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Probability Modelling of Supplier Development Investment Decisions under Uncertainty

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Declaration

This thesis is the result of the author's original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.

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Abstract

Supplier development involves activities intended to improve a supplier's performance and to add value on a buying firm's business benefit. Typically, such activities require significant resource investment and there is a risk that the benefit to be obtained from supplier development may not be enough to offset the expenses incurred. This thesis develops quantitative models to inform buyer decisions on whether it is worth investing in a supplier development activity and, if so, how much should be invested. The proposed models are built from a buyer's perspective to support analysis of the benefit obtained from the development activity. In particular, the models take account the uncertainty of the benefit and associate it with the stochastic characteristics of a supplier's performance.

More specifically, the two models are relevant for types of supplier performance data. The first model considers the case of developing a supplier whose undesirable performance is measured in categorical form, such as classes corresponding to degrees of late delivery. The multinomial distribution is used to represent the uncertainty of a supplier's performance. The second model considers a case of developing a supplier whose undesirable performance is measured in counting form, such as, the number of non-conformances. The non-homogenous Poisson process (NHPP) model is used to represent the uncertainty of a supplier's performance.

The two proposed models provide decision makers with an optimal investment level, which is the maximum amount of investment to be made for the development activity as well as an expected return. Numerical investigations are carried out to examine the behaviour of the models and to illustrate key theoretical properties. An industry case study is conducted to provide an empirical demonstration of modelling for analysis of supplier databases. This also qualitatively investigates how the models align with supplier development management decision making in practice.

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There are a number of people who I would like to acknowledge for helping me during this process. First of all, I would like to acknowledge and thank my supervisors Professor Lesley Walls and Professor John Quigley. Thank you for all your time and support that helped me complete this journey. There is no doubt that you made the process enjoyable, thought provoking and inspiring. Without you, my experience would not have been anywhere near as enjoyable. I am fortunate and honoured to have been supervised by such a wonderful team of supervisors. Secondly, I would like to say thanks to the participants during my case study. Thanks for providing me with the opportunity to get insights from the real world. Thirdly, I also want to express my gratitude to my family, who were there for me throughout it all. Last but not least, a special thank you to my fiancé, for all his love and support.

List of Symbols

Function

η	Dirichlet Prior distribution
Γ	Gamma function
π	Profit function
Ψ	Digamma function
τ	Benefit function
E	Expectation function
P_M	Multinomial distribution
P_N	Count based Non-Homogeneous Poisson Process
G	Gamma prior distribution
g	Budget constraint function
L	Likelihood function
L_a	Lagrange function
l	Log-likelihood function
u	Non-conforming cost function

u^* Non-conforming cost function under virtual age

v Cost function for virtual age

Index

ζ Index to indicate any observing point, $\zeta \in [1, K]$

i Index for the i_{th} class of performance variable $i \in [1, I]$

j Index for supplier j, $j \in [1, J]$

k Index for the k_{th} observing point, $k \in [1, K]$

q Index for the of shape parameter in power law model, $q \in [1, Q]$

s Index to indicate any suppliers$s$, $s \in [1, J]$

Parameter

α_g Parameter for Gamma prior distribution

α_i Parameter for Dirichlet prior distribution

β_g Parameter for Gamma prior distribution

δ_j Marginal cost of virtual age for supplier j

γ_{ij} Effectiveness rate of development activity at the i_{th} performance class for supplier j

γ_j Effectiveness rate of development activity for supplier j

λ_a Lagrange multiplier

Λ_j Expected number of non-conformances from supplier j

λ_j Non-conformance rate for supplier j

ω_q Weight of the shape parameter b_{jq}

a_j Scale parameter for power law model supplier j

b_j Shape parameter for power law model supplier j

b_{jq} The q_{th} type of shape parameter b_j

c_{ij} Non-conformance cost per unit at the i_{th} performance category for supplier j

c_j Non-conformance cost per unit from supplier j

c_j^T Total non-conformance cost for supplier j

c_j^V Cost of virtual age per unit for supplier j

M_j Random number of non-conformances from supplier j

m_{jk} Number of non-conformances from supplier j detected over $[t_{k-1}, t_k]$

m_j Number of non-conformances from supplier j which follow the NHPP model

N_{ij} Random number of items at the i_{th} performance class from supplier j following the multinomial distribution

n_j Number of total orders from supplier j

n_{ij} Number of items at the i_{th} performance class from supplier j following the multinomial distribution, where $\sum_{i=1}^{I} n_{ij} = n_j$.

p_j Parameter of the multinomial distribution representing the aleatory uncertainty in the i_{th} performance variable supplier j

p_{ij} The aleatory uncertainty in the frequency of the occurrences at risk category i for supplier j, $\sum_{i=1}^{I} p_{ij} = 1$

r Compounding interest rate

t_k Exposure to risk over an observing interval $[k-1, k]$

t_T Exposure to risk

z Limitation of budget

Variable

x_j Investment level for supplier j

x_j^* Optimal investment level for supplier j

x_j^{PI} Investment level under perfect information for supplier j

y_j Virtual age for supplier j

y_j^* Optimal virtual age to be invested

Vector

$\mathbf{a}$ Vector for α_i,where $\mathbf{a} = [\alpha_1 ... \alpha_I]$

$\mathbf{b_{jq}}$ Vector for b_{jq}, where $\mathbf{b_{jq}} = [b_{j1} ... b_{jQ}]$

$\mathbf{M_j}$ Vector for m_{jk}, where $\mathbf{M_j} = [m_{j1} ... m_{jK}]$

$\mathbf{N_j}$ Vector for n_{ij}, where $\mathbf{N_j} = [n_{1j} ... n_{Ij}]$

$\mathbf{P_j}$ Vector for p_{ij}, where $\mathbf{P_j} = [p_{1j} ... p_{Ij}]$

$\mathbf{w_i}$ Vector for ω_i^{α}, where $\mathbf{w_i} = [\omega_1^{\alpha} ... \omega_I^{\alpha}]$

Contents

Chapter 1

Introduction

1.1 Research Context and Motivation

To maintain a sustainable and competitive supply chain, supplier development has become an attractive strategy (Glock, 2016). Supplier development may be defined as "any efforts of a buying firm with a supplier to increase its performance and/or capabilities and meet the buying firm's short and/or long-term supply needs" (Krause and Ellram, 1997a). The core of supplier development is to improve a supplier's Key Performance Indicators (KPIs) in order to increase a buyer's competitive advantages, as the improvement of supplier performance can contribute to a number of potential benefits for a buyer. For example, an increase of suppliers' on-time delivery rate can bring down a buyer's inventory cost, and improvement of suppliers' product quality can increase a buyer's production efficiency as well as its customer satisfactory (Dalvi and Kant, 2015). Nasr and Jaber (2019) notes that supplier development has brought Toyota an 14% increase in output per worker and 50% fewer defects comparing with the suppliers of its rivals. According to Glock et al. (2017), much academic research on supplier development often differentiates indirect activities from direct activities. Indirect activities are considered as informal development which does not require a buyer to be actively involved in the process, such as, verbal/written request or guidance for suppliers' self-improvement. Direct activities are more formal and require an investment of resources from a buyer, such as, delegating a buyer's own engineers for supplier training, or financial investment (Wagner, 2009). Large organisations often adopt direct supplier development activities. For example, Intel invested in one of its suppliers, ASMI, by purchasing 4% of its common shares to facilitate material development. Walmart sent experts to help Chinese suppliers improve sustainability (Qi et al., 2015). Although direct supplier development is an effective strategy for firms to maintain a competitive supply chain, it also requires significant investment of resources and there is a risk that the benefits gained from supplier development may not be enough to offset the cost incurred. how to make decisions concerning such investments is of considerable importance (Talluri et al., 2010).

A number of quantitative decision models to support decision making in supplier development investment have been developed in previous studies. Some studies focus on developing models that provide optimal investment levels (Bhattacharyya and Guiffrida, 2015; Glock, 2016; Quigley et al., 2018; Talluri et al., 2010; Zhang and Hong, 2017; Zhu et al., 2007). For example, Talluri et al. (2010) develop a mathematical model to optimise the allocation of investment in multiple suppliers' performance development. Bhattacharyya and Guiffrida (2015) propose a budget-constrained optimisation model to obtain an optimal investment level for improving suppliers' on-time delivery. Glock (2016) develops an optimisation model which determines the optimal number of workers delegated as well as the timing and duration of a supplier training activity. Quigley et al. (2018) propose a stochastic decision support model within a Bayesian framework that provides an optimal investment level for improving supplier quality. Some studies focus on developing models that investigate the spill-over effect and seek an equilibrium profit when multiple firms invest in a shared supplier (Agrawal et al., 2016; Bai and Sarkis, 2016; Friedl and Wagner, 2016; Qi et al., 2015; Wang et al., 2014). For example, Wang et al. (2014) develop a two-stage model to explore the influences of spill-over effect on manufacturers' incentives of improving supplier's delivery reliability. Qi et al.

(2015) analyse firms' investment decisions using game theory to examine how different contractual forms affect the decision making and resulting profits and to identify when and to what extent the spill-over effect occurs. Friedl and Wagner (2016) develop an analytical model using stochastic programming to analyse the influence of cooperative investment and non-cooperative investment on optimal investment levels and expected profits.

Although quantitative studies in supplier development have been increasing since 2010, quantitative models to support decisions making in supplier development still require much attention. As noted by Glock (2016), "the majority of prior research was either conceptual or empirical in nature, and that only a few mathematical models exist that support supplier development in practice". Quantitative models attempt to express and represent the given managerial situation as accurately and faithfully as possible, and thus can be capable of supporting informed decision making for decision problems (Oral and Kettani, 1993; Pidd, 2009; Williams, 2003). A review of previous studies shows that many key decision problems in supplier development require relatively specific solutions, such as supplier investment volumes, cost savings or expected returns realised from supplier development, duration of a supplier development activity (Bai and Sarkis, 2016; Bhattacharyya and Guiffrida, 2015; Friedl and Wagner, 2012; Glock, 2016; Quigley et al., 2018; Talluri et al., 2010). Such problems can be effectively answered using quantitative models. The need for useful quantitative models to support managerial decision making in supplier development has also been highlighted in many studies (Bai and Sarkis, 2016; Friedl and Wagner, 2012; Glock, 2016). For example, Bai and Sarkis (2016) note that analytical investigations and theoretical understanding are important to guide firms in making resource investment decisions in supplier development which may not be easily achieved in qualitative studies. Anderson et al. (2015) also suggest that, "A manager can increase their decision-making effectiveness by learning more about quantitative methodology and models and by better understanding their contribution to the decision-making process."

To provide meaningful decision support, it is also important to understand the inherent uncertainties involved in the decision making process. Uncertainty is a major factor in many decision making situations and the majority of management-related investment projects involves decision making under uncertainty (Jovanović, 1999; Virlics, 2013). Kochenderfer (2015) notes that "robust decision-making system must account for these sources of uncertainty in the current state of the world and the future outcomes of events." Many previous studies in the context of supplier development have also acknowledged the issues of uncertainties involved in investment decisions making, more specifically, the uncertain returns from supplier development investment (Agrawal et al., 2016; Friedl and Wagner, 2016; Meisel, 2012; Mizgier et al., 2017; Qi et al., 2015; Quigley et al., 2018; Talluri et al., 2010; Wang et al., 2014; Zhu et al., 2007). Agrawal et al. (2016) highlight that one of the biggest challenges that make decision makers feel reluctant to conduct a supplier development activity is the uncertainty of returns obtained from the investment, such as, cost savings. As noted by Anderson et al. (2015), "the risk associated with any decision alternative is a direct result of the uncertainty associated with the final consequence."

The uncertainty of the return from supplier development investment may be analysed from different perspectives. Many authors associate it with suppliers' capability (Meisel, 2012; Mizgier et al., 2017; Qi et al., 2015; Quigley et al., 2018; Talluri et al., 2010; Wang et al., 2014; Zhu et al., 2007). For example, Talluri et al. (2010) analyse the risk of the total expected return from supplier portfolio investment using Markowitz-type mean-variance risk models, in which the variance of each supplier's return is measured using the supplier's stock market price. Quigley et al. (2018) associate the uncertainty of returns from supplier development with the uncertainty of a supplier's non-conformance rate, in which a Poisson-Gamma model is used to capture both aleatory and epistemic uncertainty in the supplier's non-conformance rate. A few authors associate the uncertainty of returns with market demand uncertainty (Friedl and Wagner, 2016; Wang et al., 2014). For example, Wang et al. (2014) consider the aleatory uncertainty in both supplier performance and demand, in which demand uncertainty is assumed to be modelled using a log-concave probability distribution, whereas the randomness of supplier performance is measured using a factor which is between 0 to 1. There are also a few studies that directly model the uncertainty of the outcomes of supplier development investment. For example, Meisel (2012) uses a random factor to represent the probabilistic realisations of different outcomes from a supplier development investment activity. Agrawal et al. (2016) use Brownian motion to model the profit generating process, in which the expected profit is associated with the type of a supplier's quality performance (high or low) and the uncertainty of the performance type is assessed based on buyers' beliefs.

We note that in the existing decisions support models for supplier development, the majority of previous studies that consider the uncertainty associated with investment decision making in supplier development have not distinguished the aleatory uncertainty from the epistemic uncertainty. Only Agrawal et al. (2016) and Quigley et al. (2018) take into account the epistemic uncertainty in their proposed decision support models. In general, aleatory uncertainty is concerned with the nature variation and randomness which is regarded as irreducible whereas epistemic uncertainty is concerned with the degree of incomplete information regarding the aspects of the system of interest, which can be reduced as more information is collected (Hartley and French, 2018). Radford (1989) has noted that, "uncertainty arises when we have incomplete information about the factors involved in these decision situations". Epistemic uncertainty in the context of supplier development investment may be considered as a buyer's prior state of knowledge about a supplier's capability which is reducible by knowing more about a supplier's true capability. As epistemic uncertainty is expressed before making the investment decision, gaining additional information to reduce the epistemic uncertainty also helps identify a supplier's improvement potential which in turn reduces a buyer's risk of investing in suppliers whose performance may already be up to the desired standards.

1.2 Research Aim and Objectives

The aim of this research is to develop quantitative decision support models to inform buyer decisions on supplier development investments given an uncertain return. We consider returns from the investment vary from suppliers. We associate the uncertainty of the benefit from supplier development with the stochastic characteristics of supplier performance. We assume that buyers have access to databases regarding the records of suppliers' orders for analysis, such as, order size, dates, on-time deliveries, non-conformances. The following objectives are to be achieved:

1. Develop a modelling framework to analyse the financial benefit from supplier development investment where both the aleatory and epistemic uncertainty of suppliers' KPIs are taken into account;

2. Construct two probabilistic models to capture the uncertainty of suppliers' KPIs of which the data are in categorical form and in count form respectively;

3. Determine a means of providing decision support under each probabilistic model, such as:

    - Assess the value of information to obtain the highest amount of investment for learning more about suppliers' true performance to buy down the epistemic uncertainty before making an investment decision;

    - Obtain the optimal investment levels and corresponding expected profits to determine the highest amount of investment that should be made on a supplier development activity;

4. Examine the theoretical properties of the proposed decision support models and illustrate the behaviour of each model through numerical experiments;

5. Investigate the empirical data analysis where the model for categorical data is applied for the decision support needs in a real industry case.

1.3 Research Outline

To achieve the research goal, the development of this research comprises four key stages:

The first stage is concerned with research objective 1. This stage focuses on reviewing literature within the research field of decision support for supplier development. The review findings are summarised based on the types of research, namely qualitative and quantitative studies. A particular focus is placed on studies that have considered the uncertainty involved in the decision making in supplier development investment. The purpose of this stage is to address the key research problems based on the identified research gaps and to further select a suitable modelling framework.

The second stage focuses on research objectives 2-4. Two probabilistic modelling techniques are used to capture different types of supplier performance. The modelling techniques employed are the multinomial distribution and the Non-Homogeneous Poisson (NHPP) model. This research embeds the probabilistic models into the decision making process and develops the decision support models by analysing the expected profit gained from supplier development investment which is measured by the difference between the expected benefit of the activity and the investment level. The outputs of the decision support models are an optimal investment level which maximises the expected profit and an expected value of perfect information (EVPI) for buying down the epistemic uncertainty in the assessment of a supplier's performance. Numerical investigations are carried out to illustrate the behaviour of the proposed decision support models.

The third stage focuses on the application of the proposed model to empirical data to achieve research objective 5. An industrial case study was conducted to assess whether the proposed model performs as expected and to validate the alignment of assumptions between decision contexts of the real and the assumed world. During the case study, three interviews with the participating firm were conducted. The first interview was an introduction meeting to determine suitable participants and to make sure that the participants would understand the process. The second interview was to collect the empirical data for model application. The third interview was to obtain the participant's feedback to validate the proposed model based on a number of pre-specified criteria. Note that due to the characteristics of the company data, only the proposed model developed in Chapter 4 was used for the analysis and the validation.

The fourth stage summarises the research findings and identifies the research limitations as well as future research work.

1.4 Thesis Structure

This section provides a structure of this thesis (see Fig.1.1) with a brief summary of the content in each chapter.

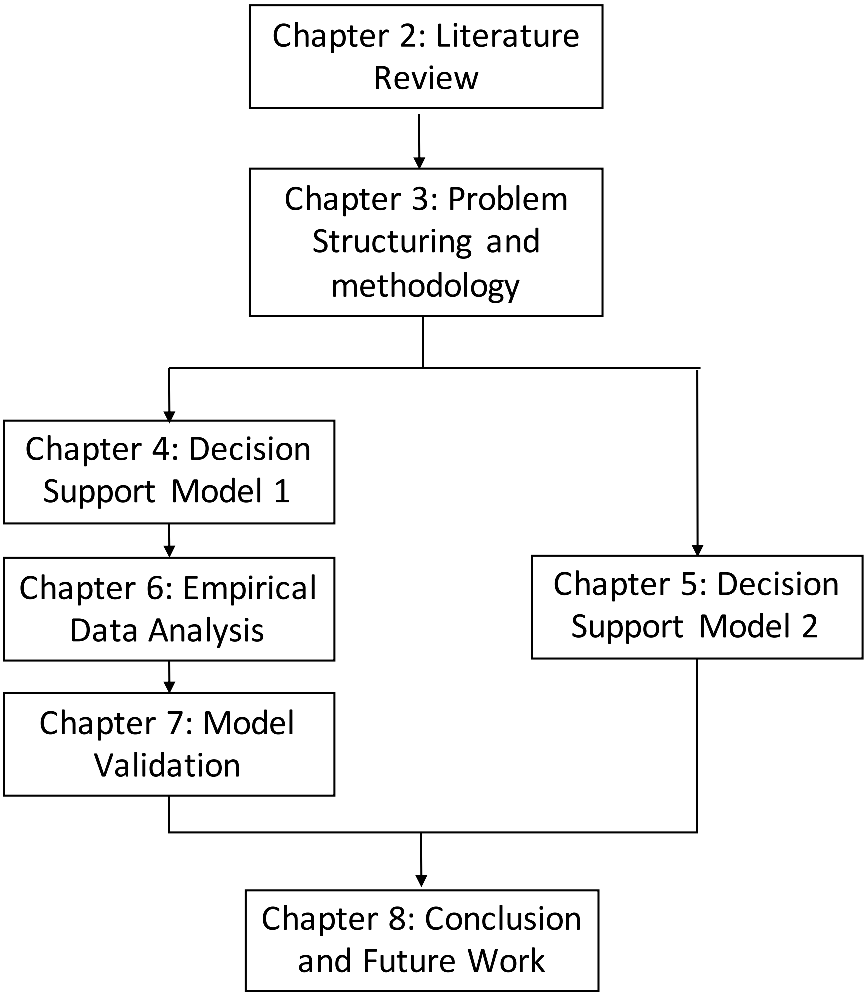

Figure 1.1: Thesis structure with key words of each chapter

Chapters 2 and 3 are concerned with the problem structuring and selection of the modelling framework which is associated with the first stage of this research. Chapter 2 provides a literature review of the previous research on supplier development and informs the selection of the modelling framework. Chapter 3 details the decision problems investigated in this research and describes the modelling framework along with explanations of the modelling techniques employed and the core modelling assumptions made. Chapters 4 and 5 develop two decision support models and conduct numerical experiments for each model which is associated with the second stage of this research.

Chapter 4 presents the first decision support model in which supplier performance data is in a categorical form and modelled by the multinomial distribution. Chapter 5 presents the second decision support model in which supplier performance data is in a count form and modelled by the power law NHPP model. Chapters 6 and 7 focus on the case study which is associated with the third stage of this research. Chapter 6 conducts an empirical data analysis where the first decision support model is applied to an industrial case. Chapter 7 validates the proposed model by obtaining feedback from the supply chain manager's perspective regarding the performance of the model based on the results of the empirical data analysis. Chapter 8 concludes the thesis by summarising this research and pointing out the direction of future work which is the final stage of this research.

Chapter 2

Literature Review

2.1 Introduction

This chapter reviews previous literature published within the field of supplier development with a particular emphasis on the studies which relate to decision making under uncertainty. The aim of this chapter is to point out popular research streams within the existing studies and to identify the research trends as well as gaps. The following questions are to be answered:

- What are the benefits and challenges of supplier development activities?
- What are the popular methods used for decision making in supplier development?
- What are the limitations in the existing decision support models?

This chapter begins by presenting a review of previous studies on supplier development and summarising the findings in Section 2.2. A discussion of decision making under uncertainty across the existing decision support models for supplier development investment is provided in section 2.3. The research trends and gaps are identified and detailed in Section 2.4. followed by a discussion regarding the focus of this research in Section 2.5.

2.2 Review of Literature for Supplier Development

To keep the review focused, it is important to first define suitable boundaries for the papers to be studied. As noted by Glock et al. (2017), too strict criteria could lead to relevant works being omitted, whereas a very broad definition may lead to excessively large literature samples which can be time-consuming to review and may cause difficulties in evaluating individual papers. In this study, we adapted the method proposed by Saunders et al. (2009) and developed a methodology for the literature review which is presented in Fig.2.1.

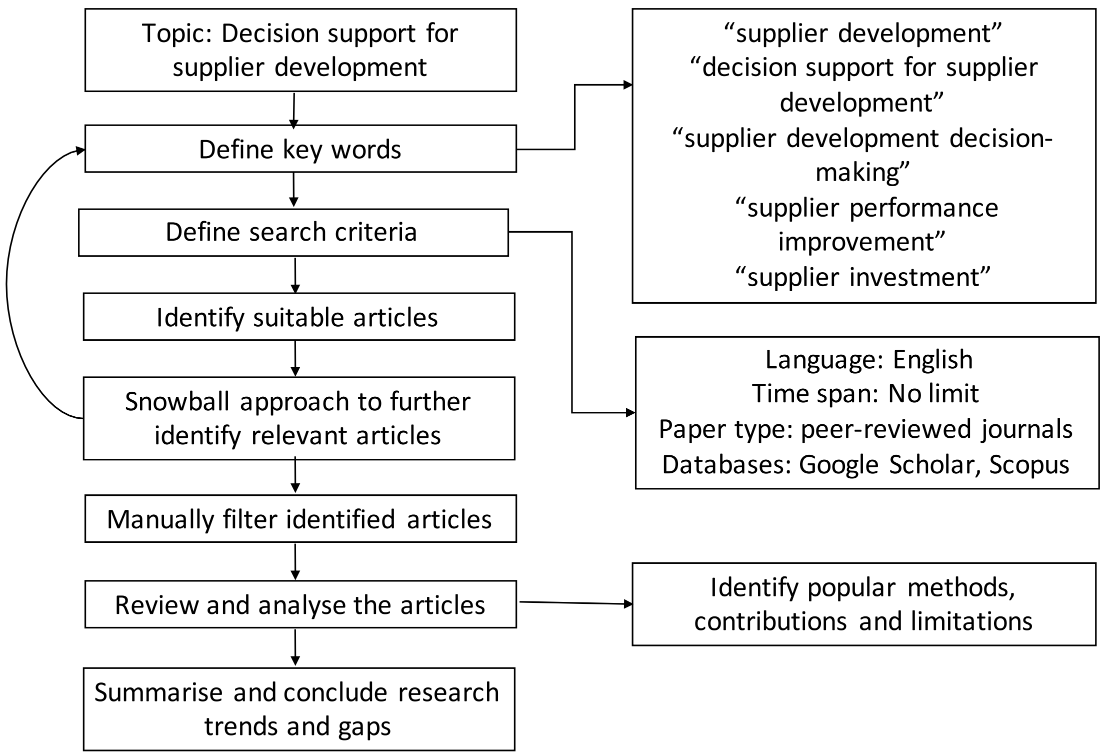

Figure 2.1: Methodology for literature review

2.2.1 Definition of Supplier Development

Krause et al. (2007) note that the term "supplier development" was first introduced by Leenders (1966) to describe the efforts made by manufacturers to improve suppliers' performance. The definition of "supplier development" then has been formally developed in a number of studies from different perspectives. This study reviewed a number of definitions for "supplier development" and summarised those that have been frequently cited in the research studies on supplier development in Table 2.1.

Table 2.1: Definition of Supplier Development

Authors	Definition
Hahn et al. (1990)	"A supplier development program, then, can be defined as any systematic organisational effort to create and maintain a network of competent suppliers. In a narrow sense, it involves adequate suppliers to meet the firm's requirements. In a broader perspective, it also involves activities designed to upgrade existing suppliers' capabilities to meet the changing competitive requirements."
Watts and Hahn (1993)	" A long-term cooperative effort between a buying firm and its suppliers to upgrade the suppliers' technical, quality, delivery and cost capabilities and to foster ongoing improvements".
Krause and Ellram (1997a)	"Any efforts of a buying firm with a supplier to increase its performance and/or capabilities and meet the buying firm's short and /or long-term supply needs".
Krause et al. (1998)	"Any set of activities undertaken by a buying firm to identify, measure and improve supplier performance an facilitate the continuous improvement of the overall value of goods and services supplied to the buying company's business unit".

According to the definitions, supplier development is an activity which may be a cooperative effort made by both suppliers and buyers but may also be an effort made only by a buying firm. The goal of the supplier development is to improve suppliers' performance and capabilities to eventually add value on the buying firm's performance. For this study, the definition proposed by Krause et al. (1998) is considered as the appropriate focus and an adapted definition of supplier development to this research is formulated as follows:

"Any set of activities undertaken by a buying firm to improve the performance and the capabilities of suppliers to meet the changing requirements and to add value on the buying firm's competitive advantages."

2.2.2 Qualitative Studies on Supplier Development

This section summarises the qualitative studies which have been highly cited within the research field of supplier development and points out the popular methods used as well as the important contributions made. The purpose of this section is to establish a comprehensive and solid knowledge and background of the research area for this research. A discussion regarding the review findings is provided in Section 2.2.2.1.

The majority of existing qualitative research for supplier development are exploratory studies which mainly focus on examining the benefits and effects of supplier development activities and identifying key factors, strategies as well as barriers for a successful supplier development activity. For example, Krause and Ellram (1997a) and Krause and Ellram (1997b) analyse the survey data collected from US buying firms to identify the critical elements and success factors for supplier development. The study finds that supplier development has a positive impact on the buyer-supplier relationship and points out that a long-term perspective and buyer's commitment are important elements for a successful supplier development activity. Krause (1999) further addresses that the degree of satisfaction regarding the supplier development results varies from firms to firms. Buying firms with high engagement into supplier development, such as supplier site visiting, or supplier training and education, are more satisfied with the results. In turn, supplier commitment and effective buyer-supplier communication may affect the degree of a buyer's engagement. Handfield et al. (2006) conduct case studies in electronic and automotive industries across multiple countries to investigate the pitfalls to be avoided in supplier development. The study also shows that the commitment and communication between suppliers and buyers play a very important role in a successful supplier development activity. Sako (2004) investigates the impact of the organisational capabilities on the effectiveness of supplier development in the Japanese automotive industry through interviews. The study finds that the corporate governance may influence the breadth and depth of supplier development. Williams (2007) conducts two case studies with Small to Medium-sized Enterprises (SMEs) in the UK to understand the effects of supplier development on smaller organisations. The study examines the effectiveness of different types of learning activities and emphasises that supplier development cannot be seen as a short-term investment with expectations of quick gains. Li et al. (2012) develop a path model to explore how supplier development practices affect buyer–supplier performance from the buying firm's perspective. The study shows that top management, supplier evaluation, and supplier strategic objectives are significant determinants of successful supplier development. Nagati and Rebolledo (2013) use a survey approach to investigate the effect and influential factors of supplier development from suppliers' perspective based on Canadian manufacturing firms. The study shows that supplier development and suppliers' performance improvement are directly related. In addition, trust is the key antecedent of suppliers' participation in a supplier development activity.

Some studies investigate the effect of supplier development activity from the types of strategies used, namely indirect and direct supplier development approach. In a supplier development activity, indirect supplier development often refers to informal strategies, such as increasing supplier performance goals or competitions, whereas direct supplier development often refers to investments of capital or human resources, such as supplier training. For example, Monczka et al. (1993) distinguish indirect supplier development from direct supplier development and conduct a survey study with US firms to explore the effects of different strategies for improving supplier performance. The study shows that firms tend to pursue indirect development strategies more than direct development strategies although direct supplier development is more effective. Similarly, Krause et al. (2000) develop two structural models to examine the relationship between development strategies and the improvement of suppliers' performance. The study suggests that direct supplier development plays a critical role in performance improvement. Wagner (2006) conducts empirical exploratory studies using a survey approach from European buying firms to explore the effects and inter-relationships between direct and indirect supplier development. The study concludes that all firms acknowledge the importance of a supplier development activity for the buying firms' business success, especially firms in an automotive industry. Wagner (2009) further explicitly investigates the effect of indirect and direct supplier development approaches. The study shows that both indirect and direct approaches are beneficial for improving suppliers' capabilities, but direct supplier development is more effective. Humphreys et al. (2011) analyse the survey data collected from firms in the Hong Kong electronic industry and examine the impact of supplier development on buyer-supplier performance based on statistical analysis. The study finds that direct supplier development has a significantly positive impact on supplier improvement. In addition, the study notes that some factors may contribute to the prediction of supplier improvement, such as long-term commitment, supplier strategic objective, effective communication.

Conceptual models are also developed to evaluate supplier development activities. For example, Hahn et al. (1990) formulate a generalised conceptual model that describes the organisational decision process associated with a supplier development activity including initiation and organisations, supplier evaluation, supplier development activities and plans, implementation and evaluation. The proposed model is used as a decision guideline for managers to evaluate activities to be conducted. A matrix is also developed to identify and design specific development activities. Modi and Mabert (2007) develop a conceptual model to evaluate the effect of knowledge transfer for supplier development based on survey data collected from US firms.

In recent years, studies on supplier development were published. Dalvi and Kant (2015) summarise the findings from these literature reviews and discuss the important aspects of benefits, criteria and activities of supplier development. The study finds that supplier development can bring improvement on both suppliers' performance and buyer's competitive advantage. The study highlights that a successful supplier development activity requires a strong and long-term supplier and buyer relationship, and also points out that more research is needed for investigating the trade-off between the supplier development activities and the risk associated with them. Glock et al. (2017) particularly focus on the studies on decision support models for supplier development and provide a systematic literature review regarding the existing decision support models. The study categorises the supplier development process into three major steps, namely "preparation phase", "development phase", "monitoring and evaluation phase". It shows that the majority of existing decision support models focus on the "preparation phase". This involves grouping or raking techniques to identify relevant suppliers or finding suitable supplier development strategies. The study also suggests that most of the research on decision making in supplier development is qualitative and there is a need of more quantitative studies.

2.2.2.1 Discussion of Findings

The review has shown that most of the previous qualitative studies for supplier development are exploratory studies, for which a survey approach is commonly used. Some studies were conducted in Western countries (Krause, 1999; Krause and Ellram, 1997a,b; Williams, 2007) and some were conducted in Asian countries (Humphreys et al., 2004; Li et al., 2012; Sako, 2004). Humphreys et al. (2011) appears to be the only study that was carried out across multiple countries including UK, US, Japan and Korea. Much research has emphasised the importance of conducting supplier development activities to ensure consistency in suppliers' capabilities and performance (Hahn et al., 1990; Monczka et al., 1993). In practice, Toyota, Honda, Nissan and General motors all have implemented supplier development programs to assist suppliers, which have led to quality improvements and cost reductions (Wagner, 2006). In particular, supplier development activities have already become a core activity for Otis Elevator's supply chain management (Modi and Mabert, 2007). There are a number of strategies that may be used for supplier development and many benefits that may be obtained from the activity. However, as supplier development is a long-term and costly activity, it also involves some challenges. A discussion about the key elements of supplier development activities is provided in the following.

Benefits

Krause et al. (2007) note that the most basic benefits of supplier development are cost reduction and improvement of on-time delivery and quality, which directly contribute to buyer's competitive advantages. A survey conducted by Krause et al. (1998) shows a more specific evidence of significant improvements of supplier performance which resulted from supplier development activity. It indicates a reduction of product defects by up to 90%, an improvement of on-time delivery by up to 15%, and a reduction of cycle time by up to 80%. Krause and Ellram (1997a) note that supplier development plays an important role in ensuring a reliable supply which in turn enhances buyers' competitive advantages. Humphreys et al. (2004) highlight that supplier development can also tackle important issues like customer satisfaction, and the uncertainty of customer demand. Bai and Sarkis (2011) argue that supplier development not only results in an improved buyer and supplier relationship but also leads to the improvement of the alignment between supplier performance and the buying firm's requirements which develops the capability of the supplier in providing customised products or services. Overall, a list of positive results of supplier development that have been widely recognised in the previous studies include:

- Cost reduction to maintain profit margin

- Quality improvement to reduce non-conformance rates
- Improvement of on-time delivery rates
- Reduction of cycle time, lead time and inventory
- Enhancement of the alignment between buyer's requirements and supplier performance
- Development of new products or services tailored to the buyer's requirements
- Improvement of productivity
- Improvement of buyer-supplier relationship
- Improvement of sustainability of supply chain

Strategies

Krause (1997) notes that "supplier development may range from limited efforts such as informal supplier evaluation and a request for improved performance, to extensive efforts, such as training of the supplier's personnel and investment in the supplier's operation." Krause et al. (1998) categorise the supplier development strategies into two types, namely reactive and strategic development strategies. Reactive development strategies focus on correcting suppliers' existing problems whereas strategic development strategies focus on enhancing the suppliers' competencies to contribute to the growth of the buying firm's competitive advantage. The study shows that firms achieve greater long-term benefit from the strategic development strategies than from using reactive development strategies. Another commonly recognised category for supplier development strategies is: indirect and direct supplier development. Indirect improvement strategies may include informal supplier evaluation and a request for improved performance in the supplier's operations, whereas direct improvement strategies may include capital or human resources investment, such as supplier personnel training, sites visiting, and knowledge transfer (Glock et al., 2017; Modi and Mabert, 2007; Wagner, 2009). Previous studies show that direct supplier development appears to be more effective than indirect supplier development, but it also requires a significant amount of human or capital resource investment from investors (Krause et al., 1998). Overall, a number of commonly used strategies for supplier development include:

- Personnel resources investment, temporary personnel transfer
- Supplier training and knowledge/experience transfer
- Information and technology sharing
- Evaluation, communication and on-site consultation

Challenges

Despite the positive results from supplier development, there are also a number of challenges in implementing a supplier development activity. One of the biggest challenges of conducting supplier development activities is the uncertainty in returns, which may lead investors to be reluctant to invest in supplier development, in particular when the activity requires lots of time and human resources (Dalvi and Kant, 2015; Krause, 1999; Wagner, 2009). In addition, a buyer may invest in developing suppliers who also provide parts to competitors (Glock et al., 2017). Therefore, the buyer's investment may involuntarily create benefits for competitors. This leads to decision making challenges. Lacking trust and motivation is another important factor that may result in unsuccessful supplier development. It is important for buyers to make sure that suppliers remain "economically viable" (Krause and Ellram, 1997a). Without a balanced benefit-sharing with suppliers, there is a lack of incentives for suppliers to actively participate in the activity which will further lead to undesirable outcomes. As noted by Handfield et al. (2006), buyers must illustrate potential benefits for suppliers in the first place, otherwise, suppliers may not be convinced that the development activity will benefit their organisations, and this will further affect suppliers' commitment to activity. Handfield et al. (2006) mentions that Honda, a Japanese automotive maker, expects suppliers to receive fair profits to make sure that suppliers are committed to the activity, even though suppliers' profit margins might be an easy area for cost reduction. Overall, a list of challenges of conducting supplier development activity include:

- Requires significant resources
- Uncertainty exists in the return to be obtained
- Requires trust, motivation and a long-term commitment
- May benefit competitors
- Incentives of suppliers' participation

2.2.3 Quantitative Studies on Supplier Development

As aforementioned, the supplier development process may be categorised into three phases: "preparation phase", "development phase" and "monitoring and evaluation phase" (Glock et al., 2017). Decision support for the "preparation phase" usually focuses on supplier evaluation and selection. Decision support for the "development phase" usually focuses on managing investment resources and providing effective resource allocation. Decision support for the "evaluation and monitoring phase" often focuses on determining actions on whether or not to modify or cancel the supplier development activity. We position the decision support provided by this research in the "development phase" as the proposed models aims to facilitate informed investment decisions in supplier development. Therefore, to keep the review focused, we only focus on the studies that develop decision support models for the "development phase" in particular, and categorise them into decision making for non-cooperative and cooperative supplier development investment.

