PREDICTIVE CONTROL OF URBAN WASTEWATER SYSTEMS

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Declaration of Authors Rights

Abstract

Within recent years, technological advances and stricter regulatory requirements have seen the increased use of automation and instrumentation within the wastewater treatment industry. As a result, advanced control strategies are required, to fully exploit the potential of these complex systems in addressing water quality concerns. Model based control strategies can be appropriate within the multivariable constrained wastewater system. In particular, the inherent model based nature of this approach can be valuable in the prediction of the treatment plant effluent quality required over a considered time period, to meet water quality standards.

Multivariable linear predictive control is implemented for a benchmark treatment plant model, demonstrating the constraint handling ability of the predictive control structure. The limitations of an effluent-based control strategy in the maintenance of river quality is discussed. A more global approach to wastewater control must be considered in order to compensate against disturbances within the system. Tackling this concern, the incorporation of receiving water quality objectives within the control strategy is proposed. To this end, the application of linear MPC to the control of dissolved oxygen concentrations in the receiving waters under storm conditions is demonstrated.

The drawbacks involved in a linear model based approach within a nonlinear urban wastewater system are considered. Several nonlinearities are present: the bioprocesses involved are by definition nonlinear, and are affected by varying wastewater load and characteristics. These can be the result of varying stormwater effects upon the treatment plant or emergency overflows to receiving waters. This therefore motivates the development of nonlinear strategies in the control of the wastewater processes. The control of SISO nonlinear processes within the urban wastewater system, such as dissolved oxygen, is demonstrated via the use of fuzzy gain-scheduled and Wiener model based predictive control. Additionally, the use of existing nonlinear process models in the control of wastewater processes is shown in the application of state dependent model predictive control.

Dedication

I dedicate this work to my parents, Eileen and Patrick, and my sisters, Róisín and Olive, for their constant support throughout the duration of my PhD.

Acknowledgements

I would like first to express my gratitude to my supervisor Prof. Andrzej Ordys, for his support, patience, ideas and knowledge, all of which were invaluable during my PhD. Thanks must also be given to Prof. Mike Grimble, whose expertise and valuable comments were most appreciated during the completion of my thesis.

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In conclusion, the most sincere gratitude is expressed to my family, to whom this thesis is dedicated and without whom this PhD would never have been possible.

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Chapter 1

Introduction

In recent years the control and the treatment of wastewater systems has become increasingly important. With the continuing increase in world population and rising health standards, water has become a valuable commodity. Within the current climate of environmental concern, the efficient performance of wastewater treatment systems is a signficant concern. The safety of water, both for municipal use and also as regards its effect on the environment is of growing interest to today's population. Recent EU directives and international changes in approach to environmental concerns signify the dynamic nature of this area of study. The aim has now become that of decreasing the effect of humans and their lifestyles upon water usable for human consumption and agricultural needs. Regulatory requirements for the treatment of wastewater are becoming increasingly strict, requiring as a consequence more efficient wastewater treatment plant (WWTP) operation. The manipulation of biological processes is of particular relevance within this area. The gradual increase in the use of mathematical and data-based models of the wastewater process, in the design and implementation of online process control, is a consequence of a gradual trend towards advanced monitoring and automation of treatment systems. Traditionally, automation has concentrated upon the wastewater treatment system, without reference to the quality of receiving waters. The urban wastewater treatment system, as considered within this thesis, is described by three components: the sewer system, the treatment plant, and the receiving waters, as shown in Figure 1-1. It is the application of control to the urban wastewater system that is the main focus of this thesis.

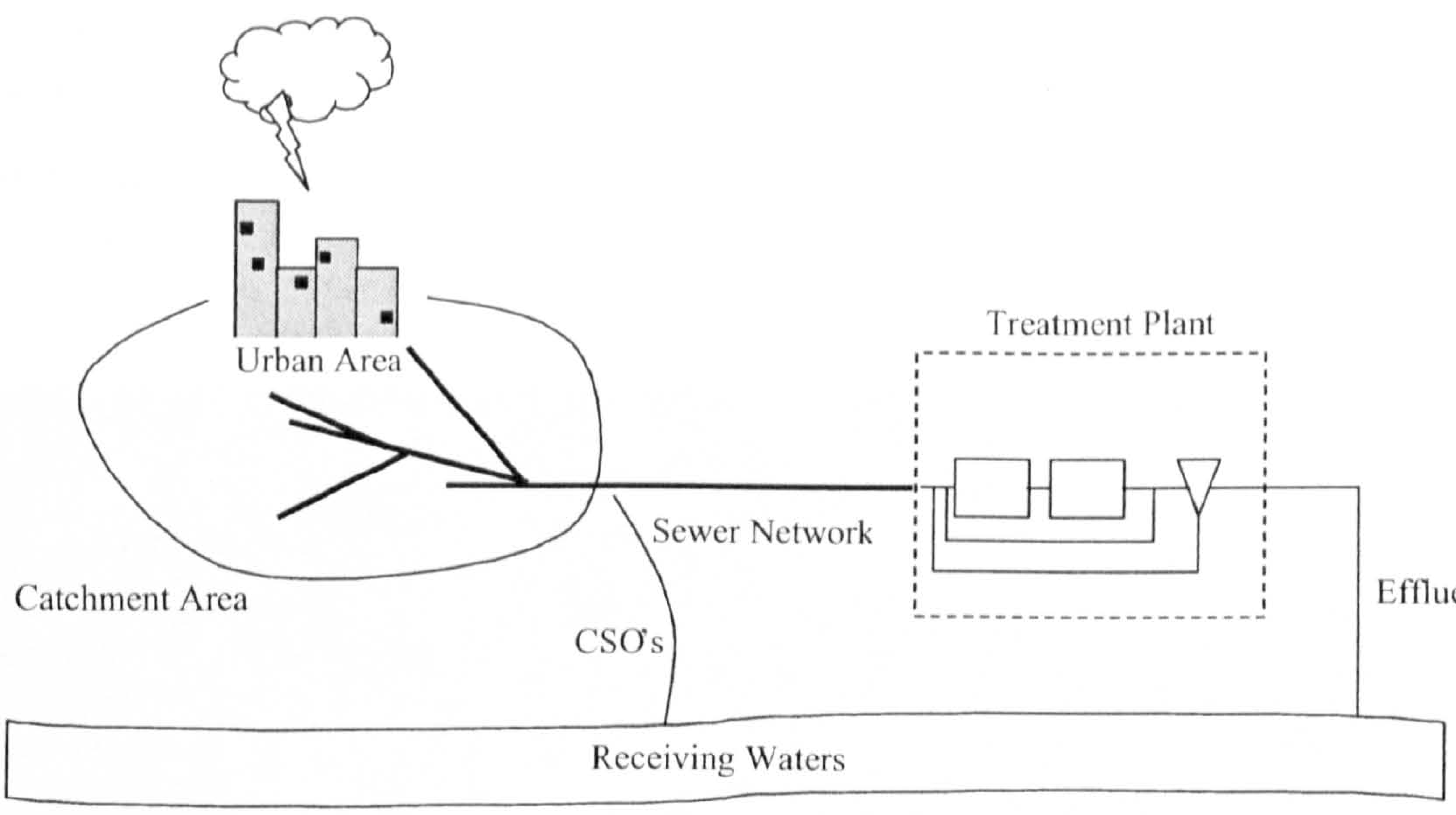

Figure 1-1: Urban Wastewater System: Sewer Network, Wastewater Treatment Plant and Receiving Waters

1.1 Historical Perspective

The Indus Valley Civilisation (present day Pakistan), which flourished from 2600 to 1900 BC was said to have produced the first flush toilets, with a sophisticated sewage system in the large city of Harappa [195], as shown in Figure 1-2. These sewer systems are the earliest recorded occurrence of sewage transportation in this manner. This civilisation was particularly advanced in several ways, with city planning (with a regular grid-like approach) and houses containing rubbish chutes. Later again in 1700 B.C., it is recorded that the Minoan Palace of Knossos (Crete) contained a total of four separate drainage systems that emptied into large sewers constructed of stone [195]. A form of a flushing "water closet", with a wooden seat and a small reservoir of water was present, a version of the flushing toilet also featured in Roman times. However, even in Ancient Egyptian times, it is speculated that suspended solids were removed using sedimentation apparatus. The sewer systems of the Roman empire were a feat of engineering of the time, small sections of the ancient systems of 'Cloaca Maxima' (literally translated as the 'Greatest Sewer') on the river Tiber still remain in use in the modern sewers of the region. This sewer system appears to have originally been built both over and underground, and became entirely beneath ground during expansion of the city of Rome, serving public toilets, baths and other public buildings. It appears that private residences could not avail of this service, instead relying on 'cess-pits'.

Figure 1-2: Ancient Urban Drainage System in Harappa, Part of the Indus Civilisation. (Image Source [195])

Interestingly, use of feedback in control can be traced to such early water devices as that of Ktesibios's (of Alexandria) water clock in the third century BC [196]. This device required an unvarying flow of water, in order to accurately mark the flow of time. However the flow of water from a container is not steady, flowing more slowly when the container is less full, and Ktesibios determined a method to maintain the constant level. The same method was used in the design of the flush toilet, wherein a float on water level operates a valve, allowing water to the container when the float is dipped, and closing the valve when sufficient water is present. The feedback is present in the form of the valve, the effect of the valve in increasing the water level is fed back to the valve and ultimately leads to its closure. The invention of the modern flush toilet, based upon a similar method, was a significant event as a waste transportation device. Early forms of water control extended to other areas, such as that of the 3rd century Funan, in southeast Asia (in modern Cambodia and Vietnam), perfomed the control of annual flooding and coastal changes with a network of waterworks and canals.

As with many inventions in the history of man, knowledge of these systems was lost or forgotten, particularly after the fall of the Roman Empire. In medieval times, sewers were mostly open waterways, which were gradually built over, thus forming covered sewer systems. In the 16th century, Sir John Harington's invention of the "washout" closet of a similar principle introduced the concept again with a flush valve and washdown approach, and is claimed to be the origin of the common slang for a toilet, the 'John'. However, it would take still another two centuries before Alexander Cumming would produce the version of the toilet used today incorporating an 'S' bend remaining filled with water between flushing, avoiding the return of air from the sewer.

By the late 19th century the majority of cities had a rudimentary underground combined sewer system. It is these systems and those built in addition to these existing systems that are still in use today. Rainwater runoff can have considerable effect upon these systems, due to the urbanisation of the surrounding areas, with roads and buildings covering most of the area through which the rainwater would have naturally drained. This wastewater then instead must be transported through the storm drains and thus through the sewer system. This high flow can lead to large overflows, which spill to receiving waters such as rivers, causing serious disturbances.

By the 1950's, the treatments for wastewater that most resemble those used today were introduced: biological treatment to tackle oxygen depletion, and, in the 1970's, combined biological-chemical treatment to reduce eutrophication of receiving waters (from phosphorus and nitrogen in wastewater). From then, the processes have been further developed, and improved, in particular with the use of automation to improve efficiency and effectiveness of the treatment process. To this end, advanced control and process optimisation of the wastewater treatment industry has become of increasing concern in recent years [20], [77][144]. The focus of this thesis is the control of wastewater treatment systems, researching advanced control strategies to compliment existing control structures and instrumentation.

1.2 Automatic Control in Wastewater Treatment

The wastewater treatment industry is becoming dependent upon automatic control systems to avoid economic, process and environmental costs associated with operator process supervision and control. The general objective of any manipulation or control in urban wastewater systems is to reduce the effect, domestic and industrial, of humans upon the environment. This involves treating the water in order to maintain the water quality. The disturbances to the system are those events which either by human error, or by extreme weather, cause effects upon concentrations within the river. This in turn has a further effect downriver in the direction of water flow, causing possible fish deaths due to toxic events and/or increase in algae growth, and thus a decrease in water quality.

Traditionally, the main occurrence of automatic control is in the manipulation of the activated sludge process, [80][94]. The objectives of the control implemented are two-fold: to reject or compensate for disturbances within the system (here this involves storm events and combined sewer overflows), and also to maintain setpoint values whilst at system steady state (for example, in normal weather conditions). In particular, the main control schemes focus upon dissolved oxygen concentrations and nutrient removal within the treatment plant, the key processes within the system. In the receiving waters, control to regulatory standards requires the measurement and control of similar concentrations in the river, such as dissolved oxygen or ammonia (plus ammonium) levels. Any disturbances within the urban wastewater system, such as sudden high flow influent to the treatment plant or overflow from the sewer network to the river (both which may occur during adverse weather conditions), will either cause a dip (for dissolved oxygen) or a peak (for ammonia) in concentration in the river. Restrictions exist upon the application of existing control technology to such a process, where there exist process model uncertainties, ill defined disturbances, nonlinear dynamics and a multivariate system. Considering the issue of tightening regulations upon treatment plant effluents and receiving water quality, there exists a demand upon the performance existing control structure. This thesis seeks to explore possible methods of exploiting the existing wastewater industry control and instrumentation in meeting these restrictions.

As stated, the regulations concerning the aforementioned concentrations, and others, have become increasingly strict in recent years, and this will continue, in particular due to recently introduced directives concerning water quality. The difficulty in meeting these demands is additionally restricted by financial and operational concerns. Changes to the physical structure of the wastewater treatment plant and the sewer network is both expensive and time consuming. The effect of wastewater treatment upon the environment limits the length of time for which the plant can viably be out of operation, untreated waste can have a severe effect. Redesign or reanalysis of the biological and chemical treatment processes can contribute an improvement to plant operation, however this biochemical research approach is not considered within the scope of this thesis, but instead improved process performance is considered by the introduction of advanced control techniques.

Additionally, the flexibility of the plant is an issue, where flexibility is defined as the level of automation present, the operational range of the equipment and structure, ability to exercise online continuous control (continuous here, as opposed to the on-off strategy) and the ability of the plant to allow for changes to the structure of the control system. The lack of an overall consistent approach to the automation of the water industry, which is managed by local authorities for the most part, provides an obstacle in the progress and improvement of the water process control. Safety margins can be necessary in wastewater treatment processes in order to allow for conditions such as disturbances caused by extreme weather events. In meeting the WFD objectives, control measures designed for normal conditions are inadequate in fulfilling the system objectives during storm events, and in addition to a nonlinear control strategy, the inclusion of safety margins would allow for the uncertainty of control models, disturbances, and variations in influent characteristics. These safety regions exist in order to maintain sustainability, and allow a balance between the safety issues and plant costs.

1.3 Contemporary Water Treatment Systems

The activated sludge process is the most common of the water treatment processes, and is defined in a general sense as a system in which wastewater is mixed with a concentrated biomass known as activated sludge, which degrades pollutants and organic carbon within the wastewater. The most popular model of this process Activated Sludge Model No.1 (ASM1) presented by the IAWQ Task Group on Mathematical Modelling for Design and Operation of Biological Wastewater Treatment Processes [61] is generally accepted as state-of-the art. ASM1 was primarily developed for municipal activated sludge wastewater treatment plants to describe the removal of organic carbon substances.. ASM1 has been extended to include biological phosphorus processes, resulting in ASM2 and ASM2d ([62],[63]). Jeppsson et al. [69] detail the applications of real-time control of treatment plants within Europe as follows:

- Dissolved Oxygen control: This type of control is the most common, implemented via air flow control, typically with the use of proportional integral (PI) control to a constant setpoint.
- Nitrate control: This commonly utilises the internal recirculation flow, however occasionally an external carbon flow may be used.
- Sludge control: This can be manipulated via either the recycle sludge flow or the waste flow, though in some cases this is merely done manually.
- Phosphate control: These levels are maintained through additions of chemicals for precipitation.
- Ammonia control: Aeration is the most common form of ammonia (plus ammonium) control.

Dissolved Oxygen control has been the most commonly implemented of all the controllable concentrations in wastewater treatment. This is due to high running costs of energy consumption during increased aeration rate, in controlling dissolved oxygen, the air flow can be used more efficiently. Many variables in water quality, both in the effluent from the treatment plant, and also in the river, depend on the dissolved oxygen level. Dissolved oxygen control therefore is an essential control mechanism within the treatment plant, and may be coupled with ammonia control, also dependent upon aeration changes. Figure 1-3 below shows the aeration detention tanks of an activated sludge wastewater treatment plant, the background shows the secondary clarifier.

For the most part, advanced control has not been widely applied in wastewater treatment systems. The most common approach to wastewater control is that of low level control, implemented via Programmable Logic Controllers (PLC), for example as detailed in [78]. The most widely used control methods are proportional control, simple on-off control, even merely manual control. Such widely used models as the ASM models, and common control practices, within research, have not been used to any considerable extent within model based control methods in the past. In recent years however, this has begun to change and many control strategies are being improved by advanced control methods. Many applications have involved constructing higher level controllers around existing low level control, resulting in more efficient and effective wastewater treatment control than before.

There are many reasons why advanced control has not been introduced into widespread use, detailed for example in [121]. Instrumentation has proved unreliable and expensive to maintain in the past. There are limited control options within wastewater treatment, and time and expense are required to develop many control approaches. Regulatory standards were not strict enough, or stringently implemented, thus there was little impetus to expend large amounts of money developing advanced control for the systems. The introduction in recent years of more sophisticated measurement methods and on-line sensors has led to a more realistic possibility of applying advanced control. Also the stricter regulations currently in place, and the fines for breaking these, have meant that control can be used in order to decrease these costs, thus proving its financial use. With this increase in control, and therefore complexity, it is obvious that manual control is no longer an option. However, operator knowledge is not lost if appropriate control methods are used, for example the fuzzy control presented later in this thesis requires operator knowledge in the design of fuzzy rules. A literature review of research in this area is presented appropriately in the following chapters: the research area of wastewater treatment is presented in Chapter 3, as well as instrumentation details, the research area of urban wastewater modelling is discussed in Chapter 4, whilst research into urban wastewater control is discussed in Chapter 5, with a brief discussion of appropriate instrumentation. A discussion of the research into nonlinear predictive control both in industry and in wastewater treatment in particular is discussed in Chapter 7.

Figure 1-3: Aerated Activated Sludge Detention Tanks in a Wastewater Treatment Plant: **Foreground:** The Aerated Activated Sludge Tanks, **Background** The Secondary Clarifier.

1.4 Legislative Issues

In the past, regulatory attitudes both locally and internationally concentrated on the human health aspects of water treatment rather than the environmental aspect. For example, legislation from the Council of European Communities provided for the protection of the quality of water intended for drinking [44] and [46], bathing [45] and fishing [46] purposes. However, recently legislative efforts have been extended to promote the environmental concerns for water quality, via two directives. The former concentrating on the pollution of water by urban waste [42] and the latter concentrating on the pollution of water by nitrates from agricultural sources [43].

Significantly, a 'Water Framework Directive' (WFD) was introduced [49] and adopted, with aim of extending protection to all waters, (including surface and ground water) including urban wastewater river basin management, where emissions and discharges are to be controlled by limits on emission values, and regulatory quality standards. An important aspect of this directive is that the quality of waters should not deteriorate upon implementation of the measures taken in accordance with the WFD. In aim of achieving river basin management, an increase in monitoring is encouraged. Importantly for the work considered here in this paper, [178] demonstrated that in order to achieve the aims set out by the WFD, the wastewater system must be considered with an integrated approach.

1.5 Objectives

The research presented within this thesis aims to investigate the control of wastewater treatment systems. This will refer to both the traditionally controlled wastewater treatment plant and also, the more recent focus of interest within the water industry, the receiving water quality. Schutze [153] defines the control objectives for an urban wastewater system under real time control as:

- Maximise the time in which river quality standards are adhered to.
- Minimise the extent by which these standards are exceeded.
- Maximise the system's ability to recover.
- Maximise the system's ability to reject disturbances.
- In a general sense, improve river water quality above minimum.

Overall, the control of the urban wastewater system is concentrated upon the maximisation of the minimum dissolved oxygen concentration in the river, though it is possible to demonstrate similar approaches to manipulation of treatment plant aeration, for control of ammonia concentrations. Butler et al.[193] describe that the above control objectives may be based for example on concentrations of dissolved oxygen or ammonia, the aims of the control concentrating on manipulating a system to, or avoiding deviation from, an ideal state (also detailed in [12]). In particular, Butler and Schutze [23] define the performance criteria used for the assessment of control scenarios to be the following

- The duration for which the dissolved oxygen (DO) concentration in the river is below a critical threshold.
- The duration of NH4 (defined here as the concentration of Ammonium plus Ammonia) concentration in the river is above a critical threshold.
- To be minimised - The maximum concentration of ammonium in the river.
- To be maximised - The minimum concentration of DO in the river.

In a practical sense, to fulfil these objectives the control approach adopted must include the following:

- Disturbance handling: This can be defined as either unmeasured or measured disturbances. The above disturbances to water quality, as a result of high loads or combined sewer overflows, can be considered as disturbances within the controlled process.
- Constraint handling: Regulatory restrictions upon water quality result in the need for a constraint handling procedure within the adopted control strategy.
- Multivariable control: The implemented control approach should allow easily for extension to the multivariable case.
- Nonlinear control: In order to incorporate the water quality objectives within the control structure, the dynamics of the receiving waters must be considered. A nonlinear strategy is thus necessary due to the nonlinear nature of this process.
- Applicability: The strategy chosen must take into account existing control structures, such as the popularly applied PI/PID control approach.

Further issues that can be considered are technological constraints, the cost of operation and the sustainability of the system, in addition to the multiple time scales present in the plant internal dynamics.

1.6 Proposed Control Methodology

The methodology implemented in this thesis can be divided into three sections: linear control, pseudo-nonlinear control and nonlinear control. The linear control is that of a state space approach to linear model based predictive control (MBPC). The pseudo nonlinear control implemented later in the thesis again uses this linear approach, however with the application of fuzzy rules, or Wiener modelling, to extend the control abilities to the nonlinear system. Finally the nonlinear predictive control is implemented using nonlinear state dependent (SD) modelling techniques in order to fully describe the system, and thus produce more accurate control. The algorithms used in this thesis have been developed from several sources. The linear predictive control algorithm used is that demonstrated by [85], and was modified using the measured and unmeasured disturbance modelling approaches and the constraint handling detailed in Maciejowski et al. [97]. This linear approach was utilised in the gain-scheduled control detailed in Chapter 5, and was modified for use with the nonlinear Wiener model. The state dependent coefficient (SDC) modelling structure used within the nonlinear GPC scheme was demonstrated in Dutka et al. [41] and [191].

An important issue in the design of a control scheme is the choice of model in the representation of the process dynamics. There are several considerations to be taken into account in this decision:

- The purpose of the model: Model choice can depend upon the application. For control purposes, many variables may be ignored.
- The dynamics of the given process: a relatively linear process dynamic can be simply represented by a linearised system model. In the case of nonlinear process dynamics, the choice of model is situation specific. The dissolved oxygen process within the treatment plant, and indeed the urban wastewater system, can be seen to be dependent upon the air flow rate into the treatment plant, and relatively decoupled from other variables. In this case, assumptions can be made in the nonlinear modelling of the process.
- The complexity required: An ASM2d model can represent up to 12 processes, within 19 concentrations as state variables. The common application of automation is usu-

ally limited to dissolved oxygen concentration and nutrient removal, thus allowing reduction in the complexity and size of the process model for control purposes.

- The accuracy required: Variability in treatment plant kinetic parameters between plants, and issues in identification of biokinetic models [2] can result in considerable modelling inaccuracy (typical activated sludge models assume temperature and kinetic parameters to be constant).

A distinction is made here between the models required for control law design (control models), and those produced for the investigation of process behaviour (simulation models). The former may however be determined from the latter through analysis of the physical laws of the system or through data driven methods. Both the 'urban wastewater system' and the 'treatment plant system' as considered within this thesis are simulation models within Matlab/Simulink, based upon complex mathematical formulations of process behaviour and include the use of information based on physical systems. Scaled models such as that of Nejjari et al. [114] and Graells and Katebi [56] utilised within the thesis allow for design of more complex nonlinear control strategies for small-scale plant models.

1.7 Thesis Summary

The remainder of the thesis is organised in the manner presented below to detail the urban wastewater system, the modelling and control approaches required, and the implementation of nonlinear predictive control to the process.

Chapter 2 Model Based Predictive Control:

A description of the linear predictive control algorithms utilised within the thesis is given, introducing the theoretical aspects of predictive control, in addition to its historical background. The assumed model structure in the initial predictive control algorithms detailed in this chapter is that of a linear time invariant (LTI) process model, identified via subspace identification, with possible unmeasured or measured disturbance modelling. Thus the structure for model based predictions of future behaviour is developed, with control optimisation in the presence of system constraints.

Chapter 3 Linear Predictive Control of a Wastewater Plant

For applications of control of the treatment plant, commonly linear control methods are sufficient. Traditionally, low level control such as PI has been popular. This chapter seeks to describe the application of the above control methodology to the control of two treatment plant processes, utilising the existing single loop control schemes. Multivariable model predictive control for these processes is compared to the original control strategy. A benchmark model is used in this implementation, to test the validity of the control designs developed. The issues concerning treatment plant based control are discussed, in particular with respect to effects upon receiving waters.

Chapter 4 Urban Wastewater Treatment Model

An introduction is given to urban water systems, discussing two urban wastewater process models. The urban wastewater system is considered as composed of: a sewer system, a wastewater treatment plant and a receiving waters. For control purposes, it is necessary to formulate these nonlinear models in a form suitable for the model predictive control structure. The transformation of these system models to the state dependent representation is shown.

Chapter 5 Nonlinear Predictive Control

An historical and theoretical background to nonlinear predictive control is detailed within this chapter. Nonlinear approaches to wastewater treatment control have traditionally been uncommon, however the extension of local and international regulations to the receiving waters has motivated the development of nonlinear strategies for the control of the wastewater process. Fuzzy gain-scheduled, Wiener model based and state dependent control algorithms are described throughout this chapter, demonstrating the model based control approaches for linear time invariant and linear time varying (LTV) model structures. A state dependent nonlinear model is used in state estimation via the use of a Kalman filter.

Chapter 6 Nonlinear Predictive Control of Wastewater Systems

A comparison of control performance and discussion of the approaches is given with the aim of demonstrating the design and application of nonlinear control for the urban wastewater system. The linear predictive control is extended via fuzzy gain-scheduling, and an approximate nonlinear modelling approach is demonstrated with the use of the Wiener modelling structure. State dependent models developed in Chapter 4 for the purpose of nonlinear predictive control are applied within the scaled urban wastewater system. The nonlinear state dependent control of the ASM2d reaction tank model utilised in the urban wastewater model is also described.

1.8 Contributions

The main contributions of the thesis can be summarised by the following

- Development of 'control models' for wastewater treatment processes

  - A novel approach to modelling for control purposes in the wastewater system was produced, in the transformation of several models to the state dependent structure. The state dependent coefficient representation of the activated sludge model no. 2d (ASM2d) of an aerobic reactor was produced, with the inclusion of a feedforward model of measured variables.

  - The state dependent coefficient representation of a reduced treatment plant model was derived, and subsequently included in the state dependent description of a related urban wastewater system (UWS) model. The control requirements for the UWS resulted in the modification of the latter SDC representation for the inclusion of a feedforward model.

  - A multivariable linear model of dissolved oxygen and nitrate/nitrite dynamics in the wastewater treatment plant was derived. SISO linear models of the dissolved oxygen and ammonia dynamics in the UWS was produced including the description of the upstream measured disturbances.

  - The Wiener model description of the dissolved oxygen process, considering the linear dynamics of the wastewater treatment plant and the nonlinear effects of the urban wastewater system, was produced to describe the effects of storm conditions upon the dissolved oxygen process.

- Demonstration of model based control strategies suitable for the wastewater treatment industry:

  - The proposal of model based control strategies suitable for the control of water quality in the urban wastewater system was given with the use of commonly available process measurements, such as dissolved oxygen and ammonia. In particular, the knowledge of influent flow levels to the treatment plant was considered as a method of prediction of water quality in the receiving waters.

  - Multiple Input/Multiple Output (MIMO) model predictive control of dissolved oxygen concentration and nitrate/nitrite within the treatment plant system under varying treatment plant influents was demonstrated and discussed.

  - Nonlinear control strategies for the urban wastewater system were developed for: linear model predictive control (MPC), fuzzy gain-scheduled control, Wiener model based predictive control (WMPC) and nonlinear predictive control based on the state dependent coefficient representation.

  - A comparison of various control strategies (Wiener MPC, Fuzzy gain-scheduled control, Linear MPC and PID control) was demonstrated and discussed in the case of dissolved oxygen control in the urban wastewater system.

- The development of model based predictive control algorithms for the above control purposes:

  - The extension of the linear predictive control algorithms was developed for the case of the constrained nonlinear model predictive control, based on the Wiener model.

  - The modification of the nonlinear predictive control algorithm based upon the state dependent representation for feedforward compensation.

- The development of Matlab based software for model based predictive control for

  - linear constrained/unconstrained predictive control, with measured and unmeasured disturbance models.

  - nonlinear model predictive control, based upon the Wiener model.

- nonlinear model predictive control, based upon the SDC linear time varying model.

- Fuzzy gain-scheduled control based upon the linear MPC software.

The research presented in this thesis has resulted in the following publications:

Conference Publications:

O' Brien, M., Pinto Castillo, S.E., Katebi, R. (2005) 'Gain Scheduled Model Based Predictive Control for Wastewater Applications'. in Proceedings of the IFAC World Congress, Prague, July 2005.

O' Brien, M., Camilleri, F., Ordys, A., (2005), 'Gain Scheduled Predictive Control for Integrated Wastewater systems in storm conditions', in Proceedings of 10ICUD, 10th International Conference on Urban Drainage, Copenhagen, Denmark.

O' Brien, M., Camilleri, F., Ordys, A., (2005), 'Predictive Fuzzy Control of Ammonia: Integrated Urban Wastewater Treatment , in Proceedings of 3rd CIWEM National Conference, Wakefield, 6th-8th September 2005.

O' Brien, M., L. Giovanini, A. Ordys. R. Katebi, Wiener modeling and predictive control for wastewater applications, in Proceedings of the International Mediteranean Modeling Conference, European Modelling and Simulation Symposium, October 20-22, Marseille, France

O' Brien, M., Ordys, A.W., (2006), Nonlinear Predictive Control of a Wastewater Model, accepted to International Conference, Control 2006, Glasgow, August 2006

Journal Publications:

M. O'Brien, A. Ordys, 'Model predictive control strategies for dissolved oxygen levels in receiving waters', submitted to the Journal of Hydroinformatics, April 2006

Balbis, L., O' Brien, M., Benazzi, F., Ordys, A.W., Katebi, R., 'Comparison of Nonlinear Predictive Control methods for Wastewater Applications', submitted to the Transactions of the Institute of Measurement and Control, June 2006

O' Brien, M., L. Giovanini, A. Ordys., 'Modelling Techniques for Predictive Control of Wastewater Applications', submitted to the Journal of Process Control, April 2006

Reports and Internal Publications:

"River water quality and river basin models", Marie O'Brien, Joanna Boguniewicz, Andrea Capodaglio, System Process Modeling Report, WWTSysEng Group November 2003

"Integrated Wastewater System Model: Case Study", M. O' Brien, January 2005, Case Study Report, WWTSysEng Group

"Predictive Control of a Simple Waste Water Plant", September 2003, Marie O' Brien, J. Wilkie, R.Katebi, 2nd Workshop of WWT&SYSENG Network, 17-20 September

"Predictive Control for Integrated Wastewater Systems", September 2004, Marie O' Brien, R. Katebi, 3rd Workshop of WWT&SYSENG Network

"Predictive Control Applications for Integrated Wastewater Treatment". M. O' Brien, F. Camilleri, R.Katebi, A. Ordys, 4th WWT & SYENG Workshop, Crete, 20-25 Aug 2005

Chapter 2

Model Based Predictive Control

2.1 Introduction

The on-line dynamic optimisation of control actions, based on predicted plant dynamics determined from an inherent process model, is defined as model based predictive control. The basis of this approach (as shown in Figure 2-1 below) is the prediction of future behaviour of the plant, starting at current time k, over the period defined as the 'prediction horizon'. The control action over a user-defined 'control horizon' is chosen as an optimised control sequence designed to produce the best predicted behaviour, according to the process model, in order to reach the required trajectory. Predictive control is one of the most widely used advanced forms of control in industry, particularly in the process industries. The popularity of this control approach results from several advantages offered by a model based control strategy. The model based technique is appropriate to the industrial requirements of nonlinear control within process boundaries for multivariable systems. Constraints are easily handled in the formulation of the optimisation sequence, and the model based strategy allows for the control of several variables, without modifications of the control algorithm.

The most significant advantage of predictive control is that it provides, within its architecture, for the inclusion of process constraints and so allows for operation in a more efficient manner. Constraint handling allows control to be implemented whilst avoiding operation in more extreme process regions, thus avoiding wear and tear. Constraints can increase the accuracy of the model being used, since actuator and plant limits can be incorporated into

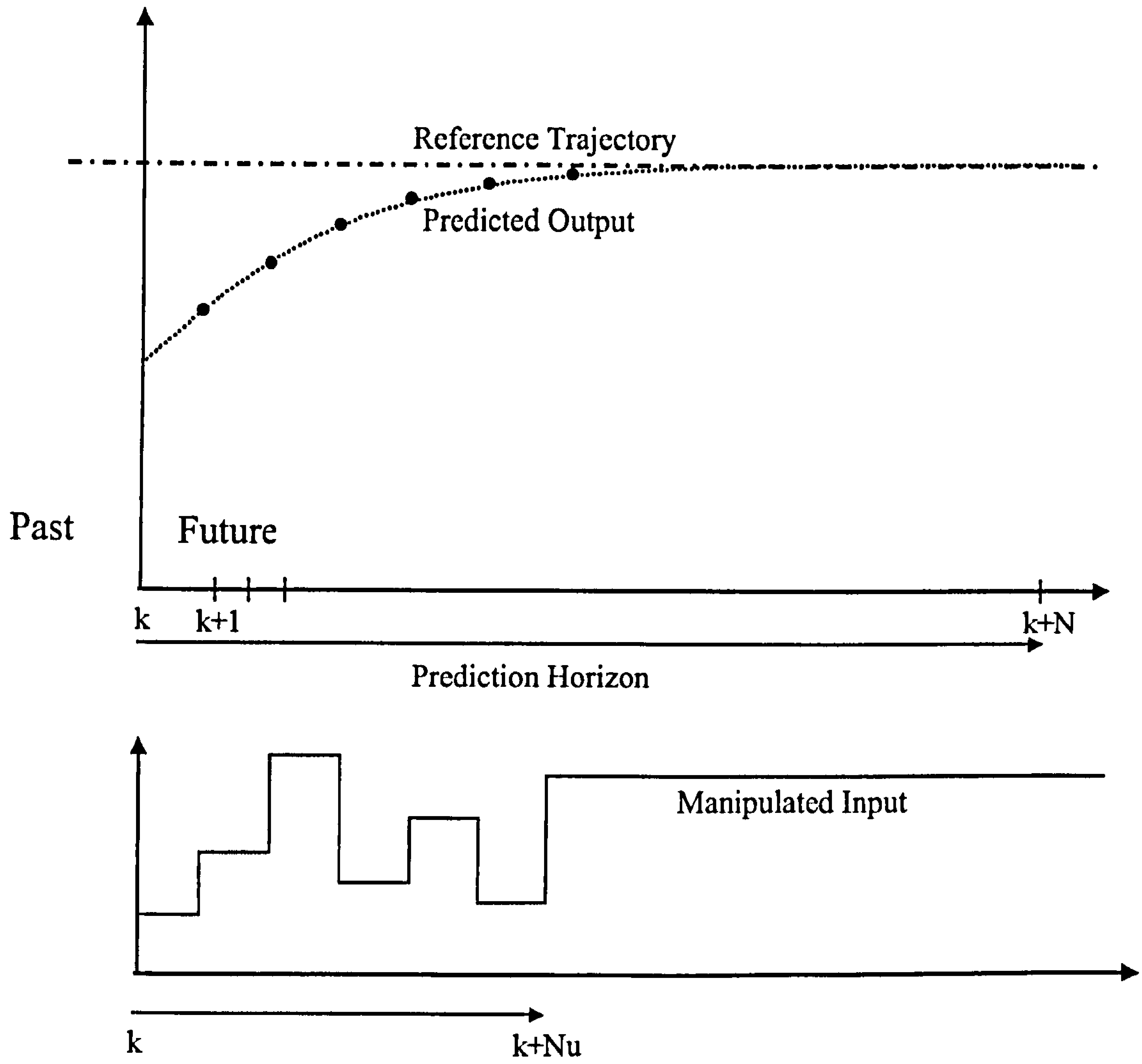

Figure 2-1: Prediction of Future Behaviour and Control Sequence from Time k

the model, an issue not considered in traditional structures such as proportional-integral (PI) control.

A sufficiently accurate process model must be determined for a model based control scheme, since the choice of process model is of most significance. State estimation, output prediction and control optimisation inherently depend upon an accurate system model. The feedback of process behaviour is obtained via the state estimation, which is based on comparison of the process model behaviour with actual measured plant output, for the given control input. The model based structure however allows for compensation in the case of modelling discrepancies such as plant-model mismatch, via the use of unmeasured disturbance modelling. Feedforward control, such as the inclusion of measured disturbances, is also accommodated, through the model-based aspect of the control.

A particularly significant advantage for larger process models is ease of application for multivariable control. In the case of multivariable control, the control methods used are the same as those used in control of single variable processes; there are no controller architecture changes required for the inclusion of multiple controlled processes. This, coupled with the intuitive nature of the controller tuning, and the advantages shown above of constraint handling and feedforward control, has resulted in the popularity of the model predictive control approach.

2.2 MPC Historical Background

Predictive control itself came from several different sources independently. All of these proposed structures had various similar elements: use of a system model in the design, the use of receding horizons, and the calculation of the control signal based on the predicted behaviour of the plant. Predictive control was designed originally for power plants and petroleum refineries, but can be found in various other areas including the chemical industry, food processing and the automotive/aerospace industry.

MPC was developed in earnest for industrial applications in the 1970s, but existed in its basic form prior to this. Researchers had described many forms of open loop optimal control, touching on the idea of receding horizon control, upon which MPC is based, for example the research presented by Lee and Markus [89]. Various other academic contributions were made in the area of receding horizon, internal model control and predicted plant behaviour, further into the 1970s, by Kleinman [79], Kwon and Pearson [86], and Rouhani and Mehra [139]. However, since model predictive control was originally popular in industrial applications, the significant developments were mostly produced in practice. A source of the modern form of predictive control was developed by Richalet et al. [135] of the French company Adersa, who proposed a form known as Model Predictive Heuristic Control (MPHC) in the product IDCOM (Identification and Command). Another proposed predictive control at that time was Dynamic Matrix Control (DMC) which originated with Cutler and Ramaker [33]. The DMC algorithm, more so than MPHC, concentrated on constraint handling (one of the most important aspects of predictive control). The first patent for this form of control was given to Sanchez [148] for a method under the name "Adaptive Predictive Control", exploiting an internal model to implement adaptive control.

The generalised predictive control approach was introduced in the late 80s, demonstrated by Clarke et al. ([28], [29]), which extended the process model structure to the use of ARMAX (Auto Regressive Moving Average Exogenous), allowing greater generality in the system model. A stable form of this algorithm was developed by Kouvaritakis et al. [82], and a continuous time based on constrained state space models was produced by Demiricioglu [36]. The discrete time state space format of this algorithm was presented by Ordys and Clarke [124]. In 1995, Chow et al. [26] proposed a gain-scheduled predictive control. Uncertainty involved in modelling of the wastewater process could require the application of a robust control strategy. Traditional robust control theory requires that the controller be linear, which is the case for the unconstrained predictive control approach. However, research by Lee and Kouvaritakis [90] demonstrated robust control for systems with input saturation, and Mayne and,Michalska [104] demonstrated robust predictive control for nonlinear systems.

The various approaches to model predictive control have a generic structure, as demonstrated in Figure 2-2 below. A state estimator allows the use of a plant knowledge. The system inputs u and plant measurements y allows the user to arrive at a state estimate $\hat{x}$. With this knowledge, the prediction algorithms detailed in the subsequent section allow the approximation of future behaviour of the process. The optimiser therefore computes future control moves according to this predicted behaviour such that the system approaches the reference (or setpoint) defined by the user. This optimisation structure must allow for the control actions to take into account plant constraints. The different approaches to predictive control, particularly commercially, may differ on some of the above details, particularly the handling of control optimisation and system constraints. A most basic part of the model based control approach is the use of a dynamic model in the control design. The conventional model used for prediction and calculation of control actions is restricted to represention of linear dynamics of the system. The structure demonstrated below however also allows the use of a representation of system nonlinearities in building accurate predictions of future behaviour.

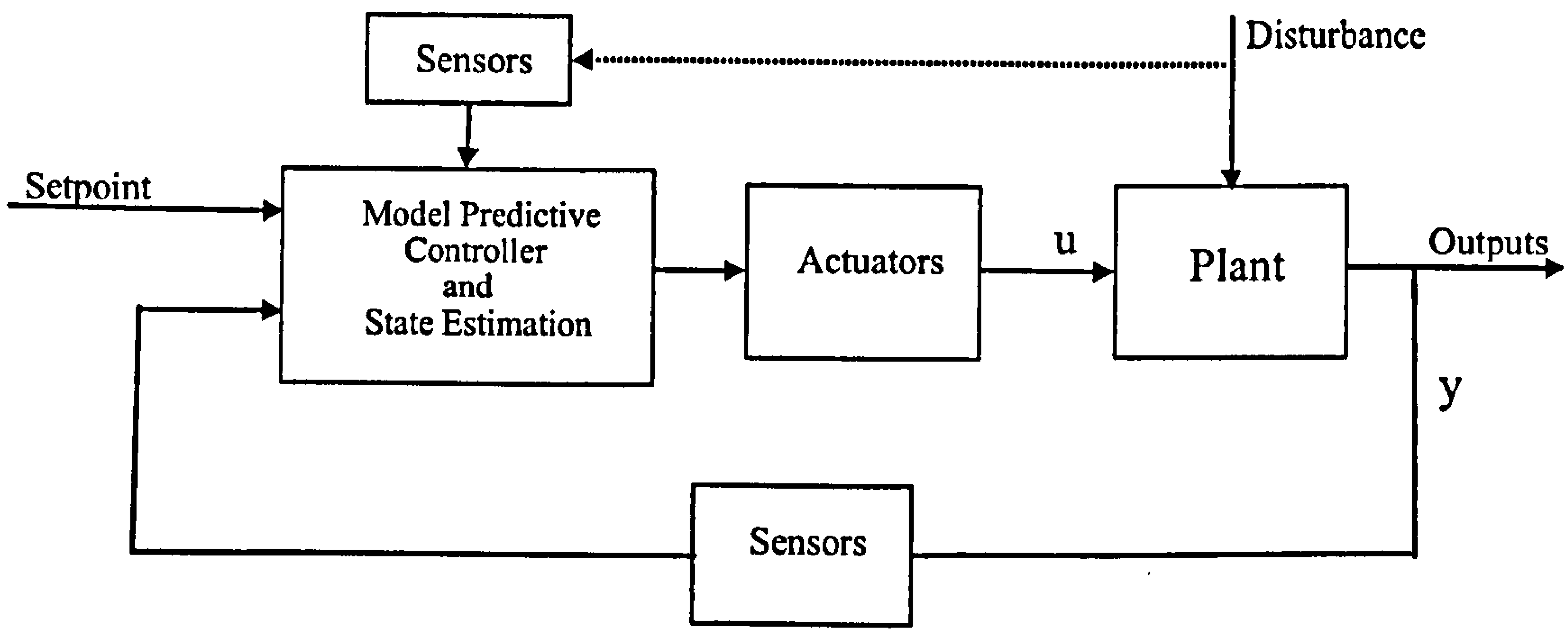

Figure 2-2: Architecture for Model Predictive Control with Measured Disturbances

The choice of model structure must have the characteristics defined by Clarke [27] as: a sufficiently accurate representation of the essential dynamics of the plant, provide the free and forced predictions of system behaviour for control use, in addition to allowing theoretical predictions of system behaviour and must be intuitive for use by plant operators.

2.3 Linear Predictive Control

From physical laws, mathematical representations of the dynamics and interactions of the process, suitable control methods can be found. The accuracy of these models is an attractive aspect, allowing for stricter regulation of the process. Amongst the issues that complex dynamic models such as these raise are controllability, observability and significant increases in control optimisation time. Alternatively then, suitable models may be identified directly from the measured experimental data of the process. These data based models are particularly useful where no physical model of the system is available.

Whilst, in both the theoretical and practical sense, the most common approach for control purposes is the use of linear modelling methods, few truly linear processes occur in nature. Despite this, linear model based control methods are significantly more popular than their nonlinear counterparts. The plant in reality behaves in a nonlinear manner, evolving in a fashion that can usually be described by the form of the equation:

$$\frac{dX}{dt} = f(X, U, t) \tag{2.1}$$

where X is defined as the state vector, U is defined as the vector of inputs to the system, with the system evolving in relation to time t. Linear MPC can be implemented to a nonlinear system effectively and has been demonstrated thoroughly in industrial applications. In the particular area of wastewater control, a practical application of a linear MPC algorithm was detailed by Sanchez et al. [145] for the control of dissolved oxygen concentrations in a wastewater treatment plant. The nonlinear process may be simplified around its operating equilibrium, simplifying to the popular state space form of a linear model shown.

2.3.1 Linear System Representation

The form most commonly used, for linear predictive control, is a linearised discrete time model in the state space form. Thus the model variables are defined to be the input (or indeed inputs) to the system $u(k) \in R^l$, the state vector $x(k) \in R^n$ and the output vector $y(k) \in R^m$. The standard state space representation is therefore simply as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{2.2}$$

where the dimensions of the state space matrices are defined as $A \in R^{nxn}$, $B \in R^{nxl}$, $C \in R^{mxn}$ and $D \in R^{mxl}$. The form of model implemented in the work presented in this thesis expresses the plant dynamics with an inherent integral effect. This is a convenient form for predictive control, which itself computes the optimal value of the control increment Δu. There are a number of ways of including this integration within the system model. The chosen method here however is that demonstrated in the work by [85], the augmentation of the state vector to include the vector of previous inputs as follows:

$$u(k) = u(k-1) + \Delta u(k) \tag{2.3}$$

and so that the discrete linear model can be considered as

$$\begin{aligned} x(k+1) &= Ax(k) + B(u(k-1) + \Delta u(k)) \\ y(k) &= Cx(k) \end{aligned} \tag{2.4}$$

Thus, the system can be equivalently defined in the following form:

$$\begin{aligned} \chi(k+1) &= \tilde{A}\chi(k) + \tilde{B}\Delta u(k) \\ y(k) &= \tilde{C}\chi(k) \end{aligned} \tag{2.5}$$

where the new state vector is defined as $\chi(k) = \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix}$, whilst the state space matrices of $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ are defined as

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} A & B \\ O & I \end{bmatrix} \tilde{B} = \begin{bmatrix} B \\ I \end{bmatrix} \\ \tilde{C} &= \begin{bmatrix} C & 0 \end{bmatrix} \end{aligned} \tag{2.6}$$

The process model equations of 2.6 are equivalent to the original state space model. The above models consider only linearised plant behaviour. The common issue for linearised models is a limited range of accuracy and validity, around the equilibrium point of the process. Once the system moves from the range of this linear model, the control may become ineffective. Several approaches may suffice in the application of linear MPC to a considerably nonlinear process. The uncertainties involved in the linear modelling process could be defined to be combined within a noise signal, a disturbance state defined as an 'unmeasured disturbance'. This disturbance state is implemented with the intention of removing the steady state offset in the control actions, introduced by plant-model mismatch.

It also introduces the ability of the MPC scheme to reject disturbances. This form of control is considered in Chapter 3 in the control of a wastewater treatment process.

The linear representation can be insufficient in describing the dynamics of the nonlinear process. Hence, some method of modelling and controlling a nonlinear system is needed. Sommer [163] states that many industrial processes have relatively linear dynamic behaviour, though not all can be approximated by a linear system description. For this reason, Sommer states that in some cases an approximated nonlinear representation can lead to better results than a linear approximation. There exists a large array of methods in the nonlinear modelling of a process, the descriptions detailed in Sommer [163] demonstrate the use of Hammerstein, Volterra and bilinear modelling techniques, and conclude that approximated nonlinear models decrease computation expense. The linear approach for which predictive control is developed can be easily extended by the use of multiple linear approximations to the nonlinear process over the operating range. The linearisations are determined off-line, thus retaining the efficiency and structure of the traditional linear MPC, whilst gaining the advantages of a nonlinear control scheme. The 'gain-scheduled' form of this approach results in a number of linear predictive controllers, scheduled for use over the nonlinear range appropriate to their equilibrium linearisation point. The 'Wiener' form of this approach results in a single linear description of the system dynamics, varying in magnitude with respect to the operating point. These two methods are demonstrated in Chapter 6 in the control of a nonlinear urban wastewater system.

On-line successive linearisation of the process can lead to further more accurate system models for control purposes. A similar method utilises the nonlinear process model, assumed constant at that sample instant, updated with predicted behaviour of the system, resulting in a time-varying model. This method retains the accuracy of the mathematical physical system description, whilst also maintaining the structure of the linear predictive control algorithm. This approach is detailed in Chapter 6 in the state dependent nonlinear predictive control of a wastewater treatment plant model, and additionally in control of the urban wastewater system.

2.3.2 Linear Model Based Predictions

In order to solve the MPC algorithm, a method must be developed for the computation of the predicted behaviour of the variables available for control. In the development of the predictive aspect of the MPC, the initial assumption is that of no disturbances, and a fully measured state vector. Later, the addition of disturbance modelling and state estimation will be introduced. The prediction equation is that demonstrated by [85], developed through iteration of the discrete linear form of the system equations in the state space. The state vector at time j=1 is defined above as:

$$\chi(k+1) = \tilde{A}\chi(k) + \tilde{B}\Delta u(k) \tag{2.7}$$

Thus iterating for the next sample instant, $j = 2$:

$$\begin{aligned} \chi(k+2) &= \tilde{A}\chi(k+1) + \tilde{B}\Delta u(k+1) \\ &= \tilde{A}^2\chi(k) + \tilde{A}\tilde{B}\Delta u(k) + \tilde{B}\Delta u(k+1) \end{aligned} \tag{2.8}$$

Iterating as above for each time instant, the predicted state at time j is defined by the following

$$\chi(k+j) = \tilde{A}^j\chi(k) + \sum_{i=0}^{j-1} \tilde{A}^{j-i-1}\tilde{B}\Delta u(k+i) \tag{2.9}$$

so that the predicted output is described by the subsequent equation for a j-step ahead predictor:

$$y(k+j) = \tilde{C}\tilde{A}^j\chi(k) + \sum_{i=0}^{j-1} \tilde{C}\tilde{A}^{j-i-1}\tilde{B}\Delta u(k+i) \tag{2.10}$$

The format of controller predictions used therefore in this thesis can be represented by the equation for the predicted output vector

$$Y(k) = F\chi(k) + H\Delta U(k) \tag{2.11}$$

where the model is iterated over a horizon known as the prediction horizon, H_p. The output vector is defined as $Y(k) = \begin{bmatrix} y(k) & y(k+1) & \dots & y(k+H_p-1) & y(k+H_p) \end{bmatrix}^T$. Since $\Delta u(k+j) = 0$ for $j \succeq Hu$, the control increment vector can therefore be defined as $\Delta U(k) = \begin{bmatrix} \Delta u(k) & \Delta u(k+1) & \dots & \Delta u(k+H_u-1) \end{bmatrix}^T$. So, in the equation above 2.11, the matrix F is found to be of dimensions $F \in R^{mH_pxn}$ and of the form, constructed from the output predictions:

$$F = \tilde{C} \begin{bmatrix} \tilde{A} \\ \vdots \\ \tilde{A}^{H_p} \end{bmatrix} \tag{2.12}$$

The matrix F is defined as the free response matrix, describing the predicted output if the control input to the system were to remain constant in the future (i.e. $\Delta u = 0$). That is, the free response F depends only upon the past. The matrix H is defined as the forced response of the system to the effect of all future control increments, and is defined by the prediction vector to be of dimensions $H \in R^{mH_pxl}$ and of the form

$$H = \tilde{C} \begin{bmatrix} \tilde{B} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \tilde{A}^{Hp-1}\tilde{B} & \cdots & \tilde{A}^{Hp-H_u}\tilde{B} \end{bmatrix} \tag{2.13}$$

where H_u is defined as the control horizon, after which the control applied to the plant is assumed constant, that is as above $\Delta u(k+j) = 0$ for $j \succeq Hu$.. The above structure demonstrates that the predicted output vector for the system is a function of the states and the changes in input.

2.3.3 Cost function and Optimisation

The control input of the system (i.e. the output of the controller) can be found by minimising the cost function of the system. The optimal control input can be found by determining the optimal value for Δu, where the single input single output (SISO) form of the cost function is

$$J = \sum_{j=H_w}^{Hp} [w(k+j) - y(k+j)]^2_{Q(j)} + \sum_{j=0}^{H_u-1} [\Delta u(k+j)]^2_{\lambda(j)} \tag{2.14}$$

where H_w is the lower cost horizon. The first term of this cost function calculates the square of the future error in the setpoint tracking, which requires knowledge of the future setpoint w. Typically, without knowledge of the setpoint trajectory in the future, the assumption is made that the variable w remains constant over the prediction horizon. The weighting factors $\lambda(j)$ and $Q(j)$ determine the importance of the two cost terms of control increments and tracking error respectively and are defined for that time j. Normalisation may be required to compute optimal control that is independent of input and output units, this may be performed via the scaling of weighting factors. The above cost function can be extended to the multivariable case by the following MIMO form, using vector notation.

$$J = (W - Y)^T Q(W - Y) + \Delta U^T \lambda \Delta U \tag{2.15}$$

where the vectors W, Y and ΔU are defined as $W = [w(k + H_w)..w(k + H_p)]^T$, $Y = [y(k + H_w)..y(k + H_p)]^T$ and $\Delta U = [\Delta u(k)....\Delta u(k + H_u - 1)]^T$ respectively. The weighting matrices Q and λ are defined by

$$Q = \begin{bmatrix} Q(H_w) & 0 & \cdots & 0 \\ 0 & Q(H_w+1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q(H_p) \end{bmatrix} \tag{2.16}$$

$$\lambda = \begin{bmatrix} \lambda(0) & 0 & \cdots & 0 \\ 0 & \lambda(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda(H_u) \end{bmatrix} \tag{2.17}$$

The vector of the free response of the system is defined as $f = Fx(k)$. The above cost function equation can be rewritten with the substitution of the equation $Y = f + H\Delta U$ to the tracking error:

$$\varepsilon = W - Y = W - f - H\Delta U \tag{2.18}$$

so that the cost function J is written as

$$\begin{aligned} J &= (W-f-H\Delta U)^T Q (W-f-H\Delta U) + \Delta U^T \lambda \Delta U \\ &= (W-f)^T Q (W-f) - H\Delta U.Q(W-f) - \\ &\quad H\Delta U.Q(W-f) + H\Delta U^T Q H \Delta U + \Delta U^T \lambda \Delta U \\ &= \Delta U^T [H^T Q H + \lambda I]\Delta U - 2(W-f)^T Q H \Delta U + (W-f)^T Q (W-f) \end{aligned} \tag{2.19}$$

yielding a quadratic minimisation problem, for which a solution is:

$$\Delta u = [H^T Q H + \lambda I]^{-1} Q H^T (W-f) \tag{2.20}$$

This minimisation problem can alternatively be performed online as a quadratic programming function, particularly in the presence of constraints. The minimisation results in a vector of future control increments, Δu, of length H_u, of which only the first element is needed. The only section of this vector being used in the control action applied is that pertaining to the next step in the control horizon, so that the control applied to the plant is:

$$u(k) = u(k-1) + K(W-f) \tag{2.21}$$

where $K = [I_l, O_l \cdots O_l][H^T Q H + \lambda I]^{-1} Q H^T$, where I_l is the lxl identity matrix and O_l is the lxl zero matrix.

2.3.4 System Modelling

A brief description of the modelling tools utilised is presented in the following section. The nonlinear models used can be impractical for control purposes. However, within certain ranges and for particular processes linear models are sufficient. The main method of identification utilised in this thesis is the approach of subspace identification, utilising data of the excited process. The scope of this thesis concentrates predominantly on the structure of model used, rather than the identification method. For this reason, the subspace identification method is briefly described here. The identification of parameters and system kinetics is not considered within the scope of the research presented, instead concentrating on practical models for process control. The detailed application of subspace identification to activated sludge systems was demonstrated by the research of Sotomayor et al. ([164],[165]). Various other process-specific identification methods such as the modelling of the dissolved oxygen process via on-line updates of the oxygen mass transfer function have been demonstrated, for example the research presented by Suescun and Ayesa [171].

Subspace Identification

The aim of using subspace identification here is to construct linear multivariable models in the state space form based on available input-output measurement data from the non-linear model. The identification of multivariable systems is of benefit here as this allows simultaneous control of several parameters (see for example the multivariable control of the COST benchmark system, in Chapter 3), or for use in modelling measured disturbances, as seen in Chapter 6 in the control of the urban wastewater network. Multivariable applications of subspace identification has been demonstrated for industrial applications, for example in Bastogne et al. [10].

The subspace algorithm used in this thesis is that developed by VanOverschee et al. [180], i.e. the SUBID algorithm within the Matlab platform. It is not the intention of this section to document the history of identification techniques, or the background theory for this algorithm, but to briefly explain the methodology involved in the use of the SUBID identification algorithm. The subspace method has been used previously with predictive control, for example Kadalia et al.[74] and Jia et al. [70] and was presented in the tuning of PID algorithms for wastewater treatment by Sanchez et al. [146]. Dorsey and Lee [39] demonstrated the use of the subspace identification methods in the online prediction of process behaviour. Lindberg [94] applied subspace identification on an ASM-based treatment plant model, for the multivariable control in a predenitrification plant (of the ammonia and nitrate processes).

The assumption is that with a sufficiently large set of data for the input and output of the system, preferably excited by a set of random signals (in this case Pseudo Random Binary Sequence (PRBS)), that the system should be describable by a state space form. The discrete-time subspace identification methods are defined as "the approximation of subspaces generated by the rows or columns of block-Hankel matrices of the input-output data, to calculate a reliable discrete-time statespace model" [127]. The form of the system identified by the subspace identification algorithm utilised is as shown below, where $u(k)$ and $y(k)$ are the input and output of the system, where the pair (A,B) are controllable and the pair (A,C) are observable

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w(k) \\ y(k) &= Cx(k) + Du(k) + v(k) \end{aligned} \tag{2.22}$$

with the state space matrices required to be both observable and controllable (by the tests shown in later sections). The signal $w(k)$ here is unobserved noise on the states and similarly for the output noise $v(k)$. The order of the model is estimated by a Singular Value Decomposition. The choice of input for excitation of the process is of most significance, depending heavily upon the typical characteristics of the signal (magnitude, mean value, discrete time step). These restrictions upon this input are the main source of the modelling error between the linearised model and the nonlinear plant, as the inputs are tailored for a particular operating state. The model is therefore valid only in the region of this operating point, and deviations from this region results in inaccuracy. In the application of this technique of identification, there exists a user decision of the trade-off between accuracy and reduction of the order of process model, resulting in the possible multiple applications of the algorithm and selection of best performance relevant to the process control.

The subspace identification approach is restricted in the case of this thesis to the modelling of multivariable processes for control purposes, consisting of model inputs of manipulated variables, and model outputs of controlled variables. Unmeasured and unestimated inputs are not considered, whilst uncontrolled processes are simply not modelled. Differences between the process and the modelled system are defined as 'plant-model mismatch' and are considered within the unmeasured disturbance handling. Any further input variables, such as measured disturbances, would be also be identified. The use of this approach within the research presented in this thesis is motivated by the need to reduce the complexity of the model, exploiting the optimisation speed, simplicity, linear nature and other advantages of a simplified model

2.4 Disturbance Modelling

The use of a disturbance model is split into two basic approaches: measured and unmeasured. The former describes the approach of including disturbances in a form of feedforward control scheme. This method assumes that the disturbance is both measurable and has known dynamics, described by the system model. Clarke [27] defines the need the disturbance modelling within model based predictive control to allow the rejection of disturbances to system performance. The second of these disturbance models, the unmeasured, is considered in the next section.

2.4.1 Unmeasured Disturbance Model

The main advantage of unmeasured disturbance modelling is the provision to the controller of the ability to reject disturbances to the process control. This consequently also allows the controller to take into account differences between the linear model and the actual process, reducing offset error in the control performance. In a practical sense, the concept of the disturbance model implemented in this thesis is that of an unmeasured but unchanging disturbance over the prediction horizon, estimated as the difference between the actual and the estimated output of the system. Thus modelling discrepancies will be included within this constant disturbance model.

The effect of disturbance on the controlled variables is removed by a change in the process model structure and thus the steady state target of the controller. The estimated disturbance state is then used with the original model states to predict outputs over the prediction horizon. This method of a constant output disturbance is a widely-used disturbance modelling approach in industry. A particular consequence of the disturbance model is the fact that the output vector can now no longer be assumed to be equal to the state vector, that is $y(k+j) \neq x(k+j)$, even if the output matrix C is equal to the identity matrix. This results in the requirement of an observer in the estimation of the state vector x, and also in determining the magnitude of the disturbance state.

The traditional MPC approaches of DMC and ID-COM used in industry both implement this form of disturbance handling, under the assumption of a noise free process. Davison and Smith [35] demonstrated the stabilising effects of the constant disturbance model and its benefits in reducing steady state offsets. Muske and Badgwell demonstrated a general state-space disturbance model for input, outputs and states, and presented conditions for which off-set free control can be guaranteed. Muske and Badgwell [109] demonstrated the requirement that the total number of disturbance states be equal to the number of outputs.

In including a constant disturbance model, the process model must be augmented to include the extra disturbance states. Thus a new augmented state vector must be defined, based on the linear state space representation found by the identification above, where $d_m(k)$ is the unmeasured disturbance state:

$$\begin{aligned} \begin{bmatrix} x(k+1) \\ d(k+1) \end{bmatrix} &= \begin{bmatrix} A & O \\ O & I \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \end{bmatrix} + \begin{bmatrix} B \\ O \end{bmatrix} u(k) \\ y(k+1) &= \begin{bmatrix} C & I \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \end{bmatrix} + Du(k) \end{aligned} \tag{2.23}$$

where the state space matrices A, B, C and D are determined via the subspace identification methods above, and 2.23 may be then substituted into 2.5 above, allowing the inclusion of the disturbance state within the predictive controller structure. In the use of a constant disturbance model, only output disturbances are modelled, acting as a constant bias upon the plant feedback. Whilst these disturbances remain constant over the sample period, other forms of disturbance can include ramp and sinusoid models. However, in the case of the fast dynamics of the controlled processes considered in this thesis, it can be seen that the constant disturbance model can be sufficient in the presence of slower effects, such as those due to diurnal variations.

2.4.2 Measured Disturbance

This method of disturbance model allows for two aspects of model based control. The former is the ability to model process dynamics produced by an input other than the specified manipulated variables. The latter is the anticipation of system disturbances and the compensation of such with suitable control actions. This can in some situations be more effective than the feedback method of disturbance rejection, as the latter has an intrinsic delay before the corrective action can be calculated. However, exact compensation for the disturbance would theoretically require an exact model of the measured disturbance transfer function, which in a practical sense is not possible to obtain. Thus the measured disturbance model is used in combination with the unmeasured disturbance model approach detailed above. This method allows the feed-forward control to anticipate the effect of the measured disturbance, whilst the feedback control can be effective in the compensation of model offsets and process disturbances.

In the work presented in this thesis, the modelling of measured disturbances was used in the control of the urban wastewater system. This allows disturbances such as the downriver effects of sewer-to-river overflows to be predicted to a certain extent, and if necessary, negated by suitable control actions by the manipulated variable. A measured disturbance model was used in this instance to model dynamics that were relevant to the accuracy of the model, but which were not controlled inputs, such as the river dynamics. The model format being used is therefore changed to include a measured disturbance.

