Application of Cournot Equilibrium for Oligopoly Electricity Market Production

Thesis Submitted for the Degree of

Master of Philosophy

by

Xiao Yang

Supervisors: Prof. K. L. Lo

Dr. Stuart Galloway

Department of Electronic and Electrical Engineering

University of Strathclyde, Glasgow

Scotland, United Kingdom

September 2009

DECLARATION

This thesis is the result of the author's original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.

ACKNOWLEDGEMENT

I would like to thank Professor K. L. Lo and Doctor Stuart Galloway for their supervision and valuable guidance during my MPhil study. I also gratefully acknowledge them for their every single help and great patience.

I would also like to give immense gratitude to my parents and deeply appreciate their support and encouragement during my MPhil study.

ABSTRACT

The electrical power industry in many countries has been undergoing significant changes since the process of deregulation was firstly implemented in the early 1990s. Traditionally, this industry was structured as a single vertically integrated company, which was exclusively responsible for scheduling and operation of power generation, transmission and distribution. Under deregulation there has been a shift from an industry with monopolistic features towards one that has moved closer to one of perfect competition. This trend within the power industry has been observed around the world (e.g. USA, Australia, UK, and Scandinavia) [1]. However, at the beginning of deregulation procedure, an electricity supply structure is often observed to be more akin to an oligopoly market than one that is perfectly competitive. For instance, at the introduction of the England and Wales Pool market in 1990 the power industry was actually a duopoly in the early period of deregulation [2]. A method, based on work by F. S. Wen and A. K. David [11], for prediction of the optimal electricity production of a for-profit power producer in such an oligopoly electricity market is presented and tested in this thesis against the back drop of deregulated electricity markets. Furthermore, this work is extended for a more realistic and complicated situation when transmission is considered. Within this method, Cournot competition game theory [10] is chosen to determine the oligopoly electricity market equilibrium state with respect to the optimal supply quantity of each power producer. Both complete and incomplete information scenarios are considered, and moreover, transmission charges and losses are also taken into account in the extended model to approach more realistic expectations.

Keywords: Electricity market, Cournot competition, market equilibrium

TABLE OF CONTENTS

DECLARATION i
ACKNOWLEDGEMENT ii
ABSTRACT iii
TABLE OF CONTENTS iv

CHAPTER 1 1
INTRODUCTION 1
1.1 Background 1
1.2 Objectives of the Thesis 5
1.3 Overview of the Thesis 6

CHAPTER 2 8
ELECTRICITY MARKET FUNDAMENTALS 8
2.1 Introduction 8
2.2 Deregulation of Electricity Market 9
2.2.1 Why deregulate? 9
2.2.1.1 Evolvement of electricity market 9
2.2.1.2 Problems with regulation 10
2.2.1.3 Problems with deregulating electricity market 11
2.2.2 What to deregulate? 12
2.2.2.1 Ancillary services and role of system operator 13
2.2.2.2 Unit commitment and congestion management 15
2.2.2.3 Transmission and distribution 16
2.3 Power Supply and Demand 16
2.3.1 Load-duration curve 16
2.3.2 Marginal cost and curves 18
2.4 Competition in Electricity Market 21
2.4.1 Benefits of perfect competition 22
2.4.2 Market equilibrium 24
2.4.3 Four Models of Competition 25
2.4.3.1 Structure 1: Monopoly 26
2.4.3.2 Structure 2: Purchasing agency 26
2.4.3.3 Structure 3: Wholesale competition 27
2.4.3.4 Structure 4: Retail competition 28
2.5 Summary 29

CHAPTER 3 32
GAME THEORY FUNDAMENTALS 32
3.1 Introduction 32
3.2 Static Game Theory 33

3.2.1 Illustration of normal form games 34
3.2.2 Illustration of extensive form games 35
3.2.3 Solution techniques for static games 36
3.2.3.1 Dominance technique 36
3.2.3.2 Equilibrium technique 40
3.3 Dynamic Game Theory 42
3.3.1 Nash equilibrium of one-off dynamic games 42
3.3.2 Subgame perfect Nash equilibrium of one-off dynamic games 44
3.3.3 Back induction of one-off dynamic games 45
3.4 Summary 47

CHAPTER 4 48
METHODOLOGY AND MATHEMATICAL MODEL OF COURNOT COMPETITION THEORY FOR OLIGOPOLY ELECTRICITY MARKET PRODUCTION 48
4.1 Introduction 48
4.2 Cournot Competition Theory in Oligopoly Markets 49
4.3 The Model to Be Investigated of an Oligopoly Electricity Market 53
4.3.1 Application of Cournot competition on an oligopoly electricity market .. 54
4.3.2 Oligopoly electricity market model to be investigated and its Cournot equilibrium under complete information 56
4.4 Cournot Equilibrium Solutions under Complete Information and Incomplete Information 58
4.4.1 Cournot equilibrium state under complete information 58
4.4.2 Cournot equilibrium state under incomplete information 59
4.4.2.1 Mathematical model for Case 1 59
4.4.2.2 Mathematical model for Case 2 61
4.4.2.3 Mathematical model for Case 3 63
4.5 Summary 65

CHAPTER 5 66
NUMERICAL EXAMPLE AND RESULTS 66
5.1 Introduction 66
5.2 Numerical Example and Results of Base Case 66
5.2.1 Numerical model for Base Case 66
5.2.2 Results of Base Case under complete information 69
5.3 Results of Case 1 under Incomplete Information 71
5.3.1 Numerical model for Case 1 71
5.3.2 Results of Case 1 under incomplete information 72
5.4 Results of Case 2 under Incomplete Information 74
5.4.1 Numerical model for Case 2 74
5.4.2 Results of Case 2 under incomplete information 75
5.5 Results of Case 3 under Incomplete Information 77
5.5.1 Numerical model for Case 3 77
5.5.2 Results of Case 3 under incomplete information 78

5.6 Summary 81

CHAPTER 6 83
EXTENSION OF THE MODEL AND TESTING RESULTS 83
6.1 Introduction 83
6.2 Numerical Model with Consideration of Wheeling Charges and Testing Results 83
6.2.1 Numerical example and results under complete information 84
6.2.1.1 Numerical model under complete information.................................84
6.2.1.2 Results of the base case under complete information.......................87
6.2.2 Numerical model and Case-study 1 under Incomplete Information.........88
6.2.2.1 Numerical model for Case 1...89
6.2.2.2 Results of Case 1 under incomplete information.............................90
6.2.3 Numerical model and Case-study 2 under Incomplete Information.........91
6.2.3.1 Numerical model for Case 2...91
6.2.3.2 Results of Case 2 under incomplete information.............................92
6.2.4 Numerical model and Case-study 3 under Incomplete Information.........93
6.2.4.1 Numerical model for Case 3...93
6.2.4.2 Results of Case 3 under incomplete information.............................94
6.3 Numerical Model with Consideration of Transmission Losses and Testing Results 95
6.3.1 Numerical example and results under complete information 96
6.3.1.1 Numerical model under complete information.................................96
6.3.1.2 Results of Base Case under complete information 97
6.3.2 Numerical model and Case-study 1 under incomplete information 98
6.3.2.1 Numerical model for Case 1...99
6.3.2.2 Results of Case 1 under incomplete information.............................99
6.3.3 Numerical model and Case-study 2 under incomplete information 101
6.3.3.1 Numerical model for Case 2...101
6.3.3.2 Results of Case 2 under incomplete information...........................101
6.3.4 Numerical model and Case-study 3 under incomplete information 103
6.3.4.1 Numerical model for Case 3...103
6.3.4.2 Results of Case 3 under incomplete information...........................103
6.4 Numerical Model with Consideration of Wheeling Charges and Transmission Losses and Testing Results...105
6.4.1 Numerical example and results under complete information 105
6.4.1.1 Numerical model under complete information...............................105
6.4.1.2 Results of Base Case under complete information 106
6.4.2 Numerical model and Case-study 1 under incomplete information 108
6.4.2.1 Numerical model for Case 1...108
6.4.2.2 Results of Case 1 under incomplete information...........................109
6.4.3 Numerical model and Case-study 2 under incomplete information 110
6.4.3.1 Numerical model for Case 2...110
6.4.3.2 Results of Case 2 under incomplete information........................... 111

6.4.4 Numerical model and Case-study 3 under incomplete information 112
6.4.4.1 Numerical model for Case 3..112
6.4.4.2 Results of Case 3 under incomplete information............................113
6.5 Summary 114

CHAPTER 7 117
CONCLUSIONS AND FUTURE WORK...117
7.1 Conclusions 117
7.2 Future Work...120

APPENDIX A: Testing Interface of Base Case...122
APPENDIX B: Testing Interface of Case 1..123
APPENDIX C: Testing Interface of Case 2 124
APPENDIX D: Testing Interface of Case 3 125
REFERENCES..126

CHAPTER 1

INTRODUCTION

The purpose of this thesis is to review a method related to game theory and apply it to optimal electricity production of each producer in an oligopoly market whilst taking account of network and related features.

1.1 Background

Electrical power industry in many countries has been deregulated with a view to increasing efficiency and cost reduction in power systems, i.e. generation, transmission, and distribution. With the process of deregulation, the electrical power industry is separated into several independent companies other than a single vertically integrated one as it used to be, which makes possible competition between companies with similar services [2]. Competition is considered as a fundamental factor that increasing the efficiency of electricity production, lowering prices but with higher quality and providing secure and reliable services as well. Simply speaking, these benefits from a competitive market can be gained through a successful process of deregulation [2].

Before going further, an argument about origin of competition in electricity power market is supposed to be clear for a better understanding. It is often misunderstood that the introduction of competition is caused by development of power generation technology but the truth is that since power transmission technology developed, possibilities for trade and competition came up [2]. Due to the emergence of extremely high voltage technology, bulk power product was able to be transferred over long distances with very little energy losses. Moreover, with development of transmission network, it made trade of electricity possible among various connected areas while power system was still a vertically integration. By

1990, due to inefficiency of regulated power systems, a general trend was encouraged towards a deregulated and competitive market which has been carried out in a number of countries [1] [3]. Nowadays, deregulation of electricity power industries has been implemented around the world, however, most of transitions were not going well at the very beginning, e.g. Power Pool in the UK [4] in which some success was achieved, such as reduction of capital costs and growth of plant efficiency, however it remained several fundamental problems, the major one is high wholesale prices. Nevertheless, initial problems do not say deregulation of electricity power market is doomed to fail. Actually, some of the improved markets are functioning well, NETA, New Electricity Trading Arrangements (and later called BETTA, British Electricity Transmission and Trading Arrangements) in the UK for example.

Development of transmission system, rather than generation technology, brings breakthrough to removal of natural monopoly feature of traditional electricity power market as interconnections between formerly monopolistic areas make trading between those areas possible. In theory, a deregulated competitive market has been proved to be more efficient than the old regulated one in abundant literatures [5] [6] [7]. In practice, California example showed improper deregulation which is even worse than formerly traditional regulation, and meanwhile, England example presented that ordinary deregulation gains more efficiency at cost-saving than a poorly regulated monopoly [8]. This is to say, successful operation on deregulating electricity power market is not achieved easily with just wishes and aspirations. Learning from these examples, a very careful design and pragmatic simulations need to be considered seriously before its implementation in the real world.

In a competitive electricity market, like any other product ones, every supplier intends to increase the market price for higher profits, in the meantime, every buyer in the market prefers cheap product but with good quality, which keeps the price at a acceptable level. In the theory of perfect competition, both desires are suppressed and it finally reaches at a state of market equilibrium. Theoretically, perfect competition exists in a market that is made up of a number of competitors selling homogeneous product to a number of customers, any of which has only a small market share. If there are suppliers which are large enough to have an impact on market prices in their own benefits, it is said that they are not price takers and market power exists [9]. The markets are also considered as oligopoly markets which are dominated by several big suppliers. It is common to see especially at the early stages of deregulation in the context of electricity sector. Due to many high barriers, such as long period constructing a new power plant, huge amounts of capital investment in entering electricity market, physical disconnection between consumers and suppliers caused by transmission constraints, at initial stages of deregulation, electricity markets are more like oligopoly ones rather than perfectly competitive ones [10]. Accordingly, in this thesis such oligopoly electricity markets are investigated in which a number of suppliers compete with each other to produce more for higher profits.

Optimal electricity production for each supplier in the oligopoly market is the main researching target in this thesis, which is often associated with decision making strategies for players involved. Non-cooperative game theory is the theoretical foundation for the analysed technique to predict how much electricity each player supplies to the market, i.e., optimal electricity production. A basic assumption about game theory is that players in the game are considered as being act rationally and game theory is always concerned with how they make decisions when they are mutually interdependent [11]. As discussed in the previous paragraphs, an oligopoly market is described as the situation that several big companies dominate a particular market which is to say they have market power. A notable feature of an oligopoly market is that these competitive big companies act interdependently which is greatly applicable to be analyzed by game theory. Interdependence is an important factor to determine whether Game Theory is applicable in a particular market structure. In the case of an oligopoly, the behavior of one company will have an effect on the profits received by other companies in the same market environment which is to say that interdependence exists. There are two other extreme examples of market structure comparing with oligopoly. In a perfect competitive market every single company is a price taker which means they are independent of each other. To the other extreme, in a monopolistic market there is only one company dominating the market, so no interdependence exist like in the perfect competitive one.

There are three classic game theory related models applying to oligopoly market namely Cournot, Stackelberg, and Bertrand competition [11]. They are all employed to analyze and predict behaviors of players in an oligopoly market which are distinguished by their underlying functions. In Cournot competition theory, quantity supplied to the market is the main object to be investigated in which companies in the market make their decisions simultaneously to determine how much they produce. In Stackelberg competition theory, one or some companies can initially decide their output level and then the rest of companies determine their own output level simultaneously based on this observation. Lastly, in Bertrand competition, similar to Cournot competition, but analytical object of quantity is replaced by price. Cournot competition model is adopted in this thesis to help suppliers decide their output level and then an optimal production for each supplier is reached and finally as is market equilibrium.

A Cournot model based on work by F. S. Wen and A. K. David [12] is presented and tested in the thesis. This model applies to prediction of optimal production in an oligopoly electricity market structure. Abundant materials about analysis of an oligopoly electricity market have been done, however most of them [13]-[16] only apply to the situation under complete information which means complete information about their rivals is known by each player involved. Apparently, it is not realistic for a deregulated market at initial stages, an oligopoly market for example. Information about the rivals is not completely known by each player, which is a major issue to be considered. It is dealt with by introducing estimated functions about the rivals to give an outline to each player, however, along with uncertainties. Cournot competition game theory as a commonly used approach for analysis of optimal production in oligopoly markets is used to determine market equilibrium under both complete information and incomplete information situations. In the model, only cost functions of generators are taken into account and transmission constraints are neglected. In this thesis, apart from uncertainties on production costs of rivals, the model is extended to the situation when transmission constraints i.e. transmission losses and wheeling charges are included which allows the model to reflect further aspects of the real world problem. Improved Cournot solutions for extended scenarios are presented and test results are given in detail.

1.2 Objectives of the Thesis

The major objectives of this thesis are summarised as follows:

a) The method, based on the work of F. S. Wen and A. K. David [12], to predict the optimal supply quantity for each producer in the same oligopoly market is reviewed in this thesis.

b) Based on the method, mathematical models under complete and incomplete information are investigated and shown in the thesis. Under incomplete information, three cases are presented, i.e., prediction based on an estimated cost function about rivals for each producer, prediction based on several estimates about rivals for each producer with probabilities, and prediction based on one estimated cost function with a distribution element about rivals for each producer.

c) The above models are simulated and tested in a repeatable environment with results derived through running separate models. The results are analysed and compared to demonstrate validity of the method.

d) At the beginning of modeling, transmission constraints are ignored for simplicity. In the extended model, these factors around transmission such as wheeling charges and losses are taken into account to capture further important aspects of the real world problem.

e) Lastly, some recommendations are given in the last chapter for the purpose of completing the model to achieve more realistic goals.

1.3 Overview of the Thesis

In Chapter 2 the basics about electricity market is reviewed to help readers to be familiar with the outline of a deregulated electricity market and necessary power system economics knowledge. It includes why and how a traditional electricity market needs to be deregulated towards a competitive one, power supply and demand in a competitive electricity market, and the way players in the market compete with each other as well as benefits received through competition is presented. The knowledge related to electricity market structure and architecture is reviewed as well for preparation of establishing mathematical models in the later chapters.

In Chapter 3, game theory fundamentals are given to provide understanding of these techniques for use in following chapters. Game theory is simply divided into two categories i.e. static game theory and dynamic game theory. Both are discussed in Chapter 3. In between, it is focused on static game theory and several solutions to static games are reviewed.

In Chapter 4, Cournot competition model is discussed in detail and used as optimal production solution to each supplier in an oligopoly market. A numerical model of an oligopoly electricity market is presented including cost functions of generators and market demand function in which simulation of proposed method is undertaken. Preparation of mathematical models and Cournot solutions is demonstrated while test results under complete and incomplete information are given in Chapter 5 as well as comments.

The initial model is extended to a wide range of new examples including those containing transmission constraints such as wheeling charges and losses. Based on simulation studies, results and discussions for this modelling extension are given in Chapter 6.

Conclusions and recommendations are given in Chapter 7.

CHAPTER 2

ELECTRICITY MARKET FUNDAMENTALS

2.1 Introduction

Electricity market is considered as being playing an important role in improving the performance of electrical power system which is encouraged by the success of privatization in most of the industries, for example, telecommunications and gas [15]. This leads to deregulation of vertically integrated utility into three main utilities which are generation companies, transmission companies and distribution companies. The success that was made in electricity deregulation in some countries like UK, USA, Norway and Australia has brought more countries to privatization of their electricity industries [16]. Electricity industries have changed significantly in its operation and management since the deregulation in the sector took place. The role of introduction of deregulation into electricity industry is to increase competition among participants of the market and to make electricity market more efficient and liquid, in the meanwhile, power system operates in a reliable and secure manner. Generally, a deregulated electricity market consists of generation companies, transmission companies, distribution companies, and an independent system operator (ISO). Generation companies try to maximize their profit by negotiating and selling the power in the market which do not have to own the generating plants. Transmission companies are responsible for delivery of power from power producers to consumers. They also maintain the transmission network and ensure reliability of power system. Similar to transmission, distribution companies transfer power to retailers or large consumers. ISO's responsibility is secure operation throughout the power system by meeting total generation and load in real time. For this purpose, ancillary services are employed by ISO to compensate the imbalances in power system. Lastly, market price is identified when market equilibrium reaches which is at the intersection point of supply and demand curve.

2.2 Deregulation of Electricity Market

2.2.1 Why deregulate?

2.2.1.1 Evolvement of electricity market

The case of initial American electricity industry is used to describe how deregulation of a monopoly market happens. Looking back on the early stages, there was competition in power industry, however, in a brutal and inefficient way. From 1887 to 1893, more than twenty power companies were set up in Chicago solely [17]. Because of overlapping distribution lines in the region, competition between these companies was fierce while costs of production were quite high. In 1907, electricity sector was commonly accepted as a monopoly body against inefficient competition through regulatory laws which was in the form of vertically integrated utility. In the early periods, electricity supply was a natural monopoly in an isolated region, Chicago for example, due to disconnection from region to region. Therefore regulation is required to ensure basic demand of consumers around the area. The state of monopoly in electricity market had been lasted for many years, during the time high voltage transmission network was established mainly for reliability purposes. With the expansion of transmission network, the entire eastern United States and eastern Canada were connected together gradually. A dramatic feature of such network is that it has the ability to deliver bulk electricity energy through long distances with only a small loss [17]. It made possible the technical foundation of electricity trading between different regions which are interconnected. From 1990, power industries have been encouraged by the initial success in other formerly state owned industries to deregulate towards more competitive ones which is a general trend and implemented in many countries. Although it might meet some problems when deregulation was undertaken in the early years, it does not to say, deregulation is doomed [4].

2.2.1.2 Problems with regulation

If a single monopolistic company can produce electricity at lower costs than a competitive market does, then motivation of deregulating electricity market is weak. Unfortunately, from experience of running electricity market as a monopoly in the past decades, it shows high production cost and low efficiency under regulation [18]. In other words, the lack of monopoly nature is a precondition to successful deregulation or to some extent that the condition of natural monopoly is held weakly. As stated, the introduction of deregulation into power industry is the result of notorious inefficiency of regulation. This has been commonly used to be the cause why a traditional vertically integrated power industry needs reforming. However, it does not mean that every electricity market deregulated would do a better job than a regulated one, for instance, California electricity crisis [19]. That is to say, deregulation is not the equivalent to perfect competition which is well recognized as to be most efficient.

To sum up, regulation has two major problems that incentives to suppliers under regulation can not be sent as efficiently and economically as under a deregulated, or more strictly speaking, under a competitive market and there are not proper incentives to regulatory authorities themselves [17]. In contrast to regulated electricity markets, perfectly competitive markets have two main functions, hold market price down to marginal cost and minimize production cost. These are generalized in figure 2.1.

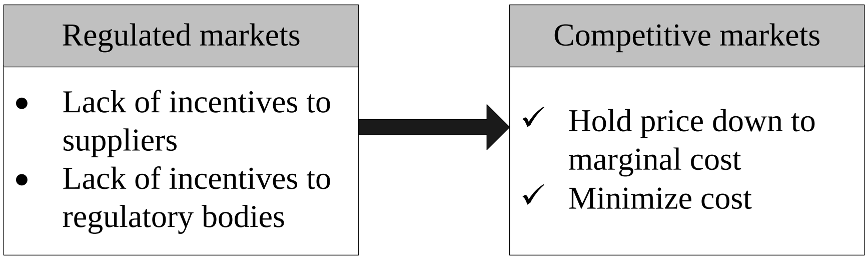

Figure 2.1 *Comparison between regulated and competitive markets*

Both of the functions can be fulfilled at the same time in a competitive market while regulation can only handle one of them each time. As a result, a trade-off is normally needed in a vertically integrated electricity market which would be costly and time consuming. In practice, regulation, at most, does a proper job on both the desires but it is not always as good as what competition does. In most of time, the practical job of regulation is to push prices down toward cost of production however incentives to minimization of cost are sometimes neglected by most regulators [17].

The above two objectives, cost minimization and holding price down to marginal cost, are also main benefits of a competitive wholesale market. For the first part, cost-saving incentives are sent to suppliers more quickly in such a market than a regulated one. Such costs include construction cost of new plants, labor cost, cost for repairs, and cost of capital investments. Lastly, incentives for better pricing are provided in a competitive market while price is minimizing towards marginal cost.

2.2.1.3 Problems with deregulating electricity market

As discussed in the previous paragraphs, electricity markets with monopolistic or vertically integrated features should be reformed and replaced by deregulated ones. Although it has been encouraged carrying out this process, there are still problems existing under deregulated market structure because of itself physical characteristics. Electricity is basically different from any other ordinary products.

It is generated and consumed continuously and simultaneously which makes trading of electricity difficult. For instance, it is normally consumed in less than a tenth of a second and another dramatic factor is that electricity can be stored in less than a tenth of a second in power system [17]. These factors can have a negative effect on cost of production and result in cost fluctuating. Besides, demand-side flaws in electricity industry are identified by Steven Stoft as lack of metering and real-time billing and lack of real-time control of power flow to specific consumers [17].

The first one indicates consumers' lack of prompt response when market price changes, or more generally this is lack of demand elasticity. The second one shows power flow is delivered in the network by its physical properties other than intention of regulators. As a result, bilateral contracts in real-time between suppliers and consumers tend to impossible to be accomplished. To sum up these two weaknesses, it might cause supply and demand curves to fail to intersect due to badly response of demand to price changes. Hence, these two demand-side flaws need considering carefully when start deregulating the electricity market.

Complexity and market power of electricity industry are another two problems when deregulating electricity market. Power system is an industry including lots of generators, transmission and distribution lines as well loads which needs to be operated reliably and securely. It can be done through well designed market rules [18]. The existence of market power is the situation that prices are influenced seriously by some large companies. That is to say, market prices are not determined by market itself but a few companies with market power. Problems presented above have to be managed successfully when deregulation is undertaken. Otherwise, regulation is still a solution for the time being.

2.2.2 What to deregulate?

Provision of electricity to consumers is actually the delivery of a package of services which includes generation, transmission, distribution, voltage support, and frequency control, etc. In theory, each service needs a separate market or several for trading under deregulated state therefore what services should be deregulated needs to be thought carefully. There is a service needed necessarily in either a regulated or a deregulated electricity market which is offered by the system operator. This service is so called coordination service that the extent of operation is different between these two structures. Reliability is a crucial indicator supplying electricity to consumers and it is achieved through a package of services identified. More details are presented in the following subsections.

2.2.2.1 Ancillary services and role of system operator

The role of the system operator is to keep the power system in balance, and provide consumers with reliable power supply. The system operator achieves these goals through purchasing a package of services that is called ancillary services. Ancillary services are supplementary services that are needed to support stable and reliable operation on power supply to keep electricity generation and load in balance in real-time. It is necessary for system operator to meet reliability standards in a power system. In other words, ancillary services market and electricity energy market are closely bound with each other for purpose of security and reliability. These services mainly include regulation and frequency response, a group of reserves, reactive power supply and voltage control [2]. In more detail these are described as

- Regulation and frequency response services:

The system frequency is an important factor for most of electrical equipments in the network and should be maintained at a constant level or within some band. Deviation of system frequency could be caused by any imbalance between power supply and demand. The task of maintaining frequency at a constant level is also one of the responsibilities of the system operator. The state of balance between generation and load can be monitored through system frequency. Accordingly, regulation and frequency services are employed to restore the system frequency in real-time when any imbalance occurs between supply and demand [20].

- Operating reserves:

Reserves can be defined as spare generating capacity that is currently available to deal with any sudden disturbance in the power system. Operating reserves are generally classified into two categories which are

  - spinning reserves and
  - supplemental reserves

Spinning reserve is described as spare generation capacity that is available within ten minutes from real-time electricity generation. Normally, it has to be required available as much as the capacity of the biggest power plant in the power system. On the other hand, supplemental reserves are extra capacity that can be used during the period ranging from minutes to hours in case disturbances occur. This kind of reserves is mainly responsible for replacement of lost generation in the system and bringing generation capacity used as spinning reserves back to normal level. Furthermore, supplemental reserves are including several types which are defined based on responding time. Non-spinning reserves are described as available generation capacity within ten minutes but it is not necessarily on-line or synchronized to the system which is different from spinning reserves. Replacement reserves are generation capacity that is available within 60 minutes which can be both on-line and off-line generation capacity. Lastly, black start is said that the capability of a power plant to restart and synchronize to the system without external support when contingency occurs. As explained, reserves can be grouped and separated by time scale or whether they are synchronized to the system.

- Reactive power supply and voltage control services:

Besides system frequency can be varied which results from imbalances, system voltage also faces the same problem. That is to say, deviation from its nominal values may pose a problem to the normal operation in part of the system if not the entire [21]. Reactive power supply and voltage control are such ancillary services for the purpose of maintaining voltage level. Due to physics of reactive power that is not suitable for delivery over long distances, this service should be located at suitable sites. Usually, generators and capacitors are used as control devices providing these services [22].

Role of system operator:

As there is little to central planning for new generation in a deregulated power industry, system operator is responsible for matching supply and demand in the system in the long run. In all kinds of deregulated markets, independent system operator is needed for coordination services and responsible for system reliability. In competitive markets, system operator needs to maintain whole power system in balance of supply and demand, for generation provision, reliability and security. The point needed to emphasize is that system operator is supposed to be fair to all participants in the market. Among lots of responsibilities of a system operator, security is the most important job during all the time. Distinction between an independent system operator (ISO) and system operator (SO) is that non-profit organization is usually called ISO and a for-profit one is SO. A for-profit system operator has to be regulated extensively and a non-profit system operator needs to be regulated lightly [17].

2.2.2.2 Unit commitment and congestion management

In a deregulated electricity market, the unit commitment service should not be provided by a central authority or a single market [17]. The provision of electricity should be determined by generation companies themselves and the unit commitment service is responsible for offering information of how much electricity will be supplied during specific periods. Decisions on this can be made by generation companies privately. Hence, generation companies need to predict the market price of electricity if they want to make a profit or predict the quantity of electricity production when market price is known.

Congestion management is one of difficulties when designing electricity market. The use of congested transmission lines will be charged in congestion rates.

Pricing of congestion makes sure that the capacity of these lines is not wasted which is considered as the only way to manage congestion efficiently [17]. Generally, congestion charges are collected from transmission line users when congestion occurs that is called congestion rent. Apart from congestion rent, transmission rights can also be transferred among parties for protection against overuse. However, provision of transmission rights has to be regulated all the time [17]. Again, this service is needed to be dealt with carefully by the system operator including both congestion rent and transmission rights.

2.2.2.3 Transmission and distribution

At early stages of deregulation of electricity industry, market for transmission should be regulated to remain monopoly features [2]. Although building of transmission lines could increase competition among suppliers, costly investment is also the reality to face. Thus building new transmission lines is a regulated and thoughtful process. It makes deregulation of market for transmission impossible due to these complexities. Compared with transmission, distribution tends to more difficult to handle towards deregulation at the beginning in practice [17].

2.3 Power Supply and Demand

In the electricity market, power supply and demand is significantly different from those in any other market due to its own physical features that production must meet consumption in real time. Several curves are given in the following paragraphs to demonstrate the relationship between supply and demand.

2.3.1 Load-duration curve

Load-duration curves represent the demand, or the total load against the number of hours per year used (or percentage). An example of load-duration curve is illustrated below in Figure 2.2.

