

# **Transmission Congestion Management and Security Cost Optimization in Deregulated Electricity Markets**

A thesis presented for the degree of

*Doctor of Philosophy*

by

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## **Declaration**

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## **Abstract**

Two sensitivity factors, namely the power transfer distribution factor (PTDF) and the impedance based sensitivity factor (ZS), are proposed respectively in the thesis to calculate the power flow contribution of each bus to the transmission network congestion. Generation re-dispatch and the corresponding security cost allocation are based on the calculated power flow contribution. The proposed generation re-dispatch method and the proposed security cost allocation method both with two sensitivity factors are validated respectively on four test systems including a modified IEEE-14 bus system, a modified IEEE-30 bus system, a modified IEEE-57 bus system and a modified IEEE-118 bus system. The experimental results are obtained including the generation re-dispatch MW amount of each generator selecting different locations of slack bus, the resulted security cost and the security cost allocation to each load. The comparisons between two sensitivity-based methods on both generation re-dispatch and security cost allocation show that the PTDF-based method and the ZS-based method are both dependent on the location of slack bus selection, which increase the risks of inaccuracy and complexity. The ZS-based method involves more generators in re-dispatch stage than the PTDF-based method, consequently results in higher security cost. Moreover, with intense level of congestion, such security cost difference between the two methods becomes larger. Compared with the ZS-based method, the PTDF-based method is more reliable on security cost allocation since in large bus system, for example, the 118-bus system, the allocation results by the ZS-based method appears unreasonable.

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# Nomenclature

NETA	New Electricity Trading Arrangement
BETTA	British Electricity Trading and Transmission Arrangements
CfD	Contract for Difference
FTR	Financial Transmission Right
PTDF	Power Transfer Distribution Factor
ZS	Bus Impedance Based Sensitivity Factor
ISO	Independent System Operator
FERC	Federal Energy Regulatory Commission
CEGB	Central Electricity Generating Board
RECs	Regional Electricity Companies
GOAL	Generating Ordering and Loading
SMP	System Marginal Price
PPP	Pool Purchase Price
SMP	System Marginal Price
CP	Capacity Payment
LOLP	Loss of Load Probability
VOLL	Value of Loss Load

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PSP	Pool Selling Price
NGC	National Grid Company
IPNs	Initial Physical Notifications
FPNs	Final Physical Notifications
SSP	System Sell Price
SBP	System Buy Price
BSUoS	Balancing Service Use of System
TSOs	Transmission System Operators
OTC	Over the Counter
RTO	Regional Transmission Organization
LMP	Locational Marginal Pricing
CPUC	California Public Utilities Commission
PX	Power Exchange
CAISO	California Independent System Operator
FACTS	Flexible AC Transmission System Devices
OPF	Optimal Power Flow
DCOPF	DC Optimal Power Flow
ACOPF	AC Optimal Power Flow
TCC	Transmission Congestion Contract
UMP	Uniform Marginal Price
LO	Level of Overloading
$C_{G_i}(P_{G_i})$	The bid-based generation cost of generator $i$

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$P_{G_i}$	The active power generated by generator $i$
$P_{L_j}$	The demand prediction of load $j$
$N_G$	The total number of generators
$N_L$	The total number of loads
$P_{G_i}^{max}$	The maximum generation capacity of generator $i$
$P_{line_k}$	The power flow on the transmission line $k$
$P_{line_k}^{max}$	The maximum power flow transmission capacity of the transmission line $k$
$bid\ price_{off}$	The bid price of the “constrained off” generator submitted to the system operator during the unconstrained dispatch
$bid\ price_{on}$	The bid price of the “constrained on” generator submitted to the system operator during the unconstrained dispatch
$P_i^g$	The real power generated at node $i$
$C_{0i}$	The no-load cost coefficient of generator $i$
$C_{1i}$	The linear cost coefficient of generator $i$
$C_{2i}$	The quadratic cost coefficient of generator $i$
$P_i^d$	The real power demand at node $i$
$P_{ij}$	The power transmission flow from node $i$ to node $j$
$P_{i,min}$	The minimum generation limitation of node $i$
$P_{i,max}$	The maximum generation limitation of node $i$
$P_l$	The real transmission power flow on the line $l$
$P_l^{max}$	The real power maximum transmission capacity of line $l$

$P_{loss}$	The transmission losses in the power system
$\lambda_0$	The Lagrangian multiplier of the whole system power balance constraint
N	The total number of nodes
L	The total number of transmission lines
$\mu_l$	The Lagrangian multiplier of transmission line constraint
$\pi_i^{min}$	The Lagrangian multipliers of minimum generation capacity of generator $i$
$\pi_i^{max}$	The Lagrangian multipliers of maximum generation capacity of generator $i$
$T_{i-1}$	The sensitivity factor for real power at node $i$ with line $l$ constraint
$LMP_i^{energy}$	The system marginal cost of node $i$
$LMP_i^{congestion}$	The cost of transmission congestion of node $i$
$LMP_i^{loss}$	The cost of transmission losses of node $i$
$Power_{contract}$	The power volume signed in the CfD contract
$price_{reference}$	The reference price which is often the spot market price
$price_{strike}$	The agreed fixed price signed in the CfD contract
$Power_{FTR}$	The active power delivered from source node to sink node
$price_{sink}$	The nodal price of the sink node
$price_{source}$	The nodal price of the source node
$PTDF_i^{j-k}$	The PTDF of the line $j-k$ respect to bus $i$
$\Delta P_{j-k}$	The active power flow variation of line $j-k$

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$\Delta P_i$	The active power injection variation at bus $i$
$P_{j-k}$	The active power flow between bus $j$ and bus $k$
$V_j$	The voltage of bus $j$
$V_k$	The voltage of bus $k$
$\theta_j$	The phase angle of bus $j$
$\theta_k$	The phase angle of bus $k$
$Z_{j-k}$	The impedance connected between bus $j$ and bus $k$
$x_{j-k}$	The reactance connected between bus $j$ and bus $k$
$b_{j-k}$	The susceptance between bus $j$ and bus $k$
$[P]$	The matrix of active power injection at buses
$[\theta]$	The matrix of phase angle of bus voltages
$[B]$	The susceptance matrix
$[B^*]$	The matrix that eliminating one row and one column of $[B]$
$[M^*]$	The invert of $[B^*]$
$[M]$	The matrix is from $[M^*]$ by adding one row and one column of 0 according to the corresponding reference bus or slack bus.
$M_{ji}$	The $j_{th}$ row and $i_{th}$ column element in the M matrix
$M_{ki}$	The $k_{th}$ row and $i_{th}$ column element in the M matrix
$C_{G_i}$	The incremental/decremental price of generator $i$
$\Delta P_{G_i}$	The incremental/decremental amount of active power injection of generator $i$

$PTDF_{G_i}^{j-k}$	The PTDF of the line $j-k$ respect to bus $i$
$P_{j-k}^{lim}$	Transmission capacity limit of line $j-k$
$P_{j-k}^{uncon}$	The active power flow of line $j-k$ in unconstraint dispatch
$P_{G_i}$	The generation amount of generator $i$ in unconstraint dispatch
$P_{G_i}^{max}$	The generation capacity of generator $i$
$P_{j-k}^{uncon}$	The active power of congested line $j-k$ in unconstrained dispatch
$P_{j-k}^{lim}$	The limit value of active power transmission capacity of congested line $j-k$
$I_i$	The nodal current in bus $i$
$Y_{ij}$	The admittance matrix of the n-bus power system
$V_j$	The voltage on bus $j$
$Z_{ij}$	The impedance matrix of the n-bus power system
$Y_{bus}^*$	The reduced admittance matrix of the n-bus power system
$S_{j-k}$	The complex power flow of line $j-k$ calculated at bus $j$
$V_j$	The bus voltage at bus $j$
$I_{j-k}^*$	The conjugate of current through line $j-k$
$Z_{ji}$	The $j_{th}$ row and $i_{th}$ column element in the impedance matrix
$I_i$	The nodal current at bus $i$
$I_{j-k}$	The current through line $j-k$
$V_k$	The bus voltage at bus $k$
$y_{j-k}$	The series admittance of the $\pi$ equivalent circuit of line $j-k$

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$y_{j-k}^{sh}$	The shunt admittance of the $\pi$ equivalent circuit of line $j-k$
$Z_{ki}$	The $k_{th}$ row and $i_{th}$ column element in the impedance matrix
$I_i^*$	The conjugate of nodal current at bus $i$
$P_i$	The active power injected at bus $i$
$Q_i$	The reactive power injected at bus $i$
$V_i$	The voltage at bus $i$
$ZS_i^{j-k}$	The bus impedance matrix sensitivity of line $j-k$ respect to real power injection at bus $i$
$\Delta P_{j-k}^i$	The portion of active power flow change through line $j-k$ only associated with bus $i$ power injection change
$\Delta P_i$	The power injection change of bus $i$
$ZS_{G_i}^{j-k}$	The bus impedance matrix sensitivity of the line $j-k$ respect to bus $i$
$SC_{total}$	The total security cost
$SC_{gens}$	The security cost portion for generators
$SC_{loads}$	The security cost portion for loads
$SC_{G_i}$	The security cost allocated to generator $i$
$P_{G_i}$	The MW power generation of generator $i$
$SC_{L_j}$	The security cost allocated to load $j$
$P_{L_j}$	The MW power consumption of load $j$
$N_G$	The number of generators in the system



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$N_L$	The number of loads in the system
$P_{j-k}$	The power flow MW value of the target transmission line $j-k$
$P_{j-k}^G$	The power flow MW value on line $j-k$ associated with all the generators
$P_{j-k}^L$	The power flow MW value on line $j-k$ associated with all the loads
$PTDF_{L_i}^{j-k}$	The PTDF of the line $j-k$ respect to load bus $i$
$P_{L_i}$	The MW power energy withdrawn by load $i$
$P_{j-k}^{L_i}$	The power flow MW value associated with load $i$
$Ctb_{L_i}$	The power flow contribution percentage of load $i$ to the congested line
$CA_{L_i}$	The allocated security cost to the load $i$
$SC_{total}$	The total security cost due to congestion management
$ZS_{G_i}^{j-k}$	The ZS of the line $j-k$ respect to generator bus $i$
$ZS_{L_i}^{j-k}$	The ZS of the line $j-k$ respect to load bus $i$

# Chapter 1

## Introduction

### 1.1. Overview

Power industry is initially controlled by one utility which is normally vertically integrated and regulated by the government. This utility controls generation, transmission and distribution services. Customers have no other choices but have to purchase electricity from the utility and the electricity price is at a set rate in the controlled area. The single utility holds all the information of power system operation, some of which are utilized to centrally dispatch and re-dispatch generation schedules. Moreover, all the planning activities of power system such as generation plant investment and maintenance of transmission network are also centrally supervised by the utility. This monopolistic utility model has been adopted for a long time. As time progresses, new requirements for power energy industry appear such as cost reducing, efficiency improvement and long-term development, which pushed the government to unbundle the vertically integrated utility into generation, transmission and distribution portions [1]. It is believed that with the restructuring of the power energy industry, a competitive atmosphere can be introduced to achieve the above requirements. Due to the market open access, incentives will be given so that investment from the society can contribute to long-term development of the power industry. After almost forty years discovery, global achievements on electricity industry restructuring have been gained such as the “England & Wales Pool” in the UK, which now is further evolved as the British Electricity Trading and Transmission Arrangements (BETTA). The original England & Wales Pool was a practical pool model electricity market while the BETTA is now an electricity market, which is built based on a bilateral market principle; the “Nord Pool” in Europe and the PJM interconnection in the US, which are both hybrid electricity models combining features of pool model and bilateral model

[2-6].

In the power system, transmission lines are restricted by several constraints such as thermal constraint, voltage constraint and stability. To ensure the reliability of the transmission network system, steady state security needs to be assessed to monitor the temperature of transmission lines [7]. When the thermal constraint of the transmission line is exceeded, the transmission line will be overloaded, which is also known as transmission congestion. Congestion can cause cascade outages, increase losses, build up barrier to energy transactions, increase electricity price and result in market power, which above are threats to the system security, so congestion issue cannot be tolerated [8]. In the traditional vertically integrated structure, resolving the congestion can be done by re-dispatching the generation schedule because it is centrally controlled [9]. However, congestion management in deregulated electricity market becomes more complex and frequent since transmission network is based on an open access principle to all the market participants so power flows are determined by power trades. Market participants only consider the maximization of their economic interests rather than the transmission network physical limitations. Hence, congestion occurs when the transmission lines have not sufficient capacity to transfer the desired power in the open market [10]. The most direct way to solve transmission congestion is to expand transmission network to provide enough power transfer capacity. But planning and construction of a transmission line lasts a long time and the investment of transmission network costs a lot. Consequently, congestion management is regarded as a practical tool for efficiently utilizing the transmission network without violating the transmission constraints.

The aim of congestion management is to avoid congestion or to relieve congestion. A. Singla et al. [11] classifies the congestion management methodologies into non-market based methods and market based methods. The market based methods are implemented according to market mechanisms so economic signals can be sent to the market while this is not the case for non-market methods. Market based methods are more acceptable as

they can enhance liquidity and transparency of the electricity market. The market based methods can be subdivided into pricing based methods, financial instruments, re-dispatch methods, load-side managements and sensitivity factors based methods. References [12] [13] and [14] give a detailed description of the locational marginal price (LMP) especially the components of the LMP. Zonal pricing method is explained in [15]. X.Ma et al. [16] conducts a comparison among the uniform marginal pricing (UMP), the LMP and the zonal pricing in the aspect of providing economic signals. To help market participants dampen the price volatility, some financial instruments are introduced into electricity market. References [17] and [18] have a description of the financial transmission rights (FTR), which is used to ensure efficient allocation of transmission access rights in the LMP-based market. Reference [19] introduces another financial instrument called contracts for differences (CfD), which is a mid or long-term two way contract between the generator and the consumer to hedge the price volatility. As known, re-dispatch method was widely used in the vertically integrated utility and the pool based market. Combined with corresponding market mechanism, it can still solve congestion issues. Reference [20] proposed a re-dispatch method combined with incremental and decremental bids submission mechanism to relieve the congestion. Apart from the generation-side congestion management methods, some methods attempt to solve the problem at the demand-side. References [21] and [22] respectively try to curtail the demand-side load to relieve congestion. Besides, some congestion management methods seek for the help of transmission network sensitivity factors to relieve congestion. In [23], the power transfer distribution factors (PTDF) is proved to be a useful index for congestion management since it can calculate the sensitivity of each bus contribution to the congestion. K. Singh et al. [24] presents another sensitivity factor for congestion management, namely bus impedance matrix based sensitivity.

This thesis firstly proposes a generation re-dispatch method to prevent the transmission congestion in deregulated electricity market. The mechanism of incremental and decremental bids submission for the generators is combined. This re-dispatch method

can be deemed as a minimum cost optimization associated with the optimal power flow (OPF) and the submitted bids from generators, at the same time, restricted by transmission capacity constraints and generation capacity constraints. During the optimization, two sensitivity factors, the power transfer distribution factor (PTDF) and the bus impedance based sensitivity factor (ZS), are introduced to respectively calculate the power flow contribution of each node in the power system to the congested transmission line. The similarity of these two sensitivity factors is that they are both trying to find out the relationship between active power change quantity of a particular transmission line and the active power change quantity at a particular bus. The difference between these two sensitivity factors is that they are derived from different algorithms. The PTDF-based factor is calculated according to the admittance matrix while the ZS-based factor is calculated according to the impedance matrix. Before tests, the connection between each sensitivity factor and the location of slack bus selection will be investigated. Then both sensitivity-based re-dispatch methods will be tested in four modified IEEE-bus systems ranging from 14-bus up to 118-bus. According to different locations of slack bus selection, results will be the re-dispatched MW amount of each generator and the total security cost due to re-dispatch. The result comparison between two sensitivity-based methods under the same slack bus selection and the same level of overloading of the congested transmission line will be obtained and discussed. Moreover, this thesis tries to balance the re-dispatch action and the resulted security cost. Hence, two security cost allocation methods respectively associated with the above two sensitivity factors in deregulated electricity market will be proposed. The resulted security cost due to the congestion will be allocated by the two methods respectively, which are based on the calculated power flow contribution by the PTDF factor and the ZS factor. Similarly, the security cost allocation results by PTDF-based method under different locations of slack bus selection will be indicated. Then the allocation performance comparison between two methods will be shown and discussed.

## 1.2. Objectives of the Thesis

The objectives of this thesis are shown as follows:

- To review former and current deregulated electricity market experiences across the world; discuss and compare the advantages and disadvantages of each deregulated electricity market model.
- To review typical transmission congestion management schemes in deregulated electricity market and to discuss and compare the pros and cons of each congestion management scheme.
- To propose a generation re-dispatch method associated with the sensitivity factor PTDF and the market bidding mechanism to relieve transmission congestion under a minimum cost principle. To investigate the influence of slack bus selection on the results of the PTDF-based method.
- To propose a generation re-dispatch method associated with the sensitivity factor ZS and the market bidding mechanism to solve the transmission congestion problem under a minimum cost principle, and to differentiate the influence of slack bus selection. Compare the re-dispatch results between the PTDF-based method and the ZS-based method.
- To develop a security cost allocation method associated with the sensitivity factor PTDF and to utilize it to calculate the power flow contribution of each node to the congested transmission line. Then allocate the security cost based on the calculated power flow contribution. To investigate the impact of slack bus selection on the results of the PTDF-based method.

- To develop a security cost allocation method associated with the sensitivity factor ZS. With the help of the ZS, to calculate the power flow contribution of each node to the congested transmission line. Then allocate the security cost based on the calculated power flow contribution. Compare the security cost allocation results between the PTDF-based method and the ZS-based method.

### **1.3. Original Contribution of the Thesis**

The major original contributions of the thesis are listed as follows:

- A proposed generation re-dispatch method associated with the sensitivity factor PTDF and the market bidding mechanism is developed, which is used to relieve the transmission congestion in deregulated electricity market. The proposed method is capable to re-dispatch generation schedule for congestion management in four test systems. The effect of location of slack bus selection on the proposed PTDF-based method is verified.
- A proposed generation re-dispatch method associated with the sensitivity factor ZS and the market bidding mechanism is developed. The aim is to relieve the transmission congestion in deregulated electricity market. The method is tested in four test systems and proved to be capable. The dependency of the location of the slack bus selection of the ZS-based method is confirmed. Comparisons with the PTDF-based method indicate the generators involved in the ZS-based method for re-dispatch are more than the PTDF-based method. Consequently, the resulted security cost in the ZS-based method is more than the PTDF-based method. Moreover, with the intense congestion, the resulted security cost difference between two methods becomes larger.

- A proposed security cost allocation method associated with the sensitivity factor PTDF is developed. The method is based on the calculated power flow contribution of each bus to allocate the security cost as a result of congestion. The proposed method is applied to four test systems and the obtained results proved that the method is working. Nevertheless, the dependency of location of slack bus selection increases the risks of inaccuracy and complexity of security cost allocation.
- A proposed security cost allocation method using sensitivity factor ZS is developed, which is based on the calculated power flow contribution of each bus to allocate the security cost as a result of congestion. The method is tested in four systems and the results prove it is capable. Parallel comparisons between the ZS-based method and the PTDF-based method indicate that the above two methods have different ideas to allocate the same value of security cost. When implemented in large power system, the ZS-based method has the risk of providing unreasonable solutions.

#### **1.4. Outline of the Thesis**

Chapter 1 gives an introduction of the whole thesis. Firstly it provides a brief explanation about electricity market deregulation, transmission congestion, transmission congestion management and the work included in this thesis. Furthermore, the objectives, the major original contributions and the structure of the thesis are presented.

Chapter 2 describes the traditional vertically integrated electricity utility and its major drawbacks which can be regarded as the motivation of electricity industry restructuring. A description of the electricity industry deregulation is given including the history of deregulation, the new structure and the expectations of the deregulation. A pool



market model, a bilateral market model and a hybrid market model are introduced as the three typically deregulated electricity market models. The role of the independent system operator is explained together with its characteristics and functions. Finally, global experiences of major deregulated electricity market are introduced one by one including the old England and Wales Pool, the British Electricity Trading and Transmission Arrangements, the Nord Pool, the PJM interconnection and the California crisis.

Chapter 3 introduces and discusses different types of transmission congestion management schemes in deregulated electricity market. Firstly three pricing schemes including the uniform marginal price, the locational marginal price and the zonal price are illustrated. Then five transaction curtailments based on five different principles are introduced. Furthermore, two financial instruments: the contract for difference and the financial transmission right are described. Finally, a case study is included to test the performances of the uniform marginal price and the locational marginal price in a modified IEEE-14 bus test system.

Chapter 4 proposes a method using sensitivity index PTDF to re-dispatch generation when a transmission congestion occurs in deregulated electricity market. Firstly, the mechanism of the proposed re-dispatch method is introduced. Secondly the algorithm of the sensitivity factor PTDF is presented. Thirdly, the mathematic model of the proposed method including an objective function and three constraints, one of which includes the sensitivity factor PTDF are described. Finally, case studies in this chapter consist of four test systems including a modified IEEE 14-bus system, a modified IEEE 30-bus system, a modified IEEE 57-bus system and a modified IEEE 118-bus system. The proposed re-dispatch method with the PTDF is tested in the above four test systems and the test results are included.

Chapter 5 proposes another generation re-dispatch method using the bus impedance matrix sensitivity factor. Firstly the methodology of the proposed method and the algorithm of the sensitivity factor  $ZS$  are described. Then the mathematic model of the proposed method is given, which is similar with chapter 4, but the only difference is it uses the bus impedance matrix sensitivity factor as the sensitivity factor. Finally, the proposed method is tested in the same four systems as chapter 4. Result comparisons between the  $ZS$ -based method and the PTDF-based method are included.

Chapter 6 proposes a security cost allocation method with the sensitivity factor PTDF. Firstly some conventional security cost allocation schemes are reviewed. Then the mathematic model of the proposed security cost allocation method using the PTDF factor is given. Finally, in the case studies, under the same four test systems previously used in chapter 4 and 5 respectively, experimental results are presented to show distinctions of allocation results by selecting different locations of slack bus in the PTDF-based method and the comparison between the pro rata method and the PTDF-based method.

Chapter 7 proposes another security cost allocation method using the sensitivity factor  $ZS$ . Firstly, the mechanism and the mathematic model of the proposed method are described. Then again the same four test systems used in chapter 4, 5 and 6 respectively are used to recur the tests, which provide a briefly comparison among three security allocation methods: the pro rata based method, the PTDF-based method with the first generator bus as the slack bus and the  $ZS$ -based method using the same slack bus.

Chapter 8 gives a summary of the thesis and the possible future research directions.

## 1.5. Publication

- Jiawei Zhao, Jianfeng Lu and K.L.Lo, “Various worldwide types of deregulated electricity markets and their respective transmission congestion management schemes”, in 51st International Universities' Power Engineering Conference (UPEC), Coimbra, Portugal, 2016.
- Jiawei Zhao, Jianfeng Lu and K.L.Lo, “Review of methods to calculate congestion cost allocation in deregulated electricity market”, in 6th World Congress on Engineering and Technology (CET): 6th Power Engineering and Automation Conference (PEAM), Shanghai, China, 2016.
- Jiawei Zhao, Jianfeng Lu and K.L.Lo, “A transmission congestion cost allocation method in bilateral trading electricity market”, in 9th Asia-Pacific Power and Energy Engineering Conference (APPEEC), Chengdu, China, 2017.
- Jiawei Zhao and K.L.Lo, “Two sensitivity-based methods to re-dispatch generation in the deregulated electricity market”, under preparation for journal submission of the IET Journals Generation Transmission & Distribution.

# Chapter 2

## Deregulated Electricity Market

### 2.1. Introduction

In modern life, electricity is a necessity. With the high speed development of human civilization, electricity capacity allocation has become a very essential topic for power system research. Several requirements came along for power energy industry such as reducing the cost, improving efficiency and maintaining the long-term development [1]. As a result, since 1980s, many countries in the world started to unbundle their vertically integrated power utility into a competitive market environment because of the development of energy industry and demand of customers. It is believed that with the introduction of competition, liberalized electricity market will theoretically be able to satisfy the above requirements thus electricity price could get close to the marginal cost of generation, companies will minimize their production cost and customers could get a cheaper price for electricity [9]. The restructuring in other industries such as railway, telecommunication and wireless communication has obtained a series of successes which also stimulates the progress of electricity industry deregulation. However, deregulation of electricity market is not simple, for example, electricity is unable to be stored easily so supply and demand must be balanced all the time. As a consequence, there are issues in the deregulation route both from technical side and economic side. The objective of energy industry restructure is to create a competitive environment for electricity trading in order to improve social welfare [25]. In fact, the motivations for electricity reform are always complex and political. In most cases, restructure progress will be more or less influenced by party politics [26]. Introducing competition into electricity market represents that government is willing to set up a sound and orderly energy system which could be utilized to stabilize power system operation, reduce the cost of power generation, decrease the

electricity price to customers, promote high technology and maintain the long-term development of electric power system.

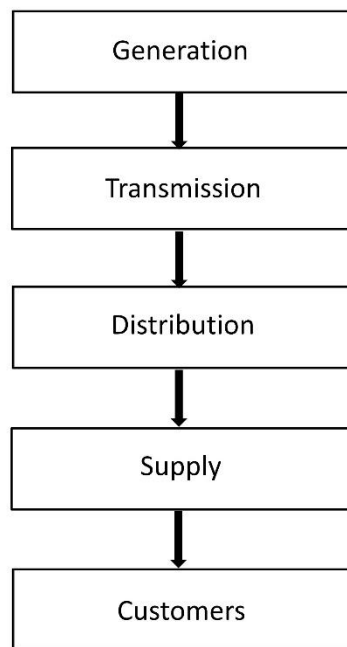
One of the earliest deregulated electricity markets in the world was set up by the United Kingdom in 1990, called the “England & Wales Pool” [2]. It has been regarded as a remarkable template for the worldwide electricity deregulation because it broke the traditional vertically integrated structure and introduced competition into the generation side. Ten years later, “English Pool” was replaced by the New Electricity Trading Arrangement (NETA), which was implemented in England and Wales and was based on bilateral principles to let generation side and demand side trade power by contracts [3]. Five years later, with the joining of Scotland electricity, NETA became the British Electricity Trading and Transmission Arrangements (BETTA) [4]. In Europe, there is another successful electricity market called “Nord Pool” which consists of several Nordic electricity markets [5]. It was the first multinational electricity market in the world. In North America, Americans also learnt to set up their own competitive electricity market. Different states adopted different approaches to manage their power system operation. One of the most famous electricity markets is the PJM interconnection which is the largest deregulated wholesale electricity market in the world dominates power systems in Pennsylvania, New Jersey, Maryland and other states in the eastern part of United States [6]. Unfortunately, all the deregulated electricity markets have to face challenges. For example, California Market, in the summer of 2001, a series of rolling blackouts were caused by electricity market crisis [27].

In this chapter, history of deregulated electricity market will be introduced including the contents such as traditional integrated electricity market in section 2.2 and motivation of reform old power utility in section 2.3. Also, categories of current electricity market models will be briefly explained in section 2.4 mainly at pool market, bilateral market and hybrid market. The role of independent system operator will be discussed in

section 2.5. An important part of this chapter is section 2.6 which includes the detailed contents about the type of deregulated market each country adopted. Finally, section 2.7 will give the summary of this chapter.

## 2.2. Traditional Vertically Integrated Electricity Utility

In many countries, prior to deregulation, generation, transmission and distribution were dominated by one large central utility which was mostly a monopoly and state-owned by government [28]. Figure 2.1 shows the structure of vertically integrated electricity utility [9].



**Figure 2.1 Vertically Integrated Utility Structure**

These central companies have authority to design and control the whole operation of power system in their designated regions. Consumers in such region only have one choice of power supply. The traditional vertically integrated utility holds all the information and data of power system operation and they dispatch and re-dispatch

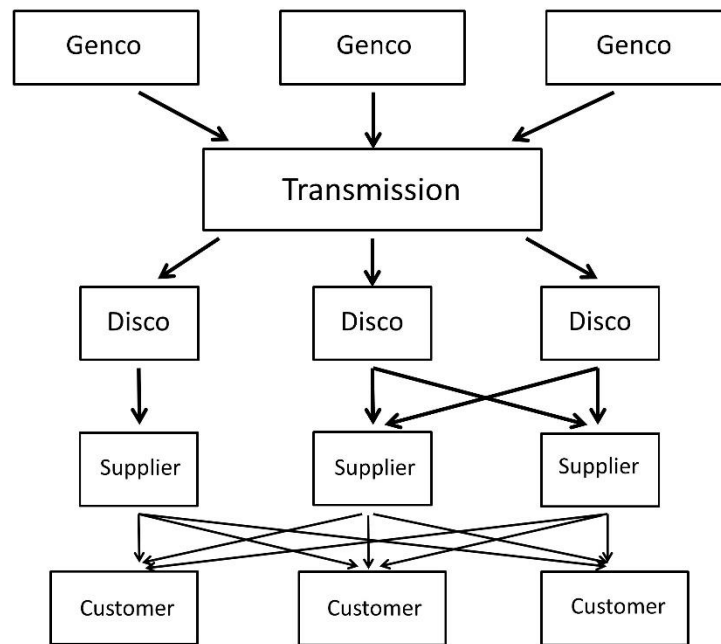
generation units by optimal power flow software to match supply and demand in order to keep power balance. In the vertically integrated mode, the setting of electricity price should be based on the overall cost of generation, transmission and distribution and price is uniform in the controlled region. Not only setting the price, the centralized company also controls the power network planning such as long-term generation investment and transmission distribution network maintenance. It is claimed that state-owned electricity utility is not as good as private ownership in the domain of efficiency improvement since state-owned companies is willing to focus more on budget maximization than costs minimization [29]. Hence, large-scale investment always leads to waste and inefficiency rather than cost reduction and better service. The risk of large investment will be placed onto consumers' shoulders. Due to lack of competition in power market, there is a high possibility that consumers would be faced with quite a high price of power supply. To sum up, electricity industry really needs to be restructured and deregulated so as to improve the efficiency and decrease the electricity price closer to marginal price.

### **2.3. Generally Understanding of Electricity Industry Deregulation**

In the early 1980s, industry restructuring, deregulation, liberalization and privatization as a combinational concept became an irresistible trend spreading all round the world. Many nationalized industries have been successfully deregulated and some achievements have been obtained so it was suggested that electricity industry should also be restructured, deregulated and liberalized therefore competitive trading can be introduced. Chile was the first country in the world to implement electricity sector reform which began in 1982 [30]. Then the consequence is, UK, United States and Nordic countries were involved in the electricity industry deregulation in 1990s to open their electricity markets.

The purpose of electricity industry deregulation is that the traditional vertically integrated industry should be unbundled into generation, transmission and distribution sections and in each section competition will exist. Figure 2.2 shows the structure of

unbundled electricity industry [3]. As a result, both buyers and sellers in electricity market can benefit from this utility structure renewal because electricity price would be pushed close to the marginal cost. Moreover, with the market opening, plenty of incentives will be created so that investments from society could ensure long-term development on power industry.



**Figure 2.2 Structure of Unbundled Power Industry [3]**

It is not a wise idea to set up a totally new structure of electricity market because it will require large amount of time and finance. As a consequence, the best thing to do is to break the former monopoly structure and introduce competition into basic components. After deregulation, the former monopoly generation company is divided into several smaller generation companies which start to compete with each other, in the meantime, some new established generation firms are able to have a chance to join in the competition. Distribution section follows the same rule for restructure. The dominated distribution company is replaced by many independent ones. However, due to inflexible feature of distribution hardware, a distribution company in a specific region operates in a



monopolistic characteristic. A distribution company can assign a right to a supplier to use the distribution network or a distribution company itself can play the role of supplier for customers' power usage. With the naturally monopolistic feature, transmission network is unable to be split up into several regional small transmission companies. Since competition cannot economically sustain, transmission network utility still stay at monopoly status. In addition to this, electricity market deregulation causes the growth of transmission congestion inescapability. An independent agency called System Operator is set up to control and supervise the operation of transmission system. In some cases, Transmission Company also undertakes the job of system operator. The independence of system operator points out an effective approach to balance the conflicts between economic interests and transmission security [31].

#### **2.4. Difference between Previous and Existing Deregulated Electricity Market Structures**

It is well known that there is no effective way to store large quantity electricity efficiently and cheaply, moreover, and hence supply and demand must be balanced all the time. As a consequence, competitive market for electricity should be distinguished from any other deregulated utility markets. Due to the trend of deregulation, restructuring and privatization, competitive electricity marketplaces are expected to be established in the way of breaking the linkage of former monopoly market structure not rebuilding a whole new electricity market [32]. The most significant issue of electricity market reform is how to effectively arrange power transaction at the meantime ensure the security of system operation. In fact, there is no such simple solution that can solve diverse problems and issues once and for all because different nations have discriminating political rules and various market situations. However, each type of electricity market deregulation should always follow one principle that generation, transmission and distribution/supply will be split up to let competition to be introduced into market environment so that electricity sellers and buyers could be benefited from the structure reform. Even though the goal is identical, routes for that by different nations appear not exactly the same. Those leading

countries of electricity reform have already obtained their own successful achievement for competitive marketplace establishment in some degree. Each competitive structure has its own feature and characteristic. Typically, there are three templated reformed electricity market structures which are pool market, bilateral market and hybrid market. It is very necessary to understand similarity and distinction between each deregulated electricity marketplace structure. Each of them has both strength and shortage and it will be a wise way to use the strong points to offset the weaknesses.

#### 2.4.1. Pool Market

The pool concept is regarded as the first innovative step which has a historical significance for electricity market restructure. In this type of deregulated marketplace, the independent system operator is called pool operator who coordinates all the power supply and manages the whole process of power trading [33]. All the generation companies need to upload their planned generation output information to the system operator normally one-day ahead. At the meantime, generation companies should submit their electricity bid price to the pool operator respectively. Then system operator starts to forecast the total electricity demand quantity on the date of generation uploaded. Based on generation cost minimization principle, pool operator sets up a list to determine which generation companies should generate electricity onto the power transmission network system. The list must be ranked to follow an increasing order from cheapest bid price to a higher price until the forecasted demand can be satisfied. The pool operator will set a series of pool prices by software calculation based on the received bids from generators and it has the authority to explain the details of these price settings. Finally generators will be paid by a uniform buying price and customers will be charged by a uniform selling price.

Technically, pool markets can be distinguished into two groups: mandatory pool and voluntary pool. In mandatory pool, all the power energy trades must be transacted in the pool; Singapore, Australia, and the former UK Pool are three typical examples of mandatory mode [9]. While, in voluntary pool, electricity trades are allowed to be made

outside the pool system. For example, the Nord pool could be considered as a typical voluntary pool market since it allows market participants make power transaction outside the pool market.

At the beginning, as the pioneer attempt of electricity utility reform, pool market has achieved some effective successes since its similarly centralized market structure allowing former regulated electricity industry to adapt to it in a short time. Nevertheless, pool structure still has two main disadvantages. Firstly, if there are several dominate generation companies in the market, they probably attempt to increase their economic profits through strategic biddings which are also considered as exercising a market power. In general, two approaches could be utilized to exercise market power: bidding high directly and withholding capacity during peak time. As a result, electricity price in the pool market will probably not decrease with the drop of generation cost so that customers are unable to benefit from industry deregulation [40]. Secondly, pool structure market only concentrates on the competition in generation end while demand side is paid much less attention [3]. Due to such above issues, a new mode of energy trading is expected to be introduced to overcome the problems which bothers the market participants in pool market.

#### 2.4.2. Bilateral Market

In an ideal bilateral marketplace, any power consumer can purchase electricity from any electricity generation company who can also sell electricity to any customer. Power trading could be done through bilateral contracts individually by buyer and seller.

By now, in real life, BETTA (British Electricity Trading and Transmission Arrangement) is a practical example of bilateral electricity market [3]. This time independent system operator only needs to concentrate on matching the real-time power balance and maintaining system operation security. Electricity price is bilaterally dealt by

generator and customer without the influence of the system operator. Contract could be signed in advance even for several years or as short as even for the last 24 hours before real delivery action. One hour before market gate closure, all the required information and data from both contractual sides should be handed in to the system operator such as power transaction quantities, transfer time and electricity offer/bid prices and such group of information is referred as the final physical notifications (FPNs). More details of the FPNs will be given in the following section. If all the power trades are accepted by the system operator, market participants who have contracts only need to follow the contractual details to pay and charge. But if any of the transactions caused transmission security problem, the system operator has the power to reject the energy trade and ask the market participants to adjust their contractual positions. The key point in bilateral market will be the imbalance settlement solution which takes place after the gate closure. System operator will make use of the above balancing mechanism to secure the power system operation [34].

In bilateral market, competition is directly introduced into the demand side which means demand-side customers can obtain more available price information and have more choices for their power source supply. With the increasing of market liquidity, the prices of electricity have been reduced which thanks to the bilateral mechanism. However, this advanced market scheme also has drawbacks. In order to avoid the penalty in balancing actions, some generators may intend to increase their power production and some suppliers may try to decrease their demand quantity. In this way, difficulty of power balance will be increased and the whole cost for power system operation will be increased. If so, imbalance settlement and penalty mechanisms will definitely be useless and the efficiency of bilateral market will no doubt drop down.

### 2.4.3. Hybrid Market

In the Hybrid market, participants process their electricity transaction inside the pool market and they can also arrange contracts with other market participants outside the pool market. Hybrid market can be regarded as the combination of pool market and bilateral market. At the moment, there are two typically successful hybrid markets among the global power markets: the Nordic market and the PJM (Pennsylvania, New Jersey, and Maryland) market.

Take Nordic market as an example, it is a multinational electricity market consists of electricity markets from Nordic and Baltic countries. Each of them has an independent system operator named differently but do the same job. Market participants can conduct trades not only in the counter market, but also in the financial market of Nord pool. In the financial market, participants sign contracts for electricity trading so as to reduce the price in the future years. Nordic system also provides physical-delivery market in order to effectively control and manage real-time power balance. Indeed, multiple liberalized electricity markets help decreased the possibility of exercising market power by dominated participants. However, due to power transmission network constraints, exercising market power behaviour can never be totally avoided.

As with the Nordic market, the PJM market also allows market participants make transaction both inside and outside the pool-type market. In fact, there are four markets exist in PJM market system which are a capacity market, an energy market, a FTR (Financial Transmission Right) auction market and an ancillary service market. Moreover, the energy market in PJM can be divided into two markets: a day-ahead market and a real-time energy balance market. Market participants can not only purchase and sell electricity in the above market they can also feel free to make bilateral contacts with each other to complete power trades.

## 2.5. Independent System Operator

After liberalisation of electricity utility, the control of the operation of the transmission system has been separated from the control of the operation of generation plants. In order to open access for market participants to the monopoly transmission system, a stand-alone independent system operator (ISO) has been created which has been regarded as the “soul of the power grid” and the “air traffic controller” of the power system [35]. The US Federal Energy Regulatory Commission (FERC) has outlined a set of characteristics and functions for the ISO which are described as follows [36]:

Characteristics:

1. Independence
2. Scope and Regional Configuration
3. Operational Authority
4. Short-term Reliability

Functions:

1. Tariff Administration and Design
2. Congestion Management
3. Parallel Path Flow
4. Ancillary Service
5. Total Transmission Capability and Available Transmission Capability
6. Market Monitoring
7. Planning and Expansion
8. Inter-regional Coordination

The ISO is an independent system operator who carry out generation schedule in electricity market [37]. By definition, all the independent system operators should undertake the task of controlling the whole operation of electricity system. However, not all the system operators perform all the above functions. They might peel off the operation of energy markets from the whole operation of the system and let a third party or their subsidiary to manage them [35]. The basic work of the system operator is to secure and manage the transmission network. In fact, ISOs in different electricity market models will perform different functions. For instance, in pool market, ISO should be responsible for energy trading, generation scheduling and prices setting. But in bilateral market, the ISO has nothing to do with energy market and the responsibility is narrowed to purely manage real-time system balance and provide ancillary service [9].

## **2.6. Global Experience of Deregulated Electricity Market**

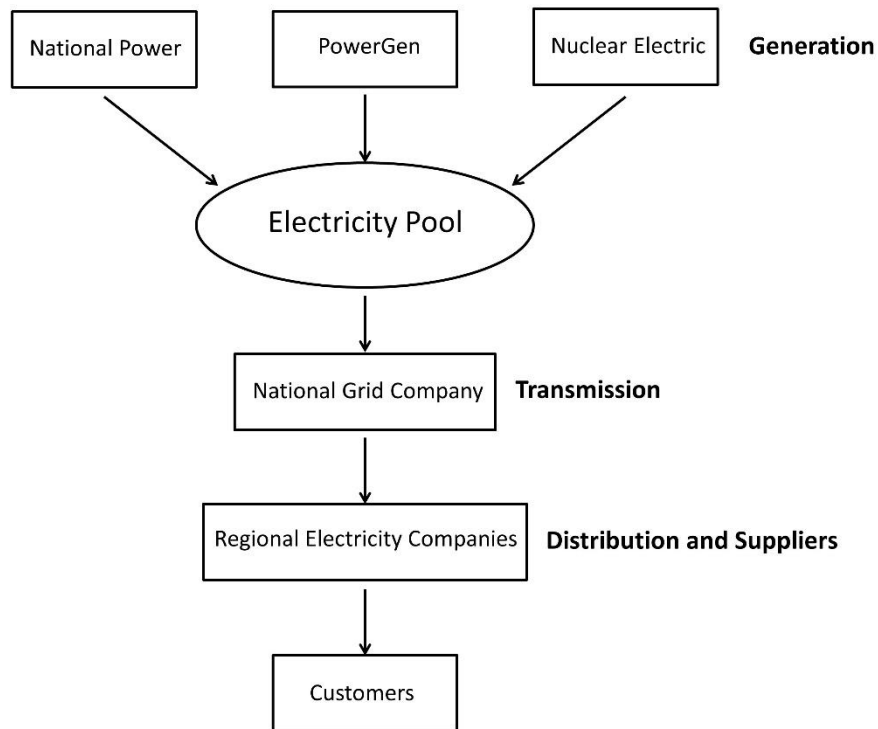
Since the concept of electricity market deregulation has been put forward around the world for more or less thirty years, more and more nations on their own have gained achievements on electricity industry restructure in various degrees. Some of those countries even successfully established sets of complete market systems in order to make full use of electricity market mechanism. In those reformed markets, rules and orders have been clearly formulated and performed. Market participants only need to follow the instruction and then to enjoy the benefits from competition mechanism. Electricity industry would obtain a long term development and improvement under new utility structures. Nevertheless, particularly several deregulated markets did not smoothly go through the tough process of reform. During the process, they suffered a series of issues, troubles and even failures. Following parts of this chapter will introduce several typical world experience of competitive electricity markets which are from UK, Nordic countries and US.

## 2.6.1. UK

### 2.6.1.1. The England & Wales Pool

Before privatization, electricity industry in England and Wales was dominated by a vertically integrated nationalised company called Central Electricity Generating Board (CEGB) which was a classic example of a cost-of-service regulated public utility [38]. On 31<sup>st</sup> March 1990, electricity industry in England and Wales started to be restructured with new mechanism [39]. The CEGB with its 75 power stations and one national grid had been split into three generating companies: National Power with 60% of conventional generating capacity (40 power stations with 30 GW capacity), PowerGen with 40% of conventional generating capacity (23 power stations with 20 GW) and Nuclear Electric (12 nuclear stations with 2 GW); and one transmission company: the National Grid Company with the high voltage grid and together with 2 GW of pumped-storage generation [40]. Twelve former area distribution boards had been replaced by 12 regional electricity companies (RECs) which were regarded as local distribution systems [1]. An electricity pool market had been established to arrange and manage electricity trades in England and Wales with wholesale market mechanism. The establishment of electricity pool in England & Wales has been treated as a remarkable innovation in electricity privatization process. The electricity pool mechanism was utilized to break the vertical linkage between generation and transmission and to introduce competition into electricity industry which requires all the generators to sell their electricity in a wholesale market, the pool. Figure 2.3 shows the structure of electricity industry at privatization [3].





**Figure 2.3 the Structure of Electricity Industry at Privatization in England & Wales**

The England & Wales Pool was a typical mandatory pool market which means every energy trade has to be transacted only inside the pool market. For the generation section, those three main generating companies competed with each other by using generation capacity and biddings. For the transmission section, the National Grid Company was not only just a transmission company who hold transmission network ownership but also an independent system operator who operated the pool market and managed the pool's settlement system on behalf of pool market participants. For the distribution section, 12 private-owned distribution/supply companies accounted for distributing and retailing electricity to customers in different regional areas. It seemed like the former electricity vertically integration was not change a much compared with the pool structure. However, the establishment of pool mechanism had a remarkable significance in electricity industry deregulation history because competition had been brought in so competitive bidding process has been created and facilitated.

In the England & Wales Pool market, by 10 a.m. each day, generation companies should send their information to the system operator including generation output quantity and bid price of electricity provided on the following day [41]. One day ahead, the independent system operator, here is the National Grid Company, already forecasted the power demand quantity in each half hour period [42]. After collected the bid information and output data from generation companies, the system operator will start to accept bids from cheapest price to more expensive price until the forecasted demand is satisfied. Then, the system operator will sort out a bid list which contains names of the generation companies who have been chosen to generate electricity on the following day [43]. Those chosen generation companies were called “in merit” generation companies which means their bids have been accepted by system operator and those whose bids have not been accepted by system operator were called “out of merit” generation companies [44]. Then the independent system operator will utilize the software called “Generating Ordering and Loading” (GOAL) to draw up an unconstrained generation schedule which follows a least cost generation principle [45]. The last job for the system operator is setting prices for all the pool market participants. As known, System Marginal Price (SMP) is defined as the price which is bidden by the most expensive generation company in the bid list [40]. In the pool market, the system operator will pay the Pool Purchase Price (PPP) to all the in merit generation companies. The Pool Purchase Price (PPP) is the System Marginal Price (SMP) plus Capacity Payment (CP) which is defined with Loss of Load Probability (LOLP) and Value of Loss Load (VOLL) [40]. On the other end of pool structure, the suppliers who is willing to purchasing the electricity power from the pool will pay for the price called Pool Selling Price (PSP). The Pool Selling Price (PSP) is the Pool Purchase Price (PPP) plus the Uplifts [1]. The equations which defined the relationships between those prices are shown as following [9]:

$$PPP = SMP + CP \quad (2.1)$$

$$CP = LOLP \times \max(0, VOLL - SMP) \quad (2.2)$$

$$PSP = PPP + Uplifts \quad (2.3)$$

$$Uplifts = \frac{\text{security constrained cost} - \text{unconstrained cost}}{\text{total load demand}} \quad (2.4)$$

CP is actually an incentive which is used to reward the generation companies who declare that their capacity is available regardless of whether they are required to guarantee or not. LOLP is the probability that electricity power capacity is unable to support the actual demand. LOLP is predicted in each half hour. VOLL is the price customers are going to pay to avoid electricity power shortage [46]. When expected electricity capacity shortage occurs, CP will climb up and when predicted electricity generation capacity exceeds, CP will drop down. Neglecting power losses and ancillary service, the Uplift can be regarded as the cost of congestion relief. It is claimed that PPP is definitely an ex-ante mechanism. The Uplift is utilized to cover the cost of system services which are important to power system security. Several aspects are taken consideration into Uplift such as transmission constraints, demand forecast errors, ancillary service and other services. The amount of the Uplift will be informed by the end of each trading day so that the Uplift will be regarded as ex-post mechanism. In the pool market, electricity price will change in every 30 minutes depending on generation, demand and generation companies bidding behaviours. Hence, electricity price is not stable. In order to hedge against the price volatility, power sellers and buyers sign a bilateral contract called Contract for Differences (CfD) outside the pool market [47].

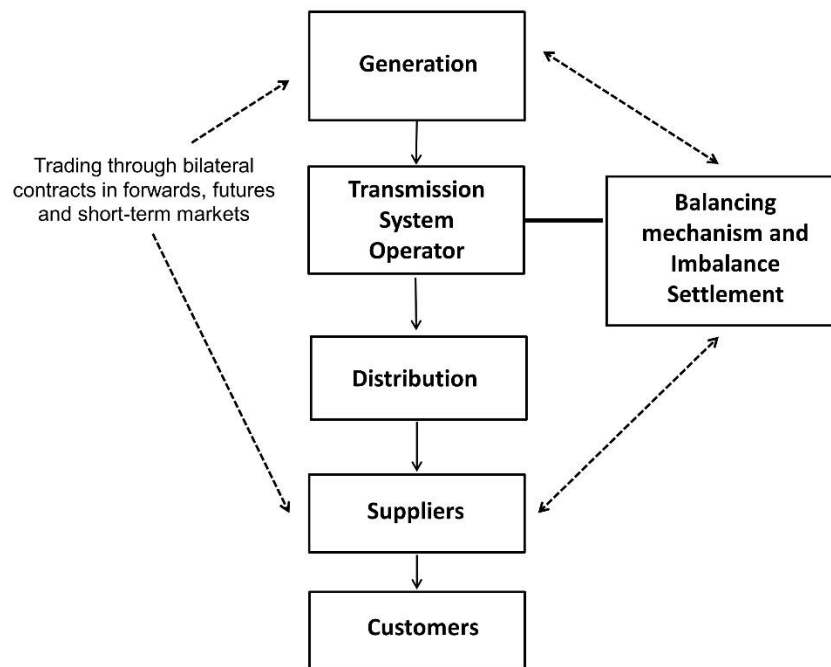
Overall, the England & Wales Pool was successful during the period from 1990 to 2001. Since the pool structure was much similar with the former vertically integrated utility, new operation mechanisms and principles were able to easily adapt to the pool system. Operation of power system during this period was almost secure and stable. Nevertheless, the original intention of ideal deregulation was not realized. The aim of introducing competition into electricity industry was to suppress electricity price for

customers but the real electricity price in pool market did not drop following with the reduction of generation cost. The electricity price's drop was not dramatic in the pool market and customers did not benefit from the liberalization of electricity industry. The electricity price in the pool was unable to truly reflect the generation marginal cost. The reason was noticed that in the pool market several dominant generation companies can easily exercise the market power by strategic bidding which will affect the efficiency of the pool market. When the forecasted electricity price is going down, dominant generation companies will use strategic bidding to raise the electricity price in the pool. Another drawback of the pool market is that demand side was lack of competition. In other words, the pool market is the market with inadequate representation of the demand side [40]. Electricity suppliers were all paid at the same pool price rather than on negotiated prices, which inhibited supply side price pressure. Moreover, a centrally forecast estimate of demand limited involvement from the demand side within the pool market. The complexity, opacity and the lack of competition in price setting inhibited the development of derivatives markets and reduced liquidity in the contracts markets. This leads to high margins on the financial contracts struck between generators and suppliers, therefore further raising prices to customers above those that would have prevailed with more competitive arrangements [48]. Electricity customers can only choose the regionally local power supplier for electricity supply and the above phenomenon is unable to satisfy the intention of restructure of electricity market.

#### **2.6.1.2. British Electricity Trading and Transmission Arrangements (BETTA)**

The England & Wales Pool had almost successfully operated for around 10 years but its performance was unable to catch up with the steps for further requirement of liberalization of electricity industry. Then the aim is to establish a power industry with whole new mechanisms which would realize the electricity free trading as far as possible like any other commodities [48]. Thus on 27<sup>th</sup> March 2001, in England and Wales, the New Electricity Trading Arrangements (NETA) were introduced to replace the pool market with the concept that market participants have rights to transact electrical power

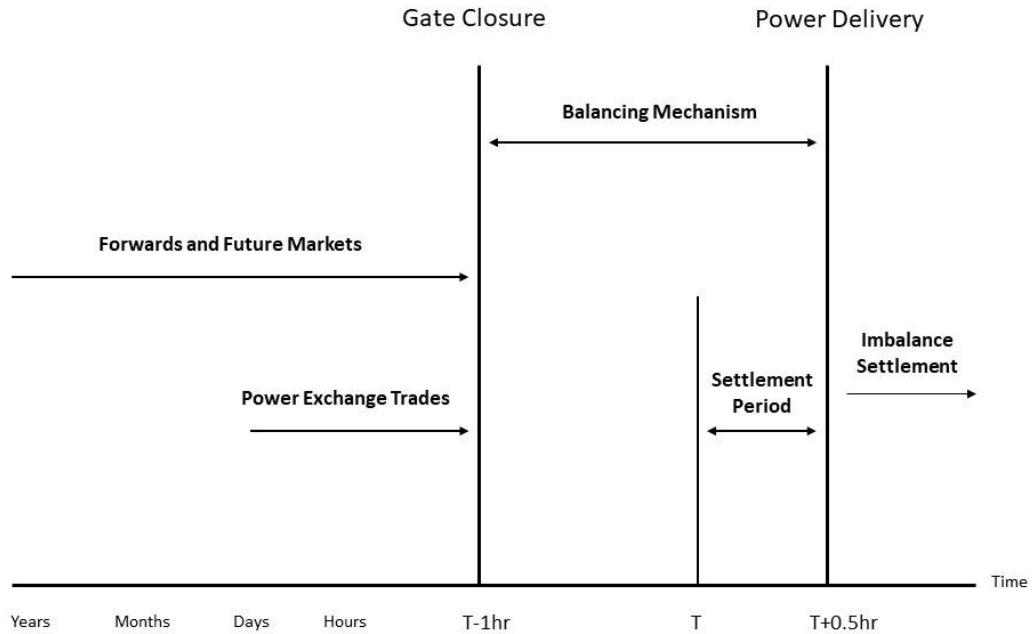
by bilateral trading [49]. On 1<sup>st</sup> April 2005, with the joining of Scottish power system network, the New Electricity Trading Arrangements (NETA) has been extended and reformed as the British Electricity Trading and Transmission Arrangements (BETTA) [50]. The structure and operational principles of new system BETTA have only small difference when compared to NETA. The BETTA can be regarded as the extension of the NETA [51]. The only difference is the BETTA covers the whole British-wide electricity industry which means all the market participants including companies from Scotland can join in this deregulated market. The vertical structure was similar with the former pool structure. However, as the independent system operator, the function of the National Grid Company (NGC) has been changed. Figure 2.4 will show the industry structure of BETTA [3]:



**Figure 2.4 the Industry Structure of the BETTA**

At this time, NGC has two roles: one is the transmission asset owner and another one is the independent system operator [3]. As the asset owner of transmission network, the NGC should ensure the maintenance, investment and long-term development of

transmission system. As the system operator, the NGC needs to keep supply-demand balance all the time and to maintain the real-time security of network operation [52]. BETTA operates the power system differently from the former pool market. Figure 2.5 expresses the diagrammatic representation of BETTA time frame process [53]:



**Figure 2.5 the Diagrammatic Representation of BETTA Time Frame Process**

In the BETTA, electricity sellers and buyers can transact power trading by bilateral contracts. The bulk of electricity power is traded by Forward/Future contracts and short-term power exchange. The Forward/Future contracts could be signed ahead of days, months or even several years before the actual electric power delivery. The forward contract market is where customers and suppliers can have contracts for electricity directly with generators. In this bilateral market, market participants negotiate with each other directly and agree to trade a quantity of electricity for physical delivery at an agreed location and over a period of time. The future contract market allows derivatives trades in

a regulated market. These are binding agreements between a buyer and a seller to deliver and take delivery on a specified future date a quantity of electricity at a price agreed today [3]. The short-term power exchange is within 24 hours before electricity delivery which offers market participants an opportunity to 'fine adjust' their contract details by a convenient way [48]. To help the system operator access to the power balance condition, market participants are required to notify their predicted physical information in each half hour period, for power seller is the planned generation output and for power buyer is the metered demand, by 11 a.m. one day ahead. This information is called initial physical notifications (IPNs) [3]. Before the gate closes, usually one hour before power delivery which was 3.5 hours prior to 12 June 2002, market participants must submit their final physical notifications (FPNs) to the system operator [54]. During the period between gate closure and real time, SO will run the Balancing Mechanism and match the balance between supply and demand so that system security could be guaranteed. Participants have an opportunity to voluntarily submit their offers and bids to let the SO know how much they are willing to pay or charge for the power quantity difference compared with their FPNs [48]. An offer is a participant's price proposal to increase generation or decrease demand and a bid is a participant's price proposal to decrease generation or increase demand. Offers and bids are requested to submit in pairs and each market participant is allowed to submit maximum of ten pairs [55]. If transmission constraint occurs, imbalance settlement will be done by the SO mandatorily and that participants whose offers have been accepted will be paid at System Sell Price (SSP) while ones whose bids have been accepted will be charged at System Buy Price (SBP) [9]. SSP and SBP are two imbalance prices which are calculated by the SO based on the collected offers and bids [55]. The original contracts position of market participants will be honored [4]. Balancing service that NGC provides is consisted of two types of balancing actions: energy imbalance actions and system imbalance actions [53]. The transmission constraint management is attached to the system imbalance actions. The SO recovers all the cost of balancing service through a Balancing Service Use of System (BSUoS) charge to all the market participants [53]. BSUoS charge is calculated in each half-hour period which divides a whole day into 48 equal intervals. The calculation of transmission congestion cost is

included inside the BSUoS charge calculation [56].

Unlike the former pool structure with system operator centrally dispatching, generators in the BETTA is able to self-dispatch according to the contracts with consumers. Moreover, demand-side in the BETTA is involved into competition mechanism so that suppliers can compete with each other. Some large customers became more active roles in the market. Since more price information is indicated, market liquidity has been improved during the new structure process. By new mechanism of the BETTA, wholesale prices and retail prices have an obvious reduction. However, the BETTA is not perfectly effective due to its dual imbalance settlement prices. Generation companies might deliberately increase electricity output to make their metered output volume larger than contracted volume so that they can escape from the penalty of the SBP or even obtain the reward from the SSP. The suppliers might deliberately lower their predicted demand value shorter than the contracted demand to get rid of the punishment from SBP or even get the advantage from the SSP. Because of above issues, generators operate below the optimum efficiency which will result in primary fuel waste and emission increase and suppliers shorten their predicted contract volume will cause the over-contract market formation hence power system reserve will always stay at a high level which will lead to the whole system operation cost increase. Another disadvantage of the BETTA is the unfairness to renewable power generation especially the wind power. All the market participants are required to have such ability of output accurately prediction, power supply guarantee and short notice flexibility. As known, intermittent is the weakest point of wind energy supply. Unable to predict the future output, wind power is the most likely to face with the penalty by imbalance settlement.

### 2.6.2. Nordic Countries

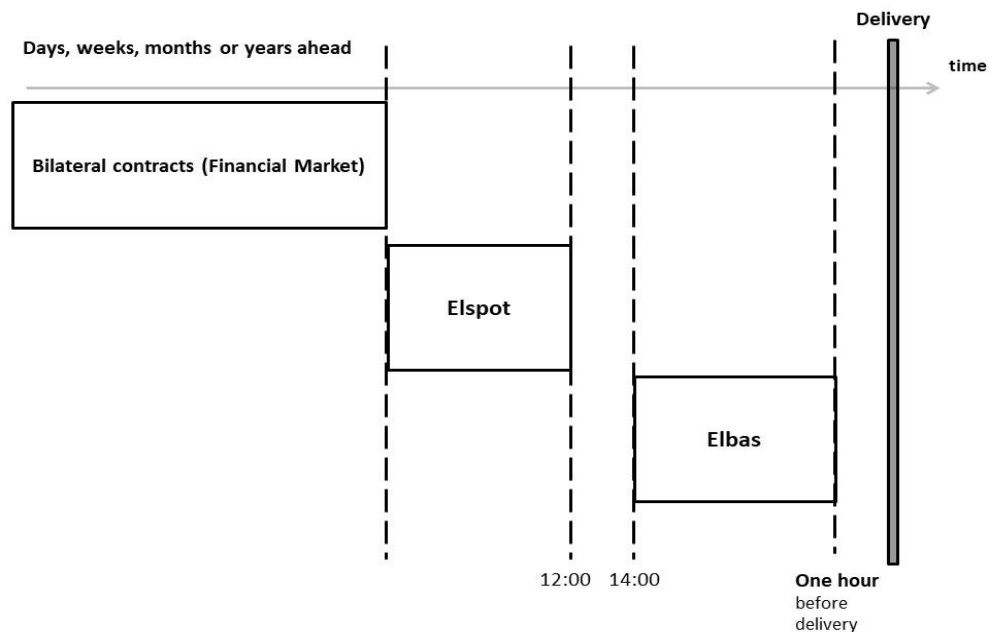
On 1<sup>st</sup> Jan 1996, Sweden and Norway joined together and established the Norwegian-Swedish Exchange called as Nord pool ASA which is the initial state of the current Nord pool [57]. Then in 1998, Finland market joined in. Two years later in 2000,



Denmark market joined in. In 2002, Nord pool spot AS referred as a separated company organized the Nord pool's spot market. Since then, a new integrated regulatory framework for electricity industry competition across Nordic countries was established which was the first truly multinational electricity market. In 2010, Estonia opened its electricity market and joined in. Then in 2012, Lithuania following the Estonia opened the power market and joined in the pool. In 2013, Latvia opened electricity market and joined in. In 2016, Nord Pool Spot is rebranded to Nord Pool [58]. Since those countries worked together to handle with the power system operation, independent system operators from different countries need to communicate and cooperate very closely. The Nord Pool is owned by the Nordic and Baltic transmission system operators (TSOs) which are Statnett SF (Norway) with 28.2% stakes, Svenska Kraftnät (Sweden) with 28.2% stakes, Fingrid (Finland) with 18.8% stakes and Energinet.dk (Denmark) with 18.8% stakes, Elering (Estonia), Litgrid (Lithuania), AST (Latvia) hold 2% stakes each of them [59]. In addition to owning and operating the high-voltage grid, TSOs are responsible for the security of supply in their countries [60].

The Nord Pool market consists of physical and financial markets which means market participants could not only trade electricity through spot market but also via bilateral contract. According to Nord Pool mechanism, Elspot market is referred as the main physical market which is a day-ahead market arranging hourly bilateral contracts of electricity power delivery [61]. Elspot market will determine the hourly spot price which will be regarded as the reference price both for physical market and financial market [62]. In 2002, approximately 30% of the electricity consumption was traded in the Nord pool and in 2017, this number has been increased to 80%. In 2002, the number of direct market participants in Elspot market was reached at two hundred including generation companies, suppliers and large customers and in 2017, number of companies involved in Nord pool was up to 380 [61, 63]. Another physical market Elbas was introduced as a supplement of Elspot market for the additional balancing services. Elbas as the intraday market offers each market participants an opportunity to adjust their physical power position such as the price and volume of supply or demand one-hour before power delivery [62]. The day-

ahead Elspot market is considered as the spot market and the intraday market Elbas plays the role of balancing mechanism in the Nord pool electricity market. For the financial section, before 2002, there were two financial markets in the Nord Pool: Eltermin and Eloption [61]. Power sellers and buyers could sign a bilateral contract in Eltermin market days, weeks, months and even up to four years ahead for hedging against the price volatility risk. Eloption market was a relatively new financial market compared with Eltermin market. It also provides contract services for market participants to prevent the electricity price risk. Since 2008, Nord pool financial market started to be organized by NASDAQ OMX Commodities exchange which is the dominant marketplace for system price contracts [64]. Besides those financial markets above, the Nord Pool also allows market participants to organise the over the counter (OTC) trading and Contracts for Differences (CfDs) to make bilateral contracts with each other. Figure 2.6 shows the diagrammatic representation of Nord pool time frame process [65].