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_d d(k) \\ y(k) &= Cx(k) + Du(k) + D_d d(k) \end{aligned} \tag{2.24}$$

where B_d is the modelled effects of disturbance and $d(k)$ is measured disturbance. This structure does not affect the identification procedure, the identified model consists of n

inputs consisting of both manipulated inputs and measured disturbance inputs. It does however result in the following changes to the controller design:

- The disturbance $d(k)$ is assumed constant over the prediction horizon, constructing the vector $D(k)$,
  i.e. $D(k) = \begin{bmatrix} d(k) & d(k+1|k) & \cdots & d(k+H_p-1|k) \end{bmatrix}^T$, where $d(k) = d(k+1|k)$, etcetera.
- The error equation changes to include the disturbance measurement, becoming $\varepsilon = W - f - H\Delta U - D_b D(k)$, where D_b is the disturbance matrix constructed as

$$D_b = \begin{bmatrix} CB_d & D_d & \cdots & \cdots & 0 \\ CAB_d & CB_d & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ CA^{Hp-1}B_d & CA^{Hp-2}B_d & \cdots & CB_d & D_d \end{bmatrix} \tag{2.25}$$

This is implemented in process simulation by replicating the input over the specified period, which in this case is the prediction horizon. Further disturbance models can represent a more accurate model of the disturbance behaviour by predicting the evolution of this variable over the prediction horizon. Bordons and Cueli [16] demonstrated the application of an auto-regressive (AR) model of the measured disturbance to a system, whilst Bodson and Douglas [15] presented the handling of a sinusoidal process disturbance. The constant disturbance model assumed here in the case of UWS control is sufficient, due to the relatively slow dynamics of the river processes.

2.5 Constraint Handling

Alvarez and de Prada [4] stated that a simple method of implementing constraints on control actions can be produced by computing the unconstrained values, and then clipping the signal according to the constraints. However, whilst this is sufficient in the handling of control input saturations, this method did not allow for the modelling of output constraints, and may drive the system to limit cycles when in closed loop. The control method produced by Richalet et al. [135] approaches constraint handling by a switching method, whereby the controller, when the constraints have been violated, switches to an alternate controller whose designed purpose is to move the system back within the constraints, and then revert to original controller. Such constraint handling methods however lead to complexities for multivariable systems. Currently, the popular approach of constraint handling is the definition of the constraints within the control objective itself; formulated within the cost function. When constraints are defined in the objective of a control scheme, the control problem is no longer a simple quadratic minimisation and cannot be simplified to the form of 2.20. Instead, the control objective is formulated as a conditional optimisation, with constraints defined by a set of inequalities. By modelling the constraints as such, the conditional optimisation results in the use of QP minimisation of the cost function of the system, with respect to this set of inequalities. In a general definition, the problem can be seen as:

$$\text{Minimise } J(t), \text{ subject to } v_{low} \leq v(t+j|t) \leq v_{high} \tag{2.26}$$

where the inequality v can be defined to contain the system constraints. For control increment, saturation and output constraints, this can be defined as

$$\begin{aligned} \Delta u_{\min} &\leq \Delta u \leq \Delta u_{\max} \\ u_{\min} &\leq u \leq u_{\max} \\ y_{\min} &\leq y \leq y_{\max} \end{aligned} \tag{2.27}$$

The above control increment, control action and output constraints can be defined in the following vector format

$$\begin{bmatrix} P & -p \end{bmatrix} \begin{bmatrix} \Delta U(k) \\ 1 \end{bmatrix} \leq 0 \tag{2.28}$$

$$\begin{bmatrix} F & \psi \end{bmatrix} \begin{bmatrix} U(k) \\ 1 \end{bmatrix} \leq 0$$

$$\begin{bmatrix} \Gamma & g \end{bmatrix} \begin{bmatrix} Y(k) \\ 1 \end{bmatrix} \leq 0$$

The simplest constraint upon the process must be that upon control increments. The constraints upon the control increments can be written as the following inequality, where $U(k) = \begin{bmatrix} u(k) & \cdots & u(k+H_u-1) \end{bmatrix}^T$ as:

$$P\Delta U(k) \leq p \tag{2.29}$$

The saturation constraint upon the control action is perhaps the most commonly applied in constrained control optimisation. The upper bound of the constraint inequalities upon the control vector $U(k)$ above can be defined as follows, where the upper bound remains unchanged over the control horizon:

$$\begin{aligned} u(k) &\leq u_{\max} && (2.30) \\ u(k+1) &\leq u_{\max} \\ &\vdots \\ u(k+H_u-1) &\leq u_{\max} && (2.31) \end{aligned}$$

By substitution as defined in 2.3

$$\begin{aligned} u(k-1)+\Delta u(k) &\leq u_{\max} && (2.32) \\ u(k-1)+\Delta u(k)+\Delta u(k+1) &\leq u_{\max} \\ &\vdots \\ u(k-1)+\Delta u(k)+\ldots+\Delta u(k+H_u-1) &\leq u_{\max} && (2.33) \end{aligned}$$

which when reformulated as follows, where $\Delta U(k) = \begin{bmatrix} \Delta u(k) & \cdots & \Delta u(k+H_u-1) \end{bmatrix}^T$ is:

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \Delta U(k) \leq u_{\text{max}} - u(k-1) \tag{2.34}$$

$$\begin{bmatrix} 1 & 1 & \cdots & 0 \end{bmatrix} \Delta U(k) \leq u_{\text{max}} - u(k-1)$$

$$\vdots$$

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} \Delta U(k) \leq u_{\text{max}} - u(k-1) \tag{2.35}$$

constructing on the left hand side of the inequality a lower triangular matrix of dimensions $R^{H_u x H_u}$

$$\mathcal{F} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 1 & 1 & \cdots & \cdots & 1 \end{bmatrix} \tag{2.36}$$

The above input constraints can be defined by the following equation therefore:

$$\mathcal{F} \Delta U(k) \leq -\mathcal{F}_1 u(k-1) + \psi \tag{2.37}$$

where $\mathcal{F} = \begin{bmatrix} \mathcal{F}_1 & \ldots & \mathcal{F}_{H_u} \end{bmatrix}$ and $\psi = \begin{bmatrix} u_{\text{max}} & \ldots & u_{\text{max}} \end{bmatrix}^T$ is of dimensions $R^{H_u x 1}$. The lower bounds of the input constraints are defined similarly, constructing the format of equation 2.28. Finally, the output constraints as defined by the following

$$y(k) \leq y_{\text{max}} \tag{2.38}$$

$$y_{\text{min}} \leq y(k)$$

Focussing upon the upper output bounds, this can be reformulated as follows

$$Fx(k) + H\Delta u(k) \leq y_{\text{max}} \tag{2.39}$$

which becomes

$$H\Delta u(k) \leq y_{\max} - Fx(k) \tag{2.40}$$

and thus can be rearranged as

$$\Gamma H\Delta u(k) \leq g - \Gamma Fx(k) \tag{2.41}$$

in the format of equation 2.28. Using the above constraint inequalities the following problem can be minimised online via a quadratic programming minimisation

$$\min_{\Theta} \frac{1}{2}\Theta^T L\Theta + M^T\Theta$$

$$\text{subject to } \Omega\Theta \leq \omega$$

where $L = 2[H^TQH + \lambda I]$ and $M = 2(w - f)^T H$, and where $\Omega\Theta \leq \omega$ can be seen to have the following form

$$\begin{bmatrix} F \\ \Gamma H \\ P \end{bmatrix} \Delta U(k) \leq \begin{bmatrix} -F_1 u(k-1) + \psi \\ -\Gamma Fx(k) + g \\ p \end{bmatrix} \tag{2.42}$$

Even in the case of a linear dynamic system, the presence of system constraints causes nonlinearities to be present, for example in actuator saturation, deadzone or backlash. The assumption of the ideal case that control action can be fully implemented upon the system can lead to degradation of performance or even stability issues. Thus the constraint handling within predictive control allows for prior knowledge of actuator nonlinearities to be included within control optimisation, embedded within the controller cost function. The constraint handling approach detailed in this case assumes an accurate model of the plant. For a more robust implementation of a constrained predictive control strategy, methods such as the dual-mode strategy have been shown to be effective. In this approach, feasible control moves are used to guide the current state to a target set, which satisfies the constraints within a finite period, after which a further control strategy can be used [92]. This can result in conservative control, and thus, methods have been developed by [34] and [101] to improve upon this through the introduction of a parameter dependent Lyapunov function.

2.6 Observer Design

In most control cases, it is not possible to measure the full state vector from the system. In subspace identification and other linearisation techniques, this is especially true, as the states are not physical states, but are instead chosen in the linearisation process to represent the model. The measured values in the system are represented by a combination of these states, and therefore states cannot be measured directly. Thus, an observer (also known as an estimator) must be used to calculate these states from the measured outputs and inputs of the real process. The process model is used to construct this state estimator, including disturbance estimation.

For both methods implemented within this thesis, pole-placement estimation and Kalman filter estimation, the technique itself is similar, differing in the choice of estimator gain. Pole-placement allows the user to define the observer gain (in placement of observer poles), whilst conversely the choice of Kalman filter gain is algorithm-based. The form of observer used is that based on a linear time invariant system model. Particularly, in the case of the state estimation with the use of the nonlinear model, such as that demonstrated in Chapter 6 for Wiener model predictive control, the nonlinear process is assumed, at a given instant, to be represented by a linear model.

The gain matrix of the observer, L , is chosen in order that the observer estimation error converges to zero. The rate of this convergence is determined therefore by the placement of the observer poles in a trade-off between rate of convergence and the limit of computation speed. The observer eigenvalues are commonly placed at a location so that the observer poles are ten times faster than the slowest system pole. The observer gain for the linear system eigenvalues is defined by the matrices, L_x and L_d, for the linearised system model and the disturbance model, respectively, so that the gain matrix above is defined as $L = [L_x \; L_d]$. The equation of the observer can thus be defined as follows

$$\begin{bmatrix} x(k+1) \\ d(k+1) \end{bmatrix} = (A - LC) \begin{bmatrix} x(k) \\ d(k) \end{bmatrix} + (B - LD)u(k) + Ly(k) \tag{2.43}$$

2.7 Summary

This chapter introduced the theoretical aspects of predictive control utilised throughout this thesis. An introduction to predictive control, in addition to its historical background, was detailed. The linear system representation and the identification of the linear model was shown, and thus the structure for model based predictions of future behaviour was demonstrated. The optimisation of system behaviour via the use of a cost function was described, and the use of inequality constraints within this cost function was given. The modelling of measured and unmeasured disturbances was also described within this chapter, with respect to the linear system model. The subsequent chapter details the application of these algorithms to the COST 624 benchmark treatment plant model.

Chapter 3

Linear Predictive Control of a Wastewater Plant

3.1 Introduction to WWTP Control

Model based control can be of particular use in the control of a multivariable constrained process such as that of wastewater treatment. Yuan et al. [192] state that the application of the model predictive control approach is still in its early stages within the wastewater treatment area. Whilst the mathematical model of plant behaviour developed in recent decades is complex, the level of measurement and the number of control handles present do not match this growth in complexity and only a few variables can be accurately measured. Long term planning in wastewater treatment processes, simulating plant behaviour over weeks, can use offline analysis of plant conditions. However, the control of dissolved oxygen and nutrient concentrations such as nitrate/nitrite within the treatment plant requires online measurements to be available.

Controllability of the wastewater treatment processes remains an issue. The restriction upon model choice exists due to the need to match process models to available technology. Typical identification procedures can utilise excitation of the system in the construction of an appropriate model, but are constrained by the lack of online measurements, and control handles by which to excite the process. A model based procedure requires access to online measurements or estimations of required model information. The amount and the quality of data available dictates the level of modelling possible. For this reason, simplified models determined by linear data-based identification techniques, whilst not suitable for advanced state or parameter estimation, can be appropriate for the control objectives of typical wastewater treatment processes.

There are several issues which motivate the application of automation within wastewater treatment. Operation costs involve the manpower expenditure (particularly in plants lacking automatic control), energy costs due to electricity and fuel usage in operation of pumps and motors, tax costs due to regulatory fines, and additionally chemical or carbon dosing costs (where present). The efficient operation of wastewater treatment plants is therefore of significant concern, and application of advanced control techniques can benefit plant performance without a proportional increase in operation costs.

In order to compare differing control strategies efficiently and effectively, a benchmark model is required and in particular a general non-situation specific model can allow control designs to be effectively compared. This model should ideally produce the same results across a number of simulation platforms. This model should also fulfil the practical requirement of efficient computation, though this is of decreasing significance as computation speed of available technology increases. The COST 624/682 research group compiled a general benchmark model [30], the Benchmark Simulation Model no. 1 (BSM1), which could be used in the area of control simulation for optimisation of wastewater treatment, using the ASM1 model developed by Henze et al. [61].

The benchmark is platform independent, in that it gives equivalent results for simulation in Matlab/Simulink (this simualtion platform is used throughout this thesis) as it does in other packages such as SIMBA [160] and STOAT [166].The accompanying basic control schemes of single variable PI loops (for the S_O and S_{NO}) have been shown to be robust for more complex and more realistic hydrodynamics [67]. Control developments for the most part concentrate on improvement in performance of these processes. Alex et al. [3] details the use of the benchmark model for evaluating effectiveness of control strategies.

3.1.1 WWTP Structure

The COST simulation benchmark provides the plant layout, the specific simulation parameters to be used, any model parameters required and provides several different simulation conditions for the plant influent characteristics (constant influent, dry weather, storm weather). The wastewater treatment plant model is comprised of:

- five biological tanks in series (of total volume of approx $6000m^3$), each using the ASM1 model, the first two tanks are unaerated, and the other three tanks are aerated.
- first two tanks (1 & 2) have a volume of $1000m^3$, and are fully mixed, although unaerated.
- tanks 3, 4 & 5 are of volume $1333m^3$, and aeration of these tanks is applied with a maximum value for K_{LA} of 360 d^{-1}.
- one non reactive secondary settler (also with a $6000m^3$ volume),which is based on the settling function by Takacs et al. [173], and has an area of 1500 m^2, depth of 4 m^2. The settler consists of 10 subdivisions, and its feed point is located $2.2m$ from the bottom of the settler.
- two recycles present internally, one nitrate/nitrite recycle from the 5^{th} tank to the first, at a rate of 55338 m^3d^{-1} and one sludge recycle (RAS) from the settler to the front end of the plant at a rate of 18446 m^3d^{-1}.
- an outflow of waste activated sludge (WAS) from the secondary settler at a rate of 385 m^3d^{-1}.

An activated sludge process is one in which organic components within wastewater are removed through biological treatment with the use of organisms within the sludge. Within the above treatment plant benchmark model, the activated sludge process model used is the Activated Sludge Model no. 1 [ASM1] [61], although there are more recent models (ASM2, ASM2d, ASM3) with further modelled processes. The model utilised represents the process conditions which exist at a temperature of 15 degrees Celsius (one of the assumptions required for parameter choices within the model). The ASM1 process model consists of 13 state variables, involving 8 processes. This model includes 3 of the most important

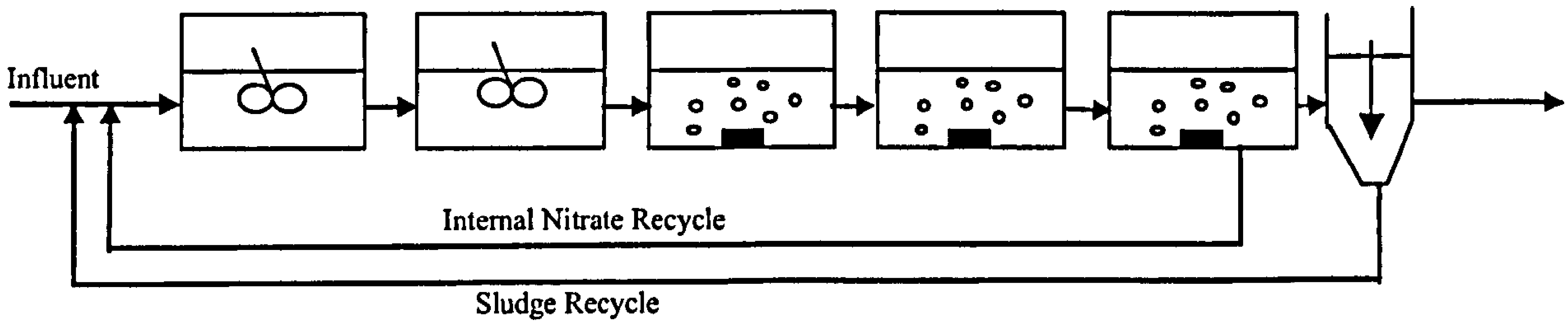

Figure 3-1: BSM1 Structure: Five Biological Tanks: Tanks 1 & 2 Fully Mixed, Tanks 3, 4 & 5 Aerated, Followed by a Non Reactive Secondary Settler

processes in activated sludge: 1) degradation of carbonaceous material 2) nitrification 3) denitrification. The later models (such as the ASM2 and ASM2d) included processes such as phosphorus removal, and the ASM3 adjusted the model again to solve numerical problems. However, for the purposes of dissolved oxygen and nitrate/nitrite control presented, the activated sludge model used is sufficient. In the COST 624 benchmark model as presented in Figure 3-1, each individual tank is represented by an ASM1 model. The aim of the settler is to separate the cleaned water from the biomass, and therefore it is assumed that there is no reaction in the settler, i.e. there are no biological changes, only physical changes and that the sediment settles in this tank due to the effect of gravity. The dynamics of the settler within the COST 624 Benchmark model are defined according to the Takacs approach [173].

The model being used, the COST benchmark, uses the ASM1 (activated sludge model no. 1) by Henze et al.[61]. This model uses 13 state variables and 8 processes. The time is given in days, the flowrate is given in m^3/day and the concentrations are given in g/m^3. The state variables of the system are as follows

- S_S, Readily biodegradable substrate.
- $X_{B,H}$, Active heterotrophic biomass.
- $X_{B,A}$ Active autotrophic biomass.
- X_S, Slowly biodegradable substrate.
- X_I, Particulate inert organic matter.

- X_P, Particulate products arising from biomass decay.

- S_O, Oxygen.

- S_{NO}, Nitrate and nitrite nitrogen.

- S_{NH}, NH_4^+ + NH_3 nitrogen.

- S_I, Soluble inert organic matter.

- S_{ND}, Soluble biodegradable organic nitrogen.

- X_{ND}, Particulate biodegradable organic nitrogen.

- S_{ALK}, Alkalinity.

The influent data defines the following variables: t (time), S_S, $X_{B,H}$, X_S, X_I, S_{NH}, S_I, S_{ND}, X_{ND} and Q (flow), and assumes that the remaining S_O, $X_{B,A}$, X_P and S_{NO} are at an influent level of zero, whilst the remaining component, S_{ALK}, is assumed to be constant at 7 $molm^{-3}$. The influent weather data considered are those of dry weather and storm weather, supplied with the BSM1 simulation package of [30], depicting diurnal variations and weekly trends in the influent data.

3.1.2 Control Structure

Traditional control of the dissolved oxygen process utilizes a feedback measurement of the concentration as the controlled variable and air flow rate as the manipulated variable. However, there exist many interconnected system requirements for control within the wastewater treatment plant, which can be decomposed within a hierarchical scheme into lower-order controlled subsystems. Galluzo et al [50] state that the dissolved oxygen can be regarded as the most important control parameter in an activated sludge process not only with respect to aeration operating costs, but also by considering the influence that the dissolved oxygen concentration has on processes within the treatment plant. Nitrification and denitrification processes, as well as phosphorous removal, are dependent upon the dissolved oxygen concentration in the aerobic area of the plant. The dynamics therefore of other processes, in addition to plantwide objectives of operation costs, effluent quality and other considerations, may be used in a hierarchical control structure for the manipulated processes in the wastewater treatment plant, wherein localised control may be coordinated by a higher level structure in the solution of a plant-wide objective.

Baeza et al [11] detail a hierarchical control structure for a wastewater treatment plant in which the supervisory control includes process control and analysis via plant computers, in addition to low-level control, failure detection and possible corrective actions within a PLC system. The upper-level PC's within the system implements PLC supervision, in addition to process control for key parameters, in which control actions are transmitted to the PLC's which determine the actuator actions for the plant. In the case of the control structure assumed within this benchmark treatment plant model of Section 3.1.1, it is taken that there exists two control loops, that of the higher level outer MPC loop manipulating the setpoint of a lower-level PID structure. It is considered that the setpoints to the MPC loops, which in practise could be provided by a hierarchical system similar to that of [11], are chosen to be constant. The dissolved oxygen process in the aerobic reactor within the treatment plant, as dictated by the ASM approach, is defined as below in equation 3.1

$$\frac{dS_0}{dt} = \frac{Q}{V}(S_{O_2,in} - S_{O_2}) + r + K_{la}(S_{O,sat} - S_O) \tag{3.1}$$

The ASM model equations are defined by the mass balance approach, with the particular behaviour of each state defined by the reaction terms of its processes. The dynamic dissolved oxygen process may be dictated by an external influence, in the presence of an aerator. It can be seen that the DO process is effectively decoupled from the other processes within the plant with the presence of this term, in that it can be strongly dependent upon the oxygen transfer characteristics. However, aeration does affect the remaining processes within the system, in particular that of ammonia and nitrate/nitrite, as it disrupts the nitrification process. Much research recently has been concentrated on the modelling of aeration, for example Gillot et al. [53], Dhanasekharan et al. [38] and Kubsad et al. [84].

A limitation on application of control within the wastewater treatment plant is the availability of online sensors. The sensors assumed available within the BSM1 are detailed in Table 3.1 Within the WWTP, the main sensors available in practice are typically flow, level and nutrient measurements. Even further limitations are enforced due to the lack of control handles, with the most common of these available being limited to aeration, recycled flow rate and chemical dosing. The BSM1 model is composed of five individual activated sludge processes, of which only two are controlled: the second tank (unaerated and fully mixed) and the fifth tank (aerated, with a maximum K_{la} of $10hr^{-1}$). In the case of the control demonstrated here, the single variable control loops considered as 'existing control' are:

- PID control for the S_{NO} nitrate/nitrite control on the 2^{nd} tank as defined in [30]
- PID control of the S_o process of the 5^{th} tank as defined by Sanchez [147]

Table 3.1 Sensors and Actuators present in BSM1 WWTP

Sensor	**Units**
Dissolved Oxygen (DO)	mg/l
Nitrate/Nitrite	mg/l
Actuator	**Output**
Blower	Aerator (m^3/d)
Pump	Flow Rate (m^3/d)

For comparison purposes, control designed for the COST benchmark must be performed and evaluated under identical conditions in each instance. For this reason, the simulation parameters used in all cases were: variable step ODE23 solver, relative tolerance 1e-4 and absolute tolerance 1e-7. The dissolved oxygen control is implemented around the last aerated tank within the wastewater treatment plant, prior to the settler, via manipulation of the airflow to the reactor, as shown in Figure 3-2.

The oxygen control loop manipulates DO to a $2gm^{-3}$ setpoint by controlling the value of the oxygen transfer coefficient (K_{LA}). The K_{la} in this compartment of the treatment plant is constrained to a maximum of 360 d^{-1}. The simulation benchmark specifications define a maximum oxygen saturation concentration of 8 $g\ O2m^{-3}$. The airflow to the fifth reactor is pumped via a blower actuator controlled by a discrete PID controller identified

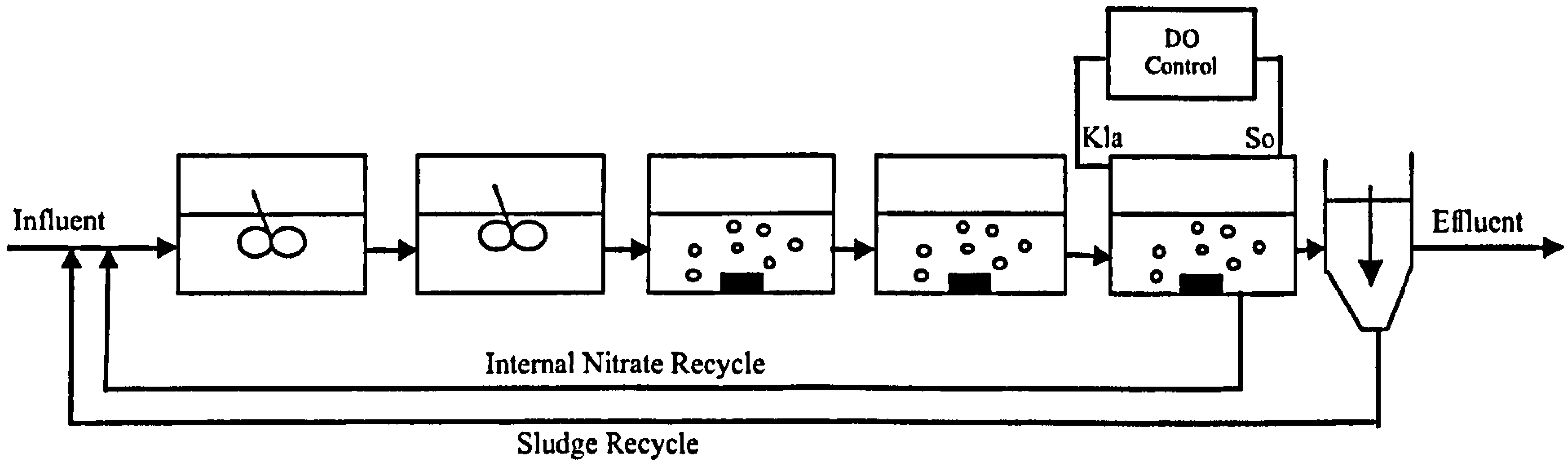

Figure 3-2: PI Control Loop of Dissolved Oxygen Concentration in Final Aerated Tank via Manipulation of the Air Flow Rate

by [147] via subspace identification of PI parameters, with the form of discrete PID transfer function $G_c(z)$:

$$G_c(z) = \frac{10000z - 9306}{z - 1} * alpha * R * h/(SOsatt * V) \tag{3.2}$$

where the oxygen transfer rate (in ratio to clear water conditions) *alpha* is defined as 0.6, the specific oxygen input as $R = 16$, the immersion depth of the air as $h = 4m$, the dissolved oxygen saturation value as $S_{O,sat}$=8g/m^3, tank volume V is defined as 1333m^3. The control of nitrate/nitrite concentrations is implemented upon the concentration measured from the output of the second anoxic compartment, which is controlled by adjusting the internal recycled flow within the system, as in Figure 3-3.

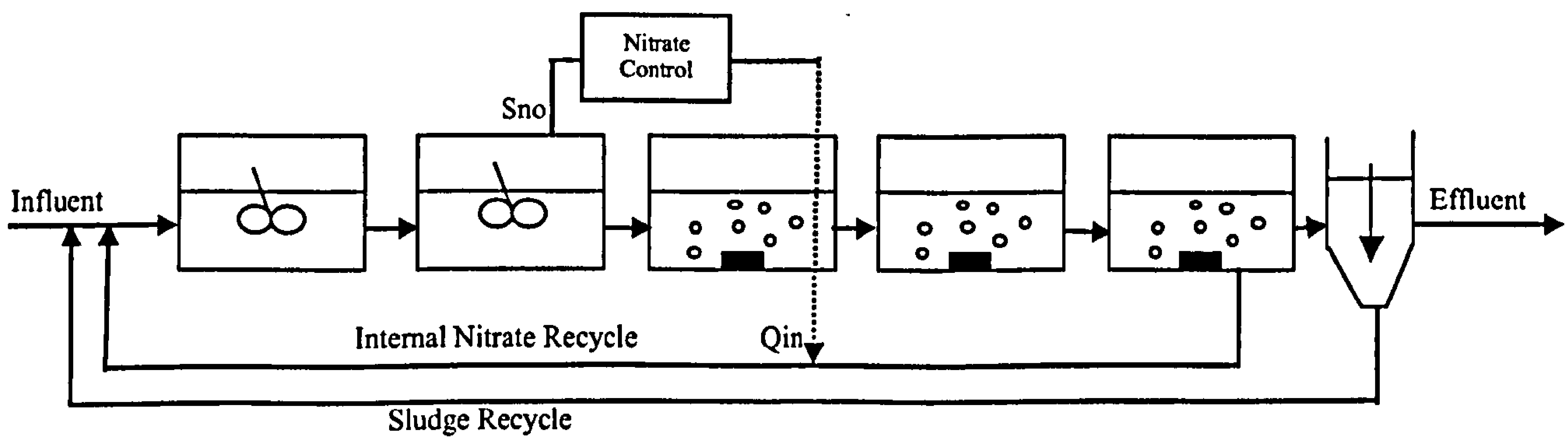

Figure 3-3: PI Control Loop of Nitrate/Nitrite Concentration in Second Non-Aerated Tank via Manipulation of the Internal Recycle Flow Rate

The nitrate/nitrite PID controller is tuned to the tracking of a setpoint of 1gm-3 and the internal recycle flow rate is constrained to a maximum of 92230 m^3d^{-1} (that is, 1.6 times the default rate). The parameters for the PI controller are defined as the proportional gain K_p, integral time constant T_i and anti-windup time constant T_d (defined by [30]):

Table 3.2 Nitrate/Nitrite Control: PID Parameters

K_p	T_i	T_d
15000	0.05	0.03

3.2 Predictive Control of a Treatment Plant

Traditionally, wastewater treatment plants have heavily relied on approaches such as PI control or simple on-off control, avoiding advanced approaches such as model based control. Reduced models of processes within the activated sludge based treatment plant have problems in their practical implementation with poor parameter identifiability and the requirement for the use of advanced estimation approaches. The use of linear models of the process, identified from online data, avoids these issues that are commonplace with ASM based models. The subspace identification approach described in Chapter 2 is used to identify a linear discrete state space model. The multivariable linear system required has two inputs of dissolved oxygen and nitrate/nitrite setpoints (for the single variable PID controllers) and two outputs of dissolved oxygen and nitrate/nitrite concentrations in effuent of the 5^{th} and 2^{nd} reactors respectively. The multivariable linear MPC is designed therefore to calculate the appropriate setpoint manipulation for dissolved oxygen and nitrate/nitrite, acting upon feedback from the plant, as shown in Figure 3-4. The inner loop setpoint is varied over time to more accurately and efficiently meet the setpoint of the outer loop.

Identification of the multivariable linear system is implemented by PRBS excitation of the above PI setpoints over a simulation period of 14 days under the constant influent wastewater conditions. The discrete time step (T_{disc}), mean value (m) and amplitude (a) of the PRBS signals chosen in the identification of the control model are as detailed in Table 3.3.

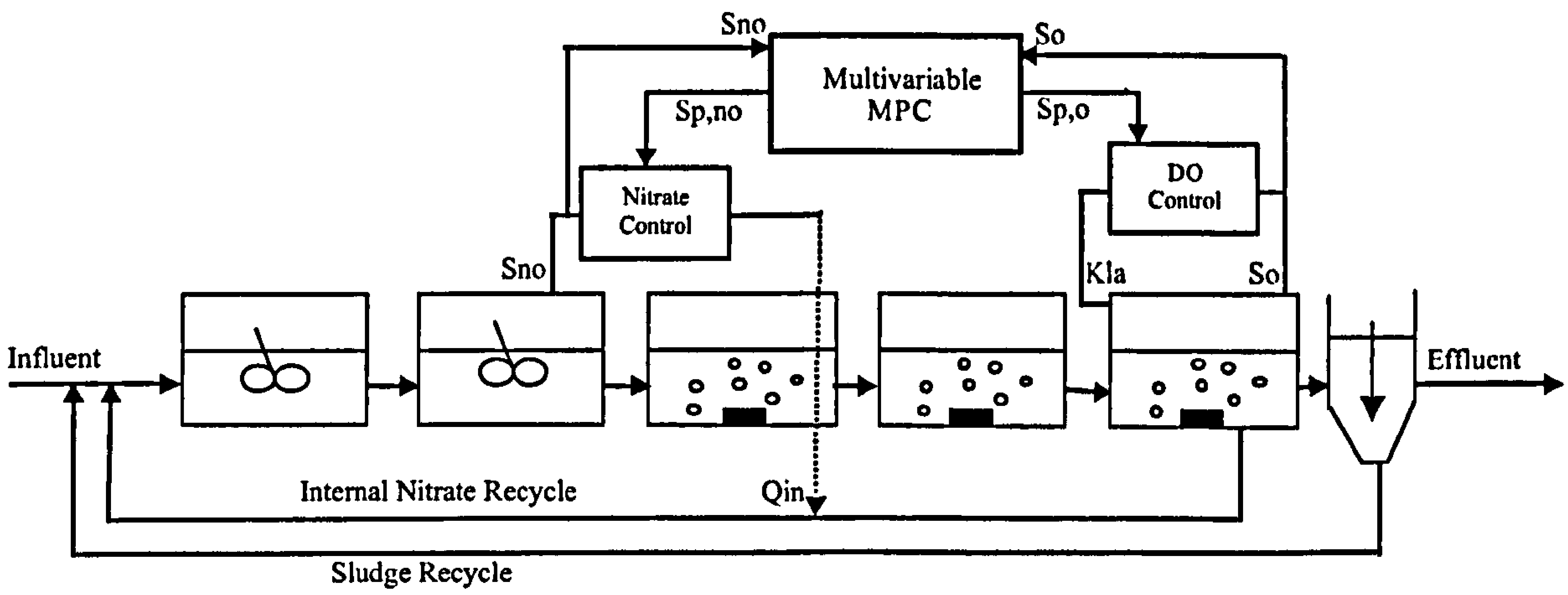

Figure 3-4: Multivariable MPC for the WWTP Plant: Dissolved Oxygen and Nitrate/Nitrite Control

Table 3.3 PRBS Parameters in Identification of DO and Nitrate/Nitrite Models

	T_{disc} (days)	m (g/m^3)	a (g/m^3)
Dissolved Oxygen	0.01	2	0.5
Nitrate/Nitrite	0.01	1	0.5

A data set of length 2 days is chosen from this, reflecting the behaviour of the above concentrations, and is utilised in the subspace identification algorithm. The model is identified under constant influent allowing the description of the dissolved oxygen and nitrate/nitrite behaviour and interactions. It does not however indicate the effect of the influent flow variations, which would negatively affect the performance of the controller. A comparison is made between the linear models, for nitrate and dissolved oxygen concentrations, and the behaviour of these processes within the treatment plant under constant influent conditions, as shown in Figure 3-5.

The linear model identified describes much of the dynamics of the dissolved oxygen process. The time constant of the dissolved oxygen process is dictated by the oxygen transfer process and thus the dissolved oxygen response is strongly coupled with aeration. The remaining mismatch is due to the nonlinearity of the overall process. The nitrate/nitrite

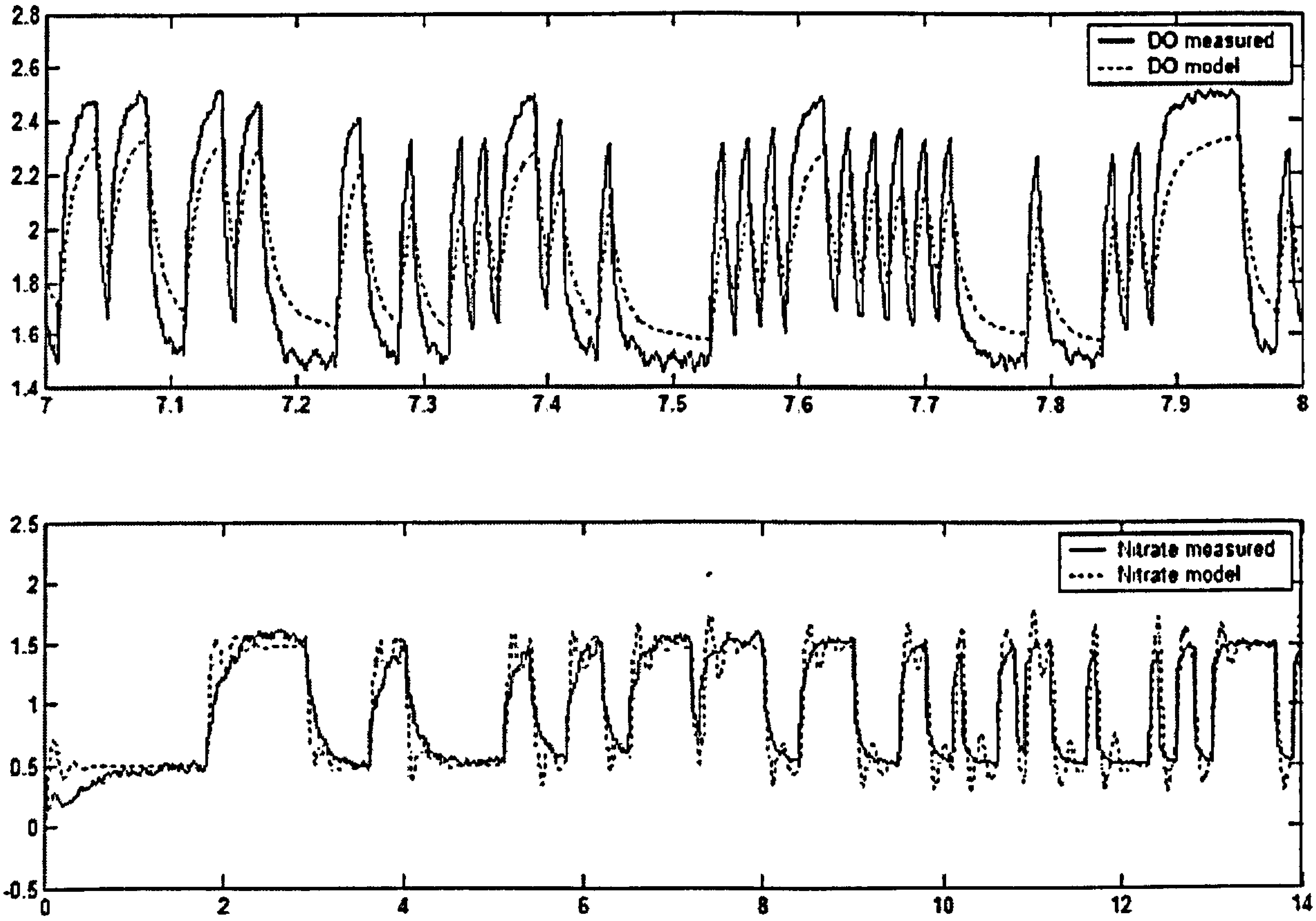

Figure 3-5: Comparison of Multivariable Linear Model with WWTP Process Behaviour

process, whilst dependent upon the internal recycle flow, is also dependent upon the process aeration. There is additionally a difference between the time constants of the nitrate/nitrite process and the faster dissolved oxygen process. This can be seen to have effects upon identification of the linear model with respect to the nitrate process. The MPC structure has the ability to compensate for this with the use of a constant unmeasured disturbance model as described in Chapter 2, as shown in Figure 3-6. The state estimation designed for the identified system will calculate the estimated disturbance state, resulting in an additional state for each modelled output of the linear state space model.

The observer poles are chosen as in Table 3.4, by convention to be ten times faster than the system eigenvalues, with two additional poles for estimation of the disturbance states.

Table 3.4 Observer and System Eigenvalues

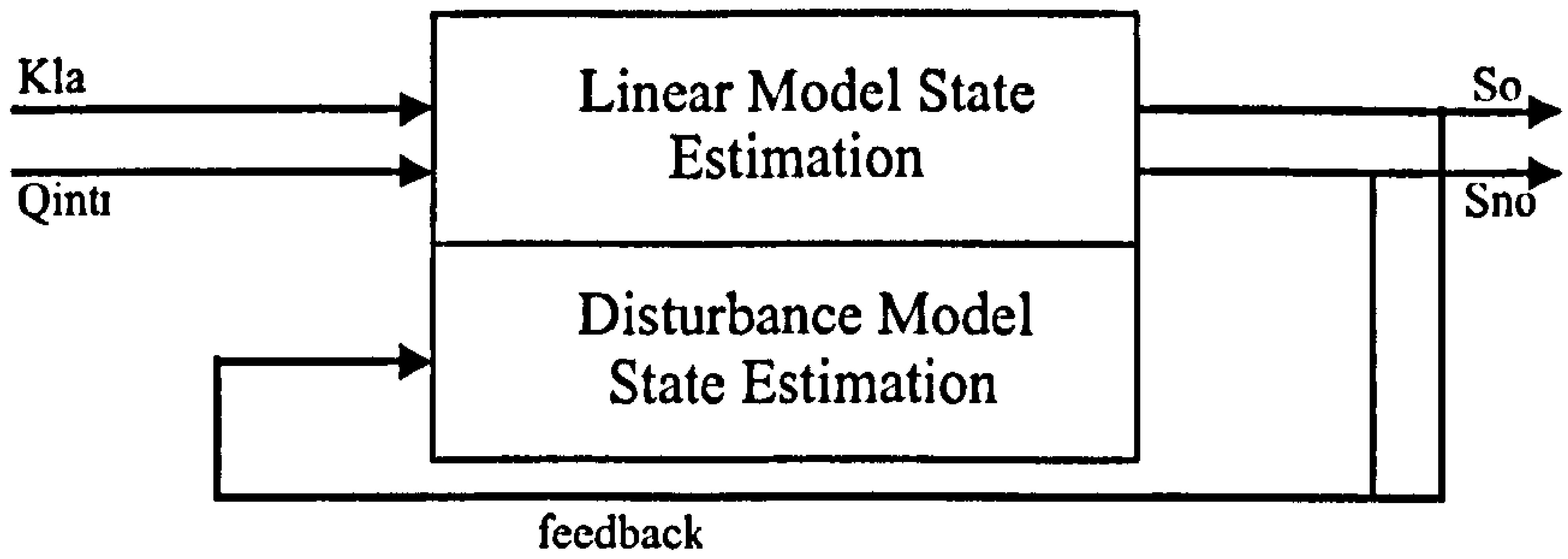

Figure 3-6: Linear Model Structure with Disturbance Estimation

System Eigenvalues	[0.9195 0.9511 0.9990 0.9952 0.9797]
Observer Poles	[0.8691 0.8690 0.8689 0.7688 0.7688 0.86 0.967]

The linear predictive control has equivalent performance to the PID control in the constant influent operating conditions. The controller performance therefore must be analysed for varying influent conditions (dry influent and storm influent characteristics). The MIMO predictive control implemented with linearised model identified is designed with the following parameters described in Table 3.5, chosen by trial and error.

Table 3.5 Linear MPC Tuning Parameters

Control Increment Weighting λ	[500 100]
Error Weighting Q	[10 1]
Prediction Horizon H_p	50
Control Horizon H_u	30
Sample Time T_s	1/1440

The main aim of the activated sludge process is to acheive a low level of biodegradable matter in the effluent from the treatment plant. The performance of the dissolved oxygen control is particularly important in acheiving this aim. Comparisons of performance of MPC and PID control are demonstrated for varying weather conditions:

- Predictive control of dissolved oxygen (as part of multivariable control in WWTP) versus the PID performance, both simulated for plant performance during dry weather conditions.

- Predictive control of dissolved oxygen versus the PID performance, both simulated for plant performance during storm weather conditions.
- Predictive control of nitrate/nitrite (as part of multivariable control in WWTP) versus the PID performance, both simulated for plant performance during dry weather conditions.
- Predictive control of nitrate/nitrite versus the PID performance, both simulated for plant performance during storm weather conditions.

The figures below demonstrate the performance of the existing constant setpoint PID control of the dissolved oxygen concentration, in comparison with MPC variable setpoint control. The figures demonstrate the response of the controlled variable (dissolved oxygen), the manipulated variable (dissolved oxygen setpoint) and the air flow rate determined by the mass transfer coefficient K_{la}. The varying dissolved oxygen setpoint allows the predictive control approach to compensate against the effect of flow variations within the treatment plant, demonstrating the advantage of a second level of control for the process. Figure 3-7 demonstrates the performance of the control under dry weather conditions.

The performance of the dissolved oxygen process shows a similar response under storm influent conditions, as seen in Figure 3-8, the disturbance model allowing for compensation against variations due to changes in influent. The flow variations do not have a significant adverse effect upon the response, the dissolved oxygen dynamics being sufficiently decoupled from the process nonlinearities by the linear effects of oxygen transfer.

Whilst the dissolved oxygen concentrations within the treatment plant are commonly controlled, the additional importance of nutrient removal from the wastewater is significant. The use of multivariable MPC control of the nitrate/nitrite levels, together with the dissolved oxygen concentration in the aerobic area, allows the process to compensate for the variations due to flow changes and varying influent wastewater characteristics. Figure 3-9 demonstrates, for dry weather conditions, the nitrate/nitrite performance, in addition to the control action of the variable nitrate setpoint, and the resulting changes in internal recycle flow rate.

Again, Figure 3-9 demonstrates, for storm weather conditions, the nitrate/nitrite per-

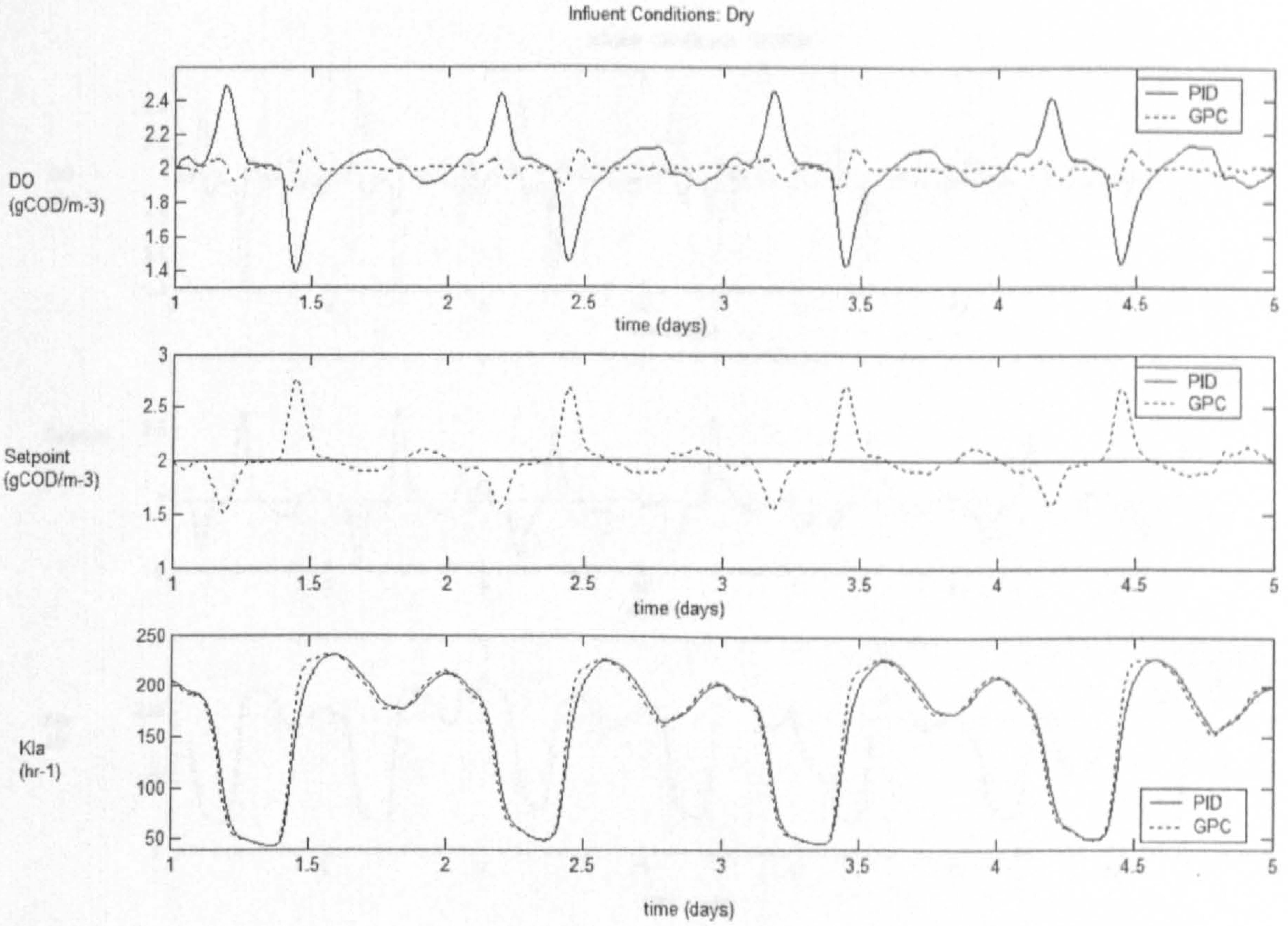

Figure 3-7: a. Dissolved Oxygen Response under MPC Control during Dry Weather Influent Conditions b. Applied Setpoint to Dissolved Oxygen PI Control Loop c. Air Flow to the Final Aerated Tank

formance in the anoxic reaction tank. The MPC control allows the reduction of the peak disturbance in the nitrate/nitrite concentration levels. However, the excessively large flow rates involved indicate the issue in the application of the above control. The restriction of control objectives to the treatment plant itself, and not the effluent concentrations or the effects upon receiving water quality, results in a control action detrimental to overall treatment plant performance.

Since the control objective concentrates only upon the performance of the chosen concentration, and not upon such issues as the concentrations of suspended solids in the effluent, the control approach under storm conditions for nitrate/nitrite would actually degrade plant performance.

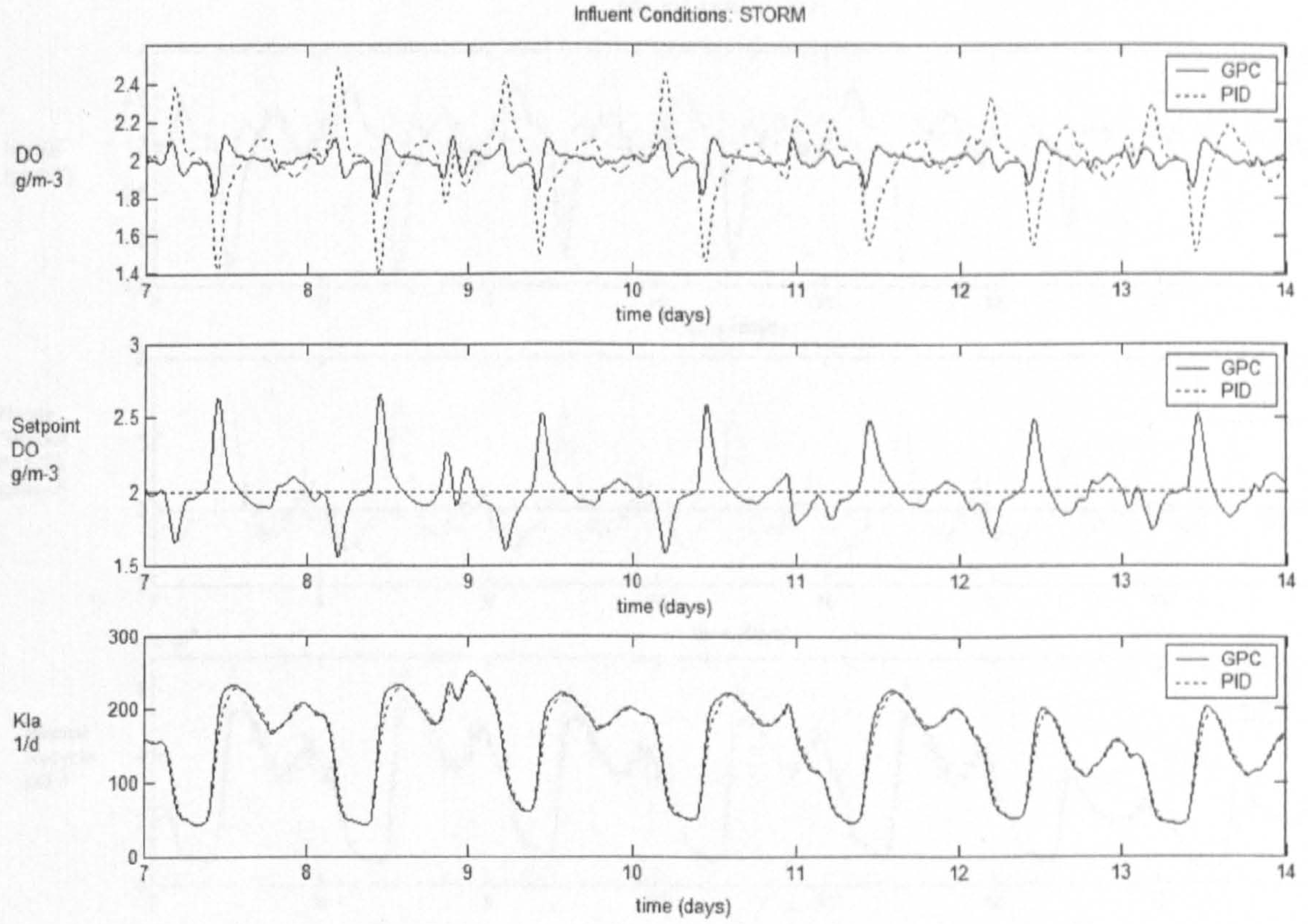

Figure 3-8: Dissolved Oxygen Response, Setpoint Applied and Air Flow to the Final Aerated Tank under MPC Control during Storm Weather Influent Conditions

3.2.1 Effluent Quality

The performance of the control strategies under differing influent conditions, as shown above, can vary. The results of different strategies may be compared via the plant performance index provided in the BSM1 [30], including such variables as:

- effluent quality (EQ) index (kg pollution units d^{-1}): this measures within one term the effluent pollution load. This is calculated using the data of the last 7 days of plant simulation under the chosen weather conditions.
- operational costs, such as pumping/aeration energy (both in units of kWh d^{-1}) and sludge production (kg d^{-1}), sludge for disposal and total sludge production).
- measures of any effluent violations, with respect to five effluent constraints, as stated

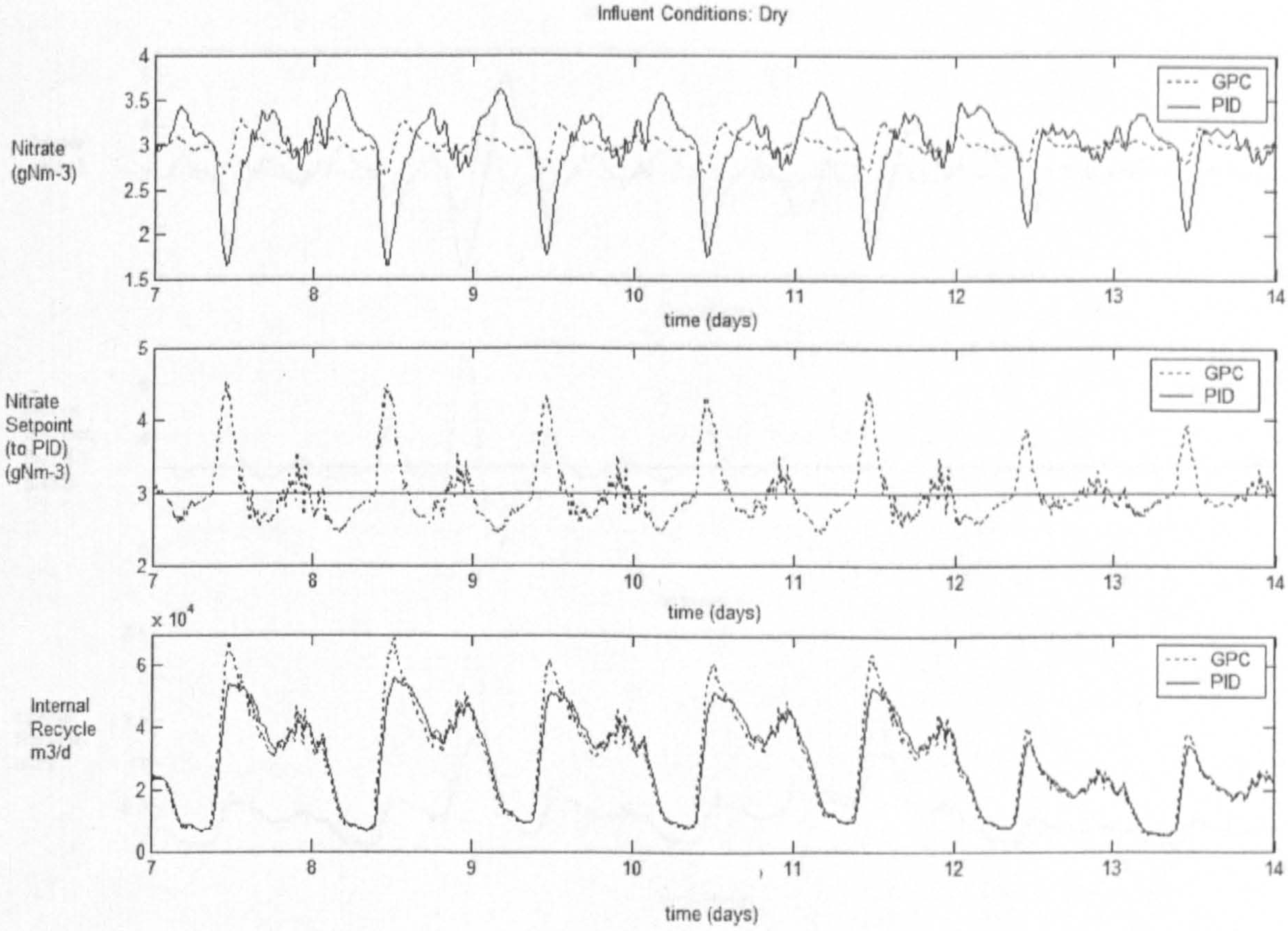

Figure 3-9: a. Nitrate/Nitrite Control under MPC Control during Dry Weather Influent Conditions b. Applied Setpoint to Nitrate/Nitrite PI Control Loop c. Internal Recycle Flow Rate

in Table 3.6. This is represented by two measures: the number of violations and the percentage of time the effluent is in violation.

Table 3.6 Effluent Limits as stated in [30], where COD is defined as Chemical Oxygen Demand

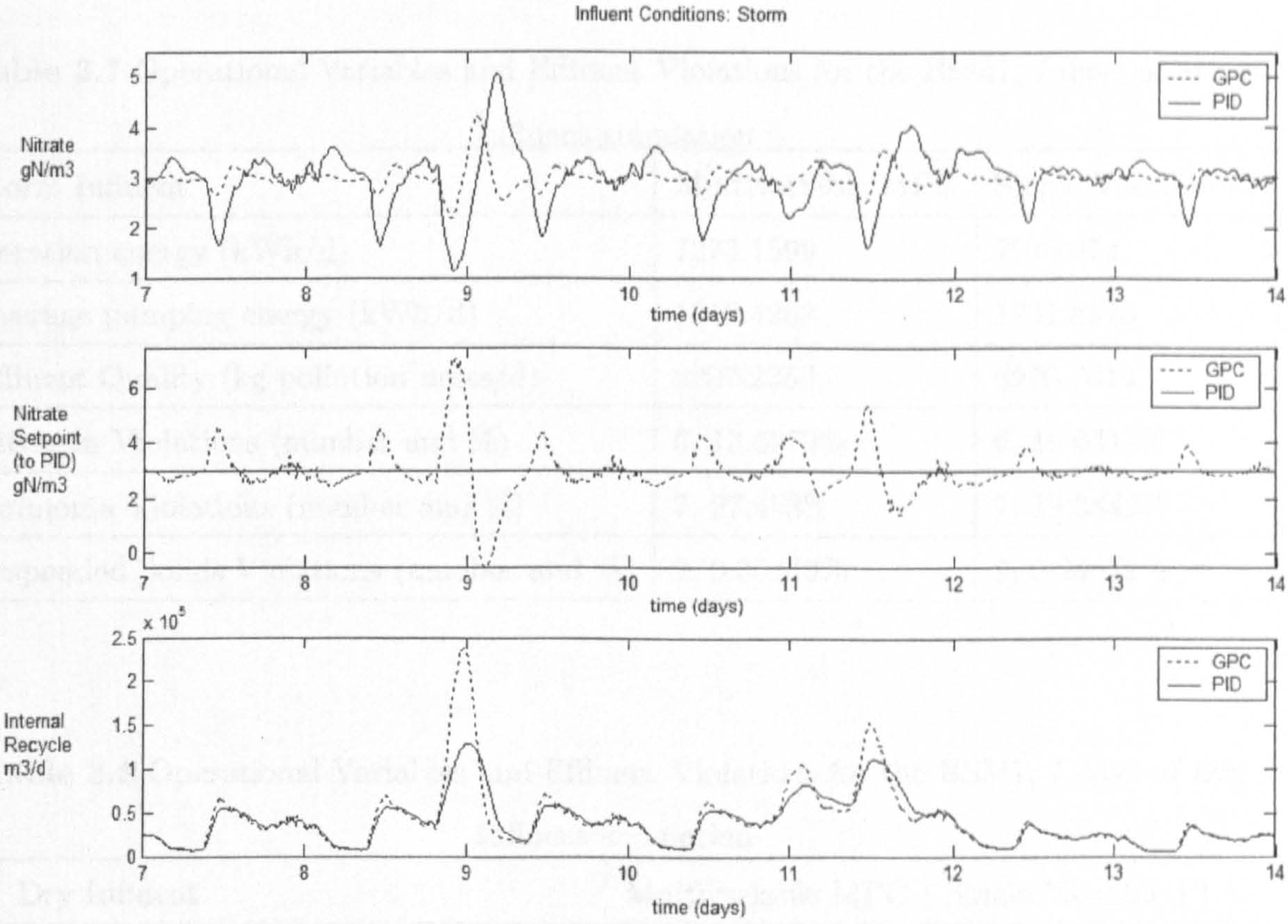

Figure 3-10: Nitrate/Nitrite Response, Setpoint Applied and Internal Recycle Flow Rate under MPC Control during Storm Weather Influent Conditions

Effluent Component	Variable	Effluent Limits	Units
Ammonia	$S_{NH,e}$	4	gNm^{-3}
Total Nitrogen	$N_{tot,e}$	18	gNm^{-3}
BOD_5	BOD_e	10	$gBODm^{-3}$
Total COD	COD_e	100	$gCODm^{-3}$
Suspended Solids	TSS_e	30	$gSSm^{-3}$

The effluent quality, limit violations and plant performance indicators are calculated for each storm condition and control approach, for comparison purposes, and are demonstrated in tables 3.7-3.9.

Table 3.7 Operational Variables and Effluent Violations for the BSM1, 7 days of Storm Influent simulation

Storm Influent	Multivariable MPC	Single Variable PI
Aeration energy (kWh/d)	7273.1599	7279.974
Average pumping energy (kWh/d)	1816.4268	1731.8525
Effluent Quality (kg pollution units/d)	8207.2352	8276.7014
Nitrogen Violations (number and %)	6, 13.6977%	6, 10.6415%
Ammonia Violations (number and %)	7, 27.453%	7, 19.3842%
Suspended Solids Violations (number and %)	2, 0.35283%	2, 0.26671%

Table 3.8 Operational Variables and Effluent Violations for the BSM1, 7 days of Dry Influent simulation

Dry Influent	Multivariable MPC	Single Variable PI
Aeration energy (kWh/d)	7227.5294	7230.8554
Average pumping energy (kWh/d)	1525.177	1494.4871
Effluent Quality (kg pollution units/d)	7474.8663	7530.8699
Nitrogen Violations (number and %)	6, 15.8922%	7, 12.8801%
Ammonia Violations (number and %)	5, 17.8566%	5, 12.5621%

Table 3.9 Average Concentrations in Effluent for Dissolved Oxygen, Nitrate/Nitrite and Ammonia

Plant Influent:	Dry		Storm	
Control Structure	PI	MPC	PI	MPC
S_O (mg (-COD)/l)	1.9641	2.0007	1.9733	2.0038
S_{NO} (mg N/l)	12.4175	12.3673	10.5572	10.4839
S_{NH} (mg N/l)	2.4772	2.3757	2.9922	2.9001

Each of these tables indicates information describing the behaviour of the components in the effluent to the receiving waters. If one inspects the behaviour graphically however (as in Figure 3.10), it can be seen that although the number of violations and length that the plant is in violation is equal or even less for the PI control, it is also clear that this description of the plant performance does not take into account the peak value of the violation. In particular, it is shown in Figure 3-11 that the predictive control application decreases the maximum nitrate/nitrite and ammonia concentration concentrations reached within the effluent. It will be seen later in the thesis that this peak value is used in the analysis of disturbance effects in the receiving waters.

