5268 2.1308*	0.7096 2.8703*	0.6624 2.6795*	0.0153 0.0617	-0.2508 -1.0146	-0.1107 -0.4478
Mar 1984	1.0751 3.5968*	0.7927 2.6521*	0.9321 3.1185*	1.2872 4.3065*	0.2916 0.9757	0.0793 0.2652	0.2918 0.9761
Jun 1984	0.6948 1.8847	0.4467 1.2118	0.4708 1.2771	1.7912 4.8589*	0.1397 0.3790	0.3658 0.9922	0.0270 0.0732
Sep 1984	0.6597 2.2074*	0.8393 2.8084*	0.4830 1.6160	1.8071 6.0466*	0.4264 1.4266	-0.1167 -0.3906	0.0099 0.0332
Dec 1984	0.8857 2.9857*	0.6342 2.1377*	0.5020 1.6921	0.5622 1.8952	-0.2685 -0.9051	-0.3971 -1.3385	-0.1114 -0.3755
Mar 1985	0.3885 0.9581	1.3068 3.2232*	1.2027 2.9666*	0.7495 1.8487	0.1869 0.4611	-0.0961 -0.2370	-0.0775 -0.1913
Jun 1985	1.0669 2.6043*	1.7037 4.1588*	0.3050 0.7446	1.5473 3.7771*	0.6114 1.4925	0.1790 0.4369	0.2791 0.6814
Sep 1985	0.2798 0.9619	0.6792 2.3349*	0.5233 1.7989	0.8807 3.0277*	0.6780 2.3309*	0.0885 0.3043	0.2392 0.8224
Dec 1985	0.4718 1.8099	0.7500 2.8771*	0.8078 3.0986*	0.5887 2.2583*	0.2165 0.8305	-0.1276 -0.4893	-0.1421 -0.5449
Mar 1986	0.6680 2.5726*	0.7943 3.0590*	0.9250 3.5621*	0.8315 3.2023*	0.3961 1.5254	0.2425 0.9338	0.2672 1.0288
Jun 1986	0.4788 1.6300	1.0622 3.6164*	0.3724 1.2679	0.8236 2.8041*	-0.1433 -0.4878	0.0338 0.1150	-0.0092 -0.0314
Sep 1986	1.0316 2.8548*	0.5871 1.6245	0.2957 0.8183	1.0830 2.9969*	1.2200 3.3760*	0.4263 1.1797	0.2098 0.5805
Dec 1986	0.5720 1.6386	0.2525 0.7234	0.2839 0.8133	1.9642 5.6269*	0.3590 1.0284	0.3546 1.0159	0.0148 0.0425
Mar 1987	0.0783 0.2578	0.0710 0.2335	0.3443 1.1330	0.8916 2.9340*	0.3517 1.1574	0.4972 1.6361	-0.1232 -0.4053
Jun 1987	0.5048 1.4373	0.8076 2.2994*	1.3224 3.7651*	2.7748 7.9003*	0.0763 0.2173	0.2602 0.7409	0.2640 0.7515

* Significant at the five percent level.

TABLE 3B

Coefficients and *t*-statistics of regression of squares of standardised residuals (Method 2 adjustment) against various dummy variables, as per Equation (10).

Contract	Θ_{t+3}	Θ_{t+2}	Θ_{t+1}	Θ_{t}	Θ_{t-1}	Θ_{t-2}	Θ_{t-3}
Sep 1983	0.5156 1.7598	1.3869 4.7340*	0.4683 1.5983	0.2340 0.7988	-0.0241 -0.0824	-0.0905 -0.3089	-0.0052 -0.0179
Dec 1983	0.5689 2.2928*	0.5635 2.2709*	0.7705 3.1054*	0.3254 1.3113	-0.0259 -0.1044	-0.2537 -1.0226	-0.1040 -0.4189
Mar 1984	1.0868 3.6341*	0.8215 2.7471*	0.9130 3.0528*	1.1527 3.8546*	0.3013 1.0075	0.0637 0.2131	0.2912 0.9736
Jun 1984	0.7370 2.4870*	0.4545 1.5337	0.5877 1.9832*	0.2626 0.8860	0.1435 0.4842	0.5011 1.6910	0.0328 0.1108
Sep 1984	0.6616 2.2370*	0.6755 2.2840*	0.2983 1.0087	0.1406 0.4754	0.0806 0.2725	-0.2717 -0.9188	0.0237 0.0802
Dec 1984	0.8916 2.9896*	0.6415 2.1510*	0.5634 1.8894	0.1099 0.3684	-0.3230 -1.0831	-0.3818 -1.2802	-0.1021 -0.3425
Mar 1985	0.4013 0.9817	1.3560 3.3169*	1.2575 3.0759*	0.2183 0.5339	0.2027 0.4958	-0.0826 -0.2021	-0.0765 -0.1872
Jun 1985	1.1572 2.7803*	1.8668 4.4850*	0.3375 0.8109	0.5014 1.2046	0.6035 1.4500	0.2096 0.5035	0.3063 0.7358
Sep 1985	0.2940 0.9964	0.7029 2.3822*	0.5546 1.8796	0.2726 0.9237	0.5129 1.7381	0.0499 0.1692	0.2423 0.8211
Dec 1985	0.4574 1.7370	0.7671 2.9133*	0.8216 3.1202*	0.3238 1.2295	0.0922 0.3500	-0.1763 -0.6695	-0.1779 -0.6757
Mar 1986	0.7075 2.6992*	0.8458 3.2265*	0.9852 3.7583*	0.2881 1.0992	0.2561 0.9771	0.2074 0.7913	0.2725 1.0395
Jun 1986	0.4210 1.4278	0.9226 3.1290*	0.2981 1.0109	0.9605 3.2577*	-0.0709 -0.2404	0.1079 0.3661	0.0344 0.1167
Sep 1986	1.2451 3.3486*	0.7278 1.9575	0.3582 0.9633	0.0828 0.2226	0.6221 1.6731	0.2492 0.6704	0.1446 0.3889
Dec 1986	0.6764 2.0490*	0.2254 0.6826	0.3018 0.9143	0.0580 0.1758	0.0910 0.2755	0.3449 1.0449	-0.0126 -0.0381
Mar 1987	0.0716 0.2278	0.0927 0.2949	0.3890 1.2371	-0.1378 -0.4382	-0.0348 -0.1107	0.3582 1.1391	-0.1938 -0.6163
Jun 1987	0.6296 1.9524	0.9041 2.8037*	1.5589 4.8343*	0.3296 1.0221	0.0623 0.1933	0.3503 1.0862	0.3488 1.0815