Non-cooperative investment means that only one investor is involved. A decision support model for non-cooperative investment is usually developed from the perspective of minimising the risk/cost or maximising the benefit from the investment. Of those studies, few focus on only the investment decision making for a single supplier. For example, Friedl and Wagner (2012) develop a formal decision model to investigate a buyer's sourcing decision regarding whether the buyer should invest in developing an existing supplier or switch to an alternative supplier in order to realise lower purchasing costs. The authors assume that the buyer is risk neutral and has a single sourcing arrangement, and the cost savings from supplier development will be equally shared with the existing supplier. The proposed decision model analyses the expected profit from supplier development and from supplier switching and then determines the optimal decisions which provide maximal expected profits. Glock (2016) develops an optimisation decision support model for supplier development investment using non-linear programming with a particular focus on supplier training activities. The study assumes that a buyer sourcing a product from a supplier has an option to delegate their workers to suppliers to help improve the supplier's performance. The proposed model maximises the profit to be obtained from the supplier's performance improvement and determines the optimal number of workers delegated as well as the starting point and the duration of the activity.

Many studies focus on the investment decision of developing multiple suppliers. For example, Talluri et al. (2010) present an analytical model to support investment decisions on multiple suppliers' investment using Markowitz's portfolio model in which both cooperative and non-cooperative investment scenarios are considered. The study assumes that the expected return from the investment is known and follows a normal distribution, and the expected return varies across suppliers. The uncertainty of the return obtained from each supplier is measured using the movement of stock market prices. Under a budget constraint, optimal investment allocations are obtained by minimising the variance of the total expected return. Trapp and Sarkis (2016) develop an optimisation model to support decision making in the dual stages of the selection of suppliers and supplier development. The study uses binary integer programming to obtain optimal decisions by maximising suppliers' sustainability performance ratings while simultaneously satisfying supply chain-related constraints. Bhattacharyya and Guiffrida (2015) develop a cost-based optimisation model to obtain an optimal investment level for improving a supplier's delivery performance. The authors assume that a supplier's untimely (late and/or early) delivery can cause penalty costs for buying firms, such as inventory costs, or suspension of manufacturing, and a buyer intends to invest in multiple suppliers. The proposed model takes into account the time value of money and obtains an optimal investment point by minimising the total cost caused by a supplier's untimely deliveries under a budget constraint. Mizgier et al. (2017) develop a decision support model for capital allocation for supplier development using stochastic programming, in which multiple decision objectives are considered, namely the squared deviation between allocated capitals and losses from investment and the total cost of capital allocation. The authors take into account the uncertainty of returns from investment which is measured using the movement of stock market prices to reflect the uncertainty of the return obtained from each supplier. The proposed model provides an informed decision by analysing the trade-offs between risk and cost of supplier development. Zhang and Hong (2017) develop an optimisation model using non-linear programming to investigate the decision made for supplier's joint investments in cost reduction and quality improvement which appears to be the first paper taking into account the improvement of binary performance characteristics. The study assumes that the marginal cost of the improvement on supplier's performance is convexly increasing with respect to the improvement level. Optimal investment strategies are obtained by maximising the improvement level under a resource constraint. Quigley et al. (2018) develop a stochastic decision support model for investment in supplier quality development under uncertainty. A probabilistic model is developed within a Bayesian framework to capture both the epistemic and aleatory uncertainties in non-conformance rates which are assumed to follow a Poisson distribution with a Gamma prior. An optimal investment level is obtained by maximising the benefit from the investment. The proposed model was also applied to a real industry context to illustrate the use of the model to support practical decision making in supplier development.

Cooperative investment refers to the investment made by multiple parties. When multiple parties invest in a supplier, a main point of concern are situations where the investors are also competitors. Therefore, dealing with the spill-over effect and finding an equal profit for all investors are the key considerations for the development of a decision support model, for which a game theory approach and dynamic programming are majorly used by a number of studies. For example, Wang et al. (2014) develop a two-stage game theoretical model to explore the influences of the spill-over effect on manufacturers' investment level on supplier development, where in the first stage the manufacturers specify investment efforts and in the second stage the improvement of supplier performance is realised and the manufacturers place orders. The study assumes that two manufacturers who are competitors and share a common supplier that produces defective items and the production process is subject to uncertainty. The proposed model shows that a manufacturer's equilibrium improvement effort usually declines in situations involving market competition, market uncertainty or spill-over effect. Bai and Sarkis (2016) develop a game theoretical model to investigate both cooperative and non-cooperative investment and the strategies used to increase supplier production capability, such as, knowledge investments, capital resources investments. The proposed model provides insights on how to determine the optimal investment strategies using dynamic programming, which also maximises the organisational profit and the overall supply chain profit. Qi et al. (2015) investigate what happens when two competing firms invest in a shared supplier based on two scenarios, namely exclusive capacity - "a firm exclusively uses the invested capacity and disallows any other use even if there is leftover" and first-priority capacity – "the investing firm demands to fulfil its own order first, but the supplier is free to use any leftover". The study assumes that both firms do not know exactly the supplier's capacity at the time of investment, and all the parties are profit-maximising and risk-neutral. The study uses dynamic programming to investigate firms' investment decision on how different contractual forms affect the decision making and resulting profits for the two firms and when and to what extent the spill-over effect occurs. A similar approach is taken by Jin et al. (2019) that assumes a maximal market demand and investigates how much to invest to reduce the costs of a shared supplier by two competing companies. Friedl and Wagner (2016) assume that two risk-neutral buyers have an option to develop a common supplier in order to reduce purchasing costs. The study develops an optimisation decision model to analyse the effects of cooperative investment versus non-cooperative investment on optimal investment levels and buyers' expected profits using stochastic programming. The study shows that the improvement of supplier performance is lower from cooperative investment compared with the performance improvement from non-cooperative investment. Agrawal et al. (2016) assume that two firms invest in a shared supplier and the quality improvement potential of the supplier has two states: high or low, which is subject to uncertainties. The study develops a game theoretical approach to obtain the optimal strategies for firms that invest in suppliers' quality improvement. In addition, Brownian motion is used to analyse the profit to be obtained from the investment. The proposed model explores the spill-over effect and provides insights for buyers to identify appropriate strategies for improving the quality of a shared supplier. Different from other studies in cooperative investment decision making for supplier development, Zhu et al. (2007) investigate the cooperative investment made by a buyer and a supplier rather than only buyers. The study considers that a buyer designs a product and owns the brand and the supplier manufactures the product. The study assumes that both a buyer and suppliers incur quality-related costs whenever a non-conforming item is sold to a customer. The study explores the roles of different parties in supplier quality improvement and develops a model to determine optimal investment options for both parties using dynamic programming which minimises the total cost within the supply chain. The aleatory uncertainty in the supplier quality control process is measured by the non-conformance rate.

2.2.3.1 Discussion of Findings

Quantitative studies for decision making in supplier development investment has attracted increasing attention in recent years. The existing decision support models may be categorised from the aspect of cooperative investment decision making and non-cooperative investment decision making. For non-cooperative investment, the objective is often to minimise supplier risks/costs or to maximise expected returns and mathematical optimisation approaches are majorly used (Bhattacharyya and Guiffrida, 2015; Glock, 2016; Mizgier et al., 2017; Talluri et al., 2010; Zhang and Hong, 2017). For cooperative investment where multiple investors are involved, dealing with the spill-over effect and finding an equilibrium profit for all investors are the key consideration taken for the development of decision support models and the commonly used approaches are game theory and dynamic programming (Agrawal et al., 2016; Bai and Sarkis, 2016; Jin et al., 2019; Qi et al., 2015; Wang et al., 2014). Alternatively, Friedl and Wagner (2012) investigate the options between supplier development and supplier switching and compare the benefit from each option. Furthermore, Friedl and Wagner (2016) compare the benefit obtained from non-cooperative and cooperative investment and conclude that the improvement of supplier performance is lower from a cooperative investment compared to a non-cooperative investment. In addition, the majority of the existing models are developed for one particular phase of supplier development. To the best of our knowledge, the study from Trapp and Sarkis (2016) is one of the few which supports decision making in both the "preparation phase" and the "development phase".

The potential improvement of suppliers' performance is a key factor for investment decisions in supplier development. In general, a supplier's key performance indicators (KPIs) include supplier cost, quality and on-time delivery. Supplier cost reduction is concerned with the reduction of buyer's purchasing price which declines as a supplier's production cost is reduced, whereas supplier quality or on-time delivery improvement is concerned with the reduction of the buyer's losses caused by supplier's undesirable performance, such as unsatisfactory quality or manufacturing delay. Most existing studies focus on the improvement of one particular aspect: cost reduction (Friedl and Wagner, 2016; Jin et al., 2019; **?**), quality improvement (Quigley et al., 2018; Zhu et al., 2007) or on-time delivery improvement (Bhattacharyya and Guiffrida, 2015). Zhang and Hong (2017) investigate the investment made for improving both supplier's cost reduction and quality improvement which appears to be the first paper considering multiple performance characteristics.

The mathematical optimisation approach is the most popular method used in the existing quantitative decision support for supplier development investment, such as, integer programming (Trapp and Sarkis, 2016) non-linear programming (Glock, 2016; Talluri et al., 2010; Zhang and Hong, 2017; Zhu et al., 2007), stochastic programming (Friedl and Wagner, 2012, 2016) or dynamic programming (Agrawal et al., 2016; Bai and Sarkis, 2016; Jin et al., 2019; Mizgier et al., 2017; Wang et al., 2014). Different from other authors, Quigley et al. (2018) consider a stochastic modelling framework and propose a novel modelling approach which provides optimal investment decisions in supplier development by examining the mathematical optimisation properties of the proposed model.

A numerical experiment is mostly used to illustrate the behaviour of the proposed model in the previous studies (Bhattacharyya and Guiffrida, 2015; Glock, 2016; Talluri et al., 2010; Trapp and Sarkis, 2016). There are very few studies that are grounded in and used the proposed model in a real-world case (Quigley et al., 2018). In addition, we found that only limited studies have investigated the inherent uncertainty existing in the supplier development investment. In particular, the majority of those studies only focus on the aleatory uncertainty whereas the epistemic uncertainty is much neglected. In the next section, a discussion about decision making in supplier development investment under uncertainty is provided.

2.3 Decision Making for Supplier Development under Uncertainty

Previous studies with consideration of decision making uncertainty analyse the uncertainty in supplier development outcomes from different perspectives. Some studies associate the uncertainty of the outcomes with the uncertainty of supplier's capability and performance (Qi et al., 2015; Quigley et al., 2018; Talluri et al., 2010; Wang et al., 2014; Zhu et al., 2007). For example, Zhu et al. (2007) associate the uncertain benefit from supplier development with the aleatory uncertainty in a supplier's nonconformance rate. Talluri et al. (2010) note that "the efficacy of supplier development programs depends on the existing capabilities of a supplier and the effectiveness with which the manufacturing firm can leverage these programs and investments. Thus, it is entirely possible that returns from these investments may vary across multiple suppliers, an indication of risk in terms of uncertain returns in supplier development investments". In a study of Talluri et al. (2010), the uncertainty of the benefit from developing a supplier is associated with the suppliers' capability for which the movement of the supplier's stock market price is used as a measurement. The study points out that such an approach relies heavily on the availability of historical data and thus cannot be used for new suppliers or suppliers who are not publicly listed. The same approach has also been taken by Mizgier et al. (2017). Qi et al. (2015) associate the uncertainty of profits with a supplier's stochastic capacity which composes of a supplier's base capacity and the capacity improvement via buyers' investment. Quigley et al. (2018) associate the decision uncertainty with the uncertainty of a supplier's nonconformance rate of which both the epistemic and aleatory uncertainty are captured using the Poisson-Gamma model. Some studies associate the investment decision with market demand uncertainty which is more concerned with the investment decision made by multiple competing buyers (Friedl and Wagner, 2016; Wang et al., 2014). In particular, Wang et al. (2014) consider the uncertainty in both supplier performance and demand, in which demand uncertainty is assumed to be modelled using a log-concave probability distribution, such as the gamma distribution, or the normal distribution, whereas the randomness of supplier performance is measured using a factor which is between 0 to 1. There are also some studies that directly model the uncertainty of the outcomes of supplier development investment (Agrawal et al., 2016; Meisel, 2012). For example, Meisel (2012) uses a random factor to represent the probabilistic realisations to different outcomes from supplier development investment activity. Agrawal et al. (2016) use Brownian motion to model the profit generating process, in which the uncertainty of the average profit to be obtained is associated with the uncertainty of the potential improvement in a supplier's capability which is measured by buyers' beliefs.

To summarise, the return from supplier development investment often involves uncertainty and measuring this uncertainty is important. To analyse the decision uncertainty, many studies argue that supplier performance plays an important for the decision making, as suppliers differ in their capabilities and the variation of supplier performance can lead to different returns under the same level of resources invested. Some studies consider that the investment decision is affected by market demands which is often concerned with investment decisions made by competing buyers. There are also a few studies that directly model the uncertainty of the outcomes of supplier development investment. We note that although the number of studies that take into account the uncertainty of the benefit from supplier development investment has been growing in recent years, the majority of previous studies only focus on the aleatory uncertainty whereas the epistemic uncertainty has been much neglected.

2.4 Research Trends and Gaps

Research on supplier development has received increasing attention in recent decades. The majority of research for supplier development at an early stage are exploratory studies which mainly focus on examining the benefits and effects of supplier development activities and identifying key factors, strategies as well as barriers for a successful supplier development activity (Hahn et al., 1990; Krause and Ellram, 1997a,b; Modi and Mabert, 2007; Monczka et al., 1993; Sako, 2004; Wagner, 2006, 2009; Williams, 2007). In recent years, the focus of research on supplier development has gradually switched from qualitative studies to quantitative studies. The number of quantitative decision support models for supplier development investment has been rising, especially since 2010. The types of these studies are diversified, which can be categorised from non-cooperative investment (Bhattacharyya and Guiffrida, 2015; Quigley et al., 2018; Talluri et al., 2010) to cooperative investment (Agrawal et al., 2016; Bai and Sarkis, 2016; Qi et al., 2015; Wang et al., 2014), from single stage to multiple stages (Trapp and Sarkis, 2016), or from single objective to multiple objectives Mizgier et al. (2017).

Despite the increasing number of decision support models that have been developed for supplier development, this research field still requires attention, as many areas have not been explored yet. Similar findings have also been addressed in other studies. For example, Dalvi and Kant (2015) highlight the need of more studies on analysing the trade-off between the supplier development activities and the risk associated with it. Glock et al. (2017) suggests more optimisation models to calculate optimal investment volumes for supplier development programs, and more applications to real world scenarios to illustrate the benefits and practicability of the proposed models. We have identified a number of limitations revealed in the existing studies regarding decision support for supplier development and and in the following we summarise several important research areas for future research:

(1) Decision Making under Uncertainty

An important aspect for future research is the uncertainty involved in supplier development investment. The sources of uncertainties may be evaluated from the several aspects, such as present condition of the events (e.g. information and understanding of objectives), or future conditions of the events (e.g. intervention actions). Existing studies that consider the decision uncertainties in supplier development investment are rather limited. More research in this regard is much needed to provide meaningful decision support for supplier development investment.

(2) Quantitative Decision Support Model

Although the number of quantitative studies in supplier development is growing, research in this field still requires attention as many important decisions made in supplier development investment require specific solutions, such as, the amount of investment, expected benefits, duration of the activity.

(3) Multi-Stage Decision Support Model

Decision making in supplier development investment in general involves three phases: the "preparation phase", the "development phase" and the "evaluation and monitoring phase". Existing decision support models focus much more on a single phase and only few studies have taken an integrated view. Therefore, more multiple-stages decision support models are needed to address this research gap.

(4) Multi-Objective or Multi-criteria Optimisation Model

Existing decision support models consider one objective in particular and the majority of them focus on maximising the benefit. Other criteria should also be taken in account, such as the duration of a supplier development activity. In addition, the impact of the supplier development activity on supplier performance may be analysed from multiple aspects. Most existing studies only focus on one particular supplier KPI, which may underestimate the total benefit from the supplier development investment.

(5) Time Value of Money

Supplier development is a long-term investment. The majority of existing studies have not taken into account the impact of the time value of money on decision making, which still requires more attention.

(6) Application in Real-World Scenarios

The majority of existing studies have not yet evaluated the performance of the proposed models in practice. More empirical applications are needed to examine the practicability of theoretical decision support models and to gain insights from real-world problems.

2.5 Discussion

In this chapter, we have reviewed a number of papers that have been published in the field of supplier development and particularly focused on the existing quantitative decision support models developed for supplier development investment (Friedl and Wagner, 2012; Glock, 2016; Quigley et al., 2018; Talluri et al., 2010; Zhang and Hong, 2017). We note that since 2010 the focus of research on supplier development has gradually switched from qualitative studies to quantitative studies. However, quantitative studies in this field still require much attention, for which a number of research gaps are identified to guide possible research directions.

This research contributes to the research field by developing quantitative decision support models that particularly take into account the inherent uncertainties in the return obtained from supplier development investment. The methodological basis is adopted from the study by Quigley et al. (2018) as we note that it is the only study that has explicitly modelled both the aleatory and the epistemic uncertainty using quantitative techniques. This research extends the model of Quigley et al. (2018) and employs different probabilistic models which allows for analysing different types of suppliers' KPIs. Based on the type of KPIs, new decision support models are proposed to provide informed decisions including an optimal investment level for developing suppliers' performance under uncertainty and a maximal amount of investment for buying down the epistemic uncertainty. In the next chapter, we will specify the key research problems and detail the modelling techniques employed as well as the modelling assumptions with their rationale.

Chapter 3

Problem Structuring and Methodology with Rationale

3.1 Problem Structuring

In this research, two analytical decision support models for supplier development investment are developed under the context that a buyer potentially faces financial losses from suppliers' undesirable performance, such as, non-conformances or late deliveries. The buyer is considering engaging in a supplier development activity to improve suppliers' KPIs. However, as a supplier development activity requires significant human and financial resources, a risk exists that the benefit obtained from the activity may not be enough to offset the cost of the activity. The buyer needs to make a decision on whether or not to invest in the activity, or whether to delay the investment decision to learn more about suppliers' true performance and then decide whether or not to invest. A diagram is presented in Fig.3.1 to show a general decision making action that may be taken by a buyer. More specifically, there are a number of different suppliers that may need to be developed, of which the level of KPIs for each supplier are different. If the buyer chooses to invest in developing a supplier, then after a development activity, the consequence of this action may be either the supplier's KPI becomes desirable or it is not. If the buyer decides to not invest in developing a supplier's KPI as he/she thinks it is not worth doing so, without a development activity, the consequence of this action may be either the supplier's KPI stays desirable or it is not. The dashed arrow represents a third action that the buyer chooses to not invest in developing a supplier for now and to learn more about the supplier's KPI as he/she has too little information about the supplier to make a decision.

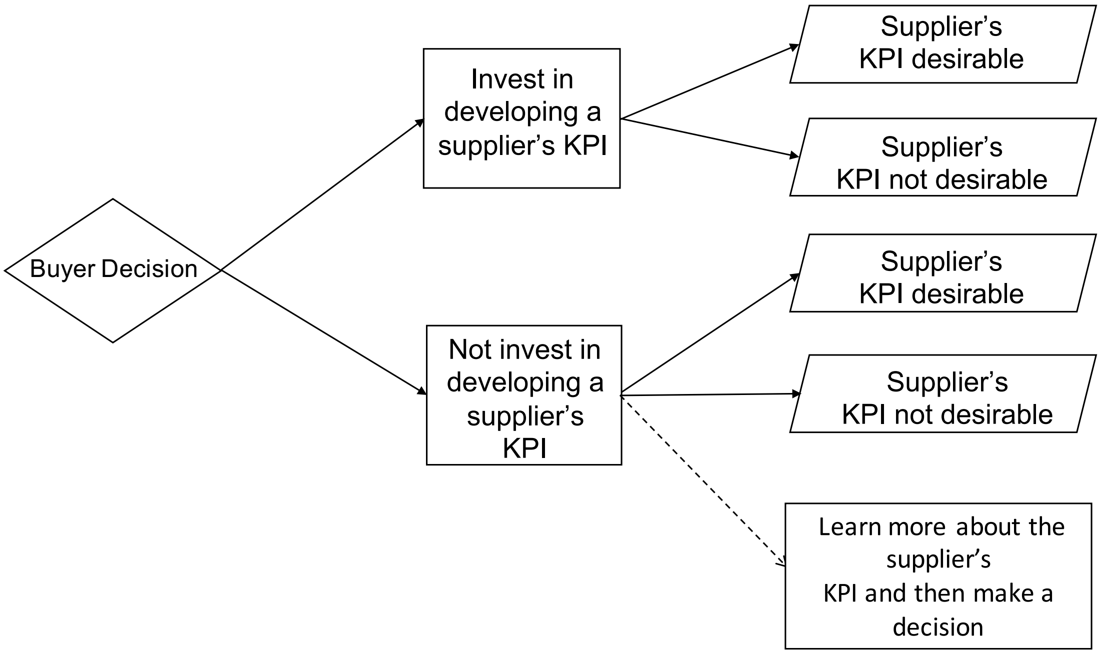

Figure 3.1: Buyer decision modelling concept where diamond node: decision; rectangle node: action; parallelogram node: output

Therefore, the key research questions to be answered in this research are as follows:

- How can a buyer decide whether it is worth investing in developing a supplier's KPI given that the outcome of the investment is uncertain; and if investing, what is the optimal investment level and what is the expected return?
- How can a buyer decide whether it is worth developing a supplier if a buyer has too little information of the supplier; should the buyer investing in gathering more

information about the supplier; and if so, how much should be invested?

3.2 Methodology with Rationale

3.2.1 Quantitative Modelling Approach for Decision Making

This research uses a quantitative approach to model the proposed decision problem. Anderson et al. (2015) note that the core of Management Science is to use rigorous and analytical methods to provide decision support for business problems. The analysis of decision problems may be differentiated based on the types of methods used, namely the qualitative approach and the quantitative approach. Decision making under qualitative analysis often focuses on collecting different viewpoints and perceptions to formulate rich descriptions of the decision making problem. By contrast, decision making under quantitative analysis primarily focuses on the application of mathematical modelling techniques for the decision problem. In this research, the key decision to be investigated is "how much it is worth for a buyer investing in a supplier development activity". Quantitative models are typically developed to deal with such problems as it allows decision makers to determine the project cost and/or profit associated with an established quantity of interest (Anderson et al., 2015). In addition, quantitative models attempt to express and represent the given managerial situation as accurately as possible and thus enables providing informed solutions (Oral and Kettani, 1993; Pidd, 2009). Therefore, we consider quantitative modelling is an appropriate approach for investigating the research question.

3.2.2 Probabilistic Models for Measuring Uncertainties

A core feature of the proposed decision support model is that it takes into account both the aleatory and epistemic uncertainty of the benefit obtained from the supplier development activity. As aforementioned, one of the biggest challenges which makes decision makers feel reluctant to conduct a supplier development activity is the uncertainty of returns (Agrawal et al., 2016). In this research, we associate the uncertainty of the benefit from supplier development investment with the uncertainty of a supplier's performance, as we consider a supplier's improvement potential has a direct impact on the benefit to be obtained. In particular, both aleatory and epistemic uncertainty are modelled. In the context of this research, aleatory uncertainty represents the uncertainty resulted by the pure randomness of suppliers' performance which cannot be reduced. Epistemic uncertainty represents the uncertainty existing in a buyer's prior knowledge about a supplier's performance before making the decision. Such uncertainty can be reduced if more information about a supplier's true capability becomes available. Therefore, modelling epistemic uncertainty is particularly useful for estimating performance of new suppliers or suppliers with scarce historical data.

To capture the aleatory uncertainty of a supplier's capability, two probabilistic models are used, namely the multinomial distribution and the Non-Homogeneous Poisson process model (NHPP). These models are relevant for contexts where supplier performance data are in either categorical form, such as classes corresponding to degrees of late delivery, or in count form, such as the number of non-conformances, respectively. In addition, the probabilistic models are developed within a Bayesian framework, in that a conjugate prior probability distribution is used to capture the epistemic uncertainty. This essentially models the uncertainty of the parameters of the probabilistic models that are estimated by a buyer's information about a supplier. We note that the epistemic uncertainty has been much neglected in the previous studies. Only Agrawal et al. (2016) and Quigley et al. (2018) take into account the epistemic uncertainty in their proposed decision support models. Agrawal et al. (2016) assume that there are two types in a supplier's improvement potential, namely "high" and "low" and epistemic uncertainty exists in the type of the improvement outcome, which is measured by buyers' beliefs and can be updated using Bayes rules. However, the way how a supplier's improvement potential is measured is not specified in the study. Quigley et al. (2018) develop a stochastic model within a Bayesian framework and explicitly capture the epistemic uncertainty in a supplier's non-conformance rate using a Gamma-Poisson distribution. A similar methodology of Quigley et al. (2018) is taken in this research. Based on the Bayes theorem, the epistemic uncertainty can be reduced as more information is obtained. If perfect information is obtained, we can consider that all the epistemic uncertainty is removed. Therefore, an expected value of perfect information (EVPI) can be measured by the difference between the expected profit under perfect information and the expected profit under uncertainty. This provides a maximal amount that a rational buyer should invest in learning more about a supplier's true performance when a buyer has too little information about the supplier to make a decision. As noted by Quigley et al. (2018), computing the EVPI "provides an assessment of how much it is worth spending, at most, to remove all epistemic uncertainty, and hence provides an upper bound on the amount it would cost to reduce uncertainty if information gained was partial and imperfect".

To estimate the prior distribution, this thesis shows how empirical Bayes inference is used. Unlike the traditional Bayes method which replies on subjective judgements, empirical Bayes primarily relies on quantitative data. It allows the quantity of interest to be described by a probability distribution and provides estimates using the pooling of empirical data from multiple events with similarities (Quigley et al., 2011; Quigley and Walls, 2018). This can be very useful for estimating a supplier whose data is not available. In the context of this research, if a new supplier's KPI is to be estimated, a pool of performance data from similar suppliers may be formed to estimate the prior empirically rather than asking a supply chain manager to make a subjective assessment of the prior probability. The estimation of the prior from an empirical Bayes approach results in an overall rate of supplier's non-conformance occurrences which is a weighted average of the frequencies of all suppliers' non-conformances across the pool. The accuracy of the estimate relies on the degree of the homogeneity of the pool of suppliers' performance data. That is, the more homogeneous the performance data is, the more accurate an estimate can be expected. Therefore, it is important to use data from suppliers who present high degree of similarity with the supplier to be estimated. Constructing such data pools could be done through elicitation. More details can be found in Quigley and Walls (2018). To estimate the prior parameters, a prior predictive distribution can be constructed which represents the distribution of the observed performance data. A number of standard parameter estimation approaches can be used for estimating the prior parameters, such as Maximum Likelihood Estimation (MLE) and Method of Moments (MoM). Then, based on the Bayes theorem, the prior distribution can be updated to obtain a posterior distribution as a new observation becomes available. Essentially, the posterior can be considered as a new prior which also implies reduced epistemic uncertainty. The use of the empirical Bayes inference approach not only distinguishes epistemic uncertainty from aleatory uncertainty but also overcomes the use of subjective bias involved in the classical Bayes approach.

3.3 Modelling Assumptions

In this research, the decision support models are developed under the following common assumptions, in which 1-7 are the assumptions of the decision making context and 8-11 are the assumptions for modelling:

1. A buyer incurs a financial loss as a consequence of suppliers' undesirable performance, such as late delivery, or poor quality;
2. The buyer intends to have a long-term relationship with the suppliers;
3. The buyer has access to databases (e.g. ERP system) regarding the records of suppliers' orders, such as, order quantity, type, due and actual delivery dates, non-conformance;
4. The buyer intends to invest capital or human resources in a supplier development activity to improve suppliers' KPI and expects to reduce the financial loss caused by suppliers' undesirable performance;

5. The buyer is risk-neutral, and the reduction of the financial loss to be obtained from the activity is uncertain;
6. A number of suppliers are pre-selected for development and do not provide parts to the buyer's competitors;
7. The buyer has an agreement with all suppliers on how the benefit obtained from the development activity is distributed;
8. The supplier's performance is stochastic, and the buyer does not have the perfect information of supplier's true performance;
9. The performance status of delivered items are statistically independent given the rate of occurrence;
10. The investment level is non-negative;
11. The marginal profit obtained from supplier development investment declines as the amount of investment increases.

Assumptions 1-4 indicate that the buyer can get access to the suppliers' database and has the motivation to invest in a supplier development activity in order to benefit from the supplier's KPI improvement. Assumption 5 assumes that the buyer acknowledges the potential risk of the investment but is indifferent to it as long as the expected return is positive. Assumption 6 implies that no competitors will benefit from the buyer's investment. Therefore, there are no spill-over effects that may affect the buyer's investment decision. Assumption 7 acknowledges any agreed form of benefit-sharing between the buyer and suppliers from the investment. This also establishes suppliers' motivation to participant in the activity. Assumption 8 recognises the randomness of suppliers' performance and the epistemic uncertainty in buyer's knowledge about suppliers' true performance. Assumption 9 indicates the occurrence of one event does not affect the probability of the occurrence of another event. This implies that the occurrence of a supplier's undesirable performance is random and independent given its true rate of occurrence. For example, a supplier's current late delivery does not have an impact on whether the next delivery will be late or on-time. Note that in this research, we do not consider events that may result in a number of consecutive undesirable performance over a fixed period of time, such as, nature disaster, labour strike, or machine breakdown. Therefore, we consider that it is reasonable to assume independent occurrence of events. We acknowledge that dependency may exist between events under certain circumstances and this will be taken it into account in future work. Assumption 10 suggests that we do not consider circumstances where a buyer asks a supplier for help, such as, borrowing some of the supplier's workforce, as the proposed model is developed under the context that suppliers may underperform. Assumption 11 is a reasonable consideration for describing a learning curve and has also been taken into account in a number of studies under the context of supplier development (Bhattacharyya and Guiffrida, 2015; Quigley et al., 2018; Zhu et al., 2007). This implies that the improvement of supplier performance eventually will reach a natural limit.

3.4 Summary

This section describes the decision problems to be investigated in this research and discusses the methodology employed. In this research, two quantitative models are developed for to provide support for the decision problems structured. The proposed models are developed from a buyer's perspective. These models analyse the expected profit from supplier development investment with a particular focus on the measurement of the uncertainty involved. The research associates the uncertainty of the benefit from the investment with a supplier's performance uncertainty, for which two different probabilistic modelling techniques are used. In particular, the epistemic uncertainty is modelled and represented using a prior distribution, and empirical Bayes inference is employed for the prior estimation. Chapter 4 and 5 will detail the development of the proposed models based on the type of supplier performance data.

Chapter 4

Multinomial-Dirichlet Distribution for Modelling Supplier KPI

4.1 Introduction

In this chapter, supplier performance is classified into different categorises based on different risk levels. The uncertainty of the occurrences of each performance category is assumed to follow the multinomial distribution. The proposed decision support model is developed from the perspective of analysing the benefit obtained from a supplier development activity. An optimal investment level is obtained by examining the optimisation property of the proposed model. The value of perfect information is also taken into account to analyse whether it is worth learning more about the supplier before making a decision. In addition, decision making in investing in multiple suppliers under a budget constraint is also investigated, for which Lagrange multiplier is used to obtain an optimal investment allocation. A numerical experiment is provided to illustrate the behaviour of the proposed model.

The remainder of this chapter is structured as follows. Section 4.2 discusses methodological issues. Section 4.3 details the use of probabilistic models on capturing the uncertainty of supplier performance along with the statistical inferences process. Section 4.4 explains the development of decision support models for supplier development investment and discusses the managerial insights obtained. Section 4.5 provides a numerical example to illustrate the behaviour of the proposed decision support model. Section 4.6 summarises this chapter.

4.2 Methodological Considerations

We note that previous studies have not categorised a supplier's performance into multiple characteristics. This is important as the level of a buyer's financial loss caused by supplier undesirable performance in fact depends on the degree of severity of the damage. For example, Bhattacharyya and Guiffrida (2015) differentiate a supplier's late delivery from early delivery. Zhang et al. (2014) categorise a supplier's late delivery based on the risk level. Damage caused by a delay between 0 to 3 days is classified as general risk whereas the damage caused by a delay between 4 to 10 days is classified as serious risk. In this research, we propose to use multiple classes to categorise the characteristics of a supplier's performance based on the corresponding risk levels. By doing so, different non-conformance cost can be assigned to different performance categorises given the risk levels. The performance data is considered in categorical form, and the uncertainty of a supplier's performance is modelled using the Dirichlet-multinomial distribution. The exposure to risk for the buyer is measured by a total number of ordered items from the supplier. The benefit of the activity is represented by the reduction of the financial loss after a supplier development activity and is assumed to be exponentially declining with respect to an investment level given an effectiveness rate. This implies that the marginal benefit declines as the investment level increases. The profit of the activity is expressed as the difference between the benefit and the investment level.

4.3 Supplier Uncertainty Modelling

4.3.1 Multinomial Distribution

Let P_M denote the multinomial distribution and n_j denote the total number of orders as the exposure to risk for the buyer from supplier j. Let p_{ij} measure the probability concerning the frequency of the occurrence of the performance category i and n_{ij} denote the corresponding number of occurrences. Given a random vector of the probabilities $\mathbf{P_j} = [p_{1j}...p_{Ij}]$, the probability distribution of the occurrence at each performance category $\mathbf{N_j} = [n_{1j}...n_{Ij}]$ for supplier j can be expressed in Eq.(4.1).

$$P_M\left(\mathbf{N_j}|\mathbf{P_j}\right) = \frac{n_j!}{n_{1j}!..n_{Ij}!}\prod_{i=1}^{I} {p_{ij}}^{n_{ij}}, \tag{4.1}$$

where, $\sum_{i=1}^{I} n_{ij} = n_j$ and $\sum_{i=1}^{I} p_{ij} = 1$.

4.3.2 Parameter Estimation

In this section, we first describe two methods for parameter estimation, namely Maximum Likelihood Estimator(MLE), and Method of Moments (MoM), and then detail the prior estimates under empirical Bayes inference using these two methods to capture the epistemic uncertainty.

