Multivariate Unobserved Components Models in a Globalised World

Ping Wu

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Department of Economics
University of Strathclyde

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Declaration

Signed:

Date:

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Abstract

This thesis is a multi-country study and takes the literature on the country-specific unobserved components (UC) model as its starting point. In three increasingly flexible essays, we gradually extend the country-specific UC model to consider the affects of globalisation. Additionally, we propose to incorporate heterogeneities across countries in a data based fashion. The contribution is provided in three essays.

In the first essay (Chapter 2) we estimate a country-specific unobserved components model with time-varying parameters and stochastic volatility. We consider 34 countries (23 advanced economies and 11 emerging market economies) and independent assumptions across countries are imposed. To consider the affects of globalisation, the model incorporates two observed global factors (oil price and global output) which account for global determinants of inflation. We find that inflation dynamics are explained by the combination of domestic factors (lagged domestic inflation and domestic output) and observed global factors (global output and oil price). Effects of these variables are constant over time. The Phillips curves are generally flat for the period under consideration (1995-2018), and different from zero. The global demand seems to matter more in emerging market economies than in advanced economies. Our results also point to the dominant role played by oil price as a key factor behind inflation dynamics over time.

In the second essay (Chapter 3) we relax the assumption that errors across countries are independent, and allow for cross-country linkages in the error covariance matrix. First, we use the factor stochastic volatility specification to allow for cross-country linkages in the error covariance matrix. This method assumes that all countries' errors are driven by latent factors. However, one practical problem is that heterogeneities are very likely to exist in errors, since we include both advanced economies and emerging market economies. For that reason, we allow for stochastic volatility in all errors and propose a method to remove stochastic volatility in a data based fashion. We rewrite the process of log-volatility using the non-centered parameterization and impose the Horseshoe prior on the coefficient that controls time-variation in the log-volatility. We apply these methods to the data in Chapter 2. We find evidence that there are global factors driving all countries' inflation (output). The estimates under this model are in line with previous studies and, for certain countries, the estimates indicate that they are influenced by both domestic factors and global factors. Allowing for cross-country linkages in the error covariance matrix will decrease the persistence and flatten the Phillips curve. We also find that this model provides a superior in-sample fit and accurate density forecasts compared to existing models in the literature, especially if the period of uncertainty is the period being forecasted.

Finally, in the third essay (Chapter 4) we further relax the assumption that the conditional mean depends on domestic factors, and allow for cross-country linkages both in the error covariance matrix and in the conditional mean. We name this model a panel unobserved components model. It extends Chapter 3 in three ways. First, it takes dynamic interdependencies into account by allowing for cross-country linkages in the conditional mean, more specifically, in coefficient matrices associated with lagged variables. Second, it takes static interdependencies into account. This is done through two blocks. One block is allowing for cross-country linkages in the error covariance matrix. Another block is allowing for cross-country linkages in the conditional mean, more specifically, in the coefficient matrix associated with the Phillips curve. Therefore, the panel unobserved components model adds a new measure of static interdependencies, by extending the traditional (own country) Phillips curve to a global Phillips curve. Third, we work with the unrestricted panel unobserved components model. One issue is that parameters can be enormous. To deal with over-parameterization concerns, we rely on the Horseshoe prior. We apply these methods to the data in Chapter 2. Estimates of coefficients and generalised impulse response functions provide evidence of interdependencies. Generalised impulse response functions also show the “fragile inflation”in emerging market economies, which means that inflation needs a longer time to settle down. We find that allowing for cross-country linkages in the error covariance matrix can provide more precise estimates of trends, while omitting cross-country linkages in the conditional mean will overestimate trend output. Additionally, our proposed model provides a superior in-sample fit and accurate density forecasts compared to existing models in the literature.

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Chapter 1

Introduction

1.1 General background

Understanding macroeconomic development has long been a subject of prolific economic research. One subject that provides a conceptual reference point is the concept of trends. For instance, trend inflation can be thought of as the rate of inflation that we would expect after temporary factors subside (Clark and Garciga, 2016). One of monetary policy aims is to maintain low and stable inflation. Hence, many central banks have set their official inflation targets (point target or target bands). For example, the Federal Reserve's Federal Open Market Committee (FOMC) has set a long-run objective for consumer price inflation of 2.0 percent. The official inflation targets in 34 countries (regions) are provided in the last column in Figure 2.2. To make the expressions easier, this thesis uses the term "country"to represent both "country"and "region".

Another subject is deviations of a variable from its trend. These deviations reflect cyclical fluctuations and are important in driving correct decisions. Families can decide which asset to invest. Employees can negotiate wage with employers. Factories can decide how many products to produce. Firms can set their prices of products. Thus, the understanding of dynamics of cyclical changes and the pace of their adjustment towards the trend is influential and is of interest to macroeconomists. We can often hear something like "*The Committee does not intend to tighten monetary policy at least until there is clear evidence that significant progress is being made in eliminating spare capacity and achieving the 2% inflation target sustainably.*"

What is *clear evidence*? One possible way is looking at both inflation and its fundamental drivers, such as output growth rate, unemployment rate, oil prices, and exchange rate. This suggests that it is important to jointly model variables. Multivariate model is preferred.

The preceding discussion suggests that researchers should be interested in a model with two characteristics. First, it should be written in terms of latent state vectors that can be given an economic interpretation, like trends. Second, it is a multivariate model. Such a model is the multivariate unobserved components (UC) model. A UC model decomposes a variable into two unobserved processes: a random walk trend component, and a stationary cyclical component. The random walk trend component is also called trend. The stationary cyclical component is also called the deviations of a variable from its trend. The three essays in this thesis extend the existing UC literature in different ways. However, one common aspect of all extensions is that they are introduced in the face of globalisation. The reason for this is discussed in the next section.

1.2 Why does globalisation matter?

Since antiquity, globalisation has been developed. The Silk Road once connected China, Central Asia, Persia, and Europe, and facilitated commercial and cultural exchange. However, the globalisation we are experiencing nowadays is the biggest and fastest in human history. Think about the products you have. Last time you went shopping, maybe you bought fruits from South America, tea from Southeast Asia, and fish from Europe. Your computer might have local components and Asian parts. Perhaps the clothes you're wearing were made in India, your car came from Japan, and you just video called a friend halfway around the world.

We live in a connected world and countries' economies are linked together. Terms like "global economies", "global interdependencies" have become part of every day articles. Countries, regions or sectors can no longer be treated in isolation and spillovers can generate influence on countries' economies. Theoretically, Canova and Ciccarelli (2013) point out that market clearing implies the excess demand in a country is compensated by excess supply in other countries. These spillovers, together with adjustments in the relative prices of goods and/or assets, imply generalized feedbacks from one country to all the others and influence the steady states (also called *trends* in this thesis). There are subtle differences between *steady state* and *trend*, but they can be interpreted as the same for the purpose of this thesis, so these two terms will be used interchangeably as equivalent through this thesis.

An evidence is from a bird's eye view of inflation globally after the Global Financial Crisis hit the economy. Both advanced economies and emerging market economies have experienced apparent decline in inflation. Credit booms can help to explain what the advanced economies have experienced. However, many of the emerging market economies did not experience similar credit booms, nor did they experience the rapid loss in employment (Jordà and Nechio, 2018). Such disconnect between inflation and its fundamental drivers is also observed in advanced economies after the Global Financial Crisis. For instance, US personal consumption expenditures inflation declined in 2017 in the midst of a low and falling unemployment rate, stable oil prices, and a depreciating dollar (Knotek II and Zaman, 2017).

This disconnect between inflation and its fundamental drivers leads to a nascent literature on inflation dynamics. In the 1970s and '80s, much research in this area was dedicated to understanding the causes and costs of high inflation and how to disinflate effectively. After the Global Financial Crisis, the focus has shifted to understanding the determinants of inflation. Many papers indicate that inflation is determined globally.

There are two issues that need to be taken care of when considering globalisation. The first issue is that every country, no matter whether there is or not theory behind it, carries some information that might be useful for explaining the dynamics of a variable. The second issue is that substantial heterogeneities remain in this globalised world (like differences in the pace of the recovery from the Global Financial Crisis). This thesis deals with both these issues. The first issue is solved by dropping the assumptions that the errors across countries are independent (see Chapter 3) and that the conditional mean is driven by domestic factors (see Chapter 4). The second issue is also taken into account in this thesis by developing a method to shrink the parameters in a data based fashion (see Chapter 3 and Chapter 4).

1.3 The contribution of this thesis

In light of the motivation outlined above, this thesis extends the existing unobserved components literature in the face of globalisation. In Chapter 2, we extend the bivariate unobserved components model to the variable-domain. Except domestic factors, we add two observed global factors as additional explanatory variables. The two global factors are global output and oil price. We allow the model to have both time-varying parameters and stochastic volatility. This leads to a high dimensional model. To ensure stationarity, we add a bound on coefficients. We apply these methods to 34 countries covering the period 1995-2018. The results show that there is stochastic volatility in inflation, and it has declined across all countries, but remained relatively high in emerging market economies and few advanced economies. One important result is that inflation dynamics are explained by the combination of domestic factors and global factors. What strikes us is that the effects of these factors are constant over time and across countries. We think the reason is that our sample is from 1995 to 2018. Oil price plays a key factor behind inflation dynamics across countries and over time. Phillips curves are generally flat, but different from zero.

Chapter 2 finds that the effects of factors are constant over time. Together with the Bayes Factor (see Chapter 3), we start from the constant coefficient unobserved components model (we allow for stochastic volatility) in Chapter 3 and Chapter 4. We relax the independent assumption in Chapter 2. The precise name of the model in Chapter 2 is multi-country unobserved components models, which means there are 34 independent models, not a multi-country unobserved components model, which is one model and has many equations (hence the term "Multivariate"). The desirable model is a multivariate unobserved components model, which allows for cross-country linkages in the error covariance matrix, or in the conditional mean, or both. As I argue in Chapter 3 and Chapter 4, I propose methods to allow for cross-country linkages in these chapters. A common feature of these methods is that we can work with the unrestricted model. There is no need to impose zero restrictions; any cross-country linkages can take place in the error covariance matrix, or in the conditional mean, or both; there is no need to develop new methods to estimate the model; all that is needed is to estimate the whole model using standard Markov Chain Monte Carlo (MCMC) algorithm; there is no need to worry whether such standard MCMC algorithm is at the expense of slower computation time; equation-by-equation estimation can be implemented to most parameters. We first allow for cross-country linkages in the error covariance matrix in Chapter 3. The reasons are that the globalisation we are experiencing nowadays is the fastest in human history and that the data we use is quarterly data. Since cross-country linkages in the error covariance matrix occur contemporaneously, we think that they are more likely to take place. Then in Chapter 4, we further allow for cross-country linkages in the conditional mean.

In Chapter 3 we use the factor stochastic volatility specification in order to allow for cross-country linkages in the error covariance matrix. Factor stochastic volatility specification is computationally efficient. Conditionally on the latent factors and their loadings, equation-by-equation estimation becomes possible within each MCMC iteration. Additionally, we allow for stochastic volatility in all equations and factors. This leads to a more flexible model. A practical problem that occurs in this case is over-parameterization. Heterogeneities across countries exist, subsequently this is a problem where data based selection is very relevant. We propose a method to remove stochastic volatility in a data based fashion. More specifically, we rewrite the process of log-volatility using the non-centered parameterization and use the Horseshoe prior to select time-variation. We apply these methods to the data in Chapter 2. The existence of global factors provides evidence of cross-country linkages in the error covariance matrix. The estimates under our model are in line with previous studies and, for certain countries, the estimates indicate that they are influenced by both domestic factors and global factors. We find that our proposed model provides a superior in-sample fit and accurate density forecasts compared to existing models in the literature, especially if period of uncertainty is the period being forecasted.

In Chapter 4 we further relax the assumption that the conditional means are driven by domestic factors in Chapter 3, and allow for cross-country linkages both in the error covariance matrix and in the conditional mean. This is done through dropping the assumption that coefficient matrices are diagonal in Chapter 3, and allowing them to be full matrices. This leads to the number of total parameters being in the order of thousands, and over-parametrization is a major concern. Such issue is further compounded when no zero restrictions are imposed. These reasons call for Bayesian shrinkage prior. We use an empirically successful prior (the Horseshoe prior), which automatically imposes the zero restriction for most elements but dropping the restriction if necessary. We apply these methods to the data in Chapter 2. The estimates of coefficients and generalised impulse response functions provide evidence of cross-country linkages. We find that allowing for cross-country linkages in the error covariance matrix can provide more precise estimates of trends, while omitting cross-country linkages in the conditional mean will overestimate trend output. Our proposed model provides a superior in-sample fit and accurate density forecasts compared to existing models in the literature.

Each chapter is self contained, so I provide specific motivation for each method used in each individual chapter. Additionally, each chapter contains the necessary information for the reader to understand the intuition behind the methods proposed in this thesis.

Chapter 2 is co-authored work with Alain Kabundi, Ayhan Kose and Aubrey Poon. All empirical work in this joint research was undertaken by myself as well as the second draft of the paper.

Chapter 2

Multi-country Time-Varying Phillips Curves with Observed Global Factors

2.1 Introduction

Recent development in inflation dynamics has raised questions as to whether the relationship between real economy and inflation has been altered and whether the Phillips curve is still valid. In the aftermath of the Global Financial Crisis (GFC), many countries experienced a sharp decline in output with mild effects on inflation (see, Simon et al., 2013). The fear of deflation shared by many policymakers and central banks did not materialize. And the post-crisis recovery, especially in advanced economies (AEs) which recorded low unemployment, was not accompanied with rising in inflation. A view widely accepted is that the slope of the Phillips curve has flattened since the early 1990s, possibly owing to both improved conduct of monetary policy with a better anchoring of inflation expectations (see, Simon et al., 2013; Kiley et al., 2015; Coibion and Gorodnichenko, 2015; Jordà et al., 2019), and a decline in inflation volatility.[1]

[1] See for example Carlstrom and Fuerst, 2008; Ball and Mazumder, 2011; Simon et al., 2013; Blanchard et al., 2015; Gillitzer and Simon, 2018; Blanchard, 2016; Chan et al., 2016; Kabundi et al., 2019 on flattening the Phillips curve.

This observation of flattening of the Phillips curve has interesting policy implications. It suggests the sacrifice ratio has increased, which means that monetary policy needs to be more aggressive to bring about a small change in inflation. This has rekindled interest about key drivers of national inflation. Two views emerge from the literature based on empirical evidence when attempting to explain this conundrum.

The first camp supports the notion that inflation is still largely explained by domestic factors. Proponents consider the credibility, achieved by many central banks in anchoring inflation expectations around the set target, as the main cause of flattening of the Phillips curve. In this environment, inflation expectations are well anchored at the central bank explicit target, which in turn stabilizes realized inflation in a way that renders it less responsive to demand pressure.

Theoretically, King and Wolman (1996) demonstrates how credible monetary policy authority is capable of stabilizing inflation, by anchoring expectations of agents around its objective. As agents form their expectations through learning, a gradual disinflation process results in slow learning, which in turn leads to adaptive inflation expectations. However, if the central bank shows a strong commitment disinflate quickly, agents will react accordingly as they believe that the central bank will achieve its objective. Hence expectations become anchored. This suggests a mild reaction of inflation to demand pressures, implying a flat Phillips curve. There is increasing empirical evidence linking the flattening of the Phillips curve to credible monetary policy especially for AEs, but also for some EMEs with inflation targeting (IT) policy framework.[2] This hypothesis is widely shared in policy circles as well as in academia. Many papers find no evidence that global factors explain largely inflation dynamics in recent two to three decades (see, Ihrig et al., 2010; Martínez-García and Wynne, 2013; Mikolajun and Lodge, 2016 and Bems et al., 2018). They think the flattening of the Phillips curve reflects policy synchronicity in AEs and EMEs, embodied wide spread use of nominal anchor. Other

[2]See for example Ball (2006), Williams et al. (2006), Simon et al. (2013), Kiley et al. (2015), Jordà et al. (2019).

factors include nonlinearity of the Phillips curve (see, Gagnon and Collins, 2019 and Hooper et al., 2020) and the composition of consumer price indices which comprises predominantly of noncyclical items (see, Stock and Watson, 2019).

The second camp highlights the role played by global factors in explaining the muted response of domestic inflation. The literature identifies several channels through which globalisation affects inflation dynamics in the last two decades. First, researchers point to the adoption of IT framework in many countries.[3] Second, rising trade integration affects domestic inflation via increase in exports and imports to GDP ratio which in turn drives up, on the one hand national income, and on the other hand, imports prices. Moreover, Gilchrist and Zakrajsek (2019) demonstrate how increased trade exposure significantly reduces the response of the US inflation to fluctuations in economic activity over time starting in the 1990s. Third, the expansion of EMEs, particularly China, contribute in various ways to recent development in countries' inflation. For example, Eickmeier and Kühnlenz (2018) show that China's demand and supply shocks affect significantly inflation in other countries. Direct channels operate via export and import prices, while indirect channels work through exposure to foreign competition and commodity prices. Furthermore, growing significance of EMEs in the world economy, coupled with rising in technology, lead to expansion of value chains to locations across borders with low cost of production. In this regard, Albuquerque and Baumann (2017) point to the essential role played by the global value chains in raising importance of the global output gap in determining domestic inflation. At the same time, there has been an increase supply of cheap labor from EMEs, which has exerted downward pressure on global wages. Fourth, one cannot undermine the role played by common shocks affecting economies simultaneously (e.g. oil or commodity prices) or spilling over from one or subset of countries to others (e.g. during the pandemic). For example, inflation volatility rose sharply among Latin American and Asian countries respectively during the Tequila and Asian crisis, which then spread globally especially in countries with

[3] See for example Gamber and Hung, 2001; Bean, 2006; Borio and Filardo, 2007; Albuquerque and Baumann, 2017; Eickmeier and Kühnlenz, 2018; Forbes, 2019; Gilchrist and Zakrajsek, 2019.

weak economic fundamentals.

Forbes (2019) demonstrate empirically, using a Phillips curve with a set of global variables, that globalisation plays an increasingly more important role in explaining inflation dynamics. While domestic forces are still relevant in driving inflation, their roles have changed over time. For example, domestic slack remains important, but it has become less so over time, whereas the role played by global slack has increased. Interestingly, she concurs with the first camp regarding the essential role played by inflation expectations and lagged inflation, which reflect strong impact of policy.

Unlike Forbes (2019) who estimates panel regressions for a dataset comprising of 43 countries, this paper estimates the Phillips curve for individual countries from a dataset with a total of 34 countries (23 AEs and 11 EMEs). Estimating the Phillips curve in this way enhances the understanding of inflation dynamics specific to each country as well as common features shared globally. Besides the time-variation in coefficients, some parameters are restricted in line with expectations. Specifically, the model constrains the inflation persistence (coefficient on lagged inflation) to be positive and less than one. This restriction allows inflation to follow a mean-reverting process in line with the data generating process. Like Kabundi et al. (2019), inflation persistence in this model is interpreted as a measure of the degree of central bank credibility. It implies that 0 means the central bank is fully credible and agents are forward looking, whereas 1 suggests complete lack of credibility and agents are backward looking. Coefficients on domestic output gap, global output gap, and oil price gap are restricted to be positive and less than one. Without this constraint, the estimation sometimes yields negative slope of the Phillips curve which is implausible. It is worth mentioning that the oil price and global out gap are estimated outside of the model. The intuition here is that each country is expected to face the same oil and global shocks. Estimating them in the model will yield different measures for each country, which is counter-intuitive.

The current paper contributes to the existing literature by attempting to answer the following questions. Are the coefficients changing over time? Does the global output gap, in addition to the domestic output and oil price gap, explain inflation dynamics? Of the four variables included in the model, which one matters the most for domestic inflation? The results can be summarized as follows.

There is evidence of changing nature of inflation volatility, which has declined across all countries but remained relatively high in EMEs and several AEs. Inflation volatility tends to rise in crisis periods compared with tranquil times. The implicit target is relatively constant in many countries at levels consistent with the set objectives, but closer to the upper bound of the target band for most of EMEs. Consistent with the literature, the results show that Phillips curves are generally flat for the period under consideration, but different from zero. They seem to be more flat in AEs than in EMEs. The global demand seems to matter more than the domestic demand in all countries, except for South Africa. The impact is constant throughout the sample and across countries. And the global demand seems to matter more in EMEs than in AEs. Finally, the results point to the dominant role played by oil prices as a key factor behind inflation dynamics across countries and over time. The coefficient on oil price gap is on average 6 times more than that of domestic output gap and 4 times more than the global output gap.

The remainder of the paper is organized as follows. Section 2.2 describes the specification of a time-varying Phillips curve with a bounded random walk process. Section 2.3 discusses the data, their transformation and empirical results. Section 2.4 concludes.

2.2 Model Specification

We start from the bivariate unobserved components model in Chan et al. (2016):

$$\pi_{i,t} - \tau^{\pi}_{i,t} = \rho_{i,t}(\pi_{i,t-1} - \tau^{\pi}_{i,t-1}) + \alpha_{i,t}(y_{i,t} - \tau^{y}_{i,t}) + \epsilon^{\pi}_{i,t} \tag{2.1}$$

$$y_{i,t} - \tau^{y}_{i,t} = \varphi_{i,1}(y_{i,t-1} - \tau^{y}_{i,t-1}) + \varphi_{i,2}(y_{i,t-2} - \tau^{y}_{i,t-2}) + \epsilon^{y}_{i,t} \tag{2.2}$$

$$\tau^{\pi}_{i,t} = \tau^{\pi}_{i,t-1} + \epsilon^{\tau\pi}_{i,t}, \quad \epsilon^{\tau\pi}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau\pi}) \tag{2.3}$$

$$\tau^{y}_{i,t} = \tau^{y}_{i,t-1} + \epsilon^{\tau y}_{i,t}, \quad \epsilon^{\tau y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau y}) \tag{2.4}$$

where i denotes country i, $i = 1, \ldots, N$. At time t, $\pi_{i,t}$ is the inflation of country i and $y_{i,t}$ is the output of country i, $\tau^{\pi}_{i,t}$ and $\tau^{y}_{i,t}$ are their trends. These trends are unobserved latent states which can be interpreted as long-run equilibrium level of inflation and output, also known as trend inflation and trend output. $\pi_{i,t} - \tau^{\pi}_{i,t}$ is the inflation gap, $y_{i,t} - \tau^{y}_{i,t}$ is the domestic output gap. $\epsilon^{\pi}_{i,t}$ is the error term with a stochastic volatility defined as:

$$h_{i,t} = h_{i,t-1} + \epsilon^{h}_{i,t}, \quad \epsilon^{h}_{t} \sim \mathcal{N}(0,\ \sigma^2_{i,h}) \tag{2.5}$$

$\rho_{i,t}$ is the inflation persistence. When expectations are well anchored, inflation is less persistent. Conversely, when expectations are adaptive, inflation tends to exhibit high persistence. $\alpha_{i,t}$ is the slope of the Phillips curve. $\rho_{i,t}$ and $\alpha_{i,t}$ are allowed to vary over time expressed by:

$$\rho_{i,t} = \rho_{i,t-1} + \epsilon^{\rho}_{i,t}, \quad \epsilon^{\rho}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{i,\rho}) \tag{2.6}$$

$$\alpha_{i,t} = \alpha_{i,t-1} + \epsilon^{\alpha}_{i,t}, \quad \epsilon^{\alpha}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{i,\alpha}) \tag{2.7}$$

The current model specification departs from Chan et al. (2016) and Kabundi et al. (2019), in that, it includes explicitly supply factor like Blanchard et al. (2015). And it includes the global output gap in addition to oil price. With these two additional variables, the first equation in (2.1) becomes:

$$\pi_{i,t} - \tau^{\pi}_{i,t} = \rho_{i,t}(\pi_{i,t} - \tau^{\pi}_{i,t}) + \alpha_{i,t}(y_{i,t} - \tau^{y}_{i,t}) + \beta_{i,t}\tilde{g}_t + \gamma_{i,t}\tilde{d}_t + \epsilon^{\pi}_{i,t} \tag{2.8}$$

where $\tilde{g}_t$ is the global output gap, $\tilde{d}_t$ is the oil price gap, $\beta_{i,t}$ and $\gamma_{i,t}$ are time-varying parameters:

$$\beta_{i,t} = \beta_{i,t-1} + \epsilon^{\beta}_{i,t}, \quad \epsilon^{\beta}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{i,\beta}) \tag{2.9}$$

$$\gamma_{i,t} = \gamma_{i,t-1} + \epsilon^{\gamma}_{i,t}, \quad \epsilon^{\gamma}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{i,\gamma}) \tag{2.10}$$

Note that each country faces the same global demand and oil price shock. It therefore makes sense to estimate them outside of the model, otherwise these shocks will be specific to each country, which is counter-intuitive. Thus, $\tilde{g}_t$ and $\tilde{d}_t$ are estimated using different filtering techniques. The baseline model uses the filtering approach developed by Grant and Chan (2017).

Matheson and Stavrev (2013), Blanchard et al. (2015), and Chan et al. (2016) constrain some parameters in line with theory. Failing to do that yields coefficients that can hardly be interpreted. For example, the inflation persistent, $\rho_{i,t}$, is restricted to be positive and strictly less than one to ensure stationarity. Similarly, inflation reacts positively to domestic and global demand pressures, and positive oil price shock. Thus, the slope of the Phillips and coefficients on global output and oil price gap are constrained to be positive and less than one. These restrictions are imposed following Chan et al. (2016), who employ a bounded random walk process. More specifically, the error terms ϵ^{ρ}_t, ϵ^{α}_t, ϵ^{β}_t, and ϵ^{γ}_t are assumed to follow a truncated normal distribution:

$$\epsilon^{\rho}_{i,t} \sim \mathcal{TN}(-\rho_{i,t}, 1-\rho_{i,t}, 0,\ \sigma^2_{i,\rho}) \tag{2.11}$$

$$\epsilon^{\alpha}_{i,t} \sim \mathcal{TN}(-\alpha_{i,t}, 1-\alpha_{i,t}, 0,\ \sigma^2_{\alpha}) \tag{2.12}$$

$$\epsilon^{\beta}_{i,t} \sim \mathcal{TN}(-\beta_{i,t}, 1-\beta_{i,t}, 0,\ \sigma^2_{i,\beta}) \tag{2.13}$$

$$\epsilon^{\gamma}_{i,t} \sim \mathcal{TN}(-\gamma_{i,t}, 1-\gamma_{i,t}, 0,\ \sigma^2_{i,\gamma}) \tag{2.14}$$

where $\mathcal{TN}$ denotes the truncated normal distribution. All coefficients, bounded and unbounded, are estimated by Bayesian method using Markov chain Monte Carlo (MCMC) algorithm. The priors are in Appendix A.1 and for further details of estimation, we refer our readers to Chan et al. (2016).

2.3 Empirical Results

2.3.1 Data

The dataset comprises of quarterly series from 34 countries, 23 advanced economies (AEs)[4] and 11 emerging market economies (EMEs)[5], observed from 1995Q1 to 2018Q1. The choice of countries and the sample size are based on data availability. The series included are the consumer price index (CPI) representing domestic inflation, the real gross domestic product (GDP) which reflects domestic demand, oil price is which used as proxy of supply shock, and global GDP as a proxy of global demand.[6] Oil price is taken from the World Bank Commodity Price Data, domestic GDPs are obtained from Haver Analytics, and the global GDP is from the St. Louis Federal Reserve Bank's database, FRED. The series are transformed into quarter-on-quarter difference of natural logarithms times 400. Note that the global output gap, obtained using the global GDP, and the detrended oil price inflation are constructed outside of the model using the filtering technique developed by Grant and Chan (2017).[7] As mentioned above, it is appropriate to estimate the global output trend and oil price trend outsied

[4]Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Latvia, Lithuania, Netherlands, Portugal, Slovakia, South Korea, Spain, Sweden, Switzerland, UK, USA.

[5]Bolivia, Brazil, China, Hungary, Indonesia, Mexico, Philippines, Russia, South Africa, Thailand, Turkey.

[6]Alternatively, import prices could be used to represent supply shocks, unfortunately this series is not available for many countries. Importantly, substituting oil price with import price yields similar results.

[7]Importantly, the results remain unchanged when using other filtering techniques such as the unobserved component with stochastic volatility (UCSV) of Beveridge and Nelson, 1981; Watson, 1986; the Hodrick and Prescott, 1997; Hodrick and Prescott, 1997 (henceforth HP), and the AR(4) filter of Hamilton (2018). Note that the Grant and Chan (2017) is flexible enough that it does not impose a constant smoothing parameter of 1600 like the HP filter and it does not suffer from end-point issue which is common in many filters. See Grant and Chan, 2017); Hamilton, 2018 for more details on the weaknesses of the HP filter.

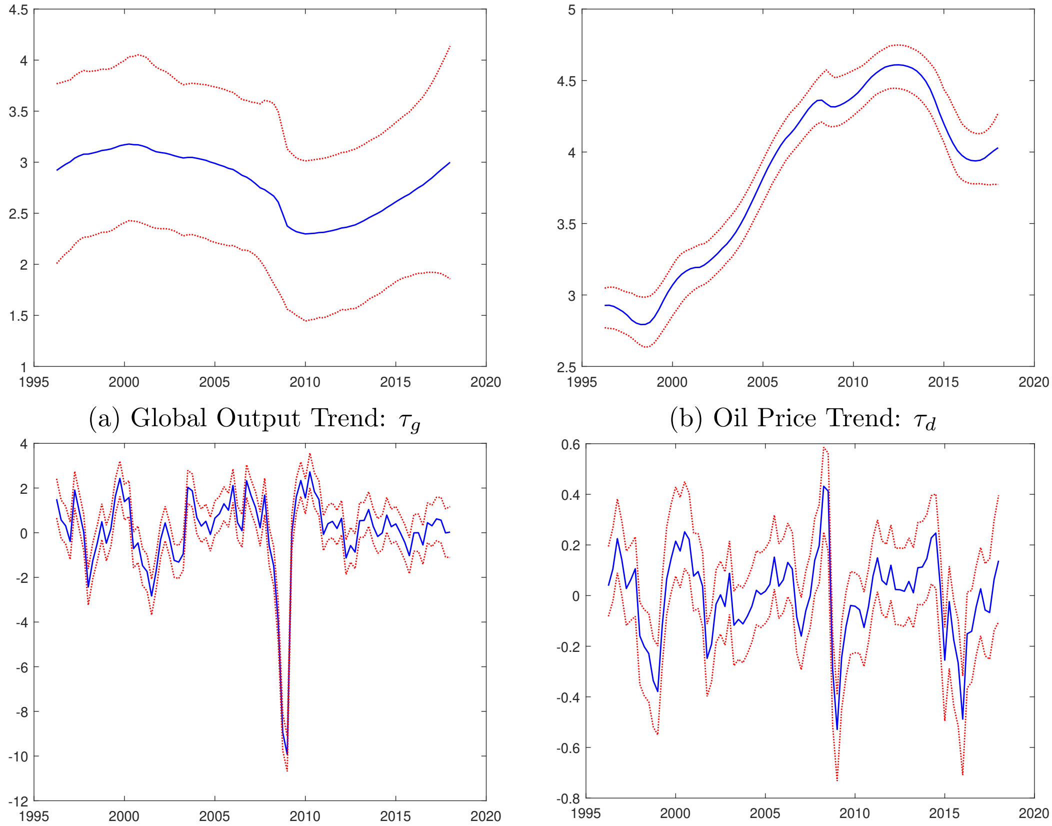
the model, given that each country faces the same global demand and supply shock. Conversely, deriving them from the model will yield different global output trend and oil price trend for each country, which is counter-intuitive.

2.3.2 Global output gap and oil price gap

Figure 2.1 presents the estimated global output trend and oil price trend, their gaps, and corresponding 84% credible intervals. The global output gap captures economic cycle in the global economy. In particular, it illustrates recessionary episodes, namely, the East Asian crisis of 1997-1998, the 2000-2001 dotcom crisis, the global financial crisis of 2007-2008, and the sovereign crisis in Europe in 2012. It then stabilizes around zero. The global output trend depicts a growth rate of 3 percent before the GFC, then drifted down to 2.3 percent before reverting back to its pre-crisis growth of 3 percent in 2018. The oil price gap captures relatively well instances where the oil price deviates from its long-term trend. Specifically, the upward movement in oil price before the GFC was driven by high global demand, particularly in emerging market economies. This demand pressure is exemplified by a steep rise in its trend starting in the late 1990s, then plateaus during and after the GFC. The cyclical component of oil price turned negative, then recovered gradually before dropping again in 2014, as a response to positive supply shock in oil market. This pushed down the trend in oil price.

2.3.3 Full Sample Results

The full sample results indicate that inflation dynamics are explained by lagged inflation, domestic output gap, global output gap, and oil price gap. Unlike Forbes (2019) who combines all countries together in a panel, interesting dynamics emerge when the Phillips curve is estimated for each country. In general, global output gap and oil price gap have larger coefficients than the domestic output gap. It is interesting to note that of the two external factors, oil price matters more. These results support the finding in the literature of a flat Phillips curve globally (see, Carlstrom and Fuerst, 2008; Ball and Mazumder, 2011; Matheson and Stavrev, 2013; Blanchard et al., 2015; Blanchard, 2016; Chan et al., 2016 and Kaihatsu and Nakajima, 2018). The coefficients on do-



(a) Global Output Trend: τ_g

(b) Oil Price Trend: τ_d

(c) Global Output Gap: $\tilde{g}_t$

(d) Oil Price Gap: $\tilde{d}_t$

Figure 2.1: **Global Output Gap and Oil Price Gap:** The solid blue line is the posterior mean, while the dotted red lines are 16% and 84% quantiles.

mestic output gap and global output gap are consistent with the full-sample results of Forbes (2019), whereas we have low inflation persistence and high coefficient on the oil price gap.

Next, we provide the details of the results. They are grouped into 5 broad categories, notably, the implicit inflation target, inflation volatility, inflation persistence, estimated coefficients on other explanatory variables, and inflation gap decomposition.

Implicit inflation target

The implicit inflation target, which is agents' beliefs about the central bank target, is represented by the mean of long-term trend inflation, i.e. $\frac{1}{T}\sum_{t=1}^{T}\tau_t^{\pi}$. Consistent with central banks objectives, on average, implicit targets are around 2 percent for AEs and above 4 percent for most of EMEs (see Table A.1). Specifically, 20 percent of countries have their average implicit targets above the official target band or point target. Figure 2.2 shows implicit inflation target of all countries. Notice a marginal decline in the trends especially for European countries following GFC, the European sovereign crisis, and the marked drop in commodity prices of 2014. In particular, the constructed target portrays a gradual decline in Belgium, Ireland, Netherlands, Denmark, Italy, Finland, France, and Germany. These countries seem to have reached a new regime of inflation below the target, hovering about one percent. Other AEs such as Australia and Canada exhibit a similar pattern with a stable trend inflation around the official target of 2 percent. Sweden and Switzerland registered extremely low implicit target, below one percent, dropping even further into negative for Switzerland amid deflationary pressure as a response to the appreciation of the currency. It is interesting to note a low implicit target for Switzerland even though it does not follow an IT regime. South Korea exhibits persistent decline in trend inflation from 3 percent to 2 percent. Conversely, inflation in the UK trended upward for prolonged periods with many instances where inflation bridges the official target. Consequently, the implicit target moves from one percent in the beginning of the sample to 2 percent and settled there until the end of the sample. Similarly, Hong Kong, which is not an inflation-targeter, has experienced an uptick in its implicit target from 1.1 percent to 2.4 percent. Finally, China and Thailand are the only EMEs with low implicit target at levels that are comparable with AEs. While the implicit target is trending up in China, it has been relatively stable in Thailand with a marginal decline from 2.2 to 1.3 percent.

Inflation volatility

Figure 2.3 presents stochastic volatility. Notice that changing nature inflation volatility over time and across countries. Interesting patterns emerge from these figures. First, there is a substantial decline in inflation volatility across countries. This can be attributed to a good policy, reflecting stable inflation dynamics which in most cases coincide with the adoption of IT policy. Besides, the literature also explains this drop

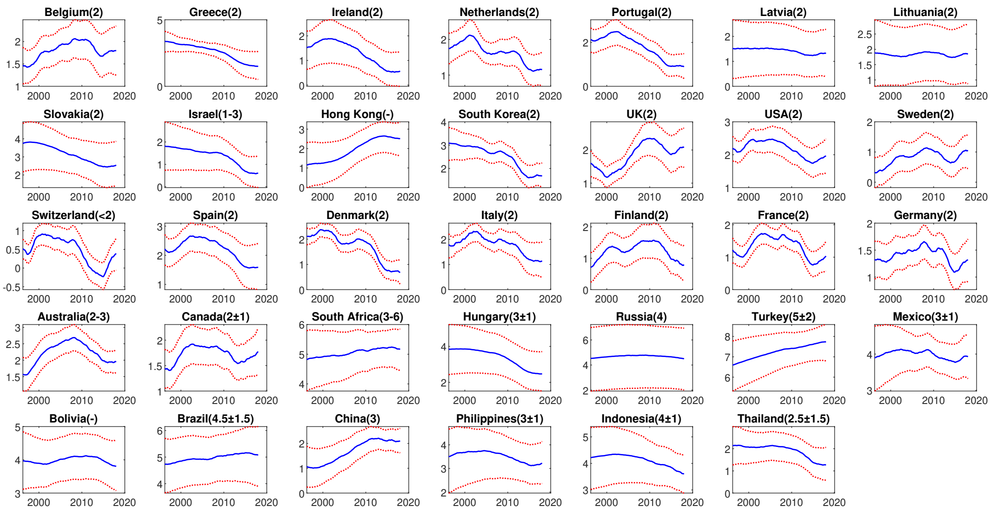


Figure 2.2: **Posterior estimates for trend inflation τ^{π}.** The title of each sub-figure is the country name, followed by the official inflation targets (point target or target bands). For Hong Kong and Bolivia, we do not find the official inflation targets, so we use "-". The solid blue lines are the means, the dotted red lines are 16% and 84% quantiles.

by a "good luck" induced by a global common shock affecting simultaneously inflation volatility in all countries. This global decline in inflation volatility also reflects the great moderation periods associated with a decrease in shock affecting the global economy. It is evident from Figure 2.3 that volatility has declined in both AEs and EMs albeit with different magnitudes. In general, volatility in AEs, which has recently been closer to zero, is lower than the levels attained in EMEs. Notice that Hong Kong is the only AE with considerably high inflation volatility, resulting from its fixed exchange rate regime. Starting initially at extremely low levels, volatility in Switzerland picks up in 2001 and remained elevated until 2011, then reverts to its pre-2001 levels. Like Hong Kong, Switzerland follows a fixed-exchange rate policy.

Even though most EMEs have witnessed a downward trend in volatility, it remains high especially in Turkey and Bolivia. Importantly, expect for Bolivia, most EMEs adopted the IT framework in the early 2000s. Even though Turkey follows an IT regime with a free-floating currency, the country has intervened several times to support its currency from depreciating, thus introducing volatility in inflation. As a result of active monetary policy, inflation trended downward from double digit (see Figure A.1, 50% in 1995 in Turkey and 20% in 1996 in Mexico) to a single digit. Since the adoption of IT, these countries managed to stabilize inflation which hovering at the upper bound of the official target band. For example, Mexico achieved price stability around four percent in 2003 while inflation has been stable around eight percent in Turkey since 2004.

