University of Strathclyde Glasgow

AN EVALUATION OF THE QUESTIONS IN THE MATHEMATICS TEXTBOOKS OF SAUDI ARABIAN SECONDARY SCHOOLS

by

Mohammed Rzgallah R. Alzahrani

A thesis submitted in fulfillment of the requirements

for the degree of

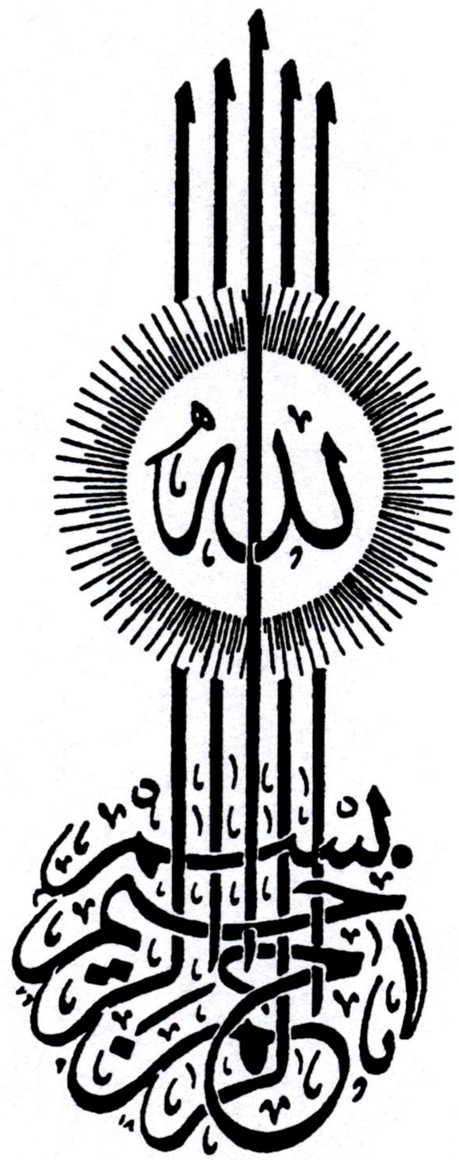
Doctor of Philosophy (Ph.D)

(Assessment and Evaluation)

Faculty of Humanities and Social Sciences, School of Education

University of Strathclyde, Glasgow

2014


بسم الله الرحمن الرحيم

Acknowledgements

In the Name of Allah, the Beneficent, the Merciful

First and foremost, I give my most profound thanks and gratitude to Allah, who granted me the opportunity, means and strength to complete my PhD. Neither this accomplishment nor any other would have been possible without His will and guidance.

I would also like to express my heartfelt thanks to my parents for their constant encouragement, support and prayers. May Allah bless them.

Next, I extend my thanks to Dr. Margaret Kirkwood, and Professor James McNally, my supervisors, for their invaluable advice and assistance. I would also like to thank Dr. Morag Findlay for her helpful comments and suggestions.

I am deeply indebted to my beloved wife, Rafeea, and my children, Asrar, Rzgallah, Fahad, Eythar, Hosam, and Rateel for their patience, support and prayers during my study.

Furthermore, I cannot forget to thank my brothers and sisters for encouraging me in my studies in the UK. I must also thank my cousin Hassan Swed, who always gave me great encouragement and advice and my good friend, Dr. Khaled Alosaimi, who has been very supportive during my stay in Glasgow. Thanks, too, to all my friends and colleagues, both in Saudi Arabia and in Scotland, for all your encouragement. In particular, I would like to thank Dr. Ghazi Almatrafy and all the others who assisted with the questionnaire distribution. Last but not least, I am so grateful to our lovely neighbours in Glasgow, Zahid Sarfraz and all his family, who made us so welcome and were so kind and generous with us.

My thanks also go to Prince Khaled Alfaisal, the Saudi Minister of Education, in the hope that this thesis may contribute to the achievement of his vision for the future of Saudi education. I am also very grateful to Professor Zayed Alharithi, the Dean of Education at Umm Al-Qura University for his great assistance. My heartfelt thanks go to Dr. Mohammed Basalamah, Dr. Ahmed Yehya Alghamdi and Dr. Ahmed Bargawi of Umm Al-Qura University for being so kind and helpful.

I would also like to thank the referees, teachers and inspectors who were involved in this study.

Abstract

This study aims to evaluate the questions in mathematics textbooks of the secondary stage, natural science section in Saudi Arabia to discover the extent to which they measure mathematical thinking skills, conform to criteria of good formulation and layout, and reinforce a positive attitude towards mathematics on the students' part, according to school teachers and inspectors. Based on a review of the literature, but moving beyond its outcomes, an analysis of what may be understood by mathematical thinking was developed. This underpinned the data analysis.

A questionnaire survey was administered to 1308 mathematics teachers and 158 inspectors from all regions of Saudi Arabia and interviews conducted with 14 teachers and 5 inspectors in order to determine their views on the questions in the selected mathematics textbooks and the extent to which they promote mathematical thinking skills. The questionnaire data were analysed quantitatively and the interview data were analysed qualitatively. In addition, content analysis of the textbooks was carried out.

In the opinion of the research participants, the questions in these mathematics textbooks did not promote mathematical thinking in the students, nor did they encourage students to have a positive attitude towards mathematics. The main aim of the textbooks appeared to be the practice of recalled skills, with little scope to apply any ideas in mathematics or to encourage thinking or questioning.

This study's weakness lies in the fact that the outcomes relied on an analysis of what people thought. It is difficult to describe mathematical thinking; responses therefore may reflect a range of perspectives on the concept. Moreover, no certain way of measuring mathematical thinking has yet been developed.

The study's strengths lie in that it goes beyond previous studies in terms of sample size, hence enhancing its reliability, and develops a taxonomy for mathematical thinking skills that can be developed through mathematics textbook questions. This taxonomy is considered to be an addition to the other taxonomies and measures addressing mathematical thinking skills.

LIST OF CONTENTS

CHAPTER 1: INTRODUCTION

CHAPTER 2: BACKGROUND TO SAUDI ARABIA

CHAPTER 3: LEARNING

CHAPTER 7: DRILL AND PRACTICE AND THE TEXTBOOK

Chapter 10: Content Analysis Data

Chapter 11: Analysing Survey Data

Chapter 12: Interview Data

List of Figures

LIST OF TABLES

CHAPTER 5: MATHEMATICAL THINKING

CHAPTER 6: LEARNING IN MATHEMATICS

CHAPTER 7: DRILL AND PRACTICE AND THE TEXTBOOK

Chapter 9: Methodology

Chapter 10: Content Analysis Data

Chapter 11: Analysing Survey Data

Chapter 12: Interview Data

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

Education relates to the passing on of knowledge, skills and attitudes from one generation to the next. In the past, such ideas were transmitted orally as young people absorbed the knowledge and skills related to living. With the advent of printing, the use of the printed text became important. At this stage, there arose the possibility of collating essential knowledge in book form and this became the school textbook. In the modern world, much communication now uses the visual and the role of the electronic transmission of information is growing steadily. Nonetheless, the place of the textbook in school and university education still appears to be secure and the focus of this study is on the textbook in the teaching and learning of mathematics.

The textbook is important in the educational process as a basic source of information for both the student and the teacher. It contains facts, information, concepts and ideas that a teacher attempts to explain to students. Thus, books in general, and textbooks in particular, are still a mainstay of education and progress in any society.

There is a need to consider the content to be covered in the textbooks. This needs planning, as there should be a balance between providing the students with information that can help them acquire new knowledge and develop understanding, while at the same time meeting the desired curricular goals (Wakeel, 1982).

However, the way that information is presented may be just as critical as content if the textbook is to be accessible for the young learners.

Specifically, mathematics textbooks are full of concepts, principles, facts, skills, algorithms, and mathematical ideas, all of which it is the teacher's role to encourage the students to learn. However, the textbook should be a tool so that the learner can develop skills related to mathematics (Reys et al., 2004). This is far more than simply presenting information but involves the development of understanding as well as building the confidence in being able to carry out mathematical procedures with competence.

One of the problems with textbooks is that, once printed, their content and presentation are fixed. They cannot be adapted readily to curricular changes or the introduction of new topics or themes. Laqani (1989) states that it is not possible to think of a static curriculum that does not respond to changing variables. We have to evaluate curricula to discover the extent to which they succeed in achieving their ultimate goals. Curriculum development is a continuous, never-ending process; it is not restricted to a specific time because it is always affected by its environment and always influenced by new research. Mathematics textbooks, therefore, it could be argued, if they are to be responsive to all the changes that take place in society, have to be checked on a regular basis to keep up-to-date with the latest developments in both mathematics and in the teaching and learning of mathematics, in order to make relevant connections to changes in society.

This chapter will continue with a discussion of the textbook in education in Arab countries, followed by a brief outline of the background to the study. Then, an overview of the research problem, and the contribution, aims and setting of the study will be presented. The statistical methods used in the study will then be mentioned, and will be discussed in greater detail later in the thesis. There will follow an explanation of the thesis methodology. Then, the research questions will be presented in detail.

1.2 The Textbook in Arab Countries

In Arab schools, the textbook represents the curriculum. It is used as the main or, in many schools, the sole educational resource. This tradition is deeply rooted. In fact, the textbook should be influenced by educational goals (Jaradat, 1986) and is one of the tools used to implement the curriculum, but not the only one. The ideal situation for implementing the curriculum requires one or more textbooks to present the elements of the syllabus to the students. However, for teachers and for school students in a country like Saudi Arabia, the textbook is prescribed and more or less defines what is to be taught, how it is to be taught and often how it is to be evaluated.

A good textbook and its questions should gradually advance with the student in an organised and coherent order, taking into consideration the students' skills, potential, and inclinations, through the use of differentiated material. The present study seeks to evaluate the questions in mathematics textbooks of the secondary stage, natural science section in the Kingdom of Saudi Arabia to discover the extent to which they measure mathematical thinking skills, conform to criteria of good formulation and good layout, and reinforce a positive attitude towards mathematics on the students' part.

It can be argued that the questions and exercises in mathematics textbooks are among the most important elements in that they allow learners to develop specific practical skills and patterns of thinking. Mathematics teachers rely on these questions to provide students with applications of taught ideas in the classroom and for homework. Thus, the questions in mathematics textbooks represent the curriculum. They may define the skills to be developed, the contexts of these skills and the ways in which mathematical problems are to be approached. For these reasons, a study of the nature of mathematics questions is a vitally important aspect of evaluating the curriculum experiences in mathematics education.

Textbook questions, in general, and mathematics textbook questions, in particular, should fulfil certain specifications. They should focus on measuring students' higher-order thinking, stimulating students' thoughts, and applying what they have learnt in real-life contexts. The questions ought also to be accurately and clearly formulated, appropriately presented, and diversified between essay and objective questions, taking individual differences into consideration in a way that reflects positively on students' attitudes towards mathematics. A lack of these standards may reduce the value of those questions for students, and limit their interest to a narrow circle of knowledge, ideas, and certain simple skills. However, it is widely recognised that this offers a picture of an ideal which is almost impossible to achieve. This is also confirmed by the researcher's own impressions gained through several years as a mathematics teacher in Saudi Arabia.

Often in school learning in Saudi Arabia, as well as in other countries in the world, particularly developing countries, the emphasis is on memorisation. In this context, Jabeer et al (1985) stressed the close link between the levels of thinking manifested in the students' answers and the types of questions they are asked; if the questions focus only on memorising facts, it is unlikely that the students will think creatively. Indeed, the questions in the textbook at the end of each lesson and chapter are a kind of summary of all the aspects of the lessons and they seek to complete all the objectives of that lesson or chapter. Therefore, looking at the nature of questions in mathematics textbooks will offer some insight into what types of thinking are given emphasis in the learning of mathematics.

Another of the aims of this study is to determine how far the textbook questions can measure the mathematical thinking skills in the mathematics textbooks of the secondary stage. The motivation for this is the fact that thinking is one of the educational issues which has been receiving considerable attention and interest in various countries of the world for some time (Marzano et al., 1988; Costa, 1985; Paul, 1993). There is an increased interest in what De Bono (1983) - one of the most prominent advocates of teaching thinking - called thinking tools or thinking skills. This is because teaching thinking is considered a way of equipping the individual with the tools needed for effectively dealing with any kind of information or variables that the future may bring (Jerwan, 1999).

Thus, the aim of the educational process is no longer limited to providing students with knowledge and facts; it is extended to developing their aptitudes for appropriate thinking (Wilson, 1993), particularly in the light of the knowledge expansion the world is currently experiencing. The importance of teaching thinking skills of all kinds is apparent. As Sternberg (cited by Jerwan, 1999) comments, knowledge is vitally important, but often it becomes outdated, whereas thinking skills remain as new as ever, and they enable us to acquire knowledge regardless of the time, the place, and the type of knowledge that thinking skills are utilised to deal with. Knowledge about when and how to use particular strategies for learning or for problem solving is known as metacognition (Flavell, 1979).

Therefore, curricula and their educational materials, it may be argued, should be oriented towards going beyond the mere provision of knowledge and skills, and pointing to some of their applications, and to empowering students to exercise thinking skills and patterns (Al-Sheikh, 2001).

This current emphasis on developing thinking skills is thought by many to be one of the main drivers of the trend towards educational development in any evolving educational system designed to effect change for the better. The Kingdom of Saudi Arabia has begun to follow this trend, particularly in view of the fact that education in the country is a national priority in the country's development projects and policies for the future. The educational process in Saudi Arabia aims to equip the individual with both traditional and modern characteristics and values, to refine learners' thinking tools, and to develop learners' capabilities for analysis, criticism, initiative and meaningful discourse. (Ministry of Education, 1986)

Accordingly, and driven by the importance of the analysis and evaluation of curricula, the quality of textbooks, and their questions, is seen as indispensable for students' development and improvement in such a way as to become more compatible with emerging societal, educational, and cognitive needs in a world of rapid change (Abu Zeenah, 1994; Posner, 1995). The focus of this study is on mathematics textbooks and their questions. The study will examine the contents of textbooks for the mathematical thinking patterns they promote at the secondary stage; the extent to which the questions meet the criteria of appropriate formulation and layout; and the extent to which they reinforce students' positive attitudes. The findings of the study should thus also offer a number of suggestions for developing and improving the mathematics textbooks and questions in such a way that they become more thinking-based and supportive of positive attitudes towards mathematics, i.e. to develop mathematical thinking at the same time as providing mathematical knowledge.

1.3 Background to the study

Educators frequently seek to embody educational objectives in the form of a textbook or curriculum statement that will serve as an aid in the realisation of these objectives, through supporting the teacher in transferring academic knowledge to the student. As Voogt and Odenthal (1997) point out, teachers use the textbook in the planning and implementation of lessons, which to a great extent contributes to the achievement of those objectives. Hence, the textbook, together with the questions it contains, has an effect on society and can be considered as a cornerstone of progress and advancement for any society (Shaheen, 1991). Therefore, attention should be given to evaluating the textbook, together with its questions, as it is a main reference for students, and, as has been shown by several studies, can also be a source of students' misunderstanding of some scientific concepts.

One such study was carried out on the physics textbooks taught to high school students in the USA, and revealed that numerous errors in concepts and forms were present in those textbooks (Danver, 1990). For all these reasons and others, the strong points of the textbooks and their questions should be identified in order to maintain and reinforce them, while the weak points must also be identified in order to amend them in the light of the research results.

The importance of evaluation increases when it deals with an important aspect of the syllabus, i.e. the formative evaluation questions within the mathematics lessons. This type of question aims to stimulate students' thinking and motivate them to learn and review what they have studied, as well as reinforcing the important aspects of the skills and information that need to be mastered. Although all mathematics textbooks contain questions related to the various lessons, no research, within the scope of what was reviewed by the researcher, has previously been conducted on them to identify the most salient positive and negative aspects of their contents. Hence the idea came of evaluating the questions of the mathematics textbooks of the natural sciences section of the secondary stage in the Kingdom of Saudi Arabia, for the purpose of identifying the extent to which these questions meet the conditions of good formulation and layout, and how far they reinforce positive attitudes in the students. This idea was further supported by the fact that the researcher was teaching mathematics with the Ministry of Education, and Assessment and Evaluation at the Faculty of Education, Umm Al-Qura University, Makkah.

Discussions were held between the researcher and his professor and teaching colleagues, as well as his graduate level students, regarding their evaluation of the questions in mathematics textbooks in general, and those in the mathematics textbooks for the natural sciences section of the secondary stage in particular, and the extent to which these questions fulfilled the conditions of appropriate formulation and layout. In the light of these discussions, opinions varied but were based only on personal views and general impressions. Thus, there is a need to carry out a more systematic and rigorous analysis of questions in mathematics textbooks.

The mathematics curriculum has general and specific objectives. It is important to explore the way the questions are formulated and designed in the light of these objectives and to determine whether the questions are advancing the learners in their mathematical skills and in their manner of thinking. This has to be set in the context of preparing the students to cope with the social, economic, scientific, and technological changes in a society of rapid and constant change.

1.4 Problem of the Study

The Kingdom of Saudi Arabia is one of the countries undergoing a phase of renewal in the development of its educational system, the core of which is the development of the syllabi around which the processes of learning and teaching are centred. The syllabi of the secondary stage are considered to be the main pathway to the university stage as, at the end of the secondary stage, which in Saudi Arabia is for students aged 16 to 18, the students are prepared to join universities. There has been a recent emphasis on abilities such as scientific and critical thinking.

It is clear that curricula need continual updating but they may also need to move away from an emphasis on the memorisation and recall of information to focus on a wider range of skills. In terms of mathematics, this can be attained through focusing on the development of students' mathematical thinking skills and providing them with a way of thinking based on a sound and accurate mathematical structure. This springs from the view of mathematics as a technique and pattern of thinking that has the advantages that render it fertile ground for training students in the patterns and methods of sound thinking and developing it, thus contributing to building their characters and creative potential, and providing them with mathematical insight and deep understanding.

The textbook and its questions are the building blocks of the learning and teaching process. For teachers, the textbook constitutes the main and very likely the only resource that they refer to when planning their lessons. For students, it is the main tool for their learning. Thus emerges the need to analyse and evaluate textbooks and their questions to identify the extent to which they cope with the desired directions in educational development.

Hence, there has emerged a growing interest in the necessity for developing students' thinking skills with their various patterns in the different grades, and the call for the necessity of developing the curricula and textbooks and directing them towards enhancing and reinforcing these thinking skills. Therefore, one of the aims of this study is to identify the extent to which mathematical thinking is represented in the questions of the mathematics textbooks of the secondary stage in the Kingdom of Saudi Arabia.

1.5 Contribution of the study

The textbook with its contents of activities and questions is an important learning tool and a major source of knowledge in an age where science appears to know no limits; it is an age characterised by the expansion and rapid spread of knowledge.

This study derives its importance from the significance attached to textbooks. This significance is particularly notable in mathematics textbooks, as mathematics is considered to be the common language of all sciences and uses specific, clearly-defined symbols and expressions, facilitating intellectual communication among people. Mathematics has always enjoyed a privileged position among the other sciences. It is regarded as a core component of the progress of civilisation and a tool for rapid technological development, as it underpins so many other disciplines. The main goal of teaching is to contribute to preparing an individual for life regardless of his/her work or future aspirations and for pursuing further studies in mathematics itself or in any other subject, whether at school or in further education (Khidr, 1988). Mathematics is often viewed as a method and pattern of thinking, and an organised way for logical proof (Abu Zeenah, 2003). However, as Thurston (1994) points out, this is not to say that there is a uniform, objective and established theory and practice of mathematical proof or that mathematics consists only of attempting to prove theorems. Goals related to instilling or improving mathematical thinking methods and problem-solving skills are considered by many to be among the most prominent aims of school mathematics (Khidr, 1988). Consequently, the development of international curricula has particularly affected school mathematics in consistency with the modern trends in education that aim to shift the focus away from just providing the student with information, facts, and skills and move towards concentrating on the way the learner can gain that information and those facts and acquire such skills (Bruner, 1963). Accordingly, the focus of this study is on evaluating mathematics textbooks and their questions, and detecting the mathematical thinking skills in them. The study also concentrates on identifying the extent to which these questions fulfil the conditions of good formulation and layout, and the extent to which they reinforce students' positive attitudes.

Moreover, the importance of this study springs from the fact that it conveys the points of view of those concerned with textbooks and their questions, namely, educational inspectors and teachers. To the best of the researcher's knowledge, this study is the first of its kind to concern itself with evaluating the questions in the secondary stage mathematics textbooks in Saudi Arabia, the extent to which they measure the contents of mathematical thinking skills, and the extent to which they fulfil the conditions of good formulation and layout, as well the extent to which they reinforce students' positive attitudes towards mathematics.

This study acquires greater importance in that it comes at a time when the Saudi Ministry of Education is working on preparing, developing, and writing new syllabi and their questions. The results of this study may shed light on important issues that need to be considered when writing the new syllabi after reviewing the strengths and weaknesses of various aspects of the textbook in order to take them into account.

The importance of the study further stems from the importance of questions as a significant tool of evaluation. Questions represent the feedback about the educational system, particularly what the students have learnt through studying the syllabus items. This is due to the fact that they are the tools most widely-used to evaluate students' achievement and measure their progress.

Moreover, the significance of this study emerges from its permitting educators to learn about the procedures used in the design, preparation, formulation, and layout of new textbook questions.

The study can also help those responsible for setting the textbook questions to develop them through identifying the mathematical thinking skills prevalent in the textbooks, as well as through discovering the extent to which they fulfil the criteria of good formulation and layout.

The study is also significant in that it is in conformity with the recommendations of Karam (2000), who indicated the necessity of directing the curricula, the textbooks and their questions towards developing thinking skills in all their various patterns and reinforcing the students' positive attitudes towards mathematics. This study is also consistent with the trend that calls for the need to teach thinking skills through their integration into the educational content of the curricula in order to achieve the thinking curriculum.

This study presents a taxonomy (a scale) for the mathematical thinking skills that can be developed through mathematics textbook questions. This taxonomy or scale is considered to be an addition to the other taxonomies and measures that address mathematical thinking skills. It is hoped that this scale will help research and studies related to mathematical thinking grow and flourish, and will open the door for researchers to address issues relevant to mathematical thinking from various other angles.

This study, together with its findings and recommendations, could help direct the attention of the experts on and authors of mathematics curricula and textbooks towards developing textbooks that have a greater capability of improving mathematical thinking skills. It could also direct the attention of mathematics teachers towards developing mathematical thinking skills through teaching and learning mathematics and through asking the kind of question that aim at giving students mathematical thinking skills.

Finally, this study can benefit all those concerned with the textbook, such as the Ministry of Education, educational inspectors, teachers, students, researchers, and all who are specifically interested in aspects of evaluating syllabi and their questions, and in the broader concept of curricula generally.

1.6 Aims of the Study

This study aims to evaluate the questions of the secondary stage mathematics textbooks in the Kingdom of Saudi Arabia from the points of view of the educational inspectors and teachers who teach these curricula. The aims of the study are to attempt to determine educational inspectors' and teachers' assessments of the conditions of good formulation and layout of these questions, and how much they reinforce the students' positive attitudes towards mathematics. It also seeks to discover the effect of a number of variables on the educational inspectors' and teachers' assessments. These variables are the following: position, academic qualifications, years of experience, training, and classes taught.

Furthermore, it aims to reveal the mathematical thinking skills prevalent in the questions of the secondary stage textbooks for detecting the effectiveness of these questions in achieving the educational development related to improving students' thinking skills.

1.7 Study Setting and Limitations

This study seeks to evaluate the questions of secondary stage mathematics textbooks, identify the extent to which these questions measure mathematical thinking skills, and discover the extent to which they reinforce students' positive attitudes. Therefore, the study context includes the following.

1.7.1 Study setting

The study setting is all regions of the Kingdom of Saudi Arabia, as the secondary stage textbooks whose questions are the subject of the evaluation are unified across the country, and are issued by one body, the General Administration for Curricula, in the Ministry of Education.

1.7.2 Study Limitations

The scope of this study is limited to evaluating the mathematics textbooks of the secondary stage, natural sciences section during the academic year of 1430/1431 Hijri (corresponding to 2009/2010). The study was applied in the second term of the same year.

In terms of methodology, the study was limited to using the analytical descriptive method. The nature and aims of the study also necessitated the researcher using the following tools to answer the questions it raises:

1. A questionnaire designed by the researcher for identifying educational inspectors’ and mathematics teachers’ assessment of the quality of mathematics textbook questions for the natural sciences section of the secondary stage, the extent to which these questions fulfil the conditions of

good formulation and layout, the extent to which they reinforce the required thinking skills, and how well they develop students' positive attitudes towards mathematics.

2. The mathematical thinking in the questions of the mathematics textbooks was measured by an analysis tool developed by the researcher and approved by a group of independent arbitrators.
3. An interview instrument designed by the researcher aimed at discovering the opinions of teachers and educational inspectors regarding the secondary stage mathematics curriculum, its questions, and the extent to which it encourages the development of students' mathematical thinking skills.

1.8 Statistical methods

For the analysis the researcher used frequency, percentages, and certain central tendency measurements, chi-square, Kendall's Tau-b and so forth, as well as the statistical software, SPSS.

The research design, methods and methodology used in the research will be discussed in detail in Chapter 9.

1.9 Study Terminology

For the purpose of this study, the following terms will be used according to their corresponding procedural definitions, as follows:

- In the first year of secondary school, students study all subjects, but in second year they must choose to specialise and join a particular section; e.g., the scientific section; the Islamic and Arabic studies; and administration so forth. The students concerned in this research belong to the scientific section.

- **Secondary Stage**: the educational stage following the elementary and middle stages, where the ages of students range between 16 and 18 years old. This stage lasts for three academic years.
- **Good question formulation**: questions written in clear, unambiguous, unequivocal language using correct and specific wording, and free from spelling errors.
- **Good question layout**: the last stage of preparing the question pages that includes typing, printing, and organising the questions asked to the students in a legible way that makes it easy for the students to answer.
- **Mathematical thinking**: the effective thinking acquired by the student cumulatively during the study of mathematics. Mathematics textbooks are considered to be a means of developing mathematical thinking if the activities and examples used in presenting the content as well as the evaluation questions at the end of each lesson or unit entail the following mathematical thinking skills:

1. **Knowledge and Recall**, which is the acquisition of mathematical knowledge represented in the concepts, symbols and algorithms by means of reading the printed text and recalling the mathematical knowledge stored in the memory. An item is classified as representing this skill if it contains mathematical knowledge, or if it prompts the student to recall knowledge from memory.
2. **Understanding and Interpretation**, which is the processing of mathematical knowledge in order to clarify its meaning and extract it. An item is classified as representing this skill if it contains a reformulation of

the given information by means of words, symbols, explanation of the relationships, clarification of the solution methods, giving examples, or if the item uses one or more of the skills of mathematical explanation, comparison, classification, condensation, expansion or justification.

3. **Modelling (Pattern Cognition)**, which is the mathematical representation of knowledge or of given data in a manner which facilitates their understanding and the perception of their relationships, in order to reach conclusions about them easily. An item is classified as representing this skill if it contains data tables, graphs, geometrical figures, illustrations, charts, or equations and so forth. The item can also be classified as representing the modelling skill if it prompts the student to represent the knowledge or the given data using any of the aforementioned mathematical models.
4. **Application**, which is the use of the learnt mathematical knowledge in new situations. An item is classified as representing this skill if it contains an analysis of the new situation in order to gain an insight into the relationship between it and previous situations, as well as to observe previously learned facts, principles and algorithms related to it, and distinguish what is not related to it, and then use this previous knowledge to formulate correct solutions for the new situations.
5. **Induction**, which means obtaining a certain result from some observations or some special examples (Abu Zeenah, 1986). An item is classified as representing this skill if it contains a number of observations that contribute

to obtaining a certain result and then records it in the form of numbers or symbols.

6. **Generalisation**, which is a general spoken formulation or written statement (Abu Zeenah, 1986). Davydov (1990) defined generalisation as a process of proceeding form the particular to the general. An item was classified as representing this skill if it contained a number of special cases, examples or observations by means of which a general spoken or written statement that is applicable to this group of special cases can be formulated.
7. **Deduction**, which means obtaining a general result based on a general or assumed principle (Abu Zeenah, 1986). Johnson-Laird (1999, p.110) defined deduction as a process that "yields valid conclusions, which must be true given that the premises are true". An item is classified as representing this skill if it contains an application of the general principles or rules to the case or to special cases of those to which the principle or rule can be applied
8. **Mathematical Proof**, which is the provision of proof or evidence for the validity of a certain statement based on a previous or accepted theory (Abu Zeenah, 1986). An item is classified as representing this skill if it contains a series of statements that aim to show the validity of a certain result by means of reasoning and logic. (Wilson,1993; Fletcher and Patty,1988)
9. **Evaluation**, which is judging the value of the mathematical knowledge or given data regarding some particular purpose (Jerwan, 1999). The evaluation includes the use of standards in order to judge or make a decision and provide proofs regarding the validity or accuracy of the

claims, and reveal the fallacies in the logical inferences and the information related to the situation.

1.10 Research Questions

The first main research question is the following

What mathematical thinking skills are emphasised in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia?

This question is further divided into sub-questions, as follows.

a) What are the mathematical thinking skills prevailing in the explanatory items of the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia?

b) What are the mathematical thinking skills prevailing in the question items of the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia?

The second main research question is the following.

What is the extent of the emphasis placed on the development of mathematical thinking in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia?

This question is further divided into sub-questions, as follows.

a) What is the extent of the attention given by the explanatory items in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia, to the development of mathematical thinking, based on the level of mathematical thinking (basic and complex) that they represent?

b) What is the extent of the attention given by the question items in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia to the development of mathematical thinking, based on the level of mathematical thinking (basic and complex) that they represent?

c) What is the extent of the attention given by the combined items in the mathematics textbooks for the scientific section at the secondary school stage in Saudi Arabia to the development of mathematical thinking, based on the level of mathematical thinking they represent?

The third main research question is the following:

What are the mathematical thinking skills prevailing in the questions in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia in the opinion of the teachers and inspectors of mathematics?

This question is further divided into sub-questions, as follows.

What are the mathematical thinking skills prevailing in the questions in the secondary stage mathematics textbooks in Saudi Arabia in the opinion of the teachers and inspectors of mathematics according to a) different grades (1st year – aged 16, 2nd year – aged 17, 3rd year – aged 18); b) post (teacher and inspector); c) qualifications (Bachelor's, diploma, Master's, PhD); d) length of experience, and; e) whether they had training in education, teaching and assessment methods and/or mathematical thinking skills?

The fourth main research question is the following:

To what extent are the criteria of good formulation and good layout of the questions in the secondary stage mathematics textbooks in Saudi Arabia fulfilled in the opinion of the teachers and inspectors of mathematics?

This question is further divided into sub-questions, as follows.

Are there any significant differences in the criteria of good formulation and good layout of the questions in the secondary stage mathematics textbooks in Saudi Arabia according to a) different grades (1st year, 2nd year, 3rd year); b) post (teacher and inspector); c) qualifications (Bachelor's, diploma, Master's, PhD); d) length of experience, and; e) whether they had training or not?

The fifth main research question is the following:

Do the questions in the mathematics textbooks in Saudi Arabia develop students' positive attitudes towards mathematics in the opinion of the teachers and inspectors of mathematics?

This question was further divided into sub-questions, as follows:

Do the opinions of the teachers and inspectors of mathematics on the ability of the questions in the mathematics textbooks in Saudi Arabia to develop students' positive attitudes towards mathematics vary according to a) different grades (1st year, 2nd year, 3rd year; b) post (teachers and inspectors); c) qualification (Bachelor's, Diploma, Master's, PhD); d) length of experience, and; e) whether they had training or not?

The sixth research question is the following:

What are the views of the teachers and inspectors concerning the textbooks, the textbook questions and mathematical thinking?

The next chapter presents some background to Saudi Arabia, so that the research context of the study may be better understood.

CHAPTER 2

BACKGROUND TO SAUDI ARABIA

2.1. Introduction

In order to provide a better and more comprehensive understanding of the main topic of this study, it is useful to present some general background information on Saudi Arabia, the context of the study. This chapter will begin with short sections on the geography and demography of Saudi Arabia, and will continue by giving a brief history of the country. The chapter ends with an overview of the history of education in Saudi Arabia and the current education system in the country.

2.2 Geography

The Kingdom of Saudi Arabia (KSA), known as Saudi Arabia, is located in the Arabian Peninsula, where three continents, Asia, Europe and Africa, meet, giving it a strategic position (Shoult, 2006). It has an area of 2,149,690 square kilometres (830,000 square miles) and is the second largest Arab country and the largest country in the Middle East. It is bordered by with Iraq, Jordan, Qatar, Kuwait, Oman, Yemen and the United Arab Emirates (Shoult, 2006) and a causeway connects it to Bahrain (OPEC, 2011). To the northeast is the Arabian Gulf, with the Red Sea to the west. Moreover, the Suez Canal lies close to its north-west border. (Ministry of Culture and Information, 2006).

Saudi Arabia is divided into several administrative districts and has over six thousand villages, towns and cities. The country is sometimes known as “The Land of the Two Holy Mosques" because of Makkah and Medina, the two holiest places in Islam. Every year many pilgrims come from all over the world to these holy places for either the Omrah or Hajj pilgrimages.

Arabic is the official language of Saudi Arabia and it uses the Arabic calendar (hegira) (Shoult, 2006). Islam is the official religion of Saudi Arabia, and it pervades all aspects of life there. Eid Al-Fitr, at the end of Ramadan, and Eid Al-Adha are the two official public holidays.

2.3 Demography

The population of Saudi Arabia is approximately 26.13 million, of whom around 5.57 million are non-Saudis (CIA World Factbook, 2011). These non-nationals come from many countries to work in Saudi Arabia. There are many Filipinos, Bangladeshis, Pakistanis, and Egyptians, as well as Europeans and North Americans working in the country (ibid). In Saudi Arabia, almost 30% of the population is aged under 15 (ibid).

Riyadh is the capital of Saudi Arabia and also its biggest city, and, at the time of the last census, had a population of 4.725 million. Other major cities in the country include Jeddah, with 3.234 million people; Makkah, with 1.484 million; Medina with 1.104 million; and Dammam, with 902,000 (CIA World Factbook, 2011).

2.4 A brief history of the country

The country is named after the royal house of Al-Saud, which dates back to Mohammed bin Saud, who was the first ruler of this royal house (Vassiliev, 2000). The establishment of the First Saudi State took place in 1744 when Sheikh

Mohammed bin Abd al Wahhab established himself in Diriyah and gained the support of Prince Mohammed bin Saud. Together with its allies, the house of Saud became dominant in Arabia, controlling most of the Najad, although not the coasts (Cordesman, 2003). They agreed to adopt Islamic legislation and call the country the Kingdom of Saudi Arabia (KSA) after the house name (Al-Rumaihi, 1997; Al-Turaaiqi, 2008). According to Hamden (2005), this Saudi state lasted for about seventy-five years until, disturbed by the increasing power of the Saudis, the Sultan of the Turkish Ottoman Empire ordered Mohammed Ali Pasha to take the area back. It was Ali's son, Ibrahim Pasha, who accomplished the task, defeating the Saudi forces in 1817.

However, a few years later, the Sauds returned to power and established the Second Saudi State in 1824. This came to an end 1891, when the country was conquered by the Al Rashid dynasty of Ha'il. Bin Saud re-took Riyadh in 1902 and, after further victories, the modern nation state of Saudi Arabia was created by King Abdulaziz in 1932.

Saudi Arabia is an absolute monarchy with no experience of democracy. Laws are based on the Quran and the Sunnah, and are issued by Royal Decree. The main legislative authorities are the Majlis al-Shura (Council of Consultation) and the Council of Ministers, of which the King is the head and the Crown Prince and Prime Minister as members. The purpose of the Council of Ministers is to help the King carry out his duties. The members of the Majlis al-Shura, based in Riyadh, are chosen by the King. These two bodies have the authority to take initiatives or endorse public policy (Basheikh, 2002). Saudi Arabia has never been colonised by any Western country. Saudi Arabia has the largest oil reserves in the world and is the world's largest exporter of oil. More than 90% of exports and 75% of government income come from oil (Cordesman, 2003).

2.5 The beginnings of education in Saudi Arabia

In the late 18th century, the Wahhabi movement promoted Islamic education for all Moslems (Metz, 1992). The purpose of Islamic education was to ensure that Muslims would know God's laws and comply with them. Classes for reading and memorising the Quran and the hadith (the sayings of Mohammed, peace be upon him) were established in towns and villages throughout the Arabian Peninsula (ibid). Teaching was done in the *khuttab,* a class of Quran recitation for children usually associated with a mosque, or, particularly in the case of girls, in private homes where instruction was given by an expert reader of the Quran (Metz, 1992). It was only in the late nineteenth century under Ottoman rule in the Hijaz and Ottoman provinces that secular subjects also began to be taught in the khuttab schools, including sometimes arithmetic, foreign languages, and Arabic reading (Metz, 1992). Students who wanted to study after the elementary level could go to informal lectures (*halaqat*) where offering instruction in Islamic jurisprudence, Arabic language and literature, Quranic commentaries *(tafsir*), hadith, rhetoric, and sometimes history and arithmetic were taught. (ibid). However, as the fundamental purpose of education was to learn the Quran, literacy was not prioritised, and illiteracy was almost universal in the peninsula. (ibid).

It was not until 1924 that formal, organised education began in the country, when King Abdulaziz became aware of the importance of education for the development of the country and set up the Directorate of Education (Al-Sadan, 2000). It was given the responsibility of the establishment of primary and secondary schools and, as well-qualified teachers were not to be found within the country, to bring in teachers from other Arab countries, Egypt in particular, to develop and execute the teaching programmes in these schools (Al-Zarah, 2008). The Directorate's task also included the registration and regulation of the few private schools which were in existence at that time in the country. In 1947, there were only 65 schools in the Kingdom, with a total of approximately 10,000 students, all of them male (ibid). The Saudi government began to pursue a new policy of educational development, and established the Ministry of Education to take the place of the Directorate General of Education (Al-Salloom and Al-Makky, 1994). With the establishment of this Ministry, more schools were opened and education became more widespread in the Kingdom (Al-Zarah, 2008).

2.6 The education system in Saudi Arabia

Education in Saudi Arabia is segregated throughout the entire education system, from elementary school to university. There are nine years of free state school education - six years in elementary school and three in intermediate school, during which time there is little difference in the curriculum for girls and boys (Badgaish, 2008). After this, students move to secondary school for three years.

In the first year of the secondary school, students all follow the same courses. For the final two years, students have a very restricted choice, having to select from three fields of study: science; Islamic and Arabic studies, or; administration (only for boys). After finishing three years in secondary school, students can move into higher education by entering university, college or an institute and choose the major that they wish.

The school years in Saudi Arabia consist of two semesters and the same subjects are studied in both semesters. Each semester lasts around 18-20 weeks, including examination time. Mathematics is compulsory in elementary and intermediate and first year of secondary school for both girls and boys. However, in the second and third years of secondary, students who opt for Islamic and Arabic studies do not have to take mathematics while those who choose natural science or administration have to study mathematics more intensively than in the previous years. There are slight differences in the mathematics curricula for boys and girls, with more emphasis on certain topics for one gender or the other. These differences may arise due to the fact that boys' and girls' schools are administered by different education departments and each can organise the educational plans that they view as being most suitable for each gender. However, both of those departments are under the supervision of the Ministry of Education. At university, there are no differences between the curricula for male and female students who specialise in mathematics, although males and females are taught separately.

In this light, in the following chapters, relevant literature will be reviewed.

CHAPTER 3

LEARNING

3.1. Introduction

As stated at the end of the last chapter, some significant literature on learning will be reviewed as a background to the main focus of the study, the questions in the given textbooks. Indeed, textbooks, and the questions contained therein, remain virtually the sole resources for learning mathematics in Saudi Arabian schools. In addition, a review of the learning literature must be carried out in order to set the framework for the review of the literature on thinking presented in Chapter 4.

3.2. The Nature of Learning

The majority of psychologists and educators view learning as a process in which experience and training modify or alter behaviour (Ali, 2008). Hamachek (1995) states that learning refers not only to an observable result, but also to elements which cannot be seen, such as attitudes, emotions and intellectual processes.

Similarly, Reid (1978) describes learning as a process leading to any change in behaviour which could not be accounted for by natural biological and cognitive development alone. Issues surrounding the conscious effort required in learning and the storage of information in the brain have been matters of discussion in the educational research domain for some time (Ali, 2008).

3.3 Learning Theories

For many years now, educational psychologists and others have developed a number of theories as to how learning occurs (Alhmali, 2007). These theories, dating from the nineteenth century up to the present day, offer a framework which can assist the educational researcher. Some of the most significant theories of thinking and research have had an influence that can be said to have completely changed how we think about the science of learning, and transformed how future educators are trained.

Various theories have been formulated as researchers have investigated different aspects of learning. For instance, some have concentrated on the acquisition of skills such as reading and writing (e.g., LaBerge and Samuels, 1974; Anderson, 1981; NRC, 2001). There are also those who have studied the emergence of new ideas by interacting with other people and contact with the world around us (e.g., Carey, 2002; Karmiloff-Smith and Inhelder, 1974; Vygotsky, 1978).

Learning theorists have also explored different settings for learning; among them, pre-school, school, laboratory, informal meeting places and normal home and workplace settings, and a number of measurements of learning (e.g., neurobiological and behavioural) have also been used. Furthermore, learning theorists work on time scales that range from milliseconds of processing time to lifelong and even inter-generational learning (e.g., Lemke, 2001; Newell et al, 2001). Making sense of these different perspectives, and giving each due consideration, is a challenging task. Although these issues are important, they are out with the scope of this research, which seeks rather to link theories of learning to the focus of the study, the questions in the Saudi Arabian mathematics textbooks. Some of these theories are discussed in the following sections.

3.3.1 Piaget

Swiss-born Piaget was best-known as a developmental psychologist, but he was also an educator and philosopher and the author of over sixty books and several hundred articles (Ali, 2008). Piaget (1952) found that there were crucial aspects of intellectual development in learning which take place with age. He suggested that up to the age of sixteen, development and progression in learning is somewhat fixed; hence, adults learn in a different manner than young children. Indeed, he proposed that human intellectual development passes through 4 basic stages, which are the following: the sensory motor stage, the pre-operational stage, the concrete operations stage, and the formal operations stage. In addition, he observed that individuals are seeking to make sense of the world around them through learning and that in this process they build their own unique models of reality (Alhmali, 2007). According to this theory, education is not concerned with the mere transmission of information from the teacher's mind to that of the student, but requires students to construct their own understanding. For this reason, Piaget's (1952) concept has become known as constructivism, and a number of subsequent researchers have expanded the concept.

According to Piaget (1952), children have to act on objects; it is thus that they acquire knowledge of these objects and this is the only way in which knowledge of the world can be discovered and constructed. Wadsworth (1984) noted that Piaget’s work was not directly concerned with predicting behaviours nor was he directly concerned with how to teach children. His work dealt mainly with systematic description and explanation of how intellectual structures and knowledge grow and develop.

However, Piaget has been criticised, notably by Ausubel (1968), for not taking children's previous experiences into consideration. Moreover, Ausubel et al (1978) disagreed with the idea that development is abrupt or occurs in jumps. They assert that it is more gradual and takes place smoothly. They also explained that intellectual functioning involves more variation at any of these stages than the concept of a stage would suggest. Ausubel's argument appears logical, although the order in which Piaget placed the developmental stages seems indisputable.

Ausubel et al (1978) also pointed out that the sample size used by Piaget in his experiments was not sufficiently large, and there was a lack of normative data on age levels. While this might affect the generalisability of his data, it does not affect the significance of what he described. It should also be noted that he never tried to claim statistical significance, but merely gave a description of what he had observed empirically, did not interpret his findings sufficiently and was inclined to simplify situations to focus on describing cognitive development in general terms (Alenezi, 2008). However, in this way, he opened the door for further researchers to extend his work (Miller, 1993; Donaldson, 1978).

There has been considerable criticism of constructivism, on several grounds; for example, it has been pointed out that learners may construct erroneous understandings (Alhmali, 2007). Kirschner et al (2006) observed that while the constructivist description of learning may be exact, the learning results proposed by constructivists may not take place. Biggs (1996) pointed out that there are varying views of constructivism, each of which differs in its implications for educational practice. There is disagreement among constructivists on the nature of knowledge and the importance of social interaction (Alenezi, 2008). However, the majority of constructivists are in broad agreement on four features that influence learning, which are that learners build their own understanding; new learning relies on present understanding; learning is facilitated by social interaction, and; meaningful learning takes placed within authentic learning tasks (Bruning et al, 2003). In this regard, Boyle (1996) criticises radical constructivists on the grounds that they over-emphasise an individual's cognition rather than his or her physiological development, based upon which cognition must be founded.

Millar (1989) and Jenkins (2000) argue that the constructivist learning theory requires a particular model of instruction or demands a progressive pedagogy. However, constructivism describes how all learning takes place: the learners seek to make sense of the world around and construct their own understanding. Learners naturally attempt to make sense of what they are taught. If they cannot understand, they may well resort to memorisation in order to pass examinations, although this is a reflection of difficulties in understanding and constructivism has nothing to say on this.

In fact, it should be noted that a great deal of the criticism levelled at constructivism is less critical of the constructivist theory of learning than of the conclusions that may be drawn from the constructivist epistemology (Ben-Ari, 2001). Matthews (1994, p. 151) states that "The one-step argument from the psychological premise (1) 'the mind is active in knowledge acquisition,' to the epistemological conclusion (2) 'we cannot know reality,' is endemic in constructivist writing." Extending this, Ben-Ari (2001) makes the point that "carried to the extreme, radical constructivism leads to solipsism, the philosophical claim that the world is one's own mental creation." He further states that this "may lead to a rejection of ethics: if the world is my own creation, why should I care what happens to others?"

None the less, many authors have suggested instructional strategies based on constructivist ideas, such as "greater emphasis on discourse relating to students' concepts; discussion in the classroom; exchange of ideas; demonstration or experience with conflict situations, and; increasing the active involvement of students" (Alenezi, 2008). Some (e.g., Garnett and Hackling, 1995) have proposed the use of modern audiovisual technologies and computer graphics to overcome difficulties with abstract, unobservable concepts such as those encountered in mathematics. However, none of those strategies and techniques is exclusive to constructivism (Alenezi, 2008). As Jenkins (2000) pointed out, "Selecting a strategy that is more, rather than less, likely to interest students and promote their learning is central to a teacher's professional competence". According to Ben-Ari (2001), the crucial question is whether constructivism needs an epistemological commitment to empiricism and idealism rather than to rationalism and realism. However, Matthews (1997, p.8) suggests that the answer to this question may be in the negative, citing the position of those "who concentrate solely on pedagogy, and improved classroom practices, for [whom] the details of epistemological psychology are unimportant, and not worth disputing."

3.3.2 Ausubel

Ausubel (1968) suggested that knowledge of an individual already exerts a powerful controlling influence on future learning. He proposed that the most important thing for teachers to establish before beginning to teach is what each student knows already. He also made a clear distinction between meaningful-rote learning and discovery-reception (Figure 3.1).

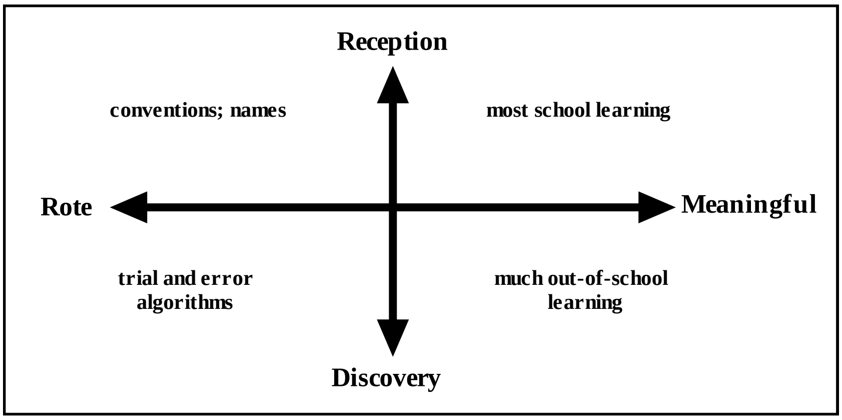


Figure 3.1 Learning Dimensions (from Ausubel et al, 1968)

Mintzes, Wandersee and Novak (1998) expanded these ideas and this shows where certain areas of school learning fit into the Ausubel model (Figure 3.2).

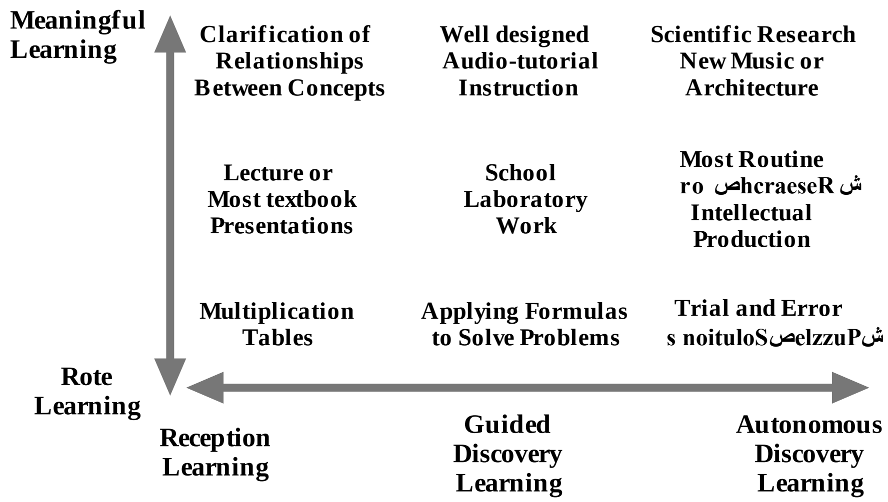


Figure 3.2 Further Dimensions of Learning (adapted from Mintzes, Wandersee and Novak, 1998)

Ausubel (1968) focused on both the presentational methods of teaching and the acquisition of knowledge. He made a major contribution to learning by studying and describing the conditions that result in 'meaningful learning'. He attempted to find 'the laws of meaningful classroom learning'.

However, Orton (2004) argued that, if it is attempted to coerce children to learn and accommodate new mathematical ideas that cannot link to knowledge which is already in an existing knowledge structure, then the ideas can only be memorised. In contrast, with meaningful learning, memorisation leads to in uninformed assimilation of new knowledge into cognitive structure. It occurs when the student cannot access any concepts in his or her cognitive structure. Rote learning also occurs when the limited working memory capacity is overwhelmed and understanding proves impossible (Jung and Reid, 2009).

Meaningful learning processes exist when the new concept can be linked to the pre-existing concept in the learners' cognitive structure (for example, already existing relevant aspects of knowledge of an image, an already meaningful symbol, a known concept or a proposition). Thus, meaning derives directly from associations that exist among ideas, events, or objects. As the new knowledge is subsumed into the existing knowledge, it interacts with and modifies it.

3.3.3 Gagné

Robert Gagné (1916-2002) was a psychologist in the USA who became involved in the development of human learning and behaviour during the Second World War in the context of training servicemen in the U.S. Air Force (Ali, 2008). His experiences with the Air Force formed the basis for his instructional theory (Ali, 2008).

Gagné (1964, p.3) described learning as “a change in human disposition or capability that persists over time that is not simply assigned to the process of growth.” By "process of growth" he meant normal biological development (Ali, 2008).

Gagné claimed that learning leads to a change in the learner’s cognitive structure, which is in continual progress. He also stated that skills should be learned one by one and that every new skill is based on skills previously learned. There are three main elements in his theory. The first is the taxonomy or classification of learning outcomes that he developed. The second is his introduction of the internal and external conditions he claimed were necessary to attain these learning outcomes. Thirdly, there are the nine events of instruction that he argued would assist in developing a unit of instruction. The nine events are "(a) gaining attention; (b) informing the learner of the lesson objective(s); (c) stimulating recall of prior learning; (d) presenting the stimulus material; (e) providing learning guidance; (f) eliciting performance; (g) providing feedback; (h) assessing performance, and; (i) enhancing retention and learning transfer" (Gagné, 1988, p.11) .

Overall, planned learning helps every learner to approach the goal, maximising the use of his/her natural abilities and experiences and assimilation with his/her physical and social environment.

Gagné (1974) discussed the fundamental supposition regarding instructional design and maintained that teaching plans must be individualised and must have the cognitive needs of the learner as its basis.

"Instructional design has phases that are both immediate (what a teacher does in preparing a lesson plan some hours before the instruction is given) and long-range (concerned with a set of topics to constitute a course sequence or perhaps with an entire instructional system, undertaken by a team of teachers, school committees, organisation of curriculum planners, text books writers and by groups of scholars representing the academic disciplines.)" (Gagné, 1974).

Instructional design plays an important part in individual development. Fundamentally, no-one should be at a disadvantage educationally and all should have an equal opportunity to use their individual talents to their full potential (Gagné, 1974).

3.4 Information Processing

Information Processing is a cognitive model that studies the way in which information enters the mind by way of the five senses and how it is stored and retrieved (Alenezi, 2008). It has been the dominant theory of learning and memory since the 1970s and it is concerned with learning processes rather than with the characteristics of the learner. The information processing approach holds that young children have a limited capacity for processing information, as opposed to simply memorising it and the average adult has a greater capacity (Sutherland, 1992).

3.4.1 Cognitive Models of Memory

According to cognitive models of memory, the brain bears a certain resemblance to a computer, as it has functions not unlike those of a computer. For example, its sensory-motor systems could be compared to a computer's input and output devices, and it also has various types of storage (Baddeley, 1990).

According to Ashcraft (1994), the modal model of human memory comprises three kinds of information storage, which are sensory memory, short-term (or working) memory and long-term memory. (Figure 3.3).

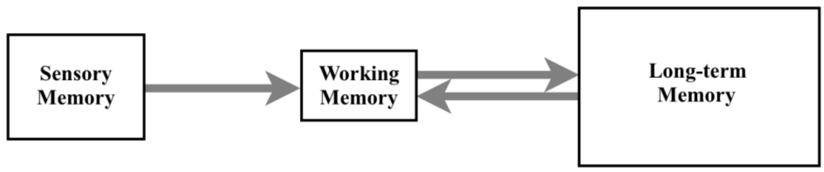


Figure 3.3   Modal Model of Memory (adapted from Ashcraft, 1994).

Sensory memory is the information store that holds stimuli from the environment for a short time until they can be processed and it consists of the sensory register (Goodenough, 1976). The sensory register is flooded with large quantities of information from the senses (sight, hearing, taste, touch and smell) and holds it only briefly. There are two kinds of sensory memory, auditory sensory memory and visual sensory memory. The former is a part of the sensory memory that is responsible for receiving auditory information from the external environment (Ashcraft, 1994). The latter refers to the part of the memory which holds visual sensations for a very short time (ibid).

The capacity of sensory memory is almost unbounded but if processing does not start almost right away, the memory trace rapidly decreases (Woolfolk, 2007). The approximate time that information can be held after the stimuli disappear varies from one second for visual information to up to four seconds for auditory information (Driscoll, 2005; Leahey & Harris, 1997; Pashler and Carrier, 1996). In these seconds, there is the opportunity to choose and organise information for further processing.

The process by which information is chosen is referred to as perception. The sensory memory is also referred to as the perception filter. The perception filter is controlled by information that is kept in long-term memory, according to the findings of Ausubel et al (1968). Previous experiences, preferences, knowledge and prejudices control the perception filter and people respond and pay attention to certain stimuli (Johnstone, 1993). The possibilities for perceiving and processing will decrease if some stimuli are given attention while others are not (Woolfolk, 2007). Woolfolk (2007) argues that "what we pay attention to is guided to a certain extent by what we already know and what we need to know, so attention is involved in and influenced by all three memory processes" (p. 252). Paying attention is considered to be the first step in learning and, if they do not pay attention, students will not be able to process information if they have no recognition or perception of it (Lachter et al, 2004). Sensory memory holds information long enough for it to be transferred to the next store, which is the working memory.

3.4.2 Working Memory

Working memory is the store where new information is held for a relatively short period and linked with knowledge from the long-term memory. In his pioneering work, Miller (1956) attempted to find a method of measuring the capacity of what was called 'short-term memory' (now known as working memory). Working memory is easily disrupted because of its limitations; it can hold only about seven plus or minus two (7 ± 2) 'chunks' of information at a time (Miller, 1956) and this information is held for a short period. The concept of chunks is hence used to refer to the limited amount of knowledge that can be dealt with by the human mind at any given time (Robillard, 1999). A chunk is a unit of information whose significance differs from individual to individual (Robillard, 1999). A chunk may be a single number or a single letter, or many pieces of information grouped together (Miller, 1956). Robillard (1999) points out that chunks are general and not related to the information content of the knowledge.

Sweller et al (1998, p.252) note that "humans are probably only able to deal with two or three items of information simultaneously when required to process rather than merely hold information". Working memory has two functions:

- Holding information temporarily (like holding a telephone number just before dialling);
- Processing information in some way (for example, in preparation for storage in long-term memory, or trying to understand information).

Information processes such as choice, comparison and organisation also take up space in the working memory and hence, the number of items that can be dealt with is fewer than the seven that can be held in working memory (Eggen and Kauchak, 2007). Baddeley and Hitch (1974) found that working memory consists of three subcomponents. The first is the central executive system (CE), which is used for starting and controlling processes, making decisions, and retrieving information from long-term memory. The second is the phonological loop (PL), which is a subsidiary system for holding and handling sound and speech, while the third, the visuo-spatial working memory (VSWM), is used for holding and manipulating non-verbal material.

Working memory encodes information from sensory memory and long-term memory (Ashcraft, 1994). Thus, when a stimulus (sight, sound, smell, touch, or taste) is encountered, if attention is paid to this stimulus, the information is transferred to the working memory. Information may be stored in the long-term memory if it is processed in the working memory.

Miller (1956) showed that almost all adults have capacities lying between five and nine. The capacity of working memory grows with age until about the age of 16. This corresponds to the age when Piaget found that formal operational thought was fully available. At age 14, the average capacity is nearer 6, and, at 12, it is nearer 5. Working memory capacity cannot be expanded. However, it can be used more efficiently. One way to do this is by what Miller (1956) called '*chunking*'. In this process, Miller (1956) showed how 'chunking' can be used to extend the restricted capacity of the working memory through the use of established memory stores to classify or encode new information. He described working memory capacity as comprising seven 'slots', with each slot able to contain one piece of information. If seven individual letters each filled a slot there would be no room for more. However, if the letters are ‘chunked’ into a word, the word would consist of a single piece of information, which would leave six slots free for more words.

3.4.3 Long-term memory

Long-term memory is where the brain stores permanent information. Eggan and Kauchak (2007) stated it has billions of entries and a network that permits these to be accessed. The long-term memory receives the information from working memory and stores it on a comparatively permanent basis for retrieval. Johnstone (1997) stated that “We store information which is potentially important, or interesting, or useful. We ignore or discard information which is more trivial or unimportant. This is a personal process and for that purpose memory uses a variety of functions such as: pattern recognition, rehearsal, elaborating, and organizing. We seek for patterns as we try to connect the new information with existing information in order to make sense. We discard the new information when it does not make sense to us.”

It has been suggested that there are three types of long-term memory storage, i.e. episodic memory, semantic memory, and procedural memory (Squire et al., 1993; Tulving, 1993).

Working memory and long-term memory differ in several ways in terms of both capacity and duration. Working memory holds the information that is recently experienced and activated. Long-term memory holds information that is understood, information simply memorised, attitudes, feelings – indeed, everything that can be stored from past experience. Whereas working memory is limited to about seven plus or minus two 'chunks', long-term memory appears to have infinite capacity. In addition, working memory holds information for a matter of seconds, but when information is safely kept in the long-term memory, it can stay there permanently.

Retention in the long-term memory can be supported by several factors. The most important of these is how well learners have learned initially (Bahrick and Hall, 1991). Higher-ability students often achieve more at the end of a course, but they often forget the same quantity of what had to be learned as students of lower ability (Slavin, 2006). According to Slavin (2006), teaching strategies that involve learners in the classes may also make a contribution to the retention of the long-term memory.

In the early 1990s, science educators (e.g. Johnstone, 1991) attempted to take into account the psychological models of learning and the cognitive structure of the learners. These approaches look at that part of the learner's brain where information is held, organised, shaped, and worked upon before it is stored and retrieved. The model he used is shown in Figure 3.4.

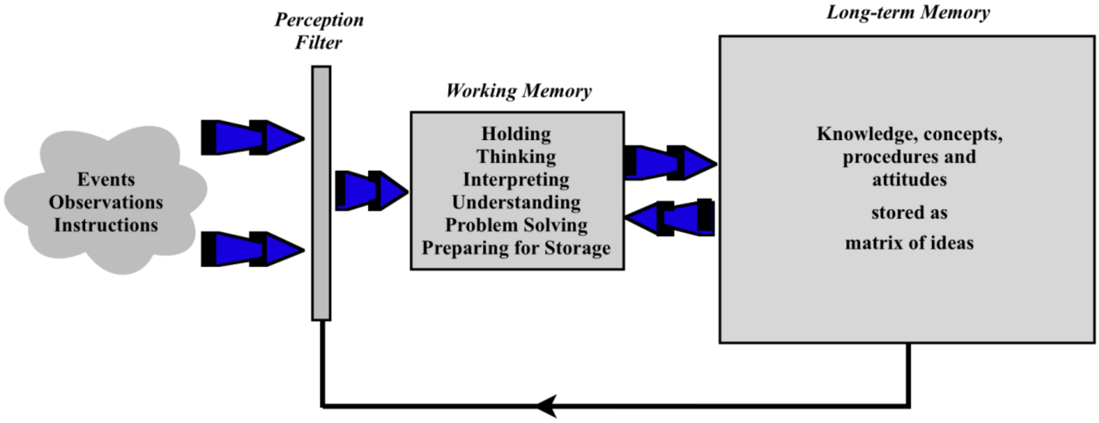


Figure 3.4 Information Processing Model (derived from Johnstone, 1997)

These models of information processing have also been studied by many researchers (e.g. Al-Naeme, 1988; Jung, 2005; Oraif, 2006). The common theme of the models is the concept of how input information is stored and processed inside the human memory and how a response to this comes into existence (Onwumere, 2009). The importance of working memory in learning has been emphasised by Kirschner et al. (2006), demonstrated by Gathercole and Alloway (2008) in their book for primary teachers, and was the subject of a complete journal issue (Research in Science and Technological Education, 2009).

3.5 Conclusion

This chapter has described the most prominent theories of learning which have been applied to education. While it should be pointed out that these learning theories are only working interpretations open to later rejection or modification, they are each supported by a considerable body of evidence. The next chapter will move from theories of learning to theories of thinking; it will discuss several theories on the nature and characteristics of thinking and various types of thinking skills.

CHAPTER 4

THINKING

4.1 Introduction

This chapter reviews some of the literature on various aspects of thinking. First, thinking skills and then various types of thinking are discussed, followed by a discussion of the various approaches to teaching thinking skills.

Recently, educational systems have placed among their top priorities the development and enhancement of students' thinking skills. There is general agreement that the development of thinking should be given a more prominent place in education at all levels, while less emphasis should be placed on memorisation and recall. In this regard, the main research question 3 in the present study seeks to determine the extent of the emphasis placed on the development of mathematical thinking in the textbooks in question. However, there is a very wide range of opinions as to what constitutes the essential characteristics of such thinking and there is little consensus about how such skills might be enhanced or, indeed, measured.

There are several classifications and taxonomies that encompass thinking skills, the most common of which are those based on Bloom's taxonomy of learning objectives in the cognitive field. These comprise knowledge and recall; understanding and comprehension; application; analysis; synthesis, and; evaluation.

Jacobsen et al (2002) classified thinking skills as comprising three levels, as follows:

1. Basic cognitive processes that include observation, deduction, generalisation, hypothesis testing, induction, and inference.
2. Higher order cognitive processes, which include problem solving, judging, critical thinking, and creative thinking.
3. Meta-cognitive processes, which include thinking about thinking.

According to Marzano et al. (1988), thinking skills could be classified as the following:

- Focusing skills: identifying problems and goals
- Information-gathering skills: observation and the formulation of questions
- Recall skills: recalling and encoding information
- Organising skills: comparison, classification, ordering, and representation
- Analysis skills: main ideas, features, components, relations, patterns and errors
- Generation skills: reasoning, prediction and clarification
- Integration skills: summarising and reconstruction
- Evaluation skills: the development of standards and verification of results

Marzano et al. (1988) also stressed the importance of mastering the following thinking skills in order to perform the following thinking processes: concept formation; principle formational comprehension; problem-solving; decision-making; research; composition, and; oral discourse.

It should be mentioned here that despite the fact that there are various definitions of thinking, and numerous taxonomies and classifications that discuss the skills and processes of thinking, in general, these all emphasise the importance of moving from a type of teaching and learning that includes a culture of memorisation and instruction, to one which embodies a culture of creativity and thinking.

4.2 Types of Thinking

There are several types of thinking, such as mathematical thinking, scientific thinking, creative thinking and critical thinking, all of which have several applications. Some of these types of thinking are discussed below.

4.2.1 Critical Thinking

Simpson and Courtney (2002) state that critical thinking cannot be learned like a method, but is a process of the mind, and therefore consists of the cognitive and affective domains. There has been great debate about defining critical thinking, with many overlapping terms and contradictory meanings. This lack of clarity led to the Delphi Project on critical thinking (Facione, 1990a). The Delphi Project offered some guidance as to what constitutes critical thinking and what does not. The concept of critical thinking has been expressed in several ways. From a philosophical standpoint, Dewey (1916) suggests that critical thinking involves "suspension of judgement and healthy scepticism". Other writers such as Ennis (1962) proposed that learners should be encouraged to engage in reflective and reasonable thinking and guided about belief or action. Ennis (1962) described critical thinking as “the correct assessing of statements” (p.83) and states that, according to this definition, a person who can think critically has the skills required for the evaluation of statements.

More recently, among critical thinking theories have been those of Watson and Glaser (1980); McPeck (1981); Paul (1982; 1983; 1985); Siegel (1991); Brookfield (1987); Kurfiss (1988); Facione (1990a and 1990b), and Boostrum (1994). Watson and Glaser (1980) saw critical thinking as going beyond a particular set of cognitive attitudes. In their view, critical thinking consists of understanding how to make inferences and generalisations, as well as an ability to reflect on proof of accuracy and rationality. Watson and Glaser (1980) also expressed the idea that being able to think critically is an important part of functioning well in contemporary society. They saw critical thinking as a prerequisite to being able to participate actively in public and private life. They also argued that attitude plays a significant role, as attitude affects the ability of an individual to query the complexities or assumptions of life in any given situations or circumstances (Watson and Glaser, 1980). This is linked to the fifth research question, which concerns whether the questions in the mathematics textbooks in Saudi Arabia develop students' positive attitudes towards mathematics in the opinion of teachers and inspectors of mathematics. The significance of students' attitudes in relation to their learning of mathematics is examined in greater depth in Chapter 8.

McPeck (1981) viewed critical thinking as involving an inclination and a skill and, therefore, learning to think critically involves both cognition and affect. McPeck's (1981) view is based on two parts of critical thinking, the first of which he describes as the 'context of discovery' and the second, the 'context of justification'. Similarly to this second aspect of McPeck's argument, Kurfiss (1988) holds that critical thinking is connected to the justification of beliefs and suggests that argumentation is the method for justification. In this respect, Bell (1991) suggests that critical thinking skills can be developed through debate.

According to Brookfield (1987), critical thinking involves more than cognitive skills, and feelings are an essential part of the critical thinking process. Anyone attempting to think critically or to teach others to do so, will soon become aware of the importance of feelings in this activity. Brookfield (1987) also suggests that critical thinkers are usually those who are productive and active and view themselves as creative. Critical thinkers also see their thinking as a procedure rather than a result. According to Brookfield (1987), critical thinkers are constantly questioning assumptions, as critical thinking is not fixed and seldom reaches a position of conclusiveness or determination.

In the literature, the number of definitions of critical thinking has been an obstacle. This motivated the American Philosophical Association in 1987 to ask Peter Facione, a philosopher, to head a systematic enquiry into critical thinking (Simpson and Courtney, 2002). Facione brought together a panel of experts from different academic disciplines from all over North America to form the Delphi Project. The panel formed a significant consensus regarding the concept of critical thinking (Simpson and Courtney, 2002) and produced a report called the Delphi Report. The definition of critical thinking in this is as follows.

"*We understand critical thinking to be purposeful, self-regulatory judgement which results in interpretation, analysis, evaluation and inference as well as explanation of the evidential, conceptual, methodological, criteriological or contextual considerations upon which that judgement was based. Critical thinking is essential as a tool of inquiry. Critical thinking is a pervasive and self-rectifying human phenomenon. The ideal critical thinker is habitually inquisitive, well-informed, honest in facing personal biases, prudent in making judgements, willing to consider, clear about issues, orderly in complex matters, diligent in seeking relevant information, reasonable in selection of criteria, focused in inquiry and persistent in* *seeking results which are as precise as the subject and the circumstances of inquiry permit*" (Facione, 1990a, p.4).

Before the Delphi Report, critical thinking had not been clearly defined, but the ideas of Ennis (1962), McPeck (1981), and Paul (1990) influenced the final consensus.

Critical thinking is associated with features such as knowledge; active argumentation; reasoning; initiative; intuition; application; analysis of complex meanings; identification of problems; thinking of alternatives, and; making contingency-related value judgements (Simpson and Courtney, 2000). According to Simpson and Courtney (2002), "*critical thinking is considerably greater than the sum of its parts, because it is a process that promotes attitudes to continuously explore, redefine or understand. All these factors contribute to a process of focused rational interaction between an individual and their environment or surrounding circumstances.*" Bittner and Tobin (1998) compare the process of critical thinking to "an umbrella under which many types of thinking flow, depending on the situation" (p. 269).

There have been numerous attempts to measure critical thinking from the philosophical and psychological areas. Among those derived from the philosophical tradition are the Watson-Glaser Critical Thinking Appraisal, the Cornell Critical Thinking Test, and the New Jersey Test of Reasoning Skills. The Triarchic Test of Intellectual Skills (Sternberg, 1986) comes from psychology, but does not attempt to disconnect critical thinking and intelligence as it interprets intelligence as a form of information processing (Li and Jin, 1995).

A number of studies have sought to investigate the development of critical thinking skills in the classroom. For instance, the aim of Al-Khuzam's (1998) study was to investigate the effect of the exploration, discussion and lecturing methods in the development of critical thinking in the case of 10^{th} grade mathematics students.

The study sample was made up of 90 students divided into three teaching groups, chosen randomly from a school in Al-Mafraq city in Jordan in the school year 1997/1998. The sample was divided into three groups: one taught by means of lecturing, one by means of discussion and the third by means of exploration.

The study instrument was a test of critical thinking that included five areas: recognition of assumptions, evaluation of arguments, interpretation, derivation and deduction.

The study findings indicated the following:

- The development of critical thinking was better in the case of the students that were taught by means of exploration compared to those in either of the other two groups.
- The discussion method was found to be better than the lecturing method in terms of the development of critical thinking.
- The lecturing method had no effect on the development of critical thinking.

The aim of Elliot's (2000) study was to investigate the effect of a university algebra course on the development of critical thinking. The study sample consisted of university students who joined an algebra course. The students were divided randomly into two equal groups: one studying a newly-developed course (the experimental group), and the other, the control group, studying the traditional course.

The study instrument was a Watson-Glaser test that measured prior and subsequent critical thinking skills. The test included the following areas: inference, hypothesis testing, deduction, interpretation, and evaluation of arguments.

The study findings indicated the absence of any statistically significant differences between the two groups in the critical thinking test. However, there was considerable development in the students' critical thinking skills in the case of the experimental group in four of the five areas, subsequent to the course. In addition, the study findings did not indicate any statistically significant differences that could have been attributed to gender.

