Two-Tiered Meta-heuristic Holistic Optimisation of Subsea Electrical Networks

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This thesis is the result of the author's original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.

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Abstract

Human thirst for energy shows no signs of abating; instead global demand for electricity is projected to increase in the coming decades. If this demand is to be supplied in a sustainable manner, a rapid uptake in renewable sourced electrical energy is required. One arena that offers significant scope for increased renewable sourced generation is the offshore domain, through the offshore wind, tidal, and wave vectors. While the energy potential of the marine domain is vast, there are a number of challenges associated with deploying machines offshore to generate electricity, and transporting this energy back to population centres where it may be utilised.

The deployment of a subsea electrical network to facilitate the extraction of energy from offshore renewable energy projects can be both logistically difficult and expensive. Consequently, the optimisation of such networks takes on critical importance if the marine domain is to make a significantly increased contribution towards the global electricity demand.

This thesis considers the application of a two-tiered meta-heuristic optimisation approach for the holistic design of subsea electrical networks for offshore renewable energy projects; accounting for in-service costs arising from electrical losses and no-service conditions, in addition to purchase and installation costs.

One current offshore wind development, Horns-Rev 1 was assessed using the optimisation framework that has been developed as part of this work. These studies demonstrate that the in-situ network at Horns-Rev 1 could have been improved upon, in terms of through-life cost minimisation while using the same design parameters, by changing the network layout. By altering the selected design parameters (voltage and frequency) the network performance, against the lifecycle cost minimisation could be further reduced.

Additionally, the optimisation framework is deployed to design an optimal network for the proposed East Anglia One offshore wind park where viable solutions were identified using the projected design parameters. Again, it is shown that by altering the voltage and frequency parameters selected in the design, the through-life cost of the network solution may be improved upon relative to the assumed parameters.

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Abbreviations

Each term that is used in abbreviated form within this thesis is written in full at the first point of use and subsequently abbreviated. Additionally, to aid readability, a complete list of abbreviations is presented as Table 1.

Abbreviation	**Represents**
A	Amperes
AC	Alternating Current
AC PF	Alternating Current Power Flow
ANM	Active Network Management
ANN	Artificial Neural Networks
DC	Direct Current
DC PF	Direct Current Power Flow
DECC	UK Department for Energy & Climate Change
DFIG	Doubly Fed Induction Generator
DG	Distributed Generation
DNO	Distribution Network Operator
DUKES	Digest of UK Energy Statistics
EA	Evolutionary Algorithm
EEA	European Energy Authority
EENP	Expected Energy Not Produced
EENS	Expected Energy Not Supplied
EU	European Union
EUE	Expected Unserved Energy
EWEA	European Wind Energy Authority
F	Farads
GA	Genetic Algorithm
GRASP	Greedy Randomised Adaptive Search Procedure
H	Henries
HAWT	Horizontal axis wind turbine
HVAC	High Voltage Alternating Current
HVDC	High Voltage Direct Current
Hz	Hertz
kV	Kilo-Volts
kW	Kilo-Watt
kWh	Kilo-Watt Hours
LCC-HVDC	Line Commutated Current High Voltage Direct Current
LFAC	Low-Frequency Alternating Current
LOLE	Loss-of-Load Expectation
LOLP	Loss-of-Load Probability

LVDC	Low Voltage Direct Current
MCT	Marine Current Turbine
MTTR	Mean Time to Repair
MVA	Mega Volt-Amperes
MW	Mega-Watt
OPF	Optimal Power Flow
PDF	Probability Density Function
PF	Power Flow
PMSG	Permanent Magnet Synchronous Generator
PSO	Particle Swarm Optimisation
pu	Per-Unit
SI	Swarm Intelligence
SVC	Static VAR Compensation
TNO	Transmission Network Operator
TSP	Travelling Salesman Problem
TWh	Tera-Watt Hours
UHVDC	Ultra-High Voltage Direct Current
V	Volts
VAr	Volt-Ampere Reactive
VAWT	Vertical axis wind turbine
VSC-HVDC	Voltage-Source Converter High Voltage Direct Current
W	Watts
XLPE	Cross-Linked Polyethylene
Ω	Ohms

Table 1 - Abbreviations

Acknowledgement

This has been easily the hardest page of this thesis to write! A few more expansive drafts have been abandoned and replaced with this short and sweet effort. I've probably missed out a number of worthwhile people; but as I'm planning to graduate this side of 2050: it's time to stop procrastinating and get something down on paper!

I'd like to thank a number of people for their efforts in relation to the successful completion of this thesis, specifically:

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Apologies to those who should be included on this list, but I've forgotten; of which I'm sure there are many.

Chapter 1: Introduction

Within this introductory chapter, the context of, and motivation for, this work is explained along with the scope of the problem being addressed; the contributions to knowledge offered herein; and an overview of how this thesis is structured.

1.1 Electrical Energy Consumption Trends

In some developed nations, such as the United Kingdom, the level of electrical power consumption, per capita, has begun to fall in recent years, after a long period of growth. This is at odds with the trend that has seen the global demand for electrical energy increase steadily over the last half century. Both of these trends are visible within Figure 1, produced by The World Bank [1], where the y-axis shows the electrical energy consumption, per capita, in thousands of kilo-Watt hours (kWh).

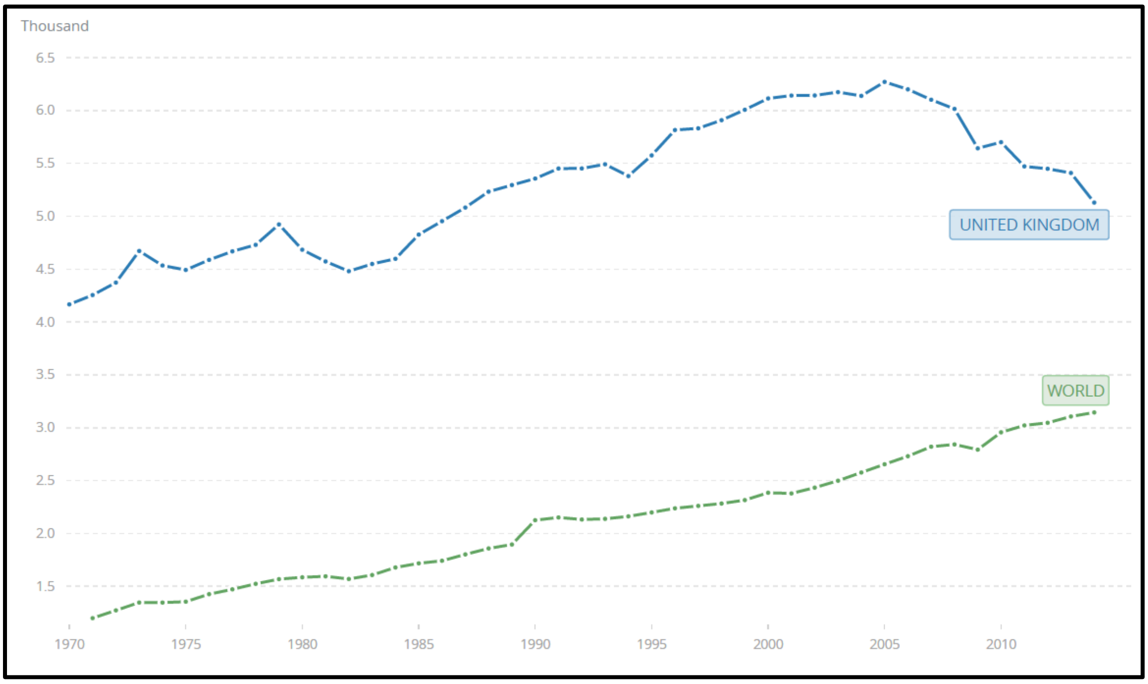


Figure 1 - Global and UK Electrical Energy Consumption (Per Capita) Trends [1]

The trends depicted in Figure 1 are based on per capita consumption. Combining UK census data [2] with the per capita consumption data shown in Figure 1 suggests that the absolute demand for electricity within the UK has fallen by 5.6 % in the intra-census period 2001-2011. There are a number of possible reasons for this reduction, including: improved efficiency of lighting, heating and consumer electronics; greater environmental awareness driving changes in societal behaviours; and increased cost of electricity. Equally, this reduction may be an anomaly driven by the corresponding global economic downturn, and demand levels may return to those observed in the early 2000s, thereafter. Whatever the enabling factors that have brought about the changes in electricity consumption habits measurable in the UK, there has been an increase in *sustainability*, as the total annual electrical demand has reduced.

While Figure 1 shows an improvement in sustainability of the UK's electricity consumption, this is not mirrored on a global basis; for which a near-linear level of growth is visible across the period 1970-2015. This will likely converge at a level similar to the UK's as developing nations transition towards developed status. This factor, combined with an expanding global population that has risen from 6.1 billion in 2000 to 7.3 billion in 2015 [3], and is projected to keep rising for decades to come [3], presents a requirement for sustainable sources of electricity to be implemented on a massive scale. Further, it is estimated that there are currently 1.2 billion people without any access to electricity at all [4], thus the challenges associated with provision of electricity for the entirety of the global population may be highly onerous.

Within developed nations, such as the UK, the per capita reduction in electrical power consumption trend visible within Figure 1 may be reversed as the electric vehicle (EV) begins to challenge the ubiquity of the internal combustion engine. While this may result in a reduction in overall energy consumption, it will increase the demand for electrical energy as vehicles are now charged at homes and workplaces, in preference to being refuelled with hydrocarbon based fuels. Figure 2 shows projections published by the International Energy Agency in 2016 that suggest that there shall be a massive increase in the number of EVs in operation in the coming years, with potentially a 4-7 fold increase between 2020 and 2030 [5].

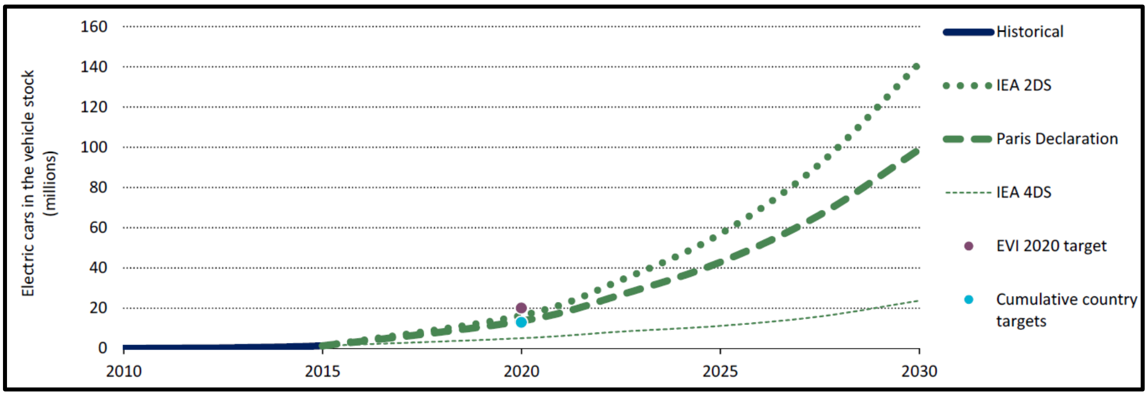


Figure 2 - Electric Vehicle Stock Projections 2010-2030 [5]

It appears unlikely, based on the evidence presented in the preceding section that the demand for electrical energy is likely to fall, either on a local or global basis, in the coming decades. The environmental impacts of such consumption depend largely on the way in which electricity is generated, as discussed in the following section.

1.2 Electrical Energy Generation Trends

Sustainability is a word that has recently entered the every-day lexicon of society, particularly in reference to energy. Electricity consumption can be made more sustainable in two distinct ways: improvements in efficiency and better utilisation of energy; and the phasing out of fossil fuels to be replaced with renewable energy sources. These options are not mutually exclusive, and the least environmentally intrusive option would be the efficient consumption of energy generated in a carbon-neutral manner. A number of developed nations have set targets to reduce their dependence on fossil fuel sourced energy, for example: in 2009 the UK government set a target that by 2020 15 % of the UK's energy consumption will be provided from renewable energy sources, representing a significant improvement on the 1.5 % level recorded in 2005 [6]. This 15 % energy figure is composed of 12 % of heating; 30 % of electricity; and 10 % of transportation. In 2018, the UK Digest of UK Energy Statistics (DUKES) report gave the realised values for renewable energy utilisation by sector, for 2017, as: 7.7 % for heating; 27.9 % for electricity; and 4.6 % for transportation. Each of these figures is an improvement upon the 2015 values of 5.64 %, 22.31 %, and 4.23 % respectively [7]. In 2016, The International Energy Authority published a breakdown on global electricity generation by fuel type, covering the period 1971 to 2014, shown as Figure 3 [8]. Within Figure 3 it may be observed that renewable energy is well represented throughout the time-period, nearly exclusively by hydroelectric power until later years. Such schemes offer significant electrical output but the range of possible deployment is limited by the geography of the land, given the need for water to be stored with a gravitational head above turbines.

In both relative and absolute terms, electricity generation from non-hydroelectric renewables, such as onshore and offshore wind, wave, tidal, and solar energies, has grown in the period covered by this report: 0.6 % of electricity generation globally in 1973 was from a non-hydroelectric renewable source, which had risen to 6.3 % by 2014. During the same timeframe, global electricity consumption has risen near four-fold such that the absolute levels of electricity generated from such sources has risen from 36.8 Tera-Watt Hours (TWh) in 1973 to 1500 TWh in 2014 [8].

While the output from renewable energy has grown forty-fold, in the period 1973 to 2014, and the percentage of global electricity generated by fossil fuels fell from 75.2 % to 66.7 %, in the same time frame; in absolute terms the consumption of electricity generated from fossil-fuels has risen by a factor of 3.5 between 1973 and 2014, from 4610 TWh to 15885 TWh [8]. In order to make global consumption of electrical energy more sustainable, more energy must be supplied from renewable sources such, thus reducing the requirement for carbon-emitting fossil fuels.

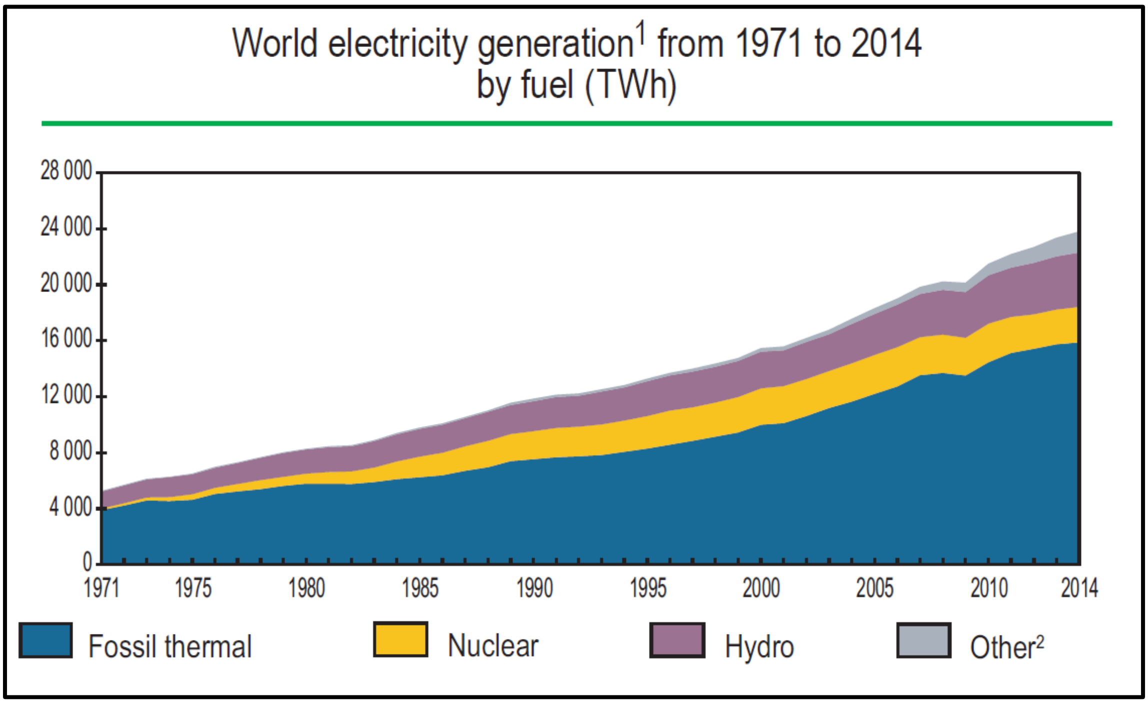


Figure 3 - Breakdown of Global Electricity Generation by Fuel Type [8]

One area which offers a significant potential source of renewable energy is the marine domain; encompassing offshore wind, tidal and wave energies. The potential of each of these energy vectors is discussed in the following section.

1.3 Potential of Offshore Generation

Offshore sourced energy offers a considerable, and currently underutilised, resource of clean, renewable energy. Offshore wind, wave and tidal generation capture naturally recurring energy above, at, and below the sea level respectively. At present, offshore wind is by far the most mature of the three generation types mentioned previously.

Comparing reports covering 2012 [9] and 2017 [10] statistics for offshore wind demonstrates substantial growth in both offshore wind capacity and total power generation achieved. In

2012, 5 GW of installed capacity yielded 18 TWh; providing 0.5 % of the total electrical demand for the European Union (EU) in that year (3600 TWh) [9]. By 2017, offshore wind generation capacity had grown to 15.8 GW, yielding 43 TWh. During the period 2012-2017, the EU wide electrical energy demand fell by almost 20 % to 2900 TWh; hence the percentage of electricity supplied by offshore wind grew from 0.5 % in 2012 to 1.48 % in 2017 [10].

These figures indicate that the Europe wide capacity factor (the percentage of actual electrical output compared with its theoretical maximum output) fell from 41.2 % in 2012 to 31.1 % in 2017; both of which are still considerably higher than the on-shore wind factor of approximately 25 % [11, 12].

A European Wind Energy Association (EWEA) report published in January 2013 indicated that there was 4,994 MW of installed offshore wind capacity in 2012, typically yielding 18 TWh annually. This is equivalent to 0.5 % of the total electricity consumption of the European Union [9]. These figures indicate a Europe wide capacity factor (the percentage of actual electrical output compared with its theoretical maximum output) of 41.2 %, which is considerably higher than the on-shore wind factor of approximately 25 % [11, 12].

The European Energy Authority (EEA) concluded in 2009 that the technical potential for European off-shore wind is approximately 30,000 TWh [13]. In 2012, the pan-European electrical demand was approximately 3600 TWh, therefore any advances towards better utilising this resource will allow renewable energy greater significance in the future electricity supply of Europe.

The mechanics by which energy is converted from waves depends largely on the location at which the device is deployed. Wave generation may occur onshore, near-shore or offshore. An onshore device may not be deployed off-shore and vice versa.

In 2010, Siemens presented to the European Future Energy Forum that the annual total European wave energy resource was approximately 1500 TWh, although only 200 TWh was economically extractable based on 2010 conditions [14].

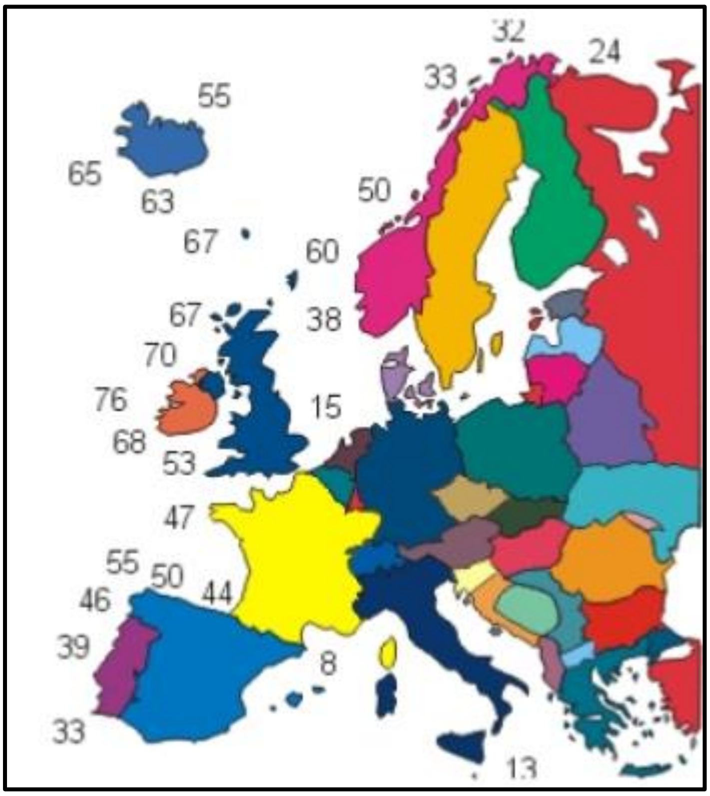


Figure 4 - European Wave Energy Resource Map [14]

If the full 200 TWh could be extracted and utilised, this would represent the equivalent of 5 % of Europe’s electricity demand in 2012. Pelamis Wave Power suggest that the recoverable wave energy around Britain is capable of supplying 14-26 % (50 – 90 TWh) of the UK’s annual electrical demand [15].

Wave generation potential is not equally distributed across Europe, with the countries that have an Atlantic coastline possessing far greater resource than those without, as depicted in Figure 4 [14]. The numbers shown in Figure 4 represent the average wave energy in kilowatts per metre (kW / m).

Pelamis Wave Power suggest that wave energy can be generated at a competitive price so long as the yearly average wave power is greater than 15 kW per metre, with many sites shown in Figure 4 meeting this minimum threshold [15]. This cost assessment is based on current economic conditions, and changes in these conditions will alter the break-even threshold.

A tidal stream system may be deployed in an undammed water channel, and generates electricity due to the natural motion of water through the channel.

There is considerable potential associated with tidal stream energy, as up to 100 GW of capacity could be installed around Europe. The geography of a given site will dictate its potential value for tidal energy generation as fast flowing water channels such as the Pentland Firth and Anglesey (UK) and Raz Blanchard (France) offer Europe's strongest resources [16].

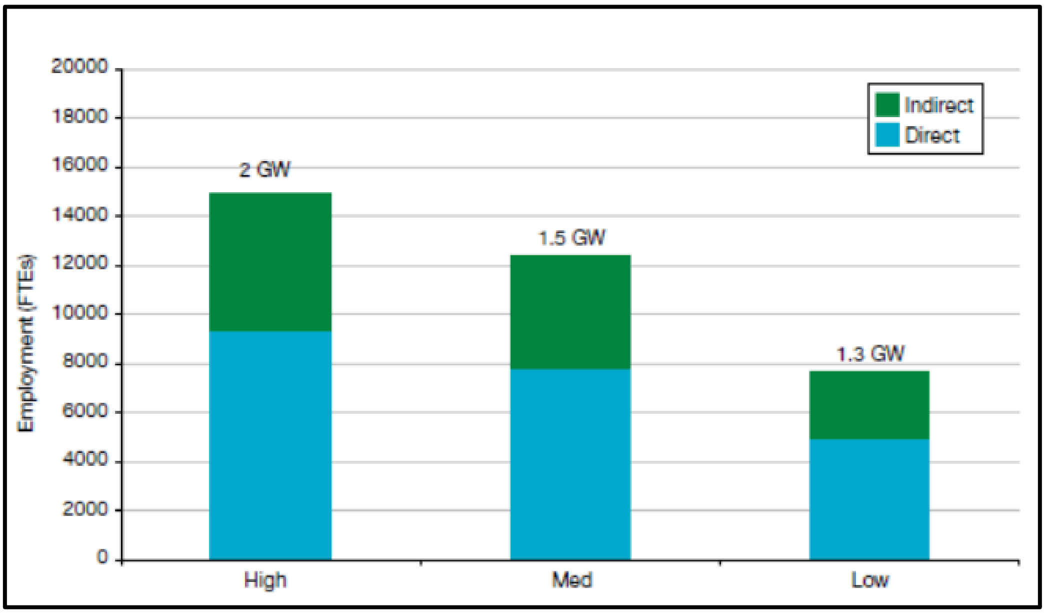


Figure 5 - Projected UK Marine Energy Employment in 2021 [17]

The implementation of offshore renewable projects is not only beneficial from environmental and energy security standpoints, but also has socioeconomic benefits. A 2011 study by Renewable UK suggested that the uptake of marine energy in the UK could see up to 9000 direct and 5600 indirect full-time equivalent jobs created by 2021, as shown in Figure 5 [17].

1.4 Optimisation of Offshore Energy Exploitation

The preceding sections have constructed a strong narrative around why optimisation of the exploitation of offshore sourced energy is desirable, in summary: the human demand for electrical energy is unlikely to dissipate in the coming decades; that despite advances in renewable energy technologies the majority of electrical generation is still fossil-fuel sourced, which is at odds with the utopia of a low-carbon, sustainable, electrical supply chain; and that significant resources of renewable energy are available in the marine domain.

Exploitation of the resource that exists in the marine domain, therefore, has a significant role to play in the global provision of sustainable electrical energy going forward. The optimisation of this exploitation is a complex and multi-faceted problem that may be approached from a number of perspectives; including, but not limited to: site selection to maximise natural resource, and minimise ecological and environmental damage; improvements in electrical machines and conversion equipment, thus they are more efficient and can provide a higher electrical output for the same input; and the development of subsea electrical networks that are reliable, efficient, and cost-effective.

This body of work approaches the optimal exploitation of marine energy problem from a network design perspective. Specifically, it pertains to the holistic optimisation of offshore electrical networks such that the through-life cost of a subsea network is minimised; thus the attraction to investors and developers to create offshore renewable projects is maximised.

This work does not seek to provide guidance on the viability or optimal site layout of a given site for deployment of offshore renewable energy generation. Instead, this is taken to be an input to the problem of network design that is considered.

1.5 Optimisation of Subsea Electrical Networks

As was discussed in the preceding section, this work tackles the open-ended problem of maximising the useful yield of marine sourced renewable energy by limiting the scope of study to the sub-problem relating to the optimal design of subsea electrical networks to support offshore energy projects.

Within this domain, there are a number of further thematic subdivisions that may be made, gathering pertinent academic literature into informal groupings of *what, why,* and *how,* and these are presented within the following sections.

1.5.1 Optimisation Technical Scope

Typically, as will be discussed later within this thesis, a subsea electrical network for the extraction of electrical energy generated in the offshore domain is comprised of two distinct stages: a lower voltage collector network; and a higher voltage transmission system; linked by an offshore substation. The scope of optimisation of a network can be limited to a single component of the network: the collection stage [18-20]; or the transmission stage [21-22].

While the approach of dividing the problem into sub-problems and optimising each separately, in this case optimising the collector and transmission networks as separate entities, before combining the results to produce a unified solution appears to have merit; the value derived from such an approach is limited due to the complex and interrelated nature of the problems.

If a collector network is said to have a *start point* at the turbines with which it interfaces, its *end point,* therefore, is the offshore substation. The offshore substation therefore forms the beginning of the transmission network which ends at the point at which it interfaces with a land based power grid. Moving an offshore substation nearer to, or farther from, the location of offshore generation has conjugate effects on the networks: less cabling required in the collector network results in more cabling required in the transmission stage; and vice-versa. Some published work has greatly reduced the range of possible solutions that may be considered within the optimisation process by mandating the position of one or more offshore substations and optimising the network around this [23-25].

The interrelation of network behaviours is not only confined to spatial considerations. If both stages of the network utilise alternating-current (AC) then there shall be reactive power interactions between both stages that govern the size of the cabling and plant required; which has financial implications also.

For these reasons, it is desirable to set the system boundaries to include both collector and transmission components of any offshore network, when its optimisation is being considered. There are many examples within the relevant academic literature that sets the system boundary in this manner [24-31].

The technical scope of the optimisation of a subsea electrical network must be bounded with respect to a number of other parameters. It is possible to include a number of different topologies with radial, ring, star, and mixed being typical topologies that are presented in the following chapter of this thesis. Again, some studies seek to limit the search space by mandating that specific topologies must be deployed at either the collector [18, 27-28, 30] or transmission network [23, 32] stages. Topological selection of collector and transmission network stages affects reactive power flows in the entire network, hence a number of system interactions are not considered if certain topologies are fixed, and thus a number of strong solutions may be inadvertently overlooked.

In addition to topology, the choice of electrical technology has a significant bearing on optimality. AC collector networks remain ubiquitous, with the concept of direct-current (DC) systems only now beginning to be assessed as an option [33, 34]. For offshore transmission, both AC and DC offer merits with the most cost-effective option depending on factors such as distance from shore and site capacity [35-39]. Some optimisation studies have further restricted the scope of design that may be produced by mandating an AC [26, 32] or DC transmission link [23]. Again, limiting the search space in this manner means that a number of strong solutions may be overlooked. Further, it may not be inferred that the optimal solution for a DC transmission based network may be arrived at by taking the optimal solution for an AC transmission based network and substituting DC transmission for AC. The interactions between an AC collector network and an AC transmission network; and between an AC collector network and a DC transmission network, are complex and optimality will differ considerably depending on which transmission technology is deployed, as is shown later in this thesis.

Voltage and frequency of both collector and transmission network components are generally fixed in order to bound the problem [18, 24-29, 39]; however some published research has included both parameters as discrete design variables [31, 41].

1.5.2 Optimisation Objective Scope

Within the previous section, some of the salient technical characteristics of optimisation studies that have been published were presented and these are discussed in greater depth later in this thesis. The technical characteristics cover *what* is to be included within the scope of the optimisation. This, however, only describes part of the optimisation process. Additionally, *why* the optimisation is being performed forms a key part of the scope.

Within the published literature, two distinct optimisation objectives have been observed: minimisation of capital expenditure [18, 26-28, 32]; and the minimisation of through-life cost [21-22, 30-31, 40, 42]. Capital expenditure minimisation is a simpler problem that is subject to fewer variables than lifecycle cost minimisation.

Within lifecycle cost minimisation, there is a range of possible implementations that account for different factors that have been published. For example, electrical losses may be calculated using an average output power [20, 24] which reduces the accuracy of the monetary values of losses, and ultimately the accuracy of the lifecycle cost of the solution, owing to the non-linearity of electrical losses. This method is advantageous in the respect that it is computationally easier to perform than a study which considers a number of electrical output states and calculates losses based on these states.

Another approach is to use a probability density function (PDF) to approximate the resource at an offshore renewable generation site; with typical examples being Weibull Distribution [25, 31] and the Rayleigh Distribution [40], both of which are introduced later in this thesis. The availability of raw energy resource at a given site may be mapped to electrical output of machines, thus losses at different resource states may be approximated, and a more accurate quantification of cost of losses may be established.

The cost of no-service conditions is also included within the minimisation of lifecycle cost objective for a number of published studies, thus reliability of the network and the value of having a redundant electrical path between a generator and the land-based network is accounted for [26, 29, 31, 40]. Zhao et al [43] defined the *generation ratio* metric that linked the electrical output at the connection point to the power generated at offshore wind turbines while network branches are taken systematically out-of-service, thus an accurate, but computationally expensive, quantification of no-service conditions may be established.

Now that context based on the academic literature around the *what* and *why* of subsea electrical network optimisation questions have been addressed, this leaves only the question of *how*, which is introduced in the following section.

1.5.3 Optimisation Methodologies

The computational optimisation of a subsea electrical network for offshore renewable energy extraction is feasibly divided into 3 sub-problems, as is discussed later in this thesis.

Specifically, these problems relate to:

1) The production of a high-level network specification, and iterative refinement of this solution.
2) The generation of connection orders for offshore cables that maximises the efficiency and / or minimises the cost of connections.
3) The quantification of network performance, either in terms of electrical performance or cost.

As shall be addressed later within this thesis, there is a limited set of candidate methodologies that may be employed for the optimisation of a high-level network specification for subsea electrical networks.

It is possible to use traditional mathematical programming techniques that ensure an optimal solution to the problem as defined, an example being Nandigam and Dhali [44]. The primary issue associated with using such a method is the reduction in fidelity that occurs from the simplifications required to express the problem in the correct format to be optimised using this technique. The optimisation requirement for a *real solution to a real problem* [45, 46] is conceptualised later in this thesis.

In order to keep the fidelity of the network model to a sufficiently high level, a technique that is capable of dealing with linear, non-linear and abstract parameters is required; hence, a number of studies in the field of computational subsea electrical network optimisation use meta-heuristic techniques. At the time of writing, the technique of choice appears to be exclusively genetic algorithms, an example of evolutionary computing [24, 26-27, 30-31, 42].

Two viable techniques have been found for the generation of efficient connection orders, namely: through the use of a modified genetic algorithm based travelling salesman problem (TSP) solver [18, 28]; and through the use of network analysis methods such as Dijkstra's Minimum Spanning Tree Algorithm [19, 20]. The merits of both methods are presented later in this thesis.

In order to address the assessment of network performance problem, the only realistic option is to use an AC power flow (PF) study. The use of a DC power flow (DC PF) does not take into account reactive power issues that are dominated by the capacitive nature of the subsea cables, thus the AC PF is the only realistic option for this task. Power flow studies, both AC and DC, are discussed in more detail later in this thesis.

1.5.4 Industrial Software Packages

In addition to academic studies described in the following sections, there are also industrial software packages that are used for the design of subsea electrical networks. An example of such software is eGrid: a component of the EMD's wider WindPro package. This package differs to the academic work in this field in the respect that it does not offer automated optimisation of subsea networks: instead it offers a graphical user interface for a user to design and specify a network, from which eGrid can assess its performance against a number of technical constraints. Consequently, this package appears to be focused more on the *tactical* level of assessing a specific network design, rather than the *strategic* level of network design optimisation [47].

Further, eGrid is limited in terms of topologies that may be assessed to radial, significantly limiting the range of network configurations that may be examined; and appears to not include the cost of cables being out of service in its operational expenditure calculations [47].

1.6 Research / Thesis Positioning

The work presented in this thesis seeks to facilitate, and ultimately conduct, the holistic optimisation of subsea electrical networks for large scale offshore renewable energy projects; typically offshore wind. Holistic, in this context, takes two distinct meanings: that the lifecycle performance of the network is accounted for; and that the optimisation of networks is performed on an *end-to-end* network design, from the turbines to the interface point with a land based network, without undue restrictions on the design space in which solutions may reside.

A number of key points pertaining to the design of the optimisation process discussed within this thesis are now presented.

This work specifically considers the design of two-stage (collector-transmission) electrical networks for the extraction of renewable energy from offshore generation projects; thus no direct connections are permitted. Both stages of the network are optimised simultaneously, as network design may not be divided into independent collector network design and transmission network design sub-problems. There is also no restrictions placed on the deployment of offshore substations, other than that they must be placed somewhere between the turbine field and the point of connection with a land based network.

Both collector and transmission network stages are free to take radial, ring, and star topologies; thus the risk of high quality solutions being overlooked due to fixed network topologies is eradicated. In the event that there are two or more collector networks under consideration as part of the same network design study, these networks may take different topologies. The optimisation tool that was developed for this work specifically caters for the design of networks that have multiple distinct collector networks, each emanating from separate offshore substations.

Given the ubiquity of AC collector networks, there is no scope for the optimisation of network that includes a DC collector stage. The transmission aspect for any network is free to be either

AC or DC, and the voltage and frequency are also adjustable; thus the sensitivity of these parameters may be investigated.

The optimisation of a potential site, from an aerodynamic and / or ecological perspective, is not considered within the scope of this work. Equally, the selection of and / or design of electrical machines and plant is not considered within the scope of this study.

This work seeks to minimise the through-life cost of subsea electrical networks, thus the cost of losses and no-service conditions across the lifecycle is normalised to reflect present day value. The optimisation objective can be reduced, through parameter selection, to minimisation of capital expenditure, such that both of objective functions prominent within the literature are covered. Quantification of losses is achieved by the application of a probabilistic model of input wind speed that allows the corresponding electrical output of each machine to be approximated. From this, the electrical losses may be calculated with much greater accuracy than calculating the average power output and computing the electrical losses based on that. No-service conditions are calculated using the Generation Ratio as defined by Zhao [43] where each network branch is systematically switched out, the power flow study re-run, and the differential losses compared with the *ideal* network calculated; thus the accuracy of no-service cost is maximised at the expense of computational effort. Network performance is assessed using an AC power flow such that the problematic reactive power issues that are likely to be experienced in a subsea electrical network may be accounted for.

In order to implement the technical requirements described in the preceding paragraphs, two meta-heuristic techniques have been selected to oversee the network specification and connection ordering problems. Specifically, swarm intelligence in the form of Particle Swarm Optimisation (PSO) has been implemented for the network specification problem, in a change from the popular method of using an evolutionary computing method such as genetic algorithm. As is discussed later in this thesis, given the similarities of both techniques, similar quality of answers and efficiency should be realised. The connection ordering problem is tackled using a second meta-heuristic method, in the form of a genetic algorithm.

The complete optimisation model is applied to two high profile offshore wind developments: Horns-Rev 1, an in-situ 160 MW wind park in Danish territorial waters; and East Anglia One, a proposed 714 MW development in UK territorial waters of the North Sea.

1.7 Research Contributions

This work presents a two-tiered meta-heuristic methodology for the optimisation of subsea electrical networks for marine sourced renewable energy. This methodology is then used to assess the optimality of an in-situ network and to suggest an optimal network configuration for an as yet unconstructed network.

Specifically, the contributions to knowledge offered within this thesis may be summarised as follows:

1. This work offers the most holistic optimisation of subsea electrical networks for offshore renewable energy developments. This is achieved by assessing contemporary studies for desirable optimisation characteristics and including **all** such characteristics; thus it is more holistic than any of the contemporary studies which only include some of these characteristics. Further, accuracy of results obtained is improved, relative to contemporary work, by calculating component sizing and associated costs directly from the power flow solution. This allows for cables and other plant to be sized in accordance with its individual requirements, in preference to the discrete sizing methodology offered elsewhere, which may significantly over-specify components.

2. This work allows for networks of larger scope and greater complexity to be analysed, relative to other work in this field. The developed optimisation process is capable of examining solutions where multiple offshore substations, and therefore multiple parallel collector networks, have been implemented. Within the specific cases considered within this thesis, 1 to 3 offshore substations have been considered, along with the use of AC and DC transmission systems. In doing so, a more rigorous examination of the feasible search space has been conducted than is achievable using contemporary methods, reducing the likelihood of optimal solutions being overlooked.

3. This thesis demonstrates the viability of swarm intelligence, namely Particle Swarm Optimisation (PSO) for the purposes of designing high-level subsea electrical networks for the connection of offshore generation. While there are many examples of meta-heuristic techniques being applied in this context through evolutionary computing methods; at the time of writing no published research using swarm intelligence has been found.

1.8 Thesis Outline

In addition to this opening chapter, this thesis contains 5 further chapters along with the list of references for this work and appendices containing simulation data and analysis of validating test cases.

Chapter 2 covers a number of introductory concepts that will offer background and aid contextualisation of the work presented thereafter. This chapter covers a number of distinct topics beginning with the approximation of resource for offshore wind generation, followed by an introduction to the generation technologies that are deployed in the offshore arena.

As this thesis pertains to the optimisation of subsea electrical networks for renewable energy, these topics are covered in some detail in this chapter. A number of salient issues relating to the design of subsea electrical networks are presented, including: choice of transmission technology (alternating or direct current); reactive power and the influence of frequency; typical network topologies that may be deployed when constructing such a network; and the choice of how and where electrical converters are utilised.

The second main topic that is covered within this chapter is optimisation techniques and methodologies. This begins with an explanation of terminology and definition of what an optimisation problem is; before the application of traditional calculus based optimisation methods are considered. Mathematical programming methods (linear; non-linear; mixed-integer) are then presented followed by a discussion around meta-heuristic optimisation techniques, and finally: network analysis methods.

Chapter 3 begins by introducing the concept of requiring both a high fidelity model and a robust solution methodology to produce *a real solution to a real problem* [45-46]. This concept shapes this chapter as two of the three headline sections within the chapter pertain to the fidelity of subsea electrical network models and to solutions methods that may be used to generate robust solutions. This chapter reviews relevant academic literature to identify how other researchers have tackled the optimisation of underwater electrical networks.

The fidelity of a subsea electrical network model for the extraction of renewable energy has many facets, including: the choice of optimisation objective; the high-level network configuration (direct connection or two-stage collector-transmission network); the choice of design variables to include within the optimisation and restrictions that are placed upon these variables; the choice of discrete or continuous component sizing and positioning; and the use of representative cost functions. These topics are all discussed within the Subsea Network Model Fidelity section of Chapter 3.

Within Chapter 3, it is identified that the optimisation of a subsea electrical network can logically be divided into three interrelated sub-problems: namely, the specification of high level network designs; efficient ordering to minimise the length and / or cost of connection, thus producing a detailed network design; and the assessment of the complete network. The second main section of this Chapter reviews the academic literature in terms of how these problems have been addressed.

In addition to the reviews of literature around fidelity and solution methods, the third offering within this chapter is the review of outcomes arising from contemporary research.

Chapter 4 details the framework that has been developed in order to construct, simulate, analyse and, ultimately, optimise offshore electrical networks. This chapter begins with justifications as to why the high-level framework design was selected from the options that were presented within the academic literature.

Once justification for the high-level design has been presented, the implementation of the design is discussed. The high-level algorithm that drives the optimisation process is presented, before the three interrelated sub-problems of the network specification problem; the connection ordering problem; and the network performance problem, are considered in greater detail.

The third core section of Chapter 4 is the Model Validation section, which examines the solutions that were obtained for simple test cases. These validation studies demonstrate that the model identifies optimal solution, or solutions that are so near to optimality that they are effectively optimal, to the network optimisation problem. Simple test cases were designed such that it was feasible to produce mathematical models of the electrical losses incurred within a network, such that optimality may be validated.

Chapter 5 considers two case studies, namely the Horns-Rev 1 offshore wind farm in Danish territorial waters which provides an *in-the-water* network for comparison and the East Anglia One development that is currently in the planning stage. This case study presents suggestions as to what the optimal network for this installation will look like. Within each case study a number of different optimisations are performed, based on the base case design parameters, each searching for solutions that involve the use of 1, 2 and 3 offshore substations.

For both case studies, key design parameters of voltage and current of both collector and transmission networks are manipulated, such that it may be assessed if the optimal solutions achieved using the base case parameters may be improved upon. Discussions around the sensitivity of each parameter, in both cases, are offered.

Chapter 6 presents a summary of this thesis and conclusions that have been drawn through the completion of research detailed therein. Additionally, further research that could be undertaken, building on work detailed in this thesis is presented.

Specifically, this chapter describes that the optimisation framework developed for this work allows for networks of larger scope and greater complexity to be analysed, relative to contemporary work. Further, the use of hybrid algorithms including the first recorded use of a swarm intelligence method for the optimisation of subsea electrical networks is presented.

A number of strands of future research are documented, including: the role that active network management and energy storage could play within the optimisation of subsea electrical networks; and extensions to the application scope to allow this framework to address further problems.

Chapter 2: Background

This chapter briefly introduces a number of strands of material that are relevant towards this work, including: why the marine energy domain offers a vast and currently underutilised resource and the means by which this energy is extracted.

This section summarises the considerable energy potential that exists in the offshore domain; the generation technologies that have been deployed in this domain; and the network technologies that allow the transportation of energy from the point of generation to the interface point with an existing on-shore network.

2.1 Wind Resource Approximation

In order to be a worthwhile development, a wind farm requires an abundant wind resource. Wind, however, is an inherently unpredictable resource, despite some observable trends. For a European development, it would be expected that there shall be a stronger wind resource available the winter months and a weaker resource during the summer months, as observable in Figure 6 [48], which depicts the wind resource measured in Ireland during 2009.

Further observable in Figure 6 is a trend for peak wind resource to be available during the mid-afternoon, with a significant increase during this period with respect to other times of the day. This trend is more noticeable for the summer months when the resource is more limited than for the winter months when the resource is more abundant.

While there is a degree of predictably of wind availability and strength based on seasonal and diurnal aspects; it remains both intermittent and subject to significant variation. This is visible within Figure 7 [48] where the resource measured at an Irish site in May 2009 is significantly different across a 5-day period. Similar variation would be expected if the typical wind resource measured on a month-by-month basis was compared over a range of years.

Wind is also an infinitesimal quantity: the wind speed at any point may take values that are measurable to levels that are beyond human comprehension. Equally, the wind resource does not conform to standardised time units: wind speed does not remain constant for any period of time; instead it is subject to fluctuation that may be infinitesimally small or more noticeable.

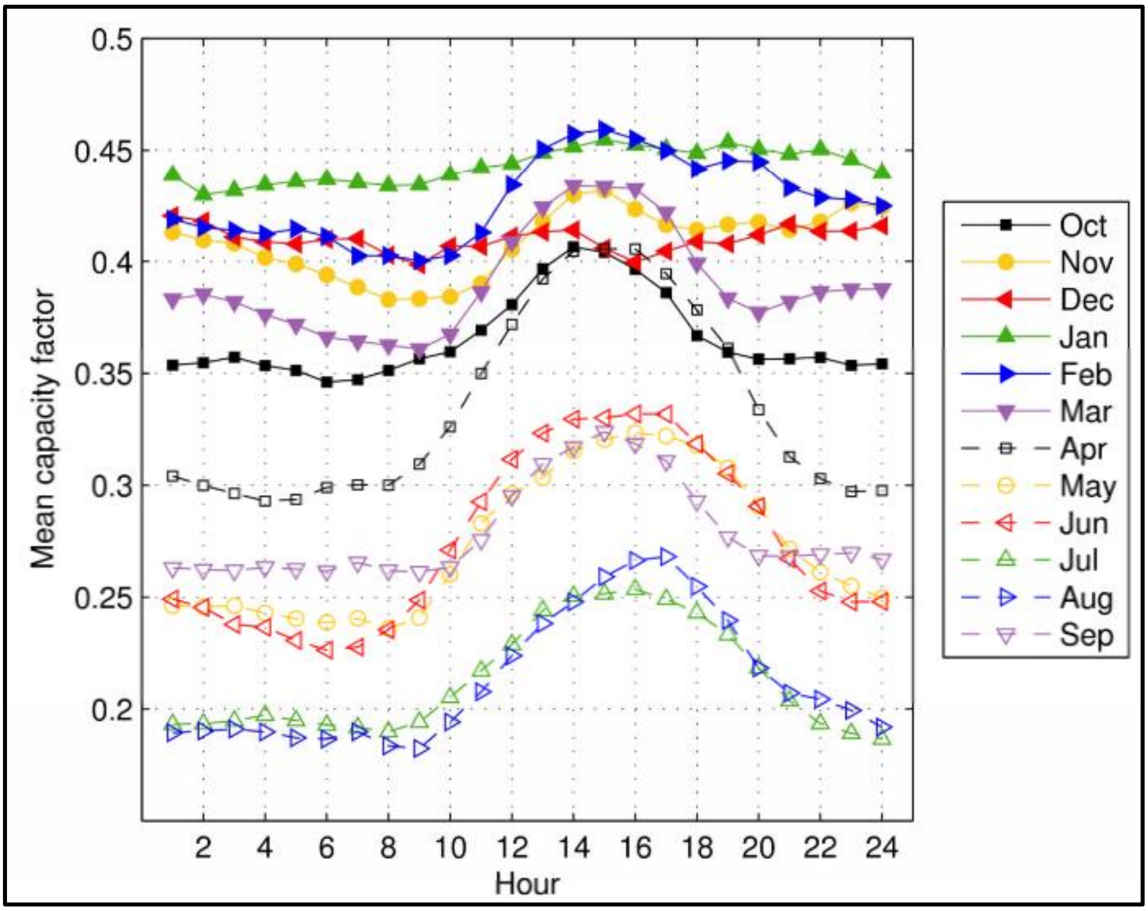


Figure 6 - Annual and Diurnal Wind Variation in Ireland in 2009 [48]

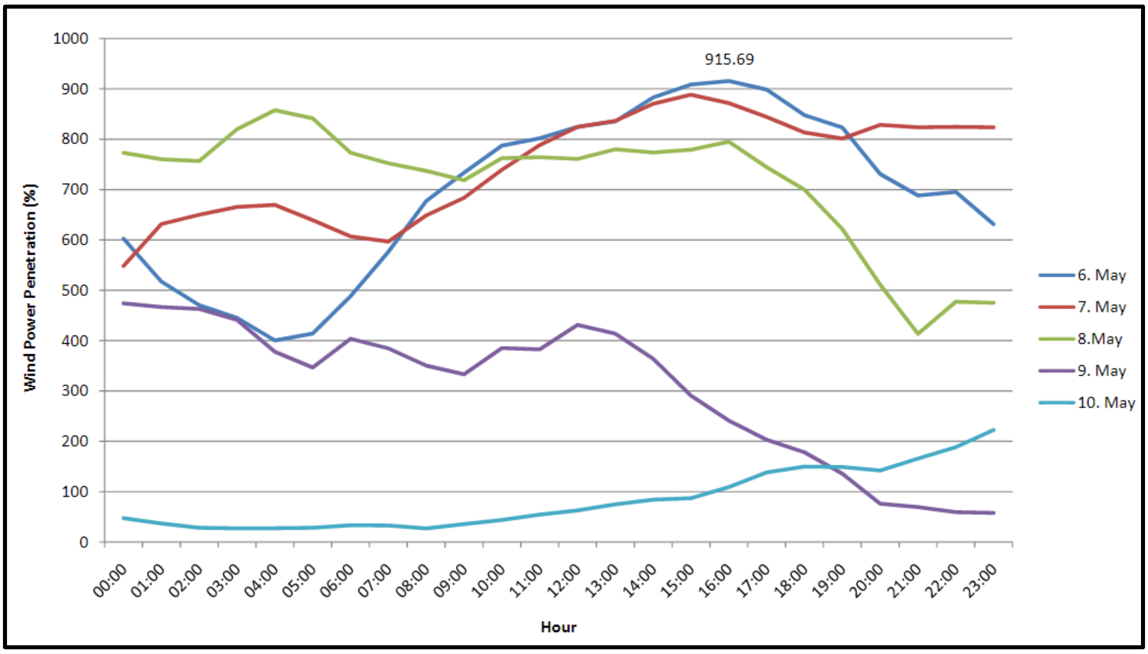


Figure 7 - Daily Wind Variation in Ireland in 2009 [48]

While taking highly granular measurements of the wind resource for a potential offshore wind farm with a high sampling rate would build up a detailed picture of the resource; typically a mathematical approximation of resource is utilised in the form of a probability density function (PDF).

A PDF may either be constructed on a quasi-continuous or a discrete basis. The PDF models the probability of the wind resource assuming a certain value. For an infinitesimal resource, such as wind, it is impossible to create a truly continuous PDF; instead, minute resolution can be employed to make an approximation of resource that is quasi-continuous. Such a distribution is depicted in Figure 8, where the wind speed is represented using discrete states of 0.01 m/s. As the wind speed steps are so small, the probability of the wind speed assuming any particular value is miniscule: 0.08 % being the highest probability.

Equally, the resource may be discretised on a less granular basis, such as 1 m/s *bins*, as depicted in Figure 9, where the peak probability of a given wind speed is now 8 %. Both the quasi-continuous and the discrete PDFs presented as Figure 8 and Figure 9 are generated using the Weibull Distribution, using the same parameters; hence both functions take the same shape.

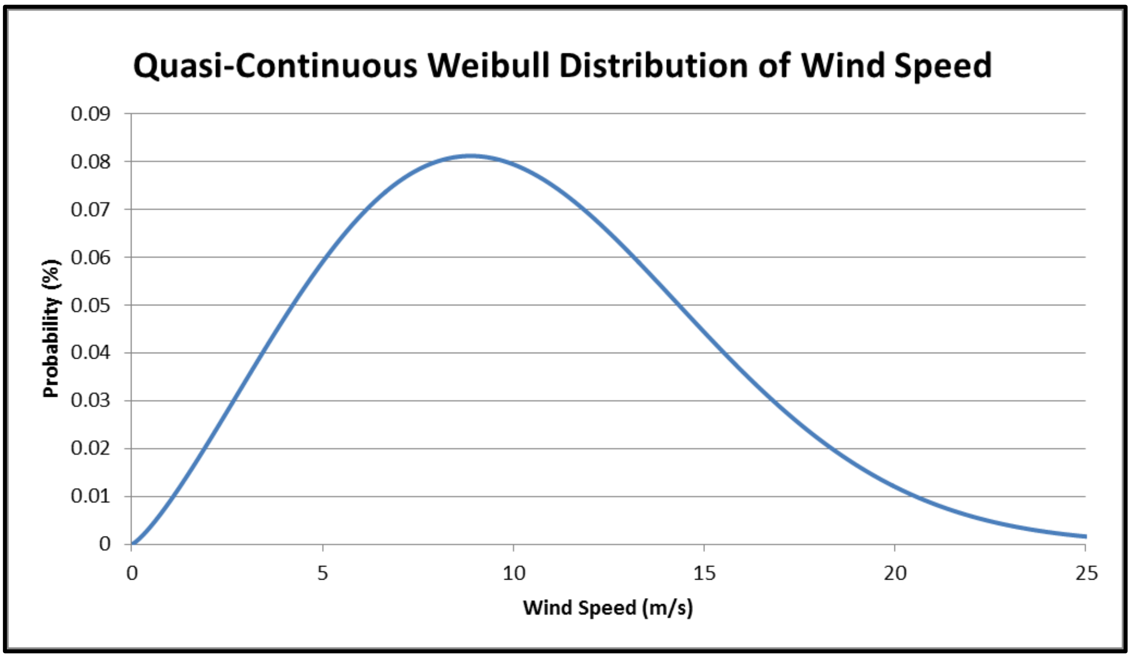


Figure 8 - Quasi-Continuous Weibull Distribution PDF

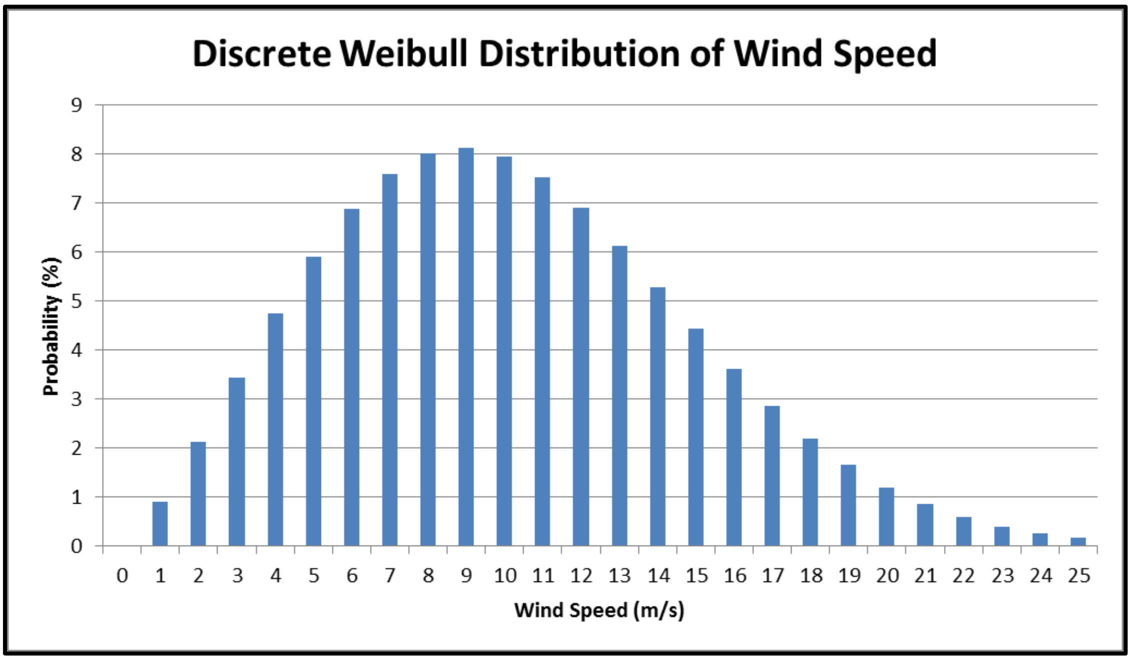


Figure 9 - 1 m/s Discretised Weibull Distribution PDF

The Weibull Distribution is a commonly utilised approximation of wind resource for the offshore domain [25, 49] and is defined mathematically as (1), where *P* is the probability of a given wind speed, *U*; *k* is a shape parameter; and *λ* a scale parameter.

$$P(U;\ \lambda, k) = \begin{Bmatrix} \frac{k}{\lambda}\left(\frac{U}{\lambda}\right)^{k-1} e^{-\left(\frac{U}{\lambda}\right)^{k}} \\ 0 \end{Bmatrix} \quad \begin{matrix} x \geq 0 \\ x < 0 \end{matrix} \tag{1}$$

The Weibull Distribution shown in Figure 8 and Figure 9 is based on a shape parameter of 2.26 and a scale parameter of 11.2, representing values typical of an offshore wind farm site in the North Sea [49].

An alternative PDF that may be employed is the Rayleigh function, as defined by (2). As with the Weibull distribution, *P* represents the probability of a given wind speed, *U*; while *σ* represents a scale parameter. Using a scale parameter of 10.152 returns the PDF as depicted in Figure 10 [40]; taken to be representative of an offshore wind farm site.

$$P(U;\ \sigma) = \begin{Bmatrix} \frac{U}{\sigma^2} e^{\left(-\frac{U^2}{2\sigma^2}\right)} \\ 0 \end{Bmatrix} \quad \begin{matrix} x \geq 0 \\ x < 0 \end{matrix} \tag{2}$$

As was the case for the Weibull Distribution, a Rayleigh Distribution may be either quasi-continuous or discrete.