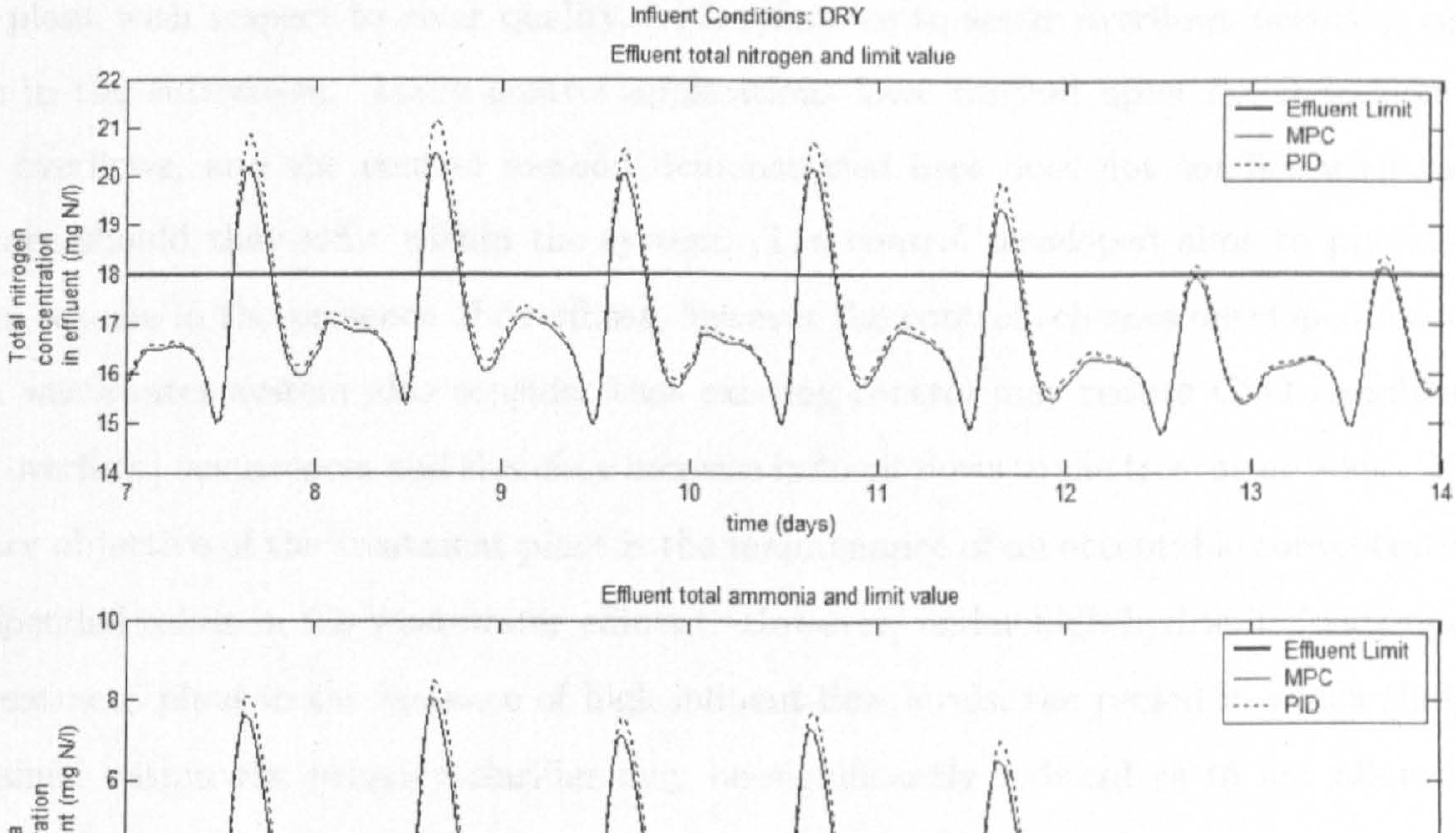

Figure 3-11: Total Nitrogen and Ammonia Concentrations in the Effluent for Single Variable and Multivariable MPC, PI Control and the Effluent Limit for Both Variables

3.3 Urban Wastewater System

The urban wastewater system consists of three subsystems: a sewer system, a treatment facility and receiving waters such as a river or a lake. The objective of the wastewater process is the removal of waste from households and industry via a sewer network, allowing its transportation to treatment facilities, after which discharge to receiving waters takes place. In certain situations, such as storm events, in which high loads of flow enter the sewer network, emergency discharges to the receiving waters may take place prior to treatment. These events are known as 'combined sewer overflows' and can have a considerable negative effect upon water quality.

The control considered within this thesis concentrates on the performance of the treatment plant with respect to river quality, with reference to sewer overflows occuring elsewhere in the subsystem. Many control applications have centred upon the reduction of sewer overflows, and the control method demonstrated here does not conflict with such measures should they exist within the system. The control developed aims to provide a control scheme in the presence of overflows, however the control schemes developed for the urban wastewater system also consider that existing control may reduce CSO (combined sewer overflow) occurrences and therefore increase influent flows to the treatment plant. The primary objective of the treatment plant is the maintenance of an acceptable concentration of suspended solids in the wastewater effluent. However, under high hydraulic loads upon the treatment plant in the instance of high influent flow levels, the period in which sludge is retained within the primary clarifier may be significantly reduced as to not allow for sufficient treatment. An additional concern may be the loss of suspended solids to the plant effluent, known as 'washout', reducing water quality and damaging plant performance. For this reason, many plants include a control strategy to bypass flow directly to the receiving waters after a threshold of maximum influent flow has been passed. Recent strategies have been suggested in avoidance of such situations, such as those described by Nielsen et al. [112] of aeration tank settling (reducing the sludge load on the settler, therefore decreasing the risk of washout of sludge).

The wastewater treatment plant is one part of a larger system, whose importance in reducing the human impact upon water, as part of this system, is considered within this thesis. The control of the wastewater treatment plant shown above aims at controlling nitrate/nitrite levels in the treatment plant and maintaining oxygen concentrations for the removal of organic substances. In the wider system, the control objectives may differ in the maintenance of water quality to regulatory levels, whilst the actuators however most commonly remain strictly within the treatment plant.

3.3.1 Effluent and Receiving Water Regulations

Water quality is defined as a description of the physical, chemical, biological and aesthetic aspects that influence fitness for use and ability to maintain the health of aquatic ecosystems. These properties can be influenced by constituents, that is the properties of water and that which is dissolved or suspended in it, i.e. its characteristic. The common regulatory approach to wastewater has concentrated on effluent quality, minimising the effect of the treatment plant subsystem instead of the effect upon the receiving waters, as shown by the approach of the Uniform Emission Standard (UES) of [182]. The implication of the 'integrated' approach adopted by the WFD is that the control schemes must be extended to include information from the other subsystems of urban wastewater treatment.

The Environmental Quality Objectives/Standards (denoted by EQO/EQS) shown in Tyson et al. [178] is focussed upon immission levels, the quality of receiving water, whilst the Uniform Emission Standard above is focussed upon emission levels, quality of effluent water, respectively. In the EQO/EQS, the capacity of receiving waters to assimilate pollutants is accounted for, in that a certain level may be assimilated and the receiving waters may still meet the water quality standards. The UES on the other hand views the objective of attaining effluent water quality without respect to issues in receiving waters due to discharges local to this point, and compliance with these regulations is more defined with respect to control technology utilised rather than the quality of the receiving waters affected [177].

The European water policy of the Water Framework Directive is established with the objectives as detailed in [17], defined as: protection of all waters (for example, groundwater and surface waters), acheiving sufficient quality status of these waters (ecologically and chemically) within the deadline set (15 years), a river basin approach to water management, control of emissions and discharges by an approach combining emission limits and quality standards, mandatory pricing for water and public participation. Tolessa [177] details that in addition to the WFD, the water quality legislation of the CWA (Clean Water Act) of the USA is of similar importance.

The CWA objectives involve the optimistic aim of eliminating the discharge of pollutants entirely, whilst in the presence of pollutants the objective is given of the restoration of, and maintenance of, the water integrity (biological, chemical and physical), in order to achcive a water quality appropriate for aquatic life and for human uses. The objectives of both the CWA and also of the WFD are based upon the river basin approach to water quality. The indicator variables of dissolved oxygen and ammonia concentrations are a popular choice. Table 3.10 below demonstrates the quality as indicated by these concentrations, for a small river receiving water.

Table 3.10 Small river receiving water: Dissolved Oxygen and Ammonia Indicator Variables

Concentration	good	sufficient	insufficient	bad
Dissolved Oxygen (duration 1h, mg/l)	4	3	2	1
Ammonia (duration 1h, mg/l)	0.1	0.2	0.3	0.4

Schilling [151] defines the possible solutions to water pollution due to urban discharges are

- reduce the amount of sewage (e.g. on a household level, and also by methods such as infiltration of stormwater into soil).
- reduce combined sewer overflow (increasing storage capacity, real time control and monitoring of sewers).
- increase WWTP capacity (physical expansion).
- increase treatment efficiency (e.g. process control, improving plant performance).

Immediate oxygen depletion in receiving water passing a CSO discharge point is considered the 'dominating acute effect in running waters' according to Schilling [151]. Large levels of easily degradable organic substances creating an oxygen demand may be discharged during CSO events. Sedimentation of slower degradable organic matter may cause a delayed oxygen depletion. Ammonia discharges are destructive, even if brief or at low levels, due to the strongly toxic nature of unionised ammonia to fish. The chemical and biochemical impacts of urban discharges on receiving waters are characterised by Schilling et al. as related to the indicator variables of toxic substances (ammonia) and oxygen depletion (dissolved oxygen) respectively. In a practical sense the above quality standards, in particular in the case of ammonia, may be too strict, especially in consideration of dynamic conditions such as the time of day or time of year at which the sample is taken, and diurnal variations of ammonia levels in the sewage. Combined sewer overflows are further discussed in Section 3.4.4.

3.3.2 Instrumentation and Actuators

Commonly, instrumentation within the wastewater treatment industry has concentrated on the treatment plant, in measurement of flows, levels, and wastewater concentrations such as dissolved oxygen, ammonia, nitrate and phosphates. Other measurements of biochemical importance, such as pH, suspended solids and COD may also be implemented. Whilst in recent decades, the bottleneck to the control and automation of wastewater treatment was considered to be instrumentation [121], this has seen a change particularly in the prevalence of the monitoring of treatment plant bioprocesses. The sensors throughout the remainder of the urban wastewater system remain as yet considerably less common.

Whilst the instrumentation in place within the treatment plant industry has increased, a similar surge has not taken place in the number of control handles available. The implementation of control strategies is limited by a lack of control handles, the existing control actions are performed by control valves (such as ball, diaphragm and plug valves), pumps (such as fans and blowers) and motors (electric motors, such as induction or DC). Control action within the sewer system differs from that of the treatment plant, based mostly upon hydraulic issues. The traditionally applied control action is that of flow manipulation (for example using gates, pumps or weirs). In monitoring of sewer conditions, water levels are measured (Hansen and Carstensen [58]), flow rates upstream and downstream. Weather systems can be of use in the measurement of rainfall, and for example in the forecast of rainfall (Adebe and Price[1]). Pleau et al. [132] demonstrated a control strategy for the sewer network based on predicted rainfall from meteorological models, and similarly for Entem et al. [48]. Whilst a significant proportion of sewer control concentrates on hydraulic problems, recent research has demonstrated control of pollutants within the sewer system, with regard to the receiving water quality. The control options within the receiving waters are defined by Schutze et al. [152] as artificial aeration, flow control and control of discharges from CSO's and also from the WWTP. On-line monitoring is possible for DO, pH, temperature, amongst others, however literature has detailed [179] the lack of use of these control handles or sensors in the control of river quality.

The aim of real time control (RTC) in the urban wastewater system should be the inclusion of river water quality as a control objective. It is for this reason that the the simulation model chosen for the urban wastewater system must allow the current state of the receiving waters to be determined, for design of possible control strategies within the subsystems and testing of their effects upon the final receiving water system. The actuators as considered within this thesis are those existing within the treatment plant. Controllability of the process depends on the number of these control handles, the manipulated variables, and the relation between these and the system control objectives. According to [7], in general the manipulated variables within an activated sludge waste treatment plant are:

- hydraulic: influent flow, wastage and recirculations.
- additions of chemical or carbon sources.
- air supply.
- pre-treatment of influent wastewater.

The actuators considered in the control applied to the urban wastewater system demonstrate in the following chapters are the those of aeration and chemical dosage, the control handles available within [31] for the manipulation of dissolved oxygen (and ammonia/ammonium) and phosphorous.

3.3.3 Real time control of Urban Wastewater Systems

Schilling et al. [150] defines real-time control of the urban wastewater system to be that where 'process data..(is)..continuously monitored in the system and, based on these measurements, regulators are operated during the actual flow and/or treatment process' and defines the 'process data' as information such as water level, flow and pollutant concentration. Meirlaen [105] details that three requirements are therefore necessary for implementation of this real time control: sensor measurement of process conditions, control computation based upon this measurement and actuator action upon the process. Schutze et al. [152] define 'integrated control' of an urban wastewater system as the 'integration of objectives' and 'integration of information'.

The former defines the control objectives of one subsystem with respect to the criteria of another subsystem, for example in the case of this thesis, the control of the treatment plant is determined by the water quality criteria of the receiving waters. The latter defines the use of information from one subsystem in the control of another, for example in this case in which the effects of combined sewer overflows upon water quality at the point of overflow are used in the control of the treatment plant performance. Schutze et al. [152] defines the control objectives within a treatment plant in a three level structure: the lowest, at plant level, in control to plant setpoints, the medium level is specified to cover time varying setpoints supplied by advanced control techniques, whilst the highest level in the control structure is the definition of plant operation objectives.

3.3.4 Combined Sewer Overflows

During rain events, a flow exceeding the hydraulic capacity of the sewer may cause the water to be diverted via 'emergency overflow' structures, in an event known as a 'combined sewer overflow'. Untreated wastewater may therefore enter the receiving waters causing detrimental effects to water quality, such as an increase in the concentration of pollutants present, or oxygen depletion. Two types of sewer structure exist in common use. The first, a separate sewer system, has a two-pipe structure: for the rain water and for the wastewater. This is more efficient for the treatment of waste, as the rain water does not dilute the wastewater, and additionally no combined overflows of waste occur. However, the 'first flush effect' of high rainfall events impacts upon seperate sewer systems, and is not transported to the wastewater plant for treatment, thus having detrimental effects upon receiving waters. The second, the combined sewer system, consists of only one pipe, containing both of the above flows, with the advantage of less construction costs. However the increased flow to the treatment plant during rain events reduces the efficiency of the treatment, and also as stated above CSO's may occur. The water quality in the sewer considered in this thesis therefore is the result of sewer mixing of the dry weather flow (household and industry wastewater flow, independent of rainfall effects) with the stormwater. The dry weather flow contains physical pollutants (e.g. suspended solids) , chemical pollutants (BOD, COD, nutrients) and microbiological pollutants dangerous to human health.

Some studies have indicated that the effects of CSO reduction in the case of dissolved oxygen control are not clearly advantageous, and that the subsequent increase in hydraulic load to the treatment plant may decrease plant performance. Research studies by Rauch and Harremoës [137] demonstrate the effects of overloading the treatment plant, causing a break-down to occur with dramatic oxygen depletion in the receiving waters. A later study by the same group [138] of real time control of CSO indicates no correlation between the minimum oxygen concentration and the volume of CSO reduction. The increased volume of wastewater stored in detention basins contributed to an increased treatment plant load, subsequently causing water pollution via the treatment plant effluent, thus working against the objectives of the water detention. The approach demonstrated within this thesis aims to allow control of the urban wastewater system both in the presence of overflows and also in the presence of increased influent flow rates to the treatment plant.

The effects of CSO's detailed by [105] are manifold, but in particular the effects upon oxygen demand can be caused by COD from CSO's, as well as NH_4, causing a reduction in dissolved oxygen, and an accumulation of biomass. Nutrients such as nitrogen and phosphorous, in their various forms, can cause environmental effects such as enrichment, which is the condition that results when the receiving water has a nutrient concentration in excess of that necessary for aquatic life. Nutrient enrichment can result in increased growth of algae and other plant life.

3.4 Summary

In this chapter, the control issues for a wastewater treatment plant with relation to linear predictive control techniques are discussed, starting with the development of a linear strategy in two-level control around single PI control loops. Two concentrations are chosen for control: the dissolved oxygen concentration and the nitrate/nitrite concentration. The control of these variables is assessed by examining the effects upon the treatment plant effluent. The performance of this control strategy under varying plant influent is demonstrated by the statistical assessment under several weather conditions. The aim of the initial section of this chapter is the introduction of predictive control in the traditional area of the maintenance of treatment plant quality. The performance of the control strategies under differing influent conditions is discussed. The analysis of treatment plant effluent behaviour with respect to specified effluent violation constraints is of particular significance.

Following this, a survey of the available sensors and actuators within the treatment plant and also the extended urban wastewater system was given. The existing real time control and its objectives is discussed, and the effect of governmental and internation legislation upon water quality control is detailed. The issue of nonlinearity within the urban wastewater system is discussed, with suggestions as to the method in which this problem should be tackled. Combined sewer overflows are also discussed, in relation to the possible effects posed. The relation between the combined sewer overflows, the treatment plant and the receiving waters must be addressed. Therefore the following chapter, the background, and the introduction to modelling, of this urban wastewater system is given.

Chapter 4

Urban Wastewater Treatment Model

4.1 Introduction

With the introduction of the Water Framework Directive in 2000 [49], an 'integrated' view of the wastewater treatment system has become more important in the automation and control of the processes. The definition of the structure of an 'integrated system' differs from source to source, Vanrolleghem et al. [181] for example define the integration of the sewer and treatment plants as the 'integrated urban waste water system'. Butler et al. [193] discusses the 'integration' involved as the exchange of information between the sections of the treatment process and the integration of control objectives. The integrated control objective can be seen to be an overall optimisation in the system, not just in one section at a time.

The area of integrated wastewater system management has been underdeveloped and research until now has, for the most part, concentrated on one aspect of wastewater treatment at a time. The modelling related to wastewater has concentrated heavily upon the development of treatment plant models. Most commonly in urban areas, wastewater (and stormwater, in the combined systems) is handled by a sewer system and a wastewater treatment plant, and later discharged to receiving waters, such as rivers. It is this complete process, the mixing, biological and chemical processes that take place in the progression of the wastewater through the system from sewer influent to receiving waters effluent, that is considered as the urban wastewater system in this thesis. The sections that are modelled within the mathematical representation of the system are: combined sewer, waste treatment plant and river.

A significant concern in the design of automation and control for an integrated approach to wastewater treatment is the issue that systems are not planned or designed specifically for integrated system control, but instead concentrate on meeting effluent regulatory standards. The need to address this problem motivates the design of control for the integrated system. This chapter therefore deals with the modelling of the urban wastewater system for the purposes of control. A general explanation of urban wastewater modelling is given, and the models used in this thesis are described. Each model is explained with regards to the individual components, and the reasoning and assumptions behind each. A model is chosen to represent the process as regards water quality criteria and control to specific objectives, thus model components unnecessary for realistic control are not examined. Motivation for reduction or simplification of the models used for control is discussed in Section 4.3.

The simplistic modelling of the urban wastewater system developed by [56] is discussed in Section 4.5. The complex mechanistic representation of the urban wastewater is detailed in Section 4.6, as developed by [31], and is demonstrated, with respect to the implementation of predictive control based upon a linearised dissolved oxygen process model in Section 4.7. The control objective, method and results, and also the limitations of linear models for a nonlinear system are discussed. The need for a nonlinear approach is detailed. In the development of a nonlinear model approach for control of the system, the state dependent format of each model is presented. The derivation of the nonlinear generalised predictive control scheme demonstrated later in Chapter 6 depends on the development of a state dependent format of the treatment plant and urban wastewater system models utilised in this thesis. The transformation of the simple wastewater treatment plant model, its related urban wastewater model and the ASM2d aerobic model to the state dependent coefficient form is shown.

4.2 Urban Wastewater System Modelling

The issue in the modelling of any biological system is the objective of the model, that is, the use for which it is designed. In the analysis of wastewater treatment and benchmarking of control schemes, a process model is required to represent the water processes affected by wastewater and changes in its composition. The modelling requirements demonstrated by Rauch et al. [136] show that the integrated models can be designed based only on those components in the receiving waters, which are affected by wastewater. Rauch et al. detailed that the state of a receiving water is not determined entirely by its chemical quality. Regulatory constraints however do not take this into account, or the fact that more often than not, local ecological conditions also affect the receiving water. This issue itself actually constrains the possibility of effectively improving ecological quality levels. It is not however the issue considered in this thesis, which concentrates on the effects of wastewater upon the receiving waters. To this end, the focus of attention within the modelling demonstrated is the representation of wastewater processes in progression from treatment plants, to the effluent to rivers, and the effects of any sewage overflow to the river; other point sources are not considered.

Rauch et al. [136] specify that the impact of wastewater on the receiving waters can be organised in the following groups: chemical, bio-chemical, physical, hygienic, aesthetic, hydraulic and hydrologic impacts. It is then the choice of the user to specify which of these impacts are to be monitored and improved, thus determining the focus of the model. The nitrate, ammonia and phosphate concentrations, in their role as nutrients and their effects upon the river life, are considered as the key factors within the nutrient control issue. In the area of wastewater treatment, nutrients present a particular problem, they are contained in domestic sewage, in argicultural run-off and are also present in industrial effluents. These nutrients, particularly nitrates and phosphates, encourage the growth of plant life in the river or receiving waters, but this can become excessive under conditions of high nutrient concentrations.

The growth and decay of algae becomes in turn an oxygen demand upon the river, in a process known as eutrophication, the over-enrichment of a body of water with nutrients with a subsequent decrease in oxygen levels. In lakes, or slow-flow rivers, this problem can become significant. The importance of nutrient control and the solution required can depend upon this signifance, and may be of no concern to local water authorities in the case of fast-flowing waters. Dissolved oxygen levels are traditionally an important issue, in that large and extended dips in oxygen in the water can lead to fish deaths. For this reason, dissolved oxygen is a commonly controlled variable.

A large issue in the management of urban water resources and treatment is the almost exponential increase in population during the last century. This population surge has also therefore resulted in increased city populations, and a proportional increase in load upon the sewer system. In addition, climate changes have taken place, as well as changes within industry, that must be taken into account by water authorities. The objectives of advanced automation and control in the increased efficiency of plant operation can allow a system to deal somewhat with the increased load by implementing changes in its mode of operation, rather than making any investment in physical changes to the system, in the form of plant expansion. Many treatment plants are overdesigned, and so, the increased load upon the wastewater treatment industry may well be within the physical constraints of the treatment system. Any investment required in the improvement of automated processes within the plant would merely take the form of the addition of extra sensors in the system, a significantly smaller cost than phsyical redesign.

4.3 Model Choices

Mathematical models of the wastewater treatment process within the control algorithm provide the ability to predict the allowable discharge of a given component to the receiving water, to meet water quality standards. The main motivation within the wastewater research area for mathematical modelling is the possibility of evaluating new designs without the need for an intrusive practical implementation of the designed strategy. The models developed therefore are sufficiently complex to describe the dynamics and interactions within the wastewater process, of which there are many. For this reason, bulky and often complex process models are produced with the aim of process representation, without reference to control requirements. Model simplification, with mechanistic models, or model reduction can be utilised to produce a 'control model'. The model simplifications result in 'surrogate models', defined by [181] to be an approximate substitute to the complex mechanistic models of the process. The complex original models represent the real process considerably more accurately, however the surrogate models can be chosen to represent only the variables under consideration, with the benefit of simplification but with the expense of accuracy. These 'surrogate' models can be defined as reduced mechanistic models, such as that of the simplified wastewater treatment plant model of [114] or more complex grey-box models such as that demonstrated by [141] in the application of neural modelling for the ASM2d process.

Model reductions utilised within this thesis are summarised as shown in Figure 4-1, similarly to the method defined by [181], involving elimination of parts of the model not necessary to the control of the process. The approach detailed by [181] utilizes the control model methodology of [106] in the relocation of system boundaries according to available actuators and sensor information. In the case of the work presented in this thesis, the control models represent only those dynamics which can be manipulated or measured, e.g. upstream and downstream concentrations of the controlled variable, the actuators and the dynamics of the treatment plant and river sections considered. For example, the parts of the sewer system upstream of the control action may be eliminated, as they may never affect the control of the system, and act only as influent behaviour and description of catchment and sewer wastewater characteristics. The river dynamics upstream prior to any sensors may not be required within the control model due to the lack of control handles, but river dynamics after any sensors may be modelled for disturbance prediction purposes. Similarly then, the downstream dynamics of the river following the last sensor may be eliminated, as any unmeasured events after this sensor do not affect the control action.

Two forms of urban wastewater model are used within the work presented. A complex mechanistic representation of the urban wastewater system based upon the ASM2d model and the QUAL2E [19] model is considered. A separate, simplified model representing the basic mass-balance equations of the urban wastewater system is presented. In the development of a 'control model' of the processes involved in both these approaches, the state dependent model representation of the nonlinear dynamics is used.

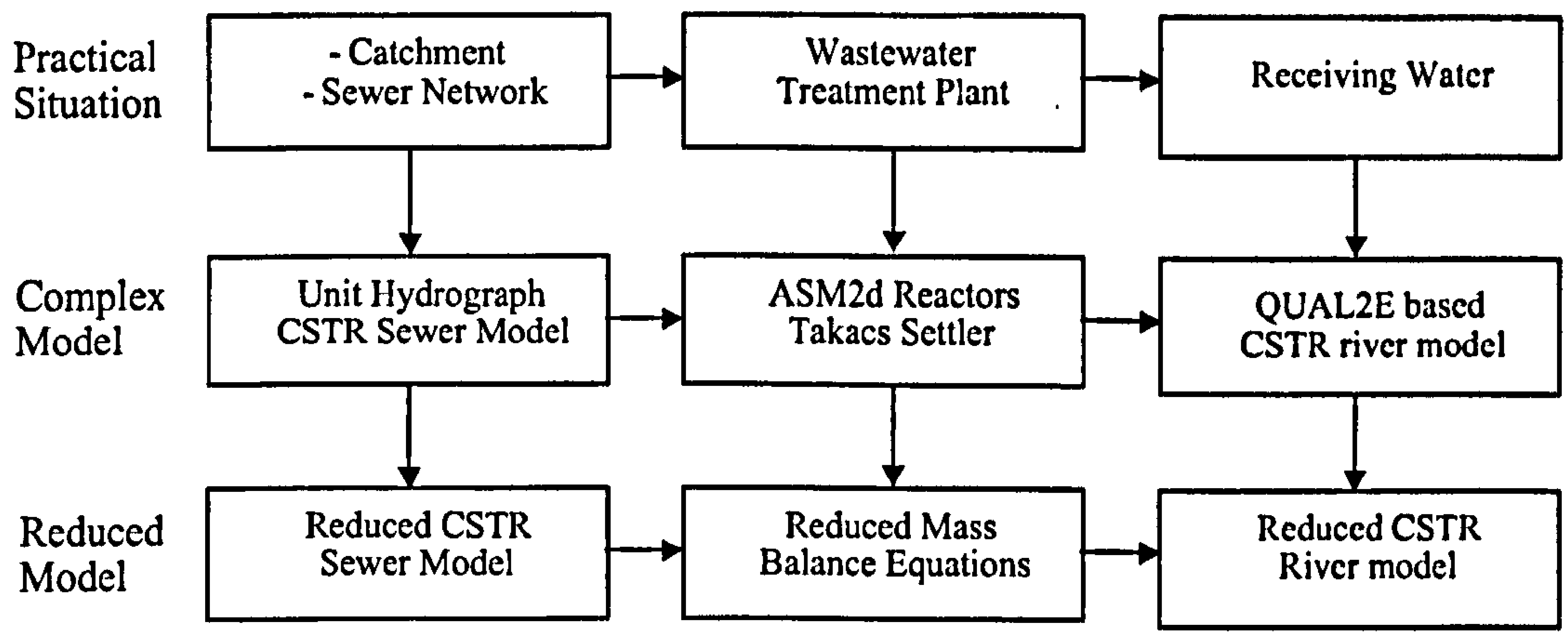

Figure 4-1: Modelling of the Urban Wastewater System: Through Complex Mechanistic Models and Reduced Representations

4.4 State Dependent Model Representation

Previously in the thesis, the state space representation of the process has taken the linear form

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{4.1}$$

However, models of this form do not fully incorporate the dynamics of a nonlinear system. In order for more accurate process representation, it is required to model the system in the form

$$\begin{aligned} x(k+1) &= A(x,u).x(k) + B(x,u)u(k) \\ y(k) &= C(x,u)x(k) + D(x,u)u(k) \end{aligned} \tag{4.2}$$

where the matrices $A(x,u)$, $B(x,u)$, $C(x,u)$ and $D(x,u)$ are 'state dependent' matrices (although in some cases these matrices are also input dependent, the model will be referred to as 'state dependent'). The nonlinear dynamics of the urban wastewater system are contained within these matrices. Thus, the model is of a format similar to that of the linear form of the state space model (thus allowing for the use of predictive control algorithms). The format of the state dependent model used here was first demonstrated by Pearson et al. in 1962 [128], and was later used in the work of Burghart [24], and Wernli et al. [185], in 1969 and 1975 respectively. The nonlinear process can be seen to be described by the general form of $\dot{x} = f(x, u)$. This must be rearranged into a state dependent coefficient (SDC) format. The method used in this case is the direct approach, where the above equation is split into two functions, $f_1(x, u)$ and $f_2(x, u)$, where $f(x, u) = f_1(x, u) + f_2(x, u)$, so that the function $f_1(x, u)$ can be represented as

$$f_1(x,u) = \begin{bmatrix} f_{11}(x_1, x_2, \cdots x_n, u_1, u_2, \cdots u_n) \\ f_{12}(x_1, x_2, \cdots x_n, u_1, u_2, \cdots u_n) \\ \vdots \\ f_{1n}(x_1, x_2, \cdots x_n, u_1, u_2, \cdots u_n) \end{bmatrix} \tag{4.3}$$

which is transformed to the required SDC format by:

$$f_1(x,u) = A(x,u) \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \tag{4.4}$$

For a multivariable system, there are a number of factorisations that will construct this. Similarly for $f_2(x, u)$ the factorisation will take the following form

$$f_2(x,u) = \begin{bmatrix} f_{21}(x_1, x_2, \cdots x_n, u_1, u_2, \cdots u_n) \\ f_{22}(x_1, x_2, \cdots x_n, u_1, u_2, \cdots u_n) \\ \vdots \\ f_{2n}(x_1, x_2, \cdots x_n, u_1, u_2, \cdots u_n) \end{bmatrix} \tag{4.5}$$

which is similarly transformed to the SDC format by:

$$f_2(x,u) = B(x,u)\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \tag{4.6}$$

4.5 Simple System Model

For the purposes of nonlinear generalised predictive control, a simpler process model than that based on the ASM standards was required. To that end, an urban wastewater system model proposed by Graells [56] is detailed briefly in this section, followed by the transformation to state dependent form of this model. The urban wastewater model presented consists of the basic hydrodynamics and biological and chemical processes in the sewer, treatment plant and river. This utilises the treatment plant model by Nejjari et al. [114] in the representation of the effects of the influent data from the sewer system upon the effluent to the river. The treatment plant is modelled as activated sludge reactors, followed by a settler.

The growth of bacteria in an aerobic environment requires the presence of a soluble substrate for carbon and energy. A simplification can be made of the complex process to two processes of the biomass increase by cell growth and decrease by decay, together with the resulting processes of the use of oxygen and removal of substrate. Therefore the simplest model of this process must include the following three components: biomass, substrate and dissolved oxygen. The following system chosen represents the dynamics of these three variables (with an additional recycled biomass description). The effect of these treatment plant dynamics upon the river is represented by biological process equations, each individual section described by a continuous stirred tank reactor (CSTR). It does not involve the description of the nutrients that would typically be present in a wastewater system (such as nitrates, ammonia, phosphorous), but it does allow the modelling and control of dissolved oxygen and substrate (or BOD) levels throughout the system.

4.5.1 Treatment Plant Model

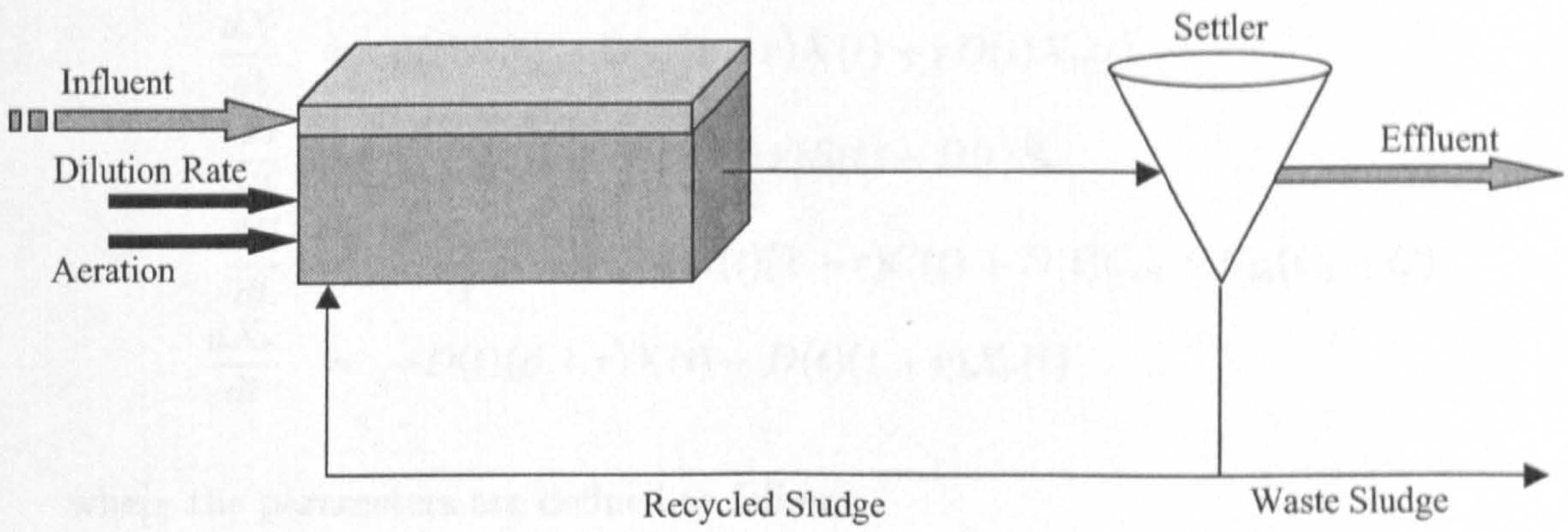

Figure 4-2: Simple Wastewater Treatment Plant: Activated Sludge Reactor and Settler, with Recycled Sludge Flow

The structure of the wastewater treatment plant system as shown in Figure 4-2, based on an activated sludge system as detailed in the work of Nejjari et al. [114], is assumed to consist of the following components:

- A bioreactor, considered here as a stirred tank, in which micro-organisms suspended there degrade the substrate, through biochemical degradation. This system is considered to be perfectly mixed, so that the concentration can be assumed to be spatially homogenous.
- A settler, in which the micro-organisms are assumed to be completely removed. No biological reactions are considered to take place within the settler. A fraction of the biomass present is recycled to the bioreactor, whilst the remainder is considered waste, in order to maintain a suitable level of organisms within the system.
- The inputs to the system are as follows: the wastewater influent, the aeration rate and the dilution rate.

An illustration of the described process is shown in Figure 4-2, where the process variables are defined as the biomass X, the recycled biomass X_r, substrate S and dissolved oxygen C, which are described by the following equations:

$$\begin{aligned}
\frac{dX}{dt} &= \mu(t)X(t) - D(t)(1+r)X(t) + rD(t)X_r(t) \\
\frac{dS}{dt} &= \frac{\mu(t)}{Y}X(t) - D(t)(1+r)S(t) + D(t)S_{in} \\
\frac{dC}{dt} &= \frac{-K_o\mu(t)}{Y}C(t) - D(t)(1+r)C(t) + D(t)C_{in} + K_{la}(C_s - C) \\
\frac{dX_r}{dt} &= -D(t)(\beta + r)X(t) + D(t)(1+r)X_r(t)
\end{aligned} \tag{4.7}$$

where the parameters are defined as follows:

- the above mentioned variables (X, S, C, X_r) are defined as the state variables
- r and β are the ratio of recycled flow to influent flow, and the ratio of waste flow to influent flow, respectively.
- $D(t)$ and $W(t)$ are defined as the manipulated variables of dilution rate and aeration rate, which control substrate and dissolved oxygen respectively.
- The constant yield coefficient is denoted as Y.
- The influent substrate and dissolved oxygen concentrations are represented by S_{IN} and C_{IN} .
- K_o is assumed to be constant, and C_S is the maximum dissolved oxygen concentration.
- The transfer of oxygen is dictated by the oxygen mass transfer function, K_{la}, which is assumed to be linearly proportional to the air flow rate $W(t)$ by the following relationship, where $\alpha > 0$: $K_{la} = \alpha W(t)$.
- The biomass specific growth rate is defined by

$$\mu(t) = \mu_{\max}\frac{S(t)}{K_S + S(t)}\frac{C(t)}{K_C + C(t)} \tag{4.8}$$

where $\mu_{\max}$ is the maximum specific growth rate, K_S is the affinity constant and K_C is the saturation constant. The chosen state dependent form for the above treatment plant process model is as follows:

$$
\begin{bmatrix} \dot{X}(t) \\ \dot{S}(t) \\ \dot{C}(t) \\ \dot{X}_r(t) \end{bmatrix} = \begin{bmatrix} \mu(t) - D(t)(1+r) & 0 & 0 & 0 \\ \frac{\mu(t)}{Y} & 0 & 0 & 0 \\ \frac{-K_o\mu(t)}{Y} & 0 & 0 & 0 \\ (1+r)D(t) & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X(t) \\ S(t) \\ C(t) \\ X_r(t) \end{bmatrix} + \begin{bmatrix} rX_r(t) & 0 \\ S_{in} - (1+r)S(t) & 0 \\ C_{in} - (1+r)C(t) & C_S - C \\ -(\beta + r)X_r(t) & 0 \end{bmatrix} \begin{bmatrix} D(t) \\ W(t) \end{bmatrix} \tag{4.9}
$$

A reduced model, such as that of the above, allows the representation of some of the nonlinear behaviour of a wastewater treatment system, in which micro-organisms biochemically degrade substrate, for the purposes of research. The limitation of this model is the opportunity for control of only two wastewater plant concentrations; the residual substrate and dissolved oxygen in the treatment plant effluent. There exists a lack of modelling of significant characteristics of the treatment plant system, such as the pollutant concentrations of nitrates, ammonia and phosphorous, discussed in Section 4.6.1 and the assumption of an ideal settler process.

4.5.2 Urban Wastewater System Model

The system modelled by [56] consists of an urban area, comprised of a catchment area nearby to a river, which uses a sewer system (here the traditional combined sewer system) to transport waste to a treatment facility. The effluent of this treatment plant flows to the receiving waters, which may be additionally affected by the CSO's from the sewer network. The structure of urban wastewater system is as shown in the block diagram in Figure 4-3. In the catchment area, a constant flow of human waste combines with a varying flow of runoff water, which together form an influent to the sewer network. In this way, the rainfall within the catchment area affects the load to the treatment plant system. The runoff is modelled by the unit hydrograph, which was originally proposed by Sherman [156]. A hydrograph is a graph that shows the change in water quantity or other characteristics (level, flow, velocity, etcetera) over time, which in this case describes the response to an inch of rainfall applied uniformly to a catchment area, at a uniform rate during its specified duration.

The flow produced then by this catchment drains into a detention tank prior to its progression through the sewer network. A simple model is used to represent the pipe network of the sewer, the method for modelling the sewer was developed in Marinaki [102]. Due to the fact that there is very little mixing in the network, it is reasonable to assume only hydraulic effects in the sewer. Thus the load from the catchment areas does not change in terms of chemical representation whilst being transported through the network. Due to the lack of control handles within this sewer network, the sewer model will not be considered within the state dependent representation of the urban wastewater model developed in this chapter.

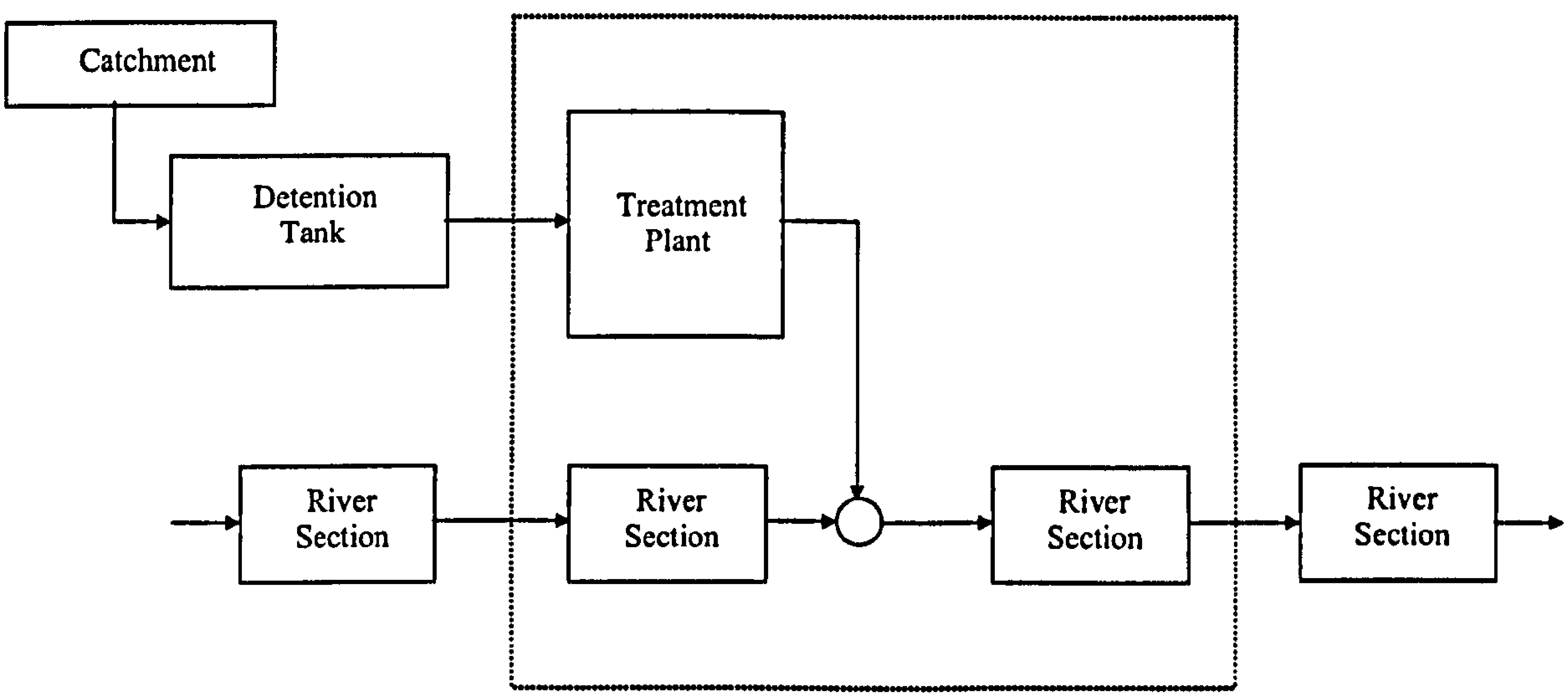

Figure 4-3: Simple Urban Wastewater System: Sewer, Treatment Plant and Receiving Water Models (Selected Area is that Considered for SDC Representation)

The river network consists of sections, each of which is modelled using the concept of a continuously stirred reactor, and some simple hydraulic dynamic equations. A key factor in determining the effect of waste upon a river is the oxygen demand upon the process. This commonly requires the use of biological treatment and traditionally was one of the main considerations in the design and operation of a treatment plant. In addition to the importance of the dissolved oxygen modelling therefore is the inclusion of both readily biodegradable and slowly biodegradable biological oxygen demand: BOD_r and BOD_s (in literature, the BOD acronym is defined as either 'biological' or 'biochemical' oxygen demand).The fractions modelled therefore within the river model are the total flow Q, dissolved oxygen C, and the two BOD fractions above.

Commonly within the wastewater treatment industry, there are existing control technologies in place, the majority of which are lower level control systems, such as on-off control or PI control, implemented via PLC. The control scheme assumed in the case of the above model is that of a PI control loop within the treatment plant only. This is a common control design traditionally, considering the treatment plant component individually, without reference to the urban wastewater system. The control therefore consists of a PI loop controlling the airflow rate for the plant $W(t)$ with respect to the dissolved oxygen level in the effluent from the treatment plant.

The treatment plant dynamic equations considered in this model are those described within the previous section. The CSTR model of the river dynamics must therefore be detailed. The state vector of the CSTR river model $[Q\ X_{DO}\ BOD_R\ BOD_S]$ differs from that of the treatment plant and thus appropriate conversions must be defined. The relationship between the substrate in the treatment plant,S, and the total BOD (the sum of the BOD_R and the BOD_S concentrations) in the receiving waters is described by

$$BOD = K * S = BOD_R + BOD_S \tag{4.10}$$

where

$$\begin{aligned} BOD_R &= K_1 * BOD \\ BOD_S &= (1 - K_1) * BOD \end{aligned} \tag{4.11}$$

The assumption is made in the treatment plant model that zero biomass is present in the effluent, and this implies therefore that the BOD is composed entirely of substrate, S. In this case, therefore, the constant K is defined as $K = 1$. The total BOD is assumed to be comprised by 80% BOD_R and 20% BOD_S, and therefore K_1 is defined as K_1=0.8. The process dynamics of the four states flow Q, dissolved oxygen X_{DO}, readily biodegradable BOD_R and slowly biodegradable BOD_S are described by the following equations:

$$\frac{dQ}{dt} = \frac{Q_{in}(t) - Q(t)}{\tau(t)} \tag{4.12}$$

$$\frac{dX_{DO}}{dt} = \frac{X_{DO,in}(t) - X_{DO}(t)}{\tau(t)} + R(t) + P(t) + SED(t) + D(t)$$

$$\frac{dX_{BOD_r}}{dt} = \frac{X_{BOD_r,in}(t) - X_{BOD_r}(t)}{\tau(t)} - (\nu_{sedR}\frac{1}{h(t)}(1 - f_{dR}) + k_{dR}\varphi_{Kd}^{T-20}\frac{X_{DO}(t)}{X_{DO}(t) + k_{DO}})X_{BOD_r}(t) \tag{4.13}$$

$$\frac{dX_{BOD_s}}{dt} = \frac{X_{BOD_s,in}(t) - X_{BOD_s}(t)}{\tau(t)} - (\nu_{sedS}\frac{1}{h(t)}(1 - f_{dS}) + k_{dS}\varphi_{Kd}^{T-20}\frac{X_{DO}(t)}{X_{DO}(t) + k_{DO}})X_{BOD_S}(t) \tag{4.14}$$

where $R(t)$ is the reaeration term, $P(t)$ is the photosynthesis term, $SED(t)$ is the sediment oxygen demand term and $D(t)$ is the deoxygenation term of the form below:

$$R(t) = k_a(DO_{sat} - X_{DO}(t)) \tag{4.15}$$

$$P(t) = a\eta(t)$$

$$SED(t) = \varphi_{SOD}^{T-20}\frac{SOD}{h_t(t)}$$

$$D(t) = -\varphi_{Kd}^{T-20}\frac{X_{DO}(t)}{X_{DO}(t) + k_{DO}}[X_{BOD_r,\infty}(t) + X_{BOD_r,\infty}(t)]$$

where $X_{BOD_{R,\infty}}$ and $X_{BOD_{S,\infty}}$: are the steady state values for X_{BOD_R} and X_{BOD_S} with zero mass flow through the tank boundaries, and where $\eta(t)$ describes the variation in sunlight intensity, which are defined by:

$$X_{BOD_{R,\infty}}(t) = X_{BOD,R}(t)\frac{k_{dR}}{1 - e^{-5k_{dR}}} \tag{4.16}$$

$$X_{BOD_{S,\infty}}(t) = X_{BOD,s}(t)\frac{k_{dS}}{1 - e^{-5k_{dS}}} \tag{4.17}$$

$$\eta(t) = \{ \begin{array}{ccc} \sin(\pi\frac{t_{day} - t_{sunrise}}{t_{daylength}}) & for & t_{day} \in [t_{sunrise}; t_{sunrise} + t_{daylength}] \\ 0 & & otherwise \end{array} \tag{4.18}$$

Hydraulic Parameters

The time constant t, volume of the tank V, cross section area A and depth h_t are defined as follows:

$$
\begin{aligned}
\tau(t) &= \frac{V(t)}{Q(t)} \\
V(t) &= A(t).l \\
A(t) &= h_t(t).b + \frac{(h_t(t))^2}{s} \\
h_t(t) &= h_w + h_m \frac{Q(t) - 111600}{Q_m - 111600}
\end{aligned}
\tag{4.19}
$$

The constant model parameters above (k_a, k_{do}, k_d, α, etcetera) are defined in the Appendix.

4.5.3 River CSTR Model Transformation to SDC

This model can be represented in the state dependent format, where the inputs to the model are upriver flow, upriver dissolved oxygen and the two oxygen demand variables upriver are modelled as the input vector $u = [Q_{in}, X_{DO,in}, X_{BOD_{R,in}}, X_{BOD_{S,in}}, \beta]^T$. The state vector x is similarly consists of $x = [Q, X_{DO}, XBOD_R, XBOD_S]^T$, which is also the output of the model, y.

$$
A(x,u) = \begin{bmatrix} -\frac{1}{\tau} & 0 & 0 & 0 \\ \frac{\varphi_{Kd}^{T-20}\frac{SOD}{h_t}}{Q} & \begin{matrix} k_a(\frac{DO_{sat}}{X_{DO}(t)} - 1) - \frac{1}{\tau} \\ -B_0[X_{BOD_r,\infty}(t) + X_{BOD_r,\infty}(t)] \end{matrix} & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau} + B_{BOD_R} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau} + B_{BODS} \end{bmatrix}
\tag{4.20}
$$

where

$$
B_{BOD_R} = -(\nu_{sedR}\frac{1}{h(t)}(1 - f_{dR}) + k_{dR}\varphi_{Kd}^{T-20}\frac{X_{DO}(t)}{X_{DO}(t) + k_{DO}})X_{BOD_r}(t) \tag{4.21}
$$

$$B_{BODS} = (\nu_{sedS}\frac{1}{h(t)}(1 - f_{dS}) + k_{dS}\varphi_{Kd}^{T-20}\frac{X_{DO}(t)}{X_{DO}(t) + k_{DO}})X_{BOD_s}(t)$$

$$B(x,u) = \begin{bmatrix} \frac{1}{\tau} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\tau} & 0 & 0 & \alpha \\ 0 & 0 & \frac{1}{\tau} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\tau} & 0 \end{bmatrix} \quad (4.22)$$

4.5.4 Urban Wastewater System Transformation to SDC

The model of Graells and Katebi [56] consists of both upriver and downriver models of up to 15 river stretches, each defined by the CSTR model above. The state dependent model considered for nonlinear control purposes consists of the following:

- Treatment Plant.
- Upriver CSTR stretch.
- Downriver CSTR stretch.

as modeled by the state dependent coefficient structures demonstrated in the sections above. The block diagram of the system is as shown in Figure 4-4.

The summation of concentrations present in the mixing between the treatment plant effluent and the concentrations already present in the receiving waters can be represented by the following equation:

$$X = \frac{1}{\sum_{j=1}^{n} Q_j}\sum_{j=1}^{n} Q_j X_j \quad (4.23)$$

where X is the concentration in question, and the flows through the point of mixing are denoted by Q. For example in this case, the dissolved oxygen mixing between the effluent ($X_{DO,1}$) and the concentration present in the river ($X_{DO,2}$) is dictated by the ratio of the magnitude of the flow from the treatment plant Q_1 and the flow through the river, Q_2, as by the above equation, where $n = 2$.

With the knowledge of the mixing equations, both the state dependent models of the river, which are of the same format, and also the knowledge of the treatment plant dynamics

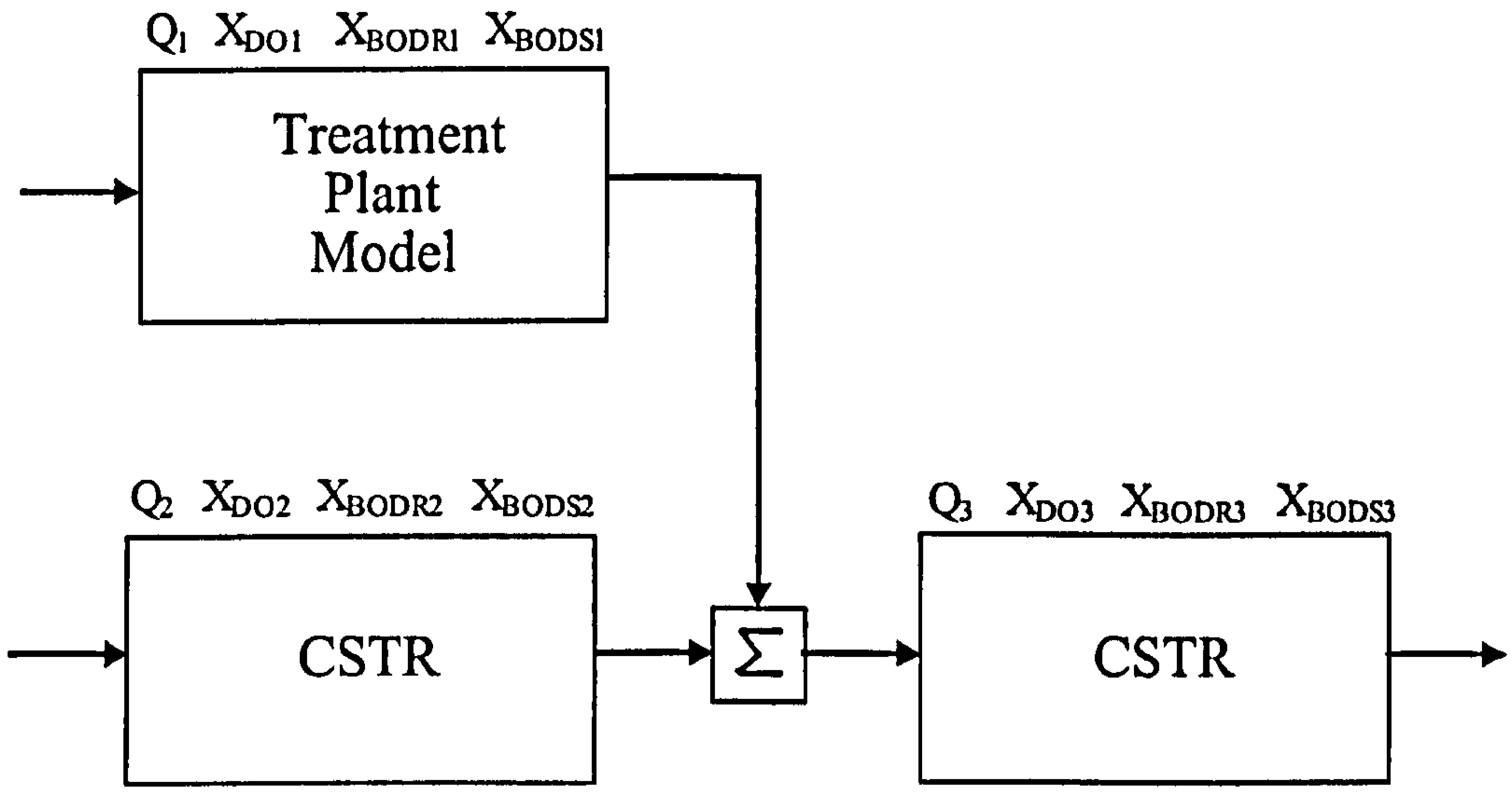

Figure 4-4: Urban Wastewater System Considered for State Dependent Model Transformation: - Two CSTR Models Representing River Dynamics - Treatment Plant Model as defined above

from the previous section, a state dependent model of the process can be built. This can be modelled by the following format, where $A_1(x)$, $B_1(x)$, $C_1(x)$ and $D_1(x)$ define the state dependent model of the treatment plant, $A_2(x)$, $B_2(x)$, $C_2(x)$ and $D_2(x)$ define the state dependent model of the upriver section and $A_3(x)$, $B_3(x)$, $C_3(x)$ and $D_3(x)$ define the state dependent model of the downriver section.

$$A_{uws} = \begin{bmatrix} \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} & 0 \\ A_{3,1} & A_{3,2} \end{bmatrix} \tag{4.24}$$

$$B_{uws} = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & \Gamma_d \\ \begin{bmatrix} 0 & B_3 \end{bmatrix} & 0 & \Gamma_d \end{bmatrix}$$

where $A_{3,1}$and $A_{3,2}$ are defined as:

$$A_{3,1} = \begin{bmatrix} 0 & 0 & 0 & 0 & -\frac{1}{\tau_{fin}} & 0 & 0 & 0 \\ 0 & 0 & \frac{Q_1}{Q_1+Q_2}\frac{1}{\tau_{fin}} & 0 & 0 & \frac{Q_2}{Q_1+Q_2}\frac{1}{\tau_{fin}} & 0 & 0 \\ 0 & (0.8 * \frac{Q_1}{Q_1+Q_2})\frac{1}{\tau_{fin}} & 0 & 0 & 0 & 0 & \frac{Q_2}{Q_1+Q_2}\frac{1}{\tau_{fin}} & 0 \\ 0 & (0.2 * \frac{Q_1}{Q_1+Q_2})\frac{1}{\tau_{fin}} & 0 & 0 & 0 & 0 & 0 & \frac{Q_2}{Q_1+Q_2}\frac{1}{\tau_{fin}} \end{bmatrix} \quad (4.25)$$

$$A_{3,2} = \begin{bmatrix} -\frac{1}{\tau_3} & 0 & 0 & 0 \\ \frac{\varphi_{Kd}^{T-20}\frac{SOD}{h_t}}{Q_3} & \begin{matrix} k_a(\frac{DO_{sat}}{X_{DO,3}(t)} - 1) - \frac{1}{\tau_3} \\ -B_0[X_{BOD_r,\infty,3}(t) \\ +X_{BOD_r,\infty,3}(t)] \end{matrix} & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau_3} + B_{BOD_R,3} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_3} + B_{BOD_s,3} \end{bmatrix}$$

and B3 is defined to be the matrix:

$$B_3 = \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{\tau_3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.26)$$

Finally, Γ_d is defined as the coefficient of the daylight measurement and is defined to be the following, where

$$\Gamma_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.27)$$

The state variables are defined by the vector

$$\begin{aligned} x = [\ & Q_1 \quad X_{DO,1} \quad X_{BOD_{R,1}} \quad X_{BOD_{S,1}} \\ & \cdots\ Q_2 \quad X_{DO,2} \quad X_{BOD_{R,2}} \quad X_{DO_{s,2}} \\ & \cdots\ Q_3 \quad X_{DO,3} \quad X_{BOD_{R,3}} \quad X_{DO_{s,3}}\ ]^T \end{aligned} \quad (4.28)$$

where:

- Q_1, Q_2, and Q_3 are the effluent flow from the treatment plant, the upriver flow and the downriver flow, respectively.

- $X_{DO,1}$, $X_{DO,2}$, and $X_{DO,3}$ are the dissolved oxygen concentration in the effluent, upriver and downriver flows respectively.

- $X_{BOD_{R,1}}$, $X_{BOD_{R,2}}$, and $X_{BOD_{R,3}}$ are the readily biodegradable biological oxygen demand in the effluent, upriver and downriver flows respectively.

- $X_{BOD_{S,1}}$, $X_{DO_{s,2}}$, and $X_{DO_{s,3}}$ are the slowly biodegradable biological oxygen demand in the effluent, upriver and downriver flows respectively.

The input variables are defined by the vector

$$u = \begin{bmatrix} D & W & Q_1 & Q_{2,in} & X_{DO2,in} & X_{BODR2,in} & X_{BODS2,in} & \eta \end{bmatrix}^T \tag{4.29}$$

where

- the time t is utilised in calculation of the daylight measurement η.

- The variables $Q_{2,in}$, $X_{DO2,in}$, $X_{BODR2,in}$ and $X_{BODS2,in}$ are defined as the upstream measurements of flow, dissolved oxygen concentration and readily and slowly biodegradable biological oxygen demand respectively.

Clearly not all of the above variables in the input vector u are used to control the downriver processes, indeed the available control actions remain the dilution rate and air flow rate, $D(t)$ and $W(t)$ respectively. Thus in the case of nonlinear control of the urban wastewater system based upon the above model, the considered structure of the process model is defined to be:

$$\dot{x} = Ax + B_o u_c + B_m d_m \tag{4.30}$$

where the control actions u_c are defined as $u_c = \begin{bmatrix} D(t) & W(t) \end{bmatrix}$ and the measured disturbance vector is defined as $d_m = \begin{bmatrix} \eta & Q_1 & Q_{2,in} & X_{DO2,in} & X_{BODR2,in} & X_{BODS2,in} \end{bmatrix}$. The A_{uws} state dependent matrix remains as defined above. The matrices B_o and B_m are defined as

$$
B_o = \begin{bmatrix} rX_r(t) & 0 \\ S_{in} - (1+r)S(t) & 0 \\ C_{in} - (1+r)C(t) & C_S - C \\ -(\beta + r)X_r(t) & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad B_m = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\tau_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\tau_2} & 0 & 0 & \alpha \\ 0 & 0 & 0 & \frac{1}{\tau_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\tau_2} & 0 \\ -\frac{1}{\tau_3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.31}
$$

4.6 Complex Urban Wastewater Model

The second model, developed by Camilleri et al. [31], consists as above of a treatment plant modelled as activated sludge processes followed by a settler. The former is represented by the ASM2d model presented by [63], and the latter by a model developed by Takacs et al. [173]. The influent to this treatment plant model is the effluent data of a simple sewer model, representing the basic physical processes of a combined sewer network. This allows the simulation of rain events which impact both the flow and concentrations into the treatment plant, and also the effects of this increased flow upon the river directly, via combined sewer overflows. As in the previous section, the sewer dynamics are considered only with respect to the effects upon the influent of the treatment plant, and the CSO's to the receiving waters. The river model is represented by a sequence of continuous stirred tank reactors, with hydraulics and dynamic chemical and biological processes based on the QUAL2E model [19].

Again, it is assumed that the unit hydrograph approach is utilised in the modelling of the characteristics of the urban catchment. The additional system variables of nutrient concentrations (ammonia, nitrates, phosphates, bicarbonates) and organic matter (biomass, particulate material, inert organics) are included in this sewer model. As previously assumed, any mixing or chemical reactions that happen in the sewer, or the CSTR detention tank, during transportation do not significantly affect the concentrations of variables in the sewer. The catchment influent data is as proposed by Camilleri [31]. The secondary settler within the treatment plant model is again defined as a ten layer clarifier, modelled by the Takacs approach. It is assumed that two recycles exist in the anoxic-aerobic treatment plant system: an internal recycle prior to the clarifier, and an external recycle from the clarifier itself.