With few exceptions, volatility in general tends to rise in crisis periods. Specifically, it increased during the Asian crisis, in EMEs and especially Asian countries, the dotcom crisis of 2001 in few AEs (Australia, Canada, France, Germany, Italy, Sweden, Switzerland, UK, US), the GFC shared globally, and the European sovereign crisis in Europe. In addition, idiosyncratic events generate remarkable spark in volatility in countries affected, notably currency crises in Mexico in 1995, depreciation of domestic currency in 2001 in South African and 2005 in Indonesia. Similarly, volatility in Italy has been on a rise since the GFC trigged by political and financial turmoil. Most countries exhibit initial high inflation volatility prior to the change in monetary policy (Australia, Bolivia, Canada, Italy, South Africa, Sweden, Switzerland, and UK). Interestingly, Blanchard et al. (2015) and Kaihatsu and Nakajima (2018) obtained the same results respectively for the AEs, the US and Japan.

Inflation persistence

Table A.2 presents the average inflation persistence together with its credible intervals. A noticeable difference is observed in the inflation persistence between AEs and EMEs. Most advanced economies exhibit a small persistence which literature attributes to a better anchoring of inflation expectations, suggesting that agents in these countries have become more forward looking. In other words, inflation process is no longer adaptive (see Cogley and Sargent, 2005; Stock and Watson, 2007; Carlstrom and Fuerst, 2008; Ball and Mazumder, 2011; Matheson and Stavrev, 2013; Blanchard et al., 2015; Gillitzer and Simon, 2018; Chan et al., 2016; Kabundi et al., 2019). The lowest persistence is found in Canada, followed by Germany, Switzerland, USA, Australia, France,

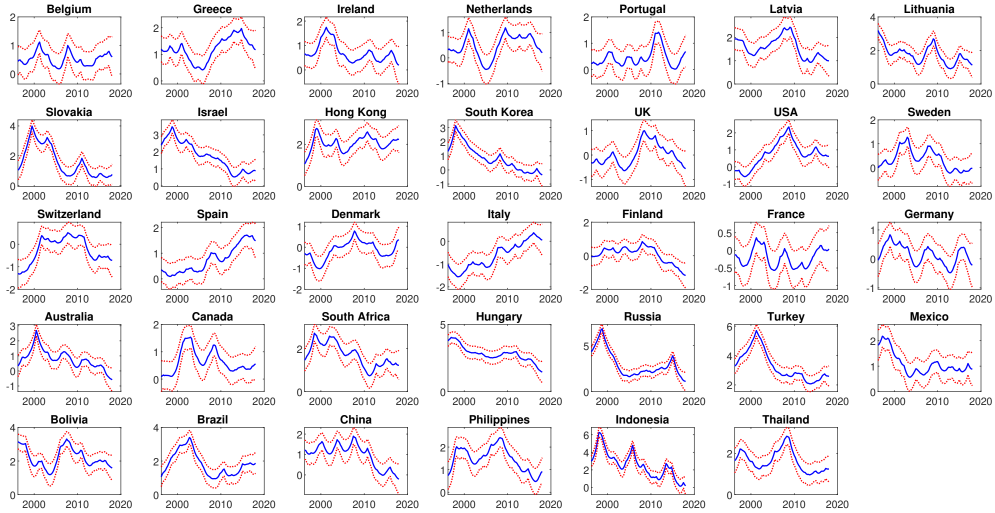


Figure 2.3: **Stochastic volatility h_t.** The solid blue line is the posterior mean, while the dotted red lines are 16% and 84% quantiles.

Denmark, the Netherlands, and South Korea. Note that these countries have implicitly or explicitly adopted the inflation targeting regime in the mid-1990s. Even though Switzerland has not explicitly adopted the IT policy, it does have a nominal anchor of maintaining inflation below 2 percent. AEs with high persistent include Italy, Ireland, Slovakia, Lithuania, and Latvia. Agents are somewhat backward looking in Sweden, Hong Kong, and Finland. It is not surprising that Hong Kong, which has a fixed exchange rate monetary policy, is the only AE with extremely high persistence.

In stark contrast, EMEs portray high persistence which suggests that expectation formation is more adaptive in these countries. Brazil, Philippines, Russia, and South Africa have the highest persistence of above 0.5, whereas it lies between 0.4 and 0.5 for the rest of EMEs.

Average values mask interesting dynamics on the evolution of inflation persistence. Next, we show the details of time variation in AEs, followed by EMEs.

The time variation in AEs shows different pictures. One group experienced a downward in inflation persistence, while another group experienced an upward. The group experiencing a downward in inflation persistence includes Hong Kong, Israel, Latvia, and Lithuania. In particular, inflation persistence in Hong Kong moves from 0.73 in 1995 to 0.27 in 2018 (see Figure 2.4). The adaptive behavior of agents in the 1990s was mainly due to the persistently high inflation between 1995 and 1997, averaging 6 percent followed by a prolonged period of negative inflation which lasted about 5 years. Since 2005, monetary policy authority has managed to stabilize inflation around 2 percent (see Figure A.1). Similar pattern in persistence is observed in Israel, however, the decline is less steep, moving from 0.78 to 0.39. High persistence at the beginning of the sample can be associated with high inflation that prevailed before the adoption of the IT regime in 1997. Latvia and Lithuania exhibit the similar pattern of inflation persistent which is consistent with the dynamics in inflation in these countries. Inflation persistence is high and constant until the GFC, then decreases slowly reaching 0.5 in 2018, suggesting that inflation is still somewhat adaptive. They started with high inflation of 20 and 29 percent, respectively. This period was followed by a rapid disinflation process, which brought inflation down to 1.49 percent in Latvia and around zero in Lithuania. However, inflation surged again after a temporary period of low inflation, attaining 19 percent in 2003 for Latvia and 13 percent in 2007 for Lithuania. Thereafter inflation plummeted and stabilized in both countries around the implicit target until the end of the sample.

The group experiencing an upward in inflation persistence includes Belgium, Finland, and the UK. The marginal increase in Belgium is associated with the deflationary process in the Europe in 2012, whereas high level attained in Finland and the UK is attributed to prolonged periods of tolerance of inflation above the target by central banks.

With regard to EMEs, Figure 2.4 divide 11 EMEs into three groups. The first group experienced a marked drop in inflation persistence. It includes China, Hungary, Mexico, Thailand, and Turkey, moving from 0.9 to 0.22, which suggests a drastic change in agents' beliefs from backward looking to forward looking. Bolivia and South Africa depict inflation persistent respectively at 0.5 and 0.6 which remains relatively constant,

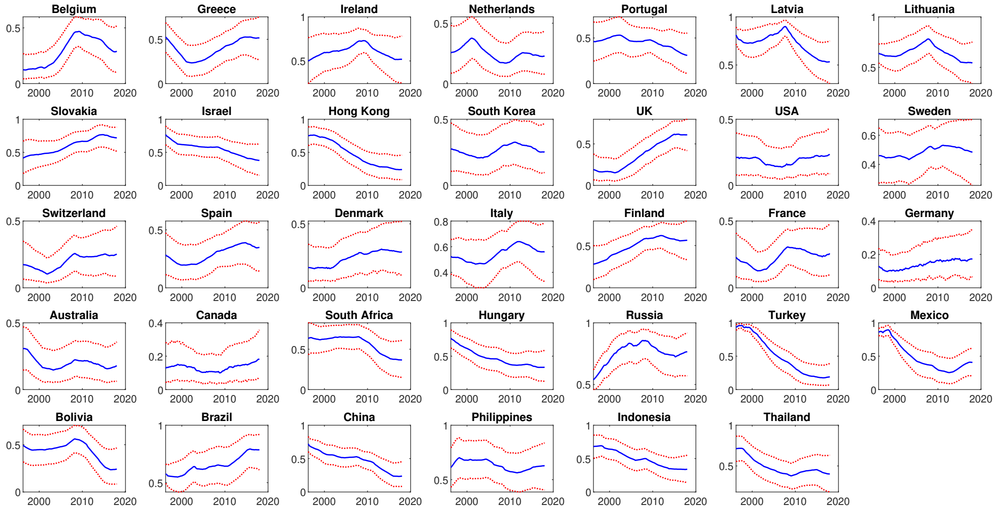


Figure 2.4: **Inflation persistence ρ_t.** The solid blue line is the posterior mean, while the dotted red lines are 16% and 84% quantiles.

then decreases after the GFC to 0.27 and 0.37, respectively. These results reflect lengthy episodes, where inflation has stayed above its long-term trend. More specifically, they support the finding of Kabundi et al. (2019), who point out that the change observed in inflation persistence in South Africa can be attributed to a combination of good policy and good luck. With regard to good policy, they argue that the central bank becomes more active in combating inflation. With regard to good luck, they argue that negative oil price shock brings inflation down.

The second group experienced a noticeable upward in inflation persistence. It includes Brazil and Russia, from 0.57 to 0.77 and 0.50 to 0.74, respectively. It is worth noting that in contrast to Brazil, inflation persistence for Russia has been trending down after reaching its peak (0.83) in 2008Q3. The two countries have tolerated prolonged periods of deviation of inflation from its long-term trend. The third group is Philippines, who has a stable inflation persistence of 0.6.

Estimates of coefficients on explanatory variables

This section contains the results of parameters of explanatory variables, namely, the domestic output gap, the global output gap, and the oil price gap. As discussed in the model, these parameters are restricted be positive in line with expectations.

With regard to the coefficients on the domestic output gap (that is, α_t), Table A.3 reports the average value over time and Figure A.3 reports the time variation aspect. On average, Table A.3 reports the mean of α_t over time. The slope of the Phillips curve varies from 0.031 (Thailand) to 0.212 (Indonesia). The small value shows that inflation is muted to domestic output gap. A pattern which emerges from the results is that inflation tend to react more to domestic demand in EMEs than in AEs. Figure A.3 plots the time varying slope of Phillips curve. We do not observe evident time variation in it. It is quite flat from 1995 to 2018. This is consistent with Blanchard et al. (2015) and Chan et al. (2016), Kabundi et al.. They use longer sample and find evidence of changing slope of the Phillips curve in the 1970s and the 1980s. But the slope has remained unchanged since early 1990s. Actually, if there is any recent change, the results point to a marginal steepness of the slope recently in most AEs like Belgium, Slovakia, South Korea, Switzerland, and the US. The recent marginal rise in the slope of the Phillips observed in the US is also documented by Gilchrist and Zakrajsek (2019).

With regard to the coefficients on the global output gap (that is, β_t), Table A.4 reports the average value over time and Figure A.4 reports the time variation aspect. Similar to the domestic demand, on average inflation in EMEs is more responsive to global demand than in AEs (Table A.4). The coefficient varies from 0.337 (China) to 0.066 (Netherlands). Figure A.3 plots the time varying β_t. Notice the general picture portrayed is that this coefficient decreases slightly until the GFC, then increases marginally until the end of the sample.[8] For example, the coefficient increases respectively from 0.19 to 0.24 for the US. This finding is comparable with Forbes (2019), who shows that the response of inflation to global demand factors increased recently. It reflects the effects of globalisation associated with increasing trade integration which started in early 1990s have intensified lately. Thus, synchronization in inflation across countries is partly a global demand phenomenon. In addition, positive credibility intervals imply

[8]In general, there is 0.5 rise in slope of global output gap from 2008 to 2018.

that the effects are not negligible, which rules out the notion that domestic inflation is only explained by domestic impact as proposed by Ihrig et al. (2010), Martínez-García and Wynne (2013), Eickmeier and Pijnenburg (2013), Mikolajun and Lodge (2016), Bems et al. (2018), and Hooper et al. (2020). Consistent with Borio and Filardo (2007) and Forbes (2019), the results indicate that global demand matters more than domestic demand in explaining dynamics in inflation in both AEs and EMEs.[9] This finding has important monetary policy implications. It suggests that monetary policy authority should monitor closely the global economy when they make decisions. Interestingly some central banks have already started incorporating the global economic outlook in their decision-making process. In its statement of October 30, 2019, the Federal Reserve Bank clearly stated that its decision to lower the federal funds rate was informed by the combination of weak global economic and muted inflationary pressure.

With regard to the coefficients on the oil price gap (that is, γ_t), Table A.5 reports the average value over time and Figure A.5 reports the time variation aspect. On average, Table A.5 reports the role played by supply shock over time. A few interesting observations can be highlighted. First, oil price gap seems to explain more dynamics in inflation than demand factors. The coefficients vary between 0.393 (Brazil) and 0.739 (Germany), compared with the maximum of 0.212 and 0.337 for domestic and global demand, respectively. Simon et al. (2013) find similar magnitudes using import prices. Second and consistent with Jordà and Nechio (2018), the impact of oil price seems to prevail more in AEs than in EMEs. This is consistent with the observation of downturn in inflation in 2014 in most AEs, which is in line with the oil price shock during the same period. Recall that implicit targets for the same countries fell as a result. Third, in line with the literature, the coefficient is constant over time during this period (see Forbes, 2019).

[9] Except for South Africa where the domestic demand exhibits a higher coefficient and Switzerland where the two demand factors yield coefficients of the same magnitude. Notice that Thailand depicts a constant coefficient on global demand, whereas South Africa is the only country with a decreasing slope of global demand.

Results in Table A.6 show coefficients of AR(2) process for each country's output gap. Notice that most AEs show high persistence in output gap, with the sum of coefficient ranging between 0.5 and 0.7. Spain depicts the highest persistence of the output gap of 0.926, which is closer to a random walk process. These results indicate that expansionary and contractionary periods are longer in AEs compared with EMEs. South Africa is the only EME country with high persistence (0.617).

Inflation Gap Decomposition

In Table A.7, we briefly show the result of inflation gap decomposition. In our model, we have five components: lagged inflation $\big(\rho_t(\pi_{t-1} - \tau^{\pi}_{t-1})\big)$, domestic output $\big(\alpha_t(y_t - \tau^{y}_{t})\big)$, global output $(\beta_t\tilde{g}_t)$, oil price $(\gamma_t\tilde{d}_t)$ and the error. The number represents the contribution of a component to the inflation gap $(\pi_t - \tau^{\pi}_t)$.

The contribution of lagged inflation has increased in 17 out of 34 countries after the GFC. An interesting finding is that in the 17 countries, 16 countries are AEs. Even if most advanced economies exhibit a small inflation persistence, the lagged inflation is still important to explain the dynamics of inflation. The contribution of domestic output has increased in 13 countries out of 34 countries after the GFC. And out of 23 AEs, only in 6 countries, the contribution of domestic output has increased after GFC. Compared with 7 countries out of 11 EMEs, this supports the finding in 2.3.3 that inflation tend to react more to domestic demand in EMEs than in AEs. The contribution of global output has increased in 9 countries. And out of 23 AEs, only in 2 countries, the contribution of domestic output has increased after GFC. Compared with 7 countries out of 11 EMEs, this supports the finding in 2.3.3 that inflation tend to react more to global demand in EMEs than in AEs. And the contribution of oil price has increased in 20 out of 34 countries after the GFC, supporting an important role played by oil prices. By comparing the magnitude of contribution, we find further evidence that the impact of oil price prevails more in AEs than in EMEs.

2.4 Conclusion

This paper estimates the Phillips for individual countries from a dataset with a total of 34 countries (23 AEs and 11 EMEs). Besides the domestic output gap, the model incorporates global output gap and oil price gap which account for global determinants of inflation. We allow for time-variation in coefficients and parameters are restricted in line with expectations. The results can be summarized as follows.

There is evidence of changing nature of inflation volatility, which has declined across all countries but remained relatively high in EMEs and several AEs. Inflation volatility tends to rise in crisis periods compared with tranquil times. Inflation persistence has declined in most AEs countries. The estimates of trend inflation are relatively constant in many countries. The levels are consistent with the official targets, but closer to the upper bound of the target band for most of EMEs. Consistent with the literature, the results show that Phillips curves are generally flat for the period under consideration, but different from zero. They seem to be more flat in AEs than in EMEs with a marginal uptick towards the end of the sample. The global demand seems to matter more in EMEs than in AEs. Finally, the results point to the dominant role played by oil prices as a key factor behind inflation dynamics across countries and over time. The coefficient on oil price gap is on average 6 times more than that of domestic output gap and 4 times more than the global output gap.

Chapter 3

A Multi-country Unobserved Components Model with Sparse Factor Stochastic Volatility

3.1 Introduction

Since the seminal work by Stock and Watson (2007), unobserved components (UC) models with stochastic volatility (SV) have been commonly used for modeling latent state vectors that can be interpreted as long-run equilibrium levels and have enjoyed great popularity. A large body of research has emerged on extending UC model. One strand of extensions has focused on introducing more indicators into the conditional mean of UC model (e.g., Stella and Stock, 2013; Chan et al., 2018; Zaman, 2021 and Kabundi et al., 2021). Another strand of extensions of UC model has focused on adding bounds on parameters (e.g., Chan et al., 2013; Chan et al., 2016; Zaman, 2021 and Kabundi et al., 2021).

Despite these popular extensions, the errors in UC model are assumed to be independent with each other. This assumption is also used in multi-country studies (e.g., Chan et al., 2018 and Kabundi et al., 2021). A priori, one would expect there exist linkages in the covariance matrix. And the linkages are more evident in multi-country studies due to globalization, omitting these linkages may affect the estimate of latent states. Inspired by these concerns, this paper proposes an approach to allow for cross-country linkages in multi-country UC models. It models all countries jointly. At the same time, this new approach allows for SV in each country, which is found to be important in the existing UC literature. This new approach also combines the two strands of extensions of UC models. Specifically, this new approach introduces the Phillips curve and constrains parameters to lie in reasonable intervals. Additionally, all of these extensions in this new approach are not at the expense of slower computation time (see below).

Coincided with the political and economic events (such as 9/11, Global Financial Crisis (GFC) of 2008, and the euro area sovereign debt crisis), an extensive empirical literature has studied the cross-country linkages in the error covariance matrix and has found evidence of commonality in international macroeconomic uncertainty. In the theoretical aspect, Cesa-Bianchi et al. (2020) assume that country-specific output growth is determined by a common persistent component with time-varying volatility, and by country-specific business-cycle components. Recent macroeconomic modelling contributions include, among others, building up a dynamic stochastic general equilibrium (DSGE) model (e.g., Mumtaz and Theodoridis, 2017 and Cross et al., 2018) and using large Bayesian vector autoregressions (BVARs) to measure international macroeconomic uncertainty. The latter includes the prominent work by Carriero et al. (2020). They assume that the error volatilities evolve over time according to a factor structure, thus successfully allowing for cross-country linkages in the error covariance matrix and estimating the linkages and its effects in one step. By studying GDP growth for 19 industrialized economies and 67 variables in quarterly data (for the U.S., euro area (E.A.), and U.K.), they obtain that there is one common factor driving strong co-movement across economies.

Inspired by the finding of commonality in international macroeconomic uncertainty, the cross-country linkages in the error covariance matrix are allowed through co-movement in volatilities. Specifically, we introduce factor stochastic volatility (FSV) specifications (e.g., Pitt and Shephard, 1999; Chib et al., 2006; Kastner, 2019 and Chan, 2021). The FSV specification assumes that correlations among the elements of innovation are induced by the latent factors. These latent factors capture the time-varying covariance structure and naturally express the commonality in uncertainty that we want to study in this paper. In addition, even small models can have a large number of parameters, estimating all countries (in our empirical application, we consider 34 countries) jointly is a high-dimensional setting and calls for techniques to avoid over-fitting problems. FSV is such a technique, because there is empirical evidence that a small number of factors would be sufficient to capture the time-varying covariance structure even when the number of dependent variables is large (see Chan, 2021). Another advantage of FSV specification is that it is computationally efficient. Conditionally on the latent factors and their loadings, equation-by-equation estimation becomes possible within each MCMC iteration (e.g., Kastner and Huber, 2020 and Chan, 2021), which substantially speeds up computations.

The second advantage of this new approach is that it is flexible enough to allow every country to have its own SV features. For example, the error for country i may support SV, while the error for country j may be homoscedastic. Then the new approach allows the data to determine whether each country variance is time-varying or time-invariant. We solve this issue by adopting the method developed for shrinking time-varying coefficients in VARs (e.g., Belmonte et al., 2014; Huber and Pfarrhofer, 2021). More specifically, we first rewrite the evolution of the log-volatility using non-centered parameterization, developed in Frühwirth-Schnatter and Wagner (2010). Non-centered parameterization enables us to decompose the time-varying parameter into a time-invariant part and a time-varying part, where the time-varying part has a constant coefficient matrix. Then, we use a Horseshoe prior specification on the constant coefficient matrix of the time-varying part. The Horseshoe prior is a global-local shrinkage prior. And it is found that if a matrix is characterized by a relatively low number of non-zero elements, a possible solution is a global-local shrinkage prior (e.g., Polson and Scott, 2010; Kastner and Huber,2020). Such advantage of global-local shrinkage prior shrinks strongly the parameter space but at the same time provides enough flexibility to allow for non-zero elements if necessary, thus removing SV for most elements but retaining SV if necessary. We refer to this specification of the log-volatility as NCP-HS-SV (Non-centered parameterization Horseshoe Stochastic Volatility).

This new approach also incorporates the two strands of extensions of UC model, that is, it introduces more indicators and constrains parameters to lie in reasonable intervals. There has been a lot of recent research devoted to introducing suitable indicators into UC model. These indicators are guided by economic theory and empirical research. For example, inspired by the Phillips curve, Stella and Stock (2013) extend the univariate UC model in Stock and Watson (2007) to bivariate UC model, and assume that it is inflation gap and unemployment gap[1] that drive the Phillips curve. Based on public commentary that central bankers pay considerable attention to measures of long-run inflation expectations, Chan et al. (2018) develop a bivariate model by introducing survey-based long-run forecasts of inflation into the UC model. To directly address critiques of omitted variable and omitted equation bias pointed out by Taylor and Wieland (2016), Zaman (2021) further extends the bivariate UC model of Stella and Stock (2013) to a large-scale UC model. In particular, they jointly estimate trends of several macroeconomic variables (they call them "stars") and build up a rich structure for each star. The observed flattening of Phillips curve has generated various explanations of this conundrum and one camp highlights the role played by global factors. Therefore, Kabundi et al. (2021) introduce global factors (global output and oil price) into the bivariate UC model of Stella and Stock (2013) to estimate trends of inflation and output. In this paper, we follow Stella and Stock (2013) to incorporate Phillips curve into UC model. One may question the existence of Phillips curve, but McLeay and Tenreyro (2020) emphasize that the Phillips curve exists and policymakers are

[1]Inflation gap is deviation of inflation from its trend, and similar interpretation of unemployment gap, deviation of unemployment rate from its trend.

completely aware of its existence. Stock and Watson (2008) also raised the point that the Phillips curve is useful for conditional forecasting. So we expect that the Phillips curve still exists, even though we are observing that it has flattened (e.g., Ball and Mazumder, 2011; Hall et al., 2013 and Blanchard et al., 2015).

To constrain parameters to lie in reasonable intervals, we follow Chan et al. (2013). They first develop a method to constrain parameters to lie in intervals to avoid them move into undesirable regions. They find the model yields more sensible measures of trends than popular alternatives. Since their seminal work, there have been many applications that have employed the use of adding bounds on parameters (e.g., Chan et al., 2016; Zaman, 2021 and Kabundi et al., 2021).

In a careful empirical analysis, involving 34 countries and two variables in each country (quarterly CPI inflation and output), we show to what extent our new approach improves upon a set of popular UC models with sensible properties.

First, we provide evidence that there is global factors driving strong co-movement across economies and spikes in the volatility associated with these global factors coincide with major economic events, which further enhances the merits of our NCP-HS-SV specification. Second, we show that allowing for contemporaneous cross-country linkages will provide higher model fit and the estimates, in certain countries, indicate that they are influenced by both domestic factors and global factors. Finally, we undertake an out-of-sample forecasting exercise where we compare our new proposed flexible model against restricted version of the model. We find allowing for contemporaneous cross-country linkages leads to improved forecast performance, both at short and long horizons. In addition, investigating the time profile of the cumulative log predictive likelihood reveals that allowing for contemporaneous cross-country linkages especially pays off when the aim is to forecast variables in times of uncertainty. And such good forecast performance is for most countries and not driven by particular countries.

This paper is organized as follows. In Section 2, we first discuss the unobserved component models for output and inflation within a single country, then introduce our new model, which models all countries jointly and allows for contemporaneous cross-country linkages through factor stochastic volatility, thus providing direct estimate of commonality in international macroeconomic uncertainty. The details of our new model also includes an elaborated account of the non-centered parameterization and the Horseshoe prior for stochastic volatility. Section 3 illustrates our modeling approach by fitting our model to 34 countries (two variables, CPI inflation and output, within each country). We divide them into three parts. In the first part, we present the estimates of commonality in international macroeconomic uncertainty (we call them global inflation uncertainty and global output uncertainty). The second part is Bayesian model comparison, where we show that allowing for contemporaneous cross-country linkages will improve in-sample fit. After justifying the importance of cross-country linkages, we present, in the third part, the estimates of country-specific parameters with the effects of cross-country linkages on them. Section 4 is the out-of-sample forecasting performance. Finally, Section 5 concludes.

3.2 Sparse Factor Stochastic Volatility for A Multi-country UC Model

3.2.1 MC-UC-FSV Model Specification

Unobserved component (UC) model is commonly used for a single economy, so we begin with the country-specific UC model for output and inflation, using the model developed in Stella and Stock (2013) and Chan et al. (2016). In particular, we start from constant coefficient UC model for output, $y_{i,t}$, and inflation, $\pi_{i,t}$ of the form:

$$\pi_{i,t} - \tau^{\pi}_{i,t} = \rho_i(\pi_{i,t-1} - \tau^{\pi}_{i,t-1}) + \alpha_i(y_{i,t} - \tau^{y}_{i,t}) + \epsilon^{\pi}_{i,t} \tag{3.1}$$

$$y_{i,t} - \tau^{y}_{i,t} = \varphi_{i,1}(y_{i,t-1} - \tau^{y}_{i,t-1}) + \varphi_{i,2}(y_{i,t-2} - \tau^{y}_{i,t-2}) + \epsilon^{y}_{i,t} \tag{3.2}$$

$$\tau^{\pi}_{i,t} = \tau^{\pi}_{i,t-1} + \epsilon^{\tau\pi}_{i,t}, \quad \epsilon^{\tau\pi}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau\pi}) \tag{3.3}$$

$$\tau^{y}_{i,t} = \tau^{y}_{i,t-1} + \epsilon^{\tau y}_{i,t}, \quad \epsilon^{\tau y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau y}) \tag{3.4}$$

where i denotes country i, $i = 1, \dots, N$. At time t, $\pi_{i,t}$ is the inflation of country i and $y_{i,t}$ is the output of country i, $\tau^{\pi}_{i,t}$ and $\tau^{y}_{i,t}$ are their trends. These trends are unobserved latent states which can be interpreted as long-run equilibrium level of inflation and output, also known as trend inflation and trend output.

This model is inspired by the Phillips curve and incorporates the properties that it is inflation gap and unemployment gap that drive the Phillips curve. These features are in common with the model of Stella and Stock (2013), Chan et al. (2013) and Chan et al. (2016).

Thus, the first equation embodies a Phillips curve, but we are assuming constant coefficients in the inflation gap equation. Many papers have emphasized that the Phillips curve has flattened post 2007 (see, Simon et al., 2013) and proposed to allow for time-variation in the coefficients to capture this behavior (see, Zaman, 2021). It seems to be more sensible to start from UC model with time-varying coefficients. However, using the data in our empirical work (observed from 1995Q1 to 2018Q1), we have considered a model where ρ_i and α_i vary over time, but find the Bayes Factor supports constant coefficients (see Appendix B.1). We think one reason is that our sample is from 1995 to 2018. This period has been associated with stable and weak Phillips curve relationship. For instance, Zaman (2021) find that post 1995, the time-varying estimate of inflation gap persistence (for the US) is stable, and the time-varying Phillips curve (for the US) is stable. Accordingly, the main model we focus on does not have time-variation in the coefficients in inflation gap equation, but it does have stochastic volatility.

To ensure stationarity, we bound ρ_i and α_i to be positive and less than one, that is $0 < \rho_i < 1$ and $0 < \alpha_i < 1$, which also ensures that the Phillips curve has a positive slope. We also impose stationary condition on the output gap equation and assume $\varphi_{i,1} + \varphi_{i,2} < 1$, $\varphi_{i,2} - \varphi_{i,1} < 1$ and $|\varphi_{i,2}| < 1$. Chan et al. (2016) and Zaman (2021) also bound the coefficients and emphasize the importance of bounding.

The second equation implies AR(2) behavior for the output. The AR(2) assumption is empirically sensible and commonly-used. Note that we are assuming constant coefficients in the output gap equation. In existing UC literature, the assumption of constant coefficients has also been used in Chan et al. (2016), Zaman (2021) and Kabundi et al. (2021). In the broader output literature, Koop et al. (2020) and Carriero et al. (2020) also assume constant coefficients.

Thus far, we have specified a country-specific UC model for output and inflation. In particular, it is a bivariate UC models used in Kabundi et al. (2021) and Chan et al. (2016), and incorporates the features from empirical findings (constant coefficients). However, conventional literature would next assume that the errors are independent across countries. It is with this assumption that we part with the existing literature.

As discussed earlier, the independent assumption across countries is not plausible when there is significant commonality across economies. To capture such cross-country linkages in uncertainty, we assume that, for all countries, their errors in the inflation gap equation are driven by several common factors and their errors in the output gap equation are also driven by common factors. This can be done through the factor stochastic volatility (FSV) specification.

To facilitate the FSV specification, at time t, we store the errors in inflation gap equation for all countries in an N-dimensional vector ϵ_t^π, that is, $\epsilon_t^\pi = (\epsilon_{1,t}^\pi, \ldots, \epsilon_{N,t}^\pi)'$. Similarly, we store the errors in output gap equation for all countries in an N-dimensional vector ϵ_t^y, that is, $\epsilon_t^y = (\epsilon_{1,t}^y, \ldots, \epsilon_{N,t}^y)'$. Through factor stochastic volatility specification, ϵ_t^π can be decomposed as:

$$\epsilon_t^\pi = L_\pi f_t + u_t^\pi \tag{3.5}$$

$$\begin{pmatrix} u_t^\pi \\ f_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0_N \\ 0_{r_\pi} \end{pmatrix} , \begin{pmatrix} \Sigma_t^\pi & 0_{r_\pi} \\ 0_N & \Omega_t^\pi \end{pmatrix} \right) \tag{3.6}$$

and ϵ_t^y can be decomposed as:

$$\epsilon_t^y = L_y g_t + u_t^y \tag{3.7}$$

$$\begin{pmatrix} u_t^y \\ g_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0_N \\ 0_{r_y} \end{pmatrix} , \begin{pmatrix} \Sigma_t^y & 0_{r_y} \\ 0_N & \Omega_t^y \end{pmatrix} \right) \tag{3.8}$$

where $f_t = (f_{1,t}, \ldots, f_{r_\pi,t})'$ is a r_π-dimensional vector of latent factors and L_π is the associated $N \times r_\pi$ loading matrix. Similarly, $g_t = (g_{1,t}, \ldots, g_{r_y,t})'$ is a r_y-dimensional vector of latent factors and L_y is the associated $N \times r_y$ loading matrix. Following Chan (2021), we assume that the factor loading matrices L_π and L_y are both a lower triangular matrix with ones on the main diagonal and $r_\pi \leq (N-1)/2$, $r_y \leq (N-1)/2$. Let $n_{l,\pi}$ denote the number of free elements in L_π, then $n_{l,\pi} = N \times r_\pi - \frac{(1+r_\pi)r_\pi}{2}$. Let $n_{l,y}$ denote the number of free elements in L_y, then $n_{l,y} = N \times r_y - \frac{(1+r_y)r_y}{2}$.

We assume that inflation gap equations across countries and output gap equations across countries are driven by different factors f_t and g_t.[2] Based on preliminary empirical work that the errors in inflation gap equation exhibit stochastic volatility, we assume that the disturbances u_t^π exhibit stochastic volatility. This is why the error variance of u_t^π is Σ_t^π. While the previous literature would assume the errors in output gap equation remain homoscedastic, that is, u_t^y are homoscedastic, we assume the disturbances u_t^y exhibit stochastic volatility. This is why the error variance of u_t^y is Σ_t^y. Such

[2]Such relations between the errors in country i and j are typically referred to as static interdependencies in the panel VAR literature. We do not incorporate, in the errors, contemporaneous relations across variables within a country, because the conditional mean has been explicitly modelled this relation in the inflation gap equation.

specification will capture time variation in a country's GDP volatility unique to that country, and is used in Carriero et al. (2020). In the theoretical part of Cesa-Bianchi et al. (2020), they also assume that country-specific business-cycle components have a conditionally heteroscedastic variance-covariance matrix. If the error is homoscedastic, our specification of the log-volatility can (nearly) remove SV (see below).

For the latent factors f_t and g_t, we assume that they exhibit stochastic volatility. This is why the error variance of f_t is Ω_t^{π}, and the error variance of g_t is Ω_t^{y}.

Note that the time-varying variance matrix $\Sigma_t^{\pi} = \text{diag}(e^{h_{1,t}}, \ldots, e^{h_{N,t}})$,
$\Sigma_t^{y} = \text{diag}(e^{h_{N+1,t}}, \ldots, e^{h_{2N,t}})$, $\Omega_t^{\pi} = \text{diag}(e^{h_{2N+1,t}}, \ldots, e^{h_{2N+r_\pi,t}})$, and
$\Omega_t^{y} = \text{diag}(e^{h_{2N+r_\pi+1,t}}, \ldots, e^{h_{2N+r_\pi+r_y,t}})$.

To allow the data to decide whether there is time-variation in their log-volatility, we model the evolution of the log-volatility according to a random walk in non-centered parameterization and then use global-local shrinkage prior (Horseshoe prior) to control time-variation. More specifically, for each $j = 1, \ldots, 2N + r_\pi + r_y$, the evolution of the log-volatility is modeled as:

$$
\begin{aligned}
h_{j,t} &= h_{j,0} + \omega_j^h \tilde{h}_{j,t} \\
\tilde{h}_{j,t} &= \tilde{h}_{j,t-1} + \epsilon_{j,t}^h, \quad \epsilon_{j,t}^h \sim \mathcal{N}(0,\ 1)
\end{aligned} \tag{3.9}
$$

The non-centered parameterization decomposes a time-varying parameter $h_{j,t}$ into two parts: a time-invariant part $h_{j,0}$ and a time-varying part $\omega_j^h \tilde{h}_{j,t}$, which has a constant coefficient ω_j^h that controls the time-variation. Then we expect that some elements in ω_j^h may be (close to) zero, which means the error is homoscedastic, but at the same time several elements in ω_j^h may be different from zero, which means the error is heteroscedastic. This case is exactly the advantage of global-local shrinkage prior. Many papers have documented that global-local shrinkage prior can cope with the case where a matrix is characterized by a small number of non-zero elements (e.g., Polson and Scott, 2010; Kastner and Huber, 2020). We use the empirically successful global-local shrinkage prior, Horseshoe prior, and consider the inverse-Gamma representation of Horseshoe prior as in Cross et al. (2020):

$$\begin{aligned}
&\omega_j^h \mid \lambda_j^{\omega^h}, \tau^{\omega^h} \sim \mathcal{N}(0,\ \lambda_j^{\omega^h}\tau^{\omega^h}),\ j = 1, \ldots, 2N + r_\pi + r_y \\
&\lambda_j^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu_j^{\omega^h}}),\quad \tau^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\omega^h}}) \\
&\nu_j^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1),\qquad \xi^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned} \tag{3.10}$$

If the error (or factor) really is homoscedastic, the Horseshoe prior will shrink ω_j^h to (nearly) zero and automatically remove (or nearly so) the SV from the error (or factor). τ^{ω^h} is the global shrinkage parameter that pushes all elements (ω_j^h) towards zero. We have assumed that all ω_j^h are forced to zero through a single global shrinkage parameter τ^{ω^h} for different factors, different countries and different equations within a given country. This is a restricted version of the Horseshoe prior in Feldkircher et al. (2021). They specify the global shrinkage parameter to differ across countries and equations within a given country. However, we notice that such a flexible prior is for the coefficients in panel VARs, and our Horseshoe prior is for the time-varying part of log-volatility. They all represent the uncertainty, so we expect that they have a single global shrinkage parameter. Our specification in Equation (10) does allow for the differences across factors, countries and equations within a given country and this is realized through the local shrinkage parameter $\lambda_j^{\omega^h}$.

To summarize the model including all countries:

$$\begin{aligned}
\pi_t - \tau_t^\pi &= P(\pi_{t-1} - \tau_{t-1}^\pi) + A(y_t - \tau_t^y) + L_\pi f_t + u_t^\pi,\ f_t \sim \mathcal{N}(0,\ \Omega_t^\pi),\ u_t^\pi \sim \mathcal{N}(0,\ \Sigma_t^\pi) \\
y_t - \tau_t^y &= \Phi_1(y_{t-1} - \tau_{t-1}^y) + \Phi_2(y_{t-2} - \tau_{t-2}^y) + L_y g_t + u_t^y,\ g_t \sim \mathcal{N}(0,\ \Omega_t^y),\ u_t^y \sim \mathcal{N}(0,\ \Sigma_t^y) \\
\tau_{i,t}^\pi &= \tau_{i,t-1}^\pi + \epsilon_{i,t}^{\tau\pi},\ \ \epsilon_{i,t}^{\tau\pi} \sim \mathcal{N}(0,\ \sigma_{\tau\pi}^2),\ i = 1, \ldots, N \\
\tau_{i,t}^y &= \tau_{i,t-1}^y + \epsilon_{i,t}^{\tau y},\ \ \epsilon_{i,t}^{\tau y} \sim \mathcal{N}(0,\ \sigma_{\tau y}^2) \\
h_{j,t} &= h_{j,0} + \omega_j^h \tilde{h}_{j,t} \\
\tilde{h}_{j,t} &= \tilde{h}_{j,t-1} + \epsilon_{j,t}^h,\ \ \epsilon_{j,t}^h \sim \mathcal{N}(0,\ 1),\ j = 1, \ldots, 2N + r_\pi + r_y
\end{aligned} \tag{3.11}$$

where $\pi_t = (\pi_{1,t}, \ldots, \pi_{N,t})'$ is an $N \times 1$ vector, $\tau_t^\pi = (\tau_{1,t}^\pi, \ldots, \tau_{N,t}^\pi)'$ is an $N \times 1$ vector, $P = \text{diag}(\rho_1, \ldots, \rho_N)$ is an $N \times N$ matrix, $A = \text{diag}(\alpha_1, \ldots, \alpha_N)$ is an $N \times N$ matrix, $y_t = (y_{1,t}, \ldots, y_{N,t})'$ is an $N \times 1$ vector, $\tau_t^y = (\tau_{1,t}^y, \ldots, \tau_{N,t}^y)'$ is an $N \times 1$ vector, L_π is an $N \times r_\pi$ matrix, f_t is a $r_\pi \times 1$ vector, u_t^π is an $N \times 1$ vector, $\Phi_1 = \text{diag}(\phi_{1,1}, \ldots, \phi_{N,1})$ is an $N \times N$ matrix, $\Phi_2 = \text{diag}(\phi_{1,2}, \ldots, \phi_{N,2})$ is an $N \times N$ matrix, L_y is an $N \times r_y$ matrix, g_t is a $r_y \times 1$ vector, u_t^y is an $N \times 1$ vector. Σ_t^π, Σ_t^y, Ω_t^π and Ω_t^π are defined previously.