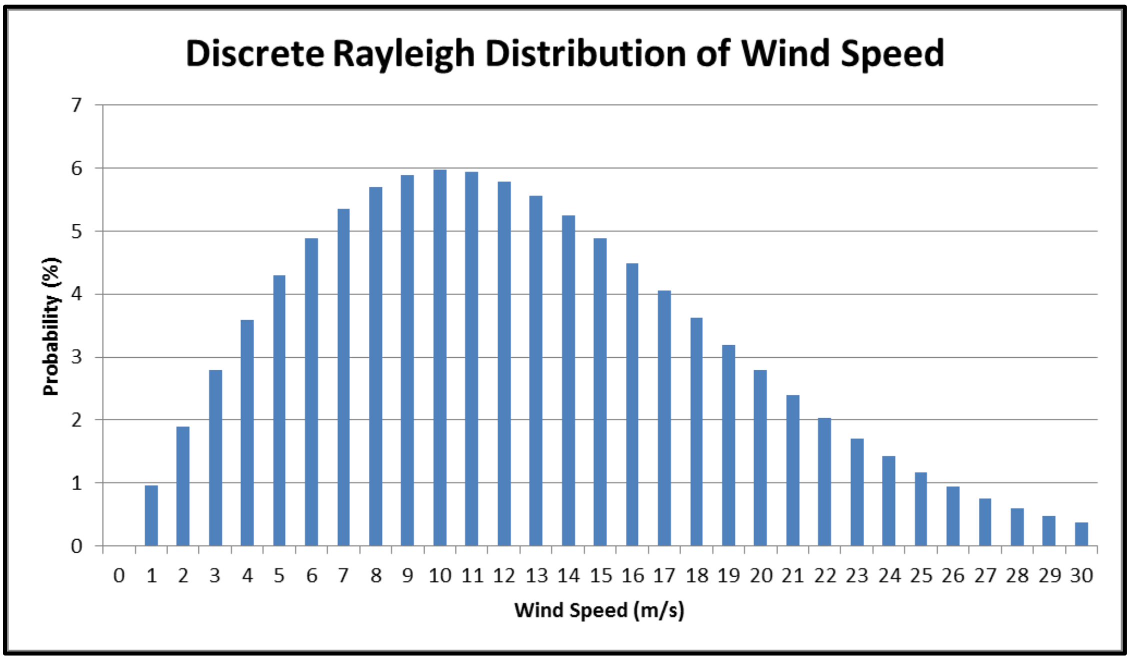


Figure 10 - 1 m/s Discretised Rayleigh Distribution PDF

Using a PDF does not take into account a number of effects such as the intra-day fluctuation shown in Figure 7. Instead, the PDF is used to calculate an approximate percentage of the lifetime of the development for which the wind resource will be at a given speed. This information is useful as this allows the anticipated electrical yield and losses to be approximated and monetised.

2.2 Generation Technologies

There are a number of generation technologies that may be deployed in the offshore arena and these are introduced within this section.

The offshore wind sector in Europe is considerably more mature than the tidal and wave sectors with large scale commercial developments in existence. Wind turbines may be either horizontal axis (HAWT) or vertical axis wind turbines (VAWT) with HAWTs being considerably more commonplace than VAWTs. Additionally, the largest commercially available VAWTs are in the tens of kilo-Watts range while modern HAWTs are in the Mega-Watt range [50-51].

Currently, the largest offshore wind turbines have capacity ratings around 7 MW [50]. Wind turbines can be connected in large arrays to form sites with generation capacities that rival large thermal power stations, such as the Borkum West wind farm in German territorial waters (80 × Areva 5 MW turbines) [52].

While offshore wind turbines may look inherently similar to on-shore turbines, they are typically low maintenance permanent magnet synchronous generator (PMSG) machines in preference to the lighter and cheaper doubly-fed induction generators (DFIG) machines utilised for most onshore developments, which require more maintenance. Typical electrical output for an offshore wind turbine will be low voltage (960 – 3300 V) 50 Hz AC with an electrical convertor capable of providing a range of power factors [50, 52].

There are a number of conceptual and functional marine current turbines (MCT) in operation around Europe. A review of the state of the art in 2009 showed 23 distinct forms of tidal turbines that were at various stages of development, with varying degrees of success [53].

An MCT looks similar to a wind turbine but with a much smaller rotor diameter. This is due to water being 800 times denser than air; hence a much smaller swept area is required to absorb the same amount of energy. In the period 2010 to 2016, the largest commercially available MCTs increased in power from around 1 MW [53] to 1.5 MW [54].

As with offshore wind turbines, an MCT will typically utilise a PMSG generator as it offers greater reliability and requires less maintenance than the cheaper, lighter DFIG equivalent [55]. Similarly to the offshore wind turbines, electrical output from an MCT is most commonly low voltage (960 – 3300 V) 50 Hz AC [55].

There are a number of available technologies to generate electricity from waves. These technologies can be broadly categorised into: oscillating water column; overtopping and oscillating bodies [14].

These technologies are at various stages of maturity, with the most developed commercial products generating around 1 MW [56]. There are prototypes that manufacturers claim will eventually have capacity in the region 4 – 10 MW, when deployed in optimal conditions [57]. If such large capacities are achieved, there is significant scope for the development of sizable *wave-parks* with similar capacity to both offshore wind developments and existing thermal plants.

As with offshore wind and tidal generation, wave generation is typically performed using a low-voltage AC machine. As wave devices can be easier to access, relative to tidal or offshore wind generation technologies; and therefore easier to maintain, DFIG machines can be used instead of the heavier, more expensive PMSGs in some cases [57].

Despite the diversity of approaches employed for power generation in the offshore domain, the vast majority of generation is performed using two types of electrical machines: the DFIG; and the PMSG.

In each case, the electrical output is of low to medium voltage AC thus from a network planning perspective, the choice of generation technology is unlikely to have a significant impact on the network design.

2.3 Offshore Electrical Networks

In order to deliver the energy from the point of generation to an existing on-shore electrical network, a subsea network is deployed. While deploying high efficiency transmission cables to each offshore machine is a technically sound solution, it is normally avoided as it would likely be prohibitively expensive and a poor utilisation of resources.

Typically, but not always, a graduated approach is used where lower-voltage lower-efficiency low-cost collector networks are deployed to link generators to offshore substations before higher-voltage, high efficiency, high cost transmission is used to transfer the energy to the shore. This is effectively the reverse of a conventional electrical power system approach where power is generated, stepped up to transmission voltage before being conveyed over long distance before being stepped down in voltage and delivered to point of consumption by a distribution network.

For both transmission and collector systems, AC or DC may be used, each with advantages and disadvantages.

2.3.1 Alternating and Direct Current

Fundamental to the design of an offshore electrical network is the selection of AC or DC for both collector and transmission stages. The choice of this technology dictates which other components must be used to complete the network, for example: if an AC collector network is to be interfaced with an AC transmission system then a transformer is required at the offshore substation, with another one potentially required at the connection point with a land based network. If a DC transmission system is employed then an AC-DC converter is required at the offshore substation and, assuming that the land based network is AC, at the connection point where the subsea and land based network meet.

AC is the more widely utilised technology for the transmission and distribution of electrical power, for a number of reasons, including but not limited to: lower installation costs than a DC equivalent system; and the ubiquity of AC networks, given that they have been in existence for many decades, while DC transmission is a relatively new technology.

The dominant cable type for subsea electrical power transfer is the cross-linked polyethylene (XLPE) [35]. Until recently, the insulation for XLPE cable would break down under DC current, greatly limiting the deployment of high-voltage direct current (HVDC) in the subsea domain. Advances in the materials used to construct XLPE cables have overcome this issue and now XLPE may be utilised for HVDC transmission applications [35].

Both AC and DC cables will experience losses, caused by resistance, where some of the energy is converted into heat. Additionally, AC cables will experience some degree of inductance and capacitance, as discussed in the following sections. In both AC and DC cases, the heat losses are in proportion to the square of the current flow. As the total electrical power may be calculated as the product of voltage and current, it follows that using a higher voltage and a lower current will yield lower losses for a given level of power. Operating at a higher voltage requires additional insulation and spacing requirements which must be taken into account.

Some more modern subsea applications now utilise DC, in the form of HVDC, in preference to an AC system, including the BARD Offshore 1 development [58]. The increased purchase cost is offset over time by the greater efficiency and HVDC cables are not subject to maximum effective length in the same manner as AC cabling. Consequently, large amounts of power may be transferred over long distance with high efficiency. For example, the Xiangjiaba – Shanghai transmission line installed by ABB utilises Ultra-High Voltage Direct Current (UHVDC) to offer a transmission capacity of 7.2 GW over 2000 kilometres [59]. This is achieved using overhead lines rather than buried cables, however, there is no precluding reason as to why a similar transmission link may not be established using subterranean or submarine cables.

HVDC has been used to connect off-shore wind farms including the BARD Offshore 1 development through the Borwin1 HVDC facility. This development is located 130 km offshore in German territorial waters. In total there are 80 machines of 5 MW, connected to a common collection point using 36 kV AC before utilising ±150 kV HVDC to link to the German transmission system at Diele, which is approximately 200 km from the wind farm (75 km of the cable is underground and on land, 125 km is subsea) [58].

HVDC offers technological advantages relative to AC resulting in greater efficiency. The costs incurred by installing HVDC over AC often make HVDC economically unviable for low power, short distance situations, with the break-even distance likely to be in the range 30 to 250 km [35-39].

ABB is an industry leader in HVDC technology and they have identified a number of possible operational configurations, each with their own advantages and disadvantages [60]. HVDC systems can present significant challenges to transformers including: combined voltage stresses; high harmonic content of operating current and DC pre-magnetisation of the transformer core [60].

In the past, HVDC technology has typically been utilised for point-to-point applications rather for networks with multiple connections. Recently, Multi-Terminal DC has allowed for multiple connections to a single HVDC converter, as shown in Figure 11 [60]. This topology, however, incurs a number of control challenges that make the deployment of multi-terminal DC both technically difficult and highly expensive.

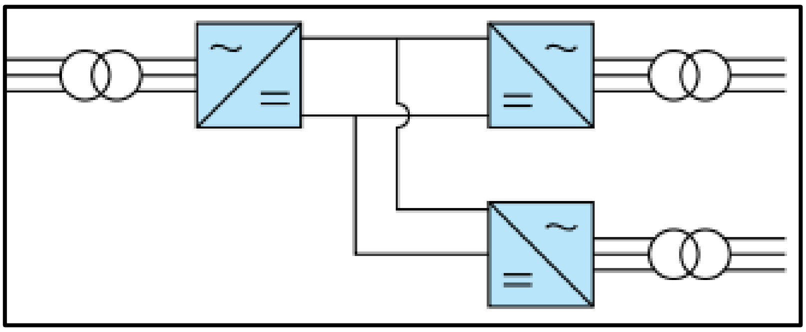


Figure 11 - Multi-Terminal High Voltage Direct Current Topology [60]

In general, a DC system will have higher investment costs but lower losses, relative to an AC equivalent system [36].

The economic arguments pointing towards an AC or DC transmission system will be dependent on the specific characteristics of the installation in question with key metrics including: the distance between the generation installation and the point of interconnection with the land based transmission or distribution system; the amount of generation capacity that is to be installed and the power quality requirements and other mandatory restrictions imposed by the grid code, or other regulations.

The purpose of a collector network is to link the transmission system with a number of generators. Typically, a collector system will be lower-voltage and less electrically efficient than an equivalent length of transmission network, but it is significantly cheaper. A transmission network allows the delivery of this electrical energy to a land based electrical grid.

A number of research papers treat the transmission component of an offshore electrical network as a single point-to-point network [18-20]; however a more comprehensive transmission grid may be employed allowing multiple clusters of offshore generation to be connected to the same transmission system.

Land based transmission of electricity is achieved wherever possible using overhead lines in preference to underground cabling for a number of reasons, including but not limited to: lower capital cost; and ease of access for repairs and greater transmission efficiency.

There are a number of reasons why overhead lines are poorly suited to the offshore environment including: the prohibitive cost of installing pylons in the sea over a distance that could be several tens or even hundreds of miles; the ability of pylons and lines to withstand the considerable wind loads associated with the offshore environment; the difficulties posed by erecting pylons in deep waters; the damage caused to conductors by salt water spray and the unsightly nature of such installations close to shore. As a result of these obstacles, sub-sea cabling is used in preference to overhead lines for offshore electrical networks.

Irrespective of whether AC or DC is used, laying cables under water presents a number of difficulties and can cost significantly more than the purchase cost of the cable. Exact costs are difficult to pinpoint as this will depend on individual circumstances but *rule of thumb* values have been offered in academic literature of between one and three times the cost of the cable in reference to American installations in 2002 [35]; approximately € 50,000 per kilometre in Europe in 2007 [19] and € 365,000 per kilometre in Europe in 2011 [61].

2.3.2 Real, Reactive, and Apparent Power

The capacity of a subsea cable is governed by the product of operating voltage and the thermal current limit of the cable. If the cable is DC then the power available at the receiving end of the cable is simply the power injected at the sending end minus the electrical losses incurred through resistance.

In an AC system, the apparent power, *S*, is defined as the instantaneous product of voltage and current. If the voltage and current sinusoids are in-phase with another then all of the apparent power is real power, *P*. However, if there is a phase angle between the voltage and current waveforms then there is reactive power, *Q*, present, in addition to real power. The relationship between apparent, real and reactive power is defined as (3).

$$S = \sqrt{P^2 + Q^2} \tag{3}$$

The thermal limitation of the cable to convey power is governed by the apparent power limit. If, therefore, there is significant levels of reactive power flow then the ability of the cable to convey real power is substantially reduced. Within the cable, the level of reactive power flow is governed by three properties: inductance, capacitance, and frequency.

Inductance is the property of a conductor by which a change in current in the conductor induces a voltage in both the conductor itself and any nearby conductors. Inductance causes a negative phase angle and consequently the current waveform *lags* the voltage waveform. This phase angle causes reactive power to flow in the cable, thus reducing the real power capacity.

Capacitance may be defined as the ability of a body to store an electrical charge. In terms of phase angle, capacitance introduces a positive phase angle such that the current waveform *leads* the voltage waveform. As with inductance, this reduces the real power output compared with a purely resistive system.

The inductance and capacitance values of a cable are independent of frequency, however; the effects of both are not. Reactance is the level to which inductance causes impedance within the cable, as defined by (4), where: X_L is the reactance; f is the operating frequency and L is the inductance of the cable. As may be observed from (4), the reactance of the cable is directly proportional to frequency.

$$X_L = 2\pi f L \tag{4}$$

The level of capacitive charging experienced in a cable is also related to frequency through susceptance, which is defined by (5). Again, the level of capacitive charging is directly proportional to the system frequency.

$$B = 2\pi f C \tag{5}$$

While the inductive and capacitive behaviours described previously are both functions of system frequency, they act in opposition to one another: in theory, the frequency can be selected for AC systems such that the effects are negated completely. In practise, it is unlikely that the effects will be entirely eradicated; instead a solution that allows effect minimisation is sought.

Cables and plant in AC systems may be said to be consuming (net inductive system) or producing (net capacitive system) a number of VArs. VAr stands for Volt-Ampere reactive and may be defined as being a volt-ampere that is 90° out of phase from the reference voltage waveform. AC cables typically experience capacitive charging whereby the cable is storing the energy rather than serving a load at the end of the line. Typical levels of capacitive charging of Cross Linked Polyethylene (XLPE) cabling, using standard frequencies of 50-60 Hz, are presented in Table 2 [35, 62]; where it may be observed that the magnitude of capacitive charging increases as the operating voltage of the cable increases.

Voltage Level (kV)	kVAr / km
33	100-150
132	1000
400	6000-8000

Table 2 - Voltage Level and Capacitive Charging Values for AC Subsea Cables [35, 62]

Power factor correction equipment allows the power quality to be increased, by bringing the voltage and current waveforms back into phase with one another. If a cable is net inductive then capacitors are used to restore the power factor back towards unity. Conversely, if the cable is net capacitive, as would be expected for a subsea electrical power cable, then inductance is used to restore the power factor. Use of this equipment, however, incurs penalties including: extra cost; additional spacing requirements and additional potential for equipment failure.

Given the nature of AC transmission with high resistive losses and capacitive charging, there is a maximum beneficial operating distance, even with power factor restoration equipment in place at both sending and receiving ends of the cable which incurs considerable expense. For example, one study into the losses incurred in HVDC and HVAC transmission for a theoretical 500 MW - 1 GW off-shore wind farm points to the maximum distances of AC transmission with power factor correction employed being as shown in Table 3 [63].

Voltage Level (kV)	Distance (km)
132	370
220	281
400	202

Table 3 - Maximum Distance / Operating Voltage Level for AC Subsea Transmission [63]

2.3.3 Network Topologies

The simplest topology that may be deployed within an offshore electrical network is the radial topology. In a radial collector network, such as that shown in Figure 12, one or more *strings* emanate from the medium voltage bus-bar of the offshore substation. Each generator has 2 electrical connections: one with the generator that precedes it in the string, or the medium voltage bus-bar in the case of the first generator; and one with the generator that follows it in the string (for example generator 2 shown in Figure 12 connects *from* generator 1 and *to* generator 3). The exception to this is the final generator in the string which only has one connection.

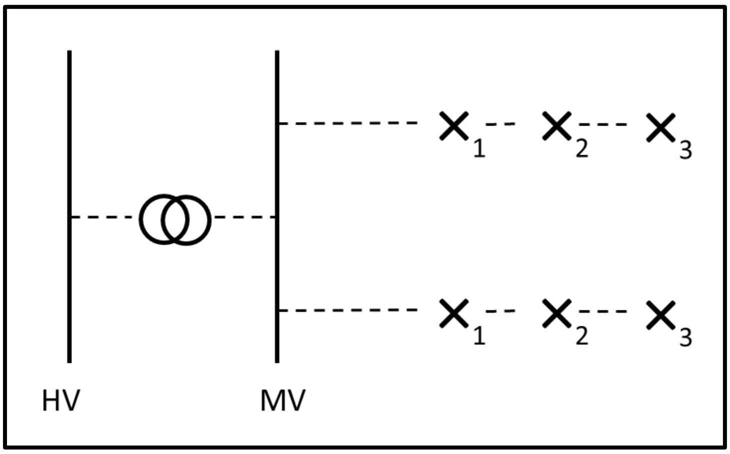


Figure 12 - Radial Collector Network [64]

The radial topology may equally be used for transmission; linking offshore substations to a point of connection with a shore based grid.

The primary advantage of the radial topology is its simplicity and minimisation of cabling. The power capacity required in a radial cable is greatest between the origin of the cable and the first generator delivering power; after this point the capacity requirements become lesser as each generator is connected, such that the final link in the cable is only required to be able to convey the power generated by the final generator. This allows the cable capacity to be tapered such that cost may be minimised.

The radial network, however, offers no inbuilt redundancy, such that a cable or switchgear failure renders any generators beyond the failure point ineffective [64].

A radial string may be reconfigured as ring string by the inclusion of a return path from the final generator in the string. This principle is depicted in Figure 13, where there are 2 parallel paths linking each generator to the medium-voltage bus-bar of the offshore substation. Configuring a network in this way incurs additional cabling cost, with respect to the radial topology. This occurs for 2 reasons: this solution uses greater length of cable than the radial topology; and there is no scope for tapering the cable capacity as each section must be capable of conveying the entirety of the connected capacity [64].

The ring topology may equally be applied to the transmission stage of the network.

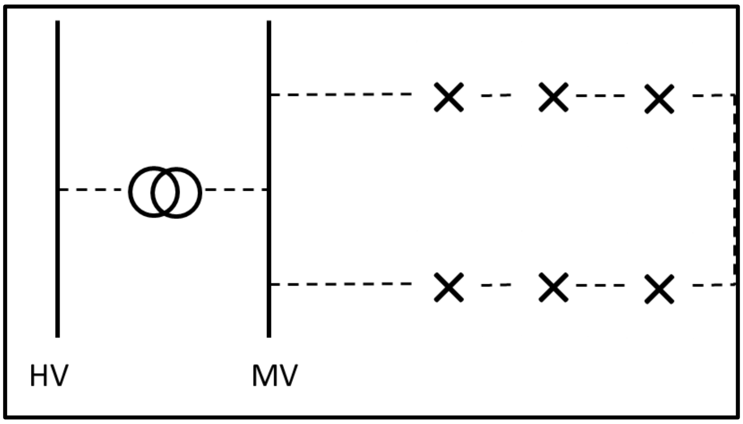


Figure 13 - Ring Collector Network [64]

If the optimisation objective is to minimise installation cost, the deployment of radial networks, for both collector and transmission stages, is attractive. If the optimisation objective is to minimise the through-life costs of the solution then the use of ring networks, for both collector and transmission stages, may become cost efficient. This occurs for 2 reasons: if there are *N* identical generators connected in a ring network, this will effectively form 2 radial networks each with *N/2* generators, thus the electrical losses are reduced relative to *N* generators connected in a single radial cable; and the inclusion of a second electrical path between each generator and the offshore substation means that a single cable failure does not preclude the delivery of electricity [64].

The star collector network topology is fundamentally different to both the radial and ring networks as each generator is connected directly to the medium-voltage bus-bar, as shown in Figure 14. The primary advantage offered by this configuration is the decrease in electrical losses, relative to radial and ring networks, as each cable is only conveying power from a single generator. This reduction in losses is achieved at a cost as a star network requires considerably more cable length to implement than a radial or ring network, albeit this cable is of lower capacity given the small power throughput [64].

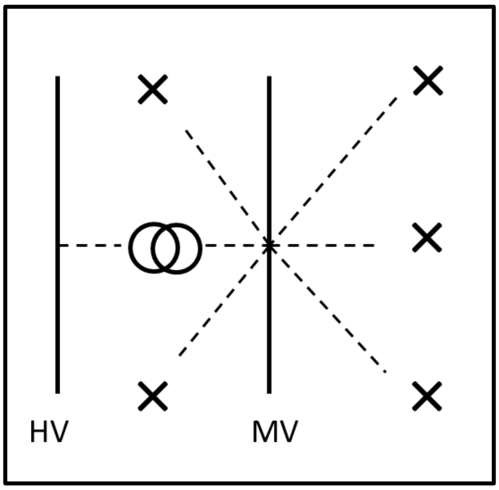


Figure 14 - Star Collector Network [64]

A direct comparison between the three collector topologies that have been introduced as stand-alone electrical systems concluded that the ring network would electrically perform better than either radial or star networks but that the higher investment cost was "*very likely to be a barrier for the deployment of these designs*" [64].

Quinonez-Varela et al concentrated on the relationship between electrical losses and the increase in costs between the different collector topologies, when using AC. It was found that a star network offered 4 % lower losses than a radial network and that a ring network offered between 18 and 45 % depending on the configuration employed. The star network was found to be marginally cheaper than the radial and the ring network was between 6 and 11 % more expensive in terms of capital expenditure than the radial network [64].This study did not, however, consider the lifetime cost of each network, which may have an operational lifetime of tens of years.

Some research within this field allows for *mixed* strings radial / star topology to be deployed as depicted in Figure 15 [26-28, 30].

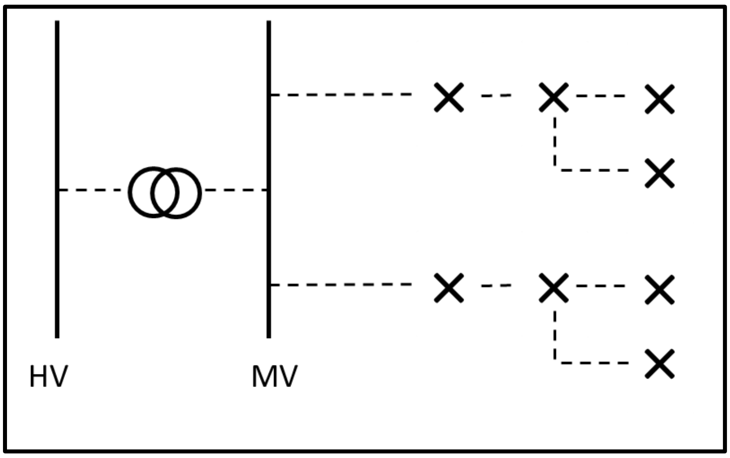


Figure 15 - Mixed (Radial-Star) Collector Network [26-28, 30]

A number of contemporary works restrict the topologies of either collector or transmission networks for their optimisations, for example: Gonzalez-Longatt et al [28] and Li et al [30] restrict the collector topology to what they term radial, but is really a mixed radial / star topology, while allowing the transmission component of the network to take radial, ring, star or mixed topologies.

Ergun et al [21] have bypassed the requirement for consideration of the collector network by taking the electrical start point of the network to be at an offshore substation, thus focussing only on the transmission aspect. In doing so, they consider radial, ring and meshed transmission grids where offshore collectors may be linked to more than 2 other network nodes, as is the case for radial and ring topologies.

The overwhelming majority of physical implementations of offshore collector systems utilise AC in preference to DC as the trade-off between the extra electrical efficiency of a DC system compared with an AC system against the reduced cost of the AC system relative to a DC system typically emerges in favour of the AC system.

Recently, research has been published examining the utilisation of DC collector systems in a DC-DC collector-transmission system. Typically, a DC collector system would be arranged in a series-parallel topology [33-34], as depicted in Figure 16.

Generators are arranged in series *strings* until the DC link voltage is established, and then these strings are connected in parallel to provide greater current. The use of a DC collector network does not infer the use of DC generators. Typically, AC generation is utilised and the output rectified to DC [33]. While there are losses associated with the rectification of the AC output for collection, it is worth noting that AC generators are normally connected to an AC collector system using an AC-DC-AC, rectification-inversion process to allow for synchronisation. Consequently, losses incurred by using an AC machine and a DC network will be less than when using AC machines and an AC network, assuming the rectification components are identical.

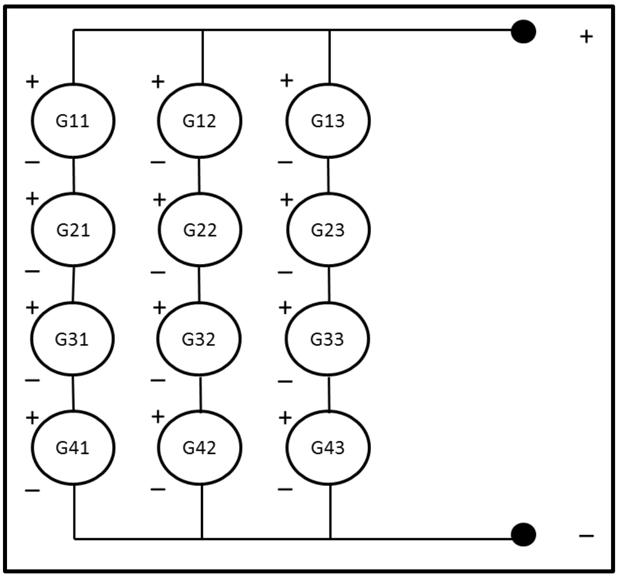


Figure 16 - Series-Parallel DC Collector System Topology [33, 34]

There is no technical impediment to the deployment of extra interconnections between strings which allow for a more flexible network which is advantageous under fault conditions. Additional cable runs and switchgear increases the cost and the economic value offered by increased life-cycle electrical output must be offset against greater initial capital costs [34].

2.3.4 Constant and Variable Speed Generators

Typically, generators used in renewable energy operations may either be constant or variable speed. If a constant speed system is employed, a gearbox is used to maintain a constant generator speed across a range of wind speeds. As the electrical frequency of each turbine is fixed, and each turbine is synchronised with one another, it is possible to link these turbines to the grid without employing power electronics. This principle is depicted in Figure 17 [31, 65].

Depending on the output voltage of the generator, a transformer may or may not be required. If a typical machine voltage of 960-3000 V is utilised, then a transformer will generally be required. However, if the generator output voltage is sufficiently high (5-11 kV), it is feasible to connect the turbines directly to the medium-voltage bus-bar of the offshore substation without the need for transformers at each turbine [65, 66]. This method is electrically advantageous as it does not require power electronic conversion between the turbine and the offshore substation, thus reducing the capital cost and losses associated with this process [31, 65]. It is, however, mechanically disadvantageous as the inclusion of a gearbox incurs cost, energy losses, weight and increased maintenance [31]. While the lack of an electrical convertor is advantageous, in terms of electrical losses, this is offset by the lack of reactive power output control, which may be particularly problematic given the highly capacitive nature of subsea cabling [31].

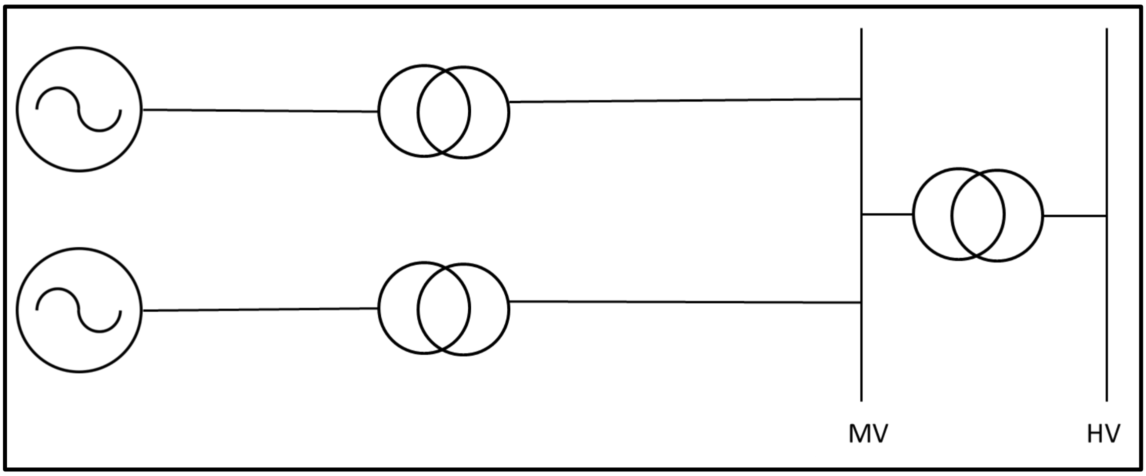


Figure 17 - Constant Speed Generator Collector Network [31, 65]

A second approach is the utilisation of individual variable speed turbines. In such a system there is no gearbox managing the relationship between wind speed and generator speed. Instead, the output frequency of each machine is kept constant through the deployment of power electronic converters; typically back-to-back AC-DC-AC conversion, as depicted in Figure 18 [31, 65]. This configuration allows for each turbine to be controlled individually and to allow the rotational speed to vary while ensuring synchronisation and constant system frequency at the medium voltage bus-bar of the offshore substation [65]. While an AC collector, AC transmission system is shown in Figure 18, the AC-DC-AC conversion process would equally work for an AC collection, DC transmission system. This configuration reduces the fatigue loading on the turbines, relative to the fixed speed approach, and eliminates the requirement for a gearbox to regulate the generator speed for a range of wind speeds.

As was the case for the fixed generator speed system, in order to facilitate efficient power transfer through the collector network, transformers are generally required unless the generator output voltage is sufficiently high. Back-to-back AC-DC-AC conversion is typically performed at low-voltage as this is more economical than performing it at medium-voltage [65].

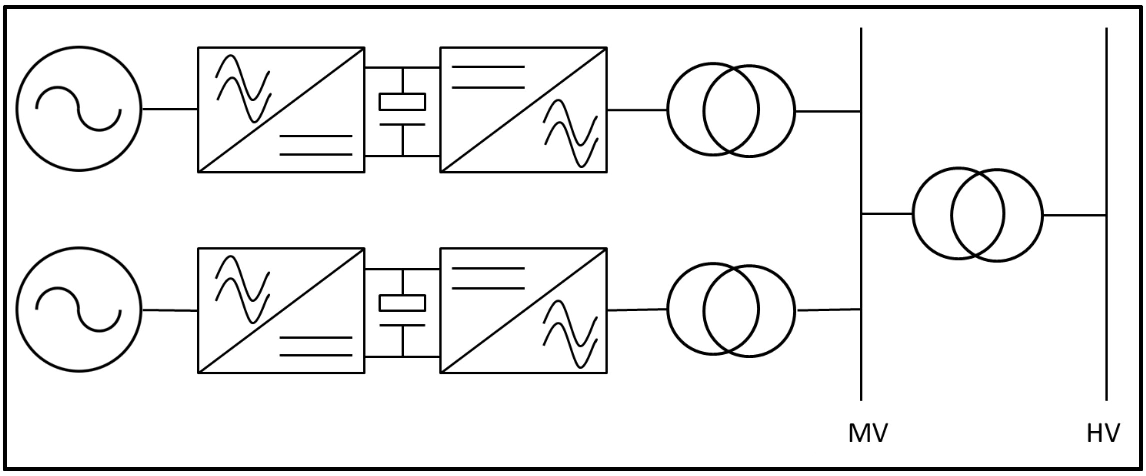


Figure 18 - Individual Variable Speed Collector System with AC Transmission [31, 65]

The converter adds costs to each turbine and its presence reduces the overall reliability of the system. Given the difficulties that can be associated with accessing turbines, plant and cables in the offshore domain, one proposed solution is to share a single converter across multiple turbines as depicted in Figure 19 [31, 65-66]. This configuration is known as cluster-coupled variable speed and in such a system, the turbines are synchronised with one another, with the generator speed, and therefore the electrical frequency of the system broadly proportional to the average wind speed within the group [65].

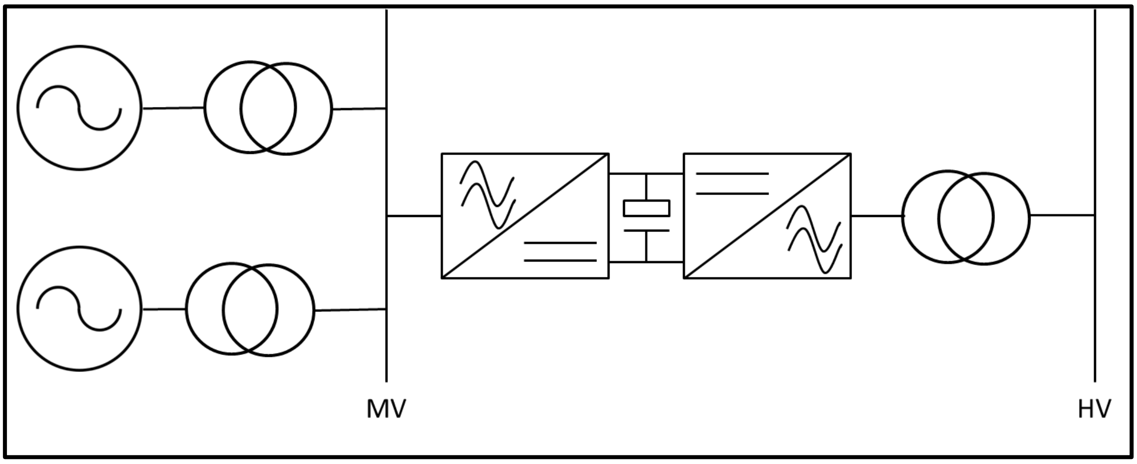


Figure 19 - Cluster Coupled Variable Speed [31, 65-66]

As with the fixed speed system discussed earlier, there is a requirement for gearboxes to regulate the relationship between turbine speed and generator speed; however, there is no requirement for power electronic conversion between the generator and the medium-voltage bus-bar of the collector system. Additionally, the fatigue loads on turbine components for this system are likely to be higher than when an individual variable speed collector system approach is utilised [31, 65].

By using an offshore substation based shared power converter in preference to turbine based individual power converters, the potential for system redundancy is increased. An example of this is the possibility of utilising a secondary share power converter that can be switched in automatically in the event of the primary converter failing [66].

The system described by Parker & Anaya-Lara [66] relies upon the use of 11 kV machines to support an efficiently high network voltage. If a more standard machine voltage range (690 – 3300 V) is used, then there would be a necessity for turbine transformers. If transformers are required at each turbine to step up the output voltage, then a number of advantages offered by the shared convertor topology, such as the ease of access at each turbine and system simplicity, are largely negated.

A shared power convertor may also be deployed at the transmission stage, as depicted in Figure 20; in which case the configuration is known as park-coupled variable speed [65]. The frequency within the collector networks may either be controlled or may be broadly proportional to wind-speed experienced. As with the other fixed generator speed network styles, deploying in this configuration incurs higher fatigue loading than a variable speed network and requires a gearbox at each turbine [31, 65].

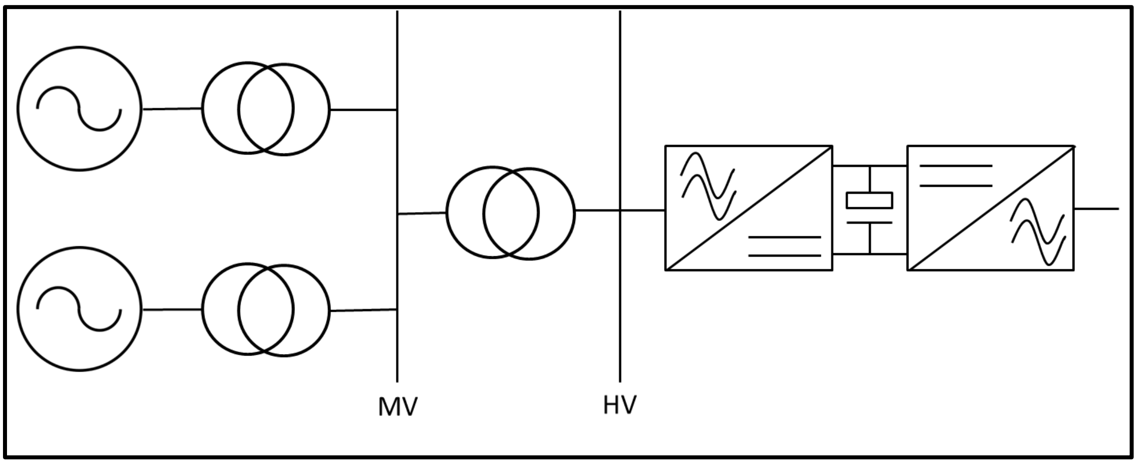


Figure 20 - Park Coupled Variable Speed [31, 65]

In 2000, Bauer et al [65] analysed the relative merits of each of the fixed and variable speed configurations discussed earlier within this section. They acknowledged that the constant speed system would offer the cheapest investment price, but this would not necessarily offer the lowest price per kWh of electricity. Further to this, they also concluded that the conversion arrangement was unlikely to have a significant bearing on the final price per kWh of electricity.

2.4 Optimisation Techniques

Within this section, a number of core concepts of optimisation are defined before a number of techniques that may be employed for specific problems are presented.

2.4.1 Terminology and Problem Definition

Optimisation, irrespective of what problem is under consideration and what method is employed to seek this solution, is concerned with the finding of the best feasible solution. Generally, this objective may be defined mathematically as:

$$\min \boldsymbol{F}(\boldsymbol{x}) = \min([f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \dots, f_m(\boldsymbol{x})]) \tag{6}$$

$$\boldsymbol{x} \in \Omega \tag{7}$$

$$\boldsymbol{g_j}(\boldsymbol{x}) = 0 \qquad j = 1,2,..m \tag{8}$$

$$\boldsymbol{h_k}(\boldsymbol{x}) \leq 0 \qquad k = 1,2,\dots n \tag{9}$$

Where the global minimiser $\boldsymbol{x^*}$ is sought such that the objective function, $\boldsymbol{F(x^*)}$, is minimised at this solution. $\boldsymbol{F(x)}$ consists of a number of objective functions, $f_1(\boldsymbol{x})$ to $f_m(\boldsymbol{x})$, and is subject to a number of equality constraints, denoted by $\boldsymbol{g(x)}$, and inequality constraints, denoted by $\boldsymbol{h(x)}$.

Within (7) it is defined that the decision vector, $\boldsymbol{x}$, must be constrained by the decision domain, Ω. The decision domain is a multi-dimensional space with the number of dimensions equal to the number of decision variables. Each decision variable within the decision vector [x_1, x_2, …, x_n] will possess its own boundaries within Ω [67].

Equations (8) and (9) refer to equality and inequality constraints which place restrictions on which solutions are feasible. These constraints may be very simple in nature, perhaps defining that a variable must take a value between a lower limit and an upper limit. Equally, the constraints may be of increased complexity; multivariable and non-linear.

Both the objective functions and the constraints around a given problem are categorised in accordance with their mathematical status. Depending on the categorisation of both functions and constraints, different techniques are employed to solve the optimisation problems.

The combination of the search space and the constraints defines one or more *feasible regions* in which viable solutions may exist. Shown in Table 4 is a representation of the feasible design space for a decision vector with two variables, under a number of conditions [67]. The feasible design spaces are coloured blue, while anywhere that is outside of the blue sections are unfeasible; hence the white space is the infeasible range.

The decision space is mapped to the objective space using the objective function, ***F(x)*** [67].

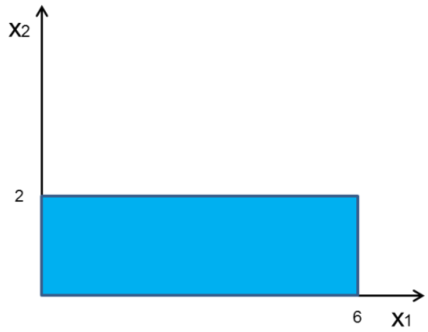
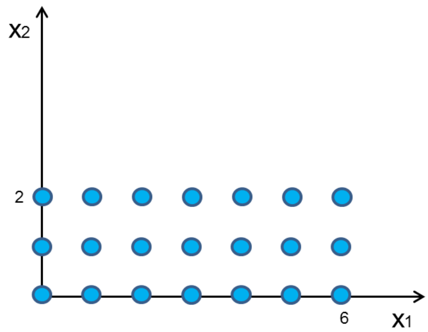
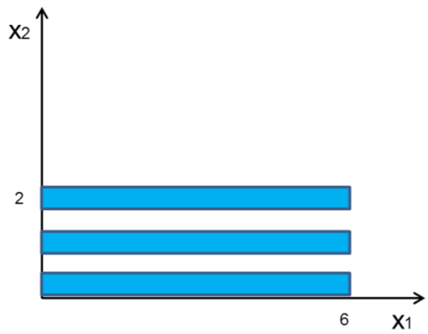
Decision Domain Ω	**Example**	**Feasible Decision Domain**
Continuous	$0 \le x_1 \le 6$ $0 \le x_2 \le 2$ $\boldsymbol{x} \in \Re$	
Discrete Integer	$0 \le x_1 \le 6$ $0 \le x_2 \le 2$ $\boldsymbol{x} \in Z$	
Mixed Integer - Continuous	$0 \le x_1 \le 6$ $0 \le x_2 \le 2$ $x_1 \in \Re$ $x_2 \in Z$	
Binary Integer	$x_1 = [0, 1]$ $x_2 = [0, 1]$	$\Omega = \begin{bmatrix} 00 \\ 01 \\ 10 \\ 11 \end{bmatrix}$

Table 4 - Decision Domain Examples [67]

Convexity is a key concept in optimisation as the difficulty in solving the optimisation problem is directly related to convexity. In a convex set, a line segment between any two points in the set lies entirely within the set. This principle is depicted in the leftmost panel of Figure 21 where it may be observed that the line segment that joins *f1(x)* and *f2(x)* is entirely within set, *C*, hence the function is convex [67].

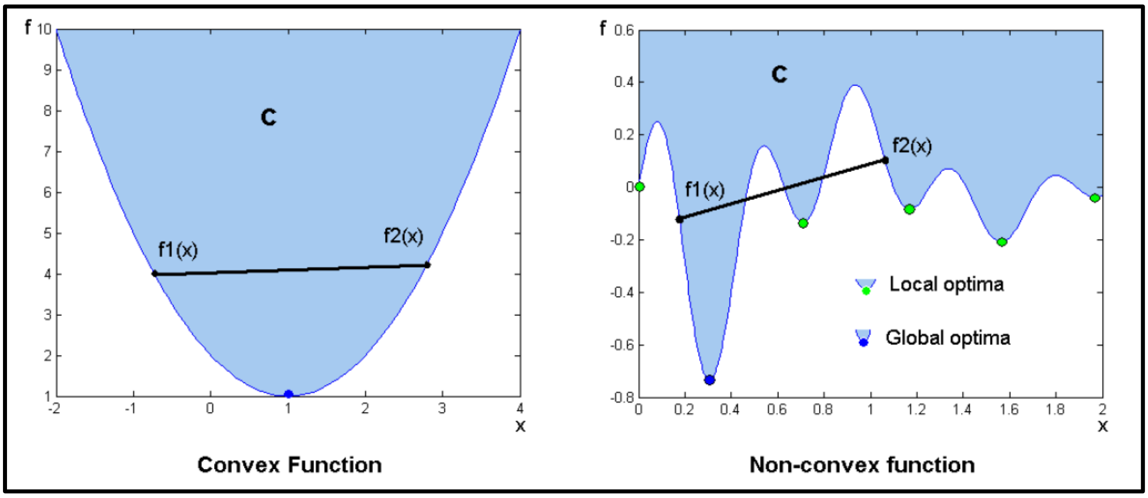


Figure 21 - Convex and Non-Convex Functions [67]

Conversely, in the rightmost panel of Figure 21, the line joining *f1(x)* and *f2(x)* is not entirely within set, *C*; hence the function is non-convex [67].

Any discontinuous set, such as those shown in the second and third rows of Table 4, is non-convex by definition. Any non-linear equality constraint is also non-convex by definition, however; non-linear functions can be either convex or non-convex [67].

When a function is convex, it has a single optimal solution. When a function is non-convex, there may be a number of local optimal solutions, in addition to a global optimal solution, hence non-convex problems are significantly more difficult to solve than convex problems [67].

2.4.2 Calculus Methods

If a problem has continuous decision variables; one or more objective functions that are continuous and double-differentiable; and constraints that are continuous and double-differentiable then calculus provides an effective methodology for finding the optimal solution.

This is achieved through differentiating the objective function(s) and solving with the derivative set to zero; allowing the position of any turning points to be identified. Each turning point may then be categorised into maximum or minimum turning points in accordance with its second-derivative. If the second derivative is positive then a minimum turning point is present; conversely, if the second derivative is negative then a positive turning point has been identified. If a second derivative is zero, then an inflection point has been recorded. All turning points present within the feasible design space must be assessed in order for global optimality to be ensured, as the double-differentiation method will also identify a number of local optimal solutions also.

Equality and inequality constraints can be incorporated into the objective function(s) using Lagrange multiplier, in the case of equality constraints; and Karush-Kahn-Tucker conditions, in the case of inequality constraints [67].

2.4.3 Mathematical Programming

Mathematical programming, a term encapsulating a number of techniques including linear, non-linear and linear programming is a method that may be used to provide optimisation solutions for specific problems [68-69].

Within the following sections, a number of mathematical programming techniques are presented and discussed.

Linear Programming

If an optimisation problem has continuous design variables; one or more linear objective functions; and one or more linear constraints then linear programming is a powerful tool for solving the problem.

In its standard form, any linear program can be expressed as:

$$optimise: z = \boldsymbol{C}^T\boldsymbol{X} \tag{10}$$

$$subject\ to: \boldsymbol{AX} = \boldsymbol{B} \tag{11}$$

$$with: \boldsymbol{X} \geq 0 \tag{12}$$

Where $\boldsymbol{X}$ is a column vector of unknowns, $\boldsymbol{C}^T$ is the row vector of corresponding costs, $\boldsymbol{A}$ is the coefficient matrix of the constraint equations, and $\boldsymbol{B}$ is the column vector of the right hand sides of the constraint equations [70].

Every *primal* linear program has associated with it another linear program with different variables that is known as the *dual*. If the primal linear program is concerned with minimisation then the dual will be seeking a maximisation and vice-versa. Both the primal and dual problems have the same optimal solution. This can be useful as it may be computationally advantageous to solve the dual over the primal program [70].

There are two primary methods used to solve linear programming problems: the Simplex method and interior point methods. The Simplex method is a matrix procedure for solving linear programs written in the standard form that has been discussed previously. It takes a basic feasible solution, X_o, to the problems and iteratively locates other feasible solutions with better values for the objective function, by resolving sets of linear equations defined by the constraints and the objective function, thus producing the optimal solution [46, 67, 70].

Simplex utilises the fact that the optimal solution will always be located at a vertex of the constraint set, thus it guarantees that the optimal solution is returned.

While the Simplex method guarantees the optimal solution, the (theoretical maximum) number of iterations required to solve the problem grows exponentially with the size of the problem being considered [46, 67].

A Dual-Simplex method can be employed which negates the need for an initial feasible solution. This method can start from an initial infeasible solution and successively move towards a feasible optimal solution [70].

When addressing significant problems, the Simplex method may be computationally inefficient. In order to address this issue, an interior point method may be utilised.

While there are a number of different variants of interior point methods, the most common are path following algorithms. These methods work by starting within the feasible region and reaching an optimal solution by iteratively computing solutions. Typically, a path following algorithm works by utilising the following concepts [71].

1. An equality constraint optimisation problem is converted to an unconstrained problem through the use of the Lagrange multiplier method.
2. The creation of a sequence of unconstrained problems, achieved through the incorporation of constraints and a logarithmic barrier function that imposes growing penalties as the boundary is approached.
3. The solution of a set of non-linear equations using Newton's method, thus solving the unconstrained optimisation problem.

Non-Linear Programming

Non-linear programming is concerned with the optimisation of problems that possess one or both of: non-linear objective functions; and linear and / or non-linear constraints, in addition to a continuous decision space. If a non-linear problem is convex, it may be readily solved using an interior point method [67], the concept of which has been discussed in the previous section.

When a non-linear problem is non-convex, it may have several local optima, hence it may become computationally difficult to optimise. Iterative solving methodologies are typically employed to produce an approximation of the optimal solution [67, 70].

If the problem is single-variable in nature, and is unimodal across the finite interval of interest, by virtue of having a single optimal solution, then it is possible to deploy a variant of sequential-search technique. Such techniques operate by assessing the value of the objective function, $f(x)$, across a range of x values and progressively reducing this range while increasing its granularity until the minimum or maximum point has been found [70].

If the problem is multi-variable in nature, the level of complexity required to obtain an optimal solution increases. A number of methods exist (Steepest Ascent, Newton-Raphson) that may be broadly categorised into *hill-climbing* methods [70].

Hill-climbing methods work by calculating the partial derivatives of the objective function with respect to each variable and combining these to make a gradient vector. The current value of each variable is then applied to the gradient vector to determine the direction of maximum increase. Considering the Steepest Ascent method specifically, the direction of maximum increase vector is multiplied by a positive scalar and added to the previous solution in order to produce the next solution. The value of the positive scalar that maximises the benefit derived from the current step can be determined using sequential-search techniques [70].

At the beginning of a hill-climbing algorithm, there is no current position from which the direction of maximum increase may be computed; consequently, an initial solution to the problem must be provided to the algorithm. This may be obtained by generating random numbers or by simply making a guess, educated or otherwise, as to where the optimal solution might lie in the decision space [70].

Hill-climbing algorithms do guarantee the discovery of local optimal solutions, however, when the problem is non-convex there may be multiple local optimal solutions, hence there are difficulties attached to using this method to return a global optimal solution. One solution to this problem is through the use of multiple starting points. Computational time and effort associated increases as the number of concurrent starting points increases, hence care must be taken to ensure an appropriate balance between accuracy of solution and the computational effort required to reach the solution [67].

Mixed-Integer Programming

As has been depicted in Table 4, the feasible decision space for a given optimisation problem may be exclusively integer based or may be mixed where some variables can only take integer values while others are continuous.

Mixed-Integer programming offers an extension of the toolsets offered by linear and non-linear programming, and it can be utilised to solve problems that are linear or non-linear in nature. By definition, any problem that has a discrete integer decision space is non-convex.

One such method to implement integer programming is the branch and bound method. When using branch and bound, all variables are initially treated as continuous. A mathematical programme is then set up based on the objective function and the, now continuous, constraints. This programme is solved using the appropriate technique, typically either linear or non-linear programming depending on the nature of the problem.

An initial solution that upholds the integer constraints for the decision variables is then generated and this becomes the incumbent *optimal* solution. In the event that the solution to the continuous mathematical program is integral then this value supersedes the incumbent solution and the algorithm is finished.

Assuming that the optimal solution to the continuous mathematical programme is not integral, additional steps are required. The branching process creates two new mathematical programmes with constraints based on the values returned by the original continuous mathematical programme. For example, if the returned value of a given parameter is 5.5 and this is not permitted as the decision space may only contain integer values then the same mathematical programme as before is now generated twice; once with a constraint that the parameter must be less than or equal to 5; and once with a constraint that it must be greater than or equal to 6. The solutions returned by these new mathematical programmes are then assessed against the incumbent optimal solution and this may be replaced accordingly.

Any solutions that produce a result which is superior to the current optimal solution are subjected to the branching approach once more, assuming that they have more than one integer variable.

Any solutions that produce a result which is inferior to the current optimal solution are immediately *bounded*, thus no future branching occurs from these solutions. The bounding process prevents wasted computational resource as solutions which are known to be suboptimal are not evaluated.

This process is performed iteratively until all solutions have either been assessed or have been ruled out by virtue of being derived from an existing suboptimal solution [67, 70].

If the problem is mixed-integer non-linear then a useful tool is Bender's decomposition which is a form of *divide-and-conquer* method. Divide and Conquer algorithms involve iteratively dividing the optimisation into sub-problems such that the sub-problems may be solved within an acceptable timeframe [72]. This involves the division of the problem into *master* and *slave* components which are solved independently of one-another. Typically, the master problem is integer or mixed integer, while the slave problem is non-linear. For each solution generated for the master problem, a corresponding optimal solution is found for the slave problem. The whole solution may then be scored against the objective function. This process is repeated iteratively until an optimal solution is reached [67].

2.4.4 Meta-Heuristic Techniques

Meta-heuristic solution finding algorithms are a relatively recent addition to the suite of tools available to those attempting to solve highly complex combinatorial optimisation problems.

The meta-heuristic approach does not guarantee an optimal solution to a given problem, instead offering a strong solution within a reasonable timeframe. In most cases, a meta-heuristic technique will either reach an optimal, or near optimal solution; however this cannot be guaranteed due to not assessing every possible combination of variables.

The umbrella term meta-heuristic represents a number of distinct techniques that share some common characteristics and ultimately attempt to achieve the same outcome. A number of techniques are based on natural phenomena including: genetics; the behaviours of ants, bees and birds in groups; bacterial foraging and immune systems [73]. Some of these techniques are introduced and briefly considered within the following sections.

All meta-heuristic methods seek to balance the exploration of the search space and the intensity with which areas with strong solutions are searched. As a general rule, single-solution meta-heuristic algorithms are concerned mainly with a very thorough search of a smaller search space while population based algorithms offer a shallower search of a broader search space [73], both of these classifications are described in the following sections.

Single-solution meta-heuristics, also termed trajectory methods, start with a single solution and move away from this solution, following a trajectory through the search space. As they traverse the search space, the current optimal solution is updated and often this forms the basis of the next change in positional space.

Examples of single solution based meta-heuristics include simulated annealing; Tabu search; the Greedy Randomised Adaptive Search Procedure (GRASP) method; variable neighbourhood search; guided local search and iterated local search [73].

While these methods have a number of differences, relative to one another; they all offer a quick solution to a given problem, which is not necessarily optimal. A number of single solution meta-heuristic methods, including simulated annealing, may overwrite the current optimal solution with a weaker solution, which simultaneously allows the algorithm scope to explore the search space but also means that a weaker solution may be returned when the algorithm has previously found a stronger solution [73].

A number of network planning papers have been published that utilise single solution meta-heuristics techniques. Mori & Iimura [74] reviewed the performance of simulated annealing, Tabu search and their proposed parallel Tabu search algorithm for distribution network expansion planning when large amounts of distributed generation is present. Each technique may be successfully employed for the purpose of network expansion, with the parallel Tabu search delivering a stronger solution than the Tabu search which in turn delivered a stronger solution, with less computational effort, than the simulated annealing method [74].

Work on transmission network planning using the Tabu search style of single solution based meta-heuristics was published by da Silva et al in 2001. They applied the Tabu search method to identify the optimal expansion of the Brazilian transmission network using two data cases and concluded that the Tabu search offered strong performance for network planning with minimum computational expense [75].

Population based meta-heuristics utilise a set of solutions in preference to the single solution based methods discussed in the previous section; and this approach is generally better suited to problems with a large search space. The two broadest groupings of population based approaches are evolutionary computing, based on Darwinian evolution of species, and swarm intelligence, based on a combination of individual cognitive abilities and social interactions between individuals acting a group [75]. For both categories, there are a number of algorithms that are slightly different to one another. Summaries of the principles of operation for an evolutionary computing method (Genetic Algorithm) and swarm intelligence (Particle Swarm Optimisation) are now considered.

Evolutionary Computing / Genetic Algorithm

Genetic or Evolutionary Algorithms (GA; EA) as the name suggests are based upon the evolution of species to suit their environment. The terms GA and EA are largely interchangeable, although some terminology differs between the two variants. The key components and concepts of a GA are now introduced.

A chromosome or individual is simply a candidate solution to a given optimisation problem. Often a solution is configured as a binary string where each element of the string may only exist as either a 0 or a 1.

A generation is a collection of individual solutions or chromosomes. The total number of solutions forming a generation is defined by the algorithm parameters. From one generation to the next, a number of individual solutions are taken forward in addition to new solutions generated to complete the generation.

Selection invokes the principle of *survival of the fittest*. Each solution in the generation is assigned a *fitness* score based on its performance and the strongest candidates are used as a basis for the next generation. The number of candidate solutions that are considered strong enough to form the next generation depends on the specified algorithmic parameters.

Mutation increases diversity within a generation by taking a number of candidate solutions from that generation and modifying some elements of the solution. Mutation introduces a stochastic change in the string representing a solution between one generation and the next. Once a candidate solution has been selected for mutation, the point at which the solution is to be mutated is selected at random and this element is altered.

Crossover or recombination involves taking elements of two or more candidate solutions from one generation to form a candidate solution in the next generation. This process increases diversity, ensuring that the GA performs a widespread and thorough search of the search space.

A simplified flowchart depicting the progression of a genetic algorithm is presented as Figure 22.

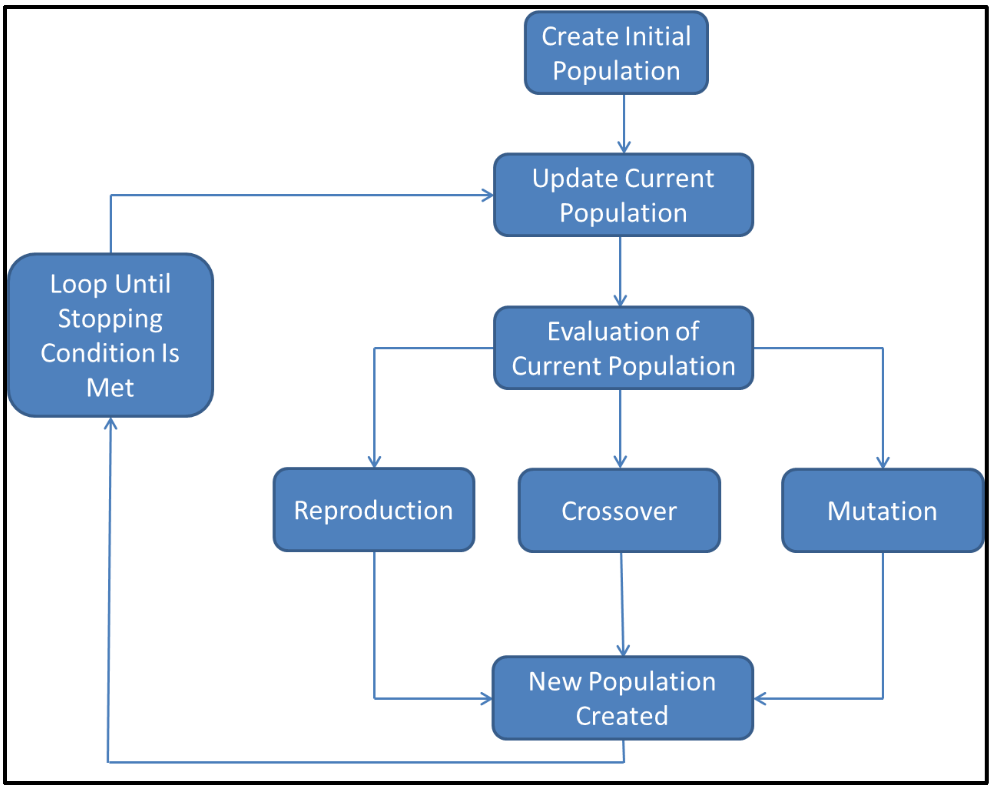


Figure 22 - Genetic Algorithm Flowchart

An initial population of solutions is randomly created within the search space and evaluated. The strongest solutions are retained and reproduced in the following generation. Other chromosomes undergo crossover and / or mutation to produce solutions for the following generation. Once the new population is complete, this becomes the current population and the evaluation process begins once more. The algorithm loops until a stopping condition is met, at which an optimal or near-optimal solution is returned.