* Significant at the five percent level.

TABLE 3C

Coefficients and *t*-statistics of regression of squares of standardised residuals (Method 3 adjustment) against various dummy variables, as per Equation (10).

Contract	Θ_{t+3}	Θ_{t+2}	Θ_{t+1}	Θ_{t}	Θ_{t-1}	Θ_{t-2}	Θ_{t-3}
Sep 1983	0.5346 1.8388	1.3292 4.5715*	0.4371 1.5033	0.2103 0.7233	-0.0464 -0.1594	-0.1416 -0.4870	-0.0210 -0.0722
Dec 1983	0.5693 2.3074*	0.5498 2.2285*	0.7391 2.9955*	0.1861 0.7541	0.0180 0.0729	-0.2608 -1.0568	-0.1169 -0.4737
Mar 1984	1.1363 3.7260*	0.8530 2.7972*	0.9359 3.0688*	0.3848 1.2619	0.3407 1.1171	0.0744 0.2440	0.3239 1.0620
Jun 1984	0.7503 2.5424*	0.4475 1.5162	0.5630 1.9077	0.2550 0.8641	0.1407 0.4766	0.4477 1.5171	0.0269 0.0910
Sep 1984	0.7659 2.6669*	1.0021 3.4894*	0.5462 1.9020	0.2227 0.7754	0.4252 1.4807	-0.1332 -0.4639	-0.0020 -0.0070
Dec 1984	0.9031 3.0463*	0.6493 2.1901*	0.5445 1.8365	0.0721 0.2434	-0.3036 -1.0240	-0.3947 -1.3313	-0.1112 -0.3751
Mar 1985	0.4043 0.9812	1.3603 3.3011*	1.2548 3.0449*	0.2302 0.5587	0.1963 0.4763	-0.0940 -0.2282	-0.0804 -0.1951
Jun 1985	1.1415 2.7616*	1.8449 4.4636*	0.3345 0.8092	0.5210 1.2606	0.6852 1.6576	0.2205 0.5334	0.3031 0.7333
Sep 1985	0.2890 0.9843	0.6899 2.3501*	0.5428 1.8491	0.3355 1.1429	0.6982 2.3782*	0.0820 0.2792	0.2476 0.8435
Dec 1985	0.4927 1.8745	0.7815 2.9729*	0.8292 3.1546*	0.1447 0.5506	0.2164 0.8233	-0.1280 -0.4868	-0.1479 -0.5625
Mar 1986	0.6911 2.6415*	0.8315 3.1780*	0.9687 3.7024*	0.2915 1.1141	0.4149 1.5859	0.2506 0.9579	0.2844 1.0871
Jun 1986	0.5088 1.7659	1.1367 3.9450*	0.3654 1.2682	0.1649 0.5722	-0.1500 -0.5205	0.0302 0.1050	-0.0161 -0.0558
Sep 1986	1.0839 2.9528*	0.6148 1.6748	0.3078 0.8386	0.3461 0.9429	1.2772 3.4795*	0.4456 1.2138	0.2299 0.6264
Dec 1986	0.6594 2.0225*	0.2319 0.7114	0.3071 0.9421	0.1038 0.3184	0.3368 1.0332	0.3971 1.2179	0.0019 0.0057
Mar 1987	0.0707 0.2287	0.0780 0.2524	0.3634 1.1759	0.0046 0.0150	0.3721 1.2041	0.5290 1.7118	-0.1418 -0.4589
Jun 1987	0.6251 2.0059*	1.0059 3.2277*	1.5265 4.8978*	0.2878 0.9235	0.0441 0.1414	0.2342 0.7514	0.2004 0.6429

* Significant at the five percent level.