4.3.2.1 Maximum Likelihood Estimator

MLE provides parameter estimates by maximising the likelihood of the occurrence of an observed event under an assumed probability distribution for the quantity of interest (Fisher, 1922). Such an approach typically provides efficient and resilient estimates if the assumptions made are valid, although it may be computationally demanding. The MLEs for the multinomial distribution (Eq.(4.1)) can be obtained as follows. Let L

denote a likelihood function, for a given $\mathbf{N_j}$, a likelihood function of $\mathbf{P_j}$ essentially can be written as $L(\mathbf{P_j}|\mathbf{N_j}) = P_M(\mathbf{N_j}|\mathbf{P_j})$. As the logarithm transforms a product of densities resulted by the likelihood function into a sum which is easy to compute, a log-likelihood function of $\mathbf{P_j}$ for a given $\mathbf{N_j}$, denoted by l, is obtained:

$$l(\mathbf{P_j}|\mathbf{N_j}) = \ln(n_j!) - \sum_{i=1}^{I} \ln(n_{ij}!) + \sum_{i=1}^{I} n_{ij} \ln p_{ij}. \tag{4.2}$$

By maximising Eq.(4.2), the MLEs of p_{ij} can be obtained:

$$\hat{p}_{ij} = \frac{n_{ij}}{n_j}. \tag{4.3}$$

4.3.2.2 Method of Moments

MoM estimates model parameters by matching the theoretical moments from the distribution with the observed moments from the data (Zhao and Ang, 2003). For a continuous distribution, the k_{th} theoretical moments are generated through $E\left(X^k\right) = \int\limits_{-\infty}^{\infty} x^k f(x)dx$, whereas for a discrete distribution, the theoretical moments are generated through $E\left(X^k\right) = \sum\limits_{x \in S_x} x^k f(x)$ where $f(x)$ is the probability distribution function and S_x is the support of x. Then, the estimates of the unknown parameters can be obtained by letting $E\left(X^k\right) = \frac{1}{n}\sum\limits_{i=1}^{n} x_i^k$, where x_i is the observed data and n is the number of observations. In general, the first four moments are often used for estimating the unknown parameters (Pearson et al., 1979; Zhao and Ang, 2003). Such an approach tends to provide unbiased estimators under weak distributional assumptions, but the error associated with this approach can be higher than MLE. However, for a wide variety of models they have closed-form solutions or are computationally easier than MLE (Lu et al., 2010; Wooldridge, 2001).

4.3.2.3 Empirical Bayes

As discussed in Chapter 3, this research uses empirical Bayes inference to estimate the prior distribution which is employed for modelling the epistemic uncertainty. In this chapter, Dirichlet distribution is employed as a generic conjugate prior representing the vector of the probabilities of the parameters of the multinomial distribution within a pool of suppliers. It is assumed that the vector of probabilities of the parameters across the pool are distributed from a Dirichlet distribution. For each supplier, a posterior distribution can be obtained by updating the empirically estimated prior through a Bayesian theorem. More specifically, let η denote the Dirichlet prior distribution function with parameters $\mathbf{a} = [\alpha_1 ... \alpha_I]$ to describe the epistemic uncertainty on the random vector $\mathbf{P_j}$ for supplier j. Mathematically, this can be expressed as:

$$\eta\left(\mathbf{P_j}|\mathbf{a}\right) = \frac{\Gamma\left(\sum\limits_{i=1}^{I} \alpha_i\right)}{\prod\limits_{i=1}^{I} \Gamma\left(\alpha_i\right)} \prod_{i=1}^{I} {p_{ij}}^{\alpha_i - 1}. \tag{4.4}$$

By taking the expectation of the multinomial distribution function (Eq.(4.1)) with respect to the Dirichlet prior (Eq.(4.4)), a predictive distribution of the random vector $\mathbf{N_j}$ under the prior parameters $\mathbf{a}$ for supplier j can be obtained:

$$P_M\left(\mathbf{N_j}|\mathbf{a}\right) = \frac{n_j!}{n_{1j}!..n_{Ij}!} \frac{\Gamma\left(\sum_{i=1}^{I} \alpha_i\right)}{\prod_{i=1}^{I} \Gamma\left(\alpha_i\right)} \frac{\prod_{i=1}^{I} \Gamma\left(\alpha_i + n_{ij}\right)}{\Gamma\left(\sum_{i=1}^{I} \alpha_i + n_{ij}\right)}. \tag{4.5}$$

Based on the Bayesian theorem, we can obtain the posterior in Eq.(4.6), which is in the same form of the prior with different parameters $\mathbf{a}$+$\mathbf{N_j}$

$$\eta\left(\mathbf{P_j}|\mathbf{N_j}\right) = \frac{\Gamma\left(\sum_{i=1}^{I} \alpha_i + n_{ij}\right)}{\prod_{i=1}^{I} \Gamma\left(\alpha_i + n_{ij}\right)} \prod_{i=1}^{I} {p_{ij}}^{\alpha_i + n_{ij} - 1}. \tag{4.6}$$

To estimate the prior parameters $\mathbf{a} = [\alpha_1 ... \alpha_I]$, the study first uses MLE. As empirical Bayes allows for pooling dataset from similar suppliers' performance, we assume that there are a number of J similar suppliers' performance data available, based on Eq.(4.5) the likelihood function of $\mathbf{a}$ can be written as:

$$
\begin{aligned}
L\left(\mathbf{a}|\mathbf{N_1}...\mathbf{N_J}\right) &= \prod_{j=1}^{J} P_M\left(\mathbf{N_j}|\mathbf{a}\right) \\
&= \prod_{j=1}^{J} \frac{n_j!}{n_{1j}!..n_{Ij}!} \frac{\Gamma\left(\sum\limits_{i=1}^{I}\alpha_i\right)}{\prod\limits_{i=1}^{I}\Gamma\left(\alpha_i\right)} \frac{\prod\limits_{i=1}^{I}\Gamma\left(\alpha_i+n_{ij}\right)}{\Gamma\left(\sum\limits_{i=1}^{I}\alpha_i+n_{ij}\right)}.
\end{aligned}
\tag{4.7}
$$

The log-likelihood function can be written as:

$$
\begin{aligned}
l\left(\mathbf{a}|\mathbf{N_1}...\mathbf{N_J}\right) = \sum_{j=1}^{J}\left[\ln\left(n_j!\right) - \sum_{i=1}^{I}\ln\left(n_{ij}!\right) + \sum_{i=1}^{I}\ln\Gamma\left(\alpha_i+n_{ij}\right) - \ln\Gamma\left(\sum_{i=1}^{I}\alpha_i+n_{ij}\right)\right] \\
+ J\left[\ln\Gamma\left(\sum_{i=1}^{I}\alpha_i\right) - \sum_{i=1}^{I}\ln\Gamma\left(\alpha_i\right)\right].
\end{aligned}
\tag{4.8}
$$

By differentiating Eq.(4.8) with respect to α_i and letting it equate to zero, we can obtain the MLE estimation of α_i. However, the estimates of prior parameters using MLE possesses a certain degree of computational difficulty, and closed-form MLE estimation equations do not exist for these parameters. Thus, we also obtain the parameter estimates using MoM, whereby a closed-form solution for the MoM estimators $\widehat{\mathbf{a}}$ is obtained as follows:

$$
\begin{aligned}
\widehat{\alpha}_{i^*} &= \frac{\sum\limits_{i=1}^{I}\omega_i^{\alpha}\left(\frac{\sum\limits_{j=1}^{J}\sum\limits_{i=1}^{I}(n_{ij})^2-\sum\limits_{j=1}^{J}n_j}{\sum\limits_{j=1}^{J}\left(n_j^2-n_j\right)}-1\right)}{\sum\limits_{i=1}^{I}\left(\omega_i^{\alpha}\right)^2-\left(\sum\limits_{i=1}^{I}\omega_i^{\alpha}\right)^2\frac{\sum\limits_{j=1}^{J}\sum\limits_{i=1}^{I}(n_{ij})^2-\sum\limits_{j=1}^{J}n_j}{\sum\limits_{j=1}^{J}\left(n_j^2-n_j\right)}} \\
\widehat{\mathbf{a}} &= \widehat{\alpha}_{i^*}\mathbf{w_i^a},
\end{aligned}
\tag{4.9}
$$

where, $\omega_i^{\alpha} = \frac{\sum\limits_{j=1}^{J}n_{ij}}{\sum\limits_{j=1}^{J}n_{i^*j}}$ and $\mathbf{w_i^a} = [\omega_1^{\alpha}...\omega_I^{\alpha}]$.

4.4 Mathematical Modelling of Decision Making

This section develops an analytical decision support model for supplier development investment in which the uncertainty of a supplier's performance is considered. The proposed decision support model allows us to determine an optimal investment level by analysing the profit from the activity. Decision making on both individual supplier and supplier portfolio investment are investigated.

4.4.1 Individual Supplier Investment

Let N_{ij} denote the random number of a buyer's ordering items at performance category i from supplier j, and c_{ij} denote the corresponding financial loss per undesirable item. Therefore, the financial loss at performance category i before improvement is $c_{ij}N_{ij}$. Note that here we assume there is no cost of desirable items, that is, $c_{ij} = 0$. Let γ_{ij} denote an effectiveness rate of the activity where a higher value reflects higher effectiveness and x_j denote the investment level. Assuming that the proportion of all undesirable performance categories exponentially decreases as the investment level increases, the financial loss after improvement at performance category i therefore can be written as $c_{ij}N_{ij}\ e^{-\gamma_{ij}x_j}$. Thus, the profit to be obtained from developing supplier j, denoted by π_j, can be expressed as:

$$\pi_j = \sum_{i}^{I} c_{ij}N_{ij}\ \left(1 - e^{-\gamma_{ij}x_j}\right) - x_j. \tag{4.10}$$

Let p_{ij} denote the probability of the occurrences at each performance category i which is assumed to be distributed from the multinomial distribution (Eq.(4.1)). Let n_j denote the number of total orders, the expected profit therefore can be expressed as:

$$E(\pi_j) = \sum_{i}^{I} c_{ij}n_jp_{ij}\left(1 - e^{-\gamma_{ij}x_j}\right) - x_j. \tag{4.11}$$

Theorem 4.1. *Under the assumption of same effectiveness rates for different performance categories, a closed form of optimal investment level for supplier j, denoted by x_j^*, is obtained:*

$$x_j^* = \max\left\{0, \frac{\ln\left(\sum_{i=1}^{I} c_{ij} n_j p_{ij} \gamma_j\right)}{\gamma_j}\right\}. \tag{4.12}$$

Proof. By taking the second derivative of the expected profit function (Eq.(4.11)) with respect to the investment level x_j, we can obtain:

$$\frac{d^2 E(\pi_j)}{dx_j^2} = -\sum_{i=1}^{I} c_{ij} n_j p_{ij} \gamma_j^2 e^{-\gamma_j x_j} - 1 < 0.$$

This implies that there exists a x^* where the profit increases as x increases and then decreases as the x_j approaches to an infinity. Let the first derivative equate to zero, we get:

$$\frac{dE(\pi_j)}{dx_j^*} = \sum_{i=1}^{I} c_{ij} n_j p_{ij} \gamma_j e^{-\gamma_j x_j^*} - 1 = 0.$$

Therefore, a closed form of x_j^* can be obtained:

$$x_j^* = \frac{\ln\left(\sum_{i=1}^{I} \gamma_j c_{ij} n_j p_{ij}\right)}{\gamma_j}.$$

As it is assumed that the optimal investment level is non-negative, therefore, x_j^* can be further formulated as follows:

$$x_j^* = \max\left\{0, \frac{\ln\left(\sum_{i=1}^{I} c_{ij} n_j p_{ij} \gamma_j\right)}{\gamma_j}\right\}.$$

□

Remark. *If* $\ln\left(\sum_{i=1}^{I} \gamma_j c_{ij} n_j p_{ij}\right) > 1$, *an increase in the effectiveness rate will result in a decrease in optimal investment, and vice versa.*

$$\frac{dx_j^*}{d\gamma_j} = \frac{1 - \ln\left(\sum_{i=1}^{I} \gamma_j c_{ij} n_j p_{ij}\right)}{\gamma_j^2}.$$

By substituting Eq.(4.12) into Eq.(4.11), an expected profit under the optimal investment level x_j^* for supplier j is obtained:

$$E\left(\pi_j, x_j^*\right) = \begin{cases} \frac{\sum\limits_{i=1}^{I} c_{ij}N_j p_{ij} - 1 - \ln\left(\sum\limits_{i=1}^{I} c_{ij}N_j p_{ij}\gamma_j\right)}{\gamma_j} & if \sum\limits_{i=1}^{I} c_{ij}N_j p_{ij}\gamma_j > 1 \\ 0 & if \sum\limits_{i=1}^{I} c_{ij}N_j p_{ij}\gamma_j \leq 1. \end{cases} \tag{4.13}$$

As aforementioned, a supplier's epistemic uncertainty can be reduced if more information about the supplier can be obtained. In decision analysis, expected value of perfect information (EVPI) is often used which indicates the expected benefit of delaying a decision until more data is obtained to enable a better decision. Perfect information assumes all relevant epistemic uncertainty is removed. In this study, let $E\left(\pi_j, x_j^{PI}\right)$ represent an expected profit under an investment level x_j^{PI} obtained under perfect information, the expression of EVPI can be written as:

$$EVPI = E\left(\pi_j; x_j^{PI}\right) - E(\pi_j; x_j^*). \tag{4.14}$$

The purpose of evaluating EVPI is to provide an upper limit of how much it is worth buying down the uncertainty before making the investment decision for supplier development. Such a consideration is not unreasonable. Assuming a buyer plans to invest a certain amount of resources to improve a supplier's performance, however, after a short period of observation before the investment it may turn out that there is no need to make the investment. EVPI provides the opportunity for buyers to know if a better decision exists if more information is gathered.

When effectiveness rates γ_{ij} are not the same for each performance category, the optimal investment level can only be obtained numerically. However, boundaries of the optimal investment level may exist if it has a monotonic relationship with the effectiveness rate.

Theorem 4.2. *When the effectiveness rates γ_{ij} of a supplier development activity are different, if there exists* $\ln\left(\sum_{i=1}^{I}\gamma_{ij}c_{ij}n_jp_{ij}\right) > 1$ *for all* γ_{ij} *, the optimal investment level* x_j^* *and its corresponding expected profit* $E\left(\pi_j; x_j^*\right)$ *have the following boundaries:*

$$
\begin{aligned}
x_0^* \leq x_j^* \leq x_{+\infty}^* \\
E\left(\pi_j; x_j^*\right) \geq \max\left\{E\left(\pi_j; x_0^*\right), E\left(\pi_j; x_{+\infty}^*\right)\right\},
\end{aligned}
\tag{4.15}
$$

where, x_0^* and $x_{+\infty}^*$ represent the optimal investment levels that are obtained under $\gamma_{\max}$ and $\gamma_{\min}$ respectively.

Proof. Let $\gamma_{\min}$ and $\gamma_{\max}$ represent the two extreme values among all γ_{ij} , where $\gamma_{\max} \geq \gamma_{\min}$. If all $\gamma_{ij} = \gamma_{\max}$, we can obtain the corresponding optimal investment level x_0^*, whereas if all $\gamma_{ij} = \gamma_{\min}$, we can obtain the corresponding optimal investment level $x_{+\infty}^*$. Assuming that for all γ_{ij} , there exists $\ln\left(\sum_{i=1}^{I}\gamma_{ij}c_{ij}n_jp_{ij}\right) > 1$. When $\ln\left(\sum_{i=1}^{I}\gamma_{j}c_{ij}n_jp_{ij}\right) > 1$, an optimal investment level decreases as an effectiveness rate increases. Therefore, the values of $x_{+\infty}^*$ and x_0^* are essentially the boundaries of the optimal investment x_j^*, where:

$$x_0^* \leq x_j^* \leq x_{+\infty}^*.$$

As x_j^* is already the optimal investment level, we can further obtain a boundary for the expected profit as follows:

$$E\left(\pi_j; x_j^*\right) \geq \max\left\{E\left(\pi_j; x_0^*\right), E\left(\pi_j; x_{+\infty}^*\right)\right\}.$$

□

4.4.2 Supplier Portfolio Investment

The decision support model for supplier development investment is developed under the context that the buyer intends to invest in multiple suppliers with a limited budget. The objective of the model is to optimally allocate the budget in order to achieve the highest return. Let z denote a budget, based on Eq.(4.11), an overall expected profit of supplier portfolio investment can be obtained in Eq.(4.16), where $\forall x_j > 0$:

$$\begin{aligned} E\left(\pi; x_1^* ... x_J^*\right) &= \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij} n_j p_{ij}\left(1 - e^{-\gamma_{ij} x_j}\right) - \sum_{j=1}^{J} x_j \\ g(x_1..x_j) &= \sum_{j=1}^{J} x_j = z. \end{aligned} \tag{4.16}$$

Assuming for each supplier, the effectiveness rates are the same for all performance categories which is denoted by γ_j, a closed form of optimal investment allocation can be obtained as shown in Theorem 4.3.

Theorem 4.3. *Under the assumption of same effectiveness rates for different performance categories, for any supplier s, a closed form of optimal investment level is obtained:*

$$x_s^* = \frac{z\frac{1}{\gamma_s} - \sum\limits_{j=1, j\neq s}^{J} \frac{\ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right)}{\gamma_j \gamma_s}}{\sum\limits_{j=1}^{J} \frac{1}{\gamma_j}}, \tag{4.17}$$

where, $c_j^T = \sum\limits_{i=1}^{I} c_{ij} n_j p_{ij}$ representing a total expected cost of supplier undesirable performance before the development activity.

Proof. Assuming for each supplier, the effectiveness rates are the same for all undesirable performance categories which is denoted by γ_j, To examine the optimisation properties of the expected profit, a Lagrange multiplier approach is used which is expressed as follows:

$$L_a(x_1..x_j, \lambda_a) = \left[\sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij} n_j p_{ij}(1 - e^{-\gamma_j x_j}) - \sum_{j=1}^{J} x_j\right] - \lambda_a\left(\sum_{j=1}^{J} x_j - z\right).$$

By differentiating $L_a(x_1..x_j, \lambda_a)$ with respect to x_j, we can obtain:

$$\frac{\partial L_a}{\partial x_j} = \sum_{i=1}^{I} c_{ij} n_j p_{ij} \gamma_j e^{-\gamma_j x_j} - 1 - \lambda_a.$$

Let x_s^* denote the optimal investment for supplier s, we have:

$$\frac{\partial L_a}{\partial x_s^*} = 0.$$

Therefore,

$$\sum_{i=1}^{I} c_{is} n_s p_{is} \gamma_s e^{-\gamma_s x_s^*} = 1 + \lambda_a^*.$$

For any supplier s and supplier j we have:

$$\frac{\sum\limits_{i=1}^{I} c_{is} n_s p_{is} \gamma_s e^{-\gamma_s x_s^*}}{\sum\limits_{i=1}^{I} c_{ij} n_j p_{ij} \gamma_j e^{-\gamma_j x_j^*}} = 1.$$

Rearrange we get:

$$e^{-\gamma_s x_s^* + \gamma_j x_j^*} = \frac{\sum\limits_{i=1}^{I} c_{ij} n_j p_{ij} \gamma_j}{\sum\limits_{i=1}^{I} c_{is} n_s p_{is} \gamma_s}.$$

Let $\sum\limits_{i=1}^{I} c_{is} n_s p_{is} = c_s^T$ and $\sum\limits_{i}^{I} c_{ij} n_j p_{ij} = c_j^T$, whereby c_s^T and c_j^T are the overall expected financial loss caused by supplier s and j respectively, we obtain:

$$\gamma_j x_j^* - \gamma_s x_s^* = \ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right).$$

Then, a relationship in the optimal investment levels between supplier s and supplier j exists:

$$x_s^* = \frac{\ln\left(\frac{c_s^T \gamma_s}{c_j^T \gamma_j}\right) + \gamma_j x_j^*}{\gamma_s}.$$

Let $z = \sum_{j=1, j\neq s}^{J} x_j^* + x_s^*$, then we have:

$$z = \sum_{j=1,j\neq s}^{J} \frac{\ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right) + \gamma_s x_s^*}{\gamma_j} + x_s^*$$
$$= \sum_{j=1,j\neq s}^{J} \frac{\ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right)}{\gamma_j} + \left(\sum_{j=1,j\neq s}^{J} \frac{\gamma_s}{\gamma_j} + 1\right) x_s^*.$$

Then we get:

$$x_s^* = \frac{z - \sum\limits_{j=1,j\neq s}^{J} \frac{\ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right)}{\gamma_j}}{\sum\limits_{j=1,j\neq s}^{J} \frac{\gamma_s}{\gamma_j} + 1} = \frac{z\frac{1}{\gamma_s} - \sum\limits_{j=1,j\neq s}^{J} \frac{\ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right)}{\gamma_j \gamma_s}}{\sum\limits_{j=1,j\neq s}^{J} \frac{1}{\gamma_j} + \frac{1}{\gamma_s}}.$$

Therefore, we can obtain:

$$x_s^* = \frac{z\frac{1}{\gamma_s} - \sum\limits_{j=1,j\neq s}^{J} \frac{\ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right)}{\gamma_j \gamma_s}}{\sum\limits_{j=1}^{J} \frac{1}{\gamma_j}}.$$

We can further examine the above mathematical form of x_s^* by checking $\sum_{s=1\ldots j}^{J} x_s^* = z$.

Let $\sum_{j=1}^{J} \frac{1}{\gamma_j} = \gamma$, as for any supplier s we have $x_s^* = \frac{z\frac{1}{\gamma_s}}{\gamma} - \frac{1}{\gamma} \sum_{j=1,j\neq s}^{J} \frac{\ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right)}{\gamma_j \gamma_s}$, therefore,

$$\sum_{s=1}^{J} x_s^* = \sum_{s=1}^{J} \left(\frac{z\frac{1}{\gamma_s}}{\gamma} - \frac{1}{\gamma} \sum_{j=1,j\neq s}^{J} \frac{\ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right)}{\gamma_j \gamma_s} \right) = z \sum_{s=1}^{J} \frac{\frac{1}{\gamma_s}}{\gamma} - \frac{1}{\gamma} \sum_{s=1}^{J} \left(\sum_{j=1,j\neq s}^{J} \frac{\ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right)}{\gamma_j \gamma_s} \right). \quad [$$

In the above equation, as $\sum_{j=1}^{J} \frac{1}{\gamma_j} = \gamma$, then $\frac{\sum_{s=1}^{J} \frac{1}{\gamma_s}}{\gamma} = 1$.

Since $\sum_{\forall j\neq s}^{J} \frac{\ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right)}{\gamma_j \gamma_s} = \sum_{\forall j\neq s}^{J} \frac{\ln(c_j^T \gamma_j) - \ln(c_s^T \gamma_s)}{\gamma_j \gamma_s}$, we can get:

$$\sum_{s=1}^{J}\left(\sum_{j=1,j\neq s}^{J}\frac{\ln\left(\frac{c_j^T\gamma_j}{c_s^T\gamma_s}\right)}{\gamma_j\gamma_s}\right)=\left(\frac{\ln\left(c_2^T\gamma_2\right)-\ln\left(c_1^T\gamma_1\right)}{\gamma_2\gamma_1}+...+\frac{\ln\left(c_J^T\gamma_J\right)-\ln\left(c_1^T\gamma_1\right)}{\gamma_J\gamma_1}\right)+$$

$$\left(\frac{\ln\left(c_1^T\gamma_1\right)-\ln\left(c_2^T\gamma_2\right)}{\gamma_1\gamma_2}+...+\frac{\ln\left(c_J^T\gamma_J\right)-\ln\left(c_2^T\gamma_2\right)}{\gamma_J\gamma_2}\right)+$$

$$...$$

$$\left(\frac{\ln\left(c_1^T\gamma_1\right)-\ln\left(c_J^T\gamma_J\right)}{\gamma_1\gamma_J}+...+\frac{\ln\left(c_{J-1}^T\gamma_{J-1}\right)-\ln\left(c_J^T\gamma_J\right)}{\gamma_{J-1}\gamma_J}\right)=0.$$

Therefore, we can prove that $\sum_{s=1...j}^{J} x_s^* = z \times 1 - \frac{1}{a} \times 0 = z$ □

Corollary 4.3.1. *A monotonic increasing relationship between an optimal for a supplier and the budget is obtained. This implies how much resources each supplier will get when extra budget is available.*

$$\frac{dx_s^*}{dz}=\frac{z\frac{1}{\gamma_s}}{\sum_{j=1}^{J}\frac{1}{\gamma_j}}.$$

Corollary 4.3.2. *If for any two suppliers we have $c_j^T\gamma_j = c_s^T\gamma_s$, a supplier with low effectiveness rates but high non-conformance costs gets more resources than a supplier with high effectiveness rates but low non-conformance costs.*

Lemma 4.4. *There is an affine relationship with a positive slope between the optimal investment level between two suppliers:*

$$x_s^*=\frac{\ln\left(\frac{c_s^T\gamma_s}{c_j^T\gamma_j}\right)+\gamma_j x_j^*}{\gamma_s}. \tag{4.18}$$

Proof. While obtaining the optimal investment allocation (Eq.(4.17)), we have obtained that a Lagrange function:

$$L_a(x_1..x_j,\lambda_a)=\left[\sum_{i=1}^{I}\sum_{j=1}^{J}c_{ij}n_jp_{ij}(1-e^{-\gamma_j x_j})-\sum_{j=1}^{J}x_j\right]-\lambda_a(\sum_{j=1}^{J}x_j-z).$$

By differentiating $L_a(x_1..x_j, \lambda_a)$ with respect to x_j, we can obtain:

$$\frac{\partial L_a}{\partial x_j} = \sum_{i=1}^{I} c_{ij} n_j p_{ij} \gamma_j e^{-\gamma_j x_j} - 1 - \lambda_a.$$

Let x_s^* denote the optimal investment for supplier s, we have:

$$\frac{\partial L_a}{\partial x_s^*} = 0.$$

Therefore, for any supplier s and supplier j we have:

$$\frac{\sum\limits_{i=1}^{I} c_{is} n_s p_{is} \gamma_s e^{-\gamma_s x_s^*}}{\sum\limits_{i=1}^{I} c_{ij} n_j p_{ij} \gamma_j e^{-\gamma_j x_j^*}} = 1.$$

Let $\sum\limits_{i=1}^{I} c_{is} n_s p_{is} = c_s^T$and $\sum\limits_{i}^{I} c_{ij} n_j p_{ij} = c_j^T$, whereby $c_s^T$and c_j^T are the overall financial loss caused by supplier s and$j$ respectively, we obtain:

$$\gamma_j x_j^* - \gamma_s x_s^* = \ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right).$$

Then, a relationship in the optimal investment levels between supplier s and supplier j exists:

$$x_s^* = \frac{\ln\left(\frac{c_s^T \gamma_s}{c_j^T \gamma_j}\right) + \gamma_j x_j^*}{\gamma_s}.$$

□

To make sure that every supplier gets a certain amount of investment, that is, $x_s^* \geq 0$, a lower bound of the budget z is obtained and formulated as Proposition 4.1.

Proposition 4.1. *The following expression gives a lower bound of the budget in order to make sure that the investment level allocated to each supplier will be non-negative.*

$$z > \max\{z_1 ... z_s\}, \tag{4.19}$$

where, $z_s = \gamma_s \sum\limits_{j=1, j\neq s}^{J} \frac{\ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right)}{\gamma_j}$.

As long as one supplier's optimal investment level x_s^* is known, a total expected profit can be expressed using x_s^*:

$$E\left(\pi; x_1^* ... x_J^*\right) = \sum_{j=1}^{J} c_j^T - \sum_{j=1, j\neq s}^{J} c_j^T e^{-\gamma_s x_s^*} \frac{c_s^T \gamma_s}{c_j^T \gamma_j} - c_s^T e^{-\gamma_s x_s^*} - z. \tag{4.20}$$

Proof. Let x_s^* to represent other supplier's optimal investment level using Eq.(4.18):

$$x_j^* = \frac{\ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right) + \gamma_s x_s^*}{\gamma_j}.$$

Substitute it into Eq.(4.16), we can obtain:

$$E\left(\pi; x_1^* ... x_J^*\right) = \sum_{j=1, j\neq s}^{J} c_s^T \left(1 - e^{-\left(ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right) + \gamma_s x_s^*\right)}\right) + c_s^T \left(1 - e^{-\gamma_s x_s^*}\right) - z.$$

As $e^{-\left(ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right) + \gamma_s x_s^*\right)} = e^{-ln\left(\frac{c_j^T \gamma_j}{c_s^T \gamma_s}\right)} e^{-\gamma_s x_s^*} = \frac{c_s^T \gamma_s}{c_j^T \gamma_j} e^{-\gamma_s x_s^*}$, therefore, we can simplify the above expression and obtain Eq.(4.20):

$$E\left(\pi; x_1^* ... x_J^*\right) = \sum_{j=1}^{J} c_j^T - \sum_{j=1, j\neq s}^{J} c_j^T e^{-\gamma_s x_s^*} \frac{c_s^T \gamma_s}{c_j^T \gamma_j} - c_s^T e^{-\gamma_s x_s^*} - z.$$

□

4.5 Numerical Investigation

This section demonstrates an application of the proposed model via a numerical experiment. The purpose is to present the behaviour of the proposed model and to provide a detailed explanation for the use of the model in supplier development decision making. Fig.4.1 summarises the modelling process for this illustrative example.

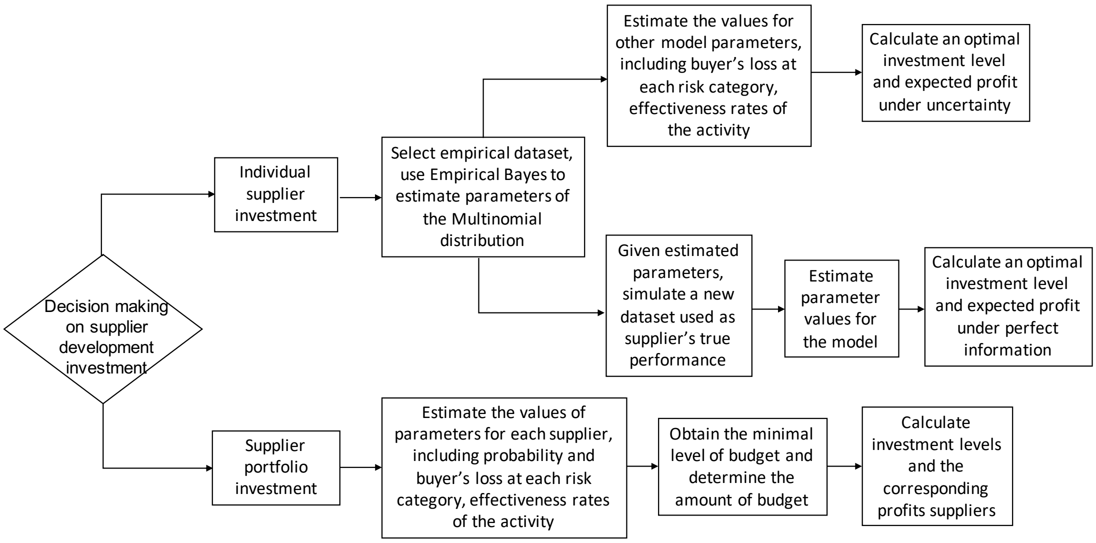

Figure 4.1: A flowchart of the data analysis process

4.5.1 Individual Supplier Investment

This numerical investigation is developed under the context where a buyer has a number of important parts to be delivered by a newly integrated supplier. If the supplier does not deliver the parts on time, the buyer may suffer a significant financial loss. To mitigate the risk, the buyer intends to conduct a supplier development activity in order to improve the supplier's rate of on-time delivery. As a supplier development activity requires investing significant resources, decisions need to be made regarding how much it is worth for the buyer to make such an investment. The supplier's undesirable delivery performance is classified into three categories based on the degree of the lateness: 1-3 days lateness, 4-10 days lateness and more than 10 days lateness, where 1-3 days lateness indicates general risk level, 4-10 days lateness indicates severe risk level and more than 10 days lateness indicates very severe risk level. We assume that the reduction of the proportion of all undesirable performance categories are added on the proportion of desirable performance, which is on-time delivery. To capture both the aleatory and epistemic uncertainty of each performance category, the Dirichlet-multinomial distribution is employed (Eq.(4.5)) for which empirical Bayes inference approach is used for prior estimates. A set of empirical data with 256 suppliers' lateness delivery records is used to construct a data pool. By using MoM (Eq.(4.9)), we obtain the estimates of the prior parameters: $\hat{\alpha}_1 = 2.20, \hat{\alpha}_2 = 1.39, \hat{\alpha}_3 = 0.10$, in which $i = 1$ indicates 1-3 days lateness, $i = 2$ indicates 4-10 days lateness and $i = 3$ indicates more than 10 days lateness. The results of the prior estimates imply that the proportion of 1-3 days lateness is larger than the other two performance categorises, and the possibility of more than 10 days lateness delivery may be rather low. The prior estimates are used in the following section along with other synthetic data to provide informed decisions using the proposed model and to obtain important managerial insights.

4.5.1.1 Numerical Specifications and Rationales

The following parameters are used in the numerical investigation:

$$n = 100, \alpha_1 = 2.20, \alpha_2 = 1.39, \alpha_3 = 0.10, c_1 = 5u, c_2 = 10u, c_3 = 20u, \gamma = [0.01...0.1]$$

n is the total number of orders from the supplier which is 100, and c_i is the cost of late delivery at each performance category. γ represents a range of possible: values $[0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1]$, which is assumed to be the same for all performance categories. The purpose of considering different levels of effectiveness rates is to evaluate the how investment decisions change under different levels of the effectiveness of the activity. Note that the value of γ is normalised as the effectiveness rate in this study is defined as the impact on a supplier's improvement per unit of an investment level.

4.5.1.2 Optimal Investment and Expected Profit

Assuming the buyer does not have the perfect information about the supplier's true delivery performance, given the pre-defined parameter values, an optimal investment level and expected profit are obtained using Eq.(4.12) and Eq.(4.13), of which the results are presented in Fig.4.2. Based on Eq.(4.11), if no development activity is conducted the buyer's expected loss will be $729u$. However, as shown in Fig.4.2, such loss may be reduced down to around $100u$ and a profit level of $430.5u$ can be expected if the buyer invests $198.7u$ on a supplier development activity with an effectiveness rate 0.01. In addition, it is interesting to observe that the optimal investment level declines as the effectiveness rate increases, which is consistent with the Corollary 4.4.1. For example, the buyer needs to invest $198.7u$ and receives profit $430.5u$ if the effectiveness rate is equal to 0.01. By contrast, the buyer only needs to invest $71.9u$ and receives profit $637.3u$ if the effectiveness rate is equal to 0.05.

