Essays on Climate Change and Political Economy

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Declaration

This thesis is the result of the author's original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.

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Synopsis

This thesis consists of three separate chapters. Chapter 1 entitled *Could climate policy be conducted through pensions?* models a novel approach to climate policy. It suggests that a tax relief on "green" pension savings could constitute a reasonable alternative to carbon taxes. Chapter 2, *Optimal policymaking across the democratic spectrum: a dynamic view*, develops a dynamic model of political economy which is used to study the impact of democracy on economic outcomes, in particular economic growth. Chapter 3, *Can autocracies save climate?*, builds on the model developed in Chapter 2 by adding a climate externality. It investigates the influence of the level of democracy on policymakers' ability to limit carbon emissions.

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Contents

Chapter 1

Could climate policy be conducted through pensions?

Abstract

Despite the theoretical effectiveness of carbon taxes as an instrument of climate policy, political constraints still halt their more common adoption. Policymakers thus may need to implement climate policy via existing policy levers that are not explicitly labelled as carbon pricing. In this chapter we evaluate a novel - potentially politically feasible - approach of conducting climate policy through the pension system. While typically policymakers grant tax relief on all pension savings, we suggest that the relief could be granted only on "green" savings. To model the policy, we rely on the Diamond-type overlapping generations framework. We find that, conditional on the unconstrained optimal policy implementing a 2°C temperature rise, our constrained optimal policy implements a 2.3°C temperature rise.

1 Introduction

The scientific consensus about the impact of anthropogenic greenhouse emissions and predictions of relatively imminent and significant damages to the economy has resulted in treaties such as the Paris Agreement. Under this, 195 countries agreed to limit the long-term global temperature rise below 2°C above pre-industrial levels. Thereby, it has led to a change in the narrative concerning the mitigation engagement from *whether* to *how*. The issue of financing the net-zero transition, however, still poses obvious economic and social obstacles. Most economic models of climate change produce optimal policy in the form of carbon pricing (e.g. Golosov et al., 2014; Barrage, 2020). This optimal policy is typically proposed to be implemented via carbon taxes or permits, however, due to political reasons, the introduction of carbon taxes remains rather slow.

According to Baranzini and Carattini (2017), public opposition against carbon taxes often stems from the scepticism regarding the issues like distributional impacts on poor households or effects on employment and competitiveness. Moreover, the authors point to the general disbelief in environmental effectiveness of carbon taxes, despite the theoretical economic rationale. Such opposition manifests especially when carbon tax policy plans are announced to the public. In practice, a vivid example of a public disapproval took place in 2016 and 2018 in the State of Washington. The citizens participating in the referendums rejected the proposal for a tax of $15 per ton of CO_2. Given the closeness of the 2050 goal declared in the Paris Agreement, effective climate policy appears to require measures that are politically feasible.

In this chapter, we examine a novel approach to climate policy, which nevertheless relies on standard instruments. We consider pension tax relief as a potential vehicle to conduct climate policy. Namely, could a policy of requiring pension funds to invest in "green projects" in exchange for their tax relief advantages constitute a reasonable alternative to carbon taxes? The primary argument favouring our approach concerns its potential political feasibility, especially for countries that already grant some form of tax relief on pension savings, such as e.g. the United Kingdom, the United States or Belgium. Utilising an existing policy might prove more acceptable to the public as it does not explicitly involve introducing new taxes.

In practice, several countries have, to some extent, incorporated pensions into broader climate policy. However, rather than using them as an actual tool, policymakers rely on the interplay between carbon tax revenue and its subsequent recycling. For instance, Norway created a public pension fund continually financed by income from oil drilling licences and carbon taxes (Sumner et al., 2009), whereas Germany reallocated the bulk of the eco-tax receipts to the existing public pension system (Weidner, 2008).

In terms of studies, perhaps the ones closest conceptually to our paper are the ones of von Below et al. (2016) and Dam (2011), both of which utilise the Diamond (1965) model. Apart from these papers (which we discuss in more detail in section 2), however, economic modelling literature oriented on exploring fiscal alternatives or complements to carbon taxes remains largely silent on the notion of pensions as a possible element of climate policy.

To address this gap, we combine the well-studied tax relief or subsidy approach with a novel way of its application. We model existing tax relief on pensions savings to see if variation in its form or rate could be used as climate policy. Specifically, we model a Diamond-type dynamic overlapping generations economy to differentiate between workers and pensioners, and thereby to directly consider pension savings. We calculate the constrained optimum policy, subject to instruments available to policymaker. The policies that we evaluate include our proposed policy of tax relief on green pension investment, a standard policy of tax relief on entire pension investment, as well as a policy mix of the latter with carbon taxes.

Given an OLG model with an environmental externality, there are two sources of inefficiency: the incomplete markets problem of being unable to trade with unborn future generations, and the global environmental externality. Clearly, therefore, implementing the socially optimal solution requires two independent policy instruments such as pensions tax relief and a carbon tax. Finding the constrained optimum subject to being able to use only pensions tax relief inevitably cannot achieve this. Nevertheless, the green pension policy, in implementing this constrained optimum, might constitute a reasonable - and potentially politically acceptable - alternative to carbon taxes. Although it is associated with a temperature rise of 2.3°C (i.e. additional 0.3°C relative to the goal consistent with the Paris Agreement) in our calibration, such a policy yields a negligible welfare loss compared to the social optimum.

Above all, our paper's primary contribution is conceptual and relates to the growing "second-best" climate change economics literature (discussed in section 2). While we rely on a well-studied tax relief approach, we model and evaluate – to the best of our knowledge – a novel proposal involving pensions as a climate policy tool.

The rest of this report is structured as follows: the next section provides an overview of the literature oriented on alternative climate policy instruments; section 3 introduces the baseline model; section 4 discusses the optimisation process related to the social planner's solution and climate policy variants; section 5 reports the numerical simulations and evaluates the policies; and section 6 concludes.

2 Alternatives to carbon taxes in the literature

2.1 Pensions

The two studies which rely on the Diamond (1965) model and tackle the issue of pensions in the environmental context are by von Below et al. (2016) and Dam (2011). The former proposes a Pareto-improving deal to resolve the trade-off between the coexisting generations. In principle, current pensioners would not experience the benefits of costly mitigation efforts (financed e.g. by carbon taxes). Therefore, to make them indifferent, the younger generation shall compensate them via pension transfers. The young, in turn, inherently save less, although expect to ease the climate damages they will experience in the future. Thus, they become better off if the discounted value of their own prospective retirement returns exceeds the loss attributable to the *total* abatement costs. What stems from such a bargain is that the economy experiences a substantial increase in overall abatement, and a higher price of carbon becomes acceptable.

Dam (2011) relates to pensions only implicitly, through attention to retirement consumption. Nevertheless, the author suggests that the intergenerational coordination problem (resulting in overaccumulation of pollution) could be resolved through the security market and socially responsible investment funds which attach environmental quality to the firm's intrinsic value. The forward-looking character of the financial market then incentivises the young to reduce pollution (to sustain firm's value and, therefore, funds available for consumption after they retire) and to indirectly consider the impact on future generations, implying no corrective policy is necessary.

2.2 Subsidies and tax reliefs

Other, possibly more diverse in design or application, prevalent instruments are subsidies and tax reliefs. As Aghion et al. (2014) recognise, while discouraging dirty production, carbon taxes alone provide limited means to induce a swift development of clean technologies. For instance, innovation may initially focus around the efficiency of combustion (however important in the green transition, too), rather than on renewable solutions. Standard climate policy, according to the authors, should therefore be reinforced with government subsidies, such as green investment tax breaks. Hoel (2012) claims that a second-best policy involving green subsidies is justified when the existing price of carbon is set below its optimal level – one of the reasons being e.g. public opposition. Moreover, Acemoglu et al. (2012) and Golosov et al. (2014) argue for the vital role of subsidies, especially for the economy to endogenously direct resources toward green technology.

The tax relief approach is among the climate tools evaluated by Monasterolo and Raberto (2016). It is used to stimulate investment in renewables but is found to comparably depress the overall economic performance (hence, the authors advocate a green monetary policy). However, the fiscal policy still performs better than the business-as-usual scenario, providing a rationale for further consideration.

Kalkuhl et al. (2013) demonstrate a comprehensive welfare analysis of four second-best regimes involving a subsidy to renewable energy: "feed-in-tariff" where subsidies are financed by taxing the energy sector (fossil and nuclear alike); "carbon trust" where carbon tax on emissions is recycled in full toward renewables; "renewable energy subsidy" where the pure subsidy is financed by lump-sum taxes on households; and "temporary subsidy policy" where initial subsidies are gradually displaced by carbon taxes (advised particularly when optimal carbon taxes are not politically viable in the short-run). Relative to the first-best optimal carbon pricing scheme, the highest consumption losses are associated with the pure subsidy policy. On the other side of the spectrum is the carbon trust policy, with the feed-in-tariff marginally more costly. The temporary subsidy approach, in turn, can be the closest to the optimum; however, only for a shorter displacement window – the longer it takes to replace the subsidy with carbon taxes, the higher the welfare losses.

2.3 Role of financial market

Subsidies are also explored in relation to another area of interest (even if our paper tackles the issue only implicitly): the financial market. Renström et al. (2021) extend Dam's (2011) framework with socially responsible investors and develop a dynamic general equilibrium model where individuals can choose between a firm's shares and green government bonds. The firm's share value is determined chiefly by its production and its "cleanness rating". Therefore, the firm can decide to abate to avoid paying higher pollution premia to the investors - which the existing system of pollution tax and abatement subsidy should further incentivise. Ultimately, the authors find higher pollution taxes to decrease pollution successfully but at the cost of the economy's performance and individual consumption. On the other hand, increased subsidies still contribute to pollution mitigation (although less effectively) while improving the scale of the economy and consumption. Their results seem to suggest that a politically feasible complement to climate policy has a potential to exert indeed positive economic impacts – in socially responsible environments, at least.

A simpler fiscal policy in a similar setting is studied by Dam and Heijdra (2011). They assess the interaction between public abatement funded by lump-sum taxes and socially responsible private investment, however without any policy that would further incentivise such efforts. The key finding from the paper is that socially responsible investment partially offsets the positive impact of public mitigation due to the crowding-out effect.

Lastly, in the contemporarily important context of developing countries, Davin et al. (2023) examine the environmental impacts of debt relief combined with public abatement. Their OLG model considers a bilateral agreement between a high and low-income country where public debt reduction of the latter (channelled through the financial market) is financed by the richer one. Although the study does not reflect on the optimality of the solution, it suggests that environmental quality can indeed improve and – depending primarily on how mobile assets are – both countries can experience welfare enhancement as a result of the debt transfer.

Overall, the literature considers various policy designs which could aid the transition towards a net-zero economy. However, it appears that the economic modelling literature has not yet properly addressed the possibility of conducting a climate policy through pensions. Moreover, a clear gap remains with respect to the evaluation of the specific policy which would grant tax relief on green pension savings only.

3 The model

In the following section, we introduce the general model based on the overlapping generations framework developed by Diamond (1965). The economy is characterised by the simultaneous existence of two finitely-lived generations of people. These individuals are assumed to live for two periods. At each period t a new generation of workers enters the labour force, earns labour income and makes consumption-saving decisions. At the beginning of the subsequent period, the generation transforms into pensioners who live on capital income. Similarly, a generation of existing pensioners dies every period and exits the model.

Firms are perfectly competitive and pay labour and capital their marginal products. The model also considers the environment: there is an externality caused by production which damages utility flows. Such negative impacts can be reduced by private abatement spending.

Below we expose the baseline model's components, establish their elemental dynamics and study the general behaviour of the key variables. Later, in section 4.2 we introduce extensions which capture specific climate policies.

3.1 Environment

To model the externality, we firstly define how environmental quality, E_t, evolves over time. Later, in section 3.3, the variable is employed to the utility function of households. After John and Pecchenino (1994), one can interpret E_t in multiple ways, ranging from the quality and cleanliness of water to certain biodiversity measures. We interpret it as some measure of the inverse of the greenhouse gases concentration, an index of climate change-related performance or simply *climate* in general. Later, in the calibration section, we translate the changes in E_t to the rise of global average temperatures.

We do not consider any source of possible natural degradation or recovery and assume the environment only to change in line with net emissions. It worsens as a result of output production, Y_t, net of private abatement a_t. The latter may include all activities aimed at reducing the potential concentration of greenhouse gases, such as energy-efficiency enhancement or shift to alternative energy sources – aggregated simply by the notion of green projects or green investment. We adapt Davin et al. (2023) specification – which itself is an incarnation of the widely recognised design of John and Pecchenino (1994) – so that it serves a single-economy case with private mitigation. The evolution of environmental quality "stock" is therefore expressed as

$$E_{t+1} = E_t - \theta Y_t + \phi a_t, \tag{1}$$

where Y_t and a_t denote global output and abatement. The actual marginal effect that contemporaneous production and mitigation activities exert on the environment is captured respectively by the emission factor $\theta > 0$ and the efficiency of abatement factor $\phi > 0$. These parameters are assumed $\phi > \theta$ and constant[1].

The benefit of formulating E's evolution as a function of net emissions and the current state of the environment is that environmental quality "memorises" the past accumulation of those emissions. This way, it can quite realistically reflect the fact of storing the greenhouse gases in the atmosphere (i.e. the gases tend to remain there for a relatively long time), consistent with e.g. Dietz et al. (2020). There is no theoretical upper bound related to the value of E, however, we assume, via preferences that are discussed in Section 3.3, that $E_t = 0$ would be associated with a climate catastrophe or human extinction.

Moreover, we can see that, holding all else constant, an additional unit of abatement is always beneficial to the environment. It is also equally beneficial across its all levels, as given by the constant returns to abatement. Likewise, output exerts a constant, negative marginal effect on environmental quality.

3.2 Firms

We assume a simple Cobb-Douglas specification for the supply side of the economy. Firms produce output using labour and capital which are inelastically provided by workers and pensioners respectively:

$$Y_t = {L_t}^{1-\alpha} K_t^{\alpha}, \qquad L, K > 0 \tag{2}$$

where α symbolises capital's share in production or, more generally, output's elasticity with respect to this input.

[1]The $\phi > \theta$ assumption seems reasonable - to fight climate change, we expect the mitigation efforts to be at least as efficient as the damages per unit of output - but follows mainly from how equation (1) is designed. To illustrate, let us imagine a hypothetical, extreme case where all output was dedicated to abatement spending, so that $a_t = Y_t$. Then, for the environmental quality to improve (i.e. $E_{t+1} > E_t$), we would still need the abatement efficiency factor to be higher than the emission factor. This can be seen by rewriting equation (1) and assuming $a_t = Y_t$: $E_{t+1} - E_t = (\phi–\theta)Y_t$. Then, for the LHS to be positive the $(\phi–\theta)$ subtraction also needs to be positive. This is ensured when $\phi > \theta$.

Profit-maximising behaviour requires firms to hire labour and capital up to the point where the marginal products of these inputs equal their prices. Hence, in the competitive economy, firms earn zero profits and pay the workers and pensioners the equivalent of their marginal products. Wage and rate of return on capital are therefore

$$W_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)\frac{K^\alpha}{L^\alpha} = (1-\alpha)\frac{Y_t}{L_t} \tag{3}$$

$$r_t = \frac{\partial Y_t}{\partial K_t} = \alpha L^{1-\alpha} K^{\alpha-1} = \alpha \frac{Y_t}{K_t}. \tag{4}$$

3.3 Preferences and budget constraints

Each period t, a homogeneous representative consumer who enters the workforce chooses – based on the current state of the economy and environment – a mix of consumption, investment and abatement which maximises their lifetime utility. The latter – which is additively separable in its arguments – is defined similarly to Davin et al. (2023) as

$$U_t^i = lnC_t + \beta lnC_{t+1} + \epsilon lnE_t + \beta\epsilon lnE_{t+1}. \tag{5}$$

Households have logarithmic preferences and β denotes the discount factor[2]. Workers care about consumption, C, in both periods of their life, but also consider the environmental quality, E, they experience, subject to the environmental sensitivity factor ϵ.

Individuals are assumed to follow a price-taking behaviour: there is a unit-mass of identical, infinitesimal households that do not internalise the possible economy-wide implications of their decisions on prices. We normalise the size of each generation to 1 and assume no growth in population. Workers receive a gross wage, W_t, which is used for current consumption and savings. Savings can be allocated in the form of either productive brown investment or abatement which will improve the environmental quality to be experienced after retiring.

Pensioners no longer earn W and consume all the proceeds from the income invested in period t while being workers (we do not consider any bequest motive so pensioners fully utilise their available budget). The said proceeds - or *pension* payments - are basically the return, r_{t+1}, realised on renting the accumulated capital to firms. The general case, therefore, results in the following budget constraints:

$$W_t = C_t + I_t + a_t \tag{6}$$

$$r_{t+1}K_{t+1} = C_{t+1}. \tag{7}$$

[2]Which formally is defined as $\frac{1}{1+\rho}$, where ρ symbolises the individual discount rate.

As implied, workers face a trade-off between saving in a way that allows consumption at old age (I_t) and a so-called "green investment" (a_t) conducive to the state of the environment. Effectively, we consider two types of assets in which workers can invest their savings: physical "brown" capital and "green projects" concerning the totality of intangible assets[3] oriented on financing emission-reducing endeavours. Investment in brown capital is more desirable in monetary terms (generates return r) but harms the environment, whereas green investment is not financially rewarding ($r = 0$) but instead improves environmental quality[4].

In regards to the dynamics of capital stock, the typical law of motion for capital applies:

$$K_{t+1} = (1 - \delta)K_t + I_t. \tag{8}$$

The above equation simply states that the stock of capital in the following period is determined by a sum of the current capital stock (subject to depreciation) and new investment. Capital is assumed to fully depreciate, what constitutes a reasonable assumption, considering that one t reflects half the lifetime of a generation. Thus, δ is set to 1 and new capital stock at $t + 1$ is determined solely by the investment outlays from the preceding period. Therefore, for the remainder of this paper, we skip the $1 - \delta$ parameter in the specifications and calculations.

4 Optimisation

In this section, we analyse the theoretical solutions to the optimisation problems faced by the agents in the model. This will serve as a groundwork for the numerical simulations discussed in section 5. Below we synthesise the baseline model - in the absence of any fiscal policy - specified in the previous section. From the perspective of an individual worker, the intertemporal relationship between the variables is primarily founded on the following equations:

$$W_t = C_t + I_t + a_t \tag{6}$$

$$r_{t+1}K_{t+1} = C_{t+1} \tag{7}$$

$$K_{t+1} = I_t \tag{8}$$

$$E_{t+1} = E_t - \theta Y_t + \phi a_t \tag{1}$$

$$Y_t = K_t^{\alpha} \tag{2}$$

In the model described above, a dynamic competitive equilibrium is characterised by a sequence of $\{K_t, E_t, C_t, I_t, a_t\}_{t=0}^{\infty}$ and a price path $\{r_t, W_t\}_{t=0}^{\infty}$ such that, for any given K_0 and E_0 , utility is maximised subject to the resource constraints, firms optimise (zero) profits and markets clear. By optimising the use of inputs, firms intrinsically dictate the equilibrium prices of inputs or income rates for

[3]Theoretically, they might be eventually realised in a physical form. Green capital formation, however, is not considered in this model.

[4]We assume the workers believe their abatement decisions indeed matter. In section 4.2, we discuss why it need not be the case.

households. Hence, firms' optimising behaviour can be implicitly evidenced by plugging the respective expressions for marginal products into wage and return identities. The interaction of the optimising behaviours then provides the general equilibrium and ensures the clearing condition.

The overall consumption-saving problem in this section is fourfold. Firstly, we characterise the solution of a central authority who is in the capacity to decide on choices on behalf of the households. Then, we turn to decentralised optimisation and solve the updated problem involving three different climate policies.

4.1 Social planner's solution

First of all, we characterise the social planner's solution, that is we maximise social welfare subject to the aggregate resource constraints, ignoring the policy levers that may implement such an allocation. The optimal plan we construct – due to the forward-looking character of the planner and ability to distribute resources in a manner not available to the market – shall also be efficient in the Pareto sense, in line with Blanchard and Fisher (1989). Therefore, the social planner overcomes the possibility of dynamic inefficiency often arising in the Diamond model and provides an idealised welfare benchmark for climate policies explored later.

The social planner shall have the entire output, Y_t, produced in the economy at their disposal, to be redistributed between consumption for both generations alive at t, investment in dirty capital (used to produce output in the following period) and abatement. The general form of the aggregate resource constraint, therefore, is written as[5]

$$Y_t = C_t^{workers} + C_t^{pensioners} + I_t + a_t. \tag{9}$$

Apart from the budget, the planner operates subject to the same laws as the rest of the economy. Namely, must follow the evolution of environmental quality (1) and the law of motion for capital (8). As of the latter, by combining it with (9) we can rewrite it in budget constraint terms, i.e.

$$K_{t+1} = Y_t - C_t - C_t' - a_t \tag{10}$$

to obtain an economy-wide capital stock accumulation identity available to the social planner, which intrinsically bounds the consumption-saving decisions. We implicitly consider brown investment, I_t, a control variable, although expressing it as a function of the resource constraint simplifies the calculations without any loss of generality. Then, by expressing Y_t as a function of capital, we eliminate the remaining superfluous variable to settle with

$$K_{t+1} = K_t^{\alpha} - C_t - C_t' - a_t \tag{11}$$

[5]Note that for clarity we add the superscripts referring to the specific generations. From this point onward, however, the notation will take the form of C_t applicable to consumption of workers and C_t' denoting consumption of pensioners.

and

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi a_t. \tag{12}$$

A feasible allocation denotes any sequence of choices for consumption and abatement $\{C_t, C'_t, a_t\}_{t=0}^{\infty}$ which satisfies the aggregate resource constraint (11). We are, however, interested in a choice rule which – for any potential value of the states – will maximise social welfare and, therefore, consider the aggregate utility. The social planner aims to maximise the joint welfare of all generations to come. This means that at each t the planner considers – subject to the social discount factor – the respective, contemporaneous utility of both generations alive at the same time. Thus, although the planner discounts future households' welfare, the concurrent generations are treated equally, without attaching weight to any specific age group. Imposing $C'_t = \alpha K_t^{\alpha}$ (such that pensioners simply consume their income), we consider the following social welfare function:

$$U_t^{social} = \sum_{s=0}^{\infty} \beta^s (lnC_{t+s} + ln(\alpha K_{t+s}^{\alpha}) + 2\epsilon lnE_{t+s}) \tag{13}$$

s.t. (11) & (12). The optimisation is solved through the dynamic programming method. By assuming that the control variables are chosen optimally, we can then write the value function in terms of the states to obtain the following Bellman equation:

$$V_t(K_t, E_t) = lnC_t + ln(\alpha K_t^{\alpha}) + 2\epsilon lnE_t + \beta V_{t+1}(K_{t+1}, E_{t+1}) \tag{14}$$

s.t.

$$K_{t+1} = K_t^{\alpha} - C_t - \alpha K_t^{\alpha} - a_t \tag{15}$$

and

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi a_t. \tag{12}$$

Based on the above, the existence and optimal characterisation of the general equilibrium can then be described by the first-order conditions taken with respect to the control variables C_t and a_t, and envelope theorem conditions obtained with respect to the state variables K_t and E_t. They, together with the description of the whole process, are found in Appendix 8.1

Knowing the initial values of the states, the 4-dimensional system of dynamical equations in four unknowns enables to numerically simulate the model forward and produce complete optimal paths. The motion of capital is denoted by an implicit equation (16); the evolution of consumption is based on the "Euler" identity (17); environmental stock is simulated forward using (18); whereas the last equation (19) allows to infer the consistent choice of abatement spending:

$$\left(\frac{1}{\beta C_t} - \frac{\alpha}{K_{t+1}}\right)\left[\alpha K_{t+1}^{\alpha-1}\left(1 - \alpha - \frac{\theta}{\phi}\right)\right]^{-1}$$
$$= \frac{1}{\beta C_t} - \frac{2\epsilon\phi}{E_t - \theta K_t^{\alpha} + \phi((1-\alpha)K_t^{\alpha} - C_t - K_{t+1})} \tag{16}$$

$$C_{t+1} = \left(\frac{1}{\beta C_t} - \frac{\alpha}{K_{t+1}}\right)^{-1}\left[\alpha K_{t+1}^{\alpha-1}\left(1 - \alpha - \frac{\theta}{\phi}\right)\right] \tag{17}$$
$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi((1-\alpha)K_t^{\alpha} - C_t - K_{t+1}) \tag{18}$$
$$a_t = (1-\alpha)K_t^{\alpha} - C_t - K_{t+1} \tag{19}$$

We can thus see that consumption allocation is dependent on the emission and abatement efficiency factor: the greater the $\frac{\theta}{\phi}$ ratio (i.e. the "more difficult" the abatement undertaking) the lower consumption at $t+1$. Put differently, a higher marginal externality shall lead to lower consumption in the next period. This is consistent with the fact that greater future consumption requires greater investment outlays, which in turn harms the environmental quality. The social planner, however, allocates the optimal consumption having incorporated the full societal impacts related to this choice, i.e. including environmental consequences.

It is evidenced at least since Zhang (1999) that non-trivial dynamics are likely to occur in environmental-growth models. Nonetheless, in the case of this relatively simple – numerically-wise – model, we can presume saddle-path stability and convergence to a steady state. This can be safely assumed to be guaranteed by infinitely negative utility as $E \longrightarrow 0$. The system (16)-(19) is solved by a forward shooting algorithm so that we tend towards the steady state given by (20)-(23). The steady state has analytic solutions given as ("star" signs denote the steady state values):

$$K^* = \left[\frac{2\alpha\beta(1 - \alpha - \frac{\theta}{\phi})}{(1+\alpha\beta)}\right]^{\frac{1}{1-\alpha}} \tag{20}$$
$$a^* = \frac{\theta}{\phi}(K^*)^{\alpha} \tag{21}$$
$$C^* = (1-\alpha)(K^*)^{\alpha} - K^* - a^* \tag{22}$$
$$E^* = \left(\frac{\beta}{1-\beta}\right)2\epsilon\phi C^*. \tag{23}$$

Our model features no ambiguity concerning the direction in which the social discount rate influences the steady state values. Therefore, a theoretical change in the discount rate does not trigger the opposing channels prevalent in some OLG models (such as Gutiérrez (2008)) in which, on the one hand, a lower discount rate translates to higher levels of capital stock provided to the future generations, and on the other, through capital's link to externality, suggests that future provision of welfare inherently reduces capital stock. Here, owing to the ability to mitigate, a lower discount rate always results in a higher level of steady state capital stock and environmental quality alike, as follows from the analytical solutions. However, the same swings in the discount rate would be associated with unequal long-run marginal effects. It can be shown by isolating the $\left[\frac{2\alpha\beta}{(1+\alpha\beta)}\right]^{\frac{1}{1-\alpha}}$ and $\left(\frac{\beta}{1-\beta}\right)$ factors from (20) and (23) respectively, that environmental quality is more sensitive to such changes than the capital stock.

Basic steady state impacts of some of the remaining parameters or variables summarise quite logically, too, if we dissect the system and analyse holding all else constant. Firstly, the greater the ratio of the emission factor θ to the efficiency of abatement ϕ, the lower the steady state stock of capital. Conversely, one can observe that abatement activity needs higher (lower) levels of spending if the said ratio is greater (smaller). At the same time, (21) highlights that, *ceteris paribus*, mitigation efforts are stronger for higher levels of the steady state capital stock. On the one hand, it reveals that higher output allows more funds to be dedicated to green projects. On the other, it suggests that greater production requires more compensation for environmental damages. Continually, consumption available to workers inevitably depends on the aggregate capital stock in the economy (in particular through steady state wages), although the abatement's opposing channel partially outweighs its amount. Lastly, the state of the environment is positively associated with the concurrent generations' sensitivity factor ϵ: it is clear that higher sensitivity requires more emphasis on climate maintenance.