4.6.1 Treatment Plant Modelling

In the treatment plant model as shown in Figure 4-5, the following processes are modelled:

- Biological processes (reduction of organic content (to avoid oxygen removal), reduction of nutrients (nitrate, phosphate)), any removed material taken as sludge.
- Secondary sedimentation, i.e. separation of activated sludge from process, removed to sludge, as above. The effluent of the secondary settler continues to the receiving water.
- Simultaneously, although not modeled here, the sludge taken off is thickened (by removal of liquid), stabilised (pathogens and odour removed, as well as further treatment), conditioned and dewatered (chemically, physically or with heat, to remove further liquid)

4.6.2 Activated Sludge Tank: Mass Balance Equations

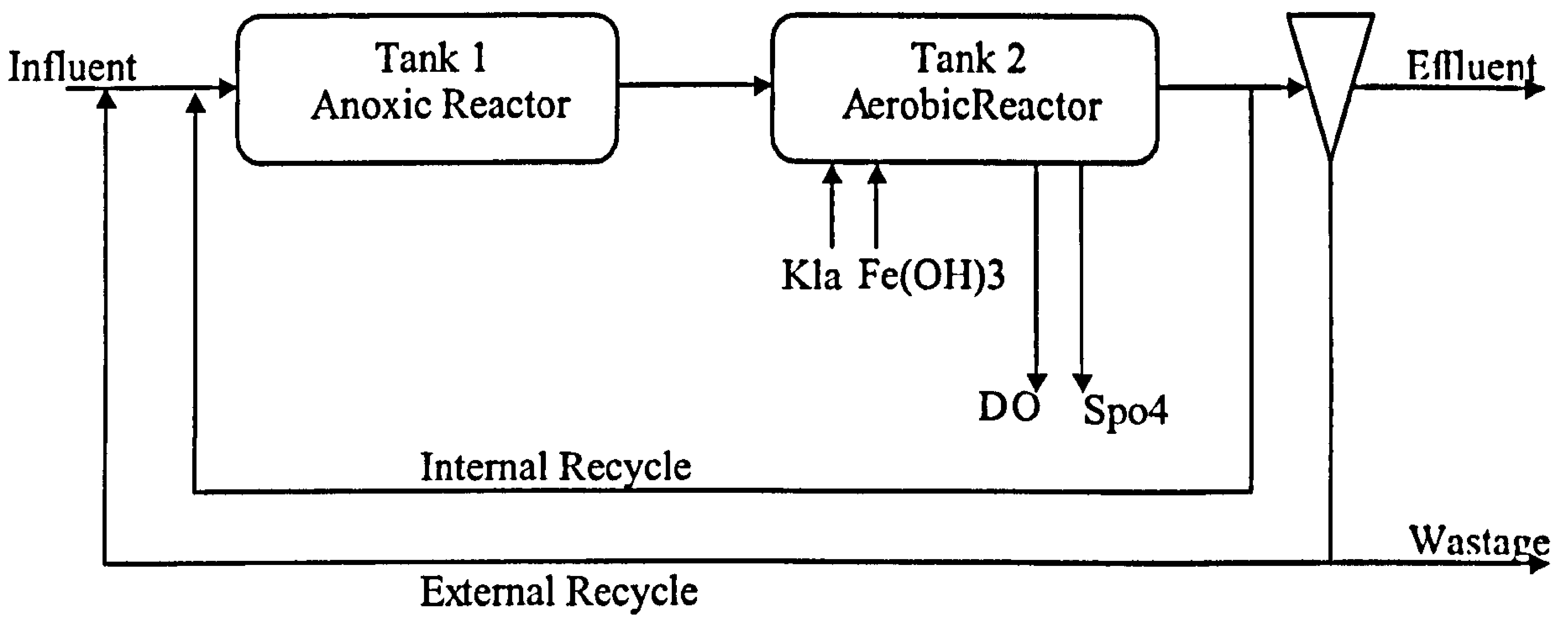

Figure 4-5: Treatment Plant Considered within the ASM2d/QUAL2E Based System: Anoxic Tank followed by Aerobic Tank, Takacs-Model Based Settler

The activated sludge tank is modelled according to the ASM2d model introduced by Henze et al. [63]. Certain processes, however, are not considered within the model considered, specifically those of the PAO (phosphorous accumulating organisms). These processes therefore are neither modelled in the original system (the process dynamics are set to a derivative of zero, i..e constant), nor in the state dependent form. The system equations (as shown in [63]) for the processes of the following are considered:

- Hydrolysis processes (anaerobic hydrolysis is not considered, the two tanks present in this model are of aerobic or anoxic processes, thus aerobic/anoxic hydrolysis is modelled).
- Fermentable growth processes (for fermentable substrates and fermentable products).
- Fermentable denitrification processes (for fermentable substrates and fermentable products) - fermentation itself however is not considered.
- Lysis.
- Aerobic, and Lysis, of X_{AUT}.
- Precipitation, and redissolution.

For each component mass balance, it can be seen that the system follows the definition as stated in [63]

$$\text{Input-Output+Reaction=Accumulation} \tag{4.32}$$

The mass balance consists of the transport terms of 'input' and 'output' which are dependent upon the system characteristics, in this case defined by the physical volume, the concentrations and the flow through the system. The inputs consist of influent concentrations into the tank (i.e. 20 state variables), as well as $FeCl_3$ concentration and aeration value. The outputs then consist of the effluent concentrations from the tank. Each reactor above can therefore be modelled with 22 inputs and 20 outputs, via the use of the ASM2d, where the mass balance approach is utilised and defined by the following equation:

$$\frac{dC}{dt} = \frac{Q_{in}}{V}C_{in} - \frac{Q_{out}}{V}C_{out} + r \tag{4.33}$$

where the concentrations C are defined to be the state variables and the rate equations are dependent upon the system processes as shown in Appendix B. The dissolved oxygen process assumes an external oxygen source, via aeration. Thus the dissolved oxygen process is represented as

$$\frac{dS_{o,2}}{dt} = \frac{Q_{in}}{V}S_{o,2,in} - \frac{Q_{out}}{V}S_{o,2,out} + r + K_{la}(S_{o,sat} - S_o) \tag{4.34}$$

Similarly for the state equation of X_{MeOH}, the addition of the external FeCl3 input results in the following process equation:

$$\frac{dX_{MeOH}}{dt} = \frac{Q_{in}}{V}X_{MeOH,in} - \frac{Q_{out}}{V}X_{MeOH,out} + r + \frac{FeCl_3}{V} \tag{4.35}$$

The components considered for the above mass balance equations are as follows:

- Organic components, excluding organic nitrogen and organic phosphorus (in gCOD/m^3): S_f readily biodegradable substrate, S_a fermentation products, S_I inert, non biodegradable organics, X_I inert, non biodegradable organics, X_S slowly biodegradable substrate, X_H heterotrophic biomass, X_{AUT} autotrophic, nitrifying biomass.

- nitrogen components in gN/m^3, S_{NH4} Ammonium, S_{NO3} Nitrate (plus nitrite), S_{N2} Dinitrogen.

- phosphorus components in gP/m^3 and gFePO$_4$/m^3 respectively, S_{PO4} Phosphate,

X_{MeP} Ferric-phosphate, $FePO_4$.

- other components: S_{O2} Dissolved Oxygen (gO_2/m^3), S_{ALK} bicarbonate alkalinity (mole HCO_3-/m^3), X_{MeOH} Ferric hydroxide, $Fe(OH)_3$ ($gFe(OH)_3/m^3$), X_{TSS} Total Suspended Solids ($gTSS/m^3$).

Matrix Representation

Any component's concentration within a system can be affected by several biological processes. The rate equations for processes in the ASM models are structured in a matrix representation, which allows for ease of description of the mass balance equations of each component. In the particular case of state dependent modelling, it provides a simpler method by which the transformation is made to the state dependent coefficient form. The stoichiometric coefficients defined for the processes allow the description of the relationships between components for particular processes. This allows the model to describe the mass relationships consistently. The system reaction term, r_i, is obtained as the summation of the product of the stoichiometric coefficients v_{ij} and the process rate expression ρ_j for the component i:

$$r_i = \sum_j v_{ij}\rho_j \tag{4.36}$$

An example: Dissolved Oxygen

Process Equation and Reaction Terms

In order to construct the state dependent equations of the treatment plant, it is required that state dependent equations of the ASM2d processes be defined. The process equation, for example for dissolved oxygen, is of the form:

$$\dot{S}_{o2} = \frac{Q_{in}}{V}S_{o,2,in} - \frac{Q_{out}}{V}S_{o,2eff} + (1-\frac{1}{Y_H})p_4 + (1-\frac{1}{Y_H})p_5 + (-Y_{PHA})p_{11} + (v_{13,02})p_{13} - (\frac{4.57 - Y_A}{Y_A})p_{18} \quad (4.37)$$

where each p product refers to specific processes. The reaction terms for each state variable are defined by the process equations p (as described in Appendix B) and the stoichiometric parameters, such as those defined by parameters K_{O2}, K_{PO4}, K_{PHA} etcetera above. Each state consists of a number of process equations, in addition to the hydraulic terms (such as $\frac{Q_{in}}{V}$). In defining the process within the state dependent form, a state dependent coefficient matrix consisting of the reaction terms can be considered. This is defined to be of a product of the following matrices:

- A constant gain matrix A_{mod} (as shown in Appendix C)
- A state dependent process matrix M_p and a state matrix x.

In this form, the resulting state dependent matrix $A(x)$ in the form $A_{mod} * M_p$. The state matrix $x(t)$ of the ASM2d model is constructed of the 20 state variables as defined above. However, for the purposes of control, several state variables are not required for the state dependent coefficient form. The variables S_i, S_{n2}, X_i, X_{pao}, X_{pp}, X_{pha} and X_{tss} are not utilised in the process equations of the remaining variables and do not affect the controlled concentrations, or prediction of future behaviour of the considered process. Therefore the state variable is defined as $x = \begin{bmatrix} S & X & Q \end{bmatrix}$, where the components are defined as

- $S = \begin{bmatrix} S_{o2} & S_f & S_a & S_{nh4} & S_{no3} & S_{p04} & S_{alk} \end{bmatrix}$
- $X = \begin{bmatrix} X_s & X_h & X_{aut} & X_{meoh} & X_{mep} \end{bmatrix}$
- flow, Q

The input vector u is defined as

$$u = \begin{bmatrix} x_{in} & K_{la} & u_{fe} \end{bmatrix} \quad (4.38)$$

where the input concentrations to the aerobic tank are defined as $x_{in} = \begin{bmatrix} S_{in} & X_{in} & Q_{in} \end{bmatrix}$ as defined by the state vector above, the air flow rate control action is defined by the mass transfer coefficient K_{la} and the dosage of $Fe(OH)_3$ is defined as u_{fe}. The state dependent matrix is chosen so that in no case should there be zero denominator.

$$
Mp = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{p(1)}{x(11)} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{p(2)}{x(5)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{p(4)}{x(1)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{p(5)}{x(3)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{p(6)}{x(2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{p(7)}{x(4)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{p(9)}{x(12)} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{p(18)}{x(6)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{p(19)}{x(16)} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{p(20)}{x(18)} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{p(21)}{x(19)}
\end{bmatrix}
$$

The tank reaction term r can be defined therefore by the following equation:

$$r = A_{\text{mod}} * M_p * x \tag{4.39}$$

Hydraulic Terms

The CSTR nature of the activated sludge reaction tank allows the assumption that components (both soluble and particulate) are dispersed evenly and immediately throughout the tank. For this reason, it can be assumed that the concentrations at the tank effluent are the concentrations present within the tank. Additionally, the influent flow is assumed equal to the effluent flow and thus the following assumption can be held

$$Q = Q_{in} = Q_{out} \tag{4.40}$$

The concentration in the effluent is the concentration of the variables within the tank, i.e. the output of the model is the value of the states. Again, in the case of the influent flow, the assumed state variables

4.6.3 State Dependent ASM2d Model

The state dependent model of an activated sludge tank can therefore be defined according the above, as follows:

$$
\begin{aligned}
A(x) &= A_{\text{mod}} M_p \\
B(x,u) &= \begin{bmatrix} Q/V & 0 & \cdots & 0 & x_1 & S_{o,sat} - x_1 & 0 \\ 0 & Q/V & \cdots & 0 & x_2 & 0 & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & 1/V \\ 0 & 0 & \cdots & Q/V & x_{12} & 0 & 0 \end{bmatrix}
\end{aligned} \tag{4.41}
$$

It can be seen from the process equation above that the inputs to the activated sludge tank are dependent upon an input variable (the flow variable, Q). As in the previous section, the input vector of the above state dependent model is composed of control actions $u_c = \begin{bmatrix} K_{la} & u_{fe} \end{bmatrix}$ and measured disturbances x_{in}.

$$
B_o = \begin{bmatrix} S_{o,sat} - x_1 & 0 \\ 0 & \vdots \\ \vdots & 1/V \\ 0 & 0 \end{bmatrix} \tag{4.42}
$$

$$
B_m = \begin{bmatrix} Q/V & 0 & \cdots & 0 & x_1 \\ 0 & Q/V & \cdots & 0 & x_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & Q/V & x_{19} \end{bmatrix} \tag{4.43}
$$

4.6.4 Storm Events

The flow in the sewer is considered to be the result of mixing of the storm water runoff and the wastewater load of the catchment within the combined sewer. The storm conditions as considered within the simulated rain events of this thesis result in an increased flow rate to the treatment plant and an overflow to the river. The characteristics of the CSO discharge, the concentrations of nutrient loads etcetera, are assumed to follow that of the wastewater characteristics of the influent flow to the treatment plant, as shown in Figure 4-6, differing only in magnitude of flow. The subsequent effect upon the receiving waters of the increased load to the treatment plant and the combined sewer overflow is shown in Figure 4-7.

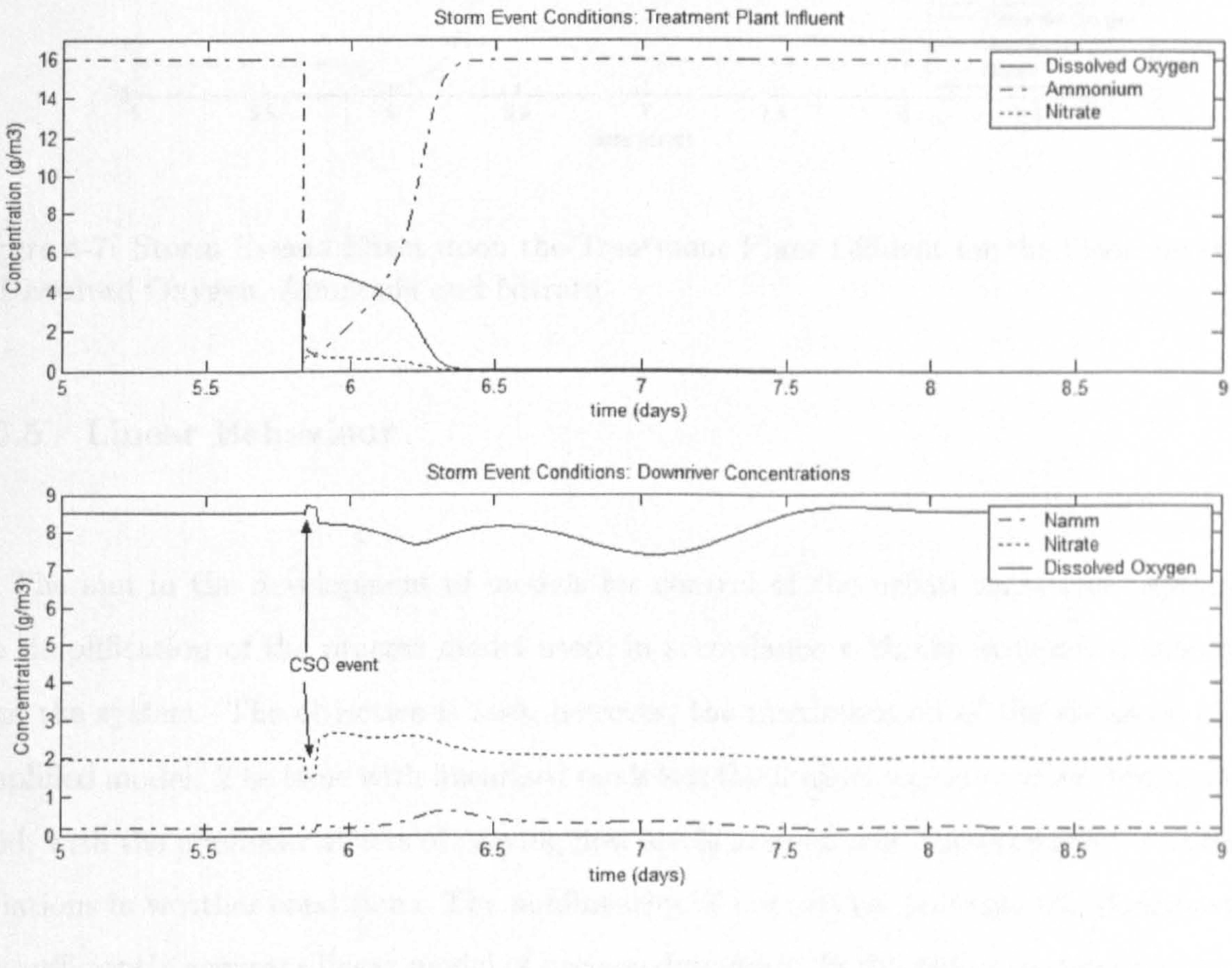

Figure 4-6: Storm Events Affect upon the Treatment Plant Influent and the Receiving Waters for the Concentrations of Dissolved Oxygen, Ammonia and Nitrate

The two responses above do not reveal the entire behaviour of the system. Instead, the effluent behaviour from the treatment plant must be inspected, wherein it can be seen that the dissolved oxygen peak in the influent flow is not present in the effluent flow. The increased flow rate through the plant instead affects the treatment plant performance and thus results in a decrease in dissolved oxygen levels in the effluent. It is considered therefore that manipulation of treatment plant performance, and thus concentrations of effluent components, could allow improvements in the quality of the receiving waters.

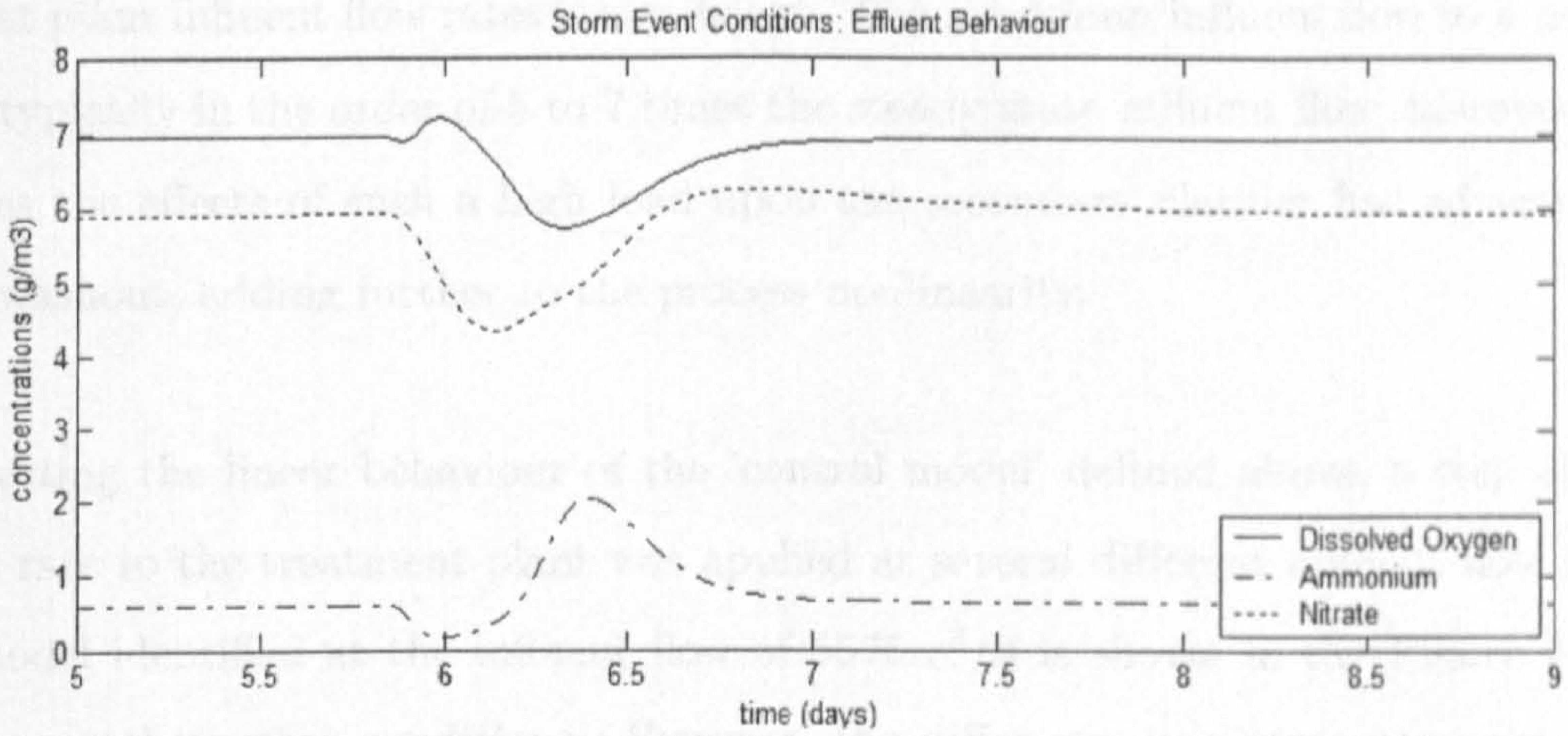

Figure 4-7: Storm Events Effect upon the Treatment Plant Effluent for the Concentrations of Dissolved Oxygen, Ammonia and Nitrate

4.6.5 Linear Behaviour

The aim in the development of models for control of the urban wastewater system is the simplification of the process model used, in accordance with the available information from the system. The objective is also, however, the maximisation of the accuracy of the simplified model. The issue with linearised models is the limited region in which the model is valid, with the nonlinear effects of varying flow levels and influent characteristics, caused by variations in weather conditions. The nonlinearity of the process prevents the identification of a sufficiently accurate linear model of process dynamics. In the urban wastewater control proposed within this thesis, the influent flow to the treatment plant is used as an indicator of the presence of an increase in load due to rainfall, and therefore the introduction of nonlinearities to the system.

In normal weather conditions, a steady state influent flow to the treatment plant of $5575\text{m}^3/d$ is present. In the identification of an appropriate 'control model', the process is considered as: a manipulated variable of aeration rate within the treatment plant, a feedforward variable of upriver dissolved oxygen measurements, and a controlled variable of the downriver dissolved oxygen concentration. Identification of the system dynamics at steady state influent flow may result in a suitably accurate system model at that given operating condition. However, as Figure 4-8 indicates, the difference of behaviour at varying treatment plant influent flow rates is significant. The maximum influent flow to a treatment plant is typically in the order of 5 to 7 times the steady state influent flow, however at such flow rates the effects of such a high load upon the secondary clarifier has adverse effects, such as washout, adding further to the process nonlinearity.

Inspecting the linear behaviour of the 'control model' defined above, a step change in aeration rate to the treatment plant was applied at several different influent flow rates. A linear model identified at the influent flow of $5575\text{m}^3/d$ is shown in the Figure 4-8 to be valid in normal weather conditions. However, the difference in system response at varying flow rates, results in a linear model unable to describe variations in dissolved oxygen concentrations during storm events. Vanrolleghem [183] states that the popular approach of linearisation of the nonlinear model around a specific operating point is not acceptable, due to the high variation in process conditions, leading to new operating points where the linearised model no longer represents process behaviour accurately. Vanrolleghem proposes that control must be adapted to accommodate these nonlinearities, and suggests the possible use of a gain scheduled approach.

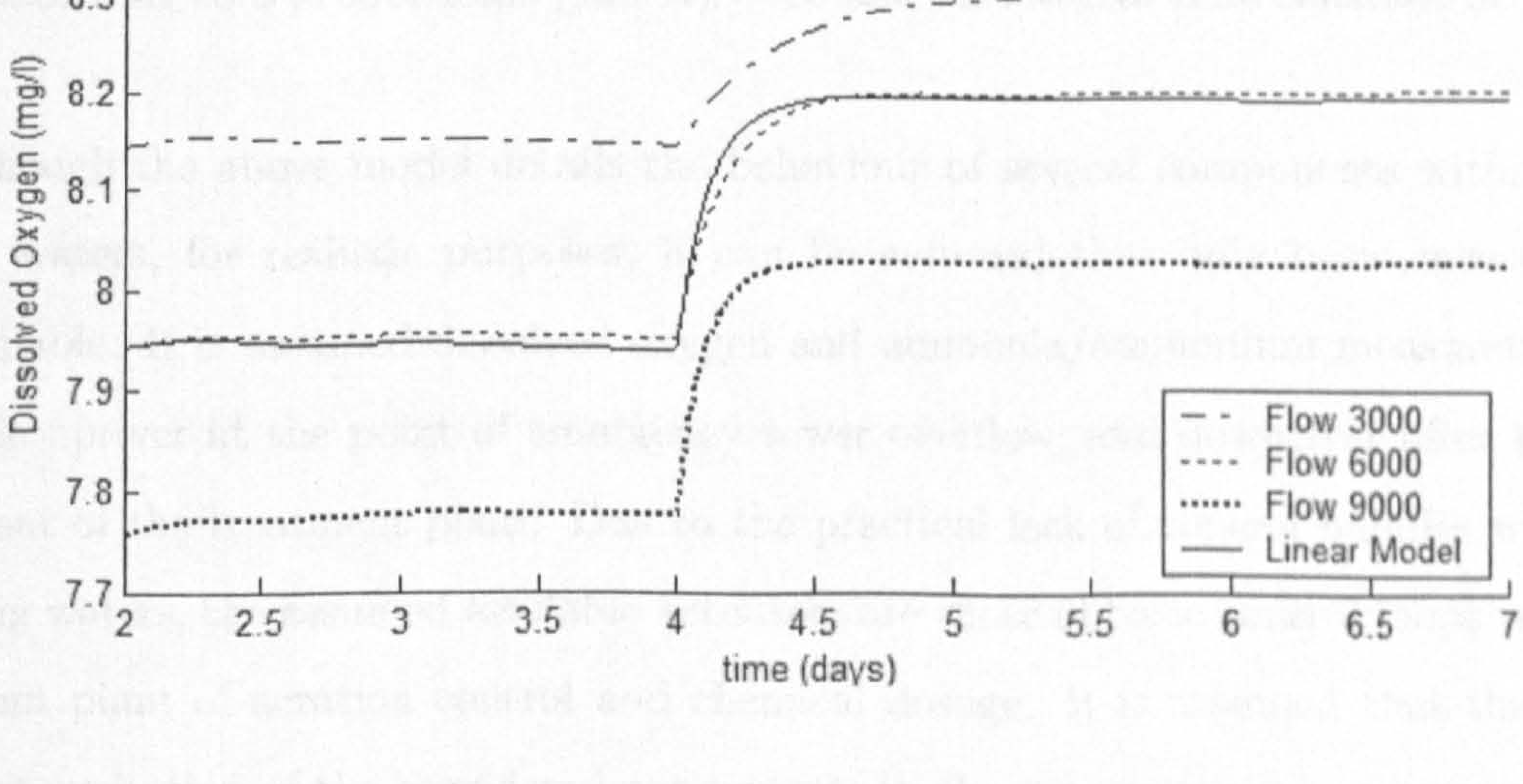

Figure 4-8: Changing Responses of Dissolved Oxygen Concentration in Receiving Waters to a Step Change in Dissolved Oxygen PI Control Setpoint within the WWTP over Varying WWTP Influent Flows

4.6.6 Linear Predictive Control

Problems in urban wastewater systems arise when storm conditions cause high influent loads to the treatment plant and indirectly therefore disturbances to river quality, and/or combined sewer overflows directly to the river itself. The depletion in concentrations such as dissolved oxygen, and an increase in nutrient loads such as ammonia, result from the above disturbances. It is the objective of a control scheme in the maintenance of river quality to meet the control objectives stated in Chapter 1. The aim is therefore, for example in the case of dissolved oxygen, to minimise the length of time for which oxygen depletion occurs, and minimise the effects, that is increase the value of minimum dissolved oxygen concentration occuring in the river. The disturbance rejection therefore should allow the system to recover to normal conditions, maximising the disturbance rejection. The process considered is as presented in Figure 4-9.

The above linear model for the steady state influent flow of $5575\text{m}^3/\text{d}$ was used in the design of a linear predictive controller for the purposes of manipulation of dissolved oxygen concentrations in the receiving waters. The predictive controller was designed to act as a higher level controller calculating an optimal setpoint for a lower level inner loop, in this case, PI control. The existing plant PI controller for dissolved oxygen concentrations in the aerobic reaction tank was used in the simulations carried out within this chapter, and was tuned according to a proportional gain $K_p = 85$ and an integral time constant of $T_i = 130$.

Although the above model details the behaviour of several components within the receiving waters, for realistic purposes, it can be assumed that only basic measurements are available. It is assumed dissolved oxygen and ammonia/ammonium measurements are available upriver at the point of emergency sewer overflow, and downriver after the point of effluent of the treatment plant. Due to the practical lack of control handles within the receiving waters, the assumed available actuators are those of basic control loops within the treatment plant of aeration control and chemical dosage. It is assumed that there exists sufficient excitation of the considered components in the process for the purposes of model identification.

In this case, the control objective for the linear predictive controller in the above system

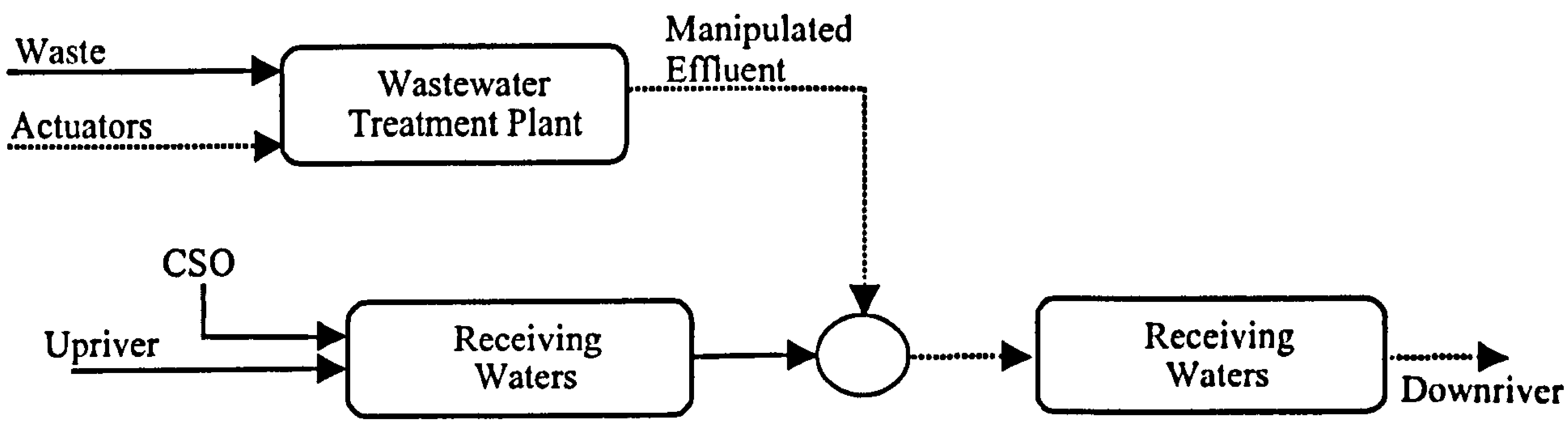

Figure 4-9: Structure of ASM2d/QUAL2E Urban Wastewater System Model as considered for control

was the maintenance of dissolved oxygen concentrations in the river, with the aim of minimising the effects of storm weather upon the receiving waters. A single linear controller was therefore designed for the flow range of $5575\text{m}^3/d$, with the following tuning parameters as shown in Table 4.1.

Table 4.1 UWS Linear MPC Tuning Parameters

T_s	Q	λ	H_u	H_p
0.05	40	0.0005	5	9

The unmeasured constant disturbance model was included in the 'control model' to allow some compensation for the modelling mismatch with the nonlinear process. State estimation was implemented with the use of a Kalman filter, with noise covariances $Qn = 0.7 * I$ where the identity matrix I is of dimension R^{nxn}, where n is the number of model states) and $Rn = 0.7$.

The Figure 4-10 compares the performance of dissolved oxygen in the receiving waters for two situations:

- The original PI control of the treatment plant: in which the control objective is the regulation of the dissolved oxygen concentration in the treatment plant effluent.
- The MPC control of the urban wastewater system: in which the control objective is the regulation of the dissolved oxygen concentration in the receiving waters, via

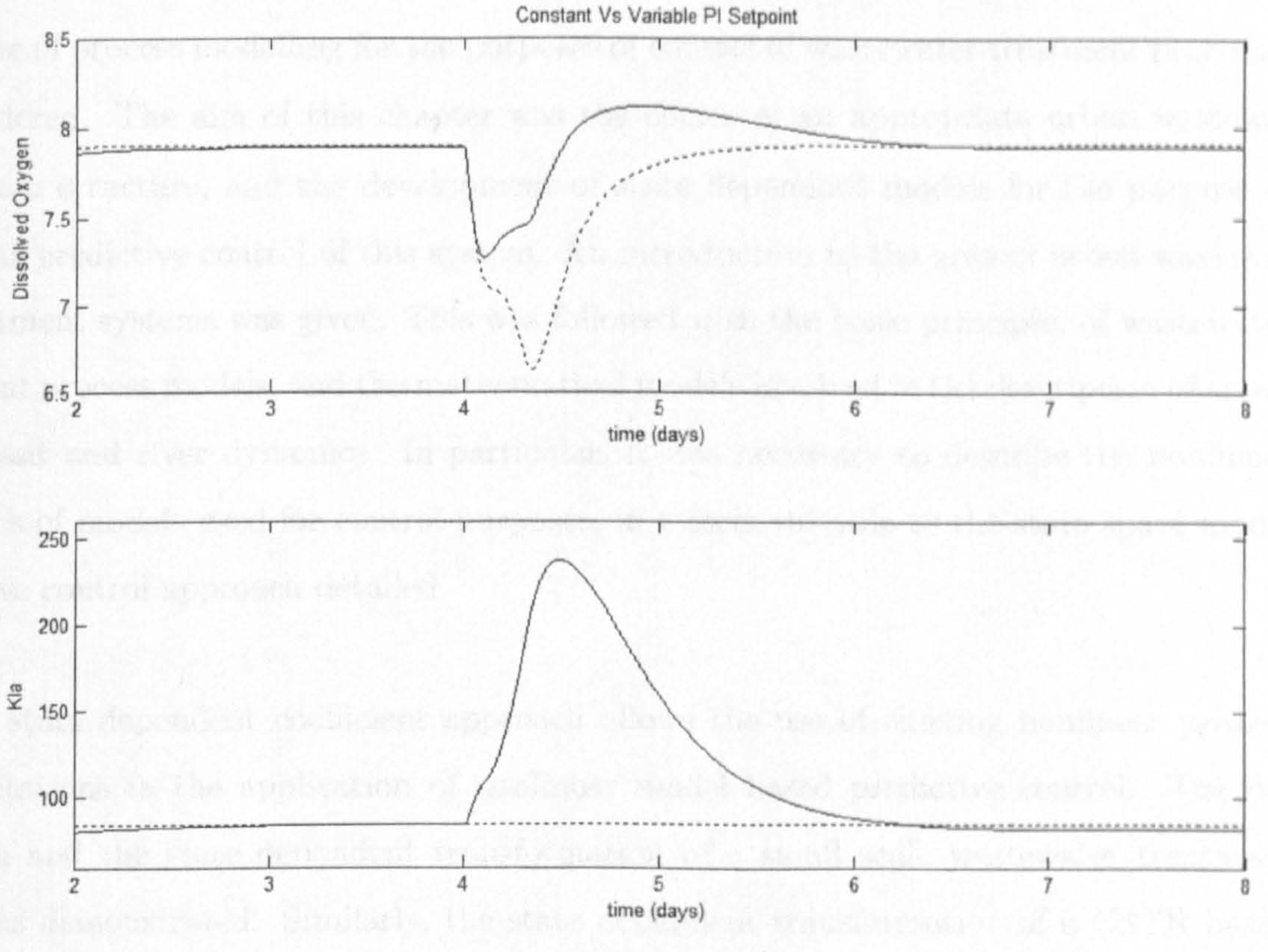

Figure 4-10: Dissolved Oxygen Concentration in the Receiving Waters: Comparison of WWTP PI Dissolved Oxygen Control with UWS MPC Dissolved Oxygen Control and Resulting Air Flow Rate to Aerobic Tank

manipulation of the above PI control and thus its concentration in the treatment plant effluent.

A change of the control, from the treatment plant objective to that of the receiving waters, allows the system to recover from storm events in a shorter time period, and decreases the maximum effect upon the dissolved oxygen concentration in the river. However, the mismatch between the linear model and the system dynamics increases with the magnitude of influent flow change from steady state. Thus it is necessary to consider further techniques in the control of the nonlinear processes within the urban wastewater system.

4.7 Summary

The issue of process modelling for the purposes of control of wastewater treatment processes is considered. The aim of this chapter was the choice of an appropriate urban wastewater system structure, and the development of state dependent models for the purpose of nonlinear predictive control of this system. An introduction to the area of urban wastewater treatment systems was given. This was followed with the basic principles of wastewater treatment process models, and the mathematical models involved in the description of treatment plant and river dynamics. In particular, it was necessary to describe the nonlinear dynamics of models used for control purposes, in a form suitable to the state space model predictive control approach detailed.

The state dependent coefficient approach allows the use of existing nonlinear process representations in the application of nonlinear model based predictive control. The description and the state dependent transformation of a small scale wastewater treatment plant was demonstrated. Similarly, the state dependent transformation of a CSTR based river model was detailed, allowing the development of a state dependent model incorporating a selection of the dynamics of the urban wastewater system. A description of the ASM2d/QUAL2E based urban wastewater model utilised within the thesis was also given, and a state dependent model was developed for a single activated sludge reactor within this system.

In discussion of the modelling approaches required for the control of the urban wastewater system, the development of a linear predictive control strategy is demonstrated. The choice of control objective, the modelling method used, the control design chosen and the performance of the controller, in addition to the limitations of linear models for a nonlinear system, are discussed.

Chapter 5

Nonlinear Predictive Control

5.1 Introduction

The approach of model predictive control clearly depends on an accurate process model for the purposes of prediction and optimisation of the performance of the controlled process. The issue then, within model based control, is one of appropriate model choice, based either upon the nonlinear system model determined from physical laws and process equations, or upon a model determined from experimental data and identification tests, or indeed a choice of model based upon a combination of these two approaches. Significant attention in control literature has been paid to the control algorithms, as opposed to the modelling and identification of the systems. This can be seen to be due to the application specific nature of the model identification. The choice of model appropriate to the application is dependent upon various factors: computation speed necessary, accuracy required, model complexity and model availability.

The existing model available for control purposes dictates the choice of control strategy. Murray-Smith and Johansen [72] state that there is little advantage in accuracy in the use of fuzzy or neural models, if there exists an accurate mathematical model of the process. In the absence of a nonlinear physical model of the process, which is commonly the case in wastewater processes, data-based representations can be appropriate. Data-based models can be of particular benefit in the description of controlled wastewater processes. These are commonly of SISO structure, and thus the complexity and computation time may be

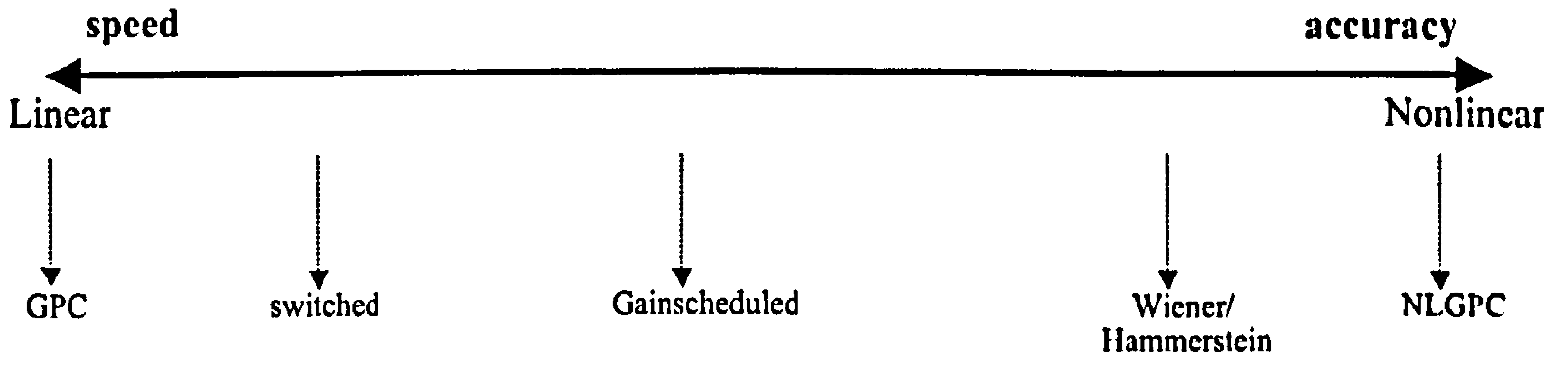

Figure 5-1: Speed Versus Accuracy: Choosing an Appropriate Model Structure

reduced, in the use of data-based models as a substitute for the nonlinear mathematical process model.

Various approaches in simplified methods have been developed for nonlinear modelling applications. The advantages of Takagi-Sugeno fuzzy modelling, ARX (Autoregressive with Exogenous Inputs) models and functional state approaches in the representation of nonlinear processes have been demonstrated, for example as discussed within [72]. Nonlinear input-output models, ARMA, ARX and Volterra models have all been implemented in the application of nonlinear predictive control (Maner et al. [98], Hernandez and Arkun, [64], Sriniwas and Arkun [168]). Wiener models have been a popular choice of process representation, being capable of arbitrary accuracy in their approximation (Gerksic et al. [52], Norquay et al. [115]). Artificial neural networks have begun to be applied in applications of nonlinear process control (Su and McAvoy [170]). Kadmiry and Driankov [40] describe how there exists no general method of nonlinear control design, but a 'collection of alternative and complimentary' approaches that are specifically advantageous for particular nonlinear systems. It can be seen that each nonlinear method presented requires the use of a different modelling technique, a modification upon the existing model for the purposes of a given control algorithm.

The models utilised for control of the nonlinear wastewater processes within this thesis extend from the Linear Time Invariant (LTI) data-based model to the Linear Time Varying (LTV) model of the process based upon the mathematical representation. The model structures considered are:

- Linear Time Invariant (LTI): This traditional approach is well researched, documented and a popular choice among process industries. The time invariant nature of the models allow for offline design of the controller, i.e. prediction equations and system control gain. Multiple LTI models can be utilised to control a process over a nonlinear operating space.

- LTI and Static Nonlinearity: The benefits of this approach can be a high level of accuracy, whilst retaining low complexity. The controller can retain the advantageous aspects of a linear approach, whilst providing the benefits of a nonlinear prediction model. This form of control can be situation and process specific.

- Linear Time Varying (LTV): This method uses existing physical models of the system to produce a nonlinear control approach, avoiding lengthy model reduction procedures. The use of the linear state space structure fits with the established linear control model structure. The approach is particularly suitable due to the time varying and nonlinear nature of bioprocesses.

System states in data based models are typically not measured and bear little resemblance to 'real' process states, being a result of the system identification procedure and structure used to represent the chosen dynamics. In the case of a mathematical system model, the issue of state estimation with the use of a complex process model arises, particularly with respect to unknown plant parameters. While research has demonstrated estimation of unknown process parameters and kinetics required in the nonlinear modelling of systems, it can be seen that a considerable difficulty lies in the lack of measurements within systems. Without sufficient plant information from sensors, the development of software sensors such as the extended Kalman filter is difficult. This can be a significant issue in the practical application of nonlinear model control techniques such as those based on the SDC approach, due to the absence of sufficient plant feedback. The accuracy of the nonlinear models however, in addition to the large number of modelled variables, and the increase in the use of sensors in the wastewater industry motivates the development of nonlinear control models, based upon the available process models.

5.2 Model Structures

Multiple model approaches are popular due to their advantages in exploiting existing modelling and control techniques. This approach produces nonlinear control of the system via a number of simpler local systems based on linear algorithms. This method has an intuitive aspect, in the definition of a nonlinear system decomposed into multiple linear equivalents. This 'divide and conquer' strategy is based on the idea that a nonlinear controller can be composed of a number of local linear controllers, effectively partitioning the control strategies. As well as the computational advantages of this methodology, the intuition involved exploits the user knowledge available. The choice of number, range and characteristics of the local models and controllers are situation specific, the aim is a balance of the trade-off between simplification and accuracy.

Fuzzy Logic allows the use of linear predictive control methodologies on a nonlinear system by the definition of fuzzy rules governing the use of controllers for a given operating point. The fuzzy gain-scheduled predictive control (FGPC) approach utilised in this thesis is similar to the functional-state approach, in that a series of states or operating regimes are defined, with an associated local model. Each linear model results in a linear controller, designed offline due to the linear time invariant nature of the model. The simple form of transitioning between these local controllers is hard-switching, such as that demonstrated by Jiang [71]. The use of fuzzy rules in the scheduling of control avoids seeks to avoid abrupt changes in the interpolation of control signals. Interpolation of control schemes leads to a smoother controller response, and the definition of FGPC transitions is dictated by the user-defined membership functions, constructing an interpolation scheme based on process knowledge. The 'scheduling variable' of the process is an indicator of the current operating point in the nonlinear range of the process, and thus the membership function should describe the choice of controller appropriate to the operating point indicated by this variable. Shamma and Athans [158] state that the scheduling variable should capture the nonlinearities of the process, since the intention of the gain-scheduling is of course to deal with these nonlinearities.

The design of nonlinear control schemes remains complex, however the divide and conquer approach of gain-scheduled systems allows the use of better developed and researched methods of analysis and design of LTI systems. The traditional popularity of linear control algorithms gives the additional benefit of 'continuity', as described by Leith et al. [93]. Existing control procedures, and the industry structure and protocols that accompany them, can make the financial, organisational and time issues of nonlinear control approaches prohibitively costly.

More accurate process models, such as the Wiener model approach, avoid the issues related with divide-and-conquer methods whilst moving further towards nonlinear control and simultaneously retaining the basic structure of the linear methods. The Wiener or Hammerstein approaches to the modelling of a process is to define the process dynamics with the use of two components: the static (memoryless) nonlinearity, and the linear dynamics. The two approaches are differentiated by the position of the static nonlinearity: the Wiener model is a linear dynamic block followed by a static nonlinear function, and the Hammerstein model has nonlinear function prior to a linear dynamic description. Zhu [194] states the advantages of the block orientated approach to be:

- The low cost in identification tests.
- The low cost in identification computation.
- The ease of incorporating a priori process knowledge.
- The ease of use for control purposes.

As the wastewater industry moves gradually towards increased use of automation and instrumentation, the mathematical process models already present within this area may shift from their function in analysis and benchmarking, and find use as 'control models' in their own right. The process knowledge available within these models has as yet remained almost entirely unexploited, restricted by the lack of control handles and sensors within the wastewater industry

5.3 Fuzzy Gain Scheduled Predictive Control

In the control of a nonlinear process, linear controllers can be implemented via gain-scheduling. Astrom and Wittenmark [6] demonstrated an early form of gain-scheduling (GS) for flight control systems, in describing the changes in system gain with differing operating conditions. Linear models are determined for certain ranges within which the behaviour may be approximated by a linear description, and are used to design individual model based predictive controllers for each. This allows for simple and quick implementation of control on a nonlinear process, though with the requirement for identification of a set of distinct linear models, and the subsequent tuning of the resulting individual controllers. The fuzzy nonlinear control implemented here can also be called fuzzy gain-scheduled (FGS) predictive control. There exists in literature and practise multiple model and multiple controller forms of fuzzy gain scheduling as in Figure 5-2, it is the latter that is considered in this thesis. This approach is similar in method to that used by Ling and Edgar [96], where model-based fuzzy gain-scheduling (MFGS) is described as the implementation of several linear model based controllers over a partitioned process operating space. In that approach, MFGS was demonstrate applied to the PID control of a water-gas shift reactor and the method was demonstrated to be comparable to nonlinear model predictive control. Johansen et al. [72] note that the Takagi-Sugeno fuzzy mechanism for multiple linear controllers can be seen as an efficient method of, and need not be distinguished from, the gain scheduling approach.

The difference between the multiple-model and the multiple-controller approach is defined by Ling and Edgar [96] as being related to the nature of the action variables (i.e. the outputs of the fuzzy interpolation): representing a model if the action variables are process outputs, but representing an aspect of control if the action variables are process inputs. Ling and Edgar also suggest that the differing size between a fuzzy controller and a fuzzy model is a limiting factor in the use of fuzzy models in predictions of future behaviour, suggesting that sufficient model accuracy for predictive purposes for a nonlinear model over a wide range would require a prohibitively large number of fuzzy rules.

Traditionally, the scheduling variable with which the control changes are referenced is considered to require a relatively slow-varying dynamic with respect to the controlled process, usually measured external from the process. Astrom and Wittenmark [6] demonstrated a slow-varying externally measured scheduling variable for the purposes of gain-scheduled control. However, Rugh [140] utilised a state internal to the process to schedule

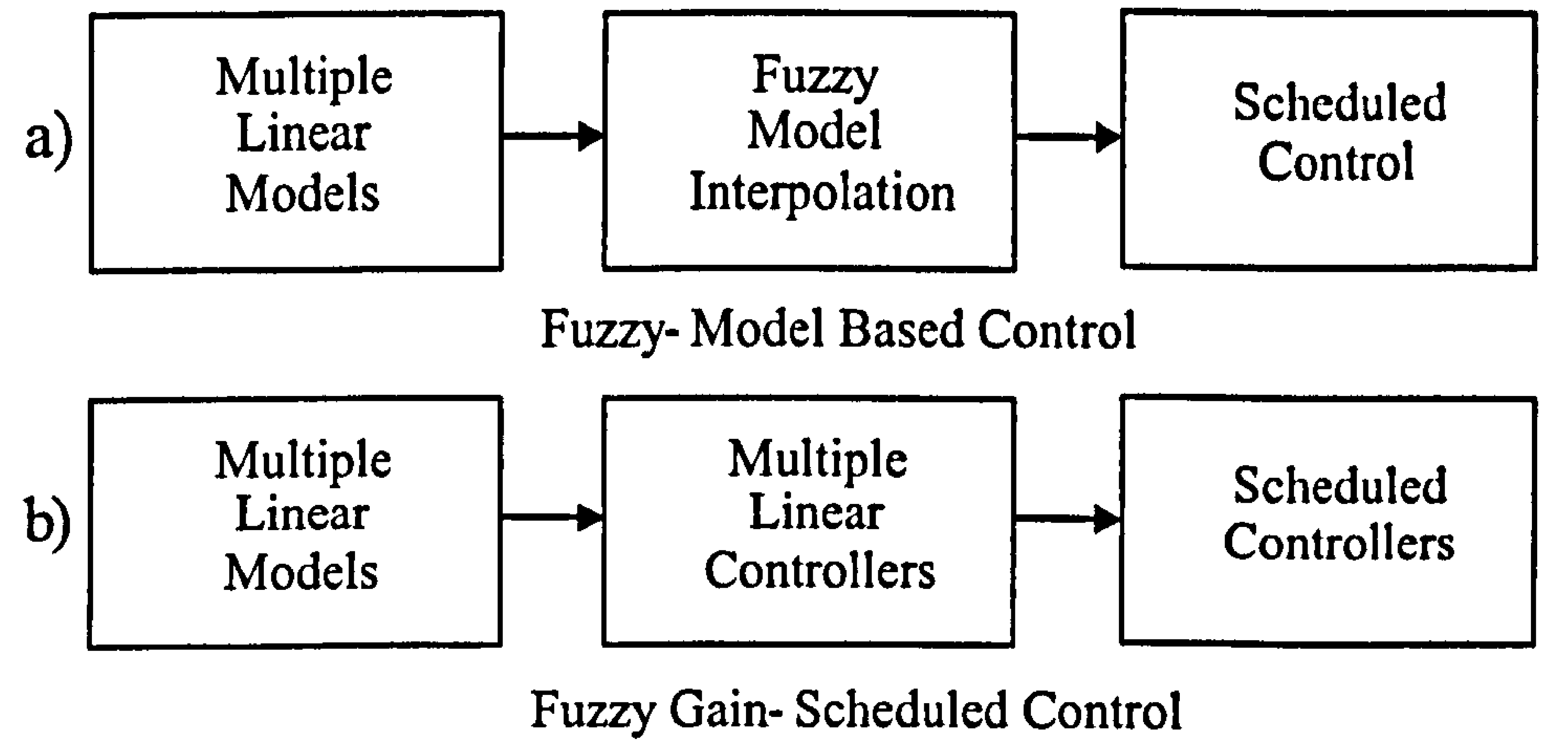

Figure 5-2: a. Fuzzy Model Based Control b. Fuzzy Gain-Scheduled Control

the controller. Additionally, Shamma and Athans [159] demonstrated the use of a rapidly changing scheduling variable.

A significant advantage of a gain-scheduled approach is the retention of the linear model structure for control design, allowing the user the choice of any appropriate linear control scheme, the simplicity and intuitive nature of a linear approach and the traditional advantages of computational efficiency. This computational efficiency allows the controller to respond quickly to changing process operating conditions. A potential difficulty of the gain-scheduled approach is the choice of a scheduling variable and the situation-specific nature of this choice, depending for the most part upon the dynamics and characteristics of the process. Additionally, the 'local' nature of the control scheme itself can lead to issues, local stability does not imply global stability for the process control. In practical applications, a major advantage lies in the ability of implementation of this form of GS control within PLC's. Fuzzy gain scheduled control implemented via a PLC of a gas-liquid seperation plant was demonstrated by Kocijan et al. [81]. The limits of PLC hardware are well known, low numerical precision, limited memory and limited sampling rate are all issues that restrict the control that can be applied. The linear representation of the system at a given operating point of the system is considered to be in the form of the state space linear model:

$$x_{k+1} = Ax_k + \begin{bmatrix} B & B_m \end{bmatrix} \begin{bmatrix} u_k \\ d_k \end{bmatrix} \quad (5.1)$$

$$y_k = Cx_k + \begin{bmatrix} D & D_m \end{bmatrix} \begin{bmatrix} u_k \\ d_k \end{bmatrix} \quad (5.2)$$

where the vector x is defined as the states of that linear model, the vector u is defined as the current control action applied to the process and the vector d is defined as the measured disturbance to the process. In the urban wastewater system considered, this allows for process knowledge due to upstream sensor measurements to be included within the process description. The nonlinear operating range of the considered process is chosen to be defined by multiple linear models of this form. Each is described by an individual state space representation:

$$\begin{aligned} x_{k+1,i} &= A_{k,i}x_{k,i} + \begin{bmatrix} B_i & B_{m,i} \end{bmatrix} \begin{bmatrix} u_k \\ d_k \end{bmatrix} \\ y_{k,i} &= C_{k,i}x_{k,i} + \begin{bmatrix} D_i & D_{m,i} \end{bmatrix} \begin{bmatrix} u_k \\ d_k \end{bmatrix} \end{aligned} \quad (5.3)$$

where $i = 1 \ldots n$, where n is the number of linear regions considered, and where the vector x_i is specific to that linear model, and similarly specific, subsequently, is the output vector y_i. The control action u is again that currently applied to the process, i.e. the output of the fuzzy controller, and the vector d remains the measured disturbance. Thus the same user-specified setpoint and feedback information from the process is supplied to each individual controllers. For each of the linear models, there are differences therefore only in individual controller and estimator parameters, in addition to state estimates. The individual prediction matrices, error vectors and calculated control action are defined by the controller structure as defined in Chapter 2 to be:

$$\begin{aligned} Y_i &= F_i x - H_i \Delta U \\ \varepsilon_i &= W - Y_i \end{aligned} \quad (5.4)$$

$$J_i = (\varepsilon_i)^T Q(\varepsilon_i) + \Delta U^T \lambda \Delta U$$

where again $i = 1 \ldots n$, where n is the number of linear models for which controllers are designed. Each linear controller is designed particular to a specific operating point of the plant, and the predictive controller and observer parameters for each controller (λ_i, Q_i, $H_{p,i}$, $H_{u,i}$, Q_n, R_n) are individually tuned to meet the control objectives local to this point, assuming the regions surrounding that operating point have similar dynamics. Optimisation of this cost function results in the computed control action for the linear model i according to the above controller tunings:

$$\Delta u_i = K_i \varepsilon_i \tag{5.5}$$

The advantages of the gain-scheduled nonlinear approach over the linear counterpart can be lost with an inappropriate choice of combination method. The fuzzy logic used here is a method of interpolation of computed control actions, utilising the Takagi-Sugeno-Kang fuzzy methods, which is based on fuzzy theory proposed by Takagi et al. [174]. In the application of fuzzy gain-scheduled control, the fuzzy function utilised is a set of linguistic rules. These describe the method in which the appropriate choice will be made of the linear controller to be used, at that specific operating point in the nonlinear process.In this format an operating point can belong entirely to one set (one controller), or partially to two sets (two controllers weighted according to operating point). At the modal value of the fuzzy set, the control action upon the process is equivalent to that of a linear predictive controller operating at the point for which the model was linearised. Thus, conversely, at non-modal values the control action upon the process is equivalent to a weighted sum of the outputs of the appropriate linear controllers. The following steps are therefore involved in the implementation of the fuzzy approach for this gain-scheduled control scheme:

1. Fuzzification: The value (here the scheduling variable, s) is fuzzified, that is, transformed into a fuzzy set specific to that variable, allowing the definition of its membership function.

2. Fuzzy rules are applied, inferring a fuzzy value, it is this that defines the action to be performed.

3. The value is de-fuzzified, in this case the fuzzy value set is defined as a set of values

to be used in the weighted summation of control actions calculated by the bank of linear MPC's.

4. The applied control is therefore defined as the variable C, determined from the sum of the weighted control values.

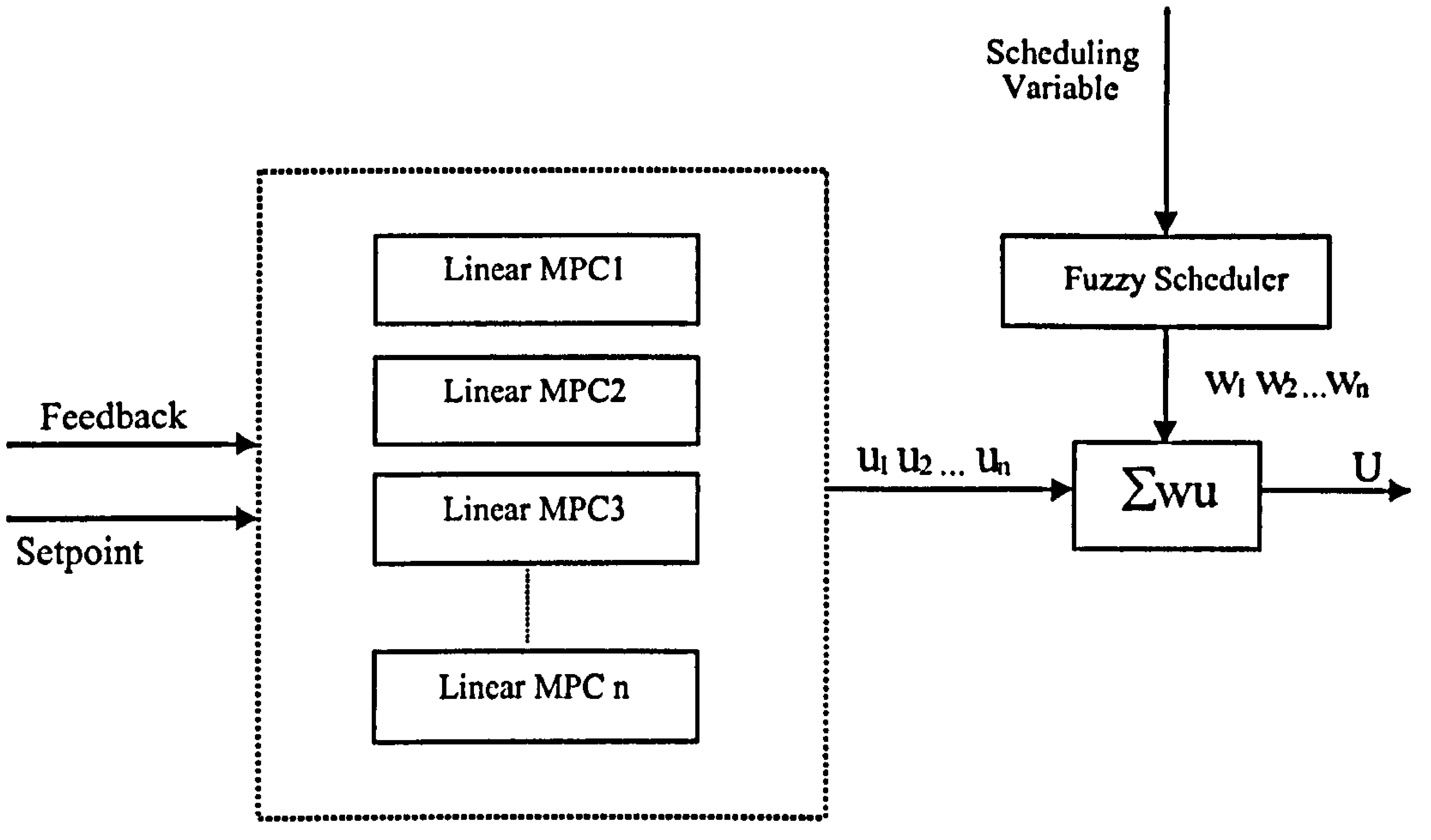

Figure 5-3: Architecture of the Fuzzy Gain Scheduled MPC, with n Linear MPC Controllers

The structure in Figure 5-3 can be expressed mathematically: the linear predictive controllers are defined by 5.4 and the control input to the plant is determined via the following general function,

$$\begin{aligned} u_i &= f(x_i, c_{k-1}, r) \\ c &= \frac{\sum w_i(\rho).u_i}{\sum w_i(\rho)} \end{aligned} \tag{5.6}$$

where $i = 1 \cdots n$, where n is the number of local controllers, and x is the vector of measured or estimated system states, c_{k-1} is the measured value of the previous control input and the setpoint is defined by vector r. For the FGS utilised in this thesis, the overall output of the composite controller (that is, the combined linear controllers) c is composed of the weighted sum of the outputs u_i of the local controllers. The weighting vector w_i is defined as being within the bounds $0 \leq w_i \leq 1$ and the sum of these weights constrained by $\sum w_i = 1$. The weightings upon the individual control signals are dictated by the scheduling variable ρ according to the fuzzy membership function. The steps involved in the choice of linear subsystems, the subsequent control design and the algorithm steps are as follows:

1. The plant local operating points (equilibrium or otherwise) are chosen according to the variable, ρ, which technically can be a plant input, output or even a system state. As stated previously, this scheduling variable must take the nonlinearities of the plant into account.
2. A fuzzy membership function is developed defining the overlapping regions above.
3. The dynamics in each individual local region are approximated via a linearisation around the given operating point.
4. For each local model, a state estimator is designed, in order that the current error and predicted future error be available to the control algorithm.
5. The prediction equations and optimal control strategy for the local controllers are determined from the system state space matrices chosen in the linearisation approach above.
6. At each controller sample instant, the individual local controllers estimate the current system states, predicted error and appropriate control response.
7. The fuzzy rules as defined by the membership function above dictates, according to the scheduling variable, the weighting function for the above vector of control responses.