**Figure 2.6 the Diagrammatic Representation of Nord Pool Time Frame Process**

In total, Nord Pool structure is quite a successful deregulated electricity market model among the whole global electricity markets. There are four main reasons for Nord Pool success. Firstly, market design is completed and mature. Secondly, significant quantity of hydropower resource enables support of the not great demand in Nordic area. Thirdly, the multiple integrated power system effectively dilutes the market power. Finally, Nord Pool obtained strong support from political side. Even though having above advantages, Nord Pool still has several threats which will affect the normal operation of Nord Pool market. One is power system faults and constraints cannot be completely avoided so that power system operation cannot be fully secured. Then the behavior of exercising market power cannot be stopped so long as competition mechanism develops. The last threat is that hydropower is highly dependent on climatic condition which will easily have influence on electric power supply.

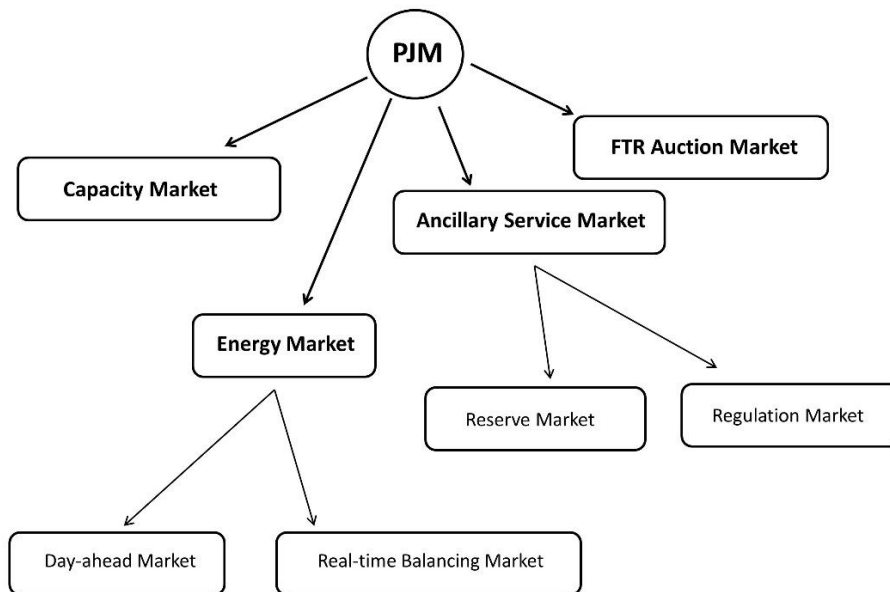
### 2.6.3. US

#### 2.6.3.1. PJM Interconnection

On 1<sup>st</sup> April 1997, PJM opened its first bid-based energy market and during that year PJM was approved as the first national fully functioning independent system operator (ISO) by the Federal Energy Regulatory Commission (FERC). On that time, PJM operated the transmission network but did not own it in order to open the access of transmission system to non-utility users. Then to develop the advanced competitive wholesale electricity market and multi-areas operation, FERC encouraged the formation of the regional transmission organization (RTO). Hence, PJM became the first national functioning RTO in 2002 [66]. Right now, PJM Interconnection is a regional transmission organization (RTO) in the United States which operates a wholesale electricity market and a power transmission network system which is part of the Eastern Interconnection grid. The PJM market is serving all or part of areas of Delaware, Illinois, Indiana, Kentucky, Maryland, Michigan, New Jersey, North Carolina, Ohio, Pennsylvania, Tennessee, Virginia, West Virginia and the District of Columbia. The head office of PJM is located in

Valley Forge, Pennsylvania [67]. Currently, PJM is operating a competitive electricity wholesale market and a high-voltage grid together serving 65 million customers. So far, PJM market has already 1032 participants with more than 178,563 megawatts of generating capacity, 165,492 megawatts peak demand and 135,252 kilometres (84,042 miles) of transmission line. In 2017, PJM has totally delivered 773,522 gigawatt-hours of power energy [68].

PJM is composed of an energy market which consists of a day-ahead market and a real-time balancing market; a capacity market; a financial transmission right (FTR) auction market and an ancillary service market which consists of a regulation market and a reserve market [69, 70, 71, 72]. In this section, only energy market in the PJM will be discussed. Figure 2.7 displays the tree structure of the PJM market:



**Figure 2.7** Tree structure of PJM markets

The day-ahead market is a forward market which will calculate the hourly clearing prices for each hour of the next operating day. These prices are calculated based on

generation companies' offers, demand side bids, virtual supply offers, virtual demand bids and bilateral transaction schedules and those data will be submitted into the day-ahead market by market participants with voluntarily basis [6]. The balancing market is a real-time market which will calculate the clearing price in every 5 minutes period with the principle of actual system operation and security constrained economic dispatch [73]. Different settlement methods are adopted in two types of markets. The day-ahead market settlement is based on the planned hourly power quantities and day-ahead hourly electricity prices. The balancing market settlement is according to the hourly quantity deviations from planned values and real-time hourly electricity prices. Both hourly prices calculation is based on the Locational Marginal Pricing (LMP) [74].

The day-ahead market offers market participants an opportunity to trade electricity power at binding day-head prices. Transmission customers can schedule bilateral transactions at binding day-ahead congestion price which is based on source and sink price difference calculated with LMPs. In the day-ahead market, generation companies who have joined in an installed capacity contract have obligation to hand in their offer schedules even if they are self-scheduled and during outage. Customers should hand in their bids to the system operator to notify whether they are willing to pay congestion charges, or they are willing to be curtailed if transmission congestion occurs in the real-time market. After the day-ahead market bid period closure, the system operator starts to make the dispatch schedule for each hour of the next operating day. The real-time energy market is based on the real-time operation conditions. The generation rebidding period from 4 p.m. to 6 p.m. is introduced for the generation companies who are not selected into day-ahead market schedules to alter their bids for use in the real-time energy market otherwise their original day-ahead bids will come into force in the real-time market [58]. Generation companies who produce more than their scheduled quantity in the day-ahead market will be paid by the real-time LMPs. For customers who consume the power volume more than the planned quantities in day-ahead market will be charged by the real-time LMPs. If market participants who are trading in spot market, their trades will be settled at

the real-time LMPs. Besides the day-ahead market and the real-time balancing market, PJM also provides financial instruments such as Financial Transmission Rights (FTRs) for energy sellers and buyers to hedge the prices variation risks.

Overall, the PJM market shows that it is quite a complete and mature competitive power market system after development of so many years. In the PJM, all the market participants have obtained an opportunity with lock in day-ahead prices for financial schedule and power trading. Of course under the functioning market principles, market liquidity and reliability have been dramatically promoted up to a whole new level. Effectively motivation has been transferred to market participants so that they are willing to take part in the day-ahead schedule submission. The PJM also provides incentives to help dispatch the real-time resources. In the PJM market, strengthened financial instrument such as the financial transmission right (FTR) is very effective for prices risk hedging. The long-term investment and development of power system can be ensured by the good market environment [73].

#### **2.6.3.2. California Crisis**

The development of deregulated competitive electricity market was not always perfectly successful sometimes even terrible. In 1992, the California Public Utilities Commission (CPUC) planned to restructure the current electricity market which is regarded as the basis of California Assembly Bill AB1890 [27]. The crisis happened in California in the beginning of 21<sup>st</sup> century would be the tough memory during the electricity market deregulation history. On one hand, the crisis was regarded as the results of multiple contributed factors including a drought that reduced the level of hydroelectric power, unexpected outages at nuclear power plants, high natural gas prices, and strong demand for power energy. On the other hand, it was because of the truly mismanagement and wrong rules establishment of market deregulation process.

Wholesale market establishment is the beginning of market deregulation so several wholesale marketplaces were created for this purpose. In order to manage and control these newborn wholesale markets, two new institutions, the Power Exchange (PX) and the California Independent System Operator (CAISO) were set up [75]. However, above two organizations were operated separately rather than one integrated utility which will lead to market inefficiency and market power manipulation. Moreover, retail price control which was created in California market cut off the relationship between consumers and power producers. It is very strange that California has controlled retail electricity prices but volatile wholesale electricity prices [76]. The negative point of applying retail price control is that cost changes in wholesale market cannot be passed to consumers end. In the California market, participants have to sell and buy electricity power only through PX and CAISO and the advance power purchase time is only maximum one day-ahead before power delivery. The current Economists believed that with the higher wholesale electricity prices, incentives will push the investors who are more likely willing to build a groups of new generation plants [27]. Nevertheless, construction of generation plant takes time and especially in California more time is needed for generation plants licensing process. In fact, the failure has been proved that few new generation was brought into the market but only a mass of applications submitted for new generation plants in 1999 and 2000 [27]. In addition, a sudden demand increasing occurred during that period without any prediction [76]. It is indicated that from 1999 to 2000 the power demand growth rate was 4% more than previous years [27]. At that time, due to climatic influence as lack of rainfall, except conventional thermal power, other new generation such as nuclear, gas-fired were already working at full capacity. Worse still, when the winter came, many generation plants needed to be shut down for repair and maintenance. On June of 2000, long-term potential problems finally caused crisis among California area. The electricity price in California market on April 2000 was around 30 dollars per megawatt hour and the price for June increased to the price interval from 250 to 450 dollars per megawatt hour [77]. The first five months of 2001 was full of high wholesale electricity prices, energy emergencies and rolling blackouts. Even worse, on December 2000, the Federal Energy Regulatory Commission (FERC) replaced the retail price control by a soft cap in wholesale market.

The further facts proved that the soft price cap was not only useless but also instigated market participants gaming behaviours which was that generation companies exported electricity from California and then reimported the power at an increased price. Lacking power supply will result in high wholesale prices combining with retail price gap will force power utilities to sell off most of their generation plants and drive retailers quit selling their products. In short, it was a vicious circle that supply shortage caused wholesale price rises that lead supply to further decrease. On June 2001, after government took over, wholesale prices dropped under 50 dollars per megawatt hour, demand declined and new generation plants got in [77]. By such above phenomenon appearance, California crisis had been finally proved was over.

The lessons from California Crisis can be obtained as follows. Firstly, connection and communication between power producers and power suppliers must be ensured. Long-term bilaterally contractual system is significantly necessary in the deregulated electricity market. Any change from wholesale level should have the channel to pass to the retail level. Secondly, strong support and appropriate intervention from governmental side are also regarded as the contributed factor which will make sure the secure and stable operation of electricity market operation.

## **2.7. Summary**

This chapter introduced the contents about the deregulated electricity market. The traditional vertically integrated electricity utility has been presented firstly and coming with the drawbacks which are also the motivation of electricity industry restructure. Then generally understanding of electricity industry deregulation has been described with the history of worldwide electricity deregulation actions, the planned new structure of electricity industry and future expectation of electricity utility deregulation. Three types of deregulated electricity market structure models have been introduced: a pool market model, a bilateral market model and a hybrid market model which can be referred as the combination of the former two models. Independent system operator as the key role of



electricity market deregulation has been recommended with its unique characteristics and important functions. Global experience of deregulated electricity market is the main content of this chapter. The description of the UK deregulated electricity market is the priority among priorities. The old England and Wales Pool as the one of the pioneers achieved a significant step in history of electricity market reform. The British Electricity Trading and Transmission Arrangements as the replacer of pool market came further close to the original intention of competition introduction. Then Nord Pool as the largest multinational electricity market has been discussed with its special features. The US also has two interesting electricity market models but one is quite successful and another one gave us some experimental results. PJM interconnection is definitely a good deregulated electricity market but California crisis reminded that electricity industry reform is a tough and complicated project.

## Chapter 3

# Transmission Congestion Management in Deregulated Electricity Market

### 3.1. Introduction

In the modern power system, transmission congestion becomes a major issue which could cause power system crisis such as cascade outages, increase in losses, electricity price rises and threats to power system security [78]. Transmission congestion is referred to the situation that operating constraints of transmission line are violated so the line transfer capacity is exceeded. Consequently, power flow going through the transmission line is restricted by its capability [79]. Transportation of electricity is constrained by several physical laws including thermal constraints which is also called overloading, voltage constraints and stability constraints [1, 35]. Congestion is also caused by unexpected events such as generation outages, load demand increase and power system equipment failure or damage [79]. In the competitive environment of deregulated electricity market, congestion occurs generally during the unconstrained dispatch stage when the system operator is not able to accommodate all the desired transactions based on the least cost principle without violating transmission system constraints [78]. At this step, the system operator must take actions to prevent actual congestion happening in the real power delivery. In the old traditional vertically integrated structure, congestion management is simple because system operator only needs to re-dispatch the generation schedule [9]. However, with the deregulation becoming mature, congestion management became more and more complex and frequent, since transmission network is on open access with non-discrimination to all the market participants. Therefore, power flows are determined by power trading rather than controlled by an integrated utility. In fact, the direct way to resolve transmission congestion is to construct new transmission lines to

provide adequate power transfer capacity. However, new line construction takes a long time and the investment of transmission network cost are huge. Additionally, there are some technical methods for congestion relieving such as transformer taps, phase shifting transformers and flexible AC transmission system devices (FACTS) [11]. But the above approaches have limited ability on transmission congestion relief. By contrast, market-based methods for congestion management have brighter future because the current congestion is mainly caused by electricity market deregulation. Market-based methods aim to resolve congestion by using the existing transmission network in an economic way which improve the efficiency of the whole power system utilization. It is a better way to go against the congestion problem with market mechanism.

In this chapter, the key content is the congestion management schemes in market environment. Each of them will be illustrated in detail. Section 3.2 includes several major market-based congestion management schemes which are utilized in various electricity market types. Section 3.2.1 explains the pricing schemes for congestion management. In pool and hybrid market, uniform marginal price in section 3.2.1.1, locational marginal price in section 3.2.1.2 and zonal price in section 3.2.1.3 are introduced in details. In bilateral market, transaction curtailments in section 3.2.2 are utilized to relieve transmission congestion. Section 3.2.3 illustrates two financial instruments: contract for difference (CfD) and financial transmission right (FTR) which have been created and developed to hedge the impact of transmission congestion and to stabilize electricity price level. Section 3.3 is a case study of a modified IEEE-14-bus system which is used for a comparison between uniform marginal price and locational marginal price. Finally, section 3.4 is the summary.

## **3.2. Market-based Transmission Congestion Management Schemes**

Power flow delivery should follow Kirchhoff's laws and a transmission line has its physical limit in aspects of heat, voltage and power angle. If market participants only

focus on economic interests maximization rather than transmission network physical limitation, issues of transmission constraints especially transmission congestion will occur inevitably. Hence, in a competitive electricity market, an independent system operator is required to control and manage the power system operation. The independent organization will not be influenced by any other market participant and its function is to balance the conflict between economic interests and power system operation security. A big challenge for the system operator is to keep system operation on a stable and reliable level, whilst at the same time, to allow market participants equal access to the transmission capacity. To accomplish such challenge, power system managers are required to establish a complete system which has specific and reasonable schemes for transmission congestion management.

There are three typical pricing schemes: uniform price, locational marginal price and zonal price. Transaction curtailment is also utilized for congestion management. Besides, several financial approaches are also applied to transmission congestion management. Congestion management methods are categorized as follows in Table 3.1 [9]:

**Table 3.1 Congestion Management Schemes in Different Market Structures**

Congestion management schemes in deregulated electricity market
Pricing Schemes: <ul style="list-style-type: none"> <li>• Uniform Marginal Price</li> <li>• Locational Marginal Price</li> <li>• Zonal Price</li> </ul> Contract for difference (CfD) Transaction curtailments Financial transmission right (FTR)

### 3.2.1. Pricing Schemes

#### 3.2.1.1. Uniform Marginal Price

As its name implies, the uniform marginal price indicates that electricity trading prices in the market will be identical for all the market participants without considering the location of power source or power sink. The pricing scheme was implemented in the former England & Wales Pool market. The pool system operator adopted this pricing scheme and is referred as “re-dispatch first, compensate later” to respond to the situation when transmission congestion happens. There are two steps the system operator needs to act: firstly unconstrained dispatch and then secondly security-constrained re-dispatch if the congestion has been triggered. In the market dispatch stage, the system operator will collect all the bids from generation companies and rank the bids from cheapest to higher prices until customer demand can be met. The highest price in the bid list will be set as the market clearing price which is also called the system marginal price (SMP) and the generator who bids the most expensive price will become the marginal generator [80]. When transmission congestion is found after the first dispatch, the system operator will launch the second stage: security-constrained re-dispatch. The system operator will re-schedule the generator list to remove the possibility of congestion, and at the same time keep the security cost at the minimum.

The algorithm of the first stage market dispatch will be showed as follows:

$$\min \sum_{i=1}^{N_G} C_{G_i}(P_{G_i}) \quad (3.1)$$

Subject to:

$$\sum_{i=1}^{N_G} P_{G_i} = \sum_{j=1}^{N_L} P_{L_j} \quad (3.2)$$

$$0 \leq P_{G_i} \leq P_{G_i}^{max} \quad (3.3)$$

Where

- $C_{G_i}(P_{G_i})$  : The bid-based generation cost of generator  $i$   
 $P_{G_i}$  : The active power generated by generator  $i$   
 $P_{L_j}$  : The demand prediction of load  $j$   
 $N_G$  : The total number of generators  
 $N_L$  : The total number of loads  
 $P_{G_i}^{max}$  : The maximum generation capacity of generator  $i$

Equation (3.1) expresses the objective function of the first stage dispatch which is subject to two constraints equation (3.2) which states the power balance between generation and demand and equation (3.3) which states the generation output of each generator should not be beyond the maximum generation capacity. At this stage, the system operator does not take transmission constraints and losses into consideration but only focuses on generation cost minimization. If the first dispatch does not trigger the transmission constraints, the initial generation schedule will be implemented.

Once the market dispatch violates the transmission constraints, an inequality constraint will be added to the dispatch algorithm by the system operator to become a security-constrained dispatch. The additional constraint is expressed as follow:

$$P_{line_k} \leq P_{line_k}^{max} \quad (3.4)$$

Where

$P_{line_k}$  : The power flow on the transmission line  $k$

$P_{line_k}^{max}$  : The maximum power flow transmission capacity of the transmission line  $k$

Equation (3.4) is the constraint which requires all the transmission lines in the re-dispatch stage should be operated within their power transmission capacities. A new schedule for re-dispatch generation will be decided by the new algorithm and the generator list is updated because some generators will be required to reduce power generation or even removed from the initial list due to transmission line constraints. The generators which were out-of-merit generators but now are on the new generation list on the second stage are called “constrained on” generators and those who were in-merit generators but now are ordered to generate lower output than their initial plans on the new schedule are called “constrained off” generators [81]. At the time of settlement, all the generation companies who participated in the power delivery are paid by the Pool Purchase Price (PPP) which is mentioned in the section 2.6.1.1 and also compensated by an adjustment payment for the difference between their bid prices and the PPP. The adjustment payments for different type of generators are expressed as follows [9]:

Adjustment for constrained off generators:

$$\begin{aligned} & Adjustment_{off} \\ & = (unconstrained\ amount - constrained\ amount) \quad (3.5) \\ & \times (PPP - bid\ price_{off}) \end{aligned}$$

Where

$bid\ price_{off}$  : The bid price of the “constrained off” generator submitted to the system operator during the unconstrained dispatch

Adjustment for constrained on generators:

$$\begin{aligned} Adjustment_{on} &= (constrained\ amount - unconstrained\ amount) \quad (3.6) \\ &\times (bid\ price_{on} - PPP) \end{aligned}$$

Where

$bid\ price_{on}$  : The bid price of the “constrained on” generator submitted to the system operator during the unconstrained dispatch

The uniform marginal price scheme is regarded as a simple approach for congestion management because there is only one price set in the whole pool market so the system operator can easily calculate and allocate the cost of congestion. However, there are several shortages indicating that such a pricing scheme needs to be improved or even replaced. The process of dispatch and re-dispatch are not transparent since all the economic and physical information is collected by the system operator so that market participants have to only obey the generation schedule in which the trading price information notified to market participants but it cannot reflect the congestion cost. The congestion cost is allocated based on an equal sharing principle which is unable to provide locational distinction between participants.



### 3.2.1.2. Locational Marginal Price

In 1982, MIT Professor Fred Schweppe proposed an hourly spot price in energy marketplace which is regarded as the origin of the locational marginal price (LMP) or nodal price [82, 83]. The definition of the locational marginal price is the minimal marginal cost of supplying the next increment of 1 megawatt hour power at a specific bus [13]. If there is no transmission congestion and transmission losses in the system, the locational marginal price of each node will be the same which is also set as the system marginal price based on the collected bids from market participants. However, in reality, transmission congestion and losses will exist during the system operation. Once congestion or power losses happens, locational marginal price between different nodes will become distinct due to different costs of power supply in each node. At the moment of settlement, generator will be paid based on the LMP at its node and customer will pay for the LMP at its node. Both actions are implemented by the independent system operator who has already run an optimal power flow to calculate the LMP of each node. Locational marginal price is utilized as the primary pricing scheme in the electricity markets of Argentina, Chile, Ireland, New Zealand, Russia, Singapore and several states in the US for electricity price calculation and transmission congestion management because of its capability of providing accurate economic signals by which market operation becomes more efficient and long-term power system investment becomes more attractive [84].

LMP is calculated based on the optimal power flow (OPF) which is the integration of the power flow calculation with the minimization of an economic objective function subject to the equality and inequality constraints of system network operation [85]. Before the calculation of LMP, the system operator will make use of OPF to calculate megawatt dispatch of each generator. There are two types of OPF model including the DC optimal power flow (DCOPF) calculation and the AC optimal power flow (ACOPF) calculation. The ACOPF is built upon an AC power flow which considers both active and reactive power flow at the same time. The ACOPF is known for the accurate simulation results which can be directly used as the benchmark to represent the real power flow. However,

because of non-linear characteristics, ACOPF calculation is always complex to simulate large scale power system, requiring large execution time and the high probability of non-convergence [86]. Compared to ACOPF, DCOPF is simpler and more convenient approach since it is linear and only considers active power flow neglecting voltage, reactive power and transmission loss [87]. DCOPF is often used for the system operator to dispatch generator and calculate LMP because of its simplicity, robustness and higher speed of convergence [86]. The algorithm of DCOPF is illustrated as follows:

In order to adapt to conventional high-voltage transmission system, in DCOPF modeling some assumptions should be formed as follows [88]:

1. Voltage angle difference  $\delta_{ij} = \delta_i - \delta_j$  between neighboring nodes is small so that  $\sin\delta_{ij} \approx \delta_{ij}$  and  $\cos\delta_{ij} \approx 1$
2. The voltage profile is flat and the value of p.u. is equal to 1 which means that the voltage amplitude is equal for all nodes
3. Transmission line resistance is negligible so that DCOPF can be considered as linear modeling

The DCOPF has the objective function which is to minimize the generation cost with several constraints such as power flow balance, generation limits and line flow limits. Assume there is a power system with N nodes and L lines, the objective function and constraints are shown as follow [89]:

$$\min f = \sum_{i=1}^N C_i(P_i^g) \quad (3.7)$$

Where

$$C_i(P_i^g) = \frac{1}{2}C_{2i}(P_i^g)^2 + C_{1i}P_i^g + C_{0i} \quad (3.8)$$

Subject to:

$$P_i^g - P_i^d - \sum_{j=1}^N P_{ij} = 0 \quad (3.9)$$

$$P_{i,min} < P_i^g < P_{i,max} \quad (3.10)$$

$$|P_l| < P_l^{max} \quad (3.11)$$

Where

- $P_i^g$  : The real power generated at node  $i$
- $C_{0i}$  : The no-load cost coefficient of generator  $i$
- $C_{1i}$  : The linear cost coefficient of generator  $i$
- $C_{2i}$  : The quadratic cost coefficient of generator  $i$
- $P_i^d$  : The real power demand at node  $i$
- $P_{ij}$  : The power transmission flow from node  $i$  to node  $j$
- $P_{i,min}$  : The minimum generation limitation of node  $i$
- $P_{i,max}$  : The maximum generation limitation of node  $i$
- $P_l$  : The real transmission power flow on the line  $l$
- $P_l^{max}$  : The real power maximum transmission capacity of line  $l$

Equation (3.7) represents the objective function of the DCOPF which is to minimize the generation cost. Equation (3.8) indicates the bid-based cost function of the generator at node  $i$ . Equation (3.9) shows the real power balance constraint at node  $i$ .

Equation (3.10) expresses the maximum and minimum output constraints of the node  $i$ . Equation (3.11) indicates the capacity constraint of the transmission line  $l$ .

For the DCOPF, the calculation of locational marginal pricing is straightforward if line losses are not included. However, the power loss in a power system network may not be negligible so many markets are moving towards marginal loss pricing. As the DCOPF is lossless by definition, it is required to develop a suitable methodology to represent line losses within the DCOPF [86]. The power loss modelling with DCOPF can be found in [87, 90]. It is considered as an equivalent formulation that will not change the optimal solution of the DCOPF. A reduced DCOPF model with losses can be derived with the replacement of real power balance constraint equation at bus  $i$  by the total real power balance equation considering power loss among all nodes in the system. As known, power injected into all the nodes is equal to power withdrawn out of all the nodes plus transmission losses. Combined with the above concept, equation (3.9) can be rewritten into such new equation as follow [90]:

$$-\sum_{i=1}^N P_i^g + \sum_{i=1}^N P_i^d + P_{loss} = 0 \quad (3.12)$$

Where

$P_{loss}$  : The transmission losses in the power system

Based on the equation (3.12), the corresponding Lagrangian equation can be defined as follows [14, 91]:

$$\begin{aligned}
 \mathcal{L} = & \sum_{i=1}^N C_i(P_i^g) + \lambda_0 \left( -\sum_{i=1}^N P_i^g + \sum_{i=1}^N P_i^d + P_{loss} \right) + \sum_{l=1}^L \mu_l (|P_l| - P_l^{max}) \\
 & + \sum_{i=1}^N \pi_i^{max} (P_i^g - P_{i,max}^g) + \sum_{i=1}^N \pi_i^{min} (P_{i,min}^g - P_i^g)
 \end{aligned} \tag{3.13}$$

Where

$\lambda_0$  : The Lagrangian multiplier of the whole system power balance constraint

$N$  : The total number of nodes

$L$  : The total number of transmission lines

$\mu_l$  : The Lagrangian multiplier of transmission line constraint

$\pi_i^{min}$  : The Lagrangian multipliers of minimum generation capacity of generator  $i$

$\pi_i^{max}$  : The Lagrangian multipliers of maximum generation capacity of generator  $i$

Based on the above Lagrangian equation, equation of the LMP of node  $i$  can be obtained as follows:

$$\begin{aligned}
 LMP_i = \frac{\partial \mathcal{L}}{\partial P_i^d} &= \lambda_0 \left( 1 + \frac{\partial P_{loss}}{\partial P_i^d} \right) + \sum_{l=1}^L \mu_l \times \frac{\partial P_l}{\partial P_i^d} \\
 &= \lambda_0 + \lambda_0 \times \left( \frac{\partial P_{loss}}{\partial P_i^d} \right) + \sum_{l=1}^L \mu_l \times T_{i-1}
 \end{aligned} \tag{3.14}$$

Where

$T_{i-1}$  : The sensitivity factor for real power at node  $i$  with line  $l$  constraint

The LMP in node  $i$  can be divided into three components as follows [16]:

$$LMP_i = LMP_i^{energy} + LMP_i^{loss} + LMP_i^{congestion} \quad (3.15)$$

Correspond with equation (3.15), equation of  $LMP_i^{energy}$ ,  $LMP_i^{loss}$  and  $LMP_i^{congestion}$  can be obtained as follows:

$$LMP_i^{energy} = \lambda_0 \quad (3.16)$$

$$LMP_i^{loss} = \lambda_0 \times \left( \frac{\partial P_{loss}}{\partial P_i^d} \right) \quad (3.17)$$

$$LMP_i^{congestion} = \sum_{l=1}^L \mu_l \times T_{i-1} \quad (3.18)$$

Where

$LMP_i^{energy}$  : The system marginal cost of node  $i$

$LMP_i^{congestion}$  : The cost of transmission congestion of node  $i$

$LMP_i^{loss}$  : The cost of transmission losses of node  $i$

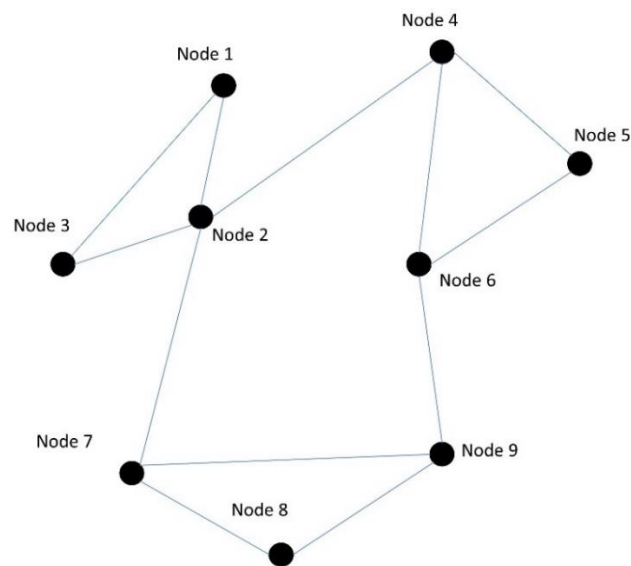
The LMP based electricity market is designed to enhance power system reliability, provide market price certainty and increase electricity market liquidity [92]. By LMP based pricing approach, economic signals are indicated to market participants so that the exercising market power is effectively prevented. The reason is that if a generator bids higher than the marginal cost, the chance of being selected for dispatch is reduced and the generator might lose the opportunity to generate. If a generator bids lower than the marginal cost, it may not receive sufficient revenue to cover its generation cost [93]. The

influence of transmission congestion and losses are reflected through the variation of LMP in nodes so that the electricity market is transparent. For long-term viewing, the LMP gives effective incentives for investment in generation and transmission assets. Nevertheless, LMP is not a perfect approach which is capable of solving all the problems in power systems. Since bids of generators submitted to the system operator are bid-based rather than cost-based, they still have the chance to enact gaming behaviors. Moreover, it is argued that the LMP only reflect short-term purchase price which is actually the spot market price expectation rather than the long-term average cost of generation. Consequently, the price signals provided by LMP are seemed as subjective, unreliable and arbitrary [94]. The other concern about LMP is that transmission congestion and losses circumstances would result in higher and more volatile nodal prices, even though LMP can be effective and workable, greater revenue collecting from ISO will cause inefficiency for economic operation of electricity market [95]. The last defect of the LMP method is that since power system networks are always big in size, the corresponding LMP specific to each node is complex which requires plenty of time and large degree of coordination.

### **3.2.1.3. Zonal Price**

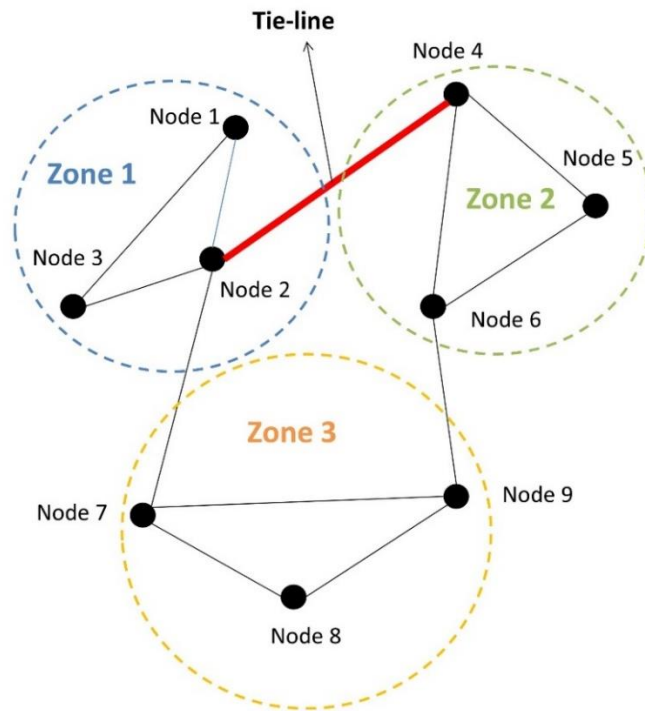
To deal with the last drawback of the nodal price scheme, some electricity markets are bundling nodes locating in neighboring regions together into a zone in which the electricity price is uniform, this can be regarded as the marginal generation cost of the zone. The pricing scheme which is used to simplify the complex process of the location marginal price is called the zonal price. In many countries, zonal price is adopted as the major approach for electricity pricing and congestion management. Take Nordic countries as a typical example, as mentioned in section 2.6.2 the Nord pool market consists of multiple electricity markets of Nordic and Baltic countries. The local system operator in each country divides the whole nation area into several bidding zones. The number of Norwegian bidding zones nowadays is five. Denmark consists of two bidding zones: eastern Denmark and western Denmark. Finland, Estonia, Lithuania and Latvia constitute one bidding area for each of them. Since 2011, Sweden has four bidding zones [96]. The

system operator splits the whole power system into zones mainly according to the historical data and the economic operation of each node. When there are no transmission constraints, all the electricity prices in each node are identical to the system marginal price which is calculated based on the bids collected from market participants [97]. Once transmission constraints occur, the whole system will be divided into different zones which were defined in advance. Inside each zone, the electricity price is uniform which is calculated according to the bids collected from participants in such single zone. Prices between zones are distinct due to the transmission constraints, for example, congestion, while the constraints inside each zone is considered as infrequent and insignificant [98]. The whole process of zonal split is illustrated in following diagrams Figure 3.1 and Figure 3.2:



**Figure 3.1 Meshed network structure of a multi-nodes power system without congestion**





**Figure 3.2 Multi-nodes power system split into three zones when congestion occurs**

From Figure 3.1 and Figure 3.2, it is noted that there is a nine-node power system which is split into three zones once transmission congestion occurs. As predefined, node 1, 2, 3, node 4, 5, 6 and node 7, 8, 9 are composed into three single zones respectively. The transmission line between node 2 and node 4 now becomes the only connection between zone 1 and zone 2 is named as “Tie-line” [9]. In a similar way, the transmission line between node 2 and node 7 and the transmission line between node 6 and node 9 are also set as Tie-lines which are responsible for the connection between zones. It can be observed that zonal price is the combination of the uniform marginal price and the locational marginal price. Inside the zone area, all the nodal prices are identical which is similar with the principle of the uniform marginal price while each different zone can be regarded as a bigger node with its own electricity price which follows the rules of the locational marginal price.

After zone partition is congested, the difference between each zone will appear on the generation capacity and zonal price. For example, assume zone 1 in Figure 3.2 has surplus generation capacity therefore the zonal price in zone 1 will be lower than other zones. Thus zone 1 will be set as the low price zone. If there is a generation shortage occurs in zone 2 then the zonal price in such area is higher than others. Thus zone 2 will be set as the high price zone. Under this circumstance, the system operator will try to figure out an effective way to balance such distinction between those two zones. As a consequence, the Tie-line between zone 1 and zone 2 will be utilized to transport the surplus electric power from zone 1 to zone 2 as long as the Tie-line is always operating under its maximum transmission capacity. Due to this action by the system operator, extra electricity from zone 1 can be sold to zone 2 at a higher price and customers in zone 2 can be benefit from the lower price electricity from zone 1. Same with the principle of the LMP, the system operator will charge the congestion cost from the customers who used the transmission network. The collected revenue due to congestion will be used for power system network investment.

In some cases, the zonal price is regarded as a simpler and lower cost approach for congestion management compared with the locational marginal price. However, the zonal pricing method cannot be simply considered as a simplification of nodal pricing since in real power system it is more complex than theory. Zonal price supposes the intra-congestion between nodes inside the zone is infrequent so that the cost of intra-congestion is too small to count. In fact, the intra-zonal congestion is frequent and the corresponding congestion cost is higher than theory [99]. Because price inside one zone is uniform for all nodes while in reality each node has different generation cost so the zonal price is unable to reflect the real cost of generation. As a result, market customers have the burden of unfair costs once intra-congestion occurs. Moreover, it is quite difficult and complicated to define a new zone. Since the boundary of a zone is defined under the assumption that inside a zone there is a low frequency of intra-congestions but the opposite occurs in reality the setting of a new zone usually lags behind the intra-zonal congestion

management. The zonal price was considered as a perfect approach for congestion management for a time. However, more electricity markets are transitioning their pricing scheme from the zonal price to the locational marginal price [3].

### 3.2.2. Transaction Curtailment

In an ideal bilateral electricity market, the power transaction is arranged by market participants themselves who do not have the obligation to submit their contract information to the system operator. However, the unique characteristic of electricity market is that power system has a series of system constraints which must not be violated. To ensure the security of power system operation, sometimes the system operator is forced to curtail the already signed transactions between participants in order to reduce the power flow on the congested transmission branches. Therefore, in bilateral electricity markets, participants still need to report the physical information of their contracts to the system operator and take the risk of transaction rejection in order to give assistance to system operation security [100]. In addition to this, power transactions are signed days, months, or even years ahead of the actual power delivery, transmission network conditions might change during that time so that the initially secure transaction may endanger the system security criterion. Due to above circumstances, the system operator should formulate the schemes of transaction curtailment to maintain the power system under a safe operational condition. By superposition theorem, the power flow in a specific transmission line can be regarded as the sum of power flow from each transaction contribution. So the system operator can relieve an overloading transmission line by power transaction curtailment which is divided into following four types [35, 83, 101, 102]:

#### 3.2.2.1. First Come First Serve

This rule of transaction curtailment is regarded as a simple and straightforward one. Once transmission congestion occurs, the system operator will firstly curtail the transaction which was the last submitted. If the congestion still exists, the system operator

will keep rejecting transactions following a reverse order of transaction submission until the congestion is relieved completely. Even though the method is simple, it may not be effective sometimes. The main reason is the system operator curtails the transactions without considering the power flow contribution of each transaction to the congested transmission line. The last submitted transaction might potentially provide the counter-flow on the congested transmission line while curtailing it arbitrarily may worsen the congestion problem.

#### **3.2.2.2. Pro Rata (Proportional)**

By this approach, once transmission congestion occurs on a transmission line, the system operator will curtail transactions to relieve the line overload based on the corresponding power flow contribution of each transaction. Based on a linear DC optimal power flow, the sensitivity of each transaction to the power flow of the overloaded transmission line can be calculated relatively accurate so that the flow contribution of each transaction to the congested line can be also found out. The curtailed volume is proportional to the transaction contribution. The power transaction which has a higher contribution to the congested line will be curtailed more power and vice versa. Different from the former method, this method takes the counter-flow into consideration. The transactions that causes counter-flow to the overloaded transmission line is regarded to have zero contribution so it will not be curtailed by the system operator.

#### **3.2.2.3. Minimum-net Curtailment**

The only difference between this method and the former one is that this method aims to restrict the totally curtailed net power volume to the minimum. Since different transactions have different power flow sensitivities, curtailment for one certain transaction which has greater contribution to the congested line is more effective than any other transaction curtailments. The system operator focuses on searching such transaction which contributes congestion more than others.

#### **3.2.2.4. Compensative price**

In this approach, if the transaction curtailment is unavoidable, the market participant whose transaction is rejected will receive a compensation payment from the system operator. Each participant is required to submit a compensative price to the system operator in advance to claim at what price they are willing to accept for the action of curtailment. Once the congestion happens, the system operator will start to curtail the transaction with the lowest compensative price until the overloaded line declines to its limit capacity. The final target for the system operator is to minimize the compensation cost while solving the transmission congestion problem. This transaction curtailment scheme requires the system operator to have income so as to compensate the market participants with their curtailed contracts. If the system operator is a nonprofit party, the compensation cost will be allocated to market participants.

#### **3.2.2.5. Willingness to Pay**

In this method, the system operator will curtail any power trade if it is necessary to relieve transmission congestion but without paying any compensation to the market participants. For the market participants who does not want their transactions to be rejected have an opportunity to bid a willingness to pay price to the system operator in order to avoid being involved in the curtailment action. If congestion occurs, during the curtailment action, the system operator will firstly curtail the transactions from the participant who did not pay for the willingness to pay price. If all the participants have submitted their willingness to pay prices, the system operator will start to curtail the transaction from the participant who bid at the lowest price until the congestion is removed. Different from the compensation price method, this method will increase income for the system operator. If the system operator is also the owner of the transmission network, this part of income will be utilized for transmission assets maintenance and reinforcement.

### 3.2.3. Financial Instruments

With the reform of electricity industry, the price of electricity is determined by a series of market conducts rather than controlled by an integrated utility. No matter in what type of electricity market model, the price volatility caused by the transmission congestion is the problem that market participants do not want to face with. For the generators, low price of electricity will raise the difficulty for them to cover the generation cost such as capital investment cost, maintenance cost and use of system charges. For the suppliers, unstable electricity price will have influence on their profit making. For the demand-side consumers, the rapid growth of electricity price will increase their production cost or life cost. Hence, some financial instruments are created to provide market participants a market-based route to hedge the impact of electricity price instability. There are two famous financial instruments: the Contract for Difference and the Financial Transmission Rights.

#### 3.2.3.1. Contract for Difference

Contract for Difference is a bilateral agreement between market participants to hedge the price volatility risk [103]. The concept of CfD was originally used in the early 1990s in London power market and utilized only in the financial field [104]. Then this financial method was introduced into the electricity market. CfD in the old England & Wales Pool is utilized to hedge the price variation between the pool purchase price and the contractual price. In the Nord pool market, CfD is aim to help power buyers and sellers to reduce the impact of the price difference between zonal price and the system marginal price [9]. The formula of CfD is defined as follows [103]:

$$CfD = Power_{contract} \times (price_{reference} - price_{strike}) \quad (3.19)$$

Where

$Power_{contract}$  : The power volume signed in the CfD contract

$price_{reference}$  : The reference price which is often the spot market price

$price_{strike}$  : The agreed fixed price signed in the CfD contract

Market spot price tends to be peaky and volatile so generation company and power supplier sign these over-the-counter contracts directly to hedge the price uncertainty. CfD provides a specific amount of electrical power at a fixed price called strike price which is target to compare with a reference price usually the market spot price. If the strike price is higher than the reference price, power buyers need to pay the price difference times the contractual power amount to the power sellers. If the strike price is lower than the reference price, the sellers should pay the difference to the buyers. The majority of CfD contracts in pool market aim to hedge against the pool purchase price volatility. If the pool price is higher than the contractual price, generator will pay for the CfD to supplier. If the pool price is lower than the contractual price, supplier will pay for the CfD to generator. Therefore, electricity price fluctuation can be hedged in the pool market.

### 3.2.3.2. Financial Transmission Right

To address the financial problem due to transmission congestion, it is a common practice for the independent system operator in the United States electricity wholesale market to issue a financial transmission right (FTR). FTR was introduced in different wholesale markets at different times and named differently. For example, FTR was introduced in PJM market on April 1998 and named Fixed Transmission Right; in New York, FTR was introduced in September 1999 and referred as Transmission Congestion Contract (TCC); FTR was introduced in California since February 2000 and named as the Firm Transmission right; in New England, FTR was referred as the Financial Transmission Right and introduced on March 2003 [105].

Because the market outcome cannot be predicted accurately in advance, market participants are likely to be exposed to congestion charges which may raise the risk of power transaction [106]. FTR is a type of financial instrument which entitle the holders to obtain a compensation for transmission congestion cost when transmission congestion occurs in the day-ahead market. In other words, FTR is an ex ante transmission capacity allocation. The compensation is based on the locational marginal pricing scheme. FTR provides a right to hedge the nodal price distinction between the node of receipt which is also called as source node and the node of delivery which is also called as sink node. FTR gives holders a benefit when the day-ahead congestion happens in the desired direction while asking holders to pay the difference when the day-ahead congestion happens in the opposite direction [106]. Currently there are three types of FTR: right for an exclusive use of the transmission capacity, right to use the transmission capacity and right to collect the congestion rents [106]. The formula of FTR is defined as follow:

$$FTR = Power_{FTR} \times (price_{sink} - price_{source}) \quad (3.20)$$

Where

- $Power_{FTR}$  : The active power delivered from source node to sink node
- $price_{sink}$  : The nodal price of the sink node
- $price_{source}$  : The nodal price of the source node

Based on equation (3.20), FTR holders will receive a positive payment according to FTR if the sink LMP is higher than the source LMP. FTR holders will have to pay the liability according to FTR if the source LMP is higher than the sink LMP. Therefore, in short-term, by purchasing FTR, the risk of LMP variation for wholesale electricity participants will be reduced. In long-term, the aim of FTR is to give the right economic signals to market participants to stimulate transmission investments.



### 3.3. Case Study

In this part of the chapter, the uniform marginal price and the locational marginal price are selected from the above mentioned congestion management schemes as two typical methods for a comparison by performing a case study. A modified IEEE-14 bus system is utilized as the base model to test and compare the performances of the UMP and LMP.

#### 3.3.1. The Modified IEEE-14 Bus System

A modified IEEE-14 bus system is chosen as the test system in this study as shown in Figure 3.3. The 14-bus system consists of 5 generators, 11 loads and 20 transmission lines. Assume all the generators have the same generation capacity. DC optimal power flow (DCOPF) is implemented by software Matpower (version 6.0) to calculate the active power flows [125]. The software Matpower is utilized due to its fast calculating speed and easy convergence, which is because when using DC network modelling, the standard OPF problem can be simplified to a quadratic program, with linear constraints and a quadratic cost function. Here the voltage magnitudes and reactive powers are eliminated from the problem completely and real power flows are modelled as linear functions of the voltage angles [126]. All transmission lines are assumed to be lossless so as to neglect the effect of power losses to active power flow so that transmission congestion is the only point that needs to be focused on. Reactive power source in the system is assumed sufficient hence all the bus voltages are fixed at one per unit. Number of congested transmission lines in this test is restricted to one only in order to reduce the complexity of the test. In the test environment, parameters of power system are set arbitrarily such as bid price of each generator, generation capacity of each generator, active power consumption of each load, and transfer capacity of each transmission line. After running an unconstrained dispatch without considering transmission constraints, it is found that power flow of transmission line 1-2 is over its transfer capacity. So here transmission congestion occurred on transmission line 1-2. The technical considerations such as ancillary service, price elasticity and reactive power effects are neglected.

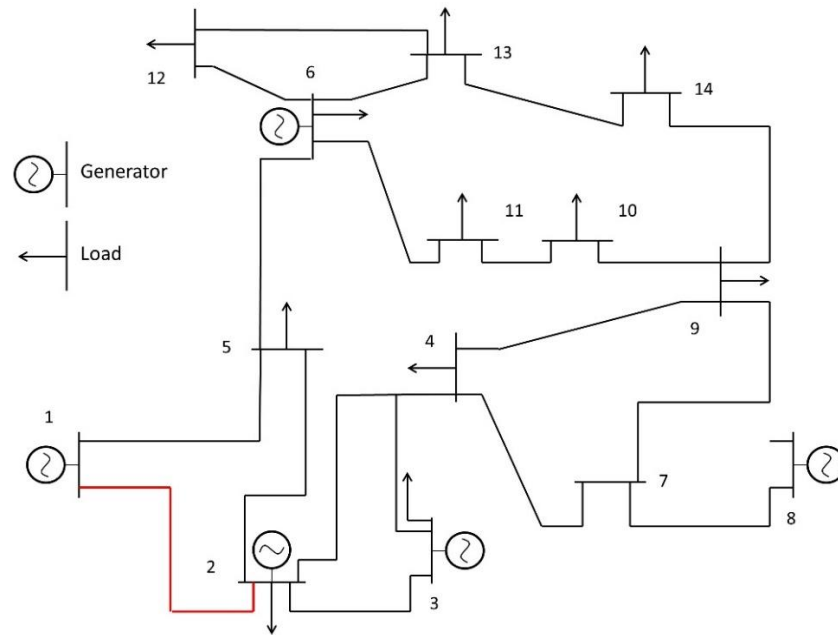


Figure 3.3 The modified IEEE-14 bus system

Before the test, some parameters of test environment are set as follows:

Table 3.2 Generation and Load Data

Bus	Generator	Bid price (£/MWh)	Min(MW)	Max(MW)	Load(MW)
1	Gen1	20	0	100	0
2	Gen2	50	0	100	24
3	Gen3	45	0	100	25
4	-	-	-	-	26
5	-	-	-	-	25
6	Gen6	35	0	100	24
7	-	-	-	-	0
8	Gen8	30	0	100	0
9	-	-	-	-	26
10	-	-	-	-	25
11	-	-	-	-	26
12	-	-	-	-	25
13	-	-	-	-	24
14	-	-	-	-	25
Total load					275

**Table 3.3 Transmission Capacity of Each Transmission Line**

Branch	From bus	To bus	Limit(MW)
1	1	2	60
2	1	5	100
3	2	3	100
4	2	4	100
5	2	5	100
6	3	4	100
7	4	5	100
8	4	7	100
9	4	9	100
10	5	6	100
11	6	11	100
12	6	12	100
13	6	13	100
14	7	8	100
15	7	9	100
16	9	10	100
17	9	14	100
18	10	11	100
19	12	13	100
20	13	14	100

In Table 3.2, bid prices, maximum and minimum MW generation outputs of five generators, MW demand of each load and total load are set as above. In Table 3.3, transmission capacity of each transmission line is also set.

### 3.3.2. Results

Assume the system operator needs to draw up a schedule of generation for a half-hour period. It has received the bid price of each generator in the power system. By implementing an unconstrained dispatch, the system operator found a congestion occurring on the transmission branch 1-2. As a result, it implements a security-constrained dispatch to satisfy the demand eliminating the transmission congestion. Using Matpower to simulate DCOPF, data of unconstrained dispatch and security-constrained dispatch is obtained as follows:

**Table 3.4 Generation Cost of Unconstrained Dispatch and Security-Constrained Dispatch**

Generator	Bid price (£/MWh)	Unconstrained dispatch (MW)	Security-constrained dispatch (MW)
Gen1	20	100	83.54
Gen2	50	0	0
Gen3	45	0	0
Gen6	35	75	91.46
Gen8	30	100	100
<b>Total (£/h)</b>		7625	7871.91
<b>Congestion cost (£/h)</b>			246.91

**Table 3.5 Power Flows of Unconstrained Dispatch and Security-Constrained Dispatch**

From bus	To bus	Unconstrained dispatch flows (MW)	Security-constrained dispatch flows (MW)	Limit (MW)
1	2	70.4	60	60
1	5	29.6	23.5	100
2	3	18.3	16.3	100
2	4	14.0	9.9	100
2	5	14.1	9.8	100
3	4	-6.7	-8.7	100
4	5	-0.5	-1.1	100
4	7	-24.4	-27.8	100
4	9	6.2	4.2	100
5	6	18.3	7.2	100
6	11	17.6	20.8	100
6	12	20.0	20.5	100
6	13	31.7	33.4	100
7	8	-100	-100	100
7	9	75.6	72.2	100
9	10	33.4	30.2	100
9	14	22.3	20.1	100
10	11	8.4	5.2	100
12	13	-5.0	-5.0	100
13	14	2.7	4.9	100

From Table 3.5, it is found that the unconstrained dispatch will cause the transmission line 1-2 congestion because power flow in that branch is 70.4 MW which exceeds the transmission capacity of 60 MW. After a security-constrained dispatch, the power flow of branch 1-2 has been reduced to the limit level. From Table 3.4, it is noted

that generator 1 is ordered to decrease its power output while generator 6 is asked to increase its power output. The security-constrained dispatch resulted in a congestion cost of 246.91 £/h. The difference between uniform marginal price and locational marginal price for congestion management is not only on the pricing mechanism but also by way of congestion cost allocation. Assume the congestion cost in both pricing methods is allocated to all the loads. Hence, different performances between the two methods on congestion cost allocation are discovered and stated as follows.

### 3.3.2.1. Congestion Cost Allocation by Uniform Marginal Price

The UMP method allocating the congestion cost based on a uniform price which is calculated as follows:

$$\text{Allocated Congestion cost per MW} = \frac{\text{Total congestion cost}}{\text{Total demand}} \quad (3.21)$$

So the allocation details is found in Table 3.6 as follows:

**Table 3.6 Congestion Cost Allocation by Uniform Marginal Price**

Load	Demand(MW)	Allocated congestion cost (£/MWh)	Allocated congestion cost (£/h)
L2	24	0.897855	21.54851
L3	25	0.897855	22.44636
L4	26	0.897855	23.34422
L5	25	0.897855	22.44636
L6	24	0.897855	21.54851
L9	26	0.897855	23.34422
L10	25	0.897855	22.44636
L11	26	0.897855	23.34422
L12	25	0.897855	22.44636
L13	24	0.897855	21.54851
L14	25	0.897855	22.44636
<b>Total</b>	<b>275</b>	<b>0.897855</b>	<b>246.91</b>

From Table 3.6, it is found that the allocation of congestion cost is based on a uniform price which is at 0.897855 £/MWh. Even though the final congestion cost allocation of each load is different but it is based on the same price to calculate.

### 3.3.2.2. Congestion Cost Allocation by Locational Marginal Price

This part of section will illustrate the congestion cost allocation by LMP method. As mentioned in section 3.2.1.2, locational marginal price is calculated based on the DC optimal power flow. In this case study, the LMP of each node is computed by the software Matpower which follows the DCOPF. Since the testing power system is assumed lossless, the LMPs in security-constrained dispatch only has two components compared with equation (3.15). Here are the system marginal cost component and the congestion component, which is indicated in following equation:

$$LMP_i = LMP_i^{energy} + LMP_i^{congestion} \quad (3.22)$$

**Table 3.7 Locational Marginal Prices in Unconstrained Dispatch and Security-Constrained Dispatch**

Bus	LMPs unconstrained (£/MWh)	LMPs security constrained (£/MWh)
1	35.0	20
2	35.0	39.98000457
3	35.0	37.79829949
4	35.0	35.91348352
5	35.0	34.55754368
6	35.0	35
7	35.0	35.67020379
8	35.0	32.9072493
9	35.0	35.53934477
10	35.0	35.44349301
11	35.0	35.22562028
12	35.0	35.04261905
13	35.0	35.07591992
14	35.0	35.33672379

From Table 3.7, it should be noted that in the unconstrained dispatch, the LMP of each node is uniform at 35 £/MWh while in the security constrained dispatch, the LMP of each node differ especially the price at node 2 and node 3 whose nodal prices are obviously higher than others. After obtained the LMP of each node, the load charge in both unconstrained dispatch and security-constrained dispatch can be known. Then the congestion cost allocated to each load can be calculated by the following equation:

$$\begin{aligned} & \text{allocated congestion cost of each load} \\ & = \text{security constrained load charge} \\ & - \text{unconstrained load charge} \end{aligned} \quad (3.23)$$

The allocated congestion cost in per MW can be obtained by the following equation:

$$\begin{aligned} & \text{allocated congestion cost of each load in per MW} \\ & = \frac{\text{allocated congestion cost of each load}}{\text{load demand}} \end{aligned} \quad (3.24)$$

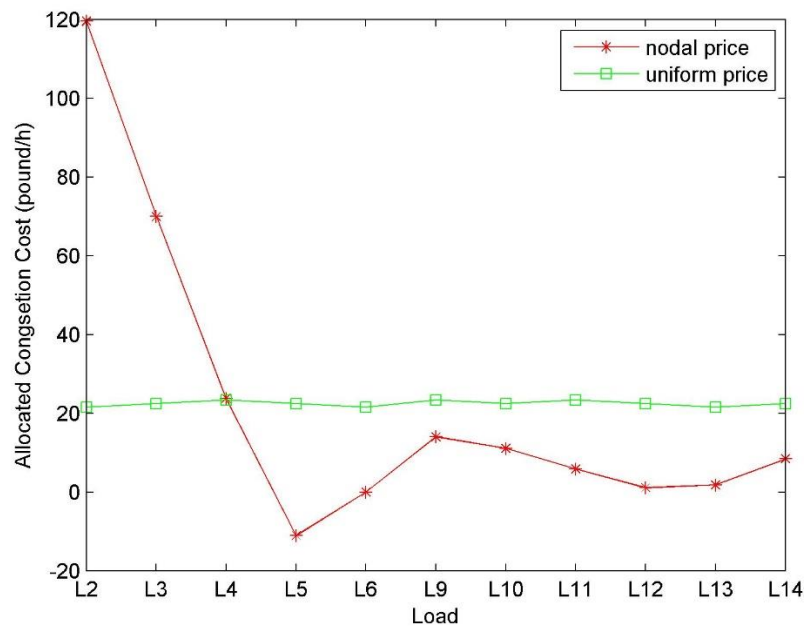
**Table 3.8 Congestion Cost Allocation by Locational Marginal Price**

Load	Demand(MW)	Load charge unconstrained (£/h)	Load charge security constrained (£/h)	Allocated congestion cost (£/h)	Allocated congestion cost (£/MWh)
L2	24	840	959.5	119.5	5.0
L3	25	875	945.0	70.0	2.8
L4	26	910	933.8	23.8	0.9
L5	25	875	863.9	-11.1	-0.4
L6	24	840	840	0	0
L9	26	910	924.0	14.0	0.5
L10	25	875	886.1	11.1	0.4
L11	26	910	915.9	5.9	0.2
L12	25	875	876.1	1.1	0.04
L13	24	840	841.9	1.8	0.08
L14	25	875	883.4	8.4	0.34
<b>Total</b>				246.9	

From Table 3.8, the allocated congestion cost and the allocated congestion cost in per MW of each load are obtained by LMP method. It is indicated that each load is allocated different congestion cost especially the node 2 and 3 whose congestion cost allocations are more than other loads.

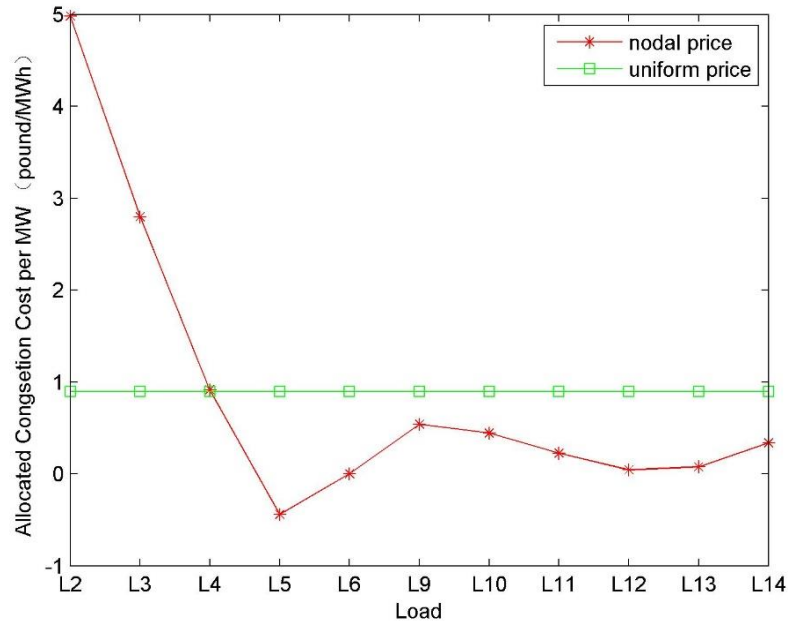
### 3.3.3. Comparison

Select the fourth column of Table 3.6 and fifth column of Table 3.8, a comparison in congestion cost allocation on each load between two methods is shown in Figure 3.4; Select the third column of Table 3.6 and the sixth column of Table 3.8, a comparison in congestion cost allocation in per MW between two methods is shown in Figure 3.5.



**Figure 3.4 Comparison in congestion cost allocation**





**Figure 3.5 Comparison in congestion cost allocation in per MW**

### 3.3.4. Discussion

From Figure 3.4, it is indicated that LMP considers the location and power flow contribution of each load so that the allocated congestion cost at different nodes is distinct. The cost allocated to Load 2 is larger than others since the transmission congestion occurred on branch 1-2. It should be noted that the congestion cost of Load 5 is a negative value which demonstrates the load does not contribute to the constraint problem, instead, it helps relieving the congestion. From Figure 3.5, it is observed that congestion cost allocation by UMP is based on one non-discriminate price. It does not reflect contribution of each load to the transmission congestion which means every load in the market shares the congestion cost uniformly. To summarise, on one hand, LMP provides economic signals to inform market participants where the congestion occurs frequently so the new generators would know where to build their new power plants. On the other hand, LMP provides a relatively fair way to allocate the congestion cost since the participant who

contributes the congestion more is required to pay for the congestion relief at a higher price.

### **3.4. Summary**

In this chapter, various types of transmission congestion management schemes in electricity market environment are introduced and discussed. In pool market and hybrid market, three types of pricing schemes: uniform marginal price, locational marginal price and zonal price are illustrated in terms of history, mechanism, pros and cons. In bilateral market, five transaction curtailments based on different principles are described. Besides, there are two financial instruments including the contract for difference and the financial transmission right which assist the market participants to hedge the impact of electricity price volatility. Finally, a case study is used for testing the performance of uniform marginal price and locational marginal price in transmission congestion management. A modified IEEE-14 bus system is built for the test. Under same testing conditions, locational marginal price provides stronger economic signals for market participants to inform the congestion frequency and is a fairer rule for congestion cost allocation.

# Chapter 4

## Re-dispatch Generation in Deregulated Electricity Market with Power Transfer Distribution Factor

### 4.1. Introduction

Due to electricity market deregulation, transmission network is open access to all the network participants. Nevertheless, the finite transmission capacity is unable to satisfy the profit maximization of all the market participants. The outcome is no doubt the transmission congestion. Re-dispatching generator output is a basic and straightforward way to solve transmission congestion. In this chapter, a generation re-dispatch method with a sensitivity index which is called power transfer distribution factor (PTDF) will be introduced. It reschedules the generation by determining the power flow contribution of each node. This chapter will describe the algorithm of this sensitivity index and the re-dispatch method mechanism which involves the sensitivity index. Four modified IEEE bus systems are used for testing the performance of this re-dispatch approach.

This chapter is structured as follows. Section 4.2 indicates the difference between conventional re-dispatch generation methods and the proposed re-dispatch method. Section 4.3 represents the algorithm of the sensitivity index PTDF. Then section 4.4 describes the mathematic model of the proposed method including the objective function and constraints which involve the sensitivity index. Section 4.5 is the case study which consists of test results from four modified IEEE bus systems including the IEEE-14 bus system, the IEEE-30 bus system, the IEEE-57 bus system and the IEEE-118 bus system. Section 4.6 is the summary.

## **4.2. Proposed Generation Re-dispatch Method in Deregulated Electricity Market**

This section will state the difference and the connection between conventional re-dispatch methods and the proposed re-dispatch method which relates to the sensitivity index PTDF.

### **4.2.1. Conventional Re-dispatch Generation in Electricity Market**

In the traditional vertically integrated power system, congestion problem can be alleviated by re-dispatching the cheapest available generation capacity because the centralized utility holds all the information to exercise the power to manage the whole power system. However, generation rescheduling will cause the cheaper electricity not to be delivered to the desired load. Hence, the total generation cost will be increased. The extra cost which is called as the security cost is put on the shoulder of the centralized utility or shared amongst consumers.