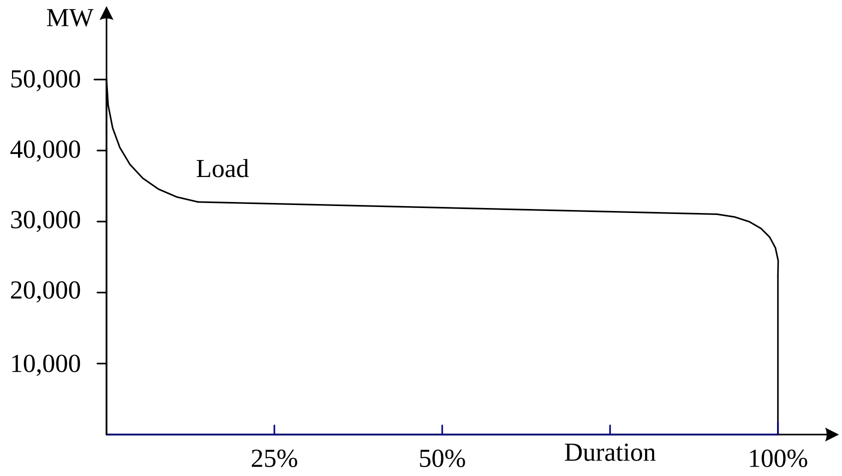

Figure 2.2 *Load-duration curve [17]*

A load-duration curve is simply illustrated as a curve that drops from the maximum load during peak hours to the minimum load and in the most of off-peak hours it is said as base load which is the smooth part of the curve. Duration can be represented in hours per year traditionally therefore it can be expressed by a ratio, or percentage that is shown in the figure 2.2. Taking the level of 35 GW as an example, 20% for duration can be found in the curve correspondingly which means the usage at load level 35 GW would be 20% of the year.

As seen in the figure 2.2, at the level of peak demand, it has to be met by the total generation in which less than 1% duration of the whole year. Therefore, generators, which are called peakers, are very different from those used for base load in terms of generation technology. Moreover, a problem is posed that how many generators should be set up by which type of generation technology. The most used technologies are coal-fired turbine or gas-fired combustion turbine in the extent of the USA [17]. Load duration curves can also be used for a regulated power market to determine how much those two generation technologies have to be allocated. When market price is fixed, then the corresponding load duration curve is fixed which is why it can be applied to determine available generation technologies. Two selected technologies based on production cost are shown in

Figure 2.3.

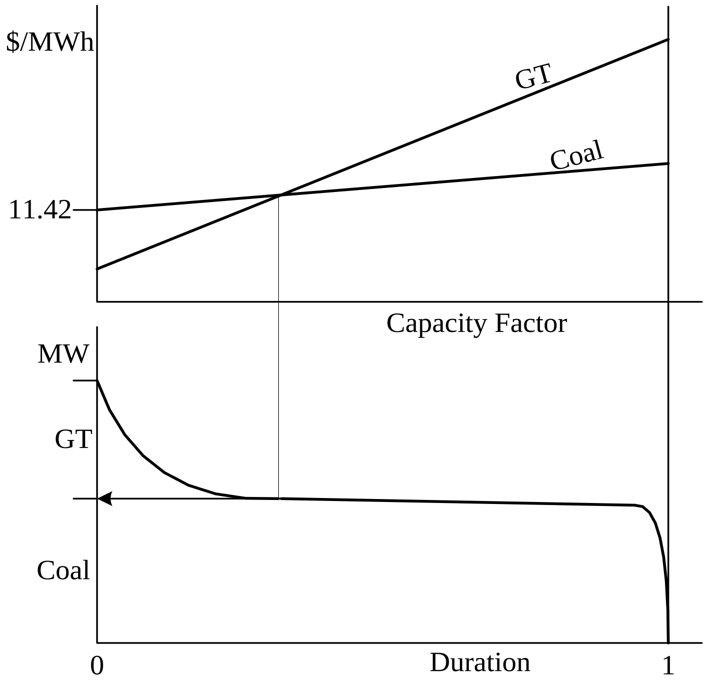

Figure 2.3 *Load duration curve using for allocation of different technologies [17]*

As seen in the graph, two linear lines, representing gas turbine and coal turbine technologies respectively, are intersected at a point to determine capacity factor which decides what technology is used. In this example, from the figure, capacity factor is approximately 30%. This is to say, load with duration more than 30% is supposed to be supplied by coal turbine technology plants. On the other hand, gas turbine technology is preferable when duration is less than 30%. This method to decide the boundary between different generation technologies is only applicable on condition that market price is fixed because load duration curve is determined by price. If market price is in the form of real time, this technique can not be used [17].

2.3.2 Marginal cost and curves

Marginal cost plays an important role in economics and it is also a key concept in electricity market. Therefore, a detailed description and application in the context of power industry is given in this section. In most electricity markets, generation companies submit their individual supply curves a day ahead and system operator determines the market price based on these data along with demand curve. Ideally, market price should be equal to marginal cost in a competitive market so that the market is cleared. It is well known that the competitive market price is derived when supply curves and demand curves are intersected. Strictly speaking, the market supply curve is supposed to be the aggregate supply curve which means the summation of all the individual generators' supply curves. Normally, a number of supply curves are collected and combined together by system operator. If there is more than one generator's supply curve is continuous in these data then it is said that the market supply curve is continuous [17]. Summation of a continuous supply curve and a discontinuous one is illustrated in Figure 2.4 below.

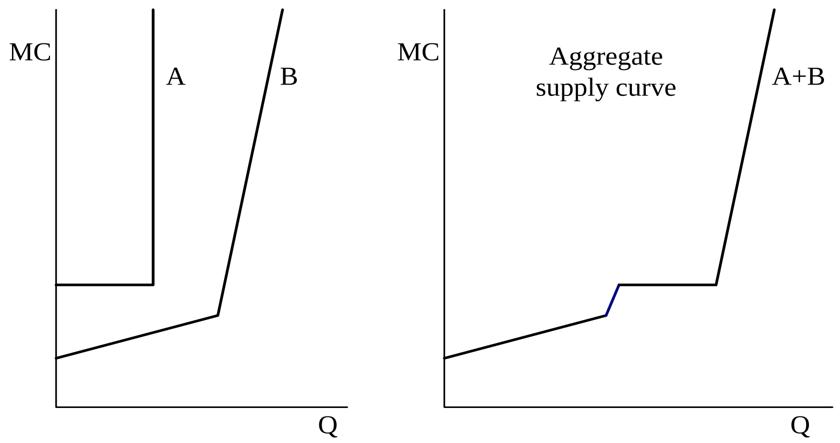

Figure 2.4 *Formation of aggregate supply curve [17]*

Supply curves without vertical element are called continuous which is named curve B while curves with the shape that A has are discontinuous. Sometimes when a generator reaches its full output, marginal cost would get to infinity with even tiny change in output. Thus the definition of marginal cost for power market needs to be reconsidered due to the introduction of discontinuous supply curves. To explain this situation, an example is illustrated in Figure 2.5 below. Demand curve is defined as a linear line with downward sloping.

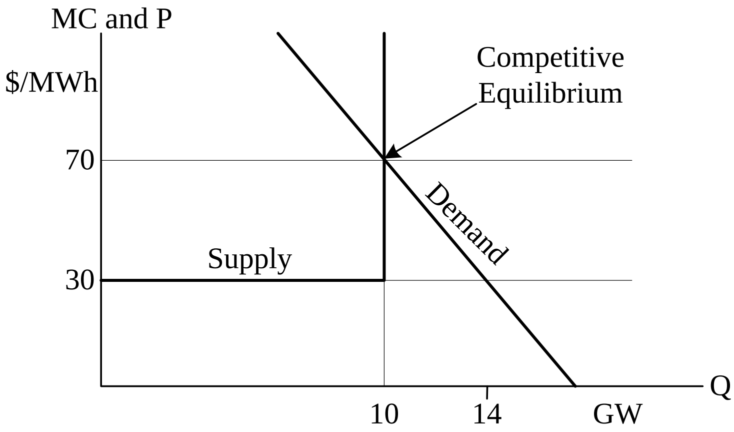

Figure 2.5 *Market equilibrium for a supply curve with vertical element [17]*

From the figure, supply curve and demand curve intersect at a point that competitive equilibrium is reached. If market price in this case is $30/MWh, the corresponding demand is 14 GW which means 4 GW of demand can not be met. As a result, some consumers would pay more, up to $70/MWh, for another MWh of supply. In this case suppliers would prefer delivering electricity to consumers at high prices above $30/MWh until the price reaches $70/MWh. At this point supply is equal to demand which are both 10 GW. If market price still goes up demand will be less than supply, and vice versa. At last the competitive market price will settle at $70/MWh.

Marginal cost is defined as “cost of producing one extra unit more (or less)” by Paul Samuelson [23]. This is true for situations when supply curves are continuous but does not apply to those of discontinuous supply curves. The definition is extended to a range of marginal cost namely left and right hand marginal costs including points of discontinuity.

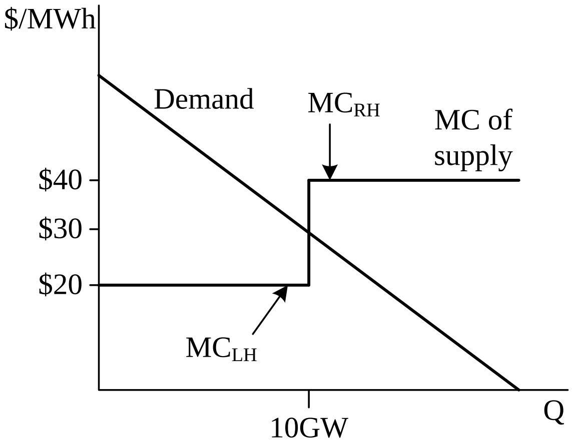

Figure 2.6 *Right and left hand marginal costs*

In figure 2.6, demand level is 10 GW however marginal cost can not be confirmed between $20.MWh and $40.MWh. In this case, marginal cost range is introduced into the definition including a range of values between MC_{LH} and $MC_{RH.}$ Marginal cost can not be fixed at a point of discontinuity but can lie in the range that is to say marginal cost range contains the market price.

To sum up, it is not necessary to analyze the marginal cost range in practical use so all marginal cost curves are normally assumed to be continuous. If supply curves of a large number of producers are aggregated, we can get a roughly smooth curve with an upward sloping. Consequently, marginal cost is always defined as a single value in practice which is recognized in most textbooks.

2.4 Competition in Electricity Market

Competition is the most commonly used word in markets and is said to be effective in electricity market as well. As discussed in previous paragraphs, a regulated electricity market should be deregulated for the purpose of introduction of competition. In any market, suppliers want to increase the market price for more profit and buyers intend to lower it for similar reasons. Both intentions can be suppressed through competition and market equilibrium is achieved at some point.

In electricity market, competition is firstly introduced into supply side and then spread to demand side. Supply side competition is the major issue to focus on in this thesis and interactions among suppliers in this kind of market is simulated in a model that is presented in the following chapters. If so-called perfect competition exists, all suppliers in the electricity market can not disturb the market price and have to accept to sell energy at this price, which are called price takers. Market price is equal to marginal cost when market equilibrium reaches. If there are any large suppliers that can affect the market price, they are not price taking and market power exists. It is noted that at the beginning of deregulation market power exists which means several big suppliers are capable of having an effect on the market.

2.4.1 Benefits of perfect competition

Perfect competition can be classified into two categories, short-run and long-run competition which are similar but with different goals. The main benefit gained from competition is the efficiency that is presented by Kenneth Arrow in his efficient-competition theory [24]. The illustration is presented in Figure 2.7.

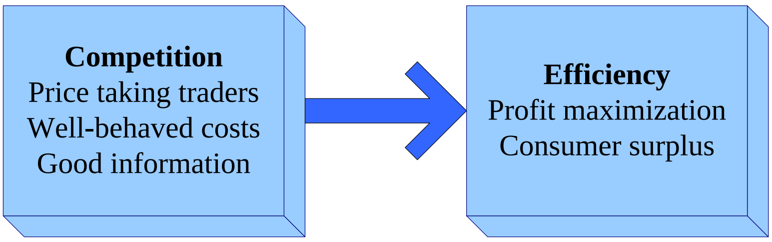

Figure 2.7 *Efficient-competition Results [24]*

From the figure above, the efficient-competition is generalized by three items other than the original theory, in which another two elements are included say well-behaved cost functions and good public information [17]. Theoretically, a competitive equilibrium can be reached only when three conditions listed in the figure are necessarily satisfied. It can be described as the situation on the supply side that suppliers do not have market power to affect market price but to accept it, and cost functions apply to marginal cost, lastly market price is known by every supplier. Only these three requirements meet will the efficiency be achieved. In the short-run, competitive equilibrium is efficient which indicates current generation resources are adequately used. In this case, suppliers maximized profit while consumers benefit from it.

Moreover, long-run competitive equilibrium is to maintain long-run investments in generation capacity which needs to be met in the future. In addition to three conditions for short-run competitive equilibrium, production cost without natural monopoly feature and free entry to the market are both necessary to guarantee a long-run equilibrium [17]. In the long-run, the economic profit level is to zero which is different to the situation of short-run.

Efficiency, or benefits, derived from competition implies that the electricity is supplied by the cheapest generators and consumed by those who want it most and finally the supply meets the demand. Through competition on the supply side, production costs can be minimized with strong incentives [2].

A better way for explanation of efficiency is through demand-supply curve in which both consumer surplus and producer surplus can be seen clearly. It is illustrate in Figure 2.8.

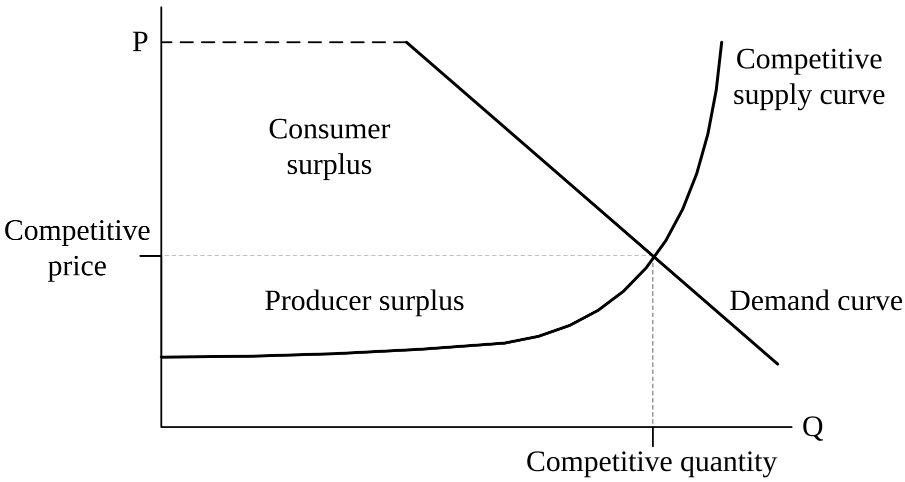

Figure 2.8 *Demand-supply curve with consumer and producer surplus [2]*

Demand curve represents how much consumers would pay for specific quantity of electricity while supply curve presents how much producers would prefer to sell at this level of quantity. From the figure above, the area under demand curve and above competitive price line indicates the consumer surplus. Producer surplus is similar to consumer surplus and it is the area above the supply curve but not exceed the competitive price. From consumers' point of view, the more consumer surplus is the more benefit they get. It applies to suppliers that they also want to increase the own surplus as consumers do. An equilibrium reaches when both surplus of consumers and producers are maximized which is said that the market is efficient [2].

2.4.2 Market equilibrium

Market equilibrium is an important concept in electricity market which determines competitive market price and the corresponding quantity to be traded. Basically there are two ways to achieve market equilibrium i.e. adjustment through price or quantity [17]. When power supply meets demand the market is said to be cleared and the price at the point of clearing is called market clearing price or competitive market price which represents market equilibrium is achieved [25].

Market equilibrium is achieved actually through interaction between producers and consumers. The stability of market equilibrium can be seen clearly from Figure 2.9. The equilibrium can be defined by market clearing price Π^* or equilibrium quantity q^* in equations 2.1 and 2.2. Symbols D and S represent the inverse demand function and supply function respectively.

$$D\left(\Pi^*\right) = S\left(\Pi^*\right) \tag{2.1}$$

$$D\left(q^*\right) = S\left(q^*\right) \tag{2.2}$$

From equations, it implies both functions are equal to each other when supply curve and demand curve are intersected. As shown in the figure 2.9, market equilibrium will always settle at equilibrium point i.e. the intersection of supply and demand curve. This process can be described in the following way. At price

Π_1 demand is not satisfied that the quantity of demand is bigger than supply. Suppliers would increase the price and sell more electricity to consumers until equilibrium is reached. Conversely, at price Π_2, there is excess supply that supply exceeds demand. As a result, to avoid a loss of money, suppliers would decrease the production of electricity until the quantity of supply meets the quantity of actual demand. Finally, the equilibrium price is equal to Π^* and equilibrium quantity is q^*.

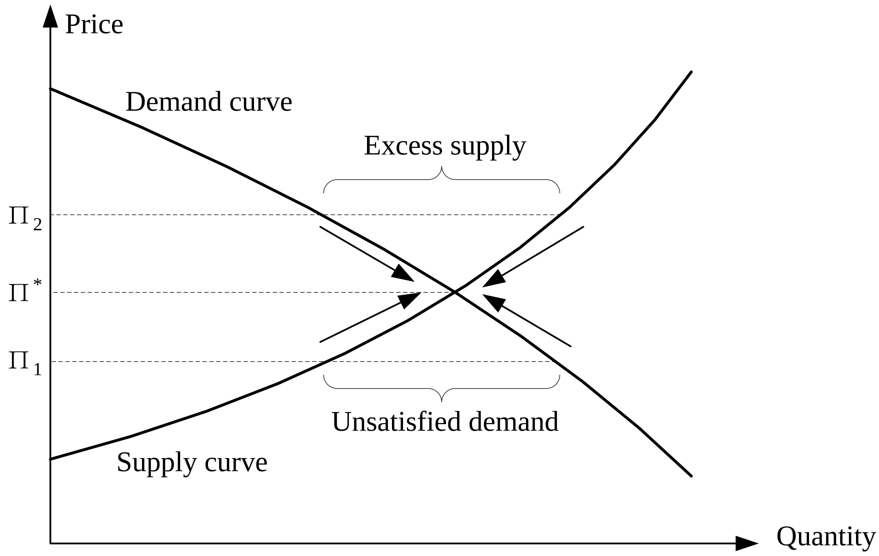

Figure 2.9 *Stability of market equilibrium [2]*

2.4.3 Four Models of Competition

Power system is basically made up of four parts i.e. generation, transmission, distribution and consumers, which is a complex system with numerous equipments and appliances. In this thesis, it is viewed from market issues other than technical ones. In this section electricity market is looked at as a whole and introduction of competition into the market is viewed in four stages that are originated by Hunt and Shuttleworth [26]. Competition is introduced into the electricity market from supply side to demand side through the progressive process of deregulation.

2.4.3.1 Structure 1: Monopoly

In Figure 2.10, for part (a), generation, transmission and distribution are vertically integrated into a big company with monopoly features while generation and transmission are included in one utility for part (b). The integrated company on the left of the figure sells electricity to consumers directly and trade with the other big company which indicates that electricity can be traded bilaterally but at transmission level [2]. On the right another big company which is shown in the block sells electricity to a distribution company that is also a monopoly.

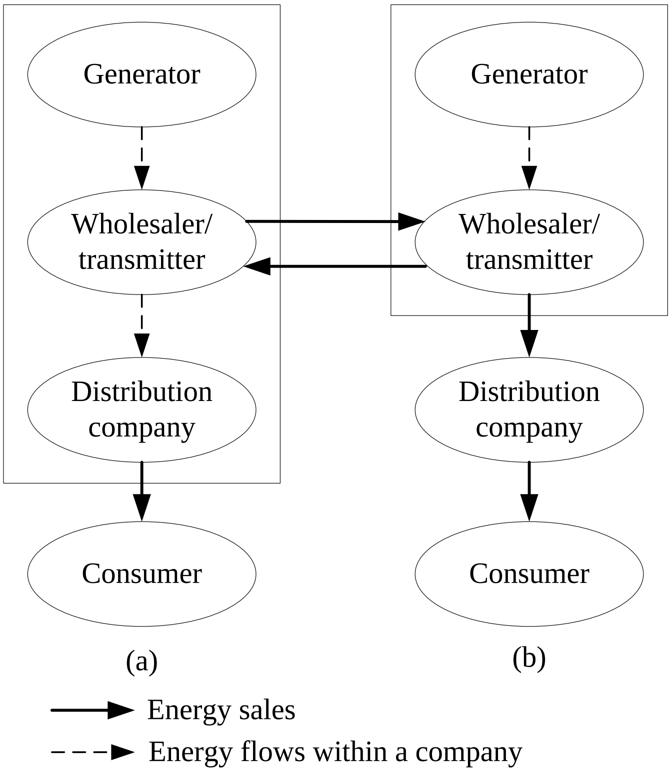

Figure 2.10 *Monopoly structure of electricity market [26]*

2.4.3.2 Structure 2: Purchasing agency

In this model, the supply side of power market is deregulated and the less monopolistic big company does not possess all the generators, which implies that independent power producers (IPP) are allowed to compete with them. However, the transmission company to which all producers sell electricity is still a part of the integrated company and this company is so-called wholesale purchasing agency. In Figure 2.11(b), the supply side is deregulated further and all generators are no longer possessed by a single company. That is to say, all the generators are IPPs and compete with each other selling electricity to a purchasing agency i.e. a monopoly transmission company. Distribution Company used to a monopoly is also disintegrated into many discos in this model and these discos buy energy through the purchasing agency. The purchasing price is set by the agency therefore it is necessary to be regulated to avoid overusing the power [2]. Although further competition is introduced into the market, competitive price is just not derived as properly as perfect competition does [26].

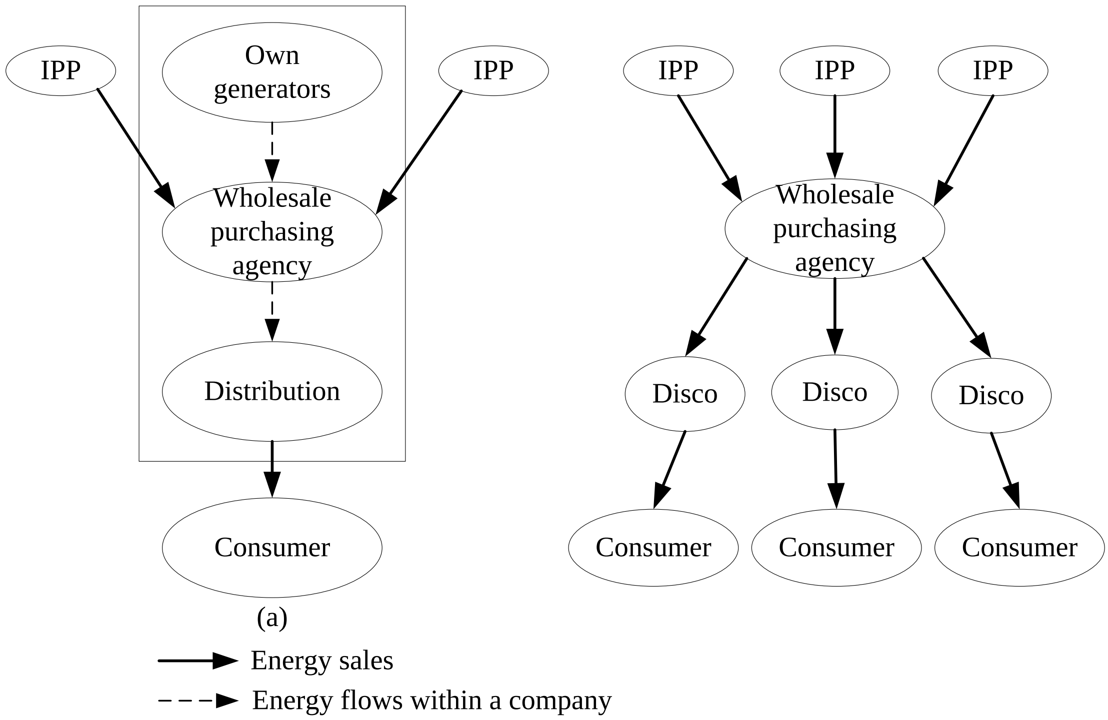

Figure 2.11 *Market structure of purchasing agency model [26]*

2.4.3.3 Structure 3: Wholesale competition

From figure 2.12, in this structure, all generators are independent and do not belong to any single central companies which is the same as the advanced purchasing agency model. There is not a purchasing agency responsible for buying energy from IPPs and selling energy to discos that serve consumers. Instead, discos can buy electricity from generators directly in a wholesale electricity market. In addition to discos, several large consumers are permitted to purchase energy from wholesale market as well. This wholesale electricity market works in terms of pool or bilateral transactions [2]. The operation of transmission network is still centralized at the wholesale level. However, distribution network belongs to discos which remains centralized. Responsibility of discos is not only serving consumers by buying electricity over wholesale market but also operating the distribution networks. Thus retail prices, which are largely determined by discos, need to be regulated to avoid high price for consumers.

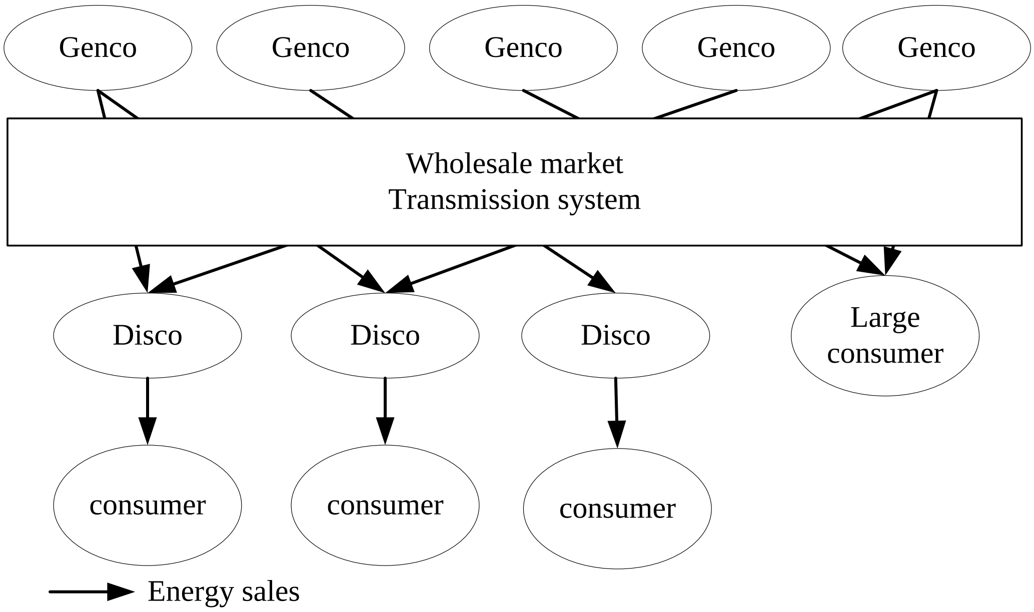

Figure 2.12 *Market structure of wholesale competition [26]*

2.4.3.4 Structure 4: Retail competition

The last one of competition is illustrated in Figure 2.13. The significant change from wholesale competition is that all consumers could freely buy electricity from their suppliers. Large consumers can still get energy in the wholesale market while other consumers are able to buy electricity from retailers. Retailers as well as such large consumers purchase energy form transmission system to meet demand of consumers. The difference between retailers and discos in the wholesale model is that the monopoly characteristics are removed from retailers since retailers do not possess the distribution network any longer. Operation of transmission and distribution networks is the only monopoly factor at this stage [26]. Consequently, retail prices need not to be regulated because consumers can choose their suppliers freely. The usage of transmission and distribution network is charged by all their users. The only thing for now to focus on is the operation cost of transmission and distribution networks which still needs to be regulated due to its monopoly feature.

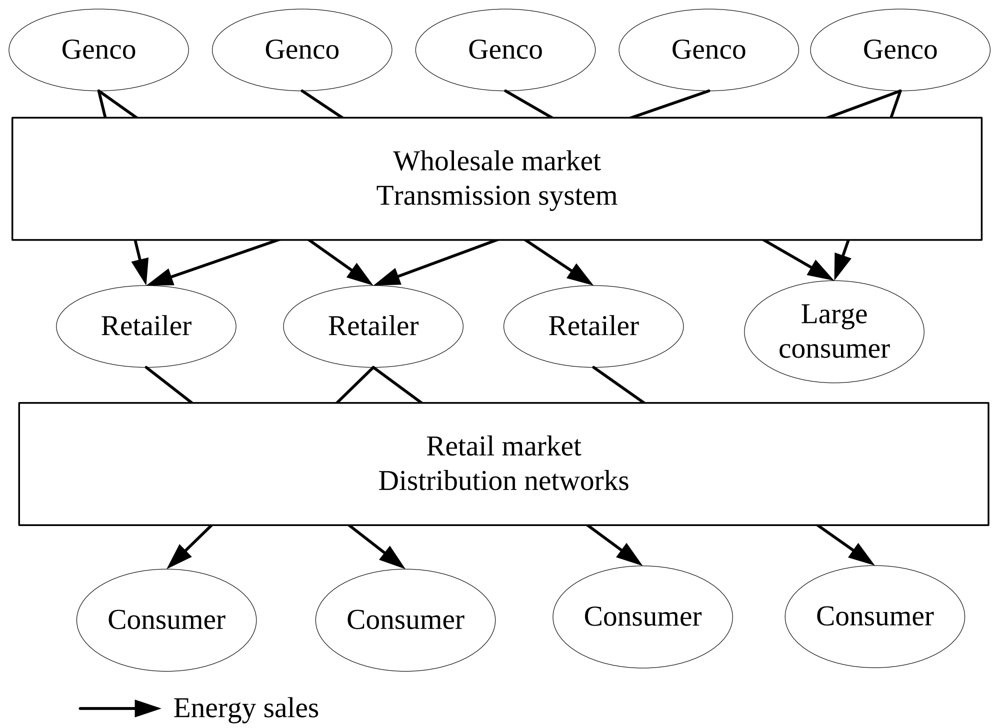

Figure 2.13 *market structure of retail competition [26]*

Evolution from monopoly to retail market, it could take a long time and might meet a lot of problems however it makes electricity market more efficient which will benefit both suppliers and consumers in the long run.

2.5 Summary

Power industries were dominated by vertically integrated utilities for a long time and have been deregulated from 1990 firstly in the UK that was encouraged by the success of privatization in other industries used to be held monopoly. To be generally accepted, the role of introducing deregulation into electricity industry is to increase competition among participants of the market and make electricity market more efficient as monopoly state of power industries took away the incentives to efficiently operation and incurred unnecessary investments [2].

From past experience, regulation has two major problems that incentives to suppliers can not be sent efficiently and lack of incentives to regulatory bodies themselves. In order to remove these negative effects, deregulation has been adopted to hold market price down to marginal cost and minimize production cost [17]. It is noted that under deregulation there are problems to handle such as demand-side flaws and market power. If these are not tackled properly, regulation is still a solution. At the beginning, the market for ancillary services should be deregulated to ensure the reliability of power system which is called by operator when imbalances occur in the system. Unit commitment and congestion management should also be considered under a deregulated market.