The aim of Teixeira's (2002) study was to compare the development of critical thinking skills and achievement in the case of two groups of students who were studying a mathematics course on quantitative reasoning, one through lectures and the other through workshops.

The study sample was made up of 150 male and female Bachelor's degree students who were studying this course. Of this sample, 83 students were taught through lectures, while the other 67 were taught through workshops. The Watson-Glaser critical thinking test was administered to the students before and after the course, and they also sat for a final achievement test at the end of the term.

The findings of the study indicated no statistically significant differences in the critical thinking test or achievement test that could be attributed to the teaching method. The findings also indicated no statistically significant differences in the critical thinking skills that could be attributed to gender, achievement or study major. However, there were statistically significant differences that could be attributed to the academic year.

Annabi (1991) studied the critical thinking aspects that appear in the classroom teaching of mathematics teachers in the secondary school stage, how widespread these are, and whether they differ depending on the different components of classroom teaching or depending on the gender of the teacher and the grade.

The study sample consisted of 38 male and female mathematics teachers of the first, second and third grades in public secondary schools in greater Amman. Classes were allotted to each male and female teacher, and then the classes were analysed in order to answer the research questions.

The study findings indicated that the mathematics teachers in general did not pay attention to the development of critical thinking while teaching mathematics. Moreover, the use of critical thinking aspects differs depending on the different elements of teaching (concepts, generalisation, skills, problem solving). The findings also indicate that the teaching of critical thinking does not vary according to the gender of the teacher or the grade he or she is teaching.

The aim of Hamadnah's (1995) study was to determine the level of critical thinking in mathematics in the case of 10^{th} grade students in Jordan and the relationship between the ability of critical thinking and gender and achievement in mathematics.

The study sample was made up of 1100 male and female students randomly chosen from the public schools of Irbid Province. The study instrument took the form of a critical thinking test devised by the researcher based on the Watson-Glaser critical thinking scale, adapted so that all its items were from the mathematical content of the 10th grade curriculum. The test included the following dimensions: inference, hypothesis testing, deduction, interpretation, and evaluation of arguments.

The findings of the study indicated a decline in the 10th grade students' level of mathematical critical thinking, and also indicated statistically significant differences between the average marks of the males and those of the females in the critical thinking test, in favour of the female students, while the performance in the critical thinking test of the students with generally higher achievement was found to be better than that of the students with lower achievement.

4.2.2 Scientific Thinking

The field of scientific thinking is undoubtedly a complex one. While many scientists have argued that scientific thinking is in some way superior to other types of thinking, those outside the scientific domain have disagreed, holding that there is nothing unique about scientific thinking (Al-Ahmadi, 2008). It should also be pointed out that there is no consensus as to what scientific thinking entails and there is a great variety of conceptions of it (Al-Ahmadi, 2008). Indeed, scientific and critical thinking have many areas of overlapping constructs, and both have come to represent subsets of what could be roughly described as 'good' thinking (Bezuidenhout, 2011).

Skills generally regarded as representative of scientific thinking include "identifying assumptions, identifying and dealing with equivocation, making value judgements, analysing arguments, asking and answering questions of clarification and/or challenge, and judging the credibility of a source" (Anderson and Soden, 2001).

As early as 1906, Sumner criticised schools as producing individuals who are all cast in the same mould. A proliferation of uncritical opinion tends to undermine scientific thinking through the perpetuation of popular perceptions which Sumner defined as containing “broad fallacies, half-truths, and glib generalisations” (1907, p. 631). However, since the 1980s, there has been greater emphasis placed on the development of scientific thinking instruction in the educational setting (Ennis, 1993). Some curricula appear to suggest that taking a course in a science or a related discipline at school level will encourage the development of scientific thinking (Al-Ahmadi, 2008).

As scientific thinking is an important skill, it is vital to consider how it might be developed in a school setting. There are two aspects to be considered. The first is to establish what can be developed at what age. Piaget (1962) described the developmental stages through which all learners progress and it is only after about the age of twelve that the cognitive skills start to develop to enable the learner to think in terms of an hypothesis as a way of considering and interpreting information. It is, therefore, unlikely that genuinely scientific thinking can be developed at a very young age, although fundamental ideas and skills can perhaps be considered. The second aspect is to consider the teaching approaches which might be most likely to bring benefit. The literature is full of suggestions on ways by which scientific thinking can be developed at school level, although there is no strong evidence to show that the suggestions do, in fact, work (Al-Ahmadi, 2008).

Dierking and Falk (1994) emphasise the importance of the developmental and state that this aspect is often not considered fully enough when considering the role that parents may play in guiding children. Womack (1988) notes that the majority of children have a natural curiosity and that this offers a starting point when developing the ideas of how to understand the world around them. Womack (1988) also argues that children should be helped to see the connections between science and other school subjects, the connections within science itself and between the ideas and inventions of one scientist and those of another, although this will be at a simple level. He goes on to say that children should be given the opportunity to practise the processes of science, such as by making hypotheses and considering the evidence for and against a particular idea. However, these arguments have little support in the literature. Young children cannot deal with many of these abstract ideas, seeing things in the physical and descriptive sense. Indeed, the work of Johnstone et al. (1997) shows very clearly that the reasoning chain of young children will not allow many of these recommendations to be fulfilled and, in fact, the idea of hypothesis formation is out of reach at a very young age.

As far as older learners are concerned, Zohar and Dori (2003) argue that students need to learn "how to read popular scientific articles written by lay people in a critical manner and how to solve complex problems that involve science, technology and society in an effective way".

Hoover (1984) is of the opinion that the best way to teach students to think is by showing them how to write things down in the way they occur in the mind in a sequence of ideas and thoughts. These can be used to frame their ideas into hypotheses and test them. However, there is little evidence to support the utility of this idea (Al-Ahmadi, 2008). It has been demonstrated very clearly that students are reluctant or unable to write down plans (Reid and Yang, 2002; Bodner 1991).

Thornton (1987) argues that computers can assist by displaying data in a manner that can be manipulated. This is seen as part of the laboratory, allowing the students to concentrate on the scientific ideas that are the goal of their investigation. This might avoid the formulaic laboratory and the approach seeks to develop an inquiring approach to science. This has considerable potential and has similar features to certain aspects of pre- and post-lab exercises used at university level (Carnduff and Reid, 2003) which have proved so successful (Johnstone et al., 1993; Johnstone et al., 1998).

Bailin (2002) suggests that students should be involved in designing an experiment to test a causal hypothesis which they have generated after making an observation. However, this assumes that the students are of sufficient age and experience to deal with such ideas. Nonetheless, with older secondary school students, the approach appears to hold considerable potential.

Many researchers have argued for scientific thinking because it makes studies in the sciences more attractive to learners or because such an approach appeals to the natural curiosity of learners (Tobias, 1993; Costello, 2003; Dierking and Falk, 1994; Zohar and Dori, 2003). Many of these stress the importance of parents in the process (e.g., Dierking and Falk, 1994).

It is highly likely that changing teaching approaches from the transmission of information into situations of enquiry and questioning will be appealing, but this is not in itself a fundamental reason for seeking to develop scientific thinking. Equally, early childhood experiences where questioning can be encouraged and developed in constructive ways will be important. However, there is still the fundamental question about the formal teaching situation: can scientific thinking be taught and if so, how and when?

Zohar and Dori (2003) suggest some types of activity for teachers to help to develop the students' scientific thinking skills; these are summarised below:

(1) Asking questions: What is it? Where does it come from? How does it happen?
(2) Discussing common scientific problems.
(3) The right answer is the one that accords most closely with the facts.
(4) Performing unusual experiments.
(5) Investigating the environment.
(6) Constructing working models.
(7) Studying interesting objects.
(8) Making connections or considering surprising facts.

This list has many positive and attractive suggestions which are integral to the practices of good and stimulating teachers. However, it is highly unlikely that, unsupported, such an approach will generate scientific thinking. Indeed, such activities raise other questions. In a curriculum where, in most countries, time is very limited in the attempt to go through the syllabus and where the rewards frequently come from the correct recall of facts or the correct application of procedures, it is difficult for a teacher to find the time to develop such activities.

Womack (1988) made some rather idealistic recommendations on this point, such as that teachers should have more practical aims; learners should be handling materials, discovering their properties and observing their behaviour under different conditions; there should be much discussing and recording of information; teachers should be encouraging learners to generalise about happenings not yet observed and to offer explanations as to why things behave as they do. He went on to suggest that learners should carry out simple experiments just to see what happens or to test an idea. However, any experiment should be carefully supervised to ensure the safety of the children, particularly at such a young age. In addition, there may be no time for such experiments due to an overcrowded curriculum. At the later primary stages, Womack (1988) argues that learners should be encouraged to make hypotheses to explain a set of results or to predict what may happen under certain conditions. All this assumes that scientific thinking is accessible at primary stages and that it is possible to carry out such activities bearing in mind these aims (Al-Ahmadi, 2008).

Zimmerman (2007) argues that an experiment aims to test an hypothesis against a substitute, be it a precise hypothesis or the complement of the hypothesis under consideration. This is widely agreed, but doing it under the pressures of a school situation is not so easy. Gold (2002) has an apparently simple answer when he emphasises that teachers must be trained to teach learners how to think. However, the evidence from research in the last decade suggests very strongly that this is likely to fail (Carroll, 2005; El-Sawaf, 2007).

The literature suggests that scientific thinking skills can be promoted, but that such a promotion is not without difficulty. As such, varying empirical results have come to define the research around the acquisition of scientific thinking skills (Bezuidenhout, 2011). Although Bruner (1966) held scientific thinking in great esteem, he was of the view that the sciences and the humanities both contribute to the development of understanding.

4.2.3 Creative Thinking

According to Marrapodi (2003), the literature on creative thinking mainly focuses on the individual and how the creative process functions, with creativity seen as a process rather than a product. Fisher (1990, in Alenazi, 2004) states that creativity is what creative individuals use to make creative products

Smith et al (2000) state that creativity is usually measured by creative results, assuming that individuals with greater creative potential have greater creative results, although they specify that the majority of very creative people are creative only within one discipline. Alenazi (2004) states that creative thinking is generally associated with the fact that not all human cognitive processes are conscious.

Michalko (1998) points out that creativity is not quite the same thing as intelligence. In this regard, Feldhusen and Goh (1995) state that *"creativity is often defined as a parallel construct to intelligence, but it differs from intelligence in that it is not restricted to cognitive or intellectual functioning or behavior. Instead, it is concerned with a complex mix of motivational conditions, personality factors, environmental conditions, chance factors, and even products*."

In addition, Richards (2003) distinguished creativity from innovation, stating that "*innovation is 'ideas to action' - taking something that seems to be a good or even exceptional idea and transforming it into something that is tangible for others to use. Innovation is an active process that has a clearly defined end or goal and that produces something that others can use and indeed want! …The goal, if not drive, of creativity is to explore beyond current reality, to realize something new. On the* *other hand, the goal of innovation is to bring those novel ideas into a tangible form that in some way conforms to what others need in the here and now*."

In terms of teaching, the creative process most often encouraged in learners by teachers involves stimulus, exploration, planning and reviewing (Alenazi, 2004). Creativity needs a response that involves both feeling and thinking and a combination of cognitive skills and creative attitudes; the former will permit the learner to generate and process ideas, while the latter will encourage the learner to be inquisitive and imaginative and use complex ideas (Alenazi, 2004).

A number of studies have examined the association between creative thinking and mathematical thinking. For example, the purpose of Al-Jassim's (1994) study was to identify the effectiveness of a training program in creative problem-solving strategy in the development of creative thinking skills with a sample of high-achieving pupils.

The researcher applied the following tools: pre- and post- Torrance verbal tests and a creative problem-solving test on a sample of 36 high-achieving pupils in third grade intermediate school in Kuwait.

The results of this study showed the effectiveness of the training program in creative problem-solving strategy to develop the skills of creative thinking among high-achieving pupils as well as the existence of a link between creativity and problem solving.

In the same vein, Kousa's (1999) study aimed to discover the effectiveness of the use of a program designed to solve problems involved in achievement and creative thinking in mathematics, using a sample of female pupils in intermediate school in Makkah in Saudi Arabia.

The study used the following tools: pre and post achievement tests and pre and post tests for mathematical creative thinking. These tests were developed by the researcher.

Among the most important findings of the study are:

• The proposed program was effective in the development of academic achievement in mathematics in the intermediate school pupils.

• The proposed program was effective in the development of creative thinking ability in mathematics in intermediate school pupils.

In this regard, the aim of Al-Hawarani's (2001) study was to explore the effect of a training programme to develop creative thinking ability in mathematics achievement in the case of 10th grade students.

The study sample consisted of 90 students from the 10th grade in Ain Al-Basha girls' secondary school, which is affiliated to the Directorate of Education in Ain Al-Basha District in Jordan. The students were divided randomly into two groups: an experimental group and a control group, with the experimental group studying the equation systems unit using the training programme, while the control group studied this unit by the traditional method. The study instrument was an achievement test in the selected mathematical unit.

The study findings indicated the presence of statistically significant differences (α = 0.05) between the average achievement of the two groups in the achievement test in favour of the experimental group, indicating the presence of a positive effect on the development of the creative thinking ability in raising the level of students' achievement in mathematics.

A similar study was carried out by Muwafi (2003), who attempted to identify the impact of Internet use on the development of some mathematical concepts and the ability to think creatively among students in the third group of the Department of Mathematics, Faculty of Education for Girls in Jeddah.

The researcher used the following tools: a test of mathematical concepts, and a Torrance test of creative thinking. The study sample was 35 students in an experimental group and 42 students in a control group in the third group of the Department of Mathematics, Faculty of Education for Girls in Jeddah. The results of the study showed that the students in the experimental group who searched for information on the Internet had a higher achievement than those in the control group who showed less improvement. In addition, there were significant differences between the mean scores of the students in the Torrance test of creative thinking for experimental group students, showing the effectiveness of Internet use in the development of the ability of creative thinking.

Another study, that of Saif's (2005), aimed to determine the effectiveness of mathematical puzzles in the development of creative thinking and attitude towards mathematics among intermediate pupils in Kuwait.

The researcher used the following tools: a Torrance's test of creative thinking, and a measure of the attitude towards mathematics. These tools were applied to a sample of 44 female third grade intermediate pupils in a public school in Kuwait. The results of this study showed that there were statistically significant differences between the mean scores of the study sample in the pre- and post-test of the creative thinking measure in the dimensions of originality, fluency, and flexibility, and in the test as a whole. There were also statistically significant differences between the mean scores of the sample in the pre- and post-measures of students' attitude towards mathematics.

Finally, the purpose of Sajjadi and Syed's (2007) study was to investigate differences in creative thinking in mathematics according to gender in Iranian students in intermediate schools and their attitudes towards mathematics.

The researchers used the Torrance test of creative thinking, and a measure of the attitudes towards mathemetics. The sample consisted of 203 intermediate school pupils in Iran who all lived in the same area, were all almost the same age and were subject to the same conditions. The results of the study showed that students with positive attitudes towards mathematics showed a remarkable development in their creative skills.

These studies suggested that there is a statistically significant positive correlation between mathematical thinking and creative thinking, and some results of these studies indicated that mathematical thinking leads to creative thinking. Therefore, mathematical thinking may be considered the more important element, although it could also be argued that this is not the case, and that creative thinking may be equally, if not more, important than mathematical thinking.

4.2.4 Inductive and Deductive Thinking

In the field of thinking skills, inductive thinking and deductive thinking are frequently mentioned. Taba (1966) introduced the concept of inductive thinking, viewing thinking as a process of interaction between the individual's mind and information toward a specific end, which the individual discerns, compares, links together, then finds the relations between, and analyses. Marten (1997) describes induction as the principle of reasoning to a conclusion about all the members of a class through examining only a few of these; in sum, reasoning from the particular to the general. He describes an inductive argument as one in which the premises give good reason to believe the conclusion, but also emphasises that modern logicians tend to look at reasoning from the general to the particular (Marten, 1997).

According to Marten (1997), deductive reasoning can be seen as deducing the particular from the general principle. He considers deductive reasoning as superior to inductive, although induction is the central feature of scientific reasoning. Nonetheless, an acceptable inductive argument with true premises may give a false conclusion. He argues that science uses inductive reasoning all the time because the corresponding deductive argument would require information that is unavailable or too costly to make it worthwhile obtaining it (Marten, 1997).

4.3 The Aesthetic Aspect of Thinking

Costa (1985) stressed the importance of the kind of thinking that is characterised by aesthetic sense, and called for the addition of an aesthetic dimension to any programme of thinking development, on the premise that the aesthetic dimension represents the sensory beginnings of rational thinking. This in turn leads to enlightened thinking, as the observations, searches and questioning skills that represent the basis of scientific investigation have their origin in the aesthetic world and arise through feelings of surprise. As students respond to its aesthetic qualities, they obtain great pleasure in thinking; this strengthens their desire for enquiry and may well lead them to pose questions such as why, how, what and when (Jabir 1997; Costa, 1985). In this context, mathematics is viewed as an art, in terms of its aesthetic qualities, consistency, and the arrangement and sequence of ideas embedded therein. This expresses the opinion of the mathematician in the most effective and concise way, and generates ideas and structures that point to the creativity of mathematicians and their imaginative and intuitive abilities (Abu Zeenah, 1994).

4.4 Approaches to Teaching Thinking Skills

There are two basic approaches to the teaching of thinking skills, each with its own philosophy and interpretation (Jones et al, 1994; Usfoor, 1999).

The first approach involves teaching thinking skills directly. This approach advocates the importance of explicit and direct teaching of thinking skills separate from the subjects of the school curriculum. It is justified by the argument that thinking processes should be taught in the same way as any other school topic, and that learning and thinking are part of the same concept. Learning uses previous knowledge and strategies by which to understand ideas and formulate them. In this way, the individual endeavours to create meaning in the same way as thinking is defined as the search for meaning and the generation and formulation of ideas based on previous knowledge.

The second approach calls for the teaching of thinking skills through the content of the curriculum. Advocates of this approach stress the importance of teaching thinking skills by merging them with the content of the curriculum at all stages of school education, in such a way that methodological learning activities are designed with the final outcome of developing thinking and gaining a deep understanding of the subject; only then can it be said that there is a thinking curriculum (Resnick and Klopfer, 1989). The advocates of this approach believe that the teaching of thinking skills as a part of the curriculum content not only contributes to the improvement and development of these skills, but also improves students' achievement. For instance, according to Gold (2002), thinking skills should be taught throughout the curriculum rather than separately, by such techniques as mind-mapping, and discussion and reflection among students. However, Al-Ahmadi (2008) points out that although this approach appears to hold potential, there is little evidence for its effectiveness.

To sum up, there are three models for teaching thinking skills: within a subject, independent of a subject, and a mixed model, where general attitudes and skills can be applied to particular knowledge and experience throughout the curriculum (McKendree et al, 2007).

This chapter has offered an overview of various types of thinking. As this study focuses on mathematical thinking skills, the next chapter will be devoted to issues surrounding mathematical thinking and the teaching of it.

CHAPTER 5

MATHEMATICAL THINKING

5.1 Introduction

In the previous chapter, several types of thinking skills such as critical thinking, creative thinking and scientific thinking were presented and discussed briefly. As mathematical thinking skills are a central concern of this research, they will be discussed in greater detail in this chapter. The chapter will begin by discussing the nature of mathematical thinking skills as seen in the literature. It will continue by reviewing the literature concerning the development of mathematical thinking skills in the classroom, the development of advanced mathematical thinking and mathematical thinking in textbook design.

5.2 The Nature of Mathematical Thinking Skills

Mathematics is essential for the practice of many other disciplines, as well as for many activities in everyday life (Charlesworth and Lind, 2011). Many sciences, such as physics, chemistry, biology and medicine depend on mathematics. Furthermore, engineers need mathematics when they construct a building, business people use arithmetic when they buy or sell goods, and pilots also need to use mathematics for navigation. Although all these benefits come directly or indirectly from mathematics, there is still a widespread perception that mathematics is useless and of little value in daily life.

Orton (2004) argued that even when mathematics "is not directly useful, it has indirect utility in strengthening the power of reasoning or in inducing a general accuracy of mind". Due to the inherent logic in the processes of mathematics, this seems plausible. It could simply be that those who possess good powers of reasoning thrive in the world of mathematics rather than that a study of mathematics develops the reasoning ability (Onwumere, 2009).

Many questions arise; for example, why mathematics is often seen as a difficult and unpopular subject by many learners, why large numbers of learners fail in mathematics and who or what is responsible for this: is it due to an inappropriate curriculum, inappropriate teaching methods or simply a lack of commitment on the part of the learners? The issues appear to be complicated and interrelated with each other. Verhoeven (2006) suggests that 'maths-phobia' and lack of interest may be two of the main factors involved in the widespread unpopularity of mathematics. Indeed, the questions at the end of the textbooks may be another issue involved in this and, as these questions are the main focus of this research, an attempt will be made to establish whether this is the case. For instance, the questions in the textbooks may not capture learners' interest sufficiently.

Whitney described the American school failure in mathematics:

> *"For several decades we have been seeing increasing failure in school mathematics education, in spite of intensive efforts in many directions to improve matters. It should be very clear that we are missing something fundamental about the schooling process. But we do not even seem to be sincerely interested in this; we push for 'excellence' without regard for causes of failure or side effects of interventions; we try to cure symptoms in*

place of finding the underlying disease, and we focus on the passing of tests instead of meaningful goals"

(cited in Skemp, 1987, p. 3)

Although the above was written in the United States over twenty years ago, in this researcher's experience, this is still the case in Saudi Arabia, where the curriculum appears to be geared almost entirely towards passing exams. Therefore, the focus of the curriculum is on rote learning rather than on the promotion of mathematical thinking skills, whereas for the future of the country, there is a requirement for Saudi citizens who possess these skills.

It is clear that although there have been many attempts to determine the meaning of thinking in general and of mathematical thinking in particular, these attempts have been hindered by the ambiguity and complexity of the concept. This is attributed to the difference in the approaches and academic interests of researchers as well as to their schools of thought. Mathematicians view thinking differently from psychologists, views differ between teachers of mathematics at the elementary school stage and those of the secondary school stage, and views also differ according to academic and professional experience (e.g., Lutfiyya, 1998; Schurter, 2002).

Therefore, the question remains: What do we mean by mathematical thinking? To answer this question, a number of researchers and mathematicians specialising in the mathematics curricula and educational psychology, have endeavoured to define mathematical thinking. Wood et al. (2006, p. 226), define mathematical thinking as "the mental activity involved in the abstraction and generalization of mathematical ideas". Mathematical thinking and reasoning are described by the Organisation for

Economic Co-operation and Development (2010, p. 106) as involving the ability to distinguish between "different kinds of statements (such as definitions, theorem, conjectures, hypotheses, examples, conditioned assertions) and understanding and handling the extent and limits of given mathematical concepts".

In addition, many attempts have been made to identify the patterns and skills of mathematical thinking clearly in order to facilitate the development of students' mathematical thinking by studying and investigating the attributes and characteristics that distinguish individuals who possess higher mathematical abilities. However, despite such endeavours, there is still no logical framework to explain all the patterns (Schurter, 2002).

In this respect, Petocz and Petocz (1997) with regard to the skills and patterns of mathematical thinking, assert that mathematical thinking includes the skills of recognising patterns, mathematical proof, inductive thinking and deductive thinking. Similarly to Petocz and Petocz (1997), Lutfiyya (1998) sees mathematical proof, induction, and deduction as being among the skills and patterns of mathematical thinking, while also identifying generalisation, symbolism, and logical thinking as being among these skills and patterns.

Schielack et al. (2000) classified mathematical thinking skills into 6 groups, the first of which is modelling, which refers to the use of tables, pictures, graphs, representations, geometric charts and so forth. The second group is inference, such as finding generalisations, while the third is symbolism and the fourth is logical analysis, which includes comparing results. This is followed by abstraction, and the sixth and final group is finding the optimum (e.g. the least costly and most effective) solution.

Wilson (1993) indicated that mathematical thinking included the use of the following mathematical skills: understanding ideas, discovering the relations between them, determining the conditions that the ideas must fulfil and the relations between them, and solving problems related to these ideas. He also identified the main skills of mathematical thinking as being estimation; mental arithmetic; problem-solving; mathematical proof; symbolism, and mathematical reasoning (including inductive and deductive reasoning). In addition, he considered the study of the structure of mathematics; that is, gaining an understanding of the structure and main ideas of the subject, and a conception of the relationships and associations between different mathematical topics, to be a mathematical skill.

Mann (2006) asserted that creativity was also involved among the skills of mathematical thinking. In addition, the higher order thinking skills classified by Bloom analysis, synthesis and evaluation have also been considered to be among mathematical thinking skills (Chancellor, 1991). Pitt (2002) indicated that mathematical thinking includes the skills of generalisation, searching for meanings between the lines, searching for patterns, and the evaluation of patterns based on the given facts. According to Carreira (2001), solving applied mathematical problems and modelling are the two main aspects that indicate the development of students' mathematical thinking and meta-cognitive thinking.

Hadamard (1945) regarded mathematical proof as the apex of the pyramid of mathematical thinking, and he described mathematical proof as the essence of mathematical thinking; hence, it is vital to include it in the mathematics curricula and in school textbooks (Hanna, 2000). Amit and Neria (2007) stressed generalisation as being among the most important skills of mathematical thinking. Dehaene and Spelke (1999) indicated that mathematical thinking includes the ability of the student to express ideas using the mathematical language that includes symbols, tables, drawings and geometrical shapes. Harte and Glover (1993) argue that the use of estimation in solving real-life problems is one of the most important skills of mathematical thinking; they also stressed the importance of integrating estimation into the mathematics curricula and school textbooks.

Mathematical problem solving has long been considered among the most important topics of the educators engaged in the development of mathematics curricula and the methods of teaching these curricula, solving mathematical problems and the strategies used in solving them is regarded as one of the fields that arouse the curiosity of students, and hence, as among the most important of mathematical thinking components (e.g., Gervasoni, 2000; Weiss, 2003).

Katagiri (2004) holds that mathematical thinking involves learning how to learn independently. Greenwood (1993) argued that mathematical thinking encompasses the following skills: finding patterns, modelling, generalisation, determining places of error, and the use of different strategies to solve the same problem. Moreover, Greenwood (1993) developed several criteria to evaluate the development of students’ mathematical thinking. One of these is the students' ability to complete a task with little or no dependence on the teacher; the teacher guides the student to find his/her own solution. Another is the ability of the student to interpret and explain the problem-solving strategies that he/she has chosen, so that they are clear and understood by others, as the student cannot explain something unless he/she is aware of it and understands it. A further criterion is the reliance of the student on him or herself and on the knowledge and skill at his/her disposal by which the tasks can be accomplished, and the ability to overcome the difficulties and obstacles that arise without resorting to the teacher for help in overcoming them. Here, the teacher helps the student by asking him/her some questions about the question or the task, and finally the student is guided to accomplish the required task. In addition, there is the ability of the student to determine the places of error in given answers, and the use of the fewest possible number of steps and calculation processes when solving the question (here the student resorts to mental arithmetic). Greenwood's (1993) criteria also include the ability of the student to provide several solutions and strategies for the same question, and to formulate additional questions about the main question or the task to be accomplished, and to put the question in contexts other than the given context, which would help him/her find the answer to the main question.

Bruner (1963) considered formulating questions as the apex of mathematical growth. That is, the student shows mathematical growth when he/she becomes capable of formulating hypotheses and posing questions about a certain issue or problem, which he/she then puts in various contexts other than the original one, and tries to answer such questions or hypotheses.

In this regard, Bruner (1963) believed that the goal of education was to help the student acquire an organised method to gain mathematical knowledge, and it was not important which knowledge was acquired by the learner, but what was most important was how this knowledge was acquired. In this respect he stated that there was no need to make students repositories of knowledge in many subjects, but they did wish to make the student use mathematical thinking because knowledge is not a product, but a process (Bruner, 1963).

Bruner (1963) distinguished between two types of mathematical thinking. One is intuitive thinking, which is developed by means of the direct experience of the learner as he/she deals directly with things, and this type of thinking is considered an important factor in building self-confidence. The other is analytical thinking, which is a type of deductive thinking based on the mathematical hypothesis and proceeds according to successive sequential steps.

It is noted that despite the varying views of researchers about the patterns and skills of mathematical thinking, there is still some consensus about several patterns and skills, which the researcher summarises as the following: inductive thinking, generalisation, searching for patterns, deductive thinking, mathematical proof, logical thinking, use of variables and symbolism, modelling, reasoning and justification, and mathematical problem solving.

In addition, despite the variation in researchers' views about the nature of mathematical thinking, there is still some consensus among educators and mathematicians involved in the development and teaching of mathematical thinking skills. They stress the need to teach the skills of mathematical thinking and provide learning opportunities that help in the development of mathematical thinking in students, and use all the available means for doing this by developing the mathematics curricula and their educational tools, or by following new teaching and evaluation methods (NCTM, 1989, 2000; Sfard, 2001; Cohen, 2002).

The review has brought together the views of many authors in seeking to try to develop a clear picture of mathematical thinking. One of the problems is that aspects of mathematical thinking suggested by many are in no way unique to mathematics. For example, deduction and induction can occur widely, while pattern seeking, logical thinking and generalisation are generic skills applying widely.

This is a problem noted by Al-Ahmadi in relation to scientific thinking (Al-Ahmadi and Reid, 2011) and Alosaimi in relation to critical thinking (Alosaimi et al., 2014). These studies are built on the analysis carried out by Chandi et al. (2009), in which she tried to develop an operational description of systems thinking. In all the analyses, the authors focused on the skills that were perceived to be unique features of systems thinking, scientific thinking and critical thinking and developed test materials based on these.

Looking at mathematical thinking, the question is to identify what might be the unique features of mathematical thinking that distinguish it from other areas of thought. The following is suggested as a feasible way forward (Figure 5.1). The key word is ‘relationships’ in that mathematics involves the study of relationships (between variables, constant, positions in space, etc.).

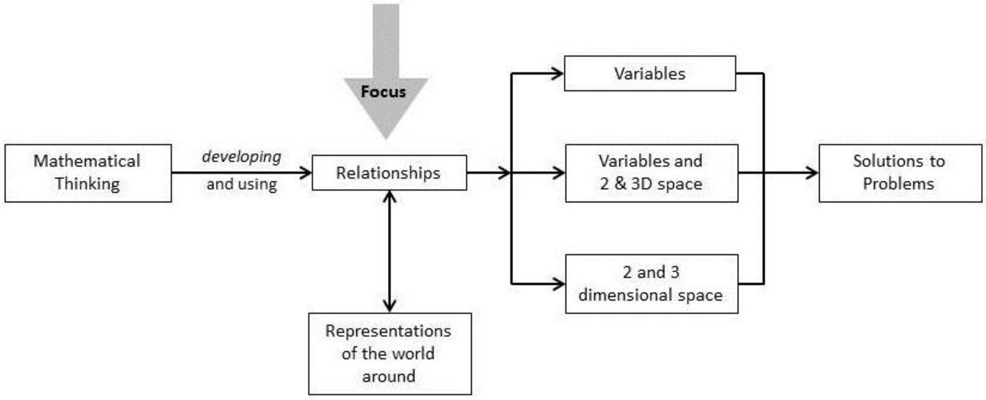


Figure 5.1: Looking at mathematical thinking (Source: Author)

Looking at the nine skills outlined on pp. 17-20, it is possible to conceptualise these in the context of relationships:

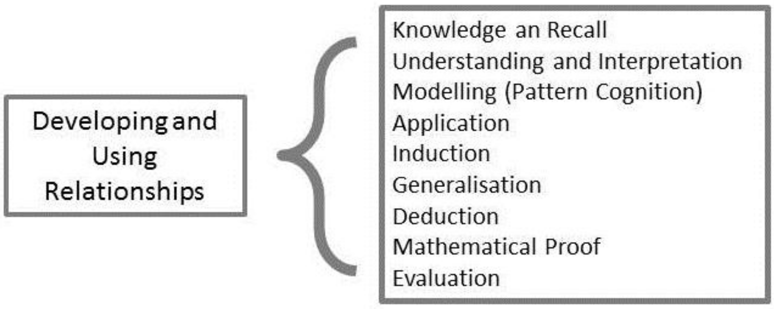


Figure 5. 2: Nine mathematical skills (Source: Author)

Thus, the nine skills can all be used as components in developing and using relationships

5.3 The Development of Mathematical Thinking

Mathematical thinking is viewed as a skill developed by training and cognitive growth as well as by experience, as it does not come from nowhere or arrive by chance. Rasmussen et al. (2005) characterise progression in mathematical thinking as active participation in a number of different socially or culturally situated mathematical practices. Hence, mathematical thinking can be developed through several procedures by developing the mathematics curriculum and its educational tools, or by following certain teaching and evaluation methods. Therefore, in this study, the focus is on the questions in the textbooks.

Mathematics curricula and questions can assist in the development of mathematical thinking by the following procedures:

- Mathematics should be presented as a sequence of structures and topics that are closely interconnected, as mathematics is a highly-structured discipline When the student becomes aware of this, his/her ability to solve mathematical problems will be enhanced and thereby his/her mathematical thinking will develop. This method is preferable to the presentation of mathematics as a sequence of separate routine processes and skills (Carpenter, 1985; Monroe and Mikovich, 1994). In this respect, in a study of the use of mathematics textbooks in English, French and German classrooms, Pepin and Haggarty (2001) found that in some textbooks, few connections were made between the concepts practised.
- Mathematical games and puzzles could be included in the mathematics textbooks, as this would help present mathematics in an interesting and amusing form. This would help maintain the students' enthusiasm for learning mathematics,, as many researchers have indicated that students start learning mathematics with great enthusiasm in the first grades of elementary school, but such interest starts to wane gradually as the students move to higher grades (e.g., Watt, 2004; Frenzel et al., 2010). Among the reasons behind this gradual loss of interest is the manner in which mathematics is presented to students, as this tends to be more abstract in the higher grades. Hence, there may be a need to include some mathematical games and

puzzles through which students will be able to discover the pleasure of the mathematical topic and which will therefore contribute to enhancing their motivation for and engagement in learning mathematics (Lewkowicz, 2003). This is likely to develop their mathematical thinking and enhance achievement (e.g., Schielack et al., 2000; Shi, 2000; Lewkowicz, 2003; Najm, 2001).

- The mathematics curriculum and its educational tools should incorporate the skills and patterns of mathematical thinking. It is important to integrate the mathematics curriculum and the curricula of other school subjects, which would allow the students to use and capitalise on mathematical thinking in the interpretation of scientific, social, and economic phenomena and so forth (Presmeg, 2006).

- Use of real-life situations and applications that are interesting to the student and make him/her aware of the importance of mathematics and its relation to everyday life (Carreira, 2001). It is also important to introduce the student to contexts that belong to his/her environment and daily life (Carreira, 2001; Swars et al., 2006). In this respect, Maaβ and Gray (2011) express the opinion that realistic settings should be used in mathematics education in order to give students an awareness of the ways in which mathematics is employed in private and professional life.

As a result of the development of educational thought, the position of the teacher and his/her role were to a great extent strengthened. Thus, teachers are no longer merely links between the school textbook and the mind of the learner, whose task is only to convey knowledge, but have started to play a major role in fulfilling the objectives of education and the attention paid to thinking. Hence, in many countries, the process of planning and development of the new curricula has begun to take into consideration ideas and opinions of teachers in their capacity as change and development makers, upon whom the burden of implementing the curricula falls (Remillard, 1991; Carl, 2005).

Modern mathematics curricula give the teacher a greater role in the direction, guidance and organisation of the learning process, in order to stimulate the students' abilities and energies in the practice of thinking (Maher, 1999; Fraivilig, 1999). As the process of mathematics teaching is a process of presenting mathematics in a form that makes the students view it as a pleasant experience (Carver, 2001), teacher preparation programmes should pay particular attention to assisting them to acquire the skills and knowledge necessary for the development and consolidation of mathematical thinking, whether such programmes are pre-service (as part of college training) or in-service. In this regard, it is very important to prepare mathematics teachers, improve their experience and upgrade their knowledge in order to assist them in becoming researchers and investigators inside the classroom, rather than limiting their role to the presentation and conveyance of knowledge (Kazemi, 2000; Frank and Kazemi 2001; Philipp et al., 2003).

The process of the consolidation and development of students' mathematical thinking entails the design of educational activities that arouse surprise, wonder and challenge in students' minds, rather than limiting the teaching process to the school textbook with all its traditional, routine problems (NCTM, 2000; Schielack et al., 2000; Resnick, 1990).

In this respect, Polya (in Ball, 2002) states that the job of the teacher is a crucial one; he/she has the choice either to kill the interests and creativity of the students, or make them practise and realise the satisfaction and pleasure of thinking with all its different patterns.

Many researchers have stressed the importance of the role played by the teacher in the development of mathematical thinking in students, and the importance of possessing particular teaching skills such as the skill of asking questions, and the ability to use new patterns of questions and activities that stimulate the students and arouse their interest in learning and practising thinking, and so forth (Vacc and Bright, 1999; Cai and Kenney, 2000; Watson, 2001; McDonough et al., 2003; Weiss, 2003). Jitendra et al (2005) suggest that teachers must have a deep understanding of their content area to prepare their students adequately to engage in complex thinking and problem solving. For example, they may need to emphasise reasoning and critical thinking, link the newly introduced concept to students' previous mathematical knowledge, and facilitate generalisable skill application. In addition, it is important that teachers attend closely to instructional design principles (e.g. specifying learning objectives that provide a direction for instruction). Providing students with carefully designed and explicit instruction that includes sufficient and varied examples and problem-solving skills is crucial (Gersten & Baker, 1998). Moreover, creating learning environments in which adequate time is devoted to unambiguous explanations and strategic application of newly learned skills to promote conceptual understanding is an essential goal for teachers (Jitendra et al, 2005). At the same time, providing instructive feedback that allows students to analyse their performance in relation to the feedback provided is critical to promote skilled, error-free performance (Jitendra et al, 2005).

Further, new teaching methods and strategies play an important role in the development of mathematical thinking and teachers may use the method of the dialogue between teacher and students in the form of a discussion that includes questions and answers leads to the stimulation of thinking. This will help the students acquire facts by themselves, while the role of the teacher will be one only of direction and guidance in the learning process (van Oers, 1996; Paul et al., 1989; 1990).

In addition, a co-operative learning approach may be adopted by teachers. Cooperative learning plays an important role in the development and consolidation of mathematical thinking due to the opportunities it creates for the students to exchange ideas, suggestions and experiences. NCTM (2000) standards stress the importance of interaction among students while they are engaged in mathematical activities through the exchange of what they have in mind, such as ideas, opinions and suggestions. The reform movement in school mathematics has stressed the importance of communication as a basic component in mathematics learning and teaching. Communication in mathematics does not only refer to the ability of students to use mathematical language to express themselves and the ideas that occur to them, but also the students' ability to think, reason and justify. Communication is another method by which views can be shared, and hence students should be given the opportunity to express themselves about the thinking processes that they use in solving mathematical problems and performing various mathematical activities, whether orally or in writing (Cai and Kenney, 2000). The result of interaction between students is that each student will acquire a set of ideas, opinions and strategies that have been exchanged among students, which would lead to the development and consolidation of a multitude of patterns of mathematical thinking (Sfard, 2001; Fraivilig et al, 1999).

Jaworsk (2006) suggested that teachers can use the exploration method in the presentation of educational material. This yields positive results in the development of mathematical thinking, because it gives students an active role in the learning and teaching of mathematics. Learning by exploration encourages the students to discover the mathematical ideas and solutions by themselves, which will generate a feeling of satisfaction in them, coupled with the desire to continue learning. Learning by exploration can result in students' conceptual learning of mathematics.

In order to develop mathematical thinking skills, teachers can also us the computer in mathematics teaching and learning. It is an effective educational tool through which to present mathematical topics in an interesting manner that helps the students in learning, particularly in subjects such as geometry, or trigonometry or in what are known as logic games (Jones, 2000; Magajna and Monaghan, 2003).

In this respect, Fraivilig et al. (1999) indicated several teaching procedures which the teacher can use in the development and consolidation of mathematical thinking when solving mathematical problems. These include providing the students with feedback about their previous work and reminding them if a problem that they have solved before is similar to the problem in hand. In addition, the teacher can instruct all the students without any exception, and irrespective of their achievement levels, to solve mathematical problems, encourage them to adopt different strategies for solving a mathematical problem, and to justify such strategies. Further, a list of all the solutions and strategies proposed by the students to solve mathematical problems could be drawn up by the teacher, in order to discuss them with the students and explain the differences in such solutions and strategies. It is also recommended that the teacher creates a healthy climate, where freedom prevails, for the students to express ideas and opinions, and creates an atmosphere of healthy challenge and competition between the students to solve mathematical problems (Fraivilig et al., 1999).

It is important to use methods that stimulate and arouse thinking in order to evaluate learning instead of restricting them to achievement tests (Aspinwall et al., 2003). Among the methods that can be used to stimulate thinking is the open-ended question (Aspinwall et al., 2003).

Students can also be instructed to make research reports, or have a bulletin board inside the classroom, which could be a window to reveal the nature of the students' thinking and their attitudes and tendencies towards mathematics. In this case, the teacher would instruct the students to write their experiences in detail while they are engaged in the process of learning and carrying out mathematical activities and tasks. The students would then indicate the nature of ideas that have occupied their minds while solving the mathematical problem, the difficulties and obstacles that stood in their way and how they managed to overcome them (Goldsby and Cozza, 2000; Weiss, 2003). In this regard, Smith (in Di Pillo and Sovchik, 1997) stressed the importance of writing, stating that we discover what we think about when we write.

In short, it can be said that educational institutions should endeavour to develop the thinking skills of the students, and should take into consideration all that has been said about this matter, including the aforementioned factors, procedures, curricula, textbooks, questions, the presence of the qualified teacher, use of modern teaching and evaluation methods, and all that stimulates thinking, as they all together constitute a single harmonious system in which each component complements the other. Hence, due care and attention must be given to all these components, rather than limiting attention to one aspect at the expense of another.

In looking at the discussion above, it is clear that much is based on opinion. Perhaps, there is a need for research to provide the evidence to support or undermine the views expressed.

5.4 The Development of Advanced Mathematical Thinking

Mathematics education research deals with the learning and teaching procedure of mathematics in primary, intermediate and secondary schools or in undergraduate and postgraduate courses in higher education establishments.

Clark and Lovric (2009) carried out research on advanced mathematical thinking and suggested that such thinking needs rigourous, deductive reasoning about mathematical ideas that are not completely accessible through the five senses. They further argue that this definition is not necessarily linked to a specific type of educational experience, or to a specific level of mathematics. They also offer examples to show how advanced mathematical thinking and elementary mathematical thinking could be distinguished. Specifically, they they examined which type of thinking may be appropriate to the length of a mathematical problem, such as problems involving infinity, and the kinds of models available.

Numerous mathematics educators have employed the term “advanced mathematical thinking” to describe particular kinds of student thinking at the higher education level of mathematics and some of the mathematical thinking conducted at the expert level of mathematics. Clark and Lovric (2009) discuss the phenomenon that appears to take place first during an undergraduate experience in undergraduate mathematics when he or she first starts to address abstract concepts and deductive proof. Students at this time frequently become aware that a number of the thinking skills that have contributed to their success in calculus courses are no longer sufficient in courses such as abstract algebra.

Clark and Lovric's (2009) study was intended to define advanced mathematical thinking in order to be able to connect it to this transitional period in a mathematics student's education. Clark and Lovric's (2009) definition is illustrated examples of mathematical situations, which allow a contrast to be seen between advanced mathematical thinking and elementary mathematical thinking.

Tall (1992) connected his concept of advanced mathematical thinking (AMT) to formal mathematics. He described AMT as comprising "two important components: precise mathematical definition (including the statement of axioms in axiomatic theories) and logical deductions of theorems based upon them."

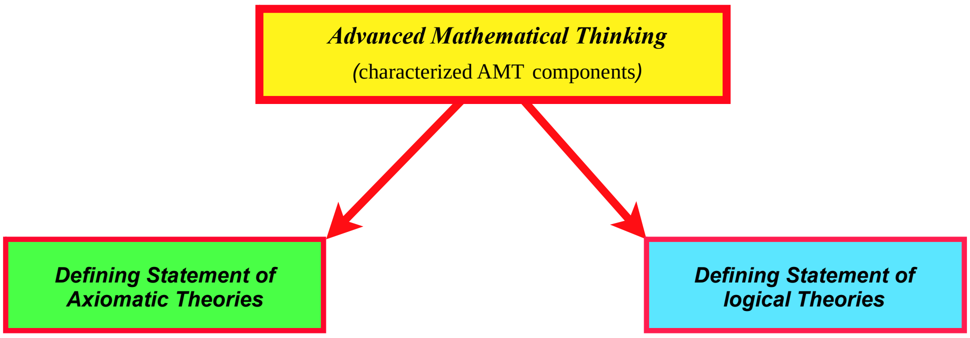


Figure 5.3: AMT components (Source: Adapted from Tall, 1992)

Tall (1992, p. 495) continued by stating that "The move to advanced mathematical thinking involves a difficult transition from a position where concepts have an intuitive basis founded on experience to one where they are specified by formal definitions and their properties constructed through logical deductions".

However, it may be true that advanced mathematics students, learners and professional mathematicians work with ideas that are "specified by formal definitions and their properties reconstructed through logical deduction" (Tall, 1992).

According to Clark and Lovric (2009), while exemplary mathematical thinking may take place at any level of mathematics and in students of all ages, what is usually considered to be advanced mathematical thinking takes place only in certain circumstances which involve rigorous deductive reasoning about mathematical objects which the five senses cannot perceive.

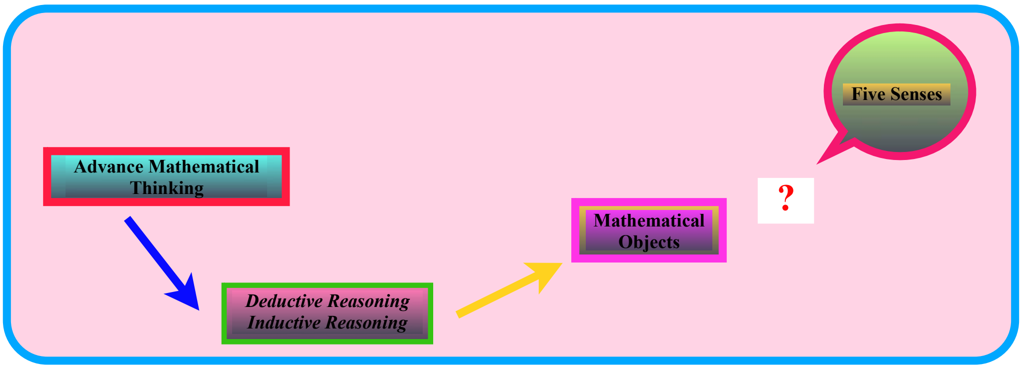


Figure 5.4: Advanced Mathematical Thinking (Source: Clark and Lovric, 2009)

This definition can help mathematics educators to be aware of the problems of the period of transition as students progress, for example, from calculus to more abstract and theoretical courses in mathematics.

Tall’s (1999) research is about the Actions Processes Objects Schemas (APOS) theory in elementary and advanced mathematical thinking. He poses several questions about mathematical thinking. Among which are the following:

- By what processes are mathematical concepts constructed?
- During this process, what types of cognitive entity are constructed?

The theories of cognitive construction developed by Piaget for younger children formed the basis for Dubinsky et al.’s (1988) APOS theory. In this, they described the way in which actions become interiorised into processes and are then captured as mental objects, subsequently taking their place in more sophisticated cognitive schemas. Dubinsky et al. (1988) thus take a construction method hypothesised in elementary mathematics and extend it to advanced mathematics.

Tall (1999)’s response to Dubinsky et al’s (1988) theory is to note the requirement for cognitive action in the production of cognitive structure, although raising questions regarding the primacy of action before object in mathematics overall. Tall (1999) argues that although APOS theory has already been shown to be strong in the design of curricula for undergraduate mathematics curricula, it may not be universally applicable.

Tall (1999) investigates the relevance of APOS in mathematics, by beginning with its source in Elementary Mathematical Thinking (EMT). In this regard, Piaget (1972) identified three modes of abstraction: empirical abstraction from objects of the environment, pseudo-empirical abstraction from actions on objects in the environment and reflective abstraction from mental objects.

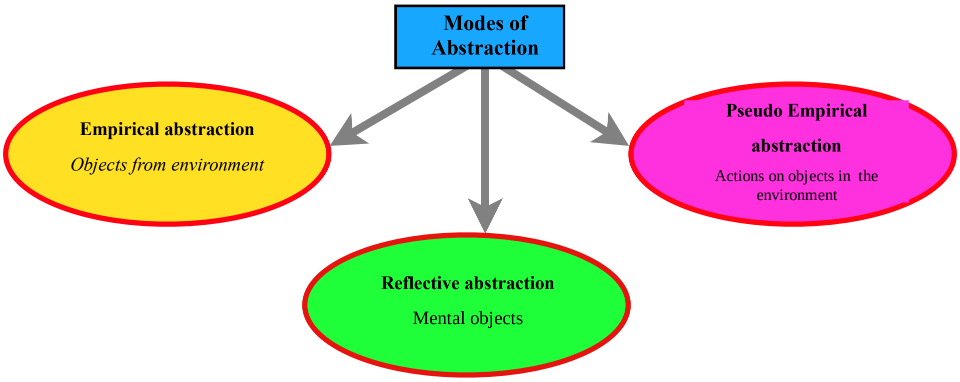


Figure 5.5: Modes of Abstraction (Source: Adapted from Piaget, 1972)

Figure 5.4 shows a sequence of the uses of processes and concepts in symbolic mathematics (Tall, 1992). Computational processes are involved in arithmetic, and potential evaluation processes in algebra, but potentially infinite computational processes are involved in manipulable concepts, the dynamic limit concept at the start of calculus, which lead to the image of quantities that may be "arbitrarily small", "arbitrarily close" or "arbitrarily large". Hence, Tall (1992) finds it unsurprising that many students adhere to the ease of finite rules of the calculus which they have learned by rote.

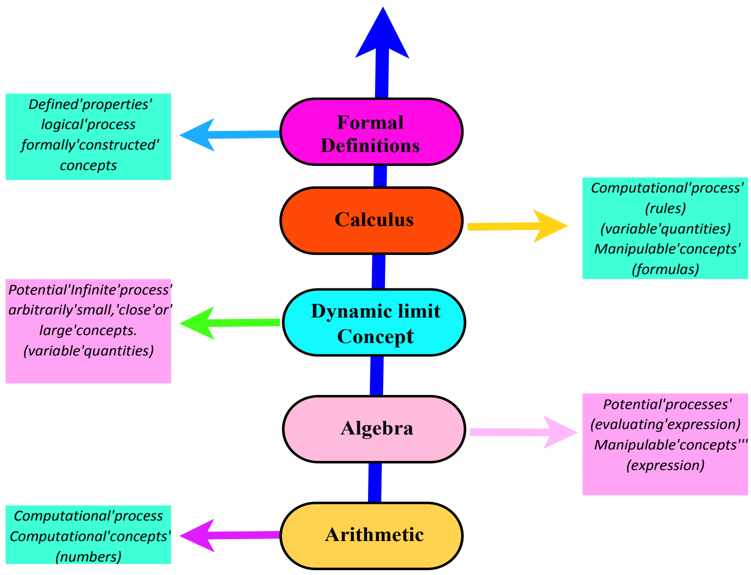


Figure 5.6: Process and concepts in symbolic mathematics (derived from Tall, 1998)

Tall (1998) describes the principal elements of arithmetic, algebra, dynamic limit concept, calculus and formal definitions which are comprised of computational process and concepts in numbers, manipulable concepts, variable quantities and defined logical and symbolic constructed concepts. Although different approaches to research both arrive at formal proof, the mathematical insights obtained differ considerably (Tall, 1998).

According to Pinto and Tall (1999), a broad range of thinking processes can be found in undergraduate mathematics students. That is, some students use their own experiences to construct meaning for definitions while others take definitions given by others and construct meaning mainly through the deduction of theorems. The latter appear to be more responsive to an action-based APOS course than the former. Pinto and Tall (1999) argue that, although cognitive actions may always be needed to build cognitive concepts, restricting the learning sequence to one way of constructing mathematical actions, mathematical processes and mathematical objects does not make any contribution to the diversity of human thought.

Tall (1999) considers APOS to be a significant contribution to the understanding of mathematical cognition, but as a useful tool, and not a universal pattern. He illustrated this in the model shown below.

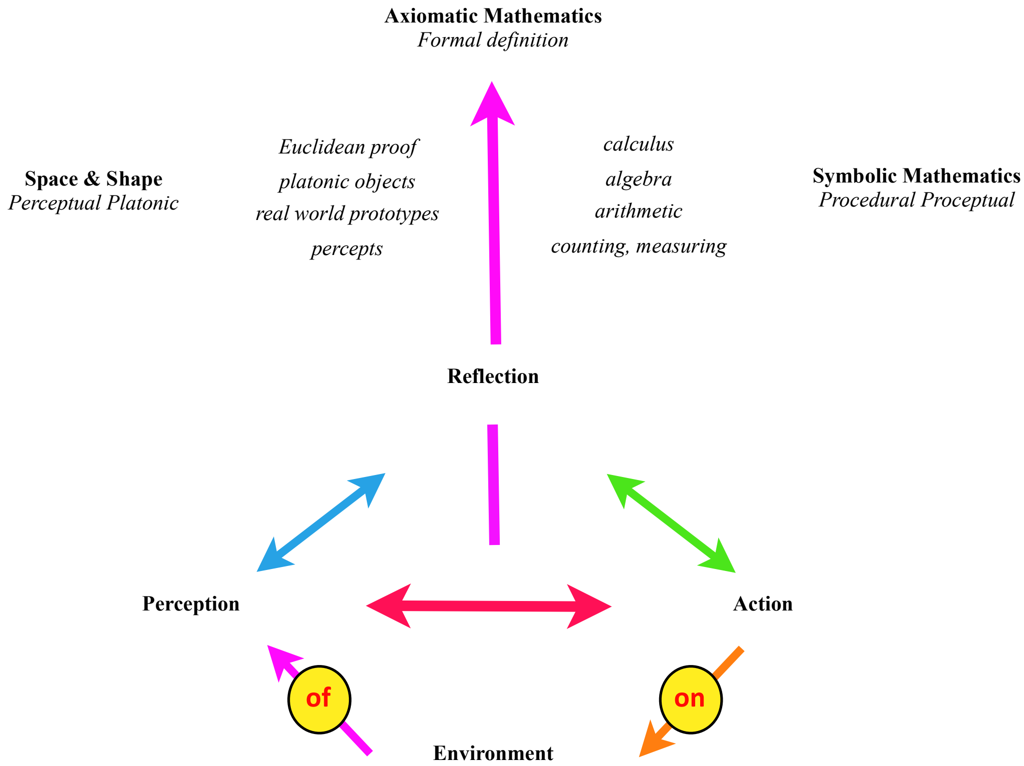


Figure 5.7: Process of Mathematical Cognition (Source: Adapted from Tall 1995; 1998)

5.5. Mathematical thinking in textbook design

Mathematical thinking in textbook design comprises mathematical questions and tasks which are intended to encourage mathematical thinking and to arouse discussion among students. It is expected of students that they explore mathematical relationships; build and justify their own rationale and strategies for problem solving; make appropriate use of problem-solving tools; and share each other's strategies (Ibrahim, 2009).

However, Rahman (2010) investigated a number of important issues which students may encounter in terms of the promotion of mathematical thinking in mathematics textbooks. Among these are the rote learning of mathematical rules and the inability of learners to cope with unfamiliar or new situations, which may lead to confusion (Rahman, 2010).

As Rahman (2010) further states, it is likely that coping effectively with novel situations will depend on which aspects of the concept/idea the learners focus their attention upon; that is, what they see as important. Activities that learners engage in can serve to disclose the structure and range of their awareness (Rahman, 2010).

In this regard, the Structure of a Topic (SoaT) framework was developed by Mason and Johnstone (2004) to depict the conception of a mathematical topic. The framework consists of three elements: behaviour, emotion and awareness, which have a close association with the more commonly used terms enaction, affect and cognition, respectively (Mason and Johnstone, 2004).

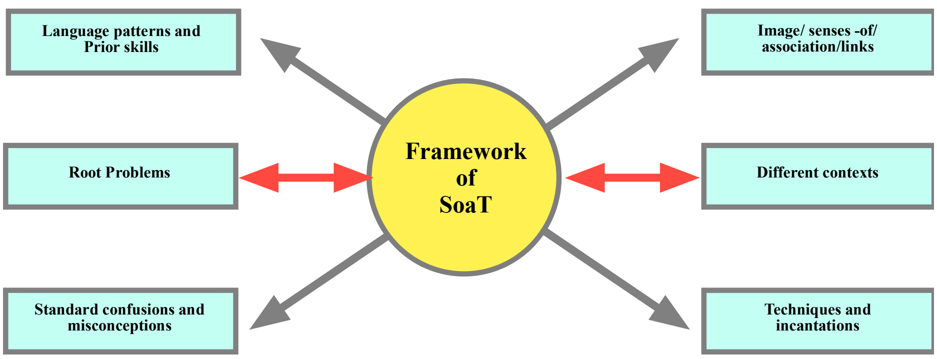


Figure 5.8: The Structure of a Topic framework (Source: Adapted from Mason and Johnstone, 2004)

Mason and Johnstone's (2004) descriptions of the concepts of behaviour, awareness and emotions are shown in the table below:

Concepts	Description
Behaviour	Practice develops behaviour but only training can make the individual inflexible.
Awareness	Learning involves raising awareness which, in turn, guides appropriate behaviour.
Emotions	Motivation and desire to learn stem from the engagement of students' emotions.

Table 5.1 Elements of the Structure of a Topic (Source: Adapted from Mason and Johnstone, 2004)

Therefore, according to this framework, flexibility stems from awareness, which guides behaviour. That is, if behaviour is to be flexible and respond to changes, it must be directed by active awareness (Mason and Johnstone, 2004).

Rahman (2010) examined the concept of mathematical thinking in terms of learning as distinguishing something from, and associating it with, a context. This supports a perception of mathematics as fundamentally concerning the study of invariance among change. The use of examples to illustrate and clarify mathematical concepts is an essential component of effective mathematics instruction (Marton and Booth, 1997). Although teachers may use examples to illustrate definitions and clarify the use of a rule or theorem, learners may concentrate on particular details of examples and may think that only those types of example are appropriate, thus leading to their thinking becoming restricted. This in turn may lead them to ignore the broader exemplification that the teacher intended (Marton and Booth, 1997).

Bills et al. (2006) suggest that for an example to be pedagogically useful, it should have two main attributes. The first is that it should be comparatively easy for the learners to determine the features that make it exemplary, while the second is to that it should also encourage generalisation; that is, it should highlight the constant features of the illustrated case, while at the same time pointing out those features which may vary (Bills et al., 2006).

Learners' comprehension of mathematical examples and their awareness of what is kept constant and what can vary in maintaining the exemplary characteristics of the examples may reveal the extent of their awareness and encourage and enhance their appreciation of mathematical topics. One method of discovering which features of a topic dominate learners' attention and whether they have understood what the teacher intended to convey is by looking at what they do with mathematical examples (Marton and Booth, 1997).

There is a clear and pressing need to exert more effort in order to develop mathematical thinking and improve it. Mathematics curricula and educational tools occupy a fundamental place in the education curriculum; hence, many countries, such as the USA with the NCTM Standards, embarked on developing the mathematics curricula and curricular materials such as the textbook and improving them in order to keep pace with contemporary requirements. An attempt has been made to have these curricula focus on the development of students' thinking, and helping them to acquire thinking methods that depend on a sound and accurate mathematical structure, in the belief that mathematics is a pattern and method of thinking. It has the qualities that make it an appropriate field in which to train students in the methods of sound thinking, and thereby contribute to students' creative ability, as well as helping them acquire mathematical insight and deep understanding.

It has to be recognised that much of the above section is simply opinion although what is being suggested may well be very reasonable.

The studies on mathematical thinking can be divided into four main categories, as follows:

(1) Studies measuring students' thinking levels in mathematics at different school stages

The aim of Younis' (1991) study was to determine the level of the following mathematical thinking patterns in students in intermediate school: generalisation, induction, deduction, symbolism, formal logic, and mathematical proof.

The study sample was made up of 600 male and female students who were randomly chosen from the 7th, 8th and 9th grades of public schools affiliated to the United Nations Relief Agency (UNRA) in Amman, Jordan.

For the purpose of the study, the researcher developed a measure of mathematical thinking made up of 40 items distributed among the aforementioned six dimensions.

The findings of the study indicated the following:

- The highest performance in mathematical thinking of the intermediate students was in deduction, while their lowest performance was in induction, and in general the performance of these students was very poor.
- There were statistically significant differences ($\alpha = 0.01$) between the performance of the three grades in induction, deduction, symbolism, formal logic and mathematical proof, in addition to the overall measure of mathematical thinking in favour of the higher grade.
- There were statistically significant differences ($\alpha = 0.01$) between the performances of the male and female students in the six patterns and the overall measure of mathematical thinking was in favour of the female students.
- There was a clear effect of gender and grade level on the performance of intermediate students in the patterns of induction, symbolism, formal logic, mathematical proof and the overall measure of mathematical thinking.

The mathematical thinking skills of university students were the focus of Stenger's (2000) study. The study sample comprised 137 male and female students majoring in mathematics, chosen from two universities in the USA during the academic year of 1998/1999. The sample represented two groups of university students: those training to be primary school mathematics teachers and those training to be secondary school mathematics teachers.

The study instrument was a test to measure the students' ability in mathematical analysis and communication. The findings of the study indicated that the level of these mathematical thinking skills in both groups of students was generally low.

The aim of Cai's (2000) study was to compare the mathematical thinking of 6th grade students in the United States of America and China. The study sample was composed of two groups: the first group was that of selected American students from Milwaukee in the state of Wisconsin, and the second group was selected Chinese students from Guiyang in the region of Guizon. The study instrument was a test of mathematical thinking that included 6 process-open problems and 6 process-constrained problems.

The study findings indicated that the Chinese students performed better than the American students in the process-constrained problems, while the American students performed better than the Chinese students in the process-open problems.

The purpose of Chap and Tee's (2007) study was to evaluate the mathematical thinking in Malaysian schools at different stages, and to analyse various mathematical thinking skills. This study also aimed to determine some of the issues and difficulties facing mathematics teachers in the application of mathematical thinking. The study sample consisted of math textbooks for various school grades and a sample of student teachers. The study used a descriptive analysis approach.

The study found that there was a tendency among student teachers to direct the application of mathematical thinking in public education through teaching and that there was little clear understanding of mathematical thinking. The limitation was that the evaluation tools were limited to the written tests.

It can be noted from the studies that discussed this skill that the students' thinking levels in mathematics at different school stages were generally low, which suggests that considerably more effort is required to develop students' mathematical thinking levels, starting from the first years of the students' enrolment in school. Efforts must also be concerted in this respect to develop and improve the mathematics curricula and textbooks and their questions by basing them on thinking skills, as well as ensuring that mathematics teachers are well trained before employment (at the university preparation stage), and continue training during their service, in the use of teaching approaches and evaluation methods and strategies that stimulate thinking. It is also vital to teach thinking skills to all students, and not to limit this to a special category of student such as the gifted and the high achievers, due to the fact that thinking is unlikely to develop or improve without constructive and organised learning, training and practice.

(2) A number of studies have discussed the relationship between mathematical thinking and certain variables such as growth, gender, achievement, educational level, type of study programme and attitude towards mathematics. Among these studies are the following:

Al-Sheikh and Abu Zeenah's (1985) study aimed to determine the development of the ability to think logically as the student advances in education from the secondary school stage to the university stage, as well as to determine the factors that affect the pattern of growth and the ability to think logically. These are the type and course of education (e.g. scientific, literary); the type of logical rules that govern logical judgments; types of situation where logical judgments take place (the formula in which the logical rule is presented, whether verbally explicit or implicit, deduced from an example of the rule), and; the type of performance required in the particular situation (choice of an example that conforms to the rule, or specifying an example that deviates from the rule).

The study sample comprised 786 male and female students, of whom 574 were from the second year of secondary school, from 16 public schools in Amman and Irbid in Jordan, and 212 male and female students from the third year in the University of Jordan and Yarmook University, majoring in science, mathematics and education. The tool of the study was a test prepared by the researchers to measure the students' ability to apply certain rules of mathematical logic.

The findings of the study indicated that the ability of third year university students to apply the rules of logic significantly exceeded the ability of the secondary school second grade students, which indicates that growth or improvement took place in the ability of logical thinking due to the progress of the academic study from the secondary stage to the university stage.

The aim of Abu Al-Huda's (1985) study was to determine the relationship between mathematical thinking on the one hand, and attitude towards mathematics and achievement in mathematics on the other hand. It also aimed at determining the effect of the educational level and the type of study (i.e., scientific, literary) on the development of the secondary school students' ability in mathematical thinking.

The study sample was made up of 799 male and female students in the first, second and third secondary school grades (scientific and literary). The study sample was chosen from the academic secondary school stage in public schools in Amman in Jordan. For the purpose of the study, the researcher used a measure for mathematical thinking that took the following skills into account: induction, generalisation, inference, symbolism, mathematical proof, and logical thinking. He also used a measure for attitude towards mathematics.

The findings of the study indicated a statistically significant positive correlation between mathematical thinking and the attitude towards mathematics, and mathematical thinking and achievement in mathematics. The findings also indicated statistically significant differences between the averages of students' performances in tests of mathematical thinking, as the highest performances were those of the students in the second and third scientific sections, while the poorest performances were those of the students in the second and third literary sections. There were no statistically significant differences between the averages of the male students' performances and the female students' performances in the mathematical thinking test.

Abu Zeenah's (1986) study aimed to determine the growth in the ability of mathematical thinking as students progress from the secondary school stage to the university stage. It also aimed at determining the effect of the students' study programme on their mathematical thinking ability.

The study sample comprised 854 male and female students in the first and second secondary school grades (scientific and literary), the students of an intermediate college (a teacher training college), and university students (second and fourth years) in the science and mathematics faculties. The study sample was chosen from male and female secondary school students, some students from intermediate colleges in Irbid and university students from the University of Yarmook in Jordan.

For the purpose of the study, the researcher used a test that he had devised for this purpose to measure the ability of the students in mathematical thinking. The test took the following skills into account: induction, generalisation, deduction, symbolism, and formal logic.

The findings of the study indicated the presence of improvement and growth in the ability of mathematical thinking as the students progressed from the secondary school stage to the university. The results of the test of mathematical thinking ability indicated better performances by the scientific section students than by the literary section students in secondary school, as well as better performances by the mathematics students in the university than by the fourth-year science students in university, while the university students performed better than the teacher training college students. This is to be expected. The findings of the study also indicated that the highest performances in the ability test of mathematical thinking were in generalisation and induction, while the lowest performances were in deduction and mathematical proof.

The aim of Nasr's (1998) study was to determine the relationship between the ability of mathematical thinking on the one hand, and the ability of mathematical problem solving and achievement in mathematics of the students majoring in science and mathematics, on the other hand. The study also aimed to determine the effect of educational level and type of study on the development of ability in mathematical thinking and mathematical problem solving. The study sample was made up of 370 male and female students chosen randomly from students majoring in science and mathematics in Al-Zarqa'a Province in Jordan.

The study used a test to measure the ability of the students in mathematical thinking, which took the following skills into account: generalisation, induction, symbolism, deduction, formal logic and mathematical proof. The study also used a test to measure ability in mathematical problem solving, including algebra and arithmetic problems.

The findings of the study indicated the presence of a statistically significant positive correlation between mathematical thinking ability and problem solving ability, between mathematical thinking ability and achievement, and between problem solving ability and achievement. The findings on mathematical thinking and problem solving ability indicated that the performance of the second year students was better than that of the first year students, and that of mathematics major students was better than that of science major students.

The aim of Bishr's (1989) study was to determine the relationship between the ability of mathematical thinking and that of creative thinking, and the progress of the students in academic study from the first to the third years of secondary school. The study also aimed at determining the effect of the student's study programme (scientific, literary) on their ability in mathematical thinking, creative thinking and achievement in mathematics.

The study sample consisted of 1160 male and female students in first, second and third grades (scientific and literary) of secondary school. The study sample was chosen from the academic secondary schools in Sana'a and Taez in Yemen.

For the purpose of the study, a test of the ability of the students in mathematical thinking was used. This took the following skills into account: generalisation, induction, symbolism, inference, logical thinking and mathematical proof. The Torrance scale for creative thinking was also used, taking into account the following skills: fluency, flexibility and originality.

The findings of the study indicated a growth in the ability of mathematical thinking and creative thinking as the students progressed in their academic study from the first year to the third year of secondary school. The performance of the students in the scientific section was found to be better than that of the literary section students in both mathematical thinking and creative thinking. It was also found that the highest performances of the students in the test of mathematical thinking were in generalisation and induction, while their poorest performances were in inference and mathematical proof. The findings of the study also indicated the presence of a statistically significant positive correlation between the ability of mathematical thinking and the ability of mathematical creativity, between mathematical thinking and achievement in mathematics, and between mathematical creativity and achievement in mathematics.

The aim of Al-Qabati's (1993) study was to determine the growth in mathematical ability and its relationship with logical thinking and achievement in mathematics during the secondary school stage and after. The study sample was composed of 774 male and female students, of whom 563 were in the first year secondary school scientific section, 106 in second year majoring in mathematics in Irbid and Hawarah colleges, and 105 in the third and fourth years, majoring in mathematics at Yarmook University in Jordan.

The study tools included a scale of logical thinking devised by the researcher, and a mathematical ability measure which took into account inferential ability, conceptual ability, numerical ability and spatial ability.

The findings of the study indicated a rise in mathematical ability as the students progressed in their academic studies, as well as a statistically significant positive correlation between mathematical ability and logical thinking, and mathematical ability and achievement in mathematics.

The aim of Lutfiyya's (1998) study was to determine the effect of grade and gender on mathematical thinking in the case of high school students in Nebraska in the United States of America, as well as to develop a tool to measure the skills and patterns of mathematical thinking of these students.

The study sample consisted of 239 male and female students in the 9^{th}-12^{th} grades chosen from 18 high schools in Nebraska State. The researcher developed a tool to measure the students' mathematical thinking ability. This included the patterns and skills of mathematical thinking that took into account the following skills: generalisation, induction, deduction, symbolism, logical thinking and mathematical proof.

The findings of the study indicated the following: A statistically significant difference in the test of mathematical thinking in favour of the higher grades (except in the 11^{th} and 12^{th} grades, where the 11^{th} grade students performed better than those in the 12^{th} grade). There was no statistically significant difference in the test of mathematical thinking between the average marks of the male students and the average marks of the female students in all the grade levels.

Abid (2004) conducted a study to determine the effect of two teaching strategies in mathematics based on investigating achievement and mathematical thinking in the case of 9^{th} grade students in Jordan. The study sample consisted of 160 female students who were distributed among 4 groups. While the school was purposefully chosen, the students were randomly distributed among the 4 groups. One group was taught using directed investigation, the second using enriched investigation, the third using a mixture of the two strategies and the fourth using the traditional method. The fourth group was regarded as a control group. All the groups were given lessons in algebra and geometry which constituted a part of the ordinary school curriculum. The researcher prepared educational materials and teaching plans based on the two patterns of investigation mentioned above in a manner appropriate to the objectives of the study. The researcher also prepared a test of achievement in the presented mathematical curriculum made up of 40 items to be answered by multiple choice. She used a modified form of the mathematical thinking test prepared by Al-Khateeb (2004), limiting the original test to six skills; namely, induction, generalisation, symbolism, deduction, modelling and speculation, so that the modified test comprised 30 items of the multiple choice and sentence completion types. The statistical analysis of the results of the students' performance in the two tests showed the existence of statistically significant differences between the results of the four groups. These can be attributed to the teaching strategy in both tests, where the directed investigation helped the students raise their achievement level, while the enriched investigation helped to raise their ability in mathematical thinking.

