STUDIES

ON

STOCK INDEX FUTURES PRICING

A UK PERSPECTIVE

Pradeep Kumar Yadav

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ABSTRACT

There has been considerable interest among market participants, market regulators and academics in the pricing of stock index futures contracts. Academic research in this area has been motivated by several considerations. First, the utility of these contracts for risk allocation and price discovery depends on the efficiency with which they are priced relative to the underlying index. Second, it has been widely believed that they have adverse impact on price dynamics in the stock market. Third, and most important, stock index futures offer the possibility of directly studying the economics of arbitrage in the context of market microstructure.

This dissertation extends the theoretical framework on stock index futures pricing in two directions. First, within the static cost of carry framework, it generalises the forward pricing formula by allowing for cash market settlement procedures. Second, it shows that in the presence of arbitrage related transaction costs, the time series of stock index futures "mispricing" can be modelled as a threshold autoregressive (TAR) process, a piecewise linear autoregressive process in which the process parameters are path dependent. The TAR model is potentially attractive for many financial applications and this dissertation appears to be the first use of the TAR model in finance.

This dissertation also provides substantial and significant new empirical evidence relevant to the theoretical issues involved. *Inter-alia,* it analyses several important aspects not adequately examined in past research, and it utilises the unique microstructural features of the London stock market to explore several major theoretical issues. The empirical analysis is based mainly on about four years of "time and sales" transactions data from the London International Financial Futures Exchange together with synchronous hourly cash index data.

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND AND MOTIVATION

Stock index futures are relatively new to the world of finance. The first contract was introduced in February 1982 by the Kansas City Board of Trade in the USA. It was based on the geometrically averaged value line index published by Arnold Bernard and Company. Two months later, the Chicago Mercantile Exchange introduced its Standard and Poor's 500 Index futures contract. The New York Stock Exchange followed in May 1982 with its NYSE Composite Index futures contract. Since then growth has been phenomenal. As Rubenstein (1989) points out, "the daily [dollar] trading volume ... in just ... the near term S&P index future, is comparable to the daily dollar trading volume on the NYSE. The resultant economies of scale and competition among hundreds of floor traders makes this market among the most liquid in the world".

The London International Financial Futures Exchange (LIFFE) introduced futures contracts on the FTSE100 index in May 1984. Volume and Open interest in these contracts has grown steadily over the years even in the UK. Average daily volume has increased from an average of about 350 contracts in 1984 to about 500 contracts in 1986, about 2000 contracts in 1988 and about 6000 contracts in 1990. Open interest in these contracts has grown even more phenomenally from about 1000 contracts in 1984, to about 2500 contracts in 1986, about 12000 contracts in 1988 and about 30000 contracts in 1990.

There are several reasons for this high level of trading activity in index futures. First, they make it possible to trade an entire basket of stocks. Second, they enable short positions to be easily taken. Third, they offer significantly lower transaction costs. In economic terms, index futures provide a convenient means for allocating risk and facilitate price discovery for the stock market as a whole, thereby allocating the resources of the economy more efficiently. Strategically, index futures are extremely useful hedging and investment management tools. In particular, they have the following uses:[1,2]

(a) Hedging price risk: Market makers and underwriters can eliminate the systematic risk component of the risk they bear when they hold unbalanced inventories of stock. Similarly, portfolio managers faced with the prospect of temporary unwanted market volatility (eg during the Gulf war) can protect their portfolios by selectively selling futures.

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1 The economic uses of futures and options, in general, have been discussed eg by Carlton (1984), Jaffee (1984) and Telser (1986).

2 The classification below is largely based on Stoll and Whaley (1988).

(b) Hedging cash flows: Pension funds and unit trusts have an irregular stream of cash flows. Many of these cash flows can be inadequate to justify purchasing/selling incremental shares of all stocks held by the fund, or it may be difficult to invest/disinvest efficiently in securities specified by the funds' investment objectives. Such cash flows can be expected (eg dividends) or unexpected (eg random changes in investor interest). Stock index futures are a suitable intermediary for market exit/entry under such circumstances.

(c) Stock selection: Security analysts who believe that they have special skills in picking undervalued stocks can totally hedge themselves against market risk and thereby earn the riskless return plus a return that represents only their (superior!) stock selection skills.

(d) Market Timing: Security analysts who believe that they have special skills in market timing, and specifically in predicting stock market returns relative to returns in the bond market and the returns from short term debt instruments, can use index futures to easily alter the beta of their portfolio and have a suitable "asset allocation".

(e) Portfolio Insurance: Portfolio insurance programs generally synthesise a put by dynamically altering the mix between a stock portfolio and a debt portfolio. The switch between stock and debt can be made much more easily and cost effectively using index futures rather than by trading underlying stocks.

It is important to note that index futures are used by investors primarily for short term trading purposes, rather than as a relatively long term alternative market basket.[3] On a typical day in the index futures market, May 11, 1989, the volume of trading across all four domestic US exchanges (NYFE, CME, KC and CBT) was equivalent to $7.2 billion, roughly one-third of that day's open interest of $22.9 billion.[4] On the other hand, only 0.5% of the shares outstanding were traded on the underlying cash market. Moreover, the outstanding market value of publicly available index funds (not including private index funds ie those with only one shareholder) was at least $175 billion, or more than 8 times the open interest in index futures.[5] Considering the largely short term nature of positions in the index futures market, the use of index futures contracts for allocating risk, or as vehicles for price discovery, depends critically on the efficiency with which these contracts are priced relative to the underlying cash index. Suppose, for example, that a market maker uses index futures to hedge a long position in a portfolio identical to

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[3] Several other market basked portfolios are feasible eg index funds, index participations and exchange stock portfolios. See Rubenstein (1989) for a comprehensive review.

[4] It is interesting to observe that the fraction of open interest traded has steadily declined in the UK from about 30% in 1984 to about 15% in 1990. The reasons for this are not pursued in this dissertation, but form an interesting avenue for future research.

[5] Figures taken from Rubenstein (1989) pp 8, footnote 10.

the underlying index. If the "mispricing"[6] of the futures contract when the hedge position is initiated is 1% and the mispricing when the hedge position is closed is -1%, then the market maker faces additional costs arising due to mispricing of 2%. Unless mispricing is identically zero, short term users of index futures markets would always face additional risk arising due to changes in mispricing. This is an important motivation for some of the published work on the relative pricing of index futures and their underlying stock indices.[7]

There is another more important reason for the academic interest in the cash futures pricing relationship. The theory of finance is based on the assumption that assets with equivalent cash flows must be identically priced to prevent riskless arbitrage. The index futures contract and the underlying cash index represent equivalent assets (except for a largely non-stochastic factor - the cost of carrying the underlying asset up to futures maturity). More importantly, the associated arbitrage strategies are easy to implement and essentially risk free[8] with clearly quantifiable transaction costs. This cannot be so in the case of pricing of primary assets like stocks where "arbitrage" can only be with reference to expectations (as discussed in Shleifer and Summers, 1990), and is not so even in the case of pricing of other derivative assets

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6 The difference between the actual index futures price and the index futures price "equivalent" to the underlying cash price (and various transformations thereof) has been labelled in the literature as "mispricing".

7 See eg Merrick (1988) and Hill *et al* (1988).

8 The arbitrage strategies are not *completely* risk free because future dividends and interest rates are not known *ex ante* with perfect certainty.

like options where arbitrage would necessitate continuous rebalancing and hence infinite transaction costs. Hence, an analysis of the cash futures relative pricing relationship is a direct study of a series of price differences between equivalent assets and can shed light on the extent to which it is reasonable to use arbitrage arguments eg for pricing more complex securities like options, where arbitrage is much more difficult, or elsewhere in corporate financial theory eg capital structure and dividend policy. More specifically, it can enable inferences relating to arbitrage in the context of market microstructure eg the profitability of arbitrage strategies[9], the effectiveness of arbitrageurs in maintaining an effective link[10], the nature of the arbitrage process ie monopolistic, imperfectly competitive or perfectly competitive[11], the effective transaction costs of different categories of arbitrageurs[12], the effects of short sale restrictions[13], and the intertemporal strategies followed by arbitrageurs from a game theoretic viewpoint or in the context of early unwinding/rollover options.[14]

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9 Eg Finnerty and Park (1988); Merrick (1989); Klemkosky and Lee (1991); and Chapters 2 and 6 in this dissertation.

10 Eg Cornell and French (1983); Modest and Sundereshan (1983); Figlewski (1984a); Mackinlay and Ramaswamy (1988); and Chapters 2 and 6 in this dissertation.

11 Eg Cooper and Mello (1990) and Holden (1990a,b,c).

12 Eg Chapter 4 in this dissertation.

13 Eg Puttonen and Martikainen (1991) and in Chapter 6 in this dissertation.

14 Eg Merrick (1989); Brennan and Schwartz (1990).

Another major reason for the interest in stock index futures pricing is that index arbitrage has been very controversial in the US. Market participants have widely believed that it increases stock price volatility.[15] But, academics have viewed it as desirable in terms of enforcing the "law of one price" and have thus "tended to dismiss the recent torrent of complaints about index arbitrage and volatility as mostly hysteria" (see eg Miller, 1990, pp 187-188). However, the critics of index arbitrage and the academic defenders have used the word *volatility* in different senses. To academics, volatility is the variability of the rate of return obtained by *holding* stocks over intervals like an hour, a day, a week or a month. On the other hand, the practitioner critics of index arbitrage are concerned with "the *velocity* of prices, in the sense of the very rapid, minute-to-minute (sometimes even second-to-second) sequences of price moves" that index arbitrage programs sometimes cause. These bursts of velocity affect the *trading* rather than the *holding* of stocks and may be regarded as potentially damaging the market, irrespective of longer term variability.

Hence, the motivation for studying the pricing of stock index futures contracts can broadly be summarised as follows: first, stock index futures are highly liquid instruments widely used for short term trading/portfolio adjustments and the utility of these contracts for risk allocation/price discovery depends on the "efficiency" with which they are priced relative to the underlying index; second, an analysis of

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15 See eg NYSE report on *Market Volatility and Investor Confidence* (1990, pp 16-19) and Miller (1991, pp 228).

this relative pricing relationship is a direct study of the economics of arbitrage in the context of market microstructure; and finally, index arbitrage has been very controversial in regard to the impact on the underlying stock market.

There is substantial empirical evidence on the pricing of index futures traded on US markets. *Inter-alia* Cornell and French (1983), Modest and Sundereshan (1983), Figlewski (1984) and Mackinlay and Ramaswamy (1988) document the existence of substantial and sustained deviations between the actual index futures price and the index futures price "equivalent" (on the basis of the forward pricing formula) to the price in the underlying cash market. Finnerty and Park (1988), Merrick (1989) and Klemkosky and Lee (1991) demonstrate the profitability of cash futures arbitrage strategies. Arditti *et al* (1986) show that passive index arbitrage has outperformed high ranking US mutual funds.[16] Saunders and Mahajan (1988) examine pricing efficiency of index futures contracts relative to the cash index using price differences instead of price levels. Merrick (1988) and Hill *et al* (1988) explore the implications of cash futures arbitrage for short term hedgers and portfolio insurers respectively. Brennan and Schwartz (1990) attempt to value the early unwinding option. Holden (1990c) tests his intertemporal arbitrage trading model based on

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[16] Such evidence has puzzled financial economists. Figlewski (1984b) has put forward disequilibrium arguments suggesting that mispricing exists, and persists, because of "unfamiliarity" with the new markets and institutional inertia in developing systems to take advantage of the opportunities they present". Rubenstein (1987) is "forced to the conclusion that ... the growth in index futures continues to outstrip the amounts of capital, that are available for arbitrage". MacKinlay and Ramaswamy (1988, pp 138) suggest that mispricing is "presumably affected by the flow of orders as well as by the difference of opinion among participants regarding parameters of the valuation market that provides 'fair values'".

hypotheses about the actions of arbitrageurs. Billingsley and Chance (1988) report on the pricing of stock index futures spreads. Cornell (1985b), Dyl and Maberly (1986a, 1986b), Junkus (1986), Maberly (1986), Keim and Smirlock (1987), Phillips-Patrick and Schneeweis (1988), and Maberly *et al* (1989) analyse intraweek seasonalities in index futures prices.

At a theoretical level, Garbade and Silber (1983) model differential price discovery and risk transfer in cash and futures markets[17] based on the assumption of a *finite* elasticity of intermarket arbitrage services (identical for all market states) and of random walk evolution in reservation prices. Brennan and Schwartz (1990) solve for the value of the early unwinding option on the assumption that cash futures mispricing follows a Brownian Bridge process. Holden (1990b) develops an intertemporal arbitrage trading model in which arbitrageurs act to maximise their profits in a Nash equilibrium. Cooper and Mello (1990) model cash futures mispricing as a path sensitive process dependent also on the number of open arbitrage positions. Finally, Miller *et al* (1991), suggest that the empirically observed mean reversion in futures mispricing, arises not because of index arbitrage activity, but because of the manifestations of non-synchronous trading in index stocks.

[17] This is a general model for commodity futures markets, applicable also to index futures markets.

On the basis of the above, the following could be identified as the *major* "gaps" in the literature, which motivated the work in this dissertation:

(a) There has been little published work on index futures markets outside the US institutional environment. In particular, there has been no (published) work on British markets.

(b) The cash-futures pricing relationships and the spread pricing relationships have been analysed in the literature without controlling for cash market settlement procedures.

(c) Though the risks necessarily involved in index arbitrage have been recognised in the literature, they have not been quantitatively estimated in terms of their effect on observed futures mispricing.

(d) Previous research has recognised that transaction costs must have a crucial role in explaining futures mispricing, but relatively little attention has been given to modelling or empirically testing the time series process governing futures mispricing in a context where transaction costs are considered explicitly and estimated accurately.

(e) Though the existence of mean reversion in futures mispricing has been recognised, there is no evidence on how this mean reversion varies with factors that are potentially relevant to the arbitrage process, eg the previous period mispricing, the time to maturity, the day of the week and hour of the day.

(f) There has been little published work on the theory and relative pricing of index futures contracts with different times to expiration even though the practical relevance for arbitrageurs, and the theoretical issues involved, are essentially similar to those involved in cash futures arbitrage.

(g) There is no evidence on intraweek and intraday seasonalities in the risk premia in the futures market *relative* to the risk premia in the cash market, particularly in the context of the differences in the microstructure of these markets.

Why are each of these "gaps" important?

British Evidence

Analysis of evidence from British markets is important for the following reasons:

(i) There are major differences in policy perceptions in the US and the UK. Index arbitrage is apparently discouraged in the USA and it is clearly encouraged in London. For example, in the context of Black

Monday, a report of the US Securities and Exchange Commission says "... futures trading, and strategies involving the use of futures, were not the 'sole cause' ... Nevertheless the existence of futures on stock indices, and the use of various strategies involving 'program trading' (ie index arbitrage, index substitution and portfolio insurance) were a significant factor in accelerating and exacerbating the decline".[18]On the other hand, Sir Nicholas Goodison, the then Chairman of the London Stock Exchange, in a widely reported letter to the Secretary of State for Trade and Industry, said that the "Quality of Markets Committee makes a good case for facilitating index arbitrage between the options and futures markets and the underlying equity market ... a conclusion ... completely at variance with the conclusions ... drawn in the United States .... Our conclusion therefore is that the US experience does not have immediate application to the management of our markets in the United Kingdom."

(ii) The microstructure of the UK cash market is very different from that of the US. The London Stock Market is a pure dealership market while the US markets are a hybrid of dealership and continuous auction systems. In this context, it is important to examine evidence

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[18] Such a conclusion is strongly disputed by some academic reviews of the crash (eg Fama, 1988; and Edwards, 1988).

on the extent to which arbitrageurs operating within different market trading systems and different institutional perspectives, have been able to ensure a "fair" spread between index futures prices and underlying cash prices.

(iii) Estimates of futures "mispricing" in the US are susceptible to measurement error in relation to non-synchronous trading in index stocks (Scholes and Williams, 1977; Cohen *et al*, 1986; Lo and Mackinlay, 1990) because cash index values based on transaction prices will, in general, not be actually tradeable values. Hence, significant mispricing values may not always represent true arbitrage opportunities. On the other hand, the UK stock index analysed in this paper is based on quotes which respective market makers are obliged to trade up to very large sizes. The stock index values thus represent actually tradeable values synchronous with futures prices. We recognise that even with such a quote based index, differences in the price adjustment delays within different index stocks will generate positive serial correlation, and result in the reported cash index value being different from the "true" value corresponding to a frictionless market. Nevertheless, the arbitrage opportunities generated with data based on such a quote based index are potentially exploitable and hence economically significant.

(iv) The London markets provide an ideal laboratory for testing the effect on futures pricing of the constraints that exist on short selling of stocks. It has been suggested that the observed preponderance of negative mispricing can be at least partially explained by the institutional restrictions and difficulties[19] that exist in selling stocks short, since the costs involved in exploiting negative mispricing are higher than the corresponding costs of exploiting positive mispricing (see eg Modest and Sundereshan, 1984; Figlewski, 1984b; Brenner *et al*, 1989; and Puttonen and Martikainen, 1991). However, it has not been possible for US based studies to formally test this hypothesis. The unique features of the settlement procedures on the London Stock Exchange can be utilised to examine the behaviour of index futures pricing when there are virtually no constraints on short selling.

Settlement Procedures

Published empirical work on index futures pricing has implicitly assumed that cash market transactions are settled immediately. This assumption is clearly incorrect. Not only that, it can lead to significantly biased inferences in regard to futures mispricing particularly in markets like those of London and Paris, where cash market settlement takes place on a fixed future date rather than within a fixed

[19] Eg margin requirements and the uptick rule in the US.

period. It is necessary to develop the settlement adjusted forward pricing formula and control for cash market settlement procedures in the empirical analysis.

Risk in Arbitrage

It has been widely recognised that index arbitrage is not riskless because of several factors - more importantly the uncertainty about the magnitude and timing of dividends, the stochastic nature of daily marking to market cash flows, and potential delays in actual execution of arbitrage trades. However, the factors which make index arbitrage risky have typically been ignored and the risk premium or the increase in the width of the arbitrage window that can be expected to exist on this account, has not been explicitly estimated. In fact, most US studies (eg Mackinlay and Ramaswamy, 1988; Klemkosky and Lee, 1991; Bhatt and Caciki, 1990) suffer from a potentially serious dividend misspecification problem. *Ex post* dividend data on the S&P500 index is not publicly available and these researchers have used dividend data from the Center for Research in Security Prices (CRSP) corresponding to the NYSE/AMEX portfolio instead of constructing a series based on individual stock dividends. Because the NYSE/AMEX portfolio contains a higher proportion of small firms than the S&P500 portfolio, it is likely that the dividend yield on the NYSE/AMEX portfolio will be different from the S&P500 dividend yield. It is important to simulate *ex post* the risks that have been involved in index arbitrage due to dividend uncertainty, interest rate uncertainty and delays in trade execution in the context of the information actually available *ex ante* to the potential arbitrageur.

Time Series Evolution and Transaction Costs

Transaction costs of different categories of arbitrageurs must clearly have a role in explaining the price differences between equivalent assets. As Mackinlay and Ramaswamy (1988, pp 137,138) emphasize, if an arbitrage "link is maintained effectively, then investors who are committed to trade will recognise these markets as perfect substitutes, and their choice between these markets will be dictated by convenience and *transaction costs*".[20] If a better understanding of the economics of intermarket arbitrage is to be obtained, it is necessary to have a model capable of explaining how mispricing evolves over time in the context of these differential transaction costs of different categories of arbitrageurs. It is also well known that new information tends to get incorporated into futures prices faster than in cash prices because of lower transaction costs and greater liquidity. Hence, a model for the time series evolution of mispricing should also allow the cash and futures markets to play different roles in price discovery.[21]

An institutionally rich model of the time series evolution of futures mispricing is important also for two other reasons. First, the stochastic process for mispricing

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[20] Italics added.

[21] Eg Garbade and Silber (1983).

is the key input in decisions relating to opening new arbitrage positions or closing existing arbitrage positions, particularly in the context of the early unwinding option and the rollover option discussed by Merrick (1989). Second, stochastic changes in mispricing are important determinants of the costs and hence the optimal strategies of several non-arbitrage categories of market participants such as short term hedgers, portfolio insurers, and those hedging written option positions in the long term OTC market.

<u>Mean Reversion</u>

An empirical investigation of the factors, that affect the behaviour of mean reversion in stock index futures mispricing is important for the following reasons:

(i) It has been presumed that mean reversion in futures mispricing is the result of the actions of index arbitrageurs (see eg Brennan and Schwartz, 1990, pp 58; Mackinlay and Ramaswamy, 1988, pp 137) since one would expect arbitrage opportunities to be rapidly eliminated in well functioning capital markets. In contrast to arbitrage-related explanations of mean reversion in mispricing, Miller *et al* (1991) have shown that the observed mean reversionary behaviour in the cash-futures basis could also be a manifestation of non-synchronous trading in the index basket of stocks, having no economic significance in terms of actual index arbitrage activity. However, the mean reversion generated by the Miller *et al* "statistical illusion" hypothesis is observationally indistinguishable from the potential mean reversion generated by actual index arbitrage activity. One way to distinguish between these competing explanations of mispricing mean reversion is to examine how the observed mean reversion varies with factors that are expected to affect the degree of index arbitrage, but which are not expected to be associated with non-synchronous trading *per se*; and vice versa.

(ii) Even within the framework of arbitrage induced mean reversion, different models of the arbitrage supply process have different implications about the variation of mean reversion with relevant factors. Hence an examination of how different factors affect mean reversion will enable inferences about the actual applicability of different models. For example, the mean reversion in the Brownian Bridge process (Brennan and Schwartz, 1990) is inversely proportional to the time to maturity, becoming infinitely large as the futures contract approaches maturity. Within the framework of the TAR model in Chapter 4 of this dissertation, mean reversion is a function of previous period mispricing. The model of Holden (1990) predicts different mean reversion behaviour under a monopolistic and an imperfectly competitive arbitrage structure. Mean reversion in a monopolistic market will increase as time to maturity decreases. On the other hand, with an imperfectly competitive arbitrage structure, Holden's model predicts that the mean reversion will be insensitive to the time to maturity. Finally, the Garbade and Silber (1983) model predicts that mean reversion should be a function of the number of active arbitrageurs, and the number of traders active in the cash market and the futures market separately. Since there is a theoretical rationale for seasonal variations in the number of active arbitrageurs/market traders (eg in the context of Admati and Pfleiderer, 1988; and Foster and Vishwanathan, 1990), this suggests potential intraweek and intraday seasonalities in the mean reversion parameter.

Pricing of Spreads

Analysis of the pricing of index futures spreads could be valuable for the following reasons:

(i) It can potentially contribute to our understanding of the economies of arbitrage in exactly the same way as the analysis of the cash-futures price difference series. Index futures contracts with different times to maturity represent equivalent assets (except for a largely non-stochastic factor - the cost of carrying the underlying asset from the maturity of the "near" contract to the maturity of the "far" contract) and the associated arbitrage strategies are easy to implement and largely risk free with clearly quantifiable transaction costs.

(ii) Spreads can be used to transfer risk from one futures trader to another. Futures markets provide a vehicle for transfer of risk to traders in the cash market. The opportunity to trade spreads enables futures traders to more easily transfer risk among themselves and hence makes them more willing to supply the price insurance demanded by hedgers in the cash market. Spread trading can contribute effectively to allocation of risk between different futures traders only when they are efficiently priced. Spread mispricing has important implications for the effectiveness and average cost of short term spread positions in the same way as cash-futures mispricing is a critical determinant of the effectiveness and average cost of short term hedging (highlighted eg by Merrick, 1988).

(iii) An examination of relative mispricing of index futures contracts with different times to maturity can also potentially contribute to the debate on whether the mean reversion observed in cash-futures arbitrage is arbitrage induced or a manifestation of non-synchronous trading (Miller *et al*, 1991). Clearly, the mispricing of one futures contract relative to another futures contract should not be influenced by factors related to non-synchronous trading in the cash index, or, more generally, any kind of measurement errors with respect to the cash index. Significant mean reversion in spread mispricing, and significant negative serial correlation in changes in spread mispricing, will provide support to the view that mean reversion in cash futures pricing is, at least not *entirely*, a consequence of non-synchronous trading.

(iv) Analysis of stock index futures spreads offers a direct test of the relevance and value of the tax timing option in relation to index futures pricing. The tax timing option is potentially valuable because stockholders have the ability to select the timing of realisation of losses and gains. Cash settlement of futures contracts implies that investors in the futures market necessarily pay taxes in the year the capital gains arise while investors holding the cash asset can defer their capital gains. On the other hand, the marginal investor may be a tax exempt institution in which case the tax timing option will have no value, or the marginal investor may be an arbitrageur/floor trader who cannot hold the cash index indefinitely in which case again the tax timing option will have no value. Clearly, the relevance of the tax timing option for index futures pricing could be different in different markets and essentially an empirical issue. *Ceteris paribus* the value of the tax timing option should be greater for longer times to expiration. Therefore, if this factor is important for index futures pricing then the far contract should be more negatively mispriced than the near contract ie the far contract should be underpriced relative to the near contract. Hence, on average, spread mispricing should be negative. This is directly testable.

(v) The use of data from the London market for analysing spread mispricing also enables a test of the effect on futures pricing of the constraints that exist on short selling of stocks. The unique features of the settlement procedures on the London Stock Exchange result in virtually no constraints on index arbitrage related short selling within the two (or three) "account" spanning futures maturity. Hence, if short selling restrictions influence futures pricing, the far contract should be significantly more underpriced relative to the near contract during the "account" spanning futures maturity than in other time periods. This will lead to spread mispricing being significantly negative, or at least lower in value, than in other periods. Again, this is directly testable.

Seasonalities: Futures Relative to Cash

The existence of seasonalities in stock markets represents an anomaly that has not yet been satisfactorily, or at least completely, explained by financial economists. Many of the "explanations" have focused on the institutional features and settlement procedures of the market such as the delay between trading and inflow/outflow of funds due to settlement rules and cheque clearing (Lakonishok and Levi, 1982), measurement error in returns (Gibbons and Hess, 1981, Keim and Staumbaugh, 1984), specialist-related biases (Keim and Staumbaugh, 1984), friction-related price adjustment delays (Theobald and Price, 1984) and "divide and conquer" pricing rules of market makers (Admati and Pfleiderer, 1989).

The possibility that the microstructure of a market might cause seasonality in prices suggests that it would be interesting to examine for seasonality in the pricing of stock index futures contracts in relation to the underlying index. In the absence of trading frictions arbitrage arguments require futures prices to equal the fair value derived from the forward pricing formula. However, the institutional features and settlement procedures of the two markets are markedly different. For example, while the London cash market is a pure dealership market (similar to NASDAQ) the LIFFE index futures market is an open outcry market with exchange members generally having relatively few open positions on their own account. Second, while the cash market is settled on the basis of a two- (or sometimes three-) week account period, trades in the index futures market are marked to market daily. Third, autocorrelation induced through friction-related price adjustment delays, or bid-ask spreads is likely to be much less important for the index futures contract than for the associated cash index. If such institutional features of markets are significant determinants of seasonalities in market prices, and the differences between markets are significant, then differences in patterns of seasonality would be expected to show up when futures market prices and cash market prices are compared.

1.2 ORGANISATION OF THIS DISSERTATION

This dissertation seeks to bridge the "gaps" identified in Section 1.1 of this chapter. A UK perspective is adopted, though most of the results are of general applicability. The studies summarised in this dissertation were undertaken from about October 1988 to about June 1992. *Inter-alia* the work involved has included building up of large datasets and this has been done progressively in stages. Hence, different studies have sometimes used slightly different datasets. Furthermore, stock index futures has been an area in which leading US academics have been very active. During 1989, 1990 and 1991 there have been several publications/working papers directly relevant to this work, eg Merrick (1989), Brenner *et al* (1989), Holden (1990a, 1990b, 1990c), Cooper and Mello (1990), Stoll and Whaley (1990), Kawaller *et al* (1990), Brennan and Schwartz (1990), Bhatt and Cakici (1990), Sofianis (1990), Hemler and Longstaff (1991), Klemkosky and Lee (1991), Puttonen and Martikainen (1991), Chung (1991), Subrahmanyan (1991), Chan (1991), Chan *et al* (1991), and Miller *et al* (1991). As a result, the studies in this dissertation undertaken later in time have been conditioned not only by the results of the studies done earlier in time, but also by the steady stream of literature which has flowed in from the USA. Hence, the studies summarised in this dissertation are presented in chronological order ie with reference to the date on which the first drafts of the relevant working papers were completed.

The rest of this dissertation is organised as follows:

(a) Chapter 2 presents preliminary UK evidence on stock index futures pricing relative to the underlying index. It is based on daily open and close data over the period June 1984 to June 1988.[22]

(b) Chapter 3 is a critical analysis of attempts to test index futures market efficiency using price *differences* rather than price levels. The empirical work is essentially illustrative and is based on daily data from June 1984 to June 1988, and hourly data from February 1986 to June 1988.[23]

(c) Chapter 4 is an attempt to model stock index futures mispricing in the context of the differential transaction costs of different categories of arbitrageurs. It is shown that mispricing should follow a path dependent threshold autoregressive (TAR) process. Tests for threshold non-linearity are conducted. Implied thresholds are calculated and compared with estimates of actual transaction costs. The elasticity of arbitrage services corresponding to different

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[22] First draft July 1989; presented to *INQUIRE Annual Conference* October 1989 and won the award for the best paper at that conference; presented also to *BAA Scottish Conference* September 1989, *BAA Annual Conference* April 1990, and to staff seminars at *London School of Economics, Warwick Financial Options Research Centre* and *Cardiff Business School*; revised version published in *Journal of Futures Markets*, Vol 10 (December 1990), pp 573-603.

[23] First draft January 1990; revised May 1990; revised version published in *Journal of Futures Markets*, Vol 11 (April 1991), pp 239-252.

transaction cost regimes is also estimated. Empirical work is based on hourly data from February 1986 to June 1988.[24]

(d) Chapter 5 compares intraweek, intraday and intra settlement period seasonalities in futures market *ex post* risk premia with corresponding seasonalities in cash market *ex post* risk premia. Hourly data from April 1986 to March 1990 is utilised. The results are analysed in the context of the differences in the microstructure between the cash and futures market.[25]

(e) Chapter 6 develops the settlement adjusted forward pricing formula and presents UK evidence on stock index futures pricing relative to the underlying index. It is based on time and sales transactions data over the period April 1986 to March 1990[26], and significantly extends the analysis in Chapter 2.

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[24] First draft September 1990; revised March 1991, October 1991 and March 1992; presented to the *ESRC Money Study Group*, October 1991 (London Business School); *Mid West Finance Association Conference*, April 1991 (USA); and *Western Finance Association Conference*, June 1991 (USA); and presented to staff seminars at *Dundee* and *Groupe HEC* Paris. Earliest version was titled "Modelling Financial Futures Mispricing using Self Exciting Threshold Autoregressive Processes" and was circulated by the *Center for the Study of Futures Markets, Columbia University*, USA as Paper # 211.

[25] First draft December 1990; revised March 1991; presented to *European Finance Association Conference*, August 1991, and the *INQUIRE Europe/UK Conference*, April 1992; revised version published in *Journal of Banking and Finance*, Vol 16 (February 1992), pp 233-270.

[26] First draft August 1991; revised March 1992; accepted for presentation at the *European Finance Association Conference*, August 1992.

(f) Chapter 7 formally tests for mean reversion in stock index futures mispricing and provides evidence on how this mean reversion varies with previous period mispricing, time to maturity, hour of the day and day of the week. The empirical evidence is based on UK hourly data from April 1986 to March 1990 and US 15 minute data from June 1983 to June 1987. The US data was very kindly provided at my request by Craig Mackinlay (Wharton).[27]

(g) Chapter 8 develops the theory for the pricing of stock index futures spreads and analyses empirical evidence in this regard based on time and sales transactions data for the period April 1986 to March 1990.[28]

(h) Chapter 9 summarises the main conclusions and outlines some directions for future research.

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27 First draft November 1991; revised March 1992; presented to staff seminars at *Dundee*, *Groupe HEC* (Paris) and Brunel; accepted for presentation at the *French Finance Association Conference*, June 1992, and the *American Finance Association Conference*, January 1993.

28 First draft June 1992.

CHAPTER 2

STOCK INDEX FUTURES PRICING

UK DAILY DATA EVIDENCE[1]

ABSTRACT

This chapter contains preliminary evidence of stock index futures mispricing and arbitrage program trading simulations based on open and close daily data for the London International Financial Futures Exchange. The time series properties of the mispricing series are described and the implications for risky arbitrage strategies examined. The impact of the 1986 Big Bang deregulation on the link between the cash and futures market is also analysed. The main conclusion of the chapter is that earlier results on index futures mispricing appear to be independent of the economic, institutional and regulatory environment.

First draft July 1989; presented to INQUIRE Annual Conference October 1989 and won the award for the best paper at that conference; presented also to BAA Scottish Conference September 1989, BAA Annual Conference April 1990, and to staff seminars at London School of Economics, Warwick Financial Options Research Centre and Cardiff Business School; revised version published in *Journal of Futures Markets*, Vol 10 (December 1990), pp 573-603.

Stock Index Futures Pricing: UK Daily Data Evidence

Recent work by Mackinlay and Ramaswamy (1988), Merrick (1988), Modest and Sudereshan (1983), Cornell and French (1983a,1983b), Figlewski (1984a, 1984b), and Arditti et al (1986) has documented the existence of substantial and sustained deviations between actual stock index futures prices and theoretical values. Based on these findings, Merrick (1989) and Finnerty and Park (1988) have attempted to demonstrate the profitability of arbitrage related program trading strategies. On the other hand, Saunders and Mahajan (1988) have adopted an alternative approach and concluded that stock index futures are priced efficiently.

One limitation of the previous empirical work in this area is that it relates only to stock index futures contracts traded within the USA. The fact that previous work involves repeated analysis of data sets pertaining to the same economic and institutional environment, albeit for different sample periods, means that whilst the work is of great interest and enables a better understanding of index futures contract pricing to be developed, it needs to be externally validated.

A second limitation of earlier empirical work in this area is that even though there appears to be a broad consensus that observed mispricing is often sufficient to span the transaction cost bounds and offer arbitrage possibilities, this is not substantiated with formal evidence on actual transaction costs. In this respect the evidence of

Stoll and Whaley (1987) is largely anecdotal, while Merrick (1989) uses an *ad hoc* estimate and Finnerty and Park (1988) ignore transaction costs.

This chapter seeks to partially bridge these "gaps" in the literature by reporting the results of a preliminary analysis of a totally new set of data relating to the UK FTSE-100 stock index futures contract traded on the London International Financial Futures Exchange (LIFFE). The results are set into perspective by an analysis of the relevant transaction costs. In order to provide direct comparability with previous work, the present study seeks to replicate, as closely as possible, many of the tests and methods developed in the US context. In the analysis we allow for institutional change during the sample period, notably the changes associated with Big Bang on 27 October 1986 when the UK stock market was substantially deregulated.

The remainder of the chapter is organised as follows: in Section 1 we outline briefly the theoretical framework within which the empirical analysis is conducted; in Section 2 we describe the UK stock index futures database and report our empirical results; and in Section 3 we present our conclusions.

1. Theoretical Framework

1.1 Futures pricing and arbitrage

Assume initially that capital markets are perfect and frictionless, ie there are no transaction costs taxes or information asymmetrics. Following Cornell and French (1983a,1983b), and Figlewski (1984a) the theoretically fair price at t of a futures contract with maturity date T will be given by

$$F^{*}_{t,T} = I_t e^{(r_{t,T}(T-t))} - \sum_{w=t+1}^{T} d_w e^{(R_{t,w,T}(T-w))} \qquad \ldots.(1)$$

where I_t is the stock index price on day t

d_t is the aggregate dividend paid by underlying stocks on day t

$r_{t,T}$ is the yield on day t of a discount bond maturing at time T

$R_{t,w,T}$ is the forward interest rate at time t for a loan that will be made at time w to mature at time T.

If the actual market price of a futures contract $F_{t,T}$ exceeds the fair value $F^{*}_{t,T}$ then, in the absence of transactions costs, an arbitrageur with capital in Treasury Bills (or equivalent fixed interest securities) should sell Treasury Bills, buy the value-weighted basket of index stocks, sell futures contracts of equivalent amount and hold the long stock - short futures position until expiration. At expiration the stock position will be liquidated and switched back into Treasury Bills (Upper arbitrage). Similarly if the market price is below the fair price, ie $F_{t,T} < F^{*}_{t,T}$, then an arbitrageur with capital in index stocks can enhance stock market yields by selling the index, buying futures contracts and Treasury Bills of equivalent amount and holding the long futures-short stock position until maturity at which time the arbitrageur could liquidate the Treasury Bill holdings and repurchase the equity index portfolio (Lower arbitrage).

1.2 Transactions costs

In equilibrium in this pure arbitrage world, systematic futures mispricing should not be observed. However, the existence of trading frictions and transactions costs will *a priori* cause futures prices to fluctuate within a band around the fair price without triggering profitable arbitrage. Denoting X_t as the percentage mispricing and comparing the relative transactions costs involved, we can see that if the futures price is above fair value the arbitrageur with capital could succeed if

$$X_t > (2T_S + T_D + T_F + T_F^*) \qquad \ldots. (2)$$

where T_S is the percentage one way transaction cost for equities, including both commissions and any potential market impact

T_D is the value of taxes (eg stamp duty) payable as a percentage of asset value

T_F is the round trip percentage commissions in the futures market

and T_F^* is the one way percentage market impact cost in the futures market

Similarly if the futures price is below fair value then the arbitrageur holding index stocks can succeed in riskless enhancement of yield if

$$X_t < -(2T_S + T_D + T_F + T_F^*)$$

This suggests that under normal circumstances the maximum observable percentage of futures mispricing should be given by

$$|X_t| = (2T_S + T_D + T_F + T_F^*) \qquad \text{..... (3)}$$

Expression (3) defines the theoretical arbitrage window, given the out-of-pocket percentage transaction costs T_S, T_D, T_F and T_{F^*}. The arbitrage window should depend on the arbitrageur with the lowest transaction costs. However it is relevant to note that in assessing the *effective* arbitrage possibilities there are potentially additional sources of transaction costs that might be relevant:

1. There will be a cost of capital associated with financing an upper arbitrage strategy and this will be equal to the spread between the borrowing and lending rates. Similarly a lower arbitrage strategy may involve additional costs associated with short-selling the index basket of stocks, including the costs of borrowing stock such as the opportunity cost of cash collateral and the broker's margin requirement. It is noted that these costs might be

partially mitigated by assuming an indirect short position by holding put options.

2. Realised dividends are uncertain and hence the mispricing calculation is conditional upon the dividend expectation, which in previous research has tended to be set equal to the actual realisation, thus introducing a potential errors-in-variables problem. However, in the UK dividends are paid semi-annually and the ex-dividend date is fairly predictable. Thus, we would not expect major problems to arise, particularly if the analysis is restricted to the near contract, which generally is the most actively traded stock index futures contract on LIFFE. Given that dividend declarations occur several weeks before a stock goes ex dividend, making them *certain* for many companies during the period of the near contract, we believe that misspecification of dividend expectation is unlikely to be a major factor in explaining any observed mispricing. Nevertheless, in the analysis which follows we explore the sensitivity of our results to conservative assumptions about expected dividends. This has the effect of increasing the size of the effective arbitrage window.

3. The fair price obtained from the forward pricing model (1) is strictly applicable to futures contracts only if interest rates are non-stochastic (Cox, Ingersoll and Ross, 1981). Otherwise, the futures price will reflect the unanticipated interest earnings or costs from financing the marking to market

cash flow in the futures position. The returns from holding the futures contract will not only be a function of the terminal value of the spot index, but will also be dependent on the path followed by the spot index in relation to the path of interest rates. In general, the uncertainty involved can lead to a wider arbitrage window. However, as Rubinstein (1987) notes, with realistic uncertain interest rates, the difference between forward and futures prices has been examined both empirically and with Monte Carlo simulations in a number of studies and found to be negligible. In particular, Modest (1984) has indicated, based on simulation analysis, that stochastic interest rates and marking to market has a minimal effect on equilibrium prices of stock index futures.

4. Arbitrage is triggered by mispricing based on the reported value of the spot index. This is not a perfect measure of the truly tradeable cash index since the constituents of the index do not trade continuously (Cohen *et al*, 1986). The risk involved in an arbitrage program due to this effect will again tend to widen the arbitrage window.

5. Issues related to the tax status of arbitrageurs complicate the situation. Cornell and French (1983) highlight the tax timing option available to stockholders due to their ability to select the timing of realisation of losses and gains. This option is not available to futures traders for whom cash settlement implies taxes fall due in the year gains arise.

The tax timing option can tilt the preferences of taxable stockholders (who arguably maintain the lower arbitrage boundary through yield-enhancing arbitrage strategies) towards keeping stocks. This would imply a further lowering of the lower side of the arbitrage window. More formally, Cornell and French (1983a, 1983b) price index futures relative to a hypothetical restricted security consisting of the underlying index minus the tax timing option, but the closed form solution is not in terms of directly measurable inputs. However, they present intuitive arguments to suggest that the value of the tax timing option will be higher when the time to expiration is greater, and hence imply that if this factor is important mispricing should be negative and converge to zero as time to expiration decreases. Cornell (1985a) has examined plots of mispricing against time to expiration and concluded that the timing option does not have a significant impact on the pricing of futures contracts.

The tax timing option could be more important in the USA than the UK. US tax law requires that the tax liability on open futures contracts be assessed by marking them to market at the end of the tax year. In the UK, the tax liability arises only when the position is closed.

6. Trading the entire basket of index stocks substantially increases the size of the minimum possible arbitrage trade (to enable trading of individual stocks in round lots). Program trading costs also increase. Index traders often

attempt to replicate the underlying basket of stocks in an index by tracking the index with a small surrogate subset of perhaps 30 stocks. The replication introduces some additional costs due to sophisticated computational techniques and additional risk due to tracking error.

There are also several relevant countervailing factors that are likely to induce a narrower arbitrage window than might otherwise be expected.

1. Arbitrageurs have the option to reverse their positions prior to the expiration date if the mispricing of the futures contract changes sign and the absolute magnitude of this mispricing exceeds T_F*, ie is sufficient to cover the additional market impact cost involved in closing the position in the futures market. Thus a "risky" arbitrage strategy may be adopted even before the mispricing reaches the boundaries of the arbitrage window in the expectation that at some time before expiration the mispricing will reverse itself sufficiently to cover and exceed the additional transaction costs involved (Arditti *et al* 1986). Such an option is particularly relevant because in practice arbitrageurs can be restricted to a fixed number of net long or short arbitrage positions at any point of time due to capital constraints or self-imposed exposure limits (Brennan and Schwartz, 1990).

2. Arbitrageurs have the option to roll forward their futures position into the next available expiration date if the direction of the mispricing on expiration day is the same as the direction when the arbitrage trade was initiated and if the extent of mispricing on expiration day exceeds (T_F + T_F*). In the new arbitrage program initiated on expiration date, there are no additional transaction costs in the stock market and significantly no additional stamp duty is payable.

   Arbitrageurs can profitably roll forward their positions even prior to expiration if

   (a) the direction of mispricing on the day the position is rolled forward is the same as the direction of mispricing on the day the position was initiated, and

   (b) the absolute magnitude of the difference between mispricing of the near contract and mispricing of the far contract exceeds the incremental transaction costs (T_F + $2T_F$*).

3. Some market participants can have special circumstances that enable them to put on arbitrage trades at a considerably lower marginal transaction cost. For example, if futures are trading above fair value, holders of index stocks such as portfolio insurers and institutional investors who are committed to reduce their exposure to equities, may use the futures market as an intermediary for market exit instead of direct sales of their equity portfolio.

Thus they can retain the equity stock, sell futures contracts of equivalent amount and hold the short futures position till expiration day, at which time the stocks can be sold as planned. Such a strategy will be profitable whenever the percentage mispricing exceeds the marginal transaction cost ($T_F + T_F^*$). It can also be repeated on expiration day for additional profits if the futures are again trading above fair value on that day, or prematurely foreclosed if there is a reversal in mispricing. Similarly, if futures are trading below fair value, investors committed to increase their equity holdings may use the futures market as an intermediary for market entry by buying futures contracts of the amount needed, buying Treasury Bills and converting to equities at expiration. Again the marginal transaction cost will be restricted to ($T_F + T_F^*$) and again the option will exist to roll over the strategy on expiration day or prematurely foreclose if conditions are "favourable". It is relevant to note that mispricing of ($T_F + T_F^*$) can often exist in well-behaved markets with arbitrage bounds ordinarily given by $|2T_S + T_D + T_F + T_F^*|$.

4. There are relevant exemption clauses for payment of stamp duty on stock purchases. Before Big Bang, stocks purchased and sold within the same settlement period did not attract stamp duty. After Big Bang market makers and broker/dealers are exempt from stamp duty if they buy and sell shares within seven days. To the extent, it is practically possible to use the exemption clauses, the arbitrage window will shrink.

5. Other derivative assets like puts and calls on the index provide new arbitrage possibilities which effectively reduce transaction costs and shrink the window of no opportunity (Gould, 1988). However in this paper such possibilities will not be investigated and hence our tests can be viewed as conservative in the sense that the transactions cost window might be overstated.

1.3 Mispricing

The major determinants at the arbitrage window should be proportional to the value of the spot index and, hence, a key variable of interest is the percentage mispricing $X_{t,T}$ defined as:

$$X_{t,T} = \frac{F_{t,T} - F_{t,T}^{*}}{I_t} \qquad \text{.... (4)}$$

where $F_{t,T}$ is the actually observed market price of the index futures contract.

Analysis of the mispricing series, in relation to transaction costs, is relevant for arbitrageurs. Of particular interest is the tendency of mispricing to persist, or reverse itself.[2]

[2] It is difficult to select a stochastic process to "model" mispricing. Brennan and Schwartz (1990) suggest a Brownian Bridge process. But this is path independent, when the empirical evidence of Mackinlay and Ramaswamy (1988) indicates that the mispricing series is path dependent.

Garbade and Silber (1983) present a model of simultaneous price dynamics in futures and cash markets. Their model essentially implies mispricing to be an AR1 process, with the value of the AR1 factor ρ representing an inverse measure of the elasticity of supply of arbitrage services.[3]

Another key time series of interest is the series of scaled first differences in mispricing, or mispricing "returns" $R^X_{t,T}$ defined as

$$R^X_{t,T} = \frac{X_{t,T}I_T - X_{t-1,T}I_{t-1}}{I_{t-1}} \qquad \text{.... (5)}$$

Even if the forward pricing formula is a biased estimator of futures prices, the existence of an arbitrage window would, given sufficiently large number of observations, result in the average of mispricing returns being constrained to zero. It is also relevant to observe that we can express $R^X_{t,T}$ as :

$$R^X_{t,T} = [R^F_{t,T} - e^{r_{t,T}(T-t)}R^I_{t,T}] + [e^{r_{t-1,T}(T-t+1)} - e^{r_{t,T}(T-t)}] \qquad \text{.... (6)}$$

where:

$$R^F_{t,T} = \frac{F_{t,T} - F_{t-1,T}}{I_{t-1}}$$ is the <u>observed</u> futures "return"

[3] Garbade and Silber (1983) call this factor δ.

and $$R^I_{t,T} = \frac{I_t + d_t - I_{t-1}}{I_{t-1}}$$ is the cash index return.

An investor, eg a market maker, wanting to hedge over one day a predetermined cash market position would on the basis of the forward pricing formula (and perfect knowledge of future absolute dividend inflows) construct a one day hedge portfolio by selling $e^{-r_{t,T}(T-t)}$ futures contracts for every unit of the cash index held by him (Merrick, 1987). The first expression in square brackets on the right hand side of Equation (6) is hence the negative of the future value at expiration of the "abnormal" return earned on this one day hedge portfolio. The second bracketed expression is the riskless return - the one day change in the discount factor (and is typically about 0.0003).

2. Empirical Results

2.1 The Data

LIFFE index futures expire four times a year in March, June, September and December, on the last business day of the month. Trading commenced in May 1984, with June 1984 being the first expiration month. In all, our data comprises 1012 daily observations on 16 different contracts spanning the period July 1, 1984 to June 30, 1988. We do not include the first contract (June 1984) because of the short period for which data is available for that contract and also because it could be unrepresentative due to possible "seasoning" effects.

Our main analysis relates to the near contract. An examination of daily trading volume reveals that the near contract is almost always the most heavily traded contract on LIFFE. Volume in the second nearest contract starts to build up about four weeks before expiration of the near contract. Hence our main data set is based on the near contract, shifting to the next contract on expiration day.[4] However we also analyse the second nearest contract for a period of four weeks prior to expiration of the near contract.

Data on LIFFE FTSE-100 futures contracts was obtained from *Datastream*. The data includes the daily settlement price, opening price, high and low prices and the volume of trading. Data on the FTSE-100 index was obtained from the *International Stock Exchange* (formerly the *London Stock Exchange*). The data includes daily opening, closing, high and low prices. Information was also obtained on the constituents of the index and how these had changed over the sample period.

Dividends and ex-dividend dates for all the relevant constituents of the index each day were collected from *Extel* cards. In addition, in order to compute the daily dividend entitlement on the FTSE-100 index, market value and unadjusted price data was taken from *Datastream* for each index constituent. The number of shares of each company outstanding at the end of each day was taken to be the closing market capitalisation divided by the closing unadjusted price of the company. The market

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[4] Opening prices on expiration day do not relate to the expiring contract.

value of the total dividend each day was then obtained by multiplying, for each company going ex-dividend on that day, the number of shares outstanding by the nominal dividend (of the company going ex-dividend) and summing over all the companies which were part of the index on that day. This was divided by the total market capitalisation of all index constituents on that day to obtain in index units the daily dividend entitlement associated with the FTSE-100 index. Daily series for one- and three-month Treasury Bill rates were also collected from *Datastream*.

Trading hours on LIFFE for stock index futures are from 9.05 hrs to 16.05 hrs. The daily closing settlement price hence reflects the value of the index future at 16.05 hrs, and the opening price reflects the value at 9.05 hrs. On the other hand, the FTSE-100 index closing series is the index value as computed at 17.00 hrs and the FTSE-100 opening series is the index value as computed at 9.00 hrs. Hence, while opening market prices are reasonably coincident with futures opening prices closing prices are less likely to be aligned and this asynchronicity will potentially produce noise in fair value estimates. Though this can be a potential source of error, it should not lead to *systematic* differences between the normative index futures price and the actual index futures price. Nevertheless, in formulating arbitrage strategies in the empirical tests which follow, some allowance is made for the timing of price observations by also considering *ex ante* strategies.

2.2 UK Transactions Costs

Transaction costs incurred in arbitrage related strategies could be expected to be higher in the UK than in the USA for several reasons. First, index futures contracts in the USA expire at the opening or closing of the market, and it is possible to unwind stock market positions with no bid-ask spreads through market on open or market on close orders. In the UK, the index futures contract expires at 11.20 hrs and the bid-ask spreads involved in unwinding stock market positions are similar to the bid-ask spreads faced when the position was initiated unless systematic changes in spreads have occurred in the interim.

Secondly, with respect to stamp duty, this is currently payable in the UK on every purchase transaction at 0.5%, except for market makers and broker/dealers buying and reselling stocks within seven days. Before Big Bang the rate was 1%, except for shares bought and sold within the same Stock Exchange account period. Market makers are granted a stamp duty exemption confined to the stocks for which they make a market. The stamp duty rate was between 0.5% and 1% over the sample period and therefore this exemption represents a significant incentive for market makers to try to track the index with the subset of stocks in which they make a market.

Thirdly, with regard to futures related transactions costs, the volume of trading in the UK futures market is relatively low and so the volume of arbitrage trade for a perceived mispricing opportunity would have to be restricted to avoid high market impact costs.

Prior to Big Bang commissions were relatively high and market wide bid-ask quotes were not computerised and hence bid-ask spreads could be expected to vary substantially across different market participants. Since Big Bang transaction costs have become very competitive. Commissions can be negotiated to very low levels by large market participants (such as index arbitrageurs) and can be expected to be insignificant in relation to bid-ask spreads which have however tended to be very volatile.

We attempt to estimate bid-ask spreads after Big Bang by using daily bid and ask price quotes of index constituents over the post-Big Bang sample period 27 October 1986 to 1 July 1988. Data was obtained from *Datastream*. The bid-ask spread for trading the index basket of stocks was calculated for each trading day in the above period by using the bid-ask spread of the different index constituents and the proportion of index value represented by the constituent. Table 1A reports the mean, standard deviation, median, and upper and lower quantiles of these daily estimates over different quarters. Bid-ask spreads declined steadily after Big Bang until the period around the market crash on 19 October 1987, after which they have tended to rise. Over the whole period, the average spread has been about 1%.

Futures related transaction costs relevant for index arbitrage are, in comparison, much smaller. Round trip commissions can be negotiated to very low levels, and even a "normal" commission of about £50 per contract represents only about 0.1% of index value. We have formally estimated bid-ask spreads by using a transaction prices database provided to us by LIFFE. For each trading day in 1987 we calculate the average of all ask quotes for the near contract and the average of all bid quotes for the near contract and used these values to estimate the bid-ask spread applicable on that day.[5] The average over the year 1987 was estimated at about 0.1%. Table 1B reports the mean, standard deviation, median, and upper and lower quantiles of these daily estimates for each quarter in the year 1987.

The costs incurred by index arbitrageurs in the USA are estimated at about 0.6% (Mackinlay and Ramaswamy, 1988). Aggregate transactions costs in the UK appear to be substantially higher in general. However, as has been mentioned previously, transaction costs will be lower for some categories of arbitrageurs. For example:

(1) Market makers and brokers/dealers recycling stocks within seven days avoid stamp duty of 0.5%.

(2) Arbitrageurs with an existing arbitrage position can seek the opportunity to profitably roll forward their position into the next available maturity.

(3) "Risk arbitrageurs" can open an arbitrage position within normal transaction cost bands in the hope of exploiting mispricing reversals.

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[5] There are about 400 bid-ask quotes every day in the database.

(4) Investors committed to market entry/exit using the futures market as an intermediary face marginal transaction costs only in the futures market.

(5) The marginal transaction costs of those market makers who track the index with stocks in which they make markets are confined to the opportunity cost of personnel time.

Consequently, in the analysis which follows we consider the implications of four different transaction cost bands - 0.5%, 1.0%, 1.5% and 2.0%. It is thought that all market participants for whom index arbitrage might have been relevant during our sample period would have faced transaction costs within this range.

2.3 The Mispricing Series

The fair value of the futures contract at the daily opening and closing of futures trading was initially computed under the following assumptions :

(a) The forecast dividend yield to maturity for each date is identical to the actual *ex post* daily cash dividend inflow for the FTSE-100 portfolio.

(b) The forward interest rate at the time t for a loan made at time w to be redeemed at time T, is identical to the interest rate at time w on a Treasury Bill maturing at time T.

(c) The value on day t of one- and three-month Treasury Bill interest rates can be used to infer a linear term structure from which the implicit interest rate for the period (T-t) can be calculated.

(d) The futures settlement price is synchronous with the closing index value and the opening futures price is synchronous with the opening index value.

The fair values at the opening and closing of trading each day were used to compute the opening and closing percentage mispricing series.

Figure 1 presents a plot of percentage mispricing based on closing prices of the LIFFE near index futures contract over the period July 1, 1984 to June 30, 1988. Figure 2 is a similar plot based on opening prices. Figures 1 and 2 are similar and suggest that asynchronicity does not appear to be a major factor in the analysis.

The figures indicate that the fair value forward pricing formula is frequently violated, and many of the violations that are observed appear too large to be accounted for solely by transaction costs. Mispricing in the pre-Big Bang period appears to have been systematically negative. Post Big Bang the tendency and frequency of (at least partial) price reversals has substantially increased, though the June 1988 contract reverted to the pre-Big Bang pattern of forward pricing fair value consistently providing a downward biased estimator of actual value. Overall the plots are consistent with the growth and improvement in the arbitrage sector, and post-Big Bang they suggest that additional potential exists for "risky" arbitrage strategies based on "predictable" mispricing reversals.

Table 2 provides summary statistics for the mispricing series of each contract expiring during the period of the study, and also for the overall sample, the sample pre-Big Bang, and the sample post-Big Bang. If we exclude the Crash period, the mean levels of mispricing do not differ significantly from the median levels of mispricing, indicating that outliers are probably not affecting our results. The maximum absolute mispricing on opening prices post-Big Bang (ignoring the December 1987 contract) is only 1.5% whereas it was 2.5% before Big Bang. The average mispricing was negative for 8 out of the 9 contracts expiring before Big Bang but for only 4 out of 7 contracts expiring after Big Bang. Systematic mispricing appears to have decreased from an average of about 0.5% in the pre-Big Bang period to below 0.2% in the post-Big Bang period. However, the standard deviation of the percentage mispricing has remained at similar levels (about 0.5%) for all contracts (except December 1987). These results can be compared with US results based on daily data reported by Merrick (1988). Over the period 1985-86 he reports the mean and standard deviation of the percentage mispricing of the S&P500 futures contract to be 0.011% and 0.411% respectively, and his plot appears to indicate that the maximum and minimum values have remained generally between +1% and -1%.

The t-statistics reported in Table 2 are for the null hypothesis that the average percentage mispricing is zero. The unadjusted t-statistics (as reported eg by Figlewski (1984a) for actual mispricing levels) are highly significant for each contract. However in computing standard errors this test ignores the high autocorrelation of the mispricing series. ARMA modelling of the percentage mispricing series shows that the series for most of the contracts are best modelled in terms of an autoregressive process of order one.[6] Table 2 gives the value of the AR(1) parameter, ρ, which best fits the model and the associated standard error, σ. The autocorrelation is uniformly positive and the coefficient is above 0.6 in most cases. Furthermore, the coefficient has been substantially higher in the Pre-Big Bang period, pointing to a marked increase in the elasticity of supply of arbitrage services after Big Bang. There thus appears to be a strong tendency for mispricing to persist, though this tendency has reduced markedly after Big Bang, or as the market has matured. The adjusted t-statistic employs a standard error calculated on the assumption that successive realisations are first order autocorrelated. Overall the adjusted t-statistics, whilst being lower, confirm the results obtained previously. At the individual contract level there are only two contracts (June and December 1987) for which the null is accepted using the adjusted t-statistic but rejected using the unadjusted statistic. However, for the post-Big Bang period the average level of mispricing based on opening prices is not significant at the 5% level with the adjusted t-statistic although without the adjustment it appears to be significant.

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[6] This is consistent with the model of Garbade and Silber (1983).

Table 3 gives the summary statistics for the series of mispricing returns (scaled first differences of the mispricing series). Average mispricing returns are indistinguishable from zero for every contract. Mispricing returns are only mildly autocorrelated as shown by the Box Pierce Q-statistic for 24 lags. The first order autocorrelation is negative for every contract, which is reasonable in the presence of an effective link between the cash and futures markets. It is pertinent to observe that the first order autocorrelation has become more negative after Big Bang.

If arbitrageurs are effective we should expect positive average mispricing returns when futures are initially underpriced and negative average mispricing returns when futures are initially overpriced. Hence Table 3 also reports the results of sub-dividing the full sample of mispricing returns into two subsets based on whether futures were initially underpriced or overpriced. As expected the average of mispricing returns with initially underpriced futures is consistently positive and the average of mispricing returns with initially overpriced futures is consistently negative, but these averages of the different series for individual contracts are significantly different from zero only after Big Bang. This clearly points to a markedly greater tendency for mispricing reversals and a greater efficiency of the arbitrage sector after Big Bang. The results also imply that a one day long-cash-short-futures hedge based on the forward pricing formula consistently earned negative "abnormal" returns if established when futures were initially underpriced and positive "abnormal" returns if established when futures were initially overpriced. This clearly limits the use that market makers can make of futures markets to hedge short term changes in stock market exposure. These results are essentially similar to those reported by Merrick (1988).

Table 4 presents summary statistics for the mispricing in the far contract. Only the four weeks prior to expiration of the near contract are considered in order to ensure that prices are based on reasonable levels of trading volume. Thus, for each contract the series consists of only m values where m = 20 (less any holidays). The absolute magnitudes of mispricing appear to be considerably larger than for the near contract and so are the extreme values. Prior to Big Bang the forward pricing formula was a downward biased estimater of futures price, just as it was for the near contract (only much more so). But after Big Bang, while the mean of the mispricing series of the near contract continues to be negative (even if we ignore the period around the October 1987 crash), the mean of the mispricing of the far contract is positive and fairly large (except for the June 1988 contract). It can be conjectured that this could be because of strong "bullish" sentiment over this period.

2.4 Arbitrage Related Ex Post Program Trading Simulations

Table 5A documents in the first column the number of mispricing violations based on the near contract for different levels of transaction cost bounds. The number of violations at all (non-zero) levels has reduced markedly after Big Bang, and it is relevant to mention that if the period around the October 1987 Crash is excluded, there has been only one violation of the 2% bound and only three violations of the

1.5% bound since October 1986. On the other hand there have been a relatively large number of violations of the 1% transactions cost bound.

We have attempted to simulate the profits that could have been earned by arbitrageurs using program trading driven by simple trading rules. Tables 5A and 5B are based on *ex post* trading rules, which assume that there is continuous trading in the market, and that it is possible to use the price at time t to execute a trade at the same price at the same time. We consider the following trading rules:

Trading Rule 1: If mispricing exceeds x%, sell one futures contract, sell Treasury Bills and buy the equivalent underlying basket of stocks, and hold the long stock-short futures position up to expiration. At expiration, sell the stock bought earlier, and reinvest in Treasury Bills. If mispricing is below -x%, buy one futures contract, sell the equivalent underlying basket of stocks, use the proceeds obtained to buy Treasury Bills, and hold the position until contract expiration, at which time the position is unwound and investment in stocks reinstated. This is the simple hold-to-expiration trading rule.

Trading Rule 2: Same as Trading Rule 1, except that, instead of waiting until contract expiration, the position is unwound as soon as mispricing changes sign and becomes at least (y% + 0.2%) in magnitude (to cover the estimated incremental transaction costs ($T_F + T^*_F$) and an incentive to trade of y%). This is the early unwinding option.

Trading Rule 3: Same as Trading Rule 1, except that during the last four weeks before maturity the positon is rolled forward to next available maturity as soon as the sign of the mispricing in the far contract is the same as the sign of the original mispricing, and when the difference in mispricing between the far contract and the near contract exceeds (y% + 0.3%) in magnitude (to cover the estimated incremental transaction costs ($T_F + 2T^*_F$) and an incentive to trade of y%). This is the rollover option. (Compound rollovers are ignored ie the rolled over position is assumed to be carried to expiration.)

Trading Rule 4: This is a combination of trading rules 1, 2 and 3. The arbitrage position is initiated as in Trading Rule 1, but is unwound early as per Trading Rule 2 or rolled forward, as per Trading Rule 3, whichever option becomes profitable at an earlier date.

Four values of x are used in each case - 0.5%, 1.0%, 1.5% and 2.0%. It is also assumed that the transaction costs being faced by the arbitrageurs are x%. In Trading Rules 2, 3 and 4 two values of y are used - 0.0% (Table 5A) and 0.5% (Table 5B). The first represents the case where a position is unwound early or rolled forward as soon as it is profitable to do so. The second represents the case where a position is unwound early, or rolled forward, only when the additional profit is at least 0.5%. In this context, when an arbitrage position is unwound early or rolled forward the arbitrageur loses the option of unwinding or rolling forward when the relevant mispricing values are more favourable to him.

Based on closing (opening) prices and prior to the consideration of transaction costs, Trading Rule 1 gives average annualised excess returns (relative to the Treasury Bill rate) of 8.0%(8.4%) with a standard deviation if 9.1%(10.4%) before Big Bang and 7.7%(7.4%) with a standard deviation of 13.4%(13.2%) after Big Bang. All excess returns were positive, and therefore, given the high standard deviation, the results suggest that there could have been many potential opportunities for arbitrage related strategies over the sample period.

Tables 5A and 5B give the profit (in pounds sterling) earned from the different Trading Rules for the various transaction cost levels. The profits reported are based on just one contract, and considering the present level of trading volumes, it would be possible to use a much larger number of contracts without tangible market impact.

Tables 5A and 5B have several interesting features. The first point to note is that significant arbitrage profits could apparently have been earned even at transaction cost levels of 1.5%, though it is relevant to emphasise that after Big Bang most of the contribution shown at transaction costs levels of 1.5% or higher is due to the December 1987 contract (spanning the October 1987 Crash). The additional profits arising out of rollover or early unwinding are a significant proportion of the total arbitrage profits and often exceed the arbitrage profits arising from the simple hold to expiration strategy. These high additional profits imply a heavy transaction cost "discount" and should generate substantial arbitrage activity even when futures prices are within transaction cost bounds. Merrick (1989) does not document additional profits arising out of rollover, but he does report profits due to early unwinding which are comparable to those in our simulations for the post-Big Bang period.

A comparison of the additional potential profit over the two sub-periods reveals that before Big Bang the additional potential profit was due mainly to rollovers whereas after Big Bang it was due mainly to early unwinding. This means that the tendency for price reversals has become substantially more pronounced after Big Bang and the systematic underpricing of the pre-Big Bang period seems to some extent to have been "corrected" by a more effective arbitrage sector. The decline in profitability of rollovers has arisen mainly because the far contract has usually been overpriced after Big Bang, while most of the arbitrage trades in the near contract have come from underpriced futures.

In Table 5B it can be seen that the option to delay early unwinding/rollover until additional profits are at least 0.5% appears to have been valuable. Higher profits would have been earned in most cases. If positions were unwound/rolled over as soon as it was profitable to do so (Table 5A, Trading Rule 4), *all* arbitrage positions based on $x \geq 1.0\%$ would have been closed prior to expiration, and even with $x = 0.5\%$ about 99% of positions would have been closed before expiration. Even if unwinding/rollover had been delayed until the additional profits were at least 0.5% (Table 5B, Trading Rule 4), less than 30% positions would have been held to expiration. This appears to indicate that arbitrage related program trading may not carry a significant risk of expiration day price, volume and volatility effects on underlying stocks.

What is the effective transactions cost "discount" created by the early unwinding/rollover option? Merrick (1989) makes a rough estimate of 27% considering only the number of trades unwound as a proportion of the total number of trades (ignoring actual magnitudes). Table 5C presents the results of simulating "risky" arbitrage strategies for our data when positions are taken within the transaction cost bounds in anticipation of additional profits due to rollover/early unwinding. The results are based on Trading Rule 4 with x = 0.5% and x = 1.0%, but with different levels of actual transaction costs up to 1% higher than x. Prior to Big Bang the strategy would have been profitable even if transaction costs exceeded the filter x% by 1.2%. After Big Bang the strategy would have been profitable for transaction costs exceeeding the filter x% by 0.8%. Thus, if an arbitrageur had actual transaction costs of 1.3% after Big Bang he could have considered initiating arbitrage trades within the bound x = 0.5%, a discount exceeding 60%.

2.5 Arbitrage Related Ex Ante Program Trading Simulations

Table 6 reports arbitrage profits based on Trading Rule 1, implemented on an *ex ante* basis. We adopt the conservative assumption that if there is an arbitrage mispricing opportunity perceived at opening prices, it will be possible to execute a strategy only at closing prices; and similarly, if there is an arbitrage mispricing opportunity perceived at closing prices one day, it will be possible to execute a strategy only at opening prices the next day. The trading rules used are the same as before, except for the delayed execution. It is also assumed that a trade decided on the basis of opening (closing) prices will be executed at the subsequent closing (opening) prices only if the mispricing has not moved to within the transaction cost bound in the intervening period. It should be noted that the profits reported in Table 6 include trades executed at both opening and closing prices, whilst Tables 5A and 5B show trades executed at closing prices separately from trades executed at opening prices.

Table 6 shows that the use of the *ex ante* trading rule considerably reduces the profits reported in Tables 5A and 5B for the *ex post* trading rules. The reduction is particularly significant after Big Bang where the average period for which profitable mispricing opportunities exist has declined markedly. If we ignore the December 1987 contract, *ex ante* profits with transactions executed with a half day lag are insignificant, even at the 1% transaction cost bound. Considering that "normal" transaction costs are estimated to be above 1%, this can be argued as evidence that the market has priced stock index futures efficiently after Big Bang. However, in the context of the high intra-day stock market volatility of the post-Big Bang period (Peel, Pope, and Yadav, 1989), the assumed execution delay of about half a day (in the absence of intra-day data) is perhaps not very realistic.

2.6 *Misspecification of Dividends*

It can be tempting to attribute systematic deviations from fair value pricing to misspecification of dividends. Table 7 gives the average percentage mispricing which can be attributed to systematic errors in forecasting dividends for different contracts. On average, systematic 10%, 25% and 50% errors in dividends result in respectively only 0.06%, 0.15% and 0.3% errors in calculated fair values. Hence dividend uncertainty does not appear to have been a significant explanation for systematic mispricing to the extent observed. Of course this does not preclude dividend uncertainty resulting in a widening of the arbitrage window. These results are broadly consistent with results for the US (Kipnis and Tsang, 1984).

2.7 *Relative Volatility*

An effective arbitrage link between the cash market and the futures market would imply the null hypothesis that the ratio of the variance of the futures market to that of the cash market should equal unity. We test this null hypothesis, using the conventional F-test for three variance estimators: the Close-to-Close estimator (based on closing prices); Open-to-Open estimator (based on opening prices); and the more efficient (Parkinson, 1980) extreme value estimator which provides an estimate of intraday volatility (based on daily high and daily low prices).

Table 8 summarises the results of the F-test. Detailed analysis of individual contracts revealed that the average intraday volatility of price changes in the futures market was higher for 15 out of 16 contracts and significantly higher at the 5% level for 8 out of these 15. The futures market intraday volatility has been significantly higher at the 1% level for both the sub-periods. The F-test is not able to distinguish so clearly between the close-to-close and open-to-open intraday volatility measures for the two markets. The close-to-close interday variance estimator for the futures market exceeds that of the cash market again for 15 out of 16 contracts, but is significantly higher at the 5% level for only 1 out of these 15 contracts. The open-to-open interday variance estimator of the futures market exceeds that of the cash market for 11 out of 16 contracts and is significantly different at the 5% level only for the contract whose observations span the period of the 1987 Crash. However, both the close-to-close estimator and the open-to-open estimator reject the hypothesis of equality of variances at the 5% level for both the overall sub-periods pre-Big Bang and post-Big Bang.

We can also draw inferences from the aggregated evidence for the 16 contracts by assuming that the variance ratios are independent across contracts. The hypothesis that the variance ratio equals one is rejected with t-values of 4.91 for the close-to-close estimator, 3.53 for the open-to-open estimator and 5.03 for the intra-day high low estimator, all of which are significant at the 1% level. The overall evidence thus appears to suggest that both interday and intraday volatility in the futures market is higher than that in the cash market. These results strongly substantiate the conclusions of Mackinlay and Ramaswamy (1988) in this regard.

2.8 *Mispricing and Time to Expiration*

The tax timing option has been argued to imply that mispricing should be negative and converge to zero as time to expiration decreases. The argument of Cornell and French (1983a, 1983b) that the value of the tax timing option will decrease with a decrease in time of expiration is not dependent on the validity of the forward pricing formula. Even if mispricing is positive because of possibly unquantified effects of other factors on index futures pricing it should still be valid to argue that mispricing will tend to decrease with time to expiration, if the tax timing option is important or relevant. A precise functional form for such a time dependence has not been suggested but it is possible to explore a linear relationship as a first approximation.

Figures 1 and 2 indicated that the mispricing might have been related to expiration time for the contracts expiring up to December 1985, but subsequently there was no obvious time related pattern of mispricing. Such a conclusion is also borne out by regressions of mispricing against time to expiration for different contracts, reported in Table 9A. OLS regression residuals displayed high autocorrelation. This is not suprising in view of the autocorrelation in the mispricing series (Table 2) and since time to expiration cannot be expected to be the major factor governing successive realisations. The regressions reported assume that the random error term is governed by an AR1 process and represent maximum likelihood estimates based on the Beach and Mackinnon (1978) iterative procedure.[7] Time to expiration is

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[7] The results are essentially similar for OLS regressions with or without lagged values of the dependent variable included as regressors.

significant as an explanatory variable with negative slope only for the first six contracts. Thereafter, the slope coefficient is either insignificant or positive. The results do not appear to provide support for the hypothesis that the tax timing option is valuable.

Time to expiration can also be associated with higher absolute magnitude of mispricing because of (i) dividend uncertainty; and (ii) uncertainty about the relative pattern of interest rates and stock prices. The results of regressing the absolute magnitude of mispricing against time to expiration are reported in Table 9B. The results are maximum likelihood estimates for the standard Tobit model with a zero lower bound censoring threshold and are based essentially on the iterative procedure of Berndt, Hall, Hall and Hansman (1974). It is interesting to observe that for the first six contracts about 50% of the variation in absolute mispricing is explained by the time to expiration and 7 out of the 9 coefficients for contracts expiring before Big Bang are significant at the 1% level. In the post-Big Bang period the coefficient of determination is much lower, but slope coefficients for *all* seven contracts are positive, and significant in 3(2) cases at the 5% level for closing (opening) prices. For both the sub periods, pre-Big Bang and post-Big Bang, the slope coefficient is highly significant. The results do appear to strongly suggest that absolute levels of mispricing increase as time to expiration increases. Hence our results support those of Mackinlay and Ramaswamy (1988) in this regard.

3.0 Conclusions

This chapter has presented empirical evidence and the results of trading simulations on the pricing of stock index futures based on a non-US data set - the UK FTSE100 contract traded on LIFFE. The results are set into perspective by an analysis of the relevant transaction costs and are reported separately for the different institutional and regulatory regimes existing before and after Big Bang when the UK stock market was substantially deregulated. Our salient results are the following:

(a) The forward pricing formula tends to provide a downward biased estimate of actual futures price, and many of the "violations" of the formula appear too large to be accounted for solely by transaction costs of the favourably positioned arbitrageurs. However, after market deregulation the extent and frequency of systematic mispricing violations has considerably decreased and the tendency for mispricing reversals has substantially increased.

(b) The mispricing series has high positive autocorrelation and tends to an AR1 process. The AR1 coefficient is high but has tended to decrease after Big Bang. Hence mispricing tends to persist, and the elasticity of supply of arbitrage services has increased after market deregulation.

(c) While the average level of mispricing has been significantly different from zero for most of the contracts, the average of mispricing "returns" has been essentially zero, apparently constrained by the actions of arbitrageurs. But the average of mispricing returns is significantly positive if futures are initially underpriced and significantly negative if futures are initially overpriced, implying that a long-cash-short-futures one day hedge portfolio based on the forward pricing formula consistently earned significant negative "abnormal" returns if established when futures were initially underpriced and significant positive "abnormal" returns if established when futures were initially overpriced.

(d) The simple hold to expiration trading rule appears to have provided only very limited opportunities for arbitrage profits, particularly after market deregulation. However, options to unwind early or rollover an arbitrage position have been very valuable and have effectively provided heavy transaction cost discounts. This has resulted in "risky" arbitrage strategies being attractive even for arbitrageurs with large transaction costs. Furthermore, the existence of these options have made expiration day price and volume effects unlikely.

(e) Dividend uncertainty does not appear to be an explanation for systematic mispricing to the extent observed.

(f) Both interday and intraday volatility of the futures market has been consistently higher than that of the cash market, thus negating the hypothesis of a perfect arbitrage link between the two markets.

(g) It is not possible to reconcile the pattern of mispricing with the hypothesis that tax timing option is valuable.

(h) The width of the arbitrage window has consistently tended to be wider for longer times to expiration.

The above results strongly substantiate earlier evidence in this regard based on US data, reported by Merrick (1988, 1989), Hill, Jain and Wood (1988), Mackinlay and Ramaswamy (1988), Arditti et al (1986), Cornell (1985a,1985b) and Figlewski (1984a). The peculiar features of stock index futures pricing, not readily explained from a theoretical standpoint, thus appear to be independent of the economic, institutional and regulatory environment. In this context it is also relevant to mention that our results are not inconsistent with those of Saunders and Mahajan (1988). Acceptance of the Saunders and Mahajan (1988) definition of pricing efficiency based on the behaviour of the first difference of cash and futures prices does *not* preclude the existence of arbitrage profits from hold-to-expiration or other trading rules. However reliance on cash and futures returns could imply *inter-alia* that the average of changes in mispricing should be constrained to zero if the market is efficient, a feature unequivocally supported by our results.