5.4 Wiener Model Predictive Control

The Wiener structure of a cascaded system of a LTI model and a memoryless nonlinear model as shown in Figure 5-4 can be valuable in the practical implementation of nonlinear model predictive control. This is due to their capability in representing systems where dynamics can be defined in a linear structure, whilst the gain curve over the nonlinear operating range can be described in a static nonlinear function. The Wiener structure has been popular in the process industries in modelling nonlinear systems, as demonstrated by the applications to distillation column control [18], pH processes ([75], [116]) and a continuous stirred tank reactor [32]. Wiener models can be determined by identification methods from input and output data, either from the process or, as in this thesis, from a nonlinear simulation model.

Recently, subspace identification methods have begun to be extended to Wiener model identification in [54] and [186]. The model found is suitable for use in the model predictive algorithm determined for linear control, using the existing optimisation techniques. In this way, Wiener model predictive control is efficient, accurate and retains the traditional MPC structure. Because of the state aspect of the process nonlinearity, it can effectively be removed from the control problem, modelled only as a gain acting upon the process. As previously described in the case of linear MPC, model disturbances and any inaccuracy introduced by mismatch with the plant will be compensated with an Kalman filter, updated at each sample instant.

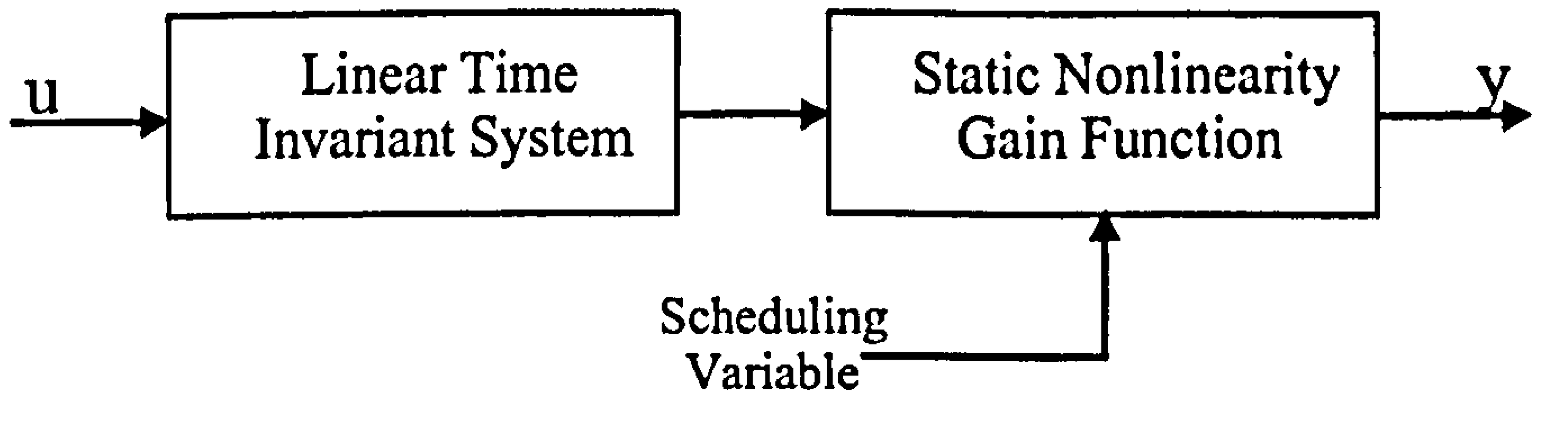

Figure 5-4: Architecture of Wiener MPC: LTI Model of System Linear Dynamics and Description of Static Nonlinearity

The representation of the linear dynamics is required to be in the format of an LTI state space description for the purposes of the MPC algorithm, thus the linear time invariant model is defined by

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ v_k &= Cx_k + Du_k \end{aligned} \tag{5.7}$$

where v is defined as the output of the linear system and thus the input to the gain function of the static nonlinearity. This results in an output vector $y = g(\rho)v$, where ρ is defined as the 'scheduling variable', and thus the static nonlinearity can be included within the state space description as

$$x_{k+1} = Ax_k + Bu_k \quad (5.8)$$

$$y_k = g(\rho)Cx_k + g(\rho)Du_k \quad (5.9)$$

so that the system can be seen to be of the format

$$x_{k+1} = Ax_k + Bu_k \quad (5.10)$$

$$y_k = C_k x_k + D_k u_k \quad (5.11)$$

where the state space matrices $C_k = g(\rho)C$ and $D_k = g(\rho)D$ are the matrices updated at each sample instant by the scheduling variable ρ. Thus at each time k there exists a linear time invariant model of the system, for which the control is recalculated. The aim of modelling in this case is the identification of both systems, in particular the measurement and analysis of the dynamic step response of the LTI subsystem, and an estimation of the characteristics of the non-linear subsystem, to produce the structure as shown in Figure 5-5.

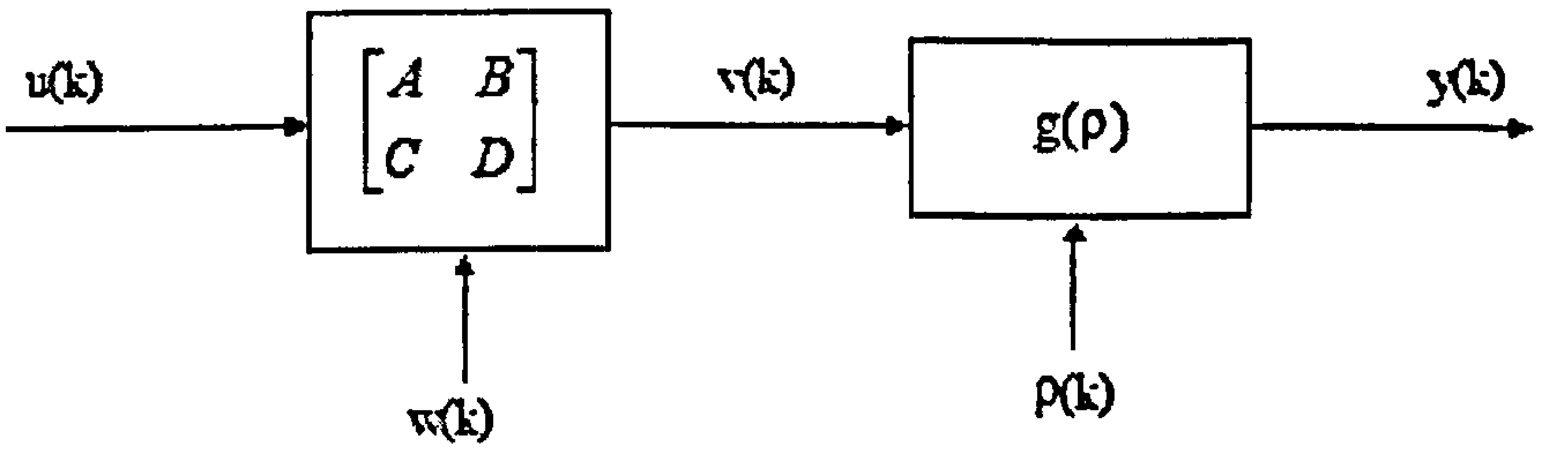

Figure 5-5: Wiener Model: Linear State Space Representation, Scaled by a Nonlinear Gain Function

The static nonlinearity in a general form can be defined as acting upon the system states with respect to this given 'scheduling' input, a variable or vector dictating the nonlinearity of the system. This can be assumed to be approximated by a linear function, though in practice this gain function can be constructed of various forms. Cervantes et al. [32] demonstrated the use of a piecewise linear function.

The Kalman filter utilised to estimate the system states (in addition to the unmeasured disturbances) assumes noise matrices giving the system the form:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k \\ y_k &= C_k x_k + D_k u_k + z_k \end{aligned} \tag{5.12}$$

where w and z are the noise vectors. The state space process model is updated at each sample instant, thus the gain of the Kalman filter L must also be again determined. The state estimator is thus used as a 'soft sensor' in the calculation of disturbances upon the system. The Kalman filter in this case is used as a state and process disturbance estimator, with gain L_k updated at each sample instant

$$\begin{aligned} x_{e,k+1} &= Ax_{e,k} + Bu_k + L_k(y_k - C_k x_{e,k} - D_k u_k) \\ y_{e,k} &= C_k x_{e,k} + D_k u_k \end{aligned} \tag{5.13}$$

The system model can be seen to be an LTI system, whose output $v(k|k)$ is transformed to $y(k|k)$ by a nonlinear function, so that at any given sample instant the model can be seen to be simply the algebraic product an LTI model and a system gain. The order of the LTI system can be chosen to be of any (practical) dimension, and together with the equally definable nonlinear function, can give the process model an arbitrary level of accuracy. The restrictions upon choice of LTI model depends on the level of accuracy required, the stability of the model and the time constraints upon the control optimisation and state estimation. Gomez et al. [54] demonstrated the ease of identification of a stable Wiener model, in comparison with a linear approach, concluding that in stability the Wiener model obtains a better performance, remaining stable for a wider range of model orders. Together with the improved predictions from a Wiener model, this illustrates the advantages of the Wiener representation based nonlinear model predictive control (NMPC).

The LTI state space representation of the nonlinear system is updated at each sample instant according to the static nonlinearity dictated by the scheduling variable. The predictive control algorithm for this Wiener-model based system is therefore similarly updated online:

- The updated Kalman filter provides state estimates $\tilde{x}_k$.
- The model representation defined by A, B, C_k, D_k dictates the prediction equation at time k $Y_k = F_k\tilde{x}_k + H_k\Delta u_k$.
- The tracking error, with respect to the user-defined setpoint, is defined as $e_k = w - Y_k$.
- The cost function to be optimised at time k is described as $J_k = (\varepsilon_k)^T Q(\varepsilon_k) + \Delta U_k^T \lambda \Delta U_k$.
- The increment of control action to the process is defined as $\Delta u_k = K_k\varepsilon_k$, where the control gain K_k is calculated with the above cost function according to the tuning parameters, $(\lambda_i, Q_i, H_{p,i}, H_{u,i}, Q_n, R_n)$.

5.5 Nonlinear Generalised Predictive Control

5.5.1 Background and History of Nonlinear GPC

The control scheme presented here implements a nonlinear form of generalised predictive control. The linear form of this control algorithm was initially developed by Clarke et al. (1987) [28] and is the form of linear control implemented earlier in this thesis. Various other forms of linear control exist such as the model based predictive control approach developed by Richalet et al. in 1978 [135], and the dynamic matrix control approach proposed by Cutler and Ramaker, in 1980 [33]. Garcia in 1984 [51] extended the latter of these approaches to a nonlinear process, in which a nonlinear model was used, with the solution online of a single quadratic program. An approach to nonlinear predictive control, utilising the optimal control calculated in the previous time step by the predictive controller was implemented by Kouvaritakis et al. [83]. It is about this optimal trajectory that the system is linearised, which is then used to recalculate the optimal control for the current time step. Youssef et al. (2003)[191] presented a similar approach for nonlinear predictive flight control. An approach similar to that used here was developed by Lee et al. [91], however this used a different model representation.

A nonlinear form of GPC utilising a nonlinear state dependent system model, was demonstrated in 2001 by Ordys and Grimble [126]. This approach was implemented in the same year by Grimble and Ordys [57] for the application of robotic control and in 2004 by Dutka et al. [41] for a helicopter control application. The approach of representation of the nonlinear discrete time model into the state and control dependent form of a state space model was demonstrated in 1998 by Mracek et al. [108] and also two years previous by Huang et al. [66]. Mutha et al. [110] describes the modification of the model based predictive equation for nonlinear control. This paper demonstrated that the control performance depends upon the ability of the prediction matrix to account for the nonlinearities of the process.

In slow changing processes, nonlinear models (based on mechanistic mathematical models) can be useful in the application of nonlinear control. The method of nonlinear generalised predictive control detailed in this chapter exploits the ability of the state space modelling approach to represent nonlinear systems, such as the state dependent model approach. The process knowledge described by the nonlinear process model can be expressed in an equivalent state space model, where the state space matrices contain the details of the nonlinear process laws. The nonlinear model can therefore be seen as a linear model at a given time instant, with the current estimated or measured states determining the characteristics of that linear model.

A model must contain sufficient description of the system process, capturing the crucial features of the plant, whilst simultaneously being adequately simple to allow for control system design. The use of a nonlinear model of the process has the obvious benefit of an increase in accuracy of estimation and predictions provided by the model. An accurate model can be seen to provide better process performance, thereby increasing efficiency and lowering costs. It provides the ability to control a complex nonlinear system. The recent surge in technological computing power has decreased the concerns of optimisation time, however the issues of observability and controllability of large complex nonlinear models still stands.

In this chapter, the nonlinear model is considered as a 'white box' model, that is, a model determined from mathematical model equations. Although these models are determined from process analysis, the nonlinear process model cannot be fully and accurately described by a mathematical model. This is due to the physical constraints of measurement and analysis, thus resulting in some, albeit minor, modelling discrepancys. The practical concerns of application of nonlinear methods, such as state dependent modelling, for control remains the aspects of controllability and observability of the nonlinear process models. It remains to be seen if future wastewater modelling will provide the requirements for control, of observable and controllable models, research such as [162] suggests that pseudo-nonlinear modelling approaches may suffice and allow for the use of more complex advanced control methods.

The assumption within nonlinear predictive control schemes is that the control model utilised has no mismatch with the plant, and that no unmeasured disturbance is acting on the system. The fidelity of identified nonlinear models to the actual physical system is an issue, thus model uncertainty is difficult to avoid. Uncertainty is a significant issue for the performance of a controller, resulting in robustness issues, such as stability and performance, and as such is a significant issue for nonlinear predictive control. Robustness analysis and synthesis for nonlinear systems is considerably more complex than in the linear case, especially in presence of state and input constraints. Several recent advances in nonlinear predictive control methods have resulted in better handling of robustness issues. The research of [190] investigated robust stability issues for uncertain nonlinear systems with disturbances and input saturation, whilst [155] considered these issues for control under perturbations. The work of [8] demonstrated a modification of the Lyapunov stability theory for nonlinear control in the situation of plant-model mismatch. Santos and Biegler [149] detailed an off-line method to determine the amount of structural and parameteric uncertainty for which robust stability can be guaranteed.

5.5.2 Nonlinear GPC Theory

The following sections will describe the approach of state dependent modelling and linear time varying models for the purpose of the nonlinear generalised predictive control (NLGPC) scheme detailed later. The approach of state dependent representation of a system model as developed by Pearson [128] avoided the linearisation of the nonlinear model by the representation of the model in the state space format, via the state dependent state space coefficients described by the process model states and parameters itself, therefore allowing a linear model to be obtained at each time step. The nonlinear system model is represented by the model:

$$\begin{aligned} \frac{dx}{dt} &= f_1(x,u) \\ y &= f_2(x) \end{aligned} \tag{5.14}$$

where x is the state vector of size n_x, u is the input vector of size n_u, y is the output vector of size n_y and f_1 and f_2 are vectors of size n_x and n_y respectively. These system equations can be rewritten in the following form of state dependent coefficients:

$$\begin{aligned} \dot{x} &= \widetilde{A}(x)x + \widetilde{B}(x)u \\ y &= \widetilde{C}(x)x + \widetilde{D}(x)u \end{aligned} \tag{5.15}$$

where $\widetilde{A}(x)$ is the state matrix of size (n_x x n_x), $\widetilde{B}(x)$ is the input matrix of size (n_x x n_u), $\widetilde{C}(x)$ is the output matrix of size (n_y x n_x) and $\widetilde{D}(x)$ is the matrix of size (n_y x n_u). There are a number of ways of reformatting the system equations 5.14 to the format of the state dependent equations 5.15, there is not a unique choice of the state dependent model. However the choice of appropriate state dependent format must allow for a controllable linear system at any given operating point. For the purposes of NLGPC, the above state-space system can be seen in the format of a linear time-varying system, at each sample point, as is shown in the next section. Note that the state dependent representation of the system is equivalent to the original nonlinear process model, and is merely an algebraic rearrangement of the initial model. For the purposes of predictive control, it is necessary for the system to be repesented by a discrete state space model, the continuous time representation of the system is therefore discretised according to sample time T_s.

5.5.3 Linear Time Varying Model

The above state dependent coefficients can be extended to described a linear time-varying system. The control input vector (of optimal control steps over the prediction horizon), computed in previous optimisations, were unused in the linear and nonlinear techniques demonstrated earlier in this thesis. However, these can be utilised with a the state dependent coefficient matrices above to predict the system behaviour for the subsequent time steps. This is updated at each sample time, with the values computed in the last optimisation routine. As before, the system is subsituted into the following form, with an incremental input to include integral action, where the new state χ_k consists of the original system states and additionally the previous input to the system $\begin{bmatrix} \chi_k & u_k \end{bmatrix}^T$:

$$\begin{aligned} \chi_{k+1} &= \begin{bmatrix} \tilde{A} & \tilde{B} \\ 0 & I \end{bmatrix} \chi_k + \begin{bmatrix} \tilde{B} \\ I \end{bmatrix} \Delta u_k \\ y_k &= \begin{bmatrix} \tilde{C} & \tilde{D} \end{bmatrix} \chi_k \end{aligned} \tag{5.16}$$

where the state space matrices of the old system model are denoted by $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$ respectively, and the new system matrices are denoted by A, B and C:

$$\begin{aligned} \chi_{k+1} &= A\chi_k + B\Delta u_k \\ y_k &= C\chi_k \end{aligned} \tag{5.17}$$

The cost function of the GPC controller here is defined as

$$J = (W - Y)^T Q (W - Y) - \Delta U^T \lambda \Delta U \tag{5.18}$$

where the vectors are defined as the setpoint vector $W = \begin{bmatrix} w(k+N_1) & \cdots & w(k+N_2) \end{bmatrix}^T$, the output vector $Y = \begin{bmatrix} y(k+N_1) & \cdots & y(k+N_2) \end{bmatrix}^T$ and the control increment vector $\Delta U = \begin{bmatrix} \Delta U(k+N_1) & \cdots & \Delta U(k+N_2) \end{bmatrix}^T$. The control weightings as denoted by Q and λ are defined as diagonal matrices of size $R^{l*H_p x l*H_p}$ and $R^{l*H_u x l*H_u}$ respectively, where l is the number of controlled outputs.

$$
\begin{aligned}
Q &= \begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & Q_{H_p} \end{bmatrix} \\
\lambda &= \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \lambda_{H_u} \end{bmatrix}
\end{aligned} \tag{5.19}
$$

The state space system matrices detailed above in 5.16 are updated, determining the predictions of future states through repeated substitution of calculated future inputs. As for the linear predictive control algorithm previously, the prediction of future behaviour is built by iteration

$$
\begin{aligned}
\chi_{k+1} &= A_k\chi_k + B_k\Delta u_k \\
\chi_{k+2} &= A_{k+1}A_k\chi_k + A_{k+1}B_k\Delta u_k + B_{k+1}\Delta u_{k+1} \\
\chi_{k+3} &= A_{k+2}A_{k+1}A_k\chi_k + A_{k+2}A_{k+1}B_k\Delta u_k + A_{k+2}B_{k+1}\Delta u_{k+1} + B_{k+2}\Delta u_{k+2}
\end{aligned} \tag{5.20}
$$

The matrices A_k, A_{k+1}, B_k, B_{k+1}, etcetera are dependent upon the values of the state vector prior to that specific sampling instant, i.e. χ_k, χ_{k+1}. Thus the calculation at time k of the state χ_{k+n} is dependent upon the calculation of the previous $n-1$ states. Therefore it is possible to build a vector of predicted behaviour at time j as in 5.21.

$$
\chi_{k+j} = \begin{bmatrix} A_{k+j-1}A_{k+j-2}\cdots A_k \end{bmatrix}\chi_k \tag{5.21}
$$

$$
\begin{aligned}
&+\left[\ A_{k+j-1}A_{k+j-2}\cdots A_{k+1}\ \right]B_k\Delta u_k \\
&+\left[\ A_{k+j-1}A_{k+j-2}\cdots A_{k+2}\ \right]B_{k+1}\Delta u_{k+1}\ldots \\
&\ldots+\left[\ A_{k+j-1}\ \right]B_{k+j-2}\Delta u_{k+j-2} \\
&+B_{k+j-1}\Delta u_{k+j-1}
\end{aligned}
\tag{5.22}
$$

Utilising the same notation as used in [191]

$$
\prod_{k=1}^{n} A = \{ \begin{matrix} A_nA_{n-1}\ldots A_1 \\ I \end{matrix} \ if \ \begin{matrix} l \leq n \\ l > n \end{matrix}
\tag{5.23}
$$

The predicted state vector at time j is defined by the following equation, wherein it is assumed again that the control increments are the control horizon are zero.

$$
\begin{aligned}
\chi_{k+j} \ = \ & \left[\prod_{i=0}^{j-1} A_{k+i}\right]\chi_k \\
& + \left[\prod_{i=1}^{j-1} A_{k+i}\right] B_k\Delta u_k \\
& + \left[\prod_{i=2}^{j-1} A_{k+i}\right] B_{k+1}\Delta u_{k+1}\ldots \\
& + \left[\prod_{i=Hu}^{j-1} A_{k+i}\right] B_{k+Hu-1}\Delta u_{k+Hu-1}
\end{aligned}
\tag{5.24}
$$

The predicted output, as determined from the above, is shown in the following, retaining the GPC notation of Krauss et al. [85]:

$$
Y = F_kA_k\chi_k + H_k\Delta U_k
\tag{5.25}
$$

where

$$
F_k = \widehat{C} * \left[\ \left[\prod_{i=1}^{0} A_{k+i}\right] \ \cdots \ \left[\prod_{i=1}^{H_p-1} A_{k+i}\right] \ \right]^T
\tag{5.26}
$$

where $\widehat{C}$ is defined as the diagonal matrix of C over the horizon H_p, and H can similarly be defined as

$$H_k = \widehat{C} * \begin{bmatrix} \left[\coprod_{i=1}^{0} A_{k+i}\right] B_k & 0 & \cdots & 0 \\ \left[\coprod_{i=1}^{1} A_{k+i}\right] B_k & \left[\coprod_{i=2}^{1} A_{k+i}\right] B_{k+1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \left[\coprod_{k=1}^{H_p-1} A_{k+i}\right] B_k & \left[\coprod_{i=2}^{H_p-1} A_{k+i}\right] B_{k+1} & \cdots & \left[\coprod_{k=Hu}^{H_p-1} A_{k+i}\right] B_{k+Hu-1} \end{bmatrix} \quad (5.27)$$

The prediction equation thus includes the extended vector of the predicted states, determined by the control vector. This allows the controller to take the dynamic nature of the process into account. Whilst the extended control vector is used for prediction, in actuality only the first element of the control vector is applied to the plant. The remaining elements of the vector are considered only as predictions of the future control actions.

The steps for the implementation of an NLGPC control scheme are as follows, for time t:

1. Measure the current states of the process - for state vector x.
2. Subsitute into system state dependent model, incorporating integral action, using system states and current control input.
3. Use the control vector calculated in the previous optimisation (without the current control input, which is already applied to the plant) to construct a prediction vector X, together with the current state vector, as detailed above.
4. Using X, the state dependent matrices are calculated, for the future state predictions, and the controller matrices.
5. The next optimal control vector is determined via optimisation of the cost function.
6. Control signal is applied to the plant, controller moves back to Step 1.

5.5.4 Measured Disturbances

Measured disturbances may be included within the plant description, allowing feedforward information about system disturbances, but also additionally including plant input signal other than those defined as process control actions. This has been shown for the linear case by several sources, for example [125]. This method, that shown in Chapter 2 for linear predictive control, is used for the nonlinear state dependent case, modifying the nonlinear algorithm. The state space state dependent model can be modified to include description of measured disturbances within the system:

$$\begin{aligned} \chi_{k+1} &= A_k\chi_k + B_k\Delta u_k + B_{d,k}d_k \\ y_k &= C\chi_k + D_d d_k \end{aligned} \tag{5.28}$$

The process model is transformed to the state dependent representation such that the nonlinearities are contained within the A_k, B_k and $B_{d,k}$ matrices. A more general approach to the modelling of measured disturbances would result in the state dependent form of the output matrices C and D_d. In the event of inclusion of measured disturbances in the system model, the above control algorithm is modified. The measured disturbances act in two ways: modelling uncontrolled input variables to the process model and secondly including measurements of disturbance effects elsewhere within the process. Thisapproach of including measured disturbances for suitable control action to be taken, is called feedforward control. The inclusion of measured disturbances in this instance affects the output prediction vector.

$$\begin{aligned} \chi_{k+1} &= A_k\chi_k + B_k\Delta u_k + B_{d,k}d_k \\ \chi_{k+2} &= A_{k+1}A_k\chi_k + A_{k+1}B_k\Delta u_k + B_{k+1}\Delta u_{k+1} \\ &\quad +A_{k+1}B_{d,k}d_k + B_{d,k+1}d_{k+1} \\ \chi_{k+3} &= A_{k+2}A_{k+1}A_k\chi_k + A_{k+2}A_{k+1}B_k\Delta u_k + A_{k+2}B_{k+1}\Delta u_{k+1} + B_{k+2}\Delta u_{k+2} \\ &\quad A_{k+2}A_{k+1}B_{d,k}d_k + A_{k+2}B_{d,k+1}d_{k+1} + B_{d,k+2}d_{k+2} \end{aligned} \tag{5.29}$$

since the measured disturbance is assumed constant over the prediction horizon, then $d_k = d_{k+1} = d_{k+2} = \ldots$. The above derivation is however also valid in the case that $d_k \neq d_{k+1} \neq d_{k+2} = \ldots$. The state dependent matrix B_d however does change over the prediction horizon, updated with changes in the state vector. Therefore the predicted state vector at time j is defined by

$$\begin{aligned}\chi_{k+j} = & \left[A_{k+j-1}A_{k+j-2}\cdots A_k \right] \chi_k \\ & + \left[A_{k+j-1}A_{k+j-2}\cdots A_{k+1} \right] B_k \Delta u_k \\ & + \left[A_{k+j-1}A_{k+j-2}\cdots A_{k+2} \right] B_{k+1} \Delta u_{k+1} \cdots \\ & \cdots + \left[A_{k+j-1} \right] B_{k+j-2} \Delta u_{k+j-2} \\ & + B_{k+j-1} \Delta u_{k+j-1} \\ & + \left[A_{k+j-1}A_{k+j-2}\cdots A_{k+1} \right] B_{d,k} d_k \\ & + \left[A_{k+j-1}A_{k+j-2}\cdots A_{k+2} \right] B_{d,k+1} d_k \\ & \cdots + \left[A_{k+j-1} \right] B_{d,k+j-2} d_k \\ & + B_{d,k+j-1} d_k \end{aligned} \tag{5.30}$$

As with its linear counterpart, the inclusion of the measured disturbance does not affect the optimisation of the cost function, but instead modifies the prediction equation, so that the predictions are defined according to the equation

$$Y = F_k A_k \chi_k + H_k \Delta U_k + D_{b,k} D(k) \tag{5.31}$$

where $D(k) = \left[\begin{array}{ccc} d(k) & d(k+1|k) & \cdots \end{array} \right]^T$. The disturbance description matrix $D_{b,k}$ is defined by:

$$D_{b,k} = \begin{bmatrix} C\left[\prod_{i=1}^{0} A_{k+i}\right] B_{d,k} & D_d & & \cdots & \cdots & & 0 \\ C\left[\prod_{i=1}^{1} A_{k+i}\right] B_{d,k} & C\left[\prod_{i=2}^{1} A_{k+i}\right] B_{d,k+1} & & \cdots & \cdots & & 0 \\ \vdots & \vdots & & \ddots & \vdots & & \vdots \\ C\left[\prod_{k=1}^{H_p-1} A_{k+i}\right] B_{d,k} & C\left[\prod_{i=2}^{H_p-1} A_{k+i}\right] B_{d,k+1} & & \cdots & C\left[\prod_{k=H_u}^{H_p-1} A_{k+i}\right] B_{d,k+H_u-1} & & D_d \end{bmatrix} \tag{5.32}$$

5.6 Summary

In this chapter, the theoretical background is given for three control schemes; that of fuzzy gain-scheduled control, Wiener model based predictive control and nonlinear state dependent predictive control. The linear predictive control algorithms detailed in Chapter 2 of this thesis were exploited in the development of a fuzzy gain-scheduled control strategy. This will be shown for the control of ammonia/ammonium concentrations and dissolved oxygen levels in the urban wastewater system within Chapter 6. The Wiener model approach of a linear dynamic model in series with nonlinear system gain was utilised in the development of an input-dependent model for nonlinear predictive control. This will be demonstrated in the subsequent chapter in the development of a dissolved oxygen control strategy for the urban wastewater system. The introduction of the state dependent nonlinear modelling approach allows the use of the subsequent nonlinear predictive control strategy, including feedforward control action via the use of a measured disturbance model. The state dependent models developed in Chapter 4 will be used in the implementation of this strategy for nonlinear predictive control, in Chapter 6.

Chapter 6

Nonlinear Predictive Control of Wastewater Systems

6.1 Introduction

The application of model based control techniques has proved appropriate for wastewater purposes. However, the majority of these applications have concentrated upon the wastewater treatment plant, and the optimisation of its performance. Linear predictive control for the extended urban wastewater system may not be suitable, in the presence of strong nonlinear effects such as storm event disturbances or varying influent wastewater characteristics. The aim therefore is the development of appropriate nonlinear control techniques with respect to the control objective of maintaining receiving water quality.

In developing nonlinear model predictive control for receiving waters, it is necessary that nonlinear model structures are utilised. The main research presented within this chapter therefore is concentrated upon appropriate modelling approaches for specific situations. The nonlinearities present in wastewater treatment result from various sources and their effects upon the system operating conditions: the plant kinetics. coupling interactions between process variables, the varying characteristics of the influent to the treatment plant, the effects of disturbances upon the system (such as toxic events and storm events) and temperature variations. The nonlinearities considered within this chapter include the effects of varying rainfall upon the characteristics of the wastewater, both in the changing treatment plant influent and also the effects of combined sewer overflows to the receiving waters.

Nonlinear control applications within wastewater industry have been uncommon, despite the nonlinear behaviour present within the biological and physical processes inherent to the process. Whilst traditionally, control on a local level of water treatment plants and individual bioreactors within these has been popular, the introduction of the recent EU directives concerning urban water systems has increased attention upon control of integrated systems. The WFD legislation requires that attention be given to interactions between subsystems, mainly towards maintenance of river quality. Meirlaen [105] describes that this in turn requires that immission concentrations (i.e. river concentrations) be included within the focus of the designed control schemes. It is the quality of the receiving water which is considered within the scope of this thesis.

6.1.1 Urban Wastewater Systems Control

As stated by Mailleret et al. [99], the trends within control of biological systems (particularly bioreactors) has concentrated upon three main methods: local, global (given full knowledge of system model), and global (accounting for some model uncertainties). The approaches considered within this thesis follow the above trends, with applications extended however from local control of bioreactions within the treatment plant, to a more global control of pollutants within urban wastewater systems, with the use of varying levels of model knowledge.

Automation of the wastewater treatment process was considered as early as the 1970's [22], however this has been slow to be implemented in practical applications. Vanrolleghem states that only minimal control devices are available in common treatment plants, with dissolved oxygen control being the most widespread application of process control. In more recent decades, the carbon pollution in wastewater is not the only objective of treatment processes but additionally nutrient removal (for example, for nitrogen and phosphorous). Olsson ([121], [123]) describes amongst other problems facing the instrumentation and control issues in the water treatment industry today are the plant constraints, insufficient sewer systems and unreliable measuring devices. Jeppsson et al. [69] states the main forces behind progress in the water industry as being: stricter regulatory standards, financial constraints, energy efficiency, increased plant monitoring (a traditonal 'bottleneck') and new technological progress.

Sensors, for example online flow measurements, redox potential and temperature sensors, are most commonly available within the treatment plant. Although there exists some equivalent technology for the monitoring of receiving waters, online river quality sensors are not frequently used, even in verification of effluent compliance with regulatory standards. This is a significant issue in the development of control schemes for receiving water quality, due to the lack of feedback measurements. More fundamentally (as stated in [69]), water treatment plants were not originally designed for real time control. This is apparent in the widespread lack of flexible control handles, and is a factor restricting the control approaches considered in this thesis. The control schemes developed here remain strictly within the existing structures of treatment plants, seeking to allow improvements upon current plant performance, and allow authorities to avoid structural modifications of existing plants.

The common approach of plant designers has been to have inbuilt safety margins such as excessive reactor volumes, whilst ignoring the necessity to include controllability and sufficient plant flexibility (as stated in [122]), thus increasing the plant construction costs whilst also limiting the future development and automation of the wastewater treatment operation. For this reason, the applicability of advanced control techniques is restricted by the controllability issue. Tables 6.1 and 6.2 demonstrate the available sensors and actuators in the urban wastewater system, as considered within this thesis.

Table 6.1 Available sensors and actuators assumed within the ASM2d/QUAL2E urban wastewater system

Section	Sensors	Actuators
Sewer	Flow to Treatment Plant	None
Treatment Plant	Dissolved Oxygen, Ammonia, Phosphorous	Aeration, $Fe(OH)_3$ dosage
River	Dissolved Oxygen, Ammonia	None

Table 6.2 Available sensors and actuators assumed within the Graells urban wastewater system model

Section	Sensors	Actuators
Sewer	Flow to Treatment Plant	None
Treatment Plant	Dissolved Oxygen, Substrate	Aeration, dilution rate
River	Dissolved Oxygen, BOD	None

6.1.2 Process Models

A model of the nonlinear dynamics, of the variables that are to be controlled, is required in the application of nonlinear control methods. Instances of calibration of ASM based models against actual system behaviour have been shown, however, in the absence of such situation-specific representations, simplified models of system dynamics may be determined. 'Black box' models of the dynamics, found by system identification techniques, may be used. In the presence of a process model, the control model structure can move from the 'black box' approach to a more 'white box' approach, based on mathematical knowledge of the system dynamic behaviour. The issue of uncertainty still remains however, even in the presence of a highly complex system model. Several assumptions are commonly made in the mathematical representation of the system dynamics. Makinia et al. [100] state that many modelling approaches consider a constant temperature. Petersen [130] described the restrictions of some of the ASM models as (among others): process parameters (which although possibly varying) are fixed at values for a chosen constant temperature, an assumed constant pH (which if varying would affect many process parameters) and nitrification parameters and correction factors are assumed constant. Whilst physical modelling of wastewater treatment systems has been the subject of much research, in practise, physical wastewater treatment models can be difficult to identify. The use of a grey-box model can be appropriate when complete physical models are not available, by retaining the basic structure of these models in the combination of fundamental and experimental modelling techniques. Whilst parameters of the physical system may not always be identifiable, parameters of grey-box models can be identified from process measurements.

The large scale of the water treatment process models and the large number of variables and uncertainties, has commonly restricted the implementation of nonlinear control schemes. System uncertainty in the activated sludge processes is heavily dependent on the identifiability of the process kinetics and parameters. Identifiability of process parameters, for example in research by [175], has been implemented. The process of identifiability commonly does not produce unique solutions, several choices of process parameters are possible, each producing valid representations of behaviour specific to the identified situation. Mailleret et al. [99] stated the 'delicate' nature of the control problem for water treatment: the approximate nature of the process models available restrict applications of nonlinear methodologies, and a large source of this lack of knowledge is based within the system kinetics. However, the presence of an identified model for a given process does provide the user with the ability to apply a more advanced nonlinear control approach than that of simplified or reduced models. The nonlinear control approaches detailed within this chapter consider, then, two situations; that in which a full process model is available and alternatively that in which there is a lack of process knowledge.

The use of mathematical models in the control of the urban wastewater system allows the integration, into the control strategy, of current and future receiving water quality dynamics. These models can be used in predicting the treatment plant effluent quality required during the subsequent operating period in order to acheive the desired quality of receiving water. By varying the performance of the treatment plant, and therefore the effluent characteristics, the quality of the receiving waters may also be varied. The research detailed in Meirlaen [105] concentrated on the use of model simplification and model reduction, and the use of artificial neural networks, in control and prediction of dissolved oxygen concentrations, and also in the minimisation of ammonia levels in receiving waters.

Linearisation of specific processes within the system is considered here, allowing the use of existing control algorithms in implementation of nonlinear control, via scheduling methods such as fuzzy gain-scheduling or the Wiener model approach demonstrated later. The lack of control handles within the overall system, and the low level of instrumentation in the receiving waters, motivates the use of simplified or reduced models such as these. The immision based strategy extending local treatment plant control to that of the receiving waters avoids the common issues of lack of presence of instrumentation and actuators within the sewer system, restricting the influence of the sewer subsystem upon the control scheme to the measurements of sewer overflows.

Vanrolleghem [183] states that the 'building blocks' of the control of a process are the following

- the process: the system that is to be controlled. In this case, the process to be controlled is that of water quality of the receiving waters.
- the actuators: the method by which the process is manipulated. In the following applications, the actuators are the basic manipulatible variables: air flow rate, dilution rate and chemical dosage.
- the control algorithm: the method by which the appropriate control action is calculated for disturbance rejection or setpoint tracking, which is in this case the model based predictive control algorithm. The aim is to minimise deviations from steady state behaviour, the objective for the water quality.
- the sensors: the devices by which measurements of output variables and disturbances are found.

Additionally, particularly in the case of predictive control, other considerations for the 'building blocks' for WWTP control could include the plant operating costs and the constraints upon the system, biological, chemical and mechanical.

6.1.3 Nonlinear Control

Nonlinear control has been recently implemented on similar biological processes, for example, Szederkényi et al. [172] demonstrated control of a fermentation process using several methods (for example LQ and input-output linearisation approaches), and adaptive control applications have been demonstrated, such as that used by Hilgert et al. [65]. Nonlinear control of a continuous bioreactor was demonstrated by Gonzalez et al. [55], via the use of a feedback linearisation controller implemented with online estimation of system uncertainties (such as process kinetics). Aoyama et al. [5] described nonlinear process control for a CSTR system via the use of a neural network approach. Maillerct states that global approaches to nonlinear bioprocess control are mainly linearising controllers (for example the control of anaerobic digestion by [129] and nonlinear bioreactor control [133]). The issue with these approaches is the use of exact linearisation and therefore the requirement of full model knowledge. Table 6.3 indicates the control strategies utilised within this chapter.

Table **6.3** Control Strategies in Chapter 6 for the Urban Wastewater System
SD - State Dependent, **WMPC** - Wiener MPC, **FGS** - Fuzzy Gain Scheduling

Section	**Control Strategy**
Sewer	None
Treatment Plant	SD, Linear
River	WMPC, FGS, Linear, SD

This chapter aims to introduce approaches in the case of the following controllable processes

- SISO Nonlinear: Clearly, the biological wastewater treatment process is multiple input multiple output (MIMO), however the common control approach is to view the controlled variables individually in single input/single output (SISO) structures. Olsson and Jeppson [122] state that the different time constants of a wastewater treatment system, ranging from minutes for the control of dissolved oxygen to days for sludge composition, decouple control actions into separate SISO controllers. Thus the initial sections concerning Fuzzy Gain-Scheduled and Wiener MPC are applicable to SISO nonlinear processes.
- MIMO Nonlinear: Although the current absence of control handles has limited control approaches to primarily SISO processes, the recent increase in instrumentation and automation of wastewater treatment, and the regulatory and environmental pressures concerning water quality, it is not unreasonable to assume that nonlinear control of MIMO processes within the urban wastewater system will become increasingly important. In this event, nonlinear modelling approaches such as state dependent techniques can be utilised in exploiting the available process descriptions.

The following control approaches are demonstrated in this chapter, in control of the urban wastewater system, in addition to the reduction of effects of storm events:

- Fuzzy Gain-Scheduled Control: The structure and algorithms developed for the purposes of linear model based control may be extended with the use of multiple linear descriptions of the nonlinear behaviour of a process. SISO nonlinear control may be produced via the scheduling of multiple linear controllers, according to the description of the process nonlinear operating range.

- Wiener based Model Predictive Control: In specific cases, the nonlinear process may be represented by a linear dynamic description, followed by a static nonlinearity. In this event, this structure may be utilised in the development of a nonlinear controller, based on an instantaneously linear process model at a given sample instant. Thus the process model is updated at each sample time, allowing for increased accuracy in predictions of system behaviour.

- State Dependent Model based Predictive Control: The nonlinear model as defined by a mathematical process description can be reformulated to the state space structure, allowing the use of the linear control algorithms. The construction of a linear time varying model, according to the calculated control sequence over the user specified control horizon, provides more accurate model predictions and control actions according to predicted behaviour.

6.2 System Conditions

The urban wastewater model utilised for the purposes of simulation of fuzzy gain scheduled (FGS) and Wiener MPC control is the ASM2d/QUAL2E model of [31]. The plant conditions used in all simulations for the control implementation are defined by the following settings. There exists a PI controller to manipulate the wastage flow, for which a constant setpoint of 150 (m^3/d) was set. The steady state influent flow can be found to be $5575m^3/d$ and the chemical dosage of $Fe(OH)_3$ is set to a value of 600000 (g/d).The temperature is assumed to be constant at a value of 20^oC. The measurements assumed available, within the treatment plant and also in the receiving waters, are as shown in the above Table 6.1. The plant PI controller for dissolved oxygen concentrations in the treatment plant aerobic reactor is as defined in Table 6.4.

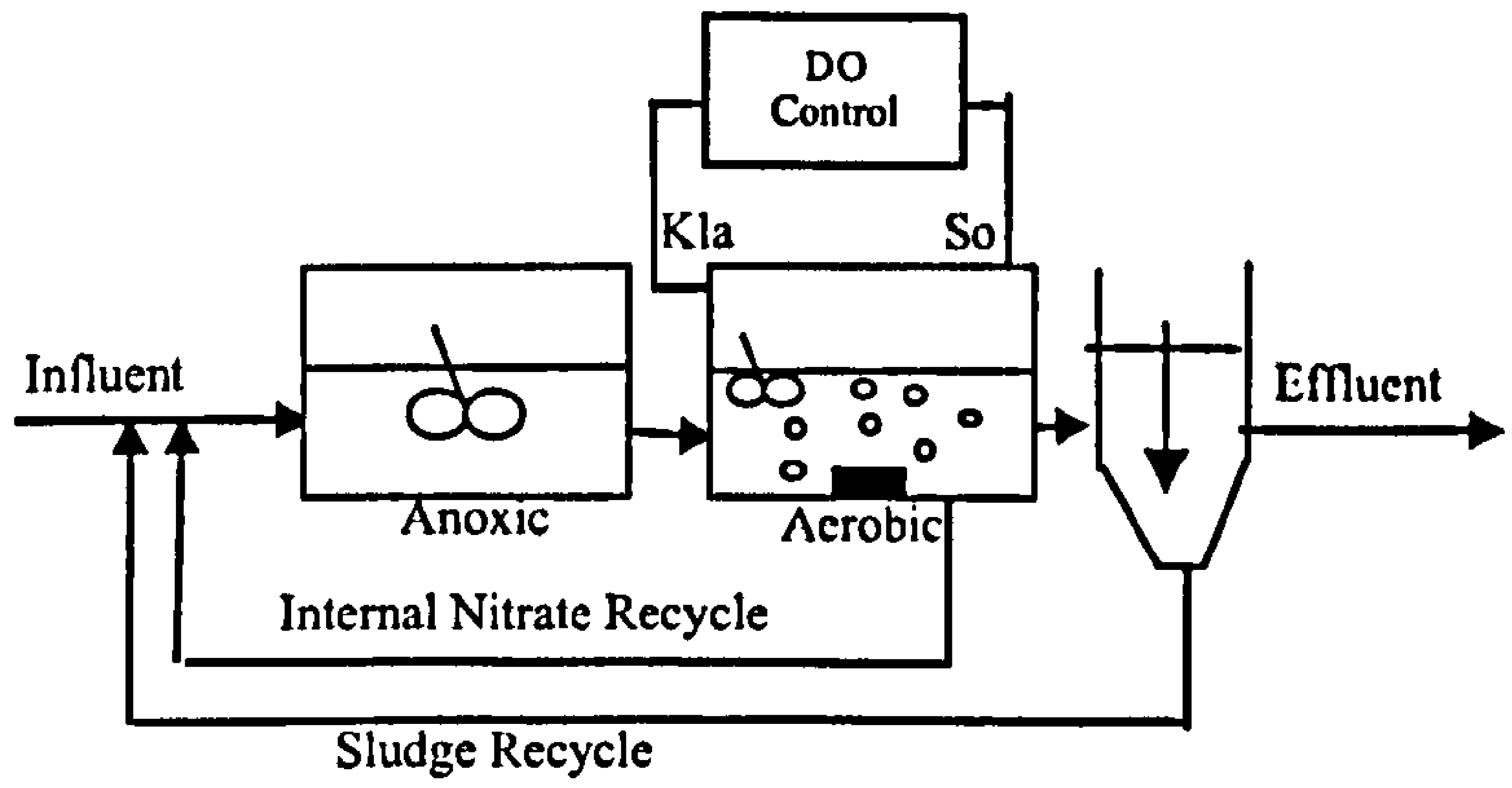

Figure 6-1: PI Control of the Aerobic Reactor within WWTP

Table 6.4 PI Tuning parameters

	K_p	T_i
Dissolved Oxygen	85	130
Ammonia	1500	500

In the event of ammonia control, the control action of aeration of the aerobic reactor is again utilised, although the above PI controller is modified to control ammonia/ammonium concentrations in the reactor effluent. However, the steady state value of ammonia in the aerobic tank is 0.53g/m^3, in comparison to a dissolved oxygen steady state value of 6.87g/m^3, and the PI control parameters are therefore scaled appropriate to the change in magnitude of the controlled variable, and are as demonstrated in Table 6.4.

6.3 Fuzzy Gain Scheduled Predictive Control

Bioprocesses are well-known within control applications for their nonlinear nature. The gain-scheduling approach allows the adaptation of the control application to the nonlinear operation, whilst retaining the common linear control. In the application considered in this section, a family of controllers are designed on the basis of a linearisation via the subspace identification algorithm. Consider that the system to be controlled consists of the following variables: a controlled variable, a measured disturbance, and a manipulated variable. The dissolved oxygen process can be considered a nonlinear SISO system with a measured disturbance model, where the manipulated variable is the air flow rate to the aerobic reaction tank within the treatment plant, and the controlled output is the dissolved oxygen concentration in the receiving waters subsequent to the treatment plant effluent point. Similarly, in the control of ammonia levels these variables are defined as downriver and upriver ammonia concentrations respectively, and, as above, the rate of air flow. The use of the feedforward approach in wastewater treatment control is stated by Vanrolleghem [183] to have the advantage of compensating for the effect of predictable disturbances in water quality. Theoretically, the compensation for system disturbances can be performed completely if it is fully measured or predicted, requiring either a perfect process model or ideal process measurements. This is clearly not currently realistic, however the feedforward approach in wastewater control, coupled with the feedback approach and model mismatch compensation may allow for some of the benefits of feedforward control.

6.3.1 Dissolved Oxygen Fuzzy Gain Scheduled Control

The efficient control of the dissolved oxygen levels in the receiving waters is one of the most significant within the urban wastewater system, making dissolved oxygen an important water quality indicator. Dissolved oxygen is essential for the survival of aquatic life (both plant and fish), and extended oxygen depletion can have extreme effects upon fish populations. In [69], the most common type of applied real-time control is stated to be the control of dissolved oxygen concentration in the aerobic reactor based on feedback of dissolved oxygen measurements. This is the case in the control, presented in this thesis, of dissolved oxygen in the receiving waters, implemented via manipulation of dissolved oxygen PI setpoint within the treatment plant, and thus the variation in aeration within this aerobic reactor.

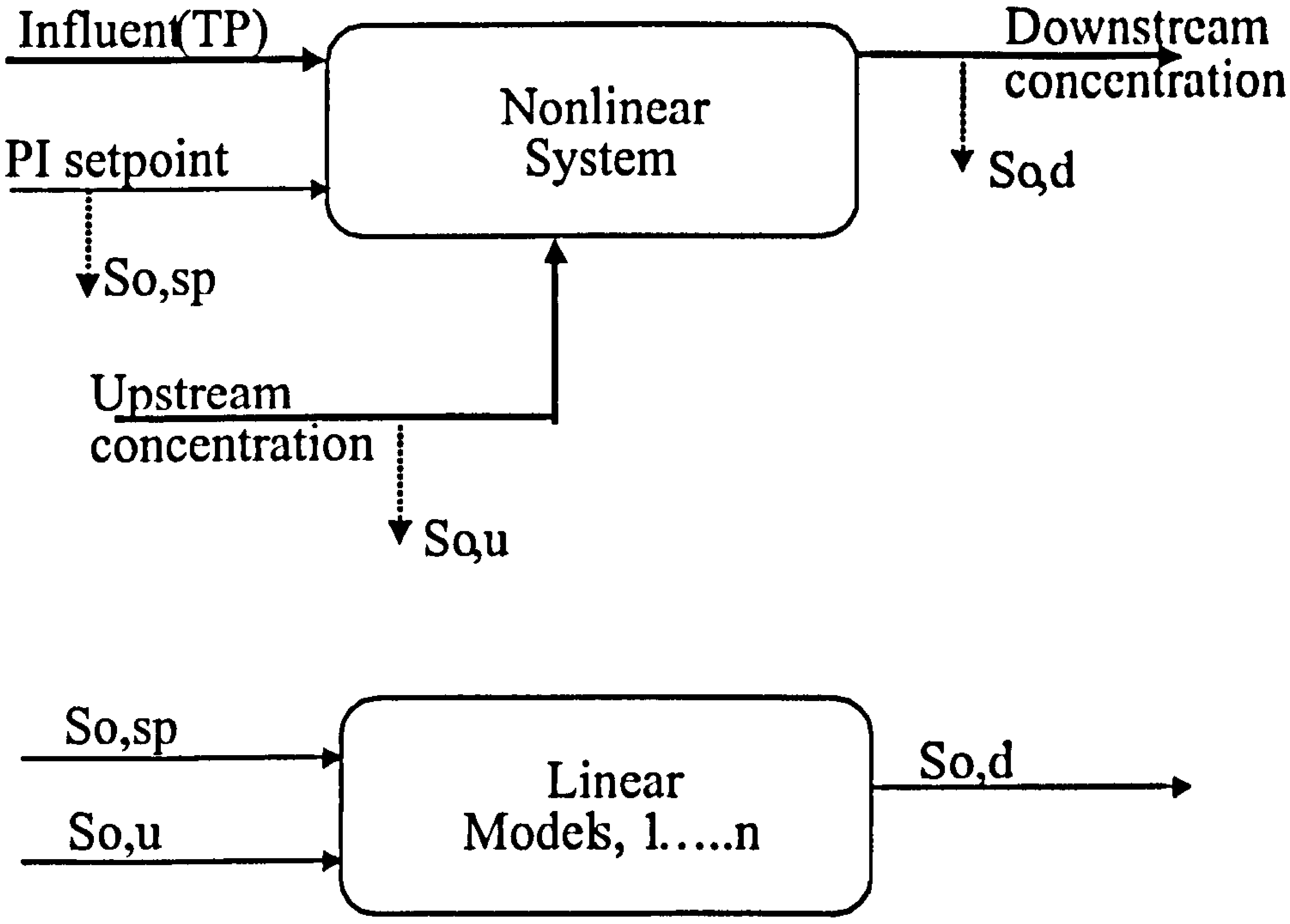

Figure 6-2: Linear model structure in description of the nonlinear process: with model inputs of dissolved oxygen setpoint $S_{o,sp}$, and upstream dissolved oxygen concentration $S_{o,u}$, and output of downstream dissolved oxygen concentration $S_{o,d}$

The decision in the design of the fuzzy gain scheduled control are dictated by three steps. The first involves the decision of the parameters that define the membership functions. The control purpose in this case is the manipulation of the urban wastewater system during changes in weather conditions, which may be characterised by the change of influent to the treatment plant. Thus the 'scheduling variable' chosen was that of treatment plant influent flow, under the assumption that changes in wastewater characteristics within the system should be indicated by variations in flow to the treatment plant. The second decision involves the choice of the number of 'descriptors' in the fuzzy scheduler. In this case the choice should cover the varying weather conditions. The approximately linear regions over the operating range in the case of nonlinear dissolved oxygen control can be divided into nine ranges of flow. The third decision in the design of the fuzzy scheduler is the choice of the shape of the membership function for each of the above. In this case, the most appropriate shape of membership function for the purpose of smooth transition between operating ranges is that of a trapezoid.

The objective of the FGS approach is to utilise the aforementioned linear model identification methods in the simplification of the control of the dissolved oxygen process in the receiving waters to a nonlinear SISO system. Subspace identification is used in the description of the nonlinear process with a set of linear models. Each linear model is identified at specific treatment plant influent flow levels, commencing at an influent flow of 3000m^3/d, and at 3000m^3/d intervals thereafter. This is implemented for a total of nine linear ranges, to a maximum influent flow of 27000m^3/d. The following PRBS parameters for discrete time step (T_{disc}), mean value (m) and amplitude (a):

Table 6.6 Dissolved Oxygen Process PRBS Identification Parameters

T_{disc}	m	a
0.05	4	4

The model structure for the predictive control algorithm assumes the following:

- Each linear system is chosen to satisfy controllability and observability requirements.
- Each linear model, of 1.....n, is considered to be of the format of: a linear model (including a measured disturbance) and a constant unmeasured disturbance state.

  - the upriver dissolved oxygen concentration is a measured disturbance, and assumed constant over the prediction horizon.

  - the linear model utilised for state estimation includes an unmeasured disturbance state, to compensate for plant-model mismatch, estimated at each sample instant.
- Kalman filters are utilised in the estimation of the linear model states, for each linear controller

The dissolved oxygen control was simulated in the presence of disturbances, during a storm event. The characteristics of the storm event are defined by its duration of 0.33h at

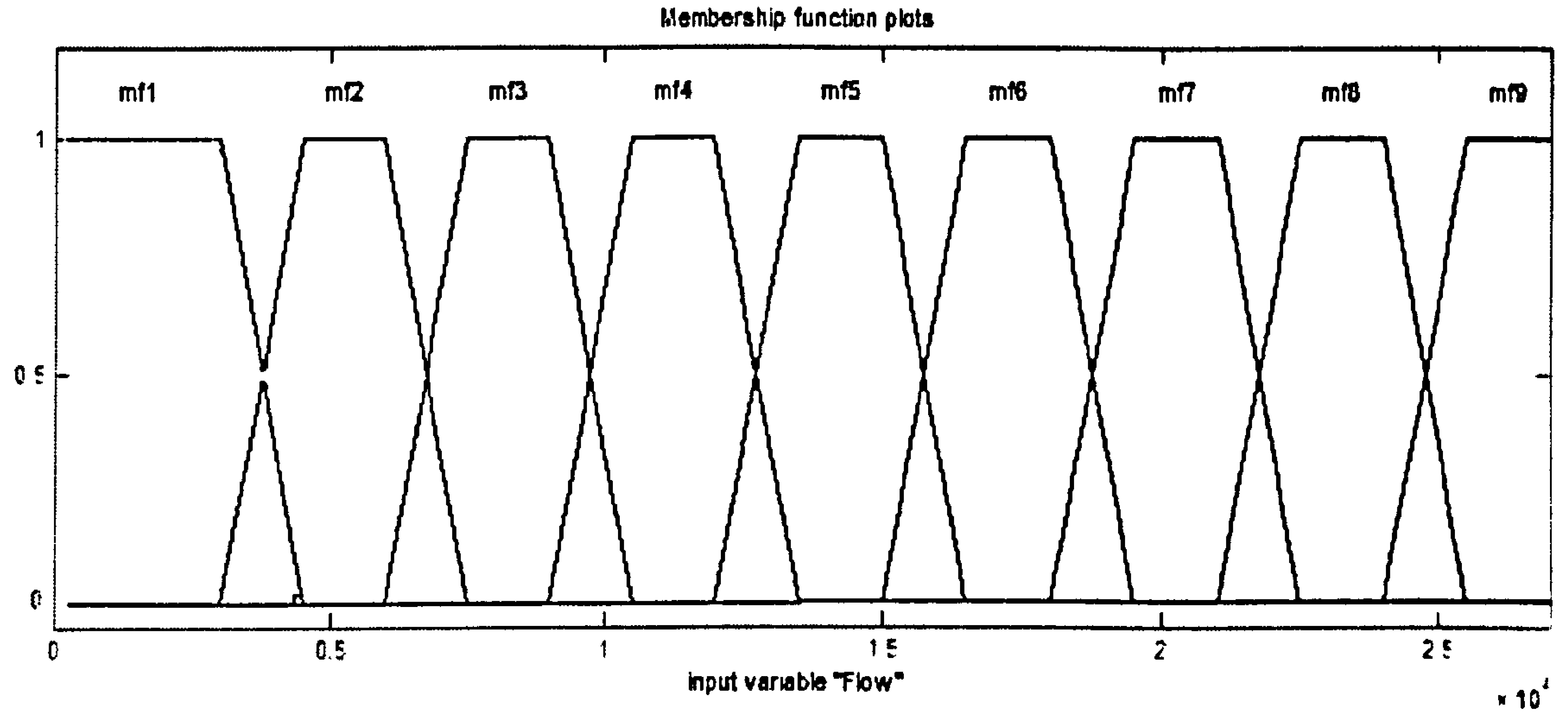

Figure 6-3: FGS Control Membership Function for the Nonlinear Dissolved Oxygen Process

time 4 *days*, and the rain intensity of $30mm/h$ causing a combined sewer overflow and a dip in the river dissolved oxygen level. This system was tuned to track a dissolved oxygen setpoint in the receiving waters of $7.5gm^{-3}$. The fuzzy membership function chosen allows for simple design and interpretation of the fuzzy rules. The fuzzy scheduler has one input, the measurement of the influent flow to the treatment plant, and $n = 9$ outputs of weighting values, each in the range $0 \leq w_n \leq 1$ and with a total sum $W = \sum_1^n w_n = 1$

The FGS approach designed was compared with a control scheme of a single linear predictive controller. The single linear controller utilised, for comparison purposes, is that controller of the FGS structure which is active at the steady state influent flow. That is, at the steady state conditions, the FGS and linear predictive control performances are equivalent. The obtained results for fuzzy gain scheduled control in Figure 6-4 show the effectiveness of the approach in the nonlinear control of dissolved oxygen concentrations in receiving waters, and its improvement upon the performance of its linear counterpart. The objectives, as stated, is the minimisation of effects of CSO's. In this case, the control objective requires that the time for which the dissolved oxygen is below a chosen threshold is minimised. By inspection of the control performance, it can be seen that the FGS approach descreases the length of time for which there exists oxygen depletion. An additional control objective is the maximisation of the minimum dissolved oxygen concentration as a result of the system disturbance, for which the FGS approach again improves upon the performance

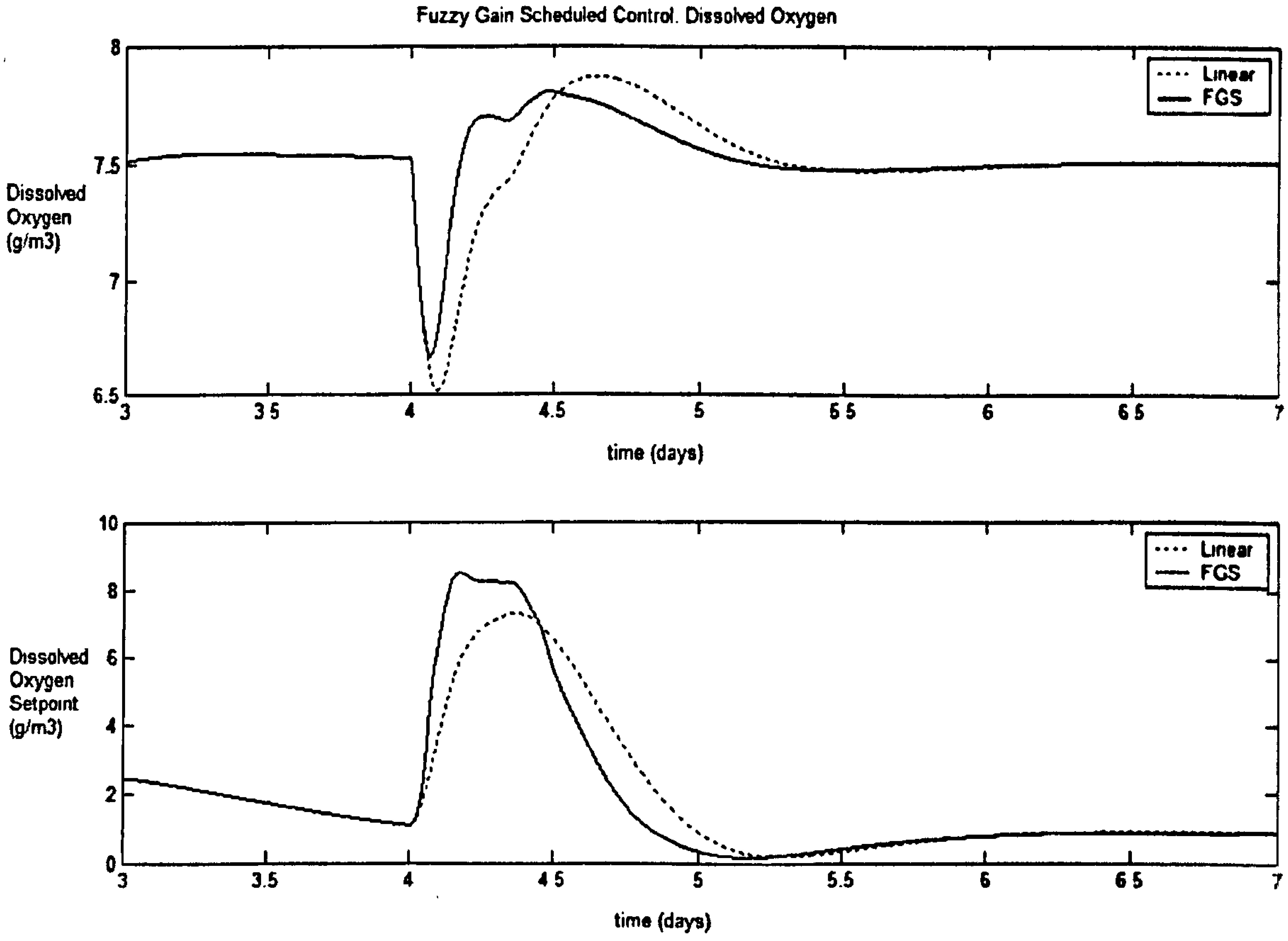

Figure 6-4: Fuzzy Gain Scheduled Predictive Control of Dissolved Oxygen in the Urban Wastewater System, versus Linear MPC

of the linear control scheme. The linear controller is based upon a linear model which is not valid during such large deviations from the steady state behaviour. On the other hand in the case of the FGS scheme, the model upon which the active controller is based provides a closer approximation to the actual plant behaviour. This approach utilises a single variable, of the treatment plant influent flow, in the scheduling of nonlinear control. However, an increase in scheduling variables could be advisable, for example, the inclusion of a pollutant concentration in the treatment plant wastewater influent. This could act as a representation of changes in the influent wastewater characteristics that may not be indicated by a change in influent flow (for example, during a high nutrient load from an industrial source).

6.3.2 Ammonia Fuzzy Gain Scheduled Control

The behaviour of ammonia is significantly more nonlinear than that of dissolved oxygen, and in addition, due to coupling, has sensitivity to variations in other system variables. In this case, the control action is defined as the variation of the setpoint of the ammonia PI control loop in the aerobic reactor in the treatment plant, additionally a measured disturbance variable is defined as the ammonia concentration upriver in the receiving waters. Dissolved oxygen control in the treatment plant based on online measurements of ammonia levels was demonstrated by Ingildsen et al. [68]. The controlled variable is the ammonia concentration in the receiving waters downriver of the treatment plant effluent.