We will use MC-UC-FSV as an acronym for this model defined through equation (11). We use $\exp(h_{j,t}/2), j = 1, \ldots, N$ to measure the **idiosyncratic inflation uncertainty**. For simplicity, we also use $\exp(h_t^\pi/2)$. We use $\exp(h_{j,t}/2), j = N + 1, \ldots, 2N$ (also $\exp(h_t^y/2)$) to measure the **idiosyncratic output uncertainty**. We use $\exp(h_{j,t}/2), j = 2N+1, \ldots, 2N+r_\pi$ (also $\exp(h_t^f/2)$) to measure the **global inflation uncertainty**. And we use $\exp(h_{j,t}/2), j = 2N + r_\pi + 1, \ldots, 2N + r_\pi + r_y$ (also $\exp(h_t^g/2)$) to measure the **global output uncertainty**. We summarize the definitions and descriptions of uncertainty in Table 3.1.

Many models can be written as restricted version of the MC-UC-FSV model and can help to investigate some aspects of our specification. These models, along with their acronyms, are as follows:

1. MC-UC-FSV-$r_y = 0$: this is the restricted version of the MC-UC-FSV where there is no common factors in the output gap equation, that is, $r_y = 0$. And the error in

Table 3.1: Definitions and descriptions of uncertainty $\exp(h_{j,t}/2)$

Definitions	descriptions of uncertainty $\exp(h_{j,t}/2)$
idiosyncratic inflation uncertainty	$\exp(h_t^\pi/2)$, the standard deviation of u_t^π
idiosyncratic output uncertainty	$\exp(h_t^y/2)$, the standard deviation of u_t^y
global inflation uncertainty	$\exp(h_t^f/2)$, the standard deviation of f_t
global output uncertainty	$\exp(h_t^g/2)$, the standard deviation of g_t
global inflation factor	f_t
global output factor	g_t

output gap equation is allowed to exhibit stochastic volatility.

2. MC-UC-FSV-$r_y, r_\pi = 0$: this is the restricted version of MC-UC-FSV where there is no common factors in inflation and output gap equations, that is, $r_\pi = 0$, $r_y = 0$. The error in inflation and output gap equation is allowed to exhibit stochastic volatility.

3. MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$: this is the restricted version of MC-UC-FSV where there is no common factors in inflation and output gap equations, that is, $r_\pi = 0$, $r_y = 0$, and the disturbances u_t^y are homoscedastic, while the disturbances u_t^π exhibit stochastic volatility. This is similar to the bivariate UC model that is used in Stella and Stock (2013), and Chan et al. (2016). They allow inflation persistence and the slope of Phillips curve to be time varying. However, we do not allow them to be time varying, that is, inflation persistence and the slope of Phillips curve are constant. Chan et al. (2016) has added a bound on them, so we also bound inflation persistence and the slope of Phillips to ensure stationarity.

We summarize the definitions and descriptions of uncertainty in Table 3.2:

Table 3.2: Models which are restricted version of MC-UC-FSV and the corresponding restrictions

Models	corresponding restrictions
MC-UC-FSV-$r_y = 0$	$r_y = 0$, no common factors in the output gap equation
MC-UC-FSV-$r_y, r_\pi = 0$	$r_y = 0$, $r_\pi = 0$, no common factors in inflation and output gap equations
MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	$r_y = 0$, $r_\pi = 0$, $\omega_j^h = 0, j = N+1, \ldots, 2N$

3.2.2 Prior

For the time-invariant part of log-volatility, $h_{j,0}$, we consider the inverse-Gamma representation of Horseshoe prior:

$$\begin{aligned} & h_{j,0} \mid \lambda_j^{h_0}, \tau^{h_0} \sim \mathcal{N}(0,\ \lambda_j^{h_0}\tau^{h_0}),\ j = 1, \ldots, 2N + r_\pi + r_y \\ & \lambda_j^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu_j^{h_0}}),\ \ \tau^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{h_0}}) \\ & \nu_j^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ 1), \qquad \xi^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ 1) \end{aligned} \tag{3.12}$$

For ω_j^h, we also consider the inverse-Gamma representation of Horseshoe prior:

$$\begin{aligned} & \omega_j^h \mid \lambda_j^{\omega^h}, \tau^{\omega^h} \sim \mathcal{N}(0,\ \lambda_j^{\omega^h}\tau^{\omega^h}),\ j = 1, \ldots, 2N + r_\pi + r_y \\ & \lambda_j^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu_j^{\omega^h}}),\ \ \tau^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\omega^h}}) \\ & \nu_j^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1), \qquad \xi^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1) \end{aligned} \tag{3.13}$$

The initial states are assumed to follow normal distribution with zero mean and variance ten. This is a relatively non-informative choice:

$$\tau_{i,1}^\pi \sim \mathcal{N}(0,\ 10),\ i = 1, \ldots, N \tag{3.14}$$

$$\tau_{i,1}^y \sim \mathcal{N}(0,\ 10),\ i = 1, \ldots, N \tag{3.15}$$

$$\tilde{h}_{j,1} \sim \mathcal{N}(0,\ 10),\ j = 1, \ldots, 2N + r_\pi + r_y \tag{3.16}$$

We use a relatively non-informative prior for the constant coefficients. We assume them to follow normal distribution with zero mean and variance ten.

$$\rho_i \sim \mathcal{N}(0,\ 10),\ i = 1, \dots, N \tag{3.17}$$

$$\alpha_i \sim \mathcal{N}(0,\ 10),\ i = 1, \dots, N \tag{3.18}$$

$$\varphi_{i,j} \sim \mathcal{N}(0,\ 10),\ i = 1, \dots, N,\ j = 1, 2 \tag{3.19}$$

$$l_m \sim \mathcal{N}(0,\ 10),\ m = 1, \dots, n_{l,\pi} + n_{l,y} \tag{3.20}$$

To ensure stationarity, we bound ρ_i and α_i to be positive and less than one, that is $0 < \rho_i < 1$ and $0 < \alpha_i < 1$. We also impose the stationary condition on the output gap equation and assume $\varphi_{i,1} + \varphi_{i,2} < 1$, $\varphi_{i,2} - \varphi_{i,1} < 1$ and $|\varphi_{i,2}| < 1$. No bounds on elements in factor loading matrix L.

The error variances are assumed to follow inverse gamma distribution. We choose relatively small values (ten) for the degrees of freedom parameters, which imply large prior variances. We then choose values for the scale parameters so that the parameters have the desired prior means. Setting it to 0.18 implies the prior mean is 0.02, while setting it to 0.09 implies the prior mean is 0.01. This choice is similar to that made in Chan et al. (2016).

$$\sigma^2_{\tau\pi} \sim \mathcal{IG}(10,\ 0.18),\ i = 1, \dots, N \tag{3.21}$$

$$\sigma^2_{\tau y} \sim \mathcal{IG}(10,\ 0.09),\ i = 1, \dots, N \tag{3.22}$$

3.3 In-sample Results on the Existence of Global Uncertainty and Model Fit

3.3.1 Data

The data are the quarterly consumer price index (CPI) and the quarterly real gross domestic product (GDP) for 34 countries, 23 advanced economies (AEs)[3] and 11 emerging market economies (EMEs)[4]. They span the period from 1995Q1 to 2018Q1. We transform the data to annualized growth rates as: $400\log(z_t/z_{t-1})$. And because the output gap equation follows an AR(2) process, our estimation start from 1995Q4. Posterior results are based on 100000 draws after a burn-in period of 20000.

3.3.2 Overview of In-sample Results

We divide our in-sample Results into three sub-sections. The first sub-section, section 3.3.3, is the MC-UC-FSV estimate of global inflation uncertainty $\exp(h_t^f/2)$ and global output uncertainty $\exp(h_t^g/2)$. With regard to the number of factors in the inflation gap equation and in the output gap equation. Identifying the optimal number is a challenging problem. In this paper, we set $r_\pi = 5$ and $r_y = 2$, that is, we include five factors in the inflation gap equation and two factors in the output gap equation. The reason for including more factors in the inflation gap equation is that we think inflation dynamics are difficult to explain [5].

The second sub-section, section 3.3.4, is about Bayesian model comparison. We compare the MC-UC-FSV to alternative models (MC-UC-FSV-$r_y = 0$, MC-UC-FSV-$r_y, r_\pi = 0$, MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) described in Table 3.2.

[3]Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Latvia, Lithuania, Netherlands, Portugal, Slovakia, South Korea, Spain, Sweden, Switzerland, UK, USA.

[4]Bolivia, Brazil, China, Hungary, Indonesia, Mexico, Philippines, Russia, South Africa, Thailand, Turkey.

[5]We first estimated a model with $r_\pi = 10$ and $r_y = 10$ but reduced the number of factors in the output gap equation upon examination of the factor loadings. This choice is discussed in more detail in the text and footnotes on the subsequent page.

After justifying that our MC-UC-FSV model provides higher model fit, we show, in the third sub-section (section 3.3.5), that persistence and the slope of the Phillips curve will decrease under the MC-UC-FSV, compared with the commonly-used MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$. And the estimates of trend inflation, trend output, idiosyncratic inflation uncertainty and idiosyncratic output uncertainty are found to be sensible.

3.3.3 MC-UC-FSV Estimates of global uncertainty

Although the MC-UC-FSV estimates of global uncertainty reflect contemporaneous effect of global factors on (the volatility of) macroeconomic data, the effect is also directly related to the loadings on the global factors. These loadings are reported in Appendix B.2. Table B.2 is the loadings on global inflation factor. We report the posterior mean of the five factors' loadings (recall that we set $r_\pi = 5$), but only the 16% and 84% quantiles of first factor's loadings for brevity. Most of the countries have sizable loadings on the first global inflation factor, and the quantiles (except Russia and Brazil) do not include zero. Table B.3 is the loadings on global output factor. We report the posterior mean and quantiles of the two factors' loadings (recall that we set $r_y = 2$). The quantiles of the first global output factor for all countries do not include zero. This provides strong evidence of significant commonality of output in the 34 countries. Carriero et al. (2020) obtain similar result in their case of the 19-country GDP dataset.

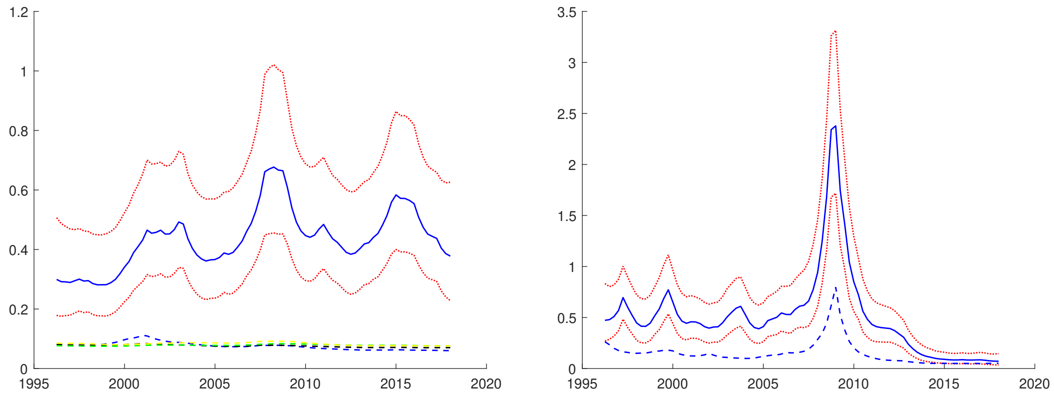
Figure 3.1 displays the posterior estimates of global uncertainty obtained from the MC-UC-FSV using the full sample. The left panel, Figure 3.1a, is the estimate of global inflation uncertainty, and the right panel, Figure 3.1b, is the estimate of global output uncertainty. In both figures, the solid lines represent the posterior means of the first uncertainty, while the dotted lines are the associated 16% and 84% quantiles. The dashed lines represent the posterior means of the remaining uncertainties. For example, with regard to global inflation uncertainty, we set $r_\pi = 5$, so we obtain the posterior estimates of the five global inflation uncertainties from MCMC, including their posterior means and quantiles. Then, in Figure 3.1a, we plot the posterior means and quantiles of the first global inflation uncertainty (see solid lines and dotted lines), but for brevity, we only plot the posterior means of the remaining uncertainties (the second, third, fourth and fifth uncertainty) using dashed lines.

As indicated in Figure 3.1a, we only observe evident and meaningful time-variation in the first global inflation uncertainty. The estimated global inflation uncertainty show significant increases around some of the political and economic events that Bloom (2009) highlights as periods of uncertainty, including 9/11, the Enron scandal, the second Gulf war, and the global financial crisis period. These spikes in the volatility associated with the global factor are documented in Kastner and Huber (2020) using US macroeconomic data. Since our data comes from 34 countries, the consistency between the estimate in Kastner and Huber (2020) and our study indicates that global macroeconomic uncertainty is closely related to uncertainty in the US, which might not seem surprising given the tie of the international economy to the US economy. One spike that is not documented in Kastner and Huber (2020) is that volatility increases from 2013 onward. This may indicate that such increase is driven countries other than US. In addition, at the end of our sample (2018Q1), the global inflation uncertainty still exists and continues to influence all countries under consideration. This is supported by a related study, Forbes (2019). They add commodity price volatility to explain inflation and find that commodity price volatility plays a large role for CPI inflation.

However, we find a different story with regard to the time-variation in the global output uncertainty from Figure 3.1b. First, the two global output uncertainties both increase during the GFC of 2008, but except this, we do not observe other meaningful time-variation from the second global output uncertainty.[6] Before the Global Financial Crisis (GFC) of 2008, there exists global output uncertainty but it does not show much time-variation. Then during the GFC, such uncertainty increases substantially. In the aftermath of the GFC, it decreases sharply. In early 2015, there is spike in inflation uncertainty,due to global oil price shock, and then towards the end of 2015 it subsides.[7]

[6]This is the first reason of including only two factors in the output gap equation.
[7]This is the second reason of including only two factors in the output gap equation

These features are documented in Carriero et al. (2020) in their 19-country GDP data set.



(a) Global inflation uncertainty

(b) Global output uncertainty

Figure 3.1: **Posterior estimates for global inflation uncertainty exp$(h_t^f/2)$ and global output uncertainty exp$(h_t^g/2)$ under MC-UC-FSV.** The solid lines represent the posterior means of the first global uncertainty, while the dotted lines are the associated 16% and 84% percentiles. The dashed lines represent the posterior means of the remaining uncertainties.

3.3.4 Bayesian Model Comparison

Before we jump into the model comparison, we assess whether the Horseshoe prior can successfully shrink strongly the parameter space (ω^h) but at the same time provides enough flexibility to allow for non-zero elements if necessary.

Evidence of NCP-HS-SV

We first investigate for time-variation in the volatility of inflation and output, then plot the estimated time-varying standard deviation $(\exp(h_{j,t}/2))$ to see whether it coincides with the test result. Of course, to test for time-variation in the volatility, a gold standard is using marginal likelihood (Bayes Factor is the ratio of two marginal likelihoods). However, in our settings where we allow for time-variation in volatility, the computation of marginal likelihood requires integrating out all the states, making it a nontrivial task. Therefore, we follow the method developed in Chan (2018). More specifically, since we notice that the model without SV is a restricted version of the model with SV, the

Bayes Factor can be calculated using the Savage-Dickey density ratio, thus avoiding the computation of marginal likelihood. The Bayes Factor in favor of the unrestricted model (model with SV) can be obtained using the Savage-Dickey density ratio as

$$\mathrm{BF}_{h_j} = \frac{p(\omega_j^h = 0)}{p(\omega_j^h = 0 |\ y)}$$

So if BF_{h_j} is larger than 1, then the Bayes Factor is in favor of the unrestricted model. In this part, the unrestricted model is time-varying h_j. For simplicity, we compare the log Bayes Factor. So a positive log Bayes Factor supports a time-varying h_j.

On the computation of posterior density ($p(\omega_j^h = 0 |\ y)$), we can obtain the posterior distribution given output from MCMC algorithm, then it is direct to compute the posterior density. On the computation of prior density ($p(\omega_j^h = 0)$), since we use the Horseshoe prior on ω_j^h, $p(\omega_j^h = 0)$ does not have a convenient analytical form. But, given the hyperparameters $(\lambda_j^{\omega^h}, \tau^{\omega^h}, \nu_j^{\omega^h}, \xi^{\omega^h})$ in Horseshoe prior, $p(\omega_j^h = 0 |\ \lambda_j^{\omega^h}, \tau^{\omega^h}, \nu_j^{\omega^h}, \xi^{\omega^h})$ is Normal. Thus, if we have output from a prior simulator, we can approximate $p(\omega_j^h = 0)$ by

$$\hat{p}(\omega_j^h = 0) = \frac{1}{S} \sum_{s=1}^{S} p(\omega_j^h = 0 |\ \lambda_j^{\omega^{h,s}}, \tau^{\omega^{h,s}}, \nu_j^{\omega^{h,s}}, \xi^{\omega^{h,s}})$$

This approximation applies for any prior which has a hierarchical form. The estimated log Bayes Factor is reported in Appendix B.4. For time-variation in the volatility of inflation in 34 countries, 12 countries are in favor of time-variation in the volatility (their log Bayes Factor are positive). For time-variation in the volatility of output in 34 countries, 14 countries are in favor of time-variation in the volatility.

To see whether the estimated time-varying standard deviation coincides with the estimated log Bayes Factor, we report the two in Figure 3.2 and Figure 3.3. Figure 3.2 depicts the estimated time-varying standard deviation for inflation. The title of each sub-figure is the country name, followed by the estimated log Bayes Factor. For example, the title of the first sub-figure is “Belgium (-6.16) ”, then the first sub-figure depicts the estimated time-varying standard deviation for Belgium inflation and the the estimated log Bayes Factor is -6.16, which is negative and implies that the log Bayes Factor does not supports time-variation in the volatility of Belgium inflation. Figure 3.3 depicts the estimated time-varying standard deviation for output. The title is named in the same way as inflation. The first sub-figure depicts the estimated time-varying standard deviation for Belgium output and the the estimated log Bayes Factor is -5.17, which is negative and implies that the log Bayes Factor does not support time-variation in the volatility of Belgium output.

We find that the estimates of both the log Bayes Factor and the time-varying standard deviation are sensible and coincide with past research. For USA, the log Bayes Factor supports time-varying volatility of inflation, while does not support time-varying volatility of output. This is consistent with what we observe from the estimated time-varying standard deviation for inflation and output. We observe marked spike in Figure 3.2, while it remains quite flat in Figure 3.3. Zaman (2021), Kabundi et al. (2021), among many others, assume that the error in the output gap equation remains homoscedastic. The consistency among the log Bayes Factor, the time-varying standard deviation and past literature implies that the Horseshoe prior can successfully remove unimportant small SV and at the same time provides enough flexibility to allow for SV if necessary.

In addition, we find that, for several countries, the log Bayes Factor supports time-varying volatility of output and we also observe marked spikes from the time-varying standard deviation. This result points towards a big advantage of our proposed model, which allows for SV in output gap equation. While past research assume the error in output gap equation is homoscedastic, such assumption displays a tendency to be over-restricted in multi-country study and ignores patterns observed under the model allowing for SV in output gap equation. Omitting the SV can severely affect the reliability of the estimates of the trend output.

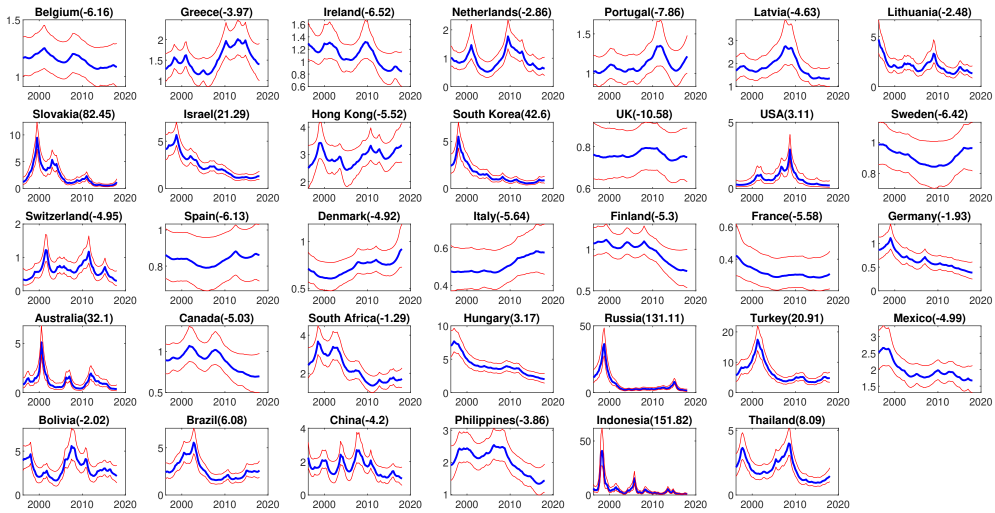


Figure 3.2: **The estimated log Bayes Factor and time-varying standard deviation in inflation gap equation.** The solid blue lines are the means, 16% and 84% quantiles of time-varying standard deviation.

In-sample fit

As discussed previously, the computation of marginal likelihood can be a challenge when there are a large number of states. Therefore, we use an approximation to the marginal likelihood (e.g., Geweke, 2001; Cross et al., 2020). They propose that conditioning on the estimation period, the sums of one-step-ahead joint log predictive likelihoods of 34 countries can be viewed as an approximation to the marginal likelihood, therefore provides a direct measure of in-sample fit. We compare four competing models: MC-UC-FSV, MC-UC-FSV-$r_y = 0$, MC-UC-FSV-$r_y, r_\pi = 0$ and MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$. Before computing the the sums of one-step-ahead joint log predictive likelihoods, we need to define some basics. Let $\hat{y}_{t+k}^{(i,j)}$ denote, at time t, the k-step-ahead forecast of the j-th variable in the i-th country, and $y_{t+k}^{(i,j)}$ denote the actual value. In our empirical work, $i = 1, \ldots, N$ with $n = 34$, $j = 1, 2$ where $j = 1$ denote inflation and $j = 2$ denote output. $\mathrm{Y}_{1:t}^{(i,j)}$ stores the data up to time t, so $\hat{y}_{t+k}^{(i,j)} = \mathrm{E}\ (y_{t+k}^{(i,j)} \mid \mathrm{Y}_{1:t}^{(i,j)})$. Then we compute the k-step-ahead log predictive likelihoods (LPL) of the j-th variable in the i-th country at time t:

$$\mathrm{LPL}_{t,i,j,k} = \log\, p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)}),\ t = T_0, \ldots, T - k$$

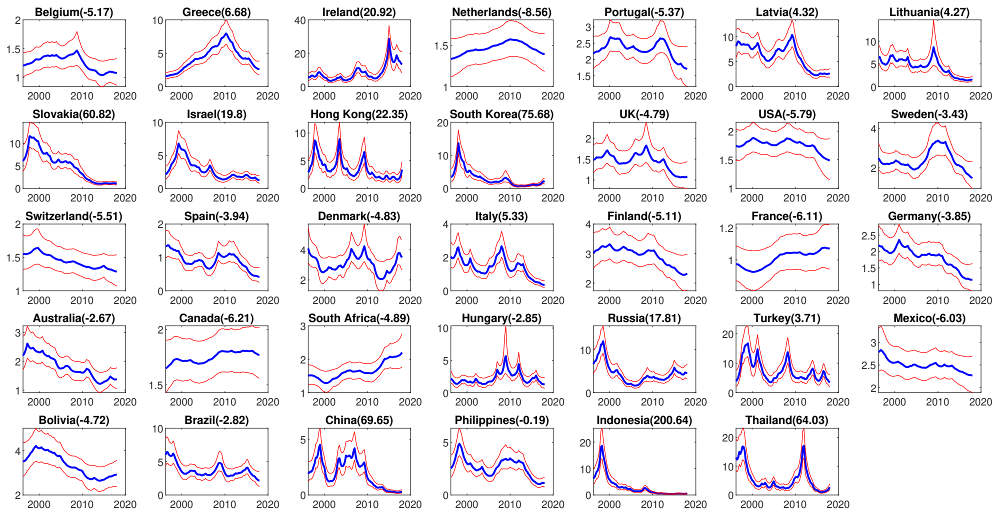


Figure 3.3: **The estimated log Bayes Factor and time-varying standard deviation in output gap equation.** The solid blue lines are the means, 16% and 84% quantiles of time-varying standard deviation.

Then the sums of one-step-ahead joint log predictive likelihoods is computed using:

$$\text{LPL}_{\cdot,\cdot,\cdot,1} = \sum_{t=T_0}^{T-1} \sum_{i=1}^{n} \sum_{j=1}^{2} \log\, p(\hat{y}_{t+1}^{(i,j)} = y_{t+1}^{(i,j)} | \text{Y}_{1:t}^{(i,j)})$$

Our estimation period starts from 1995Q4 (to 2018Q1), and the forecasting evaluation period starts from 2003Q1. We provide the sums of one-step-ahead joint log predictive likelihoods of 34 countries in Table 3.3.

In Table 3.3, results are presented relative to the forecast performance of the **UC-SV**: we take differences, so that a positive number indicates a model is forecasting better than the MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$. (Please note that we only take the sum, and no average. That may be why the number seems so large. For example, the sums of LPL under MC-UC-FSV is 895.02. If we take average over time, then it is 14.67. If we take further average across country, then it is 0.43). The results show that the MC-UC-FSV provides the best fit compared to all other models. In addition, since we find MC-UC-FSV-$r_y, r_\pi = 0$ provides higher model fit than MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$, we view this as another evidence in support of allowing for idiosyncratic stochastic volatility in output gap equation.

Table 3.3: Sum of one-step-ahead log predictive likelihood

Model	against MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$
MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0
MC-UC-FSV-$r_y, r_\pi = 0$	520.37
MC-UC-FSV-$r_y = 0$	658.57
MC-UC-FSV	883.34

3.3.5 Estimates under MC-UC-FSV

In the preceding sub-section, we provide evidence of successful shrinkage and flexibility realized by our NCP-HS-SV specification, and find that our MC-UC-FSV provides higher model fit, justifying the importance of international macroeconomic uncertainty. In this subsection, we compare the estimates of parameters produced from the MC-UC-FSV and MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$. Specifically:

1. Decrease of persistence and Flattening of the Phillips Curve

Different from Kabundi et al. (2021), which focus on the time-variation of coefficients, we focus on the effects of international macroeconomic uncertainty on the constant coefficients. This can be done through comparing the relative change of constant coefficients under MC-UC-FSV against MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$. We find that considering contemporaneous cross-country linkages will decrease the persistence and flatten the Phillips Curve.

2. Estimates of country-specific latent states

We present the posterior estimates of country-specific latent states: Trend inflation τ^π, Trend output τ^y, Idiosyncratic inflation uncertainty $\exp(h_t^\pi/2)$ (In fact, $\exp(h_t^\pi/2)$ is the standard deviation, so what we compare is the standard deviation). Idiosyncratic output uncertainty $\exp(h_t^y/2)$.

Decrease of persistence and Flattening of the Phillips Curve

We provide the posterior estimates of the coefficients in the Appendix B.3. Before describing the detailed characteristics of each constant coefficient (ρ, α, φ_1, φ_2), we first summarize the relative change of constant coefficients under MC-UC-FSV-$r_y = 0$ against MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$, to assess the effects of contemporaneous cross-country linkages on them.

In Table 3.4, the number is number of countries. For example, the "decrease" row "ρ" column is 24, then out of 34 countries, there are 24 countries whose ρ is smaller under MC-UC-FSV than the ρ under MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$. ρ is the inflation gap persistence, α is the slope of the Phillips Curve. We find, for most countries, considering global inflation uncertainty will decrease the inflation gap persistence and the slope of the Phillips Curve. Also, we find output gap persistence φ_1 decreases, so allowing for idiosyncratic stochastic volatility in output and global output uncertainty will decrease the output gap persistence.

Table 3.4: Relative change under MC-UC-FSV against MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$

	ρ	α	φ_1	φ_2	$\varphi_1 + \varphi_2$
decrease	24	25	29	9	29
no change	3	8	0	0	0
increase	7	1	5	25	5

Inflation gap persistence

Table B.4 reports the inflation gap persistence. A noticeable difference between AEs and EMEs is observed. Most AEs exhibit a smaller gap persistence, which literature attributes to a better anchoring of inflation expectations, suggesting that agents in AEs countries have become more forward looking than agents in EMEs. In other word, inflation process in AEs is no longer adaptive (see Cogley and Sargent, 2005, Stock and Watson, 2007 and Chan et al., 2016). While, expectation formation in EMEs is more adaptive. Such difference between AEs and EMEs is also documented in Kabundi et al. (2021). The new finding (compared with Kabundi et al., 2021) is that considering global inflation uncertainty will decrease inflation gap persistence in both AEs and (most) EMEs.

Output gap persistence
Table B.6 reports the output gap persistence. A similar pattern to inflation gap persistence is found for output gap persistence. We find considering global output uncertainty will decrease the AR(1) coefficient φ_1. Although the AR(2) coefficient φ_2 increases in most countries (25 out of 34 countries), the sum of the AR(1) and AR(2) coefficient decreases in most countries (29 out of 34 countries).

Phillips curve
The coefficient controlling the slope of the Phillips curve, α, has decreased in most countries (25 out of 34 countries) after taking into account global uncertainty. This provide further evidence that global uncertainty will flatten the Phillips Curve. As done in Forbes (2019), they include comprehensive controls for globalization because globalization is often cited as causing the flattening of the Phillips Curve.

Then, we report the posterior estimates of four country-specific latent states: trend inflation τ_t^π, trend output τ_t^y, idiosyncratic inflation uncertainty $\exp(h_t^\pi)$ and idiosyncratic output uncertainty $\exp(h_{j,t}^y)$. For each country, we compare their country-specific latent states under four competing models: MC-UC-FSV, MC-UC-FSV-$r_y = 0$, MC-UC-FSV-$r_y, r_\pi = 0$ and MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Estimates of Trend inflation

In Figure 3.4, we report the posterior estimates of trend inflation under the four competing models. The title of each sub-figure is the country name, followed by the official inflation targets (point target or target bands). For example, the title of the first sub-figure is "Belgium (2) ", then the first sub-figure depicts the estimate of trend inflation for Belgium and the official inflation target set by Belgium central bank is 2%. Each sub-figure plots the posterior estimates (mean, 16% and 84% quantiles) of trend inflation under MC-UC-FSV along with the posterior mean of trend inflation under three competing models. The solid blue lines are the means, 16% and 84% quantiles under MC-UC-FSV, the dotted red lines are posterior means under MC-UC-FSV-$r_y = 0$, the dashed black lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

The solid blue lines and the dotted red lines represent the estimate under the models considering global inflation uncertainty in inflation gap equation, while the dashed black lines and the dashed green lines represent the estimate under the models without global inflation uncertainty in inflation gap equation. The first 23 countries are AEs (from Belgium to Canada), followed by 11 EMEs. A pattern which emerges from the results is that considering global inflation uncertainty tend to influence the estimated trend inflation more in AEs than in EMEs. The posterior means under the four competing models are almost coincident in EMEs. However, global inflation uncertainty does generate some differences in AEs, such as Netherlands, USA, Switzerland, Denmark, Italy, France, Germany, Canada. And we observe that such differences indicate that trend inflation is driven by both domestic factors and global factors. For example, many papers without global inflation uncertainty document that trend inflation for USA has been below 2% since 2012, and this is also observed under our competing models MC-UC-FSV-$r_y, r_\pi = 0$ and MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$, in contrast, the mean estimate under model with global inflation uncertainty decreases to a higher level in 2010. Then it begins to increase, rather than decreasing until 2015.

Estimates of Trend output

In Figure 3.5, we report the posterior estimates of trend output under the four competing models. The title of each sub-figure is the country name. Each sub-figure plots the posterior estimates (mean, 16% and 84% quantiles) of trend output under MC-UC-FSV along with the posterior mean of trend output under three competing models. The meaning of each line is the same as that in Figure 3.4. The solid blue lines are the

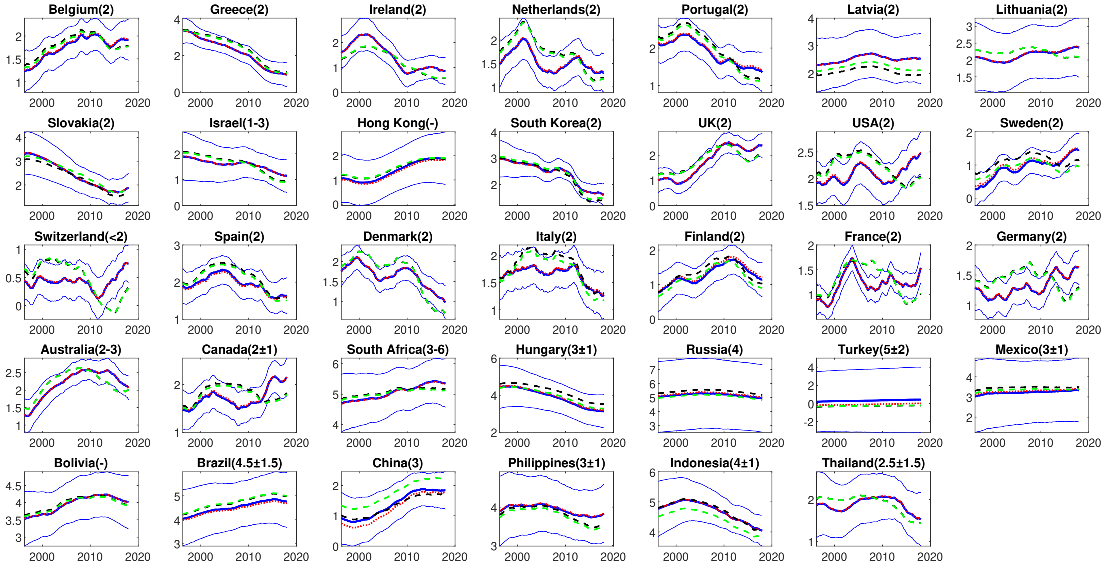


Figure 3.4: **Posterior estimates for trend inflation τ^{π}.** The title of each sub-figure is the country name, followed by the official inflation targets (point target or target bands). For Hong Kong and Bolivia, we do not find the official inflation targets, so we use "-". The solid blue lines are the means, 16% and 84% quantiles under MC-UC-FSV, the dotted red lines are posterior means under MC-UC-FSV-$r_y = 0$, the dashed black lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

means, 16% and 84% quantiles under MC-UC-FSV, the dotted red lines are posterior means under MC-UC-FSV-$r_y = 0$, the dashed black lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

We have two modifications in the output gap equation: allowing for idiosyncratic output uncertainty (MC-UC-FSV-$r_y, r_\pi = 0$ and MC-UC-FSV-$r_y = 0$) and considering global output uncertainty (MC-UC-FSV).

We first analyze the effect of idiosyncratic output uncertainty on the estimate of trend output. This is done through comparing the estimates under MC-UC-FSV-$r_y, r_\pi = 0$ (dashed black lines) with the estimate under MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ (dashed greed lines), where the error in output gap equation remains homoscedastic. We find allowing for idiosyncratic output uncertainty will provide higher estimate of trend output in many countries (roughly 20 out of 34 countries).

Then, we analyze the effect of global output uncertainty on the estimate of trend output. This is done through comparing the estimates under MC-UC-FSV (solid blue lines) with the estimate under MC-UC-FSV-$r_y = 0$ (dotted red lines). Similar to the finding in the effect of global inflation uncertainty, we find that considering global output uncertainty tend to influence the estimated trend inflation more in AEs than in EMEs.

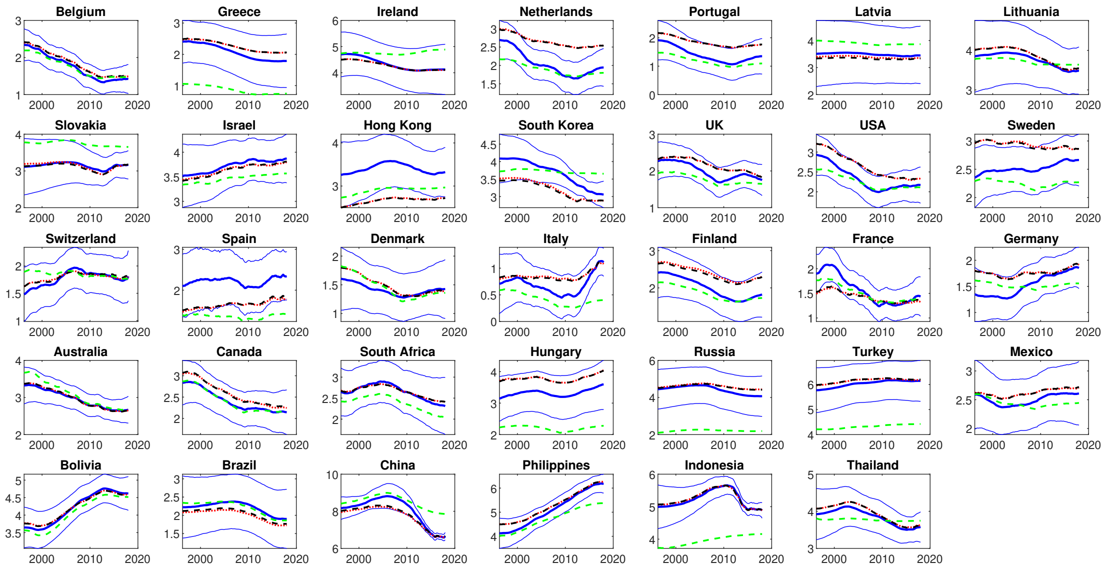


Figure 3.5: **Posterior estimates for trend output τ^y.** The solid blue lines are the means, 16% and 84% quantiles under MC-UC-FSV, the dotted red lines are posterior means under MC-UC-FSV-$r_y = 0$, the dashed black lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Estimates of idiosyncratic inflation uncertainty

In Figure 3.6, we report the idiosyncratic inflation uncertainty estimates (i.e., the standard deviation of the shocks to the inflation gap, $\exp(h_t^\pi/2)$) under the four competing models. The title of each sub-figure is the country name. Each sub-figure plots the posterior estimates (mean, 16% and 84% quantiles) under MC-UC-FSV along with the posterior mean under three competing models. The meaning of each line is the same as that in Figure 3.4. The solid blue lines are the means, 16% and 84% quantiles under MC-UC-FSV, the dotted red lines are posterior means under MC-UC-FSV-$r_y = 0$, the dashed black lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

The solid blue lines and the dotted red lines represent the estimate under the models allowing for cross-country linkages in inflation gap equation, while the dashed black lines and the dashed green lines represent the estimate under the models without cross-country linkages in inflation gap equation. A quick visual inspection shows that allowing for cross-country linkages reduces the spike of idiosyncratic inflation uncertainty. This can be regarded as an evidence supporting that there exist factors driving strong co-movement of inflation across economies. In addition, the idiosyncratic inflation uncertainty in several countries becomes quite flat after allowing for cross-country linkages, suggesting that the uncertainty in their inflation gap equation is driven by global inflation uncertainty, rather than idiosyncratic inflation uncertainty.