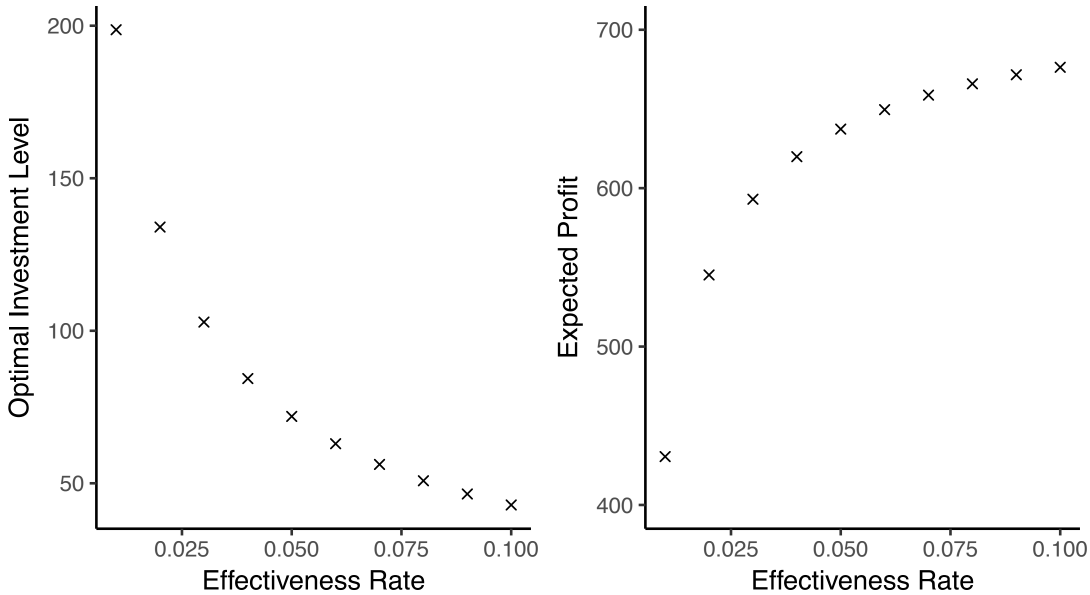

Figure 4.2: Left-to-right: numerical results of optimal investment levels and expected profits given the range of effectiveness rates are from 0.01 to 0.1, showing that the optimal investment level decreases as the effectiveness rate increases whereas the expected profit increases as the effectiveness rate increases (data provided in the Appendix A: Table A.1)

We now investigate how investment decisions change if the buyer has the perfect information about the supplier's true delivery performance. A set of performance data is simulated to represent the supplier's true performance by following the process:

(1) Set up the value of parameters:

$$runs = 1000, n = 100, \alpha_1 = 2.20, \alpha_2 = 1.39, \alpha_3 = 0.10$$

(2) Generate p_i for 1000 times, $\mathbf{p_i} \sim Dir\left(\alpha_i\right)$:

$$\mathbf{p_1} \sim Beta\left(\alpha_1, \alpha_2 + \alpha_3\right), \mathbf{p_2} \sim \left(1 - \mathbf{p_1}\right) \times Beta\left(a_2, a_3\right), \mathbf{p_3} = 1 - \mathbf{p_1} - \mathbf{p_2}$$

(3) Generate n_i for 1000 times, $\mathbf{n_i} \sim Mul\left(n, p_i\right)$:

$$\mathbf{n_1} = n\mathbf{p_1}, \mathbf{n_2} = n\mathbf{p_2}, \mathbf{n_3} = n\mathbf{p_3}$$

(4) Obtain $\mathbf{n_1}$, $\mathbf{n_2}$, $\mathbf{n_3}$.

Assuming that there is no need to make the investment if the probability of 1-3 days lateness of a supplier is higher than 0.7 as this is within the buyer's tolerance for risk. Based on Eq.(4.13), the results of the optimal investment levels and expected returns under uncertainty and under perfect information are obtained and presented in Fig.4.3. When the effectiveness rate is equal to 0.01, the expected optimal investment level under uncertainty is around $199u$ and the expected profit is around $230u$. However, the buyer only needs to invest around $128u$ and receives a higher profit of around $370u$ if the decision is made under perfect information. Notably, when the effectiveness rate is equal to 0.01, buyer should not invest more than $140u$ which is the difference between the expected profit under uncertainty and under perfect information, also referred as EVPI. Otherwise, there is no advantage of obtaining perfect information about the supplier's true performance as the expected profit will be less than directly investing in the activity without delaying the decision to learn more about the supplier.

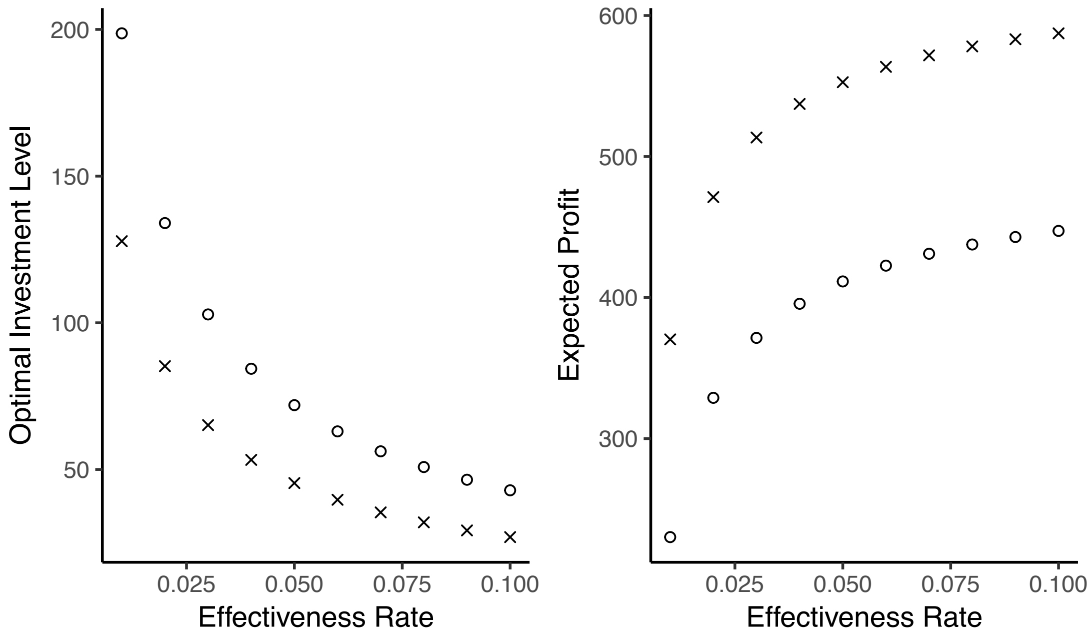

Figure 4.3: Left-to-right: a comparison between the expected optimal investment levels and the expected profits under uncertainty (circle point) and under perfect information (cross point), of which the values are averaged over all the simulated observations (data provided in the Appendix A: Table A.2)

The values of EVPI (Eq.(4.14)) obtained under different effectiveness rates are presented in Fig.4.4. It shows that the value of EVPI are all between $140u$ to $143u$.

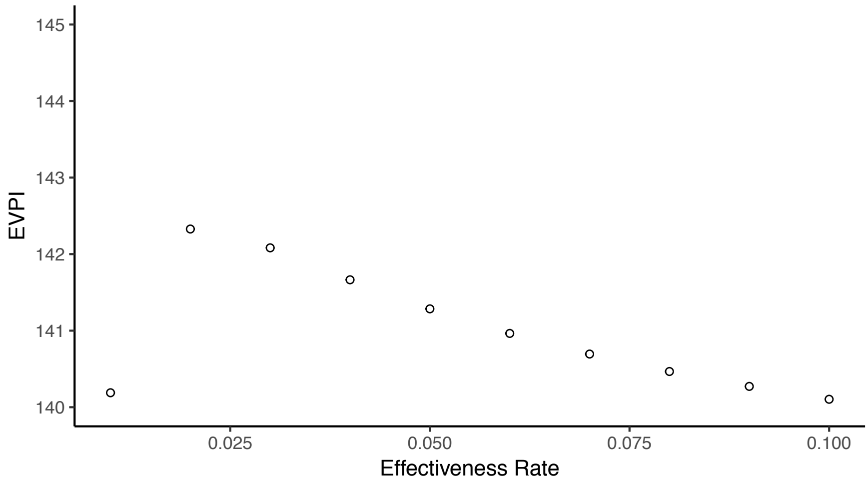

Figure 4.4: The change of EVPI with respect to the effectiveness rate (data provided in the Appendix A: Table A.3)

When effectiveness rates are not the same, three different circumstances are considered and categorised based on the difference between the effectiveness rates:

- Low (difference=0.01): $\gamma_1 = 0.02,\ \gamma_2 = 0.03,\ \gamma_3 = 0.04$
- Medium (difference=0.02): $\gamma_1 = 0.02,\ \gamma_2 = 0.04,\ \gamma_3 = 0.06$
- High (difference =0.03): $\gamma_1 = 0.02,\ \gamma_2 = 0.05,\ \gamma_3 = 0.08$

The purpose of the category is to examine how the change in effectiveness rates affect the boundaries of the optimal investment and profit. The numerical result is shown in Fig.4.5. Clearly, the optimal investment level always lies within the boundaries and the expected profit is higher than the profit obtained under the lowest and highest effectiveness rates. This shows consistency with Theorem 4.2. (Eq.(4.15)). In addition, the difference between the maximal and minimal optimal investment level is smaller under low effectiveness rates group which is when $\gamma_1 = 0.02,\ \gamma_2 = 0.03,\ \gamma_3 = 0.04$. This reflects that the closer the effectiveness rates are, the more effective the boundary will be.

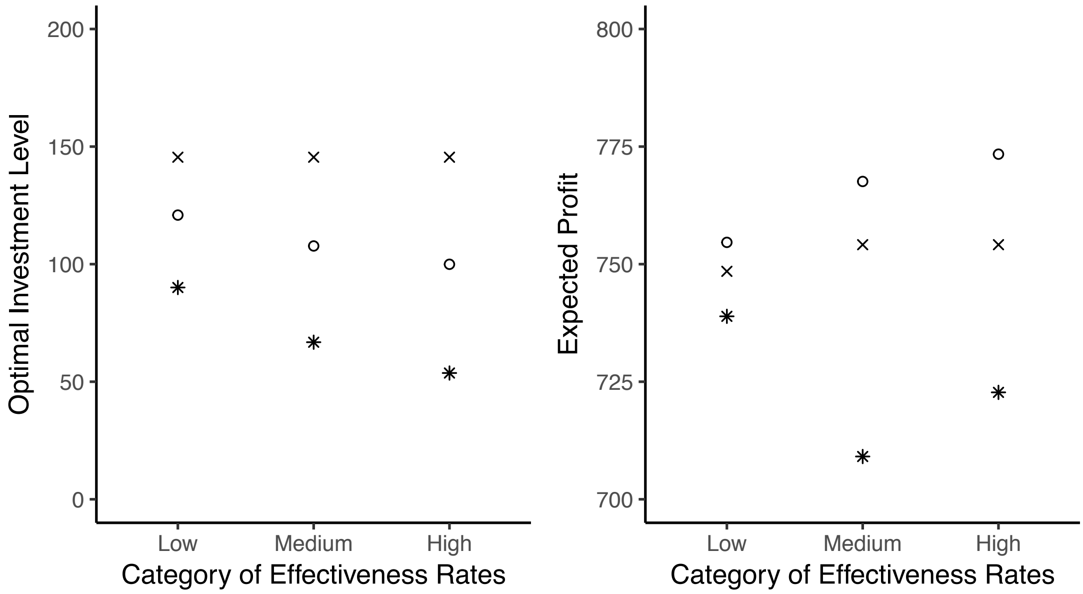

Figure 4.5: Left-to-right: the results of the optimal investment level and expected profit when the effectiveness rates are different for each risk category (circle point: the accurate value; cross and star point: the estimates boundaries; data provided in the Appendix A: Table A.4)

4.5.2 Supplier Portfolio Investment

4.5.2.1 Numerical Specifications and Rationales

For simplicity, the following supplier portfolio analysis only considers four suppliers. Assuming that the number of orders is the same for each supplier which is equal to 100, and for every supplier, the effectiveness rate γ is the same for all performance categories, the parameter settings for each supplier are specified and summarised in Table 4.1. The indicator i from 1 to 3 represents 1-3 days lateness, 4-10 days lateness and more than 10days lateness respectively.

Table 4.1: Parameter settings for the supplier portfolio investment

Suppliers	Parameter Settings
S_1	$c_1 = 3u, c_2 = 5u, c_3 = 7u, \alpha_1 = 0.5, \alpha_2 = 0.3, \alpha_3 = 0.2, \gamma = 0.08$
S_2	$c_1 = 5u, c_2 = 7u, c_3 = 10u, \alpha_1 = 0.6, \alpha_2 = 0.4, \alpha_3 = 0.0, \gamma = 0.05$
S_3	$c_1 = 5u, c_2 = 15u, c_3 = 20u, \alpha_1 = 0.6, \alpha_2 = 0.3, \alpha_3 = 0.1, \gamma = 0.03$
S_4	$c_1 = 7u, c_2 = 10u, c_3 = 15u, \alpha_1 = 0.8, \alpha_2 = 0.1, \alpha_3 = 0.1, \gamma = 0.01$

4.5.2.2 Analysis Results

A lower bound of a budget is first calculated based on Proposition 4.1. The results are presented in Fig.4.6 which is -157.8u, -125.7u, -122.8u and 85.0u. This implies that the buyer should invest no less than 85u in order to make sure that all the suppliers will receive the investment allocation. The following analysis will be based on a budget of 100u.

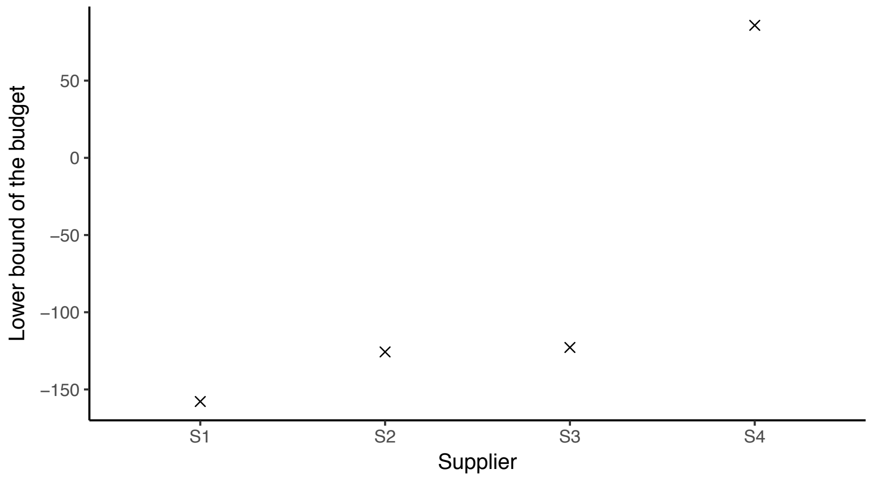

Figure 4.6: Boundaries of the budget whereby a lower bound of the budget lies at 85 so that all suppliers can obtain positive investment levels (S_1 to S_4 at the x - axis represents the supplier 1-4).

Fig.4.7 shows the results of the optimal investment level of each supplier obtained and the corresponding expected profit. The investment allocated to each supplier are $19.4u$, $27.2u$, $44.8u$ and $8.5u$ (Eq.(4.17)). The total expected profit is $1446.9u$.

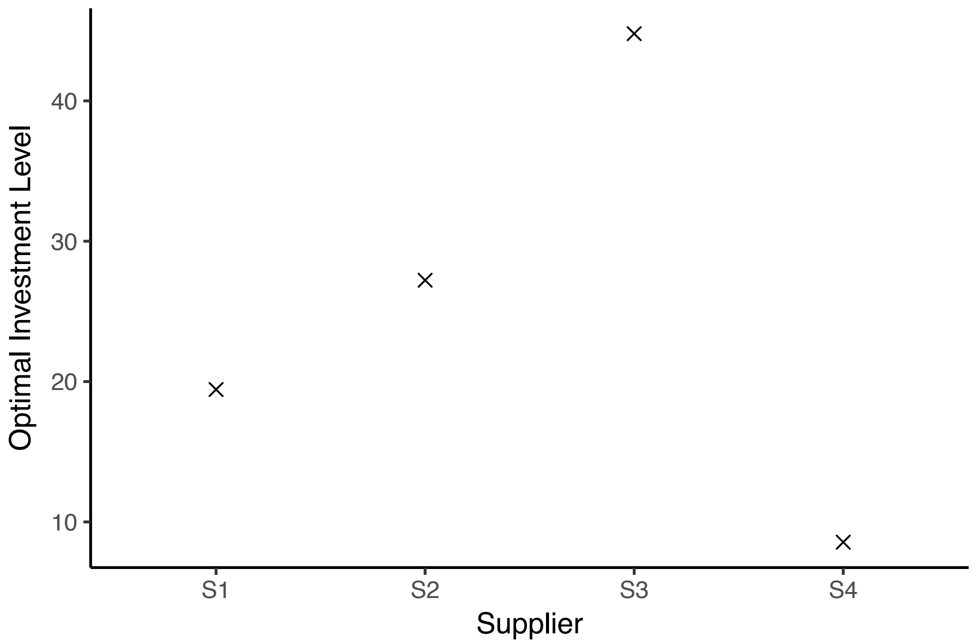

Figure 4.7: Numerical results of the optimal investment allocations for each supplier (S_1-S_4:Supplier 1-4).

Assuming that the buyer has extra budget of $50u$ available and intends to invest in the development activity, the investment allocation for each supplier given the extra budget are obtained using Corollary 4.3.1. The results are shown in Fig.4.8 in which from supplier 1 to 4 the investment allocation are: $3.77u$, $6.03u$, $10.05u$, $30.15u$.

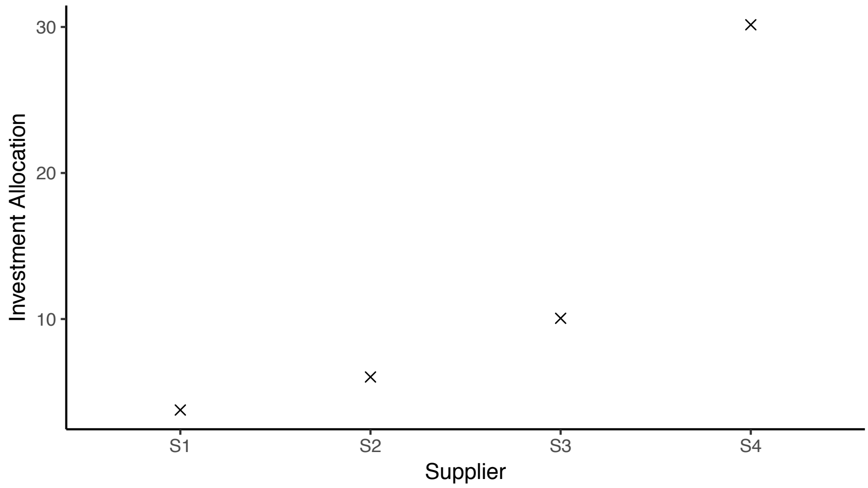

Figure 4.8: Allocation of the investment level for each supplier when extra budget is available.

4.6 Summary

This chapter develops an analytical decision support for supplier development investment under uncertainty, in which both single supplier investment and supplier portfolio investment scenarios are investigated. In the case of single supplier analysis, a closed-form solution of optimal investment level under identical effectiveness rate is obtained. In addition, a lower and upper bound for optimal investment level and expected profit are obtained under different effectiveness rates. Furthermore, the expected value of information is evaluated. In the case of supplier portfolio analysis, a closed form of an optimal investment allocation under a budget constraint is obtained using a Lagrange multiplier approach. A number of managerial insights are also obtained. For example, the model suggests that within a certain threshold the optimal investment level decreases as the effectiveness rate increases. In addition, when investing in multiple suppliers, once the optimal investment is reached, decision makers are not encouraged to take a part of the investment from one supplier to another one.

Chapter 5

Non-Homogeneous Poisson Process for Modelling Supplier KPI

5.1 Introduction

In this chapter, the decision support model is developed in the context that a buyer has a newly integrated supplier who is lacking sufficient experience but on a self-development stage, and the buyer intends to facilitate the supplier's quality improvement by delegating its own engineers to help the supplier. A power law NHPP model is employed to capture the uncertainty of the supplier's quality as the supplier's non-conformance rate is considered changing over the exposure to risk. A virtual age model is used to facilitate the elicitation of the improvement of the supplier's quality via the training. By analysing the expected benefit from the activity, an optimal investment level is obtained and interpreted as managerial insights. The time value of money is also taken into consideration. The behaviour of the proposed decision support model is demonstrated via a numerical experiment.

The remainder of this chapter is organised as follows. Section 5.2 explains the methodological considerations. Section 5.3 details the use of power law NHPP on modelling the uncertainty of supplier's undesirable performance along with the statistical inference approaches used for parameter estimation. Section 5.4 develops the decision support model and discusses the managerial insights gained. Section 5.5 gives a numerical example to evaluate the modelling techniques employed and to illustrate the performance of the proposed model. Section 5.6 summarises this chapter.

5.2 Methodological Considerations

In this chapter, NHPP model is used for capturing the uncertainty of suppliers' performance as we consider the data of supplier non-conformance is in a count form and the supplier's non-conformance rate changes over the risk exposure. NHPP model has been frequently applied in many fields of studies, in particular reliability growth modelling, such as, software reliability analysis (Kapur et al., 2010; Lai and Garg, 2012; Li and Pham, 2017; Song et al., 2018; Wang and Yu, 2013; Zhao and Xie, 1996) or repairable system reliability (Dewan and Dijoux, 2015; Dijoux, 2009; Dijoux and Idée, 2013; Doyen and Gaudoin, 2004; Kijima, 1989; Pham and Wang, 1996). This modelling technique is able to deal with failure data which is often considered non-stationary (Chiu et al., 2008; Guida et al., 1989; Hirata et al., 2009). However, we note that NHPP has not yet been used for modelling suppliers' non-conformance data. Furthermore, power law model is used as the intensity function to model a supplier's non-conformance rate. The consideration of using power law model is mainly due to its flexibilities to describe various types of supplier's performance. As Guida et al. (1989) note, power law model enabling analysing systems where the rate of occurrence of failures may be a decreasing, constant, or increasing function of time, respectively. Such characteristics might be exhibited in suppliers' KPIs.

To express a supplier's improvement potential, a virtual age model is used. The virtual age model has been used in imperfect maintenance models for assessing the maintenance effect (Dijoux and Idée, 2013). For managing important industrial systems (e.g. wind turbine generator systems), repair work is often carried out to maintain the reliability of the system and to extend the functioning life in which the assessment of the repair efficiency plays a very important role. Doyen and Gaudoin (2004) note that "The basic assumptions on repair efficiency are known as minimal repair or As Bad As Old (ABAO) and perfect repair or As Good As New (AGAN). In the ABAO case, each repair leaves the system in the same state as it was before failure. In the AGAN case, each repair is perfect and leaves the system as if it were new." The virtual age model enables describing maintenance effect and is often characterised by a sequence of effective ages to indicate the efficiency. For example, Dijoux and Idée (2013) use a bathtub shaped intensity function for describing failure rate and a virtual age to express the repair efficiency by assuming that the maintenance work enables shortening the burn-in period. A burn-in period indicates an infant stage of a system that as the operating time increases, the failure rate decreases until the system reaches a mature stage. In this research, the consideration of using the virtual age model for the proposed problem is inspired by the similarity between supplier development activity and reliability maintenance activity. First of all, both activities aim to improve the performance of a targeting entity and the average failure rate of the targeting entity is often considered non-stationary. Second, both activities require investment of resources. Thus, optimising the investment of resources to lower down the cost of the activity is important. As noted by Pham and Wang (1996) for the reliability maintenance activity, "Maintenance involves preventive (planned) and unplanned actions carried out to retain a system in or restore it to an acceptable operating condition. Optimal maintenance policies aim to provide optimum system reliability/availability and safety performance at lowest possible maintenance costs." Under the context of this research, we assume that an experienced supplier performs better than an inexperienced supplier. Similar as the use of a virtual age on reflecting the ability of shortening the burn-in period in maintenance management in the study conducted by Dijoux and Idée (2013), we use a virtual age to describe the expectation of the experience gained by the young supplier from a supplier development activity to become as good as an experienced supplier.

5.3 Supplier Uncertainty Modelling

5.3.1 Power Law NHPP Model

Let P_N denote the NHPP model and $M_j(t_T)$ denote the random number of non-conformances realised over the exposure to risk t_T from supplier j, which is assumed to be distributed from power law NHPP model. Let $\Lambda_j(t_T)$ denote the expected number of non-conformances over the exposure to risk t_T. Therefore, the probability of a given number of non-conformance m_j for supplier j can be expressed in Eq.(5.1).

$$P_N\left[M_j\left(t_T\right)=m_j\right]=\frac{\left[\Lambda_j\left(t_T\right)\right]^{m_j}e^{-\Lambda_j\left(t_T\right)}}{m_j!}. \tag{5.1}$$

Let the expected number of non-conformances $\Lambda_j(t_T)$ be expressed using power law model:

$$\Lambda_j(t_T)=a_j{t_T}^{b_j}. \tag{5.2}$$

Replacing the parameter $\Lambda_j(t_T)$ in Eq.(5.1) by Eq.(5.2), a NHPP model with a power law intensity function can be written as follows:

$$P_N\left[M_j\left(t_T\right)=m_j\right|a_j,b_j\left.\right]=\frac{\left(a_j{t_T}^{b_j}\right)^{m_j}e^{-a_j{t_T}^{b_j}}}{m_j!}. \tag{5.3}$$

The exposure to risk t_T in this study is expressed using both continuous and discrete variables, namely operating time and the number of parts ordered from the supplier. Therefore, if the exposure to risk is measured on a continuous scale, we can further obtain the expression of non-conformance rate $\lambda_j\left(t_T\right)$ at t in Eq.(5.4), where $a_j, b_j > 0$ and $t \in [0, t_T]$.

$$\lambda_j(t) = a_j b_j t^{b_j - 1}. \tag{5.4}$$

If b_j is not equal to 1, the behaviour of supplier non-conformance is a non-stationary process as the non-conformance rate changes with respect to the risk exposure t. That is, if $b_j > 1$, the non-conformance rate increases as t increases, and if $b_j < 1$, the non-conformance rate decreases as t increases. This study acknowledges the variation in supplier capacity by allowing each supplier to have its own scale and shape parameter a_j and b_j, where a_j as the scale parameter decides to stretch out or squeezes the distribution and b_j as the shape parameter affects the general shape of distribution.

5.3.2 Parameter Estimation

In this section, the MLE of the parameters in the probabilistic model (Eq.(5.3)) is first provided. Then, a mixed method of empirical Bayes inference and subjective judgement for estimating the model parameters are detailed to capture the epistemic uncertainty.

5.3.2.1 Maximum Likelihood Estimator

Let m_{jk} represent the number of non-conformances realised at the k_{th} observation over $[t_{k-1}, t_k]$ for supplier j. Assuming there are K observations in total $\mathbf{M_j} = [m_{j1}...m_{jK}]$, the likelihood function of the parameter a_j and b_j therefore can be written as in Eq.(5.5), where $t_0 = 0$ and $t_K = t_T$.

$$L\left(a_j, b_j\right) = \prod_{k=1}^{K} \frac{\left(a_j\left(t_k^{b_j} - t_{k-1}^{b_j}\right)\right)^{m_{jk}} e^{-a_j\left(t_k^{b_j} - t_{k-1}^{b_j}\right)}}{m_{jk}!}. \tag{5.5}$$

As the logarithm transforms a product of densities resulted by the likelihood function into a sum which is easier for calculation, the log-likelihood function is obtained and expressed as in Eq.(5.6).

$$l\left(a_j, b_j\right) = \sum_{K=1}^{K} m_{jk} \ln a_j + \sum_{k=1}^{K} m_{jk} \ln\left(t_k^{b_j} - t_{k-1}^{b_j}\right) - \sum_{k=1}^{K} a_j \left(t_k^{b_j} - t_{k-1}^{b_j}\right) - \sum_{k=1}^{K} \ln\left(m_{jk}!\right). \tag{5.6}$$

Although closed MLE estimations form does not exist, the estimator $\hat{a}_j$ and $\hat{b}_j$ can be easily solved in a spreadsheet using Eq.(5.7).

$$\begin{aligned}
\frac{\partial l\left(\hat{a}_j, \hat{b}_j\right)}{\partial \hat{a}_j} &= \sum_{k=1}^{K} \frac{m_{jk}}{\hat{a}_j} - \sum_{k=1}^{K} \left(t_k^{\hat{b}_j} - t_{k-1}^{\hat{b}_j}\right) = 0 \\
\frac{\partial l\left(\hat{a}_j, \hat{b}_j\right)}{\partial \hat{b}_j} &= \sum_{k=1}^{K} m_{jk} \frac{1}{t_k^{\hat{b}_j} - t_{k-1}^{\hat{b}_j}} \left(t_k^{\hat{b}_j} \ln t_k - t_{k-1}^{\hat{b}_j} \ln t_{k-1}\right) - \hat{a}_j \sum_{k=1}^{K} \left(t_k^{\hat{b}_j} \ln t_k - t_{k-1}^{\hat{b}_j} \ln t_{k-1}\right) = 0.
\end{aligned} \tag{5.7}$$

5.3.2.2 Empirical Bayes

In this study, a mixed method of empirical Bayes inference and subjective expert judgement for parameter estimation is provided to capture the epistemic uncertainty. The study proposes to use empirical Bayes method to estimate the scale parameter a_j across the data pool. Mathematically this can be accomplished by assuming the parameter a_j is realised from a common Gamma distribution denoted by G with shape parameter α_g and rate parameter β_g:

$$G\left(a_j; \alpha_g, \beta_g\right) = \frac{\beta_g^{\alpha_g} {a_j}^{\alpha_g - 1} e^{-\beta_g a_j}}{\Gamma\left(\alpha_g\right)}. \tag{5.8}$$

To provide a closed form of the predictive distribution, we assume that there are a number of possible values for the shape parameter across the data pool, denoted by $\mathbf{b_{jq}} = [b_{j1} ... b_{jQ}]$ indicating the types of suppliers in the data pool. A probability ω_q is attached to indicate the likelihood that the quantity of interest belongs to the corresponding category. The value of b_{jq} and ω_q can be obtained through expert judgement. Therefore, the predictive probability distribution can be obtained in Eq.(5.9), which results in the form of a weighted average Negative Binomial Distribution with the parameters α_g and β_g.

$$P_N[M_j(t_T) = m_j|\mathbf{b_{jq}}] = \sum_{q=1}^{Q} \omega_q \frac{\Gamma(m_j+\alpha_g)}{\Gamma(\alpha_g)\, m_j!} \left(\frac{\beta_g}{t_T{}^{b_{jq}}+\beta_g}\right)^{\alpha_g} \left(\frac{t_T{}^{b_{jq}}}{t_T{}^{b_{jq}}+\beta_g}\right)^{m_j}, \quad (5.9)$$

where, m_j represents an total observed number of non-conformances at supplier j over t_T.

To estimate the parameters, we first uses MLE method. Let m_{jk} represent the number of non-conformances at the k_{th} observation over $[t_k, t_{k-1}]$ for supplier j. Assuming there are J suppliers and for each supplier there are K observations $\mathbf{M_j} = [m_{j1}...m_{jK}]$, the likelihood function of Eq.(5.9) can be expressed in Eq.(5.10), where $t_0 = 0$, $t_K = t_T$ and $\sum\limits_{k=1}^{K} m_{jk} = m_j$

$$L(\alpha_g, \beta_g) = \prod_{j=1}^{J} \frac{\beta_g^{\alpha_g}}{\Gamma(\alpha_g)} \left(\sum_{q=1}^{Q} \omega_q \left(\frac{\Gamma(m_j+\alpha_g)}{\left(\beta_g + t_T{}^{b_{jq}}\right)^{m_j+\alpha_g}} \prod_{k=1}^{K} \frac{\left(t_k^{b_{jq}} - t_{k-1}^{b_{jq}}\right)^{m_{jk}}}{m_{jk}!} \right) \right). \quad (5.10)$$

The log-likelihood function can be obtained in Eq.(5.11).

$$l(\alpha_g, \beta_g) = \sum_{j=1}^{J} \left(\ln(\Gamma(m_j+\alpha_g)) + \ln\left(\sum_{q=1}^{Q} \omega_q \frac{\prod\limits_{k=1}^{K} \frac{\left(t_k^{b_{jq}} - t_{k-1}^{b_{jq}}\right)^{m_{jk}}}{m_{jk}!}}{\left(\beta_g + t_T{}^{b_{jq}}\right)^{m_j+\alpha_g}} \right) \right) + J\alpha_g \ln(\beta_g) - J\ln(\Gamma(\alpha_g)). \quad (5.11)$$

By differentiating Eq.(5.11) with respect to α_g and β_g and letting the differential equations equate to zero, we can obtain the MLE estimation of these parameters. However, as the MLE estimations appear to require complicated computation and do not have closed-form solutions, MoM estimators are also obtained in Eq.(5.12):

$$\begin{aligned}
\frac{\hat{\alpha}_g}{\hat{\beta}_g} &= \frac{\sum\limits_{j=1}^{J} m_j}{\sum\limits_{j=1}^{J}\sum\limits_{q=1}^{Q} \omega_q {t_T}^{b_{jq}}} \\
\frac{\hat{\alpha}_g\left(\hat{\alpha}_g+1\right)}{\hat{\beta}_g^2} &= \frac{\sum\limits_{j=1}^{J} m_j^2 - \sum\limits_{j=1}^{J} m_j}{\sum\limits_{j=1}^{J}\sum\limits_{q=1}^{Q} \omega_q {t_T}^{2b_{jq}}}.
\end{aligned} \tag{5.12}$$

5.4 Mathematical Modelling of Decision Making

This section develops an analytical decision support model for decision making in developing a supplier whose non-conformance rate changes over the risk exposure. The power law NHPP (Eq.(5.3)) is used to capture the uncertainty of a supplier's non-conformance, in which the exposure to risk is expressed using both continuous and discrete variables. The virtual age model I proposed by Kijima (1989) is adapted in this study to facilitate the assessment of the improvement of the supplier's performance. Also, the time value of the money is taken into consideration.

5.4.1 Supplier Non-Conformance and the Virtual Age Model

Let $M_j\left(t_{k-1}, t_k\right)$ denote the random number of nonconforming units realised over the exposure to risk $[t_{k-1}, t_k]$ for supplier j. As the study considers that the supplier to be developed is an inexperienced supplier who is on a self-improvement stage, this indicates that the supplier's non-conformance rate decreases as more experience is gained and leads to the shape parameter subject to $b_j \in [0, 1]$. Therefore, the expected number for supplier j can be expressed in Eq.(5.13).

$$
\begin{aligned}
E[M_j(t_{k-1}, t_k)] &= \int_{t_{k-1}}^{t_k} a_j b_j t^{b_j - 1} dt \\
&= a_j {t_k}^{b_j} - a_j {t_{k-1}}^{b_j}.
\end{aligned}
\tag{5.13}
$$

Assuming an experienced supplier performs better than an inexperienced supplier. We measure the inexperienced supplier's non-conformance after a development activity by adding a virtual age. This indicates the degree to which the inexperienced supplier can be improved "as good as" an experienced supplier through the activity. Let y_j denote the virtual age to be achieved which indicates the experience gained from the activity for the inexperienced supplier, the expected number of nonconforming items after improvement can be expressed as in Eq.(5.14).

$$
\begin{aligned}
E[M_j(t_{k-1}, t_k | y_i)] &= \int_{t_{k-1}}^{t_k} a_j b_j (t + y_j)^{b_j - 1} dt \\
&= a_j (t_k + y_j)^{b_j} - a_j (t_{k-1} + y_j)^{b_j}.
\end{aligned}
\tag{5.14}
$$

5.4.2 Decision Support Model

This section discusses the development of the decision support model based on two different types of exposure to risk: order size and and operating time.