4.2 General individual problem

The decentralised economy implicitly involves incomplete markets (i.e. current generation cannot trade with their "grandchildren"). Therefore, decentralisation of the socially optimal solution needs to include corrective measures manifested through a fiscal policy. The individual general problem is thus as follows:

$$U_t^i = lnC_t + \beta lnC'_{t+1} + \epsilon lnE_t + \beta\epsilon lnE_{t+1} \tag{24}$$

$$C_t = T_t + (1-\tau_t^p)W_t - \tau_t^c(\theta K_t^\alpha - \phi a_t) - (1-\tau_t^I)I_t - (1-\tau_t^a)a_t \tag{25}$$

$$C'_{t+1} = r_{t+1}I_t \tag{26}$$

$$E_{t+1} = E_t - \theta K_t^\alpha + \phi a_t \tag{12}$$

where transfers (taken as given) are:

$$T_t = \tau_t^p(1-\alpha)K_t^\alpha + \tau_t^c(\theta K_t^\alpha - \phi a_t) - \tau_t^I I_t - \tau_t^a a_t \tag{27}$$

In the general case, therefore, individuals may face a combination of labour income taxes and carbon taxes. The latter, τ_t^c, is set in proportion to net emissions.

Thus, the amount of the tax levied increases with production and is reduced in line with abatement undertaken during the same period. Carbon tax is refunded as part of lump sum transfers, T_t.

The labour tax in this setting is an instrument of the general pension policy[6]. The tax is reallocated as a tax relief (and, effectively, as a subsidy) to brown and green investment. Any residual balance of the tax revenue which was not used directly for the relief would be refunded through transfers to workers (pensioners are omitted for simplicity to reduce recursion in the model); this ensures that the policy is revenue-neutral to the fiscal authorities.

Regarding the evolution of the environmental quality (12), however, it might be argued that atomised individuals do not believe their mitigation efforts matter. Hence, as far as their optimisation is concerned, no τ_t would encourage it. In this regard, from the perspective of an individual worker, the actual expression for the evolution of environmental quality could resemble $E_{t+1} = E_t + \theta K_t^{\alpha} + \phi \int_0^n a_{i,t}\, di$. We do not consider such a formulation in this chapter, instead relying on assumptions which ensure abatement spending's sensibility is acknowledged.

We assume the existence of a financial intermediary (i.e. pension funds) that rewards private investment in abatement to the extent that it is valued in the aggregate. The financial sector allows individuals to make abatement decisions assuming that everyone else makes the same abatement decision, and thus that their individual abatement decision matters for the aggregate environmental outcome.

Alternatively, we can think about their preferences in (24) in terms of the warm-glow effect, similar to the specification used by Dam (2011) in his Diamond model with socially responsible investors. According to this, workers would derive satisfaction from the very fact of doing something considered ecological, rather than from affecting the environmental quality per se.

Lastly, we can simply refer to the seminal work of John and Pecchenino (1994) who also rely on the OLG specification with private mitigation and environmental quality. They optimise individual behaviour without sharing our concerns and thereby assume that workers believe their individual abatement matters. This appears in line with Fodha and Seegmuller (2012) who assume strictly positive private abatement in their model. They note that positive private abatement is supported by empirical evidence. Therefore, regardless of the specific assumption

[6] At this point, it appears worth reiterating that we refrain from modelling distinct pension market components. Instead, we benefit from the inherent design of the overlapping generations model. For instance, Blake (2006) states that a fully funded pension system formulated under this framework effectively results in an identical outcome as in the specification with no formal system whatsoever, i.e. consisting of private savings only. Owing to this, the pension tax relief policy can be implemented simply by introducing the policy variable τ^p denoting the pension tax.

(financial intermediary or the warm-glow effect), workers in our model believe their individual mitigation spending can affect the state of the environment so that (12) holds.

The individual optimisation process is described in more detail in Appendix 8.2. We impose $a_t \geq 0$ constraint[7] and obtain the following system:

$$a_t = \left[(1-\alpha)K_t^{\alpha} - \left(\frac{1-\tau_t^I+\beta}{1-\tau_t^I}\right)\frac{1}{\epsilon\beta\phi}\left(1-\tau_t^a-\tau_t^c\phi\right)\left(E_t - \theta K_t^{\alpha}\right)\right]\frac{\epsilon\beta\left(1-\tau_t^I\right)}{\epsilon\beta\left(1-\tau_t^I\right)+\left(1-\tau_t^I+\beta\right)\left(1-\tau_t^a-\tau_t^c\phi\right)} \tag{28}$$

$$C_t = \frac{1}{\epsilon\beta\phi}\left(1-\tau_t^a-\tau_t^c\phi\right)\left(E_t-\theta K_t^{\alpha}+\phi a_t\right) \tag{29}$$

$$I_t = \frac{\beta}{1-\tau_t^I}C_t \tag{30}$$

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi a_t \tag{12}$$

Equation (29) logically uncovers a negative relationship between the taxes level and consumption choice. Similarly, according to (28), higher tax rates would be associated with an increased abatement, *ceteris paribus*.

With two control variables in the social planner's solution and two sources of inefficiencies in the decentralised model (environmental externality and incomplete markets), we need two levers to fully replicate the social optimum: a labour income (pension) tax and a carbon tax. Therefore, the social planner's solution can be implemented when $\tau_t^c > 0$ & $\tau_t^p = \tau_t^I = \tau_t^a \equiv \tau_t^p$. By setting the tax rates optimally (see Appendix 8.2), the evolution of the (28)-(30) & (12) system can match the evolution of the social planner's solution.

[7] If the equation (28) yields a negative number, we impose $a_t = 0$ and split aggregate output over C_t, C_t' and I_t, s.t. $C_t' = \alpha K_t^{\alpha}$ and $C_t + I_t = (1-\alpha)K_t^{\alpha}$ and $I_t = \frac{\beta C_t}{1-\tau_t^I}$, such that $C_t = \frac{1-\tau_t^I}{1-\tau_t^I+\beta}(1-\alpha)K_t^{\alpha}$.

4.3 Green pension policy

The green pension policy discards the carbon tax and involves only the income tax. Here, however, it is used to subsidise only green investment. Individual consumption-saving decisions still follow the optimised system specified in section 4.2 by equations (28), (29), (30) and (12), although with $\tau_t^c = 0$, $\tau_t^I = 0$ & $\tau_t^p = \tau_t^a$. Accordingly, transfers become:

$$T_t = \tau_t^p (1-\alpha) K_t^\alpha - \tau_t^p a_t \tag{31}$$

Recall that we cannot replicate the social planner's solution with only a single policy variable. Hence, whereas households optimise their behaviour *subject to* the tax rate, we are interested in obtaining the solution that the planner would impose if they were subject to the constraint of only using this policy lever. The planner will take into account how workers form their decisions and, based on this, choose a sequence of tax rates τ_t^p to maximise social welfare. The planner has a single policy lever, τ_t^p, which by equations (28)-(30) uniquely determines consumption, abatement and investment. Therefore, in stating the dynamic programming problem, we can express using which control variable is most convenient. In the following, we use consumption. The planning problem is:

$$V_t(K_t, E_t) = lnC_t + ln(\alpha K_t^\alpha) + 2\epsilon lnE_t + \beta V_{t+1}(K_{t+1}, E_{t+1}) \tag{14}$$

s.t.

$$K_{t+1} = I_t = \beta C_t \tag{32}$$

and

$$E_{t+1} = E_t + \phi\left(1 - \alpha - \frac{\theta}{\phi}\right) K_t^\alpha - \phi(1+\beta) C_t. \tag{33}$$

Full details can be found in Appendix 8.3, but in principle, we rewrite the decentralised system to express it in terms of the states and only a single choice variable, C_t. Thus, rather than tax rates, the planner can equivalently construct the full optimal plan based on consumption choices[8].

4.4 Standard pension policy

The last of the evaluated policies in principle resembles the pension policy that already happens in practice, i.e. tax relief is granted on all pension investment, regardless of its "dirtiness". Here, however, we optimise the policy such that it intentionally serves the climate goals, too.

As before, we rely on the system given by equations (28), (29), (30) and (12), now with $\tau_t^c = 0$ & $\tau_t^p = \tau_t^I = \tau_t^a$. Transfers are given by:

$$T_t = \tau_t^p (1-\alpha) K_t^\alpha - \tau_t^p I_t - \tau_t^p a_t. \tag{34}$$

[8] As described in Appendix 8.3, optimal tax rates can then be inferred based on those choices.

Following the procedure from 4.3, the social planner aims to maximise social welfare, subject to the individual constraints. In this case, all savings are subsidised relative to consumption, but the planner cannot influence the relative level of brown against green investment. They construct the plan using a series of equivalent C_t choices to maximise:

$$V_t(K_t, E_t) = lnC_t + ln(\alpha K_t^{\alpha}) + 2\epsilon lnE_t + \beta V_{t+1}(K_{t+1}, E_{t+1}) \tag{14}$$

s.t.

$$K_{t+1} = \frac{1}{1+\epsilon}\left[\left(1 - \alpha - \frac{\theta}{\phi}\right)K_t^{\alpha} + \frac{E_t}{\phi} - C_t\right] \tag{35}$$

and

$$E_{t+1} = \frac{1}{1+\epsilon}\left[E_t + \epsilon\phi\left(1 - \alpha - \frac{\theta}{\phi}\right)K_t^{\alpha} - \epsilon\phi C_t\right] \tag{36}$$

where full details can be found in Appendix 8.4.

5 Numerical simulations

5.1 Calibration

The model is calibrated primarily such that the environment at its steady state - associated with the social planner's solution - is consistent with the 2°C above the pre-industrial temperature target. The parameter values that ensure this condition are found in the table below.

β	θ	ϕ	ϵ	α
0.67	2.53	50.6	0.33	0.40

Table 1: *Parameters*

The procedure which we follow to establish the stated values is as follows. Firstly, we declare the initial level of capital stock characterising the economy. This is achieved by solving for the steady state of the optimised model with no environmental problem considered. Full details are in Appendix 8.5 and the equation which is of our interest here is given by

$$K_1 = \left(\frac{2\alpha\beta(1-\alpha)}{1+\alpha\beta}\right)^{\frac{1}{1-\alpha}} \tag{37}$$

Therefore, equation (37) specifies the initial stock of capital, K_1, in our baseline model. The initial value of environmental quality, in turn, is assumed to reflect the current global warming of 1°C since the pre-industrial levels. To capture this in terms of units, we further arbitrarily assume that human extinction is associated with 10°C warming: for this level of $E_t = 0$, the agent's utility would tend to $-\infty$. Hence, the initial situation can be expressed as $E_1 = E^{max} - 1 = 9$. At the same time, we can thus define the assumed goal of 2°C which shall reflect the steady state of the social planner's solution and which is given simply by $E^* = E^{max} - 2 = 8$.

Having declared the initial states and assuming $\beta = 0.67$ (which shall reflect the choice of a discount rate of 2% and a generation's lifetime of 20 years), we can specify consistent values of the remaining parameters. Firstly, again referring to the model with no externality, we solve for the value of α. Essentially, similarly to the model variant specified in section 4.3 (however, without environment and abatement), we consider the decentralised solution with a pension tax optimised by the social planner (such that the planner corrects the incomplete markets inefficiency): α is calibrated to have an optimal labour tax, refunded on savings, of 10%[9].

Then, we note that estimates of the "carbon budget" consistent with keeping temperatures below a 2°C rise above pre-industrial levels, are typically of the order of 20 years of current emissions (see: MCC Berlin (n.d.)). Given our model has time periods of this length, we require $E^* - E_1 = -\theta K_1^{\alpha}$ (since this equates the social optimum environmental steady state with unabated production over the next 20 years). Combining equation (12) with (37) gives:

$$\begin{aligned} E^* - E_1 &= -\theta K_1^{\alpha} \\ 1 &= -\theta \left[\left(\frac{2\alpha\beta(1-\alpha)}{1+\alpha\beta} \right)^{\frac{1}{1-\alpha}} \right]^{\alpha} \end{aligned}$$

through which we obtain θ. Additionally, let us suppose that in the steady state we need to spend 5% of national output on green investment, so that:

$$\frac{\theta}{\phi} = \frac{a^*}{(K^*)^{\alpha}} = 0.05$$

which yields consistent value of ϕ. Then, using equation (23) which specifies the steady state level of environmental quality in the social planner's solution, we can solve for optimal ϵ.

5.2 Social planner's optimal plan

Below, we deterministically construct an optimal, welfare-maximising plan and thereby declare a benchmark solution for the evaluation of the chosen climate policies. We use the system specified in equations (16)-(19) and rely on the forward shooting algorithm. By adjusting the choice of C_1, we can simulate - given the assumed initial levels of the state of the economy and environment - the entire model until the steady state is reached.

[9]This is targeted based on UK tax system, in which a significant proportion of labour income is taxed at 40%, but this is refunded on savings. These savings then generate retirement incomes which attract taxes at 20%. Aggregated over whole population, this may equate to a relative tax on working age incomes of 10%.

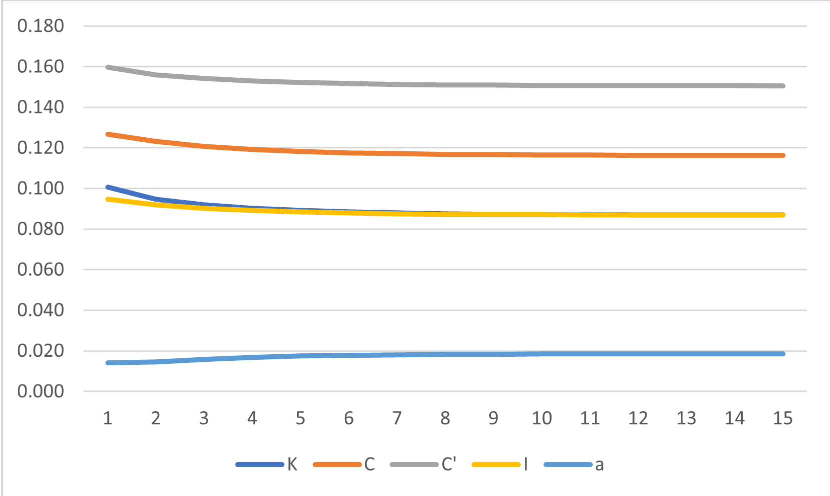

Figure 1: *Social planner's solution: paths of capital (K), worker consumption (C), pensioner consumption (C'), investment (I) and abatement (a) over time (t)*

In order to reach and keep the 2°C temperature rise target, the economy experiences relatively small but negative adjustments in the variables of interest (K falls by 14%, C by 8%, C' by 6%, and I by 8%), whereas abatement increases by as much as 32%[10]. The latter suggests that strong green action can be achieved without a proportionally large sacrifice to the economy and consumption. Regarding the overall evolution of the system, we can notice smooth and steady adjustment of the variables over time until they reach the steady state and become constant. A similar observation can be made with respect to the evolution of global temperature. Starting from the current 1°C rise, it takes roughly 11 periods to reach the assumed goal optimally and stabilise.

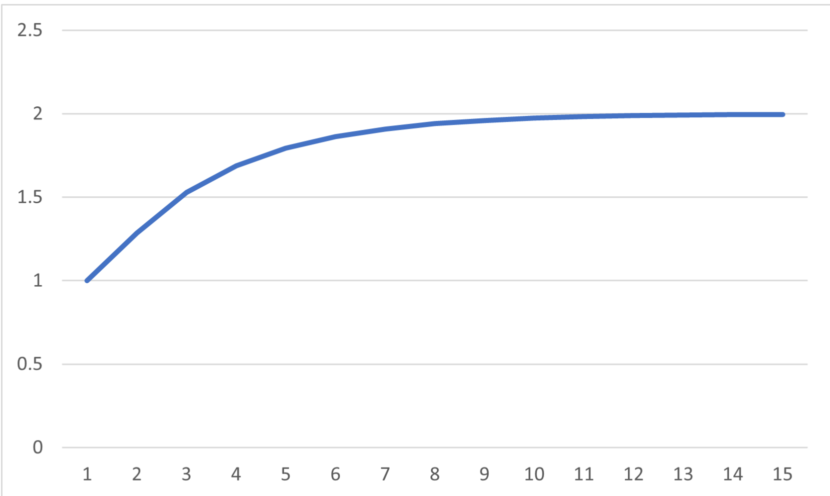

Figure 2: *Socially optimal temperature rise [°C] over time*

[10]Note that we do not show the initial period $t = 0$ associated with the no-externality steady state, in which abatement is still nonexistent.

5.3 Policy evaluation

Of the three policies specified in sections 4.2 - 4.4, only the one involving both the carbon tax and the pension tax can precisely replicate the social planner's solution. Hence, under this policy, the economy experiences the same adjustments as those shown in 5.2. Therefore, the two-taxes policy will constitute a benchmark for the remaining policies analysed in this section.

We begin the analysis by looking at paths of investment (see Figure 3). The relative levels are not surprising. Firstly, the green pension policy does not subsidise brown investment and therefore consistently features its lowest level. On the other hand, investment under the standard pension policy significantly exceeds the social optimum (by 49% in steady state). Moreover, we can notice the inverted U-shaped path of investment under the standard pension policy. This is a result of additional tax redistribution (due to the imposed $a_t \geq 0$ constraint) which further inflated brown investment[11].

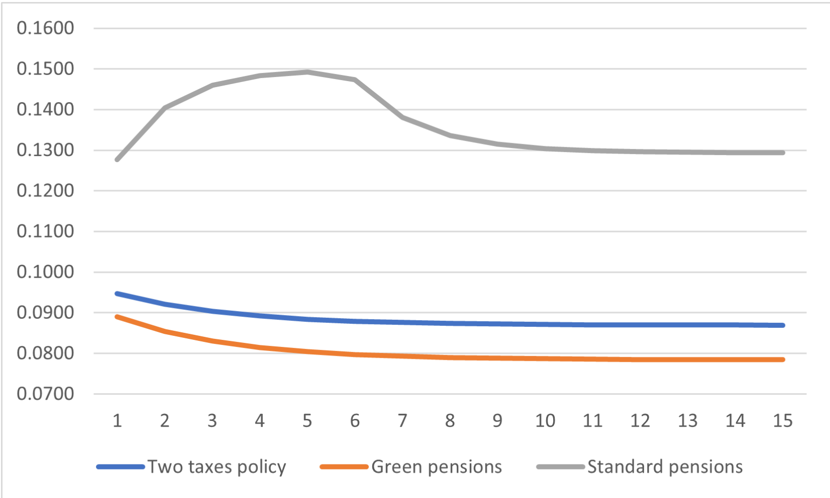

Figure 3: *Investment over time*

In terms of worker consumption, in Figure 4, we observe that initially the highest levels are associated with the green pension policy: finding investment spending not supported, individuals attach more value to current consumption. However, it gradually decreases (due to sub-optimal brown investment and, hence, output and wages "inherited" by consecutive generations) and equalises with the social optimum by the time the steady state is reached. Under the standard pension policy, initially, workers take advantage of the tax relief and prefer brown investment over consumption (hence substantial difference to the other policies). Nonetheless, consecutive generations of workers benefit from the "inherited" wages and begin to afford more consumption. The initial inverted U-shape again stems from

[11]This will become clearer once we analyse the paths of abatement. Essentially, the standard pension policy yields abatement spending non-optimal during the initial 5 periods. The entirety of the labour tax is therefore reallocated to subsidise brown investment.

the subsidy boost due to the $a_t \geq 0$ constraint. We notice that from $t = 3$, consumption surpasses the consumption levels related to the other policies. Following the halt of the extra redistribution (and positive, sizable, abatement spending commencing) at $t = 6$, however, consumption returns to significantly lower levels.

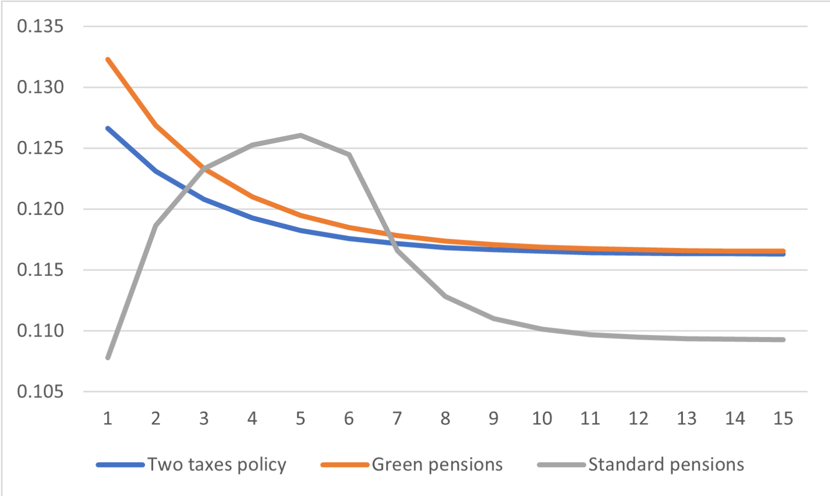

Figure 4: *Worker consumption over time*

Turning to pensioners, their consumption paths in all cases begin from the same level, which arises from the fact that the initial capital stock is not influenced by individuals in the model (i.e. pensioners at $t = 1$ "inherit" the existing capital stock and associated return on capital, irrespective of the policy). Unsurprisingly, the standard pension policy features consistently highest pensioner consumption (see Figure 5). This is a result of the higher investment, as discussed earlier. Conversely, the green pension policy - due to the lowest levels of productive investment - noticeably reduces consumption of pensioners.

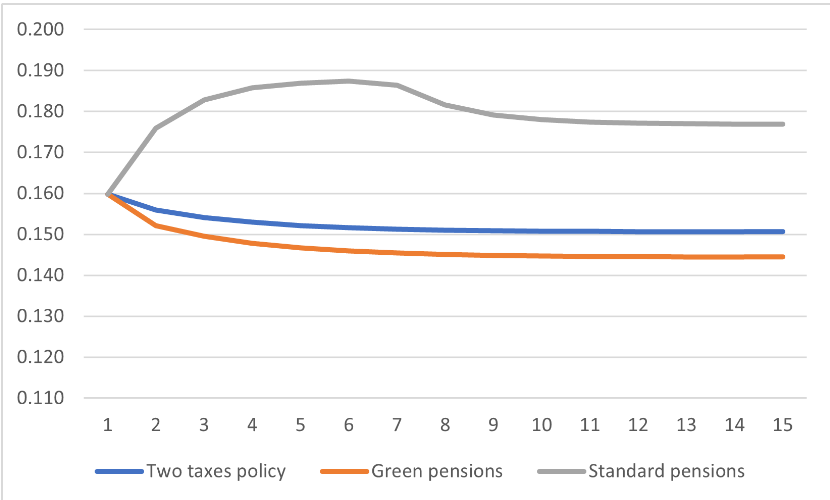

Figure 5: *Pensioner consumption over time*

Green pension policy, despite the subsidy, exhibits slightly lower abatement over the entire path, relative to the optimum (see Figure 6). We can look at this from three angles. Firstly, lower brown investment resulting from the policy translates to lower emissions. Thus, workers might be less pressed to abate, instead improving their consumption. Secondly, the policy does not feature the carbon tax, which would otherwise further incentivise abatement. Thirdly, we hypothesise (perhaps counterintuitively) that green savings are comparatively lower *because of* the sub-optimal brown capital formation. Namely, the green pensions subsidy appears to facilitate consumption and green investment at the cost of output production. The socially optimal solution, however, maintains a higher degree of brown investment (despite its negative environmental consequences) which allows to effectively dedicate more funds to future abatement.

Conversely, abatement under the standard pension policy is not incentivised sufficiently (i.e. income tax is split into two forms of investment). Thus, prior to $t = 6$, workers do not consider mitigation spending worthwhile and refrain from it completely, realising higher next-period marginal utility from consumption. After $t = 6$, when environmental quality reaches an "unacceptable" threshold, we notice a sharp increase in green investment to outweigh past negligence.

Figure 6: *Abatement over time*

Brown and green investment discussed above can be summarised by their impacts on climate. As depicted in Figure 7, whereas the optimal policy stabilises temperature at 2°C, the green pension policy produces additional 0.3°C (while abatement is lower than optimal, emission-inducing output production is lower, too. Hence, the difference in environmental performance is not striking). The standard pension policy, however, results in a global temperature growth reaching 7.8°C. Apart from the arguments discussed during the analysis of abatement paths (i.e. mitigation begins to take place too late), the decisive factor contributing to such an extreme rise is brown capital overaccumulation.

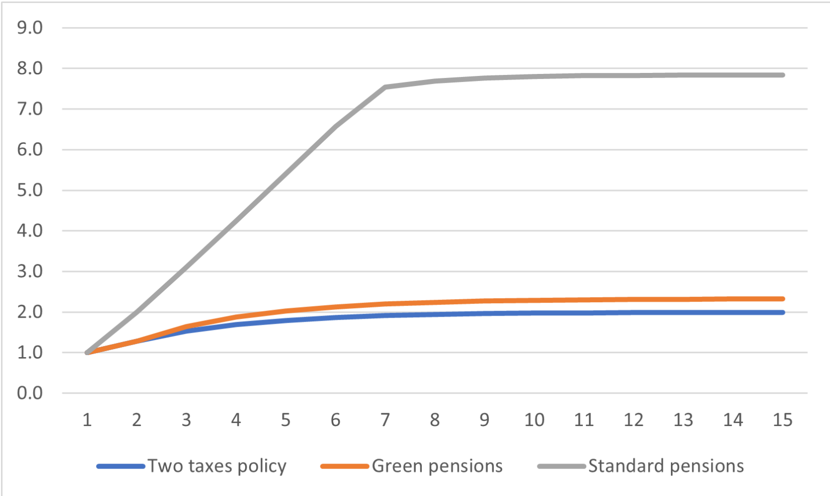

Figure 7: *Temperature rise [°C] over time*

Crucially, we are also interested in how the products of the policies translate to social welfare. Because the pension policies are not capable of fully tracking the social optimum, we might expect inevitably some degree of welfare loss, relative to the policy involving pensions and carbon taxes alike. Nonetheless, despite the noticeable differences in variables' evolution pointed out earlier, the policies deliver solutions of more comparable welfare effects. Because the social welfare measure reflects an infinite time-horizon of the aggregate utility, discounting diminishes the influence of the relative differences. Specifically, the overall social welfare loss attributable to conducting the climate policy through green pensions lays in negligible 0.1% regions. On the other hand, the standard pension policy results in a welfare loss of approximately 5%.

To conclude, we see a clear improvement related to the green pension policy, relative to the standard pension policy. Not only does it produce a nearly optimal outcome in social welfare terms, but appears to serve as a useful tool of climate policy. While, admittedly, the green pension policy produces additional 0.3°C on top of the assumed 2°C goal, it appears as a sensible - potentially socially acceptable - alternative to politically infeasible policies of carbon taxes.

6 Conclusion

This chapter has argued that the introduction of an optimal carbon tax might face political constraints: the public might be reluctant to accept an announcement of a new tax. To address this, we propose a novel approach to climate policy, such that it relies on already existing taxes. Specifically, we develop and evaluate a model of conducting the climate policy through pensions. While typically policy-makers grant tax relief on pension savings irrespective of their potential impact on climate, we suggest that the relief could be granted only on "green" savings (which would be used for emission abatement projects).

To model our green pension policy, we rely on the Diamond-type overlapping generations framework. We define the tax relief such that a labour income tax is reallocated as a subsidy to green investment. We optimise the model and compare it with a standard pension policy where tax relief is granted on all pension savings, and with a policy which involves a combination of both the standard pension policy and carbon taxes.

We assume the optimal outcome to reflect the 2°C rise in global average temperature and evidence that the socially optimal policy can be conducted only with the aid of carbon taxes. While our green pension policy produces additional 0.3°C (i.e. totalling 2.3°C), we nevertheless show that the policy results in a negligible (0.1%) welfare loss relative to the social optimum. This contrasts with the standard pension policy which induces the total temperature rise of 7.8°C and a social welfare loss of 5%.