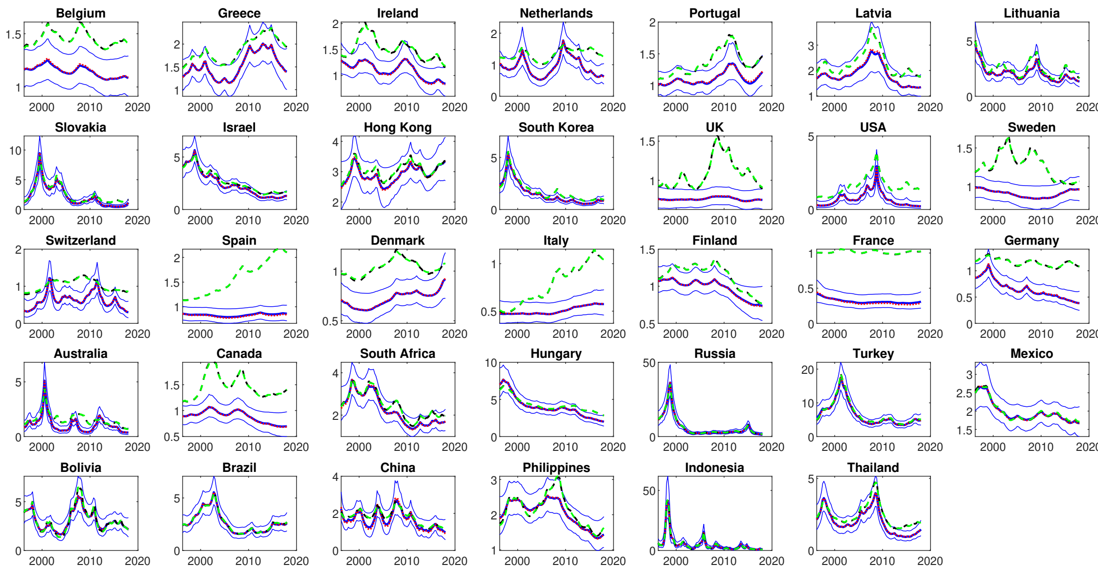


Figure 3.6: **Posterior estimates for idiosyncratic inflation uncertainty $\exp(h_t^\pi/2)$.** The solid blue lines are the means, 16% and 84% quantiles under MC-UC-FSV, the dotted red lines are posterior means under MC-UC-FSV-$r_y = 0$, the dashed black lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Estimates of idiosyncratic output uncertainty

In Figure 3.7, we report the idiosyncratic output uncertainty estimates (i.e., the standard deviation of the shocks to the output gap, $\exp(h_t^y/2)$) under the four competing models. The title and the meaning of each line is the same as that in Figure 3.6.

The solid blue lines and the dotted red lines represent the estimate under the models allowing for cross-country linkages in output gap equation, while the dashed black lines and the dashed green lines represent the estimate under the models without cross-country linkages in output gap equation. The pattern found for idiosyncratic inflation uncertainty can also be found for idiosyncratic output uncertainty. We again observe that allowing for cross-country linkages reduces the spike of idiosyncratic output uncertainty. This indicates that there exist factors driving strong co-movement of output across economies. The idiosyncratic output uncertainty in many countries becomes quite flat after allowing for cross-country linkages, suggesting that the uncertainty in their output gap equation is driven by global output uncertainty, rather than idiosyncratic output uncertainty. This number of idiosyncratic output uncertainty becoming flat is higher than the number of idiosyncratic inflation uncertainty becoming flat, which provides evidence for papers assuming that the error in output gap equation is homoscedastic, but at the same time supports the need to allow for cross-country linkages.

3.4 Out-of-sample Forecasting Results

Since our modifications are about uncertainty, we focus on the density forecast. We use the data from 1995Q4 to 2002Q4 as an initial estimation period, and use data through 2002Q4 to produce k-step-ahead forecast distributions. We consider forecast horizons of $k = 1, 4, 8, 12, 16$ quarters. So our forecast evaluation period begins in 2003Q1.

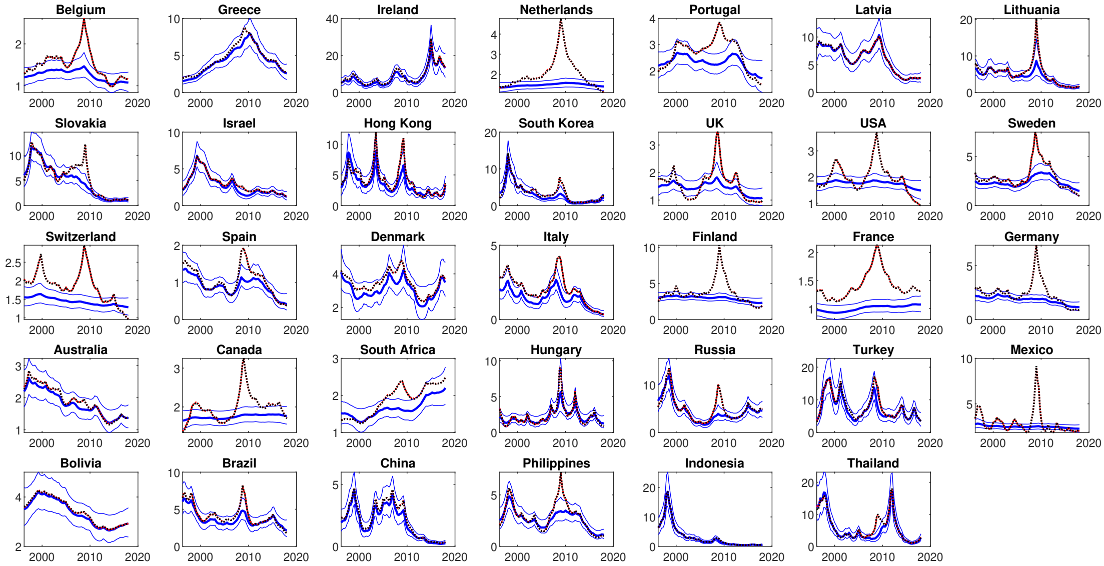


Figure 3.7: **Posterior estimates for idiosyncratic output uncertainty exp$(h_t^y/2)$.** The solid blue lines are the means, 16% and 84% quantiles under MC-UC-FSV, the dotted red lines are posterior means under MC-UC-FSV-$r_y = 0$, the dashed black lines are posterior means under MC-UC-FSV-$r_y, r_\pi = 0$.

3.4.1 Overview of Forecasting Results

We divide our out-of-sample forecasting results into three parts: forecasting inflation, forecasting output and jointly forecasting inflation and output. For each part, we discuss the results in three dimensions. The first dimension is aggregate forecasting performance over time and over countries (the aggregate LPL, by summing all countries and all time periods). Since we observe international macroeconomic uncertainty, it is natural to expect that considering such uncertainty will provide more accurate forecast in economic recession. Thus, the second dimension is about forecasting performance over time (we can study how the sums of LPL changes over time, by summing all countries at time t). After providing evidence that our MC-UC-FSV can produce more accurate forecast in economic recession, we further study whether such good forecast performance is driven by particular countries, so the third dimension is about the forecasting performance at country level. All results are presented relative to the forecast under MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$: we take differences, so a positive number indicates a model is forecasting better than the MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

3.4.2 Forecasting inflation

We first report the aggregate forecasting performance for inflation over time and over countries in Table 3.5. It is calculated by summing the LPL for the N countries over T_0 to $T-k$ (and recall that $j=1$ denote inflation):

$$\text{LPL}_{\cdot,\cdot,1,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \log\, p(\hat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \text{Y}_{1:t}^{(i,1)})$$

The results show that the model with cross-country linkages in inflation (MC-UC-FSV-$r_y = 0$ and MC-UC-FSV) provides more accurate forecast for inflation than the model without cross-country linkages (MC-UC-FSV-$r_y, r_\pi = 0$ and MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) at all horizons.

Table 3.5: Sum of k-step-ahead log predictive likelihood for 34-country inflation

Model	k=1	k=4	k=8	k=12	k=16
MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0	0	0	0	0
MC-UC-FSV-$r_y, r_\pi = 0$	-4.27	71.83	127.82	138.85	185.26
MC-UC-FSV-$r_y = 0$	98.92	**265.09**	286.02	350.53	333.11
MC-UC-FSV	**101.63**	257.39	**294.76**	**379.19**	**356.89**

The forecasting result of inflation in Table 3.5 suggests the benefits of allowing for cross-country linkages, which is done through considering the global inflation uncertainty in our paper. It is natural to expect that the good forecasting result may largely arise from periods of uncertainty. To investigate this point, we calculate the sums of LPL over time. A common method is, as done in Feldkircher et al. (2021), to sum the LPL for the N countries at time t:

$$\text{LPL}_{t,\cdot,1,k} = \sum_{i=1}^{n} \log\, p(\hat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \text{Y}_{1:t}^{(i,1)})$$

For example, suppose we are at the time point of 2007Q4, then $k=1$ means we are forecasting the data in 2008Q1, and $k=4$ means we are forecasting the data in 2008Q4. So this method helps to answer at time t, which model can provide the most accurate forecast in the future.

However, recall that global inflation uncertainty shows significant increases around 2008 and 2015 (see Figure 3.1a, and because our forecast starts from 2003Q1, so we omit the increase in 2001). Such global inflation uncertainty drives strong co-movement across countries. So a more interesting study is to investigate whether this global inflation uncertainty can improve the forecast performance during periods of uncertainty. For example, suppose that we want to know which model can provide the most accurate forecast of 2008Q1? Different forecast horizons will provide the forecast made at different time t. If $k = 1$, then this means the forecast is made at 2007Q4 (one-step-ago). If $k = 4$, then this means the forecast is made at 2007Q1 (four-step-ago). Overall, the difference is the X axis. Suppose that we are at time t, in Feldkircher et al. (2021), the X axis is t and represents when we make the forecast, but in our paper, the X axis is $t+k$ and represents the period being forecasted. That is how we produce Figure 3.8. About the starting time, since we make the first forecast at 2002Q4, if $k = 1$, the period being forecasted is 2003Q1, so in Figure 3.8, the X axis (the period being forecasted) starts from 2003Q1 when $k = 1$. If $k = 4$, the period being forecasted is 2003Q4, so in Figure 3.8, the X axis (the period being forecasted) starts from 2003Q4 when $k = 4$. Similarly, if $k = 16$, the period being forecasted is 2006Q4, so in Figure 3.8, the X axis (the period being forecasted) starts from 2006Q4 when $k = 16$.

We plot the results (against MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 3.8 (for brevity, we only plot the results of MC-UC-FSV). If the period to be forecasted is the period of uncertainty (like 2008), the MC-UC-FSV provides overall good forecast performance at all horizons, particularly at long horizons. This indicates the importance of taking into account cross-country linkages for improving forecasts of inflation, especially to forecast periods of uncertainty. To forecast more stable periods, it does not harm to take into account cross-country linkages.
The sums of LPL over time in Figure 3.8 is for the 34 countries. Someone may question whether the good forecasting result is driven by particular countries? To investigate this point, we present the forecasting result for individual countries. The LPL of inflation

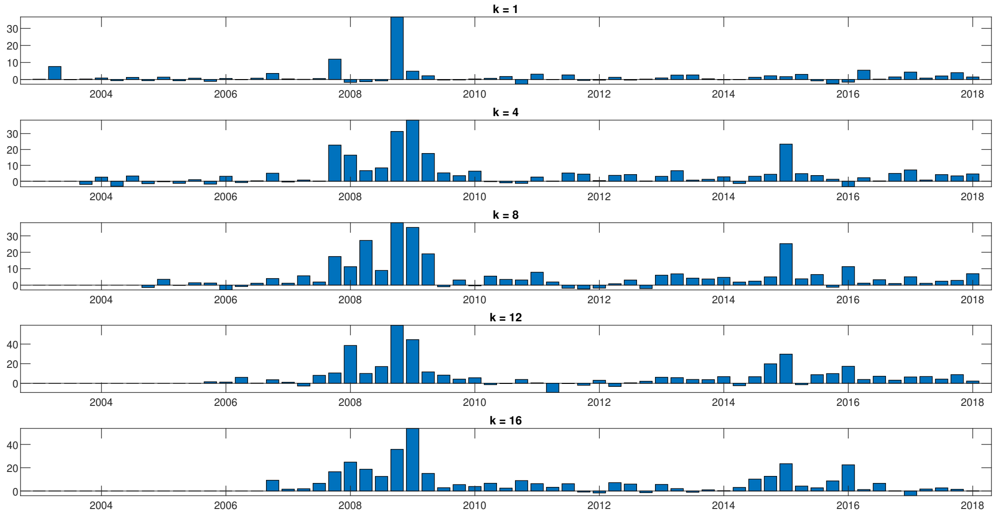


Figure 3.8: **Sums of k-step ahead LPL of inflation for MC-UC-FSV relative to MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ over time.** The X axis is $t + k$ and represents the period being forecasted.

for country i at time t, which can be calculated by:

$$\text{LPL}_{t,i,1,k} = \log p(\hat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \text{Y}_{1:t}^{(i,1)})$$

We plot the results (against MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 3.9. Here the period of uncertainty that we plot is 2008Q4, so the period being forecasted is 2008Q4 ($t+k = 2008Q4$). If $k = 1$, then the time we make forecast is 2008Q3, and we find overall good forecast performance for most countries with more pronounced gains in advanced economies (The first 23 countries are AEs, and the following 11 countries are EMEs). A similar pattern is found if $k = 16$. The time we make forecast is 2004Q4, and we also find overall good forecast performance for most countries. We also find significant gains in Spain and USA. The gain is not so significant if $k = 1$ as the gain if $k = 16$. In Figure 3.9, we only plot the shortest horizon $k = 1$ and the longest horizon $k = 16$, for middle horizons ($k = 4, 8, 12$), we find good forecasting result across most countries and did not find particular country which is important in driving good forecasting results. Overall, We find good forecast performance for MC-UC-FSV for most countries and such good forecast performance is not driven by particular countries.

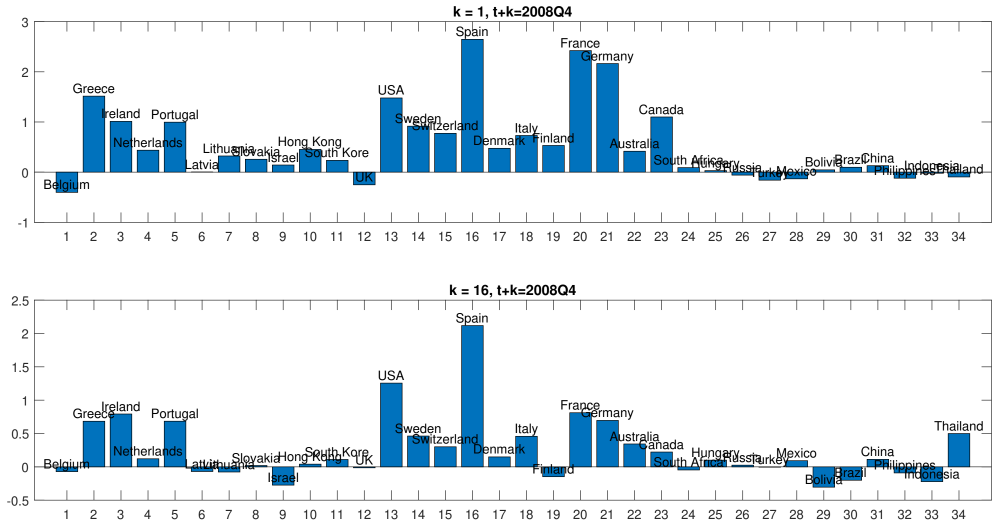


Figure 3.9: **Sums of k-step ahead LPL of inflation for country i under MC-UC-FSV relative to MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.**

3.4.3 Forecasting output

With regard to output, we report the sums of LPL of output over time and over countries in Table 3.6. It is calculated by summing the LPL for the N countries over T_0 to $T - k$ (and recall that $j = 2$ denote output):

$$\text{LPL}_{\cdot,\cdot,2,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \log\, p(\hat{y}_{t+k}^{(i,2)} = y_{t+k}^{(i,2)} | \mathrm{Y}_{1:t}^{(i,2)})$$

The results show that the model, which allows for both idiosyncratic stochastic volatility in output and cross-country linkages in output (MC-UC-FSV), provides the most accurate forecast for output at all horizons.

Table 3.6: Sum of k-step-ahead log predictive likelihood for 34-country output

Model	k=1	k=4	k=8	k=12	k=16
MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0	0	0	0	0
MC-UC-FSV-$r_y, r_\pi = 0$	577.02	694.98	811.01	797.25	684.98
MC-UC-FSV-$r_y = 0$	566.81	668.04	852.04	772.90	680.99
MC-UC-FSV	**762.93**	**1194.99**	**1211.17**	**1208.10**	**1052.36**

Similar to the analysis of inflation, the second dimension of discussion for output is sums of LPL over time (by summing all countries at time t), which can be calculated by:

$$\text{LPL}_{t,\cdot,2,k} = \sum_{i=1}^{n} \log\, p(\hat{y}_{t+k}^{(i,2)} = y_{t+k}^{(i,2)} | \text{Y}_{1:t}^{(i,2)})$$

We plot the results (against MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 3.10. If the period to be forecasted is the period of uncertainty (like 2008), the MC-UC-FSV provides overall good forecast performance. This indicates the importance of taking into account cross-country linkages for improving forecasts of output, especially to forecast periods of uncertainty. To forecast more stable periods, it does not harm to take into account cross-country linkages.

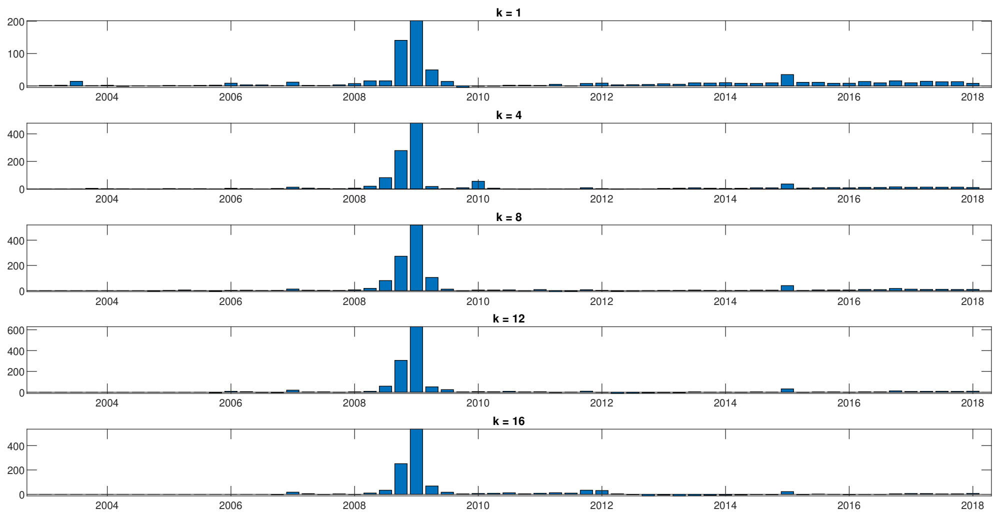


Figure 3.10: **Sums of k-step ahead LPL of output for MC-UC-FSV relative to MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ over time.** The X axis is $t + k$ and represents the period being forecasted.

To investigate whether the good forecast performance is driven by particular countries, we calculate the sums of LPL of output for country i at time t by:

$$\text{LPL}_{t,i,2,k} = \log\, p(\hat{y}_{t+k}^{(i,2)} = y_{t+k}^{(i,2)} | \text{Y}_{1:t}^{(i,2)})$$

We plot the results (against MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 3.11. We choose 2008Q4 to represent the period of uncertainty. For $k = 1$ and $k = 16$, we both find overall good forecast performance for MC-UC-FSV for all countries. The highest gain is found for Hungary, followed by Sweden. However, different from the conclusion in the case of forecasting inflation that more pronounced gains are found in AEs, we find significant gains in both AEs and EMEs. This implies that allowing for idiosyncratic stochastic volatility in output and cross-country linkages in output is important for both AEs and EMEs.

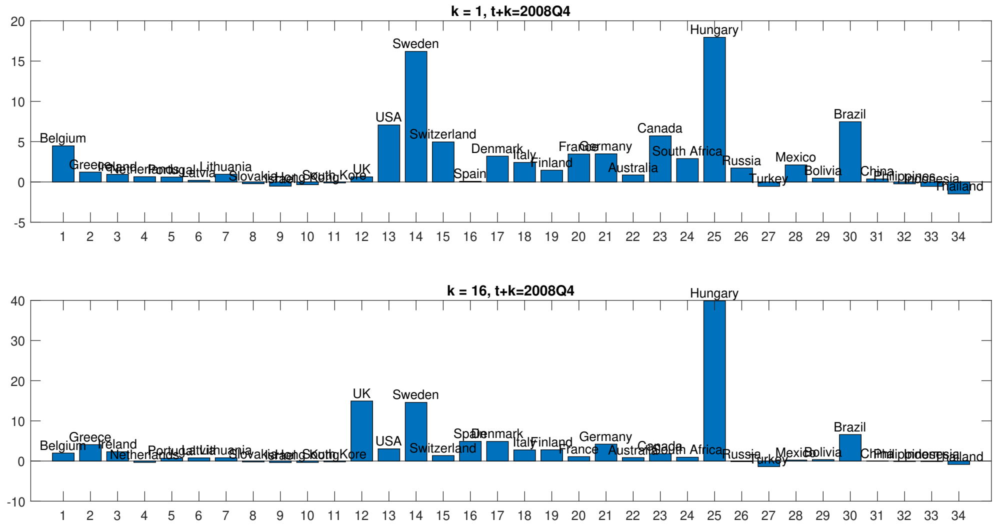


Figure 3.11: **Sums of k-step ahead LPL of output in country i for MC-UC-FSV relative to MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.**

3.4.4 Jointly Forecasting inflation and output

With regard to the joint predictive density for inflation and output, we first report the sums of joint LPL over time and over countries in Table 3.7. It is calculated by summing the LPL for the N countries over T_0 to $T - k$ (and for all j, recall that $j = 1$ denote inflation, $j = 2$ denote output):

$$\mathrm{LPL}_{\cdot,\cdot,\cdot,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \sum_{j=1}^{2} \log p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)})$$

The results show that the model, which allows for idiosyncratic stochastic volatility in output and cross-country linkages in both inflation and output (MC-UC-FSV), provides the most accurate joint forecast for inflation and output at all horizons. Next, we study

Table 3.7: Sum of k-step-ahead joint log predictive likelihood for 34-country inflation and output

Model	k=1	k=4	k=8	k=12	k=16
MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0	0	0	0	0
MC-UC-FSV-$r_y, r_\pi = 0$	520.37	679.42	751.62	794.16	615.81
MC-UC-FSV-$r_y = 0$	658.57	898.62	1084.28	1084.13	1148.35
MC-UC-FSV	**883.34**	**1513.05**	**1545.20**	**1824.70**	**1672.17**

the time-variation in forecast performance to see whether the benefits arise from the forecast during periods of uncertainty. So the second dimension of discussion for joint predictive density for inflation and output is sums of joint LPL over time (by summing all j and all countries at time t), which can be calculated by:

$$\mathrm{LPL}_{t,\cdot,\cdot,k} = \sum_{i=1}^{n}\sum_{j=1}^{2} \log\, p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)})$$

We plot the results (against MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 3.12. A similar pattern to inflation and output was found. If the period to be forecasted is the period of uncertainty (like 2008), the MC-UC-FSV provides overall good joint forecast performance. This indicates the importance of taking into account cross-country linkages (in inflation and output) for improving forecasts of inflation and output, especially during periods of uncertainty.

Finally, we investigate whether the good forecast performance of periods of uncertainty is driven by particular countries, so the third dimension of discussion for joint predictive density for inflation and output is sums of joint LPL at the country level (by

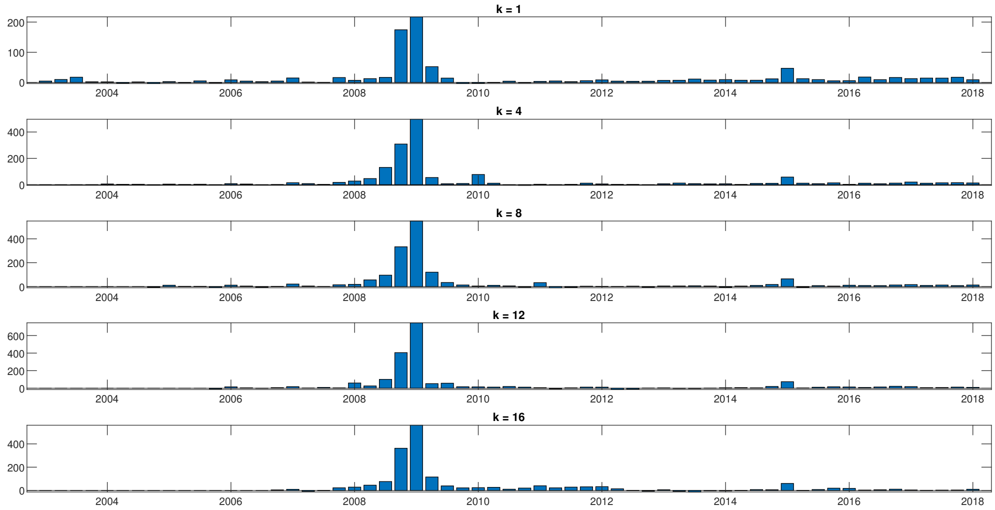


Figure 3.12: **Sums of k-step ahead joint LPL for MC-UC-FSV relative to MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ over time.** The X axis is $t+k$ and represents the period being forecasted.

summing all j for country i), which can be calculated by:

$$\text{LPL}_{t,i,\cdot,k} = \sum_{t=T_0}^{t=T-k} \sum_{j=1}^{2} \log p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \text{Y}_{1:t}^{(i,j)})$$

We plot the results (against MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 3.13. A similar pattern to output is found. (This is sensible since the gains in output are much larger than gains in inflation, see Figure 3.9 and Figure 3.11). We find overall good forecast performance for MC-UC-FSV for all countries.

3.5 Conclusion

This paper develops a multi-country unobserved components model that allows for contemporaneous cross-country linkages. The model includes 23 Advanced Economies and 11 Emerging Market Economies, with two variables (output and inflation) within each country and their relationship being inspired by the Phillips curve. An important feature of our model is that we model all countries jointly and allow for contemporaneous cross-country linkages, through factor stochastic volatility specification. Factor stochastic volatility specification enables us to study the commonality in international

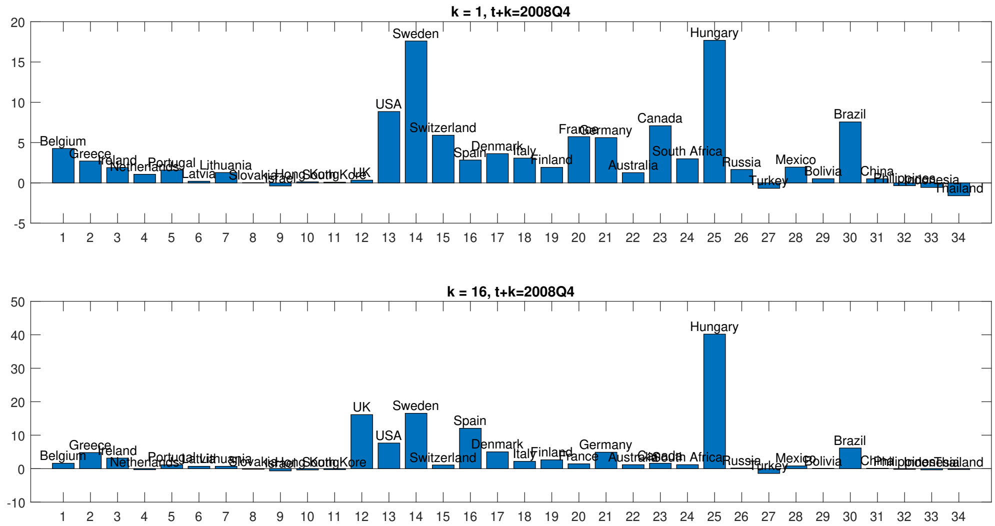


Figure 3.13: **Sums of k-step ahead joint LPL in country i for MC-UC-FSV relative to MC-UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.**

macroeconomic uncertainty (global uncertainty). Past research has highlighted importance of allowing for stochastic volatility. Accordingly, we allow for stochastic volatility in all equations and factors. To cope with over-fitting concerns, we rewrite the process of log-volatility using non-centered parameterization and use the Horseshoe prior on the coefficient that controls the time-variation in log-volatility. The Horseshoe prior is a global-local shrinkage prior, which is found to strongly shrink the parameter but at the same time provide flexibility if necessary. Since the prior density of Horseshoe prior does not have an analytical form, which makes it difficult to compute the Bayes Factor for nested models, we propose to use prior simulation to solve this problem. This method applies for any prior which has a hierarchical form. Recent research has devoted to speeding up computation and and one prominent progress is performing equation-by-equation estimation. Factor stochastic volatility specification also enables us to estimate this high dimensional model equation-by-equation.

In an empirical application we first present evidence of global uncertainty and it coincides with major economic events. Then, the Bayesian model comparison results provide strong support to our stochastic volatility specification and our model which takes into account global uncertainty. Specifically, we show that our stochastic volatility specification, written in non-centered parameterization and imposed Horseshoe prior, can successfully remove unimportant small SV and at the same time provides enough flexibility to allow for SV if necessary. Our model is supported by providing higher model fit. After justifying that our model is sensible and supported, we present an expansive set of posterior estimates that we hope would be helpful. The posterior estimates of coefficients show that considering contemporaneous cross-country linkages will decrease the persistence and flatten the Phillips curve. The posterior estimates of trends are found to be sensible and indicate that they are driven by both domestic factors and global factors. A by-product is that our model allows us to tell the uncertainty in a equation is driven by common international components, or components operating at a country level, or both. Finally, we provide a detailed forecasting exercise to evaluate the merits of our model. We find our model can provide more accurate forecasts, especially if the period being forecasted is the period of uncertainty. And such good forecast performance is for most countries and not driven by particular countries.

Chapter 4

A Panel Unobserved Components Model for Estimating Macroeconomic Trends

4.1 Introduction

The concept of trend is central to macroeconomics. Multivariate unobserved components (UC) models have been shown to provide reasonable estimates of the trend. Existing UC models in the literature either focus on one country or impose independent assumption between economies in multi-country study. But in the modern globalized economy, countries are linked together and events in one country can spill over into others. The UC model without interdependencies across countries is counter-intuitive.

In order to bridge the gap between modelling strategy and the process of globalization, this paper develops a new multi-country UC model: panel unobserved components model with factor stochastic volatility (PUC-FSV). PUC-FSV model allows for cross-country linkages both in the error covariance matrix and in conditional mean. This is inspired by large and growing panel vector autoregressive (PVAR) literature. PVAR has the same structure as VAR model, but a cross sectional dimension is added to the VAR representation (see, Canova and Ciccarelli, 2013). PUC-FSV has the same intuition as PVAR models, in the sense that PUC-FSV has the same structure as UC model, but a cross-country dimension is added to the UC representation.

Three key features characterize our PUC-FSV model. First, the PUC-FSV takes dynamic interdependencies into account by allowing for cross-country linkages in the conditional mean, more specifically, in the coefficient matrices associated with the lagged variables. Dynamic interdependencies may influence the estimates of trend through spillovers in modern globalized economies. Second, the PUC-FSV takes static interdependencies into account. This is done through two blocks. One block is allowing for cross-country linkages in the error covariance matrix. Another block is allowing for cross-country linkages in the conditional mean, more specifically, in the coefficient matrices associated with the Phillips curve. By extending the traditional (own country) Phillips curve to global Phillips curve, the PUC-FSV adds a new measure of static interdependencies. Third, we do not impose any zero restriction. This reduces the associated mis-specification risk and means that we work with unrestricted PUC-FSV model. To deal with over-parameterization concerns, we rely on a global-local shrinkage prior. Although this approach is silent as to the reasons behind the dependencies, it allows the data to tell us what the dependencies are. This is particularly important in the absence of strong a *priori* beliefs on dependencies of variables.

The first key feature of the PUC-FSV is that it takes dynamic interdependencies (DIs) into account, by allowing for cross-country linkages in the coefficient matrices associated with the lagged variables.[1] DIs are popular in PVARs (see, Canova and Ciccarelli, 2013; Koop and Korobilis, 2016 and Davidson et al., 2019). However, DIs have not been allowed for in the existing UC literature. This is the first feature that we part with them. Canova (2011) point out that the model without DIs is suitable for estimates

[1]We do not consider the dependencies (that take place with a lag) across variables within a country. The reason is from the structure of popular UC literature (see, Stella and Stock, 2013; Chan et al., 2016; Kabundi et al., 2021 and Wu, 2021). What links variables within a country together is the Phillips curve. Except the Phillips curve, one variable is driven by its own lags. The lags of other variables within a country are not included. We follow their strategy in this paper.

of trend[2] in a small open economy. But this paper includes 34 countries (23 advanced economies and 11 emerging market economies). The small open economy assumption is no longer satisfied and spillovers may influence the estimates of trend (see Canova and Ciccarelli, 2013). Taylor and Wieland (2016) also emphasize that omitting variables can affect the reliability of the estimates of trend. Therefore, in this paper, we take on the challenge of allowing for dynamic interdependencies, which drops the small open economy assumption and directly addresses critiques (pointed out by Taylor and Wieland, 2016) that are likely to appear in multi-country studies.

The second key feature of PUC-FSV is that it takes static interdependencies (SIs) into account. Generally, SIs are allowed for through the error covariance matrix. In PVARs, the off-diagonal elements of error covariance matrix determine SIs (see Koop and Korobilis, 2016; Davidson et al., 2019 and Feldkircher et al., 2021). In UC models, SIs are introduced through factor stochastic volatility (FSV), which assumes that the error covariance matrix is driven by latent factors (see Wu, 2021). The PUC-FSV follows the FSV specification, but adds a new measure of SIs. This new measure is important, because one assumption of FSV specification is that all countries' inflation are driven by common factors, while all countries' output are driven by other different factors. What links inflation and output together is their own country Phillips curve (see Stella and Stock, 2013; Chan et al., 2016; Kabundi et al., 2021 and Wu, 2021). This implies that the model does not allow for the case where one country output has a contemporaneous affect on other countries' inflation. For instance, US output has a contemporaneous affect on US inflation through own country Phillips curve, but US output does not have a contemporaneous affect on UK inflation. Such SIs are ignored, but it seems necessary to add them back. This is because that the data we have is quarterly data, while the transmission between some countries may be quick enough such that a shock originating from one country has produced an affect on another country in one quarter. To directly address this issue, we extend own country Phillips curve

[2]Precisely, Canova (2011) use the term "steady state ". We use the term "trend ". There are subtle differences between them, but they can be interpreted as the same for the purpose of this paper.

to global Phillips curve, which allows one country output to have a contemporaneous affect on other country inflation. This is the new measure of static interdependencies added in PUC-FSV.

The third key feature of PUC-FSV is that we do not impose any zero restriction on parameters, which means that we work with the unrestricted PUC-FSV. The estimation problem of panel model is related to the curse of dimensionality. One common solution is to selectively model the dynamic links across countries while imposing zero-restrictions on others (see Canova and Ciccarelli, 2009 and Canova and Ciccarelli, 2013). However, Feldkircher et al. (2021) point out that these restrictions, if wrongly chosen, potentially lead to mis-specification problems. It is clearly desirable to introduce restrictions in a data based fashion. One data-based approach is the stochastic search specification selection approach, developed by Koop and Korobilis (2016). Their approach produces posterior inclusion probabilities for every possible restrictions and these probabilities can be used to sort through restrictions in a data based fashion. Davidson et al. (2019) further extend the method of Koop and Korobilis (2016) to allow for a more detailed investigation of cross-country linkages. Another data-based approach is to rely on global-local shrinkage priors. Zero-restriction implies that the matrix is sparse. It is found that if a matrix is characterized by a relatively low number of non-zero elements, a possible solution is a global-local shrinkage prior (e.g., Polson and Scott, 2010; Kastner and Huber, 2020). Such advantage of global-local shrinkage prior shrinks strongly the parameter space but at the same time provides enough flexibility to allow for non-zero elements if necessary, thus imposing zero restriction for most elements but dropping the restriction if necessary. The global-local shrinkage prior has been applied to PVAR literature in Feldkircher et al. (2021).[3] In this paper, we follow their method, working with unrestricted PUC-FSV model and relying on the global-local shrinkage prior to deal with over-parameterization concerns.

[3] Leaving the model unrestricted will also make the computation cumbersome. Their prominent method, integrated rotated Gaussian approximation (IRGA), is powerful at computation aspect. We have not applied this new method, because our model is not so huge, compared to their model.

The only condition in this paper is the stability condition. Since we work with unrestricted PUC-FSV, the involved parameters can be enormous. To avoid them moving into undesirable regions, we first rewrite the PUC-FSV model as a VAR(1) process, conditional on the latent states. Then we impose the stability condition on the coefficient matrix. The approach of bounding the parameters has been proposed in Chan et al. (2016). They argue to bound the parameters (e.g., slope of the Phillips curve) so as ensure stationarity and find empirical importance of bounding. This bounding approach has been applied in many papers (see Zaman, 2021; Kabundi et al., 2021 and Wu, 2021). This paper takes a similar strategy to impose stability condition on the coefficient matrix.

This paper is organized as follows. In Section 2, we start from the multi-country unobserved components model that allows for cross-country linkages in the error covariance matrix, then introduce the panel unobserved components model with factor stochastic volatility (PUC-FSV), which allows for cross-country linkages both in the error covariance matrix and in the conditional mean. After introducing the PUC-FSV model, we describe the Horseshoe shrinkage priors on parameters. In Section 3, we describe the 34-country data and provide evidence of interdependencies. After showing the existence of interdependencies, we show the importance of interdependencies in the following two sections. In Section 4, we present the estimates of trend. We find that allowing for cross-country linkages in the error covariance matrix can provide more precise estimates of trend, while omitting cross-country interdependencies in the conditional mean will overestimate trend output. In Section 5, we show that the PUC-FSV model can provide higher in-sample fit and more accurate density forecasts compared to existing models in the literature. Finally, Section 6 concludes.