The behaviour of the GA depends on a number of algorithmic parameters such as the crossover rate and the mutation rate. Typically, crossover rates are between 60 and 100 % of candidate solutions and mutation rates apply to around 1 % of solutions [31, 73].

Swarm Intelligence / Particle Swarm Optimisation

Swarm Intelligence (SI) optimisation techniques can be applied to complex problems in much the same way as evolutionary computing techniques. Almost all SI methods are predicated upon the collective behaviour of swarming bodies, such as birds or ants, as they collectively and individually move towards their goal.

This principle is demonstrated with an explanation of the operation of the Particle Swarm Optimisation (PSO) algorithm. A number of other, similar, algorithms such as the Ant Colony Optimisation could equally be used, although some terminology will differ slightly between variants.

For this technique, each particle represents a candidate solution searching for the global optimal solution. If there are N variables to be considered then there are also N dimensions in the search space and the position of each particle is represented by an N dimensional vector, which may be subject to a number of value constraints.

Each particle remembers its positional coordinates that returned the most profitable outcome: its *local optimal solution*.

During each iteration of the PSO algorithm, each particle travels around the search space at a given velocity which is usually generated pseudo-randomly. The velocity vector has the same number of dimensions as the position vector described previously.

To perform a particle swarm optimisation analysis, both global and local optimal solutions must be considered. Each particle remembers its own local optimum: the position that the particle has been at in the coordinate space that returned the best solution and each future movement takes account of this location.

The globally optimal solution is the point in coordinate space that *any* particle has been, during any iteration, which returned the strongest solution. All future movements of all particles take the global optimal solution into account in addition to their local optimal solution.

The globally optimal solution will be identical to the locally optimal solution for one particle.

The PSO algorithm can be outlined as shown in Figure 23. Particles are initialised and checked to be within the specified constraints and if necessary parameters are adjusted such that they fall within the set boundaries.

The performance of each candidate solution is then considered and the locally and globally optimal solutions are identified. At this point, the iterative process of refining the solutions begins.

The velocity and the current position of the particles are used to create a new position of each particle and the performance at this new location in coordinate space is considered and local and optimal solutions updated if required.

The iterative process ends once an ending condition is reached. Ending conditions might include: total number of iterations or number of iterations without a (meaningful) change in optimal solution.

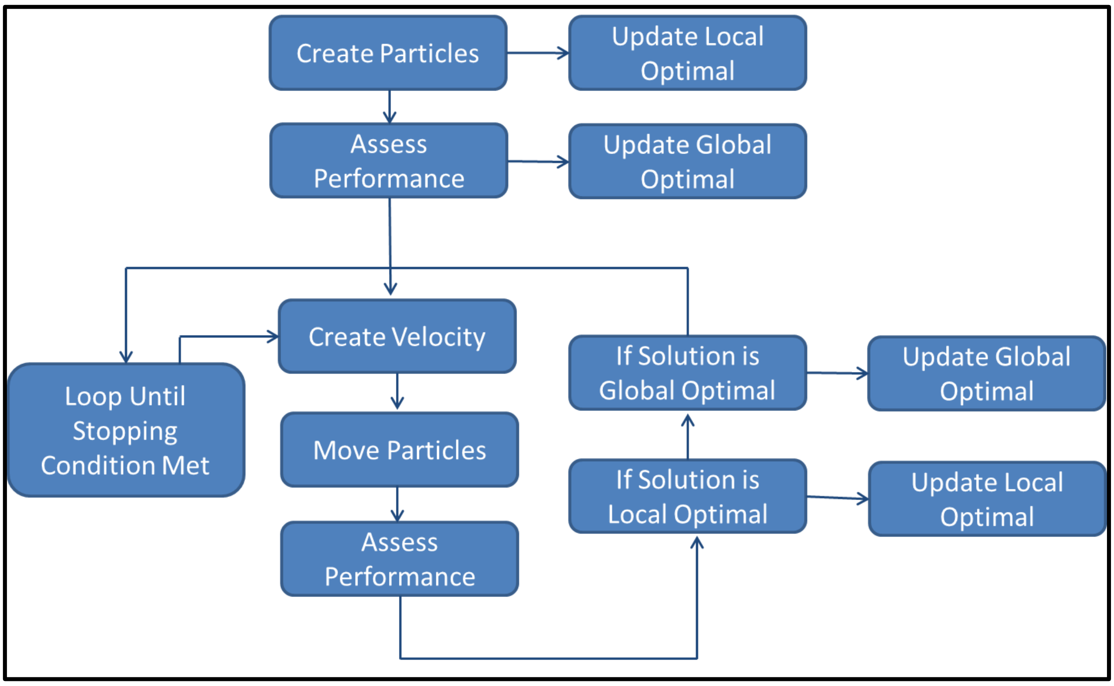


Figure 23 - Particle Swarm Optimisation Flowchart

2.4.5 Network Analysis

Network analysis can be used to seek a minimisation or a maximisation of the value associated with the traversal of a space in which a number of defined points may be visited. A *network* may be defined as being a set of *nodes* that are connected by *branches*. A branch is considered to be *oriented* if it has an associated direction, for example: a branch defined as AB permits traversal from A to B, but does not permit the reverse. A branch also has nonnegative cost associated with it. A *path* is a sequence of connected branches such that no node is visited more than once. A network may be considered to be *connected* if for each pair of nodes in the network there is at least one feasible path connecting the pair. If a path is unique for each pair of nodes, this network is called a *tree*: a connected network having *n* nodes and *n+1* branches [70].

A number of network analysis algorithms exist, following largely the same operating principles as one another. One such method is Dijkstra's Minimum Spanning Tree Algorithm (Dijkstra's Algorithm) which is now discussed [76]. While this method could be modified to seek a cost maximisation, for the purpose of this work it is considered in respect of a cost minimisation problem.

In order to apply Dijkstra's Algorithm to a network analysis problem, one node must be defined as the *initial node*. Equally, one node must be mandated as the *final node*. These nodes form the start and end points of the network. An $n \times n$ matrix, where n is the number of nodes under consideration is created, representing the cost associated with the traversal between any pair of nodes. In the first instance, the cost of traversal between the initial node and itself is set to zero, and the cost of traversal between the initial node and all other nodes is set to infinity.

Each possible branch that may potentially emanate from the initial node is then considered and the cost of each branch is calculated. If it is feasible to connect the initial node with any other node, the cost of traversal, currently set to infinity, is updated accordingly. The initial node is then added to the *visited set* and as such it may no longer be revisited.

For each subsequent node, *current node* status is applied. All of the feasible connections from the current node to other nodes that reside within the *unvisited set* are considered. The cost associated with spanning the distance from the initial node to each node in the unvisited set, via the current node is calculated, compared with the current optimal cost for that particular traversal, and updated as necessary. When all feasible connections have been considered, the node is transferred to the visited set such that it may no longer be revisited.

Once all nodes have been considered, or there is found to be no suitable solution to the problem, the algorithm is complete [76].

2.5 Power Flow Studies

Power flow (PF) simulation is a powerful tool for analysing the electrical performance of a power network.

The steady state performance of an electrical network depends upon the generators operating within specified limitations for both real and reactive power, and in doing so being able to supply the load and any losses within the network; the voltage magnitude at each bus being within statutory limits from the rated value; and lines, cables and plant within the network not being overloaded [77].

A PF study performs a steady-state analysis of a given electrical network, based on a single line diagram. It does not take account of transient behaviour, such as the starting up or shutting down of a given generator; or the sudden failure of a cable or transformer. Equally, it does not consider the cost of generation: for this an optimal power flow (OPF) simulation must be performed that seeks to minimise the total cost of generation while meeting all the constraints described previously.

In order to perform a PF analysis, the network must be configured using node-branch representation. Each bus, or node, in the network must be assigned an identifier (*1, 2, ..., N*) and a *bus type*. There are three types of buses, namely: the swing or reference bus; generator buses and load buses. The connections between each bus are *branches* [77].

Only one bus in the network may be defined as being the *swing bus*. The swing bus is considered to be entirely flexible in terms of real and reactive power output and values are calculated for these parameters that *balance* the system. The voltage magnitude and angle are defined as 1.0 pu and 0 degrees respectively, as shown in Table 5 [77].

There may be many generation buses in a network. As the name suggests, a generation bus typically hosts one or more generators, although it may equally host reactive power compensation equipment. At a generation bus, the real power output and the voltage magnitude are controlled, while the voltage angle and reactive power output are calculated.

As long as the generation bus is a net exporter of power, the value of P_N will be positive. The value of Q_N may be positive if the generation bus is exporting VArs or negative if the bus is consuming VARs. A load bus, as the name suggests hosts a load where the real and reactive power demands are fixed. Consequently, voltage magnitude and angle are calculated, as shown in Table 5. As long as the load bus is a net importer of power, the value of P_{LOAD} will be negative. The value of Q_{LOAD} may be positive if the load bus is exporting VArs or negative if the bus is consuming VArs.

Bus Type	V_N (pu)	δ_N (°)	P_N (W)	Q_N (VAr)
Swing	1.0 pu	0	Calculated	Calculated
Generator	Controlled	Calculated	Defined	Calculated
Load	Calculated	Calculated	Defined	Defined

Table 5 - Bus Types and Variables for Power Flow Analysis [77]

Every bus, irrespective of type, is modelled by four characteristics: the voltage magnitude, V_N; the voltage angle, δ_N; the real power injected, P_N; and the reactive power injected, Q_N. Each bus type has two defined attributes and two calculated attributes as shown in Table 5 [77].

Ideally, a transmission line or cable would deliver as much power to a load bus as it accepts from a generation bus. Equally, a transformer would perform with 100 % efficiency. In reality, a transmission line or cable will have some degrees of resistance, reactance and susceptance. Typically, for a PF study cables and transformers are modelled as nominal or equivalent π circuit, which is depicted as Figure 24 [77]. This model of the transmission line lumps half of the total shunt capacitance at the start point of the line and half at the end point of the line. Equally the series impedance of the line, *Z*, is modelled as a lumped parameter. In reality both the series impedance and the shunt capacitance would be distributed along the length of the line, however, for this representation makes modelling much simpler without a great loss of fidelity.

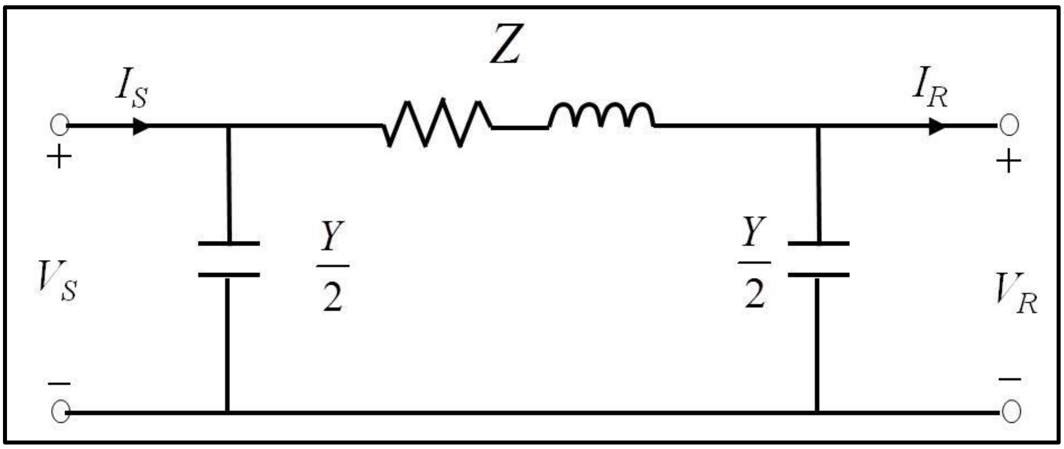


Figure 24 - Nominal / Equivalent π Transmission Line Model [77]

Mathematically, the power flow may be described as follows. The bus admittance matrix, Y_{BUS}, is a $K \times N$ matrix consisting of all line and transformer admittance data. The diagonal elements of the Y_{BUS} matrix, Y_{KK} represent the sum of admittances connected to bus K. All non-diagonal elements, Y_{KN} where $K \neq N$, contain the negative of the sum of admittances between bus K and bus N.

The Y_{BUS} matrix may now be utilised to relate the currents and voltages at each bus as described by (13).

$$\boldsymbol{I} = \boldsymbol{Y}_{BUS}\boldsymbol{V} \tag{13}$$

From (13); the current injection at bus K may be defined in terms of admittance and the sending bus voltage as shown in (14). The complex power delivered to bus K may now be defined in terms of the receiving bus voltage and current as presented in (15), which may be combined with (14) to give (16).

$$I_K = \sum_{n=1}^{N} Y_{Kn} V_n \tag{14}$$

$$S_K = P_K + jQ_K = V_K I_K \tag{15}$$

$$S_K = P_K + jQ_K = V_K \left[\sum_{n=1}^{N} Y_{Kn} V_n \right] \tag{16}$$

Now by defining Y_{Kn} and V_n as complex variables, described by (17) and (18) respectively, the complex power injected as bus K may now be expressed as shown by (19).

$$V_n = V_n e^{j\delta} \tag{17}$$

$$Y_{Kn} = Y_{Kn} e^{j\theta kn} \tag{18}$$

$$S_K = P_K + jQ_K = V_K \sum_{n=1}^{N} Y_{Kn} V_n \, e^{j(\delta_K - \delta_n - \theta_{Kn})}$$

(19)

Finally, separating (19) into real and imaginary components returns equations in terms of real and reactive power individually as shown by (20) and (21) respectively.

$$P_K = V_K \sum_{n=1}^{N} Y_{Kn} V_n \cos(\delta_K - \delta_n - \theta_{Kn})$$

(20)

$$Q_K = V_K \sum_{n=1}^{N} Y_{Kn} V_n \sin(\delta_K - \delta_n - \theta_{Kn})$$

(21)

The PF may be simplified to form a DC power flow (DC PF) study; in which case reactive power is ignored, bus voltage are always set to 1.0 pu and undesirable physical characteristics of the line are disregarded [77]. This has the advantage of being computationally less demanding than an AC PF; however this is achieved at significant loss of fidelity.

Typically, an iterative process is used to solve power flow equations with a common approach being the Newton-Raphson method which allows for non-linear AC power flow equations to be solved. This method is based on a Taylor series expansion of *f(x)* around starting, x_0 and may be expressed generically as (22), where: ***x(i+1)*** is the *next* solution; ***x(i)*** is the *current* solution or initial guess when ***i*** is zero***;*** ***y*** is the value of the non-linear equation ***f(x)***; and $\boldsymbol{J^{1}}$ ***(i)*** is the inverse of the Jacobian matrix for the current solution, as defined by (23) [77].

$$\boldsymbol{x}(i+1) = \boldsymbol{x}(i) + \mathbf{J}^{-1}(i)\{\boldsymbol{y} - f[\boldsymbol{x}(i)]\} \quad (22)$$

$$\mathbf{J}(i) = \left.\frac{d\boldsymbol{f}}{dx}\right|_{\boldsymbol{x}=\boldsymbol{x}(i)} = \begin{vmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_{n1}}{\partial x_n} \end{vmatrix} \quad (23)$$

The Newton-Raphson method for solving AC power flow equations is utilised in a number of software packages, such as Matpower [78, 79] to provide a quick and accurate approximation to a given power flow study.

2.6 Chapter Summary

Within this chapter a number of key enabling topics that facilitate the optimisation of subsea electrical networks are presented.

Probabilistic methods of quantifying the energy resource that is likely to be extractable from an offshore wind site, such as the Weibull Distribution and the Rayleigh Distribution, are extremely useful. The use of such probability density functions allows for an accurate representation of the likely power output from an offshore wind development to be developed, without the requirement for a highly onerous time based model of resource.

The enabling technologies that facilitate renewable generation in the marine domain tend to be independent of generation type: wind, wave and tidal typically utilise low-voltage electrical machines, either PMSGs or DFIGS, outputting AC at 50 or 60 Hz. Offshore wind uses the largest machines, with capacities of 7 MW being feasible [50], while tidal stream and wave fed machines are only currently available around the 1 MW range [53, 56].

Irrespective of how generation is achieved in the marine domain, an electrical network is required to facilitate the extraction of energy and supply this to a land based grid. This is typically achieved using a graduated grid with a lower-voltage collector network linking each generator to an offshore substation before a more efficient transmission link conveys this power back to shore. This approach tends to provide a strong compromise solution between cost and electrical efficiency, and both stages of the network may either be implemented in AC or DC.

AC subsea networks tend to be cheaper than DC counterparts; however, capacitive charging effects limit its usefulness when conveying large amounts of power over long distances and expensive power factor correction technology is required. Capacitive charging is a function of frequency, thus the correct selection of frequency at both collector and transmission stages of a subsea electrical network is imperative to the production of an optimal design solution. DC networks are generally more expensive to implement but do not require power factor correction and may prove cheaper on a through-life basis as the losses are lower than for an AC system.

It is possible to configure electrical networks into a number of different topologies, with radial, ring and star networks being examples of typical offshore networks. As will become evident later on in this thesis, topology selection has a significant bearing on the optimality of an offshore network.

The specific nature of a given optimisation problem governs the efficacy of a specific technique to solving that problem. An optimisation problem may be continuous, or discrete; and may be linear, or non-linear; convex, or non-convex. Each of these characteristics will drive the selection of suitable methods to solving that problem. Calculus and mathematical programming methods are powerful tools for solving problems that may be suitably defined; while modern computational optimisation techniques such as evolutionary computing and swarm intelligence can produce no guarantee of optimality but do offer additional flexibility within the problem definition.

Network analysis methods are useful tools for the analysis of specific optimisation problems, typically relating to distance and / or cost minimisation. This approach may be applied to subsea electrical network cable length minimisation which may appear desirable, as will be discussed later in this thesis.

Power flow studies allow for the power flow through each branch of an electrical network to be quantified; thus the losses incurred within a network may be calculated. A power flow study may be AC, taking account of reactive power flows; or may be of the simplified DC form which fails to account for reactive power flows. Typically, power flows are solved computationally using iterative methods, such as Newton-Raphson, which provides a quick and accurate solution.

Chapter 3: Scoping & Optimising the Subsea Electrical Network Design Problem

This chapter presents a review of relevant academic literature pertaining to the analysis and optimisation of electrical networks in the offshore domain.

Research related to the design, simulation, analysis and optimisation of offshore collector networks has largely centred on networks for offshore wind parks, where a large number of individual machines are deployed within a small geographical area.

Given the inherent characteristic similarities between the energy output generated by each of offshore wind, wave and marine energies, there is no precluding reason that conclusions drawn from research specifically relating to offshore wind may not be applied to each of the afore mentioned offshore generation types, and combinations thereof.

Figure 25 is a representation of generalised outcomes experienced with different combinations of model fidelity and quality of solution method [45, 46]. While the requirement for sufficient model fidelity paired with a high-quality solution method, thus giving a *real solution to a real problem,* is neatly articulated within Figure 25; it does not take into account the increase in optimisation effort and the potential for intractability that exists if one, or both, of these characteristics is over deployed.

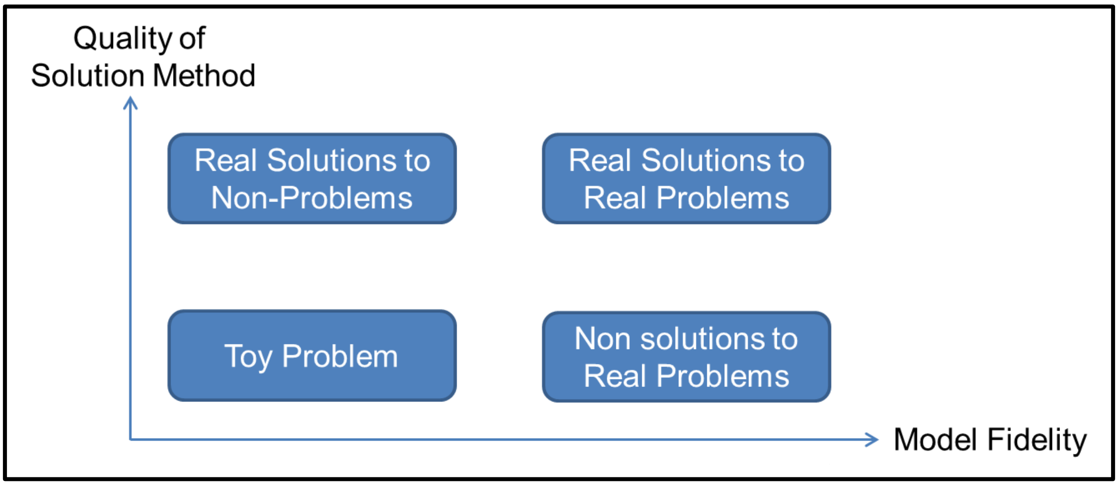


Figure 25 - Model Fidelity, Quality of Solution Method and Value of Results [45, 46]

In the following sections, a number of characteristics of subsea electrical networks are considered, along with examples of their usage in the relevant academic literature. With reference to Figure 25, this is used to analyse available options that will be used to shape the fidelity of the optimisation process utilised for this work, such that a *real problem* is established, without pushing the optimisation problem into intractability.

Additionally, as shown in Figure 25, to reach the desired upper-right quadrant there needs to be a *real solution* to the *real problem*. Within the subsequent sections, a number of solution methods and optimisation techniques which may be applied are considered; and their implementation in contemporary work discussed.

3.1 Subsea Network Model Fidelity

This section reviews the work of others who have published research relating to the optimisation of subsea electrical networks; thus the scope of what can and cannot; and should and should not, be included within a mathematical or computational representation of a subsea electrical network is considered.

3.1.1 Optimisation Objective

Fundamental to the act of optimisation is a metric for which a maximum or minimum is desired. For the design of offshore electrical networks, there are a number of possible optimisation objectives that may be selected, some of which are now presented, in addition to examples of their deployment.

Lowest electrical losses could conceivably be used for the design of offshore electrical networks; however, it is likely to be of limited value as it does not account for cost. One possible advantage of using lowest electrical losses as the optimisation objective is that the optimal solution is, generally, more readily discernible than it is for a solution that accounts for cost, thus it may be a useful method for validation of the optimisation process. Lowest losses may be considered as a constraint, rather than as the primary optimisation objectives, as evident in Nandigam and Dhali [44] where a cost minimisation is performed with maximum electrical losses and minimum reliability constraints applied.

Lowest installation cost based optimisation seeks to minimise the capital expenditure associated with an offshore development; namely the purchase and installation of cables; offshore substations; transformers or convertors; and the provision of reactive power compensation. Given that cost is likely to be a deciding factor in the design of an offshore electrical network, this approach is widely utilised. There are many examples of published work in this field, using lowest installation cost as the optimisation objective [18, 26-28, 32].

The primary disadvantage of this approach is that it fails to account for lifecycle costs thus network solutions with low capital costs and high operational costs may be returned as optimal, while in actuality such solutions may be highly cost-inefficient over the lifetime of the network.

Lowest through-life cost considers both the investment cost associated with the deployment of an offshore electrical network and an approximation of its likely operational costs, typically through the cost of electrical losses and the cost of no-service conditions. This method increases the fidelity of the optimisation process, with respect to cost minimisation, achieved at the expense of increased computational effort required. There are many examples of published work using lifecycle cost minimisation as the design objective [21-22, 30-29, 40, 42]. Zhao et al [31] consider the through-life electrical costs of the network for their work but fail to account for civil engineering costs such as the installation of cables and plant.

3.1.2 High-Level Network Configuration

An offshore electrical network for the purposes of extracting renewable energy, is typically comprised of a one or more medium voltage (typically 25-66 kV) collector networks that terminate at one or more offshore substations where a transformer or HVDC convertor is utilised in preparation for transmission at higher voltage (typically 150-400 kV). If lowest electrical losses is the design objective then this solution is inferior to a solution utilising transmission voltage cables at all points. Configuring a network in this way is, therefore, largely predicated on cost; the collector network uses cheaper cabling which incurs greater losses than the transmission cabling which is considerably more expensive. There are many studies that define the network optimisation problem using this philosophy [18, 24-30, 42].

An alternative arrangement is to bypass the requirement for an offshore substation and have the collector and transmission stages utilising the same voltage. The advantages of such a scheme include: the removal of the offshore substation and transformer, thus saving money; and the reduction of space and insulation requirement associated with a high voltage system [22]. This saving must be offset against the differential cost of cabling between this transmission scheme and a higher voltage equivalent. Furthermore, losses within a conductor are in proportion to the square of the current flowing, thus a medium voltage transmission stage will incur higher electrical losses than a higher voltage equivalent system conveying the same amount of power.

This configuration is discussed by Zubiaga et al [22], where it is noted that this approach is only useful for small capacity sites (up to ~ 60 MW) located within a short distance (less than 10 km) of the shore; by Zhao et al [31]; and by Banzo and Ramos [40] who assessed this option along with a number of higher-voltage transmission configurations within their work.

3.1.3 Design Variable Selection

The performance of a subsea electrical network, irrespective of which optimisation objective is chosen, may be thought of as a function of the following variables: number, position and capacity of generators; number and position of offshore substations; the topologies and number of concurrent cables that emanate from each offshore substation; the transmission topology; the choice of AC or DC for collector and transmission networks; the physical characteristics of collector and transmission cables; and the choice of voltage and frequency of both collector and transmission networks.

Given the number of possible design variables, there is a significant chance of the problem becoming intractable or too computationally expensive to optimise. In order to expedite the optimisation process, simplifying assumptions that *fix* one or more parameters may be made.

A number of recent studies on the computational optimisation of subsea electrical networks for offshore wind farms treat the number, position and capacity of the turbines as fixed parameters, to be determined by site evaluation and aerodynamic optimisation [18-20, 24-30]. Zhao et al [31] treat the number and position of turbines as fixed, but the capacity as a discrete variable. Given that the spacing requirements between turbines is a function of turbine diameter, which is in itself related to generator capacity, it is questionable as to whether the optimal site layout would remain the same for larger capacity machines, as it would be for smaller capacity machines.

Studies relating to the optimisation of offshore wind networks may focus solely on the collector network; solely on the transmission network; or may consider both aspects of network design. The scope of study may be limited either by eliminating the transmission system when the collector aspect of a network is under consideration; by eliminating the collector system when the transmission system is being analysed; or by fixing one or more components of the network thus restricting the search space while maintaining a realistic representation of the overall network.

When the collector portion of a network is the primary subject of study, a number of authors choose to nominate the offshore substation(s) as the *end point,* thus they fail to consider any costs or electrical behaviour associated with the transmission link [18-20]. Dutta and Overbye's work [19-20] is principally concerned with the deployment of underground collector networks for land based wind farms, which possess inherently similar technical requirements as those for offshore wind farms; albeit with different economic constraints. Within [20], they consider the use of *intermediate splices* whereby cables can be jointed between conventional nodes (wind turbines or sub-station), allowing for the creation of complex meshed electrical networks. The technical and economic feasibility of utilising such a technique in the sub-sea domain is unclear.

Equally, a number of authors focus on the collector network design while assuming one or more characteristics of the transmission system are fixed. For example, one or more substations at fixed locations may be assumed [23-25]; the topology of the transmission system might be pre-selected (typically in a radial configuration) [23, 32]; or the AC/DC status of the transmission link may be defined in advance as AC [26-27, 32] or DC [23].

It is also possible that a collector network design is being optimised subject to topological constraints; for example, the collector topology may be restricted to radial only [18, 27-28, 30, 42]. At the time of writing, no published work restricting the collector network to a non-radial topology has been found.

Another approach is to treat an agglomeration of offshore generators, typically a wind park, as a single network node with representative real and reactive power characteristics, such that the transmission system may be optimised around this. One example of this is Ergun et al [21] who have looked at the optimisation of transmission networks for multiple clusters of wind-turbines, which are modelled as single power sources at an offshore substation. Zubiaga et al [22] include the collector system within their work but do not take into account the electrical losses within the collector network, and fail to account for reactive power compensation also.

Voltage and frequency are generally set as fixed parameters within the optimisation [18, 24-29, 40]. These could be included as variables which would elevate the complexity of the problem; increase the number of solution space dimensions; and potentially push the problem towards intractability. Zubiaga et al [22] mention that voltage is an important design variable but perform their study using a fixed voltage. Zhao et al [31] include voltage as a discrete design variable (3 possible voltages for the collector network; 4 for AC transmission and 2 for DC transmission) within their optimisation process; but use a fixed, and non-stated, frequency. This approach has been further employed by Lingling and Xiaoming where there is binary selection of voltages for both collector and transmission systems [41].

3.1.4 Discrete and Continuous Component Capacity Sizing

There are two distinct ways in which cable and plant sizing may be included within an optimisation model considering the design of a subsea electrical network. The first is through the use of continuous sizing where an object may be of any capacity without restriction. The sizing of components depends on the level of real and reactive power flow throughput, both of which may be calculated using a power flow study.

The disadvantage to this approach is that the physical characteristics of the cable (resistance, inductance and capacitance; from which reactance and susceptance are derived) are functions of cable cross-section, and therefore vary with capacity. However, using an average value of these characteristics across the range of cable cross-sections produced only incurs a minimal reduction in fidelity, as this small change in cable characteristics makes minimal impact on the results of the power flow study. Computational optimisation studies that use continuous component sizing include Nandigam and Dhali [44].

Equally, discrete capacity components may be selected and then a power flow study may be run to assess their performance. This approach allows for more accurate cable characteristics to be employed during the power flow. The disadvantage to this method is the potential for components to be over-specified owing to the discrete nature of selection, thus incurring additional capital expenditure. This may be offset, in the right circumstances, as the use of a higher capacity cable may be more cost effective through-life than a lower capacity cable; particularly if the lower capacity cable is operating near its capacity limits [80]. There are many published optimisation studies that utilise discrete component sizing [25-28, 31, 42, 80].

Zhao et al [31] use only three discrete cable capacities for their optimisation of the collector network, thus there is significant potential for a cable being deployed that is considerably over-specified for its requirements. They do not mention if they use a similarly restrictive selection of discrete transmission cables.

Nedić et al [25] use discrete cable selection within their work before going on to discuss that submarine cables are often *"custom manufactured based on the needs of each project"* suggesting that there is little impediment to continuous cable sizing.

3.1.5 Discrete and Continuous Coordinate Systems

When the optimisation of an offshore network is concerned with the placement of offshore substations within sea-space, these objects may either be free to take up any position within the extremities of the deployment space [26] or there may be positional restrictions that limit the deployment to discrete points within this space [23]. This only applies to optimisation studies which consider the positional placement of offshore substations.

Discretising the deployable space has the effect of massively reducing the number of possible solutions that exist to a given network optimisation problem thus easing the computational difficulty required to reach a solution. This is potentially achieved at the expense of solution granularity, as an offshore substation may be deployed hundreds of metres from its optimal position, due to this artificial limitation on the search space.

The choice of implementing discrete or continuous positioning of objects impacts on the techniques that may be applied to address the optimisation problem, as discussed previously within this thesis.

3.1.6 Investment Costs

A large number of papers considering the costs of network components within their optimisation models reference a 2003 study by Lundberg [81] which offers cost guidelines for a number of key components [21, 24, 27-28, 31-32, 44, 61]. There appears to be a dearth of credible alternative cost functions to those presented by Lundberg; however this work, and those that derive from it, do offer approximations for almost all of the required components to construct an offshore electrical network, which are now introduced.

The cost functions that have been utilised for this work are described mathematically in the following chapter of this thesis.

The cost of AC submarine cabling is dependent on the operating voltage and the cross-sectional area of the cable, which governs the electrical throughput of the cable. The approximations offered by Lundberg are based on costs calculated at a limited number of standard voltages [81], from which voltage-dependent cost approximations may be approximated. All other publications that were found relating to this topic either use Lundberg's approximations directly, or use approximations that build upon Lundberg's work.

Similarly, the cost approximations for DC cabling that were found in published work all stem from Lundberg's work; which is again based on a small number of typical voltage levels [81], which may be interpolated and extrapolated to give a cost approximation for all voltage levels.

Transformers are deployed offshore to step-up the voltage from the medium voltage (25 – 66 kV) collector network to the high voltage (150 – 400 kV) transmission network, when an AC collector network with AC transmission system is employed. Equally, they may be employed to interface the offshore transmission network, assuming that it is an AC transmission network, with the onshore grid.

Lundberg offers an approximation for the cost of transformers, restricted in capacity to 6.3 to 150 MVA [81]. Dicorato et al present two modified version of the cost function offered by Lundberg, allowing the cost of transformers up to 150 MW and above 150 MW to be modelled [61].

AC-HVDC converters are bi-directional conversion devices allowing HVDC transmission to be interfaced with either an AC collector network or the on-shore AC electrical network. Lundberg's work suggests a value of 1 Swedish Krone per VA of rated capacity, equating to approximately 110 k€ / MVA [81]; with no rival cost approximations found.

Cost functions for AC switchgear are presented by Dicorato et al [61] that stem from the functions published by Lundberg [81]. One area in which Lundberg based cost functions are lacking is for DC switchgear. Stamatiou et al [82] in a 2011 peer-reviewed study of the economics of DC collector networks approximate the cost of DC switchgear as being double that for an AC switchgear of the same voltage. This approximation is therefore utilised to model the cost of DC switchgear.

Dicorato et al [61] present values for the various aspects of cable installation that are drawn from two sources including Lundberg's work [61, 81]. A general cost function that describes the cost of installation of cables is difficult to produce, given the inherently unique nature of different areas of the sea bed; the specifics of the cable being installed; and the proximity to other cables and other factors. Dicorato et al present an average value of 365 k€ / km as a *one-size-fits-all* approximation of cable installation cost [61]. This value is higher, but in the same order of magnitude, as the value of 256 k€ / km presented by Zubiaga et al [22]; and the value of 145 k€ / km (converted from US dollars) offered by Dahmani et al [24].

Additionally, Dicorato et al offer an approximation for the cost of an offshore substation where a transformer or HVDC converter is to be deployed [61], building upon the approximation offered by Lundberg [81]. As was the case for cable installation, this approximation may differ to empirical evidence as offshore substations may be noticeably different to one another as some include helipads, living quarters and back-up power systems including fuel reserves. Unlike the cost approximations offered for subsea cabling, the cost function for offshore substations includes installation costs.

While there is more than one technique through which reactive power compensation may be achieved, Static VAR Compensation (SVC) offers the greatest usability from a network balancing perspective. Dicorato et al present a cost approximation of 77 k€ / MVAr for SVC reactive power compensation and this value is used to approximate both net inductive and net capacitive reactive power requirements [61].

3.1.7 Cable Deployment Spatial Constraints

The costs associated with the manufacture and purchase of subsea electrical cables have been discussed already within this thesis. This aspect of the laying of cables is well defined. The installation costs are far less well defined with *one-size-fits-all* cost values being presented [22, 24, 61] which fail to account for the distance from shore, the water depth and the characteristics of the cables; all of which will induce cost variability. These cost generalisations also lack consistency, with the most expensive figure [61] amounting to 250 % of the least expensive [24].

Ideally, for ease of access relating to deployment and repair, cables would not be laid in close proximity to one another. In reality, this becomes difficult to achieve for any network of moderate, or greater, complexity. Deploying multiple cables in close proximity, particularly where they cross one-another will incur additional cost, relative to a neatly spaced solution.

Some studies assume that there are no additional cost considerations to be accounted for with the deployment of crossing cables [41]. Dahmani et al [24, 27] limit the design of collector networks within their optimisation such that a turbine may only be connected to one of its immediately adjacent neighbours in order to eliminate the crossing of cables in the intraturbine array. They do, however, allow for the cables that link offshore substations with turbines to cross cables that link turbines with turbines, thus the crossing of cables is ultimately permitted.

Within the case study presented by Dahmani et al [27] the optimal network returned has collector and transmission cables that cross one-another, however, they point towards modifications to the returned solution that could be deployed to eliminate cable crossing.

3.1.8 Model Fidelity Summary & Initial Problem Specification

Within the previous sections, a range of features that could be included within a subsea electrical network optimisation model was presented. From this review, an initial requirements specification of desirable optimisation characteristics was produced.

Of the available optimisation objectives; it would appear to be most beneficial to consider minimisation of through-life cost, particularly as this may be manipulated to utilise installation cost minimisation; and these are the two dominant objectives that have been used for a number of contemporary studies [26, 30].

While the direct connection of offshore collector network, without the need for an offshore substation, is feasible for some cases; it was decided to consider only networks with distinct collector and transmission stages, given that many new developments are significantly larger [83], and significantly farther from shore [39] than the suggested limit where direct connections may be beneficial [22].

In keeping with a number of other published papers [19, 24, 26], it was deemed desirable to treat an offshore renewable energy project, in terms of machine sizes and positions, as a fixed input. Thus the objective of this work is to maximise the network's performance for a given development, rather than to specify any aspects of the site configuration.

Some published work seeks to optimise both collector and transmission networks simultaneously [26-27]; while others only consider one aspect [18, 21]; or fix other characteristics of the network such as topology [23, 32] or the location(s) of offshore substations [23-24]. As this work is concerned with holistic optimisation, it was decided that as few as possible restrictions should be mandated; thus the locations of offshore plant, and the topologies of both collector and transmission networks should be optimisation variables, rather than fixed entities. Similarly, the location of offshore plant may be limited to discrete locations [23] or may be continuous [26]; each with their own merits. For the purposes of this work, it was concluded that continuous placement of offshore plant offered the most holistic optimisation.

Of course, there is a limit to the workable solution space thus it is not always feasible to include every possible variable within the optimisation process. In line with a number of contemporary papers, it was decided to maintain voltage and frequency, where a network is AC, as constants [18, 25, 27], within a single optimisation run. It was, however, considered desirable to be able to change these parameters between studies, such that their influence may be characterised.

While many studies relating to the optimisation of subsea electrical networks utilise component sizes as a design variable [25-26, 31]; this allows the potential for over-specification of component sizing. For this work, an alternative approach of calculating the component sizes required based on a power flow study was selected; thus eliminating the prospect of over-specifying components.

The optimisation problem, for this work, may now be qualitatively defined as:

- Minimise through-life costs associated with a subsea electrical network linking a fixed offshore renewable energy development with an existing land based network.

Where:

- Only networks encompassing both collector and transmission stages are considered.
- Both collector and transmission stages are optimised simultaneously.
- Both collector and transmission stages are free to take radial, ring, or star network topologies.
- Each collector network is free to have an independent number of parallel connection strings from its adjacencies.
- Offshore substations may be positioned anywhere within the bounds of the search space.
- Voltage and frequency, of both collector and transmission networks, are fixed for a single optimisation study, but may be manipulated between studies to characterise their influence.
- Continuous component sizing is employed, such that over-specification is eliminated.

3.2 Solution Methods

Within Figure 25, the upper right quadrant is the most desirable place for an optimisation study to reside; as the model has been defined with high fidelity and been solved using a robust solution method [45, 46]. Within this section, solutions methods for the delivery of optimal, or a near optimal, solution to a network design problem are presented and their relative merits discussed.

There are, depending on the scope of the study and the overarching methodology, up to 3 distinct aspects to the optimisation of subsea electrical networks: the specification of a network design, which may be refined iteratively until optimality is reached; the order in which connections are made within a network; and the assessment of the technical performance of the defined network which has a significant bearing on the cost performance of a candidate solution. The respective importance of each of these tasks, along with techniques that may be applied to provide solutions to these problems are now presented.

These problems are interrelated, as depicted in Figure 26. Obtaining an optimal, or near optimal, order of connection requires a high level representation of the network design as a *starting point*; hence this information is passed from the network specification aspect of the optimisation process. Quantification of network performance requires a complete network model to be present to facilitate a power flow study; hence a complete network model comprising both the specification and ordering of the network is passed from the connection ordering component of the optimisation routine to the network performance section. In order to assess the performance of the candidate solution performance and / or cost data must be fed back to the network specification aspect of the problem.

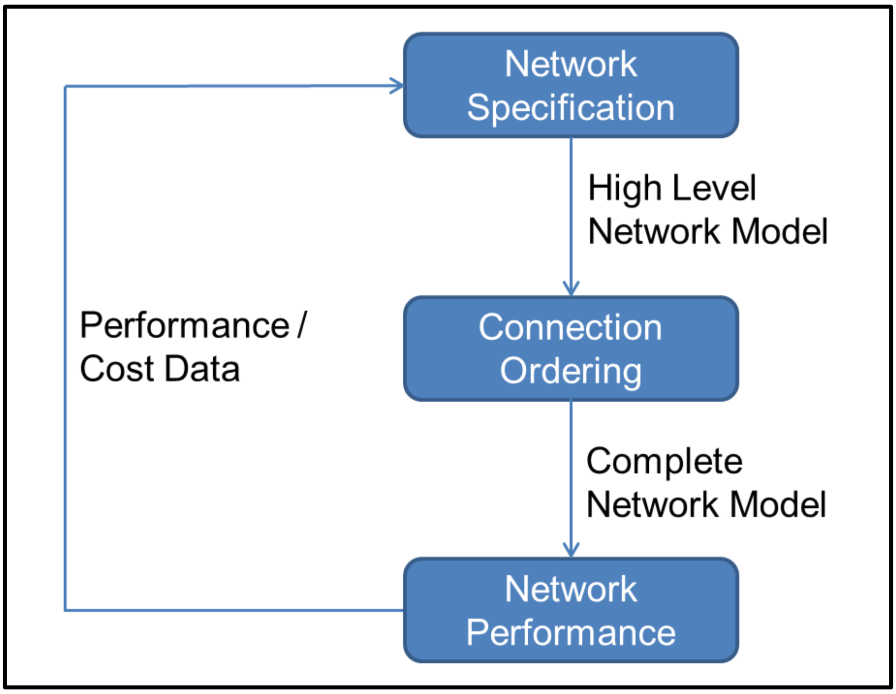


Figure 26 - Interrelated Optimisation Problems

3.2.1 Network Specification

As has been discussed previously, an offshore electrical network will typically comprise of one or more offshore substations, deployed at defined locations; which interface a number of parallel medium-voltage collector cables with a high-voltage transmission system. Consequently, a high-level network specification may possess a large number of parameters: locational co-ordinates (x and y axes), topologies and number of parallel connections for each offshore substation; the voltage and frequency of both collector and transmission aspects of the solution network and the topology of the transmission system deployed.

These parameters only describes a summary-level representation of a network, as the high number of permutations associated with connection ordering, particularly within the collector networks, ensures that *identically* specified networks may behave quite differently to one another. The issue of connection ordering is covered in the following section and, as such, only the creation and optimisation of the high-level network specification is considered at this time.

Brute force tactics, where every possible combination of inputs variables is assessed, could be employed for this problem if the physical deployment space is small and discretised. Without these restrictions, the search space will likely become too vast and, thus, this technique becomes intractable.

Earlier in this thesis, calculus methods for solving optimisation problems were discussed. In order to establish the network specification problem in such a manner than calculus may be used to return an optimal solution, the objective function must be continuous and double-differentiable; as must the constraints and decision variables. Some of the decision variables associated with the specification of an offshore network are naturally continuous: for example, the positioning of offshore substations, with respect to both x and y axes, is not naturally bound to discrete points. Some decision variables are theoretically continuous but are unlikely to be continuous in practise, for example: there is no technical impediment as to why a cable may not be configured to operate at any network voltage, but, in reality it will be difficult to obtain cables that are designed to operate at non-standard voltage. Finally, there are variables which are categorically not continuous: the number of parallel strings that are used to comprise an offshore collector network must be integral by definition.

In order to utilise a calculus method for network specification optimisation, any discontinuous variables would have to be eliminated from the optimisation. In some cases, this could be achieved quite readily: voltage, for example, could be defined firmly and an optimal solution could be sought using this fixed parameter. Equally, the number and topology of strings emanating from an offshore substation to form collector networks could be limited in the analysis to a specific value; but this would reduce the scope of the optimisation problem to consider the positioning of offshore substations only. It is unlikely that such a study would offer value as it would likely reside in the upper-left (Real-Solution to Non-Problems) or lower-left (Toy Problem) quadrants defined in Figure 25.

Even assuming that the aforementioned simplifications were not considered to be problematic, it would still be extremely challenging to define the cost of a solution into a single function that is continuous and double-differentiable.

The use of linear programming for the optimisation of the network specification problem is also challenging. As was the case when calculus was being considered as an optimal solution finding approach, to utilise linear programming the decision variables would all have to be modelled as continuous; thus it is likely that the fidelity of any network model would be too low to be useful for the same reasons that have been described previously.

Further, to use linear programming, both the objective function and the constraints must be linear. Depending on the choice of optimisation objective, this becomes problematic. If only the investment cost of the solution is being considered then it may be possible to linearise the objective function in such a way that it captures the installation cost of a network without too much of a reduction in fidelity. For some of the network components there will be no drop in fidelity as their representative functions are already linear; but for those whose cost approximation is non-linear there would some degree of fidelity loss during the linearisation process. If the optimisation objective is through-life cost minimisation, for which electrical losses must be accounted for, this presents a further barrier to the use of linear programming. Electrical losses are inherently non-linear so a further reduction in fidelity would be experienced by the linearised approximation of losses that would have to be constructed.

Non-linear programming resolves some of the issues that restrict the use of linear programming for this task as the non-linear components of the cost function can be implemented without loss of fidelity. Furthermore, this method would allow electrical losses to be quantified with much greater accuracy than a linear programming method. However, central to non-linear programming methods is the requirement for continuous decision variables so it is still likely that fidelity of any model constructed in this manner would be unacceptably low.

Mixed-Integer programming offers an extension of either linear or non-linear programming such that discretised variables may be included. This would allow the number of concurrent cable strings that comprise a subsea collector network to be included as a decision variable; similarly, voltage and frequency may now also be implemented as discrete variables, thus the potential scope of the optimisation is now greater than before.

Both the collector and transmission networks will possess a topology, with examples of typical topologies discussed previously within this thesis. One way to include topology within the optimisation would be to abstract topology to a numeric representation. For example, a radial network topology could be represented as a 1; a ring network topology as a 2; and a star network topology as a 3. In doing so, topology could theoretically be implemented into a mixed-integer programming method to optimise the network specification problem.

It would be very difficult to include topology into a mixed-integer programming method, in practise. Typically, a mixed-integer programme is solved using a method such as branch and bound, for which the first step of the algorithm is the solving of a relaxed version of the problem whereby all variables are considered to be continuous and a corresponding linear or non-linear programme is created. This approach works well when the values represented by the discrete states are related, but when they are unrelated, and abstract, such as this case then it is unclear how this could be implemented.

Using the numerical abstractions for network topologies that have been discussed previously within this section, a ring network would be represented as a 2 and a radial network as a 1. Both styles of network behave very differently from one another and have entirely distinct properties from a cost minimisation perspective. Consequently, it is unfeasible to include a network topology parameter on a continuous basis, such that the branch and bound method may be applied to optimise the problem.

Single-solution meta-heuristics could be employed for the network specification problem; and could be configured to include characteristics such as network topology that has proved problematic to interface with a mathematical programming or calculus method. There are limitations to their effectiveness and they are generally better suited to a deep exploration of a small search space in preference to a shallower exploration of a broader decision space [73], as has been described earlier in this thesis.

Population based meta-heuristics tend to be suited oppositely to single-solution meta-heuristics in the sense that they are better suited to a less-thorough exploration of a larger decision space, than to a very-thorough analysis of a smaller decision space [73].

Within population based meta-heuristic techniques, there are two primary approaches that could be utilised for this optimisation task: evolutionary computing; and swarm intelligence. Both of these methods have been presented earlier in this thesis and while they may operate in very distinct manners, from the perspective of solving the network specification problem; their credentials for this task are effectively identical.

Evolutionary computing and swarm intelligence methods, amongst other meta-heuristic methods more generally, are well suited to the network specification problem as they can operate with linear or non-linear objective functions; continuous, discrete and abstract variables simultaneously; and constraints that may be linear or non-linear. Using either of these methods allows for all of the variables that have been listed as potential decision variables previously to be included within the optimisation without loss of fidelity.

Evolutionary computing, specifically through genetic algorithms, has been used extensively for the optimisation of subsea electrical networks for offshore wind parks [21, 24, 26-27, 30-31, 41-42].

3.2.2 Connection Ordering

As is depicted by Figure 26, the connection ordering aspect of the network optimisation process has a high-level network model as an input and a detailed network model as an output. The process, therefore, involves connecting generators in an optimal, or near optimal, order to minimise cable length, electrical losses or cost.

The order in which generators are connected to an offshore substation has a significant bearing on the quality of a solution, given that the impedance of a cable is proportional to its length; electrical losses being proportional to the square of the current flowing through a cable; and cable cost being a function of capacity.

If a single cable is used to connect *n* offshore generators to a connection point then the number of possible order permutations, *P,* is defined by (24). The number of permutations grows rapidly as *n* increases.

$$P\ (n) = \prod_{l=0}^{n-1} n - l \tag{24}$$

Each possible generator connection order may be divided into separate strings of generators by applying breaks to the sequence. The first break may be applied at *n* points in the sequence, the second at *n-1* points, the third at *n-2* points and so on. Combining this with (24) returns a general function detailing the number of feasible connection orders, based upon *n* generators and *s* strings, which is presented as (25).

$$P\ (n, s) = \prod_{k=0}^{s} n - s \prod_{l=0}^{n-1} n - l \tag{25}$$

Shown in Table 6 is the number of possible permutations in which 10 strings may be used to connect *n* offshore generators to an offshore substation. Restricting the number of strings that may be deployed from an offshore collector platform has twin benefits: the search space of the external specification and positioning problem is artificially limited; and the complexity of solving the generator connection order problem is also limited. The capacity or number of turbines connected to each string does not have to be equally apportioned.

Number of Generators, n	Order Permutations
20	8.8285×10^{24}
50	1.1037×10^{71}
100	3.3866×10^{164}

Table 6 - Number of Combinations in Which *n* Generators May Be Ordered

The number of ordering permutations that would have to be assessed precludes the use of a *brute force* approach; instead more intelligent methods are required to provide a satisfactory solution. There are two distinct questions around the connection ordering problem: how the solution should be produced; and what the optimisation objective should be. Both of these points are now addressed.

Dutta & Overbye have published work relating to the optimisation of on-shore electrical wind farm collector systems with respect to minimum cabling length [19] and electrical losses [20]. This work uses Dijkstra's Minimum Spanning Tree algorithm and K-Means Clustering to minimise the distance and / or losses in the network.

Using a network analysis method, such as Dijkstra's Algorithm, to optimise the collector network portion of a solution presents both advantages and disadvantages. Such methods, in their unmodified form, are intrinsically set up to seek a minimum traversal distance or cost from a start point to an end point: consequently, it may be well suited to the optimisation of the design of a radial collector network.

A ring network may be generated from the radial solution returned by a network analysis method, simply by placing a further branch from the *final node* to the *initial node*. While this would provide solutions to the collector network optimisation problem, it is unlikely to provide strong solutions, as the optimisations are based on radial networks, rather than as ring networks.

Without modifications, a network analysis method may not design a collector network that can be defined as being of radial or ring topology. Instead, connections are assigned, at every iteration of the algorithm, on a cost or distance minimisation basis without consideration of an overarching topology; hence it is feasible that mixed topology or meshed solutions shall be returned.

Gonzalez-Longatt's research [18, 28] utilises different implementations of a genetic algorithm based *travelling salesman problem* (TSP). The TSP seeks a minimisation of the total distance travelled by one or more salesmen, while visiting all of the given cities or exactly once. The minimum travelling distance solution sought by the TSP is analogous to the minimum cabling distance of a collector network; with each generator representing a city in the classic formulation of the problem, hence this is a viable technique for the optimisation of offshore collector networks.

This approach, using a meta-heuristic technique, may not perform as well as a deterministic network analysis method, for the optimal design of radial collector networks. It does, however, offer a strong, but not necessarily optimal, solution without incurring significant computational expense. TSP approaches may be equally applied to the design of radial networks (TSP without salesmen returning to start city) and ring networks (TSP with salesmen returning to start city). Further, by their nature solutions obtained using a TSP method naturally form networks that are compliant with either radial or ring topologies; without the potential for mixed topology or meshed network solutions.

Gonzalez-Longatt uses the conventional minimisation objective of distance / cable length [18] and the minimisation of collector network implementation cost [28]. While it may appear intuitive that the collector system that utilises the least amount of cabling will be the cheapest solution to implement, this is not always the case as the cost of cabling is modelled as an exponential function of capacity and a linear function of length, as is described in the following chapter of this thesis. In addition to the implementation cost, there are through-life costs associated with the collector network design which, depending on the nature of the optimisation objective, may incentivise the use of additional cabling which offers greater electrical efficiency.

The objective function associated with a TSP solver may be configured in a number of different ways, as illustrated by Gonzalez-Longatt's use of minimisation of implementation cost in preference to the traditional distance / cable length minimisation [28]. One potential drawback to the use of through-life cost minimisation as the design objective is the requirement for a power flow study to be undertaken which requires computational time and effort to establish and execute. This may not problematic if only the collector network is being considered, as is the case in Gonzalez-Longatt's work [28]; however when many collector networks are being assessed as part of a wider ambition to optimise a holistic offshore network the simulation time associated with the additional power flow studies may make this method unfeasible.

An intermediate optimisation objective between the low-fidelity distance minimisation and the high-fidelity cost minimisation is loss minimisation. The collector network may be represented as current flowing along branches with impedance proportional to length; from which electrical losses may be readily derived. While this method does not offer a *complete picture* as the cost minimisation technique does; it offers a more robust solution than will be returned when using cable length minimisation as the optimisation objective without incurring significant simulation penalties.

3.2.3 Assessment of Network Performance

In order to quantify the electrical performance of any power network, a power flow study may be undertaken. As has been described earlier in this thesis, a power flow study may be AC or DC, with a DC analysis not accounting for the effects of reactive power. Given the significant impact of capacitive charging within a subsea electrical cable, it is unlikely that a DC PF study would offer results of any real value, as the drop in fidelity would become unacceptable. Consequently, an AC power flow analysis is the only feasible choice, and this is reflected in a number of contemporary studies [25, 31].

The operational costs associated with an offshore network fall into 3 main categories: the cost of fixed losses, which do not vary with electrical output; the cost of variable losses which vary in accordance with the square of the output current; and the cost of failing to provide a working delivery path from the point of generation to the interface point with a shore based network [84].

Depending on how a given network is assessed, it is possible that the fixed and variable losses may be combined into a single electrical losses figure, which may be readily monetised.

One approach to the monetisation of electrical losses for an offshore electrical network is to simulate a single *snapshot* of electrical network performance, typically using an average output power [22, 26]. This method reduces the computational effort associated with producing an optimal network design as fewer analyses are required relative to assessing network performance at a number of power levels; however, this is achieved with a significant drop in fidelity as, owing to the non-linearity of electrical losses as a function of power throughput, the losses calculated using average power will correspond poorly to the losses calculated at various operating points. Examples of contemporary work that utilises multiple power levels to approximate electrical losses include Nedić et al [25] and Zhao et al [31] who use 16 distinct wind states and their associated probabilities of occurrence to model the through-life behaviour of the network, without explicitly mentioning which style of probability density function they have employed. Nedić et al use a Weibull distribution with shape and scale parameters of 1.8 and 11.2 respectively [25]. Banzo & Ramos [40] have modelled the availability of wind using a Rayleigh probability density function.

Some papers do not explicitly state whether the results presented are based on a single power output level or multiple power levels [42].

In order to calculate the likelihood of a no-service condition, a number of simplifying assumptions may be made: each component is either working or is out of service (no partially working states considered) [26, 29, 31, 40, 43]; each component has a failure rate and a mean-time-to-repair (MTTR) that are average values, independent of other factors [26, 29, 31, 43]; no overload conditions are permitted [26, 29, 43]; and that all component failures are independent [26, 29-31].

There are a number of possible metrics for quantification of losses arising from no-service conditions, including: Loss-of-Load Probability (LOLP); Loss-of-Load Expectation (LOLE); Expected Energy not Supplied (EENS); Expected Unserved Energy (EUE); Expected Energy not Produced (EENP). These metrics were deemed inadequate by Zhao et al who deemed that a more *network specific* metric of the generation ratio relating the output power to the grid from the input power to the turbines was desirable [43]. Calculation of the generation ratio for out-of-service conditions requires the use of power flow studies with each component systematically set to an out-of-service condition, as required by the condition of component failures being independent [29-31, 43, 82]. This makes using the generation ratio more computationally expensive than the metrics listed above, but does increase the accuracy of the results obtained.

3.3 Outcomes from Contemporary Research

Within this section, results and conclusions from contemporary research work are discussed; covering both computational design and optimisation of subsea electrical networks, and other topics including: the economics of AC and DC transmission, and the scope for sharing of electrical grids for multiple energy vectors.

3.3.1 Efficacy of Techniques

A number of contemporary research papers pertaining to the use of meta-heuristics to design and optimise subsea electrical networks use genetic algorithms [21, 24, 26-27, 30-31, 41-42] and this has been shown in these papers to be viable and powerful technique for obtaining optimal, or near optimal, solutions. This appears to be the most utilised methodology in recent times for this task. The ability of this technique to allow networks to be modelled in high fidelity suggests that the upper-right quadrant (real solution to a real problem) shown in Figure 25 can be reached.

Genetic algorithms may also be highly effective for the connection ordering problem, defined previously, which forms the primary contribution offered by Gonzalez-Longatt, where they point to this being *"fast and effective"* while reducing the cost of the collector portion of an offshore wind park electrical network [18,28].