Due to its environmentally superior outcome, we advocate the solution involving carbon taxes. However, the approach of conducting the climate policy through green pensions shows potential to be politically feasible. It does not significantly reduce the overall social welfare and allows the individuals to sustain relatively high consumption levels. If proven to be indeed acceptable by the public, the additional 0.3°C rise in temperature still appears to outweigh the costs of inaction.

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8 Appendix

8.1 Social planner's solution

The planner maximises

$$V_t(K_t, E_t) = lnC_t + ln(\alpha K_t^\alpha) + 2\epsilon lnE_t + \beta V_{t+1}(K_{t+1}, E_{t+1}) \tag{14}$$

s.t.

$$K_{t+1} = K_t^\alpha - C_t - \alpha K_t^\alpha - a_t \tag{15}$$

and

$$E_{t+1} = E_t - \theta K_t^\alpha + \phi a_t. \tag{12}$$

First-order conditions are taken with respect to the control variables C_t and a_t, and envelope theorem conditions are obtained with respect to the state variables K_t and E_t:

F.O.C.s

$$w.r.t.\ \ C_t, \qquad \frac{\partial V_{t+1}}{\partial K_{t+1}} = \frac{1}{\beta C_t} \tag{38}$$

$$w.r.t.\ \ a_t, \qquad \phi\frac{\partial V_{t+1}}{\partial E_{t+1}} = \frac{\partial V_{t+1}}{\partial K_{t+1}} \tag{39}$$

E.T.s

$$w.r.t.\ \ K_t, \qquad \frac{\partial V_t}{\partial K_t} = \frac{\alpha}{K_t} + \alpha\beta K_t^{\alpha-1}\left[(1-\alpha)\frac{\partial V_{t+1}}{\partial K_{t+1}} - \theta\frac{\partial V_{t+1}}{\partial E_{t+1}}\right] \tag{40}$$

$$w.r.t.\ \ E_t, \qquad \frac{\partial V_t}{\partial E_t} = \frac{2\epsilon}{E_t} + \beta\frac{\partial V_{t+1}}{\partial E_{t+1}} \tag{41}$$

The next step involves combining the results just obtained, iterating them accordingly and eliminating marginal values. This way we reach the system of 4 difference equations in 4 unknowns which characterise the optimised model:

$$K_{t+1} = (1-\alpha)K_t^\alpha - C_t - a_t \tag{42}$$

$$E_{t+1} = E_t - \theta K_t^\alpha + \phi a_t \tag{12}$$

$$\frac{1}{\beta C_t} = \frac{\alpha}{K_{t+1}} + \alpha K_{t+1}^{\alpha-1}\left(1 - \alpha - \frac{\theta}{\phi}\right)\frac{1}{C_{t+1}} \tag{43}$$

$$\frac{1}{\phi\beta C_t} = \frac{2\epsilon}{E_{t+1}} + \frac{1}{\phi C_{t+1}} \tag{44}$$

Rearranging the above equations yields the system given by:

$$\left(\frac{1}{\beta C_t} - \frac{\alpha}{K_{t+1}}\right)\left[\alpha K_{t+1}^{\alpha-1}\left(1-\alpha-\frac{\theta}{\phi}\right)\right]^{-1} = \frac{1}{\beta C_t} - \frac{2\epsilon\phi}{E_t - \theta K_t^{\alpha} + \phi((1-\alpha)K_t^{\alpha} - C_t - K_{t+1})} \tag{45}$$

$$C_{t+1} = \left(\frac{1}{\beta C_t} - \frac{\alpha}{K_{t+1}}\right)^{-1}\left[\alpha K_{t+1}^{\alpha-1}\left(1-\alpha-\frac{\theta}{\phi}\right)\right] \tag{17}$$

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi((1-\alpha)K_t^{\alpha} - C_t - K_{t+1}) \tag{18}$$

$$a_t = (1-\alpha)K_t^{\alpha} - C_t - K_{t+1} \tag{19}$$

By dropping the time subscripts and rearranging, we obtain the following analytic steady state solutions:

$$K^* = \left[\frac{2\alpha\beta(1-\alpha-\frac{\theta}{\phi})}{(1+\alpha\beta)}\right]^{\frac{1}{1-\alpha}} \tag{20}$$

$$a^* = \frac{\theta}{\phi}(K^*)^{\alpha} \tag{21}$$

$$C^* = (1-\alpha)(K^*)^{\alpha} - K^* - a^* \tag{22}$$

$$E^* = \left(\frac{\beta}{1-\beta}\right)2\epsilon\phi C^* \tag{23}$$

8.2 General decentralised solution

$$U_t^i = lnC_t + \beta lnC'_{t+1} + \epsilon lnE_t + \beta\epsilon lnE_{t+1} \tag{24}$$

$$C_t = T_t + (1-\tau_t^p)W_t - \tau_t^c(\theta K_t^{\alpha} - \phi a_t) - (1-\tau_t^I)I_t - (1-\tau_t^a)a_t \tag{25}$$

$$C_{t+1} = r_{t+1}I_t \tag{26}$$

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi a_t \tag{12}$$

where transfers (taken as given) are:

$$T_t = \tau_t^p(1-\alpha)K_t^{\alpha} + \tau_t^c(\theta K_t^{\alpha} - \phi a_t) - \tau_t^I I_t - \tau_t^a a_t \tag{27}$$

To maximise (24) s.t. the constraints, we differentiate (24) with respect to I_t and a_t. This gives:

$$0 = \frac{\partial U_t}{\partial I_t} = -\frac{1-\tau_t^I}{C_t} + \frac{\beta}{I_t} \tag{46}$$

$$0 = \frac{\partial U_t}{\partial a_t} = \frac{\tau_t^c\phi - (1-\tau_t^a)}{C_t} + \frac{\epsilon\beta\phi}{E_t - \theta K_t^{\alpha} + \phi a_t} \tag{47}$$

Merging with the aggregate budget constraint and rearranging then gives:

$$a_t = \left[(1-\alpha)K_t^\alpha - \left(\frac{1-\tau_t^I+\beta}{1-\tau_t^I}\right)\frac{1}{\epsilon\beta\phi}\left(1-\tau_t^a-\tau_t^c\phi\right)\left(E_t - \theta K_t^\alpha\right)\right]\frac{\epsilon\beta\left(1-\tau_t^I\right)}{\epsilon\beta\left(1-\tau_t^I\right)+\left(1-\tau_t^I+\beta\right)\left(1-\tau_t^a-\tau_t^c\phi\right)} \tag{48}$$

$$C_t = \frac{1}{\epsilon\beta\phi}\left(1-\tau_t^a-\tau_t^c\phi\right)\left(E_t-\theta K_t^\alpha\right) \tag{29}$$

$$I_t = \frac{\beta}{1-\tau_t^I}C_t \tag{30}$$

$$E_{t+1} = E_t - \theta K_t^\alpha + \phi a_t \tag{12}$$

Assuming $\tau_t^c > 0$ & $\tau_t^p = \tau_t^I = \tau_t^a$, this solution can replicate the social planner's optimal plan. Therefore, we can use specific values of the steady state variables from the social planner's solution (i.e. given by (20)-(23)) and obtain consistent tax rates which ensure the above conditions are met from the following:

$$\tau^p = 1 - \beta\frac{C}{I}$$

$$\tau^c = \frac{1-\tau^p}{\phi} - \epsilon\beta\frac{C}{E}$$

8.3 Green pension solution

Individual consumption-saving decisions still follow the optimised system specified in section 4.2 by equations (28), (29), (30) and (12), although with $\tau_t^c = 0$, $\tau_t^I = 0$ & $\tau_t^p = \tau_t^a$, i.e.:

$$I_t = \beta C_t \tag{32}$$

$$C_t = \frac{1}{\epsilon\beta\phi}\left(1-\tau_t^p\right)\left(E_t - \theta K_t^\alpha + \phi a_t\right) \tag{49}$$

$$a_t = \frac{\epsilon\beta}{\epsilon\beta + \left(1+\beta\right)\left(1-\tau_t^p\right)}\left[(1-\alpha)K_t^\alpha - \left(\frac{1+\beta}{\epsilon\beta\phi}\right)\left(1-\tau_t^p\right)\left(E_t - \theta K_t^\alpha\right)\right] \tag{50}$$

When $a_t = 0$:

$$C_t = \frac{1-\alpha}{1+\beta}K_t^\alpha \tag{51}$$

$$I_t = \beta C_t \tag{52}$$

By using a series of equivalent consumption choices, the planner aims to maximise:

$$V_t(K_t, E_t) = lnC_t + ln(\alpha K_t^{\alpha}) + 2\epsilon lnE_t + \beta V_{t+1}(K_{t+1}, E_{t+1}) \tag{14}$$

s.t.

$$K_{t+1} = I_t = \beta C_t \tag{32}$$

and

$$E_{t+1} = E_t + \phi\Big(1 - \alpha - \frac{\theta}{\phi}\Big)K_t^{\alpha} - \phi(1+\beta)C_t. \tag{33}$$

The resulting first-order and envelope theorem conditions are then given by:

$$F.O.C.$$

$$w.r.t.\;\; C_t, \qquad \frac{1}{\beta C_t} = \phi(1+\beta)\frac{\partial V_{t+1}}{\partial E_{t+1}} - \frac{\partial V_{t+1}}{\partial K_{t+1}} \tag{53}$$

$$E.T.s$$

$$w.r.t.\;\; K_t, \qquad \frac{\partial V_t}{\partial K_t} = \frac{\alpha}{K_t} + \beta\phi\Big(1 - \alpha - \frac{\theta}{\phi}\Big)\alpha K_t^{\alpha-1}\frac{\partial V_{t+1}}{\partial E_{t+1}} \tag{54}$$

$$w.r.t.\;\; E_t, \qquad \frac{\partial V_t}{\partial E_t} = \frac{2\epsilon}{E_t} + \beta\frac{\partial V_{t+1}}{\partial E_{t+1}} \tag{55}$$

Therefore, based on (32), (33) and (53)-(55), we can drop the time subscripts and rearrange to obtain the following steady state system:

$$K = \left[\frac{\beta(1 - \alpha - \frac{\theta}{\phi})}{1+\beta}\right]^{\frac{1}{1-\alpha}} \tag{56}$$

$$C = \frac{K}{\beta} \tag{57}$$

$$\frac{\partial V}{\partial E} = \Big(\frac{\alpha}{K} + \frac{1}{\beta^2 C}\Big)\frac{1}{\phi}\left[\frac{1+\beta}{\beta} - \beta\Big(1 - \alpha - \frac{\theta}{\phi}\Big)\alpha K^{\alpha-1}\right]^{-1} \tag{58}$$

$$E = \frac{2\epsilon}{(1-\beta)\frac{\partial V}{\partial E}} \tag{59}$$

Then, from (49) we can obtain the consistent steady state pension tax rate:

$$\tau^p = 1 - \frac{\epsilon\beta\phi C}{E - \theta K^{\alpha} + \phi a} \tag{60}$$

8.4 Standard pension solution

Individual consumption-saving decisions still follow the optimised system specified in section 4.3 by equations (28), (29), (30) and (12), although with $\tau_t^c = 0$ & $\tau_t^p = \tau_t^I = \tau_t^a$, i.e.:

$$I_t = \frac{\beta}{1-\tau_t^p} C_t \tag{61}$$

$$C_t = \frac{1}{\epsilon\beta\phi}\left(1-\tau_t^p\right)\left(E_t - \theta K_t^\alpha + \phi a_t\right) \tag{62}$$

$$a_t = \frac{\epsilon\beta}{\epsilon\beta + 1 - \tau_t^p + \beta}\left[(1-\alpha)K_t^\alpha - \left(\frac{1-\tau_t^p+\beta}{\epsilon\beta\phi}\right)\left(E_t - \theta K_t^\alpha\right)\right] \tag{63}$$

When $a_t = 0$:

$$C_t = \frac{1-\tau_t^p}{1-\tau_t^p+\beta}(1-\alpha)K_t^\alpha \tag{64}$$

$$I_t = \frac{\beta}{1-\tau_t^p} C_t \tag{65}$$

By using a series of equivalent consumption choices, the planner aims to maximise

$$V_t(K_t, E_t) = lnC_t + ln(\alpha K_t^\alpha) + 2\epsilon lnE_t + \beta V_{t+1}(K_{t+1}, E_{t+1}) \tag{14}$$

s.t.

$$K_{t+1} = \frac{1}{1+\epsilon}\left[\left(1-\alpha-\frac{\theta}{\phi}\right)K_t^\alpha + \frac{E_t}{\phi} - C_t\right] \tag{35}$$

and

$$E_{t+1} = \frac{1}{1+\epsilon}\left[E_t + \epsilon\phi\left(1-\alpha-\frac{\theta}{\phi}\right)K_t^\alpha - \epsilon\phi C_t\right] \tag{36}$$

The resulting first-order and envelope theorem conditions are then given by:

F.O.C.

$$w.r.t.\ C_t, \qquad \frac{1}{\beta C_t} = \frac{1}{1+\epsilon}\left(\frac{\partial V_{t+1}}{\partial K_{t+1}} + \epsilon\phi\frac{\partial V_{t+1}}{\partial E_{t+1}}\right) \tag{66}$$

$$E.T.s$$

$$w.r.t.\ K_t, \qquad \frac{\partial V_t}{\partial K_t} = \frac{\alpha}{K_t} + \beta\Bigg(\frac{\partial V_{t+1}}{\partial K_{t+1}}\frac{\alpha}{1-\epsilon}\Big(1-\alpha-\frac{\theta}{\phi}\Big)K_t^{\alpha-1} + \frac{\alpha\phi}{1+\epsilon}\Big(\epsilon(1-\alpha)-\frac{\epsilon\theta}{\phi}\Big)K_t^{\alpha-1}\frac{\partial V_{t+1}}{\partial E_{t+1}}\Bigg) \tag{67}$$

$$w.r.t.\ E_t, \qquad \frac{\partial V_t}{\partial E_t} = \frac{2\epsilon}{E_t} + \frac{\beta}{1+\epsilon}\Bigg(\frac{1}{\phi}\frac{\partial V_{t+1}}{\partial K_{t+1}} + \epsilon\frac{\partial V_{t+1}}{\partial E_{t+1}}\Bigg) \tag{68}$$

Then, we consider the system of (35), (36) and (66)-(68) which we solve numerically. The optimal steady state pension tax rate can then be inferred from:

$$\tau^p = 1 - \beta(1+\epsilon)\Bigg(\frac{C}{1-\alpha-\frac{\theta}{\phi}}\Bigg)K^{\alpha} + \frac{E}{\phi - C} \tag{69}$$

8.5 Initial level of capital

$$V_t(K_t) = lnC_t + ln(\alpha K_t^{\alpha}) + \beta V_{t+1}(K_{t+1}) \tag{70}$$

s.t.

$$K_{t+1} = K_t^{\alpha} - C_t - \alpha K_t^{\alpha} \tag{71}$$

We differentiate to obtain the following first-order and envelope theorem conditions:

$$F.O.C.$$

$$w.r.t.\ C_t, \qquad \frac{\partial V_{t+1}}{\partial K_{t+1}} = \frac{1}{\beta C_t} \tag{72}$$

$$E.T.$$

$$w.r.t.\ K_t, \qquad \frac{\partial V_t}{\partial K_t} = \frac{\alpha}{K_t} + (1-\alpha)\alpha\beta K_t^{\alpha-1}\frac{\partial V_{t+1}}{\partial K_{t+1}} \tag{73}$$

what eventually yields the following steady state:

$$K^* = \Bigg(\frac{2\alpha\beta(1-\alpha)}{1+\alpha\beta}\Bigg)^{\frac{1}{1-\alpha}} \tag{74}$$

$$C^* = \Big((1-\alpha)(K^*)^{\alpha-1} - 1\Big)K^* \tag{75}$$

Chapter 2

Optimal policymaking across the democratic spectrum: a dynamic view

Abstract

In today's world, the theme of differing political systems - whether a democracy, a hybrid regime or a dictatorship – remains as relevant as ever. The following chapter considers economic consequences associated with the extent of democratic accountability. Specifically, it studies how the level of democracy influences policymakers' objectives and optimal policy choices. We adapt the influential model of political economy by McGuire and Olson (1996) by adding the time dimension and reframing the original problem as an intertemporal one. This allows us to show how the differences in the level of democracy contribute to long-run economic outcomes. Furthermore, incorporation of the time dimension enables us to investigate the impact of regime shocks: countries may move in more or less democratic directions. Overall, our dynamic model retains the key predictions of the original: the results say that policymakers operating under a higher level of democracy set lower taxes and extract less from the society; more democratic economies feature greater output growth due to larger public good provision; and the level of democracy contributes positively to societal consumption and welfare.

1 Introduction

The theme of democracy is still relevant nowadays. The not-so-distant events throughout the world have echoed considerably not only in the realm of politics or human rights but also of economics. Particularly vivid examples can be found in the Venezuelan crisis and Nicolas Maduro's uncompromising struggle to maintain power, Turkey's downgraded rating by S&P in 2016 (motivated partially by the erosion of democratic accountability) or, more recently, the sudden takeover of the Afghan government by the Taliban and Vladimir Putin's continuous imperialistic ambitions. The following chapter explores how institutional characteristics reflected in the degree of democracy shape policymakers' economic objectives and optimal policy choices.

The interest certainly concerns the developing world: countries in most need of economic growth also often happen to be nondemocratic. However, developed countries that shape the global policies are not free from populist upswings either. In fact, Treisman (2023) establishes that in recent years the world has experienced a slight decrease in the overall quality of democratic institutions[1].

In the economic context, the level of democracy undoubtedly matters. We can expect that differing regimes will have different economic objectives: a self-oriented, all-powerful dictator should not be assumed to exhibit the same behaviour as a democratic government, which - at least to a certain extent - manifests the will of the voters. Therefore, including regime characteristics in economic analyses may be important. At the same time, most political and economic problems are processes. Hence, the time dimension and associated dynamics can play a vital role in economic planning and policy enactment.

Looking at data and specific examples more closely - as we do in section 3 of this chapter - appears to demonstrate that similar countries, which nonetheless contrast with respect to democratic quality, experience different GDP growth over time. Our survey of the data also highlights another issue. Many countries in the world feature changes (positive and negative alike) in the level of democracy over time, whether through continuous evolution or sudden regime shifts (e.g. following coups). In such a dynamic context, Acemoglu et al. (2019) suggest the importance of the time-horizon: short-run impacts of democratic transitions on economic growth can be different from the long-run implications.

This chapter aims to develop a dynamic model that embodies political regime characteristics denoted by a possibly variable level of democracy. The research intends to offer a simple and relatively flexible framework with the potential to be applied to more specific policy contexts. Therefore, we adapt the seminal, static model of political economy by McGuire and Olson (1996) by introducing dynamics: we add the time dimension and reframe the original problem as an

[1]It appears relatively consistent with the impression resulting from the Democracy Index (The Economist Intelligence Unit, 2024): only 74 of 167 countries are considered democratic.

intertemporal one. We show that key predictions of the original (discussed in section 2) hold in our setting as well. Furthermore, we simulate full optimal paths related to the model's variables to show adjustments in the economy over time. We evidence that a higher level of democracy contributes to larger economic growth and increased societal consumption. Moreover, we visualise the theoretical impact of a possible regime shock on short-run and long-run output evolution.

The choice of McGuire and Olson's (1996) design is mainly based on its established position in the literature and the influence it exerted in political economics, particularly in the regime-studies area (see e.g. Papaioannou and van Zanden (2015)). However, an equally important argument concerns the fact of its relative simplicity: the model integrates political and economic considerations into a single dimension. Representing the degree of democracy as a parameter, and thus nesting democracy and autocracy within a single model, is crucial if we are to tractably address questions of how different regime types deal with dynamic policy questions[2]. Moreover, McGuire and Olson's framework allows valid comparisons *across* the political spectrum - this is essential if we consider the fact that most countries fall into the hybrid regime or flawed democracy category and thus lie on the neither extreme side of the said spectrum. We discuss McGuire and Olson (1996) in more detail, together with other established approaches to modelling political regimes, in section 2.

This chapter's contribution is twofold. Primarily, we dynamise the McGuire and Olson (1996) model: to the best of our knowledge, this has never been done[3]. The second key contribution refers to our model's potential application and usability: we develop a simple, tractable, and extendable tool which can be used to visualise short-run and long-run adjustments in the variables' optimal values, including the impact of democracy shocks. The model can be straightforwardly mapped onto common democracy indicators such as the Democracy Index developed by The Economist Intelligence Unit or political rights and civil liberties ratings provided by Freedom House.

The remainder of this chapter is structured as follows. The next section explains the notion of democracy and authoritarianism and discusses traditional approaches used to model political regimes. Section 3 outlines a data-based motivation for our contribution; we refer to specific countries to show how democratic dynamics correlate with economic growth. Subsequently, section 4 introduces our dynamic extension of the McGuire and Olson (1996) model and section 5 provides complete analytical solutions to the optimisation problem. In section 6, we display and analyse our dynamic simulations. The last section concludes.

[2] Chapter 3 of this thesis does exactly this: we use the dynamic extension developed here and apply it to study the impact of democracy in relation to climate change mitigation.

[3] The closest attempt probably can be found in Castellucci and Gorini (2019). However, their McGuire-Olson-Solow model relies mostly on McGuire and Olson's production properties and largely ignores the original's political dimension.

2 Modelling a regime

What do we mean by democracy or authoritarianism? In this section we review the literature on modelling political regimes. Firstly, we define the underlying concepts. Then, largely following the analysis by Persson and Tabellini (2000)[4], we provide a general overview of political economics' perspective in modelling a political system, and subsequently, we survey the traditional approaches to modelling the democratic process. Those are then supplemented by specific economic models capturing the characteristics of authoritarian systems[5]. Lastly, we discuss how McGuire and Olson (1996) framework relates to the aforementioned methods and describe the model's key features.

Typically, a regime or a political system can be defined as institutionalised, habitually practised political norms. These norms regulate the means of admission to public offices and constitute the rules associated with the development of publicly binding resolutions (Schmitter and Karl, 1991). Democracy, in a such context, is a system which, according to Schmitter and Karl (1991), features both competition and collaboration of elected officials whose actions are accountable to citizens. Bailey (2018) suggests thinking in terms of (good-quality) political and civil liberties. In the context of flawed democracies or hybrid regimes, it is therefore intuitive to consider the extent of those rights and accountability. On the other hand, authoritarian systems exhibit limited pluralism, low political participation and control over public policy assumed by a narrow clique (Li and Resnick, 2003). Throughout this chapter, we will use words such as "autocracy/autocrat", "dictatorship/dictator", "authoritarian regime" or "absolute ruler" interchangeably.

In the context of this paper, we point to the role of political economics which inherently strives to establish how conflicting preferences are aggregated, resolved, and turned into specific policies. Despite the wide range of political economy models, they typically share certain common underpinnings. Usually, citizens and politicians act both as economic and political agents. In doing so, they exhibit policy preferences: models tend to identify an agent's preferred policy (an ideal "bliss point") simply as the one which yields the highest utility[6]. Furthermore, an environment which shapes the corresponding behaviour (through the system of incentives and constraints) can be described by economic and political institutions. The political institutions appear particularly relevant for modelling a regime, given that they govern the voting and policy selection process.

[4]Unless specified otherwise, Persson and Tabellini (2000) is this section's primary source.

[5]In both cases, the selection is oriented on uncovering the mechanisms ultimately translatable into economic outcomes, as opposed to the literature focused on the contrary, i.e. how economic outcomes contribute to office or regime survival.

[6]Utility function may include traditional variables such as consumption, but can consider individual features, such worldview preferences, too.

Acemoglu and Robinson (2006) suggest the importance of political power and its dichotomy. Whereas institutions specify the formal rules and contribute to *de jure* political power, specific agents might also possess *de facto* political power. Because the latter reflects the ability of a group to mobilise (including a possibility of armed intervention), it might prove decisive for a policy choice. In such considerations, equilibrium outcomes would derive from the *total* political power.

Regarding politicians specifically, political economics traditionally assumes two different motivations: opportunistic or partisan. The former can reflect a purely office-seeking intent or be oriented on capturing resources via a rent-seeking behaviour. Partisan motivations, on the other hand, are linked to the well-being of specific groups in society.

2.1 Democratic process

Most models of democracy can be divided into pre-election and post-election models. Crucially, models of post-election politics assume that promises made to prospective voters are not binding. Office holders are therefore selected without a specific mandate. Instead, voters rely on more behavioural considerations, such as ideology, perceived competence, etc. Following election, politicians can implement policy as they see fit. Policy-setting motivations are, however, often linked to the occurrence of subsequent elections and associated accountability with respect to voters, i.e. retrospective voting, which generally constrains incumbents' rent-seeking motive (as in the career concern model; see e.g. Seabright (1996)).

Another example of post-election politics can be found in legislative bargaining (see e.g. Baron and Ferejohn (1989)), where there are a number of distinct officials or parties, each with individual policy preferences. The outcome is a result of a game – specified in line with the presumed procedures or constitution – between the elected parties. Their bargaining power strongly depends on specific costs related to the policy implementation or other factors contributing to the strategic location in the bargaining process.

Overall, post-election theories appear relatively realistic. Nonetheless, being more game-theoretic in nature, post-election models may prove complex to apply to macroeconomically advanced sides of the model. Hence, the popular – potentially due to their flexibility – tools of pre-election politics receive more attention here.

Models of pre-election politics are based on a presumption that officials are elected with expectations that announced policy platforms will be subsequently delivered, i.e. policy commitment. A relatively straightforward and broadly utilised method applicable to pre-election politics is found in the median voter theorem, according to which the median voter's preferences prove critical to the policy choice. The concept in its early incarnation dates back to Hotelling (1929), although the following discussion focuses on the influential preposition by Black (1948).

In the simplest case, assuming the pure majority system and single-peaked preferences (i.e. voters rank alternative policies in line with the relative proximity from their bliss points), it is possible to ascendingly order the citizens' bliss points across a single-dimensional spectrum[7] and, afterwards, juxtapose specific policy proposals against said bliss points. What follows is that the policy satisfying the median preferences constitutes a so-called Condorcet winner. To reinstate, even though all voters are perceived equally by the candidates, the median voter has a decisive impact on the outcome of elections. Therefore, in equilibrium, the policies will be postulated with the intention to persuade this particular voter: namely, such that they will coincide with the median voter's preferred policy.

Regardless of the conceptual simplicity of the political process, the theorem is commonly employed to analyse general interest problems. In fact, the simplicity potentially allows for additional freedom in regards to the development of a more advanced economic side of the model. Moreover, the theorem can be successfully applied to dynamic settings: a particularly relevant example is demonstrated in Azzimonti et al. (2006) who establish the median voter equilibria in a neoclassical growth model.

The second model of pre-election (popular especially due to its ability to tackle multidimensional problems) described here is the probabilistic voting model (e.g. Lindbeck and Weibull, 1987) which introduces voting uncertainty. When announcing the policy platforms, candidates consider the *expected* amount of votes and voters' responsiveness, i.e. how likely an increase in welfare will be accompanied by a vote for such policy. In doing so, candidates maximise a weighted social welfare function, where the weights depend predominantly on the size and density of specific voting groups. This implicitly leads to a situation in which – rather than to satisfy the median individual – certain ideologically-neutral[8], "mobile" groups can be perceived as more appealing, and thereby policy might be adjusted in their favour. This logic can potentially extend to special-interest problems, such as lobbying, too. If the citizens' voting responsiveness is not known to the candidates at the time of platform announcement, politicians might be tempted to secure votes or relative popularity via policy favours for organised groups. Probabilistic voting models thus offer a route to study practical issues present in democracies.

[7]Well-defined equilibrium might be more complex to obtain if policies are multidimensional. Hence, typically, the median voter theorem is used to tackle unidimensional problems.