TABLE 3D

Coefficients and *t*-statistics of regression of squares of standardised residuals (Method 4 adjustment) against various dummy variables, as per Equation (10).

Contract	Θ_{t+3}	Θ_{t+2}	Θ_{t+1}	Θ_{t}	Θ_{t-1}	Θ_{t-2}	Θ_{t-3}
Sep 1983	0.3727 1.4163	0.9437 3.5860*	0.3354 1.2746	0.2287 0.8692	0.6148 2.3364*	0.0920 0.3495	0.2231 0.8480
Dec 1983	0.3501 1.3798	0.3451 1.3598	0.4570 1.8010	0.2101 0.8281	0.7370 2.9043*	-0.0628 -0.2475	0.1015 0.4000
Mar 1984	0.9926 3.2969*	0.7275 2.4162*	0.8358 2.7762*	0.3990 1.3254	0.5682 1.8873	0.2022 0.6717	0.4485 1.4896
Jun 1984	0.7477 2.5340*	0.4474 1.5163	0.5625 1.9063	0.2581 0.8748	0.1728 0.5857	0.4556 1.5440	0.0282 0.0955
Sep 1984	0.5799 2.0801*	0.7341 2.6333*	0.3189 1.1439	0.2318 0.8313	0.8177 2.9331*	0.0817 0.2929	0.1462 0.5246
Dec 1984	0.7480 2.4748*	0.6548 2.1664*	0.7192 2.3795*	0.2116 0.7001	0.3920 1.2969	-0.0842 -0.2787	0.1020 0.3375
Mar 1985	0.2140 0.5836	0.9537 2.6004*	0.8404 2.2915*	0.2527 0.6890	0.7331 1.9988*	0.1837 0.5010	0.1016 0.2771
Jun 1985	0.9539 2.6441*	1.4121 3.9144*	0.2962 0.8210	0.5112 1.4171	1.1320 3.1378*	0.4475 1.2406	0.4958 1.3745
Sep 1985	0.3180 1.1133	0.7386 2.5860*	0.5713 2.0001*	0.3214 1.1254	-0.2229 -0.7802	0.0996 0.3487	0.2560 0.8962
Dec 1985	0.4801 1.8445	0.7477 2.8725*	0.7976 3.0640*	0.1865 0.7165	0.4451 1.7100	-0.0215 -0.0824	-0.0758 -0.2913
Mar 1986	0.7054 2.6914*	0.8446 3.2226*	0.9835 3.7526*	0.2817 1.0747	0.2729 1.0411	0.2112 0.8059	0.2735 1.0435
Jun 1986	0.3450 1.2045	0.8092 2.8253*	0.1903 0.6644	0.1867 0.6517	0.1842 0.6432	0.3244 1.1325	0.1794 0.6265
Sep 1986	1.2442 3.3484*	0.7058 1.8996	0.3489 0.9391	0.2684 0.7223	0.3475 0.9351	0.1565 0.4211	0.1243 0.3345
Dec 1986	0.6772 2.0502*	0.2287 0.6923	0.3082 0.9331	0.0749 0.2267	0.0305 0.0923	0.3454 1.0455	-0.0099 -0.0300
Mar 1987	0.0713 0.2269	0.0856 0.2726	0.3910 1.2446	-0.0645 -0.2054	-0.1706 -0.5430	0.2835 0.9023	-0.2083 -0.6631
Jun 1987	0.6179 1.9909*	0.9996 3.2206*	1.5181 4.8912*	0.2970 0.9569	0.1272 0.4099	0.2687 0.8657	0.2109 0.6795

* Significant at the five percent level.

TABLE 4

Log-likelihood values and diagnostic tests of Equation (11).

Contract	Log-likelihood	$Q(10)$	$Q^2(10)$
Sep 1983	782.211	12.990	5.193
Dec 1983	1013.414	11.589	6.277
Mar 1984	897.383	7.090	2.589
Jun 1984	846.115	9.689	19.393
Sep 1984	856.441	13.797	8.737
Dec 1984	906.474	8.270	4.378
Mar 1985	836.918	17.749	7.136
Jun 1985	1203.209	14.126	11.237
Sep 1985	1147.181	14.131	2.831
Dec 1985	1114.249	12.308	10.152
Mar 1986	842.942	10.215	11.372
Jun 1986	814.994	22.500	12.768
Sep 1986	799.272	12.690	8.663
Dec 1986	826.310	6.549	3.041
Mar 1987	801.422	4.781	12.845
Jun 1987	591.432	5.588	7.911

Note: $Q(10)$ and $Q^2(10)$ denote the Ljung-Box (1978) test for up to 10th-order serial correlation in the standardised residuals and squares of standardised residuals respectively. Under the null hypothesis of no serial correlation, the test statistic is distributed as a chi-square with 10 degrees of freedom. Critical values at the 1 and 5 percent significance level are 23.209 and 18.307 respectively.