From this review of the studies that were conducted on the various skills of mathematical thinking, the following conclusions can be drawn.

1- There are statistically significant positive correlations between mathematical thinking and achievement in mathematics (e.g., Nasr, 1988; Hassan, 1999; Kousa, 1999; Assaedi, 2006), attitude towards mathematics (e.g., Abu Al-Huda, 1985; Saif, 2005; Sajjadi and Syed, 2007) and the ability to solve mathematical problems (e.g., Al-Jassim, 1994; Abid, 2004; Al-Khateeb, 2004). This is an indicator of the importance of the development and consolidation of students' abilities in mathematical thinking in order to improve their achievement in mathematics and attitude towards mathematics. However, this implies cause and effect and correlation cannot show this.

2- Most of the studies focused on investigating six skills of mathematical thinking, namely, generalisation, induction, deduction, symbolism, formal logic, and mathematical proof (e.g. Shantawi, 1982; Abu Al-Huda, 1985; Abu Zeenah, 1986; Nasr, 1988; Bishr, 1989; Younis, 1991; Daghlas, 1991; Lutfiyya, 1998). Al-Khateeb (2004) added two other skills, namely, modelling and speculation, and Abid (2004) selected six of the skills in Al-Khateeb's (2004) study, which were induction, generalisation, symbolism, deduction, modelling and speculation. Kosa (2001) chose six skills of mathematical thinking different from these; that is, inferential, inductive, structural, reflective, and relational thinking and problem solving. Abu Al-Jidyan (1999) limited his study to the three abilities of inferential thinking, which are inference, induction and deduction, while Almedia (2001) focused on only one skill of mathematical thinking; that is, mathematical proof.

3- Some of these studies investigated the effect on the development of mathematical thinking ability of various factors, such as educational level, course of study (whether scientific or literary) (Shantawi, 1982; Abu Al-Huda, 1985; Nasr, 1988; Bishr, 1989; Abu Al-Jadyan, 1999; Lutfiyya, 1998), the effect on students' progress from one grade to another (e.g,. Shantawi, 1982), the effect of students' progress from the secondary stage to university (e.g., Abu Zeenah, 1986), the effect of achievement level (e.g., Abu Al-Jadyan, 1999; Kousa, 1999; Assaedi, 2006), the effect of gender (e.g., Shantawi, 1982; Daghlas, 1991; Abu Al-Jadyan, 1999; Al-Khateeb, 2004; Lutfiyya, 1998), and some of the students' personal characteristics and socio-economic status (e.g, Daghlas, 1991). It is difficult for the students to acquire all the different thinking skills and patterns at one time, but an ongoing effort must be made in teaching and presenting thinking skills right from the first year of enrolment in school, and this should gradually continue in the more advanced stages of the student's study.

4- These studies indicated a general weakness in the students' mathematical thinking skills at all educational levels (e.g., Lutfiyya, 1998; Almedia, 2001). However, this assumes some knowledge of what is possible – a baseline of success.

5- Some studies tried to relate mathematical thinking to other types of thinking, such as creative or innovative thinking (e.g., Muwafi, 2003; Saif, 2005).

The current study benefited from the previous studies in terms of lists and classifications of mathematical thinking skills and patterns including knowledge and recall, understanding and interpretation, modelling, application, generalisation, induction, deduction, mathematical proof, and evaluation.

(3) Studies that investigate the cognitive aspects of mathematics teachers, and the extent to which they employ methods and strategies to develop thinking while teaching students mathematics.

Among these studies was that of Vacc and Bright (1999), which aimed to investigate the change in the beliefs of primary school teachers in Wisconsin, USA, before employment (at the university preparation stage), about mathematics teaching and learning, as they progress in the university preparation programme, and the extent to which they are able to employ teaching methods that help develop students' mathematical thinking.

The study sample consisted of 34 male and female teachers in the pre-employment (university preparation) stage. The study instruments comprised a measurement of attitude consisting of 48 items that took the following aspects into account: the role of the teacher, the role of the student, the relationship between skills and understanding, and the sequence of topics. A video tape and a teacher observation paper were used during teaching, together with interviews with the teachers.

The study findings indicated that statistically significant changes took place in the teaching and learning of mathematics as the teachers progressed in the university preparation programme from one year to another. The findings also indicated that the teachers paid little attention to developing the students' mathematical thinking skills, whether when planning lessons or giving them.

The aim of Kazemi's (2000) study was to determine the effectiveness of a training programme in the USA to enhance mathematics teachers' understanding of the nature of students' mathematical thinking through research and investigation by the mathematics teachers both inside and outside of the classroom. The study sample comprised mathematics teachers in one school, who were distributed into research workgroups. These workgroups met on a monthly basis throughout the school year in order to exchange experiences, opinions and suggestions and to discuss the students' performances in the mathematical tasks assigned to them, as all the students were assigned to do the same tasks.

Information for this study was collected by analysing the documents of the performance of each workgroup as a group and those related to the performance of individual teachers.

The study findings indicated the success of the programme in the development of the understanding of the mathematics teachers, who were considerably enlightened about the nature of students' mathematical thinking, and about how to develop and advance thinking.

While reviewing the studies that investigate this aspect, the role that should be played by mathematics teachers in the development and consolidation of students' mathematical thinking is apparent. The teacher shoulders the greatest responsibility for stimulating and strengthening students' thinking. It is the teacher who encourages their abilities and motivates them to learn and practise thinking. Hence, particular care must be given to the preparation programmes of mathematics teachers, whether before employment (at the university preparation stage) or during their service, in order to refine their experience, develop their knowledge and help them to acquire the necessary academic and professional skills and knowledge.

(4) Studies investigating the development of students' mathematical thinking through a preparation programme or the use of certain teaching strategies

One study which was carried out to explore the development of students' mathematical thinking through the use of certain teaching strategies was that of Ali (1991), which aimed to discover upon what foundations mathematical computer games can be designed to suit the mathematical background of students, and to develop mathematical innovation; to what extent mathematical computer games contribute to students' mathematical improvement; whether mathematical computer games or entertaining computer games are more effective in the development of students' mathematical improvement.

The study sample was made up of three groups: the first used mathematical computer games, the second used entertaining computer games, and the third was taught using the traditional method. Each group comprised 10 students from the 4th grade and 10 from the 5th grade in various primary schools in Cairo in Egypt.

The study found statistically significant differences in the development of mathematical improvement in favour of the group that used the mathematical computer games, whose performance was better than that of either of the other two groups. However, there were no statistically significant differences between the second and third groups.

The purpose of the study of Hassan (1999) was to identify the impact of the use of a problem-solving method to increase achievement and mathematical thinking skills (induction, deduction, generalisation, mathematical proof, and mathematical logic) in a geometry unit. The study sample consisted of third grade pupils in intermediate school in Abha City, Saudi Arabia, and was divided into an experimental and a control group. The study found that there were significant differences (at the level of 0.01) between the mean scores of the pupils of the experimental group and the control group in the test of mathematical thinking, in favour of the former. In addition, there were significant differences at the level of 0.01 between the mean scores of the pupils of the experimental group and the control group in the mathematical achievement test, again in favour of the experimental group. There was also a positive correlation between mathematical achievement and mathematical thinking among students in the whole sample with a correlation coefficient of 0.64.

The aim of Al-Esawi’s (2001) study was to investigate the effect of a proposed training programme in meta-cognitive thinking skills to improve achievement in mathematics in the case of 9th grade students. It also aimed to investigate the effect of achievement level (high, low) and gender on the level of achievement in mathematics.

The study sample comprised 168 male and female students divided into four groups: two of these were the male students' experimental group and the male students' control group, and the other two were the female students' experimental group and the female students' control group. These groups were chosen purposively from two schools in North Amman, Jordan, affiliated to the United Nations Relief Agency (UNRA) in the school year 2000/2001.

The study instrument was an achievement test on the circle tangential unit and tetragonal inscribed shapes, to measure the students' achievement in mathematics.

The study findings indicated the following: The presence of statistically significant differences (α = 0.05) in achievement between the groups in favour of the experimental group that had studied using meta-cognitive thinking skills; the absence of any statistically significant differences attributed to gender in the achievement of the experimental groups; the presence of statistically significant differences in the students' categories of achievement (high, low) in favour of the experimental groups compared to their peers in the control groups, and; the presence of statistically significant differences in achievement between the experimental and the control groups, which was attributed to the common interaction between the group and achievement level, and gender and achievement level, and the group, gender and achievement level.

The aim of Allison's (2001) study was to investigate the effect of the use of the graphing calculator on the development of high school students' mathematical thinking when solving mathematical problems.

The sample comprised students from four high schools in the USA and each student was assigned some mathematical tasks using the graphing calculator. The mathematical tasks included non-routine mathematical problems which required the use of symbols and tables to solve, and exploratory problems, the solving of which required graphs.

The study tool was a model prepared expressly to test the development of the students' mathematical thinking when solving mathematical problems. According to this model, the required information was collected by means of observation and interviews that were conducted with the students solving the problems. Concluding interviews were also conducted with the students after they had finished the mathematical tasks in order to discover their opinions about the use of the graphing calculator in solving mathematical problems.

The findings of the study indicated that the use of the graphing calculator led to the development of the students' mathematical thinking, and improved their abilities in reasoning and justification and revision of the solution. The students agreed on the importance of the graphing calculator in increasing the speed and accuracy of solving the mathematical problems, and in increasing their motivation to perform the tasks of the mathematical problems assigned to them.

The aim of Lewkowicz's (2003) study was to determine the effect of using mathematical puzzles in the development of mathematical thinking and motivation in the case of intermediate level university students in an algebra course.

The study sample was made up of students from the Georgia Perimeter College, USA, and the regular students in Spring 2000, when these students studied an algebra course using a set of mathematical puzzles that required the use of algebraic concepts and processes. The study tools included a mathematical thinking test (algebra concepts), and questionnaires and interviews to measure their attitudes and motivation towards mathematics in general and algebra in particular. The study findings indicated that the use of mathematical puzzles led to the development of the students' mathematical thinking, their motivation towards mathematics in general and algebra in particular.

The aim of Harries' (2001) study was to develop mathematical thinking in the case of students who were slow learners in mathematics using Logo computer language to teach some algebra topics such as use of variables and moving from the specific to the general (generalisation).

The study sample was made up of eight slow learners of mathematics from the 9^{th} grade in a comprehensive school. The students were selected by the school itself. The study used a case study of the eight students for nine months, during which information was collected, interviews conducted and observations made. The computer processes done by the students to accomplish the mathematical tasks were also recorded, in addition to a written evaluation. The mathematical tasks performed by the students on the computer included drawing some shapes and angles, and calculation of the numerical values of some algebra amounts.

The study findings indicated the importance of the computer in creating a motivational environment for learning mathematics in the case of slow learners. The study recommended that the mathematical tasks should be broken down into simple steps, with feedback for each step of the task, which would help to develop the slow learners' mathematical thinking. This simply allows for the limited capacity of working memory (Danili and Reid, 2004).

The aim of Schoenberger and Liming's (2001) study was to develop students' mathematical thinking skills by means of a programme prepared expressly for this purpose, relying on the use of mathematical terms and processes.

The study sample consisted of students in the 6^{th} and 9^{th} grades of an intermediate school and a secondary school in the USA. The researchers developed a programme for the development of mathematical thinking skills, following an investigation of the factors that cause a decline in the level of mathematical thinking of the 6^{th} and 9^{th} grade students, including: weakness in linguistic skills, decline in the prior knowledge (prior learning), and those related to mathematical concepts, and decline in participation in mathematical activities. The study findings indicated the success of the programme in improving the students' skills of mathematical thinking.

The aim of Teixeira's (2002) study, conducted in the USA, was to compare the development of critical thinking skills and achievement in the case of two groups of students who were studying a mathematics course on quantitative reasoning, one through lectures and the other through workshops.

The study sample was made up of 150 male and female Bachelor's degree students who were studying this course. Of this sample, 83 students were taught through lectures, while the other 67 were taught through workshops. The Watson-Glaser critical thinking test was administered to the students before and after the course, and they also sat for a final achievement test at the end of the term.

The findings of the study indicated no statistically significant differences in the critical thinking test or achievement test that could be attributed to the teaching method. The findings also indicated no statistically significant differences in the critical thinking skills that could be attributed to gender, achievement or study major. However, there were statistically significant differences that could be attributed to the academic year.

This study of Abdullatif and Alwakeel (2006) aimed to determine the effectiveness of a programme based on educational activities in mathematics in developing communication skills and mathematical thinking skills, such as inductive and deductive thinking, in pupils in the fifth grade of a primary school in Egypt. The sample was divided into an experimental and a control group. The study found that the programme was highly effective in the development of communication skills in general. There were significant differences between the mean scores of students of the experimental group and the control group in the pre- and post tests of mathematical thinking skills, with higher scores for the experimental group in the latter. In addition, the programme was highly effective in the development of communication skills in general. There was a statistically significant positive correlation at a level of 0.01 between the scores of students in the evaluation of communication skills and mathematical thinking test after the completion of the programme.

In her study, Assaedi (2006) aimed to determine the effectiveness of a program in mathematics for the development of creative mathematical thinking, academic achievement and decision-making in high-achieving pupils in intermediate schools in Makkah in Saudi Arabia.

The researcher used the following tools:

- An achievement test for high-achieving intermediate-level pupils.
- A test to measure creative thinking in high-achieving intermediate-level pupils.
- A measurement of decision-making in high-achieving intermediate-level pupils.

These tools were applied to all high-achieving pupils in public schools in Makkah.

The findings were that there were significant differences between the mean scores of the experimental group and control group pupils in each of the achievement tests and the tests of mathematical creativity and decision-making, in favour of the experimental group.

The purpose of Marsigit and Mahmudi's (2007) study was to investigate the effectiveness of educational aids in improving the mathematical thinking skills used by students in the concept of the least common multiple. The sample consisted of fourth grade pupils in a primary school in Indonesia, who were divided into an experimental and a control group. Those in the experimental group were taught by teachers trained in the use of educational aids. The study found that representing problems in the form of models of thinking was useful for students to use mathematical thinking as a bridge between the abstract and the concrete and helped students to learn least common multiples at higher levels of abstraction. In addition, students used their own experience of the formulas and strategies when using mathematical thinking to solve the mathematical problems.

Ulep's (2007) study aimed to develop mathematical thinking (interpretation, modelling, and pattern recognition) through solving non-routine problems, and measuring the degree to which the study sample took responsibility for self-study using interactive activities. The study sample consisted of intermediate school pupils in the Philippines, who were taught using a non-routine problem-solving method. The study found that students are able to think on their own, so the teacher's role is limited to guidance and advice only.

It is noted from the review of the studies that investigated this aspect that the use of teaching strategies that help in the development of students' mathematical thinking, such as explorative learning, mathematical games and puzzles, and learning aids such as the computer and the graphing calculator, is of great importance, given their effective role in the development of students' mathematical thinking, in addition to increasing their motivation and making their attitude towards mathematics more positive.

Some of these studies indicated the importance of preparing specific programmes with a view to developing students' thinking, and the positive effect of increasing mathematical achievement in these cases.

In the next chapter, literature on learning in mathematics will be reviewed.

CHAPTER 6
LEARNING IN MATHEMATICS

6.1 Introduction

This chapter reviews the literature on learning in mathematics. Following the introduction, it will review the literature on such aspects of learning in mathematics as reasons for learning mathematics, and misconceptions and mistakes that may occur in mathematics instruction. It then presents the maths tetrahedron model, and discusses Bloom's well-known taxonomy of educational objectives and its subsequent influence on mathematics instruction. A number of approaches to mathematics learning are then discussed. These are the constructivist, the ethnomathematics, the investigative and the problem-solving approaches.

Research in education is unable to prove with certainty that one approach to teaching or to formulating a curriculum is superior to another. This is because of the complexity of the phenomena that are studied in educational research. Moreover, there is disagreement as to the best method of research in education and in mathematics education research; there is no consensus as to a single paradigm (Coupland, 2006).

Researchers have adopted paradigms from other disciplines, in particular psychology, as well as sociology and anthropology (Coupland, 2006). Keeves and Stacey (1999) assert that in recent years there has been "an emerging awareness that mathematics education is influenced by the social and cultural context in which it takes place and that the social and cultural characteristics of teachers and students affect outcomes” (p. 209). In this respect, the social and cultural characteristics of the context, i.e. Saudi Arabia, may well have some bearing on the teaching and learning of mathematics, particularly in respect of attitudes to mathematics. These attitudes, positive or negative, may be influenced by the form and content of the questions in the textbook. The fifth research question seeks to determine whether, and if so, in what way, these questions assist in developing a positive attitude to mathematics, in the opinions of the teachers and inspectors.

6.2 Acquisitive and Participatory Models

Empirical research in mathematics education can be seen as taking either of two perspectives of learning, which Sfard (2001) called “acquisitive” and “participatory” models. Among the “acquisitive” models are those that have information processing as their basis, which views knowledge as something that may be acquired through learning; the skills and concepts of which can be transferred from one subject to another. Conversely, “participatory” models see the context in which knowledge is acquired as of great importance (Coupland, 2006). This context comprises the classroom environment, the teacher's influence, and the learner's perception of the reward gained for effort. It is argued much of learners' enthusiasm comes from these contexts (Coupland, 2006).

In the acquisitive model, researchers have tried to identify the ideas that people are taken to build as they to make personal meaning from their experiences (Davis, 1984; Evans, 1991; Skemp, 1987). The mechanism of metacognition offers guidance for the process of concept acquisition. Kilpatrick (1985, p.13) describes metacognition as “knowledge about how one thinks, knowledge of how one is thinking at the moment (monitoring), and control over one's thinking". Schoenfeld (1987) offers an expanded definition of metacognition in mathematics education to include the intuition about mathematics that an individual brings to their study, and how these form the manner in which they undertake mathematical work.

Coupland (2006) states that "in the acquisitive model of learning, the emphasis is on the individual learner and on the individual control that the learner can exercise over their actions", such as making the decision as to whether to make an effort to perform a task and if so, how much. However, Coupland (2006) points out that the learner's social context, and issues around who selects the tasks and the reasons for selecting them, remain unquestioned for the most part.

6.3 Reasons and motivation for learning mathematics

The Cockcroft Report (1982) for England and Wales on the effects of teaching and learning for meaning and understanding offers some reasons why people should learn mathematics, among which are that "it is useful for everyday life, for science, for commerce and for industry", and also offers a strong, succinct and unequivocal method of communication. Moreover, the report claims that mathematics develops logical thinking, and it has aesthetic appeal. However, there has been criticism of some of the Cockcroft suggestions. For instance, most people only need basic arithmetic and perhaps a basic understanding of probability in their everyday lives (Onwumere, 2009). It is paradoxical that, although mathematics can solve practical problems, it is still seen, perhaps with some justification, as an 'abstract' subject (Onwumere, 2009). Indeed, Donovan and Bransford (2005) point out that "for many people, free association with the word 'mathematics' would produce strong negative images".

According to Onwumere (2009), it is the teacher's task to lead learners through the hierarchy of mathematical abstraction while emphasising the connection of mathematics with the real world. In order to do this, the teacher must understand how the learner grows in ability to encounter and understand mathematics, and how the educational and environmental climates affect mathematical growth. More explicitly, the teacher must decide what learners are able to learn, what learners should learn, what techniques best bring about learning and how these issues can best be reflected in the textbooks and their questions. This means that there needs to be a sound understanding of underpinning educational principles which govern effective and efficient learning of mathematical skills and ideas.

Hiebert and Carpenter (1992, p.187) stated that

"Because the goal of mathematics education should be the development of understanding by all students, the majority of the curriculum should be composed of tasks that provide students with problem situations. Two reasons support this claim. The first is the mathematics that is worth learning is most closely represented in problem solving tasks. The second is that students are more apt to engage in the mental activities required to develop understanding when they are confronted with mathematics embedded in problem situations".

Onwumere (2009) points out that mathematics does not involve the same type of social issues which are obvious in the sciences. However, it is vital that students see where and how mathematics fits into everyday life and society, and to appreciate that, without mathematical insights, many of the greatest and most significant developments in society could not have occurred (Onwumere, 2009).

Discussions between teacher and students and among the students themselves can considerably improve the quality of students' mathematical thinking as well as their ability to express themselves (Cockcroft, 1982).

6.4 Misconceptions and Mistakes in Mathematics Instruction

The literature indicates that misconceptions in mathematics are widespread (e.g., Helm & Novak, 1983). According to Schoenfeld (1988), there are two important implications from this. The first is that one of the traditional principles on which a great deal of current teaching is based is, if not completely erroneous, then at least insufficient. Currently, the principal model of teaching is founded on what Romberg and Carpenter (1985) have named the absorption theory of learning. "The traditional classroom focuses on competition, management, and group aptitudes; the mathematics taught is assumed to be a fixed body of knowledge, and it is taught under the assumption that learners absorb what has been covered" (Romberg and Carpenter, 1985, p. 26). According to this perspective, an effective teacher is one who repeats the same thing in several different ways, on the premise that the students are bound to understand at least one of the explanations (Schoenfeld, 1988). However, the recent literature shows that the students may have understood something quite different and that it may be difficult to remove or correct this misconception later. According to Schoenfeld (1988, p.4), "dealing with this reality calls for a significantly different perspective on the part of the teacher. It also calls for different perspectives regarding the appropriate domain of study of research on teaching, and different measures of competence."

The second implication is that it is necessary to examine the subject matter in detail, as misconceptions in arithmetic are different from those in algebra and physics, for example, and we can understand these only by studying each subject on its own terms (Schoenfeld, 1988). Hence, Schoenfeld (1988) argues that "studies of learning and teaching in particular subject areas must be grounded in analyses of what it means to understand the subject matter being taught."

According to Donovan and Bransford (2005), many students show the following four inadequacies when solving problems related to equations and formulae and therefore there should be attempts to correct and encourage the development of these areas that may lead to mathematical proficiency. These inadequacies are conceptually-based mistakes; procedural fluency mistakes; strategic competence mistakes, and; adaptive reasoning-based mistakes. The study showed that the ideas learners have about mathematics frequently influence their understanding; these perceptions also have an important part in students' attitude toward mathematics, as do their experiences of mathematics instruction (Donovan and Bransford, 2005). Attitude towards mathematics will be explored in greater depth in Chapter 8.

A study by the Human Sciences Research Council (HSRC, 2008) examined the crisis in the school mathematics education curriculum in schools in South Africa. Their findings confirmed that problems often observed in students are connected to aspects such as algebraic manipulation in terms of simplifications, formulae, equations and so forth; numeracy, i.e. basic number relationships, place value, decimals, measurement.; graphs of functions; and limited understanding of the nature of concepts such as the properties of numbers and inverses; trigonometry; computational skills (many students were unable to work without a calculator); and space and measurement (HSRC, 2008). In implementing this curriculum, teachers were expected to make significant changes to their more familiar ways of working. However, almost all teachers received very limited support in terms of training and there also had been complaints that the short training provided for teachers had not been effective (HSRC, 2008). The study's recommendations were that teaching and learning should promote general appreciation of mathematics in terms of its philosophical ideas, cognitive aspects, logic, and principles of construction of mathematical objects and theories. It also suggested that teacher education programmes should attempt to familiarise prospective teachers with common, sometimes erroneous, cognitive processes used by students. While their study seemed to be suggesting that much in mathematics gave rise to problems, it also identified that teachers perceived that they were being asked to do something for which they had very little experience or training. This will affect the attitude of students, as, if the teachers are unsure of exactly what they are supposed to be teaching, the students are likely to feel bored or confused by the lessons. However, Onumwere (2009) points out that this study assumes that training would be helpful, whereas it is possible that even with effective training, the teachers would not be able to overcome the cognitive demands made on the students.

6.5 The Maths Tetrahedron

Onumwere (2009) states that it can assist in understanding the concepts of numbers and number operations (addition, subtraction and so forth) to think of them as consisting of the construction of a network of cognitive links between the four types of experience in mathematics, i.e. concepts, procedures, symbolic representations and applications.

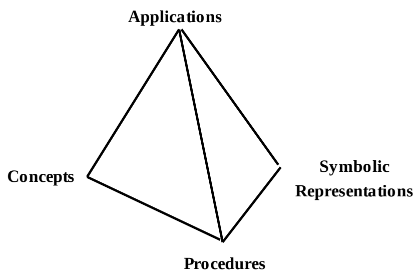


Figure 6.1: The Mathematics Tetrahedron (sources: Alenezi, 2008; Ali, 2008; Ali and Reid, 2012)

The tetrahedron model shown in Figure 6.1 serves to illustrate that understanding can be thought of as constructing cognitive connections between these four components (applications, concepts, procedures and symbols). The main point is that the learner will almost certainly experience working memory overload if required to think of ideas drawn from all four, or even from three, vertices of the tetrahedron (Onumwere, 2009). This presents a dilemma for the educator, but the general principle, particularly with young pupils, is to work at as few vertices as possible simultaneously (Onumwere, 2009).

6.6 Bloom's Taxonomy and Beyond

In 1956, Bloom et al. published their "Taxonomy of Educational Objectives". A taxonomy consists of groups of objects of study categorised according to their similarities and differences. According to Bloom's taxonomy, objectives are "explicit formulations of the ways in which learners are expected to be changed by the educative process".

The original model had a taxonomy with six skills placed in hierarchical order:

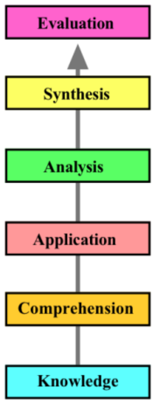


Figure 6.2: The Bloom Taxonomy

However, this view has now progressed to include learning outcomes (Smith, 2007). Bloom's taxonomy offered a language which was easily understood by practitioners and those designing assessments. Essentially, it only involved the cognitive area of mental or intellectual activity involving remembering, thinking, problem-solving, logical argument, decision making, creativity, and so forth (Onumwere, 2009).

Researchers faced difficulties in this field, such as theories not being fully developed; ambiguous terminology; and the variations in research instruments, often leading to difficulties in interpreting the literature and leaving researchers open to criticism (Onumwere, 2009). These factors make effective assessment difficult. In addition, only what is examinable tends to be taught in the school setting.

Yang (2000) noted a problem in Bloom's taxonomy in that the six cognitive skills listed by Bloom et al. were almost certainly not hierarchical. Yang (2000) proposed an alternative model (Figure 6.3)

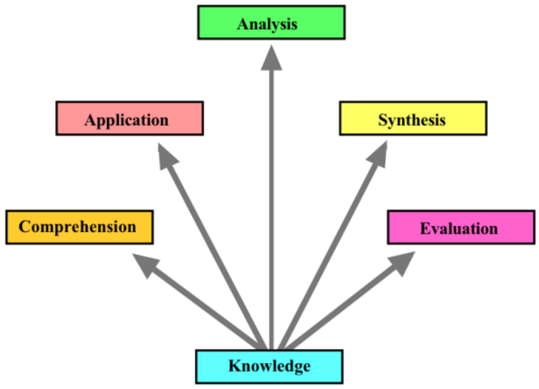


Figure 6.3: An Alternative Model (adapted from Yang, 2000)

The main difference between this model and that of Bloom et al. is that Yang's (2000) model does not assume that there is an evolutionary structure (i.e., that evaluation builds on an ability to synthesise and that synthesis builds on an ability to analyse and so forth). She only assumes that the five skills depend on knowing something or having access to that knowledge. However, the taxonomy encourages the use of precise statements of educational aims, although a term such as 'analysis' may have various meanings in different contexts. Potentially, it is applicable in all contexts of teaching and learning (Onwumere, 2009).

6.7 Constructivist Approach

Alenezi's (2008) tetrahedron theory has already been described in detail. Alenezi (2008) described the concept of tetrahedron theory and its application in terms of working memory. This present study focuses on certain novel approaches, which are of considerable significance in the area of mathematical thinking and learning.

In Verschaffel et al.'s (2010) study on mathematical thinking and the learning process, they review recent significant themes and developments in research on elements of the teaching and learning of mathematical knowledge and mathematical thinking. De Corte et al. (2004) developed a model for use in the design of strong settings for the teaching and learning of mathematics. This model has four interrelated elements, which are competence, learning, intervention, and assessment (CLIA). De Corte et al.'s (2004) CLIA model is shown in the figure below.

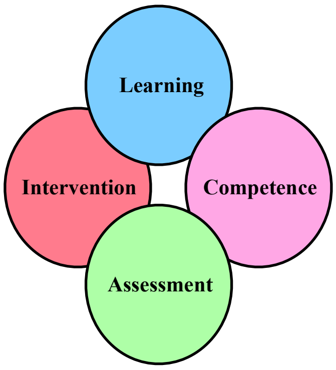


Figure 6.4: CLIA Model (Adapted from De Corte et al., 2004)

In the past few decades, a considerable body of research has emerged which adheres to the view that mathematics learning is an active and collective construction of mathematical meaning, understanding, skills and knowledge, situated within a group of learners but also within a wider socio-cultural context (DeCorte et al., 1996; Verschaffel et al., 2007).

In the opinion of De Corte et al. (1996) and Verschaffel et al. (2007), mathematical thinking is underpinned by three factors, i.e., understanding, skills and knowledge.

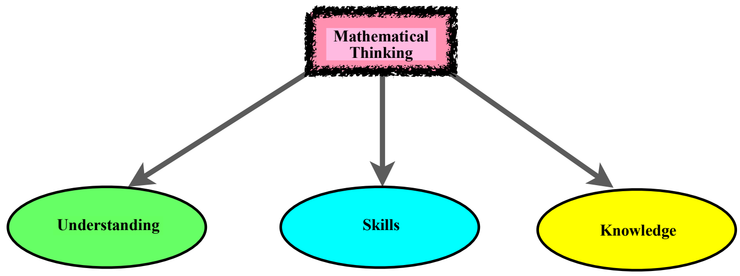


Figure 6.5: Mathematical Thinking (Sources: Adapted from De Corte et al. (1996) and Verschaffel et al. (2007)

It has become widely accepted among mathematics educators, that mathematical thinking and learning involve an active and constructive process (National Research Council, 2001), and there is considerable empirical evidence to support this contention. Ginsburg et al. (1998) and Nunes (1992) argue that this process can be observed in the accurate procedures of calculation that are used by students both within and outside the school setting. De Corte (2010, p. 90) holds that what is essential in the constructivist view of learning is the

"*mindful and effortful involvement of learners in the processes of knowledge and skill acquisition in interaction with the environment (physical and social) and building on their prior knowledge*."

The constructive nature of learning mathematical thinking can also be seen in a negative sense in misconceptions, and flawed procedures which are acquired by many learners (De Corte et al., 1996; Verschaffel et al., 2007).

It is argued by Verschaffel et al. (2007) that the cumulative feature of learning, which has a very close association with its constructive nature, refers to the central part played by previous knowledge, in ways which can be either positive or negative. They further explain that research on conceptual change has resulted in considerable evidence that, similarly to scientific reasoning, students first build a framework theory of numbers, which is arranged according to certain basic central principles or presuppositions, from their experience out of school with natural numbers, and that these framework theories encourage certain types of learning while restricting others. This concept may be very clearly understood from this example; an early understanding of natural numbers and their properties lends great support to further learning in the field of natural numbers. However, it may act as an obstacle to learners' comprehension of properties and operations of numbers beyond the natural ones, e.g., rational numbers, integers, and real numbers. Verschaffel et al.'s (2007) contention is illustrated in the figure below.

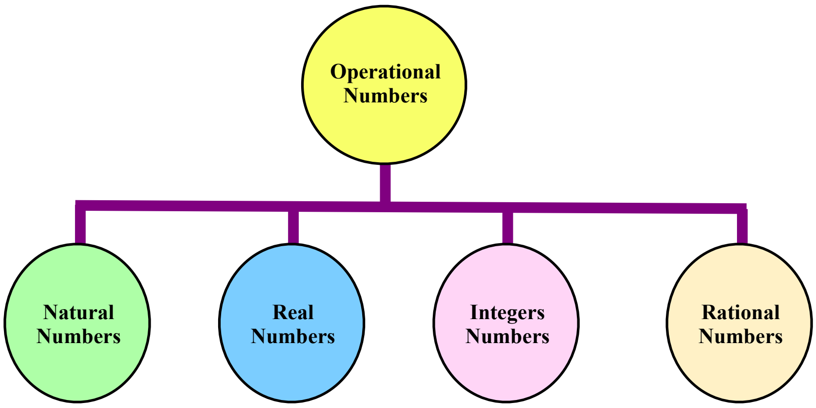


Figure 6.6: Properties and operation of Number (Source: Adapted from Verschaffel et al., 2007)

The view of Verschaffel et al. (2007) that the cumulative feature of learning, which is very closely associated with its constructive nature, refers to the central role of prior knowledge, again in both positive and negative ways, is supported by Moscardini (2009). In constructivist theory, knowledge is held to be actively constructed by the learner (Moscardini, 2009). Moreover, Moscardini (2009) reports that there is a persuasive argument, underpinned by constructivist theory, that mathematics learning should be a process of sense-making (Twomey-Fosnot and Dolk, 2001; Anghileri, 2000; Fennema and Romberg, 1999; Hiebert et al., 1997). Moscardini (2009) further states that research in classroom practice (e.g., Watson, 1996) has shown the effectiveness of constructivist approaches with learners with moderate learning difficulties. Indeed, Watson (2001) calls for constructivist practices to be developed across the curriculum. In terms of mathematics learning, the constructivist approach involves children constructing mathematical relationships for themselves (Twomey-Fosnot and Dolk, 2001; Carpenter et al., 1999; Askew et al., 1997; Hiebert et al., 1997).

Hence, Carpenter et al. (1999) contend that when concrete materials are used by learners to make sense of mathematical problems, this is consistent with a constructivist approach. Teachers' knowledge and beliefs about learners (Yackel and Rasmussen, 2003; Franke and Kazemi, 2001; Carpenter et al., 1989) and about pedagogy (Carpenter et al., 1988; Shulman, 1986) are associated with the extent to which all learners are offered opportunities to use concrete materials to support the construction of mathematical relationships.

6.8 Ethnomathematics Approach

The term 'ethno' describes "all of the ingredients that make up the identity of a group: language, codes, values, beliefs, community, class, food and dress, habits, and physical traits" (D'Ambrosio, 1987, pp. 2- 3). The term 'mathematics' is used to expresses a "broad view of mathematics which includes arithmetic, classifying, ordering, inferring, and modeling" (D'Ambrosio, 1987, pp. 2-3). Hence, the term 'ethnomathematics' is used to express the relationship between culture and mathematics (D'Ambrosio, 2001, p. 308). An understanding of these terms allows teachers to expand their mathematical perceptions and more effectively instruct students in a climate of increasing diversity in schools, and in the wider society.

According to Sfard (1998), the cognitivist view holds that the acquisition metaphor of learning focuses on individual enrichment through the development of knowledge, skills and attitude. In contrast, the situated perspective converges with the participation metaphor that Sfard (1998) identifies, stressing that learning takes place essentially in interaction with social and cultural contexts, and in particular, through participation in cultural activities (Schoenfeld, 2006; Sfard, 1998; Verschaffel et al., 2007). The ethnomathematics perspective has come to be widely used in the conception of learning and cognition as socio-culturally situated in the community of mathematics (D'Ambrosio, 2006). D'Ambrosio (2006) further explained that the outcomes of a large body of ethnomathematical studies of the informal calculation procedures, problem-solving strategies, and learning mechanisms of particular groups of children and adults involved in everyday cultural practices such as carpentry, tailoring, cooking, etc. have contributed to the popularity of the situated approach (e.g., De Corte et al., 1996; Nunes, 1992; Schoenfeld, 2006). Although the situated nature of learning has been documented particularly thoroughly within out-of-school contexts, it is clear that this situatedness is also applicable to school learning (D'Ambrosio, 2006).

For instance, Verschaffel et al. (2000) argue that students' ''suspension of sense-making'' when doing school word problems can be considered as another line of evidence for the importance of the situatedness of mathematical thinking and learning.

Verschaffel et al. (2000) pointed out that the particular significance of the situative view from an educational perspective is that it implies the importance of, and thereby reinforces the case for, interaction and collaboration in learning. There is a broad consensus among mathematics educators that learning is not an isolated and internal activity but that it is an activity involving the individual student, his/her associates in the learning context, and the available resources and tools (Verschaffel et al., 2000).

Yackel and Cobb (1996) view social interaction as essential for mathematics learning, with the construction of individual knowledge taking place through processes of interaction, negotiation, and cooperation. While the situated approach has played an important part in de-emphasising individual internal processes and emphasising the sociocultural and collaborative characteristics that had previously been ignored, it also led to certain ill-advised claims about mathematics learning and unsuitable educational recommendations (Vosniadou and Vamvakoussi, 2006). Among these are that learning is a key that is always based on real situations, hat the importance of knowledge has diminished, that knowledge does not transfer between tasks and that there is little to be gained from simulating abstraction (Vosniadou and Vamvakoussi, 2006).

Hence, a number of researchers have cautioned against abandoning the individual acquisition perspective of (mathematics) learning and have called for an appropriate balance between the individual and social aspects (Sfard, 1998; Vosniadou and Vamvakoussi, 2006). In this respect, Gray (2008), states that, while the relationship between teacher and learner is central, "pedagogy as a system or field is co-constructed with others, including teacher educators, parents, researchers and educational authorities or policymakers". Further, Gray (2008) contends that education is a feature of the political landscape in a way that other bodies of knowledge, such as psychology, are not, although they may be similarly influenced by socio-cultural interactions.

Schoenfeld (2006) argues that one of greatest challenges for the future is to determine how to integrate a number of elements and all that is linked to the individual, in the sense of knowledge, identity, and so forth, as well as in terms of that individual's relationship with various communities and the communities to which the individual belongs.

6.9 The Investigative Approach

The National Council of Teachers of Mathematics (NCTM) (1989, 1991, 2000) called for educational reforms to encourage the mathematical power of all learners. According to Baroody and Coslick (1998), mathematical power involves

(a) a positive attitude towards the learning and use of mathematics, including the confidence to face new and challenging problems;
(b) understanding, including the ability to explain and justify the rationale for a procedure; and
(c) inquiry skills, such as the ability to solve problems

According to Baroody and Coslick (1998), the best method of fostering all aspects of mathematical power may be to use what they term the "investigative approach". This approach sees instruction as purposeful, meaningful, and inquiry-based. Purposeful instruction starts with a task that is interesting and challenging and gives rise to a desire to learn or practise mathematics.

The traditional skills approach to mathematics instruction is based on the view that mathematics learning and thinking is an essentially individual activity, involving mainly the memorisation decontextualised and fragmented knowledge and procedural skills transmitted by the teacher (De Corte, 1995). However, it should be noted that these characteristics can also be distinguished from the *laissez-faire* approach to mathematics education, which emerged at the beginning of the period of the implementation of the reform in a number of countries such as the USA and the Netherlands. Rather, they generally fit with the investigative approach, i.e., a combination of a conceptual and a problem-solving approach, aiming at the mastery of basic skills, conceptual learning, and mathematical thinking and characterised by both meaningful and inquiry-based instruction and purposeful learning and practice (Baroody, 2003).

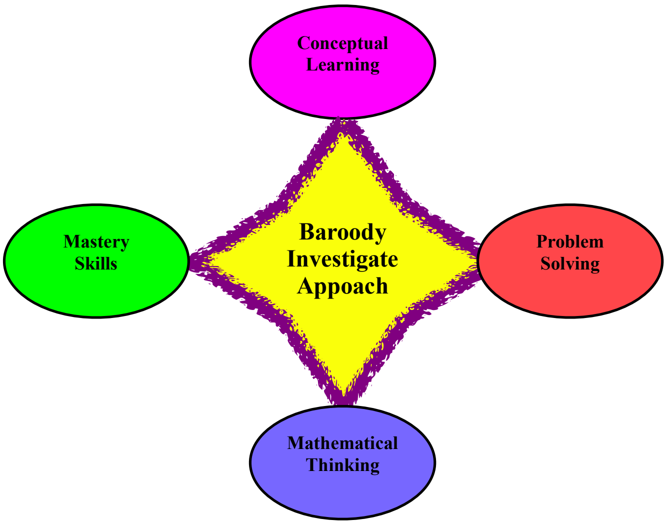


Figure 6.7: The Investigative Approach (Source: Adapted from Baroody and Coslick, 1998)

6.10 Problem-Solving Approach

Lee (2011) conducted her study on how alternative solutions affected the performance of problem solving. She suggested that problem solving is generally accepted as a way of promoting thinking skills (e.g. Schoenfeld 1985). For example, the National Council of Teachers of Mathematics (NCTM) Standards state: *"Solving problems is not only a goal of learning mathematics but also a major means of doing so. ... In everyday life and in the workplace, being a good problem solver can lead to great advantages. ... Problem solving is an integral part of all mathematics learning"* (NCTM, 2000). Further, the NCTM recommends that students should develop their "*flexibility in exploring mathematical ideas and trying alternative solution paths."*

Hiebert et al. (1997) proposed that students should be given problem solving tasks for which they *"have no memorized rules, nor for which they perceive there is one right solution method"* (p. 8). However, Silver et al. (2005) state that in spite of the general agreement that students should consider more than one way to solve mathematics problems, this practice is seldom used in US classrooms.

Alternative solutions are a valuable method in the instruction of effective problem-based mathematics (Cai, 2003). The teaching of mathematics through problem solving offers a learning environment for students to investigate problems alone and to find ways to solve them. Such activities permit them to connect related concepts, to integrate their mathematical knowledge, and to use creative thinking (Polya, 1973; Kalman, 2004; Krulik & Rudnick, 1994). Schroeder & Lester (1989) suggest that teaching through problem solving has the potential to encourage student learning.

Problem solving with alternative solutions may encourage students' mathematics learning, but there are few empirical studies focusing directly on the ways in which mathematical problem solving with alternative solutions could influence the problem solving performance of students (Groβe & Renkl, 2006; Silver et al., 2005). However, contradictory results have been found regarding the relationship between problem solving performance and alternative solutions. Although some studies found that multiple solutions or representations improved students' problem solving performance (Fouche, 1993; Brenner et al., 1997), others found no indication of this (Brenner & Moseley, 1994). This investigation was an attempt to understand better how teaching alternative solutions could influence students' problem solving performance.

The discussion above demonstrates there is considerable evidence in the literature to support the view that effective mathematics learning is a constructive process involving building knowledge and acquiring skills. It also involves many opportunities to interact, negotiate and collaborate. Hence, this novel, integrated approach to learning mathematics has encouraged both academics and practitioners to take these basic features of learning as principal guidelines to design of new curricula, textbooks, and assessment instruments which could assist in encouraging students to acquire a mathematical disposition.

With regard to assessing learners, in mathematics education, examinations and tests verify what they have acquired in terms of knowledge, understanding and, occasionally, thinking skills. Decisions about assessments and future learning are based on cognitive skills: knowledge, comprehension, application, analysis, synthesis and evaluation. However, attitudinal objectives are general statements of syllabus philosophy that stress the importance of what students bring to the learning situation or derive from the learning situation - sets of attitudes, perspectives, values and beliefs. These elements of affective abilities are rarely measured at all; indeed, measurement may be impossible or inappropriate. However, that does not mean that such aspects are unimportant. It is useful to investigate how attitudes shape educational planning and practice. It is known that they are important influences in learning behaviour and performance.

As an example of this, in a study of factors affecting more than 500 first-year Australian students, Cretchley et al. (2000) found attitudes to mathematics a considerable influence on their option to take mathematics at university. In almost all countries, there is a view that mathematics is a ‘difficult’ subject and that students do not often have a positive attitude towards it. In this respect, the textbook and its questions may be one of the influences on the formation of a positive attitude to mathematics on the part of learners, in particular if they assist learners in becoming aware of the usefulness of mathematics in their daily lives. Educational research in a wide variety of contexts indicates considerable failure of students to perform in mathematics (e.g., Haylock, 1991; Schoenfeld 1994; Christou, 2001; Alenezi, 2008; Ali, 2008). The fundamental question that arises is why mathematics is perceived in this way. Indeed, students' attitudes towards mathematics and how, according to teachers and inspectors of mathematics, they may be influenced, are the subject of the fifth research question in this present study.

6.11 Conclusion

The concept of thinking has been discussed and the way learning is understood with a connection to the learning of mathematics has been reviewed.

Mathematics can assist us in understanding our environment, in managing data and measurement, as well as being indispensable in science and other disciplines. The teaching of mathematics as an integrated subject has been generally recognised, including in Saudi Arabia. Giving students a mathematical education is a more complex proposition than simply teaching them mathematics. However, in many countries, such as Saudi Arabia, mathematics teaching in schools is simply a question of methods, procedures, rules, and algorithms. The approach taken is one of ‘doing’ mathematics rather than ‘thinking’ mathematics. In Saudi Arabia, as in many other countries, traditional approaches to teaching, curriculum development and assessment are adopted.

Learning has been explored by many and the insights gained can be related to learning mathematics. Some of the key understandings have been reviewed in this thesis. Learning refers to the acquisition new knowledge, behaviours, skills, or values or the modification of previous ones. This process may involve the synthesis of various types of information.

Johnstone (1997) described learning thus:

> *"Learning is the reconstruction of material, provided by the teacher, in the mind of the learner. It is an idiosyncratic reconstruction of what the learner understands, or thinks he/she understands of the new material provided, tempered by the existing knowledge, beliefs, biases, and misunderstandings in the mind of the learner."*

Although it has fallen out of favour with the majority of modern-day educationalists, one general learning approach that has a specific application to mathematics is the behaviourist approach. According to Orton (2004), there is a clear difference between necessary repetition, and rote learning. In this regard, Dienes (1979) stated that *"... no one today doubts any more the fact that the stimulus-response relation leads to a training which most of the time induces mental blockages".* Nonetheless, repetition has been shown to play an important part in fixing knowledge in the mind (Dienes, 1979). Indeed, in mathematics, certain aspects simply need to be memorised: tables, axioms etc.

Piaget saw learning as very similar to biological growth and showed the way cognitive skills developed with age. He also established that the natural way for the child was that of trying to make sense of the environment and experience. This led to the ideas of constructivism and the whole area of information processing. This can be illustrated as in Figure 6.8.

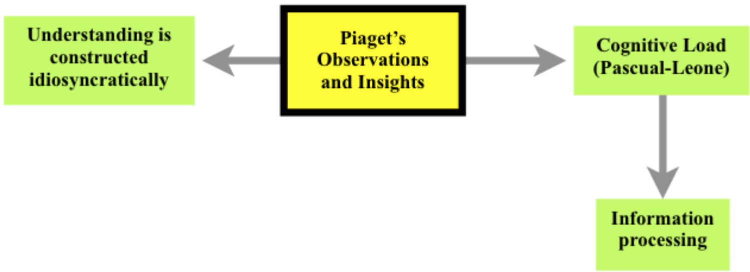


Figure 6.8: Relationship of Cognitive Theories (Source: Author)

According to DeVries (1997), constructivism, introduced in Piaget's early work, is a perspective of cognitive development in which students actively construct systems of meaning and understandings of reality through their experience and interactions.

Ausubel (1968) introduced the assimilation model of learning in which meaningful learning and rote learning were contrasted. Although incorporating Piaget's basic concept, he was critical of the emphasis placed on the effectiveness of discovery learning.

Gagné (1988) viewed learning as a process of alteration in human ability, although he saw this in terms of instruction in the context of the armed forces rather than schools. However, his findings may be very important in a subject such as mathematics. He took the view that the learning process does not depend on the growth process and this alteration in human ability applies to different performances of individuals that are of long duration. According to Gagné (1988), although learning is an internal process which cannot be observed, what takes place within the learner can be inferred from the outcomes of the learning. Gagné's five categories of learning outcomes can be seen in the figure below.

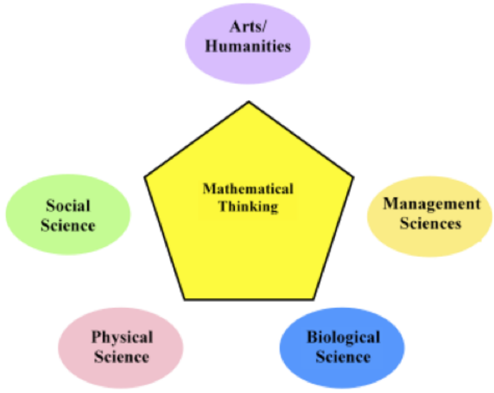


Figure 6.9: Categories of learning outcomes (Source: Author, based on Gagné, 1988)

Pascual-Leone, a student of Piaget, developed Piaget's ideas by asking why Piaget had observed what he observed as the child developed. He appreciated that the explanation lay in information load and this laid the basis for later insights form information processing (Pascual-Leone, 1970).

Information processing considers the information flows through the brain: the manner in which information enters the mind through the senses, and how it is stored in and retrieved from memory. It is an attractive theory as it provides useful experimental methodology and an accessible language (Miller, 1993). The structure of effective learning is seen in information processing models as the ability to store useful knowledge in the long-term memory. Knowledge is viewed as being coherent and holistic and as providing a basis for subsequent learning (Atkins *et al*, 1992). Indeed, all cognitive learning processes occur in the working memory. While the part played by the working memory cannot be ignored in learning mathematics, the way mathematical understandings are organised, stored, and then employed, may be of enormous importance.

The general concept of thinking connected to learning mathematics has also been examined in this thesis. There are three main thinking skills involved, which are critical thinking, scientific thinking and creative thinking. There have been a great number of studies examining the meaning of critical thinking. There is general agreement that such skills are valuable and should be an essential part of the school and university curricula. However, fewer studies have discussed methods by which this could be achieved and fewer still have provided evidence as to the assessment or achievement of such methods.

Regarding scientific thinking, this centres on the role of the experimental as a source of evidence and the hypotheses can be tested experimentally. Scientific thinking, the scientific method and the scientific attitude have been the subjects of a considerable body of literature, although it should be noted that there is a considerable overlap between these concepts (Al-Ahmadi, 2008). It is stated in numerous curriculum documents that the development of such skills is involved in teaching scientific subjects but little evidence is available on how to do this or how to measure its success, the one exception found being the study of Al-Ahmadi (Al-Ahmadi and Reid, 2011, 2012).

The principal goal of the sciences could be viewed in terms of providing insight into and understanding of the world around and the scientific method could be seen as the approach used in the sciences for that purpose. Scientific thinking is intrinsic, although not exclusive to, the scientific method, while critical thinking could be viewed as considerably broader and of significance in all disciplines. It is also problematic to offer a precise description of creative thinking (Alenezi, 2004).

Mathematical thinking is the focus of this present study, rather than the three other aforementioned thinking skills (i.e., critical thinking, scientific thinking and creative thinking.)

Schielack et al. (2000) classified mathematical thinking skills into six categories, the first of which is modelling (i.e., the use of tables, diagrams, graphs, charts and so forth). The second category is inference, which refers to, for example, making generalisations. The third is symbolism, while the fourth is logical analysis, which includes the comparison of results. The fifth category is abstraction, and the sixth is finding the optimum (i.e. the most effective) solution. In this present study, mathematical thinking is held to comprise the skills of knowledge and recall; understanding and interpretation; modelling; application; induction; generalisation; deduction; mathematical proof, and; evaluation.

There appears to be general agreement that two characteristics are salient: newness and the perception of usefulness. Table 6.1 offers brief definitions of the thinking skills described above.

Skill	***Definition***
Critical Thinking	*"Active, persistent and careful consideration of belief or supposed form of knowledge in the light of the grounds which support it and the further conclusions to which it tends."* (Dewey, 1933)
Scientific Thinking	*"The unique characteristics of scientific thinking relating to the nature, place and handling of experimentation, including the place of hypothesis formation"* (Al-Ahmadi, 2008)
Creative Thinking	*"The utilisation of remote relationships with ideas in cognitive structure to produce new products."* (Ausubel and Robinson, 1969)
Mathematical Thinking	*"A style of thinking that is a function of particular operations, processes, and dynamics recognizably mathematical"* (Burton, 1984).

Table 6.1: Definitions of Thinking Skills (Source: Author)

The problem lies with the description of mathematical thinking for it is almost a cyclical argument. It describes mathematical thinking in terms of 'recognizably mathematical'. It is perhaps better to see mathematical thinking in terms centring around the understanding of relationships, involving variables and spatial relationships (or both).

This addresses some of the issues in the following questions:

- What is the nature of mathematical thinking?
- How does mathematical thinking differ from scientific, creative, or other types of thinking?
- What are the main characteristics of mathematical thinking?
- Does a mathematical habit of thinking exist?
- If so, can this habit be learned by students?
- Do mathematicians also think mathematically about non-mathematical problems?
- To what extent is mathematical thinking relevant to problems in everyday life?

In this thesis, further explanations regarding these questions have been given in the chapter on mathematical thinking.

It appears that understanding the nature of mathematical thinking should assist in understanding the processes of thinking in physical science and biological science as well as in the social and management sciences, the arts and humanities and in everyday life. Hence, every student should be educated to understand the nature of mathematical thinking (Kapur, 1997). This is illustrated in the figure below.

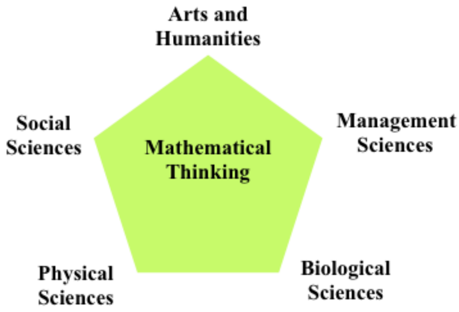


Figure 6.10: Mathematical thinking and other disciplines (Source: Author, adapted from Kapur, 1997)

Maki and Thompson (1973) assert that mathematics is useful in physics, biology and in the social and management sciences, as these sciences use mathematical techniques, and, more importantly, the mathematical habit of reasoning. The involvement of the mathematical thinking habit in these sciences is clear, although they may require the use of only a few mathematical symbols and only elementary mathematics.

Stein (1963) contends that both mathematics and the sciences are both products and processes, but that the process aspect of mathematics is even more important than the product aspect. This is because, while the product may be basic to modern society, the process is essential for its further growth. However, it is frequently the product aspect of mathematics that is imparted to students. They should be apprised of the process aspect, which essentially refers to the mathematical thinking process (Stein, 1963). Although students are informed they are learning the mathematical habit of thinking as a corollary to their learning mathematics, they are not informed what this thinking is.

Mathematics is essential for the understanding and further development of science and technology. The place of mathematics in education must be determined by the society and the culture which the education is intended to serve.

Mathematics holds a central position in the curriculum and has gained the status of a universal language, which enables individuals to express their concepts of quantity, shapes, and relationships (Mahdi, 2006). The advent of the computer has resulted in a change in the traditional theories of mathematics, as the extent of applications of mathematics was previously limited. Mathematics should be conceived of as integrated with the students' environment. In addition, mathematics plays a key role in science, technology, industry, business, and agriculture (Gall and Hicks, 1964). The study of mathematics has been linked to intellectual independence, effective thinking habits, and creative expression; however, it could be argued that these objectives had been neglected until modern technology aroused awareness of them once again (AlAbsi, 2009).

Indeed, Onwumere (2009) argues that the technological and scientific advances of recent years make it necessary for schools to emphasise the development of a learner's understanding and appreciation of mathematical procedures and methods of reasoning. Imparting knowledge and understanding to students should be done in such a way that expert knowledge can be built upon to solve new problems and to take meaningful decisions as citizens in a great variety of applications (Onumwere, 2009). If learners possess adequate cognitive skills and positive attitudes, then it is likely that the learning of mathematics will be successful and/or fulfilling.

The focus of the next chapter will be drill and practice in the textbook.

CHAPTER 7

DRILL AND PRACTICE AND THE TEXTBOOK

7.1 Introduction

In this chapter, literature on ways of learning mathematics will be reviewed. The chapter begins by discussing various ways of learning mathematics and continues by discussing curricula, drill and practice, conceptual understanding and reasoning. It continues by discussing the nature and concept of the textbook, the requirments for textbooks, the development of the concept of the textbook, criteria for a good textbook, the responsibility for textbook development and the key elements of a good textbook. This is followed by a section on the role of the textbook and its questions in mathematics instruction and the ways in which textbooks can be evaluated.

7.2 Ways of Learning Mathematics

Niss (1996) identifies knowledge of content matter, skills, understanding, attitudes and degree of independence of study as the basic aims of mathematics instruction in upper secondary school and early university or college education. Niss (1996) also discusses the goals of upper secondary and lower tertiary education as they appear to be perceived in many countries around the world. He contends that

> *higher level mathematics education should be provided for large groups and prepared for private and social life in society. Students should be able to exercise representative mathematical thinking and creativity and engage in*

> *non-routine, open situations - in exploration; representation; conjecturing; problem posing/formulation/solving; reasoning and proving, all in relation to mathematical facts, concepts, methods and theories relevant to each educational level's goals*.

It is important to convey to students that mathematics is based on reasoning and is not merely a collection of random rules; therefore, reasoning, explanation and proof at an age-appropriate level should be a major part of learning mathematics for students of all ages (Vincent and Stacey, 2008). Research has shown that in reality mathematical reasoning is uncommon in many mathematics classrooms (Vincent and Stacey, 2008) and in this regard, the second research question in the present research addresses the extent of the emphasis placed on the development of mathematical thinking in the questions in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia. Similarly, as Lithner (2004) points out, there are problematic discrepancies between goals and practice, and teaching may frequently place too strong an emphasis on knowledge of facts and procedural skills while neglecting understanding and creativity. In addition, teaching without a sound knowledge of mathematics may also be a common problem.

Ali (2008) states that difficulties in learning in particular areas of mathematics are very widespread and can be found in many countries, and argues that it is therefore unlikely that teachers are the cause of this. She points out that teachers do not make the curricula, but are obliged to follow the syllabus, textbooks and other resources supplied by the education authorities and seldom have much freedom to make any modifications or changes in the curricula. This reflects the situation in Saudi Arabia, where it is the Ministry of Education that makes all decisions concerning the curriculum and textbooks, which are the same for the whole country, and teachers have to teach what is prescribed.

7.3 Curricula

Teaching approaches, teaching strategies and the order of topics covered are usually decided upon by the logic of mathematics as a discipline and educationalists usually determine what they view as the logical order for courses in mathematics (Ali, 2008). Ali (2008) suggests that problems may arise because topics are not presented at the appropriate time for students. That is to say, curricula, textbooks and textbook questions may not always be in alignment with students' cognitive development. Piaget viewed cognitive development as an extension of biological development (Wadsworth, 1979, p. 2). Hence, just as certain physical tasks are suited only to children at particular stages of biological development, if students are presented with topics that are above the level of their cognitive development this may well make them feel frustrated with their lack of success. This in turn can lead to them developing a negative attitude towards mathematics. Oraif (2006) found that success engenders confidence and it therefore follows that a lack of success will give rise to a lack of confidence and hence a lack of achievement and enjoyment in mathematics. However, at the same time, it should be pointed out that, according to Reid (1978), difficulty does not necessarily lead to negative attitudes; and, in the same vein, Reid and Skryabina (2002) found that a challenging task *per se* does not lead to a perception of difficulty. In this respect, the present research will attempt to determine the effect on students' attitudes of the questions in the mathematical textbooks through answering research question 5.

Similarly, Alenezi (2008) points out that mathematics curricula are seldom designed by practising teachers but rather by mathematicians who may fail to appreciate the difficulties many students have in learning mathematics. This can lead to the inclusion of material which poses problems for learners, and the teachers must then find ways to teach material which is not easily accessible to the students (Alenezi, 2008). Ali (2008) argues that this can lead to further problems in terms of assessment, as teachers do not determine national certification but it is they who may be held to be at fault if their students are not successful. This can result in teachers' relying on rote learning and the memorisation of procedures, which often means that students' deeper understanding of what they are studying is neglected (Ali, 2008).

Al-Ajroush (1980) argues that the curriculum reform process in Saudi Arabia must involve identifying the problems in the curriculum, the development of the syllabus and formulation of the textbook. In line with this, the present research attempts to identify problems in the textbook questions from the perspectives of teachers and inspectors, with the aim of improving the textbook questions in order to encourage students' mathematical thinking skills.

The Saudi Ministry of Education states that curriculum reform should be done by a specially appointed committee. However, Al-Ajroush (1980) states that at all levels, the curriculum is not suitable to the needs of development because it is prepared by the committee, without the participation of the main stakeholders (teachers, parents and students). The main function of the district educational authorities is only to act as an intermediary in the transmission of regulations from the Ministry of Education to schools (Al-Ajroush, 1980).

According to Al-Ajroush (1980), significant failings in the provision of the curriculum in Saudi education are that the decision-making is removed from daily life and teachers, head teachers, parents and students do not take part in curriculum formulation. Although Al-Ajroush (1980) was writing about the situation in Saudi schools more than thirty years ago, it is arguable that the situation he describes has not altered since that time. As mentioned previously, the role of teachers in matters regarding the curriculum is to teach the syllabus to the students, but they are not involved in curriculum development.

In this respect, Al Sadan (2000) suggests that schools would perform better if teachers worked with a curriculum that they had participated in developing, as this would give them a greater sense of engagement in the work they were doing. Al-Saif (1996) reported that the involvement of teachers in creating and making decisions about the curricula they were delivering was found to have a strong positive relationship with better student performance.

In this respect, this present study has sought the opinions of the teachers and inspectors of mathematics in Saudi Arabia, as they are most closely involved with daily educational activities, on the premise that if their opinions are taken into account, this is likely to lead to an overall improvement in textbooks, curriculum, teaching, learning, and assessment.

A number of researchers have found that many mathematics students at all school levels frequently use mathematically superficial reasoning for solving tasks (e.g. Schoenfeld, 1991; Tall, 1996; Verschafel et al, 2000). Similarly, studies of upper secondary and undergraduate students indicate that reasoning about what is familiar and remembered on a surface level takes precedence over reasoning based on mathematical properties of the components involved, even when the latter could result in considerable advance (Bergqvist et al, 2003). The students' perspectives seldom appear to include this mathematical reasoning as a main approach, even if they have grasped the requisite knowledge base. Palm et al (2005) contend that, since students seldom make any attempts to construct their own solution reasoning, it is crucial for them to find solution procedures to copy and the choice of procedure to use is often made on mathematically superficial grounds. The reliance on such reasoning is unlikely to be efficient for the learning of advanced mathematical thinking or for achieving relational understanding (Skemp, 1978) of basic mathematical concepts and ideas. In addition, it is likely to be wholly unproductive in terms of finding solutions for non-routine tasks for which there are no ready-made procedures for finding the solution readily available to the students. Indeed, a large body of research has shown that many students of various age groups have considerable difficulties solving non-routine tasks (Schoenfeld, 1985; Selden et al., 1994; Verschafel et al., 2000). Moreover, the focus on remembering procedures superficially related to the task at hand also limits the possibilities of success in routine tasks when the procedure is forgotten or an error in the procedure is made (Palm et al, 2005).

The reasons for this focus on mathematically superficial reasoning may be found in the students' learning environment. Indeed, there has been a great deal of criticism of the programme of stereotypical tasks that the students in elementary and lower secondary school encounter (e.g. Reusser, 1988; Verschafel et al., 2000).

In recent years many countries have attempted to develop mathematics curricula and have exerted considerable efforts to enhance their teaching methods. Among these countries is the USA, which started to reform its mathematics and science curricula in the late 1950s after the then-Soviet Union launched an unmanned spaceship in 1957. This led the USA to mobilise its full energies and capabilities in order to catch up with the Russians. They decided that one way to reach this goal was by developing the mathematics and science curricula. The development of the mathematics curricula in the US was followed by a strong international movement in other advanced countries to develop the methods of mathematics teaching and learning in what was known as the New Math movement.

At that time, special attention was paid to the new mathematics curricula in many countries, to developing mathematical thinking in the students' minds in these countries, and helping them to acquire a thinking technique that relied on a sound mathematical foundation (Abu Zeenah, 1986). This was because mathematics is a unique discipline and hence a suitable milieu in which to train students in the appropriate patterns and methods of thinking. According to this rationale, the habit of fruitful and effective thinking is essential for mathematicians.

The curricula of modern mathematics stressed the need to help students acquire and develop the ability of logical thinking (Al-Sheikh and Abu Zeenah, 1985; Jansson, 1986), as logical thinking is a necessity and indispensable in the processes of knowledge acquisition, problem solving and decision making, as well as playing an important role in mathematical debate and mathematical proof. Some of the mathematics curricula development committees were so concerned with the development of mathematical thinking that they unequivocally called for the necessity of teaching students the rules and basics of mathematical logic and thinking directly (Al-Sheikh and Abu Zeenah, 1985).

On reviewing the list of objectives in the modern mathematics curricula, some items that involve certain skills of mathematical thinking can be seen. According to the results of the Third International Mathematics and Science Study (TIMSS), mathematics textbooks in use in the USA, in the 1980s, when compared to textbooks from other countries, appeared to lack “focus and coherence” and failed to provide “meaningful connections between the big ideas of mathematics” (Valverde and Schmidt, 1997/1998, p. 63). The increasing dissatisfaction with traditional mathematics textbook instruction in the USA and the mathematics performance of US students led to a new approach to conceptualising the teaching and learning of mathematics (Sood and Jitendra, 2007). Reform efforts at national level to improve the content and quality of the mathematics curriculum and instruction were undertaken (Jitendra et al., 2005). As a result, the National Council of Teachers of Mathematics (NCTM) issued Curriculum and Evaluation Standards for School Mathematics (the Standards) in 1989 followed by the Principles and Standards for School Mathematics (Principles and Standards) in 2000 (Schoenfeld, 2002). The Standards emphasised a move from direct instruction, rote learning, drill, and practice toward more active student engagement with mathematical ideas (Goldman et al., 1997). Accordingly, new mathematics textbooks were designed with the aim of developing conceptual understanding by engaging students in problem-solving, with an emphasis on reasoning and mathematical thinking (Remillard, 2005).

Mathematics curricula aim to create a mathematical and logical way of thinking in the minds of students, and to develop their problem-solving ability, use of scientific methods in thinking, and higher order thinking skills such as formulating and testing hypotheses, analysis, induction, deduction, recognition of patterns, reasoning, application, evaluation, proof, questioning, representation and interpretation of data, and linking of concepts.

In this respect, mathematics curricula should help students acquire the following skills:

- The habit of effective thinking, which encompasses analytical thinking, critical thinking, inferential hypothetical thinking, deduction by similarity, and the development of mental enquiry.
- The communication of ideas to others using symbols and graphs.
- Development of the ability to pass suitable judgments and decisions, and come to the right conclusion.
- Distinguishing between relevant and irrelevant data.
  (Abu Zeenah, 1986).

β and Rossman (1997) stressed the importance of the mathematics curriculum in creating mathematical thinkers by developing students' abilities in problem solving, inference, and logical thinking, and by presenting mathematical topics to them in an interesting and pleasant manner. To this end, they suggested that the mathematics curriculum do the following:

Draw the attention of the students to their role in learning mathematics, by making them the centre of teaching and learning.

Encourage students to make sense of what they have already learned before making efforts to increase their knowledge, both quantitative and qualitative.

Pay particular attention to mathematical problems, giving the students the opportunity to immerse themselves in the thinking process by using different strategies to solve the problems.

Martin (1996) suggested that the characteristics of the effective mathematics curriculum should include the following:

Present the mathematical topics in a coherent manner to help students link different mathematical topics and see the relations between them.

Give students opportunities to use mathematics in their daily lives.

Integrate mathematics with other subjects taught to the students.

Based on the above, effective educational mathematical experiences and activities must be designed to motivate the students to generate knowledge based on their prior knowledge and by using thinking skills they have already developed. Such activities and experiences must fulfil the desired objectives and make the students think about what they have learnt from such activities and experiences with respect to the thoughts and feelings it adds to their consciousness and behaviour. The understanding and development of thinking will only be achieved when students are given the opportunity to add meaning to activity, and when they use their brains to interpret the activity, reflect on its meaning and link it with their cognitive structures (Al-Sheikh, 1999).

Again on the topic of the curriculum, Al-Rasheed's (1987) study aimed to identify the extent to which the mathematics curricula in upper primary school conform to the thinking levels of the students at this stage. Their main research question was: To what extent does the content of the mathematics textbooks in upper primary school conform to the cognitive development of the students at this stage?

From this question, two sub-questions were derived:

What is the distribution of the upper primary school students in terms of Piaget's cognitive levels?

What cognitive levels should be represented in the textbook content in the upper primary school mathematics curricula?

The study sample was made up of 2889 male and female students from the 4^{th}, 5^{th}, and 6^{th} grades of primary school. The sample was chosen randomly from 22 schools, some of which were affiliated to the Directorate of Education, some of which were private, and some of which were schools affiliated to the United Nations Relief Agency (UNRA), all in Amman, Jordan.

In order to determine the distribution of these students in terms of the stages of cognitive development, the researcher used three thinking tasks: a task of preserving quantity, which distinguishes between the stages before the processes and the early material cognitive stage; a task of preserving size, which distinguishes between the early material thinking stage and the late material thinking stage; and a proportionality (simple balance) task, which distinguishes between the late material thinking stage and the early abstract thinking stage.

The determination of the thinking levels by means of the cognitive development stages that are required by the subject of mathematics was made by developing standards to determine these levels through the cognitive development stages in this subject as presented in the mathematics textbooks in upper primary school. The mathematics material that was included in the study was the number theory unit in the upper primary school mathematics textbooks.

The study used the method of presentation of each task to the students as a group. The students were given papers on each task, and were asked to answer the task items after seeing the presentation. The answers were then classified according to their types in the three tasks and according to the proportion of the students at each stage of cognitive development. The findings of the study indicated that the upper primary school students were at cognitive development stages ranging from the pre-process stage to the late material process stage.

The study findings also indicated that the determination of the thinking levels by means of the cognitive development stages where the mathematical concepts and processes of this unit exist, were distributed among the cognitive development stages starting from the stage before the processes and proceeding up to the early abstract thinking process. The comparison between the thinking levels that represent the mathematical concepts and processes of the number theory unit, with the cognitive development stages among which the upper primary school students were distributed, indicated that the students' cognitive structure at this stage does not enable them to understand all the mathematical concepts and processes presented to them in this unit (Al-Rasheed, 1987).

The aim of Brown's (2002) study was to investigate the effect on students' achievement of using a mathematics curriculum developed according to NCTM standards. The sample comprised students from schools in two school districts in Western Michigan, USA. The group in one district used the developed curriculum, while the other used the traditional curriculum. The study instrument was an achievement test to measure the students' achievement in mathematics. The study findings indicated the absence of any statistically significant differences between the achievement of the students who used the developed curriculum and the achievement of those who used the traditional curriculum.

Many studies have explored the decisions teachers make concerning the mathematics tasks they give the students and how the curriculum materials they use act as an intermediary in administering these (Romberg, 1992; Schmidt et al., 1997; Nathan, 2001). The aim of instruction should be to emphasise meaningful learning, which can be seen as reflecting the need for learners to understand sufficiently to be able to use what they have learned, to be able to make sense of the procedures they have been taught and gain insight into how they relate to real-life situations (Onwumere, 2009).

7.4 Drill and Practice

There are various views found in the research concerning the role of drill and practice in mathematics learning. Lithner (2004) states that a primary aim in learning mathematics is to learn what he calls the ‘tools of the trade’, which can be seen as a set of methods that in an algorithmic manner help to make mathematical work more routine in order to save time and effort. These are closely connected to mathematical subject areas: algorithms for arithmetical operations on multi-digit numbers, constructions of geometrical objects, algebraic equation-solution procedures, methods in calculus for maximising functions, proof techniques in analysis, and so forth (Lithner, 2004). One difficulty is that there are so very many methods and procedures it appears to be no easy task to learn them all. One approach that appears to be fairly common, not least among students, is to learn by practising all the various methods and procedures for all special task types they are likely to encounter (Lithner, 2004). Another position is that exercises should enhance understanding of more general methods and ideas, which are applied to special cases. In this sense, the two functions are similar tasks, since they are both composite functions and differentiated by the same general method (Lithner, 2003). Lithner (2003) suggests that the ideal is probably not to teach either procedures or general goals, but both.