[8]Probabilistic voting model conveniently enables to extend the preferences to an ideological dimension. Hence, models can incorporate the effect of a bias towards a specific candidate. Ideology, then, might technically prove more decisive than economic considerations as far as voting motives are concerned.

2.2 Autocracies

To address the research question and complete the picture more thoroughly, however, attention must be turned more directly to the opposite side of the political spectrum. Below, we introduce chosen models that aim to capture specific characteristics of autocracies.

Wintrobe (2004) focuses on the "power" aspect of autocratic governments. He defines a dictator's utility as a function of two arguments, consumption and power (i.e. "the desire for power"). Apart from being an argument, however, power serves also as a tool of policy implementation. Namely, power is implicitly linked to ways of acquiring resources through its (inefficient) effects on the economy, e.g. taxes or regulation in general. The budget, therefore, is a function of power. The key trade-off relates to the use of obtained proceeds: the dictator can fund their consumption or "invest" the proceeds in power accumulation.

Further, Wintrobe introduces a money-to-power relation and an additional term called "price of power" which captures the regime's productivity (through building loyalty or repression) in generating power, reflected in monetary terms. Unsurprisingly, the author highlights that vital economic implications of the ruler's actions derive from the marginal effects of interventions on the budget. In principle, the dictator will strive for more power until they cannot attain extra revenue, therefore intrinsically posing a limit to the use of power on the society.

Deacon (2009) proposes a framework which enables the contrasting of outcomes in a democratic system with an autocracy. The model's design resembles the probabilistic voting approach discussed earlier. The key role in election outcome (and thereby policy choice) is attributed to specific groups of voters. Both groups - the elite and the disadvantaged - have an identical utility function consisting of private consumption (financed by targeted transfers) and universal public good level. However, their influence over the eventual election result is different.

Similarly to probabilistic voting, Deacon assumes that the groups' political influence is reflected by the weights attached to responsiveness to consumption-oriented policy proposals. Effectively, an even distribution of the per capita influence indicates a democracy, whereas a nondemocratic system is denoted by an uneven distribution. The case of a dictatorship is illustrated when one group – the disadvantaged – is assumed to exert zero political influence. In such a case, the equilibrium typically implies the dictator provides a lower level of public good. This is chiefly because a universal public good provision is costly to the ruler (i.e. the public good is distributed to all citizens). On the other hand, targeted transfers to the elite prove more efficient given that the elite is crucial to the election outcome.

2.3 McGuire and Olson (1996)

Relative to the political economics considerations discussed earlier, the central argument for McGuire and Olson (1996) framework (henceforth abbreviated "MOF") usability concerns the fact that it simplifies and aggregates political and economic interactions into a single objective function. Moreover, it offers a consistent framework which allows meaningful comparisons across the democratic spectrum. This contrasts with methods oriented on either the democratic process (e.g. median voter theorem) or a dictatorship (e.g. Wintrobe, 2004).

The framework assumes that, irrespective of the actual regime type, the authorities face only two choices. Firstly, they choose an optimal income tax rate, such that it maximises their prospective revenue. Once the tax rate is set, they decide on the level of public good provision. The society as a whole earns disposable market income, reduced in line with said income tax. In principle, they do not influence the economy.

In MOF, public good expenditure decreases the government's rents (they aim to maximise the difference between the tax revenue and public good spending). However, public good is critical (e.g. through maintenance of social order) to the production of *potential* output, i.e. before deadweight losses are accounted for. Such losses, in turn, are a result of incentive-distorting taxation. Thus, policymakers' decisions regarding the tax rate will incorporate the extent of possible inefficiencies.

An autocrat does not sell labour and does not earn any income in the market. Hence, they only aim to maximise the rents from extraction and face the following objective function:

$$tr(t)Y(G) \; - \; G, \; s.t. \; G < tr(t)Y(G) \tag{MOF:1}$$

where

$$\begin{aligned} t &= constant\ average\ income\ tax\ rate \\ G &= amount\ of\ the\ pure\ public\ good\ input\ (price = 1) \\ Y(G) &= potential\ output;\ Y'(G) > 0;\ Y''(G) < 0;\ Y(0) = 0 \\ r(t) &= \%\ of\ potential\ Y\ produced\ for\ given\ t;\ r'(t) < 0;\ r(0) = 1 \\ r(t)Y(G) &= I = \ actual\ income \end{aligned}$$

Such conceptualisation of autocracy yields interesting theoretical predictions. The critical aspect here is that a dictator, whose self-interest in principle leads them to extract resources from the society, benefits from the productivity of their citizens. Therefore, the rational autocrat would limit the "tax-theft" inclinations because of the incentive-distorting taxation's deadweight losses related to $r(t)$ (which decrease the output level and inherently decrease the tax revenues). In essence, the dictator increases the tax rate until marginal tax revenue equals marginal deadweight costs. A similar logic applies to the public good provision. Although the ruler wants to minimise expenditure on the public good, its provision contributes to the higher income of the society and, therefore, larger tax receipts. To sum up, such "encompassing interest" implicitly limits the predominantly bandit motivations of an absolute ruler[9]. Mathematically, these conditions (associated with maximisation of (MOF:1)) are summarised by:

$$t_A^* = -\frac{r\left(t_A^*\right)}{r'\left(t_A^*\right)} \tag{MOF:2}$$

$$r_A^* Y'(G) \equiv I'\left(t_A^*, G\right) = \frac{1}{t_A^*} \tag{MOF:3}$$

where (MOF:2) gives the optimal tax rate and where the optimal public good provision stems from (MOF:3). The latter simply states that "the autocrat provides G until the marginal increase in society's actual realized income from public goods equals the reciprocal of his share of national income" (McGuire and Olson, 1996, p. 45).

McGuire and Olson extend this theory to reflect on redistributive majoritarian democracies[10]. The authors assume that such a democratic government represents only a part of the wider society and hence leaves out those who do not support it (referred to as a "minority"[11]). In essence, policymakers act in the sole interest of the ruling majority and redistribute income from the minority to themselves through taxes. At the same time, however, they earn market income: MOF introduces a parameter F, which captures the fraction of the ruling interest's stake in market income. The parameter takes values between 0 and 1, where 0 indicates full autocracy (and therefore implies a logic identical to the one described earlier, related to equation (MOF:1)) and 1 suggests a consensual democracy (i.e. the entire society is included in the ruling interest). Thus, the objective function of democratic (or nonautocratic) authorities becomes:

$$(1-t)r(t)FY(G) \ + \ [tr(t)Y(G) \ - \ G], \ s.t. \ G < tr(t)Y(G) \tag{MOF:4}$$

The ruling majority will raise taxes for redistribution to itself until "the reduction in its share of market income is exactly as large as what it gains at the margin

[9]MOF assumes that autocrat's planning problem has a "long-horizon". This ensures that autocrats would not simply seize capital goods. The alternative assumption is that there are simply no capital goods.

[10]The authors consider also special cases of consensual and non-redistributive democracies. These are, however, beyond the spectrum of our paper.

[11]Although in the case of oligarchy or other hybrid regimes, the "voiceless" part could constitute a majority of the society.

from redistribution" (McGuire and Olson, 1996, p.86). Put differently, the democratic government's direct stake in the society's income further moderates the tax-related efficiency distortions and, thus, the extent of extraction from the minority. Similarly, a higher degree of encompassing interest in the market income incentivises the policymakers to provide more public good. Hence, compared to autocratic governments, policymakers whose interests are more aligned with the society's perspective (i.e. higher F) set lower taxes, impose smaller deadweight losses, extract less from the society, provide a higher level of public good and, effectively, contribute to a greater production and income. These conclusions follow from maximising (MOF:4) and the resulting first-order conditions:

$$F[-r + (1-t)r'] + (r + tr') = 0 \tag{MOF:5}$$

which gives the optimal tax rate

$$t_R^* = -\frac{r}{r'} - \frac{F}{1-F} \tag{MOF:6}$$

and

$$\{(1-t)rF + tr\}Y' - 1 = SrY' - 1 = 0 \tag{MOF:7}$$

where $S = (1-F)t + F$ and denotes the share of total actual output production $rY = I$ that the ruling interest receives from redistribution plus its market earnings (i.e. share of social income). SrY' thus refers to the marginal private benefit from G and -1 stands for the cost of G. Optimal G is therefore chosen such that:

$$Sr\left(t_R^*\right) Y'(G) \equiv SI'\left(t_R^*, G\right) = 1 \tag{MOF:8}$$

or equivalently

$$I'\left(t_R^*, G\right) = \frac{1}{S} \tag{MOF:9}$$

i.e. the authorities provide public good until its marginal private benefit equals 1 or, equivalently, until the society's marginal social return is $\frac{1}{S}$.

To conclude, MOF points to the crucial role of F in determining economic outcomes. Namely, higher F means a larger proportion of the society is acknowledged by the government. This leads them to produce policies more aligned with the overall social consensus. In our adaptation, we will proxy this parameter by the "level of democracy".

However, MOF does not feature any intertemporal choices, nor does it exhibit adjustments in the economy over time. Hence, the framework cannot be perceived as dynamic. Therefore, given that our research operates on the premise that most economic policy problems are processes, we aim to extend this model in a dynamic direction. Moreover, MOF omits the importance of political dynamics. As we show in the next section, both the level, and the change in the level, of democracy matter for the rate of economic growth. Hence, our model intends to demonstrate theoretical adjustments in the economy over time, subject to the level of democracy. In this context, we also allow for a possibility of a *regime shock*.

3 Democracy and economic growth

Suppose we reasonably assume – as we do in our model to be described in section 4 – that McGuire and Olson's (1996) parameter F reflects the level of democracy[12]. In that case, we will accordingly expect more democratic economies, *ceteris paribus*, to demonstrate a higher production and output. We are not testing this hypothesis, but looking at the most apparent real-world examples of highly democratic countries appears to paint a similar picture.

Recognising Costa Rica as a country consistently considered one of the most democratic in Latin America (see e.g. Freedom House (2024) data on political rights and civil liberties since 1972), we can juxtapose its GDP growth rates with the mean growth rates of the entire region. As depicted below, Costa Rica tends to outperform an average Latin American economy.

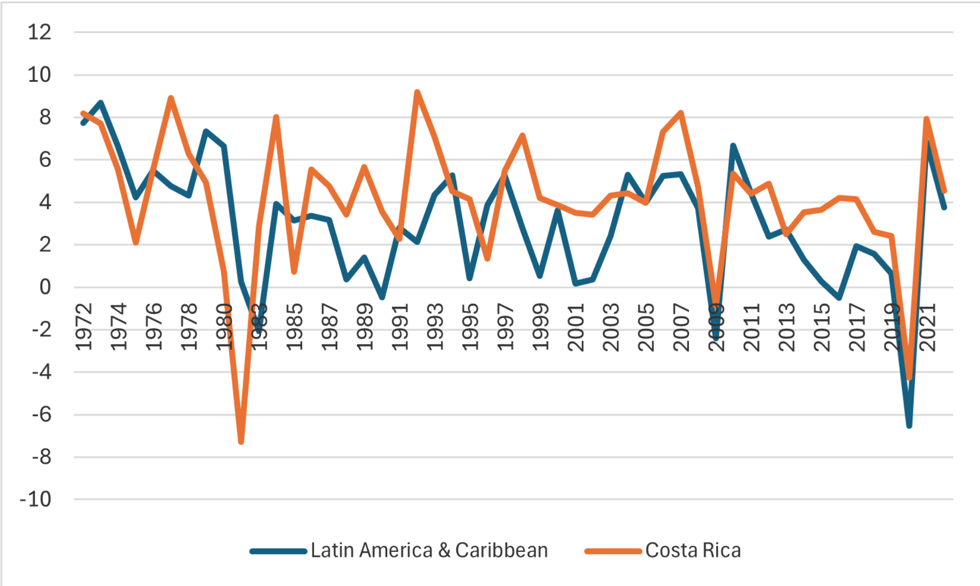

Figure 1: *Costa Rica vs Latin America: GDP growth (annual %); source: World Bank (2024a)*

The matter of economic growth, however, points us to an additional direction: time dimension and dynamics. Economies are not static and feature adjustments over time. Similar reasoning concerns politics: countries can democratise

[12] McGuire and Olson (1996) state that higher value of F in a majoritarian democracy is associated with a higher degree of encompassing interest as a larger part of the society is included in the calculus. This way, policymakers' objectives are closer to the social consensus. In practice, better political and civil rights allow citizens to influence the policymaking (e.g. through elections or civil pressure). Hence, it seems reasonable to assume that more democratic countries internalise the overall will of the society to a higher extent. Moreover, MOF makes a direct reference to specific regime types. Namely, $F = 0$ implies an autocratic system, whereas $F = 1$ implies a consensual democracy. This allows us to relatively safely assume that the overall logic of MOF's predictions applies to the extent of democratic institutions.

or consolidate power in the hands of a few. In fact, Acemoglu et al. (2019), who empirically find a positive impact of democracy level on GDP per capita growth, stress the importance of proper modelling of the democratisation process and GDP growth dynamics (especially to uncover long-term effects of democratic transitions). Critically for our chapter's contribution, however, the model by McGuire and Olson (1996) fails to account for the possibility of an institutional change just as it fails to show the impact of intertemporal choices. Below, we demonstrate a simple empirical rationale for our attention to political dynamics.

3.1 Democracy level vs economic growth

We consider a panel of 169 countries between 1972 and 2022 and make reference to their respective time-varying democracy levels and GDP growth rates. Relying on Freedom House (2024) data on "civil liberties" and "political rights", we sum both indices together so that they reflect a country's aggregate level of democracy. Freedom House assigns values ranging from 1 to 7, where those closer to 1 denote more or better-quality rights (thus, we expect a negative coefficient). We regress such aggregate democracy levels (variable "Demo0" below) against contemporaneous growth rates (sourced from World Bank (2024a)) while controlling for country and time fixed effects. Referring to Table 1, we notice a strong and significant (at 5%) correlation between the extent of democratic rights and economic growth.

Coefficients:

	Estimate	Std. Error	t value	$Pr(>\|t\|)$	
(Intercept)	4.201393	1.595080	2.634	0.008458	**
Demo0:	-0.121145	0.044201	-2.741	0.006145	**

Table 1: *Regression results: democracy against GDP growth*

To reinforce this result, we directly compare[13] two countries, democratic and nondemocratic, which otherwise appear quite similar. To strengthen the validity of such a comparison, we consider Cabo Verde and Guinea-Bissau. Both countries share Portuguese "heritage" and even considered unification in the 1970s. Both states are also longstanding members of the Economic Community of West African States, which facilitates economic integration[14]. We focus on the post-1999 period to avoid the confounding impact of the civil war in Guinea-Bissau[15]. Conveniently, in 1999, the two countries featured nearly identical GDP figures

[13]We do not provide a graph because their relative growth rates do not summarise intuitively. Therefore, we rely on average growth rates over the given period.

[14]Moreover, after 1993, the organisation formally strives to maintain peace between and in its member countries – this should enable more meaningful comparison considering the role of stability in economic growth.

[15]Although a longer period would further inflate the differences. For instance, starting from 1991 (when Cabo Verde held its first multi-party elections and became democratic) the difference in average growth between the countries is 3.92 percentage points.

(World Bank, 2024b), which further adds to the potential validity of our comparison. Throughout the said period, the average sum of liberty scores in Cabo Verde stood at only 2.13, whereas Guinea-Bissau featured a significantly higher mean level of 9. Accordingly, based on World Bank (2014a) data, more democratic Cabo Verde demonstrated an average growth rate higher by 1.75 percentage points.

3.2 Regime shocks vs economic growth

Moreover, it appears that the very fact of a change in democratic quality carries a significant (at 1%) and sizable impact on economic growth. To evidence this, we regress recent changes in the level of democracy (variable "DemoChg", defined as a difference between a current level and a value lagged by one year) over the growth rates. Table 2 below implies that a move towards democracy tends to be associated with positive growth effects.

Coefficients:

	Estimate	Std. Error	t value	$Pr(>\|t\|)$	
(Intercept)	2.229255	1.484203	1.502	0.133148	
DemoChg:	-0.329455	-0.329455	-3.639	0.000276	***

Table 2: *Regression results: democracy change against GDP growth*

The above regression results can be somewhat compellingly illustrated when analysing individual examples over time and identifying growth patterns prior and after regime shocks. Since 1972, our chosen countries have remained relatively free from armed conflicts or violent coups, which offers a clean route to the comparison of more and less democratic periods.

Firstly, we use the case of Benin to illustrate the hypothetical impact of a clear single, strong democracy shock. After Mathieu Kérékou seized power in 1972, Benin's combined Freedom House score varied between 12 and the most extreme 14. In 1990 - a year preceding the ultimate loss of Kérékou's power in 1991 – the country began a sudden democratisation process by adopting a constitution. Following the free election next year, Benin obtained the aggregate Freedom House score of 5 (i.e. the transition from a complete dictatorship to a fairly democratic state occurred within only 2 years, hence we can indeed claim a "shock"). In Figure 2 we notice that the positive democracy shock coincides with a clear cut-off in terms of the relative stability of growth. Similarly, the average growth rate in the 1972-1989 period amounts for 2.56%, while the average growth rate in the post-1990 period is 4.76%.

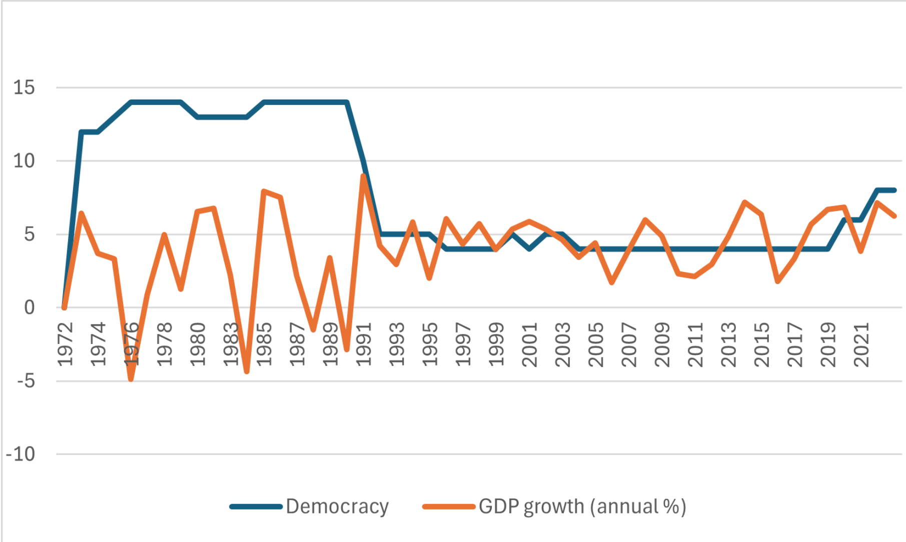

Figure 2: *Benin: political rights and civil liberties (Freedom House, 2024) vs GDP growth (annual %) (World Bank, 2024a)*

To complete the picture, we examine the examples of Turkey and Jordan which feature multiple significant but milder regime shocks, as well as slower transitions. As seen in Figure 3, Jordan constitutes a particularly vivid illustration as it features several periods where the inverse relationship between the aggregate Freedom House score and GDP growth is particularly noticeable.

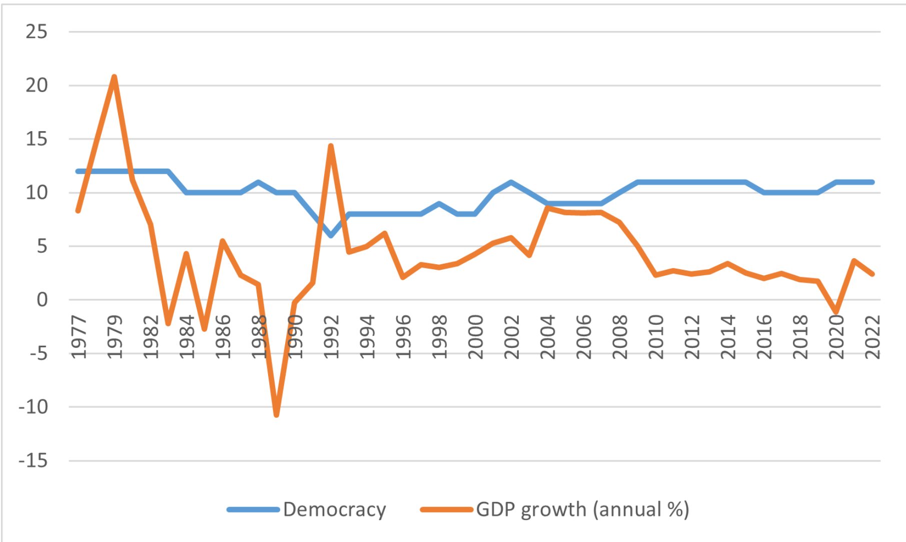

Figure 3: *Jordan: political rights and civil liberties (Freedom House, 2024) vs GDP growth (annual %) (World Bank, 2024a)*

In the case of Turkey, we arbitrarily divide the timeline (depicted in Figure 4) into relatively democratic and nondemocratic periods and show that the latter exhibit lower mean growth. We define the democratic periods as 1973-79, 1986-92, 2002-2015 and nondemocratic as 1980-85, 1993-2001, 2016-2022. Correspondingly, higher levels of democracy are linked to average growth rates of 4.4%, 4.88% and 5.92%. On the other hand, less democratic periods feature mean GDP growth of 3.41%, 2.9% and 4.78%.

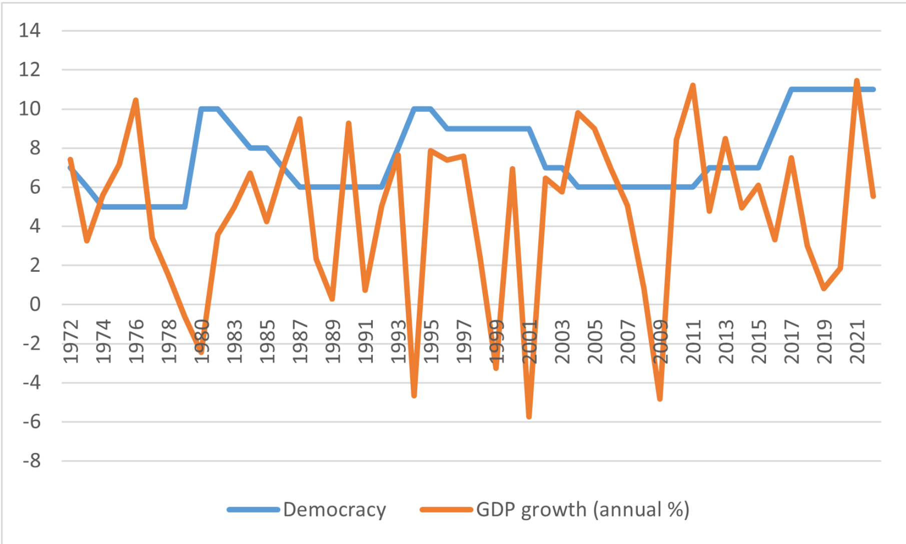

Figure 4: *Turkey: political rights and civil liberties (Freedom House, 2024) vs GDP growth (annual %) (World Bank, 2024a)*

To conclude, a priori, it seems reasonable to expect the level of democracy to play a role in the economic motivations of policymakers, A model that incorporates this, such as MOF, predicts higher levels of economic output in more democratic regimes. Here, we see that this conclusion from MOF is data consistent. However, we also see that changes in the degree of democracy have dynamic effects on economic activity, which, as well as our view that economic policy problems are processes and so dynamics matter, motivates us to extend the MOF. Our dynamic adaption of the McGuire and Olson (1996) model is introduced in the next section.

4 The model

The following section presents the dynamic adaptation of the McGuire and Olson (1996) model. The fundamental theoretical assumptions of the original remain largely unaltered. As well as introducing the time dimension and the associated intertemporal trade-offs, we write the model using specific functional forms to facilitate simulation. Below, we begin with a specification of the production side of the model. This is followed by the statement of the objective function which incorporates the political dimension.

4.1 Production

Irrespective of the political regime, production of the closed economy modelled here relies on the provision of public good, B. We assume that potential output (i.e. before efficiency losses are accounted for) is created in accordance with the Cobb-Douglas function with the labour input is normalised to 1:

$$Y_{t+1} = AB_t^{\alpha}, \tag{1}$$

where A and α denote total factor productivity and output elasticity of the public good, respectively. Consistent with MOF's restriction, marginal product of the public good is positive and diminishing.

At the same time, our production function serves as a key tool through which dynamics are introduced to the original model. Apart from the time dimension enforced by the subscripts characterising the variables, it should be noted that (potential) output production is not realised until the next period. Public good provision, therefore, can be interpreted to some extent as public investment. Such as in the original framework, however, capital formation is not considered. On dynamic grounds, such omission is explained by full depreciation. Namely, we assume that one time period is associated with an entire, two-term office tenure (+/-10 years), which shall allow us to assume complete depreciation.

The level of potential output formed according to the production function (1), however, will be affected by taxation. *Actual* output will be, therefore, reduced due to deadweight losses implied by the efficiency-distorting tax. The proportion of potential income which remains available in the economy after deadweight losses are accounted for is given by $e^{-\gamma\tau_t}$. It means we assign a specific, exponential, form to MOF's function $r(t)$ and ensure the conditions $r' < 0$ and $r(0) = 1$. In essence, higher tax rates always contribute to increased efficiency losses, and the γ parameter signifies the relative strength of the tax's impact. As we will see, such distortions are the very reason for self-imposed limits on the rulers' extraction inclinations.

4.2 Objective function and constraints

Having specified the production side of the economy, we introduce the criteria by which policymakers make decisions. To accentuate the presumption regarding the influence of democracy's level, we amalgamate the perspective of an autocrat and a democracy to a single dimension captured by parameter F, i.e. we do not separate the framework into "autocratic" and "democratic" models. Below, we firstly declare the policymaker's objective function. Further, we analyse economic choices and constraints faced by the agents. Lastly, we discuss the associated political dimension.

Overall, the government wants to maximise the sum of their discounted consumption flows C. Additionally, they care to a certain extent, F, about societal consumption, S. We assume each group's consumption is of logarithmic preferences:

$$U_t = \sum_{s=0}^{\infty} \beta^s (lnC_{t+s} + FlnS_{t+s}) \tag{2}$$

where authority consumption

$$C_t = \tau_t e^{-\gamma\tau_t} Y_t - B_t \tag{3}$$

and societal consumption

$$S_t = (1 - \tau_t) e^{-\gamma\tau_t} Y_t, \tag{4}$$

where β denotes the discount factor[16].

At any point in time, the authority extracts resources directly from the economy by setting tax rate τ_t, conditional on the deadweight loss. This tax revenue is subsequently used for elite[17] consumption and public good provision. Policymakers want to maximise the difference between the tax receipts they collect $\tau_t e^{-\gamma\tau_t} Y_t$ and the provision of productive public good B_t.

Clearly, the elite faces an intertemporal trade-off: whenever they are tempted to provide more B_t (to increase future output and thus future tax receipts), their current net benefit falls (i.e. $\frac{\partial C_t}{\partial B_t} = -1$ which is synonymous with the price of the public good). In terms of the tax, whereas the overall tax revenue increases C_t, it is not necessarily achieved by raising the tax rate as such. The government would increase the tax rate until the marginal benefit of such an increase equals marginal efficiency-loss. Precisely this key feature of MOF implies the limits to autocrats' tax-theft inclinations.

The society, in turn, consists of a unit-mass, price-taking agents whose actions are passive and cannot influence the equilibrium. Their consumption is synonymous with the disposable (market-earned) income leftover after the efficiency losses are accounted for. Higher tax rates, therefore, always reduce societal consumption:

[16] Which is a function of a discount rate, i.e. $\beta = \frac{1}{(1+\rho)^n}$.

[17] Throughout the remainder of this chapter, terms such as "elite", "authorities", "policymakers", etc. will be used interchangeably.

this is channelled both through associated efficiency damages to taxable income and the income tax itself. Similarly to the elite, however, they will benefit from public good provision which increases their prospective net-income residual: in terms of the future, the interests of both groups are aligned.