In pool electricity market, the solution of transmission congestion is similar to the route in the traditional structure of power industry in which the problem is solved by re-dispatching generator output. But this time, the information of generation cost of each generator is bid-based rather than cost-based which means the system operator reschedule the generation based on a bid-based cost minimization [35]. The security cost due to the transmission congestion is allocated to the demand-side customers by uniform marginal price, locational marginal price or zonal price.

In a bilateral electricity market, the generation re-dispatching is not simple since the system operator is unable to access the price information of the bilateral contract so it has no way to economically control the generator output. The fast way to solve congestion

problem is to curtail the bilateral transactions. This approach has been described in section 3.2.2 which introduced five types of transaction curtailment schemes: first come first serve, pro rata, minimum-net curtailment, compensative price and willingness to pay. An alternative way to achieve to relieve the congestion is to provide generators a chance to allow them to submit incremental or decremental bids which indicate the price they are willing to increase or decrease their specific output quantities. As mentioned in the section 2.6.1.2, in practical bilateral market BETTA, the incremental bid is referred as the offer and the decremental bid is referred as the bid. The system operator is allowed to re-dispatch generator outputs by using these bids as the judgment criteria when it faces the congestion problem. But this method is on a voluntary principle so it is dependent on the submitted bids from the generators. If there are few bids submitted by generators, the system operator may not have sufficient governable power resource to deal with the congestion problem.

#### 4.2.2. Adjustments in the Proposed Re-dispatch Scheme

The inspiration of the proposed re-dispatch method is from the alternative way of congestion management in bilateral market mentioned above. Because this method is more aligned with the market-based principle which allows market participants access to the competition of electricity market in an economic way. This method is also more acceptable to the original intension of electricity industry reform. Generators state their acceptable price for power output variation through a transparent channel to help the system operator reschedule the generation plan to relieve the inevitable congestion in an efficient way, in the meantime, with the minimal security cost.

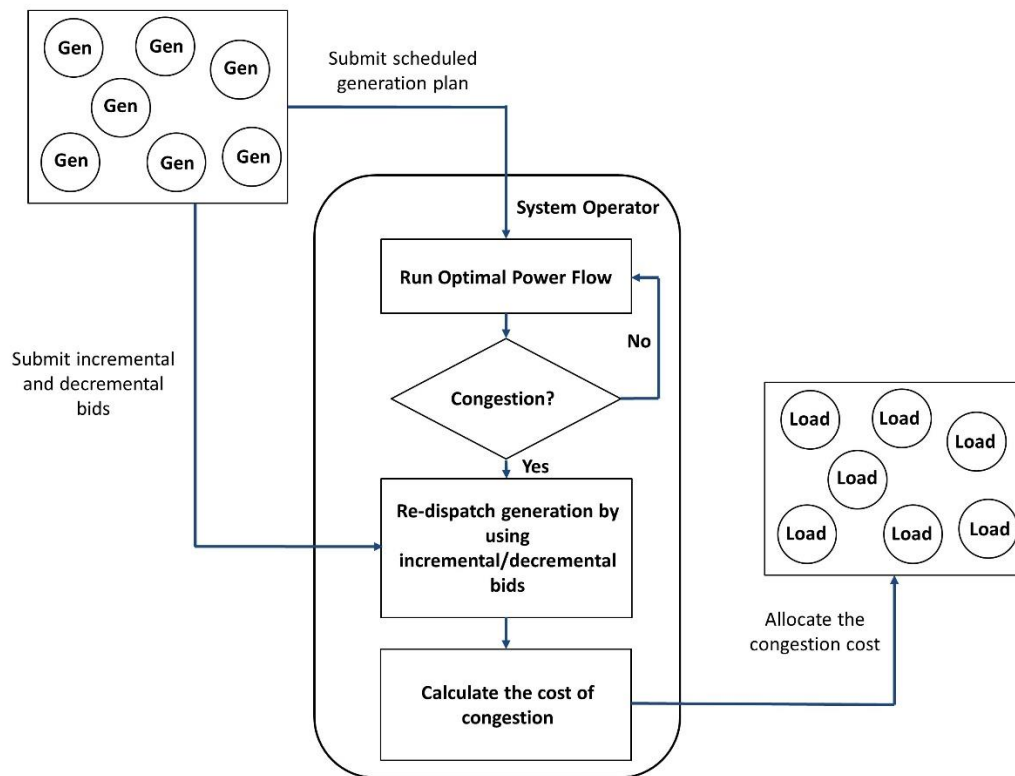
To make the proposed method more feasible and more effective, some adjustments to the alternative method need to be put forward as follows:

1. In the alternative method, the incremental and decremental bid submission for generators is voluntary. But in the proposed method, this bid submission becomes compulsory for generators. The main reason is to guarantee the system operator has sufficient amount of adjustable power resource to solve the transmission congestion issue.
2. A generator is permitted to submit a number of pairs of incremental or decremental bids. But the total power volume of its incremental bids should be limited to its generation capacity minus its scheduled power output and the total power volume of its decremental bids should be limited to its scheduled power output.
3. The congestion cost or called security cost calculation of the proposed method is based on both costs from incremental bids and decremental bids. Once the congestion occurs, the system operator will accept the incremental and decremental bids from generators to adjust generation plant to relieve congestion. If an incremental bid from a generator is accepted by the system operator, the generator will increase its output to fulfill the promise. Then the generator will get an earning which is equal to the bid price times the incremental power volume. If a decremental bid from a generator is accepted by the system operator, the generator will decrease its output. Then the generator will get a compensation which is equal to the bid price times the decremental power volume. The total cost of congestion management is same as the total cost of incremental power plus the total cost of decremental power.
4. In the proposed method, the cost of congestion is also named as the security cost and should be allocated to all the loads. All the generators are required to submit their incremental and decremental bids which is to assist the system operator to resolve congestion problem. Consequently, generators have no reason to shoulder the

congestion cost. The congestion cost allocation will be done according to the power flow contribution of each load, which will be illustrated in the next few sections.

### 4.2.3. Illustration of the proposed re-dispatch generation scheme

Figure 4.1 shows a diagram to illustrate the proposed re-dispatch method.



**Figure 4.1 Illustration of the proposed generation re-dispatch scheme**

### 4.3. The Algorithm of Power Transfer Distribution Factor

Power transfer distribution factor (PTDF) is considered as the sensitivity of power flow of a transmission line when there is a power injection change at a bus [107, 127]. In other words, it indicates the relationship between the active power flow change quantity of a particular transmission line from bus  $j$  to bus  $k$  and the active power injection change quantity at bus  $i$  [35]. PTDF can be defined as follow:

$$PTDF_i^{j-k} = \frac{\Delta P_{j-k}}{\Delta P_i} \quad (4.1)$$

Where

$PTDF_i^{j-k}$  : The PTDF of the line  $j-k$  respect to bus  $i$

$\Delta P_{j-k}$  : The active power flow variation of line  $j-k$

$\Delta P_i$  : The active power injection variation at bus  $i$

To derive the expression of PTDF, DC power flow is introduced to simplify the decoupled power flow which comes from the Newton-Raphson power flow calculation method [108]. The Jacobian matrix is simplified based on assumptions which make DC load flow linear so that power flow problems can be solved relatively quickly and simply [109]. Under rules of DC power flow, the active power flow between bus  $j$  and bus  $k$  can be expressed as:

$$P_{j-k} = \frac{V_j V_k}{Z_{j-k}} \sin(\theta_j - \theta_k) \quad (4.2)$$



Where

$P_{j-k}$  : The active power flow between bus  $j$  and bus  $k$

$V_j$  : The voltage of bus  $j$

$V_k$  : The voltage of bus  $k$

$\theta_j$  : The phase angle of bus  $j$

$\theta_k$  : The phase angle of bus  $k$

$z_{j-k}$  : The impedance connected between bus  $j$  and bus  $k$

Here these assumptions are adopted such as  $x_{j-k} \gg r_{j-k}$ ,  $V_j = V_k \approx 1$  and  $(\theta_j - \theta_k)$  is very small. So equation (4.2) can be rewrote as:

$$P_{j-k} = \frac{1}{x_{j-k}} (\theta_j - \theta_k) = b_{j-k} (\theta_j - \theta_k) \quad (4.3)$$

Where

$x_{j-k}$  : The reactance connected between bus  $j$  and bus  $k$

$b_{j-k}$  : The susceptance between bus  $j$  and bus  $k$

Then a matrix form can be expressed associated with equation (4.3):

$$[P] = [B][\theta] \quad (4.4)$$

Where

$[P]$  : The matrix of active power injection at buses

$[\theta]$  : The matrix of phase angle of bus voltages

$[B]$  : The susceptance matrix

However,  $[B]$  is a singular matrix with rank one deficiency so it is unable to be inverted [110]. Hence a slack bus is selected as a solution to find the inverse of  $[B]$ . The whole solution process is shown as:

Firstly, choose a reference bus or called the slack bus. Then eliminate the row and the column which are based on the reference bus so  $[B]$  becomes  $[B^*]$ . The eliminated row and column are associated with the selection of slack bus. For example, if the slack bus is assigned as bus number  $n$ , the  $n_{th}$  row and  $n_{th}$  column of  $[B]$  will be eliminated.  $[B^*]$  is able to be inverted. Get  $[M^*]$  by inverting  $[B^*]$  which is shown in equation (4.5):

$$[B^*]^{-1} = [M^*] \quad (4.5)$$

Where

$[B^*]$  : The matrix formed eliminating one row and one column of  $[B]$

$[M^*]$  : The inverse of  $[B^*]$

Then add one row and one column of 0 to  $[M^*]$  based on the reference bus. The added row and column are also associated with the selection of slack bus. For example, if the slack bus is assigned as bus number  $n$ , the  $n_{th}$  row 0 and  $n_{th}$  column 0 will be added back to  $[M^*]$ . Since  $[M]$  is obtained, the relation between  $[\theta]$  and  $[P]$  can be expressed as follows:

$$[\theta] = [M][P] \quad (4.6)$$

Where

$[M]$  : The matrix is from  $[M^*]$  by adding one row and one column of 0 according to the corresponding reference bus or slack bus.

Then equation (4.6) can be rewritten as follow:

$$[M] = \frac{[\theta]}{[P]} \quad (4.7)$$

The equation (4.3) can be also expressed to indicate the change quantity of power on line  $j-k$  as follow:

$$\Delta P_{j-k} = b_{j-k}(\Delta\theta_j - \Delta\theta_k) \quad (4.8)$$

Then let both sides of equation divide  $\Delta P_i$ , which leads to the following equation:

$$\frac{\Delta P_{j-k}}{\Delta P_i} = b_{j-k} \left( \frac{\Delta\theta_j}{\Delta P_i} - \frac{\Delta\theta_k}{\Delta P_i} \right) \quad (4.9)$$

Based on former definition of PTDF in equation (4.1) and the relationship between  $\Delta\theta$  and  $\Delta P$  in equation (4.7), the expression of PTDF can be calculated from equation (4.9) as follows [110]:

$$PTDF_i^{j-k} = \frac{\Delta P_{j-k}}{\Delta P_i} = b_{j-k} [M_{ji} - M_{ki}] \quad (4.10)$$

Where

$M_{ji}$  : The  $j_{th}$  row and  $i_{th}$  column element in the M matrix

$M_{ki}$  : The  $k_{th}$  row and  $i_{th}$  column element in the M matrix

Since the added row and column are both consisted of 0, the PTDF of slack bus to any congested transmission line is always 0 [110]. Different slack bus selections would cause different PTDF values, which means this method is dependent on the selection of slack bus and might produce different results [108].

#### **4.4. Mathematical Model for the Proposed Method with Power Transfer Distribution Factor**

Once the sensitivity index PTDF is obtained, the corresponding mathematical model is able to be established. In this chapter, when a single transmission line congestion occurs, the system operator will re-dispatch generation schedules based on the above sensitivity factor and incremental/decremental bids submitted by generation units. The system operator is required to minimize the total cost of rescheduling generation, in the meantime, it should obey the physical rules of power system such as balancing of supply and demand, transmission line constraints and generator capacity [111]. In order to strengthen the achievement of sensitivity factor, all the generators in the system are required to be involved in incremental/decremental bids submission stage [24]. To simplify calculation process, incremental and decremental bids from each generator are assumed at the same price. The objective function and corresponding constraints are shown as:

$$\min \sum_{i=1}^{n_g} C_{G_i} \cdot |\Delta P_{G_i}| \quad (4.11)$$

Subject to:

$$\sum_{i=1}^{n_g} \Delta P_{G_i} = 0 \quad (4.12)$$

$$\sum_{i=1}^n PTDF_{G_i}^{j-k} \cdot \Delta P_{G_i} \leq P_{j-k}^{lim} - P_{j-k}^{uncon} \quad (4.13)$$

When

$$P_{j-k}^{lim} - P_{j-k}^{uncon} < 0$$

$$\sum_{i=1}^n PTDF_{G_i}^{j-k} \cdot \Delta P_{G_i} \geq P_{j-k}^{lim} - P_{j-k}^{uncon} \quad (4.14)$$

When

$$P_{j-k}^{lim} - P_{j-k}^{uncon} > 0$$

$$-P_{G_i} \leq \Delta P_{G_i} \leq P_{G_i}^{max} - P_{G_i} \quad (4.15)$$

Where

$C_{G_i}$  : The incremental/decremental price of generator  $i$

$\Delta P_{G_i}$  : The incremental/decremental amount of active power injection of generator  $i$

$PTDF_{G_i}^{j-k}$  : The PTDF of the line  $j-k$  respect to bus  $i$

$P_{j-k}^{lim}$  : Transmission capacity limit of line  $j-k$

$P_{j-k}^{uncon}$  : The active power flow of line  $j-k$  in unconstraint dispatch

$P_{G_i}$  : The generation amount of generator  $i$  in unconstraint dispatch

$P_{G_i}^{max}$  : The generation capacity of generator  $i$

Equation (4.11) represents minimization of the total cost of generation re-dispatching. No matter if the cost is from incremental bids or decremental bids, they are all counted into the cost of congestion. Equation (4.12) indicates that the sum of power output variation from all the bids should be equal to 0 in order to keep the balance of the whole power system. Equation (4.13) and equation (4.14) express that the re-dispatching of generation is based on the sensitivity factor PTDF. The sum of generation variation in re-dispatch should result in the power flow on the congested line reduce below its transmission capacity. Equation (4.15) requests that the total incremental power volume of a generator should be limited to its generation capacity minus its scheduled power output and the total decremental power volume should be limited to its scheduled power output.

## 4.5. Case Studies

The case studies are consisted of the testing results from four modified IEEE bus systems including an IEEE-14 bus system, an IEEE-30 bus system, an IEEE-57 bus system and an IEEE-118 bus system. The test is targeted at the performance of the proposed re-dispatch method associated with the sensitivity factor PTDF. The results are mainly about the generation variations in re-dispatch and the corresponding congestion costs.

### 4.5.1. Test Description

In the case studies, in order to make power flow contribution more distinct and straightforward on transmission network, each bus is considered only has either a generator or a load. All the generators are assumed to have the same generation capacity. DC optimal power flow (DCOPF) is implemented by software Matpower to calculate the active power flows due to its fast calculating speed and easy convergence [35]. All transmission lines are assumed to be lossless so as to neglect the effect of power losses to active power flow so that transmission congestion will be the only consideration that needs to be focused on [112]. Reactive power source in the system is assumed to be sufficient hence all the bus voltages are fixed at one per unit [113]. For demonstration purpose, it is assumed that there is only one bottleneck for transmission in the network. This is the congested line. Number of congested transmission line in each test scenario is restricted to one only in order to highlight the sensitivity method efficiency on one particular line. In the test environment, parameters of power system are set arbitrarily such as bid price of each generator, generation capacity of each generator, active power consumption of each load, and transfer capacity of each transmission line. After running an unconstrained dispatch without considering transmission constraints, it is found that power flow of one particular transmission line is over its transfer capacity. So here transmission congestion occurred on this transmission line. Since different slack bus selections lead to different PTDF values, test associated to PTDF method is aim to find the different results about re-dispatched generation volume and congestion cost when selecting different slack bus. Each of the generator bus is proposed as the slack bus for each case [114].

Besides, the level of overloading (LO) is introduced in the test scenario to find the limitation of the proposed method with PTDF when transmission congestion become increasingly severe. LO is defined as:

$$LO = \frac{P_{j-k}^{uncon} - P_{j-k}^{lim}}{P_{j-k}^{uncon}} \times 100\% \quad (4.16)$$

Where

$LO$  : The level of the overloading of the congested transmission line

$P_{j-k}^{uncon}$  : The active power of congested line  $j-k$  in unconstrained dispatch

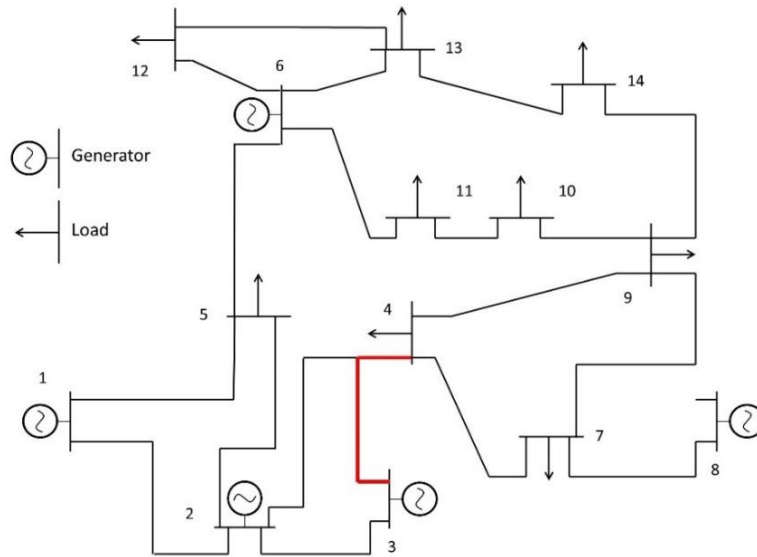
$P_{j-k}^{lim}$  : The limit value of active power transmission capacity of congested line  $j-k$

LO equals zero means there is no congestion on line  $j-k$ . When LO is above 0% and less than 100%, it shows the line  $j-k$  is congested to some degree. If LO reaches 100%, which means the transfer capacity limitation of this particular transmission line reaches zero, the transmission line is totally overloaded.

#### 4.5.2. Modified IEEE-14 Bus System

A modified IEEE-14 bus system is chosen as the first test system in this case study as shown in Figure 4.2.





**Figure 4.2 The modified IEEE-14 bus system**

The 14-bus system consists of 5 generators, 9 loads and 20 transmission lines. Before the test, some parameters of the test environment are set as follows:

**Table 4.1 Generator and Load Data in 14 Bus System**

Bus	Generator	Incremental Bid price (£/MWh)	Decremental Bid price (£/MWh)	Max power output (MW)	Load(MW)
1	Gen1	30	30	200	0
2	Gen2	30	30	200	0
3	Gen3	40	40	200	0
4	-	-	-	-	50
5	-	-	-	-	50
6	Gen6	40	40	200	0
7	-	-	-	-	60
8	Gen8	45	45	200	0
9	-	-	-	-	50
10	-	-	-	-	60
11	-	-	-	-	110
12	-	-	-	-	40
13	-	-	-	-	40
14	-	-	-	-	40
Total load					500

In Table 4.1, incremental/decremental bid prices, maximum MW generation outputs of five generators, MW demand of each load and total load are set in the table.

After a DC optimal power flow without considering the transmission line constraints, the generation schedule for each generator in the unconstrained dispatch is obtained. Then it is found that the power flow of transmission line 3-4 in the unconstrained dispatch is more than its transmission capacity limit. Therefore, congestion occurs on the line 3-4 which is regarded as the congested line in 14-bus case. Such above information is found in Table 4.2.

**Table 4.2 Generation Schedule and the Congested Line Power Flow in the Unconstrained Dispatch in 14 Bus System**

<b>Generator</b>	<b>Generation Schedule in the Unconstrained dispatch (MW)</b>
<b>Gen1</b>	153.41
<b>Gen2</b>	26.40
<b>Gen3</b>	160.10
<b>Gen6</b>	160.10
<b>Gen8</b>	0
<b>Transmission line</b>	<b>Unconstrained power flow (MW)</b>
<b>3-4</b>	121.74
	<b>Transmission line limit (MW)</b>
	66.00

Since the power flow of transmission line 3-4 in unconstrained dispatch is 121.74 MW which exceeds its transmission capacity limit 66 MW. According to the equation 4.14, the level of overloading of the congested line 3-4 is 45%. Based on the different slack bus selections, the re-dispatch generation variation of each generator calculated by the proposed method when congested line's LO equals to 45% is obtained and the results are shown in Table 4.3. In order to make it more straightforward, contents of Table 4.3 are visualized in Figure 4.3. Finally, the congestion costs of different slack bus cases are indicated in Figure 4.4.

**Table 4.3 Generator Output Variation in Each Slack Bus Selection Due to Re-dispatch in 14-Bus System when LO=45%**

Generator	$\Delta P_G$ Slack bus 1 (MW)	$\Delta P_G$ Slack bus 2 (MW)	$\Delta P_G$ Slack bus 3 (MW)	$\Delta P_G$ Slack bus 6 (MW)	$\Delta P_G$ Slack bus 8 (MW)
<b>G1</b>	0	0	0	0	+10.11
<b>G2</b>	0	0	0	0	0
<b>G3</b>	-92.80	-92.80	-92.79	-92.79	-95.16
<b>G6</b>	+38.85	+38.88	+38.84	+38.84	+38.86
<b>G8</b>	+53.94	+53.91	+53.95	+53.95	+46.19

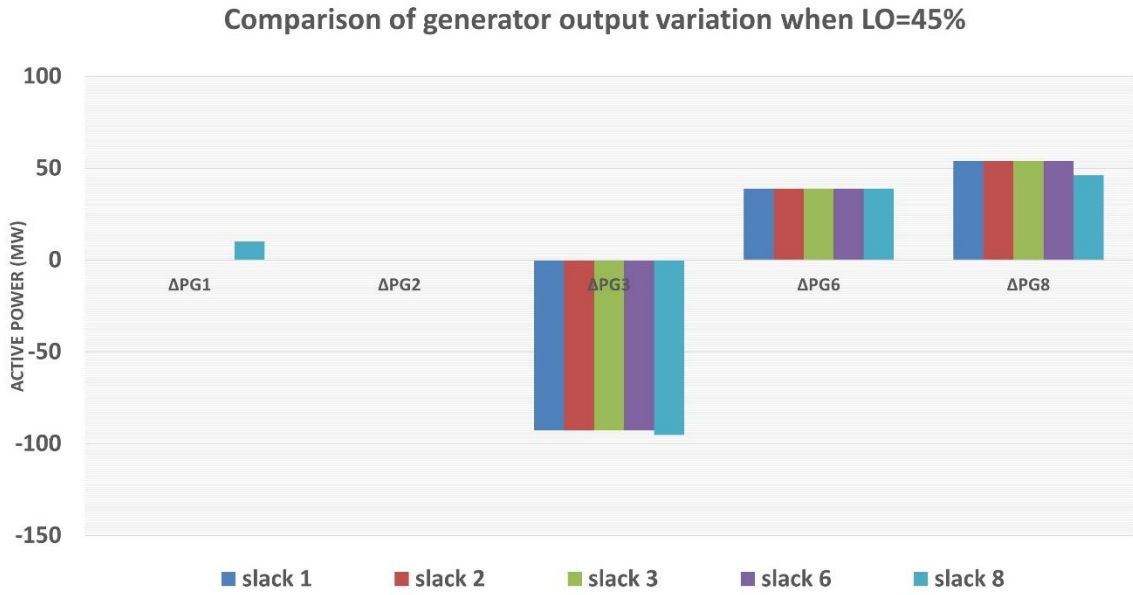


Figure 4.3 14-bus system generator output variation in re-dispatch when LO=45%

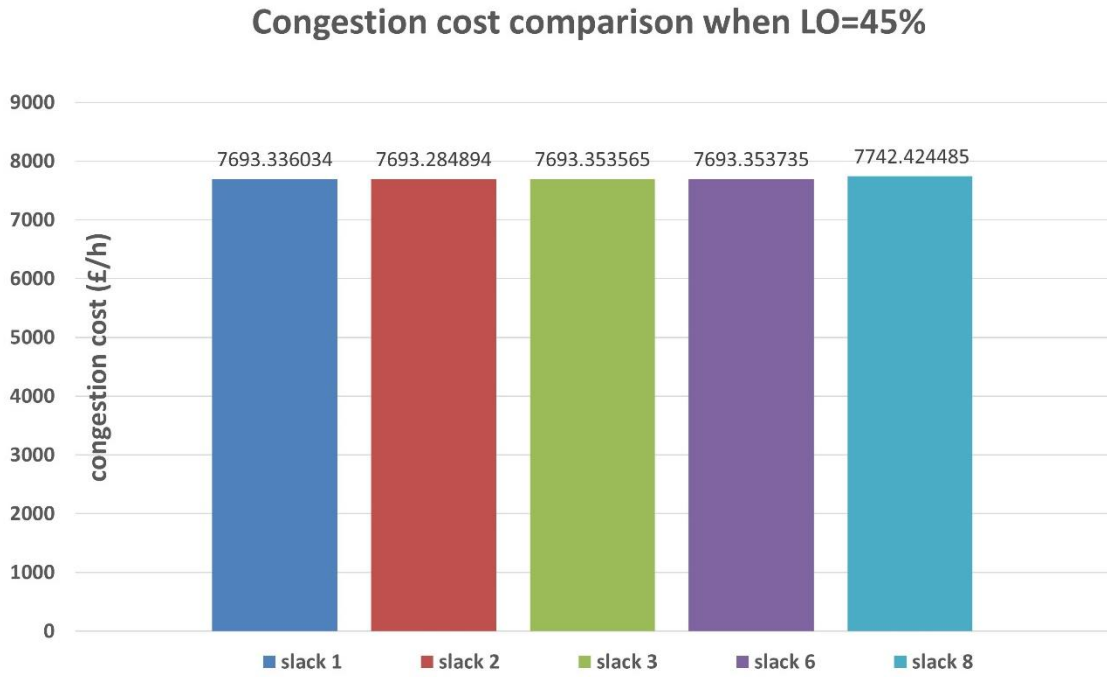


Figure 4.4 14-bus system congestion costs of re-dispatch in five slack bus cases when LO =45%

From the Figure 4.3, by using the proposed method with PTDF sensitivity factor to relieve the congestion under the level of overloading of target line at 45%, for different slack bus selection, the different re-dispatch generation variation of each generator is obtained. Generation re-dispatching in the first four slack bus selections are almost the same because generator 3 in these four cases is all asked to decrease its output and generator 6 and 8 are all ordered to increase their initial outputs. Furthermore, their output variation volumes are nearly the same. As a result, from the Figure 4.4, the corresponding congestion costs of these four cases only have slight differences which could be neglected. But when bus 8 is selected as the slack bus, it is found that the increase of generator 1's output leads to generator 3 output being decreased more than the first four cases and generator 8 increasing its output less than in the other four cases. This is the reason why the congestion cost in slack bus 8 case 7742 £/h is slightly more than other slack bus cases 7693 £/h.

#### 4.5.3. Modified IEEE-30 Bus System

A modified IEEE-30 bus system is the second test system in this case study as shown in Figure 4.5. The 30-bus system consists of 6 generators, 18 loads and 41 transmission lines.

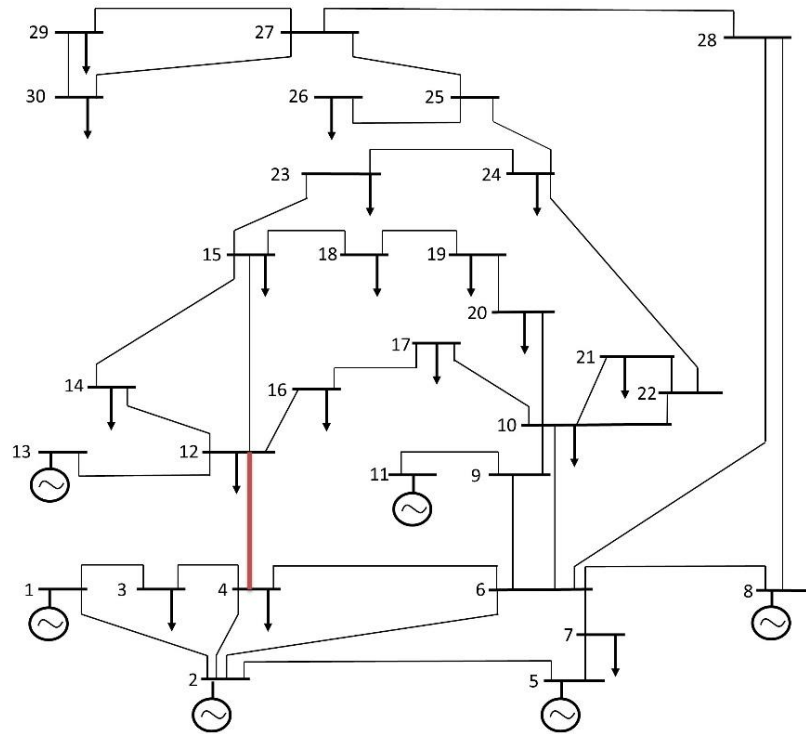


Figure 4.5 The modified IEEE-30 bus system

Before the test, some parameters of the test environment are set as follows:

Table 4.4 Generator Data in 30 Bus System

Bus	Generator	Incremental Bid price (£/MWh)	Decremental Bid price (£/MWh)	Max power output (MW)
1	Gen1	35	35	300
2	Gen2	35	35	300
5	Gen5	45	45	300
8	Gen8	45	45	300
11	Gen11	45	45	300
13	Gen13	50	50	300

**Table 4.5 Load Data in 30 Bus System**

<b>Bus</b>	<b>Load(MW)</b>	<b>Bus</b>	<b>Load(MW)</b>	<b>Bus</b>	<b>Load(MW)</b>
<b>1</b>	-	<b>11</b>	-	<b>21</b>	30
<b>2</b>	-	<b>12</b>	60	<b>22</b>	-
<b>3</b>	30	<b>13</b>	-	<b>23</b>	50
<b>4</b>	40	<b>14</b>	50	<b>24</b>	60
<b>5</b>	-	<b>15</b>	50	<b>25</b>	-
<b>6</b>	-	<b>16</b>	60	<b>26</b>	50
<b>7</b>	50	<b>17</b>	70	<b>27</b>	-
<b>8</b>	-	<b>18</b>	30	<b>28</b>	-
<b>9</b>	-	<b>19</b>	50	<b>29</b>	30
<b>10</b>	60	<b>20</b>	30	<b>30</b>	50
<b>Total load</b>					850

In Table 4.4, incremental/decremental bid prices, maximum MW generation outputs of six generators are set. In Table 4.5, each load MW and total load MW are set as above.

After running a DC optimal power flow without considering the transmission line constraints, the generation schedule for each generator in the unconstrained dispatch is obtained. Then it is shown that the power flow of transmission line 4-12 in the unconstrained dispatch is more than its transmission capacity limit. Therefore, congestion occurs on the line 4-12 which is the congested line in 30-bus case. Such above information is shown in Table 4.6 as follows:

**Table 4.6 Generation Schedule and the Congested line power flow in the Unconstrained Dispatch in 30 Bus System**

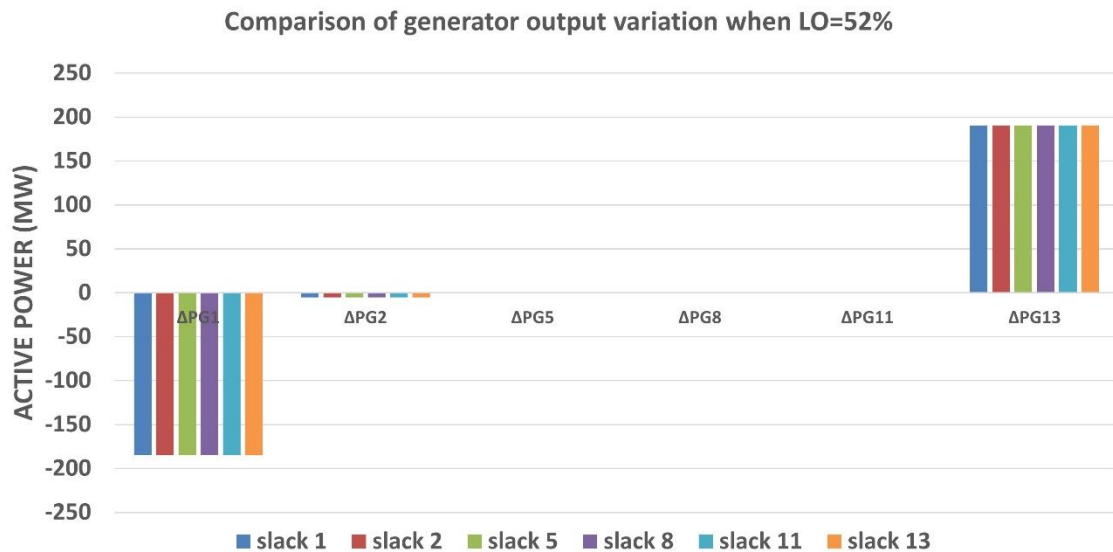
<b>Generator</b>	<b>Generation Schedule in the Unconstrained dispatch (MW)</b>
<b>Gen1</b>	185.28
<b>Gen2</b>	28.48
<b>Gen5</b>	212.08
<b>Gen8</b>	212.08
<b>Gen11</b>	212.08
<b>Gen13</b>	0
<b>Transmission line</b>	<b>Unconstrained power flow (MW)</b>
<b>4-12</b>	224.66
	<b>Transmission line limit (MW)</b>
	107.84

Because the power flow of transmission line 4-12 in unconstrained dispatch is 224.66 MW which is over its transmission capacity limit 107.84 MW. According to the equation 4.14, the level of overloading of the congested line 4-12 reaches to 52%. According to the six slack bus selections, the re-dispatch generation variation of each generator calculated by the proposed method with the LO of congested line equals to 52% is acquired and the results are shown in Table 4.7. To make it more comparable, contents of Table 4.7 is visualized in Figure 4.6. Finally, the corresponding congestion costs of six slack bus cases are indicated in Figure 4.7.



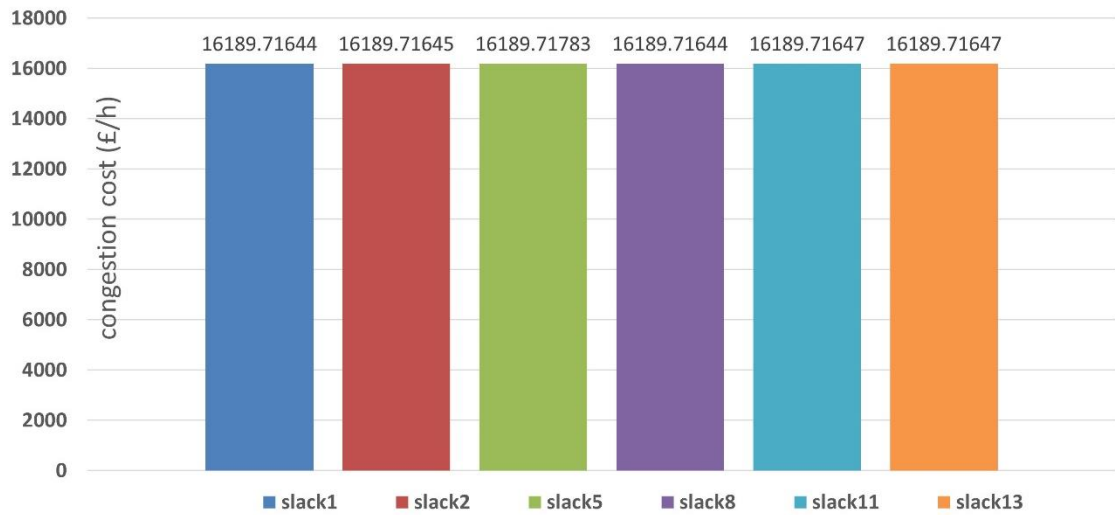
**Table 4.7 Generator Output Variation in Each Slack Bus Selection Due to Re-dispatch in 30-Bus System when LO=52%**

Generator	$\Delta P_G$ Slack bus 1 (MW)	$\Delta P_G$ Slack bus 2 (MW)	$\Delta P_G$ Slack bus 5 (MW)	$\Delta P_G$ Slack bus 8 (MW)	$\Delta P_G$ Slack bus 11 (MW)	$\Delta P_G$ Slack bus 13 (MW)
<b>G1</b>	-185.15	-185.15	-185.15	-185.15	-185.15	-185.15
<b>G2</b>	-5.32	-5.32	-5.32	-5.32	-5.32	-5.32
<b>G5</b>	0	0	0	0	0	0
<b>G8</b>	0	0	0	0	0	0
<b>G11</b>	0	0	0	0	0	0
<b>G13</b>	+190.47	+190.47	+190.47	+190.47	+190.47	+190.47



**Figure 4.6 30-bus system generator output variation in re-dispatch when LO=52%**

### Congestion cost comparison when LO=52%



**Figure 4.7 30-bus system congestion costs of re-dispatch in six slack bus cases when LO=52%**

From Figure 4.6, by selecting different buses as slack bus, the generation variation of each generator in re-dispatch stage under the congestion level of the congested line equals to 52% in 30 bus system is calculated by the PTDF method. Generation re-dispatching in all the slack bus selections are identical since in all cases generator 1 and 2 are asked to decrease their outputs and generator 13 is ordered to increase its initial outputs. Furthermore, their output variation volumes in all cases are nearly the same. So in Figure 4.7, the corresponding congestion costs of six cases are nearly identical at 16189 £/h.

#### 4.5.4. Modified IEEE-57 Bus System

A modified IEEE-57 bus system is the third test system in this case study as shown in Figure 4.8. The 57-bus system consists of 7 generators, 35 loads and 80 transmission lines.

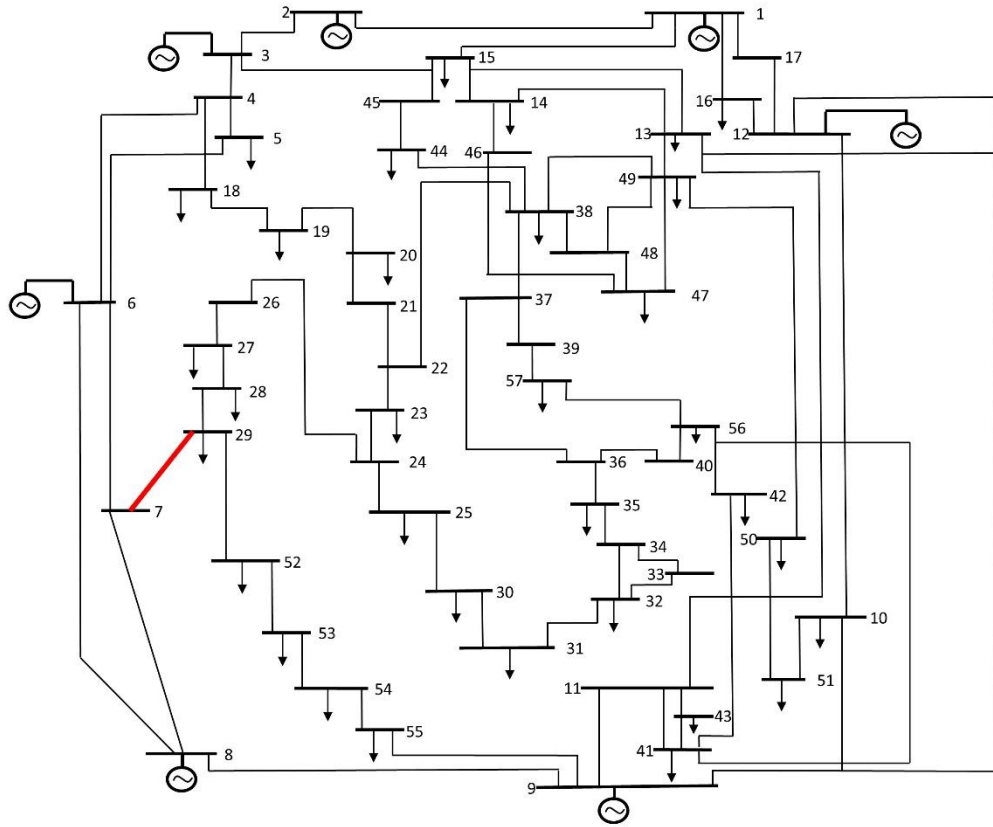


Figure 4.8 The modified IEEE-57 bus system

Before the test, some parameters of the test environment are set as follows:

Table 4.8 Generator Data in 57 Bus System

Bus	Generator	Incremental Bid price (£/MWh)	Decremental Bid price (£/MWh)	Max power output (MW)
1	Gen1	30	30	300
2	Gen2	35	35	300
3	Gen3	30	30	300
6	Gen6	45	45	300
8	Gen8	45	45	300
9	Gen9	45	45	300
12	Gen12	40	40	300

**Table 4.9 Load Data in 57 Bus System**

Bus	Load(MW)	Bus	Load(MW)	Bus	Load(MW)
1	-	20	60	39	-
2	-	21	-	40	-
3	-	22	-	41	40
4	-	23	30	42	30
5	60	24	-	43	30
6	-	25	50	44	20
7	-	26	-	45	-
8	-	27	30	46	-
9	-	28	20	47	50
10	30	29	50	48	-
11	-	30	30	49	20
12	-	31	30	50	30
13	20	32	20	51	30
14	20	33	30	52	40
15	40	34	-	53	30
16	20	35	50	54	50
17	50	36	-	55	50
18	60	37	-	56	30
19	30	38	20	57	50
<b>Total load</b>					1250

In Table 4.8, incremental/decremental bid prices, maximum MW generation outputs of seven generators are set as above. In Table 4.9, each load demand MW and the total load MW are shown.

After running a DC optimal power flow without considering the transmission line constraints, the generation schedule for each generator in the unconstrained dispatch is gotten. Then power flow results indicate that the power flow of transmission line 7-29 under the unconstrained dispatch exceeds its maximum transmission capacity. Therefore,

in 57-bus case, transmission congestion occurs on the line 7-29. Information about generation schedules and the power flow of the congested line in the unconstrained dispatch are shown in Table 4.10 as follows:

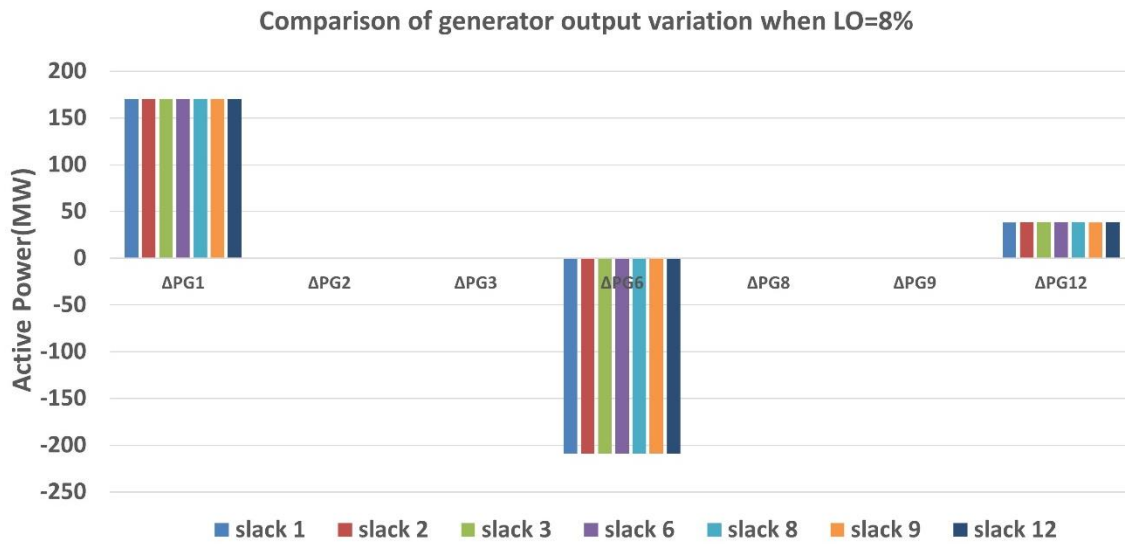
**Table 4.10 Generation Schedule and the Congested Line Power Flow in the Unconstrained Dispatch in 57 Bus System**

<b>Generator</b>	<b>Generation Schedule in the Unconstrained dispatch (MW)</b>
<b>Gen1</b>	129.50
<b>Gen2</b>	300
<b>Gen3</b>	40.19
<b>Gen6</b>	254.64
<b>Gen8</b>	114.59
<b>Gen9</b>	254.64
<b>Gen12</b>	156.44
<b>Transmission line</b>	<b>Unconstrained power flow (MW)</b>
<b>7-29</b>	213.05
	<b>Transmission line limit (MW)</b>
	196.01

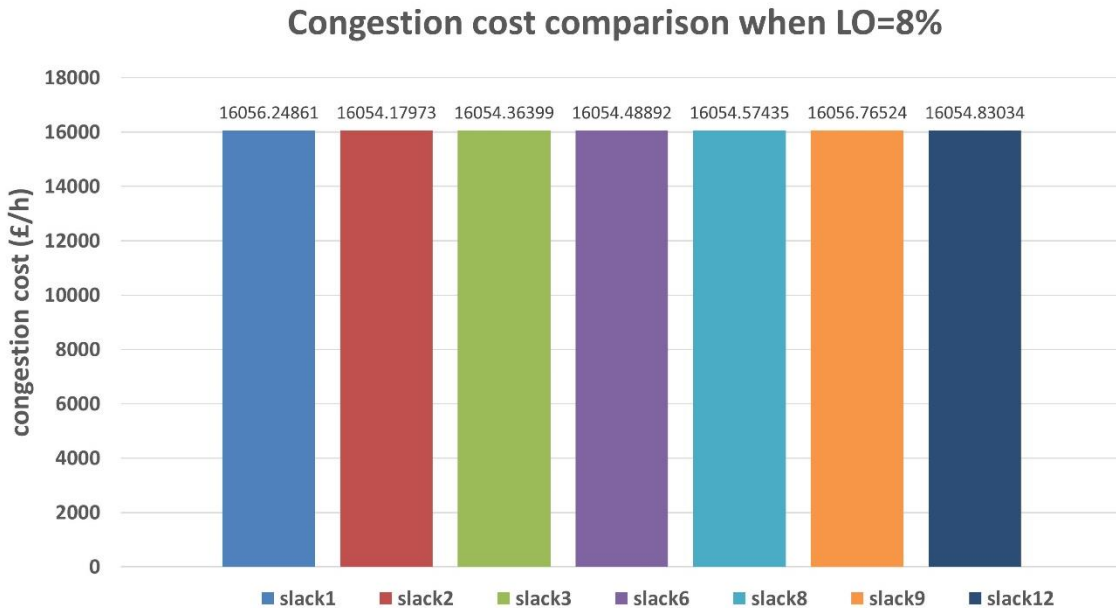
Since the power flow of transmission line 7-29 in unconstrained dispatch is 213.05 MW which is more than its transmission capacity limit 196.01 MW. By using the equation 4.14, the level of overloading of the congested line 7-29 is 8%. According to situations of selecting the different seven generator buses as slack bus, the re-dispatch generation variation of each generator calculated by the proposed method with the LO of congested line equals to 8% is obtained and the results are shown in Table 4.11. To make it more comparable, contents of Table 4.11 is visualized in Figure 4.9. Finally, the corresponding congestion costs of six slack bus cases are indicated in Figure 4.10.

**Table 4.11 Generator Output Variation Due to Re-dispatch in 57-bus System when LO=8%**

Generator	$\Delta P_G$	$\Delta P_G$	$\Delta P_G$	$\Delta P_G$	$\Delta P_G$	$\Delta P_G$	$\Delta P_G$
	Slack bus 1 (MW)	Slack bus 2 (MW)	Slack bus 3 (MW)	Slack bus 6 (MW)	Slack bus 8 (MW)	Slack bus 9 (MW)	Slack bus 12 (MW)
<b>G1</b>	+170.47	+170.47	+170.47	+170.47	+170.47	+170.47	+170.47
<b>G2</b>	0	0	0	0	0	0	0
<b>G3</b>	+0.21	0	0	0	0	+0.21	0
<b>G6</b>	-208.98	-208.93	-208.93	-208.93	-208.93	-208.98	-208.93
<b>G8</b>	0	0	0	0	0	0	0
<b>G9</b>	0	0	0	0	0	0	0
<b>G12</b>	+38.31	+38.46	+38.46	+38.46	+38.46	+38.31	+38.46



**Figure 4.9 57-bus system generator output variation in re-dispatch when LO=8%**

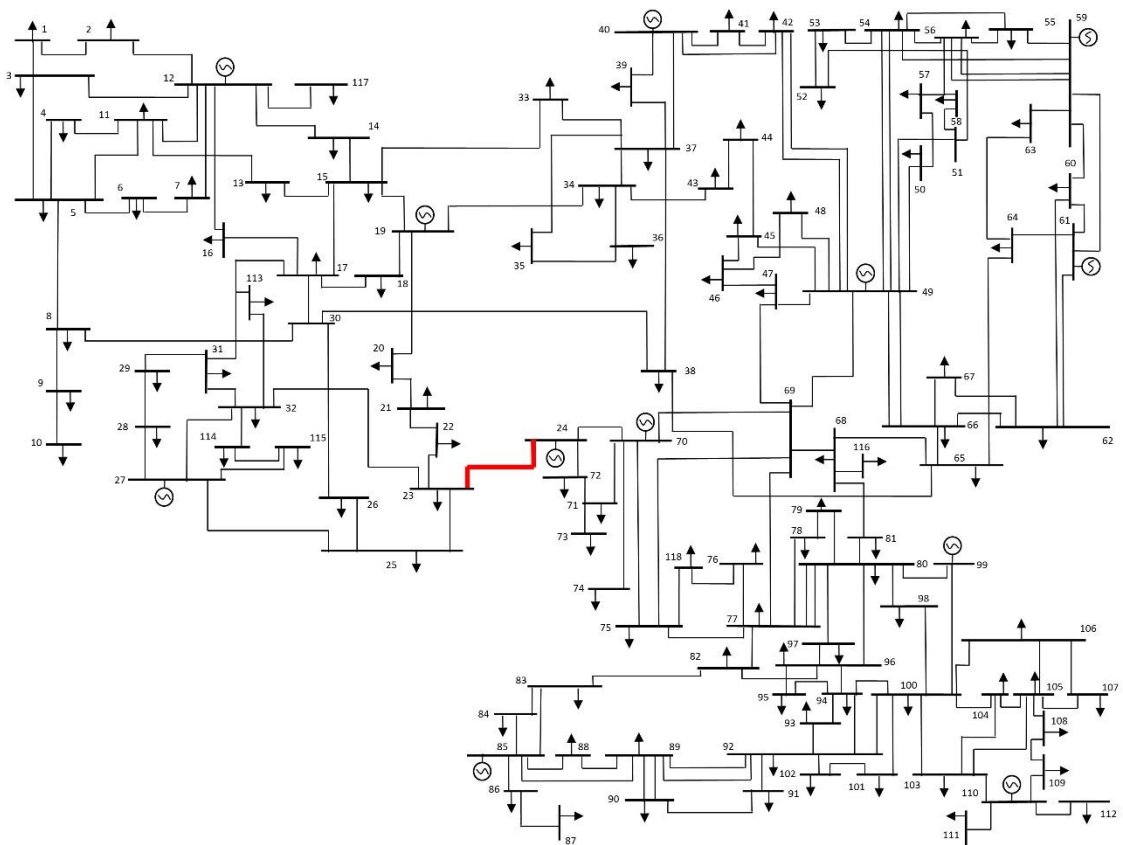


**Figure 4.10 57-bus system congestion costs of re-dispatch in seven slack bus cases when LO=8%**

From Figure 4.9, in the 57 bus system, by selecting different generator buses as slack bus, generation variation of each generator in the re-dispatch stage under the level of overloading of the congested line 7-29 equals to 8% is calculated by the proposed PTFDF sensitivity method. Generation reschedules in all the slack bus cases are all very similar since generator 1 and generator 12 are all asked to increase their outputs and generator 6 is asked to decrease its output in all cases. It is also found that the output variation volumes in all cases are very close. As a consequence, in Figure 4.10, the corresponding congestion costs of seven cases are nearly equal. However, it was found that generator 3 is involved in the re-dispatch when the slack bus was selected at bus 1 and bus 9 (highlighted in Table 4.11). This is the reason why the congestion costs of case 1 and case 9 which are both around 16056 £/h are a little bit more than other cases which are 16054 £/h. But since the involved power volume increase of generator 3 is so small compared with the total power variation volume, which is 0.1%, so it can be neglected by the system operator.

### 4.5.5. Modified IEEE-118 Bus System

A modified IEEE-118 bus system is the last test system in this case study as shown in Figure 4.11. It is noticed that the 118-bus system consists of 12 generators, 104 loads and 186 transmission lines. Before the testing, some parameters of the power system need to be set up. In Table 4.12, incremental/decremental bid prices, maximum MW generation outputs of the twelve generators can be found. In Table 4.13, each load demand MW and the total load MW are displayed.



**Figure 4.11** The modified IEEE-118 bus system



**Table 4.12 Generator Data in 118 Bus System**

<b>Bus</b>	<b>Generator</b>	<b>Incremental Bid price (£/MWh)</b>	<b>Decremental Bid price (£/MWh)</b>	<b>Max power output (MW)</b>
<b>12</b>	Gen12	30	30	300
<b>19</b>	Gen19	30	30	300
<b>24</b>	Gen24	35	35	300
<b>27</b>	Gen27	40	40	300
<b>40</b>	Gen40	30	30	300
<b>49</b>	Gen49	35	35	300
<b>59</b>	Gen59	35	35	300
<b>61</b>	Gen61	40	40	300
<b>70</b>	Gen70	30	30	300
<b>85</b>	Gen85	35	35	300
<b>99</b>	Gen99	35	35	300
<b>110</b>	Gen110	40	40	300

**Table 4.13 Load Data in 118 Bus System**

<b>Bus</b>	<b>Load(MW)</b>	<b>Bus</b>	<b>Load(MW)</b>	<b>Bus</b>	<b>Load(MW)</b>
<b>1</b>	30	<b>41</b>	30	<b>81</b>	10
<b>2</b>	20	<b>42</b>	60	<b>82</b>	50
<b>3</b>	30	<b>43</b>	20	<b>83</b>	20
<b>4</b>	30	<b>44</b>	15	<b>84</b>	10
<b>5</b>	10	<b>45</b>	50	<b>85</b>	-
<b>6</b>	30	<b>46</b>	20	<b>86</b>	10
<b>7</b>	20	<b>47</b>	30	<b>87</b>	10
<b>8</b>	30	<b>48</b>	20	<b>88</b>	40
<b>9</b>	10	<b>49</b>	-	<b>89</b>	10
<b>10</b>	20	<b>50</b>	15	<b>90</b>	10
<b>11</b>	50	<b>51</b>	15	<b>91</b>	10
<b>12</b>	-	<b>52</b>	15	<b>92</b>	60

<b>13</b>	30	<b>53</b>	20	<b>93</b>	15
<b>14</b>	15	<b>54</b>	10	<b>94</b>	30
<b>15</b>	60	<b>55</b>	60	<b>95</b>	40
<b>16</b>	20	<b>56</b>	80	<b>96</b>	30
<b>17</b>	10	<b>57</b>	15	<b>97</b>	15
<b>18</b>	10	<b>58</b>	15	<b>98</b>	30
<b>19</b>	-	<b>59</b>	-	<b>99</b>	-
<b>20</b>	15	<b>60</b>	70	<b>100</b>	30
<b>21</b>	15	<b>61</b>	-	<b>101</b>	20
<b>22</b>	10	<b>62</b>	70	<b>102</b>	15
<b>23</b>	40	<b>63</b>	10	<b>103</b>	20
<b>24</b>	-	<b>64</b>	20	<b>104</b>	30
<b>25</b>	50	<b>65</b>	10	<b>105</b>	30
<b>26</b>	30	<b>66</b>	30	<b>106</b>	40
<b>27</b>	-	<b>67</b>	20	<b>107</b>	50
<b>28</b>	10	<b>68</b>	10	<b>108</b>	20
<b>29</b>	20	<b>69</b>	-	<b>109</b>	10
<b>30</b>	-	<b>70</b>	-	<b>110</b>	-
<b>31</b>	40	<b>71</b>	10	<b>111</b>	10
<b>32</b>	50	<b>72</b>	10	<b>112</b>	6
<b>33</b>	20	<b>73</b>	10	<b>113</b>	10
<b>34</b>	50	<b>74</b>	60	<b>114</b>	10
<b>35</b>	30	<b>75</b>	50	<b>115</b>	20
<b>36</b>	10	<b>76</b>	60	<b>116</b>	10
<b>37</b>	10	<b>77</b>	60	<b>117</b>	20
<b>38</b>	10	<b>78</b>	70	<b>118</b>	30
<b>39</b>	20	<b>79</b>	30		
<b>40</b>	-	<b>80</b>	10		
<b>Total load</b>					2736

After a DC optimal power flow without considering the transmission line constraints, the initial generation schedule for each generator in the unconstrained dispatch stage is given. Then results of optimal power flow indicate that the power flow of

transmission line 23-24 in the unconstrained dispatch is above its maximum transmission capacity. Therefore, in 118-bus case, transmission congestion occurs on the line 23-24. Information about generation schedules and the power flow of the congested line in the unconstrained dispatch are shown in Table 4.14 as follows:

**Table 4.14 Generation Schedule and the Congested Line Power Flow in the Unconstrained Dispatch in 118 Bus System**

<b>Generator</b>	<b>Generation Schedule in the Unconstrained dispatch (MW)</b>
<b>Gen12</b>	71.17
<b>Gen19</b>	300
<b>Gen24</b>	300
<b>Gen27</b>	300
<b>Gen40</b>	300
<b>Gen49</b>	119.82
<b>Gen59</b>	91.04
<b>Gen61</b>	53.97
<b>Gen70</b>	300
<b>Gen85</b>	300
<b>Gen99</b>	300
<b>Gen110</b>	300
<b>Transmission line</b>	<b>Unconstrained power flow (MW)</b>
<b>23-24</b>	-211.43
	<b>Transmission line limit (MW)</b>
	2.11

Since the power flow of transmission line 23-24 in the unconstrained dispatch stage is -211.43 MW which is more than its transmission capacity limit 2.11 MW. It should be noted that the negative value means the power flow of the congested line is on an opposite direction. Based on the equation 4.14, the level of overloading of the congested line 23-24 is obtained at 99%. By selecting the different twelve generator buses

as the slack bus, the re-dispatch generation variation of each generator calculated by the proposed method with the LO of congested line equals to 99% is obtained and the results are shown in Table 4.15. To make it more comparable, contents of Table 4.15 is visualized in Figure 4.12. Finally, the corresponding congestion costs of twelve slack bus cases are shown in Figure 4.13.