The basic facts of arithmetic are sometimes called the “primary facts” (Ashlock, 2009). Grasp of a fact means that a student can give a rapid answer without using a less efficient way, for example, counting (Van de Walle, 1990). According to Van der Walle (1990), "*a developmental approach to the mastery of basic facts involves three components: (1) a strong development of number relationships and operation meanings before mastery activities; (2) the introduction of thinking strategies to help children develop ways to use their conceptual ideas to master facts; and (3) an adequate time allotment for children to develop effective use of conceptual strategies and relationships to master facts.*"

An understanding of the operations and a rapid recall of number facts are necessary to carry out estimation, maths computation, and algorithms; but these skills are just as necessary when using calculators and computers. Rapid recall of the basic number facts for each operation is essential (Reys et al, 2004). According to Phillips (2003), “developing computational fluency is a multifaceted task that underlies all further work with numbers”. Similarly, Burns (2005) stated that in order to master mathematical skills, a student should be able automatically to calculate mathematical facts.

The National Council of Teachers of Mathematics (NCTM, 2000) gave fluent computation as an aim of mathematics teaching instruction, and failing to remember basic facts is frequently linked with lack of mathematics ability (Miller and Mercer, 1997). This is a problem which is met with often when a student is learning to compute (Ashlock, 2009).

Studies have shown that the teaching of basic skills by drill tasks results in students remembering better and therefore performing better in advanced skills (Burns, 2005). Cook and Reichard (1996) stated that knowledge of basic mathematical facts is the basis of complex mathematical skills and that progression to higher skills is made easier by easy recall of facts; hence, time must be allocated to students for mathematics fact practice if they are to achieve a level of automaticity. Rapid and easy recall of basic facts makes problem-solving faster (Shaw and Blake, 1998). For instance, the lack of mastery of basic facts results in the learner being distracted from the mathematical procedure. They become unsure of where they are in the process and proceed aimlessly; it is therefore important that they grasp the basic facts of mathematics (Ashlock, 2009).

For a long time, teachers have debated whether learners should learn the basic facts by rote or otherwise (Riedesel and Schwartz, 1999). If students learn by rote they will be able to remember basic facts quickly (Riedesel and Schwartz, 1999). Eventually they will have to memorise basic facts (Shaw and Blake, 1998). Similarly, Burns (2005) stated that it is quicker to solve a problem by memory than it is to perform a mental algorithm and hence drill and practice can be an effective way to improve learning.

According to Shaw and Blake (1998), memorisation is best performed in brief, regular periods, and timed tests should be readministered on a number of occasions to ensure higher validity. It has been demonstrated that learners who have been taught basic concepts of addition and subtraction using a problem-solving approach are just as knowledgeable about basic facts as learners who have been taught using a fact-memorisation programme (Riedesel and Schwartz, 1999). This may appear surprising, as the learners in the group learning basic facts spent much more time practising basic facts than did those in the problem-solving group (Riedesel and Schwartz, 1999). Students require experience with a number of methods to solve problems and time to discuss their findings. As students use flexible strategies and show greater speed and accuracy, computational fluency will emerge (Phillips, 2003). "Maths is a sense-making, problem-solving mental activity; it is not rote memorisation of isolated facts" (Jones, 1995). Teaching time spent on developing thinking strategies will assist the acquisition of mastery and develop students' confidence in their ability to find the answer when rapid recall is not possible (Hatfield et al., 1993). Memory improvement has three principal components: (1) teaching to all sensory modalities; (2) interesting or useful information is best retained; and (3) new information is best retained if it can be connected to something already stored in the memory (Jones, 1995).

However, Lim (2009) points out that a great deal of drill and practice does not necessarily signify learning. Students should be given various types of problems to solve so as to provide them with various learning experiences about a concept (Lim, 2009). In this regard, this present research seeks to determine the extent of the emphasis on the development of mathematical thinking skills in the mathematics textbooks which are the focus of this study through research question 2.

7.4.1 Conceptual Understanding

Related to learning general methods is the idea of exercises as a means to understanding mathematical concepts, e.g., the concept of function or infinity. To give an example, a salient point in calculus is that calculations related to complicated analysis concepts such as infinity, limits, differentiation, integration, and, perhaps most importantly, differential equations, may often be handled with algorithmic tools that use only elementary algebra (Lithner, 2004). This objective includes making what is non-routine, more routine. For instance, the basic theorem of calculus together with some integration techniques makes the integration of many function types into comparatively easy routine procedures. Integration by other methods, for example, through Riemann sums and limits, leads to procedures that are much more complicated and difficult to make routine (Lithner, 2004). Analysis is more of a theoretical mathematical area, not mainly dealing with calculations and routine procedures. Bodner (1991) states that the difference between problems and exercises concerns familiarity rather than difficulty. Smith (1991) states that although both need similar cognitive tools to reach a solution, exercises are familiar and routine to the learner but problems are usually unfamiliar and non-routine. A completely different question is whether it is possible, as is sometimes advocated in the educational debate, to master the procedures without conceptual understanding and non-routine problem solving abilities, if indeed such abilities exist.

However, the converse may be true; Lim (2009) asserts that merely understanding mathematical concepts is not sufficient, as it needs practice to reinforce and to enhance learning. Therefore, conceptual understanding and practice are both closely related and reinforce each other (Lim, 2009). In addition, the ability to reason mathematically is often emphasised as a goal, but seldom defined unless restricted to the context of proof. One characterisation of skilled reasoning in problem solving is provided by Schoenfeld (1985), based on the mastery of resources, heuristics and control. This mastery is not innate, but needs practice to acquire.

7.5 The Nature and Concept of the Textbook

The textbook is a manual of instruction in any discipline and subject in academic institutions, produced according to the demands of educational institutions. Although the majority of textbooks are only published in printed format, many are now available online in electronic format and increasingly, although illegally, in scanned format on file sharing networks (Al-Bakr, 2009).

Textbooks are one of most important elements of the teaching of mathematics due to their close connection to classroom instruction and teaching strategy. Textbooks classify the topics and also make suggestions as to how classroom lessons can be planned and structured with appropriate exercises and activities. Hence, textbooks are designed to assist teachers to plan their teaching and also assist the student to understand their content. Furthermore, they present the exercises and questions that are included in their various units and chapters.

According to Stray (1994), textbooks are *"designed to provide an authoritative pedagogic version of an area of knowledge"*. They are a specific type of book, intended to be used in education, having a unique and important social function in relation to other texts as they *"represent to each generation of students an officially sanctioned, authorized version of human knowledge and culture"* (Luke et al., 1989). The textbook is an artifact in that it is man-made. Thus, there is an author or a group of authors and a producer of the textbook, whom, it may be assumed, aim to offer a well-made, and carefully prepared pedagogical version of a school topic. However, as Stray (1994), points out, publishing is an industry and therefore there are both pedagogical and economic drivers of the design and production of textbooks.

Various studies on textbooks and teachers' use of textbooks have found the following:

Authors	Findings
Reys et al., 2003	***"Mathematical topics in textbooks are most likely presented by teachers"***
Reys et al., 2003	***"Mathematical topics not included in textbooks are most likely not presented by teachers."***
Reys et al., 2003	***"Teachers' pedagogical strategies are often influenced by the instructional approach of the material."***
Freeman & Porter 1989	***Teachers' sequence of instructions is often parallel to that of the textbook.***
Schmidt et al., 2001	***Teachers report that textbooks are a primary information source in deciding how to present content.***

Table 7.1 Findings in the literature on mathematics textbooks and their uses (Source: Author)

The textbook is an important reading material (print or non-print) prepared for learners and specifically intended for their use. In the curriculum model, textbooks are regarded as the potentially implemented curriculum, the link between aims and reality (Schmidt, et al., 1997; Valverde et al., 2001). Therefore, the textbook is a source of knowledge that can be imparted in the classroom in accordance with the curriculum.

7.6 Requirement for Textbooks

Textbooks are an integral part of the curriculum implementation process. Indeed, textual materials play a critical part in achieving the objectives of the curriculum. In an appropriate curriculum, elements of learning materials, including textbooks, attainable goals, learning materials, effective instruction, teaching strategies and assessment processes are considered crucial (Taba, 1962).

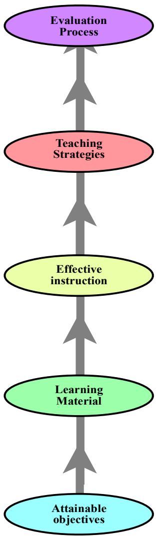


Figure 7.1: Curriculum implementation process (Source: Adapted from Taba, 1962)

The answer to the question as to the extent to which a textbook is necessary depends on the teacher's particular teaching style, the resources available to him/her, the accepted standards of teaching in schools, and so forth. However, this is not the case in Saudi Arabia, where teachers have no leeway to use their own teaching styles, but must simply follow the curriculum. The importance of textbooks and their effectiveness in a sound teaching/learning process is discussed below.

Stray (1994) states that *"Textbooks have been associated with schools for as long as schools have been known. ...Undoubtedly the textbook is the core around which subjects are taught. In a very real sense the textbook is the curriculum."*

Sharma (1983) also places considerable value on the role of the textbook in teaching-learning, asserting that the *"role of the textbook becomes more significant when there is a dearth of adequately trained teachers"*. Singh (1984) considers the textbook to be the most economic, easily accessed and widely used tool of education.

Hence, effective learning occurs only when there is a combination of good teachers, motivated students, appropriate teaching methods and a well-designed curriculum, an important part of which is the textbook, including its exercises and questions. However, Jitendra et al. (2005) conclude that when textbooks do not explicitly adhere to principles of effective instruction, this is likely to result in a gap between the intention and the implementation.

Textbooks are important in classroom interaction as they determine the subject matter or course content and, frequently, the method of teaching. “Textbooks are the most explicit manifestation of national education philosophy and the expression of national political orientation” (Williams, 2002). Textbooks should guide both teachers and students by giving them an appropriate direction for what they should do in the lessons. However, this is unlikely to occur in Saudi Arabia unless the teachers are consulted with regard to the textbook design. The role of textbooks should be considered as one of mediator between teachers and learners.

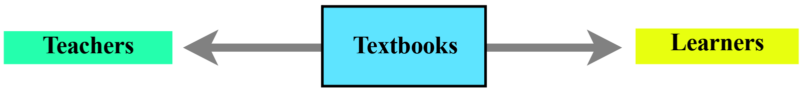


Figure 7.2: Role of textbook as mediator between teachers and learners (Source: Author)

It should be noted that those teachers who rely most heavily on textbooks are frequently those least qualified to realise its intentions or assess its method and content (Williams, 2002). However, there appear to be three choices for teachers with regard to the use or otherwise of a particular textbook in a classroom:

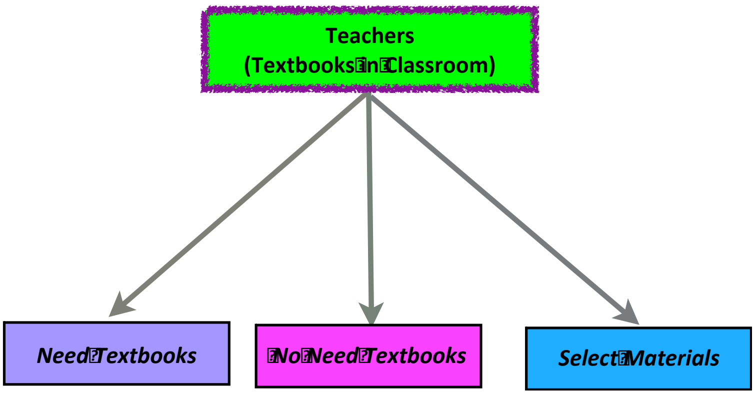


Figure 7.3: Role of the textbook in teaching (Source: Adapted from Williams, 2002)

Therefore, the teacher should choose the textbooks but supplement them with other complementary materials. However, in Saudi Arabia, teachers have no choice in the matter and are obliged to follow the textbook.

7.6.1 Development of the concept of the textbook

The development of the concept of the textbook is still in progress, with the addition of various aspects and forms of textbooks, due to the rapid advances in technology which have altered the means by which it is possible to store and present knowledge resources (AlAbsi, 2009).

In this respect, the process of curriculum development comprises various phases or elements, which the curriculum planners have to take into consideration while engaged in this process.

According to Johansson (2003), "*the elements of a curriculum are the goals, objectives, content, processes, resources, and means of evaluation.*" Warren (1981) identified a number of key elements of the curriculum, one being an analysis of the current situation, another being the formulation of objectives and aims and a third the formulation of learning experiences to achieve these objectives. Content selection is a further element of the curriculum, as are the organisation and integration of learning experiences and subject matter and the evaluation of students' performance (Warren, 1981).

In view of the aforementioned elements and taking into account the education system in Saudi Arabia, this present thesis contends that these are also the most important basic factors for the development of a standard quality of textbook and textbook questions.

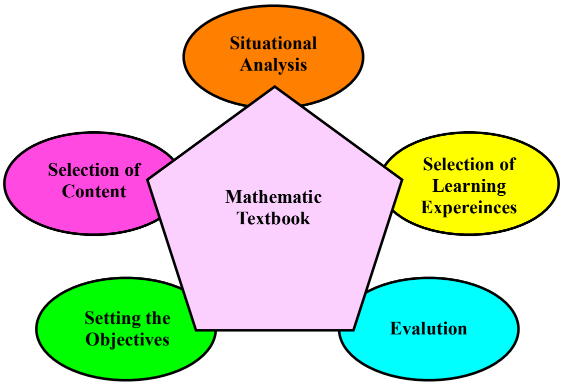


Figure 7.4: Elements of textbook development (Source: Adapted from Warren, 1981)

Thus, the conclusion is reached that the main components of textbook development are situational analysis, choice of objectives and course contents, choice of learning experience and assessment of students' performance through evaluation.

7.6.2 Criteria for a good textbook

In order to achieve the aims of a mathematics textbook, a set of criteria should be fulfilled. These are the following:

7.6.2.1 Qualified and experienced authors

The authors of mathematics textbooks should be known for their competence in both mathematics and education. In addition, he or she should have teaching experience. The textbooks should also be characterised by the accuracy, impartiality and integrity of the author. Moreover, the author should be fully aware of the norms and customs of society and of the objectives of the educational level corresponding to the textbook (Khalifa and Shobiak, 2007).

7.6.2.2 The content of the textbook

There must be a clear relationship between the topics and objectives of the textbook. The contents should be up-to-date and comprehensive and the information, facts, concepts and terminology appropriate to the students' cognitive, cultural, social and linguistic levels. The textbook should also take into consideration the facts, information, experience, skills, questions and exercises in the textbook, as well as the needs and interests of the students. In addition, the textbook should draw a clear association between mathematics and the reality of the students' lives and experiences. Moreover, the textbook should use a variety of topics, examples, facts, concepts, terminology, definitions, skills, exercises, and questions. These should be clear and unambiguous, with no linguistic or mathematical errors. A variety of tools such as illustrations, maps, models and charts should also be used with the aim of clarifying the concepts and information in the textbook and thus improving students' understanding.

The content of each textbook should relate to the content of both the previous and subsequent textbooks. That is, they should reinforce what has been previously learnt and adequately prepare the student for what will be learnt next. The sequence of the topics and subject matter in the textbooks should be coherent.

At the end of each chapter, a list of references and sources as well as a bibliography of recommended works, both national and international, should be provided. Reading these will expand the students' knowledge and expertise. A glossary of unfamiliar terms should also be included (Mahmoud and Aloubeid, 2010).

7.6.2.3 Language, Appearance and Format

Overall, the textbook should be written in clear and comprehensible language, appropriate to the students' cognitive and linguistic level, be attractively presented and the information should be imparted gradually.

Regarding format, the textbook should have an attractive appearance, including an attractive cover, and be of convenient size and shape. It should be printed on good quality paper in clear characters, with appropriate spacing between lines and words, be free from grammatical errors and misprints, have clear illustrations and diagrams, and a sturdy binding. In addition, the headings and sub-titles of each chapter should be appropriate (Mahmoud and Aloubeid, 2010).

7.6.3 Responsibility for Textbook Development

There is no doubt that the opinions of teachers and inspectors are important, but they are not sufficient, as textbook development should be an integrated effort involving academics, experts, parents and administrators.

The Ministry of Education has assigned the matter of textbook development to academics in the field of education. However, from the above discussion, the researcher recommends that three aspects should be combined in textbook development, one of which is the academic aspect, represented by academics and experts in the subject. The second aspect is the educational one, represented by teachers, inspectors and others related to the education field, while the third aspect is the technical one, represented by those responsible for the artistic design and technical production of the textbook.

7.6.4 Key elements of a good textbook

From the literature review, the researcher has identified the following elements which are crucial to a good textbook:

- It fulfils educational objectives.
- It considers learning mathematics to be a constructive activity.
- It recognises the contribution of students' prior knowledge to their future learning.
- It recognises the usefulness of students' collaborating and interacting with one another when learning mathematics.
- It recognises the role of the teacher in the process of learning mathematics.
- It combines theoretical and practical information.
- It helps the students to develop their knowledge.
- It trains students in various thinking skills.
- It assists students to associate mathematics with their everyday lives.
- It is up-to-date and in line with technological progress.
- Its content should be linked to the content of previous textbooks.
- It should encourage students to acquire knowledge and apply it in their everyday lives.
- It should contain questions and exercises to help students retain what they have learned, to ascertain their level and to develop their abilities.
- Its presentation and language are clear and unambiguous.
- It is developed in the light of modern educational and psychological theories.
- Its questions and exercises should help to develop students' higher order thinking skills rather than focusing on lower order ones.
- It should be appropriate to students' cognitive levels and culture.

7.7 The Role of the Textbook and its Questions in Mathematics Instruction

Educators frequently seek to embody educational objectives in the form of a textbook or curriculum statement that will serve as an aid in the realisation of these objectives, through supporting the teacher in transferring academic knowledge to the student. As

Voogt and Odenthal (1997) point out, teachers use the textbook and its questions in the planning and implementation of lessons, which to a great extent contributes to the achievement of those objectives. Hence, the textbook, together with the questions it contains, has an effect on society and can be considered as a cornerstone of progress and advancement for any society (Shaheen, 1991). This relates directly to the research questions in the present study, as the textbooks and their questions are the building-blocks of teaching and learning and hence of an educated society and this study seeks to determine the extent to which the mathematics textbooks and their questions, which are the focus of the research, are effective in this function.

Textbooks and other teaching materials have been described by Ball and Cohen (1996, p.6) as "the stuff of lessons and units, of what teachers and students do". However, according to Lithner (2004), most mathematics students seem to spend most of their study time doing exercises. Lithner (2004) states that "this is the way students are supposed to practise and learn mathematics in order to be able to apply their knowledge in other situations; for example in their further studies, in their future professional life, or in their everyday life as members of modern society."

Love and Pimm (1996) offer another reason why textbooks are studied in the classroom:

"The book is still by far the most pervasive technology to be found in use in mathematics classrooms. Because it is ubiquitous, the textbook has profoundly shaped our notion of mathematics and how it might be taught. By its use of the 'explanation - example – exercises' format, by the way in which it addresses both teacher and learner,

in its linear sequence, in its very conception of techniques, results and theorems, the textbook has dominated both the perceptions and the practices of school mathematics".

The term 'exercise' can be defined in various ways, but is most frequently taken to include any task type that is normally encountered in textbooks (Lithner, 2004). One essential distinction is between routine tasks and 'creative' problems (Schoenfeld, 1985, 1992). "A routine task is one where a complete solution method is readily available to the solver, and the solution is carried out in an algorithmic way by following a set of familiar procedures. The term 'algorithmic' includes all types of sequential, well-defined procedures, not only calculational ones. In the literature, the 'term' problem has been used in many different ways, to indicate any mathematical task or to indicate the kind of task which is normally only met with by mathematicians doing advanced research (Schoenfeld, 1992). Schoenfeld (1985, p. 74) describes problems thus:

"The same tasks that call for significant efforts from some students may well be routine exercises for others, and answering may just be a matter of recall for a given mathematician. Thus 'being a problem' is not a property inherent in a mathematical task. Rather, it is a particular relationship between the individual and the task that makes the task a problem for that person. The word 'problem' is used here in this relative sense, as a task that is difficult for the individual who is trying to solve it. Moreover, that difficulty should be an intellectual impasse rather than a computational one".

In Lithner's (2004) study of undergraduate calculus textbooks in Swedish universities, he found that most of the tasks could be solved, and in fact were solved, by looking for and copying procedures to be found previously in the same textbook section without considering relevant mathematical properties. Lithner (2003) found that such characteristics of the tasks included in the textbooks may be one influential factor in the development of mathematically superficial reasoning.

In intermediate mathematics, the traditional textbook and its questions have long been a principal reference for teachers and the main resource for learners. Results from the TIMSS study (Martin et al, 2002) showed that in Australia, 95% of year 8 mathematics teachers used a textbook, and that about 50% of them used a textbook as the principal lesson resource (Thomson and Fleming, 2004).

Indeed, there have been many proposals for evaluating mathematics textbooks (e.g. Shield, 1998). However, many evaluations have not paid attention to the mathematical ideas and the ways these are developed for learners. Shield (1998) explored how the textbooks conveyed the nature of mathematics and its teaching and learning. Over the past three decades, many studies have taken a deeper approach to evaluating mathematics textbooks by developing strategies that connect their content to the teaching practices laid out by authorities. One example of this was the long-term implementation of the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) in the USA (Martin et al, 2000).

The relationships between the curriculum, the teacher, and the textbook are complicated (Remillard, 1991). Tornroos (2004, p.2) used the term “potentially implemented curriculum” to describe the part played by the textbook in the mathematics classroom. As the textbooks are very important in mathematics teaching, it is an important aim to establish an effective way that they can be evaluated by their users.

This present study focuses on the questions at the end of every section and chapter to discover what type of thinking skills are required to answer them. Research questions 1 and 3 relate particularly to this issue.

7.8 Comparative aspects of mathematics textbook development

7.8.1 Translation of textbooks

National and regional cultures can have a strong influence on textbooks and their development. For example, in Saudi Arabia all education is gender-segregated, and indeed, until a few years ago, there were separate mathematics textbooks for male and female students. The main difference between them was that there was no geometry in the female students' textbooks. However, this is no longer the case and both males and females use the same mathematics textbook.

By a critical understanding of the differences between different cultures it is possible for cultures to learn from one other. However, in spite of the potential insights to be obtained through the exploration of how ideas may be translated for use in other settings, van den Akker et al. (2006) note such a task is far from easy. Although Loucks-Horsley and Roody (1990) found that practices or programs developed in one setting can often be used successfully in other places, Fullan (1998) argues that "There is no (and never will be any) silver bullet" for educational change in varying contexts". Similarly, Guthrie (1986) asserts that neglecting to understand and consider the fit of an innovation is a common cause for failure.

Once instance of this took place in Saudi Arabia in 2011, when an attempt was made by the Ministry of Education to revise the mathematics curriculum in Saudi schools. In order to accelerate the proposed changes, mathematics textbooks from the USA were translated directly from English into Arabic. This was done without consulting mathematics teachers and without any evaluation being carried out. However, the direct translations led to problems, as many of the things referred to in both the text and illustrations in these textbooks were either unknown to Saudi readers or culturally inappropriate for them. Hence, they had no association with the students' everyday lives. After some time, all these textbooks were withdrawn and pulped, at considerable expense.

The following are the negative aspects of the translated textbooks.

- No training courses were given to teachers on how to teach from the new textbook.
- The textbook was not suitable for the number of mathematics classes given in schools.
- Due to their other responsibilities, the teachers did not have time to give the new textbook their full attention.
- The textbooks required the use of equipment or facilities which were either inadequate or absent in Saudi schools.
- Some topics in the textbook required the use of a computer and many teachers, particularly older ones, had poor computer skills.
- There was inadequate technical support for the treatment of many of the problems faced by students and teachers during the implementation of the new textbook.
- Insufficient copies of the teachers' book were distributed, which did not help the teachers in planning the lessons from the new textbook.
- The social and cultural differences between the USA and Saudi Arabia were not taken into account in the translation of the textbook.
- There were a number of errors and misjudgements in the layout and presentation of the new textbook. For example, there was a picture of donkey on the cover, which caused great controversy in the school community and the Ministry of Education. It cost a considerable amount of money when the Ministry decided to withdraw all copies with that cover from schools and replace them with the same textbook in a different cover.
- There was some confusion in the translation which led to some lessons being repeated in the textbooks for different years. This shows that the textbook was printed and distributed without being adequately reviewed.

- Only the intermediate level textbooks were translated and some of the topics were based on information that the students had not previously been taught as they had not had the corresponding primary level textbooks.
- Some of the topics and questions in the textbook needed the students to use a computer, but not only were computers not available for every student in Saudi Arabia, in some villages there was not even electricity.
- Parents could not help their children with the work in the new textbooks as it was unfamiliar to them.
- The time scale set out by the Ministry of Education to complete the textbooks in class was inadequate.
- The textbook required students to engage in collaborative learning, with which students in Saudi Arabia were unfamiliar and for which the classrooms in Saudi schools were not adapted.
- There are usually between 45 and 50 students in a Saudi classroom, whereas the new textbooks were written for classes of between 20 and 25 students.
- There were many linguistic and scientific errors in the new textbooks due to the translations not having been sufficiently well reviewed.
- There was no pilot testing of the textbooks before they were distributed to all Saudi schools.

However, the translated textbooks also had certain positive aspects, which were the following:

- The new textbook was modern and in line with technological progress.
- The new textbook contained a broad spectrum of information.
- It was linked to technology, encouraging students to use it.
- It helped the students to understand the link between mathematics and everyday life better.
- It encouraged autonomous learning.
- It contained useful skills and information that were not in the old textbooks.

The experience described above demonstrates that it is not possible to take a foreign textbook or curriculum and apply it directly if it is not modified to suit the local culture. If this is done, it will be inappropriate for the students and it will therefore be much more difficult for them to make a connection between mathematics and their daily lives.

7.8.2 Saudi textbooks compared to US textbooks

Textbooks are a major, or indeed, frequently the sole, element in mathematics classrooms in Saudi Arabia, as they are in many other countries. Consequently, they often determine what school mathematics is for both students and teachers. They can also occupy a prominent position in any reform of a mathematics curriculum as the development of textbooks and other curriculum materials can be viewed as a simple and rapid to change teaching.

In the next section, a comparison is made between the mathematics education systems and mathematics textbooks in Saudi Arabia and the United States. From a review of the relevant literature and interviews with experts, indicators for various aspects of a quality textbook were identified in order to clarify the key indicators of high quality textbooks, how to develop standard questions in the exercises in the mathematics textbooks, and effective evaluation and review processes. Policies for the development and publication of textbooks vary from country to country, as does the centralisation or decentralisation of the curriculum. For example, Saudi Arabia follows a centralised curriculum and the USA follows a decentralised curriculum.

7.8.3 The decentralised curriculum and textbook development in the USA

The National Council of Teachers of Mathematics (NCTM) (1989; 1991; 2000), United States mathematics education reform has been attempting to encourage students to gain an understanding of important mathematics concepts or ideas and their relationships, as well as how to use these ideas for problem solving. This was done on the premise that number sense is one of the important ideas about which students should develop a sound understanding in their early schooldays (Sood and Jitendra, 2007).

The American Textbook Council (2009) reviews educational materials and review guidelines that are often requested by curriculum inspectors, school boards, teachers and others in charge of the selection of textbooks. Some reviewers seek fully quantifiable standards, exact readability gauges, and "scientific" formulas. However, some basic points are borne in mind while reviewing and evaluating the textbook. These are summarised in the table below.

Basic Parameters Textbook	*Content and style*	*Instructional Activities*	*Evaluation and Assessment Materials*
Accurate information	Systematic development of ideas	Opportunities provided for students to be actively engaged	Both formal and informal assessment strategies suggested
Fair and unbiased treatment of various groups in society	Depth of topics	Variety of activities	Provide positive feedback
Appropriate reading level for the students who will be using the material	Narrative styles and real-life experience	Students of differing abilities can succeed	Enable students to hypothesise and analyse.
Written in a clear and comprehensible manner	Clear link between narrative in style and illustration	Questions are provided that call on students to analyse information and to think critically	Develop mathematical thinking
Written in a style that will be interesting and hold the student's attention	Literature included or referenced	Activities provide for curriculum integration	Assessment method based on multiple objective and subjective patterns
Questions and other end-of-chapter exercises	Variety of primary sources included	Students have the opportunity to discuss ideas presented in the textbook	Draw conclusions about the subject matter they are studying
Illustrations and sidebar materials are relevant to subject matter	Meaningful excerpted passages	Activities become more challenging as the textbook progresses	Assessment strategies include student writing exercises

Table 7.2: Key factors of high quality textbooks (Source: Adapted from the American Textbooks Council, 2009)

Therefore, it can be concluded that an attempt has been made to write the mathematics textbook in a clear and understandable manner and to hold the students' attention by the inclusion of mathematical thinking and various teaching strategies. Mathematics education has developed and has been encouraging the systematic development of ideas and activities on the part of students as well as the integration of the curriculum. Evaluation is carried out by assessment strategies, both formal and informal.

7.8.4 The centralised curriculum and textbook development in Saudi Arabia

In Saudi Arabia, a centralised school curriculum is followed throughout the country. The Ministry of Education is the authority in Saudi Arabia that issues textbooks which are used in all non-higher education systems, even those that do not fall under the Ministry's jurisdiction. One or more authors write the textbook, and subsequent to the Ministry's approval, it is published at the government's expense and issued free of charge to the students (Ministry of Education, 2013).

The Ministry of Education (2013) reports that the body in charge of textbook publication within the Ministry of Education is known as the Center for Educational Development. Each textbook carries its stamp, as well as the following phrase: *The Ministry of Education has made the decision to use this textbook for teaching and to cover the costs of its publication.* In addition, on the cover page of each textbook there is the logo of the Saudi Ministry of Education, while at the bottom right of the cover page, the date is given according to both the Hijri and the western calendars. Most textbooks start with a brief introduction by the Minister of Education, with his signature, followed by an introduction to both students and teachers by the author/s.

The series of textbooks studied in this thesis is called 'Mathematics textbooks for Saudi Arabia.' These are for the classes in the first, second and third years of secondary school (i.e., for learners aged 16, 17 and 18, respectively). There are two mathematics textbooks for first, second and third years in secondary schools; one for first semester and another for the second semester (Sedgwick, 2001). They reflect the national curriculum as do all textbooks in Saudi Arabia. The Ministry of Education is in charge of setting the objectives and designing these textbooks.

The government holds that the teacher is crucial to the learning process and can benefit from the use of the various facilities such as curricula, textbooks, laboratories and audio-visual aids; however, the facilities available are inadequate. The key differences between the USA and Saudi Arabian textbooks are summarised in the table below.

Factors	*United States*	*Saudi Arabia*
Textbook development and publication policies	o *Diversity in curricula* o *No constraint by Ministry of Education.* o *Different series of mathematic textbooks.*	o *Uniform Curricula* o *Published by Ministry of Education.* O *One Series of Mathematic Textbooks.*
Choice of content	o *Discretionary Curricula* O *Number of different curricula to cater for the different needs and interests of the students.*	o *Essential Curricula* o *Follow same curricula and to learn the same content.*
The role of textbooks in teaching and learning	o *Relative Knowledge.* O *Focused on course and activity-based learning*	o *Absolute Knowledge.* O *Focused on course content.*
Physical appearance of textbooks:	o *Colourful* O *Attractive*	o *Plain or colourful* O *Normal, black and white*

Table 7.3: Comparison between Saudi and US textbooks (Source: Author)

7.9 Textbook Evaluation

The focus of Project 2061 (Kulm et al., 2000) was the development of a method to evaluate middle-grade mathematics textbooks in the USA with a focus on “their effectiveness in helping students to achieve important mathematical learning goals for which there is broad national consensus” (Kulm et al., 2000, p. 1). The evaluation criteria belonged to several categories including engaging students in mathematics; developing mathematical ideas; and encouraging student thinking about mathematics.

Pepin and Haggarty (2001) studied the use of mathematics textbooks in English, French and German school classrooms, and found that in many textbooks there is a predomination of exercises, with few links made between the concepts practised, although others encouraged the acquisition of new knowledge and attempted to motivate students. Brändström (2005), after an analysis of three Swedish primary school mathematics textbooks, found that very few of the questions presented any challenge to students apart from the use of procedures.

Mayer et al. (1995) compared lessons in three Japanese primary school mathematics textbooks with similar lessons in four mathematics textbooks in the USA. All of the Japanese textbooks, but only one of the four US ones prominently featured multiple representations (e.g., words, symbols, and diagrams) in worked examples. The Japanese textbooks made close links between the three representations, giving support to the findings of the TIMSS Video Study data, that over fifty per cent of the problems in the Japanese lessons focused on 'making connections’ (Hiebert et al., 2003).

According to Love and Pimm (1996, p.398), “the teacher normally acts as a mediator between the student and the text” offering an interpretation of the text that is “based not only on her constructions of the intention of the author, but on her accumulated experience of teaching”. However, Pehkonen (2004, p.519) concludes that “teachers want the mathematics textbooks to concentrate on the basics, since they believe the basics constitute good and proper mathematics teaching”.

Regarding the “back to basics” approach in mathematics in the 1970s in the USA, Schoenfeld (2004, p.258) states that “not surprisingly, students showed little ability at problem solving - after all, curricula had not emphasized aspects of mathematics beyond mastery of core mathematical procedures. But performance on the 'basics' had not improved either…” Schoenfeld (2004 pp. 280-281) argues that “an exclusive focus on basics leaves students without the understandings that enable them to use mathematics effectively. A focus on 'process' without attention to skills deprives students of the tools they need for fluid, competent performance".

Ideally, mathematics textbooks should provide a balance of skills and process (Vincent and Stacey, 2008). The style of language selected in a textbook could also have an effect on what teachers and students hold to be important in mathematics. In her study of the language of student texts used in the Connected Mathematics Project, which was an intermediate school problem-centred curriculum in the USA, Herbel-Eisenmann (2007, p.354) found that what the authors frequently referred to as 'questions', were "actually imperatives, which were instructions to direct actions" (p. 354). By this, she means that the students receive instructions such as 'make' or 'draw'. She states that the language used by textbook authors can affect students' ideas concerning mathematics (Herbel-Eisenmann, 2007).

Stein and Lane (1996) explored the widespread belief that learners who have not had a good basic grounding in mathematics cannot proceed to more interesting teaching. They argue that this belief is responsible for an "even greater tendency for middle school instruction to focus on procedural skill" (p. 52). Reporting on the Australian QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) Project, Stein and Lane (1996) found that the learners benefited most when students were given tasks that required non-algorithmic forms of thinking. However, the benefits were much smaller when tasks were "procedurally based and able to be solved with a single, easily accessible strategy, single representations, and little or no mathematical communication" (Stein and Lane, 1996, p. 74).

Stein and Lane (1996) note that teachers tend "to proceduralize tasks due to time constraints" and to "perform the most demanding parts of tasks for students" (p. 60). "*When problems classified on the basis of their implied solution processes ('using a procedure', 'stating a concept' or 'making connections') were followed through to their public solution, only 8% of 'making connections' problems in the Australian lessons were actually solved in this way. The remaining problems were reduced to the use of a procedure or the stating of a concept*" (Hiebert et al., 2003, p.104). This contrasted sharply with the findings for the Czech Republic, Hong Kong and Japan, where only around half of 'making connections' problems were observed to be explicitly solved by making connections (Hiebert et al, 2003).

In a similar study carried out in Indonesia, Harta (2001) investigated the reality of the mathematics problems in the elementary school mathematics textbooks from the 3rd grade to the 6th grade over the previous 40 years. The following information related to the areas of mathematical problems was collected: number and topics of problems, the roles, tasks and ages of students in relation to such problems, presence of key words, the extent to which the most recent problems resemble previous ones, quantity of data present, type of numbers and units used, number of mathematical processes needed to find a solution, and the presence of subtraction and division problems. The study findings indicated that mathematics textbooks in Indonesia were based on the example of pre-1980s American textbooks. The Indonesian textbooks contain fewer mathematical problems than the previous Indonesian textbooks and the American textbooks, as there are fewer than 500 problems in the current Indonesian textbooks compared to 2500 problems in those of the 1960s, and 2500 problems in the American textbooks. Similarly, O'Keeffe and O'Donoghue (2011) analysed the textbooks used in Irish Junior Cycle learning, with a focus on how they developed students' conceptual understanding. They identified an "over dominant influence" on the textbook in Irish classrooms and asserted that reliance on substandard textbooks may have a harmful effect on students' learning (O'Keeffe and O'Donoghue, 201, p. 304). Their analysis was based on the work of Valverde et al (2002), whose framework for text analysis comprises three main stages: Content, Structure and Expectation. To these three, O'Keeffe and O'Donoghue (2011) added another element, that of language, based on the work of Halliday (1973) and Morgan (2004). Based on this anlaysis, they found that the textbooks in question failed to motivate pupils, or to provide for the comprehension and processing of the information provided. O'Keeffe and O'Donoghue (2011) subsequently developed a model textbook chapter on fraction addition designed to enhance the students' conceptual thinking. In order to do this, they drew on the frameworks of the Adult Numeracy Network (ANN) (Curry et al., 1996), the Adult Based Education curriculum framework (Massachussets Departments of Education, 2005) and the PISA (Organisation for Economic Cooperation and Development, 2003). O'Keeffe and O'Donoghue's (2005) findings revealed that the implementation of the model chapter in the Irish Junior Cycle classroom resulted in positive changes in students' conceptual understanding of mathematics.

In a similar vein, a number of studies have attempted to evaluate the mathematics textbooks in several Arab countries, notably Jordan. Some of these are worth noting, as the context is similar to that of the present study. The aim of Abu Ali's (1989) study was to evaluate the secondary school mathematics textbooks prescribed for students in Jordan during the school year 1988/1989. The study sample comprised 43 male and female teachers and 286 male and female students who were randomly selected from 15 secondary schools affiliated to Irbid Province Directorate of Education. The study instruments consisted of two questionnaires: one for the teachers and the other for the students. The questionnaires covered the following areas of evaluation: the general appearance of the textbooks; the preface; the content; the illustrations, methods and activities; the evaluation methods applied in the textbook, and; the extent to which the textbook assisted in developing positive attitudes towards mathematics on the part of the students.

The findings for the textbook indicated the following:

The teachers evaluated four areas highly; namely, the general appearance of the textbooks; the content; the illustrations, methods and activities, and; the evaluation methods applied in the textbook, while the evaluation of the textbook preface was average, and regarding the positive attitudes towards mathematics developed by the textbook, which was evaluated as being low.

The students evaluated all areas highly apart from the development of positive attitudes towards mathematics, which they evaluated as being low.

In another Jordanian study, Al-Sir (1994) aimed to evaluate the mathematics textbooks of the 9th grade from the point of view of the teachers and students in the schools of the First Education Region of Amman by eliciting their evaluations of four aspects of the textbook: the content; the illustrations, activities and materials that assist in the use of the textbook; the evaluation tools; and the general appearance of the textbook. The study sample consisted of 64 male and female teachers teaching mathematics to the 9th grade, and 520 male and female 10th grade students. The sample was chosen from 50 schools of the First Education Region of Amman. The study tools included two questionnaires: one for the teachers with 63 items, and the other for the students with 42 items, both distributed over the four aspects under evaluation.

The study findings indicated some weaknesses in the textbook, foremost among which were the facts that the presentation method of the textbook material was not sufficiently interesting to encourage the students to learn independently, the activities contained in the textbook were neither diversified nor sufficient and did not propose a specific teaching method, and the textbook exercises and problems did not contain real-life problems.

Similarly, Al-Alem's (1994) study was carried out to evaluate the effectiveness of the mathematics textbook prescribed for the 6th grade in schools in Jordan, by determining the extent to which the textbook met the objectives of the curriculum. Specifically, the study attempted to answer the following questions:

To what extent are the learning objectives met by the mathematics textbook of the 6th grade, measured by the students' performance in an achievement test?
What is the evaluation of the teachers and 6th grade students of the mathematics textbook prescribed for the 6th grade?
The study sample consisted of 70 male and female teachers of the 6th grade, and 706 male and female students from the first and second education regions of Greater Amman and from private schools, in the school year 1993/1994.

In order to answer the research questions, an achievement test was developed to measure the main aims of mathematics teaching in the 6th grade in order to determine which goals had been achieved. Two questionnaires were also developed: one for the teachers and the other for the students, in order to discover their views about the textbook.

The study findings indicated the presence of general satisfaction on the part of the teachers and the students about the areas of the textbook that they evaluated. However, the results of the students in the achievement test indicated that few of the educational objectives of the 6th grade mathematics textbook were achieved.

Again in Jordan, Al-Dowaikat's (1996) study aimed to evaluate the mathematics textbook prescribed for the 9th grade in Jordan in the school year of 1994/1995, according to the opinion of the teachers of this subject and mathematics inspectors.

The study sample was made up of two groups: a group of 120 male and female teachers who were teaching mathematics to the 9th grade in the schools of the Directorate of Education of Irbid First Region during the school year of 1994/1995, and a group of 35 mathematics inspectors working in all the directorates of education in Jordan during the school year of 1994/1995. The research instrument was a questionnaire of 92 items distributed over six areas, namely: the preface; the objectives; the content; the activities and illustrations; the evaluation questions; and the technical production of the textbook. The study findings indicated that overall, the teachers and inspectors evaluated the textbook as being average.

A slightly later study, that of Al-Lahawiyah (1999), aimed to evaluate the mathematics textbook prescribed for second grade students in the scientific section in South Jordan schools from the viewpoint of the teachers, in order to determine whether it was suitable for its teaching purposes by identifying its strengths and weaknesses. The sample of this study consisted of 61 male and female teachers who were teaching mathematics to the scientific section of the second grade in the public schools of South Jordan during the school year of 1998/1999. The researcher developed a questionnaire whose final form comprised 78 items distributed among six areas, namely: the preface; the objectives; the content; the activities and illustrations; the evaluation tools, and; the technical production and general appearance of the textbook.

The study findings indicated the following:

The evaluation of the textbook was average except for the technical production and the aims, which were evaluated as being high.

The findings indicated the following weaknesses:

The textbook content cannot be taught thoroughly in the number of classes allocated.

The textbook does not mention the contributions of Arab and Muslim scholars to the development of mathematics.

The textbook does not contain the references used by its authors.

Al-Zughbi's (2001) study also examined Jordanian school textbooks, aiming to determine the readability level of the mathematics textbooks for the elementary school stage, and to discover the difficulties that face the students while reading these textbooks. The study also aimed to develop strategies to improve the readability level of the mathematics textbooks and measure their effect on achievement.

The study was confined to the 5^{th} grade mathematics textbook, and to the students of this grade in the Directorate of Education in Bani Kinanah Region in Irbid province, Jordan, during the school year of 1999/2000. Two schools were chosen: one for males and the other for females, and 3 groups chosen randomly from the 5^{th} grade of the two schools. These were the first experimental group (25 male students and 25 female students) who studied using a reading strategy; the second experimental group (26 male students and 25 female students) who studied using a reformulation strategy; and a control group (25 male students and 26 female students) who studied using the textbook strategy.

The difficulties of reading the mathematics textbook were recorded during the interviews and oral tests, and a Clauze test for the degree of readability of the textbook was used, together with an achievement test at the end of the year in order to measure the students' achievement.

The study findings indicated the following:

According to the students, the readability of the mathematics textbook was poor, as they reported difficulties while reading it.

The performance of the two experimental groups was better than that of the control group in terms of readability level and achievement.

There was a difference attributed to gender in the three groups in favour of the female students in terms of readability level and achievement.

The scope of Abu Mousa's (1997) study was somewhat wider, as it aimed to analyse the content of the mathematics textbooks prescribed to the students of the 5^{th} to 8^{th} grades in Jordan, in order to determine the extent to which the basic standards conformed to the international standards of foundation stage mathematics curricula. The population of the study consisted of the mathematics textbooks prescribed to the students of the 5th to 8th grades during the school year of 1991/1992. The study sample was the same as the population. The researcher developed an analysis instrument consisting of the standards of the learning material, in which he included specific standards for problem solving, mathematical communication, mathematical coherence, and mathematical inference.

The findings indicated that the number of routine problems was comparable with the number of non-routine problems in the textbooks that were subject to analysis. Problems derived from real life constituted a large proportion of these textbooks. However, the analysis results did not reveal any creative problems in any of the analysed textbooks, while very few problems of the type that require the student to formulate questions were recorded. The findings of the study also indicated that the questions were not used as a general framework for the presentation and exploration of the mathematical content. In the textbooks, the correctness of the solutions was verified by ticking the right answer. No activity that would help develop the ability of mental calculation, whether as a mental skill or strategy to verify the correctness of the solution, was found in the textbooks. Regarding the use of mathematical language as an activity to enhance the ability of mathematical communication, there were no clear activities involving translation. Further, the findings indicated some methods of inductive and deductive inference, as well as some methods of mathematical proof in the textbooks of the 7th and 8th grades, but all the methods of proof that were contained in the textbooks were of the direct type. The textbooks focused on reasons and justified the procedures and steps that were embodied therein.

It was as part of the evaluative study of an educational development programme supervised by the National Center for the Development of Human Resources in Jordan that Al-Sheikh (2001) conducted a study of the school curricula and textbooks. Among the goals of the study was to evaluate the quality of the school curricula and textbooks that had been developed as part of the educational development plan of 1989-1998. The study defined quality by the degree of success of the school curricula and textbooks in incorporating the four educational elements that were targeted by the development plan, foremost among which was the development of higher order thinking abilities and various patterns of thinking. The study was confined to the evaluation of the school curricula and textbooks of the foundation and secondary school stages in four subjects, one of which was mathematics. The evaluation was made of only the 3^{rd}, 6^{th}, and 9^{th} foundation grades, and the 1^{st} and 2^{nd} secondary school grades in both the scientific and literary sections. The study used several methods, activities and evaluation tools based on the questions that were developed to fulfil the objectives. For instance, for the evaluation of the school textbooks, some evaluation standards were derived for each of the four educational development elements, and a model was designed to analyse each chapter in the school textbooks. For each foundation school grade, the relevant textbook was analysed according to the model. The study found that the textbooks embody the traditional perspective of the organisation of the content, as they tend to present and disseminate the information with little opportunity to generate knowledge, although these textbooks appear to be full of activities. Furthermore, they do not develop the higher order thinking skills of the students. The textbooks also suffer from a lack of linkage between the information and ideas in the same lesson or unit, although they provide some real-life applications in the content topics.

7.10 Conclusion

The literature shows that many teachers are dependent on textbooks and that they may or may not add to these to make links and highlight that mathematics can go further than basic skills. It offers examples of textbooks that provide problems which challenge students considerably and stretch their abilities beyond the routine use of procedures, but shows that, in general, textbooks and their questions do not do this. The literature also stresses the importance of providing problems that go beyond the routine use of procedures in order to encourage deeper student learning. It is noted, based on the review of the studies that investigated this subject, that most of them directed their attention to the evaluation of the following aspects of mathematics textbooks: goals, content, activities and illustrations, general appearance and technical production, evaluation tools, and development of students' positive attitudes towards the subject of mathematics. The studies also relied in the process of evaluation on questionnaires whose items generally concerned the aforementioned aspects. Hence, responses were required on the part of concerned individuals (e.g. teachers, students, and/or educational inspectors) to these questionnaires in order to determine their views on mathematics textbooks and curricula.

Some of the studies aimed at investigating certain aspects of the school mathematics curricula and textbooks, such as the extent to which the mathematics curricula and textbooks conformed to the cognitive levels of the students, the readability levels of the mathematics textbooks, and the degree to which the mathematics curricula fulfilled their goals by measuring students' achievement. Other studies compared two mathematics curricula in order to determine which was better in terms of the fulfillment of the desired educational goals and improving students' achievement.

The current study goes beyond the previous studies as it endeavours to investigate the reality of mathematical thinking in the questions of secondary school mathematics textbooks. This was not done by any of the previous studies, whether those that studied mathematical thinking, or those that studied the mathematics curricula and textbooks. The current study seeks to analyse the mathematics questions in order to determine the extent to which they contribute to the development of students' mathematical thinking, through the use of content analysis.

The current study also differs from most of the previous studies in terms of its investigation of a number of secondary school mathematics textbooks of more than one secondary school grade. The majority of previous studies focused on one mathematics textbook and one grade only. The current study also has a very large sample, which enhances reliability.

However, the previous studies also stressed the importance of investigating the school curricula and textbooks by means of analysis and evaluation, as such efforts are indispensable to their development and improvement to make them more appropriate to meet both students' cognitive needs and society's needs in an ever-changing world, as evaluation is the foundation of the development of educational work.

Attitude has an important part to play in learning mathematics and for this reason, it is the focus of the next chapter.

Chapter 8

Attitude

8. 1 Introduction

In recent years, attitude has been recognised as a significant factor in students' achievement in mathematics (Singh et al, 2002). However, it appears that many students view mathematics as meaningless, complex and difficult (Sharples, 1969; Carpenter et al, 1981; Dossey et al, 1988). This chapter begins with a number of definitions of attitudes, and then discusses their significance in terms of both learning and teaching. Further, the literature surrounding attitudes towards mathematics and the development of these is reviewed.

8.2 Definitions of Attitude

There have been numerous attempts to define the term 'attitude' over the years. As early as 1929, Thurstone described an attitude as "the affect for or against the psychological object". However, this definition is limited as it concerns only the affective element of attitude. Allport's (1935, p.10) definition of attitude as a "degree of affect for or against an object or a value" is similarly limited.

A number of years later, Katz (1960) defined an attitude as "the predisposition of an individual to evaluate some symbol or object or aspect of his world in a favourable or unfavourable manner." This definition contains the concept of evaluation as a component of attitude.

Krech (1960, p.177) defined attitude as *"*an enduring system of positive or negative evaluation, emotional feeling and pro or con action tendencies, with respect to a social object". He identified three necessary components of attitudes, the first being the cognitive component, concerning beliefs about an object, such as evaluative beliefs that an object is good or bad. The second component, according to Krech (1960), is the affective component, which concerns likes and dislikes. The third component, the behavioural component, includes the apparent behaviour that individuals exhibit with regard to the object of the attitude, which can range from very positive to very negative (Krech, 1960).

However, many psychologists, such as Bagozzi and Burnkrant (1979) and McGuire (1985), while concurring that attitudes are composed of the three components identified by Krech (1960), have suggested that these may be intertwined and may therefore not be separate factors. In this respect, Halloran (1970, p. 22) stated that "*in any given situation an individual may be shown to select some of the available stimuli and neglect others. He processes or interprets the selected stimuli in certain ways, and reacts to the interpreted stimuli affectively and by behavioural tendencies which will emerge as behaviour under appropriate environmental conditions*." Reid (1978) described this interlinking of the components of attitudes succinctly, stating that "Attitudes are a network of cognitive, affective and behavioural elements with an evaluative dimension and they are learned and they can develop with new input of cognitive, affective or behavioural nature.*"* In a similar vein, Oppenheim (1992, pp. 174-175) described attitude as "a state of readiness, a tendency to respond in a certain manner when confronted with certain stimuli… attitudes are reinforced by beliefs (the cognitive component), and often attract strong feelings (the emotional component), which may lead to particular behavioural intents (the action tendency component)". However, it should be noted that Oppenheim's (1992) definition includes the cognitive, emotional, and behavioural components, but it omits the evaluative dimension mentioned by Reid (1978). This evaluative dimension was also emphasised by Chaiken and Eagly (1993), who described attitude as a psychological inclination that finds its expression in the evaluation of a particular entity with a certain degree of either favour or disfavour. This definition is now widely accepted. The key element is that of evaluation. Although the definition does not specifically mention the cognitive, affective or behavioural aspects of attitude, the approach adopted by the authors encompasses all three.

8.3 The Significance of Attitudes

According to Bohner and Wanke (2002), the importance of attitudes is evident at various levels of analysis, in that all are subjects of social and socio-psychological research.

"*At the individual level, attitudes influence perception, thinking, others' attitudes and behaviour. Accordingly, attitudes contribute heavily to a person's psychological make-up. At the interpersonal level, information about attitudes is routinely requested and communicated. If we know others' attitudes, the world becomes a more predictable place. At the social level, attitude toward one's own groups and other groups are at the core of intergroup cooperation and conflict."*

Numerous psychologists have attempted to determine whether attitudes play a major part in determining behaviour (Fishbein and Ajzan, 1976; Fazio, 1990, Chaiken and Eagly, 1993). This issue concerns the relation between an individual's attitudes and his/her action or behaviour. In this respect, Fazio (1990) stated that there could be no doubt that attitudes are often linked to subsequent behaviour and that the field had arrived at some comprehension of when this is likely to occur. More specifically, Chaiken and Eagly (1993) noted that "response to an inquiry about an attitude toward a specific behaviour directed toward a given target in a given context at a given time should predict the specific behaviour quite well because this attitude exactly corresponds to the specific behaviour".

8.4 The Theory of Reasoned Action

Ajzen and Fishbein (1980) clarified the relationship between attitude and behaviour with their theory of reasoned attitude, which dealt with behaviour over which individuals have control, i.e. rational decision-making. This theory arose from empirical data, was tested by empirical data and was found to work. However, it was also found to have a gap and therefore it was later modified to the Theory of Planned Behaviour.

This model proposes that an individuals' overt behaviour (B) depends on his/her behavioural intentions (BI), the weakness or strength of which will accordingly affect the performance of the behaviour (Ajzen and Fishbein, 1980). Therefore, the stronger the intention, the more likely it is that the individual will perform that behaviour and vice versa. An individual's behavioural intention can be predicted by two factors. The first predictor is the individual's attitude towards the behaviour (AB). That is, when an individual acquires some information about an attitude object, this leads to either positive or negative feelings about that object. The other predictor is the individual's subjective norm (SN), a concept which had been previously developed by Fishbein and Ajzen (1976). Generally, an individual will be inclined to perform behaviours that they value highly and behaviours that they think are acceptable to other people whose opinion they value. However, it was later concluded by the authors that this model did not adequately explain behavioural intentions and Ajzen (1985) extended it to the Theory of Planned Behaviour.

8.5 The Theory of Planned Behaviour

Ajzen (1985) added another factor which he named perceived behaviour (PB) to the theory of reasoned action. This refers to the perception as to whether the behaviour is possible. This theory is illustrated in Figure 8.1 below.

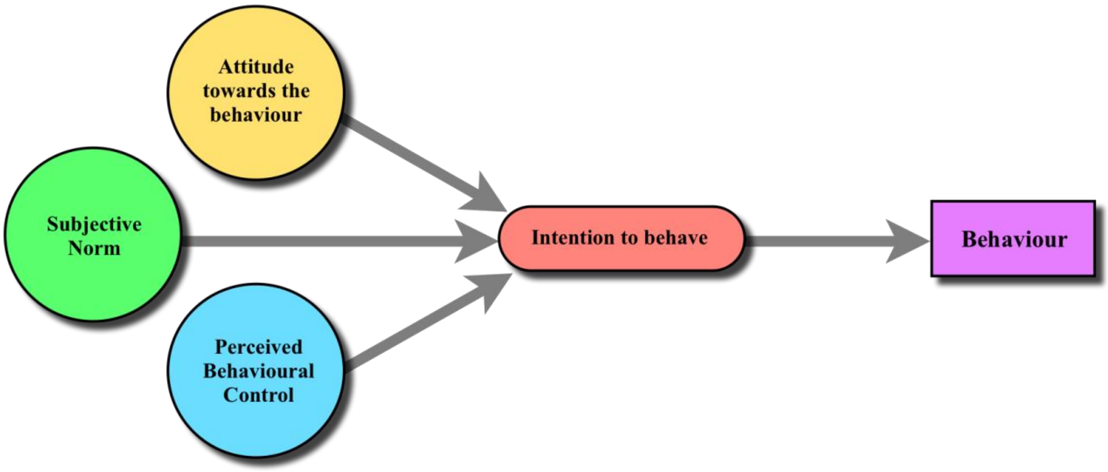


Figure 8.1: The Theory of Planned Behaviour (adapted from Ajzen, 1985).

A number of studies (Ajzen and Madden, 1985; Crawley, 1990; Crawley and Black, 1992) have suggested that this predictor has improved behavioural intention prediction, but that the attitude towards behaviour remains the most influential factor (Ali, 2008).

8.6 Measuring Attitudes

The importance of attitudes in the education process reflects the need for attitude measurement. According to Chaiken and Eagly (1993, p.23) "The aim of measurement is to assign numbers to objects so that the properties of the numbers that are assigned reflect the relations of the objects to each other on the attribute being measured". However, an attitude cannot be observed and measured in the same way as an object which is physically present (Badgaish, 2008). Attitudes can only be measured indirectly and the only way to do this is by observation of words and actions (Henerson et al, 1987). It is important to be aware that attitudes cannot be measured in any absolute sense. Moreover, it is not possible to measure an individual's attitude with any degree of certainty (Badgaish, 2008). Hence, Reid (2006) argues that all that can be done is to compare the pattern of attitudes of one group with another.

8.7 Attitudes to Mathematics

Attitudes are a very important factor in the learning and teaching process. Reid (2006, p. 33) stresses that attitudes "*allow us to:*

Make sense of ourselves;
Make sense of the world around us;
Make sense of relationships."

With regard to attitudes to learning and teaching mathematics, Hannula (2002) states "A lot of research has been done on attitudes toward mathematics, but theoretically the concept needs to be developed." Badgaish (2008) points out that a good deal of effort is spent in considering the cognitive outcomes related to mathematics classroom learning, such as how well the learners can remember, understand or use their knowledge. However, attitudes related to mathematics are less frequently studied and are not a major focus of interest for mathematics teachers (Badgaish, 2008).

Di Martino and Zan (2007, p. 166) argue that "*For a description of a pupil's attitude towards mathematics, it is not enough to highlight his/her (positive/negative) emotional disposition towards the discipline: it is necessary to point out what vision of mathematics and what self-efficacy belief this emotional disposition is associated with.*"

This raises two significant issues. The first is that attitudes towards mathematics are likely to be highly multi-dimensional and not easily measured as a number, score or grade. Secondly, attitudes towards mathematics will almost certainly affect future learning while experiences in learning mathematics will generate attitudes related to mathematics (Badgaish, 2008).

8.8 Developing Positive Attitudes towards Mathematics

Johnstone and Reid (1981) used the word "development" rather than "change" when referring to attitudes in an educational context, arguing that although social psychologists generally refer to change, this term might generate connotations of manipulation when used in an educational context.

Concerning this development, Suydam and Weaver (1975) stress the importance of developing positive attitudes and their effect on the learning of mathematics. They state that *"Teachers and other mathematics educators generally believe that children learn more effectively when they are interested in what they learn and that they will achieve better in mathematics if they like mathematics. Therefore, continual attention should be directed towards creating, developing, maintaining* *and reinforcing positive attitudes*." (Suydam and Weaver, 1975, p.45). This raises the issue of the influence of teaching on students' attitudes to mathematics. In this respect, Jung (2005) emphasises the importance of developing students' positive attitudes towards a subject, as, if their attitudes are negative, it is unlikely that they will be stimulated to learn.

What is to be taught and how it is taught might be the two major influences on students' attitudes importance that mathematics teachers are aware of the attitudes of their students toward mathematics, as these are likely to influence their selection of subjects, their future studies and indeed their future careers. Morrisett and Vinsonhaler (1965) assert that students' attitudes toward mathematics may be drawn or established from their childhood experiences. In this regard, Banks (1964, pp. 16-17) states

"An unhealthy attitude toward arithmetic may result from a number of causes …. But by far the most significant contributing factor is the attitude of the teacher. The teacher who feels insecure, who dreads and dislikes the subject, for whom arithmetic is largely rote manipulation, devoid of understanding, cannot avoid transmitting her feelings to the children … on the other hand, the teacher who has confidence, understanding, interest, and enthusiasm for arithmetic has gone a long way toward ensuring success."

More recently, in a similar vein, Lim and Ernest (2000) expressed their concern that negative images of mathematics may have contributed to the fall in the number of students studying mathematics and science at higher education level and suggest that those negative images may have developed through the influence of their school, parents, or friends.

It has been suggested that teachers are to blame for students' poor attitudes. For instance, according to the NRC (2001, p.132). *"Most US children enter school eager to learn and with positive attitudes toward mathematics. It is critical that they encounter good mathematics teaching in the early grades. Otherwise, those positive attitudes may turn sour as they come to see themselves as poor learners and mathematics as nonsensical, arbitrary, and impossible to learn except by rote memorization. Such views, once adopted, can be extremely difficult to change."* However, as Badgaish (2008) points out, this view does not take into account the nature of mathematics itself, the curriculum, the examination system, or the types of textbooks and resources which are provided for teaching mathematics, over which teachers have little or no control and which may be far more powerful influences. In this regard, the questions in the textbooks, on which this present study focuses, can also be influential in forming students' attitudes towards mathematics.

In one of the few studies on attitudes to mathematics in the Middle East, Alenezi (2008) examined attitudes to mathematics of junior secondary school students and teachers in Kuwait. She found clear evidence that students' attitudes to mathematics become increasingly negative with age and that this was mainly due to an excessively heavy curriculum, combined with the perception that some topics were irrelevant. Alenezi (2008) also highlighted the vital part played by the teacher in forming students' attitudes towards mathematics. Alenezi (2008) concludes that mathematics teachers should have a greater say in the process of making decisions about mathematics curricula, as it is they who are closest to the students and hence in a position to know their requirements.

The situation regarding mathematics education is similar in Saudi Arabia and in Kuwait, as in Saudi Arabia, the teachers have no say in the process of deciding on the curriculum. One of the objectives of this present study is to determine the perspective of Saudi mathematics teachers and inspectors regarding how the textbooks and their questions in Saudi schools affect students' attitudes towards mathematics.

The next chapter will describe the methodology adopted for this study and justify its selection.

Chapter 9

Methodology

9.1 Introduction

This chapter describes the methodology used in the study. It begins by discussing the research design, after which the choice of approach is discussed and justified. The instruments used in the study are presented, as are the study population and sample, and the verification of the validity and reliability of the study.

It also describes the analysis units and the steps and procedures carried out in the study to evaluate the questions of the mathematics textbooks in the secondary school stage, scientific section, in Saudi Arabia and the extent to which such questions measure the various skills of mathematical thinking, conform to criteria of good formulation and good layout, and reinforce a positive attitude towards mathematics on the students' part.

9.2 Research design

A research design is the plan for deciding what data to collect, what sources to use and what methods to use in the collection of the data. The research design forms the framework for gathering and analysing the data, and the selection of the research design should be informed by the research questions (Bryman, 2004).

In this study, the research design consists of four main elements, which are strategy, conceptual framework, sampling, and the tools and processes of data collection and data analysis.

De Vaus (2002) asserts that the function of a research design is to allow the researcher to answer the research question(s) without ambiguity. Therefore, according to Yin (2003), the purpose of the research design is to help to achieve the research aim and objectives. Hence, the selection of a research design should match the overall research strategy, as the selected methodology guides the methods used and the way in which each is used (Silverman, 2000). Therefore, it is vital to carry out careful planning in order to elicit precise answers to the research questions and thus accomplish the research objectives (Saunders et al, 2000). Three main concerns comprise the basis of any research strategy: the object of the research, the aims of the research and the data collection methods (Saunders et al, 2000).

The choice of a suitable methodology is basic to any research design. A research considers four main issues. The first issue refers to the 'logical rationale' for answering research questions and regards which strategy to follow (Punch, 2009, p.113). The main strategy used in this study is survey research, in which the researcher asks participants a series of questions, summing up their answers with percentages and frequencies, from which inferences may be drawn about a specific population from the sample's responses (Leedy and Ormrod, 2001). The survey is complemented by structured interviews with a selection of teachers and inspectors. These will be described in greater detail in Chapter 12.

The second issue regarding framework involves the conceptual position of who or what is being studied and their relationship with each other. Since the main issue in this research project is the questions in the mathematics textbooks of the secondary school stage, scientific section prescribed by the Ministry of Education in Saudi

Arabia, previous studies in this field have been reviewed to formulate the conceptual framework for this research. This study seeks to determine education inspectors' and teachers' evaluations of the conditions of good formulation and layout of these questions, and the extent to which they reinforce the students' positive attitudes towards mathematics. It also seeks to discover the effect of a number of variables on the educational inspectors' and teachers' evaluations.

The third issue refers to the research sampling and concerns from whom the data will be gathered. In accordance with the aim of the study, two types of respondent (school mathematics teachers and school inspectors) were the participants in this research project. In order to have representative respondents, a random sampling was employed (Stouthamer-Loeber and van Kammen, 1995). The fourth issue involves the instruments and procedures employed in the data collection and analysis. This chapter addresses the methodological approaches used for carrying out the survey and analysing the data obtained from the questionnaire and interviews in this study.

According to Kumar (2005), when conducting research in education, researchers should choose a type of research in accordance with the statement of the research problem they are investigating (Figure 9.1).

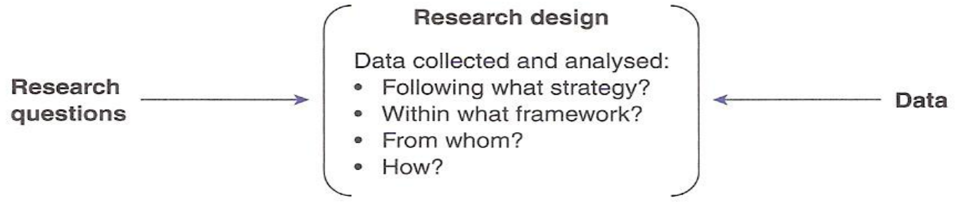


Figure 9.1: Research design linking research questions to data (Source: Punch, 2009, p. 114).

9.3 Mixed Method Approach

A mixed method approach was employed in this study. Such an approach involves both quantitative and qualitative research. According to Johnson and Onwuegbuzie (2004), mixed method research has the advantage of being able to add to the meaning of numbers through the use of words and images and vice versa. Similarly, Cresswell (2005) states that a mixed method approach is appropriate if the researcher wishes to make use of the strengths of both qualitative and quantitative data. Use of a mixed method approach also helps to ensure validity (Fraser and Tobin, 1991).

According to Bryman (2004, p.8), "[T]he way in which people being studied understand and interpret their social reality is one of the central motifs of qualitative research." However, there is no single, accepted method of carrying out qualitative research. If data analysis methods are applied correctly, this ensures that the results are not just intuitive, albeit that a strong element of empiricism is involved in qualitative research. Snape and Spencer (2004) state that the most crucial factors in qualitative research are having a clear aim for the research; using a suitable research strategy; selecting appropriate methods of analysing and interpreting data; permitting theories to emerge naturally from data, and; attempting to answer the questions, 'what?', 'how' and 'why?' .

According to Wall (2001), there should be a number of valid reasons for the decision as to which methodological approach is appropriate. Further, Gall et al. (2006) assert that qualitative and quantitative research can both be of use to educational researchers. Therefore, the researcher deemed it appropriate to use a combination of qualitative and quantitative methods for this research, with the quantitative element being based on the data collected from the questionnaire, and the qualitative one coming from the interpretation of the data collected through the interviews. This approach permitted the researcher to gain a deeper understanding and to undertake a more thorough interpretation of the data than would have been provided by using only statistical analysis.

9.4 Study Tools

In order to answer the questions of the study, the researcher prepared three tools to evaluate the questions of the three mathematics textbooks and the extent to which they measure the various skills of mathematical thinking represented therein, conform to criteria of good formulation and good layout, and reinforce a positive attitude towards mathematics on the students' part (see Appendices 2 and 3). While building these tools, the researcher utilised educational literature including books, journals and studies, many of which are reviewed in the literature review, by means of which he evaluated the questions and the various skills of mathematical thinking. He also utilised the mathematics curricula and their broad guidelines in the secondary school stage, including the general and specific objectives of the curricula that are related to the development of mathematical thinking in the students' minds.

To achieve these objectives, the researcher conducted a theoretical analytic study, together with a field study to discover the views of the inspectors and teachers regarding the questions of the textbooks which are the subject matter of the study.

A questionnaire was used in the analysis of the questions in the textbooks. The underpinning of the questionnaire was the theoretical background that the researcher took from theoretical references and previous studies. This was presented to the questionnaire respondents in order to discover their views and their evaluative ratings of these questions (see Appendix 2).

An analysis model was developed to analyse the subject matter of the items in the textbooks being studied (see Appendix 1).

Interviews were conducted with 14 teachers and 5 inspectors in order to discover their views on the questions in the textbooks and the extent to which they promote mathematical thinking skills (see Appendix 3).

The researcher chose these tools for the following reasons:

Content analysis appeared likely to be the most suitable tool to analyse the questions in the textbooks and to be capable of revealing their positive and negative features.

The questionnaire was regarded as the most suitable instrument to elicit the views of the inspectors and teachers in order to answer the research questions. The respondents can complete the questionnaire at length and without haste.

The interview schedule consists of open-ended questions, i.e. without fixed answers. The interviews were conducted in order to complement the information obtained from the questionnaire.

9.5 Questionnaire design

The main instrument used to collect data for the study is that of a questionnaire, which includes questions and statements to which the participants respond. Cohen et al. (2007) state that surveys are the most widely used method in educational research. Wiersma and Jurs (2005, p.195) indicate that the purpose of a survey is to obtain a 'snapshot of conditions, attitudes, and/or events at a single point in time.' This agrees with Leedy and Ormrod (2001), who suggest that the main purpose of a survey is to depict the characteristics of a group of individuals about a particular subject or issue by asking them questions. According to Rea and Parker (2005, pp. 3-4), another primary function of using survey research is to present an 'accurate representation of information' by the means of gathering primary data. By the 'accurate representation of information', they mean accurate generalisation about the viewpoints, attitudes, and ideas on the issues involved of a large population by studying a small section of that population (Rea and Parker, 2005, p.4). Hence, educational researchers sample a population through a survey, generally by the random choice of a small group from a large population.

Based on the aforementioned definition and purpose of surveys in educational research, this study employs a questionnaire survey, details of which were given in section 9.4, as the main research method to investigate the issues on which the thesis focuses.

9.6 Pilot Study

According to van Teijlingen and Hundley (2001), "[P]ilot studies are a crucial element of a good study design. Conducting a pilot study does not guarantee success in the main study, but it does increase the likelihood”. As De Vaus (1993, p. 54) recommends, "Do not take the risk; pilot test first." A pilot study enhances the internal validity of a questionnaire (Peat et al., 2002). The questionnaire should be administered to pilot participants in precisely the manner in which it will be administered in the principal study. They are then asked for comments in order to identify any ambiguities, repetition or questions they do not fully understand; in this way, the researcher can modify, re-word or delete questions if necessary (Saunders and Lewis, 2012). In addition, a pilot study allows the researcher to determine that responses can be interpreted in terms of the information that is required and re-word or re-scale any questions to which the responses are not as expected (Peat et al., 2002). The researcher followed this procedure, choosing as his subjects for the pilot study of the questionnaire twelve teachers and five inspectors in Jeddah. These pilot subjects indicated that two of the questions were slightly ambiguous; therefore these were re-worded for greater clarity. In addition, they indicated that six of the items were repetitious and another three were not entirely relevant to the topic. Therefore, the researcher deleted these items, reducing the number of items from 62 to 53.

9.7 Response rate

The researcher initially distributed 3409 questionnaires, of which 1466 useable questionnaires were returned. This represents a response rate of 43%, which is considered very high.

9.8 Study population

The study population consists of all the teachers and inspectors of mathematics in the secondary schools in all the regions of the Kingdom of Saudi Arabia. The sample consists of 1466 respondents in total; that is, 1308 mathematics teachers and 158 inspectors were taken from the following education directorates and departments:

9.8.1 Central Region

The General Directorates of Education in Riyadh.