Before we move on to discuss the political component in more detail, we want to briefly point to Castellucci and Gorini (2019) who - to the best of our knowledge - were the only authors to attempt the dynamisation of MOF. Relative to our adaptation, they, too, treat public good provision to some extent as public investment. Similarly, they consider intertemporal impacts related to incentive-distorting taxation[18]. Their model, however, draws from MOF only superficially and ignores the key, political, aspect of the original. Namely, the authors consider only the "production" properties of MOF (i.e. public good as a factor of potential output production and taxation as a factor which effectively decreases actual output). Crucially, therefore, their model foregoes the objective function and motivations of the authorities, which we established in the current section.

4.3 Political dimension

Consistent with MOF, we assume $F \in \langle 0, 1 \rangle$ to be an exogenous parameter. Values closer to 0 thus imply more autocratic regimes, while values closer to 1 denote more democratic systems.

Nevertheless, in our adaptation of MOF, we extend the original idea behind F and simplify the mechanism associated with the ruling interest's redistribution. First of all, we interpret F as an index related to the level of democracy (rather than a "fraction of the total income produced and earned in the market accruing to the redistributive ruling interest" (McGuire and Olson, 1996, p. 54))[19]. Hence, we treat such level of democracy as a degree of policymaker's responsiveness or sensitivity to the society's welfare. Moreover, we simplify the measurement of relative rents: we explicitly differentiate between the direct interests of the elite, C, and the wider society, S. Our analysis, therefore, is oriented on clear depiction of consumption and associated welfare effects. This contrasts with McGuire and Olson (1996) who focus simply on income (see section 2.3).

According to our specification, an autocrat ($F = 0$) would simply ignore societal consumption when making optimal decisions and focus only on the impact of public investment on future output. However, it does not automatically imply $S_t = 0$ (i.e. there are limits to the extraction). Citizens' consumption needs are just not considered by the dictator and thus are not reflected in optimal choice. Nevertheless – as a byproduct of the autocrat's mutually beneficial decisions – S_t will virtually always be positive.

[18] Castellucci and Gorini (2019) focus on the assessment of the impact of a policy change. Specifically, they consider how intertemporal distribution is affected by the change from tax to deficit financing of public investment.

[19] Using MOF's logic more directly, F in our specification could be interpreted as a fraction of "the utility derived from consumption". MOF, instead, focuses on market income.

On the other hand, even full democracies will "suffer" from a positive net extraction (here, thought of as the difference between the tax receipts and public investment back into the economy) to some extent. Technically, it derives from the said separability of interests. Nonetheless, we can relatively safely presume that even the most responsive governments are not free from pursuing the self-interest of the officeholders or their accountability to the elites. This is consistent with the assumption of opportunistic behaviour prevalent in political economy models. Alternatively, we can think of such C_t for $F = 1$ as funds needed to cover costs of running the party (e.g. campaign costs), which are not productive and hence not reflected in public good provision.

5 Optimisation

To optimise the model, we rely on the standard dynamic programming method. To sum up, given the current state of the economy and political climate, the authorities will decide on an optimal forward plan by choosing a sequence of tax rates and level of public good provision, such that they maximise the (infinite) lifetime objective (2). Due to the forward-looking character, their utility is discounted subject to the discount factor β whose value reflects the weight attached to future consumption. The dynamic problem thus is specified with the following Bellman equation:

$$V_t(Y_t) = ln\big(\tau_t e^{-\gamma\tau_t} Y_t - B_t\big) + Fln\big((1-\tau_t)e^{-\gamma\tau_t}Y_t\big) + \beta V_{t+1}\big(Y_{t+1}\big) \quad (5)$$

s.t.

$$Y_{t+1} = AB_t^{\alpha}, \quad (1)$$

where optimisation yields first-order (taken with respect to the control variables τ_t and B_t) and envelope theorem conditions (taken with respect to the state variable Y_t) included in Appendix 9.1. These allow us to obtain the following dynamic system with which we can generate complete optimal paths of τ_t, B_t, Y_t, C_t and S_t, assuming specific values attached to the exogeneous parameters and initial state of the economy Y_1:

$$(1+\alpha\beta(1+F))(1-\gamma\tau_t)(1-\tau_t) = F(1+\gamma(1-\tau_t))\tau_t \quad (6)$$

$$B_t = \frac{\alpha\beta(1+F)}{1+\alpha\beta(1+F)}\tau_t e^{-\gamma\tau_t}Y_t \quad (7)$$

$$C_t = \tau_t e^{-\gamma\tau_t}Y_t - B_t \quad (3)$$

$$S_t = (1-\tau_t)e^{-\gamma\tau_t}Y_t \quad (4)$$

$$Y_{t+1} = AB_t^{\alpha} \quad (1)$$

An optimal tax rate is found by solving the implicit equation (6), which is a function of only parameters. Subsequently, based on such tax rate, we can establish the amount of the public good to be optimally provided using (7). Having obtained the choice of B_t it is then possible to simulate the system forward using the production function equation (1). At the same time, the system allows to track the contemporaneous consumption levels given by (3) and (4).

5.1 Comparative statics

The evolution of the optimised system will virtually always tend towards the steady state. Accordingly, the model's variables have complete analytic solutions associated with the steady state given by:

$$(1 + \alpha\beta(1 + F))(1 - \gamma\tau^*)(1 - \tau^*) = F(1 + \gamma(1 - \tau^*))\tau^* \tag{8}$$

$$B^* = \left[\frac{A\alpha\beta(1 + F)}{1 + \alpha\beta(1 + F)}\tau^* e^{-\gamma\tau^*}\right]^{\frac{1}{1-\alpha}} \tag{9}$$

$$Y^* = A(B^*)^{\alpha} \tag{10}$$

$$C^* = \tau^* e^{-\gamma\tau^*} Y^* - B^* \tag{11}$$

$$S^* = (1 - \tau^*)e^{-\gamma\tau^*} Y^* \tag{12}$$

what allows us to conduct a general comparative statics analysis. This way, we can compare the predictions of our dynamic model with those of MOF.

5.1.1 Tax

First of all, given that equations (6) and (8) yield the same value for the dynamic and steady state tax rates (and so are synonymous) it follows that the tax rate is fixed across all periods. Moreover, our optimal tax rate is a function of parameters only; therefore, the decision regarding its level is independent of the prospective public good provision. This evidences that the dynamic model implicitly replicates the characteristic of MOF (i.e. rulers first decide on the tax level and only then on public good provision). The second fundamental property is also maintained: a higher degree of democracy is always associated with a lower tax rate (see equation (MOF:6)). Considering the special case of full autocracy ($F = 0$), equation (8) simplifies to $\tau = \frac{1}{\gamma}$. This further highlights the importance of the inefficiency parameter in tax rate determination. Namely – again, in alignment with the original model; see equation (MOF:2) – deadweight loss is the only limiting factor for an absolute ruler when deciding on the level of tax.

The specific impact of the deadweight loss parameter and democracy level on tax choice can be observed on the graph below[20]. Although fundamentally nothing precludes $\gamma > 10$, we arbitrarily focus only on the $\gamma \in \langle 0, 10\rangle$ interval[21]. It is clear that greater γ is always associated with a lower tax rate; however, its marginal effect is diminishing[22]. Regarding the level of democracy, the evident negative relationship with the tax rate is also confirmed. Somewhat similarly, however, we can see that democracy's effect is more apparent in its lower regions.

[20] Assuming $\alpha = 0.3$ and a discount rate of 1%.

[21] While the lower bound ensures that even full dictatorships guarantee a positive level of societal consumption, the upper bound reflects the fact that marginal distortions beyond this point become nearly insignificant.

[22] It can be observed on the graph that the change from 5 to 10 does not decrease the optimal tax rate to the extent comparable with the change from 1 to 5.

Additionally, it is less pronounced when γ becomes sufficiently large. Put differently, a higher deadweight loss factor keeps the autocrat more "in line" and closer to the societal optimum.

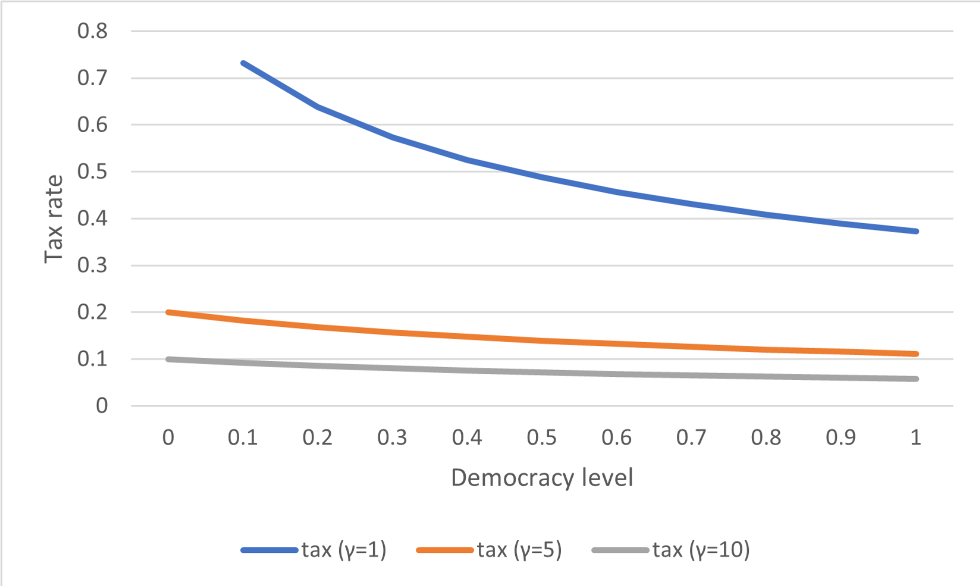

Figure 5: *Tax rate as a function of democracy, subject to* γ

5.1.2 Public good and output

Turning the attention towards the public good, equation (9) also confirms that mechanisms present in MOF apply in our adaptation. According to the original paper, the key aspects which contribute to the optimal choice of the public good provision are F, productivity of public goods (in our model determined jointly by A and α) and the efficiency-distortions associated with taxation: all of which constitute the parameters affecting optimal public investment in our setting.

Considering the association between tax and deadweight loss, we can look at equation (9) from two perspectives. Firstly, we can focus on the sole impact of the inefficiencies. It is clear that greater deadweight losses associated with $e^{-\gamma\tau}$ decrease public good provision. On the other hand, we can see by looking at $\tau e^{-\gamma\tau}$ that the tax itself positively contributes to B's level. Again, this is logical given that higher tax revenue *allows* dedicating more funds to the public good spending. However, the proportion of these tax collections that will be actually dedicated for public good provision depends predominantly on F and β.

To recollect, higher levels of democracy are associated with lower tax rates. At the same time, however, despite the government's lower tax revenue, more democratic authorities manage to provide a greater amount of public good. The "balancing item" is simply found in lower elite consumption what enables to dedicate more funds to B to the benefit of the society. While a specific amount of public good provision is less straightforward to establish under MOF, greater F is associated with higher levels of public good as well (see equations (MOF:8)-(MOF:9)). Comparably, our dynamic model clearly links a higher extent of democratic institutions with an increased level of steady-state public good provision.

The discount factor highlights another dimension through which forward-looking regimes would restrain themselves from being tempted to underprovide the public good. Although the output level is not the rulers' goal per se, it contributes to consumption: the more weight is attached to future consumption, the higher the incentive to invest in future output. Therefore, in the steady state, a lower degree of impatience results in a higher provision of the public good and, by extension, a higher level of output.

Having analysed the statics related to the fundamental variables of the model, the next section proceeds to dynamic evaluations. There, we also reflect on consumption and wider welfare consequences related to optimal plans.

6 Dynamic simulations

The most apparent or practical benefit of the model's dynamic aspect is the possibility of the system's simulation to evidence the adjustments over time. In section 6.1, we show how relative differences in the level of democracy (even if constant over time) contribute to dissimilar economic outcomes, both in the short-term and eventual steady state. Although our analysis focuses on a comparison of the two extreme solutions (full autocracy vs full democracy), we nevertheless provide graphs for the intermediate level of democracy ($F = 0.5$), too. In section 6.2, we analyse a hypothetical impact of a one-off democracy shock on output's evolution.

For illustration, we assume a sufficiently low initial state of output Y_1 (i.e. set below Y^*), so that it is possible to visualise growth as the variables tend towards the steady state. Furthermore, we arbitrarily set $A = 1$, $\alpha = 0.3$ and $\beta = 0.9$, where the latter corresponds to an annual discount rate of 1% over a 10-year period. Lastly, we choose γ such that the optimal tax rate is calibrated to match the base rate of Norwegian income tax, 22%. This is because Norway is the most democratic country according to the most recent Democracy Index (The Economist Intelligence Unit, 2024), with a score of 9.81 out of 10. We normalise this score to match our assumed boundaries $F \in \langle 0, 1\rangle$ so that $F = 0.981$. The assumed tax rate for such a parameter is ensured when $\gamma = 2.27$.

6.1 Regime comparison

Referring to Figures 6-7, we can see that both systems follow a very similar adjustment trend, although differing in relative levels. The autocrat consistently chooses a lower level of public investment, B (Figure 6), and a higher tax rate at 44%, relative to the democratic regime's 22%. Eventually, the two economies reach their respective steady states, with the democracy landing on the actual output level 14% above its autocratic counterpart. This results principally from a stronger commitment to public investment (higher by as much as 54%).

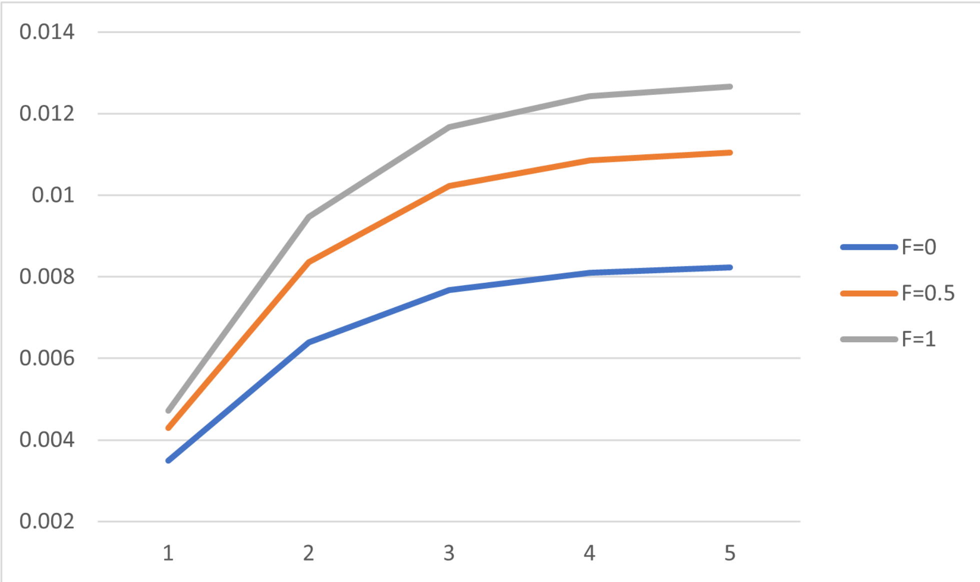

Figure 6: *Public good provision over time*

The differences in output levels appear to fit the narrative described in section 3. Comparing GDP paths of democratic Cabo Verde and nondemocratic (or semidemocratic) Guinea-Bissau depicted in Figure 8, we notice similar patterns to those predicted by our model in Figure 7. On both graphs, the two economies begin with virtually identical output levels. Distinct growth magnitudes, however, become apparent very quickly in both diagrams[23]. Admittedly, we do not observe convergence to a steady state in our real-world example but, by 2022, democratic Cabo Verde achieves GDP higher by 12% compared to Guinea-Bissau. Assuming Guinea-Bissau's democracy level would be reflected by $F \in \langle 0, 0.5 \rangle$, such GDP spread is surprisingly close to our model's theoretical outcome. The predicted steady state difference in production for $F = 1$ and $F = 0$ is 14%, whereas for $F = 1$ and $F = 0.5$ it is 4%. The factual 12% therefore remains well within our model's implied boundaries.

[23]The two diagrams, however, follow a different timescale. Our simulation assumes adjustments every decade over the total period of 50 years, while Figure 8 shows annual evolution over the period of 23 years. The intention of the comparison is to provide the overall intuition, rather than numerical accuracy.

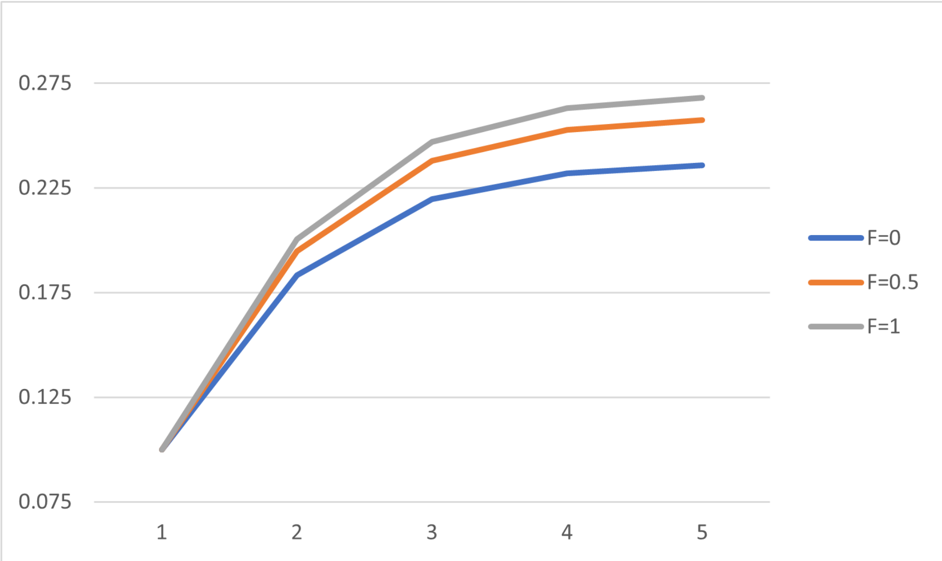

Figure 7: *Actual output over time*

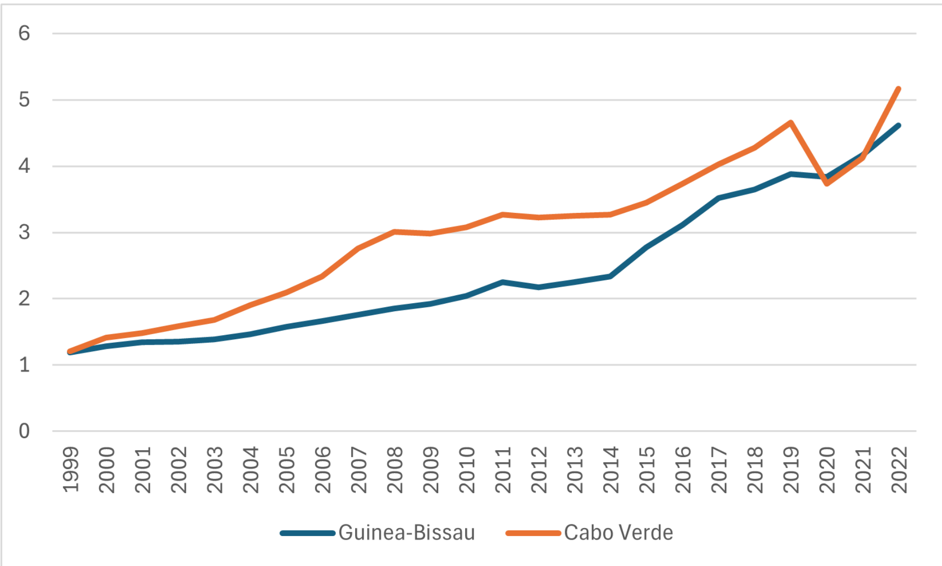

Figure 8: *Cabo Verde vs Guinea-Bissau: GDP, PPP (current int. $)[billions]; source: World Bank (2024b)*

Even though the contrast in our model's output production (despite significant differences in B allocation) is not striking, political characteristics remain critical to consumption levels and broader social welfare. An initial (at $t = 1$) swing from a full autocracy to a full democracy would decrease the elite's steady state consumption by 23% (see Figure 9). At the same time, societal consumption (Figure 10) would increase by as much as 164%. This suggests the relative inefficiency of nondemocratic states: the society bears unproportionally extensive costs to increase its rulers' well-being even slightly. Over the entire path, the welfare effects related to democratic institutions are also evident. Specifically, complete democracy induces an additional 29% in societal welfare (measured as discounted consumption flows, subject to the logarithmic preferences). The same difference would cause the elite to incur a relative loss of 8%.

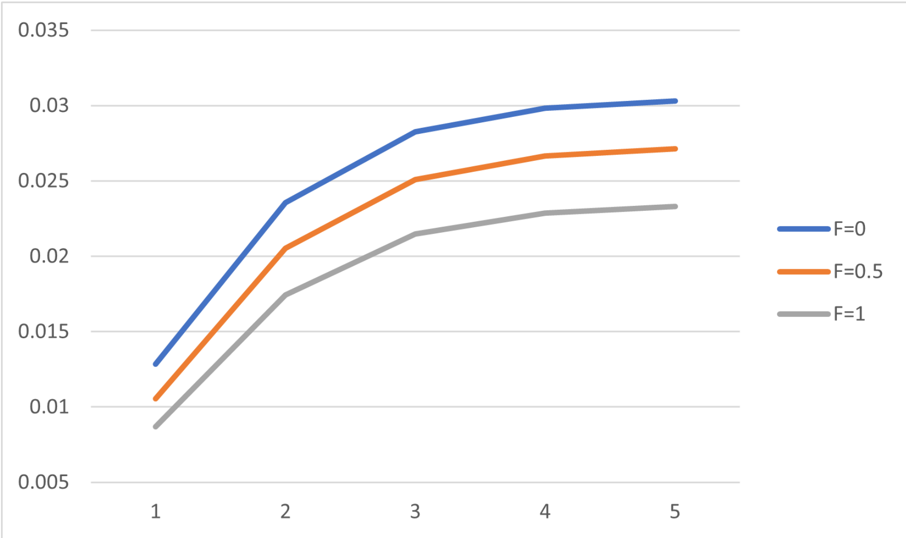

Figure 9: *Elite consumption over time*

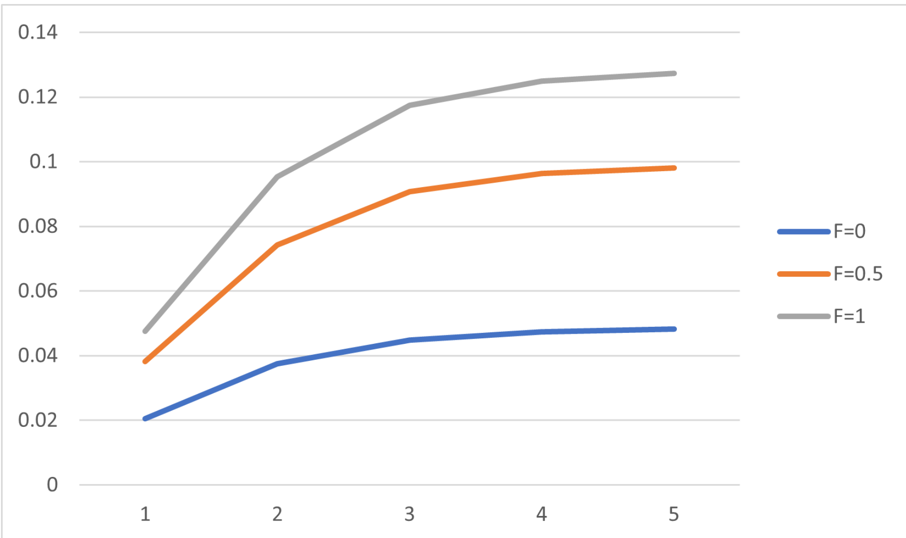

Figure 10: *Societal consumption over time*

6.2 Democracy shock

The dynamic setting also allows us to show another aspect inherently not capable of being applied in the static model: a regime shock. In this section, we make a direct reference to the case of Benin discussed in section 3 (although now we consider nominal GDP values, rather than growth rates). We assume our modelled economy begins in 1970 with the initial level of output set at its steady state value. The economy represents a complete autocracy ($F = 0$) until the decade beginning in 1990. After this, we introduce a positive democracy shock, consistent with the one experienced by Benin[24], i.e. parameter F changes to 0.75.

As seen in Figure 11, initially, the autocratic economy's output level remains at its steady state. Following the democratic transition, however, output begins to significantly grow (consistent with our regression results (Table 2) from section 3). The (now democratic) economy sustains such positive - but continually decreasing - growth until it reaches a new, higher, steady state.

We don't argue for a numerical precision of our simulation but we do show that the general direction of the prediction holds: a positive democracy shock enhances growth potential. This evidently correlates with what can be observed based on Benin's GDP data in Figure 12[25]. Between 1972 and 1990, the country grows relatively slowly. Following the rapid democratisation in the early 1990s, it appears clear that the pace of growth begins to accelerate. This factual GDP evolution of Benin seems to follow a rather exponential pattern. Our model predicts a sharper growth initially, which then gradually fades away (graphically, it resembles a logarithmic function), but otherwise fits the Benin's case relatively well.

According to our result depicted in Figure 11, the "democratic" steady state would be associated with a higher level of output, compared to the initial, "autocratic" steady state. Such difference seems also in line with Acemoglu et al. (2019) who confirm the general long-run positive effects of democracy. Specifically, they estimate that 25 years following a democratisation, "GDP increases gradually until it reaches a level 20–25 percent higher than what it would reach otherwise." (p. 50). Although our result suggests the difference of only 12%, the overall logic is reproduced quite precisely (i.e. growth potential improves).

[24] Preceding the shock in 1990, Benin stood at the joint Freedom House score of 14. In two years, the country managed to democratise and achieved the score of 5. If 14 on a $\langle 2, 14 \rangle$ scale reflects $F = 0$, then the score of 5 must reflect $F = 0.75$.

[25] We work out the best fit exponential model for the nondemocratic period, and use this to detrend the whole series. This will then be akin to an autocracy at steady state, followed by a democracy shock.