5.4.2.1 Order Size as Exposure to Risk

Let c_j denote the cost of non-conformance per unit and r denotes a continuous compounding interest rate. Assuming the supplier provides parts for the buyer permanently and the interval between observing points indicated by k is measured on a year-based unit, if no virtual age is considered, based on Eq.(5.13) the net present value (NPV) of the expected total cost of non-conformances for supplier j over $[0, \infty]$, denoted by u_j, can be expressed in Eq.(5.15).

$$
\begin{aligned}
E\left[u_j\left(0,\infty\right)\right] &= \sum_{k=1}^{\infty} E\left[u_j\left(t_{k-1},t_k\right)\right] \\
&= \sum_{k=1}^{\infty} c_j a_j \left(t_k^{b_j} - t_{k-1}^{b_j}\right) e^{-rk}.
\end{aligned}
\tag{5.15}
$$

If a virtual age is considered, let $v_j\left(y_j\right)$ denote the cost of virtual age y_j for supplier j meaning that how much the buyer needs to invest in order to increase the supplier's experience by y_j. Based on Eq.(5.14), the NPV of the expected total cost under a virtual age y_j over $[0,\infty]$, denoted by u_j^*, can be expressed in Eq.(5.16).

$$
\begin{aligned}
E\left[u_j^*\left(0,\infty\right)|y_j\right] &= \sum_{k=1}^{\infty} E\left[u_j^*\left(t_{k-1},t_k\right)|y_j\right] \\
&= \sum_{k=1}^{\infty} c_j a_j \left[\left(t_k + y_j\right)^{b_j} - \left(t_{k-1} + y_j\right)^{b_j}\right] e^{-rk} + v_j\left(y_j\right).
\end{aligned}
\tag{5.16}
$$

Therefore, the expected benefit obtained from conducting a supplier development activity for supplier j activity over $[0,\infty]$, denoted by π_j, can be written in Eq.(5.17).

$$
E\left[\pi_j\left(0,\infty\right)\right] = \sum_{k=1}^{\infty} c_j a_j \left({t_k}^{b_j} - {t_{k-1}}^{b_j}\right) e^{-rk} - \sum_{k=1}^{\infty} c_j a_j \left[\left(t_k + y_j\right)^{b_j} - \left(t_{k-1} + y_j\right)^{b_j}\right] e^{-rk} - v_j\left(y_j\right).
\tag{5.17}
$$

Proposition 5.1. *Assuming that a virtual age of y is applied on a supplier j via a supplier development activity which is equivalent to gaining ζ years of experience, we can obtain an affine function between the expected cost under no virtual age and the expected cost under a virtual age:*

$$
c_j^0 = \mu_j + \theta_j c_j^y,
\tag{5.18}
$$

where, c_j^0 is the expected total cost caused if no virtual age is applied on a supplier j, which is $c_j^0 = \sum_{k=1}^{\infty} E\left[u_j\left(t_{k-1},t_k\right)\right]$; c_j^y is the expected total cost if a virtual age y is applied on a supplier j, which is $c_j^y = \sum\limits_{k=1}^{\infty} E\left[u_j^*\left(t_{k-1}, t_k\right)|y\right]$; $\mu_j = \sum\limits_{k=1}^{\zeta} c_j d_{jk} e^{-rk} - v_j\left(y\right) e^{-r\zeta}$ and $\theta_j = e^{-r\zeta}$.

Proof. Let d_{jk} denote the number of nonconforming items detected at the k_{th} year for supplier j, which can be written as:

$$d_{jk} = a_j\left(t_k^{b_j} - t_{k-1}^{b_j}\right).$$

Assuming the supplier j will provide parts for the buyer permanently and no external development activity is conducted, we can construct a function of the total cost caused by supplier j:

$$\begin{aligned}
E\left[u_j\left(0,\infty\right)\right] &= \sum_{k=1}^{\infty} E\left[u_j\left(t_{k-1}, t_k\right)\right] \\
&= \sum_{k=1}^{\infty} c_j d_{jk} e^{-rk} \\
&= c_j d_{j,1} e^{-r} + c_j d_{j,2} e^{-2r} + c_j d_{j,3} e^{-3r} + ...
\end{aligned}$$

Assuming supplier j can gain ζ years of experience after development activity, the expected total cost under a virtual age y can be written as:

$$\begin{aligned}
\sum_{k=1}^{\infty} E\left[u_j^*\left(t_{k-1}, t_k\right)|y\right] &= \sum_{k=1}^{\infty} c_j d_{j,k+\zeta} e^{-rk} + v_j\left(y\right) \\
&= c_j d_{j,1+\zeta} e^{-r} + c_j d_{j,2+\zeta} e^{-2r} + c_j d_{j,3+\zeta} e^{-3r} + ... + v_j\left(y\right).
\end{aligned}$$

Therefore, we can obtain:

$$\sum_{k=1}^{\infty} E\left[u_j^*\left(t_{k-1}, t_k\right)|y\right] - v_j\left(y\right) = c_j d_{j,1+\zeta} e^{-r} + c_j d_{j,2+\zeta} e^{-2r} + c_j d_{j,3+\zeta} e^{-3r} + ...$$

Multiplying with $e^{-r\zeta}$ for both sides we get:

$$\left[\sum_{k=1}^{\infty} E\left[u_j^*\left(t_{k-1}, t_k\right)|y\right] - v_j\left(y\right)\right] e^{-r\zeta} = c_j d_{j,1+\zeta} e^{-(1+\zeta)r} + c_j d_{j,2+\zeta} e^{-(2+\zeta)r} + c_j d_{j,3+\zeta} e^{-(3+\zeta)r} + ...$$

Thus, a linear relationship between the total cost without a virtual age and the total cost under a virtual age can be obtained:

$$\begin{aligned}\sum_{k=1}^{\infty} E\left[u_j\left(t_{k-1}, t_k\right)\right] &= \sum_{k=1}^{\zeta} c_j d_{jk} e^{-rk} + \left(\sum_{k=1}^{\infty} E\left[u_j^*\left(t_{k-1}, t_k\right)|y\right] - v_j\left(y\right)\right) e^{-r\zeta} \\ &= \left(\sum_{k=1}^{\zeta} c_j d_{jk} e^{-rk} - v_j\left(y\right) e^{-r\zeta}\right) + \sum_{k=1}^{\infty} E\left[u_j^*\left(t_{k-1}, t_k\right)|y\right] e^{-r\zeta}.\end{aligned}$$

Let $\mu_j = \sum\limits_{k=1}^{\zeta} c_j d_{jk} e^{-rk} - v_j\left(y\right) e^{-r\zeta}and\theta_j = e^{-r\zeta}$, we can obtain:

$$\sum_{k=1}^{\infty} E\left[u_j\left(t_{k-1}, t_k\right)\right] = \mu_j + \theta_j \sum_{k=1}^{\infty} E\left[u_j^*\left(t_{k-1}, t_k\right)|y\right],$$

where, $\sum\limits_{k=1}^{\infty} E\left[u_j\left(t_{k-1}, t_k\right)\right]$ is the expected total cost under no virtual age, and $\sum\limits_{k=1}^{\infty} E\left[u_j^*\left(t_{k-1}, t_k\right)|y\right]$ is the expected total cost if a virtual age y is applied.

Let c_j^0 represent $\sum\limits_{k=1}^{\infty} E\left[u_j\left(t_{k-1}, t_k\right)\right]$ and c_j^y represent $\sum\limits_{k=1}^{\infty} E\left[u_j^*\left(t_{k-1}, t_k\right)|y\right]$, we have:

$$c_j^0 = \mu_j + \theta_j c_j^y.$$

□

In Eq.(5.18), as $0 < \theta_j < 1$, if $\mu_j > 0$, we can draw such a relationship between the expected cost under a virtual age c_j^y and the expected cost under no virtual age c_j^0 as shown in Fig.4.1. The black point is an intersection between the linear function and the reference line which indicates $c_j^0 = c_j^y$. The buyer should consider the supplier development activity if the total cost of the activity lies on the green line as $c_j^0 > c_j^y$, whereas supplier development activity should not be considered. Clearly, if $\mu_j < 0$, a virtual age is too expensive. Such a relationship allows us to identify a right investment decision quickly. In addition, it is worth mentioning that this proposition also holds when the exposure to risk is measured on a continuous scale.

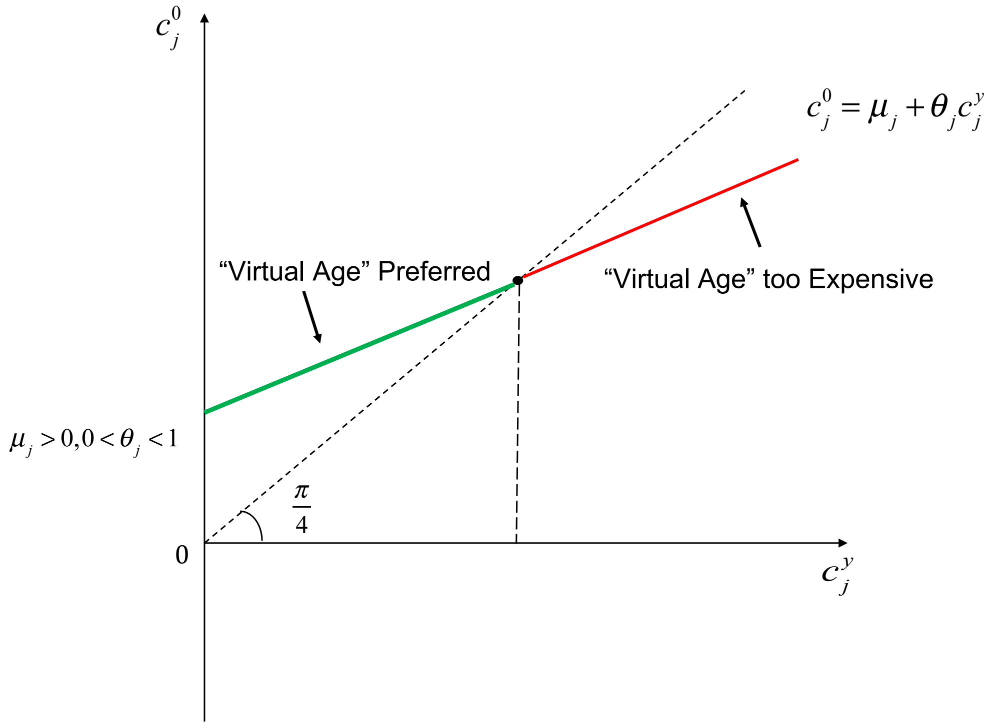

Figure 5.1: An affine relationship between the cost under a virtual age and the cost with no virtual age used, where points lie on the green line indicates “virtual age preferred” and whereas points lie on the red line indicates “virtual age not recommended”

5.4.2.2 Operating Time as the Exposure to Risk

Same as in section 5.4.2.1, c_j denotes the cost of non-conformance per unit and r denotes a continuous compounding interest rate. Assuming the supplier operates permanently and the non-conformance cost is paid immediately at each time point, if no virtual age is considered, a closed form of the expression for the NPV of total expected non-conformance cost from supplier j over the operating time $t \in [0, \infty]$ can be obtained in Eq.(5.19):

$$
\begin{aligned}
E\left[u_j\left(0,\infty\right)\right] &= \int\limits_0^\infty c_j a_j b_j t^{b_j-1} e^{-rt} dt \\
&= c_j a_j b_j \frac{\Gamma\left(b_j\right)}{r^{b_j}}.
\end{aligned} \tag{5.19}
$$

Proof.

$$
\begin{aligned}
E\left[u_j\left(0,\infty\right)\right] &= \int\limits_0^\infty c_j a_j b_j t^{b_j-1} e^{-rt} dt \\
&= c_j a_j b_j r^{-b_j} \int\limits_0^\infty (rt)^{b_j-1} e^{-rt} d(rt) \\
&= c_j a_j b_j \frac{\Gamma\left(b_j\right)}{r^{b_j}}.
\end{aligned}
$$

□

If a virtual age is considered, then we obtain the NPV of total non-conformance cost from supplier j over the operating time $t \in [0,\infty]$ is:

$$
\begin{aligned}
E\left[u_j^*\left(0,\infty\right)|y_j\right] &= \int\limits_0^\infty c_j a_j b_j (t+y_j)^{b_j-1} e^{-rt} dt + v_j\left(y_j\right) \\
&= c_j a_j b_j \frac{e^{y_j r}\Gamma\left(b_j, y_j r\right)}{r^{b_j}} + v_j\left(y_j\right).
\end{aligned} \tag{5.20}
$$

Proof.

$$
\begin{aligned}
E\left[u_j^*\left(0,\infty\right)|y_j\right] &= \int\limits_0^\infty c_j a_j b_j (t+y_j)^{b_j-1} e^{-rt} dt + v_j\left(y_j\right) \\
&= c_j a_j b_j r^{-b_j} \int\limits_0^\infty \left(tr+y_j r\right)^{b_j-1} e^{-rt} d\left(tr\right) + v_j\left(y_j\right) \\
&= c_j a_j b_j r^{-b_j} e^{y_j r} \left(\int\limits_{y_j r}^\infty \left(tr+y_j r\right)^{b_j-1} e^{-(tr+y_j r)} d\left(tr+y_j r\right) \right) + v_j\left(y_j\right) \\
&= c_j a_j b_j \frac{e^{y_j r}\Gamma\left(b_j, y_j r\right)}{r^{b_j}} + v_j\left(y_j\right).
\end{aligned}
$$

□

Assuming that the total cost of the virtual $v_i(y_i)$ increases linearly with the achieving virtual age, let δ_j denote the cost of the virtual age per unit, $\delta_i > 0$, then a mathematical expression for the cost of virtual age is written as Eq.(5.21):

$$v_j(y_j) = \delta_j y_j. \tag{5.21}$$

The expected profit over $t \in [0, \infty]$ therefore can be expressed as Eq.(5.22):

$$\begin{aligned} E[\pi_j(0,\infty)|y_j] &= E[u_j(0,\infty)] - E\left[u_j^*(0,\infty)|y_j\right] \\ &= c_j a_j b_j \frac{\Gamma(b_j) - e^{y_j r}\Gamma(b_j, y_j r)}{r^{b_j}} - \delta_j y_j. \end{aligned} \tag{5.22}$$

Based on the expression of the expected profit, an upper and lower bound of the optimal investment solution is obtained and formulated in Proposition 5.2.

Proposition 5.2. *We can obtain an upper and lower bound of the optimal virtual age y_j^*, which provides boundaries for the investment level.*

$$y_j^* \in \left(\sqrt[b_j-1]{\frac{\delta_j}{c_j a_j b_j} + \frac{\Gamma(b_j)}{r^{b_j-1}}},\ \sqrt[b_j-1]{\frac{\delta_j}{c_j a_j b_j}} \right). \tag{5.23}$$

Proof. By differentiating Eq.(5.22) with respect to y_j, we get:

$$\frac{dE[\pi_j(0,\infty)|y_j]}{dy_j} = c_j a_j b_j \frac{-e^{y_j r}\Gamma(b_j, y_j r) + (y_j r)^{b_j-1}}{r^{b_j-1}} - \delta_j.$$

Let $\frac{dE[\pi_j(0,\infty)|y_j]}{dy_j} = 0$ and re-arranging it, we have:

$$\left(y_j^*\right)^{b_j-1} = \frac{\delta_j}{c_j a_j b_j} + \frac{e^{y_j^* r}}{r^{b_j-1}}\Gamma\left(b_j, y_j^* r\right).$$

Then we can write such an expression:

$$y_{j,\varsigma+1}^* = \sqrt[b_j-1]{\frac{\delta_j}{c_j a_j b_j} + \frac{e^{y_{j,\varsigma}^* r}}{r^{b_j-1}}\Gamma\left(b_j, y_{j,\varsigma}^* r\right)}.$$

Let $h(y^*_{j,\varsigma}) = \frac{e^{y^*_{j,\varsigma}r}}{r^{b_j-1}}\Gamma\left(b_j, y^*_{j,\varsigma}r\right)$, by differentiating $h(y^*_{j,\varsigma})$ with respect to $y^*_{j,\varsigma}$, we get:

$$\frac{dh(y^*_{j,\varsigma})}{dy^*_{j,\varsigma}} = r^{2-b_j}\left[e^{y^*_{j,\varsigma}r}\Gamma\left(b_j, y^*_{j,\varsigma}r\right) - \left(y^*_{j,\varsigma}r\right)^{b_j-1}\right].$$

For $\Gamma\left(b_j, y^*_{j,\varsigma}r\right)$ we have:

$$\Gamma\left(b_j, y^*_{j,\varsigma}r\right) = \int\limits_{y^*_{j,\varsigma}r}^{\infty} \rho^{b_j-1}\mathrm{e}^{-\rho}d\rho.$$

As $0 < b_j \leq 1$, we can get: for any $\rho \geq y^*_{j,\varsigma}r$, $\rho^{b_i-1} \leq \left(y^*_{j,\varsigma}r\right)^{b_i-1}$. Therefore, we have:

$$\begin{aligned}\Gamma\left(b_j, y^*_{j,\varsigma}r\right) &\leq \left(y^*_{j,\varsigma}r\right)^{b_j-1}\int\limits_{y^*_{j,\varsigma}r}^{\infty} e^{-\rho}d\rho \\ &\leq \frac{e^{-y^*_{j,\varsigma}r}}{\left(y^*_{j,\varsigma}r\right)^{b_j-1}}.\end{aligned}$$

Thus, $e^{y^*_{j,\varsigma}r}\Gamma\left(b_j, y^*_{j,\varsigma}r\right) \leq e^{y^*_{j,\varsigma}r}\frac{e^{-y^*_{j,\varsigma}r}}{\left(y^*_{j,\varsigma}r\right)^{b_j-1}} \leq \left(y^*_{j,\varsigma}r\right)^{b_j-1}$. Then, we can obtain:

$$\frac{dh(y^*_{j,\varsigma})}{dy^*_{j,\varsigma}} = r^{b_j}\left[e^{y^*_{j,\varsigma}r}\Gamma\left(b_j, y^*_{j,\varsigma}r\right) - \left(y^*_{j,\varsigma}r\right)^{b_j-1}\right] \leq 0.$$

This indicates that $h(y^*_{j,\varsigma})$ decreases as $y^*_{j,\varsigma}$ increases. As $y^*_{j,\varsigma} \in [0, \infty]$ of $y^*_{j,\varsigma}$, we can obtain:

$$\lim_{y^*_{j,\varsigma}\to\infty} h(y^*_{j,\varsigma}) = \lim_{y^*_{j,\varsigma}\to\infty} \frac{e^{y^*_{j,\varsigma}r}\Gamma\left(b_j, y^*_{j,\varsigma}r\right)}{r^{b_j-1}} \approx 0.$$

And,

$$\lim_{y^*_{j,\varsigma}\to 0} h(y^*_{j,\varsigma}) = \lim_{y^*_{j,\varsigma}\to 0} \frac{e^{y^*_{j,\varsigma}r}\Gamma\left(b_j, y^*_{j,\varsigma}r\right)}{r^{b_j-1}} \approx \frac{\Gamma\left(b_j\right)}{r^{b_j-1}}.$$

Thus, according to the Brouwer fixed-point theorem, there exists an optimal investment solution that lies within such an interval:

$$y^*_j \in \left(\sqrt[b_j-1]{\frac{\delta_j}{c_j a_j b_j} + \frac{\Gamma\left(b_j\right)}{r^{b_j-1}}},\ \sqrt[b_j-1]{\frac{\delta_j}{c_j a_j b_j}}\right).$$

□

Furthermore, a lower bound of an expected profit can also be obtained and formulated as proposition 5.3.

Proposition 5.3. *The following expression provides a lower bound of an expected profit.*

$$E\left[\pi_j\left(0,\infty\right)|y_j\right] \geq c_j a_j b_j {y_j}^{b_j}\left(\frac{e^{-y_j r}-1}{y_j r}+\frac{1}{e^{y_j r} b_j}\right)-\delta_j y_j. \tag{5.24}$$

Proof. As $\Gamma\left(b_j\right)=\Gamma\left(b_j, y_j r\right)+\gamma\left(b_j, y_j r\right)$, from Eq.(5.22) we can obtain:

$$\begin{aligned}
E\left[\pi_j\left(0,\infty\right)|y_j\right] &= c_j a_j b_j \frac{\Gamma\left(b_j\right)-e^{y_j r}\Gamma\left(b_j, y_j r\right)}{r^{b_j}}-\delta_j y_j \\
&= c_j a_j b_j \frac{\Gamma\left(b_j, y_j r\right)+\gamma\left(b_j, y_j r\right)-e^{y_j r}\Gamma\left(b_j, y_j r\right)}{r^{b_j}}-\delta_j y_j \\
&= c_j a_j b_j \frac{\left(1-e^{y_j r}\right)\Gamma\left(b_j, y_j r\right)+\gamma\left(b_j, y_j r\right)}{r^{b_j}}-\delta_j y_j.
\end{aligned}$$

As while proving the Proposition 5.2 we have obtained that when $0 < b_j \leq 1$, $\Gamma\left(b_j, \kappa\right) \leq \frac{e^{-\kappa}}{\kappa^{1-b_j}}$, thus we have:

$$\Gamma\left(b_j, y_j r\right) \leq \left(y_j r\right)^{b_j-1} e^{-y_j r}$$

Multiplying $1-e^{y_j r}$ for both sides, where $1-e^{y_j r}<0$, we can obtain:

$$\begin{aligned}
\left(1-e^{y_j r}\right)\Gamma\left(b_j, y_j r\right) &\geq \left(1-e^{y_j r}\right)\left(y_j r\right)^{b_j-1} e^{-y_j r} \\
&\geq \left(e^{-y_j r}-1\right)\left(y_j r\right)^{b_j-1}.
\end{aligned}$$

Similarly, for $\gamma\left(b_j, y_j r\right)$ we have:

$$\begin{aligned}
\gamma\left(b_j, y_j r\right) &= \int\limits_0^{y_j r} \rho^{b_j-1} \mathrm{e}^{-\rho} d\rho \\
&\geq e^{-y_j r} \int\limits_0^{y_j r} \rho^{b_j-1} d\rho \\
&\geq e^{-y_j r} \frac{\left(y_j r\right)^{b_j}}{b_j}.
\end{aligned}$$

As we know that the expected profit is expressed as:

$$E\left[\pi_j\left(0,\infty\right)|y_j\right] = c_j a_j b_j \frac{\left(1-e^{y_j r}\right)\Gamma\left(b_j, y_j r\right) + \gamma\left(b_j, y_j r\right)}{r^{b_j}} - \delta_j y_j,$$

where,

$$\begin{aligned}
& c_j a_j b_j \frac{\left(1-e^{y_j r}\right)\Gamma\left(b_j, y_j r\right) + \gamma\left(b_j, y_j r\right)}{r^{b_j}} - \delta_j y_j \\
& \geq c_j a_j b_j \frac{\left(e^{-y_j r}-1\right)\left(y_j r\right)^{b_j-1} + e^{-y_j r}\frac{\left(y_j r\right)^{b_j}}{b_j}}{r^{b_j}} - \delta_j y_j \\
& \geq c_j a_j b_j {y_j}^{b_j}\left(\frac{e^{-y_j r}-1}{y_j r} + \frac{e^{-y_j r}}{b_j}\right) - \delta_j y_j.
\end{aligned}$$

Therefore, we can get an upper bound of the total expected profit:

$$E\left[\pi_j\left(0,\infty\right)|y_j\right] \geq c_j a_j b_j {y_j}^{b_j}\left(\frac{e^{-y_j r}-1}{y_j r} + \frac{1}{e^{y_j r} b_j}\right) - \delta_j y_j.$$

□

We further examined the optimisation properties of the expected profit (Eq.(5.22)) which is formulated in Proposition 5.4.

Proposition 5.4. *As the benefit obtained from buying a virtual age of y_j for a supplier j is measured by a reduction of non-conformance costs, a maximal expected profit exists if MB is smaller than $\frac{c_j a_j b_j y_j^{b_j-2}(1-b_j)}{r}$, and a minimal expected profit exists if the marginal benefit (MB) is larger than $\frac{c_j a_j b_j y_j^{b_j-2}(1-b_j)}{r}$.*

Proof. Let ω denote the benefit obtained from supplier development activity. Mathematically, the benefit can be formulated as follows:

$$\begin{aligned}
E\left[\tau\left(0,\infty\right)|y_j\right] &= \int_0^\infty c_j a_j b_j t^{b_j-1} e^{-rt} dt - \int_0^\infty c_j a_j b_j (t+y_j)^{b_j-1} e^{-rt} dt \\
&= c_j a_j b_j \frac{\Gamma\left(b_j\right) - e^{y_j r}\Gamma\left(b_j, y_j r\right)}{r^{b_j}}.
\end{aligned}$$

Then the marginal benefit (MB) is:

$$\begin{aligned}
MB &= \frac{dE\left[\tau\left(0,\infty\right)|y_j\right]}{dy_j} \\
&= c_j a_j b_j r \frac{-e^{y_j r}\Gamma\left(b_j, y_j r\right) + \left(y_j r\right)^{b_j-1}}{r^{b_j}}.
\end{aligned}$$

To examine the optimisation properties of the expected profit, we can take the second partial derivative of Eq.(5.22) with respect to y_j which holds a relationship with MB:

$$\begin{aligned}
\frac{d^2E\left[\pi_j\left(0,\infty\right)|y_j\right]}{dy_j^2} &= c_j a_j b_j r^2 \frac{-e^{y_j r}\Gamma\left(b_j, y_j r\right) + \left(y_j r\right)^{b_j-1} + \left(y_j r\right)^{b_j-2}\left(b_j-1\right)}{r^{b_j}} \\
&= c_j a_j b_j r^2 \frac{-e^{y_j r}\Gamma\left(b_j, y_j r\right) + \left(y_j r\right)^{b_j-1}}{r^{b_j}} + c_j a_j b_j y_j^{b_j-2}\left(b_j-1\right) \\
&= rMB - c_j a_j b_j y_j^{b_j-2}\left(1-b_j\right).
\end{aligned}$$

As $0 < b_j \leq 1$, then $c_j a_j b_j y_j^{b_j-2}\left(1-b_j\right) \geq 0$. In addition, while proving Proposition 5.2 we have obtained that $\Gamma\left(b_j, y_j r\right) \leq \left(y_j r\right)^{b_j-1} e^{-y_j r}$ when $0 < b_j \leq 1$, therefore, we can further get:

$$\begin{aligned}
MB &= c_j a_j b_j r \frac{-e^{y_j r}\Gamma\left(b_j, y_j r\right) + \left(y_j r\right)^{b_j-1}}{r^{b_j}} \\
&= \frac{c_j a_j b_j}{r^{b_j-1}} e^{y_j r}\left(\frac{e^{-y_j r}}{\left(y_j r\right)^{1-b_j}} - \Gamma\left(b_j, y_j r\right)\right) \geq 0.
\end{aligned}$$

This shows consistency with our model assumption. Therefore, a local maximum can be obtained if:

$$\frac{d^2E\left[\pi_j\left(0,\infty\right)|y_j\right]}{dy_j^2} = rMB - c_j a_j b_j y_j^{b_j-2}\left(1-b_j\right) < 0.$$

Re-arranging, we have:

$$MB < \frac{c_j a_j b_j y_j^{b_j-2}\left(1-b_j\right)}{r}.$$

Accordingly, the opposite holds for a local minimum. □

5.5 Numerical Investigation

The purpose of this section is to illustrate the behaviour of the proposed model through numerical investigations. Fig.5.2 provides a profile for the analysis process.

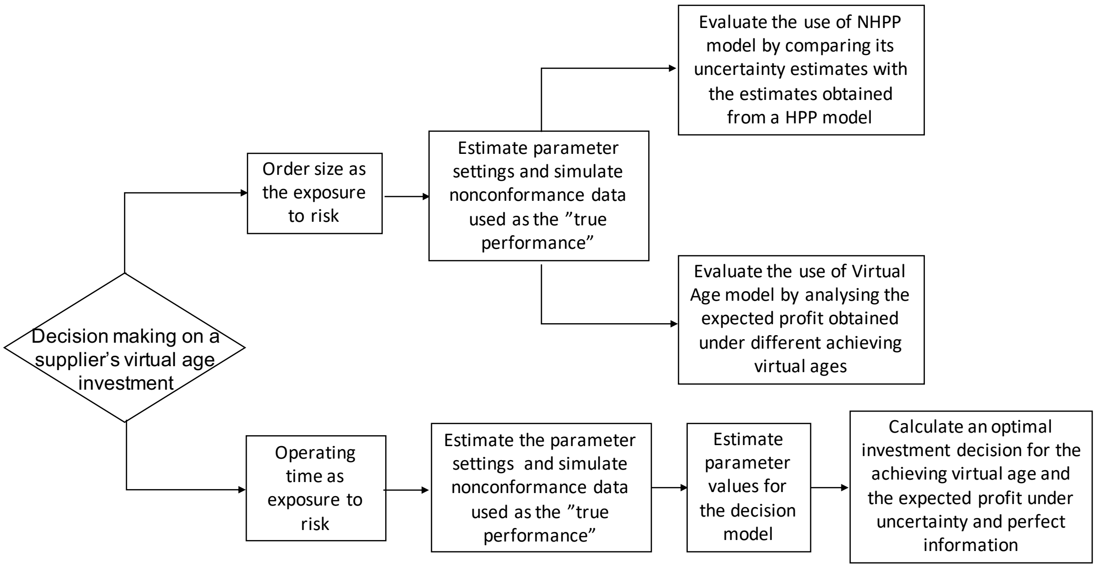

Figure 5.2: A flowchart of the data analysis process

5.5.1 Order Size as the Exposure to Risk

This section first demonstrates the consequence of using HPP to model changing rates over time in order to validate the assumption of using NHPP to analyse suppliers' performance data under the assumed decision context, and then evaluates the use of a virtual age for decision making under different non-conformance costs.

5.5.1.1 Simulation of Non-Conformance Data

We simulate 6 different datasets based on Eq.(5.3) to represent different types of suppliers' true performance by assigning different values to the parameters: $a = 5, 10$, and $b = 0.25, 0.5, 0.75$. Let t_T denote the exposure to risk, assuming that the buyer has a 10-year contract with 1000 items ordered per year from the supplier, the total number of orders over 10 years is expressed as: $t_T = 10000$. Therefore, a number of non-conforming items at the end of each year can be obtained by following this simulation algorithm:

(1) Set up the value of parameters: $a = [5, 10], b = [0.25, 0.5, 0.75], t_T = 10000$;

(2) Choose a pair of parameter values for a and b, generate a random number N from $Pois(\Lambda(t))$, where, $\Lambda\left(t\right)=a{t_T}^b$;

(3) Simulate N random variate from $u_i \sim U\left[0,1\right]$;

(4) Let $u_i=\frac{\int_0^{t_i}\lambda(t)dt}{\int_0^{t_T}\lambda(t)dt}=\left(\frac{t_i}{t_T}\right)^b$ and obtain t_i;

(5) Sort t_i with a sequence of integers from 1 to N and count the cumulative number of non-conforming items at the end of each year;

(6) Repeat the above process for 1000 times.

5.5.1.2 Evaluation of NHPP Model

HPP is used to demonstrate the consideration for the use of NHPP model when a supplier's non-conformance data changes over time. MLE is used to estimate the parameters for both NHPP and HPP models (Eq.(5.7)). We assume that the supplier has already provided parts for 3 years and use the first 3-year of "true" performance records for estimating the model parameters. The results of the parameter estimates along with the corresponding set-up values are provided in the Table 5.1, where $a^*$and b^* are the estimated parameters for the NHPP model and λ^* is the estimated parameter for the HPP model.

Table 5.1: Parameters settings and the estimates for NHPP and HPP

Dataset	Set-up Values		Estimates		
	a	b	a^*	b^*	λ^*
1	5	0.25	5.87	0.257	12.21
2	5	0.5	5.31	0.501	91.38
3	5	0.75	5.12	0.749	676.21
4	10	0.25	10.71	0.253	24.58
5	10	0.5	10.2	0.502	182.06
6	10	0.75	10.03	0.751	1351.49

Given the estimated parameters, the performance of NHPP and HPP for modelling such dataset is assessed by comparing the expected number of non-conformances at the end of the 4th year to the 9th year obtained from these two models with the "true value". The results are provided in Table 5.2. It is easy to observe that the estimated number of non-conformances from NHPP model are much closer to the true value than the estimates from HPP model. That implies that if HPP is used to model such dataset in reality, this may lead to ineffective decisions as the number of non-conformances is much overestimated.

Table 5.2: Comparison between the expected number of non-conformances under NHPP and HPP models with the true values of the number of non-conformances from T4 to T9

Dataset		1	2	3	4	5	6
	NHPP	3.52	45.62	495.74	6.13	87.72	985.57
T4 (t_T=4000)	HPP	12.21	91.38	676.21	24.58	182.06	1351.49
	True Value	2.84	42.48	488.6	5.53	84.39	975.72
	NHPP	2.92	40.2	465.25	5.07	77.32	925.33
T5 (t_T=5000)	HPP	12.21	91.38	676.21	24.58	182.06	1351.49
	True Value	2.33	37.34	459.04	4.6	74.9	915.55
	NHPP	2.51	36.36	442.31	4.36	69.93	880
T6 (t_T=6000)	HPP	12.21	91.38	676.21	24.58	182.06	1351.49
	True Value	1.93	33.7	436.03	3.88	67.81	871.18
	NHPP	2.22	33.44	424.1	3.85	64.32	844
T7 (t_T=7000)	HPP	12.21	91.38	676.21	24.58	182.06	1351.49
	True Value	1.7	31.28	417.15	3.41	62.26	833.97
	NHPP	1.99	31.13	409.11	3.46	59.88	814.36
T8 (t_T=8000)	HPP	12.21	91.38	676.21	24.58	182.06	1351.49
	True Value	1.51	28.64	403.17	3.08	57.78	807.21
	NHPP	1.82	29.24	396.44	3.15	56.25	789.3
T9 (t_T=9000)	HPP	12.21	91.38	676.21	24.58	182.06	1351.49
	True Value	1.37	26.87	390.03	2.76	53.7	780.55

The errors of the estimates under the NHPP and HPP models are obtained and presented the distributions of the errors using box plots (Fig.5.3). In general, the error distribution of the estimates under the NHPP model are close to zero, whereas the error distribution of the estimates under the HPP model appears to be relatively large.

In particular, the estimate error from HPP model becomes larger as the total order size increases from T4 (t_T=4000) to T9 (t_T=9000).

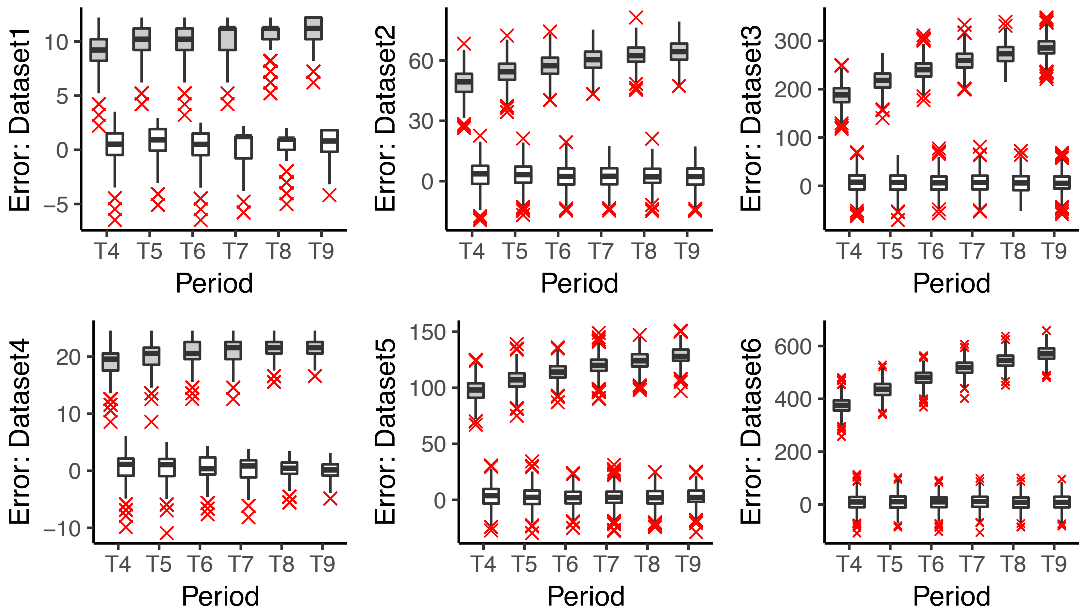

Figure 5.3: Distribution of estimated errors under NHPP (no shadow) and HPP (grey shadow) from dataset 1 to 6 (T4 indicates t_T=4000). The error distribution of the estimates under NHPP model are close to zero, whereas the error distribution of the estimates under HPP models appears to be larger as the total order size increases.