The Education Department in the Province of Al-Kharj.

9.8.2 Western Region

The General Directorate of Education in Makkah.

The General Directorate of Education in Al-Madinah.

The Education Department in the Province of Jeddah.

The Education Department in the Province of Al-Taif,

The Education Department in the Province of Al-Qunfudhah.

The Education Department in the Province of Al-Laith.

The Education Department in the Province of Yanbu.

9.8.3 Northern Region

The General Directorate of Education in Al-Bahah.

The General Directorate of Education in Jazan.

The Education Department in the Province of Al-Mikhwah.

The Education Department in the Province of Khamis Mishait.

The Education Department in the Province of Abha

9.8.4 Eastern Region

The General Directorate of Education in the Eastern Region.

The General Directorate of Education in Al-Qaseem.

The Education Department in the Province of Unayzah

9.8.5 Southern Region

The General Directorate of Education in Al-Jouf.

The General Directorate of Education in Northern Borders.

The General Directorate of Education in Tabuk.

The Education Department in the Province of Qirayat

9.9 Ethical Considerations

Having obtained the requisite ethical permission from the University of Strathclyde to carry out the field work, the researcher made every effort to observe the ethical considerations involved in this work. The research participants were informed of the reason for the research and that their answers would be used only for academic purposes. They were also assured of the confidentiality of their responses and that their identities would not be revealed to any third party. Moreover, the interviewees were asked for their permission to audio-record the interviews and informed that they were free to withdraw from the interviews at any time without having to give a reason. All but one of the interviewees declined to have the interview recorded, although none of them expressed any objection to the interviewer making notes during the interviews. The interviewer respected their wishes.

9.10 Administration of the questionnaire

In order to distribute the questionnaire to the sample, the researcher, who was employed as a university lecturer at Umm Al-Qura University, first obtained a letter explaining about his research from his supervisor at the University of Strathclyde, which he then took to the Saudi Cultural Attaché in London. The Attaché gave the researcher a letter for the Dean of the Education College of Umm Al-Qura University and the Dean wrote to the managers of the General Directorates of Education. These managers wrote to the managers of the Education Departments of each province, who in turn wrote letters of authorisation to inspectors and headteachers and the headteachers requested the co-operation of the teachers. Therefore, although obtaining the authorisation did not present any difficulties *per se*, it was a rather lengthy process.

9.11 Selection of method of analysis

The researcher considered the possibility of using discourse analysis to analyse the textbook data. Discourse analysis is a complex data analysis method. In discourse analysis, the researcher identifies themes, categories, views, ideas, and so forth, in the text (Fulcher, 2012). The researcher attempts to answer questions such as how the discourse assists an understanding of the issue under study, and how individuals build their own account of an event. "The question of reliability in discourse analysis concerns whether different researchers would interpret the text in similar ways" (Fulcher, 2012). Stratton (1997, p.116) states that researchers are likely to vary in their "motivational factors, expectations, familiarity, and avoidance of discomfort". Therefore, it must be accepted that the interpretation of the data is subjective and the data may be interpreted differently by another researcher. Content analysis is a data analysis method that may be employed with either qualitative or quantitative data and in an inductive or deductive manner (Elo and Kyngäs, 2008). According to Cole (1998), it is a way of analysing written, visual or spoken communication. Content analysis permits the researcher to improve understanding of the data. Through content analysis, words may be reduced into fewer content-related categories. "It is assumed that when classified into the same categories, words, phrases, etc. share approximately the same meaning" (Cavanagh, 1997). According to Krippendorff (1980), "Content analysis is a method for making replicable and valid inferences from data to their context, with the purpose of providing knowledge, new insights, a representation of facts and a practical guide for action". The goal is to gain a succinct and broad description of the phenomenon, and the result of the analysis is concepts or categories describing the phenomenon. The main advantage of the method is that it allows the researcher to deal with a large amount of data (Elo and Kyngäs, 2008). After careful consideration, the researcher decided to adopt content analysis, as he considered it more suitable for analysing quantitative data. In addition, there was a large amount of data to deal with and this is facilitated by content analysis. Moreover, he considered content analysis to be more objective and hence more reliable than discourse analysis.

9.12 Content Analysis

An analysis model was developed to analyse the textbook items which are the subject matter of the study. The process of development of the analysis model passed through several stages that can be summarised as follows:

1. Literature dealing with the thinking process in general and mathematical thinking in particular, was reviewed, in addition to previous studies on the analysis and evaluation of mathematics textbook questions.

2. The fundamental mathematical thinking skills that were mentioned in the various sources were noted, and a preliminary model comprising six mathematical thinking skills (generalisation, pattern logic, induction, deduction, expression with symbols and mathematical proof) was prepared.

3. The preliminary model of analysis was presented to 29 referees and experts in the field of assessment and evaluation, as well as in mathematics teaching methods. These included seven university teachers, six PhD students at Um Al-Qura University in Makkah preparing dissertations in the fields of mathematics curricula and methods of teaching, six PhD students at Um Al-Qura University in Makkah preparing dissertations in the fields of assessment and evaluation, four inspectors in the Ministry of Education who had Master's degrees in mathematics curricula and methods of teaching, and three holders of Master's degrees in the fields of assessment and evaluation. The rest of the referees were experienced secondary school mathematics teachers. The preliminary analysis model was then revised and expanded to include nine mathematical thinking skills (knowledge and recall, understanding and interpretation, modelling, application, induction, generalisation, deduction, mathematical proof, and evaluation).

4. The nine aforementioned thinking skills were distributed on two levels. These two levels of thinking skills were derived from the researcher, the literature review, the referees, and the teachers and inspectors who participated in the research.

a- Four basic cognitive skills, namely: knowledge and recall, understanding and interpretation, modelling, and application.

b- Five complex cognitive skills, namely: induction, generalisation, deduction, mathematical proof, and evaluation.

5. An initial analysis of some random samples from the mathematics textbooks of the secondary school stage, was carried out. Based on the results of this analysis, the analysis model was approved in its final form (Appendix 1).

9.13 The method and units of analysis

The following procedure was adopted in the analysis of the mathematics textbooks, which form the subject matter of this study:

1. Each textbook was analysed separately.
2. All the mathematics textbooks of the secondary school stage were used in the analysis.
3. Each unit was broken down into lessons and then into items, with each item being regarded as a unit of analysis. An item is a group of statements that convey a single idea with a complete meaning, whether consisting of one sentence or several sentences.
4. The analysed items were classified into two types; namely, explanatory items and question items. The explanatory items provide the student with mathematical knowledge such as concepts, generalisations and algorithms organised in various ways, while the question items require the students to use this acquired knowledge together with their previous experience *vis-à-*

vis new situations, or provide them with skills in the use of algorithms that they have already learnt.

5. A table showing the cognitive skills used by the textbooks in the explanatory items or the question items was drawn up. The solved examples in the textbook, as well as the teacher's manual, were used to determine the cognitive skills required to solve the questions in the textbooks.
6. The representation percentages in the previous tables for each of the nine skills of mathematical thinking were used to indicate the degree of attention shown in the analysed textbooks to the development of each thinking skill. The aforementioned representation percentages were classified into four levels: very little; little, considerable and; very considerable attention. Then the value of each level was determined as follows:

a) The difference between the skill with the highest representation percentage, which was 43%, and the lowest, which was 0%, was calculated. Hence, the difference was 43%.

b) The above difference was divided by the number of levels of degree of attention (i.e., four). The result was 11%.

c) The difference in step "b" above was added to the lowest representation percentage of all four levels. Thus the four levels appeared as follows:

i. Very little attention level is between the representation levels of 0% and 11%

ii. Little attention level is between the representation levels of above 11% and 22%

iii. Considerable attention level is between the representation levels of above 22% and 33%.

iv. Very considerable attention level is between the representation levels of above 33% and 44%.

7. Other tables were made for the classification of the analysed items based on the mathematical thinking level they represented (basic, or complex). The representation percentages extracted in the previous tables for each mathematical thinking level were used to determine the extent of the attention given in the analysed textbooks to the development of each level. The same procedures indicated in Step 6 above were used to classify the thinking levels in four levels (very little, little, considerable and very considerable), and then the extent for each attention level was determined as follows:

i. Very little attention level - between the representation levels of 0% and 22%

ii. Little attention level - between the representation levels of above 22% and 44%

iii. Considerable attention level - representation levels of over 44% to 66%.

iv. Very considerable attention level - over 66%

9.14 The mathematics textbooks

The textbooks which comprised the object of the study were the following:

The mathematics textbook of the first secondary school grade, 2009/10 edition.

The mathematics textbook of the second secondary school grade, scientific section, 2009/10 edition.

The mathematics textbook of the third secondary school grade, scientific section, 2009/10 edition.

Each mathematics textbook comprises several chapters. Each textbook discusses several study topics, as follows:

First: The mathematics textbook of the first secondary school grade consists of the following chapters:

Mathematical logic.

Relations and applications.

Plane geometry.

Equations and analytic geometry.

Inequalities.

Trigonometry.

Exponential and logarithmic functions.

Statistics.

Second: The mathematics textbook of the second secondary school grade, scientific section, consists of the following chapters:

Binary operations and groups.

Matrices and determinants.

Trigonometry.

Complex numbers.

Polynomials.

Solid geometry (1).

Vector analysis.

Binomial theory.

Probabilities.

Third: The mathematics textbook of the third secondary school grade, scientific section, consists of the following chapters:

Conic sections.

Sequences and series.

Limits and continuity.

Differentiation.

Applications of differentiation.

Integration.

Applications of definite integration.

Solid geometry (2).

In the organisation of each unit in each of the aforementioned textbooks, the following can be observed:

The order of the mathematical content is organised by headings and subheadings and each is followed by drills.

There is a group of evaluation questions for each unit.

There is a group of drills at the end of each unit for the purpose of accumulative revision with a view to consolidating skills.

9.15 Validity and Reliability

In order to ensure the validity of the analysis model that was prepared to fulfil one of the objectives of this study, the preliminary model was presented to twenty-nine referees who were expert in the fields of assessment and evaluation as well as in mathematics teaching methods, of whom 7 were university lecturers, 6 were Ph.D. students preparing theses in the fields of mathematics curricula and methods of teaching in Um Al-Qura University in Makkah, 6 were Ph.D. students preparing dissertations in the fields of assessment and evaluation in Um Al-Qura University in Makkah, 4 were inspectors in the Ministry of Education who hold Master's degrees in mathematics curricula and methods of teaching, and 3 were Master's degree holders in the fields of assessment and evaluation. The rest of referees were experienced mathematics teachers at the secondary school stage. All of these referees were asked to indicate their views regarding the items of the study instruments in general and the items of the analysis model in particular, and to suggest any changes, additions or omissions in the items. The initial analysis model was then revised and expanded to include nine skills of mathematical thinking.

Reliability of an analysis refers to the matching of the results of a single analysis when conducted by several analysts (Alam, 1991). Burns (2000) asserts that reliability may be established by decisions taken concerning categories and data, the reporting of any personal bias, and a combination of two or more data collection strategies. Reliability means the presence of a close match in terms of the following two dimensions: The matching between the researchers conducting the analysis and the temporal matching which is when one single analyst reaches the same results after having analysed the content more than once (Husein, 1983).

The reliability of the analysis was confirmed by means of these two dimensions together, as follows:

9.15.1 First: Reliability of the researcher's analysis

The researcher analysed the content twice for each of the aforementioned textbooks separated by a time interval of thirty days. There was an indication that the results of the content analysis match, for the results of the analysis were very similar apart from negligible differences.

The researcher applied the following agreement equation in order to check the agreement between two applications for each textbook in the case of the analysis of both the explanatory items and the question items:

Agreement Coefficient = Number of Agreed Upon Skills / Number of Judged Skills

Agreement (First Grade, Explanatory Items) = 730/737≈ 0.99

Agreement (First Grade, Question Items) = 2105/2420≈ 0.87

Agreement (Second Grade, Explanatory Items) = 841/946≈ 0.89

Agreement (Second Grade, Question Items) = 1961/2283≈ 0.86

Agreement (Third Grade, Explanatory Items) =803/824≈ 0.97

Agreement (Third Grade, Question Items) =2098/2110≈ 0.99

As can be seen from the above, the reliability for the textbooks that were analysed was very high.

9.15.2 Second: Reliability of the analysis according to the referees

The researcher used the agreement coefficient between his analysis and the analysis of certain referees. The following table shows the agreement coefficients between the researcher and the referees regarding the analysis of the school curricula.

Table 9.1: The agreement coefficients between the researcher and the referees

Referees	1st grade		2nd grade		3rd grade	
Item	**Explanatory**	**Question**	**Explanatory**	**Question**	**Explanatory**	**Question**
First referee	**0.91**	**0.82**	--	--	--	--
Second referee	**0.87**	**0.93**	--	--	--	--
Third referee	--	--	**0.79**	**0.88**	--	--
Fourth referee	--	--	**0.84**	**0.84**	--	--
Fifth referee	--	--	--	--	**0.81**	**0.94**
Sixth referee	--	--	--	--	**0.93**	**0.78**
Average	**0.89**	**0.88**	**0.82**	**0.86**	**0.87**	**0.86**

The table above shows agreement coefficients between the researcher and the referees in the analysis of the explanatory and question items in the textbooks for all three grades. It can be seen that the average coefficients between the researcher and the referees were very high.

As can been seen from the above, the reliability of the analysis of the textbooks was very high. Accordingly, the researcher was assured as to the reliability of the analysis of the content of all the curricula.

9.16 Conclusion

The key focus of this enquiry is to look at several aspects (mathematical thinking, formulation, layout, attitudes) used in the selected mathematics textbooks and their questions. It is important to look at current textbooks in Saudi Arabia and see what is happening. It is also important to see how those who use the textbooks (especially in teaching) find the questions.

In looking at the nature of the mathematics textbook questions, two sources of evidence were considered:

The views of those who are 'expert' in mathematics as a discipline;

The views of those who have expertise in the teaching of mathematics at this level.

In order to consider these views, it was decided to focus on an analysis of content. In this, the actual nature of the mathematics questions and the subject content matter of these questions were analysed. The nature of mathematics textbooks and their questions led to the conclusion that a content analysis approach was the best way forward.

Two major user groups were considered: teachers of mathematics, and school inspectors in mathematics. It was not possible to ask the students as, at their age and levels of experience, they are as yet unable to comment on the nature of the way mathematics questions are being asked. Respondents can offer their insights only in terms of writing or talking.

Questionnaires have the advantage of gaining an overall picture of respondent views quickly while interviews allow for more detailed insights. An alternative approach might have used focus groups but it was not possible to have groups of the potential respondents together in one place at the one time.

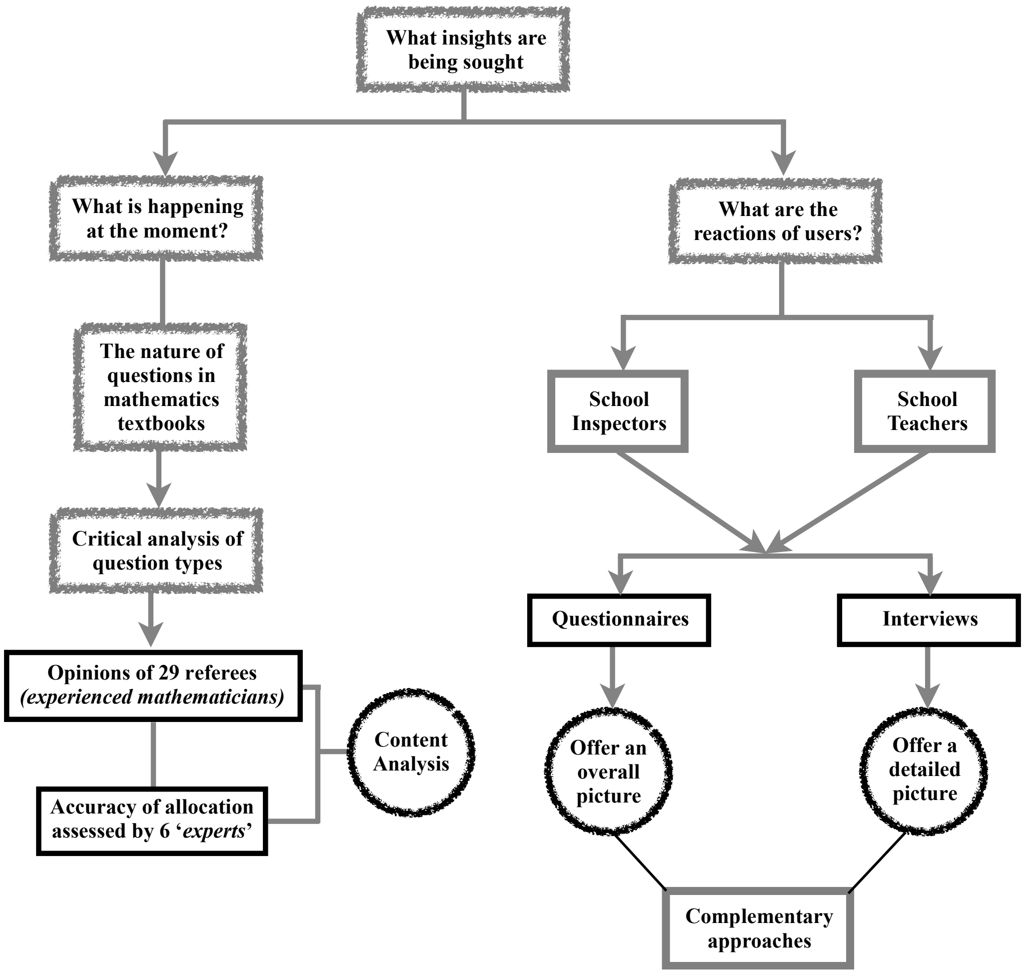


Figure 9.2: Research Procedure (Source: Author)

The numbers obtained from questionnaires are ordinal in nature. Ordinal numbers cannot be legitimately added or subtracted. Thus, in taking means of these distributions, the 'means' obtained must be seen as merely indicative, thus giving a pointer to a kind of 'average' view.

Chapter 10

Content Analysis Data

10.1 Introduction

This chapter will present the results of the content analysis of the mathematics textbooks for the first, second and third grades in the scientific section in secondary schools in Saudi Arabia. In this chapter, the first and second research questions will be answered. In all the tables, mathematical skills are described under nine headings, this being based on the analysis which can be seen in Appendix 1.

10.2 Section One

This section looks at the results of the analysis related to the answer of the first main question:

What are the skills of mathematical thinking prevailing in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia?

10.2.1 First Sub-Question

The analysis of results related to the answer of the first sub-question are considered here. The sub-question is:

What are the skills of mathematical thinking prevailing in the explanatory items of the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia?

Tables 10.1-10.3 show the classification of the explanatory items that were chosen for analysis from the items of the secondary school grade textbooks, based on the mathematical thinking skills included in each item.

Table 10.1: The classification of the explanatory items that were chosen for analysis from the items of the secondary school first-grade textbook based on the mathematical thinking skills included in each item.

N	**Title of the unit**	Number of items	Know-ledge and recall	Understa-nding and Interpret-Ation	Modell-ing	Applic-Ation	Induction	General-isation	Deduct-ion	Mathemat ical proof	Evaluat-ion	Number of skills included
1	*Mathematical logic*	108	71	61	10	0	0	3	0	13	1	159
2	*Relations and applications*	41	21	20	12	0	0	3	2	4	3	65
3	*Plane geometry*	41	15	21	9	0	0	4	8	5	0	62
4	*Equations and analytic geometry*	54	26	31	19	0	0	6	2	3	0	87
5	*Inequalities*	30	16	27	9	0	0	1	1	2	1	57
6	*Trigonometry*	45	14	39	17	0	0	1	9	10	6	96
7	*Exponential, logarithmic*	134	28	98	8	0	0	14	5	13	0	166
8	*Statistics*	25	11	20	9	0	0	1	1	0	3	45
Total		**478**	**202**	**317**	**93**	**0**	**0**	**33**	**28**	**50**	**14**	**737**
%			**27**	**43**	**13**	**0**	**0**	**5**	**4**	**7**	**2**	**100**

Table 10.2: The classification of the explanatory items that were chosen for analysis from among the items of the secondary school second grade textbook, based on the skills of mathematical thinking included in each item.

N	Title of the unit	Number of items	Know-ledge and recall	Understa-nding and Interpret-Ation	Modell-ing	Applic-Ation	Induction	General-isation	Deduct-ion	Mathemat ical proof	Evaluat-ion	Number of skills included
1	*Binary operations and groups*	59	20	48	7	0	1	6	3	0	2	87
2	*Matrices and determinants*	66	23	54	8	0	1	3	1	0	8	98
3	*Trigonom'y*	192	39	80	41	0	0	5	19	29	15	228
4	*Complex numbers*	81	12	60	10	0	0	8	7	4	2	103
5	*Polynomials*	36	22	33	6	0	0	0	4	2	0	67
6	*Solid geometry (1)*	46	8	27	13	0	2	4	2	12	1	69
7	*Vector analysis*	29	11	20	10	0	0	5	7	4	2	59
8	*Binomial theory*	80	15	65	2	0	1	5	4	3	2	97
9	*Probabilities*	97	41	74	12	0	0	1	2	5	3	138
Total		**686**	**191**	**461**	**109**	**0**	**5**	**37**	**49**	**59**	**35**	**946**
%			**20**	**49**	**12**	**0**	**1**	**4**	**5**	**6**	**4**	**100**

Table 10.3. The classification of the explanatory items chosen for analysis from the items of the secondary school, third grade textbook based on the mathematical thinking skills included in each item.

N	Title of the unit	Numbr of items	Know-ledge and recall	Understa-nding and Interpret-ation	Modell-ing	Applic-Ation	Induction	General-isation	Deduct-ion	Mathematical proof	Evaluat-ion	Number of skills included
1	*Conic sections*	42	12	49	44	0	2	5	10	1	0	123
2	*Sequences and series*	69	18	49	5	0	4	8	4	3	0	91
3	*Limits and continuity*	114	29	75	21	0	0	16	0	1	5	147
4	*Different'n*	69	29	49	5	0	1	2	5	8	2	101
5	*Applications of different'n*	33	14	29	21	0	1	3	1	1	2	72
6	*Integration*	69	24	55	11	0	0	9	3	1	1	104
7	*Appl'ns of def integr'n*	63	24	52	13	0	0	3	3	1	5	101
8	*Solid geometry (2)*	58	21	29	11	0	1	5	4	13	1	85
Total		**517**	**171**	**387**	**131**	**0**	**9**	**51**	**30**	**29**	**16**	**824**
%			**21**	**47**	**16**	**0**	**1**	**6**	**4**	**4**	**2**	**100**

In all three grade textbooks, most of the explanatory items relate to understanding and interpretation (43-49%) and knowledge and recall (20-27%). In many of the other areas, there are very few explanatory items at all.

This is illustrated in Figures 10.1 and 10.2.

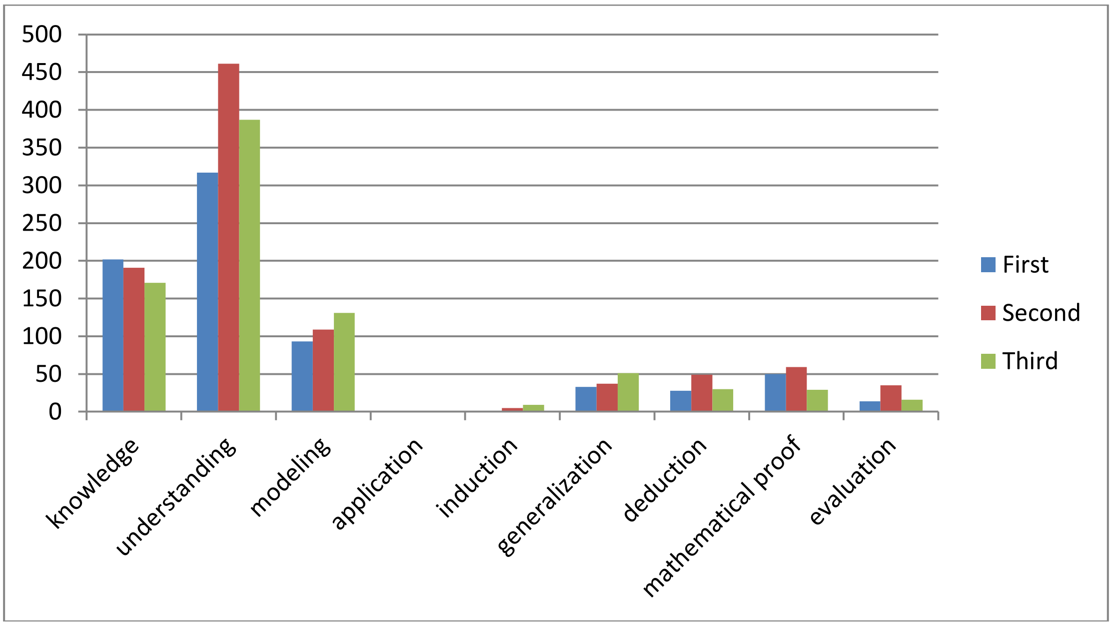


Figure 10.1: Number of explanatory items in textbooks

It is clear that the general pattern is similar for all three grade textbooks. This is a matter of concern, for it might have been expected that there would be some graduation and development with age, as new skills were encouraged.

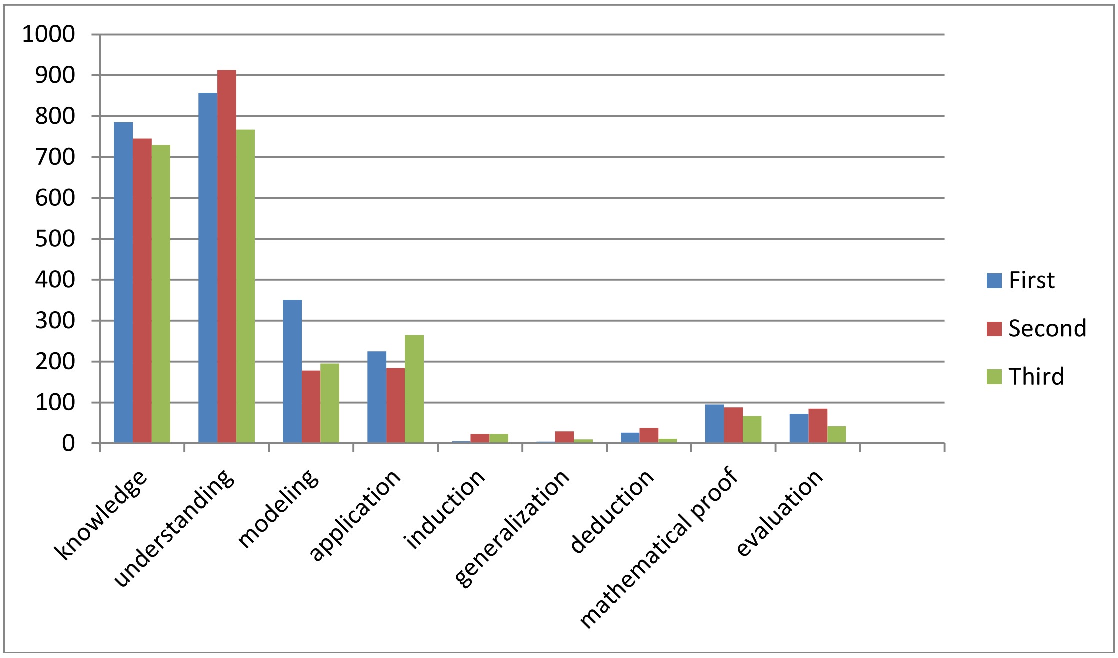


Figure 10.2: Number of question items in textbooks

Again, the balance of the various skills is very similar for all three age groups.

10.2.2 Second Sub-Question

The results of the analysis related to the answer to the second sub-question:

What are the mathematical thinking skills prevailing in the question items of the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia?

Tables 10.4 to 10.6 show the classification of the question items that were chosen for analysis from among the items of the secondary school grade textbooks, based on the mathematical thinking skills included in each item.

Table 10.4. The classification of the question items that were chosen for analysis from among the items of the secondary school first grade textbook, based on the mathematical thinking skills included in each item.

N	Title of the unit	Number of items	Know-ledge and recall	Understa-nding and Interpret-ation	Modell-ing	Applic-Ation	Induction	General-isation	Deduct-ion	Mathemat ical proof	Evaluat-ion	Number of skills included
1	*Mathematical logic*	250	149	131	19	29	1	1	1	24	9	364
2	*Relations and applications*	174	59	77	64	36	0	1	1	4	5	283
3	*Plane geometry*	125	55	76	34	41	0	0	15	3	2	226
4	*Equations and analytic geometry*	222	99	120	59	39	1	0	1	15	14	348
5	*Inequalities*	43	30	53	16	7	0	0	0	5	4	115
6	*Trigonometry*	127	77	98	34	11	1	1	1	29	11	263
7	*Exponential and logarithmic functions*	314	179	181	27	51	2	1	1	6	4	452
8	*Statistics*	110	101	121	98	11	0	0	6	9	23	369
Total		**1365**	**785**	**857**	**351**	**225**	**5**	**4**	**26**	**95**	**72**	**2420**
%			**32**	**35**	**15**	**9**	**0**	**0**	**1**	**4**	**3**	**100**

Table 10.5: The classification of the question items that were chosen for analysis from the items of the secondary school second grade textbook, based on the mathematical thinking skills included in each item.

N	Title of the unit	Number of items	Know-ledge and recall	Understa-nding and Interpret-ation	Modell-ing	Applic-Ation	Induction	General-isation	Deduct-ion	Mathemat ical proof	Evaluat-ion	Number of skills included
1	*Binary operations and groups*	125	58	136	13	52	3	4	3	5	2	276
2	*Matrices and determinants*	169	102	109	31	6	2	1	3	3	20	277
3	*Trigonometry*	288	129	143	51	29	3	4	3	13	11	386
4	*Complex numbers*	112	89	96	4	5	0	1	1	5	2	203
5	*Polynomials*	81	54	89	29	13	3	1	1	6	1	197
6	*Solid geometry (1)*	129	51	55	20	46	4	10	16	33	9	244
7	*Vector analysis*	96	69	75	15	9	2	3	5	7	9	194
8	*Binomial theory*	115	101	111	4	5	3	3	4	11	15	257
9	*Probabilities*	102	92	99	11	19	3	2	2	5	16	249
Total		**1217**	**745**	**913**	**178**	**184**	**23**	**29**	**38**	**88**	**85**	**2283**
%			**33**	**40**	**8**	**8**	**1**	**1**	**2**	**4**	**4**	**100**

Table 10.6: The classification of the question items that were chosen for analysis from the items of the secondary school third grade textbook, based on the mathematical thinking skills included in each item.

N	Title of the unit	Number of items	Know-ledge and recall	Understa-nding and Interpret-ation	Modell-ing	Applic-Ation	Induction	General-isation	Deduct-ion	Mathematical proof	Evaluat-ion	Number of skills included
1	*Conic sections*	62	69	70	35	20	1	2	1	1	0	199
2	*Sequences and series*	73	71	70	1	11	21	1	1	1	1	178
3	*Limits and continuity*	153	76	99	10	45	1	1	2	3	21	258
4	*Differentiation*	176	141	152	5	39	0	0	0	3	6	346
5	*Applications of differentiation*	110	73	86	77	29	0	0	0	7	5	277
6	*Integration*	162	120	126	19	51	0	0	0	10	3	329
7	*Applications of definite integration*	149	115	101	26	44	0	0	0	9	2	297
8	*Solid geometry (2)*	106	65	63	22	26	0	6	7	33	4	226
Total		**991**	**730**	**767**	**195**	**265**	**23**	**10**	**11**	**67**	**42**	**2110**
%			**35**	**36**	**9**	**13**	**1**	**1**	**1**	**3**	**2**	**100**

Tables 10.4 to 10.6 show similar patterns, with the majority of question items reflecting assessment of knowledge and recall, as well as understanding and interpretation.

10.3 Section Two

This looks at the results of the analysis related to the answer to the second main question:

What is the extent of the attention paid to the development of mathematical thinking in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia?

In order to indicate the extent of the attention paid to the development of each level of mathematical thinking (basic and complex) by the mathematics textbooks for the scientific section of secondary schools, the instrument described in Chapter 9 was used. The percentages of representation in the range up to 44% indicate very little to little attention, while the percentages of representation above 44% indicate considerable to very considerable attention.

10.3.1 First Sub-Question

The results of the analysis related to the answer to the first sub-question:

What is the extent of the attention paid by the explanatory items in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia, to the development of mathematical thinking based on the level of mathematical thinking (basic and complex) that they represent?

Tables 10.7 to 10.9 show the classification of the explanatory items that were chosen for analysis from the items of the secondary school textbooks for grades one to three, based on the level of mathematical thinking (basic and complex) represented by each item.

Table 10.7: The classification of the explanatory items that were chosen for analysis from the items of the secondary school first grade textbook based on the level of mathematical thinking (basic and complex) represented by each item.

N	Title of the unit	*Basic Level*		*Complex Level*		*Total*
		Frequency	***%***	***Frequency***	***%***	***Frequency***
1	*Mathematical logic*	142	89	17	11	159
2	*Relations and applications*	53	82	12	19	65
3	*Plane geometry*	45	73	17	27	62
4	*Equations and Analytic geometry*	76	87	11	13	87
5	*Inequalities*	52	91	5	9	57
6	*Trigonometry*	70	73	26	27	96
7	*Exponential and logarithmic functions*	134	81	32	19	166
8	*Statistics*	40	89	5	11	45
Total		**612**	**83**	**125**	**17**	**737**

Table 10.8: The classification of the explanatory items that were chosen for analysis from the items of the secondary school second grade textbook, based on the level of mathematical thinking (basic and complex) represented by each item.

N	Title of the unit	Basic Level		Complex Level		Total
		Repetition frequency	*%*	*Repetition frequency*	*%*	*Repetition frequency*
1	*Binary operations and groups*	75	86	12	14	87
2	*Matrices and determinants*	85	87	13	13	98
3	*Trigonometry*	160	70	68	30	228
4	*Complex numbers*	82	80	21	20	103
5	*Polynomials*	61	91	6	9	67
6	*Solid geometry (1)*	48	70	21	30	69
7	*Vector analysis*	41	70	18	31	59
8	*Binomial theory*	82	85	15	16	97
9	*Probabilities*	127	92	11	8	138
Total		**761**	**80**	**185**	**20**	**946**

Table 10.9: The classification of the explanatory items that were chosen for analysis from the items of the secondary school third grade textbook, based on the level of mathematical thinking (basic and complex) represented by each item.

N	Title of the unit	Basic Level		Complex Level		Total
		Frequency	*%*	*Frequency*	*%*	*Frequency*
1	*Conic sections*	105	85	18	15	123
2	*Sequences and series*	72	79	19	21	91
3	*Limits and continuity*	125	85	22	15	147
4	*Differentiation*	83	82	18	18	101
5	*Applications of differentiation*	64	89	8	11	72
6	*Integration*	90	87	14	14	104
7	*Applications of definite integration*	89	88	12	12	101
8	*Solid geometry (2)*	61	72	24	28	85
Total		**689**	**84**	**115**	**16**	**824**

In all three tables, representing the three age groups, the vast majority of the explanatory items (80-84%) belong to the basic level of thinking. This indicates that this level of thinking is given very considerable attention.

10.3.2 Second sub-question

Results of analysis related to the answer to the second sub-question:

What is the extent of the attention paid by the question items in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia, to the development of mathematical thinking based on the level of mathematical thinking (basic and complex) they represent?

Tables 10.10 to 10.12 show the classification of the question items that were chosen for analysis from the items of the secondary school textbooks at three grade levels based on the level of mathematical thinking (basic or complex) represented by each item.

Table 10.10: The classification of the question items that that were chosen for analysis from the items of the secondary school first grade textbook, based on the level of mathematical thinking (basic or complex) represented by each item.

N	Title of the unit	Basic Level		Complex Level		Total
		Frequency	**%**	***Frequency***	**%**	***Frequency***
1	*Mathematical logic*	328	90	36	10	364
2	*Relations and applications*	272	96	11	4	283
3	*Plane geometry*	206	91	20	9	226
4	*Equations and analytic geometry*	317	91	31	9	348
5	*Inequalities*	106	92	9	8	115
6	*Trigonometry*	220	84	43	16	263
7	*Exponential and logarithmic functions*	438	97	14	3	452
8	*Statistics*	331	90	38	10	369
Total		**2,218**	**92**	**202**	**8**	**2,420**

Table 10.11: The classification of the question items that were chosen for analysis from the items of the secondary school second grade textbook based on the level of mathematical thinking (basic or complex) represented by each item.

N	Title of the unit	Basic Level		Complex Level		Total
		Frequency	%	Frequency	%	Frequency
1	*Binary operations and groups*	259	94	17	6	276
2	*Matrices and determinants*	248	90	29	11	277
3	*Trigonometry*	352	91	34	9	386
4	*Complex numbers*	194	96	9	4	203
5	*Polynomials*	158	94	12	6	197
6	*Solid geometry (1)*	172	71	72	30	244
7	*Vector geometry*	168	87	27	13	194
8	*Binomial theory*	221	86	36	14	257
9	*Probabilities*	221	89	28	11	249
Total		**2,020**	**89**	**263**	**12**	**2,283**

Table 10.12: The classification of the question items that were chosen for analysis from the items of the secondary school third grade textbook based on the level of mathematical thinking (basic or complex) represented by each item.

N	Title of the unit	Basic Level		Complex Level		Total
		Frequency	*%*	*Frequency*	*%*	*Frequency*
1	*Conic sections*	194	98	5	3	199
2	*Sequences and series*	153	86	25	14	178
3	*Limits and continuity*	230	89	28	11	258
4	*Differentiation*	337	97	9	3	346
5	*Applications of differentiation*	265	96	12	4	277
6	*Integration*	316	96	13	4	329
7	*Applications of definite integration*	286	96	11	4	297
8	*Solid geometry (2)*	176	78	50	22	226
Total		**1,957**	**93**	**153**	**7**	**2,110**

Again, in all three tables, representing the three age groups, the vast majority of question items (89% to 93%) belong to the basic level of thinking. Therefore, this again indicates that very considerable attention is paid to this level of thinking.

10.4 Third Sub-Question

Results of the analysis related to the answer to the third sub-question:

What is the extent of the attention paid by the combined question and explanatory items in the mathematics textbooks for the scientific section at the secondary school stage in Saudi Arabia to the development of mathematical thinking based on the level of mathematical thinking they represent?

In order to indicate the extent of the attention paid by the mathematics textbooks for the scientific section at the secondary school stage to each of the nine mathematical thinking skills in this study, the percentages taken from the results of the analysis for each analysed skill were used. The methodology and procedures indicated in Chapter 9 were used; i.e. the representation percentages that are in the range of 0% – 22% indicate very little to little attention, while the representation percentages that are in the range of above 22% – 44% indicate considerable to very considerable attention.

10.4.1 Skill of Knowledge and Recall

Tables 10.13 to 10.15 show the percentages of the representation of the skill of knowledge and recall in the combined questions and explanatory items of the mathematics textbook of grades one to three of secondary school.

Table 10.13: The percentages of the representation of the skill of knowledge and recall in the combined questions and explanatory items of the mathematics textbook of the first grade of secondary school.

N	Title of the unit	Expla-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions /textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Mathematical logic*	71	159	45	149	364	41	220	523	42
2	*Relations and applications*	21	65	32	95	283	34	116	348	33
3	*Plane geometry*	15	62	24	55	226	24	70	288	24
4	*Equations and Analytic geometry*	26	87	30	99	348	28	125	435	29
5	*Inequalities*	16	57	28	30	115	26	46	172	27
6	*Trigonometry*	14	96	15	77	263	29	91	359	25
7	*Exponential and logarithmic functions*	28	166	17	179	452	40	207	618	34
8	*Statistics*	11	45	24	101	369	27	112	414	27
Total		**202**	**737**	**27**	**785**	**2,420**	**32**	**987**	**3,157**	**31**

Overall, 31% (lowest right-hand box) of the total of combined question and explanatory items represent the skill of knowledge and recall relative to the representation of other skills in these items, indicating that the attention paid to this skill of thinking in these items is considerable. However, the percentage varies from topic to topic (24% to 42%), indicating that the attention paid to this skill is considerable or very considerable according to the topic.

Looking at explanatory items (explanations) on their own shows variation from 15% to 45% from topic to topic, while the question items vary from 24% to 41%, indicating a similar range as the explanatory items.

Table 10.14: The percentages of the representation of the skill of knowledge and recall in the combined questions and explanatory items of the mathematics textbook of the second grade of secondary school.

N	Title of the unit	**Explan-ations / unit**	**Explain-ations / textbook**	**Explan-ations**	**Questions / unit**	**Questions / textbook**	**Questions**	**Total in the unit**	**Total in the textbook**	**Total**
1	*Binary operations and groups*	20	87	23	58	276	21	78	363	22
2	*Matrices and determinants*	23	98	24	102	277	37	125	375	33
3	*Trigonometry*	39	228	17	129	386	33	168	614	27
4	*Complex numbers*	12	103	12	89	203	44	101	306	33
5	*Polynomials*	22	67	33	54	197	27	76	264	29
6	*Solid geometry (1)*	18	69	12	51	244	21	59	313	19
7	*Vector analysis*	11	59	19	69	194	36	80	253	32
8	*Binomial theory*	15	97	16	101	257	39	116	354	33
9	*Probabilities*	41	138	30	92	249	37	133	387	34
Total		**191**	**946**	**20**	**745**	**2,283**	**33**	**936**	**3,229**	**29**

The pattern is similar here, with 29% of the total of combined questions and explanatory items in the secondary school second grade textbook representing the skill of knowledge and recall, indicating that the attention paid to this skill of thinking in these items is considerable. Again, there is some variation from topic to topic

Table 10.15: The percentages of the representation of the skill of knowledge and recall in the combined questions and explanatory items of the mathematics textbook of the third grade of secondary school.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Conic sections*	12	123	10	69	199	35	81	322	25
2	*Sequences and series*	18	91	20	71	178	40	89	269	33
3	*Limits and continuity*	29	147	20	76	258	30	105	405	26
4	*Differentiation*	29	101	29	141	346	41	170	447	38
5	*Applications of differentiation*	14	72	19	73	277	26	87	349	25
6	*Integration*	24	104	23	120	329	37	144	433	33
7	*Applications of definite integration*	24	101	24	115	297	39	139	398	35
8	*Solid geometry (2)*	21	85	25	65	226	29	86	311	28
Total		**171**	**824**	**21**	**730**	**2,110**	**35**	**901**	**2,934**	**31**

The third grade textbook shows a similar pattern, with the table showing that 31% of the total combined questions and explanatory items in the textbook represent the skill of knowledge and recall, indicating that the attention paid to this skill of thinking in these items is considerable. As before, there is variation from topic to topic.

10.4.2 Skill of Understanding and Interpretation

Tables 10.16 to 10.18 show the percentages of the representation of the skill of understanding and interpretation in the combined questions and explanatory items of the mathematics textbook of the three grades of secondary school.

Table 10.16: The percentages of the representation of the skill of understanding and interpretation in the combined questions and explanatory items of the mathematics textbook of the first grade of secondary school.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Question s / unit	Question s / textbook	Question s	Total in the unit	Total in the textbook	Total
1	*Mathematical logic*	61	159	38	131	364	36	192	523	37
2	*Relations and applications*	20	65	31	77	283	27	97	348	28
3	*Plane geometry*	21	62	34	76	226	34	97	288	34
4	*Equations and Analytic geometry*	31	87	36	120	348	35	151	435	35
5	*Inequalities*	27	57	47	53	115	46	80	172	47
6	*Trigonometry*	39	96	41	98	263	37	137	359	38
7	*Exponential and logarithmic functions*	98	166	59	181	452	40	279	618	45
8	*Statistics*	20	45	44	121	369	33	141	414	34
Total		**317**	**737**	**43**	**857**	**2,420**	**35**	**1,174**	**3,157**	**37**

Table 10.17: The percentages of the representation of the skill of understanding and interpretation in the combined questions and explanatory items of the mathematics textbook of the secondary school second grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-Ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Binary operations and groups*	48	87	55	136	276	49	184	363	51
2	*Matrices and determinants*	54	98	55	109	277	39	163	375	44
3	*Trigonometry*	80	228	35	143	386	37	223	614	36
4	*Complex numbers*	60	103	58	96	203	47	156	306	51
5	*Polynomials*	33	67	49	89	197	45	122	264	46
6	*Solid geometry (1)*	27	69	39	55	244	23	82	313	26
7	*Vector analysis*	20	59	34	75	194	39	95	253	38
8	*Binomial theory*	65	97	67	111	257	43	176	354	50
9	*Probabilities*	74	138	54	99	249	40	173	387	45
Total		461	946	49	913	2,283	40	1,374	3,229	43

Table 10.18: The percentages of the representation of the skill of understanding and interpretation in the combined questions and explanatory items of the mathematics textbook of the secondary school third grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Question s / unit	Question s / textbook	Question s	Total in the unit	Total in the textbook	Total
1	*Conic sections*	49	123	40	70	199	35	119	322	37
2	*Sequences and series*	49	91	54	70	178	39	119	269	44
3	*Limits and continuity*	75	147	51	99	258	38	174	405	43
4	*Differentiation*	49	101	49	152	346	44	201	447	45
5	*Applications of differentiation*	29	72	40	86	277	31	115	349	33
6	*Integration*	55	104	53	126	329	38	181	433	42
7	*Applications of definite integration*	52	101	52	101	297	34	153	398	38
8	*Solid geometry (2)*	29	85	34	63	226	28	92	311	30
Total		**387**	**824**	**50**	**767**	**2,110**	**36**	**154**	**2,934**	**39**

The three tables show similar patterns, with the total (lowest right hand boxes) varying between 37% and 43%, indicating that the attention paid to this skill is very considerable, while the totals for the explanatory items (explanations) varies between 43% and 50%, rising slightly with age. However, this did not apply to the third grade textbooks, in which the question items ranged between 35% and 40%.

10.4.3 Skill of Modelling

Tables 10.19 to 10.21 show the percentages of the representation of the skill of modelling in the combined questions and explanatory items of the mathematics textbook in the three grades of secondary school.

Table 10.19: The percentages of the representation of the skill of modelling in the combined questions and explanatory items of the mathematics textbook of the secondary school first grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Question s / unit	Question s / textbook	Question s	Total in the unit	Total in the textbook	Total
1	*Mathematical logic*	10	159	6	19	364	52	29	523	6
2	*Relations and applications*	12	65	19	64	283	23	76	348	22
3	*Plane geometry*	9	62	15	34	226	15	43	288	15
4	*Equations and Analytic geometry*	19	87	22	59	348	17	78	435	18
5	*Inequalities*	9	57	16	16	115	14	25	172	15
6	*Trigonometry*	17	96	18	34	263	13	51	359	14
7	*Exponential and logarithmic functions*	8	166	5	27	452	6	35	618	6
8	*Statistics*	9	45	20	98	369	27	107	414	26
Total		**93**	**737**	**13**	**351**	**2,420**	**15**	**444**	**3,157**	**14**

Table 10.20: The percentages of the representation of the skill of modelling in the combined questions and explanatory items of the mathematics textbook of the secondary school second grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Binary operations and groups*	7	87	8	13	276	5	20	363	6
2	*Matrices and determinants*	8	98	8	31	277	11	39	375	10
3	*Trigonometry*	41	228	18	51	386	13	92	614	15
4	*Complex numbers*	10	103	10	4	203	2	14	306	5
5	*Polynomials*	6	67	9	29	197	15	35	264	13
6	*Solid geometry (1)*	13	69	19	20	244	8	33	313	11
7	*Vector analysis*	10	59	17	15	194	8	25	253	10
8	*Binomial theory*	2	97	2	4	257	2	6	354	2
9	*Probabilities*	12	138	9	11	249	4	23	387	6
Total		**109**	**946**	**12**	**178**	**2,283**	**8**	**287**	**3,229**	**9**

Table 10.21: The percentages of the representation of the skill of modelling in the combined questions and explanatory items of the mathematics textbook of the secondary school third grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Conic sections*	44	123	36	35	199	18	79	322	25
2	*Sequences and series*	5	91	6	1	178	1	6	269	2
3	*Limits and continuity*	21	147	14	10	258	4	31	405	8
4	*Differentiation*	5	101	5	5	346	1	10	447	2
5	*Applications of differentiation*	21	72	29	77	277	28	98	349	28
6	*Integration*	11	104	11	19	329	6	30	433	7
7	*Applications of definite integration*	13	101	13	26	297	9	39	398	10
8	*Solid geometry (2)*	11	85	13	22	226	10	33	311	11
Total		**131**	**824**	**16**	**195**	**2,110**	**9**	**326**	**2,934**	**11**

The total percentages are much lower, varying between 9% and 14%. This means that very little to little attention is given to this skill. Explanations are also much lower and, in some topics, the percentages are very low indeed, indicating that this type of item is rare. However, question items were even rarer.

10.4.4 Skill of Application

Tables 10.22-10.24 show the percentages of the representation of the skill of application in the combined questions and explanatory items of the mathematics textbook of the three grade textbooks of secondary school.

Table 10.22: The percentages of the representation of the skill of application in the combined questions and explanatory items of the mathematics textbook of the secondary school first grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-Ations	Question s / unit	Question s / textbook	Question s	Total in the unit	Total in the textbook	Total
1	*Mathematical logic*	0	159	0	29	364	8	29	523	6
2	*Relations and applications*	0	65	0	36	283	13	36	348	10
3	*Plane geometry*	0	62	0	41	226	18	41	288	14
4	*Equations and Analytic geometry*	0	87	0	39	348	11	39	435	9
5	*Inequalities*	0	57	0	7	115	6	7	172	4
6	*Trigonometry*	0	96	0	11	263	4	11	359	3
7	*Exponential and logarithmic functions*	0	166	0	51	452	11	51	618	8
8	*Statistics*	0	45	0	11	369	3	11	414	3
Total		**0**	**737**	**0**	**225**	**2,420**	**9**	**225**	**3,157**	**7**

Table 10.23: The percentages of the representation of the skill of application in the combined questions and explanatory items of the mathematics textbook of the secondary school second grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Question s / unit	Question s / textbook	Question s	Total in the unit	Total in the textbook	Total
1	*Binary operations and groups*	0	87	0	52	276	19	52	363	14
2	*Matrices and determinants*	0	98	0	6	277	2	6	375	2
3	*Trigonometry*	0	228	0	29	386	8	29	614	5
4	*Complex numbers*	0	103	0	5	203	3	5	306	2
5	*Polynomials*	0	67	0	13	197	7	13	264	5
6	*Solid geometry (1)*	0	69	0	46	244	19	46	313	15
7	*Vector analysis*	0	59	0	9	194	5	9	253	4
8	*Binomial theory*	0	97	0	5	257	2	5	354	1
9	*Probabilities*	0	138	0	19	249	8	19	387	5
Total		**0**	**946**	**0**	**184**	**2,283**	**8**	**184**	**3,229**	**6**

Table10.24: The percentages of the representation of the skill of application in the combined questions and explanatory items of the mathematics textbook of the secondary school third grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Conic sections*	0	123	0	20	199	10	20	322	6
2	*Sequences and series*	0	91	0	11	178	6	11	269	4
3	*Limits and continuity*	0	147	0	45	258	17	45	405	11
4	*Differentiation*	0	101	0	39	346	11	39	447	9
5	*Applications of differentiation*	0	72	0	29	277	11	29	349	8
6	*Integration*	0	104	0	51	329	16	51	433	12
7	*Applications of definite integration*	0	101	0	44	297	15	44	398	11
8	*Solid geometry (2)*	0	85	0	26	226	12	26	311	8
Total		**0**	**824**	**0**	**265**	**2,110**	**13**	**265**	**2,934**	**9**

The proportion of items relating to applications is very low (from 7% to 9%), which is even lower than for modelling, although this varies slightly from topic to topic.

10.4.5 Skill of Induction

Tables 10.25 to 10.27 show the percentages of the representation of the skill of induction in the combined questions and explanatory items in the mathematics textbooks of the three grades of secondary school.

Table 10.25: The percentages of the representation of the skill of induction in the combined questions and explanatory items of the mathematics textbook of the first grade of secondary school.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Question s / unit	Question s / textbook	Question s	Total in the unit	Total in the textbook	Total
1	*Mathematical logic*	0	159	0	1	364	0	1	523	0
2	*Relations and applications*	0	65	0	0	283	0	0	348	0
3	*Plane geometry*	0	62	0	0	226	0	0	288	0
4	*Equations and Analytic geometry*	0	87	0	1	348	0	1	435	0
5	*Inequalities*	0	57	0	0	115	0	0	172	0
6	*Trigonometry*	0	96	0	1	263	0	1	359	0
7	*Exponential and logarithmic functions*	0	166	0	2	452	0	2	618	0
8	*Statistics*	0	45	0	0	369	0	0	414	0
Total		**0**	**737**	**0**	**5**	**2,420**	**0**	**5**	**3,157**	**0**

Table 10.26: The percentages of the representation of the skill of induction in the combined questions and explanatory items of the mathematics textbook of the second grade of secondary school.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Binary operations and groups*	1	87	1	3	276	1	4	363	1
2	*Matrices and determinants*	1	98	1	2	277	1	3	375	1
3	*Trigonometry*	0	228	0	3	386	1	3	614	1
4	*Complex numbers*	0	103	0	0	203	0	0	306	0
5	*Polynomials*	0	67	0	3	197	2	3	264	1
6	*Solid geometry (1)*	2	69	3	4	244	2	6	313	2
7	*Vector analysis*	0	59	0	2	194	1	2	253	1
8	*Binomial theory*	1	97	1	3	257	1	4	354	1
9	*Probabilities*	0	138	0	3	249	1	3	387	1
Total		**5**	**946**	**1**	**23**	**2,283**	**1**	**28**	**3,229**	**1**

Table 10.27: The percentages of the representation of the skill of induction in the combined questions and explanatory items of the mathematics textbook of the third grade of secondary school.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Conic sections*	2	123	2	1	199	1	3	322	1
2	*Sequences and series*	4	91	4	21	178	12	25	269	9
3	*Limits and continuity*	0	147	0	1	258	0	1	405	0
4	*Differentiation*	1	101	1	0	346	0	1	447	0
5	*Applications of differentiation*	1	72	1	0	277	0	1	349	0
6	*Integration*	0	104	0	0	329	0	0	433	0
7	*Applications of definite integration*	0	101	0	0	297	0	0	398	0
8	*Solid geometry (2)*	1	85	1	0	226	0	1	311	0
Total		**9**	**824**	**1**	**23**	**2,110**	**1**	**32**	**2,934**	**1**

The presence of items relating to the skill of induction is vanishingly small.

10.4.6 Skill of generalisation

Tables 10.28 to 10.30 shows the percentages of the representation of the skill of generalisation in the combined questions and explanatory items of the three mathematics textbooks of the secondary school grades.

Table 10.28: The percentages of the representation of the skill of generalisation in the combined questions and explanatory items of the mathematics textbook of the secondary school first grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Mathematical logic*	3	159	2	1	364	0	4	523	1
2	*Relations and applications*	3	65	5	1	283	0	4	348	1
3	*Plane geometry*	4	62	7	0	226	0	4	288	1
4	*Equations and Analytic geometry*	6	87	7	0	348	0	6	435	1
5	*Inequalities*	1	57	2	0	115	0	1	172	1
6	*Trigonometry*	1	96	1	1	263	0	2	359	1
7	*Exp'l and log'ic functions*	14	166	8	1	452	0	15	618	2
8	*Statistics*	1	45	2	0	369	0	1	414	0
Total		**33**	**737**	**4**	**4**	**2,420**	**0**	**37**	**3,157**	**1**

Table 10.29: The percentages of the representation of the skill of generalisation in the combined questions and explanatory items of the mathematics textbook of the secondary school second grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Binary operations and groups*	6	87	7	4	276	1	10	363	3
2	*Matrices and determinants*	3	98	3	1	277	0	4	375	1
3	*Trigonometry*	5	228	2	4	386	1	9	614	2
4	*Complex numbers*	8	103	8	1	203	1	9	306	3
5	*Polynomials*	0	67	0	1	197	1	1	264	0
6	*Solid geometry (1)*	4	69	6	10	244	4	14	313	5
7	*Vector geometry*	5	59	9	3	194	2	8	253	3
8	*Binomial theory*	5	97	5	3	257	1	8	354	2
9	*Probabilities*	1	138	1	2	249	1	3	387	1
Total		**37**	**946**	**4**	**29**	**2,283**	**1**	**66**	**3,229**	**2**

Table 10.30: The percentages of the representation of the skill of generalisation in the combined questions and explanatory items of the mathematics textbook of the third grade of secondary school.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Conic sections*	5	123	4	2	199	1	7	322	2
2	*Sequences and series*	8	91	9	1	178	1	9	269	3
3	*Limits and continuity*	16	147	11	1	258	0	17	405	4
4	*Differentiation*	2	101	2	0	346	0	2	447	0
5	*Applications of differentiation*	3	72	4	0	277	0	3	349	1
6	*Integration*	9	104	9	0	329	0	9	433	2
7	*Applications of definite integration*	3	101	3	0	297	0	3	398	1
8	*Solid geometry (2)*	5	85	6	6	226	3	11	311	4
Total		**51**	**824**	**6**	**10**	**2,110**	**1**	**61**	**2,934**	**2**

Moreover, the skill of generalisation is not very common in the items in any topic area at any level.

10.4.7 Skill of deduction

Tables 10.31-10.33 show the percentages of the representation of the skill of deduction in the combined questions and explanatory items of the mathematics textbook of the three school grades.

Table 10.31: The percentages of the representation of the skill of deduction in the combined questions and explanatory items of the mathematics textbook of the secondary school first grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Question s / unit	Question s / textbook	Question s	Total in the unit	Total in the textbook	Total
1	*Mathematical logic*	0	159	0	1	364	0	1	523	0
2	*Relations and applications*	2	65	3	1	283	0	3	348	1
3	*Plane geometry*	8	62	13	15	226	7	23	288	8
4	*Equations and analytic geometry*	2	87	2	1	348	0	3	435	1
5	*Inequalities*	1	57	2	0	115	0	1	172	1
6	*Trigonometry*	9	96	9	1	263	0	10	359	3
7	*Exponential and logarithmic functions*	5	166	3	1	452	0	6	618	1
8	*Statistics*	1	45	2	6	369	2	7	414	2
Total		**28**	**737**	**4**	**26**	**2,420**	**1**	**54**	**3,157**	**2**

Table 10.32: The percentages of the representation of the skill of deduction in the combined questions and explanatory items of the mathematics textbook of the secondary school second grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Binary operations and groups*	3	87	3	3	276	1	6	363	2
2	*Matrices and determinants*	1	98	1	3	277	1	4	375	1
3	*Trigonometry*	19	228	8	3	386	1	22	614	4
4	*Complex numbers*	7	103	7	1	203	1	8	306	3
5	*Polynomials*	4	67	6	1	197	1	5	264	2
6	*Solid geometry (1)*	2	69	3	16	244	7	18	313	6
7	*Vector analysis*	7	59	12	5	194	3	12	253	5
8	*Binomial theory*	4	97	4	4	257	2	8	354	3
9	*Probabilities*	2	138	1	2	249	1	4	387	1
Total		**49**	**946**	**5**	**38**	**2,283**	**2**	**87**	**3,229**	**3**

Table 10.33: The percentages of the representation of the skill of deduction in the combined questions and explanatory items of the mathematics textbook of the secondary school third grade.

N	Title of the unit	Explanantions/unit	Explanations / textbook	Explanantions	Questions / unit	Questions / textbook	Questions	Total of the unit	Total of the textbook	Total
1	*Conic sections*	10	123	8	1	199	1	11	322	3
2	*Sequences and series*	4	91	4	1	178	1	5	269	2
3	*Limits and continuity*	0	147	0	2	258	1	2	405	1
4	*Differentiation*	5	101	5	0	346	0	5	447	1
5	*Applications of differentiation*	1	72	1	0	277	0	1	349	0
6	*Integration*	3	104	3	0	329	0	3	433	1
7	*Applications of definite integration*	3	101	3	0	297	0	3	398	1
8	*Solid geometry (2)*	4	85	5	7	226	3	11	311	4
Total		**30**	**824**	**4**	**11**	**2,110**	**1**	**41**	**2,934**	**1**

The proportion of the items that relate to deduction is very small, although, as might be expected, they do appear a little more in plane geometry, reflecting the nature of that topic. This indicates that this skill is very infrequent.

10.4.8 Skill of mathematical proof

Tables 10.34 to 10.36 show the percentages of the representation of the skill of mathematical proof in the combined questions and explanatory items of the mathematics textbook of the three secondary school grades.

Table 10.34: The percentages of the representation of the skill of mathematical proof in the combined questions and explanatory items of the mathematics textbook of the secondary school first grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Question s / unit	Question s / textbook	Question s	Total in the unit	Total in the textbook	Total
1	*Mathematical logic*	13	159	8	24	364	7	37	523	7
2	*Relations and applications*	4	65	6	4	283	1	8	348	2
3	*Plane geometry*	5	62	8	3	226	1	8	288	3
4	*Equations and Analytic geometry*	3	87	3	15	348	4	18	435	4
5	*Inequalities*	2	57	4	5	115	4	7	172	4
6	*Trigonometry*	10	96	10	29	263	11	39	359	11
7	*Exponential and logarithmic functions*	13	166	8	6	452	1	19	618	3
8	*Statistics*	0	45	0	9	369	2	9	414	2
Total		**50**	**737**	**7**	**95**	**2,420**	**4**	**145**	**3,157**	**5**

Table 10.35: The percentages of the representation of the skill of mathematical proof in the combined questions and explanatory items of the mathematics textbook of the secondary school second grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Binary operations and groups*	0	87	0	5	276	2	5	363	1
2	*Matrices and determinants*	0	98	0	3	277	1	3	375	1
3	*Trigonometry*	29	228	13	13	386	3	42	614	7
4	*Complex numbers*	4	103	4	5	203	3	9	306	3
5	*Polynomials*	2	67	3	6	197	3	8	264	3
6	*Solid geometry (1)*	12	69	17	33	244	14	45	313	14
7	*Vector geometry*	4	59	7	7	194	4	11	253	4
8	*Binomial theory*	3	97	3	11	257	4	14	354	4
9	*Probabilities*	5	138	4	5	249	2	10	387	3
Total		**59**	**946**	**6**	**88**	**2,283**	**4**	**147**	**3,229**	**5**

Table 10.36: The percentages of the representation of the skill of mathematical proof in the combined questions and explanatory items questions of the mathematics textbook of the secondary school third grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Conic sections*	1	123	1	1	199	1	2	322	1
2	*Sequences and series*	3	91	3	1	178	1	4	269	2
3	*Limits and continuity*	1	147	1	3	258	1	4	405	1
4	*Differentiation*	8	101	8	3	346	1	11	447	3
5	*Applications of differentiation*	1	72	1	7	277	3	8	349	2
6	*Integration*	1	104	1	10	329	3	11	433	3
7	*Applications of definite integration*	1	101	1	9	297	3	10	398	3
8	*Solid geometry (2)*	13	85	15	33	226	15	46	311	15
Total		**29**	**824**	**4**	**67**	**2,110**	**3**	**96**	**2,934**	**3**

The skill of mathematical proof is only seen in specific topic areas, notably in some areas of trigonometry and geometry.

10.4.9 Skill of evaluation

Tables 10.37 to 10.39 show the percentages of the representation of the skill of evaluation in the combined questions and explanatory items of the mathematics textbook of the three secondary school grades.

Table 10.37: The percentages of the representation of the skill of evaluation in the combined questions and explanatory items of the mathematics textbook of the secondary school first grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Question s / unit	Question s / textbook	Question s	Total in the unit	Total in the textbook	Total
1	*Mathematical logic*	1	159	1	9	364	3	10	523	2
2	*Relations and applications*	3	65	5	5	283	2	8	348	2
3	*Plane geometry*	0	62	0	2	226	1	2	288	1
4	*Equations and Analytic geometry*	0	87	0	14	348	4	14	435	3
5	*Inequalities*	1	57	2	4	115	4	5	172	3
6	*Trigonometry*	6	96	6	11	263	4	17	359	5
7	*Exponential and logarithmic functions*	0	166	0	4	452	1	4	618	1
8	*Statistics*	3	45	7	23	369	6	26	414	6
Total		**14**	**373**	**2**	**72**	**2,420**	**3**	**86**	**3,157**	**3**

Table 10.38: The percentages of the representation of the skill of evaluation in the combined questions and explanatory items of the mathematics textbook of the secondary school second grade.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Binary operations and groups*	2	87	2	2	276	1	4	363	1
2	*Matrices and determinants*	8	98	8	20	277	7	28	375	8
3	*Trigonometry*	15	228	7	11	386	3	26	614	4
4	*Complex numbers*	2	103	2	2	203	1	4	306	1
5	*Polynomials*	0	67	0	1	197	1	1	264	0
6	*Solid geometry (1)*	1	69	1	9	244	4	10	313	3
7	*Vector geometry*	2	59	3	9	194	5	11	253	4
8	*Binomial theory*	2	97	2	15	257	6	17	354	5
9	*Probabilities*	3	138	2	16	249	6	19	387	5
Total		**35**	**946**	**4**	**85**	**2,283**	**4**	**120**	**3,229**	**4**

Table 10.39: The percentages of the representation of the skill of evaluation in the combined questions and explanatory items of the mathematics textbook of the third grade of secondary school.

N	Title of the unit	Explan-ations / unit	Explain-ations / textbook	Explan-ations	Questions / unit	Questions / textbook	Questions	Total in the unit	Total in the textbook	Total
1	*Conic sections*	0	123	0	0	199	0	0	322	0
2	*Sequences and series*	0	91	0	1	178	1	1	269	0
3	*Limits and continuity*	5	147	3	21	258	8	26	405	6
4	*Differentiation*	2	101	2	6	346	2	8	447	2
5	*Applications of differentiation*	2	72	3	5	277	2	7	349	2
6	*Integration*	1	104	1	3	329	1	4	433	1
7	*Applications of definite integration*	5	101	5	2	297	1	7	398	2
8	*Solid geometry (2)*	1	85	1	4	226	2	5	311	2
Total		**16**	**824**	**2**	**42**	**2,110**	**2**	**58**	**2,934**	**2**

The skill of evaluation does not occur frequently but is seen more in some specific topics: statistics, matrices and determinants, and limits and continuity.

10.5 Overall Discussion

All the data shown in the tables involve the professional judgement of six referees, who looked at the textbooks and whose adjudication was compared carefully to that of the researcher. There was a high measure of consistency (see page 230), giving some confidence that the final judgements are reliable.

However, these are professional judgements and there can be no certainty that they give a totally accurate picture. Nonetheless, there is a large measure of consistency in the findings.

Overall, the professional judgement of a group of ‘*experts*’ suggests that the main emphasis in the items in all three textbooks relates to knowledge and recall, as well as understanding and interpretation, mostly at a basic level. There was little or no evidence of any development across the textbooks, whereas it might have been expected that other skills would be introduced more as the learners became more experienced.

Having seen the broad pattern of questions used in the textbooks, Chapter 11 will move on to look at the views of the teachers and school inspectors about the nature of the textbook questions.

Chapter 11

Analysing Survey Data

11.1 Introduction

In this chapter, the third, fourth and fifth research questions will be answered. As indicated in Chapter 9, a questionnaire was designed and distributed to 1466 participants to gather the required information. The data were analysed with IBM SPSS Statistics Version 18.

11.2 Demographic profile of the respondents

The study sample consisted of 1466 participants, of whom 158 (11%) were inspectors and 1308 (89%) were teachers. Thus, the sample consisted of relatively junior employees. Information on grade, qualification, years of experience and whether the participants had attended any training courses was also obtained. 521 (36%) participants were involved with the 1st grade mathematics textbook, 543 (37%) with the 2nd grade mathematics textbook, and 402 (27%) with the 3rd grade mathematics textbook. The majority (1152, or 79%) had a Bachelor’s Degree as their highest qualification; 164 (11%) had a Postgraduate Higher Diploma as their highest qualification, 126 (9%) had a Master’s Degree as their highest qualification, and 24 (2%) had Doctor of Philosophy as their highest qualification. 250 (17%) had up to 5 years of teaching experience, 489 (33%) had been teaching for 5 to 9 years, 507 (35%) had teaching experience of between 10 and 15 years, and 220

(15%) had teaching experience of over 15 years. The majority of the participants, 829 (57%) had not attended any training course.

Table 11.1 summarises the participants' demographic characteristics.

Table 11.1: Characteristics of the study population (n=1466)

Characteristics		Sample	% of population
Current Position	Inspector	158	11
	Teacher	1308	89
Grade	1st	521	36
	2nd	543	37
	3rd	402	27
Qualification	Bachelor's Degree	1152	79
	Postgraduate Higher diploma	164	11
	Master's degree	126	9
	PhD	24	2
Years of experience	Less than 5 years	250	17
	5-9 years	489	33
	10-14 years	507	35
	15 years or more	220	15
Participated in training courses	No	829	57
	Yes	637	44

11.3 Mathematical skills in the questions in the mathematics textbooks

What are the mathematical thinking skills prevailing in the questions in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia in the opinion of the teachers and inspectors of mathematics?

To answer this question, frequency and percentages for each item measuring mathematical thinking skills were analysed with IBM SPSS Statistics Version 18.

11.4 Sub-question 1

This question was stated as follows:

What are the mathematical thinking skills prevailing in the questions in the secondary stage mathematics textbooks in Saudi Arabia in the opinion of the teachers and inspectors of mathematics according to different classes?

Nine mathematical thinking skills were analysed. The nine skills were knowledge and recall, understanding and interpretation, modelling, application, induction, generalisation, deduction, mathematical proof, and evaluation.

The tables below show the overall results for each skill. The most popular response is shown in colour. The following section provides further detail of the combined results for each item of each of these skills in all three of the selected textbooks.

11.4.1 Skill of Knowledge and Recall

Three items (23, 24 and 25) were used to identify knowledge and recall. Table 11.2 shows that the majority of the participants agree with each item of knowledge and recall.

Table 11.2: Percentage for each item of Knowledge and Recall

ITEMS	Frequency (%) n=1466				
	SD	D	N	A	SA
23. Acquire the mathematical knowledge represented in the concepts and symbols	1	2	21	51	26
24. Recall mathematical knowledge stored in the memory	1	2	15	53	30
25. Organise knowledge in a logical manner	1	11	31	37	21

Key: SD – Strongly Disagree; D – Disagree; N – Neutral; A – Agree; SA – Strongly Agree

This means that in the opinion of the majority of the teachers and inspectors, the representation of the skill of knowledge and recall in the questions of the selected textbooks is satisfactory.

11.4.2 Skill of Understanding and Interpretation

Three items (26, 27 and 28) were used to identify understanding and interpretation. Table 11.3 shows that the majority of the participants disagreed with each item of understanding and interpretation

Table 11.3: Percentage for each item of understanding and interpretation

ITEMS	Frequency (%) n=1466				
	SD	D	N	A	SA
26. Formulate the given information by means of new words or symbols	1	6	28	38	27
27. Interpret the various relationships in mathematical problems	1	7	28	37	28
28. Use more than one skill, such as mathematical reasoning, comparison, classification, justification, etc	0	5	28	41	26

It can be seen that almost two-thirds of the respondents agreed or strongly agreed with all three items. This means that in the opinion of the majority of the respondents, the representation of the skill of understanding and interpretation in the questions of the selected textbooks is satisfactory.

11.4.3 Skill of Modelling

Three items (29, 30 and 31) were used to identify modelling. Table 11.4 shows that the majority of the participants disagreed with each item of modelling.