**Table 4.15 Generator Output Variation in Each Slack Bus Selection Due to Re-dispatch in 118 Bus System at LO=99%**

Generator	$\Delta P_G$ Slack bus 12 (MW)	$\Delta P_G$ Slack bus 19 (MW)	$\Delta P_G$ Slack bus 24 (MW)	$\Delta P_G$ Slack bus 27 (MW)	$\Delta P_G$ Slack bus 40 (MW)	$\Delta P_G$ Slack bus 49 (MW)	$\Delta P_G$ Slack bus 59 (MW)	$\Delta P_G$ Slack bus 61 (MW)	$\Delta P_G$ Slack bus 70 (MW)	$\Delta P_G$ Slack bus 85 (MW)	$\Delta P_G$ Slack bus 99 (MW)	$\Delta P_G$ Slack bus 110 (MW)
<b>Gen12</b>	+228.83	+228.83	+228.83	+228.83	+228.83	+228.83	+228.83	+228.83	+228.83	+228.83	+228.83	+228.83
<b>Gen19</b>	0	0	0	0	0	0	0	0	0	0	0	0
<b>Gen24</b>	-296.55	-296.55	-296.50	-296.50	-296.55	-296.55	-296.50	-296.50	-296.55	-296.50	-296.50	-296.55
<b>Gen27</b>	0	0	0	0	0	0	0	0	0	0	0	0
<b>Gen40</b>	0	0	0	0	0	0	0	0	0	0	0	0
<b>Gen49</b>	+62.65	+62.65	+67.68	+67.68	+62.65	+62.65	+67.68	+67.67	+62.65	+67.68	+67.66	+62.67
<b>Gen59</b>	+5.09	+5.09	0	0	+5.09	+5.09	0	0	+5.09	0	0	+5.09
<b>Gen61</b>	0	0	0	0	0	0	0	0	0	0	0	0
<b>Gen70</b>	0	0	0	0	0	0	0	0	0	0	0	0
<b>Gen85</b>	0	0	0	0	0	0	0	0	0	0	0	0
<b>Gen99</b>	0	0	0	0	0	0	0	0	0	0	0	0
<b>Gen110</b>	0	0	0	0	0	0	0	0	0	0	0	0

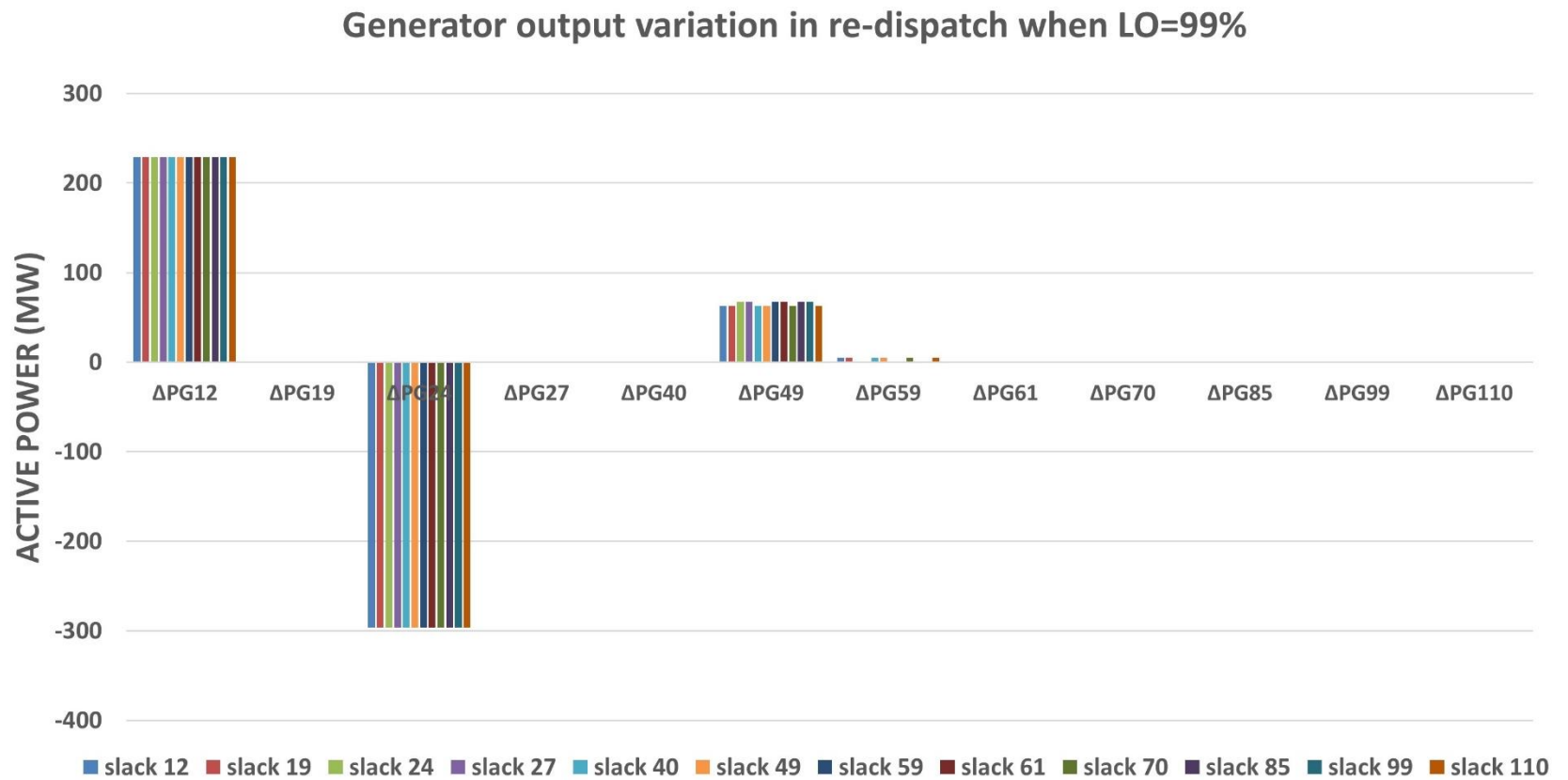


Figure 4.12 118-bus system generator output variation in re-dispatch when LO=99%

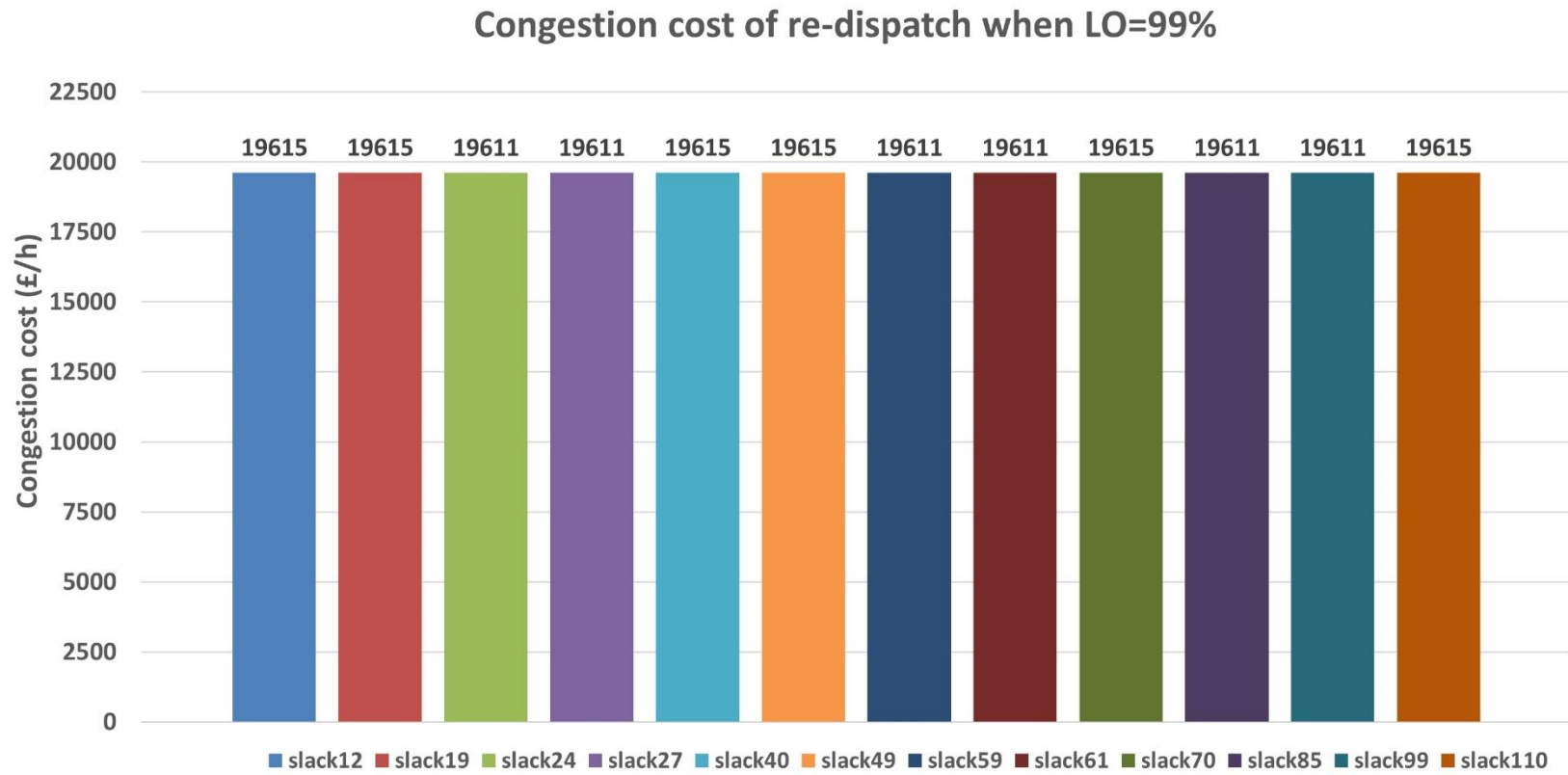


Figure 4.13 118-bus system congestion cost of re-dispatch in all slack bus cases when LO=99%

From Figure 4.12, by selecting different generator buses as the slack bus, the re-dispatch generation variation of each generator when the level of overloading of the congested transmission line 23-24 equals to 99% in 118 bus system is decided by the proposed sensitivity method. As noticed that there are eight generators have already output their maximum capacities in the unconstrained dispatch stage. So there are only four generators left still have extra capacity to allow the system operator to re-dispatch. In all twelve cases, the variation output of generator 12 and 24 are same. The difference is generator 59 is asked to increase its output when the slack bus is selected on bus 12, 19, 40, 49, 70 and 110. Furthermore the increased output variation volume in these cases are nearly the same as 5.09 MW. As a result, the power change volumes of generator 49 in such cases are different from other cases in which generator 59 is not involved in re-dispatching. So in Figure 4.13, the corresponding congestion costs of the twelve cases are slightly different. The congestion cost of the cases with generator 59 participation is 19615 £/h while the cost of the cases without generator 59 participation is 19611 £/h.

#### 4.5.6. Discussion

The proposed re-dispatch scheme with the sensitivity index PTDF is tested under four different IEEE modified power systems. Overall, the method is capable to re-dispatch the generation schedule based on the objective function with three constraints. The sensitivity factor PTDF is able to find the power flow contribution of each generator to the congested line so the re-dispatching is become more effective and accurate. Based on the rules of the incremental and decremental bids, the corresponding cost due to transmission congestion can be obtained. Additionally, the introduction of the incremental and decremental bids improves the motivation of market participants to be involved in the re-dispatching action so the system operator is allowed to remove the transmission congestion risk with plenty of power resource under a minimum cost.



Four modified test system results demonstrate the capability of the proposed PTDF method for generation re-dispatching. From 14-bus test system up to 118-bus test system, the proposed method can all provide a solution to relieve transmission congestion issue by re-dispatching initial generation schedules. With the help of the concept that the level of overloading (LO) of a congested line, the performance of the proposed method is further demonstrated. In the 14-bus case, with the overloading level of the congested line 3-4 reaching 45%, the PTDF method solved the congestion problem properly. In the 30-bus case, the LO of the congested line 4-12 is 52% but the method still works. In the 57-bus case, the LO of the congested line 7-29 is only at 8% and the proposed method fulfilled the rescheduling of the generation plan by asking three generators to change their initial generation outputs. In the 118-bus case, the LO of the congested line 23-24 came up to 99%. It should be noted that this case is very extreme and might be unrealistic but technically it can reflect the proposed method's performance in utmost conditions. The outcome is the PTDF sensitivity method provided a solution to handle this severe congestion problem.

However, the potential weakness is that the PTDF index method is dependent on the slack bus selection. The experimental outcomes proved that the results are indeed different when choosing different generator bus as the slack bus. In the 57-bus case, a difference in the results existed but the distinction is very small so it can be ignored. In the 14-bus case and 118-bus case, the outcome differences between different slack bus selections are granted. Consequently, it may make the PTDF method become inaccurate if only consider one generator bus as the slack bus. Whereas, if all the generator buses are respectively chosen as the slack bus plus the power system has a number of generators, the result calculation process will become complex. The case of 118-bus system is a difficult example to test it because there are twelve possible choices for the slack bus which need to be considered.

## 4.6. Summary

This chapter introduced a proposed method with the sensitivity index PTDF to re-dispatch generation when transmission congestion occurs in the power system. Some conventional re-dispatch generation methods are described before the content of the proposed method. The differences between the conventional method and the proposed method are indicated, which makes the latter one more effective and reasonable. The core concept of this proposed method is a sensitivity index PTDF whose algorithm is described with a series of formulae. Then the mathematic model of the proposed method including an objective function and three constraints one of which includes the sensitivity factor are displayed in detail. Case studies consist of four test systems which are modified IEEE systems respectively with 14 buses, 30 buses, 57 buses and 118 buses. Experimental results show that the proposed method is capable under most random circumstances. However, some results proved that it still has drawback. Dependence of slack bus selection lead to the inaccuracy and the complexity of the PTDF based method. This disadvantage should be addressed or another approach will come to replace this method as a solution.

## Chapter 5

# Re-dispatch Generation in Deregulated Electricity Market with Bus Impedance Matrix Sensitivity

### 5.1. Introduction

In chapter 4, a proposed generation re-dispatch method with the power transfer distribution factor is introduced, described and tested. Through experimental results from four test systems, it is proved that the weakness of the method is the dependence of slack bus selection. Choosing different generator bus as the slack bus leads to the different outcomes of generation rescheduling and the corresponding security costs become more or less distinct as well. Hence the method might be inaccurate on the generation re-dispatching if there is a transmission congestion problem occurs and generators would be in front of an unfair command to be asked to adjust their generation outputs. In this chapter, another sensitivity factor is recommended to apply in the proposed re-dispatching method, which is called the bus impedance matrix sensitivity. As its name implies, the sensitivity index is related to the bus impedance matrix of the power flow system and is able to determine the sensitivity of a particular transmission line power flow with respect to power injection variation at a particular bus. This chapter is aim to test the performance of the re-dispatch scheme with above sensitivity factor and compare it with the PTDF based method under the same test conditions.

In this chapter, the proposed generation re-dispatch method associated with the bus impedance matrix sensitivity will be represented in detail including the methodology of the proposed re-dispatch method in section 5.2. The descriptions of the admittance matrix and the impedance matrix are in the section 5.3. The algorithm of the sensitivity

index is in the section 5.4. Section 5.5 will give the mathematic model of the proposed method including an objective function and three constraints one of which contains the bus impedance matrix sensitivity. Section 5.6 is the case study which consisted of experimental results from four modified IEEE testing systems including 14-bus, 30-bus, 57-bus and 118-bus systems. Comparison between the bus impedance matrix sensitivity method and the power transfer distribution factor method will also be indicated in this section. Section 5.7 is the summary.

## **5.2. Proposed Generation Re-dispatch Method in Deregulated Electricity Market**

The mechanism of the proposed generation re-dispatch method in this chapter is as same as the method in Chapter 4. The details of the proposed method principle can be found in section 4.2. The only difference is the proposed method in this chapter adopts two different sensitivity factors to calculate the power flow contribution of each node to the targeted transmission line. This time is the bus impedance matrix sensitivity which will be explained in the following section.

## **5.3. The Admittance Matrix and the Impedance Matrix**

This section will introduce the admittance matrix and the impedance matrix including their formulations and functions.

### **5.3.1. The Admittance Matrix**

In power engineering, the nodal admittance matrix which is also called admittance matrix, Y matrix or Ybus is regarded as a mathematical model to express the power system network through a simplified calculation [115]. Power system transmission requires optimized management for real and reactive power flows. When a set of load demands, voltages and currents are given, the necessary real and reactive power flows in the power

system needs to be determined. In most realistic power systems, situations are always very complex, which is difficult to deal with. Therefore, computer simulation is required to solve above problems. The admittance matrix is an effective tool which provides a route to simplify a complex power system to a matrix format so the power system problem can be worked out by a computer program.

From load flow equations, a nodal current equation for an n-bus power system is shown as follows [116]:

$$I_i = \sum_{j=1}^n Y_{ij} V_j, i = 1, 2, \dots, n \quad (5.1)$$

Where

$I_i$  : The nodal current in bus  $i$

$Y_{ij}$  : The admittance matrix of the n-bus power system

$V_j$  : The voltage on bus  $j$

Equation (5.1) can be expressed in matrix form as follows:

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \quad (5.2)$$

Hence, in general the format of the Ybus matrix for an n-bus power system is as follows:

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{bmatrix} \quad (5.3)$$

It should be noted that the admittance matrix is a singular matrix [117]. Consequently, it is necessary to select one of the buses in the system as the slack bus or called reference bus, otherwise, a singular admittance matrix is difficult to be utilized in the power system calculation. By choosing one of the buses as the slack bus, the corresponding row and column in Ybus matrix are deleted so that the reduced admittance matrix turns into a non-singular [116]. Hence, the reduced Ybus matrix can be easily dealt with.

### 5.3.2. The Impedance Matrix

Unlike the admittance matrix which is used very frequently in power flow study, the impedance matrix was not encouraged initially due to time, memory and computational requirements [118]. However, the impedance matrix becomes an important tool for power system analysis nowadays. If the bus impedance matrix of an n-bus system is available, such a matrix can be utilized for power flow study, short-circuit calculation and fault study [119]. There are a number of algorithms for building up a Zbus matrix including directly building up, non-singular transformation, graph theoretic approach and inversion of bus admittance matrix [118]. In this thesis, Zbus matrix is derived from Ybus matrix inversion. Hence, the Ybus inversion method will be concentrated on in the following content.

From load flow equations, a nodal voltage equation for an n-bus power system is shown as follows to express the relation between bus voltage and bus current:

$$V_j = \sum_{i=1}^n Z_{ij} I_i, j = 1, 2, \dots, n \quad (5.4)$$

Where

$Z_{ij}$  : The impedance matrix of the n-bus power system

Equation (5.4) can be expressed in matrix form as follows:

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \dots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} \quad (5.5)$$

Hence, in general the formulation of the Zbus matrix for an n-bus power system is as follows:

$$Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \dots & Z_{nn} \end{bmatrix} \quad (5.6)$$

Based on the Ybus inversion algorithm, the relation between Zbus matrix and Ybus matrix should be represented as follows [116]:

$$Z_{bus} = Y_{bus}^{-1} \quad (5.7)$$

However, as mentioned in section 5.3.1, the Ybus matrix is singular and cannot be inverted. Hence the Ybus matrix must be turned into a non-singular matrix by deleting the corresponding row and column of the slack bus. For example, if bus 1 is selected as the slack bus, the corresponding row and column of bus 1 will be deleted so that the Ybus matrix becomes a reduced Ybus matrix.

$$Y_{bus}^* = \begin{bmatrix} - & - & \dots & - \\ - & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ - & Y_{n2} & \dots & Y_{nn} \end{bmatrix} = \begin{bmatrix} Y_{22} & \dots & Y_{2n} \\ \vdots & \ddots & \vdots \\ Y_{n2} & \dots & Y_{nn} \end{bmatrix} \quad (5.8)$$

Where

$Y_{bus}^*$  : The reduced admittance matrix of the n-bus power system

Now the reduced Ybus matrix has become a non-singular matrix so it is able to be inverted. The inverse of the reduced Ybus matrix, a reduced Zbus matrix can be obtained.

$$Z_{bus}^* = Y_{bus}^{*-1} \quad (5.9)$$



Then by adding one row 0 and one column 0 into the reduced Zbus matrix, the Zbus matrix can be found.

$$Z_{bus} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \\ 0 & & Z_{bus}^* \end{bmatrix} \quad (5.10)$$

In conclusion, the Zbus matrix is formulated by inverting the Ybus matrix with selecting a slack bus. It should be noted that the Zbus matrix derived via the inversion of Ybus matrix relies on slack bus selection and elements associated with slack bus are zero.

#### 5.4. The Algorithm of Bus Impedance Matrix Sensitivity

Bus impedance matrix sensitivity (ZS) is the second sensitivity factor to detect the sensitivity of a particular transmission line power flow with respect to a bus power injection. The derivation of this sensitivity factor originated from the expression of the complex power flow of a particular transmission line from bus  $j$  to bus  $k$  [24, 112, 120]. A transmission line can be represented by the  $\pi$  equivalent circuit, as shown in Figure 5.1 [121], and the algorithm of the bus impedance matrix sensitivity is displayed as follows:

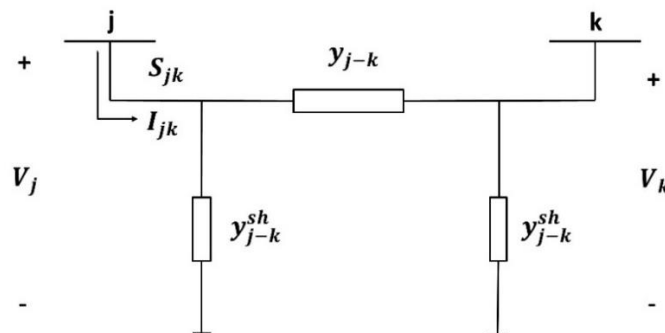


Figure 5.1  $\pi$  Equivalent Circuit of Transmission Line j-k

The expression of the complex power flow over transmission line  $j-k$  from bus  $j$  is shown as:

$$S_{j-k} = V_j \cdot I_{j-k}^* \quad (5.11)$$

Where

$S_{j-k}$  : The complex power flow of line  $j-k$  calculated at bus  $j$

$V_j$  : The bus voltage at bus  $j$

$I_{j-k}^*$  : The conjugate of current through line  $j-k$

Using the impedance matrix, the voltage at bus  $j$  is shown as:

$$V_j = \sum_{i=1}^n Z_{ji} \cdot I_i \quad (5.12)$$

Where

$Z_{ji}$  : The  $j_{th}$  row and  $i_{th}$  column element in the impedance matrix

$I_i$  : The nodal current at bus  $i$

The current through the line  $j-k$  is shown as:

$$I_{j-k} = (V_j - V_k) \cdot y_{j-k} + V_j \cdot y_{j-k}^{sh} \quad (5.13)$$

Where

$I_{j-k}$  : The current through line  $j-k$

$V_k$  : The bus voltage at bus  $k$

$y_{j-k}$  : The series admittance of the  $\pi$  equivalent circuit of line  $j-k$

$y_{j-k}^{sh}$  : The shunt admittance of the  $\pi$  equivalent circuit of line  $j-k$

Substituting equation (5.12) and equation (5.13) into equation (5.11), the complex power flow over transmission line  $j-k$  is rewritten as:

$$S_{j-k} = \sum_{i=1}^n V_j \cdot [ (Z_{ji} - Z_{ki}) \cdot y_{j-k} + Z_{ji} \cdot y_{j-k}^{sh} ]^* \cdot I_i^* \quad (5.14)$$

Where

$Z_{ki}$  : The  $k_{th}$  row and  $i_{th}$  column element in the impedance matrix

$I_i^*$  : The conjugate of nodal current at bus  $i$

The active power through line  $j-k$  is the real component of  $S_{j-k}$ :

$$P_{j-k} = \sum_{i=1}^n Re \{ V_j [ (Z_{ji} - Z_{ki}) \cdot y_{j-k} + Z_{ji} \cdot y_{j-k}^{sh} ]^* \cdot I_i^* \} \quad (5.15)$$

Where

Re: Taking the real component only

Since  $I_i^*$  can be expressed as:

$$I_i^* = \frac{P_i + jQ_i}{V_i} \quad (5.16)$$

Where

$P_i$  : The active power injected at bus  $i$ ,

$Q_i$  : The reactive power injected at bus  $i$ ,

$V_i$  : The voltage at bus  $i$ .

So the active power through line  $j-k$  can be rewritten as:

$$P_{j-k} = \sum_{i=1}^n Re \left\{ V_j [(Z_{ji} - Z_{ki}) \cdot y_{j-k} + Z_{ji} \cdot y_{j-k}^{sh}]^* \cdot \frac{P_i + jQ_i}{V_i} \right\} \quad (5.17)$$

Neglecting the contribution of  $Q_i$  to  $P_{j-k}$ , the expression can be further rewritten as:

$$P_{j-k} = \sum_{i=1}^n Re \left\{ \frac{V_j}{V_i} [(Z_{ji} - Z_{ki}) \cdot y_{j-k} + Z_{ji} \cdot y_{j-k}^{sh}]^* \cdot P_i \right\} \quad (5.18)$$

The relation between  $P_{j-k}$  and  $P_i$  has been found and simplified as:

$$P_{j-k} = \sum_1^n ZS_i^{j-k} \cdot P_i \quad (5.19)$$

$$ZS_i^{j-k} = \text{Re} \left\{ \frac{V_j}{V_i} \left[ (Z_{ji} - Z_{ki}) \cdot y_{j-k} + Z_{ji} \cdot y_{j-k}^{sh} \right]^* \right\} \quad (5.20)$$

Where

$ZS_i^{j-k}$  : The bus impedance matrix sensitivity of line  $j-k$  respect to real power injection at bus  $i$ .

The total active power flow through a particular transmission line can be split into portions and each portion of the active power is associated with one particular bus power injection. The relation is shown as:

$$\Delta P_{j-k}^i = ZS_i^{j-k} \cdot \Delta P_i \quad (5.21)$$

Where

$\Delta P_{j-k}^i$  : The portion of active power flow change through line  $j-k$  only associated with bus  $i$  power injection change

$\Delta P_i$  : The power injection change of bus  $i$ .

As mentioned in section 5.3.2, the Zbus matrix is dependent on slack bus selection. Hence, the impedance matrix sensitivity method is also dependent on slack bus selection.

## 5.5. Mathematical Model for the Proposed Method with Bus Impedance Matrix Sensitivity

The mathematical model for the proposed method is the same as the one in chapter 4 and the details can be obtained in section 4.4. The only different point is that in the second constraint the PTDF sensitivity factor is replaced by the bus impedance matrix sensitivity. The mathematical model with the bus impedance matrix sensitivity is showed as follows:

$$\min \sum_{i=1}^n C_{G_i} \cdot |\Delta P_{G_i}| \quad (5.22)$$

Subject to:

$$\sum_{i=1}^n \Delta P_{G_i} = 0 \quad (5.23)$$

$$\sum_{i=1}^n ZS_{G_i}^{j-k} \cdot \Delta P_{G_i} \leq P_{j-k}^{lim} - P_{j-k}^{uncon} \quad (5.24)$$

When

$$P_{j-k}^{lim} - P_{j-k}^{uncon} < 0$$

$$\sum_{i=1}^n ZS_{G_i}^{j-k} \cdot \Delta P_{G_i} \geq P_{j-k}^{lim} - P_{j-k}^{uncon} \quad (5.25)$$

When

$$P_{j-k}^{lim} - P_{j-k}^{uncon} > 0$$

$$-P_{G_i} \leq \Delta P_{G_i} \leq P_{G_i}^{max} - P_{G_i} \quad (5.26)$$

Where

$C_{G_i}$  : The incremental/decremental bid price of generator  $i$

$\Delta P_{G_i}$  : The incremental/decremental amount of active power injection of generator  $i$

$ZS_{G_i}^{j-k}$  : The bus impedance matrix sensitivity of the line  $j-k$  respect to bus  $i$

$P_{j-k}^{lim}$  : Transmission capacity limit of line  $j-k$

$P_{j-k}^{uncon}$  : The active power flow of line  $j-k$  in unconstrained dispatch

$P_{G_i}$  : The generation amount of generator  $i$  in unconstrained dispatch

$P_{G_i}^{max}$  : The generation capacity of generator  $i$

Same with the contents of section 4.4, equation (5.22) represents minimization of the total cost of generation re-dispatching. No matter if the cost is from incremental bids or decremental bids, they are all counted into the cost of congestion or called security cost. Equation (5.23) indicates that the sum of power output variations from all the bids should be equal to 0 in order to keep the balance of the whole power system. Equation (5.26) requests that the total incremental power volumes of a generator should be limited to its generation capacity minus its scheduled power output and the total decremental power volumes should be limited to its scheduled power output. In this chapter, equation (5.24) and equation (5.25) express that the re-dispatching of generation is based on the sensitivity

factor ZS rather than the PTDF. But the aim is identical that the sum of generation variations in re-dispatch should result in the power flow on the congested line reducing below its transmission capacity.

## **5.6. Case studies**

In a similar way, the case studies also consist of the testing results from four modified IEEE bus systems including an IEEE-14 bus system, an IEEE-30 bus system, an IEEE-57 bus system and an IEEE-118 bus system. These tests are intended to find the performance of the proposed generation re-dispatch method associated with the sensitivity factor ZS. The results are about the generation variations in re-dispatch stage and the corresponding congestion costs or called security costs. This chapter is focus on the comparison between the PTDF based method and the ZS based method under the same test environment.

### **5.6.1. Test Description**

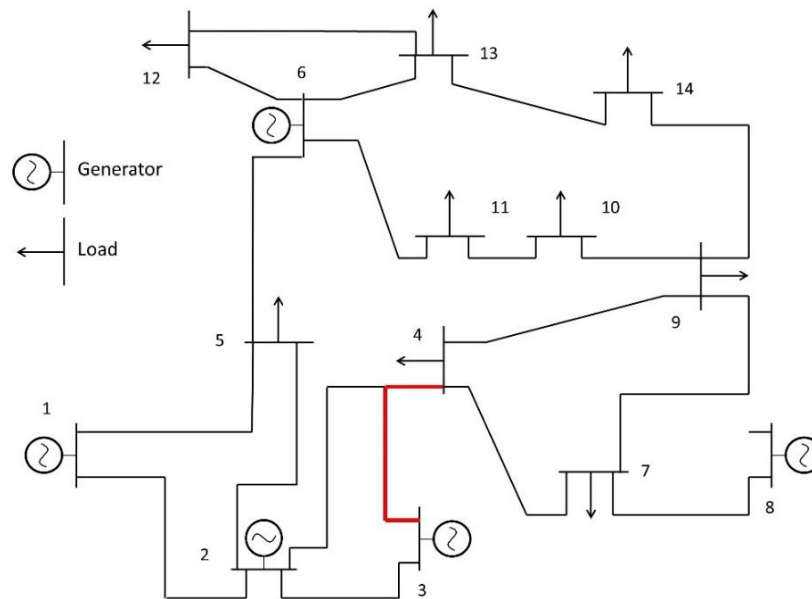
The test condition is same with the chapter 4 and the details can be found in section 4.5.1. The concept the level of overloading (LO) is continued to be used. The content of the test in this chapter is to calculate the rescheduled generation for each generator and the total congestion cost under a same LO condition by using the PTDF method and the ZS method. Compare the results from both methods by tables and graphs. Due to the dependence of slack bus selection, to simplify the comparison process, both methods only consider the first generator bus as the slack bus. Additionally, tests of both the PTDF method and the ZS method under totally five LO scenarios to discover the performances of both two methods when dealing with different congestion situations, especially when the congestion of the transmission line becomes severer. Then compare the security costs caused by both the PTDF method and the ZS method under these five congestion situations. Two curves will be found to represent the difference between two methods. It



should be noted that in order to simplify the comparison process, the PTDF method and the ZS method in this comparison both only select the first generator bus as the slack bus.

### 5.6.2. Modified IEEE-14 Bus System

The modified IEEE-14 bus test system is same with the system in chapter 4, which is showed in Figure 5.2 as follow:



**Figure 5.2 The modified IEEE-14 bus system**

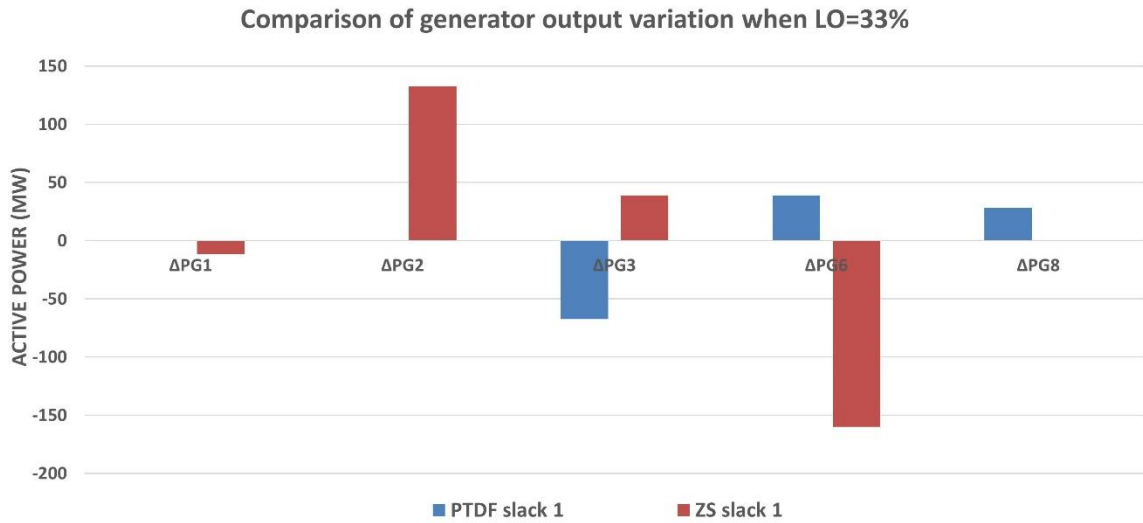
The parameters of the generators and loads in the test environment are set up the same, which are displayed in Table 4.1. The generation schedule in the unconstrained dispatch is showed in Table 4.2.

After a DC optimal power flow without considering the transmission line constraints, the generation schedule for each generator in the unconstrained dispatch is

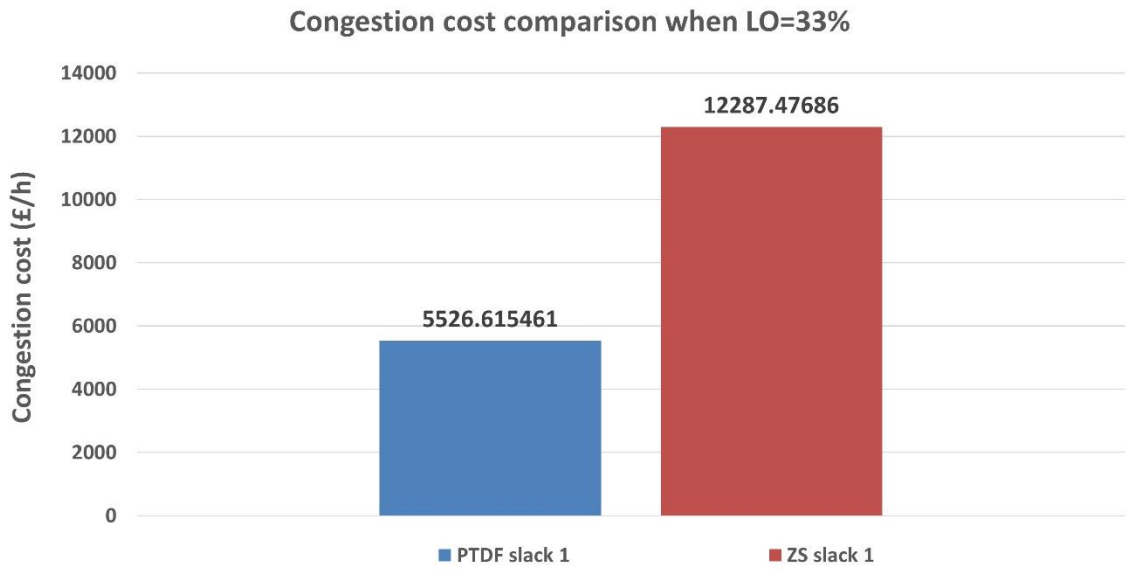
obtained, which is as same as the section 4.5.2. Then it is found that the power flow of transmission line 3-4 in the unconstrained dispatch is more than its transmission capacity limit and the level of overloading (LO) of the congested line is 33%. Then use two sensitivity methods to provide solutions for this particular congestion problem. The re-dispatch generation variation of each generator calculated by two sensitivity-based methods when the LO equals to 33% is obtained and the results are shown in Table 5.1. To make it more straightforward, the information of Table 5.1 is visualized in Figure 5.3. The corresponding congestion costs calculated by two methods are indicated in Figure 5.4. Follow the similar way, other four scenarios with the LO of 1%, 7%, 14%, 21% are tested respectively. Finally, the corresponding security costs in five scenarios calculated by two methods are obtained and displayed in Figure 5.5 by two curves to represent the different outcomes by two methods.

**Table 5.1 Generator Output Variation Due to Re-dispatch by Two Methods in 14-Bus System When LO=33%**

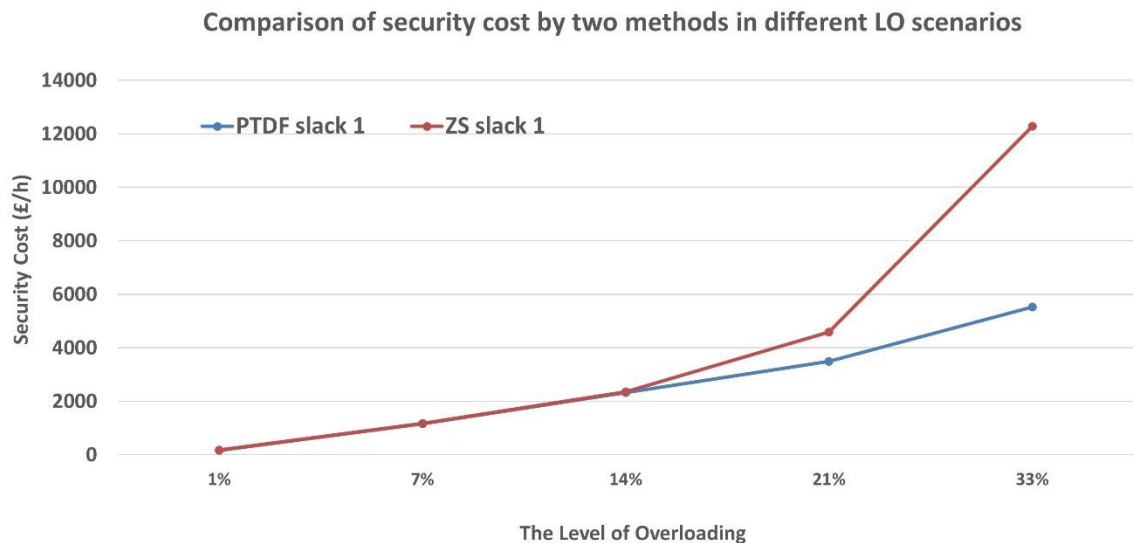
Generator	$\Delta P_G$ PTDF slack bus 1 (MW)	$\Delta P_G$ ZS slack bus 1 (MW)
<b>G1</b>	0	-11.52
<b>G2</b>	0	+132.72
<b>G3</b>	-67.31	+38.90
<b>G6</b>	+38.90	-160.10
<b>G8</b>	+28.41	0



**Figure 5.3 14-bus system generator output variation in re-dispatch by two methods when LO=33%**



**Figure 5.4 14-bus system congestion cost of re-dispatch by two methods when LO=33%**

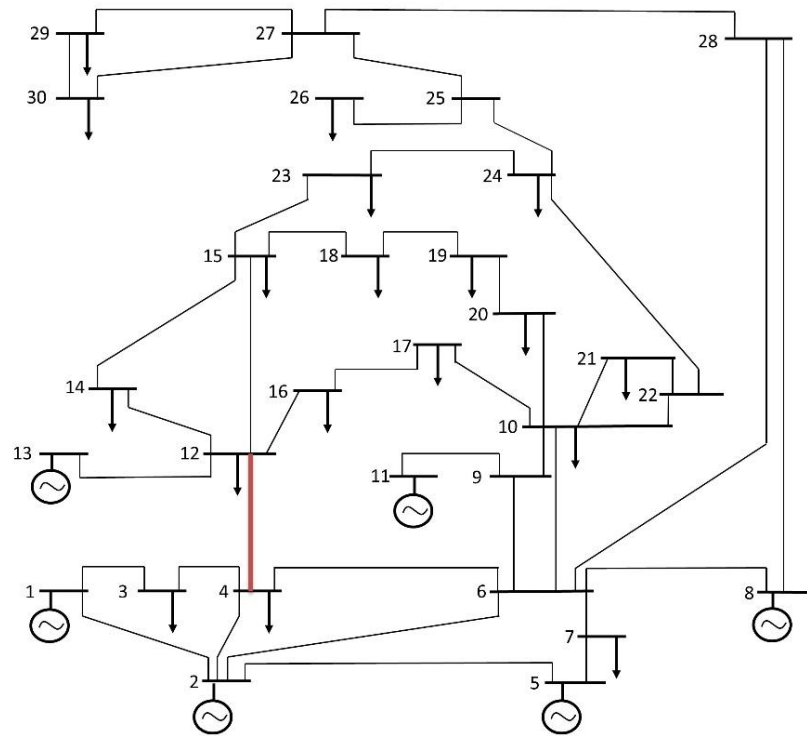


**Figure 5.5 Comparison of security cost by two methods in five different LO scenarios in 14-bus test system**

From Figure 5.3, the rescheduled generation for each generator accomplished by two methods is obtained. It is indicated that the ZS based method asks generator 1, 2, 3, and 6 to adjust their power outputs while only three generators are involved in re-dispatch in the PTDF based method, which are generator 3, 6 and 8. From Figure 5.4, it illustrates that the security cost due to congestion management by the ZS based method, which is £12287/h, is more than twice the PTDF based method, which is £5527/h. The reason can be found in Figure 5.3 that not only the number of generator involvement in re-dispatch by two methods is different, but also the output volume variations of generator 2 and 6 are large plus the decremental bid price of generator 6 is high. From Figure 5.5, it is found that when the level of overloading of the congested line is 1%, 7% and 14%, the security costs due to congestion management by two methods are nearly the same. When the LO is 21%, the security costs by two methods start to be distinct. When the LO condition is at 33%, the distinction of security cost calculation by two methods become larger. It can be concluded that with severer degree of congestion, the security costs caused by two sensitivity-based methods will become more different.

### 5.6.3. Modified IEEE-30 Bus System

The modified IEEE-30 bus test system is as same as the test system in chapter 4, which is showed in Figure 5.6. The parameters of the generators and loads in the test environment are set up the same, which are displayed in Table 4.4 and Table 4.5. In the unconstrained dispatch, generation schedules of six generators are showed in Table 4.6.



**Figure 5.6 The modified IEEE-30 bus system**

After a DC optimal power flow without taking the transmission line constraints into consideration, the generation schedule for each generator in the unconstrained dispatch is obtained, which is as same as the section 4.5.3. Then it is found that the power

flow of transmission line 4-12 in the unconstrained dispatch exceeds its transmission capacity limit and the level of overloading (LO) of the congested line is 19%. Then use the two sensitivity-based methods to find solutions for this congestion problem. The re-dispatch generation variation of each generator based on two methods when LO equals to 19% is obtained and the results are shown in Table 5.2 and visualized in Figure 5.7. The congestion costs calculated by two methods are indicated in Figure 5.8. In the same way, other four scenarios with the LO of 1%, 4%, 8%, 13% are tested respectively. Therefore, the corresponding security costs in totally five scenarios calculated by two methods are obtained and displayed in Figure 5.9 by two curves to represent the different results by two methods.

**Table 5.2 Generator Output Variation Due to Re-dispatch by Two Methods in 30-Bus System When LO=19%**

Generator	$\Delta P_G$ PTDF slack bus 1 (MW)	$\Delta P_G$ ZS slack bus 1 (MW)
G1	-69.55	0
G2	0	+271.50
G5	0	+73.97
G8	0	-133.39
G11	0	-212.08
G13	+69.58	0

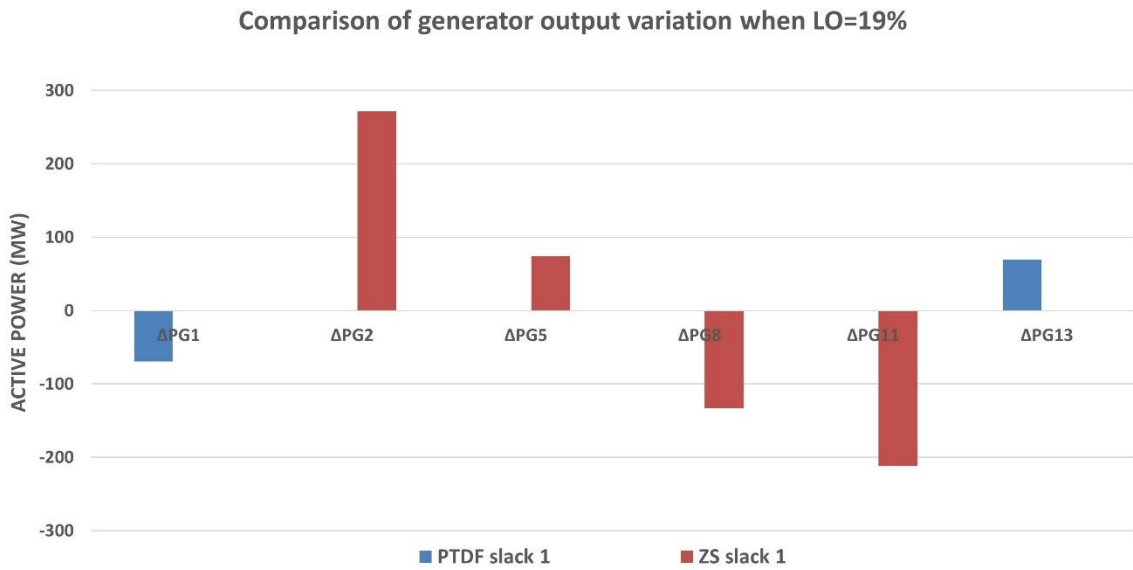


Figure 5.7 30-bus system generator output variation in re-dispatch when LO=19%

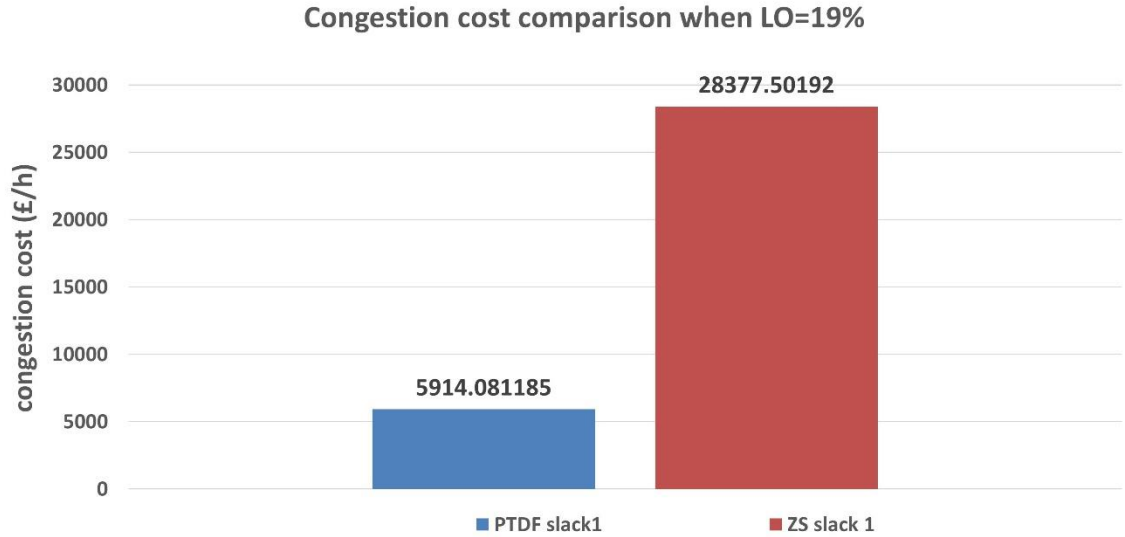
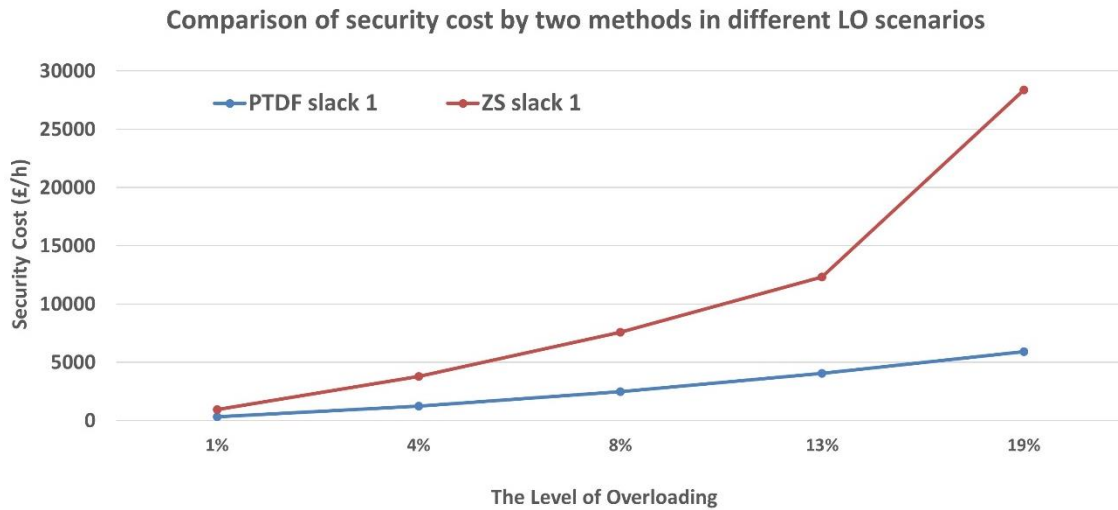


Figure 5.8 30-bus system congestion cost of re-dispatch when LO=19%



**Figure 5.9 Comparison of security cost by two methods in five different LO scenarios in 30-bus test system**

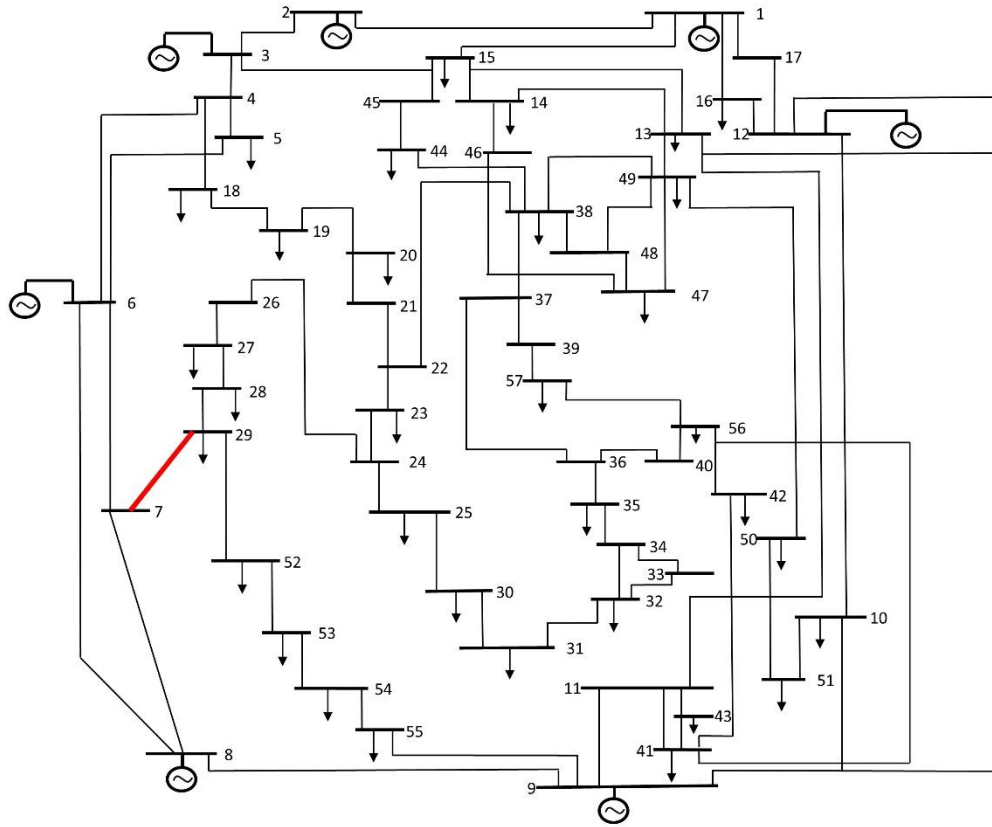
From Figure 5.7, it is indicated that the ZS based method asks generator 2, 5, 8, and 11 to adjust their power outputs while only two generators are involved in re-dispatch in the PTDF based method, which are generator 1 and 13. From Figure 5.8, it illustrates that the security cost due to congestion management by the ZS based method, which is £28378/h, is more than five times as the PTDF based method, which is £5914/h. It is because the number of generator involvement in re-dispatch by the ZS method is doubled the PTDF method, and the incremental/decremental volumes of generators involved in the ZS method are larger than the PTDF method plus the incremental/decremental bid prices of generators involved in the ZS method are higher than the ones in the PTDF method. According to the Figure 5.9, it is showed that with higher level of overloading of the congested line, the security costs distinction based on two sensitivity-based methods become larger. It should be noted from Table 5.2 that when dealing with the congestion at LO equals to 19%, the PTDF-based method involves G1 and G13 into re-dispatch stage while both of them are not asked to change their generation output in the ZS-based method. Instead, the ZS-based method asked other four generators to change their outputs while



they are not involved in re-dispatch stage in PTDF-based method. It is very clear to say that under this congestion scenario, two sensitivity-based method have totally different idea to re-dispatch generation output. Consequently, the security cost in ZS-based method is around five times to the PTDF-based method, which can be seen from Figure 5.8. Although, in the first four congestion scenarios, the security cost values in the ZS-based method are all more or less three times to the PTDF-based method, the sudden jump at LO equals to 19% is still obviously observed. The direct reason for this has been explain as the ZS-based method involves more generation power in re-dispatch. To dig in, the deeper reason may be obtained from Appendix Table B. It is found that when selecting bus 1 as the slack bus, two methods calculated totally two different sensitivity factors for six generator buses respectively because two types of sensitivity factors are calculated based on different algorithms. As a result, the ZS-based method may not be regarded as a more appropriate approach than the PTDF-based method under this particular circumstance, not only it may produce more security cost but also the instability.

#### 5.6.4. Modified IEEE-57 Bus System

The modified IEEE-57 bus test system is same with the system in chapter 4, which is showed in Figure 5.10. The parameters of the generators and loads in the test environment are set up the same, which are displayed in Table 4.8 and Table 4.9. In the unconstrained dispatch, generation schedules of seven generators are showed in Table 4.10.



**Figure 5.10 The modified IEEE-57 bus system**

After a DC optimal power flow without transmission line constraints, the generation schedule for each generator in the unconstrained dispatch is obtained, which is as same as in section 4.5.4. Then it is found that the power flow of transmission line 7-29 in the unconstrained dispatch is above its transmission capacity limit and the level of overloading (LO) of the congested line is 8%. Then the two sensitivity-based methods are utilized to find out solutions for this transmission congestion problem. The generation output variation of each generator in rescheduling based on two sensitivity methods when LO equals to 8% is obtained and the results are shown in Table 5.3 and Figure 5.11. The security costs caused by two methods are indicated in Figure 5.12. Similarly, tests of other four scenarios with the LO of 1%, 3%, 5%, 7% are accomplished respectively. Therefore,

the corresponding security costs in totally five scenarios in two methods are obtained and displayed in Figure 5.13 by two curves to represent the different results by two methods.

**Table 5.3 Generator Output Variation Due to Re-dispatch by Two Methods in 57-Bus System When LO=8%**

<b>Gen</b>	$\Delta P_G$ <b>PTDF slack bus 1</b> (MW)	$\Delta P_G$ <b>ZS slack bus 1</b> (MW)
<b>G1</b>	+170.47	-129.50
<b>G2</b>	0	-70.36
<b>G3</b>	+0.21	+125.53
<b>G6</b>	-208.98	+45.36
<b>G8</b>	0	+185.41
<b>G9</b>	0	0
<b>G12</b>	+38.31	-156.44

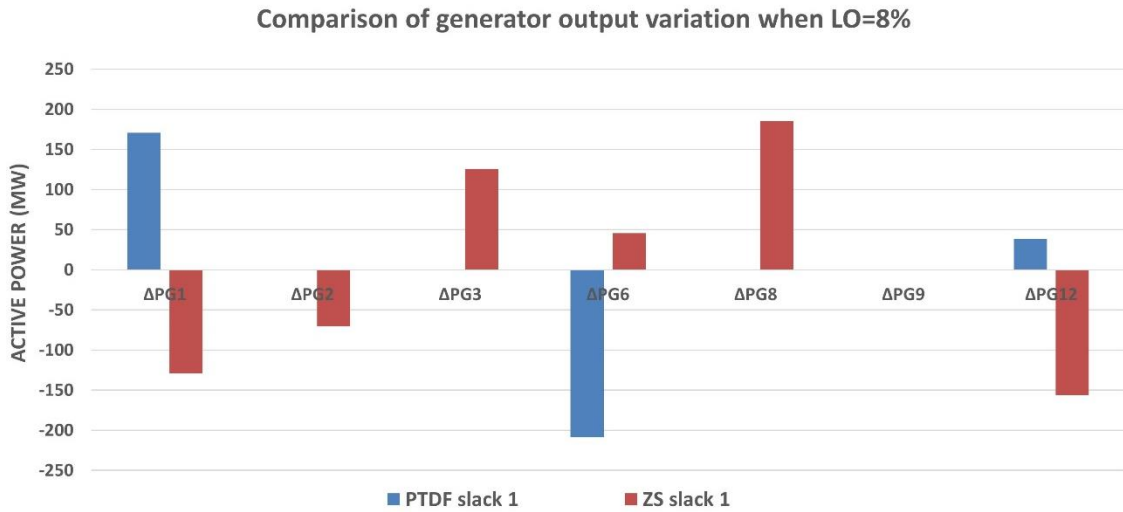


Figure 5.11 57-bus system generator output variation in re-dispatch when LO=8%

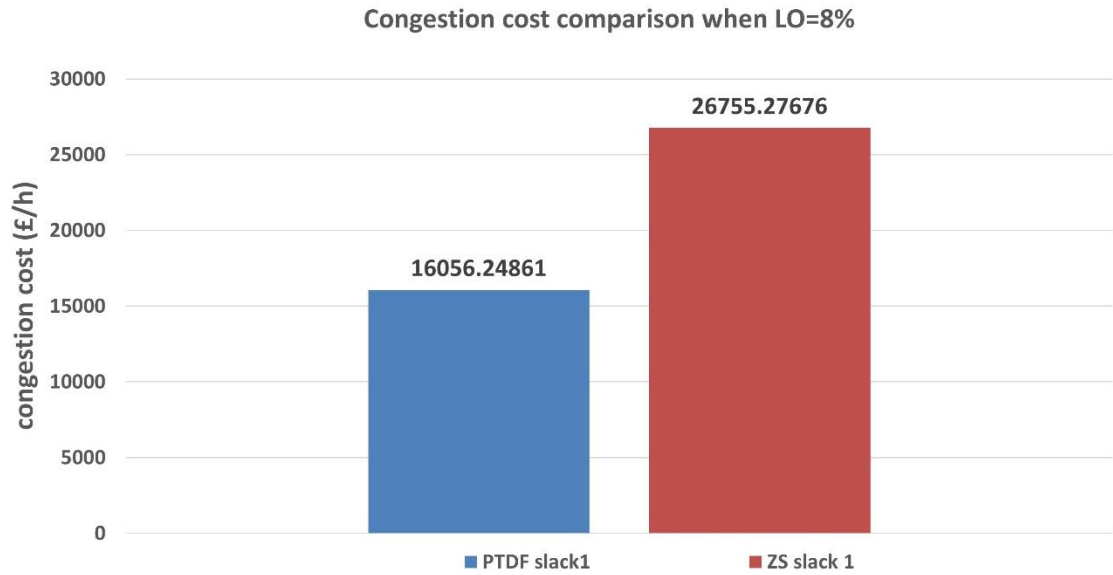
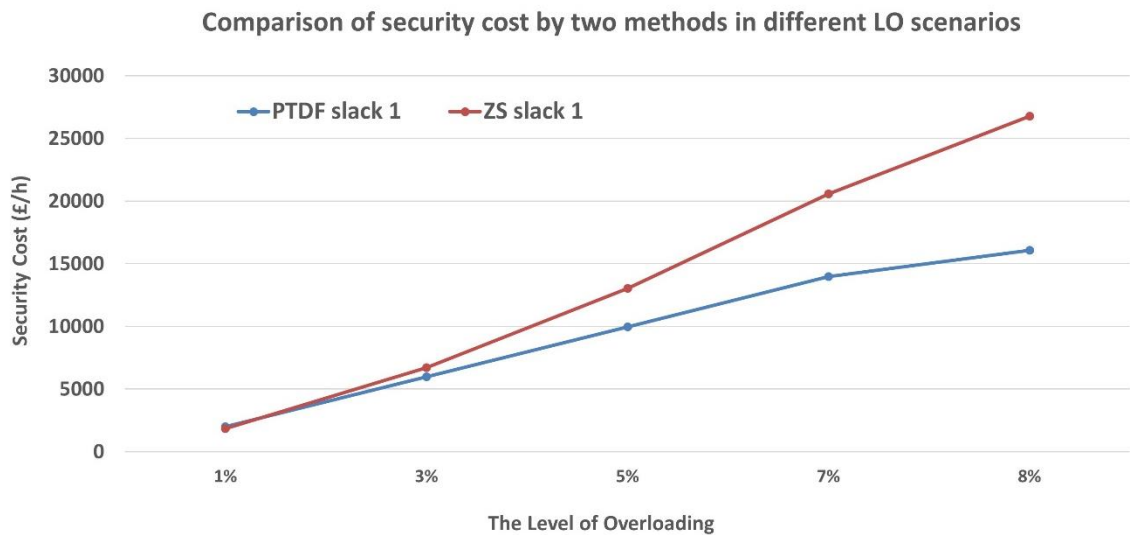


Figure 5.12 57-bus system congestion cost of re-dispatch when LO=8%

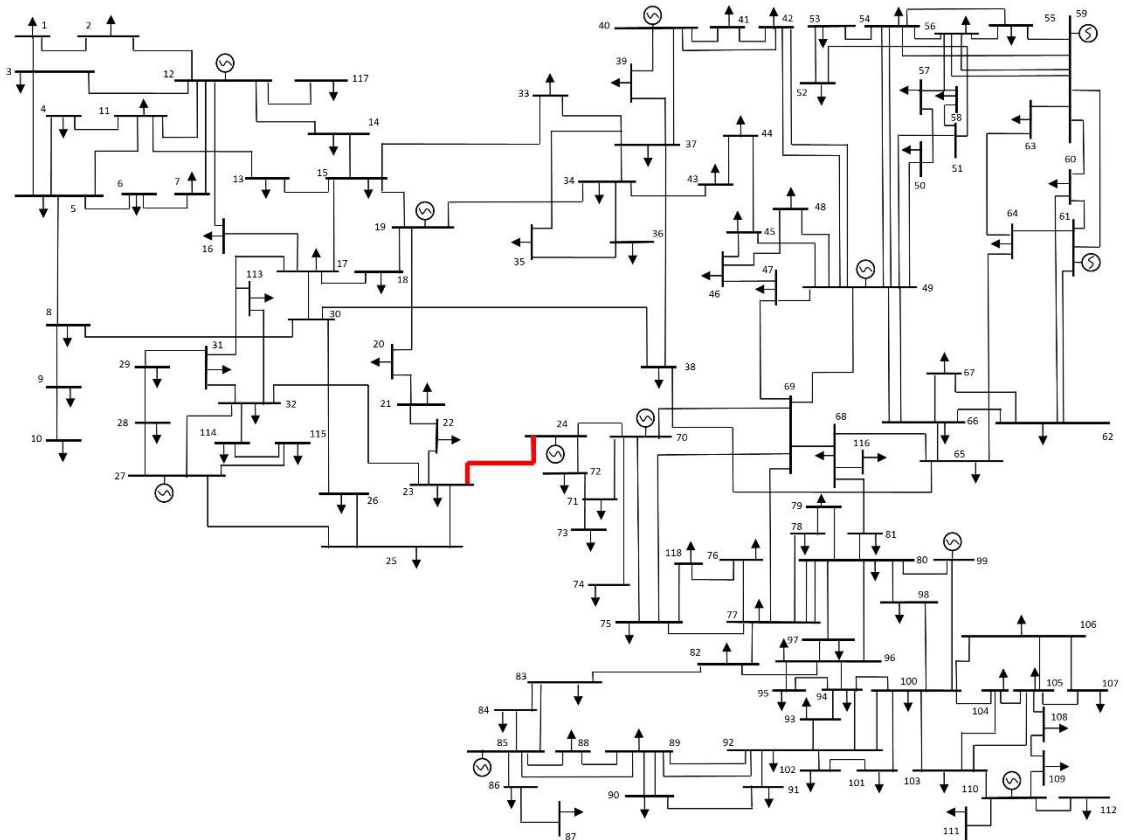


**Figure 5.13 Comparison of security cost by two methods in five different LO scenarios in 57-bus test system**

From Figure 5.11, it is noted that the ZS based method asks all the generators in the system except generator 9 to adjust their power outputs while only three generators are involved in re-dispatch in the PTDF based method, which are generator 1, 6 and 12. Figure 5.12 illustrates that the security cost due to congestion management by the ZS based method, which is £26755/h, is about £10000/h more than the PTDF based method, which is £16056/h. The reason is the number of generator involved in the re-dispatch by the ZS method is doubled the number in the PTDF method. From Figure 5.13, it is found that when the level of overloading of transmission line 7-29 is 1%, the security costs due to congestion management by two methods are roughly close and in fact the security cost caused by the PTDF based method is slightly higher than the ZS based method. When the LO is 3%, the security costs caused by the two methods start to be distinguished. Based on the results of the LO interval between 3% and 8%, it is showed that with severer degree of transmission congestion occurs on the congested line, the security cost distinction of two sensitivity-based methods becomes larger.

### 5.6.5. Modified IEEE-118 Bus System

The modified IEEE-118 bus test system is same with the test system in chapter 4, which is showed in Figure 5.14. The parameters of the generators and loads in the test environment are set up as the same in chapter 4, which are displayed in Table 4.12 and Table 4.13. Generation schedule in the unconstrained dispatch is showed in Table 4.14.