Power supply and demand is different from ordinary product market due to its physical features that production is equal to consumption in real time. Marginal cost in an electricity market is the cost when aggregated supply curve and aggregated demand curve are intersected. At this point, it is said that market equilibrium is achieved and market price equals to marginal cost in a perfect marketplace. In a market, efficient competition exists when three conditions are satisfied that players are price taking, costs are well behaved, and free access to good public information [17]. Through competition, efficiency is gained in terms of profit maximization and rise of consumer surplus.

Development in electricity market is described as four stages by Hunt and Shuttleworth [26] which is classified by the degree of competition. They are monopoly, purchasing agency, wholesale competition, and retail competition. In monopoly, there are one or several vertically integrated companies selling electricity to consumers. In purchasing agency model, generation is unbundled and generators compete to sell electricity to transmission which functions as purchasing agency. In wholesale competition, transmission's responsibility is only the operation of network. Electricity trading between generators and discos is completed in the wholesale market. In last one, distribution focuses on operation of network and consumers buy electricity from retailers directly through retail market.

CHAPTER 3

GAME THEORY FUNDAMENTALS

3.1 Introduction

In this thesis, non-cooperative game theory is used as researching method to analyze the optimal production of each player in an oligopoly electricity market. Before organizing the model some basics of game theory are given in this chapter. It gives the elementary knowledge of non-cooperative game theory for preparation of modelling oligopoly electricity with specific parameters.

Game theory is concerned with "how rational individuals make decisions when they are mutually interdependent" [10]. Another way to describe it is that game theory is related to the actions of decision makers who are aware that their actions can influence the decisions of others [27]. Game theory is not applicable when players in the game make decisions when ignoring the actions of others. This is to say such players are not strategically thinking [28] or they are not rational players.

Game theory is usually classified into two types that are co-operative and non-cooperative. The definition given earlier is only applied to non-cooperative game theory. In this thesis only non-cooperative one is discussed and adopted as researching method for studying in the following chapters. Strictly speaking, in non-cooperative game theory the individuals or players in a game are not allowed to access to "binding and enforceable agreements" with one another [29]. Non-cooperative game theory has individualistic features. On the other hand such agreements are allowed to enter for players involved in the environment where co-operative game theory is applicable. It is noted that in non-cooperative game theory it is not against the cooperation of players however cooperation takes place only in their own self-interest [29].

As described, the definition of Game theory that is given in Graham Romp's book contains three elements which are individualism, rationality, and mutual interdependence [10]. These factors are basic assumptions applied to game theory, which helps to understand the researching category. Players in a game make decisions individually that means they are individual decision makers and all their decisions are in their own self-interest. Along with individualism, rationality is an assumption that players in a game are rational individuals and they make decisions rationally. However, it is impossible to think rationally for individuals in practice [10]. For purpose of analysis, some necessary assumptions and simplifications are made to let models more easily to handle and in meantime without losing possible features of reality. For example, complex organizations are assumed to act as individual decision makers, which is impossible in reality.

Mutually interdependence is an important concept in game theory as game theory is not applicable when players do not interact with each other. In this condition the benefits of players are dependent on the possible actions of other players in the game. It is commonly assumed that players in a game act strategically [28] and have incentives to forecast the actions of others for better welfare. If each player acts rationally competitive equilibrium is finally achieved. Pareto Efficiency is used to examine the competitive equilibrium. The situation when "no one individual can be made better off without making someone else worse off" is said to be Pareto efficient [10]. In other words, when Pareto inefficiency occurs there is at least one player can be made better off without making any other worse off in a game.

3.2 Static Game Theory

Static games are presented and examined in this section. So-called static games are some one-off games in which the players are supposed to make their actions at the same time [30]. It is noted that players do not know exactly how others will do next in static games. Seeing as non-cooperative game theory is considered solely in the thesis, the way represents a game can be categorized into normal form (or strategic form) and extensive form [31]. Organization and solution to a static game are discussed in details in the following paragraphs.

3.2.1 Illustration of normal form games

Basically, a normal form game is including three elements which are players, strategies, and pay-offs [31]. Explanations are given below.

- **Players**

The players are participants in a game who decide what to do next step. At least two players are needed in the game due to the interdependent characteristics of game theory. There is an exception say ‘nature’ which determines the type of players in the game [31]. Actually it is not a real player making decisions but further information or limit to participants.

- **Strategies**

For each player, a list of strategies is available. It shows how a player would make decisions and take actions. It is noted that such strategies describe the possible actions of each player which depend on what others will do.

- **Pay-offs**

For each list of strategies, there exists a set of pay-offs to show what a player will receive after taking particular action [31]. It is believed that players in the game always prefer higher pay-offs. That is to say, such players are rational.

An example of normal form game is illustrated in Figure 3.1 which is taken as a well known static game called ‘the Prisoners’ Dilemma’.

		Prisoner 2	
		Confess	Don't confess
Prisoner 1	Confess	-6,-6	0,-9
	Don't confess	-9,0	-1,-1

Figure 3.1 *The Prisoners' Dilemma in normal form [10]*

There are two players (prisoner 1 and prisoner 2), each of them has two strategies (confess or don't confess), and payoffs for each possible combination of strategies (represented in numbers).

3.2.2 Illustration of extensive form games

Comparing to normal form games, in extensive games, it is concerned with the timing of actions players may take and the information they have when they take particular actions. Similarly, an extensive form game has four elements commonly say nodes, branches, vectors and information sets. Descriptions are following:

- **Nodes**

It is the position in a game at which players have to make decisions and take actions. It is noted that the first position is presented by an open dot which called the initial node and all others are represented in dots. Each one is named by numbers to show which player is making decision.

- **Branches**

It represents the actions which are available for players to take.

- **Vectors**

It represents the pay-offs for each player when they take particular action.

- **Information sets**

It is common to see that dots in an extensive form game are connected by a dashed line which means they are in the same information set [31]. The player who has to make decision does not know what the other might do.

Still, the Prisoners' Dilemma game is presented to illustrate how an extensive form game is structured. It is shown in Figure 3.2.

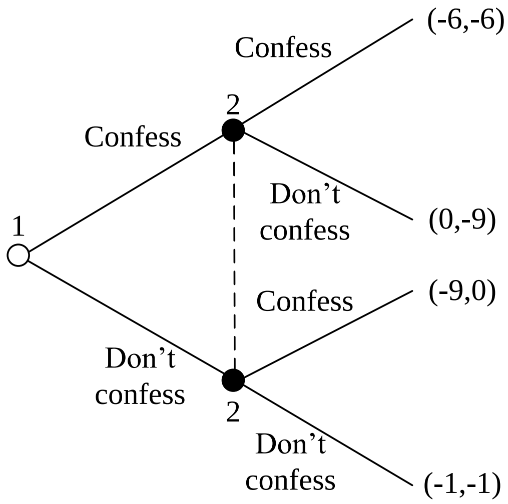

Figure 3.2 *The Prisoners' Dilemma Game in extensive form [10]*

3.2.3 Solution techniques for static games

A solution to a game is the prediction how players in the game make decisions and what they will do. The solution is normally an optimal outcome for every player and unique as well. In non-cooperative game theory, two solution techniques are popularly used i.e. dominance and equilibrium. For dominance technique, it is the way ruling out strategies that rational player would never play [31]. For equilibrium technique, equilibrium is the situation when "no players have an incentive to deviate from the predicted solution" [10].

3.2.3.1 Dominance technique

Dominance can be further divided into two types, strict dominance and weak dominance. Both techniques of dominance are discussed briefly in the subsections.

- **Strict dominance**

In this technique, it is assumed that a strictly dominated strategy will never be chosen to play by a rational player [31]. A strictly dominated strategy occurs when other strategies always give better pay-offs. All strictly dominated strategies may be ruled out until one strategy left for each player. The left strategy is the solution to such games. The game of “Prisoners’ Dilemma” is regenerated and used to illustrate the process of strict dominance solution.

		Prisoner 2	
		Confess	Don’t confess
Prisoner 1	Confess	**-6,-6**	0,-9
	Don’t confess	-9,0	-1,-1

Figure 3.3 *The Prisoners’ Dilemma game for illustration of strict dominance*

In figure 3.3, for prisoner 1, it can be seen that strategy of “confess” would bring him better pay-offs as pay-offs of (-6, 0) when he confesses is better than those of (-9, -1) when he does not confess. This is to say “don’t confess” for prisoner 1 is a strictly dominated strategy and should not be considered corresponding to the concept of exclusion of all strictly dominated strategies. In the same way, for prisoner 2, strategy of “don’t confess” should be disregarded as pay-offs of (-6, 0) to “confess” are better than those of (-9,-1) to “don’t confess”. As a result, “confess” i.e. pay-offs of (-6,-6) for both players is the final strategy. Until now the game is said to be solved.

- **Weak dominance**

In the technique, a rational player will never play a weakly dominated strategy [31]. Such weakly dominated strategies come up when other strategies give the player better pay-offs in some situations and leave them indifferent in all others. It is illustrated in Figure 3.4 below.

	Player 2	
	Left	Right
Player 1 Up	**7,2**	5,2
Player 1 Down	7,4	2,0

Figure 3.4 *An example for illustration of weak dominance*

Similar to strict dominance, however with slight difference, in this game, if player 1 moves "up" pay-offs would be (7, 5) when player 2 moves left and right respectively and if player 1 chooses "down", pay-offs would be (7, 2) when player 2 chooses left and right. Comparing two set of pay-offs, it is indifferent at the pay-off of (7), and weakly dominated strategy can be identified through pay-off of (5) against (2) when player 2 moves right. The situation of "down" for player 1 is weakly dominated strategy which should be disregarded. In the same way, for player 2, the strategy of "right" is weakly dominated and should be disregarded. Therefore, the combination of "up" for player 1 and "left" for player 2 is the solution to the game.

- **Iterated strict/weak dominance**

When there are more than two strategies, iterated strict/weak dominance techniques might be used. It is only applied to the particular situations not to all. Examples are given below.

	Player 2		
	Left	Middle	Right
Player 1 Up	1,0	**1,2**	0,1
Player 1 Down	0,3	0,1	2,0

Figure 3.5 *An example for illustration of iterated strict dominance*

In this example, shown in Figure 3.5, player 2 has 3 possible strategies. It is impossible to solve the game by looking at player 1 firstly as it is not clear from the comparison between pay-offs of (1, 1, 0) when he moves up and (0, 0, 2) when he moves down. In this case, looking at player 2 first, the strategy of “right” is omitted as pay-offs of “middle”, (2, 1) are better than those of “right”, (1, 0). The strategy of “down” for player 1 is omitted in the same way. Lastly, the solution is settled at the combination of “up” for player 1 and “middle” for player 2 and pay-off is (1,2) for player 1 and player 2 respectively.

		Player 2		
		Left	Middle	Right
Player 1	Up	10,0	**5,1**	4,-2
	Down	10,1	5,0	1,-1

Figure 3.6 *An example for illustration of iterated weak dominance*

Similar to iterated strict dominance, the game from Figure 3.6 has to be solved successively. Simply, the unique solution would be the combination of “up” for player 1 and “middle” for player 2 and pay-off is (5, 1) for players respectively.

From these two examples, it can be seen that the sequence of solving the game affects the outcome of the game. However, in some situations, shown in the example in figure 3.7, a game can not be tackled by dominance techniques and imprecise predictions are involved [10]. For this reason, a stronger solution technique is needed that is so called equilibrium technique.

Player 2

Player 1		Left	Middle	Right
	Up	0,4	4,0	5,3
	Centre	4,0	0,4	5,3
	Down	3,5	3,5	6,6

Figure 3.7 *An example for illustration of failure of dominance techniques*

3.2.3.2 Equilibrium technique

Cournot (1838) originated the method of solving static games using equilibrium thoughts and then it was developed by John Nash (1951), which is called Nash equilibrium [31]. Nash equilibrium is popularly applied to games which are considered difficult to handle by dominance techniques. It is described that in Nash equilibrium the combination of each player's selected strategy is optimal and the strategy every player selected is so-called equilibrium strategy [31]. Similar to dominance, in Nash equilibrium, it is assumed that players in the game are rational.

- **Steps of finding Nash equilibrium**
  1. Identifies every player's optimal strategy separately (receiving better pay-off) based on observation what the other players might do.
  2. Nash equilibrium is settled in the situation that every single player is playing their optimal strategies at the same time. (combination of the strategies for each player gives optimal pay-off)

Strictly speaking, the steps above are only applied to pure strategy Nash equilibrium other than mixed strategy Nash equilibrium [10]. Looking at the outcome, it is said that there exists unique equilibrium in a pure strategy Nash equilibrium game while there are more than one equilibrium in a mixed strategy one. In this thesis only the pure strategy one is discussed in detail.

- **Illustration of finding Nash equilibrium**

The example of Prisoners' Dilemma game in Figure 3.1 is regenerated and used to describe the Nash equilibrium solution technique. It is shown in Figure 3.8.

Prisoner 2

		Confess	Don't confess
Prisoner 1	Confess	-6,-6	0,-9
	Don't confess	-9,0	-1,-1

Figure 3.8 *The Prisoners' Dilemma game for illustration of Nash equilibrium*

In step one, if prisoner 2 chooses to "confess", the optimal strategy for prisoner 1 would be "confess" (as pay-off of -6 is better than -9). Similarly, if prisoner 2 chooses to "don't confess", the optimal strategy for prisoner 1 would be "confess" as well (0 is better than -1). Looking at prisoner 2, if prisoner 1 decides to "confess", the best choice for him would be "confess" (pay-off of -6 is better than -9) and if prisoner 1 decides to "don't confess" then strategy of "confess" for him receives better pay-off (0 is better than -1).

In step two, as all players play optimal strategies in a Nash equilibrium simultaneously, from the figure it is clearly seen that the combination of both "confess" for prisoner 1 and prisoner 2 is Nash equilibrium (-6,-6). The outcome derived is the same as the one when using dominance technique (in Figure 3.3).

Now looking at the game that dominance technique can not work out (in figure 3.7), the solution is given to handle the problem that is illustrated in Figure 3.9.

Player 2

Player 1		Left	Middle	Right
	Up	0,4	4,0	5,3
	Centre	4,0	0,4	5,3
	Down	3,5	3,5	6,6

Figure 3.9 *A further example for illustration of Nash equilibrium technique*

Applying Nash equilibrium technique to the example, by finding optimal strategies of every player, unique solution is finally found settling at (6, 6) when player 1 moves "down" and player 2 moves "right". From this example, it is seen that Nash equilibrium technique could solve more complicated case where dominance technique does not work. It is also noted that examples of static games are all presented in normal form because the outcome of the game does not depend on information and timing of decisions [30].

3.3 Dynamic Game Theory

Dynamic games are different from static games in terms of the number of interactions [27]. Dynamic games are more close to the real world than static ones as in practice players interact with one another repeatedly other than just one time. In this section it presents how dynamic games can be examined and solved through predictions. Only dynamic one-off games are considered to give a brief introduction to dynamic game theory.

3.3.1 Nash equilibrium of one-off dynamic games

Comparing with static games, timing that players make decisions and information sets are principal factors for dynamic games. Due to these features, dynamic games are normally represented in extensive forms. In a dynamic game, players could observe actions of the others before they make their decisions while players are only able to make decisions simultaneously in a static game. From this point of view, it can be seen that it determines whether players in the game observe or not to distinguish dynamic games and static games. The number of actions and strategies are the same for players in a static game, the characteristic of observation in a dynamic game makes the number of strategies for the players observing not equal to the number of actions anymore [31]. In contrast, this feature makes the number of strategies largely greater than the number of actions for the observing players.

An example of one-off dynamic game is chosen to present how a dynamic game can be solved by using Nash equilibrium theory. It is given in Figure 3.10 in extensive form.

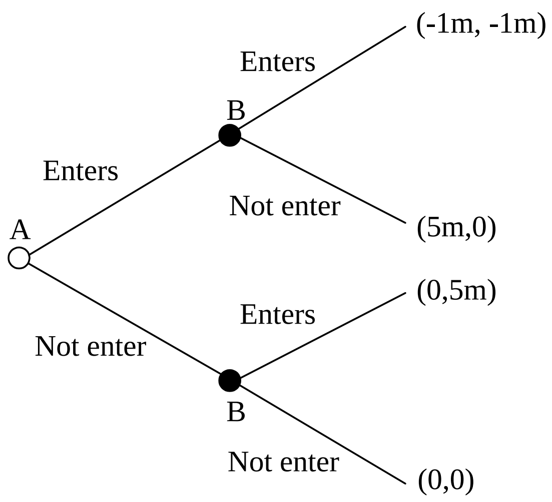

Figure 3.10 *A two-period one-off entry dynamic game in extensive form [10]*

As timing and information are involved, extensive form is considered better to represent the dynamic game. There are two players in the game (A and B), player A moves first and then player B makes his decision based on observation of what A does, which is so called two periods game. Different combinations of strategies of players determine different pay-offs. It is assumed that both players are keen on bigger results, which is said that players are rational. As shown in the figure, a major difference made from static game is that the points for player B are not connected by a dashed line. This means player B's nodes are separate information sets and player B can observe the actions of player A before making his decision which increases B's strategies. Correspondingly, the game can be reproduced in normal form in Figure 3.11.

		B			
		Enters	Not enter	Same as A	Opposite of A
A	Enters	-1m -1m	5m 0	-1m -1m	5m 0
	Not enter	0 5m	0 0	0 0	0 5m

Figure 3.11 *The two-period one-off entry dynamic game in normal form [10]*

Applying the steps of finding Nash equilibrium, the multiple Nash equilibria are identified. Since players play optimal strategies at the same time under Nash equilibrium, three Nash equilibria are targeted. They are the situations when A "not enter" and B "enters", when A "enters" and "not enter", and when A "enters" and B does "opposite of A". Among the Nash equilibria, an important issue is needed to be considered is whether B's strategies are credible [27]. An assumption is made that players only believe credible statements because it is in their own best interests. Consequently, Nash equilibrium needs improving to suit this case.

3.3.2 Subgame perfect Nash equilibrium of one-off dynamic games

As discussed previously, the solution of Nash equilibrium to a dynamic game can be multiple Nash equilibria, which means that incredible statements may involve. For this reason, Nash equilibrium is refined to subgame perfect Nash equilibrium to rule out incredible statements [10]. Under subgame perfect Nash equilibrium, the solution should be Nash equilibrium in every single subgame [31]. The example of entry dynamic game is used to present the process of finding subgame perfect Nash equilibrium.

Each Nash equilibrium that found previously is examined one by one here to identify whether it is a subgame perfect Nash equilibrium.

1. In the first equilibrium, B “enters” the market ignoring what A does. However, if A enters the market, B will not enter, as it is not in B’s interest to carry it out. Therefore, B’s statement is not credible. That is to say, the Nash equilibrium is not a subgame perfect Nash equilibrium.
2. In the second equilibrium, B does “not enter” the market ignoring what A does. However, if A chooses to not enter the market, then the answer to B would be “enter”, as it is not in his interest to stay out when A have decided “not enter”. So, this Nash equilibrium is also not a subgame perfect Nash equilibrium.
3. In the final equilibrium, B always does “opposite of A” no matter what A does. This is a subgame perfect Nash equilibrium because if A “enters”, B will stay out, and if A does “not enter” B will choose to “enter”. B always does the opposite what A does which is in his own interests to do it. Hence, it is believed that the statement is credible and the solution is a subgame perfect equilibrium.

Finally, the unique subgame perfect Nash equilibrium is identified that A “enters” and B does “not enter” and pay-offs are (5m, 0).

3.3.3 Back induction of one-off dynamic games

Subgame perfect Nash equilibrium technique takes a lot of time to solve such dynamic games. Back induction is therefore introduced to apply to such games however it is subject to strict assumptions [27]. The same as subgame perfect method, this technique gives unique solution which is subgame perfect [27]. The process of finding subgame perfect Nash equilibrium is shown in Figure 3.12.

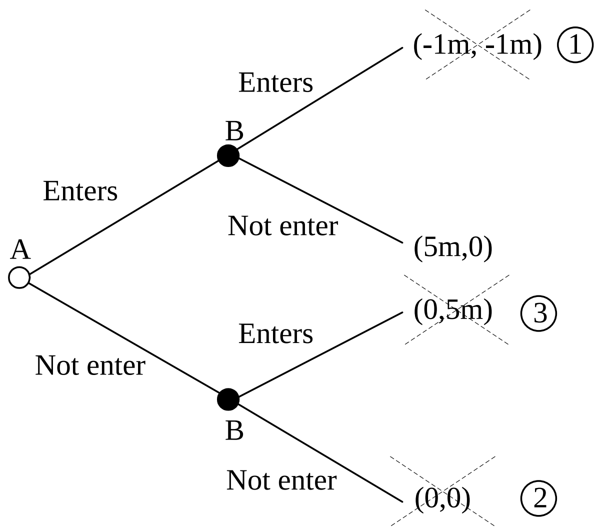

Figure 3.12 *Back induction of two-period one-off entry dynamic game [10]*

The principle of Back induction is to rule out actions other than strategies that players would not play in their own interests, starting from last period to first period and from last nodes to initial node [10].

For the first branch, if A enters, B has two actions “enter” or “not enter”. If B enters, the pay-off would be -1m and if B does not enter, the pay-off would be 0. In this situation, B would not enter the market, and then the possibility of B entering the market can be deleted as shown in the figure. Now looking at the situation when A does not enter the market, B also has two possible actions “enter” or “not enter”. Similarly, the action of B not entering the market could be deleted because the action B enters the market obtains higher pay-off say 5m against 0. According to the principle of Back induction, initial node would be considered at final stage where player A locates at. For A, if he enters, he will receive 5m, and if he does not enter he will gain nothing. Apparently, the action A enters the market is in his best interest to do and vector (0, 5m) can be deleted as described. The unique subgmae perfect Nash equilibrium (5m, 0) is determined, which is the same as the solution shown in the subgame Nash equilibrium technique.

3.4 Summary

In static games, players make decisions simultaneously for which such games are represented in normal form other than extensive form. They can be solved through two major techniques “dominance” and “equilibrium”. Imprecise predictions may occur when applying dominance techniques to games when more than two players are involved. Accordingly, a stronger solution Nash equilibrium is introduced to meet the situation.

In dynamic games, the ordering that players make decision makes the game dynamic and expands the available strategies more greatly than actions. Because of issues of timing and information sets, dynamic games are normally represented in extensive form. Dynamic games are also able to be solved by Nash equilibrium however situation of multiple equilibria might happen in which incredible statements among the equilibria exist. Subgame perfect Nash equilibrium method is adopted to refine it until unique subgame perfect Nash equilibrium is derived. Backward induction is a quicker way to solve the game than subgame perfect Nash equilibrium method does but with strict assumptions.

For preparation of applying Cournot equilibrium to an oligopoly electricity market, basic knowledge of non-cooperative game theory is reviewed briefly. As Cournot equilibrium is discussed in depth in chapter 3 and 4, it is not mentioned in this chapter. Moreover, both methods of Cournot and Nash bring the same equilibrium which is often called the Cournot-Nash equilibrium [10]. It is noted that examples of games presented in this chapter are all related to complete information which means pay-offs of each player are common knowledge to all players in the game. This concept will be frequently quoted in the following chapters.

CHAPTER 4

METHODOLOGY AND MATHEMATICAL MODEL OF COURNOT COMPETITION THEORY FOR OLIGOPOLY ELECTRICITY MARKET PRODUCTION

4.1 Introduction

In this chapter, it is discussed in details that Cournot competition method in determining optimal production for suppliers in an oligopoly market. In addition, both situations under complete information and incomplete information are investigated and mathematical models are established in the environment that specific number of suppliers compete to maximize their production. Cournot equilibrium solutions originated by F. S. Wen and A. K. David in different situations under complete and incomplete information are derived as well and given in equations in this chapter.

The evolution of electricity market towards a deregulated and competitive one has become a trend from around the world which is discussed in abundant materials. At the early stage of deregulation, an electricity supply structure is more like an oligopoly market other than a perfect competitive one. This is shown and examined in the example of initial deregulating process in England and Wales [1]. As a result, market suppliers are able to increase their own profits through strategic bidding [32]. The oligopoly feature of electricity market occurring in early period is mainly caused by long construction cycle of power plants, large capital investment for new entrants as well as the isolation between consumers and generators that arises from transmission constraints and transmission line losses [1]. Accordingly, in such oligopoly electricity markets, there are only a few suppliers that dominate the market and compete with each other which is commonly quoted oligopoly.

The feature of this market structure is that the suppliers are interdependent players to maximize their profits through strategically bidding. This is to say, profits of the suppliers are affected and determined by actions of others which makes possible to apply game theory to oligopoly markets [10]. In comparison with oligopoly market structure, there are two extreme situations, monopoly and perfect competition. Under monopoly that power industry held for a long time, there is only one suppler dominating the market in which interdependence does not exist and therefore game theory can not be applied to. Under perfect competition market structure, the interdependence does not exist either as all players are assumed to be price takers, which means their profits are not dependent on actions of others. Accordingly, game theory is applicable to the situation of oligopoly market other than these two structures.

Abundant literatures have been presented to analyze the issues relating to oligopoly electricity market [3][13][14]. However, these methods are only applied to handle oligopoly cases under complete information which implies every supplier has complete information about any other players. This is not realistic in practice because suppliers would never have accurate information about their competitors to decide how much they produce, which is called competition under incomplete information. In order to meet practical goals, estimates of this vague information are introduced to help players to decide their optimal production. As a popular method, Cournot competition theory is selected in this thesis for the purpose that analyzes oligopoly markets under incomplete information.

4.2 Cournot Competition Theory in Oligopoly Markets

There are three classical models used for analysis of oligopoly markets i.e. Cournot, Stackelberg and Bertrand. All three models are applied to predict the possible actions of oligopolistic players. They have similarities in each other however researching target is different. In Cournot competition players interact with each other at the same time to decide how much quantity of products supplied to the market i.e. optimal production. In Stackelberg competition, it is still related to quantity of production but it is allowed that one or some players set their output level initially which makes the game dynamic. For Bertrand competition, price is the major researching issue which has to be determined by all players at the same time. This thesis is focused on the application of Cournot competition model as the optimal production quantity is the researching target. In this section, one-off games are considered when examining the theory of Cournot competition.

The process of Cournot competition model is that once the total output is determined and then market is cleared at the market price which is derived from demand curves. Some assumptions are made before looking further. It is assumed that players supplying the same product to the market are rational players. For explanation, a duopoly game is given below that shows in an extensive form. In the game, two big firms, firm 1 and firm 2, dominate the market and interact with each other determining their supply quantity simultaneously. Dotted line represents simultaneous decisions.

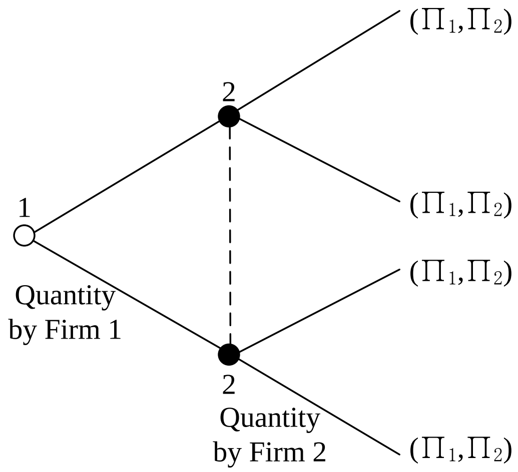

Figure 4.1 *Extensive form game of Cournot Competition for duopoly*

From the figure, this is a static game that strategies are equal to actions and q_1

and q_2 are any quantities supplying to the market. The pay-offs for firm 1 and firm 2 are Π_1 and Π_2 respectively which are profits each firm earns. In fact, there are a lot of available actions and pay-offs for each firm which are not shown in the figure.. According to the theory, suppliers determine their optimal supply level simultaneously, and then market price is settled, and finally these firms receive their profits while the market is cleared [10].

Considering the duopoly Cournot competition as a static game, it can be solved by techniques discussed in last chapter. Nash equilibrium method is adopted to find the outcome of the game since it reaches the same result as Cournot competition method does. Before doing this, the reaction function of each player is introduced to show their optimal supply level which indicates that the output of suppliers is dependent on outputs of others. The derivation of such reaction functions for both firms is represented and explained as follows.

Inverse demand function:

$$P = a - Q \tag{4.1}$$

Where P represents market price, Q is the total supply that is a sum of the outputs of firm 1 and firm 2 (i.e. $Q = q_1 + q_2$) and a, a positive constant.

For Firm 1

Profit that firm 1 earns is

$$\Pi_1 = Pq_1 - cq_1 \tag{4.2}$$

Where Π_1 is the pay-off of firm 1, c is the cost of production

Substituting 4.1 and $Q = q_1 + q_2$ into equation 4.2 derives

$$\Pi_1 = (a - q_1 - q_2)q_1 - cq_1$$

For maximization of output of firm 1, q_1, by setting

$$\frac{d\Pi_1}{dq_1} = a - 2q_1 - q_2 - c = 0$$

Obtains reaction function of firm 1

$$q_1 = \frac{a - q_2 - c}{2} \tag{4.3}$$

Maximization happens when second order derivative is less than zero

$$\frac{d^2\Pi_1}{dq_1^{\,2}} = -2 < 0$$

For Firm 2

The profit of firm 2 is given in

$$\Pi_2 = Pq_2 - cq_2 \tag{4.4}$$

Together with 4.1 and $Q = q_1 + q_2$ obtains

$$\Pi_2 = \left(a - q_1 - q_2\right)q_2 - cq_2$$

For maximization of output of firm 2, q_2, by setting

$$\frac{d\Pi_2}{dq_2} = a - q_1 - 2q_2 - c = 0$$

It derives reaction of firm 2

$$q_2 = \frac{a - q_1 - c}{2} \tag{4.5}$$

Maximization happens when

$$\frac{d^2\Pi_2}{dq_2^{\,2}} = -2 < 0$$

Equations 4.3 and 4.5 are the so called reaction functions which are derived by differentiating the profit functions of both firms concerning the corresponding supply quantities and setting this to zero [10]. In the second-order derivative, the outcome is less than zero that means maximum of outputs of the firms is valid on a mathematical basis. The Cournot-Nash equilibrium can be illustrated through a diagram including curves of reaction functions of each firm. It is illustrated in Figure 4.2.