Table 11.4: Percentage for each item of Modelling

ITEMS	Frequency (%) n=1466				
	SD	D	N	A	SA
29. Mathematically represent the given data in an easy to understand manner	20	42	30	6	2
30. Perceive the relationships between the given data in order to obtain the required deductions	16	38	33	11	2
31. Make tables and graphs of the given data	19	39	31	9	2

Thus, in the opinion of the majority of the teachers and inspectors, the representation of the skill of modelling in the questions of the selected textbooks is unsatisfactory, although over 30% expressed no definite opinion on this matter.

11.4.4 Skill of Application

Two items (32 and 33) were used to identify application. Table 11.5 shows that the majority of the participants disagreed with each item of application.

Table 11.5: Percentage for each item of Application

ITEMS	Frequency (%) n=1466				
	SD	D	N	A	SA
32. Use mathematical knowledge in new situations	18	38	32	10	2
33. Analyse the new mathematical knowledge in order to perceive its relationships with the previous mathematical knowledge	18	41	31	9	2

It can be seen that over half of the participants disagreed or strongly disagreed for both items, while almost one-third was neutral. This means that in the opinion of the majority of the respondents, the representation of the skill of Application in the questions of the selected textbooks is unsatisfactory, although over 30% expressed no definite opinion on this matter.

11.4.5 Skill of Induction

Two items (34 and 35) were used to identify Induction. Table 11.6 shows that the majority of the participants disagreed with both of these items.

Table 11.6: Percentage for each item of Induction

ITEMS	Frequency (%) n=1466				
	SD	D	N	A	SA
34. Arrive at a new result based on particular examples or observations	18	40	29	11	2
35. Indicate relationships between introductions and results	19	38	32	10	2

The pattern of responses means that, in the opinion of the majority of the respondents, the representation of the skill of Induction in the questions of the selected textbooks is unsatisfactory, although around 30% expressed no definite opinion on this matter.

11.4.6 Skill of Generalisation

Three items (36, 37 and 38) were used to identify Generalisation. Table 11.7 shows that the majority of the participants disagreed for all the items of Generalisation.

Table 11.7: Percentage for each item of Generalisation

ITEMS	Frequency (%) n=1466				
	SD	D	N	A	SA
36. Use statements to describe particular cases	18	40	34	7	1
37. Formulate general statements that include multiple features of cases	23	43	28	5	1
38. Express the general rule using precise language	26	41	26	6	1

The pattern is for disagreement, with between a quarter and a third remaining neutral. This means that in the opinion of the majority of the respondents, the representation of the skill of Generalisation in the questions of the selected textbooks is unsatisfactory, although a considerable minority expressed no definite opinion on this matter.

11.4.7 Skill of Deduction

Three items (39, 40 and 41) were used to identify deduction. Table 11.8 shows that the majority of the participants disagreed with all three items of Deduction.

Table 11.8: Percentage for each item of Deduction

ITEMS	Frequency (%) n=1466				
	SD	D	N	A	SA
39. Indicate the particular cases that follow the general rule	23	41	29	8	0
40. Indicate the relationship between the particular and general mathematical cases.	26	39	31	3	0
41. Apply the general rule to a particular case	27	39	29	5	0

The data suggest that, in the opinion of the majority of the respondents, the representation of the skill of deduction in the questions of the selected textbooks is unsatisfactory, although a considerable minority were neutral.

11.4.8 Skill of Mathematical Proof

Three items (42, 43 and 44) were used to identify Mathematical Proof. Table 11.9 shows that the majority of the participants were neutral for each item of Mathematical Proof.

Table 11.9: Percentage for each item of Mathematical Proof

ITEMS	Frequency (%) n=1466				
	SD	D	N	A	SA
39. Indicate the particular cases that follow the general rule	5	26	37	24	8
40. Indicate the relationship between the particular and general mathematical cases.	5	31	44	17	2
41. Apply the general rule to a particular case	11	34	38	14	4

The response patterns are close to a normal distribution. This might indicate a spread of views or it might indicate that the respondents were not completely clear as to what they understood by the term 'mathematical proof'. However, while many respondents were neutral, more found it unsatisfactory than satisfactory.

11.4.9 Skill of Evaluation

Three items (45, 46 and 47) were used to identify evaluation. Table 11.10 below shows that the majority of the participants disagreed with all three of the items of evaluation.

Table 11.10: Percentage for each item of evaluation

ITEMS	Frequency (%) n=1466				
	SD	D	N	A	SA
45. Use criteria to pass judgment	28	38	30	4	0
46. Prove the validity of mathematical rules	24	38	35	3	0
47. Discover mistakes in mathematical relationships	25	38	33	3	0

In the opinion of the majority of the respondents, the representation of the skill of evaluation in the questions of the selected textbooks is unsatisfactory, although a considerable minority were neutral.

11.5 Further Analysis

The data obtained are ordinal in nature and, therefore, the computation of means is inadmissible. However, to gain an overall impression, the average means were calculated for each skill at each grade level. The numbers obtained are only indicative of the general pattern of responses and are not being used in any integer sense here. The numbers are shown in table 11.11.

Thus, the researcher counted the number of times the participant chose each of the five answers (strongly disagree (SD), disagree (D), neutral (N), agree (A), and strongly agree (SA), and the results are shown in table 11.11 below. In this case, the maximum number of times the participant could choose each of the five answers is three (this occurs when the participant chooses the same answer for all three items) and the minimum is zero (this occurs when the participants did not choose any answer for any of the three items). For each participant "x" we obtain T_x= (t1, t2, t3, t4, t5), where t1 is the number of times the participant answered strongly disagree, t2 is the number of times the participant answered disagree, t3 is the number of times the participant answered neutral, t4 is the number of times the participant answered agree and t5 is the number of times the participant answered strongly agree, such that t1+t2+t3+t4+t5 = 3 for each participant, and all t1,t2,t3,t4,t5 $\leq$ 3. For example, participant 1 has answered 'agree' twice and 'strongly agree' once.

Table 11.11: The number of times each participant chose each of the five answers for knowledge and recall questions

No	*Question 23*	*Question 24*	*Question 25*	*Strongly Disagree*	*Disagree*	*Neutral*	*Agree*	*Strongly Agree*
1	4	5	4	0	0	0	2	1
2	3	4	3	0	0	2	1	0
3	4	5	4	0	0	0	2	1
4	3	5	2	0	1	1	0	1
5	3	5	5	0	0	1	0	2
6	4	5	3	0	0	1	1	1
7	5	4	4	0	0	0	2	1
8	3	5	4	0	0	1	1	1
9	4	3	3	0	0	2	1	0
10	4	4	5	0	0	0	2	1
11	2	4	2	0	2	0	1	0
12	3	4	4	0	0	1	2	0
13	3	4	3	0	0	2	1	0
14	4	5	3	0	0	1	1	1
15	3	5	5	0	0	1	0	2
16	3	5	4	0	0	1	1	1
17	5	5	4	0	0	0	1	2
18	4	5	4	0	0	0	2	1
19	4	4	5	0	0	0	2	1
1466	4	2	3	0	1	1	1	0

For each participant T_x is ranked. The mean ranks of strongly disagree, disagree, neutral, agree and strongly agree were calculated for each of the three classes.

It has to be recognised that this only gives a picture of the pattern of responses but this may still be useful.

Table 11.12: Mean rank of responses of mathematical thinking skills according to grade

Class	Answers	Skill 1 Knowledge and recall	Skill 2 Understanding and interpretation	Skill 3 Modelling	Skill 4 Application	Skill 5 Induction	Skill 6 Generalisation	Skill 7 Deduction	Skill 8 Mathematical proof	Skill 9 Evaluation
1st	*Strongly Disagree*	1.9	2.0	3.0	3.0	3.0	3.2	3.4	2.3	3.4
	Disagree	2.2	2.3	3.9	3.7	3.7	4.0	3.9	3.5	3.8
	Neutral	3.3	3.5	3.6	3.5	3.5	3.5	3.5	3.9	3.7
	Agree	4.2	3.9	2.5	2.6	2.6	2.3	2.3	3.0	2.1
	Strongly Agree	3.4	3.4	2.1	2.2	2.2	2.0	2.0	2.2	1.9
2nd	*Strongly Disagree*	2.0	1.9	3.1	2.9	3.0	3.2	3.3	2.4	3.4
	Disagree	2.2	2.3	3.8	3.8	3.7	3.9	3.9	3.6	3.9
	Neutral	3.3	3.5	3.6	3.5	3.4	3.6	3.5	3.9	3.7
	Agree	4.1	3.9	2.5	2.6	2.7	2.3	2.3	3.0	2.2
	Strongly Agree	3.4	3.5	2.0	2.2	2.3	1.9	1.9	2.2	1.9
3rd	*Strongly Disagree*	2.0	1.9	2.9	3.0	3.0	3.3	3.3	2.3	3.5
	Disagree	2.3	2.4	4.0	3.7	3.8	4.0	3.9	3.6	3.9
	Neutral	3.2	3.4	3.6	3.5	3.4	3.5	3.6	3.8	3.6
	Agree	4.2	3.9	2.4	2.6	2.6	2.3	2.3	3.0	2.1
	Strongly Agree	3.4	3.4	2.0	2.2	2.2	2.0	1.9	2.2	1.9

In simple terms, the data in table 11.12 indicate the predominant views of the respondents on the nine skills. The nine skills can be grouped into a small number of categories:

(a) Where the respondents consider that the skill is emphasised: knowledge and recall, understanding and interpretation.

(b) Where the respondents have considerable doubts that the skill is being emphasised: modelling, application, induction, generalisation, deduction, evaluation.

(c) Where the respondents hold a range of views: mathematical proof.

It is interesting to note that there appear to be very few differences in the way they view the textbooks at the three levels. This is a matter for concern for it implies that mathematical education is not developing an increasing range of skills with age. Thus, the respondents found the representation in all three textbooks of the skills of knowledge and recall, and understanding and interpretation satisfactory, they were neutral regarding the representation of the skill of mathematical proof, and found the representation of all the other 6 skills unsatisfactory.

11.5.1 Teachers and Inspectors

The research question is:

What are the mathematical thinking skills prevailing in the questions in the secondary stage mathematics textbooks in Saudi Arabia in the opinion of the respondents according to post (i.e., teachers and inspectors)?

Table 11.13 shows the pattern of responses of teachers (post 1) and inspectors (post 2).

Table 11.13: Mean rank of responses of mathematical thinking skills according to post

Post	Answers	Skill 1	Skill 2	Skill 3	Skill 4	Skill 5	Skill 6	Skill 7	Skill 8	Skill 9
		Knowledge and recall	*Understanding and interpretation*	*Modelling*	*Application*	*Induction*	*Generalisation*	*Deduction*	*Mathematical proof*	*Evaluation*
1.0	*Strongly Disagree*	2.0	1.9	2.9	2.9	3.0	3.2	3.3	2.4	3.4
	Disagree	2.4	2.3	4.0	3.7	3.8	4.0	4.0	3.5	3.9
	Neutral	3.2	3.5	3.5	3.6	3.5	3.5	3.5	3.8	3.6
	Agree	4.1	3.9	2.5	2.6	2.6	2.3	2.2	3.1	2.1
	Strongly Agree	3.3	3.4	2.1	2.2	2.2	2.0	2.0	2.1	1.9
2.0	*Strongly Disagree*	2.0	1.9	3.0	3.0	3.0	3.2	3.3	2.3	3.4
	Disagree	2.2	2.3	3.9	3.7	3.7	4.0	3.9	3.6	3.9
	Neutral	3.2	3.5	3.6	3.5	3.4	3.5	3.5	3.9	3.7
	Agree	4.2	3.9	2.5	2.6	2.6	2.3	2.3	3.0	2.1
	Strongly Agree	3.4	3.4	2.0	2.2	2.2	2.0	1.9	2.2	1.9

The clear pattern in table 11.13 is that the two groups (teachers and inspectors) seem to hold very similar views on all nine skill areas.

11.5.2 Qualifications

The research question is:

What are the mathematical thinking skills prevailing in the questions in the secondary stage mathematics textbooks in Saudi Arabia in the opinion of teachers and inspectors of mathematics according to qualifications?

Table 11.14 below shows that the mathematical thinking skills prevailing are knowledge and recall, and understanding and interpretation for each of the qualifications. The qualifications are coded as follows:

1= Bachelor, 2 = Postgraduate Higher diploma, 3 = Master's degree, and 4 = PhD

Table 11.14: Mean rank of responses of mathematical thinking skills according to qualification

Qual'n	Answers	Skill 1 *Knowledge and recall*	Skill 2 *Understanding and interpretation*	Skill 3 *Modelling*	Skill 4 *Application*	Skill 5 *Induction*	Skill 6 *Generalisation*	Skill 7 *Deduction*	Skill 8 *Mathematical proof*	Skill 9 *Evaluation*
1	*Strongly Disagree*	2.0	1.9	3.0	3.0	3.0	3.2	3.3	2.3	3.4
	Disagree	2.2	2.3	3.9	3.8	3.7	4.0	3.9	3.6	3.9
	Neutral	3.2	3.5	3.6	3.4	3.4	3.5	3.5	3.9	3.7
	Agree	4.2	3.9	2.5	2.6	2.6	2.3	2.3	3.0	2.1
	Strongly Agree	3.4	3.4	2.0	2.2	2.2	2.0	1.9	2.2	1.9
2	*Strongly Disagree*	2.0	2.0	2.9	3.1	2.9	3.2	3.4	2.3	3.4
	Disagree	2.3	2.3	4.0	3.6	3.8	4.0	3.8	3.6	4.0
	Neutral	3.2	3.4	3.6	3.5	3.5	3.5	3.6	3.9	3.6
	Agree	4.2	3.9	2.5	2.6	2.6	2.4	2.3	3.0	2.1
	Strongly Agree	3.4	3.3	2.1	2.2	2.3	2.0	1.9	2.2	2.0
3	*Strongly Disagree*	1.9	1.9	3.1	3.0	2.8	3.2	3.4	2.3	3.5
	Disagree	2.3	2.3	3.9	3.7	3.8	3.9	3.9	3.6	3.7
	Neutral	3.4	3.4	3.4	3.6	3.7	3.6	3.6	3.9	3.8
	Agree	4.0	3.9	2.4	2.5	2.7	2.3	2.2	3.0	2.1
	Strongly Agree	3.5	3.5	2.1	2.2	2.1	2.0	1.9	2.2	1.9
4	*Strongly Disagree*	2.0	2.0	3.2	3.3	3.0	3.4	3.5	2.6	3.4
	Disagree	2.1	2.3	3.7	3.7	3.8	3.9	4.0	3.8	3.9
	Neutral	2.8	3.7	3.4	3.5	3.0	3.5	3.3	3.7	3.4
	Agree	4.5	3.8	2.6	2.3	2.9	2.3	2.2	2.8	2.4
	Strongly Agree	3.5	3.2	2.1	2.2	2.3	1.9	2.1	2.2	1.9

The clear pattern is that there is very little difference in the views of the four groups on any of the nine skills.

11.5.3 Experience

What are the mathematical thinking skills prevailing in the questions in the secondary stage mathematics textbooks in Saudi Arabia in the opinion of teachers and inspectors of mathematics according to length of experience?

Table 11.15 looks at the pattern of responses in relation to experience. Here:

1 = less than 5 years, 2 = 5-9 years, 3 = 10-14 years, and 4 = 15 years or more

Table 11.15: Mean rank of responses of mathematical thinking skills according to length of experience

Exp.	Answers	Skill 1	Skill 2	Skill 3	Skill 4	Skill 5	Skill 6	Skill 7	Skill 8	Skill 9
		Knowledge and recall	*Understanding and interpretation*	*Modelling*	*Application*	*Induction*	*Generalisation*	*Deduction*	*Mathematical proof*	*Evaluation*
1.0	*Strongly Disagree*	2.0	1.9	3.0	3.0	3.0	3.2	3.3	2.3	3.5
	Disagree	2.4	2.3	3.9	3.8	3.7	4.0	4.0	3.5	3.9
	Neutral	3.1	3.4	3.6	3.4	3.5	3.5	3.5	3.9	3.6
	Agree	4.2	4.0	2.5	2.5	2.6	2.3	2.3	3.2	2.1
	Strongly Agree	3.4	3.4	2.0	2.2	2.2	2.0	1.9	2.2	1.9
2.0	*Strongly Disagree*	2.0	1.9	3.0	3.0	3.0	3.2	3.3	2.4	3.4
	Disagree	2.2	2.3	4.0	3.7	3.7	3.9	3.9	3.6	3.9
	Neutral	3.3	3.5	3.6	3.4	3.4	3.6	3.6	3.9	3.7
	Agree	4.2	3.8	2.4	2.6	2.6	2.3	2.3	3.0	2.1
	Strongly Agree	3.4	3.5	2.0	2.2	2.2	2.0	1.9	2.2	1.9
3.0	*Strongly Disagree*	2.0	1.9	3.0	2.9	2.9	3.3	3.4	2.3	3.4
	Disagree	2.2	2.4	3.9	3.8	3.8	4.0	3.9	3.6	3.9
	Neutral	3.3	3.5	3.6	3.5	3.5	3.5	3.5	3.9	3.6
	Agree	4.1	3.9	2.4	2.6	2.6	2.3	2.3	3.0	2.1
	Strongly Agree	3.4	3.4	2.1	2.2	2.2	2.0	1.9	2.3	2.0
4.0	*Strongly Disagree*	2.0	2.0	3.0	3.0	3.0	3.2	3.4	2.4	3.3
	Disagree	2.2	2.3	3.9	3.6	3.7	3.9	4.0	3.5	3.9
	Neutral	3.2	3.4	3.5	3.5	3.4	3.6	3.5	3.9	3.8
	Agree	4.2	4.0	2.5	2.6	2.7	2.3	2.2	3.0	2.2
	Strongly Agree	3.4	3.3	2.0	2.2	2.2	1.9	1.9	2.1	1.9

Again, it does not appear that length of experience changes the views of the respondents.

11.5.4 Training

The research question is:

What are the mathematical thinking skills prevailing in the questions in the secondary stage mathematics textbooks in Saudi Arabia in the opinion of teachers and inspectors of mathematics according to whether they had had training or not?

From table 11.16 below it can be seen that the prevailing mathematical thinking skills are knowledge and recall, and understanding and interpretation, regardless of whether the participant had any training in education, teaching and assessment methods and/or mathematical thinking skills.

Table 11.16: Mean rank of responses for mathematical thinking skills according to training

Training	Answers	Skill 1	Skill 2	Skill 3	Skill 4	Skill 5	Skill 6	Skill 7	Skill 8	Skill 9
		Knowledge and recall	*Understanding and interpretation*	*Modelling*	*Application*	*Induction*	*Generalisation*	*Deduction*	*Mathematical proof*	*Evaluation*
No	*Strongly Disagree*	2.0	1.9	3.0	3.0	3.0	3.2	3.3	2.4	3.4
	Disagree	2.3	2.3	4.0	3.8	3.7	4.0	3.9	3.6	3.9
	Neutral	3.2	3.5	3.6	3.4	3.5	3.5	3.6	3.9	3.7
	Agree	4.2	3.9	2.5	2.6	2.6	2.3	2.3	3.0	2.1
	Strongly Agree	3.4	3.4	2.0	2.2	2.2	2.0	1.9	2.2	1.9
Yes	*Strongly Disagree*	2.0	1.9	3.0	2.9	3.0	3.2	3.4	2.3	3.4
	Disagree	2.2	2.3	3.9	3.7	3.7	4.0	3.9	3.6	3.9
	Neutral	3.2	3.4	3.6	3.5	3.4	3.6	3.5	3.9	3.6
	Agree	4.2	3.9	2.5	2.6	2.7	2.3	2.3	3.0	2.1
	Strongly Agree	3.4	3.4	2.1	2.2	2.2	2.0	1.9	2.2	1.9

11.6 Conclusion

There are no obvious differences between subgroups. In the views of all, knowledge and recall, and understanding and interpretation are strongly emphasised, while they are ambivalent about the emphasis on mathematical proofs.

All other skills are regarded as strongly underemphasised in the questions in the selected mathematics textbooks.

Another way to look at this is to correlate the questions against code numbers used for qualifications, experience and training. With ordinal numbers, Kendall's Tau-b is used.

11.7 Correlations

It is possible to explore the way different sub-groups respond to the questions by using Kendall's Tau-b correlation. As the data are ordinal and distributions do not correspond closely to normal distributions, correlation has to be conducted using Kendall's Tau-b. The way response patterns relate to qualifications, experience and training is now considered. Only correlations which are significant are considered.

11.7.1 Qualification

The highest numerical value obtained is r = -0.05, showing that qualifications make almost no difference in the way they respond, consistent with the data in table 11.14.

11.7.2 Experience

The highest numerical correlation value is 0.06 (table 11.17).

Table 11.17 Kendall's Tau-b correlations (r): experience and response patterns

Item	N = 1466	r	P
36	Use statements to describe particular cases.	+0.05	< 0.05
47	Discover mistakes in mathematical relationships.	+0.05	< 0.05
50	Questions encourage students to apply what they learn in a practical way in their daily lives.	-0.06	< 0.01
51	Questions and exercises contribute to developing positive attitudes towards mathematics.	+0.05	< 0.05

This shows that length of experience makes almost no difference in the way they respond, consistent with the data in table 11.15.

11.7.3 Training

Again values are numerically very low, the highest numerical value being 0.05. This confirms the picture in table 11.16.

11.8 Formulation and Layout

The research question is:

To what extent are the criteria of formulation and good layout of the questions in the secondary stage mathematics textbooks fulfilled?

Formulation is considered first. Here, thirteen items (item 1 to item 13) were used to identify formulation, as shown in table 11.18 below. Again, the data are shown as percentages for clarity.

Table 11.18: Percentage for each item of formulation

ITEMS	Frequency (%) n=1466				
	SD	D	N	A	SA
1.The language in the questions is clear and easily understood.	2	8	29	38	24
2.The mathematical problems vary in terms of words and symbols.	7	26	33	24	10
3.Questions are free of spelling and printing mistakes.	1	3	24	44	29
4.Questions are free of factual errors.	1	2	24	43	30
5.Symbols and terms used are similar to those used in the textbook's content.	1	5	28	42	25
6.Questions are brief, but contain all necessary information.	2	10	31	37	21
7.Questions do not include hints to make answering easier.	0	3	24	46	27
8.Questions are accurately formulated.	4	29	32	24	11
9.The formulation of the exercises at the end of each lesson are similar to those in the textbook's content.	0	2	19	48	31
10.Questions are varied to include questions requiring student-produced responses together with objective questions.	75	15	10	0	0
11.The exercises include suitable and logical tables, figures and numbers.	0	3	21	44	32
12.The directions of the exercises are formulated in such a way as to need no inquiry from the students.	0	3	19	47	31
13.The exercises in the textbook are characterised by academic accuracy.	0	1	15	56	28

In almost all the items, the respondents show positive views about the way questions are formulated. However, it is clear that mixtures of questions requiring student-produced responses and objective questions (e.g., true or false, multiple choice) do not often occur.

Layout is now considered.

Nine items (item 14 to item 22) were used to identify good layout, as illustrated in table 11.19 below.

Table 11.19: Percentage for each item of layout

ITEMS	Frequency (%) n=1466				
	SD	D	N	A	SA
14.Questions in the textbook are displayed in a manner that is interesting and encouraging for the readers.	14	43	36	6	1
15.There is enough space between each exercise and the next.	1	4	19	48	28
16.There is enough space between the main part of the exercises and the secondary parts.	0	2	14	45	40
17. Figures and diagrams related to the exercises are positioned appropriately.	0	0	0	33	67
18. Exercises are accurately numbered.	0	0	0	14	86
19. The items of the questions and exercises are accurately numbered.	0	0	0	16	84
20. No paragraph of any exercise is divided between two pages.	0	0	0	9	90
21. Exercises are not crowded on each page.	0	0	6	13	80
22. Exercises are written in clear fonts.	0	0	0	16	84

In general, the respondents are very positive about layout, although they are not convinced that layout encourages interest.

11.9 Formulation and layout – various related factors

The research sub-questions 1- 4 were stated as follows:

Are there any significant differences in the criteria for good formulation and good layout of the questions in the secondary stage mathematics textbooks according to grade, role, qualification, or experience?

Again, the data are presented in terms of the average frequency of choice of the various options (strongly agree to strongly disagree), shown in tables 11.20 to 11.24.

Table 11.20 Formulation and layout related to Grade

Measure	Grade	*Strongly disagree*	*Disagree*	*Neutral*	*Agree*	*Strongly agree*
Formulation	1st	1.7	1.9	3.5	4.5	3.5
	2nd	1.7	1.9	3.5	4.5	3.5
	3rd	1.7	1.8	3.6	4.4	3.4
Layout	1st	1.7	2.2	2.5	3.7	4.9
	2nd	1.7	2.3	2.5	3.6	4.9
	3rd	1.7	2.3	2.5	3.6	4.9

Table 11.21 Formulation and layout related to Role

Measure	Role	*Strongly disagree*	*Disagree*	*Neutral*	*Agree*	*Strongly agree*
Formulation	Inspector	1.7	2.0	3.4	4.4	3.5
	Teacher	1.7	1.9	3.5	4.5	3.5
Layout	Inspector	1.6	2.3	2.5	3.6	4.9
	Teacher	1.7	2.2	2.5	3.7	4.9

Table 11.22 Formulation and layout related to Qualification

Measure	Qualification	*Strongly disagree*	*Disagree*	*Neutral*	*Agree*	*Strongly agree*
Formulation	Bachelors	1.7	1.9	3.5	4.5	3.5
	Diploma	1.7	1.9	3.6	4.4	3.4
	Masters	1.7	2.0	3.4	4.5	3.4
	PhD	1.8	2.1	3.3	4.4	3.4
Layout	Bachelors	1.7	2.2	2.5	3.6	4.9
	Diploma	1.7	2.2	2.5	3.6	4.9
	Masters	1.7	2.2	2.4	3.8	4.9
	PhD	1.7	2.5	2.4	3.5	4.9

Table 11.23 Formulation and layout related to Length of Experience

Measure	Length of Experience	*Strongly disagree*	*Disagree*	*Neutral*	*Agree*	*Strongly agree*
Formulation	< 5 years	1.6	1.9	3.5	4.6	3.4
	5-9 years	1.7	1.9	3.5	4.4	3.5
	10-14 years	1.7	1.9	3.5	4.4	3.5
	≥ 15 years	1.7	1.9	3.5	4.5	3.4
Layout	< 5 years	1.7	2.4	2.4	3.7	4.9
	5-9 years	1.7	2.2	2.5	3.7	4.9
	10-14 years	1.7	2.2	2.5	3.6	4.9
	≥ 15 years	1.7	2.2	2.6	3.6	4.9

Table 11.24 Formulation and layout related to Training

Measure	Training attended	*Strongly disagree*	*Disagree*	*Neutral*	*Agree*	*Strongly agree*
Formulation	No	1.7	1.9	3.5	4.5	3.5
	Yes	1.7	1.9	3.5	4.4	3.5
Layout	No	1.7	2.2	2.5	3.7	4.9
	Yes	1.7	2.2	2.5	3.7	4.9

It is possible to explore this statistically using Kendall’s Tau-b where the response pattern of each of the items is correlated with grade, qualification, length of experience and training.

It is found that all the correlations are extremely low (most below 0.05 numerically). Thus, there is no difference in the criteria of good formulation and layout of questions in the secondary stage mathematics textbooks in Saudi Arabia according to different grades, according to participants' qualifications, length of experience or whether they had training or not.

11.10 Comparison between Inspectors and Teachers

It might be expected that the views of the two groups would differ. However, when their distributions for each item are compared using chi-square as a contingency test, in only 4 of the items is a significant difference observed (table 11.25).

Table 11.25 Chi-square comparisons between teachers and inspectors

No.	Item		Responses (%)					Comparisons	
	$N_{(inspectors)} = 158$, $N_{(teachers)} = 1308$		SD	D	N	A	SA	χ2 (df)	P
5	Symbols and terms used are similar to those used in the textbook's content.	*Insp*	1	5	34	43	17	7.8(2)	< 0.05
		Teach	0	5	27	42	26		
15	There is enough space between each exercise and the next.	*Insp*	1	10	17	44	29	9.5(3)	< 0.05
		Teach	1	4	19	49	28		
16	There is enough space between the main part of the exercises and the secondary parts.	*Insp*	0	6	15	47	32	6.3 (2)	< 0.05
		Teach	0	2	14	44	41		
25	Organise knowledge in a logical manner.	*Insp*	1	17	33	32	17	9.4(3)	< 0.05
		Teach	§	10	31	37	21		

Thus, inspectors are slightly less confident that the symbols and terms used are similar to those used in the textbook's content, that spaces between exercises and secondary parts are adequate and that knowledge is organised in a logical manner. However, even in these questions, the differences are not large.

11.11 Attitudes

The key research question here is:

Do the questions in the mathematics textbooks develop positive attitudes towards mathematics in the opinion of the teachers and inspectors of mathematics?

Six items (48 to 53) were used to identify good attitude, as shown in table 11.26 below.

Table 11.26: Percentage for each item of good attitude

ITEMS	Frequency (%) n=1466				
	SD	D	N	A	SA
48.Questions include emotional aspects (tendencies, attitudes, values, etc).	18	52	26	4	0
49.Questions are relevant to situations in the students' daily lives.	3	27	40	24	6
50.Questions encourage students to apply what they learn in a practical way in their daily lives.	8	33	39	16	4
51.Questions and exercises contribute to developing positive attitudes towards mathematics.	18	45	29	6	2
52.Questions and exercises indicate the role of mathematics in daily life.	16	47	33	4	1
53.Questions and exercises indicate the role played by mathematics in other subjects.	14	32	32	16	6

Here, opinions are much less positive. It is possible that the respondents do not see textbook questions as really making any major contribution to the development of positive attitudes. This is entirely consistent with the evidence (in physics) which shows the key role of the teacher and the actual subject matter of the curriculum (Reid and Skryabina, 2000) in encouraging positive attitudes towards the subject.

When the response patterns from the attitude items are correlated (Kendall's Tau-b), there are no correlations above 0.06. When the response patterns of teachers and inspectors are compared, chi-square shows no significant differences.

11.12 Conclusions

It is possible to explore the way different sub-groups respond to the items by using Kendall's Tau-b correlation. As the data are ordinal and distributions do not correspond closely to normal distributions, correlation has to be conducted using Kendall's Tau-b. The way response patterns relate to qualifications, experience and training is now considered. Only correlations which are significant are considered.

11.12.1 Qualifications

The highest numerical value obtained is r = -0.05 (p < 0.05) and this is for item 32.

'*Use of mathematical knowledge in new situations*'

11.12.2 Experience

The highest numerical correlation value is -0.1 but several items show values between ±0.05 and ±0.10 (table 11.27).

Table 11.27: Kendall's Tau-b correlation for length of experience

No.	**N = 1466**	**R**	**P**
6	Questions are brief, but contain all necessary information.	-0.07	< 0.01
10	Questions are varied to include topic questions along with objective questions.	-0.10	< 0.001
14	Questions in the textbook are displayed in a manner that is interesting and encouraging for the readers.	+0.06	< 0.01
16	There is enough space between the main part of the exercises and the secondary parts.	-0.07	< 0.01
36	Use statements to describe particular cases.	+0.05	< 0.05
47	Discover mistakes in mathematical relationships.	+0.05	< 0.05
50	Questions encourage students to apply what they learn in a practical way in their daily lives.	-0.06	< 0.01
51	Questions and exercises contribute to developing positive attitudes towards mathematics.	+0.05	< 0.05

11.12.3 Training

Again values are numerically very low (table 11.28).

Table 11.28: Kendall's Tau-b correlation for training

	N = 1466	**r**	**P**
6	Questions are brief, but contain all necessary information.	-0.05	< 0.05
10	Questions are varied to include topic questions along with objective questions.	-0.05	< 0.05
16	There is enough space between the main part of the exercises and the secondary parts.	-0.08	< 0.001
31	Make tables and graphs of the given data.	+0.05	< 0.05

Chapter 12
Interview Data

12.1 Introduction

Chapter 11 provides the quantitative data analysis of the data gathered from the teachers and inspectors through the administration of a questionnaire survey. This chapter presents the analysis of the primary data which were obtained through interviews with nineteen participants, five inspectors and fourteen teachers. The sixth research question (What are the views of the teachers and inspectors concerning the textbooks, the textbook questions and mathematical thinking?) will be addressed in this chapter.

The following sections present the coding analysis and the related results through a thematic analysis.

12.2 Sample Selection

Several criteria were used to select the interviewees. Firstly, the researcher approached 51 teachers who were known to be excellent teachers of mathematics, having won local and/or national teaching awards. As the researcher was based in Jeddah, it was judged practical to approach teachers in this group who were in Jeddah or within a convenient travelling distance from it; therefore, he approached teachers who were in Jeddah, Makkah and Taif. The majority of those approached did not wish to be interviewed, mainly citing lack of time as the reason for this. Fourteen of those approached accepted. Regarding the inspectors, 27 inspectors in

Jeddah, Makkah and Taif were approached, but only five accepted to be interviewed. All the interviewees had previously responded to the questionnaire.

There were, therefore, a total of nineteen interviewees. According to Ghauri and Gronhaug (2010), use of a program for analysing qualitative data is beneficial when there is a large quantity of data to be coded, annotated and linked. However, if there is only a small quantity of data, it is not necessary to use such a program and data can be analysed manually. As the sample in this research consists of only nineteen interviewees, it was considered that manual analysis was appropriate.

To analyse the data, focused coding was used. "The objectives of focused coding are to identify recurrent patterns and multiple layers of meaning, and to delineate variations and interconnections among sub-themes within the general topic." (Hsiung, 2010). According to Charmaz (2006), the use of focused coding to develop potential conceptual categories permits the researcher to obtain a framework for the analysis. In accordance with Charmaz's (2006) recommendation, this researcher developed the conceptual categories, expressed in short phrases, to best represent what he saw as the salient points in the data.

12.3 Interview Schedule and Procedure

The interviews were intended to obtain information which was complementary to that obtained from the questionnaires. According to Gill and Johnson (1991), an adept interviewer uses interviews to discuss questions in greater depth, thus gaining more than a superficial response. Similarly, Cohen et al. (2007) state that face-to-face interviews permit the interviewer to gain an insight into the interviewees' opinions. They also assist the interviewer in probing into an issue in greater depth.

The interviewer initially considered using focus group interviews which, according to Wilkinson (1998) are used frequently as a research tool in education, as in other disciplines. However, Smithson (2000) argues that the focus group method is not in fact a rapid way to develop pertinent themes around a topic, and can degenerate into a type of social gathering. In addition, a focus group is frequently limited by having one or more dominant individuals who tend to allow only their own opinions to be heard (Smithson, 2000). Therefore, it was decided that face-to-face interviews would be more appropriate for this research.

The content validity of the interview schedule was assessed primarily by discussion with the researcher's supervisor. This discussion focused on the wording and sequences of the questions with the purpose of clarifying any ambiguity. Wellington (2000) recommends a careful use of language in interview questions, stating that "The questions need to make sense and be unambiguous" (p.76). After this discussion, the schedule was pilot-tested on five teachers and two inspectors in Jeddah. No modifications were indicated by the pilot study.

Structured interviews were chosen, as they offer the researcher greater control and reliability than do semi-structured or unstructured interviews (Smith and Osborn, 2008). That is "the interview will be reliable in the sense that the same format is being used with each respondent, and that the identity of the interviewer should have minimal impact on the responses obtained." (Smith and Osborn, 2008, p.58).

Only three of the interviewees were known personally to the researcher. He had originally intended to audio-record the interviews and, in accordance with ethical practice, asked the interviewees for their consent to do this, assuring them of confidentiality and that they could ask for the recording to be stopped at any time during the interview. However, only one of them, a teacher who was well-acquainted with the researcher, consented to the interview being recorded. According to Al-Zarah (2008), this reluctance to be recorded reflects the conservative nature of Saudi society. Hence, the interviewer had to rely on notes made during interviews, which he transcribed as soon as possible afterwards.

It is important that the interviewees feel relaxed with the interviewer and with the setting in which the interviews are conducted. It is also essential that they are assured of the confidentiality of their responses (Bell, 1999). Hence, the interviewees were interviewed in their place of work and assured that their responses would be used only for academic purposes and their names would not appear. In addition, to set them at their ease, the researcher began by asking a few general questions such as whether they enjoyed their work and found it satisfying before beginning to ask the interview questions.

The responses were coded into categories, these categories being determined by the actual responses made. In the following sections, the overall pattern of categories is summarised, followed by an analysis of the participants who actually responded under each category.

It has to be stressed that the data from the interviews reflects what the interviewees thought. It does not indicate what is possible or what would actually be effective. However, it does offer a picture of what teachers and inspectors of varied experience think.

12.4 Knowledge of Mathematical Thinking

The participants were asked about their knowledge of mathematical thinking. For this question, the teachers and inspectors were divided into two groups, those with 15 years or more experience as teachers, and those with fewer than 15 years experience. There were 9 teachers and 3 inspectors in the former group and 5 teachers and 2 inspectors in the latter. Table 12.1 below shows the coded answers of both groups.

Table 12.1: Knowledge of Mathematical Thinking

Question 1	What is your knowledge of mathematical thinking?
Focused Coding	
1	Teachers and inspectors with 15 years' experience or more No experience Vague idea through self-study
2	Teachers and inspectors with fewer than 15 years experience Some theoretical knowledge, but no practical application Short course on thinking in general, but not specifically on mathematical thinking
Theme 1	The knowledge of mathematical thinking of all the teachers and inspectors is non-existent or basic.

Table 12.2: Focused Coding Number 1 for Question 1

Teachers and inspectors with 15 years' experience or more		
Participants	***Total***	***Response***
1,5,6,9,10, 12, 18, 19	8	Do not have any experience
2,3,7, 16	4	Have a vague idea through self-study

Table 12.3: Focused Coding Number 2 for Question 1

Teachers with less than 15 years' experience		
Participants	***Total***	***Response***
4, 13, 14, 15	4	Some theoretical knowledge, but no practical application
8, 11, 17	3	Short course on thinking in general, but not specifically on mathematical thinking

From the tables above it can be seen that teachers and inspectors with more experience either do not have any knowledge of mathematical thinking, or have a little knowledge which they gained through their own reading, rather than through official channels. The teachers and inspectors with less experience either had been given a short course on mathematical thinking but had never applied it in the classroom or had had some training in thinking in general but not mathematical thinking in particular. However, perhaps the greatest problem in considering the responses is that the way '*mathematical thinking*' is conceptualised by different interviewees may vary considerably. In addition, it could be argued that the responses of the interviewers are of little value given their relative lack of knowledge about mathematical thinking. In this respect, it should be noted that the interview sample was small and therefore not easily generalizable, and that data triangulation was used; i.e., content analysis and questionnaires were also used to gather data. Thus, confidence in the study findings is enhanced.

12.5 Views of mathematical thinking

In this question, the participants were asked about their views on mathematical thinking. The coding is shown below in table 12.4.

Table 12.4: Views of mathematical thinking

Question 2	**What is your view of mathematical thinking?**
Focused Coding	
1	I don't have any view on it.
2	It sounds good, but I don't know enough about it to be sure.
3	I think it could be useful, but it would be difficult to apply.
4	It is very important and it should be taught in all mathematics classrooms.
Theme 2	Varied answers, from no view to thinking it is very important.

Table 12.5: Focused Coding Number 1 for Question 2

I don't have any view on it.		
Participants	*Total*	*View*
1, 5, 6, 12, 14, 15, 18	7	I don't have any view on it.

Table 12.6: Focused Coding Number 2 for Question 2

It sounds good, but I don't know enough about it to be sure.		
Participants	***Total***	***View***
2, 3, 8, 15, 16	5	It sounds good, but I don't know enough about it to be sure.

Table 12.7: Focused Coding Number 3 for Question 2

I think it could be useful, but it would be difficult to apply.		
Participants	***Total***	***View***
4, 7,18	3	I think it could be useful, but it would be difficult to apply.

Table 12.8: Focused Coding Number 4 for Question 2

It is very important and it should be taught in all mathematics classrooms.		
Participants	***Total***	***View***
9, 10, 11, 17	4	It is very important and it should be taught in all mathematics classrooms.

It is clear that the teachers and inspectors are not fully aware of the implications of teaching mathematical thinking, with only 4 thinking that it should be taught in all classrooms. This clearly implies that the teachers and inspectors in general do not have a great deal of knowledge about mathematical thinking.

An interesting question arises about whether mathematical thinking can be taught in the formal sense. In an interesting set of studies, scientific thinking was explored and measured (Reid and Serumola, 2006, 2007; Al-Ahmadi and Reid, 2011, 2012). The studies found that scientific thinking could only be developed in learners aged about 16 or over and that it did not seem to develop unless it was overtly embedded in the class lessons. However, Fleer (1992) disagrees, claiming that scientific thinking can be instilled in much younger children. Critical thinking was also found to be '*teachable*' at a younger age (Al-Osaimi, 2013). How mathematical thinking might relate to these findings is not known.

12.6 Mathematical thinking as a goal of mathematics education

Table 12.9: Mathematical thinking as a goal of mathematics education

Question 3	**Do you think that developing mathematical thinking should be one of the goals of mathematics education in Saudi Arabia? Please give a reason for your answer.**
Focused Coding	
1	Yes, it will be of great benefit to students' learning of mathematics.
2	Yes, it will enable the students to solve a problem by several different approaches.
3	Yes, if the students learn mathematical thinking, they will come to have a more positive attitude to mathematics.
4	Yes, it will help students to solve problems in other subjects and in daily life as well.
Theme 3	Mathematical thinking should be one of the goals of mathematics education and it would benefit the students in several ways.

Table 12.10: Focused Coding Number 1 for Question 3

Yes		
Participants	***Total***	***Reason***
2, 3, 7, 8, 10, 17	6	Great benefit to their learning of mathematics.

Table 12.11: Focused Coding Number 2 for Question 3

Yes		
Participants	***Total***	***Reason***
6, 11, 16,	3	To solve a problem by several different approaches.

Table 12.12: Focused Coding Number 3 for Question 3

Yes		
Participants	***Total***	***Reason***
1, 4, 13, 14, 15	5	More positive attitude to mathematics.

Table 12.13: Focused Coding Number 4 for Question 3

Yes		
Participants	***Total***	***Reason***
5, 9, 12, 18, 19	5	To solve problems in other subjects and in daily life as well.

There is general agreement that developing mathematical thinking will bring benefits although there is no clear agreement on the nature of these benefits.

12.7 Mathematics textbooks and mathematical thinking

This looks at textbook questions in relation to mathematical thinking.

Table 12.14: Results of Question 4

Question 4	**Do you believe that the questions in the mathematics textbooks encourage mathematical thinking? Please give a reason for your answer.**
Focused Coding 4	
1	No. Focus on memorisation and rote learning only.
2	No. Focus on quantity not quality,
3	No. Number of students in class
4	No. Teachers' lack of experience.
5	No. Increase teachers' responsibilities.
6	No. No relation to real life.
Theme 4	Consensus that the questions in the mathematics textbooks do not encourage mathematical thinking.

Table 12.15: Focused Coding Number 1 for Question 4

No.		
Participants	Total	Reason
1, 3, 6, 9, 10, 17	6	Focus on memorisation and rote learning only.

Table 12.16: Focused Coding Number 2 for Question 4

No.		
Participants	***Total***	***Reason***
2, 4, 15	3	Focus on quantity not quality,

Table 12.17: Focused Coding Number 3 for Question 4

No.		
Participants	***Total***	***Reason***
12, 13,	2	Number of students in class

Table 12.18: Focused Coding Number 4 for Question 4

No.		
Participants	***Total***	***Reason***
16, 18, 19	3	Teachers' lack of experience.

Table 12.19: Focused Coding Number 5 for Question 4

No.		
Participants	***Total***	***Reason***
5, 14	2	Increase teachers' responsibilities.

Table 12.20: Focused Coding Number 6 for Question 4

No.		
Participants	***Total***	***Reason***
7, 8, 11	3	No relation to real life.

Focused coding 2 can be attributed to such answers being easier for the teachers to mark. Focused coding 3 refers to the fact that classes in Saudi Arabia are generally very large, with more students than are to be found in classrooms in developed countries. Focused coding 4 implies that the teachers do not have enough experience or ability to match questions to students. Focused coding 5 indicates that teachers already have many responsibilities and this would increase their load, while focused coding 6 reveals that students do not see the relation between the questions and their everyday lives.

Perhaps the key lies in the first response coding (table 12.15). As long as examinations give rewards on the basis of the recall and use of information and procedures, then other skills will never be emphasised.

12.8 Obstacles to Mathematical Thinking

This poses the question about what might hold up the development of mathematical thinking.

Table 12.21: Results of Question 5

Question 5	What are the obstacles to students learning to think mathematically?
Focused Coding 5	
1	Focus on passing exams.
2	No time – too much information.
3	Lack of equipment or teachers' ability to use it
4	Students' negative attitude
5	Maths not attractive
6	Examples and questions
Theme 5	Numerous obstacles.

Table 12.22: Focused Coding Number 1 for Question 5

Focus on passing exams.		
Participants	***Total***	***Obstacle***
2, 7, 8, 10, 18	5	Focus on passing exams.

Table 12.23: Focused Coding Number 2 for Question 5

No time – too much information.		
Participants	***Total***	***Obstacle***
1, 5, 13, 19	4	No time – too much information.

Table 12.24: Focused Coding Number 3 for Question 5

Lack of equipment or teachers' ability to use it.		
Participants	***Total***	***Obstacle***
3, 15	2	Lack of equipment or teachers' ability to use it

Table 12.25: Focused Coding Number 4 for Question 5

Students' negative attitude		
Participants	***Total***	***Obstacle***
4, 12, 14	3	Students' negative attitude

Table 12.26: Focused Coding Number 5 for Question 5

Maths not attractive		
3.03	***Total***	***Obstacle***
9, 11, 17	3	Maths not attractive

Table 12.27: Focused Coding Number 6 for Question 5

Examples and questions		
Participants	***Total***	***Obstacle***
6, 16	2	Examples and questions

It is clear that due to the time that must be spent to get through the curriculum, teachers are unable to teach mathematics in depth. The focus is on passing exams rather than acquiring knowledge. Similarly, there is a lack of equipment in the classroom or teachers are unable to use what equipment there is. Students generally have a negative attitude to mathematics. Usually the examples given in the textbook are easy, but the questions are difficult.

12.9 Encouragement from Inspectors

This question was asked of teachers only: Does your inspector encourage you to develop your students' mathematical thinking skills? Why?

Table 12.28: Results of Question 6

Question 6	**Does your inspector encourage you to develop your students' mathematical thinking skills? Give a reason for your answer.**
Focused Coding 6	
1	No: Focus on keeping to lesson schedule. Contradiction between inspectors' recommendations and students' requirements.
2	Yes: Discussion with students in class. More practical work on the board Following homework Use of technology
Theme 6	Most inspectors do not encourage the development of mathematical thinking skills, but some do.

Table 12.29: Focused Coding Number 1 for Question 6

No		
Participants	***Total***	***Reasons***
2, 3, 7, 9, 10, 12	6	Focus on keeping to lesson schedule.
1, 5, 6	3	Contradiction between inspectors' recommendations and students' requirements.

Table 12.30: Focused Coding Number 2 for Question 6

Yes		
Participants	***Total***	***Reasons***
14	1	Discussion with students in class.
11	1	More practical work on the board
8	1	Following homework
4, 13	2	Use of technology

The answers revealed that the majority of teachers thought that inspectors did not encourage the development of mathematical thinking skills, as they focused only on the lesson schedule. However, a minority of teachers felt their inspectors did encourage them to develop their students' mathematical thinking skills, mainly through the use of modern technology.

Perhaps the correct picture is seen best in table 12.29. The risk is that inspectors may lose the reality of the classroom very quickly. They then resort to an emphasis on efficiency of procedures and lose sight of the real needs of young people in learning situations.

12.10 The Importance of Short Training and Workshops (in Developing Mathematical Thinking Skills) for Teachers

There is an assumption in education that issues can be addressed simply by putting teachers through courses. This question explores the views of interviewees on this issue.

Table 12.31: Results of Question 7

Question 7	**Do you think short courses for teachers are important to develop students' mathematical thinking skills? Please give a reason for your answer.**
Focused Coding 7	
1	Yes Innovation and creativity Enhance the teachers' experience Reduce time and effort and improve performance Address the teachers' shortcomings Inform teachers of modern methods Discover new talent Break the routine and increase motivation
2	No Insufficient time No incentives Irrelevant to curriculum
Theme 7	Short courses useful to most teachers, not useful according to others.

Table 12.32: Focused Coding Number 1 for Question 7

Yes		
Participants	***Total***	***Reasons***
2, 14	2	Enhancing the teachers' experience
5, 11, 13	3	Innovation and creativity
4, 9	2	Reduce time and effort and improve performance
16, 18	2	Address the teachers' shortcomings
10	1	Inform teachers of modern methods
15	1	Discover new talent
3, 7	2	To break the routine and increase motivation

Table 12.33: Focused Coding Number 2 for Question 7

No		
Participants	***Total***	***Reasons***
1, 12	2	Insufficient time
6	1	No incentives
8, 17, 19	3	Irrelevant to curriculum

Focused coding 1 reveals that, according to the interviewees, short courses would be useful to increase teachers' ability in teaching mathematical thinking skills. It indicates that the courses would enhance the teachers' experience to face challenges regarding mathematical thinking skills. It also implies that such courses would help teachers to learn to improve their performance in teaching mathematical thinking skills. It refers to most teachers having no qualifications in mathematical thinking skills and needing to address this deficiency. It also refers to the need to inform teachers about new approaches to improve the teaching of mathematical thinking skills. There is also a need to discover teachers who are talented at developing mathematical thinking skills, improving the curriculum, and producing teaching aids to facilitate the teaching of mathematical thinking skills, to break the routine and increase motivation.

On the other hand, focused coding 2 indicates that teachers already have a heavy workload (planning lessons, marking papers, etc.), do not have enough time in the day and are too tired at night. In addition, there are no incentives, as teachers are not paid to take such courses; they will only be given a certificate. It could be inferred that some teachers also feel there is no point to these courses, as the curriculum will not change.

This last point is important. In two studies, it was shown that quality training courses failed to bring about the desired changes, not because the teachers were in any way unconvinced or unwilling (Carroll, 2005; Elsawaf, 2007). In the latter study, the reasons (derived from two countries: one developed, one in the Middle East) were clearly shown to be because of the stranglehold of the curriculum and assessment, both being controlled outside the schools. In addition, this is supported by the study of HSRC (2008), which found that almost all teachers obtained very little support in terms of training and frequently complained that the brief training course for teachers had not been useful.

12.11 Improving Textbook Questions

This looks at the actual mathematics questions in relation to mathematical thinking.

Table 12.34: Results of Question 8

Question 8	**What are the obstacles to developing the questions in the maths textbooks in terms of the students' mathematical thinking?**
Focused Coding	
1	Financial constraints
2	Expertise
3	Unwillingness of some officials
4	Administrative obstacles
Theme 8	Several obstacles, mainly administrative.

Table 12.35: Focused Coding Number 1 for Question 8

Financial constraints		
Participants	***Total***	***Obstacle***
5, 9, 18, 19	4	Financial constraints

Table 12.36: Focused Coding Number 2 for Question 8

Expertise		
Participants	***Total***	***Obstacle***
3.8	5	Expertise

Table 12.37: Focused Coding Number 3 for Question 8

Unwillingness of some officials		
Participants	***Total***	***Obstacle***
3, 7, 8, 11	4	Unwillingness of some officials

Table 12.38: Focused Coding Number 4 for Question 8

Administrative obstacles		
Participants	***Total***	***Obstacle***
4, 6, 10, 12, 13, 16	6	Administrative obstacles

They raise numerous obstacles but there is no evidence that these actually hinder the development of mathematical thinking in questions.

12.12 Difficulties of Understanding and Applying Mathematics

This explores their views on a major issue.

Table 12.39: Results of Question 9

Question 9	**How can we overcome the difficulties that students face in understanding and applying mathematics?**
Focused Coding	
1	Reformulation of material and questions
2	More attractive textbooks
3	Examples from real life
4	Use modern technology
Theme 9	Change textbooks to facilitate students' understanding.

Table 12.40: Focused Coding Number 1 for Question 9

Reformulation of material and questions		
Participants	***Total***	***Suggestion***
1, 3, 4, 7, 10, 12, 13, 14, 18	9	Reformulation of material and questions

Table 12.41: Focused Coding Number 2 for Question 9

More attractive textbooks		
Participants	***Total***	***Suggestion***
2, 6, 8	3	More attractive textbooks

Table 12.42: Focused Coding Number 3 for Question 9

Examples from real life		
Participants	***Total***	***Suggestion***
5, 16	2	Examples from real life

Table 12.43: Focused Coding Number 4 for Question 9

Use modern technology		
Participants	***Total***	***Suggestion***
9, 11, 15, 17 19	5	Use modern technology

Three of the responses to question 9 indicate that the textbooks could be made more attractive using examples from real life that students can relate to and using more modern language which will appeal to the students more. In addition, the responses indicated that better use could be made of modern technology.

However, there is little evidence that any of these will make any difference. A number of researchers (e.g. Kirschner et al. 2006; Prasad, 2006; Alenezi, 2004, 2008; Reid, 2009 a,b; Ali and Reid, 2012) are of the view that limited working memory space has a critical role in mathematics frequently being considered difficult.

12.13 Encouraging Positive Attitudes

Attitudes in relation to studies are very important and this seeks to explore the views of interviewees on that topic.

Table 12.44: Results of Question 10

Question 10	**How can we encourage students to have a positive attitude towards mathematics?**
Focused Coding	
1	Prizes
2	Collaborative learning activities
3	Relate to real life
4	Relate to other subjects
Theme 10	Incentives

Table 12.45: Focused Coding Number 1 for Question 10

Prizes		
Participants	***Total***	***Suggestion***
1, 3, 9, 10, 11, 14, 17, 18,19	9	Prizes

Table 12.46: Focused Coding Number 2 for Question 10

Collaborative learning activities		
Participants	***Total***	***Suggestion***
2, 4, 6, 7, 12	5	Collaborative learning activities

Table 12.47: Focused Coding Number 3 for Question 10

Relate to real life		
Participants	***Total***	***Suggestion***
5, 8, 16	3	Examples from real life

Table 12.48: Focused Coding Number 4 for Question 10

Relate to other subjects		
Participants	***Total***	***Suggestion***
13, 15	2	Relate to other subjects

The responses indicate that the interviewees argue for some maths competitions and prizes and that students can be put in groups to discuss and work together. In addition, maths should be made less abstract and more relevant to the students' everyday lives.

Of course, these are their views. The evidence in relation to the development of positive attitudes towards school subjects very much relates to a curriculum which learners see as related to their lifestyle and context as well as to teachers who not only know their subjects and are enthusiastic but also show a genuine empathy towards learners. This evidence is typified by the study by Reid and Skryabina (2000).

12.14 Suggestions

In question 11, the teachers and inspectors were asked to give any suggestions they might have regarding the evaluation and development of the questions in the mathematics textbooks.

Table 12.49: Results of Question 11

Question 11	**Do you have any suggestions regarding the evaluation and development of the questions in the mathematics textbooks?**
Focused Coding	
1	No suggestion.
2	Reduce the contents of the textbook and increase the questions
3	Use problem solving and relate questions to students' real lives
4	Use more varied types of question (e.g. multiple choice)
5	Ministry of Education should consult teachers about question development
Theme 11	Several ways to improve the questions.

Table 12.50: Focused coding 1 for Question 11

No suggestion		
Participants	***Total***	***Suggestion***
1, 6, 14, 15, 18, 19	6	No Suggestion

Table 12.51: Focused coding 2 for Question 11

Reduce the contents of the textbook and increase the questions		
Participants	***Total***	***Suggestion***
2, 4, 10	3	Reduce the contents of the textbook and increase the questions

Table 12.52: Focused coding 3 for Question 11

Use problem solving and relate questions to students' real lives		
Participants	***Total***	***Suggestion***
5, 8, 16	3	Use problem solving and relate questions to students' real lives

Table 12.53: Focused coding 4 for Question 11

Use more varied types of question (e.g. multiple choice)		
Participants	***Total***	***Suggestion***
9, 12, 17	3	Use more varied types of question (e.g. multiple choice)

Table 12.54: Focused coding 5 for Question 11

Ministry of Education should consult teachers about question development		
Participants	***Total***	***Suggestion***
3, 7, 11, 13	4	Ministry of Education should consult teachers about question development

It can be seen from the above that while four teachers and two inspectors had no suggestions to make as to the evaluation and development of the questions in the mathematics textbooks, the remainder had various suggestions regarding this matter. Focused coding 2 reveals that three participants thought that the content in the textbook should be reduced and the number of questions increased. This suggests that the curriculum tries to cover too much in a limited period of time, without giving them sufficient time to reflect on and practise what they have learned.

Focused coding 3 echoes the suggestion made in focused coding 3 of question 10, that the questions should relate more closely to students' real lives. This is not as easy as it sounds and, again, the problems of limited working memory capacity can make this counterproductive, as Alenezi (2008) has shown.

From focused coding 4, it can be seen that two teachers and one inspector are of the opinion that it would be beneficial to use more varied types of question, such as multiple choice, in the textbook. This would perhaps make the questions less tedious and more attractive to the students, hence encouraging them to have a more positive attitude to mathematics. Good multiple choice questions are notoriously difficult to develop, and if poorly developed, are flawed, invalid and unreliable (Friel and Johnstone, 1978 a,b, 1979). However, well-developed multiple choice questions have proved very effective and for this reason have been adopted with increasing frequency in recent years in a number of international examinations.

From table 12.54 it can be seen that four teachers (although, perhaps significantly, no inspectors) took the view that the Ministry of Education should consult with teachers as to the development of textbook questions. This would appear reasonable, as the teachers may be assumed best placed to know what type of questions are most appropriate for the students. This is an interesting suggestion and is consistent with the findings of Ali (2008), where she showed that a mathematics curriculum designed by teachers was much more effective than that designed nationally by those outside the classroom. This finding also supported Al Saif (1996) who found a strong positive relationship between improved student performance and teachers' involvement in the the development of the curriculum. Similarly, Al Sadan (2000) suggested that there would be better performance in schools if teachers could work with a curriculum that they had helped to develop, as in this case they would feel more committed to their work.

12.15 Conclusion

In this chapter, the primary data which were obtained through interviews with nineteen participants, five inspectors and fourteen teachers, were analysed and the sixth research question (What are the views of the teachers and inspectors concerning the textbooks, the textbook questions and mathematical thinking?) addressed.

It was found that teachers and inspectors with longer experience either have no knowledge of mathematical thinking, or have a little knowledge which they gained through their own reading, rather than through official channels. On the other hand, those teachers who began teaching more recently had either been given a short course on mathematical thinking but had never applied it in the classroom or had had some training in thinking in general but not mathematical thinking in particular. However, it was clear that none of the teachers or inspectors has an in-depth knowledge of mathematical thinking. Perhaps because of this, the teachers and inspectors do not appear to be aware of the implications of teaching mathematical thinking, as only a few thought that it should be taught in all classrooms.

Another finding which emerged is that, due to the time that must be spent to complete as much of the curriculum as possible, teachers cannot teach maths in depth even if they wished to. The focus in Saudi classrooms is explicitly on passing exams rather than acquiring deeper knowledge. There is a lack of equipment in the classroom or teachers are unable to use what equipment there is. In view of this, it is perhaps not surprising to find that students generally have a negative attitude to mathematics. In this vein, it was found that the majority of teachers thought inspectors did not encourage the development of mathematical thinking skills, as they focused only on the lesson schedule.

It was revealed that short courses would be useful to increase teachers' ability in teaching mathematical thinking skills, as most teachers have no qualifications in mathematical thinking skills. However, it was also highlighted that teachers already have a heavy workload, do not have enough time in the day and are too tired at night to pursue additional courses, particularly as they are not paid to take them. It may be that some teachers feel there is no point to these courses, as the curriculum will not change.

There is also the problem about who will offer these courses if neither teachers nor school inspectors are confident in this area. It is of limited value bringing in mathematical '*experts*' who have no direct classroom experience.

Most of the teachers and inspectors had several suggestions to make concerning how to improve the textbooks and their questions. For example, they suggested that the textbooks could be made more attractive by using examples from real life that students can relate to and more modern language which will appeal to the students. In addition, several interviewees indicated that better use could be made of modern technology. It was found that three participants thought that the content in the textbook should be reduced and the number of questions increased. This suggests that the curriculum tries to teach too much to the students in a limited period of time, without giving them sufficient time to reflect on and practise what they have learned. In addition, three interviewees were of the opinion that it would be beneficial to use more varied types of question, such as multiple choice, in the textbook. They suggested that this could make the questions less tedious and more attractive to the students, and that this might encourage them to have a more positive attitude to mathematics. A number of the teachers, although none of the inspectors, felt that the Ministry of Education should consult with teachers as to the development of textbook questions.

Overall, some interesting insights have been revealed on the opinions of teachers and school inspectors. While there are useful suggestions to pursue further, much that they reveal is not supported in the research literature.

In the next chapter, the findings from the interviews are discussed further in linking them to the findings from the questionnaire.

Chapter 13

Discussion of Results

13.1 Discussion of Results

This chapter discusses the results of the study related to answering its main research questions. Each research question is now considered in the light of all the data obtained.

13.2 First Question

What mathematical thinking skills are emphasised in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia?

This question is further divided into sub-questions:

(a*)* *What are the mathematical thinking skills prevailing in the explanatory items of the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia?*

(b) *What are the mathematical thinking skills prevailing in the question items of the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia?*

On the basis of the literature review, nine mathematical thinking skills were proposed.

These can be arranged in descending order based on their representation in the total of the items in the textbooks that were analysed by the researcher with expert opinion (column 1 in table 13.1). When the explanatory items were analysed separately, the skills that were represented were reduced to eight, as the skill of application was not represented in any of these items (column 2, table 13.1). When the question items were analysed separately, the analysis indicated a slight change in the order (column 3, table 13.1).

Table 13.1 Skills of mathematical thinking

Total of the items in the textbooks	Explanatory items	Question items
Understanding and interpretation	Understanding and interpretation	Understanding and interpretation
Knowledge and recall	Knowledge and recall	Knowledge and recall
Modelling	Modelling	Modelling
Application	Mathematical proof	Application
Mathematical proof	Generalization	Mathematical proof
Evaluation	Deduction	Evaluation
Deduction	Evaluation	Deduction
Generalisation	Induction	Induction
Induction		Generalisation

The results of the analysis of the questions in the mathematics textbooks showed that the emphasis was almost entirely on knowledge and recall, understanding and interpretation. There were very few questions that related to modelling, application, induction, generalisation, deduction, mathematical proof, and evaluation. In addition, the questions tended to be simple and basic, with little opportunity to develop skills at a more advanced level.

There was no obvious attempt at the development of higher order or complex operations such as critical thinking, creative thinking, problem solving and decision making. From this, we can deduce that the lack of attention given in the textbooks to such higher patterns of thinking definitely contributes to focusing the efforts on the consolidation of the traditional educational curriculum that is based on memorisation and recalling information, along with a poor ability to process this information in order to use it creatively in finding solutions, decision making or problem solving. Hence, the textbooks rarely create opportunities for students to practise autonomous learning activities, carry out practical projects based on what they have learned, or discover knowledge by themselves.

From the above, we can deduce that the mathematics textbooks of the scientific section of secondary schools in the Kingdom of Saudi Arabia generally provide the students with ready-made information and knowledge, and then require them to memorise it, and finally check this memorisation by means of routine mechanical questions and problems. This lack of mathematical thinking in the analysed textbooks contradicts the international objectives of teaching mathematics in general (NCTM, 2000) and with the recommendations of the educational development plan in Saudi Arabia in particular. To change all this would entail a radical change in the concepts and philosophy of evaluation in Saudi Arabia (Ministry of Education, 1992).

13.3 Second Question

What is the extent of the emphasis placed on the development of mathematical thinking in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia?

This question is further divided into sub-questions, as follows.

(a) What is the extent of the attention given by the explanatory items in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia, to the development of mathematical thinking, based on the level of mathematical thinking (basic and complex) that they represent?

(b) What is the extent of the attention given by the question items in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia to the development of mathematical thinking, based on the level of mathematical thinking (basic and complex) that they represent?

(c) What is the extent of the attention given by the combined items in the mathematics textbooks for the scientific section at the secondary school stage in Saudi Arabia to the development of mathematical thinking, based on the level of mathematical thinking they represent?

The results of the analysis indicated that little attention was paid in the mathematics textbooks of the scientific section in secondary schools in the Kingdom of Saudi Arabia to the development of mathematical thinking skills at the complex level, and focus their attention on the students' acquisition of knowledge in the conceptual and procedural form represented by the two skills of knowledge and recall, and understanding and interpretation. This is despite the recommendations for mathematical curricula in Saudi Arabia clearly emphasising the development of mathematical thinking as one of the objectives of these curricula (Ministry of Education, 1970).

On the other hand, the results of the analysis indicated the popularity of the skill of understanding and interpretation. This can be attributed to the reliance of these textbooks on the memorisation and rote learning method in the presentation of the mathematical content. In this method, the targeted knowledge is presented and then illustrations, examples and questions about the concepts or principles are given when the knowledge presented is conceptual, or the methods for solving the problems are given when the presented knowledge is procedural.

This result can also be attributed to the different methods used to instil the skills of the understanding and interpretation skill of thinking, including formulation, coding, justification, giving examples, problem solving, comparison, explanation and summarisation. It was noted that the textbooks and their questions placed great emphasis on training the students to reformulate the mathematical knowledge or the presented information by alternating between symbolic and verbal forms. As an example, the coordinates of points are sometimes changed from the Descartes system to the polar system or vice versa, the quantities changed from the exponential form to the logarithmic one or vice versa, or a series written using the sum symbol or expanded or vice versa. It was also noted that the textbooks used symbols as a mathematical language in order to present the content, such as using the symbols of the limit, change in relationship, derivative, integration, sum, combination, and permutation. However, the textbooks in this regard can be criticised for not giving sufficient opportunity to the students to use the symbols by themselves to express their ideas verbally. Data, such as the questions in the textbooks that were used to develop this skill, focused on the transformation from a verbal statement to an equation or an association, as in the case of differentiation and integration and exponents and logarithms. Conversely, a shortcoming was noted in the presentation in the textbooks of cases that required the students to translate a symbolic statement into a real-life situation.

The textbooks, while presenting the mathematical content, tried to enrich the main ideas and the present the information with details and additions, but failed to develop these ideas in terms of encouraging the students, when they read them, to extract any important outcome from them. The textbooks also showed an obvious lack in the training of the students to summarise the given facts by divesting them of the secondary ideas that they contain and keeping the main ideas alone, as the textbooks seldom resorted to summarising, except in the cases of summarising algorithms by giving solutions through examples.

Furthermore, the textbooks rarely draw students' attention to the use of comparison as a method of organisation of information that would lead them to useful deductions. Comparison is used in rare cases in order to note the similarities or differences between two or more things, such as in the case of curves of conical sections, between trigonometric identities, or between the different methods for the solution of the same question in trigonometry. Moreover, the textbooks do not direct the student to use the skill of arrangement as a method of organisation of information, or as a method of aligning the items in a certain system or context based on a specific criterion. This skill was only required in one topic, the calculation of a Spearman correlation coefficient for a group of values in statistics.

In this respect, it is clear that the questions in the textbooks rely on routine drills, exercises and problems that aim to help the student acquire skills in carrying out algorithms, for which it is sufficient to use only basic thinking skills such as knowledge and recall or understanding and interpretation.

Some situations in the analysed textbooks can be processed by more than one method, and at times the students were reminded that they could solve a given problem using more than one method. However, cases such as these are very few, and are rendered less effective due to the dominance of the single solution method in most of the examples, exercises and problems.

In terms of using knowledge and recall as one of the skills of mathematical thinking, the textbooks focused on building a cognitive structure made up of mathematical concepts, symbols, principles and algorithms in the student's mind, by means of the direct method used in presenting the content. However, they confined themselves to providing ready-made knowledge. This can be clearly seen from the first glance at these textbooks, as they employ the information narrative method, by distributing information in various items, but offering few opportunities to the students to generate knowledge by themselves.

While presenting the mathematical content, the textbooks repeatedly direct the attention of the student to the use of conscious observation of the written text in order to obtain information. This is more obvious when examples are given, in the case of interpretation of mathematical models, definition of links between the constituents, or distinguishing between concepts and familiarising the students with their properties and components. The textbooks and their questions, in most of their items, require the students to recall the mathematical knowledge they stored in their memory and their previous experiences, and to use this previous knowledge in the acquisition and usage of new knowledge. This is very obvious when there is preparation for the presentation of new concepts and principles, or preparation for solving routine exercises and problems. Conversely, the textbooks do not draw the attention of the students to questioning the facts, although questioning the facts on a continuous basis is a skill necessary for the collection of information. No instances were found in the textbooks where the students would use this skill, except in a few items on the topics of integration and statistics.

In other instances, the textbooks often use mathematical models in order to present and use mathematical knowledge, and attempt, whenever possible, to present the given data in a manner which will facilitate understanding the relationships that link them together in order to reach deductions about them easily. Such presentation of data is made in different forms, including graphs, where association curves and conical sections are presented, or when presenting vectors in a plane or in space, and in number lines in real numbers, differentiation and integration. The textbooks also use tables in order to organise mathematical knowledge, as well as data on the topics of exponents, logarithms, probabilities, statistics, differentiation and integration, and limits and continuity. The textbooks focus on using geometrical models for the representation of the given data in the topics of trigonometry, vector geometry, solid geometry, and integration applications. There are few instances in the analysed textbooks in which illustrative figures are used to model the examples and problems on the topics of probabilities and statistics. The instances of modelling in which matrices are used are confined in the textbooks that were analysed to a single topic, namely, solving equations by means of matrices in one unit in the secondary school second grade textbook. Despite all of the above, the results of the analysis indicate that the representation of the modelling skill in all the items of the analysed textbooks combined did not exceed 11.4%, indicating the poor development in the textbooks of the modelling skill of thinking in which the student is given the opportunity to make models that represent real-life situations. This is contrary to the objectives of learning mathematics at the secondary school stage, as well as contrary to the directives of the principles and standards of school mathematics, that call for the school mathematics curricula to be used in an effort to enable students to build and use mathematical models in order to consolidate and organise mathematical knowledge, and communicate mathematical ideas to others, as well as interpret natural, social and mathematical phenomena (NCTM, 2000).

Furthermore, the analysed textbooks did not present sufficient situations to assist the students in choosing the mathematical models suitable for solving the problems. Several researchers have indicated that the use of mathematical models to present problems increases the student's ability to solve problems. Instead, the student's attention is focused on the application and translation of given models. This result is again contrary to the directives of the document of the principles and standards of school mathematics (NCTM, 2000). In short, the results of the analysis indicated that the analysed textbooks and their questions did not offer sufficient activities devoted to mathematical modelling which could develop the students' ability in mathematical thinking.

The results related to the application skill of thinking embody the traditional perspective in the evaluation of the student, as the percentage representing mathematical application in all the textbooks was only 21.8%. The result achieved by the secondary school third grade textbook in this regard was 9%, which was better than the result achieved by the secondary school first grade, which was only 7.1%, and that achieved by the secondary school second grade, with just 5.7%. It is clear that the questions written in the textbooks mainly revolve around measuring the extent to which the students acquire mathematical concepts, or the extent to which they possess the skills related to performing mathematical algorithms, and solving routine problems. Very few questions direct the students to use the mathematical knowledge previously learnt in new situations not encountered before by them. These application questions focus on some topics of differentiation and integration, probabilities, conical sections, and exponents and logarithms. The reason behind focusing on applications in these topics rather than in others may be because these topics are new to the students, as they had not been exposed to them in previous grades.