TABLE 5A

Coefficients and *t*-statistics of regression of standardised residuals from Equation (11) against various dummy variables, as per Equation (10).

Contract	θ_{t+3}	θ_{t+2}	θ_{t+1}	θ_{t}	θ_{t-1}	θ_{t-2}	θ_{t-3}
Sep 1983	-0.0114 -0.0729	-0.0933 -0.5943	-0.0140 -0.0894	-0.0853 -0.5435	0.0009 0.0056	-0.2603 -1.6581	-0.0656 -0.4178
Dec 1983	-0.0128 -0.0818	-0.0023 -0.0148	0.0364 0.2321	0.0146 0.0929	0.0101 0.0644	0.0478 0.3048	0.0256 0.1629
Mar 1984	-0.0713 -0.4515	-0.0305 -0.1930	-0.0343 -0.2172	-0.0456 -0.2886	-0.0038 -0.0243	-0.0363 -0.2297	0.0650 0.4118
Jun 1984	-0.0076 -0.0470	-0.0103 -0.0638	-0.0059 -0.0367	-0.0582 -0.3613	-0.0040 -0.0250	-0.0267 -0.1658	-0.0286 -0.1775
Sep 1984	-0.0688 -0.4586	-0.0078 -0.0520	-0.0825 -0.5498	0.0733 0.4885	-0.0128 -0.0853	-0.0189 -0.1258	-0.0320 -0.2133
Dec 1984	-0.0803 -0.5158	-0.0531 -0.3413	-0.0337 -0.2163	-0.1155 -0.7419	-0.0552 -0.3545	-0.0643 -0.4132	-0.0539 -0.3460
Mar 1985	-0.0311 -0.1876	-0.0311 -0.1877	-0.0311 -0.1873	-0.0311 -0.1878	-0.0311 -0.1874	-0.0311 -0.1877	-0.0310 -0.1872
Jun 1985	0.0048 0.0319	-0.0360 -0.2404	-0.0627 -0.4186	-0.0240 -0.1599	-0.0347 -0.2317	-0.0680 -0.4540	-0.0198 -0.1318
Sep 1985	-0.0366 -0.2323	-0.0367 -0.2329	-0.0397 -0.2518	-0.0536 -0.3398	-0.0411 -0.2604	-0.0140 -0.0891	-0.0298 -0.1892
Dec 1985	-0.0795 -0.5405	-0.0052 -0.0356	-0.1149 -0.7815	0.0046 0.0310	-0.0425 -0.2893	-0.0852 -0.5794	-0.0630 -0.4285
Mar 1986	0.0016 0.0106	-0.0132 -0.0889	-0.0249 -0.1680	-0.0042 -0.0284	0.0153 0.1032	-0.0127 -0.0855	0.0041 0.0275
Jun 1986	-0.0191 -0.1299	-0.0607 -0.4121	-0.0331 -0.2245	-0.0242 -0.1644	-0.0340 -0.2305	-0.0684 -0.4641	-0.0761 -0.5167
Sep 1986	0.0491 0.3343	0.1630 1.1092	0.0837 0.5693	0.0728 0.4957	0.1392 0.9468	0.0548 0.3730	0.0696 0.4733
Dec 1986	0.0405 0.2778	-0.0424 -0.2910	-0.0812 -0.5570	-0.0408 -0.2798	-0.0448 -0.3073	-0.1043 -0.7155	-0.4200 -2.8814*
Mar 1987	0.0198 0.1335	-0.0743 -0.5014	0.0938 0.6331	0.0578 0.3902	-0.0022 -0.0151	-0.1443 -0.9733	-0.0141 -0.0948
Jun 1987	-0.0987 -0.6644	-0.1238 -0.8333	-0.1462 -0.9839	-0.2903 -1.9535	-0.0655 -0.4407	-0.1091 -0.7341	-0.0903 -0.6073

* Significant at the five percent level.

TABLE 5B

Coefficients and *t*-statistics of regression of squares of standardised residuals from Equation (11) against various dummy variables, as per Equation (10).