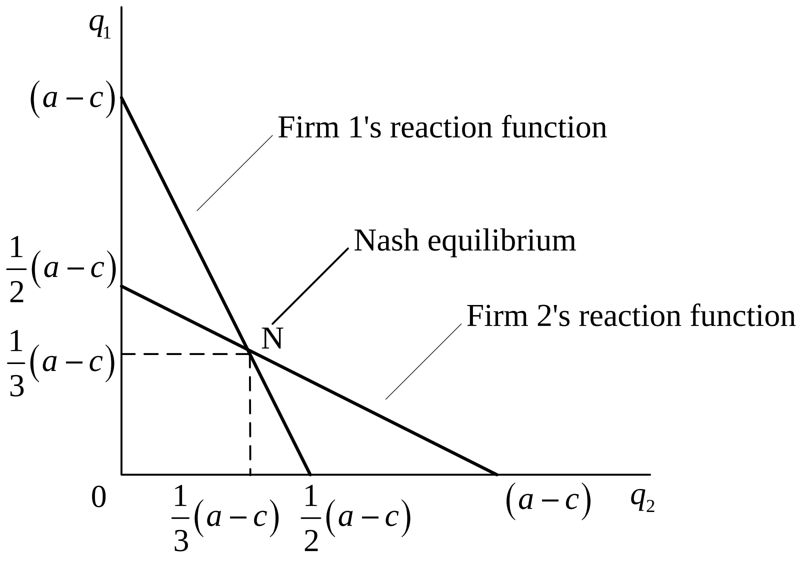

Figure 4.2 *Cournot-Nash equilibrium for two Duopoly firms [10]*

For firm 1, the maximum profit occurs when its output is maximized while the other firm's output is nothing and vice versa. The corresponding quantity would be $\frac{1}{2}(a-c)$ supplying to the market. This means the further the output of firm supplying to the market is away from this value the smaller its profit. In Nash equilibrium, players decide their optimal strategies at the same time which is described in figure 4.2 at point N. Therefore, the output of either firm is $\frac{1}{3}(a-c)$ at Nash equilibrium in this situation. This equilibrium is also called Cournot-Nash equilibrium [10] as Cournot found this result originally and Nash developed it. Different from the way Nash does, Cournot found this equilibrium "by analyzing how the firms would react when they were out of equilibrium" [10]. Both methods can reach the same outcome that it is often called the Cournot-Nash equilibrium.

4.3 The Model to Be Investigated of an Oligopoly Electricity Market

As stated, the purpose that electricity markets to be reformed towards deregulated competitive ones is increasing efficiency and decreasing market price through introduction of competition. Due to special features that investment barrier to new entrants, transmission constraints and losses, the earlier electricity market looks more like an oligopoly market other than competitive one. An oligopoly market is defined as a market that a few big firms supplying a homogeneous product and dominating it. Based on this, a model of oligopoly electricity market is presented and Cournot solutions are given in this section.

4.3.1 Application of Cournot competition on an oligopoly electricity market

The relationship of demand of a large amount of consumers and the market price can be seen in an inverse demand function and represented in an inverse demand curve. It is drawn in figure 4.3.

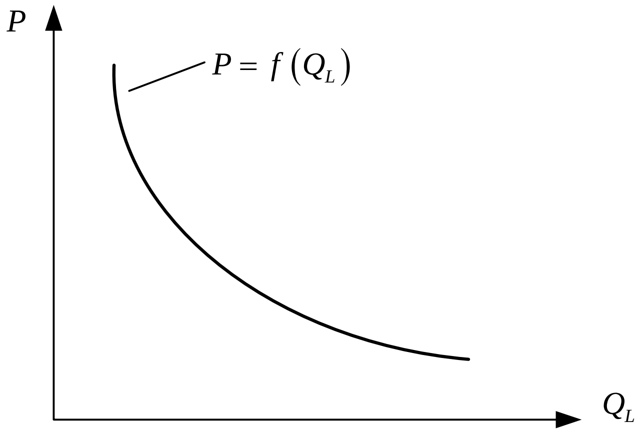

Figure 4.3 *The Inverse Demand Curve of* $P = f(Q_L)$

In the inverse demand function, P is the market price and Q_L denotes the total output of suppliers in the market. Assuming that q_i is the output of the i th supplier, the total load is then represented as $Q_L = \sum_{i=1}^{n} q_i$. Each supplier has a cost function $C_i(q_i)$.

Therefore, the profit function for each supplier can be defined as

$$U_i(Q) = q_i f(Q_L) - C_i(q_i) \tag{4.6}$$

Where $Q=\left(q_1,q_2,...,q_n\right)$

The profit functions are also called pay-off functions in the following sections. This is extracted from the model of oligopolistic electricity market simulated later on.

Generally game theory has two categories, cooperative and non-cooperative game theory which is determined by whether players in a game communicating with each other. The model under oligopoly market structure in the thesis is analysed based on non-cooperative game theory. As discussed in the previous chapter, due to features of interdependence, oligopoly markets can be analysed by game theory method. In the thesis, Cournot competition model is used to fulfill this task and find market equilibrium state in an oligopoly market. A common method of Cournot game theory is presented in literatures [32] [33] to reach equilibriums in which players make decisions simultaneously without communicating with each other. According to the literatures, a Cournot equilibrium state is a set of optimal supply quantities, $Q^*=\left(q_1^*,q_2^*,...,q_i^*,...,q_n^*\right)$ which are derived from each supplier's maximization function. Cournot equilibrium state can be found by differentiating the profit functions of each supplier's, which is $\frac{dU_i(Q)}{dq_i}=0 \quad i=1,2,n.$ [10] [32] [33]. Hence a Cournot equilibrium can be defined as a state when profits of each supplier are maximized through finding optimal production of them. The process of reaching such equilibrium states is generalized in figure 4.4 below.

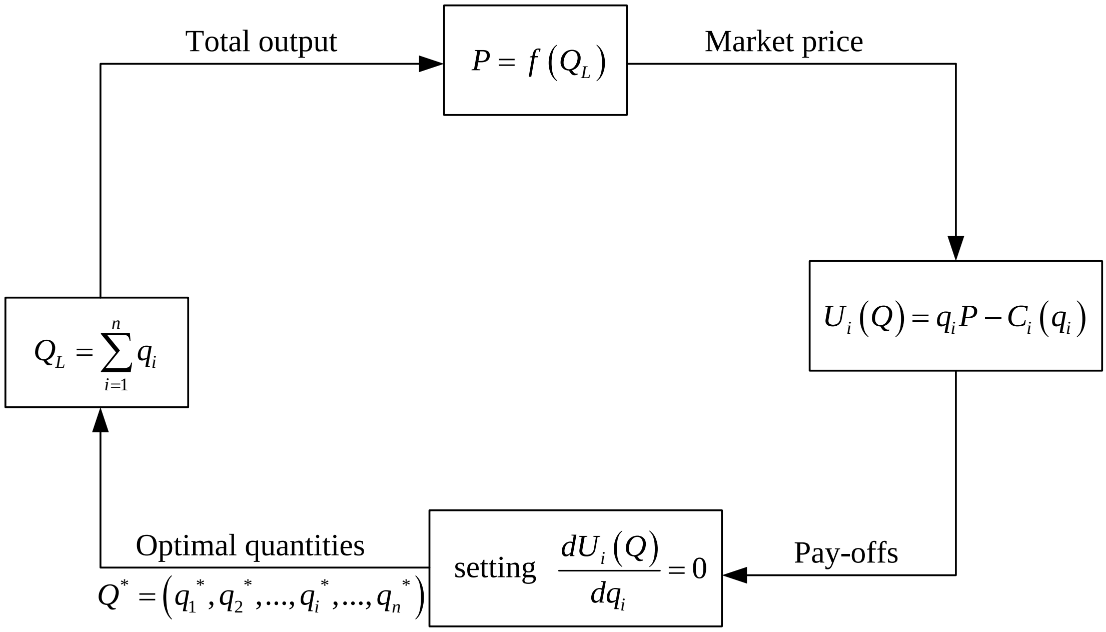

Figure 4.4 *Cournot equilibrium state in an Oligopoly electricity market*

It is noted that optimal quantities are valid when $\frac{d^2U_i(Q)}{dq_i^2} < 0$ [10] [32] [33].

4.3.2 Oligopoly electricity market model to be investigated and its Cournot equilibrium under complete information

A model of an oligopoly electricity market with basic components is established in this section in consistent with the one used in literature [11]. In the model, the demand function is a linear one that is defined as follows.

$$Q_L = Q_{L\max} - P\left(\frac{Q_{L\max}}{P_{\max}}\right) \tag{4.7}$$

Where Q_L is the total production,

P is the market price,

$Q_{L\max}$ is the maximum quantity of total available output when $P = 0$,

$P_{\max}$ is the upper limit of the market price that means there is no consumers who want to buy electricity when market price is higher than $P_{\max}$.

Rearranging the equation 4.6, the inverse demand function is in the form with a gradient K and market price P is described as a function of Q_L.

$$P = P_{\max} - KQ_L = f(Q_L) \tag{4.8}$$

Where, $K = \dfrac{P_{\max}}{Q_{L\max}}$

The linear feature between market price and production can be shown in figure 4.5, in which intercepts of axes are maximum production and price respectively.

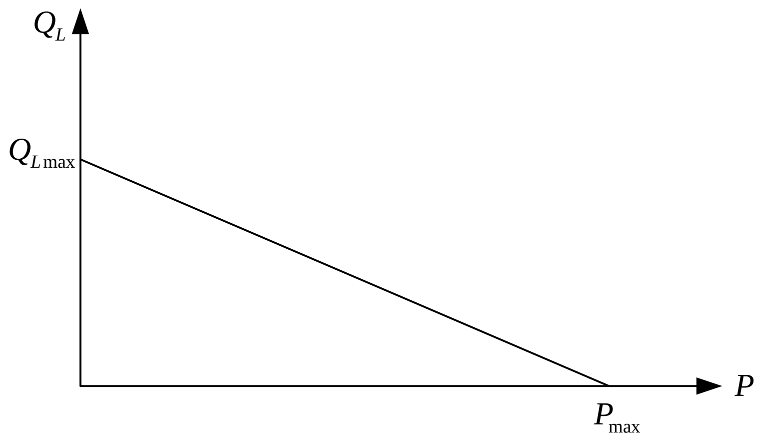

Figure 4.5 *A Linear Demand Curve of* $P = f\left(Q_L\right)$

In this model, cost functions of each supplier are defined as quadratic ones. Cost functions of each supplier are represented in equation 4.9.

$$C_i\left(q_i\right) = a_i + b_i q_i + c_i q_i^{\,2} \qquad i = 1, 2, \ldots, n \tag{4.9}$$

Where a_i, b_i and c_i are cost coefficients of the i th supplier which are constants. Profit function of the i th supplier is presented in equation 4.6 that regenerated below

$$U_i\left(Q\right) = q_i P - C_i\left(q_i\right) \qquad i = 1, 2, \ldots, n \tag{4.10}$$

In the model it is assumed that each generator is a single supplier selling electricity to the market. Transmission constraints and losses are ignored in the preliminary model. Improvements are made in the extension model in chapter 6. Lastly, regulatory policies are known that may have an effect on electricity trading which is beyond the scope of the thesis.

4.4 Cournot Equilibrium Solutions under Complete Information and Incomplete Information

Based on the model of oligopoly electricity market in section 4.3, Cournot equilibrium solutions when considering complete information and incomplete information are derived and presented in equations in this section.

4.4.1 Cournot equilibrium state under complete information

In the established model of an oligopoly electricity market, complete information represents that each supplier knows their own cost functions as well as their rivals'. Accordingly, market price-demand function is known by all suppliers as well. Ignoring transmission losses, the total load is simply the aggregation of outputs of all suppliers.

$$Q_L = \sum_{i=1}^{n} q_i \tag{4.11}$$

Substituting equation 4.9 and 4.11 into pay-off function 4.10, it obtains

$$U_i(Q) = -(K + c_i)q_i^2 + \left(P_{\max} - K\sum_{\substack{j=1 \\ j \neq i}}^{n} q_i - b_i \right) q_i - a_i \qquad i = 1, 2, ..., n \tag{4.12}$$

Market equilibrium state is reached by differentiating equation 4.12 and letting it equal to zero, gives

$$\frac{dU_i(Q)}{dq_i} = -2(K + c_i)q_i - K\sum_{\substack{j=1 \\ j \neq i}}^{n} q_j + P_{\max} - b_i = 0 \qquad i = 1, 2, ..., n \tag{4.13}$$

Cournot equilibrium solution under complete information is derived from equation 4.13 through mathematical manipulation. Equation 4.14 gives the optimal production for the i th supplier.

$$q_i = \frac{P_{max} - b_i - K\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{b_i - b_j}{K + 2c_j}}{2\left(K + c_i\right) + K\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{K + 2c_i}{K + 2c_j}} \qquad i = 1, 2, ..., n \tag{4.14}$$

In equation 4.14, the optimal production of each supplier is associated with cost coefficients and the ratio K. It is noted that coefficient a_i has no influence on the result. The case under complete information is described as the base case which is seen as a reference.

4.4.2 Cournot equilibrium state under incomplete information

Comparing with the situation under complete information, Cournot equilibrium state under incomplete information is more complicated to achieve as uncertainties about cost functions of their rivals are introduced. In the circumstances, suppliers in the market do not know exactly their rivals' cost functions. Due to these uncertainties, the process of estimation is introduced trying to find the equilibrium under incomplete information.

In situations of incomplete information, suppliers have to estimate their rivals' cost functions to decide their supplying level in order to maximize their profits. Here, suppliers are assumed as rational players that in their own best interests to compete. Three methods to estimate cost functions of the rivals that presented in F. S. Wen and A. K. David's literature [11] will be reviewed in this section. These methods are included in three cases under incomplete information and corresponding Cournot equilibrium solutions are given at the end of the case.

4.4.2.1 Mathematical model for Case 1

In this case, each suppler has only one estimated cost function for each of their rivals.

The cost function of the i th supplier is

$$C_i\left(q_i^{(i)}\right)=a_i+b_iq_i^{(i)}+c_i\left(q_i^{(i)}\right)^2 \qquad i=1,2,...,n \tag{4.15}$$

Where $q_i^{(i)}$ is the output of the i th supplier which is known already.

The cost function of the j th supplier that is estimated by the i th supplier is

$$C_j^{(i)}\left(q_j^{(i)}\right)=a_j^{(i)}+b_j^{(i)}q_j^{(i)}+c_j^{(i)}\left(q_j^{(i)}\right)^2 \qquad i,j=1,2,...,n \tag{4.16}$$

Where $q_j^{(i)}$ is the output of the j th supplier which is estimated by the i th supplier. It is noted that subscript j represents the estimated object and superscript i represents the supplier who estimates.

The total load that is estimated by the i th supplier is

$$Q_L^{(i)}=\sum_{j=1}^{n}q_j^{(i)} \qquad i=1,2,...,n \tag{4.17}$$

Together with equation 4.10 and 4.17, the pay-off function of the i th supplier will be

$$U_i\left(Q\right)=q_i^{(i)}f\left(Q_L^{(i)}\right)-C_i\left(q_i^{(i)}\right) \qquad i=1,2,...,n \tag{4.18}$$

The pay-off function of the j th supplier that is estimated by the i th supplier will be

$$U_j^{(i)}\left(Q\right)=q_j^{(i)}f\left(Q_L^{(i)}\right)-C_j^{(i)}\left(q_j^{(i)}\right) \qquad i,j=1,2,...,n \quad i\neq j \tag{4.19}$$

Cournot equilibrium state of the i th supplier is gained by setting

$$\frac{dU_i\left(Q\right)}{dq_i^{(i)}}=0 \qquad i=1,2,...,n \tag{4.20}$$

Similarly, Cournot equilibrium state of the j th supplier is gained by setting

$$\frac{dU_j^{(i)}\left(Q\right)}{dq_j^{(i)}}=0 \qquad i,j=1,2,...,n \quad i\neq j \tag{4.21}$$

It is noted that the latter equilibrium is determined by the estimates of i th supplier. Substituting equation 4.8, 4.15 and 4.18 into 4.20 it derives

$$\frac{dU_i(Q)}{dq_i^{(i)}} = -2(K+c_i)q_i^{(i)} - K\sum_{\substack{j=1\\j\neq i}}^{n} q_j^{(i)} + P_{\max} - b_i = 0 \qquad i=1,2,...,n \tag{4.22}$$

Similarly substituting equations 4.8, 4.16 and 4.19 into 4.21 it derives

$$\frac{dU_j^{(i)}(Q)}{dq_j^{(i)}} = -2\left(K+c_j^{(i)}\right)q_j^{(i)} - K\sum_{\substack{l=1\\l\neq i\\l\neq j}}^{n} q_l^{(i)} - Kq_i^{(i)} + P_{\max} - b_j^{(i)} = 0$$

$$i, j = 1,2,...,n \quad i \neq j \tag{4.23}$$

Together with equation 4.22 and 4.23, the optimal production for the i th supplier is obtained in equation 4.24 through mathematical manipulation. Cournot equilibrium solution for case 1 is represented in equation 4.24.

$$q_i^{(i)} = \frac{P_{\max} - b_i - K\sum\limits_{\substack{j=1\\j\neq i}}^{n}\dfrac{b_i - b_j^{(i)}}{K+2c_j^{(i)}}}{2(K+c_i) + K\sum\limits_{\substack{j=1\\j\neq i}}^{n}\dfrac{K+2c_i}{K+2c_j^{(i)}}} \qquad i=1,2,...,n \tag{4.24}$$

Based on the expected optimal output for the i th supplier, the total expected load when Cournot equilibrium reaches would be

$$Q_L^* = \sum_{i=1}^{n} q_i^{(i)} \tag{4.25}$$

The equation 4.24 is used to determine the optimal output for the i th supplier and it will be tested in an numerical model in chapter 5.

4.4.2.2 Mathematical model for Case 2

In this case, each suppler has more than one estimated cost function for each of their rivals however one is selected from these functions that is based on a function of probability.

The cost function of the i th supplier is the same as equation 4.15. The cost function of the j th supplier that is estimated by the i th supplier is shown in equation 4.26.

$$C_{j,t}^{(i)}\left(q_{j,t}^{(i)}\right)=a_{j,t}^{(i)}+b_{j,t}^{(i)}q_{j,t}^{(i)}+c_{j,t}^{(i)}\left(q_{j,t}^{(i)}\right)^2$$
$$i,j=1,2,...,n \quad j\neq i \quad t=1,2,...,T_j^{(i)} \tag{4.26}$$

Where $T_j^{(i)}$ is the total number of the cost functions for the j th supplier, which is estimated by the i th supplier. $q_{j,t}^{(i)}$ is the output of the j th supplier that is estimated by the i th supplier.

Different from case 1, selection of cost functions are associated with a probability factor. Therefore, the expected total production that is estimated by the i th supplier would be

$$Q_L^{(i)}=q_i^{(i)}+\sum_{t_1\in\left[1,T_1^{(i)}\right]}\cdots\sum_{\substack{t_l\in\left[1,T_l^{(i)}\right]\\ l\neq i}}\cdots\sum_{t_n\in\left[1,T_n^{(i)}\right]}r_{1,t_1}^{(i)}...r_{l,t_l}^{(i)}...r_{n,t_n}^{(i)}$$
$$\times\left(q_{1,t_1}^{(i)}+...+q_{l,t_1}^{(i)}+...+q_{n,t_n}^{(i)}\right) \tag{4.27}$$

Where $r_{l,t_l}^{(i)}$ is the probability of selecting the cost function of the l th supplier which is estimated by the i th supplier. Accordingly the cost function will be $c_{l,t_l}^{(i)}\left(q_{l,t_l}^{(i)}\right)$.

Moreover

$$\sum_{t_l=1}^{T_l^{(i)}}r_{l,t_l}^{(i)}=1 \qquad l,i=1,2,...,n \quad l\neq i \tag{4.28}$$

Substituting 4.28 into 4.27, it derives

$$Q_L^{(i)}=q_i^{(i)}+\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{t=1}^{T_j^{(i)}}r_{j,t}^{(i)}q_{j,t}^{(i)} \qquad i=1,2,...,n \tag{4.29}$$

The pay-off function of the i th supplier is the same as 4.18 and the pay-off function of the j th supplier that is estimated by the i th supplier will be

$$U_{j,t}^{(i)}(Q)=q_{j,t}^{(i)}f\left(Q_L^{(i)}\right)-C_{j,t}^{(i)}\left(q_{j,t}^{(i)}\right) \qquad i,j=1,2,...,n \quad i\neq j \quad t=1,2,...,T_j^{(i)} \tag{4.30}$$

The Cournot equilibrium state of the i th supplier can be reached in this case by setting

$$\frac{dU_i(Q)}{dq_i^{(i)}} = 0 \qquad i = 1,2,...,n$$

For the j th supplier, by setting

$$\frac{dU_{j,t}^{(i)}(Q)}{dq_{j,t}^{(i)}} = 0 \qquad i = 1,2,...,n \quad i \neq j \quad t = 1,2,...,T_j^{(i)}$$

After mathematical manipulation, the optimal production for the i th supplier is derived that is shown in 4.31. It is the Cournot equilibrium solution to case 2.

$$q_i^{(i)} = \frac{P_{max} - b_i - K\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{t=1}^{T_j^{(i)}} r_{j,t}^{(i)} \frac{b_i - b_{j,t}^{(i)}}{K + 2c_{j,t}^{(i)}}}{2(K + c_i) + K\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{t=1}^{T_j^{(i)}} r_{j,t}^{(i)} \frac{K + 2c_i}{K + 2c_{j,t}^{(i)}}} \qquad i = 1,2,...,n \tag{4.31}$$

Comparing with case 1, the Cournot equilibrium state in this case is associated with coefficients of cost function of themselves and their rivals as well as corresponding probability factor.

4.4.2.3 Mathematical model for Case 3

In this case, each suppler has only one estimated cost function for each of their rivals however a distribution function is considered when estimating.

The cost function of the i th supplier is the same as 4.15 while the cost function of the j th supplier that is estimated by the i th supplier is shown as follows

$$C_j^{(i)} = a_j^{(i)} + b_j^{(i)} q_j^{(i)} + c_j^{(i)} \left(q_j^{(i)}\right)^2 + \xi_j^i \qquad i, j = 1,2,...,n \quad j \neq i \tag{4.32}$$

Where $\xi_j^{(i)} \in N\left[0, \sigma^2\left(C_j^{(i)}\right)\right] \qquad i, j = 1,2,...,n \quad j \neq i$

$N\left[0, \sigma^2\left(C_j^{(i)}\right)\right]$ is a normal distribution (or a Gaussian distribution) in which mean is 0 and standard deviation σ is defined as $\sigma\left(C_j^{(i)}\right)$. Let $\sigma\left(C_j^{(i)}\right) = gq_j^{(i)}$, where g is a specified positive constant [11]. In this case, the total estimated production function that is estimated by the i th supplier is the same as equation

4.17. Pay-off functions of the i th supplier and the j th supplier are still the same as those in equation 4.18 and 4.19.

The mean and standard deviation functions are given below

$$\overline{U}_j^{(i)}(Q) = q_j^{(i)} f\left(Q_L^{(i)}\right) - a_j^{(i)} - b_j^{(i)} q_j^{(i)} - c_j^{(i)}\left(q_j^{(i)}\right)^2 \qquad i, j = 1, 2, \ldots, n \quad j \neq i \qquad (4.33)$$

$$\sigma\left[U_j^{(i)}(Q)\right] = g q_j^{(i)} \qquad i, j = 1, 2, \ldots, n \quad j \neq i \qquad (4.34)$$

In literature [34], this problem can be solved by using the weighted sum maximization.

$$\text{Maximize} \quad \Psi_j^{(i)}(Q) = \overline{U}_j^{(i)}(Q) - \alpha\sigma\left[U_j^{(i)}(Q)\right] \qquad i, j = 1, 2, \ldots, n \quad j \neq i \qquad (4.35)$$

Where α is a specified constant between 3 and 4 [34]

The Cournot equilibrium state can be found by setting

$$\frac{dU_i(Q)}{dq_i^{(i)}} = 0 \qquad i = 1, 2, \ldots, n$$

And

$$\frac{d\Psi_j^{(i)}(Q)}{dq_j^{(i)}} = 0 \qquad i, j = 1, 2, \ldots, n \quad j \neq i$$

After mathematical manipulation, the optimal production for the i th supplier can found in equation 4.36. It is the Cournot equilibrium solution to case 3.

$$q_i^{(i)} = \frac{P_{\max} - b_i - K\sum\limits_{\substack{j=1 \\ j \neq i}}^{n} \dfrac{b_i - b_j^{(i)} - \alpha g}{K + 2c_j^{(i)}}}{2\left(K + c_i\right) + K\sum\limits_{\substack{j=1 \\ j \neq i}}^{n} \dfrac{K + 2c_i}{K + 2c_j^{(i)}}} \qquad i = 1, 2, \ldots, n \qquad (4.36)$$

It is seen that the optimal output level is determined by not only cost coefficients but also coefficients α and g which are extracted from the distribution function.

4.5 Summary

Interdependence is the feature of an oligopoly electricity market that players interact with each other constantly. This makes possible the application of game theory on oligopoly electricity markets as game theory is to analyze how rational players make decisions when they are mutually interdependent [10]. Cournot competition theory is related to non-cooperative game theory that is used to determine optimal production level of each player involved. It is seen as a common method for oligopolistic markets studying which is used to find equilibrium state in terms of quantity. It is demonstrated in the section by using a duopoly electricity market in which the equilibrium state is the intersected point of curves of suppliers' reaction functions.

In the early periods of deregulation, electricity market has oligopolistic features other than perfect competition pattern. A mathematical model of this structure to be investigated in next chapter is organized that is based on a linear demand function. In the model transmission constraints and losses are neglected that will be included in chapter 6. Cost functions of suppliers are defined in quadratic form and pay-off functions to each supplier are given as well. The Cournot equilibrium state is worked out by setting derivative of such pay-off functions to zero.

Situations under complete information and incomplete information are both studied in this chapter and corresponding mathematical models are established. It is found that the outcome in Cournot equilibrium state is mainly dependent on the cost functions of the rivals. In complete information, such cost functions are known by all suppliers, so there is no problem with acquiring them. However, under incomplete information these functions have to be estimated therefore the outcome is also related to the accuracy of estimation. Three methods of estimation are introduced in the chapter and Cournot equilibrium solutions for these situations are given in equations correspondingly in this chapter. Numerical examples and simulation results of these models are discussed in Chapter 5.

CHAPTER 5

NUMERICAL EXAMPLE AND RESULTS

5.1 Introduction

A numerical example based on the mathematical model presented previously is given in this chapter in which six generators compete with each other for optimal production and maximization of profits. It is assumed that no transmission constraints and losses are involved and the target market uses a single sided (competition in generation only) power exchange. Six generators supply and sell electricity to the market, which are also called suppliers in this case. Besides it assumes that generators could produce and supply electricity to the market as much as they want in their capability without worrying about the demand. Numerical models under compete information and incomplete information are established and simulation results are given accordingly with a brief discussion.

5.2 Numerical Example and Results of Base Case

Base case under complete information is regarded as a benchmark for comparison with various situations. In this section, the mathematical model established in chapter 4 with specific market parameters under complete information is presented and simulation results are presented.

5.2.1 Numerical model for Base Case

The target oligopoly market where six generators compete with each other is illustrated in figure 5.1.

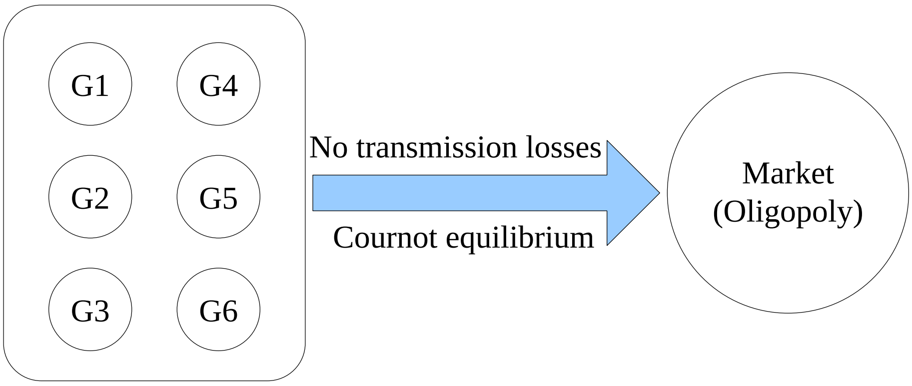

Figure 5.1 *The target oligopoly market with six generators*

Cost function coefficients of the six generators are given in table 5.1, it is assumed that generators have a quadratic cost function of the form $a+bx+cx^2$ that is shown in equation (4.9). The values of $P_{\max}$ and $Q_{L\max}$ in equation (4.7) are defined as 20 and 500 respectively and therefore K in equation 4.7 is 0.04 which are consistent with literature [11].

Coefficients	a	b	c	$P_{\max}$	$Q_{L\max}$	K
G1	0	2	0.00375	20	500	0.04
G2	0	1.75	0.0175			
G3	0	1	0.0625			
G4	0	3.25	0.00834			
G5	0	3	0.025			
G6	0	3	0.025			

Table 5.1 *Cost coefficients and basic parameters for base case*

It is noted that transmission constraints and losses are ignored in this market model and coefficient a is zero for all generators, which means it has no influence on results and all cost functions pass through the origin. The cost coefficients of generators **G5** and **G6** are the same values in order to distinguish the behaviors under complete and incomplete information in the later sections. The cost curves for six generators are given in figure 5.2. It is noted that there is a mixture of costs associated with the generators, in particular is noted that come of these curves cross which leads to least costs solutions being exchanged between generators.