Every new situation contains a problem that requires study and thinking. If application is regarded as one of the thinking skills that consolidate the ability of the student to solve problems, then the general context of these textbooks and their questions would not help consolidate this ability. Not only does this contradict the objectives of learning mathematics in secondary schools, but it also contradicts the objectives of primary education (Al-Sadan, 2000), as well as the international orientation of focusing on the consolidation of the student's ability to solve problems as a main objective of school mathematics (NCTM, 1989; Ministry of Education, 1992) .

Regarding the mathematical proof skill of thinking, the presentation context of this skill in the analysed textbooks was used to prove the validity of trigonometric rules and identities, rules of exponents and logarithms, rules of probabilities, theories of progressions and series, properties of operations of real and complex numbers, properties of operations of vectors, rules of limits and continuity, and rules of differentiation and integration. However, there are no examples in the textbooks regarding the use of mathematical proof when other topics such as matrices, statistics and conical sections are presented, even when the student is asked to use mathematical proof to solve some problems on these topics.

Generally speaking, it was noted that the use of diverse skills belonging to the logic of mathematical proof is poor in the textbooks. This characteristic is augmented by the lack of topics in the textbooks that discuss mathematical logic, except for some in the secondary school first grade textbook. It appears as though the textbooks depend in most cases on providing the students with ready-made patterns made up of successive statements to prove the validity of the given result. The students are then required to use these in similar situations that appear within the exercises and problems, which affects the development of the students' skills related to the mathematical proof skill of thinking, and makes them lose confidence in using their abilities in new situations. Moreover, the reliance of the textbooks on describing the proof steps in order to arrive at the final solution, without paying attention to the justification of such steps, precludes these steps becoming fixed in the minds of the students (Abu Al-Huda, 1985). Some studies have pointed to the fact that the students' abilities of mathematical proof decline if care is not taken to build this proof on inferential judgment.

Moreover, it is striking to note that the textbooks seldom give the mathematical justification, as they confine themselves to using the justification in the context of the mathematical proof, in the interpretation of hypotheses and probabilities, when presenting new concepts and principles, or when justifying the procedures followed in solving the examples of solutions in topics such as probabilities, statistics, limits and continuity, trigonometry and vectors. The textbooks never ask the students to justify the methods that they use to solve the questions, except in a very few instances such as in the topic of probabilities. The cases where the student is required to elicit the deductions and mathematical phenomena, or to carry out such tasks, are very few. Among these are the topics of probabilities and statistics and limits and continuity.

Regarding the skill of deduction in thinking, the results of the analysis indicated an obvious weakness in the use of this in the presentation of the content of the analysed textbooks and their questions, as its representation did not exceed 1.9%. This percentage was mainly concentrated in the explanatory items, while only 1.1% of the total number of questions was intended to develop the students' ability to use deduction. Developing students' deductive capacities involves giving them some mathematical concepts and principles with which they are familiar and then encouraging them to derive mathematical information not previously known to them. It should be noted that the textbooks require a certain amount of deduction in some lessons, and attempt to encourage the students to arrive at a particular result based on a general or presumed principle. However, these attempts are not sufficient to develop the student's abilities in deduction, as they involve limited situations, such as the derivation of rules and relationships in the topics of trigonometry, probabilities and statistics, and conical sections, in the derivation of the operational properties in complex numbers and matrices, or in the derivation of rules and theories in the topics of differentiation and integration, and in the exponential and logarithmic functions. It is therefore clear that the textbooks do not use the deductive method as an effective means of presentation to develop the students' deductive ability. This leads to a general weakness in the development of mathematical thinking, as deduction is regarded as one of the most important of its skills (Lutffiya, 1998). It may be assumed that the method currently followed in the content of the textbooks and their questions is one of the factors involved in the decline of students' abilities to use the skill of deduction effectively.

Similarly to the skill of deduction, the results of the analysis indicated a deficiency in the textbooks in the development of the student's abilities in generalisation and induction, despite the fact that these are among the main skills of mathematical thinking. In the textbooks, only 1.8% and 0.07% of the combined analysed items were representative of generalisation and induction respectively. These results indicate that the skills of generalisation and induction were poorly represented, as the representation of the two skills together was as follows: 1.4% in the secondary school first grade textbook, 2.9% in the second grade textbook and 3.2% in the third grade textbook.

Apart from in a few questions, the textbooks gave little attention to the use of generalisation and induction to evaluate the students' development, nor did they focus on the development of the inductive thinking that incorporates these two skills, as the inductive method is not used as a general framework for the presentation of the mathematical content in these textbooks. The use of this method in the textbooks is found in the topics of exponential and logarithmic functions, vector geometry, and limits and continuity, while its use in other topics is either limited or absent. In this respect, the textbooks and their questions, apart from in the unit of mathematical logic in the first grade textbook, can also be criticised for being devoid of any mathematical content oriented towards the teaching of the concepts of logic, which have been shown by some studies to have a positive effect on students' abilities to formulate the generalisations that they discover during their learning.

Regarding the total of results related to the three skills of inferential thinking, namely, deduction, generalisation and induction, it can be deduced that the mathematics textbooks, in the scientific section of secondary school in Saudi Arabia, are weak in the use of inference as a method of presenting mathematical content. Despite the fact that there are several examples in the textbooks of the use of modelling the presented mathematical knowledge, the use of mathematical proof to indicate the validity of certain theories, rules and relationships, the use of inference in presenting the mathematical content, and the use of evaluation in certain cases, and the examples and the questions that represent the complex level of thinking are quantitatively and qualitatively limited, compared to the obvious focus on students' memorising mathematical knowledge and training them to carry out the algorithms skillfully.

Furthermore, the analysis results revealed an obvious shortcoming in developing the student's abilities to make judgments regarding the value of the given information with respect to a certain objective. The analysis results showed that no more than 2.8% of the total number of items analysed consisted of a certain type of evaluation, and that there was a somewhat higher percentage of them in the question items than in the explanatory items. Evaluation was used particularly in the analysed textbooks to develop the ability of the student to verify the validity of a certain claim in topics such as probabilities, vector geometry, solid geometry, statistics, matrices, exponential and logarithmic functions, differentiation, and complex numbers. Evaluation was also used in the analysed textbooks to develop the ability of the student to identify errors by verifying the validity of the solutions in the topics of progressions and series and trigonometry. However, it is noted that the analysed textbooks did not train the students to establish the necessary criteria to enable them to make judgments about the ideas and data, but provided them only with ready-made criteria, and asked them to use these to fulfil the required objectives. The textbooks and their questions did not contain enough instances to show the development of the student's ability to detect the fallacies or weaknesses in the logical inferences, or weaknesses in the information related to the situation or topic, or to distinguish between opinions and facts. It is almost certain that such a method cannot support the students' ability to evaluate, and will not enhance their competency in decision making, of which evaluation is one of the main components. Such a method reduces the students' ability to deal with tasks that require the use of critical thinking, of which evaluation is also one of the main components. This again indicates the lack of attention paid by the analysed textbooks and their questions to developing higher mathematical thinking.

To sum up, the results of the analysis clearly indicate that the mathematics textbooks of the scientific section of the secondary school stage are weak in the development of mathematical thinking, apart from the two skills of knowledge and recall, and understanding and interpretation. In view of this, the current textbooks and their questions cannot be said to develop mathematical thinking sufficiently and in a manner which conforms to the objectives of the educational development plan and the school mathematics curriculum (Ministry of Education, 1984). These and the official directions concerning the principles and standards of school mathematics (NCTM, 2000) emphasise the development of mathematical thinking as the main objective for learning and teaching mathematics.

Furthermore, the textbooks and their questions analysed in this study do not offer a gradual presentation of these skills of thinking. The reason for this may be the fact that these textbooks focus only on the presentation of mathematical knowledge without giving similar attention to the development of the skills of mathematical thinking. To sum up, it can be said that the mathematics textbooks and their questions in Saudi Arabian secondary schools are in need of revision which would orient them towards the presentation of all types of mathematical knowledge, and to make an effort to develop the skills of mathematical thinking.

In this regard too, the mechanism by which the school curricula are planned and implemented in general should also be reconsidered, together with the process by which school textbooks are prepared and written. Al-Sheikh (2001) pointed to a dichotomy between the processes of the design of the school curricula and that of the preparation of the school textbooks, as well as a dichotomy between these processes of preparation and their implementation, which may well have had an impact on the failure of the school textbooks to embody the directives of educational development. Hence, the various processes by which the school curricula and textbooks and their questions are developed should be reconsidered in terms of an integrated process that requires concerted efforts, as well as interaction and communication between the multiple processes and inputs, both human and material.

13.4 Third Question

What are the mathematical thinking skills prevailing in the questions in the mathematics textbooks for the scientific section in secondary schools in Saudi Arabia in the opinion of the teachers and inspectors of mathematics?

This question is further expanded:

What are the mathematical thinking skills prevailing in the questions in the secondary stage mathematics textbooks in Saudi Arabia in the opinion of the teachers and inspectors of mathematics according to:

different grades (1st year – aged 16, 2nd year – aged 17, 3rd year – aged 18);
post (teacher and inspector);
qualifications (Bachelor's, diploma, Master's, PhD);
length of experience, and;
whether they had training or not?

Similarly to the previous two questions, the skills of knowledge and recall, and understanding and interpretation were found to be represented in the textbook questions to a greater extent than the other skills. For modelling, application, induction, generalisation, deduction, and evaluation, the teachers and inspectors tended to disagree that they were present, while they were more neutral about mathematical proof. In general, these results are similar to those in the second question. For the second question, content analysis was used and for this question the questionnaire was used; the results of both bear each other out.

13.5 Fourth Question

To what extent are the criteria of good formulation and good layout of the questions in the secondary stage mathematics textbooks in Saudi Arabia fulfilled in the opinion of the teachers and inspectors of mathematics?

This question is further expanded:

Are there any significant differences in the criteria of good formulation and good layout of the questions in the secondary stage mathematics textbooks in Saudi Arabia according to:

different grades (1st year – aged 16, 2nd year – aged 17, 3rd year – aged 18);
post (teacher and inspector);

qualifications (Bachelor's, diploma, Master's, PhD);
length of experience, and;
whether they had training or not?

In thinking of good formulation and good layout of the questions, there were almost no differences which related to grades, posts, qualifications, length of experience or extent of training. Thus, in the opinion of the respondents, regardless of variable, the criteria for good formulation and good layout of the textbook questions were fulfilled. This indicates that the writers of the textbook questions have assisted students to read the questions in the textbooks in all three grades by providing textbook questions that are clearly formulated and laid out. They may have taken great care with the formulation and layout of questions as they were aware that these textbooks would be used by students throughout Saudi Arabia.

13.6 Fifth Question

Do the questions in the mathematics textbooks in Saudi Arabia develop students' positive attitudes towards mathematics in the opinion of the teachers and inspectors of mathematics?

This question was further expanded:

Do the opinions of the teachers and inspectors of mathematics on the ability of the questions in the mathematics textbooks in Saudi Arabia to develop students' positive attitudes towards mathematics vary according to:

different grades (1st year – aged 16, 2nd year – aged 17, 3rd year – aged 18);
post (teacher and inspector);
qualifications (Bachelor's, diploma, Master's, PhD);
length of experience, and;
whether they had training or not?

It can be concluded that, according to the opinions of the teachers and inspectors of mathematics, regardless of the variables, the questions in the mathematics textbooks do not develop students' positive attitudes towards mathematics. It can be seen that the questions in the textbook are deficient in terms of including emotional aspects (tendencies, attitudes, values, etc)., developing positive attitudes towards mathematics and emphasising the role played by mathematics in other subjects.

In some cases, the textbooks used examples borrowed from real-life situations in order to present the mathematical content, particularly in the topics of statistics, probabilities and real numbers. However, this use was confined to the explanation of the given concepts and principles, and was not developed to the level of relating mathematics to real life in order to improve the students' attitude towards mathematics and make it more meaningful. In fact, these textbooks focus their attention on the presentation of mathematical knowledge without being concerned about presenting it in a real-life context, and without presenting sufficient applications to link real life to work. The scarcity of real-life problems in the analysed textbooks indicates that the textbooks do not focus on complex thinking skills.

However, bringing in real-life problems raises a very fundamental issue. It immediately places the limited resources of working memory under greater stress. This has been shown in much research and is best seen in the mathematics tetrahedral model (See Figure 6.1, page 121) developed by Alenezi (2008) and Ali and Reid (2012). The key to success may well lie in following up the suggestion of Alenezi (2008) that procedures and symbolism need to be automated before understanding and applications can be introduced.

13.7 Sixth Question

What are the views of the teachers and inspectors concerning the textbooks, the textbook questions and mathematical thinking?

It was found that teachers and inspectors with longer experience either do not have any knowledge of mathematical thinking, or have a little knowledge which they gained through their own reading. However, those teachers who began teaching more recently had either been given a short course on mathematical thinking but had never applied it in the classroom or had had some training in thinking in general but not mathematical thinking in particular. None the less, it was clear that none of the teachers or inspectors had a deep knowledge of mathematical thinking. The participants did not appear to be aware of the implications of teaching mathematical thinking, and only a few thought that it should be taught in all classrooms.

However, it appears that teachers could not teach maths in-depth even if they wished to. The focus in Saudi classrooms is on passing exams rather than acquiring knowledge. The majority of teachers thought inspectors did not encourage the development of mathematical thinking skills, as they focused only on the lesson schedule.

While many thought that training would help, it is likely that this will only be effective if the teachers are given enough freedom in the curriculum and the way they teach to enable them to apply what they have learned. Indeed, if assessment continues to reward recall, then it is difficult to move beyond an emphasis on recall.

Suggestions that textbooks could be made more attractive and use examples from real life that students can relate to and more modern language are not supported by any evidence although they are not necessarily bad ideas.

Three participants thought that the content in the textbook should be reduced and the number of questions increased. In addition, three interviewees were of the opinion that it would be beneficial to use more varied types of questions in the textbook. They suggested that could make the questions less tedious and more attractive to the students, hence encouraging them to have a more positive attitude to mathematics. A number of the teachers, although none of the inspectors, felt that the Ministry of Education should consult with teachers as to the development of textbook questions. Some of these ideas may prove helpful.

13.8 Conclusion

This chapter has discussed the six main research questions in the light of the data analysis and content analysis. Moreover, there was no apparent attempt to develop complex operations such as creative or critical thinking, decision making or problem solving. Hence, it can be inferred that the lack of attention paid in the textbooks to such higher levels of thinking contributes to focusing on the consolidation of the conventional educational curriculum which prioritises the memorisation and recall of information, while not promoting the ability to process this information in order to use it creatively in problem solving, finding solutions, or decision making.

For the second research question, the results of the analysis indicated that little attention was paid in the selected mathematics textbooks to the development of mathematical thinking skills at the complex level, but instead they focus their attention on the students' acquisition of knowledge in the conceptual and procedural form represented by knowledge and recall, and understanding and interpretation. The textbooks present the mathematical content and attempt to enrich the main ideas and present the information with details and additions, but fail to develop these ideas in terms of encouraging the students to derive any significant outcome from them. In this respect, it is clear that the questions in the textbooks depend on routine drills, exercises and problems that aim to assist the student to acquire skills in carrying out algorithms, for which only basic thinking skills such as knowledge and recall or understanding and interpretation are required.

For the third research question, the answers were similar to those of the second question. In the views of the teachers and inspectors, the skills of knowledge and recall, and understanding and interpretation were found to be more frequently represented in the textbook questions than the other skills. Regarding modelling, application, induction, generalisation, deduction, and evaluation, there was disagreement among the teachers and inspectors about their presence, although they were more neutral about mathematical proof.

For the fourth research question, regarding good formulation and good layout of the questions, there were virtually no differences relating to grades, posts, qualifications, length of experience or extent of training. Thus, in the opinion of the respondents, regardless of variable, the criteria for good formulation and good layout of the textbook questions were satisfied.

For the fifth research question, in the opinions of the teachers and inspectors of mathematics, regardless of the variables, the questions in the mathematics textbooks do not develop students' positive attitudes towards mathematics. Their opinions are that the questions in the textbook are inadequate in terms of including emotional aspects (attitudes, values, tendencies, and so on), developing positive attitudes towards mathematics and highlighting the part played by mathematics in other subjects.

Regarding the sixth research question, it was found that teachers and inspectors with longer experience either have no knowledge of mathematical thinking, or have a little knowledge which they have gained through their own reading. However, those teachers who began teaching more recently had either been given a short course on mathematical thinking but had never applied it in the classroom or had had some training in thinking in general but not mathematical thinking in particular. However, it was apparent that none of the teachers or inspectors had an in-depth knowledge of mathematical thinking.

Chapter 14

Conclusion and Recommendations

14.1 The problem in its context

In Saudi Arabia, there have been sustained attempts to re-think school education to move it away from its traditional emphasis on memorisation and recall towards the achievement of wider goals and, specifically, thinking skills (Ministry of Education, 1992). In this context, mathematics education at school level must play its part.

Much learning in mathematics centres on the use of textbooks, not only to give worked examples of procedures but also to give the learner exercises to practise the skills. The danger is that the emphasis still remains on the correct application of memorised procedures. O'Keeffe and O'Donoghue's (2011) findings suggest that, while a number of factors may influence students' learning, if effective changes were applied to mathematics textbooks, positive changes in students' conceptual understanding of mathematics would be brought about. This project focused on the nature of what the textbooks and their questions were seeking to develop in learners. This was explored from the perspective of a team of '*experts*' as well as teachers and school inspectors.

14.2 The Measurements Made

In order to answer the questions of the study, the researcher prepared three tools to evaluate the questions of the three mathematics textbooks and the extent to which

they measure the various skills of mathematical thinking represented therein, conform to criteria of good formulation and good layout, and reinforce a positive attitude towards mathematics on the students' part (see Appendices 1, 2 and 3). The researcher relied on the following instruments to fulfil the objectives of the study:
A questionnaire (see Appendix 2) was used in the analysis of the questions in the textbooks. The underpinning of the questionnaire was the theoretical background that the researcher took from theoretical references and previous studies. This was presented to the questionnaire respondents in order to discover their views and their evaluative ratings of these questions.

An analysis model (Appendix 1) was developed to analyse the subject matter of the items in the textbooks being studied.

Structured interviews (see Appendix 3) were conducted with 14 teachers and 5 inspectors in order to discover their views on the questions in the textbooks and the extent to which they promote mathematical thinking skills.
While building these tools, the researcher utilised educational literature including books, journals and studies, many of which are reviewed in the literature review, by means of which he evaluated the questions and the various skills of mathematical thinking. He also utilised the mathematics curricula and their broad guidelines in the secondary school stage, including the general and specific objectives of the curricula that are related to the development of mathematical thinking in the students' mind.

The data from the questionnaires were analysed using a quantitative approach, while the data from the interviews were analysed using a qualitative approach. The textbooks were analysed using content analysis. The use of the mixed method was intended to enhance the reliability and validity of the study.

14.3 The Key Findings

The key findings of the study are the following. It was found that the textbook questions focused almost exclusively on the skills of knowledge and recall, and understanding and interpretation. In addition, there were no apparent attempts at the development of higher order or complex skills such as critical thinking, creative thinking, and so forth in the textbooks or their questions. Rather, the focus was on a basic level of thinking. Moreover, it was found that in the textbooks, emphasis was placed on memorisation and rote learning, checked by routine problems and questions. Further, it was found that the textbooks and their questions offer the students few opportunities to generate knowledge by themselves and do not encourage the students to question the facts given in the textbooks.

The teachers and inspectors who participated in the research were in general agreement that the textbooks and their questions do not develop mathematical thinking adequately or in a manner that complies with the objectives of the Saudi educational development plan. However, regarding the layout and formulation of the textbook questions, both the teachers and inspectors, regardless of variable, found them to be generally satisfactory. Regarding attitude towards mathematics, there was agreement among the teachers and inspectors that the textbooks and their questions do not encourage students to have a positive attitude towards mathematics, nor do they highlight the role played by mathematics in other subjects. Moreover, in the opinion of the teachers and inspectors, the textbooks and their questions make little connection between mathematics and situations which the students are likely to meet in their daily lives.

The majority of teachers felt that inspectors did not encourage mathematical thinking skills, but focused only on the lesson schedule. It was also found that none of the teachers and inspectors had a deep knowledge of mathematical thinking. This being the case, their responses to the interviews could be considered as of little value, hence calling the study findings into question. However, it should be borne in mind that the interview sample was small (19) compared to the number of questionnaire respondents (1466) and therefore may not be entirely representative of the population. Moreover, this potential shortcoming was avoided by the triangulation of the data through the use of content analysis and questionnaires in addition to the interviews. That all three methods yielded similar results increases confidence in the findings.

14.4 Strengths and weaknesses of the study

The limitation of the study relates to the fact that the researcher explored people's opinions. As these rely on human judgement and are therefore essentially subjective, they may or may not reflect reality. To date, no viable, objective method of measuring mathematical thinking has been developed and it is unlikely that it will be in view of the nature of thinking *per se*.

However, this study's strengths lie in the fact that it has multiple sources of evidence that broadly agree. In addition, there were very large samples for the questionnaire and the samples were also geographically distributed across the whole country. In addition, the questonnaire was constructed and applied in such a way that there is every possibility that the respondents answered in the way intended. All these factors are important in achieving good reliability in the test-retest sense (Reid, 2006). Of greater importance is questionnaire validity and this is supported by evidence from the interviews.

14.5 Recommendations for Future Research

The results of this study reveal that there are inadequacies in the questions in the mathematics textbooks in Saudi Arabia in terms of the mathematical skills they incorporate. These are likely to reflect inadequacies in the textbook overall and, indeed, in the curriculum. However, this research has taken only the first steps in identifying them and in making some suggestions as to how they may be rectified. There is a need for more in-depth research. In particular, there is a need to focus on the learner.

The fundamental problem is being able develop an agreed operational description of mathematical thinking and then developing ways to measure its achievement or otherwise. By operational description, it is meant that the description allows measurement to be made. A way of looking at this would be to try to develop a mathematics test where success in answering the questions is more likely for those students who can think mathematically. Thus, it would be useful to develop some type of test material for upper school levels where success is enhanced if the students can think mathematically or, perhaps, in some specific area of mathematical thinking. This has already been done for scientific thinking (Al-Ahmadi and Reid, 2011, 2012) and critical thinking (Alosaimi, 2013). However, achieving this for mathematical thinking may prove even more demanding.

It is therefore recommended that future research focus on students entering university to study mathematics and to attempt to determine in what way they think differently from other students of the same age who are studying other subjects.

14.6 Implications and Recommendations

It is clear from the results of this study that there is a need to address the questions in the textbook. In the light of this, the following recommendations can be made.

It is recommended that applied (real-life) mathematical problems be presented in the textbooks, in particular, problems in geometry, as the results of the study have shown that there was a failure in the school mathematics textbooks to give sufficient attention to applied (real-life) mathematical problems in this field. Such mathematical problems should also be linked to the other topics that exist in the curriculum (NCTM, 2000; Brahier and Olson, 1999; Stubbs, 1996; Turner and Rossman, 1997; Carreira, 2001; Cerrito, 1996; Pacyga, 1994). It is also recommended that the students be given some mathematical puzzles in the textbooks, the solutions to which require the use of mathematical knowledge that is the subject matter of the lesson.

In addition, the recommendation is made for the students to be given some practical activities that break the daily routine of the classroom, such as visits to particular places of interest in order to perform some measurements of dimensions, distances and volumes, collect some statistical information or make different models and find their volumes and surface areas by means of experimentation and measurements and other such activities (NCTM, 2000; Brahier and Olson, 1999; Stubbs, 1996; Turner and Rossman, 1997; Carreira, 2001.)

It is recommended that there be a discussion with examination boards to see if assessment can be broadened. In addition, working groups of '*able*' mathematics teachers could be delegated to develop teaching materials and questions which will give greater scope for mathematical thinking. Academics and practitioners alike could develop a way to give students experience of mathematical thinking. Teacher training programmes could be examined with a view to integrating the development of maths skills into them.

Perhaps the place to start is to look at national assessment. If this continues only to measure the accurate recall of mathematical procedures, then little progress can be made. Teachers will only teach towards the skills which enable their students to gain the greatest credit. There are examples of assessment in mathematics which move well beyond this type of recall approach (Scottish Qualifications Authority, undated).

Once the national assessment started to move towards assessing wider skills, there may be opportunities to re-consider the mathematics content to be taught. It would be useful to select the content in such a way that the development of mathematical thinking was encouraged. The curriculum will naturally follow the style of the assessments, and textbooks and their questions will in turn follow the curriculum.

14.7 Practical guidance

The recommendations in this section offer guidelines to the processes of designing new and improved textbooks in Saudi Arabia, both from a process point of view (e.g., arranging for the involvement of teachers) and from a pedagogical one.

In Saudi Arabia, there are no panels of qualified and experienced authors of mathematical textbooks. The result is that traditionally prepared mathematical textbooks fail to attract the interest of the students. It is therefore recommended that the Ministry of Education choose the most suitable authors for the textbook, with the relevant qualifications and experience, even if they are not in the Ministry of Education. In addition, these authors should go into the field and consult with teachers, inspectors and other stakeholders concerning the preparation of the textbooks.

It is also imperative that, to assist the authors of textbooks, a set of guidelines be prepared. This was found to be an issue in the USA, where the National Council for the Teaching of Mathematics (NCTM) issued Curriculum and Evaluation Standards for School Mathematics (the Standards) in 1989, followed by the Principles and Standards for School Mathematics (Principles and Standards) in 2000 (Schoenfeld, 2002). However, while the Standards provided a framework for curriculum development, they omitted to offer specific guidelines for designing new materials, such as textbooks (Jitendra et al., 2005).

The model below, developed by the researcher, is put forward as offering practical guidance for the process of designing new and improved textbooks and textbook questions in Saudi Arabia.

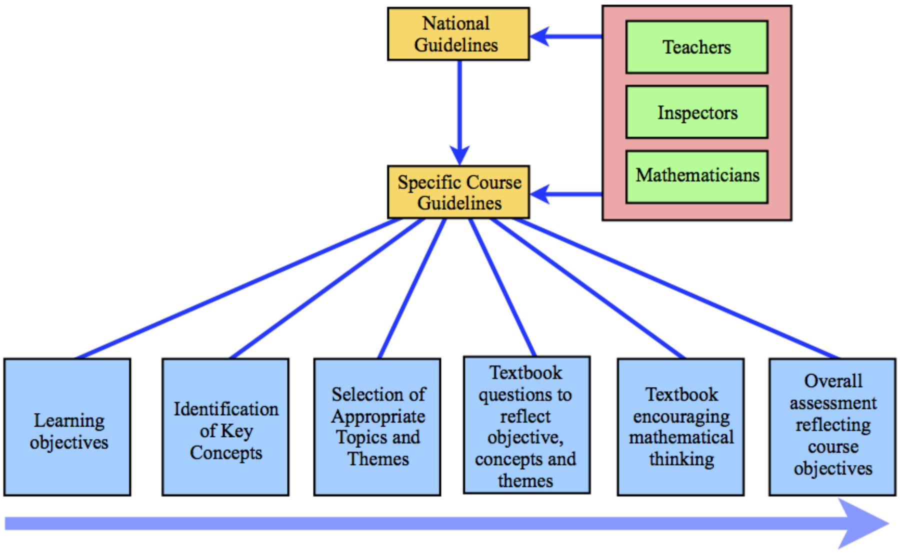


Figure 14.1: The process of designing new and improved textbooks in KSA. (Source: Author)

The model very clearly identifies the main areas for the improvement of textbooks in Saudi Arabia. Textbooks should be developed in the light of the national curriculum with the input of expert mathematic teachers, as they will play a main role in the implementation of this textbook, using the appropriate methods and procedures presented in the model. This is what has been done in Scotland, with the development of the Curriculum for Excellence (Education Scotland, n.d.). The development process involved close engagement with teachers and other practitioners. It has built upon the existing good practice across all sectors of Scottish education and takes account of research and international comparisons.

Textbook objectives should be related to mathematical thinking, while the subject matter of the contents should be conceptual and understanding-based. The questions and assessment procedure of textbooks should therefore be conceptual and should be relevant to national and local interest of the national agenda and policy.

Indeed, among the key issues in models of curriculum development and implementation is that particular attention should be paid to the curriculum being related to the needs of the learners in their own environment, as far as this is possible.

It is clear that a balance of national and local needs and interests should be reflected in the curriculum in order to achieve an acceptable and effective implementation. Furthermore, there are challenges and opportunities to be faced in the process of curriculum localisation which lead to consideration of the factors that limit and also enable the localisation processes.

These guidelines may also be of some assistance to teachers and inspectors in the assessment of the educational value of various teaching materials. For the effective writing of textbooks and their questions, it is necessary to have an insight into the teaching/learning situation, the specific learning objectives at a particular level of development.

The area of assessment is critical. If national assessment does not reflect the agreed goals for the mathematics curriculum, then schools will seek to maximise learner performance as required by the national assessment. This may over-emphasise skills of recall of information and procedures, rather than skills of understanding and thinking

The process of textbook design should ensure that teachers are sufficiently well-trained in the new materials and their requirements. This will require a manual/guide with specific information about how to initiate, deliver, and end each lesson in the textbook. Indeed, it may be necessary for teachers to follow a training course to instruct them in the correct approach to the new textbooks.

14.8 Provision of improved textbooks within an overall vision for the future

The improvement of textbook and curriculum development has considerable implications for the future development of mathematics education in Saudi Arabia. This study has shown that the current textbooks and their questions, in terms of subject, content, and knowledge focus almost completely on memorisation and rote learning rather than on higher order or complex mathematical thinking. Moreover, there is no apparent attempt to develop higher order or complex skills such as critical thinking or creative thinking in the textbooks or their questions.

It was found that the content and knowledge contained in the secondary school mathematics textbooks and their questions in Saudi Arabia did not assist students to develop problem solving skills or the habit of logical reasoning. Therefore, it is recommended that the content of these mathematics textbooks and the formulation of the questions in them be revised and updated accordingly in order to conform to the criteria of appropriate formulation and layout of questions in the secondary stage mathematics textbooks. Moreover, the subject matter may be enriched in order to arouse students' interest in and positive attitudes towards mathematics. This means that understanding must be an overt goal and that the learners can see the mathematics as being meaningful to them in their attempt to make sense of *their* world.

Of course, seeking to develop their skills of critical thinking, logical reasoning, problem solving, and creativity will be useful targets as well although these will not be easy to achieve. Further, the questions in the mathematics textbooks of the secondary stage should be revised to develop students' mathematical thinking skills.

To achieve excellence both in the textbook and curriculum, there should be a focus on the type of skills that students will use throughout their lives and the evaluation of mathematical knowledge and skills is therefore likely to become ever more important.

Mathematical thinking is generally considered to be a skill developed by appropriate training and cognitive growth as well as by experience. This type of thinking does not spring from nowhere or by chance. The student has to be involved in learning situations and activities that enhance his/her thinking in a number of ways.

It should be borne in mind that there are a number of thinking skills, such as critical thinking, scientific thinking, creative thinking and mathematical thinking. However, it may be difficult to apply the descriptions as given in Figure 14.3.

In addition, curriculum developers generally attempt to formulate curricula in terms of the development of thinking skills. The Curriculum for Excellence (Education Scotland, n.d.) is one example of this and it is apparent that the development of thinking skills appears to be an important feature in this curriculum. However, it is vital to define precisely what is understood by such skills and to outline how they could be assessed.

Figure 14.3 below shows the four thinking skills that are a part of the model.

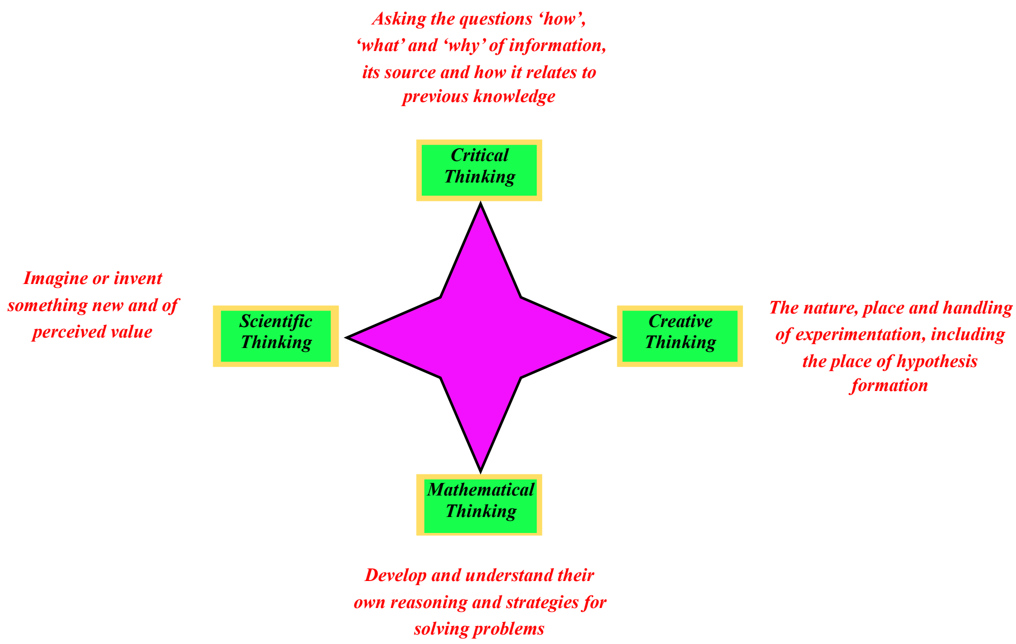


Figure 14.2: Thinking skills in textbooks and their questions (Source: Adapted by the author from Al-Ahmadi and Reid, 2011.)

Further, mathematical thinking can be developed through the development of the mathematics textbook and its educational tools, as well as by following certain teaching and evaluation methods.

14.9 Endpiece

This study has aimed to look briefly at the textbook questions in mathematics at three school levels in the scientific section of secondary schools in Saudi Arabia and explored the extent to which mathematical skills are being developed. It has revealed some major inadequacies and it is hoped that this will stimulate developments and further enquiry so that the next generation in Saudi Arabia can become more mathematically skilled.

References

Abid, E. (2004). *The Effect of Two Mathematics Teaching Strategies on the Investigation of the Achievement and Mathematical Thinking in the Case of the Ninth Basic Grade Students in Jordan*. Unpublished Doctoral Dissertation, Amman Arab University for Postgraduate Studies, Amman, Jordan.

Abdullatif, M. and Al-Wakeel, A. (2006). The effectiveness of the proposed program in mathematics based on educational activities in the development of communication skills and mathematical thinking among primary school pupils, *Journal of Maths Education,* Volume IX, November, pp. 131-169.

Abu Al-Huda, R., (1985). *Mathematical Thinking and its Relations with Attitudes towards Mathematics and Achievement in Mathematics in the Case of Secondary School Students in Jordan.* Unpublished Master's Dissertation, Jordanian University, Amman, Jordan

Abu Al-Jadyan, M. (1999). *Abilities of Inferential Thinking in the Case of High and Medium Achievement Secondary School Students* Unpublished Master's Dissertation, Islamic University, Gaza, Palestine.

Abu Ali, S. (1989). *Evaluation of the Mathematics textbooks in the Secondary School Stage in Jordan*. Unpublished Master's Dissertation, Al-Yarmook University, Irbid, Jordan.

Abu Mousa, M. (1997). *Analysis of the Developed MathematicsTextbooks of the Fifth to the Eighth Basic Grades in Jordan, in the Light of the International Standards of Mathematics Curricula.* Unpublished Master's Dissertation, Al-Yarmook University, Irbid, Jordan.

Abu Zeenah, F. (1986). Development of Students' Mathematical Thinking Abilities in the Secondary School and After. *Arab Journal for Human Sciences*, Vol. 21, pp. 147 – 165.

Abu Zeenah, F. (1994). *School Mathematics Curricula and their Teaching.* First Edition. Kuwait: Al-Falah Bookshop for Publishing and Distribution.

Abu Zeenah, F. (2003). *School Mathematics Curricula and their Teaching*. Second Edition. Kuwait: Al-Falah Bookshop for Publishing and Distribution.

Ajzen, I. (1985). From intentions to actions: A theory of planned behavior. In J. Kuhl & J. Beckmann (Eds.), *Action-control: From cognition to behavior* (pp. 1 l-39). Heidelberg: Springer

Ajzen, I. and Fishbein, M. (1980). *Understanding Attitudes and Predicting Social Behavior*. New Jersey: Pearson

Ajzen, I. and Madden, T. J. (1986). Prediction of goal-directed behavior: Attitudes, intentions, and perceived behavioral control. *Journal of Experimental Social Psychology*, Vol. 22, pp. 453-474.

AlAbsi, M. (2009). *Games and Thinking in Mathematics*. Jordan: Dar Al Masireh

Al-Ahmadi, F. M. (2008). *The Development of Scientific Thinking with Senior School Physics Students*. PhD Thesis, University of Glasgow.

Al-Ahmadi, F. and Reid, N. (2011) Scientific Thinking - What is it and can it be measured? *Journal of Science Education*, 11(2), 53-59.

Al-Ahmadi, F. and Reid, N. (2012) Scientific Thinking – Can it be Taught? *Journal of Science Education*, 13(1), 18-23.

Al-Ajroush, H. (1980). *Historical development of the public secondary school curriculum in Saudi Arabia from1930 to the present.* Unpublished PhD thesis from Oklahoma University, Oklahoma, USA

Al-Alem, M. (1994). *Evaluation of the Mathematics Textbook Prescribed for the Basic Sixth Grade in Jordan as Indicated by the Students' Achievement Level, Fulfillment of the Objectives of the Curriculum and the Opinion of the Teachers and Students about the Textbook.* Unpublished Master's Dissertation, Jordanian University, Amman, Jordan.

Al-Bakr, R. (2009). *Developing Thinking through the Textbook*. Saudi Arabia: Alroshd

Al-Dowaikat, A. (1996). *Evaluative Study of the Mathematics Textbook Prescribed for the Basic Ninth Grade Students in Jordan.* Unpublished Master Dissertation, Al-Yarmook University, Irbid, Jordan.

Alenezi, D.F. (2004). *Difficulties Associated with Teaching and Learning Mathematics: A Study of Psychological Factors Affecting Pupils' Performance.* MSc Thesis, Glasgow: University of Glasgow.

Alenezi, D.F. (2008). *A Study of Learning Mathematics Related to some Cognitive Factors and to Attitudes*. PhD Thesis, Glasgow: University of Glasgow. [http://theses.gla.ac.uk/333/]

Al-Esawi, S. (2001). *Effect of a Training Program for the Use of Meta-Cognitive Thinking Skills on Achievement in Mathematics in the Case of Sixth Grade Students*. Unpublished Master's Dissertation, Jordanian University, Amman, Jordan.

Alhmali, R.J. (2007*). Student Attitudes in the Context of the Curriculum in Libyan Education in Middle and High Schools*. PhD thesis. University of Glasgow, Glasgow.

Al-Jamal, N. (1983). *Towards a Modern Educational Curriculum*. Second Edition. Amman, Jordan: People's House Presses.

Al-Jassim, F. A. (1994). *The impact of a training program in the strategy to solve problems creatively on the development of creative thinking among a sample of high-achieving students.* Unpublished Master's dissertation submitted to Arabian Gulf University, Bahrain.

Al-Khateeb, K. (2004). *Investigation of the Effectiveness of a Training Program for the Mathematics Teachers in the Development of Students' Abilities in Mathematical Thinking and Achievement in Mathematics in the Higher Basic Stages*. Unpublished Master's Dissertation, Amman Arab University for Postgraduate Studies, Amman, Jordan.

Al-Khuzam, A. (1998). *Effect of Discovering, Discussion and Lectures in the Development of Critical Thinking in the Case of Basic Tenth Grade Students.* Unpublished Master's Dissertation, Al Al-Bait University, Al-Mifraq, Jordan.

Al-Lahawiyah, M. (1999). *Evaluation of the Second Grade Scientific Section Mathematics Textbook from the Point of View of the Teachers in South Jordan*. Unpublished Master's Dissertation, Al-Yarmook University, Irbid, Jordan.

Allam, S. A. M. (1991). *Content analysis and evaluation in schools*, Tunisia: Arab Organization for Education, Culture and Science.

Al-Masad, M. (2002). *Guidance Evidence for the Mathematics Teachers in order to Deal with the Students' Learning Mistakes in the Light of their Results of the Third International Mathematics and Science Study* (TIMSS-R). Amman: National Center of Human Resources Development.

Al-Naeme, F. F. A. (1988). *A study of some psychological factors affecting performance in chemistry at secondary and tertiary levels.* PhD thesis, University of Glasgow.

Alosaimi, K.H.M. (2013). *The development of critical thinking in the sciences with young adolescents*, PhD Thesis, University of Dundee, Dundee.

Al-Osaimi, K.H., Reid, N., Rodrigues, S. (2014) Critical Thinking - Can it be Measured? *Journal of Science Education*, Vol. 15, No. 3, pp. 30-36.

Al-Qabati, A. (1993). *Mathematical Ability and its Relationship with Logical Thinking and Achievement in Mathematics in the Secondary School and After*. Unpublished Master's Dissertation, Al-Yarmook University, Irbid, Jordan.

Al-Rasheed, S. (1987). *The Extent of Matching between the Mental Development Level of the Higher Elementary School Students and the Mathematics Curricula of this Stage*. Unpublished Master's Dissertation, Jordanian University, Amman, Jordan.

Al-Rumaihi, J. S. H. (1997*). Setting accounting standards in a non-western environment with special reference to the Kingdom of Saudi Arabia*. PhD thesis, University of Dundee, Scotland.

Al-Sadan, I.A. (2000). Profile of Educational Assessment Systems Worldwide. *Assessment in Education*, Vol.7, No.1.

Al-Saif, A. (1996). *The current experience of Islamic education in Saudi secondary schools*. Unpublished PhD thesis submitted to Manchester University, UK.

Al-Saloom, H. I. (1991). *History of Educational Movement in the Kingdom of Saudi Arabia*, Washington D.C.: International Grafeix

Al-Sheikh, O. (2001). *Evaluation of School Curricula and Textbooks*. Report, No. 5. Amman, National Center of Human Resources Development.

Al-Sheikh, O. and Abu Zeenah, F. (1985). Development of the Ability of Students' Logical Thinking in the Secondary School and University Stages. *Al-Yarmook Researches: Series of Human and Social Sciences*, Vol.1, pp. 17 – 44.

Al-Shibli, H. (1998). *Levels of Satisfaction of the Educators and Teachers of Governmental Schools in the Provinces of Jarsh and Ajloon Regarding the Status of Educational Activities*. Unpublished Master's Dissertation, Al-Yarmook University, Irbid, Jordan.

Al-Sir, K. (1994). *Evaluation of the Ninth Grade Mathematics Textbook from the Point of View of the Teachers and Students in the First Greater Amman Area*. Unpublished Master's Dissertation, Jordanian University, Amman, Jordan

Al-Turaiqi, A. (2008). *The Political System of Saudi Arabia*. London: Ghainaa Publications.

Al-Zarah, L. N. (2008) *An investigative study of the female initial teacher preparation programme in Saudi Arabia in an Islamic context: perceptions of the key participants*. PhD thesis, University of Durham.

Al-Zughbi, A. (2001). *Development of the Reading Ability of Mathematics Textbooks and its Effect on Achievement.* Unpublished Doctoral Dissertation, Baghdad University, Baghdad, Iraq.

Ali, A.A. (2008) *Perceptions, difficulties and working memory capacity related to mathematics performance*, MSc Thesis, University of Glasgow, Glasgow. [http://theses.gla.ac.uk/441/]

Ali, A.A. and Reid, N. (2012) Understanding Mathematics Some Key Factors, *European Journal of Educational Research*, 1(3), in press.

Ali, M. (1991). *Design of Mathematics Computer Games as a Method for the Mathematics Creativity Development for the Basic Education Students*. PhD thesis, Ain Shams University, Cairo, Egypt.

Allison, J.A. (2001). High School Students' Problem Solving with a Graphing Calculator . *D.A.I-A* , Vol. 61, No. 6, pp. 4314

Allport, G. (1935). Attitudes, in: C. M. Murchison (Ed.) *Handbook of Social Psychology*, London: Oxford University Press, pp. 798–844

Almedia, D. (2001). Pupils' Proof Potential. *International Journal of Mathematics Education in Science and Technology*, Vol. 32, No. 1, pp. 53-60.

American Association for the Advancement of Science (2001). *Atlas of science literacy*. Washington, DC: AAAS.

American Textbooks Council (2009). Review Guidelines. Available from http://www.historytextbooks.org/review.htm. Accessed 21st October, 2011.

Amit, M., and Neria, D. (2007). "Rising to the challenge": using generalization in pattern problems to unearth the algebraic skills of talented pre-algebra students. *ZDM*, Vol. 40, No. 1, pp. 111-129.

Annabi, H. (1991). *Aspects of Critical Thinking in the Instruction of Secondary School Mathematics Teachers*. Unpublished Master's Dissertation, Jordanian University, Amman, Jordan.

Anderson, J. R. (1981). *The Architecture of Cognition*. Cambridge: MA: Harvard University Press.

Anderson, L., Karthwohl, D., Airasian, P., Cruikshank, k., Mayer, R., Pintrich, P., Raths, J., and Wittrock, M (Eds.). (2001). *A Taxonomy for learning, Teaching and Assessing - A Revision of Bloom's Taxonomy of Educational Objectives*. Addison Wesley Longman

Anderson, T. and Soden, R. (2001). Peer interaction and the learning of critical thinking skills. *Psychology Learning and Teaching*, Vol. 1, No. 1, pp. 37-40.

Andrews, G. E. (1996). Mathematical Thinking and Problem Solving. *American Scientist,* 84(3), 302-304

Anghileri, J. (2000) *Teaching Number Sense*. London: Continuum.

Carl, A. (2005). The\"voice of the teacher\'in curriculum development: a voice crying in the wilderness. *South African Journal of Education*, Vol. 25, No. 4, pp. 223-228.

Arnold, A., Shiu, C. & Ellerton, N. (1996) 'Critical issues in the distance teaching of mathematics and mathematics education'. In A. J. Bishop K. Clements, C. Keitel, J. Kilpatrick, and C. Laborde (Eds.), *International handbook of mathematics education* Dordrecht, The Netherlands: Kluwer, pp. 701-754.

Ashcraft, M. (1994). *Human Memory and Cognition* (2nd ed.). New York: Harper Collins College Publishers

Askew, M., Brown, M., Rhodes, V., Wiliam, D. and Johnson, D. (1997) *Effective Teachers of Numeracy – Final Report*: *Report of a study carried out for the Teacher Training Agency 1995–1996 by the School of Education, King's College London*. London: King's College.

Aspinwall, L., Aspinwall, J. S. and Bay-Williams, J. (2003) Investigating Mathematical Thinking Using Open Writing Prompts**.** *Mathematics Teaching in the Middle School***,** Vol. 8, No. 7, pp. 350-354

Australian Education Council (1991). *A National Statement on Mathematics for Australian Schools.* Carlton, Victoria: Curriculum Corporation

Ausubel, D.P. (1968) *Educational Psychology: A Cognitive View*. New York: Holt, Rinehart and Wilson

Ausubel, D. P., Novak, J. D., and Hanesian, H. (1978). *Educational Psychology: A Cognitive View*. (2nd ed.) New York: Holt, Rinehart and Winston.

Baddeley, A. (1990). Human Memory: Theory and Practice. London: Lawrence Erlbaum Associates Ltd.

Baddeley, A. D. and Hitch, G. (1974) Working Memory, in G. H. Bower (ed.), *Recent Advances in Learning and Motivation*, Volume 8, New York: Academic Press.

Badgaish, M. (2008) *Attitudes and Extent of Divergency and Convergency related to Performance of Saudi Pupils in Mathematics*, MSc Thesis, University of Glasgow, Glasgow.

Bagozzi, R. P. and Burnkrant, R. E. (1979). Attitude organization and the attitude-behaviour relationship. *Journal of Personality and Social Psychology*, Vol. 37, pp. 913-929.

Bahrick, H. P. and Hall, L. K. (1991). Lifetime maintenance of high school mathematics content. Journal of *Experimental Psychology*, Vol. 120, Issue 1, pp. 20-33.

Bailin, S. (2002). Critical Thinking and Science Education. *Science and Education*, Vol. 11, No. 4, pp. 361-375.

Baki, R., (2004). Gender-segregated education in Saudi Arabia: Its impact on social norms and the Saudi labor market. *Education Policy Analysis Archives*, Vol.12, No. 28, pp. 28-43

Ball, B. (2002). What is Mathematical Thinking? *Mathematics Teaching,* Vol. 181, pp. 17-20

Ball, D. and Cohen, D. (1996). Reform by the book: What is—or might be—the role of curriculum materials in teacher learning and instructional reform? *Educational Researcher,* Vol. 25, Issue 9, pp. 6-14.

Banks, J. H. (1964) *Learning and Teaching Arithmetic*, Boston, MA: Allyn and Bacon.

Baroody, A. J. (2003). 'The development of adaptive expertise and flexibility: The integration of conceptual and procedural knowledge'. In Baroody, A. J. and Dowker, A. (eds.) *The Development of Arithmetic Concepts and Skills*, Mahwah, NJ: Erlbaum, pp 1–34.

Baroody, A. J. and Coslick, R. T. (1998). *Fostering Children's Mathematical Power: An Investigative Approach to K–8 Mathematics Instruction*. Mahwah, N.J.: Erlbaum Associates.

Basheikh, A. M. (2002). *Financial reporting in Saudi Arabia and bank lending decisions*, PhD thesis, submitted to University of Portsmouth, U.K

Bell, A. W., Costello, J. and Küchemann, D. E. (1983). *A review of research in mathematical education, prepared for the committee of inquiry into the teaching of mathematics in school: Part A, Research on learning and teaching.* Windsor, U.K.: NFER-Nelson.

Bell, E.A. (1991) Debate: a strategy for teaching critical thinking. *Nurse Educator*, Vol. 16, No. 2, pp. 6-7.

Bell, J. (1999). *Doing Your Research Project*, Basingstoke, UK: Open University Press

Ben-Ari, M. (2001). Constructivism in Computer Science Education. *Journal of Computers in Mathematics and Science Teaching*, Vol. 20, No. 1, pp. 45-73. Norfolk, VA: AACE

Ben–Zeev, T. and Star, J. R. (2001). Spurious Correlation in Mathematical Thinking. *Cognition and Instruction,* Vol. 19, No. 3, pp. 253-276.

Bergqvist, T., Lithner, J., Sumpter, L. (2003). *Reasoning characteristics in upper secondary school students' task solving.* Technical report, University of Luleä, Sweden

Bezuidenhout, S. (2011). *Toward Assessing Scientific Thinking: A qualitative analysis of student reasoning among psychology undergraduates.* Thesis submitted to University of Stellenbosch, Cape Town, South Africa.

Biggs, J. (1996). Enhancing teaching through constructive alignment. *Higher Education*, pp. 347-346.

Bills, L., Dreyfus, T., Mason, J., Tsamir, P., Watson, A. and Zaslavsky, O. (2006). Exemplification in mathematics education. In J. Novotna, H. Moraova, M. Ktatka, and N. Stehlikova (eds.), *Procedings of the 30th conference of the international group for the psychology of mathematics education,* Vol. 1, pp. 126-154.

Bishr, M. (1989). *Development of the Abilities of Mathematical Thinking and Creative Thinking and their Relationship with Achievement in Mathematics in the Case of Secondary*

School Students in Yemen, Unpublished Master's Dissertation, Al-Yarmook University, Irbid, Jordan.

Bittner, N. P. and Tobin, D. (1998) Critical thinking: strategies of clinical practice. *Journal for Nurses in Staff Development*, Vol. 14, No. 6, pp. 267-272.

Bloom, B.S., Bertram, B.M. and Krathwohl, D.R. (1956). *Taxonomy of Educational Objectives: The classification of educational goals.* New York, N.Y: McKay.

Bodner, G. M. (1991). I have found you an argument: the conceptual knowledge of beginning chemistry graduate students. *Journal of Chemical Education*, Vol. 86, No. 5, pp. 384-388.

Bohner, G. and Wanke, M. (2002). *Attitudes and Attitudes Change*, Taylor & Francis.

Boostrum, R. (1994). *Developing creative and critical thinking: An integrated approach*. Lincolnwood, IL: National Textbook Company.

Boyle, T. (1996). *Design for multimedia learning*. Hemel Hempstead: Prentice-Hall.

Brahier, D. J. and Olson, M. (1999). The World's Largest Math Event: Promoting Mathematical Thinking. *Teaching Children Mathematics***,** Vol. 5, No. 7, pp. 430-432.

Brändström, A. (2005). *Differentiated Tasks in Mathematics Textbooks*. PhD thesis presented to Luleä University of Technology, Sweden.

Brenner, M. E. and Moseley, B. (1994). Preparing students for algebra: The roles of multiple representations in problem solving. In *Proceedings of the Sixteenth Annual Conference for the Psychology of Mathematics Education*. Baton Rouge, LA: Louisiana State University, pp. 138-144.

Brenner, M. E., Mayer, R. E., Moseley, B., Brar, T., Duran, R., Reed, B., and Webb, D. (1997). Learning by understanding: The role of multiple representations in learning algebra. *American Educational Research Journal*, Vol. 34, Issue 4, pp. 663-689.

Brookfield, S. D. (1987). *Developing critical thinkers: Challenging adults to explore alternative ways of thinking and acting*. San Francisco: Jossey-Bass

Brown, M. R. (2002). The Effects of Reform Mathematics on Student Achievement. *M.A.I* , Vol. 40, No. 1, pp. 26

Brown, J. S., Collins, A., and Duguid, P. (1989). *Situated cognition and the culture of learning. Educational Researcher*, Vol. 18, No. 1, pp.32–42.

Bruner, J. S. (1966). *Towards the Theory of Instruction*. Cambridge, Mass: Harvard University Press

Bruner, J. S. (1996). *Towards the Theory of Instruction*. Cambridge, Mass: Harvard University Press

Bruning, R. H., Schraw, G. J., Norbey, M. M. and Ronning, R. R. (2003). *Cognitive Psychology and Instruction.* 4th edition. New Jersey: Prentice- Hall.

Bryman, A. (2004). *Research Design Methods*. 2nd edition. Oxford: Oxford University Press

Burkhardt, H. and Resnick, L. B. (1991) *A Balanced Assessment Package*. Nottingham, England: Shell Centre for Mathematical Education.

Burns, M. K. (2005). Using incremental rehearsal to increase fluency of single-digit multiplication facts with children identified as learning disabled in mathematics computation. *Education & Treatment of Children*, Vol. No. 3, pp. 237-250

Burton, L. (1984). Mathematical Thinking: The Struggle for Meaning. *Journal for Research in Mathematics Education,* Vol. 15, No. 1, pp. 35-49

Burton, L. (1996). Assessment of mathematics - what is the agenda? In M. Birenbaum and F. Dochy (eds.), *Alternatives in Assessment of Achievements, Learning Processes and Prior Knowledge*. Dordrecht, The Netherlands: Kluwer.

Cai, J. (2000). Mathematical Thinking Involved in U.S. and Chinese Students` Solving of Process-Constrained and Process-Open Problems. *Mathematical Thinking and Learning*, Vol. Issue 4, pp. 309-40.

Cai, J. (2003). What research tells us about teaching mathematics through problem solving. In F. Lester (Ed.), *Research and issues in teaching mathematics through problem solving* (pp. 241-254). Reston, VA: National Council of Teachers of Mathematics.

Cai, J. and Kenney, P. A. (2000). Fostering Mathematical Thinking through Multiple Solutions. *Mathematics Teaching in the Middle School*, Vol. 5, Issue 8, pp. 534-540

Carey, S. (2002). The origin of concepts: Continuing the conversation. In L. Stein, P. Bauer and M. Rabinowiz. (Eds.), *Representation, memory, and development: Essays in honor of Jean Mandler* Mahwah: NJ: Erlbaum, pp. 43- 52.

Carnduff, J. and Reid, N. (2003). *Enhancing Undergraduate Chemistry Laboratories*. London: The Royal Society of Chemistry.

Carpenter, T. P. (1985). Research on the Role of Structure in Thinking. Arithmetic Teacher, Vol. 32, No. 6, pp. 58-66

Carpenter, T. P., Corbitt, M. K., Kepnner, H. S., Lindquist, M. M. & Reys, R. E (1981). *Results from the second mathematics assessment of the national Assessment of Educational Progress*. Reston, VA: National Council of Teachers of Mathematics.

Carpenter, T.P, Fennema, E., Franke, M. L., Levi, L. and Empson, S.B. (1999) *Children's Mathematics – cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carpenter, T.P, Fennema, E., Peterson, P.L., and Carey, D.A. (1988) Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education*, Vol.19, No. 5, pp. 385–401.

Carpenter, T.P, Fennema, E., Peterson, P.L., Chiang, C.P. and Loef, M. (1989) Using knowledge of children's mathematical thinking in classroom teaching; an experimental study. *American Educational Research Journal*, Vol. 26, No. 4, pp. 499–531.

Carreira, S. (2001). Where There's a Model, There's a Metaphor: Metaphorical Thinking in Students` Understanding of a Mathematical Model. *Mathematical Thinking and Learning***,** Vol. 3, No. 4, pp. 261-287

Carroll, M. (2005) *Experience, intention and practice in the teaching of 5-14 primary science*, PhD Thesis, University of Glasgow, Glasgow.

Carver, E. C. (2001). *A Case Study: A Fifth-Grade Teacher's Commitment to Facilitating Change in Her Mathematics Curriculum*. PhD thesis submitted to Oklahoma State University, Oklahoma, USA.

Cavanagh, S. (1997). Content Analysis: Concepts, Methods and Applications. *Nurse Researcher*, Vol. 4, pp. 5-15.

Cerrito, P. B. (1996). Mathematics Across the Curriculum. *College Teaching*, Vol. 44, No. 2, pp. 48-51

Chaiken, S. H. and Eagly, A. (1993) *The Psychology of Attitudes*, Harcourt Brace College Publishers, New York.

Chancellor, D. (1991). Higher-Order Thinking: a "Basic" Skill for Everyone. *Arithmetic Teacher*, Vol. 38, No. 6, pp. 48-50.

Chandi, S.S., Reid, N., McWilliam. A., and Gray, D. (2009). Exploring the usefulness of a systems-based model from the perspective of biology students, *Scottish Educational Studies*, Vol. 41, No. 1, pp. 68-85.

Chap, L. and Tee, H. (2007). Promoting Mathematical Thinking in the Malaysian Classroom: Issues and Challenges. *The APEC Project Collaborative Studies on Innovation for Teaching and Learning Mathematics in Cultures (II).* Centre for Research on International Cooperative Educational Development (CRICED). University of Tsukuba, Japan.

Charlesworth, R. and Lind, K. (2011). *Math & science for young children*. 7th ed. Belmont, CA: Wadsworth.

Charmaz, K. (2006) *Constructing grounded theory: a practical guide through qualitative analysis*, London: Sage

Christou, K. (2001). *Difficulties in Solving Algebra Story Problems in a Greek Secondary Centre* for *Science Education*. Unpublished PhD thesis. Glasgow, UK: University of Glasgow.

CIA World Factbook (2011). *Saudi Arabia*. Available from https://www.cia.gov/library/publications/the-world-factbook/geos/sa.html Accessed 18th August, 2012

Clark, M., & Lovric, M. (2009). Understanding secondary–tertiary transition in mathematics. *International Journal of Mathematical Education in Science and Technology*, Vol. 40, No. 6, pp. 755-776.

Clarke, D. (1996). ‘Assessment’. In A. J. Bishop K. Clements, C, Keitel, J. Kilpatrick, and C. Laborde (Eds.), *International handbook of mathematics education*. Dordrecht, The Netherlands: Kluwer, pp. 327-370

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., and Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, Vol. 32, No. 1, pp. 9–13.

Cockcroft Report (1982). *Mathematics Counts*. London: HMSO

Cohen, D. (2002). Adults` Mathematical Thinking and Emotions. *Studies in the Education of Adults*, Vol. 34, No. 1, pp. 88-91.

Cohen, L., Manion, L. and Morrison, K. (2007). *Research Methods in Education.* London: Routledge.

Cole, F. L. (1988). Content analysis:process and application: *Clinical Nurse Specialist*, Vol. 2, No. 1, pp. 53-57.

Cooke, N.L. and Reichard, S.M. (1996). The effects of different interspersal drill ratios on acquisition and generalization of multiplication and division facts. *Education & Treatment of Children*, 19(2), 124-143.

Cordesman, A. H. (2003). *Saudi Arabia Enters the 21st Century*. Westwood, CT: Grenwood Publishing Group.

Costa, A. (1985). *Developing Minds: A Resource Book for Teaching Thinking*. Alexandria, Egypt: ASCD .

Costello, P. J. (2003). *Thinking Skills and Early Childhood Education*. London: David Fulton Publishers Ltd.

Coupland, M. (2006). *Literature Review and Annotated Bibliography to Inform the Stage 6 Mathematics Review.* New South Wales, Australia: Board of Studies

Cowan, P., Morrison, H. and McBride, F. (1998). Evidence of a Spiral Curriculum Using a Mathematical Problem-Solving Tool. *Interactive Learning Environments*, 6(3), pp. 205-224

Crawley, F. (1990). Intentions of science teachers to use investigative teaching methods: a test of the theory of planned behaviour. *Journal of Research in Science Teaching*, Vol. 27, No. 7, pp. 685-697.

Crawley, F. and Black, C. (1992). *Casual modeling of secondary school chemistry: A study of psychological factors affecting pupil performance*. MSc thesis. Science Education Centre, University of Glasgow, UK.

Creswell J.W (2005). *Educational Research*. 2nd edition. New York: Prentice Hall.

Cretchley, P., Fuller, A. and McDonald, C. (2000). Influences on the Choice to Study Mathematics at University, *Proceedings of MERGA 23*, Freemantle: MERGA Inc.

Curry, D., Schmitt, M., and Waldron, S. (1996). A framework for adult numeracy standards: The mathematical skills and abilities adults need to be equipped for the future", The Adult Numeracy Practitioners Network System Reform Planning Project, July 2006

D'Ambrosio, U. (1987). Reflections on Ethnomathematics. *International Study Group on Ethnomathematics Newsletter* Vol. 3, No. 1

D'Ambrosio, U. (2001). What Is Ethnomathematics, and How Can It Help Children in Schools? *Teaching Children Mathematics*, Vol., No. 6, pp. 308-10.

D'Ambrosio, U. (2006). *Ethnomathematics: Link between traditions and modernity*. Rotterdam: Sense Pub

Daghlas J. (1991). *Effect of the Mathematical Thinking Ability and Some Personal Traits and Socio-Economic Status and Gender on Creative Thinking in the Case of Secondary School Students*. Unpublished Master's Dissertation, Jordanian University, Amman, Jordan.

Darouza, A. N. (1995). *Procedures in the Design of Curricula;* 2nd edition, Nablus, Palestine: Documentation, Manuscripts and Publishing Center.

Davis, J. K. and Frank, B. M. (1979). Learning and memory of field-dependent individuals. *Journal of Research in Personality,* Vol. 13, pp. 469-479.

Davis, R. B. (1984). *Learning mathematics: The cognitive science approach to mathematics education*. London: Croom Helm.

Davydov, V.V. (1990). Types of Generalization in Instruction; Logical and Psychological Problems in the Structure of School Curricula. *Soviet Studies in Mathematics Education Series,* Vol. 2. Reston VA: National Council of Teachers of Mathematics.

De Corte, E. (1995). Fostering cognitive growth: A perspective from research on mathematics learning and instruction. *Educational Psychologist*, Vol. 30, No. 1, pp. 37-46.

De Corte, E., Greer, B., and Verschaffel, L. (1996). Mathematics teaching and learning. In Berliner, D. C. and Calfee, R. C. (eds.) *Handbook of Educational Psychology*, pp. 491–549. New York: Macmillan.

De Corte, E., Verschaffel, L., and Masui, C. (2004). The CLIA-model: A framework for designing powerful learning environments for thinking and problem solving. *European Journal of Psychology of Education,* Vol. 19, pp. 365–384.

Delaney, S., Ball, D. L., Hill, H. C., Schilling, S. G., & Zopf, D. (2008). Mathematical knowledge for teaching: Adapting US measures for use in Ireland. *Journal of Mathematics Teacher Education*, Vol. 11, No. 3, pp. 171-197.

De Vaus, D.A. (1993), *Surveys in Social Research* (3rd edn.), London: UCL Press.

De Vaus, D. A. (2002) *Research Design in Social Research*. London: SAGE Publications.

DeVries, R. (1997). *"Piaget's Social Theory."* Educational Researcher 26(2): 4-17.

De Walle J. V. and Thompson, C.S. (1985). Let's Do It – Promoting Mathematical Thinking. *Arithmetic Teacher,* Vol. 32, No. 6, pp. -13.

Dehaene, S. and Spelke, E. (1999). *Sources of Mathematical Thinking: Behavioral and Brain-Imaging Evidence*. *Science,* Vol. 284, No. 5416, pp. 970-974

Denzin, N. K. and Lincoln, Y. S. (eds.) (1994*). Handbook of Qualitative Research*, London: Sage Publications, Inc

Dewey J. (1916) *Democracy and education: An introduction to the philosophy of education*. New York: Macmillan, 1916.

Dewey, J. (1933). *How We Think*. Boston: Mifflin Company.

Diamond, R. M (2008). *Designing and Assessing Courses and Curricula*: A Practical Guide (3rd Ed). San Francisco: Jossey-Bass.

Di Martino, P., and Zan, R. (2007). *Attitude toward Mathematics. Overcoming the Positive/Negative Dichotomy*. Montana, USA: Council of Teachers of Mathematics

Dienes, Z. P. (translated by P. Seaborne) (1973). *The six stages in the process of learning mathematics*. Windsor, NFER.

Dierking, L. D. and Falk, J. H. (1994). Family behavior and learning in informal science settings: A review of the research. *Science Education*, Vol. 78, No. 1, pp. 57–72.

Di Pillo, M. L. and Sovchik, R. (1997). Exploring Mathematical Thinking Through Journals. *Mathematics Teaching in Middle School.* Vol. 2, No. 5, pp. 308-314.

Donaldson, M. (1978). *Children's minds.* London: Biddles.

Donovan, M. S. and Bransford, J. D. (2005). *How Students Learn Mathematics in the Classroom*. Washington, D.C: National Academic Press.

Dossey, J. A., Mullis, I. V., Lindquist, M. M. and Chambers, D. L. (1988). *The mathematics report card. Are we measuring up? Trends and achievement based on the 1986 national assessment.* Princeton, NJ: Educational Testing Service.

Dreyfus, T, Hershkowitz, R & Schwarz, B. (2001). Abstraction in Context: Epistemic Actions. *Journal for Research in Mathematics Education*, Vol. 32, No. 2. pp. 195-222.

Driscoll, M. (2005). *Psychology of Learning for Instruction*. Boston, MA.: Allyn & Bacon.

Dubinsky, E., Elterman, F. & Gong, C. (1988). The Student's Construction of Quantification. *For the Learning of Mathematics*, Vol. 8, pp. 44–51.

Eckstein, M. A. and Noah, H. J. (1993). *A comparative study of secondary school examinations. Research Working Paper No. 7*, London: International Centre for Research on Assessment.

Education Scotland (n.d.) What is Curriculum for Excellence? Available from http://www.educationscotland.gov.uk/thecurriculum/whatiscurriculumforexcellence/howwasthecurriculumdeveloped/index.asp Accessed 19th December, 2013.

Eggen, P. & Kauchak, D. (2007) *Educational Psychology: Windows on the classroom*. 7th Ed.

Elo, S. and Kyngäs, H. (208). The qualitative content anlaysis process. *Journal of Advanced Nursing.* Vol. 61, No. 1, pp. 107-115.

El-Sawaf, M.M.F. (2007), *Educational beliefs development with pre- and in-service teachers using Perry's model: a cross-cultural study*, PhD Thesis, University of Glasgow, Glasgow.

Elliott, B. M. (2000). The Influence of an Interdisciplinary Course on Critical Thinking Skills . *D.A.I-A* , Vol. 60, No. 9, pp. 3287

Ennis, R.H. (1962). A Concept of Critical Thinking. *Harvard Educational Review*, Vol. 22, No. 1, pp.81-111.

Ennis, R.H. (1993). Critical Thinking Assessment. *Theory into Practice* 32 (3), pp. 179-186

Evans, J. S. (1983). *Thinking and Reasoning: Psychological Approaches*. London: Routledge and Kegan Paul.

Evered, L. J. and Gningue, S. (2001). Developing Mathematical Thinking Using Codes and Ciphers. *Teaching Children Mathematics*, Vol. 8, No. 1, pp. 8-16 .

Facione, P. A. (1990a). *Critical thinking: A statement of expert consensus for purposes of educational assessment and instruction: The Delphi Report*. Milbrae, C.A: California Academic Press.

Facione, P. A. (1990b). The California Critical Thinking Skills Test: College level. Available from http://www.eric.ed.gov/ERICWebPortal/contentdelivery/servlet/ERICServlet?accno=ED327549 Accessed 29th May, 2011

Fazio, R. (1990). Multiple processes by which attitudes guide behavior: The MODE model as an integrative framework. *Advances in Experimental Social Psychology*, Vol. 23, pp. 75-109.

Fennema, E. and Romberg, T.A. (eds.) (1999) *Mathematics Classrooms that Promote Understanding*. Mahwah, NJ: Lawrence Erlbaum Publishers.

Fishbein, M. and Ajzen, I. (1976). *Belief, Attitude, Intention and Behaviour*. Addison-Wesley.

Flavell, J.H. (1979). Metacognition and cognitive monitoring. A new area of cognitive-development inquiry. *American Psychologist*, Vol. 34, No. 10, pp. 906–911

Fleer, M. (1992). Identifying teacher-child interaction which scaffolds scientific thinking in young children. *Sci. Ed*., Vol. 76, pp. 373–397.

Fletcher, P. and Patty, C. W. (1988). *Foundations of Higher Mathematics*. Boston, Mass: PWS-KENT Publishing Co.

Fouche, K. K. (1993). *Problem solving and creativity:* Multiple solution methods in a cross -cultural study in middle level mathematics. PhD thesis submitted to University of Florida.

Fraivillig, J.L., Murphy, L.A. and Fuson, K.C. (1999). Advancing children’s mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematics Education*, Vol. 30, pp. 148-170.

Franke, M.L. and Kazemi, E. (2001) Learning to teach mathematics: focus on student thinking. *Theory into Practice*, Vol. 40, No. 2, pp.102–109

Fraser, B.J. and Tobin, K. (1991). Combining qualitative and quantitative methods in classroom environment research. In B.J. Fraser and H. J. Walberg (Eds.), *Educational*

Environments: evaluation, antecedents and consequences, Oxford: Pergamon, (pp. 271–292).

Freeman, D. J., & Porter, A. C. (1989). Do Textbooks Dictate the Content of Mathematics Instruction in Elementary Schools? *American Educational Research Journal*, Vol. 26, No. 3, pp. 403-421.

Frenzel, A. C., Goetz, T., Pekrun, R., & Watt, H. M. (2010). Development of mathematics interest in adolescence: Influences of gender, family, and school context. *Journal of Research on Adolescence*, Vo. 20, No. 2, pp. 507-537.

Friel, S and Johnstone, A.H. (1978a) A Review of the Theory of Objective Testing, *School Science Review*, Vol. 59, pp. 733-8.

Friel, S and Johnstone, A.H. (1978b) Seconds Thoughts in Multiple Choice Tests in Science, *Journal of Chemical Education*, Vol. 56, pp. 326.

Friel, S and Johnstone, A.H. (1979) Does the Position of the Answer in a Multiple Choice Test Matter, *Education in Chemistry,* Vol. 16, No. 6, pp. 175.

Fulcher, E. (2012). What is Discourse Analysis? Available from http://www.eamonfulcher.com/discourse_analysis.html Accessed 11th December, 2012.