4.2 A Panel Unobserved Components model with Factor Stochastic Volatility

In this section, we develop the panel unobserved components model with factor stochastic volatility. We do not impose any zero restrictions. To deal with the over-parameterization concerns, we rely on a global-local shrinkage prior (the Horseshoe prior). The priors are described after introducing the PUC-FSV model.

4.2.1 PUC-FSV Model Specification

We begin with the multi-country unobserved component (MC-UC-FSV) model developed in Wu (2021). They allow for cross-country linkages in the error covariance matrix, through factor stochastic volatility specification. In particular, for country i, $i = 1, \dots, N$, $\pi_{i,t}$ is the inflation of country i at time t and $y_{i,t}$ is the output of country i, $\tau^{\pi}_{i,t}$ and $\tau^{y}_{i,t}$ are their trends. The MC-UC-FSV model for N-country inflation and output is defined as:

$$
\begin{aligned}
\pi_t - \tau^{\pi}_t &= P(\pi_{t-1} - \tau^{\pi}_{t-1}) + A(y_t - \tau^{y}_t) + L_{\pi} f_t + u^{\pi}_t,\ f_t \sim \mathcal{N}(0,\ \Omega^{\pi}_t),\ u^{\pi}_t \sim \mathcal{N}(0,\ \Sigma^{\pi}_t) \\
y_t - \tau^{y}_t &= \Phi(y_{t-1} - \tau^{y}_{t-1}) + \Theta(y_{t-2} - \tau^{y}_{t-2}) + L_y g_t + u^{y}_t,\ g_t \sim \mathcal{N}(0,\ \Omega^{y}_t),\ u^{y}_t \sim \mathcal{N}(0,\ \Sigma^{y}_t) \\
\tau^{\pi}_{i,t} &= \tau^{\pi}_{i,t-1} + \epsilon^{\tau\pi}_{i,t},\quad \epsilon^{\tau\pi}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau\pi}),\ i = 1, \dots, N \\
\tau^{y}_{i,t} &= \tau^{y}_{i,t-1} + \epsilon^{\tau y}_{i,t},\quad \epsilon^{\tau y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau y}) \\
h_{j,t} &= h_{j,0} + \omega^h_j \tilde{h}_{j,t} \\
\tilde{h}_{j,t} &= \tilde{h}_{j,t-1} + \epsilon^h_{j,t},\quad \epsilon^h_{j,t} \sim \mathcal{N}(0,\ 1),\ j = 1, \dots, 2N + r_{\pi} + r_y
\end{aligned}
\tag{4.1}
$$

where $\pi_t = (\pi_{1,t}, \dots, \pi_{N,t})'$ is an $N \times 1$ vector, $\tau^{\pi}_t = (\tau^{\pi}_{1,t}, \dots, \tau^{\pi}_{N,t})'$ is an $N \times 1$ vector, $P = \text{diag}(\rho_1, \dots, \rho_N)$ is an $N \times N$ matrix, $A = \text{diag}(\alpha_1, \dots, \alpha_N)$ is an $N \times N$ matrix, $y_t = (y_{1,t}, \dots, y_{N,t})'$ is an $N \times 1$ vector, $\tau^{y}_t = (\tau^{y}_{1,t}, \dots, \tau^{y}_{N,t})'$ is an $N \times 1$ vector, L_{π} is an $N \times r_{\pi}$ matrix, f_t is a $r_{\pi} \times 1$ vector, u^{π}_t is an $N \times 1$ vector, $\Phi = \text{diag}(\phi_1, \dots, \phi_N)$ is an $N \times N$ matrix, $\Theta = \text{diag}(\theta_1, \dots, \theta_N)$ is an $N \times N$ matrix, L_y is an $N \times r_y$ matrix, g_t is a $r_y \times 1$ vector, u^{y}_t is an $N \times 1$ vector. $\Sigma^{\pi}_t = \text{diag}(e^{h_{1,t}}, \dots, e^{h_{N,t}})$, $\Sigma^{y}_t = \text{diag}(e^{h_{N+1,t}}, \dots, e^{h_{2N,t}})$, $\Omega^{\pi}_t = \text{diag}(e^{h_{2N+1,t}}, \dots, e^{h_{2N+r_{\pi},t}})$, and

$\Omega_t^y = \text{diag}(e^{h_{2N+r_\pi+1,t}}, \dots, e^{h_{2N+r_\pi+r_y,t}})$.

The assumption that the errors are driven by latent factors (f_t and g_t) allows for cross-country linkages in the error covariance matrix. However, Wu (2021) assume the coefficient matrices P, A, Φ and Θ are diagonal. It is with this assumption that we part with them. One would expect that country i variables depend on other countries' variables, either contemporaneously or with a lag. Therefore, we relax this diagonal assumption to allow for a more comprehensive investigation of interdependencies. Specifically, we assume that the coefficient matrices P, A, Φ and Θ are full matrices:

$$\begin{aligned}\pi_{i,t} - \tau_{i,t}^{\pi} =& \rho_{i,1}(\pi_{1,t-1} - \tau_{1,t-1}^{\pi}) + \rho_{i,2}(\pi_{2,t-1} - \tau_{2,t-1}^{\pi}) + \cdots + \rho_{i,N}(\pi_{N,t-1} - \tau_{N,t-1}^{\pi}) \\ &+ \alpha_{i,1}(y_{1,t} - \tau_{1,t}^{y}) + \alpha_{i,2}(y_{2,t} - \tau_{2,t}^{y}) + \cdots + \alpha_{i,N}(y_{N,t} - \tau_{N,t}^{y}) \\ &+ L_{i,\pi} f_t + u_{i,t}^{\pi}\end{aligned} \tag{4.2}$$

$$\begin{aligned}y_{i,t} - \tau_{i,t}^{y} =& \phi_{i,1}(y_{1,t-1} - \tau_{1,t-1}^{y}) + \phi_{i,2}(y_{2,t-1} - \tau_{2,t-1}^{y}) + \cdots + \phi_{i,N}(y_{N,t-1} - \tau_{N,t-1}^{y}) \\ &+ \theta_{i,1}(y_{1,t-2} - \tau_{1,t-2}^{y}) + \theta_{i,2}(y_{2,t-2} - \tau_{2,t-2}^{y}) + \cdots + \theta_{i,N}(y_{N,t-2} - \tau_{N,t-2}^{y}) \\ &+ L_{i,y} g_t + u_{i,t}^{y}\end{aligned} \tag{4.3}$$

where $\rho_{i,j}$ for $i, j = 1, \dots, N$ represents the affect of country j inflation gap on country i inflation. Similarly, $\alpha_{i,j}$ for $i, j = 1, \dots, N$ represents the affect of country j output gap on country i inflation. $\phi_{i,j}$ and $\theta_{i,j}$ represent the affect of country j output gap on country i output. Equation (4.2)-(4.3) specify the model for country i.

Written in matrix, we can obtain the multi-country PUC-FSV model specification:

$$
\begin{aligned}
\pi_t - \tau_t^{\pi} &= P(\pi_{t-1} - \tau_{t-1}^{\pi}) + A(y_t - \tau_t^{y}) + L_\pi f_t + u_t^{\pi},\ f_t \sim \mathcal{N}(0,\ \Omega_t^{\pi}),\ u_t^{\pi} \sim \mathcal{N}(0,\ \Sigma_t^{\pi}) \\
y_t - \tau_t^{y} &= \Phi(y_{t-1} - \tau_{t-1}^{y}) + \Theta(y_{t-2} - \tau_{t-2}^{y}) + L_y g_t + u_t^{y},\ g_t \sim \mathcal{N}(0,\ \Omega_t^{y}),\ u_t^{y} \sim \mathcal{N}(0,\ \Sigma_t^{y}) \\
\tau_{i,t}^{\pi} &= \tau_{i,t-1}^{\pi} + \epsilon_{i,t}^{\tau\pi},\quad \epsilon_{i,t}^{\tau\pi} \sim \mathcal{N}(0,\ \sigma_{\tau\pi}^2),\ i = 1, \ldots, N \\
\tau_{i,t}^{y} &= \tau_{i,t-1}^{y} + \epsilon_{i,t}^{\tau y},\quad \epsilon_{i,t}^{\tau y} \sim \mathcal{N}(0,\ \sigma_{\tau y}^2) \\
h_{j,t} &= h_{j,0} + \omega_j^h \tilde{h}_{j,t} \\
\tilde{h}_{j,t} &= \tilde{h}_{j,t-1} + \epsilon_{j,t}^{h},\quad \epsilon_{j,t}^{h} \sim \mathcal{N}(0,\ 1),\ j = 1, \ldots, 2N + r_\pi + r_y
\end{aligned}
\tag{4.4}
$$

We refer to this specification as Panel Unobserved Components model with Factor Stochastic Volatility (PUC-FSV). Our PUC-FSV model has three important features.

First, full matrices (P, Φ and Θ) allow for dynamic interdependencies (DIs) (see, Canova and Ciccarelli, 2013 and Davidson et al., 2019). More specifically, a full matrix P allows that country i inflation depends on the first lag of other countries' inflation gap. For instance, if US inflation gap last quarter has an affect on UK this quarter, then we say there is an inflation DI from the US to UK. The magnitude of inflation DI is measured by the off-diagonal elements of matrix P. If the corresponding coefficient is (or close to) zero, then there is no inflation DI from the US to UK. Full matrices Φ and Θ allow that country i output depends on the first and second lag of other countries' output gap. For instance, if US output gap last two quarters have an affect on UK output this quarter, then we say there is an output DI from the US to UK. The magnitude of output DI is measured by the off-diagonal elements of matrices Φ and Θ. The magnitude measured by matrix Φ represents a faster transmission between two countries and we call this as output DI1st. The magnitude measured by matrix Θ represents a slower transmission between two countries and we call this as output DI2nd. If the corresponding coefficients are (or close to) zero, then there is no output DI from the US to UK.

Second, full matrix A and factor stochastic volatility allow for static interdependencies (SIs). More specifically, full matrix A allows that country i inflation depends on contemporaneous output gap, including own country output gaps and other countries' output gap. This is the second new feature of PUC-FSV. For instance, if US output gap this quarter has an affect on UK inflation this quarter, then we say there is a Phillips SI from the US to UK. The magnitude of Phillips SI is measured by the off-diagonal elements of matrix A. If the corresponding coefficient is (or close to) zero, then there is no Phillips SI from the US to UK. Factor stochastic volatility assumes that all countries' errors in the inflation gap equation are driven by common factors f_t and all countries' errors in the output gap equation are driven by g_t. And we call such SIs as inflation error SIs and output error SIs. The magnitude of two error SIs is measured by factor loading matrices L_π and L_y.

Third, we do not impose any zero restriction on the coefficient matrices, which means that we work with the unrestricted PUC-FSV. Leaving panel model unrestricted can lead to enormous parameters to estimate. And to deal with over-parameterization concerns, we rely on a global-local shrinkage prior. The Horseshoe prior is a global-local shrinkage prior and empirically successful. Next, we describe the Horseshoe prior and the priors for all other parameters.

4.2.2 Priors

We first introduce the prior for full matrices P, A, Φ and Θ. Then we introduce the prior for other parameters.

The prior for full matrices is the Horseshoe prior. The Horseshoe prior was proposed by Carvalho et al. (2010). It involves a global shrinkage parameter (τ) and a local shrinkage parameter (λ). Carvalho et al. (2010) have shown that the induced distributions over the global and local shrinkage parameters allow for optimal rates of shrinkage near zero, while having sufficiently thick tails (see Cross et al. (2020), 2020).

More specifically, we use the inverse-Gamma representation of Horseshoe prior for elements in the full matrix P, A, Φ and Θ. We assume that the global shrinkage parameter (τ) is specified to differ across types of parameters, that is, each full matrix has two global shrinkage parameters: one for own country coefficients and one for other country coefficients. For instance, suppose that $\rho_{i,m}$ is an element in P, if $i = m$, then $\rho_{i,i}$ is own country coefficient, and if $i \neq m$, then $\rho_{i,m}$ is other country coefficient, then the Horseshoe prior for own country coefficient $\rho_{i,i}$ is:

$$\begin{aligned}
&\rho_{i,i} \mid \lambda^{\rho}_{i,i}, \tau^{\rho,d} \sim \mathcal{N}(0,\ \lambda^{\rho}_{i,i}\tau^{\rho,d}),\ i = 1, \ldots, N \\
&\lambda^{\rho}_{i,i} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu^{\rho}_{i,i}}),\quad \tau^{\rho,d} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\rho,d}}) \\
&\nu^{\rho}_{i,i} \sim \mathcal{IG}(\frac{1}{2},\ 1),\qquad \xi^{\rho,d} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned} \tag{4.5}$$

the Horseshoe prior for other country coefficient $\rho_{i,m}$ is:

$$\begin{aligned}
&\rho_{i,m} \mid \lambda^{\rho}_{i,m}, \tau^{\rho,nd} \sim \mathcal{N}(0,\ \lambda^{\rho}_{i,m}\tau^{\rho,nd}),\ i = 1, \ldots, N,\ m = 1, \ldots, N,\ i \neq m \\
&\lambda^{\rho}_{i,m} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu^{\rho}_{i,m}}),\quad \tau^{\rho,nd} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\rho,nd}}) \\
&\nu^{\rho}_{i,m} \sim \mathcal{IG}(\frac{1}{2},\ 1),\qquad \xi^{\rho,nd} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned} \tag{4.6}$$

To ensure stationarity, we impose condition on P, A, Φ and Θ. Specifically, we first rewrite the first two equations in Equation (4) as a VAR(1):

$$\begin{pmatrix} \pi_t - \tau^{\pi}_t \\ y_{t+1} - \tau^{y}_{t+1} \\ y_t - \tau^{y}_t \\ y_{t-1} - \tau^{y}_{t-1} \end{pmatrix} = \begin{pmatrix} P & A & 0 & 0 \\ 0 & \Phi & \Theta & 0 \\ 0 & I_N & 0 & 0 \\ 0 & 0 & I_N & 0 \end{pmatrix} \begin{pmatrix} \pi_{t-1} - \tau^{\pi}_{t-1} \\ y_t - \tau^{y}_t \\ y_{t-1} - \tau^{y}_{t-1} \\ y_{t-2} - \tau^{y}_{t-2} \end{pmatrix} + \begin{pmatrix} L_{\pi} f_t + u^{\pi}_t \\ L_y g_{t+1} + u^{y}_{t+1} \\ 0 \\ 0 \end{pmatrix} \tag{4.7}$$

Then we obtain the VAR(1) representation. The stability condition requires that all the eigenvalues of coefficient matrix are smaller than one in modulus. And we use the command "eig" in Matlab to compute eigenvalues. The diagonal elements in A is the slope of Phillips curve, so we also constrain them to be positive.

The priors on other parameters are the same as that in Wu (2021). More specifically, we model the evolution of the log-volatility according to a random walk in non-centered parameterization and then use the Horseshoe prior to control time-variation. For each $j = 1, \ldots, 2N + r_\pi + r_y$, the evolution of the log-volatility is modeled as:

$$
\begin{aligned}
h_{j,t} &= h_{j,0} + \omega_j^h \tilde{h}_{j,t} \\
\tilde{h}_{j,t} &= \tilde{h}_{j,t-1} + +\epsilon_{j,t}^h, \quad \epsilon_{j,t}^h \sim \mathcal{N}(0,\ 1)
\end{aligned}
\tag{4.8}
$$

The non-centered parameterization decomposes a time-varying parameter $h_{j,t}$ into two parts: a time-invariant part $h_{j,0}$ and a time-varying part $\omega_j^h \tilde{h}_{j,t}$, which has a constant coefficient ω_j^h that controls the time-variation. For constant parameters ω_j^h and $h_{j,0}$, we use the Horseshoe prior:

$$
\begin{aligned}
&\omega_j^h \mid \lambda_j^{\omega^h}, \tau^{\omega^h} \sim \mathcal{N}(0,\ \lambda_j^{\omega^h} \tau^{\omega^h}),\ j = 1, \ldots, 2N + r_\pi + r_y \\
&\lambda_j^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu_j^{\omega^h}}), \quad \tau^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\omega^h}}) \\
&\nu_j^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1), \qquad \xi^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned}
\tag{4.9}
$$

$$
\begin{aligned}
&h_{j,0} \mid \lambda_j^{h_0}, \tau^{h_0} \sim \mathcal{N}(0,\ \lambda_j^{h_0} \tau^{h_0}),\ j = 1, \ldots, 2N + r_\pi + r_y \\
&\lambda_j^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu_j^{h_0}}), \quad \tau^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{h_0}}) \\
&\nu_j^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ 1), \qquad \xi^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned}
\tag{4.10}
$$

The initial states are assumed to follow normal distribution with zero mean and variance ten. This is a relatively non-informative choice:

$$\tau_{i,1}^{\pi} \sim \mathcal{N}(0,\ 10),\ i = 1, \ldots, N \tag{4.11}$$

$$\tau_{i,1}^{y} \sim \mathcal{N}(0,\ 10),\ i = 1, \ldots, N \tag{4.12}$$

$$\tilde{h}_{j,1} \sim \mathcal{N}(0,\ 10),\ j = 1, \ldots, 2N + r_\pi + r_y \tag{4.13}$$

The elements in factor loading matrices are assumed to follow a normal distribution with zero mean and variance ten, that is:

$$l_m \sim \mathcal{N}(0,\ 10),\ m = 1, \ldots, n_{l,\pi} + n_{l,y} \tag{4.14}$$

where $n_{l,\pi}$ denotes the number of free elements in matrix L_π, and $n_{l,y}$ denotes the number of free elements in matrix L_y.

The error variances are assumed to follow inverse gamma distribution. We choose relatively small values (ten) for the degrees of freedom parameters, which imply large prior variances. We then choose values for the scale parameters so that the parameters have the desired prior means. Setting it to 0.18 implies the prior mean is 0.02, while setting it to 0.09 implies the prior mean is 0.01. This choice is similar to that made in Chan et al. (2016).

$$\sigma^2_{\tau\pi} \sim \mathcal{IG}(10,\ 0.18),\ i = 1, \ldots, N \tag{4.15}$$

$$\sigma^2_{\tau y} \sim \mathcal{IG}(10,\ 0.09),\ i = 1, \ldots, N \tag{4.16}$$

We use the Markov Chain Monte Carlo (MCMC) algorithm to sample all parameters. More specifically, to sample $\tau^{\pi}_{i,t}$, the prior still follows a random walk process, but the likelihood will come from N equations and each equation is defined through Equation (2). To sample $\tau^{y}_{i,t}$, the prior still follows a random walk process, but the likelihood will come from two parts: the first part is N equations in Equation (2), the second part is N equations in Equation (3). It is standard to sample other parameters and we refer readers to Chan et al. (2016) and Chan (2021) for details.

4.3 Data and Evidence of Interdependencies

In this section, we first introduce the data, then we provide evidence of interdependencies in two ways. The first way is through analyzing the matrices that measure interdependencies. The second way is through impulse response analysis.

4.3.1 Data

The data are the quarterly consumer price index (CPI) and the quarterly real gross domestic product (GDP) for 34 countries, 23 advanced economies (AEs)[4] and 11 emerging market economies (EMEs)[5]. They span the period from 1995Q1 to 2018Q1. We transform the data to annualized growth rates as: $400\log(z_t/z_{t-1})$. And because the output gap equation follows an AR(2) process, our estimation start from 1995Q4. We assume that there is one common factor driving 34-country inflation, that is $r_\pi = 1$. We assume that there is one common factor driving 34-country output, that is $r_y = 1$. This assumption comes from the empirical results in Wu (2021). Having estimated a model with $r_\pi = 10$ and $r_\pi = 10$, they find there is one global factor driving 34-country inflation and one global factor driving 34-country output. Posterior results are based on 100000 draws after a burn-in period of 20000.

4.3.2 Evidence of Interdependencies

In this section, we present evidence of interdependencies across countries in two different ways. The first way analyzes matrices that measure interdependencies, discussing estimates of coefficients, which countries support interdependencies and whether these interdependencies occur contemporaneously or with a lag. The estimates show that the Horseshoe prior can achieve sensible shrinkage. Events in both AEs and EMEs can spill over into AEs, contemporaneously and with a lag. By contrast, there are only several spillovers into EMEs. We find more evidence of static interdependencies (SIs) than dynamic interdependencies (DIs). The reason may be that we are using quarterly data.

Then we move to impulse response analysis, discussing how a shock affects the 34 countries analyzed. We do so by computing the generalised impulse response functions

[4] Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Latvia, Lithuania, Netherlands, Portugal, Slovakia, South Korea, Spain, Sweden, Switzerland, UK, USA.

[5] Bolivia, Brazil, China, Hungary, Indonesia, Mexico, Philippines, Russia, South Africa, Thailand, Turkey.

(GIRFs) for each shock. We find the GIRFs are much larger when a global shock hits the system than when a US shock hits the system. The GIRFs also show that whatever the shock is, the GIRFs of EMEs inflation go back to zero more slowly than AEs. We call this “fragile inflation”in EMEs.

We want to emphasize that the above features are supported by data, and it is impossible to know all of them before estimating the model. They show the power of working with unrestricted model and selecting the appropriate restrictions in a data based manner.

Estimates of coefficients

For each of 34 countries, we have two blocks of interdependencies: one takes place contemporaneously (Static Interdependency, SI), the other takes place with a lag (Dynamic Interdependency, DI). Within each block, we have three matrices measuring the corresponding affect. Matrices A, L_π and L_y measure SIs, while matrice P, Θ, Φ measure DIs. We thus have six matrices summarizing the affects corresponding to the SIs (Figure 4.1 and Table C.1) and the DIs (Figure 4.2-4.4). The table presents the estimates of factor loading matrices L_π and L_y. The figures plot the four full matrices A, P, Θ and Φ. The Y axis in each figure is the left hand in the equation (see Equation (4.4)). The X axis is the corresponding right hand in the equation. For instance, matrix P is the coefficient matrix for lag of inflation gap in the inflation gap equations. We plot the estimates of matrix P in Figure 4.2. Then, the Y axis in Figure 4.2 is the 34-country inflation gap this quarter ($\pi_t - \tau_t^\pi$), and the X axis in Figure 4.2 is the 34-country inflation gap last quarter ($\pi_{t-1} - \tau_{t-1}^\pi$).

The general pattern is that our modelling strategy induces a high degree of sparsity, since most elements in full matrices (A, P, Θ and Φ) are close to zero. More importantly, the induced shrinkage is sensible, as evidenced by that own country information are treated more important than other countries' information. For instance, the diagonal elements of P are larger than the off-diagonal elements (see Figure 4.2), which means that our model believes own country inflation lag is more important than other countries' inflation lag. Similar pattern is observed from Phillips SIs (matrix A in Figure 4.1), output DI1st (matrix Φ in Figure 4.3) and output DI2nd (matrix Θ in Figure 4.4).

We observe evidence of interdependencies. First, we consider 23 AEs and 11 EMEs. We find events in both AEs and EMEs can spill over into AEs, contemporaneously and with a lag. By contrast, there are only several spillovers into EMEs. Second, we allow for both static interdependencies and dynamic interdependencies. We find more evidence of SIs than DIs. We observe considerable evidence of SIs from Figure 4.1 and Table C.1. They provide evidence that some countries' information is treated as important as own country information, and there is a common factor that drives all countries' inflation (output). However, most off-diagonal elements, in DI figures (Figure 4.2-4.4), are close to zero. Several non-zero off-diagonal elements seem to support that DIs occur within AEs and within EMEs. We describe the details of SIs and DIs below.

The first evidence of SIs comes from matrix A in Figure 4.1. Matrix A is the coefficient matrix of output gap in the inflation gap equations. It links all countries' inflation gap and output gap together and measures the Phillips SIs. The diagonal element is the slope of own country Phillips curve. Except the diagonal elements, we find some off-diagonal elements have a value that is quite comparable to the diagonal elements. This implies that our model deems these information (from other countries) as important as own country information. "Belgium" row to "Canada" row show the spillovers into AEs. We find events in both AEs and EMEs can spill over into AEs. However, in terms of spillovers into EMEs (see "South Africa" row to "Thailand" row), events in EMEs can spill over into EMEs. There is only one small degree of positive spillover from AEs into EMEs, from USA into China. Except this, we do not see other evident spillovers from AEs into EMEs. The second evidence of SIs comes from factor loading matrices L_π and L_y (see Table C.1). We find the posterior means of factor loading matrices are quite large and 84% credible intervals do not include zero, supporting that 34-country inflation are driven by a common factor f_t and that 34-country output are driven by a common factor g_t.

With respect to DIs, we have three matrices (P, Θ and Φ) measuring DIs and we find the three matrices have different characteristics. Figure 4.2 plots the estimates of matrix P. Matrix P is associated with the lag of inflation gap and measures inflation DIs. First, we find the diagonal cells for EMEs are darker green than the diagonal cells for AEs, which implies that EMEs exhibit a higher inflation gap persistence, while AEs exhibit a lower inflation gap persistence. In other words, inflation process in AEs is no longer adaptive (see, Cogley and Sargent, 2005; Stock and Watson, 2007 and Chan et al., 2016). While expectation formation in EMEs is more adaptive. Second, several non-diagonal elements are green, describing inflation DIs. Most spillovers are positive (green cells) except that events in France have a small negative effect on inflation in Turkey (red cell).

Figure 4.3-4.4 plot the output DIs, which are measured by matrices Φ and Θ. The two matrices are the coefficients in output gap equation. Figure 4.3 the estimates of matrix Φ, which is associated with the first lag of output gap and measures output DI1st. We notice that the output DI1st occurs within AEs and within EMEs, while we do not observe evident output DI1st between AEs and EMEs.

Figure 4.4 plots Θ, which is associated with the second lag of output gap and measures output DI2nd. Non-diagonal elements also describe spillovers, but it has a different pattern from Φ. “Belgium” row to “Canada” row show the spillovers into AEs. We find events in both AEs and EMEs can spill over into AEs. However, in terms of spillovers into EMEs (see “South Africa” row to “Thailand” row), there is only one small degree of negative spillover into EMEs, from Switzerland into Thailand. Except this, we do not see other evident spillovers into EMEs.

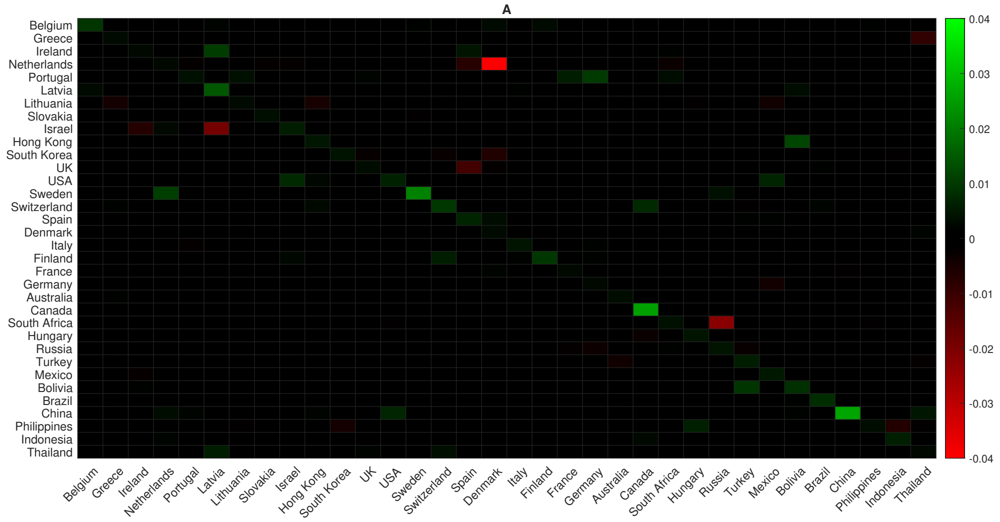


Figure 4.1: **Heatmap of posterior means for the coefficients of output gap in inflation equations.** The Y axis is the left hand in the equation. The X axis is the right hand in the equation. Then the Y axis denotes the 34-country inflation equations (this quarter), that is, $\pi_t - \tau_t^{\pi}$. The X axis denotes the 34-country output gap (this quarter), that is, $y_t - \tau_t^{y}$.

Impulse Response Analysis

The preceding discussion suggests the existence of interdependencies. An excellent aspect of panel model is that, after allowing for interdependencies, it can model the manner in which shocks are transmitted across countries (see Dees et al., 2007 and Canova and Ciccarelli, 2009). We will show that PUC-FSV can consider the effects of both variable-specific shocks and, more importantly, global shocks. With regard to global shocks, as pointed out by Dees et al. (2007), it is possible to view US shock as global shock in the case of a US equity market shock, but it might be less defensible for other types of shocks. Therefore, it might be desirable to consider the effects of global shocks which might not necessarily originate from a particular country, but could be common to the world economy as a whole. Examples of such shocks include major developments in technology. By using the factor stochastic volatility (FSV) specification, the proposed PUC-FSV directly allows us to consider the effects of global shocks.

Since PUC-FSV allows for dynamic dependencies, static dependencies and time varia-

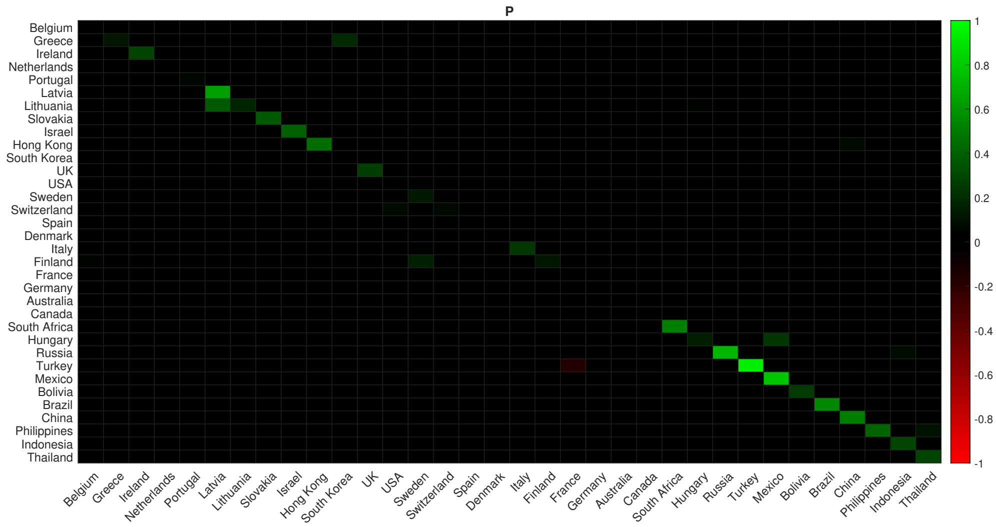


Figure 4.2: **Heatmap of posterior means for the coefficients of inflation gap lag.** The Y axis is the left hand in the equation. The X axis is the right hand in the equation. Then the Y axis denotes the 34-country inflation equations (this quarter), that is, $\pi_t - \tau_t^{\pi}$. The X axis denotes the 34-country inflation gap (last quarter), that is, $\pi_{t-1} - \tau_{t-1}^{\pi}$.

tion in the parameters (e.g., stochastic volatility), we are interested in computing the responses of the endogenous variables to shocks in the variables/global factors and in describing their evolution over time. In this situation, we use the Generalized Impulse Response Functions (GIRFs), proposed in Koop et al. (1996).

With 68 endogenous variables (34 countries and 2 variables of each country) and time-varying parameters, there will be a different set of generalised impulse response functions (GIRFs) at each time in the sample period. However, for our study, we focus on the GIRFs for the end of sample period (2018Q1). we investigate the implications of four different shocks: (a) a 1-standard-deviation positive shock to US inflation; (b) a 1-standard-deviation positive shock to US output; (c) a 1-standard-deviation positive shock to global inflation; and (d) a 1-standard-deviation positive shock to global output.

More specifically, the computation of generalised impulse response function in Koop et al. (1996) is: given the posterior draws, the GIRF is obtained from the difference between two alternative paths: in one a shock hits the system, and in the other this

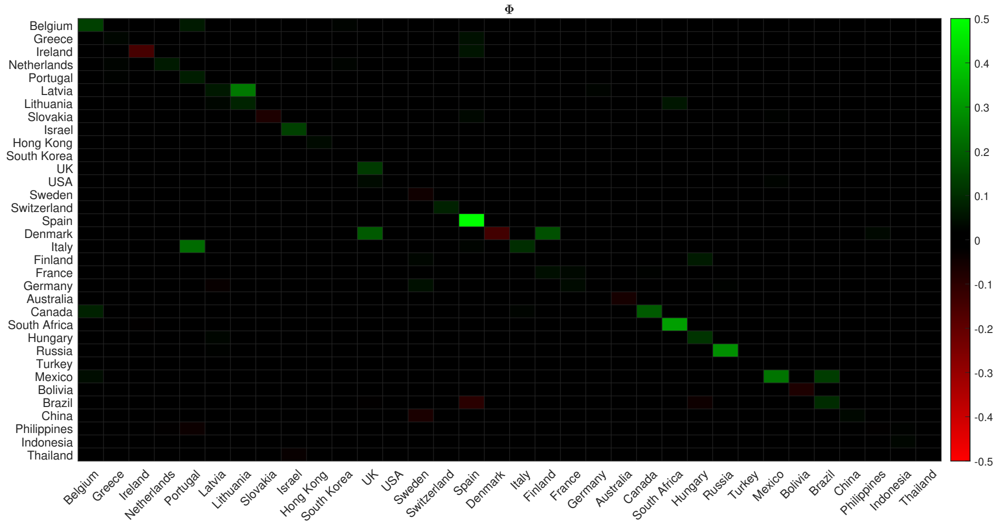


Figure 4.3: **Heatmap of posterior means for the coefficients of output gap lag.** The Y axis is the left hand in the equation. The X axis is the right hand in the equation. Then the Y axis denotes the 34 output equations (this quarter), that is, $y_t - \tau_t^y$. The X axis denotes the first lag of 34-country output gap (last quarter), that is, $y_{t-1} - \tau_{t-1}^y$.

shock is absent:

$$GIRF_{t+k} = \mathbb{E}[\mathbf{z}_{t+k}|\mathbf{u}_t, \mathbf{I}_t] - \mathbb{E}[\mathbf{z}_{t+k}|\mathbf{I}_t] \tag{4.17}$$

where $\mathbf{z}_{t+k}$ is the forecast of the endogenous variables at the horizon k, $\mathbf{I}_t$ represent the current information set and $\mathbf{u}_t$ is the current structural disturbance terms. The computation of the generalised impulse response functions for a horizon k can be summarised in 4 steps:

Step1 : We first draw parameters from the posterior distributions within the Gibbs Sampler.

Step2 : We rewrite the errors in Equation (4.7). Suppose u_t is a vector and each element in u_t follows standard normal distribution $\mathcal{N}(0,\ 1)$, then:
$u_t^\pi = \Sigma_t^\pi u_t$ where $\Sigma_t^\pi = \mathrm{diag}(e^{h_{1,t}}, \ldots, e^{h_{N,t}})$,
$u_t^y = \Sigma_t^y u_t$ where $\Sigma_t^y = \mathrm{diag}(e^{h_{N+1,t}}, \ldots, e^{h_{N+N,t}})$,

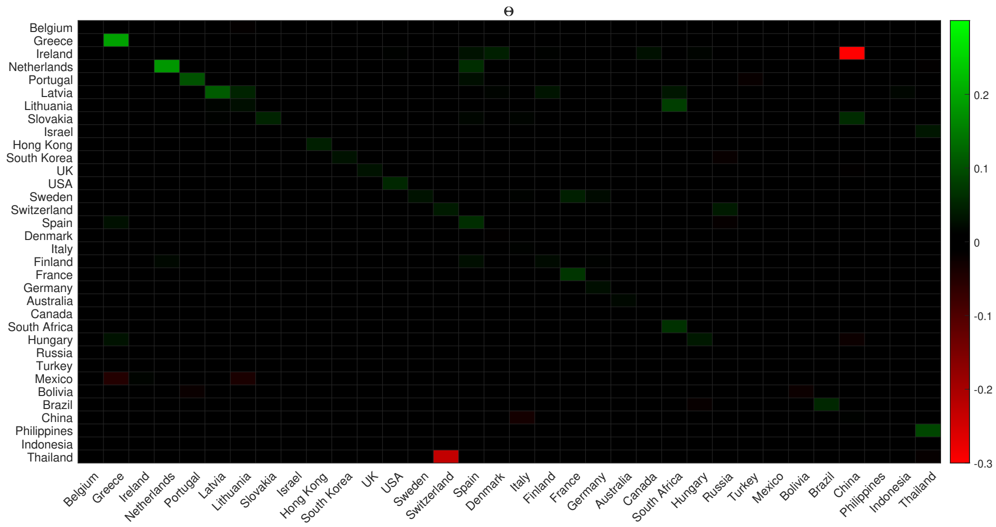


Figure 4.4: **Heatmap of posterior means for the coefficients of output gap lag.** The Y axis is the left hand in the equation. The X axis is the right hand in the equation. Then the Y axis denotes the 34 output equations (this quarter), that is, $y_t - \tau_t^y$. The X axis denotes the second lag of 34-country output gap (the quarter before last), that is, $y_{t-2} - \tau_{t-2}^y$.

$$f_t = \Omega_t^\pi u_t \text{ where } \Omega_t^\pi = \text{diag}(e^{h_{2N+1,t}}, \ldots, e^{h_{2N+r_\pi,t}}),$$
$$g_t = \Omega_t^y u_t \text{ where } \Omega_t^y = \text{diag}(e^{h_{2N+r_\pi+1,t}}, \ldots, e^{h_{2N+r_\pi+r_y,t}}).$$

Step3 : We draw u_t from the standard normal distribution, then generate two paths: one with the shock and the other without shock. For the latter case, we just compute the errors in *Step2* using u_t and then stochastically simulate a random path of length k using the coefficients drawn from *Step1*. For the former case, we set $u_{i,t}$ to the corresponding shock that we are interested in. For example, say we are interested in USA inflation shock (we consider 34 countries and the order for USA is 13), then the disturbance term will be $\tilde{u}_t = (u_{1,t}, u_{2,t}, \ldots, u_{13,t} + 1, \ldots, u_{N,t})'$. Thus, we compute the errors in *Step2* using $\tilde{u}_t$ and then stochastically simulate another random path of length k. If we are interested in global inflation shock (we consider one global factor), then the disturbance term will be $\tilde{u}_t = u_t + 1$.

Step4 :To compute the impulse response function, we take difference between the two paths.

We report the generalised impulse response functions in Figure 4.5-4.10. Figure 4.5 is about the US inflation shock, Figure 4.6-4.7 are about the US output shock, Figure 4.8 is about global inflation shock and Figures 4.9-4.10 are about global output shock. A quick visual inspection shows that the GIRFs go back to zero over the next 20 quarters and most GIRFs settle down quickly, possibly resulting from the stability condition we impose on the coefficient matrices (see Equation (4.7)).