Contract	Θ_{t+3}	Θ_{t+2}	Θ_{t+1}	Θ_{t}	Θ_{t-1}	Θ_{t-2}	Θ_{t-3}
Sep 1983	0.2050 0.7877	0.1969 0.7568	0.2223 0.8542	0.1185 0.4553	0.1071 0.4116	0.3841 1.4761	0.3286 1.2628
Dec 1983	0.0781 0.3177	0.0954 0.3882	0.0928 0.3775	0.1042 0.4240	0.0268 0.1091	0.2192 0.8917	0.2570 1.0454
Mar 1984	0.1865 0.6599	0.2255 0.7979	0.3779 1.3371	0.1985 0.7026	0.1045 0.3698	0.1195 0.4230	0.3413 1.2078
Jun 1984	0.2512 0.8037	0.2979 0.9534	0.2674 0.8558	0.2551 0.8164	0.2232 0.7144	0.2421 0.7746	0.3936 1.2594
Sep 1984	0.0701 0.2654	-0.0132 -0.0499	0.0484 0.1833	0.0200 0.0757	0.1844 0.6982	-0.1021 -0.3866	0.0033 0.0124
Dec 1984	-0.1763 -0.6359	-0.1790 -0.6455	-0.1217 -0.4391	-0.1377 -0.4967	-0.2415 -0.8712	-0.2168 -0.7821	-0.2066 -0.7453
Mar 1985	-0.0415 -0.1283	-0.0415 -0.1284	-0.0416 -0.1286	-0.0416 -0.1287	-0.0417 -0.1291	-0.0418 -0.1294	-0.0040 -0.0122
Jun 1985	0.3582 1.1222	0.3108 0.9737	0.3025 0.9479	0.3332 1.0437	0.2730 0.8551	0.2668 0.8359	0.3445 1.0791
Sep 1985	0.3693 1.2625	0.3396 1.1609	0.3571 1.2208	0.3232 1.1047	0.3106 1.0617	0.3026 1.0345	0.5991 2.0479*
Dec 1985	-0.1027 -0.3832	-0.0756 -0.2822	0.0097 0.0363	-0.0548 -0.2043	-0.0692 -0.2581	-0.1501 -0.5601	-0.2397 -0.8940
Mar 1986	0.0129 0.0470	0.0482 0.1752	0.1125 0.4089	0.0551 0.2002	0.0183 0.0664	0.0124 0.0451	0.0303 0.1103
Jun 1986	0.1776 0.5987	0.1190 0.4010	0.2137 0.7203	0.0850 0.2864	0.0671 0.2262	0.1458 0.4916	0.0408 0.1375
Sep 1986	0.0133 0.0454	0.0034 0.0117	-0.0005 -0.0016	0.0649 0.2223	0.0496 0.1697	-0.0535 -0.1833	0.0630 0.2155
Dec 1986	-0.0707 -0.2098	-0.0549 -0.1630	-0.0950 -0.2821	-0.1308 -0.3883	-0.1272 -0.3776	-0.1058 -0.3140	-0.1852 -0.5498
Mar 1987	0.0887 0.2853	0.0159 0.0510	-0.0991 -0.3188	-0.1374 -0.4419	-0.0782 -0.2515	-0.1631 -0.5247	-0.0642 -0.2065
Jun 1987	0.1380 0.5144	0.1811 0.6752	0.1793 0.6684	0.1211 0.4515	0.1348 0.5024	0.0014 0.0051	0.2785 1.0382

* Significant at the five percent level.

TABLE 6A

Coefficients and robust t-statistics of dummy variables in the mean equation of Equation (11), estimated by means of quasi-maximum likelihood (Bollerslev and Wooldridge, 1992).

Contract	η_{t+3}	η_{t+2}	η_{t+1}	η_{t}	η_{t-1}	η_{t-2}	η_{t-3}
Sep 1983	0.0341	0.0637	-0.0454	0.1046	-0.0254	0.0242	0.0042
	1.0980	1.3494	-1.0960	2.3978*	-0.8321	1.3976	0.1395
Dec 1983	0.0187	0.0275	-0.0075	-0.0018	-0.0041	-0.0160	0.0095
	0.5291	0.8161	-0.1177	-0.0316	-0.1056	-0.6168	0.2982
Mar 1984	-0.0287	0.0251	-0.0354	0.0669	0.0446	-0.0149	-0.0199
	-1.1825	0.8850	-1.2687	2.1790*	1.8425	-0.7028	-0.9613
Jun 1984	0.0447	0.0001	-0.0023	0.0255	0.0039	-0.0712	0.0024
	1.6624	0.0064	-0.0859	0.8410	0.1646	-3.0804*	0.1381
Sep 1984	0.0235	-0.0049	-0.0802	-0.0343	0.0155	-0.0164	-0.0700
	0.8448	-0.1753	-1.9633*	-0.9911	0.6309	-0.7293	-3.1458*
Dec 1984	-0.0370	-0.0686	0.0350	0.0784	0.0038	-0.0113	-0.0051
	-1.1060	-2.0511*	0.9842	2.1136*	0.1557	-0.5655	-0.2206
Mar 1985	0.0249	0.0260	-0.0267	0.0057	0.0346	-0.0101	0.0607
	0.8818	0.6746	-0.7155	0.1689	1.3672	-0.4710	2.9028*
Jun 1985	-0.0129	0.0286	0.0070	0.0612	-0.0023	0.0052	-0.0062
	-0.5228	1.0451	0.3871	2.2125*	-0.1033	0.3098	-0.3822
Sep 1985	0.0174	-0.0462	0.0145	0.0348	0.0049	-0.0161	-0.0031
	1.1476	-2.3585*	0.9001	1.5414	0.2389	-0.7881	-0.1652
Dec 1985	-0.0071	0.0126	0.0127	0.0173	0.0070	-0.0055	0.0001
	-0.3005	0.4857	0.4568	0.6570	0.3126	-0.2972	0.0025
Mar 1986	0.0072	0.0481	0.0324	0.0431	0.0178	-0.0006	0.0387
	0.1763	1.4757	0.6061	1.2125	0.5611	-0.0206	1.3698
Jun 1986	0.0571	0.0487	0.0151	0.0685	0.0157	0.0217	-0.0258
	1.8173	1.2186	0.4556	1.8356	0.5826	0.7549	-0.8840
Sep 1986	-0.0294	0.0471	-0.0738	0.0042	-0.0318	-0.0175	0.0752
	-0.9978	1.5683	-2.8009*	0.1310	-0.8412	-0.5759	3.1463*
Dec 1986	-0.0297	0.0008	0.0339	-0.0608	0.0833	-0.0054	0.0456
	-0.8289	0.0259	0.9061	-1.6449	2.6998*	-0.1753	1.0451
Mar 1987	-0.0254	0.0267	-0.0748	0.0146	0.0085	0.0157	-0.0167
	-0.8115	0.7700	-2.0050*	0.3655	0.2354	0.3958	-0.5697
Jun 1987	0.0377	-0.0382	0.0329	0.1851	0.0253	-0.0056	0.0826
	0.7976	-0.7136	0.4534	2.2677*	0.4977	-0.1472	2.6018*