We further obtain the RMSE (Root Mean Square Error) which provides a relatively high weight to large errors to express the average error of estimates under these two models. The results are presented in Fig.5.4. In general, the RMSE under NHPP model is much smaller than the RMSE under HPP model. For NHPP model, the RMSE decreases as the order size increases, whereas for HPP model the RMSE increases as the order size increases. In addition, we can observe that the value of RMSE is larger for suppliers with higher number of non-conformances. Given the analysis of the error distribution and RMSE of the estimates obtained from NHPP model and HPP model, we conclude that for modelling non-conformance dataset which has similar characteristics with the simulated data, using NHPP model is more appropriate than using HPP model.

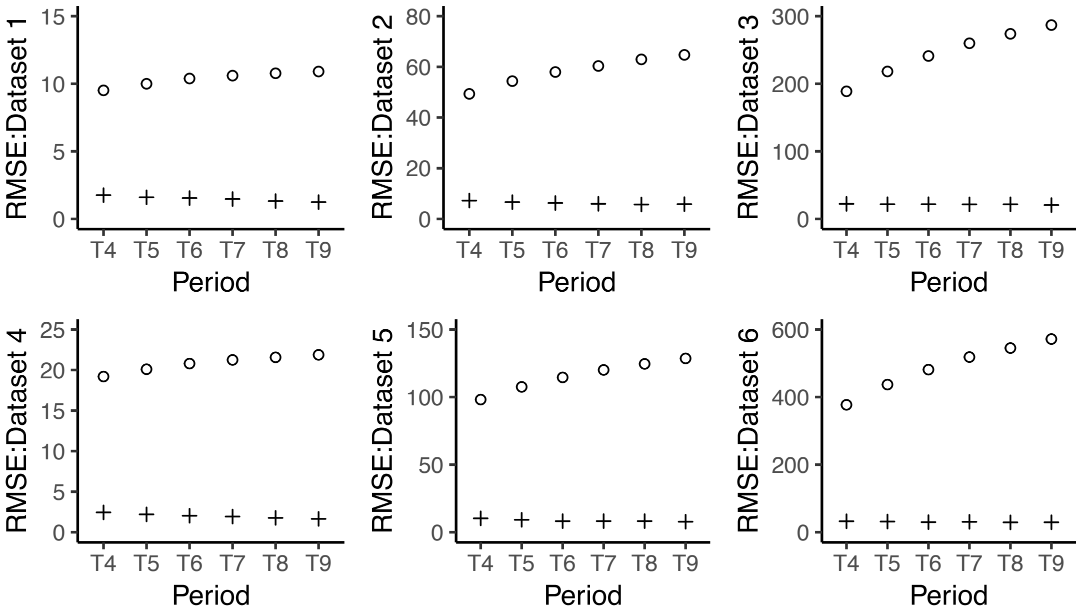

Figure 5.4: RMSE of the average estimated error under NHPP (marked as a cross) and HPP model (marked as a circle). The RMSE of estimates from NHPP model decreases as the order size increases, whereas the RMSE of estimates from HPP model increases as the order size increases (data provided in the Appendix A: Table A.5)

5.5.1.3 Evaluation of the use of a Virtual Age on Decision Making

We now evaluate the use of a virtual age by analysing the change in the expected profit in relation to the level of the virtual age achieved. The following values for an achieving virtual age are considered to represent the manufacturing experience to be gained: $Y = 1000, 2000, 3000, 4000, 5000$, where Y represents the number of items manufactured. Assuming that the marginal cost of the virtual age are $1u$, $5u$ and $10u$ and the cost of non-conformances is $100u$ per order and the annual compounding interest rate is 0.5%, under the different achieving virtual ages expected profits are calculated based on the proposed decision support model (Eq.(5.17)), where the expected profit is obtained under the simulated data of non-conformances. The results are presented Fig.5.5-Fig.5.7 which provide a number of interesting findings. First of all, the profit increases as the cost of virtual age decreases, therefore, a virtual age may be considered if the marginal cost is low. For example, for dataset 1, under the virtual age Y=1000, when the marginal cost of a virtual age is $10u$ (Fig.5.5) the profit is -$6715u$, however, when the marginal cost of a virtual age is $1u$ (Fig.5.7) the profit is $2285u$. Similar findings can also be found in other datasets. Second, the type of suppliers has a significant impact on the profit obtained from an achieving virtual age. That is, the profit is high if the expected number of non-conformances is high. For example, for dataset 1 and 4, as the expected number of non-conformances is already rather low, no profit is obtained if a virtual age is expensive. In contrast, for dataset 3 and 6, where the expected number of non-conformances is high, the profit grows as the achieving virtual age increases. Third, an optimal investment solution of the virtual age to be invested exists. For example, for dataset 5, when the marginal cost of a virtual age is $5u$ (Fig.5.6), an optimal achieving virtual age may be obtained at Y=3000.

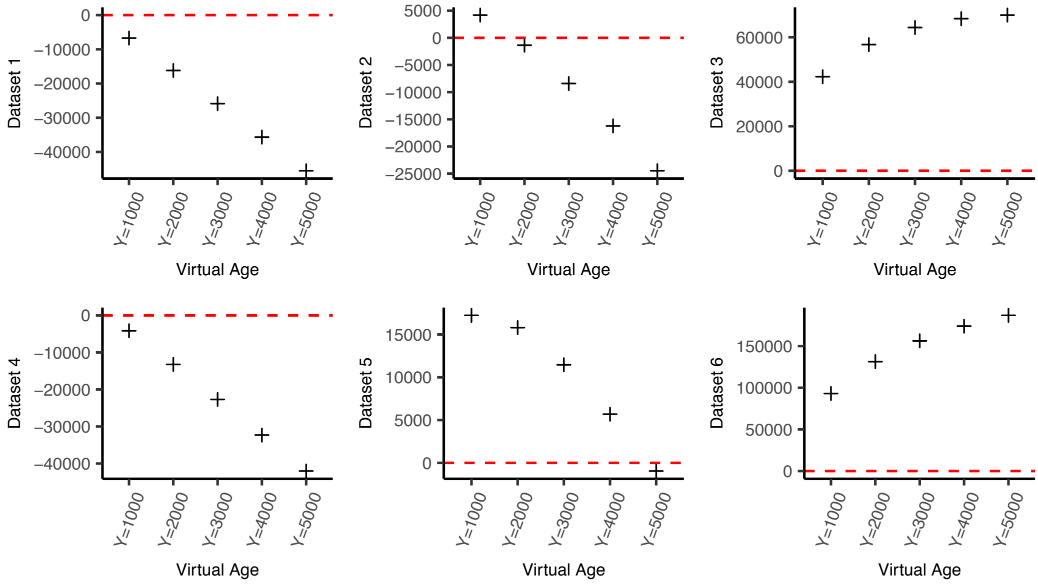

Figure 5.5: NPV of expected profit when the marginal cost of virtual age is 10u (Dataset 1-6: left-to-right, top-to-bottom). In general, the profit increases as the cost of virtual age decreases and the profit is high if the expected number of non-conformance is high (data provided in the Appendix A: Table A.6)

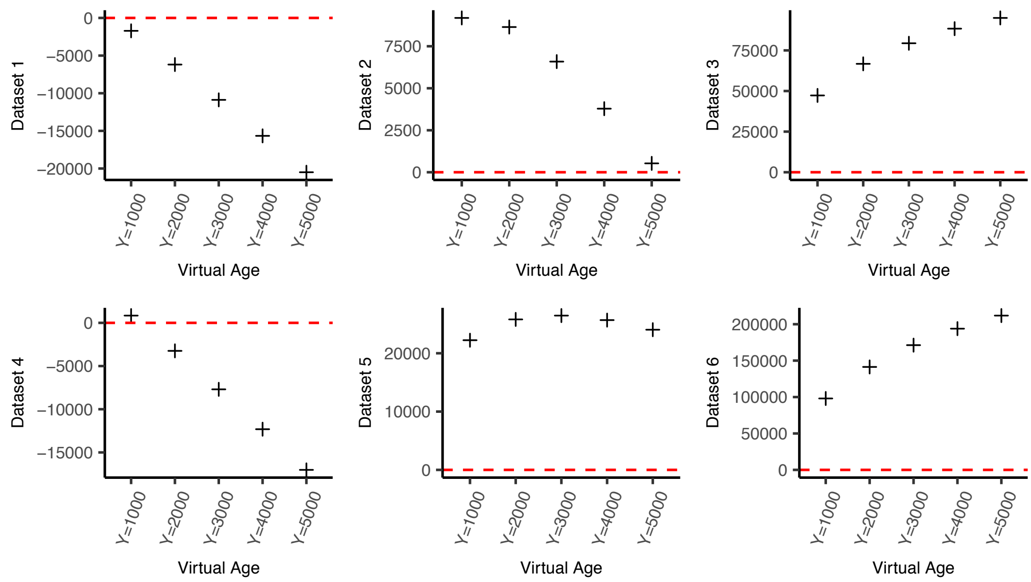

Figure 5.6: NPV of expected profit when the marginal cost of virtual age is 5u (Dataset 1-6: left-to-right, top-to-bottom; data provided in the Appendix A: Table A.7)

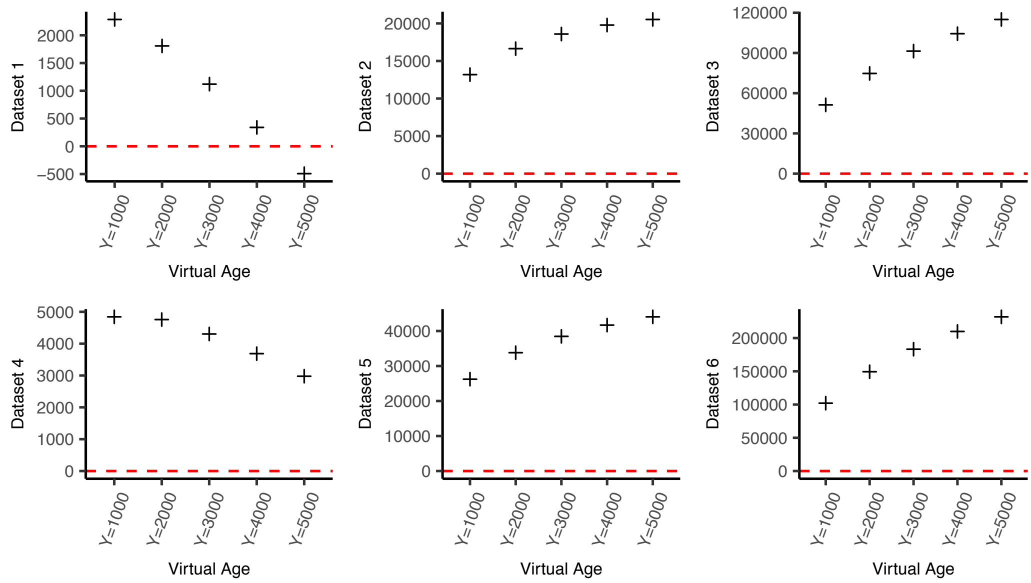

Figure 5.7: NPV of expected profit when the marginal cost of virtual age is 1u (Dataset 1-6: left-to-right, top-to-bottom; data provided in the Appendix A: Table A.8)

5.5.2 Operating Time as the Exposure to Risk

The purpose of this illustrative example is to demonstrate the behaviour of the proposed decision support model (Eq.(5.22)) and to also assess the value of information.

5.5.2.1 Simulation of Non-Conformance Data

We simulate a dataset to represent a supplier's non-conformances which follows the power law NHPP (Eq.(5.3)). The parameters used for the simulation include:

$$a = 1.4375, b = 0.5, t_T = 500$$

Where, a is the scale parameter which is assumed to follow the Gamma distribution (Eq.(5.8)), for which empirical Bayes inference is used and the prior parameters are estimated using MoM (Eq.(5.12)): $\alpha = 2.3$ and $\beta = 1.6$. Note that the estimate of a is an expected value. b is the shape parameter for which the true value is assumed to be 0.5, and the operating time t_T is assumed to be 500 days. Based on the specified values of parameters, the supplier's non-conformance rate and the number of supplier's non-conformances detect at each time point are presented in Fig.5.8.

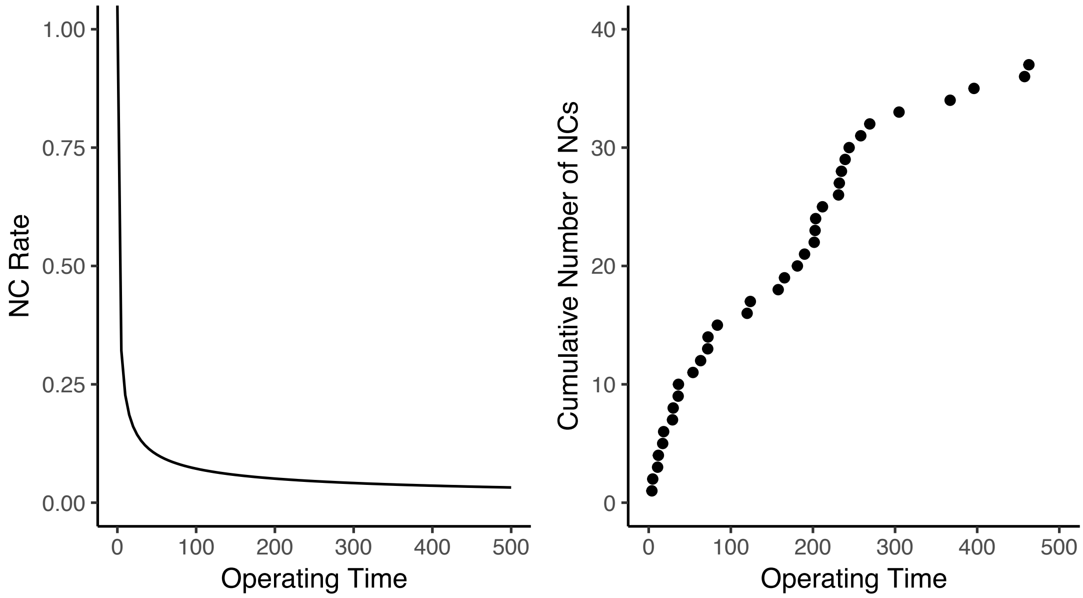

Figure 5.8: Left to right: a simulated supplier's non-conformance rate and the number of non-conformance occurrences cumulated over 500 days of manufacturing. The non-conformance rate is assumed to follow a power law model with scale parameter a=1.4375, shape parameter b=0.5, and the number of non-conformances detected is assumed to follow NHPP

Fig.5.8 shows that the non-conformance rate declines as the supplier gains more supply experience. In particular, the non-conformance rate declines dramatically within the first 100 days, which means that the supplier's performance can already be improved once it gains 100-days of experience. For this supplier, if no external help is provided from the buyer, the expected total number of nonconformances within the next 500 days is 37.

5.5.2.2 Model Implementation and Interpretation

Assuming that the financial loss caused by supplier non-conformance is $100u$ per unit and the annual compounding interest rate is 0.5%, the NPV of a total non-conformance cost with respect to an achieving virtual age can be obtained. In Fig.5.9, The dashed line represents that the expectation of the non-conformance cost can be as high as $34420u$ if no virtual age is considered. Clearly, the non-conformance cost becomes less as virtual age is applied.

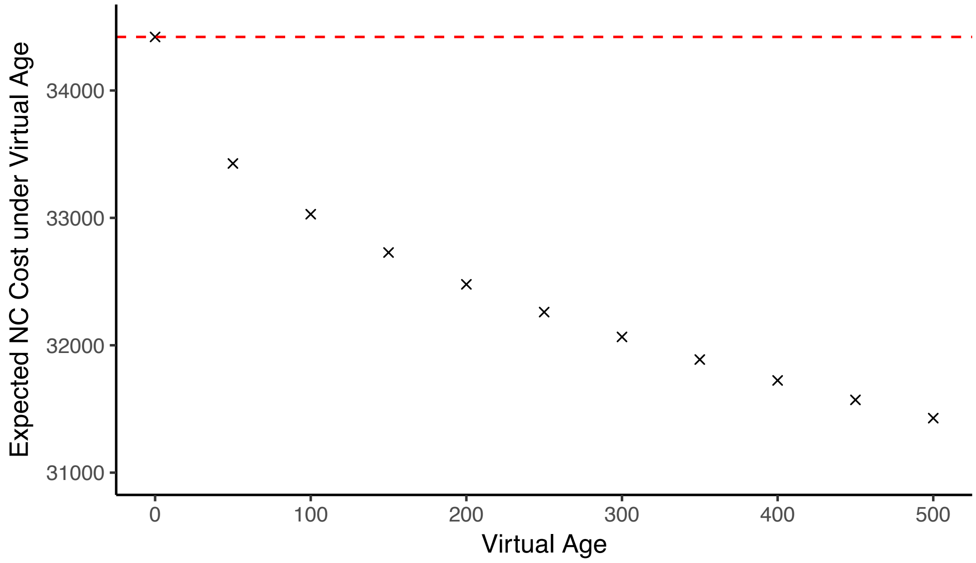

Figure 5.9: The expected non-conformance cost decreases as the achieving age increases. The red dashed line indicates the total non-conformance costs if no virtual age is applied (data provided in the Appendix A: Table A.9)

Assuming that the marginal cost of virtual experience is $5u$, meaning that gaining one more day of experience for the supplier costs $5u$, the expected profit of the supplier development activity with respect to the achieving virtual age is obtained using Eq.(5.22) and presented in Fig.5.10. The upper bound and the lower bound of the optimal investment decision on the virtual experience are also obtained using Eq.(5.23) and represented by the blue dashed lines respectively. It shows that an optimal virtual age is 174 which lies within the estimated boundaries 172.6 and 206 and reaches a maximal expected profit at around $947u$.

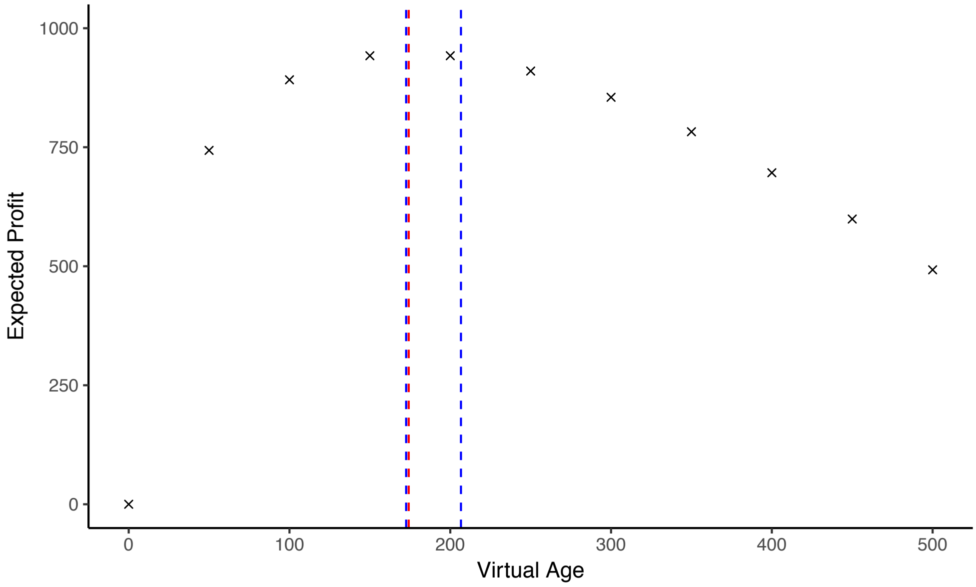

Figure 5.10: The expected profit with respect to the virtual age where an optimal virtual age is 174 which lies within the estimated boundaries 172.6 and 206 and achieves a maximal expected profit at around 947u. The optimal investment level is indicates by the red dashed line and the boundaries are indicated by the blue vertical dashed lines (data provided in the Appendix A: Table A.9)

We simulate the supplier's cumulative number of non-conformances after gaining 174 days of experience which is an optimal virtual age to be invested and compare the results with the supplier performance with no virtual age applied (Fig.5.11). Clearly, the supplier performs better if more experience is gained. The total number of non-conformances can be down to 16 as opposed to the original number of 37.

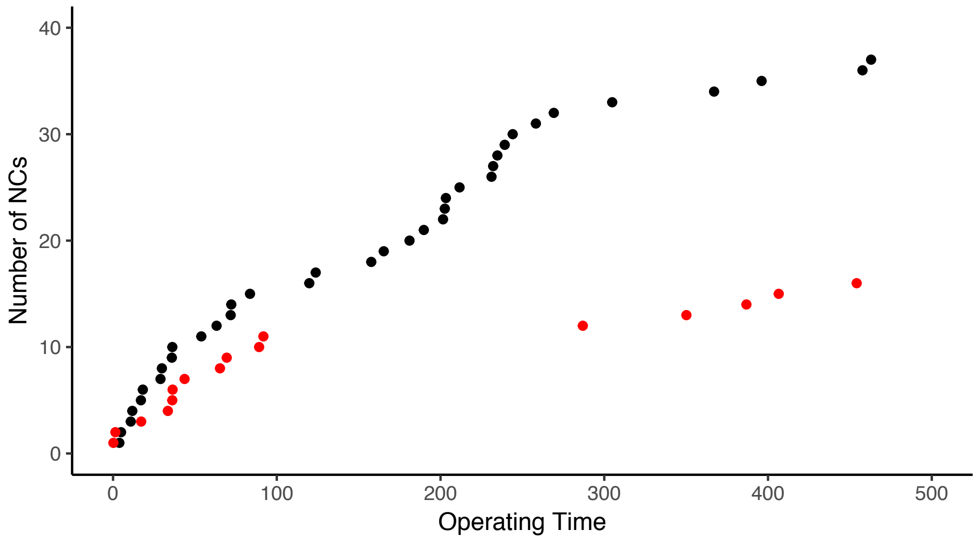

Figure 5.11: Comparation between the cumulative number of non-conformance under no virtual age (black points) and under the optimal virtual age (red points)

5.5.2.3 Value of Information

We have also evaluated the expected value of perfect information (EVPI) which provides an upper bound of the investment level for gaining information of suppliers' true performance. Assuming that the buyer can tolerate if the non-conformance rate is lower than 0.01, thus there is no need to invest in a supplier activity. As the scale parameter a in the non-conformance rate function (Eq.(5.4)) is assumed to be distributed from a Gamma distribution with estimated prior parameters $\alpha = 2.3$ and $\beta = 1.6$, we simulate 50 observations to represent the supplier's true non-conformance rate and obtain the profits from the decisions made under perfect information and under uncertainty using Eq.(5.22) (see Fig.5.12). Clearly, the profit obtained under perfect information is often higher than the profit obtained under uncertainty. By averaging all the profits, we obtain that the expected profit under perfect information is around $1499u$ whereas the expected profit obtained under uncertainty is around $1092u$. Thus, the EVPI is $407u$ which is the difference between the expected profit under perfect information and under uncertainty. It means that the buyer should invest no more than $407u$ to know about the supplier's true performance.

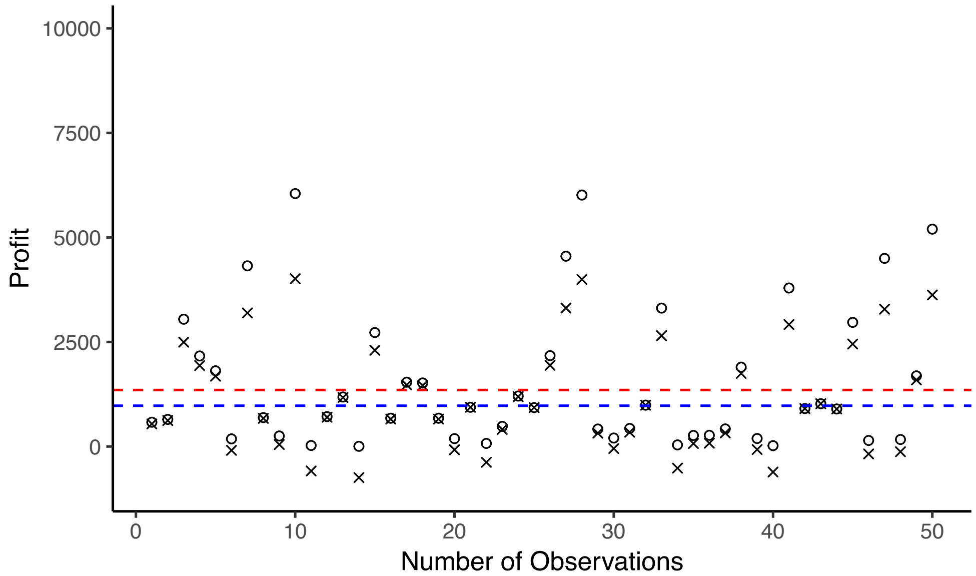

Figure 5.12: Profits under uncertainty (marked as cross points) and under perfect information (marked as circle points). The expected profit obtained under uncertainty is around $1092u$ (blue dashed line), whereas the expected profit obtained under perfect information is around $1499u$ (red dashed line)

5.6 Summary

In this chapter, a quantitative decision support model is developed under the consideration that how to provide investment decision support of conducting a supplier development activity to improve a supplier who is already at a self-improvement stage. The major contributions are summarised as follows. First of all, a power law NHPP model is used to capture the stochastic behaviour of supplier quality performance which enables non-stationary data to be analysed. In addition, the use of power law allows various types of suppliers to be investigated which provides a certain degree of flexibility. Secondly, we assume that for young suppliers, the performance may be better as more experience is gained, and we propose to use a "virtual age" to indicate the difference in experience between young suppliers and senior suppliers in order to support the decision making that if it is worth investing in gaining a virtual age for the young supplier. Such a modelling technique has not been used under the context of supplier development. Thirdly, an affine relationship exists between the benefit of using a virtual age and the benefit of not using a virtual age is demonstrated in Proposition 5.1 which can help decision makers quickly identify the "right" decision. When the risk exposure is measured on a continuous scale, an exact value of the optimal investment solution can be obtained numerically to maximise the financial benefit for the buyer. In addition, an analytical expression of the boundaries of optimal investment is also obtained which also provides a quicker way to support investment decision making. Finally, the behaviour of the proposed decision support models is demonstrated through numerical investigations. It shows that the use of a virtual age has a very close relationship with the level of supplier's non-conformances.

Chapter 6

Empirical Data Analysis: An Industrial Case Study

6.1 Introduction

In this research, we conducted an industrial case study with a Chinese automotive firm to evaluate the performance of the proposed decision support model using empirical data. In this chapter, we focus on the empirical data analysis using the model developed in Chapter 4 (Eq.(4.10)) as the collected data is in a categorical form and the multinomial distribution is considered appropriate. Given the result of the analysis, we also obtained the feedback from the participant regarding the performance of the model used for the analysis and summarised the findings in Chapter 7. Additional information of the case study is provided in the Appendix B, including an overview of the case study process, the selection of the participating firm and participants, a timeline of conducting the study, data documentation and ethical considerations.

The remainder of this chapter is organised as follows. To begin with, a profile of the modelling process is presented in Section 6.2. Then, the uncertainties of supplier performance and other input variables are assessed in Section 6.3. Next, optimal investment levels and expected returns are obtained for both individual and multiple suppliers' investment. Then, the results are compared with the actual decisions made by the company in Section 6.4. Finally, a summary of this chapter is provided in Section 6.5.

6.2 A Profile of the Modelling Process

The protocol of the data analysis process is as follows. First, the input values of the proposed model are assessed including the uncertainty of supplier non-conformances, total number of orders, non-conformance costs and effectiveness rates of the supplier development activity. Then, the Dirichlet-multinomial distribution is used to model the suppliers' non-conformance rates for which empirical Bayes inference is employed to estimate the model parameters. The suitability of the selected modelling technique is examined via P-P (Probability-Probability) plot. Optimal investment levels and the corresponding expected profits are calculated for both individual supplier investments and supplier portfolio investments. To evaluate the performance of the proposed model, optimal investment decisions are compared with the actual decisions made by the participating firm.

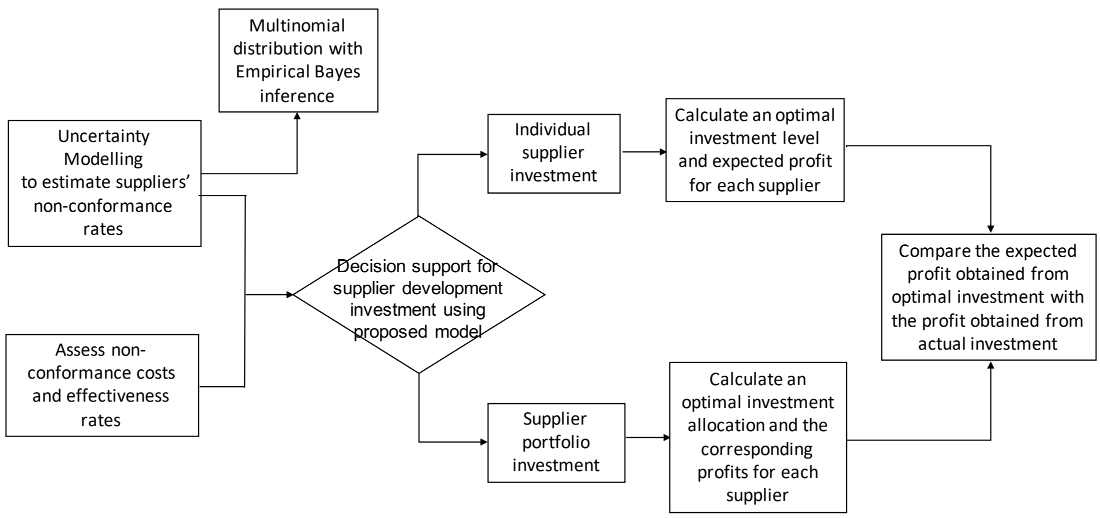

Figure 6.1: A profile of the modelling process

6.3 Assessment of Model Inputs

This section assesses the model inputs based on the secondary data collected and the managerial knowledge gained during the visit to the participating firm from 05/03/2019 to 18/03/2019. Confidential data are all desensitised.

6.3.1 Supplier Quality Uncertainty

In total, empirical data from 9 suppliers were used for the analysis. During the visit, the participant mentioned that the firm generally categorises supplier quality performance into three classes, namely "non-conforming items detected when parts arrive", "non-conforming items detected during manufacturing" and "conforming items". We consider the collected data is in a categorical form for which the multinomial distribution (Eq.(4.1)) is a reasonable and appropriate model to capture the uncertainty of each performance category. To capture suppliers' epistemic uncertainty, we assume that all suppliers' priors follow the Dirichlet distribution (Eq.(4.4)). We pool all suppliers' quality data together and use the pooled empirical data to estimate the prior parameter α_i using MoM (Eq.(4.9)), in which $i = 1$ indicates non-conforming items type 1 (NC1) that are detected when parts arrive, $i = 2$ indicates non-conforming items type 2 (NC2) that are detected during the manufacturing, $i = 3$ indicates conforming items. Table 6.1 gives the estimates of the prior parameter $\hat{\alpha}_i$ for each performance category.

Table 6.1: Parameter estimates of the prior distribution before development

Parameter Estimates			
Prior	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$
Value	0.4501	0.3383	277.5584

To assess the fit of the Dirichlet-Multinomial model (Eq.(4.5)) to the empirical data, we obtained the posterior distribution for each supplier based on the Bayes theorem

(Eq.(4.6)) which provides a model estimate for the proportion of each performance category of each supplier. We used a P-P plot to compare the estimated proportion of each performance category with the actual proportion which indicates a good fit of the model (see Fig.6.2). The data of both the estimates tayes and the actual rates are provided in the Appendix A: Table A.10. Clearly, the NC1 rate and NC2 rate for all suppliers are much lower than their conformance rates.

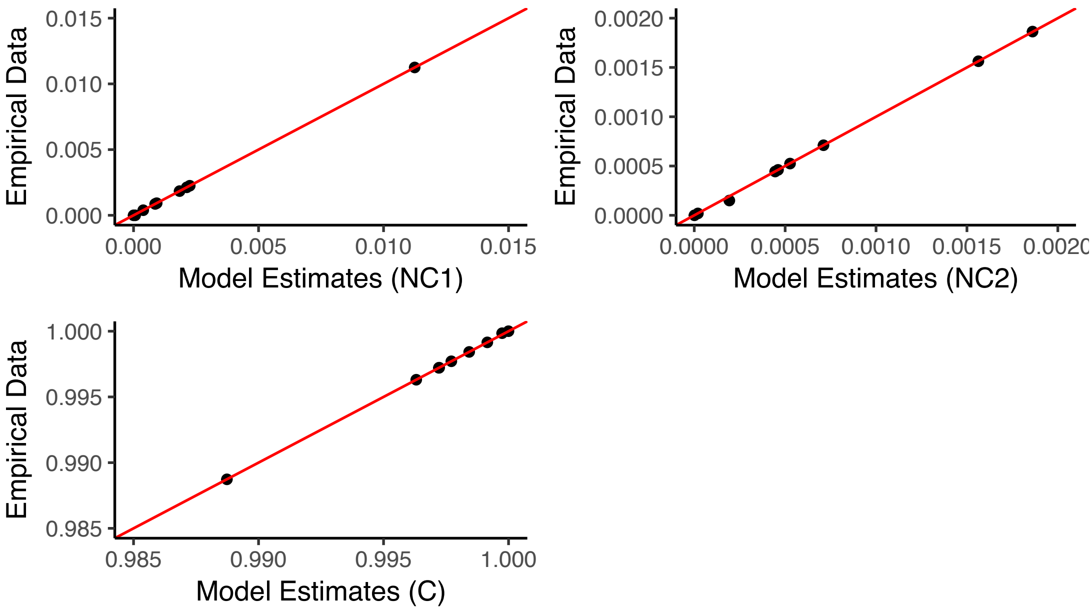

Figure 6.2: P-P plot of the model estimates of the proportion of each performance category for each supplier before supplier development activity against the empirical data (NC1: non-conformance type 1, NC2: non-conformance type 2, C: conformance)

6.3.2 Determinant Variables

6.3.2.1 Total Number of Orders

We obtained the average monthly orders for each supplier based on the historical data and we assume that all suppliers will deliver parts for the participating firm for 10 years. The amount of 10-year orders for each supplier are provided in Table 6.2.

Table 6.2: Total number of orders for each supplier (k=1000)

Suppliers	S1	S2	S3	S4	S5	S6	S7	S8	S9
10-year of orders	1200k	2400k	6000k	6000k	720k	6000k	4800k	600k	240k

6.3.2.2 Non-Conformance Cost

During the semi-structured interview, we noted that the company does not have an explicit approach to calculate non-conformance costs. The participant mentioned that non-conformance cost of each supplier depends on the purchase price of the item. In addition, the participant also mentioned that depending on the circumstances, the non-conformance cost also varies, as at other times a supplier's non-conformances may not cause any financial loss for the firm whereas sometimes they may cause a significant loss, such as when the non-conformances lead to the suspension of manufacturing. In this analysis, we assume that the non-conformance cost per unit is the same for both non-conformance categories, namely NC1 and NC2. We use a ratio, denoted by k, to reflect the relationship between the non-conformance cost and the purchasing price per item. For example, k equal to 5 means that the cost of non-conformance is 5 times the purchase price. We desensitise the purchase price per item using u, and consider two ranges of non-conformance costs: (1) u,$5u$,$10u$,$15u$,$20u$, indicating "high" non-conformance costs as $k \geq 1$; (2) $1/5u$,$1/10u$,$1/15u$,$1/20u$, indicating "low" non-conformance costs as $0 < k < 1$

6.3.2.3 Effectiveness Rates

In the proposed decision support model (Eq.(4.11)) the number of non-conformance items exponentially decreases as the total effective investment level increases, the effectiveness rate per unit of investment at each undesirable performance category i for supplier j can be calculated using Eq.(6.1).

$$\gamma_{ij} = \frac{-\ln\left(\frac{c_{ij}n_{ij}^{'}}{c_{ij}n_{ij}}\right)}{x_j} \tag{6.1}$$

$n_{ij}^{'}$ - Realised number of non-conformances after the improvement activity at undesirable performance category i

n_{ij} - Realised number of non-conformances before the improvement activity at undesirable performance category i

x_j - The actual investment level for supplier j

γ_{ij} - Effectiveness rate per investment unit

The results of the effectiveness rates for each supplier are presented in Fig.6.3. These are normalised values given the total actual investment level. We have also obtained the average value of the effectiveness rate for NC type1 which is around 4.5×10^{-5} represented by the red dashed line, and the average value of the effectiveness rate for NC type2 which is around 2.7×10^{-5} represented by the blue dashed line. This shows that on average the effectiveness rate for the NC type1 is higher than the NC type2.