We observe two notable differences. The first difference is between US shocks and global shocks. Figure 4.5-4.7 are about US shocks, while Figure 4.8-4.10 are about global shocks. The GIRFs are much larger when a global shock hits the system than when a US shock hits the system. We think this is because almost all countries load on the global factors and the loadings are quite large (see Appendix C.2). This provides evidence that it is important to allow for SIs (static interdependencies) as is done in this paper, where we assume that the covariance matrices are driven by latent factors. This possibly further confirms that it is not defensible to view US inflation shock as global shock (see Dees et al., 2007).

The second difference is the GIRFs of inflation between AEs and EMEs. Whatever the shock is, the GIRFs of EMEs inflation go back to zero more slowly than AEs, which implies that a shock has on average a longer-term effect on EMEs inflation. We call this “fragile inflation”EMEs. We also observe some other differences among different shocks and we describe them below.

US Inflation Shock:

Figure 4.5 report the GIRFs of 34-country inflation to a 1-standard-deviation increase in US inflation. We have in total 68 endogenous variables (34-country inflation and 34-country output), but We find that, from Equation (4.7), output only depends on the lag of output and does not depend on inflation. This means that US inflation shock will affect 34-country inflation, while does not affect 34-country output, so we only plot the GIRFs of 34-country inflation to a 1-standard-deviation increase in US inflation over the next 20 quarters. We report the posterior means and the 84% credible intervals.

US inflation increases by 0.4 on impact, and it quickly goes back to zero, which implies that US inflation shock has on average only a short-term effect on US inflation. For the cross-country spillovers, we observe some differences across the remaining 33 countries. First, US inflation shock has positive effects on 11 countries (Belgium, South Korea, Sweden, Switzerland, Italy, Finland, France, Australia, Mexico, Bolivia and China). In these 11 countries, the strongest affect occurs in Switzerland. The mean impact is 0.03. Second, US inflation shock has negative effects on 13 countries (Ireland, Netherlands, Latvia, Lithuania, Israel, Spain, Denmark, Germany, Canada, South Africa, Hungary, Turkey and Thailand) and the absolute effects are quite small (absolute values are smaller than 0.003). Finally, the GIRFs of the remaining 9 economies (Greece, Portugal, Slovakia, Hong Kong, UK, Russia, Brazil, Philippines and Indonesia) oscillate between positive and negative territories and then converge back to zero.

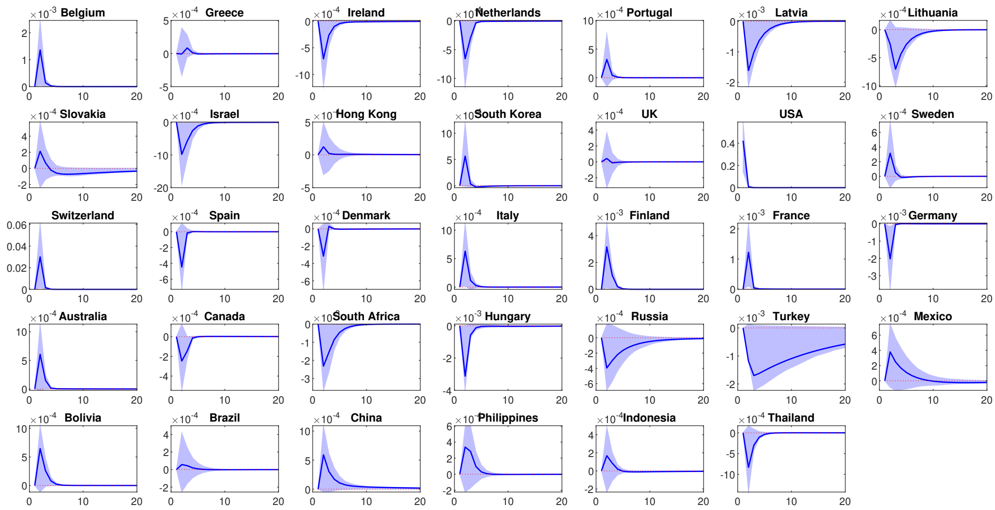


Figure 4.5: **The mean generalised impulse responses of inflation to a 1-standard-deviation increase in US inflation rate over the next 20 quarters.** The shaded areas indicate the 84% posterior credible intervals.

US Output Shock:

Figure 4.6-4.7 report the GIRFs of 34-country inflation and output to a 1-standard-deviation increase in USA output respectively. We report the posterior means and the 84% credible intervals.

Figure 4.6 reports the GIRFs of 34-country inflation to a 1-standard-deviation increase in US output. US inflation increases by 0.01 on impact, and it quickly goes back to zero, which implies that US output shock has on average only a short-term effect on US inflation. For the cross-country spillovers, we find the largest impact on China inflation, which is close to 0.01 at the beginning and takes almost 10 quarters to go back to zero. The large and immediate impact provides evidence of interdependencies.

Figure 4.7 reports the GIRFs of 34-country output to a 1-standard-deviation increase in US output. US output increases by 1.7 on impact, and it quickly goes back to zero, which implies that US output shock has a large but short-term effect on US output. For the cross-country spillovers, we find US output shock has a positive effect on most AEs, while US output shock has a negative effect on most EMEs. This provides evidence of dependencies between AEs, and more importantly, between AEs and EMEs. Imposing the cross-sectional homogeneity restriction may lead to mis-specification.

Global Inflation Shock:

Figure 4.8 reports the generalised impulse response functions of 34-country inflation to a 1-standard-deviation increase in global inflation. The impacts are unambiguously positive in all countries. The GIRFs go back to zero over the next 20 quarters, except one country, Turkey. The GIRFs in Turkey decrease slowly and arrives at 0.76 after 20 quarters.

Global Output Shock:

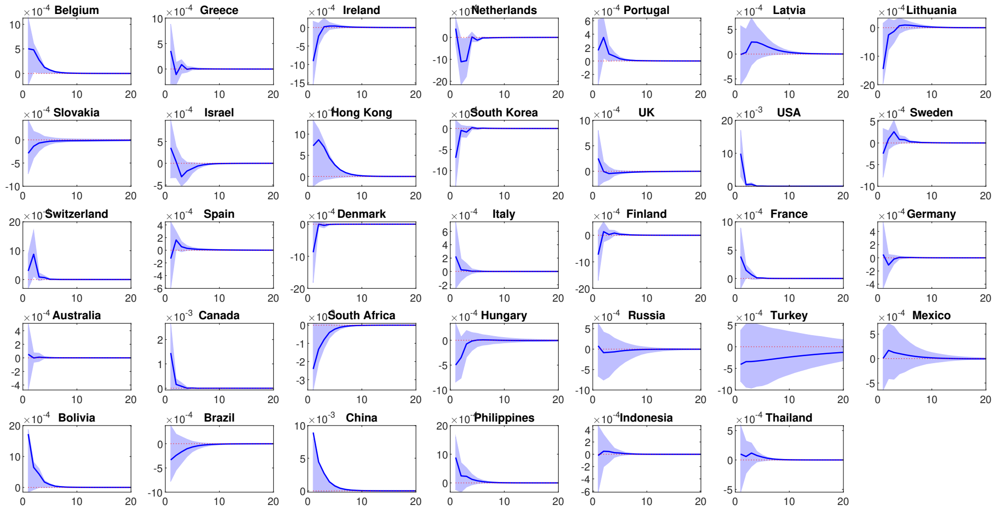


Figure 4.6: **The mean generalised impulse responses of inflation to a 1-standard-deviation increase in US output over the next 20 quarters.** The shaded areas indicate the 84% posterior credible intervals.

Figures 4.9 and 4.10 report the generalised impulse response functions of inflation and output to a 1 standard deviation increase in global output. Similar to the impacts of global inflation shock on 34-country inflation, the impacts of global inflation shock on output are positive and quickly goes back to zero. By contrast, the impacts on inflation need more time to converge back to zero compared to the impacts on output.

4.4 Estimates of Trends

Figure 4.11-4.12 plot the posterior estimates for trend inflation and trend output respectively, estimated using the full sample. We consider 34 countries: the first 23 countries are AEs (from Belgium to Canada) and the following 11 countries are EMEs (from South Africa to Thailand). We compare three models: (a) Bi-UC-SV (This is similar to the bivariate UC model that is used in Stella and Stock (2013), and Chan et al. (2016). They allow inflation persistence and the slope of Phillips curve to be time varying. However, we do not allow them to be time varying, that is, inflation persistence and the slope of Phillips curve are constant. We have allowed for stochastic volatility in inflation equations); (b) MC-UC-FSV (the model in Wu, 2021. The

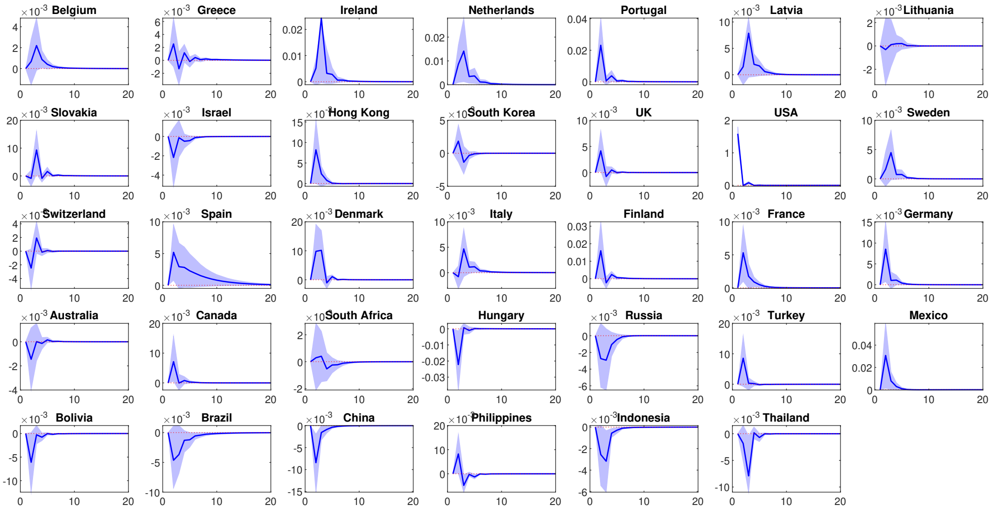


Figure 4.7: **The mean generalised impulse responses of output to a 1-standard-deviation increase in US output over the next 20 quarters.** The shaded areas indicate the 84% posterior credible intervals.

number of common factors is set to one, that is, $r_\pi = 1$, $r_y = 1$); and (c) PUC-FSV model. The differences are: the Bi-UC-SV model does not allow for any interdependencies across countries, the MC-UC-FSV model allows for cross-country linkages in the error covariance matrix, while the PUC-FSV model allows for cross-country linkages both in the error covariance matrix and in conditional mean. In both figures, the solid blue lines are the posterior means under PUC-FSV. The dotted blue lines are the 16% and 84% quantiles under PUC-FSV, the solid black lines are posterior means under MC-UC-FSV, while the solid red lines are posterior means under Bi-UC-SV.

Figure 4.11 plots the posterior estimates for trend inflation τ^π. The title of each sub-figure is the country name, followed by the official inflation targets (point target or target bands). For example, the title of the first sub-figure is “Belgium (2) ”, then the first sub-figure depicts the estimates of trend inflation for Belgium and the official inflation target set by Belgium central bank is 2%.

The broad contours reflected in the posterior mean from the three models are similar. However, we observe some interesting differences. First, we find that both the

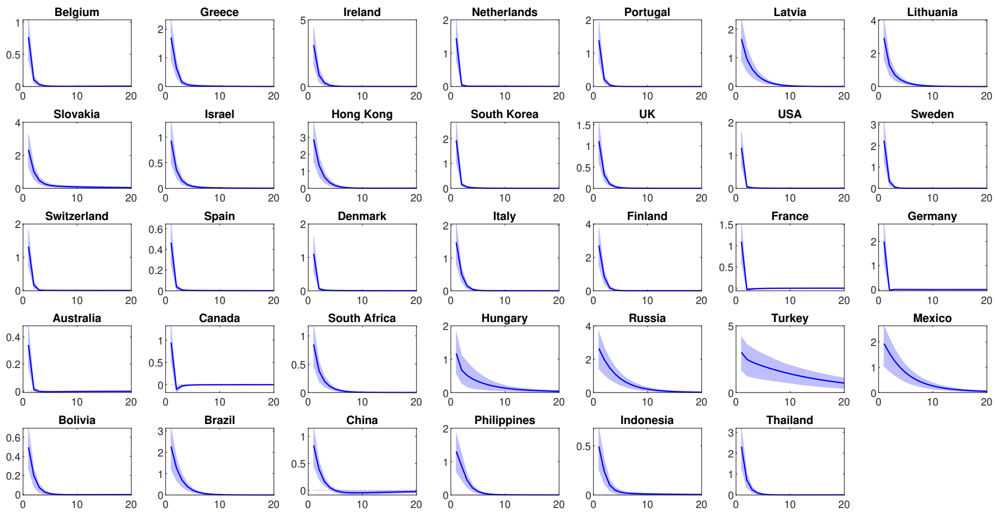


Figure 4.8: **The mean generalised impulse responses of inflation to a 1-standard-deviation increase in global inflation over the next 20 quarters.** The shaded areas indicate the 84% posterior credible intervals.

MC-UC-FSV model and the PUC-FSV model can provide narrower width than the Bi-UC-SV model. The precision of the estimates, as measured by the width of the 84% credible intervals. This indicates that allowing for cross-country linkages in the error covariance matrix can provide more precise estimates of trend inflation. We report the width of the 84% credible intervals in Appendix C.1.

Second, we find that allowing for cross-country linkages has more effects on AEs than on EMEs. This is in line with the analysis of cross-country spillovers (Through analyzing P, A, Φ and Θ, we find evident spillovers into AEs, while only several spillovers into EMEs. See Section 4.3.2). We take the US for example. The official inflation target is 2%. Trend inflation from the model without any cross-country interdependencies (that is the Bi-UC-SV model, plotted using solid red lines) increases to 2.5% in 2000s and has decreased to 1.8% in 2015. However, our PUC-FSV model, which allows for cross-country interdependencies (solid blue lines), chooses to estimate trend inflation as a slight increase to 2.3%in 2000s. After the financial crisis has hit, trend inflation decreases to 1.9% in 2010 and then it begins to recover (that is, increase). The effect of cross-country interdependencies in the conditional mean is analyzed by

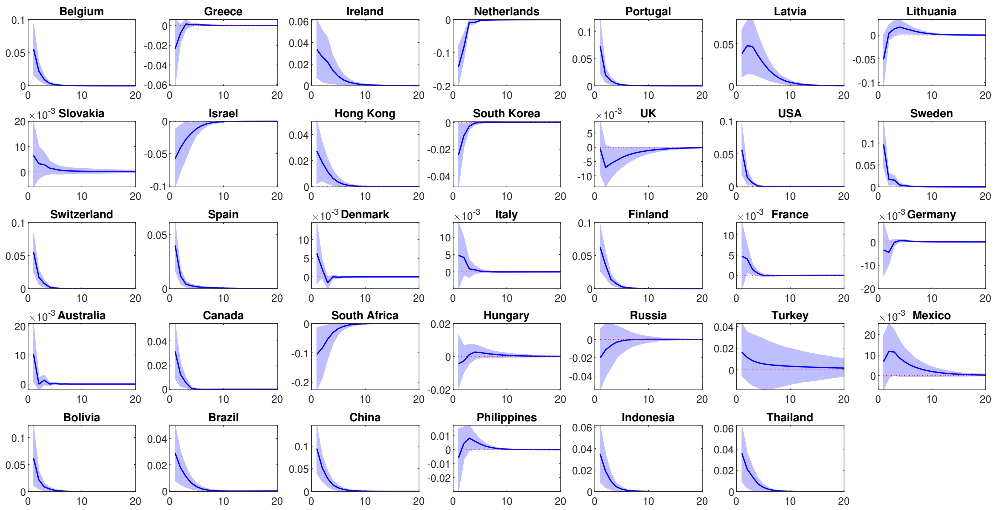


Figure 4.9: **The mean generalised impulse responses of inflation to a 1-standard-deviation increase in global output over the next 20 quarters.** The shaded areas indicate the 84% posterior credible intervals.

comparing the PUC-FSV (the solid blue line) to the MC-UC-FSV (the solid black line). The effect is observed before 2002 and allowing for cross-country interdependencies in conditional mean will generate a higher estimates of trend inflation. Trend inflation under PUC-FSV fluctuates in a range between 2.0% and 2.3%, while trend inflation under MC-UC-FSV fluctuates in a range between 1.9% and 2.2%.

Figure 4.12 plots the posterior estimates for trend output τ^y. First, a similar pattern to trend inflation is observed. We find that both the MC-UC-FSV and the PUC-FSV can provide narrower width than the Bi-UC-SV. This indicates that allowing for cross-country linkages in the error covariance matrix can provide more precise estimates of trend output. We report the width of the 84% credible intervals in Appendix C.1. Second, a general effect of cross-country linkages in the conditional mean is observed. Note that the MC-UC-FSV model allows for cross-country linkages in the error covariance matrix, while the PUC-FSV model allows for cross-country linkages both in the error covariance matrix and in the conditional mean. We find the estimates of trend output under MC-UC-FSV (the solid black line) is higher than the estimates under PUC-FSV (the solid blue line). Although the two models can both provide precise

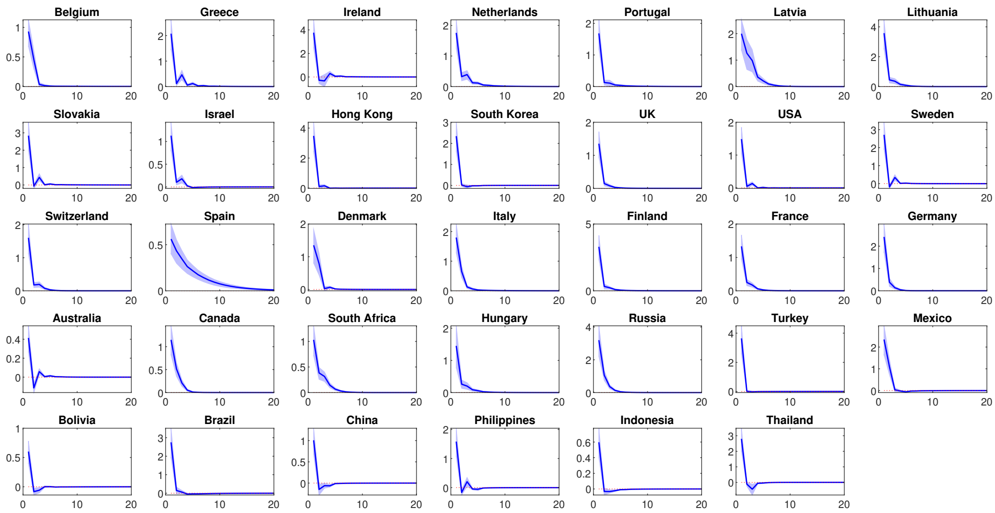


Figure 4.10: **The mean generalised impulse responses of output to a 1-standard-deviation increase in global output over the next 20 quarters.** The shaded areas indicate the 84% posterior credible intervals.

estimates, we find the MC-UC-FSV provides a lower model fit than the PUC-FSV (as shown in Table 4.1, 885.51 against 918.44). This suggests that the PUC-FSV fits the data better and omitting cross-country interdependencies in the conditional mean will overestimate trend output.

4.5 Model Comparison

We divide this section into two sub-sections. In both sub-sections, we compare our PUC-FSV to the Bi-UC-SV (the model in Chan et al., 2016. The coefficients are constant, but we allow for stochastic volatility in inflation gap equations) and the MC-UC-FSV (the model in Wu, 2021. The number of common factors is set to one, that is, $r_\pi = 1$, $r_y = 1$). The first sub-section, section 4.5.1, reports the in-sample fit results. The second sub-section, section 4.5.2, reports the out-of-sample forecasting results.

4.5.1 In-sample fit

The gold standard is using marginal likelihood, however, in our settings where we allow for time-variation in volatility, the computation of marginal likelihood requires integrating out all the states, making it a nontrivial task. Therefore, we use an approximation

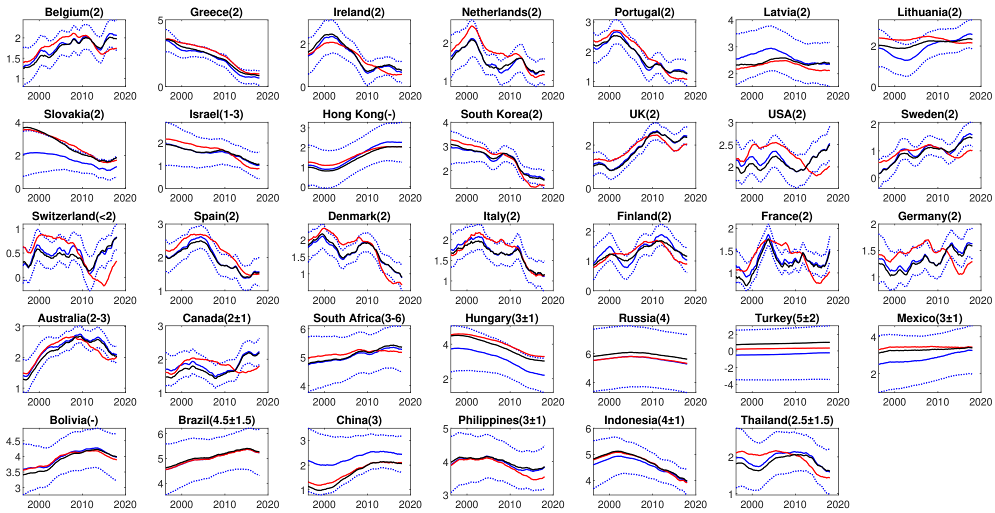


Figure 4.11: **Posterior estimates for trend inflation τ^{π}.** The title of each subfigure is the country name, followed by the official inflation targets (point target or target bands). For Hong Kong and Bolivia, we do not find the official inflation targets, so we use "- ". The solid blue lines are the posterior means under PUC-FSV. The dotted blue lines are the 16% and 84% quantiles under PUC-FSV, the solid black lines are posterior means under MC-UC-FSV, while the solid red lines are posterior means under Bi-UC-SV.

to the marginal likelihood (e.g., Geweke, 2001; Cross et al., 2020). They propose that conditioning on the estimation period, the sums of one-step-ahead joint log predictive likelihoods of 34 countries can be viewed as an approximation to the marginal likelihood, therefore provides a direct measure of in-sample fit.

Before computing the the sums of one-step-ahead joint log predictive likelihoods, we need to define some basics. Let $\hat{y}_{t+k}^{(i,j)}$ denote, at time t, the k-step-ahead forecast of the j-th variable in the i-th country, and $y_{t+k}^{(i,j)}$ denote the actual value. In our empirical work, $i = 1, \ldots, N$ with $n = 34$, $j = 1, 2$ where $j = 1$ denote inflation and $j = 2$ denote output. $\mathrm{Y}_{1:t}^{(i,j)}$ stores the data up to time t, so $\hat{y}_{t+k}^{(i,j)} = \mathrm{E}\,(y_{t+k}^{(i,j)} \mid \mathrm{Y}_{1:t}^{(i,j)})$. Then we compute the k-step-ahead log predictive likelihoods (LPL) of the j-th variable in the i-th country at time t:

$$\mathrm{LPL}_{t,i,j,k} = \log\, p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)}),\ t = T_0, \ldots, T-k$$

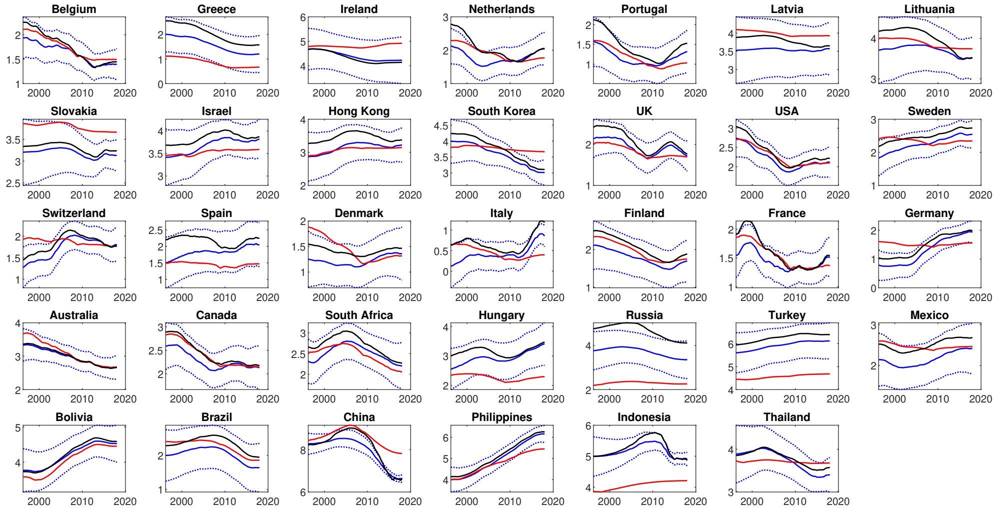


Figure 4.12: **Posterior estimates for trend inflation τ^y.** The solid blue lines are the posterior means under PUC-FSV. The dotted blue lines are the 16% and 84% quantiles under PUC-FSV, the solid black lines are posterior means under MC-UC-FSV, while the solid red lines are posterior means under Bi-UC-SV.

Then the sums of one-step-ahead joint log predictive likelihoods is computed using:

$$\text{LPL}_{\cdot,\cdot,\cdot,1} = \sum_{t=T_0}^{T-1} \sum_{i=1}^{n} \sum_{j=1}^{2} \log p(\hat{y}_{t+1}^{(i,j)} = y_{t+1}^{(i,j)} | \text{Y}_{1:t}^{(i,j)})$$

Our estimation period starts from 1995Q4 (to 2018Q1), and the forecasting evaluation period starts from 2003Q1. We provide the sums of one-step-ahead joint log predictive likelihoods of 34 countries in Table 4.1.

In Table 4.1, results are presented relative to the forecast performance of the Bi-UC-SV: we take differences, so that a positive number indicates a model is forecasting better than Bi-UC-SV. (Please note that we only take the sum, and no average. That may be why the number seems so large. For example, the sums of LPL under PUC-FSV is 918.44. If we take average over time, then it is 15.06. If we take further average across country, then it is 0.44). The results show that the PUC-FSV provides the highest model fit.

Table 4.1: Sum of one-step-ahead log predictive likelihood

Model	against Bi-UC-SV
Bi-UC-SV	0
MC-UC-FSV	885.51
PUC-FSV	918.44

4.5.2 Out-of-sample Forecasting

Having shown that PUC-FSV provides competitive in-sample fit, we now compare the out-of-sample forecast performance of the three models. We use the data from 1995Q4 to 2002Q4 as an initial estimation period, and use data through 2002Q4 to produce k-step-ahead forecast distributions. We consider forecast horizons of $k = 1, 2, 3, 4, 6$ quarters. So our forecast evaluation period begins in 2003Q1. We divide our out-of-sample forecasting results into three parts: forecasting inflation, forecasting output and jointly forecasting inflation and output. For each part, we discuss the results in three dimensions. The first dimension is aggregate forecasting performance over time and over countries (the aggregate LPL, by summing all countries and all time periods). The second dimension is about forecasting performance over time (we can study how the sums of LPL changes over time, by summing all countries at time t). After providing evidence that PUC-FSV can produce more accurate estimate in economic recession, we further study whether such good forecast performance is driven by particular countries, so the third dimension is about the forecasting performance at country level. All results are presented relative to the forecast under Bi-UC-SV: we take differences, so a positive number indicates a model is forecasting better than Bi-UC-SV.

Forecasting inflation

We first report the aggregate forecasting performance for inflation over time and over countries in Table 4.2. It is calculated by summing the LPL for the N countries over

T_0 to $T-k$ (and recall that $j=1$ denote inflation):

$$\text{LPL}_{\cdot,\cdot,1,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \log p(\hat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \text{Y}_{1:t}^{(i,1)})$$

First, the positive values provide evidence that PUC-FSV forecasts inflation more accurately at all horizons. Second, except the case when $k=1$, PUC-FSV provides the most accurate forecast of inflation at other (longer) horizons.

Table 4.2: Sum of k-step-ahead log predictive likelihood for 34-country inflation

Model	k=1	k=2	k=3	k=4	k=6
Bi-UC-SV	0	0	0	0	0
MC-UC-FSV	**124.57**	210.18	216.28	223.02	286.31
PUC-FSV	114.75	**252.52**	**280.23**	**286.59**	**387.72**

The second dimension of discussion for inflation is sums of LPL over time (by summing all countries at time t), which can be calculated by:

$$\text{LPL}_{t,\cdot,1,k} = \sum_{i=1}^{n} \log p(\hat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \text{Y}_{1:t}^{(i,1)})$$

We plot the results (against the Bi-UC-SV) in Figure 4.13. If the period to be forecasted is the period of uncertainty (like 2008), the MC-UC-FSV and the PUC-FSV provide overall good forecast performance at all horizons. And the PUC-FSV forecasts better than MC-UC-FSV at long horizons ($k=4$ and $k=6$).

The third dimension of discussion for inflation is the forecasting result for individual countries. The LPL of inflation for country i at time t, which can be calculated by:

$$\text{LPL}_{t,i,1,k} = \log p(\hat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \text{Y}_{1:t}^{(i,1)})$$

We plot the results (against Bi-UC-SV) in Figure 4.14. Here the period of uncertainty that we plot is 2008Q4, so the period to be forecasted is 2008Q4 ($t+k = 2008Q4$). If $k=1$, then the time we make forecast is 2008Q3, and we find overall good forecast performance for most countries with more pronounced gains in advanced economies

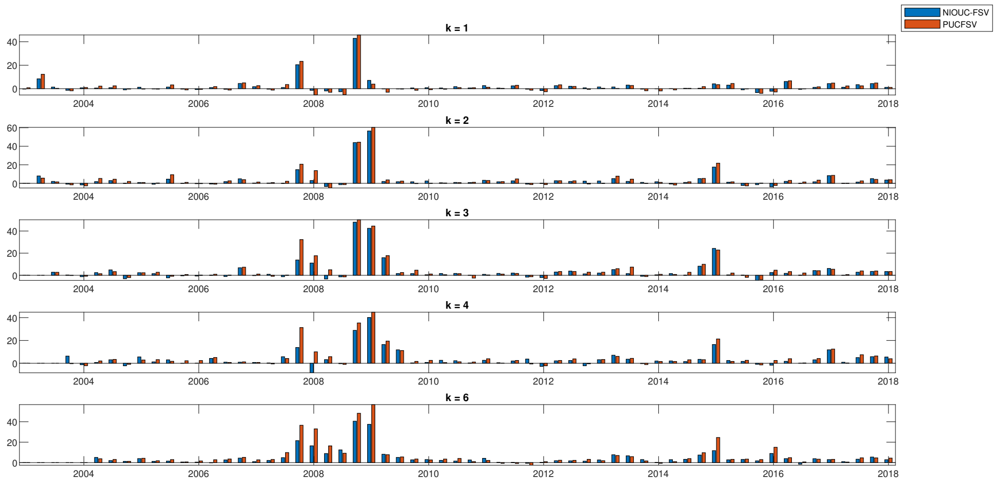


Figure 4.13: **Sums of k-step ahead LPL of inflation for PUC-FSV and MC-UC-FSV relative to Bi-UC-SV over time.** The X axis is $t+k$ and represents the period being forecasted.

(The first 23 countries are AEs, and the following 11 countries are EMEs). A similar pattern is found if $k=6$. The time we make forecast is 2007Q2, and we also find overall good forecast performance for most countries. In Figure 4.14, we only plot the shortest horizon $k=1$ and the longest horizon $k=6$, for middle horizons ($k=2,3,4$), we find good forecasting result across most countries and did not find particular country which is important in driving good forecasting results. Overall, We find good forecast performance for most countries and such good forecast performance is not driven by particular countries.

Forecasting output

With regard to output, we report the sums of LPL of output over time and over countries in Table 4.3. It is calculated by summing the LPL for the N countries over T_0 to $T-k$ (and recall that $j=2$ denote output):

$$\mathrm{LPL}_{\cdot,\cdot,2,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \log p(\hat{y}_{t+k}^{(i,2)} = y_{t+k}^{(i,2)} | \mathrm{Y}_{1:t}^{(i,2)})$$

The results show that PUC-FSV provides the most accurate forecast for output at all horizons.

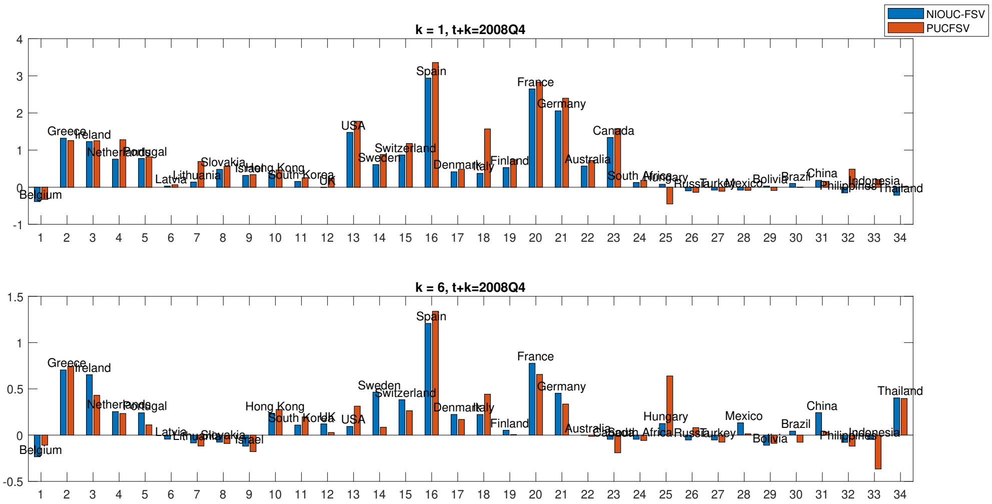


Figure 4.14: **Sums of k-step ahead LPL of inflation for country i under PUC-FSV and MC-UC-FSV relative to Bi-UC-SV.**

Table 4.3: Sum of k-step-ahead log predictive likelihood for 34-country output

Model	k=1	k=2	k=3	k=4	k=6
Bi-UC-SV	0	0	0	0	0
MC-UC-FSV	750.25	979.29	969.58	830.92	942.70
PUC-FSV	**791.86**	**1010.76**	**1010.26**	**1041.74**	**1112.64**

Similar to the analysis of inflation, the second dimension of discussion for output is sums of LPL over time (by summing all countries at time t), which can be calculated by:

$$\mathrm{LPL}_{t,\cdot,2,k} = \sum_{i=1}^{n} \log\, p(\hat{y}_{t+k}^{(i,2)} = y_{t+k}^{(i,2)} | \mathrm{Y}_{1:t}^{(i,2)})$$

We plot the results (against Bi-UC-SV) in Figure 4.15. If the period to be forecasted is the period of uncertainty (like 2008), the MC-UC-FSV and the PUC-FSV provide overall good forecast performance at all horizons.

To investigate whether the good forecast performance is driven by particular coun-

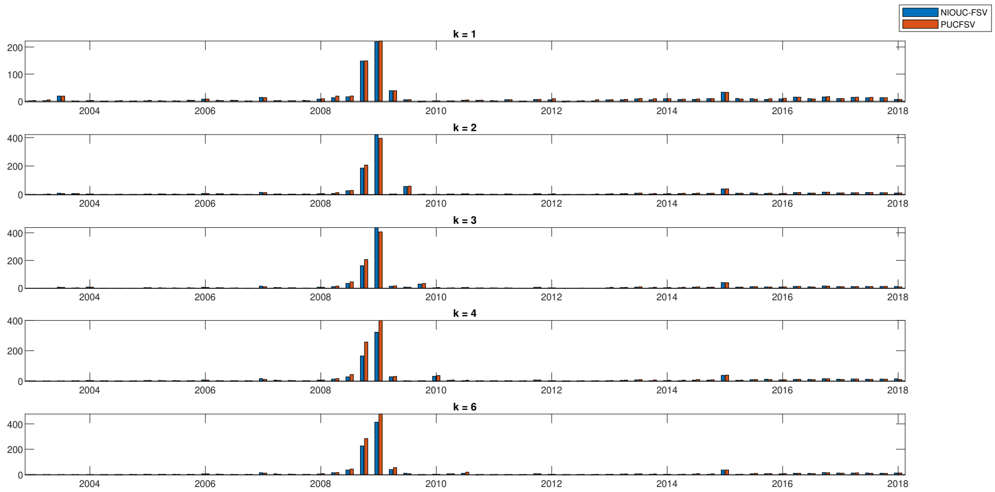


Figure 4.15: **Sums of k-step ahead LPL of output for PUC-FSV and MC-UC-FSV relative to Bi-UC-SV over time.** The X axis is $t + k$ and represents the period being forecasted.

tries, we calculate the sums of LPL of output for country i at time t by:

$$\text{LPL}_{t,i,2,k} = \log p(\hat{y}^{(i,2)}_{t+k} = y^{(i,2)}_{t+k} | \text{Y}^{(i,2)}_{1:t})$$

We plot the results (against Bi-UC-SV) in Figure 4.16. We choose 2008Q4 to represent the period of uncertainty. For $k = 1$ and $k = 6$, we both find overall good forecast performance for PUC-FSV and MC-UC-FSV for all countries. For several countries (like Spain, Italy and Germany), PUC-FSV forecasts better than MC-UC-FSV.

Jointly Forecasting inflation and output

With regard to the joint predictive density for inflation and output, we first report the sums of joint LPL over time and over countries in Table 4.4. It is calculated by summing the LPL for the N countries over T_0 to $T - k$ (and for all j, recall that $j = 1$ denote inflation, $j = 2$ denote output):

$$\text{LPL}_{\cdot,\cdot,\cdot,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \sum_{j=1}^{2} \log p(\hat{y}^{(i,j)}_{t+k} = y^{(i,j)}_{t+k} | \text{Y}^{(i,j)}_{1:t})$$

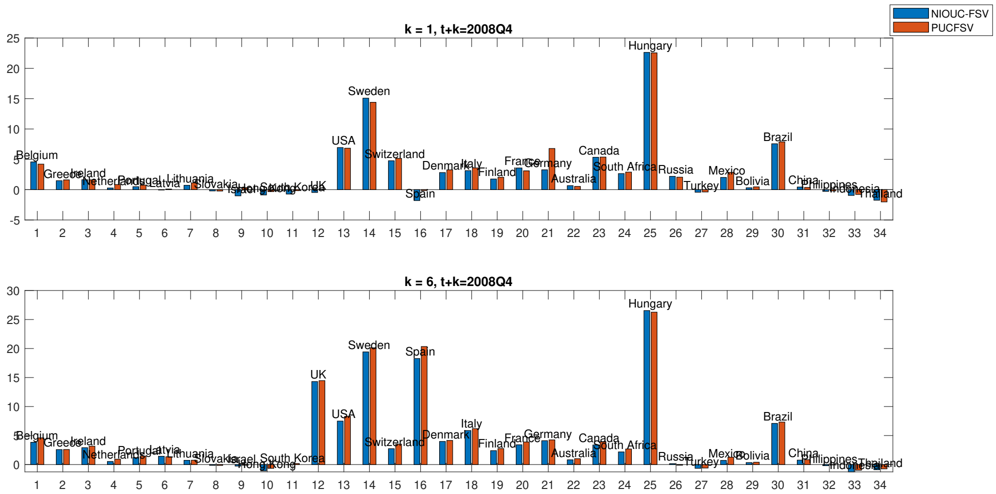


Figure 4.16: **Sums of k-step ahead LPL of output in country i for PUC-FSV and MC-UC-FSV relative to Bi-UC-SV.**

The results show that PUC-FSV provides the most accurate joint forecast for inflation and output at all horizons. Next, we study the time-variation in forecast performance

Table 4.4: Sum of k-step-ahead joint log predictive likelihood for 34-country inflation and output

Model	k=1	k=2	k=3	k=4	k=6
Bi-UC-SV	0	0	0	0	0
MC-UC-FSV	885.51	1111.32	1221.00	1044.97	1093.88
PUC-FSV	**918.44**	**1169.58**	**1291.92**	**1318.78**	**1253.67**

to see whether the benefits arise from the forecast during periods of uncertainty. So the second dimension of discussion for joint predictive density for inflation and output is sums of joint LPL over time (by summing all j and all countries at time t), which can be calculated by:

$$\mathrm{LPL}_{t,\cdot,\cdot,k} = \sum_{i=1}^{n} \sum_{j=1}^{2} \log\, p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)})$$

We plot the results (against Bi-UC-SV) in Figure 4.17. A similar pattern to inflation and output was found. If the period to be forecasted is the period of uncertainty (like 2008), the MC-UC-FSV and the PUC-FSV provide overall good joint forecast performance at all horizons.