TABLE 1A : QUOTED BID-ASK SPREAD IN TRADING THE INDEX BASKET OF STOCKS

Period	Mean (%)	SD (%)	Median (%)	Q1 (%)	Q3 (%)
Oct 86 - Dec 86	1.19	0.12	1.22	1.19	1.25
Jan 87 - Mar 87	1.03	0.11	1.04	1.02	1.09
Apr 87 - Jun 87	0.67	0.15	0.62	0.59	0.65
Jul 87 - Sep 87	0.66	0.09	0.66	0.62	0.69
Oct 87 - Dec 87	1.33	0.35	1.43	1.17	1.61
Jan 88 - Mar 88	1.05	0.08	1.04	0.98	1.11
Apr 88 - Jun 88	0.98	0.08	0.97	0.94	1.01

Notes : SD = standard deviation

Q1 = lower quartile

Q3 = upper quartile

TABLE 1B : QUOTED BID-ASK SPREAD IN TRADING LIFFE INDEX FUTURE

Period	Mean (%)	SD (%)	Median (%)	Q1 (%)	Q3 (%)
Jan 87 - Mar 87	0.12	0.13	0.12	0.10	0.13
Apr 87 - Jun 87	0.11	0.04	0.11	0.09	0.12
Jul 87 - Sep 87	0.09	0.06	0.09	0.08	0.10
Oct 87 - Dec 87	0.21	0.21	0.17	0.12	0.25

TABLE 2 : LEVEL OF FUTURES MISPRICING - SUMMARY STATISTICS

Contract		n	<0	>0	mean (%)	median (%)	SD (%)	min (%)	max (%)	t-value	ρ	σ	adjusted t-value
9/84	C	64	63	1	-0.99	-0.92	0.50	-1.91	0.05	-15.90	0.83	0.08	-5.04
	O	64	64	0	-1.04	-0.93	0.54	-2.42	-0.05	-15.26	0.73	0.09	-6.24
12/84	C	64	63	1	-0.99	-0.91	0.63	-2.20	0.01	-12.58	0.95	0.05	-2.39
	O	64	62	2	-0.97	-0.88	0.66	-2.54	0.04	-11.74	0.82	0.07	-3.85
3/85	C	63	62	1	-0.53	-0.54	0.31	-1.17	0.25	-13.75	0.68	0.10	-6.05
	O	63	58	5	-0.50	-0.51	0.38	-1.33	0.47	-10.44	0.27	0.12	-7.95
6/85	C	61	57	4	-1.12	-1.32	0.66	-2.37	0.34	-13.30	0.96	0.05	-2.38
	O	61	55	6	-1.12	-1.34	0.63	-2.15	0.22	-13.93	0.96	0.06	-2.50
9/85	C	65	64	1	-0.71	-0.72	0.37	-1.52	0.28	-15.33	0.74	0.09	-6.14
	O	65	64	1	-0.70	-0.65	0.39	-1.77	0.12	-14.58	0.64	0.10	-6.90
12/85	C	64	50	14	-0.66	-0.73	0.64	-1.73	0.50	-8.21	0.95	0.05	-1.57
	O	64	52	12	-0.69	-0.79	0.65	-1.91	0.59	-8.53	0.92	0.06	-1.92
3/86	C	61	51	10	-0.37	-0.45	0.40	-1.16	0.59	-7.06	0.59	0.11	-3.72
	O	61	46	15	-0.31	-0.37	0.44	-1.08	0.82	-5.44	0.51	0.12	-3.17
6/86	C	63	46	17	-0.31	-0.37	0.54	-1.32	1.27	-4.50	0.70	0.09	-1.96
	O	63	44	19	-0.23	-0.22	0.54	-1.32	1.31	-3.35	0.41	0.12	-2.20
9/86	C	65	65	9	0.66	0.71	0.56	-0.77	2.06	9.45	0.75	0.08	3.69
	O	65	65	9	0.68	0.67	0.51	-0.19	2.32	10.69	0.61	0.10	5.37
12/86	C	64	45	19	-0.29	-0.28	0.52	-1.42	0.88	-4.37	0.73	0.09	-1.81
	O	64	47	17	-0.26	-0.23	0.52	-1.29	0.94	-3.93	0.63	0.10	-1.94
3/87	C	63	24	39	0.22	0.17	0.43	-0.65	1.40	4.04	0.30	0.12	3.00
	O	63	17	46	0.30	0.26	0.38	-0.56	1.45	6.29	0.24	0.13	4.93
6/87	C	61	24	37	0.15	0.24	0.56	-1.13	1.16	2.05	0.66	0.10	0.97
	O	61	24	37	0.21	0.42	0.57	-1.18	1.15	2.83	0.50	0.11	1.68
9/87	C	65	29	36	0.01	0.04	0.61	-1.41	1.61	0.17	0.73	0.09	0.05
	O	65	26	39	0.12	0.21	0.57	-1.20	1.51	1.77	0.76	0.08	0.64
12/87	C	64	36	28	-0.50	-0.06	1.38	-5.83	1.15	-2.89	0.64	0.10	-1.38
	O	64	35	29	-0.44	-0.07	1.47	-7.37	1.35	-2.40	0.41	0.12	-1.56
3/88	C	64	45	19	-0.28	-0.30	0.54	-2.04	1.38	-4.21	0.47	0.11	-2.51
	O	64	41	23	-0.08	-0.12	0.39	-0.83	0.88	-1.67	0.39	0.12	-1.10
6/88	C	61	53	8	-0.60	-0.69	0.50	-1.36	0.68	-9.38	0.70	0.10	-4.03
	O	61	51	10	-0.50	-0.58	0.45	-1.15	0.72	-8.68	0.58	0.11	-4.54

TABLE 2 : LEVEL OF FUTURES MISPRICING - SUMMARY STATISTICS [cont]

	n	<0	>0	mean (%)	median (%)	SD (%)	min (%)	max (%)	t-value	ρ	σ	adjusted t-value
Total pre-Big Bang												
C	589	472	117	-0.55	-0.59	0.74	-2.42	2.06	-18.06	0.88	0.02	-4.53
O	589	463	126	-0.53	-0.57	0.76	-2.57	2.31	-17.06	0.83	0.02	-5.23
Total post-Big Bang												
C	423	248	175	-0.20	-0.13	0.78	-5.85	1.58	-5.38	0.66	0.04	-2.43
O	423	233	190	-0.11	-0.05	0.78	-7.39	1.49	-2.90	0.55	0.04	-1.58
Overall Sample												
C	1012	720	292	-0.40	-0.40	0.77	-5.85	2.06	-16.63	0.80	0.02	-5.63
O	1012	696	316	-0.35	-0.32	0.79	-7.39	2.31	-14.25	0.73	0.02	-5.68

Note : C denotes closing price series
O denotes opening price series

TABLE 3 : MISPRICING "RETURNS"

Contract		n	<0	>0	mean (%)	SD (%)	t-value	BP_{24}	ρ_1	ρ_2	ρ_3
9/84	C	64	29	35	0.03	0.32	0.72	19.2	-0.15	-0.09	-0.18
	O	64	29	35	0.03	0.42	0.51	34.5	-0.13	-0.40	0.11
12/84	C	64	35	29	-0.00	0.38	-0.10	19.4	-0.14	-0.11	0.14
	O	64	29	35	-0.01	0.45	-0.10	16.9	-0.21	-0.12	0.07
3/85	C	63	30	33	0.00	0.26	0.02	23.5	-0.22	-0.20	-0.09
	O	63	32	31	0.00	0.46	0.05	38.3	-0.51	0.08	-0.14
6/85	C	61	28	33	-0.01	0.41	-0.18	14.3	-0.18	-0.10	-0.01
	O	61	25	36	-0.00	0.39	-0.01	22.8	-0.10	-0.10	-0.15
9/85	C	65	28	37	0.00	0.28	0.01	19.6	-0.23	-0.16	0.03
	O	65	31	34	-0.00	0.34	-0.07	15.7	-0.25	-0.03	-0.00
12/85	C	64	32	32	-0.00	0.30	-0.03	20.4	-0.07	-0.04	0.02
	O	64	33	31	-0.00	0.33	-0.06	33.2	-0.08	-0.13	0.17
3/86	C	61	29	32	0.00	0.37	0.05	25.6	-0.39	-0.09	0.10
	O	61	33	28	0.00	0.44	0.07	39.5	-0.38	0.02	-0.19
6/86	C	63	30	33	-0.01	0.46	-0.11	40.2	-0.16	-0.16	-0.11
	O	63	30	33	-0.01	0.59	-0.16	28.4	-0.48	0.17	-0.18
9/86	C	65	36	29	-0.00	0.41	-0.01	18.4	-0.22	-0.09	-0.11
	O	65	33	32	-0.00	0.47	-0.03	45.8	-0.25	-0.14	-0.17
12/86	C	64	34	30	-0.00	0.38	-0.07	24.4	-0.18	-0.20	-0.09
	O	64	27	37	0.00	0.45	0.07	30.2	-0.37	-0.08	-0.01
3/87	C	63	33	30	-0.00	0.51	-0.02	82.7	-0.43	0.09	-0.21
	O	63	32	31	-0.00	0.47	-0.06	42.5	-0.38	-0.02	-0.02
6/87	C	61	28	33	-0.01	0.49	-0.13	24.8	-0.33	-0.08	-0.04
	O	61	30	31	-0.01	0.58	-0.10	37.5	-0.42	-0.01	-0.08
9/87	C	65	31	34	0.01	0.53	0.09	26.8	-0.28	-0.07	-0.10
	O	65	31	34	0.00	0.43	0.02	42.2	-0.25	-0.09	-0.18
12/87	C	64	31	33	0.01	1.15	0.09	46.4	-0.35	-0.20	0.09
	O	64	29	35	0.02	1.52	0.13	69.1	-0.57	0.02	0.27
3/88	C	64	30	34	0.01	0.59	0.12	23.9	-0.18	-0.32	0.04
	O	64	28	36	-0.00	0.43	-0.03	25.5	-0.19	-0.36	0.07
6/88	C	61	33	28	-0.00	0.40	0.00	32.0	-0.33	0.04	-0.10
	O	61	31	30	-0.01	0.42	-0.16	29.8	-0.32	-0.04	-0.07

TABLE 3 : MISPRICING "RETURNS" [cont]

Contracts Expiring*		Sample Subset of Mispricing Returns with Initially Underpriced Futures Mean μ_1	$H_0: \mu_1 = 0$ T-stat	P value	Sample Subset of Mispricing Returns with Initially Overpriced Futures Mean μ_2	$H_0: \mu_2 = 0$ T-stat	P value	Difference $H_0: \mu_1 = \mu_2$ T-stat	P value
Dec 85	C	0.008	0.17	0.870	-0.030	-1.09	0.300	0.69	0.490
	O	0.021	0.51	0.610	-0.099	-0.78	0.450	0.90	0.390
Mar 86	C	0.035	0.70	0.480	-0.184	-1.34	0.220	1.50	0.160
	O	0.066	1.12	0.270	-0.225	-1.78	0.099	2.08	0.052
Jun 86	C	0.047	0.75	0.460	-0.133	-1.04	0.310	1.26	0.220
	O	0.080	1.04	0.310	-0.251	-1.58	0.130	1.87	0.071
Sep 86	C	0.199	0.97	0.380	-0.019	-0.37	0.710	1.04	0.350
	O	0.403	2.44	0.059	-0.041	-0.70	0.490	2.53	0.045
Dec 86	C	0.074	1.33	0.190	-0.184	-2.35	0.031	2.68	0.011
	O	0.121	2.40	0.020	-0.345	-2.49	0.025	3.17	0.005
Mar 87	C	0.307	3.48	0.002	-0.190	-2.53	0.016	4.29	0.000
	O	0.271	2.55	0.021	-0.104	-1.60	0.120	3.01	0.005
Jun 87	C	0.227	2.25	0.035	-0.154	-2.12	0.041	3.06	0.004
	O	0.270	2.46	0.022	-0.180	-2.01	0.052	3.18	0.003
Sep 87	C	0.221	2.26	0.031	-0.187	-2.53	0.016	3.33	0.002
	O	0.213	2.58	0.016	-0.147	-2.46	0.019	3.53	0.001
Dec 87	C	0.286	1.19	0.240	-0.313	-2.92	0.007	2.28	0.027
	O	0.446	1.68	0.100	-0.481	-1.98	0.057	2.58	0.012
Mar 88	C	0.178	2.18	0.035	-0.422	-3.99	0.001	4.49	0.000
	O	0.116	1.99	0.058	-0.211	-2.20	0.039	2.91	0.006
Jun 88	C	0.05	0.94	0.350	-0.300	-1.82	0.110	2.02	0.078
	O	0.068	1.21	0.230	-0.340	-3.18	0.010	3.38	0.004
Pre-Big Bang	C	0.029	1.87	0.062	-0.103	-2.53	0.013	3.03	0.003
	O	0.043	2.21	0.028	-0.153	-3.36	0.001	3.96	0.000
Post-Big Bang	C	0.168	3.90	0.000	-0.230	-6.50	0.000	7.14	0.000
	O	0.182	3.89	0.000	-0.216	-4.52	0.000	5.95	0.000

* Contracts expiring up to September 1985 are omitted since the sample subset of mispricing returns with initially overpriced futures contained inadequate data.

TABLE 4 : MISPRICING IN THE FAR CONTRACT - SUMMARY STATISTICS

Levels of Mispricing

		Mean	SD	Min	Max	%<0	%>0
Pre-	C	-1.21	1.21	-3.45	2.55	82	18
Big Bang	O	-1.20	1.25	-3.95	2.02	79	21
Post-	C	0.47	0.81	-1.63	2.05	32	68
Big Bang	O	0.58	0.86	-1.15	2.66	26	74

Changes in Mispricing

		Mean	SD	Min	Max	%<0	%>0
Pre-	C	0.03	0.35	-0.82	1.20	44	56
Big Bang	O	0.01	0.62	-2.23	2.13	49	51
Post-	C	-0.01	0.45	-1.44	1.06	46	54
Big Bang	O	-0.02	0.59	-1.82	1.88	47	53

TABLE 5A : EX POST ARBITRAGE RELATED TRADING SIMULATION WITH y = 0

Positions unwound/rolled over as soon as unwinding/rollover becomes profitable

	Trading Rule 1		Trading Rule 2		Trading Rule 3		Trading Rule 4		
Cont-acts xpiring	No of Mispricing Violations	Base Arbitrage Profits/ Contract (£'000)	No of Early Un-windings	Addl Arbitrage Profits/ Contract (£'000)	No of Rollovers	Addl Arbitrage Profits/ Contract (£'000)	No of Early Un-windings	No of Rollovers	Addl Arbitrage Profits/ Contract (£'000)
= 0.5%									
re-Big C	374	65.8	271	12.1	345	87.6	41	331	86.7
ang O	367	65.0	266	13.3	339	88.3	46	320	85.5
ost-Big C	201	41.6	180	31.8	92	17.3	175	24	38.3
ang O	193	35.4	180	25.8	97	29.4	151	38	38.2
= 1.0%									
re-Big C	164	22.3	108	5.2	160	37.5	8	156	37.5
ang O	161	23.0	109	5.5	157	39.5	7	154	38.3
ost-Big C	60	14.2	59	11.1	12	1.8	59	1	11.2
ang O	45	11.6	45	7.0	10	2.6	41	4	7.5
= 1.5%									
re-Big C	68	5.4	39	1.8	68	17.2	0	68	17.2
ang O	67	5.8	35	1.7	67	16.7	0	67	16.7
ost-Big C	14	7.9	14	4.6	2	0.3	14	0	4.6
ang O	9	7.0	9	3.3	0	0.0	9	0	3.3
= 2.0%									
re-Big C	10	0.4	3	0.1	10	1.2	0	10	1.2
ang O	11	0.9	6	0.2	11	1.4	3	8	1.2
ost-Big C	9	5.3	9	3.0	1	0.2	9	0	3.0
ang O	7	5.2	7	2.6	0	0.0	7	0	2.6

TABLE 5B : EX POST ARBITRAGE RELATED TRADING SIMULATION WITH y = 0.5%

Positions unwound/rolled over as soon as unwinding/rollover yields additional profit of 0.5%

		Trading Rule 1		Trading Rule 2		Trading Rule 3		Trading Rule 4		
Contracts Expiring		No of Mispricing Violations	Base Arbitrage Profits/ Contract (£'000)	No of Early Un-windings	Addl Arbitrage Profits/ Contract (£'000)	No of Rollovers	Addl Arbitrage Profits/ Contract (£'000)	No of Early Un-windings	No of Rollovers	Addl Arbitrage Profits/ Contract (£'000)
x = 0.5%										
Pre-Big	C	374	65.8	66	15.7	268	88.6	30	266	94.5
Bang	O	367	65.0	28	6.1	269	91.4	28	265	95.1
Post-Big	C	201	41.6	102	37.0	49	18.2	102	18	43.6
Bang	O	193	35.4	88	31.7	82	34.7	78	49	48.2
x = 1.0%										
Pre-Big	C	164	22.3	23	5.9	127	39.3	7	125	40.6
Bang	O	161	23.0	5	1.1	130	42.1	5	129	41.9
Post-Big	C	60	14.2	31	11.5	6	2.3	31	1	11.8
Bang	O	45	11.6	22	7.7	10	3.8	20	4	8.2
x = 1.5%										
Pre-Big	C	68	5.4	4	1.0	54	17.4	0	54	17.4
Bang	O	67	5.8	0	0.0	52	17.2	0	52	17.2
Post-Big	C	14	7.9	14	5.3	0	0.0	14	0	5.3
Bang	O	9	7.0	9	3.3	0	0.0	9	0	3.3
x = 2.0%										
Pre-Big	C	10	0.4	1	0.3	3	0.9	0	3	0.9
Bang	O	11	0.9	3	0.9	3	1.0	3	3	1.9
Post-Big	C	9	5.3	9	3.5	0	0.0	9	0	3.5
Bang	O	7	5.2	7	2.6	0	0.0	7	0	2.6

TABLE 5C : "RISKY" ARBITRAGE STRATEGIES

Ex Post Arbitrage Profits using Trading Rule 4 with x = 0.5% and x = 1.0% and different transaction costs.

x = 0.5% — Arbitrage Profits per contract with transaction costs of

Contracts Expiring		0.7%	0.9%	1.1%	1.3%	1.5%
Pre-Big Bang	C	135.5	110.8	86.1	61.4	36.7
	O	135.8	111.5	87.3	63.0	38.8
Post-Big Bang	C	66.2	47.2	28.2	9.3	-9.7
	O	65.1	46.6	28.1	9.6	-8.9

x = 1.0%

		1.2%	1.4%	1.6%	1.8%	2.0%
Pre-Big Bang	C	52.4	41.9	31.4	20.9	10.5
	O	54.7	44.4	34.1	23.8	13.5
Post-Big Bang	C	20.6	15.1	9.7	4.3	-1.2
	O	15.8	11.7	7.5	3.4	-0.7

TABLE 6 : ARBITRAGE PROFITS FROM EX ANTE STRATEGIES

Contract Expiring	Ex-ante Profit (£'000) per contract at transaction cost level of 0.0%	0.5%	1.0%	1.5%	2.0%
September 1984	34.1	18.0	6.9	2.0	0.0
December 1984	35.7	19.8	9.0	3.2	0.7
March 1985	21.0	5.4	0.3	0.0	0.0
June 1985	45.0	27.9	13.3	2.9	0.2
September 1985	30.0	10.8	1.9	0.1	0.0
December 1985	35.3	16.5	5.1	0.4	0.0
March 1986	19.4	3.4	0.0	0.0	0.0
June 1986	23.6	5.6	0.3	0.0	0.0
September 1986	35.0	13.5	3.6	0.2	0.0
December 1986	21.7	6.2	0.8	0.0	0.0
March 1987	18.3	2.7	0.1	0.0	0.0
June 1987	29.7	5.5	0.0	0.0	0.0
September 1987	30.8	7.7	0.8	0.0	0.0
December 1987	45.5	24.8	16.0	11.4	7.8
March 1988	18.4	1.3	0.0	0.0	0.0
June 1988	31.8	11.7	0.7	0.0	0.0
Pre-Big Bang	279.1	120.9	40.4	8.8	0.9
Post-Big Bang	196.2	59.8	18.5	11.4	7.8

TABLE 7 : MISSPECIFICATION OF DIVIDENDS

		% effect on fair value of misspecification equal to		
		10%	25%	50%
September 84	C	0.08	0.20	0.40
	O	0.08	0.20	0.40
December 84	C	0.04	0.10	0.20
	O	0.04	0.10	0.20
March 85	C	0.11	0.28	0.57
	O	0.11	0.28	0.57
June 85	C	0.05	0.11	0.23
	O	0.05	0.11	0.23
September 85	C	0.07	0.17	0.33
	O	0.07	0.17	0.33
December 85	C	0.05	0.13	0.27
	O	0.05	0.13	0.27
March 86	C	0.06	0.16	0.32
	O	0.06	0.16	0.32
June 86	C	0.04	0.09	0.18
	O	0.04	0.09	0.18
September 86	C	0.07	0.17	0.34
	O	0.07	0.17	0.34
December 86	C	0.06	0.14	0.28
	O	0.06	0.14	0.28
March 87	C	0.07	0.19	0.37
	O	0.07	0.19	0.37
June 87	C	0.04	0.10	0.20
	O	0.04	0.10	0.20
September 87	C	0.06	0.14	0.28
	O	0.06	0.14	0.28
December 87	C	0.05	0.13	0.26
	O	0.05	0.13	0.26
March 88	C	0.08	0.21	0.42
	O	0.08	0.21	0.42
June 88	C	0.05	0.12	0.24
	O	0.05	0.12	0.24
Pre-Big Bang	C	0.06	0.16	0.32
	O	0.06	0.16	0.32
Post-Big Bang	C	0.06	0.15	0.29
	O	0.06	0.15	0.29

TABLE 8 : RELATIVE VOLATILITY

	n	Ratio of Interday Variance Closing Prices	P value of F stat	Ratio of Interday Variance Opening Prices	P value of F stat	Ratio of Intraday Variance	P value of F stat
ep 84	64	1.12	0.32	1.16	0.28	1.73	0.01
ec 84	64	0.83	0.77	0.99	0.51	1.31	0.14
ar 85	63	1.10	0.36	1.28	0.17	1.59	0.03
une 85	61	1.02	0.47	0.91	0.64	1.22	0.22
ep 85	65	1.02	0.47	1.19	0.24	1.41	0.09
ec 85	64	1.12	0.32	1.22	0.21	0.85	0.74
ar 86	61	1.03	0.45	1.14	0.30	1.44	0.08
une 86	63	1.44	0.08	1.43	0.08	1.73	0.02
ep 86	65	1.25	0.18	1.07	0.40	1.64	0.02
ec 86	64	1.38	0.10	1.27	0.17	1.81	0.01
ar 87	63	1.16	0.28	0.74	0.88	1.09	0.37
ne 87	61	1.49	0.06	0.95	0.58	1.47	0.07
p 87	65	1.47	0.06	1.08	0.38	1.45	0.07
ec 87	64	1.29	0.16	1.70	0.02	3.41	0.00
ar 88	64	1.72	0.02	1.36	0.11	2.73	0.00
ne 88	61	1.30	0.15	0.98	0.53	1.55	0.05
e.	589	1.15	0.05	1.18	0.02	1.47	0.00
.g	423	1.33	0.00	1.43	0.00	2.75	0.00

TABLE 9A : REGRESSIONS OF MISPRICING VS TIME TO EXPIRATION*

$$X^i_{t,T} = a^i + b^i(T-t) + \epsilon^i_t \qquad i = 1, ... 16$$

		b^i	T-stat	Sig.level	DW of Transformed Residuals
September 84	C	-0.015	-5.33	0.000	1.91
	O	-0.015	-5.04	0.000	1.81
December 84	C	-0.021	-7.27	0.000	2.27
	O	-0.021	-7.24	0.000	2.06
March 85	C	-0.008	-4.38	0.000	1.98
	O	-0.007	-4.12	0.000	2.02
June 85	C	-0.022	-6.67	0.000	2.10
	O	-0.022	-6.81	0.000	2.02
September 85	C	-0.010	-5.27	0.000	2.18
	O	-0.010	-4.84	0.000	2.06
December 85	C	-0.017	-3.40	0.001	2.26
	O	-0.018	-3.88	0.000	2.11
March 86	C	-0.001	-0.31	0.760	2.26
	O	-0.002	-0.58	0.563	2.18
June 86	C	-0.003	-0.55	0.586	2.24
	O	-0.003	-0.77	0.443	2.33
September 86	C	0.002	0.43	0.670	2.08
	O	0.004	0.94	0.349	2.08
December 86	C	0.003	0.61	0.544	2.04
	O	0.004	0.94	0.348	2.22
March 87	C	0.001	0.28	0.780	2.06
	O	-0.000	-0.12	0.908	2.00
June 87	C	-0.005	-1.00	0.319	2.34
	O	-0.005	-1.24	0.215	2.29
September 87	C	0.012	2.72	0.007	2.19
	O	0.013	3.55	0.000	2.21
December 87	C	-0.011	-0.90	0.368	2.24
	O	-0.011	-1.13	0.257	2.30
March 88	C	-0.004	-0.93	0.354	1.84
	O	-0.000	-0.08	0.936	1.87
June 88	C	-0.006	-1.12	0.264	2.22
	O	-0.002	-0.63	0.527	2.11

* Beach and Mackinnon (1978) Maximum Likelihood procedure used to transform autocorrelated OLS residuals.

TABLE 9B : REGRESSIONS OF ABSOLUTE MAGNITUDE OF MISPRICING VS TIME TO EXPIRATION*

$$|X_{t,T}^{i}| = a^i + b^i\,(T\text{-}t) + \epsilon_t^i \quad i = 1, \ldots 16$$

		b^i	T-statistic	Sig.level	R^2 (%)
September 84	C	0.014	8.95	0.000	55.6
	O	0.014	7.73	0.000	48.3
December 84	C	0.021	14.53	0.000	76.7
	O	0.021	11.52	0.000	67.4
March 85	C	0.008	6.79	0.000	42.3
	O	0.007	4.43	0.000	23.8
June 85	C	0.021	14.00	0.000	76.3
	O	0.019	13.30	0.000	74.4
September 85	C	0.010	8.47	0.000	52.5
	O	0.010	7.00	0.000	43.0
December 85	C	0.013	8.12	0.000	50.7
	O	0.012	7.50	0.000	46.8
March 86	C	-0.000	-0.22	0.829	0.1
	O	-0.002	-1.48	0.139	3.9
June 86	C	0.005	3.64	0.000	17.4
	O	0.004	2.84	0.005	12.2
September 86	C	-0.000	-0.02	0.986	0.0
	O	0.002	1.01	0.311	1.6
December 86	C	0.001	0.59	0.554	0.5
	O	0.001	0.85	0.393	1.1
March 87	C	0.001	0.95	0.344	1.4
	O	0.001	0.54	0.590	0.5
June 87	C	0.002	1.69	0.092	4.4
	O	0.002	1.16	0.245	2.2
September 87	C	0.003	1.85	0.064	5.0
	O	0.004	3.04	0.002	12.4
December 87	C	0.014	2.84	0.004	11.2
	O	0.012	2.13	0.033	6.6
March 88	C	0.004	2.14	0.033	6.7
	O	0.002	1.52	0.129	3.5
June 88	C	0.004	2.42	0.016	8.8
	O	0.003	1.95	0.052	5.8
Pre-Big Bang	C	0.009	13.01	0.000	22.3
	O	0.009	12.13	0.000	20.1
Post-Big Bang	C	0.005	4.92	0.000	5.4
	O	0.004	3.91	0.000	3.5

* Maximum likelihood estimates for the Standard Tobit Model with a zero lower bound censoring threshold and based essentially on the iterative procedure of Berndt, Hall, Hall and Hansman (1974).

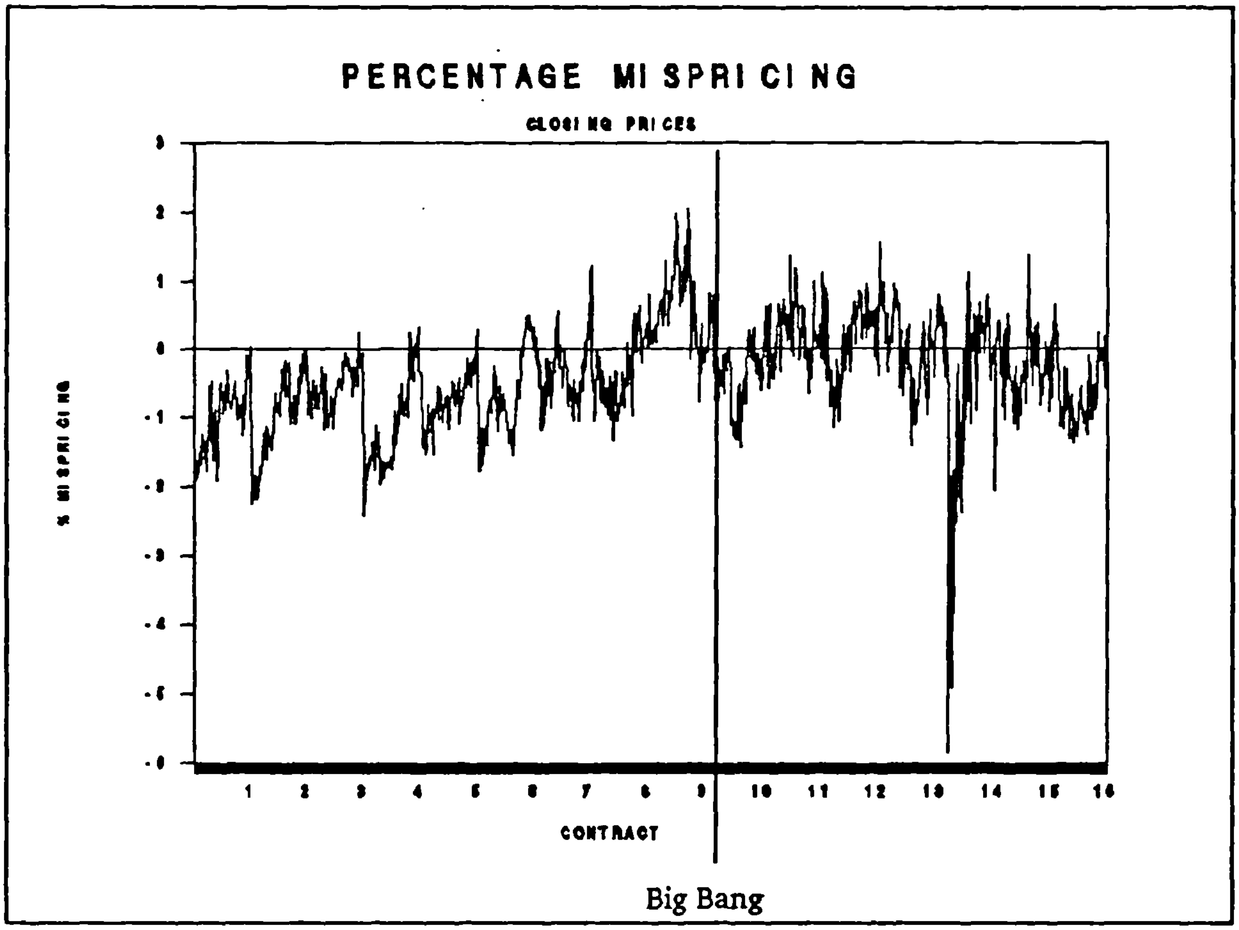
PERCENTAGE MISPRICING
CLOSING PRICES
% MISPRICING
CONTRACT
Big Bang

Figure 1

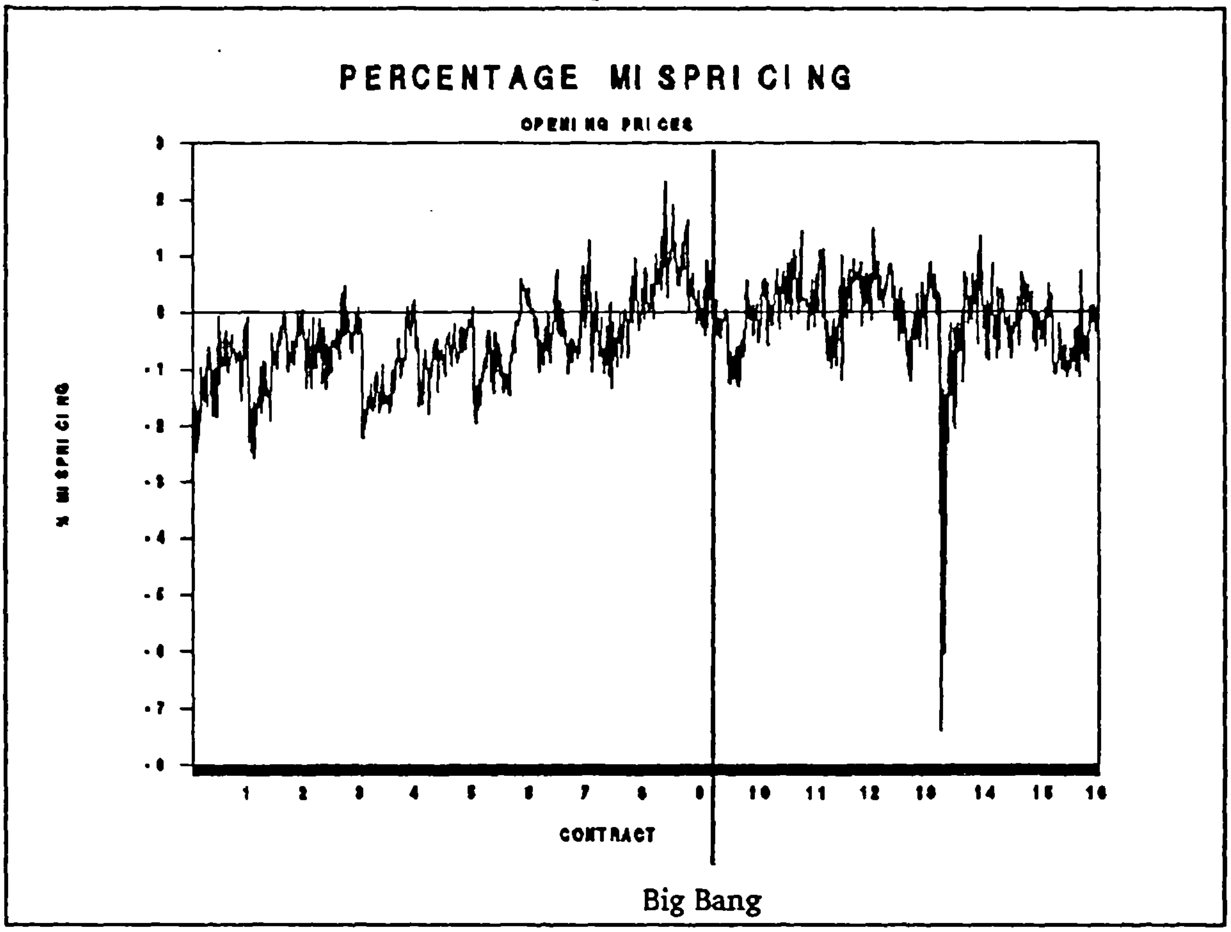
PERCENTAGE MISPRICING
OPENING PRICES
% MISPRICING
CONTRACT
Big Bang

Figure 2

CHAPTER 3

TESTING INDEX FUTURES MARKET EFFICIENCY USING PRICE DIFFERENCES: A CRITICAL ANALYSIS[1]

ABSTRACT

In this chapter the tests of stock index futures pricing efficiency introduced by Saunders and Mahajan (1988) are reappraised. Firstly, it is pointed out that the tests do not preclude the existence of arbitrage profits due to mispricing. Secondly, given the existence of mispriced futures, the validity of the SM slope test is questioned. Finally, it is shown that the intercept test is misspecified and that it's power is so low that under normal circumstances, it will almost never reject efficiency. The arguments are illustrated with empirical results from the London markets.

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[1] First draft January 1990; revised May 1990; revised version published in *Journal of Futures Markets*, Vol 11 (April 1991), pp 239-252.

1. Introduction

Saunders and Mahajan (1988) (hereafter SM) suggest an alternative approach for analysing pricing efficiency of stock index futures contracts that is based not on the levels of cash and futures prices, but on their first differences. The approach is *a priori* attractive since it does not price futures contracts as forward contracts and hence obviates the need to explicitly assume deterministic interest rates or dividend payouts beyond the next period. On the basis of their empirical results SM conclude that they cannot reject the hypothesis that index futures pricing is "efficient". This conclusion is in apparent contrast with the evidence on substantial forward pricing formula "mispricing" and the simulated profitability of arbitrage-related trading rules reported *inter alia* by Merrick (1988, 1989), Mackinlay and Ramaswamy (1988) and Yadav and Pope (1990).

This paper examines the tests proposed by SM in greater detail. Firstly, we point out that the failure to reject "efficiency" using the SM tests does not necessarily preclude the existence of arbitrage profits from hold to expiration or other trading rules. Secondly, we question the validity of the SM slope test, given that in the absence of perfectly elastic arbitrage, the futures contract will exhibit at least some mispricing relative to the cash index. Finally, we show that the SM intercept test is misspecified and that its power is so low that under normal circumstances it will almost never reject efficiency. Our arguments are illustrated with evidence based on data relating to the UK FTSE100 stock index futures contract traded on the

London International Financial Futures Exchange (LIFFE).

The chapter is organised as follows. Section 2 documents the basic features of the SM tests relevant to this paper. Section 3 presents our critique of the SM tests. Section 4 provides illustrative empirical results. Section 5 summarises the conclusions.

2. The SM Tests - Basic Features

SM show that under their stated assumptions it is reasonable to write

$$E(R^F_t) \left[\prod_{w=t}^{T} e^{-E(r_{w,w+1})} \right] = E(R^I_t) \quad (1)$$

In the above equation $r_{w,w+1}$ is the one period risk free rate at time w; T is the value of the time parameter at futures maturity; and R^F_t and R^I_t are the period t futures "return" and cash return respectively defined in terms of the t period futures price $F_{t,T,}$ the t period cash index price I_t, and the t period dividend d_t as[2]:

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[2] The reason for dividing by I_{t-1} is to avoid potential problems related to heteroskedascity.

$$E(R^F_t) = \frac{\{E(F_{t,T}) - F_{t-1,T}\}}{I_{t-1}}$$

$$E(R^I_t) = \frac{\{E(I_t) - I_{t-1} + d_t\}}{I_{t-1}}$$

SM convert the *ex ante* equation (1) into *ex post* form by assuming that (i) actual realised values can be taken as surrogates for expected values; and (ii) the best proxy for the product of expected one period interest rates can be taken to be the interest rate observed at t for an instrument maturing at T.

SM's suggestion is that if pricing mechanisms are efficient, the following regression equation (2) should have OLS estimated coefficients of a = 0 and b = 1.

$$R^F_t \, e^{-r_{t,T}(T-t)} = a + bR^I_t + u_t \qquad (2)$$

where $E(u_t) = 0$

and $Cov(R^I_t, u_t) = 0$

It is important to highlight the following observations of SM in regard to equation (2):

(i) SM state that "... the prevalence of a significant intercept parameter would support the hypothesis that the arbitrage relationship was systematically violated" (SM p 214). They actually find that the null hypothesis a=0 is "... unambiguously accepted ..." and infer that "... no systematic excess returns were possible by maintaining a position in any index futures contract implying that the market was in equilibrium and pricing efficiently" (SM p 220).

(ii) SM also state that "... finding that the slope parameter was significantly different from one would support the hypothesis that the arbitrage relationship was consistently violated unsystematically for short periods of time" (SM p 214). They find that the null hypothesis b=1 is rejected for the early contracts and not rejected for the later contracts and conclude that "... systematic and significant arbitrage opportunities have disappeared ..." (SM p 226).

It should also be noted that :

(i) SM also explicitly recognise that "the *ex post* empirical counterpart to .... (their) .... normative equilibrium equation (1) .... is identical to that derivable under the assumptions that forward prices are not different from futures prices and that future dividends are certain ...." (SM p 213).

(ii) Regression equation (2) requires SM to assume that cash returns are independent of the error term. They do not consider the alternative possibility of a reverse regression of (time adjusted) cash returns on futures returns (also consistent with equation (1)) which would require the assumption that futures returns are independent of the error term.

(iii) The normative equilibrium equation (1) ignores the one day interest r_o. If this is included (but time variation in this factor is ignored) then equation (1) can be written as :

$$E(R^F_t) \left[\prod_{w=t}^{T} e^{-E(r_{w,w+1})} \right] = E(R^I_t) - r_o$$

3. The SM Tests - A Critique

3.1 SM "efficiency" and profitability of trading rules

The claim made by SM is that their results support the "efficiency" of the index futures market and hence conflict with previous studies (SM, p 211). However, it is important to realise that failure to reject "efficiency" as implicitly defined by SM does not necessarily preclude the existence of riskless arbitrage profits from hold to expiration or other trading rules.

Firstly it is relevant to highlight the difference between the implicit definition of the term "efficiency" used by SM and that used by earlier studies on index futures markets.[3] Studies on index futures markets based on the levels of futures prices (like Merrick, 1988, 1989; Mackinlay and Ramaswamy, 1988; Yadav and Pope, 1990) have attempted to identify opportunities for riskless excess returns using trading rules which exploit the known change in cash futures basis between the day of the trade and the expiration day. The relevant measure of efficiency in these studies has implicitly or explicitly been the number of cases in which the deviation of actual prices from non-arbitrageable prices has exceeded transactions cost based thresholds. These studies have hence adopted a perspective which is consistent with, or at least subsumes, Jensen's (1978) trading rule orientation to testing efficiency. On the other hand, transactions costs are not directly relevant in the SM notion of efficiency. The SM tests imply a definition of efficiency equivalent to the unbiased realisation, on average, of expectations of changes in futures prices, conditional upon changes in cash prices (where expectations are based on infinitely elastic arbitrage). In that sense, it is comparable with the formulation of Fama (1970, 1976), and not to Jensen's trading rule approach to testing efficiency.

Secondly, what is relevant from the SM viewpoint is whether the OLS regression of equation (2) indicates bias. Inferences based only on the regression line can obviously mask significant features of the data - in particular the systematic patterns in mispricing returns (ie the regression residuals). For example, Merrick (1988)

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[3] For an excellent review of the different conceptual definitions of market efficiency see Ball (1988).

(for US data) and Yadav and Pope (1990) (for UK data) report that the returns on one day hedges are significant and positive (negative) if such hedges are established when mispricing is initially positive (negative) even though the average returns on one day hedges are zero. However, the only information which the SM regression line can provide in this context is on the *average* returns on one day hedges. The SM tests thus appear capable of overlooking systematic and potentially exploitable mispricing (inefficiency) because of their focus on overall regression line rather than on individual observations.

Finally, it should also be realised that inferences from the SM tests are based only on changes in cash and futures prices and not on the levels of these prices. These inferences would not be affected if all the prices used in the computations displayed the same bias, however large. Such a scenario is conceivable if expiration day observations are not included in the data.[4] Under such circumstances the SM tests would not be capable of providing information about levels of mispricing and the implied profitability of arbitrage related trading rules.

3.2 Validity of the SM Tests

In this section we question the validity of the SM tests, given that in the absence of perfectly elastic arbitrage, futures prices will in general exhibit some mispricing.

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[4] In any case, expiration day observations are usually not included in studies of pricing efficiency in view of expiration day distortions documented *inter alia* by Stoll and Whaley (1987). Furthermore, it is sometimes difficult to include expiration day/time observations as eg in the UK where futures contracts mature at 11.20 am!

Let $F_{t,T}*$, $F_{t-1,T}*$ be theoretical futures prices in the SM framework, ie prices generated when the expectations contained in equation (1) are exactly realised in every period. This is feasible if intermarket arbitrage is perfectly elastic so that deviations from equilibrium due to noise are continually and instantaneously arbitraged away.[5] Let $W_{t,T}$, $W_{t-1,T}$ be the corresponding futures "mispricing" ie ($F_{t,T}$ - $F_{t,T}*$), ($F_{t-1,T}$ - $F_{t-1,T}*$). Then following Merrick (1988) the scaled change in futures mispricing or mispricing "return" R^x_t can be written as:

$$R^x_t = \frac{W_{t,T} - W_{t-1,T}}{I_{t-1}}$$

$$= \frac{F_{t,T} - F_{t,T}^*}{I_{t-1}} - \frac{F_{t-1,T} - F_{t-1,T}^*}{I_{t-1}}$$

$$= \frac{F_{t,T} - F_{t-1,T}}{I_{t-1}} - \frac{F_{t,T}^* - F_{t-1,T}^*}{I_{t-1}}$$

$$= R^F_t - R^F_t*$$

where R^F_t* is the "theoretical" futures return given by :

$$R^F_t* \; e^{-r_{t,T}(T-t)} = R^I_t - r_o$$

[5] Since daily cash and futures return in an SM framework are interrelated in exactly the same way as they would be if the forward pricing formula was valid, and since cash and futures prices are necessarily identical on expiration day, $F_{t,T}*$ and $F_{t-1,T}*$ would also be the forward pricing formula futures prices.

Hence[6]

$$(R^F_t \; e^{-r_{t,T}(T-t)} - R^I_t) \quad = \quad R^X_t \, e^{-r_{t,T}(T-t)} - r_o \tag{3}$$

In the presence of stochastic changes in futures mispricing, the estimated values of the OLS coefficients in the SM regression (2) will be :

$$\hat{b} \quad = \quad 1 + b_{xc} \tag{4a}$$

$$\hat{a} \quad = \quad \text{Mean}\,(R^X_t \; e^{-r_{t,T}(T-t)}) - b_{xc}\,\text{Mean}\,(R^I_t) - r_o \tag{4b}$$

$$\text{where} \quad b_{xc} \quad = \quad \frac{\text{Cov}(R^X_t \; e^{-r_{t,T}(T-t)}, R^I_t)}{\text{Var}\,(R^I_t)} \tag{4c}$$

ie b_{xc} is the OLS estimate of the slope of the regression line of (time adjusted) mispricing returns on cash returns.

3.2.1 The SM Slope Test

The existence of mispriced futures has several important implications for the SM slope test.

Firstly, an OLS regression of equation (2) estimates $\hat{a}$ and $\hat{b}$ so as to minimise the variance of $\{R^F_t \, e^{-r_{t,T}(T-t)} - \hat{a} - \hat{b}\, R^I_t\}$. In other words, we are constructing a risk

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6 An investor, eg a market maker, wanting to hedge a predetermined long (short) cash market position over one period, would on the basis of infinitely elastic arbitrage and perfect knowledge of future absolute dividend inflows, construct a one period hedge portfolio by selling (buying) exp $\{-r_{t,T}(T-t)\}$ futures contracts for every unit of the cash index held. The mispricing return thus has a direct economic interpretation in terms of the abnormal return earned on such a one day hedge portfolio.

minimising hedge for an investor who is buying *spot* to hedge a fixed quantity commitment in the *futures* market. In this context, most of the issues raised by Merrick (1988) are valid, *mutatis mutandis*. In particular, the estimated slope $\hat{b}$ has two components. The first is the equilibrium component arising from equation (1). The second is an adjustment factor equal to the slope between the changes in futures mispricing and the contemporaneous change in cash index prices. This adjustment factor is zero only when period t cash returns are uncorrelated with period t changes in mispricing.

Neither the forward pricing formula nor the theory underlying the SM tests can tell us anything about the size and sign of this slope adjustment factor. Guidance in this respect can only come from a *dynamic* model which specifically incorporates inelastic intermarket arbitrage, such as the model of Garbade and Silber (1983). More generally, if futures prices and cash equivalent futures prices are modelled in an error correction model framework with a cointegrating parameter of unity[7], the mispricing variable X_t follows an autoregressive process of the form[8]:

$$X_t = \alpha + \sum_{k=1}^{p} \phi_k X_{t-k} + e_t \tag{5}$$

[7] See Engle and Granger (1987) for details of models with cointegrated variables.

[8] The Garbade and Silber (1983) model is a special case of such an error correcting model.

where $|\phi_1| < 1$

$p \geq 1$

This implies that mispricing tends to regress towards its equilibrium value of zero and is consistent with the evidence of mispricing reversals document by Merrick (1988) and Yadav and Pope (1990). Mispricing reversals (or arbitrage related restoration of equilibrium) means that long futures/short cash hedges begun with overpriced (underpriced) futures can be predicted *ex ante* to earn less (more) than the riskless return. In other words, conditional upon information about mispricing in period (t-1), the *ex ante expected* (time adjusted) futures return in period t will be equal not just to the cash index return in period t (as provided in equation (1)) but will also include a component reflecting the correction of mispricing in period (t-1) and, possibly, earlier periods. Hence, if efficiency is considered relative to a pricing model which incorporates inelastic intermarket arbitrage, the slope adjustment factor b_{xc} in equations (4a), (4b) and (4c) need not necessarily be equal to zero. In particular, it would depend on whether *on average* as mispricing changes from period to period in accordance with equation (5), cash prices move further towards futures prices than futures prices move towards cash prices - ie it would depend specifically on the relative roles of the cash market and the futures markets in price discovery. As such, with mispriced futures, the hypothesis $b=0$ does not necessarily reflect an unbiased realisation of expectations and is hence not necessarily a valid null hypothesis.

Secondly, it is also relevant to observe that :

$$\hat{b} \quad = \quad \rho_{fc} \quad \frac{\sigma_f}{\sigma_c} \tag{6}$$

where ρ_{fc} is the correlation between (time adjusted) futures returns and cash returns; and σ_f and σ_c are thus the respective standard deviations.

If the arbitrage link is perfect, the futures contract is a perfect substitute for the cash index, mispricing is identically zero and hence

$$\rho_{fc} = 1, \ \sigma_f = \sigma_c, \text{ and } b = 1$$

With mispriced futures $0 < \rho < 1$ and $\hat{b}$ can be equal to unity (as hypothesised by SM) only if $\sigma_f > \sigma_c$, ie only if the perfect equivalence of cash and futures markets (or the perfect arbitrage link) is destroyed by futures prices systematically overshooting cash equivalent prices.

There is evidence to show that futures prices exhibit greater volatility than cash prices (see eg Yadav and Pope, 1990 and Mackinlay and Ramaswamy, 1988). Hence, the SM "acceptance" of the null hypothesis $\hat{b}=1$ is consistent with this excess volatility of futures relative to cash, and instead of indicating efficiency, supports, on the contrary, the rejection of the hypothesis of a perfect arbitrage link.

Finally, it is also important to note that in moving from equation (1) to equation (2), SM assume that cash returns are independent of the error term and that the error term is not autocorrelated. In frictionless markets the error term in equation (2) should hence represent the effect of randomly arriving new information, which though incorporated (instantaneously) into cash prices is not yet reflected in futures prices, since futures prices are still "catching up". In other words, equation (2) is based on price discovery taking place entirely in the cash market. In fact, it is often argued that because of lower transaction costs, mature futures markets tend to play the more active role in price discovery (Hill, Jain and Wood, 1988). Merrick (1987) reports evidence consistent with this argument. This alternative possibility of futures returns being independent of the error term is also equally consistent with equation (1) and implies a reverse regression of the form

$$R^I_t \quad = \quad a^1 + b^1 R^F_t e^{-r_{t,T}(T-t)} + u_t^1 \tag{7}$$

The OLS estimates of b^1 will be:

$$\hat{b}^1 \quad = \quad \rho_{fc} \quad \frac{\sigma_c}{\sigma_f} \tag{8}$$

Thus if $\hat{b} = 1$ (the SM null hypothesis) and $\rho_{fc} \neq 1$ then $\hat{b}^1$ will *definitely* be less than unity. With mispriced futures one of $\hat{b}$ or $\hat{b}^1$ will *always* be different from unity. $\hat{b}$ by itself does not provide a meaningful picture of the extent of integration between the markets. Information on $\hat{b}^1$ (or equivalently the R^2 of the regression based on equation (2) or ρ_{fc}) is also required.

To summarise, with mispriced futures, SM equation (1) does not necessarily reflect an unbiased realisation of expectations. Furthermore even if we start from SM equation (1), it is not possible to cogently argue for either $\hat{b}$ or $\hat{b}^1$ to be *the* measure to assess "efficiency" unless we assume that one market is completely irrelevant from the viewpoint of price discovery. Not only that, at least one of $\hat{b}$ or $\hat{b}^1$ is always different from unity. As such the SM slope test (or its variant based on the reverse regression equation (7)) cannot result in a clear statement about "efficiency".

3.2.2 The SM Intercept Test

The SM null hypothesis $\hat{a} = 0$ will be rejected at the $\alpha\%$ level when

$$\left| \frac{\hat{a}}{SE(\hat{a})} \right| > t_\alpha \tag{9}$$

If n observations are used in the SM regression (2), then since $[Mean(R^I_t)]^2 \lll Var(R^I_t)$ it follows that

$$SE(\hat{a}) = \frac{\sigma_u}{\sqrt{n}}$$

Condition (9) can hence be rewritten as

$$\frac{\sqrt{n}}{\sigma_u t_\alpha} \left| \text{Mean}\,(R^X_t \cdot e^{-r_{t,T}(T-t)}) - b_{xc}\,\text{Mean}\,(R^I_t) - r_o \right| > 1$$

which implies that the following expression is greater than one:

$$\left| \frac{\sqrt{n}}{\sigma_u t_\alpha} \text{Mean}\,(R^X_t\ e^{-r_{t,T}(T-t)}) \right| + \left| \frac{\sqrt{n}}{\sigma_u t_\alpha} b_{xc}\,\text{Mean}(R^I_t) \right| + \left| \frac{\sqrt{n}\, r_o}{\sigma_u t_\alpha} \right| \tag{10}$$

Note that under the null hypothesis σ_u can be approximated by the standard deviation of forward pricing formula mispricing returns.[9] Yadav and Pope (1990), using four years of daily UK data, estimate this standard deviation to be about 0.5%. The estimate made by Hill, Jain and Wood (1988) using three years of daily US data is very similar.

Under the SM null hypothesis b_{xc} should be zero. In any case, it is likely to be small enough to make the second term in equation (10) much smaller than the third term. With daily data, r_o, the one period interest, will also be less than 0.05%. Hence, with daily data, and at the 5% significance level, the second and third terms will be important in relation to unity only when $n \geq 400$.

If there are no missing values in the time series of cash and futures returns, with n

[9] It will actually be slightly less depending on the extent to which b_{xc} is different from zero.

observations the first term in expression (10) above will be approximately[10]

$$\frac{|X_n - X_1|}{\sigma_u \, t_\alpha / \sqrt{n}}$$

With daily data and transaction costs of about 1%-2%, this can be important in relation to unity only when

$$n \leq 20.$$

Thus, considering the "normal" levels of transaction costs in the US and the UK markets, and the implied "normal" variation in mispricing, the SM intercept test using daily data could reject the null hypothesis only when the number of observations used is below about 20 or exceeds about 400. With the use of about three to twelve months of daily data, it is unlikely to ever reject "efficiency".

It should also be noted that ignoring the one day interest factor (as SM have done) results in a serious misspecification of the SM null hypothesis relating to the intercept since with reasonably large sample sizes and reasonably SM "efficient" markets, the third term rapidly becomes the dominant term in equation (10). For

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[10] In the general case (ie with possible missing values), the results of Yadav and Pope (1990) show that with the strong tendency for mispricing reversals, the mean of mispricing returns has been much smaller than the one day interest in the UK. Though not specifically reported, the results of Merrick (1988) point to a similar conclusion for daily US data.

example, if we consider three months' daily data, interest rates of about 10% and highly SM "efficient" markets (ie $b_{xc} = 0$ and mispricing returns averaging zero), then the null hypothesis will be incorrectly rejected whenever the transaction costs are low enough to make the standard deviation of mispricing returns below about 0.1%.

Equation (4b) also shows that, with mispriced futures, the value of the OLS intercept depends on the *ex post* cash index returns. Thus, even if the markets are "efficient" enought to result in zero average mispricing returns, the *ex ante* expected value of the intercept depends on the expected market returns. The null hypothesis is to that extent misspecified, though b_{xc} is likely to be small enough to make the effect of this factor much smaller than the effect of ignoring the one day interest factor.

To summarise, with mispriced futures the SM intercept test is misspecified to the extent that the "null" value of the intercept depends on average market returns and, more importantly, on the average one day interest factor. Furthermore, the power of the test is so low that under normal circumstances it will almost never reject "efficiency".

4. Empirical Results

This section illustrates the arguments of Section 3 with empirical evidence based on the FTSE100 index futures contract traded on the London International Financial Futures Exchange (LIFFE). Our empirical analysis includes 1012 trading days over 16 different contracts spanning the period July 1, 1984 to June 30, 1988. The first 9 contracts expiring up to September 1986 constitute evidence for the pre Big Bang (ie market deregulation)[11] period, while the last 7 contracts constitute evidence for the deregulated post Big Bang period. The analysis is based on the near contract, shifting to the next contract on expiration day.

"Time and Sales" data on FTSE100 index futures was obtained from LIFFE. The data includes all bid and ask quotes and all transaction prices relating to the FTSE100 index futures contract over our period of study. The data was used to infer the mid market quotes valid at 10.00 am, 11.00 am, 12 noon, 1.00 pm, 2.00 pm, 3.00 pm, 3.30 pm and 4.00 pm[12] each day. Data on FTSE100 index was obtained from the International Stock Exchange (formerly the London Stock Exchange). This included opening, 3.30 pm, and closing values of the index over the entire period of study. Hourly data on the FTSE100 index was collected from the *Financial Times*. This data was available only from February 18, 1986,

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[11] Market deregulation took place from October 27, 1986.

[12] Prior to April 28, 1986, LIFFE index futures traded only up to 3.30 pm each day. Thus 4.00 pm values are used only after April 28, 1986.

onwards, and hence the aggregated results of analysing hourly data are reported only for contracts expiring after Big Bang. Information on the constituents of the index, and how these constituents had changed over the sample period, was obtained from the International Stock Exchange. The daily dividend entitlement on the FTSE100 index was calculated as in Yadav and Pope (1990). Daily series for one- and three-month Treasury Bill rates were also collected from *Datastream*, and used to infer a linear term structure over the relevant period.

The results of regressions based on Equation (2) or Equation (7) are likely to be dependent on the time period over which first differences in cash and futures prices are measured. Table 1 is based on weekly intervals (coincident 3.30 pm prices Thursday to Thursday), Table 2 on daily intervals (coincident 3.30 pm prices each day), and Table 3 on hourly intervals (using coincident prices at 10.00 am, 11.00 am, 12 noon, 1.00 pm, 2.00 pm, 3.00 pm and 4.00 pm each day).[13] However, all computations for weekly and daily price intervals were repeated with coincident prices at 10.00 am, 11.00 am, 12 noon, 1.00 pm, 2.00 pm, 3.00 pm and 4.00 pm[14] and all computations for weekly intervals were repeated with different days of the week. The results of these computations were essentially identical to the results reported.

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[13] 4.00 pm values not used before April 28, 1986.

[14] LIFFE index futures open at 9.05 am and close at 4.05 pm (3.30 pm before April 28, 1986). Cash index values were available, at best hourly. Results based on closing and opening prices are not reported in view of this misalignment.

In all cases of daily intervals, Friday-to-Monday, and other holiday gapped returns are not used in the regressions. In all cases of weekly intervals returns over weeks which included holidays are not used. For hourly price differences 4.00 pm to 10.00 am returns are not used.[15]

In almost all cases, OLS regressions revealed that the residuals were autocorrelated and also that the residuals could be best modelled as AR(1) processes. Hence, the regression results reported are based on GLS estimation, with the assumption that residuals are an AR(1) process. The actual computation uses the Beach and Mackinnon (1978) maximum likelihood procedure to estimate coefficients and standard errors. In <u>all</u> cases, the autocorrelation of the transformed residuals was insignificantly low, and the Durbin Watson Statistic for the transformed residuals was within satisfactory limits.

The essential features of the results reported in Table 1 (weekly data), Table 2 (daily data) and Table 3 (hourly data) are :

(1) Neither the hypothesis $a=0$ nor the hypothesis $a^1=0$ can be rejected at the 5% level.[16]

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15 3.00 pm to 10.00 am returns were not used before April 28, 1986.

16 Both a and a^1 are insignificant for both sub periods with weekly data, for 15 out of 16 contracts with daily data and for all contracts with hourly data.

(2) For weekly and daily data :

(a) In the pre-Big Bang period, the hypothesis $b=1$ is conclusively rejected but the hypothesis $b^1=1$ cannot be rejected.[17]

(b) In the post-Big Bang period, the hypothesis $b=1$ cannot be rejected, but the hypothesis $b^1=1$ is conclusively rejected.[18]

(3) With hourly data both the hypotheses $b=1$ and $b^1=1$ are conclusively rejected.[19] Both b and b^1 are significantly below unity, but this arises because of the relatively lower overall correlation between cash returns and (time adjusted) futures returns.[20] Furthermore, for every contract, the implied correlation between futures returns and mispricing returns is consistently greater in magnitude than the implied correlation between cash returns and mispricing returns. Futures returns are significantly more volatile than cash returns, but because of the lower correlation between them, b remains below unity.

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[17] b is significantly below unity (at 5% level) for weekly data and for 8 out of 9 contracts with daily data. On the other hand b^1 is neither significantly different from unity with weekly data, nor for any of the 9 contracts using daily data.

[18] b is neither significantly different from unity (at 5% level) for weekly data nor for 5 out of 7 contracts using daily data. On the other hand, b^1 is significantly different from unity at 5% level for weekly data, and for 6 out of 7 contracts with daily data.

[19] b is significantly below unity for 6 out of 7 contracts and b^1 is significantly below unit for each of the 7 contracts.

[20] The overall correlation drops from above 0.9 with weekly/daily returns to about 0.7 with hourly returns.

The above results clearly illustrate the arguments of Section 3. The hypothesis $a=0$ is almost never rejected. And whenever the hypothesis $b=1$ is not rejected, the hypothesis $b^1=1$ is conclusively rejected. As pointed out in Section 3.2.1, whenever $b=1$ the volatility of futures returns is greater than that of cash returns, contrary to the hypothesis of a perfect arbitrage link between the two markets. Note that even for US markets, over the period that SM were unable to reject the hypothesis $b=1$, Merrick (1988) reported significantly high correlation between mispricing returns and futures returns. This implies that over SM's sample period b^1 was significantly less than unity.

5. Conclusions

This paper has examined the nature and validity of the tests for pricing efficiency proposed by Saunders and Mahajan (1988). Firstly, it has pointed out that the failure to reject "efficiency" using the SM tests does not necessarily preclude the existence of arbitrage profits from hold to expiration or other trading rules. Secondly, it has questioned the validity of the SM slope test, given that in the absence of perfectly elastic arbitrage the futures contract will, in general, exhibit at least some mispricing relative to the cash index. Thirdly, it has argued that the SM intercept test is seriously misspecified. Finally, it has shown that the power of the SM intercept test is so low that under normal circumstances it will almost never reject "efficiency". The arguments presented in this paper have been illustrated with empirical results from the London markets.

TABLE 1

WEEKLY DATA

Contracts Expiring	a $x10^3$	t-stat $H_o{:}a=0$	b	t-stat $H_o{:}b=1$	a^1 $x10^3$	t-stat $H_o{:}a^1=0$	b^1	t-stat $H_o{:}b^1=1$
Pre Big Bang	-0.69	-1.41	0.915	-3.37**	0.96	1.92	1.023	0.92
Post Big Bang	-0.16	-0.16	1.010	0.19	0.87	1.00	0.877	-2.85**

** Significant at the 1% level

TABLE 2

DAILY DATA

Contracts Expiring	a $x10^3$	t-stat H_o:a=0	b	t-stat H_o:b=1	a^1 $x10^3$	t-stat H_o:a^1=0	b^1	t-stat H_o=ρ=0
September 84	0.73	1.28	0.921	-1.26	-0.06	-0.09	0.926	-1.16
December 84	0.44	1.23	0.819	-3.54**	-0.49	-1.26	1.057	0.87
March 85	0.47	1.35	0.913	-2.03*	-0.31	-0.82	0.991	-0.19
June 85	0.17	0.55	0.871	-2.65**	-0.41	-1.13	0.974	-0.46
September 85	-0.02	-0.08	0.840	-2.78**	0.14	0.35	0.967	-0.50
December 85	1.02	4.24**	0.840	-3.36**	-0.77	-2.47*	1.020	0.33
March 86	0.81	1.72	0.855	-2.53*	-0.27	-0.54	0.979	-0.32
June 86	0.08	0.14	0.884	-2.01*	-0.11	-0.18	0.942	-0.94
September 86	-0.44	-1.02	0.888	-2.38*	0.29	0.64	0.993	-0.14
December 86	0.08	0.23	0.949	-0.74	0.01	0.05	0.855	-2.35**
March 87	0.23	0.61	0.853	-2.88**	0.35	0.96	0.988	-0.22
June 87	0.21	0.33	0.984	-0.28	0.15	0.26	0.886	-2.24*
September 87	-0.02	-0.05	1.043	0.93	0.07	0.17	0.886	-2.92**
December 87	-0.00	-0.01	1.024	0.66	-0.04	0.07	0.927	-2.25*
March 88	-0.17	-0.46	1.093	1.93	0.28	0.78	0.822	-4.80**
June 88	-0.26	-0.61	1.147	2.31*	0.42	1.20	0.785	-4.93**
Pre Big Bang	0.33	2.57*	0.890	-6.43**	-0.21	-1.56	0.974	-1.39
Post Big Bang	-0.04	-0.27	1.014	0.80	0.18	1.18	0.904	-6.23**

** Significant at the 1% level
* Significant at the 5% level

TABLE 3

HOURLY DATA

Contracts Expiring	a $x10^3$	t-stat $H_o:a=0$	b	t-stat $H_o:b=1$	a^1 $x10^3$	t-stat $H_o:a^1=0$	b^1	t-stat $H_o:b^1=0$
March 86	0.23	1.35	0.782	-2.48*	0.00	0.02	0.449	-10.11**
June 86	0.03	0.27	0.710	-5.26**	-0.18	-1.58	0.393	-17.26**
September 86	0.08	0.66	0.580	-7.42**	-0.15	-1.28	0.362	-16.22**
December 86	-0.00	-0.02	0.822	-3.32**	0.04	0.69	0.479	-16.13**
March 87	-0.03	-0.41	0.881	-3.46**	-0.07	0.98	0.705	-10.24**
June 87	-0.07	-0.90	0.784	-5.61**	0.05	0.64	0.668	- 9.72**
September 87	-0.16	-2.02	0.929	-2.12*	0.06	0.80	0.723	-10.02**
December 87	0.00	0.00	0.832	-3.42**	-0.09	-0.34	0.544	-14.25**
March 88	-0.06	-0.64	1.131	3.13**	-0.01	-0.11	0.596	-18.06**
June 88	-0.05	-0.71	1.020	0.53	0.00	0.08	0.675	-12.63**
Post Big Bang	-0.06	-1.05	0.864	-7.82**	-0.01	0.25	0.580	-35.75**

** Significant at the 1% level
* Significant at the 5% level

CHAPTER 4

TRANSACTION COST THRESHOLDS, ARBITRAGE ACTIVITY AND INDEX FUTURES PRICING[1]

ABSTRACT

This chapter explores the effect of transaction costs on the time series process followed by stock index futures mispricing. It is argued that the market demand schedule of arbitrageurs in financial futures markets is unlikely to vary linearly with the level of mispricing. Instead market demand can be expected to vary with mispricing in a non-linear fashion - specifically in the form of a step function. Consequently, the time series of mispricing should follow a self exciting threshold autoregressive (SETAR) process. This is a piecewise linear autoregressive process in which the process parameters describing the evolution of mispricing are path-dependent. Empirical tests of the hypothesis of SETAR type non-linearity in the time series of stock index futures mispricing indicate that the hypothesis of a linear AR model is conclusively rejected in favor of threshold non-linearity. The threshold estimates are consistent with the marginal transaction cost levels faced by the different categories of arbitrageurs expected to be active in the markets. Furthermore, estimates of a measure of the elasticity of arbitrage services corresponding to different transaction cost regimes are also consistent with the model.

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[1] First draft September 1990; revised March 1991, October 1991 and March 1992; presented to the ESRC Money Study Group, October 1991 (London Business School); Mid West Finance Association Conference, April 1991 (USA); and Western Finance Association Conference, June 1991 (USA); and presented to staff seminars at Dundee and Groupe HEC Paris. Earliest version was titled "Modelling Futures Mispricing by Self Exciting Threshold Autoregressive Processes" and was circulated by the Center for the Study of Futures Markets, Columbia University, USA as Paper # 211.

TRANSACTION COST THRESHOLDS, ARBITRAGE ACTIVITY AND INDEX FUTURES PRICING

I. INTRODUCTION

Systematic and significant deviations of index futures prices from their fair value and the profitability of arbitrage-related program trading have been documented, *inter-alia*, by Figlewski (1984a), Merrick (1989), Brenner *et al* (1989), Yadav and Pope (1990) and others. There has also been extensive discussion of the implications of stochastic changes in this futures mispricing by Merrick (1988), Hill *et al* (1988), Yadav and Pope (1991) and others. However, to date relatively little attention has been devoted to modeling or empirically testing the stochastic process governing futures mispricing, particularly in a context where transaction costs and market microstructure characteristics are considered explicitly. Previous research has recognized that transaction costs must have a role in explaining the magnitude of price differences between equivalent assets, but there is a need for a model capable of explaining how mispricing evolves over time, if a better understanding of the economics of intermarket arbitrage is to be obtained. Furthermore, an institutionally rich model of the stochastic process for mispricing is important in view of the need for arbitrageurs to evaluate the early unwinding option and the rollover option that they face.[2] Similarly, stochastic changes in mispricing are important determinants of the cost of various non-arbitrage participants in the

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[2] New arbitrage positions are typically opened within the transaction cost window because of the heavy transaction cost discounts provided by early unwindings and rollovers. The results of Sofianos (1990) show that most index arbitrage positions are not held to expiration but are closed early.

futures market including short term hedgers, portfolio insurers and those hedging written OTC option positions. Richer models will also potentially assist in the formulation of optimal strategies for such groups.

The aim of this chapter is to explicitly incorporate the impact of *differential* transaction costs on the time series evolution of futures mispricing by extending the model proposed by Garbade and Silber (1983). One attraction of the Garbade and Silber model is that it explicitly allows the cash and futures markets to assume different roles in price discovery. However, a limitation of the model is that it does not allow for the potential presence of differential marginal transaction costs for different market participants. This chapter recognises that different categories of arbitrageurs face different marginal transaction costs. The model used in the chapter maintains the feature of different price discovery roles for cash and futures markets found in the Garbade and Silber model but also allows the potential impact of differential transaction costs on the time series evolution of mispricing to be estimated.

The starting position of this chapter is the model of Garbade and Silber, which leads to futures mispricing following an autoregressive process, with process parameters dependent on the nature of intermarket arbitrage activity. Following Gould (1988) it is assumed that several different categories of arbitrageurs exist and that marginal transaction costs differ for each category. In practice arbitrageurs face capital constraints and self-imposed position limits and as a result the lowest marginal transaction cost arbitrageur will not always be able to initiate new positions and so prevent higher marginal cost arbitrageurs from entering the market. This characterization of the arbitrage market implies that the arbitrage demand schedule will be a step function in lagged mispricing. The hypothesis which follows is that the time series of mispricing will follow a non-linear, self-exciting threshold autoregressive (SETAR) process. This is a piecewise linear autoregressive process in which the process parameters describing the evolution of mispricing are path-dependent.[3] Empirical tests of this hypothesis, using an intraday time series of stock index futures mispricing and tests proposed recently by Tsay (1989) and Petrucelli and Davies (1986), are found to strongly support the hypothesis of SETAR type non-linearity. The results include estimates of the number and values of the thresholds and these are found to be consistent with the transaction cost levels of the different categories of arbitrageurs believed to be active in the markets considered over the sample period. The estimates of a measure of the elasticity of arbitrage services corresponding to different transaction cost regimes are also found to be consistent with the model.

The remainder of the chapter is organized as follows : Section II reviews the previous literature concerned with modeling futures mispricing; Section III develops the model of mispricing evolution incorporating transaction costs; Section IV outlines the empirical methods used for testing the model and establishing parameter

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[3] It is important to note that Mackinlay and Ramaswamy (1988) provide evidence of path dependence in the time series of index futures mispricing.

estimates; Section V documents the empirical results; and Section VI presents the conclusions.

II. PREVIOUS LITERATURE

Garbade and Silber (1983) (henceforth GS) provide a model of concurrent price changes in cash and futures markets which explicitly recognizes that the elasticity of intermarket arbitrage services is not infinite because of: (i) the transaction costs involved in arbitrage; and (ii) the risks inherent in arbitrage transactions due to non-stochastic net storage costs and the constrained availability of arbitrage capital. Based on simple assumptions about the behavior of economic agents in the cash and futures markets and about the evolution of reservation prices, the model leads to price dynamics which imply that futures mispricing follows an AR(1) process. Parameter estimates from the model allow inferences to be drawn about the respective roles of the cash and futures markets in price discovery. However, a significant limitation of the GS model is that transaction costs are not specifically incorporated, except in so far as their existence leads to the elasticity of arbitrage services being finite. The GS model is based on linear demand functions whereas, as we will show below, the presence of different discrete levels of marginal transaction costs for different groups of market participants will lead to a step-function form of non-linearity in demand schedules.

An alternative modeling approach is adopted in a recent paper by Brennan and Schwartz (1990). They assume that mispricing follows a Brownian Bridge process, which has the property in common with a mean-zero stationary autoregressive process that it tends to return to zero, but additionally it constrains the value of mispricing to be zero at the futures contract maturity date with probability one. The maximum likelihood estimates of the process parameters reported by Brennan and Schwartz display considerable variation across contracts, but no explanation for this variability in process parameters is offered. Furthermore, Brennan and Schwartz were forced to omit the observations relating to last five trading days of each contract from their empirical analysis because ".. parameter estimates were very sensitive to these dates .." (p S15), even though the key reason for the choice of a Brownian Bridge process is that it ensures expiration day convergence. Apparently, as the time to maturity approached zero, the time series did not display the infinite rate of mean reversion predicted by the Brownian Bridge process. Thus there are reasons to believe that the use of a pure Brownian Bridge process might not be an entirely reliable way of modeling futures mispricing.

The modeling approach adopted by Cooper and Mello (1990) is to assume that, in the absence of arbitrage, mispricing follows a mean reverting Ornstein-Üllenbeck process about a mean of zero. They use stochastic calculus and numerical methods to solve for the change in mispricing due to the optimal actions of arbitrageurs, which are determined by the number of open arbitrage positions and hence the past history of mispricing. Cooper and Mello assume only one category of arbitrageur and also assume that price discovery occurs only in the futures market. Their model appears difficult to test formally, but in principle it provides a framework within which to test whether empirical mispricing behavior corresponds to a monopolistic or a competitive arbitrage structure.

The work by Holden (1990a, 1990b, 1990c) represents an important attempt to model endogenously the stochastic process governing futures mispricing. Holden develops an "intertemporal arbitrage trading model" based on the following four main assumptions: (i) the price of the underlying asset in the cash market and the price of its synthetic equivalent in the futures market differ as a result of the demands of liquidity traders, "time-series" market makers (trading only in their respective market) and "cross-sectional" market makers or arbitrageurs (trading in both markets); (ii) cumulative liquidity shocks in each market, arising from the stochastic total demand of the respective liquidity traders, mean revert to zero; (iii) cash and futures asset prices, subject to these zero mean reverting liquidity shocks, follow processes that mean revert to a common underlying fundamental value; and (iv) prices always reflect all information and hence arbitrage opportunities arise because of the price differences generated by liquidity shocks. Each period every arbitrageur (assumed to be risk neutral) calculates the optimal quantity of arbitrage trading so as to maximize his individual profit, holding fixed his expectations concerning arbitrage trading by others. All arbitrageurs are assumed identical, and hence all quantity decisions are also identical.

Holden's model has several implications, of which two are particularly relevant to this chapter. First, in the absence of arbitrage mispricing follows a linear AR(1) process because liquidity shocks are transitory and both futures prices and futures-equivalent cash prices mean revert to the underlying fundamental value. The implied mispricing process is hence similar to that implied by the GS model discussed earlier. Second, in the presence of a non-monopolistic arbitrage structure the mispricing variable follows an ARMA(2,1) process if arbitrage trades are limited to one per period.[4] The first- and second-order autoregressive parameters are functions of the mean reversion parameters for liquidity trading and arbitrage trading. However, the empirical results in Holden (1990c), show that the mean reversion due to arbitrage trading is far greater than the mean reversion due to liquidity trading. The aggressive trading of arbitrageurs appears to be the major force pulling mispricing back to zero.

Holden's work represents a significant advance in the modeling of futures mispricing, but his model has two important limitations. First, it assumes that both cash and futures prices instantaneously reflect all information. No consideration is given to possible differences in the speeds of adjustment to new information of cash and futures prices. Second, transaction costs are assumed away completely,[5] even though Holden (1990b,1990c) recognizes that incremental transaction costs are

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[4] This result is valid when the number of arbitrageurs is greater than one and when "the last couple of periods before maturity" are excluded so that "the optimal finite horizon arbitrage strategy can be approximated to double precision accuracy by the optimal infinite horizon arbitrage strategy" (Holden, 1990c, p 148).

[5] Holden (1990b) attempts to extend the model by including transaction costs in a two period model, but the solution in this extended framework is not in a closed analytic form, and requires numerical techniques. Given this, the implications are not pursued.

possibly the key input to decisions concerning the initiation of new arbitrage positions and the closure of existing positions. These limitations are avoided in the model proposed in this chapter. However, as discussed in Section III below, the model proposed here is not inconsistent with, and can be viewed as an extension of the work of Holden.

Finally, Miller *et al* (1991) suggest that predictable changes in the observed cash-futures *basis* (as distinct from the value basis or mispricing) could potentially arise because of non-synchronous trading in index stocks. They argue that mean reversion in the basis could be just a statistical illusion that has no economic significance in terms of index arbitrage activity. By assuming that price changes in the cash market follow a "modified" AR(1) process, and that price changes in the futures market follow a MA(1) process, they show that negative first order serial correlation in basis changes can be expected for a wide range of realistic parameter combinations. However, it is important to recognize that the analysis and evidence of Miller *et al* shows *at best* that observed basis predictability could *also* be explained in terms of non-synchronous trading in index stocks. In other words, the non-synchronous trading explanation and the arbitrage explanation of reversals are observationally equivalent. Miller *et al*'s empirical work does not exclude the possibility that basis predictability is caused, at least in part, by periodic realignments of prices due to the actions of index arbitrageurs. It is argued below that evidence in support of the model proposed in this chapter could be regarded as evidence in favor of the presence of arbitrage-related activity.

III. MODEL DEVELOPMENT

The starting point of our model is the Garbade and Silber (1983) model. The assumptions and analysis are also identical to those of GS with just one important difference[6]: GS assume that the elasticity of arbitrage services is constant - and in particular independent of the lagged value of futures mispricing - whereas we assume that the elasticity of arbitrage services depends on the categories of arbitrageurs active in the market at a point in time. This in turn depends on the lagged value of futures mispricing in relation to the marginal transaction cost thresholds of different categories of arbitrageurs.

Following GS, we assume that the aggregate market demand schedule of arbitrageurs in period t can be written as $H.X_t$, where $H(>0)$ is the elasticity of arbitrage services and X_t is the scaled "mispricing" of the futures contract relative to cash prices, defined as:

$$......(1)\quad X_t = \frac{\text{Observed Futures Price at t - Futures Equivalent Cash Price at t}}{\text{Cash Price at t}}$$

[6] There is another relatively minor difference. The GS analysis is based on cash prices and cash-equivalent futures prices. Our empirical work uses index futures prices, and consistent with the literature on index futures mispricing (eg Mackinlay and Ramaswamy, 1988 and Merrick, 1988) we base our analysis on futures prices and futures-equivalent cash prices. In principle, the two formulations are equivalent.