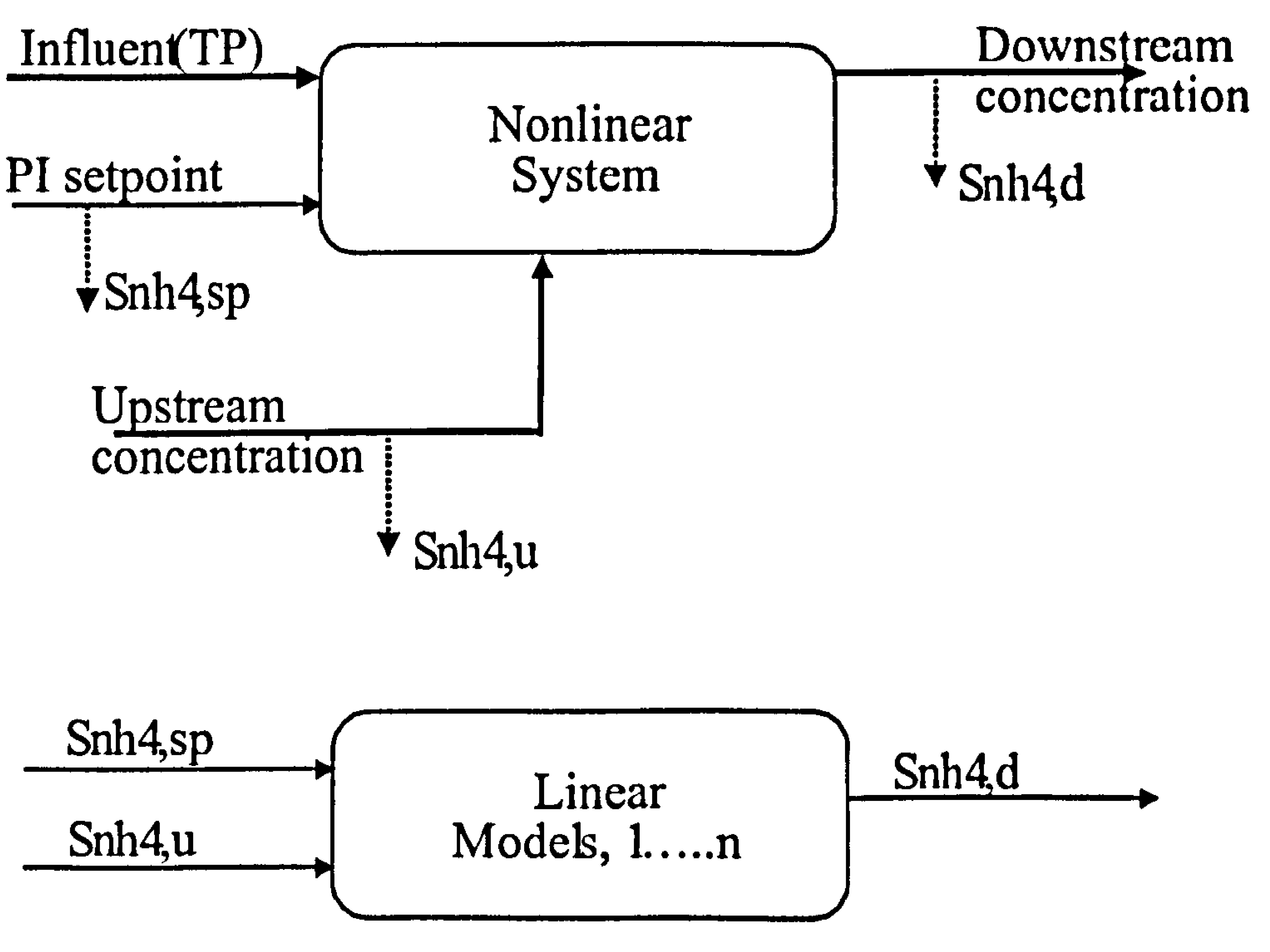

Figure 6-5: Linear model structure in description of the nonlinear process: with model inputs of nitrate/nitrite setpoint $S_{nh4,sp}$, and upstream nitrate/nitrite concentration $S_{nh4,u}$, and output of downstream nitrate/nitrite concentration $S_{nh4,d}$

The approximately linear regions over the operating range in the case of nonlinear ammonia control can be divided into five regions, dictated again by linear operating ranges of flow of $3000m^3$ width each. Due to the 'washout effect' of the hydraulic load upon the treatment plant, the effects of aeration changes upon ammonia dynamics at high flows is not of significant magnitude for control use. This restricted the range of identification of linear models to a maximum flow of $15000m^3/d$. The choice of membership function is again trapezoidal for smooth transitions between linear controllers. As before, therefore, the chosen ranges for the linear controllers are defined by the scheduler membership function as shown below

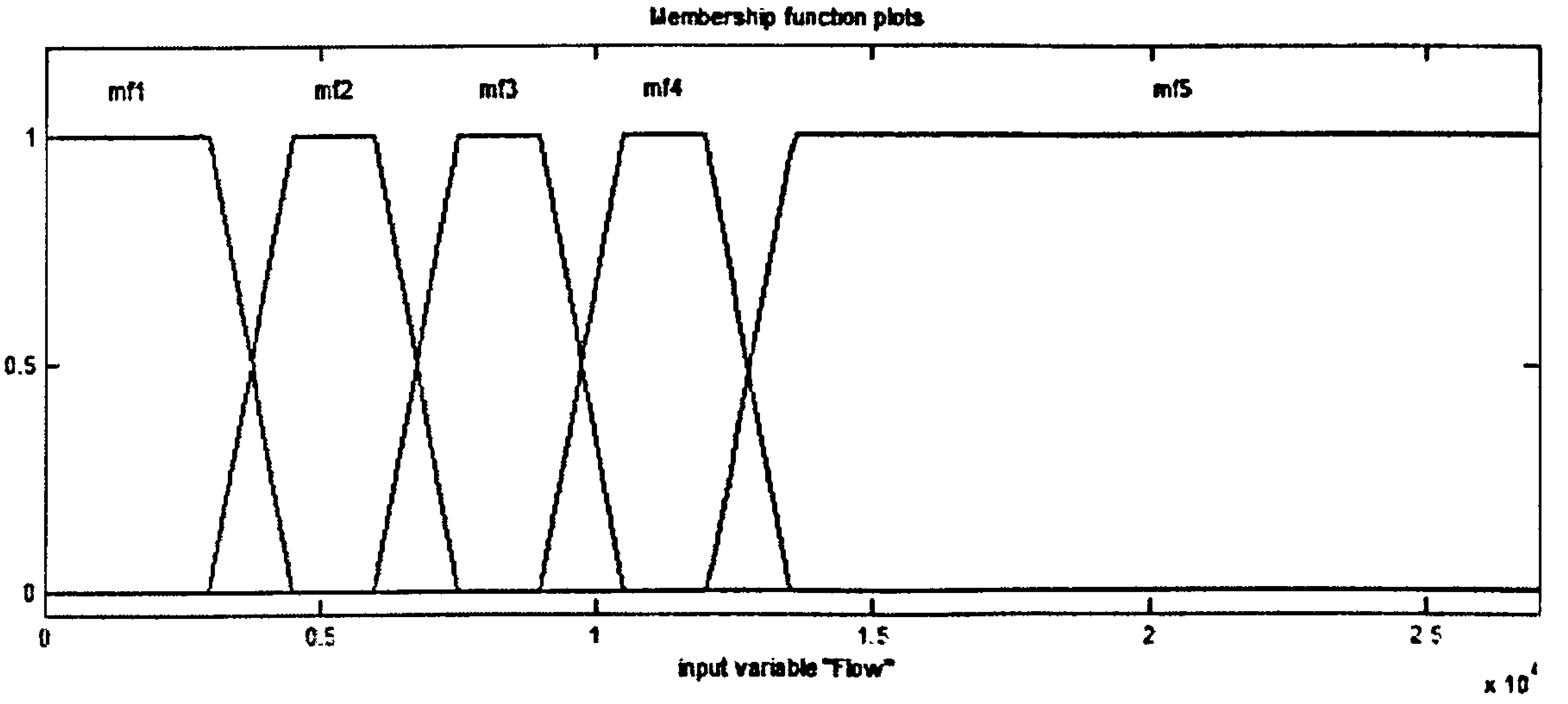

Figure 6-6: Ammonia Membership Function

Table 6.7 Ammonia Process PRBS Identification Parameters

T_{disc}	m	a
0.05	0.4	0.2

Again, the urban wastewater model was simulated in the presence of a storm event defined by a duration of $0.33h$ at time 4 *days*, and the rain intensity of $30mm/h$ causing a combined sewer overflow and an increase in the river ammonia level. This system was tuned to track an ammonia setpoint in the receiving waters of $0.4gm^{-3}$, that is, at the outer limit of the water quality regulations. The aim is therefore to return to within this limit as quickly as possible, that is, to reduce the period for which the system is outside the regulatory constraints. The unconstrained case of the model predictive control is demonstrated. The fuzzy membership function chosen for the scheduling of the linear controllers is demonstrated by the graph above.

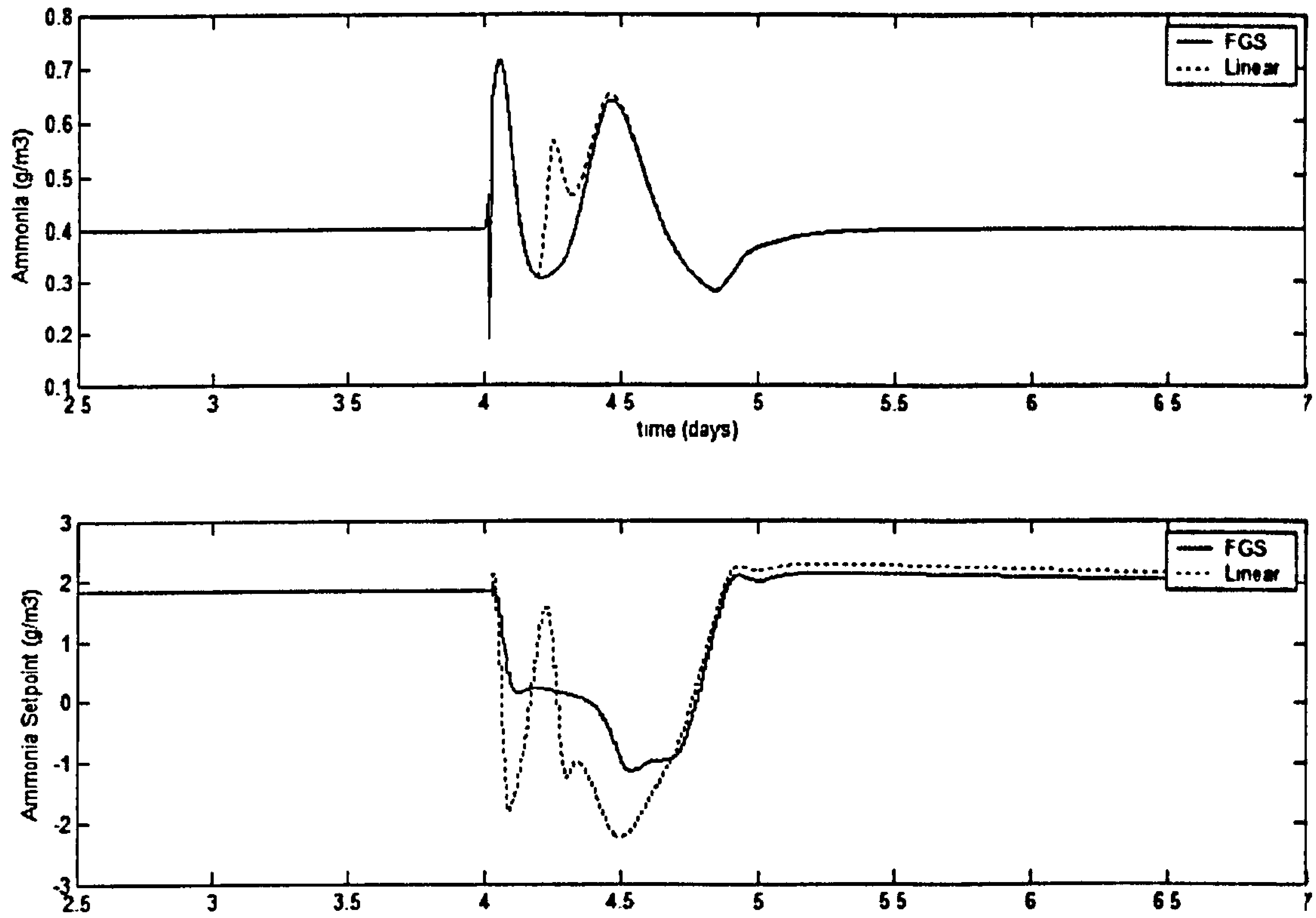

Figure 6-7: Fuzzy Gain Scheduled Control of Ammonia concentration in receiving water

Again the linear control scheme used for comparison purposes was that of the controller identified for a steady state treatment plant influent flow, $5575m^3/d$, and therefore has the equivalent control performance as the FGS at that steady state. The result of the FGS control performance is shown on the graph above and demonstrates the nonlinear control of ammonia concentrations in receiving waters, and the improved performance over its linear counterpart. The issue in this case however, is that the maximum ammonia concentration reached is the same in both cases, the treatment plant is simply unable to compensate further for this ammonia load upon the system. The period of time for which the ammonia concentration is over a specified threshold value (which in this case is set to a value of 0.5g/m3) is less in the case of the FGS control, spending a total of 7.5 hours over the threshold value, in comparison with the linear control scheme for which the period of time spent in violation of the specified threshold was 10.1 hours.

6.4 Nonlinear Behaviour

In order to analyse the behaviour of both dissolved oxygen and ammonia concentrations in the receiving waters, varying hydraulic loads upon the treatment plant were simulated. The effects of a step change in aeration rate of the aerobic reactor within the WWTP during these various load scenarios is demonstrated in Figure 6-8. The dissolved oxygen demonstrates a similar response to the step change at each point over the nonlinear range, indicating an almost linear behaviour scaled by a nonlinear system gain. Conversely, the ammonia process is significantly nonlinear within the treatment plant, and at low flows demonstrates a response to air flow changes that is both slow and of low magnitude. The use of existing low level control structures such as PI control can linearise the behaviour of the treatment plant subsystem further in the case of dissolved oxygen control, allowing the steady state behaviour of the treatment plant to be replicated over changing influent flow. The linear dynamics of the dissolved oxygen process are a result of the effects of the oxygen transfer function for aeration. This can be exploited in the modelling of the dissolved oxygen process via linear dynamics and nonlinear system gain, in the format of the Wiener model approach detailed in Chapter 5.

6.5 Wiener Model Predictive Control

The Wiener model structure describes the relation between input aeration rate K_{la}, and output dissolved oxygen concentration in the receiving water, S_O. The dissolved oxygen process is suitable for the Wiener model approach due to the linearising effect of the oxygen transfer of the treatment plant process aeration. The manipulated variable in this system model is defined to be the above aeration rate, and therefore a subsequent variable must be included to 'schedule' the static nonlinear gain function. The transfer function of the system is intended to represent the dissolved oxygen process and the effects of the treatment plant influent flow dynamics upon this. The magnitude of influent flow is chosen for this purpose, as the influent flow variable covers the nonlinearity of the process. It

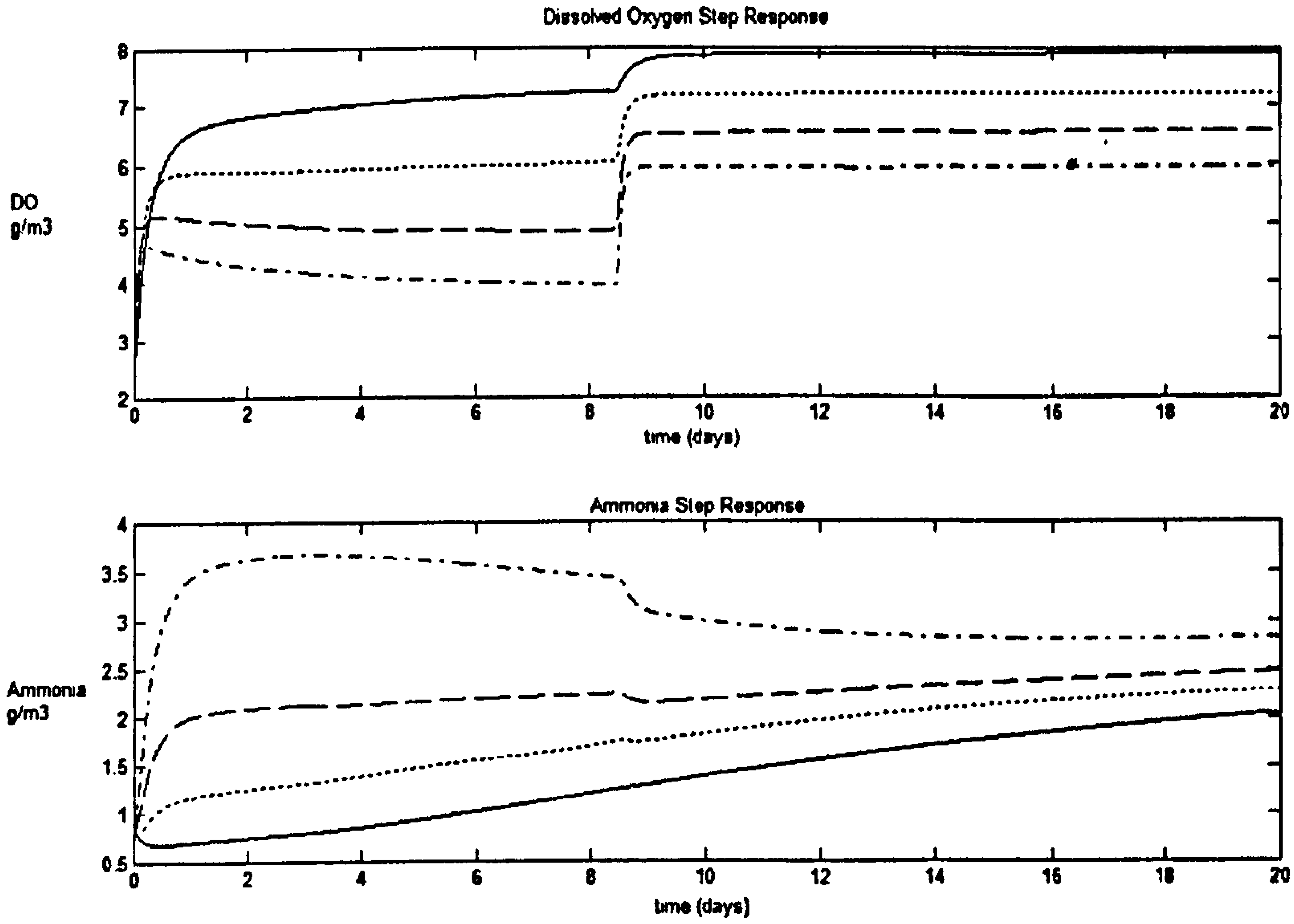

Figure 6-8: Dissolved Oxygen and Ammonia responses to step change in air flow within WWTP

can be defined using previously detailed concepts, as the 'measured disturbance' variable defining unmanipulated but modelled system dynamics or, equivalently, could be described similarly to the 'scheduling variable' as utilised in the fuzzy gain-scheduled approach. The Wiener model identified is valid over a larger operating range than the linear time invariant representation, allowing for more accurate predictions.

An element of the model based predictive control approach is the reduction of the effect of the plant-model mismatch by the implementation of only the first action of the calculated control sequence, subsequent to which the sequence is recalculated. However, this can be further exploited by updating the process model at each sampling point. For the control of the dissolved oxygen process, the process model may be simplified to the nonlinear SISO representation of aeration effects upon the receiving water dissolved oxygen dynamics. The Wiener model approach realises the nonlinear dynamics via the separation of linear dynamics and static nonlinearities. From the viewpoint of identification, the dissolved oxygen process within treatment plant systems can be regarded as a Wiener structure; an LTI system cascaded with a static nonlinearity.

In particular, within the urban wastewater system the effect of the major nonlinearity in the dissolved oxygen process, that is variable flow, can be modelled within a nonlinear gain function, acting upon the dynamics of the oxygen process itself. Lindberg et al. [94] demonstrated the representation of this process via the knowledge of oxygen transfer. The control approach of [25] et al utilises the dissolved oxygen reference trajectory, using the structure of the dissolved oxygen dynamics and the two time scales (fast and slow) that exist in this process.

The LTI transfer function and static nonlinearity as determined by the Wiener model approach are not directly used in the predictive control algorithm, but are instead formulated together within the input dependent state space model of Chapter 6. This approach utilises the idea of a state space model, similar to that of the state dependent approach, updating the model at each sample point.

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= C(u_k)x_k + D(u_k)u_k \end{aligned} \tag{6.1}$$

As can be seen in the above diagram, the Wiener model approach is a block oriented method of modelling system nonlinearities, consisting of a dynamic block and a static block. The Wiener model in this case is utilised to represent the significant dynamics with respect to the control of the dissolved oxygen concentration and the model is developed by performing step tests on the nonlinear system within the operating space of the process. It is assumed that system physical parameters are constant (or at least changing slowly), though this may introduce some uncertainty in the model in the case of changing parameters. The system is treated as a two input, one output system. The inputs are defined as the dissolved oxygen PI loop control in the treatment plant (with the manipulated variable K_{la} (the mass transfer coefficient, related to the air flow rate)) and the measured disturbance (or scheduling variable) Q. The output (or controlled variable) is defined as the dissolved oxygen

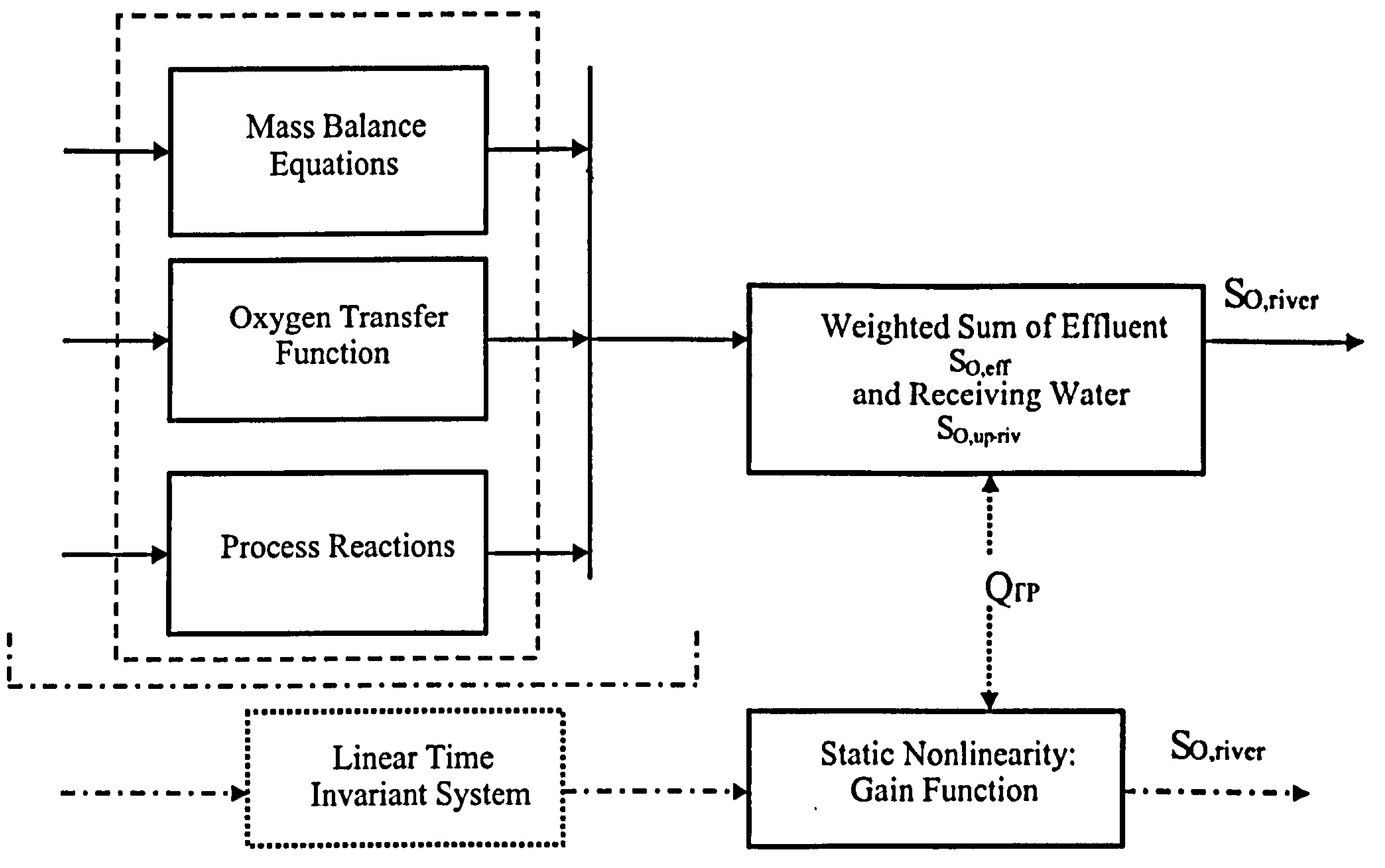

Figure 6-9: Receiving water Dissolved Oxygen Dynamics with respect to Changes in WWTP PI Setpoint Represented by Wiener Model Structure

concentration in the river, $S_{o,river}$. In this case, the upriver concentration of dissolved oxygen is not considered, in that the feedforward information concerning CSO's is not included in the process model. The nonlinear model is assumed to take into account changes in influent flow levels to the plant, and to consider any additional oxygen depletion as a disturbance event (caused by combined sewer overflows). The nonlinear state space model of this process is developed according to the following steps:

1. Step tests are performed upon the process. The same step change in PI setpoint is applied for varying flow loads to the treatment plant.

2. The analysis of the steady state values of the system is made. The system is analysed via the step response from steady state of the system at that given operating point (which varied at each flow rate). The steady state gain of the system, varying over the operating range, is described by a nonlinear gain function dependent upon the influent flow to the treatment plant $g(\rho)$, where ρ is defined as the scheduling variable, Q_{in}..

The dynamics of the system are described by a transfer function.

3. Subsequent to the steady state gain analysis, the response are scaled to a unity steady state gain, for the analysis of the dynamics of the step response. The settling time of the step response, for a system of linear dynamics, should be similar for each flow range, allowing a low-order description of the system to be determined.

4. The nonlinear representation of the system can be seen in the format scaled by the gain function $g(\rho)$

$$G(s) = \frac{as+1}{bs+1} g(\rho) \tag{6.2}$$

which is converted to the state space representation for the purposes of the use in the model predictive control structure, producing an input dependent state space description of the process. The identification problem is considered as: in the presence of changes in influent flow levels to the treatment plant and variations in the air flow rate, to produce a state space realisation of the system dynamics, and an estimate of the system nonlinearity. In this case, the model identification is determined by analytical methods, via step response tests upon the process model. The data is divided into two components: the component that changes over the flow range (the steady state gain) and the component which does not (the transfer function of the step response).

Step 1: The former (the static nonlinearity) was determined by calculating the steady state gain of the system at different values of flow. This was implemented by applying a step change to the system and analysing the responses. It is necessary to analyse the changes, from the steady state, of the dissolved oxygen concentration at each given flow level. The required information in constructing a function of the nonlinear system gain is the magnitude of the step response at each given influent flow. The offset (from zero) at steady state for each operating point, prior to this step change, was therefore removed rom each set of step response data. In this manner, the magnitude of step response could be determined. The steady state offset at any given operating point within this flow range may be established via interpolation of the data described within the Table below. Thus, the step responses, with zero initial steady state offset, are shown in the following figure.

Table 6.8 Steady State Offsets for Wiener model over flow range

Flow (m3/d)	3000	6000	9000	12000	15000	18000	21000	24000
Steady State Offset	8.0850	7.8350	7.6100	7.4100	7.2250	7.0700	6.9250	6.7970

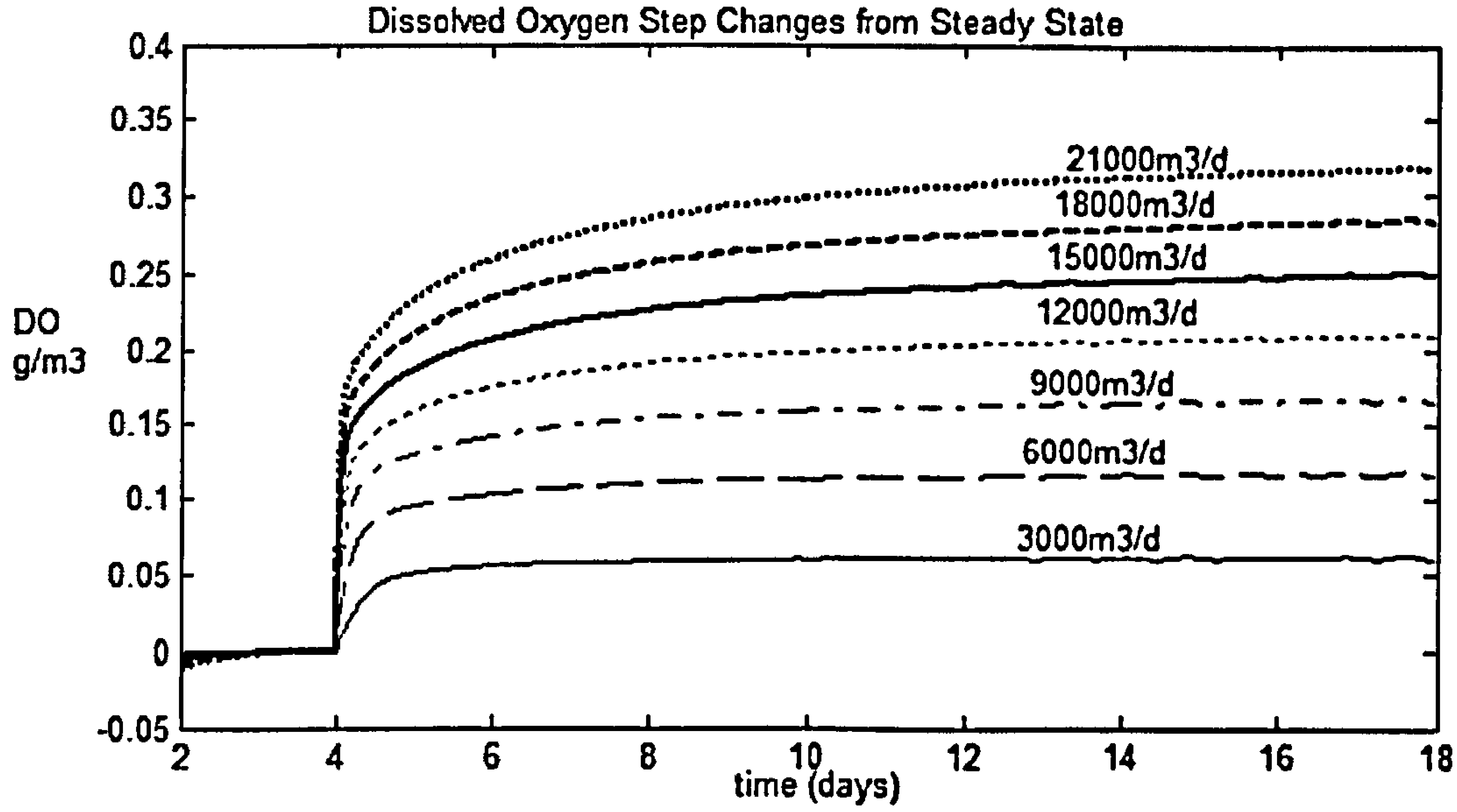

Figure 6-10: Step responses of dissolved oxygen in receiving waters for varying influent WWTP flows

Step 2: In determining the static nonlinearity of the process, the system gain with respect to the changing influent flow must be analysed. The effects of flow variations upon the dissolved oxygen process shown in the table below can be represented by a nonlinear function.

Table 6.9 Nonlinear system gains for Wiener model over flow range

Flow (m3/d)	3000	6000	9000	12000	15000	18000	21000	24000
Nonlinear gain	0.061	0.1180	0.1650	0.2070	0.2450	0.2800	0.3120	0.3350

However, it can be observed that the static nonlinearity of the process is approximately linear. The behaviour of the nonlinear gain (y) with respect to changing influent flow (ρ) is therefore defined by

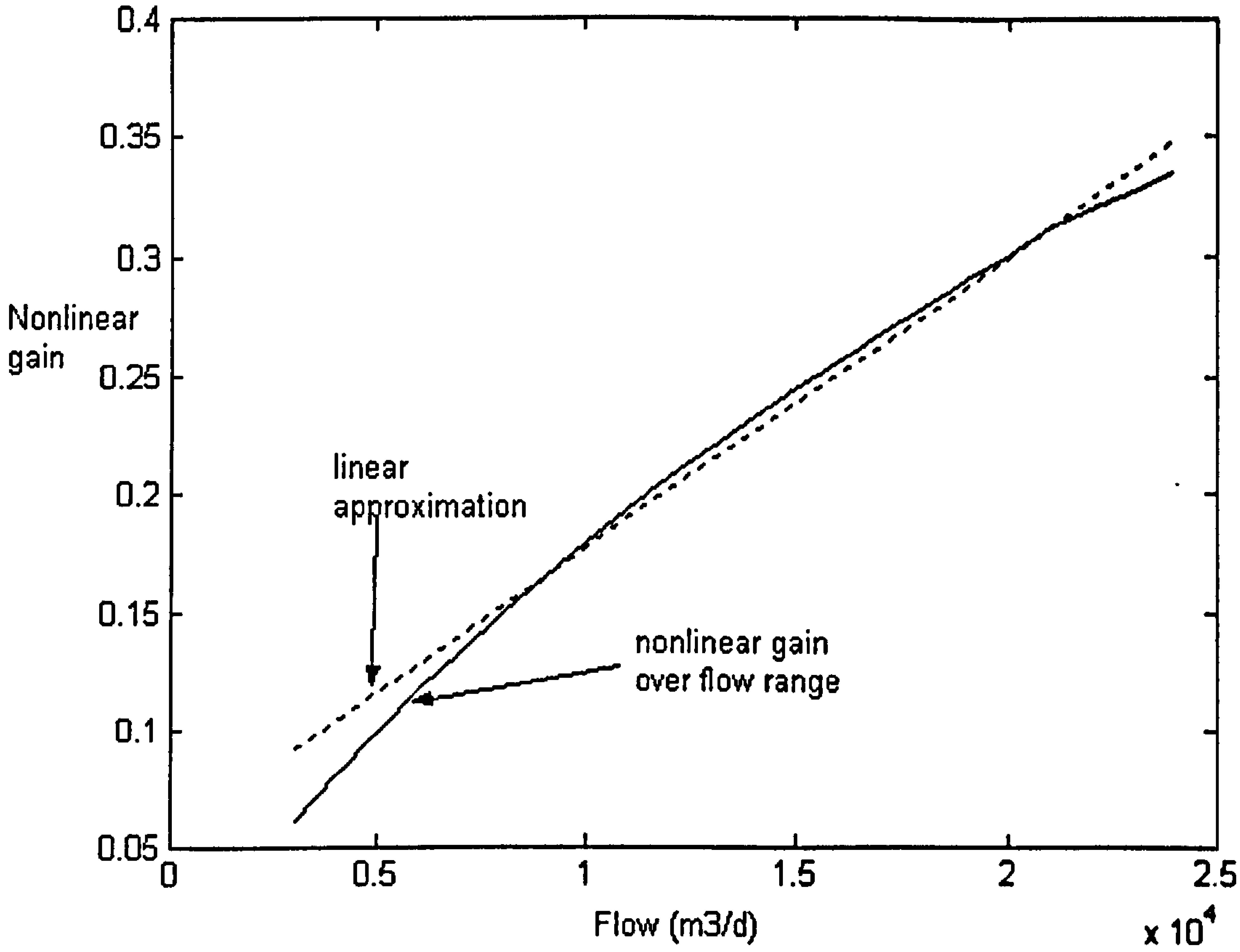

Figure 6-11: Nonlinear steady state system gain response with respect to changing WWTP influent flow rate

$$y = g(\rho) \tag{6.3}$$
$$= 1.2250x10^{-005}\rho + 0.05475 \tag{6.4}$$

Step 3: It can be seen that the dynamics of the system are approximately constant over the nonlinear range. Tian and Fugii [176] state that the gain can arbitrarily be fixed in either subsystem, affecting the scaling upon the other, without affecting the input output characteristics of the nonlinear model. The steady state gain at each influent flow level is used to produce the nonlinear gain function, as demonstrated in Steps **1** and **2**. Therefore the steady state responses above are adjusted to a unity steady state gain (that is, normalised with respect to the input step change), eliminating the effects of the static nonlinearity, so that it may be seen that the responses behave almost identically at each flow level.

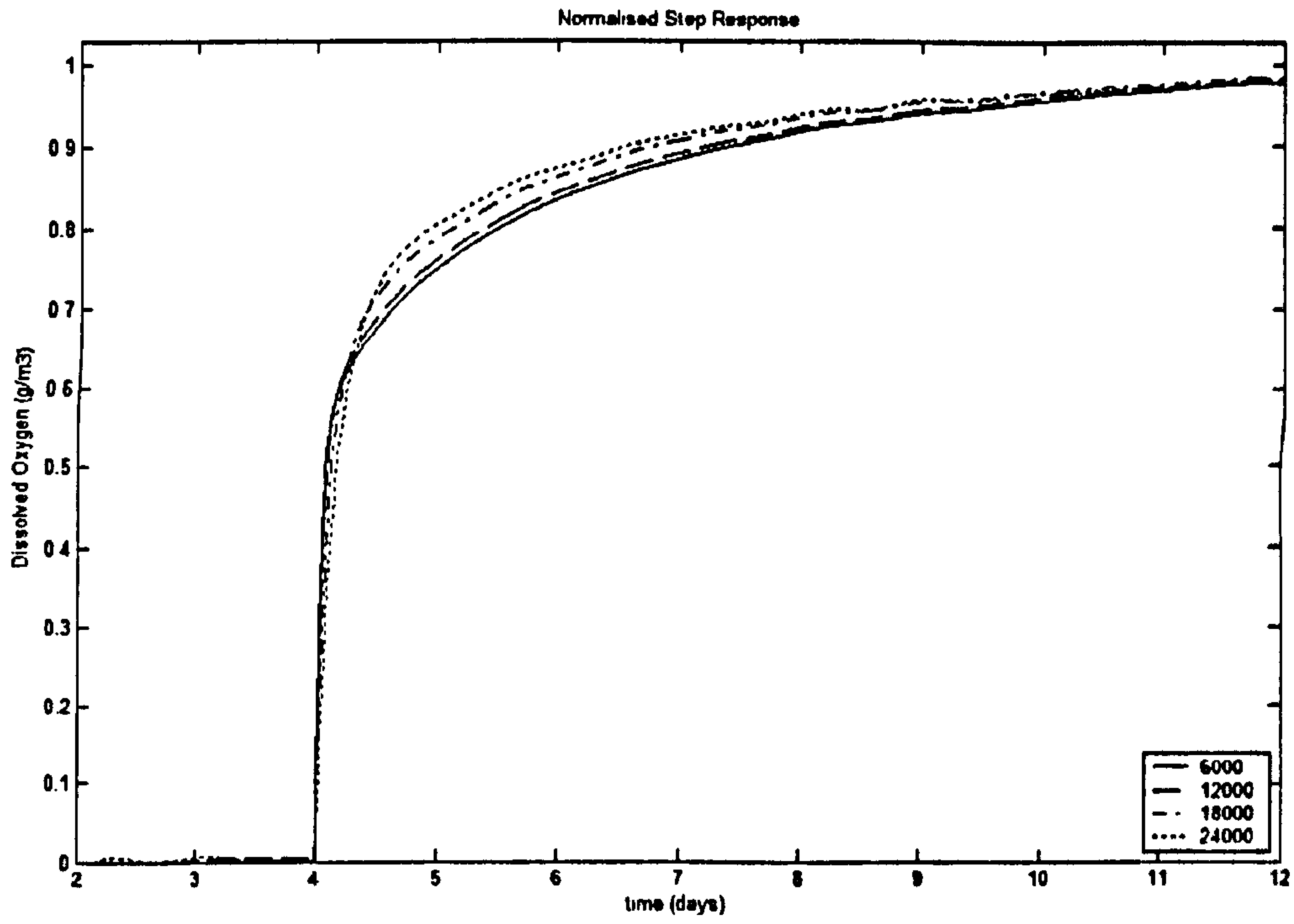

Figure 6-12: Unity gain step responses of dissolved oxygen in receiving waters

Step 4: The LTI transfer function of the system dynamics determined from the step tests above is defined by the following transfer function as

$$G(s) = \frac{0.45s+1}{s+1} \tag{6.5}$$

The state space model representation of this transfer function is required and thus a conversion to the state space domain produces the form as follows:

$$\begin{aligned} A &= [-1] \qquad B=[1] \\ C &= [0.55] \qquad D=[0.45] \end{aligned} \tag{6.6}$$

so that the full state space description of the Wiener process is as follows:

$$A = [-1] \qquad B=[1] \tag{6.7}$$

$$C = [0.55g(\rho)] \qquad D = [0.45 * g(\rho)]$$

The behaviour of the Wiener model of the dissolved oxygen process considered is compared with the behaviour of the 'real' process for a storm event. The figure below demonstrates that the Wiener model response is comparable to that of the 'real' system model.

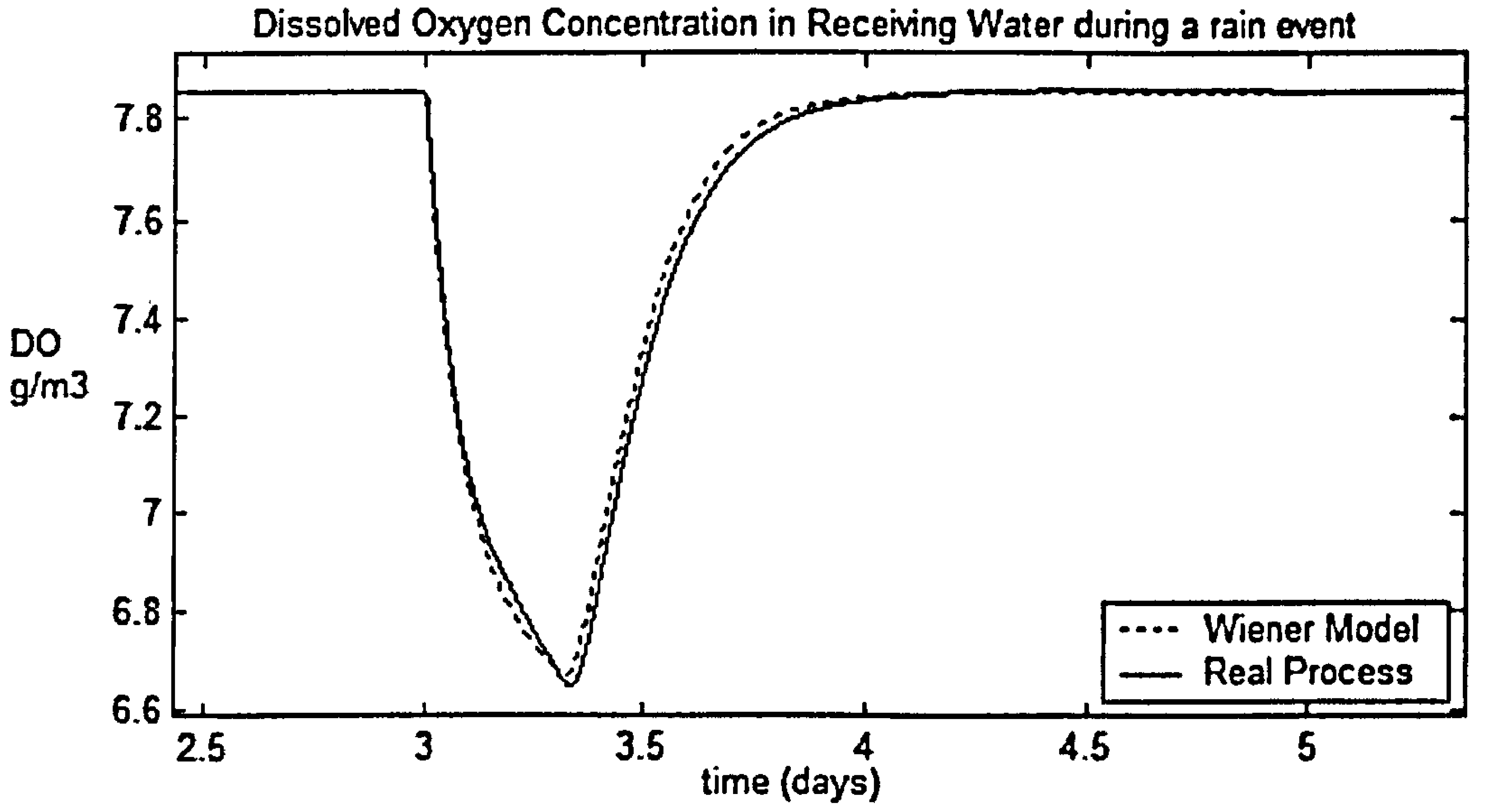

Figure 6-13: Comparison of urban wastewater model and wiener model response during a storm event

The controller design is based around the state space system model as constructed in Equation 6.7 above. Whilst the system model is updated at each sample instant, the GPC algorithm formulation however remains unchanged from the original structure demonstrated in Chapter 2. The difference however, especially in a comparison with the gain-scheduled control, is in the online calculation of the controller characteristics: the prediction equation, the controller gain matrices and the optimisation of the updated cost function. The nonlinear controller designed follows at each sample instant, these steps:

1. Update input dependent model matrices C_k and D_k.
2. State estimation, in this case using the Kalman approach, using the model as defined above.
3. Calculation of the system predictions, using the states calculated in Step 2.
4. Optimisation of the cost function as defined by the matrices of Step 1.

5. Application of the initial element of this control vector, return to Step 1 at next sample instant.

The state estimation is found using an Kalman filter, where the state space equations are updated at each sampling instant. The Kalman gain is determined from these state space equations, and thus is also updated. The predictions are found using the approach as demonstrated by Krauss et al. [85], and the control actions are calculated to minimise a user-specified cost function. The responses demonstrated in the following figures show a moderate rain event: intensity 5mm/hr, duration 1hr, at time 3 days. Two control approaches are shown: a linear controller identified for a steady state influent flow of 5575m$^3/d$ in the treatment plant effluent and a Wiener model predictive controller defined over the nonlinear operating range up to a flow of 27000m$^3/d$. The tuning as specified in the above table is utilised for both controllers for comparison purposes. The objective of the control approach, as previously, is the minimisation of the effects of combined sewer overflow and increased influent flow to the treatment plant. In this case, the measured variable is the dissolved oxygen downriver in the receiving waters only, eliminating the need for a sensor upstream of the treatment plant effluent. The design requirement is reduced to that of a single controller, updated online with each sample instant by a measured disturbance variable, of one control input, and one feedback measurement. This single controller is tuned according to the following parameters:

Table 6.10 MPC Tuning Parameters for Wiener Model approach

T_s	Q	λ	H_u	H_p
0.05	40	0.0005	5	10

The control signal computed by the Wiener and linear predictive controllers developed are constrained by the following inequality:

$$0 \leq u \leq 10gO_2m^{-3} \tag{6.8}$$

The performance as indicated in Figure 6-14 demonstrates the unconstrained case of linear and Wiener model predictive control, whilst Figure 6-15 demonstrates the constrained

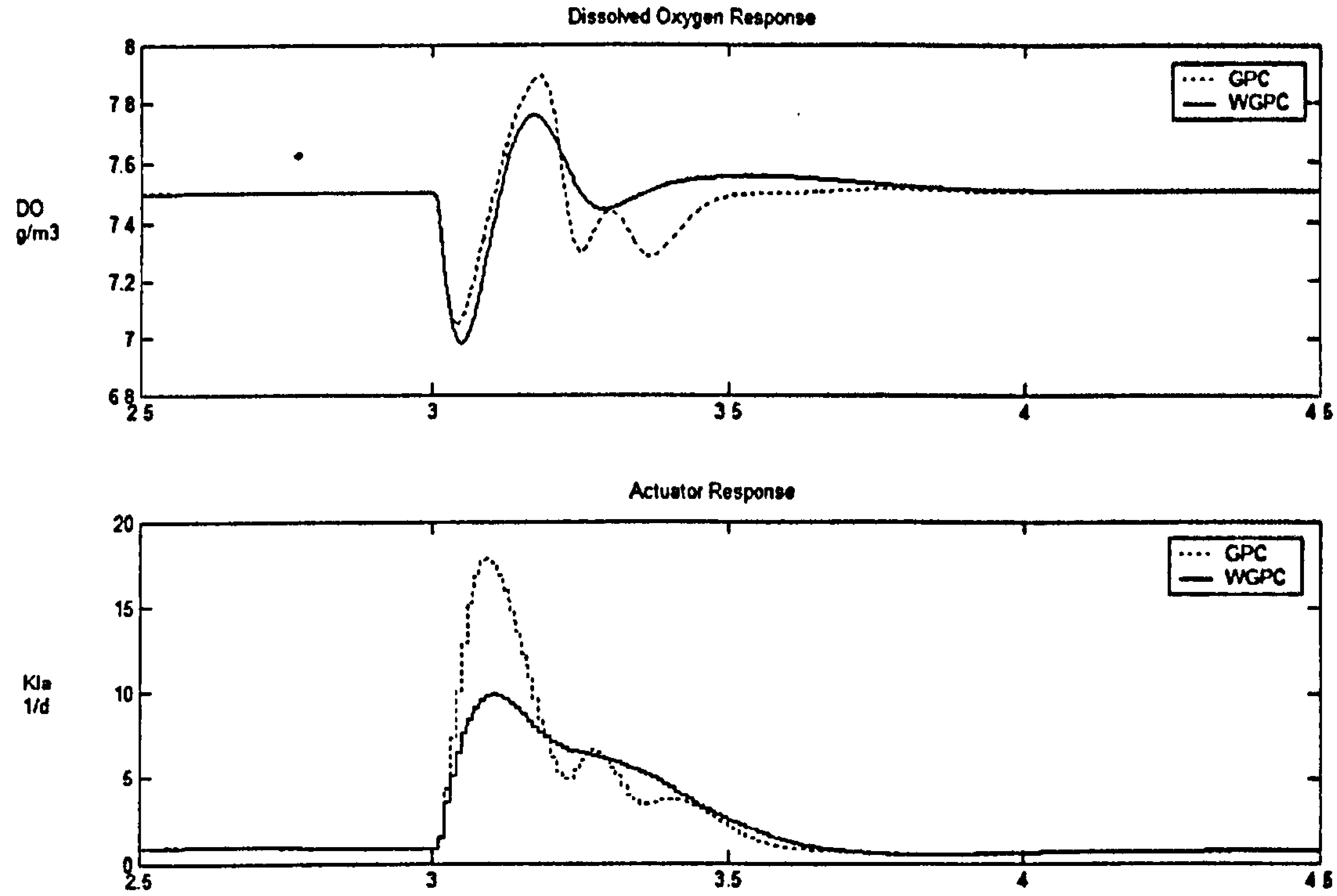

Figure 6-14: Wiener Model Predictive Control of Dissolved Oxygen in the Urban Wastewater System, versus Linear MPC, during a moderate rain event, no CSO

control of both controllers, according to the above inequality. Both cases indicated the improved disturbance rejection abilities of the Wiener MPC technique, reducing the oxygen depletion effects of the storm event. The Wiener MPC approach additionally shows a less oscillatory response than its linear counterpart. The increased accuracy of the Wiener model provides improved predictions of future plant behaviour in the presence of varying treatment plant influent, over the approximated predictions of the linear process model.

Whilst the above Wiener state space model was determined offline via analysis of step responses, online identification of Wiener models has been demonstrated for other applications. The issue concerning an application of such a method to the control of ammonia levels in the receiving waters is the nonlinearity of the process. Whilst the dynamics of the dissolved oxygen process over changing flows can be modelled approximately by a simple first or second order transfer function, and thus a linear time invariant model, the dynamics of the ammonia process are considerably more nonlinear over similar flow variations.

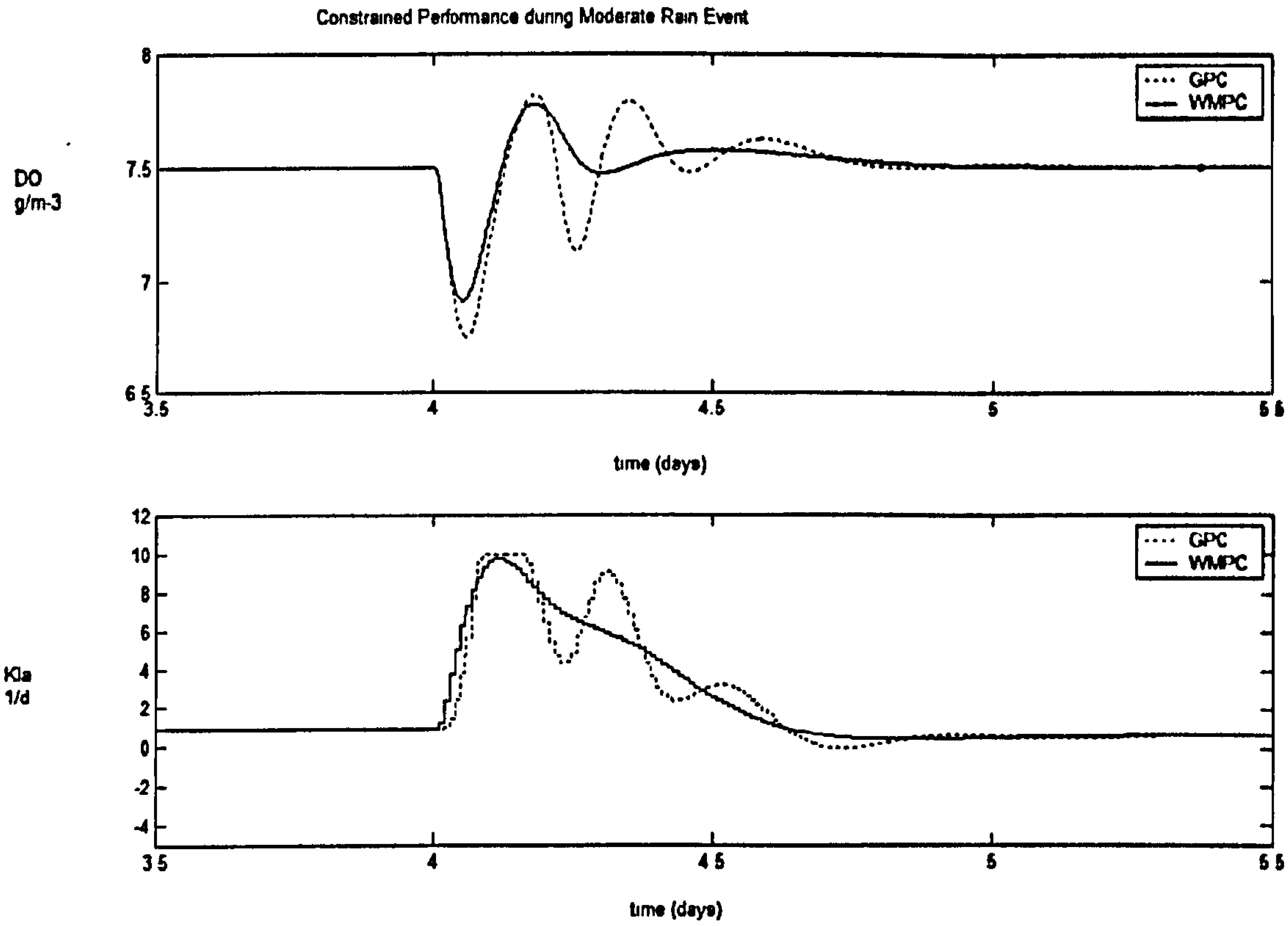

Figure 6-15: Wiener Model Predictive Control of Dissolved Oxygen in the Urban Wastewater System, versus Linear MPC, during a moderate rain event, no CSO: In the presence of actuator constraints

6.6 Comparison of Control Approaches

To discuss the control approaches, in terms of effectiveness and applicability, the responses of the designed controllers to the disturbance of a rain event must be examined. The four following controllers are therefore evaluated in these simulation studies:

- fuzzy gain scheduled control utilising feedforward control via measured disturbances, which considers individual linear models (nine in total) to define system dynamics over the nonlinear operating conditions of a storm event, identified via subspace identification. Nine linear model predictive controllers are simulated in parallel with the measured disturbance variable of dissolved oxygen concentration in the upstream receiving water, for the controlled output of dissolved oxygen concentration in the receiving waters. The treatment plant influent flow measurement is considered as a scheduling variable, in the fuzzy interpolation of linear controllers according to operating point.

- Wiener model based control, which considers the controlled process to be a SISO system of one manipulated input of airflow rate and one controlled output of dissolved oxygen concentration in the receiving waters, with knowledge of the treatment plant influent flow as a measurable disturbance variable, identified through system step test analysis.

- linear model based predictive control, which considers a linear model with state variables and inputs as in the case of the fuzzy gain scheduled control, for the steady state plant conditions (that is, a treatment plant influent flow of $5575m^3/d$). This linear controller is equivalent to that of the membership function of the FGS controller for this flow range.

- the PID control for the receiving waters is a modification of the original PI control whose control objective was the setpoint tracking for dissolved oxygen concentrations in the aerobic tank of the treatment plant, but which in this case instead considers the dissolved oxygen concentration in the downstream receiving water.

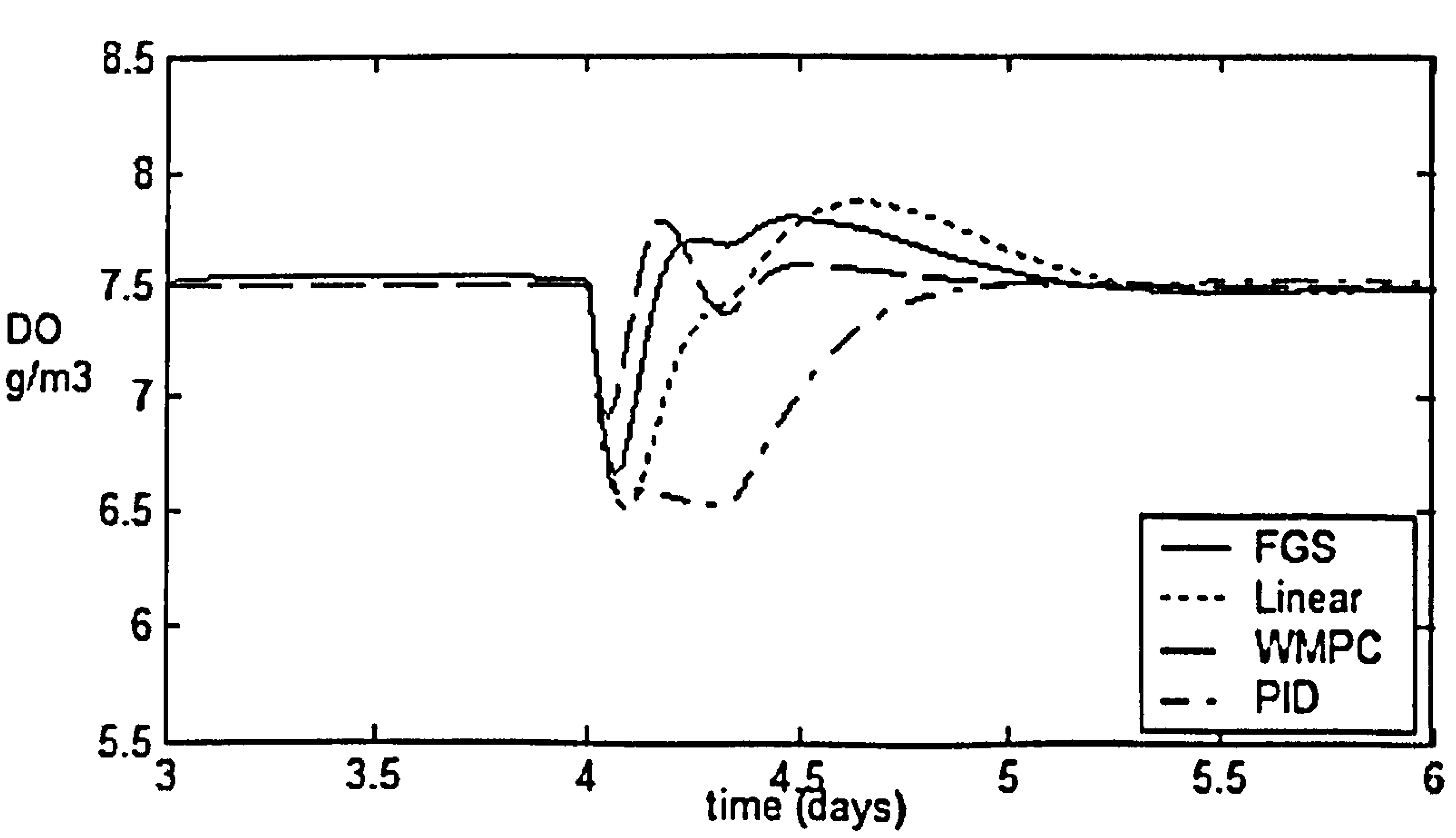

Figure 6-16: Comparison of Predictive Control Approaches during a rain event: WMPC, FGS, Linear and PI

PI control is included for comparison purposes, tuned via the trial and error method. The simplicity and popularity of the PI approach has led to its common application for wastewater applications, and with advanced tuning methods (for nonlinear control) could perform adequately, although lacking several of the advantages of the model based control structure. MPC has the advantage of ease of application for multivariable processes, multivariable nonlinear MPC control of the urban wastewater system is demonstrated later in this chapter. Additionally, the PI structure does not easily allow for constraint handling, where the model based control approach excels, for example in the constraint handling demonstrated in the WMPC application in the previous section.

The criteria by which these control schemes are compared is defined by two values: the minimum dissolved oxygen concentration for a given event, and the length of time that the dissolved oxygen concentration was below a certain threshold value. Since the storm event does not cause oxygen depletion below the regulatory levels, a threshold value will be chosen, for comparison purposes, that all of the control schemes exceed. The threshold dissolved oxygen concentration of 7g/m3 is chosen. The performance of the control schemes are therefore as detailed in Table 6.11 below. The reason for the improvement of performance over that of PI control can be explained by several factors: the inclusion of a feedforward mechanism for control in the case of FGS, the accuracy of the Wiener model in the case of WMPC, or indeed the inclusion of unmeasured disturbance modelling in the model based control, including that of the linear case. Particularly, the intuitive nature of the MPC tuning allowed for better performance.

Table 6.11 Comparison of Control Schemes

Control Scheme	Period of time below threshold (days)	Minimum DO (g/m^3)
PID	0.4725	6.52
WMPC	0.04	6.91
Linear	0.086	6.67
FGS	0.16	6.51

Closer inspection of the behaviour of the system over extended storm events, such as the following event of intensity 10 at a time of 4 days, for a duration of 3 hours as shown in Figure 6-17, indicates the issues involved in the use of the PI control. The extended nature of the storm event allows the PI to more closely follow the responses of the model based control techniques, with respect to the period of time for which oxygen depletion occurs. However, the PI controller requires an extended period of time to return to steady state, and in addition results in the application of an excessively large control action. The model based control approaches, if compared, can be seen to improve in performance proportionally to the level of accuracy in the control models utilised. In particular, the Wiener MPC approach results in a minimum dissolved oxygen concentration of $6g/m^3$, in comparison with the PI control case, in which the minimum dissolved oxygen concentration is $5g/m^3$. It can be seen therefore that the nonlinear approach of the Wiener model based control scheme improves considerably, with respect to water quality objectives, over the traditional PI approach.