Solutions may be validated against other published work [21] or against a real-world network design.

Nadingham & Dhali [44] conclude that their use of geometric programming is an effective technique for subsea electrical network design. While this technique may be effective, it does not lend itself to reaching the desired upper-right quadrant depicted in Figure 25, as there have been too many simplifications, as discussed earlier in this thesis. Instead, using this approach is likely to reside in the upper-left quadrant (real solution to non-problem) of Figure 25.

3.3.2 Optimal Design Outcomes

As has been discussed previously, a number of studies were undertaken with restrictions around a number of network characteristics. Such studies do not offer much insight into the complex relationships between transmission technology, voltage and power throughput. Ergun et al [21] do allow for different transmission technologies (AC or DC); transmission topology types and voltages to be considered and they conclude that the voltage and technology will affect the optimal transmission topology. Thus, they demonstrated that there is no *one-size-fits-all* solution to subsea network design.

Zubiaga et al [22] conclude that up to a cable length of 20 km, an offshore substation should not be utilised when designing a network for an offshore wind park; instead a medium-voltage system should be deployed. They further point to the distance from shore at which it becomes worthwhile to deploy an offshore substation and operate a two-voltage network as being up to 60 km; however, this is under very specific operating conditions and is not considered to be a general case [22].

While they do not offer any form of insight as to what an optimal electrical system design may be, Hopewell et al [80] highlight the importance of a rigorous analysis of the available technical options and their associated economics.

3.3.3 Cost Parametric Sensitivity

It has been presented earlier in this thesis that there is a dearth of credible information relating to the cost modelling of subsea electrical network components, with only the equations presented by Lundberg [81] and those that derive from this work [21, 24, 27, 31, 32, 44, 61].

A number of papers stress the importance of these cost functions in determining optimality; implying that any error or limitation in these functions may skew the results towards a sub-optimal solution [21, 44].

In addition to the optimal outcome being highly sensitive to the accuracy of the cost functions employed, there is further cost sensitivity around other financial parameters such as the price paid per unit of electricity, and the discount rate applied for through-life cost calculations. Such parameters are discussed within the Case Studies chapter of this thesis.

3.3.4 Analysis on Using AC or DC for Point-to-Point Transmission

A number of studies exist that have examined the most suitable technologies for use in point-to-point transmission in the subsea environment. A number of these studies present *break-even* distances in reference to the cable length at which the economic penalties incurred by using the more expensive DC option are outweighed by the decreased losses.

Academic literature on this subject offers a broad spectrum of distances at which point DC transmission becomes more economically viable than AC: between 30 and 250 km [35]; 60 to 80 km [36]; approximately 90 km assuming converter stations are working perfectly [37]; 100 to 150 km [38]; and circa 100 km [39]. The break-even distance depends on the specific voltage and power level of the system under consideration; hence there is a fairly broad range of values and no generalised conclusions may be made.

With any economic analysis of equivalent options, the conclusions drawn are based upon a number of variables including but not limited to: cost of equipment; conversion efficiencies; and reliability, and any change in any of these factors may skew the analysis towards a different conclusion.

Current trends in offshore wind parks, one of the primary users of subsea electrical networks, have been towards larger developments that are farther offshore. Rodrigues et al in their 2012 paper [39] made a comparison of the energy losses experienced when using HVAC (150 kV) and HVDC (320 kV) for the purposes of the transmission of 500 MVA over a range of distances. Their results pointed to a break even distance, where an AC and DC systems experience equal losses, of 100 km. Losses for both AC and DC systems can be readily modelled as first-order functions of length, with the AC system having a much larger gradient but lower losses at short distances and the DC system experiencing much higher losses at short distances but a much smaller increase in losses as the cable length increases [39].

This analysis is particularly relevant for large offshore wind parks, as the capacity is similar to large completed projects such as the Bard Offshore 1 development (400 MW) and the length range considered takes in the likely distance-to-shore of modern developments (currently 33 km average for projects under construction) [39].

In addition to their analysis of losses as a function of transmission cable length, Rodrigues et al also considered the relationship between transmission system length and cost, for both AC and DC technologies for the proposed 500 MVA system.

The HVDC technology system cost may be modelled as a first order relationship with a low gradient and high fixed costs. The HVAC technology system cost also possessed two distinct sections where the cost is linearly related to system length; with an asymptotic rise between 110 and 120 km [39].

Barberis Negra et al produced a comparison of HVAC; Line Commutated Current HVDC (LCC-HVDC) and Voltage-Source Converter HVDC (VSC-HVDC) in 2005, for use in offshore wind farms in the capacity range of 500 – 1000 MW at distances from shore up to 200 km [85].

The submarine cables used for this study for the HVAC analysis are of the XLPE type that is ubiquitous in modern offshore developments. For the HVDC variants considered, older mass impregnated cables are modelled. Until recently, XLPE cables would break down under DC current flow hence this study uses the older technology. An updated study using the most up-to-date XLPE DC cabling might yield significantly different results; nonetheless, the key findings of this work are now presented.

Barberis Negra et al found that the HVAC system performed best up-to 50 km, but for longer distances LCC-HVDC offered the most efficient transmission technology [85]. Additionally, they presented a breakdown of losses for a 132 kV 500 MW 100 km HVAC system and 150 kV 500 MW 100 km LCC-HVDC and VSC-HVDC systems are presented as Table 7 [85]. It is worth noting that the total losses for the HVAC and HVDC systems presented in Table 7 are not identical, as shown in the final row.

One observation from examining Table 7 is the considerable difference in cable losses between the HVAC and HVDC systems. While the cable losses for the HVDC systems are considerably lower than those for the HVAC system while using mass impregnated cables; this would likely further reduce if the system was remodelled using the modern XLPE cables. As the cable itself is only responsible for a low percentage of overall losses, it follows that longer distances will benefit more from the use of HVDC, over HVAC, than shorter distances. This agrees with the conclusion Barberis Negra et al arrived at that HVAC performed best up-to 50 km, but for longer distances LCC-HVDC offered the most efficient transmission technology [85].

Component	**AC Losses (%)**	**DC Losses (LCC) (%)**	**DC Losses (VSC) (%)**
Cables	87	13	18
Offshore Transformer / Converter	5	44	41
Onshore Transformer / Converter	4	43	41
Reactive Power Compensation	4	N/A	
Total Losses (%)	**4.77**	**1.98**	**4.87**

Table 7 - Losses Breakdown for HVAC, LCC-HVDC and VSC-HVDC Systems [85]

Since this paper was published, advances in VSC-HVDC have seen it overtake LCC-HVDC as the most likely technology to be deployed for high-power, long distance transmission. Advantages of VSC-HVDC over LCC-HVDC include smaller converter stations; no requirement for an AC reference voltage for commutation and support for weak grids [23, 82, 86].

Stoutenburg & Jacobson point to the benefits of VSC-HVDC over HVAC including reduced cable losses over long distance; improved fault response; and reactive power control. Conversely, they offer an approximation of losses incurred in each converter station as being 2 % of the power delivered [86].

Additionally, the use of HVDC allows the collection grid and the shore based grid to be operated asynchronously, which allows both networks to be completely electrically decoupled [23, 82].

3.3.5 Transmission System Sharing Across Generation Types

While sources occurring in the marine domain offer a more reliable supply of renewable energy than land based sources, renewable energy is a fundamentally intermittent and inconsistent resource. Consequently, there has been research conducted into the utilisation of sites for multiple generation types and the sharing of sub-sea network infrastructure. Given the inherent similarities between commonly used generation technologies and the fact that generators are linked to a subsea network using power electronics, from an electrical engineering perspective the connection of wave, tidal and offshore wind generation at the same site presents no significant challenges.

The suitability of using a given area for a combination of offshore wind and wave development simultaneously will depend on a number of factors. However, there have been studies that point to this technique having significant potential. One such study considered the simultaneous deployment of both offshore wind and wave converters off the coast of California, USA, and reached a number of telling conclusions. It is worth noting that the sites considered are considered *"abundant"* in terms of both resources [87].

The Californian combined site usage study concluded that utilising both wind and wave farms simultaneously at the same location achieved *"reductions in variability equivalent to aggregating power from two offshore wind farms approximately 500 km apart or two wave farms approximately 800 km apart"* and that the reduction in zero output hours from a combined system could be reduced from over 1000 hours for wind and 200 hours for wave to under 100 for a combined system [87]. Additionally, it was concluded that due to the inextricable link between wind and wave availabilities; the deployment of a 1000 MW combined wind and wave farm could be linked optimally using a reduced transmission capacity of 920 MW, as the combined output will seldom be above this threshold. As the capital cost of transmission for an offshore wind project is typically around 16 %, it follows that any potential savings pertaining to the transmission can be worth a considerable amount of money [86, 88].

3.3.6 Optimisation of Electrical Systems for Onshore Wind Developments

The optimisation of electrical networks for onshore wind developments has produced less research interest than those for offshore renewable projects. Largely, a core assumption is made that a collector network designed for a large wind farm site will be sufficiently close to a transmission system that there is no requirement to include transmission within the optimisation; consequently, typically only the collector network is optimised [19-20, 89-90].

While onshore wind generation is typically performed using smaller machines than for offshore developments, the aggregate capacity for onshore sites can rival that of offshore projects. For example, the UK's largest onshore wind farm: Whitelee, operated by Scottish Power; has an aggregate capacity of 539 MW [91]. Given the capacity of such onshore developments, it is surprising that the scope of network topologies under consideration for network optimisation research appears to be limited to radial only [19-20, 89-90]; as there may be scope for high-redundancy higher-cost networks outperforming low-redundancy lower-cost networks, against an optimisation objective of through-life cost minimisation. Equally, it may also be the case that the lower strike price for electricity generated from onshore wind farms (approximately £80 / MWh for the period 2016-19 [92]) compared with offshore wind; the lower likelihood of cable damage; and reduced time and cost associated with repairing damaged electrical cabling makes capital expenditure minimisation a more useful optimisation objective than through-life cost minimisation. This is visible in the literature, where no published research has been identified that utilise lifecycle cost minimisation; instead, capital expenditure minimisation appears to be the most commonly utilised objective [19-20, 89-90].

Methodologies that have been employed to optimise the performance of onshore electrical networks for large renewable developments include: network analysis methods [19-20, 90];

and integer linear programming methods [89]. As has been discussed previously within this chapter, network analysis methods offer a deterministic and efficient optimisation for the design of radial networks; however, they are poorly suited to the optimisation of ring networks due to their inherently unidirectional nature. The use of an integer linear programming method makes the inclusion of network topology within the optimisation algorithm very difficult, as has been considered previously within this thesis. It is unclear if the cost functions associated with the deployment of an onshore electrical network can be linearised without incurring an unacceptable drop in fidelity.

No research has been identified that considers the deployment of parallel collector networks for onshore renewable energy developments.

3.4 Optimisation Positioning

Within this chapter, a review of academic literature pertinent to this thesis has been reported where a number of papers have been assessed based on which desirable optimisation characteristics have been included. Further to *what* is included, is another question relating to *how* it is included. For example, some authors have completely disregarded the transmission aspect of a subsea electrical network from their studies [19, 20]; while others include this component but place restrictions on the form it may take, for example: mandating AC transmission [32, 40]; or having radial topology [23, 32]. In both of these examples, the rationale for excluding, or limiting, transmission aspects is to focus more intently upon the collector network optimisation. Given the interrelated nature of collector and transmission network design, optimising in such a manner may scope the problem in such a way that does not return the best possible network design for a given site.

Each of the papers that have been published within the field of optimisation of subsea electrical networks has *strengths*, where the fidelity and / or quality of solution methods are high; or there are no artificial restrictions placed upon the optimisation process. Equally, each has *limitations* where simplifications have been made in order to limit the scope of the problem.

By assessing the strengths of each study, a number of desirable characteristics have been identified. It has become evident that there is no published work that has been found, at the time of writing, which simultaneously addresses **all** of the following, in relation to the optimisation of two-stage collector-transmission subsea electrical networks for offshore renewable energy projects:

- Use of through-life cost as the headline optimisation objective, considering the purchase and installation of equipment; cost of electrical losses; and no-service condition costs.
- Optimisation of both collector and transmission stages simultaneously.
- Allows for the implementation of multiple simultaneous collector networks.
- Allows collector and transmission networks to take radial, ring, or star topologies.
- Allows transmission to take AC or DC status.
- Allows voltage and frequency of both collector and transmission stages to be manipulated, such that their influence may be ascertained.
- Allows offshore plant to be positioned on a continuous basis, in preference to a discrete positioning system, or fixed positioning.
- Calculates component sizes based on required capacity, rather than on the basis of standard component sizes.
- Calculates the cost of installation of the network based on cost functions that have been widely accepted, and utilised within a number of contemporary studies.
- Calculates the cost of electrical losses on the basis of a number of discrete input energy states, in preference to an average state which incurs fidelity losses due to non-linearity of losses.
- Calculate the cost of no-service conditions through assessing the differential losses incurred by the loss of each subsea cable, and the probability of each going out of service.

By considering all of the above requirements simultaneously, there exists an opportunity for this work to be more holistic in its optimisation of subsea electrical networks than all of the contemporary studies, from which the desirable characteristics are drawn.

While this work incorporates a number of desirable features that have been identified in the work of others, it is not without limitations, for example:

- Collector and transmission network frequencies and voltages are included as discrete variables. Including these parameters as continuous design variables would allow their impact to be more fully characterised than what may be achieved within this work.
- For adjacent collector networks, network topology and number of parallel strings of connections are specified on a network-by-network basis. The voltage and frequency at

which collector networks operate is specified on a collective basis. The inclusion of individual collector network frequencies and voltages as design variables would allow for a more thorough examination of the design space than is currently possible.

- The inclusion of DC collector networks and mixed radial-star network topologies would allow for a broader range of network types to be assessed.

While it is recognised that the inclusion of the preceding points would make the developed optimisation process more holistic; this must be offset against the exponentiation of computational resource required as the number of design space dimensions increases.

3.5 Chapter Summary

When attempting to optimise the design of a subsea electrical network, it is desirable that the combined model and solution generation method resides in the upper right quadrant depicted in Figure 25; thus a real solution to a high-fidelity problem has been obtained [45, 46]. The offshore network design optimisation problem has significant scope for intractability, given the number of potential variables that exist; which in turn creates a multi-dimensional search space that becomes extremely computationally expensive to fully traverse. Consequently, there are challenges associated with defining the problem in such a manner that captures the requisite detail without over complication.

In order to place some restrictions on the possible search space, simplifications have been applied in the pertinent academic literature. For example: voltage and frequency may be treated as constant values [18, 24-29, 40]; there may be one or more offshore substations placed at defined locations [23-25]; the system topology may be pre-selected, most likely as a radial configuration [23, 32]; or the transmission technology might be restricted to either AC only [26-27, 32] or DC only [23]. It is also possible to focus efforts only on optimising either the collector portion of the network as a stand-alone system [18, 28] or the transmission aspect as a distinct entity [21].

Figure 26 shows the interrelated problems that must be solved to optimise an offshore electrical network. The network specification problem can be non-convex, non-linear and may combine continuous object positioning variables with discrete variables representing the number of parallel cable connections, voltage, and frequency. Further, if network topologies are to be included within the optimisation, the range of possible solution techniques is further reduced as a number of methods are not compatible with the abstraction of network topology into a numerical parameter.

Meta-heuristic methods are well suited to the optimisation of subsea electrical networks. Single solution based meta-heuristics techniques tend to perform best where a detailed search of a small search space is required [73]. For the purposes of optimising a subsea electrical network, with many variables, a shallower examination of a wider solution space is preferable; hence these techniques are of limited value. Population based meta-heuristics, on the other hand, are well suited to the design and optimisation of subsea electrical networks with evolutionary computing methods such as a genetic algorithm and swarm intelligence methods such as PSO being strong candidates to drive this part of the optimisation process [73].

The connection ordering problem has a number of possible permutations that precludes the use of a *brute force* approach and consequently this must be achieved in a more intelligent fashion, with Dijkstra's Minimum Spanning Tree algorithm [19-20] being one possible approach and using a genetic algorithm being another viable option [18, 28].

In addition to *how* the problem is solved for connection ordering, there is an issue around *what* the optimisation objective should be. A power flow study could be carried out on each solution to the connection ordering problem that would allow investment or lifecycle costs to be calculated. As this is likely to prohibitively slow to execute, quicker approaches would be appropriate. A modified travelling salesman problem solver allows for the optimisation objective to be manipulated: shortest distance, minimum electrical losses or lifecycle cost may all be integrated into the fitness function.

The third and final interrelated problem, as defined in Figure 26, centres on quantification of network performance. While a DC power flow option is considered, it lacks suitability for the study of subsea electrical networks where capacitive charging has a significant and undesirable impact; hence the more computationally expensive AC power flow is the only realistic tool that may be used.

It is possible to use an average power output to calculate the electrical losses; however, owing to the inherent non-linearity this reduces the accuracy of the losses relative to performing multiple state simulations. The advantage to this approach is that there is a reduction in computational effort arising from fewer power flows being executed.

In order to quantify the cost of no-service conditions, the Generation Ratio [43] offers a high level of accuracy by assessing the differential losses incurred when each network component is systematically withdrawn from service. Combining the capital expenditure of a network with its operational costs of electrical losses and no-service conditions allows for the lifecycle cost of a network to be calculated, and normalising operational costs to present day values ensures that the solution reached has a high level of accuracy.

Chapter 4: Simulation and Optimisation Model

Within the preceding chapter of this thesis, options pertaining to modelling and optimisation of subsea electrical networks have been identified and discussed. Within this chapter the techniques and design choices used to optimise network performance for the work considered in this thesis are presented and justified.

4.1 Optimisation Scope

For this research, the boundaries of the optimisation scope have been defined as depicted in Figure 27; such that the optimal design parameters are sought and applied to a subsea electrical network to maximise its performance at interfacing a given offshore renewable energy site layout and a land based electrical power network, subject to a number of fixed design parameters, and case-specific constraints.

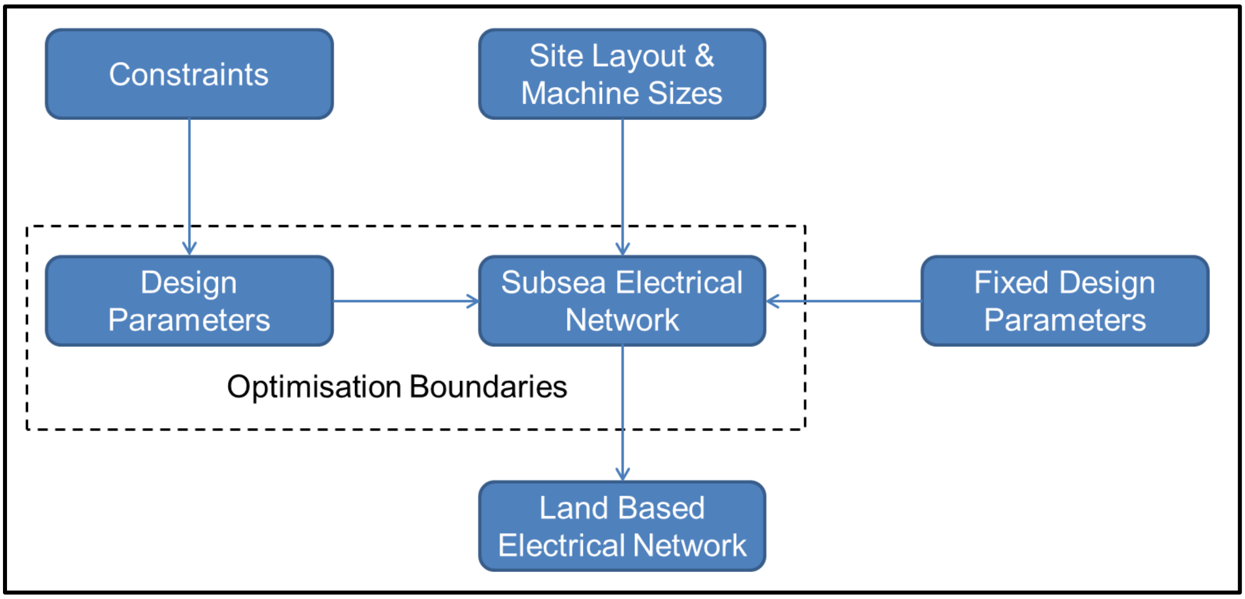


Figure 27 - Optimisation Boundaries

As has been discussed within the previous chapter of this thesis, there are a number of possible design parameters for the optimisation of subsea electrical networks, including:

- The positioning (x and y co-ordinates) of all offshore substations.
- The topologies and number of concurrent implementations of the chosen topology at each substation.
- The order in which the electrical generators are connected to form collector networks.

- The order in which the offshore substations are connected to form the transmission network.
- The collector network voltage and frequency.
- The transmission network voltage and frequency.
- The choice of an AC or DC network within both the collector networks, and the transmission network.

Parameters may be constrained, thus restricting the values of given design parameters that are acceptable. For example, it may be mandated that any offshore substations must be placed a minimum distance offshore for environmental reasons; or a constraint may arise from the maximum voltage at which subsea electrical cabling may be produced and procured.

There are also a number of fixed design parameters, which may be included within the optimisation process, but are not manipulated. Examples of this may include the anticipated development lifetime; the discount rate applied to future costs and earnings; and the value assigned to electricity generated. It is difficult to justify the inclusion of such parameters as design variables as they are externalities to the network design problem, thus they are not governed by the network designer.

The primary input to the optimisation process considered within this thesis is the site layout and machine sizes. This work, along with a number of contemporary studies [19-20, 24-25, 28-30], does not attempt to optimise the performance of either the site layout, or the machines deployed. This allows a prospective offshore wind farm developer to select a site based on the aerodynamic performance alone, with the logistics of how energy is extracted from the site to be considered thereafter. The output from the optimisation process is specification of a network which is to be interfaced with a land based electrical grid, assumed to be capable of conveying the power delivered from the offshore generation without requiring upgrades, in all cases.

Within the following sections, a number of modelling and implementation aspects of this optimisation process are detailed and defined.

4.2 High Level Design

In order to better define how an offshore electrical network may be modelled and optimised, some high level design choices must be made. Key design decisions relating to the optimisation objective and high-level network configuration are discussed and defined in the following sections.

4.2.1 Optimisation Objective

Within the previous chapter of this thesis, two possible optimisation objectives have been identified as being potentially suitable for this work, namely: minimisation of capital expenditure; and minimisation of lifecycle expenditure.

The primary optimisation objective selected for this work is the minimisation of through-life costs, in keeping with a number of contemporary studies [22, 31, 42]. It is possible to discount the through-life aspects of the lifecycle cost, thus the implementation cost alone may be returned. The optimisation objective for this body of work may be generalised mathematically as (26), where: C_{LIFE} is the through-life cost associated with a given network; C_{INST} is the installation cost of the network; C_{LOSS} is the through life cost of the electrical losses; and $C_{N\text{-}S}$ is the through life cost of no-service conditions.

$$\min C_{LIFE} = C_{INST} + C_{LOSS} + C_{N-S} \quad (26)$$

The generalised equation presented in (26) will be formalised once the design parameters and fixed design parameters have been discussed and defined in the following sections.

4.2.2 High-Level Network Configuration

Previously in this thesis, two distinct high-level network designs were discussed: direct connection single voltage solutions where there is no offshore substation; and two-stage collector and transmission network solutions which are interfaced using an offshore substation.

Zubiaga et al [22] identified that there was value in direct connection of collector networks to shore, under a limit range of circumstances: the site must be less than 10 km from shore, and the overall capacity of the site less than 60 MW. At present, the average distance from shore of a new offshore wind development is 33 km [39], and the capacity of modern offshore wind parks may be many times higher than the 60 MW limit identified by Zubiaga at el [22], for example: East Anglia One is proposed to be 714 MW [83]. Given the trends observed in the industry, it was decided that the optimisations performed for this work would be configured to use distinct collector and transmission network stages with no scope for direct connection of collector networks to the onshore grid. Consequently, the number of offshore substations, n, must be greater than or equal to 1, as defined in (27), where n_{max} is the upper bound on the number of offshore substations.

$$1 \leq n \leq n_{max} \tag{27}$$

Previously in this thesis, constant and variable speed generator operations were considered, along with different converter positioning strategies. The only paper within this field that was found that discussed converter strategies was Zhao et al [31], who discussed the relative merits of each style of convertor deployment but do not articulate which style they have employed for their optimisation. It is assumed, for purposes of this work, that the output of each turbine is controlled on an individual variable speed basis, as conceptualised in Figure 18, as this configuration:

- Allows the rotational speed of each machine to vary, while maintaining a constant bus-bar frequency [65].
- Reduces fatigue loading on the turbines [31, 65].
- Eliminates the possibility of a single converter failure precluding the delivery of electrical power from all turbines [31, 65].

4.2.3 Discrete and Continuous Capacity Sizing

One approach which marks this research apart from the majority of its contemporaries is the use of continuous cable and plant sizing, and ultimately pricing, in preference to discrete component sizing. A number of genetic algorithm based research papers include component selection as a design variable within the algorithm where cables and plant may be specified from a list of candidates [31, 41, 42]; thus the number of number of search space dimensions increases. This research tackles this problem in a different fashion as a power flow study is undertaken with all turbines outputting at rated capacity and the capacity required for each component in the system is derived from the results, including provision for reactive power flows. This approach incurs a minor reduction in fidelity as the physical characteristics of nominally similar cables will vary slightly with capacity; however, this increases the accuracy of the optimal solution as the potential for over specification of components is eradicated.

4.2.4 Cable Deployment Spatial Constraints

In order to minimise the network design complexity, and therefore cost, of installation and repair; it is desirable to have all subsea cables arranged neatly with no cables crossing one another. In practise, this may become difficult as the number of cables increases.

Of the academic literature reviewed for this work, the only papers that addressed the issue of crossing cables did so by limiting the scope of possible connections from any given turbine to its immediate adjacencies only; thus the crossing of collector cables may be avoided [24, 27]. In both cases, the crossing of transmission cables and collector cables was permitted.

While it may be desirable to implement minimum spacing constraints between cables, there are three primary reasons why this was not implemented within this work. Firstly, the cost associated with installation of subsea cables is generally approximated as being constant [22, 24, 61]: irrespective of water depth, distance from shore, and other factors; thus the complexity of installation is not taken into account. Secondly, the additionally complexity involved in capturing the sea and cable run influences without seabed information and embedding this within the optimisation process requires operational data sets that are not available at this time. Finally, even if the required sea-bed information was available, this would add a significant layer of complexity to a problem that already has a vast design space, which may push the problem into intractability.

4.3 Design Variable Selection

In order to characterise the design vector, ***x***, a number of design variables were selected, with the justifications for each selection, along with the mathematical characteristics of the selection, presented in this section.

This work is focussed on the holistic optimisation of subsea electrical networks, hence as few restrictions as possible are implemented.

Holistic optimisation of subsea electrical networks is achieved through the concurrent optimisation of both collector and transmission stages, without imposing restrictions on one component which may impact upon the optimal design of the other component. There are many examples of simplifications that have been made by others working in this field including: considering only the collector network design within the optimisation [18-20], or only the transmission network design [21]; fixing one or more network characteristics, such as topology, for the collector network [27-28, 42], or the transmission network [23, 26-27, 32]; mandating the position of one or more offshore substations [23-25]; and the inclusion of voltage and frequency as either discrete variables [31, 41] or fixed parameters [24-29].

As this work is concerned with holistic optimisation, it follows logically that the *end-point* of the network is taken to be the interface with the land-based network, such that both collector and transmission network components are considered on a simultaneous basis.

Unlike studies that define the position of one or more offshore substations, for this work these may be positioned at any point within definable bounds over a given area using both x and y co-ordinates systems, in line with (28-29), where: x_{MIN} and x_{MAX} are the lower and upper bounds on the x-coordinate location; and y_{MIN} and y_{MAX} are the corresponding bounds for the y-coordinate location.

$$x_{MIN} \leq x_i^P \leq x_{MAX} \tag{28}$$

$$y_{MIN} \leq y_i^P \leq y_{MAX} \tag{29}$$

Only a small minority of research papers pertaining to computational optimisation of subsea electrical networks for offshore renewable energy projects make mention of whether the positioning of components is discrete [23] or continuous [26].

The primary advantage of using discrete positioning of offshore substations over continuous positioning is the reduction in possible solutions that may exist for a given network optimisation problem. This may be of benefit if a brute-force method is employed where all possible solutions to a combinatorial problem are considered. This has the disadvantage of placing artificial constraints into the solution, which may return a suboptimal solution when compared with continuous component placement.

In order to maximise the accuracy of the solutions, it was decided to implement continuous object positioning such that offshore substations are free to take up any position that falls within the specified upper and lower bounds. While this approach vastly increases the number of potential solutions that exist to the network specification problem; this is not deemed to be problematic so long as a suitable optimisation technique is employed to efficiently traverse the search space. Consequently, the boundaries that were applied to the positioning of offshore substations as (28-29) can now be defined as being real numbers, in accordance with (30-31).

$$x_i^P \in \Re \quad (30)$$

$$y_i^P \in \Re \quad (31)$$

Further to this aim, both collector and transmission networks are free to take either radial, ring, or star topologies; with collector network topologies deployed on an individual basis rather than the same topology being mandated for each collector network present within the wider network. The topologies are abstracted into numerical parameters such that 1 represents a radial system; 2 represents a ring connection; and 3 represents a star network, and amalgamated into a set: *Top*. Now, the topology of each collector network, $CTop_{i,}$ may be defined by (32).

$$CTop_i \ \in Top \quad (32)$$

A number of offshore renewable energy developments use multiple instances of the same topology; for example, the Horns-Rev 1 project uses 10 parallel radial connections to interface 80 turbines with a single offshore substation [93]. Within this work, it was decided to implement a number of concurrent *strings* of connections that emanate from an offshore substation. This is presented mathematically as (33) where the number of strings emanating from a given offshore substation, $CStr_i$, must always be at least one, otherwise there is no collector network, and has an upper limit, $CSTR_{MAX}$, that is situation dependent.

$$1 \leq CStr_i \leq CStr_{MAX} \quad (33)$$

Again, it not feasible to have a non-integer number of parallel strings emanating from an offshore substation, hence (34) is true.

$$CStr_i \ \in Z \quad (34)$$

The transmission network, in addition to all collector networks, must have a topology and a number of concurrent strings. As within this thesis, small numbers of offshore substations are under consideration (n_{max} typically being 3), it was decided to limit the number of concurrent strings to form this stage of the network to one. The transmission topology, *TTop*, of a given network is subject to the same constraints as the collector topology, in accordance with (35).

$$TTop \ \in Top \quad (35)$$

As has been discussed in the preceding chapter of this thesis, there may be a vast number of possible connection orders for interfacing a number of turbines to a number of offshore substations; and a substantial range of possible connection orders when constructing a transmission network. As the cost of a subsea cable; impedance, and ultimately: losses, of a cable are functions of length, it follows that the connection order deployed at collector and transmission network levels will have a significant bearing on optimality. Consequently, the following parameters were selected for inclusion within the design: CO_i representing the connection order emanating from the i^{th} offshore substation and *TO* which represents the connection order within the transmission network.

Specific details relating to how the connection orders are optimised is covered later within this chapter.

Now, the decision vector, ***x***, may be formally defined as (36). The decision vector must be defined within a decision domain, ***Ω***, as has been previously formalised in (7); and this is defined as (37).

$$\boldsymbol{x} = \begin{bmatrix} x_i^P \\ y_i^P \\ CTop_i \\ CStr_i \\ T_{TOP} \\ CO_i \\ TO \end{bmatrix} \tag{36}$$

$$\boldsymbol{\Omega} = \begin{bmatrix} x_{MIN} \leq x_i^P \leq x_{MAX} \\ y_{MIN} \leq y_i^P \leq y_{MAX} \\ 1 \leq CStr_i \leq CStr_{MAX} \end{bmatrix} and \begin{bmatrix} x_i^P \in \Re \\ y_i^P \in \Re \\ CTop_i \in Top \\ CStr_i \in Z \\ TTop \in Top \end{bmatrix} \tag{37}$$

The variables defined within (36), and their associated limits defined with (37) are factors over which a subsea electrical network designer shall have control, thus they are appropriate as design variables. Further to these variables, there are also a number of fixed design parameters that must be included.

4.4 Fixed Design Parameter Selection

It is not realistic to include all parameters that pertain to the optimisation of subsea electrical networks as design variables, for two primary reasons: firstly, a number of these are not controlled by the designer, such as the price paid for electricity, and the discount rates applied; and the exponentiation of solution space that occurs when the number of search dimensions increases has the potential to push the problem to intractability.

As has been identified earlier in this chapter, both collector and transmission voltages may be treated as design variables. Given that voltage based approximations of component cost function have been presented earlier in this thesis, there is no impediment to optimising network design at a range of voltages. Further, any non-zero voltage could, theoretically, be used for both collector and transmission networks, thus an infinite range of voltages could be considered.

While the collector and transmission voltages are continuous variables, they are generally discretised [31] to take standard cable voltages or are kept constant [24-25, 28-29]. In order to bound the search space, and in keeping with published literature, within this research it was decided to keep both collector and transmission voltages constant during single optimisation studies; however, these parameters may be altered between simulations such that their influence may be characterised. It was also decided to implement collector network voltage on a collective basis, such that this parameter may not differ between adjacent collector networks within a wider network, as no examples of deployed networks with different collector network voltages could be found.

As was the case with collector and transmission voltage, from a *pure* optimisation perspective there is no reason why AC collector network and transmission frequency may not take any non-zero value, and therefore could be included as a design variable. Within the published literature, frequency is either discretised at standard operating frequencies [26, 27], or is kept constant [24-25, 28-29]. For the purposes of this research, both collector and transmission frequencies have been treated as constants during single optimisation studies, however, both parameters may be changed between runs in order to assess the influence of these variables.

Finally, both collector and transmission networks may use AC or DC. These technologies may be abstracted into numerical representations in a similar manner to the abstraction used for network topologies, for example: AC may be represented by 1; and DC by 2. It is not feasible for a network to *partially* utilise one of these technologies, hence both collector and transmission technology types must either be 1 or 2, in accordance with (38). As the use of AC collector systems is ubiquitous, it was decided to fix the technology choice for collector networks to AC, but leave the transmission network parameter, T_{TECH}, free to take AC or DC. During any given optimisation study, the AC or DC transmission technology status is treated as a constant and is not included within the design vector.

$$T_{TECH} \in 1,2 \tag{38}$$

The parameters that have been previously considered for inclusion within the fixed design parameters vector could feasibly be included within the optimisation as design variables, or as fixed design parameters. In extension to these parameters, a number of externalities require to be included as fixed design parameters, and these are now discussed.

The planned lifetime of a development has potential to greatly influence optimality in respect of the network design. A longer lifetime may incentivise the use of networks which incur lower operational costs, but with higher capital costs, than those which incur higher operational costs with lower capital costs. Equally, a shorter anticipated lifetime may place greater emphasis on minimising the capital costs associated with the network as there is less time for the operational costs to accrue.

The development or project lifetime variable is included within a number of published studies, as a fixed parameter [21-22, 31]. No published work has been found, at the time of writing that included this parameter as a design variable. In keeping with this, it was decided to include a fixed development lifetime, *DevLife*, within the fixed design parameters vector, ***y***, which may be altered between studies but is constant for each study.

In order to normalise the costs associated with an offshore wind farm project that occur over many years into an equivalent present day value, a discounted cash flow calculation is used. The value of the discount rate parameter has significant potential to skew optimality as a higher discount rate will reduce the value of operational costs incurred over the lifetime of the development more than a lower discount rate.

Mathematically, the discount rate applied to the life of an offshore electrical network may be expressed as (39), where: *PV* is the normalised or present value of future transactions; *DR* is the interest or discount rate; n is the year; *DevLife* is the project lifetime (in years); and FV_n is the future value of transactions in year, n [21].

$$PV(DR, n) = \sum_{n=1}^{DevLife} \frac{FV_n}{(1 + DR)^n} \tag{39}$$

This parameter has been used within a number of contemporary research publications [21-22, 28], and as a result the discount rate, *DR*, has been chosen for inclusion within this work as a fixed design parameter.

The price paid per MWh of electricity generated has no effect on the capital cost associated with a solution, but has a significant impact on the losses and no-service costs and thus this parameter has the potential to skew optimality. A high price of electricity results in higher operational costs, thus it may be beneficial to incur higher capital costs in order to minimise the operational costs. Equally, a low electricity price might incentivise the use of cheaper to implement systems as the operational costs through-life will be lower.

This parameter has been included as a fixed design variable for a number of published studies [21, 40, 94], and has been included within this body of work on the same basis, under the symbol P_{ELEC}. Similarly, the cost of reactive power compensation has been included, using the symbol P_{Q_SVC}.

While cable failures do not affect the installation cost of a given network, they do influence the operational cost, specifically the no-service costs. The more frequently cables fail and the longer they are out of service for increases the operational cost, and ultimately total lifecycle cost; for which a minimisation is being sought.

The inclusion of cable failures and associated mean time to repair (MTTR) parameters, which approximates the length of time a cable will be unavailable for in the event of a failure, as fixed design parameters is visible within a number of published studies [22, 40, 95]. Similarly, an offshore substation platform may have an expected failure rate and an anticipated MTTR value that may be included as fixed design parameters within this optimisation [95].

Failure rate parameters for both subsea cables ($FailRate_{CABLES}$) and offshore substations ($FailRate_{OFFSUB}$) have been included within the fixed design parameter vector, **y**, along with corresponding MTTR parameters ($MTTR_{CABLES}$, $MTTR_{OFFSUB}$).

The submarine cables that constitute a subsea electrical network also have a cost associated with them that may be defined on a collective or area-by-area basis to reflect differing costs associated with environmental factors such as water depth. It is commonplace within the published literature to include the cost of cable installation as a constant fixed design variable [22, 24, 61]; hence this is included, on a constant basis, within the fixed design parameter vector as $Cable_{INST}$.

This work is concerned with through-life cost minimisation of subsea electrical networks for offshore renewable energy projects, which is an inherently intermittent resource. The electrical losses of a subsea network such as this are non-linearly dependent on the input energy, thus it is useful to calculate the losses incurred at given input energy levels and combining these losses with their probability of occurrence to characterise the through-life electrical losses.

Consequently, it was decided to utilise a PDF, as described earlier within this thesis, to represent the renewable energy input to the electrical machines. As most of the literature available on the topic of subsea electrical network optimisation for offshore renewable energy projects pertains to wind, most authors who have employed PDFs as fixed design parameters use either Weibull [25, 49] or Reynolds [40] distributions.

It was decided to use a Weibull distribution to represent energy input at the machines, in keeping with a number of contemporary studies [25, 49]; hence the shape (PDF_{SHAPE})and scale (PDF_{SCALE}) parameters defined by (1) are included within the fixed design parameters vector, ***y***. In order to map the wind energy input to an electrical output from a given machine, the cut-in ($V_{CUT\text{-}IN}$), rated (V_{RATED}), and cut-out ($V_{CUT\text{-}OUT}$) speeds of the machines are also required for inclusion as fixed design parameters.

The complete fixed design parameters vector, ***y***, may now be defined as (40).

$$
\boldsymbol{y} = \begin{bmatrix} V_C \\ f_C \\ V_T \\ f_T \\ T_{TECH} \\ DevLife \\ DR \\ P_{ELEC} \\ P_{Q\_SVC} \\ FailRate_{CABLES} \\ MTTR_{CABLES} \\ FailRate_{OFFSUB} \\ MTTR_{OFFSUB} \\ CableInst \\ PDF_{SHAPE} \\ PDF_{SCALE} \\ V_{CUT-IN} \\ V_{RATED} \\ V_{CUT-OUT} \end{bmatrix} \tag{40}
$$

Now that both the design variable and fixed design parameter vectors have been defined, the optimisation function for this work may now be formalised, and this is presented in the following section.

4.5 Definition of Optimisation Function

A high level generalisation of the optimisation objective for this work was presented in (26), and may now be formalised in (41) where $\boldsymbol{x}$ is the design vector defined in (36) and is subject to being a member of the decision domain set, $\boldsymbol{\Omega}$, defined in (37), by (42); and $\boldsymbol{y}$ is the fixed design parameters vector defined in (40).

$$
\min C_{LIFE}(\boldsymbol{x},\boldsymbol{y}) = C_{INST}(\boldsymbol{x},\boldsymbol{y}) + C_{LOSS}(\boldsymbol{x},\boldsymbol{y}) + C_{N-S}(\boldsymbol{x},\boldsymbol{y}) \tag{41}
$$

$$
\boldsymbol{x} \in \boldsymbol{\Omega} \tag{42}
$$

Now that the optimisation problem has been defined, the following sections cover how the problem may be delineated into sub-problems, and ultimately solved.

4.6 Delineation of Sub-Problems

As has been considered previously within this thesis, it is feasible to split the holistic optimisation of a subsea electrical network into three interrelated sub-problems, as depicted in Figure 26. Equally, the connection ordering problem may be thought of as an *internal* problem residing entirely within the *external problem* of network specification, as conceptualised in Figure 28. Within this abstraction of the optimisation process, a connection order is optimised as the *internal problem,* for a specific high level network specification, before the performance of a complete network model is assessed; thus the information exchanges between the processes depicted by Figure 26 and Figure 28 are identical.

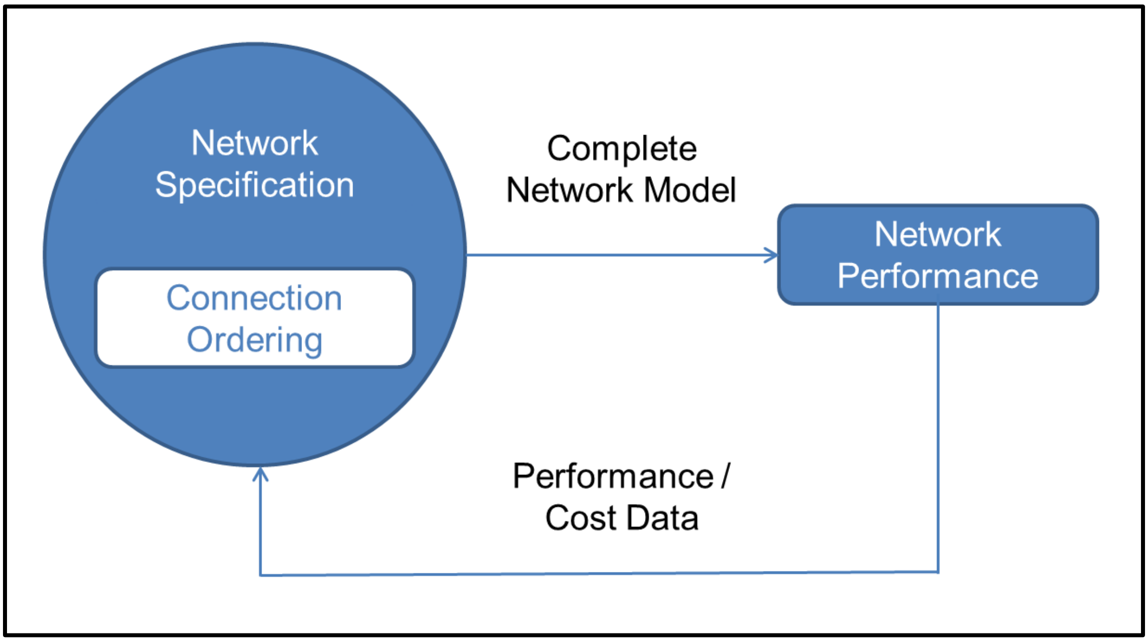


Figure 28 - *Inner* and *Outer* Network Design Problems

This approach, with its inherent modularity, has been utilised for this work. Each of these sub-problems combines to form the high-level optimisation algorithm as conceptualised by Figure 29.

The positioning of offshore substations, along with the specification of collector network topologies and number of parallel connection strings emanating from these offshore substations forms the network specification problem. These aspects of the overall optimisation process are considered within the *Create Initial Solutions* and *Modify Solutions* blocks presented in Figure 29. The development of efficient collector networks (subject to the topology and number of strings inherited from the network specification problem), represented within the *Optimise Connection Orders* blocks in Figure 29, forms the connection ordering problem. The solution to the connection ordering problem feeds into the network specification problem solution and allows the overall quality of the solution to be assessed as part of the network performance problem; carried out within the *Assess Solutions* blocks depicted in Figure 29.

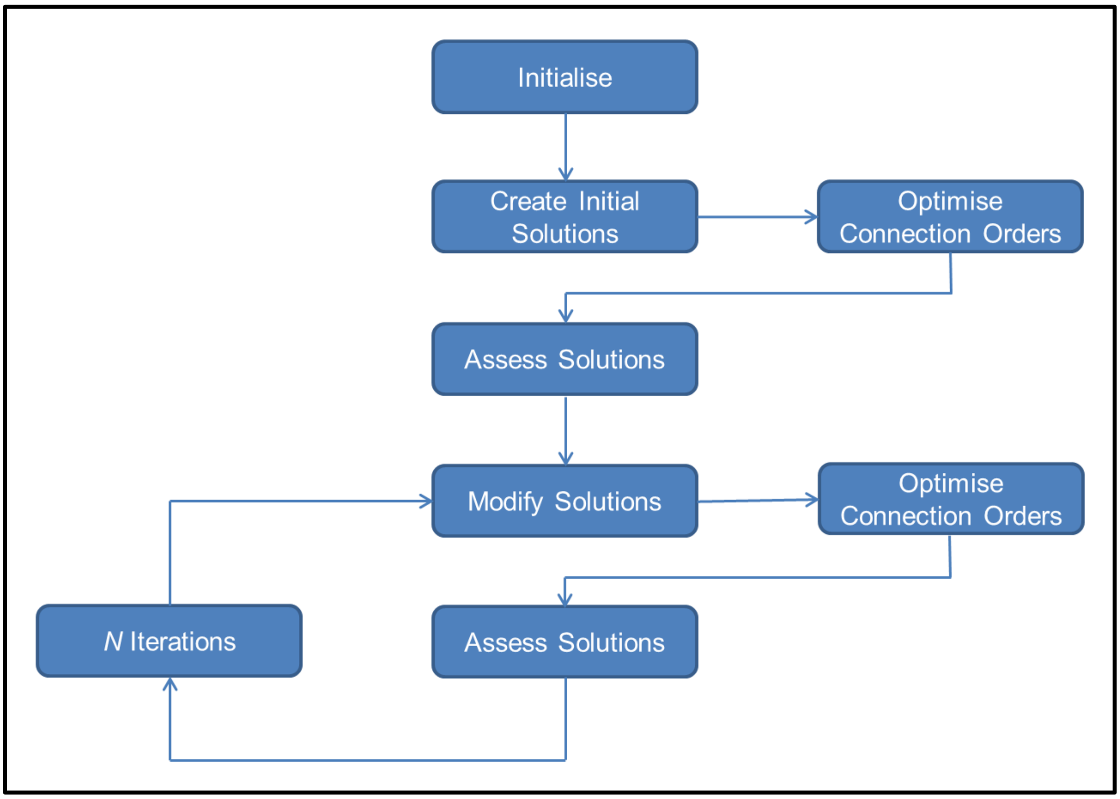


Figure 29 - Optimisation Algorithm

4.6.1 Network Specification Problem

Within Chapters 2 and 3 of this thesis, a number of techniques that may be applied to the network specification problem have been presented and their suitability for this application discussed. The selection of a technique for this problem may not be made without consideration of a number of factors which limit the range of potential options. The primary factor which restricts the choice of technique for this application is the inclusion of topology within the optimisation process, abstracted into an integer representation. This approach precludes the use of a number of techniques, as has been discussed earlier in this thesis.

Consequently, a meta-heuristic technique that is capable of operating with the abstracted topology representation was sought. This category of technique may be divided into single-solution meta-heuristics and population based meta-heuristics, with single-solution techniques generally superior for thorough searches of a narrower search space; and population based techniques better suited to a shallower search of wider search space [73]. Owing to the multi-dimensional nature of feasible search space, population based meta-heuristic methods have strong applicability for this problem.

Population based techniques may be further subdivided into evolutionary techniques, such as genetic algorithms, and swarm intelligence, such as PSO. Despite the difference in how these methods reach a solution, both are capable of operating with all desired design variables, including the abstracted topology parameter.

The prevalent trend within published research into computational optimisation of subsea electrical networks has been for the use of evolutionary algorithm techniques to optimise the network specification problem, mainly in the form of genetic algorithms [21, 26, 42]. It was decided to demonstrate the viability of a swarm intelligence method, as a rival technique to evolutionary algorithm methods. At the time of writing, no published work within this field had been found that utilised this technique, for the purposes of subsea electrical network optimisation.

The network specification problem is optimised meta-heuristically through the use of PSO; the concept of which has previously been discussed within this thesis. While PSO has not been used for the purposes of high-level network design for offshore renewable projects; it has been utilised effectively in many adjacent fields, for example: the optimisation of an offshore wind farm site layout [96]; the optimisation of control of DFIG based wind generation [97]; and for a number of power system optimisation applications [98].

The number of parameters within the optimisation, and consequently the number of dimensions in the PSO search space, for the problems considered within this thesis, is governed by the number of offshore substations. Each offshore substation and associated collector network included in the system increases the number of parameters, and consequently the number of search space dimensions, by 4 (x-coordinate, y-coordinate, network topology, number of concurrent strings) while the transmission network always has a single parameter (network topology).

Each dimension within the search space requires values for the minimum and maximum acceptable position, and the maximum velocity that a particle may possess. The minima and maxima values are typically user specified, while the maximum dimensional velocity is calculated as half of the range between the minimum and maximum acceptable position for that dimension. This results in the minimum and maximum position vectors and the maximum velocity vector possessing the same dimensions and configuration as described in the previous paragraph.

A number of particles are created, each of which is a solution to the network optimisation problem. Current position, current velocity, last position and last velocity vectors are created for each particle with each of these vectors being of the same dimensions and following the same configuration as described previously within this section.

A particle's initial position, for each parameter or search space dimension, is generated randomly between the minimum and maximum acceptable values.

At this stage, positional and topological information is provided as an input to the connection ordering problem which seeks to return an optimal collector network arrangement. This arrangement, in parallel with the network specification created feeds into the network performance problem which calculates the reactive power compensation required, component capacities and the losses incurred within the network. The optimal collector network arrangement problem and the power flow and cost models are discussed in the following sections.

Each particle is then assessed against the global optimal solution, and the local optimal solution for that particle. These values, along with the particle's current position, are used along with a random component to calculate a velocity vector, in accordance with (43), where: $\boldsymbol{V_{NEW}}$ represents the vector of new velocity of the particle; $\boldsymbol{V_{CURRENT}}$ represents the previous velocity of the particle; *C1* is the weighting applied to the differential between a particle's local optimal solution and its current position; *r* is a random component between zero and one; $\boldsymbol{P_{BEST}}$ is the (current) local best position of a given particle; $\boldsymbol{P_{CURRENT}}$ is the current position of the particle; *C2* is the weighting applied to the differential between the global optimal solution and the current position of a given particle; and $\boldsymbol{G_{BEST}}$ is the (current) global optimal solution.

$$\boldsymbol{V_{NEW}} = \boldsymbol{V_{CURRENT}} + C1 \times r\,(\boldsymbol{P_{BEST}} - \boldsymbol{P_{CURRENT}}) + C2 \times r(\boldsymbol{G_{BEST}} - \boldsymbol{P_{CURRENT}}) \quad (43)$$

The particle's current position and new velocity vectors are summed to create the particle's new position vector, in accordance with (44). Before the next iteration of the algorithm, the *new* velocity and position vectors overwrite the *current* velocity and position vectors.

$$\boldsymbol{P_{NEW}} = \boldsymbol{P_{CURRENT}} + \boldsymbol{V_{NEW}} \quad (44)$$

During each assessment phase, the global optimal solution and local optimal solution of each particle are updated, if the current solution performs better against the objective function than the existing optimal, as conceptualised in Figure 23. The updating of these parameters, stored within the network specification problem, requires information provided by the network performance problem, closing the inter-problem information exchange loop generalised in Figure 26 and Figure 28.

After each iteration, the maximum velocity of a given particle may be reduced by a specified percentage to encourage a more detailed analysis of the area of search space immediately surrounding strong solutions. This limited exploration of a small area of the search space can allow strong solutions to be further refined, but has the disadvantage of restraining a particle from fully traversing the search space, save for the first few iterations when its maximum velocity is still high. This gave extremely accurate solutions when test case problems were being considered, with deployment tolerances in the millimetre range. Equally, the reduction in maximum velocity can be set to zero, such that each particle is free to traverse the entire search space. This was found to be more useful for the configurations considered in the case studies where a narrower search of a broader area was more useful than a deeper search of a reduced area.

The process of assessing solutions and applying a velocity, thus moving the solution through the search space and re-assessing is repeated a number of times. The stopping criteria used within this work are: reaching the user-specified number of iterations; and reaching a user-specified number of iterations without an improvement in the globally optimal solution. Similarly, a tolerance based stopping criteria could be implemented. Under such an arrangement, the algorithm will stop if the improvement in the globally optimal solution is less than a specified tolerance over a defined number of iterations.

The PSO algorithm was implemented in the MATLAB programming language; with an implementation focused description presented as Appendix F.

4.6.2 Connection Ordering Problem

The second problem depicted in Figure 26 (the *internal problem* within Figure 28) is that of connection ordering; taking a high level network description as the input and delivering a detailed network model as the output.

Generators are fixed entities within the optimisation model; defined by their real power output capacity, reactive power output range and location. This differs in comparison with other objects, such as offshore transformers and converter stations, for which the locations are optimisation variables and capacities are calculated. Each generator is automatically assigned to its closest offshore substation.

As has been stated previously, the number of permutations that would have to be assessed precludes the use of a *brute force* approach to solving the connection ordering problem. Consequently, a more intelligent and efficient approach is required.

Potential methods that may be employed for this task that have been identified include a genetic algorithm based TSP solver [18, 28], and the use of a network analysis technique such as Dijkstra's Algorithm [19, 20]. A GA based modified-TSP solver methodology was chosen to be implemented for this work as the solutions from the algorithm naturally form radial and ring networks, while the network returned by network analysis may be of mixed or meshed topology. Further, network analysis methods require modification to create high-performing ring networks, as they naturally operate on a unidirectional basis, hence they are better suited to the creation of radial collector networks.

The travelling salesman problem (TSP) is typically a distance, or cost of travel, minimisation. It requires the production of completed directed graph such that each of *N cities* (nodes) is visited exactly once, by one or more travelling salesman making *tours.*

A matrix, $\boldsymbol{x}_{ab}$, represents the status of connections that may be made between nodes. If a connection is made between *city a* and *city b* then this element is represented by a one; otherwise all elements are zero, symbolising no connection. This is detailed mathematically as (45) [99].

$$\boldsymbol{x}_{ab} = \begin{cases} 1 \ \ if \ arc(a,b) is \ in \ tour \\ 0 \qquad\qquad otherwise \end{cases} \tag{45}$$

A further matrix, $\boldsymbol{c}_{ab}$, represents the *cost* (typically financial cost or distance) of connection that may be made between each combination of nodes. The objective function is then to minimise the product of costs and defined connections subject to the constraints that each node in the set is entered and exited precisely once. This is defined mathematically as (46-49) [99].

$$\min \sum\nolimits_{ab} \boldsymbol{c_{ab}} \boldsymbol{x_{ab}}$$

(46)

$$s.t. \sum\nolimits_{a} \boldsymbol{x_{ab}} = 1 \qquad \forall_{a}$$

(47)

$$\sum\nolimits_{b} \boldsymbol{x_{ba}} = 1 \qquad \forall_{b}$$

(48)

$$0 \leq \boldsymbol{x_{ab}} \leq 1 \qquad \boldsymbol{x_{ab}} \in Z \tag{49}$$

In addition to *how* solutions are generated, there is also a question around the optimisation objective that should be selected. As has been discussed previously, it is feasible to use a modified TSP objective function to seek minimum cabling distance; lowest electrical losses; or minimum cost.

If a single string emanates from an offshore substation, and all generators are nominally identical, then the problem becomes analogous to the traditional TSP and, as such, minimum distance may be employed as the fitness function. Utilising a TSP approach with multiple strings is problematic, however, as this will not take into account the loading on each cable, with cable cost a function of required capacity and electrical losses a function of current flow; instead it will seek the solution that utilises the shortest cable length.

Through-life cost could be used as the fitness function and this would give the most accurate reflection of the best performing network, particularly when the global ambition of the optimisation is the minimisation of through-life costs. In order to facilitate such a study, a full power flow must be established, executed and the associated costs calculated. While this solution would offer improved fidelity, it incurs significant computational cost, as networks scale, to the point where the simulation time becomes unacceptable.

Ultimately, lowest electrical losses was used as a compromise objective when optimising the connection ordering problem. This gave a more representative solution than minimum cabling distance which failed to account for loading of cables; and did not incur the significant computational expensive associated with the establishment and execution of repeated power flow studies such that minimum cost could be utilised as a solution. Mathematically, this modified TSP approach may be defined as (46-49) but now the cost associated with travelling from node to node, $\boldsymbol{c_{ab}}$, is defined in accordance with (50) where i_{ab} is the current flow into the branch linking nodes *a* and *b*; and Z_{ab} is the impedance between the same node pair.

$$c_{ab} = {i_{ab}}^2 Z_{ab} \tag{50}$$

Initially, a number of random connection orders were generated, as were randomised break points. These break points divide the connection order into distinct strings. The population of solutions is assessed against a fitness criterion of lowest electrical losses with some of the strongest maintained and most of weaker solutions subjected to crossover, mutation and reproduction in order to produce the next generation of solutions. This process is repeated a number of times, at the end of which a solution is returned.

The GA algorithm was implemented in the MATLAB programming language; with an implementation focused description presented as Appendix F.

Within the decision vector, $\boldsymbol{x}$, that was defined in (40); the final entries relate to the connection order of each collector network, CO_i, and the transmission network, *TO*. These parameters are optimised within the solution to the connection ordering problem and are fed, along with parameters generated by the network specification problem, into the network performance problem

4.6.3 Assessment of Network Performance

The final problem depicted in Figure 26 is the assessment of the candidate network's performance. For this purpose, an AC power flow study is undertaken and the solution through-life cost is calculated, based on representative approximations of probable power outputs, generated using a Weibull distribution, as has been described earlier in this thesis. A DC power flow would reduce the level of computational effort associated with the optimisation, but the drop in fidelity associated with this was deemed too severe; given the considerable reactive power compensations challenges associated with subsea electrical networks, arising from the high capacitance of the cables.