**Figure 5.14** The modified IEEE-118 bus system

After running a DC optimal power flow without considering transmission line constraints, the generation schedule for each generator in the unconstrained dispatch is obtained, which is as same as the information in section 4.5.5. Then it is found that the power flow of transmission line 23-24 in the unconstrained dispatch is over its transmission capacity limit and the level of overloading (LO) of the congested line is 99%,

which can be regarded as the transmission line is nearly blocked. Then let the two sensitivity-based methods figure out solutions for this serious transmission congestion issue. The generation output variation of each generator in re-dispatch based on two sensitivity methods when LO equals to 99% is obtained and the results are shown in Table 5.4 and Figure 5.15. The security costs caused by two re-dispatch methods are indicated in Figure 5.16. At last, tests of other four scenarios with the LO of 1%, 25%, 50%, 75% are accomplished respectively. Therefore, the corresponding security costs in totally five scenarios of two methods are obtained and displayed in Figure 5.17 by two curves to represent the different results by two methods.

**Table 5.4 Generator Output Variation Due to Re-dispatch by Two Methods in 118-Bus System When LO=99%**

Gen	$\Delta P_G$	$\Delta P_G$
	PTDF Slack bus 12 (MW)	ZS slack bus 12 (MW)
G12	+228.83	-71.17
G19	0	-241.47
G24	-296.55	0
G27	0	-300
G40	0	0
G49	+62.65	+180.18
G59	+5.09	+208.96
G61	0	+223.50
G70	0	0
G85	0	0
G99	0	0
G110	0	0

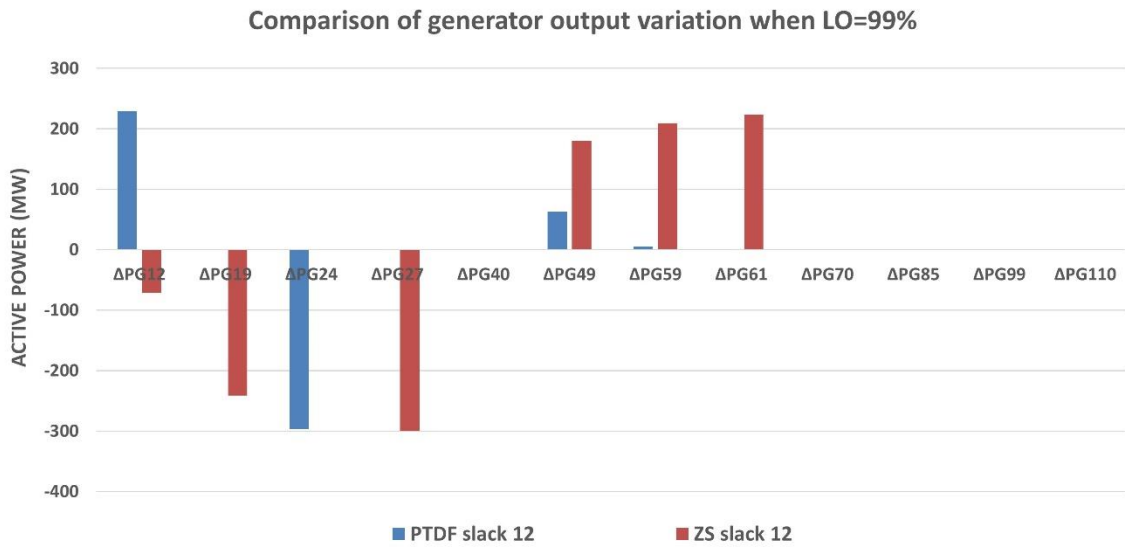


Figure 5.15 118-bus system generator output variation in re-dispatch when LO=99%

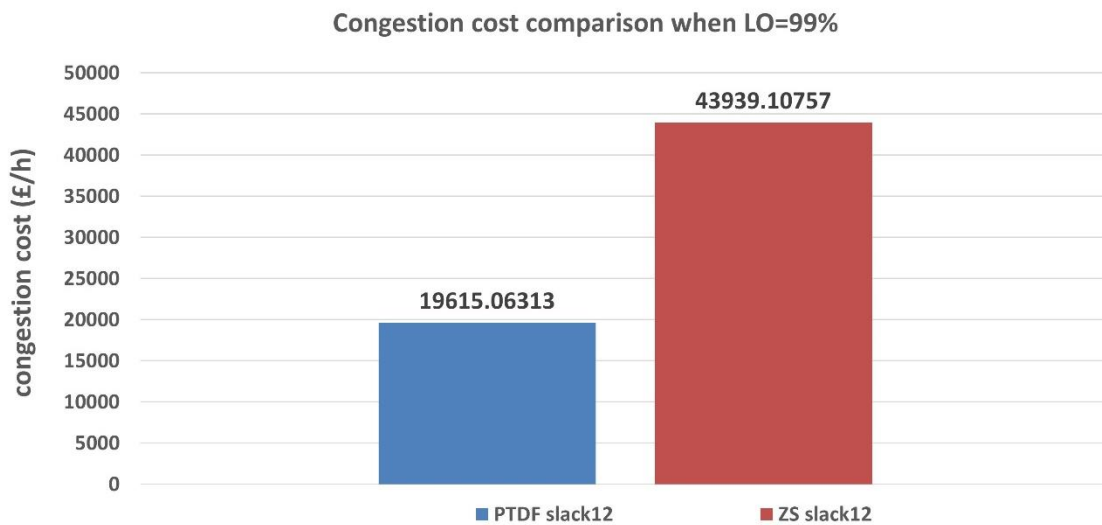
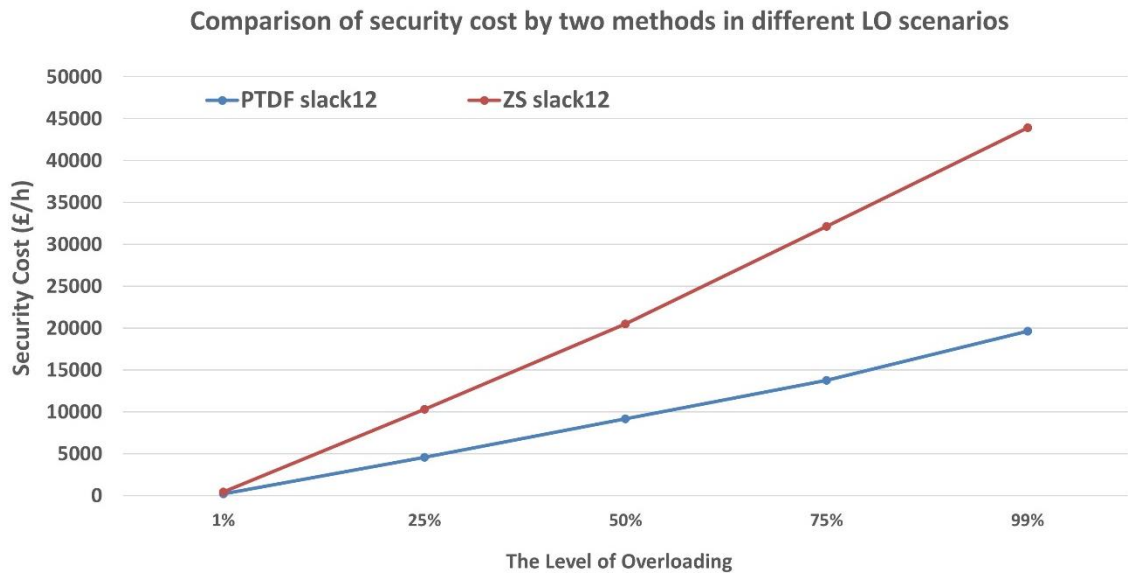


Figure 5.16 118-bus system congestion cost of re-dispatch when LO=99%





**Figure 5.17 Comparison of security cost by two methods in five different LO scenarios in 118-bus test system**

From Figure 5.15, it is shown that the ZS based method requires generator 12, 19, 27, 49, 59 and 61 to change their power outputs while only four generators which are generator 12, 24, 49 and 59 are involved in re-dispatch when using the PTDF based method. From Figure 5.16, it is illustrated that the security cost due to congestion management by the ZS based method, which is £43939/h, is more than the doubled security cost by the PTDF based method, which is £19615/h. It is due to the involved generators in re-dispatch by the ZS method is more than the ones in the PTDF method. From Figure 5.17, it is indicated that when the level of overloading of the congested line 23-24 is 1%, the security costs due to congestion management by two methods are similarly approximated but in fact, the value in the ZS-based method is doubled the PTDF-based method. From the results of the LO interval between 25% and 99%, it is demonstrated that with greater level of transmission congestion occurs on the congested line 23-24, the security costs distinction of two sensitivity-based methods becomes more distinct, with the values on the ZS-based curve all approximately twice the values on the

PTDF-based curve. The explanation can be seen inside the Appendix Table D. It is found that when selecting bus 12 as the slack bus, the two methods calculated totally different sensitivity factors for twelve generator buses respectively because the two types of sensitivity factors are calculated based on different algorithms. As a result, the ZS-based method may not be selected as the right way to re-dispatch generation under this particular circumstance since it may produce more security cost.

#### 5.6.6. Discussion

In this case study, performances of the two sensitivity-based generation re-dispatch methods are tested under four different test systems. Experimental results prove that the ZS based method can implement the generation re-dispatching with fulfilling the objective function and the three constraints.

However, the ZS method still has some disadvantages. First of all, the ZS based method is also dependent on the slack bus selection, just like the PTDF based method. Therefore, it will make the ZS method inaccurate if only consider one generator bus as the slack bus. However, if choose each the generator bus in the system as the slack bus respectively, the problem solving process will become time-consuming. Secondly, compared with the PTDF method, generation re-dispatching by the ZS method involves more generators to adjust their initial generation output schedules. In the case of 14-bus system, the comparison of the ratio of involved generators between the ZS method and the PTDF method is 4:3; in the case of 30-bus system, the ratio is 4:2; in the case of 57-bus system, the ratio is 6:3 and in the case of 118-bus system, the ratio is 6:4. Due to the number of generator involved in re-dispatching by the ZS method is more than the PTDF method, the corresponding security cost or called congestion cost caused by congestion management in the ZS method is more expensive than the PTDF method. The comparisons of the security costs between two methods in all the test systems are the evidences to prove the above conclusion. In the case of 14-bus system, the security cost

in the ZS method is £12287/h while in the PTDF method the cost is £5526/h. The PTDF-based method only produced nearly 50% security cost of the ZS-based method; in the case of 30-bus system, ZS method causes £28377/h congestion cost and PTDF method causes £5914/h congestion cost, which means the PTDF-based method only results in 21% security cost of the ZS-based method; in the case of 57-bus system, the security cost contrast is £26755/h to £16056/h. The security cost value in the PTDF-based method is only approximately 60% of the ZS-based method; in the case of 118-bus system, the cost contrast is £43939/h to £19615/h, which indicates that the security cost value the PTDF-based method produced is only around 45% of the value in the ZS-based method. It is also found that with the increasing level of overloading of the congested line, the distinction of security costs caused by two sensitivity-based methods become more distinguished.

## 5.7. Summary

In this chapter, the proposed generation re-dispatch method associated with the bus impedance matrix sensitivity is described in details including the methodology of the proposed re-dispatch method with the sensitivity factor and the algorithm of the sensitivity factor. The mathematic model of the proposed method including an objective function and three constraints one of which contains the bus impedance matrix sensitivity are given. Case studies consist of experimental results from four modified IEEE testing systems including a 14-bus, a 30-bus, a 57-bus and a 118-bus systems are implemented. Comparison between the bus impedance matrix sensitivity method and the power transfer distribution factor method indicates that the ZS based method is dependent on slack bus selection as well; number of generator involvement in re-dispatch by the ZS method is more than the PTDF method which leads to the security cost in the ZS method is higher than the PTDF method when dealing with a same level of overloading congestion problem; with the severer LO situation, the security cost difference between the ZS method and the PTDF method will become larger.

## **Chapter 6**

### **Security Cost Allocation in Deregulated Electricity market with Power Transfer Distribution Factor**

#### **6.1. Introduction**

The extra cost due to transmission congestion management through re-dispatching generation is called the congestion cost or the security cost. It is the responsibility of the system operator to balance the congestion management and the security cost. Since in the deregulated electricity market, the system operator does not have any source of income, the security cost needs to be recovered from market participants. Therefore the security cost allocation become very essential to decentralize the cost balancing problem in the electricity market. As mentioned in section 4.2.2, the security cost of the electricity market is originated from the incremental/decremental bids mechanism. Generators take part in the bid submission so as to help the system operator to relieve the congestion and the system operator will pay for generators the extra cost of the re-dispatches. Since generators have already offered assistance to manage congestion, the security cost will only be allocated to the demand side. The allocation principle is according to the power flow contribution of each load to the congested transmission line. The power flow contribution of each load will be calculated based on the power transfer distribution factor (PTDF) which was introduced in chapter 4. The proposed allocation method will be tested in four test systems under the same test conditions as the case studies of chapter 4 to allocate the already calculated security costs to the loads according to their power flow contributions to the target line.

In this chapter, conventional security cost allocation schemes will be described in section 6.2. Then the mathematic model of the proposed allocation method with PTDF will be given in section 6.3. Section 6.4 is the case study including experimental results of four modified IEEE test systems which are an IEEE-14 bus system, an IEEE-30 bus system, an IEEE-57 bus system and an IEEE-118 bus system. Section 6.5 is the summary.

## **6.2. Conventional Security Cost Allocation Schemes**

In chapter 3, various types of congestion management schemes are discussed in details. In fact, they not only just manage transmission congestion but also intend to balance the extra cost caused by congestion relief. Different electricity market models apply different schemes on security cost allocation.

### **6.2.1. Security Cost Allocation in Vertically Integrated Power Utility**

In the traditional vertically integrated power utility, transmission congestion is resolved by re-dispatching generation. The cost of re-dispatching action which is considered as the security cost will be put on the shoulder of the vertically integrated utility [110].

### **6.2.2. Security Cost Allocation in Pool Electricity Market**

In the pool-based electricity market, transmission congestion is relieved by generation re-dispatching according to market-based rules. The corresponding security cost is allocated to the loads by using uniform marginal price, locational marginal price and zonal price [107, 122]. As discovered in section 3.3, uniform marginal price and locational marginal price is different on the security cost allocation. Uniform marginal price allocates the security cost based on a non-discrimination principle which means all the loads in the system will shoulder the extra cost uniformly. The demand side customers are asked to equally share the security cost without considering their impact on the

congestion problem. By contrast, locational marginal price allocates the security cost with taking load location into consideration. As described in section 3.2.1.2, the locational marginal price is consisted of three components: marginal cost component, congestion cost component and loss cost component. If there is no congestion in the system, the locational marginal price has only the marginal cost component, which means prices on all nodes are identical. Once congestion occurs in the system, the congestion cost component will be added into nodal price calculation, which leads to different locational prices on nodes. Since the loads are charged based on their nodal prices, the security cost is already allocated to the loads through electricity price settlement. The idea of locational marginal price reflects the locational factor of load, in other words, it considers the impact of each load to the congestion. However, this method will trigger merchandising surplus issue which is due to the net income of the system operator [35]. Adopting the locational marginal price as the pricing scheme, the congestion will result in a revenue of the system operator. With more severe congestion situation, the revenue will become get larger. This revenue can be distributed back to the market participants or paid off to the holders of financial instruments [123].

### 6.2.3. Security Cost Allocation in Bilateral Electricity Market

In the bilateral electricity market, contract between generator and load does not think of congestion and the corresponding security cost. Hence the incurred security cost will be allocated to market participants. Firstly, the total security cost will be divided into generation portion and load portion on a half-half basis [35]. Secondly, each cost portion will be allocated to generators or loads based on their power energy generations/consumptions [107]. The process of the security cost allocation in bilateral market is according to the following equations [35]:

$$SC_{gens} = SC_{loads} = 0.5 \times SC_{total} \quad (6.1)$$

$$SC_{G_i} = \frac{P_{G_i}}{\sum_{i=1}^{N_G} P_{G_i}} \times SC_{gens} \quad (6.2)$$

$$SC_{L_j} = \frac{P_{L_j}}{\sum_{j=1}^{N_L} P_{L_j}} \times SC_{loads} \quad (6.3)$$

Where

$SC_{total}$  : The total security cost

$SC_{gens}$  : The security cost portion for generators

$SC_{loads}$  : The security cost portion for loads

$SC_{G_i}$  : The security cost allocated to generator  $i$

$P_{G_i}$  : The MW power generation of generator  $i$

$SC_{L_j}$  : The security cost allocated to load  $j$

$P_{L_j}$  : The MW power consumption of load  $j$

$N_G$  : The number of generators in the system

$N_L$  : The number of loads in the system

Equation (6.1) expresses the total security cost is divided equally into generator portion and load portion. Equation (6.2) and equation (6.3) show that the security cost allocated to each generator or each load is based on its power energy generation or consumption.

Nevertheless, such security cost allocation method cannot reflect the impact of each market participant to the transmission congestion. This is unfair for the participant especially the one who contributes less to the congestion to shoulder the security cost. A method should be proposed to let the security cost allocate to the participant who is more responsible for the transmission congestion rather than spread equally over all the participants [124]. Therefore the power flow contribution of each market participant to the congestion problem should be calculated in the first place.

### **6.3. Proposed Security Cost Allocation Method with Power Transfer Distribution Factor**

The proposed security cost allocation method with PTDF factor will be described in details including the mechanism and the mathematical model of the method.

#### **6.3.1. Mechanism of the Proposed Security Cost Allocation Method with Power Transfer Distribution Factor**

As mentioned in section 4.3, the power transfer distribution factor can figure out the relationship between the active power flow of a particular transmission line from bus  $j$  to bus  $k$  and the active power import or export at bus  $i$ . With the help of the PTDF factor, the power flow on a particular transmission line can be regarded as the consequence of the combined actions of all the generators and loads in the system. That is to say it is possible to find the contribution of each generator or each load to the power flow of the congested line if there is a congestion situation occurs in the power system.

In the proposed method, generators do not bear the security cost, therefore, all the loads in the system are requested to shoulder all of it. As a result, the contributions of all the generators to the power flow of the target transmission line should only be used to calculate the MW contributions of all the loads to the power flow on the congested line,



which is the total power flow MW values on the congested line minus the power flow MW values associated with generators contribution. Once the MW portion on the congested line associated with all the loads is obtained, combined with the power flow specifically associated with each load, the contribution of each load to the power flow on the congested transmission line can be captured. Finally, the produced power flow contribution of each load times the total security cost goes to the allocated security cost to each load.

### 6.3.2. Mathematical Model of the Proposed Security Cost Allocation Method with Power Transfer Distribution Factor

The following equations are the mathematical model which is aim to express the above mechanism of the proposed security cost allocation method with PTDF.

Equation (6.4) expressed the total MW power flow of the congested line is consisted of generators contribution plus loads contribution. Combined with the definition of the PTDF, which is contained in section 4.3, the relationship between the active power flow change quantity of a particular transmission line from bus  $j$  to bus  $k$  and the active power injection change quantity at bus  $i$ , the active power flow quantity of a particular transmission line in the system can be deemed as the result of joint action of all buses no matter they are injecting power into the system or withdrawing power out of the system. The above explanation can be further expressed as that each bus's contribution to the power flow of a particular transmission line can be calculated as its power input or output to the system times its sensitivity factor to the particular transmission line. It should be noted that the contribution of one particular bus to one particular transmission line could be either positive or negative. If the contribution is positive, it means this bus helps to increase the power flow of the transmission line. If the contribution is negative, it means this bus helps to decrease the power flow of the transmission line. Therefore, the power

flow of a particular transmission line  $j-k$  consists of such positive contributions and negative contributions from all the buses in the system.

$$P_{j-k} = P_{j-k}^G + P_{j-k}^L \quad (6.4)$$

Where

$P_{j-k}$  : The power flow MW value of the target transmission line  $j-k$

$P_{j-k}^G$  : The power flow MW value on line  $j-k$  associated with all the generators

$P_{j-k}^L$  : The power flow MW value on line  $j-k$  associated with all the loads

Equation (6.5) indicates that the total MW contribution of generators is the sum of the power flow contribution of each generator.

$$P_{j-k}^G = \sum_{i=1}^{N_G} PTDF_{G_i}^{j-k} \cdot P_{G_i} \quad (6.5)$$

Where

$PTDF_{G_i}^{j-k}$  : The PTDF of the line  $j-k$  respect to generator bus  $i$

$P_{G_i}$  : The MW power energy injected by generator  $i$

Equation (6.6) shows that total MW contribution of load is the sum of the power flow contribution of each load.

$$P_{j-k}^L = \sum_{i=1}^{N_L} PTDF_{L_i}^{j-k} \cdot P_{L_i} \quad (6.6)$$

Where

$PTDF_{L_i}^{j-k}$  : The PTDF of the line  $j-k$  respect to load bus  $i$

$P_{L_i}$  : The MW power energy withdrawn by load  $i$

Based on the equation (6.5) and (6.6), equation (6.4) can be further extended as follow:

$$P_{j-k} = \sum_{i=1}^{N_G} PTDF_{G_i}^{j-k} \cdot P_{G_i} + \sum_{i=1}^{N_L} PTDF_{L_i}^{j-k} \cdot P_{L_i} \quad (6.7)$$

As mentioned before, generators will not be involved in the security cost allocation, so security cost allocation here is only based on the contributions of loads to the congested transmission line. Therefore equation (6.6) is only the equation to be considered into security cost allocation calculation.

Equation (6.8) tells that the MW power flow contribution of each load is equals to the PTDF of the load times the power energy extraction of the load. Such equation is introduced to help to find out each load's contribution percentage to the total power contributions of loads to the congested transmission line.

$$P_{j-k}^{L_i} = PTDF_{L_i}^{j-k} \cdot P_{L_i} \quad (6.8)$$

Where

$P_{j-k}^{L_i}$  : The power flow MW value associated with load  $i$

Equation (6.9) is the expression of the contribution percentage of each load.

$$Ctb_{L_i} = \frac{P_{j-k}^{L_i}}{P_{j-k}^L} \times 100\% = \frac{PTDF_{L_i}^{j-k} \cdot P_{L_i}}{\sum_{i=1}^{N_L} PTDF_{L_i}^{j-k} \cdot P_{L_i}} \times 100\% \quad (6.9)$$

Where

$Ctb_{L_i}$  : The power flow contribution percentage of load  $i$  to the congested line

Equation (6.10) is the calculation of the security cost allocated to each load.

$$CA_{L_i} = Ctb_{L_i} \times SC_{total} \quad (6.10)$$

Where

$CA_{L_i}$  : The allocated security cost to the load  $i$

$SC_{total}$  : The total security cost due to congestion management

## 6.4. Case Studies

The case studies in this chapter follow the rule of the former two chapters, which is using four modified IEEE bus systems including an IEEE-14 bus system, an IEEE-30 bus system, an IEEE-57 bus system and an IEEE-118 bus system to test the performance of the proposed security cost allocation method with PTDF. In order to get a comparison of the method performance, a pro rata allocation method based on power volume will be introduced in the tests.

### 6.4.1. Test Description

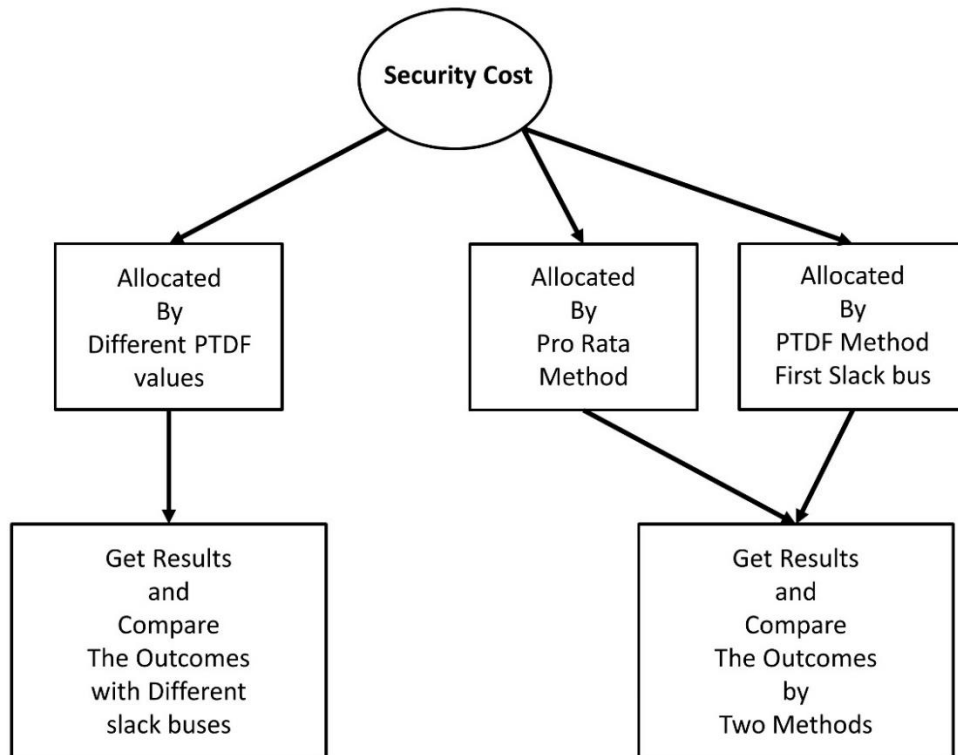
Rules and environment of the test are same with the chapter 4 and the details can be found in section 4.5.1. The test is designed for finding the outcomes of the proposed security cost allocation method with PTDF factor. The security cost will be allocated to each load by the above method and the results will be presented by tables and graphs. To get a comparison associated with the PTDF method, a pro rata based allocation method will be brought to contrast to the PTDF method. Such pro rata method is an evolution of the method mentioned in section 6.2.3. The variation is that the total security cost will be only allocated to the loads according to their power volume consumptions. Equation (6.11) describes the principle of the pro rata method as follows:

$$CA_{L_i} = \frac{P_{L_i}}{\sum_{i=1}^{N_L} P_{L_i}} \times SC_{total} \quad (6.11)$$

The value of the security cost in each testing case is original from the test results in section 5.5. On one hand, it is to guarantee the integrity of the proposed congestion management scheme because of the proposed method is aim to not only re-dispatch generation but also to balance the security cost attributed to the congestion management.

On the other hand, choosing the results from chapter 5 is intended to pave the way for the next chapter which will introduce the impedance matrix based method to allocate the security cost. In this way, it is convenient to compare the outcomes by different security cost allocation schemes.

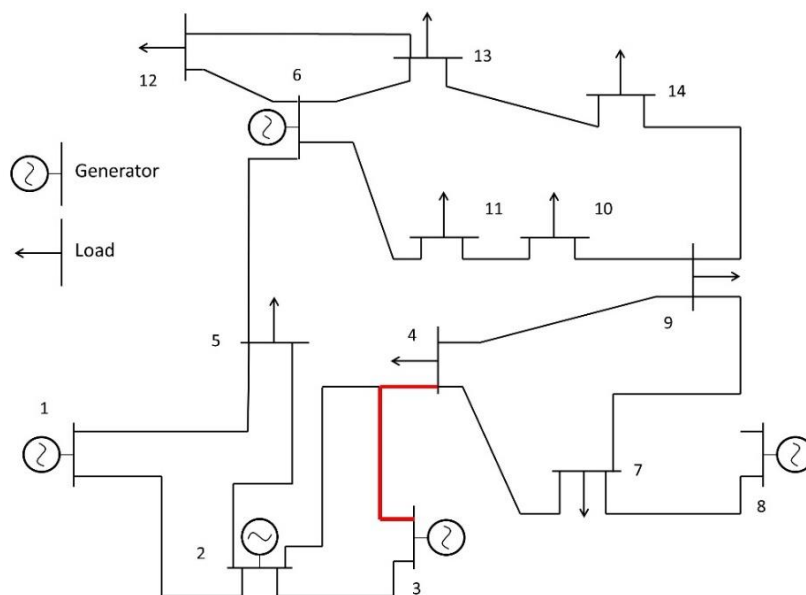
Since the PTDF based method is dependent on the slack bus selection, selecting different generator bus as slack bus might incur different experimental outcomes. To simplify the experiment, case studies are mainly focus on two comparisons. One is the comparison of different experimental results based on different PTDF values due to different slack bus selections. Another one is the outcome comparison between the pro rata method and the PTDF method when the first generator bus selected as the slack bus. The test description is visualized by Figure 6.1 as follows:



**Figure 6.1 Test description for chapter 6**

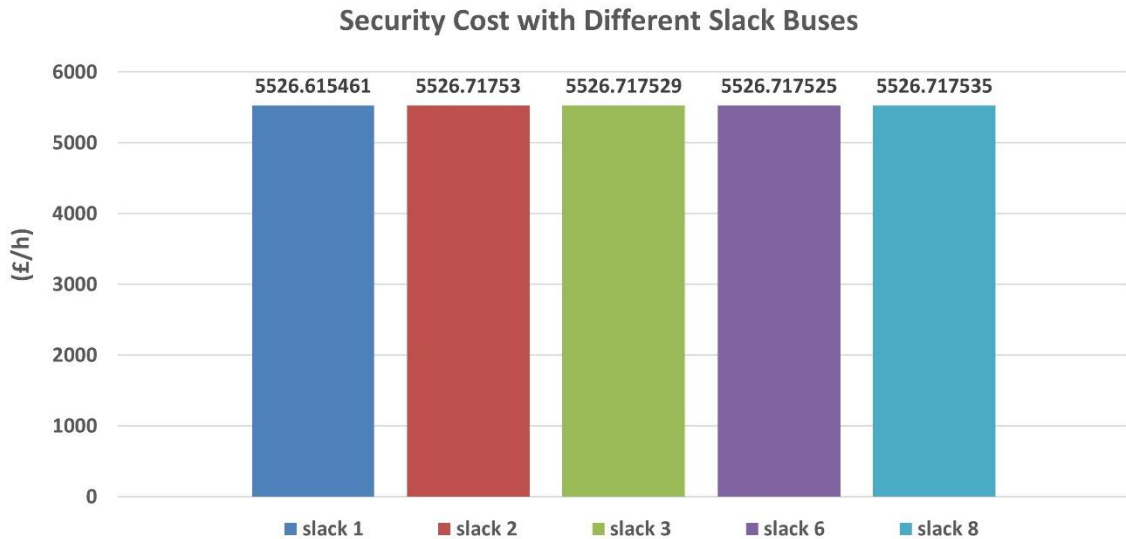
### 6.4.2. Modified IEEE-14 Bus System

The modified IEEE-14 bus test system is exactly same with the system in chapter 4 and 5, which is showed in Figure 6.2 as follow:



**Figure 6.2 The modified IEEE-14 bus system**

The parameters of the generators and loads are set up the same, which are displayed in Table 4.1. Transmission congestion still occurs on the target branch 3-4. There are 5 generators and 9 loads in the system. The security costs are picked up from the experimental results in section 5.5.2 which are displayed in Figure 6.3.



**Figure 6.3 Security cost settings in the 14-bus test system**

#### 6.4.2.1. Comparison between Security Cost Allocation with Different Slack Bus Selections in the 14-bus Test System

After used the proposed security cost allocation method with PTDF, the security cost allocated to each load in the system is presented in Table 6.1 and Figure 6.4 as follows:

**Table 6.1 Security Cost Allocated to Each Load with Different Slack Buses in the 14-bus Test System**

(£/h)	L4	L5	L7	L9	L10	L11	L12	L13	L14
<b>slack 1</b>	642.13	437.46	726.49	585.66	685.43	1184.27	408.55	412.57	444.06
<b>slack 2</b>	626.62	457.46	715.52	579.94	681.58	1189.77	414.38	417.70	443.73
<b>slack 3</b>	572.15	527.59	676.99	559.86	668.05	1209.00	434.83	435.70	442.56
<b>slack 6</b>	1574.05	-762.41	1385.81	929.36	917.03	855.30	58.75	104.66	464.17
<b>slack 8</b>	-384.74	1759.65	0.00	206.95	430.25	1546.82	794.01	751.88	421.91



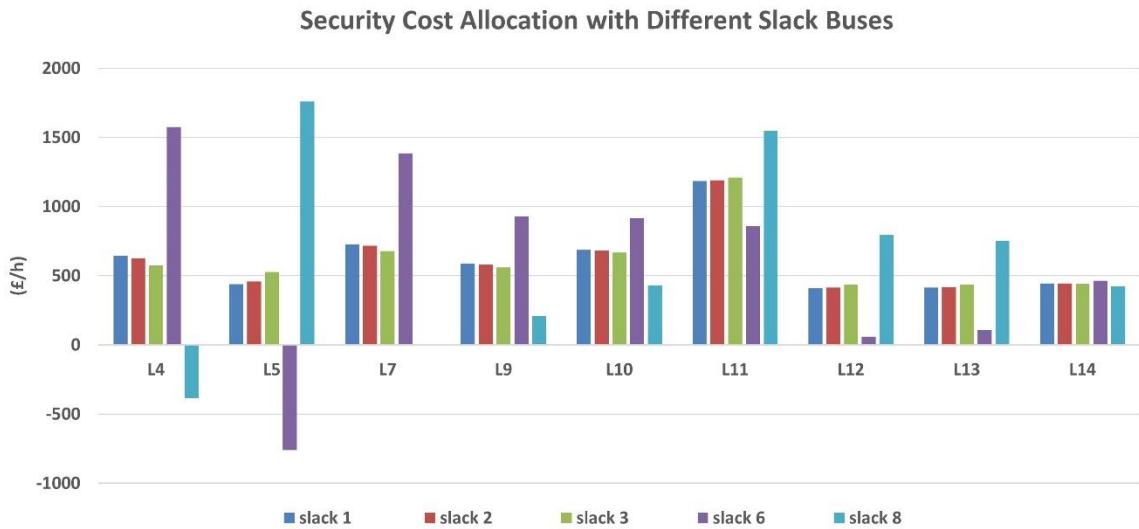


Figure 6.4 Security cost allocated to each load with different slack buses in the 14-bus test system

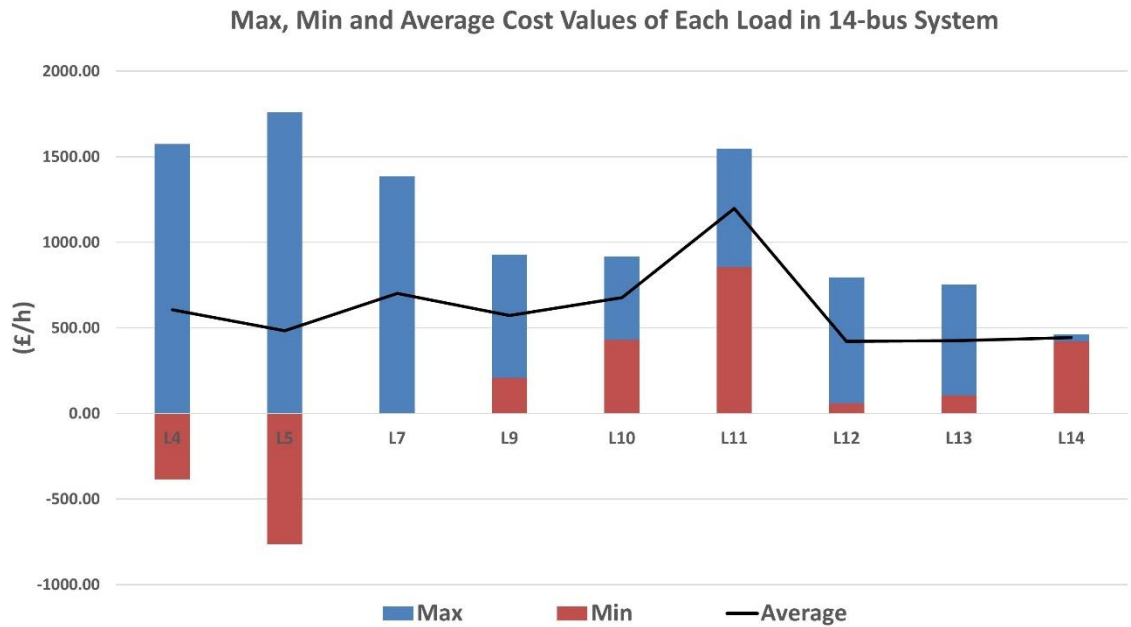


Figure 6.5 Maximum, minimum and average security cost values of each load in 14-bus system

Figure 6.3 represents the security costs originated from the experimental results in chapter 5. It is indicated that the security costs based on different slack bus selections are very close at around £5526/h.

Figure 6.4 compares the security cost allocation to each load with different slack buses. It is found that when select the generator bus 1, 2 and 3 as the slack bus respectively, the allocation results are similar. However, when bus 6 or bus 8 is regarded as the slack bus, the results of security cost allocation show quite a distinction. In the case of slack bus 6, load 4, 7, 9, 10 and 14 shoulder more security cost compared with the first three cases and the cost allocated to load 11, 12, 13 and 14 is lower than the first three cases. In the case of slack bus 8, load 5, 11, 12 and 13 accept more security cost and load 7, 9 and 10 shoulder less cost compared with the first three cases. It should be noted that load 5 in slack bus 6 and load 4 in slack bus 8 are allocated negative values of security cost, which means such two loads both have negative contributions on power flow of congested line, in other words, they are helping to relieve the congestion. As mentioned in section 6.3.2, the PTDF-based method is trying to find out each bus's real contribution to one particular transmission line's power flow no matter it is positive or negative. Therefore, based on these real contributions of loads, the system operator could take measures to do appropriate security cost allocation to loads. Here, as bus 6 is selected as the slack bus, the PTDF-based method deems load 5 should be allocated negative security cost since it helps to relieve the congestion and as bus 8 is selected as the slack bus, load 4 is considered as the one who helps to relieve the congestion. The system operator should avoid allocating security cost to such loads and even reward them. From the above test results, it should be known that the PTDF based method is indeed suffered the drawbacks of complexity and inaccuracy due to dependence on slack bus selection.

Figure 6.5 shows the maximum, the minimum and the average security cost values of each load in 14-bus system under different slack bus selections. From the figure, it can

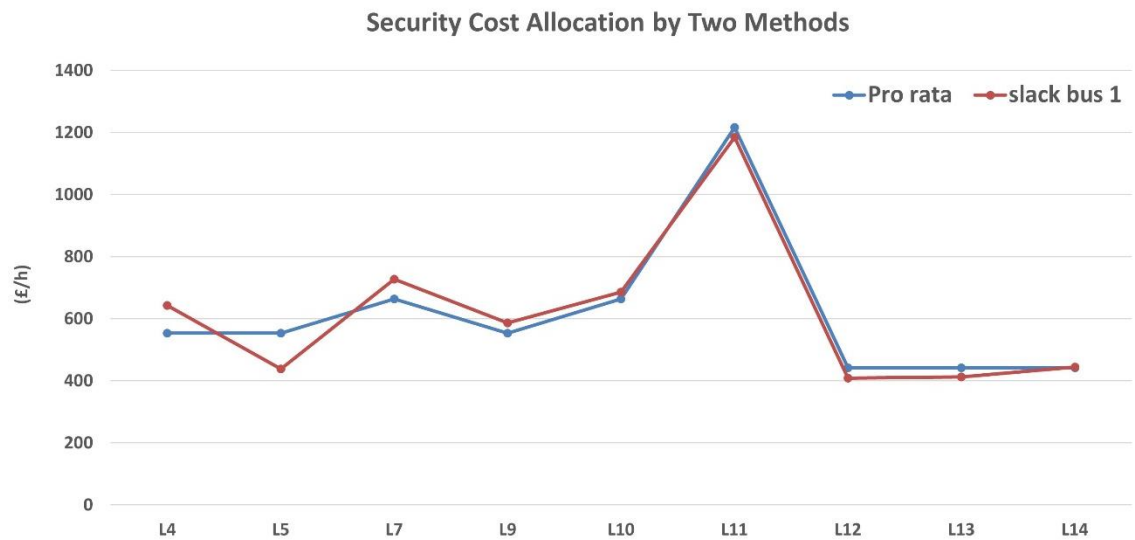
be found that load 5 has the biggest distance between the maximum value at £1759/h and the minimum value at -£762/h. The range is £2521/h. Load 14 has the smallest range between the maximum value at £464/h and the minimum value at £421/h. The range is £41/h. The black curve shows the average security cost values of each load under different slack bus selections. From the curve, it can be noted that load 11 has the highest average security cost allocation value at £1197/h and load 12 has the lowest value at £422/h.

#### 6.4.2.2. Comparison between Pro Rata Method and PTDF Method in the 14-bus Test System

Then the pro rata based method will be implemented to compare with the PTDF method when generator bus one is selected as the slack bus. The comparison will be illustrated in Table 6.2 and Figure 6.6 as follows:

**Table 6.2 Comparison between Two Methods on Security Cost Allocation in the 14-bus Test System**

(£/h)	L4	L5	L7	L9	L10	L11	L12	L13	L14
<b>Pro rata</b>	552.66	552.66	663.19	552.66	663.19	1215.86	442.13	442.13	442.13
<b>slack bus 1</b>	642.13	437.46	726.49	585.66	685.43	1184.27	408.55	412.57	444.06

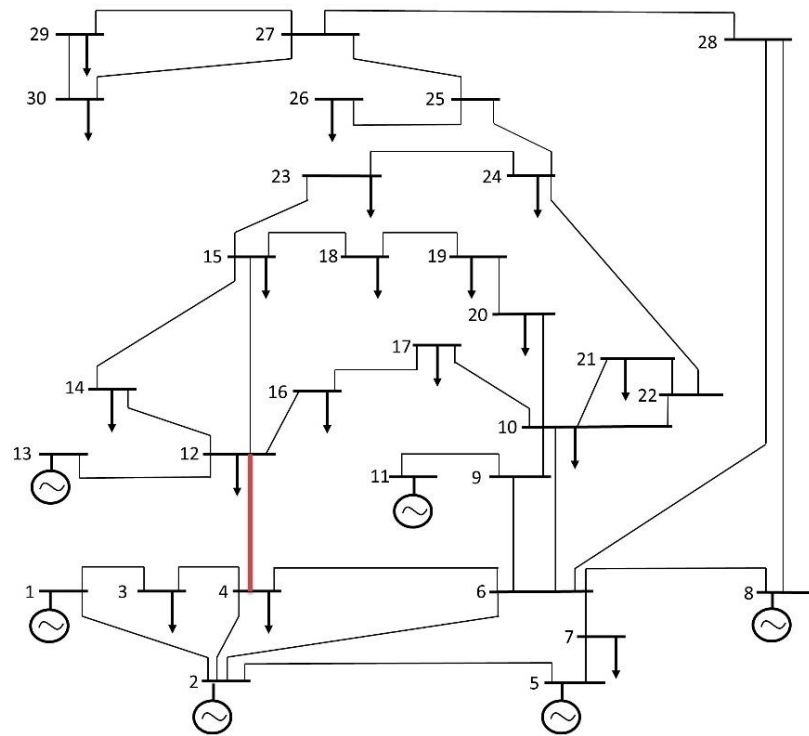


**Figure 6.6 Comparison between two methods on security cost allocation in the 14-bus test system**

From Figure 6.6, it is indicated that security cost allocation results by pro rata method and PTDF method seems very similar. Except load 4, 5 and 7 shoulder a little different security cost, the other loads are allocated approximately the same amount of cost. It is unable to indicate the theoretical advantage of the PTDF method compared to the Pro rata method. More discoveries about the comparison between two methods will be obtained in the next few systems.

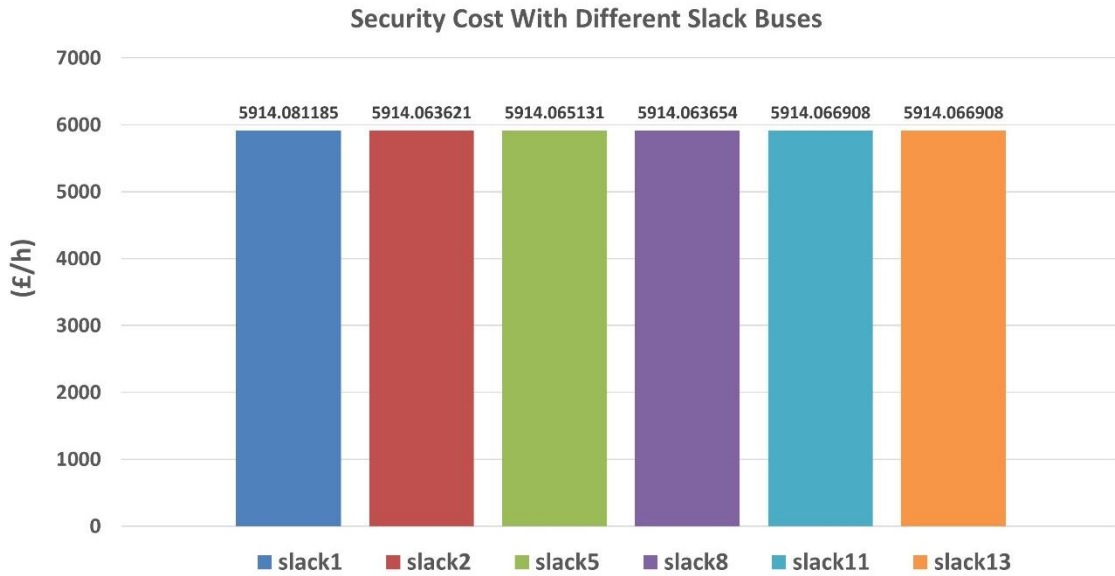
### 6.4.3. Modified IEEE-30 Bus System

The modified IEEE-30 bus test system is same with the system in chapter 4 and 5, which is shown in Figure 6.7 as follow:



**Figure 6.7 The modified IEEE-30 bus system**

The parameters of the generators and loads are set up in the same way as those the former two chapters, which are displayed in Table 4.4 and Table 4.5 respectively. There are 6 generators and 18 loads in the system. Transmission congestion are still set on the target branch 4-12. The security costs are taken from the experimental results in section 5.5.3 which are displayed in Figure 6.8.



**Figure 6.8 Security cost settings in the 30-bus test system**

#### **6.4.3.1. Comparison between Security Cost Allocation with Different Slack Bus Selections in the 30-bus Test System**

By implementing the proposed PTDF based security cost allocation method, the security cost allocated to each load with different slack buses is presented in Table 6.3 and Figure 6.9 respectively as follows:

**Table 6.3 Security Cost Allocated to Each Load with Different Slack Buses in the 30-bus Test System**

(£/h)	L3	L4	L7	L10	L12	L14	L15	L16	L17	L18	L19	L20	L21	L23	L24	L26	L29	L30
<b>slack 1</b>	-10.03	-16.44	30.30	359.03	782.51	601.21	561.51	605.55	506.67	281.85	415.51	232.17	183.93	465.85	405.58	235.65	102.47	170.78
<b>slack 2</b>	-13.75	-21.45	24.90	358.04	788.71	605.52	565.14	608.74	507.00	283.10	416.65	232.57	183.51	467.85	405.38	233.74	100.66	167.76
<b>slack 5</b>	-24.27	-35.63	9.62	355.23	806.27	617.70	575.41	617.79	507.94	286.61	419.91	233.70	182.31	473.53	404.81	228.34	95.55	159.24
<b>slack 8</b>	-37.87	-53.96	-10.12	351.59	828.97	633.45	588.69	629.48	509.16	291.16	424.11	235.15	180.77	480.86	404.07	221.37	88.94	148.23
<b>slack 11</b>	-331.69	-449.83	-436.67	273.11	1319.25	973.69	875.61	882.09	535.52	389.37	514.93	266.63	147.46	639.29	388.11	70.62	-53.79	-89.64
<b>slack 13</b>	458.91	615.39	711.08	484.29	0.00	58.18	103.59	202.37	464.60	125.11	270.56	181.93	237.10	212.99	431.05	476.24	330.26	550.43

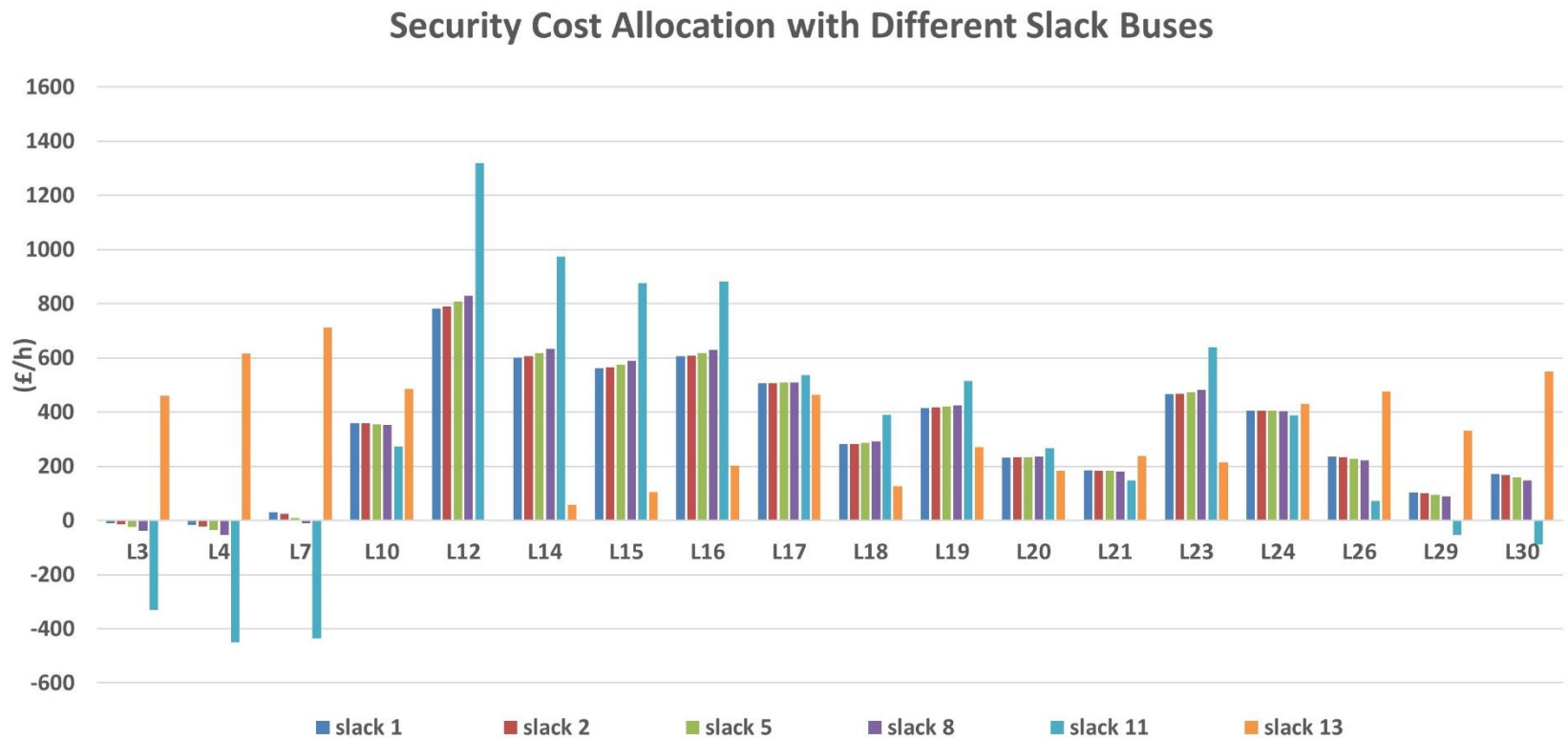


Figure 6.9 Security cost allocated to each load with different slack buses in the 30-bus test system



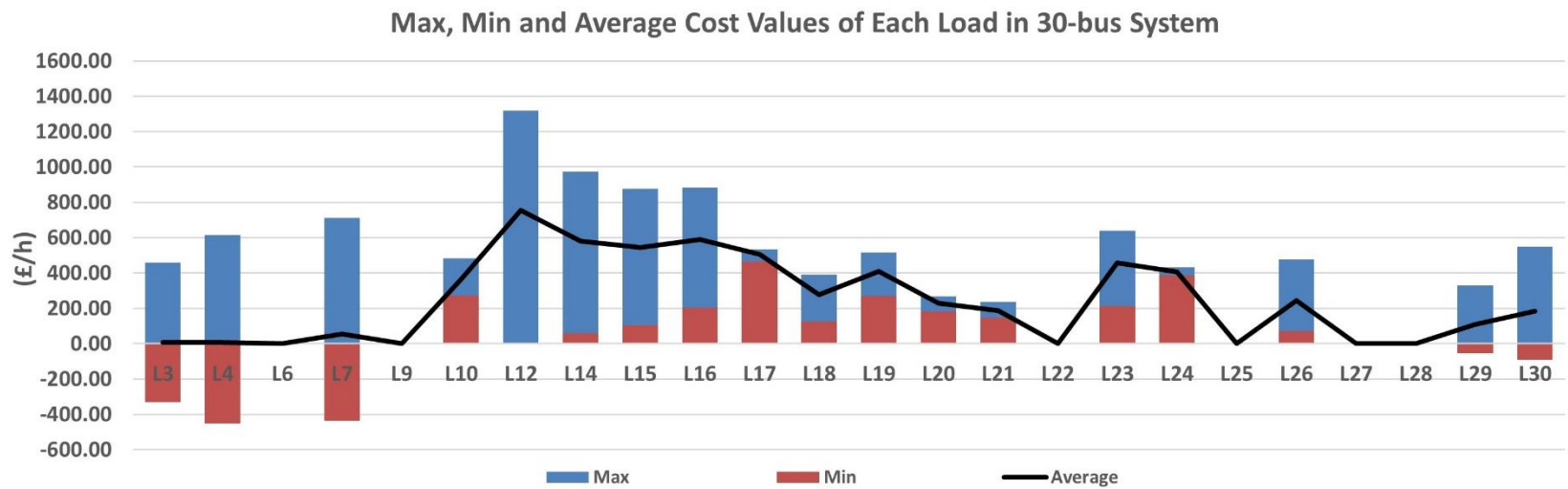


Figure 6.10 Maximum, minimum and average security cost values of each load in 30-bus system

Figure 6.8 displays the security costs originated from the experimental results in chapter 5 as the basic security cost settings in this case. It is indicated that the security costs caused by different slack bus selections are all approximated at £5914/h.

Figure 6.9 compares the security cost allocation to each load under different slack bus selections. It is found that selecting the first four generator buses respectively as the slack bus, the allocation results are close. However, when bus 11 or bus 13 is chosen as the slack bus, the outcome of security cost allocation becomes significantly different. In the case of slack bus 11, load 12, 14, 15, 16, 17, 18, 19, 20 and 23 undertake more security cost compared with the first four cases and the cost allocated to load 10, 21, 24 and 26 is lower than the first four cases. In the case of slack bus 13, load 3, 4, 7, 10, 21, 24, 26, 29 and 30 accept more security cost and load 14, 15, 16, 17, 18, 19, 20 and 23 are allocated less cost when compared with the first four cases. It is noticeable that load 3 and 4 in the first five case, load 7 in slack bus 8 and slack bus 13 cases and load 29 and 30 in slack bus 11 are allocated negative values of security cost, which means the loads at here have contributed to relieving congestion. The PTDF-based method detects each bus's real contribution to the congestion no matter it is positive or negative. Here these loads are considered as to help to relieve the congestion. In theory, they should be rewarded by the system operator rather than shoulder the extra cost. From the above results, it is obtained again that the PTDF based method could result in different security costs and is dependent on the selection of slack bus.

Figure 6.10 shows the maximum, the minimum and the average security cost values of each load in 30-bus system under different slack bus selections. From the figure, it can be found that load 7 has the biggest range between the maximum value at £711/h and the minimum value at -£436/h. The range is £1147/h. Load 24 has the smallest range between the maximum value at £431/h and the minimum value at £388/h. The range is £43/h. The black curve shows the average security cost values of each load under different

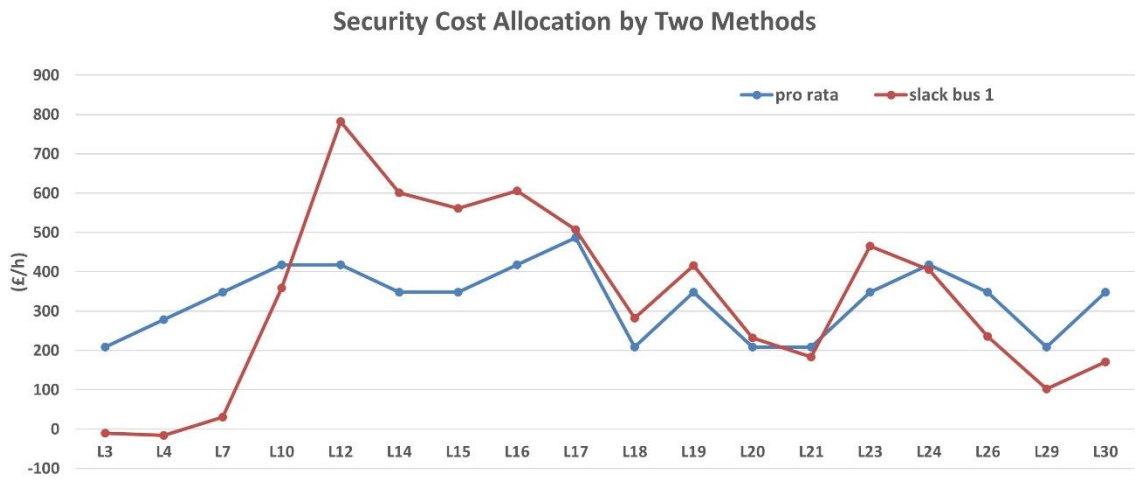
slack bus selections. From the curve, it can be noted that load 12 has the highest average security cost allocation value at £754/h and load 4 has the lowest value at £6/h except those zeros.

#### 6.4.3.2. Comparison between Pro Rata Method and PTDF Method in the 30-bus Test System

The pro rata based method is used to offer a comparison with the PTDF method with selecting generator bus one as the slack bus. The comparison will be represented in Table 6.4 and Figure 6.11 respectively as follows:

**Table 6.4 Comparison between Two Methods on Security Cost Allocation in the 30-bus Test System**

(£/h)	L3	L4	L7	L10	L12	L14
<b>Pro rata</b>	208.73	278.31	347.89	417.46	417.46	347.89
<b>slack bus 1</b>	-10.03	-16.44	30.30	359.03	782.51	601.21
	L15	L16	L17	L18	L19	L20
<b>Pro rata</b>	347.89	417.46	487.04	208.73	347.89	208.73
<b>slack bus 1</b>	561.51	605.55	506.67	281.85	415.51	232.17
	L21	L23	L24	L26	L29	L30
<b>Pro rata</b>	208.73	347.89	417.46	347.89	208.73	347.89
<b>slack bus 1</b>	183.93	465.85	405.58	235.65	102.47	170.78

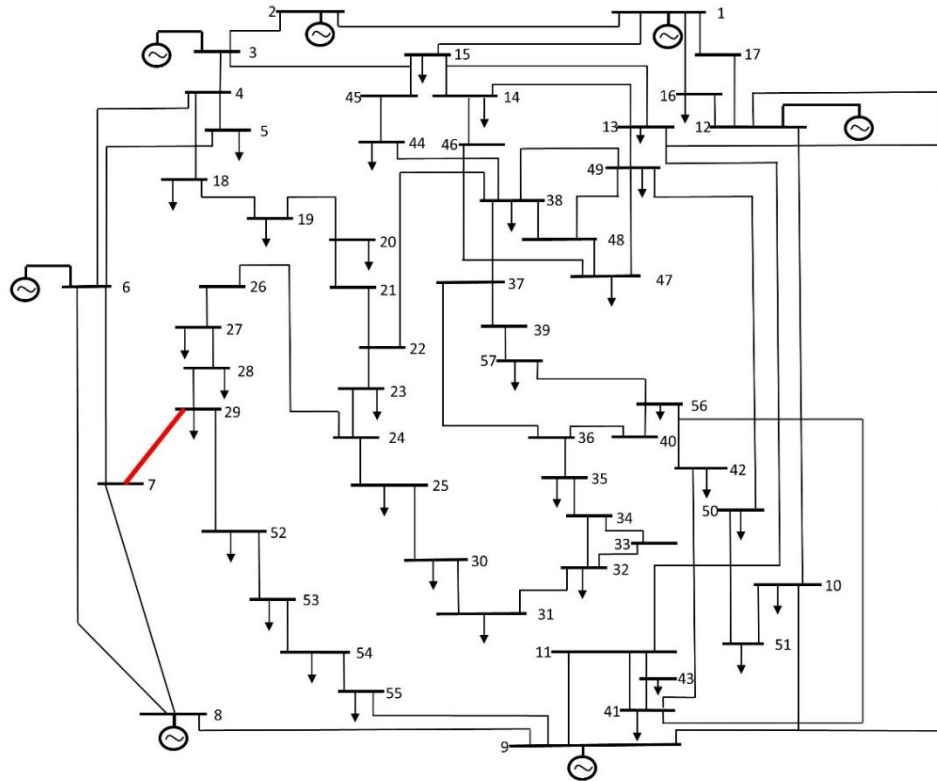


**Figure 6.11 Comparison between two methods on security cost allocation in the 30-bus test system**

Figure 6.11 visualizes the comparison of security cost allocation results between pro rata method and PTDF method. It is found that the interval of two curves from load 17 to load 30 have similar tendency of reaction. The difference between two curves is mainly at load 3, 4, 7, 12, 14, 15, 16, 23, 26, 29 and 30. Loads 3, 4, 7, 26, 29 and 30 in the PTDF method are allocated lower security cost than in pro rata method because PTDF method can be figured out the contribution of these three loads to the transmission congestion is small. Similarly, since the contributions of load 14, 15, 16, 23 and especially 12 to the congestion on the line 4-12 are large, the security cost allocated to those loads are higher than other loads.

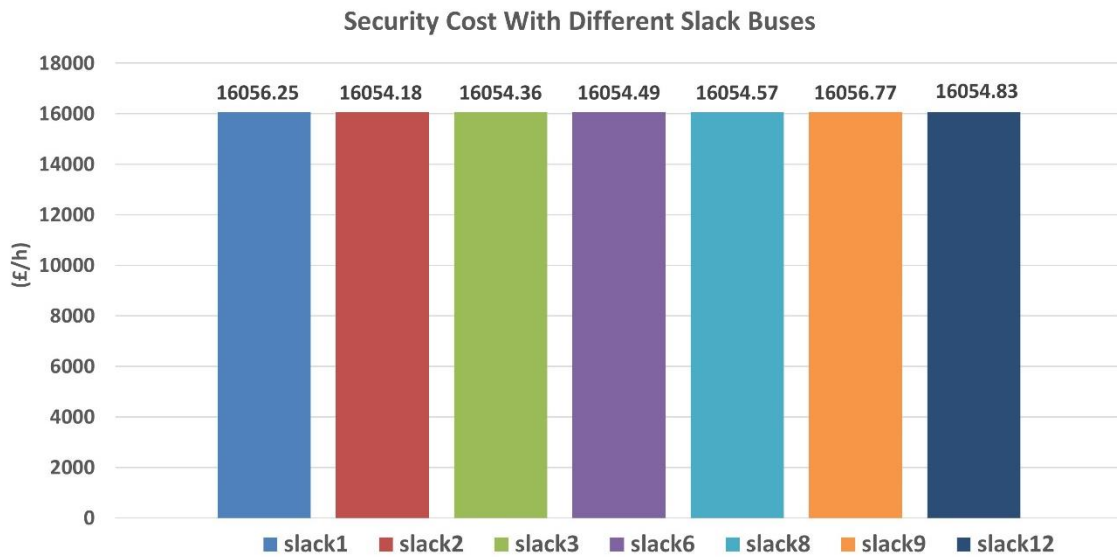
#### 6.4.4. Modified IEEE-57 Bus System

Test system in this case is used exactly the same system with chapter 4 and 5 respectively, which is shown in Figure 6.12 as follow:



**Figure 6.12 The modified IEEE-57 bus system**

The parameters of the generators and loads are set up in the same way as in the former two chapters, which are displayed in Table 4.8 and Table 4.9 respectively. There are 7 generators and 35 loads totally in the system. Transmission congestion are still set on the target branch 7-29. The security costs are taken from the experimental results in section 5.5.4 which are displayed in Figure 6.13.