* Significant at the five percent level.

TABLE 6B

Coefficients and robust *t*-statistics of dummy variables in the variance equation of Equation (11), estimated by means of quasi-maximum likelihood (Bollerslev and Wooldridge, 1992).

Contract	ζ_{t+3}	ζ_{t+2}	ζ_{t+1}	ζ_{t}	ζ_{t-1}	ζ_{t-2}	ζ_{t-3}
Sep 1983	0.0059 1.0232	0.0359 2.6545*	-0.0367 -5.2309*	0.1184 2.8992*	-0.1460 -4.1843*	-0.0231 -5.0355*	0.0041 1.3578
Dec 1983	0.0082 0.0678	-0.0015 -0.0086	0.0054 0.7161	-0.0015 -0.0167	-0.0208 -0.5273	-0.0212 -0.3222	0.0041 0.4586
Mar 1984	0.0327 7.9354*	-0.0055 -1.7436	-0.0003 -0.0987	0.0199 4.0512*	-0.0297 -4.7861*	-0.0071 -1.5548	0.0024 0.5494
Jun 1984	0.0171 1.5548	-0.0058 -0.4624	-0.0012 -0.0920	0.0561 1.7023	-0.0567 -1.9803*	0.0009 0.1138	-0.0133 -1.9913*
Sep 1984	0.0285 2.2608*	0.0188 0.7297	-0.0270 -0.9749	0.0857 2.8300*	-0.0844 -3.5635*	-0.0128 -1.1174	-0.0030 -0.4763
Dec 1984	0.0465 3.6448*	0.0039 0.2677	0.0117 0.4465	0.0103 0.3825	-0.0403 -2.4124*	-0.0085 -1.3940	0.0086 1.5723
Mar 1985	0.0162 1.8046	0.0494 2.4983*	0.0320 0.8604	0.0150 0.5857	-0.0032 -0.2270	-0.0062 -0.7824	-0.0052 -0.8772
Jun 1985	0.0206 3.0133*	0.0389 1.8027	0.0022 0.4632	0.0326 4.7456*	0.0120 2.0656*	0.0007 0.1617	0.0009 0.2591
Sep 1985	-0.0048 -0.9083	0.0061 1.1265	-0.0019 -0.3051	0.0081 0.8594	-0.0036 -0.4961	-0.0146 -2.5156*	-0.0031 -0.8520
Dec 1985	0.0171 2.9263*	0.0113 1.8951	0.0065 2.3235*	0.0009 0.1736	-0.0065 -2.5983*	-0.0073 -0.7967	0.0022 0.4458
Mar 1986	0.0368 2.1065*	0.0416 3.1618*	0.0537 2.3263*	0.0453 2.8906*	0.0239 2.0633*	0.0136 1.4561	0.0118 0.7114
Jun 1986	0.0142 0.7575	0.0406 1.9066	-0.0423 -2.1748*	0.0342 1.4055	-0.0507 -2.3175*	0.0043 0.3433	-0.0012 -0.1334
Sep 1986	0.0596 1.5294	0.0023 0.0757	-0.0078 -0.5819	0.0578 2.5977*	0.0470 1.3000	-0.0175 -0.6467	-0.0157 -1.1204
Dec 1986	0.0336 2.1767*	0.0070 0.4306	0.0158 1.0576	0.1155 3.6965*	0.0049 0.3857	0.0245 2.3460*	0.0104 0.7215
Mar 1987	0.0010 0.0846	0.0076 0.7708	0.0254 1.6223	0.0459 1.8768	-0.0289 -1.2426	0.0287 1.4546	-0.0444 -1.8608
Jun 1987	0.0474 2.2304*	0.0522 1.3753	0.1079 1.3054	0.2206 2.1373*	-0.2730 -2.4567*	0.0142 0.7298	-0.0248 -1.6240