The futures-equivalent cash price is the price which should exist in the futures market given the cash price and infinitely elastic arbitrage services. This is the theoretical futures price calculated on the basis of the cost-of-carry forward pricing formula (see eg Figlewski, 1984a p 665), if dividends and interest rates until maturity are known *ex ante* with perfect certainty, and margins attract market rates of interest.

Following the same steps as in GS, the mispricing variable X_t will follow the equation:

......(2) $$X_t = \rho_0 + \rho_1 X_{t-1} + e_t$$

where ρ_1, the autoregressive parameter, is an inverse function of H; ρ_0 reflects "secular price trends" in the data and "persistent differences between cash and futures prices" (GS, p 293); and e_t is a white noise error term.

If H is independent of the level of mispricing and constant over time, equation (2) is a simple AR(1) process for the mispricing time series. If the elasticity of arbitrage services is infinite, ρ_1 should be close to zero and X_t will be white noise if $\rho_0=0$. If the elasticity of arbitrage services is zero, ρ_1 should be close to 1, and X_t should be a martingale if $\rho_0=0$. If the elasticity of arbitrage services is significantly greater than zero (but not infinite), ρ_1 should be significantly less than unity but greater than zero.

In practice it appears unlikely that the aggregate demand schedule of arbitrageurs will be a linear function of mispricing. In real markets there are likely to be several categories of arbitrageurs with different levels of transaction costs. For example, Yadav and Pope (1990) highlight four distinct categories of arbitrageurs in the UK stock index futures market:

(a) those whose marginal costs are confined to the transaction cost in the futures market. Examples of arbitrageurs falling into this category include those who are otherwise committed to enter or exit the stock market and hence use the futures market only as an intermediary, and those arbitrageurs with existing arbitrage positions who seek opportunities to profitably exercise the rollover option or the early unwinding option.

(b) those who have capital in fixed interest instruments or a basket of index stocks and who are not liable to pay the capital transaction tax (stamp duty) levied in London.[7] Examples of arbitrageurs in this category are market makers making a market in all index stocks and recycling stocks within seven days.

(c) those who have capital in fixed interest instruments or a pool of index stocks but who have to pay the 0.5% transaction tax (stamp duty) on their share purchases. Typical arbitrageurs in this category would be fund management institutions.

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[7] In November 1991, stamp duty on stock market purchases is to be abolished.

(d) those who have to borrow capital or stocks to initiate an arbitrage position.

If there is sufficient uncommitted arbitrage capital available to the lowest marginal transaction costs arbitrageurs, the higher marginal cost arbitrage categories will never be able to enter the market (Gould, 1988). Under such circumstances the width of the arbitrage window will be governed solely by the lowest transaction costs bound. However, as has been observed by Stoll and Whaley (1987) and Brennan and Schwartz (1988), arbitrageurs function within real and self-imposed position limits. Hence, even in the highly liquid index futures markets, several different categories of arbitrageurs can be active depending on the actual level of mispricing and on the extent to which the capital available to each category of arbitrageur is committed. The availability of capital will in turn depend upon past levels of mispricing. Support for this characterization is to be found in the widespread evidence of deviations between index futures prices and their theoretical values that cannot be accounted for by the transaction costs of the *most favorably* positioned arbitrageur (Mackinlay and Ramaswamy, 1988; Merrick, 1988, 1989; Yadav and Pope, 1990).

In order to consider the effect of differential transaction costs on the time series evolution of mispricing, assume initially that mispricing is sufficiently small such that no arbitrageur in any category will be active. This implies that locally $H=0$ and that mispricing will follow a martingale if $\rho_0=0$. The change in mispricing over the following time period will depend on differences in liquidity trading, noise trading, information arrival and price discovery between the cash and futures markets. However, when mispricing evolves to the point where its absolute value exceeds the transaction costs level of the lowest marginal cost category of arbitrageur, H will become non-zero. The value of H will depend *inter alia* on the aggregate arbitrage capital available to this category of arbitrageur. In turn, if mispricing happens to exceed the marginal transaction costs level faced by the next category of arbitrageur, this latter group will also enter the arbitrage market and the value of H will change again. This argument can be extended to eventually include all groups of arbitrageurs and if mispricing is sufficiently large to attract even the least favorably positioned arbitrageur, the value of H will be at its highest. This representation of the arbitrage supply process suggests that H can be modelled as a step function whose value depends on lagged values of mispricing.

The autoregressive parameter ρ_1 in equation (2) is an inverse function of the elasticity of arbitrage services H. The arguments above suggest that ρ_1 can also be modelled as a step function whose value depends on the lagged value of the mispricing variable ie X_{t-1} and on the transaction cost thresholds of different categories of arbitrageurs. Figure 1 illustrates the implications of the arbitrage supply process, by schematically showing the assumed relationships between mispricing, the elasticity of arbitrage, the autoregressive parameter ρ_1 and the different transaction cost thresholds. (For simplicity only six finite transaction cost thresholds are illustrated.)

Let us assume that there are k transaction cost bands defined by (k-1) finite transaction cost thresholds $r_1, r_2, ... r_{k-1}$, with $r_1 < r_2 < .... r_{k-1}$. The k transaction cost bands extend from $-\infty$ to r_1, r_1 to r_2, ... r_{j-1} to r_j .... , and r_{k-1} to $+\infty$. In our model ρ_1 in equation (2) can take any one of k possible values depending on the transaction cost band corresponding to the value of lagged mispricing X_{t-1}. It will be $\rho_1^{(1)}$ if $-\infty < X_{t-1} < r_1$, $\rho_1^{(2)}$ if $r_1 \leq X_{t-1} < r_2$, . . . , $\rho_1^{(j)}$ if $r_{j-1} \leq X_{t-1} < r_j$, ... and $\rho_1^{(k)}$ if $r_{k-1} \leq X_{t-1} < \infty$. For the sake of generality we can also allow ρ_0 in equation (2) to depend on the transaction cost band corresponding to the value of lagged mispricing, X_{t-1}. Let $\rho_0^{(j)}$ be the intercept term specific to transaction cost band j just as $\rho_1^{(j)}$ is the autoregressive parameter specific to transaction cost band j. Our representation of the arbitrage supply process hence leads to the following time series process for the mispricing variable:

$$X_t = \rho_0^{(j)} + \rho_1^{(j)} X_{t-1} + e_t^{(j)} \quad ......(3)$$

where $r_{j-1} \leq X_{t-1} < r_j$ ($j=1,2,...k$, $r_0=-\infty$, $r_k=\infty$)

Equation (3) represents a self-exciting threshold autoregressive process (SETAR). Such a process partitions the one dimensional Euclidean space into k regimes, and follows a linear autoregressive model in each regime.

Equation (2) implies that arbitrage related mispricing adjustment takes place within one time period. This is appropriate in the context of GS who used daily data.

Complete adjustment of mispricing through arbitrage may actually take several time periods depending on the periodicity of the data. As such, X_t need not necessarily be a one lag autoregressive process. More generally, X_t can be an autoregressive process of order p:

......(4)
$$X_t = \rho_0 + \sum_{g=1}^{p} \rho_g X_{t-g} + e_t$$

Our representation of the arbitrage supply process, when applied to the one lag autoregressive model (equation (2)) which followed from GS, leads to a SETAR model with one autoregressive lag (equation (3)). If the autoregressive parameter ρ_1 and/or the other autoregressive parameters ρ_g $(g>1)$ in equation (4) are also taken as functions of the elasticity of arbitrage services H, then our representation of the arbitrage supply process, when applied to the more general autoregressive model with p autoregressive lags (equation (4)) leads *mutatis mutandis* to a SETAR model with p autoregressive lags[8], ie:

......(5)
$$X_t = \rho_0^{(j)} + \sum_{g=1}^{p} \rho_g^{(j)} X_{t-g} + e_t^{(j)}$$

where $r_{j-1} \leq X_{t-1} < r_j$ and r_j, $j=1....k-1$ represents the j^{th} transaction cost threshold with $r_1 < r_2 < ... < r_{k-1}$, $r_0 = -\infty$, $r_k = +\infty$; $\rho_0^{(j)}$

[8] Note that while X_t is dependent on its p lagged values, the elasticity of arbitrage services has a step function dependence only on X_{t-1}, the lagged value in the immediately preceding period, because it is assumed that it is that value which determines the decisions of arbitrageurs.

is a transaction cost regime-specific intercept term; and $\rho_g^{(j)}$ is the g^{th} lag autoregressive parameter specific to transaction cost regime j.

The general SETAR model with p autoregressive lags, equation (5), can be reconciled with the work of Holden (1990a, 1990b, 1990c). The model in Holden (1990b) leads to mispricing following an ARMA (2,1) process if there is one arbitrage trade per period and no transactions costs. The moving average error component has an equivalent infinite autoregressive representation and this is used by Holden in his empirical estimation. Therefore in the absence of transaction costs, X_t can (for sufficiently large p) be approximated by an AR(p) process. The autoregressive parameters in Holden's model depend *inter alia* on the mean reversion due to arbitrage trading. The model presented in this chapter allows for the aggressiveness of arbitrage trading, and hence the strength of mean reversion in mispricing, to vary as a result of differential transaction costs. Therefore it can consistently be viewed as an extension of Holden's (1990b) model.

In contrast it is difficult to reconcile the SETAR model with the analysis in Miller *et al* (1991). If basis predictability is *purely* a statistical illusion created by non-synchronous trading and index arbitrage is not believed to be influential, there appears to be no obvious reason to expect step function type threshold non-linearity based on the lagged value of mispricing to be present in the mispricing time series.[9] Should SETAR type non-linearity actually be observed, it could be regarded as evidence in support of the view that basis predictability is at least not *entirely* a spurious statistical phenomenon caused by non-synchronous trading in index stocks.

IV. EMPIRICAL METHODOLOGY

A. Testing the Model

The threshold autoregressive time series model was first proposed by Tong (1978) and was subsequently developed in detail by Tong and Lim (1980) and Tong (1983). Formal tests for the existence of threshold type nonlinearity in time series fall into two main categories: (a) likelihood ratio tests described in Chan and Tong (1986,1988); and (b) tests based on recursive residuals and arranged autoregressions (ie autoregressions ordered in accordance with the value of the threshold variable) proposed by Petrucelli and Davies (1986), and Tsay (1989). Problems in testing threshold non-linearity arise mainly because the threshold parameters r_1, r_2, ... r_k are seldom known. For example, in the context of futures mispricing, it is difficult to identify precise values for the marginal transaction cost bounds applicable to different categories of arbitrageurs (Yadav and Pope, 1990). With unknown threshold parameters, the null asymptotic distribution of the likelihood ratio test statistic cannot be tabulated except for the simplest cases. To use the likelihood ratio

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9 It is relevant to note that the analysis of Miller *et al* (1991) is in terms of the nominal cash-futures basis, while our analysis follows most of the literature on index futures pricing and concentrates on mispricing or what is sometimes called the "value basis". The two are not identical but are clearly related.

test in practice, one has to use computer intensive Monte Carlo methods to obtain the approximate tail area of the null distribution for the likelihood ratio test statistic. The procedure rapidly becomes intractable for multiple thresholds.

In light of the problems associated with the likelihood ratio tests, the empirical results reported in this chapter are based on the tests for threshold non-linearity proposed by Petrucelli and Davies (1986) (hereafter PD) and by Tsay (1989). Both these tests are based on the concept of an arranged autoregression. In equation (3), X_t depends on its lagged value X_{t-1} but ρ_0 and ρ_1, the coefficients describing this dependence, are themselves functions of X_{t-1}. The pair of values (X_t, X_{t-1}) will be referred to as a "case" of data, the first member of the case being a realisation of the dependent variable and the second member, a realisation of the independent variable. An arranged autoregression rearranges the cases of data in a suitable way, in this chapter on the basis of the magnitude of the lagged mispricing variable X_{t-1}. If π_i is the time index of the i^{th} smallest observation in the available set of lagged mispricing values $\{X_{t-1}\}$, and if there are s_1, s_2, ... s_j, ... s_k cases belonging to the k transaction cost regimes (and $s_0=0$), then equation (3) could also be written as

......(6) $$X_{\pi_i+1} = \rho_0^{(j)} + \rho_1^{(j)} X_{\pi_i} + e_{\pi_i+1}^{(j)}$$

where $$\sum_{\zeta=0}^{j-1} s_\zeta < i \le \sum_{\zeta=0}^{j} s_\zeta$$

The parameters s_1, s_2 .... s_k are, in general, unknown, but the arranged autoregression provides a means by which data points are grouped so that all of the observations in a group follow the same linear AR(1) model. X_t is still being regressed on its lagged value in natural time (X_{t-1}), but successive cases of data are not in their natural time order but grouped according to the time series process they are hypothesised to follow.

Similarly, for a SETAR model with p autoregressive lags, the set of values (X_t, X_{t-1}, ... X_{t-p}) would constitute a case of data, and the arranged autoregression would again order different cases of data on the basis of the value of the one period lagged mispricing X_{t-1}. If π_i is defined as before, equation (5) could then be written as

......(7) $$X_{\pi_i+1} = \rho_0^{(j)} + \sum_{g=1}^{p} \rho_g^{(j)} X_{\pi_i-g+1} + e_{\pi_i+1}^{(j)}$$

where $$\sum_{\zeta=0}^{j-1} s_\zeta < i \leq \sum_{\zeta=0}^{j} s_\zeta$$

The recursive residual $W_{\tau+1}$ is the one-step-ahead standardised prediction error computed from the OLS parameters estimated from the first τ cases of data.[10] Both the PD test and the Tsay test involve generating a sequence of recursive residuals by choosing a base of the first m cases of data in equation (7), computing the recursive residual W_{m+1}, extending the base to include (m+1) cases, computing the recursive residuals W_{m+2}, and continuing in the same way until all cases are used.

Assume, without loss of generality, that all the m cases chosen for computing the first recursive residual W_{m+1} belongs to the first transaction cost regime. By construction the recursive residuals will be white noise and orthogonal to the regressors. However, when the recursive autoregression changes to include cases belonging to the next regime, the recursive residuals become biased and the orthogonality between the recursive residuals and the regressors is destroyed. Hence, one way to test for threshold non-linearity (or the existence of different autoregression regimes), without needing to know the value of the thresholds, is to regress the recursive residuals of the arranged autoregression (equation (6) or equation (7)) on the regressors $\{X_{\pi_i-g+1} | g=1, \ldots p\}$ and then to examine the F-

[10] Recursive residuals are a member of the general class of LUS residuals ie linear, unbiased with a scaler variance matrix. If the true disturbances of a regression model are independently and identically distributed, then the set of recursive residuals are also independently and identically distributed. On the other hand, estimated regression residuals will, in general, display heteroskedasticity and non-zero covariances even when true disturbances are homoskedastic and have zero covariances. For a definition of the recursive residual, and a discussion of its properties, see eg Johnston (1984, pp 384-385).

statistic of the resulting regression. Tsay (1989) shows formally that if X_t is a linear stationary AR(p) process then running the OLS regression

$$\ldots\ldots(8) \qquad W_{\pi_i+1} = a_0 + \sum_{g=1}^{p} a_g X_{\pi_i-g+1} + \varepsilon_{\pi_i+1}$$

where a_g, $g=1..p$ are regression coefficients and $i = m+1,\ldots, n$

and then computing the associated F-statistic[11]

$$\ldots\ldots(9) \qquad F = \frac{(\sum W^2 - \sum \varepsilon^2)/(p+1)}{\sum \varepsilon^2/(n-m-p-1)}$$

will yield a test statistic distributed as an F distribution with [(p+1), (n-m-p-1)] degrees of freedom.

The choice of m in the Tsay test is subjective. The selected value for m should be small enough to ensure that all cases used to calculate the first recursive residual belong to the same transaction cost regime, but not so small so as to lead to large standard errors in calculating the OLS parameters. Tsay (1989) suggests that computation of recursive residuals should start with a minimum base of (n/10 + 1) observations. The empirical results on the Tsay test reported in this chapter are based on running Tsay-type regressions for all m > (n/10 + 1) and recording the minimum p-value of the test statistic for each contract. However, the results

[11] Time subscripts are dropped to improve clarity. Summations are over the (n-m-1) observations in regression (8).

reported are generally robust with respect to the choice of m over a wide range, often up to m representing approximately half the data points.

The PD test uses the cumulative sums (CUSUM) of the *normalised* recursive residuals, defined as the recursive residuals divided by their own standard deviation. If there is no systematic change, linked to the lagged value of mispricing, in the intercept and slope parameters of the regression model equation (7), then the expected value of CUSUM will be zero. However, if the intercept or slope parameters are dependent on the lagged values of mispricing, then since successive cases of data are ordered according to the lagged value of mispricing there will be a tendency for a disproportionate number of recursive residuals to have the same sign and CUSUM will diverge significantly from zero. The PD test statistic is given by:

......(10)

$$T = \max_{m+1 \le r \le n} \frac{\left| \sum_{g=m+1}^{r} Z_g \right|}{(n-m)^{1/2}}$$

where Z_g is the normalised recursive residual.

The choice of m in the PD test statistic is again subjective. The empirical results on the PD test reported in this chapter are again based on running PD tests for all m > (n/10 + 1) and recording the minimum p-value of the test statistic for each contract. The results reported are robust with respect to the choice of m.

B. Estimation of Thresholds

Recursive residuals may also be used to estimate the approximate value of the thresholds, or points of structural change, in the autoregressive model of equation (4). The techniques used involve a study of one or more of the following three statistics:

1. CUSUM (cumulative sum of normalised recursive residuals) as defined in the PD test. Without a change in regime the expected value of the CUSUM is zero, but with a change in regime the CUSUM will tend to diverge from the zero mean line and the bias in the recursive residuals will lead to runs (Ertel and Fowlkes, 1976). CUSUM plots can be used to estimate judgementally any changes in regime.

2. CUSUM-SQ (cumulative sum of squared recursive residuals) defined as

$$CS_t = \frac{\sum_{\tau=p+2}^{t} W_\tau^2}{\sum_{\tau=p+2}^{n} W_\tau^2} \qquad \text{......(11)}$$

$$t = p+2, \ldots n$$

   Under the null hypothesis of no change in regime CS_t follows a Beta distribution with parameters (t-p-1)/2 and (n-t)/2 (Brown, Durbin and Evans, 1975). CS_t plots can also be used to estimate regime change points.

3. MOSUM-SQ (moving sums of squared recursive residuals) defined as

   where t = G+p+1, ... n and G is the "window" used for the moving sum. Under the null hypothesis of no change in regime, MQ_t follows an F-distribution with (G, T-p-1-G) degrees of freedom (Hackl, 1980; Westland

$$\ldots\ldots(12)\quad MQ_t = \frac{\left(\sum_{\tau=t-G+1}^{t} W_\tau^2\right)}{\left(\sum_{\tau=p+2}^{t-G} W_\tau^2 + \sum_{\tau=t+1}^{n} W_\tau^2\right)} \cdot \left(\frac{(T-k-G)}{G}\right)$$

and Tornkvist, 1989).

Although each of these statistics can be used to test for the existence of different autoregression regimes, the significance levels for these statistics are best used as yardsticks against which to assess the observed plots, rather than as formal tests of significance (Brown, Durbin and Evans, 1975). CUSUM plots are very effective in locating single thresholds, but can be difficult to use for detecting multiple thresholds. In general, the power of the different methods depends on the distribution of the test statistics under the specific alternative hypothesis. Westland and Tornkvist (1989), after extensive simulations with the alternative hypothesis specified as a two-regime, one-intercept, one-regressor model, recommend the use of MOSUM-SQ.

V. EMPIRICAL RESULTS

A. The Data

The empirical tests relate to the FTSE100 index futures contract traded on the London International Financial Futures Exchange (LIFFE). The FTSE100 index is a market value weighted index of the one hundred largest market value companies traded on the London Stock Exchange. LIFFE index futures expire four times a year in March, June, September and December, on the last business day of the month. "Time and Sales" transactions data on FTSE100 index futures was obtained from LIFFE. The data includes all bid and ask quotes and all transaction prices relating to this contract. Mid-market quotes valid at 10 am, 11 am, 12 noon, 1 pm, 2 pm, 3 pm and 4 pm were extracted from the data for each day of the sample period.[12] Hourly data on the FTSE100 cash index was obtained from the *Financial Times*. The cash index is based on the average of the best bid and the best ask quotes of index stocks and is updated every minute. These quotes represent prices at which competing market makers are obliged to trade for up to fairly large contract sizes. Thus, the hourly FTSE100 index values are based on quotes which represent actually tradeable values synchronous with the hourly bid and ask quotes in the futures market. The hourly index data was available only from February 1986 onwards and hence the sample includes a total of 3679 observations covering the 9 contracts expiring between June 1986 and June 1988.[13]

Information on the constituents of the index and how these constituents changed over the sample period was obtained from the London Stock Exchange. Dividends and ex-dividend dates for all the relevant constituents of the index each day were

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[12] LIFFE index futures trade from 9.05 am to 4.05 pm each day. In order to have constant interval observations, and since index data was available only on the hour, the 9.05 am and 4.05 pm values were not used.

[13] The December 1987 contract spanned Black Monday. In order to avoid potential distortion due to outliers, a two week period from October 19, 1987 was not included in the dataset on which the reported results are based. However, inclusion of this two week period did not make a qualitative difference to the results of the threshold non-linearity tests.

collected from *Extel* cards. In addition, in order to compute the exact *ex post* daily dividend flow on the FTSE100 index, the individual constituents' dividend flows on each day were value weighted, aggregated and converted into index points using price and market value data collected from *Datastream*. Daily data on one- and three-month UK Treasury Bill rates were also collected from *Datastream*.

The futures mispricing X_t was defined as in equation (1) and the futures-equivalent cash price was computed for each hour using the cost-of-carry forward pricing formula with the following assumptions:[14] (a) the forecast dividend yield to maturity for each date is identical to the actual *ex post* daily cash dividend inflow for the FTSE100 portfolio; (b) the forward interest rate at the time t for a loan made at time w to be redeemed at time T, is identical to the interest rate at time w on a Treasury Bill maturing at time T; and (c) the value on day t of one- and three-month Treasury Bill interest rates can be used to infer a linear term structure from which the implicit spot interest rate for the period (T-t) can be calculated.

An examination of daily trading volumes reveals that the near contract is almost always the most heavily traded contract on LIFFE. Volume in the far contract starts to build up about four weeks before expiration of the near contract. The results of Yadav and Pope (1990) show that the magnitude of mispricing in the far contract is considerably higher than in the near contract, possibly because the arbitrage

[14] These assumptions are fairly standard in the literature. See, for example, Merrick (1988) and Mackinlay and Ramaswamy (1988).

window tends to be wider due to greater dividend and interest rate uncertainty. Hence, the empirical analysis is based on the near contract. Since mispricing at expiration is necessarily zero and the estimation techniques do not explicitly enforce this restriction, all expiration day hourly mispricing values were removed from the dataset.

In order to confine the study to observations over equally spaced time intervals, arranged autoregressions use only those cases of data in which successive observations do not span the overnight interval 4pm to 10am. This means that in the first order autoregressive models the 10am value was excluded and in the second order models the 11am value was excluded so as to avoid using observations from the previous day as explanatory variables. This resulted in a loss of about 14% of the total possible cases in first order models and 28% of the possible cases in second order models. For the same reason, the arranged autoregressions use only those cases of data in which successive observations do not span any of the few missing values that existed in the data. Application of these criteria left about 360 cases of data for each contract in first order models and about 300 cases of data for each contract in second order models.

B. Results

As an initial check on the stationarity characteristics of the mispricing variable the unit root property of the mispricing variable was tested using the Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) procedures (Dickey and Fuller, 1979;

Fuller, 1976). Table 1 reports the results of these tests. The hypothesis of a unit root in mispricing is rejected at the 5% significance level for eight of the nine contracts and at the 10% level for every contract. The ADF test rejected a unit root for six contracts at the 5% level and seven contracts at the 10% level.[15] In the context of the GS model, the rejection of a unit root is consistent with the existence of significant arbitrage activity. The results also provide support for the strong evidence of cointegration between futures prices and futures-equivalent cash prices found by Holden (1990c) with US data.

Table 2 reports the results of linear autoregressive modeling of the futures mispricing variable X_t. In all cases a large proportion of the variation in X_t is explained by X_{t-1} and there is only a marginal increase in explanatory power when X_{t-2} is included. Examination of the Durbin Watson statistics make it clear that all contracts can be modelled satisfactorily as AR(2) processes.[16] This is probably reasonable since any arbitrage-related price adjustment would be expected to occur within the next hourly period and delays of more than two hours would be extremely unlikely in an arbitrage context. However, the question remains as to whether the autoregressive parameter is constant, or whether it should be modeled

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15 The ADF regressions were confined to only one additional lag in ΔX_t in light of the results reported in Table 2.

16 Due to the gaps in the mispricing time series the widely used Box-Pierce statistic is inappropriate as a test for serial correlation for our data.

as being dependent on the level of lagged mispricing, as implied by the threshold autoregressive model.

Table 3 reports the results of the Tsay-test and the PD test for threshold non-linearity. Since Table 2 shows that most of the arbitrage-related mispricing adjustment takes place within the next hourly period, it should be adequate to test for SETAR with one autoregressive lag (p=1). However, we also report the results of the more conservative SETAR tests based on two autoregressive lags (p=2). The cases are ordered according to the value of one hour lagged mispricing and the minimum number of cases used as a benchmark (m) is set equal to (n/10+1). It should be emphasized that though we report the lowest Type I error probability values obtained by varying m, the results are generally robust to the choice of m over a wide range. Hence the minimum values do not represent outliers in any sense.

Using the Tsay test and a first order model (p=1), the null hypothesis of linearity in the autoregression process is conclusively rejected in favor of the hypothesis of threshold non-linearity, for five contracts at the 1% level, eight contracts at the 5% level, and all the nine contracts at the 10% level with the maximum Type I error probability being 0.062. The results of the PD test assuming a first order model are virtually identical. Using the more conservative second order model (p=2) the Tsay test rejects linearity in favor of threshold non-linearity for four contracts at the 1% level, six contracts at the 5% level and eight contracts at about the 10% level. The

PD test, even when used with a second order model, continues to reject linearity for five contracts at about the 1% level, and eight contracts at about the 5% level. Clearly there is a notable consistency in the pattern of probability values obtained from the two tests. Overall, these results provide strong support for threshold non-linearity in futures mispricing, with the first order autoregressive parameter having a step function dependence on lagged mispricing.

In our model, threshold non-linearity arises from the existence of different categories of arbitrageurs with different transaction cost thresholds. The important question at this stage is whether we have a strong case for believing that the threshold non-linearity observed in Table 3 is a manifestation of differentials in transaction costs. To attempt to answer this question we first estimated the values of the thresholds present in the mispricing time series. The estimated values reported in Table 4 are based on plots of the MOSUM-SQ statistic (and its probability value) against the case number, cases being ordered according to the value of lagged mispricing. A range of plots was prepared for each contract, corresponding to different values of G and to the two different sets of recursive residuals calculated using $p=1$ and $p=2$. Figure 2 illustrates the probability value plot for the March 1988 contract. CUSUM and CUSUM-SQ statistics were also analyzed for each contract, but although they are never inconsistent with the MOSUM-SQ plots, in most cases they did not provide as strong a resolution as the MOSUM-SQ statistic.

Table 4 also reports the upper and lower quartiles of the mispricing variable. Individual contract periods clearly display systematic tendencies towards either positive mispricing or negative mispricing and therefore *ex post* we would not expect to observe all the possible thresholds when the observed mispricing series are analyzed for individual contracts. Table 4 suggests that although there is some minor variation from contract to contract, the implied thresholds are fairly stable and consistent with the *ex post* distribution of mispricing. The minimum thresholds appears to be about ± 0.2% to ± 0.3% and the next threshold appears to be about ± 0.6% to ± 0.8%, except for the contract spanning Black Monday (October 19th 1987) when it was -1.0%.

We next attempt to estimate the actual transaction costs faced by different categories of index arbitrageurs. These estimates will serve as a benchmark for evaluating the plausibility of the estimated thresholds. Table 5 reports the average inner market spread or the market "touch"[17] for UK "alpha"[18] stocks on the basis of the values published periodically by the *Stock Exchange Quarterly*. The set of alpha stocks corresponds very closely to the index basket of stocks and hence the average market touch is a good proxy for the average bid-ask spreads ordinarily involved in trading the index basket. The average touch before Black Monday was consistently about

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[17] The "touch" is defined as the difference between the best bid and ask prices expressed as a percentage of the middle price.

[18] Stocks in London have been classified as "alpha", "beta" etc on the basis of the number of competing market makers and the trade/quote reporting regulations applicable to them. Alpha stocks are generally the most liquid and all FTSE100 constituents belong to this group.

0.8%, increasing to over 1% after Black Monday. However, a major component of the quoted bid-ask spread, namely adverse information costs, will clearly not be relevant in pricing market making services for index arbitrage. Transaction costs related to the cash market should be confined to marginal order processing costs and marginal inventory holding costs. We are not aware of any published estimate for the UK market of the percentage of the quoted spread which arises due to adverse information costs. However, Stoll (1989) finds that on NASDAQ, 43% of the quoted spread represents adverse information costs, 10% represents inventory holding costs and 47% represents order processing costs. If we use these figures as a first approximation for the London market, which has an almost identical trading structure to NASDAQ, the quoted spread for index arbitrage should be 0.4% to 0.5% before Black Monday and 0.6% to 0.7% thereafter. However, the rise in the quoted spread after Black Monday is more likely to be due to an increase in the adverse information component and the inventory holding component and is hence likely to affect index arbitrage trades to a much lesser extent. Commissions are usually on a flat rate basis and for large volume index arbitrage trades are virtually negligible when expressed as a percentage of value traded.

In Table 5 we also report estimates of the percentage market impact costs in the index futures market. A subset of LIFFE time and sales data was analyzed consisting only of cases in which ask prices and bid prices are posted within 60 seconds of each other. The average percentage spread[19] for the near futures contract varied from about 0.1% to about 0.2% over the sample. Roundtrip percentage commissions in the futures market are typically less than about 0.05%. Thus, arbitrageurs with the lowest marginal transaction costs, ie those who have only marginal costs in the futures market such as portfolio managers using the futures market as an efficient mechanism for market entry or market exit, and those rolling positions forward and those unwinding early, face marginal costs of only about 25 basis points. Hence, the innermost threshold window that we expect corresponds closely to the estimates of 0.2% to 0.3% observed in Table 4.

Arbitrageurs with the next level of transaction costs are those who have capital as cash or stocks and are not liable to pay stamp duty - essentially market makers making a market in all index stocks. Assuming that market makers charge a fair price for their services, the incremental costs faced by this group will be approximated by the quoted spread for index arbitrage trades, which we estimated to be about 50 basis points except for the contract spanning Black Monday. Hence, after adding in the marginal futures market cost of 25 basis points, a plausible value for the next threshold is approximately 75 basis points, except for the contract spanning Black Monday where it should be about 100 basis points. The thresholds that we observe in Table 4 do correspond closely to these levels.

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[19] Percentage spread $= \dfrac{2 * (\text{Ask-Bid}) * 100}{(\text{Ask+Bid})}$

The next level of transaction costs corresponds to the costs faced by arbitrageurs who also have to bear the additional 0.5% stamp duty when purchasing stocks. The next threshold that we should expect is therefore about 125 basis points. However *a priori* we expect that there would be very few cases where arbitrageurs with this level of marginal transaction costs would have opportunities to enter the riskless arbitrage market. Again, Table 4 does have 3 cases in which thresholds of approximately this magnitude are observed.

Overall, Table 5 suggests that a transaction cost-based explanation of the estimated thresholds reported in Table 4 is very plausible. Admittedly the linkage we are making between the estimated thresholds and transaction costs is based on approximations of the actual effective transaction costs. However, subject to this caveat, the magnitudes of the thresholds documented in Table 4 appear to be consistent with the transaction cost bands applicable to potential index arbitrageurs operating in the market.

To seek further evidence supporting the link between the threshold non-linearity that we observe in Table 3 and differentials in transaction costs, we estimated the differences in the implied elasticity of arbitrage services conditional on the level of one period lagged mispricing. Specifically, we assumed five transaction cost regimes corresponding to one period lagged mispricing $\{X_{\pi_i}\}$ falling within the following ranges:

Regime 1 : $\{-\infty < X_{\pi_i} < -0.75\%\}$

Regime 2 : $\{-0.75\% \leq X_{\pi_i} < -0.25\%\}$

Regime 3 : $\{-0.25\% \leq X_{\pi_i} < 0.25\%\}$

Regime 4 : $\{0.25\% \leq X_{\pi_i} < 0.75\%\}$

Regime 5 : $\{0.75\% \leq X_{\pi_i} < \infty\}$

Five corresponding dummy variables were defined as

$D_i^{(j)}$ = 1 if one period lagged mispricing (X_{π_i}) lies within regime j

= 0 otherwise, j = 1, 2 ... 5.

The elasticity of arbitrage is reflected in the first order autoregressive parameter in equation (2). Therefore the following OLS regression was estimated:

$$\ldots\ldots(14) \qquad (X_{\pi_i+1} - X_{\pi_i}) = \Phi_0 - \sum_{j=1}^{5} \Phi_1^{(j)} D^{(j)} X_{\pi_i} + \epsilon_{\pi_i}$$

Equation (14) is similar to equation (2) except that it has been cast as a Dickey-Fuller regression by defining the *change* in mispricing as the dependent variable. Since ρ_1 in equation (2) is an inverse function of the elasticity of arbitrage services, $\Phi_1^{(j)}$ in equation (14) is a direct measure of the elasticity of arbitrage services corresponding to transaction cost regime j. In the context of our model, we would expect $\Phi_1^{(j)}$ to be different for different transaction cost regimes and that $\Phi_1^{(1)} > \Phi_1^{(2)} > \Phi_1^{(3)}$ and $\Phi_1^{(3)} < \Phi_1^{(4)} < \Phi_1^{(5)}$ because arguably the elasticity of arbitrage services will be greater in higher transaction cost regimes capturing higher levels of mispricing. Consequently equation (14) was used to test the following null hypotheses:

$H_1^{(j)}$: $\Phi_1^{(j)} = 0$

against the alternative that $\Phi_1^{(j)}$ is greater than zero, j = 1, 2, 3, 4 and 5;

and H_2 : $\Phi_1^{(1)} = \Phi_1^{(2)} = \Phi_1^{(3)} = \Phi_1^{(4)} = \Phi_1^{(5)}$

against the alternative that at least one coefficient is different.

Hypotheses $H_1^{(j)}$ were tested using the Dickey-Fuller τ-statistic. Hypotheses H_2 was tested using an F-test. Table 6 reports the results of testing these hypotheses over the full sample of data. The results show that hypothesis $H_1^{(1)}$ is conclusively rejected (p-value < 0.001). Hypothesis $H_1^{(2)}$ is rejected at the 1% level and $H_1^{(5)}$ at the 5% level. Hypotheses $H_1^{(3)}$ and $H_1^{(4)}$ cannot be rejected. Hypothesis H_2 is also conclusively rejected (probability value < 0.001). The estimates reveal that $\Phi_1^{(1)} > \Phi_1^{(2)} > \Phi_1^{(3)}$ and $\Phi_1^{(5)} > \Phi_1^{(4)}$. In economic terms, the results show that the elasticity of arbitrage is an increasing function of the *absolute* level of mispricing and that the transaction cost-based thresholds we selected discriminate significantly between different regimes. Hence Table 6 lends further strong support to the model proposed in this chapter.

In order to sustain the argument developed in this chapter, it is necessary to consider whether the evidence of threshold non-linearity can be reconciled with explanations of mean reversion in mispricing that do not rely on transaction cost arguments and arbitrage. Two possible explanations are that the evolution of futures mispricing will potentially depend on non-synchronous adjustments of cash and futures prices to new information, and on differential levels of liquidity trading in the cash and futures markets (Holden, 1990b). However, it is difficult to visualise how such factors would generate threshold non-linearity (conditional on the lagged value of mispricing) in the time series process of mispricing. The arrival of new information is random and unrelated to past mispricing values. Orders for non-discretionary liquidity trading are again random in relation to past mispricing. Discretionary liquidity trading (in the sense of Admati and Pfleiderer (1988)) could conceivably be related to the lagged mispricing value because there would be incentives to trade cash (futures) when mispricing is positive (negative). However, such incentives should change gradually as mispricing changes and there does not appear to be any obvious reason for the step function type of dependence revealed in our results. Therefore we believe that the threshold non-linearity observed in the results is not directly related to non-synchronous adjustments in cash and futures prices and/or to differential levels of liquidity trading.

Finally, our results do not support the view expressed in Miller *et al* (1991) that basis predictability is *mainly* a statistical illusion created by non-synchronous trading. This view implies that index arbitrage activity simply serves to counteract the drag in index adjustment and does not impact directly on the evolution of mispricing *per se*. The obvious question raised by the results in this chapter is why we would expect the step function threshold non-linearity based on the lagged value of mispricing, if predictability in the basis is a statistical illusion. We can think of no plausible, statistical explanation capable of resolving this question. Furthermore, there is no obvious reason why we should find that $\Phi_1^{(i)}$ should differ significantly and substantially across different regimes if the interpretation of Miller *et al* accounts fully for observed behavior of the basis.[20]

VI. CONCLUSIONS

This chapter has proposed a model of futures mispricing which allows, firstly, for the marked differences that can exist in the marginal transaction costs faced by different categories of arbitrageurs and, secondly, for the constraints that are expected to exist on the supply of arbitrage capital. On the basis of these institutional features we have argued that the market demand schedule of arbitrageurs will not vary linearly with the mispricing of futures contracts relative to cash market prices. Instead, the demand schedule should vary with mispricing in a non-linear fashion and specifically in the form of a step function. As a result, the time series of futures mispricing should follow a self exciting threshold autoregressive process. This is a piecewise linear autoregressive process in which the process parameters describing the evolution of mispricing are path-dependent. The empirical results presented in the chapter are strongly suggestive of threshold

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[20] It is also relevant to note that our results are based on an index which is computed from best bid and ask quotes of constituent stocks. These quotes represent prices at which competing market makers are necessarily obliged to trade up to fairly large sizes. The cash index values thus represent actually tradable values synchronous with the hourly bid and ask quotes in the futures market. Hence, there is nothing "spurious" about these index arbitrage opportunities even though quoted cash index values may not instantaneously reflect "true" value due to delays in the speed with which new information is incorporated into quotes and these delays might vary for different index stocks, thereby raising modeling issues similar to those raised by non-synchronous *trading* (see Cohen *et al*, 1986). Furthermore, there are no explicit institutional restrictions like those highlighted by Miller *et al* (1991, pp 27) that are capable of inducing lags in the adjustment of stock prices to jumps in futures prices, thereby generating "self inflicted" gaps.

non-linearity in the time series of stock index futures mispricing. Furthermore, the estimates of the values of the thresholds appear to be consistent with the model, given the estimated transactions costs levels faced by the different categories of arbitrageurs who are potentially active in these markets. Estimates of a measure of the elasticity of arbitrage services corresponding to different transaction cost regimes are also strongly consistent with the model.

The model and the empirical tests described in this chapter should be directly relevant for modeling intermarket arbitrage and the price differences of equivalent assets in a variety of other situations. To the extent that transaction cost structures are different to those prevailing in markets examined here, we would expect these differences to be reflected in parameter estimates of the model. Of course, in markets where differential transaction cost levels and/or arbitrage capital constraints are not effectively present, threshold non-linearity should not be detectable. Future research across a variety of market settings therefore has an important role in confirming, or denying, the role of transaction costs in determining the evolution of the relative prices of equivalent assets.

TABLE 1

UNIT ROOT TESTS ON FUTURES MISPRICING USING DICKEY-FULLER AND AUGMENTED DICKEY-FULLER REGRESSIONS[a]

Futures Mispricing X_t is defined as {(Futures Price - Futures Equivalent Cash Price)/Cash Price}

Contract Expiry Date	Dickey-Fuller (DF) Test $\Delta X_t = \alpha + \beta X_{t-1} + \epsilon_t$	Augmented Dickey-Fuller (ADF) Test $\Delta X_t = \alpha + \beta X_{t-1} + \gamma_1 \Delta X_{t-1} + \epsilon_t$
	t-statistic for β	t-statistic for β
June 86	-4.61***	-3.70***
Sept 86	-4.03***	-3.35**
Dec 86	-4.44***	-2.97**
Mar 87	-5.09***	-3.97***
June 87	-2.83*	-1.63
Sept 87	-3.89***	-2.83*
Dec 87	-7.06***	-4.21***
Mar 88	-5.38***	-4.16***
June 88	-3.06**	-1.64

[a] * Denotes significance at the 10% level
** Denotes significance at the 5% level
*** Denotes significance at the 1% level
Critical values of the t-statistics are based on the tables in Fuller (1976), p 373.

TABLE 2

DURBIN WATSON AND GOODNESS OF FIT STATISTICS FOR LINEAR AUTOREGRESSIVE MODELS OF FUTURES MISPRICING

Futures Mispricing X_t is modelled as per the following specification:

$$X_t = \rho_0 + \rho_1 X_{t-1}[+\rho_2 X_{t-2}] + e_t$$

Contract Expiry Date	AR(1) model DW	AR(1) model $\bar{R}^2$(%)	AR(2) model DW	AR(2) model $\bar{R}^2$(%)
June 86	1.94	82.3	1.98	86.2
Sept 86	2.18	74.9	2.09	75.0
Dec 86	2.44	86.2	1.97	87.2
Mar 87	2.31	74.6	2.04	78.5
June 87	2.46	88.2	2.19	90.6
Sep 87	2.52	85.8	2.03	86.9
Dec 87	2.42	60.8	2.13	62.4
Mar 88	2.15	74.3	1.94	73.1
June 88	2.28	88.3	1.94	89.7

TABLE 3

TESTS FOR THRESHOLD NON-LINEARITY IN FUTURES MISPRICING

The model tested is the self exciting threshold autoregressive model:

$$X_t = \rho_0^{(j)} + \sum_{g=1}^{p} \rho_g^{(j)} X_{t-g} + e_t^{(j)}$$

The model parameters depend on lagged mispricing X_{t-1}. Hence if $r_{j-1} \leq X_{t-1} < r_j$, and r_j, $j = 1.....k-1$, represents the j'th transaction cost threshold, with $r_1 < r_2 < ... < r_{k-1}$ and $r_0=-\infty$, $r_k=\infty$; then $\rho_0^{(j)}$ is the relevant threshold-specific intercept term; and $\rho_g^{(j)}$ is the relevant g'th lag AR parameter specific to threshold j. The tests for threshold non-linearity are due to Tsay (1989) and Petrucelli & Davies (1986) and they test for the existence of one or more (unknown) thresholds r_j against the alternative of a linear autoregressive model for X_t.

	Tsay Test Probability Value of Test statistic		Petrucelli-Davies Test Probability Value of Test statistic	
Contract Expiry Date	One lag[a] model (p=1)	Two lag[b] model (p=2)	One lag[a] model p=1	Two lag[b] model p=2
June 86	0.000	0.004	0.000	0.003
Sep 86	0.000	0.021	0.000	0.009
Dec 86	0.048	0.116	0.060	0.104
Mar 87	0.023	0.016	0.016	0.011
June 87	0.000	0.000	0.001	0.019
Sep 87	0.062	0.383	0.050	0.032
Dec 87	0.000	0.000	0.000	0.000
Mar 88	0.000	0.000	0.000	0.000
June 88	0.015	0.091	0.040	0.060

a Tests based on one lag models use about 360 cases of data for each contract.

b Tests based on two lag models use about 300 cases of data for each contract.

TABLE 4

ESTIMATES OF THRESHOLD VALUES FOR SETAR MODEL OF FUTURES MISPRICING

The model estimated is the self exciting threshold autoregressive model :

$$X_t = \rho_0^{(j)} + \sum_{g=1}^{p} \rho_g^{(j)} X_{t-g} + e_t^{(j)}$$

The model parameters depend on lagged futures mispricing X_{t-1}. Hence if $r_{j-1} \leq X_{t-1} < r_j$, where r_j, j = 1.....k-1 represents the j'th transaction cost threshold, with $r_1 < r_2 < \ldots < r_{k-1}$ and $r_0 = -\infty$, $r_k = \infty$; then $\rho_0^{(j)}$ is the relevant threshold-specific intercept term; and $\rho_g^{(j)}$ is the relevant g'th lag AR parameter specific to threshold j. The number and the value of the thresholds r_j are estimated using MOSUM-SQ plots based on the ordered lagged mispricing variable.

Contract Expiry Date	Quartiles of Mispricing Variable X_t(%)		Estimated Threshold Values (%)								
	Lower	Upper	One lag model (p=1)					Two lag model (p=2)			
June 86	-0.6	0.2	-0.7	-0.2	0.2	-	-	-0.7	-	-	-
Sep 86	0.2	1.0	0.6	1.2	1.6	-	-	1.2	1.6	-	-
Dec 86	-0.6	0.1	-1.1	-0.7	-0.3	-	-	-1.1	-0.7	-	-
Mar 87	0.0	0.6	0.3	0.8	-	-	-	0.3	-	-	-
June 87	-0.3	0.6	-0.5	-0.3	0.3	0.6	1.1	-0.5	-0.3	0.6	1.1
Sep 87	-0.2	0.5	-0.4	0.3	0.7	1.1	-	-0.4	0.3	0.7	-
Dec 87	-0.9	0.4	-1.0	-	-	-	-	-1.0	-	-	-
Mar 88	-0.4	0.1	-0.6	-0.2	0.6	-	-	-0.6	0.6	-	-
June 88	-0.9	-0.2	-0.9	-0.6	-0.3	0.1	-	-0.6	-0.3	0.1	-

TABLE 5

ESTIMATED TRANSACTION COSTS IN UK STOCK MARKET AND STOCK INDEX FUTURES MARKET

Quarter Ending	Cash Market Average Inner Market Spread for "Alpha" Stocks* (%)	Futures Market Average Spread between simultaneously posted bid and ask quotes for the near contract (%)
June 86	0.75	0.14
Sep 86	0.75	0.16
Dec 86	0.74	0.14
Mar 87	0.73	0.12
June 87	0.76	0.11
Sep 87	0.84	0.09
Dec 87	1.52	0.19
Mar 88	1.27	0.12
Jun 88	1.15	0.10

* Source: *Stock Exchange Quarterly/Quality of Markets Quarterly Review*

TABLE 6

ESTIMATES OF A MEASURE OF ELASTICITY OF ARBITRAGE FOR FIVE TRANSACTION COST REGIMES

The model tested is the following:

$$(X_{\pi_i+1} - X_{\pi_i}) = \Phi_0 - \sum_{j=1}^{5} \Phi_1^{(j)} D^{(j)} X_{\pi_i} + \epsilon_{\pi_i}$$

where $D_i^{(j)}$ is equal to 1 if one period lagged mispricing X_{π_i} lies within regime j and zero otherwise corresponds to the following five transaction cost regimes:

1 : $\{-\infty < X_{\pi_i} < -0.75\%\}$
2 : $\{-0.75\% \leq X_{\pi_i} < -0.25\%\}$
3 : $\{-0.25\% \leq X_{\pi_i} < 0.25\%\}$
4 : $\{0.25\% \leq X_{\pi_i} < 0.75\%\}$
5 : $\{0.75\% \leq X_{\pi_i} < \infty\}$

π_i is the indicator variable for ordering cases according to one period lagged mispricing value.
$\Phi_1^{(j)}$ is the estimated measure of the elasticity of arbitrage services corresponding to transaction cost regime j
$H_1^{(j)}$: $\Phi_1^{(j)} = 0$
against the alternative that $\Phi_1^{(j)}$ is greater than zero.

j = 1, 2, 3, 4, 5
(Tested using Dickey Fuller τ-statistic)

H_2 : $\Phi_1^{(1)} = \Phi_1^{(2)} = \Phi_1^{(3)} = \Phi_1^{(4)} = \Phi_1^{(5)}$
against the alternative that at least one coefficient is different.
(Tested using the F-statistic)

	$\Phi^{(j)}$	τ-statistic for $H_1^{(j)}$	F-statistic for H_2
$\Phi_1^{(1)}$	0.152	11.43 ***	
$\Phi_1^{(2)}$	0.086	3.82 **	
$\Phi_1^{(3)}$	0.017	0.34	10.20 ***
$\Phi_1^{(4)}$	0.000	0.01	
$\Phi_1^{(5)}$	0.046	3.21 *	

*** p value <0.001
** p value <0.01
* p value <0.05

CHAPTER 5

INTRAWEEK AND INTRADAY SEASONALITIES IN STOCK MARKET RISK PREMIA: CASH vs FUTURES[1]

ABSTRACT

The unique nature of the settlement procedures on the London Stock Exchange are utilised to directly investigate intraweek and intraday seasonalities in *ex post* risk premia (rather than total returns) and compare them with the corresponding intraweek and intraday seasonalities in the index futures market. Seasonalities in the occurrence of extreme price changes are also examined. The results reported are based on about four years of hourly cash and futures data. The analysis is conducted using both parametric and non-parametric methods. The observed empirical regularities have implications for explanations of seasonality based on market microstructure, timing of news, and psychological behavioural patterns. It is also found that the stock market has not efficiently accounted for the interest costs inherent in its own settlement procedures.

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1 First draft December 1990; revised March 1991; presented to European Finance Association Conference, August 1991, and the INQUIRE Europe/UK Conference, April 1992; revised version published in *Journal of Banking and Finance*, Vol 16 (February 1992), pp 233-270.

INTRAWEEK AND INTRADAY SEASONALITIES IN STOCK MARKET RISK PREMIA: CASH vs FUTURES

1. INTRODUCTION

The existence of seasonalities in stock markets represents an anomaly that financial economists are still seeking to explain. Many of the explanations in the literature have highlighted the institutional features and settlement procedures of the market. Relevant factors might include the delay between trading and the inflow/outflow of funds due to settlement rules and cheque clearing (Lakonishok and Levi, 1982), measurement error in returns (Gibbons and Hess, 1981; Keim and Staumbaugh, 1984), specialist-related biases (Keim and Staumbaugh, 1984), friction-related price adjustment delays (Theobald and Price, 1984) and "divide and conquer" pricing rules of market makers (Admati and Pfleiderer, 1989).

This chapter is concerned with possible seasonality in the pricing of UK stock index futures contracts in relation to the underlying index. In the absence of trading frictions, arbitrage arguments require futures prices to equal the fair value derived from the forward pricing formula. However, the institutional features and settlement procedures of the UK equity and futures markets are markedly different. Firstly, while the London equity market is a pure dealership market (similar to NASDAQ) the LIFFE index futures market is an open outcry market with exchange members generally having relatively few open positions on their own account. Secondly, while the cash market is settled on the basis of a two- (or sometimes three-) week account period, trades in the index futures market are marked to market daily. Thirdly, autocorrelation induced through friction-related price adjustment delays or bid-ask spreads is likely to be much less significant for index futures than for the associated cash index because of lower transaction costs and higher liquidity in the futures market. If such institutional features are significant determinants of seasonalities in market prices and if the differences in this regard between the equities and futures markets are significant, then differences in patterns of seasonality would be expected to show up when futures market prices and cash market prices are compared.

There is extensive evidence on intraweek seasonalities in stock index returns from both cash and futures markets. There is also substantial evidence on intraday seasonalities, but all of it comes from the cash market. Appendix A summarises previous research in these areas. There is also evidence, albeit controversial, on seasonality in the nominal cash-futures basis (Cornell, 1985b) but seasonality in the more relevant variable, namely the value basis or futures "mispricing", does not appear to have been investigated directly. More importantly, even though the literature clearly documents seasonality in yields on fixed interest securities[2], all evidence from the stock market relates to seasonalities in *total* returns rather than

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[2] See eg Ball and Bowers (1988).

in *ex post realised risk premia.* It is possible that observed seasonalities in total stock returns depend, at least partially, on the level of the risk free rate.

A further suggested explanation of price seasonalities is that they can be related to systematic timing in the arrival of good and bad news - in particular that negative price changes over the non-trading weekend interval could arise because announcements of important bad news are systematically postponed until after the close of trading on Friday.[3] Patell and Wolfson (1982) provide evidence that corporate earnings announcements conveying bad news are more frequently delayed until after close of trading than earnings announcements conveying good news, particularly on Fridays. Such information timing would have implications for the distribution of extreme price changes. The only reported evidence that we are aware of on this issue is in Dyl and Maberly (1986, pp 518-519).

In this chapter we attempt to contribute to the study of seasonalities in several new directions. Firstly, utilising the unique features of the settlement procedures on the London Stock Exchange, we analyse seasonalities in *ex post* risk premia, rather than total returns. Secondly, we extend earlier day-of-the-week studies on index futures "returns" to also include an examination of intraday seasonalities. Thirdly, in addition to investigating seasonalities in cash and futures markets separately, we also directly analyse *differences* in the seasonal behaviour of cash and futures markets

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3 See eg Thaler (1987) and French (1980).

by examining the "mispricing" of index futures contracts. Fourthly, we investigate intraweek and intraday seasonality in extreme price changes, as represented by price changes in the top and bottom deciles and the top and bottom quartiles. Finally, our evidence is based on index futures traded on the London International Financial Futures Exchange (LIFFE) and hence on data drawn fom an economic and institutional environment outside the USA - a dataset totally different from that used by earlier studies of index futures markets.

The chapter is organised as follows: Section 2 reviews the UK institutional environment in the context of this study; Section 3 describes the data set; Section 4 outlines the methodology and the empirical tests; Section 5 documents the empirical results; and finally, Section 6 presents the main conclusions.

2. THE UK INSTITUTIONAL ENVIRONMENT AND PRICING

2.1 Equity settlement

In the context of this study the most important institutional feature of the UK (cash) stock market is the settlement system. Effectively the market is a forward market. The year is divided into (usually 24) "accounts" most of which are of two weeks duration but a few (usually 4 which span exchange holidays) are of three weeks duration. Typically, an account starts on a Monday and ends on a Friday. Over the sample period covered by this study, all transactions occurring between 3.30 pm on the Last Friday of the "previous" account and 3.30 pm on the Last Friday of the "current" account[4] are settled on a single day - the second Monday of the "subsequent" account.[5] The effect of these settlement procedures can be analysed on the lines suggested for the Paris Bourse by Crouhey, Galai and Keita (1990)[6]. Note that the cash flows associated with all trades within an account occur on the same settlement day. If prices at some future time t_2 were known *ex ante* with perfect certainty, then the absence of riskless arbitrage would require that equilibrium prices at time t_1 ($<t_2$) be such that the return R_τ between t_1 and t_2 should be zero in all cases except when the interval τ spans the interval between the end of one account period and the start of the next account period. In this special case[7]

$$R_\tau = rL \qquad (1)$$

where r = risk free interest rate for one day

and L = length of the imminent account period in days.

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[4] The dealing period is defined by Stock Exchange Rule 600.2.

[5] A trade can also be made for settlement for the "new" account period at any other time on the last two days of an account, but such trades are typically made on special terms specified by the market maker and usually involve a price adjustment of approximately 1%. Reported prices are based on the assumption that a transaction will be for the "current" account if it takes place before 3.30 pm on the last day of the account, and for the "new" account if it is made later.

[6] See also Solnik (1990) or Solnik and Bousquet (1990)

[7] When the period t_1 to t_2 spans the interval between the end of one account period and the start of the next, then a long (short) stock holding will require borrowing costs (generate lending profits) equal in magnitude to rL. If $R_\tau > rL$ (if $R_\tau < rL$) an arbitrageur can earn riskless arbitrage profits with a long (short) stock position. In all other cases, stock holdings do not involve borrowing costs or generate lending profits.

In the presence of uncertainty, the expected return should include a risk premium $E(\lambda_\tau)$. Hence $E(R_\tau)$ will be equal to $(rL+E(\lambda_\tau))$ when the interval τ spans the end of one account period and the start of the next, or $E(\lambda_\tau)$ otherwise. Hence, we can write :

$$E({}_cR_\tau) \; = \; E({}_c\lambda_\tau) + \; rLD \qquad (2)$$

where τ represents the time interval; D is a dummy variable set to unity for an interval which spans the end of one account and the start of the next, and zero otherwise; and the subscript c denotes the cash market.

Effectively, the time at which one account ends and the next account starts is 3.30 pm on the Friday before the First Monday of the Account.[8] Clearly, the settlement arrangements have no impact on returns over intervals which do not span this changeover point and, for all such intervals, a test of the equality of realised holding period returns will be equivalent to a test of the equality of *ex post* realised risk premia. However, we would expect that a test of equality of daily returns over different days of the week will be rejected, because of the settlement period effect. Nevertheless, a test of equality of *ex post* realised risk premia over different days of the week can be constructed using observed returns, if the returns over intervals

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[8] See Stock Exchange Rule 600.3(b).

spanning the changeover time are adjusted by deducting the interest component equal to rL.[9]

2.2 Futures pricing

The forward pricing formula for the fair value of the index futures contracts can be modified to allow for cash market settlement delays as follows[10]

$$F_{t,T}^{*} \quad = \quad I_t e^{-r_{s_1,s_2}(s_2-s_1)} \; - \; FV_{s_2}(Div) \qquad (3)$$

where $F_{t,T}^{*}$ = Price at time t of futures contract maturing at T

I_t = Spot price at time t

$FV_t(Div)$ = Expected future value at s_2 of all dividends on the index from t to T

s_1 = Settlement date for cash market transactions at t[11]

s_2 = Settlement date for cash market transactions at T

r_{s_1,s_2} = Forward rate of interest on a loan committed on day t, for disbursement on day s_1 and repayment on day s_2.

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9 It is important to note that earlier UK studies (eg Choy and O'Hanlon, 1989 or Board and Sutcliffe, 1988) could have *incorrectly* assumed that *all* trades which take place within an account, and not just trades before 3.30 pm on the Last Friday, are settled on the second Monday of the subsequent account. It is not clear to us whether the "closing" prices used by them are 3.30 pm prices or 5.00 pm prices. If they have used 5.00 pm prices, the close-to-close First Monday return *cannot* be used to infer the implied interest rate as they have done.

10 See eg Yadav and Pope (1990) for a discussion of the applicability of the forward pricing formula in a UK context.

11 It is important to note that the forward pricing formula assumes *inter alia* that futures are settled only at expiration and not marked daily to market. The effect of such daily marking to market cash flows on fair value are not likely to be important in the context of this study, since we are only considering *changes* in futures prices over (at most) a daily interval.

There is no "investment" in futures contracts and although there is a mark to market margining system, margin capital is in interest bearing securities. Hence it is more meaningful to examine price changes rather than returns. However, to avoid potential heteroskedasticity problems in the empirical analysis, the results we report are based on price changes deflated by the cash index level, which are sometimes referred to as futures "returns".[12] We define futures returns over an interval τ (ie from t_1 to t_2) of a futures contract maturing at T as :

$$_{F}R_{\tau} = \frac{F_{t2,T} - F_{t1,T}}{I_{t1}} \tag{4}$$

If futures are priced according to the forward pricing formula (3) and if it is assumed that dividends with an ex-date of t, are actually paid at s_1, then it can be shown that the expected futures return for period τ is[13]

$$E(_{F}R_{\tau})e^{-r_{s_1,s_2}(s_2-s_1)} = E(_{C}R_{\tau}) - rLD$$

$$= E(_{C}\lambda_{\tau})$$

Since futures markets have zero "investment", the expected risk premium in the futures market will be the same as the expected futures "return". Hence :

$$E(_{F}\lambda_{\tau}) = E(_{C}\lambda_{\tau})e^{-r_{s_1,s_2}(s_2-s_1)} \tag{5}$$

[12] All computations were repeated for price changes, with no essential differences in results. These unreported results are available from the authors on request.

[13] In particular, the problem of weekend interest outlined by Patrick-Phillips and Schneeweiss (1988) will not arise.

where τ denotes different days of the week or hours of the day.

Irrespective of the account settlement period system, seasonality in the expected index futures risk premium should reflect any seasonality in the expected risk premium relating to the underlying index.

2.3 Mispricing Returns

The "mispricing" of the futures contract is the observed futures price minus the fair value determined from the forward pricing formula (3), ie

$$W_{t,T} \quad = \quad F_{t,T} - F_{t,T}^{*} \tag{6}$$

"Mispricing" returns are defined as :

$${}_{x}R_{\tau} \quad = \quad \frac{\{W_{t2,T} - W_{t1,T}\}}{I_{t1}}$$

Here ${}_{x}R_{\tau}$ represents the one period return earned over interval τ on a MV efficient cash-futures hedge portfolio where the hedge ratio is calculated in the forward pricing formula framework. It can be shown that[14]

$$E({}_{x}R_{\tau}) \quad = \quad E({}_{F}\lambda_{\tau}) \; - \; e^{-r_{s_1,s_2}(s_2-s_1)}E({}_{c}\lambda_{\tau}) \tag{7}$$

Hence, irrespective of the account settlement period system, seasonality in mispricing returns should directly reflect any *difference* in the seasonality pattern between the expected risk premia in the futures market and the cash market.

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[14] The analysis is similar to that in Yadav and Pope (1991).

3. DATA

London has only one exchange traded stock index futures contract. This contract is traded on the LIFFE and is based on the FTSE100 index - an arithmetic average, market value weighted index of one hundred (highest capitalisation) stocks. LIFFE index futures expire four times a year in March, June, September and December on the last business day of the month. Three maturities are traded at any particular time, but the near contract is almost always the most heavily traded contract. LIFFE trading is based on an open outcry market and, in common with other futures markets, all margin accounts are marked to market on a daily basis.

The results reported in this chapter are based on hourly cash and futures data on the FTSE100 index for the period April 28, 1986 to March 23, 1990.[15] The choice of this period has been dictated largely by availability of data and changes in exchange trading hours. The beginning of the sample period corresponds with date on which LIFFE extended its trading hours from 9.35 am-3.30 pm to 9.05 am-4.05 pm. The end of the sample period corresponds to the date on which the International Stock Exchange changed its trading hours from 9.00 am-5.00 pm to 8.30 am-4.30 pm. The sample selection criteria ensure that the trading hours in each market did not change over the sample period.

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[15] The analysis of intraweek seasonality in trading period and nontrading period price changes was repeated using daily open and close data over the period July 1, 1984, to June 30, 1990, with no essential differences in results.

Information on the constituents of the FTSE100 index and how these had changed over the sample period was obtained from the International Stock Exchange. Hourly data on the FTSE100 index was collected from the *Financial Times*. "Time and Sales" data for the FTSE100 index futures market was obtained from LIFFE. The data includes all bid and ask quotes and all transaction prices relating to the contract. The data was used to identify the mid-market quotes valid at 9.06 am, 10 am, 11 am, 12 noon, 1 pm, 2 pm, 3 pm and 4 pm each day. An examination of trading volume revealed that the near contract is, almost always, the most heavily traded index contract on LIFFE. Futhermore, the results of Yadav and Pope (1990) show that the magnitude of mispricing in the far contract is considerably higher than in the near contract, possibly because the arbitrage window tends to be wider due to greater dividend and interest rate uncertainty. Our analysis is hence based on the near contract, shifting to the next contract on expiration day.

Dividends and ex-dividend dates for all constituents of the index each day were collected from *Extel* cards. In order to compute the daily dividend entitlement on the FTSE100 index, market value and unadjusted price data was taken from *Datastream* for each index constituent. The number of shares of each company outstanding at the end of each day, was taken to be the closing market capitalisation divided by the closing unadjusted price of the company. The market value of the total dividend each day was then obtained by multiplying the number of shares outstanding by the nominal dividend paid by any stock going ex dividend and summing over all stocks in the index on that day. The resulting figure is a measure of the daily dividend entitlement associated with ownership of the FTSE100 index, measured in index points. Daily series for one- and three- month Treasury Bill rates were also collected from *Datastream*.

Mispricing of the futures contract was calculated on the basis of the forward pricing formula (3). In addition to the usual assumptions of the forward pricing formula, the following additional assumptions were made : (a) forecast dividends to maturity for each date are identical to the actual *ex post* daily cash dividend inflow for the FTSE100 basket; (b) the forward interest rate at time t for a loan made at time w to be redeemed at time T, is identical to the interest rate at time w on a Treasury Bill maturing at time T; (c) the value on day t of one- and three-month Treasury Bill interest rates can be used to estimate a linear term structure from which the implied interest rate for the period (T-t) can be calculated; and (d) actual payment of dividends is made on the account period settlement date corresponding to the ex-date.[16]

The mispricing series was used to calculate mispricing "returns", as defined above. Assumptions (a), (b), (c), and (d) above can all be regarded as innocuous because

[16] This is an ad hoc assumption made essentially for convenience. It implies that actual payment of dividends is made between 10 and 28 days of the ex-date - an assumption that appears reasonable. Published studies on mispricing have typically ignored the lag between the ex date and actual payment of dividends.

the variable of interest is the *change* in mispricing over hourly, or at most, daily intervals.

During the sample period the cash market was open from 9.00 am to 5.00 pm, but the futures market was open from 9.05 am to 4.05 pm. Consequently the mispricing series is based on coincident cash and futures prices at 10.00 am, 11.00 am, 12 noon, 1.00 pm, 2.00 pm, 3.00 pm and 4.00 pm. Mispricing at the "close" is based on coincident 4.00 pm prices. However, mispricing at the "open" is not based on exactly coincident prices because 9.00 am cash prices are matched with 9.06 am futures prices. The estimates of mispricing at the open will therefore potentially be contaminated with some "noise" due to this non-synchronicity, though arguably it should not affect the results in a systematic way.

4. METHODOLOGY

This study investigates intraweek and intraday seasonality in cash, futures and mispricing series by examining changes in intraday hourly prices, price changes from previous-close-to-open, and price changes from open-to-close. In view of possible "contagion" effects related to the opening of the US market[17], the main analysis is conducted on a dataset which excludes days on which the US market was

[17] This has been documented *inter-alia* by King and Wadhwani (1990).

closed and also days on which the US market did not open at 14.30 hours UK time.[18] In addition, to avoid distortion due to large outliers, the 1987 "crash" period and the 1989 "mini-crash" period were removed from the sample.[19] Finally, the days immediately following UK exchange holidays were also removed from the sample because *a priori* such days may display atypical behaviour.

Intraweek Seasonality

To formally test for intraweek seasonality, the following model was estimated :

$$\lambda_t = \sum_{j=2}^{5} \beta_j D_{jt} + \beta_{FM} D_{FM,t} + \beta_{OM} D_{OM,t} + \tilde{\epsilon}_t \qquad \text{....(8)}$$

where D_{jt} (j=2,...5) are dummy variables for Tuesday, Wednesday, Thursday and Friday

$D_{FM,t}$ is a dummy variable for a Monday which is also the first day of an account

$D_{OM,t}$ is a dummy variable for a Monday which is *not* the first day of an account

and λ_t is the *ex post* realised risk premium on an asset in period t[20]

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[18] Over the sample period, the switch from Summer time to Winter time took place on the same date in the US and the UK, but there was a difference of one week in the switch from Winter time to Summer time. These differences were controlled for in the analysis.

[19] Specifically, all entries relating to a four week period beginning with October 19, 1987 and to the single day October 16, 1989, were removed.

[20] For the mispricing variable λ_t is the same as the mispricing return and can be interpreted as the risk premium corresponding to the forward pricing formula cash-futures hedge portfolio.

Equation (8) was estimated for the following time intervals :

(a) Previous Close to Open (non-trading period),

(b) Open to Close (trading period), and

(c) Hourly returns for each hour of the day, ie eight one-hour periods (from 9.00 am to 5.00 pm) for cash, seven one-hour[21] periods (from 9.05 am to 4.00 pm) for futures and seven one-hour periods (9.00 am to 4.00 pm)[22] for mispricing.

Based on equation (8) the following null hypotheses were tested using the conventional F-test: [23]

H_1^W : The mean *ex post* risk premia on Tuesdays, Wednesdays, Thursdays, Fridays, First Mondays and Other Mondays are equal.

H_2^W : The mean *ex post* risk premia on Tuesdays, Wednesdays, Thursdays and Fridays are equal.

H_3^W : The mean *ex post* risk premium on First Mondays is equal to the mean *ex-post* risk premium over Tuesdays, Wednesdays, Thursdays and Fridays.

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21 Since the futures market opened at 9.05 am instead of 9.00 am the first "one-hour" interval was only from 9.05 am to 10 am.

22 The 9.00 am mispricing value is based on non-coincident cash futures prices as explained earlier in Section 3 ie 9.00 am cash value and 9.05 am futures value.

23 All references to risk premia in this section and the section on empirical results relate to *ex-post* realised risk premia even when not explicitly mentioned.

H_4^W : The mean *ex post* risk premium on Other Mondays is equal to the mean *ex-post* risk premium over Tuesday, Wednesday, Thursday and Friday.

Seasonality in *ex-post* risk premia over the different weeks in the settlement period was investigated by testing the following hypotheses using a two sample T-test[24]:

H_5^W : The mean *ex post* risk premium on First Mondays is equal to the mean *ex-post* risk premium on Last Mondays.

H_6^W : The mean *ex post* risk premium on Tuesdays *before* First Mondays is equal to the mean *ex-post* risk premium on Tuesdays *after* First Mondays.

H_7^W : The mean *ex post* risk premium on Wednesdays *before* First Mondays is equal to the mean *ex-post* risk premium on Wednesdays *after* First Mondays.

H_8^W : The Mean *ex post* risk premium on Thursdays *before* First Mondays is equal to the mean *ex-post* risk premium on Thursdays *after* First Mondays.

[24] It is relevant to note that the hypotheses tested related to the *first* and *last* weeks of Account Settlement periods. There were relatively few three week accounts in the sample period and hence the few *middle* weeks of the Account were not investigated.

H_9^W : The mean *ex post* risk premium on Fridays *before* First Mondays is equal to the mean *ex-post* risk premium on Fridays *after* First Mondays.

In addition, the mean *ex post* risk premia on each of the ten days (ie the first five days and the last five days of the account) were calculated separately and t-tests conducted to see if mean *ex-post* risk premia were each significantly different from zero.

It has been suggested[25] that the negative Monday effect arises because the magnitude of negative price changes is disproportionately large in comparison to the magnitude of positive price changes. It is also conceivable that the negative mean returns typically observed are being influenced unduly by the presence of outliers. The robustness of inferences based on mean values was investigated by examining the proportion of positive risk premia on each of the ten (main) days of the Account (ie the first five days and the last five days) and testing whether the proportion was significantly different from 0.5. Subsequently the hypotheses H_1^W, H_2^W, H_3^W and H_4^W were tested using the corresponding non-parametric Kruskal Wallis test and the hypotheses H_5^W, H_6^W, H_7^W, H_8^W and H_9^W were tested using the corresponding non-parametric Mann Whitney test.

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[25] See eg Dyl and Maberly (1986).

Intraday Seasonality

Intraday seasonality is tested with the following model[26] :

$$\lambda_t = \sum_{t=1}^{n} \beta_j D_{jt} + \tilde{\epsilon}_t \qquad (9)$$

where D_t (t=1,...n) are dummy variables for the n different hourly periods during the day

n is 8 for cash returns (9.00 am to 5.00 pm), 7 for futures returns (9.05 am to 4.00 pm) and for mispricing returns (9.00 am to 4.00 pm)

Equation (9) was estimated separately for first Mondays, other Mondays, Tuesdays, Wednesdays, Thursdays and Fridays and also for the aggregated data set including all days of the week.[27]

Based on equation (9) the following null hypotheses were tested using the parametric F-test:

H_1^D : The mean hourly *ex post* risk premia are equal across all hours during the day

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[26] For all three variables - cash, futures and mispricing - intraday *ex post* risk premia were assumed to be identical to the actual return over the corresponding period.

[27] It was also investigated separately for each of the ten days of the account - first five days and last five days - with no essential differences in results.

H_2^D : The mean hourly *ex post* risk premium during each of the one-hour periods (1...8 for cash, 1...7 for futures, and 1...7 for mispricing) is equal to the mean hourly *ex post* risk premia over the other hourly periods of the day

Once again, the robustness of inferences based on mean values was also investigated by testing hypotheses H_1^D and H_2^D using the non-parametric Kruskal Wallis Test.

Extreme Price Changes

"Extreme" price changes were investigated for both the cash market and the futures market by examining for seasonality in the risk premia of top and bottom deciles and also top and bottom quartiles of price changes. The results reported are based only on top and bottom deciles. However, the results for the top and bottom quartiles were qualitatively similar.[28]

Intraweek seasonality was examined separately for each intraday hourly interval, the non trading interval (ie previous close to open) and the trading interval (ie open to close). In each case, the following hypothesis was tested using a χ^2 test :

H_{EX}^W : The proportion of first Mondays, other Mondays, Tuesdays, Wednesdays, Thursdays and Fridays in the top and bottom deciles is equal to the proportion of first Mondays, other

[28] Available on request.

Mondays, Tuesdays, Wednesdays, Thursdays and Fridays in the complete data set.

Intraday seasonality was examined separately for each day of the week (ie first Mondays, other Mondays, Tuesdays, Wednesdays, Thursdays and Fridays). In each case, the following hypothesis was tested using a χ^2 test :

H_{EX}^D : The proportion of each one-hour interval in the top and bottom deciles is equal to the proportion of the respective one-hour interval in the complete data set.

Seasonality in Correlations between US and UK Markets

Seasonality in correlations between the US and UK markets was investigated through their respective index futures markets[29] - specifically the "returns" on the S&P500 "near" futures contract traded on IMM[30] and the LIFFE FTSE100 "near" futures contract.

The effect of day t returns over interval p in Market 1 on day t^* returns over interval q in Market 2 was estimated through the following regression equation :

[29] Returns in the futures market were used because opening US cash prices were not available on *Datastream,* and 2.30 pm UK cash prices were also not readily available.

[30] Though the near contract was used, delivery month observations were excluded. Hence the series changed to the next contract on the first day of the delivery month. Furthermore, the US futures returns were defined as $Log_e(F_t/F_{t-1})$. This is not expected to be important in the context of this study.

$${}_2R_{t*}^{q} = \sum_{j=1}^{2} \gamma_j \, {}_2R_{t*}^{q}[-j] + \sum_{j=2}^{5} \alpha_j D_{jt*} + \alpha_{FM} D_{FM,t*} + \alpha_{OM} D_{OM,t*}$$

$$+ \sum_{j=2}^{5} \beta_{jp}^{p} D_{jt} \, {}_1R_t^{p} + \beta_{FM}^{p} D_{FM,t} \, {}_1R_t^{p} + \beta_{OM}^{p} D_{OM,t} \, {}_1R_t^{p}$$

$$+ \epsilon_{t*} \qquad \text{.... (10)}$$

α_j (j=FM,OM,2...5) and γ_j (j=1,2) are intended to capture serial dependence and the "normal" variations due to day of the week effects in Market 2. D's are dummy variables defined as in Eq (8).

The following null hypotheses were tested using the F-statistic :

H_0^S : Day t returns over interval p in Market 1 are not correlated with day t^* returns over interval q in Market 2, ie

$$\beta_{FM}^{p} = \beta_{OM}^{p} = \beta_2^{p} = \beta_3^{p} = \beta_4^{p} = \beta_j^{p} = 0$$

H_0^S : The correlation of day t returns over interval p in Market 1 and day t* returns over interval q in Market 2 is not significantly different on different days of the week, ie

$$\beta_{FM}^{p} = \beta_{OM}^{p} = \beta_2^{p} = \beta_3^{p} = \beta_4^{p} = \beta_5^{p}$$

Various choices of t, t*, p, and q are evidently possible. The investigation was confined to the following:

(a) Seasonality in the correlation between UK intraday returns and (overlapping) interday US returns, ie to what extent is the US previous-close-to-open return predicted by the UK market return from UK-open to US-open and is this predictability significantly different on different days of the week?

(b) Seasonality in the effect of UK intraday returns on (future) intraday US returns, ie to what extent is the US open-to-close return predicted by the UK market return from UK-open to US- open and is this predictability significantly different on different days of the week?

(c) Seasonality in the effect of US intraday returns on (overlapping) UK interday returns, ie to what extent is the UK previous-close-to-open return predicted by the US open-to-close return on the previous day and is this predictability significantly different on different days of the week?

(d) Seasonality in the effect of US interday returns on (future) UK returns, ie to what extent is the UK return from 2.30 pm to 3.00 pm UK time predicted by the US previous-close-to-open return and is this predictability significantly different on different days of the week?

5. EMPIRICAL RESULTS

Tables 1 and 2 report the results of analysing intraweek and intraday seasonality of the *ex post* realised risk premia for, respectively, the cash and futures markets and Table 3 refers to mispricing returns (ie cash-futures hedge returns).[31] In each of these tables Panel A reports the mean risk premia and the percentage of positive values for each interval. Panel B reports the results of testing the null hypotheses related to intra-week and intra-settlement-period seasonality. Panel C reports the results of testing each of the null hypotheses relating to intraday seasonality. Panel D reports the percentage frequency of different days of the week in the top and bottom deciles and the χ^2 statistic for testing the null hypothesis H_{EX}^{W}.[32] Finally, Panel E reports the percentage frequency of different hours of the day in the top and bottom deciles and the χ^2 statistic for testing the null hypothesis H_{EX}^{D}.[33]

Results for the Cash Market

An initial examination of the close-to-close cash *returns* on first Mondays of the account revealed them to be significantly positive, on average. On other Mondays they are significantly negative, on average, thus confirming earlier work based on

[31] *Ex post* returns are identical to *ex post* risk premia over all intraday intervals.

[32] The "expected" percentage in each category is not reported separately since it is not materially different from 10%, 10%, 20%, 20%, 20%, 20% for first Mondays, Tuesdays, Wednesdays, Thursdays and Fridays respectively.

[33] The "expected" percentage in each category is not reported separately since it is always equal to 12.5% for cash, 14.2% for futures, and 14.2% for mispricing.