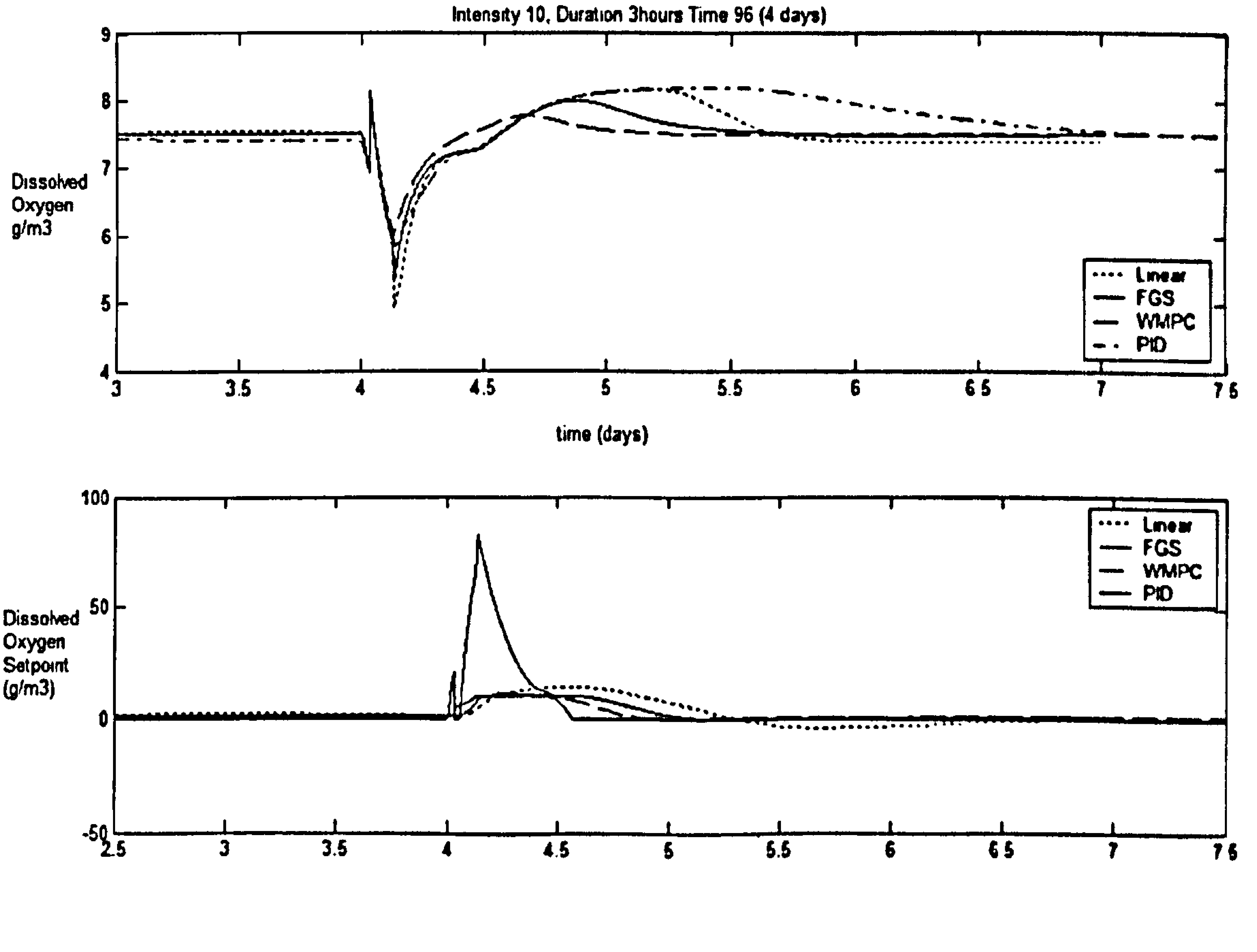

Figure 6-17: Extended Storm Event: Comparison of Control Approaches for the Urban Wastewater System

6.7 State Dependent Nonlinear Predictive Control

In the presence of an accurate nonlinear process model of a system, the above model simplifications and reductions may be made redundant. By representation of the nonlinear model in the state space format, the traditional predictive control techniques may be employed. The application of such an approach does however require several assumptions to be made, arguably making the approach unrealistic. The application of the MPC approach using a nonlinear model is investigated, and the assumptions and conclusions are discussed, in the remainder of this chapter.

Nonlinear models based on mechanistic mathematical models can be useful in the application of nonlinear control. This approach has not been widely explored in wastewater treatment control, for many reasons. The most obvious of these is the lack of accurate nonlinear models of the wastewater systems, and in addition the number of unobservable and uncontrollable processes within wastewater treatment plants themselves. The models utilised in water treatment control have been, for the most part, linear or multiple linear models, and have been sufficient for their purpose. It is the objective of this section to explore the application of a nonlinear advanced control approach to wastewater, using a mechanistic model and compare to the previous methods demonstrated in this thesis. The increasingly stringent regulatory requirements may perhaps push industry towards further accurate and possibly more complex control schemes.

The choice of model used to demonstrate nonlinear predictive wastewater control in this chapter is itself a reduced model [114]. In order to allow for controllability and observability issues, a larger (more complex) model such as the ASM based models was impractical for study in the scope of this thesis. The nonlinear GPC approach presented in the previous chapter is applied to the following wastewater treatment control problem. The urban wastewater system is not considered here, instead the objective of this control is the optimisation of the treatment system performance. Although, in a general sense, the main structure and function of a treatment plant is similar in most situations, the control developed is system specific, as the system characteristics, dynamics and kinetics in each case differ. The aim here therefore is to demonstrate the application of a nonlinear GPC strategy in the case of a known nonlinear system model. In practice, the size of the treatment plant, the influent characteristics, the control systems present and the measured variables, as well as the regulatory standards required may differ from system to system

As stated previously, the large variation in the time constants associated with sub-processes within the wastewater process allows the application of decoupled SISO control schemes as shown above. Considering the MIMO nature of the process however Nielsen and Onnerth [113] for example demonstrated the benefit of MIMO control of nitrate concentrations at the effluent of a full scale treatment plant utilising carbon addition and the oxygen supply. Lech et al. [87] demonstrated process instability caused by interactions and coupling across separate control loops. The benefit therefore of the MIMO controller design can be elimination of these issues, as shown in a decoupling MIMO control scheme for a carbon removal wastewater treatment plant application demonstrated by Vanrolleghem [184].

The use of nonlinear models explicitly within the model based predictive control applications for biological wastewater treatment processes has not been commonly applied. Vanrolleghem states that the use of nonlinear models themselves as part of control coptimisation has only been demonstrated by a few examples due to the unrealistic assumption of a perfect process model with a fixed structure, and lists simulation examples of this numerical optimisation (for example, those shown in [161],[189], [103], [73], [37]).

6.7.1 Treatment Plant Control

The application of nonlinear predictive control using SDC modelling shown here is demonstrated in O' Brien et al [120], whilst a comparison of this method with a 'linearisation around a trajectory' nonlinear predictive control approach is demonstrated with the use of the extended Kalman filter for state estimation in O' Brien et al [9].

The approach demonstrated here is the application of nonlinear predictive control to a wastewater treatment plant model, using the state dependent format of the Nejjari [114] model. The state dependent coefficients $A(x,u)$ and $B(x,u)$ defined in Chapter 5, in addition the following state dependent coefficient matrices, define the state dependent form of the treatment plant:

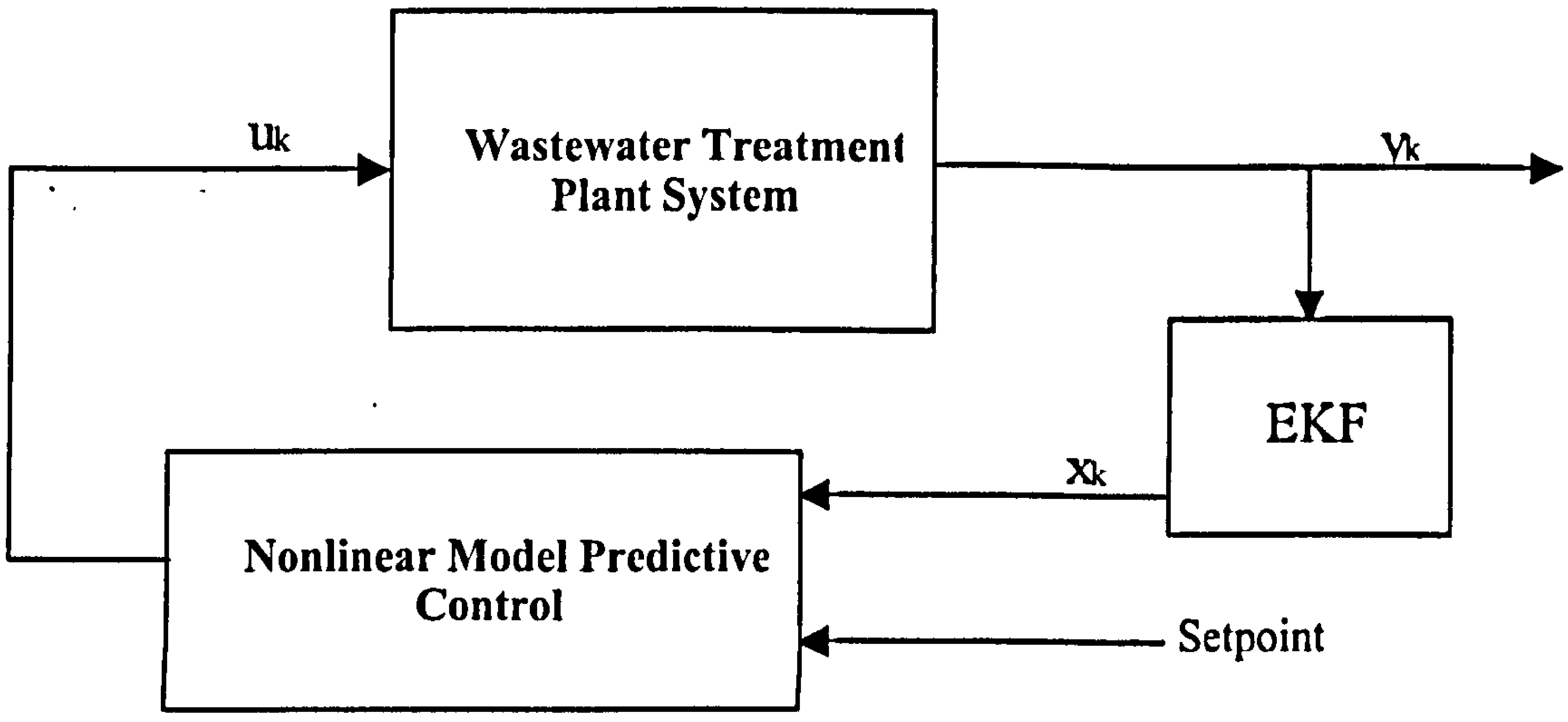

Figure 6-18: Closed Loop Architecture for Nonlinear Control of a Wastewater Treatment Plant

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{6.9}$$

where the C matrix is assumed to represent the two controlled process outputs, substrate and dissolved oxygen, whilst the zero D matrix denotes the lack of direct feedthrough from the inputs. The system state vector is therefore defined as $x = \begin{bmatrix} X(t) & S(t) & C(t) & X_r(t) \end{bmatrix}^T$, the input vector is defined as $u = \begin{bmatrix} D(t) & W(t) \end{bmatrix}^T$ and the output vector is defined as $y = \begin{bmatrix} S(t) & C(t) \end{bmatrix}^T$.

Table 6.12 Control Parameters for State Dependent Nonlinear Control of WWTP

Parameters	SD-GPC
T_s	1
Q	$[11/40^2 \ 10/7^2]$
λ	$[2/0.0825^2 \ 5/91^2]$
H_u	2
H_p	4

Table 6.13 Treatment Plant System Constants

Constant	Meaning	Value (unit)
K_o	Constant	0.5
K_s	Affinity constant	100(mg/l)
K_c	Saturation constant	2(mg/l)
$\mu_{\max}$	Maximum specific growth rate	0.15(h-1)
S_{in}	Influent substrate	200(mg/l)
C_{in}	Influent dissolved oxygen	0.5(mg/l)
Y	Yield coefficient	0.65
r	Ratio of recycled flow to influent	0.6 (-)
β	Ratio of waste-flow to influent	0.2 (-)

Available measurements within the wastewater treatment industry are often both insufficient, as detailed in Chapter 3, but also, even if available, can be of poor quality, with corruption due to noise and the possibility of sensor failure. Several state variables, which cannot be measured by direct method because there is no reliable instrumentation available, will require the use of mathematical models within "software sensors". In this case, a Kalman filter as demonstrated in O' Brien et al [9] is utilised. In the case study shown, the online measurement of biomass X, recycled biomass X_r and substrate concentrations at the plant effluent, S are assumed not to be available. Sensor measurements for the dissolved oxygen, C, are assumed to be corrupted with noise. The extended Kalman filter uses the linearisation of state and observation equations around the currently estimated plant operating states to estimate the current state without noise corruption. In order to facilitate the application of SDC nonlinear control, the assumption is made that the kinetics of this model are constant. This is usually not the case in a real process, however the lack of kinetic modelling is a common problem in wastewater applications. Research by Benazzi et al. [13] has shown a level of parameter estimation for this process with the use of an extended Kalman filter.

The nonlinear predictive control algorithm described by the state space equations and controller paramaters above is applied to an activated sludge wastewater treatment plant. The control objective of this case study is to track setpoint changes of dissolved oxygen, C, and substrate, S at the treatment plant effluent. The controller configuration contains two control actions, those of dilution rate and air flow rate into the plant. The above model is discretised,via a Tustin integration of sample time 1h. The MIMO nonlinear controller structure produced by the SDC approach is chosen with the objective of tracking setpoint changes for dissolved oxygen and substrate levels simultaneously, in the presence of process noise.

The choice of controller parameters is made through trial and error, via simulation tests of system performance, with the aim of minimisation of the settling time of the controlled process to a zero steady state error, and avoidance of excessive actuator control actions. The nonlinear discrete state space representation of the process leads to an on-line formulation of the NMPC controllers at each sampling time, where the discretisation of the nonlinear SDC model is performed via a Tustin integration, with a sample time of 1h.

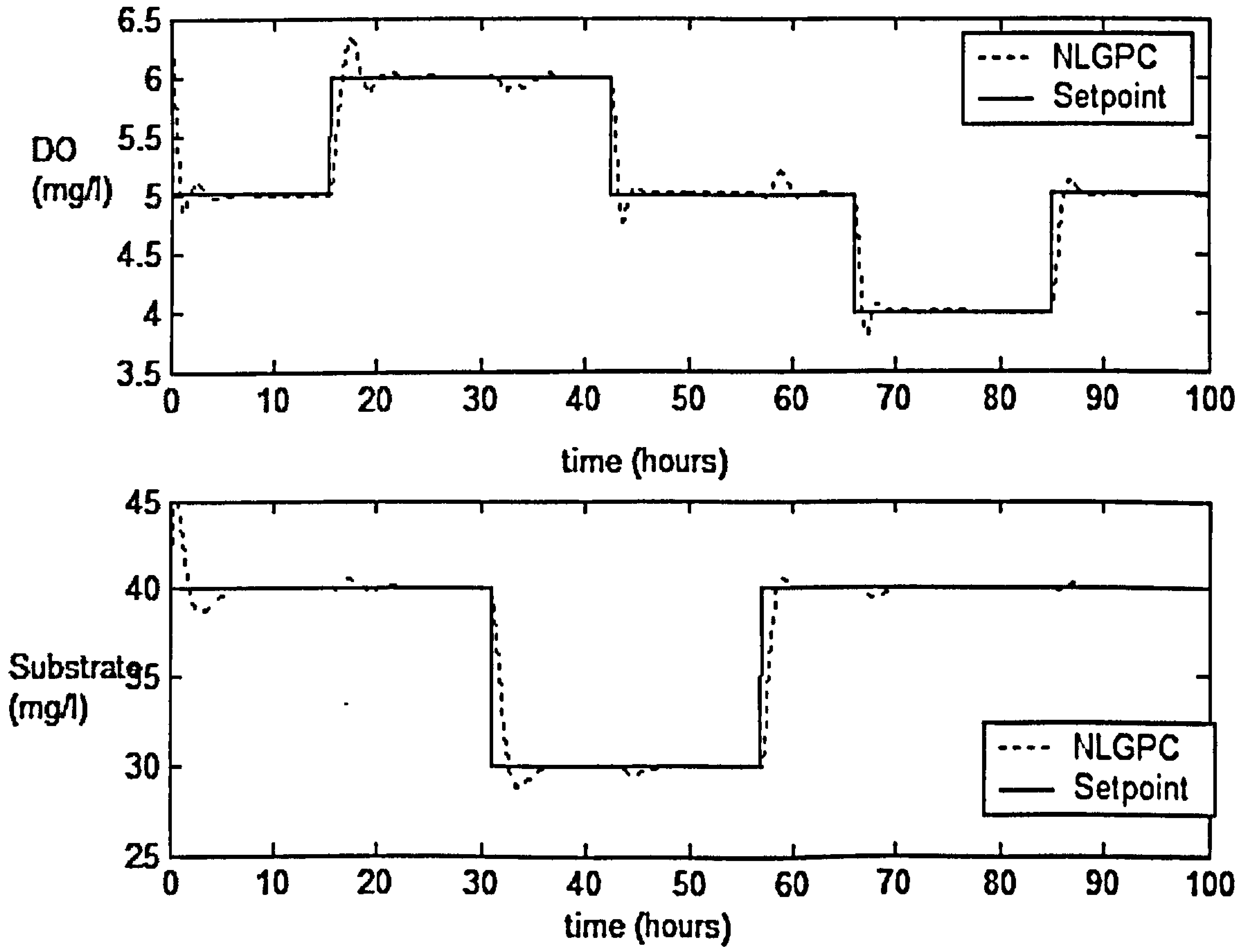

Figure 6-19: Closed Loop Control Response for the WWTP concentrations of Dissolved Oxygen and Substrate

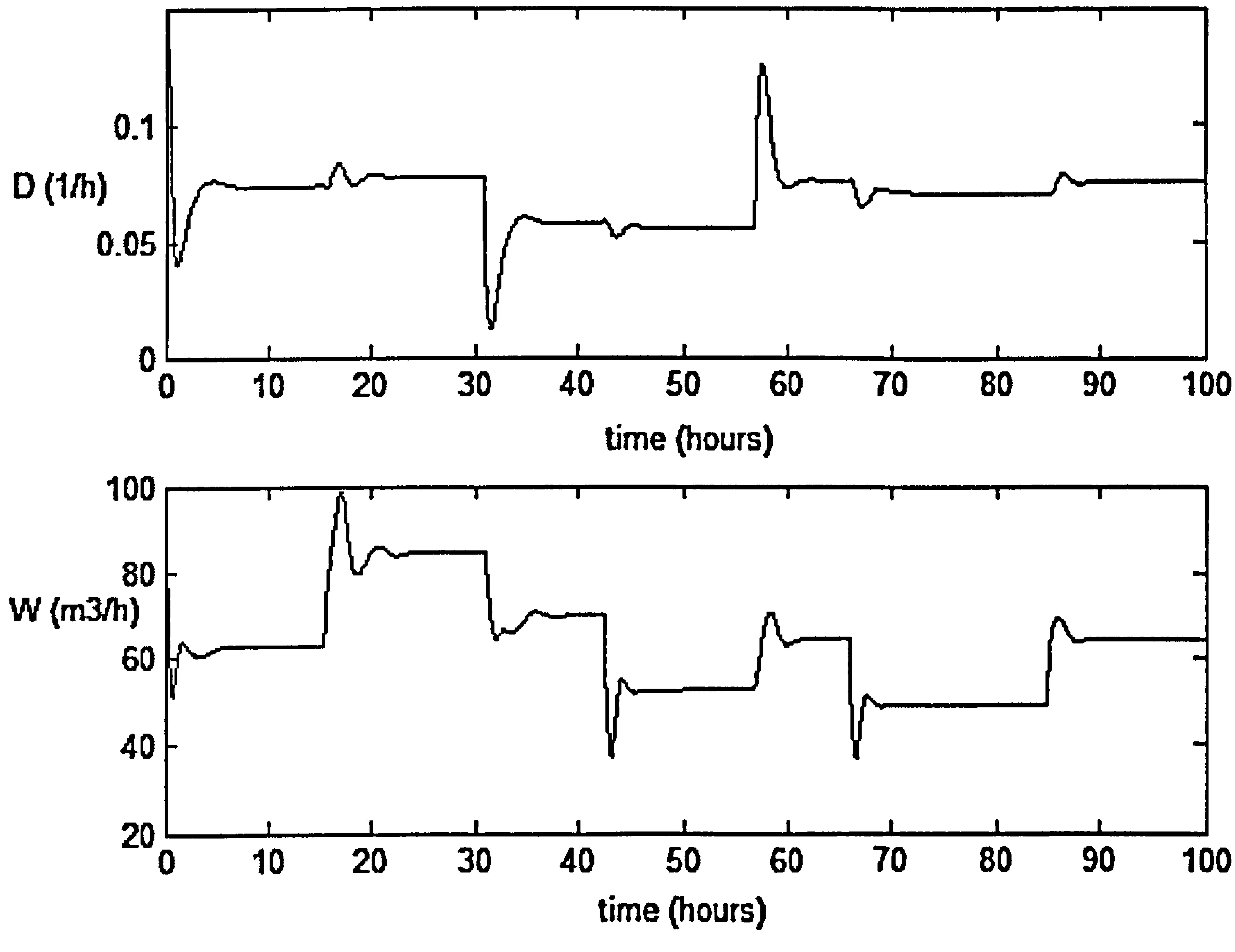

Figure 6-20: Closed Loop Actuator Response for Airflow Rate and Dilution Rate in the WWTP

The closed loop response of the process and actuators for a simulated time period of 100hours is demonstrated by 6-19 and 6-20 above. The linear time varying model produced results in the use of a linear quadratic programming algorithm in the cost function optimisation, according to the control weightings specified in Table 6.12. In steady state, there exists a negligible offset from the desired setpoint value, that is, there is effectively zero steady state error for the control response. The initial response in the nonlinear control demonstrates an oscillation at time 0→20h. The initial state estimation error in the extended Kalman filter introduces oscillations, due to a mismatch in initial conditions between estimator and plant. The setpoint changes must begin a period of time after this point for the estimated states to reach the true value.

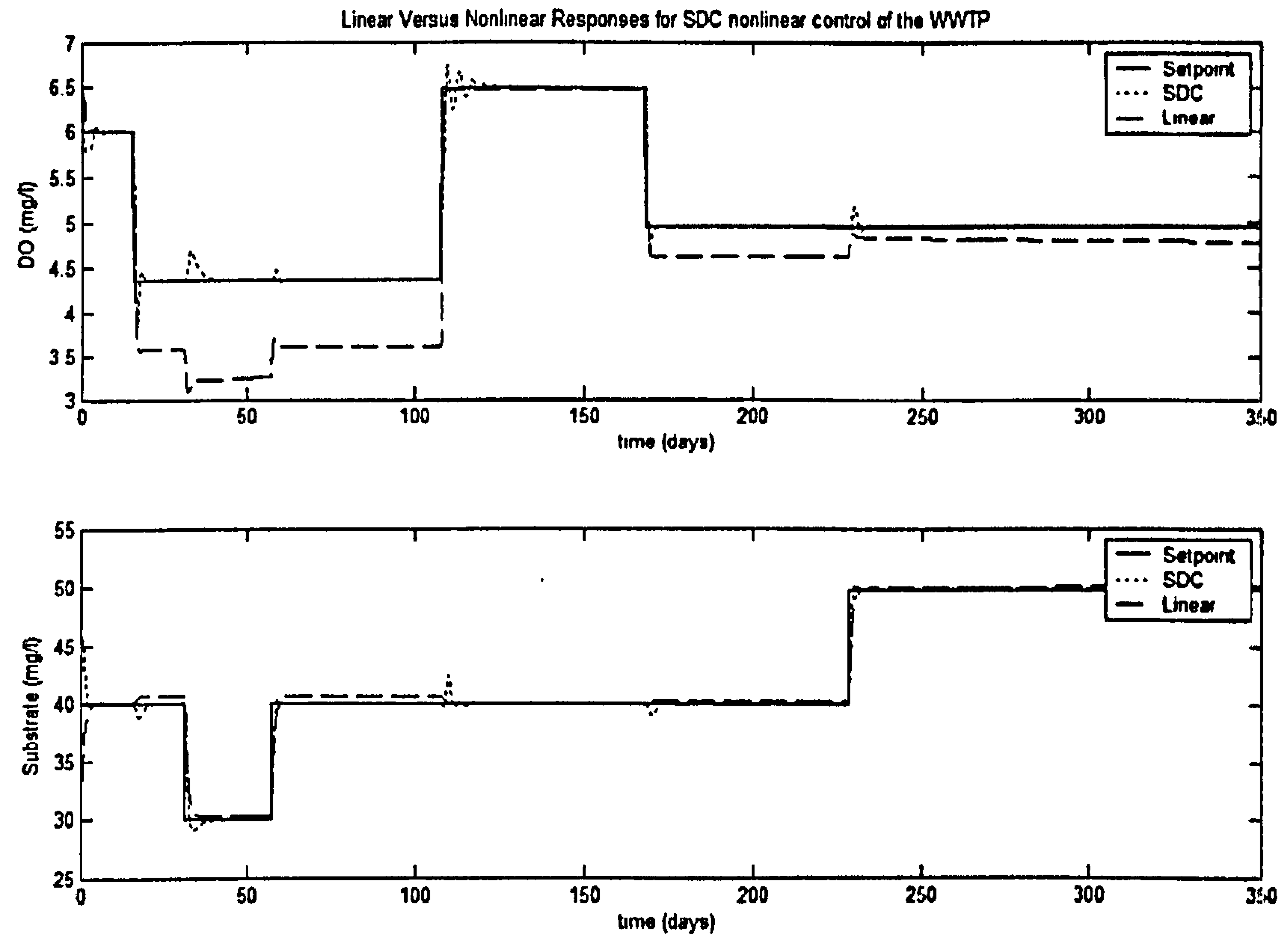

Figure 6-21: Linear MPC versus Nonlinear MPC for a Wastewater Treatment Plant Application

The application of model based methods in proposing a nonlinear control scheme for a wastewater treatment plant requires the availability of an accurate nonlinear process model. In the presence of such a model, state dependent modelling techniques can allow a nonlinear control model to be developed. The model obtained has a linear time varying model structure, allowing the prediction of future behaviour with sufficient accuracy to be used in the calculation of appropriate control actions. The traditional approach for model predictive control is the linearisation of the considered process. For the reasons of comparison, the demonstration of the linear predictive control response is as shown in 6-21, without the use of the constant disturbance model traditionally used to compensate for the modelling error. The linear approach to predictive control (without mismatch compensation) is appropriate for small operating ranges in the vicinity of the operating point at which linearisation was implemented. However, the nonlinear model approach can handle the dynamics involved in the full operating range of the process. The step response above demonstrates the limited valid region of operation of the linear GPC, in a small deviation from the steady state operating point. The linear predictive controller is unable to follow the setpoint change for the dissolved oxygen concentration, whilst the nonlinear method allows for the elimination of steady state offset throughout the nonlinear range.

6.7.2 Urban Wastewater System Control

This section focusses upon the application of nonlinear state dependent coefficient control to downstream dissolved oxygen and BOD regulation in a portion of a receiving water. The control is accomplished by means of manipulation of the treatment plant effluent, aiming to compensate for variations within the receiving waters via modification of discharges from the treatment plant. The approach above was extended to include control of the urban wastewater system according to the diagram below. The river sections were modelled as CSTR, each contained in a state dependent state space description. The model considered one river stretch prior to the addition of the treatment plant effluent, and one river stretch subsequent to the mixing. The upriver dynamics and the effects of the effluent from the treatment plant were combined via a weighted sum of the concentrations of each flow.

The two original controlled variables in the treatment plant were those of the substrate and of the dissolved oxygen concentration in the effluent. The substrate however is not considered within the CSTR river model, instead the related BOD concentrations are described, and so the BOD levels in the river are chosen as an alternative controlled variable. The state vectors at the chosen points upriver and downriver are assumed to be known, in addition to the treatment plant influent flow rate. In the case study presented here, the dependance of BOD upon dilution rate (via the substrate process) is exploited for the purposes of control.

The full state dependent representation of the above can be seen as detailed in Chapter 4, in addition to the output matrices as defined by:

$$C(x,u) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{6.10}$$

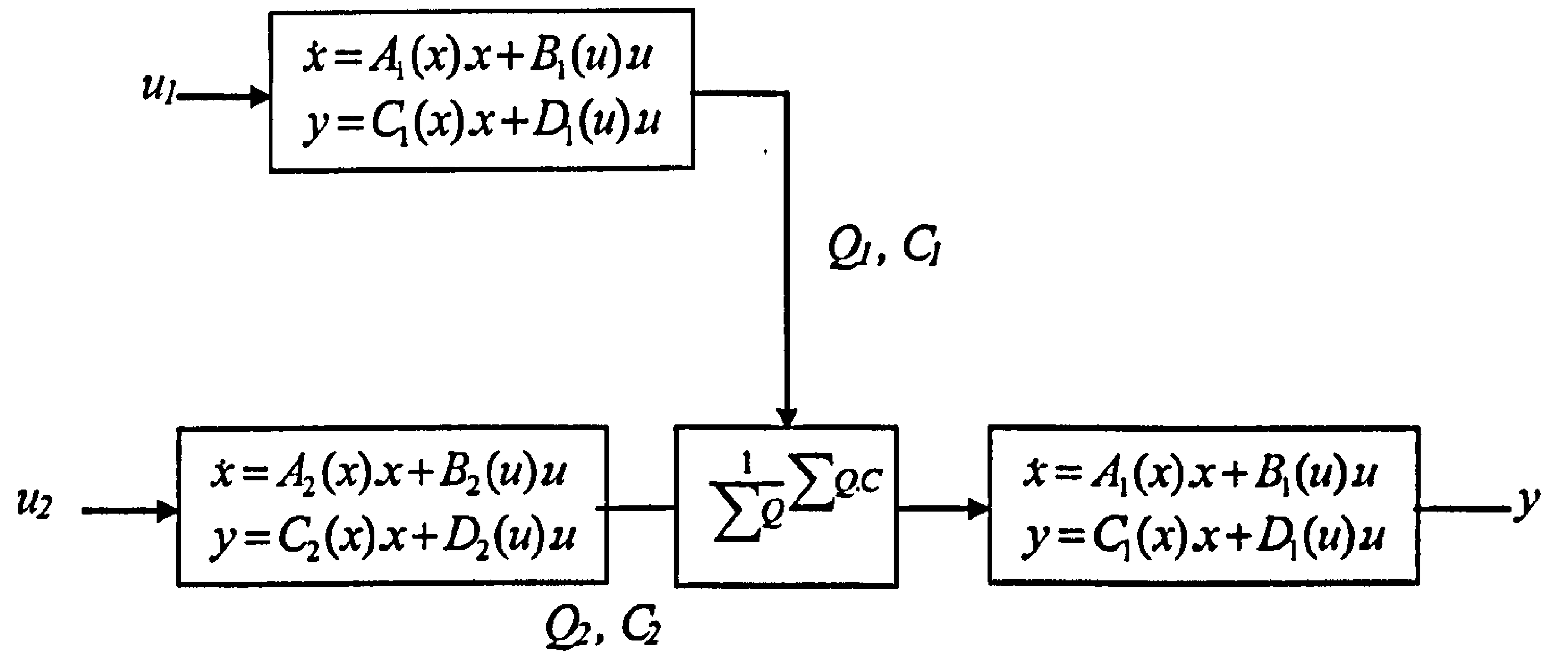

Figure 6-22: State dependent architecture for the Urban Wastewater System

$$D(x,u) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (6.11)$$

6.7.3 Application

The important control objective within this scaled system is the control of the dissolved oxygen level in the receiving waters, whilst the second objective is that of BOD levels at the same point. The system as defined above considers the dynamics of the river in two parts: a partial upriver reach consisting of one CSTR model, and similarly for a point downriver. The treatment plant is as considered in the previous application, with the inclusion of a varying treatment plant influent rate. The controller is designed as in the previous section and applied to the urban wastewater system as described in Chapter 4, and the following controller parameters are chosen as shown in Table 6.14. Additionally, the urban wastewater system constants are as shown in Appendix A.

Table 6.14 Controller Parameters for State Dependent Nonlinear Control of UWS

Parameters	SD-GPC
T_s	1
Q	[1 1]
λ	$[2/0.0825^2 \ 5/91^2]$
H_u	2
H_p	4

In demonstrating the extension of the above treatment plant control to the urban wastewater system, setpoint tracking for varying step changes in dissolved oxygen and $BOD_{S,2,in}$ in the river is shown in the Figure below, 6-23. The demonstrated control performance, whilst although demonstrating the ability of the controller to account for step changes, is not realistic as such control of the system is only possible at high flows. This is due to the magnitude of the river flow 18,000m^3/h, in comparison to the steady state influent flow to the treatment plant of 373.72m^3/h which, due to the effects of mixing with the receiving waters, has only a low magnitude effect (if any) upon the receiving waters. For the purposes of demonstration of setpoint tracking, the treatment plant influent flow is set equal to that of the receiving waters. However, considering that the objective of the control scheme is the reduction of the effects of disturbance events (for example here, high flow events), it is therefore logical that the control performance be demonstrated during these periods.

Upon inspection, it can be seen that the dissolved oxygen concentration in the river varies diurnally. The causes of these variations are defined to be primarily the processes of photosynthesis and aerobic respiration, as given by equations 4.15 in Chapter 4. The photosynthesis process is defined as the production of free oxygen in the presence of sunlight, thus resulting in an increase in dissolved oxygen during the daylight hours. The respiration process for algae and plants consumes oxygen, thus causing a decrease in dissolved oxygen, and releasing carbon dioxide during the night. The oscillation present in the dissolved oxygen response for nonlinear predictive control therefore can be seen to be a result of diurnal dissolved oxygen variations within the receiving waters. A typical diurnal cycle for dissolved oxygen would be sinusoidal in nature, with the minimum and maximum concentrations occuring early morning and late day respectively. In the case of excessive algae growth, there would occur an increase in the diurnal variation, however the case considered in this section is that of typical dissolved oxygen diurnal variations of a river, occuring within environmental safety margins. It is clear that the dissolved oxygen responds to step changes in the setpoint, the BOD_r concentration indicates a similar ability in setpoint tracking.

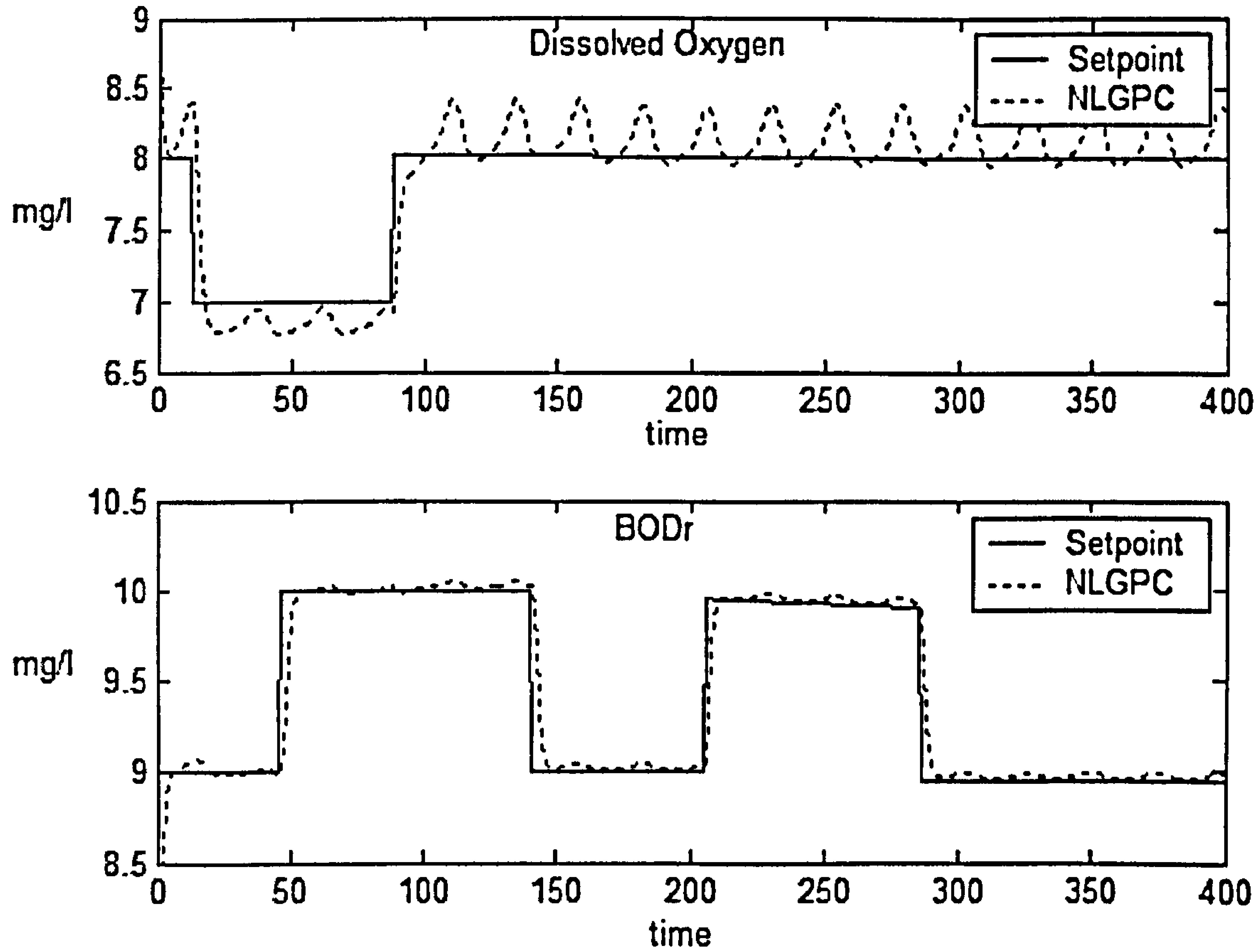

Figure 6-23: State Dependent Control of Dissolved Oxygen and BOD_R behaviour in receiving waters of UWS for setpoint tracking

The primary control objective in this case was the rejection of disturbances caused by a high flow treatment plant influent, and this was implemented for the dissolved oxygen concentration in the receiving waters, $C_{2,in}$, and slowly biodegradable BOD, $BOD_{S,2,in}$. The controller configuration contains the two original control actions for the treatment plant, those of dilution rate and air flow rate into the plant. The high flow event occurs at a time of 30 hours, with a duration of 3 hours, and an intensity of 15 mm/h, with an oxygen depletion in the receiving waters of up to 7mg/l. For comparison purposes, the control of the same system with the use of the SISO approach of the FGS scheme as demonstrated in

IFAC is shown. It is clear from Figure 6-24 that the air flow rate changes implemented by the NLGPC scheme compensate for the dissolved oxygen depletion in the receiving waters, whilst a considerable decrease in the levels of BOD disturbance in the river is also shown. The complete rejection of disturbances shown in this case in dissolved oxygen is unlikely, particularly given the constraints upon any possible compensatory action that would take place within the treatment plant. Additionally, the difference, in practise, in the time scales of the dissolved oxygen and BOD processes would also impact upon the performance of the predictive control scheme. Steffens et al [169] demonstrated the characterisation of the processes of an activated sludge process according to time scales. Fast system states, of time constants of the order of 1-10 minutes, included dissolved oxygen and ammonia, medium system states were defined as nitrate and soluble inerts, whilst the slower system states included autotrophic and heterotrophic biomass. The aim of the performances shown below however is to indicate the need for multivariable control within the urban wastewater system: the MIMO control of dissolved oxygen and BOD allows better disturbance rejection than the SISO control scheme for dissolved oxygen only.

6.7.4 ASM2d State Dependent Model

The full influent vector is considered in this control application, although realistically the characteristics of the influent will not be fully known. It can be seen therefore that the assumption must be made that measurements of the influent are available. In reality the following components are measurable online: influent flow Q, nitrate S_{no3}, ammonia S_{nh4}, dissolved oxygen S_o, phosphorous S_{PO4}. Respirometer measurements of readily biodegradable substrate and slowly biodegradable substrate are in some situations available every $30mins$. The remainder would require advanced estimation methods, such as the extended Kalman filter demonstrated for the ASM1 model, for example [13] demonstrated active heterotrophic and autotrophic biomass (demonstrated to be valid for a limited period of a fortnight) and estimation via FFT of soluble and particulate biodegradable organic nitrogen. It is assumed that these estimates/measurements would be available in this case. The control handles available within the model of [31] are those of aeration and chemical dosage. Further manipulated variables possible for wastewater treatment plants include return sludge flow rate in the control of effluent suspended solids, nitrate recycle and carbon

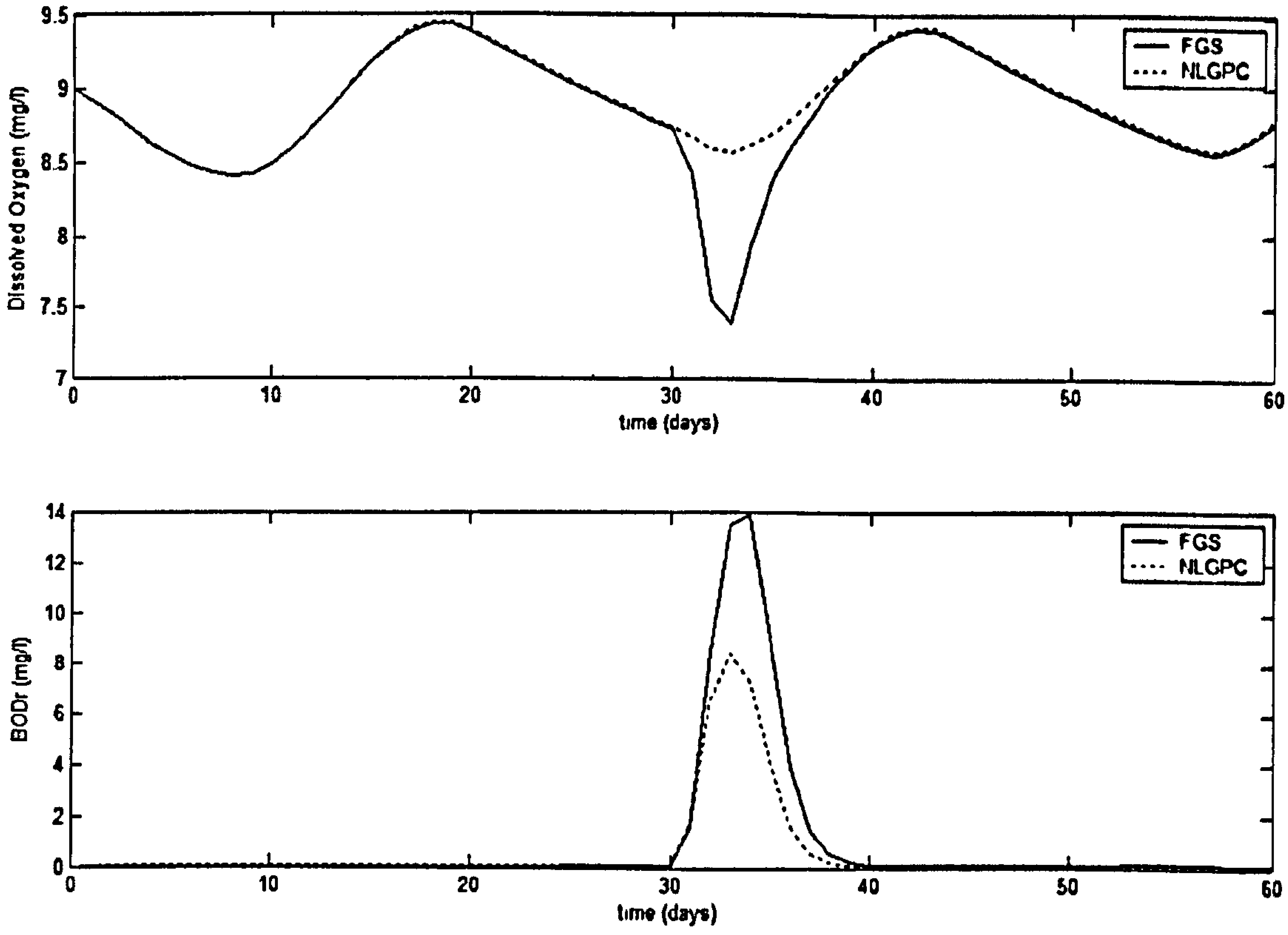

Figure 6-24: State Dependent Control versus Fuzzy Gain-Scheduled Control of Dissolved Oxygen and BOD_R behaviour in receiving waters of UWS for disturbance rejection

addition for N removal. Other assumptions in this control application are considered to be:

- The kinetic parameters of the ASM2d processes within the tank are assumed to be fixed, although in practice these could vary.
- The realistic assumption is made that the controlled outputs are measurable.

The identifiability of the ASM2d parameters is a current focus of research, the calibration of the ASM2d model is hampered by the overparameterisation of the model with respect to the data available for calibration. It is therefore possible that the identified parameters are not unique, and in response to this research has been focussed upon the development of systematic methods for parameter identification (Tapia et al. [175]). Whilst the dissolved oxygen process, as stated in section 6.7.3, has a short time constant, the slower process associated with phosphorous precipitation is of the magnitude of hours. The issue of optimisation of control for an ASM2d based wastewater treatment plant process with multiple time scales was demonstrated in the work of Rutkowski et al [142].

In the application of nonlinear generalised predictive control to the ASM2d reaction tank, the state dependent model as defined in Chapter 4 is considered. The ASM2d state dependent model considers the following definition of the system variables:

- there exist two manipulated variables: air flow rate and addition of ferric chloride ($FeCl_3$).
- The above variables manipulate the value of two controlled variables of the tank effluent: S_o (dissolved oxygen) and S_{PO4} (phosphate).
- The measured disturbances are the remaining influent concentrations into the aerobic tank.

The method by which the Ferric Chloride $FeCl_3$ affects the phosphate concentrations S_{P04} is detailed by the following expressions, as detailed in [31]. The reactions of the chemical Ferric Chloride added to the water, the Fe^{+3} ions reacting with the OH^- present, to become Ferric Hydroxide $Fe(OH)_3$, the component considered within the ASM2d model (denoted by the term MeOH, where Me is defined as the generic metal term).

$$FeCl_3 + H_20 \longleftrightarrow Fe(OH)_3 + 3H^+ + 3Cl^-$$

The general representation of metal hydroxide reaction with a phosphate, producing a metal phosphate MeP

$$MeOH + PO_4 \longleftrightarrow MeP$$

which in this case is seen to be as follows

$$Fe^{+3} + H_nPO_4^{-3-n} \longleftrightarrow FePO_4 + nH^+$$

where Fe^{+3} is dissociated according to the equation $Fe(OH)_3 \longleftrightarrow Fe^{+3} + OH^-$. The concentrations X_{MeP} and X_{MeOH} are modelled within the ASM2d process. Since ASM2d denotes the chemical reagent as $Fe(OH)_3$, the chemical dosage of $FeCl_3$ as utilised in [31] is scaled according to the definition that 1g of $FeCl_3$ produces 0.66 g of $Fe(OH)_3$. For clarity, the simulation results produced in this thesis of the system response to setpoint changes therefore demonstrate the response of the $Fe(OH)_3$ dosage directly.

6.7.5 ASM2d NLGPC Application

The state-of-the-art in terms of water treatment is currently changing with the introduction of new directives, development of the industry technology and the surge in interest and concern for environmental matters. Due to this, the control applied is also in transition, adapting to include the new constraints upon the requirements of the water industry. The control of dissolved oxygen processes is standard amongst the schemes in place within the water industry, whilst nitrification-denitrification and phosphorous removal are acquiring a similarly common place.

As with any biological process, the dynamics of the ASM2d aerobic process are intrinsically nonlinear. With increased applications of process model identifiability [175], and also research into the online estimation of plant kinetic parameters, it can be concluded that the use of a nonlinear process model for control could eliminate (or at the least, compensate) for the lack of modelling accuracy involved in the linear approaches traditionally used. The aim in this section is the application of nonlinear control with a full nonlinear process model. Realistically, a selection of variables are available from respirometers and other sensors, and estimated with the use of nonlinear state estimation techniques. Additionally, for many nonlinear control applications, reduced or scaled models are utilised. As previously, the mass balance equations of the ASM2d process reactions can be transformed to a nonlinear state dependent coefficient model, taking into account the coupled biological reactions with the aerobic tank.

The system considered is that of an aerobic activated sludge process within an anoxic-aerobic treatment plant structure, whose main control structures consist of variable aeration of the biological processes within aerobic tank and also the chemical precipitation of Phosphorous. The objective of the control application is the therefore the setpoint tracking for dissolved oxygen and concentrations of inorganic soluble phosphorous (S_{po4}) in the effluent

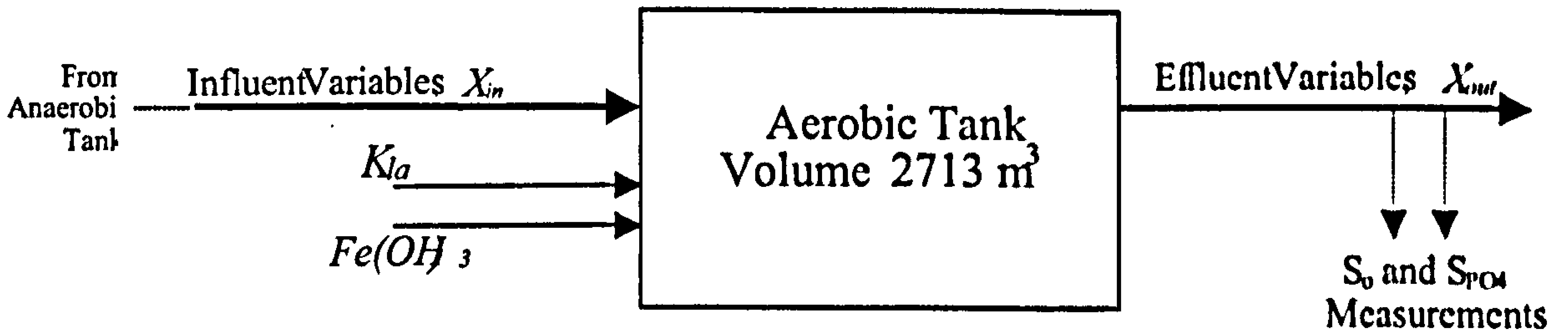

Figure 6-25: ASM2d process as considered for MPC control

of the aerobic reactor within the treatment plant as defined by [31]. The advantages of phosphorous removal by chemical precipitation can be the simpler process dynamics and the ease of operation, some wastewater compositions make the removal of phosphorous by biological methods difficult. However, the sludge resulting from the chemical precipitation method is difficult to handle and dispose of, and it is thus costly to store and treat this sludge. The added costs of chemical dosage can also be disadvantageous. There exists a third approach in the treatment of phosphorous: 'simultaneous precipitation', in which chemical and biological methods both exist. However, Camilleri [31] states, in the choice of modelling of chemical phosphorous removal in the ASM2d/QUAL2E approach, that "the reduction of the soluble Phosphorus is very important because, in absence of other methodologies, this the only way to keep the concentration of the total Phosphorus below the regulation limits without significantly decrease the flowrate entering the plant" (p. 309). Many regulatory bodies do not provide limits for Phosphorous levels, restricting the concentrations only in sensitive areas. Many plants operate with a fixed level of chemical additions, so that the need for efficient control of this process is therefore necessary, to allow minimisation of the chemical dosage applied during steady state conditions.

Table 6.15 UWS control conditions during ASM2d control

Variables	Value
Q_w	150
b	0.2
Q_{in}	5750
T_s	1/1440

The processes considered in the TPMP1 treatment plant model of [31] are not all of those considered in the ASM2d model developed by [63], the anaerobic processes are not included, and additionally the phosphorous processes, the reaction terms of lysis, aerobic/anoxic growth and storage of X_{PAO}, X_{PP} and X_{PHA}, are not considered due to the choice of simulated processes within the model, although the mass balances for the associated states are still calculated. The motivation behind this is stated by [31] as the introduction of the chemical removal of phosphorous (via the $Fe(OH)_3$ process), eliminating the use of the biological removal process.

Table 6.16 SDC control parameters for ASM2d control

Parameters	SD-GPC
T_s	1/1440
Q	[20/10^2 10]
λ	[2/90^2 0.1/1200000^2]
H_u	5
H_p	15

An open loop observer is used, that is the influent values to the treatment plant are used in calculating the state variables with no reference to the values of the effluent of the aerobic reactor, to calculate the states according to the state dependent model and initial plant conditions at time t=0. In reality, this would be unsuitable, and a similar Kalman filter approach as that used in the control of the UWS above would be required. This is however outside the scope of the thesis. It is assumed therefore that the method used in the following application utilises accurate knowledge of plant behaviour. The time period of the simulation examples described is 6 days. Constraints exist upon the manipulated variables, with a saturation value for airflow, denoted by a maximum mass transfer coefficient K_{la} of 250 $1/d$, and a maximum dosage of $Fe(OH)_3$ of $15x10^5$ g/d

$$\begin{aligned} 0 &\leq K_{la} \leq 250\ 1/d \\ 0 &\leq Fe(OH)_3 \leq 15x10^5\ g/d \end{aligned} \quad (6.12)$$

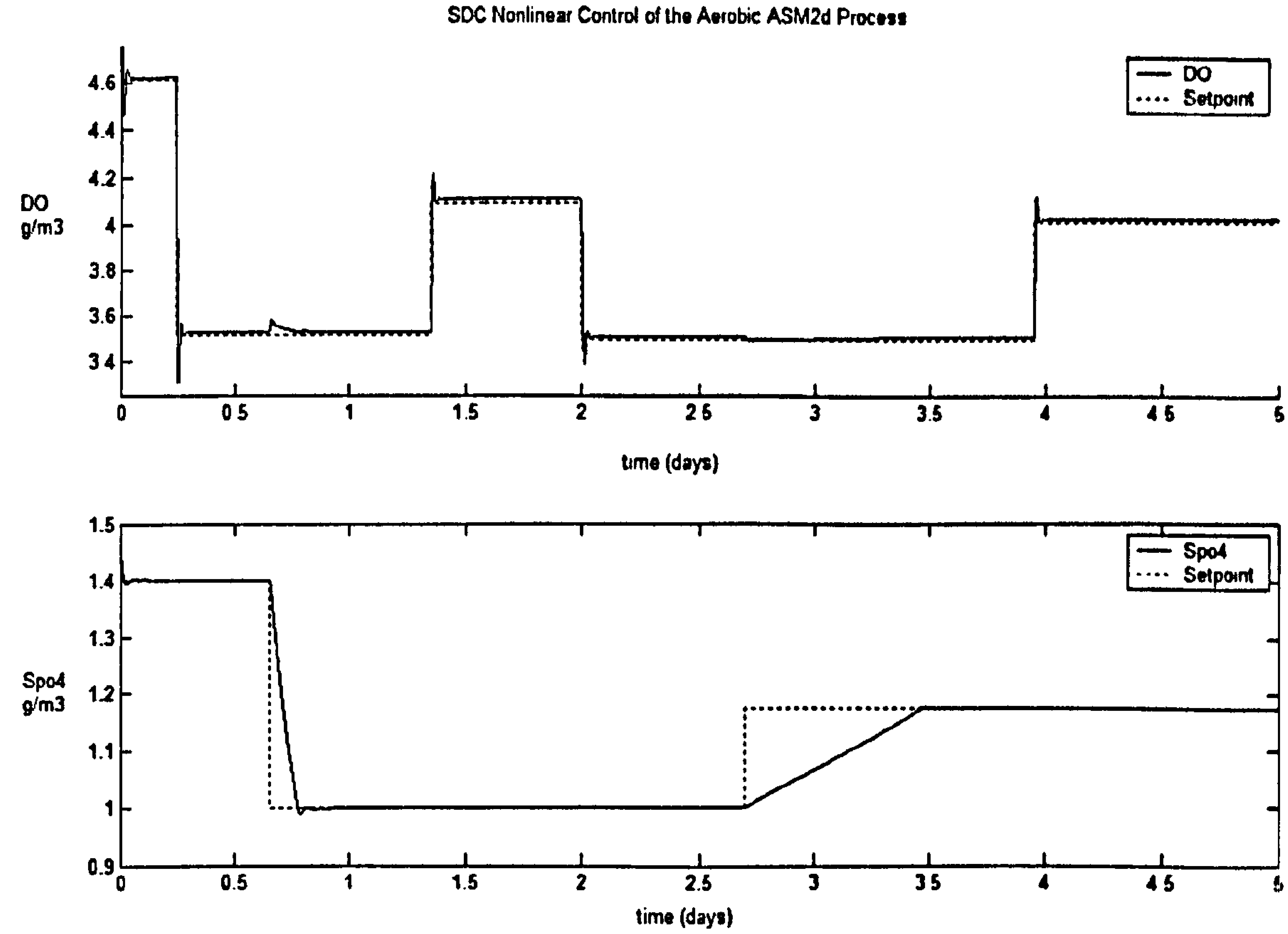

Figure 6-26: ASM2d Step Setpoint Change Responses for Dissolved Oxygen and Inorganic Soluble Phosphorous

In simulations of closed loop behaviour of this system under SDC control, it can be seen that both dissolved oxygen and phosphorous concentrations converged to, and reacted to changes in, the specified setpoints. This behaviour was produced during steady state operation of the plant, at a treatment plant influent flow of $5745m^3/d$. It may be observed that the constraints upon the dosage of $Fe(OH)_3$ slow the response of S_{PO4} to changes in setpoint, the delay introduced in the reaction follows the period of between 12 and 24 hours after step changes, for which the manipulated input constraints are in effect. The response of the dissolved oxygen process to setpoint changes, in comparison, has a settling time of approximately 1.2 hours. A setpoint reduction for S_{PO4} of $0.4g/m^3$ results in a significant increase in the chemical dosage required, over 7 times the steady state dosaged required. Conversely, the control action required for manipulation of the dissolved oxygen concentration does reach the air flow constraints, but however has behaviour of transients large increases in airflow rate during step changes in setpoint. The control responses and also the control actions performed indicates a level of coupling of the behaviour of the controlled variables.

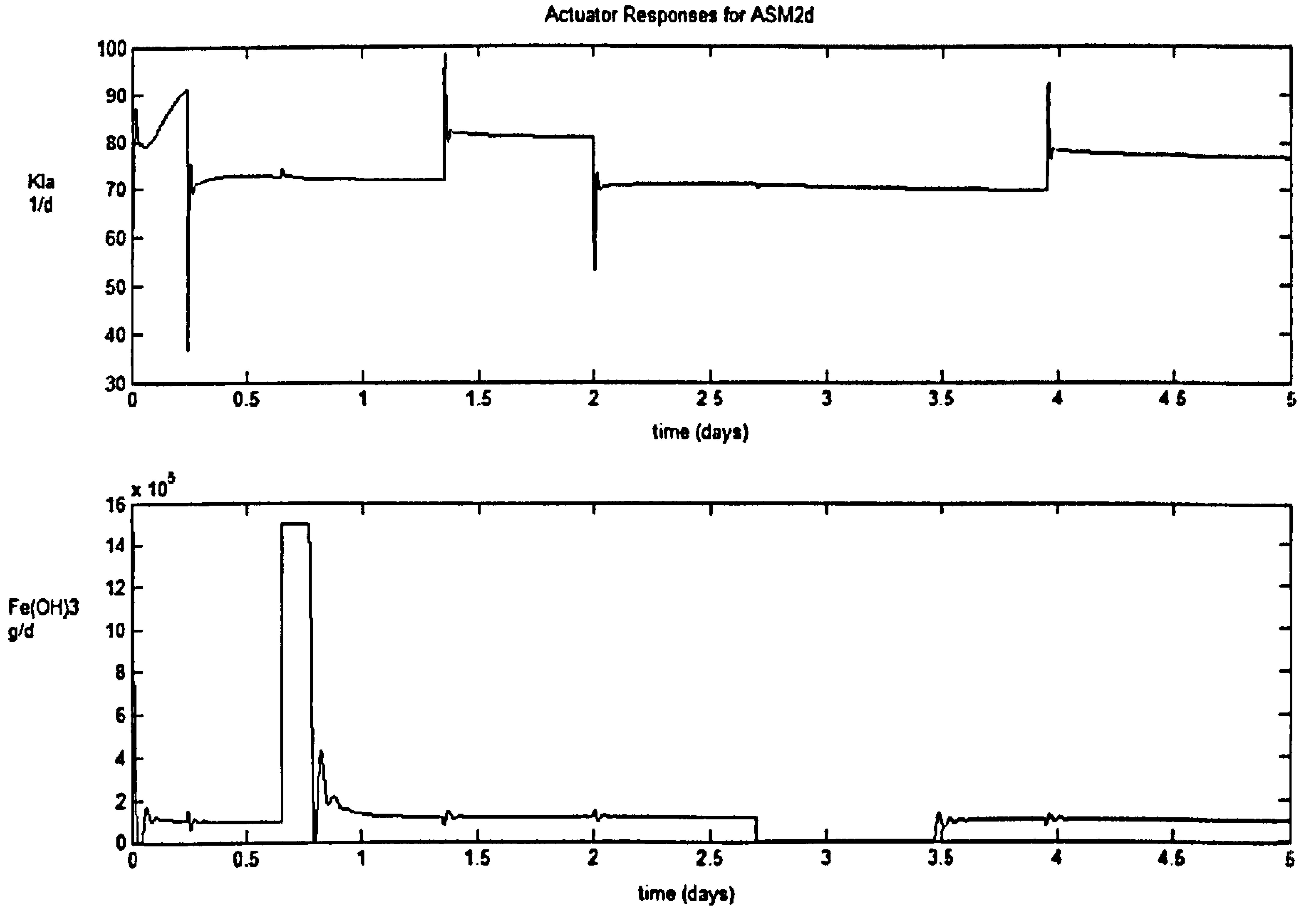

Figure 6-27: ASM2d Actuator Responses for Airflow Rate and Fe(OH)3

The above responses make a large assumption, namely that the influent variables are both measurable and also available with little delay. Substrate (readily and slowly biodegrable) can be measured via respirometer, usually with a minimum delay of 30 minutes. Dissolved oxygen concentration measurements are available with little or no delay, whilst nitrate/nitrite levels are commonly measurable with a 15 minute delay. The use of Kalman filter estimation has been demonstrated for the ASM1 model, for example Benazzi et al. [13] detailed the application of Kalman filters in the estimation of soluble and particulate biodegradable organic nitrogen, and partially in the case of estimation of active heterotrophic and autotrophic biomass. The application of nonlinear state dependent control in the case of the ASM2d model produces many issues:

- The parameters of the plant are situation specific, and must be identified from plant data. However, the extended kalman filter is sufficient for estimation if these parameters are regularly and well calibrated, and was demonstrated for the ASM2d model by [21]
- the kinetic parameters of the plant are not constant, as assumed in the application. Biological processes are time varying and nonlinear. This results in inaccuracies in the system model. This control approach relies heavily upon an accurate process model.
- In the case of a known model, there still exists a lack of controllability of the system.
- Many variables are not measured, and although some are estimatable from the system model, not all information about the system is available

The advantages on the other hand in the application of the state dependent approach are the following

- In the case of accurate nonlinear model, system dynamics and interactions can be accurately represented
- The linear time varying modelling approach allows for more accurate predictions of future behaviour
- The state space representation allows the traditional MPC technique to be extended to nonlinear systems
- The model linearisation techniques commonly used are inaccurate outside of the linear operating range

Considering the current regulatory climate of 'integrated' control, it may be assumed that future research may make way for identifiability of parameters for complex models (such as ASM2d), improvements in river water quality measurements and estimation, the provision of further control handles and indeed therefore the underlying issue of controllability and observability. In that case, the introduction of nonlinear control such as that based upon the state dependent state-space model representation of nonlinear processes, is possible.

6.8 Summary

The aim of this chapter was the design and application of nonlinear control approaches for the urban wastewater system. An introduction to the area of urban wastewater treatment system control was given. The linear predictive control detailed in previous chapters was extended for the control of river quality via the use of gain-scheduling with fuzzy methods and alternatively by the use of the Wiener modelling approach. In particular, this latter approach was applied in the case of dissolved oxygen control in the urban wastewater system. The linear dynamics of the dissolved oxygen process and its varying system gain over the nonlinear range was exploited in the production of a Wiener model of this process, allowing the development of a input dependent nonlinear state space model of the dissolved oxygen process to be developed.