* Significant at the five percent level

CHAPTER 6

CONCLUDING REMARKS

1. SALIENT CONCLUSIONS

In a relatively short span of time, ARCH models have grown from being an obscure econometric alternative to an almost universally accepted technique for modelling financial time series. Unlike some other modelling techniques, which seldom find their way out of the academic world into the practitioner's world, ARCH models are gaining acceptance as useful methods for predicting changes in volatility. In fact, an article in *Euromoney* term them as "*a revolution in techniques ... to address the problems created by changing volatility in the prices of financial assets*"[1].

This dissertation serves to extend the current literature on ARCH models in various directions. First, it explore empirically the volatility structure as proposed by different versions of these models. Second, it employs these models to investigate various related dynamics, namely trading and nontrading volatility, and volatility and volume relationship. Finally, in the absence of theoretical models of volatility, it suggests a framework in which such models can be applied to empirical research.

The findings of this dissertation can be summarised as follows:

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[1] "The Search for a Better Model of Volatility" by Martin Brookes, *Euromoney*, March 1993.

(1) ARCH models are generally good descriptions of the temporal variation in volatility. Over the 20 year period that we have examined, we do not find evidence of misspecification in the models, despite using it for out-of-sample predictions. This indicates that ARCH models are unbiased estimators of the actual variance.

(2) The choice of a conditional normal distribution appears to be inadequate for the models that we have examined. This is because the standardised residuals still exhibit skewness and kurtosis that are significantly different from normal.

(3) The stock returns appear to display some asymmetry in volatility. As a result, models that account for this asymmetric effect have significantly higher log-likelihood values.

(4) The recent diagnostic test introduced by Engle and Ng (1993) appears to have low power to distinguish between the various parametric ARCH models that we have investigated.

(5) The performance of the ARCH models are not consistent across time, since certain models perform better than others in one period, but not in another period. Their performance also do not appear to be size related.

(6) Superior performance appears to be related to the ability to predict large volatility shocks. The choice of a benchmark that gives greater weight to predicting smaller shocks results in similar prediction performances.

(7) The volatility shocks of large companies appear to affect the future volatility of small companies, but the opposite effect is not present.

(8) There are also differences between trading and nontrading shocks. Trading shocks appear to have a more pronounced effect on future volatility than nontrading shocks. This suggest that volatility is related to the trading activities of informed investors.

(8) Volatility shocks appear to be related to contemporaneous changes in trading volume rather than volume levels. This finding is not inconsistent with the information models that relate stock prices to volume, since changes in levels can be interpreted as a proxy for new information.

(9) A simple modification to the standard GARCH model that incorporate this joint volatility-volume relationship is proposed. This model appears to perform better than the standard GARCH model.

(10) Financial data is often contaminated by imperfections, such as holidays, weekends, calendar effects etc. Given the dynamic nature of ARCH models, failure to account for these effects may lead to incorrect inferences.

(11) Our examination of some of the current methods of adjusting for nontrading effects reveals that they are inadequate.

(12) A simple diagnostic test to detect omitted dummy variables in the conditional mean and variance equation is proposed. It is suggested that this more objective test be used instead of ad hoc adjustments.

(13) Using intraday futures data, our analysis of the behaviour of conditional volatility around a nontrading period reveals a pattern similar to that of unconditional volatility. Conditional volatility appears to increase prior to market closure, with the greatest increase occurring when the market is closed. Once the market reopens, a downward shift in conditional volatility is observed.

2. GENERAL DIRECTIONS FOR FUTURE RESEARCH

As far as modelling techniques go, ARCH models are still relatively young in terms of chronological age. Despite their youthfulness, ARCH models have gained almost universal acceptance in the academic community. Nevertheless, significant progress can still be made in various areas. We identify them along the following lines:

(a) Theoretical specifications of ARCH models

(b) Empirical specifications of ARCH models

(c) Applications of ARCH models

2.1 Theoretical Specifications of ARCH Models

In Chapter 2, we have illustrated how easy it is to combine salient features from different specifications into a more general model. Indeed, one can look upon this as a search for a "better model of volatility"[2]. Unfortunately, traditional economic theories fail to provide any guiding principles. In spite of this, there are several potential aspects which merit some investigation.