UK data.[34] Table 1 enables the timing of this seasonality to be identified. Positive close-to-close First Monday risk premia accrue during the trading period on Mondays and not over the weekend non-trading interval. On Other Mondays also negative risk premia accrue during the trading period on Monday, but there is no evidence of systematic negative price changes over the preceding weekend. In fact, a significant proportion of the close to open price changes on both First Mondays and Last Mondays are *positive*. Clearly, there is no *weekend effect*, but a Monday trading period effect.[35]

There is a significant rise in cash prices on Last Fridays between 3 pm and 4 pm, and also between 4 pm and 5 pm, and there is no associated rise in mean futures prices over the period, suggesting that some of the interest costs inherent in settlement procedures are being reflected in cash prices during these intervals. However, the magnitude of the average price change is much smaller than the average value of the interest which should be reflected in cash prices. This is evident from the large magnitude of mispricing returns which accrue over the 3 pm to 4 pm interval - about 26 basis points. Hence, consistent with the results of Solnik (1990) and Solnik and Bosquet (1990) for the Paris Bourse, the market does not appear to efficiently incorporate into prices the *entire* effect of interest costs

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34 See eg Board and Sutcliffe (1988).

35 Since the index being investigated comprises large capitalisation stocks, this result contrasts with Harris (1986) who found that negative returns accrue over the weekend for large stocks and that it is only small stocks that display negative returns during the trading period on Monday.

inherent in its settlement procedures. The large positive risk premia during the trading period on First Mondays is also consistent with the large positive returns observed by Solnik (1990) and Solnik and Bosquet (1990) on the second day of a settlement period. However, these positive trading period First Monday risk premia carry over into the futures market, and hence it is difficult to regard them as delayed manifestations of settlement period interest accruing during the account changeover.

The average hourly risk premia on first Mondays are positive for each hour from 9.00 am to 2.00 pm (and significantly so for three out of five of those periods). However, the risk premia become negative as soon as the US market opens in the hourly periods 2.00 pm- 3.00 pm and 3.00 pm-4.00 pm. Even on last Mondays, while average returns are negative in every hourly period during the day, they are most negative in the two hourly periods immediately after the US market opens. This is consistent with intermarket "contagion" discussed later in this section.

Harris (1986) observes significant intraweek seasonality in 15 minute US returns only during the first 45 minutes of trading. In the UK, intraweek seasonality in hourly risk premia is significant at the 5% level for 5 out of 8 hourly intervals[36] - in particular for the first two hours and the last two hours of the day. During the first two hourly periods, the hypothesis of no intraweek seasonality is rejected

[36] And significant for all hourly intervals with a maximum p-value of 16.6%.

primarily because first Monday risk premia are strongly positive and last Monday risk premia are strongly negative. During the last two hourly periods the hypothesis of no intraweek seasonality is rejected because Friday risk premia are strongly positive and last Monday risk premia are strongly negative.

The results in Table 1 also show that prices tend to rise systematically during the first hour of trading. The average *ex post* realised risk premium in the first hour of trading is positive and the proportion of positive returns is more than 50% on all days of the account except last Mondays. The average risk premium during the first hour is positive and highly significant and the hypothesis that this first hour average is equal to the average over other hours of the day is conclusively rejected. These results are consistent with Harris (1986) and are established despite the fact that the US market opens through a specialist controlled auction procedure whereas the UK market has the same pure dealership market structure throughout the day. Hence it appears unlikely that the observed first hour positive returns are attributable to the market trading mechanisms *per se*.

Another important result relates to risk premia during the period 2.00 pm-3.00 pm, the interval spanning the opening of the US market. The average risk premium during this period is negative on each day of the week except last Fridays and the hypothesis that it is equal to the average risk premium over other hours of the day is conclusively rejected. King and Wadhwani (1990) document evidence of greater volatility in the UK market during the 2.00 pm-3.00 pm interval, arising presumably because of the opening of the US market. This result is intuitively reasonable because the UK market will incorporate the US market's assessment of value-relevant information generated in the (US) overnight period into local prices. However there appears to be no obvious reason why the UK market should have significantly and systematic negative *mean* returns coinciding with the opening of the US market. If anything, it could be argued that higher *ex-ante* volatility should lead to a higher risk premium during this one-hour interval in comparision to the other one-hour intervals during the day. To explicitly control for the US market assessment of overnight information the 2.00 pm to 3.00 pm hourly return was regressed on the US previous close-to-open return with dummy variables to allow for differences in the dependence of both slopes and intercepts across different days of the week. However, the pattern of negative risk premia was found to exist even after this control was introduced.[37]

One explanation for the price behaviour between 2.00 pm and 3.00 pm is that the UK market tends to be too "bullish" about the US market and needs a "correction" when the US opening market price is actually observed. However, interestingly Harris (1986) observes a price decline between 2.30 pm and 3.15 pm (US time) and it is possible that the UK market displays a similar intraday effect independent of any contagion effect. To help distinguish between these two explanations, a subset

[37] Results are available on request. Interestingly, there is significant intraweek seasonality in the effect of the US previous close-to-open return on the UK 2.00 pm to 3.00 pm return. In particular, there is almost no dependence on Mondays!

of data consisting only of days on which the US market was closed was analysed.[38] It was found that returns from 2.00 pm to 3.00 pm are *not* negative on average for this subset and *not* significantly different from the returns over other intervals during the day. This suggests that the UK market could tend to be relatively optimistic in its assessment of US overnight information and the observed seasonality could be due to a contagion effect.[39] [40]

The analysis also revealed that the mean (median) risk premium during the first week of an account is 0.98% (1.16%) and the proportion of positive values is 73.9% and is greater than 50% for each day of the first account week. In contrast the average risk premium during the last week is slightly negative and significantly different from the mean risk premium during the first week of the account. Significant differences exist between the average open-to-close risk premia in first and last weeks of the account for Mondays and Wednesdays. Differences also exist between the mean (median) close-to-open premia in the first and last weeks of the Account for Mondays and also for Tuesdays.

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[38] As pointed out earlier, the results in the Tables relate to a data subset which does not include the days on which the US market was closed, or the days on which it did not open at 2.30 pm UK local time.

[39] In view of the limited number of data points, these regressions were run not for each day of the week separately, but only for the complete set of hourly *ex post* risk premia ie including all days of the week.

[40] It could also be because this data subset was too small to reveal statistically significant differences.

The pattern of intraweek seasonality in the top and bottom deciles[41] of the cash *ex post* risk premia distribution is qualitatively similar to the pattern observed for the entire data set. In particular, it displays the significant negative close-to-open risk premia on first Mondays, negative open to close risk premia on other Mondays and positive 3.00 pm-4.00 pm risk premia on Fridays. The proportion of other Mondays in the bottom decile of close-to-open returns is not different from that which would be expected if bottom decile risk premia included all days of the week equally. These results do *not* support the conjecture that important bad news tends to be released more frequently over the weekend non-trading interval than on other days of the week.

The pattern of intraday seasonality in the top and bottom deciles of the cash risk premia distribution also has some interesting features. Firstly, the seasonality pattern is very pronounced. The hypothesis of no intraday seasonality is conclusively rejected at the 0.1% level for the top decile for each day of the week and at the 10% level for the bottom decile. Secondly, while the top decile has a significantly high proportion of cases relating to the first hour of trading, the bottom decile also has a significantly high proportion of cases relating to the first hour. This shows that the first hour is a period in which important news is released. On average, good news tends to dominate bad news in the first hour. Thirdly, the proportion of cases of the 2.00 pm-3.00 pm interval in the bottom decile is *not*

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[41] The pattern of intraweek seasonality in the top and bottom quartiles is essentially similar. Details are available on request.

unusual, but the proportion of cases belonging to this interval which appear in the top decile is significantly low, suggesting that average returns in this period are significantly negative because important good news fails to be released in this period rather than it being a period when bad news (in the form of the US opening price and otherwise) hits the market. Finally, the proportion of cases relating to the two hourly intervals from 12 noon to 2.00 pm, is significantly low *both* in the top decile and in the bottom decile, indicating that this is a period during which important news is not usually released.[42]

Futures Market and Futures Mispricing Results

Seasonality patterns in risk premia in the futures market are reported in Table 2 and are largely similar to the seasonality patterns in the cash market, although there are also some important differences. On Last (First) Mondays, the negative (positive) risk premia also accrue in the futures market during the trading period rather than over the weekend. Also, the average risk premium during trading on first Mondays is large and the average futures risk premium over the first week of the account is also positive and significantly higher than the negative premium over the last week of the account. Furthermore first Monday risk premia become negative as soon as the US market opens and even on last Mondays price changes are most negative in the two hourly periods after the US market opens.

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[42] The pattern of intraday seasonality in the top and bottom quartiles is essentially similar. Details are available on request.

However, as highlighted earlier, mispricing returns are large and positive between 3 pm to 4 pm on Last Fridays, and hence it appears that the *entire* interest which should accrue over this interval, is not being reflected in cash prices. Furthermore, even though the close-to-open risk premia for First Mondays are individually not significantly different from zero in both the cash market and the futures market, they is of opposite sign, and, as such, mispricing returns over this interval are large and significantly positive. A long-cash-short-futures hedge portfolio would have earned large and significantly positive returns, with positive returns accruing in 89% of cases with a mean return before transactions costs of about 21 basis points. These results again suggest that the cash market does not account efficiently for interest costs.

There is a difference between cash and futures results not only on Last Fridays but also on First Fridays. There is no rise in prices in the futures market on First Friday afternoons, whereas prices rise in the cash market. This leads to fairly significant negative mispricing returns, on average, over this interval. The divergence between cash and futures market behaviour on both First and Last Fridays is *not* consistent with explanations of seasonaltity based on cognitive biases in market behaviour, unless the two markets can be viewed as segmented.[43] However, this result is consistent with the conjecture that the cash prices rise because market makers do not

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[43] Research on experimental markets highlighted by Thaler (1987) proposes explanations of the weekend effect based on psychological factors, such as a preference for compound gambles over simple gambles. Other behavioural explanations include variation in moods of market participants (like good moods on Fridays and bad moods on Mondays).

wish to hold short open positions over the weekend, while, on the other hand, exchange members on LIFFE seldom hold large open positions in the index futures market and "locals" tend not to hold overnight positions.

It is also interesting to observe that the average close-to-open futures market *ex-post* risk premium is positive but the average trading period risk premium is not significantly different from zero. In contrast, the average close-to-open risk premium in the cash market is not significantly different from zero while the average trading period risk premium is significantly positive. In other words, the cash market rises when the market is open but the futures market rises when the market is closed! This is a somewhat surprising result, which begs an obvious explanation. It implies (as can be seen in Table 3) that a long-cash-short-futures hedge earns, on average, statistically significant positive returns of about 5 basis points during trading periods while a short-cash-long futures hedge earns, on average, statistically significant positive returns of about 5 basis points during non-trading periods.

As with the cash market, the average price change in the futures market during the first hour of trading is positive and significantly different from the average price change over other hours of the day. However, the average magnitude and frequency of such positive price changes is significantly less than in the cash market. As a result, the average mispricing return in the first hour is significantly negative and significantly different from the average mispricing return over other hours of the day.

Futures market price changes also follow the cash market during the 2.00 pm-3.00 pm interval (spanning the opening of the US market) in being significantly negative on average and significantly different from the average hourly price change over other intervals. Again, this result remains even after controlling for the US market assessment of overnight information. Furthermore the data subset consisting only of dates on which the US market was closed does not have negative average 2 pm to 3 pm price changes. The availability of futures market transactions data enabled us to separately analyse the two sub-periods 2 pm to 2.30 pm and 2.30 pm to 3 pm. It was found that while the mean and median price change from 2 pm to 2.30 pm were not significantly different from zero, the mean and median price change from 2.30 pm to 3 pm were significantly negative on each day of the week except Fridays. These results confirm that the UK market tends to fall immediately after the US market opens.

Futures risk premia are significantly negative over the 3.00 pm to 4.00 pm interval. The mean 3.00 pm to 4.00 pm price change is also significantly different from the mean price change over other hourly intervals. This feature is manifestly different from the cash market. As a result, the average mispricing return in the 3.00 pm-4.00 pm interval is significantly negative and significantly different from the average mispricing return over other hourly intervals. In fact, the average futures price change over the last two hours of the trading day (2.00 pm-4.00 pm) is negative on each day of the account settlement period.

There is little evidence of intraweek seasonality in the bottom decile of the futures risk premia distribution. For the top decile, the hypothesis of no intraweek seasonality is rejected at the 5% level only for the 1.00 pm to 2.00 pm interval and for the overall open to close interval. This suggests that the intraweek seasonality otherwise observed is not being driven by outliers and important news arrival. In particular, the proportion of different days in the top and bottom deciles[44] of the close-to-open futures risk premia is not significantly different from the "expected" proportion, showing clearly that the conjecture of systematic timing of important good and bad news is not supported by the results. Intraweek seasonality in the top and bottom deciles of mispricing returns essentially reflects the high magnitude of mispricing returns over the non-trading weekend interval before first Mondays and the 3.00 pm to 4.00 pm interval on Fridays.

The pattern of intraday seasonality in the top and bottom deciles of futures risk premia is essentially similar to the cash market except for one additional feature - that the proportion of cases relating to the last hour is significantly high in the bottom deciles of the day but *not* significantly high in the corresponding bottom quartiles. This suggests that the significantly negative last hour risk premium

44 And also the top and bottom quartiles.

highlighted earlier could arise because of outliers. Since this pattern does not exist in the cash market, it is difficult to regard these outliers as "important bad news", and their origin has to be traced to institutional factors relating to the futures market.[45]

Round trip transaction costs for institutional investors in the futures market can be as low as only about ten basis points. Hence, while the magnitude of the seasonality observed in the cash market is definitely not enough to enable the formulation of profitable trading rules, some of the seasonal features in the futures market could be regarded as being potentially economically significant, since trading rules based on them have provided *ex-post* profits after transaction costs. The *differences* in seasonality patterns between cash and futures markets could have been exploited only by those arbitrageurs whose marginal costs are confined to the futures market.[46]

Seasonality in Correlations between US and UK Markets

Table 4 documents the results of running regression equation (10) and testing hypotheses relating to intraweek seasonality in the dependence between US returns

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[45] Following Brock and Kleidon (1989), it can be conjectured that passive index tracking portfolio managers tend to trade in the cash market towards the end of the day (to minimise tracking error since such funds are typically valued at closing prices) and thus cash prices rise or are prevented from falling too steeply while futures prices have no such support.

[46] Eg arbitrageurs committed otherwise to corresponding cash market transactions.

and overlapping and past UK returns, and intraweek seasonality in the dependence between UK returns and overlapping and past US returns.[47]

Table 4 has several interesting features. First, there is strong association between returns in the UK market from UK opening time to US opening time and not only overlapping (ie previous close to open) US returns, but also future (ie open to close) US returns. Second, there is clear evidence of intraweek seasonality in the dependence between US returns from previous US close to US open and UK returns over the overlapping period of UK open to US open. On the other hand, there appears to be no statistically significant seasonality in the association between US returns (from US open to US close) and *past* UK returns (from UK open to US open). Third, non-trading period UK returns (from previous UK close to UK open) are strongly correlated with US returns over the overlapping period (of previous day US open to US close). Furthermore the hypothesis of no seasonality in this correlation is rejected at the 5% level since the dependence tends to be higher on Wednesdays, Thursdays and Fridays than on other days. Finally, the opening of the US market appears to impact significantly on the UK market. US returns (from previous US close to US open) strongly predict subsequent UK returns (from 2.30 pm to 3.00 pm), the hypothesis of no predictability being conclusively rejected. However, the correletion between the two sets of returns is consistently negative (average magnitude about -0.21). Hence, positive overnight US returns appear to

[47] Other choices of overlapping and past-future intervals are feasible. Table 4 reports what appears to be the most relevant considering that access to intraday US data was not available.

be associated with negative UK returns immediately after opening of the US market, and vice versa. Furthermore, the predictability is clearly not significantly different on different days of the week.

It thus appears that though there is statistically significant seasonality in the correletion between the returns of the two markets over overlapping periods, there is no seasonality in the effect of one market on *future* returns of the other market.

6. CONCLUSIONS

This chapter has attempted to contribute to the empirical analysis of seasonalities in several new directions. Firstly, utilising the unique features of the settlement procedures on the London Stock Exchange, it has focussed directly on seasonalities in *ex post* risk premia, rather than total returns. Secondly, it has extended earlier day-of-the-week studies on index futures "returns" by examining a new dataset and including a study of intraday seasonalities. Thirdly, besides investigating seasonalities in cash and futures markets separately, it has directly analysed the *differences* in the seasonality behaviour of cash and futures markets by examining the "mispricing" of index futures contracts. Fourthly, it has investigated intraweek and intraday seasonality in extreme price changes. Finally, it has examined *seasonalities* in the correlations between returns in the UK market and the returns in the UK market.

The results of this study suggest that the UK stock market did not appear to efficiently incorporate into prices the *entire* interest costs inherent in its settlement procedures. The market also displayed significant seasonality within a settlement cycle. This seasonality within the settlement cycle carried over to the futures market. It was also found that "abnormal" Monday returns accrued during the trading day on Monday and not over the weekend non-trading interval. This and the examination of extreme price changes suggests that the observed Monday effect cannot be explained by the conjecture that bad news tends to be released more frequently over the weekend non-trading interval than on other days. Furthermore, the divergent behaviour of cash and futures markets in certain periods is interesting in that it cannot apparently be explained by "behavioural" explanantions, unless the two markets are segmented. Some "new" empirical regularities, apparently without an obvious explanation, are also documented - eg the cash market rising when the market is open and the futures market rising when the market is closed. Finally, there is clear evidence in this study of the opening of the US market being associated with a systematic fall in the UK markets. In this context, it appears that though there is statistically significant seasonality in the correletion between the returns of the two markets over overlapping periods, there is no seasonality in the effect of one market on *future* returns of the other market.

The magnitude of the seasonalities observed in the cash market is definitely not enough to enable the formulation of profitable trading rules. However, in view of the small transaction costs in the futures market, some of the seasonal features could be regarded as being potentially economically significant, since trading rules based on them have provided *ex-post* profits after transaction costs. Low marginal cost arbitrageurs could also potentially exploit the *differences* in seasonal patterns in the two markets.

TABLE 1A - CASH RISK PREMIA

The first entry in each cell is the mean *ex post* risk premia. The second entry is the percentage of cases in which *ex post* risk premia was positive.

	Last Mon	Last Tues	Last Wed	Last Thurs	Last Fri	First Mon	First Tues	First Wed	First Thurs	First Fri	All Days
Close-to-Open	0.053	-0.032	0.069	0.021	0.004	0.052	0.084	0.064	0.030	-0.003	-0.005
	66.7**	42.1	57.3	53.9	58.2	60.0	64.4**	59.3	56.2	53.9	54.8**
9am - 10am	-0.060	0.061	0.033	0.060	0.042	0.094*	0.108**	0.050	0.043	0.028	0.045**
	37.7	61.8	52.3	60.7	60.7	67.4**	64.4**	56.0	60.0	51.2	57.1**
10am - 11am	-0.067*	0.058	-0.008	0.016	-0.003	0.024	0.049	0.048	-0.014	0.010	0.011
	34.8*	55.3	50.0	53.9	57.3	54.7	58.9	58.9	42.2	61.6	53.1
11am - 12noon	-0.035	0.059*	-0.054	-0.046	0.001	0.046*	0.002	0.006	0.020	0.042	0.007
	53.6	61.8	51.1	52.8	48.3	65.1**	51.1	51.1	63.3*	61.6*	56.6**
12noon - 1pm	-0.034	0.051*	-0.006	-0.016	-0.017	0.044**	0.023	0.013	0.016	0.025	0.009
	52.2	70.7**	51.1	56.8	47.1	66.3**	61.1	59.3	64.0*	62.8*	59.3**
1pm - 2pm	0.002	0.037*	0.014	-0.024	-0.024	0.014	-0.009	0.026	-0.044	-0.002	-0.001
	52.2	62.7	56.8	52.3	49.4	60.5	48.9	63.7*	56.2	51.8	55.5*
2pm - 3pm	-0.073**	-0.033	-0.034	-0.030	0.019	-0.033	0.002	-0.010	-0.054	-0.027	-0.025**
	37.7*	50.7	46.6	44.9	58.0	48.8	52.2	56.0	51.1	48.2	49.9
3pm - 4pm	-0.085**	-0.049*	-0.005	-0.051*	-0.295	-0.042	-0.001	0.003	-0.004	0.023	-0.005
	39.1*	46.1	51.1	39.3	10.3**	47.7	50.0	57.1	52.3	59.3	52.2
4pm - 5pm	-0.064	-0.029	-0.002	-0.031	0.055*	0.028	0.026	0.076**	0.032	0.060**	0.015
	50.7	46.1	50.0	56.2	65.5**	55.8	57.8	65.9*	67.4**	72.1**	58.7**
Open-to-Close	-0.416**	0.153	-0.056	-0.124	0.240**	0.175*	0.199**	0.211**	-0.010	0.154*	0.056*
	29.0**	64.5*	42.7	43.8	64.0*	61.6	64.4**	68.1**	52.8	60.7	55.4**

* denotes significance at the 5% level. ** denotes significance at the 1% level.

TABLE 1B - P VALUE OF HYPOTHESES TESTS ON INTRAWEEK & INTRA SETTLEMENT PERIOD SEASONALITY IN CASH RISK PREMIA[1]

The main entry is the p-value of the relevant F-test or the T-test. The parentheses contain the p-value of the Kruskal Wallis test or the Mann Whitney test.

	H_1^w	H_2^w	H_3^w	H_4^w	H_5^w	H_6^w	H_7^w	H_8^w	H_9^w
Close-to-Open	0.946	0.755	0.764	0.860	0.999	0.110	0.940	0.890	0.920
	0.743	(0.844)	0.595	(0.650)	0.777	(0.067)	(0.875)	(0.567)	(0.815)
9am - 10am	0.016	0.420	0.242	0.002	0.003	0.310	0.720	0.720	0.780
	(0.009)	(0.353)	(0.231)	(0.002)	(0.003)	(0.340)	(0.405)	(0.742)	(0.310)
10am - 11am	0.014	0.066	0.796	0.010	0.018	0.850	0.110	0.440	0.730
	(0.007)	(0.136)	(0.963)	(0.002)	(0.019)	(0.565)	(0.097)	(0.185)	(0.748)
11am - 12noon	0.166	0.222	0.141	0.343	0.041	0.110	0.076	0.110	0.240
	(0.204)	(0.334)	(0.139)	(0.263)	(0.046)	(0.088)	(0.329)	(0.135)	(0.126)
12noon - 1pm	0.031	0.205	0.091	0.049	0.006	0.270	0.390	0.300	0.190
	(0.154)	(0.337)	(0.130)	(0.173)	(0.043)	(0.349)	(0.472)	(0.609)	(0.056)
1pm - 2pm	0.115	0.074	0.490	0.562	0.470	0.062	0.560	0.680	0.610
	(0.262)	(0.113)	(0.672)	(0.943)	(0.530)	(0.191)	(0.528)	(0.405)	(0.887)
2pm - 3pm	0.140	0.411	0.555	0.022	0.180	0.270	0.380	0.510	0.110
	(0.080)	(0.709)	(0.714)	(0.004)	(0.061)	(0.329)	(0.528)	(0.607)	(0.129)
3pm - 4pm	0.000	0.000	0.868	0.464	0.240	0.210	0.790	0.220	0.000
	(0.000)	(0.000)	0.912	0.734	(0.174)	(0.247)	(0.742)	(0.093)	(0.000)
4pm - 5pm	0.001	0.023	0.815	0.002	0.041	0.220	0.018	0.052	0.860
	(0.012)	(0.028)	(0.926)	(0.019)	(0.178)	(0.072)	(0.032)	(0.062)	(0.625)
Open-to-Close	0.000	0.001	0.284	0.000	0.000	0.690	0.006	0.300	0.420
	(0.000)	(0.003)	(0.359)	(0.000)	(0.000)	(0.670)	(0.001)	(0.242)	(0.148)

TABLE 1C - P VALUE OF HYPOTHESES TESTS ON INTRADAY SEASONALITY IN CASH RISK PREMIA

The main entry is the p-value of the F-test. The parentheses contain the p-value of the Kruskal-Wallis test.

	H_1^D	H_2^D Hour 1	H_2^D Hour 2	H_2^D Hour 3	H_2^D Hour 4	H_2^D Hour 5	H_2^D Hour 6	H_2^D Hour 7	H_2^D Hour 8
First Mondays	0.001	0.001	0.915	0.282	0.324	0.719	0.015	0.004	0.795
	(0.005)	(0.004)	(0.964)	(0.127)	(0.363)	(0.378)	(0.025)	(0.014)	(0.767)
Other Mondays	0.245	0.517	0.577	0.313	0.589	0.016	0.278	0.370	0.444
	(0.042)	(0.202)	(0.247)	(0.238)	(0.211)	(0.021)	(0.054)	(0.275)	(0.475)
Tuesdays	0.000	0.000	0.021	0.656	0.472	0.469	0.028	0.005	0.153
	(0.000)	(0.000)	(0.019)	(0.388)	(0.392)	(0.355)	(0.009)	(0.001)	(0.413)
Wednesdays	0.032	0.019	0.890	0.119	0.557	0.410	0.040	0.341	0.066
	(0.067)	(0.050)	(0.553)	(0.327)	(0.486)	(0.559)	(0.031)	(0.268)	(0.071)
Thursdays	0.052	0.002	0.681	0.902	0.706	0.112	0.123	0.323	0.732
	(0.008)	(0.001)	(0.848)	(0.326)	(0.705)	(0.199)	(0.058)	(0.041)	(0.440)
Fridays	0.000	0.047	0.857	0.183	0.589	0.780	0.928	0.000	0.001
	(0.000)	(0.206)	(0.283)	(0.149)	(0.790)	(0.193)	(0.731)	(0.000)	(0.000)
All Days	0.000	0.000	0.224	0.475	0.343	0.742	0.000	0.000	0.079
	(0.000)	(0.000)	(0.208)	(0.054)	(0.312)	(0.340)	(0.000)	(0.000)	(0.002)

The hypotheses tested are as follows :

H_1^D : The mean hourly *ex post* risk premia are equal across all hours during the day

H_2^D : The mean hourly *ex post* risk premium during each of the one-hour periods is equal to the mean hourly *ex post* risk premia over the other hourly periods of the day

TABLE 1D - PERCENTAGE FREQUENCY OF DIFFERENT DAYS OF THE WEEK IN TOP AND BOTTOM DECILES OF CASH RISK PREMIA[1]

	First Monday		Other Monday		Tuesday		Wednesday		Thursday		Friday		χ^2	
	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
Close-to-Open	15.2	14.1	6.5	8.7	12.0	17.4	28.3	16.3	18.5	20.7	19.6	22.8	8.9	3.7
9 am-10 am	15.2	6.5	7.6	12.0	22.8	18.5	17.4	20.6	16.3	25.0	20.6	17.4	5.6	3.2
10 am-11 am	10.9	9.8	1.1	12.0	22.8	14.1	23.9	22.8	29.4	21.7	12.0	19.6	13.9*	2.6
11 am-12 noon	10.9	5.4	8.7	16.3	19.6	12.0	15.2	23.9	21.7	23.9	23.9	18.5	2.3	11.1*
12 noon-1 pm	10.9	5.4	2.2	10.9	20.6	12.0	15.2	22.8	27.2	28.3	23.9	20.6	8.7	7.2
1 pm-2 pm	8.7	5.4	6.5	4.3	18.5	12.0	19.6	18.5	25.0	32.6	21.7	27.2	1.5	14.6*
2 pm-3 pm	9.8	12.0	3.3	12.0	21.7	19.6	16.3	18.5	21.7	23.9	27.2	14.1	6.5	4.4
3 pm-4 pm	12.0	8.7	5.4	6.5	19.6	13.0	21.7	7.6	26.1	18.5	15.2	45.7	4.3	38.9**
4 pm-5 pm	10.9	7.6	4.3	14.1	22.8	22.8	29.4	20.6	16.3	26.1	16.3	8.7	7.4	11.6*
Open-to-Close	12.0	8.7	1.1	21.7	25.0	14.1	21.7	15.2	16.3	27.2	23.9	13.0	9.8	25.2**

TABLE 1E - PERCENTAGE FREQUENCY OF DIFFERENT HOURS OF THE DAY IN TOP AND BOTTOM DECILES OF CASH RISK PREMIA[3]

	First Monday		Other Monday		Tuesday		Wednesday		Thursday		Friday		All Days	
	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
9 am-10 am	37.7	21.7	26.6	21.9	31.7	20.4	31.4	23.1	32.0	19.0	27.2	17.2	30.2	21.4
10 am-11 am	15.9	17.4	12.5	12.5	19.7	10.6	18.0	15.4	19.6	12.4	17.2	11.9	16.8	14.0
11 am-12 noon	14.5	8.7	21.9	18.8	12.7	11.3	10.9	16.0	13.7	12.4	17.9	10.6	14.4	13.6
12 noon-1 pm	5.8	1.5	3.1	10.9	4.2	4.2	5.1	7.1	5.9	9.1	5.3	5.3	4.8	6.7
1 pm-2 pm	1.5	1.5	6.3	1.6	2.8	4.9	5.8	4.5	3.9	7.8	4.6	9.9	3.8	5.9
2 pm-3 pm	5.8	17.4	4.7	9.4	7.8	14.1	5.8	10.9	5.9	12.4	9.3	7.3	6.9	12.4
3 pm-4 pm	8.7	21.7	12.5	10.9	6.3	19.7	5.8	13.5	11.1	17.0	8.6	35.1	11.3	15.1
4 pm-5 pm	10.1	10.1	12.5	14.1	14.8	14.8	17.3	9.6	7.8	9.8	9.9	2.7	11.7	10.1
χ^2	48.8**	26.7**	24.3**	13.5	73.9**	28.9**	75.3**	29.7**	75.8**	12.5	49.7**	86.8**	296.3**	99.8**

TABLE 2A - FUTURES RISK PREMIA

The first entry in each cell is the mean *ex post* risk premia. The second entry is the percentage of cases in which *ex post* risk premia was positive.

	Last Mon	Last Tues	Last Wed	Last Thurs	Last Fri	First Mon	First Tues	First Wed	First Thurs	First Fri	All Days
Close-to-Open	0.054	-0.009	0.075	0.042	0.067	-0.085	0.212**	0.145**	0.091	0.068	0.069**
	62.7	46.6	54.0	54.7	63.1*	48.0	64.7**	57.8	57.5	52.4	56.6**
9am - 10am	-0.063	0.036	0.040	0.032	0.042	0.147**	0.003	0.032	0.002	-0.047	0.023
	48.5	55.4	47.1	60.0	56.3	64.3*	52.8	55.6	49.4	46.6	53.3
10am - 11am	-0.054	0.023	-0.008	0.014	0.000	0.038	0.047	0.030	-0.018	0.009	0.009
	50.0	50.0	52.3	56.5	55.3	63.1	62.9*	52.2	48.3	48.2	54.3
11am - 12noon	-0.020	0.060	-0.024	-0.057	-0.027	0.038	0.016	0.000	0.004	0.036	0.006**
	52.9	62.2	54.7	56.5	48.8	60.7	56.8	53.9	57.5	56.5	56.5
12noon - 1pm	-0.049	0.082**	0.021	-0.023	-0.014	0.055*	-0.003	0.014	0.004	-0.011	0.006*
	48.5	66.2*	61.6	63.1	56.3	62.2	51.2	64.0	53.5	52.4	58.1
1pm - 2pm	-0.007	0.009	-0.002	-0.021	0.026	0.014	-0.017	0.024	-0.034	0.031	0.002
	59.1	52.1	55.8	52.4	53.3	64.2	50.0	58.6	54.8	53.7	55.5
2pm - 3pm	-0.062*	-0.045	-0.053	-0.021	0.030	-0.071*	-0.011	-0.034	-0.054	-0.008	-0.030**
	36.4**	53.5	47.7	45.2	57.1	34.6**	47.7	46.0	48.2	57.3	47.7**
3pm - 4pm	-0.120**	-0.039	0.004	-0.063*	-0.033	-0.024	-0.019	0.012	0.002	-0.018	-0.028**
	35.3**	44.6	47.7	42.9	53.2	50.6	50.6	52.8	48.8	50.6	47.8*
Open-to-Close	-0.376**	0.128	-0.017	-0.139	0.056	0.200*	0.028	0.089	-0.074	-0.009	-0.004
	32.4**	52.7	50.6	45.2	56.1	58.0	55.2	58.4	49.4	52.3	51.4

* denotes significance at the 5% level. ** denotes significance at the 1% level.

TABLE 2B

P VALUE OF HYPOTHESES TESTS ON INTRAWEEK & INTRA SETTLEMENT PERIOD SEASONALITY IN FUTURES RISK PREMIA[a]

The main entry is the p-value of the F-test or the T-test. The parentheses contain the p-value of the Kruskal Wallis test or the Mann Whitney test.

	H_1^w	H_2^w	H_3^w	H_4^w	H_5^w	H_6^w	H_7^w	H_8^w	H_9^w
Close-to-Open	0.324	0.913	0.021	0.579	0.250	0.035	0.420	0.590	0.980
	(0.648)	(0.996)	(0.076)	(0.910)	(0.154)	(0.046)	(0.305)	(0.384)	(0.806)
9am - 10am	0.025	0.843	0.004	0.094	0.005	0.580	0.890	0.570	0.094
	(0.163)	(0.836)	(0.030)	(0.170)	(0.019)	(0.588)	(0.885)	(0.280)	(0.079)
10am - 11am	0.216	0.397	0.415	0.079	0.046	0.620	0.350	0.440	0.850
	(0.246)	(0.417)	(0.274)	(0.130)	(0.054)	(0.341)	(0.406)	(0.241)	(0.757)
11am - 12noon	0.295	0.199	0.265	0.861	0.170	0.320	0.480	0.240	0.110
	(0.642)	(0.523)	(0.293)	(0.964)	(0.265)	(0.187)	(0.844)	(0.522)	(0.180)
12noon - 1pm	0.014	0.208	0.052	0.028	0.004	0.008	0.830	0.510	0.940
	(0.213)	(0.596)	(0.257)	(0.067)	(0.037)	(0.023)	(0.991)	(0.336)	(0.757)
1pm - 2pm	0.422	0.254	0.715	0.862	0.330	0.530	0.420	0.800	0.930
	(0.797)	(0.752)	(0.419)	(0.656)	(0.145)	(0.919)	(0.398)	(0.431)	(0.611)
2pm - 3pm	0.164	0.295	0.098	0.185	0.830	0.410	0.620	0.410	0.340
	(0.005)	(0.104)	(0.011)	(0.022)	(0.713)	(0.999)	(0.814)	(0.999)	(0.459)
3pm - 4pm	0.197	0.635	0.909	0.017	0.053	0.670	0.850	0.180	0.730
	(0.195)	(0.652)	(0.628)	(0.021)	(0.026)	(0.469)	(0.704)	(0.219)	(0.796)
Open-to-Close	0.000	0.109	0.030	0.000	0.000	0.410	0.270	0.580	0.590
	(0.000)	(0.170)	(0.099)	(0.000)	(0.000)	(0.661)	(0.298)	(0.519)	(0.225)

TABLE 2C - P VALUE OF HYPOTHESIS TESTS ON INTRADAY SEASONALITY IN FUTURES RISK PREMIA

The main entry is the p-value of the F-test. The parentheses contain the p-value of the Kruskal-Wallis test.

	H_1^D	H_2^D Hour 1	H_2^D Hour 2	H_2^D Hour 3	H_2^D Hour 4	H_2^D Hour 5	H_2^D Hour 6	H_2^D Hour 7
First Mondays	0.000	0.000	0.760	0.758	0.406	0.641	0.001	0.087
	(0.001)	(0.009)	(0.377)	(0.469)	(0.417)	(0.836)	(0.000)	(0.418)
Other Mondays	0.255	0.702	0.968	0.114	0.899	0.139	0.617	0.046
	(0.111)	(0.871)	(0.983)	(0.062)	(0.743)	(0.136)	(0.121)	(0.051)
Tuesdays	0.058	0.645	0.104	0.184	0.250	0.505	0.081	0.029
	(0.053)	(0.658)	(0.128)	(0.132)	(0.242)	(0.473)	(0.057)	(0.040)
Wednesdays	0.283	0.101	0.969	0.905	0.760	0.621	0.018	0.969
	(0.349)	(0.310)	(0.875)	(0.776)	(0.571)	(0.650)	(0.013)	(0.983)
Thursdays	0.733	0.142	0.591	0.665	0.674	0.431	0.546	0.532
	(0.523)	(0.240)	(0.974)	(0.708)	(0.267)	(0.854)	(0.245)	(0.171)
Fridays	0.762	0.898	0.889	0.920	0.588	0.161	0.716	0.222
	(0.899)	(0.936)	(0.872)	(0.893)	(0.714)	(0.637)	(0.293)	(0.304)
All Days	0.001	0.007	0.249	0.391	0.411	0.656	0.002	0.003
	(0.000)	(0.056)	(0.331)	(0.067)	(0.134)	(0.702)	(0.000)	(0.004)

The hypotheses tested are as follows:

H_1^D : The mean hourly *ex post* risk premia are equal across all hours during the day

H_2^D : The mean hourly *ex post* risk premium during each of the one-hour periods is equal to the mean hourly *ex post* risk premia over the other hourly periods of the day

TABLE 2D - PERCENTAGE FREQUENCY OF DIFFERENT DAYS OF THE WEEK IN TOP AND BOTTOM DECILES OF FUTURES RISK PREMIA[a]

	First Monday		Other Monday		Tuesday		Wednesday		Thursday		Friday		χ^2	
	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
Close-to-Open	6.5	12.0	7.6	7.6	18.5	18.5	27.2	18.5	18.5	19.6	21.7	23.9	2.8	1.7
9 am-10 am	18.5	8.7	9.8	15.2	19.6	13.0	23.9	20.6	13.0	18.5	15.2	23.9	12.6*	7.1
10 am-11 am	10.9	7.6	6.5	10.9	23.9	12.0	18.5	22.8	21.7	23.9	18.5	22.8	2.3	4.1
11 am-12 noon	12.0	9.8	9.8	10.9	22.8	10.9	16.3	23.9	19.6	29.4	19.6	15.2	2.6	8.7
12 noon-1 pm	12.0	3.3	3.3	12.0	22.8	19.6	21.7	20.6	28.3	22.8	12.0	21.7	10.2	5.0
1 pm-2 pm	2.2	5.4	3.3	3.3	22.8	20.6	20.6	22.8	21.7	26.1	29.4	21.7	12.3*	6.2
2 pm-3 pm	10.9	14.1	5.4	7.6	16.3	19.6	16.3	21.7	19.6	18.5	31.5	18.5	8.3	2.8
3 pm-4 pm	12.0	8.7	4.3	12.0	20.6	17.4	22.8	21.7	22.8	20.6	17.4	19.6	3.5	1.4
Open-to-Close	17.4	10.9	0.0	16.3	23.9	15.2	19.6	17.4	19.6	20.6	19.6	19.6	15.6**	7.8

TABLE 2E - PERCENTAGE FREQUENCY OF DIFFERENT HOURS OF THE DAY IN TOP AND BOTTOM DECILES OF FUTURES RISK PREMIA[a]

	First Monday		Other Monday		Tuesday		Wednesday		Thursday		Friday		All Days	
	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
9 am-10 am	40.0	20.0	33.9	33.9	28.2	29.0	35.8	22.6	25.4	22.4	31.1	28.0	30.8	26.3
10 am-11 am	13.3	16.7	21.4	17.9	20.2	12.9	14.6	18.3	18.7	16.4	13.6	15.9	17.1	16.3
11 am-12 noon	11.7	13.3	17.9	10.7	15.3	6.4	10.9	12.4	9.7	15.7	11.4	9.1	12.1	11.7
12 noon-1 pm	8.3	1.7	5.4	8.9	7.3	5.7	8.8	8.8	9.0	9.7	4.6	6.8	7.5	7.5
1 pm-2 pm	1.7	5.0	5.4	3.6	6.4	12.1	8.8	8.0	11.2	11.2	13.6	8.3	88.9	8.1
2 pm-3 pm	11.7	21.7	7.1	5.4	8.9	15.3	7.3	12.4	10.4	8.2	14.4	11.4	10.3	11.8
3pm-4 pm	13.3	21.7	8.9	19.6	13.7	18.5	13.9	17.5	15.7	16.4	11.4	20.5	13.4	18.4
χ^2	36.5**	16.5*	27.0**	26.0**	32.2**	33.0**	56.1**	16.6*	20.5**	13.6*	36.3**	32.9**	170.2**	118.2**

TABLE 3A - MISPRICING RETURNS

The first entry in each cell is the mean *ex post* risk premia. The second entry is the percentage of cases in which *ex post* risk premia was positive.

	Last Mon	Last Tues	Last Wed	Last Thurs	Last Fri	First Mon	First Tues	First Wed	First Thurs	First Fri	All Days
Close-to-Open	-0.042	0.110**	0.015	0.004	0.053	0.209**	0.085**	0.048	-0.020	0.036	0.050**
	41.8	66.2**	48.9	47.7	54.7	21.3**	55.3	59.1	51.7	54.8	56.1**
9am - 10am	-0.002	-0.032	0.003	-0.020	-0.019	0.043	-0.102**	-0.023	-0.050	-0.063*	-0.025**
	56.5	46.7	51.1	48.3	50.6	52.4	38.2*	43.3	42.5	45.9	47.5
10am - 11am	0.013	-0.037	0.000	0.001	0.008	0.010	-0.004	-0.023	-0.008	0.000	-0.003
	56.5	42.7	43.2	52.9	46.0	47.6	47.2	40.5	48.3	43.5	46.7
11am - 12noon	0.016	0.007	0.022	-0.015	-0.017	-0.007	0.014	-0.009	-0.010	-0.005	0.000
	55.1	46.7	53.4	44.8	45.4	47.6	54.5	46.1	44.8	44.7	48.5
12noon - 1pm	-0.011	0.033*	0.027*	0.004	0.013	0.010	-0.022	0.001	-0.010	-0.035*	0.000
	42.0	61.6	54.5	54.7	55.0	45.1	43.0	49.4	37.7*	39.3	48.4
1pm - 2pm	-0.014	-0.031	-0.018	0.010	0.047*	0.002	-0.007	-0.001	0.014	0.029	0.003
	43.3	39.4	42.1	59.3	59.0	50.6	47.7	55.2	50.0	50.6	50.3
2pm - 3pm	0.012	-0.010	-0.025	0.008	0.008	-0.036	-0.014	-0.018	-0.011	0.017	-0.006
	49.3	46.5	45.4	52.3	46.1	42.0	44.2	42.5	51.2	53.7	47.6
3pm - 4pm	-0.034	0.008	0.017	-0.025	-0.261**	0.021	-0.021	0.006	0.013	-0.040	-0.023**
	40.6	47.3	54.5	46.5	86.3**	53.1	43.7	53.9	52.4	44.7	45.4
Open-to-Close	-0.024	-0.069	0.025	-0.036	-0.303**	0.047	-0.142**	-0.062	-0.056	-0.094*	-0.052**
	47.8	50.0	47.7	40.7	80.2**	46.9	39.1	43.8	38.8	31.8**	42.4**

* denotes significance at the 5% level. ** denotes significance at the 1% level.

TABLE 3B - P VALUE OF HYPOTHESES TESTS ON INTRAWEEK & INTRA SETTLEMENT PERIOD SEASONALITY IN MISPRICING RETURNS'

The main entry is the p-value of the F-test or the T-test. The parentheses contain the p-value of the Kruskal Wallis test or the Mann Whitney test.

	H_1^w	H_2^w	H_3^w	H_4^w	H_5^w	H_6^w	H_7^w	H_8^w	H_9^w
Close-to-Open	0.000	0.026	0.000	0.051	0.003	0.610	0.510	0.640	0.720
	(0.000)	(0.011)	(0.000)	(0.045)	(0.003)	(0.354)	(0.336)	(0.776)	(0.721)
9am - 10am	0.089	0.368	0.018	0.259	0.360	0.110	0.550	0.530	0.230
	(0.269)	(0.439)	(0.197)	(0.115)	(0.863)	(0.165)	(0.430)	(0.504)	(0.476)
10am - 11am	0.694	0.564	0.471	0.436	0.910	0.260	0.410	0.710	0.790
	(0.794)	(0.665)	(0.701)	(0.380)	(0.696)	(0.481)	(0.355)	(0.357)	(0.845)
11am - 12noon	0.481	0.327	0.763	0.307	0.450	0.790	0.170	0.870	0.640
	(0.507)	(0.352)	(0.787)	(0.316)	(0.478)	(0.636)	(0.263)	(0.938)	(0.991)
12noon - 1pm	0.704	0.600	0.507	0.452	0.350	0.003	0.110	0.530	0.059
	(0.964)	(0.880)	(0.841)	(0.605)	(0.775)	(0.005)	(0.184)	(0.334)	(0.016)
1pm - 2pm	0.018	0.013	0.813	0.189	0.370	0.320	0.450	0.810	0.570
	(0.142)	(0.089)	(0.826)	(0.226)	(0.228)	(0.244)	(0.181)	(0.780)	(0.280)
2pm - 3pm	0.330	0.468	0.156	0.359	0.100	0.890	0.820	0.460	0.770
	(0.265)	(0.276)	(0.128)	(0.719)	(0.216)	(0.559)	(0.971)	(0.488)	(0.593)
3pm - 4pm	0.000	0.000	0.917	0.044	0.110	0.500	0.740	0.210	0.000
	(0.000)	(0.000)	(0.072)	(0.273)	(0.035)	(0.820)	(0.812)	(0.184)	(0.000)
Open-to-Close	0.000	0.000	0.169	0.783	0.240	0.230	0.100	0.710	0.000
	(0.019)	(0.023)	(0.057)	(0.450)	(0.382)	(0.343)	(0.108)	(0.799)	(0.000)

TABLE 3C - P VALUE OF HYPOTHESES TESTS ON INTRADAY SEASONALITY IN MISPRICING RETURNS

The main entry is the p-value of the F-test. The parentheses contain the p-value of the Kruskal-Wallis test.

	H_1^D	H_2^D Hour 1	H_2^D Hour 2	H_2^D Hour 3	H_2^D Hour 4	H_2^D Hour 5	H_2^D Hour 6	H_2^D Hour 7
First Mondays	0.373	0.089	0.863	0.542	0.858	0.836	0.054	0.499
	(0.583)	(0.918)	(0.944)	(0.802)	(0.831)	(0.606)	(0.059)	(0.217)
Other Mondays	0.454	0.888	0.454	0.232	0.600	0.409	0.345	0.093
	(0.279)	(0.348)	(0.481)	(0.205)	(0.728)	(0.450)	(0.687)	(0.027)
Tuesdays	0.015	0.000	0.760	0.056	0.261	0.875	0.729	0.686
	(0.053)	(0.003)	(0.589)	(0.068)	(0.152)	(0.922)	(0.937)	(0.784)
Wednesdays	0.563	0.387	0.692	0.311	0.362	0.665	0.201	0.299
	(0.264)	(0.214)	(0.353)	(0.208)	(0.256)	(0.897)	(0.162)	(0.194)
Thursdays	0.481	0.042	0.788	0.636	0.673	0.213	0.601	0.938
	(0.649)	(0.104)	(0.902)	(0.511)	(0.873)	(0.230)	(0.688)	(0.665)
Fridays	0.000	0.001	0.705	0.079	0.108	0.146	0.823	0.000
	(0.000)	(0.006)	(0.450)	(0.046)	(0.195)	(0.273)	(0.837)	(0.000)
All Days	0.001	0.000	0.863	0.709	0.804	0.413	0.493	0.001
	(0.018)	(0.000)	(0.467)	(0.626)	(0.544)	(0.392)	(0.415)	(0.002)

The hypotheses tested are as follows:

H_1^D : The mean hourly *ex post* risk premia are equal across all hours during the day

H_2^D : The mean hourly *ex post* risk premium during each of the one-hour periods is equal to the mean hourly *ex post* risk premia over the other hourly periods of the day

TABLE 3D - PERCENTAGE FREQUENCY OF DIFFERENT DAYS OF THE WEEK IN TOP AND BOTTOM DECILES OF MISPRICING RETURNS[a]

	First Monday		Other Monday		Tuesday		Wednesday		Thursday		Friday		χ^2	
	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
Close-to-Open	2.2	21.5	7.5	9.7	28.0	8.6	20.4	20.4	17.2	26.9	24.7	12.9	10.3	25.0**
9 am-10 am	14.0	9.7	9.7	7.5	16.1	24.7	26.9	19.4	20.4	18.3	12.9	20.4	6.8	2.1
10 am-11 am	12.9	8.6	9.7	8.6	17.2	24.7	18.3	18.3	12.9	16.1	30.1	23.7	8.9	3.3
11 am-12 noon	10.8	11.8	9.7	8.6	21.5	16.1	18.3	16.1	23.7	24.7	16.1	22.6	2.2	3.2
12 noon-1 pm	10.8	8.6	7.5	14.0	17.2	19.4	22.6	12.9	23.7	21.5	18.3	23.7	1.3	6.4
1 pm-2 pm	5.4	4.3	6.4	8.6	17.2	24.7	18.3	28.0	19.4	18.3	33.3	16.1	10.2	7.0
2 pm-3 pm	6.4	10.8	10.8	5.4	15.1	22.6	23.7	24.7	19.4	18.3	24.7	18.3	3.2	3.1
3 pm-4 pm	6.5	9.7	5.4	12.9	12.9	17.2	16.1	19.4	10.8	23.7	48.4	17.2	44.8**	3.0
Open-to-Close	12.9	8.6	6.5	6.5	10.8	28.0	14.0	21.5	15.0	19.3	40.9	16.1	28.0**	5.3

TABLE 3E - PERCENTAGE FREQUENCY OF DIFFERENT HOURS OF THE DAY IN TOP AND BOTTOM DECILES OF MISPRICING RETURNS[a]

	First Monday		Other Monday		Tuesday		Wednesday		Thursday		Friday		All Days	
	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
9 am-10 am	30.0	26.7	24.6	24.6	24.8	37.6	25.9	30.9	22.8	30.9	11.9	32.1	22.6	31.6
10 am-11 am	20.0	15.0	19.3	14.0	16.8	16.8	13.7	14.4	13.2	13.2	18.7	16.4	15.7	15.1
11 am-12 noon	11.7	15.0	12.3	12.3	16.0	8.8	9.4	9.4	16.2	14.7	7.5	11.9	12.1	11.2
12 noon-1 pm	8.3	5.0	8.8	14.0	8.0	3.2	7.9	3.6	8.8	5.9	5.2	9.0	7.2	6.5
1 pm-2 pm	1.7	3.3	5.3	5.3	7.2	7.2	7.2	10.8	8.1	5.9	10.5	5.2	7.2	6.8
2 pm-3 pm	10.0	20.0	15.8	8.8	11.2	13.6	15.1	16.5	13.2	12.5	9.7	12.7	12.7	13.8
3pm-4 pm	18.3	15.0	14.0	21.0	16.0	12.8	20.9	14.4	17.6	16.9	36.0	12.7	22.4	15.1
χ^2	21.6**	16.5*	9.9	10.6	19.4**	66.0**	28.7**	42.3*	15.0*	40.8**	64.3**	41.5**	111.1**	196.0**

TABLE 4 - SEASONALITY IN CORRELATIONS BETWEEN US AND UK MARKETS

$$ {}_2R_{t*}^{q} = \sum_{j=2}^{2} \gamma_j R_{t*}^{q}[-j] + \sum_{j=2}^{5} \alpha_j D_{jt*} + \alpha_{FM} D_{FM,t*} + \alpha_{OM} D_{OM,t*} + \sum_{j=2}^{5} \beta_j^P D_{jt-1} R_t^P + \beta_{FM}^P D_{M,t-1} R_t^P + \beta_{OM}^P D_{OM,t-1} R_t^P + \epsilon_{t*} $$

	β_{FM}^{+}	β_{OM}^{+}	β_2^{+}	β_3^{+}	β_4^{+}	β_5^{+}	$F(H_0^S)$	$F(H_1^S)$	$R^2_{adj}(\%)$
US returns from previous close to open predicted by UK returns from open to 2.30 pm	0.135 (0.064)	0.336 (0.079)	0.232 (0.043)	0.224 (0.054)	0.209 (0.045)	0.490 (0.045)	185.93 (0.000)	6.37 (0.000)	19.3
US returns from open to close predicted by UK returns from open to 2.30 pm	0.315 (0.145)	0.263 (0.182)	0.092 (0.101)	0.174 (0.122)	0.160 (0.103)	0.156 (0.111)	44.22 (0.000)	0.39 (0.857)	1.1
UK returns from previous close to open predicted by previous day US returns from open to close.	0.306 (0.100)	0.755 (0.163)	0.119 (0.088)	0.307 (0.094)	0.266 (0.107)	0.224 (0.098)	48.93 (0.000)	2.47 (0.031)	7.4
UK returns from 2.30 pm to 3 pm predicted by US returns from previous close to open	-0.033 (0.042)	-0.106 (0.042)	-0.046 (0.038)	-0.106 (0.039)	-0.046 (0.038)	-0.056 (0.025)	49.22 (0.000)	0.67 (0.647)	7.0

H_0^S : $\beta_{FM} = \beta_{OM} = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$

H_1^S : $\beta_{FM} = \beta_{OM} = \beta_2 = \beta_3 = \beta_4 = \beta_5$

\+ Standard errors in parenthesis
* p-value of F-statistic in parenthesis

APPENDIX 1

INTRAWEEK AND INTRADAY SEASONALITY IN STOCK MARKETS: CASH + FUTURES - SUMMARY OF EVIDENCE

Fields (1931)	DJIA 1915-1930 Daily	1.	Closing prices on Saturdays significantly higher than the average of the closing prices on Fridays and Mondays.
Cross (1973)	S & P 500 1953-1970 Daily	1.	Positive Friday returns.
		2.	Negative Monday returns.
French (1980)	S & P Composite 1953-1978 Daily	1.	Negative Monday returns.
		2.	Weekend effect is not a general holiday effect.
		3.	"Trading time" and "calendar time" hypothesis are both rejected.
Gibbons and Hess (1981)	S&P 500 1962-1978 Daily	1.	Negative Monday returns.
	CRSP value weighted 1962-1978 Daily	2.	Friday returns larger for small firms than large firms.
	CRSP equally weighted 1962-1978 Daily	3.	"Measurement Error" hypothesis rejected.
	30 DJIA firms 1962-1978 Daily	4.	"Settlement Effect" hypothesis rejected.
	T-Bills 1962-1968 Daily	5.	Heteroskedasticity correction irrelevant.
		6.	Seasonality in "Market Adjusted" security returns.
		7.	No variance seasonality.
Lakonishok and Levi (1982)	CRSP value weighted 1962-1979 Daily	1.	Settlement procedures and check clearing delays provide only a partial explanation.
Lakonishok and Levi (1985)	CRSP equally weighted	2.	Negative Monday effect has disappeared in 1974-79 sub period.
Dyl and Martin (1985)			
Theobald and Price (1984)	UK FT Ordinary 1975-1981 Daily (Geometrically averaged index of 30 leading stocks)	1.	Seasonality in mean returns stronger for the more regularly traded index.
		2.	Small variance seasonality.
		3.	Effect of UK settlement procedures highlighted.
	UK FT Actuaries All Share 1975-1981 Daily (Value weighted broad based)	4.	Seasonality in trading volume.
Keim and Stambaugh (1984)	S & P Composite 1928-1982 Daily	1.	Negative Monday returns over "one day" weekends similar to negative returns over "two day" weekends.
	All NYSE/AMEX firms in CRSP 1963-1979 Daily	2.	Last trading day in the week tends to have higher return - Friday's return is significantly larger if Friday is last trading day.
		3.	Monday returns negative irrespective of size and small firms have greater tendency for high Friday returns.
		4.	"Measurement Error" hypothesis inconsistent with results.
		5.	Specialist related biases unlikely to be relevant.

		6. Settlement related effects unlikely to be important.
Rogalski (1984)	DJIA 1974-1984 Daily (Open & Close) SP500 1978-1983 Daily (Open & Close) All NYSE/AMEX firms in CRSP 1963-1982 Daily	1. Close to Close negative Monday returns accrue during the non trading weekend Friday Close to Monday Open. 2. Trading day returns do not exhibit Monday seasonality. 3. Monday and non trading weekend effects are not present in January. 4. January/turn of the year effects dominate the Monday effect.
Cornell (1985b)	SP500: Spot, Futures and Basis May 1982-July 1984 Daily (Open + Close)	1. No intraweek seasonality in futures price. 2. Intraweek seasonality in basis to reflect cash market seasonality.
Jaffe and Westerfield (1985)	Japan : Nikkei Dow 1970-1983 Daily Canada : Toronto SE 1976-1983 Daily Australia : Statex Actuaries 1973-1982 Daily UK : FT Ordinary 1950-1983 Daily US : SP500 1962-1983 Daily	1. Intraweek seasonality exists in every market even after allowing for US seasonality. 2. Japan and Australia have strongly negative Tuesday returns not satisfactorily explained by time zone differences. 3. "Measurement Error" or settlement related hypothesis unlikely. 4. Seasonality in foreign exchange market is not an explanation.
Wood, McInish and Ord (1985)	Francis Emery Fitch Co transactions database Minute by minute data on all NYSE stocks. Six months 1971-1972 and 12 months 1982.	1. Large positive returns in the first 30 minutes. 2. Positive end of day returns. 3. After excluding first 30 minutes and closing minutes, market returns are normally distributed and have substantially reduced autocorrelation. 4. Several patterns in trading frequency, number of shares per trade, size of price changes, length of time between trades, and absolute value of price changes.
Dyl and Maberly (1986a)	SP500 futures My 1982-December 1985 Daily (Open + Close)	1. Error in data and results of Cornell (1985). 2. Negative non-trading weekend returns like Rogalski (1984).
Dyl and Maberly (1986b)	SP500 Futures 1982-1985 36 months Daily (Absolute Price Changes)	1. No seasonality in close-to-close price changes but marked seasonality in close-to-open price changes with significant average negative Friday close Monday open price change and related mild seasonality in open to close price changes. 2. Large positive price changes are distributed uniformly over the weekend, but the frequency of large negative price changes is significantly higher for the weekend.
Harris (1986)	Transactions data for all NYSE stocks for 296 trading days December 1981-January 1983	1. For large firms, negative Monday close to close return accrues before the market opens, while for smaller firms most of it accrues during trading on Monday. 2. Significant intraweek seasonality 15 minute returns exists only during the first 45 minutes of trading.

		3. There is significant intraday seasonality. Mean returns are much higher in magnitude during the opening and closing periods. There is also rise of prices between 12.30 and 1.30 and a fall between 2.30 and 3.15. 4. Systematic data errors or deliberate price manipulation for high Friday close prices are unlikely.
Junkus (1986)	Spot and Futures on Value line 25 months 1982-84 Daily NYSE 23 months 1982-84 Daily SP500 23 months 1982-84 Daily	1. Hypothesis of no intraweek seasonality not rejected for either cash or futures, even within the two one year sub periods. 2. Calendar time hypothesis rejected but trading time hypothesis not rejected.
Maberly (1986)	Value Line Futures 46 months 1982-85 (Open + Close) Daily	1. Friday-Close-to-Monday-Open price changes significantly negative and different from other non trading periods.
Smirlock and Starks (1986)	DJIA 1963-1983 Hourly	1. Pattern of intraweek and intraday seasonality has been changing with time. 2. In particular, prior to 1974, close to close Monday return returns arose primarily due to negative trading period returns on Monday. 3. Trading time hypothesis rejected prior to 1974.
Condoyanni, O'Hanlon and Ward (1987)	US DJIA 1969-1984 Daily UK FT All-Share 1969-1984 Daily Toronto Composite 1969-1984 Daily Paris CAC Industrial 1969-1984 Daily Straits Times (Singapore) 1969-1984 Daily Tokyo New SE 1969-1984 Daily Australia SE All Ordinaries 1981-1984 Daily	1. Significantly negative Monday returns in US, UK and Canada and significantly Tuesday returns in markets whose index is calculated before US opens - France, Singapore, Japan and Australia. 2. Stability of results across sub periods. 3. January factor does not alter intraweek seasonality.
Keim and Smirlock (1987)	S&P500 Futures 1982-1984 Daily (Open + Close) Value Line Futures 1982-1984 Daily (Open + Close) S&P500 Cash 1982-1984 Daily Close	1. S&P500 futures rise during the trading period but the value line futures rise during the non-trading period. 2. Day of the week patterns of the stock market carry over to the futures market. 3. Prices in futures markets do not rise on Fridays as observed for cash markets.
Board and Sutcliffe (1988)	UK FT All Share 1962-1986 Daily	1. Investigation and results essentially similar to French (1980). 2. Settlement procedure provide only partial explanation for results. 3. Friday (Monday) prices not biased upwards (downwards).
Flannery and Protopapadakis (1988)	CRSP Value weighted 1977-1984 Daily CRSP Equally weighted 1977-1984 Daily SP500 1977-1984 Daily Seven Treasury securities 1988-1984 Daily	1. Clearing and payment conventions provide only a partial explanation for intraweek seasonality. 2. Intraweek seasonality differs significantly across different stock indices and across different Treasury securities.

	Overnight repurchase agreements 1977-1984 Daily	3.	Negative Monday returns differ significantly with time to maturity of Treasury securities.
Lakonishok and Smidt (1988)	DJIA 1897-1986 Daily	1.	Consistently negative Monday average returns.
		2.	Consistently large and positive average returns on the last trading day of the week.
Phillips-Patrick and Schneeweis (1988)	CRSP Index 1982-85 Daily SP500 futures Daily	1.	Dividend distribution adjustment is a partial explanation for the weekend effect in stock indices.
		2.	Interest rate adjustment is a partial explanation for the weekend effect in index futures.
Choy and O'Hanlon (1989)	Two separate four months periods covering 375 and 290 UK listed companies respectively.	1.	Day of the week effect is stronger for large firms than small firms contrary to Keim and Stambaugh (1984).
Maberly, Spahr and Herbst (1989)	S & P 500 Spot+Futures 1982-86 Hourly Value Line Spot+Futures 1982-1986 Hourly MMI (spot only) 1983-1986 Daily Close NASDAQ100 (spot only) 1983-1986 Daily Close SPOC 250 (spot only) 1983-1986 Daily Close	1.	Results largely similar to those of Harris (1986) except that the daily pattern in returns observed across firm size by Harris does not carry over to futures market.

CHAPTER 6

STOCK INDEX FUTURES MISPRICING

UK TRANSACTIONS DATA EVIDENCE[1]

ABSTRACT

The chapter analyses empirical evidence on stock index futures pricing based on about four years of synchronous hourly data from the UK where the cash index is traded in a pure dealership market and reported index values are based on quotes on which respective market makers are obliged to trade, thereby making identified arbitrage opportunities actually exploitable and economically significant. The unique features of the London cash market settlement procedures also enable examination of futures mispricing in a period during which constraints on short selling are irrelevant for index arbitrage. Several interesting results are documented after controlling for cash market settlement procedures. Consistent with earlier US evidence, both *ex post* and *ex ante* trading rules have generated attractive profits for the two more favourably positioned categories of index arbitrageurs, even after controlling for the risks of dividend uncertainties, marking to market cash flows and possible delays in execution. At any particular point of time, the far contract and the near contract have tended to be mispriced in the same direction. Restrictions on short selling clearly appear to have been an important factor influencing futures pricing. The mild tendency of futures to be mispriced in rising markets and underpriced in falling markets has been of no economic significance for index arbitrage. The absolute magnitude of mispricing has been greater for longer times to maturity. Finally, there has been a strong positive relationship between futures mispricing and the *ex ante* market volatility implied by index call option prices.

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[1] First draft August 1991; revised March 1992; accepted for presentation at the European Finance Association Conference, August 1992.

STOCK INDEX FUTURES MISPRICING

UK TRANSACTIONS DATA EVIDENCE

1. INTRODUCTION

There has been considerable interest among market participants, market regulators and academics in the pricing of stock index futures contracts and the associated profitability of index arbitrage. However, while there is substantial empirical evidence on the pricing of index futures traded on US markets,[2] there is relatively little published work on index futures markets traded outside the US institutional environment. Brenner, Subrahmanyam and Uno (1989) report on index futures markets in Japan and Singapore using closing/settlement prices and Yadav and Pope (1990) on British markets using opening and closing prices. The results reported in both these studies are based on daily data and, more importantly, not on synchronous cash and futures prices.[3] In view of the high intraday volatility of both cash and futures prices, it is difficult to draw strong conclusions based on non-synchronous data. The primary purpose of this chapter is to provide empirical evidence on stock index futures pricing based on about four years of synchronous hourly data from the London markets. This is important for several reasons. First,

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[2] *Inter-alia*, Mackinlay and Ramaswamy (1988), Figlewski (1984a), Cornell and French (1983a, 1983b) and Modest and Sundereshan (1983) document the existence of substantial and sustained deviations between actual and theoretical index futures prices while Merrick (1989) and Finnerty and Park (1988) demonstrate the *ex post* profitability of arbitrage related program trading strategies.

[3] Brenner, Subrahmanyam and Uno (1989) mention that they also examine synchronous data for 4 contracts, but do not report separately because of "negligible" differences between the results based on this alternative data set and their main data set.

there are major differences in policy perceptions in the US and the UK. Index arbitrage is apparently discouraged in the USA but it is clearly encouraged in London.[4] The microstructure of the UK cash market is also very different from that in the US. The London stock market is a pure dealership market while US markets are a hybrid of dealership and continuous auction systems. In this context, it is important to examine evidence on the extent to which arbitrageurs operating within different market trading systems and different institutional perspectives, have been able to ensure a "fair" spread between index futures prices and underlying cash prices.

Second, estimates of futures "mispricing" in the US are susceptible to measurement error in relation to non-synchronous trading in index stocks (Scholes and Williams, 1977; Cohen *et al*, 1986; Lo and Mackinlay, 1990) because cash index values based on transaction prices will, in general, not be actually tradeable values. Hence, significant mispricing values may not always represent true arbitrage opportunities. On the other hand, the UK stock index analysed in this chapter is based on quotes on which respective market makers are obliged to trade up to very large sizes. The stock index values thus represent actually tradeable values synchronous with futures prices. We recognise that even with such a quote based

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[4] Commenting in the context of Black Monday, Sir Nicholas Goodison, the then Chairman of the London Stock Exchange, in a widely reported letter to the Secretary of State for Trade and Industry, said that the "Quality of Markets Committee makes a good case for facilitating index arbitrage between the options and futures markets and the underlying equity market ... a conclusion ... completely at variance with the conclusions ... drawn in the United States ...".

index, differences in the price adjustment delays within different index stocks will generate positive serial correlation, and result in the reported cash index value being different from the "true" value corresponding to a frictionless market. Nevertheless, the arbitrage opportunities generated with data based on such a quote based index are potentially exploitable and hence economically significant.

Third, the London markets provide an ideal laboratory for testing the effect on futures pricing of the constraints that exist on short selling of stocks. It has been suggested that the observed preponderance of negative mispricing can be at least partially explained by the institutional restrictions and difficulties[5] that exist in selling stocks short, since the costs involved in exploiting negative mispricing are higher than the corresponding costs of exploiting positive mispricing. (See eg Modest and Sundereshan, 1984; Figlewski, 1984b; Brenner *et al*, 1989; and Puttonen and Martikainen, 1991). However, it has not been possible for US based studies to formally test this hypothesis. This chapter utilises the unique features of the settlement procedures on the London Stock Exchange to examine the behaviour of index futures pricing when there are virtually no constraints on short selling.

The chapter also aims to address several issues related to index arbitrage and futures pricing which do not appear to us to have been adequately examined in past research. First, it has been suggested that index futures tend to be overpriced in

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[5] Eg margin requirements and the uptick rule in the US.

sharply rising markets and underpriced in sharply falling markets[6] and hence it is difficult to execute the cash leg of the initial arbitrage trade since market makers are short of stock when the arbitrageur needs to buy stocks and have surplus stock when the arbitrageur needs to sell stocks.[7] However, this hypothesis has neither been formally tested nor its impact on index arbitrage profitability quantified. This chapter analyses the relevant empirical evidence in this regard.

Second, to the best of our knowledge, all published empirical evidence on stock index futures pricing has implicitly assumed that cash market transactions are settled immediately. This assumption can lead to significantly biased inferences in regard to futures mispricing particularly in markets like those of London and Paris, where cash market settlement takes place on a fixed future date rather than within a fixed period. This chapter develops the settlement adjusted forward pricing formula and controls for cash market settlement procedures in its empirical analysis.

Third, investors in the cash market have a tax timing option not available to investors in the futures market (see eg Cornell and French, 1983a). *Ceteris paribus*, the value of the tax timing option should be higher (and hence futures mispricing should be more negative) when the *ex ante* forecast volatility of the cash market is greater. The general equilibrium model of Hemler and Longstaff (1991) also

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[6] See eg Gould (1988).

[7] See the *Quality of Markets Quarterly*, 1989.

suggests that futures mispricing should vary systematically with *ex ante* forecast volatility of the cash market. However, we are not aware of any published empirical evidence in this regard.[8] This chapter empirically examines the variation of futures mispricing with one *ex ante* estimate of volatility - the volatility implied in index call option prices.

Finally, it has been widely recognised that index arbitrage is not riskless because of several factors -more importantly the uncertainty about the magnitude and timing of dividends, the stochastic nature of daily marking to market cash flows, and potential delays in actual execution of arbitrage trades. However, the factors which make index arbitrage risky have typically been ignored and the risk premium or the increase in the width of the arbitrage window that can be expected to exist on this account, has not been explicitly estimated. In fact, most US studies (eg Mackinlay and Ramaswamy, 1988; Klemkosky and Lee, 1991; Bhatt and Caciki, 1990) suffer from a potentially serious dividend misspecification problem. *Ex post* dividend data on the S&P500 index is not publicly available and these researchers have used dividend data from the Center for Research in Security Prices (CRSP) corresponding to the NYSE/AMEX portfolio instead of constructing a series based on individual stock dividends. Because the NYSE/AMEX portfolio contains a higher proportion of small firms than the S&P500 portfolio, it is likely that the dividend yield on the NYSE/AMEX portfolio will be different from the S&P500 dividend yield. This

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[8] In the context of their model, Hemler and Longstaff (1991) analyse the dividend adjusted futures-spot price ratio for the NYFE index futures contract based on the NYSE composite index.

chapter simulates *ex post* the risks that have been involved in index arbitrage due to dividend uncertainty, interest rate uncertainty and delays in trade execution in the context of the information actually available *ex ante* to the potential arbitrageur.

This chapter is organised as follows: Section 2 develops the forward pricing formula adjusted for cash market settlement procedures and outlines briefly the institutional and theoretical framework within which the empirical analysis is conducted; Section 3 describes the database, explains the methodology and documents the empirical results; and Section 4 presents the conclusions.

2. SETTLEMENT ADJUSTED PRICING OF INDEX FUTURES CONTRACTS AND THE UK INSTITUTIONAL FRAMEWORK

Stock index futures have been generally priced as forward contracts, ignoring the stochastic cash flows associated with daily marking to market of the futures position. The forward pricing formula is based on the existence of a portfolio of the underlying asset and Treasury Bills, which can exactly replicate the payoffs of the forward contract, given the following assumptions: (a) no transaction costs (including in particular costless short sales); (b) no taxes; (c) no spread between borrowing and lending rates; (d) interest bearing margins; and (e) dividends and interest rates up to futures maturity known *ex ante* with perfect certainty.

Published empirical work on stock index futures pricing has typically been based on the forward pricing formulation outlined by Cornell and French (1983a, Equation 11, pp 681) and Figlewski (1984a, pp 665). It has hence implicitly assumed that cash market transactions are settled immediately. In general cash market transactions are not settled on the same day and then the arbitrage free "fair" value $F^*_{t,T}$ at time t of an index futures contract maturing at time T will be given by:

$$F^*_{t,T} = S_t \exp\{r_{t,t',T'}(T'-t')\} - \sum_{w=t+1}^{T} d_w \exp\{r_{t,w',T'}(T'-w')\} \qquad \text{....(1)}$$

where S_t	=	Stock index cash value at time t
t'	=	Settlement date for cash market transactions at time t
T'	=	Settlement date for cash market transactions at time T
d_w	=	Aggregate dividend cash flows on the index associated with an ex dividend time period w
w'	=	Time at which dividend cash flow d_w is actually received
$r_{t,w',T'}$	=	Forward interest rate at time t for a loan to be disbursed at time w' for repayment at time T'

To prove this, consider the following strategy: (a) at t sell one futures contract, buy the cash index and arrange to borrow an amount S_t from t' to T' at the *forward* rate $r_{t,t',T'}$; (b) at t' receive disbursement of the borrowing S_t arranged at t and pay for the long cash index position; (c) for the index constituents going ex dividend between t and T, invest all dividends d_w (known *ex ante* and received on the actual dividend payment date w) at the corresponding forward rate $r_{t,w',T'}$ to receive an amount $\sum_{w=t+1}^{T} d_w \exp\{r_{t,w',T'}(T'-w')\}$ *at* T'; (d) at T sell the cash index for S_T and settle the futures contract carrying forward the associated cash flow ($F_{t,T}$-S_T) from T to T'; and (e) at T' collect the cash index sale proceeds S_T, collect proceeds of dividend related investments $\sum_{w=t+1}^{T} d_w \exp\{r_{t,w',T'}(T'-w')\}$ and repay $S_t \exp\{r_{t,t',T'}(T'-t')\}$ against the loan taken disbursed at t'. In the spirit of ignoring marking to market cash flows, assume that there are no costs or revenues involved in carrying the cash flows generated in (d) above (from the futures position) from T to T'. The strategy provides arbitrage profits if the actual futures price $F_{t,T}$ exceeds $F^*_{t,T}$ as determined from equation (1).

Alternatively, consider the following strategy: (a) at t buy one futures contract, sell the cash index short and arrange to lend an amount S_t from t' to T' at the forward rate $r_{t,t',T'}$; (b) at t' receive the entire proceeds from the short sale and lend the amount received S_t as arranged earlier at t; (c) for the index constituents going ex dividend between t and T, pay (to the agent from whom stock has been borrowed) all dividends due (known *ex ante*) on the corresponding actual dividend payment date using funds borrowed at the relevant forward interest rate $r_{t,w',T'}$, thereby creating a liability of $\sum_{w=t+1}^{T} d_w \exp\{r_{t,w',T'}(T'-w')\}$ at T'; (d) at T buy back the cash index for S_T, and settle the futures contract, carrying forward the associated cash flow (S_T-$F_{t,T}$) from T to T'; and (e) at T' receive an amount $S_t \exp\{r_{t,t',T'}(T'-t')\}$

as repayment against the loan given earlier at t', pay an amount S_T for the cash index bought at T and return the stocks to the agent from whom they had been borrowed earlier at t' and pay an amount $\sum_{w=t+1}^{T} d_w \exp\{r_{t,w',T'}(T'-w')\}$ to discharge dividend related liabilities created in (c) above. In the spirit of ignoring marking to market cash flows, assume that there are no costs or revenues involved in carrying the cash flows generated in (d) above (from the futures position) from T to T'. This strategy provides arbitrage profits if the actual futures price is below $F^*_{t,T}$ as determined from equation (1). Hence equation (1) gives the settlement adjusted forward pricing formula "fair" value for the futures contract.

The "mispricing" of the futures contract can be defined as[9]:

$$X_{t,T} = \frac{F_{t,T} - F^*_{t,T}}{S_t} \qquad \text{....(2)}$$

The adjustment for cash market settlement procedures is particularly important in the UK where cash settlement procedures are themselves organised as a forward market.[10] The year is divided into (usually 24) "Account settlement periods".

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9 Following Merrick (1988, 1989), Mackinlay and Ramaswamy (1988), and Yadav and Pope (1990, 1992a) we define futures mispricing as {(Futures Price-Theoretical Price)/Cash Price} where the "Theoretical Futures Price" is calculated in accordance with the cost-of-carry forward pricing formula.

10 For the US, inclusion of cash market settlement procedures in the analysis introduces only a relatively minor change: the relevant interest rate in the first term in equation (1) is the *forward* rate at t for a loan disbursed at t' and repaid at T', rather than the spot rate from t to T, as in Cornell and French (1983a, pp 681).

Most of these (usually 20) settlement procedures are of two weeks length while a few (usually 4, and spanning holidays) are of three weeks length. All transactions made within an account period are settled on the second Monday of the following account settlement period.

Even if arbitrage is otherwise perfectly riskless, the existence of transaction costs will allow futures prices to fluctuate within a band around the "fair" price without triggering profitable arbitrage. The width of the band arising due to direct out of pocket transaction costs should be:

$$|2T_S + T_D + T_F + T_{F'} + T_B| \qquad \text{....(3)}$$

where T_S = Percentage one way transaction costs for trading the index basket of stocks including both commissions and market impact costs.

T_D = Transaction tax payable as percentage of asset value transacted.[11]

T_F = Round trip percentage commissions in the futures market.

$T_{F'}$ = One way percentage market impact costs in the futures market.

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[11] In the UK, there is no transaction tax on futures market transactions. Cash market purchases currently attract 0.5% stamp duty (1% before Big Bang). No tax is payable on cash market sales.

T_B = Cost of borrowing fixed interest capital and index stocks.[12]

Different categories of market participants have different levels of transaction costs. Yadav and Pope (1990) accordingly highlight four categories of potential arbitrageurs:

Category A: Arbitrageurs whose marginal costs are confined to transaction costs relating to the futures market ie those for whom $T_B=T_D=T_S=0$. Examples of potential arbitrageurs falling in this category include

(i) those who are otherwise committed to enter or exit the market (due to eg portfolio insurance or tactical price based strategies) and use the futures market only as an intermediary

(ii) those with existing arbitrage positions who seek to profitably rollover their arbitrage position

(iii) those with existing arbitrage positions who seek to unwind early if such early unwinding is profitable.

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[12] This is faced only by arbitrageurs who do not have capital in treasury bills (for upper arbitrage) and index stocks (for lower arbitrage).

Category B: Arbitrageurs whose marginal costs include cash market related transaction costs, but those who have capital in fixed interest deposits and a pool of index stocks and who are not liable to pay transaction tax ie those for whom $T_B=T_D=0$ but $T_S \neq 0$. UK examples in this category are market makers recycling stocks within seven days as they are not liable to pay transaction taxes.