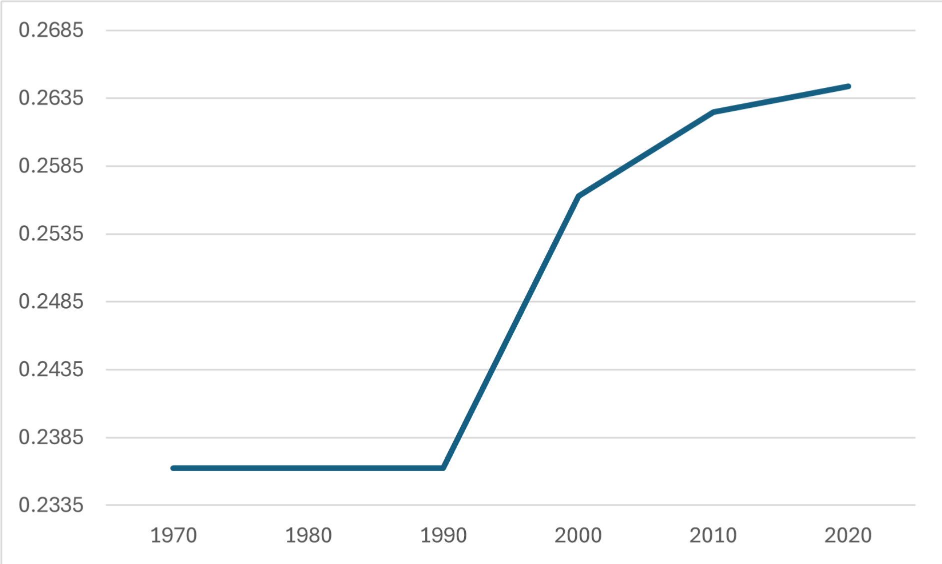

Figure 11: *Actual output: theoretical democracy shock*

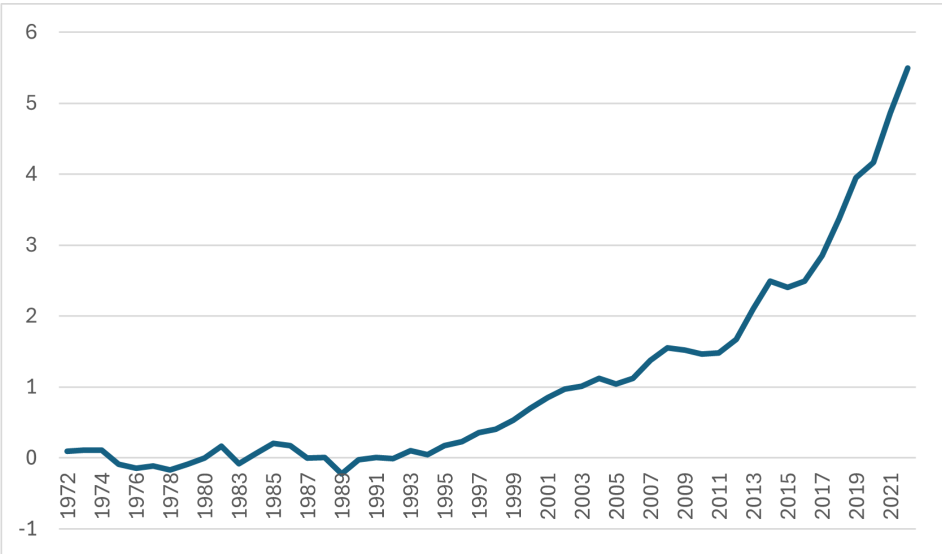

Figure 12: *Benin: GDP (constant 2015 US$)[billions]; We work out the best fit exponential model for the nondemocratic period, and use this to detrend the whole series. This will then be akin to an autocracy at steady state, followed by a democracy shock; source: World Bank (2024c)*

7 Conclusion

This chapter focused on the potential role of the level of democracy on economic objectives of the policymakers. We reviewed the political economics literature oriented on modelling different regimes. Most of these models are equipped to tackle economic problems in either a democracy or a dictatorship. One of the few consistent frameworks that can represent the level of democracy as a parameter within a single model was found in the seminal model by McGuire and Olson (1996).

The limitation of this framework, however, is that it is static and is not capable of showing adjustments in the economic processes over time. We argued that political and economic dynamics can be important in economic analyses. Through econometric exercises and specific country examples, we showed that the democracy level, as well as its changes over time, tend to correlate with economic growth.

Considering this, we adapted the McGuire and Olson (1996) model by adding the time dimension and reframing the original problem as an intertemporal one. Using our dynamic model, we simulated the optimal solutions to illustrate how differing democratic quality contributes to long-run economic outcomes. Our results are in line with the static predictions of the original: more democratic policymakers set lower tax rates and extract less from the society; democratic economies feature greater output growth due to larger public good provision; and the level of democracy contributes positively to societal consumption and welfare. Moreover, our dynamic adaptation is well-equipped to simulate the impact of regime shocks. In this context, we showed that positive democracy shocks improve growth prospects.

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9 Appendix

9.1 Optimisation

The planning problem is as follows:

$$V_t(Y_t) = ln\big(\tau_t e^{-\gamma\tau_t} Y_t - B_t\big) + Fln\big((1-\tau_t)e^{-\gamma\tau_t} Y_t\big) + \beta V_{t+1}\big(Y_{t+1}\big) \tag{5}$$

s.t.

$$Y_{t+1} = AB_t^{\alpha} \tag{1}$$

Optimisation of the Bellman equation (5) yields the following first order (i.e. taken with respect to the control variables τ_t and B_t) and envelope theorem (i.e. taken with respect to the state variable Y_t) conditions:

$$F.O.C.s$$

$$w.r.t.\ \ \tau_t, \qquad 0 = \left[\frac{1-\gamma\tau_t}{C_t} - \frac{F(1+\gamma(1-\tau_t))}{S_t}\right] e^{-\gamma\tau_t} Y_t \tag{13}$$

$$w.r.t.\ \ B_t, \qquad \frac{1}{\beta C_t} = \alpha AB_t^{\alpha-1} e^{-\gamma\tau_t} Y_t \frac{\partial V_{t+1}}{\partial Y_{t+1}} \tag{14}$$

$$E.T.$$

$$w.r.t.\ \ Y_t, \qquad \frac{\partial V_t}{\partial Y_t} = \frac{1+F}{Y_t} \tag{15}$$

Chapter 3
Can autocracies save climate?

Abstract

Climate change mitigation undoubtedly proves a political matter, thereby stalling efficient energy transition. Hence, a natural question seems to arise: are certain political systems more capable than others of conducting effective climate policy? On the one hand, authoritarian governments possess the necessary apparatus to implement unpopular but effective solutions. Yet, in practice, it appears that these tools are not utilised for environmental goals to a degree comparable with democratic states. This chapter aims to establish the theoretical impact of such institutional conditions (i.e. level of democracy) on climate change mitigation. Thus, we rely on the dynamic adaptation of the seminal model of political economy by McGuire and Olson (1996) and introduce a climate externality. The results suggest that lower democratic accountability is associated with lower cumulative emissions. This is achieved, however, by reduced economic growth and the ability to constrain societal consumption rather than higher investment in renewables. We show that a positive democracy shock contributes to increased investment in renewables, as well as fewer emissions when expressed as a percentage of output. Moreover, democratic policymakers prove more efficient in limiting emissions in the event of a climate shock.

1 Introduction

Climate change mitigation – despite the scientific consensus regarding the seriousness of the phenomenon – undoubtedly proves a political problem. It probably is not surprising: because agents in society assess policies differently, every policy instrument generates an economic conflict (Persson and Tabellini, 2000). The political aspect of the issue is well illustrated by the referendums in the State of Washington. The proposals for a carbon tax of $15 per ton of CO_2 were rejected twice (in 2016 and 2018). At the same time, it directly points to possible shortcomings of the democratic system in relation to efficient climate policy.

Similarly, it raises the question if nondemocratic systems are perhaps better suited to overcoming political constraints and producing effective solutions. Political regime characteristics, however, were not considered by any of the influential economic models of climate change (e.g. Nordhaus (2008); Golosov et al. (2014)). In this chapter, we address this gap and focus on the theoretical impact of the level of democracy on the efficiency of climate policy and the ability to limit emissions. Would a rational autocrat be more concerned about prospective climate damages to their source of income? Should countries democratise as far as climate change is concerned?

Climate change is a long-horizon problem. At the same time, the costs of its mitigation are immediate. The electoral cycle, however, appears to favour policies with quick positive impacts and minimal costs upon the voters. Budget constraints might give precedence to more urgent (and critical to economic subsistence) matters than environmental care (Midlarsky, 1998). At the same time, officials might indeed make bold climate policy proposals, but delay their implementation so that the budget consequences fall onto their successors (Sinn, 2009).

Furthermore, as von Stein (2022) points out, the fundamental issue that relates democratic quality to environmental outcomes concerns citizens' preferences: electoral accountability implies that policymakers must consider what the public actually wants. Politicians will not risk their next term by implementing unpopular decisions if voters view a climate policy as unacceptable (as was the case e.g. in Washington). Additionally, Barker (2008) stresses the importance of agents' heterogeneity when it comes to practice. Democratic policymaking inherently strives to achieve a compromise between various interest groups, which might halt the efficient development of timely solutions.

On the other hand, authoritarian[1] regimes care about their citizens' preferences and wellbeing only to a limited extent. If rulers deem a policy or an investment worthwhile, they will simply implement it without too much consideration for households' welfare. Undeniably, the "if" is critical here, but specific examples from the world – while not numerous – illustrate the efficiency argument vividly.

[1] Throughout this chapter, we will use terms like "authoritarian", "autocratic", "dictatorship", etc. interchangeably.

For instance, in 2023, China began constructing the world's largest plant to generate hydrogen from renewable sources. Kazakhstan commissioned one of its biggest solar plants (100MW) in 2020. Probably the most relevant aspect of the Kazakh undertaking in the context of efficiency is that the entire process (from the bidding and permission, followed by construction and launch of commercial operation) took only two years. Therefore, such examples might explain why the seriousness of the prospective climate crisis and lack of sufficient action in this regard gave rise to the notions of environmental authoritarianism (see e.g. Beeson (2010)) or authoritarian environmentalism (see e.g. Shen and Jiang (2020)).

As we evidence in the literature review in section 2, the literature offers more (often mutually exclusive) arguments regarding the theoretical channels relating the regime type with environmental performance. Moreover, we show that the overall political dimension of environmental performance or policy is studied rather extensively. However, the specific issue of the role of democracy's level in this context is found mostly in empirical papers. Therefore, a clear gap remains in the economic modelling literature. We describe Congleton (1992) and Eriksson and Persson (2003) - two of the very scarce models which attempt to address the matter of our interest - in the literature review, too.

In this chapter, we develop a dynamic model of political economy and climate change. The core of our motivation is that carbon emissions accumulate over time and gradually increase the global average temperature. Climate change and associated damages to the economy, therefore, constitute a long-term problem. To address this, we rely on the model presented in Chapter 2, which is a dynamic adaptation of the framework originally developed by McGuire and Olson (1996). Instead of directly modelling the electoral cycle, we focus on the interplay (channelled by the extent of democratic accountability) between office-holders' and society's objectives to see how respective consumption needs effectively shape policy enactment.

Here, we extend our dynamic model by accounting for a climate externality and two capital types: green (emission-neutral) capital and brown (more productive but emission-inducing) capital. This way, we show the theoretical impact of the level of democracy on the relative efficiency of limiting emissions, both in the short and long run. Moreover, we visualise the adjustments in the economy over time and demonstrate the impact of climate and regime shocks.

Ultimately, our model predicts that - due to lower economic growth - an autocracy tends to feature lower cumulative emissions. However, a more democratic economy produces fewer emissions when measured as percentage of output. A positive democracy shock contributes to increased emissions, but also to (more than proportionally) higher investment in renewables. In addition, democracies are more efficient in curtailing emissions when faced with a climate shock.

The contribution of this chapter is primarily theoretical as it strives to answer the following question: is democracy conducive to efficient climate policy and limiting carbon emissions? Therefore, we extend the existing, very limited, economic modelling literature that analyses the impact of a political regime on environmental regulation. Our study's contribution also touches on methodological grounds. Namely, to our knowledge, we present the first climate change adaptation of the McGuire and Olson (1996) model, especially in the dynamic variant. Consequently, our chapter has the potential to inform climate-related policymaking, with particular consideration of the political regime aspect. Recommendations of this kind could be particularly relevant to international agencies facilitating global climate transition.

The chapter is structured as follows. The next section surveys the existing literature, whereas section 3 introduces our model. Subsequently, we outline the optimisation procedure, and section 5 reports the results of dynamic simulations. The last section concludes.

2 Literature review

Regarding the theoretical channels which relate the regime type with environmental performance or the overall efficiency of policymaking, the literature offers many (often mutually exclusive) arguments. According to Li and Reuveny (2006), free media assured in more democratic countries enable raising public awareness regarding the environment (although the same freedom of speech might as well give a platform for denialist misinformation (von Stein, 2022)). Well-informed citizens might then pressure the government to act or elect suitable officials owing to their civil and political rights. Authoritarian regimes are inherently less sensitive to such pressures (Payne, 1995).

Another argument concerns policy variability. Due to frequent elections and possible government changes, democracies are prone to policy and agenda instability (Rodrik, 1991). A clear example in this context can be found in Donald Trump, who withdrew the US from the Paris Agreement following his appointment in 2017. On the other hand, autocracies also exhibit a risk of policy reversals, as well as an overall lack of credibility, thereby possibly deterring investment (Adam and Filippaios, 2007). Mobilising private investment in renewables in such an environment might, therefore, face obvious obstacles.

Lastly, following Tsebelis (2002), democratic policymaking is vulnerable to special-interest groups who can act as veto players. Considering the active role of oil sector lobbyists who oppose pro-climate legislation, this logic seems sensible. Nevertheless, autocratic regimes are not free from the influence of the elites who support or legitimise the reigns of a dictator, either (Bueno de Mesquita et al., 2003). They, too, often possess the control over the nation's natural resources.

Overall, the existing literature studied the impact of democracy on climate performance rather willingly; empirics is especially fruitful in this regard. Despite the political constraints prevalent in democracies, empirical papers mostly give a reassuring view of the positive impact of democratic quality on environmental performance. For instance, Sinha et al. (2023) find that democracies emit less CO_2 for a unit increase in per capita income. According to Lv (2017), democracies indeed curtail carbon emissions. However, this happens only once a country achieves a certain income level. Povitkina (2018), on the other hand, observes that once the influence of corruption is controlled for, the differences between regimes cease to be significant. Nevertheless, while providing us with beneficial insights, empirics – due to its backwards-looking character – does not constitute a sufficient tool for planning the climate transition. Turning to the economic modelling literature, we notice that the topic is much less prevalent.

Admittedly, environmental policy has been studied relatively extensively in political economy models. For instance, the impact of polluting producers' lobbying activity on environmental legislation – in the form of a green tax and three redistribution scenarios – is examined in the probabilistic voting model by Aidt (2010). Borissov et al. (2014), on the other hand, develop a dynamic median voter model with heterogenous households who vote for an environmental tax. Another example can be found in Tol (2020) who developed a model of climate policy with "selfish bureaucrats". However, modelling literature remains largely silent regarding the comparison of environmental policy across the democratic spectrum. Below, we present two scarce examples of such studies.

The most relevant paper is by Congleton (1992) who compares the stringency of pollution regulations between a democracy and an autocracy. In this model, environmental policy is said to follow an individual evaluation of the probability of environmental degradation. Such probability, in turn, decreases due to stricter regulations and increases with national output[2]. Furthermore, environmental standards are assumed to exert a nonlinear impact on national income. Initially, they improve the overall productivity (e.g. health) and increase income. Once a certain threshold is reached, additional regulations decrease national income "as less productive technologies are mandated and inputs are diverted from ordinary economic production to environmental improvement without offsetting productivity increases" (Congleton, 1992, p. 414).

The political component in Congleton's model is reflected chiefly by the share in the economy's income. In this sense, a democracy is governed by preferences of a median voter. Conversely, an authoritarian regime represents the choices of an agent whose share of income is necessarily larger than the median. Moreover, the author assumes the autocrat to have a time-horizon that is shorter

[2]Regarding the influence of output, however, the author does not appear to assume it to be a source of environmental deterioration (which, in fact, is not specified in the paper at all). Instead, he suggests that the individual perception of risk increases when income is higher. Therefore, environmental regulation plays a role of a social insurance.

than the median voter's. These two differences determine that the authoritarian regime would ultimately enforce a comparably less stringent environmental policy. Firstly, the autocrat would bear higher marginal costs of environmental control (i.e. would suffer "a larger fraction of associated reductions in national income"). Moreover, the costs of environmental policy are assumed to be concentrated in the initial periods, with gains manifesting in the later future. For this reason, a shorter time horizon of the autocrat also disincentivises more significant environmental protection.

The median voter theory is also used by Eriksson and Persson (2003) to examine the interplay between democracy, inequality and pollution. Compared to Congleton (1992), however, they assume the median voter to be decisive in both democracy and nondemocracy. The authors restrict the decision-making in nondemocracies to an exogenously given population subset, i.e. the policy outcome will reflect the preferences of the median voter from the privileged group only. Moreover, the model assumes heterogeneity regarding individual productivity, income and experienced environmental quality (i.e. the privileged group lives in cleaner areas).

Production-wise, the key tradeoff concerns the use of production technology: more productive technology is directly linked to higher emissions. Regarding the preferences, the voter must balance out the marginal utility of consumption with the marginal disutility of pollution. The authors find that a democracy pollutes less than a nondemocracy, assuming both regimes are characterised by a more equal income distribution. Moreover, focusing on environmental inequality, they conclude that it affects pollution levels only in nondemocracies. In such a case, higher environmental inequality contributes to greater contamination.

Relative to papers by Congleton (1992) or Eriksson and Persson (2003) which focus on air pollution, our study provides a fully dynamic perspective on the economics of climate change specifically. While the two models, to some extent, do consider the time dimension (i.e. Congleton (1992) refers to an agent's planning horizon; Eriksson and Persson (2003) - in an attempt to mimic the Environmental Kuznets Curve - assume the existence of two phases related to development), they fail to account for accumulation of emissions over time and associated temperature rise. This chapter, moreover, shows the impact of climate and democracy shocks on the level of emissions in the short and long run alike.

3 The model

In the following section, we describe the fundamental features of our model. Essentially, we rely on the characteristics related to the objective function developed in Chapter 2, which itself is a dynamic adaptation of the McGuire and Olson (1996) framework of political economy. The key aspect of our model is that it allows us to examine how the level of democracy affects the optimal intertemporal decisions of the policymakers.

We merge this baseline model with an adaptation of production and damage functions used in the analytic climate economy model by Golosov et al. (2014). The inclusion of components developed by these authors is motivated predominantly by the fact that it allows us to operate within the same class of models (i.e. analytic integrated assessment models). Moreover, the Golosov et al. (2014) model exhibits useful properties that facilitate its optimisation, such as the existence of analytic solutions and consumption being a constant fraction of output. We simplify the original production function to reduce the amount of interrelated state variables and choices. The simplifying assumptions related to our adaptation of Golosov et al. (2014) are stated in Appendix 8.1.

Overall, our model consists of a passive, price-taking society and policymakers who set a (deadweight loss-inducing) income tax rate and decide on public investment. The extent to which the government acknowledges societal consumption is denoted by the level of democracy. Furthermore, the production of *potential* output (before the deadweight loss is accounted for) relies on two inputs: brown and green capital. At the same time, carbon emissions accumulate as a result of brown investment and cause damage to output.

Below, we firstly outline the production function in more detail, as well as specify the climate externality and associated damages. In 3.2, we describe the political motivations, related economic objectives and constraints which characterise the actions of policymakers.

3.1 Production and climate damages

Golosov et al. (2014) specify production as a Cobb-Douglas function of labour, capital and energy, subject to the total factor productivity. In our model, we normalise labour and total factor productivity to 1. Secondly, we alter the original energy composite function. We still adopt the constant elasticity of substitution (CES) form; however, rather than considering energy resources, we implicitly assume that energy is a product of two capital types. This allows to amalgamate the overall energy input (as a CES function of capital) with the production function where capital features directly, so that:

$$Y_t = e^{-\xi P_t} \left(\psi K_{B,t}^{\rho} + K_{G,t}^{\rho} \right)^{\frac{\alpha}{\rho}} \tag{1}$$

Effectively, our production function features two inputs, “brown” capital K_B and “green” capital K_G. α refers to the output elasticity of total capital and ρ refers to the substitution parameter between the capital classes. A value of the latter approaching negative infinity would indicate that the capital classes are perfect complements; in contrast, a value of 1 would imply perfect substitutes. Furthermore, ψ signifies the relative efficiency advantage related to brown capital, which we assume to take values above 1.

Consistent with Golosov et al. (2014), we assume that capital fully depreciates, given that one t reflects a period of approximately 10 years. The amount of capital in the following period, therefore, depends only on respective investment:

$$K_{B,t+1} = I_{B,t} \tag{2}$$

$$K_{G,t+1} = I_{G,t}. \tag{3}$$

Potential output production is additionally affected by the climate externality. Again, drawing from Golosov et al. (2014), we assume that economic damages result from the accumulated stock of carbon emissions. According to production function (1), the proportion of potential output that remains in the economy is captured by $e^{-\xi P_t}$, an exponential function of emissions stock[3]. P_t constitutes the cumulative emission level and ξ denotes the damage parameter that enables scaling the damage function. Furthermore, we assume that emissions accumulate in line with brown investment, such that

$$P_{t+1} = P_t + \theta I_{B,t} \tag{4}$$

where θ represents a multiplier parameter associated with emissions per unit of brown investment (i.e. emission intensity) or simply the "dirtiness" of brown investment.

To sum up, it becomes apparent that production exhibits a trade-off between the input of brown and green capital. On the one hand, brown capital is more productive than its green counterpart. On the other, its stock is inherently linked to brown investment, which contributes to (damaging) emissions.

3.2 Preferences

Government's preferences are specified as in Chapter 2 of this thesis. Policymakers[4] choose tax rates, τ, and make public investment decisions - in this chapter disaggregated into I_B and I_G - such that they maximise the sum of their discounted consumption flows, C. Moreover, depending on the weight F, they internalise the impact of their decisions on societal consumption, S. The parameter F is synonymous with the level of democracy, where a value of 0 implies an autocracy and a value of 1 suggests a full democracy. Consumption of both groups is described by logarithmic preferences:

$$\sum_{t=0}^{\infty} \beta^t (lnC_t + FlnS_t) \tag{5}$$

[3]According to Golosov et al. (2014), such an exponential function relatively precisely approximates the damage function proposed by Nordhaus (2008).

[4]Throughout this chapter, we will use terms like "government", "policymakers", "authorities", "elite", etc. interchangeably.

where the authority consumption

$$C_t = \tau_t e^{-\gamma \tau_t} Y_t - I_{B,t} - I_{G,t} \tag{6}$$

and societal consumption

$$S_t = (1 - \tau_t) e^{-\gamma \tau_t} Y_t. \tag{7}$$

β in (5) denotes the discount factor. The $e^{-\gamma \tau_t}$ component present in (6) and (7) represents the proportion of potential output that remains after deadweight losses are accounted for. Such losses are attributable to incentive-distorting tax, τ_t, and γ parameter allows us to scale said distortions.

Considering specific consumption elements, for each time period, the authorities want to maximise the difference between the collected tax revenue, $\tau_t e^{-\gamma \tau_t} Y_t$, and public investment outlays I_B, I_G. At the same time, they need to weigh up the impact the tax rate exerts on deadweight losses and, therefore, taxable income. The society – lacking any impact on the economy's equilibrium – simply consumes the disposable income remaining after the income tax and deadweight loss reductions. Higher taxes always decrease the flow of current societal consumption.

4 Optimisation

The following subsection addresses the trade-offs faced by the policymakers, specifies the model's constrained optimisation problem and provides the optimal solution.

The primary tradeoff concerns the optimal tax-setting. In order to maximise tax collections, the policymakers need to balance out the marginal benefits related to a higher tax rate with its offsetting marginal costs. Namely, a higher tax rate always contributes to increased deadweight losses and thus decreases actual output (i.e. income) to be taxed. Therefore, even dictatorships will restrain the appetite for over-extraction from society. The situation is further accentuated if we consider nonautocratic governments (i.e. $F > 0$). Specifically, a higher degree of sensitivity towards society's needs inherently leads policymakers to internalise the additional impact of taxes on societal consumption. Hence, we can expect more democratic states to be associated with lower tax rates and thereby smaller deadweight losses.

The second trade-off is an intertemporal one. In principle, higher investment spending decreases current consumption of the authorities. Nevertheless, being forward-looking (subject to the discount rate), policymakers realise the need to create future output given that it will enable their prospective consumption. This fact once again aligns the interests of the society with the government and is even more pronounced for higher levels of democracy: a more substantial investment is needed to fund future consumption of *both* groups.

The issue of investment inherently brings us to the third key trade-off, which effectively constitutes our model's climate policy. Primarily, policymakers are tempted to invest in brown capital given its productivity advantage over green capital. However, they are aware of the emissions resulting from brown investment and potential damages to their future consumption. Hence, to a certain extent, the government shall mitigate the prospective climate damages by investing in green capital. Whether more democratic governments prove more sensitive to the prospective climate damages constitutes the ultimate research question of this chapter.

The above considerations are aggregated numerically in the following dynamic programming problem. Essentially, the policymakers choose series of tax rates, brown investment and green investment to maximise the infinite lifetime objective (5), subject to the initial states. The Bellman equation is as follows:

$$\begin{aligned} V_t(K_{B,t}, K_{G,t}, P_t) = ln\left(\tau_t e^{-\gamma\tau_t} Y_t - I_{B,t} - I_{G,t}\right) + Fln\left((1-\tau_t)e^{-\gamma\tau_t}Y_t\right) \\ + \beta V_{t+1}(K_{G,t+1}, K_{B,t+1}, P_{t+1}) \end{aligned} \tag{8}$$

where production is given by (1) and the state variables evolve according to (2), (3) and (4).

The entire solution to our model is described in Appendix 8.2 - 8.8. Differentiation of the objective function yields first-order and envelope theorem conditions which are available in Appendix 8.3. In the subsequent numerical process, we avail ourselves of the features of the Golosov et al. (2014) model. This provides consumption as a constant proportion of output, $C = \lambda Y$. Secondly, the optimality condition equates the marginal products of capital via an implicit carbon tax, such that $MPK_G = MPK_B - T$.

In a similar manner, firstly, we find that the authority consumption is a constant proportion, λ, of tax revenue[5]. Hence, for all periods $t > 1$ we have:

$$C_t = \lambda\tau_t e^{-\gamma\tau_t} Y_t \tag{9}$$

Secondly, also following Golosov et al. (2014), we find that the optimal carbon tax, T, is a constant multiplicity $\frac{\theta\xi}{1-\beta}$ of actual output. Therefore, accounting for the emission-inducing impacts of brown investment, the optimality requires that marginal products of capital for all periods $t > 1$ equal:

$$MPK_G(t) = MPK_B(t) - \frac{\theta\xi}{1-\beta}e^{-\gamma\tau_t}Y_t \tag{10}$$

Such an implicit carbon tax clearly reflects the importance of θ and ξ. It appears logical that a higher carbon tax would be required to balance out more significant marginal emissions and damages.

[5] In Appendix 8.8, we describe how optimal λ is calculated.

4.1 Optimal investment

The two identities established above enable us to obtain consistent investment choices. As before, full details of the solution are available in Appendix 8.5. Nonetheless, the level of green investment for all periods $t > 1$ is obtained from the implicit equation (11)

$$\begin{aligned} \alpha I_{G,t}^{\rho-1} &= \alpha\psi\left((1-\lambda)\tau_t e^{-\gamma\tau_t}Y_t - I_{G,t}\right)^{\rho-1} \\ &\quad - \frac{\theta\xi}{1-\beta}\left(\psi\left((1-\lambda)\tau_t e^{-\gamma\tau_t}Y_t - I_{G,t}\right)^{\rho} + I_{G,t>1}^{\rho}\right) \end{aligned} \tag{11}$$

while the consistent choice of brown investment follows from

$$I_{B,t} = (1-\lambda)\tau_t e^{-\gamma\tau_t}Y_t - I_{G,t} \tag{12}$$

Essentially, it appears clear that investment decisions do not *directly* depend on the level of democracy. However, similarly to the logic established in Chapter 2, we can expect more democratic authorities to impose smaller deadweight losses and consume less. Therefore, this shall leave more resources (otherwise extracted as rents) available for investment. The relative tendency with respect to a specific investment type is less straightforward to establish. Nevertheless, the output of our simulations exhibited in the section 5 will be able to aid the answer.

4.2 Optimal tax rates

As specified in Appendix 8.6, equations characterising the optimal tax rate differ between nonautocratic polity ($F > 0$) and full autocracy ($F = 0$). The former is obtained from

$$\lambda = \frac{1}{F}\left(\frac{1}{\tau(1+\gamma-\gamma\tau)} - 1\right) \tag{13}$$

whereas to consider the particular instance of full autocracy, we rely on a simple formula which depends only on the γ parameter:

$$\tau = \frac{1}{\gamma} \tag{14}$$

In both cases, irrespective of λ's value, tax rate will always be constant over time: it is a function of only parameters. This feature proves consistent with the baseline model developed in Chapter 2 and confirms that the choice of tax is independent of its prospective impacts on the remaining decisions (i.e. public investment).