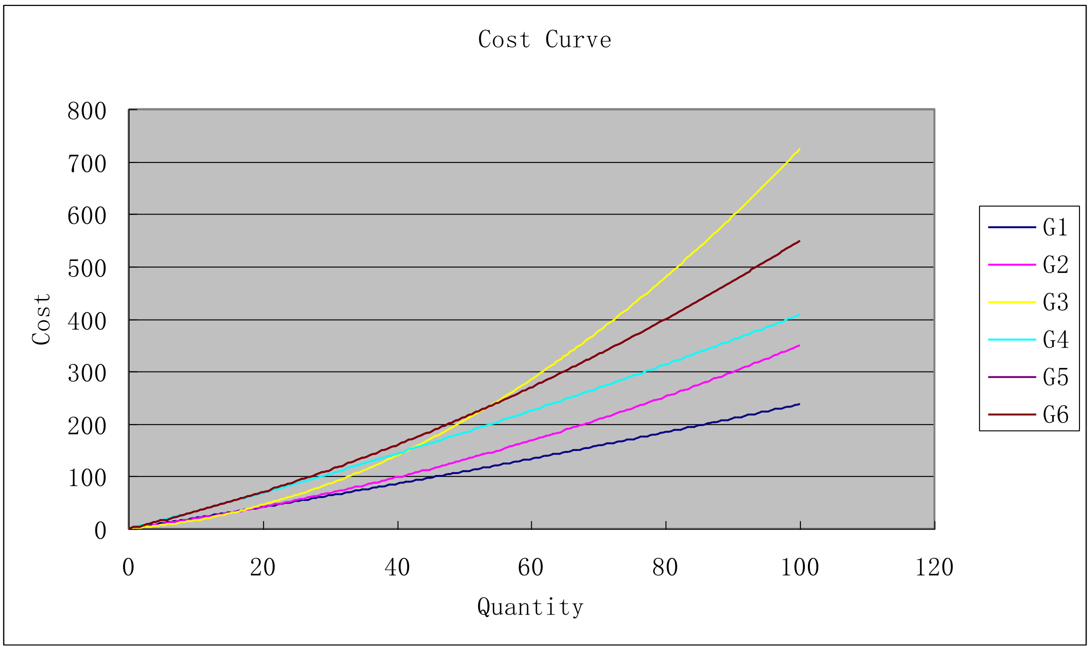

Figure 5.2 *Cost curves of six generators*

In the figure, quantity and cost are represented without units as the market model is hypothetical with a lot of assumptions which is away from the real one. Normally, electricity produced is measured in MWh while costs in $/MWh, ￡/MWh. As the cost functions of **G5** and **G6** are the same, the cost curves of them are represented by one in the line. The cost functions exhibit the quadratic component and generally the cost goes up while the output is growing. But nonetheless, how fast the cost increases depends on the cost coefficients. Taking **G3** as an example, below the output level of around 20 the cost grows slowly, however, above 20 the cost increases greatly when the output grows. It will be the most costly production when the output exceeds 60.

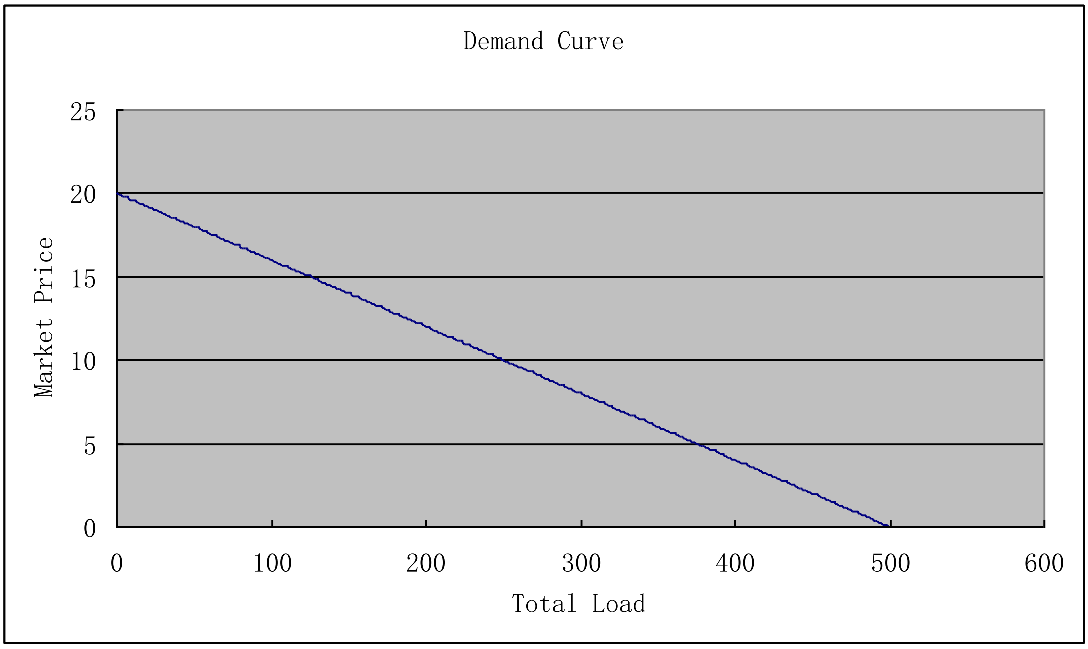

Figure 5.3 *Demand curve of the target market*

Using the parameters from table 5.1 the demand curve (derived from equation (4.8)) is shown in figure 5.3. As shown, demand curve is linear while cost curves are quadratic. Besides, load and price are represented in numbers without units.

5.2.2 Results of Base Case under complete information

The test results of base case under complete information are presented in table 5.2 below. This equilibrium state is found through the solution in equation (4.14). The testing interface is given in Appendix A.

Producer No.	Output	Profit	Price
G1	96.932	411.064	6.604
G2	64.723	240.874	
G3	33.965	118.247	
G4	59.179	169.293	
G5	40.047	104.246	
G6	40.047	104.246	
Total	334.894	1147.971	

Table 5.2 *The optimal production for base case under complete information*

The Cournot equilibrium solution, which is shown in table 5.2, is a group of supply quantities. The optimal pay-offs (profits) for the generators have been previously shown to be dependent on the cost functions of their rivals in terms of cost coefficients (see section 4.4.1). It is noted that the optimal output and profit of **G5** and **G6** are exactly the same under complete information as their cost functions are identical. For clarity, a chart about outputs and profits for each generator is provided below in figure 5.4.

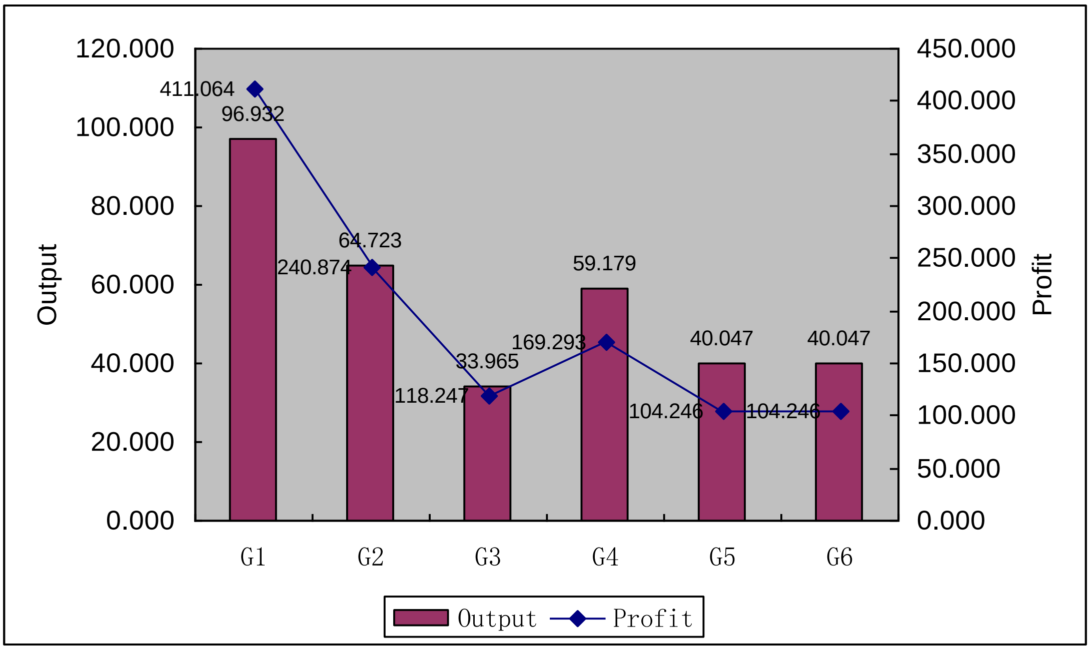

Figure 5.4 *The line-column chart of optimal outputs & profits for base case*

As the cost of **G1** is the lowest, in Cournot equilibrium solution it produces more electricity to the market and earns more than other producers. The contribution that **G5** and **G6** based on their rating and cost function means that they make the same profit. An interesting thing shown in the figure is that **G3** produces less electricity but earns more comparing with **G5** and **G6**. This is because the cost of **G3** is lower than those of **G5** and **G6** when the output is below 40 which can be seen from figure 5.2. Under complete information, suppliers know exactly the cost functions of their rivals as well as demand function of the market which helps them to calculate how much profit they gain at specific level of output by using

Cournot equilibrium solution. The complete information solution provides the theoretical equilibrium for the market place.

5.3 Results of Case 1 under Incomplete Information

In this section, the mathematical models established in chapter 4 with specific market parameters under incomplete information are presented and simulation results are presented. Under incomplete information, there are three cases to be investigated which are distinguished by estimated methods of cost functions about their rivals. In case 1, each suppler has only one estimated cost function for each of their rivals.

5.3.1 Numerical model for Case 1

In this case, each supplier knows their own cost functions however the costs about their rivals are not as clear as those in base case. The basic parameters stay the same as those used in the base case (see table 5.1).

Different from the base case, the information about the cost functions of the rivals has to be estimated so that the Cournot equilibrium state can be worked out. Consequently, the uncertainty is introduced when cost coefficients need estimating. It is assumed that the coefficients of the rivals' cost functions are obtained through the equations below.

$$a_j^{(i)} = (1-\gamma_1)a_j, \qquad b_j^{(i)} = (1-\gamma_2)b_j, \qquad c_j^{(i)} = (1-\gamma_3)c_j$$
$$i, j = 1, 2, ..., 6 \quad i \neq j \tag{5.1}$$

Where $a_j^{(i)}$ describes that the cost coefficients of the jth supplier are estimated by the ith supplier. An example of this is generated below in figure 5.5. In equation (5.1), a_j, b_j and c_j are not estimates but the 'real' values about the jth supplier itself. γ_1, γ_2 and γ_3 are uniformly distributed random numbers, which are randomly selected from the range [0, 0.2] for the example shown in table 5.3.

$b(i,j)$	**1**	**2**	**3**	**4**	**5**	**6**
1	2.00000	1.56330	0.93961	2.75553	2.97279	2.52571
2	1.65142	1.75000	0.92720	3.21522	2.82110	2.84172
3	1.67016	1.43116	1.00000	2.61300	2.93618	2.99058
4	1.95879	1.73402	0.93981	3.25000	2.75918	2.90231
5	1.83489	1.52839	0.88333	2.66128	3.00000	2.77266
6	1.74730	1.71571	0.81726	2.89682	2.59323	3.00000
$c(i,j)$	**1**	**2**	**3**	**4**	**5**	**6**
1	0.00375	0.01547	0.05282	0.00698	0.02293	0.02313
2	0.00371	0.01750	0.05594	0.00735	0.02189	0.02360
3	0.00331	0.01671	0.06250	0.00793	0.02000	0.02212
4	0.00315	0.01646	0.05064	0.00834	0.02361	0.02177
5	0.00322	0.01677	0.06149	0.00790	0.02500	0.02355
6	0.00328	0.01554	0.05206	0.00681	0.02249	0.02500

Table 5.3 *An example of cost coefficients of* $b(i,j)$ *and* $c(i,j)$

As the values of γ_1, γ_2 and γ_3 are generated randomly the cost coefficients of $b(i,j)$ and $c(i,j)$ are no longer the same as those in base case. Noted, the diagonal elements are $b(i,i)$ and $c(i,i)$ which represent the cost coefficients about the i th supplier as estimated by the i th supplier. These are the same as parameters in table 5.1. The uncertainty introduced by these estimates will be shown to influence the equilibrium away from the theoretical level.

5.3.2 Results of Case 1 under incomplete information

The test results of case 1 under incomplete information are presented in table 5.4 below. This equilibrium state is found through the solution in equation (4.24). The testing interface is given in Appendix B.

Producer No.	Output	Profit	Price
G1	89.960	469.341	7.555
G2	59.839	284.678	
G3	32.527	147.075	
G4	55.551	213.387	
G5	36.162	132.010	
G6	37.097	134.557	
Total	311.135	1381.048	

Table 5.4 *The optimal production for case 1 under incomplete information*

As random numbers γ_1, γ_2 and γ_3 lie in the range [0, 0.2], it makes the cost coefficients of the rivals be underestimated, up to 20%. On this basis, the suppliers make decisions using equation 4.24 with these estimates and given the error they would therefore produce less. As shown in table 5.4, the total output would be reduced while the price goes up, say 7.555.

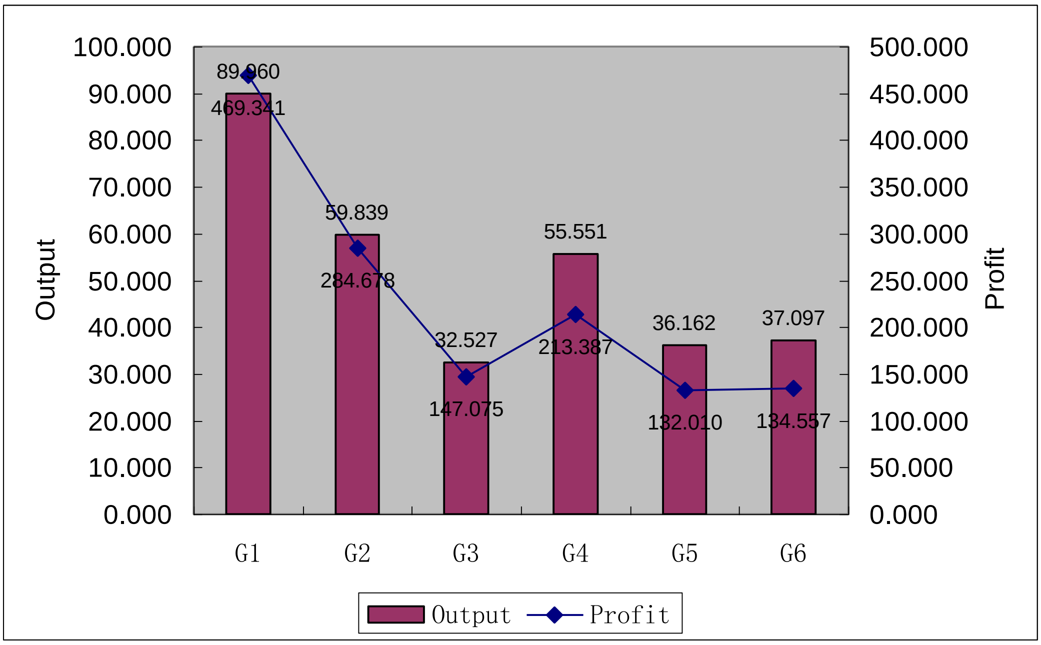

Figure 5.5 *The line-column chart of optimal outputs & profits for Case 1*

Since the uncertainty is introduced into this case the outcome of **G5** and **G6** are not symmetrical any more under incomplete information. It is noted that the total profit goes up while the total production is decreasing from table 5.4. This is because the market price increases greatly comparing with the base case. From figure 5.7, although **G3** produces less than **G5** and **G6**, it remains earning more than those two due to the high cost of **G5** and **G6** when the output is below 40. The results of others behave normally, this is to say, the more suppliers produce the more benefit they gain which can be seen from the column chart above.

5.4 Results of Case 2 under Incomplete Information

In this section, the mathematical model of case 2 established in chapter 4 with specific market parameters is presented and simulation results are given. In case 2, each suppler has more than one estimated cost function for each of their rivals however one is selected from these functions that is based on a function of probability.

5.4.1 Numerical model for Case 2

In this case, each supplier knows their own cost functions however the cost functions about their rivals are still unknown. Therefore, these need to be estimated in a way that is given in case 2. Similar to case 1, in this case, estimates about cost function coefficients are randomly selected from a range of numbers. Furthermore, it is extended that several cost functions are involved with a probability function. In this section, it is assumed that there are only two estimated cost functions for every supplier and the probability of selecting one from such functions is 50%. The parameters being used in this case are generated as those shown in table 5.1.

The cost coefficients about the suppliers themselves are represented as $a_i^{(i)}$, $b_i^{(i)}$ and $c_i^{(i)}$. It describes that the coefficients of the i th supplier are estimated by the i th supplier which is known by themselves. But the coefficients of the j th supplier need to be estimated though the equations below in this case.

$$a_{j,k}^{(i)} = (1-\gamma_{1,k})a_j \qquad b_{j,k}^{(i)} = (1-\gamma_{2,k})b_j \qquad c_{j,k}^{(i)} = (1-\gamma_{3,k})a_j$$
$$i, j = 1,2,...,6 \quad i \neq j \quad k = 1,2 \tag{5.2}$$

In the equation (5.2), $\gamma_{1,k}$, $\gamma_{2,k}$, and $\gamma_{3,k}$ are selected randomly from a uniform distribution, k is the numbering of the cost function (here labeled 1 or 2) and a_j, b_j and c_j are the cost coefficients of the jth supplier which are genuine values from table 5.1. The random number range defined for the situation when $k=1$ is within [0, 0.1] while the situation when $k=2$ within [0, 0.2]. That is to say, random numbers $\gamma_{1,1}$, $\gamma_{2,1}$ and $\gamma_{3,1}$ range between 0 and 0.1 and $\gamma_{1,2}$, $\gamma_{2,2}$ and $\gamma_{3,2}$ range between 0 and 0.2. The underestimations for these two cost functions would be up to 10% and 20% respectively.

5.4.2 Results of Case 2 under incomplete information

The test results of Case 2 under incomplete information are presented in table 5.5 below. This equilibrium state is found through the solution in equation (4.31) which is reproduced below. As some assumptions are made when simulating the Cournot equilibrium solution is rewritten below in equation (5.3). The testing interface is given in Appendix C.

$$q_i^{(i)} = \frac{P_{max} - b_i - K\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{t=1}^{2} r_{j,t}^{(i)} \dfrac{b_i - b_{j,t}^{(i)}}{K + 2c_{j,t}^{(i)}}}{2(K + c_i) + K\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{t=1}^{2} r_{j,t}^{(i)} \dfrac{K + 2c_i}{K + 2c_{j,t}^{(i)}}} \qquad i, j = 1,2,...,n \quad i \neq j \tag{5.3}$$

Where the probability of selecting one cost function from two is 0.5 which is defined in the numerical model previously

Producer No.	Output	Profit	Price
G1	91.889	450.739	7.250
G2	62.116	274.109	
G3	32.686	137.510	
G4	55.952	197.692	
G5	38.384	126.294	
G6	37.726	124.748	
Total	318.754	1311.091	

Table 5.5 *The optimal production for case 2 under incomplete information*

In this case, the cost of the jth supplier is still underestimated by the ith supplier as the cost coefficients are discounted by 10% up to 20% in either cost function. Each generator supplies electricity to the market discreetly and tends to produce less. Thus the total output in case 2 would be reduced in comparison with the base case under complete information which can be shown in table 5.5. From table 5.5, comparing to the base case, the price goes up i.e. 7.25 as well as the total pay-offs of the suppliers. What causes it is that the reduction of the total production makes the market price go up which can be seen from the linear demand curve in figure 5.3. Although the total output is less, the benefit suppliers gained increases. The chart of output and profit of six generators is presented in figure 5.6 to illustrate this visually.

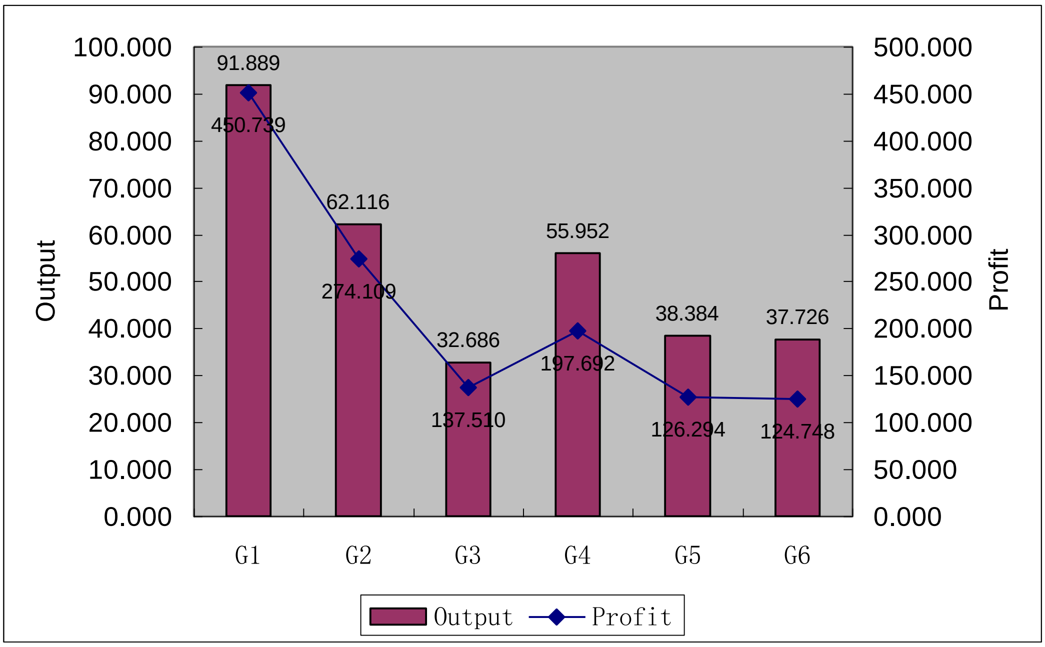

Figure 5.6 *The line-column chart of optimal outputs & profits for Case 2*

Similar to case 1, since more uncertainty factors i.e. two cost functions with probability are introduced the outcome of **G5** and **G6** are not symmetrical in this case. It can be seen from figure 5.6 that the output and profit of suppliers basically comply with the rules that the more suppliers produce the more benefits they earn. However the bar chart of **G3, G5** and **G6** shows something different which seems to beyond this. From the chart, **G3** produces less electricity than **G5** and **G6** but it earns more profit. It happens when testing of case 1 simply because the cost of producing this level of energy for **G5** and **G6** are comparatively high in comparison with **G3** which can be found in cost curves in figure 5.2. In this case, although the initial parameters for **G5** and **G6** are exactly the same the results of output and profit are no longer symmetrical due to inaccuracy of estimation.

5.5 Results of Case 3 under Incomplete Information

In this section, the mathematical model of case 3 established in chapter 4 with specific market parameters is presented and simulation results are given. In case 3, each suppler has only one estimated cost function for each of their rivals however a distribution function is considered when estimating.

5.5.1 Numerical model for Case 3

In this case, the cost function of the i th supplier is the same as the equation (4.15) while the cost function of the j th supplier is unknown. It needs to be estimated in the way that is given below. Estimation of cost functions is then associated with the process estimating the cost coefficients. Basic parameters are shown in table 5.1.

The cost coefficients for the i th supplier are given and then the cost functions are acquired. Different from the other two cases under incomplete information, the cost function of the j th supplier in this case is dependent on not only estimated cost coefficients but also a distribution function which is shown in equation (4.32). Cost coefficients of the j th supplier which are estimated by the i th supplier are still assumed to be the same as those in table 5.1. In this case, cost coefficients of the rivals are estimated in the same way as case 1 that is in equation (5.1). γ_1, γ_2 and γ_3 are randomly selected from the range of uniformly distributed numbers between [0, 0.2]. It is defined that the parameters α and g in equation (4.36) are 4 and 0.1 respectively which determines the Cournot equilibrium state. Lastly the cost functions of the i th supplier are still the same as those in equation 4.15 as well as the cost curves in figure 5.2. Due to introduction of uncertainty, the underestimation about the cost of their rivals is again existed in this case. The test results are presented in the following.

5.5.2 Results of Case 3 under incomplete information

The test results of case 3 under incomplete information are presented in table 5.6 below. This equilibrium state is gained through the Cournot equilibrium solution in equation (4.36) when the values of α and g are defined as 4 and 0.1. The testing interface is given in Appendix D.

Producer No.	Output	Profit	Price
G1	94.984	416.367	6.740
G2	64.033	247.756	
G3	33.825	122.639	
G4	58.127	174.669	
G5	39.797	109.235	
G6	40.740	110.863	
Total	331.506	1181.529	

Table 5.6 *The optimal production for Case 3 when* g *=0.1*

In this case, the cost of the j th supplier is underestimated by the i th supplier as the cost coefficients are discounted by up to 20% and a distribution function is introduced for estimation of the cost function of the j th supplier. From table 5.6, it is noted that the production for each supplier is close to the quantities under complete information, moreover, the profits each supplier earn are slightly greater than what they get in the base case. Consequently, the price of the market is reduced apparently comparing to the other two cases under incomplete information. It is the result of the introduction of the distribution function when estimating the j th suppliers' cost functions. This is different to the two cases discussed previously. It can be described that the introduction of the distribution function compensates the effect of underestimation about cost coefficients of the j th supplier and makes the estimating process towards the complete information situation. The chart of output and profit of six generators is presented in figure 5.7.

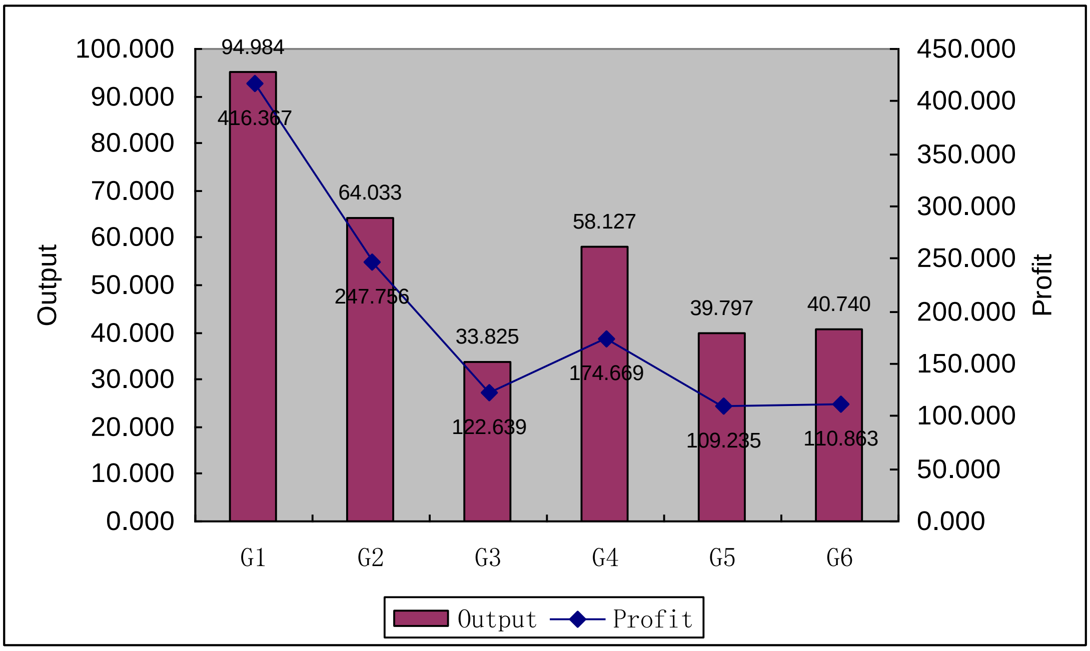

Figure 5.7 *The chart of optimal outputs & profits for Case 3 when* g *=0.1*

As the cost coefficients of the j th supplier are selected from a range of random numbers in this case, there is still a difference between output and profit of **G5** and **G6** in values. It implies that the suppliers do not know exactly the cost functions of the rest. **G3** makes more profit than **G5** and **G6** despite the higher production of **G5** and **G6** which is because the production cost of **G3** is far less than **G5** and **G6** at this specific level.

Another test on the parameter g is carried out. In equation (4.34), standard deviation σ of distribution function is defined as $gq_j^{(i)}$ and in equation (4.36) the testing results in case 3 are also affected by g. In this section g is set at 0.2 (bigger than 0.1) and the test results are presented below in table 5.7 and figure 5.8.

Producer No.	Output	Profit	Price
G1	101.609	337.160	5.699
G2	68.706	188.726	
G3	35.796	88.129	
G4	64.571	123.376	
G5	44.045	70.389	
G6	42.793	69.727	
Total	357.519	877.506	

Table 5.7 *The optimal production for Case 3 when* g *=0.2*

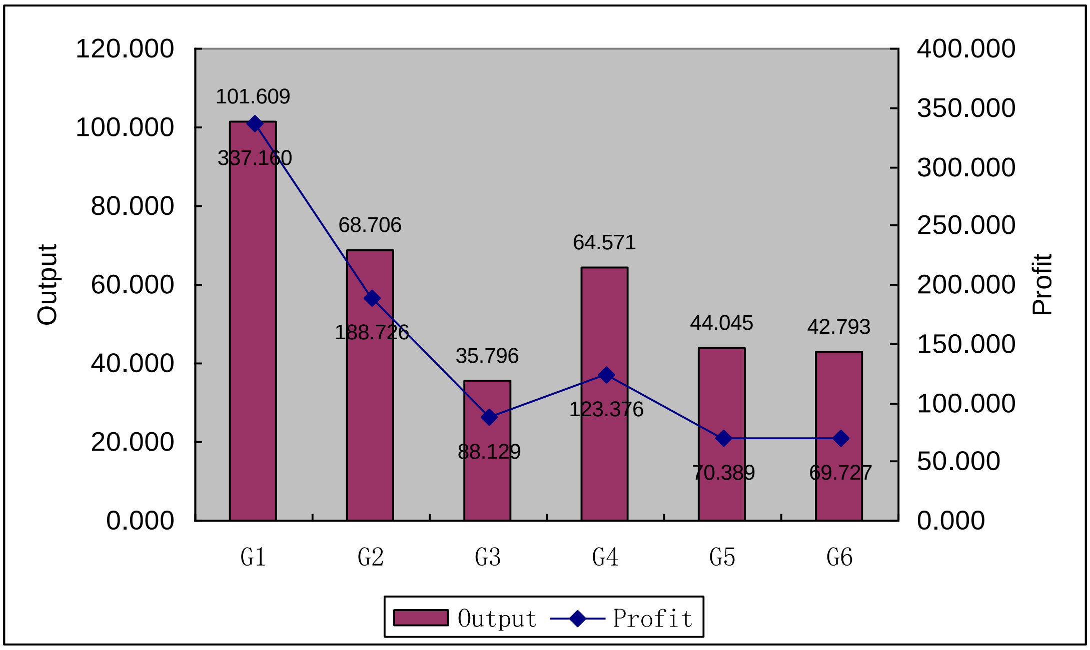

Figure 5.8 *The chart of optimal outputs & profits for Case 3 when* g *=0.2*

Comparing with the former scenario when g =0.1, the output and profit of suppliers are reduced greatly. This is as the result of the fact that standard deviation σ of the distribution function increasing brings about less accurate estimation. Besides, together with equation (4.36) it turns out that the profit increases.