Category C: Arbitrageurs who have capital in treasury bills and a pool of index stocks, but who have to pay transaction tax in their dealings ie those for whom $T_B=0$, but $T_D \neq 0$ and $T_S \neq 0$. Examples in this category will be index funds/institutions.

Category D: Arbitrageurs who have to borrow capital or stock to initiate an arbitrage position ie those for whom $T_B \neq 0$, $T_D \neq 0$ and $T_S \neq 0$.

If there is adequate uncommitted arbitrage capital available to the arbitrageur with the lowest marginal transaction costs, arbitrageurs with higher marginal costs will never be able to enter the market (Gould, 1988). However, as has been pointed out by Stoll and Whaley (1987) and Brennan and Schwartz (1990) arbitrageurs function within real or self imposed position limits. Hence, several different categories of arbitrageurs can be active depending on the actual level of mispricing and on the extent to which the capital available to each category of arbitrageur is committed.

This will, in turn, depend on the past levels of mispricing. This means that given estimates of T_S, T_F and $T_{F'}$, an examination of the mispricing time series can lead to inferences in regard to the categories of arbitrageurs that are active in the market, or conversely to the profitability of arbitrage related program trading for different categories of potential arbitrageurs.

It is important to note that in actual practice, arbitrage strategies are not perfectly riskless. First, the magnitude of future dividends, their ex-dividend dates and their actual payment dates are uncertain. Typically, market participants estimate future dividends by applying a fixed percentage growth factor to past dividends and use identical (or corresponding[13]) ex dividend/payment dates. This factor will clearly be relevant only when the time to maturity exceeds the period between the announcement date and the ex-dividend date.

Second, there is uncertainty about the level at which the cash leg of the initial arbitrage trade is actually executed in relation to the cash index value used in deciding whether to initiate the arbitrage position. In the USA, the cash index is based on the last transaction price and may not represent the value at which the cash index can actually be traded. In the UK, the reported value of the cash index is an excellent measure of the truly tradable cash index since the prices of index constituents that go into the computation of the index are firm quotes on which

[13] In the UK stocks go ex dividend only on Mondays and hence the *exact* date changes from year to year.

respective market makers are legally obliged to trade up to fairly large sizes.[14] However, there is a risk of delayed execution because:

(i) Unlike the USA, execution is not through an automated system triggered by a computerised trading program.

(ii) Execution is usually through the firm's in house equity market makers and it has been suggested that[15] they will have excess stock when there is an arbitrage related sell program and will be short of stock when there is an arbitrage related buy program since mispricing is often negative in falling markets and positive in rising markets.[16]

Third, there can be uncertainty about the level at which the cash leg of the arbitrage position can be closed at expiration. There is no uncertainty in this regard in the US since an arbitrage positon can be closed on the basis of a market-on-open (or market-on-close) order. However, in the UK, it is not possible to be certain about unwinding the cash position at *precisely* the settlement price of the futures contract

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[14] It is important to note that whilst the quotes on the basis of which the index is calculated are firm, they may not necessarily be from the same market maker for each of the 100 stocks. Therefore, extra costs may be involved in hitting the best quotes for all 100 stocks. That said, basket trading facilities are available from the large market making firms like James Capel.

[15] See eg *Quality of Markets Quarterly*, 1989.

[16] See eg Gould (1988).

at expiration, since this settlement price is based on the *average* value of the cash index between 11.10 am and 11.20 am on expiration day.

Finally, uncertainty arises because of the stochastic nature of the payoffs on account of daily marking to market of the futures position. Futures prices cannot be risklessly estimated *ex ante* in the same way as forward prices. The *ex post* difference between forward and futures prices depends on the covariation of changes in futures prices with changes in the prices of zero coupon bonds maturing with the futures contract.[17] In this context, the net cash flows on this account depends on the *ex post* path of futures price changes up to maturity.

These uncertainties lead to a risk premium and an effective increase in the width of the arbitrage band by an amount ΔT^+ reflecting possibly the worst case scenarios. *Ceteris paribus*, the magnitude of ΔT^+ is related to the level of uncertainty about dividends, interest rates and future index values and hence should be greater for longer times to expiration. While these factors have been recognised in the literature[18], their effect has not been explicitly quantified. However, the absolute magnitude of mispricing has been found to increase with the time to maturity of the futures contract by Mackinlay and Ramaswamy (1988) for the US market and by Yadav and Pope (1990) for the UK market (with daily data).

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[17] See eg Cox, Ingersoll and Ross (1981), pp 326.

[18] See eg Klemkosky and Lee (1991), Mackinlay and Ramaswamy (1988) and Yadav and Pope (1990).

However, an effective transaction cost discount ΔT^- is created by the possibility of "risky" arbitrage strategies and in particular by:

(a) The option of unwinding prior to expiration, which is profitable whenever the direction of mispricing is different from the direction of mispricing when the arbitrage position was first initiated and the absolute magnitude of mispricing exceeds the incremental marginal transaction costs involved ie $T_{F'}$.

(b) The option of rollover into the next available maturity at or prior to expiration which is profitable whenever the direction of mispricing of the far contract is the same as the direction of mispricing of the near contract when the position was first initiated and the absolute magnitude of the difference between the mispricing of the near and far contracts exceeds the incremental transaction costs involved ie $(T_F+2T_{F'})$.

The results of Sofianis (1991) show that the profitability of index arbitrage comes essentially from "risky" arbitrage strategies and hence from the existence of these transaction cost discounts ΔT^-. The evolution of mispricing over time and in particular, the tendency of mispricing to persist or reverse itself, is important for arbitrageurs since it determines the magnitude of the effective transaction cost discounts resulting from the early unwinding and rollover options and determines also the implicit cost of the risk of delayed execution.

In addition, there are institutional restrictions that inhibit the arbitrage process. Important among these restrictions are the constraints on short sales. In the US because of the "uptick" rule for short sales, arbitrageurs cannot always use short positions in index arbitrage strategies if futures are underpriced unless they employ the pools of stock they own or control . Similarly in the UK, only registered market makers have special stock borrowing privileges. It is very difficult for non-market makers to undertake arbitrage transactions involving shorting stock as a matter of normal course, unless other trading books within the same institution (eg index funds) are already long in stock. This has been suggested as a possible explanation for the predominantly negative average mispricing that has been reported in several studies.[19] The London markets provide an ideal laboratory to test such a hypothesis in view of the unique features of the cash settlement procedures. For all index arbitrage activity during the account settlement period which spans the futures maturity date, there is no need to borrow stocks in order to go short and no need for special stock borrowing privileges. T_B is effectively zero for <u>all</u> arbitrageurs, not just for arbitrageurs who have capital in index stocks or arbitrageurs with special stock borrowing privileges. Furthermore, during this period, there is no cost of carrying the cash position, and no dividend uncertainty, thereby making arbitrage virtually riskless except for the risk of non-synchronous/delayed execution.

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[19] See eg Figlewski (1984b), Cornell and French (1983a), Merrick (1988), Brenner *et al* (1989), Yadav and Pope (1990) and Puttonen and Martikainen (1991).

Another important issue is the tax timing option available to stockholders due to their ability to select the timing of realisation of losses and gains. Cash settlement of futures contracts implies that investors in the futures market necessarily pay taxes in the year the capital gains arise, while investors holding the cash asset can defer their capital gains. This should result in futures being underpriced relative to the fair value in equation (1). On the other hand, the marginal investor may be a tax exempt institution[20] in which case the tax timing option will have no value, or the marginal investor may be an arbitrageur/floor trader who cannot hold the cash index indefinitely in which case again the tax timing option will have no value. Clearly, the relevance of the tax timing option for index futures pricing could be different in different markets and essentially an empirical issue. *Ceteris paribus*, the value of the tax timing option should arguably be higher when the *ex ante* volatility of the underlying asset is greater and hence it can be argued that if the tax timing option is a relevant factor for index futures pricing, mispricing should be relatively more negative when the *ex ante* volatility of the index is greater.[21]

Recently, Hemler and Longstaff (1991) have developed a general equilibrium model with stochastic interest rates and market volatility. In the framework of this model, futures mispricing defined with reference to the forward pricing model equation (1), should vary systematically with the *ex ante* forecast volatility of the cash market.

[20] This is not conceivable in the UK since tax law has effectively prevented the use of index futures contracts by tax exempt institutions, except for hedging purposes.

[21] The value of the tax timing option should also be greater for longer times to expiration.

However, we are not aware of any studies which have empirically investigated the dependence of mispricing on an estimate of *ex ante* market volatility eg the volatility implied in index call option prices.[22]

3. EMPIRICAL EVIDENCE

3.1 Data

London has an exchange traded stock index futures contract based on the FTSE100 index - an arithmetic average, market value weighted index of one hundred (highest capitalisation) stocks. The contract is traded on the London International Financial Futures Exchange (LIFFE). LIFFE index futures expire four times a year in March, June, September and December on the last business day of the month. LIFFE trading is based on an open outcry market and, in common with other futures markets, all margin accounts are marked to market on a daily basis. Though three maturities are traded on LIFFE at any particular time, only the two earlier maturities have significant trading volume. Our analysis is hence confined to the two earliest maturity contracts. The contract nearest to maturity at any time is labelled as the "near" contract and the next maturing contract is labelled as the "far" contract. Expiration day observations are not included in the near contract.

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22 There is conflicting evidence on the information content of implied volatility as an *ex ante* predictor of future volatility. Canina and Figlewski (1991) find that implied volatility is a very poor predictor of future volatility but Day and Lewis (1992) find that implied volatility adds substantial predictive power. In this paper, we use implied call volatility as the proxy for *ex ante* market volatility.

The results reported in this chapter are based on hourly cash and futures data on the FTSE100 index for the period April 28, 1986 to March 23, 1990.[23] This corresponds to 990 trading days. During the sample period, the cash market was open from 9.00 am-5.00 pm and the futures market from 9.05 am-4.05 pm. Data on the FTSE100 cash index was collected from the *Financial Times*. Though the cash index is updated every minute, the *Financial Times* reports index values only on the hour. Consequently, our analysis is based only on synchronously available cash, near futures and far futures prices at 10.00 am, 11.00 am, 12 noon, 1.00 pm, 2.00 pm, 3.00 pm and 4.00 pm.

"Time and Sales" transactions data on FTSE100 index futures was obtained from LIFFE. The data includes all bid and ask quotes and all transaction prices relating to this contract. For each day of the sample period, the last near futures price and the last far futures price posted before 10.00 am, 11.00 am, 12 noon, 1.00 pm, 2.00 pm, 3.00 pm and 4.00 pm was extracted from the data, *provided the price was posted less than 60 seconds before the corresponding cash market quote on the hour.*

[23] The choice of this period has been dictated largely by availability of cash index data and changes in exchange trading hours. The beginning of the sample period corresponds with the date on which LIFFE extended its trading hours from 9.35 am-3.30 pm to 9.05 am-4.05 pm. The end of the sample period corresponds to the date on which the London Stock Exchange changed its trading hours from 9.00 am-5.00 pm to 8.30 am-4.30 pm.

In view of the potential impact of measurement error in US data based studies induced by the use of transaction prices and by non-synchronous trading in index stocks, the distinct properties of the particular dataset employed here are worth emphasising. The cash index is based on the average of the best bid and the best ask *quotes* and it is updated every minute. The quotes underlying the index computation represent prices at which competing market makers are obliged to trade for up to very large contract sizes. Thus, the hourly FTSE100 index values are based on prices at which an arbitrageur could actually trade, before transaction costs. The use of futures prices posted less than 60 seconds before the corresponding cash index quotes ensures that, to the extent possible, the futures prices are synchronous with the hourly mid-market index quote in the cash market.

Information on the constituents of the index and how these constituents changed over the sample period was obtained from the London Stock Exchange. Dividends and ex-dividend dates for all the relevant constituents of the index each day were collected from *Extel* cards. In addition, in order to compute the exact *ex post* daily dividend flow on the FTSE100 index, the individual constituents' dividend flows on each day were value weighted, aggregated and converted into index points using price and market value data collected from *Datastream*. Additionally, ex-dividend dates and actual dividend payment dates were collected from the *London Business School Share Price Database*. These were used to estimate the average time delay between the ex-dividend date and the dividend payment date.

Daily data on one and three-month UK Treasury Bill discount rates were also collected from *Datastream*.

London also has exchange traded stock index option contracts based on the FTSE100 index. These are traded on the London Traded Options Market. Index options expire on the last business day of every month. Over the sample period, four maturities (corresponding to the four immediately following month-end dates) and several exercise prices were available at any one time. However, at the money and relatively near maturity options tend to have the greatest liquidity. Furthermore, there is also some evidence of instability close to expiration. Hence, the implied volatility estimate used in this study is that calculated by Johnsen (1990). For each day in the sample period, daily closing index call option prices, and the corresponding synchronous cash index value, were collected from the *Financial Times*, and the implied volatility was calculated, using the dividend adjusted Black-Scholes Model, for the two call option series which were closest to being at the money and which were expiring more than one month but less than two months later. The implied volatility estimate for the day was taken to be the arithmetic average of the two implied volatilities so calculated.

3.2 UK Transaction Costs

The components that make up the total transaction costs relevant for index arbitrage are indicated in equation (3). T_B, the cost of borrowing capital or index stocks, is faced only by arbitrageurs who do not have capital in treasury bills or index stocks. T_D, the transactions tax, is not payable by market makers and brokers/dealers recycling stocks within seven days.

Table 1 reports the average inner market spread for UK "alpha" stocks[24] on the basis of the values published by the Stock Exchange Quarterly from time to time. This inner market spread has varied from about 0.7% to about 1.3% - averaging about 1.0% - except for the contract spanning Black Monday. However, the quoted bid-ask spread will be an upward biased estimate of the cash market transaction costs $2T_s$ relevant for index arbitrage, since a major component of the quoted spread, namely adverse information costs, should not be relevant in pricing market making services for index arbitrage. Transaction costs related to the cash market should be confined to marginal order processing costs and marginal inventory holding costs. We are not aware of any published estimate for the UK market of the percentage of the quoted spread which arises due to adverse information costs. However, Stoll (1989) finds that on NASDAQ, 43% of the quoted spread represents adverse information costs, 10% represents inventory holding costs and 47% represents order processing costs. If we use these figures as a first approximation for the London market, which has an almost identical trading structure to NASDAQ, the quoted spread for index arbitrage should average about 0.5% except for the

[24] Stocks in London had been classified on the basis of the number of competing market makers and the trade/quote reporting restrictions applicable to them. Alpha stocks generally had the lowest spreads, and all FTSE100 index stocks belonged to this category.

contract spanning Black Monday. Even the rise in the quoted spread after Black Monday is more likely to be due to an increase in the adverse information component and the inventory holding component and is hence likely to affect index arbitrage trades to a much lesser extent. Commissions are usually on a flat rate basis and for large volume index arbitrage trades are virtually negligible when expressed as a percentage of value traded.

To estimate the percentage market impact costs in the UK index futures market, a subset of LIFFE time and sales data was analyzed consisting only of cases in which ask prices and bid prices are posted within 60 seconds of each other. Table 1 also reports the median percentage spread[25] for the near futures contract and the far futures contract. The median percentage spread for the near futures contract has varied from 0.04% to 0.15%, averaging about 0.1%. The median percentage spread for the far futures contract has varied from 0.12% to 0.44%, averaging about 0.25%. Roundtrip percentage commissions in the futures market have been typically less than £25 per contract ie less than about 0.05% of underlying index value.

On the basis of the above, the total average arbitrage related transaction costs of the three more important categories of arbitrageurs highlighted in Section 2 - Category

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[25] Percentage spread has been defined as:

$$100 * \frac{(Ask-Bid)}{\{(Ask+Bid)/2\}}$$

A, Category B and Category C - are reported in Table 1. The total marginal transaction costs for Category A arbitrageurs average below 0.15% (0.30%) for the near (far) contract. For Category B arbitrageurs, marginal transaction costs vary from about 0.5% (0.6%) to about 0.8% (1.0%) for the near (far) contract except for the contract spanning Black Monday. For Category C arbitrageurs, the transaction costs are higher than those for Category B arbitrageurs by 1.0% for the first two contracts and 0.5% for the remaining contracts.

On the basis of the above, we take the transaction costs of Category A, Category B, and Category C arbitrageurs to be 0.25%, 0.75%, and 1.25% respectively.

3.3 Mispricing

Mispricing of the futures contract was calculated on the basis of the forward pricing formula Equation (1) and the definition of Equation (2). In addition to the usual assumptions of the forward pricing formula, the following additional assumptions were initially made: (a) forecast dividends to maturity for each date are identical to the actual *ex post* daily cash dividend inflow for the FTSE100 basket; (b) the forward interest rate at time t for a loan made at time w to be redeemed at time T, is identical to the (future) spot interest rate at time w on a Treasury Bill maturing at time T; (c) the value of day t of one- and three-month maturity Treasury Bill interest rates can be used to estimate a linear term structure from which the implied forward interest rate for the period S_1 to S_2 (in equation (1)) can be calculated; and

(d) actual payment of dividends is made 53 calendar days after the ex dividend date, this being the average *ex post* delay between the ex dividend date and the actual dividend payment date for index stocks over the sample period.

Table 2A reports some relevant descriptive statistics of the percentage mispricing variable for each of the 16 near contracts expiring during the sample period and for the aggregated data for all near contracts. Table 2B reports similar statistics for each of the 16 far contracts and for the aggregated data for all far contracts.[26] Tables 2A and 2B also report the t-statistic for the hypothesis that the average mispricing is equal to zero. The t-statistic has been calculated after controlling for the autocorrelation structure of the mispricing variable.[27]

Tables 2A and 2B have several interesting features. First, it is inappropriate to conclude that the forward pricing formula is, on average, an upward or a downward biased estimate of the actual futures price. Average mispricing is significantly negative and the proportion of negative mispricing values is significantly greater than 50% for 9(6) near (far) contracts. On the other hand, average mispricing is also significantly positive and the proportion of positive mispricing value is significantly greater than 50% for 4(5) near (far) contracts. Average mispricing is not significantly different from zero for only 3(5) near (far) contracts. Mispricing

[26] The statistics for the aggregate sample have been computed after excluding the somewhat atypical 4 week period starting with Black Monday October 19, 1987 and also the single day of the Mini Crash October 16, 1989.

[27] Standard error of the mean calculated as standard deviation divided by $\sqrt{N}$ is inappropriate in view of autocorrelation in mispricing. The standard error is calculated as:

$$SE(\bar{X}) = \sqrt{\frac{SD(X)}{N}\left(1+2\sum_{k=1}^{N-1}\left(1-\frac{k}{N}\right)\rho_k\right)}$$

where N is the number of observations.

often tends to be predominantly negative and often predominantly positive. Whether mispricing is predominantly negative or predominantly positive varies substantially from contract to contract and often from the far phase to the near phase of the same contract. The overall inference about average mispricing for the aggregate sample depends on the choice of sample period. For example, for a three year sample period starting from the third quarter of 1986 to the second quarter of 1989, mispricing is predominantly negative for 5(3), and predominantly positive for 5(5) near (far) contracts; and average mispricing is positive! This is consistent with US evidence where researchers examining different sample periods have come to different conclusions on whether average mispricing is positive or negative.[28]

Second, the 5th and 95th percentiles, and the first and third quartiles, of the mispricing variable, also vary substantially from contract to contract. If the forward pricing formula is an unbiased estimator of the fair futures price, the 5th and 95th percentiles, or the first and third quartiles, could be regarded as proxies for the lower and upper boundaries of the arbitrage windows for different categories of arbitrageurs. Similarly, the interquartile range (Q_3-Q_1) or the difference (P_{95}-P_5) could be regarded as a proxy for the corresponding overall width of the arbitrage window. In this context, the boundaries of the arbitrage window appear to be very volatile. For 8 out of 16 near contracts, both quartiles are of the same sign and for the September 88 near contract, even the 5th and 95th percentiles have the same

[28] Eg Figlewski (1984a), Cornell and French (1983a,1983b), Modest and Sundereshan (1983) and Eyton and Harpaz (1986) conclude that average mispricing is negative, while Bhatt and Cakici (1990), Klemkosky and Lee (1991) and Chung (1991) conclude that average mispricing is positive. Klemkosky and Lee (1991) also recognise that mispricing exhibited systematic trends, being predominantly positive in some periods and predominantly negative in others.

sign. The interquartile range (Q_3-Q_1), or the difference (P_{95}-P_5), appears relatively more stable. However, there appears to be no obvious relationship between the variation in the transaction cost estimates in Table 1 and the variation in average/median mispricing or variation in Q_3 or Q_1 or P_5 or P_{95} in Table 2A/2B, or even changes in the overall width of the arbitrage window proxied by, say, (Q_3-Q_1) or (P_{95}-P_5). This again suggests changing levels of systematic biases in the forward pricing formula estimate of the futures price, and persistence in mispricing, rather than rapidly changing boundaries of the arbitrage window.

Third, the minimum and maximum values, the 5th and the 95th percentile, and, in several cases, even the lower and upper quartiles suggest that the absolute magnitude of mispricing often exceeds the estimated transaction costs of Category A and Category B arbitrageurs, and sometimes even exceed the transaction costs of Category C arbitrageurs. Furthermore, there does not appear to have been any systematic reduction with the passage of time in the magnitude of average mispricing, or the standard deviation of the mispricing variable.

Fourth, the average absolute magnitude of mispricing, the standard deviation of the mispricing variable, the interquartile range (Q_3-Q_1), and the difference (P_{95}-P_5) are all larger for the far contract than for the near contract. Since each of these measures can be viewed as proxies for the overall width of the arbitrage window, this is consistent with index arbitrage being regarded as more risky when the time to maturity is longer because of greater dividend and interest rate uncertainty. (See eg Yadav and Pope, 1990, and Mackinlay and Ramaswamy, 1988.)

Fifth, out of the 9 quarters in which average mispricing is significantly different from zero for *both* the near and the far contract, there are 8 quarters in which the direction of the significant average mispricing in the near contract is the same as the direction of the significant average mispricing for the far contract. It is only for the quarter ending December 87 (ie the somewhat atypical period spanning Black Monday) that the direction of the significant average mispricing for the near contract is opposite to the direction of the significant average mispricing for the far contract. Hence, *at any particular point of time* the far contract has tended to be mispriced in the same direction as the near contract suggesting that time period specific factors (like market sentiment and market volatility) could have been important determinants of mispricing. On the other hand, there are 3 contracts in which average mispricing is significant in one direction in the far phase and also significant in the opposite direction in the near phase of the same contract; while there are 4 contracts in which average mispricing is significant in one direction in the far phase and also significant in the same direction in the near phase of that contract. Hence, *for a particular contract*, the systematic bias in mispricing has tended to persist from the far phase to the near phase (about) as often as it has tended to reverse itself. This suggests that contract specific factors (like misspecification of dividends) may not have been important determinants of systematic bias in futures pricing.

The relative direction of the mispricing in the near and far contract is examined also by analysing the subset of data observations corresponding to only those hours for which values of *both* the near contract mispricing and the far contract mispricing were simultaneously available ie those hours for which *both* near and far futures prices were posted during the last 60 seconds of an hourly interval. There are 899 such cases. In 640 (ie 71.2%) of these cases, the near contract is mispriced in the same direction as the far contract - significantly greater (p value < 0.0001) than the proportion expected in the absence of any association between the direction of near and far contract mispricing. Furthermore, of the remaining 259 cases, as many as 120 cases correspond to the account period spanning near futures maturity, where the behaviour of near contract mispricing could potentially be of a qualitatively different nature because of the complete irrelevance of short selling constraints for index arbitrage with respect to the near contract during that period. Table 3 presents the matrix of possibilities for the relative direction of near and far contract mispricing. The number of cases in both the cells along the diagonal (which correspond to near and far contract mispricing being in the same direction) are significantly greater (p value < 0.0001) than the number of cases expected if there was no relationship between near and far contract mispricing.

To further investigate the association between near and far contract mispricing, the mispricing variable X is classified into five bands as follows:

$\{-2\}$: $\{X \leq -1.0\%\}$

$\{-1\}$: $\{-1.0\% < X \leq -0.5\%\}$

$\{0\}$: $\{-0.5\% < X \leq 0.5\%\}$

$\{+1\}$: $\{-0.5\% < X \leq 1.0\%\}$

$\{+2\}$: $\{1.0\% < X\}$

Out of 899 simultaneous observations, mispricing in the near and far contracts are in the same band in 368 (ie 40.9%) cases, one band away from each other in 379 (ie 42.2%) cases, two bands away from each other in 147 (ie 16.4%) cases, and three bands away from each other in the remaining 5 (ie 0.5%) cases. There are no cases in which mispricing is in one outermost mispricing band for the near contract and in the opposite direction outermost band for the far contract.

Table 3 also presents the matrix of possibilities for the relative association between the band of near contract mispricing and the band of far contract mispricing. The number of cases in each of the cells along the diagonal, which correspond to near and far mispricing being in the same band, and most of the cells which correspond to a difference of one band, are significantly greater than the number of cases expected in the absence of any association between near and far contract mispricing. On the contrary, the number of cases in the cells which are far from the diagonal, and, in particular in the cells which correspond to differences of three or four bands, is significantly less than the number of cases expected in the absence of any association. Clearly, the mispricing of the far contract tends to be in the same direction, and "close" to the mispricing in the near contract.

Mispricing also tends to persist over long periods. The degree of this persistence can be estimated non-parametrically in terms of the average length of a run (or the average time before a mispricing reversal) and parametrically in terms of the serial correlation in the mispricing time series. Table 4 reports results in this regard for both the near and the far contracts. For the aggregate sample, the average time before mispricing reversal for the near contract is about 15 trading hours when mispricing is positive and about 19 trading hours when mispricing is negative. For each contract the "runs" test showed that the number of runs is significantly less than the number of runs expected if successive observations were independent; the hypothesis of no persistence being conclusively rejected (with p value <0.001) in every case. The first order autocorrelation is significantly greater than zero in every case. In fact it is very close to unity - varying from 0.503 (0.586) to 0.960 (0.958) for the near(far) contract. The extent of persistence in mispricing - whether measured in terms of serial correlation or the average length of a run - can be regarded as an inverse measure of the elasticity of arbitrage services.[29] Hence, it is clear that the elasticity of arbitrage services has not increased over time. The high degree of persistence in mispricing suggests that the possibility of delayed execution may not be a serious risk for index arbitrageurs. On the other hand, since the average run is less than about 3 trading days for the near contract, it appears that the early unwinding option should be significantly valuable.

3.4 Index Arbitrage Profitability Simulations

3.4.1 Ex Post Profit Simulations

Tables 5A and 5B present, *inter alia*, the results of simulating the profitability of index arbitrage based on simple *ex post* trading rules which assume that it is possible to use the prices at any time to execute a trade at the same price at the same time. Let TC% be the transaction cost relevant for the arbitrageur, and Y%

[29] If the elasticity of arbitrage services is infinitely high, the successive realisations of the mispricing variable should be independent and mispricing should be a serially uncorrelated time series. If the elasticity of arbitrage services is zero, the mispricing variable should have unit roots. Yadav and Pope (1992b) show formally through unit root tests that the hourly mispricing time series in the UK does not have unit roots: the elasticity of arbitrage services is hence significantly greater than zero.

be the additional profit required by the arbitrageur to be motivated to consider an arbitrage trade. Four trading rules are considered:

Trading Rule 1: If mispricing exceeds (TC+Y), sell one futures contract, sell treasury bills and buy the equivalent underlying basket of stocks, and hold the long stock-short futures position up to expiration. At expiration, sell the stock bought earlier, and reinvest in Treasury Bills. If mispricing is below (TC+Y), buy one futures contract, sell the equivalent underlying basket of stocks, use the proceeds obtained to buy Treasury Bills, and hold the position until contract expiration, at which time the position is unwound and investment in stocks reinstated. This is the simple hold-to-expiration trading rule.

Trading Rule 2: Same as Trading Rule 1, except that, instead of waiting until contract expiration, the position is unwound as soon as mispricing changes sign and becomes large enough in magnitude to cover the estimated incremental transaction costs ($T_F+T^*_F$) plus the required incentive to trade ie Y. This is the early unwinding option.

Trading Rule 3: Same as Trading Rule 1, except that the position is rolled forward to next available maturity as soon as the sign of the mispricing in the far contract is the same as the sign of the original mispricing, and when the difference in mispricing between the far contract and the near contract becomes large enough in magnitude to cover the estimated incremental transaction costs ($T_F+2T^*_F$) plus the required incentive to trade ie Y. This is the rollover option. (Compound rollovers are ignored ie the rolled over position is assumed to be carried to expiration.)

Trading Rule 4: This is a combination of Trading Rules 1, 2 and 3. The arbitrage position is initiated as in Trading Rule 1, but is unwound early as per Trading Rule 2 or rolled forward, as per Trading Rule 3, whichever option becomes activated at an earlier date.

Y is assumed to be 0.25%. Two values of TC are used in each case - 0.25% and 0.75%. These correspond to the estimated transaction costs of Category A and Category B arbitrageurs respectively. It is also assumed that the transaction costs being faced by the arbitrageurs is TC.

Tables 5A and 5B report the profits (in £'000) per contract earned from the different trading rules by Category A and Category B arbitrageurs. The first point to note that the *additional* profits arising from the rollover option or the early unwinding option are a significant proportion of the total arbitrage profits, and often exceed the arbitrage profits arising from the simple hold to expiration strategy. These high additional profits imply a heavy transaction cost "discount" and should generate "risky" arbitrage activity even when mispricing is within transaction cost bounds. Secondly, arbitrage positions are almost never held to expiration. With Trading Rule 4, more than 97% positions are closed before expiration and the median time to early unwinding/rollover is less than two weeks for both transaction cost levels.[30] Thus index arbitrage should not create significant expiration day price volume and volatility effects in underlying stocks. Thirdly, the median time to early unwinding is substantially greater than the median time to rollover.[31] Thus arbitrage positions are more likely to be rolled over into the next available maturity rather than being unwound early. This is because, as we have seen earlier, the far contract tends to be mispriced in the same direction as the contract.

3.4.2. Ex Ante Profit Simulations

Table 5A and Table 5B also report index arbitrage profits for transaction cost bounds of 0.25% and 0.75%, based again on Trading Rules 1, 2, 3 and 4 but implemented on an *ex ante* basis. Thus, the calculation of *ex ante* profits in Table 5A and Table 5B assumes that if there is an arbitrage opportunity perceived on the basis of cash and futures prices in hourly period t, the required arbitrage strategy is executed only on the basis of the prices in hourly period (t+1). Similarly it is assumed that if an early unwinding or rollover trade is indicated on the basis of cash and futures prices in hourly period t', the trade is actually executed only on the basis

[30] The time to early unwinding and the time to rollover indicated in Table 5A/5B is an upward biased conservative estimate of the true time to early unwinding/rollover, because, in our calculations we have assumed that mispricing is zero at the open and on the hours corresponding to our "missing" values ie whenever a futures price was not available in the 60 seconds prior to the cash index quote.

[31] The upward bias cited in the immediately preceding footnote is also likely to be greater for the time to rollover than for the time to early unwinding since there were many more "missing" values for the far contract than for the near contract.

of prices in hourly period $(t'+1)$. The assumed execution delay of one hour is clearly a conservative assumption for assessing the potential risk in index arbitrage due to the possibility of delayed execution of trades.[32] In this context, it is important to emphasise that the cash index in London is based on firm quotations on which respective market makers are obliged to trade up to fairly large sizes. Thus the cash and futures prices one hour later clearly represent actually tradable prices.

The results of *ex ante* trading rules are qualitatively similar to those of *ex post* trading rules, but the magnitude of arbitrage profits are reduced, as can be expected. The reduction in profits from Trading Rule 1 is, on average, about one half for Category A arbitrageurs and two-thirds for Category B arbitrageurs. About two-thirds of trades are still profitable in both cases. The reduction in total profits from Trading Rule 4 is, on average, of similar order of magnitude. Even on an *ex ante* basis the profits per contract are surprisingly large. Clearly, the profitability of index arbitrage has been fairly robust to delays in execution of even one hour.

3.4.3. Profit Simulations for Risky Arbitrage

The high profit generated in Tables 5A and 5B by the early unwinding option and the rollover option suggest potential for risky "arbitrage" trades which initiate the arbitrage trade with the magnitude of mispricing less than the actual transaction cost of the potential arbitrageur in the hope that the marginal profit generated by early

[32] Execution delays are likely to be at most a few minutes for basket trades, but could be somehwat longer otherwise.

unwinding/rollover will not only cover the loss involved in initiating a trade within the transaction cost window, but also generate a net profit. The results of Sofianos (1990) indicate also that most of the profit from index arbitrage comes from such risky "arbitrage" trades. There can be several forms of risky "arbitrage" trades. The profit simulations reported in Table 6 are based on implementing the following trading rule:

Risky Arbitrage Trading Rule: The arbitrage position is initiated whenever mispricing exceeds X% in magnitude (X = 0.5%, 1.0%) even when actual transaction costs exceed X% (by an amount equal to 0.25%, 0.50%, 0.75%, 1.00%); and the position is unwound early, or rolled forward, as soon as the additional profit from early unwinding, and the additional profit from rollover, makes the overall position profitable after inclusion of the incremental transaction costs involved in early unwinding/rollover.

The results in Table 6 are fairly striking. With a trading rule threshold of 0.5%, even after excluding the contract spanning Black Monday, index arbitrage has been profitable for arbitrageurs with transaction costs of 1.25% with more than 80% of positions being profitable. With a trading rule threshold of 1.0%, index arbitrage has been profitable even for arbitrageurs with transaction costs of 2.0% again with more than 80% positions being profitable. In either case, the options to unwind early or rollover have provided a transaction cost discount of about 1% in magnitude and have reduced the effective width of the arbitrage window by more than 60%.[33] Since such trades are clearly not risk free, it appears that those index arbitrageurs who do not want to take on any risk in addition to the inherently unavoidable risks arising due to stochastic interest rates and dividends, should never be able to enter the arbitrage market if market participants actively engaged in "risky" arbitrage have sufficient arbitrage capital available with them.

3.5 Execution Risks

The results of the *ex ante* profit simulations show clearly that the profitability of index arbitrage has been robust to delays in execution. This section addresses another kind of execution risk ie whether the profitability of index arbitrage is overstated on account of index futures being significantly overpriced only in sharply rising markets and underpriced only in sharply falling markets, making it difficult to execute the cash leg of the initial arbitrage trade since market makers are short of stock when the arbitrageur needs to buy stocks and have surplus stock when the arbitrageur needs to sell stocks. This possibility is highlighted *inter-alia* by Gould (1988) and the *Stock Exchange Quality of Markets Quarterly* (1989). It is relevant in the UK even in the context of obligatory market maker quotes because market making firms tend to be the most favourably positioned arbitrageurs and prefer to execute the cash leg of the trade with their own in house market making arm.

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33 Once again, the simulated profits from risky arbitrage are conservative estimates and are likely to be understated because of our including only those futures prices which were posted less than 60 seconds before the cash index quote.

To investigate this, it is necessary to examine whether arbitrage opportunities arising due to significant index futures overpricing are accompanied by significantly high cash returns and arbitrage opportunities arising due to significant index futures underpricing are accompanied by significantly low cash returns. The mispricing variable X is again classified into five bands as follows:

{-2}: $\{X \leq -1.0\%\}$

{-1}: $\{-1.0\% < X \leq -0.5\%\}$

{0}: $\{-0.5\% < X \leq 0.5\%\}$

{+1}: $\{0.5\% < X \leq 1.0\%\}$

{+2}: $\{1.0\% < X\}$

For the five sub-samples sorted on the basis of the five mispricing bands above, Table 7 reports the mean (median) cash returns and the percentage of observations in the band which also correspond to the top and bottom deciles and the top and bottom quartiles of cash returns in the overall sample. Table 7 also reports the results of testing the following hypotheses for each of these five sub samples:

(a) The hypothesis that the mean (median) cash return in the sub sample corresponding to a particular mispricing band is equal to the mean (median) cash return in the whole sample;

(b) The hypothesis that, within the sub sample corresponding to a particular mispricing band, the percentage of observations which also correspond to the *highest* decile (quartile) of cash returns is equal to 10%(25%); and

(c) The hypothesis that within the sub sample corresponding to a particular mispricing band, the percentage of observations which also correspond to the *lowest* decile (quartile) of cash returns is equal to 10%(25%).

The mean (median) cash return in the sub sample corresponding to the mispricing band {-1: -1.0% $<$ X $\leq$ -0.5%} is *lower* than the overall mean (median) cash return, and the mean (median) cash return in the sub sample corresponding to the mispricing band {+1: 0.5% $<$ X $\leq$ 1.0%} is *higher* than the overall mean (median) cash return, but the differences are not statistically significant. However, the median cash return in the sub sample corresponding to the mispricing band {-2: X $\leq$ -1.0%} is significantly *below* the overall median cash return; and the median cash return in the sub sample corresponding to the mispricing band {+2: 1.0% $<$ X} is significantly *above* the overall median cash return.

Furthermore, the percentage of observations in the negative mispricing bands {-2: X $\leq$ -1.0%} and {-1: -1.0% $<$ X $\leq$ -0.5%} which also correspond to the lowest decile (quartile) of cash returns is *higher* than 10%(25%) while the percentage of observations which also correspond to the highest decile (quartile) of cash returns is *lower* than 10%(25%). Similarly, the percentage of observations in the positive mispricing bands {+2: 1.0% $<$ X} and {+1: 0.5% $<$ X $\leq$ 1.0%} which also correspond to the highest decile (quartile) of cash returns is *higher* than 10%(25%) while the percentage of observations which also correspond to the lowest decile (quartile) of cash returns is *lower* than 10%(25%). However, the differences between the observed proportions and the expected proportion of 10%(25%) are, at best, of marginal statistical significance. In terms of execution risk faced by index arbitrageurs, the differences are of virtually no economic significance. 86.6%(67.5%) of observations in the largest magnitude positive mispricing band {2: 1.0% < X} do *not* correspond to the highest decile (quartile) of cash returns; and 89.5%(71.8%) of observations in the largest magnitude negative mispricing band {-2: X ≤ -1.0%} do *not* correspond to the lowest decile (quartile) of cash returns. While there is a tendency for significant negative (positive) mispricing to be accompanied by significant negative (positive) cash returns it cannot explain the existence of arbitrage opportunities to any economically significant extent in terms of the difficulties faced by arbitrageurs in executing the cash leg of the transaction due to market makers being short of stock when the arbitrageur needs to buy stocks and market makers having surplus stock when the arbitrageur needs to sell stocks.

3.6 Short-Selling Constraints

There is no need for an arbitrageur to borrow stocks, and hence no need for special stock borrowing privileges, to exploit negative mispricing during the account settlement period which spans the futures maturity date. This unique feature of the London Stock Exchange settlement procedures provides an opportunity to test whether short selling constraints contribute significantly to futures underpricing.

Within the account period spanning futures maturity, the average (median) mispricing of the maturing contract is found to be 0.146%(0.148%) - significantly positive (p value < 0.0001) and significantly greater than the -0.15%(-0.09%) reported in Table 2A for the aggregate sample of all near contracts. However, considering that average (median) has been both significantly positive and significantly negative in different periods, this could potentially have arisen because the account period spanning futures maturity has coincidentally corresponded to a period with positive average mispricing. To control for this possibility we analyse the mispricing variable, separately for the near contract and the far contract, in three different subperiods - sub period 0 corresponding to the account period spanning near futures maturity, sub period 1 corresponding to the account period just before the account period spanning near futures maturity, and sub period 2 corresponding to all other account periods. Since the direction of the average mispricing in the near contract has tended to be the same as the direction of average mispricing in the far contract at that time, we would expect that, if short selling constraints have no impact on futures mispricing within sub period 0, the average (median) mispricing of the near contract should be about equal to the average (median) mispricing of the far contract in that sub period. Furthermore, in view of the strong tendency of mispricing to be predominantly positive or predominantly negative over long periods (reflected in Tables 2A/2B), we would expect that, if short selling constraints have no impact on futures within sub period 0, the average (median) mispricing of the near contract in sub period 0 should be about equal to the average (median) mispricing of the near contract in sub period 1.

Accordingly, Table 8 provides descriptive statistics on the mispricing variable for each of these three sub samples and reports the results of testing the following hypotheses:

H_{N0-N1}: The average (median) mispricing of the near contract in sub period 0 is equal to the average (median) mispricing of the near contract in sub period 1.

H_{N0-N2}: The average (median) mispricing of the near contract in sub period 0 is equal to the average (median) mispricing of the near contract in sub period 2.

H_{N0-F0}: The average (median) mispricing of the near contract in sub period 0 is equal to the average (median) mispricing of the far contract in sub period 0.

H_{N1-F1}: The average (median) mispricing of the near contract in sub period 1 is equal to the average (median) mispricing of the far contract in sub period 1.

H_{N2-F2}: The average (median) mispricing of the near contract in sub period 2 is equal to the average (median) mispricing of the far contract in sub period 2.

H_{F0-F1}: The average (median) mispricing of the far contract in sub period 0 is equal to the average (median) mispricing of the far contract in sub period 1.

H_{F0-F2}: The average (median) mispricing of the far contract in sub period 0 is equal to the average (median) mispricing of the far contract in sub period 2.

Hypotheses H_{N0-N1}, H_{N0-N2}, and H_{N0-F0} are conclusively rejected each with a p value < 0.0001 while none of the other hypotheses can be rejected at the 5% level.

Within the account period spanning futures maturity, the average (median) mispricing of the expiring contract is significantly greater (p value < 0.0001) than the -0.045% (-0.177%) of the far contract at that time; significantly greater (p value < 0.0001) than the 0.026% (-0.009%) of the expiring contract in the immediately preceding account period; and significantly greater (p value < 0.0001) than the -0.158% (-0.190%) of the expiring contract in all other account periods. These results clearly provide strong support for the view that short selling constraints have a significant impact on the average behaviour of the mispricing variable.

3.7 Misspecification of Dividends

The fair value of the futures contract depends on three *ex ante* dividend *forecasts* relating to the *future* dividend profile on the index:

(a) Magnitude of dividends

(b) Ex dividend date

(c) Actual dividend payment date.

Our basic mispricing estimates are based on the assumption that the dividend forecast used by the arbitrageur is equal to the actual *ex post* dividend inflow on the index.[34] The perfect foresight assumption is clearly unrealistic. This section

[34] Most US studies (eg Mackinlay and Ramaswamy, 1988; Klemkosky and Lee, 1991; Bhatt and Caciki, 1990) have a potentially serious dividend misspecification problem since they mix cash/futures prices on the S&P500 index with dividend yields from the small stock dominated NYSE/AMEX portfolio. These studies do not even estimate actual individual stock dividends, ex dividend dates and actual dividend payment dates.

examines first whether the systematic mispricing reported in Section 3.3 could arise due to risk of misspecification of dividends.

Typically arbitrageurs use the previous year's dividend plus an ad hoc growth factor as their dividend forecast. Table 9 accordingly reports descriptive statistics on a variable $d_{div1}(y)$ defined as:

$d_{div1}(y)$ = Mispricing estimated from actual *ex post* dividend inflows minus mispricing estimated using last year's dividend plus a y% growth factor
y = 0%, 10% and 20%.

0% and 20% growth are conservative assumptions on either side so as to assess the sensitivity of mispricing estimates to the maximum dividend misspecification likely to be possible. The average/median value of d_{div1} is clearly insignificant for cash contract and cannot explain the systematic biases in observed mispricing. The magnitude of d_{div1} is below 0.1% in about two-thirds of cases. A possible measure of the risk faced by the arbitrageur on this account is the interquartile range of the $d_{div1}(y)$ variable. This risk will be higher for longer times to maturity. However, Table 9 indicates that the risk on this account is always about 0.1% and hence largely insignificant.

Our basic mispricing estimates are also based on the assumption that the average time interval between the ex dividend date and the actual dividend payment date is equal to the average *ex post* value of this time interval for all index constituents over the sample period. Variation in this time interval from year to year can result in uncertainty in estimated mispricing. Table 9 also reports descriptive statistics on a variable $d_{div2}(y)$ defined as:

$d_{div2}(y)$ = Mispricing estimated assuming that the time interval between the ex dividend date and the actual dividend payment date is equal to the actual *ex post* value of this time interval for all index constitutents minus the mispricing estimated assuming misspecification of the above time interval by a time interval y

y = 4 weeks and 8 weeks

The average value of $d_{div2}(y)$ variable, and its interquartile range, are both clearly insignificant for all contracts, showing that this factor is important neither for systematic price biases nor as a determinant of arbitrage risks.

In this context, it is relevant to note that error in forecasting the ex dividend date will affect only the values of mispricing during the interval between the forecast and actual ex dividend dates and the contribution to average mispricing *over the contract* will be much smaller. Thus a 4 week and 8 week error in forecasting the ex dividend date will have an even smaller effect than that measured by $d_{div2}(y)$ above.

Since dividends are typically announced at least a few weeks before the ex dividend date, there is essentially no uncertainty arising due to stochastic dividends if the time remaining to maturity is so small that all companies going ex dividend before futures maturity have actually declared their dividends.

3.8 Marking to Market Cash Flows

The mispricing estimates of Section 3.3 have been calculated on the assumption that futures prices are the same as forward prices ie after ignoring marking to market cash flows. Cox *et al* (1981, pp 326 Proposition 6) derive the relationship between futures prices and forward prices. Even if interest rates remain constant, the impact of marking to market cash flows depends on the path of *ex post* futures price changes up to maturity, and this makes arbitrage risky. This risk should arguably lead to an effective increase in the width of the arbitrage window, and needs to be estimated to judge the significance of the mispricing estimates in Section 3.3.

The *ex post* impact of marking to market cash flows can be estimated on the basis of Cox *et al* (1981, Proposition 6, pp 326). We accordingly estimate the following variable using day end settlement prices for each day t in the sample period:

$$d_{mm1}(t) = \frac{1}{S_t} \sum_{\tau=t}^{T-1} (F_{\tau,T}-F_{\tau+1,T}) \left\{ \frac{\exp(r_{\tau+1,\tau+1,T}(T-\tau-1))}{\exp(r_{\tau,\tau,T}(T-\tau)} - 1 \right\}$$

$d_{mm1}(t)$ is a measure of the *ex post* misspecification of the mispricing variable (as defined in equation (2)) because of the marking to market cash flows. However, it is a valid measure only if the futures position is rebalanced daily.[35] Arbitrageurs typically buy the futures contract in lieu of the forward contract, treating the two positions as equivalent except for an estimated price adjustment reflecting the additional cost of financing the futures position. In this context, the *ex post* impact on arbitrageurs arising from marking to market cash flows can be estimated by assuming that the arbitrage position is established and held to expiration as per Trading Rule 1 with futures contracts bought or sold accordingly only at the time the arbitrage position was first established and all (possibly negative) marking to market cash flows invested in zero coupon bonds maturing at futures maturity. Accordingly, to capture the *ex post* misspecification of the mispricing variable, the following variable is also estimated using day end settlement prices for each day t in the sample period:

$$d_{mm2}(t) = \frac{1}{S_t} \sum_{\tau=t}^{T-1} (F_{\tau,T}-F_{\tau+1,T}) \{\exp(r_{\tau+1,\tau+1,t}(T-\tau-1))-1\}$$

Table 10 shows that the average (median) impact of marking to market cash flows, whether measured by d_{mm1} or d_{mm2} is below 0.02%. The average (median) impact over different contracts is also not significantly correlated with the observed average

[35] As indicated in Cox *et al* (1981, Proposition 6).

systematic bias. The contribution of marking to market cash flows to the risk of the arbitrage position as measured by the interquartile range of d_{mm1} or d_{mm2} is also well below 0.1%, and hence insignificant. However, the large magnitude of the maximum and minimum values of the d_{mm2} variable shows that, if the futures position is not rebalanced with reference to daily price changes, the risk on this account can occasionally be substantial. Depending on the degree of risk aversion, the extreme values could potentially be important.

3.9 Absolute Mispricing and Time to Maturity

The impact on mispricing of the uncertainty about dividends, marking-to-market cash flows and future volatility should be greater for longer times to maturity leading to a wider arbitrage window and hence a higher absolute magnitude of mispricing. However, it is difficult to specify a precise functional form for the dependence of absolute mispricing on time to maturity. Hence, to investigate this dependence, we sort the data into 5 groups based on time to maturity (T-t) as follows:[36]

Group 1: $\{0 \leq (T\text{-}t) \leq 30\}$

Group 2: $\{31 \leq (T\text{-}t) \leq 60\}$

Group 3: $\{61 \leq (T\text{-}t) \leq 90\}$

Group 4: $\{91 \leq (T\text{-}t) \leq 120\}$

Group 5: $\{121 \leq (T\text{-}t) \leq 180\}$

[36] Groups 1, 2, 3 and 4 are monthly intervals. Group 5 is a two month interval in view of the relatively few observations for such long times to maturity.

Table 11 documents the average/median absolute magnitude of mispricing for each of the above groups. Consistent with the results of MacKinlay and Ramaswamy (1988) and Yadav and Pope (1990), the absolute magnitude of mispricing appears to be clearly greater for longer times to maturity.[37] Using a dummy variable based regression procedure, the hypothesis of equality of mean mispricing across different groups is conclusively rejected. Similarly, the hypothesis of equality of median mispricing across different groups is conclusively rejected using the Mood (1954) non-parametric test. To examine whether the relationship between the absolute magnitude of mispricing and time to maturity is monotonic in nature, the following hypothesis is tested:

H_{TIME}: Median absolute mispricing in Group 1 < Median absolute mispricing in Group 2 < Median absolute mispricing in Group 3 < Median absolute mispricing in Group 4 < Median absolute mispricing in Group 5.

H_{TIME} is tested using the Jonckheere (1954) - Terpstra (1952) non-parametric test for ordered alternatives.[38] H_{TIME} is conclusively rejected with a Z-statistic of 78.40. This is consistent with arbitrage being perceived as more risky for longer times to maturity.

[37] The average/median mispricing in each of these groups was also analysed, and consistent with the results of Cornell (1985a) and Yadav and Pope (1990) there did not appear to be a monotonic relationship between mispricing and time to maturity.

[38] This is a popular test for an application of this nature. For details see eg Daniel (1978, pp 207-210).

3.10 Mispricing and Implied Index Volatility

The value of the tax timing option highlighted by Cornell and French (1983a, 1983b) should be greater for higher implied index volatility, suggesting that, irrespective of the futures pricing model and the effects of other factors, mispricing should be lower for greater implied index volatility. The general equilibrium model of Hemler and Longstaff (1991) also predicts a dependence of futures mispricing on implied index volatility. To investigate this dependence, the data are sorted into 5 equally sized groups on the basis of the volatility implied in option prices with Group 1 corresponding to the lowest volatility and Group 5 to the highest volatility.[39]

Table 12 documents the average/median mispricing, the proportion of positive mispricing values, and the corresponding average/median implied index volatility, for each of these groups. The mispricing appears to be clearly higher for greater implied index volatility.[40]Using a dummy variable based regression procedure, the hypothesis of equality of average mispricing across different groups is conclusively rejected. Similarly, the hypothesis of equality of median mispricing across different groups is conclusively rejected using the Mood (1954) non-parametric test. To test whether the relationship between mispricing and implied volatility is monotonic in nature, the following hypothesis is tested:

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[39] To avoid potential distortion due to outliers, a period of twenty trading days starting on Black Monday, October 19, 1987, and the day of the "mini-crash", October 16, 1989, are omitted from the data.

[40] On the other hand, the average/median absolute magnitude of mispricing in each of these groups was also examined, and there did not appear to be a monotonic relationship between the absolute magnitude of mispricing and the volatility implied in call option prices.

H_{IVOL}: Median mispricing in Group 1 < Median mispricing in Group 2 < Median mispricing in Group 3 < Median mispricing in Group 4 < Median mispricing in Group 5.

H_{IVOL} is again tested using the Jonckheere (1954)-Terpstra (1952) non-parametric test for ordered alternatives, and is conclusively rejected with a Z-statistic of 33.31, confirming the systematic variation of mispricing with *ex ante* implied index volatility. However, the dependence of mispricing on implied call volatility documented in Table 12 is *opposite* to what can be expected if implied call volatility affects futures mispricing only because of the possible value of the tax timing option.

In this context, it is important to note that the futures contract on the index can be replicated by a position consisting of a long call, a short put and riskless assets. Hence the strong positive dependence of futures mispricing on call implied volatility can potentially be driven by "mispricing" of the index call (relative to the cash index) tending to be in the same direction as the "mispricing" of the futures contract relative to cash (just as the far futures contract has been found to be mispriced in the same direction as the near futures contract). Because of the difficulties involved in trading the index basket of stocks, market makers in options markets are likely to use index futures, rather than the cash index, to hedge their positions, and this can lead to index calls being mispriced in the same direction as index futures. However, this issue is not explored further in this dissertation and can be the subject of further research.

4. CONCLUSIONS

This chapter has analysed empirical evidence on stock index futures pricing based on about four years of synchronous hourly data from the UK. The London stock market is a pure dealership system and the index is based on obligatory market-maker quotes. The arbitrage opportunities identified with such a dataset are hence economically significant. The London markets are also an ideal laboratory for testing the effect on futures pricing of the constraints that exist on short selling of stocks. The chapter has also addressed several other issues not adequately analysed in past research. It has controlled for cash market settlement procedures; has assessed the difficulties that could arise because of index futures being overpriced in sharply rising markets and underpriced in sharply falling markets; has examined the systematic variation of futures mispricing with the market volatility implied in call option prices; and has simulated *ex post* the risks that have been involved in index arbitrage due to dividend uncertainty, marking-to-market cash flows, and possible delays in trade execution.

The salient results are as follows:

(a) The absolute magnitude of mispricing has often exceeded the estimated transaction costs of the more favourably positioned categories of arbitrageurs. Simulations show that even *ex ante* trading rules have provided attractive profits after transaction costs. In this context, both the early unwinding option and the rollover option have been very valuable suggesting potential for "risky arbitrage" and little possibility of expiration day effects.

The magnitude of mispricing cannot be explained by dividend uncertainties or the risk of marking-to-market cash flows.

(b) Average mispricing over a contract has often been significantly different from zero, but the direction of significant average mispricing has varied substantially from period to period. However, the direction of mispricing in the near contract at any particular time has tended to be the same as the direction of mispricing in the far contract at that time. The variation in systematic mispricing cannot be explained by variation in transaction costs.

(c) Though there has been a mild tendency for futures to be underpriced in sharply falling markets and overpriced in sharply rising markets, this has been essentially of no economic significance for index arbitrageurs.

(d) The analysis of futures mispricing in periods during which index arbitrageurs face no constraints on short selling show that short selling constraints have apparently made a very significant impact on futures pricing.

(e) The absolute magnitude of mispricing has been greater for longer times to maturity, consistent with arbitrage being perceived as more risky when time to maturity is greater.

(f) There appears to have been a strong positive relationship between futures mispricing and the *ex ante* market volatility implied by index call option prices. The direction of this relationship is found to be *opposite* to that predicted by the existence of a tax timing option, but is consistent with index calls being "mispriced" (relative to the cash index) in the same direction as index futures contracts.

TABLE 1

TRANSACTION COSTS OF DIFFERENT CATEGORIES OF ARBITRAGEURS

Category A: Arbitrageurs with marginal costs confined to transaction costs in the futures market.

Category B: Arbitrageurs with marginal costs including also cash market related transaction costs but not including transaction taxes or cost of borrowing stocks or capital.

Category C: Arbitrageurs with marginal costs also including transaction taxes.

Quarter Ending	Cash Average Inner Market Spread	Futures Median Spread Near Contract	Futures Median Spread Far Contract	Near Contract Transaction Costs of Different Categories of Arbitrageurs			Far Contract Transaction Costs of Different Categories of Arbitrageurs		
				Category A	Category B	Category C	Category A	Category B	Category C
Jun 86	0.75	0.12	0.34	0.11	0.54	1.54	0.22	0.65	1.65
Sep 86	0.75	0.13	0.35	0.12	0.55	1.55	0.23	0.66	1.66
Dec 86	0.75	0.12	0.34	0.11	0.54	1.04	0.22	0.65	1.15
Mar 87	0.73	0.10	0.31	0.10	0.52	1.02	0.21	0.63	1.13
Jun 87	0.76	0.10	0.44	0.10	0.53	1.03	0.27	0.70	1.20
Sep 87	0.84	0.08	0.40	0.09	0.57	1.07	0.25	0.73	1.23
Dec 87	1.52	0.15	0.39	0.13	1.00	1.50	0.25	1.12	1.62
Mar 88	1.27	0.11	0.44	0.11	0.83	1.33	0.27	0.99	1.49
Jun 88	1.15	0.11	0.22	0.11	0.77	1.27	0.16	0.82	1.32
Sep 88	0.80	0.08	0.22	0.09	0.55	1.05	0.16	0.62	1.12
Dec 88	0.85	0.06	0.17	0.08	0.56	1.06	0.14	0.62	1.12
Mar 89	0.84	0.05	0.22	0.08	0.56	1.06	0.16	0.64	1.14
Jun 89	0.87	0.05	0.18	0.08	0.58	1.08	0.14	0.64	1.14
Sep 89	0.87	0.05	0.18	0.08	0.58	1.08	0.14	0.64	1.14
Dec 89	1.23	0.05	0.12	0.08	0.78	1.28	0.11	0.81	1.31
Mar 90	1.24	0.04	0.14	0.07	0.77	1.27	0.12	0.82	1.32

TABLE 2A

NEAR CONTRACT MISPRICING - DESCRIPTIVE STATISTICS

Contract Expiring	No. of Obs.	% of -ve Misp. Values	Mean μ	Std. Dev.	Median	Min	Max	Quartiles Lower Q_1	Quartiles Upper Q_3	(Q_3-Q_1)	Percentiles 5th P_5	Percentiles 95th P_{95}	$(P_{95}-P_5)$	Serial Corr. adj. T-Stat $H_0:\mu=0$
Jun 86	113	80.5	-0.35	0.45	-0.34	- 1.27	0.73	-0.66	-0.07	0.59	-1.03	0.42	1.45	-2.49
Sep 86	160	20.0	0.50	0.56	0.44	- 0.82	1.93	0.09	0.88	0.80	-0.30	1.54	1.84	5.31
Dec 86	148	75.0	-0.35	0.52	-0.30	- 1.53	0.85	-0.74	0.00	0.75	-1.28	0.43	1.71	-3.35
Mar 87	235	15.7	0.39	0.36	0.41	- 0.40	1.35	0.12	0.66	0.53	-0.21	0.98	1.19	7.44
Jun 87	264	38.3	0.14	0.59	0.26	- 1.10	1.42	-0.33	0.59	0.92	-0.88	0.96	1.83	1.21
Sep 87	264	50.8	0.02	0.51	-0.00	- 1.33	1.39	-0.29	0.37	0.66	-0.82	0.82	1.64	0.15
Dec 87	272	64.7	-0.76	1.55	-0.30	-10.80	1.00	-1.28	0.21	1.49	-3.82	0.61	4.43	-2.78
Mar 88	221	69.2	-0.22	0.39	-0.26	- 1.11	0.82	-0.50	0.04	0.55	-0.83	0.49	1.32	-11.06
Jun 88	193	75.1	-0.37	0.44	-0.46	- 1.21	0.70	-0.76	0.00	0.76	-0.97	0.34	1.31	-4.18
Sep 88	241	96.7	-0.72	1.03	-0.57	- 7.04	0.26	-0.84	-0.32	0.52	-1.13	-0.03	1.10	-4.10
Dec 88	240	49.2	-0.07	0.61	0.01	- 1.47	1.12	-0.44	0.38	0.81	-1.23	0.82	2.04	-0.29
Mar 89	313	11.8	0.50	0.39	0.53	- 0.42	1.68	0.24	0.74	0.50	-0.18	1.10	1.28	8.82
Jun 89	295	23.4	0.29	0.35	0.30	- 0.48	1.29	0.02	0.51	0.49	-0.28	0.94	1.21	2.82
Sep 89	281	81.9	-0.38	0.39	-0.36	- 1.50	0.51	-0.60	-0.10	0.50	-1.14	0.23	1.37	-3.91
Dec 89	310	76.5	-0.62	0.64	-0.73	- 2.27	0.71	-1.09	-0.03	1.05	-1.61	0.42	2.02	-3.02
Mar 90	312	89.7	-0.38	0.28	-0.39	- 1.15	0.23	-0.56	-0.17	0.39	-0.86	0.07	0.93	-6.10
All	3862	56.6	-0.15	0.77	-0.09	-10.80	1.93	-0.54	0.32	0.86	-1.17	0.85	2.01	-2.54

TABLE 2B

FAR CONTRACT MISPRICING - DESCRIPTIVE STATISTICS

Contract Expiring	No. of Obs.	% of -ve Misp. Values	Mean μ	Std. Dev.	Median	Min	Max	Quartiles Lower Q_1	Quartiles Upper Q_3	(Q_3-Q_1)	Percentiles 5th P_5	Percentiles 95th P_{95}	$(P_{95}-P_5)$	Serial Corr. adj. T-Stat $H_0:\mu=0$
Jun 86	37	32.4	0.24	0.44	0.23	- 0.51	1.28	-0.15	0.55	0.70	-0.50	0.93	1.44	1.57
Sep 86	62	25.8	0.66	0.92	0.41	- 0.95	2.68	-0.08	1.42	1.50	-0.73	2.26	2.99	2.35
Dec 86	42	45.2	-0.01	0.55	0.12	- 1.01	0.88	-0.57	0.40	0.97	-0.87	0.78	1.65	-0.04
Mar 87	79	8.9	0.65	0.51	0.70	- 0.73	1.65	0.35	0.98	0.63	-0.27	1.47	1.74	6.30
Jun 87	90	1.1	1.31	0.51	1.41	- 0.07	2.53	1.00	1.67	0.67	0.10	2.04	1.94	15.32
Sep 87	62	48.4	0.02	0.52	0.07	- 1.07	0.93	-0.39	0.45	0.85	-0.86	0.85	1.71	0.13
Dec 87	65	10.8	0.92	0.79	1.10	- 1.79	2.90	0.54	1.40	0.86	-0.77	1.75	2.52	6.29
Mar 88	86	67.4	-0.16	0.44	-0.18	- 0.96	0.95	-0.50	0.13	0.62	-0.85	0.64	1.49	- 1.69
Jun 88	87	88.5	-0.27	0.26	-0.28	- 0.79	0.59	-0.47	-0.10	0.38	-0.66	0.14	0.81	-14.92
Sep 88	78	100.0	-1.53	1.03	-1.38	- 7.84	-0.79	-1.54	-1.18	0.36	-1.98	-0.98	0.99	-10.68
Dec 88	75	100.0	-0.50	0.29	-0.47	- 1.31	-0.02	-0.67	-0.27	0.40	-1.10	-0.12	0.97	-6.29
Mar 89	105	8.6	0.82	0.46	0.89	- 0.77	1.57	0.72	1.08	0.36	-0.30	1.36	1.66	8.82
Jun 89	73	63.0	-0.19	0.43	-0.13	- 1.51	0.60	-0.43	0.11	0.54	-1.10	0.33	1.43	-1.82
Sep 89	102	100.0	-0.86	0.30	-0.84	- 1.74	-0.19	-1.04	-0.62	0.42	-1.45	-0.44	1.01	-8.84
Dec 89	84	96.4	-0.89	0.68	-0.67	- 2.73	0.13	-1.49	-0.34	1.14	-2.10	-0.07	2.02	-3.41
Mar 90	45	100.0	-1.10	0.28	-1.10	- 1.84	-0.44	-1.31	-0.89	0.42	-1.61	-0.71	0.90	-21.96
All	1172	56.6	-0.05	0.98	-0.13	- 7.84	2.90	-0.68	0.70	1.37	-1.48	1.55	3.02	-0.21

TABLE 3

RELATIVE MISPRICING OF NEAR AND FAR CONTRACTS

X_{Near} denotes mispricing of the near contract. X_{Far} denotes mispricing of the far contract. Only these 899 cases are considered for which (both) near and far contract futures prices were posted within 60 seconds of the cash index quote. In each cell of each of the two matrices below, the main observation is the number of cases which fall into the joint classification corresponding to that cell. The first parenthesis in every cell contains the number of cases expected if there was no relationship between near and far contract mispricing. The second parenthesis in every cell contains Z-statistic for the hypothesis that the the difference between the actual number of cases and the expected number of cases could have arisen purely by chance.

	$\{X_{Near} < 0\}$	$\{X_{Near} > 0\}$
	363	107
$\{X_{Far} < 0\}$	(269.2)	(200.8)
	(6.83***)	(-7.51***)
	152	277
$\{X_{Far} > 0\}$	(245.8)	(183.2)
	(-7.02***)	(7.76***)

	$\{X_{Near} \leq -1.0\%\}$	$\{-1.0\% < X_{Near} \leq -0.5\%\}$	$\{-0.5\% < X_{Near} \leq 0.5\%\}$	$\{0.5\% < X_{Near} \leq 1.0\%\}$	$\{1.0\% < X_{Near}\}$
	14	9	4	2	0
$\{X_{Far} \leq -1.0\%\}$	(4.7)	(5.2)	(11.0)	(3.5)	(4.5)
	(4.26***)	(1.68*)	(-2.12**)	(-0.82)	(-2.13**)
	50	24	19	3	3
$\{-1.0\% < X_{Far} \leq -0.5\%\}$	(16.2)	(17.7)	(37.6)	(12.1)	(15.4)
	(8.48***)	(1.50)	(-3.09***)	(-2.64***)	(-3.19***)
	83	125	289	81	53
$\{-0.5\% < X_{Far} \leq 0.5\%\}$	(103.2)	(113.0)	(239.3)	(77.2)	(98.3)
	(-2.11**)	(1.21)	(3.75***)	(0.45)	(-4.84***)
	0	3	28	24	67
$\{0.5\% < X_{Far} \leq 1.0\%\}$	(19.9)	(21.8)	(46.3)	(14.9)	(19.0)
	(-4.52***)	(-4.08***)	(-2.76***)	(2.37**)	(11.13***)
	0	0	1	0	17
$\{1.0\% < X_{Far}\}$	(2.9)	(3.2)	(6.8)	(2.2)	(2.8)
	(1.72*)	(-1.80*)	(-2.24**)	(-1.49)	(8.49***)

* Significant at the 10% level ** Significant at the 5% level *** Significant at the 1% level

TABLE 4

PERSISTENCE AND REVERSALS IN MISPRICING

	NEAR CONTRACT					FAR CONTRACT				
			Average Time Before Reversal (Trading Hrs)		First order Serial Correlation in Mispricing			Average Time Before Reversal (Trading Hours)		First order Serial Correlation in Mispricing
Quarter Ending	Number of Obs.	Number of Reversals	+ve Mispricing	-ve Mispricing		Number of Obs.	Number of Reversals	+ve Mispricing	-ve Mispricing	
Jun 86	113	16	7.1	29.2	0.808	37	9	41.6	20.0	0.942
Sep 86	160	33	21.3	5.4	0.503	62	15	42.6	14.7	0.754
Dec 86	148	33	6.7	19.7	0.903	42	9	49.1	40.5	0.586
Mar 87	235	34	21.2	3.9	0.758	79	9	80.4	7.8	0.851
Jun 87	264	25	20.2	12.6	0.879	90	2	281.3	3.3	0.750
Sep 87	264	43	10.2	10.5	0.872	62	7	58.7	55.0	0.936
Dec 87	272	40	7.7	14.2	0.739	65	5	133.0	15.9	0.906
Mar 88	221	42	6.5	14.4	0.773	86	21	13.0	27.6	0.867
Jun 88	193	25	8.2	24.8	0.882	87	11	8.3	62.8	0.802
Sep 88	241	12	2.3	67.6	0.821	78	-	-	-	0.586
Dec 88	240	26	16.5	16.0	0.960	75	-	-	-	0.713
Mar 89	313	21	34.8	4.7	0.837	105	3	198.4	18.6	0.780
Jun 89	295	37	17.8	5.4	0.891	73	11	27.2	46.5	0.935
Sep 89	281	17	9.1	40.8	0.932	102	-	-	-	0.866
Dec 89	310	20	10.0	32.2	0.946	84	4	6.3	170.1	0.926
Mar 90	312	34	2.4	21.2	0.783	45	-	-	-	-
All	3862	466	14.8	19.3	0.864	1172	110	62.2	80.4	0.958

ARBITRAGE PROFIT SIMULATIONS; CATEGORY A ARBITRAGEURS

Contract Expiring	Total Number of Observations	Trading Rule 1 (Hold to Expiration) Number of Arbitrage Opportunities	Arbitrage Profits/Contract (£000) Ex post Simulations	Arbitrage Profits/Contract (£000) Ex ante Simulations	Percentage of Profitable Trades in ex ante Simulations	Trading Rule 2 (Early Unwinding) Number of Early Unwindings	Additional Arbitrage Profits/Contract (£000) Ex post Simulations	Additional Arbitrage Profits/Contract (£000) Ex ante Simulations	Average Time to Early Unwinding (Trading Hours)
Jun 86	113	48	10.3	1.4	39.6	47	5.3	-0.2	260.8
Sep 86	160	79	22.6	0.4	27.8	78	19.2	9.6	389.3
Dec 86	148	60	15.6	2.6	40.0	7	1.5	-0.1	74.8
Mar 87	235	101	23.1	8.5	59.4	0	0.0	0.0	0.0
Jun 87	264	134	36.4	14.7	62.7	81	17.7	12.8	105.5
Sep 87	264	90	26.6	12.0	66.7	88	16.2	0.2	116.5
Dec 87	272	141	91.1	59.3	65.2	140	21.6	-0.4	79.7
Mar 88	221	67	12.9	2.6	53.7	49	7.8	3.8	70.5
Jun 88	193	90	21.1	8.9	64.4	90	11.3	-8.1	335.2
Sep 88	241	138	51.8	26.4	58.0	0	0.0	0.0	-
Dec 88	240	99	27.1	9.6	55.6	57	9.9	8.6	439.2
Mar 89	313	167	45.9	31.7	79.6	0	0.0	0.0	-
Jun 89	295	78	19.4	11.4	76.9	73	9.8	8.4	261.5
Sep 89	281	109	31.1	16.1	67.0	47	8.4	0.7	150.2
Dec 89	310	188	84.2	60.5	78.7	182	55.0	54.5	302.0
Mar 90	312	100	25.3	12.5	75.0	0	0.0	0.0	-
All	3862	1689	544.4	278.7	63.9	939	183.9	89.2	258.0

ARBITRAGE PROFIT SIMULATIONS; CATEGORY A ARBITRAGEURS

Contract Expiring	Number of Rollovers	Trading Rule 3 (Rollover) Additional Arbitrage Profits/Contract (£000) Ex Post Simulations	Ex Ante Simulations	Average Time to Rollover (Trading Hrs)	Number of Early Unwindings/ Rollovers	Trading Rule 4 (Early Unwinding/Rollover) Number of Early Unwindings	Number of Rollovers	Additional Arbitrage Profits/Contract (£000) Ex Post Simulations	Ex Ante Simulations	Average Time to Early Unwinding/ Rollover
Jun 86	15	2.1	-0.9	64.1	47	32	15	5.6	-0.8	178.0
Sep 86	72	26.5	-3.8	84.2	78	9	69	27.9	-4.0	77.9
Dec 86	57	6.7	-6.2	227.6	57	7	50	7.4	-5.5	125.3
Mar 87	101	27.6	16.0	81.1	101	0	101	27.6	16.0	81.1
Jun 87	132	47.1	18.7	84.8	134	25	109	42.1	6.1	55.6
Sep 87	88	21.6	0.9	338.5	90	51	39	19.5	0.9	81.2
Dec 87	141	56.8	11.6	301.6	141	105	36	45.1	21.9	54.7
Mar 88	67	13.7	6.6	180.8	67	27	40	11.6	2.8	47.4
Jun 88	90	11.6	1.1	53.5	90	6	84	11.4	1.4	52.7
Sep 88	138	49.4	0.6	43.2	138	0	138	49.4	0.6	43.2
Dec 88	57	22.3	5.6	67.3	57	0	57	22.3	5.6	67.3
Mar 89	167	44.5	4.7	99.0	167	0	167	44.5	4.7	99.0
Jun 89	71	11.5	11.0	198.2	73	2	71	11.7	11.3	194.5
Sep 89	108	30.8	14.5	41.0	109	14	95	27.7	17.1	31.7
Dec 89	184	56.9	54.9	39.5	184	0	184	56.9	54.9	39.5
Mar 90	100	33.1	3.3	22.8	100	0	100	33.1	3.3	22.8
All	1588	462.3	138.5	136.2	1633	278	1355	443.9	136.1	81.4

ARBITRAGE PROFIT SIMULATIONS; CATEGORY B ARBITRAGEURS

Contract Expiring	Total Number of Observations	Trading Rule 1 (Hold to Expiration): Number of Arbitrage Opportunities	Trading Rule 1: Arbitrage Profits/Contract (£000) Ex post Simulations	Trading Rule 1: Arbitrage Profits/Contract (£000) Ex ante Simulations	Trading Rule 1: Percentage of Profitable Trades in ex ante Simulations	Trading Rule 2 (Early Unwinding): Number of Early Unwindings	Trading Rule 2: Additional Arbitrage Profits/Contract (£000) Ex post Simulations	Trading Rule 2: Additional Arbitrage Profits/Contract (£000) Ex ante Simulations	Trading Rule 2: Average Time to Early Unwinding (Trading Hours)
Jun 86	113	7	1.1	0.5	57.1	7	0.8	0.0	378.0
Sep 86	160	32	7.5	-4.6	28.1	32	8.0	4.2	333.3
Dec 86	148	20	3.7	-2.8	35.0	0	0.0	0.0	-
Mar 87	235	12	2.0	-1.6	41.7	0	0.0	0.0	-
Jun 87	264	20	3.8	-1.3	50.0	16	3.6	2.1	81.8
Sep 87	264	12	2.8	0.6	75.0	12	2.2	0.7	207.9
Dec 87	272	82	61.8	38.3	67.1	82	11.8	-1.0	99.3
Mar 88	221	2	0.3	-0.6	0.0	2	0.4	0.5	91.2
Jun 88	193	8	1.3	0.1	75.0	8	1.0	-0.7	410.6
Sep 88	241	26	20.3	8.2	50.0	0	0.0	0.0	-
Dec 88	240	28	5.8	-2.0	53.6	26	4.5	4.0	494.8
Mar 89	313	24	6.1	2.0	70.8	0	0.0	0.0	-
Jun 89	295	9	1.6	0.4	77.8	9	1.2	1.1	326.5
Sep 89	281	17	4.6	3.1	88.2	17	3.0	0.2	158.0
Dec 89	310	96	30.2	13.1	76.0	96	29.0	28.6	282.0
Mar 90	312	8	1.5	0.4	50.0	0	0.0	0.0	-
All	3862	403	154.5	52.0	61.8	307	65.6	39.7	288.3

ARBITRAGE PROFIT SIMULATIONS; CATEGORY B ARBITRAGEURS

Contract Expiring	Number of Rollovers	Trading Rule 3 (Rollover) Additional Arbitrage Profits/Contract (£000) Ex post Simulations	Trading Rule 3 (Rollover) Additional Arbitrage Profits/Contract (£000) Ex ante Simulations	Average Time to Rollover (Trading Hrs)	Number of Early Unwindings/ Rollovers	Trading Rule 4 (Early Unwinding/Rollover) Number of Early Unwindings	Number of Rollovers	Additional Arbitrage Profits/Contract (£000) Ex post Simulations	Additional Arbitrage Profits/Contract (£000) Ex ante Simulations	Average Time to Early Unwinding/ Rollover
Jun 86	5	0.8	-0.6	22.9	7	2	5	1.0	-0.6	107.4
Sep 86	32	16.7	-1.6	29.6	32	0	32	16.7	-1.6	29.6
Dec 86	20	2.5	-2.0	100.5	20	0	20	2.5	-2.0	100.5
Mar 87	12	3.9	2.2	50.3	12	0	12	3.9	2.2	50.3
Jun 87	20	6.8	4.7	122.0	20	8	12	4.9	0.4	56.6
Sep 87	12	3.9	1.9	230.6	12	3	9	3.7	2.3	79.8
Dec 87	82	36.2	15.7	347.2	82	62	20	32.8	22.8	65.2
Mar 88	2	0.5	0.4	68.9	2	0	2	0.5	0.4	68.9
Jun 88	8	1.0	1.0	52.0	8	0	8	1.0	1.0	52.0
Sep 88	26	8.3	-1.7	73.4	26	0	26	8.3	-1.7	73.4
Dec 88	26	10.6	4.1	65.8	26	0	26	10.6	4.1	65.8
Mar 89	24	5.5	-0.3	118.8	24	0	24	5.5	-0.3	118.8
Jun 89	9	1.5	1.5	257.7	9	0	9	1.5	1.5	257.7
Sep 89	17	3.8	6.9	17.2	17	0	17	3.8	6.9	17.2
Dec 89	96	28.4	31.2	55.5	96	0	96	28.4	31.2	55.5
Mar 90	8	2.1	0.8	4.5	8	0	8	2.1	0.8	4.5
All	399	132.4	64.1	162.0	401	75	326	127.0	67.3	81.0

RISKY ARBITRAGE STRATEGIES

The arbitrage position is initiated whenever mispricing exceeds X% in magnitude (X=0.5%,1.0%) even when actual transaction costs exceed X%. The position is unwound early, or rolled forward, as soon as the additional profit from early unwinding, or the additional profit from rollover, makes the overall position profitable after inclusion of the incremental transaction costs involved in early unwinding/rollover.