Instead, the tax rate will be affected by the democracy level and deadweight loss parameter, γ. Namely, consistent with McGuire and Olson (1996), lower F leads to a higher tax rate, what is ultimately accentuated in full autocracy. By the same token, higher inefficiency losses captured by γ constitute a limiting factor, decreasing the optimal tax rate.

5 Simulations and results

In this section, we present the output of dynamic simulations and show how our model optimally adjusts over time, focusing predominantly on the impact of the level of democracy. Our analysis concentrates on the most illustrative comparison of the two extreme solutions (full autocracy vs full democracy), although the model is well-equipped to deal with intermediate levels of democracy as well.

Firstly, we describe the calibration strategy. Afterwards, we analyse democracy's influence on the key parts of the modelled economy. Consequently, we address the underlying question posed in this chapter: is democracy conducive to combating emissions and limiting temperature rise? Having provided general results, we deliver plausible explanations.

5.1 Calibration

We assume a model period is 10 years, and we are interested in simulating the model from 2020 to 2100. Hence, $t = 1$ would be synonymous with the interval beginning in the year 2020. Secondly, we consider two cases. Both deal with the same initial state of the economy and differ only in terms of the level of democracy F, i.e. all regimes are initially equally endowed. For possibly the most effective illustration, we contrast the solution linked to $F = 0$ with the one connected to $F = 1$.

To obtain the values of the initial state variables at $t = 1$, we solve an additional model variant, with the same production structure but no environmental externality or political component (refer to Appendix 8.9 for details of the entire procedure). In this variant, policymakers have entire output at their disposal and make choices on brown and green investment, however, not realising any climate consequences. We choose the substitutability between brown and green capital such that they exist in an 80:20 ratio in the steady state of this model. This condition is imposed when $\psi = 4^{(1-\rho)}$. The remaining parameters are chosen arbitrarily and summarised in the table below:

θ	ξ	ρ	ψ	α	β	γ	F
1	0.1	0.8	1.32	0.3	0.82	2	0 vs 1

Table 1: *Parameters*

where the value of β reflects an annual discount rate of 2%.

The last aspect concerns the global temperature growth. As stated in 3.1, we assume the externality is directly mapped to emissions, not temperature. Nonetheless, to aid visualisation, we produce simulations which also reflect the temperature growth. Overall, we depict the incremental changes relative to the base year 2020 where emission stock is zero. In principle, this initial level could reflect 1°C temperature rise since pre-industrial era. The growth that we show thus would reflect the *additional* temperature rise. Then, by referring to the emission stock of a fully democratic economy in 2100, we "translate" this level such that it corresponds to the overall, additional, rise of 2°C (i.e. 3°C pre-industrial). Increments in emissions over time shall then be reflected in proportional increases in temperature, regardless of the regime type. This way, both economies shall pass the goal of the Paris Agreement (2°C above pre-industrial) around 2050/2060.

5.2 Results and discussion

5.2.1 Output and welfare

In $t = 1$ (i.e. 2020), incentive-distorting taxation appears in the economy as the policymakers' tool. The autocrat sets the optimal (fixed) tax rate at 50%, while full democracy does so with 25% [6]. Together with the taxes, deadweight losses come into existence. Thus, relative to the initial state given by $t = 0$, in Figure 1, we observe a sudden fall in actual output[7]. The fall is experienced by both regimes; however, it is more significant for a higher tax rate, i.e. in autocracy.

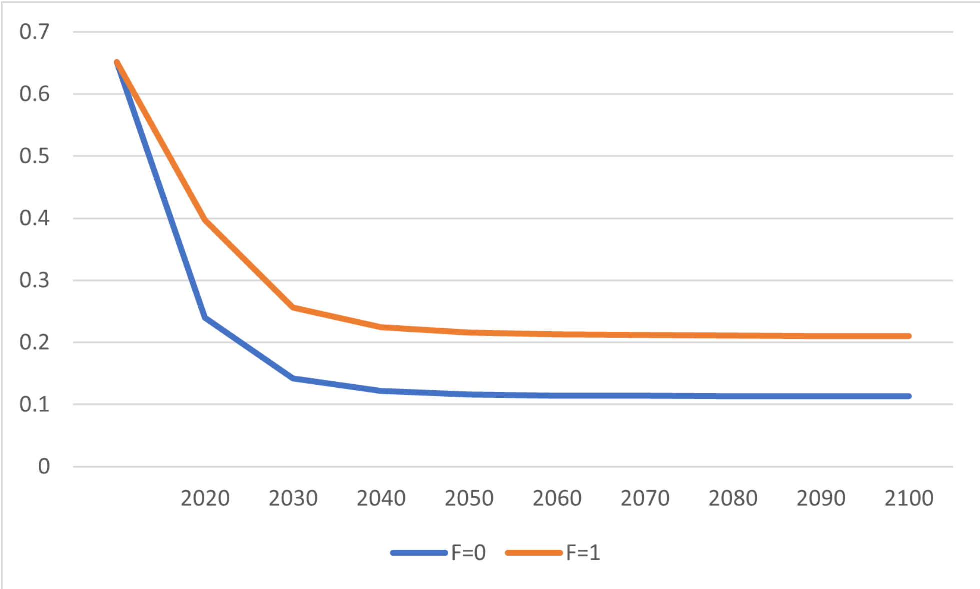

Figure 1: *Actual output over time: full autocracy vs full democracy*

This relative difference in actual output production is maintained up until 2100. By then, democracy would be able to produce 85% more than autocracy[8]. This result is inevitably connected to consistently stronger public investment: in 2100 alone, the democracy's total investment exceeds the autocracy by 46% (investment will be analysed in more detail in 5.2.2).

[6]Specific values of the tax rates are not important for interpretation of further results. However, we report them to demonstrate the significant difference in motivations and choices of the policymakers on the opposite sides of the political spectrum.

[7]Note that from this point onward, we will exclude $t = 0$ from graphs.

[8]As we will see shortly, democracy achieves higher output level despite higher emissions and thereby more significant climate damages.

Furthermore, the society living under the democratic authorities experiences relative welfare gains. Such relative difference (measured as a discounted sum of consumption flows, subject to logarithmic preferences) amounts to as much as 60%. Conversely, the authorities would face a welfare loss of 10%. Consumption paths are depicted in Figures 2 and 3.

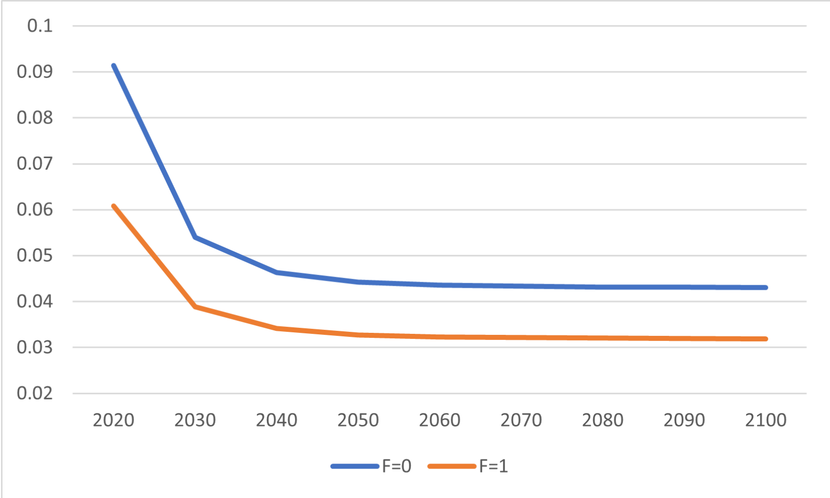

Figure 2: *Elite consumption over time: full autocracy vs full democracy*

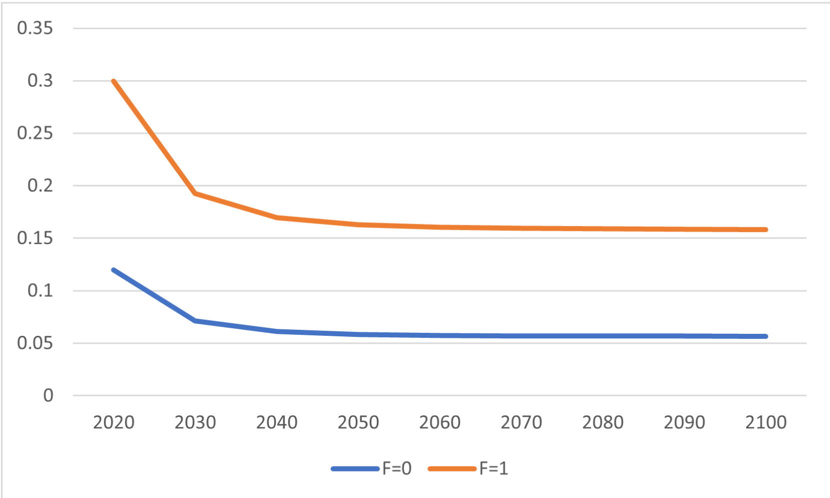

Figure 3: *Societal consumption over time: full autocracy vs full democracy*

The direction of the democracy's level impact remains consistent with the baseline model developed in Chapter 2. The overall logic can be reiterated and summarised as follows. Fundamentally, more authoritarian regimes set higher taxes. As a result of their rent-seeking, they impose more significant deadweight losses. This leaves them with even fewer resources available for public investment, considering they still want to maximise the difference between tax revenue and public spending.

In contrast, more democratic governments internalise their voters' welfare to a higher extent. Such a higher degree of the "encompassing interest" effectively translates to lower taxes and deadweight losses. Despite smaller tax collections, lower authority consumption allows for more resources to be dedicated for public investment and future production.

5.2.2 Emissions

Referring to Figures 4 and 5, we can observe that lower democratic accountability is associated with lower (26% by 2050 and 29% by 2100) cumulative emissions and a slower temperature rise. By 2050, the authoritarian regime achieves 0.66°C (0.23°C less than democracy) and 1.43°C by 2100 (0.57°C less than democracy).

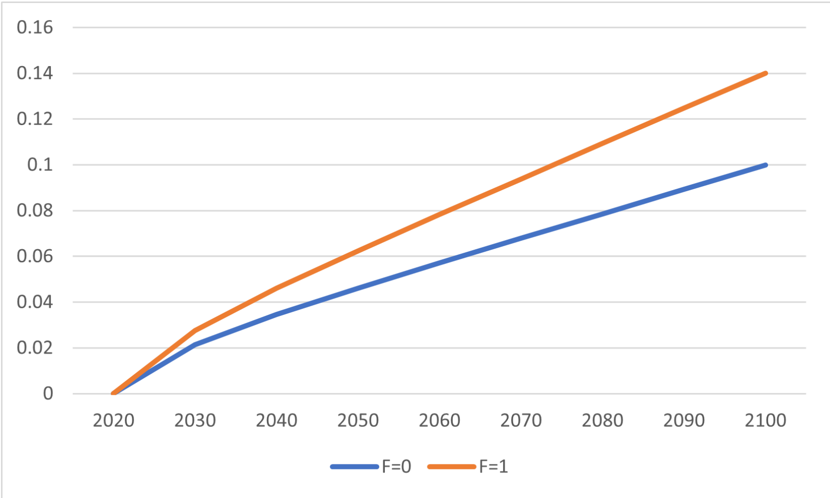

Figure 4: *Cumulative emissions relative to 2020: full autocracy vs full democracy*

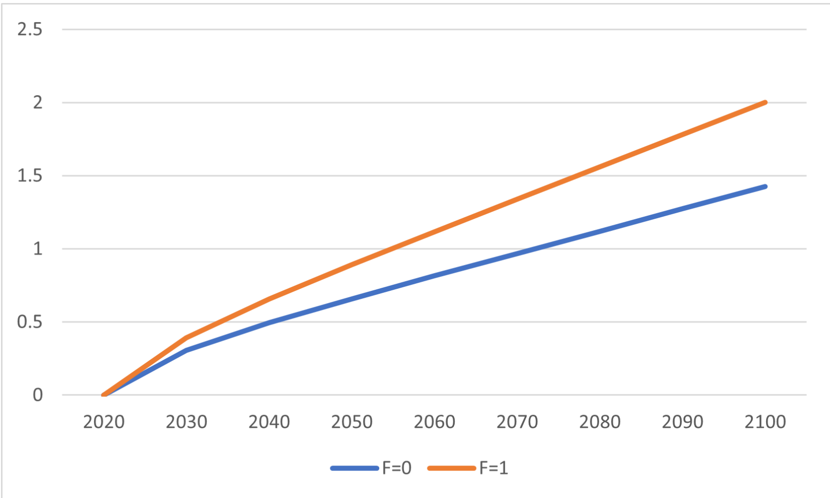

Figure 5: *Temperature rise [°C] relative to 2020: full autocracy vs full democracy*

This is achieved, however, by the ability to constrain societal consumption and reduced output production rather than increased investment in renewables. Namely, following the logic established earlier, the autocracy underprovides public investment. This includes green investment, but also the carbon-intensive brown investment (see Figures 6 and 7). Capital mix, nonetheless, still favours brown capital - although the mean capital mix is only slightly "greener" (1 percentage point difference) under full democracy. Similarly, while the democracy's brown investment in 2100 is 44% higher than the dictator's, green investment is higher by as much as 53%. Crucially for the assessment of the relative "efficiency" of climate policy, the democracy features lower emissions as % of actual output[9]. By 2100, they reach 67%, compared to autocratic 88%.

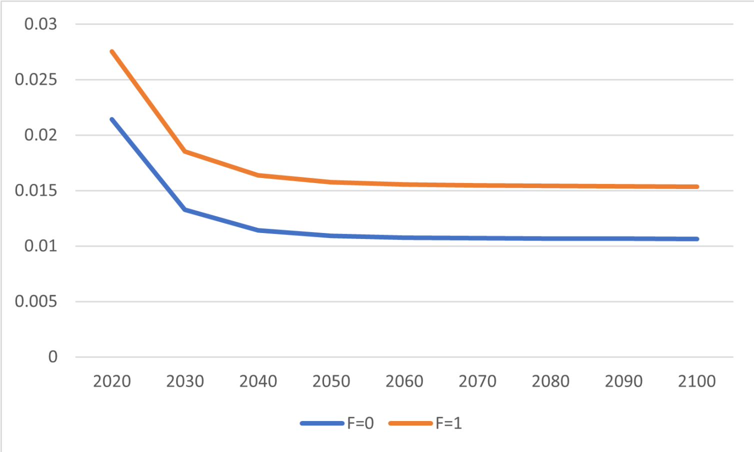

Figure 6: *Brown investment over time: full autocracy vs full democracy*

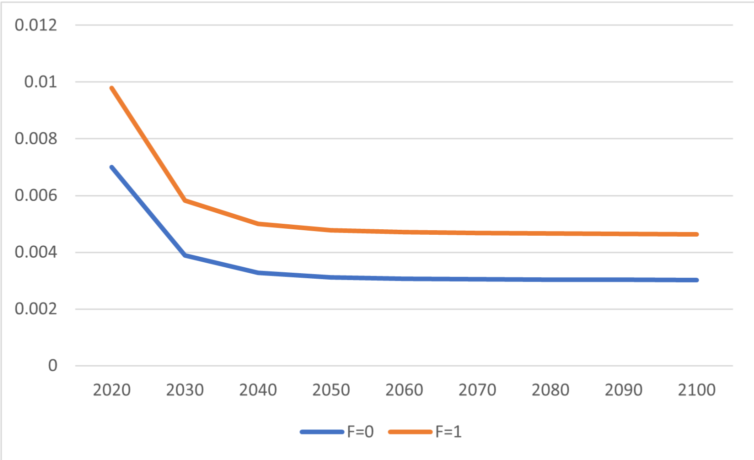

Figure 7: *Green investment over time: full autocracy vs full democracy*

[9]Typically, emission intensity is expressed as CO_2 emissions in kilograms per unit of economic output. However, for the ease of illustration and comparison, we assume emissions are expressed in the same units as output.

Therefore, to sum up, fewer emissions of the autocracy are only a welcome byproduct of lower investment in general. Democracy, in turn, produces higher actual output and invests more despite comparatively larger climate damages (which are greater by 0.4 percentage points in 2100). In the next subsections, we analyse how shocks complete the picture painted above.

5.2.3 Democracy shock

In this subsection we look at a positive democracy shock and its impact on cumulative emissions. Beginning with 2020, we consider a full autocracy what - prior to the shock - implies optimal paths identical to those described in 5.2.2. Then, by 2050 the economy experiences a sudden, moderate democratisation (i.e. $F = 0$ changes to $F = 0.5$). In Figures 8 - 10, we report emission and investment paths: we juxtapose the evolved regime against the counterfactual for a constant $F = 0$.

Following the regime shock, we begin to notice a divergence of emission paths: by 2100, the semi-democratic regime accumulates 15% more emissions. However, this is a result of increased investment overall. After the shock, policymakers reduce the extractions and efficiency losses, thus beginning to invest more. By 2100, the relative difference in brown investment amounts to 31%. The difference in green investment, however, is even higher (37%), what suggests that democratisation improves prospects of a greener capital mix. Similarly, emissions as % of actual output decrease from 88% to only 64%.

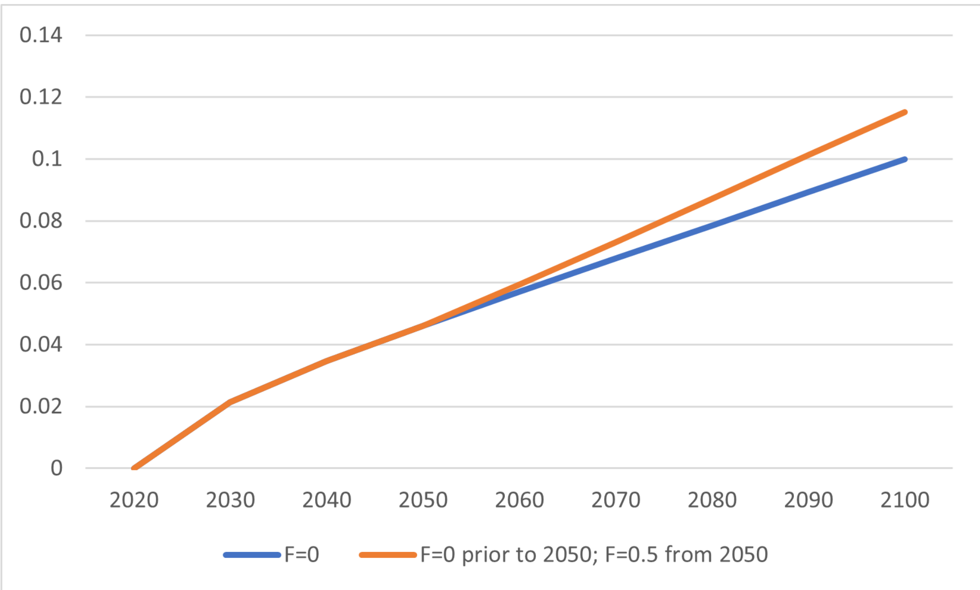

Figure 8: *Cumulative emissions over time: a path where the economy faces a democracy shock by 2050 vs a counterfactual for a constant democracy level*

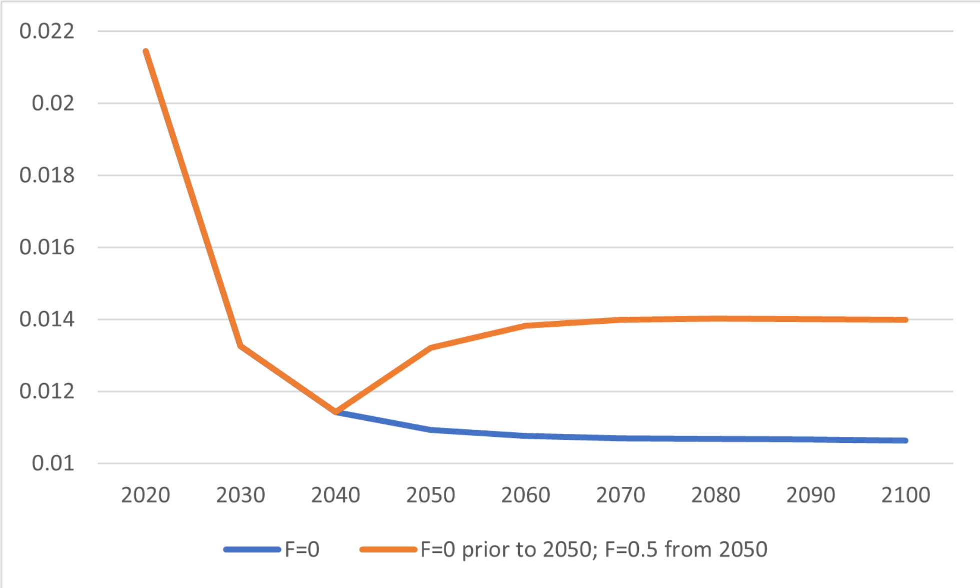

Figure 9: *Brown investment over time: a path where the economy faces a democracy shock by 2050 vs a counterfactual for a constant democracy level*

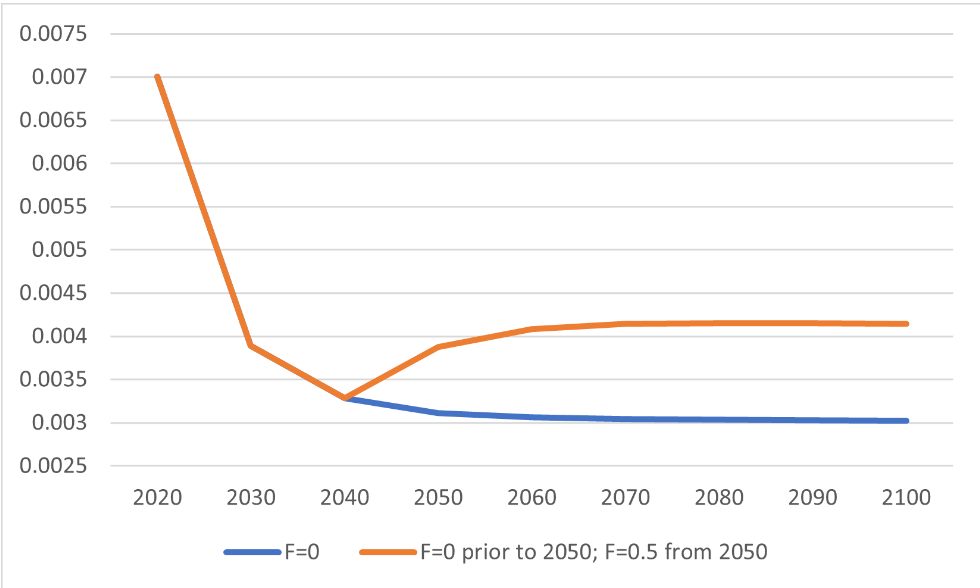

Figure 10: *Green investment over time: a path where the economy faces a democracy shock by 2050 vs a counterfactual for a constant democracy level*

5.2.4 Climate shock

Below we reflect on the relative capability of regimes to cope with a climate shock. As in 5.2.2, we compare two extreme polity cases. Both regimes initially begin with the emission intensity and the damage factor parameters assumed as earlier, i.e. $\theta = 1$ and $\xi = 0.1$. By 2050, however, they realise (we assume a new scientific evidence is available) that the climate repercussions become more significant. This is reflected by the change of said parameters to $\theta = 1.5$ and $\xi = 1.5$.

Following the shock, the pace of emissions growth noticeably decreases in both regimes. Moreover, compared to the solution in 5.2.2 where we observed a continuously increasing divergence in emission levels, here we notice that the two paths (insignificantly) converge. Put differently, compared to the solution in 5.2.2, the relative difference in 2100 cumulative emissions diminishes. Ultimately, although the democracy still emits more (18%), it also proves more efficient in curtailing emissions: compared to cumulative emissions reported in 5.2.2, the democracy reduces emissions by 19% and the autocracy by 14%.

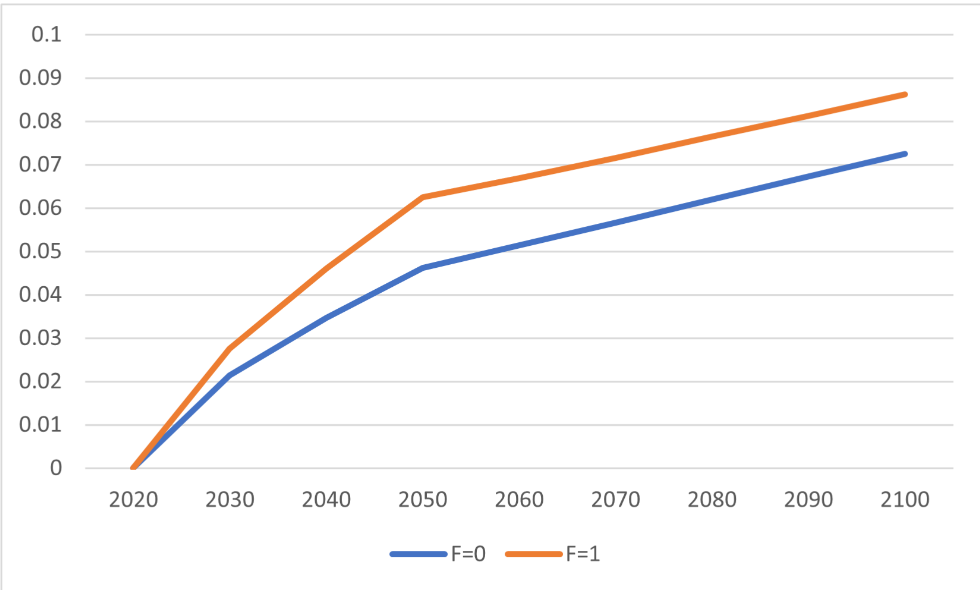

Figure 11: *Cumulative emissions relative to 2020 with a climate shock occurring in 2050: full autocracy vs full democracy*

Turning attention to investment (Figures 12 and 13), we observe that both regimes swiftly adjust to the shock. They, already in 2050, significantly increase green investment; the democracy features a higher increase and maintains the relative difference in the long run (76% by 2100). Similarly, both economies substantially decrease brown investment; the democracy exhibits a greater fall, such that the comparative levels remain negligible over the long run. By 2100, capital mix in both economies favours renewables: 25:75 in the democracy and 36:64 in the autocracy. These results, together with the significant difference in emissions as % of actual output (53% under democracy and 81% under autocracy), suggest that democracy is better equipped to transition to a low-carbon economy.