**Figure 6.13 Security cost settings in the 57-bus test system**

#### 6.4.4.1. Comparison between Security Cost Allocation with Different Slack Bus Selections in the 57-bus Test System

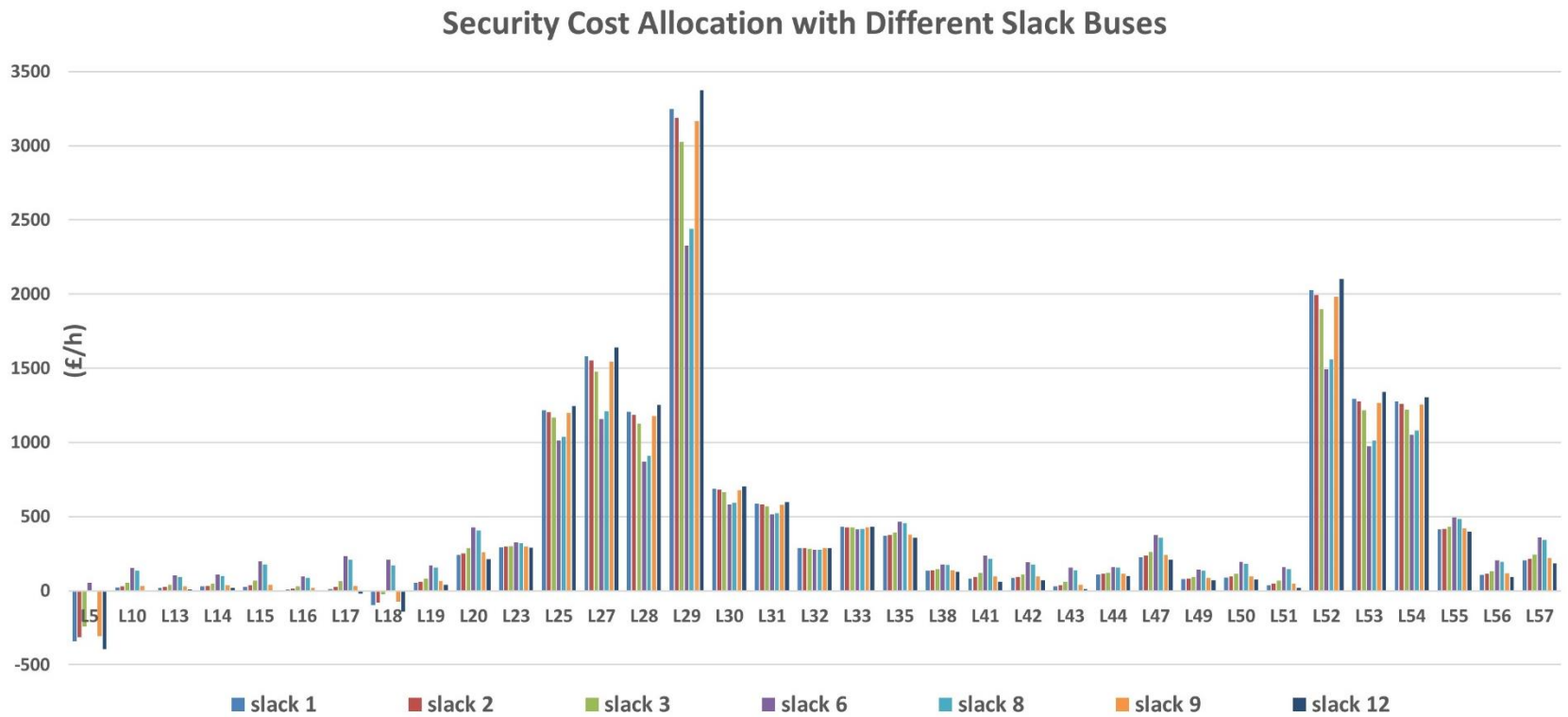
By implementing the proposed PTDF based security cost allocation method, the security cost allocated to each load with different slack bus selections is obtained and shown in Table 6.5 and Figure 6.14 respectively as follows:

**Table 6.5 Security Cost Allocated to Each Load with Different Slack Buses in the 57-bus Test System**

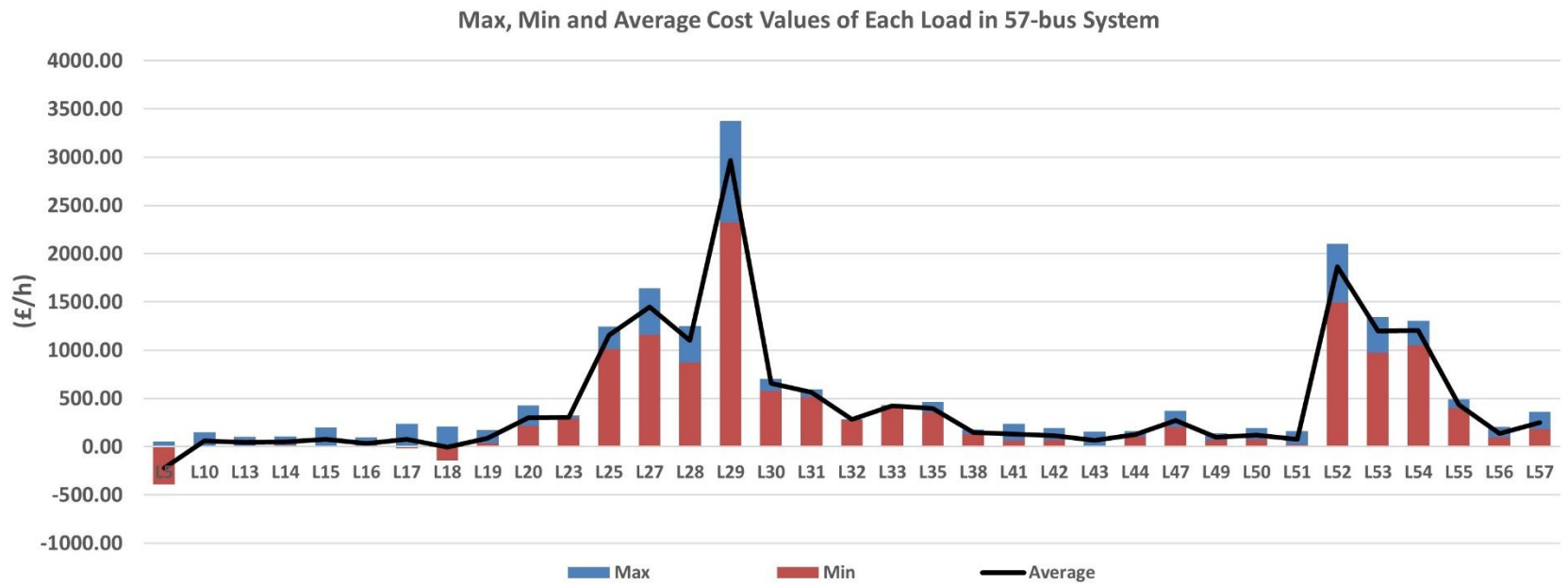
(£/h)	L5	L10	L13	L14	L15	L16	L17	L18	L19	L20	L23	L25	L27	L28	L29	L30	L31	L32
<b>slack 1</b>	-339.84	21.61	19.71	27.63	23.70	8.60	11.29	-99.07	54.14	239.64	293.39	1215.32	1579.82	1205.49	3247.97	687.52	585.00	286.17
<b>slack 2</b>	-314.51	29.89	25.11	32.85	34.85	14.25	25.65	-79.25	61.67	251.70	295.44	1202.12	1552.42	1183.74	3188.23	680.55	580.38	285.47
<b>slack 3</b>	-244.36	52.87	40.09	47.33	65.81	29.94	65.51	-24.31	82.59	285.25	301.26	1165.93	1476.98	1123.83	3023.66	661.47	567.77	283.62
<b>slack 6</b>	53.12	150.30	103.63	108.75	197.10	96.45	234.52	208.67	171.32	427.51	325.89	1012.43	1157.03	869.74	2325.69	580.54	514.30	275.78
<b>slack 8</b>	4.92	134.51	93.33	98.80	175.83	85.67	207.14	170.92	156.94	404.46	321.90	1037.31	1208.88	910.92	2438.80	593.65	522.97	277.05
<b>slack 9</b>	-305.42	32.89	27.06	34.74	38.89	16.30	30.85	-72.11	64.41	256.11	296.25	1197.59	1542.84	1176.12	3167.30	678.18	578.83	285.27
<b>slack 12</b>	-394.20	3.79	8.09	16.40	-0.31	-3.56	-19.62	-141.66	37.91	213.61	288.86	1243.28	1638.18	1251.84	3375.31	702.26	594.73	287.58

(£/h)	L33	L35	L38	L41	L42	L43	L44	L47	L49	L50	L51	L52	L53	L54	L55	L56	L57	
<b>slack 1</b>	429.25	369.15	133.37	82.73	84.69	28.03	108.18	227.08	78.13	87.66	36.89	2027.93	1295.82	1275.05	411.37	107.46	205.37	
<b>slack 2</b>	428.20	375.32	136.16	92.54	91.53	36.17	111.55	236.50	82.19	94.42	44.82	1993.20	1274.92	1260.48	416.57	113.77	215.29	
<b>slack 3</b>	425.43	392.58	143.97	119.77	110.52	58.74	120.94	262.73	93.49	113.23	66.83	1897.57	1217.42	1220.52	431.16	131.33	242.89	
<b>slack 6</b>	413.67	465.73	177.06	235.24	191.06	154.45	160.78	373.95	141.37	192.97	160.17	1492.00	973.55	1051.02	493.01	205.77	359.92	
<b>slack 8</b>	415.58	453.88	171.70	216.53	178.01	138.94	154.33	355.93	133.61	180.05	145.05	1557.72	1013.07	1078.49	482.99	193.71	340.96	
<b>slack 9</b>	427.91	377.63	137.20	96.10	94.01	39.11	112.79	239.96	83.68	96.89	47.69	1981.06	1267.64	1255.47	418.54	116.08	218.92	
<b>slack 12</b>	431.37	355.74	127.30	61.61	69.96	10.53	100.88	206.73	69.37	73.07	19.82	2101.91	1340.30	1305.93	400.02	93.84	183.96	





**Figure 6.14 Security cost allocated to each load with different slack buses in the 57-bus test system**



**Figure 6.15 Maximum, minimum and average security cost values of each load in 57-bus system**

Figure 6.13 displays the security costs which are from the experimental results in chapter 5 as the security cost resources. It is indicated that the security costs calculated using by different slack bus are very similar.

Figure 6.14 gives a comparison of the security cost allocation to each load under each slack bus selection. It is found that when select the generator bus 1, 2, 3, 9 and 12 respectively as the slack bus, the allocation results are close. However, when bus 6 or bus 8 is chosen as the slack bus, the results of security cost allocation become different. In these two slack bus selections, load 25, 27, 28, 29, 30, 31, 52, 53 and 54 are allocated smaller security cost than the other five cases and the costs allocated to load 10, 13, 14, 15, 17, 18, 20, 35, 41, 42, 43, 44, 47, 49, 50, 51, 55, 56 and 57 are more than the other five slack bus selections. It is noticeable that load 5 and 18 in these five slack bus selections are allocated negative security cost, which means such loads have positive impact on congestion relief. The PTDF-based method is used to find out each bus's real contribution to the congestion no matter it is positive or negative. Here these two loads are considered as the ones who are helping to relieve the congestion under these particular slack bus selections. Theoretically, they should be rewarded by the system operator rather than shoulder the security cost. From the above results, it is also obtained that the PTDF based method could result in different allocation of security cost of individual load and is dependent on the selection of slack bus.

Figure 6.15 shows the maximum, the minimum and the average security cost values of each load in 57-bus system under different slack bus selections. From the figure, it can be found that load 29 has the biggest range between the maximum value at £3375/h and the minimum value at £2325/h. The range is £1050/h. Load 32 has the smallest range between the maximum value at £287/h and the minimum value at £275/h. The range is £12/h. The black curve shows the average security cost values of each load under different

slack bus selections. From the curve, it can be noted that load 29 has the highest average security cost allocation value at £2966/h and load 5 has the lowest value at -£220/h.

#### 6.4.4.2. Comparison between Pro Rata Method and PTDF Method in the 57-bus Test System

The pro rata based method is used to compare with the PTDF method with selecting generator bus one as the slack bus. The comparison will be represented in Table 6.6 and Figure 6.16 as follow respectively:

**Table 6.6 Comparison between Two Methods on Security Cost Allocation in 57-bus Test System**

(£/h)	L5	L10	L13	L14	L15	L16	L17
<b>Pro rata</b>	770.70	385.35	256.90	256.90	513.80	256.90	642.25
<b>slack bus 1</b>	-339.84	21.61	19.71	27.63	23.70	8.60	11.29
	L18	L19	L20	L23	L25	L27	L28
<b>Pro rata</b>	770.70	385.35	770.70	385.35	642.25	385.35	256.90
<b>slack bus 1</b>	-99.07	54.14	239.64	293.39	1215.32	1579.82	1205.49
	L29	L30	L31	L32	L33	L35	L38
<b>Pro rata</b>	642.25	385.35	385.35	256.90	385.35	642.25	256.90
<b>slack bus 1</b>	3247.97	687.52	585.00	286.17	429.25	369.15	133.37
	L41	L42	L43	L44	L47	L49	L50

<b>Pro rata</b>	513.80	385.35	385.35	256.90	642.25	256.90	385.35
<b>slack bus 1</b>	82.73	84.69	28.03	108.18	227.08	78.13	87.66
	<b>L51</b>	<b>L52</b>	<b>L53</b>	<b>L54</b>	<b>L55</b>	<b>L56</b>	<b>L57</b>
<b>Pro rata</b>	385.35	513.80	385.35	642.25	642.25	385.35	642.25
<b>slack bus 1</b>	36.89	2027.93	1295.82	1275.05	411.37	107.46	205.37

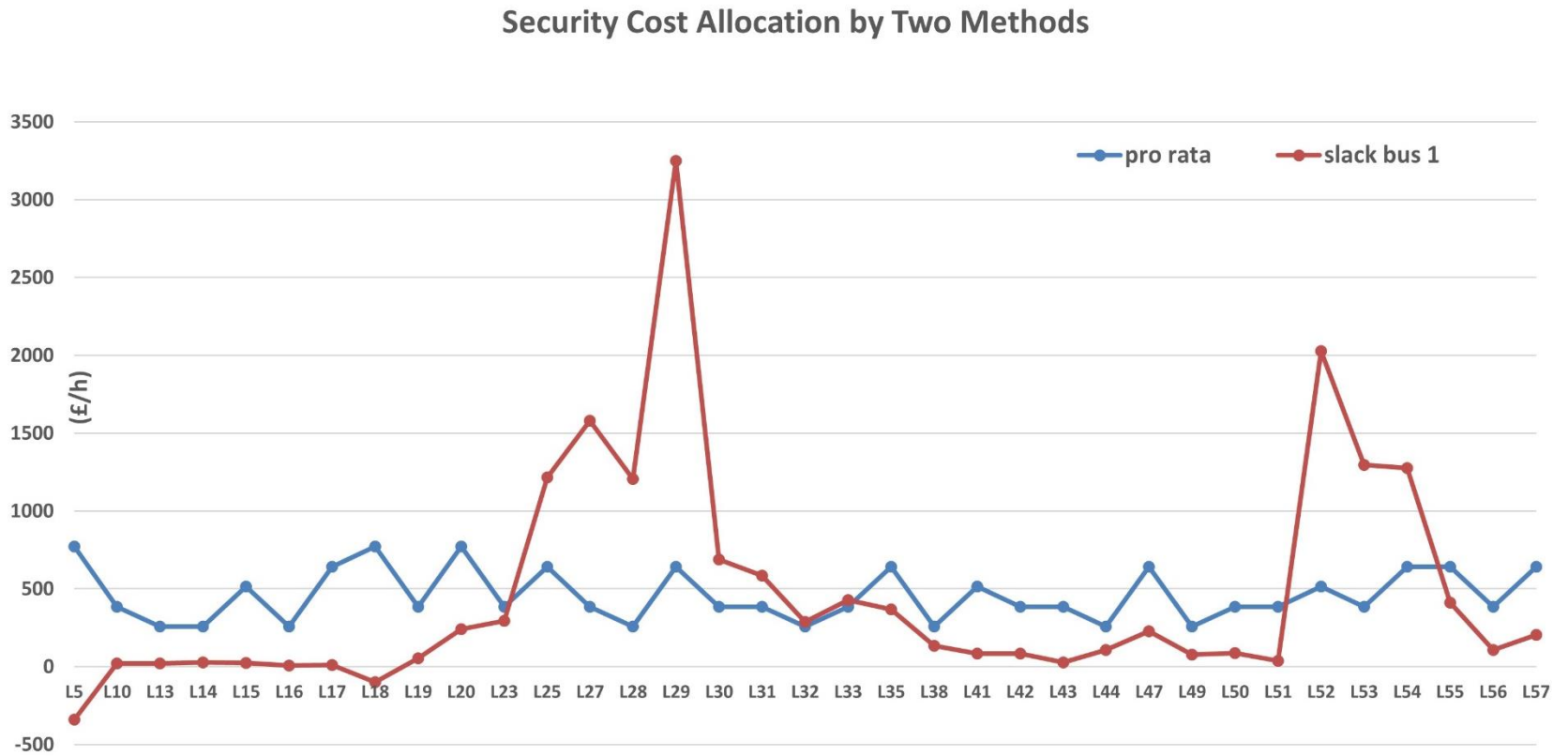


Figure 6.16 Comparison between two methods on security cost allocation in the 57-bus test system

Figure 6.16 indicates the comparison of security cost allocation results between pro rata method and PTDF method back on slack bus 1. The curve of the pro rata method varies between the vertical coordinate intervals between £ 0/h to £ 1000/h. The slack bus 1 curve is apparently above the pro rata curve only at load 25, 27, 28, 30, 31, 52, 53, 54 and especially 29 and the rest of them are below the pro rata curve. It is explained that the above loads particularly load 29 have higher contribution to the congestion occurs on the line 4-29 so they should bear more security cost than other loads.

#### 6.4.5. Modified IEEE-118 Bus System

The modified IEEE-118 bus test system in this case is same with the system in chapter 4 and 5 respectively, which is shown in Figure 6.17 as follow:

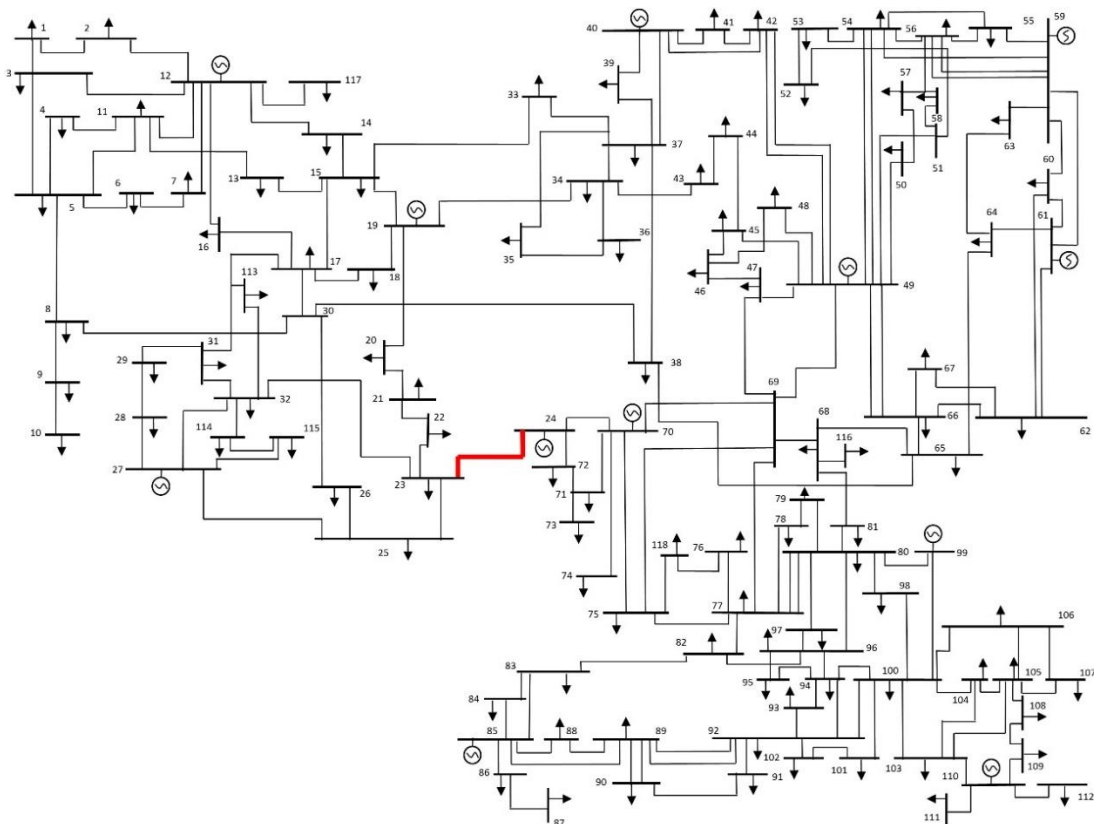


Figure 6.17 The modified IEEE-118 bus system

The parameters of the generators and loads are set up as the same as the former two chapters, which are displayed in Table 4.12 and Table 4.13 respectively. There are 12 generators and 104 loads in the test system. Transmission congestion are still set on the target branch 23-24. The security costs are taken from the experimental results in section 5.5.5 which are displayed in Figure 6.18.



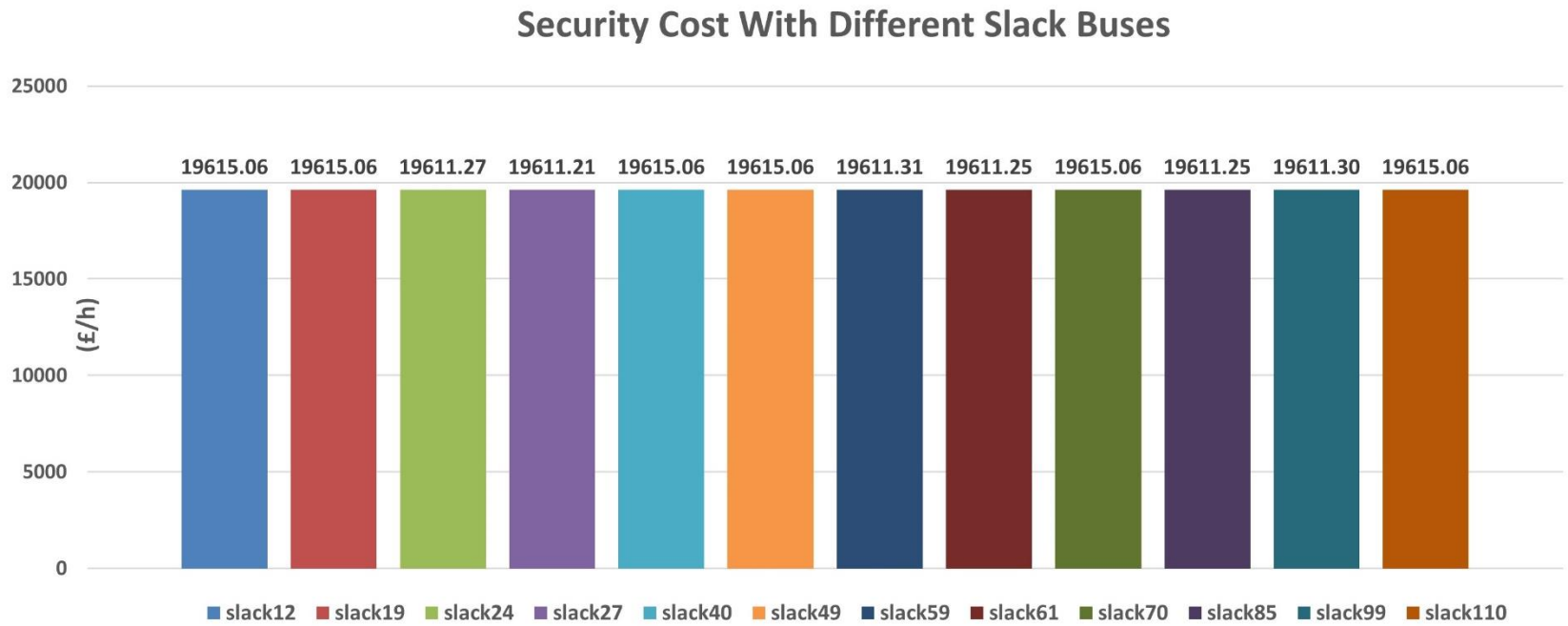


Figure 6.18 Security cost settings in the 118-bus test system

#### **6.4.5.1. Comparison between Security Cost Allocation with Different Slack Bus Selections in the 118-bus Test System**

By implementing the proposed PTDF based security cost allocation method, the security cost allocated to each load in the test system with twelve generator buses selected as the slack bus respectively is shown in Table 6.7 and Figure 6.19 respectively as follows:

**Table 6.7 Security Cost Allocated to Each Load with Different Slack Buses in the 118-bus Test System**

(£/h)	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L13	L14	L15	L16	L17	L18	L20
<b>slack 12</b>	0.74	0.19	0.93	1.91	0.69	1.04	0.43	3.88	1.29	2.58	1.04	0.06	-0.25	-3.70	-3.02	-4.77	-3.52	-32.63
<b>slack 19</b>	7.40	4.64	7.59	8.53	2.89	7.69	4.87	10.44	3.48	6.96	12.15	6.74	3.10	9.78	1.53	-2.40	-1.18	-28.28
<b>slack 24</b>	267.43	178.36	267.39	267.15	89.04	267.36	178.30	266.67	88.89	177.78	445.77	267.60	133.87	536.13	179.15	90.37	90.07	141.78
<b>slack 27</b>	79.40	52.74	79.53	80.14	26.75	79.59	52.90	81.39	27.13	54.26	132.22	78.97	39.31	155.53	50.71	23.29	24.08	18.82
<b>slack 40</b>	-464.03	-310.32	-463.41	-460.32	-153.28	-463.09	-309.56	-454.09	-151.36	-302.73	-773.99	-466.20	-233.97	-944.46	-320.49	-170.58	-166.61	-336.58
<b>slack 49</b>	960.38	641.32	959.71	956.31	318.60	959.35	640.48	949.48	316.49	632.98	1601.30	962.76	482.33	1938.77	652.47	337.58	333.23	594.95
<b>slack 59</b>	878.73	586.77	878.13	875.11	291.55	877.81	586.02	869.02	289.67	579.35	1465.14	880.85	441.27	1773.49	596.70	308.45	304.57	541.56
<b>slack 61</b>	865.30	577.80	864.71	861.75	287.10	864.40	577.06	855.79	285.26	570.53	1442.75	867.38	434.52	1746.32	587.53	303.66	299.86	532.78

<b>slack 70</b>	354.19	236.32	354.06	353.43	117.78	353.99	236.17	352.15	117.38	234.77	590.44	354.63	177.49	711.74	238.41	121.32	120.51	198.51
<b>slack 85</b>	523.80	349.64	523.52	522.11	173.96	523.37	349.29	519.28	173.09	346.18	873.27	524.78	262.78	1055.05	354.26	181.83	180.02	309.44
<b>slack 99</b>	532.17	355.23	531.88	530.43	176.74	531.73	354.87	527.53	175.84	351.68	887.23	533.18	267.00	1072.00	359.98	184.81	182.96	314.92
<b>slack 110</b>	530.16	353.89	529.87	528.44	176.07	529.72	353.53	525.55	175.18	350.37	883.88	531.17	265.99	1067.93	358.61	184.10	182.26	313.60
<b>(£/h)</b>	<b>L21</b>	<b>L22</b>	<b>L23</b>	<b>L25</b>	<b>L26</b>	<b>L28</b>	<b>L29</b>	<b>L31</b>	<b>L32</b>	<b>L33</b>	<b>L34</b>	<b>L35</b>	<b>L36</b>	<b>L37</b>	<b>L38</b>	<b>L39</b>	<b>L41</b>	<b>L42</b>
<b>slack 12</b>	-53.81	-52.01	-313.79	-226.92	-93.25	-37.38	-65.48	-124.44	-214.33	31.83	171.76	103.61	34.48	34.78	35.23	87.75	159.65	388.34
<b>slack 19</b>	-48.80	-48.16	-295.13	-208.73	-83.67	-33.99	-58.99	-111.67	-196.53	35.29	177.56	107.07	35.64	35.93	36.36	89.48	161.37	389.64
<b>slack 24</b>	146.96	101.92	433.53	501.50	290.41	98.34	194.42	387.24	498.42	170.63	404.04	242.29	80.77	80.70	80.59	156.96	228.59	440.29
<b>slack 27</b>	5.42	-6.60	-93.32	-12.03	19.93	2.66	11.19	26.50	-4.07	72.76	240.25	144.50	48.13	48.32	48.60	108.15	179.96	403.61
<b>slack 40</b>	-403.69	-320.23	-1616.05	-1496.27	-761.83	-273.88	-518.37	-1016.12	-1456.37	-210.08	-233.09	-138.11	-46.20	-45.26	-43.84	-32.89	39.46	297.68

<b>slack 49</b>	668.60	501.82	2375.07	2393.99	1287.21	450.95	869.65	1716.66	2350.20	531.30	1007.68	602.69	201.07	200.04	198.49	336.84	407.81	575.54
<b>slack 59</b>	607.14	454.71	2146.35	2171.03	1169.77	409.41	790.10	1560.03	2132.04	488.80	936.52	560.21	186.89	185.97	184.59	315.63	386.67	559.54
<b>slack 61</b>	597.03	446.96	2108.72	2134.36	1150.46	402.58	777.01	1534.27	2096.15	481.81	924.82	553.23	184.56	183.66	182.31	312.15	383.20	556.92
<b>slack 70</b>	212.26	151.98	676.55	738.39	415.19	142.48	278.94	553.66	730.22	215.79	479.64	287.43	95.84	95.65	95.36	179.49	251.05	457.29
<b>slack 85</b>	339.95	249.87	1151.84	1201.65	659.19	228.79	444.23	879.08	1183.51	304.06	627.34	375.61	125.28	124.85	124.21	223.50	294.88	490.30
<b>slack 99</b>	346.25	254.70	1175.30	1224.52	671.23	233.05	452.39	895.14	1205.89	308.42	634.64	379.97	126.73	126.29	125.63	225.67	297.05	491.93
<b>slack 110</b>	344.73	253.53	1169.61	1218.99	668.33	232.03	450.42	891.27	1200.48	307.38	632.92	378.94	126.39	125.95	125.30	225.17	296.56	491.62
<b>(£/h)</b>	<b>L43</b>	<b>L44</b>	<b>L45</b>	<b>L46</b>	<b>L47</b>	<b>L48</b>	<b>L50</b>	<b>L51</b>	<b>L52</b>	<b>L53</b>	<b>L54</b>	<b>L55</b>	<b>L56</b>	<b>L57</b>	<b>L58</b>	<b>L60</b>	<b>L62</b>	<b>L63</b>
<b>slack 12</b>	101.56	112.14	417.83	180.89	287.68	183.83	138.98	139.60	139.75	186.90	93.66	562.56	749.65	139.90	140.01	666.84	666.35	95.33
<b>slack 19</b>	102.86	112.00	415.99	179.72	285.43	182.58	138.00	138.60	138.75	185.55	92.98	558.45	744.18	138.89	139.00	661.71	661.24	94.60

<b>slack 24</b>	153.58	106.39	343.88	134.19	197.29	133.47	99.83	99.68	99.64	132.72	66.31	397.70	530.38	99.61	99.58	461.42	461.54	65.90
<b>slack 27</b>	116.89	110.43	395.96	167.09	260.98	168.95	127.41	127.81	127.90	170.89	85.58	513.86	684.87	127.99	128.06	606.15	605.84	86.64
<b>slack 40</b>	10.86	122.12	546.57	262.20	445.12	271.54	207.16	209.12	209.61	281.27	141.30	849.68	1131.52	210.06	210.42	1024.61	1023.06	146.60
<b>slack 49</b>	288.82	91.53	152.02	12.98	-37.39	2.74	-1.79	-3.95	-4.48	-7.94	-4.70	-30.26	-38.83	-4.98	-5.37	-71.87	-70.17	-10.52
<b>slack 59</b>	272.87	93.27	174.56	27.23	-9.78	18.11	10.16	8.24	7.77	8.60	3.65	20.07	28.12	7.32	6.97	-9.14	-7.63	-1.53
<b>slack 61</b>	270.25	93.55	178.28	29.58	-5.24	20.64	12.13	10.25	9.79	11.33	5.02	28.36	39.15	9.35	9.01	1.19	2.67	-0.05
<b>slack 70</b>	170.53	104.55	319.93	119.05	167.96	117.13	87.13	86.73	86.63	115.14	57.43	344.22	459.24	86.54	86.46	394.76	395.08	56.35
<b>slack 85</b>	203.61	100.89	272.87	89.34	110.45	85.09	62.23	61.33	61.11	80.67	40.03	239.33	319.74	60.90	60.74	264.08	264.78	37.62
<b>slack 99</b>	205.24	100.71	270.55	87.87	107.61	83.51	61.00	60.08	59.85	78.97	39.17	234.16	312.86	59.64	59.48	257.63	258.36	36.70
<b>slack 110</b>	204.87	100.77	271.19	88.26	108.35	83.93	61.32	60.41	60.18	79.41	39.40	235.51	314.66	59.97	59.81	259.31	260.02	36.94

(£/h)	L64	L65	L66	L67	L68	L71	L72	L73	L74	L75	L76	L77	L78	L79	L80	L81	L82	L83
<b>slack 12</b>	191.20	96.33	284.07	189.85	107.38	197.92	277.86	197.92	940.21	744.46	832.70	745.13	865.37	367.55	119.32	111.78	610.07	243.69
<b>slack 19</b>	189.72	95.56	281.93	188.41	106.27	194.00	271.45	194.00	924.37	732.47	820.19	735.35	854.08	362.81	117.84	110.53	602.25	240.57
<b>slack 24</b>	131.67	65.66	198.17	132.00	62.95	40.82	21.28	40.82	305.38	264.03	331.66	353.07	412.88	177.76	60.04	61.88	296.88	118.84
<b>slack 27</b>	173.61	87.27	258.70	172.76	94.26	151.55	202.14	151.55	752.83	602.65	684.80	629.38	731.79	311.52	101.81	97.05	517.61	206.83
<b>slack 40</b>	294.89	149.74	433.69	290.60	184.77	471.63	724.91	471.63	2046.22	1581.45	1705.58	1428.12	1653.64	698.16	222.58	198.71	1155.65	461.18
<b>slack 49</b>	-22.89	-13.96	-24.84	-18.18	-52.40	-367.23	-645.20	-367.23	-1343.44	-983.74	-969.59	-665.09	-762.22	-315.10	-93.91	-67.70	-516.43	-205.39
<b>slack 59</b>	-4.71	-4.60	1.39	-0.52	-38.83	-319.18	-566.71	-319.18	-1149.32	-836.83	-816.41	-545.25	-623.90	-257.09	-75.79	-52.45	-420.70	-167.23
<b>slack 61</b>	-1.71	-3.06	5.71	2.39	-36.59	-311.28	-553.80	-311.28	-1117.37	-812.66	-791.19	-525.52	-601.13	-247.53	-72.80	-49.94	-404.94	-160.94
<b>slack 70</b>	112.35	55.70	170.30	113.23	48.53	-10.23	-62.11	-10.23	99.12	107.94	168.89	225.73	265.91	116.12	40.78	45.67	195.16	78.29

<b>slack 85</b>	74.48	36.19	115.64	76.42	20.27	-110.16	-225.31	-110.16	-304.68	-197.64	-149.81	-23.66	-21.92	-4.60	3.08	13.93	-4.05	-1.13
<b>slack 99</b>	72.61	35.23	112.95	74.61	18.88	-115.09	-233.36	-115.09	-324.60	-212.72	-165.53	-35.96	-36.12	-10.56	1.22	12.37	-13.88	-5.05
<b>slack 110</b>	73.09	35.48	113.65	75.08	19.23	-113.87	-231.38	-113.87	-319.65	-208.96	-161.60	-32.87	-32.55	-9.06	1.68	12.76	-11.41	-4.06
<b>(£/h)</b>	<b>L84</b>	<b>L86</b>	<b>L87</b>	<b>L88</b>	<b>L89</b>	<b>L90</b>	<b>L91</b>	<b>L92</b>	<b>L93</b>	<b>L94</b>	<b>L95</b>	<b>L96</b>	<b>L97</b>	<b>L98</b>	<b>L100</b>	<b>L101</b>	<b>L102</b>	<b>L103</b>
<b>slack 12</b>	121.58	121.45	121.45	484.86	121.05	121.01	120.96	725.35	181.23	362.27	483.29	362.73	180.20	359.24	361.39	241.29	181.22	240.92
<b>slack 19</b>	120.03	119.91	119.91	478.70	119.52	119.48	119.43	716.18	178.94	357.69	477.19	358.14	177.94	354.76	356.84	238.25	178.93	237.89
<b>slack 24</b>	59.48	59.51	59.51	238.29	59.61	59.62	59.63	357.91	89.50	179.05	238.67	178.94	89.75	179.79	179.27	119.42	89.51	119.51
<b>slack 27</b>	103.25	103.17	103.17	412.06	102.91	102.89	102.86	616.87	154.15	308.18	411.07	308.47	153.50	306.26	307.62	205.31	154.14	205.08
<b>slack 40</b>	229.75	229.35	229.35	914.38	228.07	227.95	227.80	1365.46	341.02	681.43	909.44	682.89	337.76	671.85	678.64	453.60	340.98	452.43
<b>slack 49</b>	-101.78	-101.33	-101.33	-402.02	-99.93	-99.80	-99.64	-596.32	-148.70	-296.74	-396.59	-298.34	-145.13	-286.22	-293.67	-197.07	-148.65	-195.78



<b>slack 59</b>	-82.80	-82.40	-82.40	-326.66	-81.15	-81.04	-80.89	-484.01	-120.66	-240.73	-321.82	-242.16	-117.48	-231.37	-238.01	-159.81	-120.62	-158.67
<b>slack 61</b>	-79.67	-79.28	-79.28	-314.25	-78.06	-77.95	-77.80	-465.51	-116.04	-231.51	-309.51	-232.91	-112.93	-222.34	-228.84	-153.68	-116.01	-152.56
<b>slack 70</b>	39.31	39.40	39.40	158.21	39.66	39.68	39.71	238.56	59.71	119.55	159.22	119.25	60.38	121.51	120.12	79.84	59.72	80.08
<b>slack 85</b>	-0.18	0.00	0.00	1.37	0.58	0.63	0.70	4.83	1.37	3.01	3.62	2.34	2.85	7.36	4.27	2.32	1.38	2.85
<b>slack 99</b>	-2.13	-1.94	-1.94	-6.37	-1.35	-1.29	-1.22	-6.70	-1.51	-2.74	-4.06	-3.42	0.01	1.73	-1.44	-1.51	-1.49	-0.96
<b>slack 110</b>	-1.64	-1.45	-1.45	-4.42	-0.86	-0.81	-0.74	-3.79	-0.79	-1.29	-2.13	-1.97	0.72	3.15	0.00	-0.54	-0.77	0.00
<b>(£/h)</b>	<b>L104</b>	<b>L105</b>	<b>L106</b>	<b>L107</b>	<b>L108</b>	<b>L109</b>	<b>L111</b>	<b>L112</b>	<b>L113</b>	<b>L114</b>	<b>L115</b>	<b>L116</b>	<b>L117</b>	<b>L118</b>				
<b>slack 12</b>	361.39	361.39	481.85	602.31	240.92	120.46	120.46	72.28	-9.69	-42.33	-84.47	107.38	0.00	432.44				
<b>slack 19</b>	356.84	356.84	475.79	594.73	237.89	118.95	118.95	71.37	-7.16	-38.78	-77.39	106.27	4.45	425.69				
<b>slack 24</b>	179.27	179.27	239.03	298.78	119.51	59.76	59.76	35.85	91.57	99.55	199.06	62.95	178.41	161.90				

<b>slack 27</b>	307.62	307.62	410.16	512.70	205.08	102.54	102.54	61.52	20.18	-0.47	-0.83	94.26	52.62	352.58				
<b>slack 40</b>	678.64	678.64	904.85	1131.07	452.43	226.21	226.21	135.73	-186.16	-289.56	-578.55	184.77	-310.92	903.78				
<b>slack 49</b>	-293.67	-293.67	-391.56	-489.45	-195.78	-97.89	-97.89	-58.73	354.68	468.16	935.69	-52.40	641.97	-540.76				
<b>slack 59</b>	-238.01	-238.01	-317.34	-396.68	-158.67	-79.34	-79.34	-47.60	323.68	424.74	848.90	-38.83	587.35	-458.04				
<b>slack 61</b>	-228.84	-228.84	-305.12	-381.40	-152.56	-76.28	-76.28	-45.77	318.58	417.59	834.63	-36.59	578.37	-444.42				
<b>slack 70</b>	120.12	120.12	160.16	200.20	80.08	40.04	40.04	24.02	124.51	145.69	291.27	48.53	236.45	74.00				
<b>slack 85</b>	4.27	4.27	5.70	7.12	2.85	1.42	1.42	0.85	188.91	235.93	471.59	20.27	349.91	-98.09				
<b>slack 99</b>	-1.44	-1.44	-1.92	-2.40	-0.96	-0.48	-0.48	-0.29	192.09	240.38	480.49	18.88	355.51	-106.58				
<b>slack 110</b>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	191.33	239.30	478.34	19.23	354.17	-104.46				

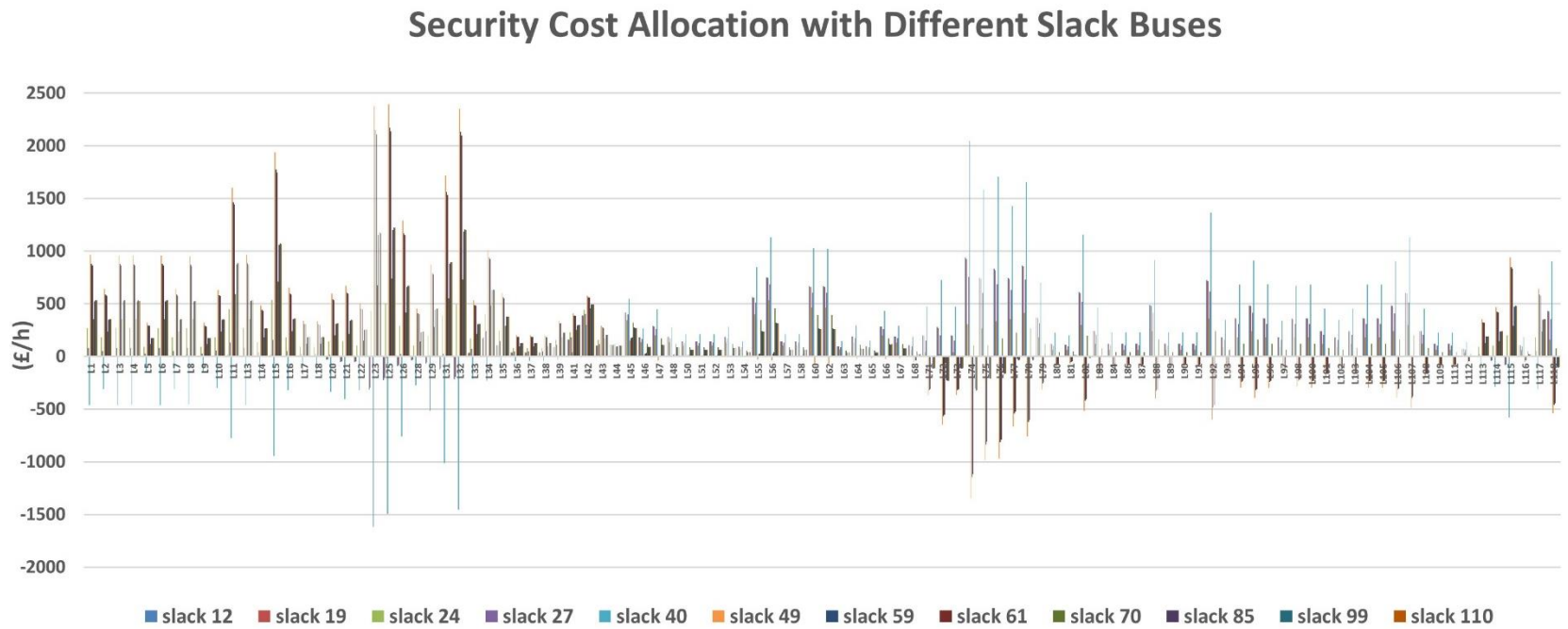


Figure 6.19 Security cost allocated to each load with different slack buses in the 118-bus test system

Since there are totally 104 loads in the system, it is not practical to use only one graph to contain such large density of security cost allocation results. Hence the Figure 6.19 is divided into three sections which are Figure 6.20 contains results from load 1 to load 41, Figure 6.21 contains results from load 42 to load 81 and Figure 6.22 contains results from load 82 to load 118. Those three figures are shown successively as follows:

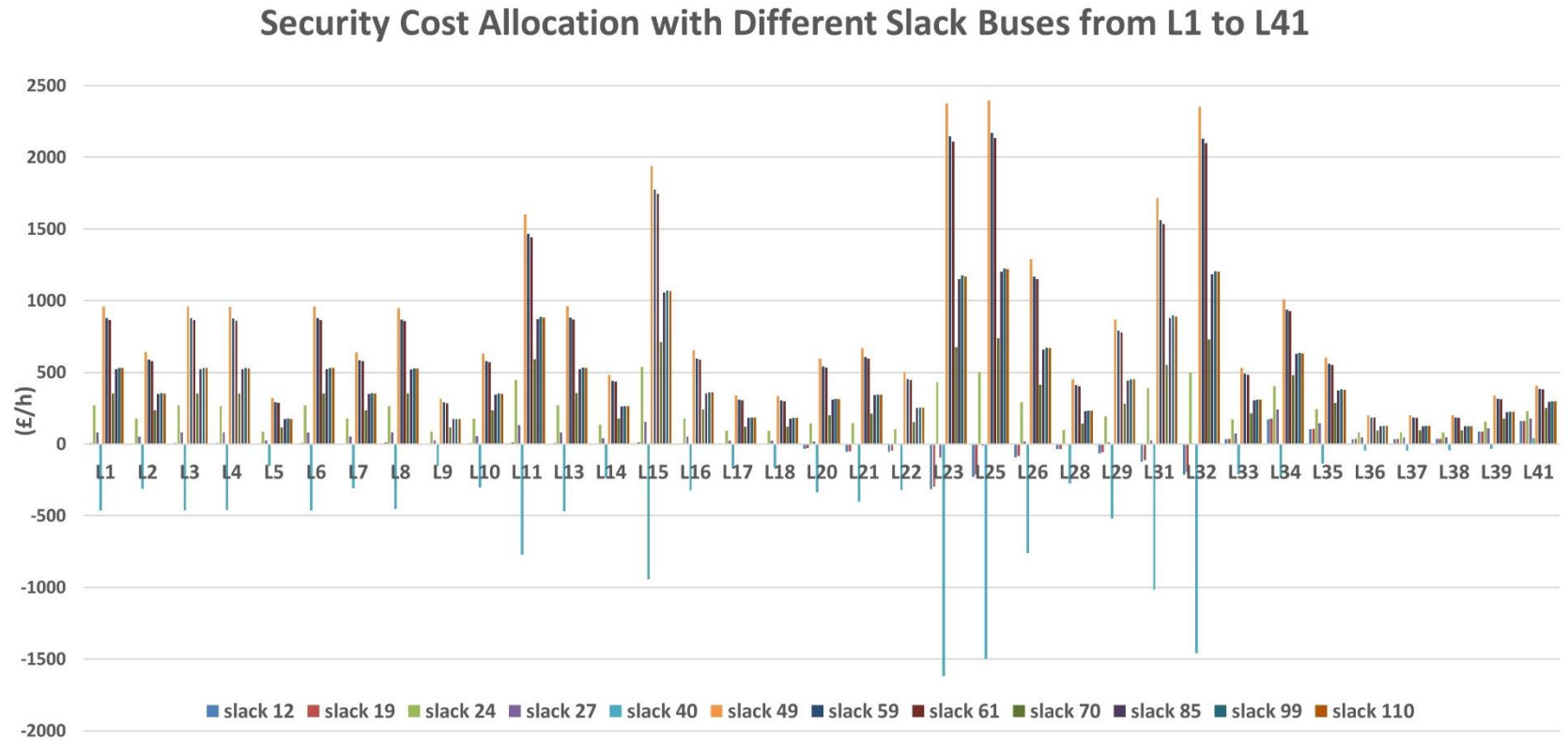


Figure 6.20 Security cost allocated to each load with different slack buses in the 118-bus test system from L1 to L41

### Security Cost Allocation with Different Slack Buses from L42 to L81

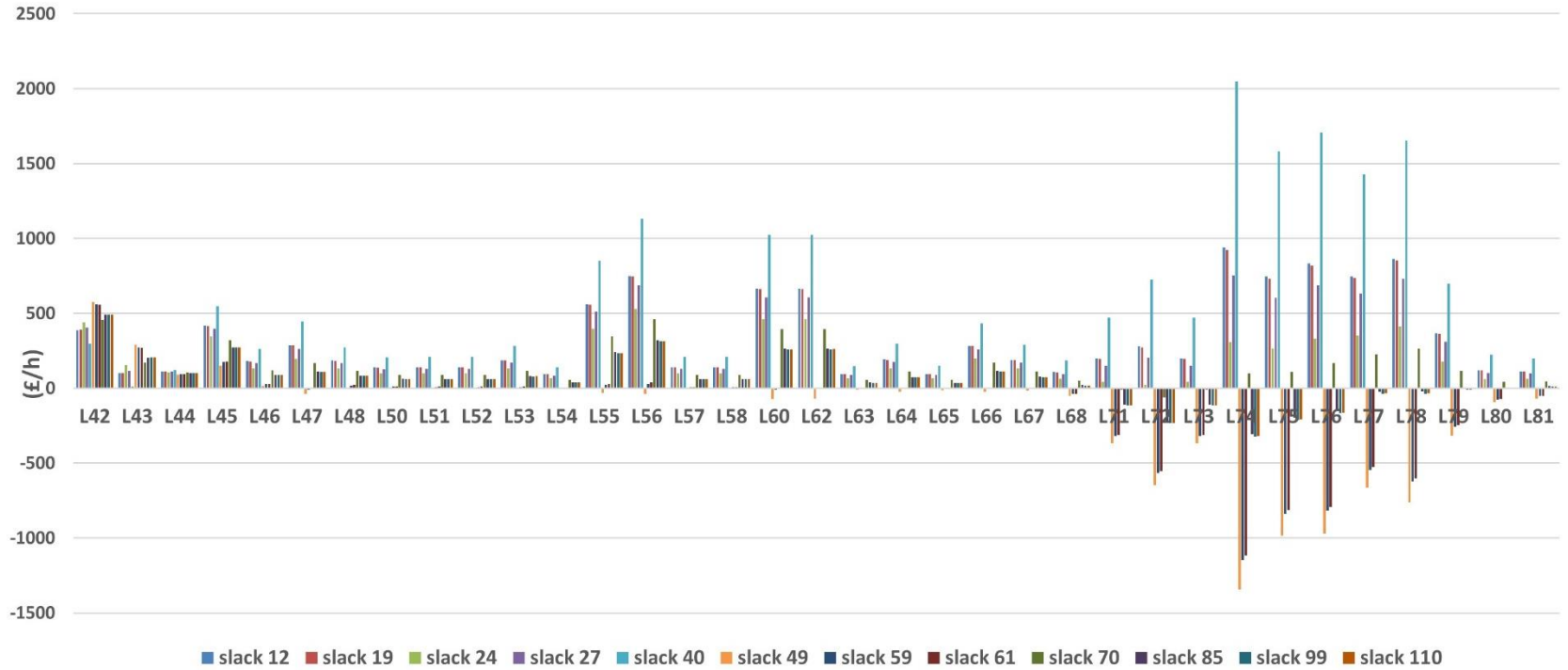


Figure 6.21 Security cost allocated to each load with different slack buses in in the 118-bus test system from L42 to L81

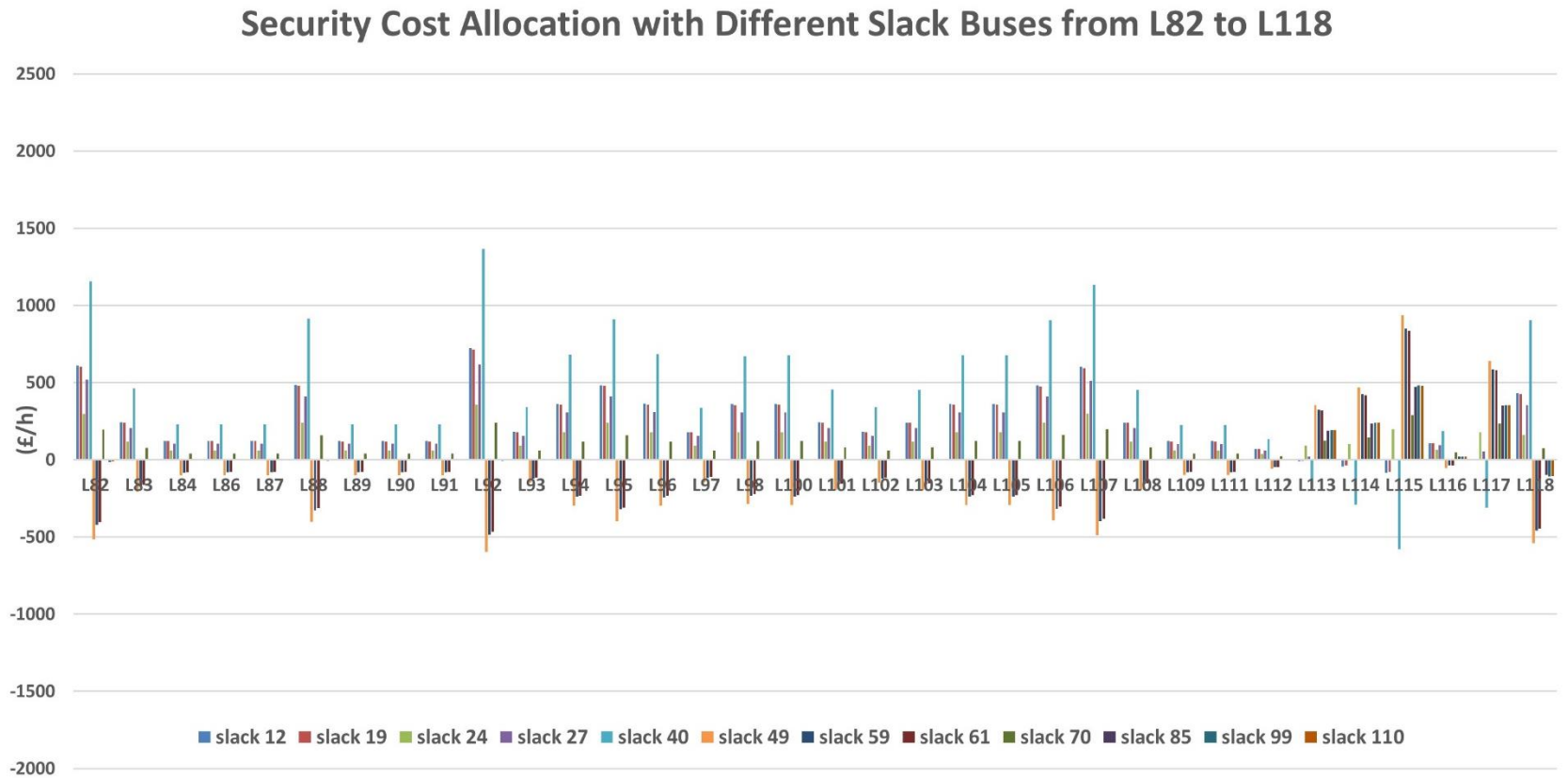
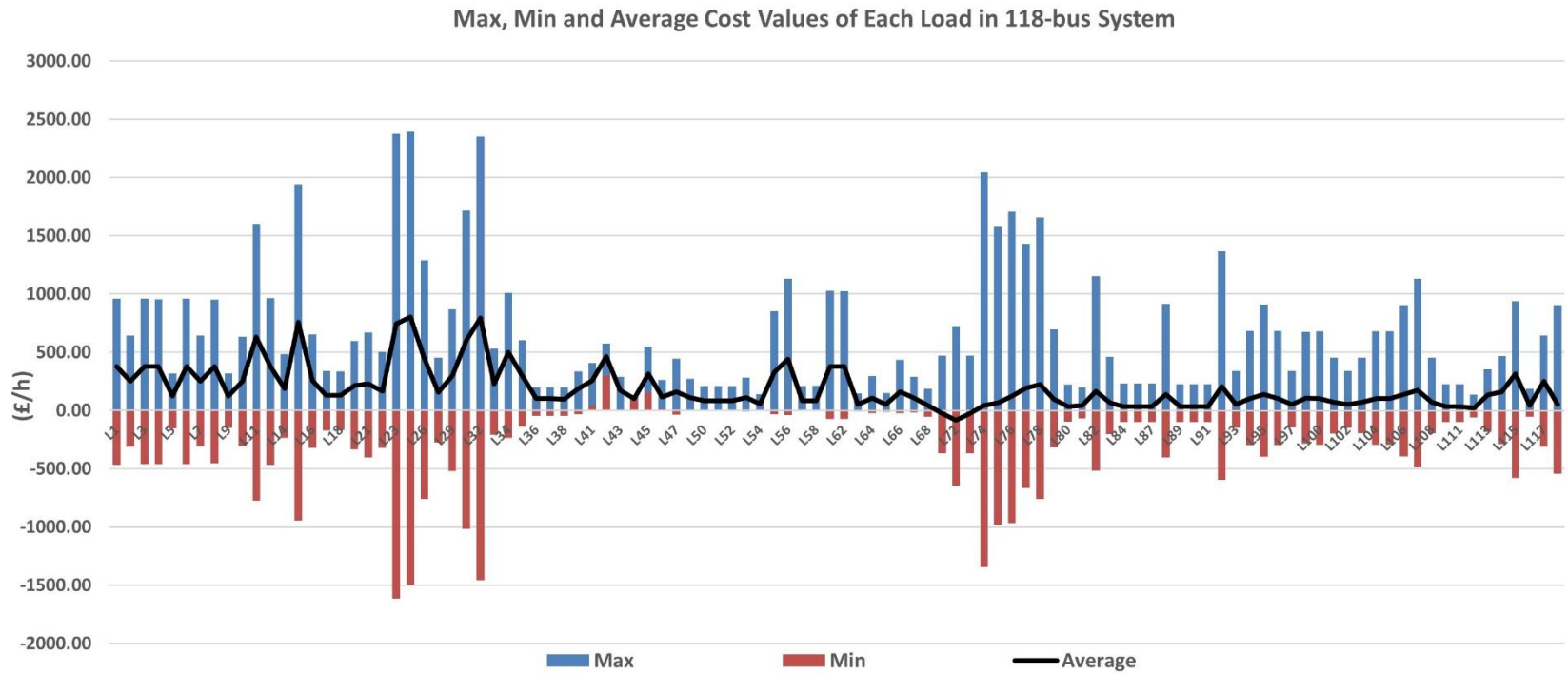


Figure 6.22 Security cost allocated to each load with different slack buses in in the 118-bus test system from L82 to L118



**Figure 6.23 Maximum, minimum and average security cost values of each load in 118-bus system**



Figure 6.18 shows the security costs from the experimental results in chapter 5 as the basic security cost. Due to there are 12 generators in the system, slack bus selection have twelve possibilities, which is so say that the whole security cost allocation results should be based on twelve groups of PTDF factors respectively. After observed the contents from Figure 6.20, 6.21 and 6.22 respectively, the following regular patterns are found. Security cost allocations with bus 12, 19 and 27 as the slack bus respectively have similar results. Selecting bus 49, 59 and 61 respectively as the slack bus causes similar allocation outcomes. Selections of slack bus 89, 99 and 110 have similar results for security cost allocation. While selections of slack bus 24, 40 and 70 have their own characteristic respectively to allocate the security cost to the loads. Slack bus 40 as one of the unique cases has very different idea for security cost allocation, which can be observed in the above three graphs. It should be noted that many loads are allocated negative values of security cost, which means such loads have negative contributions on power flow of congested line, in other words, they are helping to relieve the congestion. As mentioned in section 6.3.2, the PTDF-based method is trying to detect each bus's real contribution to one particular transmission line's power flow no matter it is positive or negative. Therefore, based on these real contributions of loads, the system operator could take measures to do appropriate security cost allocation to loads. The system operator should avoid allocating security cost to such loads who have negative contributions to the power flow of the congested line and even reward them. The 118-bus case is a convincing example to illustrate that the PTDF based method could result in different allocation of security cost to load and can be highly dependent on slack bus position.

Figure 6.23 shows the maximum, the minimum and the average security cost values of each load in 118-bus system under different slack bus selections. From the figure, it can be found that load 23 has the biggest range between the maximum value at £2375/h and the minimum value at -£1616/h. The range is £3991/h. Load 44 has the smallest range between the maximum value at £122/h and the minimum value at £91/h. The range is

£31/h. The black curve shows the average security cost values of each load under different slack bus selections. From the curve, it can be noted that load 25 has the highest average security cost allocation value at £803/h and load 72 has the lowest value at -£85/h.