The use of through-life costing based on probabilistic output differs from a number of contemporary studies that calculate the losses based on a single representative average power output [22, 26], which incurs a loss in fidelity due to the non-linearity of electrical losses.

While the network specification and connection ordering problems are related to the creation of solutions, neither of these aspects of the optimisation provides any assessment of the generated solution. The purpose of network performance problem section of the optimisation process is to generate a model of the network under consideration, execute the power flow and return the results and associated costs to the PSO algorithm, as depicted in Figure 29.

Offshore substations are deployed at specified positions, and the optimal connection order at each offshore substation is used to calculate the physical cable characteristics (resistance, reactance, susceptance and number of failures per year) of the collector networks. The transmission network is implemented at this stage in a similar fashion. Once the network model is complete, the power flow analysis may be undertaken. The power flow analyses undertaken for this work simulate a number of *snapshots* of network performance where each generator is outputting power based on a Weibull distribution.

If the solution uses an AC collector network with an AC transmission component, a complete power flow study using the entire network model is executed. If the solution uses AC collector networks with a DC transmission component, power flows are executed for each collector network before the transmission performance is considered using a separate DC network model.

Once the power flow studies have been completed, the capacity required at each component may be calculated. A number of the relevant components that have cost functions that are dependent on capacity; hence the requirement to cost the network after the power flow study has been undertaken.

Where a radial network is employed, the cable capacity is tapered such that sections of cable conveying less power are not as capacious, and therefore expensive, as those conveying higher power levels. Conversely, for a ring network, each section of the cable must be capable of conveying the total power output of all generators attached to that cable.

Installation Costs

As has been discussed previously within this thesis, there is a limited amount of component cost models available, with most published work in this field [21, 31, 61] building upon or directly utilising, cost approximation functions developed by Lundberg [81].

In a number of cases, Lundberg's approximations are based on a limited range of discrete voltage ratings. In order to make these models more widely applicable, lines of best fit for the discrete data were calculated and a more general approximation offered. Where cost approximations are offered in Swedish Krone, an exchange rate of 8.98:1 was applied to convert to Euros.

The cost of AC submarine cabling is dependent on the operating voltage and the cross-sectional area of the cable, which governs the electrical throughput of the cable. This may be observed in (51), where the AC cost function, in Swedish Krone per kilometre, that is used to approximated the cost of AC cabling for this work is presented as a function of voltage dependent coefficients (A_p, B_p and C_p; approximated by (52-54)), and cable capacity, S_n. The A_P, B_P and C_P parameters have R-squared values of 0.9921, 0.9404 and 0.96 respectively, thus allowing for the cost of any cable, of any voltage to be approximated with a high degree of accuracy.

$$C_{CABLE,AC} = A_P + B_P e^{\left(\frac{C_P S_n}{10^8}\right)} \tag{51}$$

$$A_P(V) = 15.132\, V - 131227 \tag{52}$$

$$B_P(V) = 860735\, e^{-9\times10^{-6}\, V} \tag{53}$$

$$C_P(V) = 45.713\, \left(\frac{V}{1000}\right)^{-0.693} \tag{54}$$

Similarly, Lundberg presents the cost function approximation for DC submarine cabling as shown in (55), and this cost function has been utilised for this research [81]. The cost is given in Swedish Krone per km with A_p and B_p voltage depended coefficients, as approximated by (56-57) and P_n is the rated power of the cable in VA. The R-squared values for A_P and B_P are both 0.9998.

$$C_{CABLE,DC} = A_P + B_P P_n \tag{55}$$

$$A_P(V) = 3.8982\, \left(\frac{V}{1000}\right)^2 + 0.972\, V - 354504 \tag{56}$$

$$B_P(V) = 1.8036\, \left(\frac{V}{1000}\right)^{-0.92} \tag{57}$$

Irrespective of whether an installed cable is AC or DC, an approximation for the cost of installation is required. A range of disparate constant values to approximate the cost of installation were presented in the literature, as described earlier in this thesis, hence it was chosen to utilise an installation cost per km variable, $Cable_{INST}$, in preference to a hard-coded approximation for this work.

Dicorato et al's transformer cost approximations have been used for this work, with distinct models used depending on the size of the transformer. This is presented as (58) where $C_{TRANSFORMER}$ represents the cost of the transformer in k€ and P_{RATED} is the rated power of the transformer in MVA [61].

$$C_{TRANSFORMER} = \begin{Bmatrix} = 153.05 + 131.1 P_{RATED}{}^{0.4473} \\ 42.688\ P_{RATED}{}^{0.7513} \end{Bmatrix} \quad \begin{matrix} P_{RATED} \leq 150 \\ P_{RATED} > 150 \end{matrix} \tag{58}$$

Where an HVDC convertor is employed in preference to a transformer, a cost of 110 k€ / MVA is employed. This is approximated from Lundberg's value of 1 Swedish Krone per VA of rated capacity [81].

In order to link subsea cables with other plant, switchgear is required. Where this switchgear is AC, Dicorato et al's approximation, as presented in (59), is used for this work. For (59), $C_{SWITCHGEAR,AC}$ represents the cost of the switchgear in k€; and Vn represents the nominal voltage in Volts.

$$C_{SWITCHGEAR,AC} = 40.543 + 0.00076\, V_n \tag{59}$$

In the absence of an explicit DC switchgear cost approximation being found in the relevant literature, Stamatiou et al's assertion that DC switchgear is typically twice as expensive as AC switchgear was used for this research [82].

The cost of offshore substations, exclusive of transformers or HVDC convertors, has been modelled using Dicorato's approximation, as defined by (60), where $C_{SUBSTATION}$ is the cost of the substation in k€, and S_{TOTAL} is the combined (apparent) power throughput in MVA [61].

$$C_{SUBSTATION} = 2534 + 88.7\, S_{TOTAL} \tag{60}$$

The power flow results also indicate where reactive power compensation is required to maintain system voltage; the cost of which may then be calculated. Dicorato's approximation of 77 k€ / MVAr for Static VAR Compensation (SVC) was utilised within this work to approximate this cost [61].

It has been assumed, for this work, that the land based network with which the subsea network will interface is AC and operating at a standard frequency of either 50 or 60 Hz. If the subsea network's transmission system is also AC and operating at a standard frequency, they may be interfaced using a transformer, and this is costed accordingly. If the subsea transmission system is AC but operating at an LFAC frequency, typically one-third of a standard AC frequency, then back-to-back AC-DC-AC conversion is employed, and the interface cost is calculated based on this. Finally, if DC is utilised for subsea transmission then a single DC-AC conversion stage is required to interface with the land based network, and this is priced accordingly.

Losses Cost

Once the capital costs have been identified for the solution, losses costs are then calculated. A Weibull Distribution is used to approximate the proportion of time, across the lifetime of the site, for which each generator is receiving discrete levels of input energy resource. It is assumed that the input to each generator (wind for this work) is identical across a given site, such that the output of each generator is identical. The cut-in, rated, and cut-out speed of the turbines is then used to approximate the output power of each turbine that corresponds to each wind level.

A power flow study is undertaken at each wind state and the losses associated with that wind state, L_{WS}, are calculated. It is not feasible to work out an average wind speed and calculate losses based on that, owing to the non-linearity of electrical losses within a network. Combining the losses experienced at a given power output with the anticipated proportion of time spent at that wind state, P_{WS}, allows the magnitude of losses to be approximated, from which the cost of these losses may be derived by multiplying by the price per unit of electricity, *EP*. As the losses will occur throughout the lifetime of the project, *PL* years, these must be normalised to reflect the value of money at the time of network installation, using an appropriate discount rate, *DR*. Mathematically, the cost of through-life losses may be defined as (61); and this value directly feeds into the through-life cost minimisation objective defined as (41).

$$C_{LOSSES} = \sum_{n=0}^{PL} \frac{EP \times \sum L_{WS} P_{WS}}{(1 + DR)^n}$$

(61)

No-Service Costs

While it is not feasible to use an average power output for each generator to calculate through-life losses, this is acceptable for the quantification of lifecycle no-service costs. Consequently, an average power output of the turbines is derived, and the losses experienced within the network, L_{AVP}, with all cables and plant operational, is calculated.

The failure rate of each cable type is defined on a per year, per kilometre basis. By considering the failure rate and the length of the cable, the number of outages for each cable, FR_m, may be approximated. Each cable is systematically withdrawn from service and a power flow study run where the losses incurred by operating without cable *m* in service, L_{AVm} may be calculated. These differential losses may be multiplied by the likelihood of failure; the anticipated time to repair (*MTTR*) and the electricity price to give a total value of electrical losses per year. As the electricity price is usually defined on an hourly basis while MTTR is usually defined in days, this value requires to be multiplied by 24 to maintain consistency of units.

As was the case with the monetisation of electrical losses, cost of no-service conditions must be normalised into *Year 0* values, hence a discount rate, *DR*, is used to calculate the current value of no-service conditions across the life time of the project, *PL* years. Mathematically, the cost of no-service conditions may then be expressed as (62); which feeds directly into the lifecycle cost minimisation objective defined within (41).

$$C_{N-S} = \sum_{n=0}^{PL} \frac{24 \times EP \times MTTR \times \sum FR_m (L_{AVP} - L_{AVm})}{(1+DR)^n} \tag{62}$$

Failures of offshore substations are considered within the same process as described previously, with the appropriate MTTR value for offshore substations being used in preference to that for cables.

In keeping with a number of peer-reviewed studies, only independent component failures are considered [29-31, 43, 82].

The outcome of the network performance problem is fed back into the network specification PSO algorithm such that the quality of the solution may be assessed and the local and global optimal solutions updated if necessary.

4.7 Conceptual Solution

Within the preceding sections, the optimisation performed is described and defined; both mathematically and in terms of technique based implementation. Ultimately, the solution that is returned, at the end of the optimisation process, takes the form of the decision vector, $\boldsymbol{x}$, as defined in (37). To aid contextualisation of the decision vector, it has been overlaid onto a conceptual network, as shown in Figure 30.

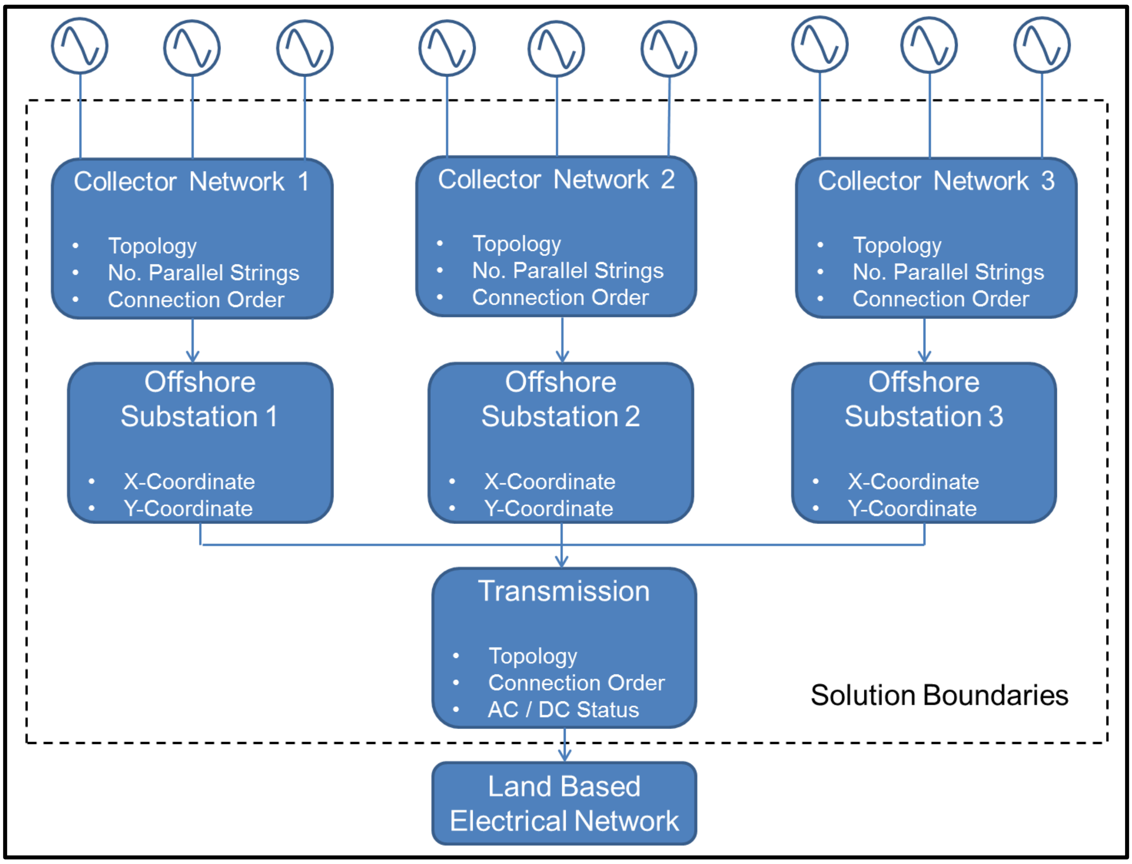


Figure 30 - Decision Vector Overlaid on Network Abstraction

Figure 30 abstracts the most complicated case that is considered within the Case Studies section of this thesis, where 3 parallel substations are utilised to interface collector networks with the transmission system.

Boundaries, defined by the decision domain: $\boldsymbol{\Omega}$, as defined in (37); that have been applied to the decision vector, $\boldsymbol{x}$, do not form part of the final solution, hence they are not shown within Figure 30. Similarly, fixed design parameters: $\boldsymbol{y}$, as defined in (40); shape the final solution but do not form part of it, hence they are not shown in Figure 30.

4.8 Model Validation

In order to validate the developed optimisation model, a number of test cases with readily discernible solutions were simulated and the results analysed. As such, it was decided to conduct the model validation studies as minimum loss simulations, as the optimal solution is much easier to discern than for a through-life cost solution.

For these test cases, networks were sought for the implementation of 1, 2 and 3 offshore generators, positioned as detailed in Table 8 and depicted in Figure 31. Test case 1 seeks to implement a network for the conveyance of power from generator 1, as defined in Table 8; test case 2 for generators 1 and 2; and test case 3 for all 3 generators defined in Table 8.

Generator ID	X Pos (km)	Y Pos (km)	Pout (MW)	Reactive Power Limits (MVAr)	
1	-5	10	4	-2	2
2	5	10	4	-2	2
3	0	15	4	-2	2

Table 8 - Generator Data for Model Validation Test Cases

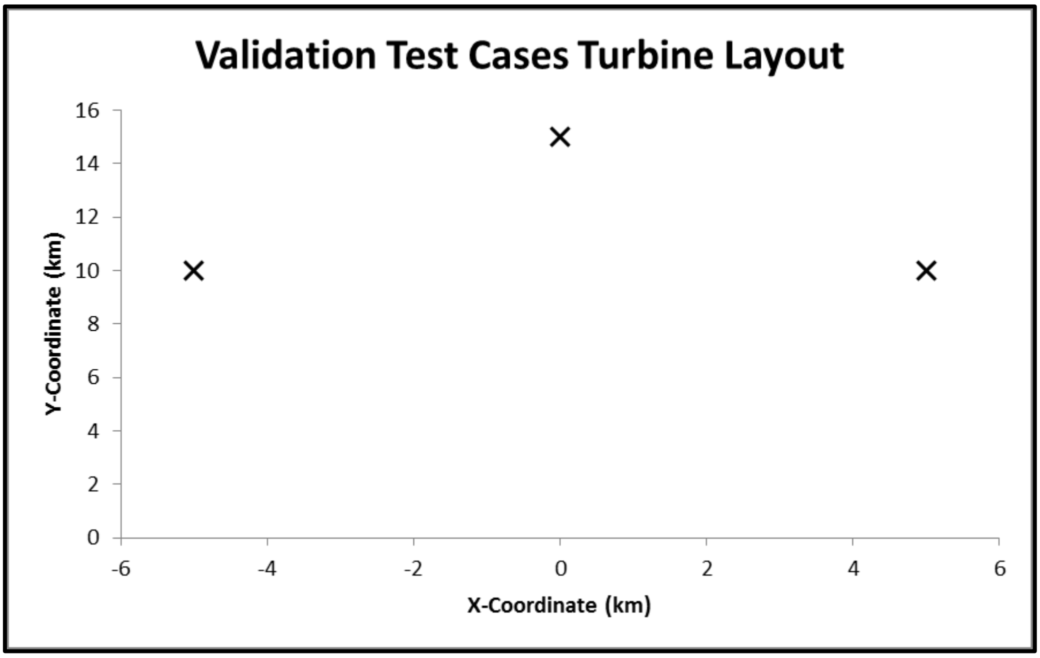


Figure 31 - Validation Test Cases Turbine Layout

The cable data used for this test case is presented as Table 37 within Appendix A. The physical characteristics (columns 3-5) of both cable types are taken from a prominent manufacturer of subsea electrical cables [100].

For each case, the extremities of the sea space in which components could be placed was -6.5 to +6.5 km on the x-axis and 0 to 20 km on the y-axis, yielding a total sea space of 260 km^2. The costs associated with installation of cables are constant across the possible sea space.

Each test case was considered firstly on a *sub-problem* basis, where each combination of topologies of both transmission and collector systems was specified and analysed sequentially. The optimisation, for each test case, was then performed again with the topology selection to be mandated by the optimisation algorithm.

For each test case, the developed optimisation process may produce solutions which are within millimetres of the known optimal solutions. In reality, such a level of accuracy would not be required as the tolerance that may be achieved for object positioning is likely to be several orders of magnitude greater than this.

The complete list of optimisation parameters used in this case is presented in Appendix A.

4.8.1 Test Case 1: Single Collector, Single Generator AC-AC System

The simplest test case involves seeking an optimal solution to a single collector network, single generator AC-AC system. It is recognised that such a case would not require the presence of an offshore substation and the generator would be connected directly to the on-shore network. However, as a test of the optimisation process, with a readily discernible expected outcome, it was considered worthwhile. The total cost of the solution is ignored for the purposes of this assessment, as is the reliability of the network, such that an optimal solution is sought based only on minimising electrical losses.

Consequently, it was expected that the returned outcome would be an optimal offshore substation position with x and y co-ordinates exactly matching the generator location. Each combination of transmission and collector topologies was considered sequentially. As identified in the previous section, the optimal location for each scenario is identical and is located at (-5, 10). The positional error between the solution returned and the known optimal solution, is presented in Table 9. After each iteration the maximum positional change (in both x and y directions) that a collector may be moved is reduced by 1 % such that good solutions are obtained.

Topologies		Positional Error wrt Optimal (m)	Iterations
Transmission	Collection		
Radial	Radial	3.33×10^{-3}	1006
Radial	Ring	3.97×10^{-4}	1294
Radial	Star	1.51×10^{-4}	1812
Ring	Radial	1.50×10^{-4}	1396
Ring	Ring	3.02×10^{-4}	1939
Ring	Star	2.53×10^{-3}	1045
Star	Radial	1.51×10^{-4}	2168
Star	Ring	3.14×10^{-4}	1569
Star	Star	2.58×10^{-4}	1201

Table 9 - Optimisation Results for Test Case 1

For each possible combination of transmission and collector topologies, the offshore substation is positioned within millimetres of the optimal. As has been described previously within this thesis, meta-heuristic optimisation techniques offer a compromise between the quality of solution and the time taken to arrive at the solution; with a high quality but not necessarily optimal solution arrived at within an acceptable simulation time.

The distance between the obtained collector position and the optimal position range from 0.15 mm to 3.33 mm against an optimal transmission cable length of 11.18 km and a search space of 260 km^2. This demonstrates that the algorithm works effectively at returning strong solutions to this simple optimisation problem.

Given the reliance of the PSO algorithm on stochastic elements to generate movements within the search space, repeating the test runs that were used to produce the data shown in Table 9 would return results that are slightly different, but of a similar standard.

For this case, given that the optimisation objective is lowest losses, it follows logically that this may be achieved by minimising the current flow on each branch; thus the use of a ring network for both transmission and collector stages will minimise the electrical losses incurred. When this simulation was re-run with the optimisation algorithm left to define the optimal topology, the returned solution was as is shown in Table 10. The solution is of slightly lesser quality than that returned when both collector and transmission networks are defined as ring based; however, the positional error of the offshore substation with respect to the known optimal position is 0.92 cm which is still likely to be within the deployment tolerance of offshore plant.

Parameter	**Value**	
	X	**Y**
Returned Substation Position (km)	-5.0000	10.0000
Transmission Network Type	Ring	
Transmission Connection Order	Col 1	
Collector Network Type	Ring	
Positional Error wrt Optimal (m)	0.0092	
Number of Iterations	2067	

Table 10 - Test Case 1 Optimisation Result

The solution detailed in Table 10 took 2067 iterations to reach, representing a 6.6 % increase compared with the case where the optimisation was only to calculate the position of the offshore substation to minimise electrical losses.

4.8.2 Test Case 2: Two Generator AC-AC System with Radial Transmission

The second test case considers a two generator AC-AC system with generators as shown in the first 2 rows of Table 8. A number of simulations were run, using loss minimisation as the objective function, under various constraints and these are now presented and selected results discussed.

In order to reduce the number of variants to be simulated, in each case the transmission system is restricted to being radial only. The transmission stage of the network is optimised using the same process as the collector network; hence it is safe to assume that if the optimisation works as intended for the more complex collector networks, it will also work correctly for the simpler transmission stage.

Shown in Table 11 is a summary of the total positional errors associated with the deployment of offshore collector platforms. The positioning of the collector platforms only provides partial optimality; this must be coupled with the correct connection order to ensure that the solution is optimal. In each of the cases summarised in Table 11, the solution returned has the optimal connection order.

Each of the solutions detailed in Table 11 are discussed in Appendix B. A number of these cases (Variants 1, 3, 6, 7 and 8) have optimal solutions that require the placement of offshore substations at zero distance from one, or more, of the offshore substations. In each of these cases, the positional error in the deployment of these offshore substations, in the range of fractions of a millimetre, is of significantly smaller magnitude than the achievable deployment tolerance.

For variants 2, 4 and 5, the optimal deployment positions have been calculated using the connection order that has been returned for that solution, along with the levels of reactive power mandated to balance the system. Models of the losses, as functions of collector y-coordinate, were then produced. For these cases, the positional error returned is several orders of magnitude higher than for the other variants; however, these values are still small when compared to the 260 km^2 deployable space.

Variant	Collectors	Strings	Collector Topology	Positional Error (m)	Iterations
1	1	1	Radial	7.54×10^{-5}	2976
2	1	2	Radial	2.3	1929
3	1	1	Ring	2.69×10^{-7}	2885
4	1	2	Ring	0.67	2652
5	1	N/A	Star	2.3	1756
6	2	1	Radial	1.75×10^{-5}	1936
7	2	1	Ring	2.65×10^{-5}	1936
8	2	N/A	Star	4.37×10^{-5}	1545

Table 11 - Summary of Optimisation Results for Test Case 2

Validation of the result obtained for Variant 4 is shown in Figure 32. While the solution returned is 0.67 m from the calculated optimal deployment position, there is only an insignificant impact in terms of differential electrical losses. As may be observed in Figure 32, the differential losses incurred by deploying the offshore substation anywhere in the y-coordinate range of 9.3 km to 9.4 km is 2.5 W (0.01 % of losses when the offshore substation is optimally deployed); demonstrating the sensitivity and accuracy of the utilised optimisation process.

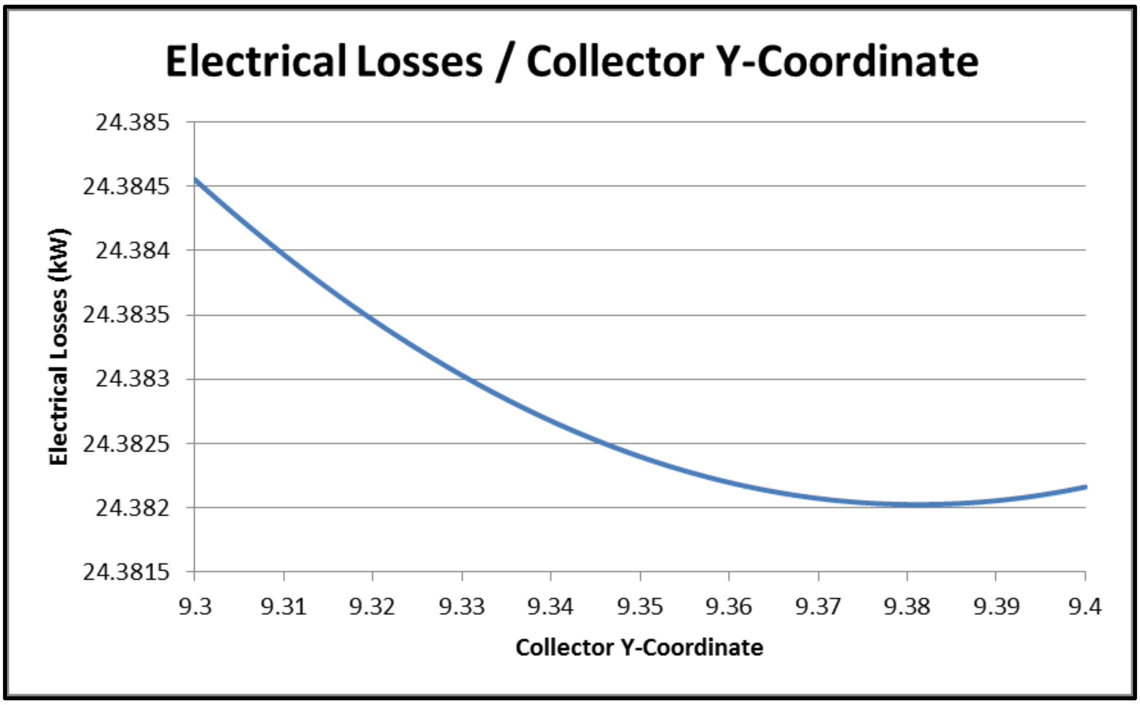


Figure 32 - Electrical Losses and Collector Y-Coordinate for Test Case 2 - Variant 4

In addition to testing each variant defined in Table 11 sequentially, this simulation was re-run with the optimisation algorithm left to mandate the optimal network configuration, in addition to the position of the offshore substation. Given that a radial transmission system has been mandated, it follows that the electrical losses may be minimised by placing offshore substations at zero distance from each of the turbines, and constructing ring based collector networks. This expected outcome was borne out, as detailed in Table 12, where it may be observed that this network configuration was returned, albeit with a positional error that is orders of magnitude higher than when the collector network topology was mandated. That said, the positional error with respect to the known optimal locations is still less than 3 cm, which is likely to be within the achievable positional tolerance of offshore plant.

Further, it may be observed in Table 12 that the number of iterations required to converge on a solution increased by approximately 19 %, relative to the case where the network topologies were specified.

Parameter	Value	
	X	Y
Substation Position 1 (km)	-5.0000	10.0000
Substation Position 2 (km)	5.0000	10.0000
Transmission Network Type	Ring	
Transmission Connection Order	Col 1	
Collector Network 1 Type	Ring	
Collector Network 2 Type	Ring	
Positional Error wrt Optimal (m)	0.029	
Number of Iterations	2303	

Table 12 - Test Case 2 Optimisation Result

4.8.3 Test Case 3: Three Generator AC-AC System with Radial Transmission

The third test case involves the construction of networks around three identical offshore generators, the details of which are presented in Table 8.

Within this test case, a number of variants with specified constraints were considered. The solutions are presented, and analysed, in Appendix C, and summarised within Table 13. As was the case for Test Case 2, there are two components that must both be present for optimality to be achieved: the positioning of components at the correct location(s); and the most efficient ordering of connection. For all the cases summarised within Table 13, the connection orders are optimal, as may be observed in Appendix C.

Given the inherent symmetry of the turbine layout considered for this test case, it follows that in a number of cases there are *mirrored* optimal solutions.

In the majority of cases summarised within Table 13, the optimal solution involves the placement of offshore collectors at zero distance from one or more generators. The only variants where this did not provide the optimal solution, from a loss minimisation perspective, were where there were 3 strings emanating from a single offshore substation (Variants 3, 6 and 7). In this instance, irrespective of collector network topology, it became more efficient to deploy an offshore substation positioned towards the centre of the turbine cluster.

Variant	Collectors	Strings	Collector Topology	Positional Error (m)	Iterations
1	1	1	Radial	2.764×10^{-5}	2972
2	1	2	Radial	0.205	1925
3	1	3	Radial	0.64	1925
4	1	1	Ring	0.183	2020
5	1	2	Ring	0.04	2335
6	1	3	Ring	0.16	1828
7	1	N/A	Star	0.59	1884
8	2	1	Radial	0.05	1744
9	2	1	Ring	0.024	1355
10	2	N/A	Star	0.01	2237
11	3	1	Radial	0.048	1794
12	3	1	Ring	0.063	1995
13	3	N/A	Star	0.052	1646

Table 13 - Summary of Optimisation Results for Test Case 2

In order to validate variants 3, 6 and 7, mathematical models of electrical losses as functions of collector y-coordinate were developed. These models used the generator connection order returned, along with the levels of reactive power compensation mandated. In reality, the reactance of the network is a function of cable length, and as such the levels of reactive power compensation will change as the offshore substation moves, as the transmission and collector cables lengths will alter.

The result of the mathematical modelling validation of Variant 3 of Test Case 3 is presented as Figure 33. For this case, the positional error between the deployed position and the calculated optimal position was 0.64 m, in a 260 km^2 possible deployment space.

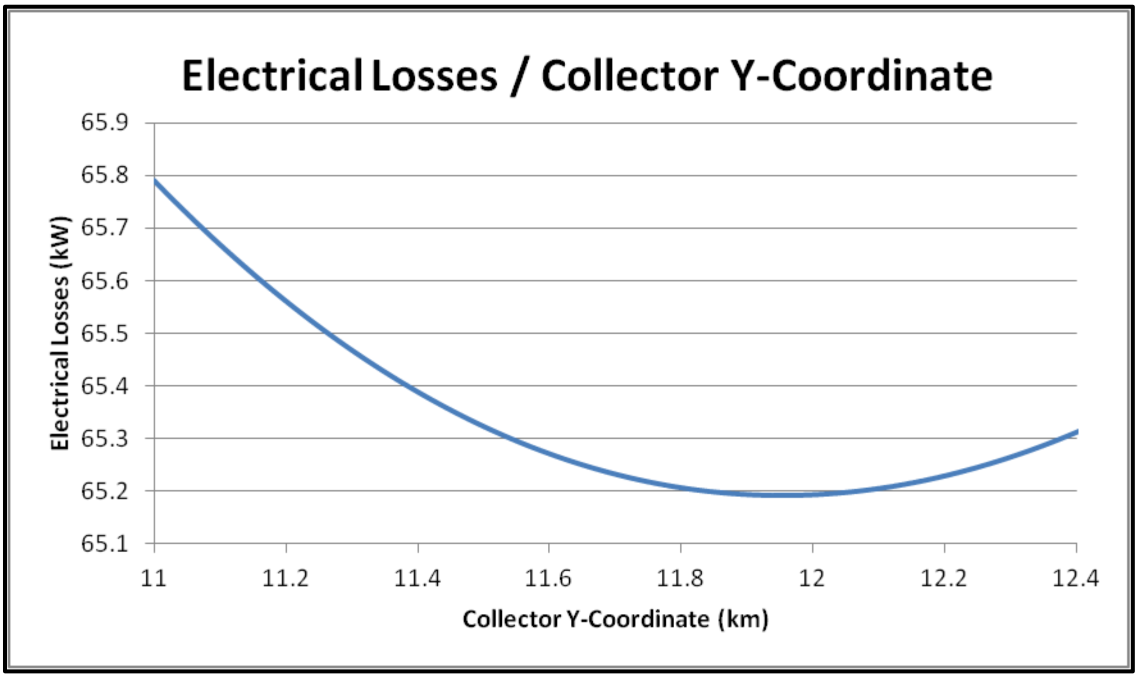


Figure 33 - Electrical Losses and Collector Y-Coordinate for Test Case 3 – Variant 3

The optimal offshore substation y-coordinate for Test Case 3, Variant 3 was calculated to be 11.953 km, which was returned with an error of 0.005 %. From Figure 33, it may be observed that deploying the offshore substation anywhere within 0.4 km of the optimal solution (in either direction) incurs additional losses of no more than 100 W, or 0.15 % of the losses recorded when the offshore substation is deployed at its calculated optimal position.

Test Case 3 was re-simulated with the network topology selection left to the optimisation framework to mandate. In a similar fashion to Test Case 2, losses may be minimised by placing the offshore substations at zero distance from the turbines, and using ring collector networks. This result may be observed in Table 14, where it may be further observed that the positional error of the substation deployment has increased by 12.7 %, but is still a combined 7.1 cm for the positioning of 3 offshore substations within a 260 km^2 search space. The number of iterations required to converge on this solution is approximately 33 % higher for the open topology simulation than when the topology is mandated.

Parameter	Value	
	X	Y
Substation Position 1 (km)	0.0000	15.0000
Substation Position 2 (km)	-5.0000	10.0000
Substation Position 3 (km)	5.0000	10.0000
Transmission Network Type	Ring	
Transmission Connection Order	Col 1	
Collector Network 1 Type	Ring	
Collector Network 2 Type	Ring	
Collector Network 3 Type	Ring	
Positional Error wrt Optimal (m)	0.071	
Number of Iterations	2652	

Table 14 - Test Case 3 Optimisation Result

4.8.4 Discussion of Test Case Results

The aim of the this body of work is to return a network design that is optimal for the specific inputs and constraints and the method by which this network is returned is through the specification of offshore substations positions in the search space, and the connection order that links each generator to a substation and each substation to the connection point with a land based network.

The test cases considered to validate the results returned range from overly simple to mildly complex. In each case, there is a positional error in the deployment of collector(s) that is typically in the range of millimetres or centimetres. Without expert knowledge of the deployment positional tolerance achievable when installing an offshore platform; it is likely that the overwhelming majority, if not all, of the positional errors returned for these test cases fall within the working tolerance of offshore plant deployment.

It is assumed that each cable linking nodes on the network is perfectly straight (minimum possible length) while in reality there will be a degree of curvature in each cable that will add some degree of additional length that will increase the resistance, reactance and susceptance of the cable. For this reason, it becomes impossible, in a real world context, to optimise an offshore substation position down to the levels returned. Cable lengths are also calculated only using two-dimensions (x and y) without considering a z-axis component that may be in the order of several metres. Without including this component, it is futile to specify the collector positions to centimetre or millimetre accuracy.

For each test case, there is no tolerance based stopping criteria applied. In cases such as this, where an optimal solution may be readily derived, the algorithm could be stopped when the returned solution falls within a specified positional error of the calculated optimal. Instead, the stopping criteria applied to these cases was reaching the maximum number of iterations, or reaching a specific number of iterations without an improvement in the returned solution. Additionally, the use of tolerance based stopping conditions may only be applied when an optimal solution is known, and it was decided that the test cases should be simulated in the same manner as the case studies.

Test Case 2, and the scenarios considered within, demonstrates that the solution that delivers the minimum cable length is not necessarily the optimal solution in terms of electrical losses. Including component costs and reliability within the optimisation will significantly alter the results obtained. The reason that only electrical losses were considered for these test cases was to allow for readily discernible optimal solutions that could be compared with delivered results.

Within Test Cases 2 and 3, the losses incurred within the network are often modelled as functions of collector y-coordinate (with collector x-coordinate set to zero). Such loss models are based upon the assumptions that the order of connections remains the same irrespective of collector positions and that the reactive power injections at each node remain the same as those returned for the optimal solution. As such the networks returned as optimal for each scenario may be validated only against these assumptions.

Where the collector position(s) remain within a small distance of the returned optimal solution, the connection order generated will remain constant and the reactive power injections will not be greatly different. When the collector position(s) are at a larger distance to the returned optimal solution, the connection order generated will be different and the reactive power injections may be considerably different to those obtained for the returned optimal solution. Nevertheless, this method demonstrates the credibility of results obtained in both Test Cases 2 and 3 as it shows the optimality of the network subject to the above assumptions.

Modelling electrical losses as a function of collector position also shows the sensitivity at which the underlying power flow engine has to operate to return optimal solutions. Test Case 2: Variant 4 returns a result that is 0.67 m from the calculated optimal solution; however the difference in electrical losses between placing the collector optimally and at the returned position is less than 1 W which represents a tiny proportion of the real power throughput of the system.

Within Test Case 3, it is evident that many solutions which are extremely *transmission heavy* are returned, in the respect that the collector is often placed at the farthest away generator. As all of the test cases are set-up to minimise electrical losses without considering either cost or reliability, it follows that networks utilising more of the higher efficiency transmission system and less of the lower efficiency (but cheaper) collector system will perform best.

The test cases presented were simulated on a *full-problem* basis where the network topologies were not mandated and on a *sub-problem* basis where the network topologies were specified sequentially from the available options. As expected, where the selection of topology is left to the optimisation algorithm, there is a slight increase in positional error of offshore substations; however, this is still likely to be within feasible deployment tolerance of offshore plant. Further, optimising in this way increased the number of iterations required to reach the returned solution, as would be expected.

4.9 Optimisation Framework Positioning

Within the preceding chapter of this thesis, a number of contemporary studies were assessed and critiqued against which optimisation characteristics were included, and how they were included. The purpose of this exercise was to define how this work could encompass all of the desirable characteristics present within the published literature; thus making the most holistic optimisation of subsea electrical networks for renewable energy.

The developed framework, described in this chapter, allows for all of the following criteria and in doing so includes **all** of the desirable characteristics that have been included within the work of others, thus producing a more holistic subsea electrical network optimisation method, than any of the contemporary studies.

- Lifecycle cost minimisation has been included as the primary optimisation objective, as generalised by (26) and defined by (51-62). Specifically, this includes the purchase and installation of equipment; the cost of electrical losses; and the cost of no-service conditions.

- Both collector and transmission network portions are optimised on a simultaneous basis, with no artificial restrictions placed on the position, technology, or topology of any component part of the network. In doing so, the full solution space is open to traversal; which is not the case when stipulations are applied to one or more network characteristics (for example, if a particular network topology, or offshore substation position is mandated, this eliminates large sections of solution space). Further, offshore plant is positioned on a continuous basis, in preference to a discretised positioning approach, which greatly limits the range of possible solutions.
- Both collector and transmission networks are free to adopt: radial, ring, or star topologies. Further, the order in which turbines are connected to form collector networks, or offshore substations are connected to form a transmission network is optimised on a loss minimisation basis.
- Subsea networks with parallel collector portions may be implemented, such that networks of higher complexity may be studied, compared with contemporary published work in this field. Further, each collector network is unique, with each free to adopt its own characteristics in terms of topology and number of parallel strings of connections.
- Transmissions network are free to take AC or DC status, in keeping with observed trends relating to the use of HVDC transmission where large offshore wind parks are being developed. Given the ubiquity of AC collector networks, there is no scope for the use of DC collector networks.
- Operating voltage and frequency of both collector and transmission stages are free to be manipulated, such that their influence may be ascertained. Within a single optimisation these parameters are fixed, in order to limit a highly expansive solution space; however, they may be varied between optimisations.
- The calculation of component sizes, and therefore cost, are based on required capacity; rather than on the basis of standard component sizes, thus eradicating the possibility of over-specification. Calculation of component costs, relating to both purchase and installation of the network, is based on cost functions that have been widely accepted, and utilised within a number of contemporary studies.
- Electrical losses, and associated cost, are calculated on the basis of a number of discrete input energy states, in preference to an average state which incurs fidelity losses due to non-linearity of losses; thus maximum accuracy is achieved.

- The cost of no-service conditions is calculated through assessing the differential losses incurred by the loss of each subsea cable, and the probability of each going out of service. While this approach requires many extra power flow studies to be conducted, it does allow for the most accurate quantification of additional losses that are incurred when a given cable is incapacitated.

By including all of the above characteristics, this body of work takes a more holistic approach to the optimisation of subsea electrical networks than any contemporary study; as no published work contains **all** of these characteristics.

4.10 Chapter Summary

This chapter begins by defining the boundaries of the optimisation; which limits the scope to optimising parameters that directly relate to the design of a subsea electrical network for a fixed offshore renewable energy project design that interfaces with an existing land-based network. This scope definition shapes the high-level design of the optimisation process used for this work which is introduced within this chapter.

Once the high-level design of the optimisation process has been established, possible design variables are discussed and their merits for selection analysed. Equally, a number of fixed design parameters are defined; thus the design vector; the decision space; and the fixed design parameters vector are promulgated. This allows the optimisation objective to be defined mathematically, forming a starting point for efforts to optimise the performance of the network.

This work seeks to deliver optimality of through-life cost minimisation of subsea electrical networks. This is realised by the division of this task into 3 interrelated problems: network specification; connection ordering; and network performance, as generalised in Figure 26 and depicted at a high-level in Figure 29. This high-level design is justified against possible rival approaches in the opening sections of this chapter.

The network specification problem is tackled using particle swarm optimisation which is used to define, and optimise, a number of high-level candidate solution network descriptions. A high-level network summary is exported to the connection ordering problem which utilises a genetic algorithm based travelling salesman problem solver with the objective function specified as minimum losses in preference to the more conventional minimum cable length objective. Once the connection order for the network has been established, a complete network model may be passed to the network performance problem.

The network performance problem utilises a number of power flow studies on the network. The results obtained from the power flow are used to calculate the levels of reactive power required to balance the network and the component capacities required. This information is used to calculate the investment costs associated with a given solution.

Wind states are modelled using a probability density function with the power generation level of each machine at each wind speed used to approximate the electrical losses in the network. Using a single average speed and calculating the losses based on that value would reduce the fidelity of the result, given the inherent non-linearity of losses in an electrical network.

The cost of no-service is then found by analysing the network performance with each cable section within the network sequentially taken out of service. This information is then combined with the probability of each cable experiencing a failure, which is proportional to cable length, to quantify the cost of no-service conditions.

The investment costs and the operational costs associated with losses and no-service conditions are normalised to present day value and this information is fed back into the network specification model, completing the information exchange loop shown in Figure 26.

Test cases varying from simple to mildly complex are presented to validate that the optimisation performs as expected. In each case, effectively optimal solutions are returned and in a number of cases the positioning of offshore substations is specified to within a fraction of a millimetre of the known optimal solution.

Where the optimal solution does not involve deploying an offshore substation at zero-distance from a generator, simple mathematical models have been developed to show that the solution is effectively optimal. In such cases, the positional error between the returned offshore substation position and the calculated optimal incurs miniscule increases (a few Watts in a multi MW system) in electrical losses, demonstrating a high-level of working sensitivity.

Chapter 5: Case Studies

Within this chapter, the developed optimisation framework described in the preceding chapter is used to analyse and optimise networks for specific offshore wind developments. This framework could equally be applied for the purposes of optimising networks for wave or tidal networks in preference to offshore wind; however, with wind being the most mature of the offshore generation vectors, it offers the most scope for real world comparison.

Within Chapter 3 of this thesis, desirable characteristics of subsea network optimisation drawn from contemporary studies have been identified and in Chapter 4, a framework that includes all of the identified desirable optimisation characteristics identified is described. Within this Chapter, the framework described within Chapter 4 is applied to 2 distinct case studies.

The first case study considers an *in-the-sea* network for an existing offshore wind farm such that the optimality of the in-situ network may be assessed, and improved upon. Specifically, Horns-Rev 1 in Danish territorial waters has been selected for consideration. The second case study considers, at the time of writing, the proposed East Anglia One development, with the objective being to offer a recommendation as to what the network should look like.

For both case studies, *base case* optimisations were performed using a number of assumed parameters and the utilised or proposed network design variables. Further, the optimisation process is reapplied to assess the impact, on the optimal solutions returned for each case, of voltage and frequency selection at both collector and transmission stages of the network.

5.1 Common Fixed Design Parameters

Within the preceding chapter of this thesis, a vector, $\boldsymbol{y}$, containing a number of fixed design parameters was defined as (40). Values of these parameters that were utilised for both case studies, considered within this chapter, are presented in this section.

5.1.1 Wind Speed & Corresponding Power Output

The sites selected for both case studies are located within the North Sea: Horns-Rev 1 is in Danish territorial waters; and East Anglia One is in UK waters. Typical shape and scale parameters for a Weibull distribution for the North Sea (shape = 2.26, scale = 11.2) [49] have been utilised to approximate the wind resource for both case studies; producing the distribution depicted in Figure 34. For purposes of both case studies the distribution was gathered into 1 m/s resolution discrete states.

Wind speed data, from the Weibull distribution, is combined with the cut-in, rated, and cut-out speeds of the turbines deployed to calculate the output power that corresponds to each wind speed. As this work is concerned with planning rather than operational performance, it is assumed that all turbines within the cluster are subject to identical wind speeds and thus, given the identical nature of the machines, generate the same power output [40].

At each wind speed, and associated output power level, a power flow study was undertaken. The results of this, combined with the probability of each state occurring, allowed the through-life electrical losses to be calculated. It is possible to calculate an average power output and subsequently perform the through-life losses calculation; however this incurs a significant drop in accuracy, as the losses are non-linearly related to power throughput.

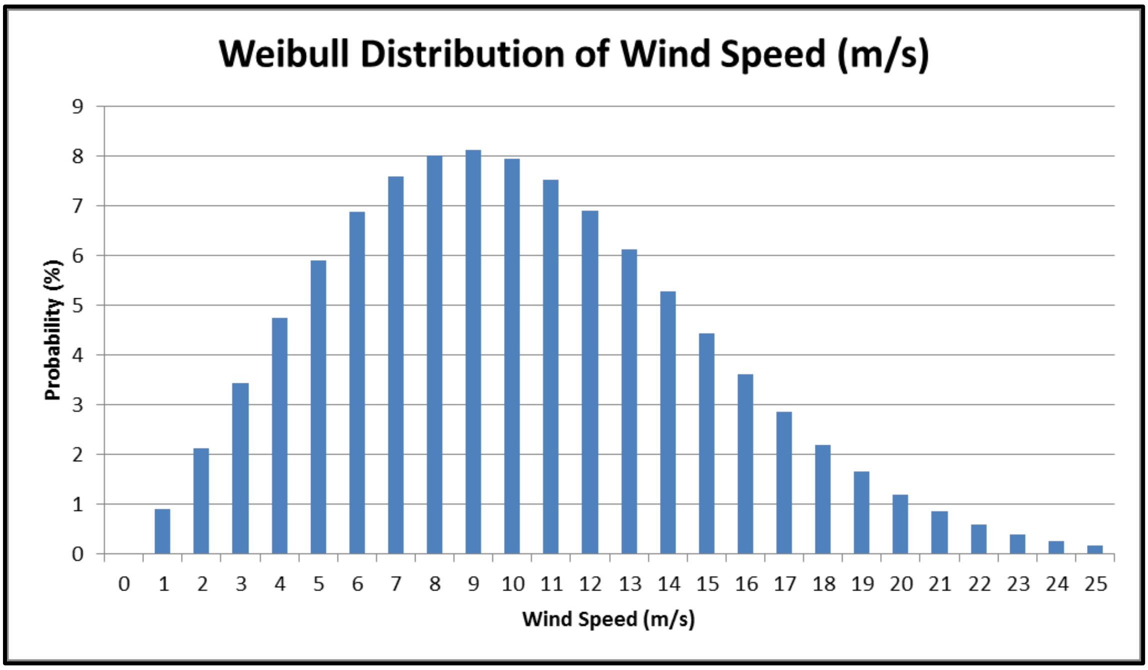


Figure 34 - Assumed Weibull Wind Speed Distribution for Case Studies 1 and 2 [49]

5.1.2 Reactive Power Compensation Cost

In both case studies, the turbines may generate at a lagging power factor, to compensate for the highly capacitive subsea cabling. It is assumed that additional variable reactive power compensation may be installed at each turbine, subject to the cost of 77,000 k€ / MVAr [61] presented earlier in this thesis.

Reactive power compensation may also be employed at offshore substations and at the connection point where the offshore network meets its onshore counterpart. This is assumed to be carried out using static VAR compensation as described previously. Within the developed optimisation framework, it is possible for an offshore substation to have no turbines connected to it and may exist solely to provide reactive power compensation.

Reactive power compensation is always applied such that the voltages at each bus within the network (the turbines, the offshore collectors and the connection point) are at 1 per-unit voltage. Variable compensation is required to ensure that this voltage regulation can be achieved.

5.1.3 Cable Installation Cost

As has been discussed previously, there is a broad range of cable installation cost approximations presented in the literature: 145 k€ / km [61]; 256 k€ / km [22]; and 365 k€ / km [24]. Each of these values is a single approximation that does not account for location, sea depth or cable type. As the networks returned may be complex with some degree of cables crossing one another, it has been chosen to attempt to include this network complexity by using the most expensive figure of 365 k€ / km, offered by Dicorato et al [61].

5.1.4 Development Lifetime

The planned lifetime of a development has potential to greatly influence optimality in respect of the network design. A longer lifetime may incentivise the use of networks which incur lower operational costs, but with higher capital costs, than those which incur higher operational costs with lower capital costs. Equally, a shorter anticipated lifetime may place greater emphasis on minimising the capital costs associated with the network as there is less time for the operational costs to accrue.

Typically, the expected lifetime of an offshore renewable energy development will be in the order of a few tens of years, with 20 [22, 28, 31, 40] and 25 years [21] featuring in the published literature. For both case studies, 25 years has been chosen as the anticipated lifetime of the development.

5.1.5 Failure Rates & Mean Time to Repair

While cable failures do not affect the installation cost of a given network, they do influence the operational cost, specifically the no-service costs. The more frequently cables fail and the longer they are out of service for increases the operational cost, and ultimately total lifecycle cost; for which this work seeks a minimisation.

In the absence of specific cable failures rates, a general failure rate was applied to all subsea cables. The value of 1.1114×10^{-3} cable failures per kilometre per year was taken from a 2015 study undertaken by the European Marine Energy Test Centre (EMEC) in collaboration with the UK Crown Estate [88]. This value is approximately double the failure rate considered by Banzo & Ramos in their 2011 stochastic optimisation of offshore wind farm networks study [40].

A mean time to repair (MTTR) parameter is used to approximate the length of the time the cable is unavailable for, in the event of a cable failure. The value used for both case studies is 90 days [22, 88, 95].

Offshore substations are assumed to have a failure rate of 3.2×10^{-3} failures per year, with a mean time to repair of 30 days [95].

5.1.6 Electricity Price

The price paid per MWh of electricity generated has no effect on the capital cost associated with a solution, but has a significant impact on the losses and no-service costs and thus this parameter has the potential to skew optimality. A high price of electricity results in higher operational costs, thus it may be beneficial to incur higher capital costs in order to minimise the operational costs. Equally, a low price of electricity might incentivise the use of cheaper to implement systems as the operational costs through-life will be lower.

A number of diverse values for this parameter were found. In 2013, the UK's Department for Energy and Climate Change (DECC) published strike prices to be paid for various types of renewable sourced energy in the period 2014 to 2019, which suggested that at the time of writing, the price paid for offshore wind energy would be £ 150 / MWh (approximately € 185 / MWh) [94]. A similar value is applied to the cost of electricity in Nedić et al's work [25] where £ 150 / MWh was assumed.

In Ergun et al's analysis of transmission network optimisation for large wind farms published in 2012, a value of € 50 / MWh [21] is utilised. This is broadly similar to the value that Banzo and Ramos published of € 80 / MWh [40] in 2011.

For the purposes of the case studies considered for this thesis, a constant value of € 185 / MWh is used throughout; as this figure represents the UK strike price at the time of writing [94] and also matches the value utilised by Nedić et al [49].

In 2016, the UK Department of Business, Energy & Industrial Strategy published proposed strike prices of £ 105 / MWh for 2021/22 and £ 100 / MWh for 2022/23 [101]; representing reductions of 30 % and 33.33 % against the electricity price used for this work. Utilising either of these lower figures would reduce the impact of losses and no-service conditions; thus incentivising capital expenditure minimisation over operational expenditure minimisation. Consequently, it is likely that the solutions produced by the developed optimisation framework would be markedly different if such a strike price had been utilised.

5.1.7 Discount Rate

In order to normalise the costs associated with an offshore wind farm project that occur over many years into an equivalent present day value, a discounted cash flow calculation is used. The value of the discount rate parameter has significant potential to skew optimality as a higher discount rate will reduce the value of operational costs incurred over the lifetime of the development more than a lower discount rate.

For the purposes of the case studies considered for this thesis, a 5 % discount rate has been utilised [21]. Equally, a value of 4 % may have been deployed [22, 28].

5.1.8 Partial Fixed Design Parameters Vector

Within the preceding sections, values for a number of fixed design parameters have been discussed, and values common to both case studies may now be used to partially define the fixed design variable vector, $\boldsymbol{y}$, as shown by (63). The parameters within (63) that remain undefined will be defined using parameters specific to each case study.

$$
\boldsymbol{y} = \begin{bmatrix} V_C \\ f_C \\ V_T \\ f_T \\ T_{TECH} \\ DevLife \\ DR \\ P_{ELEC} \\ P_{Q\_SVC} \\ FailRate_{CABLES} \\ MTTR_{CABLES} \\ FailRate_{OFFSUB} \\ MTTR_{OFFSUB} \\ CableInst \\ PDF_{SHAPE} \\ PDF_{SCALE} \\ V_{CUT-IN} \\ V_{RATED} \\ V_{CUT-OUT} \end{bmatrix} = \begin{bmatrix} V_C \\ f_C \\ V_T \\ f_T \\ T_{TECH} \\ 25\ years \\ 5\ \% \\ £\ 185\ per\ MWh \\ €\ 77{,}000\ per\ MVAr \\ 0.001114\ per\ km\ per\ year \\ 90\ days \\ 0.0032\ per\ year \\ 30\ days \\ €\ 365{,}000\ per\ km \\ 2.26 \\ 11.2 \\ V_{CUT-IN} \\ V_{RATED} \\ V_{CUT-OUT} \end{bmatrix} \quad (63)
$$

5.2 Algorithmic Parameters

As has been described within the previous chapter of this thesis: during an optimisation, two meta-heuristic algorithms are utilised: a particle swarm optimisation algorithm positions offshore substations at locations in the search space and assigns topology and number of string parameters to these objects. The second algorithm takes the set of generators; the topology employed at the collector network and the number of strings that are to be deployed and seeks to order the generators in the most efficient manner. This is achieved using a Genetic Algorithm based travelling salesman problem solver that utilises electrical losses minimisation, in preference to cable length minimisation; as discussed previously within this thesis.

The selection of algorithmic parameters represents a trade-off between solution quality and computational effort required to reach convergence on a solution. Increasing the values used for the number of particles and iterations used for the PSO increase the likelihood of an optimal, or near-optimal, position being found, achieved at the expense of computation time and effort. Similarly, increasing the number of concurrent solutions within the GA and the number of iterations will increase the likelihood of achieving a strong solution to the connection ordering problem.

The values chosen for both case studies for algorithmic parameters are as shown in Table 15. These values were chosen empirically after analysis of quality of solutions returned and the computational time required to produce these solutions. The weightings applied to the local differential and global differential positions for PSO were both set to the default value of 2, such that a particle's own (local) optimal solution and the global optimal solution are given equal weighting for the creation of new velocities to traverse the search space.

Parameter	**Value**
PSO – Particles	50
PSO – Number of iterations	1000
PSO – Number of iterations without global optimal solution improvement to stop algorithm	1000
PSO – Reduction in maximum velocity after each iteration (%)	0
PSO – Local Differential Weighting (C1)	2
PSO – Global Differential Weighting (C2)	2
GA – Population size	80
GA – Number of iterations	10000

Table 15 - Algorithmic Parameters for Case Studies

For the PSO algorithm, restricting the maximum particle velocity after each iteration was found to be useful for *honing in* on a solution. If the search space is limited and the objective is simply to optimise the positioning of offshore substations, without considering topologies and number of strings to be deployed, this feature offered value. This may be observed in the test cases considered previously in this work, where it was shown possible to position an offshore substation within millimetres of the known optimal. When the search space is less restricted, and topologies, number of strings and the positioning of multiple offshore substations must be considered simultaneously, it was found that reducing the search space, for each dimension, after each iteration produced low quality results.

5.3 Case Study 1: Horns-Rev 1

Horns-Rev 1 is a 160 MW wind farm situated approximately 18 km from the Danish coast in the North Sea, and has been deliberately selected for examination as the results obtained may be directly compared with a tangible physical network.

5.3.1 Wind-Farm Configuration

The Horns-Rev 1 wind farm contains 80 identical Vestas V80 2 MW wind turbines giving a total power capacity of 160 MW [102]. The magnitude of both the individual machine capacity and the aggregate capacity of the development are small by modern standards but represented a large development when Horns-Rev was constructed in 2002.

The turbines are organised into 10 radial cables, each consisting of 8 turbines, as depicted in Figure 35 [93]. The coordinates have been normalised to place the origin of the x-axis at the centre of the development. While the wind farm is positioned approximately 18 km offshore, the transmission cable length is given as 34 km, suggesting that the interface point with the onshore grid is 16 km in-land. For this reason, the y-coordinates of each turbine are given relative to the interface point in preference to the shoreline [103].

Each string of turbines emanates from a common point where an offshore transformer is deployed to link collector and transmission components of the system. While the location of the offshore substation is not given, it is assumed to be at the origin on the x-axis and at 33.44 km on the y-axis, as this would be 7 turbine diameters (560 m) distant from the first row of turbines, in keeping with the general layout of the development [93].