5.6 Summary

A numerical example with six generators is simulated in this chapter under both complete information and incomplete information situation by using the Cournot equilibrium methods presented in Chapter 4. The producers supply electrical energy to a single sided (competition in generation only) power exchange.

Four cases are simulated in this chapter under complete and incomplete information. In base case, suppliers know exactly cost functions about themselves and their rivals' while they only have their own cost coefficients and the others' have to be estimated on the basis of corresponding formula under incomplete information. From the base case the optimal production and profit for each supplier are basically dependent on the cost of their rivals. It is not a problem knowing such costs of their rivals under complete information. The state that each supplier has optimal production can be gained through the relevant Cournot equilibrium solution.

Uncertainty is introduced into the solutions under incomplete information due to underestimation about the rivals' costs. Three methods estimating such costs are suggested in this chapter when incomplete information situation is considered. In case 1, one estimated cost function is regarded with random numbers for cost coefficients which represents discount of estimation. In case 2, two estimates of cost functions are incorporated and lastly in case 3 a distribution function of the cost of suppliers' rivals is employed. Adoption of these estimating functions results in errors on behalf of generator estimates being introduced which in turn means that generators deviate from the theoretical optimum of the complete information case. The simulation results show that producers supply is largely dependent on how they estimate costs about their rivals and the accuracy of estimation is an important factor in such oligopoly markets.

CHAPTER 6

EXTENSION OF THE MODEL AND TESTING RESULTS

6.1 Introduction

The model without consideration of transmission system is examined and the results are given in chapter 5. In this chapter a model with consideration of both transmission losses and wheeling charges is investigated and the simulation results are given as well. Due to the introduction of new elements, the equations in determining the optimal production of suppliers are not valid any more and the Cournot equilibrium solution equations need to be reorganized to fulfill this task in various cases. The model reviewed in chapter 5 has no constraints involved which can be seen as an ideal situation. The objective of this chapter is to improve the model features to reflect some of the additional features of the real world. Thus in this chapter transmission constraints are included in the model.

6.2 Numerical Model with Consideration of Wheeling Charges and Testing Results

In this section, improved mathematical methods in determining optimal production when considering wheeling charges are presented and tested in various numerical scenarios. Because of deregulation of vertically integrated power system, wheeling has become an important issue which is defined as "the use of a utility's transmission facilities to transmit power for other buyers and sellers" [35]. Due to unbundling of transmission services, there is not a direct interconnection between power sellers (i.e. suppliers) and power buyers (i.e. consumers) [36]. As a result, power sellers and buyers have to pay wheeling charges for the access and use of transmission network. The Cournot equilibrium solutions with associated network constraints under complete information and three cases under incomplete information are presented and simulation results are given and discussed in this section.

6.2.1 Numerical example and results under complete information

Derivation of improved Cournot equilibrium formula is presented and simulation results and comments are given in this section.

6.2.1.1 Numerical model under complete information

Taking wheeling charges into account, the Cournot equilibrium solution has to be reset to accommodate to the situation. The cost of wheeling charges for each supplier, $C_{W_i}(q_i)$, is defined in equation (6.1).

$$C_{W_i}(q_i) = P_X \times q_i \qquad i = 1, 2, ..., n \tag{6.1}$$

Where P_X is a constant coefficient of wheeling charges for the i th supplier. The pay-off function for the i th supplier is thenrewritten in equation (6.2).

$$U_i(Q) = q_i P - C_i(q_i) - C_{W_i}(q_i) \qquad i = 1, 2, ..., n \tag{6.2}$$

The cost functions of each supplier remain the same as those in equation 4.9. Substituting equation (4.7), (4.8), (4.9) and (6.1) into (6.2), it derives

$$U_i(Q) = -(K + c_i)q_i^2 + \left(P_{\max} - P_X - K\sum_{\substack{j=1 \\ j \neq i}}^{n} q_i - b_i \right) q_i - a_i \qquad i = 1, 2, ..., n \tag{6.3}$$

The Cournot equilibrium solution is acquired by setting $\frac{dU_i(Q)}{dq_i} = 0$. After mathematical manipulation it obtains

$$q_i = \frac{P_{max} - P_X - b_i - K\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{b_i - b_j}{K+2c_j}}{2\left(K+c_i\right) + K\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{K+2c_i}{K+2c_j}} \qquad i, j = 1, 2, \ldots, n \quad i \neq j \tag{6.4}$$

The optimal production when including wheeling charges, $C_{W_i}\left(q_i\right)$, for the i th supplier is shown in equation (6.4). Below, in figure 6.1, it illustrates such an oligopoly market that six generators competing with each other to supply electricity to customers through a transmission network which takes charges of using the line into consideration.

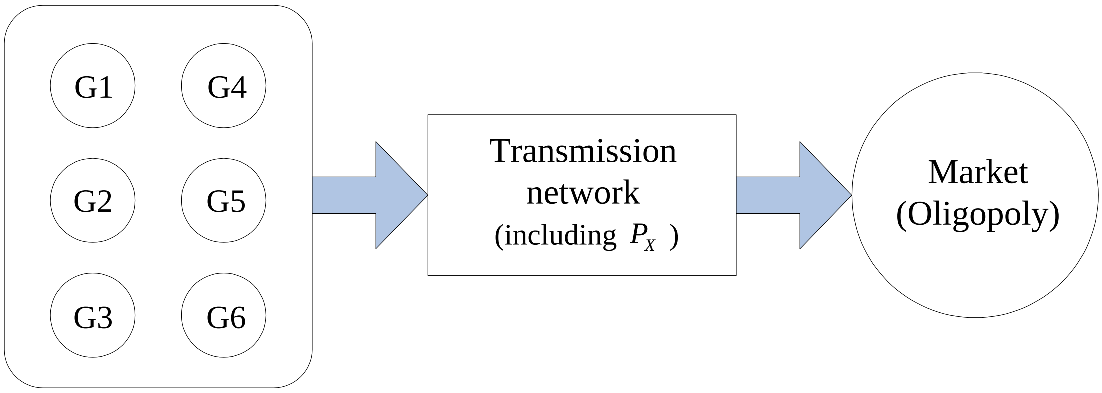

Figure 6.1 *The target oligopoly market considering transmission constraints* P_X

Cost coefficients of generators and basic market parameters are shown in table 6.1 which is consistent with Chapter 5.

Coefficients	a	b	c	$P_{\max}$	$Q_{L\max}$	K
G1	0	2	0.00375	20	500	0.04
G2	0	1.75	0.0175	P_X		
G3	0	1	0.0625	0.7		
G4	0	3.25	0.00834			
G5	0	3	0.025			
G6	0	3	0.025			

Table 6.1 *Cost coefficients and basic parameters including* P_X *for base case*

Main parameters are kept the same as those in chapter 5 in order that it is convenient to compare results between different scenarios. The coefficient P_X of wheeling charges is set at 0.7 for testing purposes. Cost coefficient a is ignored and coefficients b and c for **G5** and **G6** are still defined as the same values in Chapter 5. Cost functions of six generators are in accordance with those in previous models as well as demand function. Curves for the cost functions and demand are regenerated below in figure 6.2 and 6.3 respectively.

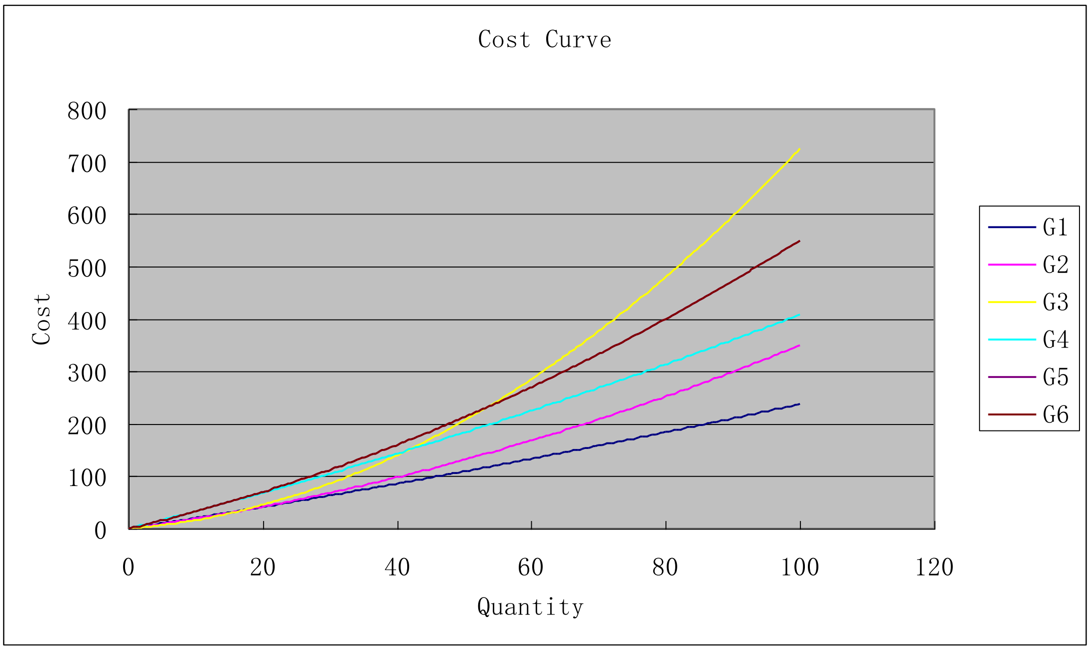

Figure 6.2 *Cost curves of six generators*

Generally, the cost increases while the output is growing. Furthermore, the growing speed of cost of production is closely dependent on the values of cost coefficients, which can be seen from figure 6.2.

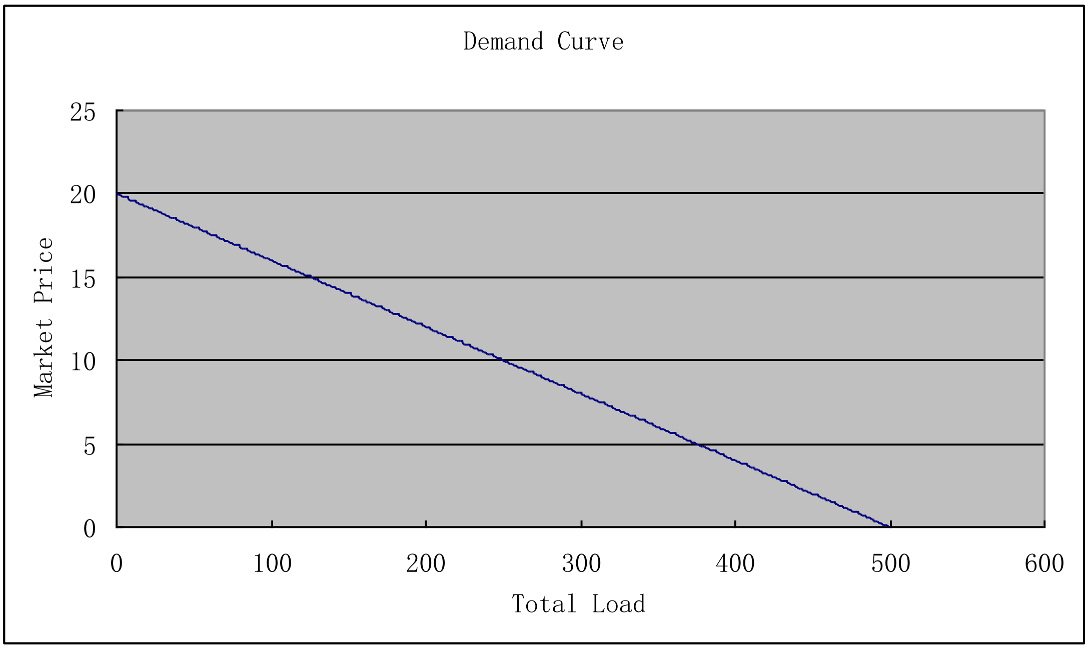

Figure 6.3 *Demand curve of the numerical model*

In figure 6.3, the curve of demand curve shows linear features according to demand function in equation (4.8).

6.2.1.2 *Results of the base case under complete information*

The test results of the base case under complete information are presented in table 6.2 which considers wheeling charges. The optimal production for each producer is achieved through the Cournot equilibrium solution in equation (6.4).

Producer No.	Output	Profit	Price
G1	93.433	381.928	7.138
G2	62.508	224.665	
G3	32.958	111.339	
G4	56.247	152.935	
G5	38.201	94.855	
G6	38.201	94.855	
Total	321.548	1060.577	

Table 6.2 *The optimal production for base case including* P_X

The Cournot equilibrium state is found for the i th supplier on the basis of cost functions of their rivals. Under complete information, each supplier knows cost coefficients of their rivals clearly and makes decisions decisively. The identical results of **G5** and **G6** indicate that they make exactly the same decision as result of the same cost function they have. When wheeling charges are involved, comparing the results with the model without wheeling in table 5.2, it can be seen that output of each producer reduces and profit they earn inclines. Correspondingly the market price goes up as the cost of electricity is not only including production cost but including transmission cost in this scenario.

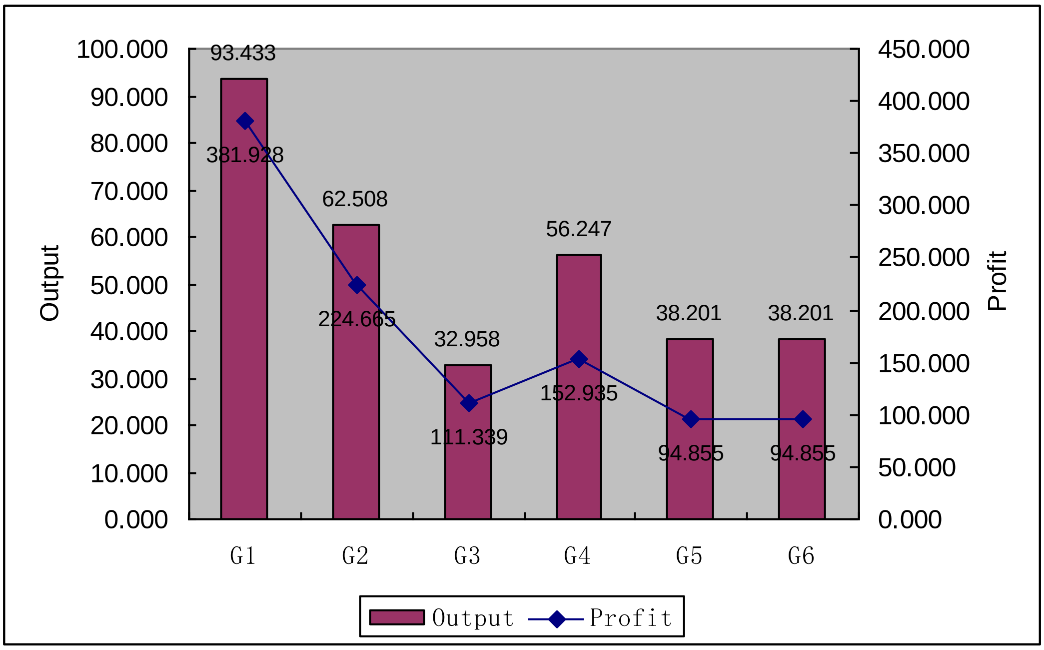

Figure 6.4 *The chart of optimal outputs & profits for base case including* P_X

The feature that **G5** and **G6** supplying the same quantity of electricity to the market is clearly reflected in figure 6.4. The situation between G3 and G5 (or G6), in which higher production makes less profit, indicate that profit suppliers earn is related to their cost functions.

6.2.2 Numerical model and Case-study 1 under Incomplete Information

Under incomplete information, suppliers do not know exactly how their rivals make decision therefore the supplier has to estimate the cost functions of their rivals. There are three scenarios of estimation about their rivals' cost functions being considered. In this part the case of prediction based on one estimated function about their rivals is presented and tested.

6.2.2.1 Numerical model for Case 1

In this case, each supplier knows exactly their own cost functions however they are not sure what strategies their rivals take. An estimated cost function is introduced to predict the quantity of production of their rivals. When considering wheeling charges, the Cournot equilibrium solution for the i th supplier needs to be reset. It is shown in equation (6.5).

$$q_i^{(i)} = \frac{P_{\max} - P_X - b_i - K\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{b_i - b_j^{(i)}}{K + 2c_j^{(i)}}}{2\left(K + c_i\right) + K\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{K + 2c_i}{K + 2c_j^{(i)}}} \qquad i, j = 1, 2, ..., n \quad i \neq j \tag{6.5}$$

The estimation of cost coefficients of the j th supplier is rewritten in equation (6.6).

$$a_j^{(i)} = \left(1-\gamma_1\right)a_j, \qquad b_j^{(i)} = \left(1-\gamma_2\right)b_j, \qquad c_j^{(i)} = \left(1-\gamma_3\right)c_j$$
$$i, j = 1, 2, ..., 6 \quad i \neq j \tag{6.6}$$

Where γ_1, γ_2 and γ_3 are randomly selected from the range [0, 0.2].

Together with equation (6.5) and (6.6), it represents the Cournot equilibrium solution of optimal production for each supplier when wheeling charges are included. The basic parameters of numerical model for case 1 are shown in table 6.1.

The cost coefficients a, b, and c in the table 6.1 represent $a\left(i,i\right)$, $b\left(i,i\right)$ and $c\left(i,i\right)$ which are the cost coefficients for the i th supplier. The coefficient for wheeling charges, P_X, is set at 0.7. Cost curves and demand curve are given in figure 6.2 and 63 respectively.

6.2.2.2 Results of Case 1 under incomplete information

The test results are presented in table 6.3 below when considering wheeling charges.

Producer No.	Output	Profit	Price
G1	88.342	436.153	7.968
G2	58.141	261.688	
G3	31.155	134.628	
G4	51.965	186.295	
G5	35.393	119.755	
G6	35.794	120.752	
Total	300.790	1259.271	

Table 6.3 *The optimal production for case 1 including* P_X

γ_1, γ_2 and γ_3 in equation 6.6 are randomly selected from [0, 0.2] it implies that underestimation is made to cost coefficients of the j th supplier. This brings about less production of the i th supplier and profit reduction accordingly. Comparing with table 5.4, not including wheeling charges, each supplier in this case produces even less and earns less while market price is increasing as the result of introduction of transmission cost.

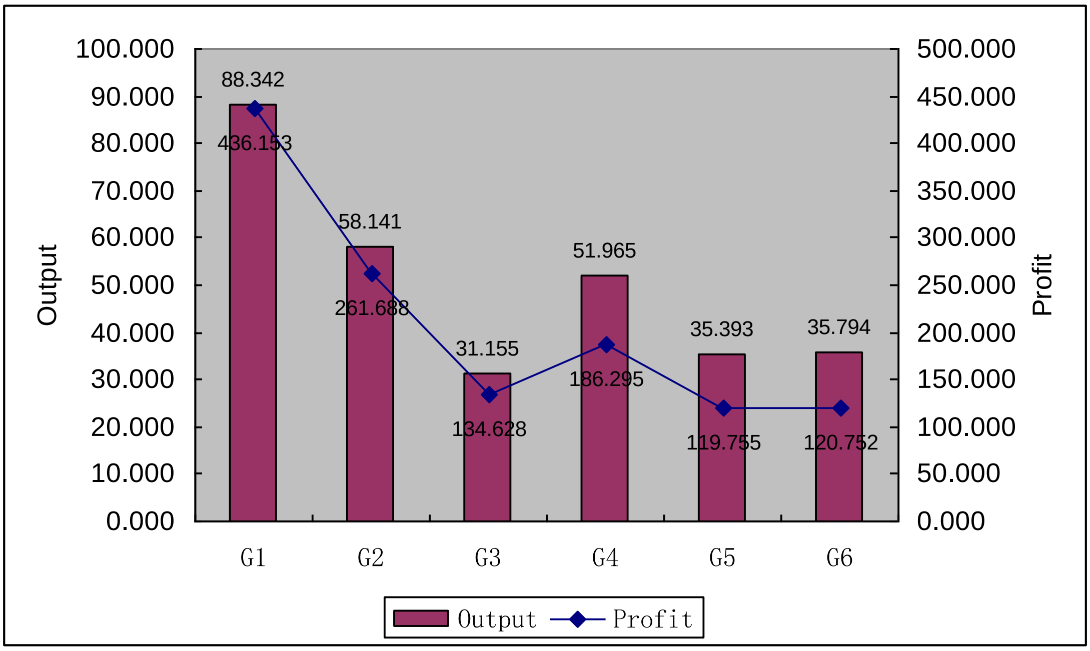

Figure 6.5 *The chart of optimal outputs & profits for case 1 including* P_X

In figure 6.5 it is illustrated that the more production the more profit for most generators except for the instance between **G3** and **G5** (or **G6**) due to their cost functions. In addition, the results of **G5** and **G6** are not symmetrical which is caused by estimating uncertainties.

6.2.3 Numerical model and Case-study 2 under Incomplete Information

In this part the case of prediction based on several estimated functions with a probability about their rivals is presented and tested.

6.2.3.1 Numerical model for Case 2

In case 2, prediction of each supplier about cost functions of their rivals is based on several estimated cost functions and a probability that selecting one from these functions. The Cournot equilibrium solution for the i th supplier is given in equation (6.7) when considering wheeling charges.

$$q_i^{(i)} = \frac{P_{max} - P_X - b_i - K\sum_{\substack{j=1 \\ j\neq i}}^{n}\sum_{t=1}^{T_j^{(i)}} r_{j,t}^{(i)} \frac{b_i - b_{j,t}^{(i)}}{K + 2c_{j,t}^{(i)}}}{2(K + c_i) + K\sum_{\substack{j=1 \\ j\neq i}}^{n}\sum_{t=1}^{T_j^{(i)}} r_{j,t}^{(i)} \frac{K + 2c_i}{K + 2c_{j,t}^{(i)}}}$$

$$i, j = 1, 2, \ldots, n \quad i \neq j \quad t = 1, 2, \ldots, T_j^{(i)} \qquad (6.7)$$

Estimated functions in this case are associated with the process estimating cost coefficients which are determined by equation (6.8).

$$a_{j,k}^{(i)} = (1 - \gamma_{1,k})a_j \qquad b_{j,k}^{(i)} = (1 - \gamma_{2,k})b_j \qquad c_{j,k}^{(i)} = (1 - \gamma_{3,k})a_j$$
$$i, j = 1, 2, \ldots, 6 \quad i \neq j \quad k = 1, 2 \qquad (6.8)$$

Where $\gamma_{1,1}$, $\gamma_{2,1}$ and $\gamma_{3,1}$ when $k = 1$ are randomly selected from range [0, 0.1] while $\gamma_{1,2}$, $\gamma_{2,2}$ and $\gamma_{3,2}$ from range [0, 0.2]. As two cost functions are involved in this scenario, the probability is 0.5. Together with equation (6.7) into (6.8), it determines the optimal production for the i th supplier in case 2.

The cost coefficients and parameters used in this case are extracted from table 6.1.

6.2.3.2 Results of Case 2 under incomplete information

Test results of case 2 under incomplete information are presented in table 6.4 below.

Producer No.	Output	Profit	Price
G1	88.628	420.380	7.776
G2	59.828	255.978	
G3	32.034	130.488	
G4	52.151	176.823	
G5	36.879	116.301	
G6	36.091	114.527	
Total	305.611	1214.497	

Table 6.4 *The optimal production for case 2 including* P_X

In table 6.4, the cost of the j th supplier is still underestimated by the i th supplier due to underestimation of cost coefficients. Consequently, each supplier observes this and makes decisions conservatively which leads to reduction of production and profit. After introducing wheeling charges, comparing with table 5.5, suppliers produce electricity even less and earn less profit accordingly. Market price goes up because of extra cost of using transmission network.

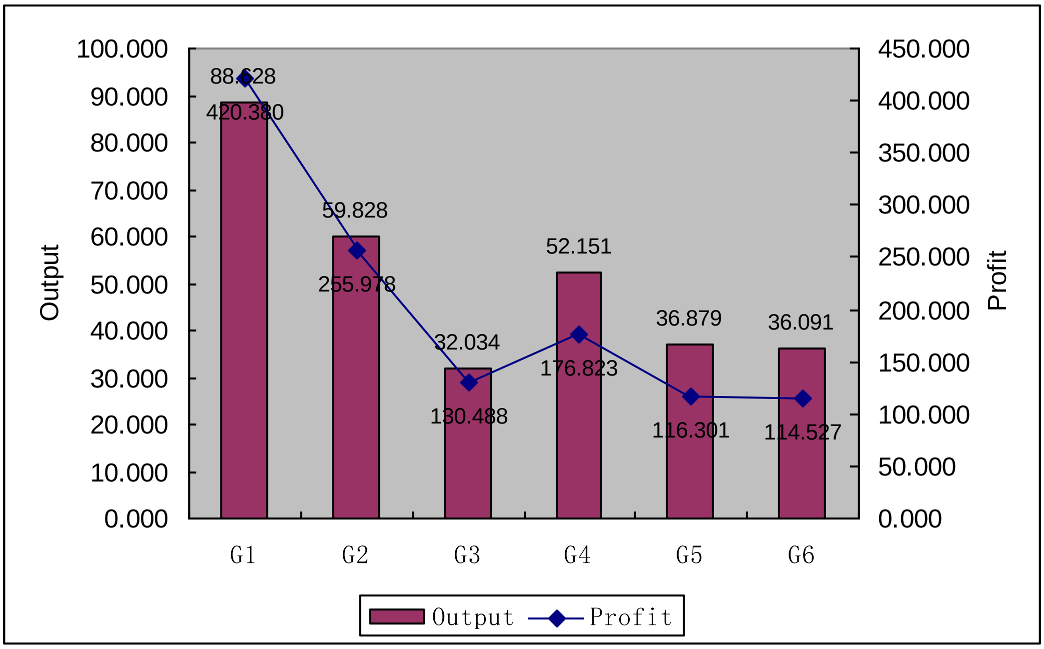

Figure 6.6 *The chart of optimal outputs & profits for case 2 including* P_X

Corresponding results of this case are shown graphically in figure 6.6.

6.2.4 Numerical model and Case-study 3 under Incomplete Information

In this part each supplier predicts the cost of their rivals through an estimated cost function with a probability distribution element.

6.2.4.1 Numerical model for Case 3

The Cournot equilibrium solution of case 3 for the i th supplier is given in equation 6.9 when considering wheeling charges.

$$q_i^{(i)} = \frac{P_{\max} - P_X - b_i - K\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{b_i - b_j^{(i)} - \alpha g}{K + 2c_j^{(i)}}}{2(K + c_i) + K\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{K + 2c_i}{K + 2c_j^{(i)}}} \qquad i, j = 1, 2, \ldots, n \quad i \neq j \tag{6.9}$$

Cost coefficients for the i th supplier and basic parameters are shown in table 6.1. Together with equation (6.6) and (6.9), it determines the optimal production for the i th supplier in case 3. In this case, γ_1, γ_2 and γ_3 are still randomly selected from the range [0, 0.2] and parameters α and g are defined by 4 and 0.1 respectively.

6.2.4.2 *Results of Case 3 under incomplete information*

Test results considering wheeling charges of case 3 are presented in table 6.5 below.

Producer No.	Output	Profit	Price
G1	91.691	387.953	7.275
G2	61.434	230.367	
G3	33.546	116.683	
G4	55.889	159.778	
G5	37.495	98.896	
G6	38.073	99.870	
Total	318.126	1093.547	

Table 6.5 *The optimal production for case 3 including* P_X

Each supplier's output and profit reduce due to underestimation when comparing with the case without consideration of wheeling charges. As extra cost of transmission is included, the cost of selling electricity goes up as well as the market price. With introduction of a distribution function, the results are close to those under complete information in table 6.2 which compensates the effect of the underestimates. In addition, similar to results in table 5.6, the output and profit reduce while the parameter g is getting bigger. Results of case 3 are also shown graphically in figure 6.7.

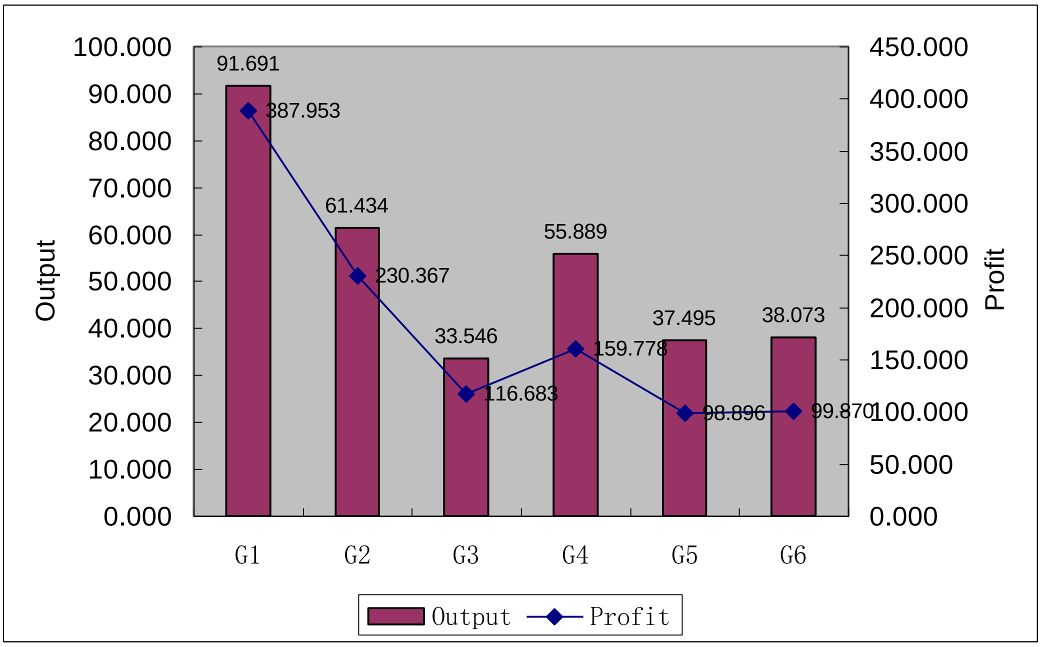

Figure 6.7 *The chart of optimal outputs & profits for case 3 including* P_X

6.3 Numerical Model with Consideration of Transmission Losses and Testing Results

In this section, improved mathematical solutions in determining optimal production with consideration of transmission losses are presented and tested in various numerical scenarios. With the process in the delivery of electricity from generators to consumers by transmission network, a fraction of electricity energy loses due to the effect of Joule heat. The transmission losses were estimated at 7.2% in the USA in 1995 [37] and at 7.4% in the UK in 1998 [38]. Here, the effect of transmission losses is taken into account when organizing the Cournot equilibrium solutions. Both situations under complete information and incomplete information are investigated and simulation results are given in the following.