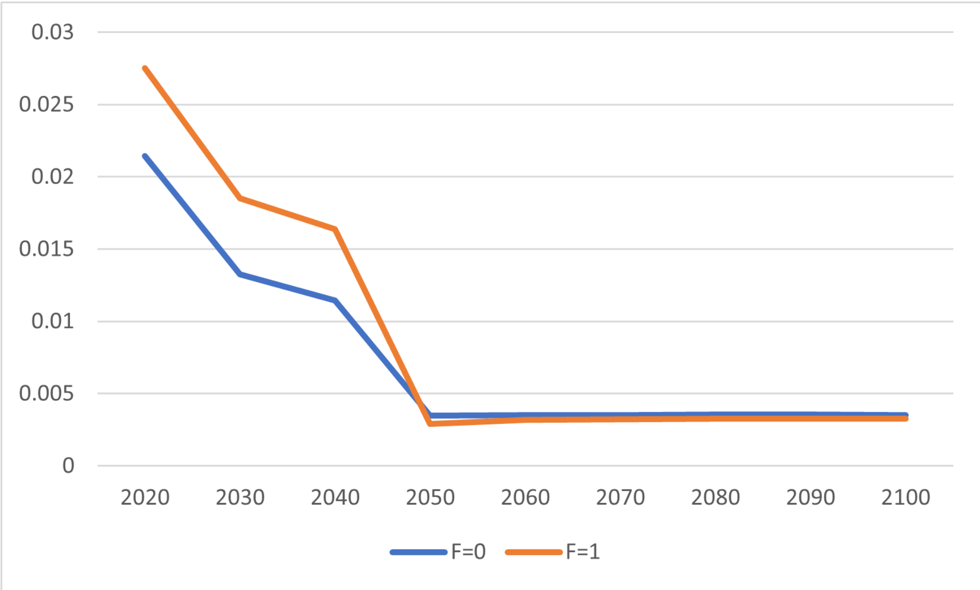

Figure 12: *Brown investment over time with a climate shock occurring in 2050: full autocracy vs full democracy*

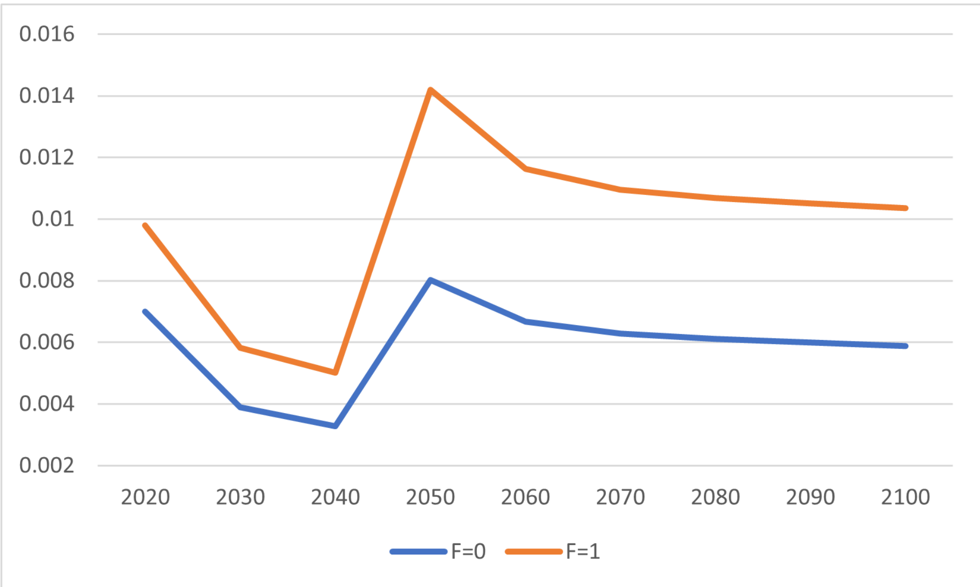

Figure 13: *Green investment over time with a climate shock occurring in 2050: full autocracy vs full democracy*

5.2.5 Emissions for inflated values of θ and ξ

To complete the picture, we show an additional case where we assume that both regimes realise higher climate consequences already in 2020. This analysis highlights the utmost importance of time and early action in climate change mitigation. In 5.2.4, the democracy demonstrated a better efficiency of investment adjustments, but still ended up with higher cumulative emissions.

By inflating the climate consequences already in 2020, we obtain a more meaningful image of policymakers' actions. If rational authorities are aware of higher environmental and economic implications related to brown investment, it becomes clear that democracies internalise such an externality to a greater extent. Namely, their cumulative emissions by 2100 amount to 16% less than under an autocrat (see Figure 14). This further highlights the relative inability of autocracies to combat climate change when emission intensity is stronger. Assessment of the paths of investment (to follow on the next page) provides an intuition for this development.

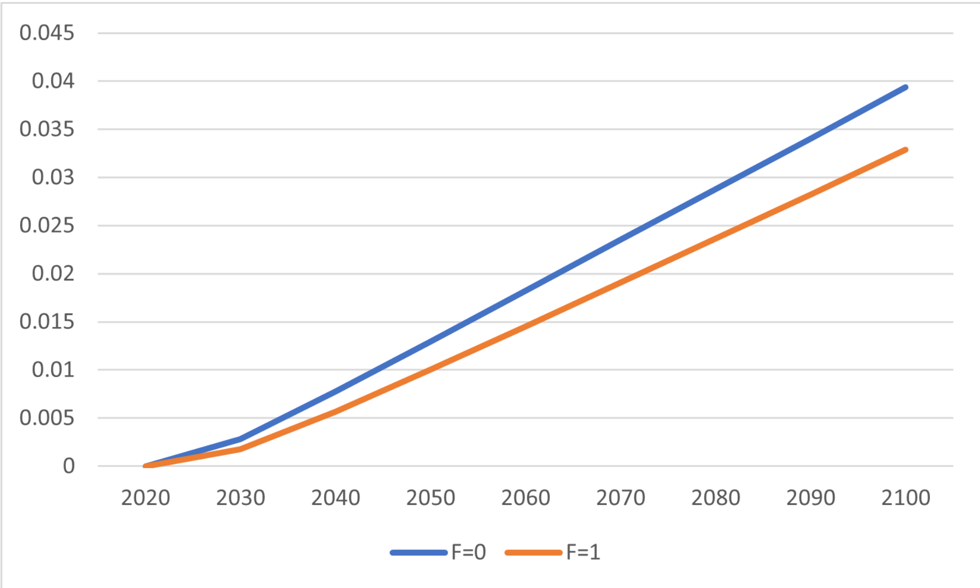

Figure 14: *Cumulative emissions relative to 2020 for* $\theta = 1.5$ *and* $\xi = 1.5$*: full autocracy vs full democracy*

Initially, both regimes react quickly to the perspective of serious environmental impacts and adjust their investment accordingly: green investment significantly exceeds brown investment. The democracy, from the start, invests comparably even more in green capital. Interestingly, however, both regimes gradually decrease green investment over time and increase accompanying brown investment (see Figures 15 and 16). While both economies follow a similar trend, the initial difference in magnitudes is maintained over time. Specifically, by 2100, the complete democracy invests 88% more into green sources when juxtaposed with the autocracy. At the same time, it constrains its brown investment: compared to the authoritarian policymakers, it invests 12% less. Relative differences in capital mix also become more apparent. While the average ratio of brown to green capital is now 29:71 in the autocracy, the democracy achieves a ratio of 17:83. Lastly, by 2100, the democracy features the figure of emissions as % of actual output of only 18%, compared to 41% under the dictatorship.

To explain the overall logic behind our results, we suggest two complementary arguments. Firstly, more authoritarian governments are simply inefficient in the deadweight losses sense. This leaves them with fewer resources available for investment in general. To fund their future consumption, they would then rely on more productive (brown) investment to a higher degree, compared to democracies. Secondly, a higher level of democracy strengthens the encompassing interest. Namely, not only do policymakers care about their own consumption, but also about future societal consumption. Therefore, they will internalise the long-term damaging impacts of emissions on *both* groups and try to limit them more substantially.

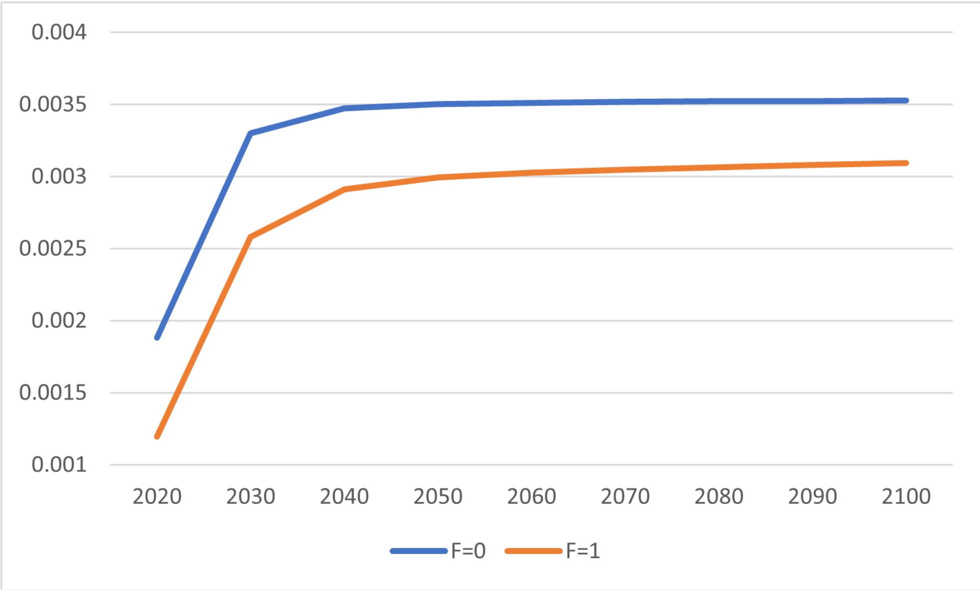

Figure 15: *Brown investment over time for* $\theta = 1.5$ *and* $\xi = 1.5$*: full autocracy vs full democracy*

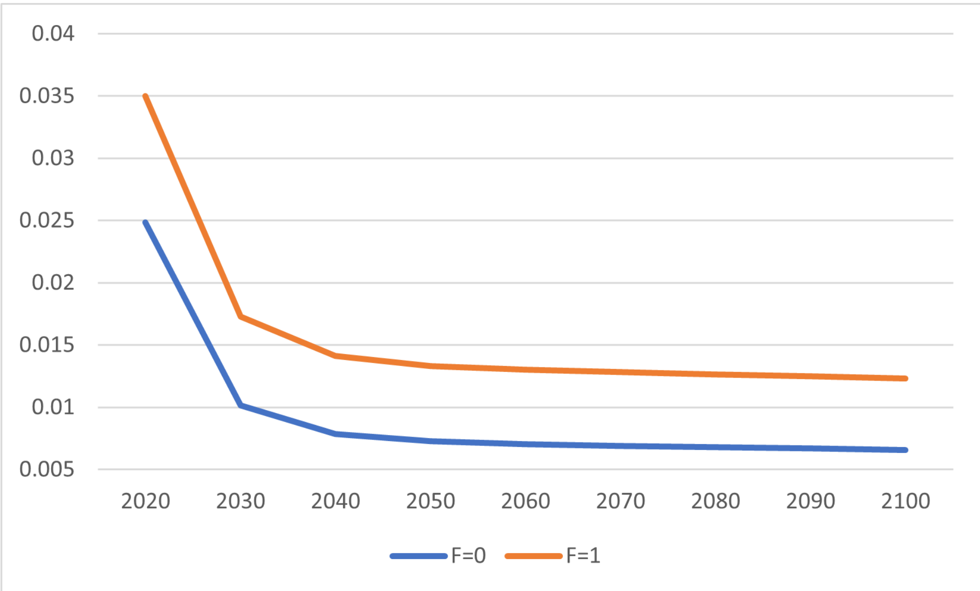

Figure 16: *Green investment over time for* $\theta = 1.5$ *and* $\xi = 1.5$*: full autocracy vs full democracy*

6 Conclusion

In this chapter, we provided a positive view of the regime type's (denoted by the level of democracy) influence on the policymakers' willingness to limit carbon emissions. The literature offers various, but often contradictory, theoretical channels for the potential relationship between democratic quality and environmental performance. Nevertheless, economic models of climate change very rarely deal with political issues of this kind. We fill this gap by adding a climate externality to the dynamic variant of the McGuire and Olson (1996) model of political economy.

We find that less democratic economies are associated with lower cumulative emissions. However, this can be perceived as a byproduct of lower economic growth. Autocrats extract a large proportion of income from the society to themselves, thereby imposing higher deadweight losses. Such inefficiency losses leave the policymakers with fewer resources available for investment regardless of its "dirtiness".

Conversely, more democratic economies push for higher societal consumption. This results in smaller extractions from the society and lower deadweight losses. Therefore, democratic regimes possess more resources that are used for investment. This logic concerns both the emission-heavy investment and investment in renewables: compared to autocracies, democracies would invest more in both capital types. Considering the ratio of emissions to output, however, higher levels of democracy are characterised by a less emission-intensive production.

We also analyse the impact of democracy and climate shocks. Considering the former, a positive regime shock contributes to more emissions. However, the associated increase in brown investment is smaller than the increase in green investment. Thus, democratisation stimulates a slightly greener capital mix. Regarding the climate shock, we find that democratic economies are better equipped to limiting emissions (although, cumulatively, they still emit more). Specifically, they decrease brown investment to levels comparable with autocracies and increase investment in renewables significantly more. Therefore, democracies maintain higher economic growth while featuring a greener capital mix in the long run.

Lastly, we show that timing is essential to the effectiveness of limiting emissions. If policymakers are aware of more significant climate consequences early on, it becomes clear that democracies produce fewer cumulative emissions by switching to renewables more swiftly. Overall, because autocracies are inefficient in the deadweight loss sense, such governments prefer more productive (i.e. brown) investment. Moreover, democratic policymakers - by caring about their citizens' future consumption - internalise the prospective climate damages to a greater extent.

Our results offer some political insights into the wider climate change debate and associated policymaking. While it might seem that the political transition to a less democratic system could save climate, one might object to such a view on ethical grounds (i.e. we show that social welfare is radically worse under an authoritarian system). Moreover, we show that democracies are nevertheless more apt and efficient in adjusting their climate policy and limiting emissions.

7 References

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8 Appendix

8.1 Simplifying assumptions

To simplify the energy inputs used by Golosov et al. (2014) so that output is a product of capital, we rely on the following assumptions:

$$Y_t = F_0(\underline{E}_{0,t}, P_t) = e^{-\xi P_t} \tilde{F}_0(\underline{E}_{0,t}) = e^{-\xi P_t} \left(\frac{\psi}{\theta \rho}\right)^{\frac{\alpha}{\rho}} \left(E^{\rho}_{0,B,t} + E^{\rho}_{0,G,t}\right)^{\frac{\alpha}{\rho}}$$

$$\underline{E}_{0,t} = \left(E_{0,B,t}, E_{0,G,t}\right)$$

$$E_{0,B,t} = E_{B,t} = F_1(K_{B,t}) = \theta K_{B,t}$$

$$E_{0,G,t} = E_{G,t} = F_2(K_{G,t}) = \left(\frac{\theta}{\psi^{\frac{1}{\rho}}}\right) K_{G,t}$$

$$K_t = K_{B,t} + K_{G,t}$$

Moreover, emissions evolve according to:

$$P_t = \tilde{P}\left(\sum_{s=0}^{t-1} E_{1,t-s}\right) = \sum_{s=0}^{t-1} E_{B,t-s} = E_{B,t-1} + \sum_{s=1}^{t-1} E_{B,t-s} = E_{B,t} + P_{t-1}$$

i.e.

$$P_t - \overline{P} = \sum_{s=0}^{t+T} (1-d_s) E_{B,t-s} \quad with\ \overline{P} = 0,\ T = -1, (1-d_s) = 1\ \forall s$$

i.e.

$$1 - d_s = \phi_L + (1-\phi_L)\phi_0(1-\phi)^s = 1\ \forall s\ \Rightarrow \phi = 0, \phi_0 = 1$$

which means there is no depreciation of the emissions stock.

8.2 Marginal products of capital

Differentiating the production function (1) with respect to capital yields the following marginal products of capital:

$$
\begin{aligned}
MPK_B &= \frac{Y_t}{\partial K_{B,t}} \left[e^{-\gamma \tau_t} e^{-\xi P_t} \left(\psi K_{B,t}^{\rho} + K_{G,t}{}^{\rho} \right)^{\frac{\alpha}{\rho}} \right] \\
&= e^{-\gamma \tau_t} Y_t \frac{\alpha \psi K_{B,t}^{\rho-1}}{\psi K_{B,t}^{\rho} + K_{G,t}{}^{\rho}}
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
MPK_G &= \frac{Y_t}{\partial K_{G,t}} \left[e^{-\gamma \tau_t} e^{-\xi P_t} \left(\psi K_{B,t}^{\rho} + K_{G,t}{}^{\rho} \right)^{\frac{\alpha}{\rho}} \right] \\
&= e^{-\gamma \tau_t} Y_t \frac{\alpha K_{G,t}^{\rho-1}}{\psi K_{B,t}^{\rho} + K_{G,t}{}^{\rho}}
\end{aligned} \tag{16}
$$

8.3 Optimisation

Optimisation of the Bellman equation (8) subject to (2)-(4) yields the following first order (i.e. taken with respect to the control variables τ_t, $I_{B,t}$ and $I_{G,t}$) and envelope theorem (i.e. taken with respect to the state variables $K_{B,t}$, $K_{G,t}$ and P_t) conditions:

F.O.C.s

$$w.r.t.\ \tau_t,\ \ for\ F > 0 \qquad C_t = \frac{(1-\tau_t)(1-\gamma\tau_t) e^{-\gamma \tau_t} Y_t}{F(1+\gamma(1-\tau_t))} \tag{17}$$

$$w.r.t.\ \tau_t,\ \ for\ F = 0 \qquad \tau_t = \frac{1}{\gamma} \tag{18}$$

$$w.r.t.\ I_{B,t}, \qquad \frac{1}{C_t} = \beta \left[\frac{\partial V_{t+1}}{\partial K_{B,t+1}} + \theta \frac{\partial V_{t+1}}{\partial P_{t+1}} \right] \tag{19}$$

$$w.r.t.\ I_{G,t}, \qquad \frac{1}{C_t} = \beta \frac{\partial V_{t+1}}{\partial K_{G,t+1}} \tag{20}$$

E.T.s

$$w.r.t.\ K_{B,t}, \qquad \frac{\partial V_t}{\partial K_{B,t}} = \frac{\tau_t}{C_t} MPK_B(t) + \frac{F}{e^{-\gamma \tau_t} Y_t} MPK_B(t) \tag{21}$$

$$w.r.t.\ K_{G,t}, \qquad \frac{\partial V_t}{\partial K_{G,t}} = \frac{\tau_t}{C_t} MPK_G(t) + \frac{F}{e^{-\gamma \tau_t} Y_t} MPK_G(t) \tag{22}$$

$$w.r.t.\ P_t, \qquad \frac{\partial V_t}{\partial P_t} = \beta \frac{\partial V_{t+1}}{\partial P_{t+1}} - \xi \frac{\tau_t e^{-\gamma \tau_t} Y_t}{C_t} - F\xi \tag{23}$$

8.4 Carbon tax

Merging and rearranging conditions (20) and (22) yields the following Euler equation:

$$\frac{1}{C_t} = MPK_G(t+1)\left[\beta\frac{\tau_{t+1}}{C_{t+1}} + \beta\frac{F}{e^{-\gamma\tau_{t+1}}Y_{t+1}}\right] \tag{24}$$

Moreover, merging and rearranging conditions (19) and (21) gives:

$$\frac{1}{C_t} = \beta\left[\frac{\tau_{t+1}}{C_{t+1}}MPK_B(t+1) + \frac{F}{e^{-\gamma\tau_{t+1}}Y_{t+1}}MPK_B(t+1) + \theta\frac{\partial V_{t+1}}{\partial P_{t+1}}\right] \tag{25}$$

Combining (24) with (25) then yields

$$MPK_G(t+1) = MPK_B(t+1) + \frac{\theta}{\frac{\tau_{t+1}}{C_{t+1}} + \frac{F}{e^{-\gamma\tau_{t+1}}Y_{t+1}}}\frac{\partial V_{t+1}}{\partial P_{t+1}} \tag{26}$$

This must mean that the carbon tax that equalises the marginal products of capital is given by

$$\frac{\theta}{\frac{\tau_{t+1}}{C_{t+1}} + \frac{F}{e^{-\gamma\tau_{t+1}}Y_{t+1}}}\frac{\partial V_{t+1}}{\partial P_{t+1}} = -Te^{-\gamma\tau_{t+1}}Y_{t+1} \tag{27}$$

i.e. carbon tax is a constant proportion of actual output.

Subsequently, we plug condition (23) to (27) and obtain:

$$\frac{\xi\theta}{(1-\beta)} = T \tag{28}$$

i.e.

$$MPK_G(t+1) = MPK_B(t+1) - \frac{\theta\xi}{1-\beta}e^{-\gamma\tau_{t+1}}Y_{t+1} \tag{29}$$

8.5 Optimal investment for $t > 1$

Assuming $C_{t>1} = \lambda \tau_{t>1} e^{-\gamma \tau_{t>1}} Y_{t>1}$, it must mean that $I_{B,t>1} + I_{G,t>1} = (1-\lambda)\tau_{t>1} e^{-\gamma \tau_{t>1}} Y_{t>1}$. Thus:

$$I_{B,t>1} = (1-\lambda)\tau_{t>1} e^{-\gamma \tau_{t>1}} Y_{t>1} - I_{G,t>1} \tag{30}$$

Expressing (29) in terms of investment allows us to use (30) and rearrange the equation so that:

$$\begin{aligned} \alpha I_{G,t>1}^{\rho-1} &= \alpha\psi \left((1-\lambda)\tau_{t>1} e^{-\gamma \tau_{t>1}} Y_{t>1} - I_{G,t>1} \right)^{\rho-1} \\ &\quad - \frac{\theta\xi}{1-\beta} \left(\psi \left((1-\lambda)\tau_{t>1} e^{-\gamma \tau_{t>1}} Y_{t>1} - I_{G,t>1} \right)^{\rho} + I_{G,t>1}^{\rho} \right) \end{aligned} \tag{31}$$

from which we can implicitly obtain $I_{G,t>1}$. Once $I_{G,t>1}$ is obtained, we refer back to (30) to get the consistent choice of $I_{B,t>1}$.

8.6 Optimal tax rates for $t > 1$

To obtain the optimal tax rates, we rely on condition (17) and (9) such that

$$\lambda \tau_{t>1} e^{-\gamma \tau_{t>1}} Y_{t>1} = \frac{(1-\tau_{t>1})(1-\gamma\tau_{t>1}) e^{-\gamma \tau_t} Y_{t>1}}{F(1+\gamma(1-\tau_{t>1}))} \tag{32}$$

i.e.

$$\lambda = \frac{1}{F} \left(\frac{1}{\tau_{t>1}(1+\gamma-\gamma\tau_{t>1})} - 1 \right) \tag{33}$$

where $F > 0$.

The full autocracy case, $F = 0$, relies on condition (18), i.e.

$$\tau_{t>1} = \frac{1}{\gamma} \tag{34}$$

8.7 Optimal choices at $t = 1$

At $t = 1$ we are still choosing optimally such that

$$MPK_G(2) = MPK_B(2) - \frac{\theta\xi}{1-\beta} e^{-\gamma\tau_2} Y_2 \tag{35}$$

i.e.

$$\alpha I_{G,1}^{\rho-1} = \alpha\psi I_{B,1}^{\rho-1} - \frac{\theta\xi}{1-\beta}\left(\psi I_{B,1}^{\rho} + I_{G,1}^{\rho}\right) \tag{36}$$

Using the budget constraint (i.e. $\tau_t e^{-\gamma\tau_t} Y_t - I_{B,t} - I_{G,t} = C$), we can express the above as

$$\begin{aligned}\alpha I_{G,1}^{\rho-1} = {} & \alpha\psi\left(\tau_1 e^{-\gamma\tau_1} Y_1 - C_1 - I_{G,1}\right)^{\rho-1} \\ & - \frac{\theta\xi}{1-\beta}\left(\psi\left(\tau_1 e^{-\gamma\tau_1} Y_1 - C_1 - I_{G,1}\right)^{\rho} + I_{G,1}^{\rho}\right)\end{aligned} \tag{37}$$

to solve for $I_{G,1}$. Consistent $I_{B,1}$ is then taken from the budget constraint.

Furthermore, the optimal initial tax rate for $F > 0$ is given by condition (17) so that

$$C_1 = \frac{(1-\tau_1)(1-\gamma\tau_1)e^{-\gamma\tau_1} Y_1}{F(1+\gamma(1-\tau_1))} \tag{38}$$

and for $F = 0$ by condition (18):

$$\tau_1 = \frac{1}{\gamma} \tag{39}$$

Lastly, C_1 is chosen such that the equations (37) and (38) or (39) hold and that the Euler equation (24) implies:

$$\frac{1}{C_1} = \beta\frac{\tau_2}{C_2} MPK_G(2) + \beta\frac{F}{e^{-\gamma\tau_2} Y_2} MPK_G(2) \tag{40}$$

i.e.

$$\frac{1}{C_1} = \frac{\beta}{\lambda}\frac{\alpha I_{G,1}^{\rho-1}}{\psi I_{B,1}^{\rho} + I_{G,1}^{\rho}} + \beta F \frac{\alpha I_{G,1}^{\rho-1}}{\psi I_{B,1}^{\rho} + I_{G,1}^{\rho}} \tag{41}$$

8.8 Optimal λ

Having specified how all control variables of the model are optimally chosen assuming the validity of (9), the only remaining matter is to solve for a value of λ that would indeed guarantee the above conditions are met. Given the optimal choices at $t = 1$, we specify the entire optimised system for any $t > 1$ relying on the control variables given by (11)-(14) and evolution of the state variables given by (2)-(4). Ultimately, we solve for the value of λ that ensures the Euler equation (24) holds for every $t > 1$, subject to tolerance error.

8.9 Initial state variables

Assume our model starts from the no-externality, no-politics steady state where production is given by:

$$Y_t = \left(\psi K_{B,t}^{\rho} + K_{G,t}^{\rho}\right)^{\frac{\alpha}{\rho}} \tag{42}$$

The Bellman Equation is specified as:

$$\begin{aligned} V_t(K_{B,t}, K_{G,t}) = ln\left(\left(\psi K_{B,t}^{\rho} + K_{G,t}^{\rho}\right)^{\frac{\alpha}{\rho}} - I_{B,t} - I_{G,t}\right) \\ + \beta V_{t+1}(K_{B,t+1}, K_{G,t+1}) \end{aligned} \tag{43}$$

where:

$K_{B,t+1} = I_{B,t}$

$K_{G,t+1} = I_{G,t}$

Optimisation of the Bellman equation (43) yields the following first order (i.e. taken with respect to the control variables $I_{B,t}$ and $I_{G,t}$) and envelope theorem (i.e. taken with respect to the state variables $K_{B,t}$ and $K_{G,t}$) conditions:

F.O.C.s

$$w.r.t.\ \ I_{B,t}, \qquad \frac{1}{C_t} = \beta \frac{\partial V_{t+1}}{\partial K_{B,t+1}} \tag{44}$$

$$w.r.t.\ \ I_{G,t}, \qquad \frac{1}{C_t} = \beta \frac{\partial V_{t+1}}{\partial K_{G,t+1}} \tag{45}$$

E.T.s

$$w.r.t.\ \ K_{B,t}, \qquad \frac{\partial V_t}{\partial K_{B,t}} = \frac{1}{C_t} MPK_B(t) \tag{46}$$

$$w.r.t.\ \ K_{G,t}, \qquad \frac{\partial V_t}{\partial K_{G,t}} = \frac{1}{C_t} MPK_G(t) \tag{47}$$

Rearranging the conditions yields the Euler equation

$$\frac{C_{t+1}}{C_t} = \beta MPK_G(t+1) \tag{48}$$

and the identity

$$MPK_G(t+1) = MPK_B(t+1) \tag{49}$$

i.e.

$$K_{G,t+1}^{\rho-1} = \psi K_{B,t+1}^{\rho-1} \tag{50}$$

Using (48) we guess that $C_t = \lambda Y_t$. Then:

$$\frac{1}{Y_t} = \frac{\alpha\beta K_{G,t+1}^{\rho-1}}{\psi K_{B,t+1}^{\rho} + K_{G,t+1}^{\rho}} \tag{51}$$

i.e.

$$K_{B,t+1} + K_{G,t+1} = \alpha\beta Y_t \tag{52}$$

This means that

$$Y_t - C_t = (1-\lambda)Y_t = \alpha\beta Y_t \tag{53}$$

i.e. validating our guess.

Finally, we assume the steady state reflects $\dfrac{K_{G,1}}{K_{B,1} + K_{G,1}} = 20\%$, which implies $K_{B,1} = 4K_{G,1}$.

However, we need to ensure $MPK_B = MPK_G$, from which follows:

$$K_{B,1}^{\rho-1} = \psi(4K_{G,1})^{\rho-1} \Rightarrow \psi = 4^{1-\rho} \tag{54}$$

and

$$K_{G,1} = \left[\alpha\beta\left(\psi 4^{\rho} + 1\right)^{\frac{\alpha-\rho}{\rho}}\right]^{\frac{1}{1-\alpha}} \tag{55}$$