State dependent models developed in previous chapters for the purpose of nonlinear predictive control are applied in the control of dissolved oxygen concentrations and substrate levels within the WWTP, and dissolved oxygen and BOD levels in the urban wastewater system. The nonlinear state dependent control of the dissolved oxygen and inorganic soluble phosphorous processes within ASM2d reaction tank model utilised in the urban wastewater model was also demonstrated, via the manipulation of airflow rate and chemical dosage of Ferric Hydroxide.

Chapter 7

Conclusions and Future Work

7.1 Thesis Summary

This thesis investigates the control techniques applicable for the maintenance of water quality in urban wastewater systems. Real-time control of a wastewater treatment system allows for optimisation of existing systems, technology and infrastructure, without the need for physical extension of the system or excessive financial investment. Traditionally, linear control techniques have proved popular and efficient in the control of wastewater treatment processes, concentrating on emission based strategies of effluent water quality control from treatment plants. Recently, the drive towards a more 'integrated' approach, focussed upon imission strategies, has motivated research into schemes whose primary objective is maintenance of river water quality.

Whilst reliable research has been presented for the minimisation of CSO events and their subsequent effects upon river quality, an alternative approach considers the extension of the existing control structures within treatment plants to incorporate river quality objectives; the latter approach is utilised within this thesis. The choice of model and control scheme for a wastewater treatment process must take into account the limits and constraints of that industry: the choice of control must be practical and easy to implement, it must be applicable with the existing technology, actuators and sensors and it should not require an excessive financial expenditure. It is proposed within this thesis that a model based predictive control approach is sufficient to fulfil the requirements of the wastewater industry.

Chapter 2 introduced the concepts of the Linear Model Predictive Control technique used within this thesis, with a brief historical background of the control strategy. The theoretical background to the MPC algorithm structure for linear state space system models was introduced. The inclusion of disturbance rejection within the control method chosen is of signifance, both in compensation for plant-model mismatch and also in the rejection of system disturbances to the desired performance. In addition, the inclusion of feedforward process knowledge in the handling of disturbances can be of advantage. To this end, the formulation of measured and unmeasured disturbance modelling for this control approach was detailed. The constraint handling and subsequent structure of the cost function optimisation for the predictive control algorithm was described.

The application of linear MPC to a wastewater treatment plant simulation case study was demonstrated within Chapter 3. An overview of the Benchmark Simulation Model 1 (BSM1) was given, and the development of a MIMO control application was detailed for the nitrate/nitrite and dissolved oxygen processes. This required the subspace identification of linear models of both processes. The control strategies developed from these linear models were tested for two treatment plant influent conditions: in dry conditions and in storm conditions. It is shown that the simple PI control approach, combined with the advantages of the MPC strategy, allow for exploitation of the best aspects of both forms of control. The simplicity of the PI structure is maintained, and the ease of design, implementation and use of the MPC strategy complements this existing control. The issues involved in control based within the wastewater treatment plant were discussed, in particular with regards to the effluent and receiving water quality requirements. Optimisation of plant performance, such as setpoint tracking for the chosen variables, is shown to be inadequate with respect to the effects upon receiving water quality. This motivated the development of an imission based control approach, considering the urban wastewater system.

The introduction of the process models utilised in the description of the urban wastewater system is shown in Chapter 4, with respect specifically to the representation of treatment plant and receiving water dynamics. In the particular case of nonlinear modelling for control purposes, the state dependent formulation of several wastewater models is demonstrated: a wastewater treatment plant process model, a model of receiving water dynamics and subsequently the urban wastewater model based upon these approaches. The state dependent coefficient representation was also produced for the ASM2d process model of an aerobic reactor. These representations were additionally modified for the inclusion of a feedforward model of measured variables

Two distinct forms of nonlinear control were presented within Chapter 5 of this thesis. The initial control approaches demonstrated within this chapter extended the linear MPC algorithm to the nonlinear process, via gain-scheduled and Wiener model based techniques. The later control approach utilised an inherently nonlinear control model based upon a Linear Time Varying representation of the process, with the state dependent coefficient modelling technique. The fuzzy approach to gain scheduled MPC introduced within this chapter describes the nonlinear range with a finite set of linear models, to be chosen appropriate to the given operating condition by a fuzzy membership function. The Wiener model approach described considers the nonlinearities of the process as definable by two components: the dynamic linear process and the static nonlinear system gain. Thus at any given operating point, the process can be considered as defined by a linear representation, and is seen as an approximate model of the nonlinear process over the operating range. In the presence of an existing process model, it may be chosen to include the full nonlinear process description within the model based control algorithm. The nonlinear control approach based upon the SDC process representation retains the features of the linear predictive control algorithm whilst gaining the beneficial accuracy of the nonlinear process models.

Chapter 6 demonstrates the application of the above control methodologies to the urban wastewater system. Two implementations of fuzzy gain scheduled control were demonstrated in Chapter 6 in the maintenance of water quality: for the dissolved oxygen concentration and for the ammonium concentration in the receiving waters. The FGS control was demonstrated for disturbance rejection in the event of storm weather and combined sewer overflows. Analysis of the behaviour of these processes with respect to air flow rate changes within the treatment plant indicated the nonlinearity of the ammonia process. Additionally however, this analysis demonstrated the linear nature of the dissolved oxygen response. This behaviour was thus exploited in a Wiener model description of the effect of air flow rate changes upon the dissolved oxygen levels in the receiving waters. The implementation of Wiener MPC was demonstrated, again in the case of a storm weather event.

A comparison of the FGS and WMPC approaches with their linear counterpart, in addition to the traditional PI control approach was shown. The nonlinear generalised predictive control algorithm of Chapter 6 was utilised in three simulation case study control applications: the control of dissolved oxygen and substrate concentrations in a WWTP, the control of dissolved oxygen and BOD in an UWS and the control of dissolved oxygen and phosphorous levels in an aerobic activated sludge process. The control approaches detailed within Chapter 6 aim to demonstrate the necessity for and advantages in the use of advanced control in the automated operation of the wastewater treatment process. In particular, the use of control in the compensation for disturbances within the system due to high wastewater loads and storm weather events was considered, with respect to the control objectives proposed within Chapter 1. Advanced control in the current climate benefits the water industry in its efforts to comply with water quality standards. Stricter control will be a necessity in the event of more stringent environmental quality criterion.

7.2 Future Work

Possible future work can be summarised by the following points

- Application of nonlinear generalised predictive control to the ASM2d/QUAL2E urban wastewater model, requiring the SDC model representation of the complex settler dynamics. The CSTR nature of the receiving waters allows for the relative ease of transformation of the nonlinear river system model to the state dependent structure, thus providing the description of the nonlinear dynamics of the urban wastewater system within the state dependent format, allowing nonlinear control of several river water quality variables.
- The complexity of the settler dynamics limits the applicability of the SDC to the full complex urban wastewater model. Recent research in modelling of the settler has

further increased this complexity, with the most accurate clarifier effluent predictions from 2d and 3d hydrodynamic models.These models however may not be suited to online control applications due both to their complexity and also to the large amount of data necessary for their calibration. A formulation of the SDC for the settler model would be necessary to fully describe the complex behaviour of the process.

- Expansion of nonlinear methods to include more controlled variables, such as state dependent control of BOD, Ammonia, Nitrate and Suspended Solids..
- Use of extended Kalman filter estimation methods to provide non-measured variables, with nonlinear GPC - demonstrating possible practical application of nonlinear control. In the application of control using the SDC modelling approach for the urban wastewater system and also for the ASM2d treatment plant model, the assumption was made that full state information can be measured. In application, the system state may not be fully measured, and significantly only the controlled output and manipulated input measurements may be available. The application of state and parameter estimation would be necessary for the practical application of this approach to the real nonlinear system.
- Online identification, in conjunction with the above developed methods, would reduce the uncertainty involved in the practical implementation of control based upon process models.
- Extension of nonlinear state dependent, to allow control to adapt to changing parameters, and/or robust to system uncertainties.
- Use of sewer advanced techniques during and after rain events, such as prediction of future flow rates or influent loads, allowing further nonlinear control and/or extension to include emergency procedures such as stormtank emptying to avoid washout effects.
- Inclusion of further objectives in the control optimisation, such as ecological quality standards, effluent quality requirements, chemical and carbon dosing, aeration energy and pumping energy costs

Bibliography

[1] A. J. Abebe and R. K. Price, "Decision Support System for Urban Flood Management", *Journal of Hydroinformatics*, vol. 7, 2005, pp. 3-15.

[2] P. Afonso and M. Da Conceicao Cunha, "Assessing Parameter Identifiability of Activated Sludge Model Number 1", *Journal of Environmental Engineering*, vol. 128, no. 8, 2002, pp. 748-754.

[3] J. Alex, J.F. Beteau, J.B. Copp, C. Hellinga, U. Jeppsson, S. Marsili Libelli, M.N. Pons, H. Spanjers and H. Vanhooren, "Benchmark for Evaluating Control Strategies in Wastewater Treatment Plants", *Proc. European Control Conference (ECC 99)*, 1999.

[4] T. Alvarez and C. de Prada, "Handling Infeasibilities in Predictive Control", *Computers and Chemical Engineering*, vol. 21, no. 1, 1997, pp. 577-582.

[5] A. Aoyama, F.J. Doyle and V. Venkatasubramanian, "Control Affine Neural Network Approach for Non-Minimum-Phase Nonlinear Process Control", *Journal of Process Control*, vol. 6, no. 1, 1996, pp 17-26.

[6] K.J. Astrom and B. Wittenmark, *Adaptive Control*, New York, Addison Wesley, 1989.

[7] E. Ayesa, J. Carstensen, U. Jeppsson and P. Vanrolleghem, *Identification of the Dynamic Processes in WWTP and control of WWTP*, Report of Research Group COST 682: Environment, (D. Dochain, P. Vanrolleghem, M. Henze, eds.), European Commission, Directorate-General XII: Science, Research and Development, Luxembourg, 1995, pp 89-104.

[8] T.A. Badgwell, "Robust Model Predictive Control for Nonlinear Plants", Presented at the *Annual AIChE Meeting*, Chicago, 1996.

[9] L. Balbis, M. O' Brien, F. Benazzi, A.W. Ordys and R. Katebi, "Comparison of Nonlinear Predictive Control methods for Wastewater Applications", submitted to the *Transactions of the Institute of Measurement and Control*, June 2006

[10] T. Bastogne, H. Noura, P. Sibille and A. Richard, "Multivariable Identification of a Winding Process by Subspace Methods for Tension Control", *Control Engineering Practice*, vol. 6, 1998, pp.1077-1088.

[11] J.A. Baeza, E.C. Ferreira and J. Lafuente, "Knowledge-based Supervision and Control of Wastewater Treatment Plant: a Real-time Implementation", *Water Science and Technology*, vol. 41, No 12, 2000, pp 129-137.

[12] M.B Beck, "Engineering Control and the Dynamics of Environmental Systems", *Proc. European Conference on Biotechnology*, Brussels 1999.

[13] F. Benazzi, K. V. Gernaey, U. Jeppsson and R. Katebi, "On-line Estimation and Detection of Abnormal Substrate Concentrations in WWTPs Using a Software Sensor: A Benchmark Study", *Proc. 2^{nd} IWA Conference, Instrumentation, Control and Automation for Water and Wastewater Treatment and Transport Systems*, Busan, Korea, 2005.

[14] S. Billings, and S. Fakhouri, "Identification of Systems Containing Linear Dynamic and Static Nonlinear Elements', *Automatica*, vol. 18, no. 1, 1982, pp. 15-26.

[15] M. Bodson and S.C. Douglas, "Adaptive Algorithms for the Rejection of Sinusoidal Disturbances with Unknown Frequency", *Automatica*, vol. 33, no. 12, 1997, pp. 2213-2221.

[16] C. Bordons and J.R. Cueli, "Predictive Controller with Estimation of Measurable Disturbances: Application to an Olive Oil Mill", *Journal of Process Control*, vol. 14, 2004, pp. 305-315.

[17] H. Bloch, "EU Policy on Nutrients Emissions: Legislation and Implementation", *Water Science and Technology*, vol. 44, no. 1, 2001, pp. 1-6.

[18] H. Bloemen, C. Chou, T. van den Boom, V. Verdult, M. Verhaegen, and T. Backx, "Wiener Model Identification and Predictive Control for Dual Composition Control of a Distillation Column', *Journal of Process Control*, vol. 11, 2001, pp. 601-620.

[19] L.C. Brown and T.O. Barnwell Jr., *The Enhanced Stream Water Quality Models QUAL2E and QUAL2E-UNCAS: Documentation and User Manual*, Environmental Research Laboratory, Office of Research and Development. United States Environmental Protection Agency, Athens, GA, 1987.

[20] M. Brdys and Y.Zhang, "Modelling and Control of Wastewater Treatment Plants", *Proc. 3rd National Conference: Technology and Automation of Water Supply and Sewerage Systems*, Stawisk/Gdansk, Poland, 1999.

[21] M. Brdys, T. Chang and K. Konarczak, "Estimation of Wastewater Treatment Plant State for Model Predictive Control of N-P Removal at Medium Time Scale", *Proc. IFAC 10th Symposium Large Scale Systems: Theory and Applications*, July, 2004, Japan.

[22] H.O. Buhr, J.F. Andrews and T.M. Keinath, "Research Needs for Automation of Wastewater Treatment Systems", *Proc. U.S. Environmental Protection Agency Workshop*, Clemson University, South Carolina, 1974, pp. 135.

[23] D. Butler and M. Schutze, "Integrating Simulation Models with a View to Optimal Control of Urban Wastewater Systems", *Environmental Modelling and Software*, vol. 20, 2005, pp. 415-426.

[24] J.H. Burghart, "A Technique for Sub-optimal Feedback Control of Nonlinear Systems", *IEEE Transactions on Automatic Control*, vol. 14, 1969, pp. 530-533.

[25] W.Chotkowski, M.A.Brdys and K.Konarczak, "Dissolved Oxygen Control for Activated Sludge Processes", *International Journal of Systems Science*, Vol. 36, No. 12, 2005, pp. 727-736.

[26] C. Chow, A. Kuznetsov, and D. Clarke, "Using Multiple Models in Predictive Control", *Proc. 3^{rd} European Control Conference*, 1995, pp. 1732-1737.

[27] D.W. Clarke, "Advances in Model-Based Predictive Control', *Advances in Model-Based Predictive Control*, Oxford University Press, 1994, pp. 69.

[28] D. W. Clarke, C. Mohtadi, and P.S. Tuffs, "Generalised Predictive Control. Part 1: The Basic Algorithms", *Automatica*, vol. 32, no. 2, 1987, pp. 137-148.

[29] D. W. Clarke, C. Mohtadi, and P.S. Tuffs, "Generalised Predictive Control. Part 2: Extensions and Interpretations", *Automatica*, vol. 32, no. 2, 1987, pp. 149-160.

[30] J. Copp, *The COST Simulation Benchmark - Description and Simulator Manual*, COST (European Cooperation in the field of Scientific and Technical Research), Brussels, Belgium, 2001.

[31] F. Camilleri. "Modelling and Optimisation of Integrated Wastewater Systems", M.Phil. Thesis, EEE Department, University of Strathclyde, Glasgow, UK, 2004.

[32] A.L. Cervantes, O.E. Agamennoni and J.L. Figueroa, "A Nonlinear Model Predictive Control System Based on Wiener Piecewise Linear Models", *Journal of Process Control*, vol.13, no.7, 2003, pp. 655-666.

[33] C. R. Cutler, and B.L. Ramaker, "Dynamic Matrix Control: A Computer Control Algorithm", *Proc. Joint Automatic Control Conference*, San Francisco, CA, 1980.

[34] F.Cuzzola, J.C. Geromel and M.Morari, "An Improved Approach for Constrained Robust Model Predictive Control", *Automatica*, vol. 38, 2002, pp. 1183-1189.

[35] E.J. Davison and H.W. Smith, "Pole Assignment in Linear Time-Invariant Multivariable Systems with Constant Disturbances", *Automatica*, vol. 7, 1971, pp. 489-498.

[36] H. Demircioglu, and D. Clarke, "CGPC with Guaranteed Stability Properties", *IEE Proceedings*, vol. 139, no. 4, 1992, pp. 371-380.

[37] C. Demuynck, P.A. Vanrolleghem, C. Mingneau, J. Liessens and W. Verstraete, "ND-BEPR Process Optimization in SBRs: Reduction of External Carbon Source and Oxygen Supply, *Water Science and Technology*, vol. 30, no. 4, 1994, pp. 169-179.

[38] K.M. Dhanasekharan, J. Sanyal, A. Jain and A. Haidari, "A Generalized Approach to Model Oxygen Transfer in Bioreactors Using Population Balances and Computational Fluid Dynamics", *Chemical Engineering Science*, vol. 60, no. 1, 2005, pp. 213-218.

[39] A.W. Dorsey and J.H. Lee, "Building Inferential Prediction Models of Batch Processes Using Subspace Identification", *Journal of Process Control*, vol. 13, 2003, pp. 397-406.

[40] D. Driankov and B. Kadmiry, "A Fuzzy Flight Controller Combining Linguistic and Model-Based Fuzzy Control", *Fuzzy Sets and Systems*, vol. 146, no. 3, 2004, pp. 313-347.

[41] A. Dutka, A. Ordys and M.J. Grimble, "Nonlinear Predictive Control of a 2dof Helicopter Model" *42nd IEEE Control and Decision Conference Proceedings*, Maui, Hawaii, 2003.

[42] *Directive Concerning Urban Wastewater Treatment (91/271/EEC)*, Council of the European Communities, Official Journal L135, 1991, pp. 40-52.

[43] *Directive Concerning Protection of Water Against Pollution by Nitrates from Agriculture (91/676/EEC)*, Council of the European Communities, Official Journal L375, 1991, pp. 1.

[44] *Directive Concerning the Quality of Surface Waters Intended for the Abstraction of Drinking Water (75/440/EEC)*, Council of the European Communities, Official Journal L194, 1975, pp. 26.

[45] *Directive Concerning the Quality of Bathing Waters (76/160/EEC)*, Council of the European Communities, Official Journal L31, 1976, pp. 32.

[46] *Directive Concerning the Quality of Fish Waters (78/659/EEC)*,Council of the European Communities ,Official Journal L222, 1978, pp. 1.

[47] *Directive Concerning the Quality of Water for Human Consumption (80/778/EEC)*, Council of the European Communities, Official Journal L229, 1980, pp. 11.

[48] S. Entem, A. Lahoud, L. Yde and B. Bendsen, "Real Time Control of the Sewer System of Boulogne Billancourt - A Contribution to Improving the Water Quality of the Seine", *Water Science Technology*, vol. 37, no. 1, 1998, pp. 327-332.

[49] *Directive of the European Parliament and the Council 2000/60/EC Establishing a Framework for Community Action in the Field of Water Policy*, The European Parliament, The Council, PE-CONS 3639/1/00/REV.1 EN, 2000.

[50] M. Galluzzo, R. Ducato, V. Bartolozzi and A. Picciotto, "Expert Control of DO in the Aerobic Reactor of an Activated Sludge Process", *Computers and Chemical Engineering*, 25, 2001, pp. 619-625

[51] C.E. Garcia, "Quadratic/Dynamic Matrix Control of Nonlinear Processes: An Application to a Batch Reaction Process", *Proc. AIChE Annual Meeting*, San Francisco, CA, 1984.

[52] S. Gerksic, D. Juricic, S. Strmcnik and M. Matko, "Wiener Model Based Nonlinear Predictive Control", *International Journal of Systems Science, vol.* 31, 2000, pp. 189-202.

[53] S. Gillot, S. Capela-Marsal, M. Roustan and A. Héduit, "Predicting Oxygen Transfer of Fine Bubble Diffused Aeration Systems-Model Issued from Dimensional Analysis", *Water Research*, vol. 39, no. 7, 2005, pp. 1379-1387.

[54] J.C. Gomez, and E. Baeyens, "Subspace Identification of Multivarial Hammerstein and Wiener Models", *Proc. 15th International Federation of Automatic Control World Congress*, Barcelona, 2002, pp. 2849-2854.

[55] J. Gonzalez, R. Aguilar, J. Alvarez-Ramirez and M.A. Barren, "Nonlinear Regulation for a Continuous Bioreactor via a Numerical Uncertainty Observer", *Chemical Engineering Journal*, vol. 69, no. 2, 1998, pp. 105-110.

[56] F. Graells and R. Katebi, "Framework for Modelling, Simulation and Control of Integrated Urban Waste water Systems', *16th IFAC World Congress*, Prague, 2005.

[57] M. J. Grimble and A. Ordys, "Non-linear Predictive Control for Manufacturing and Robotic Applications", *IEEE Conference on Methods and Models in Automation and Robotics*, Miedzyzdroje, Poland, 2001.

[58] M.M. Hansen and J. Carstensen, "Continuous Quality Control of Measurements in a Real Time Controlled Sewer System in the Municipality of Copenhagen", *Water Science Technology*, vol. 36, no. 8-9, 1997, pp. 349-353.

[59] M. Henson, and D. Seborg, "Adaptive Nonlinear Control of a pH Neutralization Process", *IEEE Transactions on Control System Technology*, vol. 2, no. 3, 1994, pp. 169-182.

[60] M. Henson,"Nonlinear Model Predictive Control: Current Status and Future Directions", *Computers and Chemical Engineering*, vol. 23, 1998, pp. 187-202.

[61] M. Henze, C.P.L. Grady Jr., W. Gujer, G.v.R. Marais and T. Matsuo, *Activated Sludge Model No. 1. Scientific and Technical Report No. 1*, IAWQ, London, Great Britain, 1987.

[62] M. Henze, W. Gujer, T. Mino, T. Matsuo, M. C. Wentzel and G. v. R. Marais, *Activated Sludge Model No. 2. Scientific and Technical Report No. 3*, IAWQ, London, Great Britain, 1995.

[63] M. Henze, W.Gujer, T. Mino and M. van Loosdrecht, *Activated Sludge Models ASM1, ASM2, ASM2d and ASM3*, Scientific and Technical Report No. 9, IWA Publishing, London, UK, 2000.

[64] E. Hernandez and Y. Arkun, "Control of Nonlinear Systems Using Polynomial ARMA Models", *AIChE Journal*, vol. 39, 1993, 446-460.

[65] N. Hilgert, J. Harmand, J. Steyer and J. Vila, "Nonparametric Identification and Adaptive Control of an Anaerobic Fluidized Bed Digester", *Control Engineering Practice*, vol. 8, no. 4, 2000, pp. 367-376.

[66] Y. Huang and W.M. Lu, " Nonlinear Optimal Control: Alternatives to Hamilton-Jacobi Equation", *Proc. 35th IEEE Conference on Decision and Control*, 1996, pp. 3942-3947.

[67] M. Pons and O. Potier, "Control Strategy Robustness with Respect to Hydraulics", *Control Engineering Practice*, vol. 12, 2004, pp.349-356.

[68] P. Ingildsen, U. Jeppsson and G. Olsson, "Dissolved Oxygen Controller Based on On-line Measurements of Ammonium Combining Feedforward and Feedback", *Water Science and Technology*, vol. 45, no. 4-5, 2002, pp. 453-460.

[69] U. Jeppsson, J. Alex, M.N. Pons, H. Spanjers and P.A. Vanrolleghem, "Status and Future Trends of ICA in Wastewater Treatment - A European Perspective", *Water Science and Technology*, vol. 47, no. 4-5, 2002, pp. 485-494.

[70] C. Jia, S. Rohani and A. Jutan, "FCC Unit Modeling, Identification and Model Predictive Control, a Simulation Study", *Chemical Engineering and Processing*, vol. 42, 2003, pp. 311-325.

[71] J. Jiang, "Optimal Gain Scheduling Controller for a Diesel Engine", *IEEE Control Systems Magazine*, vol. 14, no. 4, 1994, pp. 42-48.

[72] T.A. Johansen and R. Murray-Smith, "The Operating Regime Approach', *Multiple Model Approaches to Modelling and Control*, Taylor and Francis, 1997.

[73] J.C. Kabouris, A.P. Georgakakos and A. Camara. "Optimal Control of the Activated Sludge Process: Effect of Sludge Storage", *Water Research*, vol. 26, 1992, pp. 507-517.

[74] R.Kadalia, B. Huanga and A. Rossiter, "A Data Driven Subspace Approach to Predictive Controller Design", *Control Engineering Practice*, vol. 11, 2003, pp. 261-278.

[75] A. Kalafatis, N. Arifin, L. Wang, and W. Cluett, "A New Approach to the Identification of pH Processes Based on the Wiener Model", *Chemical Engineering Science*, vol. 50, no. 23, 1995, pp. 3693-3701.

[76] R.E. Kalman, "A New Approach to Linear Filtering and Prediction Theory", *Transactions of the ASME - Journal of Basic Engineering*, vol. 82, 1960, pp 35-45.

[77] M.R. Katebi, M.A.Johnson and J.Wilkie, "*Control and Instrumentation of Wastewater Treatment Plants*", Advances in Industrial Control Monographs Series, Springer Verlag, London, 1999

[78] P.Y. Keskar, "Structured Approach in PLC Programming for Water/Wastewater Applications", *ISA Transactions*, vol. 29, no. 2, 1990, pp. 47-56.

[79] D. Kleinman, "An Easy Way to Stabilise a Linear Constant System", *IEEE Transactions on Automatic Control*, vol. 15, 1970.

[80] H. Kim, T.J.McAvoy, J.S.Anderson and O.J.Hao, "Control of an Alternating Aerobic-Anoxic Activated Sludge System, Part 2: Optimisation Using a Linearised Model", *Control Engineering Practise*, vol. 2, 2000, pp. 279-289.

[81] J. Kocijan, G. Zunic, S. Strmcnik and D. Vrancic, "Fuzzy Gain-Scheduling Control of a Gas-Liquid Separation Plant Implemented on a PLC", *International Journal of Control*, vol. 75, no. 14, 2002, pp. 1082-1091.

[82] B. Kouvaritakis, J.A. Rossiter, and A. Chang, "Stable Generalised Predictive Control: An Algorithm with Guaranteed Stability", *IEE Proceedings*, vol. 139, no. 4, 1992, pp. 349-362.

[83] B. Kouvaritakis, M. Cannon and J.A. Rossiter, "Non Linear Model Based Predictive Control", *International Journal Control*, vol. 72, no. 10, 1999, pp. 919-928.

[84] V. Kubsad, S. Chaudhari and S.K. Gupta, "Model for Oxygen Transfer in Rotating Biological Contactor", *Water Research*, vol. 38, no. 20, 2004, pp. 4297-4304.

[85] P. Krauss, K. Dass and H. Rake, "Model Based Predictive Controller with Kalman Filtering for State Estimation", *Advances in Model-Based Predictive Control*, Oxford University Press, 1994, pp. 69.

[86] W.H. Kwon and A.E. Pearson, "A Modified Quadratic Cost Problem and Feedback Stabilization of a Linear System", *IEEE Transactions on Automatic Control*, vol. 22, 1977, pp. 838-842.

[87] R.F. Lech, C.P. Grady, H.C. Lim and L.B. Koppel, "Automatic Control of the Activated Sludge Process II, Efficacy of control strategies", *Water Research*, vol. 12, 1978, pp. 91-99.

[88] J.H. Lee, "Modelling and Identification for Nonlinear Model Predictive Control: Requirements, Current Status and Future Research Needs", *Proc. Nonlinear Model Predictive Control Workshop - Assessment and Future Directions*, Ascona, Switzerland, June, 1998.

[89] E.B. Lee and L. Markus, *Foundations of Optimal Control Theory*, John Wiley and Sons, New York, 1967.

[90] Y.I. Lee and B. Kouvaritakis, "Robust Receding Horizon Predictive Control for Systems with Uncertain Dynamics and Input Saturation", *Automatica*, vol. 36, 2000, pp.1497-1504.

[91] Y.I. Lee, B. Kouvaritakis and M. Cannon, "Constrained Receding Horizon Predictive Control for Nonlinear Systems", *Automatica*, vol. 38, no. 12, 2002, pp. 2093-2102.

[92] Y.I.Lee, B.Kouvaritakis, "Constrained Robust Model Predictive Control based on Periodic Invariance", *Automatica Online*, Short Communication, 1 September 2006

[93] D. Leith, A. Tsourdosb, B.A. Whiteband and W.E. Leithead, "Application of Velocity-Based Gain-Scheduling to Lateral Auto-Pilot Design for an Agile Missile", *Control Engineering Practice*, vol. 9, 2001, pp. 1079-1093.

[94] C.F. Lindberg, "Control and Estimation Strategies Applied to the Activated Sludge Process", PhD thesis, Uppsala University, 1997

[95] C. Ling and T.F. Edgar, "Real-Time Control of a Water-Gas Shift Reactor by a Model-Based Fuzzy Gain Scheduling Technique", *Journal of Process Control*, vol. 7, no. 4, 1997, pp. 239-253.

[96] C. Ling and T.F. Edgar, "Experimental Verification of Model-Based Fuzzy Gain Scheduling Technique", *Proc. American Control Conference*, 1994, pp. 2475-2480.

[97] J.M. Maciejowski, *Predictive Control with Constraints*, Prentice Hall, 2002

[98] B.R. Maner, F.J. Doyle, B.A. Ogunnaike and R.K. Pearson, "Nonlinear Model Predictive Control of a Simulated Multivariable Polymerization Reactor Using Second-Order Volterra Models", *Automatica*, vol. 32, 1996, pp. 1285-1301.

[99] L. Mailleret, O. Bernard and J. Steyer, "Nonlinear Adaptive Control for Bioreactors with Unknown Kinetics", *Automatica*, vol. 40, 2004, pp. 1379 1385.

[100] J. Makinia, and S. Wells, "Temperature Modeling in Activated Sludge Systems: A Case Study", *Water Environment Research*, vol. 77, no. 5, 2005, pp. 525-532.

[101] J.W. Mao, "Robust Stabilisation of Uncertain Time-varying Discrete Systems and Comments on an Improved Approach for Constrained Robust Model Predictive Control", *Automatica*, 39, 2003, 1109-1112

[102] M. Marinaki, "Optimal Real-Time Control of Sewer Networks", PhD thesis, Technical University of Crete, 2002

[103] S. Marsili-Libelli, "Optimal Control Strategies for Biological Wastewater Treatment", *Environmental Systems Analysis and Management*, North-Holland, Amsterdam, 1982, pp. 279-287.

[104] D.Q. Mayne and H. Michalska, "Robust Receding Horizon Control of Constrained Nonlinear Systems", *IEEE Transactions on Automatic Control*, vol. 38, no.11, 1993, pp.1623-1633.

[105] J. Meirlaen, "Immission Based Real-Time Control of the Integrated Urban Wastewater System", PhD thesis, Ghent University, 2002.

[106] J. Meirlaen, J. Van Assel and P. Vanrolleghem, "Real time Control of the Integrated Urban Wastewater System using Simultaneously Simulating Surrogate Models", Water Science and Technology, vol. 45, no. 3, 2002, pp. 109-116.

[107] J.D. Morningred, B.E. Paden, D.E. Seborg and D.A. Mellichamp, "An Adaptive Nonlinear Predictive Controller", *Chemical Engineering Science,* vol. 47, 1992, pp. 755-762.

[108] C.P. Mracek and J.R. Cloutier, "Control Designs for the Nonlinear Benchmark Problem via the State Dependent Riccati Equation", *International Journal of Robust and Nonlinear Control*, vol. 8, 1998, pp. 401-433.

[109] K.R. Muske and T.A. Badgwell, "Disturbance Modelling for Offset Free Linear Model Predictive Control", *Journal of Process Control*, vol. 12, no. 5, 2002, pp. 617-632.

[110] R.K. Mutha, W.R. Cluett and A. Pendilis, "Modifying the Prediction Equation for Nonlinear Model-Based Predictive Control', *Automatica*, vol. 34, no. 10, 1998, pp. 1283-1287.

[111] M. K. Nielsen and A. Lynggaard, "Superior Tuning and Reporting (STAR): a New Concept for On-line Process Control of Wastewater Treatment Plants", *Instrumentation, Control and Automation of Water and Wastewater Treatment and Transportation Systems*, B. Jank, ed., IAWQ, London, 1993, pp. 560-574

[112] M.K. Nielsen, H. Bechman and M. Henze, "Modelling and Test of Aeration Tank Settling (ATS)", *Water Science Technology*, vol. 41, no. 9, 2000, pp. 179-184.

[113] M.K. Nielsen and T.B. Onnerth, "Improvement of a Recirculating Plant by Using STAR Control", *Water Science Technology*, vol. 31, no. 2, 1995, pp. 171-180.

[114] F. Nejjari, G. Roux, B. Dahhou and A. Benhammou, "Estimation and Optimal Control Design of a Biological Wastewater Treatment Process", *Mathematics and Computers in Simulation,* vol. 48, 1999, pp. 269-280.

[115] S. Norquay, A. Palazoglu and J. Romagnoli, "Model Predictive Control Based on Wiener Models", *Chemical Engineering Science*, vol. 53, no. 1, 1998, pp. 75-84.

[116] S. Norquay, A. Palazoglu and J. Romagnoli, "Application of Wiener Model Predictive Control (WMPC) to a pH Neutralization Experiment", *IEEE Transactions Control System Technology*, vol. 7, no. 4, 1999, pp. 437-445.

[117] M. O' Brien, S.E. Pinto Castillo and R. Katebi, "Gain Scheduled Model Based Predictive Control for Wastewater Applications", *Proc. IFAC World Congress*, Prague, 2005.

[118] M. O' Brien, F. Camilleri and A. Ordys, "Gain Scheduled Predictive Control for Integrated Wastewater Systems in Storm Conditions', *Proc. 10th International Conference on Urban Drainage (10ICUD)*, Copenhagen, Denmark, 2005.

[119] M. O' Brien, F. Camilleri and A. Ordys, "Predictive Fuzzy Control of Ammonia: Integrated Urban Wastewater Treatment", *Proc. 3^{rd} CIWEM National Conference*, Wakefield, 2005.

[120] M. O' Brien and A. Ordys, "Nonlinear Predictive Control of a Wastewater Model", *Proc. Control 2006*, Glasgow, 2006.

[121] G. Olsson, "Advancing ICA Technology by Eliminating the Constraints', *Water Science Technology*, vol. 28, no. 11-12, 1993, pp. 1-7.

[122] G. Olsson and U. Jeppsson, "Establishing Cause-Effect Relationships in Activated Sludge Plants -What Can Be Controlled?", *Proc. Workshop Modelling, Monitoring and Control of Wastewater Treatment Plants*, Univ. Gent, vol. 59, pp. 2057-2070.

[123] G. Olsson, H. Aspegren, and M.K. Nielsen, "Operation and Control of Wastewater Treatment a Scandinavian Perspective Over 20 Years", *Water Science Technology,* vol. 37. no. 12, 1998, pp. 113.

[124] A. W. Ordys and D. W Clarke,"A State-Space Description for GPC Controllers", *International Journal of Systems Science*, vol. 23, no. 2, 1993.

[125] A.W.Ordys, A.W. Pike and M.A. Johnson, "Optimisation of Set-point Transition in Combined Cycle Power Plants using Predictive Control Techniques", Proc. IFAC Symposium on Adaptive Systems in Control and Signal Processing, Budapest, Hungary, June 1995.

[126] A. W. Ordys and M.J. Grimble, "Predictive Control Design for Systems with the State-Dependent Nonlinearities", *Proc. SIAM Conference on Control and its Applications*, San Diego, California, 2001.

[127] O. A.Z. Sotomayor, S. Won Park and C. Garcia, "Model Reduction and Identification of Wastewater Treatment Plants - a Subspace Approach", *Latin American Applied Research*, vol. 33, 2003, pp. 135-140.

[128] J. D. Pearson, "Approximation Methods in Optimal Control", *Journal of Electronics and Control*, vol. 13, 1962, pp. 453-465.

[129] M. Perrier and D. Dochain, "Evaluation of Control Strategies for Anaerobic Digestion Processes", *International Journal of Adaptive Control and Signal Processing*, vol. 7, 1993, pp. 309-321.

[130] B. Petersen, "Calibration, Identifiability and Optimal Experimental Design of Activated Sludge Models", PhD. Thesis. Faculty of Agricultural and Applied Biological Sciences. Ghent University, 2000.

[131] A. Petruck, A. Cassar and J. Dettmar, "Advanced Real Time Control of a Combined Sewer System", *Water Science Technology*, vol. 37, no. 1, 1998, pp. 319-326.

[132] M. Pleau, G. Pelletier, H. Colas and P. Lavallée, "Global Optimal Real-Time Control of the Quebec Urban Drainage System", *Environmental Modelling and Software*, vol. 20, no. 4, 2005, pp. 401-413.

[133] T. Proell and N. Karim, "Nonlinear Control of a Bioreactor Using Exact and I/O Linearization", *International Journal of Control*, vol. 60, , 1994, pp. 499-519.

[134] A.I. Propoi, "Use of Linear Programming Methods for Synthesizing Sampled-Data Automatic Systems", *Automation and Remote Control*, vol. 24, no. 7, 1963, pp. 837-844.

[135] J. Richalet, A. Rault, J. L. Testud and J. Papon, "Model Predictive Heuristic Control Applications to Industrial Processes", *Automatica*, vol. 14, 1978, pp. 414-428.

[136] W. Rauch, H. Aalderink, P. Krebs, W. Schilling and P. Vanrolleghem, "Requirements for Integrated Wastewater Models- Driven by Receiving Water Objectives", *Proc. 19th IAWQ Biennial Conference*, Vancouver, Canada, 1998.

[137] W. Rauch, and P. Harremoës, "The Importance of the Treatment Plant Performance During Rain to Acute Water Pollution", *Water Science and Technology*, vol. 34, no. 3-4, 1996, pp. 18.

[138] W. Rauch, and P. Harremoës, "Correlation of Combined Sewer Overflow Reduction due to Real-Time Control and Resulting Effect to the Oxygen Concentration in the River", *Proc. 7^{th} International Workshop on Instrumentation, Control and Automation of Water and Wastewater Treatment and Transport Systems*, Brighton, UK, 1997.

[139] R. Rouhani and R.K.Mehra, "Model Algorithmic Control (MAC): Basic Theoretical Properties", *Automatica*, vol. 18, 1982, pp. 401-414.

[140] W. Rugh, "Analytical Framework for Gain Scheduling", *IEEE Control Systems Magazine*, vol. 11, no. 1,1991, pp. 79-84.

[141] T.Rutowski, M. Grochowski, M.A. Brdys, J.Makinia and K.Duzinkiewicz, "Robust Recursive On-line Estimation of Variables and Parameters in Grey-Box Models of Biological Reactor in Activated Sludge Wastewater Treatment Plants", *Proc. of IFAC on Technology, Automation and Control of Wastewater and Drinking Water Systems*, Poland, 2002.

[142] T.Rutowski, M.A. Brdys, K. Konarczak and T. Gminski, "Set-Bounded Joined Parameter and State Estimation for Model Predictive Control of Integrated Wastewater Treatment Plant Systems at Medium Time Scale", *Proc. of 10th IFAC/IFORS/IMACS/IFIP Symposium on Large Scale Systems: Theory and Applications*, 2004, Japan.

[143] M.K. Sain and S. Yurkovich, "Controller Scheduling: A Possible Algeraic Viewpoint", *Proc. American Control Conference*, 1982, pp. 261-269.

[144] A. Sanchez, M.R.Katebi, and M.A.Johnson, "Optimal Control of an Alternating Aerobic-Anoxic Wastewater Treatment Plant", *Proc. 15th IFAC World Congress*, Barcelona, 2002.

[145] A. Sanchez and M.R.Katebi, "Predictive Control of Dissolved Oxygen in an Activated Sludge Wastewater Treatment Plant", *Proc. European Control Conference (ECC 2003)*, Cambridge, UK, 2003.

[146] A. Sanchez, M.R.Katebi and M.A.Johnson, "Subspace Identification Based PID Control Tuning", *Proc. 13th IFAC Symposium on System Identification*, Rotterdam, 2003.

[147] A. Sanchez, "Data-driven Control Design of Wastewater Treatment Systems", PhD Thesis, University of Strathclyde, 2004.

[148] J.M. Sanchez, *Adaptive Predictive Control System*, US Patent No. 4197576, 1976.

[149] L. Santos. and T. Biegler, "A tool to analyze robust stability, Journal of Process Control", vol. 9, no. 3, 1999, pp. 233-246.

[150] W. Schilling, B. Andersson, U. Nyberg, H. Aspegren, W. Rauch and P. Harremoës, "Real Time Control of Wastewater Systems", *Journal Hydraulic Research*, vol. 34, no. 6, 1996, pp. 785-795.

[151] W. Schilling, W. Bauwens, D. Borchardt, P. Krebs, W. Rauch and P. Vanrolleghem, "On the Relation Between Urban Wastewater Management Needs and Receiving Waters Objectives", *Proc. 27th IAHR Conference*, San Francisco. vol. 1, 1997, pp 510-515.

[152] M. Schütze, D. Butler and M.B. Beck, *Modelling, Simulation and Control of Urban Wastewater Systems*, Springer Verlag, London, UK, 2002.

[153] M. Schütze, D. Butler, M.B. Beck and H.R. Verworn, "Criteria for Assessment of the Operational Potential of the Urban Wastewater System, *Water Science Technology*, vol. 45, no. 3, 2002, pp. 141-148.

[154] M. Schütze, A. Campisano, H. Colas, W. Schilling and P. Vanrolleghem, "Realtime Control of Urban Wastewater Systems - Where Do We Stand Today?", *Proc. 9th International Conference on Urban Drainage*, Portland, 2002.

[155] P. Scokaert and J. B. Rawlings, "Stability of model predictive control under perturbations", *Proc. of IFAC Symposium on Nonlinear Control System Design*, NOLCOS '95, Tahoe City, CA, 1995.

[156] L.K. Sherman, "Streamflow from Rainfall by the Unitgraph Method", *Engineering News Record,* vol. 108, 1932, pp. 501.

[157] J.S. Shamma, "Analysis and Design of Gain-Scheduled Control Systems", PhD Thesis, Department of Mechanical Engineering, Massachusetts Institute of Technology, May, 1988.

[158] J.S. Shamma and M. Athans, "Analysis of Gain Scheduled Control for Nonlinear Plants", *IEEE Transactions Automatic Control,* vol. 35, 1990, pp. 898-907.

[159] J.S. Shamma and M. Athans, "Gain Scheduling: Potential Hazards and Possible Remedies", *IEEE Control Systems Magazine*, vol. 12, no. 3, 1992, pp. 101-107.

[160] *SIMBA 4.0, SIMBA-Sewer 4.0 Users Guide*, Institut für Automation und Kommunikation ifak Magdeburg and Barleben, Germany, 2001.

[161] D. Sincic and J.E. Bailey, "Optimal Periodic Control of Activated Sludge Processes. I. Results for the Base Case with Monod/Decay Kinetics", *Water Research*, vol. 12, 1978, pp. 47-53.

[162] I.Y. Smetsa, J.V. Haegebaerta, R. Carretteb and J.F. Van Impea, "Linearization of the activated sludge model ASM1 for fast and reliable predictions", *Water Research*, vol. 37, 2003, pp. 1831-1851.

[163] St. Sommer, "MBPC Methods Based on Nonlinear and Bilinear Parametric System Descriptions", *Advances in Model-Based Predictive Control,* Oxford University Press, 1994, pp. 69.

[164] O. Sotomayor and C. Garcia, "Model Based Predictive Control of a Pre-Denitrification Plant: a Linear State Space Model Approach", *Proc. 15th IFAC Triennial World Congress*, Vol. Q. Modelling and Control of Agricultural, Biological and Chemical Systems, Barcelona, 2002.

[165] O. Sotomayor, S.W. Park and C. Garcia, "Multivariable Identification of an Activated Sludge Process with Subspace-Based Algorithms", *Control Engineering Practice*, vol. 11, no. 8, 2003, pp. 961-969.

[166] *STOAT Manual*, WRc plc, 1994.

[167] M. Soroush and C. Kravaris, "Multivariable Nonlinear Control of a Continuous Polymerization Reactor", *Proc. American Control Conference (ACC'92)*, vol.1, San Francisco, USA, 1992, pp. 607.

[168] G. Sriniwas and Y. Arkun, "A Global Solution to the Nonlinear Model Predictive Control Algorithms using Polynomial ARX Models", *Computers and Chemical Engineering*, vol. 21, 1997, pp. 431-439.

[169] M.A.Steffens, P.A.Lant and R.B.Newell, "A Systematic Approach for Reducing Complex Biological Wastewater Treatment Models", *Water Research*, vol. 31, no. 3, 1997, pp. 590-606.

[170] H.T. Su and T.J. McAvoy, "Artificial Neural Networks for Nonlinear Process Identification and Control", *Nonlinear Process Control*, M.A. Henson and D.E. Seborg, eds., Prentice-Hall, NJ, 1997, pp. 371-428.

[171] J. Suescun, E. Ayesa, "Practical Identification of the Dissolved Oxygen Dynamic in Activated Sludge Plants", *Water Science Technology*, vol. 45, no.4-5, 2002, pp. 397-404.

[172] G. Szederkényi, N. R. Kristensen, K. M. Hangos and S. Bay Jørgensen, "Nonlinear Analysis and Control of a Continuous Fermentation Process", *Computers & Chemical Engineering*, vol. 26, no. 4-5, 2002, pp. 659-670.

[173] I. Takács, G.G. Patry and D. Nolasco, "A Dynamic Model of the Clarification-Thickening Process", *Water Research*, vol. 25, 1991, pp. 1263-1271.

[174] T. Takagi and M. Sugeno, "Fuzzy Identification of Systems and Its Application to Modelling and Control", *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 15, no.1, 1985, pp. 116-132.

[175] G. Tapia, D. Gabriel, J.A. Baeza and J. Lafuente, "Model Based Calibration for a Wastewater Treatment Pilot Plant", *Proc. European Modelling and Simulation Symposium* (EMSS2005), 2005.

[176] C. Tian and T. Fujii, "Nonlinear System Identification of Rapid Thermal Processing", *Control Engineering Practice,* vol. 13, 2005, pp. 681-687.

[177] D. Tolessa, "Dynamic Integrated Modelling of Basic Water Quality and Fate and Effect of Organic Contaminants in Rivers", Phd Thesis, Ghent University, 2004.

[178] J. Tyson, C.F. Guarino, H.J. Best and K. Tanaka, "Management and Institutional Aspects", *Water Science Technology,* vol. 27, no. 12, 1993, pp. 159-172.

[179] A. van Griensven, V. Vandenberghe, J. Bols, N. De Pauw, P. Goethals, J. Meirlaen, P.A. Vanrolleghem, L. Van Vooren and W. Bauwens, "Experience and Organisation of Automated Measuring Stations for River Water Quality Monitoring", *Proc. 1st World Congress of the International Water Association,* Paris, 2000.

[180] P. VanOverschee and B. De Moor, *Subspace Identification for Linear Systems: Theory - Implementation - Applications,* Kluwer Academic Publishers, 1996.

[181] P.A. Vanrolleghem, L. Benedetti and J. Meirlaen, "Modelling and Real-Time Control of the Integrated Urban Wastewater System", *Environmental Modelling and Software,* vol. 20, 2005, pp. 427-442.

[182] P. Vanrolleghem, C. Fronteau and W. Bauwens, "Evaluation of the Design and Operation of the Sewage Transport and Treatment System by an EQO/EQS Based Analysis of the Receiving Water Immission Characteristics", *Proc. WEF Conference Urban Water Wet Weather Pollution: Controlling Sewer Overflows and Stormwater Runoff,* Quebec, Canada, 1996.

[183] P. Vanrolleghem, "Model Based Control of Wastewater Treatment Plants", *Proc. ESF Workshop Integrated Environmental Bioprocess Design,* Obernai, France, 1995.

[184] P. Vanrolleghem, "On-line Modelling and Control of Activated Sludge Processes: Development of an Adaptive Sensors", PhD thesis, Faculty of Agricultural and Applied Biological Sciences, University Ghent, Belgium, 1994, pp 201.

[185] A. Wernli and G. Cook,"Sub-optimal Control for the Nonlinear Quadratic Regulator Problem", *Automatica*, vol. 11, 1975, pp. 75-84.

[186] D. Westwick and M. Verhaegen, "Identifying MIMO Wiener Systems using Subspec Model Identification Methods", *Signal Processing*, vol. 52, 1996, pp. 235-258.

[187] T. Wigren, "Recursive Prediction Error Identification Using the Nonlinear Wiener Model", *Automatica*, vol. 29, no. 4, 1993, pp. 1011-1025.

[188] R. Wright and C. Kravaris, "Nonlinear Control of pH Processes Using the Strong Acid Equivalent", *Industrial and Engineering Chemistry Research*, vol. 30, no. 7, 1991, pp. 1561-1572.

[189] S. Yeung, D. Sincic and J.E. Bailey, "Optimal Periodic Control of Activated Sludge Processes II. Comparison with Conventional Control for Structured Sludge Kinetics", *Water Research*, vol. 14, 1980, pp. 77-83.

[190] T.H. Yang and E. Polak, "Moving Horizon Control of Nonlinear Systems with Input Saturation, Disturbances and Plant Uncertainty", *International Journal of Control*, vol. 58, no. 4, 1993, pp. 875-903.

[191] A. Youssef, M.J. Grimble, A.W. Ordys, A. Dutka and D. Anderson, "Robust Nonlinear Predictive Flight Control", *European Control Conference*, Cambridge, UK, 2003

[192] Z. Yuan, J. Keller and P. Lant, *Optimization and Control of Nitrogen Removal in Activated Sludge Processes: a View of Recent Developments*, Technical report 2, IWA, Instrumentation, Control and Automatisation Specialist Group, 2001

[193] A.I. Zacharof , D. Butler, M. Schutze and M.B. Beck, "Screening for Real-time Control Potential of Urban Wastewater Systems", *Journal of Hydrology*, vol. 299, 2004, pp. 349-362.

[194] Y. Zhu, "Estimation of an NLN Hammerstein Wiener model", *Automatica*, vol. 38, 2002, pp. 1607-1614

[195] J.Schladweiler, "The History of Sanitary Sewers", Revision: 1 May 2006, Accessed: 19 May 2006, http://www.sewerhistory.org

[196] K. Higgins, D.Miner, C.N.Smith and D.B. Sullivan, "Earliest Clocks: A Walk through Time", revision: Version 1.2.1, Accessed: 19 May 2006, http://physics.nist.gov/GenInt/Time/early.html

Appendix A

UWS Constant Parameters

Symbol	Definition	Value (units)
v_{sedR}	BOD_R sedimentation rate	0.0416666 (m/h)
v_{sedS}	BOD_S sedimentation rate	0.0083333 (m/h)
k_{dR}	BOD_R decomposition rate	0.025 (1/h)
k_{dS}	BOD_S decomposition rate	0.00625 (1/h)
f_{dR}	Dissolved fraction of BOD_R	1 (-)
f_{dS}	Dissolved fraction of BOD_S	1 (-)
k_d	Temperature coefficient for BOD de-oxygenation	1.047 (-)
k_{DO}	BOD half-saturation constant	0.5 (mgO_2/l)
k_a	Reaeration rate	0.0403333 (1/h)
α	Weight factor for the photosynthetic activity	0.2
SOD	Sediment oxygen demand	0.104166666 g/(m^2h)
l	Length of reach	2000 (m)
b	Base width	21.1 (m)
s	Bank slope	1.5 (-)
h_m	Flow dependent parameter	1 (m)
Q_m	Maximum Flow before Flooding	239760 (m^3/h)
h_w	Minimum reach depth	1.58 (m)
DO_{sat}	Dissolved Oxygen Saturation	9.6055 (-)

Initial Values of State Variables of Single River Reach

Symbol	Definition	Value (units)
Q	Base flow	18000 (m3/h)
DO	Initial dissolved oxygen	9 (mg/l)
BOD_R	Initial BOD_R	0 (mg/l)
BOD_S	Initial BOD_S	0 (mg/l)

Appendix B

ASM2D Processes

Aerobic hydrolysis:

$p(1) = K_H \frac{So2}{Ko2+So2} \frac{Xs/Xh}{Kx+Xs/Xh} . X_H$

Anoxic hydrolysis:

$p(2) = K_H \mu_{NO3} \frac{Ko2}{Ko2+So2} \frac{Sno3}{Kno3+Sno3} \frac{Xs/Xh}{Kx+Xs/Xh} X_H$

Anaerobic hydrolysis:

$p(3) = K_H \mu_{fe} \frac{Ko2}{Ko2+So2} \frac{Kno3}{Kno3+Sno3} \frac{Xs/Xh}{Kx+Xs/Xh} X_H$

Growth on fermentable substrate, S_f:

$p(4) = \mu_H \frac{So2}{Ko2+So2} \frac{Sf}{Kf+Sf} \frac{Sf}{Sf+Sa} \frac{Snh4}{Knh4+Snh4} \frac{Spo4}{Kp+Spo4} \frac{Salk}{Kalk+Salk} X_H$

Growth on fermentation product, S_a:

$p(5) = \mu_H \frac{So2}{Ko2+So2} \frac{Sa}{Ka+Sa} \frac{Sa}{Sf+Sa} \frac{Snh4}{Knh4+Snh4} \frac{Spo4}{Kp+Spo4} \frac{Salk}{Kalk+Salk} X_H$

Denitrification with fermentable substrates, S_f:

$p(6) = \mu_H \eta_{no3} \frac{Ko2}{Ko2+So2} \frac{Sno3}{Kno3+Sno3} \frac{Sf}{Kf+Sf} \frac{Sf}{Sf+Sa} \frac{Snh4}{Knh4+Snh4} \frac{Spo4}{Kp+Spo4} \frac{Salk}{Kalk+Salk} X_H$

Denitrification with fermentation products, S_a:

$p(7) = \mu_H \eta_{no3} \frac{Ko2}{Ko2+So2} \frac{Sno3}{Kno3+Sno3} \frac{Sa}{Ka+Sa} \frac{Sa}{Sf+Sa} \frac{Snh4}{Knh4+Snh4} \frac{Spo4}{Kp+Spo4} \frac{Salk}{Kalk+Salk} X_H$

Fermentation:

$p(8) = q_{fe} \frac{Ko2}{Ko2+So2} \frac{Kno3}{Kno3+Sno3} \frac{Sf}{Kfe+Sf} \frac{Salk}{Kalk+Salk} X_H$

Lysis :

$p(9) = b_h X_h$

Storage of X_{pha} :

$p(10) = q_{pha} \frac{Sa}{Ka+Sa} \frac{Salk}{Kalk+Salk} \frac{Xpp/Xpao}{Kpp+Xpp/Xpao} X_{PAO}$

Aerobic storage of X_{pp} :

$$p(11) = q_{pp} \frac{So2}{Ko2+So2} \frac{Spo4}{Kps+Spo4} \frac{S_{ALK}}{K_{ALK}+S_{ALK}} \frac{Xpha/Xpao}{Kpp+Xpha/Xpao} \frac{K_{max}-Xpp/Xpao}{Kipp+K_{max}-Xpp/Xpao} X_{PAO}$$

Anoxic storage of X_{pp} :

$$p(12) = p(11) * \eta_{no3} \frac{Ko2}{So2} \frac{Sno3}{Kno3+Sno3}$$

Aerobic growth on X_{pha}:

$$p(13) = \mu_{PAO} \frac{So2}{Ko2+So2} \frac{Snh4}{Knh4+Snh4} \frac{Spo4}{Kp+Spo4} \frac{Salk}{Kalk+Salk} \frac{Xpha/Xpao}{Kpp+Xpha/Xpao} Xpao$$

Anoxic growth on X_{pha}:

$$p(14) = p(12) * \eta_{no3} \frac{Ko2}{So2} \frac{Sno3}{Kno3+Sno3}$$

Lysis of Xpao:

$$p(15) = b_{pao} X_{pao} \frac{S_{ALK}}{K_{ALK}+S_{ALK}}$$

Lysis of Xpp:

$$p(16) = b_{pp} X_{pp} \frac{S_{ALK}}{K_{ALK}+S_{ALK}}$$

Lysis of Xpha:

$$p(17) = b_{pha} X_{pha} \frac{S_{ALK}}{K_{ALK}+S_{ALK}}$$

Aerobic growth of autotrophic organisms:

$$p(18) = \mu_{AUT} \frac{So2}{Ko2+So2} \frac{Snh4}{Knh4+Snh4} \frac{Spo4}{Kp+Spo4} \frac{Salk}{Kalk+Salk} X_{AUT}$$

Lysis

$$p(19) = b_{AUT} X_{AUT}$$

Precipitation of Phosphorous

$$p(20) = k_{pre} S_{po4} X_{meoh}$$

Redissolution of Phosphorous

$$p(21) = k_{red} X_{mep} \frac{Salk}{Kalk+Salk}$$

Appendix C

A-mod Matrix

$$A_{\text{mod}}= \begin{bmatrix} 0 & 0 & -0.6 & -0.6 & 0 & 0 & 0 & -18 & 0 & 0 & 0 \\ 1 & 1 & -1.6 & 0 & -1.6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.6 & 0 & -1.6 & 0 & 0 & 0 & 0 & 0 \\ 0.01 & 0.01 & -0.022 & -0.07 & -0.022 & -0.07 & 0.031 & -4.24 & 0.031 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.21 & -0.21 & 0 & 4.17 & 0 & 0 & 0 \\ 0 & 0 & -0.04 & -0.02 & -0.004 & -0.02 & 0.01 & -0.02 & 0.01 & -1 & 1 \\ 0.001 & 0.001 & -0.001 & 0.021 & 0.014 & 0.036 & 0.002 & -0.6 & 0.002 & 0.048 & -0.048 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0.9 & 0 & 0.9 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ -0.75 & -0.75 & 0.9 & 0.9 & 0.9 & 0.9 & -0.15 & 0.9 & -0.15 & 1.42 & -1.42 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3.45 & 3.45 \end{bmatrix}$$

Appendix D

Abbreviations

AR: Auto Regressive

ARMA: Autoregressive Moving Average

ARMAX: Auto Regressive Moving Average Exogenous

ARX: Autoregressive with Exogenous Inputs

ASM: Activated Sludge Model

BOD: Biochemical Oxygen Demand

BSM: Benchmark Simulation Model

CSTR: Continuous Stirred Tank Reactor

COD: Chemical Oxygen Demand

CSO: Combined Sewer Overflow

CWA: Clean Water Act

DMC: Dynamic Matrix Control

DO: Dissolved Oxygen

EQ: Effluent Quality

EQO/EQS: Environmental Quality Objectives/Standards

EU: European Union

FGS: Fuzzy Gain Scheduling

GPC: Generalised Predictive Control

GS: Gain Scheduling

LTI: Linear Time Invariant

LTV: Linear Time Varying

MBPC: Model Based Predictive Control

MIMO: Multiple Input Multiple Output

MPC: Model Predictive Control

MPHC: Model Predictive Heuristic Control

NH4: Ammonium plus Ammonia

NLGPC: Nonlinear Generalised Predictive Control

NMPC: Nonlinear Model Predictive Control

PAO: Phosphorous Accumulating Organisms

PI: Proportional Integral

PID: Proportional Integral Derivative

PLC: Programmable Logic Controller

PRBS: Pseudo Random Binary Sequence

RTC: Real Time Control

SDC: State Dependent Coefficient

SD: State Dependent

SISO: Single Input Single Output

UES: Uniform Emission Standard

UWS: Urban Wastewater System

WFD: Water Framework Directive

WMPC: Wiener Model Predictive Control

WWTP: Wastewater Treatment Plant

Appendix E

List of Notation

a, amplitude

A_{mod}, constant gain matrix

A, B, C, D, State Space Matrices

B_d, D_d, Disturbance Matrices

b, ratio of waste flow to influent flow

C, Dissolved Oxygen

Cin, dissolved oxygen concentrations

Cs, Maximum dissolved oxygen concentration

D, dilution rate

D, deoxygenation term

d, Disturbance Vector

F, Free Response Matrix

F, ψ Input Constraints

f, Free response vector

Γ, g, Output Constraints

H, Forced Response Matrix

H_p, Prediction Horizon

H_u, Control Horizon

J, Cost Function

K_C, saturation constant

K_{la}, oxygen mass transfer function

Kp, Proportional Gain

K_S, affinity constant

L, Observer Gain

m, mean value

M_p, state dependent process matrix

P, photosynthesis term

P, p, Rate Constraints

Q, Error Weighting

Q, flow

R, reaeration term

r, ratio of recycled flow to influent flow

S, Substrate

S_{ALK}, Alkalinity.

SED, sediment oxygen demand term

S_I, Soluble inert organic matter.

Sin, influent substrate

S_{NH}, $N_{H4} + N_{H3}$ nitrogen.

S_{NO}, Nitrate and nitrite nitrogen.

S_O, Oxygen.

$S_{O,sat,}$ dissolved oxygen saturation

S_{ND}, Soluble biodegradable organic nitrogen.

S_S, Readily biodegradable substrate.

t, time

Td, derivative time constant

$Tdisc$, discrete time step

Ti, Integral time constant

Ts, Sample Time

U, Input Vector

ΔU, Control Increment Vector

W, aeration rate

X, State Vector

χ, Augmented State Vector

X, Biomass

X_{bodr}, River readily biodegradable concentration

X_{bods}, River slowly biodegradable concentration

$X_{BODR,\infty}$, steady state value for XBODR

$X_{BODS,\infty}$, steady state value for XBODR

$X_{B,A}$, Active autotrophic biomass

$X_{B,H}$, Active heterotrophic biomass.

X_{do}, River dissolved oxygen concentration

X_I , Particulate inert organic matter.

X_{ND}, Particulate biodegradable organic nitrogen.

X_P, Particulate products arising from biomass decay

X_r, Recycled Biomass

X_S, Slowly biodegradable substrate.

Y, Predicted Output Vector

Y, constant yield coefficient

ε, Tracking Error

λ, Control Increment Weighting

Γ_d, coefficient of the daylight measurement

p_j, process rate

r_i, reaction term

μ, biomass specific growth rate

μmax, maximum specific growth rate

v_{ij}, stoichiometric coefficients

in practice this gain function can be constructed of various forms. Cervantes et al. [32] demonstrated the use of a piecewise linear function.

The Kalman filter utilised to estimate the system states (in addition to the unmeasured disturbances) assumes noise matrices giving the system the form:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k \\ y_k &= C_k x_k + D_k u_k + z_k \end{aligned} \tag{5.12}$$

where w and z are the noise vectors. The state space process model is updated at each sample instant, thus the gain of the Kalman filter L must also be again determined. The state estimator is thus used as a 'soft sensor' in the calculation of disturbances upon the system. The Kalman filter in this case is used as a state and process disturbance estimator, with gain L_k updated at each sample instant

$$\begin{aligned} x_{e,k+1} &= Ax_{e,k} + Bu_k + L_k(y_k - C_k x_{e,k} - D_k u_k) \\ y_{e,k} &= C_k x_{e,k} + D_k u_k \end{aligned} \tag{5.13}$$

The system model can be seen to be an LTI system, whose output $v(k|k)$ is transformed to $y(k|k)$ by a nonlinear function, so that at any given sample instant the model can be seen to be simply the algebraic product an LTI model and a system gain. The order of the LTI system can be chosen to be of any (practical) dimension, and together with the equally definable nonlinear function, can give the process model an arbitrary level of accuracy. The restrictions upon choice of LTI model depends on the level of accuracy required, the stability of the model and the time constraints upon the control optimisation and state estimation. Gomez et al. [54] demonstrated the ease of identification of a stable Wiener model, in comparison with a linear approach, concluding that in stability the Wiener model obtains a better performance, remaining stable for a wider range of model orders. Together with the improved predictions from a Wiener model, this illustrates the advantages of the Wiener representation based nonlinear model predictive control (NMPC).