6.3.1 Numerical example and results under complete information

Derivation of improved Cournot equilibrium formula is presented and corresponding simulation results are given in this part.

6.3.1.1 Numerical model under complete information

Derivation of Cournot equilibrium solution under complete information is described in the following paragraphs. When considering transmission losses, the pay-off function for the i th supplier is rewritten in equation (6.10).

$$U_i(Q) = (1-L)q_i P - C_i(q_i) \qquad i = 1, 2, ..., n \tag{6.10}$$

Where the coefficient L is the transmission loss factor and is defined as the percentage of energy lost over total energy transferred

Substituting equation (4.7), (4.8), (4.9) into (6.10), it derives

$$U_i(Q) = -\left[(1-L)K + c_i\right]q_i^2 + \left[(1-L)P_{\max} - (1-L)K\sum_{\substack{j=1\\ j\neq i}}^{n} q_i - b_i\right]q_i - a_i$$

$$i = 1, 2, ..., n \tag{6.11}$$

The Cournot equilibrium solution is derived by setting $\frac{dU_i(Q)}{dq_i} = 0$. After mathematical manipulation it obtains

$$q_i = \frac{(1-L)P_{\max} - b_i - (1-L)K\sum\limits_{\substack{j=1\\ j\neq i}}^{n}\frac{b_i - b_j}{(1-L)K + 2c_j}}{2\left[(1-L)K + c_i\right] + (1-L)K\sum\limits_{\substack{j=1\\ j\neq i}}^{n}\frac{(1-L)K - 2c_i}{(1-L)K + 2c_j}}$$

$$i, j = 1, 2, ..., n \quad i \neq j \tag{6.12}$$

The optimal production for the i th supplier including transmission losses is shown in equation (6.12). The target market to be simulated is illustrated in figure 6.8 in which six generators interact with each other to optimize their production and profit.

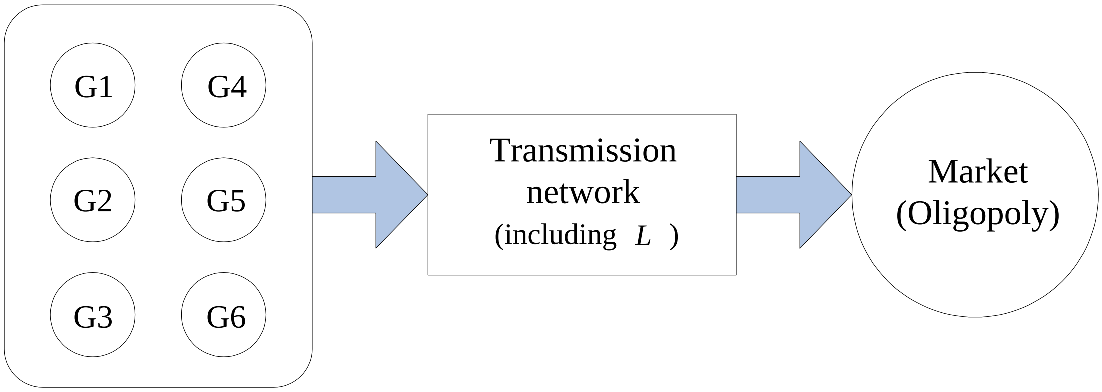

Figure 6.8 *The target oligopoly market considering transmission constraints L*

Cost coefficients of generators and basic market parameters are given in table 6.6.

Coefficients	a	b	c	$P_{\max}$	$Q_{L\max}$	K
G1	0	2	0.00375	20	500	0.04
G2	0	1.75	0.0175	L		
G3	0	1	0.0625	0.1		
G4	0	3.25	0.00834			
G5	0	3	0.025			
G6	0	3	0.025			

Table 6.6 *Cost coefficients and basic parameters including L for base case*

The coefficient L of transmission losses is set at 0.1 (10%) for testing later. Other parameters remain the same. Cost curves of the i th supplier and demand curve can be found in figure 6.2 and 6.3 respectively.

6.3.1.2 Results of Base Case under complete information

Simulation results of base case under complete information are presented in table 6.7 below where transmission loss is considered. The optimal production for each producer is settled through equation (6.12).

Producer No.	Output	Profit	Price
G1	97.736	379.702	6.946
G2	63.401	215.056	
G3	32.618	104.798	
G4	56.976	143.940	
G5	37.808	87.197	
G6	37.808	87.197	
Total	326.347	1017.888	

Table 6.7 *The optimal production for base case including* L

Under complete information, suppliers are able to make their decisions precisely. It is demonstrated that **G5** and **G6** with identical cost functions produce same quantity and earn same profit which is shown in table 6.7. Comparing with table 5.2, market price goes up when output and profit decrease as the result of introduction of transmission losses. The results are depicted in figure 6.9.

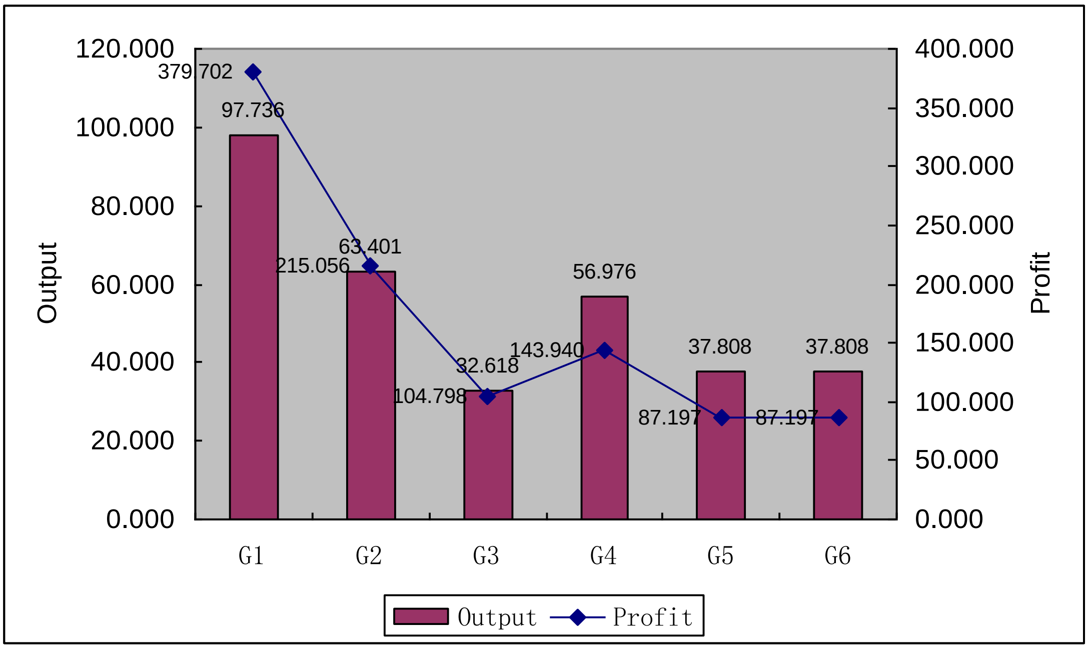

Figure 6.9 *The chart of optimal outputs & profits for base case including* L

6.3.2 Numerical model and Case-study 1 under incomplete information

From this section and forward, three cases under incomplete information are presented. Under incomplete information, the cost function of the j th supplier has to be estimated by the i th supplier based on various estimated methods.

6.3.2.1 Numerical model for Case 1

In this case, only one estimated cost function about the rivals is related. When the transmission loss is taken into account, the Cournot equilibrium solution for the i th supplier has to be reorganized which is shown below.

$$q_i^{(i)} = \frac{(1-L)P_{\max} - b_i - (1-L)K\sum_{\substack{j=1\\j\neq i}}^{n}\frac{b_i - b_j^{(i)}}{(1-L)K + 2c_j^{(i)}}}{2\left((1-l)K + c_i\right) + K\sum_{\substack{j=1\\j\neq i}}^{n}\frac{(1-L)K + 2c_i}{(1-L)K + 2c_j^{(i)}}} \tag{6.13}$$

$$i, j = 1, 2, \ldots, n \quad i \neq j$$

The estimated cost coefficients of the j th supplier are derived from equation (6.6) where γ_1, γ_2 and γ_3 are randomly selected from the range [0, 0.2]. Together with (6.6) and (6.13), it represents the Cournot equilibrium solution of optimal production for the i th producer when transmission losses are taken into consideration. The basic parameters for the numerical model of case 1 are given in table 6.6 where transmission loss factor L is equal to 0.1. Cost curves and demand curves are given in figure 6.2 and 6.3 respectively.

6.3.2.2 Results of Case 1 under incomplete information

The simulation results of case 1 are presented in table 6.8 where transmission losses are considered.

Producer No.	Output	Profit	Price
G1	92.513	438.533	7.875
G2	60.371	258.430	
G3	30.700	127.972	
G4	50.969	173.911	
G5	34.795	111.947	
G6	33.785	109.550	
Total	303.134	1220.344	

Table 6.8 *The optimal production for case 1 including* L

Uncertainty caused by introduction of random numbers γ_1, γ_2 and γ_3 implies that the cost of the j th supplier is underestimated by the i th supplier. Seeing this, producers tend to reduce the production which leads to the rise of market price. As a result, each supplier earns more profit. However, with consideration of transmission losses, both output and profit decrease while market price is higher. It is noted that although production does not change much and market price goes up comparing with table 5.4, the profit each supplier earns reduce due to the loss of transferring. The results are reproduced in figure 6.10.

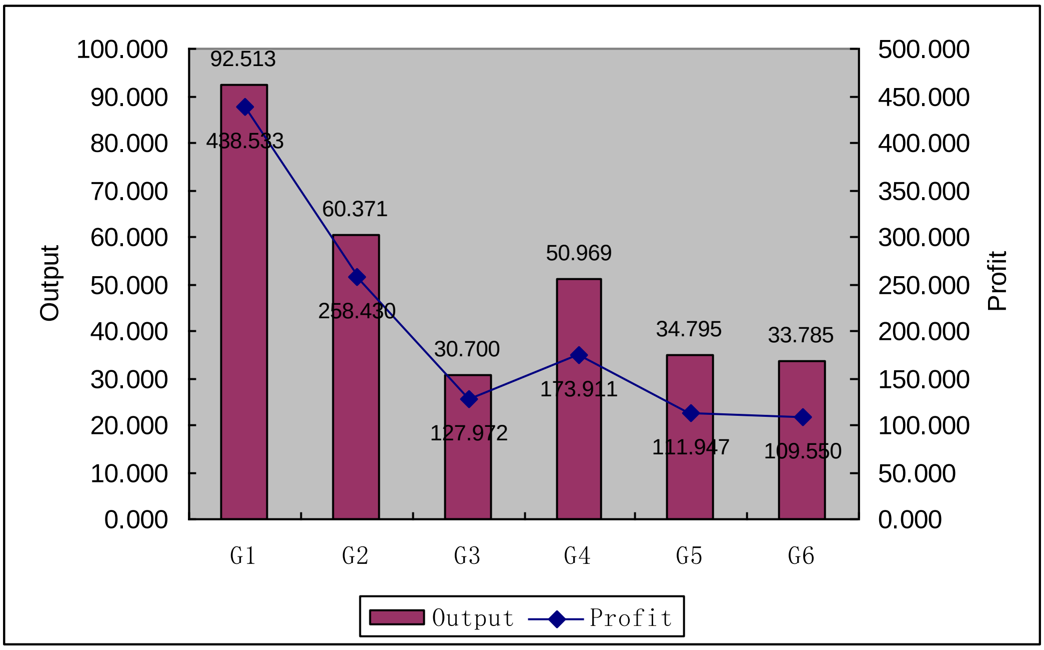

Figure 6.10 *The chart of optimal outputs & profits for case 1 including* L

6.3.3 Numerical model and Case-study 2 under incomplete information

In this part, cost functions of the rivals are predicted through several estimated functions with a probability. Improve d Cournot equilibrium solution is presented and testing results are given where transmission losses are considered.

6.3.3.1 Numerical model for Case 2

The Cournot equilibrium solution for the i th supplier is given in equation (6.14) where transmission losses are considered.

$$q_i^{(i)} = \frac{(1-L)P_{max} - b_i - (1-L)K\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{t=1}^{T_j^{(i)}} r_{j,t}^{(i)} \frac{b_i - b_{j,t}^{(i)}}{(1-L)K + 2c_{j,t}^{(i)}}}{2\left[(1-L)K + c_i\right] + (1-L)K\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{t=1}^{T_j^{(i)}} r_{j,t}^{(i)} \frac{(1-L)K + 2c_i}{(1-L)K + 2c_{j,t}^{(i)}}}$$

$$i, j = 1, 2, ..., n \quad i \neq j \quad t = 1, 2, ..., T_j^{(i)} \qquad (6.14)$$

Together with equation (6.8) and (6.14), it determines the optimal production for each producer. In equation 6.8, $\gamma_{1,1}$, $\gamma_{2,1}$ and $\gamma_{3,1}$ are randomly selected from range [0, 0.1] while $\gamma_{1,2}$, $\gamma_{2,2}$ and $\gamma_{3,2}$ from range [0, 0.2]. As two cost functions are involved in this scenario, the probability of selection is 0.5.The basic parameters for the numerical model of case 1 are given in table 6.6 where transmission loss factor L is equal to 0.1. Cost curves and demand curves are given in figure 6.2 and 6.3 respectively.

6.3.3.2 Results of Case 2 under incomplete information

Test results of case 2 under incomplete information are presented in table 6.9 below.

Producer No.	Output	Profit	Price
G1	91.574	413.059	7.616
G2	60.247	243.983	
G3	32.852	124.865	
G4	53.497	168.939	
G5	35.149	104.580	
G6	36.290	106.940	
Total	309.609	1162.367	

Table 6.9 *The optimal production for case 2 including* L

In this case, the cost of the j th supplier is still underestimated by the i th supplier as cost coefficients are underestimated. It causes the reduction of production for each producer and rise of market price and then profit increases. When transmission loss is taken into account, comparing with table 5.5, the output of each supplier does not change much and market price goes up, however, profit of each producer earned decline which can be acquired by equation (6.10). Simulation results are reproduced in form of diagram in figure 6.11.

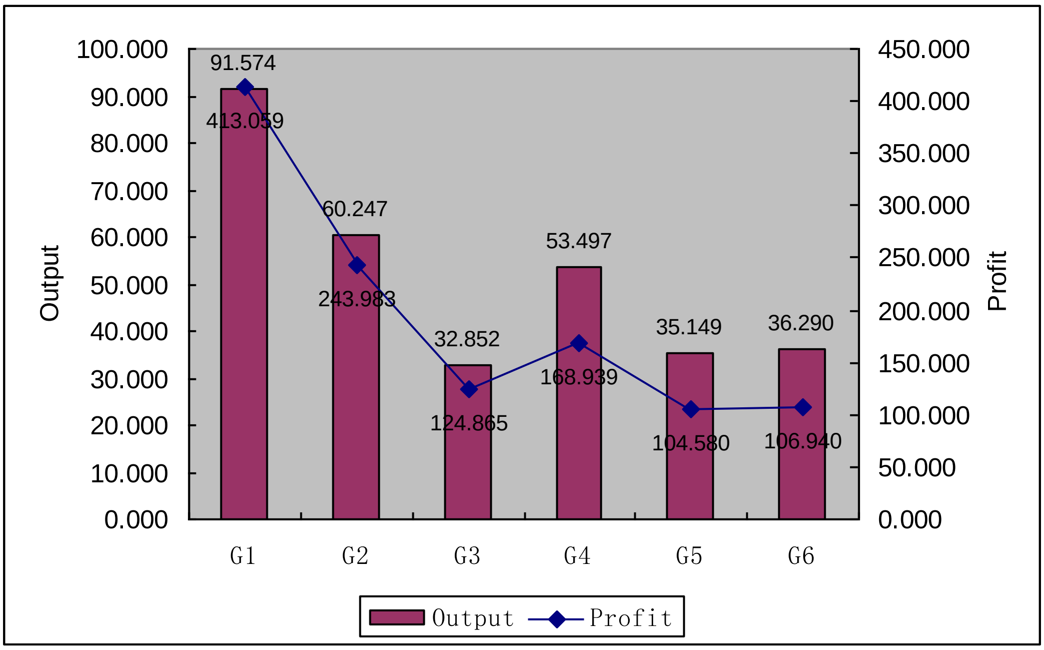

Figure 6.11 *The chart of optimal outputs & profits for case 2 including* L

6.3.4 Numerical model and Case-study 3 under incomplete information

In this part, improved Cournot equilibrium solution is presented and testing results are given.

6.3.4.1 Numerical model for Case 3

The Cournot equilibrium solution of case 3 for the i th supplier is presented in equation (6.15) where transmission losses are considered.

$$q_i^{(i)} = \frac{(1-L)P_{\max} - b_i - (1-L)K\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{b_i - b_j^{(i)} - \alpha g}{(1-L)K + 2c_j^{(i)}}}{2\left[(1-L)K + c_i\right] + (1-L)K\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{(1-L)K + 2c_i}{(1-L)K + 2c_j^{(i)}}}$$

$$i, j = 1, 2, \ldots, n \quad i \neq j \tag{6.15}$$

Where $L = 0.1$ and α and g are defined by 4 and 0.1 respectively.

Together with (6.6) and (6.15) it derives the optimal production for the i th supplier in this case. The basic parameters for the numerical model are given in table 6.6. Cost curves and demand curve are shown in figure 6.2 and 6.3 respectively.

6.3.4.2 Results of Case 3 under incomplete information

Test results of case 3 are presented in table 6.10 below where transmission losses are considered.

Producer No.	Output	Profit	Price
G1	96.392	377.199	6.972
G2	62.915	215.397	
G3	32.329	105.201	
G4	57.700	146.756	
G5	37.382	87.477	
G6	38.987	89.668	
Total	325.704	1021.699	

Table 6.10 *The optimal production for case 3 including L*

In this case, the results are close to those in base case in table 6.7 due to introduction of distribution element. With consideration of transmission losses, the output of each supplier is reduced a bit and market price goes up however the profits they earned drop a lot. It can be found in equation (6.10) that the actual selling energy is cut down due to transmission losses. Corresponding results are regenerated in figure 6.12.

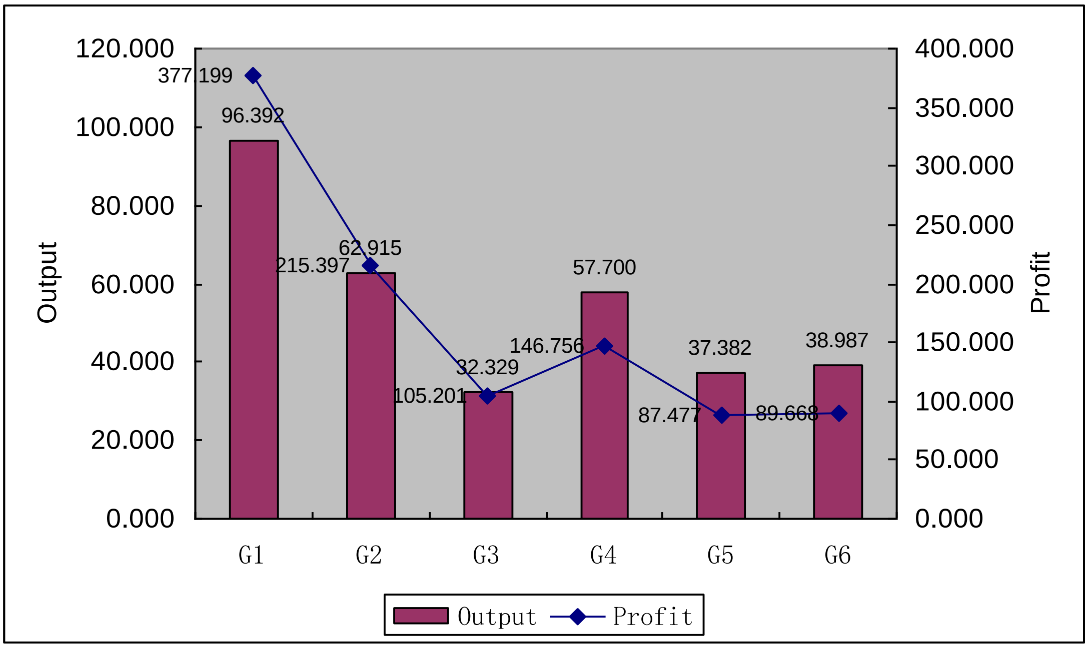

Figure 6.12 *The chart of optimal outputs & profits for case 3 including L*

6.4 Numerical Model with Consideration of Wheeling Charges and Transmission Losses and Testing Results

In this section, improved mathematical equations in determining optimal production are presented and tested in situations under complete information and incomplete information where both wheeling charges and transmission losses are included. Numerical results of each case are shown in both table and diagram in the following.

6.4.1 Numerical example and results under complete information

Derivation of improved equilibrium formula is presented and simulation results based on this are given in form of table and diagram where both wheeling charges and transmission loss are considered.

6.4.1.1 Numerical model under complete information

When considering both wheeling charges and transmission loss, the Cournot equilibrium solution has to be reformed to fulfill the task. The pay-off function for the i th supplier is then reorganized in equation (6.16).

$$U_i(Q) = (1-L) q_i P - C_i(q_i) - C_{W_i}(q_i) \qquad i = 1, 2, \ldots, n \tag{6.16}$$

Substituting equation (4.7), (4.8), (4.9) and (6.1) into (6.16), it gets

$$U_i(Q) = -\left[(1-L)K + c_i\right] q_i^2 + \left[(1-L)(P_{\max} - P_X) - (1-L)K \sum_{\substack{j=1 \\ j \neq i}}^{n} q_i - b_i\right] q_i - a_i$$

$$i = 1, 2, \ldots, n \tag{6.17}$$

The Cournot equilibrium solution is derived by setting $\dfrac{dU_i(Q)}{dq_i} = 0$. After mathematical manipulation it obtains

$$q_i = \frac{(1-L)(P_{\max} - P_X) - b_i - (1-L)K\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{b_i - b_j}{(1-L)K + 2c_j}}{2\left[(1-L)K + c_i\right] + (1-L)K\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{(1-L)K - 2c_i}{(1-L)K + 2c_j}}$$

$$i, j = 1, 2, \ldots, n \quad i \neq j \tag{6.18}$$

Equation (6.18) represents the optimal production for the i th supplier where both wheeling charges and transmission losses are considered. In figure 6.13 it describes such an oligopoly market that six generators competing with each other to supply electricity though a transmission network with constraints of wheeling charges and transmission losses.

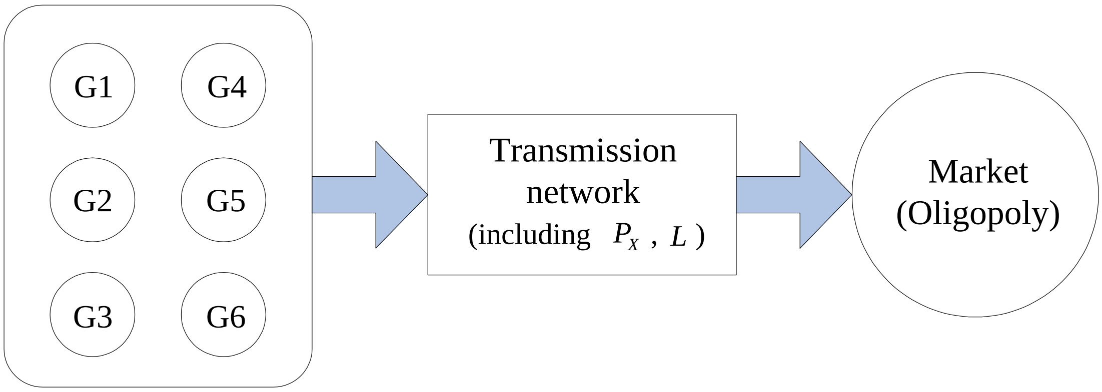

Figure 6.13 *The target oligopoly market with* P_X *and* L

Cost coefficients and initial market parameters are the same as shown in table 6.1. Coefficient P_X of wheeling charges is set at 0.7 and transmission loss factor L is 0.1 (10%). As cost functions of six generators and demand function remain the same, curves for them are given in figure 6.2 and 6.3 respectively. Curves of cost have quadratic features while curve of demand is linear.

6.4.1.2 Results of Base Case under complete information

Test results of base case are presented in table 6.11 below where both wheeling charges and transmission loss are included.

Producer No.	Output	Profit	Price
G1	94.185	352.614	7.474
G2	61.226	200.551	
G3	31.659	98.724	
G4	54.044	129.507	
G5	36.012	79.109	
G6	36.012	79.109	
Total	313.138	939.613	

Table 6.11 *The optimal production for base case including* P_X *and* L

In comparison with base case in an ideal situation, table 5.2, in this case, the output of each supplier is reduced and market price is relatively high while profit each one earned decline considerably which is caused by mutual effect of wheeling charges and transmission losses. It is noted that profit each supplier gets is determined by pay-off function in equation (6.16). Although the market price is high, introduction of cost of transmission use and losses compensate the actual profit earned by suppliers. In addition, under complete information, suppliers make their decisions decisively which can be seen from the symmetrical results of **G5** and **G6** with the same cost function. This feature is also shown in figure 6.14.

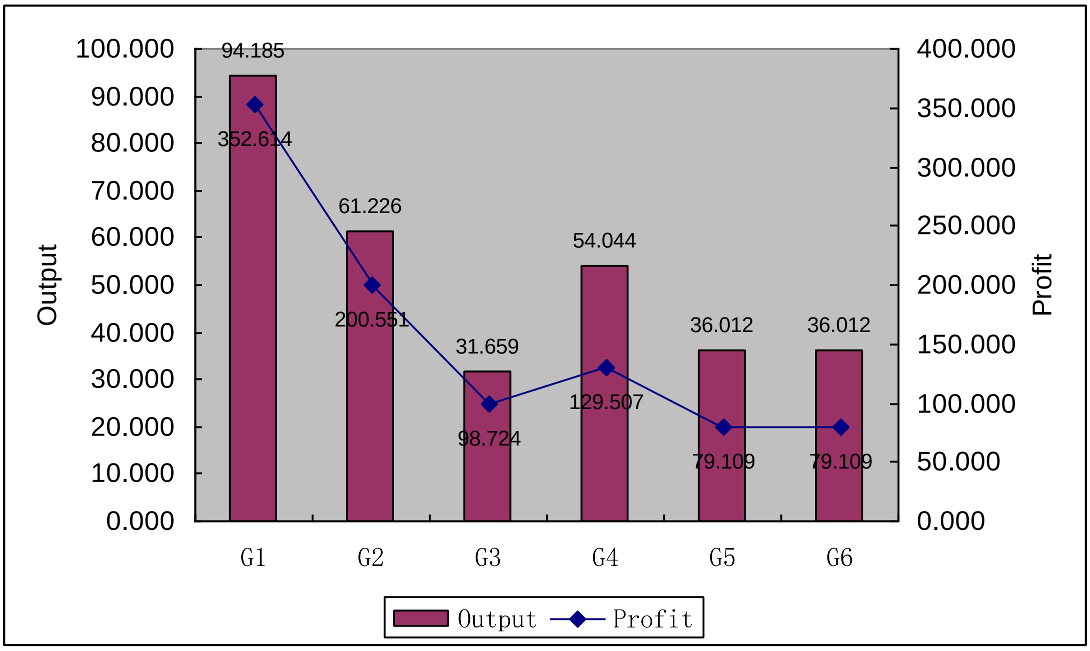

Figure 6.14 *The chart of optimization for base case including* P_X *and* L

6.4.2 Numerical model and Case-study 1 under incomplete information

In this part, prediction of cost functions of rivals is based on an estimated cost function. Due to consideration of wheeling charges and transmission loss, the Cournot equilibrium solution has to be revised. Improved one is presented and testing results are given in the following paragraphs.

6.4.2.1 Numerical model for Case 1

The Cournot equilibrium solution of case 1 for the i th supplier is presented where wheeling charges and transmission losses are included. It is shown in equation (6.19).

$$q_i^{(i)} = \frac{(1-L)(P_{\max} - P_X) - b_i - (1-L)K\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{b_i - b_j^{(i)}}{(1-L)K + 2c_j^{(i)}}}{2\left((1-l)K + c_i\right) + K\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{(1-L)K + 2c_i}{(1-L)K + 2c_j^{(i)}}}$$

$$i, j = 1, 2, \ldots, n \quad i \neq j \tag{6.19}$$

Estimated cost coefficients of the j th supplier are derived through equation (6.6) where γ_1, γ_2 and γ_3 are randomly selected from the range [0, 0.2].

Together with equation (6.6) and (6.19), it derives the optimal production for the i th supplier when considering wheeling charges and transmission losses. Cost coefficients of six generators and initial parameters of numerical model for case 1 are given in table 6.1. The coefficient of wheeling charges, P_X, is defined by 0.7 and transmission loss factor, L, is 0.1 (10%). Corresponding cost curves and demand curve are given in figure 6.2 and 6.3 respectively.

6.4.2.2 Results of Case 1 under incomplete information

Test results are given in table 6.12 where wheeling charges and transmission losses are considered.