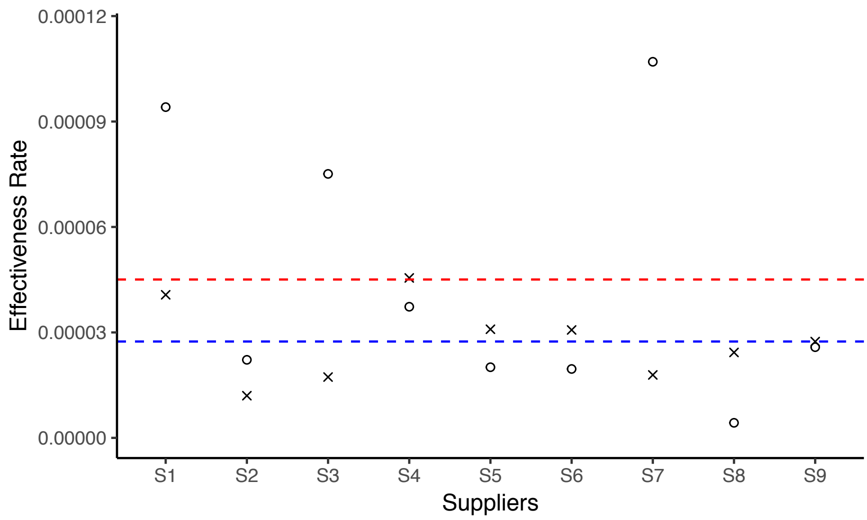

Figure 6.3: Effectiveness rates of supplier development activity for each supplier per unit of investment (circle: NC type 1, cross: NC type 2; data provided in the Appendix A: Table A.11)

6.4 Decision Analysis

In the following data analysis, the logarithm transformation is applied to all the results and all cost and investment figures are expressed in terms of an unspecified monetary unit.

6.4.1 Individual Supplier Analysis

Fig.6.4 and Fig.6.5 show the results of the optimal investment level obtained by maximising the expected return in Eq.(4.11) for each supplier under different levels of non-conformance costs. The data for this are provided in Table A.12 and Table A.13 of Appendix A. Notably, the optimal investment levels are mostly higher than the actual investment level when non-conformance cost is high (Fig.6.4), whereas the optimal investment level is generally lower than the actual investment level when the non-conformance is low (Fig.6.5). More specifically, as the non-conformance cost increases, the optimal investment level also increases, which implies that a decision maker should consider high levels of investment when the non-conformance cost is high, and vice versa. There are a few occasions where the optimal investment level is almost the same as the actual investment level, such as for supplier S1 at the cost level $5u$, or for supplier S4 at the cost level $1/10u$. We have also obtained zero values for optimal investment levels which implies that the investment in improving such suppliers is not recommended as it is not financially beneficial. For suppliers whose optimal investment level is zero, data has been omitted in the figures as all the results are log-transformed. This applies, for example, supplier S2 (see, Fig.6.4 and Fig.6.5), or supplier S8 (see, Fig.6.5). In particular, for supplier S2, we note that its expected non-conformance rates have remained very low even before the development activity (see Table A.10). This implies that decision makers might not have the perfect information about the suppliers' true performance before making the investment decision and accordingly made unnecessary efforts on developing the suppliers who already performed well.

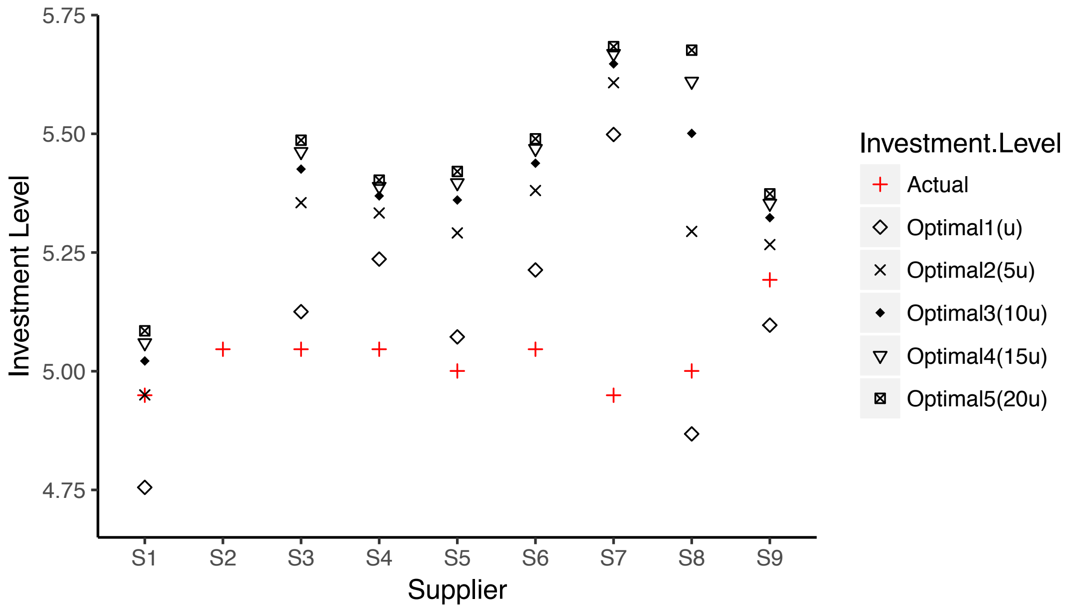

Figure 6.4: A comparison between the optimal investment levels and the actual investment level at high non-conformance costs (from u to $20u$)

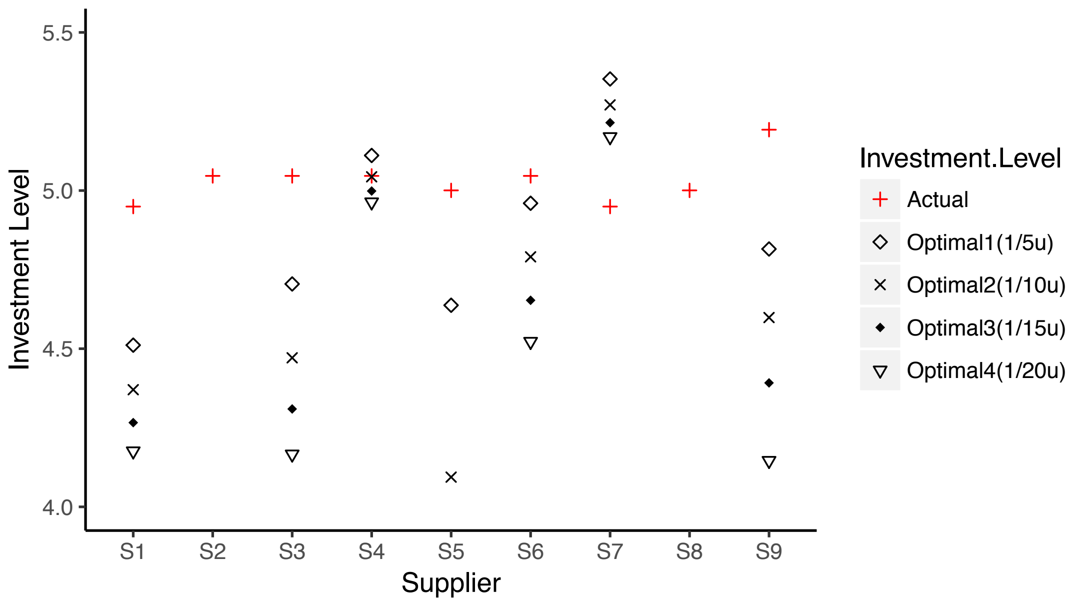

Figure 6.5: A comparison between the optimal investment levels and the actual investment level at low non-conformance costs (from $1/20u$ to $1/5u$)

We calculate the differences between the expected return from the optimal investment levels and the actual return from actual investment levels using Eq.(4.11), referred as "increased return". As all increased returns are positive values, a log-transformation can be applied (see Fig.6.6 and Fig.6.7). The data are provided in Table A.14 and Table A.15 of Appendix A. This also implies that the proposed model provides a better decision making, as the returns obtained under optimal investments are all higher than the returns obtained under the actual investment. In addition, we note that when the optimal investment levels are higher than the actual investment level, the increased return increases as the non-conformance cost increases (see, S3 - S7 in Fig.6.6, or S7 in Fig.6.7) and vice versa (see, S1,S3, S6 and S9 in Fig.6.7). When the optimal investment level is the same as the actual investment level, the increased return reaches the lowest value (see, S1 at the cost level $5u$, and S4 at the cost level $1/10u$).

The boundaries for the optimal investment level and expected returns are also obtained at the cost level $10u$ which shows consistency with the Theorem 4.2 (see Fig.6.8 and Fig.6.9). In particular, the boundaries for the expected return are very close to the true value. Note that the analysis of supplier S2 is not considered as it has shown that the optimal investment level for supplier S2 is zero in any case.

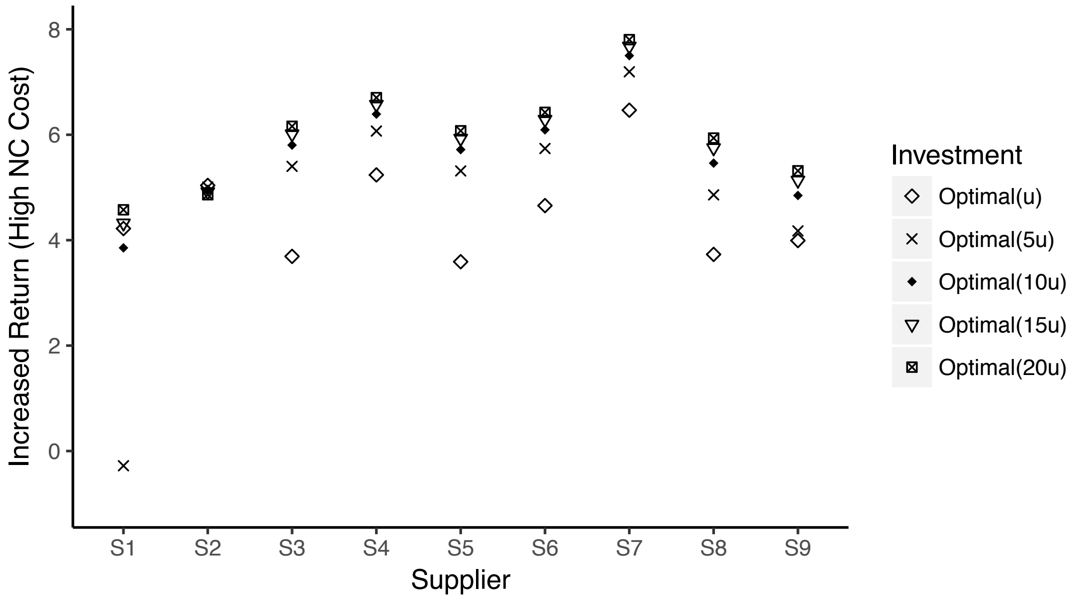

Figure 6.6: The increased returns (log-transformed value) obtained from optimal investment at high non-conformance costs (from u to $20u$)

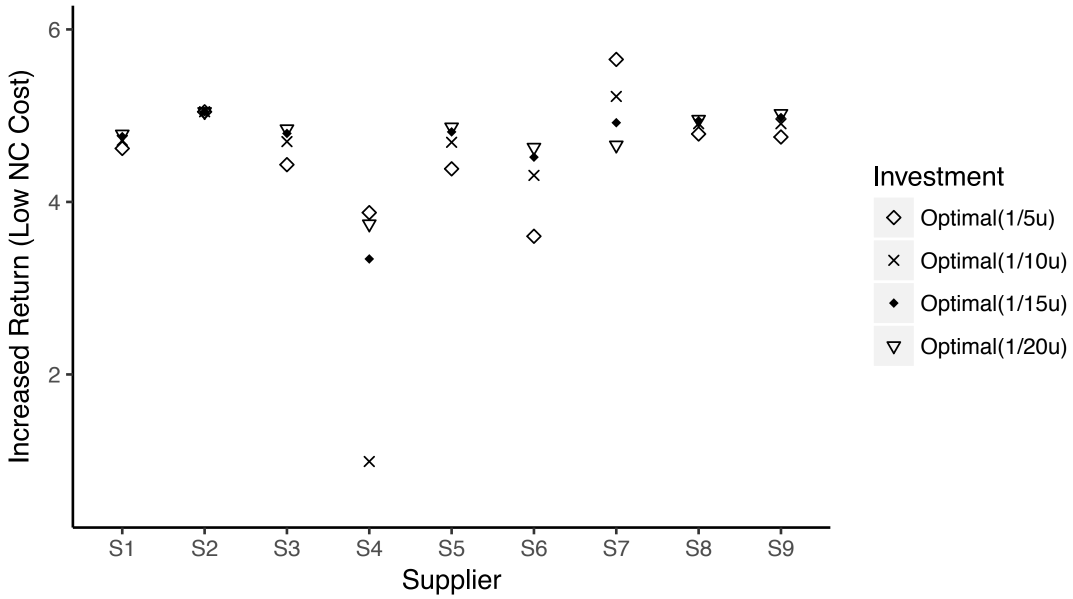

Figure 6.7: The increased returns (log-transformed value) obtained from optimal investment at low non-conformance costs (from $1/20u$ to $1/5u$)

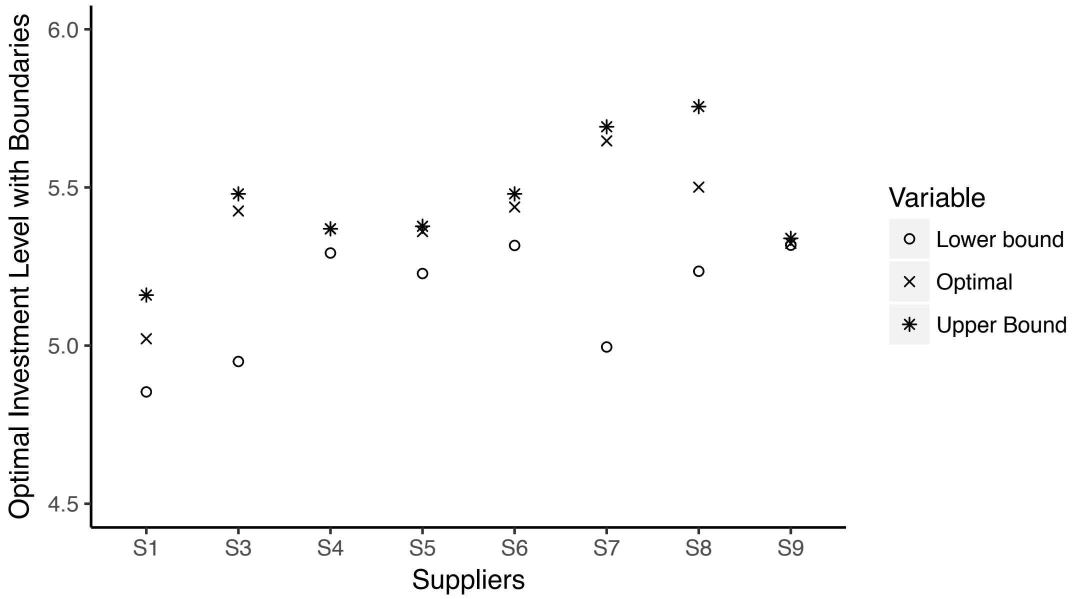

Figure 6.8: The boundaries and the exact value of the optimal investment level under different effectiveness rates (data provided in the Appendix A: Table A.16)

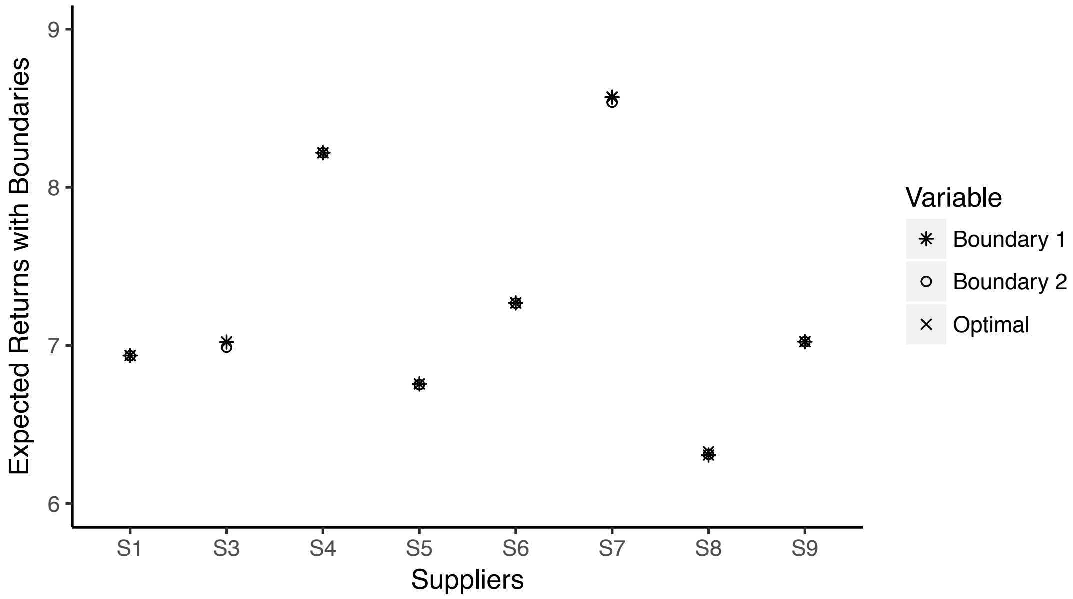

Figure 6.9: The boundaries and the exact value of the expected return under different effectiveness rates (data provided in the Appendix A: Table A.17)

6.4.2 Supplier Portfolio Analysis

We now evaluate the optimal decisions for supplier portfolio investment, for which a budget constraint is considered and measured by the total actual investment level. Note that suppliers whose optimal investment levels are obtained as zero in the previous individual investment analysis will not be considered in the portfolio analysis. Therefore, under different levels of non-conformance costs, different budgets are considered. That is, when the non-conformance cost is high (from u to $20u$), the budget is 5.94 whereas when the non-conformance cost is low (from $1/20u$ to $1/5u$), the budget is 5.89.

We obtain the investment allocation by maximising the total expected return in Eq.(4.16) and compare the results of optimal investment allocation with the actual investment allocation. As shown in Fig.6.10, the investment allocations for supplier S4 and S7 are much higher than the actual allocation, and the optimal investment allocations for supplier S6 are close to the actual investment level. For most suppliers, the investment levels are less than the actual ones. More interestingly, we found that the non-conformance cost has no impact on the optimal investment allocation under the same amount of budget. This is because when the budget is fixed, the non-conformance cost for each supplier changes at the same ratio which results in the same investment allocation.

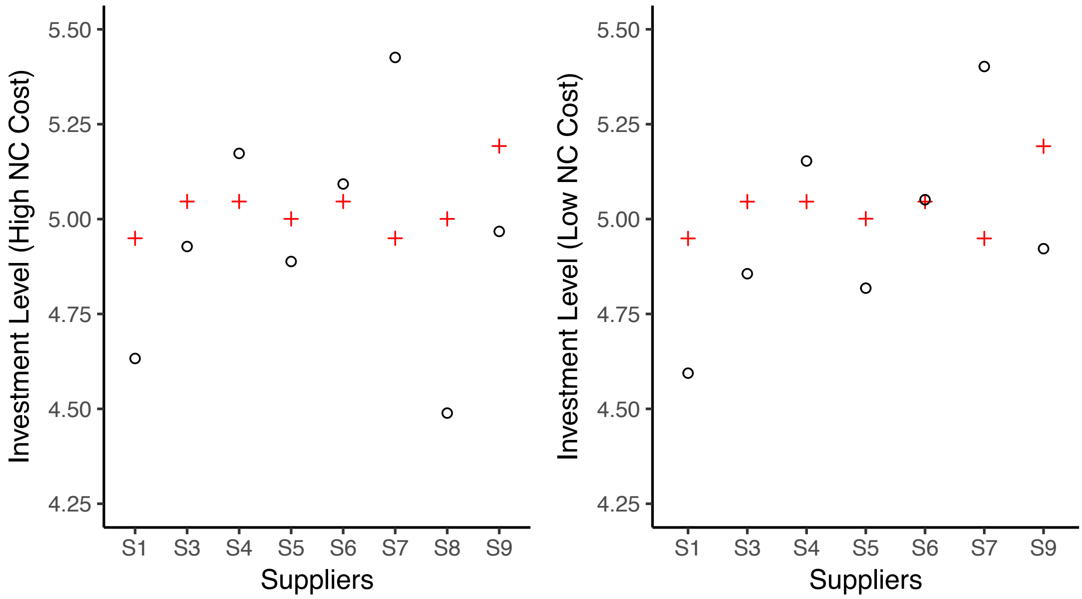

Figure 6.10: Comparison between optimal and actual investment allocation on supplier portfolio investment under high (left) and low (right) non-conforming costs (cross: optimal investment, circle: actual investment; data provided in the Appendix A: Table A.18)

The total increased returns under the optimal investment allocation are also obtained and presented in Fig.6.11. As the total increased returns are all positive, log-transformation can be applied. This also indicates that the returns obtained from optimal investment decisions are higher than the returns obtained from the actual investment levels. It is interesting to find that the total increased return increases as the non-conformance cost increases under both high and low non-conformance costs. This is because when the optimal investment level is fixed, the increased returns has a monotonic relationship with the non-conformance cost. A mathematical explanation is provided in the Appendix D.1.

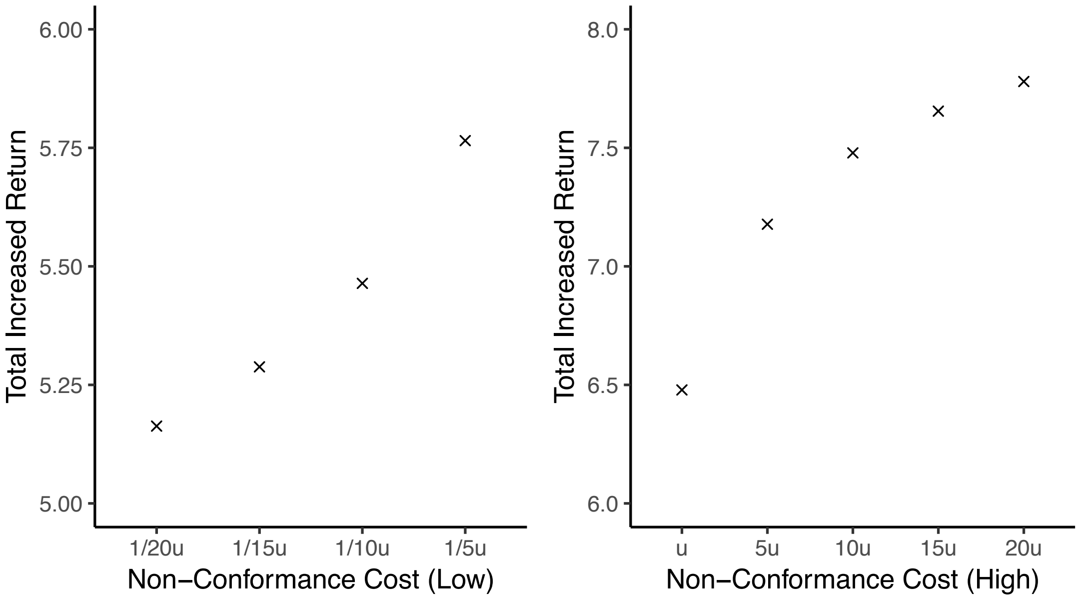

Figure 6.11: Total increased returns from the optimal investment allocation at all non-conformance costs (from $1/20u$ to $20u$); data provided in the Appendix A: Table A.19.

We consider that the maximal total investment is essentially the sum of the optimal investment levels of all suppliers to be developed. We obtain the expected returns from a number of possible amounts of total investment level under the non-conformance cost of $10u$ and $1/10u$ (see, Fig.6.12). It shows that when the non-conformance is $10u$, in total the firm should invest no more than 6.3, and when the non-conformance is $1/10u$, in total the firm should invest no more than 5.75.

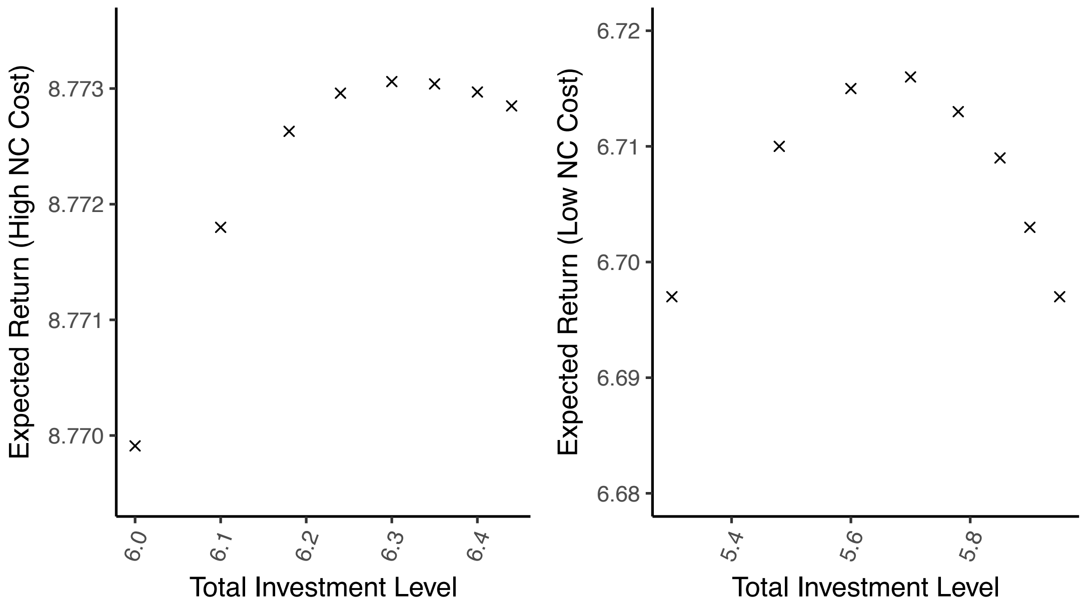

Figure 6.12: The expected returns with respect to the total investment level under high non-conformance cost $10u$ (left) and low non-conformance cost $1/10u$ (right)

6.5 Summary

The purpose of this chapter is to apply the proposed decision support model developed in Chapter 4 using the empirical data provided by the participating company. The results of the analysis show that the proposed model has performed reasonably well, as the optimal investment decisions obtained from the model provide higher expected returns than the actual investment decisions. In addition, the analysis also shows that a few suppliers should not have been invested in, as no financial benefit was achieved. For supplier portfolio investment, the model suggests that the investment allocation is not affected by the level of non-conforming cost under the same amount of budget. More specifically, if the budget is fixed and the non-conformance cost for each supplier changes on the same ratio, the investment allocation will not be affected. The next chapter will validate the decision support model used for this analysis and discuss the findings.

Chapter 7

Model Validation and Reflections on the Industrial Case Study

7.1 Introduction

Based on the results of the data analysis in Chapter 6, we have also obtained feedback from the participant via a semi-structured interview approach to validate the model from three different perspectives: model output, data requirements and modelling process. The purpose is to assess whether the model performs as expected and also to identify whether there are limitations in the model application or any room for improvement. This chapter summarises the findings from the interview and discusses the reflections and some practical issues encountered during the case study. To begin, Section 7.2 discusses the methodological issues, including methods and rationales for the model validation. Then, Section 7.3 details the model validation process and the findings from the semi-structured interviews. Finally, Section 7.4 summarises this chapter.

7.2 Methodological Considerations

7.2.1 Qualitative Approach for Model Validation

Model validation may be considered as a process of examining the degree to which the model represents the reality in the domain of applicability given a set of specified criteria. The purpose of validation is to ensure that the model developed meets the goal of the intended uses (Ling and Mahadevan, 2013). Depending on the purpose of the model, the method used for validation varies (Scandizzo, 2016). In this research, a qualitative approach is employed for model validation. There are several reasons for this consideration. First of all, one of the important aspects is the validity of the model output. Although numerical validation (e.g. statistical analysis) has been often used for validating the model output by comparing it to the output generated from a real system, such an approach may not be possible for use when observable data is unavailable (Anderson et al., 2015; Sankararaman and Mahadevan, 2011). In this research, experiments for obtaining data from an "actual system" may be extremely expensive and time-consuming. In addition, the purpose of the model is to provide decision support. The validity of the model output should be assessed in relation to its ability of being supportive rather than being accurate, which is considered as a subjective criterion. As Virlics (2013) argues, "the decision of the investor to invest is often subjective." Therefore, a qualitative approach is a preferable option for the model validation in this research. Furthermore, the examination of the model validity also involves the assumptions made for the decision making context, for which decision makers play an important role. Also, from researchers' perspective, subjective assessment for model validation also has the advantage that it allows for discovering unintended consequences by obtaining an "insider" view.

7.2.2 Semi-Structured Interview

We consider that the data to be obtained for model validation are qualitative. To obtain the data, a semi-structured interview approach is employed. Saunders et al. (2009) identify three types of interviews, namely structured interviews, semi-structured interviews and unstructured interviews. Structured interview is typically used for quantitative data collection often requiring a predetermined list of questions, like a survey, whereas an unstructured interview allows participants to talk freely about facts and beliefs in relation to the research topic, which is typically used to gather qualitative information. A semi-structured interview is a mixed approach which not only involves pre-defined questions but also allows participants to respond freely to provide rich information. This study considers a semi-structured interview as appropriate as the validation requires the participants to provide "answers" with rich explanations based on a number of pre-defined questions. The flexibility of semi-structured interview allows participants to provide plenty of information which they consider is relevant.

7.3 Model Validation

This section validates the proposed model based on the participant's feedback on the results of the data analysis in Chapter 6. The key objective is to validate whether the proposed model enables the provision of providing meaningful decision support on supplier development investment and to examine the operational considerations made for the assumed decision making context in Chapter 3. We developed a questionnaire (see Appendix C) and conducted a semi-structured interview to obtain the participant's feedback on whether the model meets the pre-specified validation criteria. This section first explains the rationales of the criteria used for validation in Section 7.3.1, and then presents the findings from the semi-structured interview in Section 7.3.2. A discussion and reflection on the case study is also provided in Section 7.3.3 from the modeller's perspective.

7.3.1 Criteria

Oral and Kettani (1993) highlight that the criteria for model validation should be different for different kinds of purposes. In this research, model validation is carried out based on three different focuses: model output, modelling process and data requirement. The "model output" mainly focuses on validating the usefulness of the decision support provided by the proposed model. The "modelling process" and "data requirement" focus on examining the operational assumptions made for the decision making context. For each category, different criteria are identified and provided in Table 7.1.

Table 7.1: Model Validation Criteria

Focus	Criteria
Model Output	a. Useful
	b. Acceptable/Realistic
Modelling Process	a. Reasonable
	b. Practical
Data Requirement	a. Available
	b. Easy to get

The first consideration is to validate the model output, for which two criteria used, namely "useful" and "acceptable". To be a meaningful decision support model, the proposed model needs to provide supportive information for decision makers. A key criterion is whether or not the model output is "useful" for the decision maker. This means that the decision maker who uses the model considers that the model is able to help them make better decisions. Another important criterion is if the model output is "acceptable" or "realistic". This is mainly concerning with whether an optimal investment level provided is usable or not. For instance, if the optimal investment level obtained from the proposed model is much higher than the amount that the firm could reasonably afford, the output of the model may not be supportive.

The second consideration is to validate the modelling process, for which two criteria are identified, namely "reasonable" and "practical". Both criteria are concerned with the operational assumptions made in the proposed model, as these are the basis for the development of the model. In addition, it can also further evaluate the model's ability of representing the decision making context in the real world.

The third consideration is to validate the availability of the input data required in the model, for which two key criteria are identified, namely "available" and "easy to obtain". We argue that the efforts needed to obtain the required data is important, as the degree of difficulty in obtaining the data may affect the use of the model.

7.3.2 Validation Process

The result of the empirical data analysis was sent back to the participant in the form of a PowerPoint presentation via email on 18 April 2019. The interview for the model validation was held on 30 August 2019. This section summarises the findings from the interview with respect to the identified criteria.

7.3.2.1 Model Output

The participant considers the model output to be acceptable, as the majority of the optimal investment levels provided are within the firm's budget. Also, the model output is considered useful. The participant was much interested in the optimal investment level, as this had never been considered previously. In addition, the participant mentioned that the firm did not pay attention to the uncertainty of suppliers' true performance which had resulted in investing in suppliers who already performed well. As the participant argued, "when we make the investment decision, we just used the same investment levels for many suppliers, we didn't consider an optimal investment level. At that time, we did not have such tools. If we did, we may have reduced some of our investments in particular suppliers." More importantly, the participant expressed an interest in using the proposed model as a supportive tool for the decision making in supplier investment in the future.

7.3.2.2 Modelling Process

The interview with the participant reveals that most of the operational assumptions made in the proposed model in Chapter 3 are reasonable and practical. For example, for assumptions 1 and 4, suppliers' poor quality did cause financial losses for the buyer which was also the key reason that the participating firm conducted this activity. As noted by the participant, "our main motivation (of conducting the supplier development activity) was that when we compared our current suppliers with the suppliers of our competitors, we found that our suppliers were less competitive. Suppliers' non-conformance had caused significant damage on our performance. That is why we wanted to start this supplier development program". In addition, for assumption 2 and 6, the participant mentioned that all suppliers have been providing parts for the participating firm for more than 20 years and participated in the activity voluntarily. Thus, both the participating firm and suppliers were willing to have a long-term relationship and were motivated for conducting the activity.

7.3.2.3 Data Requirement

In total, there are four key input data that are required in the model, namely suppliers' performance data, total number of orders, non-conformance costs and effectiveness rates. The participant considers that all the input data required for analysis is generally available in the firm and easy to be obtained. Although a systematic approach of calculating non-conformance costs is not available within the firm, the participant is able to provide a way of approximating the non-conformance costs used within the firm. Furthermore, the participant certainly had no difficulty providing the suppliers' database, such as supplier non-conformances, or total number of orders, as the firm has been keeping recording. This also validates assumption 3 made in Chapter 3. As for the effectiveness rates, the participant appeared to have some concerns providing the exact number but had no problem to express their expectations on the improvement of suppliers' performance. This can be indirectly used to obtain the effectiveness rates using Eq.(6.1). The participant mentioned that the firm has five categories from A to E for suppliers which is rated using the company's own formula. This had been used as a way of setting up the goal of a supplier's improvement during the time when the supplier development activity was conducted, as noted by the participant: "we were able to provide such estimates as we have five categories for our suppliers from A to E, A is the best, E is the worst. We expressed our goal for suppliers' improvement based on the category. For example, one supplier was at the level C, and after the activity, we expected this supplier to be improved to the level A."

7.3.3 Discussion and Reflection

This section discusses the main findings from the semi-structured interview including the key merits and the recommendations on improvement of the proposed model, and the challenges encountered during the case study.