#### **6.4.5.2. Comparison between Pro Rata Method and PTDF Method in the 118-bus Test System**

Using the results from the PTDF method of case of slack bus 12 to contrast with the pro rata based method. The comparison is obtained in Table 6.8 and Figure 6.24 respectively as follows:

**Table 6.8 Comparison between Two Methods on Security Cost Allocation in the 118-bus Test System**

(£/h)	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L13	L14
<b>Pro rata</b>	215.08	143.38	215.08	215.08	71.69	215.08	143.38	215.08	71.69	143.38	358.46	215.08	107.54
<b>slack bus 12</b>	0.74	0.19	0.93	1.91	0.69	1.04	0.43	3.88	1.29	2.58	1.04	0.06	-0.25
	L15	L16	L17	L18	L20	L21	L22	L23	L25	L26	L28	L29	L31
<b>Pro rata</b>	430.15	143.38	71.69	71.69	107.54	107.54	71.69	286.77	358.46	215.08	71.69	143.38	286.77
<b>slack bus 12</b>	-3.70	-3.02	-4.77	-3.52	-32.63	-53.81	-52.01	-313.79	-226.92	-93.25	-37.38	-65.48	-124.44
	L32	L33	L34	L35	L36	L37	L38	L39	L41	L42	L43	L44	L45
<b>Pro rata</b>	358.46	143.38	358.46	215.08	71.69	71.69	71.69	143.38	215.08	430.15	143.38	107.54	358.46
<b>slack bus 12</b>	-214.33	31.83	171.76	103.61	34.48	34.78	35.23	87.75	159.65	388.34	101.56	112.14	417.83
	L46	L47	L48	L50	L51	L52	L53	L54	L55	L56	L57	L58	L60
<b>Pro rata</b>	143.38	215.08	143.38	107.54	107.54	107.54	143.38	71.69	430.15	573.54	107.54	107.54	501.85
<b>slack bus 12</b>	180.89	287.68	183.83	138.98	139.60	139.75	186.90	93.66	562.56	749.65	139.90	140.01	666.84
	L62	L63	L64	L65	L66	L67	L68	L71	L72	L73	L74	L75	L76
<b>Pro rata</b>	501.85	71.69	143.38	71.69	215.08	143.38	71.69	71.69	71.69	71.69	430.15	358.46	430.15
<b>slack bus 12</b>	666.35	95.33	191.20	96.33	284.07	189.85	107.38	197.92	277.86	197.92	940.21	744.46	832.70

	<b>L77</b>	<b>L78</b>	<b>L79</b>	<b>L80</b>	<b>L81</b>	<b>L82</b>	<b>L83</b>	<b>L84</b>	<b>L86</b>	<b>L87</b>	<b>L88</b>	<b>L89</b>	<b>L90</b>
<b>Pro rata</b>	430.15	501.85	215.08	71.69	71.69	358.46	143.38	71.69	71.69	71.69	286.77	71.69	71.69
<b>slack bus 12</b>	745.13	865.37	367.55	119.32	111.78	610.07	243.69	121.58	121.45	121.45	484.86	121.05	121.01
	<b>L91</b>	<b>L92</b>	<b>L93</b>	<b>L94</b>	<b>L95</b>	<b>L96</b>	<b>L97</b>	<b>L98</b>	<b>L100</b>	<b>L101</b>	<b>L102</b>	<b>L103</b>	<b>L104</b>
<b>Pro rata</b>	71.69	430.15	107.54	215.08	286.77	215.08	107.54	215.08	215.08	143.38	107.54	143.38	215.08
<b>slack bus 12</b>	120.96	725.35	181.23	362.27	483.29	362.73	180.20	359.24	361.39	241.29	181.22	240.92	361.39
	<b>L105</b>	<b>L106</b>	<b>L107</b>	<b>L108</b>	<b>L109</b>	<b>L111</b>	<b>L112</b>	<b>L113</b>	<b>L114</b>	<b>L115</b>	<b>L116</b>	<b>L117</b>	<b>L118</b>
<b>Pro rata</b>	215.08	286.77	358.46	143.38	71.69	71.69	43.02	71.69	71.69	143.38	71.69	143.38	215.08
<b>slack bus 12</b>	361.39	481.85	602.31	240.92	120.46	120.46	72.28	-9.69	-42.33	-84.47	107.38	0.00	432.44

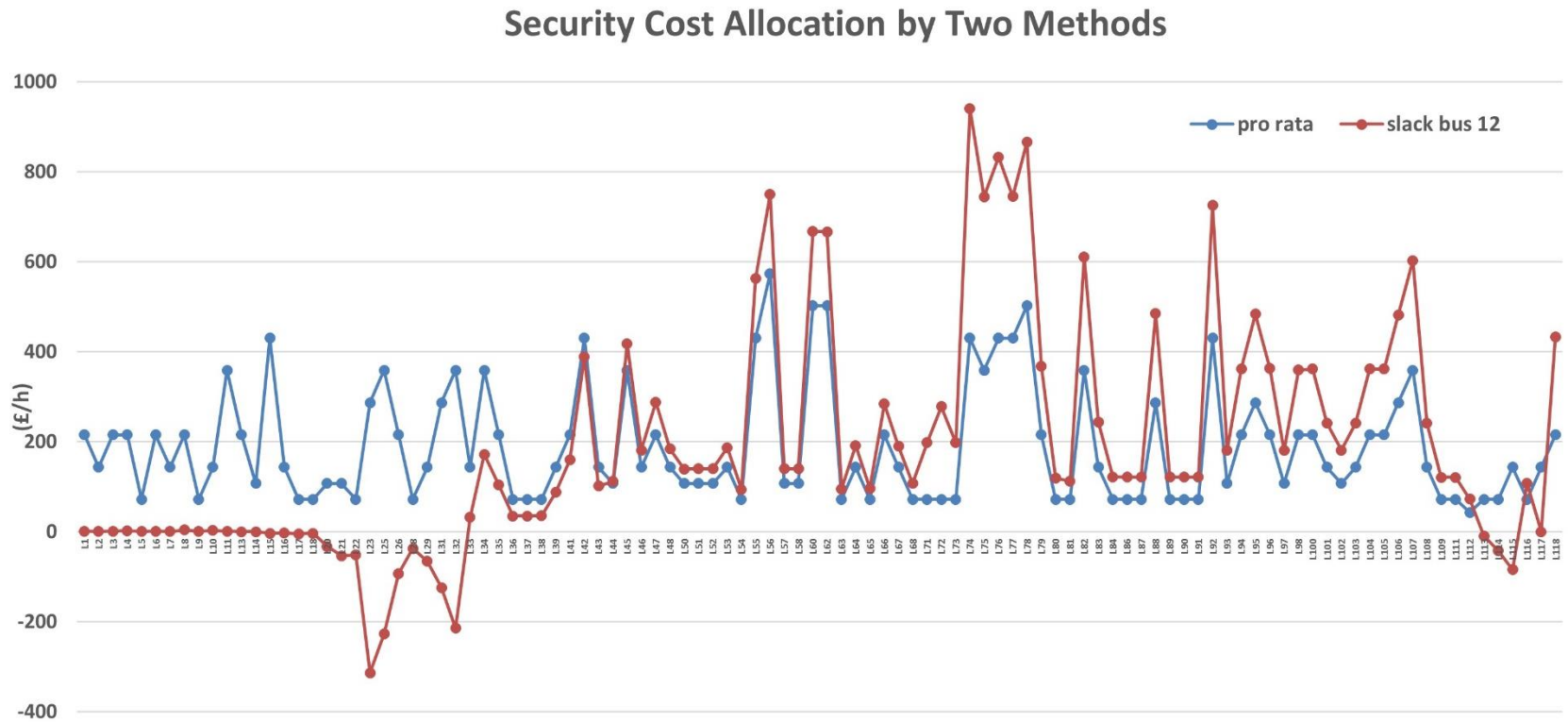


Figure 6.24 Comparison between two methods on security cost allocation in the 118-bus test system

Figure 6.24 indicates the comparison of security cost allocation results between pro rata method and PTDF method. The curve of the pro rata method varies between in the vertical coordinate interval between £ 0/h to £ 600/h. Before load 33, from load 1 to load 18, the PTDF method considers these loads have no contribution to the congested line 23-24, the security cost allocated to them are nearly zero. From load 20 to load 32, the PTDF method treats these loads have negative contributions to the power flow of congested transmission line so their allocated costs are negative values. Based on the same reason, load 113 to load 115 have negative security cost allocations. Beginning from load 33 to load 118, two curves have the similar track but the PTDF curve is higher than the pro rata curve all alone except load 113, 114, 115 and 117. The slack bus 12 curve is apparently higher than the pro rata curve at load 56, 60, 62, 74, 75, 76, 77, 78, 82, 88, 92 and 107. It is because the above loads have higher contribution to the congestion occurs on the line 23-24 hence they shoulder more security cost than other loads. The only difference of judgement on security cost allocation between two method is that the pro rata method is based on the power consumption ratio of loads in the system to allocate the security cost, which is so say that the load with higher active power consumption will be allocated more security cost by the system operator. In the pro rata method, even the load who has a least consumption as long as it withdraws positive volume of power from the system, it will definitely shoulder the cost of congestion. While the PTDF-based method digs in and tries to find out the power flow contribution each load makes to the congested transmission line's power flow. The load with higher power flow contribution to the congested transmission line will be allocated more security cost by the system operator. The negative allocated security costs to the loads in the PTDF-based method is actually to indicate the absolute power flow contribution of load to the congested transmission line, which can inform that such loads help to relieve the congestion. The consumptions of power at such buses are actually decreasing the power flow of the congested transmission line. In theory, negative security cost allocation means that the system operator should reward the loads who helps to relieve the congestion. But it depends on the specific mechanism in each electricity market. The PTDF-based method is only to give a real experimental results to the system operator.

#### 6.4.6. Discussion

By contrasting the PTDF based security cost allocation method with the pro rata based method, no matter in which test systems it is proved that PTDF method is able to calculate the power flow contribution of each load to the congested line. Even though the comparison in 14-bus test system is not distinct, in other three test systems, the comparisons show that the PTDF method allocates higher security cost to the loads who have more contribution to the transmission congestion, at the meantime, allocates lower security cost to the loads who have little impact on the congestion, which seems like fairer and more convincing than the pro rata method.

Nevertheless, through the testing results from four cases, the common drawback of the PTDF method is exposed, which is the complexity and inaccuracy on security cost allocation. It is discovered that the PTDF based method is dependent on slack bus selection, which means choosing different generator bus as the slack bus leads to different outcomes. In each test case, every generator bus is selected as the slack bus respectively and the corresponding experimental results on security cost allocation are obtained. The results indicate that although some of outcomes with different slack buses are similar, there are always one or some loads that have apparently different outcomes on security cost allocation. Consequently, when utilize the PTDF method, the system operator should consider all the slack bus selections in order to guarantee the preciseness of the method, which causes the complexity of the process, for example, in 118-bus case, the system operator should conduct 12 times for calculation to see all the possible results. However, if the system operator only considers one slack bus choice, the result on security cost allocation is only based on the slack bus position.

## 6.5. Summary

In this chapter, conventional security cost allocation schemes are described and discussed. The mathematic model of the proposed allocation method with PTDF factor is given. Experimental results about security cost allocation of four modified IEEE test systems are represented through tables and figures. The comparisons between the pro rata method and the PTDF method by four test systems are obtained. The advantage of the PTDF method is that it can calculate the power flow contribution of each load to the congested transmission line so as to provide a fair way to allocate the security cost. The disadvantage of the PTDF method is that it is dependent on slack bus selection.



# Chapter 7

## Security Cost Allocation in Deregulated Electricity Market with Bus Impedance Matrix Sensitivity

### 7.1. Introduction

In chapter 6, the security cost allocation method associated with the PTDF is demonstrated in four modified IEEE test systems respectively. To provide an alternative security cost allocation method, the bus impedance matrix sensitivity will be introduced, which is indicated and discussed in chapter 5. As another power flow sensitivity factor similar with the PTDF, the ZS factor is also able to seek out the power flow contribution of each load in the system to the congested transmission line. The security cost allocation with bus impedance matrix sensitivity is based on the power flow contribution of each load. To find out the advantages and disadvantages of the ZS based allocation method, the result of security cost allocated to each load in the system by three methods which are the pro rata method, the PTDF method and the ZS method will be compared and discussed in the case studies.

In this chapter, the mathematic model of the proposed security cost allocation method with the ZS will be expressed in section 7.2. Section 7.3 indicates the case studies including experimental results by the ZS method in four modified IEEE test systems including: an IEEE-14 bus system, an IEEE-30 bus system, an IEEE-57 bus system and an IEEE-118 bus system. Then combined with the results from chapter 6, the comparison among three security cost allocation methods which are pro rata method, the PTDF method with the first generator bus as the slack bus and the ZS method with the first generator bus as the slack bus will be obtained. Section 7.4 is the summary.

## 7.2. Proposed Security Cost Allocation Method with Bus Impedance Matrix Sensitivity

The proposed security cost allocation method with the ZS factor will be described in details including the mechanism and the mathematical model of the method.

### 7.2.1. Mechanism of the Proposed Security Cost Allocation Method with Bus Impedance Matrix Sensitivity

The mechanism of the proposed security cost allocation method with the ZS is almost as same as the PTDF based method, which is described in section 6.3.1. But this time it is with the use of the ZS factor to calculate the power flow contribution percentage of each load to the congested transmission line and the security cost allocation is conducted according to the above contribution percentage.

### 7.2.2. Mathematical Model of the Proposed Security Cost Allocation Method with Bus Impedance Matrix Sensitivity

The following equations are the mathematical model of the proposed security cost allocation method with bus impedance matrix sensitivity:

Equation (7.1) expressed the total MW power flow of the congested line is due to generators contribution plus loads contribution.

$$P_{j-k} = P_{j-k}^G + P_{j-k}^L \quad (7.1)$$

Where

$P_{j-k}$  : The power flow MW value of the target transmission line  $j-k$

$P_{j-k}^G$  : The power flow MW value on line  $j-k$  associated with all the generators

$P_{j-k}^L$  : The power flow MW value on line  $j-k$  associated with all the loads

Equation (7.2) indicates that the total MW contribution of generators is the sum of the power flow contribution of each generator.

$$P_{j-k}^G = \sum_{i=1}^{N_G} ZS_{G_i}^{j-k} \cdot P_{G_i} \quad (7.2)$$

Where

$ZS_{G_i}^{j-k}$  : The ZS of the line  $j-k$  respect to generator bus  $i$

$P_{G_i}$  : The MW power energy injected by generator  $i$ .

Equation (7.3) shows that the total MW contribution of loads is the sum of the power flow contribution of each load.

$$P_{j-k}^L = \sum_{i=1}^{N_L} ZS_{L_i}^{j-k} \cdot P_{L_i} \quad (7.3)$$

Where

$ZS_{L_i}^{j-k}$  : The ZS of the line  $j-k$  respect to load bus  $i$

$P_{L_i}$  : The MW power energy withdrawn by load  $i$ .

Equation (7.4) shows that the MW power flow contribution of each load is equals to the ZS of the load  $i$  times the power energy extraction of the load  $i$ .

$$P_{j-k}^{L_i} = ZS_{L_i}^{j-k} \cdot P_{L_i} \quad (7.4)$$

Where

$P_{j-k}^{L_i}$  : The power flow MW value associated with load  $i$ .

Equation (7.5) calculates of the power flow contribution percentage of each load.

$$Ctb_{L_i} = \frac{P_{j-k}^{L_i}}{P_{j-k}^L} \times 100\% = \frac{ZS_{L_i}^{j-k} \cdot P_{L_i}}{\sum_{i=1}^{N_L} ZS_{L_i}^{j-k} \cdot P_{L_i}} \times 100\% \quad (7.5)$$

Where

$Ctb_{L_i}$  : The power flow contribution percentage of load  $i$  to the congested line

Equation (7.6) finds out the security cost allocated to each load.

$$CA_{L_i} = Ctb_{L_i} \times SC_{total} \quad (7.6)$$

Where

$CA_{L_i}$  : The allocated security cost to the load  $i$

$SC_{total}$  : The total security cost due to congestion management.

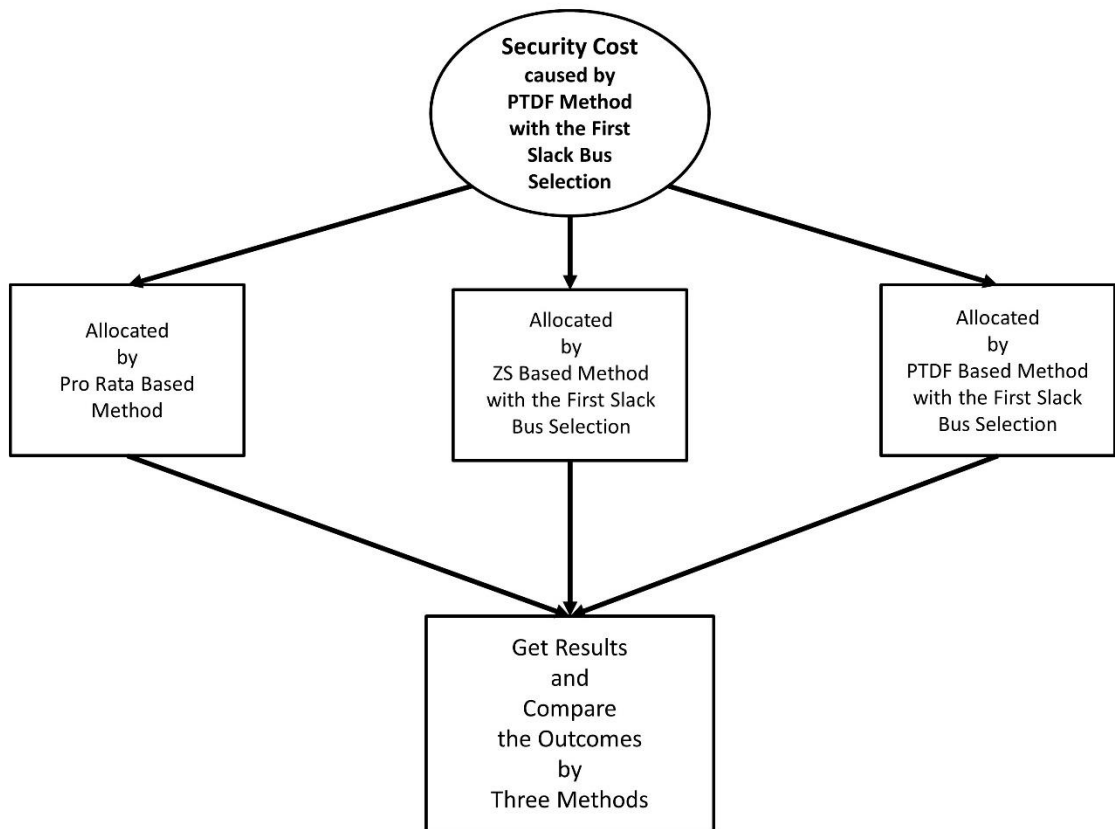
### 7.3. Case Studies

Similar with previous three chapters, the case studies in this chapter use four modified IEEE bus systems including an IEEE-14 bus system, an IEEE-30 bus system, an IEEE-57 bus system and an IEEE-118 bus system to test the performance of the proposed security cost allocation method with the ZS. The comparison between the ZS method and the other two methods including the pro rata method and the PTDF method will be obtained and discussed.

#### 7.3.1. Test Description

Based on the contents in section 5.5, the PTDF based method and ZS based method are both utilized to re-dispatch generation to relieve the transmission congestion. The results indicate that the security costs caused by two congestion management schemes in all four test systems are all apparently different. However, in order to make the comparison of security cost allocation by above two methods more direct viewing, the security cost they are aim to allocate in these case studies should be assumed at a same amount. The security cost of the first slack bus selection in each test system of the section 6.4 will be chosen as the security cost amount in each corresponding test system of these case studies.

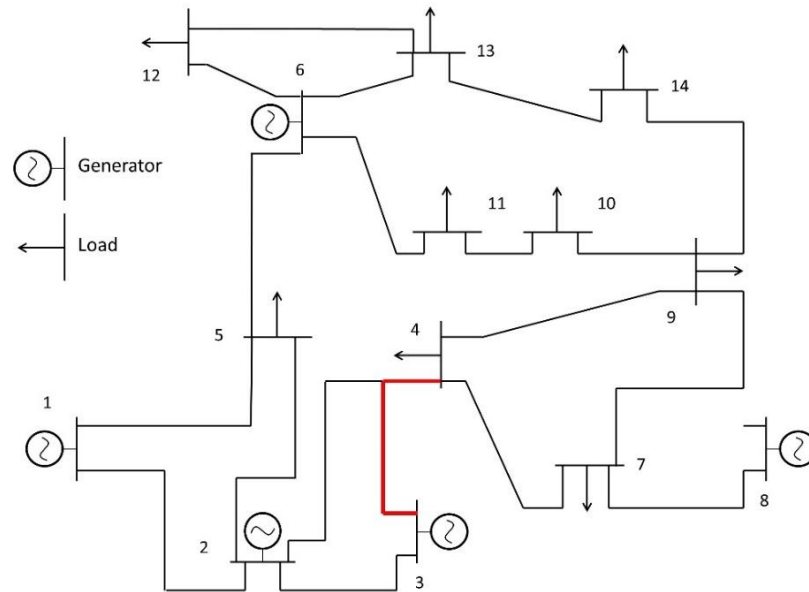
It should be noted that since the ZS method is also dependent on the slack bus selection, here the ZS method only considers the first generator bus as the slack bus, which is same as the slack bus selection in the PTDF method. The pro rata method which is mentioned in section 6.4.1 will continue to be used to provide the third set of comparable result. Additionally, rule and environment of the test in this chapter are same with the chapter 4 and the details can be found in section 4.5.1. The test description is visualized by Figure 7.1 as follows:



**Figure 7.1** Test description for chapter 7

### 7.3.2. Modified IEEE-14 Bus System

The modified IEEE-14 bus test system is exactly same with the system in section 6.4.2, which is showed in Figure 7.2 as follow:

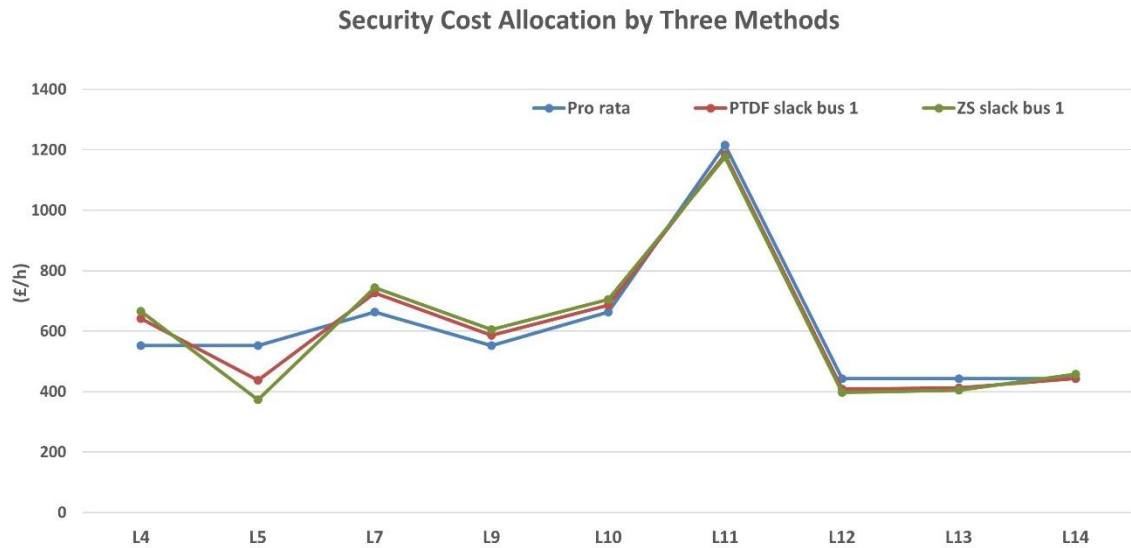


**Figure 7.2 The modified IEEE-14 bus system**

The parameters of the generators and loads are set up the same, which are displayed in Table 4.1. Transmission congestion occurs on the transmission branch 3-4. There are 5 generators and 9 loads in the system. The security cost amount is picked up from the experimental result shown in Figure 6.3, which is £5526.62/h. Then implement three security cost allocation methods respectively including the pro rata method, the PTDF method with the first slack bus selection and the ZS method with the first slack bus selection to allocate the above security cost to the loads in the system. The comparison will be illustrated in Table 7.1 and Figure 7.3 as follows:

**Table 7.1 Comparison between Three Methods on Security Cost Allocation in the 14-bus Test System**

(£/h)	L4	L5	L7	L9	L10	L11	L12	L13	L14
<b>Pro rata</b>	552.66	552.66	663.19	552.66	663.19	1215.86	442.13	442.13	442.13
<b>PTDF slack bus 1</b>	642.13	437.46	726.49	585.66	685.43	1184.27	408.55	412.57	444.06
<b>ZS slack bus 1</b>	665.08	372.62	744.06	605.45	704.43	1175.73	396.82	405.21	457.23

**Figure 7.3 Comparison between three methods on security cost allocation in the 14-bus test system**

From Figure 7.3, it can be obtained that security cost allocation results by three methods are very close since the tendencies of the above three curves are similar. By contrast with the curve of pro rata method, curves of the PTDF method and the ZS method are even closer. The reason behind this can be found out through the details of the calculation process of load's contributions by three methods. The pro rata method calculates each load's contribution based on their active power consumptions, which uses each load's MW consumption divided by the demand-side total MW. The other two

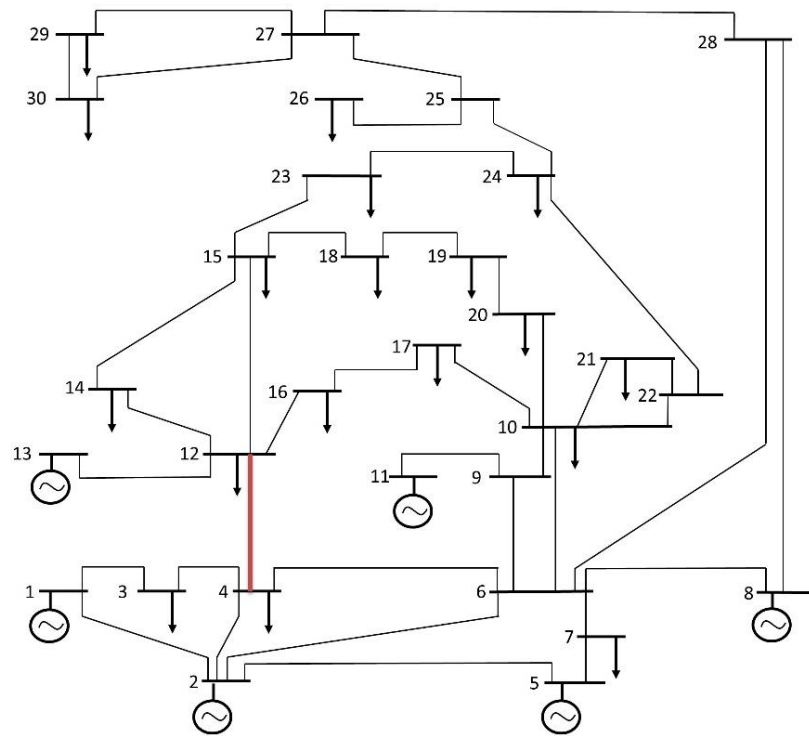


sensitivity-based methods calculate each load's contribution according to the sensitivity-based contribution of each load, which is calculated by the sensitivity factor of each load times its MW consumption and then divided by the total sensitivity-based contributions of loads. It should be noted that the only difference is the sensitivity factor used, one is the PTDF and the other is the ZS. Take load 14 as an example since the three methods calculated the most similar results on it: It can be found that for the contribution of load 14, the pro rata method calculation is 8%, the PTDF-based method calculation is 8.03% and the ZS-based method calculation is 8.27%.

But they still have differences in details. For example, ZS method allocates the slightly higher cost to load 4 and allocates lower cost to load 5 compared with other two methods. But the difference is so tiny, it is not convincing to conclude anything. More comparisons among three methods will be obtained in the next three test systems.

### 7.3.3. Modified IEEE-30 Bus System

The modified IEEE-30 bus test system is as same as the system in section 6.4.3, which is showed in Figure 7.4 as follow:

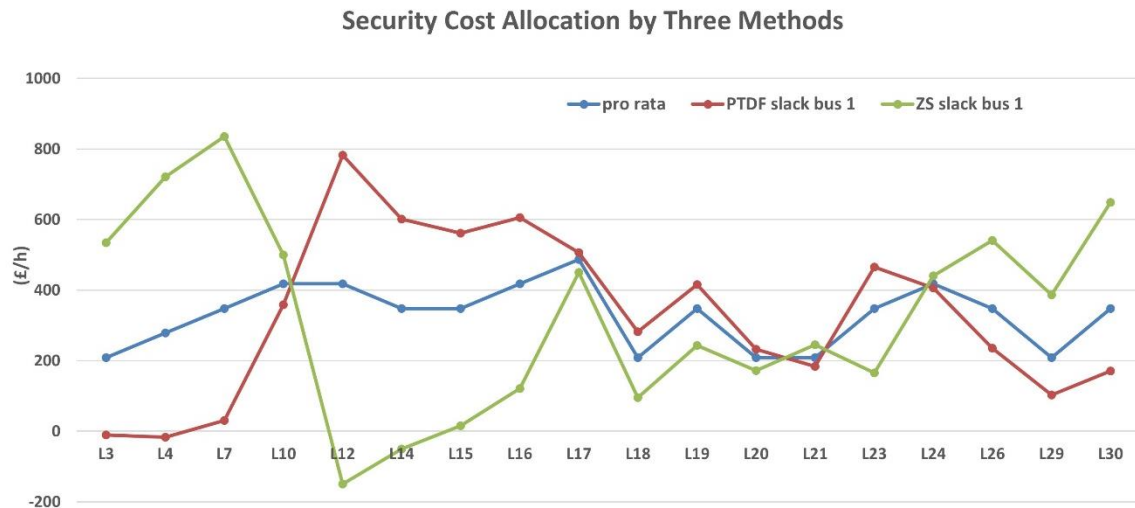


**Figure 7.4 The modified IEEE-30 bus system**

The parameters of the generators and loads are set up as same as the former two chapters, which are displayed in Table 4.4 and Table 4.5. There are 6 generators and 18 loads in the system. Transmission congestion occurred on the transmission branch 4-12. The security cost amount is picked up from the experimental result in Figure 6.8, which is £5914.08/h. Then implement three security cost allocation methods respectively including the pro rata method, the PTDF method with the first slack bus selection and the ZS method with the first slack bus selection to allocate the above security cost to the loads in the system. The comparison will be illustrated in Table 7.2 and Figure 7.5 as follows:

**Table 7.2 Comparison between Three Methods on Security Cost Allocation in the 30-bus Test System**

(£/h)	<b>L3</b>	<b>L4</b>	<b>L7</b>	<b>L10</b>	<b>L12</b>	<b>L14</b>
<b>Pro rata</b>	208.73	278.31	347.89	417.46	417.46	347.89
<b>PTDF slack bus 1</b>	-10.03	-16.44	30.30	359.03	782.51	601.21
<b>ZS slack bus 1</b>	533.95	720.90	835.60	500.32	-149.74	-50.66
	<b>L15</b>	<b>L16</b>	<b>L17</b>	<b>L18</b>	<b>L19</b>	<b>L20</b>
<b>Pro rata</b>	347.89	417.46	487.04	208.73	347.89	208.73
<b>PTDF slack bus 1</b>	561.51	605.55	506.67	281.85	415.51	232.17
<b>ZS slack bus 1</b>	15.60	120.75	450.05	95.07	243.03	171.53
	<b>L21</b>	<b>L23</b>	<b>L24</b>	<b>L26</b>	<b>L29</b>	<b>L30</b>
<b>Pro rata</b>	208.73	347.89	417.46	347.89	208.73	347.89
<b>PTDF slack bus 1</b>	183.93	465.85	405.58	235.65	102.47	170.78
<b>ZS slack bus 1</b>	245.70	165.39	440.80	541.26	386.21	648.33



**Figure 7.5 Comparison between three methods on security cost allocation in the 30-bus test system**

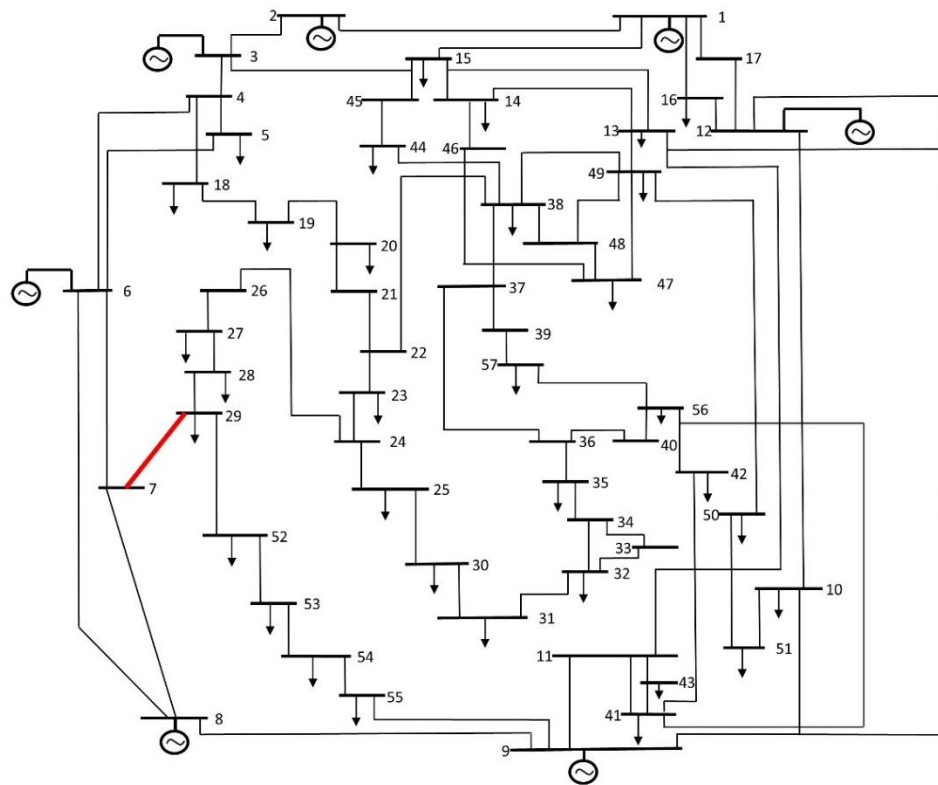
From Figure 7.5, it can be seen that three curves are apparently different, which means three methods have different ideas for security cost allocation. Among all the loads, except load 10, 17, 20, 21 and 24, the other loads shoulder distinctly different security costs in three allocation schemes. For example, load 3, 4 and 7 in the PTDF method are considered as the low impact loads to the congested line so they are allocated very few security cost respectively while in the ZS method they are considered as the high impact loads so they are allocated more security cost respectively. For load 12 to load 16, the security cost allocated to them is quite higher than other loads in the PTDF method especially the load 12 which is considered as the load who has the most power flow contribution to the congested line so it is allocated the most security cost. However, these above loads in the ZS method have much lower cost allocation especially the load 12 who is regarded as the load with the smallest impact on the congested line. Additionally, load 26, 29 and 30 are allocated more security cost in the ZS method than the PTDF method.

The reason to explain the above results can be found out from the details of the calculation process of load's contributions by three methods. Here two typical examples

are taken, one is load 12, where the most obvious difference among three methods occurs, the other one is load 24, where the smallest difference occurs. It can be found that for the contribution of load 12, the pro rata method calculation is 7.06%, the PTDF-based method calculation is 13.23% and the ZS-based method calculation is -2.5%. For the contribution of load 24, the pro rata method calculation is 7.06%, the PTDF-based method calculation is 6.86% and the ZS-based method calculation is 7.45%.

### 7.3.4. Modified IEEE-57 Bus System

The modified IEEE-57 bus test system is as same as the system in section 6.4.4, which is showed in Figure 7.6 as follow:



**Figure 7.6** The modified IEEE-57 bus system

The parameters of the generators and loads are set up the same with the previous two chapters, which are displayed in Table 4.8 and Table 4.9. There are 7 generators and totally 35 loads in the system. Transmission congestion occurred on the transmission branch 7-29. The security cost amount is picked up from the experimental result shown in Figure 6.13, which is £16056.25/h.

Then implement three security cost allocation methods respectively including the pro rata method, the PTDF method with the first slack bus selection and the ZS method with the first slack bus selection to allocate the above security cost to the loads in the system. The comparison between three methods on security cost allocation will be shown in Table 7.3 and Figure 7.7 as follows:

**Table 7.3 Comparison between Three Methods on Security Cost Allocation in the 57-bus Test System**

(£/h)	L5	L10	L13	L14	L15	L16	L17
<b>Pro rata</b>	770.70	385.35	256.90	256.90	513.80	256.90	642.25
<b>PTDF slack bus 1</b>	-339.84	21.61	19.71	27.63	23.70	8.60	11.29
<b>ZS slack bus 1</b>	1268.35	551.57	366.19	365.69	739.42	363.85	913.16
	L18	L19	L20	L23	L25	L27	L28
<b>Pro rata</b>	770.70	385.35	770.70	385.35	642.25	385.35	256.90
<b>PTDF slack bus 1</b>	-99.07	54.14	239.64	293.39	1215.32	1579.82	1205.49
<b>ZS slack bus 1</b>	1205.92	561.97	1055.82	446.12	389.13	-246.56	-248.62

	<b>L29</b>	<b>L30</b>	<b>L31</b>	<b>L32</b>	<b>L33</b>	<b>L35</b>	<b>L38</b>
<b>Pro rata</b>	642.25	385.35	385.35	256.90	385.35	642.25	256.90
<b>PTDF slack bus 1</b>	3247.97	687.52	585.00	286.17	429.25	369.15	133.37
<b>ZS slack bus 1</b>	-748.84	260.34	322.38	267.65	402.26	831.99	328.22
	<b>L41</b>	<b>L42</b>	<b>L43</b>	<b>L44</b>	<b>L47</b>	<b>L49</b>	<b>L50</b>
<b>Pro rata</b>	513.80	385.35	385.35	256.90	642.25	256.90	385.35
<b>PTDF slack bus 1</b>	82.73	84.69	28.03	108.18	227.08	78.13	87.66
<b>ZS slack bus 1</b>	737.08	560.54	556.39	337.76	866.79	351.10	544.85
	<b>L51</b>	<b>L52</b>	<b>L53</b>	<b>L54</b>	<b>L55</b>	<b>L56</b>	<b>L57</b>
<b>Pro rata</b>	385.35	513.80	385.35	642.25	642.25	385.35	642.25
<b>PTDF slack bus 1</b>	36.89	2027.93	1295.82	1275.05	411.37	107.46	205.37
<b>ZS slack bus 1</b>	550.85	-297.46	-99.03	323.93	766.66	551.05	909.75

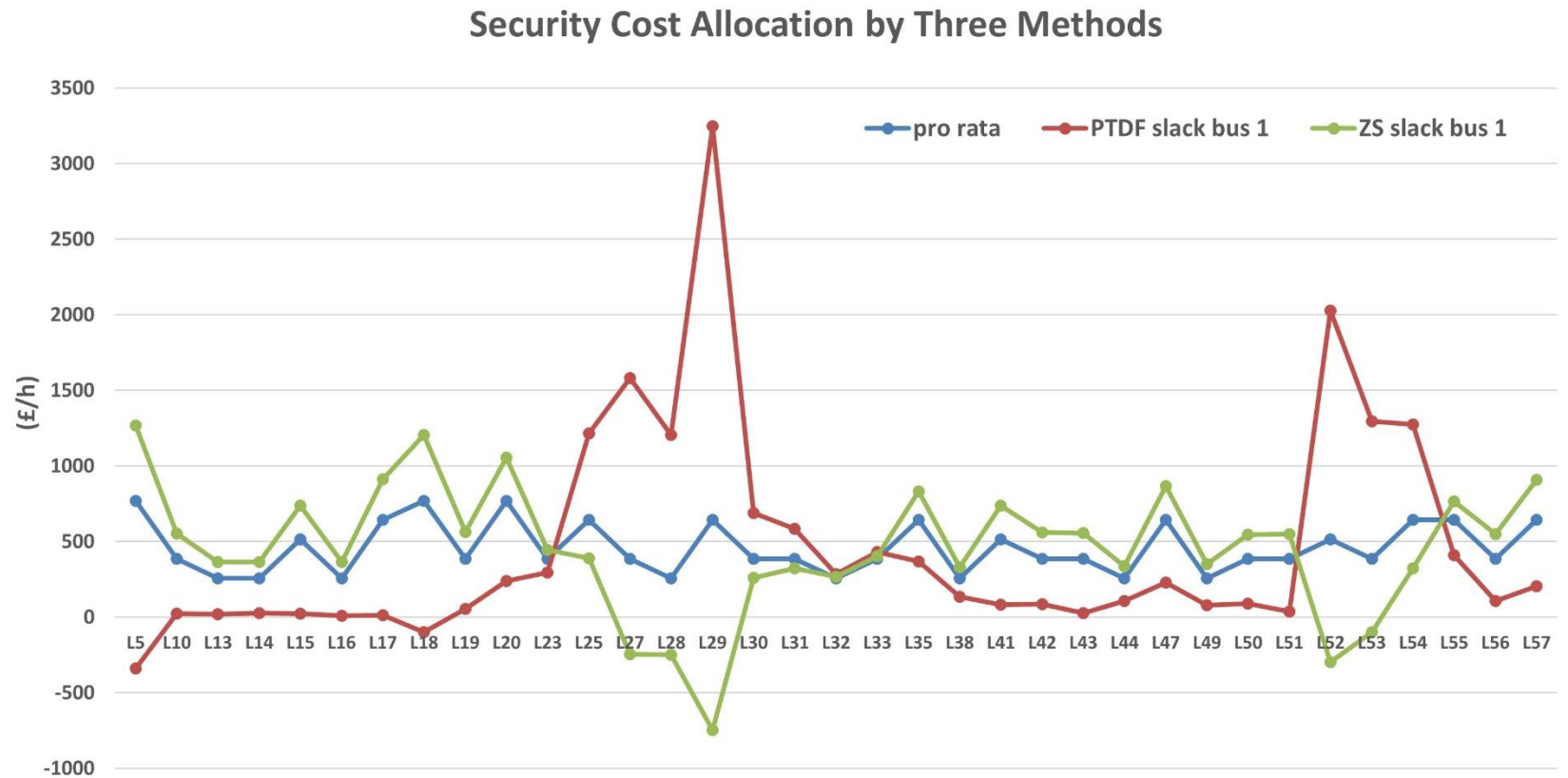


Figure 7.7 Comparison between three methods on security cost allocation in the 57-bus test system

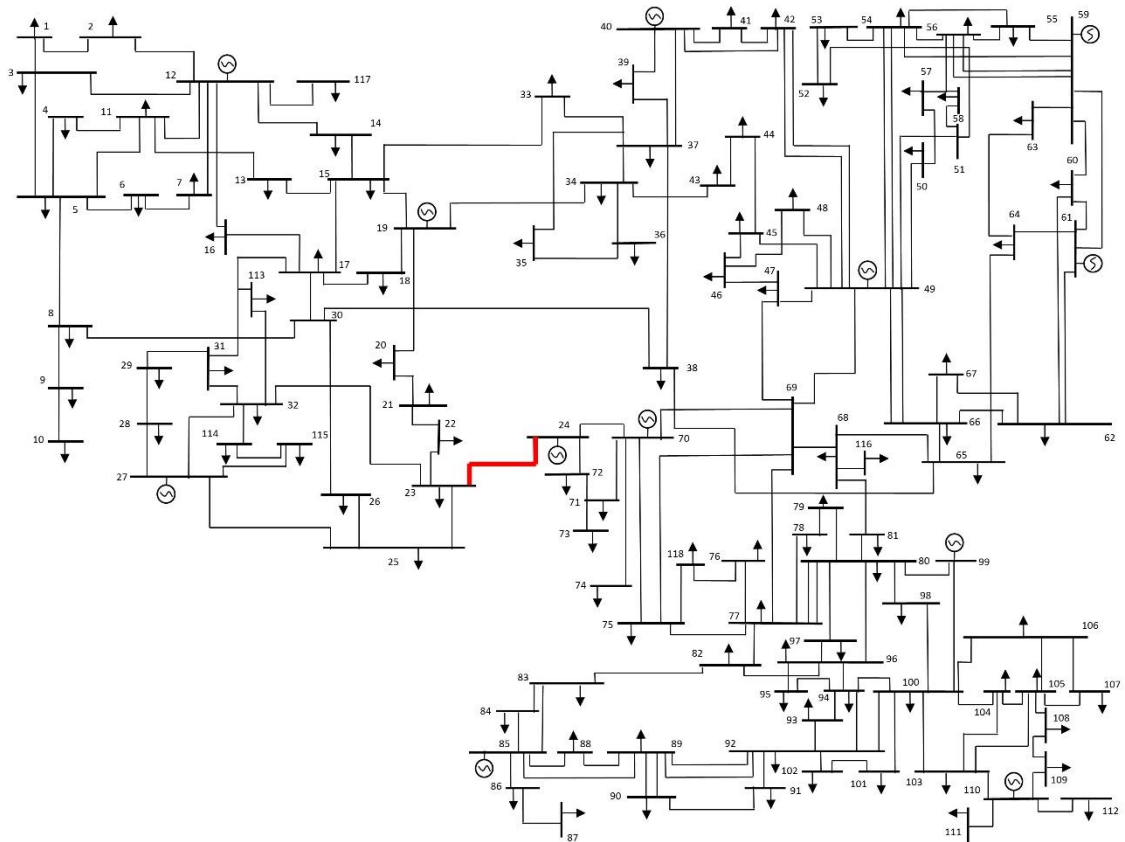


Figure 7.7 indicates that except the load interval from load 23 to 30 and load interval from load 52 to 54, the curve of the ZS method has the similar tendency with the pro rata curve but a little higher than the pro rata curve. There are two remarkable differences between the ZS method curve and the PTDF method curve, which are the load interval from load 25 to 29 and the load interval from load 52 to 54. In the PTDF method, security cost allocation on load 29 is the first peak point and load 52 is the second peak point. However, the ZS method obtains the opposite results. Security cost allocation on load 29 is the first nadir point and load 52 is the second lowest point. The explain is that in the PTDF method, the power flow contribution percentages of load 29 and 52 respectively to the congested line 7-29 are higher than other loads while in the ZS method, the power flow contribution percentages of load 29 and 52 respectively to the congested line 7-29 are lower than other loads so security allocated to the load 29 is the lowest point and load 52 is the second lowest point.

To explain the above results, the detailed calculation process of load's contributions by three methods will be discussed. As known, three methods calculate each load's contribution to the congested transmission line based on their own calculation principles respectively. This is the reason why in figure 7.7, three curves are showing differently even though there are still some close points. Here two typical examples are taken to show the details, one is load 29, where the most obvious difference among three methods occurs, the other one is load 32, where the smallest difference occurs. It can be found that for the contribution of load 29, the pro rata method calculation is 4%, the PTDF-based method calculation is 20.2% and the ZS-based method calculation is -4.7%. For the contribution of load 32, the pro rata method calculation is 1.6%, the PTDF-based method calculation is 1.8% and the ZS-based method calculation is 1.7%.

### 7.3.5. Modified IEEE-118 Bus System

The modified IEEE-118 bus test system is as same as the system in section 6.4.5, which is showed in Figure 7.8 as follow:



**Figure 7.8 The modified IEEE-118 bus system**

The parameters of the generators and loads are set up as the same as the former two chapters, which are displayed in Table 4.12 and Table 4.13. There are 12 generators and 104 loads in the test system. Transmission congestion occurred on the transmission branch 23-24. The security cost amount needs to be allocated is picked up from the experimental result in Figure 6.18, which is £19615.06/h.

Then three security cost allocation methods are implemented respectively including the pro rata method, the PTDF method with the first slack bus selection and the ZS method with the first slack bus selection to allocate such above security cost to the loads in the system. The comparison between three methods on security cost allocation will be shown in Table 7.4 and Figure 7.9 as follows:

**Table 7.4 Comparison between Three Methods on Security Cost Allocation in the 118-bus Test System**

(£/h)	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L13	L14
<b>Pro rata</b>	215.08	143.38	215.08	215.08	71.69	215.08	143.38	215.08	71.69	143.38	358.46	215.08	107.54
<b>PTDF slack bus 12</b>	0.74	0.19	0.93	1.91	0.69	1.04	0.43	3.88	1.29	2.58	1.04	0.06	-0.25
<b>ZS slack bus 12</b>	-2391.29	-1566.04	-2363.96	-2302.58	-765.03	-2311.52	-1539.97	-2261.45	-764.80	-1497.79	-3859.58	-2322.71	-1139.56
	L15	L16	L17	L18	L20	L21	L22	L23	L25	L26	L28	L29	L31
<b>Pro rata</b>	430.15	143.38	71.69	71.69	107.54	107.54	71.69	286.77	358.46	215.08	71.69	143.38	286.77
<b>PTDF slack bus 12</b>	-3.70	-3.02	-4.77	-3.52	-32.63	-53.81	-52.01	-313.79	-226.92	-93.25	-37.38	-65.48	-124.44
<b>ZS slack bus 12</b>	-4400.37	-1539.53	-755.16	-752.16	-1327.53	-1477.04	-1085.55	-4887.77	-4895.11	-2660.20	-1020.42	-1966.25	-3865.39
	L32	L33	L34	L35	L36	L37	L38	L39	L41	L42	L43	L44	L45
<b>Pro rata</b>	358.46	143.38	358.46	215.08	71.69	71.69	71.69	143.38	215.08	430.15	143.38	107.54	358.46
<b>PTDF slack bus 12</b>	-214.33	31.83	171.76	103.61	34.48	34.78	35.23	87.75	159.65	388.34	101.56	112.14	417.83
<b>ZS slack bus 12</b>	-5256.30	-1119.64	-1810.25	-1085.73	-362.71	-356.46	-350.84	-517.53	-458.06	-139.70	-367.70	124.06	896.45
	L46	L47	L48	L50	L51	L52	L53	L54	L55	L56	L57	L58	L60
<b>Pro rata</b>	143.38	215.08	143.38	107.54	107.54	107.54	143.38	71.69	430.15	573.54	107.54	107.54	501.85
<b>PTDF slack bus 12</b>	180.89	287.68	183.83	138.98	139.60	139.75	186.90	93.66	562.56	749.65	139.90	140.01	666.84
<b>ZS slack bus 12</b>	499.88	894.85	526.14	415.28	441.83	449.60	615.08	306.54	1852.45	2459.05	443.53	451.55	2127.64

	<b>L62</b>	<b>L63</b>	<b>L64</b>	<b>L65</b>	<b>L66</b>	<b>L67</b>	<b>L68</b>	<b>L71</b>	<b>L72</b>	<b>L73</b>	<b>L74</b>	<b>L75</b>	<b>L76</b>
<b>Pro rata</b>	501.85	71.69	143.38	71.69	215.08	143.38	71.69	71.69	71.69	71.69	430.15	358.46	430.15
<b>PTDF slack bus 12</b>	666.35	95.33	191.20	96.33	284.07	189.85	107.38	197.92	277.86	197.92	940.21	744.46	832.70
<b>ZS slack bus 12</b>	2113.24	304.41	601.61	292.35	839.67	585.55	399.77	1046.69	1535.94	1042.96	4972.80	3845.36	4408.78
	<b>L77</b>	<b>L78</b>	<b>L79</b>	<b>L80</b>	<b>L81</b>	<b>L82</b>	<b>L83</b>	<b>L84</b>	<b>L86</b>	<b>L87</b>	<b>L88</b>	<b>L89</b>	<b>L90</b>
<b>Pro rata</b>	430.15	501.85	215.08	71.69	71.69	358.46	143.38	71.69	71.69	71.69	286.77	71.69	71.69
<b>PTDF slack bus 12</b>	745.13	865.37	367.55	119.32	111.78	610.07	243.69	121.58	121.45	121.45	484.86	121.05	121.01
<b>ZS slack bus 12</b>	3631.07	4236.14	1789.34	553.56	457.26	3214.64	1305.12	658.80	661.90	645.70	2582.30	618.09	660.27
	<b>L91</b>	<b>L92</b>	<b>L93</b>	<b>L94</b>	<b>L95</b>	<b>L96</b>	<b>L97</b>	<b>L98</b>	<b>L100</b>	<b>L101</b>	<b>L102</b>	<b>L103</b>	<b>L104</b>
<b>Pro rata</b>	71.69	430.15	107.54	215.08	286.77	215.08	107.54	215.08	215.08	143.38	107.54	143.38	215.08
<b>PTDF slack bus 12</b>	120.96	725.35	181.23	362.27	483.29	362.73	180.20	359.24	361.39	241.29	181.22	240.92	361.39
<b>ZS slack bus 12</b>	662.21	3886.28	982.19	1953.12	2608.39	1902.85	897.23	1785.82	1933.55	1319.95	981.55	1354.56	2133.30
	<b>L105</b>	<b>L106</b>	<b>L107</b>	<b>L108</b>	<b>L109</b>	<b>L111</b>	<b>L112</b>	<b>L113</b>	<b>L114</b>	<b>L115</b>	<b>L116</b>	<b>L117</b>	<b>L118</b>
<b>Pro rata</b>	215.08	286.77	358.46	143.38	71.69	71.69	43.02	71.69	71.69	143.38	71.69	143.38	215.08
<b>PTDF slack bus 12</b>	361.39	481.85	602.31	240.92	120.46	120.46	72.28	-9.69	-42.33	-84.47	107.38	0.00	432.44
<b>ZS slack bus 12</b>	2166.39	2890.88	3703.41	1449.22	725.87	717.58	434.62	-791.20	-1054.99	-2109.70	399.66	-1559.10	2277.88

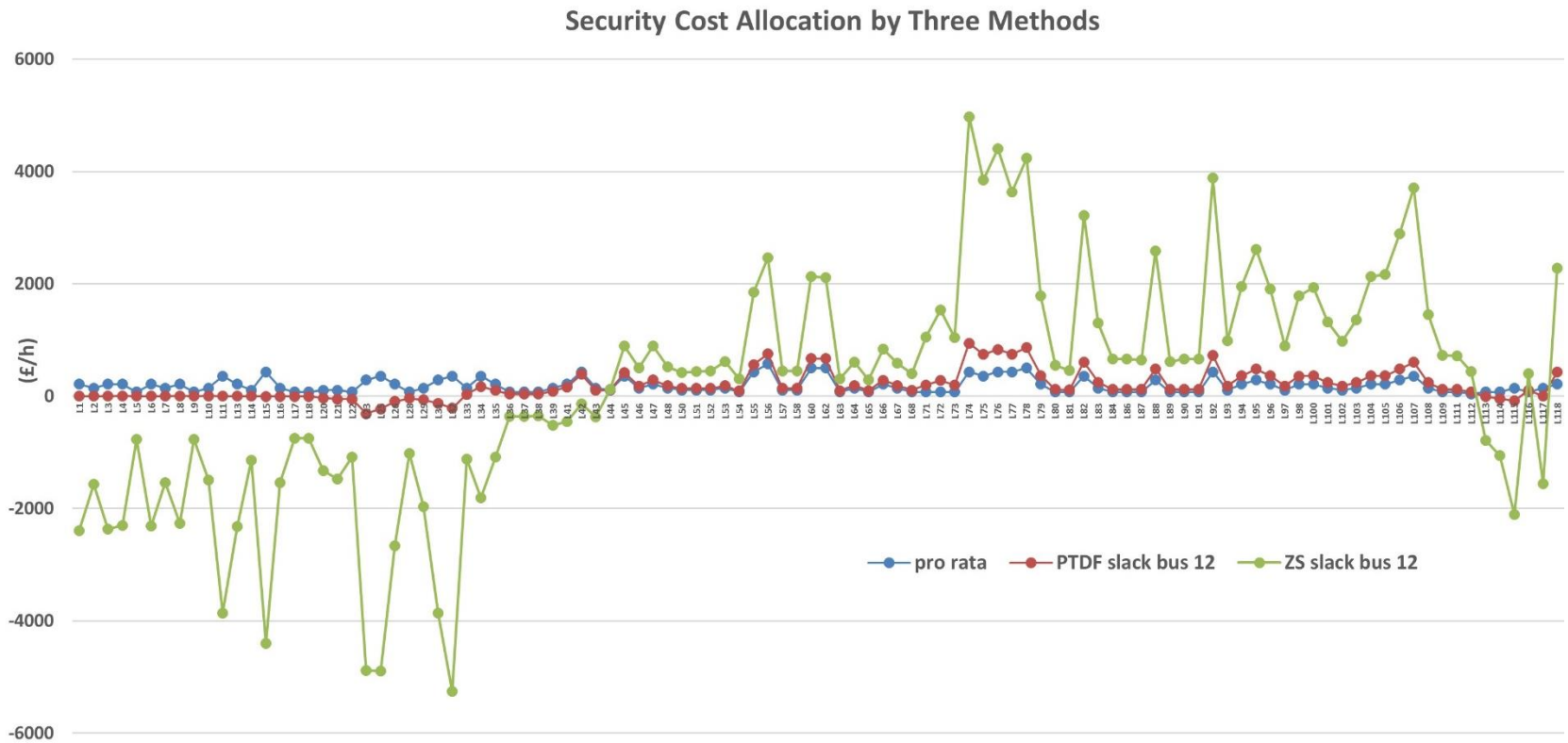


Figure 7.9 Comparison between three methods on security cost allocation in the 118-bus test system

Figure 7.9 indicates that the ZS method gives a distinctly different result on the security cost allocation compared with other two methods. The amounts of the allocated security cost to loads by the ZS method is not at the same order of magnitude with the other two methods. But if observe very carefully, there are still some noticeable details in such above figure. One is that in the PTDF method, allocated security cost on load 23 and load 32 are two negative peaks on the curve. In the ZS method, load 23 and load 32 are also the negative peaks showed on the ZS curve. Another one is from the load interval from load 45 to load 118, the curves of both the PTDF method and the ZS method actually have similar tendencies. The ZS method curve is up and down following the curve of the PTDF method. But the allocated security cost specifically to each load in the ZS method is almost a few times of the values in the PTDF method.

### 7.3.6. Discussion

In the case studies of this chapter, the bus impedance matrix sensitivity factor is utilized to allocate the security cost due to transmission congestion management. Through four test systems including a 14-bus, a 30-bus, a 57-bus and a 118-bus system, the performance of the ZS based security cost allocation method is found out, which is then compared with the former two methods in chapter 5 under the same testing conditions. Discussions about the four groups of testing results are informed as follows:

In the 14-bus test system case, it is found that plans of security cost allocation through three methods are almost the same. But in the 30-bus test system case, results of security cost allocation by three methods start to be distinct. It can be observed that the ZS method has another idea to consider the power flow contribution percentage of each load to the congested line compared with the PTDF method. Things turn more complicated in the 57-bus test system case because except for the two negative peaks, the ZS method curve is following the tendency of the pro rata method curve. However, the positions of the two negative peaks in the ZS method curve are the two positive peaks of the PTDF

method curve. It can be concluded that in the 57-bus test system the ZS method has the opposite logic with the PTDF method on power flow contribution calculation. If it is acceptable for security cost allocation results by the ZS method in the first three test systems, the allocation result of the ZS method in the 118-bus test system seems unreasonable. It is because the power flow contribution percentage of each load to the congestion calculated by the ZS method is so exaggerated no matter if positive values or the negative values, which leads to the values of security cost allocated to load to become exaggerated as well. Since the ZS method considers nearly half of loads have contribution to congestion relief so they should be rewarded. The loads who are considered by the ZS factors as the contributors of the transmission congestion should pay to recover not only the security cost but also the rewards of the loads which have relieved congestion. Consequently, the security cost allocated to the loads who have contribution to the congestion looks unreasonably high.

The comparisons between the three security cost allocation methods show the advantages and disadvantages of each method. The pro rata method is simple but not valid because it is unable to find out the contribution of each load to the transmission congestion. The PTDF method can find the power flow contribution percentage of each load to the transmission congestion. Nevertheless, since the dependency of slack bus selection, both the PTDF method and the ZS method will result in different allocation of security cost to the load and can be highly dependent on slack bus position. Furthermore, as the power system becomes large, the security cost allocation results of the ZS method appear unreasonable and unacceptable. Hence the system operator should decide to choose the method to allocate the security cost according to different power system conditions.



## 7.4. Summary

In this chapter, the mechanism and the mathematic model of the proposed allocation method with the bus impedance matrix based sensitivity factor are described. Case studies including experimental results by the ZS method in four modified IEEE test systems: an IEEE-14 bus system, an IEEE-30 bus system, an IEEE-57 bus system and an IEEE-118 bus system are shown. The comparisons among three allocation methods which are pro rata method, PTDF method with first generator bus as the slack bus and the ZS method with first generator bus as the slack bus are obtained and discussed. The conclusion is that each above method has pros and cons on the security cost allocation and there is no a perfect way to solve all the security cost allocation. It should be based on the specific condition when choose the method to allocate security cost due to congestion management.

# Chapter 8

## Conclusion and Future Work

### 8.1. Conclusion

This thesis has achieved the goals of proposing a generation re-dispatch method to relieve transmission congestion and developing a security cost allocation method to balance the congestion management and the resulted security cost in deregulated electricity market. The re-dispatch method is actually a minimum cost optimization combined with the incremental and decremental bids submission mechanism for the generators, at the same time, restricted by transmission constraints and generation constraints. Two transmission sensitivity factors, namely the power transfer distribution factor (PTDF) and the bus impedance based sensitivity factor (ZS), are involved in the transmission capacity constraint calculation, which are based on the power flow contribution of each node to the congested transmission line calculated respectively by above two factors. The security cost allocation method allocates the security cost, which is the result of the congestion, to the loads based on the power flow contribution calculated respectively by above two factors as well.

The algorithm of the PTDF has proved that this sensitivity factor is dependent on the location of the slack bus selection. Case studies in chapter 4 attempt to verify the re-dispatch results are affected by the location of slack bus selection. Results show that the proposed method is capable under each circumstance. However, different results by selecting different generator bus as the slack bus have been obtained including the re-dispatched MW amount of each generator and the corresponding total security cost due to the congestion management. It is proved that the PTDF-based re-dispatch method has the risk of inaccuracy if there is only one bus chosen as the slack bus and if select all the generators as the slack bus respectively, the complexity of the method will increase.

The impedance matrix has been proved to be dependent on the location of slack bus selection. Consequently, the bus impedance based sensitivity factor, which is based on the impedance matrix, is also relying on the location of slack bus selection. Case studies in chapter 5 try to investigate the performance comparison between the ZS-based re-dispatch method and the PTDF-based re-dispatch method, with selecting the same location of slack bus and facing with the same level of overloading of the congested transmission line. Experimental results indicate that the number of generators involved in the ZS-based re-dispatch method is more than the number in the PTDF-based method, which eventually leads to the total resulted security cost due to the ZS-based re-dispatch method greater than the PTDF-based method. It is also showed that in all test systems, with the intense level of overloading of congestion, the security cost calculated by the ZS-based method is always beyond the PTDF-based method, and the distinction between their resulted security costs becomes larger.

The PTDF-based security cost allocation method also results in different allocation results with selecting different locations of slack bus. Case studies in chapter 6 aim to verify it. A pro rata method is added to the tests to provide a parallel comparison to highlight the advantage of the PTDF-based method. Experimental results indicate that one particular load is allocated different security costs when selecting different locations of slack bus. It is proved that the dependency of location of slack bus selection enhances the risks of inaccuracy and complexity of the security cost allocation by the PTDF-based method. Only considering one slack bus selection is not convincing while taking all the slack buses locations into consideration is time-consuming. Nevertheless, compared with the pro rata method, which is only based on the MW amount percentage, the PTDF-based method is still a fair way to allocate the security cost since it can find out the power flow contribution of each load particular to the congested transmission line.