Producer No.	Output	Profit	Price
G1	88.657	401.300	8.321
G2	57.616	236.260	
G3	29.842	119.182	
G4	49.584	158.439	
G5	33.445	101.096	
G6	32.831	99.745	
Total	291.975	1116.021	

Table 6.12 *The optimal production for case 1 including* P_X *and* L

The output of each supplier is reduced and profit increases due to high market price. This is the result of underestimation of cost coefficients of the rivals. When introducing wheeling charges and transmission losses, although market price is at a high level, the outputs of suppliers decline further as well as profit each supplier earns This is different to the case in table 5.4. Comparing with table 6.3 including wheeling charges and table 6.8 including transmission losses, the results in this case represent that mutual effect of wheeling charges and transmission losses make the output drop more as well as profit while market price is at a very high level. It is shown graphically in figure 6.15.

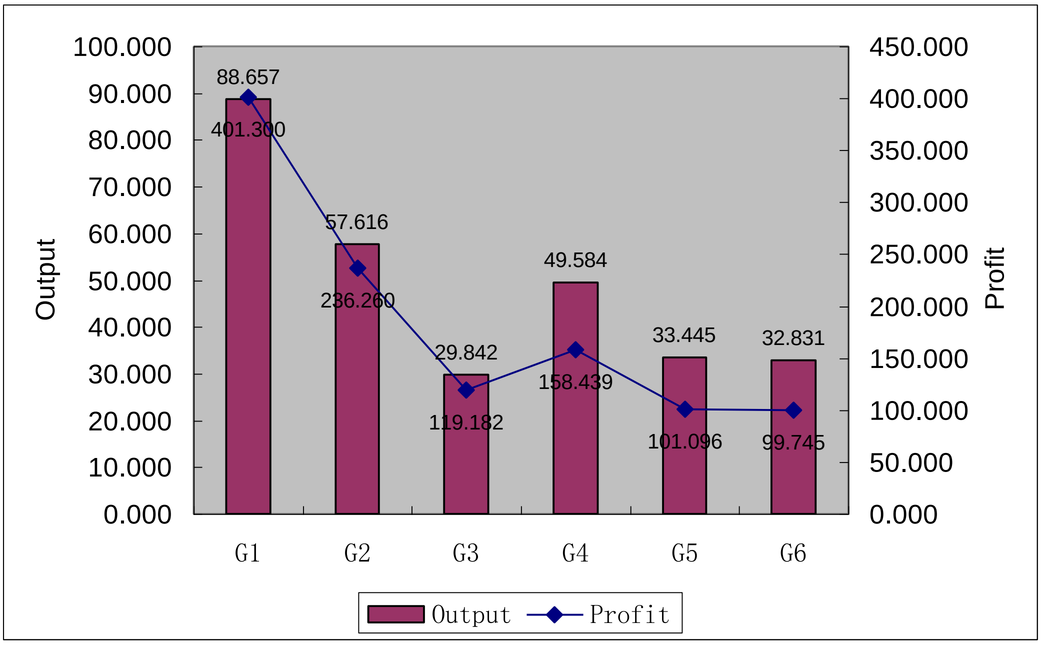

Figure 6.15 *The chart of optimization for case 1 including* P_X *and* L

6.4.3 Numerical model and Case-study 2 under incomplete information

In this part, prediction of cost functions of rivals is based on several estimated cost functions with a probability. With consideration of wheeling charges and transmission losses, the Cournot equilibrium solution has to be revised. Improved one is presented and testing results are given in the following paragraphs.

6.4.3.1 Numerical model for Case 2

The Cournot equilibrium solution for the i th supplier is presented in equation (6.20) where both wheeling charges and transmission losses are included.

$$q_i^{(i)} = \frac{(1-L)(P_{max}-P_X)-b_i-(1-L)K\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{t=1}^{T_j^{(i)}} r_{j,t}^{(i)}\frac{b_i-b_{j,t}^{(i)}}{(1-L)K+2c_{j,t}^{(i)}}}{2\left[(1-L)K+c_i\right]+(1-L)K\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{t=1}^{T_j^{(i)}} r_{j,t}^{(i)}\frac{(1-L)K+2c_i}{(1-L)K+2c_{j,t}^{(i)}}}$$

$$i,j=1,2,\ldots,n \quad i\neq j \quad t=1,2,\ldots,T_j^{(i)} \qquad (6.20)$$

Estimated coefficients of cost functions are derived in equation (6.8) where $\gamma_{1,1}$, $\gamma_{2,1}$ and $\gamma_{3,1}$ are randomly selected from range [0, 0.1] while $\gamma_{1,2}$, $\gamma_{2,2}$ and $\gamma_{3,2}$ from range [0, 0.2]. Probability is defined by 0.5 as two cost functions are involved in this case.

Together with equation (6.8) and (6.20) it determines the optimal production for the i th supplier in case 2 when considering wheeling charges and transmission losses. Cost coefficients and parameters of numerical model are extracted from table 6.1. In addition, the coefficient of wheeling charges, P_X, is set at 0.7 and transmission loss factor, L, is 0.1 (10%).

6.4.3.2 Results of Case 2 under incomplete information

Simulation results of case 2 are given in table 6.13 where both of wheeling charges and transmission losses are included.

Producer No.	Output	Profit	Price
G1	89.347	384.840	8.080
G2	59.080	227.955	
G3	31.142	115.099	
G4	50.735	150.641	
G5	34.010	94.959	
G6	33.677	94.310	
Total	297.991	1067.805	

Table 6.13 *The optimal production for case 2 including* P_X *and* L

In this case, the cost of the j th supplier is still underestimated by the i th supplier. As a result, the output of each supplier decreases and the profit increases due to the rise of market price when comparing with the ideal situation. However, with introduction of wheeling charges and transmission losses, the optimal production is reduced. Although market price increases greatly, the profit each supplier earned declines which can be found in pay-off function in equation (6.16).

Comparing with the results in table 6.4 with wheeling charges and table 6.9 with transmission losses, the output decreases further as well as the profit while market price is even higher which is caused by mutual influence of wheeling charges and transmission losses. Corresponding results are reproduced in figure 6.16.

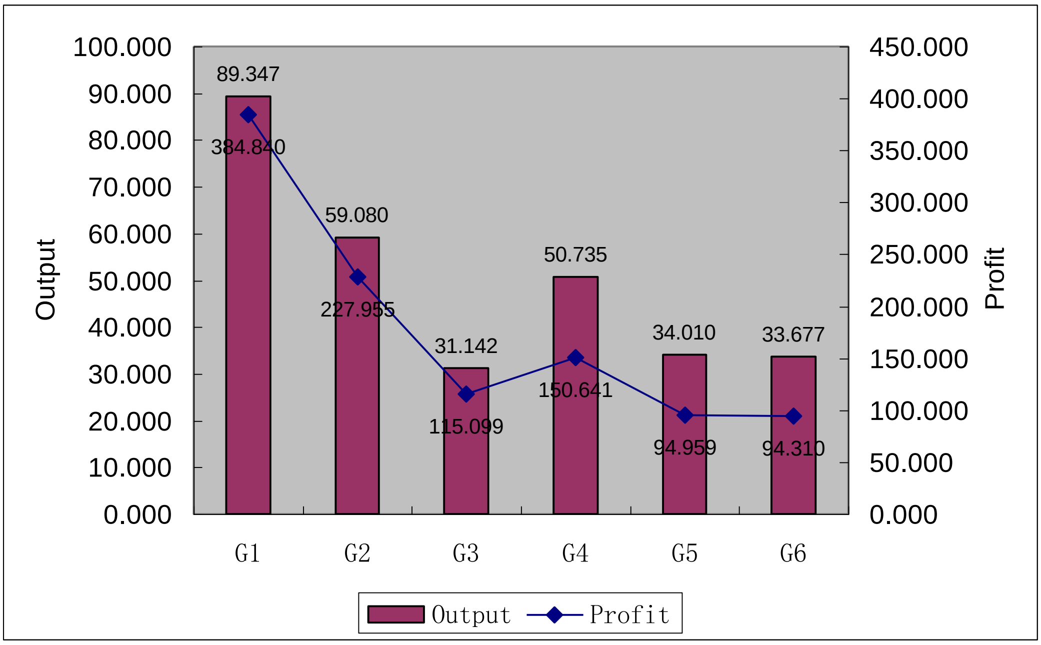

Figure 6.16 *The chart of optimization for case 2 including* P_X *and* L

6.4.4 Numerical model and Case-study 3 under incomplete information

In this case, in addition to estimated cost functions, a distribution function is introduced when predicting the optimal production for each supplier. As wheeling charges and transmission losses are considered, an improved Cournot equilibrium solution is presented to fulfill the task and testing results are given as well.

6.4.4.1 Numerical model for Case 3

The Cournot equilibrium solution of case 3 for the ith supplier is given in equation (6.21) where both wheeling charges and transmission losses are included.

$$q_i^{(i)} = \frac{(1-L)(P_{\max} - P_X) - b_i - (1-L)K\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{b_i - b_j^{(i)} - \alpha g}{(1-L)K + 2c_j^{(i)}}}{2\left[(1-L)K + c_i\right] + (1-L)K\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{(1-L)K + 2c_i}{(1-L)K + 2c_j^{(i)}}} \tag{6.21}$$

$$i, j = 1, 2, \ldots, n \quad i \neq j$$

Cost coefficients of the j th supplier are estimated through equation (6.6) where γ_1, γ_2 and γ_3 are randomly selected from the range [0, 0.2].

Together with equation (6.6) and (6.21), it determines the optimal production for the i th supplier in case 3 where wheeling charges and transmission losses are considered. Cost coefficients and initial parameters of numerical model are extracted from table 6.1. In addition, coefficient of wheeling charges, P_X, is set at 0.7 and transmission loss factor, L, is 0.1 (10%). Parameters α and g are defined by 4 and 0.1 respectively.

6.4.4.2 Results of Case 3 under incomplete information

Simulation results of case 3 are given in table 6.14 below where wheeling charges and transmission losses are included.

Producer No.	Output	Profit	Price
G1	93.540	357.393	7.557
G2	61.115	204.857	
G3	31.819	101.274	
G4	53.366	132.158	
G5	36.099	81.910	
G6	35.129	80.562	
Total	311.069	958.154	

Table 6.14 *The optimal production for case 3 including* P_X *and* L

In this case, the cost of the j th supplier is still underestimated by the i th supplier. As a result, the output of each supplier decreases and the profit increases due to the rise of market price when comparing with the ideal situation. However, when wheeling charges and transmission losses are considered, the optimal production is reduced. Although market price increases greatly, the profit each supplier earned declines which can be found in pay-off function in equation (6.16). Comparing with the results in table 6.5 with wheeling charges and table 6.10 with transmission losses, the output decreases further as well as the profit while market price is even higher which is the result of mutual influence of wheeling charges and transmission losses. However, with introduction of distribution elements, the results tend to be close to those in base case in table 6.11. Corresponding results are reproduced in figure 6.17.

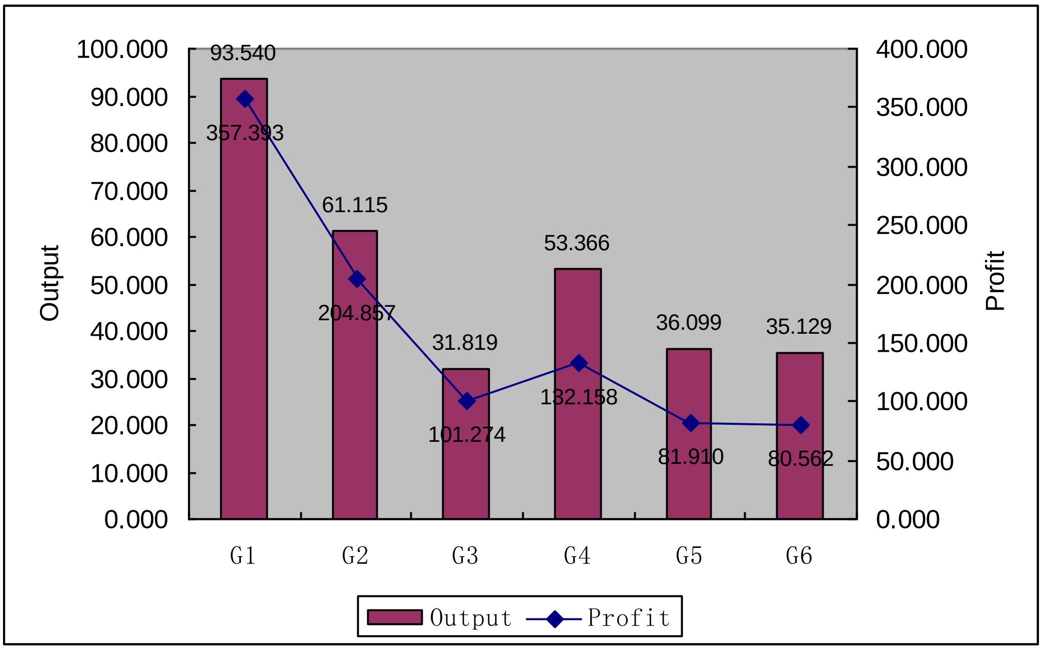

Figure 6.17 *The chart of optimization for case 3 including* P_X *and* L

6.5 Summary

Improved Cournot equilibrium solutions on various cases are presented and tested in this chapter under both complete information and incomplete information. Three conditions are investigated with simulation results which are wheeling charges included, transmission losses included and both wheeling charges and transmission losses considered. In each condition, a base case under complete information and three cases under incomplete information are tested and results are given in form of table and diagram.

In condition including wheeling charges, comparing with the preliminary modelling in Chapter 5, the optimal production of each supplier is reduced as well as the profit they earned while the market price is comparatively high. This can be seen from the results of base case and other three cases under incomplete information. It is noted that the fact of reduction of profits is not only determined by market price but also the pay-off function. Under incomplete information, for case 1 and case 2, the optimal production is reduced due to underestimates and profits increase caused by high market prices, for case 3, the results are close to those in base case which is the result of introduction of distribution elements.

In condition including transmission losses, comparing with the preliminary modelling, the outcomes are similar to those in situations considering wheeling charges that production and profit decline while market price goes up. In contrast to the base case, three cases under incomplete information act similarly to those when wheeling charges are considered. However, there are slight differences between the conditions of wheeling charges and transmission losses with same values of parameters that reduction of production and rise of market price are less while profit falls further where only transmission losses are considered.

In condition including both wheeling charges and transmission losses, in contrast to the preliminary modelling, production and profit decrease further while market price is even higher which is the result of mutual effect of introduction of wheeling charges and transmission losses. The results of cases under incomplete information indicate the same trend in accordance with the findings in previous conditions.

Cournot competition is the model to present a market structure where players compete on the quantity of output which they make decisions independently and simultaneously. As discussed Cournot equilibrium solution in determining the optimal production for each supplier is providing the theoretical equilibrium for the oligopoly market place. The objective of this chapter is to improve the model to reflect some of the additional features of the real world. Although even constraints mentioned in this chapter are included, it is still away from the reality. However, it is getting closer to the real world as more features are taken into consideration, such as transmission constraints and generation constraints, which could help market participants to better understand the market interaction.

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

This thesis has considered the application of Cournot equilibrium theory to an oligopoly electricity market. In this case each supplier involved has to decide their optimal production, which is a process of decision making. In order to help market participants to better understand market interaction, a method based on that developed by F. S. Wen and A. K. David is implemented as preliminary model in this thesis. This model has been reviewed and validated against the results of [13] This model is then extened such that a simple model of transmission constraints are taken into account and thereby to reflect some additional features of the real world trading prolem (i.e. wheeling charges and transmission losses) and accordingly improved Cournot equilibrium solutions have been obtained to fulfill the task finding market equilibrium in more complicated circumstances.

Chapter 2 of the thesis gives a review of deregulation of the vertically integrated electricity sector. The role of introducing deregulation into the electricity market is to increase competition among participants, e.g. suppliers on supply side, to hold down market price to marginal cost and minimize production cost. This is because, under regulation in a traditional market, it lacks of incentives to suppliers and incentives to regulatory bodies which results in incentives to efficiently operation and incurred unnecessary investments are removed. Due to physical features of electricity, power supply has to meet demand in real time which is the responsibility of system operator. In view of business, market equilibrium is achieved when aggregated supply curve and demand curve are intersected. Development of electricity market is described as four stages, monopoly, purchasing agency, wholesale competition and retail competition.

Chapter 3 of the thesis reviews game theory fundamentals to better understand the Cournot competition theory. Game theory is concerned with how rational individuals make decisions when they are mutually interdependent [10]. It can be classified into cooperative and non-cooperative game theory by seeing if players communicate with each other and static and dynamic game theory by seeing how many times the game is played and if they make decisions simultaneously. In this thesis, the game to be investigated is associated with non-cooperative and static game theory. Based on this, the game can be solved by two techniques, i.e. dominance and equilibrium. Equilibrium method is used in extensive situations comparing with dominance one.

Chapter 4 of the thesis provides methodology and mathematical models of Cournot competition for oligopoly electricity market production under complete information and incomplete information. As presented, Cournot competition is a model to describe market structure where companies compete on the quantity of output, which they make decisions independently and simultaneously. The features of Cournot competition are summarized as follows:

- more than one company produces a homogeneous product
- there is no communication and cooperation among companies
- companies have market power
- the number of companies is fixed
- companies compete in quantity and make decisions simultaneously
- companies are rational players in their own interest to maximize profit

Given these features, Cournot competition model is suitable to study the oligopoly electricity market to help the suppliers to decide their optimal production level. The preliminary market model is organized with cost functions and pay-off functions of producers as well as demand function of the market, however transmission losses are ignored. Based on pay-off functions of suppliers and estimated cost functions of the rivals if needed (under incomplete information), Cournot equilibrium solutions are derived.

Chapter 5 of the thesis gives simulation results of the preliminary model in which six producers compete to supply electrical energy to a single sided (competition in generation only) power exchange. They have different quadratic cost functions producing electricity energy. Both situations under complete information and incomplete information are considered and simulated. In base case, under complete information, suppliers know their own cost functions as well as cost functions about their rivals. Therefore, the derivation of Cournot equilibrium solution complies with Cournot competition theory. However, under incomplete information, cost functions of the rivals are not known exactly. Cournot equilibrium solutions have to be derived along with estimated cost functions of the rivals where estimating inaccuracy occurs. In the thesis three different estimating methods are presented. From simulation results, it is found that the optimal production for each supplier is mainly dependent on the cost coefficients of their rivals (cost functions) under complete information. It is also related to the accuracy of estimation on the cost functions of the rivals when considering incomplete information situations.

Lastly, Chapter 6 presents an extended model that includes transmission use cost (wheeling charges) and transmission losses as well. Due to introduction of these new features, the Cournot equilibrium solutions have been improved to fulfill the task. Three conditions have been investigated with simulation results which are wheeling charges included, transmission losses included and both wheeling charges and transmission losses considered. In comparison with the preliminary model, the presence of wheeling charges or transmission losses results in reduction of each supplier's production as well as profit, although market price goes up. Furthermore, presence of both wheeling charges and transmission losses has a further influence on the results.

To sum up, the optimal production of each supplier in the extended model is then determined by mutual effect of cost functions of suppliers, pay-off functions of theirs, market demand function, wheeling charges function and transmission losses factor. When considering incomplete information situations, estimated methods of cost functions of the rivals are needed to be taken into account and the outcome of Cournot equilibrium is associated with accuracy of methods of estimation. The thesis gave a view of Cournot competition to reflect some of the features of real world including transmission constraints to help market participants to better understand market interaction in an oligopoly electricity market and give them a rough sketch of their best production level and maximum profits when they do not have generation capacity limits and the network have some limited transmission constraints. It is also useful for market operator to observe the aggregated supply level and decide the way the market is cleared and the level of market price where it is assumed that total supply always meets demand.

7.2 Future Work

This section suggests possible future work to improve and to expand the application of the proposed Cournot equilibrium solution for the oligopoly electricity market production.

- **Enhancing wheeling charges functions:**

The wheeling charges function considered in the thesis indicates aggregated cost with a constant wheeling charge coefficient. It can be enhanced to reflect more features of real world by using embedded cost methods in determining wheeling charges for each player who uses the network. For example, methods may include postage stamp method, contract path method, distance based MW-mile method, and power flow based MW-mile method [39].

- **Considering transmission capacity:**

In the thesis, it is assumed that transmission network has infinite transmission capacity and generators can supply electricity energy to the demand-side as much as they could. When transmission network has a limited capacity, the transmission capacity may turn out to be scarce which results in supply may not meet the demand. Therefore, pricing allocation of this capacity should be taken into account.

- **Considering capacity limits of generators:**

It is assumed that generators produce as much electricity as they could to meet demand in the market place. However, in reality, generators have their own generation capacity which means production of electricity can not exceed this limit. Therefore, the limit of generation capacity needs to be treated carefully in the future work, until then the model reflects major realistic facts.

- **Expanding the model to a multi-market environment:**

A single electricity market is considered in the thesis in which the proposed approach has been studied and tested. At next stage, the model can be expanded to a multi-market one that suppliers could choose to supply electricity through more than one transmission line to specific markets for higher profits. Thus functions of transmission congestion and costs need to be organized.

APPENDIX A: Testing Interface of Base Case

In this appendix the inputs and outputs associated with the base case model are given.

- **Testing interface of Base Case in Spreadsheet:**

Coefficients	a	b	c	Pmax	QLmax	K
G1	0	2	0.00375	20	500	0.04
G2	0	1.75	0.0175			
G3	0	1	0.0625			
G4	0	3.25	0.00834			
G5	0	3	0.025			
G6	0	3	0.025			

Producer No.	Output	Profit	Price
G1	96.932	411.064	6.604
G2	64.723	240.874	
G3	33.965	118.247	
G4	59.179	169.293	
G5	40.047	104.246	
G6	40.047	104.246	
Total	334.894	1147.971	

Results

Reset

APPENDIX B: Testing Interface of Case 1

In this appendix the inputs and outputs associated with the model for Case 1are given.

- **Testing interface of Case 1 in Spreadsheet:**

Coefficients	a	b	c	Pmax	QLmax	K
G1	0	2	0.00375	20	500	0.04
G2	0	1.75	0.0175			
G3	0	1	0.0625			
G4	0	3.25	0.00834			
G5	0	3	0.025			
G6	0	3	0.025			

Producer No.	Output	Profit	Price
G1	89.960	469.341	7.555
G2	59.839	284.678	
G3	32.527	147.075	
G4	55.551	213.387	
G5	36.162	132.010	
G6	37.097	134.557	
Total	311.135	1381.048	

Results

Reset

APPENDIX C: Testing Interface of Case 2

In this appendix the inputs and outputs associated with case 2 model are given.

- **Testing interface of Case 2 in Spreadsheet:**

Coefficients	a	b	c	Pmax	QLmax	K
G1	0	2	0.00375	20	500	0.04
G2	0	1.75	0.0175			
G3	0	1	0.0625			
G4	0	3.25	0.00834			
G5	0	3	0.025			
G6	0	3	0.025			

Producer No.	Output	Profit	Price
G1	91.889	450.739	7.250
G2	62.116	274.109	
G3	32.686	137.510	
G4	55.952	197.692	
G5	38.384	126.294	
G6	37.726	124.748	
Total	318.754	1311.091	

Results

Reset

APPENDIX D: Testing Interface of Case 3

In this appendix the inputs and outputs associated with the case 3 model are given.

- **Testing interface of Case 3 in Spreadsheet:**

Coefficients	a	b	c	Pmax	QLmax	K
G1	0	2	0.00375	20	500	0.04
G2	0	1.75	0.0175	Alpha	g	
G3	0	1	0.0625	4	0.1	
G4	0	3.25	0.00834			
G5	0	3	0.025			
G6	0	3	0.025			

Producer No.	Output	Profit	Price
G1	94.984	416.367	6.740
G2	64.033	247.756	
G3	33.825	122.639	
G4	58.127	174.669	
G5	39.797	109.235	
G6	40.740	110.863	
Total	331.506	1181.529	

Results

Reset

REFERENCES

[1] R. J. Green and D. M. Newbery, *Competition in the British electricity spot market*, H. Political Economy, vol. 100, no.5, pp. 929-953, 1992

[2] Daniel S. Kirschen and Goran Strbac, *Fundamentals of Power System Economics*, UMIST, John Wiley & Sons, Ltd, UK, November 2005

[3] Severin Borenstein, James B. Bushnell and Frank A. Wolak, *Measuring Market Inefficiencies in California's Restructured Wholesale Electricity Market*, The American Economic Review, Vol. 92, No. 5, pp. 1376-1405, American Economic Association, Dec. 2002.

[4] Richard D. Tabors, *Lessons from the UK and Norway*, IEEE Spectrum, August 1996.

[5] W. J. Lee and C. H. Lin, *Utility Deregulation and Its Impact on the Industrial Power Systems*, Industrial and Commercial Power systems Technical Conference, IEEE, 1997

[6] Gregory J. Miranda, *Be Prepared, An overview of process industry options in the deregulated era*, IEEE Industry Applications Magazine, Mar/Apr 2003.

[7] Narinder K. Trehan and Rohit Saran, *Electric Utility Deregulation: Failure or Success*, Nuclear Science Symposium Conference Record, IEEE, 2004

[8] George G. Djolov, *The Economics of Competition, the race to monopoly*, the Haworth Press Inc., USA, 2006

[9] J. D. Liu, T. T. Lie, and K. L. Lo, *An Empirical Method of Dynamic Oligopoly Behavior Analysis in Electricity Markets*, IEEE, Transactions on Power Systems, vol. 21, No. 2, may 2006

[10] Graham Romp, *Game Theory: Introduction and Applications*, Oxford University Press, 2005

[11] F. S. Wen and A. K. David, *Oligopoly Market Production under Incomplete Information*, vol 21, pp. 58-61, IEEE Power Engineering Review, April 2001

[12] S. Borenstein and J. Bushnell, *An Empirical Analysis of the Potential for Market Power in California's Electricity Industry*, vol. 47, no. 3, pp. 285-323, J. Ind. Econ., September 1999

[13] Z. Younes and M. Ilic, *Generation Strategies for Gaming Transmission Constraints: Will the deregulated electric power market e an oligopoly?*, vol. 24, no. 3-4, pp. 207-222, Decision Support Syst., 1999

[14] A. Rudkevich, M. Duckworth, and R. Rosen, *Modeling Electricity Pricing in a Deregulated Generation Industry: The potential for oligopoly pricing in a poolco*, vol. 19, no. 3, pp. 19-48, Energy J., 1998

[15] Erik R Larsen and Derek W Bunn, *Modelling Electricity Privatization in the UK*, International System Dynamics Conference, UK, 1994

[16] N S Modi and B R Parekh, *Transmission Network Congestion in Deregulated*

Wholesale Electricity Market, Proceedings of the International MultiConference of Engineers and Computer Scientists, Hong Kong, 2009

[17] Steven Stoft, *Power System Economics: Designing Markets for Electricity*, IEEE/Wiley, USA, February 2002

[18] Fereidoon P. Sioshansi and Wolfgang Pfaffenberger, *Electricity Market Reform, An international perspective*, Elsevier Ltd., UK, 2006

[19] Jerry Taylor and Peter VanDoren, *California's Electricity Crisis, What's going on, Who's to blame, and What to do*, Cato Institute, Policy Analysis, Jul 2001

[20] Siddhartha Kumar Khaitan, Yuan Li, and Chen-Ching Liu, *Optimization of Ancillary Services for System Security: Sequential vs. Simultaneous LMP calculation*, IEEE International Conference, May 2008

[21] Yuan-Kang Wu, *System Operation and Risk Management in Deregulated Electricity Markets*, PhD thesis in department of EEE, University of Strathclyde, 2004

[22] I. El-Samahy, K. Bhattacharya, and C. A. Canizares, *A Unified Framework for Reactive Power Management in Deregulated Electricity Markets*, Power Systems Conference and Exposition, IEEE, 2006

[23] Paul A. Samuelson and William D. Nordhaus, Economics – the 18^{th} Edition, McGraw-Hill Press, Jul 2004

[24] Kenneth J. Arrow and Gerard Debreu, *Existence of an Equilibrium for a Competitive Economy*, Econometrica, vol. 22, No. 3, pp. 265-290, Jul 1954

[25] Ross Baldick, *Computing the Electricity Market Equilibrium: Uses of Market Equilibrium Models*, Power Systems Conference and Exposition, IEEE, 2006

[26] Sally Hunt and Graham Shuttleworth, *Competition and Choice in Electricity*, Chichester, UK, John Wiley & Sons, 1996

[27] Eric Rasmusen, *Games and Information, Second Edition, an Introduction to Game Theory*, Blackwell publishers Ltd, Oxford, UK, Printed in the USA, 1994

[28] Avinash K. Dixit, Barry J. Nalebuff, *Thinking Strategically, the Competitive Edge in Business, Politics, and Everyday Life*, W. W. Norton & Company Ltd, New York, USA, 1991

[29] H. Scott Bierman and Luis Florentin Fernandez, *Game Theory with Economic Applications*, Addison-Wesley, Reading, 1993

[30] Robert J. Aumann and Sergiu Hart, *Handbook of Game Theory with Economic Applications*, Elsevier Ltd, UK, 1994

[31] David M. Kreps, *Game Theory and Economic Modelling*, Oxford University Press Inc., New York, USA, 1990

[32] J. W. Friedman, *Oligopoly and the Theory of Games*, North Holland, New York, 1977

[33] J. W. Friedman, *Game Theory with Applications to Economics*, Oxford University Press, New York, 1990

[34] R. Billinton and R.N. Allan, *Reliability Evaluation of Engineering Systems, Concepts and Techniques*. White Plains, NY: Longman/New York: Plenum, 1984

[35] “*U.S. Congress, Office of Technology Assessments, Electric power wheeling and dealing: Technological consideration for increasing competition*” U.S. Government Printing Office, Washington, D.C., Rep. OTA-E-409, May 1989

[36] Wei Jen Lee, C. H. Lin and Larry D. Swift, *Wheeling Charge under a Deregulated Environment*, IEEE Transactions on Industry Applications, Vol. 37, No. 1, Jan/Feb 2001

[37] Report: *Price Manipulation in Western Market Docket*, U.S. Department of Energy, Federal Energy Regulatory Commission, Mar. 2003

[38] Graham Philips, *Domestic Energy Use in the UK–Power Conversion, Transport, and Use*, 2000

[39] Mohammad Yusri Hassan, *MW-mile Charging Methodology for Wheeling Transaction*, PhD Thesis of University of Strathclyde, Jun 2004