Trading Rule Threshold = 0.5%

	Transaction costs = 0.75%		Transaction Costs = 1.0%		Transaction Costs = 1.25%		Transaction Costs = 1.5%	
Quarter Ending	% of Profitable Positions	Total Profit/Contract (£000)	% of Profitable Positions	Total Profit/Contract (£000)	% of Profitable Positions	Total Profit/Contract (£000)	% of Profitable Positions	Total Profit/Contract (£000)
Jun 86	97.9	4.7	97.9	2.4	58.3	- 3.8	29.2	-10.2
Sep 86	100.0	25.5	97.5	28.7	91.1	29.4	87.3	28.0
Dec 86	95.0	9.3	85.0	3.7	55.0	- 2.9	41.7	- 8.4
Mar 87	100.0	23.0	100.0	18.5	100.0	13.0	100.0	10.6
Jun 87	100.0	35.8	100.0	24.5	100.0	21.9	100.0	22.4
Sep 87	100.0	15.8	94.4	7.9	71.1	3.5	62.2	- 8.3
Dec 87	100.0	103.2	100.0	90.3	100.0	83.0	100.0	77.5
Mar 88	100.0	7.2	95.5	3.5	55.2	- 7.4	7.5	-22.8
Jun 88	100.0	10.5	92.2	2.3	51.1	- 10.5	13.3	-25.8
Sep 88	100.0	69.5	100.0	53.7	100.0	42.4	100.0	33.8
Dec 88	57.6	27.1	57.6	15.9	57.6	4.8	51.5	- 8.5
Mar 89	100.0	21.4	100.0	22.4	100.0	22.1	100.0	12.8
Jun 89	94.9	10.7	46.2	- 5.4	16.7	- 19.3	1.3	-31.5
Sep 89	100.0	27.0	100.0	19.5	99.1	13.8	73.4	-10.2
Dec 89	98.9	88.7	97.3	63.1	96.3	40.8	96.3	27.7
Mar 90	100.0	29.3	100.0	16.2	99.0	11.0	57.0	-17.8
All	96.9	508.7	93.1	367.3	84.0	241.7	72.9	69.3

TABLE 6 (CONT'D)

RISKY ARBITRAGE STRATEGIES

Trading Rule Threshold = 1.0%

Quarter Ending	Transaction costs = 1.25% % of Profitable Positions	Total Profit/Contract (£000)	Transaction Costs = 1.5% % of Profitable Positions	Total Profit/Contract (£000)	Transaction Costs = 1.75% % of Profitable Positions	Total Profit/Contract (£000)	Transaction Costs = 2.0% % of Profitable Positions	Total Profit/Contract (£000)
Jun 86	100.0	0.6	100.0	0.2	28.6	- 1.3	0.0	- 2.5
Sep 86	100.0	12.1	100.0	15.9	100.0	14.5	100.0	14.7
Dec 86	100.0	1.7	90.0	0.3	35.0	- 3.3	10.0	- 6.1
Mar 87	100.0	2.7	100.0	2.1	100.0	1.1	100.0	1.0
Jun 87	100.0	2.9	100.0	1.9	100.0	1.2	100.0	1.8
Sep 87	100.0	2.9	83.3	1.1	75.0	0.1	50.0	- 2.5
Dec 87	100.0	76.2	100.0	68.2	100.0	61.8	100.0	58.8
Mar 88	100.0	0.3	100.0	0.1	0.0	- 0.6	0.0	- 0.8
Jun 88	100.0	0.3	87.5	- 0.1	0.0	- 2.3	0.0	- 3.2
Sep 88	100.0	22.5	100.0	19.5	100.0	17.0	100.0	22.0
Dec 88	92.9	10.1	92.9	6.9	92.9	3.7	75.0	- 1.5
Mar 89	100.0	2.7	100.0	2.0	100.0	2.6	100.0	1.7
Jun 89	100.0	0.7	11.1	- 1.7	0.0	- 3.1	0.0	- 4.2
Sep 89	100.0	3.7	100.0	1.6	100.0	1.7	88.2	0.3
Dec 89	100.0	32.1	100.0	20.0	100.0	15.1	100.0	15.4
Mar 90	100.0	1.3	100.0	0.4	50.0	- 1.1	12.5	- 3.7
All	99.5	172.8	96.3	138.4	88.6	107.1	83.6	91.3

TABLE 7

ARBITRAGE OPPORTUNITIES AND CASH RETURNS

Mispricing Bands	Percentage of observations which also correspond to deciles/quartiles of cash returns Lowest Decile[a]	Lowest Quartile[a]	Highest Decile[a]	Highest Quartile[a]	Mean Cash Return[b] (%)	Median Cash Return[c] (%)
{-2: X ≤ - 1.0%}	10.5	28.2	7.6	21.8	-0.031	-0.013
	(0.717)	(0.138)	(0.104)	(0.137)	(0.076)	(0.043)
{-1: -1.0% < X ≤-0.5%}	10.1	26.0	9.2	22.0	-0.013	0.000
	(0.938)	(0.482)	(0.423)	(0.035)	(0.230)	(0.112)
{0: -0.5% < X ≤ 0.5%}	10.0	24.6	9.9	24.9	0.006	0.011
	(0.953)	(0.657)	(0.850)	(0.936)	(0.760)	(0.711)
{+1: 0.5% < X ≤ 1.0%}	10.0	25.0	11.3	28.0	0.027	0.006
	(0.990)	(0.983)	(0.256)	(0.060)	(0.130)	(0.324)
{2: 1.0% < X}	9.1	20.4	13.4	32.5	0.022	0.039
	(0.628)	(0.081)	(0.034)	(0.004)	(0.740)	(0.010)

(a) In parentheses is the p-value for the hypothesis that, in the sub sample corresponding to that mispricing band, the percentage of observations which also correspond to the particular decile (quartile) of cash returns is equal to 10% (25%).

(b) In parentheses is the p-value for the hypothesis that the mean cash return in the sub sample of that mispricing band is equal to the mean cash return in the whole sample ie 0.003%.

(c) In parentheses is the p-value for the hypothesis that the median cash return in the sub sample of that mispricing band is equal to the median cash return in the whole sample ie 0.006%.

Descriptive Statistics on Mispricing

	Sub Sample 0 ie Within Account Period Spanning Futures Maturity		**Sub Sample 1 ie Within Account Period just before Account Period Spanning Futures Maturity**		**Sub Sample 2 ie All other Account Periods**	
	Near Contract	Far Contract	Near Contract	Far Contract	Near Contract	Far Contract
Number of observations	404	374	646	363	2697	427
Proportion of positive mispricing observations	0.688	0.388	0.481	0.466	0.404	0.440
Mean (%)	0.146	-0.045	0.026	0.016	-0.158	-0.119
Standard Deviation (%)	0.323	0.806	0.415	0.856	0.713	1.174
Median (%)	0.148	-0.177	-0.009	-0.081	-0.190	-0.151
Lower Quartile (%)	-0.047	-0.580	-0.254	-0.679	-0.624	-0.905
Upper Quartile (%)	0.347	0.725	0.310	0.586	0.344	0.712

p Value of Hypotheses Tests

H_{N0-N1}:	Near Contract mean in sub-sample 0	=	Near Contract mean in sub-sample 1	p value $\ll$ 0.0001
	Near Contract median in sub-sample 0	=	Near Contract median in sub-sample 1	p value $\ll$ 0.0001
H_{N0-N2}:	Near Contract mean in sub-sample 0	=	Near Contract mean in sub-sample 2	p value $\ll$ 0.0001
	Near Contract median in sub-sample 0	=	Near Contract median in sub-sample 2	p value $\ll$ 0.0001
H_{N0-F0}:	Near Contract mean in sub-sample 0	=	Far Contract mean in sub-sample 0	p value $\ll$ 0.0001
	Near Contract median in sub-sample 0	=	Far Contract median in sub-sample 0	p value $\ll$ 0.0001
H_{N1-F1}:	Near Contract mean in sub-sample 1	=	Far Contract mean in sub-sample 1	p value = 0.84
	Near Contract median in sub-sample 1	=	Far Contract median in sub-sample 1	p value = 0.08
H_{N2-F2}:	Near Contract mean in sub-sample 2	=	Far Contract mean in sub-sample 2	p value = 0.50
	Near Contract median in sub-sample 2	=	Far Contract median in sub-sample 2	p value = 0.69
H_{F0-F1}:	Far Contract mean in sub-sample 0	=	Far Contract mean in sub-sample 1	p value = 0.32
	Far Contract median in sub-sample 0	=	Far Contract median in sub-sample 1	p value = 0.56
H_{F0-F2}:	Far Contract mean in sub-sample 0	=	Far Contract mean in sub-sample 2	p value = 0.29
	Near Contract median in sub-sample 0	=	Far Contract median in sub-sample 2	p value = 0.40

TABLE 9

MISPRICING AND MISSPECIFICATION OF DIVIDENDS

d_{div1} (y%) = Mispricing estimated from actual *ex-post* dividend inflows minus mispricing estimated by using previous year's dividend plus a y% growth factor.

d_{div2} (y weeks) = Mispricing estimated by assuming that the time interval between the ex-dividend date and the actual dividend payment date is equal to the actual *ex-post* value of this interval for all index stocks minus mispricing assuming misspecification of this time interval by y weeks.

	d_{div1} (0%)	d_{div1} (10%)	d_{div1} (20%)	d_{div2} (4 weeks)	d_{div2} (8 weeks)
Mean (%)	0.051	-0.004	-0.059	0.005	0.010
Standard Deviation (%)	0.093	0.090	0.101	0.004	0.007
Median (%)	0.055	0.000	-0.041	0.005	0.009
Minimum	-0.151	-0.245	-0.371	0.000	0.000
Maximum	0.356	0.242	0.155	0.015	0.029
Lower Quartile Q_1	0.000	-0.056	-0.121	0.002	0.004
Upper Quartile Q_3	0.112	0.058	0.000	0.008	0.015
Percentage of cases > |0.1%|	38.4	24.0	33.7	0.0	0.0

TABLE 10

MISPRICING AND MARKING-TO-MARKET CASH FLOWS

$$d_{mm1}(t) = \frac{1}{S_t} \sum_{\tau=t}^{T-1} (F_{\tau,T} - F_{\tau+1,T}) \left\{ \frac{\exp(r_{\tau+1,\tau+1,t}(T-\tau-1))}{\exp(r_{\tau,\tau,T}(T-\tau))} - 1 \right\}$$

$$d_{mm2}(t) = \frac{1}{S_t} \sum_{\tau=t}^{T-1} (F_{\tau,T} - F_{\tau+1,T}) \left\{ \exp(r_{\tau+1,\tau+1,t}(T-\tau-1)) - 1 \right\}$$

	$d_{mm2}(t)$ (%)	$d_{mm2}(t)$ (%)
Mean	0.000	0.014
Standard Deviation	0.004	0.085
Median	0.000	0.004
Minimum	-0.009	-0.482
Maximum	0.033	0.351
Lower Quartile	-0.002	-0.007
Upper Quartile	0.001	0.033
Percentage of cases > |0.1%|	0.0	12.1

TABLE 11

ABSOLUTE MAGNITUDE OF MISPRICING AND TIME TO MATURITY

The data are sorted into 5 groups based on time to maturity (T-t) as follows:

Group 1: $\{0 \leq (T\text{-}t) \leq 30\}$
Group 2: $\{31 \leq (T\text{-}t) \leq 60\}$
Group 3: $\{61 \leq (T\text{-}t) \leq 90\}$
Group 4: $\{91 \leq (T\text{-}t) \leq 120\}$
Group 5: $\{121 \leq (T\text{-}t) \leq 180\}$

Group	Mean Absolute Mispricing	Median Absolute Mispricing	F-stat for equality of means[a]	Chi-sq stat for equality of medians[b]	Z-stat for H_{Time}
1	0.330	0.273	151.6***	458.4***	78.40***
2	0.592	0.539			
3	0.555	0.512			
4	0.709	0.624			
5	0.907	0.820			

(a) A dummy variable based regression is used to test the hypothesis of equality of means across different groups.

(b) The Mood (1954) non parametric test is used to test the hypothesis of equality of medians across different groups.

H_{Time}: Median Absolute Mispricing of Group 1 < Median Absolute Mispricing of Group 2 < Median Absolute Mispricing of Group 3 < Median Absolute Mispricing of Group 4 < Median Absolute Mispricing of Group 5

The Jonckheere (1954) - Terpstra (1952) non parametric test for ordered alternatives is used to test H_{Time}.

TABLE 12

MISPRICING AND INDEX VOLATILITY IMPLIED IN CALL OPTION PRICES

The data are sorted into 5 equally sized groups on the basis of the volatility implied in call option prices with Group 1 corresponding to the lowest volatility and Group 5 to the highest volatility. The call implied volatility estimate is the average of the implied volatility calculated, using the dividend adjusted Black Scholes Model, for the two call option series which were closest to being at the money and which were expiring more than one month but less than two months later.

	Implied Volatility		Mispricing		Proportion of	F-stat for equality of	Chi-sq stat for equality of	Z-stat for
up	Mean	Median	Mean	Median	+ ve values	Means[a]	medians[b]	H_{IVOL}
1	0.105	0.115	-0.558	-0.501	0.155	260.9***	619.9***	33.31***
2	0.144	0.143	-0.246	-0.212	0.360			
3	0.174	0.174	-0.123	-0.085	0.418			
4	0.206	0.206	0.058	0.065	0.530			
5	0.275	0.259	0.399	0.412	0.714			

(a) A dummy variable based regression is used to test the hypothesis of equality of means across different groups.

(b) The Mood (1954) non parametric test is used to test the hypothesis of equality of medians across different groups.

H_{IVOL}: Median Mispricing of Group 1 < Median Mispricing of Group 2 < Median Mispricing of Group 3 < Median Mispricing of Group 4 < Median Mispricing of Group 5

The Jonckheere (1954) - Terpstra (1952) non parametric test for ordered alternatives is used to test H_{IVOL}.

CHAPTER 7

MEAN REVERSION IN STOCK INDEX FUTURES MISPRICING: EVIDENCE FROM THE US AND THE UK[1]

ABSTRACT

This chapter presents evidence on mean reversion in index futures mispricing using high frequency intraday data from both the US and the UK markets. The results from the two markets are remarkably consistent and have several interesting features. First, the existence of mean reversion in the overall mispricing series is firmly established. The change in mispricing depends significantly on the *level* of mispricing in the previous period. This is consistent with the existence of significant arbitrage activity. Second, the mean reversion parameter is a systematic function of the time to maturity of the futures contracts, increasing as the time to maturity decreases. This is consistent with arbitrage being considered more risky when time to maturity is higher. It is also inconsistent with the view that basis predictability is *entirely* a statistical illusion created by non-synchronous trading. Third, mean reversion appears to depend significantly on the value of mispricing in the previous period. It is not significantly different from zero when the magnitude of the previous period mispricing is so small that no arbitrageurs are likely to be active, but becomes significant in magnitude when the magnitude of the previous period mispricing becomes large enough to exceed the estimated marginal transaction costs of arbitrageurs. This supports the view that mean reversion is arbitrage induced. Fourth, mean reversion appears to be significantly lower on Mondays than on other days of the week. Finally, mean reversion also exhibits significant intraday seasonality, following a U-shaped intraday pattern. However, these seasonalities cannot be explained by corresponding patterns in lagged mispricing and volatility.

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[1] First draft November 1991; revised March 1992; presented to staff seminars at Dundee, Groupe HEC (Paris) and Brunel; accepted for presentation at the French Finance Association Conference, June 1992, and the American Finance Association Conference, January 1993.

MEAN REVERSION IN STOCK INDEX FUTURES MISPRICING: EVIDENCE FROM THE US AND THE UK

1. INTRODUCTION

Mean reversion in stock index futures mispricing,[2] whereby mispricing changes depend upon the *level* of mispricing in the previous period, has been documented by Merrick (1988) and Brennan and Schwartz (1990) for US data and by Pope and Yadav (1991) for UK data. It has been presumed that this mean reversion is the result of the actions of index arbitrageurs (see eg. Brennan and Schwartz, 1990 pp 58; Yadav and Pope, 1990 pp 583). In fact, as noted by Holden (1990b pp 1) it is surprising that the extent of this mean reversion is, in actual practice, fairly small when one would expect arbitrage opportunities to be rapidly eliminated in well functioning capital markets. In contrast to arbitrage-related explanations of mean reversion in mispricing, Miller *et al* (1991) have suggested that the observed mean reversionary behaviour in the cash-futures basis could just be a manifestation of non-synchronous trading in the index basket of stocks, having no economic significance in terms of actual index arbitrage activity. However, the mean reversion generated by the Miller *et al* "statistical illusion" hypothesis is observationally indistinguishable from the potential mean reversion generated by actual index

[2] Mispricing has been defined slightly differently by different researchers. Merrick (1988,1989), MacKinlay and Ramaswamy (1988) and Yadav and Pope (1990, 1992a) use {(Futures Price-Theoretical Futures Price)/Cash Price}. Holden (1990c), Brennan and Schwartz (1990) and Sofianis (1991) use {Futures Price - Theoretical Futures Price}. Cooper and Mello (1990) effectively use {(Theoretical Futures Price - Futures Price)/ Futures Price}. Miller *et al* use the nominal cash futures basis {(Futures Price - Cash Price)}. In this paper, mispricing is defined as {log (Futures Price) - log (Theoretical Futures Price)}.

arbitrage activity. The analysis and evidence of Miller *et al* shows that observed basis predictability could *also* be explained in terms of non-synchronous trading in index stocks. The only apparent way to distinguish between these competing explanations of mispricing mean reversion is to examine how the observed mean reversion varies with factors that are expected to affect the degree of index arbitrage, but which are not expected to be associated with non-synchronous trading *per se*; and vice versa.

An empirical investigation of the factors that affect the behaviour of the mean reversion parameter is potentially important, even within the framework of arbitrage induced mean reversion. Different models of the arbitrage supply process have different implications. The mean reversion in the Brownian Bridge process (Brennan and Schwartz, 1990) is inversely proportional to the time to maturity, becoming infinitely large as the futures contract approaches maturity. An alternative approach adopted by Pope and Yadav (1991) is to model the evolution of index futures mispricing on the assumption that there is a hierarchy of different categories of arbitrageurs with different arbitrage related marginal transaction costs. Their model predicts that the mean reversion parameter would be zero if mispricing in the previous period is smaller in magnitude than the transaction cost of the lowest cost arbitrageur and would successively increase in magnitude as the magnitude of the previous period mispricing crosses the transaction cost thresholds of higher cost arbitrageurs. The models of Holden (1990c) and Cooper and Mello (1990) predict different levels of mean reversion under a monopolistic and an imperfectly competitive arbitrage structure. This again suggests different levels of mean reversion for different levels of the lagged mispricing variable since, as Cooper and Mello (1990, pp.19) point out, the lowest cost arbitrageur will be monopolistic within the range of the basis that is small enough not to attract the arbitrageur with the next lowest cost into the market. Additionally, in Holden's model, the monopolistic strategy is highly sensitive to the time to maturity, implying that mean reversion in a monopolistic market will increase as time to maturity decreases. On the other hand, in an imperfectly competitive arbitrage structure, Holden's model predicts that the strategy of arbitrageurs, and the corresponding mean reversion, will be insensitive to the time to maturity. Finally, uncertainty about dividends and interest rates and hence the risk in arbitrage is less for shorter times to maturity. This again suggests that mean reversion in mispricing should increase as time to maturity decreases. Clearly, empirical evidence on how the mean reversion parameter actually varies with lagged mispricing, time to maturity and other relevant variables can potentially lead to a better understanding of the economics of index arbitrage.

The behaviour of the mean reversion parameter is also of significant practical importance for index arbitrageurs. Merrick (1989) (for US data) and Yadav and Pope (1990) (for UK data) show that the early unwinding option is potentially valuable and arbitrage positions should seldom be held till maturity. The evidence of Sofianos (1991) shows that arbitrageurs put on arbitrage positions within their transaction cost window but typically earn high early unwinding returns due to mispricing reversals before maturity. The value of the early unwinding option is clearly a key input in decisions relating the opening of new arbitrage positions and the closing of existing arbitrage positions. Hence the factors affecting the mean reversion parameter are critically relevant.

Mean reversion in index futures mispricing is also an important determinant of the costs of various non-arbitrage participants in the futures market such as short term hedgers, portfolio insurers and those hedging OTC option positions (see eg. Merrick (1988), Hill *et al* (1988) and Yadav and Pope (1991)). A better understanding of the mean reversionary behaviour in mispricing will assist in the formulation of optimal strategies for such groups of market participants.

The aim of this chapter is to provide empirical evidence on how mean reversion in stock index futures mispricing varies with time to maturity of the futures contract and the value of mispricing in the previous period. Intraweek and intraday seasonality in the mean reversion parameter is also investigated. The evidence is based on about four years of high frequency intraday data from both the US and the UK markets. The MacKinlay and Ramaswamy (1988) 15-minute-interval dataset has been used for the US market and the Yadav and Pope (1992a) hourly interval dataset for the UK market.

The results relating to the US market and the UK market are remarkably consistent with each other and have several interesting features. First, the time series of mispricing do not have unit roots. They are stationary I(0) series. This is consistent with the model of Holden (1990b) and also formally establishes the existence of mean reversion in the overall mispricing series. Second, the mean reversion parameter appears to be a systematic function of the time to maturity, increasing as the time to maturity decreases. This is consistent with arbitrage being considered more "risky" when time to maturity is higher. More importantly, since non-synchronous trading in index stocks is clearly not related to the time to maturity of the futures contract, this shows that basis predictability is not *entirely* a statistical illusion created by non-synchronous trading. Third, mean reversion appears to depend significantly on the value of mispricing in the previous period. It is not significantly different from zero when the magnitude of the previous period mispricing is so small that no arbitrageurs are likely to be active, but becomes significant in magnitude when the magnitude of the previous period mispricing becomes large enough to exceed the estimated marginal transaction costs of arbitrageurs. This supports the view that mean reversion is arbitrage induced and in particular, is consistent with the TAR model of Pope and Yadav (1991). Fourth, mean reversion appears to be significantly lower on Mondays than on other days of the week. Finally, mean reversion also exhibits significant intraday seasonality. It follows a U-shaped intraday pattern. However, the seasonalities cannot be explained in terms of corresponding patterns in lagged mispricing and volatility.

The rest of the chapter is organised as follows: Section 2 reviews the previous literature relevant to this study; Section 3 describes the datasets; Section 4 outlines the methodology and documents the empirical results; and finally Section 5 presents the main conclusions.

2. PREVIOUS LITERATURE

The static cost-of-carry model for the pricing of index futures contracts[3] has been the basis of extensive empirical research. Systematic and significant deviations of futures prices from their fair values have been documented *inter alia* by MacKinlay and Ramaswamy (1988) for US data and Yadav and Pope (1990) for UK data. These two studies also report large and significant negative autocorrelation in the first differences of the mispricing time series and large and significant positive autocorrelation in the time series of mispricing levels. Large negative autocorrelation in the first differences of mispricing shows a tendency for mispricing reversals if the time series of mispricing levels is mean reverting; while the large and positive autocorrelation in mispricing levels shows a tendency for mispricing to persist. Neither of these studies analyzed clearly the economic implications of these autocorrelations. However, both these studies also reported that the absolute magnitude of mispricing tends to increase significantly with time to maturity and this was explained in terms of the greater risk involved in arbitrage for longer times to maturity due to the greater uncertainty about dividends and interest rates.

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3 See eg Cornell and French (1983a) or Figlewski (1984a).

Merrick (1988) discussed mispricing reversals and its implications from the perspective of short term hedging. As a direct consequence of negative autocorrelation in first differences of mispricing and mean reversion in the time series of mispricing levels, he finds that long-cash-short-futures one period hedges earn significantly more (less) than the relevant risk free rate when futures are initially overpriced (underpriced) and short-cash-long-futures one period hedges earn significantly less (more) than the risk free rate when futures are initially overpriced (underpriced)[4]. More importantly, he reports direct evidence of mispricing mean reversion. The change in mispricing over the next period is strongly and negatively related to the current level of the mispricing variable[5]. Merrick (1988) pursues this result only to the extent that it impacts on short term hedging but in Merrick (1989) he highlights an important implication of mispricing reversals for index arbitrageurs - the early unwinding option. His simulations on US data - and those of Yadav and Pope (1990) on UK data - show that arbitrage positions are not likely to be held to expiration and hence the early unwinding option, created by mean reversion in mispricing, is potentially very valuable. Sofianos (1990) analyses data on 2659 actual index arbitrage transactions reported to the NYSE and confirms that most arbitrage positions were not held to expiration but were unwound early following profitable mispricing reversals.

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[4] Yadav and Pope (1990) find similar results with UK data.

[5] However, the statistical tests used to judge significance are not appropriate in the context of the vast literature on unit roots processes.

All the above studies have adopted the static perspective of the cost-of-carry model even when reporting mean reversion in mispricing and time varying mispricing behaviour. In contrast, Brennan and Schwartz (1990) analyze futures mispricing and the early unwinding option from a dynamic perspective. They model mispricing as a Brownian Bridge process which has the property (in common with a mean-zero stationary autoregressive process) that it tends to return to zero, but additionally it constrains the value of mispricing to be zero at the future maturity day with probability one. This is done by explicitly making the mean reversion parameter inversely proportional to the time to maturity. Thus as time to maturity approaches zero, it displays larger and larger (and eventually infinite) mean reversion. The critical assumption in the model is that the mean reversion parameter is constant except for its inverse dependence on the time to maturity. However, Brennan and Schwartz were forced to omit the observations relating to the last five trading days of each contract from their empirical analysis because " ...parameter estimates were very sensitive to these data..." (p 515). Apparently, as the time to maturity approached zero, the time series did not display the infinite rate of mean reversion predicted by the Brownian Bridge process. The mean reversion in the Brennan and Schwartz (1990) model is, by assumption, independent of the actions of arbitrageurs, implying that the behaviour of arbitrageurs must be constrained to prevent the realization of infinite arbitrage profits, for example by position limits. Clearly an investigation of the variation of the mean reversion parameter with time to maturity, and with other relevant factors proxying for the actions of arbitrageurs is particularly relevant in the context of this model.

Cooper and Mello (1990) attempt to endogenously model the mispricing process. They assume that, in the absence of arbitrage, mispricing follows a mean reverting Ornstein-Üllenbeck process about a mean of zero. In the presence of arbitrage, the change in mispricing is dependent not only on the current level of mispricing, as in the Brownian Bridge process, but also on the volume of open arbitrage positions since arbitrage traders are assumed to behave differently depending on the size of their open positions. In particular, if mispricing has crossed one arbitrage bound, arbitrageurs will have open positions which they will close down before the opposite arbitrage bound is reached. The model apparently leads to path dependency of the type found by MacKinlay and Ramaswamy (1988), but its implications are difficult to test with only times series data on mispricing.

The work by Holden (1990a, 1990b, 1990c) represents an important attempt to model endogenously the stochastic process governing futures mispricing. Holden develops an "intertemporal arbitrage trading model" based on the following four main assumptions: (i) the price of the underlying asset in the cash market and the price of its synthetic equivalent in the futures market differ as a result of the demands of liquidity traders, "time-series" market makers (trading only in their respective market) and "cross-sectional" market makers or arbitrageurs (trading in both markets); (ii) cumulative liquidity shocks in each market, arising from the stochastic total demand of the respective liquidity traders, mean revert to zero; (iii) cash and futures asset prices, subject to these zero mean reverting liquidity shocks, follow processes that mean revert to a common underlying fundamental value; and

(iv) prices always reflect all information and hence arbitrage opportunities arise because of the price differences generated by liquidity shocks. Each period every arbitrageur (assumed to be risk neutral) calculates the optimal quantity of arbitrage trading so as to maximise his individual profit, holding fixed his expectations concerning arbitrage trading by others. All arbitrageurs are assumed identical, and hence all quantity decisions are also identical.

With a non-monopolistic arbitrage trading structure, the mispricing variable in Holden's model follows a stationary time series process. The result is valid when the number of arbitrageurs is greater then one and when "the last couple of periods before maturity" are excluded so that "the optimal finite horizon arbitrage strategy can be approximated to double precision accuracy by the optimal infinite horizon arbitrage strategy" (Holden, 1990c, p.148). If futures prices and futures-equivalent cash prices[6] are assumed to be individually non-stationary I(1) processes, this implies that futures prices and futures-equivalent cash prices are cointegrated with a cointegrating vector of [1,-1][7]. However, the mean reversion in the mispricing time series is a function of both liquidity trading and arbitrage trading, though the empirical results in Holden (1990c) show that, within the framework of the model, the mean reversion due to arbitrage trading is far greater than the mean reversion

6 Futures equivalent cash price is defined as the price which should exist in the futures market, given the cash price and infinitely elastic arbitrage services. Typically, this would be the price determined on the basis of the cost-of-carry forward pricing formula (see eg Figlewski, 1984).

7 See eg Engle and Granger (1987) for a formal discussion on cointegration.

due to liquidity trading.

In Holden's model, arbitrage opportunities are created by stochastic liquidity trading and not by differences in the speed with which cash and futures prices adjust to new information. New information is assumed to be reflected instantaneously in both cash and futures prices. Since there is no "catching up" involved, the pool of potential gains available to arbitrageurs do not "evaporate" if not exploited immediately. Consequently, a monopolistic arbitrageur acting optimally will spread out his trading over all periods to the maturity of the futures contract, thereby exploiting arbitrage opportunities slowly. In contrast, in a non-monopolistic arbitrage market, arbitrageurs will trade on arbitrage opportunities quickly. While the strategy of arbitrageurs (and the resulting mean reversion in mispricing) does not depend on the time to maturity in a non-monopolistic market, the mean reversion in a monopolist market should increase as the time to maturity decreases. In any case, an investigation of the variation of the mean reversion parameter in relation to relevant variables, such as the time to maturity and lagged mispricing, can provide valuable insights into the applicability of the assumptions in Holden's model, and their implications.

An important early attempt to analyze futures mispricing from a dynamic perspective is that of Garbade and Silber (1983). They provide a model of concurrent price changes in cash and futures markets which explicitly recognizes that the elasticity of intermarket arbitrage services is not infinite because of: (i) the transaction costs involved in arbitrage; and (ii) the risk inherent in arbitrage transactions due to non-stochastic net storage costs and the constrained availability of arbitrage capital. Based on simple assumptions about the behaviour of economic agents in the cash and futures markets and about the evolution of reservations prices, the model leads to price dynamics which imply that futures mispricing follows an AR(1) process. Parameter estimates from the model allow inferences to be drawn about the relative speed with which new information is incorporated into cash and futures prices. The autoregressive parameter is a function of the elasticity of arbitrage services. In the absence of intermarket arbitrage, the elasticity of arbitrage services is zero and the autoregressive parameter is equal to unity, implying that the mispricing series is non-stationary. On the other hand, if the elasticity of arbitrage services is infinite, the autoregressive parameter will be zero and the mispricing series will be white noise. In actual practice, in the presence of arbitrageurs the elasticity of arbitrage services will be significantly greater than zero, but will not be infinite because of transaction costs and stochastic carrying costs. Therefore, the autoregressive parameter should be significantly less than unity but greater than zero.

If the time series of asset prices (cash prices and cash equivalent futures prices in the case of Garbade and Silber, 1983) are assumed to be individually non-stationary I(1) processes, the model of Garbade and Silber (1983) also implies that if intermarket arbitrage is significant, then cash prices and cash equivalent futures prices are cointegrated. This is intuitively reasonable. In the presence of differential price discovery and transaction costs, these prices could be different in the short run. However if they move too far apart, the actions of arbitrageurs should tend to pull them together towards restoration of the equilibrium. It is important to note that in this model the opportunities for arbitrage are created only by differential price discovery in the two markets. Liquidity trading is not considered. Furthermore, the mean reversion in mispricing is a function of the elasticity of arbitrage services which is assumed to be constant. An empirical analysis of the dependencies of this mean reversion on arbitrage related factors is clearly relevant in the context of this model.

Pope and Yadav (1991) extend the model of Garbade and Silber (1983) by introducing the possibility that several categories of arbitrageurs exist, each having different levels of transaction costs. It is argued that if the magnitude of mispricing is sufficiently small, no arbitrageur in any category will be active and mispricing will follow a martingale process, with the change in mispricing over the following period depending only on (random) differences in liquidity trading, noise trading, information arrival and price discovery between the two markets. However, when mispricing evolves to a point where its absolute value exceeds the transaction costs level of the lowest marginal cost category of arbitrageurs, the elasticity of arbitrage services will become greater than zero, and will depend *inter alia* on the aggregate arbitrage capital available to this category of arbitrageurs. If mispricing happens to exceed the marginal transaction costs faced by the next category of arbitrageurs, this latter group will also enter the market, and the elasticity of arbitrage services will again increase. Following this line of argument, market demand can consequently be expected to vary with mispricing in a non-linear fashion -specifically in the form of a step function. Furthermore, the time series of mispricing should follow a (self exciting) threshold autoregressive (TAR) process. The TAR process has been described recently by Hsieh (1991) and is a piecewise linear autoregressive process in which the process parameters describing the evolution of mispricing are path-dependent. In particular, the mean reversion in the mispricing variable is a function of magnitude of mispricing in the previous period, because that determines the categories of arbitrageurs who are likely to be attracted to arbitrage away the mispricing.

Finally, Miller *et al* (1991) suggest that predictable changes in the observed cash-futures *basis* (as distinct from the value basis or mispricing) could potentially arise because of non-synchronous trading in index stocks. They argue that mean reversion in the basis could be just a statistical illusion that has no economic significance and is unconnected with index arbitrage activity. By assuming that price changes in the cash market follow a "modified" AR(1) process, and that price changes in the futures market follow a MA(1) process, they show that negative first order serial correlation in basis changes can be expected for a wide range of realistic parameter combinations[8]. However, it is important to recognise that the analysis and evidence of Miller *et al* shows that observed basis predictability could *also* be

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[8] Miller *et al* do not appear to be concerned about the extent to which the *level* of mispricing at time t predicts the change in mispricing from time t to time (t+1).

explained in terms of non-synchronous trading in index stocks. The non-synchronous trading explanation and the arbitrage explanation of reversals are observationally equivalent and Miller *et al's* empirical work does not exclude the possibility that basis predictability is caused, at least in part, by periodic realignments of prices due to the actions of arbitrageurs. The only way to distinguish between these competing hypotheses is to examine how the observed rate of mean reversion varies with factors that are expected to affect the extent of index arbitrage, but which are also expected to be independent of non-synchronous trading; or vice versa.

In each of the dynamic arbitrage related models we have outlined - Brennan and Schwartz (1990), Cooper and Mello (1990), Holden (1990a, 1990b, 1990c), Garbade and Silber (1983) and Pope and Yadav (1991) - an important aspect of the modelling process, or an important implication, is the dependence between the level of mispricing at time t and the change in mispricing from time t to time (t + 1). This dependence reflects mean reversion in mispricing and is the primary focus of the empirical analysis in this chapter. First, we test whether mean reversion in the overall time series is significantly greater than zero. This is equivalent to testing for the absence of unit roots in the mispricing time series. Then, we investigate the dependence of the mean reversion parameter Φ on:

(a) Time to maturity of the futures contract,

(b) Mispricing level in the previous period,

(c) Day of the week, and,

(d) Hour of the day.

3. THE DATA SETS

3.1 US Data

The US dataset analyzed in this study is that described and used in MacKinlay and Ramaswamy (1988, pp 143-144). It consists of 15 minute interval cash and futures data on the S&P 500 index from June 17, 1983 to June 18, 1987. The data covers 16 contracts expiring over the period September 1983 to June 1987. The total data consist of 26,070 15-minute observations over 1008 trading days. The data relates only to the near contract from the expiration date of the previous contract until its own expiration date. The cash prices and the futures prices used were supplied by the Chicago Mercantile Exchange and correspond to the nearest quote available *after* every quarter hour mark; forecast dividends yield were proxied by the realised daily dividend yield of the value weighted index of all NYSE stocks supplied by the Centre for Research in Security Prices (CRSP) at the University of Chicago and Kidder, Peabody and Co; and interest rates implied by certificates of deposit expiring around contract expiration had also been supplied by Kidder, Peabody and Co. Over the sample period, the trading hours were from 9am Central Time to 3pm Central Time up to September 27, 1985; and 8.30am Central Time to 3am Central Time thereafter.

The futures equivalent cash price is taken as identical to the forward pricing formula fair value calculated as in MacKinlay and Ramaswamy's equation (1) (1988 pp 140).[9]

3.2 UK Data

The UK dataset analysed in this study is that described and used in Yadav and Pope (1992a pp 238-240). It consists of hourly interval cash and futures data on the FTSE100 index from April 28, 1986 to March 23, 1990. The data covers 16 contracts expiring over the period June 1986 to March 1990. The total data consist of 7920 hourly observations over 990 trading days. The data relates only to the near contract, shifting to the next contract on the expiration date. The cash prices were collected from the *Financial Times*. The futures prices were extracted from the "time and sales" data supplied by the London International Financial Futures Exchange. Both these prices correspond to the average of the last bid and the last ask quotes available before the hourly mark. Forecast dividends were proxied by the exact *ex post* daily dividend flow on the FTSE100 index, computed with data from *Extel* cards and *Datastream*. UK Treasury Bill rates were collected from *Datastream*. During the sample period the cash market was open from 9.00am to 5.00pm, but the futures market was open from 9.05am to 4.05pm. Consequently the mispricing series is based on coincident cash and futures prices at 10.00am, 11.00am, 12.00 noon, 1.00pm, 2.00pm, 3.00pm and 4.00pm. Mispricing at the

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[9] The forward pricing formula makes several simplifying assumptions, eg that futures are not marked to market daily and dividends are known *ex ante* with perfect certainty. The effect of such simplifying assumptions on fair value are not likely to be important in the context of the study since we are only considering *changes* in futures prices over (at most) an overnight interval.

"close" is based on coincident 4.00pm prices. However, mispricing at the "open" is not based on exactly coincident prices because 9.00am cash prices are matched with 9.06am futures prices. The estimates of mispricing at the open will therefore potentially be contaminated with some "noise" due to this non-synchronicity, though arguably it should not affect the results in a systematic way.

The futures equivalent cash price is taken as identical to the settlement adjusted forward pricing formula fair value calculated as in Yadav and Pope's equation (3) (1992a pp 237).

In view of the potential impact of measurement error induced by non-synchronous trading in index stocks on the time series process of mispricing, the distinct properties of the UK dataset employed here are worth emphasising. The cash index is based on the average of the best bid and the best ask *quotes* and it is updated every minute. The quotes underlying the index computation represent prices at which competing market makers are obliged to trade for up to fairly large contract sizes. Thus, the hourly FTSE100 index values are based on prices at which an arbitrageur could actually trade (before transaction costs) and the values are synchronous with the hourly bid and ask quotes in the futures market. Therefore, the index arbitrage opportunities generated with this dataset are not "spurious" but potentially economically significant even within the context of possible differential price adjustment delays within index stocks.

3.3 The Mispricing Series

Mispricing has been defined slightly differently by different researchers[10]. In this chapter the mispricing X_t at time t of a future contact maturing at T is defined as:

......(1)

where $F_{t,T}$ = Observed price at t of a futures contract maturing at T

and $F_{t,T}^{*}$ = Futures equivalent cash price ie. the theoretical price at t of a futures contract maturing at T given the cash price at t.

The measure X_t, defined as in equation (1), has been preferred to a simple price difference measure so as to avoid heteroskedasticity problems that are likely with the use of four years of unnormalised data.

4. METHODOLOGY AND EMPIRICAL RESULTS

4.1 Unit roots in the mispricing time series

The basic model estimated in this chapter is the following:

$$\Delta X_t = -\Phi X_{t-1} + \beta_o + \sum_{g=1}^{p} \beta_g \Delta X_{t-g} + \epsilon_t \qquad ......(2)$$

where $\Delta X_t = (X_t - X_{t-1})$ ......(3)

[10] See note #1.

In equation (2), Φ is taken as the measure of mean reversion in the mispricing time series and is the focus of the empirical analysis. In view of the established empirical evidence on negative serial correlation in mispricing changes persisting over several time periods (see eg MacKinlay and Ramaswamy (1988) for the US and Yadav and Pope (1990) for the UK), it is necessary to add lagged terms in mispricing changes ΔX_{t-g} ($g=1,2,\ldots p$) so as to ensure that the estimate of Φ reflects only the dependence on the *level* of mispricing in the previous period and is not biased by inclusion of components representing mispricing changes over previous periods. Accordingly, in the empirical tests, lagged terms are added until the estimated regression residuals ϵ_t are purged of significant serial correlation. The empirical analysis is confined to Φ and hence to "first order" mean reversion.

All estimations exclude observations relating to the expiration day. This is done for two reasons. First, there is well documented evidence of "abnormal" behaviour related to expiration (see eg Stoll and Whaley, 1987). Second, the implications of Holden's model relating to an imperfectly competitive arbitrage structure are based on excluding "the last couple of periods before maturity" (Holden, 1990c, pp 148). In addition, to avoid distortion due to large outliers, all estimations also exclude the 1987 "Crash" period and the 1989 "mini-crash" period.[11]

[11] This is relevant only for UK data since the US dataset does not span these two periods. For the UK, all observations relating to a four week period beginning with October 19, 1987 and all observations relating to October 16, 1989, are coded as missing values.

The hypothesis that $\Phi=0$ is equivalent to the presence of unit roots in futures mispricing. There is a vast literature on testing for unit roots. (See Diebold and Nerlove, 1990, for a selective survey.) The results reported by us in this chapter are based on augmented Dickey-Fuller type regressions.[12] The appropriate testing procedure depends on the choice of the maintained model and the form of the alternative hypothesis. Besides equation (2), which is a stationary model without any time trend, we also consider the possibility of the following stationary model with a time trend:

$$\Delta X_t = -\Phi X_{t-1} + \alpha(T-t) + \beta_o + \sum_{g=1}^{p} \beta_g \Delta X_{t-g} + \epsilon_t \qquad \ldots\ldots(4)$$

A time trend of this nature can exist for two reasons. First, futures prices could be lower than the prices estimated on the basis of the forward pricing formula because cash holdings provide a tax timing option which futures holdings do not provide (Cornell and French, 1983); and, as argued by Cornell (1985), the mispricing on this account should be more negative for longer times to maturity. Second, since mispricing is constrained to be zero at maturity and is generally different from zero at the time period corresponding to the start of the data sample, a time trend would

12 An alternative approach is that pioneered by Phillips (1987) and extended to a variety of related problems by Durlauf and Phillips (1988), Perron (1989), Perron and Phillips (1987), Phillips (1988) and Phillips and Perron (1988). The basic idea is to estimate a non-augmented Dickey-Fuller regression (ie without using lagged terms in ΔX_t) and then to "correct" the Dickey-Fuller studentised statistic τ for general forms of serial correlation and heteroskedasticity that may be present in the remaining error term, using semi-parametric methods. The Phillips-Perron statistic can be computed in two ways, using $\epsilon_t^1 = (X_t - X_{t-1})$ or using $\epsilon_t^2 = \{X_t-(1-\Phi)X_{t-1}\}$. Stock and Watson (1988) show that tests based on ϵ_t^1 are inconsistent while the results of Schwert (1989) show that the performance of tests based on the augmented Dickey-Fuller framework is distinctly better than Phillips-Perron tests based on ϵ_t^2. Hence, our results are based on the augmented Dickey-Fuller framework.

be expected if there are no missing values in the data.[13]

An intercept term is included in both equation (2) and equation (4) in view of the evidence of significant deviations of futures prices from their fair value estimates.[14]

The following null hypothesis is tested:

H_0: $\Phi = 0$

against the alternative that $\Phi > 0$

For equation (2), H_0 is tested using the tables for τ_μ in Fuller (1976).[15] For equation (4), H_0 is tested using the tables for τ_t in Fuller (1976).

Both models equation (2) and equation (4) above are estimated separately for each contract. In every case, p is chosen to be the minimum value that ensures that serial correlation in the regression residuals, as indicated by the Box-Pierce Q-statistic, is statistically insignificant. For US data, every contract contains about 1500 observations (25 or 27 quarter hour observations per day over approximately 60 trading days). For UK data, every contract contains about 480 observations (8

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[13] However, the results of Cornell (1985a) for US data and Yadav and Pope (1990) for UK data do not provide any support for the existence of significant time trends in mispricing.

[14] See eg Figlewski (1984a) and Yadav and Pope (1990).

[15] Schmidt (1990) shows that with an intercept term, the relevant critical values are lower than in Fuller (1976). However, in our case, the estimated intercepts are small in relation to the minimum value used in the simulations of Schmidt (1990). In any case, we preferred to do a more conservative test.

hourly observations per day again over approximately 60 trading days). The Box-Pierce Q-statistic is computed using 50 lags for US data and 24 lags for UK data. In certain cases it was found that the Box-Pierce Q-statistic remains statistically significant even for very large values of p eg the contracts expiring in December '83 and March '86 for US data and the contracts expiring in March '88 and December '89 for UK data. Closer examination of the autocorrelation function revealed that in each such case, there were spikes in the autocorrelation function at a few specific large lags. For example, for US data, the contract expiring in December 1983 had spikes at lags 28, 29, 37 and 48 and the contract expiring in March 1986 had a spike at lag 31. For UK data, the contract expiring in March 1988 had spikes at lags 11 and 16 and the contract expiring in December 1989 had spikes at lags 19 and 23. These spikes can be regarded as being spurious and economically irrelevant. However, in view of this problem, an alternative criteria for selection of p was also introduced. This involves choosing the lag length that minimises the Schwarz Information Criteria (see Schwarz (1978)). Essentially, additional regressors reduce bias in the estimated value of Φ (due to the omission of potentially relevant variables) but also reduce the power of the unit root tests and make it more difficult to distinguish between competing hypotheses (by including unnecessary coefficients with insignificant t-statistics). Information criteria, like Schwarz (1978), add regressors until the estimated gain from reduced bias offsets the estimated loss of power. Alternative information criteria like the Akaike Information Criteria (Akaike, 1974) could also have been used. But, it was found that, in every problem case, the results were robust to the addition of more lags and hence the Schwarz

Information Criteria which imposes a stiffer penalty on additional regressors in comparison to the Akaike Information Criteria, was preferred.

Table 1A reports the results of the unit root tests for US data. The null hypothesis of a unit root is rejected for every contract. For the model of equation (2), the null hypothesis is rejected at the 1% level for 12 contracts, at the 5% level for a further 2 contracts and at the 10% level for the remaining 2 contracts. For the model of equation (4), the null hypothesis is rejected at the 1% level for 14 contracts and at the 10% level for the remaining 2 contracts. For the overall dataset, the null hypothesis is conclusively rejected.

Table 1B reports the results of the unit root tests for UK data. The null hypothesis of a unit root is rejected for 15 contracts out of 16. For the model of equation (2), the null hypothesis is rejected at the 1% level for 9 contracts, at the 5% level for a further 3 contracts and at the 10% level for another 2 contracts. For the model of equation (4), the null hypothesis is rejected at the 1% level for 7 contracts, at the 5% level for a further 5 contracts and at the 10% level for another 2 contracts. For the overall dataset, the null hypothesis is conclusively rejected once again.

Clearly, the time series of mispricing levels does not have unit roots and appears to be a stationary I(0) series. This is important for several reasons. First, it formally establishes the existence of mean reversion in the overall mispricing series in as much as the change in mispricing depends, in part, on the *level* of mispricing in the previous period.[16] Second, it is consistent with the existence of significant index arbitrage activity.[17] In particular, it is consistent with the inter-temporal arbitrage trading model of Holden (1990b) with an imperfectly competitive arbitrage supply structure. Third, if futures prices and futures-equivalent-cash prices are non-stationary I(1) processes (as they are *a priori* expected to be in an efficient market), this result would imply that futures prices and futures-equivalent-cash-prices are cointegrated time series.

For US data, the coefficient of the time to maturity variable in equation (4) is found to be significantly positive for 5 contracts, significantly negative for 3 contracts and not significantly different from zero for 8 contracts.[18] For the overall dataset time to maturity is totally insignificant. For UK data, the time to maturity variable in equation (4) is found to be significantly positive for 1 contract, significantly negative for 5 contracts and not significantly different from zero for 10 contracts. For the overall dataset, it is only marginally significant. Estimation in subsequent sections is based on the overall datasets, aggregated over all the 16 contracts, and on the basis of the above results, there did not appear to be any need to include time to

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[16] It thereby confirms the earlier results of Merrick (1988) (in a different context) but after using econometrically appropriate testing procedures and high frequency and economically diverse datasets.

[17] As mentioned earlier, Miller *et al* do not appear to model the extent to which the *level* of mispricing at t predicts the *change* in mispricing from t to (t+1). They appear to be concerned essentially with negative autocorrelation in "basis changes". Therefore, it is not clear whether their arguments and simulations lead to the same implications in respect of Φ, as they do for negative autocorrelation in basis changes.

[18] This is not reported in Table 1A. Actual values are available on request.

maturity as an additional explanatory variable. Therefore, further analysis - ie analysis of the variation of Φ with time to maturity, value of mispricing in the previous period, day of the week and hour of the day - is done only on the basis of equation (2).

For US data, p, the necessary number of lags of ΔX_t required varies from 3 to 7 (quarter hour) intervals in individual contract estimations, but for the overall dataset 23 lags in ΔX_t are necessary to minimise the Schwarz Information Criteria as a result of the very large sample size. For UK data, p varies from 1 to 8 hourly intervals in the case of individual contracts, and is 8 for the overall dataset. Since the study of the variation of Φ with time to maturity, value of mispricing in the previous period, day of the week, and hour of the day is based on the overall datasets, the analysis is based on the inclusion in equation (2) of 23 quarter hour lags of ΔX_t for US data and 8 hourly lags of ΔX_t for UK data. In both cases, this corresponds to a "memory" of about one trading day.

4.2 Time to maturity and mean reversion in mispricing

The variation of the mean reversion parameter Φ with time to maturity of the futures contract is estimated using the following model:

$$\Delta X_t = \sum_{j=1}^{6} \Phi^{(j)} D^{(j)} X_{t-1} + \beta_o + \sum_{g=1}^{p} \beta_g \, \Delta X_{t-g} + \epsilon_t \qquad \ldots\ldots(5)$$

where, in the context of the results of Section 4.1, p=23 quarter hour intervals for US data and p=8 hourly intervals for UK data and where $D^{(j)}$ is equal to 1 if time to maturity (T-t) lies within the window j and zero otherwise. The windows are defined as follows:

j = 1 :	{0 days ≤ (T-t) ≤ 15 days}
j = 2 :	{16 days ≤ (T-t) ≤ 30 days}
j = 3 :	{31 days ≤ (T-t) ≤ 45 days}
j = 4 :	{46 days ≤ (T-t) ≤ 60 days}
j = 5 :	{61 days ≤ (T-t) ≤ 75 days}
j = 6 :	{76 days ≤ (T-t) ≤ 95 days}

This classification divides the dataset into 6 windows each of 15 calendar days.[19] $\Phi^{(j)}$ is the estimated mean reversion measure corresponding to time-to-maturity window j.

The following hypotheses are tested:

(a) $H_{TIME1}{}^{(j)}$: $\Phi^{(j)} = 0$

against the alternative that $\Phi^{(j)}$ is greater than zero, j = 1, 2, 3, 4, 5, 6. This is tested using Fuller (1976) τ_μ statistic.

(b) H_{TIME2} : $\Phi^{(1)} = \Phi^{(2)} = \Phi^{(3)} = \Phi^{(4)} = \Phi^{(5)} = \Phi^{(6)}$

against the alternative that at least one coefficient is different. This hypothesis is tested using the F-statistic.

Table 2 reports the results. The hypothesis H_{TIME2} that the $\Phi^{(j)}$ (j=1,2,3,4,5,6) are equal is conclusively rejected for both US data and UK data. For US data $\Phi^{(1)} > \Phi^{(2)} > \Phi^{(3)} > \Phi^{(4)} > \Phi^{(5)} > \Phi^{(6)}$. The mean reversion parameter Φ (and also its

[19] The last window will have slightly more or less than 15 calendar days per contract depending on the time interval between corresponding expiration dates.

associated τ_μ statistic) declines monotonically with an increase in average time to maturity. For the farthest window corresponding to time-to-maturity in excess of 75 calendar days, the mean reversion parameter is not significantly different from zero even at the 10% level. The mean reversion parameter declines with an increase in average time to maturity even for UK data ($\Phi^{(1)} > \Phi^{(2)} > \Phi^{(3)}$) but the variation appears to level off between about 30-45 calendar days.

The systematic increase in mean reversion as time to maturity decreases has several important implications. First, the results are consistent with the risks associated with arbitrage due to uncertainty about future dividends and interest rates being a significant factor in arbitrage related decision making. These risks typically increase as time to maturity increases. For example, future dividends become essentially certain over the approximately last 30 days before maturity because of the usual lag between the dividend declaration date and the ex-dividend date. In the UK context, within the last approximately two weeks before maturity, when the cash settlement date becomes identical for all trading days up to and including the futures maturity date, arbitrage can become almost completely riskless, having no associated-cost-of-carry, no short selling constraints and no dividend uncertainty. Second, while the Brownian Bridge process may not be appropriate[20], our results show that models for mispricing should incorporate a decrease in mean reversion with an increase in time to maturity. Third, since non-synchronous trading in index stocks is clearly

[20] In particular, the process is path independent, while the available evidence clearly indicates path dependence.

not related to time to maturity of the futures contract, this result does not support the view that basis predictability is *entirely* a statistical illusion created by non-synchronous trading without any economic basis in arbitrage related activities.

4.3 Previous period mispricing and mean reversion

As noted earlier, the potential relevance of the variation in the mean reversion parameter with the value of mispricing in the previous period is motivated by the existence of different levels of marginal transaction costs faced by different categories of arbitrageurs. Thus, tests of the variation in mean reversion with the value of previous period mispricing are most likely to identify such variation if data are sorted on the basis of previous period mispricing is such a way that the different bands correspond (approximately) to the estimated transaction costs of different categories of arbitrageurs.

It is difficult to accurately estimate transaction costs relevant to arbitrage, particularly because the results of Sofianos (1990) show that early unwinding returns are high and arbitrageurs could be initiating arbitrage trades within their transaction cost window. However, since early unwinding is clearly important, at least two broad categories of arbitrageurs with different arbitrage related marginal transaction costs will arguably be present:

(a) those whose marginal costs are confined to the transaction costs in the futures market. Examples of arbitrageurs falling into this category include those who are otherwise committed to enter or exit the stock market and hence use the futures market only as an intermediary, and those arbitrageurs with existing arbitrage positions who seek opportunities to profitably exercise the rollover option or the early unwinding option.

(b) those whose marginal costs include also the transaction costs in the cash market. These would typically be arbitrageurs initiating new positions.

For the US, arbitrage related transaction costs have been estimated *inter alia* by Stoll and Whaley (1987) and Sofianos (1990). The recent estimates by, for example Sofianos (1990), are much lower than the earlier estimates by, for example Stoll and Whaley (1987). Considering that arbitrage is not actually risk free, and our data corresponds to a period much earlier than Sofianos (1990) when institutional index arbitrage trading procedures could have been less developed than they are today, we lean towards the more conservative estimates and take the magnitude of futures market transaction costs likely to constrain arbitrage related decisions in category (a) above as 0.1% of futures fair value and the magnitude of total transaction costs (including both cash and futures) likely to constrain arbitrage decisions in category (b) above as 0.5% of futures fair value.

In the UK, the average inner market spread for UK "alpha"[21] stocks, as reported by the *Stock Exchange Quarterly* from time to time, has averaged about 0.8%, except for a few quarters after Black Monday. However arbitrage trades usually take place within the spread. To determine the applicable spread, it is relevant to note that a major component of the quoted spread, namely adverse information costs, should not be relevant in pricing market making services for index arbitrage. Transaction costs related to the cash market should be confined to marginal order processing costs and marginal inventory holding costs. We are not aware of any published estimate for the UK market of the percentage of the quoted spread which arises due to adverse information costs. However, Stoll (1989) finds that on NASDAQ, 43% of the quoted spread represents adverse information costs, 10% represents inventory holding costs and 47% represents order processing costs. If we use these figures as a first approximation for the London market, which has an almost identical trading structure to NASDAQ, the quoted spread for index arbitrage should average about 0.4% to 0.5% except for a few quarters after Black Monday. Even the rise in the quoted spread after Black Monday is more likely to be due to an increase in the adverse information component and the inventory holding component and is hence likely to affect index arbitrage trades to a much lesser extent. Commissions are usually on a flat rate basis and for large volume index arbitrage trades are virtually negligible when expressed as a percentage of value

[21] Stocks in London have been classified on the basis of the number of competing market makers and the trade/quote reporting restrictions applicable to them. Alpha stocks generally have the lowest spreads and all FTSE100 index constituents belong to this category.

traded. To estimate the percentage market impact costs in the UK index futures market, a subset of LIFFE time and sales data was analyzed consisting only of cases in which ask prices and bid prices are posted within 60 seconds of each other. The average percentage spread for the near futures contract varied from about 0.1% to about 0.2% over the sample. Roundtrip percentage commissions in the futures market are typically less than about 0.05%. On the basis of the above, for the UK, the futures market transaction costs likely to constrain arbitrage related decisions in category (a) above were taken as 0.25% of futures fair value and the magnitude of total transaction costs (including both cash and futures) likely to constrain arbitrage decisions in category (b) above were taken as 0.75% of futures fair value.

The variation of the mean reversion parameter Φ with the value of previous period mispricing is hence estimated using the following model:

$$\Delta X_t = \sum_{j=1}^{5} \Phi^{(j)} D^{(j)} X_{t-1} + \beta_0 + \sum_{g=1}^{p} \beta_g \Delta X_{t-g} + \epsilon_t \qquad \ldots\ldots(6)$$

where p=23 quarter hour intervals for US data and p=8 one hour intervals for UK data, and where $D^{(j)}$ is equal to 1 if one period lagged mispricing X_{t-1} lies within the lagged mispricing window j and zero otherwise. The lagged mispricing windows are defined as follows:

	US Data	UK Data
j = 1 :	$\{-\infty < X_{t-1} < -0.50\%\}$	$\{-\infty < X_{t-1} < -0.75\%\}$

$j = 2$:	$\{-0.50\% \leq X_{t-1} < -0.10\%\}$	$\{-0.75\% \leq X_{t-1} < 0.25\%\}$
$j = 3$:	$\{-0.10\% \leq X_{t-1} < 0.10\%\}$	$\{-0.25\% \leq X_{t-1} < 0.25\%\}$
$j = 4$:	$\{0.10\% \leq X_{t-1} < 0.50\%\}$	$\{0.25\% \leq X_{t-1} < 0.75\%\}$
$j = 5$:	$\{0.50\% \leq X_{t-1} < \infty\}$	$\{0.75\% \leq X_{t-1} < \infty\}$

$\Phi^{(j)}$ is the estimated mean reversion measure corresponding to lagged mispricing window j.

The following hypotheses are tested:

(a) $H_{LMIS1}{}^{(j)}$: $\Phi^{(j)} = 0$

against the alternative that $\Phi^{(j)}$ is greater than zero, j = 1, 2, 3, 4, 5. This hypothesis is tested using Fuller (1976) τ_μ statistic.

(b) H_{LMIS2} : $\Phi^{(1)} = \Phi^{(2)} = \Phi^{(3)} = \Phi^{(4)} = \Phi^{(5)}$

against the alternative that at least one coefficient is different. This hypothesis is tested using the F-statistic.

Table 3 reports the results. The hypothesis H_{LMIS2} that the $\Phi^{(j)}$ (j=1,2,3,4,5) are equal, is rejected for both US data and UK data. In both cases, mean reversion in mispricing is not significantly different from zero in the innermost lagged mispricing window, corresponding to values of lagged mispricing where no arbitrageurs are likely to be active. And, in both cases, mean reversion in mispricing is highly significant in the outermost lagged mispricing windows corresponding to values of lagged mispricing where arbitrageurs are likely to be most active. Furthermore, in both cases, the marginal significance level of $\Phi^{(1)}$ is greater than the marginal significance level of $\Phi^{(2)}$ and the marginal significance level of $\Phi^{(5)}$ is greater than the marginal significance level of $\Phi^{(4)}$, showing that mean reversion is more significant in windows corresponding to larger magnitudes of previous period mispricing.

Clearly, the US results and the UK results are consistent with each other and show that mean reversion appears to depend significantly on the value of mispricing in the previous period. Furthermore, since the lagged mispricing windows have been defined on the basis of the estimated marginal transaction costs of different categories of arbitrageurs, the results support the view that mean reversion is, at least partially, arbitrage induced. The results in Table 3 also provide support for the model of Pope and Yadav (1991) in as much as the difference in marginal transaction costs of different categories of arbitrageurs is associated with the parameters of the stochastic process governing futures mispricing.

4.4 Intraweek seasonality in mean reversion

Intraweek seasonality in mean reversion is estimated with the following model:

$$\Delta X_t = \sum_{j=1}^{5} \Phi^{(j)} D^{(j)} X_{t-1} + \beta_0 + \sum_{g=1}^{p} \beta_g \Delta X_{t-g} + \epsilon_t \qquad \ldots\ldots(7)$$

where p=23 quarter hour intervals for US data and p=8 one hour intervals for UK data, and where $D^{(j)}$ is equal to 1 if t lies within the day of the week j and zero otherwise. Here j=1,2,3,4 and 5 correspond to Mondays, Tuesdays, Wednesdays, Thursdays and Fridays respectively $\Phi^{(j)}$ is the estimated mean reversion measure corresponding to day of the week j.

The following hypotheses are tested:

(a) $H_{IW1}^{(j)}$: $\Phi^{(j)} = 0$

against the alternative that $\Phi^{(j)}$ is greater than zero, j = 1, 2, 3, 4, 5. This hypothesis is tested using Fuller (1976) τ_μ statistic.

(b) H_{IW2} : $\Phi^{(1)} = \Phi^{(2)} = \Phi^{(3)} = \Phi^{(4)} = \Phi^{(5)}$

against the alternative that at least one coefficient is different. This hypothesis is tested using the F-statistic.

Table 4 reports the results. For US data, hypothesis H_{IW2} is rejected at the 5% level. Whereas the mean reversion is highly significant on Tuesday, Wednesday and Friday, it is not significant on Monday and Thursday. For UK data, hypothesis H_{IW2} is not rejected (p value 0.18). Whereas the mean reversion is highly significant on Tuesday, Wednesday, Thursday and Friday, it is not significant on Monday. In common with many other day-of-the-week effects, we have no immediate explanation for this difference. Considering that there appears to be a further example of the Monday effect, we also tested the following hypothesis:

H_{IW3} : $\Phi^{(1)} = \Phi^{(0)}$

against the alternative that

$\Phi^{(1)} \neq \Phi^{(0)}$

where $\Phi^{(0)}$ was the mean reversion observed over the sample for Tuesday, Wednesday, Thursday and Friday. This hypothesis was tested using the F-statistic.

Hypothesis H_{IW3} is rejected for both US and UK data albeit only at the 10% level. The mean reversion on Monday appears to be lower than the mean reversion on the remaining days of the week.

In light of the results in section 4.3, the day of the week seasonality in mean reversion could potentially arise if the frequencies of cases in the highest magnitude lagged mispricing windows ie windows 1 and 5, were substantially lower on Mondays than on other days of the week. In other words, it is possible that the day of the week could be proxying for the magnitude of previous period mispricing. However, for US data the proportion of cases in the highest magnitude lagged mispricing windows is *highest* on Mondays ie 23.6% - significantly higher than the 19.2% expected on the basis of the population mean if there are no differences among different days of the week. For UK data, the proportion of cases in the highest magnitude lagged mispricing windows is 19.6% - again higher than the expected value of 18.6%. Thus it would appear to be unlikely that the day of the week dependency of mean reversion is proxying for the magnitude of lagged mispricing.

4.5 Intraday seasonality in mean reversion

Intraday seasonality in mean reversion is estimated with the following model:

$$\Delta X_t = \sum_{j=1}^{q} \Phi^{(j)} D^{(j)} X_{t-1} + \beta_0 + \sum_{g=1}^{p} \beta_g \Delta X_{t-g} + \epsilon_t \qquad ......(8)$$

where p=23 quarter hour intervals and q=6 intraday intervals for US data and p=8 one hour intervals and q=8 intraday intervals for UK data, and where $D^{(j)}$ is equal to 1 if t lies within the intraday interval j and zero otherwise. For UK data, intraday interval 0 corresponds to the overnight interval and intraday intervals 1,2,3,4,5,6 and 7 correspond to the successive hourly intervals after the opening of the market. For US data, if the overnight interval is labelled 0 and the successive 15-minute intervals after market opening are labelled as 1,2,...24 on or before September 27 1985 and 1,2,...26 thereafter, then intraday intervals corresponding to j=0,1,2,3,4 and 5 are defined as follows:

	September 27 1985 and Before	After September 27 1985
j = 0 :	$\{0 \leq t \leq 3\}$	$\{0 \leq t \leq 3\}$
j = 1 :	$\{4 \leq t \leq 7\}$	$\{4 \leq t \leq 7\}$
j = 2 :	$\{8 \leq t \leq 11\}$	$\{8 \leq t \leq 12\}$
j = 3 :	$\{12 \leq t \leq 16\}$	$\{13 \leq t \leq 17\}$
j = 4 :	$\{17 \leq t \leq 20\}$	$\{18 \leq t \leq 22\}$
j = 5 :	$\{21 \leq t \leq 24\}$	$\{23 \leq t \leq 26\}$

$\Phi^{(j)}$ is the estimated mean reversion measure corresponding to intraday interval j.

The following hypotheses are tested:

(a) $H_{ID1}^{(j)}$: $\Phi^{(j)} = 0$

against the alternative that $\Phi^{(j)}$ is greater than zero, j = 0, 1, 2, 3, 4, 5 for the US and j = 0, 1, 2, 3, 4, 5, 6, 7 for the

UK. This hypothesis is tested using Fuller (1976) τ_μ statistic.

(b) H_{ID2} : $\Phi^{(0)} = \Phi^{(1)} = \Phi^{(2)} = \Phi^{(3)} = \Phi^{(4)} = \Phi^{(5)}$

for the US;

and $\Phi^{(0)} = \Phi^{(1)} = \Phi^{(2)} = \Phi^{(3)} = \Phi^{(4)} = \Phi^{(5)} = \Phi^{(6)} = \Phi^{(7)}$

for the UK;

against the alternative that at least one coefficient is different. This is tested using the F-statistic.

Table 5 reports the results. The hypothesis H_{ID2} is conclusively rejected for both US and UK data. There appears to be a strong intraday pattern in mean reversion. In both cases, the pattern is U-shaped. For US data, mean reversion is not significantly different from zero in approximately the second and the third hour of trading. It is maximum at the market open, then declines to about zero, and again rises steadily towards the market close. For UK data, mean reversion is not significantly different from zero, except in the first two hours of trading and during the last hour before the close. Again, in light of the results in section 4.3, there is need to examine whether the intraday periods are proxying for the value of lagged mispricing. This would be revealed if the frequencies of cases in the highest magnitude lagged mispricing windows ie windows 1 and 5, was found to be much higher in the opening hour and towards the close, than at midday. For the US, it was found that while the proportion of cases in the highest magnitude lagged mispricing windows in the opening hour was 18.9% - significantly higher than the expected 15.5% - it was lower than the expected proportion of cases towards the close. For the UK, the proportion of cases in the highest magnitude lagged mispricing windows were approximately evenly spread over the day, and, in particular, not significantly different from the expected proportion of cases during the opening hour and towards the close. Thus, whilst it is likely that the stronger mean reversion at the US opening is at least partially due to the higher lagged mispricing at that time, it would appear that the magnitude of previous period mispricing cannot completely explain the intraday patterns of variation in mean reversion.

The U-shaped intraday pattern in mean reversion is similar to the U-shaped intraday pattern in intraday one period volatility in the datasets (see eg McInish and Wood (1990) and Yadav and Pope (1992a)). However, the significantly lower mean reversion on Mondays does not appear to be related to the intraweek seasonality in intraday one period volatility in the datasets. Therefore, we did not pursue explanations of this seasonality related to cash/futures one period volatility.

5. CONCLUSIONS

This chapter has presented evidence on mean reversion in index futures mispricing based on about four years of high frequency intraday data from both the US and the UK markets. The results from the two markets are remarkably consistent and have several interesting features. First, the existence of mean reversion in the overall mispricing series has been firmly established in as much as the change in mispricing depends significantly on the *level* of mispricing in the previous period. This is consistent with the existence of significant arbitrage activity and, in particular, with the intertemporal arbitrage trading model of Holden (1990b). Second, the mean reversion parameter is a systematic function of the time to maturity, increasing as the time to maturity decreases. This is consistent with arbitrage being considered more risky when time to maturity is higher. More importantly, it does not support the view that basis predictability is *entirely* a statistical illusion created by non-synchronous trading. Third, mean reversion appears to depend significantly on the value of mispricing in the previous period. It is not significantly different from zero when the magnitude of the previous period mispricing is so small that no arbitrageurs are likely to be active, but becomes significant in magnitude when the magnitude of the previous period mispricing becomes large enough to exceed the estimated marginal transaction costs of arbitrageurs. This supports the view that mean reversion is arbitrage induced, and, in particular is consistent with the TAR model of Pope and Yadav (1991). Fourth, mean reversion appears to be significantly lower on Mondays than on other days of the week. Finally, mean reversion also exhibits significant intraday seasonality, following a U-shaped intraday pattern. However, the seasonalities cannot be explained in terms of corresponding patterns in lagged mispricing and volatility.

TABLE 1A

UNIT ROOT TESTS ON FUTURES MISPRICING USING AUGMENTED DICKEY-FULLER REGRESSIONS - US DATA[1]

Futures Mispricing X_t is defined as {Log (Futures Price) - Log (Futures Equivalent Cash Price)}[2]

Two alternative models are estimated, one with a time trend and one without a time trend.

Model without time trend:

Model with time trend:[3]

p is chosen in each case as the minimum value that ensures that serial correlation in the regression residuals is statistically insignificant. Serial correlation is estimated by the Box-Pierce Q statistic for 50 lags[4]

	Model without time trend				Model with time trend			
Contract Expiring	Number of lags p	P-value of Q Statistic	Φ	T-Statistic Null:Φ=0[5]	Number of lags p	P-value of Q Statistic	Φ	T-Statistic Null:Φ=0[6]
September 83	5	0.376	0.087	5.48***	5	0.471	0.106	6.03***
December 83	6	0.025	0.034	3.33**	6	0.025	0.034	3.31*
March 84	7	0.227	0.029	2.87**	7	0.239	0.036	3.27*
June 84	4	0.150	0.079	4.93***	4	0.163	0.098	5.52***
September 84	7	0.189	0.054	4.12***	7	0.246	0.090	5.43***
December 84	4	0.108	0.020	2.71*	3	0.173	0.045	4.40***
March 85	3	0.054	0.014	2.56*	4	0.147	0.054	4.60***
June 85	3	0.153	0.031	4.15***	3	0.192	0.056	5.16***

TABLE 1A (CONTINUED)

Contract Expiring	Model without time trend: Number of lags p	P value of Q Statistic	Φ	T-Statistic Null:Φ=0[5]	Model with time trend: Number of lags p	P-Value of Q Statistic	Φ	T-Statistic Null:Φ=0[6]
September 85	4	0.074	0.033	3.96***	4	0.150	0.066	5.39***
December 85	4	0.456	0.032	3.63***	4	0.542	0.052	4.81***
March 86	7	0.014	0.050	4.15***	7	0.014	0.054	4.36***
June 86	5	0.133	0.056	4.61***	5	0.146	0.061	4.73***
September 86	7	0.109	0.079	5.03***	7	0.115	0.085	5.17***
December 86	7	0.115	0.044	3.83***	4	0.212	0.183	8.73***
March 87	4	0.229	0.133	6.74***	4	0.234	0.137	6.86***
June 87	5	0.190	0.266	9.11***	5	0.218	0.271	9.21***
All Contracts	23	-	0.018	8.59***	23	-	0.018	8.57***

(1) Mackinlay and Ramaswamy (1988) 15 minute interval dataset is used.

(2) Futures Equivalent Cash Price is taken as identical to the price determined in accordance with the cost-of-carry forward pricing formula.

(3) T is the time subscript corresponding to futures maturity.

(4) In some cases the Box Pierce Q statistic remains statistically significant even for very large values of p, due to spikes in serial correlation at long lags. The Schwartz (1978) Information Criteria is used to choose p in such cases.

(5) The τ_μ statistic in Fuller (1976) and Schmidt (1990) is used to flag significance
***: p-value <0.01; **: p value <0.05; *: p value <0.10.

(6) The τ_t statistic in Fuller (1976) is used to flag significance.
***: p value <0.01; **: p value <0.05; *: p value <0.10.

TABLE 1B

UNIT ROOT TESTS ON FUTURES MISPRICING USING AUGMENTED DICKEY-FULLER REGRESSIONS - UK DATA[1]

Futures Mispricing X_t is defined as {Log (Futures Price) - Log (Futures Equivalent Cash Price)}[2]

Two alternative models are estimated, one with a time trend and one without a time trend.

Model without time trend:

Model with time trend:[3]

p is chosen in each case as the minimum value that ensures that serial correlation in the regression residuals is statistically insignificant. Serial correlation is estimated by the Box-Pierce Q statistic for 24 lags[4]

	Model without time trend				Model with time trend			
Contract Expiring	Number of lags p	P-value of Q Statistic	Φ	T-Statistic Null:Φ=0[5]	Number of lags p	P-value of Q Statistic	Φ	T-Statistic Null:Φ=0[6]
June 86	1	0.259	0.130	4.31***	1	0.573	0.237	5.79***
September 86	2	0.134	0.101	3.83***	2	0.133	0.105	3.92**
December 86	1	0.131	0.073	3.76***	1	0.131	0.074	3.77**
March 87	2	0.517	0.130	4.64***	2	0.517	0.130	4.63***
June 87	5	0.121	0.055	2.57*	5	0.119	0.066	2.85
September 87	6	0.214	0.072	3.38**	6	0.222	0.104	3.80**
December 87	5	0.094	0.132	3.60***	5	0.088	0.130	3.54**
March 88	8	0.000	0.135	4.14***	8	0.000	0.135	4.14***

TABLE 1B (CONTINUED)

Contract Expiring	Number of lags p	Model without time trend: P value of Q Statistic	Φ	T-Statistic Null:Φ=0[5]	Number of lags p	Model with time trend: P-Value of Q Statistic	Φ	T-Statistic Null:Φ=0[6]
June 88	2	0.148	0.071	3.17**	2	0.149	0.071	3.16*
September 88	1	0.383	0.062	2.84*	1	0.794	0.207	6.03***
December 88	1	0.131	0.023	2.15	1	0.255	0.071	3.20*
March 89	1	0.221	0.086	3.94***	1	0.229	0.091	3.97**
June 89	2	0.115	0.081	3.80***	2	0.170	0.121	4.50***
September 89	1	0.277	0.052	3.17**	1	0.351	0.086	4.08***
December 89	3	0.002	0.023	1.73	3	0.003	0.047	2.52
March 90	1	0.840	0.169	5.56***	1	0.923	0.217	6.38***
All Contracts	8	-	0.038	8.43***	8	-	0.039	8.65***

(1) Yadav and Pope (1992a) hourly interval dataset is used.

(2) Futures Equivalent Cash Price is taken as identical to the price determined in accordance with the cost-of-carry forward pricing formula.

(3) T is the time subscript corresponding to futures maturity.

(4) In some cases the Box Pierce Q statistic remains statistically significant even for very large values of p, due to spikes in serial correlation at long lags. The Schwartz (1978) Information Criteria is used to choose p in such cases.

(5) The τ_μ statistic in Fuller (1976) and Schmidt (1990) is used to flag significance
***: p-value <0.01; **: p value <0.05; *: p value <0.10.

(6) The τ_t statistic in Fuller (1976) is used to flag significance.
***: p value <0.01; **: p value <0.05; *: p value <0.10.

TABLE 2

TIME TO MATURITY AND MEAN REVERSION IN MISPRICING

The model estimated is the following:

$$\Delta X_t = \sum_{j=1}^{6} \Phi^{(j)} D^{(j)} X_{t-1} + \beta_0 + \sum_{g=1}^{p} \beta_g \Delta X_{t-g} + \epsilon_t$$

where p=23 quarter hour intervals for US data and p=8 one hour intervals for UK data, and $D^{(j)}$ is equal to 1 if time to maturity (T-t) lies within the window j and zero otherwise and where the windows are defined as follows:

j = 1 : {0 days ≤ (T-t) ≤ 15 days}
j = 2 : {16 days ≤ (T-t) ≤ 30 days}
j = 3 : {31 days ≤ (T-t) ≤ 45 days}
j = 4 : {46 days ≤ (T-t) ≤ 60 days}
j = 5 : {61 days ≤ (T-t) ≤ 75 days}
j = 6 : {76 days ≤ (T-t) ≤ 95 days}

$\Phi^{(j)}$ is the estimated mean reversion measure corresponding to time-to-maturity window j

$H_{TIME1}^{(j)}$: $\Phi^{(j)} = 0$
against the alternative that $\Phi^{(j)}$ is greater than zero.
j = 1, 2, 3, 4, 5, 6
(Tested using Fuller (1976) τ_μ statistic)

H_{TIME2} : $\Phi^{(1)} = \Phi^{(2)} = \Phi^{(3)} = \Phi^{(4)} = \Phi^{(5)} = \Phi^{(6)}$
against the alternative that at least one coefficient is different.

	US Data[1]			UK Data[2]		
Time-to-Maturity Window	$\Phi^{(j)}$	τ_μ-statistic[3] for $H_{TIME1}^{(j)}$	F-statistic[3] for H_{TIME2}	$\Phi^{(j)}$	τ_μ-statistic[3] for $H_{TIME1}^{(j)}$	F-statistic[3] for H_{TIME2}
1	0.069	7.95***	11.21***	0.102	5.81***	5.29***
2	0.042	5.68***		0.064	5.33***	
3	0.024	4.76***		0.032	3.45***	
4	0.016	4.20***		0.021	2.34	
5	0.014	3.52**		0.031	3.17**	
6	0.008	1.99		0.035	3.44**	

1 The MacKinlay and Ramaswamy (1988) 15 minute interval dataset.

2 The Yadav and Pope (1992a) hourly interval dataset.

3 *** Significant at the 1% level
** Significant at the 5% level
* Significant at the 10% level

TABLE 3

PREVIOUS PERIOD MISPRICING AND MEAN REVERSION

The model estimated is the following:

$$\Delta X_t = \sum_{j=1}^{5} \Phi^{(j)} D^{(j)} X_{t-1} + \beta_0 + \sum_{g=1}^{p} \beta_g \Delta X_{t-g} + \epsilon_t$$

where p=23 quarter hour intervals for US data and p=8 one hour intervals for UK data, and $D^{(j)}$ is equal to 1 if time to maturity (T-t) lies within the lagged mispricing window j and zero otherwise and where the lagged mispricing windows are defined as follows:

	US Data	UK Data
j = 1 :	$\{-\infty < X_{t-1} < -0.50\%\}$	$\{-\infty < X_{t-1} < -0.75\%\}$
j = 2 :	$\{-0.50\% \leq X_{t-1} < -0.10\%\}$	$\{-0.75\% \leq X_{t-1} < -0.25\%\}$
j = 3 :	$\{-0.10\% \leq X_{t-1} < 0.10\%\}$	$\{-0.25\% \leq X_{t-1} < 0.25\%\}$
j = 4 :	$\{0.10\% \leq X_{t-1} < 0.50\%\}$	$\{0.25\% \leq X_{t-1} < 0.75\%\}$
j = 5 :	$\{0.50\% \leq X_{t-1} < \infty\}$	$\{0.75\% \leq X_{t-1} < \infty\}$

$\Phi^{(j)}$ is the estimated mean reversion measure corresponding to lagged mispricing window j

$H_{LMIS1}^{(j)}$: $\Phi^{(j)} = 0$
against the alternative that $\Phi^{(j)}$ is greater than zero.
j = 1, 2, 3, 4, 5
(Tested using Fuller (1976) τ_μ statistic)

H_{LMIS2} : $\Phi^{(1)} = \Phi^{(2)} = \Phi^{(3)} = \Phi^{(4)} = \Phi^{(5)}$
against the alternative that at least one coefficient is different.