The ZS-based security cost allocation method also has the dependency of the location of slack bus selection. Case studies in chapter 7 are conducted to provide comparisons among three security cost allocation methods: the pro rata method, the PTDF-based method with the first generator bus as the slack bus and the ZS-based method with the same slack bus. All target security cost amounts are chosen from the costs calculated by the PTDF-based re-dispatch method with the first slack bus selection in chapter 4. Experimental results show that three allocation methods have different ideas to allocate security cost to the loads. It should be noted that in the large power system, the ZS-based method calculates unreasonable allocation results since some of the loads are allocated so much costs while some other loads are allocated too much costs in negative values. The negative values of allocated costs can be regarded as the rewards to the loads by the system operator, but the extreme numbers are unacceptable. It is proved that when implemented in large power systems, the ZS-based security cost allocation method has the risk of providing incomprehensible solutions.

## 8.2. Future Work

This thesis has developed a proposed congestion management scheme with two different power flow sensitivity factors for both generation re-dispatching and corresponding security cost allocation. However, due to the time constraint, some further issues are not considered in this research and some mechanisms of the proposed method can be further investigated.

- In this thesis, all experimental simulations of power system environment are based on the software package Matpower (version 6.0), which is implemented in the software Matlab. Power flow optimization problems are solved by using it no matter if it is in 14-bus system or 118-bus system. For the experimental purpose, it may be a good choice to use but for the industry standard studies, its limitation might be exposed since it is usually used to solve problems under ideal and steady conditions

while in reality, power system is always complex and varying. In the future, some state-of-the-art tools which could meet industrial requirements should be utilized to simulate power system environment and to solve power flow problems. For example, the software CPLEX which is an optimizer implemented in the C programming language to solve mixed-integer programming problems.

- As known, in this thesis, four modified IEEE-bus test systems are used to build as the power system bases for power flow optimizations and congestion management implementations. But these IEEE-bus models have been used for many years and may not model modern power systems with sufficient accuracy. Furthermore, some parameters of power system simulations in this thesis are set arbitrarily such as bid price of each generator, generation capacity of each generator, active power consumption of each load, and transfer capacity of each transmission line. For a more convincing approach, in the future, a realistic power system case will be found and built as an experimental model. Parameters of such power system model will be more close to the real situation, which will also become more logical and real.
- The experiments in this thesis are only based on the DC optimal power flow since its fast calculating speed and easy convergence. All the transmission power flows are considered as lossless since the DC assumptions. In the future work, the AC optimal power flow will be utilized to simulate a more realistic power system environment. The power losses and reactive power will be taken into considerations. Two transmission congestion management schemes which are proposed in this thesis to re-dispatch generation and allocate security cost will be upgraded to adapt to the AC power flow situations.
- In all the test scenarios, there is only one transmission congestion occurs a time. Number of congested transmission line in each test scenario is restricted to one only in order to highlight the sensitivity method efficiency on one particular line. However,

in the reality, there will be multi-congestions occur at the same time during the power system operation. In the future work, both the PTDF-based method and the ZS-based method will be improved to deal with multi-congestion problems including the generation re-dispatching and the corresponding security cost allocation. Under the multi-congestion situations, power flow contribution of each generator and each load will be re-evaluated in order to decide new generation reschedules and new allocation of security cost.

- In the proposed congestion management scheme mechanism, only generators have the opportunity to submit incremental and decremental bids, who will be involved in the congestion relief action because the demand side is assumed to be inelastic. In the future work, loads will be considered as elastic, which means loads in the power system enable to take part in the adjustment bids submission to help the system operator has more options to adjust the power flow on the congested line from both ends of the power system. The corresponding security cost allocation scheme should be designed as, for example, the load who helped the system operator relieve the congestion should not be allocated any security cost but even received rewards from system operator.
- As known, the PTDF method has the drawbacks of inaccuracy and complexity due to the dependency of slack bus selection. But there are still some specific slack bus selections, which are seemed as reasonable solutions on generation re-dispatching and security cost allocation. In the future work, how to correctly choose the generator bus as the slack bus in the PTDF method can be considered as an investigative topic to improve the exist PTDF-based method on congestion management. If the appropriate slack bus can be chosen in a quick way, the PTDF-based method can still be regarded as a suitable approach to manage transmission congestion.

- It is found that the ZS-based method always involves more generators into re-dispatch stage, which leads to corresponding security cost more than the PTDF-based method. In the aspect of security cost allocation, when implemented in large power system, the ZS-based method gives unreasonable outcomes. For the moment, this problems are attributed to the ZS-based sensitivity factors because both generation re-dispatching and security cost allocation in the ZS-based method are based on these sensitivity factors. In the future work, the deeper reason behind this will be researched and discovered so the ZS-based method could be improved to obtain a better performance on congestion management. Since the ZS-based method is also dependent on the location of slack bus selection, if its performance is better than the PTDF-based method, it can become a preferred method for the system operator to solve the transmission congestion issues.

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# Appendix

## A. PTDF and ZS Sensitivity Factors in Modified IEEE-14 Bus System

**Table A. PTDF and ZS Sensitivity Factors Associated with Congested Transmission Line 3-4 in Modified IEEE-14 Bus System**

<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b><u>4</u></b>	<b><u>5</u></b>	<b><u>6</u></b>	<b><u>7</u></b>
<b>PTDF slack 1</b>	0	0.02735	0.467992	-0.151329	-0.103095	-0.118834	-0.142675
<b>Bus</b>	<b><u>8</u></b>	<b><u>9</u></b>	<b><u>10</u></b>	<b><u>11</u></b>	<b><u>12</u></b>	<b><u>13</u></b>	<b><u>14</u></b>
<b>PTDF slack 1</b>	-0.142675	-0.1380197	-0.13461	-0.12686	-0.12035	-0.12153	-0.13081
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b><u>4</u></b>	<b><u>5</u></b>	<b><u>6</u></b>	<b><u>7</u></b>
<b>PTDF slack 2</b>	-0.02735	0	0.440642	-0.178679	-0.130445	-0.146184	-0.170025
<b>Bus</b>	<b><u>8</u></b>	<b><u>9</u></b>	<b><u>10</u></b>	<b><u>11</u></b>	<b><u>12</u></b>	<b><u>13</u></b>	<b><u>14</u></b>
<b>PTDF slack 2</b>	-0.170025	-0.1653696	-0.16196	-0.15421	-0.1477	-0.14888	-0.15816
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b><u>4</u></b>	<b><u>5</u></b>	<b><u>6</u></b>	<b><u>7</u></b>
<b>PTDF slack 3</b>	-0.46799	-0.44064	0	-0.619321	-0.571087	-0.586826	-0.610667
<b>Bus</b>	<b><u>8</u></b>	<b><u>9</u></b>	<b><u>10</u></b>	<b><u>11</u></b>	<b><u>12</u></b>	<b><u>13</u></b>	<b><u>14</u></b>
<b>PTDF slack 3</b>	-0.610667	-0.6060119	-0.602602	-0.594852	-0.588342	-0.58953	-0.5988
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b><u>4</u></b>	<b><u>5</u></b>	<b><u>6</u></b>	<b><u>7</u></b>
<b>PTDF slack 6</b>	0.118834	0.146184	0.586826	-0.032495	0.0157391	0	-0.023841

<b>Bus</b>	<b><u>8</u></b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>PTDF slack 6</b>	-0.023841	-0.0191856	-0.015776	-0.008026	-0.001516	-0.0027	-0.01198
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b>4</b>	<b>5</b>	<b><u>6</u></b>	<b>7</b>
<b>PTDF slack 8</b>	0.142675	0.170025	0.610667	-0.008654	0.0395796	0.0238405	0
<b>Bus</b>	<b><u>8</u></b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>PTDF slack 8</b>	0	0.00465493	0.0080646	0.0158148	0.0223245	0.02114	0.011863
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b>4</b>	<b>5</b>	<b><u>6</u></b>	<b>7</b>
<b>ZS slack 1</b>	0	-0.065096	-0.502803	0.1088657	0.0609936	0.078889	0.1014944
<b>Bus</b>	<b><u>8</u></b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>ZS slack 1</b>	0.0988872	0.09910399	0.0960888	0.0874781	0.0811924	0.082909	0.093553

## B. PTDF and ZS Sensitivity Factors in Modified IEEE-30 Bus System

**Table B. PTDF and ZS Sensitivity Factors Associated with Congested Transmission Line 4-12 in Modified IEEE-30 Bus System**

<b>Bus</b>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>
<b>PTDF slack 1</b>	0	-0.005474107	0.01572735	0.019335496	-0.0200072	-0.034518376	-0.028508711
<b>Bus</b>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>
<b>PTDF slack 1</b>	-0.036955563	-0.194812341	-0.281490099	-0.194812341	-0.613507156	-0.613507156	-0.565639667
<b>Bus</b>	<u>15</u>	<u>16</u>	<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	<u>21</u>
<b>PTDF slack 1</b>	-0.528284695	-0.4747652	-0.340492078	-0.44196387	-0.390922	-0.364057844	-0.288409
<b>Bus</b>	<u>22</u>	<u>23</u>	<u>24</u>	<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>
<b>PTDF slack 1</b>	-0.290589	-0.438285424	-0.31798937	-0.22171	-0.22171	-0.16067	-0.04856
<b>Bus</b>	<u>29</u>	<u>30</u>					
<b>PTDF slack 1</b>	-0.16067	-0.16067					
<b>Bus</b>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>
<b>PTDF slack 2</b>	0.005474107	0	0.02120145	0.024809603	-0.0145331	-0.029044269	-0.023034605
<b>Bus</b>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>
<b>PTDF slack 2</b>	-0.031481456	-0.189338234	-0.276015992	-0.189338234	-0.608033049	-0.608033049	-0.560165561
<b>Bus</b>	<u>15</u>	<u>16</u>	<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	<u>21</u>
<b>PTDF slack 2</b>	-0.522810588	-0.4692911	-0.335017972	-0.436489763	-0.3854479	-0.358583737	-0.2829349
<b>Bus</b>	<u>22</u>	<u>23</u>	<u>24</u>	<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>
<b>PTDF slack 2</b>	-0.2851149	-0.432811318	-0.312515263	-0.21624	-0.21624	-0.1552	-0.04309
<b>Bus</b>	<u>29</u>	<u>30</u>					
<b>PTDF slack 2</b>	-0.1552	-0.1552					
<b>Bus</b>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>



PTDF slack 5	0.020007234	0.014533128	0.03573458	0.039342731	0	-0.014511141	-0.008501477
Bus	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>
PTDF slack 5	-0.016948328	-0.174805106	-0.261482864	-0.174805106	-0.593499921	-0.593499921	-0.545632433
Bus	<u>15</u>	<u>16</u>	<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	<u>21</u>
PTDF slack 5	-0.50827746	-0.454758	-0.320484844	-0.421956635	-0.3709148	-0.34405061	-0.2684017
Bus	<u>22</u>	<u>23</u>	<u>24</u>	<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>
PTDF slack 5	-0.2705818	-0.41827819	-0.297982136	-0.2017	-0.2017	-0.14066	-0.02855
Bus	<u>29</u>	<u>30</u>					
PTDF slack 5	-0.14066	-0.14066					
Bus	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>
PTDF slack 8	0.036955563	0.031481456	0.05268291	0.056291059	0.0169483	0.002437187	0.008446852
Bus	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>
PTDF slack 8	0	-0.157856778	-0.244534536	-0.157856778	-0.576551593	-0.576551593	-0.528684104
Bus	<u>15</u>	<u>16</u>	<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	<u>21</u>
PTDF slack 8	-0.491329132	-0.4378097	-0.303536515	-0.405008307	-0.3539664	-0.327102281	-0.2514534
Bus	<u>22</u>	<u>23</u>	<u>24</u>	<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>
PTDF slack 8	-0.2536334	-0.401329862	-0.281033807	-0.18475	-0.18475	-0.12372	-0.01161
Bus	<u>29</u>	<u>30</u>					
PTDF slack 8	-0.12372	-0.12372					
Bus	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>
PTDF slack 11	0.194812341	0.189338234	0.21053969	0.214147837	0.1748051	0.160293965	0.16630363
Bus	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>
PTDF slack 11	0.157856778	0	-0.086677758	0	-0.418694815	-0.418694815	-0.370827326
Bus	<u>15</u>	<u>16</u>	<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>	<u>21</u>
PTDF slack 11	-0.333472354	-0.2799529	-0.145679737	-0.247151529	-0.1961096	-0.169245503	-0.0935966
Bus	<u>22</u>	<u>23</u>	<u>24</u>	<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>

<b>PTDF slack 11</b>	-0.0957767	-0.243473083	-0.123177029	-0.0269	-0.0269	0.034141	0.146251
<b>Bus</b>	<b>29</b>	<b>30</b>					
<b>PTDF slack 11</b>	0.034141	0.034141					
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b>3</b>	<b>4</b>	<b><u>5</u></b>	<b>6</b>	<b>7</b>
<b>PTDF slack 13</b>	0.613507156	0.608033049	0.6292345	0.632842652	0.5934999	0.57898878	0.584998444
<b>Bus</b>	<b><u>8</u></b>	<b>9</b>	<b>10</b>	<b><u>11</u></b>	<b>12</b>	<b><u>13</u></b>	<b>14</b>
<b>PTDF slack 13</b>	0.576551593	0.418694815	0.332017057	0.418694815	0	0	0.047867488
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 13</b>	0.085222461	0.1387419	0.273015077	0.171543286	0.2225852	0.249449312	0.3250982
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 13</b>	0.3229181	0.175221731	0.295517786	0.391798	0.391798	0.452835	0.564946
<b>Bus</b>	<b>29</b>	<b>30</b>					
<b>PTDF slack 13</b>	0.452835	0.452835					
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b>3</b>	<b>4</b>	<b><u>5</u></b>	<b>6</b>	<b>7</b>
<b>ZS slack 1</b>	0	-0.518185671	-0.5471995	-0.55408986	-0.5191326	-0.502721889	-0.513800764
<b>Bus</b>	<b><u>8</u></b>	<b>9</b>	<b>10</b>	<b><u>11</u></b>	<b>12</b>	<b><u>13</u></b>	<b>14</b>
<b>ZS slack 1</b>	-0.500350201	-0.338042623	-0.256367645	-0.328463384	0.076725495	0.075841548	0.031153106
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>ZS slack 1</b>	-0.009589601	-0.0618713	-0.197662755	-0.097427353	-0.1494352	-0.175784547	-0.2517981
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>ZS slack 1</b>	-0.2499219	-0.101699336	-0.225869378	-0.32787	-0.33282	-0.39007	-0.49044
<b>Bus</b>	<b>29</b>	<b>30</b>					
<b>ZS slack 1</b>	-0.39579	-0.39865					

## C. PTDF and ZS Sensitivity Factors in Modified IEEE-57 Bus System

**Table C. PTDF and ZS Sensitivity Factors Associated with Congested Transmission Line 7-29 in Modified IEEE-57 Bus System**

<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b>4</b>	<b>5</b>	<b><u>6</u></b>	<b>7</b>
<b>PTDF slack 1</b>	0	0.003417	0.013791	0.032453	0.064671	0.080317	0.146625
<b>Bus</b>	<b><u>8</u></b>	<b><u>9</u></b>	<b>10</b>	<b>11</b>	<b><u>12</u></b>	<b>13</b>	<b>14</b>
<b>PTDF slack 1</b>	0.066028	0.004692	-0.00822	-0.00606	-0.00685	-0.01125	-0.01578
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 1</b>	-0.00677	-0.00491	-0.002577	0.018853	-0.020605	-0.045604	-0.092267
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 1</b>	-0.09901	-0.11166	-0.32485	-0.27753	-0.36981	-0.60128	-0.68822
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 1</b>	-0.74171	-0.26167	-0.22265	-0.16337	-0.16337	-0.09042	-0.0843
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 1</b>	-0.08008	-0.07841	-0.07614	-0.07754	-0.07855	-0.02362	-0.03223
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 1</b>	-0.01067	-0.061758	-0.031227	-0.033567	-0.05186	-0.05812	-0.04461
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 1</b>	-0.03336	-0.01404	-0.57887	-0.49319	-0.29117	-0.09394	-0.0409
<b>Bus</b>	<b>57</b>						
<b>PTDF slack 1</b>	-0.0469						
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b>4</b>	<b>5</b>	<b><u>6</u></b>	<b>7</b>
<b>PTDF slack 2</b>	-0.00342	0	0.010374	0.029035	0.061254	0.0769	0.143208
<b>Bus</b>	<b><u>8</u></b>	<b><u>9</u></b>	<b>10</b>	<b>11</b>	<b><u>12</u></b>	<b>13</b>	<b>14</b>
<b>PTDF slack 2</b>	0.062611	0.001275	-0.01164	-0.00948	-0.01027	-0.01467	-0.01919
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>

<b>PTDF slack 2</b>	-0.01018	-0.00833	-0.005994	0.015435	-0.024022	-0.049021	-0.095684
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 2</b>	-0.10242	-0.11508	-0.32827	-0.28095	-0.37323	-0.6047	-0.69163
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 2</b>	-0.74513	-0.26509	-0.22607	-0.16679	-0.16679	-0.09384	-0.08772
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 2</b>	-0.0835	-0.08183	-0.07956	-0.08096	-0.08197	-0.02703	-0.03565
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 2</b>	-0.01409	-0.065175	-0.034644	-0.036985	-0.05527	-0.06154	-0.04802
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 2</b>	-0.03678	-0.01746	-0.58229	-0.49661	-0.29459	-0.09736	-0.04432
<b>Bus</b>	<b>57</b>						
<b>PTDF slack 2</b>	-0.05032						
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b><u>4</u></b>	<b><u>5</u></b>	<b><u>6</u></b>	<b><u>7</u></b>
<b>PTDF slack 3</b>	-0.01379	-0.01037	0	0.018662	0.05088	0.066526	0.132834
<b>Bus</b>	<b><u>8</u></b>	<b><u>9</u></b>	<b>10</b>	<b>11</b>	<b><u>12</u></b>	<b>13</b>	<b>14</b>
<b>PTDF slack 3</b>	0.052237	-0.0091	-0.02202	-0.01985	-0.02064	-0.02504	-0.02957
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 3</b>	-0.02056	-0.0187	-0.016368	0.005062	-0.034395	-0.059394	-0.106057
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 3</b>	-0.1128	-0.12545	-0.33865	-0.29132	-0.3836	-0.61507	-0.70201
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 3</b>	-0.7555	-0.27546	-0.23644	-0.17717	-0.17717	-0.10421	-0.09809
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 3</b>	-0.09387	-0.0922	-0.08993	-0.09133	-0.09234	-0.03741	-0.04602
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 3</b>	-0.02446	-0.075548	-0.045018	-0.047358	-0.06565	-0.07191	-0.0584

<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 3</b>	-0.04715	-0.02783	-0.59266	-0.50698	-0.30496	-0.10773	-0.05469
<b>Bus</b>	<b>57</b>						
<b>PTDF slack 3</b>	-0.06069						
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b>4</b>	<b>5</b>	<b><u>6</u></b>	<b>7</b>
<b>PTDF slack 6</b>	-0.08032	-0.0769	-0.06653	-0.04786	-0.01565	0	0.066308
<b>Bus</b>	<b><u>8</u></b>	<b><u>9</u></b>	<b>10</b>	<b>11</b>	<b><u>12</u></b>	<b>13</b>	<b>14</b>
<b>PTDF slack 6</b>	-0.01429	-0.07562	-0.08854	-0.08638	-0.08717	-0.09157	-0.09609
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 6</b>	-0.08708	-0.08523	-0.082894	-0.06146	-0.100921	-0.12592	-0.172583
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 6</b>	-0.17932	-0.19198	-0.40517	-0.35785	-0.45013	-0.6816	-0.76853
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 6</b>	-0.82203	-0.34199	-0.30297	-0.24369	-0.24369	-0.17074	-0.16462
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 6</b>	-0.1604	-0.15873	-0.15645	-0.15786	-0.15887	-0.10393	-0.11255
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 6</b>	-0.09099	-0.142074	-0.111544	-0.113884	-0.13217	-0.13844	-0.12492
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 6</b>	-0.11368	-0.09436	-0.65919	-0.57351	-0.37149	-0.17426	-0.12122
<b>Bus</b>	<b>57</b>						
<b>PTDF slack 6</b>	-0.12722						
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b>4</b>	<b>5</b>	<b><u>6</u></b>	<b>7</b>
<b>PTDF slack 8</b>	-0.06603	-0.06261	-0.05224	-0.03358	-0.00136	0.014289	0.080597
<b>Bus</b>	<b><u>8</u></b>	<b><u>9</u></b>	<b>10</b>	<b>11</b>	<b><u>12</u></b>	<b>13</b>	<b>14</b>
<b>PTDF slack 8</b>	0	-0.06134	-0.07425	-0.07209	-0.07288	-0.07728	-0.0818

<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 8</b>	-0.07279	-0.07094	-0.068605	-0.04718	-0.086632	-0.111632	-0.158294
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 8</b>	-0.16503	-0.17769	-0.39088	-0.34356	-0.43584	-0.66731	-0.75424
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 8</b>	-0.80774	-0.3277	-0.28868	-0.2294	-0.2294	-0.15645	-0.15033
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 8</b>	-0.14611	-0.14444	-0.14217	-0.14357	-0.14458	-0.08964	-0.09826
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 8</b>	-0.0767	-0.127786	-0.097255	-0.099595	-0.11788	-0.12415	-0.11063
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 8</b>	-0.09939	-0.08007	-0.6449	-0.55922	-0.3572	-0.15997	-0.10693
<b>Bus</b>	<b>57</b>						
<b>PTDF slack 8</b>	-0.11293						
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b>4</b>	<b>5</b>	<b><u>6</u></b>	<b>7</b>
<b>PTDF slack 9</b>	-0.00469	-0.00127	0.009099	0.027761	0.059979	0.075625	0.141933
<b>Bus</b>	<b><u>8</u></b>	<b><u>9</u></b>	<b>10</b>	<b>11</b>	<b><u>12</u></b>	<b>13</b>	<b>14</b>
<b>PTDF slack 9</b>	0.061336	0	-0.01292	-0.01076	-0.01154	-0.01594	-0.02047
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 9</b>	-0.01146	-0.0096	-0.007269	0.014161	-0.025297	-0.050296	-0.096959
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 9</b>	-0.1037	-0.11636	-0.32955	-0.28222	-0.3745	-0.60597	-0.69291
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 9</b>	-0.7464	-0.26636	-0.22734	-0.16807	-0.16807	-0.09512	-0.08899
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 9</b>	-0.08478	-0.08311	-0.08083	-0.08223	-0.08324	-0.02831	-0.03693
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>

<b>PTDF slack 9</b>	-0.01536	-0.06645	-0.035919	-0.038259	-0.05655	-0.06282	-0.0493
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 9</b>	-0.03806	-0.01873	-0.58357	-0.49788	-0.29586	-0.09863	-0.04559
<b>Bus</b>	<b>57</b>						
<b>PTDF slack 9</b>	-0.05159						
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b>4</b>	<b>5</b>	<b><u>6</u></b>	<b>7</b>
<b>PTDF slack 12</b>	0.006849	0.010266	0.020639	0.039301	0.07152	0.087165	0.153474
<b>Bus</b>	<b><u>8</u></b>	<b><u>9</u></b>	<b>10</b>	<b>11</b>	<b><u>12</u></b>	<b>13</b>	<b>14</b>
<b>PTDF slack 12</b>	0.072877	0.011541	-0.00138	0.000785	0	-0.0044	-0.00893
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 12</b>	8.37E-05	0.001938	0.004271	0.025701	-0.013756	-0.038755	-0.085418
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 12</b>	-0.09216	-0.10482	-0.31801	-0.27068	-0.36296	-0.59443	-0.68137
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 12</b>	-0.73486	-0.25482	-0.2158	-0.15653	-0.15653	-0.08357	-0.07745
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 12</b>	-0.07323	-0.07156	-0.06929	-0.07069	-0.0717	-0.01677	-0.02538
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 12</b>	-0.00382	-0.054909	-0.024378	-0.026719	-0.04501	-0.05127	-0.03776
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 12</b>	-0.02651	-0.00719	-0.57203	-0.48634	-0.28432	-0.08709	-0.03405
<b>Bus</b>	<b>57</b>						
<b>PTDF slack 12</b>	-0.04005						
<b>Bus</b>	<b><u>1</u></b>	<b><u>2</u></b>	<b><u>3</u></b>	<b>4</b>	<b>5</b>	<b><u>6</u></b>	<b>7</b>
<b>ZS slack 1</b>	0	-0.43762	-0.45725	-0.47381	-0.49952	-0.50805	-0.54726
<b>Bus</b>	<b><u>8</u></b>	<b><u>9</u></b>	<b>10</b>	<b>11</b>	<b><u>12</u></b>	<b>13</b>	<b>14</b>

<b>ZS slack 1</b>	-0.47657	-0.44238	-0.43445	-0.43838	-0.42475	-0.43265	-0.43207
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>ZS slack 1</b>	-4.37E-01	-0.42989	-0.431562	-0.47493	-0.442649	-0.415818	-0.372272
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>ZS slack 1</b>	-0.36433	-0.35139	-0.12318	-0.1839	-0.08028	0.194211	0.293751
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>ZS slack 1</b>	0.353904	-0.20506	-0.25393	-0.31623	-0.31685	-0.38876	-0.3932
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>ZS slack 1</b>	-0.39421	-0.39329	-0.3878	-0.39478	-0.39631	-0.43543	-0.44152
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>ZS slack 1</b>	-0.43825	-0.399062	-0.418448	-0.423371	-0.40965	-0.404	-0.41483
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>ZS slack 1</b>	-0.42916	-0.43388	0.175723	0.078001	-0.15309	-0.36233	-0.43404
<b>Bus</b>	<b>57</b>						
<b>ZS slack 1</b>	-0.42995						



## D. PTDF and ZS Sensitivity Factors in Modified IEEE-118 Bus System

**Table D. PTDF and ZS Sensitivity Factors Associated with Congested Transmission Line 23-24 in Modified IEEE-118 Bus System**

Bus	1	2	3	4	5	6	7
PTDF slack 12	-0.000506564	-0.00019322	-0.00063956	-0.00130797	-0.00141	-0.00071019	-0.00044063
Bus	8	9	10	11	<u>12</u>	13	14
PTDF slack 12	-0.00265335	-0.0026533	-0.00265335	-0.00042824	0	-3.84E-05	0.00033664
Bus	15	16	17	18	<u>19</u>	20	21
PTDF slack 12	0.001265137	0.00310089	0.00979718	0.007227716	0.004719312	0.044668464	0.073657207
Bus	22	23	<u>24</u>	25	26	<u>27</u>	28
PTDF slack 12	0.106777444	0.161067317	-0.74920996	0.093181956	0.063820318	0.085372375	0.076739505
Bus	29	30	31	32	33	34	35
PTDF slack 12	0.067218105	-0.00503605	0.06387602	0.08801095	-0.0326731	-0.0705309	-0.0709073
Bus	36	37	38	39	<u>40</u>	41	42
PTDF slack 12	-0.0708035	-0.0714129	-0.0723287	-0.0900837	-0.1007402	-0.1092636	-0.132891
Bus	43	44	45	46	47	48	<u>49</u>
PTDF slack 12	-0.1042598	-0.1534989	-0.1715773	-0.1856969	-0.1968885	-0.18872184	-0.18953008
Bus	50	51	52	53	54	55	56
PTDF slack 12	-0.1902348	-0.1910837	-0.19129264	-0.19187355	-0.192307	-0.1925082	-0.1923959
Bus	57	58	<u>59</u>	60	<u>61</u>	62	63
PTDF slack 12	-0.1914906	-0.1916436	-0.1947263	-0.19559232	-0.19570744	-0.19544898	-0.19573972
Bus	64	65	66	67	68	69	<u>70</u>
PTDF slack 12	-0.19628671	-0.19777535	-0.19442001	-0.194898	-0.22047231	-0.2540757	-0.3739976
Bus	71	72	73	74	75	76	77
PTDF slack 12	-0.40636704	-0.57049397	-0.406367	-0.321739	-0.305702	-0.28494842	-0.2549823
Bus	78	79	80	81	82	83	84
PTDF slack 12	-0.253825	-0.2515479	-0.24497793	-0.2295062	-0.25051721	-0.25016723	-0.24962509
Bus	<u>85</u>	86	87	88	89	90	91
PTDF slack 12	-0.2493618	-0.24936182	-0.24936182	-0.24887509	-0.2485353	-0.24845956	-0.24836232
Bus	92	93	94	95	96	97	98
PTDF slack 12	-0.24821437	-0.24806378	-0.2479338	-0.24807325	-0.248249	-0.2466575	-0.2458634
Bus	<u>99</u>	100	101	102	103	104	105

<b>PTDF slack 12</b>	-0.2466652	-0.24733107	-0.2477101	-0.2480465	-0.24733107	-0.24733107	-0.24733
<b>Bus</b>	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b>110</b>	<b>111</b>	<b>112</b>
<b>PTDF slack 12</b>	-0.24733	-0.24733	-0.24733	-0.24733	-0.24733	-0.24733	-0.24733
<b>Bus</b>	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>	
<b>PTDF slack 12</b>	0.019897	0.086903	0.086714	-0.22047	0	-0.29596	
<b>Bus</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
<b>PTDF slack 19</b>	-0.005225877	-0.00491253	-0.00535887	-0.00602729	-0.0061293	-0.0054295	-0.00515994
<b>Bus</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>PTDF slack 19</b>	-0.00737266	-0.0073727	-0.00737266	-0.00514755	-0.00471931	-4.76E-03	-0.00438267
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 19</b>	-0.00345418	-0.0016184	0.00507786	0.002508403	0	0.039949152	0.068937895
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 19</b>	0.102058132	0.156348005	-0.75392927	0.088462644	0.059101006	0.080653063	0.072020193
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 19</b>	0.062498793	-0.00975537	0.05915671	0.08329164	-0.0373924	-0.0752502	-0.0756266
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 19</b>	-0.0755229	-0.0761323	-0.077048	-0.094803	-0.1054595	-0.1139829	-0.1376103
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 19</b>	-0.1089791	-0.1582182	-0.1762966	-0.1904162	-0.2016078	-0.19344115	-0.1942494
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 19</b>	-0.1949541	-0.195803	-0.19601195	-0.19659287	-0.1970263	-0.1972275	-0.1971152
<b>Bus</b>	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>	<b>61</b>	<b>62</b>	<b>63</b>
<b>PTDF slack 19</b>	-0.1962099	-0.196363	-0.1994456	-0.20031163	-0.20042676	-0.20016829	-0.20045903
<b>Bus</b>	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>	<b>70</b>
<b>PTDF slack 19</b>	-0.20100602	-0.20249466	-0.19913932	-0.19961731	-0.22519162	-0.258795	-0.3787169
<b>Bus</b>	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>
<b>PTDF slack 19</b>	-0.41108635	-0.57521328	-0.4110864	-0.3264583	-0.3104213	-0.28966773	-0.2597016
<b>Bus</b>	<b>78</b>	<b>79</b>	<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>
<b>PTDF slack 19</b>	-0.2585444	-0.2562673	-0.24969725	-0.2342255	-0.25523652	-0.25488654	-0.2543444
<b>Bus</b>	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>	<b>91</b>
<b>PTDF slack 19</b>	-0.2540811	-0.25408113	-0.25408113	-0.2535944	-0.2532547	-0.25317887	-0.25308163
<b>Bus</b>	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>
<b>PTDF slack 19</b>	-0.25293368	-0.2527831	-0.25265311	-0.25279256	-0.2529683	-0.2513768	-0.2505827
<b>Bus</b>	<b>99</b>	<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>	<b>105</b>

<b>PTDF slack 19</b>	-0.2513845	-0.25205038	-0.25242941	-0.2527658	-0.25205038	-0.25205038	-0.25205
<b>Bus</b>	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b>110</b>	<b>111</b>	<b>112</b>
<b>PTDF slack 19</b>	-0.25205	-0.25205	-0.25205	-0.25205	-0.25205	-0.25205	-0.25205
<b>Bus</b>	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>	
<b>PTDF slack 19</b>	0.015178	0.082183	0.081995	-0.22519	-0.00472	-0.30068	
<b>Bus</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
<b>PTDF slack 24</b>	0.748703396	0.749016745	0.748570404	0.747901986	0.74779995	0.74849977	0.748769332
<b>Bus</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>PTDF slack 24</b>	0.746556614	0.74655661	0.746556614	0.748781722	0.749209961	7.49E-01	0.749546601
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 24</b>	0.750475097	0.75231085	0.75900714	0.756437677	0.753929273	0.793878425	0.822867168
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 24</b>	0.855987405	0.910277278	0	0.842391917	0.813030279	0.834582336	0.825949466
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 24</b>	0.816428066	0.744173908	0.81308598	0.83722091	0.71653684	0.67867907	0.67830265
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 24</b>	0.67840642	0.67779702	0.6768813	0.6591262	0.6484698	0.6399464	0.6163189
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 24</b>	0.6449501	0.5957111	0.5776327	0.563513	0.5523215	0.56048812	0.55967988
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 24</b>	0.5589751	0.5581262	0.55791732	0.55733641	0.5569029	0.5567018	0.556814
<b>Bus</b>	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>	<b>61</b>	<b>62</b>	<b>63</b>
<b>PTDF slack 24</b>	0.5577193	0.5575663	0.5544837	0.55361764	0.55350252	0.55376099	0.55347024
<b>Bus</b>	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>	<b>70</b>
<b>PTDF slack 24</b>	0.55292326	0.55143461	0.55478995	0.55431196	0.52873765	0.4951343	0.3752124
<b>Bus</b>	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>
<b>PTDF slack 24</b>	0.34284292	0.17871599	0.3428429	0.4274709	0.443508	0.46426154	0.4942277
<b>Bus</b>	<b>78</b>	<b>79</b>	<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>
<b>PTDF slack 24</b>	0.4953849	0.497662	0.50423203	0.5197037	0.49869275	0.49904273	0.49958487
<b>Bus</b>	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>	<b>91</b>
<b>PTDF slack 24</b>	0.4998481	0.49984814	0.49984814	0.50033487	0.5006746	0.5007504	0.50084764
<b>Bus</b>	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>
<b>PTDF slack 24</b>	0.50099559	0.50114618	0.50127616	0.50113671	0.500961	0.5025524	0.5033465
<b>Bus</b>	<b>99</b>	<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>	<b>105</b>

<b>PTDF slack 24</b>	0.50254478	0.501878894	0.501499864	0.5011635	0.501878894	0.50187889	0.501879
<b>Bus</b>	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b>110</b>	<b>111</b>	<b>112</b>
<b>PTDF slack 24</b>	0.501879	0.501879	0.501879	0.501879	0.501879	0.501879	0.501879
<b>Bus</b>	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>	
<b>PTDF slack 24</b>	0.769107	0.836113	0.835924	0.528738	0.74921	0.453247	
<b>Bus</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
<b>PTDF slack 27</b>	-0.08587894	-0.08556559	-0.08601193	-0.08668035	-0.0867824	-0.08608257	-0.085813
<b>Bus</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>PTDF slack 27</b>	-0.08802572	-0.0880257	-0.08802572	-0.08580061	-0.08537238	-8.54E-02	-0.08503574
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 27</b>	-0.08410724	-0.0822715	-0.0755752	-0.07814466	-0.08065306	-0.04070391	-0.01171517
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 27</b>	0.021405069	0.075694942	-0.83458234	0.007809581	-0.02155206	0	-0.00863287
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 27</b>	-0.01815427	-0.09040843	-0.0214964	0.00263858	-0.1180455	-0.1559033	-0.1562797
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 27</b>	-0.1561759	-0.1567853	-0.1577011	-0.1754561	-0.1861125	-0.1946359	-0.2182634
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 27</b>	-0.1896322	-0.2388712	-0.2569496	-0.2710693	-0.2822609	-0.27409422	-0.27490246
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 27</b>	-0.2756072	-0.2764561	-0.27666502	-0.27724593	-0.2776794	-0.2778805	-0.2777683
<b>Bus</b>	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>	<b>61</b>	<b>62</b>	<b>63</b>
<b>PTDF slack 27</b>	-0.276863	-0.277016	-0.2800986	-0.28096469	-0.28107982	-0.28082135	-0.28111209
<b>Bus</b>	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>	<b>70</b>
<b>PTDF slack 27</b>	-0.28165908	-0.28314772	-0.27979238	-0.28027037	-0.30584469	-0.3394481	-0.4593699
<b>Bus</b>	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>
<b>PTDF slack 27</b>	-0.49173941	-0.65586634	-0.4917394	-0.4071114	-0.3910744	-0.37032079	-0.3403546
<b>Bus</b>	<b>78</b>	<b>79</b>	<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>
<b>PTDF slack 27</b>	-0.3391974	-0.3369203	-0.33035031	-0.3148786	-0.33588958	-0.33553961	-0.33499746
<b>Bus</b>	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>	<b>91</b>
<b>PTDF slack 27</b>	-0.3347342	-0.33473419	-0.33473419	-0.33424747	-0.3339077	-0.33383193	-0.3337347
<b>Bus</b>	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>
<b>PTDF slack 27</b>	-0.33358674	-0.33343616	-0.33330617	-0.33344563	-0.3336214	-0.3320299	-0.3312358
<b>Bus</b>	<b>99</b>	<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>	<b>105</b>

PTDF slack 27	-0.3320376	-0.33270344	-0.33308247	-0.3334189	-0.33270344	-0.33270344	-0.3327
Bus	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b>110</b>	<b>111</b>	<b>112</b>
PTDF slack 27	-0.3327	-0.3327	-0.3327	-0.3327	-0.3327	-0.3327	-0.3327
Bus	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>	
PTDF slack 27	-0.06548	0.00153	0.001342	-0.30584	-0.08537	-0.38134	
Bus	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
PTDF slack 40	0.100233594	0.100546943	0.100100601	0.099432184	0.09933015	0.100029968	0.100299529
Bus	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
PTDF slack 40	0.098086812	0.09808681	0.098086812	0.10031192	0.100740159	1.01E-01	0.101076798
Bus	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
PTDF slack 40	0.102005295	0.10384105	0.11053733	0.107967874	0.105459471	0.145408623	0.174397366
Bus	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
PTDF slack 40	0.207517603	0.261807476	-0.6484698	0.193922114	0.164560477	0.186112534	0.177479664
Bus	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
PTDF slack 40	0.167958264	0.095704106	0.16461618	0.18875111	0.06806703	0.03020927	0.02983284
Bus	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
PTDF slack 40	0.02993661	0.02932722	0.0284115	0.0106564	0	-0.0085234	-0.0321509
Bus	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
PTDF slack 40	-0.0035197	-0.0527587	-0.0708371	-0.0849568	-0.0961483	-0.08798168	-0.08878993
Bus	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
PTDF slack 40	-0.0894947	-0.0903436	-0.09055248	-0.0911334	-0.0915669	-0.091768	-0.0916558
Bus	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>	<b>61</b>	<b>62</b>	<b>63</b>
PTDF slack 40	-0.0907505	-0.0909035	-0.0939861	-0.09485216	-0.09496729	-0.09470882	-0.09499956
Bus	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>	<b>70</b>
PTDF slack 40	-0.09554655	-0.09703519	-0.09367985	-0.09415784	-0.11973215	-0.1533355	-0.2732574
Bus	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>
PTDF slack 40	-0.30562688	-0.46975381	-0.3056269	-0.2209989	-0.2049618	-0.18420826	-0.1542421
Bus	<b>78</b>	<b>79</b>	<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>
PTDF slack 40	-0.1530849	-0.1508078	-0.14423777	-0.1287661	-0.14977705	-0.14942707	-0.14888493
Bus	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>	<b>91</b>
PTDF slack 40	-0.1486217	-0.14862166	-0.14862166	-0.14813493	-0.1477952	-0.1477194	-0.14762216
Bus	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>
PTDF slack 40	-0.14747421	-0.14732363	-0.14719364	-0.14733309	-0.1475089	-0.1459174	-0.1451233
Bus	<b>99</b>	<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>	<b>105</b>

<b>PTDF slack 40</b>	-0.145925	-0.14659091	-0.14696994	-0.1473063	-0.14659091	-0.14659091	-0.14659
<b>Bus</b>	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b>110</b>	<b>111</b>	<b>112</b>
<b>PTDF slack 40</b>	-0.14659	-0.14659	-0.14659	-0.14659	-0.14659	-0.14659	-0.14659
<b>Bus</b>	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>	
<b>PTDF slack 40</b>	0.120637	0.187643	0.187454	-0.11973	0.10074	-0.19522	
<b>Bus</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
<b>PTDF slack 49</b>	0.18902352	0.189336868	0.188890527	0.18822211	0.18812007	0.188819893	0.189089455
<b>Bus</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>PTDF slack 49</b>	0.186876737	0.18687674	0.186876737	0.189101845	0.189530084	1.89E-01	0.189866724
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 49</b>	0.190795221	0.19263097	0.19932726	0.1967578	0.194249396	0.234198548	0.263187291
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 49</b>	0.296307528	0.350597401	-0.55967988	0.28271204	0.253350402	0.274902459	0.266269589
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 49</b>	0.256748189	0.184494031	0.25340611	0.27754104	0.15685696	0.11899919	0.11862277
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 49</b>	0.11872654	0.11811714	0.1172014	0.0994464	0.0887899	0.0802665	0.0566391
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 49</b>	0.0852703	0.0360312	0.0179528	0.0038332	-0.0073584	0.00080824	0
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 49</b>	-0.0007047	-0.0015536	-0.00176256	-0.00234347	-0.0027769	-0.0029781	-0.0028658
<b>Bus</b>	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>	<b>61</b>	<b>62</b>	<b>63</b>
<b>PTDF slack 49</b>	-0.0019605	-0.0021136	-0.0051962	-0.00606223	-0.00617736	-0.00591889	-0.00620963
<b>Bus</b>	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>	<b>70</b>
<b>PTDF slack 49</b>	-0.00675662	-0.00824526	-0.00488993	-0.00536791	-0.03094223	-0.0645456	-0.1844675
<b>Bus</b>	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>
<b>PTDF slack 49</b>	-0.21683695	-0.38096389	-0.216837	-0.1322089	-0.1161719	-0.09541834	-0.0654522
<b>Bus</b>	<b>78</b>	<b>79</b>	<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>
<b>PTDF slack 49</b>	-0.064295	-0.0620179	-0.05544785	-0.0399761	-0.06098712	-0.06063715	-0.060095
<b>Bus</b>	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>	<b>91</b>
<b>PTDF slack 49</b>	-0.0598317	-0.05983173	-0.05983173	-0.05934501	-0.0590053	-0.05892948	-0.05883224
<b>Bus</b>	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>
<b>PTDF slack 49</b>	-0.05868428	-0.0585337	-0.05840372	-0.05854317	-0.0587189	-0.0571274	-0.0563333
<b>Bus</b>	<b>99</b>	<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>	<b>105</b>

PTDF slack 49	-0.0571351	-0.05780098	-0.05818001	-0.0585164	-0.05780098	-0.05780098	-0.0578
Bus	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b>110</b>	<b>111</b>	<b>112</b>
PTDF slack 49	-0.0578	-0.0578	-0.0578	-0.0578	-0.0578	-0.0578	-0.0578
Bus	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>	
PTDF slack 49	0.209427	0.276433	0.276244	-0.03094	0.18953	-0.10643	
Bus	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
PTDF slack 59	0.19421969	0.194533039	0.194086698	0.19341828	0.19331624	0.194016064	0.194285626
Bus	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
PTDF slack 59	0.192072908	0.19207291	0.192072908	0.194298016	0.194726255	1.95E-01	0.195062894
Bus	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
PTDF slack 59	0.195991391	0.19782714	0.20452343	0.20195397	0.199445567	0.239394719	0.268383462
Bus	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
PTDF slack 59	0.301503699	0.355793572	-0.55448371	0.287908211	0.258546573	0.28009863	0.27146576
Bus	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
PTDF slack 59	0.26194436	0.189690202	0.25860228	0.28273721	0.16205313	0.12419536	0.12381894
Bus	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
PTDF slack 59	0.12392271	0.12331331	0.1223975	0.1046425	0.0939861	0.0854627	0.0618352
Bus	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
PTDF slack 59	0.0904664	0.0412274	0.023149	0.0090293	-0.0021623	0.00600441	0.00519617
Bus	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
PTDF slack 59	0.0044914	0.0036425	0.00343361	0.0028527	0.0024192	0.0022181	0.0023303
Bus	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>	<b>61</b>	<b>62</b>	<b>63</b>
PTDF slack 59	0.0032356	0.0030826	0	-0.00086606	-0.00098119	-0.00072272	-0.00101346
Bus	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>	<b>70</b>
PTDF slack 59	-0.00156045	-0.00304909	0.00030625	-0.00017174	-0.02574606	-0.0593494	-0.1792713
Bus	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>
PTDF slack 59	-0.21164078	-0.37576771	-0.2116408	-0.1270128	-0.1109757	-0.09022216	-0.060256
Bus	<b>78</b>	<b>79</b>	<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>
PTDF slack 59	-0.0590988	-0.0568217	-0.05025168	-0.03478	-0.05579095	-0.05544098	-0.05489883
Bus	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>	<b>91</b>
PTDF slack 59	-0.0546356	-0.05463556	-0.05463556	-0.05414884	-0.0538091	-0.0537333	-0.05363607
Bus	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>
PTDF slack 59	-0.05348811	-0.05333753	-0.05320754	-0.053347	-0.0535228	-0.0519313	-0.0511372
Bus	<b>99</b>	<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>	<b>105</b>

<b>PTDF slack 59</b>	-0.0519389	-0.05260481	-0.05298384	-0.0533202	-0.05260481	-0.05260481	-0.0526
<b>Bus</b>	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b>110</b>	<b>111</b>	<b>112</b>
<b>PTDF slack 59</b>	-0.0526	-0.0526	-0.0526	-0.0526	-0.0526	-0.0526	-0.0526
<b>Bus</b>	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>	
<b>PTDF slack 59</b>	0.214623	0.281629	0.281441	-0.02575	0.194726	-0.10124	
<b>Bus</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
<b>PTDF slack 61</b>	0.19520088	0.195514228	0.195067887	0.19439947	0.19429743	0.194997254	0.195266815
<b>Bus</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>PTDF slack 61</b>	0.193054098	0.1930541	0.193054098	0.195279205	0.195707444	1.96E-01	0.196044084
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 61</b>	0.196972581	0.19880833	0.20550462	0.20293516	0.200426756	0.240375908	0.269364652
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 61</b>	0.302484888	0.356774761	-0.55350252	0.2888894	0.259527762	0.281079819	0.272446949
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 61</b>	0.262925549	0.190671391	0.25958347	0.2837184	0.16303432	0.12517655	0.12480013
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 61</b>	0.1249039	0.1242945	0.1233787	0.1056237	0.0949673	0.0864439	0.0628164
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 61</b>	0.0914476	0.0422086	0.0241302	0.0100105	-0.0011811	0.0069856	0.00617736
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 61</b>	0.0054726	0.0046237	0.0044148	0.00383389	0.0034004	0.0031993	0.0033115
<b>Bus</b>	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>	<b>61</b>	<b>62</b>	<b>63</b>
<b>PTDF slack 61</b>	0.0042168	0.0040638	0.0009812	0.00011513	0	0.00025847	-3.23E-05
<b>Bus</b>	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>	<b>70</b>
<b>PTDF slack 61</b>	-0.00057926	-0.0020679	0.00128743	0.00080945	-0.02476487	-0.0583683	-0.1782901
<b>Bus</b>	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>
<b>PTDF slack 61</b>	-0.21065959	-0.37478653	-0.2106596	-0.1260316	-0.1099946	-0.08924098	-0.0592748
<b>Bus</b>	<b>78</b>	<b>79</b>	<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>
<b>PTDF slack 61</b>	-0.0581176	-0.0558405	-0.04927049	-0.0337988	-0.05480976	-0.05445979	-0.05391764
<b>Bus</b>	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>	<b>91</b>
<b>PTDF slack 61</b>	-0.0536544	-0.05365437	-0.05365437	-0.05316765	-0.0528279	-0.05275212	-0.05265488
<b>Bus</b>	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>
<b>PTDF slack 61</b>	-0.05250692	-0.05235634	-0.05222635	-0.05236581	-0.0525416	-0.0509501	-0.050156
<b>Bus</b>	<b>99</b>	<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>	<b>105</b>



<b>PTDF slack 61</b>	-0.0509577	-0.05162362	-0.05200265	-0.052339	-0.05162362	-0.05162362	-0.05162
<b>Bus</b>	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b>110</b>	<b>111</b>	<b>112</b>
<b>PTDF slack 61</b>	-0.05162	-0.05162	-0.05162	-0.05162	-0.05162	-0.05162	-0.05162
<b>Bus</b>	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>	
<b>PTDF slack 61</b>	0.215604	0.28261	0.282422	-0.02476	0.195707	-0.10026	
<b>Bus</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
<b>PTDF slack 70</b>	0.373490996	0.373804345	0.373358003	0.372689586	0.37258755	0.37328737	0.373556931
<b>Bus</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>PTDF slack 70</b>	0.371344214	0.37134421	0.371344214	0.373569322	0.37399756	3.74E-01	0.3743342
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 70</b>	0.375262697	0.37709845	0.38379474	0.381225276	0.378716873	0.418666024	0.447654768
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 70</b>	0.480775005	0.535064877	-0.3752124	0.467179516	0.437817879	0.459369936	0.450737065
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 70</b>	0.441215666	0.368961507	0.43787358	0.46200851	0.34132443	0.30346667	0.30309025
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 70</b>	0.30319402	0.30258462	0.3016689	0.2839138	0.2732574	0.264734	0.2411065
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 70</b>	0.2697377	0.2204987	0.2024203	0.1883006	0.1771091	0.18527572	0.18446748
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 70</b>	0.1837627	0.1829138	0.18270492	0.18212401	0.1816905	0.1814894	0.1816016
<b>Bus</b>	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>	<b>61</b>	<b>62</b>	<b>63</b>
<b>PTDF slack 70</b>	0.1825069	0.1823539	0.1792713	0.17840524	0.17829012	0.17854858	1.78E-01
<b>Bus</b>	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>	<b>70</b>
<b>PTDF slack 70</b>	0.17771086	0.17622221	0.17957755	0.17909956	0.15352525	0.1199219	0
<b>Bus</b>	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>
<b>PTDF slack 70</b>	-0.03236948	-0.19649641	-0.0323695	0.0522585	0.0682956	0.08904914	0.1190153
<b>Bus</b>	<b>78</b>	<b>79</b>	<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>
<b>PTDF slack 70</b>	0.1201725	0.1224496	0.12901963	0.1444913	0.12348035	0.12383033	0.12437247
<b>Bus</b>	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>	<b>91</b>
<b>PTDF slack 70</b>	0.1246357	0.12463574	0.12463574	0.12512247	0.1254622	0.125538	0.12563524
<b>Bus</b>	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>
<b>PTDF slack 70</b>	0.12578319	0.12593378	0.12606376	0.12592431	0.1257486	0.12734	0.1281341
<b>Bus</b>	<b>99</b>	<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>	<b>105</b>

PTDF slack 70	0.12733238	0.126666494	0.126287464	0.1259511	0.126666494	0.12666649	0.126666
Bus	106	107	108	109	110	111	112
PTDF slack 70	0.126666	0.126666	0.126666	0.126666	0.126666	0.126666	0.126666
Bus	113	114	115	116	117	118	
PTDF slack 70	0.393894	0.4609	0.460712	0.153525	0.373998	0.078035	
Bus	1	2	3	4	5	6	7
PTDF slack 85	0.248855253	0.249168601	0.24872226	0.248053843	0.2479518	0.248651626	0.248921188
Bus	8	9	10	11	12	13	14
PTDF slack 85	0.24670847	0.24670847	0.24670847	0.248933578	0.249361817	2.49E-01	0.249698457
Bus	15	16	17	18	19	20	21
PTDF slack 85	0.250626954	0.25246271	0.25915899	0.256589533	0.254081129	0.294030281	0.323019024
Bus	22	23	24	25	26	27	28
PTDF slack 85	0.356139261	0.410429134	-0.49984814	0.342543773	0.313182135	0.334734192	0.326101322
Bus	29	30	31	32	33	34	35
PTDF slack 85	0.316579922	0.244325764	0.31323784	0.33737277	0.21668869	0.17883092	0.1784545
Bus	36	37	38	39	40	41	42
PTDF slack 85	0.17855827	0.17794887	0.1770331	0.1592781	0.1486217	0.1400983	0.1164708
Bus	43	44	45	46	47	48	49
PTDF slack 85	0.145102	0.095863	0.0777846	0.0636649	0.0524733	0.06063998	0.05983173
Bus	50	51	52	53	54	55	56
PTDF slack 85	0.059127	0.0582781	0.05806917	0.05748826	0.0570548	0.0568537	0.0569659
Bus	57	58	59	60	61	62	63
PTDF slack 85	0.0578712	0.0577182	0.0546356	0.0537695	0.05365437	0.05391284	5.36E-02
Bus	64	65	66	67	68	69	70
PTDF slack 85	0.05307511	0.05158647	0.05494181	0.05446382	0.0288895	-0.0047139	-0.1246357
Bus	71	72	73	74	75	76	77
PTDF slack 85	-0.15700522	-0.32113215	-0.1570052	-0.0723772	-0.0563402	-0.0355866	-0.0056205
Bus	78	79	80	81	82	83	84
PTDF slack 85	-0.0044632	-0.0021861	0.00438388	0.0198556	-0.00115539	-0.00080541	-0.00026327
Bus	85	86	87	88	89	90	91
PTDF slack 85	0	0	0	0.00048673	0.0008265	0.00090226	0.0009995
Bus	92	93	94	95	96	97	98
PTDF slack 85	0.00114745	0.00129803	0.00142802	0.00128857	0.0011128	0.0027043	0.0034984
Bus	99	100	101	102	103	104	105

<b>PTDF slack 85</b>	0.00269664	0.00203075	0.00165172	0.0013153	0.00203075	0.00203075	0.002031
<b>Bus</b>	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b>110</b>	<b>111</b>	<b>112</b>
<b>PTDF slack 85</b>	0.002031	0.002031	0.002031	0.002031	0.002031	0.002031	0.002031
<b>Bus</b>	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>	
<b>PTDF slack 85</b>	0.269259	0.336264	0.336076	0.02889	0.249362	-0.0466	
<b>Bus</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
<b>PTDF slack 99</b>	0.246158614	0.246471962	0.246025621	0.245357204	0.24525516	0.245954988	0.246224549
<b>Bus</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>PTDF slack 99</b>	0.244011832	0.24401183	0.244011832	0.24623694	0.246665178	2.47E-01	0.247001818
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 99</b>	0.247930315	0.24976607	0.25646235	0.253892894	0.251384491	0.291333642	0.320322386
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 99</b>	0.353442623	0.407732495	-0.50254478	0.339847134	0.310485496	0.332037554	0.323404683
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 99</b>	0.313883284	0.241629125	0.3105412	0.33467613	0.21399205	0.17613429	0.17575786
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 99</b>	0.17586163	0.17525224	0.1743365	0.1565815	0.145925	0.1374016	0.1137741
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 99</b>	0.1424053	0.0931663	0.0750879	0.0609682	0.0497767	0.05794334	0.05713509
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 99</b>	0.0564304	0.0555815	0.05537254	0.05479162	0.0543582	0.054157	0.0542693
<b>Bus</b>	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>	<b>61</b>	<b>62</b>	<b>63</b>
<b>PTDF slack 99</b>	0.0551746	0.0550215	0.0519389	0.05107286	0.05095773	0.0512162	5.09E-02
<b>Bus</b>	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>	<b>70</b>
<b>PTDF slack 99</b>	0.05037847	0.04888983	0.05224517	0.05176718	0.02619287	-0.0074105	-0.1273324
<b>Bus</b>	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>
<b>PTDF slack 99</b>	-0.15970186	-0.32382879	-0.1597019	-0.0750738	-0.0590368	-0.03828324	-0.0083171
<b>Bus</b>	<b>78</b>	<b>79</b>	<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>
<b>PTDF slack 99</b>	-0.0071599	-0.0048828	0.00168725	0.0171589	-0.00385203	-0.00350205	-0.00295991
<b>Bus</b>	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>	<b>91</b>
<b>PTDF slack 99</b>	-0.0026966	-0.00269664	-0.00269664	-0.00220991	-0.0018702	-0.00179438	-0.00169714
<b>Bus</b>	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>
<b>PTDF slack 99</b>	-0.00154919	-0.00139861	-0.00126862	-0.00140807	-0.0015838	7.65E-06	0.0008017
<b>Bus</b>	<b>99</b>	<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>	<b>105</b>

<b>PTDF slack 99</b>	0	-0.00066589	-0.00104492	-0.0013813	-0.00066589	-0.00066589	-0.00067
<b>Bus</b>	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b>110</b>	<b>111</b>	<b>112</b>
<b>PTDF slack 99</b>	-0.00067	-0.00067	-0.00067	-0.00067	-0.00067	-0.00067	-0.00067
<b>Bus</b>	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>	
<b>PTDF slack 99</b>	0.266562	0.333568	0.333379	0.026193	0.246665	-0.0493	
<b>Bus</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
<b>PTDF slack 110</b>	0.246824502	0.247137851	0.24669151	0.246023092	0.24592105	0.246620876	0.246890438
<b>Bus</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>PTDF slack 110</b>	0.24467772	0.24467772	0.24467772	0.246902828	0.247331067	2.47E-01	0.247667707
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>PTDF slack 110</b>	0.248596203	0.25043196	0.25712824	0.254558783	0.252050379	0.291999531	0.320988274
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>PTDF slack 110</b>	0.354108511	0.408398384	-0.50187889	0.340513023	0.311151385	0.332703442	0.324070572
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>PTDF slack 110</b>	0.314549172	0.242295014	0.31120709	0.33534202	0.21465794	0.17680017	0.17642375
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>PTDF slack 110</b>	0.17652752	0.17591812	0.1750024	0.1572473	0.1465909	0.1380675	0.11444
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>PTDF slack 110</b>	0.1430712	0.0938322	0.0757538	0.0616341	0.0504426	0.05860923	0.05780098
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>PTDF slack 110</b>	0.0570962	0.0562473	0.05603842	0.05545751	0.055024	0.0548229	0.0549351
<b>Bus</b>	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>	<b>61</b>	<b>62</b>	<b>63</b>
<b>PTDF slack 110</b>	0.0558404	0.0556874	0.0526048	0.05173875	0.05162362	0.05188209	5.16E-02
<b>Bus</b>	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>	<b>70</b>
<b>PTDF slack 110</b>	0.05104436	0.04955572	0.05291106	0.05243307	0.02685875	-0.0067446	-0.1266665
<b>Bus</b>	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>
<b>PTDF slack 110</b>	-0.15903597	-0.3231629	-0.159036	-0.074408	-0.0583709	-0.03761735	-0.0076512
<b>Bus</b>	<b>78</b>	<b>79</b>	<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>
<b>PTDF slack 110</b>	-0.006494	-0.0042169	0.00235313	0.0178248	-0.00318614	-0.00283616	-0.00229402
<b>Bus</b>	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>	<b>91</b>
<b>PTDF slack 110</b>	-0.0020308	-0.00203075	-0.00203075	-0.00154402	-0.0012043	-0.00112849	-0.00103125
<b>Bus</b>	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>
<b>PTDF slack 110</b>	-0.0008833	-0.00073272	-0.00060273	-0.00074218	-0.0009179	6.74E-04	0.0014676
<b>Bus</b>	<b>99</b>	<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>	<b>105</b>

<b>PTDF slack 110</b>	0.00066589	0	-0.00037903	-0.0007154	0	0	0
<b>Bus</b>	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b>110</b>	<b>111</b>	<b>112</b>
<b>PTDF slack 110</b>	0	0	2.41E-35	1.20E-35	0	0	0
<b>Bus</b>	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>	
<b>PTDF slack 110</b>	0.267228	0.334234	0.334045	0.026859	0.247331	-0.04863	
<b>Bus</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
<b>ZS slack 12</b>	-0.229468006	-0.22541591	-0.22684564	-0.22095527	-0.2202362	-0.22181306	-0.22166374
<b>Bus</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>ZS slack 12</b>	-0.21700847	-0.2201711	-0.21559244	-0.22221924	0	-2.23E-01	-0.21870422
<b>Bus</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>
<b>ZS slack 12</b>	-0.21112969	-0.2216001	-0.2173941	-0.21653133	-0.2136824	-0.25478029	-0.28347339
<b>Bus</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>
<b>ZS slack 12</b>	-0.31250845	-0.35177285	0.581722946	-0.28184065	-0.25527284	-0.30094506	-0.29375801
<b>Bus</b>	<b>29</b>	<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>
<b>ZS slack 12</b>	-0.28302169	-0.20220587	-0.2781919	-0.3026363	-0.1611607	-0.1042271	-0.1041867
<b>Bus</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>	<b>41</b>	<b>42</b>
<b>ZS slack 12</b>	-0.104418	-0.1026171	-0.1009993	-0.0744939	-0.057291	-0.0439554	-0.0067029
<b>Bus</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>ZS slack 12</b>	-0.0529269	0.0238101	0.051614	0.0719533	0.0858693	0.07573218	0.07599278
<b>Bus</b>	<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>
<b>ZS slack 12</b>	0.0797009	0.0847963	0.08628733	0.08853439	0.0882456	0.0888806	0.0884888
<b>Bus</b>	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>	<b>61</b>	<b>62</b>	<b>63</b>
<b>ZS slack 12</b>	0.0851215	0.0866623	0.0887703	0.08750072	0.08712248	0.08690864	8.76E-02
<b>Bus</b>	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>	<b>70</b>
<b>ZS slack 12</b>	0.08659565	0.08416061	0.08057457	0.08428337	0.11508629	0.1526866	0.2744479
<b>Bus</b>	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>
<b>ZS slack 12</b>	0.30132103	0.44216469	0.3002473	0.2385946	0.2214001	0.21153287	0.1742184
<b>Bus</b>	<b>78</b>	<b>79</b>	<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>
<b>ZS slack 12</b>	0.174214	0.171705	0.15935886	0.1316361	0.18508597	0.18785845	0.18965545
<b>Bus</b>	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>	<b>91</b>
<b>ZS slack 12</b>	0.1883848	0.19054819	0.1858847	0.18584795	0.1779356	0.19007944	0.19063796
<b>Bus</b>	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>
<b>ZS slack 12</b>	0.18646348	0.18850214	0.18742126	0.18772556	0.1825975	1.72E-01	0.1713672
<b>Bus</b>	<b>99</b>	<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>	<b>105</b>

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<b>ZS slack 12</b>	0.18153122	0.185543625	0.189993047	0.1883795	0.194975869	0.20471157	0.207887
<b>Bus</b>	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>	<b><u>110</u></b>	<b>111</b>	<b>112</b>
<b>ZS slack 12</b>	0.208056	0.213228	2.09E-01	2.09E-01	0.208291	0.206575	0.20853
<b>Bus</b>	<b>113</b>	<b>114</b>	<b>115</b>	<b>116</b>	<b>117</b>	<b>118</b>	
<b>ZS slack 12</b>	0.208056	0.213228	2.09E-01	2.09E-01	0.208291	0.206575	