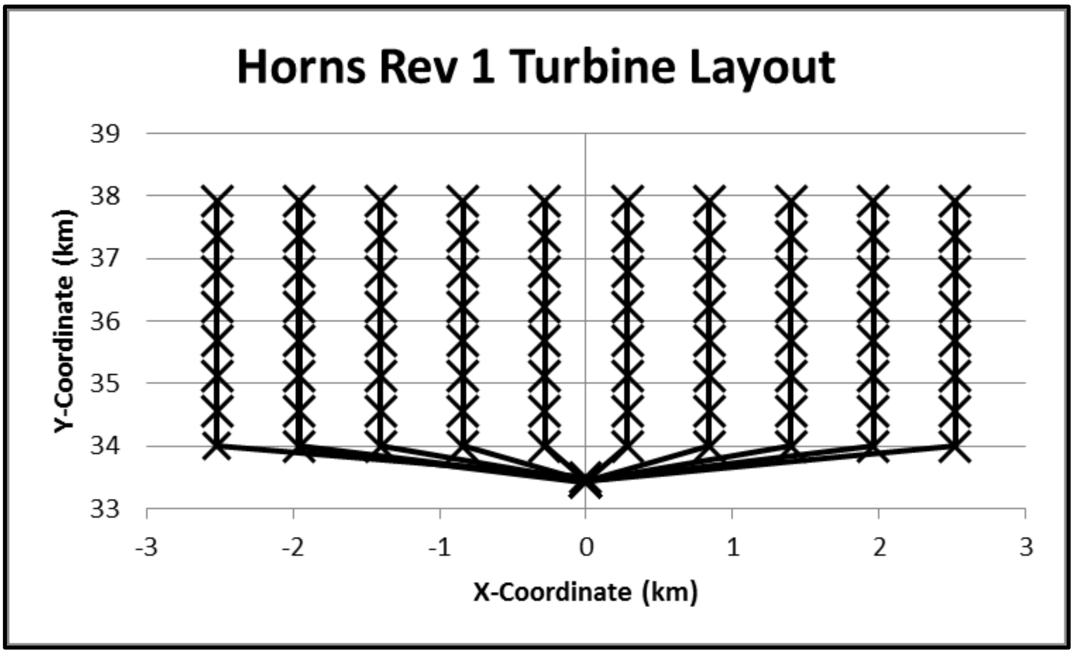


Figure 35 - Horns Rev 1 Turbine Layout

Each of the Vestas V-80 2 MW machines is capable of generating, at rated power, at a power factor of 0.98 capacitive to 0.96 inductive [102]; hence it has a maximum reactive power consumption of 0.58 MVAr and maximum reactive power generation of 0.4 MVAr. Additional reactive power generation or consumption capacity may be prescribed at each turbine if necessary. Each turbine is located 7-turbine diameters apart from its neighbour in both x and y directions [93]. The complete generator data set for Case Study 1 is presented as Table 62 [93] in Appendix D.

The collector stage of the network is comprised of ten strings of 8 × 2 MW machines, giving a total output of 16 MW per string. Each of these radial strings operates at 33 kV and utilises Nexans 36 kV PEX cables [103]. The technical parameters of the collector cables are presented in Table 63 [104] in Appendix D. The exact cable type used for the transmission stage has not been identified, and as such a 220 kV HVAC cable by ABB is used to represent the cable characteristics for the transmission stage. The characteristics of this cable are presented in Table 63 in Appendix D [100].

5.3.2 Modelled Representation of In-Situ Network

The Horns-Rev 1 development was specifically chosen for examination as it allows comparison between the solutions returned and an *in-the-sea* network. In order to make such a comparison the Horns-Rev 1 network has been modelled, utilising: the site layout [93]; the real and reactive power output of each turbine [102]; and the cable characteristics of both the collector [104] and transmission [100] aspects of the offshore network.

Cable lengths were calculated for the links between each node, and the corresponding resistance, reactance and susceptance values derived and per-unitised. This data is presented as Table 66, Table 68 and Table 69 in Appendix D.

A power flow analysis on this network, with all turbines generating at rated capacity, returned real power losses of 4.168 MW giving an efficiency of approximately 97.4 %. Significant levels of reactive power compensation are required to maintain nominal voltage at each bus. The generators themselves provide some of this reactive power compensation, providing power at a lagging power factor to compensate for the highly capacitive cabling. The rest of the reactive power required to balance the system is provided by SVC.

Using the cost guidelines presented earlier in this thesis, costs for reactive power compensation and all other components of the network were calculated and are presented in Table 16. In addition to the capital expenditure of the project, cost of losses and no-service conditions were calculated, allowing a total lifetime cost to be calculated. These values, along with the total lifecycle cost, are also presented in Table 16.

Component	**Cost (M€)**	**Cost (%)**
Reactive Power Compensation	23.42	12.98
Collector Cables	30.32	16.80
Transmission Cable	27.65	15.32
Offshore Substation	26.50	14.68
Transformers	5.72	3.17
Switchgear	10.92	6.05
Losses	37.18	20.60
No-Service	18.77	10.40
Capital Expenditure	**124.53**	**69.00**
Operational Expenditure	**55.95**	**31.00**
Through-Life Cost	**180.48**	**100.00**

Table 16 - Through-Life Cost Breakdown for Horns-Rev 1 Subsea Electrical Network

5.3.3 Mathematical Definition of Optimisation

This case study was optimised using through-life cost minimisation as the optimisation objective, as defined by (41) where: the decision vector, $\boldsymbol{x}$, is defined by (36); the decision domain, $\boldsymbol{\Omega}$, is defined by (37); and the fixed design parameters vector, $\boldsymbol{y}$, defined by (40).

For the purposes of this case study, the number of offshore collectors, n, is set to a minimum of one and a maximum of three. The lower limit precludes the implementation of direct turbine to connection point cables, as discussed earlier in this thesis. While offshore wind parks utilising a single collector network configuration is commonplace, the proposed East Anglia One development is an example of a network expected to use twin offshore substations [83]. By setting the maximum number of collector networks to three, this allows networks of higher complexity than are currently being deployed to be considered.

From an offshore substation, several subsea cables may emanate. Horns-Rev 1, in as example of this as 10 radial strings link 8 turbines each to the offshore substation. For the purposes of this case study, the minimum number of strings emanating from an offshore substation is 1 and the maximum is 10. This upper limit is largely arbitrary but is included to provide some form of limit to the vast number of connection order permutations that have been discussed earlier in this thesis

As shown in Figure 35, the turbines are positioned between negative and positive 2.52 km on the x-axis; and between 34 and 37.92 km on the y-axis. Consequently, the positional deployment limits for offshore substations have been set at -6.5 to 6.5 km on the x-axis; and 0 to 40 km on the y-axis. This allows the decision domain, $\boldsymbol{\Omega}$, to be parametrically defined as (64). The set of topologies abstracted into numerical representation, *Top*, is as defined earlier within this thesis.

$$\boldsymbol{\Omega} = \begin{bmatrix} -6.5 \leq x_i^P \leq 6.5 \\ 0 \leq y_i^P \leq 40 \\ 1 \leq CStr_i \leq 10 \end{bmatrix} and \begin{bmatrix} x_i^P \in \Re \\ y_i^P \in \Re \\ CTop_i \in Top \\ CStr_i \in Z \\ TTop \in Top \end{bmatrix} \quad (64)$$

For the base-case, the real-world network voltages and frequency parameters were used, specifically: 33 kV for the collector network; 220 kV for the transmission network; and a frequency of 50 Hz used throughout [93]. The cut-in, rated and cut-out speeds of the turbines employed at Horns-Rev 1 were used, specifically: 4 m/s; 14 m/s; and 25 m/s [102]. The fixed design parameters vector, $\boldsymbol{y}$, which was partially defined in (63) may now be fully defined as (65). As both AC and DC transmission networks are under consideration, T_{TECH}, is not defined as a singular numerical value.

In the first instance, optimisations were performed using the base-case voltage and frequency parameters discussed previously. Further to that, optimisations were re-run to assess the effects of manipulating these parameters.

$$\boldsymbol{y} = \begin{bmatrix} V_C \\ f_C \\ V_T \\ f_T \\ T_{TECH} \\ DevLife \\ DR \\ P_{ELEC} \\ P_{Q\_SVC} \\ FailRate_{CABLES} \\ MTTR_{CABLES} \\ FailRate_{OFFSUB} \\ MTTR_{OFFSUB} \\ CableInst \\ PDF_{SHAPE} \\ PDF_{SCALE} \\ V_{CUT-IN} \\ V_{RATED} \\ V_{CUT-OUT} \end{bmatrix} = \begin{bmatrix} 33\ kV \\ 50\ Hz \\ 220\ kV \\ 50\ Hz \\ T_{TECH} \\ 25\ years \\ 5\ \% \\ £\ 185\ per\ MWh \\ €\ 77{,}000\ per\ MVAr \\ 0.001114\ per\ km\ per\ year \\ 90\ days \\ 0.0032\ per\ year \\ 30\ days \\ €\ 365{,}000\ per\ km \\ 2.26 \\ 11.2 \\ 4\ m/s \\ 14\ m/s \\ 25\ m/s \end{bmatrix} \quad (65)$$

5.3.4 Base-Case Optimisation

Using the base-case voltage and frequency variables, as defined within the previous section, optimal networks were sought based on utilising 1 to 3 offshore substations with both AC and DC transmission networks. The through-life costs associated with each returned network are presented in Table 17.

Network Configuration			**Expenditure (M€)**		
Collector	**Transmission**	**Substations**	**Capital**	**Operational**	**Total**
Real World Network			124.53	55.95	**180.48**
AC	AC	1	138.94	36.24	**175.18**
AC	AC	2	144.91	36.59	**181.50**
AC	AC	3	147.95	36.83	**184.78**
AC	DC	1	192.25	14.58	**206.83**
AC	DC	2	178.38	13.43	**191.81**
AC	DC	3	180.41	12.28	**192.69**

Table 17 - Through-Life Costs of Base-Case Solutions for Case Study 1

Within Table 17, it may be observed that the only solution that was returned that performed better than the real world network, against an objective of through-life cost minimisation, utilises a single offshore substation, an AC collector network, and an AC transmission system.

For this case, the complete network configuration returned is as shown in Table 18. The layout of the collector network is further depicted in Figure 36, with a radial transmission cable linking the offshore substation to the on-land network interconnector. Unlike the real world network, the returned solution does not connect the turbines in columns, nor does it have strings connecting equal numbers of turbines. Of the 10 strings deployed: 4 interface with 9 turbines; 2 with 8 turbines; and 4 with 7 turbines.

Parameter	**Value**	
	X	**Y**
Returned Collector Position (km)	0.1908	36.0670
Transmission Network Type	Radial	
Transmission Connection Order	Col 1	
Collector 1 Network Type	Ring	
Number of Strings @ Collector 1	10	

Table 18 - Optimal Solution for Case Study 1: Single Collector AC-AC Network

Another primary difference between the returned solution and the deployed solution is the location of the offshore substation. For the real world solution, the collector network is assumed to be originate at (0, 33.44), in keeping with the spacing regime of the turbines in the development. For the returned solution, the offshore substation is positioned within the turbine cluster. Like the real world network, this solution utilises a radial transmission link.

Table 19 shows the breakdown of capital costs by component for this solution. By comparing Table 16 and Table 19, it may be observed that the 1 collector solution returned is approximately € 11.5 million more expensive, in terms of capital expenditure, than the real world solution.

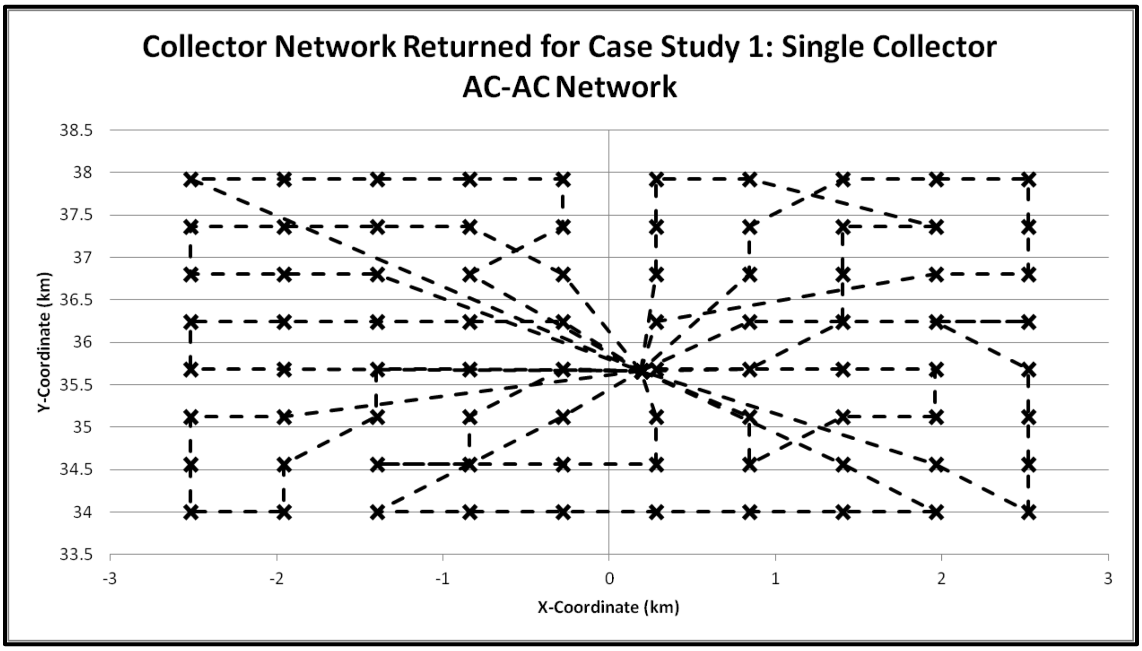


Figure 36 - Collector Network Solution for Case Study 1: Single Collector AC-AC Network

Component	Cost (M€)	Cost (%)
Reactive Power Compensation	23.84	13.61
Collector Cables	40.81	23.30
Transmission Cable	29.66	16.93
Offshore Substation	26.64	15.21
Transformers	5.76	3.29
Switchgear	12.23	6.98
Losses	18.41	10.51
No-Service	17.83	10.18
Capital Expenditure	**138.94**	**79.31**
Operational Expenditure	**36.24**	**20.69**
Through-Life Cost	**175.18**	**100.00**

Table 19 - Through-Life Cost Breakdown for Case Study 1: Single Collector AC-AC Network

The capital, losses and no-service costs of the solution, along with the total cost, are presented in Table 19. While this solution is more expensive to implement, relative to the real world network, the losses and the no-service costs are greatly reduced by the use of ring based collector networks, compared with the radial networks of the real world system, thus the lifecycle cost is approximately € 8.2 million, or 4.5 %, cheaper.

Based on the costs presented, it would appear that the Horns-Rev 1 electrical network may have been deployed with the aim of minimising implementation cost, as this network is considerably cheaper to implement than any of the returned solutions. The Horns-Rev 1 electrical network could be improved upon, in terms of through-life cost minimisation without deviating from the base case design variables, by utilising a modified single offshore substation with AC collector and transmission networks approach.

5.3.5 Effects of Network Voltage Selection on Optimality

In addition to the base case collector network voltage of 33 kV, studies seeking optimal networks for 1-3 AC collector networks with AC and DC transmission system were sought using an increased and decreased collector network voltage, with the effect on through-life cost summarised in Table 20. For this case, the increased voltage was 45 kV and the decreased voltage was 20 kV, as these are the next available standard cable voltages [100, 104].

Network Configuration			Through-Life Cost (M€)		
Collector	Transmission	Substations	20 kV	33 kV	45 kV
AC	AC	1	223.90	175.18	167.10
AC	AC	2	207.25	181.50	159.82
AC	AC	3	206.66	184.78	170.47
AC	DC	1	253.32	206.83	183.36
AC	DC	2	257.03	191.81	178.45
AC	DC	3	231.26	192.69	182.08

Table 20 - Effect on Through-Life Cost of Collector Network Voltage Selection for Case Study 1

In each of the cases presented in Table 20, reducing the collector network voltage to 20 kV increases the through-life cost of the returned solutions. Conversely, increasing the collector network voltage to 45 kV reduces the lifecycle cost associated with each solution. As may be observed in Table 20, the solution that performs best against the optimisation objective of lifecycle cost minimisation utilises 2 offshore substations, and therefore 2 AC collector networks, with AC transmission.

In addition to the base case collector transmission voltage of 220 kV, studies seeking optimal networks for 1-3 AC collector networks with AC and DC transmission system were sought using a lower voltage of 150 kV and a higher voltage of 275 kV. These voltages were selected as they represent the next highest and next lowest readily available submarine cable voltage [100, 104].

As may be observed from Table 21, there is no case whereby the reduction in transmission network voltage from 220 kV to 150 kV is advantageous, on a through-life cost minimisation basis. Further, it can be seen that in every instance that was considered, increasing the transmission network voltage from 220 kV to 275 kV produced a reduction in the through-life cost associated with the solution.

Network Configuration			Through-Life Cost (M€)		
Collector	**Transmission**	**Substations**	**150 kV**	**220 kV**	**275 kV**
AC	AC	1	184.40	175.18	175.10
AC	AC	2	191.66	181.50	178.75
AC	AC	3	196.03	184.78	180.78
AC	DC	1	211.02	206.83	202.72
AC	DC	2	199.42	191.81	187.62
AC	DC	3	199.46	192.69	191.06

Table 21 - Effect on Through-Life Cost of Transmission Network Voltage Selection for Case Study 1

5.3.6 Effects of Network Frequency Selection on Optimality

Four cases were considered in order to analyse the effect of network frequency selection on optimality, namely: the collector network operating at 50 Hz while the transmission network operates at 50 Hz, or DC (Case 1 / Base case); the collector network operating at 150 Hz while the transmission network operates at 50 Hz, or DC (Case 2); the collector network operating at 50 Hz while the transmission network operates at 16.66 Hz, or DC (Case 3); and the collector network operating at 150 Hz while the transmission network operates at 16.66 Hz, or DC (Case 4).

The effect of frequency selection, at both collector and transmission networks, on the through-life cost of each solution is presented in Table 22. From Table 22 it may be observed that increasing the collector network frequency reduces the lifecycle cost of each solution. This may be seen by comparing Case 1 (50 Hz collector network frequency with 50 Hz transmission network frequency) with Case 2 (150 Hz collector network frequency with 50 Hz transmission network frequency); and by comparing Case 3 (50 Hz collector network frequency with 16.66 Hz transmission network frequency) with Case 4 (150 Hz collector network frequency with 16.66 Hz transmission network frequency).

Network Configuration			Through-Life Cost (M€)			
Collector	Transmission	Substations	Case 1	Case 2	Case 3	Case 4
AC	AC	1	175.18	153.73	269.79	244.29
AC	AC	2	181.50	150.27	272.27	248.01
AC	AC	3	184.78	152.95	277.57	255.19
AC	DC	1	206.83	154.80	206.83	154.80
AC	DC	2	191.81	151.35	191.81	151.35
AC	DC	3	192.69	153.87	192.69	153.87

Table 22 - Effect on Through-Life Cost of Network Frequency Selection for Case Study 1

In order to interface a collector network that uses a higher frequency with a transmission stage using a lower frequency, a cycloconverter is employed [105]; with the cost approximated using the cost function for an equivalent rated AC-DC converter presented earlier in this thesis. In order to interface a lower frequency transmission network with a higher frequency land based network, back-to-back AC-DC-AC conversion is required. The significant level of cost associated with implementing back-to-back conversion is the primary reason why the 16.66 Hz transmission cases are less cost economical than the 50 Hz transmission cases, where a simple, and comparatively cheap, transformer may be used to interface the transmission aspect of the subsea network with the onshore network. There is no case where the reduction in transmission frequency reduces the through-life cost of a solution as may be observed by comparing Case 1 (50 Hz collector network frequency with 50 Hz transmission network frequency) with Case 3 (50 Hz collector network frequency with 16.66 Hz transmission network frequency); and by comparing Case 2 (150 Hz collector network frequency with 50 Hz transmission network frequency) with Case 4 (150 Hz collector network frequency with 16.66 Hz transmission network frequency).

Based on the limited range of collector and transmission network frequencies considered, it appears that the most advantageous frequency configuration is to use 150 Hz for collection and 50 Hz for transmission.

5.3.7 Effects of Combined Voltage and Frequency Selection on Optimality

Within the preceding sections, it has been shown that increasing the collector network voltage from 33 kV to 45 kV; increasing the transmission voltage from 220 kV to 275 kV; and increasing the collector network operating frequency from 50 Hz to 150 Hz all induce improvements in the returned solutions against the objective of through-life cost minimisation, with respect to the base case solutions. These improvements applied to all cases: one to three AC collector networks with AC transmission; and one to three AC collector networks with DC transmission.

Within this section, the collector network voltage and frequency, and transmission network voltage changes that brought about improvements in the solutions when enacted on an individual basis are now enacted in parallel. The results derived from these studies are presented in Table 23, along with the results generated using the base case values for each parameter.

Table 23 summarises the through-life costs associated with each case using the base case design variables; the optimal solution that was achieved by increasing the collector network voltage as stand-alone change; the optimal solution that was achieved by increasing the transmission voltage while keeping all other variables at their base case values; and the through-life cost minimisation that was achieved by manipulating the network frequencies.

The combined parameter change case produced the through-life cost minimisation results as presented in the final column of Table 23. To produce these results, optimisation studies were undertaken that operated the AC collector networks at 45 kV 150 Hz, and the AC transmission network at 275 kV 50 Hz or the DC transmission network at 275 kV, as applicable.

The headline outcome that can be observed from the final column of Table 23 is that the modified parameters that were used to generate the results for these cases, produces better performing solutions than both the base case parameters; and the manipulation of any single parameter as a stand-alone change.

Network Configuration			Through-Life Cost (M€)				
Collector	**Transmission**	**Substations**	**Base**	**V_{CN}**	**V_{TN}**	**Freq**	**Combined**
AC	AC	1	175.18	167.10	175.10	153.73	134.56
AC	AC	2	181.50	159.82	178.75	150.27	139.42
AC	AC	3	184.78	170.47	180.78	152.95	145.22
AC	DC	1	206.83	183.36	202.72	154.80	150.55
AC	DC	2	191.81	178.45	187.62	151.35	149.73
AC	DC	3	192.69	182.02	191.06	153.87	150.51

Table 23 - Effects on Through-Life Cost of Combined Network Changes for Case Study 1

From Table 23, it may be observed that the most cost-effective solution once the voltage and frequency parameters have been manipulated remains the single AC collector network with AC transmission system. The through-life cost of the solution is now 23.2 % cheaper than the solution returned using the base-case parameters, and 25.4 % more cost-efficient than the real world network. While this is obviously encouraging, a subsea electrical network generally only constitutes a small part of the total cost of a development; thus a dramatic cost saving may be diluted when considered in the wider context of a complete offshore wind farm.

In 2015, the Oxford Institute for Energy Studies produced a generalised breakdown for the cost of offshore wind installations, in which they suggest that *electrical infrastructure* accounts for 10 % of a typical installation. It is not clear if this grouping includes installation and infrastructure costs or if some proportion of *transportation and installation* (20 %) and *foundation* (18 %) costs are attributable to the deployment of a subsea electrical network [106]. If it is assumed that the cost of a subsea network accounts for between 10 and 20 % of an offshore development, a 25 % network saving may only account for between 2.5 % and 5 % of the complete project cost.

The solution returned using the modified voltage and frequency parameters is detailed as Table 24, with the collector network layout depicted in Figure 37.

Parameter	**Value**	
	X	**Y**
Returned Substation Position (km)	-0.1407	34.9228
Transmission Network Type	Star	
Transmission Connection Order	Col 1	
Collector 1 Network Type	Radial	
Number of Strings @ Collector 1	10	

Table 24 - Modified Design Parameters Optimal Solution for Case Study 1

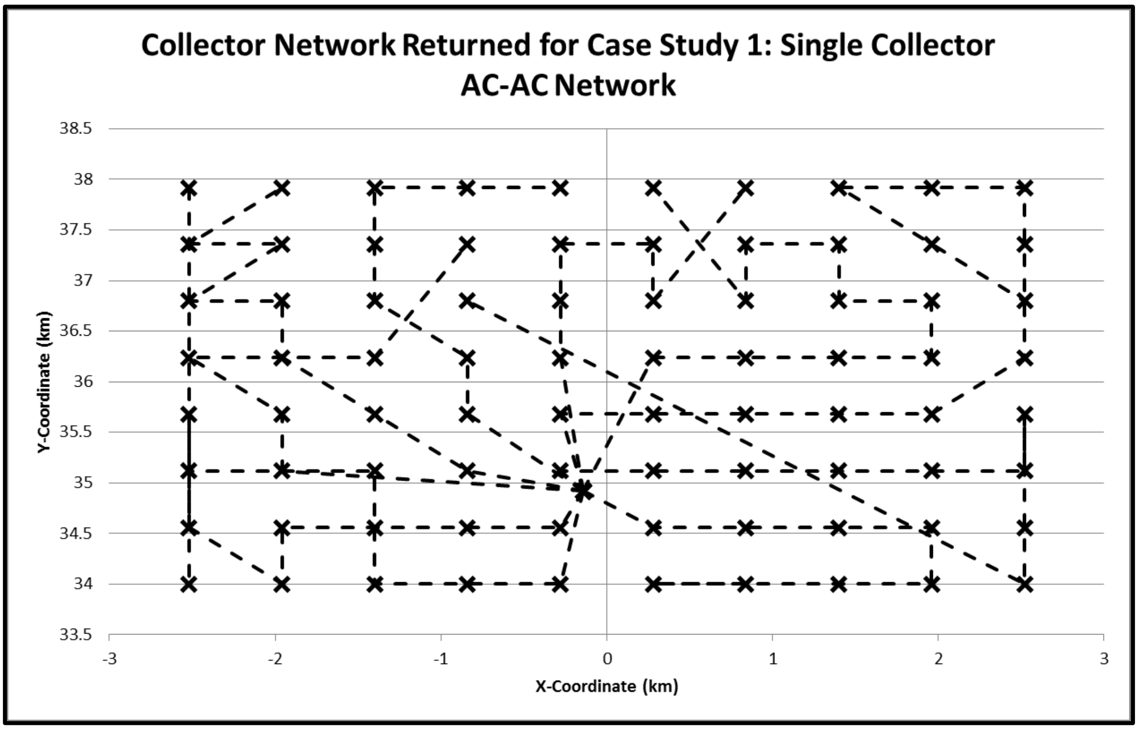


Figure 37 - Collector Network Solution for Case Study 1: Single Collector AC-AC Network

The through-life cost breakdown associated with this solution is presented as Table 25.

Component	Cost (M€)	Cost (%)
Reactive Power Compensation	9.20	6.83
Collector Cables	32.81	24.38
Transmission Cable	30.13	22.39
Offshore Substation	17.74	13.18
Transformers	4.07	3.03
Switchgear	12.46	9.26
Losses	8.55	6.35
No-Service	19.60	14.57
Capital Expenditure	**106.40**	**79.08**
Operational Expenditure	**28.15**	**20.92**
Through-Life Cost	**134.56**	**100.00**

Table 25 - Through-Life Cost Breakdown for Case Study 1: Optimal Network Using Modified Voltage and Frequency Parameters

Comparing this breakdown with that for the optimal solution returned using the base case voltage and frequency parameters, detailed in Table 19, shows significant cost savings in a number of areas: reactive power compensation costs are down by over 60 %; the cost of an offshore substation is 33 % lower; and the cost of collector cables, now using radial topology instead of ring as before, is down by nearly 20 %. Finally, the cost of electrical losses achieved by manipulating the network voltages and frequency and is reduced by over 50 % relative to the optimal solution returned using the base case design parameters.

5.3.8 Discussion of Parametric Sensitivity on Optimality

Table 17 summarises the through-life costs associated with the deployment of one to three AC collector networks with either AC or DC transmission, for the purposes of extracting energy from the Horns-Rev 1 offshore wind park. In order to assess the sensitivity of specific parameters on the optimality of solution, voltages and frequencies for both collector and transmission networks were varied on a stand-alone basis before the optimal configurations for each were combined, and more solutions generated.

When enacted as stand-alone changes each of the considered parameters induced a reduction in through-life cost of the returned solutions. The parameter that brought about the smallest reduction in lifecycle cost was the transmission network voltage. Increasing this voltage from 220 kV to 275 kV did result in a reduction in the cost of the optimal solution for each case, but only by an average of 1.45 %, with the solution for single collector AC network with AC transmission saving only € 80,000 against a € 175 million project by configuring the network in this way.

When the collector network voltage was increased from 33 kV to 45 kV, with all parameters left at their base case values, a reduction in through-life costs for all considered solutions was observed. This reduction in cost was much more significant with between 4.6 % and 11.94 % savings, depending on the configuration that was deployed. The average reduction in through-life cost by configuring the network in this manner, for the specific cases considered, was 8.03 %.

For each network configuration, four different combinations of collector and transmission network frequencies were considered: 50 Hz collector networks with 50 Hz transmission as the base case; 150 Hz collector networks with 50 Hz transmission; 50 Hz collector networks with 16.66 Hz transmission; and 150 Hz collector networks with 16.66 Hz transmission. For each of the considered network configurations, the combination that found to offer the most significant cost savings saw the collector networks operated at 150 Hz and the transmission network at 50 Hz. Operating in this manner saw the lifecycle cost of each solution drop by between 12.24 % and 25.16 %, depending on configuration, with an average reduction of 18.85 %.

Within the developed optimisation framework, the *end point* of the network is taken to be after any interfacing with an existing onshore grid has been performed, thus the cost of interfacing is included within the lifecycle cost. It is assumed that the land-based networks operate at 50 Hz AC at an unknown transmission voltage. This gives 50 Hz transmission networks a massive advantage as the cost of a transformer is insignificant compared to the cost of AC-DC-AC conversion that must be employed to interface a low-frequency AC (LFAC) transmission link with a 50 Hz transmission system. If the subsea network was to be interfaced with an on-shore LFAC grid, there is a strong chance that operating the transmission component of the subsea network at lower frequency will offer superiority from a cost minimisation perspective.

For all of the considered network configurations, the parameters that offered the most value from the studies that examined single variable manipulation were: 45 kV and 150 Hz for the collector networks; and 275 kV and 50 Hz or DC for the transmission networks. Consequently, these were the values implemented for the combined parameters study described in the previous section. Configuring the networks in this way was shown to reduce the through-life cost of the returned solutions by an average of 23.14 %, with the range of value being 21.41 % to 27.21 %, relative to the base case solutions.

In each of the considered cases, the combined modified parameters case performs better than the cases where a single parameter is manipulated, by quite some margin. This suggests that the benefits obtained by manipulating these parameters are complimentary rather than contradictory.

Based on the evidence presented, the Horns-Rev 1 network could have been designed better by increasing the collector network voltage and frequency; and increasing the transmission voltage. With these changes enacted, the optimal solution was found to be using a single offshore substation positioned towards the centre of the turbine cluster that interfaces 10 radial strings of turbines and utilises a radial AC transmission link back to shore. This essentially mirrors the network design of Horn Rev 1, albeit with a revised collector network arrangement.

5.4 Case Study 2: East Anglia One

The second case study centres on a future development: East Anglia One in British territorial waters; thus the objective of this case study is to suggest a number of optimal networks with respect to specific constraints.

5.4.1 Wind-Farm Configuration

As this wind farm is in the planning stage, there are a number of details that are yet to be defined or published, such as the exact layout of the site. It is widely accepted that the site will comprise 102 7 MW Siemens wind turbines, giving a total capacity of 714 MW [83]. While the capacity of the site is significantly larger than for the Horns-Rev 1 development considered in the previous case study; from a network design perspective, these cases are similar as they contain a similar number of machines (80 & 102 respectively).

The rotor diameter of each turbine is 154 m. Combining this value with the 7-diameters (in both x and y directions) minimum spacing requirements that is utilised in both the Horns-Rev 1 development and a number of other deployments suggests a minimum distance between turbines of 1078 m [83, 93]. Consequently, this distance will be assumed to be the spacing between turbines in both x and y directions.

If the turbines are to be laid out in an orthodox rectangular fashion in a similar manner to the Horns-Rev 1 site, the only suitable configurations, that respect the spatial constraints discussed previously, are 6 rows of 17 turbines or 17 rows of 6 turbines.

One source of information detailing a number of offshore wind developments, suggests that the nearest point of the development to the shore is 45.4 km and the centre is 53.8 km offshore [83]. If this information is accurate, this suggests that the proposed layout is 15 turbines deep in the y-direction, perpendicular to the shoreline. 7 rows of 15 turbines would total 105 machines, so it is assumed that the deployment is configured as 3 rows of 14 turbines and 4 rows of 15 turbines, totalling 102 machines, and as depicted in Figure 38.

It is also suggested that the export transmission cables are 85 km in length, and as such the interface point with the land-based network occurs approximately 40 km from the shoreline [83]. As such when the turbine layout is reference to the coordinate system required for analysis, the closest turbines are located at 85.4 km, as shown in Figure 38.

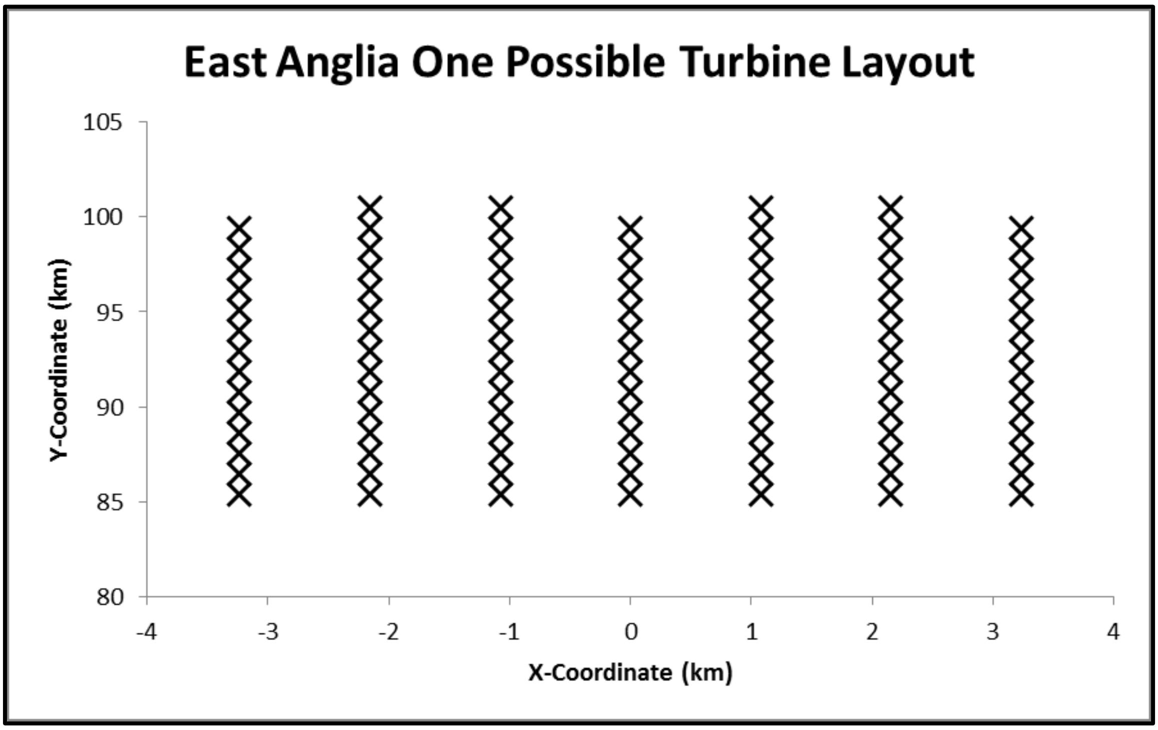


Figure 38 - Possible East Anglia One Turbine Layout

It was specified within the optimisation parameters that all offshore substations are installed at least 40 km from the interface point with the land based network.

The proposed machine type to be used in this development is the Siemens SWT-7.0-154 which is a 7 MW machine that can operate at wind speeds between 3 and 25 m/s and outputs electrical power at 690 V, 50 Hz AC [107]. As Siemens do not state the reactive power output range of this machine, it is assumed that these values are the same as for the 2 MW machines used for the Horns-Rev development and as such the power factor limits are 0.96 inductive and 0.98 capacitive. Consequently, the limits are taken to be 2.04 MVAr for reactive power consumption and 1.42 MVAr for reactive power generation.

It is believed that the collector networks will operate at 66 kV and the transmission aspect of the network will operate at 400 kV AC [83]. In the absence of knowing the specific cable types, ABB 66 kV and 400 kV submarine cables have been used to provide the requisite parameters [100]. The same collector and transmission voltages have been utilised for the AC collector networks with AC transmission cases; and AC collector networks with DC transmission scenarios.

5.4.2 Mathematical Definition of Optimisation

As with Case Study 1, this case study was optimised using through-life cost minimisation as the optimisation objective, as defined by (41) where: the decision vector, $\boldsymbol{x}$, is defined by (36); the decision domain, $\boldsymbol{\Omega}$, is defined by (37); and the fixed design parameters vector, $\boldsymbol{y}$, is defined by (42).

Utilising the same rationale as Case Study 1, the number offshore substations, *n*, is limited between 1 and 3; and the number of concurrent strings of connections is bounded between 1 and 10.

As shown in Figure 38, the turbines are assumed to be positioned between negative and positive 3.234 km on the x-axis; and between 85.4 and 100.492 km on the y-axis. Consequently, the positional deployment limits for offshore substations have been set at -3.5 to 3.5 km on the x-axis; and 40 to 100 km on the y-axis. This allows the decision domain, $\boldsymbol{\Omega}$, to be parametrically defined as (66). The set of topologies abstracted into numerical representation, *Top*, is as defined earlier within this thesis.

$$\boldsymbol{\Omega} = \begin{bmatrix} -3.5 \leq x_i^P \leq 3.5 \\ 40 \leq y_i^P \leq 100 \\ 1 \leq CStr_i \leq 10 \end{bmatrix} \; and \; \begin{bmatrix} x_i^P \in \Re \\ y_i^P \in \Re \\ CTop_i \in Top \\ CStr_i \in Z \\ TTop \in Top \end{bmatrix} \tag{66}$$

For the base-case, the real-world network voltages and frequency parameters were used, specifically: 66 kV for the collector network; 400 kV for the transmission network; and a frequency of 50 Hz used throughout [83]. The cut-in, rated and cut-out speeds of the turbines employed for East Anglia One were used, specifically: 3 m/s; 13 m/s; and 25 m/s [107]. The fixed design parameters vector, $\boldsymbol{y}$, which was partially defined in (63) may now be fully defined as (67). As both AC and DC transmission networks are under consideration, T_{TECH}, is not defined as a singular numeric value.

In the first instance, optimisations were performed using the base-case voltage and frequency parameters discussed previously. Further to that, optimisations were re-run to assess the effects of manipulating these parameters.

$$\boldsymbol{y} = \begin{bmatrix} V_C \\ f_C \\ V_T \\ f_T \\ T_{TECH} \\ DevLife \\ DR \\ P_{ELEC} \\ P_{Q\_SVC} \\ FailRate_{CABLES} \\ MTTR_{CABLES} \\ FailRate_{OFFSUB} \\ MTTR_{OFFSUB} \\ CableInst \\ PDF_{SHAPE} \\ PDF_{SCALE} \\ V_{CUT-IN} \\ V_{RATED} \\ V_{CUT-OUT} \end{bmatrix} = \begin{bmatrix} 66\ kV \\ 50\ Hz \\ 400\ kV \\ 50\ Hz \\ T_{TECH} \\ 25\ years \\ 5\ \% \\ £\ 185\ per\ MWh \\ €\ 77{,}000\ per\ MVAr \\ 0.001114\ per\ km\ per\ year \\ 90\ days \\ 0.0032\ per\ year \\ 30\ days \\ €\ 365{,}000\ per\ km \\ 2.26 \\ 11.2 \\ 3\ m/s \\ 13\ m/s \\ 25\ m/s \end{bmatrix} \quad (67)$$

5.4.3 Base Case Optimisation

As with Case Study 1, optimal networks were sought using the base-case voltage and frequency variables based on utilising 1 to 3 offshore substations with both AC and DC transmission networks. The through-life costs associated with each returned network are presented in Table 26, from which it may be observed that the most cost-efficient solution utilises 3 offshore substations and DC transmission.

Network Configuration			**Expenditure (M€)**		
Collector	**Transmission**	**Substations**	**Capital**	**Operational**	**Total**
AC	AC	1	508.13	360.74	**868.87**
AC	AC	2	477.10	339.94	**817.04**
AC	AC	3	739.73	91.40	**831.13**
AC	DC	1	647.63	88.72	**736.35**
AC	DC	2	574.15	87.44	**661.59**
AC	DC	3	576.62	73.12	**649.74**

Table 26 - Through-Life Costs of Base-Case Solutions for Case Study 2

The three collector networks with DC transmission solution returned effectively divides the turbine cluster into a nearest to shore third; a middle third; and a farthest from shore third, with each offshore substation conveying power from turbines located within those areas, through a ring transmission network to the connection point, as detailed in Table 27, and depicted in Figure 39.

Parameter	Value	
	X	Y
Collector 1 Position (km)	-0.1549	87.9876
Collector 2 Position (km)	0.1786	92.9462
Collector 3 Position (km)	0.1251	97.9810
Transmission Network Type	Ring	
Transmission Connection Order	Col 1, Col 2, Col 3	
Collector 1 Network Type	Radial	
Number of Strings @ Collector 1	10	
Collector 2 Network Type	Radial	
Number of Strings @ Collector 2	10	
Collector 3 Network Type	Radial	
Number of Strings @ Collector 3	10	

Table 27 - Returned Solution for Case Study 2: Three Collector AC-DC Network

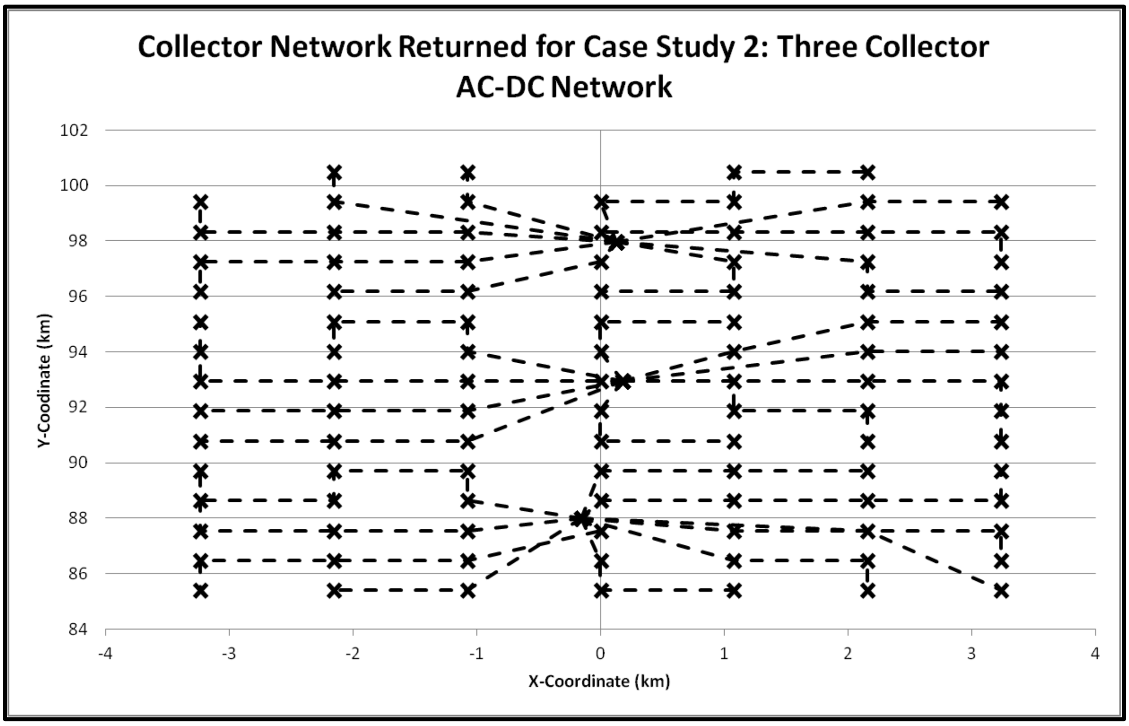


Figure 39 - Returned Collector Network Solution for Case Study 2: Three Collector AC-DC Network

The allocation of turbines to offshore substations for this solution is 35 turbines linked to substation 1, 35 to substation 2 and 32 to substation 3. At substation 1, each string links between 3 and 5 turbines. At substation 2, the division of turbines to strings varies between 3 and 6; and at substation 3, it is between 2 and 5.

Component	Cost (M€)	Cost (%)
Reactive Power Compensation	26.42	4.07
Collector Cables	67.56	10.40
Transmission Cable	199.16	30.65
Offshore Substation	85.26	13.12
AC/DC Conversion	174.20	26.81
Switchgear	24.02	3.70
Losses	64.34	9.90
No-Service	8.78	1.35
Capital Expenditure	**576.62**	**88.75**
Operational Expenditure	**73.12**	**11.25**
Through-Life Cost	**649.74**	**100.00**

Table 28 - Through-Life Cost Breakdown for Case Study 2: Three Collector AC-DC Network

In total, 6 distinct solutions, optimised subject to constraints around network configuration, have been generated using the base case design parameters, and a summary of the expenditure associated with each network is presented in Table 26.

For the solutions that utilise AC for both collector and transmission aspects of the network, the cost of transmission cabling, modelled as an exponential function, dominates the cost of the overall solution and solutions that utilise less transmission cabling, thus reducing capital expenditure, but incur significant amounts of operational losses are returned as optimal. This is borne out in the first two rows of Table 26.

DC transmission cables are significantly cheaper to purchase than their AC counterparts, as characterised by the cost approximations presented earlier in this thesis; however this cost saving is offset by the requirement for an AC-DC converter to be placed at an offshore substation and for a corresponding DC-AC inverter to be installed at the interface point with the land-based network. While AC-DC (and DC-AC) conversion is typically an order of magnitude more expensive than a transformer; for this case the cost savings introduced by utilising DC transmission cabling outweigh the additional costs incurred by the conversion process, hence the AC-DC solutions are significantly cheaper, from a through-life perspective, than the AC-AC networks.

There are a number of distinct network designs returned for this case study, with the cheapest through-life solution depending on a number of cost functions that may be sensitive to marginal changes. There are cases whereby the trade-off between capital costs and operational costs emerges in favour of a low-redundancy or high loss network; equally, there are also cases where the trade-off emerges in favour of a more resilient network that incurs lower costs in the future but higher costs in the immediate term.

Based on the analysis conducted, the most efficient through-life solution that could be implemented from the options summarised in Table 26 for the East Anglia One offshore wind farm is the 3 collector, AC collector, DC transmission network.

The results presented in the previous sections were based on running optimality studies using base case design variables, which were derived from the best available information about the proposed network design. Within the following sections, the results obtained from running a number of other studies with altered design variables are presented, in order to highlight the sensitivity of key parameter selection.

5.4.4 Effects of Network Voltage Selection on Optimality

The collector networks for the East Anglia One development are proposed to be 66 kV 50 Hz AC; hence this has been used for the base case optimality studies. Studies were undertaken using 45 kV and 110 kV as alternative collector network design voltages. These values were selected as they are *standard voltages* for subsea cables, one voltage rating higher and lower than the proposed network voltage [100, 104].

Through-life cost calculations for each collector network voltage, for AC and DC transmission systems with 1-3 AC collector networks are presented in Table 29, and discussed thereafter.

Network Configuration			Through-Life Cost (M€)		
Collector	Transmission	Substations	45 kV	66 kV	110 kV
AC	AC	1	1036.04	868.87	769.41
AC	AC	2	959.64	817.04	763.22
AC	AC	3	946.98	831.13	730.02
AC	DC	1	912.73	736.35	640.22
AC	DC	2	821.36	661.58	603.16
AC	DC	3	789.67	649.74	599.90

Table 29 - Effect of Collector Network Voltage Selection for Case Study 2

As may be observed in Table 29, there is no case where reducing the collector network operating voltage is beneficial, at least from a through-life cost minimisation perspective. The reduction in collector network voltage fundamentally alters all of the solutions returned as star topology collector networks are mandated in all cases. Conversely, each of the scenarios considered benefits from an increase in collector network voltage to 110 kV; with through-life costs reducing in the order of tens of millions of Euros.

The transmission network for the East Anglia One development is proposed to be 400 kV, and this value forms part of the base case design variables. Studies were also undertaken using 330 kV and 420 kV as alternative transmission design voltages, based on typically available cable voltages [100, 104]. For both alternative transmission voltages, optimisations studies were run using 1-3 offshore substations and using AC and DC transmission systems. The through-life cost returned for each study is presented in Table 30, in addition to the costs produced when running the simulation using the base case transmission voltage.

As may be observed within Table 30, there is no case where reducing the transmission network voltage is advantageous; instead the through-life cost is raised by tens or hundreds of millions of Euros.

Network Configuration			Through-Life Cost (M€)		
Collector	Transmission	Substations	330 kV	400 kV	420 kV
AC	AC	1	1038.62	868.87	867.92
AC	AC	2	933.62	817.04	791.94
AC	AC	3	932.62	831.13	822.86
AC	DC	1	752.59	736.35	720.28
AC	DC	2	676.06	661.58	639.39
AC	DC	3	667.57	649.74	630.84

Table 30 - Effect of Transmission Network Voltage Selection for Case Study 2

While there is no advantage to be gained by reducing the transmission network operating voltage to 330 kV, increasing the network voltage to 420 kV reduces the through-life cost in each of the considered cases; as may be observed in Table 30. The reduction in through-life costs may be more pronounced if the increase in voltage was greater than the 5 % differential between 400 and 420 kV; however, finding manufacturers that offered submarine cables of greater than 420 kV proved to be challenging.

5.4.5 Effects of Network Frequency Selection on Optimality

Four cases were considered in order to analyse the effect of network frequency selection on optimality, namely: the collector network operating at 50 Hz while the transmission network operates at 50 Hz, or DC (Case 1 / Base case); the collector network operating at 150 Hz while the transmission network operates at 50 Hz, or DC (Case 2); the collector network operating at 50 Hz while the transmission network operates at 16.66 Hz, or DC (Case 3); and the collector network operating at 150 Hz while the transmission network operates at 16.66 Hz, or DC (Case 4).

The results produced from running studies seeking the optimal single, two, and three AC collector networks with AC and DC transmission for each combination of system frequencies are presented as Table 31.

From Table 31 it may be observed that for the AC transmission cases, the only solution that sees a reduction in through-life cost when the base case frequencies are deviated from is the three AC collector network case where it is most beneficial to operate the network using a collector network frequency of 150 Hz and a transmission network frequency of 50 Hz.

Network Configuration			Through-Life Cost (M€)			
Collector	Transmission	Substations	Case 1	Case 2	Case 3	Case 4
AC	AC	1	868.87	898.59	1000.70	946.63
AC	AC	2	817.04	847.14	889.82	865.17
AC	AC	3	831.13	820.05	898.84	822.89
AC	DC	1	736.35	647.84	736.35	647.84
AC	DC	2	661.58	587.81	661.58	587.81
AC	DC	3	649.74	583.78	649.74	583.78

Table 31 - Effect on Through-Life Cost of Network Frequency Selection for Case Study 2

It may be further observed from Table 31 that an increase in collector network frequency is beneficial, from a through-life cost minimisation perspective, for each of the cases that utilises DC transmission.

For the single and two AC collector networks with AC transmission cases, the most cost efficient solution is to utilise the base case design frequencies of 50 Hz for both collector and transmission stages. The addition of a cylcoconverter to interface a higher frequency collector network with a lower frequency transmission system is highly expensive. The benefits derived from operating the collector networks at this frequency fail to offset the significant expenditure associated with this converter. Further, the requirement for back-to-back AC-DC-AC conversion to interface a lower frequency transmission system with a standard frequency onshore network is prohibitively expensive, hence the cases where 16.66 Hz transmission is used (Cases 3 & 4) are more expensive than those using 50 Hz, which may be interfaced with the land based grid using only a transformer.

Based on the limited assessment of operating frequency combinations assessed, it appears to be worthwhile to operate both collector and transmission stages at 50 Hz when AC transmission is being utilised; and to operate collector networks at 150 Hz when DC transmission is being utilised.

5.4.6 Effects of Combined Voltage and Frequency Selection on Optimality

Within the previous sections, it has been seen that increasing the voltage of both collector (66 kV to 110 kV) and transmission stages (400 to 420 kV) has produced better performing solutions for all considered cases. In terms of system frequencies, no such blanket conclusions around optimal operating conditions could be drawn; with 50 Hz collector and transmission producing the most cost efficient solutions when AC transmission is employed; and 150 Hz producing the most cost efficient solutions when DC transmission is utilised.

Within this section, the optimal results drawn from the previous studies, considering only a single aspect of the network design, are now combined and the effects assessed. The results obtained from these studies are summarised in Table 32 before being discussed thereafter.

Within Table 32, the *Base* column refers to the solutions produced for a given network configuration using the base case design variables. The V_{CN} column refers to the optimal results produced by manipulating the collector network voltage while maintaining all variables at their base case values. Similarly, the V_{TN} column in Table 32 refers to the optimal result produced by adjusting the transmission voltage while maintaining all other variables at their base case values; and the *Freq* column refers to the optimal result produced by manipulating the collector and transmission network frequencies while using base case voltages throughout. Finally, the *Combined* column refers to the results produced when the optimal conditions returned for collector and transmission network voltage and frequency manipulations are enacted simultaneously.

Network Configuration			Through-Life Cost (M€)				
Collector	Transmission	Substations	Base	V_{CN}	V_{TN}	Freq	Combined
AC	AC	1	868.87	769.41	867.92	868.87	755.07
AC	AC	2	817.04	763.22	791.94	817.04	755.49
AC	AC	3	831.13	730.02	822.86	820.05	739.65
AC	DC	1	736.35	640.22	720.28	647.84	616.08
AC	DC	2	661.58	603.16	639.39	587.81	582.82
AC	DC	3	649.74	599.90	630.84	583.78	586.89

Table 32 - Effects on Through-Life Cost of Combined Network Changes for Case Study 2

As may be observed in Table 32, the optimal solution returned for the single AC collector network with AC transmission case has a through-life cost of € 868.87 million. The optimal collector network voltage option, while maintain all other variables at base case values for this case was found to be 110 kV and increasing this voltage is shown to reduce the through-life cost of this solution by 11.44 %. Increasing the transmission voltage from 400 kV to 420 kV, with all design variables at their base case values, was shown to reduce the lifecycle cost of the solution by the margin of 0.1 %. The base case frequencies of 50 Hz for both collector and transmission networks was found to be the most cost-effective, hence the through-life cost in the adjusted frequency column is identical to that in the base case column.

As a consequence of the results obtained by manipulating single values, the combined scenario for the single AC collector network with AC transmission case considered operating the collector network at 110 kV 50 Hz and the transmission network at 420 kV 50 Hz. Operating the network in this manner was shown to reduce the through-life cost of the solution by € 113.8 million (13.10 %), largely driven by the reduced costs associated with increased collector network voltage.

The second case presented in Table 32 is the two AC collectors with AC transmission case. As was shown for the previous case, increasing the collector network voltage to 110 kV while keeping all other design parameters as they were has a beneficial impact on through-life cost with a reduction of € 53.8 million (6.59 %). Similarly, increasing the transmission network voltage to 420 kV as a stand-alone change sees the lifecycle cost reduce by € 25.1 million (3.07 %). As was evident in the single AC collector network with AC transmission case, none of the frequency deviations considered produce a favourable outcome, relative to the base case solution.

For the two AC collector networks with AC transmission scenario, the combined scenario refers to operating the network using 110 kV 50 Hz for the collector network and 420 kV 50 Hz for the transmission component. This combination is shown to reduce the through-life cost of the solution by € 61.55 million or 7.53 %, relative to the base case design parameters. Again, this is only marginally more than the reduction in lifecycle cost associated with increasing the collector network voltage as a stand-alone change.

The third case presented in Table 32 covers the three AC collector networks with AC transmission case. As was the case for the single and double AC collector network with AC transmission scenarios considered previously, increasing the collector network voltage reduces the lifecycle cost associated with the solution by € 101.1 million or 12.17 %. Similarly, increasing the transmission network voltage to 420 kV as a stand-alone change induces a reduction in through-life cost of € 8.27 million (1.00 %). Unlike the cases considered previously, manipulating the network frequencies for this case can yield a lower lifecycle cost than using the base case variables: € 11.08 million or 1.33 % reduction is achieved by operating the collector network at 150 Hz with the transmission system left at 50 Hz.

The combined scenario for the three AC collector networks with AC transmission case sees the collector network operated at 110 kV 150 Hz and the transmission stage operated at 420 kV 50 Hz. Operating in this manner presented considerable savings (€ 91.48 million or 11.01 %) relative to the base case design variables, but this solution is not as cost efficient as increasing the collector network voltage while keeping all other variables at their base case values.

The first DC transmission case presented in Table 32 considers a single AC collector network with DC transmission. The solution returned when the base case design variables are utilised was calculated to cost, through life, € 736.35 million. When the collector network voltage was raised from 66 kV to 110 kV, with all other design variables remaining at their base case value, this was found to reduce the cost of the optimal solution by € 96.13 million or 13.05 %. Similarly, increasing the transmission voltage was shown to reduce the lifecycle cost of the solution but to a lesser extent than increasing the collector voltage: € 16.07 million (2.18 %). Increasing the collector network frequency from 50 Hz to 150 Hz was calculated to save € 88.51 million (12.02 %) on a through-life basis, relative to the base case solution.

The combined parameters study for this case, therefore, considered the optimisation of a single AC collector network operated at 110 kV 150 Hz, interfaced with a 420 kV DC transmission link. Operating in this manner returned a solution that is € 120.27 million (16.33 %) cheaper than the solution obtained using the base case design variables and is also cheaper than any of the solutions obtained by manipulating any single design variable.

The two AC collector networks with DC transmission case presented in Table 32 has a base case design variable through-life cost of € 661.58 million. Increasing the collector network voltage from 66 kV to 110 kV, while maintaining all other design variables at their base case values, was shown to reduce the lifecycle cost of the solution by € 58.42 million or 8.83 %. Similarly, increasing the transmission network voltage from 400 kV to 420 kV, as a stand-alone change, was shown to reduce the through-life cost of the solution by € 22.19 million (3.35 %). Increasing the collector network frequency from 50 Hz to 150 Hz reduces the through-life cost more substantially than either of the voltage manipulations: € 73.77 million (11.15 %) is now saved relative to the base case design variables.

As was the case for the single AC collector with DC transmission scenario, the combined parameters case sees the AC collector network operated at 110 kV 150 Hz, and the DC transmission link operated at 420 kV. Configuring the network in this manner sees the through-life cost of the solution drop by € 78.76 million (11.90 %), relative to the base case solution, and this solution is also more cost efficient than all of the single design variable manipulation cases considered previously.

The final case presented in Table 32 covers the optimisation of a three AC collector networks with DC transmission system, under a range of operating parameters. The base case design variable solution in this instance was calculated to cost € 649.74 million, on a through-life basis. Increasing the collector network voltage from 66 kV to 110 kV, while keeping all other design variables at their base case values, may be observed to reduce the lifecycle cost by € 49.84 million (7.67 %). Similarly, increasing the transmission network voltage from 400 kV to 420 kV was found to reduce the through-life cost of the optimal solution by € 18.9 million, or 2.91 %. Finally, increasing the collector network frequency from 50 Hz to 150 Hz, as a stand-alone change, reduced the lifecycle cost of the solution by € 65.96 million (10.15 %).