Fuson, K., Stigler, J., and Bartsch, K., (1988), Grade Placement of Addition and Subtraction Topics in Japan, Mainland China, the Soviet Union, Taiwan, and the United States, *Journal for Research in Mathematics Education*, Vol. 19, pp. 449-456.

Gagné, R.M. (1964). Problem solving. In A.W. Melton (ed.). *Categories of Human Learning*. New York: Academic Press

Gagné, R.M. (1974). *Principles of Instructional Design*. New York: Holt, Rinehart and Wilson

Gagné, R.M. (1977). *The Conditions of Learning*. 3rd ed. New York: Holt, Rinehart and Wilson.

Gagné, R.M. (1988). Mastery Learning and Instructional Design. *Performance Improvement Quarterly*, Volume 1, Issue 1, pp. 7–18.

Gall, M.D. Borg, J.P. and Gall, W.R. (2006). *Educational Research: An Introduction*, 8th ed. Boston, MA:Allyn and Bacon

Gall, M. and Hicks, W.V. (1964). *Modern Secondary Education*, New York: American Book Company.

Garnett, P. J. and Hackling, M. W. (1995). Students' alternative conceptions in chemistry. *Studies in Science Education* Vol. 25, pp. 69-95.

Gathercole, S. E. and Alloway, T. P. (2008). *Working Memory and Learning: A Practical Guide for Teachers*. Thousand Oaks, CA: SAGE

Gay, L. R., Mill, G. E. and Airasian, P. W. (2003). *Educational Research: Competencies for Analysis and Applications*, Upper Saddle River, New Jersey: Merrill/Prentice Hall.

Gervasoni, A. (2000). Problem Solving Enhances Children's Numeracy Learning. Australian Primary Mathematics Classroom, 5 (4), pp. 12-14.

Ghanim, M. (1995). *Child Thinking: Development and Learning Methods.* First Edition. Amman, Jordan: Fikr Publishing and Distribution House.

Ghauri, P. and Gronhaug, K. (2010). *Research methods in business studies* (4th ed.). London: Pearson Education Limited.

Gifford, B. R. and O'Connor, M.C. (1992). *Achievement and Instruction*. Boston: Kluwer

Gill, J. and Johnson, P. (1991), *Research Methods*, Paul Chapman, London.

Ginsburg, H. P., Klein, A., and Starkey, P. (1998). The development of children's mathematical thinking: Connecting research with practice. In Sigel, I. E. and Renninger, K. A. (eds.) *Handbook of Child Psychology: Volume. 4. Child Psychology in Practice*, 5th ed. New York: Wiley, pp 401–476.

Gold, K. (2002). Thinking: the next big idea. Times Educational Supplement, Available from: http://www.tes.co.uk/section/story/?story_id=365216 Accessed 30th April, 2011.

Goldman, S. R., Hasselbring, T. S., & the Cognition and Technology Group at Vanderbilt (1997). Achieving meaningful mathematics literacy for students with learning disabilities. *Journal of Learning Disabilities*, Vol. 30, pp. 198-208.

Goldsby, D.S. and Cozza, B. (2002). Writing Samples to Understand Mathematical Thinking. *Mathematical Teaching in the Middle School.* Vol. 7, No. 9, pp. 517-521

Goodenough, D. R. (1976). The role of individual differences in field dependence as a factor in learning and memory. *Psychological Bulletin*, Vol. 83, pp.675-694.

Gray, P. (2008). *Pedagogy and the Scottish Education System: an overview*. Report prepared for the Norwegian Association of Higher Education Institutions. Available from http://www.uhr.no/documents/PedagogyScot1.pdf Accessed 14th January, 2014

Greenwood, J. J. (1993). On the Nature of Teaching and Assessing Mathematical Power and Mathematical Thinking. *Arithmetic Teacher*. Vol. 41, No. 3, pp. 144-152.

Groβe, C. S., & Renkl, A. (2006). Effects of multiple solution methods in mathematics learning. *Learning and Instruction*, Vol. 16, pp. 122-138.

Grouws, D. A. (ed.). (1992). *Handbook of research on mathematics teaching and learning*. New York: Macmillan.

Guthrie, G. (1986). Current research in developing countries: The impact of curriculum reform on teaching. *Teaching and Teacher Education*, Vol. 2, pp. 81–89

Hadamard, J. (1945). *The Psychology of Invention in the Mathematical Field.* Princeton, USA: Princeton University Press

Halliday, M. (1973). Explorations in the Functions of Language, London: Edward Arnold.

Halloran, J. D. (1970) (Ed.). *Attitude Formation and Change*, Leicester: Leicester University Press.

Halpern, D. F. (ed.) (1992). *Enhancing Thinking Skills in the Science and Mathematics*. New York: Routledge

Hamachek, D. (1995). *Psychology in Teaching, Learning and Growth*. Boston, MA: Allyn and Bacon

Hamadnah, A. (1995). *Level of Critical Thinking in Mathematics of the Tenth Grade Students in Jordan*. Unpublished Master's Dissertation, Al-Yarmook University, Irbid, Jordan.

Hamden, A. (2005). Women and Education in Saudi Arabia: Challenges and Achievements. *Institute of Education Journal*, Vol. 1, pp. 42-64.

Hanna, G. (2000). Proof , Explanation and Exploration: An Overview. *Educational Studies in Mathematics.* 44(1-2) , 5-23

Hannula, M. S. (2002) Attitude towards Mathematics: Emotions, Expectations and Values. *Educational Studies in Mathematics*, Vol. 49, pp. 25-46.

Harries, T. (2001). Working through Complexity: an Experience of Developing Mathematical Thinking through the Use of Logo with Low Attaining Pupils. *Support for Learning*, 16(1), 23-27

Harta, I. (2001). The Nature of Story Problems in Indonesian Elementary Mathematics Textbooks During the Last Forty Years . *D.A.I-A* , 61(12) , 4659

Harte, S. W. and Glover, M. J. (1993). Estimation in Mathematical Thinking. *Arithmetic Teacher* , 41(2) ,75-78

Hassan, M. M. (1999). The impact of the use of problem-solving on academic achievement and mathematical thinking among middle school students in Saudi Arabia, *Journal of the College of Education in Assiut*, Vol. 15, No.1, pp. 15-41.

Hatfield, M.M., Edwards, N.T., & Bitter, G.G. (1993). *Mathematics methods for the elementary and middle school* (2nd ed.). Boston Heights, MA: Allyn & Bacon.

Haylock, D. (1991). *Teaching Mathematics to Low Attainers.* London: Paul Chapman Publishing ltd.

Healy, L. and Hoyles, C. (1999). Visual and Symbolic Reasoning in Mathematics : Making Connections with Computers? *Mathematical Thinking and Learning*, Vol. 1, No. 1, pp. 59-84 .

Hegarty, M., Mayer, R. and Monk, C. (1995). Comprehension of arithmetic word problems: a comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology*, Vol. 87, No. 1, pp. 1995

Helm, H. and Novak, J. D. (Eds.) (1983). *Proceedings of the international seminar on misconceptions in science and mathematics.* June 20-22, 1983, Department of Education, Cornell University, Ithaca, NY, USA

Henerson, M., E, Morris, L. Fitz-Gibbon, C. (1987). *How to Measure Attitudes*. Newbury Park, CA: SAGE

Herbel-Eisenmann, B. (2007). From intended curriculum to written curriculum: Examining the “voice” of a mathematics textbook. *Journal for Research in Mathematics Education*, Vol. 38, No. 4, pp. 344-369.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., Chui, A.Y., Wearne, D., Smith, M., Kersting, N., Manaster, A., Tseng, E., Etterbeck, W., Manaster, C., Gonzales, P. and Stigler, J. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 Video Study*. Washington, DC: National Centre for Education Statistics, U.S. Department of Education.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., and Murray, H. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Holton, D. and Anderson, J. (1999). Mathematical Problem Solving in Support of the Curriculum? *International Journal of Mathematical Education in Science and Technology*, Vol. 30, No. 3, pp. 351-370

Hoover, K. (1984). *The elements of social scientific thinking*. Clearwater, FL, U.S.A.

Hsiung, P.-C. (2010). Lives and Legacies: A Guide to Qualitative Interviewing. Available from http://www.utsc.utoronto.ca/~pchsiung/LAL/ Accessed 15th December, 2012.

HSRC (Humans Sciences Research Council) (2008), Business Day, 14 July 2008 (Linda Chisholm and Martin Carnoy), DST/SAMF/SAMS Workshop 31 July 2008. Available from http://02e6f35.netsolvps.com/files/document/file/assessing_exams.pdf Accessed 9th August, 2012

Hussein, S. M. (1983). *Content analysis – definition of concepts and basic determinants, its uses, groups, methodology, and its association with media and advertising research and public opinion*, Cairo: The World of Books.

Ibrahim, M. (2002). *Logic and Proof in the Teaching of Mathematics*. Cairo: Nahdhat Al-Sharq House

Ibrahim, M. (2009). *Mathematical Thinking and Problem Solving.* Cairo: Book World

Jabir, J. (1997). *Readings in the Instruction of Thinking and Curriculum*. Cairo: Al-Nahdhat Al-Arabiyah House.

Jacobsen, D., Eggen, P. and Kauchak, D. (2002). *Methods for Teaching*. 6th ed. Upper Saddle River, NY: Merrill.

Jansson, L. C. (1986). Logical Reasoning Hierarchies in Mathematics. *Journal for Research in Mathematics Education*, Vol. 17, No. 1, pp. 3-20

Jaradat, E. (1986). School Curricula and the Needs of the Arab Society, *The Teacher Mission*, Jordan, Vol. 27, Issue 3, p. 17.

Jaradat, E. (1992). Philosophy and Orientation of Educational Development in Jordan, *Risalat Al-Mu'alim Journal*, Vol. 33, No. 2, pp. 5-15.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, Vol. 9, No. 2, pp. 187-211.

Jenkins, E. W. (2000). What can physics teachers learn from research in physics education? *Physics Education*, Vol. 34, No. 4, pp. 245-250.

Jerwan, F. (1999). *Teaching Thinking: Concepts and Applications*. Al-Ain: University Book House.

Jitendra, A. K., Deatline-Buchman, A., & Sczesniak, E. (2005). A comparative analysis of third-grade mathematics textbooks before and after the 2000 NCTM standards. *Assessment for Effective Intervention*, Vol. 30, No. 2, pp. 47-62.

Johansson, M. (2003). *Textbooks in mathematics education: a study of textbooks as the potentially implemented curriculum* Licentiate thesis sumitted to Department of Mathematics, Luleå University of Technology, Luleå, Sweden.

Johnson, R. B. and Onwuegbuzie, A.J. (2004). Mixed Methods Research: A Research Paradigm whose Time has Come, *Educational Researcher*, Vol. 33, No. 7, pp. 14-26.

Johnson-Laird, P. N. (1999). Deductive Reasoning. *Annual Review of Psychology*, Vol. 50, pp. 109-135.

Johnstone, A. H. (1991) Why is Science Difficult to Learn? Things are Seldom What They Seem, *Journal of Computer Assisted Learning*, Vol. 7, pp. 75-83.

Johnstone, A. H. (1993) The Development of Chemistry Teaching - A Changing Response to Changing Demand, *Journal of Chemical Education*, Vol. 70, No. 9, pp. 701-705.

Johnstone, A. H. (1997) Chemistry Teaching, Science or Alchemy? *Journal of Chemistry Education*, Vol. 74, Issue 3, pp. 262-268.

Johnstone, A. H. and Reid, N. (1981). Toward a model of attitude change. *European Journal of Science Education*. Vol. 3, No. 2, pp. 205-212.

Johnstone, A. H., Hogg, W.R. and Ziane, M. (1993). A working memory model applied to physics problem solving. *International Journal of Science Education*, Vol. 15, No. 6, pp. 663-672.

Johnstone, A.H., Watt, A. and Zaman, T. U. (1998). The Students' Attitude and Cognition Change to a Physics Laboratory. *Physics Education*, Vol. 1, No. 1, pp. 9-15.

Jones, B., Balniskar, A., Oagle, D. and Karr, E. (1994). *Strategic Education and Learning: Cognitive Teaching in the Areas of Content*. Translated by Omar Al-Sheikh, Amman: College of Education, UNRWA/UNESCO.

Jones, K. (2000). Providing a Foundation for Deductive Reasoning: Students` Interpretations When Using Dynamic Geometry Software and Their Evolving

Mathematical Explanations. *Educational Studies in Mathematics***.** Vol. 44, No. 1-2, pp. 55-85

Jordan, N. C. and Hanich, L. B. (2000). Mathematical Thinking in Second Grade Children with Different Forms of LD. *Journal of Learning Disabilities*, Vol. 33, No. 6, pp. 567-579

Judd, C. H. (1908). The Relation of Special Training to General Intelligence, *Educational Reviews*, Vol. 36, pp. 28-42.

Jung, E. S. (2005) *Attitudes and Learning Difficulties in Middle School Science in South Korea*, MSc Thesis, Glasgow, University of Glasgow.

Jung, E. S. and Reid, N. (2009), Working Memory and Attitudes, *Research in Science and Technological Education*, Vol. 27, Issue 2, pp. 205-224.

Kalman, R. (2004). The value of multiple solutions. *Mathematics Teaching in the Middle School*, Vol. 10, No. 4, pp. 174-179.

Kang, W. (1990). *Didactic Transposition of Mathematical Knowledge in Textbooks*. Unpublished PhD Dissertation, submitted to University of Georgia, USA.

Kapur, J. N. (1976), *Culture, Excitement and Relevance of Mathematics*. New Delhi: S. Chand & Co.

Karam, I. (2000). *Textbooks and the Development of Thinking Skills*. Paper presented at 12th Conference on Education and Teaching Methods, Cairo, Egypt, 25-26th July, 2000.

Karmiloff-Smith, A. and Inhelder, B. (1974). If You Want to Get Ahead, Get a Theory. *Cognition*, Vol. 3, Issue 3, pp. 195–212.

Katagiri, S. (2004). Mathematical thinking and how to teach it. CRICED, University of Tsukuba, Japan.

Katz, D. (1960). The Functional Approach to the Study of Attitudes. In Halloran, J.D. (1970) (ed.), *Attitude Formation and Change*. Leicester: Leicester University Press.

Kazemi, E. (2000). Teacher Learning within Communities of Practice: Using Student's Mathematical Thinking to Guide Teacher Inquiry. PhD thesis submitted to University of Califronia, Los Angeles, USA.

Keeves, J. P. and Stacey, K. (1999) Research into Mathematics Education. In J.P. Keeves & Kevin Marjoribanks (Eds.) *Australian Education: Review of Research 1965 – 1998*. Melbourne: ACER, pp. 205 – 242.

Khalifa, A. and Shobiak, W. (2007). *The Quality of Textbooks in Basic Education from the Standpoint of Supervisors*. Paper presented at The 3rd Educational Conference on the Quality of Public Education, Islamic University of Palestine, 2007.

Khidr, N. (1988). *Origins of Mathematics Instruction*. Cairo: Book World.

Kilpatrick, J. (1985). Reflection and Recursion. *Educational Studies in Mathematics*, Vol. 16, No. 1, pp. 1-26.

Kirschner, P. E., Swell, J. and Clark, R.E. (2006). Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experiential, and Inquiry-Based Teaching. *Educational Psychologist,* Vol. 41, Issue 2, pp. 75-86

Kosa, S. (2001). *Mathematical Thinking and Achievement in Mathematics in the Case of the Female Students of the Elementary Schools in Makkah*. Annual Academic Conference, Egyptian Society of Female Mathematics Educators, Cairo, pp. 583-605.

Koussa, S. A. H. (1999). *The effective use of a program prepared to solve problems in the development of creative and innovative thinking in mathematics among middle school students in Makkah*, Unpublished Ph.D. thesis submitted to Jeddah Education College, Saudi Arabia.

Krech, D. (1960). Cognitive, Affective and Behavioural Components of Attitudes, in Halloran, J.D. (1970) (ed.), *Attitude Formation and Change*. Leicester: Leicester University Press.

Krippendorff K. (1980). *Content Analysis: An Introduction to its Methodology*. Newbury Park: Sage Publications.

Kulm, G. (1993). *A theory of classroom assessment and teacher practice in mathematics*. Washington, DC: Department of Education. ERIC Document Reproduction Service No. ED 358 129.

Kumar, R. (2005). *Research Methodology: A Step-by-Step Guide for Beginners* London: Sage Publications Ltd.

Kurfiss, J. G. (1988) *Critical thinking theory, research, practice and possibilities*. (ASHE-ERIC Higher Education Report No.2). Washington DC: Association for the Study of Higher Education.

LaBerge, D. and Samuels, S. J. (1974). Toward a theory of automatic information-processing in reading. *Cognitive Psychology*, Vol. 6, pp. 293-323.

Lachter, J., Forster, K. I., & Ruthruff, K. I. (2004). Forty-five years after Broadbent (1958): Still no identification without attention. In Woolfolk, A. (2007). *Educational Psychology*, London: Pearson International Edition.

Laqani, A. H. (1989). *Curricular Theory and Applications*. 3rd ed. Cairo: World of Books.

Leahey, T. and Harris, R. (1997). Learning and cognition. In Eggen, P. and Kauchak, D. (2007) *Educational Psychology: Windows on the classroom*. 7th Ed. Upper Saddle River, N.J.: Prentice Hall.

Leedy, P. D. and Ormrod, J. E. (2001). *Practical research: planning and design*, Upper Saddle River, N.J.: Merrill Prentice-Hall.

Lee, S.-Y. (2011). The Effect of Alternative Solutions on Problem Solving Performance, *International Journal for Mathematics Teaching & Learning*; December 2011, p.1.

Lemke, J.L. (2001). Articulating communities: Sociocultural perspectives on science education, *Journal of Research in Science Teaching*, 38 (3), pp. 296–316.

Lewkowicz , M. L. (2003). The Use of Intrigue to Enhance Mathematical Thinking and Motivation in Beginning Algebra. *Mathematics Teacher,* Vol. 96, No. 2, pp. 92-96

Li, Q. and Jin, Y. (1995). Comments on R. J.Sternberg's Componential Theory of Intelligence. *Journal of Psychological Science*. June 1995.

Lim, C. S. (2009). Assessment in Malaysian School Mathematics: Issues and Concerns. Available from http://www.criced.tsukuba.ac.jp/math/apec/apec2009/doc/pdf_20-21/LimChapSam-paper.pdf Accessed 18th August, 2012

Lim, C.S. and Ernest, P. (2000). A survey of public images of mathematics. *Research in Mathematics Education*, 2:1, pp. 193-206

Lingefjärd, T. (1997). *Assessment and Mathematics Examinations in the CDIO project*. Göteborg University and Chalmers University of Technology. Sweden.

Lingefjärd, T. (2002) Teaching and Assessing Mathematical Modelling. *Teaching Mathematics and Its Applications*, Vol. 21, No. 2, pp. 75-83

Lithner, J. (2000). Mathematical reasoning in school tasks. *Educational Studies in Mathematics*, Vol. 41, No. 2, pp. 165–190.

Lithner. J. (2003). Students' mathematical reasoning in university textbook exercises. *Educational Studies in Mathematics*, Vol. 52, No. 1, pp. 55, 2003.

Lithner, J. (2004) Mathematical reasoning in calculus textbook exercise. *Journal of Mathematical Behavior*, Vol. 23, No. 4, pp. 371–496.

Love, E. and Pimm, D. (1996). 'This is so': A text on texts. A. J. Bishop K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp.701-754). Dordrecht, The Netherlands: Kluwer.

Luke, C., de Castell, S. and Luke, A. (1989) Beyond criticism: the authority of the school textbook. In De Castell, S. et al (eds.) *Language, authority and criticism: readings on the school textbook*. London: Falmer Press.

Lutfiyya, L. (1998). Mathematical Thinking of High School Students in Nebraska. *International Journal of Mathematical Education in Science and Technology*, Vol. 29, No. 1, pp. 55-56.

Magajna, Z. and Monaghan, J. (2003). Advanced Mathematical Thinking in a Technological Workplace. *Educational Studies in Mathematics***,** Vol. 52, No. 2, pp. 101-122.

Mahdi, Y. (2006). *High Achievement and the Development of Thinking Skills in Mathematics*. Jordan: Dar De Bono

Maher, C. A., Davis, R. B. and Alston, A. (1992). Teachers Paying Attention to Students` Thinking. *Arithmetic Teacher,* Vol. 39, No. 9, pp. 34-37

Maher, C.A. (1991). Implementing a "Thinking Curriculum" in Mathematics. *Journal of Mathematical Behavior*, Vol. 10, No. 3, pp.219-224.

Mahmoud, A. and Aloubeid, M. (2010). *A Textbook in Elementary School: Content, Method and Layout.* Paper presented at Conference on Arabic Teaching, Riyadh, Saudi Arabia, 21-23 February, 2010.

Maki, D. P., and Thompson, M., 1973, *Mathematical Models and Applications* (New York: Prentice Hall).

Mann, E. L. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted*, Vol. 30, No. 2, pp. 236-260.

Marrapodi, J. (2003). Critical thinking and creativity, an overview and comparison of the theories. ED7590 Critical Thinking and Adult Education Paper.

Marsigit, D. A. and Mahmudi, A. (2007). *Lesson study: Promoting Student on the Concept of Least Common Multiple (LCM) through a Realistic approach in the 4th Grade of Primary Mathematics Teaching*. The APEC Project Collaborative Studies on Innovation for Teaching and Learning Mathematics in Different Cultures (II). Center for Research on International Cooperation Education Development (CRICED), University of Tsukuba, Japan, Vol. 2, pp. 179-193.

Marton, F. and Booth, S. (1997). *Learning and Awareness*. Mahwah, NJ: Erlbaum.

Marten, R. (1997). *Scientific thinking*. Canada: Broadview Press.

Martin, H. (1996). *Integrating Mathematics Across the Curriculum NCTM-Aligned Activities*. Arlington Heights, IL: Skylight Training and Publishing, Inc.

Martin, M. O.; Mullis, I. V. S.; Gonzalez, E. J.; Gregory, K. D.; Smith, T. A.; Chrostowski, S. J.; Garden, R. A. and O'Connor, K. M. (2000). *TIMSS 1999 international science report: findings from IEA's Repeat of the Third International Mathematics and Science Study at the eighth grade*. Chestnut Hill, MA: Boston College Lynch School of Education.

Marzano, R. J. , Brandt, R. S. , Hughes, C. S. , Jones, B. F., Presseisen, B. Z. , Rankin, S. C. and Suhor, C. (1988) *Dimensions of Thinking : A Framework for Curriculum and Instruction.* Virginia, USA: ASCD

Mason, J., Burton, L and Stacy, K. (1991). *Thinking mathematically*. Wokingham, UK: Addison-Wesley.

Mason, J. and Johnston-Wilder, S. (2004). *Fundamental Constructs in Mathematics Education.* London: RoutledgeFalmer.

Maaβ, K. and Gray, P. (2011) ProCoNet Education. Available from http://ncm.gu.se/media/ncm/dokument/Flyer_111015.pdf Accessed 13th January, 2014

Massachusetts Department of Education (2005). Massachusetts adult basic education curriculum framework for mathematics and numeracy", Adult and Community Learning Services, October, 2005.

Massialas, B.G. and Jarrar, S. A. (1983) *Education in the Arab World*. New York: Praeger Publishers.

Matthews, M.R. (1994). *Science teaching: The role of history and philosophy of science*. New York: Routledge.

Matthews, Y. A. (2000) . A Comparison of a Constructivist Mathematics Curriculum and an Instructivist Mathematics Curriculum in Relation to the NCTM Curriculum Standards and the Delaware State Standards. *D.A.I-A*, 61(3), 923

McDonough, A. Clarke, B. and Clarke, D.M (2003). Understanding, Assessing and Developing Children's Mathematical Thinking: the Power of a One-to-One Interview for Preservice Teachers in Providing Insights into Appropriate Pedagogical Practices . *International Journal of Educational Research*, Vol. 37, No. 2, pp. 211-227.

McGuire, W. J. (1985). Attitudes and attitude change. *Advances in Experimental Social Psychology,* Vol. 16, pp. 1-47.

McKendree, J., Small, C., Stenning, K., and Conlon, T. (2007). The Role of Representation in Teaching and Learning Critical Thinking. *Education Review*, 54(1), 57-67.

McPeck, H. (1981). *Critical thinking and education*. New York: St. Martin's Press.

Metz, H. C. (ed.) (1992). *Saudi Arabia: A Country Study*. Washington, USA: GPO for the Library of Congress.

Michalko, M. (1998). Thinking like a genius: Eight strategies used by the supercreative, from Aristotle and Leonardo to Einstein and Edison. *The Futurist*, Vol. 32, No. 4, p. 21.

Millar, R. (1989). Constructive criticisms. *International Journal of Science Education,* 11(Special Issue), pp. 83-94

Miller, G. D. (1956). The Magical Number Seven Plus or Minus Two: Some Limits on Our Capacity for Processing Information, *Psychological Reviews*, Vol. 63, No. 81, pp. 81-97.

Miller, P.H. (1993). *Theories of Developmental Psychology*. New York: W.H. Freeman and Company.

Miller, S. and Mercer, C. (1997). Educational aspects of mathematical disabilities. *Journal of Learning Disabilities*, 30, 47-56

Ministry of Education (1984). The Arab Group on the Planning and Development of the School Curricula, *Risalat Al-Mu'alim Journal*, Vol. 25, No. 2, pp. 109 – 117.

Ministry of Education. (1970). Educational Policy in the Kingdom of Saudi Arabia. Riyadh, Saudi Arabia: Ministry of Education.

Ministry of Education. (1984). System of Education in the Kingdom of Saudi Arabia. Riyadh: Center of Statistical Data and Educational Documentation.

Ministry of Education (1986) Development of Education in the Kingdom of KSA, Riyadh: Data Centre.

Ministry of Education (1992). Development of Education in the Kingdom of Saudi Arabia. Riyadh: Data Centre.

Ministry of Education (2013). Available from http://www.moe.sa/madiv.htm Accessed 12th December, 2013.

Mintzes, J.J., Wandersee, J.H. and Novak, J.D. (1998). *Teaching science for understanding: A human constructivist view*. San Diego, CA: Academic Press.

Monroe, E. E. and Mikovich, A. K. (1994). Making Mathematical Connections Across the Curriculum : Activities to Help Teachers Begin. *School Science and Mathematics*, Vol. 94, No. 7, pp. 371-376

Morgan, C. (2004). *Writing Mathematically: The Discourse of Investigation*. London: Falmer Press.

Morgan, D. (2010). Saudi Arabia Building a Knowledge-based Economy. Global Arab Network [Online]. Available from: http://www.english.globalarabnetwork.com/201006226317/Culture/saudi-arabia-building-a-knowledge-based-economy.html [Accessed 2 July 2012].

Morrisett, L. M., and Vinsonhaler, J. (1965). *Mathematical Learning*. Monographs of the Society for Research in Child Development, Vol. 3, pp. 132.

Moscardini, L. (2009). Tools or crutches? Apparatus as a sense-making aid in mathematics teaching with children with moderate learning difficulties. *Support for Learning*, Vol. 24, No. 1, pp. 35-41.

Musser, G. L. and Burger, W. (1988). *Mathematics for Elementary Teachers*. Macmillan Publishing Company

Muwafi, S. M. (2003). The impact of Internet use on the development of some mathematical concepts and the ability of creative thinking among students at the, Faculty of Education for Girls in Jeddah. The Egyptian Society of Curricula and Teaching Methods, *Studies in the Curriculum and Teaching Methods*, Issue 90.

Najm, K. (2001). *Effect of Mathematics Educational Games in the Case of the Seventh Grade Students on their Achievement and Attitude towards Mathematics*. Unpublished Master's Dissertation, Jordanian University, Amman, Jordan.

Nasr, F. (1998). *Mathematical Thinking and its Relationship to the Solving of Mathematical Problems and Achievement in Mathematics in the Case of the Science and Mathematics Students in the Community Colleges*. Unpublished Master's Dissertation, Jordanian University, Amman, Jordan.

Nathan, M. J. (2001). *Implicit Views of Mathematical Development within Algebra Textbooks: Implications for Educational Reform*. Boulder, Colorado: Institute of Cognitive Science

NCTM (National Council of Teachers of Mathematics (1989). Curriculum and Evaluation Standards for School Mathematics. Reston , Virginia, USA.

NCTM (National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics: Reston, VA: NCTM.

NCTM (National Council of Teachers of Mathematics) (2000). Principles and Standards for Mathematics. Reston, Virginia, USA

National Research Council. (2001). *Investigating the influence of standards: A framework for research in mathematics, science, and technology education*. Washington, DC.: The National Academic Press.

NRC (National Research Council) (2001). *Adding It Up: helping children learn mathematics*, Washington, D C: National Academy Press.

Newell, K., Liu, Y. and Meyer-Kress, G. (2001). Time scales in motor learning and development. *Psychological Review*, Vol. 108, No. 1, pp. 57-82.

Niss, M. (1996). Assessment in mathematics education and its effects: An introduction. In M. Niss (ed.) *Investigation into Assessment in Mathematics Education* (pp. 1-30). Dordrecht: Kluwer

Nissen. P. N. (2000). Textbooks and the National Council of Teachers of Mathematics Curriculum Standards for Geometry. *D.A.I-A*, Vol. 6, No. (6), p. 2226

Noonan, J. (1990). Readability Problems Presented by Mathematics Text. *Early Child Development and Care*, Vol. 54, No.1, pp, 57-81.

Nunes, T. (1992). Ethnomathematics and everyday cognition. In Grouws, D. A. (ed.) *Handbook of Research on Mathematics Teaching and Learning*, pp 557–574. New York: MacMillan.

O'Keefe, L. and O'Donoghue, J. (2011). The use of evidence based research on mathematics textbooks to increase student conceptual understanding. *International Journal for Cross-Disciplinary Subjects in Education (IJCDSE*), Vol. 2, No. 1, pp. 304-311.

Organisation for Economic Cooperation and Development (2006). Assessing scientific, reading and mathematical literacy a framework for PISA 2006. Paris: OECD.

Organisation for Economic Co-operation and Development. (2010). *PISA 2009 Assessment Framework: Key Competencies in Reading, Mathematics and Science*. Paris: OECD Publications.

Onwumere, O. (2009) *Difficulties in understanding mathematics: an approach related to working memory and field dependency*. Unpublished PhD thesis. Glasgow: University of Glasgow

Oppenheim, A. N. (1992). *Questionnaire design, interview and attitude measurement*. London & New York: Continuum.

Oraif, F. A. (2006). *An Exploration of Confidence Related to Formal Learning in Saudi Arabia.* PhD Thesis, Glasgow: University of Glasgow.

Organization for Economic Cooperation and Development (OECD) (1998). *Measuring Students' Knowledge and Skills*. Paris: OECD

Orton, A. (2004). *Learning Mathematics: Issues, Theory, and Classroom Practice*. 3rd edition, London: Continuum.

Pacyga, R. (1994) Implementing the Curriculum and Evaluation Standards. *Mathematics Teacher*. Vol. 87, Issue 1, pp. 43-47.

Palm, K. (2005) *The realism of mathematical school tasks features and consequences*. Doctoral thesis, submitted to University of Ume°a, Department of Mathematics.

Pascual-Leone, J. (1970). A mathematical model for the transition rule in Piaget's developmental stages, *Acta Psychologica*, Vol. 32, pp. 301-345.

Pashler, H. and Carrier, M (1996). Structures, Processes, and the flow of information. In Eggen, P. & Kauchak, D. (2007) *Educational Psychology: Windows on the classroom*. 7th Ed.

Paul, R. (1982) Teaching critical thinking in the 'strong' sense: A focus on self-deception, coordinating views and a dialectical mode of analysis. *Informed Logic Newsletter*, Vol. 4, No. 2, pp. 2-7.

Paul, R. (1983) An agenda item of the informal logic/critical thinking movement. *Informed Logic Newsletter*, Vol. 5, No. 2, pp. 23-4.

Paul, R. (1985). The critical thinking movement. *National Forum*, Vol. 65, No. 1, pp. 32.

Paul, R. (1993). *Critical Thinking: What Every Person Needs to Survive in a Rapidly Changing World.* Santa Rosa, CA: Foundation for Critical Thinking

Paul, R., Binker, A.J., Jensen, K. and Kreklau, H. (1990). Critical Thinking Handbook: 4th- 6th Grades. Sonoma State University, CA, USA: Foundation for Critical Thinking.

Paul, R., Binker, A.J, Martin, D. and Adamson, K. (1989). *Critical Thinking Handbook: High School.* Santa Rosa, CA, USA: Foundation for Critical Thinking

Peat, J., Mellis, C., Williams. K. and Xuan, W. (2002). *Health Science Research: A Handbook of Quantitative Methods.* London: Sage

Pehkonen, E. 2004. State-of-the-Art in Problem Solving: Focus on Open Problems. In H. Rehlich and B. Zimmermann (eds.) *Problem Solving in Mathematics Education*, Hildesheim: Verlag Franzbecker, pp. 93–111.

Pepin, B. and Haggarty, L. (2001). Mathematics textbooks and their use in English, French and German classrooms: A way to understand teaching and learning cultures. *Zentralblatt für Didaktik der Mathematik*, Vol. 33, No. 5, pp. 158-175.

Perry, B. (1998). Children's Mathematical Thinking in the Primary Years. *International Journal of Early Years Education*, Vol. 6, No. 2, pp. 245-247

Peterson, P. L., Fennema, E., and Carpenter, T. (1992). Using children's mathematical knowledge. In B. Means, C: Chelemer, & M. S. Knapp (Eds.), *Teaching advanced skills to at-risk children*. San Francisco: Jossey-Bass, pp. 68-111

Petocz, P. and Petocz, D. (1997). Pattern and Proof: The Art of Mathematical Thinking . *Australian Mathematics Teacher*, 12-15 Vol. 53, No. 3, pp. 12-15.

Philipp, R. A., Thanheiser, E. and Clement, L. (2003). The Role of a Children's Mathematical Thinking Experience in the Preparation of Prospective Elementary School Teachers. *International Journal of Educational Research*, Vol. 37, No. 2, pp. 195- 211

Phillips, D. C. (1995). The Good, the Bad and the Ugly. The Many Faces of Constructivism. *Educational Research,* Vol. Issue 7, pp. 5-12.

Piaget, J. (1952). *The origins of intelligence in children*. New York: International Universities.Press.

Piaget, J. (1963) *The Child's Conception of the World*, N. J.: Little Field.

Piaget, J. (1972). *The Principles of Genetic Epistemology*. London: Routledge & Kegan Paul.

Pinto, M. & Tall, D. (1999), *Student constructions of formal theory:* giving and extracting meaning, Proceedings of PME 23, Vol. 4, pp. 4-65.

Pitt, A. (2002). Mathematical Thinking? *Mathematics Teaching***,** Vol. 181, pp. 3-6.

Polya, G. (1973). *How to solve it.* Princeton, NJ: Princeton University Press.

Posner, G. J. (1995). *Analyzing the Curriculum*. Second Edition. New York: McGraw-Hill, Inc .

Post, S. (1994). Teaching the Art of Problem Solving. *Primus***,** Vol. 4, No. 4, pp. 359-368.

Powell, A. B. (1997). Capturing, Examining, and Responding to Mathematical Thinking through Writing. *Clearing House*, Vol., No. 1, pp. 21-25

Prasad, P.N. (2006) *Performance of secondary students in India related to working memory with reference to some learning styles*, MSc Thesis, University of Glasgow, UK.

Presmeg, N. (2006). Semiotics and the "connections" standard: Significance of semiotics for teachers of mathematics. *Educational Studies in Mathematics*, Vol. 61, No. 1-2, pp. 163-182.

Punch, K. F. (2009). *Introduction to Research Methods in Education.* London:SAGE Publications Ltd.

Putnam, R. T. and Reineke, J. W. (1993). *Learning to Attend to Students` Mathematical Thinking: Case Study of a Collaboration*. East Lansing, MI: Center for the Learning and Teaching of Elementary Subject.

Rahman, S (2010). The Development of a Framework to Assess Mathematical Thinking of Stage One Pupils. *Procedia-Social and Behavioral Sciences*, Vol. 8, pp. 307–311.

Rasmussen, C., Zandieh, M., King, K., and Teppo, A. (2005). Advancing mathematical activity: A practice-oriented view of advanced mathematical thinking. *Mathematical thinking and learning*, Vol. 7, No. 1, pp. 51-73.

Rathmell, E. C. (1994). Planning for Instruction Involves Focusing on Children's Thinking. *Arithmetic Teacher,* 41(6), pp. 290-291

Rea, L. M. and Parker, R. A. (2005). *Designing and Conducting Survey Research: A Comprehensive Guide*, San Francisco, Jossey Bass.

Reardon, S.F., Scott, K. and Verre, J. (1994) Symposium: equity in educational assessment: editors' introduction. *Harvard Educational Review*, Vol. 64, No. 1, pp. 1-4.

Reid, N. (1978) *Attitude Development through a Science Curriculum*. PhD thesis. Glasgow: University of Glasgow

Reid, N. (2006). Thoughts on Attitude Measurement. *Research in Science and Technological Education*, Vol. 21, No. 1, pp. 3-27.

Reid, N. (2009a) The Concept of Working Memory, *Research in Science and Technological Education,* 27(2), 131-138.

Reid, N. (2009b) Working Memory and Science Education, *Research in Science and Technological Education,* 27(2), 245-250.

Reid, N. and Serumola, L. (2006) Scientific Enquiry: The Nature and Place of Experimentation: some recent evidence, *Journal of Science Education*, 7(2), 88-94.

Reid, N. and Skryabina, E (2002) Attitudes towards Physics, *Research in Science and Technological Educatio*n, Vol. 20, No. 1, 67-81.

Reid, N. and Yang, M. J. (2002) Problem Solving in Chemistry: The More Open Ended Problem, *Research in Science and Technological Education*, 20(1), 83-98.

Remillard, J. (1991). Abdicating Authority for Knowing: A Teacher's Use of an Innovative Mathematics *Review of Educational Research. Vol.* 75, No. 2, pp. 211- 246

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research*, Vol. 5, No. 2, pp. 211-246.

Resnick, L. (1987). *Education and learning to think*. Washington, DC: National Academy.

Resnick, L. and Klopfer, L. (eds) (1989). *Toward the Thinking Curriculum*. Virginia, USA: ASCD

Resnick, L. B. (1990). Thinking in Arithmetic. In B. Means and M.S. Knapp (eds.) *Teaching Advanced Skills to Educationally Disadvantaged Students*. Pittsburgh, PA: Learning Research and Development Center

Reusser, K. (1988). Problem solving beyond the logic of things: Contextual effects on understanding and solving word problems. *Instructional Science*, Vol. 17, pp. 309–338

Reys, B. J., Reys, R.E., Chavez, O. (2004). Why Mathematics Textbooks Matter. *Educational Leadership*, 61(5), pp. 61-66

Reys, R.E., Lindquist, M.M., Lambdin, D.V., Smith, N.L.and Suydam, M.N. (2004). *Helping children learn mathematics* (7th ed.). Hoboken, NJ: John Wiley & Sons

Reys, R., Reys, B., Lapan, R., Holliday, G., and Wasman, D. (2003). Assessing the impact of standards-based middle grades mathematics curriculum materials on student achievement. *Journal for Research in Mathematics Education*, Vol. 34, No. 1, pp. 74-95.

Richards, B. (2003). Intelligent innovation: Ideas to action. *The Journal for Quality and Participation*, Vol. 26, No. 2, p. 14.

Riedesel, A.C. and Schwartz, J. E. (1999). *Essentials of elementary mathematics*. Boston, Mass: Allyn and Bacon

Robillard, P. N. (1999). The role of knowledge in software development. *Communications* *of the ACM*, Vol. 42, No. 1, pp. 87-92.

Sedgwick, R. (2001). Education in Saudi Arabia, *World Education News and Reviews*, Vol.14, Issue 6, pp. 1-8

Saedi, L. S. (2006). *Effectiveness of the proposed program in mathematics in the development of mathematical creativity and academic achievement and decision-making in high-achieving intermediate stage pupils in Makkah*. Unpublished Ph.D. thesis, submitted to Makkah Education College, Saudi Arabia

Saif, R. (2005). The effectiveness of educational puzzles in the development of creative thinking and attitude towards mathematics in middle school pupils in Kuwait. *The Journal of Gulf Studies and the Arabian Peninsula*, No. 117, pp. 51-84.

Salem, T. (2001). *Geometry Thinking Levels in the Case of the Higher Basic Elementary Stage Students in Jarsh Province and its Relationship with Gender and Achievement in Mathematics.* Unpublished Master's Dissertation, Hashemite University, Al-Zarqa'a, Jordan.

Saunders, M., Lewis, P. and Thornhill, A. (2000). *Research Methods for Business Students*, London: Pearson Education Ltd.

Saunders, M. and Lewis, P. (2012). *Doing Research in Business and Management*. London: Pearson.

Sajjadi, B. and Sayed, H. (2007). Sex and grade differences in verbal creative thinking amomng Iranian middle school children. *Psychological Reports,* Vol. 1000, No. 3, pp. 759-767.

Schielack, J. F., Chancellor, D. and Childs, K. M. (2000) Designing Questions to Encourage Children's Mathematical Thinking. *Teaching Children Mathematics,* Vol. 6, No. 6, pp. 398-402

Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H., Wiley, D. E., Cogan, L. S., et al. (2001). *Why schools matter: a cross-national comparison of curriculum and learning*. San Francisco: Jossey-Bass.

Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E. (1997). *Many visions, many aims: a cross-national investigation of curricular intentions in school mathematics* (Vol. 1). Dordrecht: Kluwer.

Schoenberger, K. M. and Liming, L. A. (2001). Improving Students` Mathematical Thinking Skills through Improved Use of Mathematics Vocabulary and Numerical Operations. MA Action Research Project, Saint Xavier University, Chicago, USA. Available from http://eric.ed.gov/PDFS/ED455120.pdf Accessed 6th January, 2012.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando: Academic Press.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* Hillsdale, N.J.: Erlbaum, pp. 189-215

Schoenfeld, A.H. (1988) When Good Teaching Leads to Bad Results: The Disasters of "Well-Taught" Mathematics Courses, *Educational Psychologist*, Vol. 23, No. 2, pp. 1-22.

Schoenfeld, A.H. (1991). On mathematics as sense-making: An informal attack on the unfortunate divorce of formal and informal mathematics. In J. Voss, D. Perkins, and J. Segal (eds.), *Informal Reasoning and Education*, pp. 311–344. Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning*. New York: MacMillan, pp. 334-370.

Schoenfeld, A. H. (1994). Learning to think mathematically: problem-solving, metacognition, and sense-making in mathematics. In D. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillan, pp. 334-370

Schoenfeld, A. (2004). The math wars. *Educational Policy*, Vol. 18, No. 1, pp. 253-286.

Schoenfeld, A. H. (2006). Mathematics teaching and learning. In Alexander, P. A. and Winne, P. H. (eds.) *Handbook of Educational Psychology*, 2nd edn. pp 479–510. New York: Macmillan.

Schroeder, T. L. and Lester, F. K. (1989), *Developing understanding in mathematics via problem solving.* In P. R. Trafton (ed.), New directions for elementary school mathematics: 1989.

Scottish Qualifications Authority (undated) Advanced Higher Grade Mathematics, past papers. Available from http://www.sqa.org.uk/pastpapers/findpastpaper.htm?subject=Mathematics&level=AH Accessed 4st December, 2012.

Schurter, W. A. (2002). Comprehension Monitoring: An Aid to Mathematical Problem Solving. *Journal of Developmental Education*, Vol. 26, No. 2, pp. 22-33.

Selden, J., Selden, A. and Mason, A. (1994). Even good students can't solve non-routine problems. In J Kaput and E Dubinsky, (eds.), *Research Issues in Undergraduate Mathematics Learning*, pages 19–26. Washington, D.C.: Mathematical Association of America.

Sfard, A. (2001). There is More to Discourse than Meets the Ears: Looking at Thinking as Communicating to Learn More about Mathematical Learning. *Educational Studies in Mathematics*, Vol. 46, No. 1-3, pp. 13-57

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher*, Vol. 27, No. 2, pp. 4–13.

Sharma, I. S. (1983). *A Study in the Comprehensibility of Language Used in Science*, Social Science and Language Textbooks at Primary Level. New Delhi: NCERT.

Sharples, D. (1969). Attitudes to Junior School Activities. *British Journal of Educational Psychology* Vol. 39, pp. 72-77.

Shatnawi, F. (1982). *Development of Mathematical Thinking in the Case of Secondary School Students in Jordan.* Unpublished Master's Dissertation, Al-Yarmook University, Irbid, Jordan

Shaw, J. M. and Blake, S. (1998). *Mathematics for Young Children*. Upper Saddle River NJ: Merrill

Shi, Y. (2000). The Game PIG: Making Decisions Based on Mathematical Thinking. *Teaching Mathematics and Its Applications*, Vol. 19, No. 1, pp. 30-34.

Shield, M. J. (1998). *Mathematics textbooks: Messages to students and teachers*. Proceedings of the 21st Annual Conference of the Mathematics Education Research Group of Australasia, 21, pp. 516-523

Shoult, A. (2006) *Doing Business with Saudi Arabia*. 3rd edition, London: GMB Publishing

Shulman, L.S. (1986). Those who understand: knowledge growth in teaching, *Educational Researcher*, Vol. 5, No. 2, pp. 4–14.

Siegel, H. (1991). The generalizability of critical thinking. *Educational Philosophy and Theory*, Vol. 23, No. 1, pp. 18-30

Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C. and Strawhun, B. T. F. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *Journal of Mathematical Behavior*, Vol. 24, No. 3, pp. 287-301.

Silverman, D. (2000). *Doing qualitative research: A practical handbook*. London: Sage.

Simon, M. (1995) Reconstructing Mathematics Pedagogy from a Constructivist Perspective, *Journal of Research in Mathematics Education*, Vol. 26, No. 2, pp. 114-145.

Simpson, E. and Courtney, M. (2002) Critical Thinking in Nursing: A Literature Review. *International Journal of Nursing Practice*, Vol. 8, pp. 89-98

Singh, K., Granville, M. and Dika, S. (2002). Mathematics and science achievement: effects of motivation, interest, and academic engagement. *Journal of Educational Research*, Vol. 95, No. 6, pp. 323-332.

Singh, M. (1984). *A Critical Evaluation of Nationalized English Textbook for Classes VI to X of Haryana*, PhD. Thesis submitted to Kurukshetra University, India.

Skemp, R. (1978) Relational understanding and instrumental understanding. *Arithmetic Teacher*, Vol. 26, No. 3, pp. 9–15.

Skolverket. (2001). *Syllabuses for the compulsory school*. Stockholm: Fritzes.

Slavin, R. E. (2000) *Educational Psychology* (6th Edition), London: Allyn and Bacon.

Smith, G. H., Wood, L. N., Coupland, M., Stephenson, B., Crawford, K., & Ball, G. (1996). Constructing mathematical examinations to assess a range of knowledge and skills. *International Journal for Mathematical Education in Science and Technology,* Vol. 27, No. 1, pp. 65-77.

Smith, J. A. and Osborn, M. (2008) Interpretative phenomenological analysis. In J. A. Smith (ed).*Qualitative Psychology: A Practical Guide to Methods*. London: Sage

Smith, T. (2000). Bridging the Research Practice Gap: Developing a Pedagogical Framework That Promotes Mathematical Thinking and Understanding. *Mathematics Teacher Education and Development*, Vol. 2, pp. 4-16.

Smithson, J. (2000). Using and analysing focus groups: limitations and possibilities, *International Journal of Social Research Methodology*. Vol. 3, No. 2, pp. 103-119

Snape, D. and Spencer, L. (2004). The Foundations of Qualitative Research. In J. Ritchie and J. Lewis (eds.), *Qualitative Research Practice*, 2nd ed., London, Sage Publications, p. 4

Sood, S., and Jitendra, A. K. (2007). A comparative analysis of number sense instruction in reform-based and traditional mathematics textbooks. *The Journal of Special Education*, Vol. 41, No. 3, pp.145-157.

Squire, L. R., Knowlton, B. and Musen, G. (1993). The structure and organization of memory. *Annu. Rev. Psychol*. Vol. 44, pp. 453-95

Stake, R. E., Hood, S. and O'Sullivan, R. G. (eds.) (1991). *Advances in Program Evaluation: A Research Annual: Using Assessment Policy to Reform Education,* Part A, Vol. 1. Greenwich, CT: JAI Press

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, 2, 50-80.

Stein, M. K., Grover, B. W. and Henningsen, M. (1996). Building Student Capacity for Mathematical Thinking and Reasoning : An Analysis of Mathematical Tasks Used in Reform Classrooms. *American Educational Research Journal*, Vol. 33, No. 2, pp. 455-488

Stein, S. K. (1963), *Mathematics, the Man-Made Universe*. San Francisco: W. H. Freeman.

Stenger, C. L. (2000) . Characterization of University Students` Mathematical Thinking. *D.A.I-A*, Vol. 60, No. 11, pp. 39-44.

Sternberg, R. J. (1986). *Intelligence applied*. Orlando, FL: Harcourt Brace Jovanovich.

Stouthamer-Louber, M. and Van Kammen, W. B. (1995). *Data Collection and Management: A Practical Guide*, London: Sage Publications, Inc.

Stratton, P. (1997). Attributional coding of interview data: Meeting the needs of longhaul passengers. In N. Hayes (Ed.), *Doing qualitative analysis in psychology*. Hove, UK: Psychology Press, pp.115-142.

Stray, C. (1994). Paradigms regained: Towards a historical sociology of the textbook. *Journal of Curriculum Studies*, Vol. 26, No. 1, pp. 1-29

Stubbs, M. H. (1996). Using Rocketry in the Curriculum: Some Mathematical Applications. *Australian Science Teachers Journal,* Vol. 42, No. 4, pp. 51-53

Sumner, W. G. (1907). *Folkways: A study of the sociological importance of usages, manners, customs, mores, and morals.* Boston: Ginn, p. 631

Sutherland, P. (1992). *Cognitive development today: Piaget and his critics*. London: Paul Chapman Publishing Ltd.

Suydam, M. N. and Weaver, J. F. (1977). Research on Problem-solving: Implications for elementary school classrooms. *The Arithmetic Teacher* Vol. 25, No. 2, pp. 40-42

Sweller, J., van Merrienbeer, J. and Pass, F.G. (1998). Cognitive architecture and instructional design. *Educational Psychology Review*, Vol. 10, pp. 251-296.

Taba, H. (1966). *Teaching Strategies and Cognitive Functioning in Elementary School Children.* San Francisco: San Francisco State College

Taba, H. (1962). *Curriculum Development: Theory and Practice*. Harcourt, Brace and Company, New York. p.7

Tall, D. (1991). *Advanced Mathematical Thinking*. Dordrecht, Netherlands: Kluwer Academic Publishers.

Tall, D. (1992). The transition to advanced mathematical thinking: Functions, limits, infinity and proof. In D. A. Grouws (Ed.), *NCTM handbook of research on mathematics teaching and learning.* NewYork: Macmillan, pp. 495–511.

Tall, D. (1996) Functions and calculus. In. A. J. Bishop K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp.289-325). Dordrecht, The Netherlands: Kluwer

Tall, D. (1998) 'Symbols and the Bifurcation between Procedural and Conceptual Thinking'. Paper presented at the International Conference on Teaching Mathematics at Pythagorion, Samos, Greece in July 1998.

Tall, D. (1999). Reflections on APOS theory in elementary and advanced mathematical thinking. In *Proceedings of the PME Conference*, Vol. 1, pp. 1-111.

Teixeira, K. (2002). An Experimental Study Comparing Critical Thinking Growth and Learning Styles in a Traditional and Work Shop Based Introductory Mathematics Course *D.A.I-A* , 62 (10), 3327 .

Thompson, A. G. (1985). On Patterns, Conjectures, and Proof: Developing Students' Mathematical Thinking. *Arithmetic Teacher*, Vol. 33, No. 1, pp. 20-23

Thomson, S. and Fleming, N. (2004). *Summing it up: Mathematics achievement in Australian Schools in TIMSS 2002.* Camberwell, VIC: Australian Council for Educational Research

Thornton, R. (1987). Tools for scientific thinking-microcomputer-based laboratories for physics teaching *Physics Education*, Vol. 22, pp. 230-238.

Thurstone, L. L. (1929). Attitudes can be measured. *American Journal of Psychology,* Vol. 33, pp. 529-554.

Tobias, S. (1993). What Makes Science Hard? A Karplus Lecture. *Journal of Science Education and Technology*, Vol. 2, No. 1, pp. 297-304.

Törnroos, J. (2004). *Finnish Mathematics Textbooks in Grades 5-7.* Paper presented at the 2nd Scandinavian Symposium on Research Methods in Science and Mathematics Education.

Torrance, H. (1995) The role of assessment in education reform. In H. Torrance (ed.), *Evaluating Authentic Assessment: Problems and Possibilities in New Approaches to Assessment*. Buckingham, UK: Open University Press, pp. 144–156.

Tulving, E. (1993). What is episodic memory? *Current Directions in Psychological Science*, 2, pp. 67-70.

Turner, J. C. and Rossman, K. (1997). Encouraging Mathematical Thinking. *Mathematics Teaching in Middle School*, 3(1), pp. 66 -72

Twomey-Fosnot, C. and Dolk, M. (2001). *Constructing Number Sense, Addition and Subtraction*. Portsmouth, NH: Heinemann.

Ulep, S. (2007). Enhancing elementary school mathematics teachers' mathematical thinking through in-service training. *The APEC Project Collaborative Studies in Innovations for Teaching and Learning Mathematics in Different Cultures (II),* Center for Research on International Cooperation Education Development (CRICED), University of Tsukuba, Japan, Vol. 2, pp. 91-105

U.G.C. (1974), *Mathematics in India, Meeting the Challenge*. New Delhi: University Grants Commission.

Unabi, H. (1991). *Aspects of Critical Thinking in the Instruction of Secondary School Mathematics Teachers.* Unpublished Master's Dissertation, Jordanian University, Amman, Jordan.

Usfoor, W. (1998), Development of Students' Thinking. Study Topic EP 31, Amman, College of Education, UNRWA, Jordan.

Usfoor, W. (1999). Explicit Teaching of the Thinking Skills. *Al-Mu'alim and Student Journal*, Issue 3/4, pp. 23 – 28.

Usnick, V. E. and Brown, S. (1992). Assessing Students` Mathematical Thinking Interpersonal Process Recall . *Focus on Learning Problems in Mathematics,* 14 (41), 56-62

Vacc, N. N. and Bright, G. W. (1999). Elementary Preservice Teachers` Changing Beliefs and Instructional of Children's Mathematical Thinking. *Journal for Research in Mathematics Education,* Vol. 30, No. 1, pp. 89-111

Valverde, G. A., & Schmidt, W. H. (1997/1998). Refocusing U. S. math and science education. *Issues in Science and Technology,* Vol. 14, pp. 60-66.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., and Houang, R. T. (2002). *According to the Book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht: Kluwer Academic Publishers.

Van den Akker, J., Gravemeijer, K., McKenney, S. and Nieven, N. (2006). Introducingeducational design research. In J. van den Akker, N., K. Gravemeijer, S. McKenney and N. Nieven (Eds.) *Educational Design Research*,. London: Routledge, pp. 3-7.

Van Oers, B. (1996). Are You Sure? Stimulating Mathematical Thinking during Young Children's Play. *European Early Childhood Education Research Journal*, Vol. 4, No. 1, pp. 71-87.

van Teijlingen, E. R. and Hundley, V. (2001). *The Importance of Pilot Studies.* Social Scence Update No. 35, Univeisity of Surrey: Surrey, UK

Vassiliev, A. (1998). *The History of Saudi Arabia*. London: Saqi

Verhoeven, P. (2006). Statistics education in the Netherlands and Flanders: An outline of introductory courses at Universities and Colleges. In *ICOTS-7 Conference Proceedings.* Available from https://www.stat.auckland.ac.nz/~iase/publications/17/3A4_VERH.pdf Accessed 31st January, 2014

Verschaffel, L., Greer, B., and De Corte, E. (2007). *Whole number concepts and operations.* In Lester, F. K. (ed.) *Second Handbook of Research on Mathematics Teaching and Learning*, Greenwich, CT: Information Age Publishing, pp. 557–628.

Verschaffel, L. Greer, B. and DeCorte, E. (2000) *Making sense of word problems*. Lisse, The Netherlands: Swets and Zeitlinger Publishers.

Villarreal, M. (2000). Mathematical Thinking and Intellectual Technologies: The Visual and the Algebraic. *For the Learning of Mathematics,* Vol. 20, No. 2, pp**.** 2-7

Vincent, J. and Stacey, K. (2008) Do Mathematics Textbooks Cultivate Shallow Teaching? Applying the TIMSS Video Study Criteria to Australian Eighth-grade Mathematics Textbooks. *Mathematics Education Research Journal*, Vol. 20, Issue 1, pp. 82-87

Vincent, J. and Stacey, K. (2008). Do Mathematics Textbooks Cultivate Shallow Teaching? Applying the TIMSS Video Study Criteria to Australian Eighth-grade Mathematics Textbooks. *Mathematics Education Research Journal*,20 (1), pp.82- 107

Voogt, J. M. and Odenthal L. E. (1997). *A portrait of emerging practices: A conceptual framework*. Enschede, the Netherlands: University of Twente

Vosniadou, S. and Vamvakoussi, X. (2006). *Examining mathematics learning from a conceptual change point of view*. In Verschaffel, L., Dochy, F., Boekaerts, M., and Vosniadou, S. (eds.) *Instructional Psychology: Past, Present and Future Trends*, pp 55–72. Oxford: Elsevier Ltd.

Vygotsky, L. S. (1978). *Mind in Society*. Cambridge, MA: Harvard University Press.

Wadsworth, B. J. (1979). *Piaget's theory of cognitive development*. New York: D. McKay.

Wadsworth, B. J. (1984). *Piagets Theory of Cognitive and Affective Development* (Vol. 3). USA: Longman Inc.

Wakeel, H. A. (1982). *Curricular Development*, 7^{th} ed. Cairo: Anglo-Egyptian Library.

Wakefield, A. P. (1998). Support Math Thinking. *Education Digest*, 63(5) , 59-56 .

Wall, A. L. (2001), Evaluating an Undergraduate Unit Using a Focus Group, *Quality Assurance in Education*, Vol. 9, No. 1, pp. 23-31.

Warren, C. (1981). Adopting Textbooks. In J. Cole and T. Sticht (eds.) *The Textbook in American Society*. Washington, DC: Library of Congress.

Watson, A. (2001). Low Attainers Exhibiting Higher-Order Mathematical Thinking *Support for Learning,* Vol. 16, No. 4, pp. 179-184.

Watson, G. and Glaser, E.M. (1980). *Watson–Glaser Critical Thinking Appraisal: Forms A and B.* San Antonio, TX: PsychCorp.

Watson, J. (1996). Reflection through Interaction: the classroom experience of pupils with learning difficulties. London: Falmer.

Watson, J. (2001) Social constructivism in the classroom. *Support for Learning,* Vol. 16, No. 3, pp. 140–148.

Watt, H. M. G. (2004). Development of adolescents' self-perceptions, values, and task perceptions according to gender and domain in 7th through 11th grade Australian students. *Child Development*, No. 75, pp. 1556–1574.

Webb, N. L. (1992). Assessment of students' knowledge of mathematics: Steps toward a theory. In D.A. Grouws (ed.), *Handbook of research on mathematics teaching and learning* (pp. 661-683). New York: Macmillan.

Webb, N. L. (1997) Determining alignment of expectations and assessments in mathematics and science education. *NISE Brief*, Vol. 1, Issue 2, pp. 1-8

Weiss, S. (2003). *Mathematical Thinking. Teaching Children Mathematics* , Vol. 9, No. 7, pp. 431-433

Wellington, J. (2000). *Educational Research: Contemporary Issues and Practical Approaches*. London: Continuum

Wertheimer, M. (1959). *Productive Thinking*, New York: Harper and Row

Westwood, P. (1993). *Commonsense Methods for Children with Special Educational Needs.* London: Routledge.

Wiersma, W. and Jurs, S. G. (2005). *Research Methods in Education: An Introduction*, Boston, Mass.; London, Pearson Education

Wilkinson, S. (1998), Focus group methodology: a review. *International Journal of Social Research Methodology, Theory and Practice*, Vol.1, pp.181–204.

Williams, D. D. (2002). Evaluation of learning objects and instruction using learning objects. In D. A. Wiley (ed.), *The instructional use of learning objects*. Bloomington: Agency for Instructional Technology/Association for Educational Communications & Technology.

Wilson, P. S. (1993). *Research Ideas for the Classroom: High School Mathematics*. New York: Macmillan Publishing Company.

Womack, D. (1988). *Developing Mathematical and Scientific Thinking in Young Children*. London: Cassell Education Limited.

Wood, T., Williams, G. and McNeal, B. (2006). Children's Mathematical Thinking in Different Classroom Cultures. *Journal for Research in Mathematics Education*, Vol. 37, No. 3, pp. 222-255

Woolfolk, A. (2007). *Educational Psychology*, London: Pearson International Edition: Pearson.

Yackel, E. and Rasmussen, C. (2003) Beliefs and norms in mathematical classrooms. In G.C. Leder, E. Pehkonen & G. Törner (eds.), *Beliefs: a hidden variable in mathematics education?* Dordrect, Netherlands: Kluwer Academic Publishers.

Yackel, E. and Cobb, P. (1996*).* Sociomathematical norms, argumentations, and autonomy in mathematics*. Journal for Research in Mathematics Education,* Vol. 27, pp. 458–477

Yang, M. J. (2000). *Problem Solving in Chemistry at Secondary School*. Unpublished PhD thesis. Glasgow: University of Glasgow, Glasgow.

Yelinik, M. (1998). *Inductive Mathematical Thinking.* Translated by the College of Education. Amman: Ministry of Education – UNRWA / UNESCO.

Younis, M. (1991). Patterns of *Mathematical Thinking in the Case of the Intermediate School Students.* Unpublished Master's Dissertation, Jordanian University, Amman, Jordan.

Zimmerman, B. and Schunk, D. (Eds.). (1989). *Self-regulated learning and academic achievement: Theory, research, and practice.* New York: Springer-Verlag.

Zimmerman, C. (2007). The development of scientific thinking skills in elementary and middle school. *Developmental Review*, Vol. 27, No. 2, pp. 172-223.

Zohar, A. and Dori, Y. (2003). Higher Order Thinking Skills and Low-Achieving Students: Are they Mutually Exclusive? *Journal of the Learning Science*, Vol. 12, No. 2, pp. 145 - 181.

Appendices

Appendix 1

Appendix 1

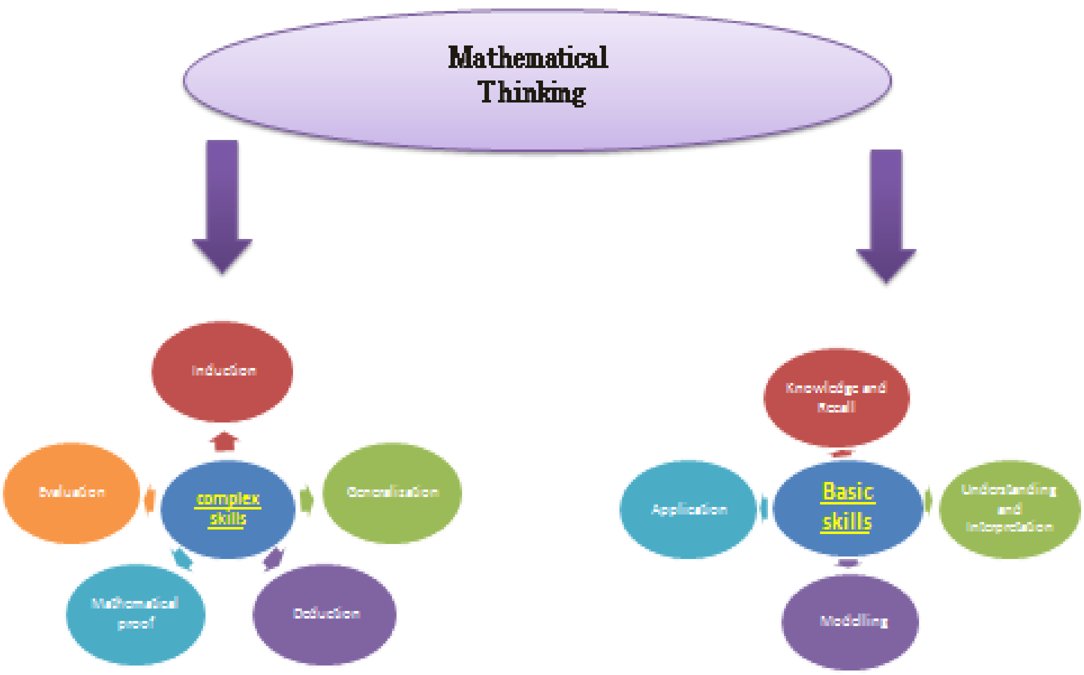
An analysis model

The nine aforementioned thinking skills were distributed on two levels as follows.

These were divided after reviewing the literature and following the advice of the expert referees.

1- Four basic cognitive skills, namely:
- Knowledge and Recall
- Understanding and Interpretation,
- Modelling
- Application.

2- Five complex cognitive skills, namely:
- Induction
- Generalization
- Deduction
- Mathematical proof
- Evaluation.



Appendix 2

Appendix 2

The questionnaire

Dear Teacher/Inspector,

This questionnaire is on the evaluation of the questions in the mathematics textbooks in Saudi schools in terms of formulation, layout, the extent to which they encourage mathematical thinking, and the extent to which they promote positive attitudes towards mathematics.

There are 53 items in total. Please take the time to read each one carefully before giving your answer. There are no right or wrong answers. Just put a tick beside the answer that corresponds most closely to your own opinion.

This research is being undertaken as part of the research for the degree of Ph.D. at the University of Strathclyde in the UK. Its aim is to discover the opinions of teachers and inspectors on the textbook questions and the questionnaire is being given to you as you are a suitable person to complete it.

You may rest assured that all your answers will be held in complete confidence, so you may feel free to respond honestly. At no time will your identity be revealed to any third party and your answers will be used only for academic purposes. Your co-operation is greatly appreciated.

Thank you very much.

Mohammed Alzahrani

Please give the following information, which will help with the organisation and analysis of the responses.

- **Directorate of Education**

☐ Riyadh ☐ Makkah ☐ Al-Madinah ☐ azan ☐ ubouk ☐ l-Jouf ☐ bha

☐ Al-Qaseem ☐ Northern Borders ☐ Eastern Region ☐ eddah ☐ l-Qunfudah

☐ Al-Laith ☐ Al-Baha ☐ l-Taif ☐ hamis Mishait ☐ -Kharj ☐ nbu

☐ Unayzah ☐ Al-Mikhwah ☐ Al-Qurayat ☐ nother (please specify) ………………….

- **Current position**

☐ Inspector ☐ Teacher

- This questionnaire is to evaluate the questions in the mathematics textbook for grade:

☐ 1st ☐ 2nd ☐ 3rd

- **Qualification**

☐ Bachelor's Degree

☐ Postgraduate Higher Diploma

☐ Master's Degree

☐ Ph.D.

- **Years of experience**

☐ Less than 5 years

☐ 5-9 years

☐ 10-14 years

☐ 15 years or more

- Have you ever participated in training courses in the fields of measurement and evaluation or in the fields of curricula and teaching methodology?

☐ Yes ☐ No

N	Statements	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	*The language in the questions is clear and easily understood.*					
2	*The mathematical problems vary in terms of words and symbols.*					
3	*Questions are free of spelling and printing mistakes.*					
4	*Questions are free of factual errors.*					
5	*Symbols and terms used are similar to those used in the textbook's content.*					
6	*Questions are brief, but contain all necessary information.*					
7	*Questions do not include hints to make answering easier.*					
8	*Questions are accurately formulated.*					
9	*The formulation of the exercises at the end of each lesson are similar to those in the textbook's content.*					
10	*Questions are varied to include questions requiring student-produced responses together with objective questions.*					
11	*The exercises include suitable and logical tables, figures and numbers.*					
12	*The directions of the exercises are formulated in such as way as to need no inquiry from the students.*					
13	*The exercises in the textbook are characterised by academic accuracy.*					
14	*Questions in the textbook are displayed in a manner that is interesting and encouraging for the readers.*					
15	*There is enough space between each exercise and the next.*					
16	*There is enough space between the main part of the exercises and the secondary parts.*					
17	*Figures and diagrams related to the exercises are positioned appropriately.*					
18	*Exercises are accurately numbered.*					
19	*The items of the questions and exercises are accurately numbered.*					
20	*No paragraph of any exercise is divided between two pages.*					
21	*Exercises are not crowded on each page.*					
22	*Exercises are written in clear fonts.*					
(Items 23-47) The questions in the textbooks help students to						
23	*Acquire the mathematical knowledge represented in the concepts and symbols.*					
24	*Recall mathematical knowledge stored in the memory.*					
25	*Organise knowledge in a logical manner.*					
26	*Formulate the given information by means of new words or symbols.*					
27	*Interpret the various relationships in mathematical problems.*					
28	*Use more than one skill, such as mathematical reasoning, comparison, classification, justification, etc.*					
29	*Mathematically represent the given data in an easy to understand manner.*					
30	*Perceive the relationships between the given data in order to obtain the required deductions.*					
31	*Make tables and graphs of the given data.*					
32	*Use mathematical knowledge in new situations.*					
33	*Analyse the new mathematical knowledge in order to perceive its relationships with the previous mathematical knowledge.*					
34	*Arrive at a new result based on particular examples or observations.*					
35	*Indicate relationships between introductions and results.*					
36	*Use statements to describe particular cases.*					
37	*Formulate general statements that include multiple features of cases.*					
38	*Express the general rule using precise language.*					
39	*Indicate the particular cases that follow the general rule.*					
40	*Indicate the relationship between the particular and general mathematical cases.*					
41	*Apply the general rule to a particular case.*					
42	*Distinguish between given data and what is required.*					

43	*Link the results to each other in order to reach the required goal.*					
44	*Use more than one mathematical proof method to solve the problem.*					
45	*Use criteria to pass judgment.*					
46	*Prove the validity of mathematical rules.*					
47	*Discover mistakes in mathematical relationships.*					
48	*Questions include emotional aspects (tendencies, attitudes, values … etc).*					
49	*Questions are relevant to situations in the students' daily lives.*					
50	*Questions encourage students to apply what they learn in a practical way in their daily lives.*					
51	*Questions and exercises contribute to developing positive attitudes towards mathematics.*					
52	*Questions and exercises indicate the role of mathematics in daily life.*					
53	*Questions and exercises indicate the role played by mathematics in other subjects.*					

Appendix 3

Appendix 3

Interview Schedule

1. What is your knowledge of mathematical thinking?

2. What is your view of mathematical thinking?

3. Do you think that developing mathematical thinking should be one of the goals of mathematics education in Saudi Arabia? Please give a reason for your answer.

4. Do you believe that the questions in the mathematics textbooks encourage mathematical thinking? Please give a reason for your answer.

5. What are the obstacles to students learning to think mathematically?

6. Does your inspector encourage you to develop your students' mathematical thinking skills? Give a reason for your answer.

7. Do you think short courses for teachers are important to develop students' mathematical thinking skills? Please give a reason for your answer.

8. What are the obstacles to developing the questions in the maths textbooks in terms of the students' mathematical thinking?

9. How can we overcome the difficulties that students face in understanding and applying mathematics?

10. How can we encourage students to have a positive attitude towards mathematics?

11. Do you have any suggestions regarding the evaluation and development of the questions in the mathematics textbooks?

The questionnaire and interview schedule are shown above in an English translation solely for the benefit of the reader. It should be noted that in the research, the Arabic questionnaire and interview schedule only were applied.