	US Data[1]			UK Data[2]		
Lagged Mispricing Window	$\Phi^{(j)}$	τ_μ-statistic[3] for $H_{LMIS1}^{(j)}$	F-statistic[3] for H_{LMIS2}	$\Phi^{(j)}$	τ_μ-statistic[3] for $H_{LMIS1}^{(j)}$	F-statistic[3] for H_{LMIS2}
1	0.055	8.54***	11.20***	0.035	4.58***	2.39**
2	0.035	4.55***		0.026	2.11	
3	-0.008	-0.26		0.034	1.13	
4	0.014	1.86		0.026	2.00	
5	0.008	3.05**		0.060	5.99***	

1 The MacKinlay and Ramaswamy (1988) 15 minute interval dataset.

2 The Yadav and Pope (1992a) hourly interval dataset.

3 *** Significant at the 1% level
** Significant at the 5% level
* Significant at the 10% level

TABLE 4

MEAN REVERSION IN INDEX FUTURES MISPRICING: INTRAWEEK SEASONALITY

The model estimated is the following:

$$\Delta X_t = \sum_{j=1}^{6} \Phi^{(j)} D^{(j)} X_{t-1} + \beta_0 + \sum_{g=1}^{p} \beta_g \Delta X_{t-g} + \epsilon_t$$

where p=23 quarter hour intervals for US data and p=8 one hour intervals for UK data, and $D^{(j)}$ is equal to 1 if t lies within the day of the week j and zero otherwise. j=1,2,3,4 and 5 correspond to Mondays, Tuesdays, Wednesdays, Thursdays and Fridays respectively.

$\Phi^{(j)}$ is the estimated mean reversion measure corresponding to day of the week j and $\Phi^{(0)}$ is the estimated mean reversion over Tuesdays, Wednesdays, Thursdays and Fridays.

$H_{IW1}^{(j)}$: $\Phi^{(j)} = 0$
against the alternative that $\Phi^{(j)}$ is greater than zero.

j = 1, 2, 3, 4, 5
(Tested using Fuller (1976) τ_μ statistic)

H_{IW2} : $\Phi^{(1)} = \Phi^{(2)} = \Phi^{(3)} = \Phi^{(4)} = \Phi^{(5)}$
against the alternative that at least one coefficient is different.

H_{IW3} : $\Phi^{(1)} = \Phi^{(0)}$ against the alternative that $\Phi^{(1)} \neq \Phi^{(0)}$

	US Data[1]				UK Data[2]			
Day of the Week	$\Phi^{(j)}$	τ_μ-statistic[3] for $H_{IW1}^{(j)}$	F-statistic[3] for H_{IW2}	F-statistic[3] for H_{IW3}	$\Phi^{(j)}$	τ_μ-statistic[3] for $H_{IW1}^{(j)}$	F-statistic[3] for H_{IW2}	F-statistic[3] for H_{IW3}
Monday	0.011	2.67*	2.42**	2.95*	0.023	2.29	1.60	2.80*
Tuesday	0.023	5.32***			0.032	3.56***		
Wednesday	0.023	5.21***			0.054	5.69***		
Thursday	0.010	2.32			0.041	4.32***		
Friday	0.022	5.05***			0.036	3.68***		

[1] The MacKinlay and Ramaswamy (1988) 15 minute interval dataset.

[2] The Yadav and Pope (1992a) hourly interval dataset.

[3] *** Significant at the 1% level
** Significant at the 5% level
* Significant at the 10% level

TABLE 5 - MEAN REVERSION IN INDEX FUTURES MISPRICING: INTRADAY SEASONALITY

The model estimated is the following:

$$\Delta X_t = \sum_{j=1}^{q} \Phi^{(j)} D^{(j)} X_{t-1} + \beta_0 + \sum_{g=1}^{p} \beta_g \Delta X_{t-g} + \epsilon_t$$

where p=23 quarter hour intervals and q=6 intraday intervals for US data and p=8 one hour intervals and q=8 intraday intervals for UK data, and $D^{(j)}$ is equal to 1 if t lies within the intraday interval j and zero otherwise. For UK data, intraday interval 0 corresponds to the overnight interval and intraday intervals 1,2,3,4,5,6 and 7 correspond to the successive hourly intervals after the opening of the market. For US data, if the overnight interval is labelled 0 and the successive 15-minute intervals after market opening are labelled as 1,2,...24 on or before September 27 1985 and 1,2,...26 thereafter, then intraday intervals corresponding to j=0,1,2,3,4 and 5 are defined as follows:

	September 27 1985 and Before	After September 27 1985
1 :	$\{0 \le t \le 3\}$	$\{0 \le t \le 3\}$
2 :	$\{4 \le t \le 7\}$	$\{4 \le t \le 7\}$
3 :	$\{8 \le t \le 11\}$	$\{8 \le t \le 12\}$
4 :	$\{12 \le t \le 16\}$	$\{13 \le t \le 17\}$
5 :	$\{21 \le t \le 24\}$	$\{23 \le t \le 26\}$

$\Phi^{(j)}$ is the estimated mean reversion measure corresponding to intraday interval j

$H_{ID1}^{(j)}$: $\Phi^{(j)} = 0$
against the alternative that $\Phi^{(j)}$ is greater than zero.
j = 0, 1, 2, 3, 4, 5 for the US and j = 0, 1, 2, 3, 4, 5, 6, 7 for the UK
(Tested using Fuller (1976) τ_μ statistic)

H_{ID2} : $\Phi^{(0)} = \Phi^{(1)} = \Phi^{(2)} = \Phi^{(3)} = \Phi^{(4)} = \Phi^{(5)}$ for the US
and $\Phi^{(0)} = \Phi^{(1)} = \Phi^{(2)} = \Phi^{(3)} = \Phi^{(4)} = \Phi^{(5)} = \Phi^{(6)} = \Phi^{(7)}$ for the UK
against the alternative that at least one coefficient is different.

	US Data[1]			UK Data[2]		
Intraday Interval	$\Phi^{(j)}$	τ_μ-statistic[3] for $H_{ID1}^{(j)}$	F-statistic[3] for H_{ID2}	$\Phi^{(j)}$	τ_μ-statistic[3] for $H_{ID1}^{(j)}$	F-statistic[3] for H_{ID2}
0	0.038	8.10***	9.13***	0.111	9.44***	17.57***
1	-0.002	-0.43		0.101	8.62***	
2	0.008	1.73		0.009	0.75	
3	0.014	3.47***		0.008	0.70	
4	0.024	4.98***		-0.008	-0.70	
5	0.024	4.81***		0.004	0.38	
6	-	-		0.027	2.22	
7	-	-		0.044	3.71***	

1 The MacKinlay and Ramaswamy (1988) 15 minute interval dataset.

2 The Yadav and Pope (1992a) hourly interval dataset.

3 *** Significant at the 1% level
** Significant at the 5% level
* Significant at the 10% level

CHAPTER 8

PRICING OF STOCK INDEX FUTURES SPREADS: THEORY AND EVIDENCE[1]

ABSTRACT

This chapter develops the theoretical framework for pricing of stock index futures spreads after adjusting for cash market settlement procedures, and provides empirical evidence in this regard based on about four years of "time and sales" transactions data from the London International Financial Futures Exchange (LIFFE). It also simulates the profitability of spread arbitrage strategies (in particular in the context of the early unwinding option), analyses the effect of spread mispricing on short term spread positions, and explores mean reversion in the time series of spread mispricing. The results have implications for the use of arbitrage related arguments in the context of market microstructure, the ability of futures traders to transfer risk within themselves, the debate on whether cash-futures basis behaviour is arbitrage induced or a manifestation of non-synchronous trading, the market value of the futures related tax timing option and the effect of short selling constraints on futures pricing.

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[1] First draft June 1992.

PRICING OF STOCK INDEX FUTURES SPREADS:
THEORY AND EVIDENCE

1. INTRODUCTION

There has been considerable interest in the pricing of stock index futures contracts relative to the underlying cash index. *Inter-alia* Mackinlay and Ramaswamy (1988), Figlewski (1984a), Cornell and French (1983a, 1985a), Modest and Sundereshan (1983), Brenner, Subrahmanyam and Uno (1989), and Yadav and Pope (1990, 1992c) document the existence of substantial and sustained deviations between the actual index futures price and the index futures price "equivalent" (on the basis of the forward pricing formula) to the price in the underlying cash market. Finnerty and Park (1988), Merrick (1989), Yadav and Pope (1990, 1992c), and Klemkosky and Lee (1991) demonstrate the profitability of cash futures arbitrage strategies. Saunders and Mahajan (1988) and Yadav and Pope (1991) examine pricing efficiency of index futures contracts relative to the cash index using price differences instead of price levels. Merrick (1988) and Hill *et al* (1988) explore the implications of cash-futures mispricing for short term hedgers and portfolio insurers respectively. Brennan and Schwartz (1990), Holden (1990), Cooper and Mello (1990) and Pope and Yadav (1991) analyse the stochastic process followed by cash-futures mispricing in the context of market structure, transaction costs and the optimal actions of arbitrageurs. Finally, Yadav and Pope (1992b) and Miller *et al* (1991) investigate specifically the mean reversionary behaviour of cash-futures mispricing.

However, there has been little published work on the relative pricing of index futures contracts with different times to expiration even though the practical relevance for arbitrageurs and the theoretical issues involved are essentially similar to those involved in cash futures arbitrage. Specifically, an analysis of the pricing of index futures "spreads" can be valuable for several reasons. First, it can potentially contribute to our understanding of the economics of arbitrage. Index futures contracts with different times to maturity represent equivalent assets (except for a largely non-stochastic factor - the cost of carrying the underlying asset from the maturity of the "near" contract to the maturity of the "far" contract) in the same way as cash and futures represent equivalent assets. An important reason for the academic interest in the cash futures pricing relationship has been that the associated arbitrage strategies are easy to implement and largely risk free[2] with clearly quantifiable transaction costs. This cannot be so in the case of pricing of primary assets like stocks, where "arbitrage" can be only with reference to expectations (Scheifer and Summers, 1990) and is not so even in the case of pricing of other derivative assets like options where arbitrage would necessitate continuous rebalancing of the arbitrage position and hence infinite transaction costs. A study of the price difference between index futures contracts with different times to maturity, and how it evolves over time is direct study of a series of price differences between equivalent securities and can shed light on the extent to which it is reasonable to use arbitrage arguments eg for pricing options, where arbitrage is

[2] Arbitrage is not totally riskless because future dividends and interest rates are not known *ex ante* with perfect certainty.

much more difficult, or elsewhere in corporate financial theory eg capital structure or dividend policy. More specifically it can enable inferences relating to arbitrage in the context of market microstructure eg about effective transaction costs of different categories of arbitrageurs (as in Pope and Yadav, 1991), the nature of the arbitrage structure ie monopolistic, imperfectly competitive or perfectly competitive (as in Cooper and Mello, 1990; and Holden, 1990c), the effects of short sale restrictions (as in Yadav and Pope, 1992c, and Puttonen and Martikainen, 1991) and the intertemporal strategies followed by arbitrageurs (like the use of the early unwinding option highlighted in Merrick, 1989; and Brennan and Schwartz, 1990).

Second, spreads perform an important economic function. They can be used to transfer risk from one futures trader to another and thereby facilitate allocation of risk between different futures traders. Futures markets provide a vehicle for transfer of risk to traders in the cash market. The opportunity to trade spreads enables futures traders to more easily transfer risk among themselves and hence makes them more willing to supply the price insurance demanded by hedgers in the cash market. Spread trading can contribute effectively to allocation of risk between different futures traders only when they are efficiently priced. Spread "mispricing"[3] has important implications for the effectiveness and average cost of short term spreading in the same way as cash futures mispricing is a critical

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[3] Spreads can be mispriced due to transaction costs, short sale restrictions, difficulties of trading the cash index and non-stochastic carrying costs (due to uncertainty about future dividends and interest rates).

determinant of the effectiveness and average cost of short term hedging (highlighted by Merrick, 1988).

Third, an examination of relative mispricing of index futures contracts with different times to maturity can potentially contribute to the debate on whether the mean reversion observed in cash futures arbitrage is arbitrage induced or a "statistical illusion" created by non-synchronous trading in index stocks. It had been presumed that this mean reversion is a result of the actions of arbitrageurs (see eg Brennan and Schwartz, 1990, pp 58; Mackinlay and Ramaswamy, 1988, pp 137) since one would expect arbitrage opportunities to be rapidly eliminated in well functioning capital markets. However, Miller *et al* (1991) have suggested that the observed mean reversionary behaviour in the cash futures basis could just be a manifestation of non-synchronous trading (differential speeds of price adjustments) in the index basket of stocks, having no economic significance in terms of actual index arbitrage activity. The mean reversion generated by this "statistical illusion" hypothesis is observationally indistinguishable from the potential mean reversion generated by actual index arbitrage activity. The analysis and evidence of Miller *et al* only shows that the observed basis predictability could *also* be explained in terms of non-synchronous trading in index stocks. Clearly, the mispricing of one futures contract relative to another futures contract should not be influenced by factors related to non-synchronous trading in the cash index, or, more generally, any kind of measurement errors with respect to the cash index. Significant mean reversion in spread mispricing, and significant negative serial correlation in changes in spread mispricing, will provide support to the view that mean reversion in cash futures pricing is, at least not *entirely*, a consequence of non-synchronous trading.

Fourth, analysis of stock index futures spreads offers a direct test of the relevance and value of the tax timing option in relation to index futures pricing. The tax timing option is potentially valuable because stockholders have the ability to select the timing of realisation of losses and gains. Cash settlement of futures contracts implies that investors in the futures market necessarily pay taxes in the year the capital gains arise while investors holding the cash asset can defer their capital gains. On the other hand, the marginal investor may be a tax exempt institution in which case the tax timing option will have no value, or the marginal investor may be an arbitrageur/floor trader who cannot hold the cash index indefinitely in which case again the tax timing option will have no value. Clearly, the relevance of the tax timing option for index futures pricing could be different in different markets and essentially an empirical issue. *Ceteris paribus* the value of the tax timing option should be greater for longer times to expiration. Therefore, if this factor is important for index futures pricing then the far contract should be more negatively mispriced than the near contract ie the far contract should be underpriced relative to the near contract. Hence, on average, spread mispricing should be negative.

To the best of our knowledge, Billingsley and Chance (1988) is the only published evidence on the pricing of stock index futures spreads. It is based on weekly data, assumes instantaneous settlement in cash markets, and is largely confined to a description of the basic statistics. This chapter formally develops the theoretical framework for pricing of stock index futures spreads after adjusting for cash market settlement procedures, and provides empirical evidence in this regard based on about four years of "time and sales" transactions data from the London International Financial Futures Exchange (LIFFE). It also simulates the profitability of spread arbitrage strategies (in particular in the context of the early unwinding option), analyses the effect of spread mispricing on short term spread positions, and explores mean reversion in the time series of spread mispricing.

The use of data from the London market also enables a test of the effect on futures pricing of the constraints that exist on short selling of stocks. It has been suggested that the observed preponderance of negative mispricing can be at least partially explained by the institutional restrictions and difficulties that exist in selling stocks short, since the costs involved in exploiting futures underpricing (relative to cash) are higher than the corresponding costs of exploiting futures overpricing relative to cash (see eg Modest and Sundereshan, 1984; Figlewski, 1984b; Brenner *et al*, 1989; and Puttonen and Martikainen, 1991.) However, it has not been possible for US based studies to formally test this hypothesis. The unique features of the settlement procedures on the London Stock Exchange result in virtually no constraints on index arbitrage related short selling within the two (or three) "account" spanning futures maturity. Hence, if short selling restrictions influence futures pricing, the far contract should be significantly more underpriced relative to the near contract during the "account" spanning futures maturity than in other time periods.

This chapter is organised as follows: Section 2 provides the institutional and theoretical framework within which the empirical analysis is conducted; Section 3 describes the database, explains the methodology and documents the empirical results; and Section 4 presents the conclusions.

2. THEORETICAL AND INSTITUTIONAL FRAMEWORK

2.1 Settlement Adjusted Pricing of Stock Index Futures Spreads

Index futures have been generally priced as forward contracts ignoring the stochastic cash flows associated with daily marking to market of the futures position. The forward pricing formula is based on the existence of a portfolio of the underlying asset and treasury bills which can exactly replicate the payoffs of the forward contract given the following assumptions: (a) no transactions costs (including in particular costless short sales); (b) no taxes; (c) no spread between borrowing and lending rates; (d) interest bearing margins; and (e) dividends and interest rates up to futures maturity known *ex ante* with perfect certainty.

Billingsley and Chance (1988, pp 304-305) provide the spread pricing relationship assuming a constant dividend yield. They also implicitly assume that cash market transactions are settled immediately. However, dividend yields are not constant and cash market transactions are not settled on the same day. In general, the time t arbitrage free "fair" value $F_{t,T_2}{}^*$ of an index futures contract maturing at T_2 will be given by:

$$F^*_{t,T_2} = F_{t,T_1}\exp\{r_{t,T_1',T_2'}(T_2'-T_1')\} - \sum_{w=T_1+1}^{T_2} d_w \exp\{r_{t,w',T_2'}(T_2'-w')\} \qquad \text{....(1)}$$

where

F_{t,T_1}	=	Value at time t of an index futures contract maturing at T_1
T_1'	=	Settlement date for cash market transactions made at T_1
T_2'	=	Settlement date for cash market transactions made at T_2
d_w	=	Aggregate dividend cash flows on the index associated with an ex-dividend time period w
w'	=	Time at which dividend cash flow d_w is actually received.
$r_{t,T_1',T_2'}$	=	Forward interest rate at time t for a loan to be disbursed at time T_1' for repayment at time T_2'
$r_{t,w',T_2'}$	=	Forward interest rate at time t for a loan to be disbursed at time W' for repayment at time T_2'

To prove this, consider the following strategy: (a) at t sell one far futures contract, buy one near futures contract, and arrange to borrow an amount F_{t,T_1} from T_1' to T_2' at the forward rate $r_{t,T_1',T_2'}$; (b) at T_1 buy the cash index for S_{T_1} and settle the near futures contract (carrying over associated cash flows (S_{T_1}-F_{t,T_1}) from T_1 to T_1'); (c) at T_1' receive disbursement of the borrowing F_{t,T_1} arranged at t and pay S_{T_1} for the long cash index position after adding the cash flow (S_{T_1}-F_{t,T_1}) received at (b) from settlement of the near futures contract; (d) for the index constituents going ex-dividend between T_1 and T_2, invest all dividends d_w (known *ex ante* and received on the actual dividend payment date w') at the corresponding forward rate $r_{t,w',T_2'}$ to receive an amount $\sum_{w=T_1+1}^{T_2} d_w \exp\{r_{t,w',T_2'}(T_2'-w')\}$ at T_2'; (e) at T_2 sell the cash

index for S_{T_2} and settle the far futures contract carrying forward the associated cash flow (F_{t,T_2}-S_{T_2}) from T_2 to T_2'; and (f) at T_2', collect the cash index sale proceeds

S_{T_2}, collect proceeds of dividend related investments $\sum_{w=T_1+1}^{T_2} d_w \exp\{r_{t,w',T_2'}(T_2'-w')\}$

and repay $F_{t,T_1}\exp\{r_{t,T_1',T_2'}(T_2'-T_1')\}$ disbursed at T_1' (after including the cash flow (F_{t,T_2}-S_{T_2}) carried from T_2 to T_2'. In the spirit of ignoring marking to market cash flows, assume that there are no costs or revenues involved in carrying the cash flows generated in (b) above from T_1 to T_1' and the cash flows generated in (e) above from T_2 to T_2'. This strategy provides arbitrage profits if the far futures price F_{t,T_2} exceeds F_{t,T_2}^* as determined from equation (1).

Alternatively, consider the following strategy: (a) at t buy one far futures contract, sell one near futures contract and arrange to lend an amount F_{t,T_1} from T_1' to T_2' at the forward rate $r_{t,T_1',T_2'}$; (b) at T_1 sell the cash index short for S_{T_1} and settle the near futures contract, carrying over associated cash flows (F_{t,T_1}-S_{T_1}) from T_1 to T_1'; (c) at T_1' receive the entire proceeds from the short sale of the cash asset and lend the amount F_{t,T_1} from T_1' to T_2' (as arranged earlier at t) after adding the cash flow (F_{t,T_1}-S_{T_1}) received at (b) from settlement of the near futures contract; (d) for the index constituents going ex-dividend between T_1 and T_2, pay (to the agent from whom the stock has been borrowed) all dividends due (known *ex ante*) on the corresponding actual ex-dividend date using funds borrowed at the relevant forward interest rate $r_{t,w',T_2'}$, thereby creating a liability of $\sum_{w=T_1+1}^{T_2} d_w \exp\{r_{t,w',T_2'}(T_2'-w')\}$ at T_2'; (e) at T_2 buy the cash index for S_{T_2} and settle the far futures contract carrying forward the associated cash flow (S_{T_2}-F_{t,T_2}) from T_2 to T_2'; and (f) at T_2', receive the amount $F_{t,T_1}\exp\{r_{t,T_1',T_2'}(T_2'-T_1')\}$ as repayment against the loan disbursed earlier at T_1', pay an amount S_{T_2} for the cash index bought at T_2, return the stocks to the agent from whom they had been borrowed earlier at T_1' and pay the amount $\sum_{w=T_1+1}^{T_2} d_w \exp\{r_{t,w',T_2'}(T_2'-w')\}$ to discharge dividend related liabilities created in (d) above (after including the cash flow (S_{T_2}-F_{t,T_2}) carried from T_2 to T_2'). In the spirit of ignoring marking to market cash flows, assume that there are no costs or revenues involved in carrying the cash flows generated in (b) above from T_1 to T_1' and the cash flows generated in (e) above from T_2 to T_2'. This strategy provides arbitrage profits if the far futures price is below F_{t,T_2}^* as determined from equation (1). Hence, equation (1) gives the settlement adjusted forward pricing formula "fair" value for the far futures contract relative to the near futures contract.

The adjustment for cash market settlement procedures is particularly important in the UK where cash settlement procedures are themselves organised as a forward market.[4] The year is divided into (usually 24) "Account settlement periods". Most of these (usually 20) settlement procedures are of two weeks length while a few (usually 4, and spanning holidays) are of three weeks length. All transactions made within an account period are settled on the second Monday of the following account settlement period.

The mispricing of the far contract relative to the near contract, hereafter called spread mispricing, can be defined as:

$$Y_t = \log\left(\frac{F_{t,T_2}}{F^*_{t,T_2}}\right) \qquad \text{....(2)}$$

The measure Y_t defined in equation (2) has been preferred to a simple price difference measure (F_{t,T_2}-F_{t,T_2}^*) so as to avoid heteroskedasticity problems that are likely with the use of four years of unnormalised data.[5]

4 For the US, inclusion of cash market settlement procedures in the analysis introduces only a relatively minor change: the relevant interest rate in the first term in equation (1) is the forward rate at t for a loan disbursed at T_1' and repaid at T_2', rather than the forward rate at t for a loan from T_1 to T_2.

5 It has also been preferred to a measure similar to the one used for cash futures mispricing by eg Merrick (1988) or Mackinlay and Ramaswamy (1988) or Yadav and Pope (1990,1992c), because the measure defined in equation (2) making it possible to directly relate unit root tests on spread mispricing to cointegration between near futures prices and far futures prices, since these near and futures prices are *a priori* expected (and actually found) to be I(1) variables.

Even if arbitrage is otherwise perfectly riskless, the existence of transaction costs will allow the far futures prices to fluctuate within a band around the "fair" price of equation (1) without triggering profitable arbitrage. The width of the band arising due to direct out of pocket transaction costs should be:

$$|2T_S + T_D + T_{FF} + T_{FF'} + T_{NF} + T_{NF'} + T_B| \qquad \text{....(3)}$$

where T_S = Percentage one way transaction costs for trading the index basket of stocks including both commissions and market impact costs.

T_D = Transaction tax payable as percentage of asset value transacted.[6]

T_{FF} = Round trip percentage commissions for the far futures contract.

T_{NF} = Round trip percentage commissions for the near futures contract.

$T_{FF'}$ = One way percentage market impact costs for the far futures contract.

$T_{NF'}$ = One way percentage market impact costs for the near futures contract.

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[6] In the UK, there is no transaction tax on futures market transactions. Cash market purchases currently attract 0.5% stamp duty (1% before Big Bang). No tax is payable on cash market sales.

T_B = Cost of borrowing fixed interest capital and index stocks.[7]

Different categories of market participants have different levels of transaction costs. Yadav and Pope (1990) highlight four categories of potential arbitrageurs:

Category A: Arbitrageurs whose marginal costs are confined to transaction costs relating to the futures market ie those for whom $T_B=T_D=T_S=0$. Examples of potential arbitrageurs falling in this category are those with existing spread arbitrage positions who seek to unwind early and close their existing spread position if such early unwinding is profitable.

Category B: Arbitrageurs whose marginal costs include cash market related transaction costs, but those who have capital in fixed interest deposits and a pool of index stocks and who are not liable to pay transaction tax ie those for whom $T_B=T_D=0$ but $T_S \neq 0$. In this category are arbitrageurs initiating new spread arbitrage positions but those who are market makers recycling stocks within seven days and hence not liable to pay transaction taxes.

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[7] This is faced only by arbitrageurs who do not have capital in treasury bills (for upper arbitrage) and index stocks (for lower arbitrage).

Category C: Arbitrageurs who have capital in treasury bills and a pool of index stocks, and who are initiating a new spread arbitrage position but who have to pay transaction tax in their dealings ie those for whom $T_B=0$, but $T_D \neq 0$ and $T_S \neq 0$. Examples in this category will be index funds/institutions.

Category D: Arbitrageurs who have to borrow capital or stock to initiate a new spread arbitrage position ie those for whom $T_B \neq 0$, $T_D \neq 0$, $T_S \neq 0$.

If there is adequate uncommitted arbitrage capital available to the arbitrageur with the lowest marginal transaction costs, arbitrageurs with higher marginal costs will never be able to enter the market (Gould, 1988). However, as has been pointed out (with reference to cash futures arbitrage) by Stoll and Whaley (1987) and Brennan and Schwartz (1990) arbitrageurs function within real or self imposed position limits. Hence, several different categories of arbitrageurs can be active depending on the actual level of spread mispricing and on the extent to which the capital available to each category of arbitrageur is committed. This will, in turn, depend on the past levels of spread mispricing. This means that given estimates of the different components of transaction costs, an examination of the spread mispricing time series can lead to inferences in regard to the categories of arbitrageurs that are active in this market, or conversely about the profitability of spread arbitrage related trading for different categories of potential arbitrageurs.

It is important to note that in actual practice, arbitrage strategies are not perfectly riskless. First, the magnitude of future dividends, their ex-dividend dates and their actual payment dates are uncertain. Typically, market participants estimate future dividends by applying a fixed percentage growth factor to past dividends and use identical (or corresponding[8]) ex dividend/payment dates.

Second, uncertainty arises because of the stochastic nature of the payoffs on account of daily marking to market of the futures position. Futures prices cannot be risklessly estimated *ex ante* in the same way as forward prices. The *ex post* difference between forward and futures prices depends on the covariation of changes in futures prices with changes in the prices of zero coupon bonds maturing with the futures contract.[9] In this context, the net cash flows on this account depends on the *ex post* path of futures price changes up to maturity. Clearly, the risk in this regard will be negligible if the spread arbitrage position is closed prior to near contract expiration since the near and far contract marking to market cash flows would tend to offset each other.

These uncertainties lead to a risk premium and an effective increase in the width of the arbitrage band. Yadav and Pope (1990c) also discuss several other factors

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[8] In the UK stocks go ex dividend only on Mondays and hence the *exact* date changes from year to year.

[9] See eg Cox, Ingersoll and Ross (1981), pp 326.

which could potentially be relevant for the cash holding phase of spread arbitrage position.[10] However, in general, they find through *ex post* simulations that the effect of these factors is largely negligible.

It is also important to appreciate that an effective transaction cost discount is created by the possibility of "risky" arbitrage strategies and in particular by the option of unwinding prior to near contract expiration. This option is profitable whenever the direction of spread mispricing is different from the direction of spread mispricing when the arbitrage position was first initiated and the absolute magnitude of spread mispricing exceeds the incremental marginal transaction costs involved ie $T_{FF'}$ + $T_{NF'}$. Hence, the evolution of mispricing over time and in particular, the tendency of mispricing to persist or reverse itself, is important for spread arbitrageurs since it determines the magnitude of the effective transaction cost discounts resulting from the early unwinding options.[11]

Restrictions on short sales of stock inhibit cash futures arbitrage. In the US because of the "uptick" rule for short sales, arbitrageurs seldom use short positions in index arbitrage strategies; they can only employ the pools of stock they own or control if futures are underpriced. Similarly in the UK, only registered market makers have

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[10] Eg in the context of spread positions there is uncertainty about the level at which the cash lag of the arbitrage trade is executed (at near contract expiration) and uncertainty about the level at which the cash lag of the arbitrage position can be closed at (far contract) expiration.

[11] It also determines the implicit cost of delayed execution.

special stock borrowing privileges. It is very difficult for non-market makers to undertake cash futures arbitrage transactions involving shorting stock as a matter of normal course, unless other divisions within the same institution (eg index funds) are already long in stock. Hence, it can be argued that this can explain the predominantly negative average cash futures mispricing that has been reported in several studies.[12] The London markets provide an ideal laboratory to test such a hypothesis in view of the unique features of the cash settlement procedures. For all cash futures arbitrage activity during the account settlement period which spans the futures maturity date, there is no need to borrow stocks in order to go short and no need for special stock borrowing privileges. T_B is effectively zero for all arbitrageurs, not just for arbitrageurs who have capital in index stocks, or arbitrageurs with special stock borrowing privileges. Furthermore, during this period, there is no cost of carrying the cash position, and no dividend uncertainty, thereby making cash futures arbitrage virtually riskless except for the risk of non-synchronous/delayed execution. Clearly, if restrictions on short selling cause futures to be underpriced, then during the account period spanning near futures maturity, the far contract should be underpriced, but the expiring near contract should not be underpriced since the near contract is not subject to these restrictions on short selling. Hence, if short selling restrictions influence futures pricing, the far contract should be significantly more underpriced relative to the near contract

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[12] See eg Figlewski (1984b), Cornell and French (1983a), Merrick (1988), Brenner *et al* (1989) and Yadav and Pope (1990).

during the account spanning futures maturity than in other accounts. This is directly testable.

Another important issue is the tax timing option available to stockholders due to their ability to select the timing of realisation of losses and gains. Cash settlement of futures contracts implies that investors in the futures market necessarily pay taxes in the year the capital gains arise, while investors holding the cash asset can defer their capital gains. This should result in futures being underpriced relative to their forward pricing formula fair value. On the other hand, the marginal investor may be a tax exempt institution[13] in which case the tax timing option will have no value, or the marginal investor may be an arbitrageur/floor trader who cannot hold the cash index indefinitely in which case again the tax timing option will have no value. Clearly, the relevance of the tax timing option for index futures pricing could be different in different markets and essentially an empirical issue. *Ceteris paribus*, the value of the tax timing option should be higher for longer times to expiration. Therefore, if the tax timing option is a relevant factor for index futures pricing, the far contract should be more underpriced than near contract at all times, and, hence, spread mispricing should be negative in all periods. Again, this is directly testable.

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[13] This is not conceivable in the UK since tax law has effectively prevented the use of index futures contracts by tax exempt institutions, except for hedging purposes.

2.2 Mean Reversion in Spread Mispricing

Spread mispricing as defined by equation (2) should be identically zero if interest rates[14] and dividends[15] on index stocks are known *ex ante* with perfect certainty, and both cash and futures markets are perfectly frictionless, because, under these circumstances, the supply of arbitrage services between the near and far contracts should be infinitely elastic. In actual practice, transaction costs do exist, and arbitrage is not perfectly riskless.[16] As a result, the elasticity of arbitrage services is not infinite. However, if arbitrageurs are active, one would expect the elasticity of arbitrage services to be greater than zero. Hence, if for some reason spread mispricing becomes non-zero, the actions of arbitrageurs should pull it back towards zero. In other words, the spread mispricing series should display mean reversion whereby changes in spread mispricing will be negatively related to the *level* of spread mispricing in the previous period.

Mean reversion in spread mispricing follows formally from a framework similar to that used for cash-futures arbitrage by Garbade and Silber (1983). Assume that:

(a) The demand schedule of a trader trading only in the near (far) contract is linearly proportional to the difference between the near (far) contract price and the reservation price of that trader for the

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[14] Interest rates up to far contract maturity.

[15] Dividends between near and far contract maturity.

[16] Not only because interest rates and dividends are not know *ex ante* with perfect certainty, but also because of microstructural factors like non-simultaneous or delayed execution of trades etc.

near (far) contract, the elasticity of demand being the same for all such traders.

(b) The demand schedule of arbitrageurs seeking to profit from spread mispricing is linearly proportional to the extent of spread mispricing.[17]

(c) The change in the reservation price of a trader reflects *random* arrival of new information. (This new information is allowed to have a component common to all traders, and a component specific to a particular trader.)

Then, following steps similar to those in Garbade and Silber (1983) for cash futures pricing, it can be shown that the mispricing variable Y_t will follow the equation:

$$\Delta Y_t = \rho_0 - \Phi\, Y_{t-1} + e_t \qquad \text{....(4)}$$

where $\Delta Y_t = (Y_t - Y_{t-1})$, Φ is a function of the elasticity of associated arbitrage services, ρ_0 reflects "persistent differences" (Garbade and Silber, 1983, pp 293) between near and far contract prices and e_t is a white noise error term.

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[17] More sophisticated modelling can allow for non-linear dependence of arbitrage demand on spread mispricing similar to that followed eg by Pope and Yadav (1991).

If the elasticity of associated arbitrage services is infinite, then Φ should be close to unity and Y_t will be white noise if $\rho_0=0$. If the elasticity of arbitrage services is zero, Φ should be close to zero, and Y_t should be a martingale if $\rho_0=0$. In actual practice, one would expect that the activities of arbitrageurs would result in the elasticity of arbitrage services being significantly greater than zero (but not infinite) and, hence, Φ being significantly greater than zero but less than unity. In other words, without significant arbitrage activity, the Y_t series should have unit roots, but with active arbitrageurs, the Y_t series should be mean reverting and not have unit roots.[18] Clearly, this mean reversion is directly testable.

It is also important to note that both near contract and far contract futures prices should be non-stationary I(1) series (Samuelson, 1965). Equivalently, both $\log(F_{t,T2_1})$ and $\log(F_{t,T_2}^*)$ (as defined in equation (1) in terms of a non-stochastic transformation of F_{t,T_1}) should be I(1) series. Y_t is just the difference between $\log(F_{t,T_2})$ and $\log(F_{t,T_2}^*)$. Hence, the absence of unit roots in Y_t is equivalent to cointegration between F_{t,T_2} and F_{t,T_2}^*. Therefore, if the elasticity of arbitrage between the near and far contract is significant, F_{t,T_2} and F_{t,T_2}^* should be cointegrated with a cointegrating vector of [1,-1]. Such cointegration is equivalent to an "error correction model"[19] in which price growth in the far futures contract,

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18 Mean reversion will also exist if spread mispricing is modelled to follow a Brownian Bridge process (in the same way as Brennen and Schwartz (1990) model cash futures mispricing).

19 Equation (10) in Garbade and Silber (1983, pp 292) is a special case of a general "error correction model".

and/or price growth in the near futures contract is predicted *inter alia* by previous period spread mispricing. It also raises the possibility that far futures contracts and near futures contracts are not equal in their capacity to discover new information about asset prices. If spread mispricing history predicts far futures price growth but does not predict near futures price growth, then it is the far contract which always adjusts towards the near contract behaving like a pure satellite without any role in price discovery. The opposite is true if spread mispricing history predicts near futures price growth but does not predict far futures price growth.

2.3 Spread Mispricing and Short Term Spread Positions

Spread positions can contribute most effectively to short term allocation of risk between different futures traders when spread mispricing is identically zero. Otherwise, the mean reversion in spread mispricing will introduce an important "mispricing return" component in the total spread return. The existence of this mispricing return affects the effectiveness and average cost of short term spreads in the same way as cash futures mispricing return is a critical determinant of the effectiveness and average cost of short term cash futures hedges. Hence, the discussion in this section follows *mutatis mutandis* the analysis of short term hedging with mispriced futures in Merrick (1988).

Consider the one period problem of minimising the return variance of a spread portfolio consisting of one near futures contract and h_t far futures contracts. The one period rate of return on this spread portfolio can be expressed as:

$$R^s_{t+1} = R^n_{t+1} + h_t R^f_{t+1} \qquad \text{....(5)}$$

where

$$R^n_{t+1} = \frac{F_{t+1,T_1} - F_{t,T_1}}{F_{t,T_1}} \qquad \text{....(6a)}$$

and

$$R^f_{t+1} = \frac{F_{t+1,T_2} - F_{t,T_2}}{F_{t,T_1}} \qquad \text{....(6b)}$$

represent the one period return on the near and far contract respectively.

The conditional variance of R^s_{t+1} given all available information at t is:

$$VAR(R^s_{t+1}) = VAR(R^n_{t+1}) + h_t^2\ VAR(R^f_{t+1}) + 2h_t\ cov(R^n_{t+1}, R^f_{t+1})$$

Hence, the value of h_t which minimises the risk of the spread is given by:

$$h_t = \frac{-Cov(R^n_{t+1}, R^f_{t+1})}{VAR(R^f_{t+1})} \qquad \text{....(7)}$$

It is easily shown that if spread mispricing is identically zero ie if far futures prices are always given by equation (1), then the risk (and return) of the spread can be reduced to zero by choosing the "spread ratio" h_t to be equal to ${}_{cc}h_t$ where[20]:

$${}_{cc}h_t = -\exp\{r_{t,T_1',T_2'}(T_2'-T_1')\} \qquad \text{....(7a)}$$

If spread mispricing is not identically zero, then the one period spread position can never be made completely riskless. The return on the cost of carry spread portfolio becomes[21]:

$${}_{cc}R^s_{t+1} = -\exp\{-r_{t,T_1',T_2'}(T_2'-T_1')\}R^Y_{t+1} \qquad \text{....(8)}$$

where R^Y_{t+1} is the spread "mispricing return" defined as the difference between the far futures return and the "equivalent" near futures return.

$$R^Y_{t+1} = \frac{(F_{t+1,T_2}-F_{t,T_2})}{F_{t,T_1}} - \frac{(F^*_{t+1,T_2}-F^*_{t,T_2})}{F_{t,T_1}} \qquad \text{....(8a)}$$

Cost of carry spread portfolio return depends on the stochastic spread mispricing return. The cost of carry spread portfolio variance is equal to

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20 The prescript in ${}_{cc}h_t$ stands for strict cost of carry pricing.

21 It is assumed that there is no difference between the forward interest rates $r_{t+1,T_1',T_2'}$ and $r_{t,T_1',T_2'}$.

$$[\exp\{r_{t,T_1',T_2'}(T_2'-T_1')\}^2]\ VAR(R^y_{t+1})$$

More importantly, mean reversion in spread mispricing implies that the change in spread mispricing, and hence spread mispricing return[22], depends on the spread mispricing in the previous period. Therefore, the expected return on a cost of carry spread portfolio will vary according to the state of initial spread mispricing. Spreads begun with overpriced far futures *predictably* earn positive returns while those begun with underpriced far futures *predictably* earn a negative return. With non-zero spread mispricing, the risk minimising spread ratio ${}_{m}h_t$ ie the value of h_t which minimises the risk of the spread, need not be equal to ${}_{cc}h_t$. It will be given by:

$$_{m}h_t = -[\exp\{r_{t,T_1',T_2'}(T_2'-T_1')\}]\ (1-\beta^Y_t) \qquad \text{....(9)}$$

$$where\ \ \beta^Y_t = \frac{Cov(R^Y_{t+1},R^f_{t+1})}{VAR(R^f_{t+1})} \qquad \text{....(9a)}$$

${}_{m}h_t$ is a sum of two components. The first is the normal cost of carry component from equation (7a). The second is an adjustment factor reflecting the correlation between spread mispricing return and far futures return. If this correlation is zero, the risk minimising spread ratio and the spread return is given by equations (7a) and (8) respectively. However, in general, the return and variance of the risk minimising spread position is given by:

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22 The two are easily shown to be directly related.

$$ {}_{m}R^{s}_{t+1} = -[\exp\{r_{t,T_1',T_2'}(T_2'-T_1')\}]\ (R^{Y}_{t+1}-\beta^{Y}_{t}R^{f}_{t+1}) \qquad \text{....(10a)} $$

$$ VAR({}_{m}R^{s}_{t+1}) = [\exp\{r_{t,T_1',T_2'}(T_2'-T_1')\}]\ [1-(\varphi^{Y}_{t})^{2}]\ VAR(R^{Y}_{t+1}) \qquad \text{....(10b)} $$

where φ^{Y}_{t} is the correlation at time t between spread mispricing returns and far futures returns.

The risk minimising spread return consists of the equilibrium cost of carry return, the spread mispricing return and a term reflecting the correlation between the spread mispricing return and the far futures return. The variance of this risk minimising portfolio is lowered by the existence of this correlation.

3. EMPIRICAL EVIDENCE

3.1 Data

London has only one exchange traded stock index futures contract. This contract is traded on the LIFFE and is based on the FTSE100 index - an arithmetic average, market value weighted index of one hundred (highest capitalisation) stocks. LIFFE index futures expire four times a year in March, June, September and December on the last business day of the month. LIFFE trading is based on an open outcry market and, in common with other futures markets, all margin accounts are marked to market on a daily basis. Though three maturities are traded on LIFFE at any particular time, only the two earlier maturities have significant trading volume. Our analysis is hence confined to the two earliest maturity contracts. The contract nearest to maturity at any time is labelled as the "near" contract and the next maturing contract is labelled as the "far" contract. Expiration day observations are not included in the near contract.

The results reported in this chapter are based on hourly cash and futures data on the FTSE100 index for the period April 28, 1986 to March 23, 1990.[23] This corresponds to 990 trading days. During the sample period, the cash market was open from 9.00 am-5.00 pm and the futures market from 9.05 am-4.05 pm.

"Time and Sales" transactions data on FTSE100 index futures was obtained from LIFFE. The data included all bid and ask quotes posted by traders in the pit and all transaction prices relating to this contract. A subset of this data was analysed consisting only of (a) those cases in which the ask prices of the near contract and the ask prices of the far contract were posted within 60 seconds of each other; and (b) the bid prices of the near contract and the bid prices of the far contract were posted within 60 seconds of each other. This subset of data was sampled at hourly intervals, using the first set of such "synchronous" near and far futures prices in

[23] The choice of this period has been dictated largely by changes in exchange trading hours. The beginning of the sample period corresponds with the date on which LIFFE extended its trading hours from 9.35 am-3.30 pm to 9.05 am-4.05 pm. The end of the sample period corresponds to the date on which the London Stock Exchange changed its trading hours from 9.00 am-5.00 pm to 8.30 am-4.30 pm.

each hourly interval 9.05 am-10.00 am, 10.00 am-11.00 am, ..., 3.00 pm-4.00 pm.[24] [25] The use of near and far futures prices posted within 60 seconds of each other, ensured that, to the extent possible, the near and far futures prices used were "synchronous". Furthermore, the use of *either* ask prices *or* bid prices ensured that the set of prices being used were directly comparable, differing only because of equation (1) and the transaction cost components in equation (3).

Information on the constituents of the index and how these constituents changed over the sample period was obtained from the London Stock Exchange. Dividends and ex-dividend dates for all the relevant constituents of the index each day were collected from *Extel* cards. In addition, in order to compute the exact *ex post* daily dividend flow on the FTSE100 index, the individual constituents' dividend flows on each day were value weighted, aggregated and converted into index points using price and market value data collected from *Datastream*. Additionally, ex-dividend dates and actual dividend payment dates were collected from the *London Business School Share Price Database*. These were used to infer the average time delay between the ex-dividend date and the dividend payment date.

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[24] Since the futures market was open from 9.05 am to 4.05 pm, the first "hourly" interval was only 55 minutes, and the last hourly interval ended 5 minutes before the close.

[25] Daily price series are also generated by sampling the hourly series at hourly intervals using the first non-missing hourly observation on each trading day.

Daily data on one and three-month UK Treasury Bill discount rates were also collected from *Datastream.*

3.2 UK Transaction Costs

The components that make up the total transaction costs relevant for index arbitrage are indicated in equation (3). T_B, the cost of borrowing capital or index stocks, is faced only by arbitrageurs who do not have capital in treasury bills or index stocks. T_D, the transactions tax, is not payable by market makers and brokers/dealers recycling stocks within seven days.

Table 1 reports the average inner market spread for UK "alpha" stocks[26] on the basis of the values published by the Stock Exchange Quarterly from time to time. This inner market spread has varied from about 0.7% to about 1.3% - averaging about 1.0% - except for the contract spanning Black Monday. However, the quoted bid-ask spread will be an upward biased estimate of the cash market transaction costs $2T_s$ relevant for index arbitrage, since a major component of the quoted spread, namely adverse information costs, should not be relevant in pricing market making services for index arbitrage. Transaction costs related to the cash market should be confined to marginal order processing costs and marginal inventory holding costs. We are not aware of any published estimate for the UK market of

[26] Stocks in London have been classified on the basis of the number of competing market makers and the trade/quote reporting restrictions applicable to them. Alpha stocks generally have the lowest spreads, and all FTSE100 index stocks belong to this category.

the percentage of the quoted spread which arises due to adverse information costs. However, Stoll (1989) finds that on NASDAQ, 43% of the quoted spread represents adverse information costs, 10% represents inventory holding costs and 47% represents order processing costs. If we use these figures as a first approximation for the London market, which has an almost identical trading structure to NASDAQ, the quoted spread for index arbitrage should average about 0.5% except for the contract spanning Black Monday. Even the rise in the quoted spread after Black Monday is more likely to be due to an increase in the adverse information component and the inventory holding component and is hence likely to affect index arbitrage trades to a much lesser extent. Commissions are usually on a flat rate basis and for large volume index arbitrage trades are virtually negligible when expressed as a percentage of value traded.

To estimate the percentage market impact costs in the UK index futures market, a subset of LIFFE time and sales data was analyzed consisting only of cases in which ask prices and bid prices are posted within 60 seconds of each other. Table 1 also reports the median percentage spread[27] for the near futures contract and the far futures contract. The median percentage spread for the near futures contract has varied from 0.04% to 0.15%, averaging about 0.1%. The median percentage spread for the far futures contract has varied from 0.12% to 0.44%, averaging about

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27 Percentage spread has been defined as:

$$100 * \frac{(Ask-Bid)}{\{(Ask+Bid)/2\}}$$

0.25%. Roundtrip percentage commissions in the futures market have been typically less than £25 per contract for both near and far contracts ie less than about 0.05% of underlying index value.

On the basis of the above, the total average arbitrage related transaction costs of the three more important categories of spread arbitrageurs highlighted in Section 2 - Category A, Category B and Category C - are reported in Table 1. The total marginal transaction costs for Category A spread arbitrageurs vary from about 0.2% to 0.4%. For Category B spread arbitrageurs, marginal transaction costs vary from about 0.7% to about 1% except for the contract spanning Black Monday. For Category C spread arbitrageurs, the transaction costs are higher than those for Category B spread arbitrageurs by 1.0% for the first two contracts and 0.5% higher for the remaining contracts.

On the basis of the above, we take the transaction costs of Category A, Category B, and Category C spread arbitrageurs to be 0.5%, 1.0% and 1.5% respectively.

3.3 Spread Mispricing

Mispricing of the futures contract was calculated on the basis of the forward pricing formula Equation (1) and the definition of Equation (2). In addition to the usual assumptions of the forward pricing formula, the following additional assumptions were initially made: (a) forecast dividends to maturity for each date are identical to the actual *ex post* daily cash dividend inflow for the FTSE100 basket; (b) the forward interest rate at time t for a loan made at time w to be redeemed at time T, is identical to the (future) spot interest rate at time w on a Treasury Bill maturing at time T; (c) the value of day t of one- and three-month maturity Treasury Bill interest rates can be used to estimate a linear term structure from which the implied forward interest rate for the period s_1 to s_2 (in equation (1)) can be calculated; and (d) actual payment of dividends is made 53 calendar days after the ex dividend date, this being the average *ex post* delay between the ex dividend date and the actual dividend payment date for index stocks over the sample period.

Table 2A reports some relevant descriptive statistics of the spread mispricing variable for each of the 16 contracts expiring during the sample period and for the aggregated data for all contracts. Table 2B reports the results of two hypotheses tests; first, that the proportion of negative (or positive) observations is significantly different from 50%; and second that the average spread mispricing is equal to zero. The unadjusted t-statistic in Table 2B assumes that successive observations are independent. The adjusted t-statistic has been calculated after controlling for the autocorrelation structure of the mispricing variable.[28]

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[28] Standard error of the mean calculated as standard deviation divided by $\sqrt{N}$ is clearly inappropriate in view of autocorrelation in mispricing. The standard error is calculated as:

$$SE(\bar{X}) = \sqrt{\frac{SD(X)}{N}\left(1+2\sum_{k=1}^{n-1}\left(1-\frac{k}{n}\right)\rho_k\right)}$$

Tables 2A and 2B have several interesting features. First, it is inappropriate to infer that equation (1) is, on average, an upward or downward biased estimate of the actual far futures price, relative to the near futures price. Average spread mispricing is significantly negative, and the proportion of negative mispricing values is significantly greater than 50%, for 6 contracts. On the other hand, average spread mispricing is also significantly positive for 7 contracts and the proportion of positive mispricing values is significantly greater than 50% for 9 contracts. Average spread mispricing is not significantly different from zero for only 3 contracts. Spread mispricing tends to be either predominantly negative or predominantly positive. Whether spread mispricing is predominantly negative or predominantly positive has varied sharply from the first half of the sample period to the second half. The overall inference about average spread mispricing depends on the choice of sample period. During the sub-period April 1986 to June 1988, spread mispricing is predominantly positive for all contracts; and average spread mispricing is, of course, significantly positive. On the other hand, during the sub-period July 1988-March 1990, spread mispricing is predominantly negative for 6 out of 7 contracts, and average spread mispricing is significantly negative.

Second, the 5th and 95th percentiles, and the first and third quartiles, of the mispricing variable, also vary substantially from contract to contract. If equation (1) is an unbiased estimator of the far futures price relative to the near futures price, the 5th and 95th percentiles, or the first and third quartiles, could be regarded as proxies for the lower and upper boundaries of the arbitrage windows for different categories of arbitrageurs. Similarly, the interquartile range (Q_3-Q_1) or the difference (P_{95}-P_5) could be regarded as a proxy for the corresponding overall width of the arbitrage window. However, for 13 out of 16 near contracts, both quartiles are of the same sign and for the 9 out of 16 contracts, even the 5th and 95th percentiles have the same sign! The interquartile range (Q_3-Q_1), and the difference (P_{95}-P_5), are also very unstable. Furthermore, there appears to be no obvious relationship between the variation in the transaction cost estimates in Table 1 and the variation in average/median spread mispricing or variation in Q_3 or Q_1 or P_5 or P_{95} or (Q_3-Q_1) or (P_{95}-P_5) or even quarterly returns.

Third, the minimum and maximum values, the 5th and the 95th percentile, and, in several cases, even the lower and upper quartiles suggest that the absolute magnitude of spread mispricing often exceeds the estimated transaction costs of Category A and Category B arbitrageurs, and sometimes even exceed the transaction costs of Category C arbitrageurs. Furthermore, there does not appear to have been any systematic reduction with the passage of time in the magnitude of average spread mispricing, or the standard deviation of the spread mispricing variable.

Can the systematic bias in far futures pricing be explained by misspecification of dividends? The magnitude of future dividends, their ex-dividend dates and their actual payment dates are uncertain. Typically, market participants estimate future dividends by applying a fixed percentage growth factor to past dividends and use identical (or corresponding) ex-dividend/payment dates. Yadav and Pope (1992c) show that the potential effect of misspecification in ex-dividend dates and actual payment dates is absolutely trivial (<0.01%). To assess the impact of misspecification in the magnitude of dividends, the following variable was estimated:

$d_{divl}(y\%)$ = Spread mispricing estimated from actual *ex post* dvidend inflows minus spread mispricing estimated by using previous year's dividend plus a y% growth factor.

Table 3B reports the descriptive statistics of the $d_{divl}(y)$ variable for y = 0%, 10% and 20%. Since it is realistic to assume a non-zero growth factor, the *average* effect of misspecification in dividends could clearly not have been greater than about 0.1% and cannot explain the magnitude of *average* spread mispricing observed. Similarly, the maximum possible affect of dividend misspecification could also not have been greater than 0.3% and again cannot explain the potential arbitrage opportunities for Category A and Category B arbitrageurs that appear to have existed.

The risk arising on account of daily marking to market is likely to have been negligible if the spread arbitrage position is closed prior to near contract expiration, since the near and far contract marking to market cash flows would tend to offset each other. After near contract expiration, a spread arbitrage position becomes identical to a cash futures arbitrage position, and, for such cases, the simulations of

Yadav and Pope (1990c) show that the average impact in this regard has been negligible (<0.02%) with about 90% of observations corresponding to an impact of less than 0.1%. Once again, the risk arising on account of daily marking to market of the futures position cannot explain the systematic biases observed in spread mispricing.

It is important to note that the results in Table 2A/Table 2B are not consistent with the view that the tax timing option is a significant factor in futures pricing. The existence of a valuable tax timing option should have caused spread mispricing to be consistently negative. However, about 50% of spread mispricing observations are positive, and during the first half of the sample period, spread mispricing is positive in more than 85% hourly periods.

Spread mispricing also tends to persist over long periods. The degree of this persistence can be non-parametrically estimated in terms of the average length of a run (or the average time before a mispricing reversal) and parametrically in terms of the serial correlation in the mispricing time series. Table 4 reports results in this regard. The average time before spread mispricing reversal is about 22 trading hours for the aggregate sample, but varies from 6 trading hours to infinity for different contracts. For 3 contracts, spread mispricing does not reverse at all! For each contract the "runs" test showed that the number of runs is significantly less than the number of runs expected if successive observations were independent; the hypothesis of no persistence being conclusively rejected (with p value <0.001) in every case. The first order autocorrelation is significantly greater than zero in every case. The extent of persistence in mispricing - whether measured in terms of serial correlation or the average length of a run - can be regarded as an inverse measure of the elasticity of arbitrage services. Hence, it is clear that the elasticity of arbitrage services has not increased over time. The high degree of persistence in mispricing suggests that the possibility of delayed execution may not be a serious risk for arbitrageurs. It also suggests that the early unwinding option should not be significantly valuable.

3.4 Spread Arbitrage Profitability Simulations

3.4.1 Ex Post Profit Simulations

Tables 5A and 5B present *inter alia* the results of simulating the profitability of spread arbitrage based on simple *ex post* trading rules which assume that it is possible to use the prices at any time to execute a trade at the same price at the same time. Let TC% be the transaction cost relevant for the arbitrageur. Two trading rules are considered:

Trading Rule 1: If spread mispricing exceeds TC, sell one far futures contract and buy one near futures contract. Hold this position till near contract expiration. At near contract expiration, settle the near contract, sell treasury bills and buy the equivalent underlying basket of stocks, and hold this position up to far contract expiration. At far contract expiration, sell the stock bought earlier, and reinvest in Treasury Bills. If mispricing is below TC, buy one far futures contract and sell one near futures contract. Hold this position till near contract expiration. At near contract expiration, settle the near contract, sell the equivalent underlying basket of stocks, use the proceeds obtained to buy Treasury Bills, and hold the position until far contract expiration, at which time the position is unwound and investment in stocks reinstated. This is the simple hold-to-expiration trading rule.

Trading Rule 2: Same as Trading Rule 1, except that, instead of waiting until near contract expiration, the position is unwound as soon as spread mispricing changes sign and becomes large enough in magnitude to cover the incremental transaction costs involved. This is the early unwinding option.

Two values of TC are used - 0.5% and 1.0%. This corresponds to the estimated transaction costs of Category A and Category B arbitrageurs respectively.

Tables 5A and 5B report the profits (in £'000) per contract earned from the different trading rules for transaction cost levels of 0.5% and 1.0%. The tables have several interesting features. First, there have been significant number of arbitrage opportunities and significant arbitrage profits for Category A arbitrageurs, but these arbitrage opportunities and profits have been substantially less for Category B arbitrageurs. Second, early unwinding is possible in many cases but the *additional* profits arising from the early unwinding option are a very small proportion of the total arbitrage profits, even for Category A arbitrageurs. This suggests that there is relatively little potential for risky arbitrage strategies and transaction cost "discounts" on that basis. Third, more than 80% of arbitrage positions are held to near contract expiration. This is in sharp contrast to the case of cash futures arbitrage positions which are not likely to be held to expiration (Merrick, 1989; Yadav and Pope, 1990c). This means that though expiration day price and volume effects in the cash market are not likely because of cash futures arbitrage related unwindings, they are certainly possible due to spread arbitrageurs initiating new cash market positions at near contract settlement.

3.4.2. Ex Ante Profit Simulations

Table 5A and Table 5B also report index arbitrage profits for transaction cost bounds of 0.5% and 1.0%, based again on Trading Rules 1 and 2 but implemented on an *ex ante* basis. Thus, the calculation of *ex ante* profits in Table 5A and Table 5B assumes that if there is a spread arbitrage opportunity perceived on the basis of prices in hourly period t, the required spread arbitrage strategy is executed only on the basis of the prices in hourly period (t+1). Similarly it is assumed that if an early unwinding trade is indicated on the basis of prices in hourly period t', the trade is actually executed only on the basis of prices in hourly period $(t'+1)$. Since the delay in execution of spread arbitrage trades is likely to be only of the order of a few minutes, the assumed execution delay of one hour is clearly a conservative assumption for assessing the potential risk in spread arbitrage due to the possibility of delayed execution of trades.

The results of *ex ante* trading rules are qualitatively similar to those of *ex post* trading rules, but the magnitude of arbitrage profits are reduced, as can be expected. The reduction in profits from Trading Rule 1 is, on average, about one third for Category A arbitrageurs and two-thirds for Category B arbitrageurs. About 70% of trades are still profitable in both cases. However, early unwinding profits disappear completely.

3.4.3. Profit Simulations for Risky Arbitrage

The high profits generated in the simulations of Merrick (1989) and Yadav and Pope (1990, 1992c) by the early unwinding option in cash futures arbitrage, motivated the investigation of the potential for risky *spread* arbitrage trades. In such cases, the arbitrage trade is initiated with the magnitude of spread mispricing less than the actual transaction cost of the potential arbitrageur in the hope that the marginal profit generated by early unwinding will not only cover the loss involved in initiating a trade within the transaction cost window, but also generate a net profit. There can be several forms of risky arbitrage trades. The profit simulations reported in Table 6 are based on implementing the following trading rule:

Risky Spread Arbitrage Trading Rule: The arbitrage position is initiated whenever mispricing exceeds Y% in magnitude (Y = 0.5%, 1.0%) even when actual transaction costs exceed Y% (by an amount equal to 0.25%, 0.50%, 0.75%, 1.00%); and the position is unwound early as soon as the additional profit from early unwinding makes the overall position profitable after inclusion of the incremental transaction costs involved in early unwinding.

With a trading rule threshold of 0.5%, risky spread arbitrage has been profitable for arbitrageurs with transaction costs of 0.75% in 15 out of 16 quarters in the sample period, even though only 27% positions could be unwound early. With a trading rule threshold of 0.5% and transaction costs of 1%, profits have been confined to 9 out of the 16 quarters, even though there has been substantial overall profit. Clearly, the transaction cost discount at this trading rule threshold is about 0.25% in magnitude. With a trading rule threshold of 1.0%, risky arbitrage profits have disappeared in 9 out of 16 quarters, even for an arbitrageur with transaction costs of only 1.25%. Risky arbitrage potential has declined sharply with an increase in transaction costs. This in sharp contrast to cash futures arbitrage over this period, where the results of Yadav and Pope (1990c) have shown that similar risky arbitrage strategies with a 1% threshold, have been largely profitable even for arbitrageurs with transaction costs of 2.0%, thereby providing heavy transaction cost discounts.

3.5 Short selling constraints and futures pricing

There is no need for an arbitrageur to borrow stocks, and hence no need for special stock borrowing privileges, to exploit negative cash futures mispricing during the account settlement period which spans the futures maturity date. This unique feature of the London Stock Exchange settlement procedures provides an opportunity to test whether short selling constraints contribute significantly to futures pricing.

Within the account period spanning futures maturity, the average (median) spread mispricing is found to be -0.166%(-0.295%) ie significantly negative (p value <0.0001) and significantly lesser than the -0.02%(0.01%) reported in Table 2A for the aggregate sample. However, considering that average (median) has been both significantly positive and significantly negative in different periods, this could potentially have arisen because the account period spanning futures maturity has coincidentally corresponded to a period with positive average spread mispricing. To control for this possibility we analyse the spread mispricing variable in three different subperiods - sub period 0 corresponding to the account period spanning near futures maturity, sub period 1 corresponding to the account period just before the account period spanning future maturity, and sub period 2 corresponding to all other account periods. In view of the strong tendency of spread mispricing to persist and be predominantly positive or predominantly negative over long periods (reflected in Tables 2A/2B and 4) we would expect that, if the absence of short selling constraints has no impact on futures pricing within sub period 0, the average (median) spread mispricing in sub period 0 should be about equal to the average (median) spread mispricing in sub period 1.

Accordingly, Table 7 provides descriptive statistics on the mispricing variable for each of these three sub samples and reports the results of testing the following hypotheses:

$_{ss}H_{01}$: The average (median) spread mispricing in sub period 0 is equal to the average (median) spread mispricing in sub period 1.

$_{ss}H_{02}$: The average (median) spread mispricing in sub period 0 is equal to the average (median) spread mispricing in sub period 2.

$_{ss}H_{12}$: The average (median) spread mispricing in sub period 1 is equal to the average (median) spread mispricing in sub period 2.

Hypotheses H_{01} and H_{02} are conclusively rejected each with a p value <0.0001 while hypothesis H_{12} cannot be rejected. Within the account period spanning near futures maturity, the average (median) spread mispricing is significantly lesser (p value <0.0001) than the -0.026%(0.047%) in the immediately preceding account period; and significantly lesser (p value <0.0001) than the -0.017%(0.03%) in all other account periods. These results clearly provide strong support for the view that the absence of short selling constraints in the account period spanning futures maturity has a significant impact on the average futures pricing.

3.6 Mean Reversion in Spread Mispricing

The hypotheses that $\Phi=0$ in equation (4) is equivalent to the presence of unit roots in the spread mispricing series. There is a vast literature on testing for unit roots. (See Diehold and Nerlove, 1990, for a selective survey.) The results reported in Table 8 are based on augmented Dickey-Fuller type regressions.[29] The appropriate testing procedure depends on the choice of the maintained model and the form of the alternative hypothesis.

Two models are estimated, one with a time trend and one without a time trend.

[29] An alternative approach is that pioneered by Phillips (1987) and extended to a variety of related problems by Durlauf and Phillips (1988), Perron (1989), Perron and Phillips (1987), Phillips (1988) and Phillips and Perron (1988). The basic idea is to estimate a non-augmented Dickey-Fuller regression (ie without using lagged terms in ΔX_t) and then to "correct" the Dickey-Fuller studentised statistic τ for general forms of serial correlation and heteroskedasticity that may be present in the remaining error term, using semi-parametric methods. The Phillips-Perron statistic can be computed in two ways, using $\epsilon_t^1 = (X_t - X_{t-1})$ or using $\epsilon_t^2 = \{X_t-(1-\Phi)X_{t-1}\}$. Stock and Watson (1988) show that tests based on ϵ_t^1 are inconsistent while the results of Schwert (1989) show that the performance of tests based on the augmented Dickey-Fuller framework is distinctly better than Phillips-Perron tests based on ϵ_t^2. Hence, our results are based on the augmented Dickey-Fuller framework.

$$\Delta Y_t = -\phi Y_{t-1} + \rho_0 + \sum_{g=1}^{p} \rho_g \, \Delta Y_{t-g} + \epsilon_t \qquad \text{....(11a)}$$

$$\Delta Y_t = -\phi Y_{t-1} + \rho_0 + \sum_{g=1}^{p} \rho_g \, \Delta Y_{t-g} + \alpha(T_1 - t) + \epsilon_t \qquad \text{....(11b)}$$

Table 4 shows that there is significant negative serial correlation in the time series of changes in spread mispricing. This is similar to the negative serial correlation in changes in cash futures mispricing documented by Mackinlay and Ramaswamy (1988) for the US (which persists over several time periods) and Yadav and Pope (1990) for the UK. Hence, it is necessary to add lagged terms in spread mispricing changes ΔY_{t-g} (g=1,2,...p) so as to ensure that the estimate of Φ reflects only the dependence on the *level* of spread mispricing in the previous period and is not biased by inclusion of components representing spread mispricing changes over previous periods. Accordingly, in the empirical tests, lagged terms are added until the estimated regression residuals ϵ_t are purged of significant serial correlation.

A model with a time trend is also considered because far futures prices could potentially be systematically lower than the "equivalent" near futures prices in view of the tax timing option (highlighted in Section 2.1) and the systematic bias on this account should decrease as time to near contract maturity decreases.

An intercept term is included in both models in view of the evidence in Table 2A/2B on the existence of significant systematic biases in spread mispricing.

Both models, equations (11a) and (11b), are estimated separately for each contract. In every case p is chosen to be the minimum value that ensures that serial correlation in the regression residuals, as indicated by the Box-Pierce Q-statistic, is statistically insignificant.[30] For 12 out of 16 contracts, just one lag in (ΔY_t) is required.[31]

For every contract, the following null hypotheses are tested:

(a) $\Phi = 0$ against the alternative that $\Phi > 0$,

(b) $\rho_1 = 0$ against the alternative that $\rho_1 \neq 0$, and

(c) $\alpha = 0$ against the alternative that $\alpha \neq 0$.

For the model of equation (11a), the hypothesis that $\Phi=0$ is tested using the tables for τ_μ in Fuller (1976).[32] For the model of equation (11b), the hypothesis that $\Phi=0$ is tested using the tables for τ_t in Fuller (1976).

Table 8 shows that the null hypothesis of a unit root is rejected in 12 out of 16 quarters for the model with a time trend (in each case with a p-value <0.01); and is rejected for 11 out of 16 quarters for the model without a time trend (in 10 cases

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[30] RATS econometric package is used and this reports the Q-statistic for $3\sqrt{N}$ lags, where N is the number of observations.

[31] The maximum number of lags required were three.

[32] Schmidt (1990) shows that with an intercept term, the relevant critical values are lower than in Fuller (1976). However, in our case, the estimated intercepts are small in relation to the minimum value used in the simulations of Schmidt (1990). In any case, we preferred to do a more conservative test.

with a p-value <0.01 and in 11 cases with a p-value <0.05). The null hypothesis that $\rho_1=0$ is conclusively rejected (with a p value $\ll 0.01$) in 15 out of 16 quarters for both models. However, the inferences regarding α are ambiguous. α is significantly less than zero (p-value <0.05) in 6 quarters, significantly greater than zero (p-value <0.10) in 5 quarters, and not significantly different from zero (p-value <0.10) in 5 quarters.

The hourly spread mispricing series is also sampled at daily intervals, using the first non-missing hourly observation on each trading day. The models in equations (11a) and (11b) are estimated for the overall daily spread mispricing series. For both models, three lags in ΔY_t are required to purge the residuals of serial correlation (Box Pierce Q-statistic >0.20). In both cases, the hypothesis that $\Phi=0$ is rejected against the alternative that $\Phi>0$ (τ_μ statistic = 3.39, p-value <0.01; τ_t statistic = 3.43, p-value <0.05). The hypothesis that $\rho_1=0$ is also conclusively rejected for both models (t-statistic >12.00; p-value $\not\leq 0.01$). The hypotheses that $\alpha=0$ cannot be rejected (t-statistic = -0.70; p-value = 0.48).

Clearly, the change in spread mispricing in period t is strongly dependent on the change in spread mispricing in period (t-1). Furthermore, the time series of spread mispricing levels appears to have been a stationary I(0) series over most of the quarters in the sample period; in other words, spread mispricing has been mean reverting (even over hourly intervals) in as much as the change in spread mispricing has depended, in part, on the *level* of spread mispricing in the previous period.

These results are important for several reasons. First, in the context of a model similar to Garbade and Silber (1983), they are consistent with the existence of significant spread arbitrage activity. Second, they show that spread mispricing return[33], and hence the return on a cost-of-carry spread portfolio[34], is *predictable* based on the level of spread mispricing, and the spread mispricing return in the previous period. The regression R^2 (not reported in Table 8) has varied from about 25% to 50%! This is pursued in greater depth in the next section. Third, they are very similar to the results reported in Yadav and Pope (1992b) for cash-futures mispricing. They are, hence, directly relevant to the debate on whether the mean reversion observed in cash futures mispricing, and the negative serial correlation observed in cash-futures mispricing changes, are a manifestation of index arbitrage activity (as had always been assumed eg by Brennan and Schwartz, 1990, pp 58; and Mackinlay and Ramaswamy, 1988, pp 137), or whether they are a "statistical illusion" created by the effects of non-synchronous trading in index stocks having no economic significance in terms of index arbitrage activity (as has recently been suggested by Miller *et al*, 1991). Clearly, spread mispricing, ie the mispricing of one futures contract relative to another futures contract, should not be influenced by factors related to non-synchronous trading in the cash index, or more generally, any kind of measurement errors with respect to the cash index. The evidence on significant mean reversion in spread mispricing, and significant negative serial

[33] Spread mispricing return is defined in equation (8a).

[34] In view of the equivalence in equation (8).

correlation in changes in spread mispricing, provides strong support to the view that the significant mean reversion in cash-futures mispricing, and the significant negative serial correlation in changes in cash-futures mispricing, is, at least not *entirely*, a consequence of non-synchronous trading. The evidence hence suggests that there is economically significant cash-futures (and spread) arbitrage activity influencing the time series behaviour of the cash-futures (and spread) mispricing.

The following time series were also tested for the existence of unit roots using augmented Dickey-Fuller regressions in exactly the same way as outlined above for the spread mispricing series:

(a) Logs of far futures prices ie $\{Log(F_{t,T_2})\}$,

(b) Logs of "equivalent" near futures prices ie $\{Log(F_{t,T_2}^*)\}$,

(c) Growth in far futures prices ie $\{Log(F_{t,T_2})-Log(F_{t-1,T_2})\}$,

(d) Growth in equivalent near futures prices ie $\{Log(F_{t,T_2}^*)-Log(F_{t-1,T_2}^*)\}$.

In each case, the results were unequivocally clear and totally consistent over all quarters in the sample period. For the time series of the logs of far futures prices, and the time series of the logs of equivalent near futures prices, the null hypothesis of a unit root could not be rejected (p-value $\gg 0.10$) for any quarter. On the other hand, for the time series of growth in far futures prices, and the time series of growth in equivalent near futures prices, the null hypothesis of a unit root is conclusively rejected (p-value $\ll 0.01$) for every quarter. This shows that the series of logs of far futures prices, and the series of logs of equivalent near futures prices, are both I(1) series.[35] Since Y_t, the difference between these two I(1) series, is found to be I(0), these results suggest that far futures prices and equivalent near futures prices are cointegrated with a cointegrating vector of [1,-1].[36,37] This is intuitively reasonable because both near and far futures prices are subject to identical stochastic economy-wide shocks. In the presence of differential price discovery and transaction costs, prices could be different in the short run. However, if they move too far apart, the actions of arbitrageurs will tend to pull them together and restore equilibrium.

3.7 Predictability of Future Returns

Table 9 reports the results of regressions to estimate the ability of spread mispricing Y_t, and changes in spread mispricing ΔY_t[38], to predict period (t+1) far futures return, near futures return and spread mispricing return defined as in equations (6b), (6a) and (8a). Time to near contract maturity is included as an additional "conditioning" variable in view of the potential impact of the tax timing option. Specifically, the following models are estimated:

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[35] This is, *a priori*, expected on theoretical grounds. See eg Samuelson (1965).

[36] Cointegration between two series is equivalent to cointegration between the logs of the two series.

[37] This is not a formal test for cointegration, and, in any case, not a formal test for the cointegrating vector being [1,-1]. Such formal tests were not considered necessary in view of the underlying theme of this research.

[38] A simple error correction model needs only Y_t. But ΔY_t is included in view of the results in Table 8.

$$R^{f}_{t+1} = \alpha^{f} + \beta^{f}_{1} Y_{t} + \beta^{f}_{2} \Delta Y_{t} + \beta^{f}_{3}(T_{1}-t) + u^{f}_{t+1} \qquad \text{....(12a)}$$

$$R^{n}_{t+1} = \alpha^{n} + \beta^{n}_{1} Y_{t} + \beta^{n}_{2} \Delta Y_{t} + \beta^{n}_{3}(T_{1}-t) + u^{n}_{t+1} \qquad \text{....(12b)}$$

$$R^{Y}_{t+1} = \alpha^{Y} + \beta^{Y}_{1} Y_{t} + \beta^{Y}_{2} \Delta Y_{t} + \beta^{Y}_{3}(T_{1}-t) + u^{Y}_{t+1} \qquad \text{....(12c)}$$

The following hypothesis is tested for each model:

$$H_0\colon\ \beta^{(i)}_{1} = \beta^{(i)}_{2} = \beta^{(i)}_{3}; \quad i = f,n,Y$$

against the alternative that at least one of the betas is not equal to zero.

These regressions are estimated both with daily data and hourly data. The results of the daily data analysis and hourly data analysis are consistent. Time to near contract maturity does not play a significant role in predicting far futures, near futures or mispricing return. Spread mispricing history strongly predicts far futures return (p-value $\ll 0.001$) but cannot predict near futures return (p-value > 0.10) at conventional significance levels. This suggests that "price discovery" is taking place in the near contract, since with deviation from equilibrium in the form of spread mispricing, it is the far contract which is adjusting towards the near contract but not vice versa. Furthermore, even with daily data, mispricing return (and hence spread portfolio return) is strongly predicted by spread mispricing history ($R^2 > 20\%$, p-value $\ll 0.001$). Clearly, this generates important externalities for market participants with short term spread positions.

3.8 Spread Mispricing and Short Term Spread Positions

The implications of spread mispricing for short term spread positions should arguably be similar to the implications of cash futures mispricing for short term cash futures hedges. Evidence on the latter has been provided by Merrick (1988). Hence, in order to enable direct comparability of results, this section attempts to replicate, *mutatis mutandis*, the empirical analysis of Merrick (1988) in this context.

The results of Section 3.7 show that spread mispricing history significantly predicts far futures return, near futures return and spread portfolio return. Spread mispricing history should accordingly be important information for those initiating spread positions. Therefore, it is necessary to include spread mispricing history in the conditioning information set. Since one lag in ΔY_t is adequate to purge the residuals in the regression equations (11a) and (11b) in 12 out of 16 quarters, only Y_t and ΔY_t are included as conditioning variables. Following Merrick (1988), time to near contract expiration is also included in the conditioning information set along with previous period spread mispricing.[39]

Table 10 reports the results of estimating the following regression for in-sample selection of the risk minimising spread ratio given a predetermined position in the near futures contract, and with the spread ratio and the expected spread return being

[39] Merrick (1988) includes only mispricing and time to expiration in the conditioning information set. However, Yadav and Pope (1992b) show that with both US and UK data change in mispricing is an important predictor variable.

both linearly conditional upon the three information variables, spread mispricing, change in spread mispricing from the previous period, and time to near contract expiration:

$$R^{n}_{t+1} = a + a_1 Y_t + a_2 \Delta Y_t + a_3(T_1 - t) + bR^{f}_{t+1} + b_1 Y_t R^{f}_{t+1} + b_2 \Delta Y_t R^{f}_{t+1} + b_3(T_1 - t) R^{f}_{t+1} + \qquad \textbf{....(13)}$$

Such a specification models both the risk minimising spread ratio and the risk minimising spread return as linear functions of the three conditioning variables Y_t, ΔY_t and $(T_1\text{-t})$.

$$_m h_t = {}_m h_t \,(Y_t, \Delta Y_t, (T_1 - t))$$

$$_m R^{s}_{t+1} = {}_m R^{s}_{t+1} \,(Y_t, \Delta Y_t, (T_1 - t))$$

The results for hourly data and daily data are largely consistent with each other. First, the estimate of b (which represents the spread ratio when the near contract is very close to expiration and the far contract is correctly priced) is significantly less than unity. It is closer to unity with daily data than with hourly data. This is intuitively reasonable since disequilibrium manifestations should be arguably greater for hourly data. Second, b_2 is significantly greater than zero. Hence, the time to near contract expiration is a significant determinant of the risk minimising spread ratio. Third, spread mispricing history is a significant determinant of the spread ratio (through b_1 and b_2), and more importantly, of the expected return of a risk minimising spread a_1 and a_2 are both significantly different from zero (p-value

$\ll$0.0001 in both cases). In particular, a one day (one hour) risk minimising long spread earned, over the sample period, earned a 0.072% (0.020%) premium if far futures were initially overpriced by 1%, and underperformed by the same amount if far futures were initially underpriced by 1%. This is a manifestation of the mean reversion documented in Section 3.6. Finally, b_3 is significantly greater than zero, but a_3 is not significantly different from zero. As a result, time to near contract expiration appears to have been a significant determinant of the spread ratio, but not of the overall spread return.