7.3.3.1 Key Merits and Recommendations on Improvement

One of the key merits discovered is that the optimal investment level generated from the proposed model provides the participant with new insights into the investment decision making, as this was not considered previously. The participant showed an interest in the decision support for supplier portfolio investment in particular, as the model provides information to guide investment allocation under limited resources for supplier development if the modelling assumptions made are reasonable for the decision context. The participant mentioned that the previous decision on the amount of investment for each supplier may be changed if they had such tools, as it is likely that investment may be reduced for particular suppliers. Another key merit of the proposed model is its ability of analysing the benefit of supplier development under uncertainty to inform meaningful decision support. As shown in the data analysis, there exist suppliers who had already performed well, but still were involved in the development activity. Therefore, investment may have not been needed if the participating firm had known more about these suppliers. The use of probabilistic models to analyse suppliers' quality data not only enables the participant to develop a deeper understanding of stochastic behaviour of supplier performance, but also helps them determine whether it is worth investing in a supplier or not. As noted by the participant, "we did not (consider the risk), we just chose the suppliers who wanted to participate in this activity voluntarily."

To further improve the model, the participant mentioned that apart from financial returns, the participating firm also cares about the improvement of other aspects, such as buyer-supplier relationship, supplier's organisational management capacity, which have not been considered in the proposed model. Therefore, more decision criteria may be have to be taken into consideration. Also, the participant noted that apart from the investment level, they also would like to know the optimal duration for conducting a development activity. This is because the company needs to delegate their own engineers to the suppliers' sites to supervise and to assess suppliers' improvement progress from time to time which may be costly.

Overall, the model shows the ability of providing meaningful decision support for supplier development investment, and the participant also expressed an interest in using the proposed model as a supportive tool for supplier development in the future.

7.3.3.2 Challenges during the Case Study

There are also some challenges that were encountered during the case study. First of all, one of the major challenges is that data transformation was needed for quantifying effectiveness rates. We found that directly eliciting the quantitative value of the effectiveness rate of a supplier development activity from decision makers appears to be difficult. In the analysis in Chapter 6, as historical data of the supplier performance before and after intervention was available, we were able to calculate the value of the observed historical effectiveness rates. We acknowledge that this could be challenging when obtaining effectiveness rates for suppliers without prior development experience. In addition, the participant expressed concerns about implementing the proposed model within the company. As a certain degree of knowledge regarding probabilistic models is required, training may be needed if staff do not possess such knowledge, which may be time-consuming and discourages staff from learning the model. Furthermore, the model validation process requires the participants to not only understand the function of the model and but also to provide meaningful comments. Due to the time constraint, we were not able to find more suitable experts to participate in this case study in order to obtain more diversified opinions for the model validation.

7.4 Summary

This chapter discusses the feedback from the industry manager based on the application of the model developed in Chapter 4 to the company data analysed in Chapter 6. The key objective of the case study is to assess whether the model performs as expected and enables the provision of providing meaningful decision support. The model is validated based on three focuses, namely model output, modelling process and data requirement. For each category, different criteria are specified. Overall, the proposed model meets the specified criteria. The participant mentioned that the proposed model provides the company with new insights on the decision making process which has not been considered previously. In addition, the participant expressed an interest in using the proposed model as a supportive tool for supplier development in the future and also provided suggestions for further improvement of the model.

Chapter 8

Conclusion and Future Work

8.1 Summary of Research

This research proposes two quantitative models to support decision making in supplier development investment from a buyer's perspective with a particular emphasis on the epistemic uncertainty involved in the return. The proposed decision support models are developed in the context that a buyer faces financial losses caused by suppliers undesirable performance and intends to conduct a supplier development activity to improve the suppliers' performance and to add value to the buying firm's business benefit. However, uncertainty exists in the return to be obtained from the activity. The aim of the research is to investigate whether it is worth investing in the activity and if so how much should be invested, or whether it is worth delaying the investment decision to learn more about suppliers' true performance and then decide to invest or not. A summary of this research is detailed in relation to the objectives set in Chapter 1.

To achieve objectives 1 and 2, we associate the uncertainty of the benefit from supplier development investment with the stochastic characteristics of supplier performance. Two probabilistic models are developed and evaluated based on the multinomial distribution and the Non-Homogeneous Poisson Process (NHPP) within a Bayesian framework. The rationales for the use of these models are relevant to the type of supplier performance data. More specifically, the decision support model developed in Chapter 4 is concerned with the situation that the amount of a buyer's financial loss caused by supplier undesirable performance may depend on the degree of severity of the damage. Therefore, the supplier performance data are modelled in a categorical form. The multinomial distribution is used for capturing the frequency of the occurrences of supplier undesirable performance at different risk categorises. The Dirichlet distribution is employed as a prior to model the epistemic uncertainty. The decision support model developed in Chapter 5 is concerned with the situation that a buyer intends to facilitate the improvement of the performance of a newly integrated supplier who is on a self-development stage. The supplier's non-conformance rate is considered changing over the risk exposure which is expressed using both a continuous and a discrete variable. The power law NHPP model is employed to capture the aleatory uncertainty of the supplier's performance data. To capture the epistemic uncertainty, we use the Gamma distribution as a prior for estimating the scale parameter and we use expert judgement for estimating the shape parameter, by which the predictive model is resulting in a form of Negative Binomial distribution.

To achieve objectives 3 and 4, we develop the decision support models by analysing the profit from supplier development investment which is measured by the difference between the cost reduction of supplier undesirable performance after the activity and the investment level. An optimal investment level is obtained as a managerial insight which gives a maximal investment level that the buyer should invest in supplier development. An expected return is also obtained under the optimal investment level which is the highest amount of profit that the buyer may expect. More specifically, the model developed in Chapter 4 investigates the investment decision making in both a single supplier and multiple suppliers. When the effectiveness rates are the same, a closed solution of the optimal investment levels can be obtained. We found that under certain conditions the optimal investment level declines as the effectiveness rate increases. In addition, we have also obtained an expression that shows how much investment each supplier may be allocated if the extra budget is available. The model developed in Chapter 5 employs a virtual age model to describe the expectation of the improvement gained by a newly integrated supplier, for which the boundaries of an optimal virtual age to be invested are provided. We have also obtained an affine relationship between the benefit of investing in a virtual age and the benefit of not investing in a virtual age. Such a relationship can help decision makers quickly identify the "right" investment decision. In addition, we found that a higher profit may be obtained from a supplier with a higher non-conformance rate. In particular, an expected value of perfect information (EVPI) was also evaluated under both proposed models. It provides an upper bound of the investment level for learning more about suppliers' true performance to buy down the epistemic uncertainty before making the investment decision.

To achieve objective 5, an industrial case study was carried out to validate the model developed in Chapter 4 based on a number of pre-specified criteria. An empirical data analysis was conducted to evaluate the optimal investment levels by comparing the results with the actual investments made by the participating firm. The empirical analysis shows that the participating firm may obtain higher profits under the optimal investment levels. In particular, the model findings suggest that there exist a few suppliers that should not be invested in, as no financial benefit can be obtained. The result of the data analysis was shared with the participant for a model validation, for which a semi-structured interview approach was used. The model validation focuses on three aspects, namely model output, modelling process and data requirement. The validation process shows that the proposed model met all the pre-specified criteria. That is, the model output is considered useful and acceptable, and the operational assumptions of the model appear well aligned with the real work of the participating company. Overall, the proposed decision support model has been highly appreciated by the participating firm.

8.2 Research Contribution

Many existing decision support models have not investigated the impact of epistemic uncertainty on the return from the investment in detail. This research addresses this gap by making a number of methodological, theoretical and practical contributions which are summarised as below.

Methodologically, this research associates the uncertainty of the benefit with suppliers' capabilities and captures both the aleatory and epistemic uncertainties by developing stochastic models within a Bayesian framework. We extended the Quigley et al. (2018) class of models to diversify the ways for modelling different types of supplier performance data, in which two probabilistic models are used, namely the multinomial distribution and the NHPP model. The use of the multinomial distribution allows for modelling a wider class of performance data, such as, classes corresponding to degrees of late delivery. The use of the NHPP model enables modelling a supplier's performance whose non-conformance rate changes over the risk exposure. Such modelling techniques are useful for analysing suppliers' performance data but have not been used in the previous studies. In addition, prior distributions are used for modelling the epistemic uncertainty, namely the Dirichlet distribution and the Gamma distribution. Based on the Bayes theorem, the epistemic uncertainty can be reduced as more data is gathered. To estimate the parameters, empirical Bayes inference is employed. Furthermore, the Lagrangian multiplier is used to obtain the optimal investment allocation for supplier portfolio investment. Also, the virtual age model is used to facilitate expert judgment on suppliers' improvement potentials. Such approaches have not been used in the context of supplier development in the previous studies.

Theoretically, we generate a number of new theorems as managerial insights. First of all, a closed form of optimal investment levels is obtained to provide decision makers an upper bound of the investment level which can be used for drawing up a budget. In addition, the expected value of perfect information (EVPI) is provided, which gives decision makers a maximal amount of investment for buying down the epistemic uncertainty before making investment decisions. The proposed decision support models not only specify how much it is worth investing in a supplier but also provide insights on whether it is worth delaying the investment decision to know more about suppliers' true performance and if so how much should be invested.

Practically, we conducted an industrial case study with a Chinese automotive company to evaluate and validate one of the proposed models. The managerial insights provided by the proposed model were highly appreciated by the participating firm. As noted by the participant, "when we make the investment decision, we just used the same investment levels for many suppliers even though the efforts made for improving these suppliers were very different, we didn't consider an optimal investment level. At that time, we did not have such tools to provide an optimal investment level. If we did, we might have reduced some of our investment in particular suppliers". In addition, the participant also expressed an interest in using the proposed model in the future and mentioned that "we would like to use this model as a supportive tool for decision making if there is an opportunity in the future. It did provide us with new insights on decision making which we did not consider before". As aforementioned, most of existing models have not been empirically evaluated. Empirical studies allow us to not only further validate the proposed model but also explore more research opportunities to develop solutions for real-world problems. We bridged this gap and also obtained insights from the practitioners for future research opportunities. For example, during the semi-structured interview the participant expressed their interest in making investment decisions based on multiple criteria. In addition, we noted that eliciting the effectiveness rates directly from the practitioners appears to be difficult and thus, there is a need of a method for estimating the effectiveness of supplier development strategies.

8.3 Research Limitations

This research also involves several limitations. First of all, some modelling assumptions still require validation. For example, we assume that a supplier's learning curve follows an exponential decay function with respect to the investment level. Although this assumption has been used in many previous studies, it has not been validated empirically. In addition, in the empirical data analysis, we used empirical Bayes for parameter estimation by pooling all suppliers' performance data together, but we have not examined the degree of the similarities among the suppliers. Also, we have not evaluated the effectiveness of a virtual age as a way of facilitating the expectation of supplier's performance improvement. Furthermore, the EVPI is evaluated under the assumption that perfect information is obtained which is an ideal scenario that may not be obtained in reality. Secondly, some modelling techniques used could still be further improved. For example, a prior distribution may be developed for the shape parameter in the power law NHPP model instead of using expert judgement. In addition, the use of the exponential decay function also restricts the possibility of allowing a buyer to move away some investment from one supplier to another supplier to increase the total profit. Thirdly, the use of empirical Bayes for data analysis is also subject to a number of limitations. Although empirical Bayes allows us to estimate the rate of a rare event using data from similar events, the degree of similarities among the events is important as a lower level of similarities may result in lower level of accuracy in estimates, and empirical Bayes for data analysis may become less effective accordingly. In addition, identifying similar suppliers can also be difficult and such similar suppliers may not always be available. Furthermore, the use of empirical Bayes inference also leads to a certain degree of computational difficulty for the estimates of prior parameters. In fact, this thesis only shows how empirical Bayes is used to reduce the epistemic uncertainty but a full Bayes approach can also be used for the same purpose which may be subject to less computational complexity. Fourthly, the proposed models only focus on one decision criterion. As noted, the proposed models were developed from the perspective of maximising the financial benefit. This could be one of the key criteria for making an investment decision. Also, multiple suppliers' KPIs may be modelled rather than focusing on one particular characteristic. Other factors which may also result in an uncertain benefit have not been considered, such as, the uncertainty in the effectiveness of intervention strategies. Fifthly, due to the time and resource constraint, only one case study was conducted. More empirical case studies are still needed, in particular, validation is still needed for the decision model developed in Chapter 5.

8.4 Future Work

One aspect for future work is the improvement of the current proposed models. First of all, a method for estimating the effectiveness rate of a supplier development activity needs to be developed. As aforementioned, during the industrial case study we noted that directly eliciting the effectiveness rates from participants seems to be challenging. Therefore, a systemic approach to facilitate the process is needed. Then, the cost function for a virtual age may be modelled in a more complex way rather than as linear relationship as assumed in this research. In addition, a noise variable may be built into the expression of EVPI to take into account a scenario where perfect information cannot be obtained. Also, the supplier's learning curve may be modelled differently in future, such as a S-shaped function. The prior probability distribution for the shape parameter in the power law NHPP model may be further investigated. Furthermore, more practical applications need to be conducted. As aforementioned, a few modelling assumptions made in the current proposed models still need to be examined empirically.

Another aspect for future work is to explore different decision problems. For example, different decision criteria for supplier development investment may be taken into consideration, such as optimal duration to conduct the activity. As aforementioned, the industry manager also noted that apart from financial considerations, the company also has concerns about the time required for the activity. In addition, models to support different decision making stages may be investigated, as this study has only focused on the supplier development stage. We may further integrate the decision problem of supplier selections. Alternatively, there may exist a possibility that a supplier development activity may be terminated during supplier development. This may also be taken into account in the decision making process. Furthermore, this research has only focused on a single buyer's decision. In the future, a cooperative investment by multiple buyers or both buyers and suppliers may be investigated.

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Appendices

Appendix A

Tables of Experimental Data

Table A.1: Data for Fig.4.2

Effectiveness Rate	Optimal Investment Level	Expected Return
$\gamma = 0.01$	198.67754	430.5206
$\gamma = 0.02$	133.99613	545.2021
$\gamma = 0.03$	102.84626	593.0186
$\gamma = 0.04$	84.32674	619.8714
$\gamma = 0.05$	71.92427	637.2739
$\gamma = 0.06$	62.97558	649.5559
$\gamma = 0.07$	56.18122	658.7312
$\gamma = 0.08$	50.82771	665.8705
$\gamma = 0.09$	46.48889	671.5982
$\gamma = 0.10$	42.89360	676.3046

Table A.2: Data for Fig.4.3

Effectiveness Rate	Optimal Investment Level		Expected Return	
	Under Uncertainty	Under Perfect Information	Under Uncertainty	Under Perfect Information
$\gamma = 0.01$	198.67754	127.81365	230.1566	370.3453
$\gamma = 0.02$	133.99613	85.22110	328.9158	471.2437
$\gamma = 0.03$	102.84626	65.12610	371.4249	513.5070
$\gamma = 0.04$	84.32674	53.26769	395.6241	537.2888
$\gamma = 0.05$	71.92427	45.35882	411.4343	552.7195
$\gamma = 0.06$	62.97558	39.66781	422.6549	563.6190
$\gamma = 0.07$	56.18122	35.35530	431.0720	571.7665
$\gamma = 0.08$	50.82771	31.96241	437.6426	578.1092
$\gamma = 0.09$	46.48889	29.21588	442.9280	583.1998
$\gamma = 0.10$	42.89360	26.94226	447.2806	587.3841

Table A.3: Data for Fig.4.4

Effectiveness Rate	EVPI
$\gamma = 0.01$	140.1888
$\gamma = 0.02$	142.3280
$\gamma = 0.03$	142.0821
$\gamma = 0.04$	141.6647
$\gamma = 0.05$	141.2852
$\gamma = 0.06$	140.9641
$\gamma = 0.07$	140.6945
$\gamma = 0.08$	140.4666
$\gamma = 0.09$	140.2718
$\gamma = 0.10$	140.1036

Table A.4: Data for Fig.4.5

Category	Optimal investment Level		
	Exact Value	Upper Bound	Lower Bound
Low	120.86900	145.48415	90.07076
Medium	107.73381	145.48415	66.80492
High	99.94241	145.48415	53.69972
Category	Corresponding Profit		
	Exact Value	Boundary 1	Boundary 2
Low	754.6340	748.4746	738.9054
Medium	767.5806	754.1375	709.1000
High	773.3891	754.1375	722.7379

Table A.5: Data for Fig.5.4

Dataset	Model	T4	T5	T6	T7	T8	T9
1	NHPP	1.758	1.599	1.541	1.470	1.319	1.242
	HPP	9.511	9.996	10.388	10.600	10.774	10.904
2	NHPP	7.240	6.620	6.272	5.961	5.693	5.821
	HPP	49.332	54.365	57.958	60.348	62.947	64.720
3	NHPP	22.274	21.723	21.717	21.532	21.600	20.469
	HPP	188.793	218.166	241.082	259.861	273.828	286.845
4	NHPP	2.445	2.200	2.033	1.932	1.769	1.647
	HPP	19.195	20.095	20.794	21.246	21.564	21.873
5	NHPP	10.265	9.189	8.195	8.221	8.227	7.780
	HPP	98.148	107.527	114.519	120.065	124.537	128.573
6	NHPP	32.461	31.604	29.836	30.765	29.169	29.425
	HPP	377.034	436.972	481.148	518.333	545.015	571.625

Table A.6: Data for Fig.5.5

Virtual Age	Dataset1	Dataset2	Dataset3	Dataset4	Dataset5	Dataset6
Y=1000	-6715	4185	42278	-4158	17240	93050
Y=2000	-16192	-1357	56724	-13243	15808	131338
Y=3000	-25880	-8414	64391	-22696	11465	156256
Y=4000	-35660	-16216	68378	-32313	5688	173911
Y=5000	-45493	-24472	69982	-42022	-958	186862

Table A.7: Data for Fig.5.6

Virtual Age	Dataset1	Dataset2	Dataset3	Dataset4	Dataset5	Dataset6
Y=1000	-1715	9185	47278	842	22240	98050
Y=2000	-6192	8643	66724	-3243	25808	141338
Y=3000	-10880	6586	79391	-7696	26465	171256
Y=4000	-15660	3784	88378	-12313	25688	193911
Y=5000	-20493	528	94982	-17022	24042	211862

Table A.8: Data for Fig.5.7

Virtual Age	Dataset1	Dataset2	Dataset3	Dataset4	Dataset5	Dataset6
Y=1000	2285	13185	51278	4842	26240	102050
Y=2000	1808	16643	74724	4757	33808	149338
Y=3000	1120	18586	91391	4304	38465	183256
Y=4000	340	19784	104378	3687	41688	209911
Y=5000	-493	20528	114982	2978	44042	231862

Table A.9: Data for Fig.5.8 and Fig.5.9

Virtual Age	NC Cost under Virtual Age	Expected Profit from Virtual Age
0	34420.27	0.00
50	33426.92	743.3467
100	33028.64	891.6301
150	32728.08	942.1852
200	32478.05	942.2178
250	32260.26	910.0037
300	32065.35	854.9115
350	31887.76	782.5038
400	31723.86	696.4035
450	31571.14	599.1257
500	31427.77	492.5009

Table A.10: Data for Fig.6.2

Suppliers	NC1 Estimate	NC1 Real	NC2 Estimate	NC2 Real	C Estimate	C Real
S1	0.001842	0.001843	0.000445	0.000444	0.997712	0.997713
S2	0.000002	0.000000	0.000001	0.000000	0.999997	1.000000
S3	0.000388	0.000388	0.000461	0.000461	0.999151	0.999151
S4	0.011245	0.011250	0.000020	0.000019	0.988735	0.988731
S5	0.002248	0.002249	0.000527	0.000525	0.997225	0.997225
S6	0.000864	0.000864	0.000711	0.000711	0.998425	0.998425
S7	0.002129	0.002129	0.001563	0.001563	0.996308	0.996308
S8	0.000926	0.000922	0.001861	0.001864	0.997213	0.997213
S9	0.000065	0.000000	0.000193	0.000150	0.999743	0.999850
Pool	0.001617	0.002446	0.001215	0.000688	0.997168	0.996866

Table A.11: Data for Fig.6.3

Suppliers	Effectiveness Rate for NC Type1	Effectiveness Rate for NC Type2
S1	0.00009414	0.00004071
S2	0.00002217	0.00001198
S3	0.00007511	0.00001733
S4	0.00003729	0.00004549
S5	0.00002010	0.00003092
S6	0.00001962	0.00003066
S7	0.00010699	0.00001791
S8	0.00000428	0.00002434
S9	0.00002582	0.00002737

Table A.12: Data for Fig.6.4

Suppliers	Actual Investment	Optimal Investment				
		u	$5u$	$10u$	$15u$	$20u$
S1	4.949	4.755	4.950	5.022	5.060	5.085
S2	5.046	0	0	0	0	0
S3	5.046	5.125	5.355	5.426	5.462	5.486
S4	5.046	5.236	5.333	5.369	5.389	5.402
S5	5.001	5.072	5.291	5.361	5.397	5.421
S6	5.046	5.213	5.380	5.438	5.469	5.489
S7	4.949	5.499	5.608	5.647	5.669	5.684
S8	5.001	4.868	5.294	5.501	5.610	5.676
S9	5.192	5.097	5.267	5.323	5.353	5.373

Table A.13: Data for Fig.6.5

Suppliers	Actual Investment	Optimal Investment			
		$1/5u$	$1/10u$	$1/15u$	$1/20u$
S1	4.949	4.511	4.370	4.266	4.176
S2	5.046	0	0	0	0
S3	5.046	4.705	4.471	4.309	4.166
S4	5.046	5.111	5.043	4.998	4.166
S5	5.001	4.637	4.094	0	0
S6	5.046	4.960	4.790	4.653	4.523
S7	4.949	5.353	5.271	5.215	5.170
S8	5.001	0	0	0	0
S9	5.192	4.815	4.598	4.392	4.146

Table A.14: Data for Fig.6.6

Suppliers	Increased Return (High NC Cost)				
	u	$5u$	$10u$	$15u$	$20u$
S1	4.2203	-0.2790	3.8539	4.3222	4.5760
S2	5.0387	5.0071	4.9640	4.9162	4.8624
S3	3.6925	5.4004	5.8033	6.0134	6.1597
S4	5.2382	6.0695	6.3908	6.5743	6.7032
S5	3.5911	5.3135	5.7177	5.9306	6.0749
S6	4.6555	5.7385	6.0931	6.2889	6.4243
S7	6.4658	7.1945	7.5002	7.6779	7.8037
S8	3.7301	4.8595	5.4615	5.7508	5.9366
S9	3.9925	4.1755	4.8475	5.1351	5.3159

Table A.15: Data for Fig.6.7

Suppliers	Increased Return (Low NC Cost)			
	$1/5u$	$1/10u$	$1/15u$	$1/20u$
S1	4.6200	4.7131	4.7572	4.7851
S2	5.0448	5.0455	5.0458	5.0459
S3	4.4318	4.6993	4.7942	4.8461
S4	3.8760	0.9907	3.3390	3.7440
S5	4.3837	4.6913	4.8118	4.8672
S6	3.6020	4.3073	4.5188	4.6321
S7	5.6520	5.2228	4.9184	4.6583
S8	4.7877	4.9070	4.9405	4.9563
S9	4.7525	4.9065	4.9783	5.0233

Table A.16: Data for Fig.6.8

Suppliers	Optimal Investment Level		
	Exact Value	Upper Bound	Lower bound
S1	5.022	5.160	4.854
S3	5.426	5.480	4.950
S4	5.369	5.369	5.293
S5	5.361	5.377	5.228
S6	5.438	5.480	5.317
S7	5.647	5.692	4.996
S8	5.501	5.756	5.235
S9	5.323	5.339	5.317

Table A.17: Data for Fig.6.9

Suppliers	Corresponding Returns		
	Exact Value	Boundary1	Boundary2
S1	6.937	6.936	6.935
S3	7.023	7.022	6.989
S4	8.218	8.218	8.218
S5	6.757	6.757	6.752
S6	7.269	7.269	7.267
S7	8.570	8.570	8.538
S8	6.328	6.307	6.312
S9	7.024	7.024	7.024

Table A.18: Data for Fig.6.10

Suppliers	Original Investment	Investment (High Cost)	Investment (Low Cost)
S1	4.949	4.633	4.594
S2	5.046	0	0
S3	5.046	4.928	4.856
S4	5.046	5.173	5.153
S5	5.001	4.888	4.818
S6	5.046	5.092	5.051
S7	4.949	5.426	5.402
S8	5.001	4.489	0
S9	5.192	4.967	4.922

Table A.19: Data for Fig.6.11

Non-Conformance Cost	Total Increased Return
$1/20u$	5.163
$1/15u$	5.288
$1/10u$	5.464
$1/5u$	5.765
u	6.479
$5u$	7.178
$10u$	7.479
$15u$	7.655
$20u$	7.780

Appendix B

Additional Information for the Case Study

B.1 Case Study Process

The procedure of the industrial case study consists of four steps. The first step is the preparation for semi-structured interview that is sending invitations to seek a participating company. Once the participating firm was confirmed. An introduction meeting was arranged to give a detailed explanation regarding the project and to seek appropriate participants. The second step is concerned with designing interview questions and then conducting semi-structured interviews for data collection. The interview content was transcribed and safely documented along with the secondary data obtained from the participating firm. The third step is to select a suitable model to fit the collected data and analyse the data using an appropriate model. The final step is to validate the proposed model based on a set of pre-specified criteria. The results of the data analysis were summarised in a report and were shared with the participant. A semi-structured interview was conducted at the participant's convenience to gather feedback.

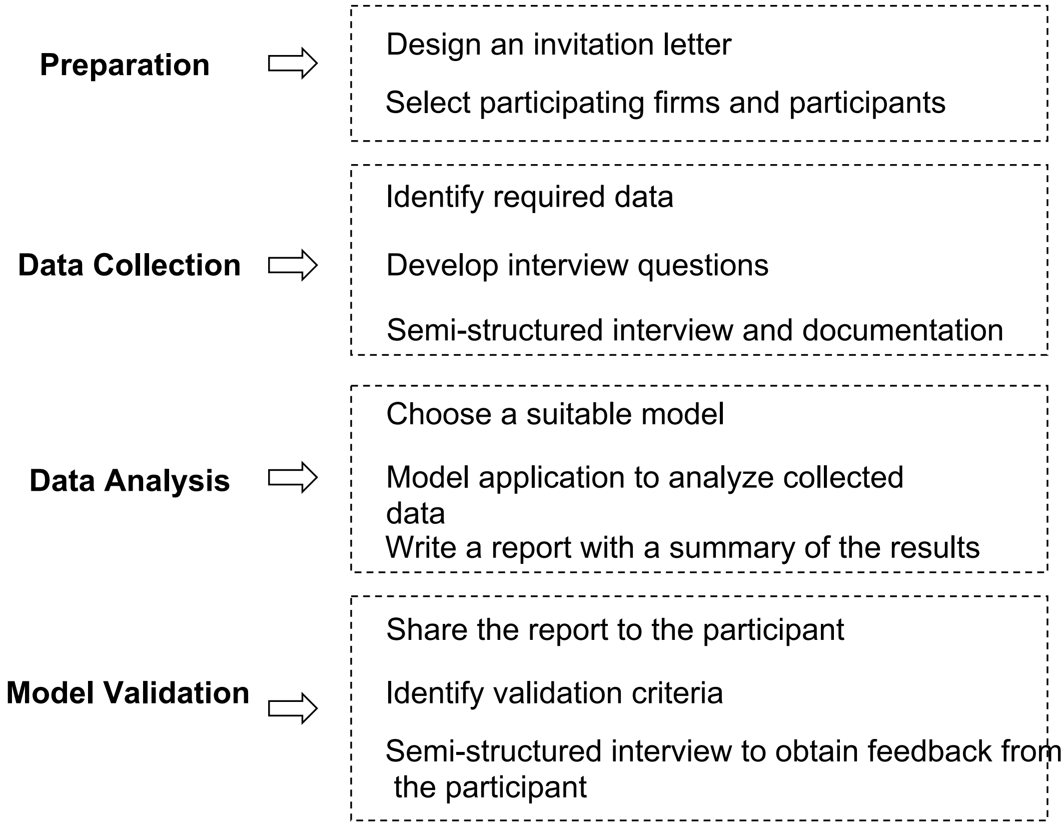

Figure B.1: Case study process

B.2 Timeline of Conducting the Case Study

Time	Activity
15/11/2018	Sending an invitation letter to the potential participating firm
20/11/2018	Confirmation of participation from the participating firm
04/01/2019	The first visit to the participating firm to give an introduction of the research and to identity suitable participants
05-18/03/2019	The second visit to collect data and managerial knowledge for empirical data analysis
18/04/2019	Sharing the data analysis results with the participant
30/08/2019	The third visit to feedback on the data analysis result from the participant

B.3 Selection of the Participating Company

In this study, a Chinese automotive company was selected for the case study. The participating company is principally involved in the design, development. It is a leading car manufacturer and provides a broad line of automotive components. The company has extensive supply bases across mainland China and procures parts from both external and internal suppliers which provides sufficient database for our research. We consider the participating firm appropriate for our research with several reasons. First, the participating firm is within an automotive industry which is highly competitive and involves high changes and large size of suppliers. Such characteristics force buying firms to continuously improve their suppliers' performance to enhance product quality and to reduce production costs (Handfield et al., 2006). Therefore, firms in such industries are more motivated to commit into supplier development and to participate into this research. Second, much previous work on supplier development mainly focused on European or US firms (Humphreys et al., 2011). A few previous studies for supplier development were conducted in the context of Chinese industry. To the best our knowledge, empirical investigations for supplier development have not yet been conducted in Chinese automotive industry. In particular, very few existing decision support models have been evaluated empirically. We bridge this research gap and believe that findings and implications of the study can also be applicable for the similar industrial environment.

B.4 Selection of Participants

The quality of participants plays an important role within the data collection process to ensure that reliable and valid information is obtained. To select appropriate participants, an initial contact with the participating firm was established. A snowballing approach was utilised to identify suitable participants. An initial meeting was set up on the 4th of January 2019 with a manager of Logistic Management. At the meeting, the objectives of the study and requirements for participants were explained. The manager was able to identify suitable participants across the firm and provide contacts. The researcher then contacted each of these people individually. In the end, the manager of Quality Management Department was selected as the most appropriate participant for this case study, who has more than 10 years of working experience in supply chain management and in particular was also the leader and the supervisor of the supplier development activity.

B.5 Data Documentation

To document the collected data properly, interview audio-recordings was transcribed shortly after each interview along with detailed notes taken during the interview. In addition, transcription of interviews was carried out as soon as possible after the interview as memory can assist in understanding the transcription. All the interview recordings are kept on a university computer until the PhD is completed, and then deleted. Any information identifying the participant is anonymised. Data is also kept on the computer.

In total, this case study involved face-to-face interviews with two participants. The roles of participants in the organisation included the manager of Logistic Planning and Management and the manager of Quality Management. The interviews for quantitative data collection and model validation were conducted mainly with the manager of Quality Management who has more than 10 years of working experience in supply chain management and is responsible for supplier quality management in particular. In addition, the participant was also the manager of the supplier development project investigated for this study. Therefore, we argue that this participant possesses considerable knowledge regarding the planning, implementation and progress monitoring of the activity. All interviews were conducted on a one-to-one basis. Each interview lasted for 90 minutes in average (with the minimum and maximum time of 60 and 120 minutes, respectively). The time and location of interviews was arranged by the participants to best suit their availability. All interviews were held in a meeting room provided by the organisation.

B.6 Ethical Considerations

According to the University Ethics Policy, this case study is considered to be a "standard" project, as it does not involve any "at risk" investigation, such as personal confidential data, vulnerable groups with patients, children or drug users, although there were still several common ethical issues to consider. The first ethical consideration is related to the participants' agreement to share their beliefs and opinions given the research questions. The study obtained ethical approval from the departmental ethics committee before conducting the interview. As required by the University of Strathclyde's ethical standards, all participants were provided an information sheet stating the purposes of the study as well as requirements. Interviewees have confirmed their interests and agreed to participate in the study. Although the participant was able to withdraw from the study without giving any reason prior to interviews, the participant did not. The participant was also signed consent forms before the interview started. The second ethical consideration is the interview questions. The researcher was aware that no questions regarding personal details or out of the scope of the research were asked. The third ethical consideration is the use of data. The researcher ensured that all data of audio-recordings was anonymous and confidential. No information identifying the participants was publicly available. Confidential information to be used for analysis was desensitised. Overall, the researcher has highly valued the participant's time and expressed gratitude for the participation.

Appendix C

Semi-Structured Interview Questions

C.1 Questionnaire for Model Validation

This feedback questionnaire is in response to the report on the case study of the PhD project: Stochastic Modelling for Decision Support in Supplier Development. Please provide the rationale for your answer as any information gathered may help improve the proposed decision support model. Your response to this questionnaire will be used anonymously and are for the evaluation of the case study only.

Model Output

- In the analysis report, which part you are interested most?
- From your perspective, is there any insight obtained from the outputs of the model that could be used to support decision making?
- Which information obtained from the model you feel that would help most in decision making?
- Will you use the optimal investment level provided from the decision model?

- Do you feel well-informed by the model?
- Any more information you would like to know through decision support tools?

Modelling Process

- What are the incentives to conduct the supplier development activity?
- What do you care most in the supplier development activity?
- What risks have you anticipated?
- Would you still invest in the supplier development activity if there is a high possibility that the project may fail?
- Is there any objective variable which you think may have significant impact on the decision making but has been neglected in the proposed model?
- How different is this decision making approach from what you have done for decision making?
- Any adjustment of the model?
- Any recommendation for the improvement of the model?
- Will you consider using the proposed model to support decision making on supplier development in the future?

Requirement of Data

- How do you think about the availability of the data? Do you feel difficult to obtain the data of the input variables?
- Are you able to anticipate the effectiveness of the intervention strategies?
- Are you able to anticipate the expected supplier performance after development using analogies?

Appendix D

Mathematical Explanations for Empirical Data Analysis

D.1 Proof for Fig.6.11

Assuming there are a number of J suppliers and the profit obtained under the actual investment level made by the firm is the base line, when the non-conformance is c_{ij}, according to Eq.(4.11), the total increased return is expressed as:

$$\begin{aligned}
\sum_{j=1}^{J} \Delta\pi_{ij} &= \sum_{j=1}^{J} \left(\pi_{ij}^{optimal} - \pi_{ij}^{actual}\right) \\
&= \sum_{j=1}^{J} \left[\left(c_{ij} n_j p_j \left(1 - e^{-rx_j^*}\right) - x_j^*\right) - \left(c_{ij} n_j p_j \left(1 - e^{-rx_j}\right) - x_j\right)\right] \\
&= \sum_{j=1}^{J} \left[c_{ij} n_j p_j \left(e^{-rx_j} - e^{-rx_j^*}\right) - \left(x_j^* - x_j\right)\right] \\
&= \sum_{j=1}^{J} c_{ij} n_j p_j \left(e^{-rx_j} - e^{-rx_j^*}\right) - \sum_{j=1}^{J} \left(x_j^* - x_j\right)
\end{aligned}$$

Where, $\pi_{ij}^{optimal}$ is the profit from the optimal investment level x_j^* for supplier j under c_{ij}, and π_{ij}^{actual} is the profit from the actual investment level x_j for supplier j under c_{ij}.

As $\sum\limits_{j=1}^{J} x_j^* = \sum\limits_{j=1}^{J} x_j$, then we can get:

$$\sum_{j=1}^{J} \Delta\pi_{ij} = \sum_{j=1}^{J} c_{ij} n_j p_j \left(e^{-rx_j} - e^{-rx_j^*} \right)$$

As we have obtained that under the same budget, the optimal investment allocation does not change as the non-conformance cost changes, therefore, we can write:

$$\sum_{j=1}^{J} \Delta\pi_{ij} = \sum_{j=1}^{J} c_{ij} A$$

Where $A = n_j p_j \left(e^{-rx_j} - e^{-rx_j^*} \right)$ is a constant value. This implies that the total increased return has a monotonic relationship with the non-conformance cost.