The first comes from the literature on stochastic volatility. Several papers have recently attempted to reinterpret ARCH models as approximations of continuous time processes (see Nelson, 1990, and Bollerslev, Engle, and Nelson, 1993). Generally, stochastic volatility models are algebraically more tractable but computationally less feasible. Establishing a link between the two not only increases our understanding of the dynamics of asset prices, but has practical implications as well (for example in computing forecasts of volatility).

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[2] See Footnote (1) of this Chapter.

The second area worth exploring is in the incorporation of specific regularities of financial data. A notable example is the leverage effect, which has motivated various asymmetric models such as EGARCH, TGARCH and QGARCH. We could think of one other regularity: conditional skewness. It is generally accepted that investors prefer a skewed distribution. Future research could focus on how to *meaningfully* model conditional third (or even higher) moments.

Another potential research topic concerning the theoretical specification of ARCH models lies in the cross-sectional properties of ARCH models. A well known fact from portfolio theory is that the *unconditional* variance of a portfolio is usually less than the sum of the unconditional variance of the constituent securities. It remains to be seen if this property carries over to the conditional distribution. We believe that this can be important, especially in hedging decisions.

2.2 Empirical Specification of ARCH Models

A researcher using ARCH models in empirical work is often required to make judgment calls regarding the empirical specification of the model. Depending on the purpose of the research, a wrong specification may or may not be important. For example, if one is using the model to forecast volatility, in most cases, the forecast will still be accurate, despite the model being misspecified (Nelson, 1992a). On the other hand, if the model is used to infer the significance of a variable, a misspecified model (either in the mean/volatility equation or in the choice of the distribution) may lead to an incorrect inference. Given the contaminations of financial data, this further complicates matter. In light of this, investigating the properties of ARCH models and introducing new diagnostic tests represent an avenue for future research.

2.3 Applications of ARCH Models

Perhaps the biggest potential of ARCH models lies in its empirical applications. Chapter 3 of this dissertation uses a conditional volatility model to explore the dynamics of volatility between large and small firms, and between nontrading and trading periods. Similar techniques can be used to explore other relationships. We cite two potential area of applications.

First, ARCH models can be used to investigate situations in which information flow is expected to affect future pricing dynamics. For example, earnings announcements may affect the volatility of the stock and induce persistence in the variance. The use of ARCH models will allow one to dynamically infer whether or not the announcement is anticipated.

Second, we can use ARCH models to examine volatility dynamics. Antoniou and Foster (1992) use the GARCH model to examine the impact of futures trading on the volatility of the underlying asset; Cheung and Ng (1990) examine the lead-lag relationship in volatility between futures contract and the underlying asset with a GARCH model. Other relationships that can be explored include the lead-lag relationship between the volatility in options market and the volatility of the underlying asset, the intraday pricing dynamics of different assets and their interrelationships, the relationship between conditional volatility of an index and the conditional volatility of the constituent of the index, etc. Given the ease in which ARCH models can be implemented, the list here is limited only by one's imagination.

3. EXTENSIONS OF DISSERTATION

In the previous section, we suggest some general directions for future research in conditional volatility. In this section, we identify specific areas for future research arising from this dissertation.

a) In Chapter 2, we compare the performance of various parametric ARCH specifications in modelling and forecasting volatility. However, there are other measures of volatility (e.g. historical volatility, implied volatility etc) which could potentially offer similar performance. It would be interesting therefore to compare the performance of ARCH models in relation to these alternative measures of volatility.

b) Our comparison of the performance of ARCH models uses daily UK stock returns data. It is not inconceivable that some models will perform better using higher (or lower) frequency data than other models. One may therefore wish to compare the performance of the different ARCH specifications in relation to the frequency of the dataset.

c) Another area which merits investigation is the performance of the different ARCH specifications in forecasting volatility. Certain models may be better at forecasting short-term volatility, while other models may be better at forecasting medium or long-term volatility.

d) Chapter 3 documents an asymmetric predictability in the daily conditional volatilities of firms of different sizes. One may also wish to look at the extent of this predictability in intradaily (e.g. hourly, half-hourly etc) conditional volatilities.

e) The results of Bollerslev and Domowitz (1991) show that the trade execution process can significantly alter the intertemporal patterns in volatility. An extension of Chapter 3 can be made by looking at the relationship between the trade execution process and the degree of asymmetric predictability in the conditional variances.

f) The trade execution process can also affect the relationship between volatility and volume. In that respect, the results of Chapter 4 may be applicable only to a quote-driven trading system, as opposed to an order-driven trading system. It would therefore be interesting to compare the empirical relationship between volatility and volume across different markets with alternative trading mechanisms.

g) Given that volatility and volume are jointly determined, Chapter 4 can be extended by using techniques that simultaneously estimate both the volatility and volume parameters (e.g. VAR, multivariate GARCH etc).

h) The test developed in Chapter 5 can be applied to examine the conditional volatility of asset returns surrounding specific macroeconomic and microeconomic events, such as announcements of interest rates changes, earnings announcements, introduction and expiration of options and futures contracts etc.

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