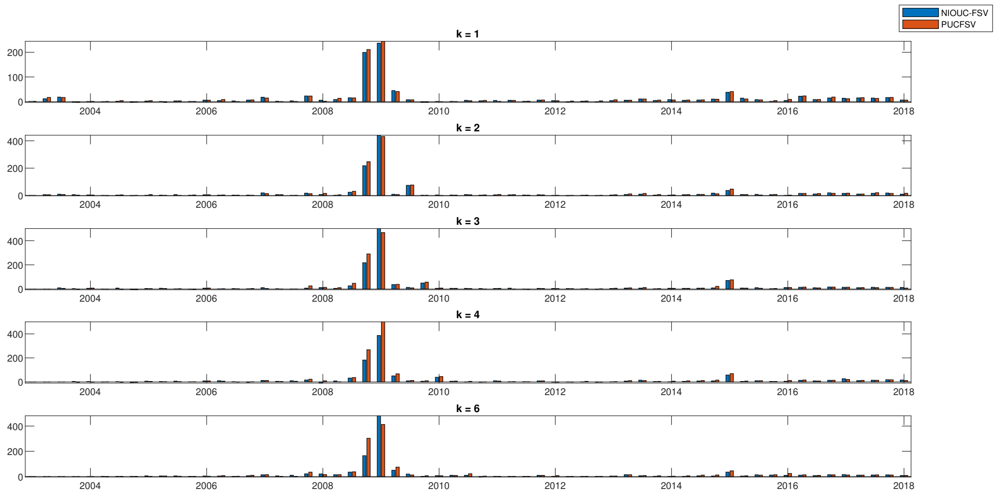


Figure 4.17: **Sums of k-step ahead joint LPL for PUC-FSV and MC-UC-FSV relative to Bi-UC-SV over time.** The X axis is $t+k$ and represents the period being forecasted.

Finally, we investigate whether the good forecast performance of periods of uncertainty is driven by particular countries, so the third dimension of discussion for joint predictive density for inflation and output is sums of joint LPL at the country level (by summing all j for country i), which can be calculated by:

$$\text{LPL}_{t,i,\cdot,k} = \sum_{t=T_0}^{t=T-k} \sum_{j=1}^{2} \log p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \text{Y}_{1:t}^{(i,j)})$$

We plot the results (against Bi-UC-SV) in Figure 4.18. A similar pattern to output is found. (This is sensible since the gains in output are much larger than gains in inflation, see Figure 4.14 and Figure 4.16). We find overall good forecast performance for PUC-FSV for all countries.

4.6 Conclusions

In a globalized world, countries are linked together and cross-country linkages may influence the estimates of trend. However, such influence has not been considered in

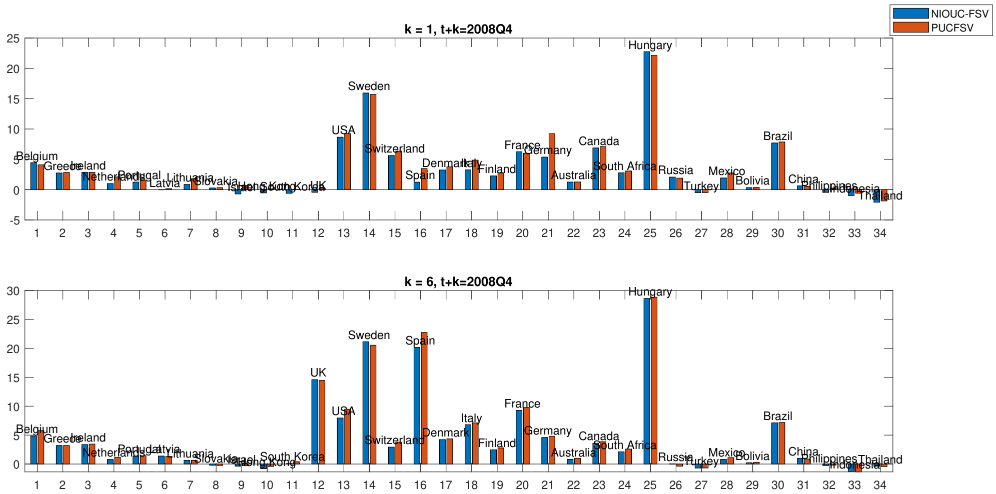


Figure 4.18: **Sums of k-step ahead joint LPL in country i for PUC-FSV and MC-UC-FSV relative to Bi-UC-SV.**

unobserved components models, which are popular to estimate the trend. In this paper, we develop a panel unobserved components model that allows for cross-country linkages both in the error covariance matrix and in the conditional mean. We extend the existing unobserved components literature in three ways. First, we take dynamic interdependencies into account by allowing for cross-country linkages in the conditional mean, more specifically, in the coefficient matrices associated with the lagged variables. Second, we takes static interdependencies into account. This is done through two blocks. One block is allowing for cross-country linkages in the error covariance matrix. Another block is allowing for cross-country linkages in the conditional mean, more specifically, in the coefficient matrices associated with the Phillips curve. By extending the traditional (own country) Phillips curve to global Phillips curve, the PUC-FSV adds a new measure of static interdependencies. Third, we work with unrestricted panel unobserved components model. To deal with over-parameterization concerns, we rely on a global-local shrinkage prior.

We demonstrate the merits of our model through a multi-country study involving 34 countries. The estimates of coefficients and generalised impulse response functions (GIRFs) provide evidence of interdependencies. The estimates of coefficients show that the Horseshoe prior can achieve sensible shrinkage. Events in both AEs and EMEs can spill over into AEs, contemporaneously and with a lag. By contrast, there are only several spillovers into EMEs. We find more evidence of static interdependencies than dynamic interdependencies. The GIRFs are much larger when a global shock hits the system than when a US shock hits the system. The GIRFs also show that whatever the shock is, the GIRFs of EMEs inflation go back to zero more slowly than AEs. We call this “fragile inflation”in EMEs. These features show the power of working with unrestricted model and selecting the appropriate restrictions in a data based manner.

We also present the importance of interdependencies. First, we find that allowing for cross-country linkages in the error covariance matrix can provide more precise estimates of trend, while omitting cross-country interdependencies in conditional mean will overestimate trend output. Second, our proposed model provides a superior in-sample fit and accurate density forecasts compared to existing models in the literature.

Chapter 5

Conclusion

5.1 Summary & policy implications

Globalisation has been accompanied by a weakening in the relationship between domestic slack and domestic inflation, and by a corresponding strengthening in the relationship between global forces and domestic prices (see, Carney, 2017b). Countries, regions or sectors can no longer be treated in isolation and spillovers can take time to affect countries' economies at the policy-relevant horizon, which result in an increasing importance of both static and dynamic interdependencies. (see, Canova and Ciccarelli, 2013 and Carney, 2017b). Additionally, multi-country studies lead to working with high-dimensional models, which suggests that an efficient algorithm is desirable.

This thesis deals with all these three important modeling issues. In three distinct - but increasingly flexible - settings, I show how globalisation can be taken into account through i) introducing observed global factors into bivariate unobserved components models with time-varying parameters and stochastic volatility; ii) allowing for cross-country linkages in the error covariance matrix using factor stochastic volatility specification; iii) allowing for cross-country linkages both in the error covariance matrix and in the conditional mean. Additionally, all these methods do not come at the computational cost. Standard Markov Chain Monte Carlo (MCMC) algorithm can be used to sample all parameters and, more importantly, equation-by-equation estimation can be implemented to most parameters.

In Chapter 2, we extend the country-specific bivariate unobserved components model to the variable-domain. Except domestic factors, we add two observed global factors as additional explanatory variables. We allow the model to have both time-varying parameters and stochastic volatility. We apply these methods to 34 countries covering the period 1995-2018. The results show that there is stochastic volatility in inflation, and it has declined across all countries, but remained relatively high in emerging market economies and few advanced economies. One important result is that inflation dynamics are explained by the combination of domestic factor and global factors. What strikes us is that the effects of these factors are constant over time and across countries. We think the reason is that the sample period is from 1995 to 2018. This, together with the Bayes Factor (see Chapter 3), builds the starting point in Chapter 3 and Chapter 4. More specifically, we start from the constant coefficient unobserved components model (we allow for stochastic volatility).

In Chapter 3 we develop a multi-country unobserved components model. It allows for cross-country linkages in the error covariance matrix through factor stochastic volatility specification. Additionally, we allow for stochastic volatility in all equations and factors. To deal with over-parameterization concerns, we propose a method to remove stochastic volatility in a data based fashion. More specifically, we rewrite the process of log-volatility using the non-centered parameterization and use the Horseshoe prior to select time-variation. We apply these methods to the data in Chapter 2. The existence of global factors provides evidence of cross-country linkages in the error covariance matrix. The estimates under our model are in line with previous studies and, for certain countries, the estimates indicate that they are influenced by both domestic factors and global factors. We find that our proposed model provides a superior in-sample fit and accurate density forecasts compared to existing models in the literature, especially if period of uncertainty is the period being forecasted.

In Chapter 4 we develop a panel unobserved components model. It allows for cross-country linkages both in the error covariance matrix and in the conditional mean. This is done through dropping the assumption of diagonal coefficient matrices in Chapter 3 and allowing them to be full matrices. No zero restrictions are imposed. Over-parametrization concern is solved by the Horseshoe prior, which automatically imposes zero restriction for most elements but dropping the restriction if necessary. We apply these methods to the data in Chapter 2. The estimates of coefficients and generalised impulse response functions provide evidence of cross-country linkages. We find that allowing for cross-country linkages in the error covariance matrix can provide more precise estimates of trend, while omitting cross-country linkages in the conditional mean will overestimate trend output. Our proposed model provides a superior in-sample fit and accurate density forecasts compared to existing models in the literature.

5.2 Further research

This thesis focuses on how to consider globalisation in multivariate unobserved components models. Given that globalisation seems to have a different pattern (e.g., Brexit), we may need to consider the implications for modelling if the process of globalisation were to slow or go into reverse. In this case, the models in Chapter 3 and Chapter 4 with constant coefficients (even if stochastic volatility has been considered) may be not sensible. Time-variation is a possible extension. Time-varying parameters models are very popular in modern macroeconomics since they are able to capture many important features of the observed data (see, Cogley and Sargent, 2005; Primiceri, 2005; Hauzenberger et al., 2019 and Chan et al., 2020). Structural change is another possible extension (see, Sims and Zha, 2006 and Jochmann and Koop, 2015). Or even more complicated methods, like nonlinear state space models.

This thesis uses aggregate data. However, different barriers are imposed on different trades. For instance, barriers to services trade are estimated currently to be up to three times higher than those for goods trade (see, Carney, 2017a). Nowadays, many papers propose to use the components of inflation instead of the standard aggregates (see Stock and Watson, 2020; Zaman, 2019 and Tallman and Zaman, 2017). They find that some inflation components have strong and stable correlations with the cyclical component of real activity, while other components have weak and/or unstable correlations with cyclical activity. It would be useful to develop methods to estimate a model using components of inflation.

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Appendix A

Chapter 2 Appendix

A.1 Priors

$$
\begin{aligned}
\varphi_1 &\sim \mathcal{N}(0,\ 10)\\
\varphi_2 &\sim \mathcal{N}(0,\ 10)\\
\tau_1^{\pi} &\sim \mathcal{N}(3,\ 10)\\
\tau_1^{y} &\sim \mathcal{N}(5,\ 10)\\
h_1 &\sim \mathcal{N}(0,\ 1)\\
\rho_1 &\sim \mathcal{N}(0,\ 10)\\
\alpha_1 &\sim \mathcal{N}(0,\ 10)\\
\beta_1 &\sim \mathcal{N}(0,\ 10)\\
\gamma_1 &\sim \mathcal{N}(0,\ 10)\\
\sigma^2_{\tau\pi} &\sim \mathcal{IG}(10,\ 0.18)\\
\sigma^2_{\tau y} &\sim \mathcal{IG}(10,\ 0.09)\\
\sigma^2_{y} &\sim \mathcal{IG}(10,\ 4.5)\\
\sigma^2_{h} &\sim \mathcal{IG}(10,\ 0.9)\\
\sigma^2_{\rho} &\sim \mathcal{IG}(10,\ 0.018)\\
\sigma^2_{\alpha} &\sim \mathcal{IG}(10,\ 0.009)\\
\sigma^2_{\beta} &\sim \mathcal{IG}(10,\ 0.009)\\
\sigma^2_{\gamma} &\sim \mathcal{IG}(10,\ 0.009)
\end{aligned}
$$

A.2 Supplementary material: Tables

Table A.1: Mean of Trend Inflation over time

countries	posterior mean over time	16% quantile	84% quantile
Belgium	1.787	1.376	2.198
Greece	2.592	1.943	3.248
Ireland	1.303	0.605	1.988
Netherlands	1.645	1.256	2.033
Portugal	1.781	1.282	2.278
Latvia	1.427	0.411	2.441
Lithuania	1.839	0.893	2.785
Slovakia	3.074	1.921	4.220
Israel	1.390	0.592	2.183
Hong Kong	1.841	0.943	2.766
South Korea	2.509	2.017	3.002
UK	1.782	1.355	2.209
USA	2.172	1.782	2.563
Sweden	0.794	0.303	1.285
Switzerland	0.467	0.144	0.790
Spain	2.202	1.647	2.759
Denmark	1.727	1.388	2.064
Italy	1.777	1.339	2.228
Finland	1.225	0.734	1.721
France	1.309	0.979	1.639
Germany	1.387	1.075	1.699
Australia	2.196	1.794	2.597
Canada	1.713	1.349	2.077
South Africa	5.018	4.260	5.786
Hungary	3.349	2.157	4.548
Russia	4.585	2.118	7.112
Turkey	7.171	6.237	8.128
Mexico	3.971	3.391	4.559
Bolivia	3.990	3.283	4.696
Brazil	4.867	3.902	5.842
China	1.659	1.075	2.243
Philippines	3.526	2.543	4.506
Indonesia	4.079	3.175	4.993
Thailand	1.936	1.267	2.606

Table A.2: Inflation persistence

countries	posterior mean over time	16% quantile	84% quantile
Belgium	0.303	0.156	0.447
Greece	0.399	0.217	0.577
Ireland	0.577	0.381	0.777
Netherlands	0.268	0.105	0.429
Portugal	0.441	0.257	0.623
Latvia	0.714	0.576	0.855
Lithuania	0.639	0.492	0.790
Slovakia	0.586	0.405	0.769
Israel	0.552	0.386	0.718
Hong Kong	0.487	0.321	0.649
South Korea	0.282	0.112	0.450
UK	0.372	0.218	0.528
USA	0.209	0.072	0.346
Sweden	0.486	0.316	0.656
Switzerland	0.207	0.072	0.342
Spain	0.300	0.135	0.462
Denmark	0.252	0.095	0.407
Italy	0.543	0.372	0.716
Finland	0.489	0.317	0.659
France	0.235	0.091	0.379
Germany	0.160	0.054	0.266
Australia	0.229	0.091	0.366
Canada	0.159	0.052	0.265
South Africa	0.559	0.388	0.732
Hungary	0.483	0.308	0.656
Russia	0.734	0.604	0.865
Turkey	0.518	0.390	0.644
Mexico	0.501	0.342	0.657
Bolivia	0.433	0.269	0.597
Brazil	0.650	0.508	0.796
China	0.479	0.335	0.622
Philippines	0.623	0.451	0.801
Indonesia	0.497	0.330	0.660
Thailand	0.479	0.311	0.646

Table A.3: Coefficient on domestic output gap

Countries	posterior mean over time	16% quantile	84% quantile
Belgium	0.150	0.046	0.250
Greece	0.042	0.016	0.069
Ireland	0.035	0.020	0.043
Netherlands	0.045	0.016	0.072
Portugal	0.078	0.030	0.124
Latvia	0.101	0.037	0.165
Lithuania	0.105	0.034	0.174
Slovakia	0.091	0.031	0.151
Israel	0.114	0.035	0.197
Hong Kong	0.096	0.032	0.163
South Korea	0.112	0.045	0.178
UK	0.098	0.036	0.162
USA	0.103	0.036	0.173
Sweden	0.108	0.038	0.178
Switzerland	0.125	0.048	0.201
Spain	0.119	0.039	0.201
Denmark	0.074	0.034	0.117
Italy	0.080	0.027	0.132
Finland	0.093	0.033	0.151
France	0.092	0.032	0.150
Germany	0.043	0.013	0.074
Australia	0.100	0.073	0.138
Canada	0.142	0.052	0.232
South Africa	0.133	0.048	0.210
Hungary	0.153	0.053	0.255
Russia	0.107	0.036	0.180
Turkey	0.110	0.042	0.177
Mexico	0.060	0.024	0.098
Bolivia	0.121	0.040	0.203
Brazil	0.107	0.042	0.174
China	0.167	0.061	0.274
Philippines	0.068	0.031	0.103
Indonesia	0.212	0.069	0.360
Thailand	0.031	0.013	0.044

Table A.4: Coefficient on global output gap

countries	posterior mean over time	16% quantile	84% quantile
Belgium	0.161	0.056	0.267
Greece	0.113	0.035	0.192
Ireland	0.162	0.075	0.245
Netherlands	0.066	0.020	0.112
Portugal	0.176	0.067	0.284
Latvia	0.169	0.046	0.298
Lithuania	0.151	0.043	0.266
Slovakia	0.171	0.056	0.286
Israel	0.169	0.051	0.290
Hong Kong	0.205	0.058	0.354
South Korea	0.120	0.033	0.208
UK	0.122	0.040	0.205
USA	0.210	0.077	0.343
Sweden	0.174	0.049	0.297
Switzerland	0.123	0.041	0.206
Spain	0.203	0.080	0.323
Denmark	0.125	0.043	0.210
Italy	0.094	0.030	0.156
Finland	0.152	0.052	0.254
France	0.117	0.040	0.195
Germany	0.136	0.044	0.230
Australia	0.138	0.055	0.219
Canada	0.179	0.061	0.298
South Africa	0.120	0.042	0.200
Hungary	0.221	0.063	0.385
Russia	0.160	0.052	0.259
Turkey	0.249	0.078	0.425
Mexico	0.068	0.025	0.112
Bolivia	0.337	0.126	0.542
Brazil	0.166	0.053	0.287
China	0.337	0.170	0.499
Philippines	0.187	0.060	0.319
Indonesia	0.254	0.088	0.421
Thailand	0.322	0.122	0.517

Table A.5: Coefficient on oil price gap

countries	posterior mean over time	16% quantile	84% quantile
Belgium	0.613	0.314	0.871
Greece	0.556	0.251	0.849
Ireland	0.602	0.306	0.876
Netherlands	0.581	0.296	0.844
Portugal	0.502	0.164	0.835
Latvia	0.484	0.167	0.810
Lithuania	0.528	0.207	0.844
Slovakia	0.738	0.469	0.901
Israel	0.545	0.240	0.837
Hong Kong	0.560	0.252	0.860
South Korea	0.590	0.260	0.879
UK	0.570	0.261	0.861
USA	0.698	0.510	0.906
Sweden	0.476	0.168	0.787
Switzerland	0.718	0.502	0.924
Spain	0.589	0.293	0.866
Denmark	0.596	0.303	0.869
Italy	0.628	0.349	0.884
Finland	0.680	0.408	0.916
France	0.708	0.499	0.910
Germany	0.739	0.534	0.915
Australia	0.657	0.401	0.872
Canada	0.527	0.144	0.837
South Africa	0.477	0.154	0.795
Hungary	0.474	0.169	0.791
Russia	0.477	0.141	0.834
Turkey	0.534	0.310	0.702
Mexico	0.533	0.214	0.839
Bolivia	0.513	0.175	0.845
Brazil	0.393	0.109	0.699
China	0.490	0.164	0.836
Philippines	0.463	0.109	0.811
Indonesia	0.532	0.220	0.831
Thailand	0.598	0.189	0.880

Table A.6: Autoregressive coefficients

countries	AR(1)			AR(2)		
	mean	16% quantile	84% quantile	median	16% quantile	84% quantile
Belgium	0.686	0.588	0.784	-0.161	-0.259	-0.063
Greece	0.193	0.102	0.282	0.435	0.346	0.525
Ireland	-0.114	-0.208	-0.020	0.100	0.007	0.193
Netherlands	0.377	0.278	0.476	0.188	0.088	0.287
Portugal	0.311	0.213	0.407	0.275	0.178	0.373
Latvia	0.302	0.210	0.392	0.356	0.265	0.447
Lithuania	0.202	0.108	0.297	0.212	0.119	0.305
Slovakia	-0.104	-0.200	-0.009	0.060	-0.034	0.154
Israel	0.210	0.113	0.308	0.163	0.070	0.256
Hong Kong	0.241	0.145	0.338	0.212	0.116	0.308
South Korea	0.366	0.268	0.464	0.003	-0.092	0.099
UK	0.605	0.504	0.706	0.070	-0.030	0.170
USA	0.318	0.217	0.419	0.211	0.111	0.312
Sweden	0.293	0.196	0.390	0.153	0.056	0.252
Switzerland	0.491	0.392	0.590	0.039	-0.059	0.138
Spain	0.837	0.736	0.939	0.089	-0.013	0.190
Denmark	0.042	-0.058	0.141	0.118	0.019	0.219
Italy	0.540	0.440	0.640	0.056	-0.043	0.155
Finland	0.245	0.145	0.344	0.118	0.021	0.213
France	0.526	0.424	0.629	0.141	0.039	0.243
Germany	0.343	0.246	0.440	0.035	-0.062	0.132
Australia	-0.121	-0.218	-0.023	0.078	-0.020	0.175
Canada	0.522	0.422	0.622	-0.088	-0.188	0.012
South Africa	0.514	0.410	0.618	0.102	-0.001	0.205
Hungary	0.085	0.000	0.351	0.008	0.000	0.000
Russia	0.541	0.445	0.638	-0.042	-0.144	0.060
Turkey	0.017	-0.080	0.114	0.121	0.029	0.214
Mexico	0.476	0.385	0.567	-0.100	-0.178	-0.021
Bolivia	-0.169	-0.266	-0.073	-0.167	-0.263	-0.070
Brazil	0.247	0.150	0.344	-0.012	-0.107	0.083
China	0.178	0.075	0.283	0.106	0.008	0.204
Philippines	0.065	-0.035	0.163	0.058	-0.040	0.154
Indonesia	0.515	0.417	0.613	-0.017	-0.114	0.080
Thailand	-0.007	-0.105	0.091	0.114	0.020	0.207

Table A.7: Inflation Gap Decomposition

	Country		lagged inflation	domestic output	global output	oil price	Other
North	USA	pre GFC	0.223	0.085	0.223	0.086	0.945
		post GFC	0.178	**0.098**	0.181	0.062	0.976
America	Canada	pre GFC	0.110	0.211	0.175	0.077	0.945
		post GFC	**0.136**	0.123	0.100	**0.094**	0.996
Latin	Bolivia	pre GFC	0.623	0.155	0.179	0.036	1.059
		post GFC	0.376	0.085	0.076	0.021	1.006
America	Mexico	pre GFC	0.890	0.044	0.018	0.020	0.812
		post GFC	0.374	**0.046**	**0.032**	**0.054**	1.070
	Brazil	pre GFC	0.569	0.160	0.049	0.020	1.107
		post GFC	**0.775**	0.098	0.033	**0.024**	1.151
Europe	UK	pre GFC	0.232	0.096	0.166	0.135	1.011
		post GFC	**0.572**	0.057	0.076	0.070	1.020
	Belgium	pre GFC	0.124	0.175	0.139	0.088	0.763
		post GFC	**0.464**	0.153	0.070	0.056	0.567
	Greece	pre GFC	0.426	0.099	0.124	0.063	1.114
		post GFC	**0.479**	0.066	0.031	0.035	0.730
	Italy	pre GFC	0.505	0.202	0.169	0.171	1.051
		post GFC	**0.691**	0.140	0.066	0.090	1.007
	Spain	pre GFC	0.252	0.282	0.161	0.065	0.883
		post GFC	**0.341**	0.132	0.088	0.049	1.114
	Sweden	pre GFC	0.480	0.177	0.159	0.070	1.298
		post GFC	**0.562**	0.122	0.080	0.053	0.995

Table A.7: Inflation Gap Decomposition

Country		lagged inflation	domestic output	global output	oil price	Other
Switzerland	pre GFC	0.142	0.269	0.199	0.157	0.953
	post GFC	**0.222**	0.256	0.140	0.109	0.776
Denmark	pre GFC	0.159	0.255	0.176	0.089	0.951
	post GFC	**0.276**	0.187	0.118	**0.144**	1.145
Finland	pre GFC	0.363	0.328	0.199	0.080	1.202
	post GFC	**0.588**	0.238	0.106	**0.105**	0.968
Germany	pre GFC	0.133	0.101	0.149	0.094	1.017
	post GFC	**0.167**	0.086	0.127	**0.124**	0.935
Slovakia	pre GFC	0.522	0.144	0.065	0.025	0.964
	post GFC	**0.662**	0.069	0.048	**0.047**	1.037
France	pre GFC	0.189	0.156	0.121	0.164	0.945
	post GFC	**0.246**	**0.172**	0.098	0.126	0.972
Ireland	pre GFC	0.522	0.060	0.103	0.048	0.937
	post GFC	**0.616**	**0.111**	0.095	**0.069**	1.309
Portugal	pre GFC	0.511	0.121	0.141	0.070	1.158
	post GFC	0.412	**0.134**	**0.143**	0.049	1.106
Lithuania	pre GFC	0.738	0.116	0.068	0.022	1.068
	post GFC	0.715	**0.129**	0.040	**0.035**	1.063
Hungry	pre GFC	0.603	0.057	0.032	0.009	0.579
	post GFC	0.439	0.057	**0.057**	**0.018**	0.826
Latvia	pre GFC	0.752	0.164	0.050	0.012	0.801
	post GFC	0.716	0.128	0.038	**0.021**	0.991

Table A.7: Inflation Gap Decomposition

	Country		lagged inflation	domestic output	global output	oil price	Other
	Netherlands	pre GFC	0.308	0.077	0.067	0.083	0.988
		post GFC	0.184	0.042	0.032	0.063	0.960
Asia	Indonesia	pre GFC	0.640	0.097	0.044	0.014	0.870
		post GFC	0.369	**0.244**	**0.110**	**0.051**	0.968
	Philippines	pre GFC	0.585	0.079	0.054	0.032	0.939
		post GFC	0.552	**0.106**	**0.077**	**0.040**	0.947
	China	pre GFC	0.589	0.089	0.175	0.035	1.095
		post GFC	0.477	**0.234**	**0.180**	**0.046**	0.825
	Hong Kong	pre GFC	0.756	0.080	0.051	0.021	0.914
		post GFC	0.244	0.074	0.049	**0.039**	1.012
	Thailand	pre GFC	0.690	0.065	0.202	0.036	1.090
		post GFC	0.440	0.050	0.113	0.035	0.980
	South Korea	pre GFC	0.200	0.181	0.074	0.048	1.122
		post GFC	**0.263**	**0.214**	0.067	**0.120**	0.900
Other	Australia	pre GFC	0.177	0.048	0.133	0.085	0.986
		post GFC	**0.191**	0.046	0.092	0.079	1.033
	Israel	pre GFC	0.548	0.117	0.055	0.028	1.247
		post GFC	0.449	0.112	**0.063**	**0.041**	1.029
	Russia	pre GFC	0.837	0.057	0.015	0.004	0.645
		post GFC	0.824	**0.068**	**0.031**	**0.016**	1.175
	Turkey	pre GFC	0.842	0.025	0.010	0.001	0.491
		post GFC	0.219	**0.148**	**0.063**	**0.020**	1.011

Table A.7: Inflation Gap Decomposition

	Country		lagged inflation	domestic output	global output	oil price	Other
	South Africa	pre GFC	0.701	0.090	0.053	0.019	1.269
		post GFC	0.497	**0.112**	0.053	**0.037**	1.022
End							

Note that the increasing contribution is indicated in Bold.

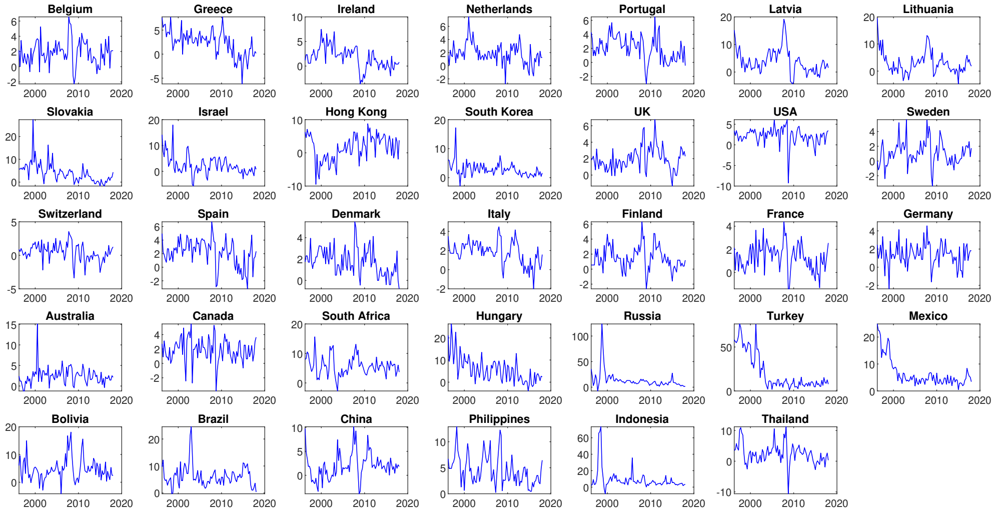

Figure A.1: **Inflation π.**

A.3 Supplementary material: Figures

Figure A.1 plots the inflation data that we use in the model. Figure A.2 plots posterior estimates for trend output τ^y. Figure A.3 plots coefficient on domestic output gap α_t. Figure A.4 plots coefficient on global output gap β_t. Figure A.4 plots coefficient on oil price gap γ_t.

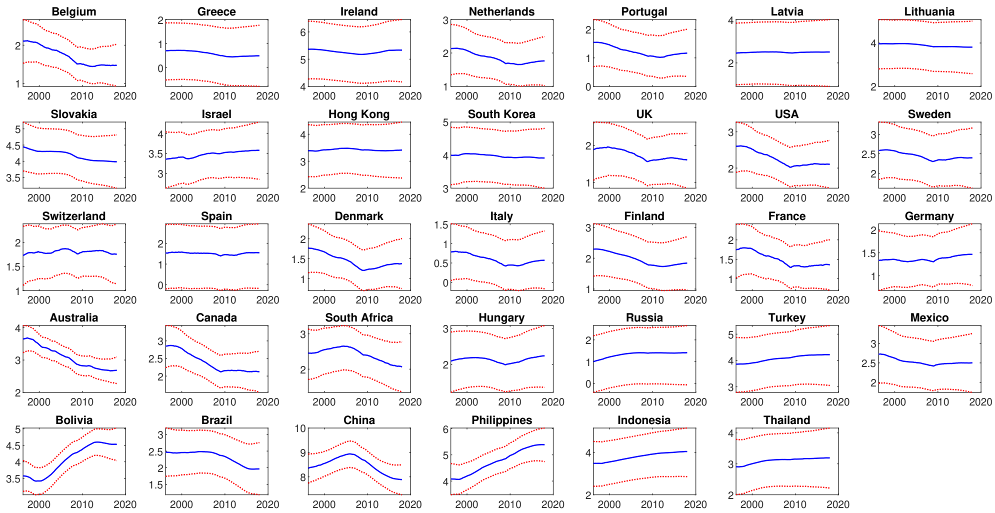

Figure A.2: **Posterior estimates for trend output τ^y.** The solid blue line is the posterior mean, while the dotted red lines are 16% and 84% quantiles.

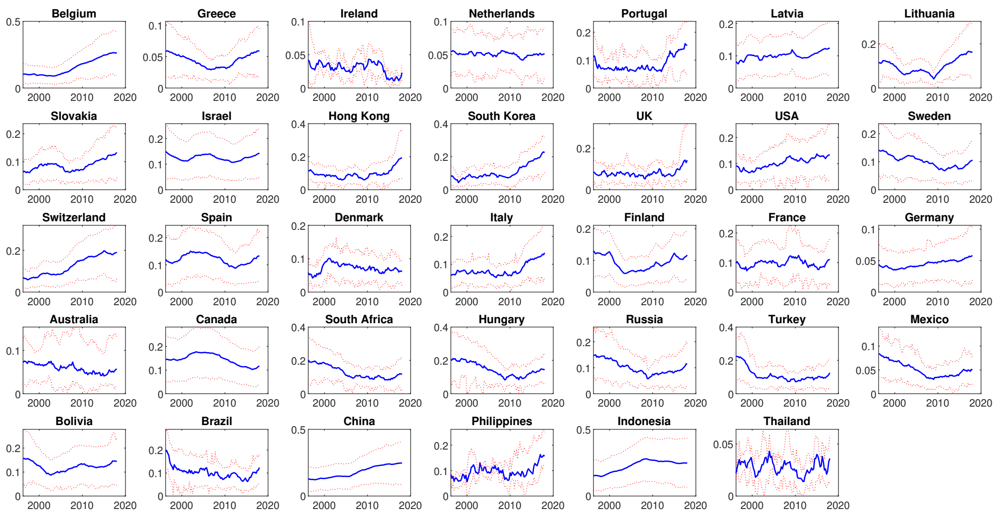

Figure A.3: **Coefficient on domestic output gap α_t.** The solid blue line is the posterior mean, while the dotted red lines are 16% and 84% quantiles.

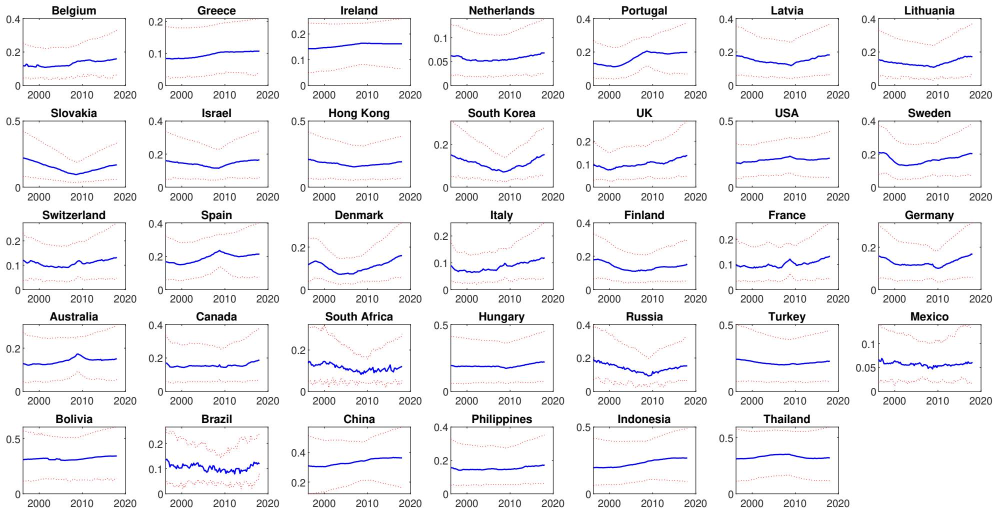

Figure A.4: **Coefficient on global output gap β_t.** The solid blue line is the posterior mean, while the dotted red lines are 16% and 84% quantiles.

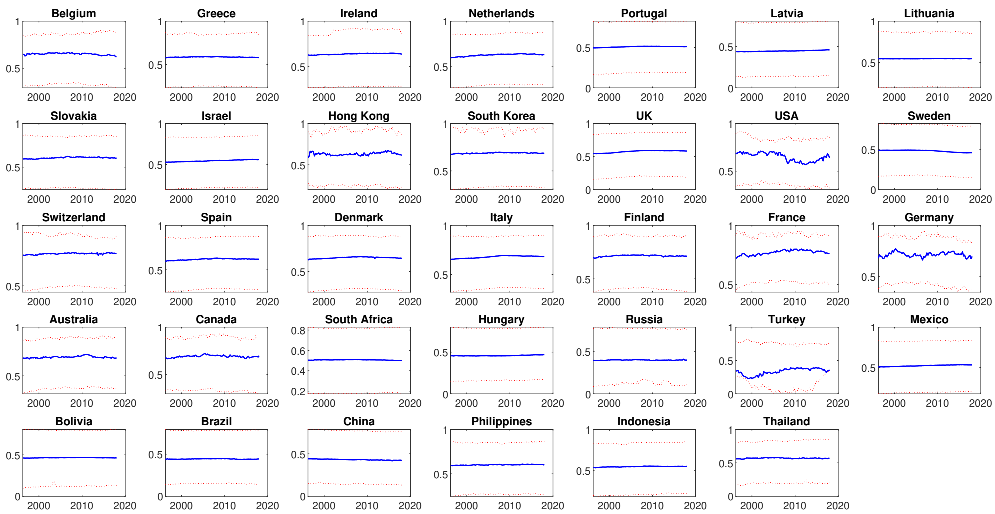

Figure A.5: **Coefficient on oil price gap γ_t.** The solid blue line is the posterior mean, while the dotted red lines are 16% and 84% quantiles.

Appendix B

Chapter 3 Appendix

B.1 Testing for Time-Variation in Coefficients

In this appendix, we illustrate the method to test for time-variation in coefficients and report the estimated Bayes Factor, which support the constant coefficients model (we allow for stochastic volatility in the model).