As a result of the single variable manipulations described previously, the combined parameters case for this scenario involved operating the AC collector network at 110 kV 150 Hz and the DC transmission link at 420 kV. The optimal solution returned, for this network configuration was calculated to be € 62.85 million (9.67 %) cheaper, on a through-life basis, than the base case solution. Unlike the single and two AC collector networks with DC transmission cases, the combined solution does not perform better than all of the single-variable manipulation cases, as this is less cost-efficient than the increased collector network frequency with all other design variables left at their base case values.

As may be observed within Table 32, through-life cost minimisation is achieved using a two-offshore substations and collector networks with DC transmission. The solution returned for this network configuration is presented as Table 33 and the collector network layout is as depicted in Figure 40. Finally, the cost breakdown for this solution is as shown in Table 34.

Parameter	**Value**	
	X	**Y**
Collector 1 Position (km)	-0.0323	87.4386
Collector 2 Position (km)	0.0000	94.7338
Transmission Network Type	Ring	
Transmission Connection Order	Col 2, Col 1	
Collector 1 Network Type	Radial	
Number of Strings @ Collector 1	7	
Collector 2 Network Type	Radial	
Number of Strings @ Collector 2	10	

Table 33 - Returned Solution for Case Study 2: Three Collector AC-DC Network

When the base case voltage and frequencies were employed, the optimal solution comprised of three radial collector networks that effectively divided the turbine field into a *nearest-to-shore third*; *a middle third*; and *a farthest-from-shore third.* The prescribed transmission link for this solution utilised a DC ring system.

Using the modified parameters, the most economical solution now uses two radial collector networks, dividing the turbine field on a similar basis to before; with a ring DC transmission system. In both of these cases, the cost associated with incorporating greater redundancy into the collector networks fails to offset the additional costs incurred to implement such networks.

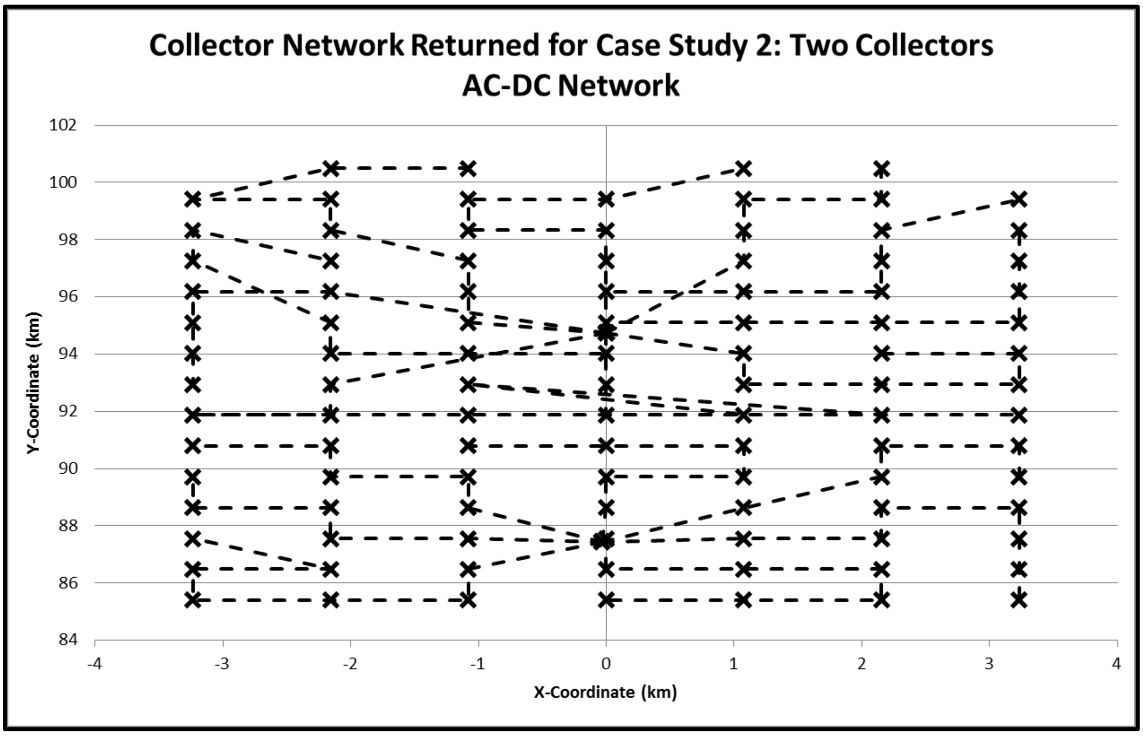


Figure 40 - Returned Collector Network Solution for Case Study 2: Two Collector AC-DC Network

Component	Cost (M€)	Cost (%)
Reactive Power Compensation	11.68	2.00
Collector Cables	75.00	12.87
Transmission Cable	189.59	32.53
Offshore Substation	68.57	11.77
AC/DC Conversion	156.90	26.92
Switchgear	29.64	5.09
Losses	39.47	6.77
No-Service	11.97	2.05
Capital Expenditure	531.39	91.17
Operational Expenditure	51.43	8.83
Through-Life Cost	**582.82**	**100.00**

Table 34 - Through-Life Cost Breakdown for Case Study 2: Two Collector AC-DC Network

5.4.7 Discussion of Parametric Sensitivity on Optimality

From the results presented in Table 32 and subsequently discussed, it is clear that the manipulation of design variables plays an important role in the design of a cost-efficient network. For the AC transmission cases, the solutions appear to be most sensitive to the manipulation of the collector network voltage. While the transmission network voltage and collector frequency can be adjusted to reduce the through-life cost of the solution, the magnitude of the savings achieved is less significant than for the collector network voltage.

Further, the cost savings achieved by manipulating collector network voltage and frequency, and transmission voltage simultaneously are little more than those achieved by manipulating the collector network voltage alone, for the single and two AC collector networks with AC transmission cases. For the three AC collector networks with AC transmission case, the simultaneous manipulation of the collector network voltage and frequency, in addition to the transmission network voltage produces an inferior solution, at least from a cost minimisation perspective, to that obtained when the collector network voltage is manipulated alone.

For the DC transmission cases, there is no single *key* parameter as there is for the AC transmission cases. Both collector network voltage and frequency are shown to reduce the through-life cost of the solutions significantly, with a more limited reduction achieved by increasing the transmission network voltage, as a stand-alone change. For the single and two AC collector networks with DC transmission cases, the optimal results are obtained by manipulating collector network voltage and frequency, and transmission network voltage simultaneously. For the three AC collector networks with DC transmission case, through-life cost minimisation is achieved by manipulating the collector network frequency as a stand-alone change.

Frequency manipulation is more beneficial for the DC cases than the AC cases as there is no cost implication incurred by increasing the collector network frequency. For both 50 Hz and 150 Hz collector network frequencies, offshore rectification must be employed to permit DC transmission. For the AC cases, operating at a frequency that is different to the land based network frequency, assumed to be 50 Hz, requires interfacing equipment that is typically an order of magnitude more expensive than a transformer, which allows for different voltages to be interfaced but does not allow for frequency conversion, and in the considered cases this was found to be uneconomical.

The lifecycle cost results obtained for both AC and DC transmission cases when considering the impact of increasing the transmission network voltage do not differ massively in comparison those obtained using the base case design variables. This is not a great surprise given that the voltage differential between 400 and 420 kV is only 5 %. If physical characteristic data for higher voltage cables, perhaps in the region of 500-600 kV, could be obtained, the conclusions drawn might be quite different as to the value of transmission network voltage selection.

5.5 Discussion of Case Study Results

The optimisation framework developed for this research allows for the assessment of a broad range of network configurations, including: multiple parallel collector networks; and a range of technologies and topologies.

The values derived from considering a broad range of possible network configurations is demonstrated in Table 35, where the network topologies returned for scenarios considering 1 to 3 offshore substations, with both AC and DC transmission systems; for both case studies considered within this thesis, are presented. For brevity, only solutions generated using the base case design variables, described earlier in this thesis are presented.

Case Study	Transmission Type	Offshore Substations	Topology			
			Transmission	Collector 1	Collector 2	Collector 3
1	AC	1	Radial	Ring	N/A	N/A
1	AC	2	Star	Ring	Ring	N/A
1	AC	3	Star	Ring	Ring	Ring
1	DC	1	Ring	Ring	N/A	N/A
1	DC	2	Ring	Radial	Radial	N/A
1	DC	3	Ring	Radial	Radial	Radial
2	AC	1	Radial	Ring	N/A	N/A
2	AC	2	Radial	Ring	Ring	N/A
2	AC	3	Ring	Ring	Power Factor Correction	
2	DC	1	Ring	Ring	N/A	N/A
2	DC	2	Ring	Radial	Radial	N/A
2	DC	3	Ring	Radial	Radial	Radial

Table 35 - Summary of Returned Topologies Using Base-Case Design Variables

In terms of collector networks, there is no dominant topology returned with radial and ring networks being mandated on a near equal basis; as may be observed in Table 35. For transmission, ring networks are often superior to non-redundant topologies within the optimal solutions identified by this work. This may be indicative that the inclusion of redundancy and associated lower electrical losses is worth the additional expenditure relative to radial connected transmission in the majority of developments, irrespective of size. Ultimately, more studies would have been carried out and analysed to accept that this is the case.

The lack of any high-level specification characteristics dominating the range of optimal solutions, as shown in Table 35, demonstrates the value that an optimisation framework that can consider a broad range of configurations offers over methods that place more restrictions on the solution space, such as: mandating radial transmission [23, 32]; or enforcing AC in preference to DC for transmission [32, 40].

For Case Study 1, offshore plant may be deployed anywhere within a 520 km^2 area. Within the optimisation framework, offshore plant may be placed on a continuous basis; however if it was assumed that substations could only be deployed discretely at 10 m intervals, 5.2 million possible locations for offshore substations would exist. Each collector network, emanating from an offshore substation, is free to take radial, ring, or star topology; and may interface with turbines using 1 to 10 parallel strings (where radial or ring topologies are selected). Additionally, the transmission network is free to take either radial, ring, or star topologies. On this basis, the number of possible high-level solutions, without considering the number of turbine connection order permutations which may exceed 10^{100} as described in Chapter 3 of this thesis, is as shown in Table 36.

Offshore Substations	High-Level Solutions
1	3.276×10^{8}
2	3.578×10^{16}
3	3.909×10^{24}

Table 36 - Possible High Level Solutions for Case Study 1

The sheer number of feasible high-level solutions to this problem, as shown in Table 36, precludes their direct enumeration: even if it was possible to assess 1 million solutions per second it would require over 1000 years to directly assess all high-level solutions for the 2 collector network case; and over 100 billion years for the 3 collector network case.

Swarm intelligence and evolutionary computing were utilised together for this work to provide a strong solution to the specific optimisation problem within a workable timescale.

For the Case Studies, high-level network designs were produced using PSO with 50 concurrent solutions, and 1000 iterations of the algorithm; thus after a maximum of 50,000 candidate solutions have been examined, a strong solution was returned. For each candidate solution, a number of solutions were sought to the *internal* problem of connection ordering which was tackled using a genetic algorithm where 80 parallel solutions were iteratively improved 10,000 times; thus a maximum of 800,000 possible orders were considered.

For the single offshore substation case, with a comparatively modest 327.6 million high-level solutions, the devised framework returned a strong solution while examining only 0.015 % of the total high-level solutions. For each high-level solution, the 800,000 assessed solutions represented an infinitesimal proportion of the possible connection orders that exist (in excess of 10^{100}).

For each of the considered cases, the optimisation algorithm took around 80-120 hours to converge on a solution; demonstrating the ability of this method to deliver a robust solution in a timely manner.

5.5.1 Complexity of Optimised Networks

Within this chapter, a number of networks have been optimised subject to a range of operating conditions and constraints. Within the collector networks depicted in Figure 36, Figure 37, Figure 39, and Figure 40 there is a high degree of complexity: with a number of cables laid in close proximity to one another; and examples of crossed cables. This differs considerably to the Horns-Rev 1 network depicted in Figure 35, where there is no crossing of cables, and strings of connections are neatly arranged.

Owing to the complexity of the networks that may be generated using the developed optimisation framework, the highest value of cable installation cost found in the relevant literature was utilised; in attempt to account for such complexity. While this approach works well from a theoretical perspective; it is unclear, however, if this approach would be sufficient to work in practise, as:

- The cost of installing cables in an already crowded sea bed may be prohibitive. The generalised cost models that have been presented may only account for idealised installation conditions.
- It may be unfeasible to accurately lay cables in the specified manner, due to the crowded nature of the sea floor.

- The difficulties associated with accessing cables in such complex arrangements for repairs may cause cables to be out-of-service for considerably longer than the MTTR associated with subsea cabling.

Such logistical challenges may have to be addressed; and modifications may have to be made either to the optimisation framework that has been created for this work; or to the solutions returned by this framework, in order to allow for practical implementation of optimal solutions.

5.6 Chapter Summary

The developed optimisation framework was used to return optimal solutions for two developments: the Horns-Rev 1 offshore wind park in Denmark which provides a real world network as a comparative base case; and the proposed East Anglia One wind farm in the UK.

A number of optimal networks for each site have been sought for each case, using the base case design variables. For the first case study, which focusses on the Horns-Rev 1 development, solutions comprising AC collection systems with AC transmission; and AC collection systems with DC transmission were generated using the base case design variables. For each category of solution, solutions were obtained that utilised one, two and three offshore substations.

Using the base case parameters, a solution was returned that offered greater cost efficiency, on a through-life basis, than the real world network which was modelled for comparison. A remodelled single collector network solution was found to offer a through-life saving of 4.5 %, relative to the in-situ network.

In order to assess the effect of key design parameters, studies were undertaken which performed the optimisations under altered collector and transmission voltages, and collector and transmission frequencies. The conclusion drawn from the sensitivity analyses was that the Horns-Rev 1 network could be made to perform better, from a through-life cost minimisation perspective, if the collector voltage was increased from 33 kV to 45 kV; the collector frequency increased from 50 Hz to 150 Hz; the transmission voltage increased from 220 kV to 275 kV; and the transmission frequency remaining at 50 Hz. Implementation of such design parameter changes reduced the through-life cost of the optimal solution by 25.4 % compared with the real world network.

The proposed East Anglia One development in the UK has, at the time of writing, no network available for performance comparison; instead the optimisation framework was mandated with providing an optimal design that could be deployed. Two varieties of network configuration were assessed using the base case design variables: AC collector networks with AC transmission; and AC collector networks with DC transmission. For each variety, single, double and triple offshore substations were considered.

Viable solutions for each network type and number of offshore substations were returned. The DC transmission solutions were found to be the most cost efficient set as they were ranked first to third in terms of through-life cost efficiency, thus AC collector networks with AC transmission were ranked fourth to sixth in terms of cost efficiency.

Within the DC transmission solutions generated using the base case design parameters, the most cost efficient solution was found to be the 3 offshore substation solution that utilised radial collector networks with a ring transmission system.

As was the case for the first case study, parametric sensitivity studies were undertaken. For both AC and DC transmission cases, it was found to be worthwhile to increase the collector network voltage from 66 kV to 110 kV; and the transmission network voltage from 400 to 420 kV. When AC transmission is deployed, it was found that maintaining a 50 Hz collector network performed best against the optimisation objective of through-life cost minimisation; however, when DC transmission is utilised, it was found that 150 Hz for collector network frequency offered the strongest performance.

Chapter 6: Conclusions and Further Work

This chapter is divided into three distinct sections, covering: a summary of this thesis; the conclusions drawn from the research described within this thesis; and the identification of potential streams of future research.

6.1 Thesis Summary

As was introduced during Chapter 1 of this thesis, there is a global requirement for the provision of sustainable electrical energy. Sustainability of supply can be improved through the use of renewable energy sources; with offshore renewable energy offering vast potential. For example: the potential annual output of offshore wind around Europe (30,000 TWh) greatly exceeds the entire pan-European electricity demand for 2012 (3,600 TWh) [2]. Maximising the exploitation of offshore renewable energy generation is a complex and multi-faceted problem covering: site selection and machine layout to maximise input energy; design of machines and converters to maximise the efficiency with which input energy is converted into electrical energy; and the optimisation of the electrical networks required to convey energy from the point of generation, to the population centres where it is consumed.

This body of work is concerned with the optimisation of subsea electrical networks to interface proposed renewable energy developments with existing land-based electrical networks, as conceptualised in Figure 27. Consequently, it may be of use to developers who have identified a suitable site and optimised the site layout and machine selection to best utilise the available resource. Such developers may then utilise the developed optimisation methodology to minimise either the capital expenditure associated with the project; or the through-life cost associated with the development. Similarly, this framework may be used for *screening* potential sites through the generation of an approximate cost of a network for a development; which may eliminate the requirement for a potentially expensive and time-consuming site assessment.

Within Chapter 3 of this thesis, academic literature pertaining to the optimisation of subsea electrical networks is reviewed, and a number of desirable optimisation characteristics identified. Equally, a number of limitations or simplifications that have been included within contemporary studies were also discussed, such as: limiting the scope of optimisation to the collector networks only [19-20]; specifying the transmission technology type [32, 40] or topology [23, 32]; and fixing the location(s) of one or more offshore substations [23-25].

Further, a number of studies seek to minimise the installation cost of a subsea electrical network only [21-22, 30-31, 40, 42], failing to account for operational costs; or account for these costs using methods such as calculating losses based on average electrical output [22, 26] which fails to account for the inherent non-linearity of losses incurred in an electrical network.

This work has sought to make the most holistic optimisation of subsea electrical networks, through the parallel implementation of **all** the desirable characteristics detailed in Chapter 3, specifically through:

- The use of lifecycle cost minimisation as the principal optimisation objective; covering the installation cost, the cost of electrical losses, and additional costs incurred by cables being out-of-service.
- The simultaneous optimisation of both collector and transmission stages of a network. Given their high-level of interdependence on one-another, it is problematic to attempt to optimise collector and transmission networks separately.
- The inclusion of multiple offshore substations, thus allowing for multiple collector networks to be deployed. The topology and number of concurrent strings of connections that makes up the high-level design of each collector network is defined on an individual basis; allowing for asymmetric network layouts to be considered.
- Allowing both collector and transmission networks to take radial, ring, or star topologies; thus both cheaper to implement low redundancy networks; and high redundancy networks with increased capital expenditure may be considered.
- Permitting transmission networks to utilise either AC or DC transmission.
- The inclusion of voltage and frequency variables for both collector and transmission stages of the network design, such that the influence of these parameters may be characterised.
- Allowing offshore plant to be positioned on a continuous basis, in preference to either a fixed position or discretised basis.
- The calculation of cable sizes, and ultimately quantification of their cost, based on required capacity, in preference to the inclusion of cable types as a design variables.
- The quantification of network component costs using approximation functions that have been widely accepted within the relevant literature, and are utilised within a number of contemporary studies.

- Calculating the cost of electrical losses through probabilistic modelling of energy input resource, and calculating the energy losses associated with such conditions; thus the inherent non-linearity of electrical losses is accounted for.
- Computing the cost of no-service conditions, through assessing the differential losses associated with each cable being out-of-service, along with the probability of the cable being unavailable. By replicating the *Generation Ratio* [43] calculation, the additional losses that are incurred when a component section of a network is removed from service are accounted for.

No published work has been found that includes **all** of the characteristics listed above, thus this is the most holistic optimisation of subsea electrical networks for offshore renewable energy developments. It allows the most expansive selection of network configurations to be examined. Consequently, an optimal solution may be returned that would be infeasible where other toolsets place restrictions on the search space. An example of this occurs when the optimal solution returned for the East Anglia One development, using the base case design variables, has 3 radial AC collector networks with a ring DC transmission system. This solution could not be returned by methodologies that restrict the search space through mandating AC transmission over DC [32, 40] or by enforcing radial transmission over a ring system [23, 32]. Further, this solution is obviously unfeasible when a methodology that assumes that a single offshore substation is used to interface all turbines [23-25].

For the Horns-Rev 1 case study, the developed optimisation framework returned a network design is returned that is shown to be more cost efficient, through life, than the real world network, while using the base case design variables. For the East Anglia One development, there is no deployed network for comparison; only some indications as to how the network might be configured [83]. The solution returned using the base case design variables, suggests that it would be beneficial to include a third offshore substation, relative to the proposed 2 offshore substation design, connected in an HVDC ring network; utilising radial collector networks throughout.

In both cases, through-life cost minimisation can be improved upon by altering the voltage and frequency characteristics associated with both collector and transmission network components. Three of the four voltage and frequency design variable selections made by the designers of the Horns-Rev 1 network could be changed to improve lifecycle cost minimisation. Specifically, the changing of the collector network voltage from 33 kV to 45 kV; the collector network frequency from 50 Hz to 150 Hz; and the transmission voltage from 220 kV to 275 kV all yield improvements. Similarly, it appears that three out of the four voltage and frequency design variable selections for East Anglia One, considered as Case Study 2 for this thesis, could be improved upon. It was found to be most beneficial to select 150 Hz 110 kV AC for collector networks, and 50 Hz 420 kV AC or 420 kV DC for transmission.

The two-tiered meta-heuristic approach combining swarm intelligence with evolutionary computing techniques has been demonstrated to provide strong solutions to the subsea electrical network optimisation problem: the solution returned for the Horns-Rev 1 site outperforms the in-situ network in terms of through-life cost minimisation, while only directly assessing an infinitesimal number of possible solutions to both the network specification and connection ordering problems. While there are a number of published studies that utilise meta-heuristic optimisation, in the form of evolutionary computing methods [24, 26-27, 30-31, 42] to optimise high-level network solutions; this is the first instance of swarm intelligence being demonstrated as a viable alternative.

6.2 Conclusions

The conclusions that have been drawn from the completion of work described in this thesis are outlined in the following sections.

6.2.1 Networks of Larger Scope & Greater Complexity

The optimisation framework developed for this research allows for a broader range of subsea electrical network configurations to be examined, in high-fidelity, than other published methods. This has been achieved through the implementation of **all** of the desirable optimisation characteristics identified with the relevant academic literature, specifically allowing for:

- Networks that have multiple collector stages interfacing with a single transmission system, by means of one or more offshore substations.
- AC or DC transmission systems, in radial, ring, and star topologies.
- Collector networks which have topologies and number of concurrent strings of connections prescribed on an individual basis, such that asymmetric networks are feasible.
- Adjustable voltage and frequency parameters, for both collector and transmission networks.

The optimisation framework developed for this work is capable of assessing networks of larger scope (number of parallel offshore substations and associated collector networks) and greater complexity (range of topologies, numbers of parallel connections for collector networks; AC and DC transmission; adjustable voltage and frequency parameters) than any published work may consider. An example of this is demonstrated where the optimal network design for the proposed East Anglia One development, using the anticipated design voltages and frequencies, is shown to have 3 offshore substations, rather than the proposed 2 offshore substation design, connected in an HVDC ring transmission system; with all collector networks using a radial configuration. When the voltage and frequency parameters, for both the collector and transmission networks, were manipulated, as described in Chapter 5; the optimal solution returned for the East Anglia One development had 2 offshore substations interfacing with radial AC collector networks of asymmetric design (7 and 10 parallel connection strings); and connected to the land based network using an HVDC ring transmission system.

The lack of any discernible *one-size-fits-all* network configurations demonstrates the value of the optimisation framework and the underlying swarm intelligence search technique as it allows for a vast range of high-level design options to be considered. In some cases, as shown in Table 32, the differential cost between solutions may amount to hundreds of millions of Euros, for a large offshore development. Consequently, the consideration of the broadest range of possible solutions, achieved through an optimisation framework that is capable of considering networks of larger scope and greater complexity than any contemporary study, is of vital significance to those looking to develop offshore renewable energy generation projects, as they seek to maximise the utility of their investment.

The accuracy of optimisation results obtained was improved, relative to contemporary work, by calculating component sizing and associated costs directly from the power flow simulation. This allows for cables and other plant to be sized in accordance with its individual requirements, in preference to the discrete sizing methodology offered elsewhere [25-28, 31, 42, 80].

6.2.2 Hybrid Algorithms

Figure 26 depicts an abstracted 3 sub-problem representation of a subsea network optimisation problem delineated as: network specification; connection ordering; and quantification of network performance, and ultimately cost.

The network specification aspect of the complete optimisation process concerns itself with the production of high-level candidate solutions, refined iteratively to produce an optimal result. This high-level solution contains the locations of all offshore substations; the topologies and number of parallel connections utilised for each collector network; and the transmission system topology. This alone does not provide an assessable network model, as the same high-level solution may possess over 10^{100} possible detailed solutions once orders of machine connection are considered, as discussed in Chapter 3 of this thesis; thus an *inner* problem of connection ordering is established to support the *outer* problem of network specification. This concept is depicted within Figure 28 where connection ordering is shown to be residing entirely within the network specification problem.

A number of published studies exist that seek to optimise both collector and transmission stages of subsea electrical concurrently. Within such papers, little mention is made of how the network specification and connection ordering problems are delineated, defined mathematically, and ultimately addressed. Often, the issue is either not discussed at all [25-26, 29, 31]; or only a token reference is made to connection ordering being undertaken: in an undisclosed manner [23], using minimum spanning tree methods [27, 30, 32], or using travelling salesman methods [28]. For each of these cases, only radial connections are catered for.

Wu et al [42] presents delineated processes: with an *optimal line connection topology* (analogous to connection ordering) being sought for a specific *layout* (analogous to high-level network specification). This work, however, is based on cable length minimisation and may only consider radial connection of machines [42].

This work builds on the delineation of problems offered by Wu [42]. The delineation is formally articulated; and its problem scope is established and defined mathematically. This work addresses the issue that is present in the relevant literature where other published methods that seek to simultaneously optimise collector and transmission systems are limited to radial topology only. By including the ability of networks to adopt ring and star topologies, this optimisation framework can consider a broader range of solutions than existing methods: the value of which is observable in the solutions returned for both Case Studies considered for this thesis.

This work also demonstrates the feasibility of utilising distinct algorithms to perform the optimisations required for address the *inner* and *outer* problems, as conceptualised in Figure 28. Swarm intelligence, through Particle Swarm Optimisation, is used to optimise high-level network specifications; while evolutionary computing, through a Genetic Algorithm, is utilised to provide an efficient connection order. In both cases, the algorithms may directly assess an infinitesimally small amount of the feasible solutions, as described in the previous section; thus demonstrating their efficacy at minding strong solutions in an acceptable timeframe.

This work is the first example of swarm intelligence being utilised for the optimisation of high-level solutions describing subsea electrical networks, for the connection of offshore generation. It has been shown to be a viable and credible alternative to evolutionary computing methods which have been frequently utilised for this task [24, 26-27, 30-31, 42].

6.3 Future Research

A number of avenues that could be explored that build upon the work described within this thesis have been identified and are now presented.

6.3.1 Active Network Management

Throughout this work, a simplifying assumption has been made that the energy resource, in this case wind, is applied consistently across all machines in an offshore development. As a consequence of this assumption, returned networks are scoped to handle the situation where all turbines are simultaneously outputting maximum power.

In reality, there will be some degree of variability between the wind strength at each turbine. Applying some degree of stochastic variability to the wind resource observed at each turbine may demonstrate that catering for the simultaneous maximum power condition, considered previously, is not financially viable. It may be that this condition is experienced so infrequently that it is more financially prudent to operate with a reduced network capacity and switch out turbines if this capacity is exceeded.

Active Network Management (ANM) is concerned with real-time generation output control, such that thermal limitations and / or voltage constraints of the network are not breached. Typically, ANM is applied to distribution networks where there is a significant penetration of distributed generation (DG) or there are constraints limiting the reinforcement of specific circuits [108]. In such networks, DG is often connected on a *non-firm* (managed) basis such that their output is controllable by the Distribution Network Operation (DNO) [108, 109]; as opposed to utilising a *firm* connection where the output is not subject to DNO control. Connecting in this manner limit allows for a cost-effective means of connecting DG into existing distribution networks, as the existing circuits can be operated nearer to capacity more of the time, without the need for scoping all circuits to handle the simultaneous maximum power output of all DG [109].

By including ANM within subsea electrical networks, it may be beneficial to limit capacity, such that the network may not simultaneously convey the full power of all turbines. This condition may occur sufficiently infrequently that the cost of increased losses associated with curtailment of generation at peak output conditions is offset in its entirety by the reduced capital expenditure of a lower capacity network.

ANM could be paired with strategic deployment of energy storage within a subsea electrical network to further reduce the requirement for the rated capacity to match the aggregate capacity of all offshore generation. Where energy storage is typically used for peak demand reduction on a conventional distribution network [108]; this may be reversed within an offshore renewable energy extraction network, such that ANM instructs batteries to charge when the power output of the machines exceeds the network capacity; and initiates a discharge when network capacity exceeds generation output. Energy storage could be employed at the turbines themselves; at offshore substations; or at both locations simultaneously; with the economics of ANM and energy storage offering vast research potential.

In addition to the strategic level benefits that may be derived from the inclusion of ANM and energy storage, there may also be operation level benefits. When energy storage is interfaced with the network using 4-quadrant power conversion systems, they may deliver real and reactive power independently [108], thus they may provide rapid power factor correction response. Further, the inclusion of energy storage devices may limit the requirements for reactive power compensation technologies, thus offsetting some or all of the cost of including energy storage. The inclusion of managed energy storage within subsea electrical networks, therefore, offers research scope from both economic and technical performance perspectives.

6.3.2 Optimisation Framework Extensions

DC collector networks were not considered within the developed optimisation framework due to the overwhelming prevalence of AC collector networks, primarily due to the historically prohibitive costs associated with DC collector systems.

DC collector networks could be included in order to allow the examination of their performance when deployed with both AC and DC transmission systems. Low-Voltage DC (LVDC) distribution networks are beginning to emerge as a viable alternative to AC with trials in Finland showing that LVDC (less than 1500 V) was economical for the replacement of medium voltage (1-36 kV) AC cabled sections of the distribution network [110, 111]. Given this voltage range is consistent with collector network voltages used for large offshore wind farms; it is worth investigating if the same conclusions with respect to economic viability can be made when LVDC is used in preference to a medium voltage AC subsea collector network.

The developed optimisation framework was created to address *blank canvas* situations where an entirely new network is being designed. At present, it has no value for the expansion of an existing network. With some modifications, the scope of the optimisation framework could be changed to allow the consideration of network expansion problems.

6.3.3 Exhaustive Parametric Sensitivity Analysis

Within both of the two case studies considered within this thesis, a limited sensitivity analysis is performed that examines the effect of increasing and decreasing the collector and transmission network voltage to the nearest commonly available voltage. Similarly, the frequency of collector networks was increased to an MFAC value of 150 Hz and decreased for transmission to an LFAC value of 16.66 Hz.

For both case studies, it was found that the selection of design variables pertaining to voltage and frequency could be improved upon. In both cases, it was found to be beneficial to increase the collector network voltage and frequency, and the transmission voltage while maintaining an AC transmission frequency equal to that of the grid with which the subsea network is to interface.

In order to gain a more solid understanding of the effects of these design variables on optimality, a more exhaustive sensitivity analysis could be undertaken, considering a broader range of operating voltages and frequencies.

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Appendices

Appendix A: Test Case 1 Input Data

A.1 Cable Data

The cable data used for Test Case 1 is presented as Table 37.

Type	Voltage (kV)	R (Ω / km)	L (mH / km)	C (μF / km)
AC Collector	25	0.1	0.46	0.18
AC Transmission	150	0.1	0.4	0.16

Table 37 - Test Case 1: Cable Data [36]

A.2 Optimisation Parameters

The complete set of optimisation parameters for Test Case 1 is presented as Table 38.

As the purpose of the optimisation in Test Case 1 is to minimise the electrical losses without taking cost into account, the cost of SVC is set to be free for both reactive power consumption and generation; hence the values in Table 38 for MVAr Consumption Cost and MVAr Generation Cost are set to zero.

Parameter	Value	Units
General Parameters		
Network Life Time	25	Years
Meant Time To Repair – Cables	90	Days
Mean Time To Repair – Offshore Substations	30	Days
Cable Failure Rate	0.001114	Per Km / Per Year
Offshore Substation Failure Rate	0.0032	Per Year
Cost Per MWh Received	185	€
Discount Rate	5	%
MVAr Consumption Cost	0	€ / MVAr
MVAr Generation Cost	0	€ / MVAr
Collector Parameters		
Min Number of Collectors	1	Km
Max Number of Collectors	1	Km

Min Collector X Position	-6.5	Km
Max Collector X Position	6.5	Km
Min Collector Y Position	0	Km
Max Collector Y Position	20	Km
Collector Operating Frequency	50	Hz
Radial Collector Topology Acceptable	Configuration Dependent	
Ring Collector Topology Acceptable		
Star Collector Topology Acceptable		
Min Number of Strings	1	
Max Number of Strings	1	
Collector Voltage	25	kV
Transmission Parameters		
Transmission Operating Frequency	50	Hz
Radial Transmission Topology Acceptable	Variant Dependent	
Ring Transmission Topology Acceptable		
Star Transmission Topology Acceptable		
Transmission Voltage	150	kV
PSO Parameters		
Particles in Swarm	20	
Number of Iterations	10000	
Number of Iterations without improvement to stop	200	
Reduction in maximum velocity after each iteration	1	%
GA Modified TSP Solver Parameters		
Population Size	80	
Number of Iterations	10000	
Weibull Wind Distribution Parameters		
Shape (k)	2.26	
Scale (A)	11.5	
Cut-In Speed	3	m/s
Rated Speed	13	m/s
Cut-Out Speed	25	m/s

Table 38 - Test Case 1: Optimisation Parameters

Appendix B: Test Case 2 Input Data

This appendix contains the input data used for Test Case 2.

B.1 Cable Data

The cable data used for Test Case 1 is presented as Table 39.

Type	Voltage (kV)	R (Ω / km)	L (mH / km)	C (μF / km)
AC Collector	25	0.1	0.46	0.18
AC Transmission	150	0.1	0.4	0.16

Table 39 - Test Case 2: Cable Data [36]

B.2 Optimisation Parameters

The complete set of optimisation parameters for Test Case 2 is presented as Table 40.

As the purpose of the optimisation in Test Case 2 is to minimise the electrical losses without taking cost into account, the cost of SVC is set to be free for both reactive power consumption and generation; hence the values in Table 40 for MVAr Consumption Cost and MVAr Generation Cost are set to zero.

Parameter	Value	Units
General Parameters		
Network Life Time	25	Years
Meant Time To Repair – Cables	90	Days
Mean Time To Repair – Offshore Substations	30	Days
Cable Failure Rate	0.001114	Per Km / Per Year
Offshore Substation Failure Rate	0.0032	Per Year
Cost Per MWh Received	185	€
Discount Rate	5	%
MVAr Consumption Cost	0	€ / MVAr
MVAr Generation Cost	0	€ / MVAr
Collector Parameters		
Min Number of Collectors	Variant Dependent	
Max Number of Collectors		

Min Collector X Position	-6.5	Km
Max Collector X Position	6.5	Km
Min Collector Y Position	0	Km
Max Collector Y Position	20	Km
Collector Operating Frequency	50	Hz
Radial Collector Topology Acceptable	Variant Dependent	
Ring Collector Topology Acceptable		
Star Collector Topology Acceptable		
Min Number of Strings		
Max Number of Strings		
Collector Voltage	25	kV
Transmission Parameters		
Transmission Operating Frequency	50	Hz
Radial Transmission Topology Acceptable	Variant Dependent	
Ring Transmission Topology Acceptable		
Star Transmission Topology Acceptable		
Transmission Voltage	150	kV
PSO Parameters		
Particles in Swarm	20	
Number of Iterations	5000	
Number of Iterations without improvement to stop	200	
Reduction in maximum velocity after each iteration	1	%
GA Modified TSP Solver Parameters		
Population Size	80	
Number of Iterations	10000	
Weibull Wind Distribution Parameters		
Shape (k)	2.26	
Scale (A)	11.5	
Cut-In Speed	3	m/s
Rated Speed	13	m/s
Cut-Out Speed	25	m/s

Table 40 - Test Case 2: Optimisation Parameters

B.3 Analysis of Returned Solutions

Variant 1: Single Collector, Single String, Radial Collector Network

By placing the collector hub at a distance of 0 km from either generator, the total length of the collector cable is minimised and fixes the transmission cable length at 11.18 km. Either of these solutions will minimise the electrical losses associated with the network.

The solution returned for this scenario is detailed in Table 41. This solution is technically sub-optimal as the collector is placed at a non-zero distance from the optimal location. As has been discussed previously, this error is orders of magnitude smaller than the positional tolerance of offshore plant; hence this placement is effectively optimal.

This solution is only optimal as the generators are connected in the correct order. As generator 2 is positioned at (-5, 10) and generator 1 at (5, 10), this is the optimal connection order as detailed in Table 41.

Owing to the symmetry of the network, an equally valid solution would be the deployment of the offshore subsation at zero-distance to generator 2 (-5, 10) with a generator connection order of 2, 1.

Parameter	Value	
	X	Y
Optimal Position (km)	-5 OR +5	10
Returned Position (km)	5.0000	10.0000
Error wrt Optimal (m)	7.54×10^{-5}	
Connection Order	Gen 1, Gen 2	

Table 41 - Returned Solution for Test Case 2 - Variant 1

Variant 2: Single Collector, Two Strings, Radial Collector Network

This simulation was setup identically to Variant 1 of Test Case 2, except now two strings are used to link the generators and the offshore substation. By configuring as a 2 string radial network, this is analogous to a star collector network.

Positioning the offshore substation with at (0, 10) allows for the length, and therefore the losses, within the collector network to be minimised. By reducing the value of the y-coordinate of the offshore substation, the length of transmission cabling, and associated losses, which must be used may be reduced, while incurring additional cabling and losses in the collector network. In order to identify the optimal position, a simple mathematical model of electrical losses as a function of collector y-coordinate was developed, with the result depicted in Figure 41.

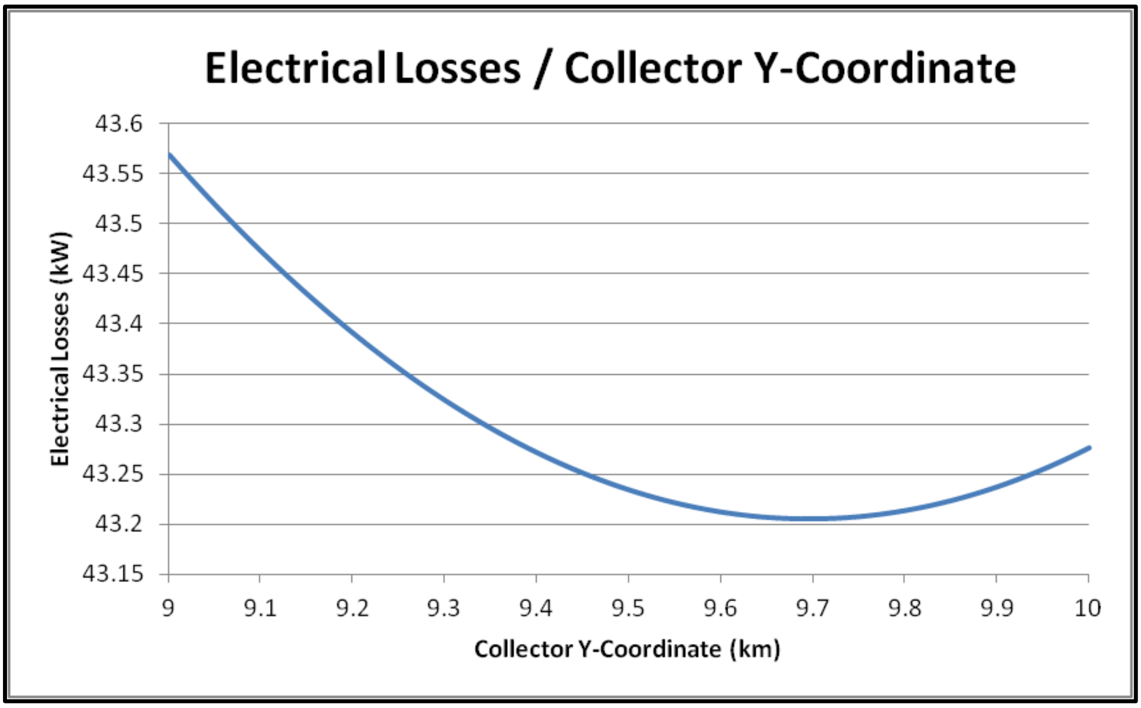

Figure 41 - Electrical Losses and Collector Y-Coordinate for Test Case 2 - Variant 2

Parameter	Value	
	X	Y
Calculated Optimal Position (km)	0.0000	9.6950
Returned Position (km)	0.0000	9.6927
Error wrt Optimal (m)	2.3	

Table 42 - Returned Solution for Test Case 2- Variant 2

The solution returned is presented as Table 42 is 2.3 m displaced from the calculated optimal solution. While this positional error is greater than for a number of other test cases, it is worth noting from Figure 41 that the losses incurred by positioning the collector with a y-coordinate in the range 9.45 and 9.9 km alters the losses incurred in the system by a maximum of 50 W, which is negligible against both the total losses (0.1 %) and the power throughput of the system (0.000625 %). As a result, the differential losses between the returned position and the calculated optimal position will be smaller than the losses described previously that are considered negligible.

Variant 3: Single Collector, Single String, Ring Collector Network

For this case, the lowest loss solution and the minimum cabling solution are distinct. Positioning the offshore substation at (0, 10) provides the total shortest distance of collector and transmission cabling. If the collector cabling was identical to the transmission cabling, this would be the optimal solution. As the collector cabling is less efficient than the higher-voltage transmission cabling; by positioning the offshore substation at a zero distance to either generator, the electrical losses are minimised.

The solution returned for this scenario is presented in Table 43. As with solutions to previous scenarios, the collector placement is effectively, if not strictly, optimal.

Parameter	**Value**	
	X	**Y**
Optimal Position (km)	-5 OR +5	10
Returned Position (km)	-5	10
Error wrt Optimal (m)	2.69×10^{-7}	
Connection Order	Gen 2, Gen 1	

Table 43 - Returned Solution for Test Case 2 - Variant 3

Variant 4: Single Collector, Two Strings, Ring Collector Network

This scenario is setup with optimisation parameters identical to those used for Test Case 2 - Variant 3 with the exception that two collector strings are used in preference to one.

In a similar fashion to Test Case 2 – Variant 2, the electrical losses may be mathematically modelled as a function of y-coordinate, assuming a fixed x-coordinate of 0. The resulting losses, focussing on the region of the optimal y-coordinate, are as shown in Figure 42.

The solution returned for this scenario is presented as Table 44. While the difference in collector positions between the optimal solution and that returned is larger than some other test case at 0.67 m, this can still be taken as being optimal due to the tolerances to which an offshore collector may be positioned.

It is also worth noting that the difference in electrical losses between positioning the offshore collector with a y-coordinate of 9.3 and 9.3811 is 2.5 W which is negligible in terms of both the magnitude of losses (~24.4 kW) and the real power delivered through the network (8 MW). As such, a range of solutions with an offshore collector positioned around the mathematical optimal solution will deliver an effectively optimal solution, in terms of electrical power losses.

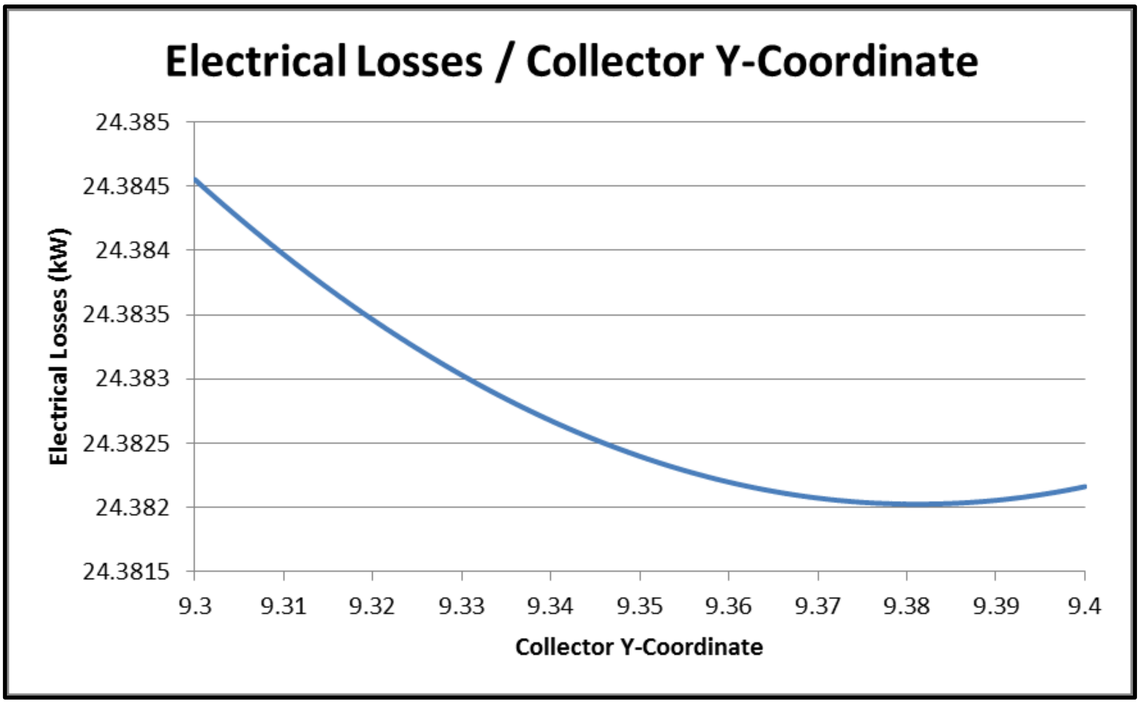

Figure 42 - Electrical Losses and Collector Y-Coordinate for Test Case 2 - Variant 4

Parameter	**Value**	
	X	**Y**
Optimal Position (km)	0.0000	9.3811
Returned Position (km)	0.0000	9.3818
Error wrt Optimal (m)	0.67	

Table 44 - Returned Solution for Test Case 2 - Variant 4

Variant 5: Single Collector, Star Collector Network

When a star collector network is deployed the number of strings is always identical to the number of generators connected to the offshore substation. Consequently, this network is analogous to two-string radial network considered in Test Case 2 – Variant 2, and as such the optimal collector position occurs at (0, 9.695).

The solution returned for the scenario is presented as Table 45. As was the case with Variant 2, there is moderate discrepancy between the calculated optimal solution and the observed optimal solution. The solutions returned for Variants 2 and 5 are identical to 5 decimal places.

Parameter	**Value**	
	X	**Y**
Calculated Optimal Position (km)	0.0000	9.6950
Returned Position (km)	0.0000	9.6927
Error wrt Optimal (m)	2.3	

Table 45 - Returned Solution for Test Case 2 - Variant 5

Variant 6: Two Collectors, Single String, Radial Collector Network

By using two collectors, each with a single string emanating from it, and transmission cables that are more electrically efficient than collector cables, it follows that the minimum losses solution occurs when the collectors are placed at zero distance from each generator and as such the optimal collector positions are (-5, 10) and (5, 10). The solution returned for this scenario is detailed in Table 46. As with a number of variants of Test Case 2 the delivered solution is not strictly optimal, but is effectively optimal.

Parameter	**Value**
Collector 1 X-Coordinate (km)	-5.0000
Collector 1 Y-Coordinate (km)	10.0000
Collector 1 Topology	Radial
Number of Strings @ Collector 1	1
Collector 2 X-Coordinate (km)	4.9999
Collector 2 Y-Coordinate (km)	9.9999
Collector 2 Topology	Radial
Number of Strings @ Collector 2	1
Transmission Topology	Radial
Positional Error wrt Optimal (m)	1.75×10^{-5}

Table 46 - Returned Solution for Test Case 2 - Variant 6

As was the case with Test Case 1, in this scenario a direct connection would be made between each generator and the connection point, and this scenario is optimised only to demonstrate that the optimisation is performing as it should.

Variant 7: Two Collectors, Single String, Ring Collector Network

This scenario differs from that described in Test Case 2 - Variant 6 only in the respect that a return cable links each collector with each generator. If the collectors are placed at zero distance to the generators, then the impedance of the collector network is zero and the optimal collector positioning is identical to that considered for Variant 6.

The solution that was delivered for this scenario is presented as Table 47. As was the case for Variant 6, the solution returned is effectively optimal with the collectors positioned a total of 2.65×10^{-5} m from their ideal locations, against a search space of 260 km^2.

Parameter	**Value**
Collector 1 X-Coordinate (km)	-4.9999
Collector 1 Y-Coordinate (km)	9.9999
Collector 1 Topology	Ring
Number of Strings at Collector 1	1
Collector 2 X-Coordinate (km)	4.9999
Collector 2 Y-Coordinate (km)	9.9999
Collector 2 Topology	Ring
Number of Strings at Collector 2	1
Transmission Topology	Radial
Positional Error wrt Optimal (m)	2.65×10^{-5}

Table 47 - Returned Solution for Test Case 2 - Variant 7

Variant 8: Two Collectors, Star Collector Network

Parameter	**Value**
Collector 1 X-Coordinate (km)	5.0000
Collector 1 Y-Coordinate (km)	9.9999
Collector 1 Topology	Star
Number of Strings at Collector 1	1
Collector 2 X-Coordinate (km)	-5.0000
Collector 2 Y-Coordinate (km)	9.9999
Collector 2 Topology	Star
Number of Strings at Collector 2	1
Transmission Topology	Radial
Positional Error wrt Optimal (m)	4.37×10^{-6}

Table 48 - Returned Solution for Test Case 2 - Variant 8

This scenario is electrically equivalent to Variant 6 as the star connection of each generator is analogous to a single generator being connected in a radial fashion. Consequently, the optimal solution occurs when the collectors are placed at zero distance from the generators, at (5, 10) and (-5, 10) respectively. As may be observed in Table 48, the total positional error with respect to the optimal position of the collectors is 4.37×10^{-6} m, which is several orders of magnitude smaller than the achievable tolerance of collector placement.

Appendix C: Test Case 3 Input Data

This appendix contains the input data used for Test Case 3.

C.1 Cable Data

The cable data used for Test Case 1 is presented as Table 49.

Type	Voltage (kV)	R (Ω / km)	L (mH / km)	C (μF / km)
AC Collector	25	0.1	0.46	0.18
AC Transmission	150	0.1	0.4	0.16

Table 49 - Test Case 3: Cable Data [36]

C.2 Optimisation Parameters

The complete set of optimisation parameters for Test Case 3 is presented as Table 50.As the purpose of the optimisation in Test Case 3 is to minimise electrical losses without accounting for cost, SVC is set to be free for both reactive power consumption and generation; hence the values in Table 50 for MVAr Consumption Cost and MVAr Generation Cost are set to zero.

Parameter	Value	Units
General Parameters		
Network Life Time	25	Years
Meant Time To Repair – Cables	90	Days
Mean Time To Repair – Offshore Substations	30	Days
Cable Failure Rate	0.001114	Per Km / Per Year
Offshore Substation Failure Rate	0.0032	Per Year
Cost Per MWh Received	185	€
Discount Rate	5	%
MVAr Consumption Cost	0	€ / MVAr
MVAr Generation Cost	0	€ / MVAr

Collector Parameters		
Min Number of Collectors	Variant Dependent	
Max Number of Collectors		
Min Collector X Position	-6.5	Km
Max Collector X Position	6.5	Km
Min Collector Y Position	0	Km
Max Collector Y Position	20	Km
Collector Operating Frequency	50	Hz
Radial Collector Topology Acceptable	Variant Dependent	
Ring Collector Topology Acceptable		
Star Collector Topology Acceptable		
Min Number of Strings		
Max Number of Strings		
Collector Voltage	25	kV
Transmission Parameters		
Transmission Operating Frequency	50	Hz
Radial Transmission Topology Acceptable	Variant Dependent	
Ring Transmission Topology Acceptable		
Star Transmission Topology Acceptable		
Transmission Voltage	150	kV
PSO Parameters		
Particles in Swarm	20	
Number of Iterations	5000	
Number of Iterations without improvement to stop	200	
Reduction in maximum velocity after each iteration	1	%
GA Modified TSP Solver Parameters		
Population Size	80	
Number of Iterations	10000	
Weibull Wind Distribution Parameters		
Shape (k)	2.26	
Scale (A)	11.5	
Cut-In Speed	3	m/s

Rated Speed	13	m/s
Cut-Out Speed	25	m/s

Table 50 - Test Case 3: Optimisation Parameters

C.3 Analysis of Returned Solutions

Variant 1: Single Collector, Single String, Radial Collector Network

This scenario is identical to Test Case 2: Variant 1 with the exception that now three generators are considered instead of two. Consequently, the optimal location of the collector occurs at the same location(s) as Test Case 2: Variant 1. The solution returned for this scenario is as shown in Table 51.

Parameter	**Value**	
	X	**Y**
Optimal Position (km)	-5 OR +5	10
Returned Position (km)	5.0000	10.0000
Error wrt Optimal (m)	2.764×10^{-5}	
Connection Order	Gen 2, Gen 3, Gen1	

Table 51 - Returned Solution for Test Case 3 - Variant 1

The collector position is only one aspect of the optimal solution for this test case. Additionally, the offshore generators must be connected in the order that allows the collector cable length to be minimised. As the offshore substation is placed at zero distance to Gen 2, it follows that the optimal connection order is as shown in the bottom row of Table 51.

Given the inherent symmetry of the generator layout, an equally valid optimal solution occurs when the collector is placed at (5, 10) with a connection order of 1, 3, 2.

Variant 2: Single Collector, Two Strings, Radial Collector Network

This is the first test case where there is a requirement for an asymmetrical collector network, in terms of the number of generators connected to each string. As there are three generators to be linked to the offshore substation it follows that there should be one string linking one generator to the collector and one string connecting two generators to the collector.

The string linking the single generator and the collector will experience a constant current at all points in the cable. The string linking two generators and the collector will experience differing levels of current, due to the injection of power at distinct points. As electrical losses are proportional to the square of the current flow, it follows that the losses may be minimised by minimising the length of cable carrying *double* current.

The solution returned for this test case is as detailed in Table 52. The placement of the offshore substation at zero-distance from the generator at (0, 15) allows the heaviest losses in the collector network, between the nearest generator and the collector hub on the two-generator string to be mitigated.

Additionally, by placing the collector at (0, 15); the total length of collector cable is reduced to 14.04 km which represents a significant reduction on the 17.07 km that would be required if the collector was placed at zero distance to either of the other two generators in the system. This reduction in collector cabling required is offset against the requirement for additional transmission cabling which is more expensive. This analysis, however, considers only the electrical losses and is not concerned with financial aspects of the network optimisation.

Parameter	**Value**	
	X	**Y**
Optimal Collector Position (km)	0	15
Returned Collector Position (km)	1.79×10^{-4}	14.9999
Error wrt Optimal (m)	0.205	
String 1 Connection Order	Gen 3, Gen 2	
String 2 Connection Order	Gen 1	

Table 52 - Returned Solution for Test Case 3 - Variant 2

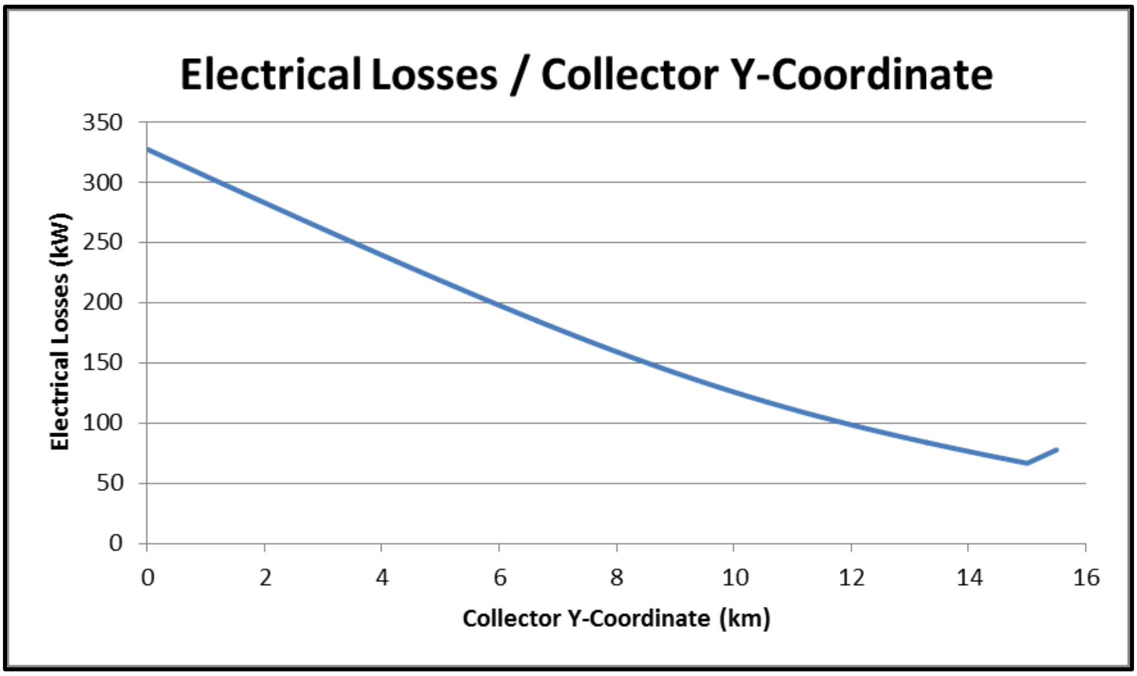

Figure 43 - Electrical Losses and Collector Y-Coordinate for Test Case 3 - Variant 2

By mathematically modelling the electrical losses in the system as the y-coordinate of the offshore collector is varied, while the x-coordinate is constant at 0, the validity of the solution returned may be verified. As may be observed in Figure 43, the electrical losses reduce as the collector is positioned farther offshore and reaches its electrically optimal position at (0, 15). The trend depicted in Figure 43 is generated based on the assumptions that the order in which nodes in the network remains the same and that the levels of reactive power injected or consumed at each node remain identical as the collector position is adjusted.

Variant 3: Single Collector, Three Strings, Radial Collector Network

As this case considers three string emanating from the collector, it follows that each string will link one generator to the collector hub; and as such this network is electrically identical to a star connected collector network.

The solution returned is as detailed in Table 53. Figure 44 shows the relationship between electrical losses and the y-coordinate of the collector placement, with minimum losses occurring when the collector is positioned 11.953 km offshore. As may be observed in Table 53, the returned optimal solution, and the calculated optimal solution differ by 0.64 m, in terms of collector position, in a search space of 260 km^2.