As discussed in Section 2.3, the significant difference between the risk minimising (regression selected) spread ratio ${}_{m}h_t$ and the cost-of-carry spread ratio ${}_{cc}h_t$, can be interpreted in terms of an adjustment factor representing the covariation between spread mispricing returns R^Y_{t+1} and the far futures return R^f_{t+1}. Table 11 reports the results of estimating this adjustment factor, conditional upon spread mispricing, change in spread mispricing from the previous period, and time to near contract expiration, through the following regression:

$$R^Y_{t+1} = a^Y + a_1^Y Y_t + a_2^Y \Delta Y_t + a_3^Y (T_1 - t) + b^Y R^f_{t+1} + b_1^Y Y_t R^f_{t+1} + b_2^Y \Delta Y_t R^f_{t+1} + b_3^Y (T_1 - t) l \qquad \text{....(14)}$$

Once again, the results of both hourly data and daily data are largely consistent with each other, and also consistent with the results in Table 10. First, the estimate of b^Y (which represents the value of the adjustment factor when the near contract is close to expiration and the far contract is correctly priced) is significantly greater than zero, and greater for hourly data than daily data. Second, the adjustment factor depends significantly (through b_1^Y and b_2^Y) on spread mispricing history. Third, the adjustment factor depends significantly (through b_3^Y) on time to near contract expiration.

Finally, Table 12 presents comparative statistics on average returns and residual risk for three alternative spread portfolios: first, the "naive" spread ie one long near contract and one short far contract; second, the cost of carry spread portfolio ie one long near contract and ${}_{cc}h_t$ far contracts where ${}_{cc}h_t$ is given by equation (7a); and third, the regression selected in-sample risk minimising spread portfolio ie one long near contract and ${}_{m}h_t$ far contracts where ${}_{m}h_t$ is given by equation (9). Furthermore, to highlight the effect of the initial spread mispricing on subsequent average spread returns, the data is divided into subsamples which group observations according to whether the far futures was initially overpriced ($Y_t > 0$) or underpriced ($Y_t < 0$).

The results for both daily data and hourly data in Table 12 are mutually consistent and have two major features. First, the risk of the cost-of-carry spread portfolio (with time varying spread ratio) is only marginally ($<2\%$) lower than the risk of a naive spread portfolio (with a consistent spread ratio of -1). However, the risk of a risk minimising spread portfolio which "underhedges" by taking into account the consistently positive correlation between spread mispricing returns and far futures returns, is significantly (about 10%) lower than the corresponding risk of a cost-of-carry spread portfolio. Second, the effect of initial spread mispricing on average spread return is highly significant (p-value $\ll 0.001$) for all the three alternative spread portfolios. Long spread portfolios with initially positive spread mispricing earned significantly positive returns and those with initially negative spread mispricing earned significantly negative returns; and vice versa for short spread portfolios.[40]

The results in this section for short term spread portfolios correspond very closely with the results of Merrick (1988) for short term cash-futures hedge portfolios. Spread mispricing has significant implications for spread ratio selection and for the risk and return of spread portfolios, in the same way as cash-futures mispricing has significant implications for short term hedge ratio selection and for the risk and return of cash futures short term hedge portfolios.

4. CONCLUSIONS

This chapter develops the theoretical framework for pricing of stock index futures spreads after adjusting for cash market settlement procedures, and provides empirical evidence in this regard based on about four years of "time and sales" transactions data from the London International Financial Futures Exchange (LIFFE). It also simulates the profitability of spread arbitrage strategies (in particular in the context of the early unwinding option), analyses the effect of spread mispricing on short term spread positions, and explores mean reversion in the time

40 A long spread involves a long near contract position and a short far contract position. A short spread involves a short near contract position and a long far contract position.

series of spread mispricing. The results have implications for the use of arbitrage related arguments in the context of market microstructure, the ability of futures traders to transfer risk within themselves, the debate on whether cash-futures basis behaviour is arbitrage induced or a manifestation of non-synchronous trading, the market value of the futures related tax timing option and the effect of short selling constraints on futures pricing.

The salient results of this chapter are as follows:

(a) The absolute magnitude of spread mispricing has often exceeded the estimated transaction costs of the more favourably positioned categories of arbitrageurs. The magnitude of mispricing cannot be explained by dividend uncertainties or the risk of marking-to-market cash flows.

(b) Average spread mispricing over a contract has often been significantly different from zero, but the direction of significant average mispricing has varied substantially from the first to the second half of the sample period. The variation in systematic mispricing cannot be explained by variation in transaction costs.

(c) Simulations show that arbitrage related trading rules have provided attractive arbitrage related profits after transaction costs. However, the early unwinding option has not been very valuable, and hence risky arbitrage strategies have not provided significant transaction cost discounts, like they have for cash futures arbitrage. This creates the possibility of expiration day price and volume effects arising due to new cash positions being initiated by spread arbitrageurs at near contract expiration.

(d) It is not possible to reconcile the pattern of spread mispricing with the hypothesis that the tax timing option is valuable.

(e) The analysis of spread mispricing in periods during which index arbitrageurs face no constraints on short selling shows that short selling constraints have apparently made a very significant impact on futures pricing.

(f) The spread mispricing series displays significant mean reversion. As a result, far futures prices and equivalent near futures prices are cointegrated with a cointegrating vector of [1,-1]. Furthermore, this suggests that the significant mean reversion in *cash-futures* mispricing, and the significant negative serial correlation in *cash-futures* mispricing, is, at least not *entirely*, a consequence of non-synchronous trading.

(g) Spread mispricing has significant implications for spread ratio selection and for the risk and return of spread portfolios, in the same way as cash futures mispricing has significant implications for short term hedge ratio selection and for the risk and return of cash futures short term hedge portfolios. In particular, short term spreads with initially over priced far futures have earned significantly positive returns and short term spreads with initially underpriced far futures have earned significantly negative returns.

TABLE 1

TRANSACTION COSTS OF DIFFERENT CATEGORIES OF ARBITRAGEURS

Category A: Arbitrageurs with marginal costs confined to transaction costs in the futures market.

Category B: Arbitrageurs with marginal costs including also cash market related transaction costs but not including transaction taxes or cost of borrowing stocks or capital.

Category C: Arbitrageurs with marginal costs also including transaction taxes.

Quarter Ending	Cash Average Inner Market Spread	Futures Median Spread Near Contract	Futures Median Spread Far Contract	Transaction Costs of Different Categories of Arbitrageurs: Category A	Category B	Category C
Jun 86	0.75	0.12	0.34	0.33	0.76	1.76
Sep 86	0.75	0.13	0.35	0.35	0.78	1.78
Dec 86	0.75	0.12	0.34	0.33	0.76	1.26
Mar 87	0.73	0.10	0.31	0.31	0.73	1.23
Jun 87	0.76	0.10	0.44	0.37	0.80	1.30
Sep 87	0.84	0.08	0.40	0.34	0.82	1.32
Dec 87	1.52	0.15	0.39	0.38	1.25	1.75
Mar 88	1.27	0.11	0.44	0.38	1.10	1.60
Jun 88	1.15	0.11	0.22	0.27	0.93	1.43
Sep 88	0.80	0.08	0.22	0.25	0.71	1.21
Dec 88	0.85	0.06	0.17	0.22	0.70	1.20
Mar 89	0.84	0.05	0.22	0.24	0.72	1.22
Jun 89	0.87	0.05	0.18	0.22	0.72	1.22
Sep 89	0.87	0.05	0.18	0.22	0.72	1.22
Dec 89	1.23	0.05	0.12	0.19	0.89	1.39
Mar 90	1.24	0.04	0.14	0.19	0.89	1.39

TABLE 2A

SPREAD MISPRICING - DESCRIPTIVE STATISTICS

Near Contract Expiring	Total No. of Obs.	No. of Pos. Obs.	No. of Neg. Obs.	Mean μ	Std. Dev.	Median	Min	Max	Quartiles Lower Q_1	Quartiles Upper Q_3	(Q_3-Q_1)	Percentiles 5th P_5	Percentiles 95th P_{95}	$(P_{95}-P_5)$
Jun 86	243	7	236	0.35	0.26	0.37	- 1.19	1.14	0.21	0.50	0.30	0.04	0.70	0.65
Sep 86	321	33	288	0.33	0.38	0.35	- 1.58	4.14	0.16	0.51	0.35	-0.21	0.75	0.97
Dec 86	285	5	280	0.40	0.21	0.40	- 1.47	1.06	0.32	0.48	0.16	0.16	0.75	0.59
Mar 87	374	170	204	0.10	0.42	0.04	- 1.68	1.49	-0.15	0.26	0.41	-0.39	0.97	1.36
Jun 87	343	3	340	0.89	0.31	0.97	- 0.13	1.64	0.73	1.12	0.39	0.33	1.31	0.99
Sep 87	243	62	181	0.12	0.24	0.08	- 1.50	0.78	-0.00	0.25	0.26	-0.17	0.63	0.81
Dec 87	306	7	299	1.18	0.65	1.14	- 1.17	3.08	0.84	1.59	0.74	0.09	2.44	2.35
Mar 88	331	49	282	0.20	0.31	0.15	- 0.97	1.39	0.06	0.27	0.21	-0.27	0.86	1.12
Jun 88	320	132	188	0.16	0.32	0.10	- 0.41	0.86	-0.14	0.47	0.61	-0.28	0.64	0.92
Sep 88	326	326	0	-0.97	0.20	-0.93	- 1.83	-0.38	-1.03	-0.85	0.18	-1.39	-0.67	0.71
Dec 88	339	339	0	-0.79	0.19	-0.80	- 1.41	-0.19	-0.95	-0.65	0.30	-1.06	-0.51	0.56
Mar 89	457	95	362	0.22	0.33	0.29	- 1.14	0.94	0.05	0.44	0.39	-0.45	0.63	1.08
Jun 89	327	283	44	-0.17	0.19	-0.14	- 1.52	0.32	-0.25	-0.05	0.20	-0.49	0.07	0.56
Sep 89	341	340	1	-0.58	0.16	-0.61	- 1.00	0.25	-0.69	-0.51	0.18	-0.79	-0.24	0.56
Dec 89	395	394	1	-0.63	0.14	-0.62	- 1.19	0.03	-0.73	-0.54	0.19	-0.85	-0.44	0.41
Mar 90	386	386	0	-0.72	0.13	-0.71	- 1.19	-0.10	-0.84	-0.62	0.22	-0.92	-0.54	0.38
All	5337	2631	2706	-0.02	0.66	0.01	- 1.83	4.14	-0.60	0.40	1.00	-0.94	1.12	2.06

TABLE 2B

SPREAD MISPRICING: HYPOTHESES TESTS

Near Contract Expiring	No. of observations	Percentage of negative observations p (%)	p value H_0: p=0	Mean	Unadjusted t-statistic	t-statistic adjusted for serial correlations
Jun 86	243	2.9	0.000	0.35	20.86	5.52
Sep 86	321	10.3	0.000	0.33	15.57	6.17
Dec 86	285	1.8	0.000	0.40	32.09	22.85
Mar 87	374	45.5	0.079	0.10	4.77	0.73
Jun 87	343	0.9	0.000	0.89	53.82	16.70
Sep 87	243	25.5	0.000	0.12	7.70	2.43
Dec 87	306	2.3	0.000	1.18	31.57	9.50
Mar 88	331	14.8	0.000	0.20	11.69	2.10
Jun 88	320	41.3	0.002	0.16	9.03	1.31
Sep 88	326	100.0	0.000	-0.97	-85.78	-29.27
Dec 88	339	100.0	0.000	-0.79	-77.76	-33.53
Mar 89	457	20.8	0.000	0.22	14.03	1.74
Jun 89	327	86.5	0.000	-0.17	-16.01	-5.42
Sep 89	341	99.7	0.000	-0.58	-65.96	-12.67
Dec 89	395	99.7	0.000	-0.63	-87.55	-46.90
Mar 90	386	100.0	0.000	-0.72	-105.30	-30.77
All	5337	49.3	0.305	-0.02	-2.18	-0.11

TABLE 3

SPREAD MISPRICING AND MISSPECIFICATION OF DIVIDENDS

d_{divl} (y%) = **Mispricing estimated from actual *ex-post* dividend inflows minus mispricing estimated by using previous year's dividend plus a y% growth factor.**

	d_{divl} (0%)	d_{divl} (10%)	d_{divl} (20%)
Mean (%)	0.128	0.032	-0.065
Standard Deviation (%)	0.086	0.089	0.095
Median (%)	0.104	0.024	-0.055
Minimum	0.007	-0.118	-0.243
Maximum	0.359	0.243	0.128
Lower Quartile Q_1	0.073	-0.039	-0.154
Upper Quartile Q_3	0.184	0.087	0.008

TABLE 4

SPREAD MISPRICING: PERSISTENCE AND REVERSALS

Near Contract Expiring	Number of Obs.	Number of Reversals	Average Time Before Reversal (Trading Hrs) +ve Mispricing	-ve Mispricing	First order Serial Correlation in Mispricing ρ	Standard Error of ρ	First order Serial Correlation in Mispricing Changes ρ'	Standard Error of ρ
Jun 86	243	13	48.8	1.4	0.401	0.071	-0.598	0.076
Sep 86	321	43	21.2	2.4	0.406	0.057	-0.488	0.061
Dec 86	285	11	83.8	1.5	0.240	0.049	-0.568	0.055
Mar 87	374	74	7.3	6.1	0.782	0.037	-0.466	0.055
Jun 87	343	7	120.9	1.1	0.667	0.044	-0.556	0.053
Sep 87	243	67	11.4	3.9	0.591	0.061	-0.496	0.070
Dec 87	306	10	91.0	2.1	0.921	0.023	-0.468	0.061
Mar 88	331	30	28.1	4.9	0.909	0.030	-0.394	0.054
Jun 88	320	28	19.8	13.9	0.922	0.027	-0.594	0.053
Sep 88	326	-	-	-	0.835	0.039	-0.385	0.062
Dec 88	339	-	-	-	0.864	0.034	-0.452	0.056
Mar 89	457	30	25.3	6.7	0.893	0.021	-0.427	0.044
Jun 89	327	41	3.2	20.8	0.749	0.043	-0.419	0.050
Sep 89	341	3	0.8	255.2	0.586	0.045	-0.493	0.038
Dec 89	395	3	0.6	251.4	0.629	0.045	-0.519	0.053
Mar 90	386	-	-	-	0.654	0.039	-0.426	0.049
All	5337	352	22.8	22.1	0.955	0.005	-0.492	0.014

TABLE 5A

ARBITRAGE PROFIT SIMULATIONS; CATEGORY A ARBITRAGEURS

Near Contract Expiring	Total Number of Observations	Trading Rule 1 (Hold to Expiration): Number of Arbitrage Opportunities	Trading Rule 1: Arbitrage Profits/Contract (£000) Ex post Simulations	Trading Rule 1: Arbitrage Profits/Contract (£000) Ex ante Simulations	Percentage of Profitable Trades in ex ante Simulations	Trading Rule 2 (Early Unwinding): Number of Early Unwindings	Trading Rule 2: Additional Arbitrage Profits/Contract (£000) Ex post Simulations	Trading Rule 2: Additional Arbitrage Profits/Contract (£000) Ex ante Simulations	Average Time to Early Unwinding (Trading Hours)
Jun 86	243	64	4.0	- 1.3	45.3	8	1.5	0.1	18.8
Sep 86	321	101	7.7	- 1.1	49.5	101	35.3	2.0	45.0
Dec 86	285	62	4.0	- 2.4	29.0	26	2.7	1.4	34.6
Mar 87	374	63	12.4	8.7	76.2	9	0.2	0.4	12.8
Jun 87	343	289	78.4	55.6	77.2	0	0.0	0.0	0.0
Sep 87	243	18	2.3	0.4	66.7	2	0.0	-0.1	11.0
Dec 87	306	260	92.9	72.1	85.0	260	29.4	-7.9	146.6
Mar 88	331	45	6.6	- 1.3	37.8	43	12.6	-1.4	88.3
Jun 88	320	66	3.0	- 1.9	50.0	66	0.1	-1.5	186.2
Sep 88	326	324	68.8	44.0	77.5	0	0.0	0.0	0.0
Dec 88	339	323	45.4	31.7	84.2	0	0.0	0.0	0.0
Mar 89	457	91	5.6	3.3	79.1	9	1.1	0.1	59.8
Jun 89	327	15	1.5	0.3	26.7	4	0.3	-0.2	102.0
Sep 89	341	271	23.4	17.8	87.5	3	0.1	1.3	24.3
Dec 89	395	342	31.7	21.0	82.5	0	0.0	0.0	0.0
Mar 90	386	379	49.9	34.7	84.2	0	0.0	0.0	0.0
All	5337	2713	437.6	281.5	77.0	531	83.2	-5.8	114.8

TABLE 5B

ARBITRAGE PROFIT SIMULATIONS; CATEGORY B ARBITRAGEURS

		Trading Rule 1 (Hold to Expiration)				Trading Rule 2 (Early Unwinding)			
Near Contract Expiring	Total Number of Observations	Number of Arbitrage Opportunities	Arbitrage Profits/Contract (£000)		Percentage of Profitable Trades in ex ante Simulations	Number of Early Unwindings	Additional Arbitrage Profits/Contract (£000)		Average Time to Early Unwinding (Trading Hours)
			Ex post Simulations	Ex ante Simulations			Ex post Simulations	Ex ante Simulations	
Jun 86	243	3	0.2	-0.8	0.0	2	0.0	0.1	6.5
Sep 86	321	3	1.6	-0.8	0.0	3	1.1	-0.1	14.7
Dec 86	285	2	0.2	-0.4	0.0	1	0.2	-0.1	8.0
Mar 87	374	17	2.7	0.8	76.5	2	0.1	-0.0	2.0
Jun 87	343	149	12.8	-1.7	59.1	0	0.0	0.0	0.0
Sep 87	243	1	0.3	-0.5	0.0	1	0.0	-0.0	19.0
Dec 87	306	201	43.0	24.1	74.1	201	22.1	-6.1	167.3
Mar 88	331	11	0.9	-2.7	0.0	11	3.7	-0.6	83.4
Jun 88	320	0	0.0	0.0	*	0	0.0	0.0	0.0
Sep 88	326	110	8.9	-1.3	52.7	0	0.0	0.0	0.0
Dec 88	339	48	1.2	-2.1	47.9	0	0.0	0.0	0.0
Mar 89	457	1	0.1	-0.4	0.0	1	0.0	0.0	3.0
Jun 89	327	2	0.3	-0.5	0.0	1	0.1	-0.1	108.0
Sep 89	341	0	0.0	0.0	*	0	0.0	0.0	0.0
Dec 89	395	2	0.2	-0.3	0.0	0	0.0	0.0	0.0
Mar 90	386	9	0.3	-1.6	0.0	0	0.0	0.0	0.0
All	5337	559	72.8	11.5	59.2	223	27.4	-6.8	155.8

TABLE 6

RISKY ARBITRAGE STRATEGIES

The arbitrage position is initiated whenever mispricing exceeds Y% in magnitude (Y=0.5%,1.0%) even when actual transaction costs exceed X%. The position is unwound early as soon as the additional profit from early unwinding makes the overall position profitable after inclusion of the incremental transaction costs involved in early unwinding.

Trading Rule Threshold = 0.5%

	Transaction costs = 0.75%		Transaction Costs = 1.0%		Transaction Costs = 1.25%		Transaction Costs = 1.5%	
Quarter Ending	% of Positions Unwound	Total Profit/Contract (£000)	% of Positions Unwound	Total Profit/Contract (£000)	% of Positions Unwound	Total Profit/Contract (£000)	% of Positions Unwound	Total Profit/Contract (£000)
Jun 86	20.6	9.5	20.6	-7.2	16.5	- 26.0	14.1	-44.0
Sep 86	98.6	79.4	94.8	79.1	87.8	60.4	85.4	39.2
Dec 86	16.3	0.7	8.5	-26.6	5.0	- 53.4	3.5	-80.3
Mar 87	47.6	18.9	47.6	13.4	47.6	0.3	47.6	-16.1
Jun 87	0.0	75.6	0.0	29.4	0.0	- 16.9	0.0	-63.1
Sep 87	35.3	7.1	35.3	2.0	29.4	- 9.0	29.4	-18.9
Dec 87	97.5	125.1	96.8	95.4	92.1	64.4	76.3	29.9
Mar 88	86.0	20.3	44.7	3.4	36.8	- 10.0	34.2	-23.3
Jun 88	79.7	1.6	36.1	-15.2	0.6	- 37.7	0.0	-55.6
Sep 88	0.0	68.7	0.0	32.1	0.0	- 4.5	0.0	-41.0
Dec 88	0.0	44.9	0.0	6.9	0.0	- 31.0	0.0	-68.9
Mar 89	47.8	40.7	47.8	11.5	47.8	- 28.9	18.2	-110.1
Jun 89	15.7	-2.8	3.6	- 13.9	3.6	- 25.0	2.4	-36.6
Sep 89	1.9	20.8	0.6	- 27.2	0.3	- 75.1	0.0	-123.5
Dec 89	0.0	30.1	0.0	- 25.1	0.0	- 80.3	0.0	-135.5
Mar 90	0.0	49.8	0.0	- 5.6	0.0	- 61.0	0.0	-116.4
All	27.0	590.4	23.0	152.4	20.1	-333.5	16.0	-864.2

TABLE 6 (CONT'D)

RISKY ARBITRAGE STRATEGIES

Trading Rule Threshold = 1.0%

	Transaction costs = 1.25%		Transaction Costs = 1.5%		Transaction Costs = 1.75%		Transaction Costs = 2.0%	
Quarter Ending	% of Positions Unwound	Total Profit/Contract (£000)	% of Positions Unwound	Total Profit/Contract (£000)	% of Positions Unwound	Total Profit/Contract (£000)	% of Positions Unwound	Total Profit/Contract (£000)
Jun 86	44.4	0.2	44.4	- 0.3	44.4	- 1.0	44.4	- 1.8
Sep 86	100.0	10.1	100.0	8.2	100.0	6.3	100.0	4.5
Dec 86	13.3	- 0.4	6.7	- 2.1	6.7	- 3.6	6.7	- 5.2
Mar 87	5.0	1.3	5.0	- 3.8	5.0	- 8.9	5.0	-13.9
Jun 87	0.0	6.9	0.0	-27.3	0.0	- 61.4	0.0	-95.6
Sep 87	20.0	- 0.3	20.0	- 1.7	20.0	- 1.7	20.0	- 2.1
Dec 87	99.2	67.7	81.7	37.7	53.3	4.9	35.0	-25.2
Mar 88	100.0	7.8	100.0	5.3	95.7	2.6	47.8	- 3.7
Jun 88	33.3	- 0.2	0.0	- 0.6	0.0	- 0.9	0.0	- 1.3
Sep 88	0.0	- 0.7	0.0	- 34.7	0.0	- 68.8	0.0	-102.8
Dec 88	0.0	- 6.6	0.0	- 28.8	0.0	- 50.9	0.0	-73.1
Mar 89	20.0	- 0.3	20.0	- 1.5	20.0	- 2.7	10.0	- 4.3
Jun 89	33.3	0.2	33.3	- 0.2	33.3	- 0.6	0.0	- 1.0
Sep 89	0.0	- 3.1	0.0	- 7.3	0.0	- 11.6	0.0	-15.8
Dec 89	0.0	- 8.1	0.0	- 19.4	0.0	- 30.6	0.0	-41.9
Mar 90	0.0	- 13.0	0.0	- 35.9	0.0	- 58.7	0.0	-81.5
All	21.5	61.5	18.3	-111.6	13.1	-287.6	8.9	-464.6

TABLE 7

SPREAD MISPRICING AND CASH MARKET SHORT SELLING CONSTRAINTS

	Descriptive Statistics on Mispricing		
	Sub Sample 0 ie Within Account Period Spanning Futures Maturity	Sub Sample 1 ie Within Account Period just before Account Period Spanning Futures Maturity	Sub Sample 2 ie All other Account Periods
Number of observations	823	1243	3211
Proportion of positive mispricing observations	0.354	0.557	0.519
Mean (%)	-0.166	-0.026	-0.017
Standard Deviation (%)	0.665	0.635	0.616
Median (%)	-0.295	0.047	0.030

p Value of Hypotheses Tests

H_{01}:	Mean in sub-sample 0	=	Mean in sub-sample 1	p value $\lessdot$ 0.0001
	Median in sub-sample 0	=	Median in sub-sample 1	p value $\lessdot$ 0.0001
H_{02}:	Mean in sub-sample 0	=	Mean in sub-sample 2	p value $\lessdot$ 0.0001
	Median in sub-sample 0	=	Median in sub-sample 2	p value $\lessdot$ 0.0001
H_{12}:	Mean in sub-sample 1	=	Mean in sub-sample 2	p value $\lessdot$ 0.660
	Median in sub-sample 1	=	Median in sub-sample 2	p value $\lessdot$ 0.355

TABLE 8

UNIT ROOT TESTS ON SPREAD MISPRICING

Spread Mispricing Y_t is defined as {Log(Far Futures Price)-Log(Equivalent Near Futures Price)}.

Augmented Dickey Fuller Regressions are estimated with two alternative models

Model without time trend:

$$\Delta Y_t = -\phi Y_{t-1} + \rho_o + \sum_{g=1}^{p} \rho_g \Delta Y_{t-g} + \epsilon_t$$

Model with time trend:

$$\Delta Y_t = -\phi Y_{t-1} + \rho_o + \sum_{g=1}^{p} \rho_g \Delta Y_{t-g} + \alpha(T_1 - t) + \epsilon_t$$

where $\Delta Y_t = (Y_t - Y_{t-1})$ and T_1 is the time corresponding to near contract expiration.

In each case, p is chosen as the minimum value that ensures that serial correlation in the regression residuals is statistically insignificant. Serial correlation is estimated by the Box Pierce statistic for $3\sqrt{N}$ lags where N is the number of observations. The p-value of Q statistic is reported below.

The hypothesis $\Phi=0$ is tested in the model without the time trend by using the τ_μ statistic in Fuller (1976).

The hypothesis $\Phi=0$ is tested in the model with the time trend by using the τ_t statistic in Fuller (1976).

TABLE 8 (CONT'D)

UNIT ROOT TESTS ON SPREAD MISPRICING

Contract	Model without Time Trend					Model with Time Trend					
		τ_μ-statistic		t-statistic	p-value of		T_τ statistic		t-statistic	p-value of	t-statistic
Expiring	Φ	null $\Phi=0$	ρ_1	null: $\rho_1=0$	Q-statistic	Φ	null: $\Phi=0$	ρ_1	null: $\rho_1=0$	Q-statistic	null: $\alpha=0$
Jun 86	0.474	5.11***	0.057	0.52	0.97	0.642	6.08***	0.087	0.83	0.98	-2.83***
Sep 86	0.430	5.87***	-0.278	- 4.16***	0.49	0.494	6.29***	-0.239	-3.48***	0.80	2.12**
Dec 86	0.393	5.68***	-0.361	- 5.75***	0.99	0.403	5.71***	-0.358	-5.70***	1.00	-0.72
Mar 87	0.124	3.20***	-0.407	- 7.13***	0.65	0.254	4.87***	-0.340	-5.78***	0.86	-3.63***
Jun 87	0.206	4.44***	-0.459	- 8.27***	0.30	0.273	5.38***	-0.426	-7.63***	0.56	-3.03***
Sep 87	0.235	3.53***	-0.376	- 4.99***	0.51	0.349	4.64***	-0.331	-4.40***	0.70	-3.00***
Dec 87	0.080	2.45	-0.415	- 6.32**	0.56	0.072	2.01	-0.421	-6.32***	0.53	0.60
Mar 88	0.090	2.97**	-0.345	- 6.32***	0.49	0.229	5.11***	-0.277	-4.93***	0.68	4.13***
Jun 88	0.003	0.14	-0.592	-10.83***	0.55	0.155	4.21***	0.526	-9.77***	0.91	5.00***
Sep 88	0.071	2.14	-0.342	- 5.26***	0.99	0.098	2.60	-0.328	-5.01***	1.00	1.51
Dec 88	0.009	0.33	-0.597	- 8.86***	0.59	0.012	0.44	-0.614	-9.15***	0.58	-2.38**
Mar 89	0.069	3.20***	-0.394	- 8.76***	0.11	0.232	5.04***	-0.286	-6.02***	0.25	-5.62***
Jun 89	0.048	1.27	-0.367	- 5.82***	0.53	0.102	2.45	-0.352	-5.65***	0.62	2.85***
Sep 89	0.159	3.73***	-0.502	- 8.01***	0.51	0.232	4.07***	-0.457	-6.87***	0.43	1.92*
Dec 89	0.270	5.35***	-0.382	- 6.68***	0.96	0.274	5.18***	-0.380	-6.59***	0.96	0.81
Mar 90	0.170	4.26***	-0.354	- 7.05***	0.84	0.188	4.42***	-0.346	-6.83***	0.87	1.19

*** : p value <0.01; ** : p-value <0.05; * : p-value <0.10.

TABLE 9

SPREAD MISPRICING AND FUTURE RETURNS

OLS regressions are run to estimate the ability of spread mispricing Y_t and changes in spread mispricing ΔY_t to predict far futures "return", near futures "return" and spread mispricing "return". Time to near contract expiration is included as an additional "conditioning" variable.

The following reversions are estimated:

$$R^f_{t+1} = \alpha^f + \beta^f_1 Y_t + \beta^f_2 \Delta Y_t + \beta^f_3 (T_1 - t) + u^f_{t+1}$$

$$R^n_{t+1} = \alpha^n + \beta^n_1 Y_t + \beta^n_2 \Delta Y_t + \beta^n_3 (T_1 - t) + u^n_{t+1}$$

$$R^Y_{t+1} = \alpha^Y + \beta^Y_1 Y_t + \beta^Y_2 \Delta Y_t + \beta^Y_3 (T_1 - t) + u^Y_{t+1}$$

where R^f_{t+1}, R^n_{t+1}, *and* R^Y_{t+1} are the far futures return, near futures return and mispricing return defined as in equations (6b), (6a) and (8a) respectively.

The F-statistic is for the null hypothesis that $\beta_1 = \beta_2 = \beta_3 = 0$.

Coefficient standard errors are in parentheses.

TABLE 9 (CONT'D)

SPREAD MISPRICING AND FUTURE RETURNS

	α	β_1	β_2	β_3 (x 10^3)	F-statistic Null: $\beta_1 = \beta_2 = \beta_3 = 0$	R^2 (%)
Hourly Data Results						
Far Futures Returns	-0.000	-0.012	-0.518	0.43	83.45***	5.9
	(0.010)	(0.010)	(0.034)	(0.33)		
Near Futures Returns	0.001	0.011	-0.028	0.39	1.19	0.1
	(0.010)	(0.009)	(0.030)	(0.30)		
Mispricing Returns	-0.002	-0.024	-0.490	0.04	446.89***	25.2
	(0.005)	(0.004)	(0.014)	(0.14)		
Daily Data Results						
Far Futures Returns	-0.021	-0.015	-0.387	2.74	4.69***	1.7
	(0.075)	(0.064)	(0.125)	(1.56)		
Near Futures Returns	-0.028	0.061	-0.009	3.02	1.61	0.6
	(0.073)	(0.062)	(0.120)	(1.50)		
Mispricing Returns	0.006	-0.083	-0.395	0.39	75.46***	21.5
	(0.019)	(0.017)	(0.032)	(0.40)		

*** p-value <0.01

TABLE 10

SELECTION OF RISK MINIMISING SPREAD RATIO

The following regression is estimated:

$$R^{n}_{t+1} = a + a_1 Y_t + a_2 \Delta Y_t + a_3(T_1 - t) + bR^{f}_{t+1} + b_1 Y_t R^{f}_{t+1} + b_2 \Delta Y_t R^{f}_{t+1} + b_3(T_1 - t) R^{f}_{t+1} + u_{t+1}$$

where R^{n}_{t+1}, R^{f}_{t+1} are the near and far futures return defined as in equations (6a) and (6b) respectively, Y_t is the spread mispricing, $\Delta Y_t = (Y_t - Y_{t-1})$, and $(T_1\text{-}t)$ is the time to near contract expiration.

The basic regression is of R^{n}_{t+1} *on* R^{f}_{t+1} . Y_t, ΔY_t and $(T_1\text{-}t)$ are "conditioning" variables.

Coefficient standard errors are in parentheses.

TABLE 10 (CONT'D)

SELECTION OF RISK MINIMISING SPREAD RATIO

	Hourly Data	Daily Data
Intercept(s)		
a	0.001	-0.002
	(0.004)	(0.017)
a_1	0.020	0.072
	(0.004)	(0.015)
a_2	0.389	0.345
	(0.013)	(0.029)
$a_3(\text{x } 10^3)$	0.003	0.300
	(0.125)	(0.365)
Slope(s)		
b	0.797	0.888
	(0.012)	(0.015)
b_1	-0.019	-0.026
	(0.007)	(0.012)
b_2	-0.026	0.074
	(0.011)	(0.020)
$b_3(\text{x } 10^3)$	0.939	1.157
	(0.303)	(0.311)
Regression R^2(%)	83.3	94.3

TABLE 11

ADJUSTMENT FACTOR IN SPREAD RATIO SELECTION

The following regression is estimated:

$$R^Y_{t+1} = a^Y + a^Y_1 Y_t + a^Y_2 \Delta Y_t + a^Y_3 (T_1 - t) + b^Y R^f_{t+1} + b^Y_1 Y_t R^f_{t+1} + b^Y_2 \Delta Y_t R^f_{t+1} + b^Y_3 (T_1 - t) R^f_{t+1} + u^Y_{t+1}$$

where R^f_{t+1} *and* R^Y_{t+1} are the far futures return and mispricing return defined as in equations (6b) and (8a) in the text, Y_t is the spread mispricing, $\Delta Y_t = (Y_t - Y_{t-1})$, and (T_1-t) is the time to near contract expiration.

The basic regression is of R^Y_{t+1} *on* R^F_{t+1}. Y_t, ΔY_t and (T_1-t) are "conditioning" variables.

Coefficient standard errors are in parentheses.

TABLE 11 (CONT'D)

ADJUSTMENT FACTOR IN SPREAD RATIO SELECTION

	Hourly Data	Daily Data
Intercept(s)		
a^Y	-0.002	0.001
	(0.004)	(0.019)
a_1^Y	-0.021	-0.079
	(0.004)	(0.016)
a_2^Y	-0.400	-0.373
	(0.013)	(0.032)
$a_3^Y(x\ 10^3)$	0.018	0.347
	(0.130)	(0.391)
Slope(s)		
b^Y	0.184	0.087
	(0.012)	(0.016)
b_1^Y	0.024	0.028
	(0.008)	(0.013)
b_2^Y	0.022	-0.078
	(0.012)	(0.022)
$b_3^Y(x\ 10^3)$	1.105	1.126
	(0.315)	(0.333)
Regression R^2(%)	36.4	26.2

TABLE 12

RISK-RETURN OF ALTERNATIVE SPREAD PORTFOLIOS

Sample	Naive Spread[a]			Cost-of-carry Spread[b]			Risk-Minimising Spread[c]		
	Mean Return μ_s	t-statistic Null: $\mu_s=0$	Standard Deviation	Mean Return μ_s	t-statistic Null: $\mu_s=0$	Standard Deviation	Mean Return μ_s	t-statistic Null: $\mu_s=0$	Standard Deviation
Hourly Data Results									
Total Sample	-0.0019	-0.63	0.203	-0.0014	-0.46	0.200	0.0011	0.43	0.179
Sub-sample with initially underpriced far futures	-0.0208	-5.86***	0.167	-0.0203	-5.81***	0.164	-0.0151	-4.64***	0.153
Sub-sample with initially overpriced far futures	0.0166	3.38***	0.233	0.0172	3.57***	0.228	0.0171	4.04***	0.201
Daily Data Results									
Total Sample	-0.0008	-0.07	0.324	0.0036	0.33	0.330	0.005	0.49	0.311
Sub-sample with initially underpriced far futures	-0.0520	-3.81***	0.287	-0.0492	-3.55***	0.292	-0.0440	-3.31***	0.280
Sub-sample with initially overpriced far futures	0.0525	3.09***	0.350	0.0588	3.38***	0.358	0.0563	3.49***	0.333

(a) Naive Spread: Spread ratio = -1

(b) Cost-of-Carry Spread: Spread ratio determined by the forward pricing formula.

(c) Risk-Minimising Spread: Spread which minimises *ex post* the risk of the spread portfolio.

*** p-value <0.01

CHAPTER 9

CONCLUDING REMARKS

9.1 SALIENT CONCLUSIONS

There has been considerable interest among market participants, market regulators and academics in the pricing of stock index futures contracts. Academic research in this area has been motivated by several considerations. First, the utility of these contracts for risk allocation and price discovery depends on the efficiency with which they are priced relative to the underlying index. Second, it has been widely believed that they have adverse impact on price dynamics in the stock market. Third, and most important, stock index futures offer the possibility of directly studying the economics of arbitrage in the context of market microstructure.

This dissertation extends the theoretical framework on stock index futures pricing in two directions. First, within the static cost of carry framework, it generalises the forward pricing formula by allowing for cash market settlement procedures. Second, it shows that in the presence of arbitrage related transaction costs, the time series of stock index futures "mispricing" can be modelled as a threshold autoregressive (TAR) process, a piecewise linear autoregressive process in which the process parameters are path dependent.

This dissertation also provides substantial and significant new empirical evidence relevant to the theoretical issues involved. *Inter-alia,* it analyses several important aspects not adequately examined in past research, and it utilises the unique microstructural features of the London stock market to explore several major theoretical issues. The empirical analysis is based mainly on about four years of "time and sales" transactions data from the London International Financial Futures Exchange together with synchronous hourly cash index data.

The salient results of this research can be summarised as follows:

(a) The absolute magnitude of cash futures mispricing has often exceeded the estimated transaction costs of the more favourably positioned categories of arbitrageurs. The magnitude of mispricing cannot be explained by dividend uncertainties or the risk of marking-to-market cash flows.

(b) Average mispricing over a contract has often been significantly different from zero, but the direction of significant average mispricing has varied substantially from period to period. However, the direction of mispricing in the near contract at any particular time has tended to be the same as the direction of mispricing in the far contract at that time. The variation in systematic mispricing cannot be explained by variation in transaction costs.

(c) Simulations show that even *ex ante* trading rules have provided attractive arbitrage related profits after transaction costs. In this context, the tendency for mispricing reversals has made the early unwinding option very valuable. Similarly, the fact that the near and far contract tend to be mispriced in the same direction, has made the rollover option very valuable. This leads to potential for risky arbitrage and little feasibility of expiration day price and volume effects of arising from cash-futures arbitrage related unwinding of positions.

(d) The cash futures mispricing series displays significant mean reversion. This results in one day cash-futures hedge portfolios based on the forward pricing formula earning significantly negative abnormal returns if established when futures are initially underpriced and significantly positive abnormal returns when futures are initially overpriced.

(e) The nature of spread mispricing has been largely similar to cash futures mispricing. The magnitude has often exceeded arbitrage related transaction costs and cannot be explained by dividend uncertainties. The average mispricing has varied substantially from period to period. Mean reversion has been statistically significant. As a result, short term spreads with initially overpriced far futures contracts have earned significantly positive returns and short term spreads with initially underpriced far futures contracts have earned significantly negative returns. However, there have been relatively fewer mispricing reversals, and hence the early unwinding option has not been valuable for spread arbitrageurs. As such, expiration day price and volume effects could potentially arise due to new cash positions being initiated by spread arbitrageurs at near contract expiration.

(f) It is not possible to reconcile the pattern of cash futures or spread mispricing with the hypothesis that the tax timing option is valuable.

(g) Though there has been a mild tendency for futures to be underpriced in sharply falling markets and overpriced in sharply rising markets, this has been essentially of no economic significance for index arbitrageurs.

(h) The analysis of cash-futures mispricing and spread mispricing in periods during which index arbitrageurs face no constraints on short selling show that short selling constraints have apparently made a very significant impact on futures pricing.

(i) The absolute magnitude of cash futures mispricing has been greater for longer times to maturity, consistent with arbitrage being perceived as more risky when time to maturity is greater.

(j) There appears to have been a strong positive relationship between cash-futures mispricing and the *ex ante* market volatility implied by index call option prices. The direction of this relationship is found to be *opposite* to that predicted by the existence of a tax timing option, but is consistent with index calls being "mispriced" (relative to the cash index) in the same direction as index futures contracts.

(k) The time series behaviour of spread mispricing has significant implications for selection of appropriate risk minimising spreads for reallocation of risk among different futures traders.

(l) A model of futures mispricing has been proposed which allows, firstly, for the marked differences that can exist in the marginal transaction costs faced by different categories of arbitrageurs and, secondly, for the constraints that are expected to exist on the supply of arbitrage capital. On the basis of these institutional features it has been argued that the market demand schedule of arbitrageurs will not vary linearly with the mispricing of futures contracts relative to cash market prices. Instead, the demand schedule should vary with mispricing in a non-linear fashion and specifically in the form of a step function. As a result, the time series of futures mispricing should follow a self exciting threshold autoregressive process. This is a piecewise linear autoregressive process in which the process parameters describing the evolution of mispricing are path-dependent. The empirical results presented in the dissertation are strongly suggestive of threshold non-linearity in the time series of stock index futures mispricing. Furthermore, the estimates of the values of the thresholds appear to be consistent with the model, given the estimated transactions costs levels faced by the different categories of arbitrageurs who are potentially active in these markets. Estimates of a measure of the elasticity of arbitrage services corresponding to different transaction cost regimes are also strongly consistent with the model.

(m) Evidence on mean reversion in US and UK cash futures mispricing data is remarkably similar and has several interesting features. First, mean reversion is statistically significant in almost all contracts. This is consistent with the existence of significant arbitrage activity. Second, the mean reversion parameter is a systematic function of the time to maturity of the futures contracts, increasing as the time to maturity decreases. This is consistent with arbitrage being considered more risky when time to maturity is higher. It is also not consistent with the view that basis predictability is mainly a statistical illusion created by non-synchronous trading. Third, mean reversion appears to depend significantly on the value of mispricing in the previous period. It is not significantly different from zero when the magnitude of the previous period mispricing is so small that no arbitrageurs are likely to be active, but becomes significant in magnitude when the magnitude of the previous period mispricing becomes large enough to exceed the estimated marginal transaction costs of arbitrageurs. This supports the view that mean reversion is arbitrage induced and also supports the TAR model of Chapter 4. Fourth, mean reversion appears to be significantly lower on Mondays than on other days of the week. Finally, mean reversion also exhibits significant intraday seasonality, following a U-shaped intraday pattern. However, these seasonalities cannot be explained by corresponding patterns in lagged mispricing and volatility.

(n) Analysis of seasonalities has shown that the UK stock market did not appear to efficiently incorporate into prices the *entire* interest costs inherent in its settlement procedures. The market also displayed significant seasonality within a settlement cycle. This seasonality within the settlement cycle carried over to the futures market. It was also found that "abnormal" Monday returns accrued during the trading day on Monday and not over the weekend non-trading interval. This and the examination of extreme price changes suggests that the observed Monday effect cannot be explained by the conjecture that bad news tends to be released more frequently over the weekend non-trading interval than on other days. Furthermore, the divergent behaviour of cash and futures markets in certain periods is interesting in that it cannot apparently be explained by "behavioural" explanations, unless the two markets are segmented. Some "new" empirical regularities, apparently without an obvious explanation, have also been detected - eg the cash market rising when the market is open and the futures market rising when the market is closed. Finally, there is clear evidence of the opening of the US market being associated with a systematic fall in the UK markets.

(o) The magnitude of the seasonalities observed in the cash market has not been enough to enable the formulation of profitable trading rules. However, in view of the small transaction costs in the futures market, some of the seasonal features could be regarded as being potentially economically significant, since trading rules based on them have provided *ex post* profits after transaction costs. Low marginal cost arbitrageurs could also have exploited the *differences* in seasonal patterns in the two markets.

9.2 DIRECTIONS FOR FUTURE RESEARCH

This dissertation appears to be the first use of the TAR model in finance. The TAR model, and the associated empirical tests, should be directly relevant for modeling intermarket arbitrage and the price differences of equivalent assets in a variety of other situations. To the extent that transaction cost structures are different to those prevailing in markets examined here, we would expect these differences to be reflected in parameter estimates of the model. Of course, in markets where differential transaction cost levels and/or arbitrage capital constraints are not effectively present, threshold non-linearity should not be detectable. Future research across a variety of market settings therefore has an important role in confirming, or denying, the role of transaction costs in determining the evolution of the relative prices of equivalent assets. The TAR model is also potentially attractive for many other financial applications.

The studies summarised in this dissertation have essentially focused on:

(a) Contemporaneous relationships,

(b) Pricing,

(c) First moments of the variables involved, and

(d) The index as a whole, rather than component stocks.

Clearly, future work can focus on lead lag relationships, volume, volatility, and the impact on component index stocks. In this context, there are several other areas of interest relating to stock index futures markets which follow on from this work and could be explored further utilising the special features of UK cash market microstructure.

(a) Price discovery and lead lag relationships in price changes

Our results formally establish the existence of mean reversion in stock index futures mispricing. The overall mispricing series does not have unit roots ie mispricing is an I(0) variable. However, the logs of futures prices and futures equivalent cash prices are *a priori* likely to be non-stationary I(1) variables. Hence, since their difference is I(0), futures prices and futures equivalent cash prices are likely to be cointegrated. This necessarily implies an error-correction model in which price changes in one market provide predictive information about price changes in the other market. The existence of such lead lag relationships also follows from the theoretical models of Subrahmanyam (1991, pp 44) and Holden (1990b). It also raises the possibility that futures and cash markets are not equal in their capacity to discover new information about asset prices. This has been recognised, and allowed for, in the modelling undertaken in Chapters 4 and 7, but the results have not been presented and analysed from this perspective. Clearly, this is a possible direction for future research.

Indeed, several US based studies (eg Stoll and Whaley, 1990; Lo, 1988; Cheung and Ng, 1990; Kawaller *et al*, 1987; and Chan, 1992) find that futures returns significantly lead cash index returns, though there is also some evidence (particularly Chan, 1992) that cash index returns also have some predictive ability about futures returns. This suggests that the futures market serves as the primary market for price discovery, with information being incorporated faster into futures prices than cash prices.[1]

In this context, it is relevant to mention that there are several issues regarding the lead lag relationship which deserve examination in a UK context. First, is the lead lag relationship observed in the US based on infrequent trading of component stocks? It has already been highlighted in Chapters 4, 6 and 7 that UK cash index values represent actually tradeable values synchronous with futures prices, and hence, though the index is not free from the effects of differential price adjustment delays within index stocks, the actionable implications are economically significant. Second, is the lead lag relationship different under good news than under bad news? It could be argued that, since it is easier to take short positions in the futures market than in the cash market, the futures lead over cash should be stronger under bad news than under good news. However, in the UK, because of the account settlement period system, it is costless to sell short if the investment horizon does not extend beyond the end of the account. This feature could potentially be utilised to design interesting tests of the hypothesis. Third, it has been suggested by theoretical studies (eg Subrahmanyam, 1991), and could also be intuitively expected, that fixed costs of trading and budget constraints cause futures market traders to

[1] This is inconsistent with the theoretical model of Subrahmanyam (1991, Proposition 11, pp 43).

collect more market wide information and cash market traders to collect more firm specific information. This is directly testable since this implies that under economy wide information (proxied eg by, say, greater comovement across stocks) the futures lead should be stronger than under firm specific information. Finally, a study of lead lag relationships in volatility follows on from a study of lead lag relationships in price changes. It can be argued that information flows between the two markets should be measured by time varying conditional volatility of price changes, rather than actual price changes. Ross (1989) shows formally that it is the volatility of an asset's price, and not the asset's price change, that is related to the flow of information to the market. Furthermore, considering the large volume of recent literature on time varying volatility (see eg Bollerslev *et al*, 1992, for a review), tests of lead lag specifications which do not incorporate this time varying volatility could lead to incorrect inferences. This is apparently borne out by the results of Chan *et al* (1991).

(b) Volume

Chapter 4 has presented a model which the demand schedule of arbitrageurs, and hence the extent of arbitrage activity, varies as a step function in cash futures mispricing. Chapter 7 has interpreted mean reversion as being "consistent" with the existence of significant arbitrage activity. Inferences in relation to arbitrage activity are not being made directly but through the behaviour of the price series. Arbitrage activity will necessarily be accompanied by arbitrage related trading volume. Clearly, it is possible to directly test many of the issues discussed in this dissertation by examining actual arbitrage trades (as has been done by Sofianos, 1990), and if data on actual arbitrage trades is not available, the next best alternative is to analyse trading volume in cash market component stocks and in futures markets in relation to the mispricing series. This is an important avenue for future research.

(c) Volatility

Chapter 2 has shown that the volatility of futures prices exceeds the volatility of corresponding cash prices. This is consistent with the results of Edwards (1988) and Mackinlay and Ramaswamy (1988). Mackinlay and Ramaswamy (1988) argue that if arbitrageurs maintain the link between the two markets, the variances of spot and futures prices should be equal. However, as pointed out by Schwert (1990), the volatility of futures prices could be higher due to several reasons. Firstly, there could be variation in dividend yields on the index. Secondly, non-synchronous trading among index constituents results in a downward biased estimate of true cash volatility. Thirdly, lower transaction costs in futures markets can lead to situations where macroeconomic information is such that it is attractive to trade in the futures market, but not in the cash market. Finally, there can be greater speculative noise trading in futures market. Future research can undertake an in depth study of the futures cash volatility ratio and attempt to isolate the factors which cause overall futures market volatility to exceed cash market volatility.

(d) Index Arbitrage and Underlying Stocks

(i) *Expiration Day Impact on Volatility and Volume*

Stoll and Whaley (1986) in a heavily publicised exchange commissioned study documented highly significant expiration day impact on volatility and volume. Volatility of price change in the last hour was significantly higher (on the days the S&P500 futures contract expired) for stocks which were in the S&P500 index, but not for other stocks. The volume of trading in the last hour was also substantially higher than normal. The impact was much weaker on days on which only options expired but futures did not expire. This was interpreted as a manifestation of unwinding of futures related arbitrage positions just prior to futures expiration. However, the results in Chapters 2 and 6 in this dissertation show that arbitrage positions should almost never be held to expiration, and either be unwound early or rolled forward. Hence, expiration day volume-volatility effects are unlikely. In this context it is important to note that futures contract expirations in the US have been accompanied also by expirations of individual options and of index options leading to the "triple witching hour" which could potentially be caused by interaction effects. In the UK, individual stock options and futures contracts expire on different days. The effect of futures contract expiration on underlying stocks can hence be completely isolated. This isolation is important, particularly because Pope and Yadav

(1992) report significant price and volume effects associated with expiration days of options on individual stocks.

(ii) *Impact on Underlying Volatility and "Velocity"*

Index arbitrage has been very controversial in the US. Market participants have widely believed that it increases stock price volatility.[2] However, academics have viewed it as desirable in terms of enforcing the "law of one price" and have thus "tended to dismiss the recent torrent of complaints about index arbitrage and volatility as mostly hysteria" (see eg Miller, 1990, pp 187-188). There have been some US based academic studies (eg Edwards, 1988; Harris *et al*, 1990; and Damodaran, 1990[3]) which provide evidence on this issue. However, "there continues to be disagreement about the causal relationship between index arbitrage and price volatility, and more research is needed to resolve this issue" (NYSE Report on *Market Volatility and Insider Confidence*, 1990, pp 19). There are several aspects which are important. First, there is need for evidence from important non-US markets, like the UK. Second, as Miller (1991, pp 187-188) rightly points out, the critics of index arbitrage

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2 See eg NYSE report on *Market Volatility and Investor Confidence* (1990, pp 16-19) and Miller (1991, pp 228).

3 Also relevant in this context is the effect of options trading on underlying stocks. See eg Watt *et al* (1992).

and the academic defenders have used the word *volatility* in different senses. To academics, volatility is the variability of the rate of return obtained by *holding* stocks over intervals like an hour, a day, a week or a month. On the other hand, the practitioner critics of index arbitrage are concerned with "the *velocity* of prices, in the sense of the very rapid, minute-to-minute (sometimes even second-to-second) sequences of price moves" that index arbitrage programs sometimes cause. These bursts of velocity affect the *trading* rather than the *holding* of stocks and may be regarded as potentially damaging the market, irrespective of longer term variability. Third, there could be concern about "episodic volatility". For example, Schwartz (1990) argues that heavy index arbitrage activity in times of market stress may create confusion, thus contributing to panic and stock mispricing. Finally, correlation between index arbitrage activity and stock price changes does not necessarily make index arbitrage a villain. Index arbitrage could be the messenger through which new information gets incorporated into stock prices rather than the primary cause of stock price changes. Without index arbitrage prices could still change - though possibly more slowly - with the arrival of new information. Consistent with this messenger scenario is the evidence of Froot *et al* (1991) which suggests that, over the past few years, there has been an increase in the speed with which information gets incorporated into stock prices.

This dissertation has found that the mispricing variable in the UK is not qualitatively different from the mispricing variable in the US. However, as emphasized in the introduction, policy perspectives appear to have been different. In this context, an examination of the impact of index arbitrage on volatility and "velocity" is an important area for future research.

REFERENCES

Admati A R and Pfleiderer P (1988) "A Theory of Intraday Patterns: Volume and Price Variability", *Review of Financial Studies 1*, pp 3-40.

Admati, A R and P Pfleiderer (1989), "Divide and Conquer: A Theory of Intraday and Day-of-the-Week Mean Effects", *Review of Financial Studies* 2, pp 189-223.

Akaike H (1974) "A New Look at Statistical Model Identification" *IEEE Transactions on Automatic Control* AC-19, pp 716-723.

Arditti F, Ayadin S, Mattu R and Rigskee S, (1986) "A Passive Futures Strategy that Outperforms Active Management", *Financial Analysts Journal*, pp 63-66.

Ball R (1988) "What do we know about stock market efficiency?" in *A Reappraisal of the Efficiency of Financial Markets*, Guimaraes R M C, Kingsman B G and Taylor S J (eds). Springer Verlag Note A51 Series, Vol F54, pp 25-55.

Ball, R and J Bowers (1988), "Daily Seasonals in Equity and Fixed Interest Returns: Australian Evidence and Tests of Plausible Hypotheses" in "Stock Market Anomalies" edited by E Dimson Cambridge University Press, pp 74-90.

Beach C and Mackinnon J, (1978) "A Maximum Likelihood Procedure for Regression with Autocorrelated Errors", *Econometrica*, pp 51-58.

Berndt E R, Hall B H, Hall P E and Hausman J A, (1974) "Estimation and Inference in Non-linear Structural Models", *Annals of Economic and Social Measurement* 3(4), pp 653-65.

Bhatt S and Cakici N (1990) "Premiums on Stock Index Futures - Some Evidence", *Journal of Futures Markets* 10, pp 367-375.

Billingsley R S and D M Chance (1988) "The Pricing and Performance of Stock Index Futures Spreads", *Journal of Futures Markets* 8, pp 303-318.

Board, J L G and C M S Sutcliffe (1988), "The Weekend Effect in UK Stock Market Returns", *Journal of Business Finance and Accounting* 15, pp 199-213.

Bollerslev T, R Y Chou and K F Kroner (1992), "ARCH Modelling in Finance: A Review of the Theory and Empirical Evidence", *Journal of Econometrics* 52, pp 5-59.

Brennan M J and Schwartz E S (1990) "Arbitrage in Stock Index Futures", *Journal of Business* 63, pp S7-S31.

Brenner M, Subrahmanyam M G and Uno J (1989) "The Behavior of Prices in the Nikkei Spot and Futures Market", *Journal of Financial Economics* 23, pp 363-383.

Brock, W A and A W Kleidon (1989), "Exogenous Demand Shocks and Trading Volume: A Model of Intraday Bids and Asks", Working Paper Graduate School of Business, Stanford University.

Brown R L, Durbin J and Evans J M (1975) "Techniques for Testing the Constancy of Regression Relationships Over Time", *Journal of the Royal Statistical Society*, Series B, 37, pp 149-192.

Canina L and Figlewski S (1991) "The Information Content of Implied Volatility", New York University Stern School of Business Working Paper.

Carlton D W (1984) "Futures Markets: Their Purpose, Their History, Their Growth, Their Successes and Failures", *Journal of Futures Markets* 4, pp 237-271.

Chan K (1991) "A Further Analysis of the Lead-lag Relationship between the Cash Market and Stock Index Futures Market", *Review of Financial Studies* 4, pp 123-152.

Chan K S and Tong H (1986) "On Tests for Non-linearity in Time Series Analysis", *Journal of Forecasting* 5, pp 217-228.

Chan K S and Tong H (1988) "On Likelihood Ratio Tests for Threshold Autoregression", Technical Report, Institute of Mathematics, University of Kent.

Chan K, K C Chan and G A Karolyi (1991) "Intraday Volatility in the Stock Index and Stock Index Futures Markets", *Review of Financial Studies* 4, pp 657-684.

Cheung Y and L Ng (1990) "The Dynamics of S&P500 Index and S&P500 Futures Intraday Price Volatilities", Working Paper, University of California at Santa Cruz.

Choy, A Y F and J O'Hanlon (1989), "Day of the Week Effects in the UK Equity Market: A Cross Sectional Analysis", *Journal of Business Finance and Accounting* 16, pp 89-104.

Chung Y P (1991) "A Transactions Data Test of Stock Index Futures Market Efficiency and Index Arbitrage Profitability", *Journal of Finance* 46, pp 1791-1809.

Cohen K J, Maier S R, Schwartz R A and Whitcomb D K, (1986) "The Microstructure of Securities Markets", Prentice Hall.

Condoyani, L, J O'Hanlon and C W Ward (1987), "Day of the Week Effects on Stock Returns: The International Evidence", *Journal of Business Finance and Accounting* 14, pp 159-174.

Cooper I A and Mello A S (1990) "Futures/Cash Arbitrage with Early Unwinding Opportunities and Inelastic Liquidity Demand", MIT/LSE Working Paper.

Cornell B, (1985a) "Taxes and the pricing of stock index futures", *Journal of Futures Markets* 5, pp 89-101.

Cornell, B, (1985b) "The Weekly Pattern in Stock Returns: Cash versus Futures: A Note", *Journal of Finance* 40, pp 583-588.

Cornell B and French K R, (1983a) "Taxes and the pricing of Stock Index Futures", *Journal of Finance* 38, pp 675-694.

Cornell B and French K R, (1983b) "The Pricing of Stock Index Futures", *Journal of Futures Markets*, pp 1-14.

Cox J C, Ingersoll J E Jr and Ross S A, (1981) "The Relation between Forward Prices and Futures Prices", *Journal of Financial Economics* 9, pp 321-346.

Cross, F, (1973) "The Behaviour of Stock Prices on Fridays and Mondays", *Financial Analysts Journal* 29 (1973), pp 67-69.

Crouhey, M, D Galai and B Keita, (1990) "The Settlement Day Effect in the French Bourse", French Finance Association (AFFI) Conference, Joey-en-Josas (June)

Damodaran A (1990) "Index Futures and Stock Market Volatility", Working Paper, Stern School of Business, New York University.

Daniel W W (1978) *Applied Nonparametric Statistics*, Houghton Miffin Company.

Day T E and Lewis C M (1992) "Stock Market Volatility and the Information Content of Stock Index Options", *Journal of Econometrics* 52, pp 267-287.

Dickey D A and Fuller W A (1979) "Distribution of Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association* 74, June, pp 427-31.

Diebold F X and Nerlove M (1990) "Unit Roots in Economic Time Series: A Selective Survey", *Advances in Econometrics* 8, pp 3-69.

Durlauf S N and Phillips P C B (1988) "Trends versus Random Walks in Time Series Analysis", *Econometrica* 56, pp 1033-1044.

Dyl, E A and S A Martin, Jr, (1985) "Weekend Effects on Stock Returns: A Comment", *Journal of Finance* 40, pp 347-349.

Dyl, E A and E D Maberly, (1986a) "The Daily Distribution of Changes in the Price of Stock Index Futures", *Journal of Futures Markets* 6, pp 513-521.

Dyl, E A and E D Maberly, (1986b) "The Weekly Pattern in Stock Index Futures: A Further Note", *Journal of Finance* 41, pp 1149-52.

Edwards F R (1988) "Does Futures Trading Increase Cash Market Volatility?", *Financial Analysts Journal* 44, pp 63-69.

Engle E G and Granger C W J (1987) "Cointegration and Error Correction: Representation, Estimation and Testing", *Econometrica* 55, pp 251-276.

Ertel J E and Fowlkes E B (1976) "Some Algorithms for Linear Spline and Piecewise Multiple Linear Regression", *Journal of the American Statistical Association* 71, pp 640-648.

Eytan T and Harpaz G (1986) "The Pricing of Futures and Option Contracts on the Value Line Index", *Journal of Finance* 41, pp 843-857.

Fama E F (1970) "Efficient Capital Markets: A review of theory and empirical work", *Journal of Finance* 25, pp 383-417.

Fama E F (1976) *Foundations of Finance*, Basic Books, New York.

Fama E F (1988) "What did we learn from the Crash?", Centre for Research in Security Prices, University of Chicago Working Paper #122.

Fields, M J (1931), "Stock Prices: A Problem in Verification", *Journal of Business* 4, pp 169-177.

Figlewski S, (1984a) "Hedging performance and basis risk in Stock Index Futures", *Journal of Finance*, Vol 39, pp 657-669.

Figlewski S, (1984b) "Explaining the early discounts on Stock Index Futures: The case for disequilibrium", *Financial Analysis Journal*, 40, pp 43-47.

Finnerty J E and Park H Y, (1988) "How to profit from program trading", *Journal of Portfolio Management*, pp 41-46.

Flannery, M J and A A Protopapadakis, (1988) "From T-Bills to Common Stocks: Investigating the Generality of Intraweek Return Seasonality", *Journal of Finance* 43, pp 431-450.

Foster F D and S Vishwanathan (1990) "A Theory of the Interday Variations in Volume, Variance and Trading Costs in Securities Markets", *Review of Financial Studies* 3, pp 593-624.

French, K, "Stock Returns and the Weekend Effect", (1980), *Journal of Financial Economics* 8, pp 55-69.

Froot K A, J F Gammill Jr and A F Perold (1991) "New Trading Practices and the Short Run Predictability of the S&P500", NBER Working Paper.

Fuller W (1976) *Introduction to Statistical Time Series*, New York: John Wiley & Sons.

Garbade K D and Silber W L, (1983), "Price movements and Price discovery in Futures and Cash Markets", *The Review of Economics and Statistics* 65, pp 289-297.

Gibbons, M and P Hess, (1981) "Day of the Week Effects and Asset Returns", *Journal of Business* 54, pp 579-596.

Gould F J, (1988) "Stock Index Futures: The Arbitrage Cycle and Portfolio Insurance", *Financial Analysis Journal*, 44, pp 44-54.

Hackl P (1980) "Testing the Constancy of Regression Models over time", Göttingen: Vanderhoeck and Ruprecht.

Harris L (1986) "A Transaction's Data Study of Weekly and Intradaily Patterns in Stock Returns", *Journal of Financial Economics* 16, pp 99-117.

Harris L, G Solianos and J E Shapiro (1990) "Program Trading and Intraday Volatility", New York Stock Exchange Working Paper #90-03.

Hemler M L and Longstaff F A (1991) "General Equilibrium Stock Index Futures Prices: Theory and Empirical Evidence", *Journal of Financial and Quantitative Analysis* 26, pp 287-308.

Hill J M, Jain A and Ward R A (1988) "Insurance: Volatility Risk and Futures Mispricing", *Journal of Portfolio Management*, Winter, pp 23-29.

Holden C W (1990a) "A Theory of Arbitrage Trading in Financial Market Equilibrium", UCLA Graduate School of Management Working Paper.

Holden C W (1990b) "Intertemporal Arbitrage Trading: Persistence, Empirical Implications, Cointegration and Commission Costs", UCLA Graduate School of Management Working Paper.

Holden C W (1990c) "An Empirical Test of the Intertemporal Arbitrage Trading Model", Draft PhD Dissertation, Chapter 4 (draft dated September 14, 1990).

Hsieh D (1991) "Chaos and Nonlinear Dynamics: Application to Financial Markets", *Journal of Finance* 46, pp 1839-1877.

Jaffe, J and R Westerfield (1985), "The Weekend Effect in Common Stock Returns: The International Evidence", *Journal of Finance* 40, pp 433-454.

Jaffee D M (1984) "The Impact of Financial Futures and Options on Capital Formation", *Journal of Futures Markets* 4, pp 417-447.

Jain, P C and G H Joh (1988), "The Dependence between Hourly Prices and Trading Volume", *Journal of Financial and Quantitative Analysis* 23, pp 269-283.

Jensen M C (1978) "Some Anomalous Evidence Regarding Market Efficiency", *Journal of Financial Economics* 6, pp 95-101.

Johnsen O (1991) "The Impact of Economic News on Volatility Implied in Option Prices", Honours Dissertation, University of Strathclyde.

Johnston J (1984), Econometric Methods Third Edition, McGraw Hill.

Jonckheere A R (1954) "A Distribution Free k-sample Test Against Ordered Alternatives", *Biometrika* 41, pp 133-145.

Junkus, J C (1986), "Weekend and Day-of-the-week Effects in Returns on Stock Index Futures", *Journal of Futures Markets* 6, pp 397-407.

Kawaller I G, P D Koch and T W Koch (1987) "The Temporal Price Relationship between the S&P500 Futures and the S&P500 Index", *Journal of Finance* 42, pp 1309-1329.

Kawaller I G, P D Koch and T W Koch (1990) "Intraday Relationships between Volatility in S&P500 Futures Prices and Volatility in the S&P500 Index", *Journal of Banking and Finance* 14, pp 373-398.

Keim, D and R Stambaugh (1984), "A Further Investigation of the Weekend Effect in Stock Returns", *Journal of Finance* 39, pp 819-834.

Keim, D B and M Smirlock (1987), "The Behaviour of Intraday Stock Index Futures Prices", *Advances in Futures and Options Research* 2, pp 143-166.

Kipnis G and Tsang S, (1984) "Significance of Errors in Estimating Dividends", *Stock Market Strategist*, Donaldson, Lutken and Jennette.

Klemkosky R C and Lee J H (1991) "The Intraday *ex post* and *ex ante* Profitability of Index Arbitrage", *Journal of Futures Markets* 11, pp 291-311.

Lakonishok, J and M Levi (1982), "Weekend Effects on Stock Returns: A Note", *Journal of Finance* 37, pp 883-889.

Lakonishok, J and M Levi (1985), "Weekend Effects on Stock Returns: A Reply", *Journal of Finance* 40, pp 351-352.

Lakonishok, J and S Smidt (1988), "Are Seasonal Anomalies Real? A Ninety Year Perspective", *Review of Financial Studies* 1, pp 403-425.

Lo N N (1988) "An Econometric Analysis of the Role of Price Discovery in Futures Markets", University of Pennsylvania PhD Dissertation.

Lo A and MacKinlay C (1990) "An Econometric Analysis of Non-Synchronous Trading", *Journal of Econometrics* 45, pp 181-211.

McInish T H and Wood R A (1990) "A Transactions Data Analysis of the Variability of Common Stock Returns During 1980-1984", *Journal of Banking and Finance* 14, pp 99-112.

Maberly, E D (1986), "The Informational Content of the Interday Price Change with respect to Stock Index Futures", *Journal of Futures Markets* 6, pp 385-395.

Maberly, E D (1987), "An Analysis of Trading and Non-trading Period Returns for the Value Line Composite Index Spot versus Futures: A Note", *Journal of Futures Markets* 7, pp 497-500.

Maberly, E D, R W Spahr and A F Herbst (1989), "An Analysis of Daily Patterns in Stock Returns across Indices: Spot versus Futures", *Quarterly Journal of Business and Economics*, pp 55-67.

Mackinlay C and Ramasway K (1988) "Program Trading and the behaviour of Stock Index Futures Prices", *Review of Financial Studies* 1, pp 137-158.

Merrick J J Jr (1987) "Price Discovery in the Stock Market", Solomon Brothers Centre for the Study of Financial Institutions Working Paper Series #416.

Merrick J J Jr (1988) "Hedging with Mispriced Futures", *Journal of Financial and Quantitative Analysis* 23, pp 451-464.

Merrick J J Jr (1989) "Early Unwindings and Rollovers of Stock Index Futures Arbitrage Programs: Analysis and Implications for Predicting Expiration Day Effects", *Journal of Futures Markets* 9, pp 101-110.

Miller M H (1991) *Financial Innovations and Market Volatility*, Blackwell.

Miller M H, Muthuswamy J and Whaley R E (1991) "Predictability of S&P500 Index Basis Changes: Arbitrage Induced or Statistical Illusion?", Center for Research in Security Prices, Graduate School of Business, University of Chicago Working Paper #331, Version June 26, 1991.

Modest D, (1984) "On the pricing of stock index futures", *Journal of Portfolio Management*, pp 51-57.

Modest M and Sunderashen M (1984) "The Relationship between Spot and Futures in Stock Index Futures: Some Preliminary Evidence", *Journal of Futures Markets*, pp 15-41.

Mood A M (1954) "On the Asymptotic Efficiency of Certain Nonparametric Two-Sample Tests", *Annals of Mathematical Statistics* 25, pp 514-522.

Parkinson M, (1980) "The extreme value method for estimating the variance of the rate of return", *Journal of Business* 53, pp 61-65.

Patell J M and M A Wolfson, (1982) "Good news, Bad news, and the Interday timing of Corporate Disclosures", *Accounting Review* 57, pp 509-527.

Peel D A, Pope P F and Yadav P K, (1989) "Deregulation and UK Stock Market Volatility", *Journal of Business Finance and Accounting* (forthcoming).

Perron P (1989) "The Great Crash, The Oil Price Shock, and the Unit Root Hypothesis", *Econometrica* 57, pp 1361-1401.

Perron P and Phillips P C B (1987) "Does GNP have a Unit Root? A Re-evaluation", *Economics Letters* 23, pp 139-145.

Petruccelli J D and Davies N (1986) "A Portmanteau Test for Self Exciting Threshold Autoregression Type Non-linearity in Time Series", *Biometrika* 73, pp 687-694.

Phillips P C B (1987) "Time Series Regression with a Unit Root", *Econometrica* 55, pp 277-301.

Phillips P C B (1988) "Regression Theory for Near-Integrated Time Series", *Econometrica* 56, pp 1021-1044.

Phillips P C B and Perron P (1988) "Testing for a Unit Root in Time Series Regression", *Biometrika* 75, pp 335-346.

Phillips-Patrick, F J and T Schneeweis (1988), "The Weekend Effect for Stock Indexes and Stock Index Futures: Dividend and Interest Rate Effects", *Journal of Futures Markets* 8, pp 115-121.

Pope P F and Yadav P K (1991)[1] "Transaction Costs and Threshold Non-linearity in Futures Mispricing", *Western Finance Association* 1991 Meeting Conference Paper.

Pope P F and Yadav P K (1992) "The Impact of Option Expiration on Underlying Stocks: The UK Evidence", *Journal of Business Finance and Accounting* 19, pp 329-344.

Puttonen V and Martikainen T (1991) "Short Sales Restrictions: Implications for Stock Index Arbitrage", *Economics Letters* 37, pp 159-163.

Rogalski, R, (1984) "New Findings Regarding Day-of-the-week Returns over Trading and Non-trading Periods, *Journal of Finance* 39, pp 1603-1614.

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[1] This is based on Chapter 4 in this dissertation.

Ross S A (1989) "Information and Volatility: The No-Arbitrage Martingale Approach to Timing and Resolution Irrelevancy", *Journal of Finance* 44, pp 1-17.

Rubenstein M (1987) "Derivative Assets Analysis", UCLA Working Paper.

Rubenstein M (1989) "Market Basket Alternatives", *Financial Analysts Journal* 45, pp 20-29.

Samuelson P A (1965) "Proof that Properly Anticipated Prices Fluctuate Randomly", *Industrial Management Review* 6, pp 41-49.

Saunders E M and Mahajan A, (1988) "An Empirical Examination of Composite Stock Index Futures Pricing", *Journal of Futures Markets* 8, pp 211-228.

Schleifer A and L H Summers (1990) "The Noise Trader Approach to Finance", *Journal of Economic Perspectives* 4, pp 19-33.

Scholes M and Williams J (1977) "Estimating Betas from Non-Synchronous Data", *Journal of Financial Economics* 5, pp 309-328.

Schmidt P (1990) "Dickey Fuller Tests with Drift", *Advances in Econometrics* 8, pp 161-200.

Schwarz G (1978) "Estimating the Dimension of a Model", *Annals of Statistics* 6, pp 461-464.

Schwartz R A (1990) "Price Discovery, Instability and Market Structure", in NYSE Report on Market Volatility and Investor Confidence.

Schwert G W (1989) "Tests for Unit Roots: A Monte Carlo Investigation", *Journal of Business and Economic Statistics* 7, pp 147-160.

Schwert G W (1990) "Stock Volatility and the Crash of '87", *Review of Financial Studies*, 3, pp 77-102.

Sofianos G (1990) "Index Arbitrage Profitability", NYSE Working Paper #90-04 (draft dated November 27, 1990).

Smirlock, M and L Starks (1986), "Day of the Week and Intraday Effects in Stock Returns", *Journal of Financial Economics* 17, pp 197-210.

Solnik, B (1990), "The Distribution of Daily Stock Returns and Settlement Procedures : The Paris Bourse", *Journal of Finance* 45, pp 1601-1609.

Solnik, B and L Bousquet (1990), "Day of the Week Effect on the Paris Bourse", *Journal of Banking and Finance* 14, pp 461-468.

Stock J H and Watson M W (1988) "Testing for Common Trends", *Journal of the American Statistical Association* 83, pp 1097-1107.

Stoll H R (1989) "Inferring the Components of the Bid-Ask Spread: Theory and Empirical Tests", *Journal of Finance* 44, pp 115-134.

Stoll H R and R E Whaley (1986) "Expiration Day Effects of Index Options and Futures", Monograph Series in Finance and Economics, Salomon Brothers Center for the Study of Financial Institutions, New York University.

Stoll H R and Whaley R E (1987) "Program Trading and Expiration Day Effects", *Financial Analysts' Journal* March, pp 16-27.

Stoll H R and R E Whaley (1988) "Futures and Options on Stock Indexes: Economic Purpose, Arbitrage and Market Structure", *Review of Futures Markets* 7, pp 224-248.

Subrahmanyam A (1991) "A Theory of Trading in Stock Index Futures", *Review of Financial Studies* 4, pp 17-51.

Teber L G (1986) "Futures and Actual Markets: How they are Related", *Journal of Business* 59, pp 5-21.

Terpstra T J (1952) "The Asymptotic Normality and Consistency of Kendall's Test Against Trend, When Ties are Present in One Ranking", *Indag Math* 14, pp 327-333.

Thaler, R (1987), "Seasonal Movements in Security Prices II: Weekend, Holiday, Turn of the Month and Intraday Effects", *Journal of Economic Perspectives* 1, pp 169-177.

Theobald, M and V Price (1984), "Seasonality Estimation in Thin Markets", *Journal of Finance* 39, pp 377-392.

Tong H (1978) "On the Threshold Model" in C H Chen (ed) "Pattern Recognition and Signal Processing" (Amsterdam, Sijthoff and Noordhoff).

Tong H (1983) "Threshold Models in Non-linear Time Series Analysis", Lecture Notes in Statistics No 21, (Heidelberg, Springer Verlag).

Tong H and Lim K S (1980) "Threshold Autoregression, Limit Cycles, and Cyclical Data", *Journal of the Royal Statistical Society*, Series B, 42, pp 245-292.

Tsay R S (1989) "Testing and Modeling Threshold Autoregressive Processes", *Journal of the American Statistical Association* 84, pp 231-240.

Watt W H, P K Yadav and P Draper (1992) "The Impact of Option Listing on Underlying Stocks: The UK Evidence", *Journal of Business Finance and Accounting* (forthcoming June 1992 issue).

Westland A H and Törnkvist B (1989) "On the Identification of Time for Structural Changes by MOSUM-SQ and CUSUM-SQ Procedures" in Hackl P (ed) *Statistical Analysis and Forecasting of Economic Structural Change*, Springer-Verlag.

Wood, T A, T J McInish and J K Ord (1985), "An Investigation of Transactions Data for NYSE Stocks", *Journal of Finance* 40, pp 723-741.

Yadav P K and Pope P F (1990)[2] "Stock Index Futures Pricing: International Evidence", *Journal of Futures Markets* 10, pp 573-603.

Yadav P K and Pope P F (1991)[3] "Testing Index Futures Market Efficiency using Price Differences: A Critical Analysis", *Journal of Futures Markets*, 11, pp 239-252.

Yadav P K and Pope P F (1992a)[4] "Intraweek and Intraday Seasonalities in Stock Market Risk Premia: Cash vs Futures", *Journal of Banking and Finance* 16, pp 233-270.

Yadav P K and Pope P F (1992b)[5] "Mean Reversion in Stock Index Futures Mispricing: Evidence from the US and the UK", University of Strathclyde Working Paper. To be presented to the *French Finance Association* Conference June 1992 and the *American Finance Association* Conference January 1993.

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2 This is based on Chapter 2 in this dissertation.

3 This is based on Chapter 3 in this dissertation.

4 This is based on Chapter 5 in this dissertation.

5 This is based on Chapter 7 in this dissertation.

Yadav P K and Pope P F (1992c)[6] "Stock Index Futures Mispricing: Profit Opportunities or Risk Premia?", University of Strathclyde Working Paper. To be presented to the *European Finance Association* Conference August 1992.

[6] This is based on Chapter 6 in this dissertation.