What we did is to allow the coefficients in UC-SV to be time-varying in the noncentered parameterization as follows:

$$\pi_{i,t} - \tau^{\pi}_{i,t} = (\rho_{i,0} + \omega^{\rho}_{i}\tilde{\rho}_{i,t})(\pi_{i,t-1} - \tau^{\pi}_{i,t-1}) + (\alpha_{i,0} + \omega^{\alpha}_{i}\tilde{\alpha}_{i,t})(y_{i,t} - \tau^{y}_{i,t}) + \epsilon^{\pi}_{i,t}, \quad \epsilon^{\pi}_{i,t} \sim \mathcal{N}(0,\ e^{h_{i,t}}) \tag{B.1}$$

$$y_{i,t} - \tau^{y}_{i,t} = \varphi_{i,1}(y_{i,t-1} - \tau^{y}_{i,t-1}) + \varphi_{i,2}(y_{i,t-2} - \tau^{y}_{i,t-2}) + \epsilon^{y}_{i,t}, \quad \epsilon^{y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{y}) \tag{B.2}$$

$$\tau^{\pi}_{i,t} = \tau^{\pi}_{i,t-1} + \epsilon^{\tau\pi}_{i,t}, \quad \epsilon^{\tau\pi}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau\pi}) \tag{B.3}$$

$$\tau^{y}_{i,t} = \tau^{y}_{i,t-1} + \epsilon^{\tau y}_{i,t}, \quad \epsilon^{\tau y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau y}) \tag{B.4}$$

$$h_{i,t} = h_{i,t-1} + \epsilon^{h}_{i,t}, \quad \epsilon^{h}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{h}) \tag{B.5}$$

$$\tilde{\rho}_{i,t} = \tilde{\rho}_{i,t-1} + \epsilon^{\rho}_{i,t}, \quad \epsilon^{\rho}_{i,t} \sim \mathcal{N}(0,\ 1) \tag{B.6}$$

$$\tilde{\alpha}_{i,t} = \tilde{\alpha}_{i,t-1} + \epsilon^{\alpha}_{i,t}, \quad \epsilon^{\alpha}_{i,t} \sim \mathcal{N}(0,\ 1) \tag{B.7}$$

We assume a normal prior with zero mean and variance ten for $\rho_{i,0}$, ω_i^{ρ}, $\alpha_{i,0}$, ω_i^{α}. The prior for other parameters are kept the same as NIOUC-FSV.

Next, we calculate the Bayes factor in favor of the unrestricted model against the restricted version where $\omega_i^{\rho} = 0$ as:

$$BF_{\rho_i} = \frac{p(\omega_i^{\rho} = 0)}{p(\omega_i^{\rho} = 0|\ y)} \tag{B.8}$$

So if BF_{ρ_i} is larger than 1, then the Bayes Factor is in favor of the unrestricted model. In this part, the unrestricted model is time-varying ρ_i. For simplicity, we compare the log Bayes Factor. So a positive log Bayes Factor supports the time-varying coefficient ρ_i. We can calculate the log Bayes Factor for ω_i^{α} similarly.

Using the data in the empirical section, we report the log Bayes Factor in Table B.1. We find most log Bayes Factor are negative (except for 3 cases: log BF_{ρ_i} for Latvia, Turkey and Mexico), so we think this result strongly supports constant coefficients models.

Table B.1: The estimated log Bayes factors for ω_i^ρ and ω_i^α

countries	log BF_{ρ_i}	log BF_{α_i}
Belgium	-2.83	-1.03
Greece	-2.42	-0.99
Ireland	-2.00	-0.86
Netherlands	-2.21	-2.23
Portugal	-2.61	-1.52
Latvia	0.08	-1.81
Lithuania	-2.20	-2.01
Slovakia	-2.64	-2.33
Israel	-3.30	-3.48
Hong Kong	-2.54	-3.65
South Korea	-1.71	-2.63
UK	-1.44	-3.37
USA	-2.86	-3.63
Sweden	-2.79	-2.28
Switzerland	-2.80	-2.15
Spain	-3.15	-2.54
Denmark	-3.40	-2.45
Italy	-2.30	-2.85
Finland	-2.95	-3.02
France	-2.72	-2.82
Germany	-3.07	-2.94
Australia	-3.13	-3.09
Canada	-3.02	-1.27
South Africa	-1.97	-3.45
Hungary	-3.00	-3.57
Russia	-1.41	-3.37
Turkey	1.08	-3.42
Mexico	2.99	-3.44
Bolivia	-1.02	-3.68
Brazil	-2.18	-3.10
China	-2.09	-2.87
Philippines	-2.99	-2.81
Indonesia	-2.91	-2.39
Thailand	-3.37	-1.41

B.2 Estimates of factor loading matrices

In this appendix, we report the posterior estimates of factor loading matrices under NIOUC-FSV. Basically, we have two classes of factors:

(1): global inflation factor f_t, and its loading matrix is L_π, L_π is $n \times r_\pi$, in our empirical application, $n = 34$, $r_\pi = 5$. Table B.2 is the loadings of global inflation factor. We report the posterior mean of the five factors' loadings, but only the quantiles of first factor's loadings for brevity.

(2): global output factor g_t, and its loading matrix is L_y, L_y is $n \times r_y$, in our empirical application, $n = 34$, $r_y = 2$. Table B.3 is the loadings of global output factor. We report the posterior mean and quantiles of the two factors' loadings.

And for identification, we assume the factor loading matrices are lower triangular matrices with ones on the main diagonal, so some elements in L_π and L_y are 1 or 0.

Table B.2: Posterior Estimates of factor loading matrix L_π

	1st factor			2nd factor	3rd factor	4th factor	5th factor
country	mean	16%	84%	mean	mean	mean	mean
Belgium	1	1	1	0	0	0	0
Greece	2.29	1.52	3.01	1	0	0	0
Ireland	2.09	1.40	2.74	1.05	1	0	0
Netherlands	1.79	1.17	2.39	-0.97	-0.69	1	0
Portugal	1.57	0.98	2.15	-0.71	-0.17	0.51	1
Latvia	1.96	1.17	2.75	0.66	0.28	0.20	-0.71
Lithuania	2.26	1.41	3.08	1.00	0.35	0.19	-0.82
Slovakia	2.05	1.35	2.72	-0.09	-0.07	0.50	0.23
Israel	2.06	1.31	2.81	-0.29	0.02	-0.12	0.34
Hong Kong	0.93	0.15	1.74	0.60	0.34	-0.90	-0.73
South Korea	1.53	0.99	2.05	-0.18	-0.21	-0.24	-0.63
UK	1.83	1.24	2.38	0.09	-0.13	0.00	-0.22
USA	2.94	2.02	3.81	-0.73	-0.45	0.29	-0.34
Sweden	1.98	1.32	2.61	0.03	0.15	-0.44	-0.59
Switzerland	1.73	1.17	2.26	-0.16	0.07	-0.27	0.34
Spain	3.08	2.11	3.97	-0.03	-0.01	0.71	0.87
Denmark	1.69	1.14	2.21	0.04	0.03	0.21	-0.03
Italy	1.32	0.87	1.74	-0.42	-0.11	0.11	0.75
Finland	1.23	0.77	1.68	0.42	0.38	-0.18	-0.31
France	2.10	1.46	2.68	0.69	0.59	-0.25	0.26
Germany	2.09	1.43	2.68	0.36	0.15	0.46	-0.23
Australia	2.16	1.47	2.80	0.01	0.29	-0.21	0.44
Canada	2.44	1.65	3.19	-0.08	-0.01	-0.54	-0.45
South Africa	1.72	1.01	2.43	-0.79	-0.68	0.45	-0.27
Hungary	3.00	1.86	4.15	0.14	0.11	0.46	-0.05
Russia	0.26	-0.65	1.17	0.48	0.52	-0.57	-0.04
Turkey	3.04	1.79	4.31	0.15	0.13	-0.12	0.04
Mexico	0.49	0.02	0.97	0.41	0.22	0.07	0.18
Bolivia	0.82	-0.04	1.69	0.65	0.25	-0.52	-0.85
Brazil	-0.36	-1.02	0.30	0.17	0.15	-0.50	-0.43
China	0.38	-0.11	0.89	0.50	0.29	-0.62	-0.86
Philippines	1.33	0.68	1.98	-0.25	-0.14	-0.19	-0.12
Indonesia	0.61	0.05	1.18	-0.26	-0.04	-0.16	0.12
Thailand	2.52	1.67	3.33	0.23	-0.04	-0.16	-0.70

Table B.3: Posterior Estimates of factor loading matrix L_y

	1st factor			2nd factor		
country	mean	16%	84%	mean	16%	84%
Belgium	1	1	1	0	0	0
Greece	2.11	1.35	2.86	1	1	1
Ireland	3.91	2.61	5.21	-2.28	-5.00	0.40
Netherlands	1.75	1.23	2.28	2.61	1.59	3.89
Portugal	1.60	1.14	2.06	1.04	-0.15	2.22
Latvia	1.39	0.26	2.52	0.46	-2.13	3.04
Lithuania	3.70	2.22	5.18	5.54	1.06	10.09
Slovakia	2.99	1.84	4.12	4.63	1.79	7.37
Israel	1.08	0.74	1.41	-1.02	-2.19	0.03
Hong Kong	3.57	2.63	4.51	-0.18	-2.42	1.98
South Korea	2.93	2.20	3.66	-2.03	-4.41	0.21
UK	1.34	0.94	1.74	-1.19	-2.81	0.24
USA	1.60	1.20	1.99	-0.68	-2.00	0.57
Sweden	2.91	2.24	3.57	-0.12	-2.33	1.99
Switzerland	1.74	1.37	2.10	-0.95	-2.30	0.28
Spain	0.70	0.48	0.93	0.12	-0.65	0.84
Denmark	2.12	1.46	2.78	0.40	-1.49	2.26
Italy	2.08	1.62	2.54	-0.23	-1.65	1.05
Finland	3.28	2.28	4.27	4.23	2.38	6.28
France	1.41	1.13	1.70	-0.12	-1.00	0.72
Germany	2.56	1.95	3.18	2.23	0.91	3.72
Australia	0.58	0.23	0.93	-1.62	-2.80	-0.62
Canada	1.34	0.98	1.70	0.81	-0.24	1.87
South Africa	1.04	0.73	1.34	0.60	-0.27	1.49
Hungary	1.31	0.45	2.24	-1.53	-4.37	1.17
Russia	3.36	2.48	4.24	-0.62	-3.09	1.76
Turkey	3.83	2.65	5.00	1.05	-1.50	3.55
Mexico	2.52	1.79	3.26	3.37	2.01	5.04
Bolivia	0.83	0.34	1.33	-0.48	-2.01	0.98
Brazil	3.02	2.20	3.83	-2.27	-4.93	0.22
China	0.95	0.51	1.39	-1.15	-2.63	0.23
Philippines	1.37	0.69	2.05	3.03	1.29	4.94
Indonesia	0.60	0.26	0.94	-0.08	-1.39	1.14
Thailand	2.87	2.12	3.63	0.74	-1.21	2.63

B.3 Estimates of constant coefficients

In this appendix, we report the posterior estimates of constant coefficients: ρ, α, φ_1, φ_2.

Table B.4: Posterior estimates of inflation persistence ρ

country	NIOUC-FSV mean	NIOUC-FSV 16%	NIOUC-FSV 84%	NIOUC-FSV- $r_y = 0$ posterior mean	NIOUC-FSV- $r_y, r_\pi = 0$ posterior mean	NIOUC-FSV- $r_y, r_\pi = 0, \omega_y^h = 0$ posterior mean
Belgium	0.28	0.19	0.37	0.28	0.33	0.33
Greece	0.36	0.27	0.45	0.36	0.41	0.41
Ireland	0.46	0.35	0.57	0.46	0.59	0.59
Netherlands	0.29	0.18	0.39	0.29	0.28	0.28
Portugal	0.36	0.27	0.46	0.36	0.47	0.47
Latvia	0.65	0.59	0.71	0.65	0.70	0.70
Lithuania	0.62	0.54	0.70	0.62	0.66	0.66
Slovakia	0.54	0.45	0.62	0.54	0.63	0.63
Israel	0.47	0.38	0.56	0.47	0.53	0.53
Hong Kong	0.56	0.45	0.68	0.57	0.58	0.58
South Korea	0.22	0.12	0.31	0.22	0.31	0.31
UK	0.43	0.34	0.52	0.43	0.41	0.41
USA	0.22	0.14	0.29	0.22	0.28	0.28
Sweden	0.37	0.28	0.45	0.37	0.52	0.52
Switzerland	0.34	0.26	0.42	0.34	0.28	0.28
Spain	0.24	0.17	0.30	0.23	0.39	0.40
Denmark	0.23	0.13	0.32	0.23	0.25	0.25
Italy	0.49	0.41	0.57	0.49	0.59	0.59
Finland	0.49	0.40	0.57	0.49	0.56	0.56
France	0.15	0.09	0.21	0.15	0.26	0.26
Germany	0.07	0.02	0.13	0.08	0.11	0.11
Australia	0.16	0.08	0.24	0.16	0.20	0.20
Canada	0.10	0.03	0.16	0.10	0.11	0.11
South Africa	0.55	0.46	0.65	0.55	0.56	0.56
Hungary	0.40	0.30	0.50	0.40	0.49	0.50
Russia	0.80	0.71	0.89	0.80	0.79	0.79
Turkey	0.93	0.90	0.97	0.94	0.94	0.94
Mexico	0.81	0.75	0.88	0.81	0.80	0.81
Bolivia	0.33	0.23	0.44	0.33	0.32	0.33
Brazil	0.63	0.52	0.74	0.63	0.60	0.61
China	0.53	0.44	0.62	0.52	0.58	0.58
Philippines	0.57	0.47	0.67	0.57	0.61	0.61
Indonesia	0.36	0.26	0.47	0.36	0.35	0.35
Thailand	0.41	0.31	0.51	0.41	0.53	0.53

Table B.5: Estimates of slope of Phillips Curve α

country	NIOUC-FSV mean	NIOUC-FSV 16%	NIOUC-FSV 84%	NIOUC-FSV- $r_y = 0$ posterior mean	NIOUC-FSV- $r_y, r_\pi = 0$ posterior mean	NIOUC-FSV- $r_y, r_\pi = 0, \omega_y^h = 0$ posterior mean
Belgium	0.16	0.09	0.23	0.16	0.22	0.22
Greece	0.02	0.00	0.03	0.02	0.02	0.02
Ireland	0.01	0.00	0.02	0.01	0.01	0.01
Netherlands	0.01	0.00	0.03	0.01	0.03	0.03
Portugal	0.05	0.01	0.08	0.05	0.05	0.05
Latvia	0.07	0.04	0.10	0.07	0.10	0.10
Lithuania	0.02	0.00	0.04	0.02	0.04	0.04
Slovakia	0.03	0.01	0.06	0.03	0.08	0.08
Israel	0.07	0.02	0.12	0.07	0.09	0.09
Hong Kong	0.05	0.01	0.09	0.05	0.06	0.06
South Korea	0.03	0.01	0.06	0.04	0.08	0.08
UK	0.02	0.00	0.04	0.02	0.04	0.04
USA	0.05	0.01	0.08	0.05	0.05	0.06
Sweden	0.09	0.06	0.13	0.09	0.14	0.14
Switzerland	0.07	0.03	0.10	0.07	0.15	0.15
Spain	0.08	0.03	0.13	0.07	0.12	0.11
Denmark	0.03	0.01	0.05	0.03	0.05	0.05
Italy	0.04	0.02	0.06	0.04	0.09	0.09
Finland	0.08	0.05	0.10	0.08	0.11	0.11
France	0.02	0.00	0.04	0.02	0.11	0.11
Germany	0.02	0.00	0.03	0.02	0.05	0.05
Australia	0.02	0.00	0.04	0.02	0.03	0.03
Canada	0.10	0.05	0.16	0.10	0.20	0.20
South Africa	0.04	0.01	0.07	0.04	0.06	0.06
Hungary	0.05	0.01	0.10	0.05	0.08	0.08
Russia	0.04	0.01	0.07	0.04	0.04	0.05
Turkey	0.06	0.02	0.11	0.07	0.08	0.09
Mexico	0.04	0.01	0.08	0.04	0.05	0.05
Bolivia	0.09	0.02	0.15	0.09	0.08	0.08
Brazil	0.09	0.03	0.15	0.09	0.08	0.08
China	0.22	0.13	0.30	0.21	0.23	0.22
Philippines	0.02	0.00	0.04	0.02	0.03	0.03
Indonesia	0.15	0.03	0.26	0.14	0.12	0.15
Thailand	0.01	0.00	0.02	0.01	0.02	0.02

Table B.6: Estimates of output persistence φ_1

country	NIOUC-FSV mean	NIOUC-FSV 16%	NIOUC-FSV 84%	NIOUC-FSV- $r_y = 0$ posterior mean	NIOUC-FSV- $r_y, r_\pi = 0$ posterior mean	NIOUC-FSV- $r_y, r_\pi = 0, \omega_y^h = 0$ posterior mean
Belgium	0.37	0.27	0.47	0.58	0.58	0.62
Greece	0.09	-0.02	0.20	0.17	0.17	0.10
Ireland	-0.17	-0.29	-0.05	-0.03	-0.03	-0.21
Netherlands	0.14	0.06	0.22	0.31	0.31	0.27
Portugal	0.12	0.02	0.23	0.28	0.28	0.21
Latvia	0.23	0.11	0.34	0.25	0.25	0.17
Lithuania	0.19	0.09	0.28	0.22	0.22	0.06
Slovakia	-0.06	-0.15	0.03	-0.06	-0.06	-0.21
Israel	0.28	0.16	0.39	0.34	0.34	0.12
Hong Kong	0.10	-0.03	0.23	0.39	0.39	0.13
South Korea	-0.06	-0.18	0.06	0.23	0.24	0.27
UK	0.22	0.10	0.35	0.38	0.38	0.53
USA	0.04	-0.07	0.16	0.26	0.26	0.23
Sweden	-0.07	-0.17	0.04	0.07	0.07	0.19
Switzerland	0.21	0.11	0.31	0.46	0.46	0.39
Spain	0.58	0.47	0.71	0.72	0.72	0.73
Denmark	-0.13	-0.24	-0.01	-0.02	-0.02	-0.05
Italy	0.29	0.19	0.39	0.54	0.54	0.44
Finland	0.05	-0.03	0.13	0.16	0.16	0.14
France	0.15	0.06	0.24	0.46	0.45	0.43
Germany	0.06	-0.02	0.14	0.20	0.20	0.25
Australia	-0.08	-0.20	0.03	-0.09	-0.09	-0.20
Canada	0.33	0.23	0.43	0.43	0.42	0.42
South Africa	0.39	0.28	0.50	0.57	0.57	0.39
Hungary	0.20	0.09	0.32	0.24	0.24	0.39
Russia	0.38	0.27	0.48	0.52	0.52	0.43
Turkey	-0.04	-0.14	0.06	0.03	0.03	-0.02
Mexico	0.30	0.22	0.39	0.29	0.29	0.38
Bolivia	-0.18	-0.30	-0.06	-0.17	-0.17	-0.28
Brazil	0.26	0.13	0.38	0.39	0.39	0.13
China	0.08	-0.05	0.22	0.19	0.18	0.07
Philippines	-0.06	-0.16	0.05	-0.02	-0.02	0.00
Indonesia	0.11	-0.03	0.26	0.13	0.14	0.43
Thailand	-0.02	-0.13	0.09	0.10	0.10	-0.10

Table B.7: Estimates of output persistence φ_2

	NIOUC-FSV			NIOUC-FSV- $r_y = 0$	NIOUC-FSV- $r_y, r_\pi = 0$	NIOUC-FSV- $r_y, r_\pi = 0, \omega_y^h = 0$
country	mean	16%	84%	posterior mean	posterior mean	posterior mean
Belgium	-0.03	-0.12	0.06	-0.07	-0.07	-0.28
Greece	0.35	0.24	0.45	0.39	0.38	0.35
Ireland	0.10	0.01	0.21	0.15	0.15	-0.05
Netherlands	0.30	0.22	0.37	0.36	0.36	0.11
Portugal	0.27	0.17	0.37	0.33	0.33	0.18
Latvia	0.31	0.21	0.42	0.33	0.33	0.29
Lithuania	0.17	0.08	0.26	0.12	0.12	0.07
Slovakia	0.23	0.15	0.31	0.18	0.18	0.02
Israel	0.04	-0.06	0.13	0.06	0.06	0.08
Hong Kong	0.17	0.08	0.26	0.23	0.23	0.12
South Korea	0.12	0.03	0.21	0.18	0.18	-0.10
UK	0.16	0.05	0.27	0.10	0.09	-0.05
USA	0.22	0.12	0.31	0.15	0.15	0.12
Sweden	0.19	0.10	0.28	0.16	0.16	0.05
Switzerland	0.17	0.09	0.26	0.05	0.05	-0.08
Spain	0.22	0.11	0.32	0.18	0.18	-0.01
Denmark	0.07	-0.04	0.17	0.12	0.12	0.04
Italy	0.08	-0.01	0.16	0.01	0.01	-0.04
Finland	0.19	0.11	0.27	0.24	0.24	0.04
France	0.24	0.16	0.32	0.21	0.21	0.03
Germany	0.16	0.09	0.23	0.09	0.09	-0.05
Australia	0.10	-0.01	0.20	0.10	0.10	-0.04
Canada	0.04	-0.05	0.14	-0.02	-0.02	-0.18
South Africa	0.18	0.07	0.28	0.05	0.05	0.02
Hungary	0.16	0.06	0.25	0.12	0.13	-0.06
Russia	0.05	-0.04	0.15	0.07	0.07	-0.11
Turkey	0.06	-0.03	0.14	0.11	0.11	0.06
Mexico	-0.06	-0.14	0.02	0.03	0.03	-0.23
Bolivia	-0.13	-0.24	-0.01	-0.15	-0.15	-0.24
Brazil	0.14	0.03	0.24	0.08	0.08	-0.05
China	0.05	-0.05	0.16	0.10	0.09	0.01
Philippines	0.09	-0.01	0.18	0.09	0.10	-0.06
Indonesia	-0.01	-0.12	0.10	-0.05	-0.05	-0.11
Thailand	-0.05	-0.14	0.04	0.00	-0.01	0.08

B.4 Testing for Time-Variation in volatilities

In this appendix, we report the estimated log Bayes factors to test for time-variation in volatilities.

Table B.8: The estimated log Bayes factors for ω_i^h

countries	log BF_{h_i} for inflation	log BF_{h_i} for output
Belgium	-6.16	-5.17
Greece	-3.97	6.68
Ireland	-6.52	20.92
Netherlands	-2.86	-8.56
Portugal	-7.86	-5.37
Latvia	-4.63	4.32
Lithuania	-2.48	4.27
Slovakia	82.45	60.82
Israel	21.29	19.80
Hong Kong	-5.52	22.35
South Korea	42.60	75.68
UK	-10.58	-4.79
USA	3.11	-5.79
Sweden	-6.42	-3.43
Switzerland	-4.95	-5.51
Spain	-6.13	-3.94
Denmark	-4.92	-4.83
Italy	-5.64	5.33
Finland	-5.30	-5.11
France	-5.58	-6.11
Germany	-1.93	-3.85
Australia	32.10	-2.67
Canada	-5.03	-6.21
South Africa	-1.29	-4.89
Hungary	3.17	-2.85
Russia	131.11	17.81
Turkey	20.91	3.71
Mexico	-4.99	-6.03
Bolivia	-2.02	-4.72
Brazil	6.08	-2.82
China	-4.20	69.65
Philippines	-3.86	-0.19
Indonesia	151.82	200.64
Thailand	8.09	64.03

Appendix C

Chapter 4 Appendix

C.1 Width of Credible Intervals for Trends

In this appendix, we report the width of 84% credible intervals for trends under three models: Bi-UC-FSV, MC-UC-FSV and PUC-FSV. Figure C.1 reports the width of credible interval for trend inflation. Figure C.2 reports the width of credible interval for trend output.

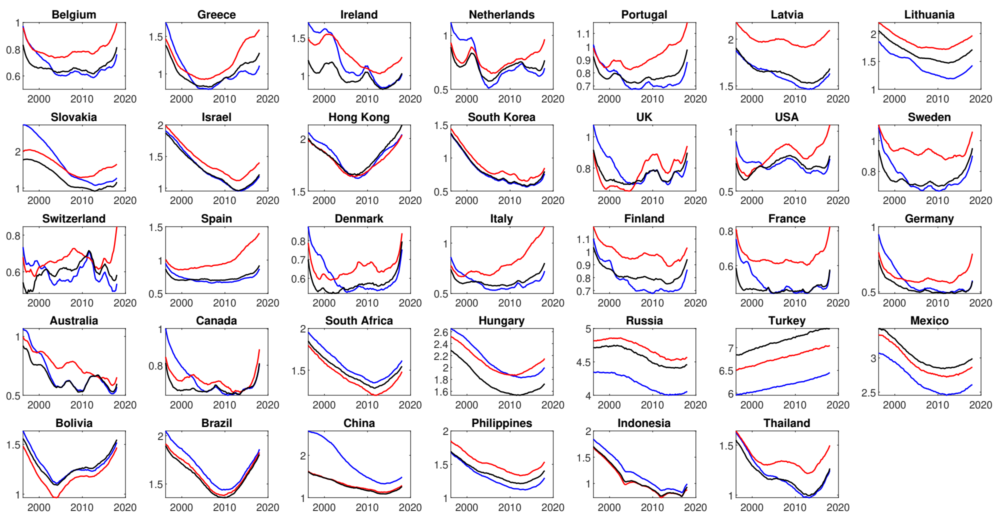

Figure C.1: **The width of 84% credible interval for trend inflation under three models: Bi-UC-SV, MC-UC-FSV and PUC-FSV.** The red lines are the width under Bi-UC-SV. The black lines are the width under MC-UC-FSV. The blue lines are the width under PUC-FSV.

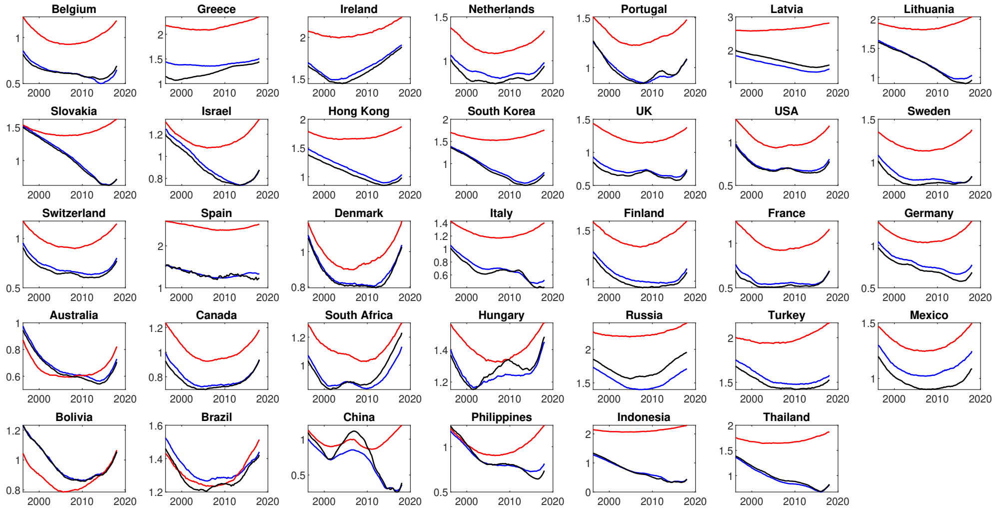

Figure C.2: **The width of 84% credible interval for trend output under three models: Bi-UC-SV, MC-UC-FSV and PUC-FSV.** The red lines are the width under Bi-UC-SV. The black lines are the width under MC-UC-FSV. The blue lines are the width under PUC-FSV.

C.2 Estimates of factor loading matrices

In this appendix, we report the posterior estimates of factor loading matrices under PUC-FSV. Basically, we have two factors:

(1): global inflation factor f_t, and its loading matrix is L_π, L_π is $n \times r_\pi$, in our empirical application, $n = 34$, $r_\pi = 1$. (2): global output factor g_t, and its loading matrix is L_y, L_y is $n \times r_y$, in our empirical application, $n = 34$, $r_y = 1$.

We report the posterior mean and the 84% quantiles of both L_π and L_y. And for identification, we assume the factor loading matrices are lower triangular matrices with ones on the main diagonal, so some elements in L_π and L_y are 1 or 0.

Table C.1: Posterior Estimates of factor loading matrix L_π and L_y

	L_π			L_y		
country	mean	16%	84%	mean	16%	84%
Belgium	1	1	1	1	1	1
Greece	1.54	1.03	2.11	2.28	1.54	3.01
Ireland	1.42	0.96	1.93	3.94	2.62	5.27
Netherlands	1.21	0.82	1.66	1.91	1.50	2.32
Portugal	1.18	0.77	1.64	1.83	1.41	2.26
Latvia	1.37	0.85	1.92	2.18	1.16	3.21
Lithuania	1.31	0.77	1.85	3.91	2.70	5.13
Slovakia	1.58	1.07	2.17	3.11	2.16	4.08
Israel	1.38	0.87	1.91	1.23	0.86	1.60
Hong Kong	0.72	0.17	1.27	3.77	2.91	4.63
South Korea	1.08	0.71	1.48	2.54	1.87	3.21
UK	1.21	0.83	1.66	1.47	1.07	1.87
USA	1.96	1.34	2.67	1.62	1.23	2.01
Sweden	1.51	1.03	2.08	2.95	2.33	3.57
Switzerland	1.22	0.84	1.66	1.74	1.39	2.10
Spain	2.09	1.45	2.86	0.61	0.42	0.81
Denmark	1.17	0.80	1.60	1.49	0.85	2.11
Italy	0.92	0.63	1.26	1.96	1.56	2.36
Finland	1.04	0.69	1.43	3.59	2.82	4.35
France	1.44	1.00	1.96	1.45	1.17	1.73
Germany	1.39	0.96	1.91	2.64	2.13	3.16
Australia	1.45	1.00	1.98	0.45	0.15	0.75
Canada	1.72	1.16	2.36	1.26	0.90	1.62
South Africa	1.18	0.67	1.69	1.12	0.84	1.40
Hungary	2.27	1.42	3.16	1.54	0.67	2.37
Russia	0.27	-0.28	0.83	3.46	2.67	4.24
Turkey	2.16	1.21	3.15	3.94	2.87	5.02
Mexico	0.38	0.08	0.68	2.58	2.02	3.14
Bolivia	0.48	-0.05	1.02	0.65	0.18	1.12
Brazil	0.08	-0.32	0.47	3.02	2.22	3.81
China	0.49	0.14	0.83	1.11	0.69	1.52
Philippines	0.87	0.45	1.30	1.73	1.11	2.35
Indonesia	0.41	0.09	0.73	0.65	0.36	0.94
Thailand	1.71	1.11	2.34	3.05	2.40	3.71

C.3 Estimates of Uncertainty

In this appendix, we report the posterior estimates of uncertainty. There are four uncertainties in PUC-FSV:

(1): Idiosyncratic Inflation Uncertainty: the idiosyncratic inflation uncertainty is the standard deviation of the shocks to the inflation gap, $\exp(h_t^{\pi}/2)$. We report this in Figure C.3. The title of each sub-figure is the country name. Each sub-figure plots the posterior estimates under three competing models (PUC-FSV, MC-UC-FSV and Bi-UC-SV). The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, the red lines are posterior means under Bi-UC-SV, while the black lines are posterior means under MC-UC-FSV.

(2): Idiosyncratic Output Uncertainty: the idiosyncratic output uncertainty is the standard deviation of the shocks to the output gap, $\exp(h_t^{y}/2)$. We report this in Figure C.4. The title of each sub-figure is the country name. Each sub-figure plots the posterior estimates under two competing models (PUC-FSV and MC-UC-FSV). The stochastic volatility is not allowed in Bi-UC-SV, so we only have the estimates under two models. The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, while the black lines are posterior means under MC-UC-FSV.

(3): Global Inflation Uncertainty: the global inflation uncertainty is the standard deviation of the shocks to the global inflation factor, $\exp(h_t^{f}/2)$. We report this in Figure C.5. The figure plots the posterior estimates under two competing models (PUC-FSV and MC-UC-FSV). There is no global factors in Bi-UC-SV model, so we only have the estimates under two models. The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, while the black lines are posterior means under MC-UC-FSV.

(4): Global Output Uncertainty: the global output uncertainty is the standard deviation of the shocks to the global output factor, $\exp(h_t^g/2)$. We report this in Figure C.6. The figure plots plots the posterior estimates under two competing models (PUC-FSV and MC-UC-FSV). The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, while the black lines are posterior means under MC-UC-FSV.

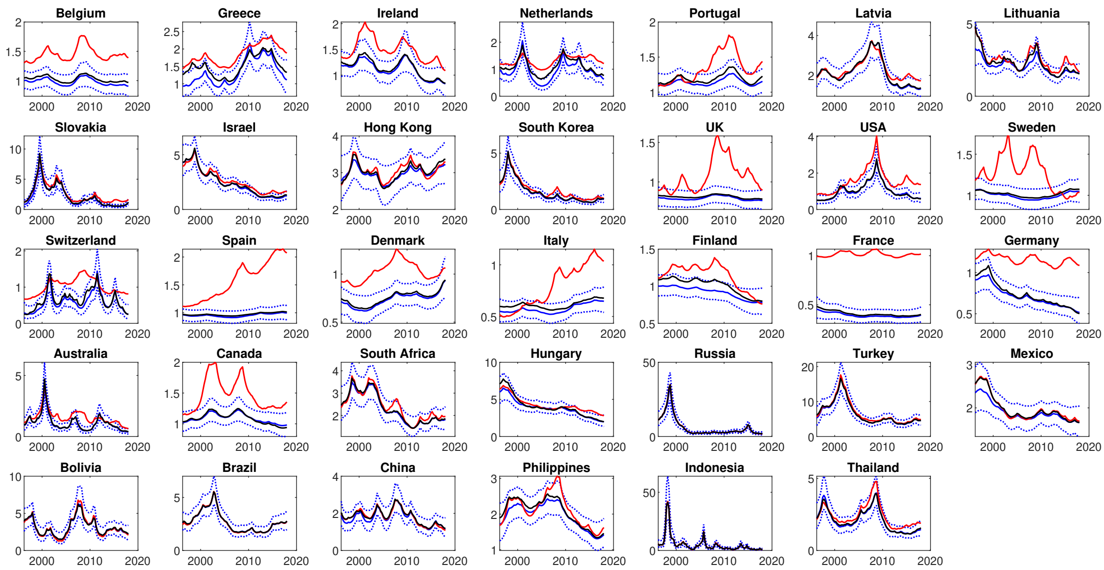

Figure C.3: **Posterior estimates for idiosyncratic inflation uncertainty $\exp(h_t^\pi/2)$.** The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, the red lines are posterior means under Bi-UC-SV, while the black lines are posterior means under MC-UC-FSV.

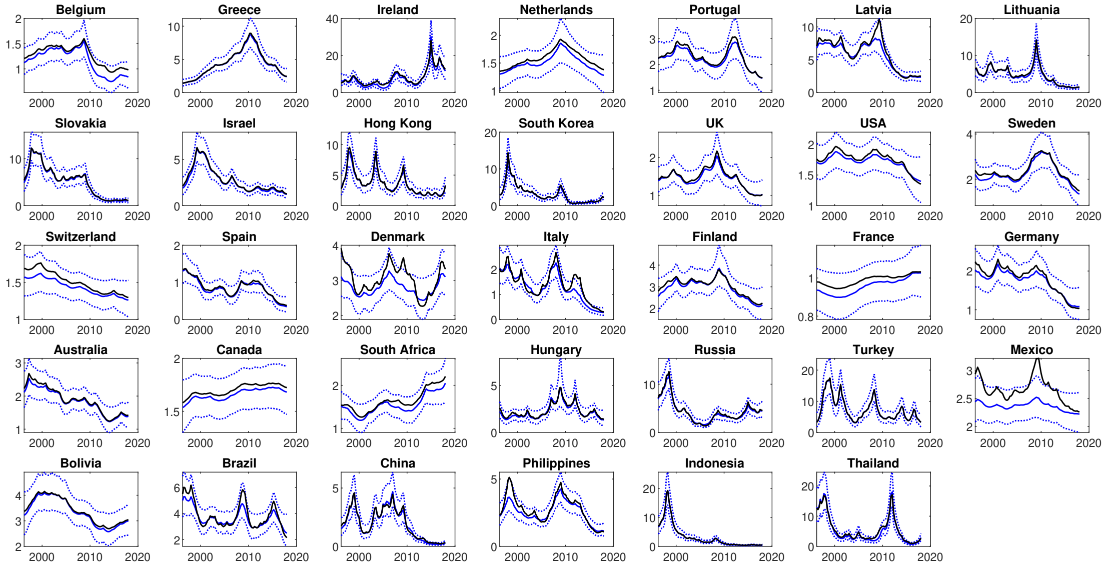

Figure C.4: **Posterior estimates for idiosyncratic inflation uncertainty $\exp(h_t^y/2)$.** The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, the red lines are posterior means under Bi-UC-SV, while the black lines are posterior means under MC-UC-FSV.

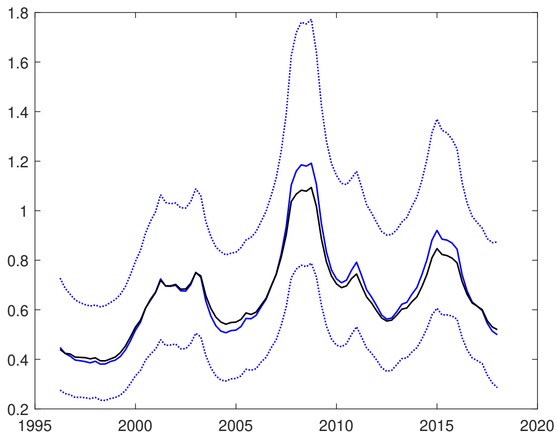

Figure C.5: **Posterior estimates for global inflation uncertainty $\exp(h_t^f/2)$.** The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, while the black lines are posterior means under MC-UC-FSV.

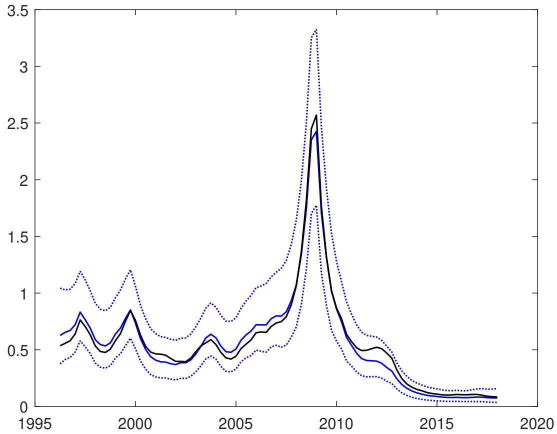

Figure C.6: **Posterior estimates for global output uncertainty exp**$(h_t^g/2)$**.** The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, while the black lines are posterior means under MC-UC-FSV.