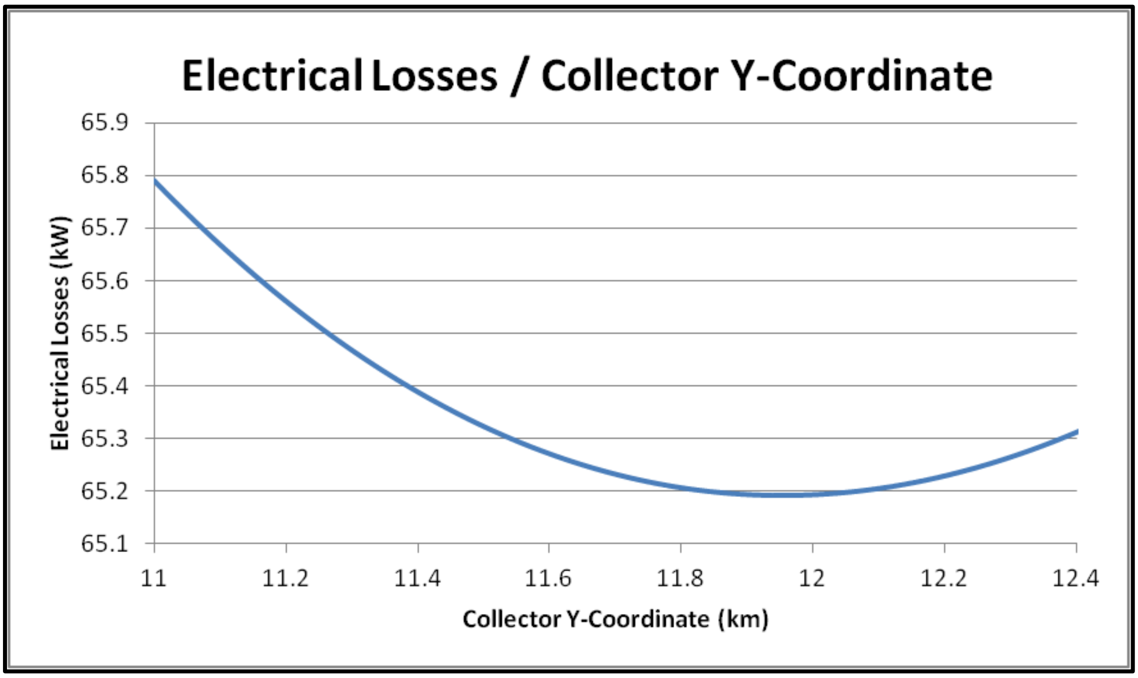

Figure 44 - Electrical Losses and Collector Y-Coordinate for Test Case 3 - Variant 3

The trend depicted in Figure 44 is generated based on the assumptions that the order in which nodes in the network remains the same and that the levels of reactive power injected or consumed at each node remain identical as the collector position is adjusted.

Parameter	**Value**	
	X	**Y**
Optimal Collector Position (km)	0	11.953
Returned Collector Position (km)	0.0000	11.9536
Error wrt Optimal (m)	0.64	
String 1 Connection Order	Gen 1	
String 2 Connection Order	Gen 2	
String 3 Connection Order	Gen 3	

Table 53 - Returned Solution for Test Case 3 - Variant 3

Variant 4: Single Collector, Single String, Ring Collector Network

In this scenario, the offshore network behaves in a very similar fashion to Test Case 3: Variant 2. By positioning the offshore collector at zero distance to one of the generators, the *double current* situation, and the associated heavy losses, described in the analysis of Variant 2 is negated. As generators 1 and 2 are identical, some current from both machines will flow along the cable linking them, effectively cancelling one another out. Instead all current will flow from these machines back to the collector hub. As there is no current flow on this cable section, the network becomes electrically analogous to that considered in Test Case 3: Variant 2 with generator 1 effectively connected to the offshore subation by a radial string and generators 3 and 2 linked to the offshore substation by another.

Given the analogous nature of this network with that considered in Test Case 3: Variant 2; it follows that for both cases the optimal collector position returned should be identical. The difference in collector position returned for these cases is 2.2 cm, which is likely to be orders of magnitude smaller than the possible positional deployment tolerance.

The complete solution returned for this scenario is given as Table 54. The electrical losses as a function of collector position on the y-axis, assuming that the network retains the same nodal connection order and the same reactive power output at each node, is as presented in Figure 43.

Parameter	Value	
	X	Y
Optimal Collector Position (km)	0	15
Returned Collector Position (km)	1.62×10^{-4}	14.9999
Error wrt Optimal (m)	0.183	
String 1 Connection Order	Gen 1, Gen 2, Gen 3	

Table 54 - Returned Solution for Test Case 3 – Variant 4

Variant 5: Single Collector, Two Strings, Ring Collector Network

While this network is asymmetrically loaded as one string is linking two generators with the collector hub while the second string links only the one; strategic placement of the collector at zero distance to generator 3 at (0, 15) allows the network to become, effectively, a symmetrical network with generators 1 and 2 being linked to the collector using ring cables. The *double current* situation described previously is negated by placing the collector in this position.

The solution returned for this test case is as detailed in Table 55. As with a number of other test scenarios, there is a marginal discrepancy between the optimal collector position for this scenario and the calculated optimal (4 cm); however as this is likely below the deployment positional tolerance achievable, this solution may be taken as optimal.

Owing to the inherent symmetry of the considered test case, an equally valid optimal solution would be to connect generators 2 and 3 using a single string and generator 1 using the second string.

Parameter	**Value**	
	X	**Y**
Optimal Collector Position (km)	0	15
Returned Collector Position (km)	0.0000	14.9999
Error wrt Optimal (m)	0.04	
String 1 Connection Order	Gen 1, Gen 3	
String 2 Connection Order	Gen 2	

Table 55 - Returned Solution for Test Case 3: Variant 5

Variant 6: Single Collector, Three Strings, Ring Collector Network

This scenario is broadly similar to Test Case 3: Variant 3 as each generator is connected to the collector hub using its own string. The only difference between Variants 3 and 6 is the presence of the return cable forming ring network for each string. As these cases are inherently similar, it would be naturally expected that the optimal network configuration for Variant 6 is similar to that returned for Variant 3.

Parameter	**Value**	
	X	**Y**
Optimal Collector Position (km)	0	11.152
Returned Collector Position (km)	0.0000	11.1513
Error wrt Optimal (m)	0.16	
String 1 Connection Order	Gen 1	
String 2 Connection Order	Gen 2	
String 3 Connection Order	Gen 3	

Table 56 - Solution for Test Case 3: Variant 6

The optimum solution returned is detailed in Table 56. The electrical losses, modelled as a function of collector y-coordinate, are presented as Figure 45. The difference between the calculated optimal solution for this scenario, assuming the connection order and reactive power output at each node is constant for all collector placements, is 0.16 m.

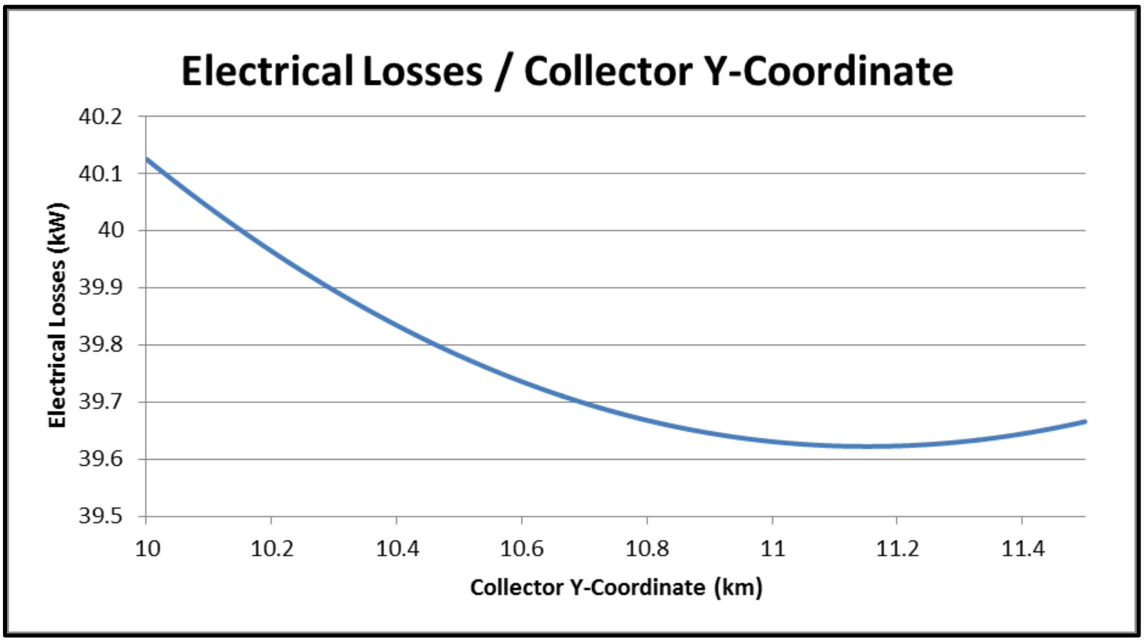

Figure 45 - Electrical Losses and Collector Y-Coordinate for Test Case 3 - Variant 6

It may be observed in Figure 45 that the differential value of losses between placing the collector with a y-coordinate anywhere in the range 10 to 11.15 km is approximately 500 W which is small in comparison with the overall losses (~1.25 %) and negligible in comparison with the overall power of the system (~0.004 %).

Variant 7: Single Collector, Star Collector Network

Parameter	**Value**	
	X	**Y**
Optimal Collector Position (km)	0	11.9530
Returned Collector Position (km)	0.0000	11.9536
Error wrt Optimal (m)	0.59	
Positional Difference wrt Variant 3 (m)	0.048	

Table 57 - Returned Solution for Test Case 3: Variant 7

This case is electrically analogous to Variant 3 where a three-stringed radial collector network effectively formed a star collection system. As would be expected, the results obtained for Variants 3 and 7 are very similar, with the losses being infinitesimally different to one another and the returned optimal collector positions being less than 5 cm apart, in a potential deployment space of 260 km^2.

The solution returned for this scenario is detailed in Table 57.

Variant 8: Two Collectors, Single String, Radial Collector Network

In this case, there are two collectors with a single radial string emanating from each. The collectors themselves are connected radially to the interface point with the shore based network. As the transmission cables are more efficient than the collector cables, it follows that the optimal solution, from an electrical losses perspective, will be *transmission heavy* and *collector light*. The solution returned for this scenario is as detailed in Table 58.

This solution returned minimises the losses incurred within the collector network as only 7.07 km of low efficiency collector cable is required to form the radial strings from each collector. This is achieved through positioning the collectors at zero distance from generators 1 and 2. If the collectors were positioned at zero distance from one of generator 3 and one of generator 1 and 2 then the collector cable distance would be 10 km.

Parameter	**Value**	
	X	**Y**
Optimal Collector 1 Position (km)	-5.0000	10.0000
Optimal Collector 2 Position (km)	5.0000	10.0000
Returned Collector 1 Position (km)	-5.0000	10.0000
Returned Collector 2 Position (km)	5.0000	10.0000
Total Positional Error wrt Optimal (m)	0.05	
Transmission Connection Order	Col 2, Col 1	
Connection Order Collector 1	Gen 2	
Connection Order Collector 2	Gen 1, Gen 3	

Table 58 - Returned Solution for Test Case 3: Variant 8

As has been considered before, the electrical losses within a cable are in proportion to the square of the current flow. While the transmission cables are more efficient than the collector cables, it is still desirable to minimise the cable lengths through which large currents must flow. The one generator that is not at zero distance from an offshore substation, generator 3, is connected to collector 2. This greatly reduces the losses incurred within the network compared to generator 3 being connected to collector 1.

As with all other test scenarios, the network is being optimised only to minimise electrical losses and an analysis considering reliability and costs would return a different network.

It is not possible to have two or three strings emanating from each collector as there would need to be at least 4 and 6 generators respectively in the system for these to be feasible scenarios.

Variant 9: Two Collectors, Single String, Ring Collector Network

This case differs to Test Case 3: Variant 8 only in the respect that ring strings now emanate from each collector, in preference to radial strings. Due to the inherent similarities between these cases, it might be expected that a similar network would be optimal for each case; however the returned solution is quite different.

As with other test scenarios there is a slight discrepancy in the collector positions between that returned and the calculated optimal solution, in this case 2.4 cm. The full solution returned is as detailed in Table 59.

Parameter	Value	
	X	Y
Optimal Collector 1 Position (km)	-5.0000	10.0000
Optimal Collector 2 Position (km)	0.0000	15.0000
Returned Collector 1 Position (km)	-5.0000	10.0000
Returned Collector 2 Position (km)	0.0000	15.0000
Total Positional Error wrt Optimal (m)	0.024	
Transmission Connection Order	Col 1, Col 2	
Connection Order Collector 1	Gen 2	
Connection Order Collector 2	Gen 3, Gen 1	

Table 59 - Returned Solution for Test Case 3: Variant 9

It is unfeasible to specify more than one string from each collector given that there are only 3 generators in the system modelled.

Variant 10: Two Collectors, Star Collector Network

The optimal solution for this scenario is essentially identical to that observed for Test Case 3: Variant 8. The full solution returned for this scenario is presented as Table 60.

Parameter	**Value**	
	X	**Y**
Optimal Collector 1 Position (km)	-5.0000	10.0000
Optimal Collector 2 Position (km)	5.0000	10.0000
Returned Collector 1 Position (km)	-5.0000	10.0000
Returned Collector 2 Position (km)	5.0000	10.0000
Total Positional Error wrt Optimal (m)	0.01	
Transmission Connection Order	Col 2, Col 1	

Table 60 - Returned Solution for Test Case 3: Variant 10

As this scenario considers only star connected collectors, there is no connection order to be defined at each collector as each generator has its own connection cable.

Variant 11: Three Collectors, Single String, Radial Collector Network

As there are three collectors specified, three generators in the system and the optimisation is only considering electrical losses, it follows logically that for this case, the three collectors will be placed at zero distance to each generator and the network will be configured exclusively from higher efficiency transmission cables in preference to lower efficiency collector cables.

In addition the collector networks having (effectively) zero length, the optimal network for this configuration requires the transmission network length to be minimised. Both of these facets of optimality are included within the solution returned for this scenario, which is detailed in Table 61.

Due to the inherent symmetry of the system, an equally valid optimal solution would occur if the offshore collector hubs were linked in the order 2-3-1, so long as each collector still maintained a zero distance connection to its adjacent generator.

As with other solutions obtained, there is a minor discrepancy between the optimal collector positions and those returned. In this case, the total positional error is less than 5 cm, and as such this solution may be taken as optimal.

Parameter	Value	
	X	Y
Optimal Collector 1 Position (km)	5.0000	10.0000
Optimal Collector 2 Position (km)	-5.0000	10.0000
Optimal Collector 3 Position (km)	0.0000	15.0000
Returned Collector 1 Position (km)	5.0000	10.0000
Returned Collector 2 Position (km)	-5.0000	10.0000
Returned Collector 3 Position (km)	0.0000	15.0000
Total Positional Error wrt Optimal (m)	0.048	
Transmission Connection Order	Col 1, Col 3, Col 2	
Connection Order Collector 1	Gen 1	
Connection Order Collector 2	Gen 2	
Connection Order Collector 3	Gen 3	

Table 61 - Returned Solution for Test Case 3: Variant 11

Variant 12: Three Collectors, Single String, Ring Collector Network

This scenario is identical to that considered for Test Case 3 – Variant 11 with the exception that ring networks are used to link the collectors with the generators. As there are three collectors and only three generators and only losses are being considered within the optimisation, the optimal solution lies with a minimal length transmission network that allows for the each collector network to be zero length – just as it was for Test Case 3 – Variant 11.

In this case, the solution returned was the *mirror* network of that returned for the previous variant, with the collectors linked in the order 2-3-1 based on the collector positions detailed in Table 61. As would be expected, the total positional error in the solution returned was similar to that experience for the previous scenario (6.3 cm).

Variant 13: Three Collectors, Star Collector Network

This scenario is electrically analogous to Test Case 3 – Variant 11 and consequently, it follows that the optimal solutions should be identical for these cases. For this case, the solution returned is identical to that for Variant 11, with the exception that the total positional error is 5.2 cm.

Appendix D: Case Study 1 Input Data

This appendix contains the input data used for Case Study 1.

D.1 Generator Data

The generator data used for Case Study 1 is presented as Table 62, and the layout of the wind farm is as depicted in Figure 35.

Gen ID	X Pos (km)	Y Pos (km)	Pout (MW)	Reactive Power Limits (MVAr)	
1	-2.52	34	2	-0.58	0.4
2	-1.96	34	2	-0.58	0.4
3	-1.4	34	2	-0.58	0.4
4	-0.84	34	2	-0.58	0.4
5	-0.28	34	2	-0.58	0.4
6	0.28	34	2	-0.58	0.4
7	0.84	34	2	-0.58	0.4
8	1.4	34	2	-0.58	0.4
9	1.96	34	2	-0.58	0.4
10	2.52	34	2	-0.58	0.4
11	-2.52	34.56	2	-0.58	0.4
12	-1.96	34.56	2	-0.58	0.4
13	-1.4	34.56	2	-0.58	0.4
14	-0.84	34.56	2	-0.58	0.4
15	-0.28	34.56	2	-0.58	0.4
16	0.28	34.56	2	-0.58	0.4
17	0.84	34.56	2	-0.58	0.4
18	1.4	34.56	2	-0.58	0.4
19	1.96	34.56	2	-0.58	0.4
20	2.52	34.56	2	-0.58	0.4
21	-2.52	35.12	2	-0.58	0.4
22	-1.96	35.12	2	-0.58	0.4
23	-1.4	35.12	2	-0.58	0.4
24	-0.84	35.12	2	-0.58	0.4
25	-0.28	35.12	2	-0.58	0.4
26	0.28	35.12	2	-0.58	0.4
27	0.84	35.12	2	-0.58	0.4
28	1.4	35.12	2	-0.58	0.4
29	1.96	35.12	2	-0.58	0.4
30	2.52	35.12	2	-0.58	0.4

31	-2.52	35.68	2	-0.58	0.4
32	-1.96	35.68	2	-0.58	0.4
33	-1.4	35.68	2	-0.58	0.4
34	-0.84	35.68	2	-0.58	0.4
35	-0.28	35.68	2	-0.58	0.4
36	0.28	35.68	2	-0.58	0.4
37	0.84	35.68	2	-0.58	0.4
38	1.4	35.68	2	-0.58	0.4
39	1.96	35.68	2	-0.58	0.4
40	2.52	35.68	2	-0.58	0.4
41	-2.52	36.24	2	-0.58	0.4
42	-1.96	36.24	2	-0.58	0.4
43	-1.4	36.24	2	-0.58	0.4
44	-0.84	36.24	2	-0.58	0.4
45	-0.28	36.24	2	-0.58	0.4
46	0.28	36.24	2	-0.58	0.4
47	0.84	36.24	2	-0.58	0.4
48	1.4	36.24	2	-0.58	0.4
49	1.96	36.24	2	-0.58	0.4
50	2.52	36.24	2	-0.58	0.4
51	-2.52	36.8	2	-0.58	0.4
52	-1.96	36.8	2	-0.58	0.4
53	-1.4	36.8	2	-0.58	0.4
54	-0.84	36.8	2	-0.58	0.4
55	-0.28	36.8	2	-0.58	0.4
56	0.28	36.8	2	-0.58	0.4
57	0.84	36.8	2	-0.58	0.4
58	1.4	36.8	2	-0.58	0.4
59	1.96	36.8	2	-0.58	0.4
60	2.52	36.8	2	-0.58	0.4
61	-2.52	37.36	2	-0.58	0.4
62	-1.96	37.36	2	-0.58	0.4
63	-1.4	37.36	2	-0.58	0.4
64	-0.84	37.36	2	-0.58	0.4
65	-0.28	37.36	2	-0.58	0.4
66	0.28	37.36	2	-0.58	0.4
67	0.84	37.36	2	-0.58	0.4
68	1.4	37.36	2	-0.58	0.4
69	1.96	37.36	2	-0.58	0.4
70	2.52	37.36	2	-0.58	0.4
71	-2.52	37.92	2	-0.58	0.4
72	-1.96	37.92	2	-0.58	0.4

73	-1.4	37.92	2	-0.58	0.4
74	-0.84	37.92	2	-0.58	0.4
75	-0.28	37.92	2	-0.58	0.4
76	0.28	37.92	2	-0.58	0.4
77	0.84	37.92	2	-0.58	0.4
78	1.4	37.92	2	-0.58	0.4
79	1.96	37.92	2	-0.58	0.4
80	2.52	37.92	2	-0.58	0.4

Table 62 - Case Study 1: Generator Data [47]

D.2 Cable Data

The cable data used for Case Study 1 is as presented in Table 63.

Type	Voltage (kV)	R (Ω / km)	L (mH / km)	C (µF / km)
AC Collector	33	0.17	0.39	0.24
AC Transmission	220	0.05	0.41	0.17

Table 63 - Case Study 1: Cable Data [36, 56]

D.3 Optimisation Parameters

The complete set of optimisation parameters for Case Study 1 is presented as Table 64.

Parameter	Value	Units
General Parameters		
Network Life Time	25	Years
Meant Time To Repair – Cables	90	Days
Mean Time To Repair – Offshore Substations	30	Days
Cable Failure Rate	0.001114	Per Km / Per Year
Offshore Substation Failure Rate	0.0032	Per Year
Cost Per MWh Received	185	€
Discount Rate	5	%
MVAr Consumption Cost	77,000	€ / MVAr
MVAr Generation Cost	77,000	€ / MVAr
Collector Parameters		
Min Number of Collectors	Variant Dependent	
Max Number of Collectors		

Min Collector X Position	-6.5	Km
Max Collector X Position	6.5	Km
Min Collector Y Position	0	Km
Max Collector Y Position	40	Km
Collector Operating Frequency	60	Hz
Radial Collector Topology Acceptable	1	
Ring Collector Topology Acceptable	1	
Star Collector Topology Acceptable	1	
Min Number of Strings	1	
Max Number of Strings	10	
Collector Voltage	33	kV
Transmission Parameters		
Transmission Operating Frequency	60 / DC	Hz
Radial Transmission Topology Acceptable	1	
Ring Transmission Topology Acceptable	1	
Star Transmission Topology Acceptable	1	
Transmission Voltage	220	kV
PSO Parameters		
Particles in Swarm	50	
Number of Iterations	1000	
Number of Iterations without improvement to stop	1000	
Reduction in maximum velocity after each iteration	0	%
GA Modified TSP Solver Parameters		
Population Size	80	
Number of Iterations	10000	
Weibull Wind Distribution Parameters		
Shape (k)	2.26	
Scale (A)	11.5	
Cut-In Speed	4	m/s
Rated Speed	14	m/s
Cut-Out Speed	25	m/s

Table 64 - Case Study 1: Optimisation Parameters

D.4 Real World Network Model

In order to provide comparison with the real world subsea network utilised at the Horns-Rev 1 installation, a model of this network covering its electrical performance was developed. In this section each of the data sections that comprise the MATPOWER case file are presented.

D.4.1 Bus Data

Within the developed framework, when a model is being constructed, each generator is assigned a unique bus identifier, based on the number of offshore collectors in the system as shown by (68). In this case, the bus identifiers are assigned in exactly the same way with bus identifier 1 representing the connection point with the land based network and bus 2 representing the offshore collector. Each generator listed in Table 62 with generator identifiers 1 through 80 is assigned bus identifiers of 3 to 82.

$$Bus\ ID = Generator\ ID + Number\ of\ Collectors + 1 \tag{68}$$

The connection point, bus 1, is set up at a swing bus for the purposes of the power flow analysis and consequently has a bus type of 3. A real power demand equal to the summation of the generation capacity is defined at this bus.

The offshore collector, bus 2, is set up as a generator bus that has no real or reactive power demand. By setting up this node as a generator bus, in preference to a load bus, a virtual generator with zero real power capacity but infinite reactive power output range may be established, simulating the placement of reactive power compensation at this node.

Each generator, buses 3 to 82 in this case, is set up as a generator bus, such that the real power output of each is fixed but the reactive output is variable. All generator buses are assigned a bus type value of 2, to indicate to MATPOWER that this is a generator bus.

The complete bus data array that is fed into MATPOWER for the power flow study of the real world network is presented as Table 66. Contextualisation of Table 66 is offered in Table 65.

Column Number	Column Name	Meaning	Units
1	Bus ID	Unique identifier of bus	
2	Type	Bus type (1 = Load; 2 = Generator; 3 = Swing)	
3	PD	Real power demand at bus	MW
4	QD	Real power demand at bus	MVAr
5	GS	Shunt conductance at bus	MW
6	BS	Shunt susceptance at bus	MVAr
7	Bus Area	Area number	
8	VM	Voltage Magnitude	per-unit
9	VA	Voltage angle	degrees
10	kV Base	Base voltage	kV
11	Zone	Loss Zone	
12	VMax	Maximum voltage magnitude	per-unit
13	VMin	Minimum voltage magnitude	per-unit

Table 65- Column Context for Table 66

Bus ID	Type	PD	QD	GS	BS	Area	VM	VA	kV Base	Zone	VMax	VMin
1	3	160	0	0	0	1	1	0	150	1	1.05	0.95
2	2	0	0	0	0	1	1	0	150	1	1.05	0.95
3	2	0	0	0	0	1	1	0	25	1	1.05	0.95
4	2	0	0	0	0	1	1	0	25	1	1.05	0.95
5	2	0	0	0	0	1	1	0	25	1	1.05	0.95
6	2	0	0	0	0	1	1	0	25	1	1.05	0.95
7	2	0	0	0	0	1	1	0	25	1	1.05	0.95
8	2	0	0	0	0	1	1	0	25	1	1.05	0.95
9	2	0	0	0	0	1	1	0	25	1	1.05	0.95
10	2	0	0	0	0	1	1	0	25	1	1.05	0.95
11	2	0	0	0	0	1	1	0	25	1	1.05	0.95
12	2	0	0	0	0	1	1	0	25	1	1.05	0.95
13	2	0	0	0	0	1	1	0	25	1	1.05	0.95
14	2	0	0	0	0	1	1	0	25	1	1.05	0.95
15	2	0	0	0	0	1	1	0	25	1	1.05	0.95
16	2	0	0	0	0	1	1	0	25	1	1.05	0.95
17	2	0	0	0	0	1	1	0	25	1	1.05	0.95
18	2	0	0	0	0	1	1	0	25	1	1.05	0.95
19	2	0	0	0	0	1	1	0	25	1	1.05	0.95
20	2	0	0	0	0	1	1	0	25	1	1.05	0.95
21	2	0	0	0	0	1	1	0	25	1	1.05	0.95
22	2	0	0	0	0	1	1	0	25	1	1.05	0.95

23	2	0	0	0	0	1	1	0	25	1	1.05	0.95
24	2	0	0	0	0	1	1	0	25	1	1.05	0.95
25	2	0	0	0	0	1	1	0	25	1	1.05	0.95
26	2	0	0	0	0	1	1	0	25	1	1.05	0.95
27	2	0	0	0	0	1	1	0	25	1	1.05	0.95
28	2	0	0	0	0	1	1	0	25	1	1.05	0.95
29	2	0	0	0	0	1	1	0	25	1	1.05	0.95
30	2	0	0	0	0	1	1	0	25	1	1.05	0.95
31	2	0	0	0	0	1	1	0	25	1	1.05	0.95
32	2	0	0	0	0	1	1	0	25	1	1.05	0.95
33	2	0	0	0	0	1	1	0	25	1	1.05	0.95
34	2	0	0	0	0	1	1	0	25	1	1.05	0.95
35	2	0	0	0	0	1	1	0	25	1	1.05	0.95
36	2	0	0	0	0	1	1	0	25	1	1.05	0.95
37	2	0	0	0	0	1	1	0	25	1	1.05	0.95
38	2	0	0	0	0	1	1	0	25	1	1.05	0.95
39	2	0	0	0	0	1	1	0	25	1	1.05	0.95
40	2	0	0	0	0	1	1	0	25	1	1.05	0.95
41	2	0	0	0	0	1	1	0	25	1	1.05	0.95
42	2	0	0	0	0	1	1	0	25	1	1.05	0.95
43	2	0	0	0	0	1	1	0	25	1	1.05	0.95
44	2	0	0	0	0	1	1	0	25	1	1.05	0.95
45	2	0	0	0	0	1	1	0	25	1	1.05	0.95
46	2	0	0	0	0	1	1	0	25	1	1.05	0.95
47	2	0	0	0	0	1	1	0	25	1	1.05	0.95
48	2	0	0	0	0	1	1	0	25	1	1.05	0.95
49	2	0	0	0	0	1	1	0	25	1	1.05	0.95
50	2	0	0	0	0	1	1	0	25	1	1.05	0.95
51	2	0	0	0	0	1	1	0	25	1	1.05	0.95
52	2	0	0	0	0	1	1	0	25	1	1.05	0.95
53	2	0	0	0	0	1	1	0	25	1	1.05	0.95
54	2	0	0	0	0	1	1	0	25	1	1.05	0.95
55	2	0	0	0	0	1	1	0	25	1	1.05	0.95
56	2	0	0	0	0	1	1	0	25	1	1.05	0.95
57	2	0	0	0	0	1	1	0	25	1	1.05	0.95
58	2	0	0	0	0	1	1	0	25	1	1.05	0.95
59	2	0	0	0	0	1	1	0	25	1	1.05	0.95
60	2	0	0	0	0	1	1	0	25	1	1.05	0.95
61	2	0	0	0	0	1	1	0	25	1	1.05	0.95
62	2	0	0	0	0	1	1	0	25	1	1.05	0.95
63	2	0	0	0	0	1	1	0	25	1	1.05	0.95
64	2	0	0	0	0	1	1	0	25	1	1.05	0.95

65	2	0	0	0	0	1	1	0	25	1	1.05	0.95
66	2	0	0	0	0	1	1	0	25	1	1.05	0.95
67	2	0	0	0	0	1	1	0	25	1	1.05	0.95
68	2	0	0	0	0	1	1	0	25	1	1.05	0.95
69	2	0	0	0	0	1	1	0	25	1	1.05	0.95
70	2	0	0	0	0	1	1	0	25	1	1.05	0.95
71	2	0	0	0	0	1	1	0	25	1	1.05	0.95
72	2	0	0	0	0	1	1	0	25	1	1.05	0.95
73	2	0	0	0	0	1	1	0	25	1	1.05	0.95
74	2	0	0	0	0	1	1	0	25	1	1.05	0.95
75	2	0	0	0	0	1	1	0	25	1	1.05	0.95
76	2	0	0	0	0	1	1	0	25	1	1.05	0.95
77	2	0	0	0	0	1	1	0	25	1	1.05	0.95
78	2	0	0	0	0	1	1	0	25	1	1.05	0.95
79	2	0	0	0	0	1	1	0	25	1	1.05	0.95
80	2	0	0	0	0	1	1	0	25	1	1.05	0.95
81	2	0	0	0	0	1	1	0	25	1	1.05	0.95
82	2	0	0	0	0	1	1	0	25	1	1.05	0.95

Table 66 - Bus Data for Case Study 1: Real World Network Model

D.4.2 Generator Data

The relevant generator data array that is fed into MATPOWER for the power flow study of the real world network is presented as Table 68. There are a number of other columns that are not required for MATPOWER to perform a power flow study and all of these values are left as zeros. Contextualisation of Table 68 is offered in Table 67.

There are *"virtual generators"* placed at bus 1 (the interface point with an onshore network) and bus 2 (the offshore substation). The virtual generator positioned at the offshore collector allows (effectively) infinite reactive power to be deployed in order to balance the system. This is achieved in conjuction with setting the bus type for the offshore collector to a generator bus where the real power and voltage are fixed and the voltage angle and reactive power and the voltage angle are variables. The cost of installing this reactive power compensation may then be approximated using the guidelines presented earlier in this thesis. There is no real power generating capacity at the offshore collector.

At the connection point there is also a virtual generator positioned. As this bus is a swing bus, the voltage magnitude and angle are assumed to be 1.0 pu and 0 degrees respectively. The real and reactive power *output* of this generator represents the real power losses within the system and the reactive power compensation required to balance the system.

All other generators are conventional generators with fixed real power output and voltage magnitude and variable reactive power and voltage angle. While the reactive power limits of each generator is specified, additional reactive power may be deployed at each generator and associated costs may be calculated.

Column Number	Column Name	Meaning	Units
1	Bus	Generator bus location identifier	
2	PG	Real power output	MW
3	QG	Reactive power output	MVAr
4	QMax	Maximum reactive power output	MVAr
5	QMin	Minimum reactive power output	MVAr
6	VG	Voltage magnitude set-point	per-unit
7	MBase	MVA base of machine	
8	Status	Machine operational status	Binary

Table 67 - Column Context for Table 68

Bus	PG	QG	QMax	QMin	VG	MBase	Status
1	0	0	9999	-9999	1.0	100	1
2	0	0	9999	-9999	1.0	100	1
3	2	0	0.4	-0.5	1.0	100	1
4	2	0	0.4	-0.5	1.0	100	1
5	2	0	0.4	-0.5	1.0	100	1
6	2	0	0.4	-0.5	1.0	100	1
7	2	0	0.4	-0.5	1.0	100	1
8	2	0	0.4	-0.5	1.0	100	1
9	2	0	0.4	-0.5	1.0	100	1
10	2	0	0.4	-0.5	1.0	100	1
11	2	0	0.4	-0.5	1.0	100	1
12	2	0	0.4	-0.5	1.0	100	1
13	2	0	0.4	-0.5	1.0	100	1
14	2	0	0.4	-0.5	1.0	100	1
15	2	0	0.4	-0.5	1.0	100	1
16	2	0	0.4	-0.5	1.0	100	1
17	2	0	0.4	-0.5	1.0	100	1
18	2	0	0.4	-0.5	1.0	100	1

19	2	0	0.4	-0.5	1.0	100	1
20	2	0	0.4	-0.5	1.0	100	1
21	2	0	0.4	-0.5	1.0	100	1
22	2	0	0.4	-0.5	1.0	100	1
23	2	0	0.4	-0.5	1.0	100	1
24	2	0	0.4	-0.5	1.0	100	1
25	2	0	0.4	-0.5	1.0	100	1
26	2	0	0.4	-0.5	1.0	100	1
27	2	0	0.4	-0.5	1.0	100	1
28	2	0	0.4	-0.5	1.0	100	1
29	2	0	0.4	-0.5	1.0	100	1
30	2	0	0.4	-0.5	1.0	100	1
31	2	0	0.4	-0.5	1.0	100	1
32	2	0	0.4	-0.5	1.0	100	1
33	2	0	0.4	-0.5	1.0	100	1
34	2	0	0.4	-0.5	1.0	100	1
35	2	0	0.4	-0.5	1.0	100	1
36	2	0	0.4	-0.5	1.0	100	1
37	2	0	0.4	-0.5	1.0	100	1
38	2	0	0.4	-0.5	1.0	100	1
39	2	0	0.4	-0.5	1.0	100	1
40-	2	0	0.4	-0.5	1.0	100	1
41	2	0	0.4	-0.5	1.0	100	1
42	2	0	0.4	-0.5	1.0	100	1
43	2	0	0.4	-0.5	1.0	100	1
44	2	0	0.4	-0.5	1.0	100	1
45	2	0	0.4	-0.5	1.0	100	1
46	2	0	0.4	-0.5	1.0	100	1
47	2	0	0.4	-0.5	1.0	100	1
48	2	0	0.4	-0.5	1.0	100	1
49	2	0	0.4	-0.5	1.0	100	1
50	2	0	0.4	-0.5	1.0	100	1
51	2	0	0.4	-0.5	1.0	100	1
52	2	0	0.4	-0.5	1.0	100	1
53	2	0	0.4	-0.5	1.0	100	1
54	2	0	0.4	-0.5	1.0	100	1
55	2	0	0.4	-0.5	1.0	100	1
56	2	0	0.4	-0.5	1.0	100	1
57	2	0	0.4	-0.5	1.0	100	1
58	2	0	0.4	-0.5	1.0	100	1
59	2	0	0.4	-0.5	1.0	100	1
60	2	0	0.4	-0.5	1.0	100	1

61	2	0	0.4	-0.5	1.0	100	1
62	2	0	0.4	-0.5	1.0	100	1
63	2	0	0.4	-0.5	1.0	100	1
64	2	0	0.4	-0.5	1.0	100	1
65	2	0	0.4	-0.5	1.0	100	1
66	2	0	0.4	-0.5	1.0	100	1
67	2	0	0.4	-0.5	1.0	100	1
68	2	0	0.4	-0.5	1.0	100	1
69	2	0	0.4	-0.5	1.0	100	1
70	2	0	0.4	-0.5	1.0	100	1
71	2	0	0.4	-0.5	1.0	100	1
72	2	0	0.4	-0.5	1.0	100	1
73	2	0	0.4	-0.5	1.0	100	1
74	2	0	0.4	-0.5	1.0	100	1
75	2	0	0.4	-0.5	1.0	100	1
76	2	0	0.4	-0.5	1.0	100	1
77	2	0	0.4	-0.5	1.0	100	1
78	2	0	0.4	-0.5	1.0	100	1
79	2	0	0.4	-0.5	1.0	100	1
80	2	0	0.4	-0.5	1.0	100	1
81	2	0	0.4	-0.5	1.0	100	1
82	2	0	0.4	-0.5	1.0	100	1

Table 68 - Generator Data for Case Study 1: Real World Network Model

D.4.3 Branch Data

The third and final data table that must be supplied to MATPOWER for the power flow simulation to be run is the branch data. As was the case with the generator data table, not all variables are required for a power flow analysis and consequently these are left as zeros.

From Bus	**To Bus**	**R (pu)**	**X (pu)**	**B (pu)**
1	2	0.00345455	0.008899275	0.864391374
2	3	0.040298468	0.029043844	0.002119613
3	13	0.008741965	0.006300494	0.000459809
13	23	0.008741965	0.006300494	0.000459809
23	33	0.008741965	0.006300494	0.000459809
33	43	0.008741965	0.006300494	0.000459809
43	53	0.008741965	0.006300494	0.000459809
53	63	0.008741965	0.006300494	0.000459809
63	73	0.008741965	0.006300494	0.000459809

2	4	0.031821233	0.022934146	0.001673728
4	14	0.008741965	0.006300494	0.000459809
14	24	0.008741965	0.006300494	0.000459809
24	34	0.008741965	0.006300494	0.000459809
34	44	0.008741965	0.006300494	0.000459809
44	54	0.008741965	0.006300494	0.000459809
54	64	0.008741965	0.006300494	0.000459809
64	74	0.008741965	0.006300494	0.000459809
2	5	0.023538461	0.0169646	0.001238072
5	15	0.008741965	0.006300494	0.000459809
15	25	0.008741965	0.006300494	0.000459809
25	35	0.008741965	0.006300494	0.000459809
35	45	0.008741965	0.006300494	0.000459809
45	55	0.008741965	0.006300494	0.000459809
55	65	0.008741965	0.006300494	0.000459809
65	75	0.008741965	0.006300494	0.000459809
2	6	0.015759802	0.011358378	0.000828932
6	16	0.008741965	0.006300494	0.000459809
16	26	0.008741965	0.006300494	0.000459809
26	36	0.008741965	0.006300494	0.000459809
36	46	0.008741965	0.006300494	0.000459809
46	56	0.008741965	0.006300494	0.000459809
56	66	0.008741965	0.006300494	0.000459809
66	76	0.008741965	0.006300494	0.000459809
2	7	0.009773814	0.007044167	0.000514082
7	17	0.008741965	0.006300494	0.000459809
17	27	0.008741965	0.006300494	0.000459809
27	37	0.008741965	0.006300494	0.000459809
37	47	0.008741965	0.006300494	0.000459809
47	57	0.008741965	0.006300494	0.000459809
57	67	0.008741965	0.006300494	0.000459809
67	77	0.008741965	0.006300494	0.000459809
2	8	0.009773814	0.007044167	0.000514082
8	18	0.008741965	0.006300494	0.000459809
18	28	0.008741965	0.006300494	0.000459809
28	38	0.008741965	0.006300494	0.000459809
38	48	0.008741965	0.006300494	0.000459809
48	58	0.008741965	0.006300494	0.000459809
58	68	0.008741965	0.006300494	0.000459809
68	78	0.008741965	0.006300494	0.000459809
2	9	0.015759802	0.011358378	0.000828932
9	19	0.008741965	0.006300494	0.000459809

19	29	0.008741965	0.006300494	0.000459809
29	39	0.008741965	0.006300494	0.000459809
39	49	0.008741965	0.006300494	0.000459809
49	59	0.008741965	0.006300494	0.000459809
59	69	0.008741965	0.006300494	0.000459809
69	79	0.008741965	0.006300494	0.000459809
2	10	0.023538461	0.0169646	0.001238072
10	20	0.008741965	0.006300494	0.000459809
20	30	0.008741965	0.006300494	0.000459809
30	40	0.008741965	0.006300494	0.000459809
40	50	0.008741965	0.006300494	0.000459809
50	60	0.008741965	0.006300494	0.000459809
60	70	0.008741965	0.006300494	0.000459809
70	80	0.008741965	0.006300494	0.000459809
2	11	0.031821233	0.022934146	0.001673728
11	21	0.008741965	0.006300494	0.000459809
21	31	0.008741965	0.006300494	0.000459809
31	41	0.008741965	0.006300494	0.000459809
41	51	0.008741965	0.006300494	0.000459809
51	61	0.008741965	0.006300494	0.000459809
61	71	0.008741965	0.006300494	0.000459809
71	81	0.008741965	0.006300494	0.000459809
2	12	0.040298468	0.029043844	0.002119613
12	22	0.008741965	0.006300494	0.000459809
22	32	0.008741965	0.006300494	0.000459809
32	42	0.008741965	0.006300494	0.000459809
42	52	0.008741965	0.006300494	0.000459809
52	62	0.008741965	0.006300494	0.000459809
62	72	0.008741965	0.006300494	0.000459809
72	82	0.008741965	0.006300494	0.000459809

Table 69 - Branch Data for Case Study 1: Real World Network Model

Appendix E: Case Study 2 Input Data

This appendix contains the input data used for Case Study 2.

Unlike for Case Study 1, there is no real world network model for comparison purposes.

E.1 Generator Data

The generator data used for Test Case 1 is presented as Table 70, and the assumed layout of the wind farm is as depicted in Figure 38.

Gen ID	X Pos (km)	Y Pos (km)	Pout (MW)	Reactive Power Limits (MVAr)	
1	-3.234	85.4	7	-2.04	1.42
2	-2.156	85.4	7	-2.04	1.42
3	-1.078	85.4	7	-2.04	1.42
4	0	85.4	7	-2.04	1.42
5	1.078	85.4	7	-2.04	1.42
6	2.156	85.4	7	-2.04	1.42
7	3.234	85.4	7	-2.04	1.42
8	-3.234	86.478	7	-2.04	1.42
9	-2.156	86.478	7	-2.04	1.42
10	-1.078	86.478	7	-2.04	1.42
11	0	86.478	7	-2.04	1.42
12	1.078	86.478	7	-2.04	1.42
13	2.156	86.478	7	-2.04	1.42
14	3.234	86.478	7	-2.04	1.42
15	-3.234	87.556	7	-2.04	1.42
16	-2.156	87.556	7	-2.04	1.42
17	-1.078	87.556	7	-2.04	1.42
18	0	87.556	7	-2.04	1.42
19	1.078	87.556	7	-2.04	1.42
20	2.156	87.556	7	-2.04	1.42
21	3.234	87.556	7	-2.04	1.42
22	-3.234	88.634	7	-2.04	1.42
23	-2.156	88.634	7	-2.04	1.42
24	-1.078	88.634	7	-2.04	1.42
25	0	88.634	7	-2.04	1.42
26	1.078	88.634	7	-2.04	1.42
27	2.156	88.634	7	-2.04	1.42
28	3.234	88.634	7	-2.04	1.42
29	-3.234	89.712	7	-2.04	1.42

30	-2.156	89.712	7	-2.04	1.42
31	-1.078	89.712	7	-2.04	1.42
32	0	89.712	7	-2.04	1.42
33	1.078	89.712	7	-2.04	1.42
34	2.156	89.712	7	-2.04	1.42
35	3.234	89.712	7	-2.04	1.42
36	-3.234	90.79	7	-2.04	1.42
37	-2.156	90.79	7	-2.04	1.42
38	-1.078	90.79	7	-2.04	1.42
39	0	90.79	7	-2.04	1.42
40	1.078	90.79	7	-2.04	1.42
41	2.156	90.79	7	-2.04	1.42
42	3.234	90.79	7	-2.04	1.42
43	-3.234	91.868	7	-2.04	1.42
44	-2.156	91.868	7	-2.04	1.42
45	-1.078	91.868	7	-2.04	1.42
46	0	91.868	7	-2.04	1.42
47	1.078	91.868	7	-2.04	1.42
48	2.156	91.868	7	-2.04	1.42
49	3.234	91.868	7	-2.04	1.42
50	-3.234	92.946	7	-2.04	1.42
51	-2.156	92.946	7	-2.04	1.42
52	-1.078	92.946	7	-2.04	1.42
53	0	92.946	7	-2.04	1.42
54	1.078	92.946	7	-2.04	1.42
55	2.156	92.946	7	-2.04	1.42
56	3.234	92.946	7	-2.04	1.42
57	-3.234	94.024	7	-2.04	1.42
58	-2.156	94.024	7	-2.04	1.42
59	-1.078	94.024	7	-2.04	1.42
60	0	94.024	7	-2.04	1.42
61	1.078	94.024	7	-2.04	1.42
62	2.156	94.024	7	-2.04	1.42
63	3.234	94.024	7	-2.04	1.42
64	-3.234	95.102	7	-2.04	1.42
65	-2.156	95.102	7	-2.04	1.42
66	-1.078	95.102	7	-2.04	1.42
67	0	95.102	7	-2.04	1.42
68	1.078	95.102	7	-2.04	1.42
69	2.156	95.102	7	-2.04	1.42
70	3.234	95.102	7	-2.04	1.42
71	-3.234	96.18	7	-2.04	1.42

72	-2.156	96.18	7	-2.04	1.42
73	-1.078	96.18	7	-2.04	1.42
74	0	96.18	7	-2.04	1.42
75	1.078	96.18	7	-2.04	1.42
76	2.156	96.18	7	-2.04	1.42
77	3.234	96.18	7	-2.04	1.42
78	-3.234	97.258	7	-2.04	1.42
79	-2.156	97.258	7	-2.04	1.42
80	-1.078	97.258	7	-2.04	1.42
81	0	97.258	7	-2.04	1.42
82	1.078	97.258	7	-2.04	1.42
83	2.156	97.258	7	-2.04	1.42
84	3.234	97.258	7	-2.04	1.42
85	-3.234	98.336	7	-2.04	1.42
86	-2.156	98.336	7	-2.04	1.42
87	-1.078	98.336	7	-2.04	1.42
88	0	98.336	7	-2.04	1.42
89	1.078	98.336	7	-2.04	1.42
90	2.156	98.336	7	-2.04	1.42
91	3.234	98.336	7	-2.04	1.42
92	-3.234	99.414	7	-2.04	1.42
93	-2.156	99.414	7	-2.04	1.42
94	-1.078	99.414	7	-2.04	1.42
95	0	99.414	7	-2.04	1.42
96	1.078	99.414	7	-2.04	1.42
97	2.156	99.414	7	-2.04	1.42
98	3.234	99.414	7	-2.04	1.42
99	-2.156	100.492	7	-2.04	1.42
100	-1.078	100.492	7	-2.04	1.42
101	1.078	100.492	7	-2.04	1.42
102	2.156	100.492	7	-2.04	1.42

Table 70 - Case Study 2: Generator Data [50]

E.2 Cable Data

The cable data used for Test Case 2 is presented as Table 63.

Type	Voltage (kV)	R (Ω / km)	L (mH / km)	C (µF / km)
AC Collector	66	0.0843	0.0371	0.2545
AC Transmission	400	0.0309	1.3467	0.1667

Table 71 - Case Study 2: Cable Data [36]

E.3 Optimisation Parameters

The complete set of optimisation parameters for Case Study 2 is presented as Table 72.

Parameter	**Value**	**Units**
General Parameters		
Network Life Time	25	Years
Meant Time To Repair – Cables	90	Days
Mean Time To Repair – Offshore Substations	30	Days
Cable Failure Rate	0.001114	Per Km / Per Year
Offshore Substation Failure Rate	0.0032	Per Year
Cost Per MWh Received	185	€
Discount Rate	5	%
MVAr Consumption Cost	77,000	€ / MVAr
MVAr Generation Cost	77,000	€ / MVAr
Collector Parameters		
Min Number of Collectors	Variant Dependent	
Max Number of Collectors		
Min Collector X Position	-3.5	Km
Max Collector X Position	3.5	Km
Min Collector Y Position	40	Km
Max Collector Y Position	100	Km
Collector Operating Frequency	50 / 16.66	Hz
Radial Collector Topology Acceptable	1	
Ring Collector Topology Acceptable	1	
Star Collector Topology Acceptable	1	
Min Number of Strings	1	
Max Number of Strings	10	
Collector Voltage	66	kV
Transmission Parameters		
Transmission Operating Frequency	50 / 16.66 / DC	Hz
Radial Transmission Topology Acceptable	1	
Ring Transmission Topology Acceptable	1	
Star Transmission Topology Acceptable	1	
Transmission Voltage	400	kV

PSO Parameters		
Particles in Swarm	50	
Number of Iterations	1000	
Number of Iterations without improvement to stop	1000	
Reduction in maximum velocity after each iteration	0	%
GA Modified TSP Solver Parameters		
Population Size	80	
Number of Iterations	10000	
Weibull Wind Distribution Parameters		
Shape (k)	2.26	
Scale (A)	11.5	
Cut-In Speed	3	m/s
Rated Speed	13	m/s
Cut-Out Speed	25	m/s

Table 72 - Case Study 2: Optimisation Parameters

Appendix F: Algorithm Implementation

For this work, two distinct algorithms are used in conjunction with one another to produce an overarching optimisation process which is generalised in Figure 29 and is described conceptually within Sections 4.6.1 and 4.6.2 of this thesis. This appendix contains details of the implementation of both primary algorithms used within this work, namely: swarm intelligence / PSO for the generation of high-level network solutions; and evolutionary computing / GA for optimising the connection ordering of generators.

F.1 High Level Network Specification Optimisation using PSO

A flowchart depicting the principles of operation for the PSO algorithm is presented as Figure 23, earlier within this thesis. This process was implemented, for this work, in the MATLAB programming language, as described in this section.

The algorithm is fully parameterised, with the relevant parameters loaded from an external data store into MATLAB during the initialisation aspect of execution. As has been described during the generalised presentation of the PSO algorithm, a number of vectors are required to store critical parameters. Each required vector is of the same size, with the number of dimensions governed by the number of offshore substations. Each substation, and associated collector network, has 4 high level parameters (x-coordinate; y-coordinate; network topology; and number of concurrent strings of connections). The transmission system requires only a single topology parameter to design a network. Consequently, if there is a single offshore substation, the PSO algorithmic vectors have 5 dimensions; if there are 2 offshore substations the vectors have 9 dimensions, and so on.

Maximum and minimum values for each parameter are loaded into MATLAB from the external data source, and corresponding vectors *Min* and *Max* created, as defined by (69-70) where: x_{MIN} and x_{MAX} are the lower and upper bounds on the x-coordinate that an offshore substation may take; y_{MIN} and y_{MAX} are the lower and upper bounds on the y-coordinate that an offshore substation may take; $CTop_{MIN}$ and $CTop_{MAX}$ are the lower and upper bounds on the topology that may be implemented for associated collector networks; $CStr_{MIN}$ and $CStr_{MAX}$ are the lower and upper bounds on the number of parallel strings of connections that may be implemented for associated collector networks; and $TTop_{MIN}$ and $TTop_{MAX}$ are the lower and upper bounds on the topology that may be implemented for the transmission network.

$$\boldsymbol{Min} = \begin{bmatrix} x_{MIN} \\ y_{MIN} \\ CTop_{MIN} \\ CStr_{MIN} \\ TTop_{MIN} \end{bmatrix} \quad (69)$$

$$\boldsymbol{Max} = \begin{bmatrix} x_{MAX} \\ y_{MAX} \\ CTop_{MAX} \\ CStr_{MAX} \\ TTop_{MAX} \end{bmatrix} \quad (70)$$

A user-specified number of particles, solutions to the optimisation problem, are created. Each particle has a number of vectors with dimensions corresponding to the *Min* and *Max* vectors defined by (69-70), for the purposes of storing: the particle's current position; its new position; its current velocity; its new velocity; and its (local) optimal position. The first position attributed to a particle is assigned randomly between the corresponding minimum and maximum values, in accordance with (71) where *rand* is a random number between 0 and 1.

$$\boldsymbol{Current\ Position} = \begin{bmatrix} x_{MIN} + rand(x_{MAX} - x_{MIN}) \\ y_{MIN} + rand(y_{MAX} - y_{MIN}) \\ CTop_{MIN} + rand(CTop_{MAX} - CTop_{MIN}) \\ CStr_{MIN} + rand(CStr_{MAX} - CStr_{MIN}) \\ TTop_{MIN} + rand(TTop_{MAX} - TTop_{MIN}) \end{bmatrix} \quad (71)$$

As has been described previously, the high level description of a solution contains both continuous (offshore substation positional co-ordinates) variables; integral variables (number of concurrent strings of connections emanating from an offshore substation); and abstract variables (collector and transmission network topologies). Consequently, the *Current Position* vector defined by (71) must be updated to take account of this: the number of concurrent strings of connections is rounded to the nearest integer; and the network topologies are assessed against the list of permitted topologies and assume the value of the closest acceptable topology, where required.

Now the population of particles may be assessed, fulfilling the *Create Initial Solutions* step depicted in the optimisation algorithm flowchart presented as Figure 29. Each high-level network solution is then assigned a connection order, and is assessed against the optimisation objective function. As this is the first iteration of the algorithm, the first position assumed by each particle is, by default, the local optimal solution for that particle. Further, one particle's position is also the global optimal solution and that position is stored in the *Global Optimal Position* vector.

In order for particles to traverse the search space, velocity is required. Velocity is added to the current position of the particle to determine its next position, in accordance with (44). In the first instance, velocity is generated randomly between zero and half of the range of acceptable values permitted for a given dimension. The first velocity calculation for a given particle is presented as (72) where *rand* is a random number between 0 and 0.5.

$$\boldsymbol{New\,Velocity} = \begin{bmatrix} rand(x_{MAX} - x_{MIN)} \\ rand(y_{MAX} - y_{MIN)} \\ rand(CTop_{MAX} - CTop_{MIN)} \\ rand(CStr_{MAX} - CStr_{MIN)} \\ rand(TTop_{MAX} - TTop_{MIN)} \end{bmatrix} \tag{72}$$

The *New Velocity* vector, defined as (72) is then added to the *Current Position* vector, defined as (71); in order to form the *New Position* vector for a given particle, in accordance with (44). If the value ascribed to any dimension of the *New Position* vector lies outside the acceptable domain defined by the *Min* and *Max* vectors defined as (69) and (70), then the appropriate maximum or minimum value is used in place of the calculated value. Again, the number of concurrent strings and network topology variables present with the *New Position* vector must be modified to ensure that they are both integral, and in-line with acceptable network topologies. The *New Velocity* vector parameters are then transferred to the *Current Velocity* vector.

For all subsequent iterations of the algorithm, the *New Velocity* vector is calculated in accordance with (43). After each iteration, the global optimal solution is compared with the current population of solutions and the *Global Optimal Position* vector updated, as required.

The stopping criteria that may be utilised for this algorithm are either: reaching an absolute number of iterations; or reaching a user-specified number of iterations without there being an improvement in the performance of the global optimal solution.

F.2 Connection Ordering Optimisation using GA

Optimal connection ordering is achieved through a modified travelling salesman problem solver that utilises a GA, implemented in the MATLAB programming language. As was the case for the PSO algorithm that addresses the high level network design problem, a GA utilises a number of parallel solutions. A solution to this problem has 2 components: a string of unique integers corresponding to the identifiers of turbines (or offshore substations); and the locations of breaks in this connection sequence, corresponding to the allocation of turbines to specific connections strings. A single matrix, *Sequences*, of dimensions *M×N* is created, where: *M* is the solution population size; and *N* is the number of turbines to be ordered, to store all connection sequences. This principle is depicted as (73), where the first 3 rows contain the first 3 elements of connection ordering sequences.

$$\boldsymbol{Sequences} = \begin{bmatrix} 1 & 2 & 3 & \dots \\ 2 & 5 & 7 & \dots \\ 3 & 4 & 6 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix} \tag{73}$$

In parallel to the *Sequences* matrix, there is also the *Breaks* matrix; which stores the breaks in a connection sequence which corresponds to the array index of the last machine connected to a given string of connection. For example, if a sequence is *1,3,5,2,4,6,8,10,9,7* with a single break index of *5* then the first connection string order is *1,3,5,2,4* and the second string order is *6,8,10,9,7.* The *Breaks* matrix is of dimensions *M×P-1,* where: *M* is the solution population size; and P is the number of concurrent strings of connections. An example *Breaks* matrix is defined by (74), where the break indices are after positions 3 and 7 for the first solution; 2 and 8 for the second; and so forth.

$$\boldsymbol{Breaks} = \begin{bmatrix} 3 & 7 \\ 2 & 8 \\ 3 & 5 \\ \dots & \dots \end{bmatrix} \tag{74}$$

Each of the initial solutions is assessed against the optimisation objective, and a performance ranking produced. The best performing solution is recorded as the global optimal. A user-specified proportion of the strongest solutions are automatically taken forward from this population generation to the next. Solutions not automatically taken forward to the next generation; are subjected to one or both of *crossover* and *mutation.*

The sequencing of connections may be subject to *mutation* but may not be subject to *crossover*, given the unique nature of the elements stored within the connection sequence. For example, if one sequence of connections is *1,2,3,4* and a second is *4,3,2,1* then applying crossover after the second element gives a sequence of *1,2,2,1*; which breaches the requirement for each turbine to be included only once and only within a single connection string. The mutation operator may be used freely to swap elements in the connection sequence without breaching the need for unique elements, for example: *1,2,3,4* can be mutated to *1,3,2,4* while keeping each turbine within a single connection string. Crossover may be applied to the combination of sequence and break-points.

Once the *crossover* and *mutation* operators have been applied, a new generation of solution is finalised. These are, once again, assessed against the optimisation objective and ranked by performance. If the best performing solution from the new generation performs better than incumbent global optimal solution, then this solution is updated. Again, the strongest performing solutions are carried forward to form part of the next generation of solutions; while the weaker solutions are subject to crossover and mutation.

While it is feasible to include a stopping condition based on there being no improvement in the global optimal solution over a number of iterations; the only stopping methodology that was employed for this algorithm was a user-specified number of iterations being reached.