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A Reliability Rating System for Flood Embankments in the United Kingdom

by

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Signed Marco Pedrazzoli

Date 4 June 2009

To Carlo and Flora

Considerate la vostra semenza:
fatti non foste a viver come bruti,
ma per seguir virtute e canoscenza[1]

Dante Alighieri
La Divina Commedia
Inferno, Canto XXVI, vv 118-120

[1] Consider well the seed that gave you birth: you were not made to live your lives as brutes, but to be followers of worth and knowledge

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Abstract

The probability of structural failure of flood defences is an important component in the modelling of flood risk. Flood embankments are a very common type of defence whose reliability assessment shows some problematic aspects. In particular it is difficulty to model the process of piping through the earthfill and, when assessing the safety of an extended flood defence network, a diffuse lack of knowledge (epistemic uncertainty) about some of the characteristics of the structures has to be addressed.

This thesis presents a new methodology, named the Reliability Rating System, which makes possible the rapid quantification of the expected performance of flood embankments in extreme hydraulic conditions. The proposed methodology ranks and compare flood embankments according to their proneness to breaching using a performance indicator, called the Probability of Breaching in Extreme hydraulic conditions P_{BE}. This performance indicator is related to probability of structural failure for a limited range of water levels above and immediately below the crest. The Reliability Rating System takes into account three failure modes: breaching initiated by grass cover failure in overtopping conditions, breaching induced by piping through the earthfill (through-piping) and breaching due to piping in the founding soil (under-piping).

Credible mathematical models are available for the grass cover failure and under-piping and the probability of breaching induced by these failure modes can be calculated with the methods of reliability analysis. By contrast no credible process-based mathematical model is currently available for through-piping. In this work the probabilities of breaching by through-piping have been estimated with a rigorous process of subjective judgement elicitation.

In the case of uncertainty on some of the relevant characteristics the final users can rapidly study different possible scenarios thanks to the tabulated solutions presented here. Upper and lower bounds on the performance indicator P_{BE} can be readily determined in order to handle the epistemic uncertainty typical of flood defences safety assessments.

Notation

The following are the main symbols used throughout his volume. Other symbols have just a local meaning and are defined where they occur.

AEP	annual exceedance probability of a water level
AEP_{crest}	annual exceedance probability of the crest level
AEP_{ot}	annual exceedance probability of the reference level for overtopping conditions
C	consequence of an undesired event
$\mathbf{C}$	covariance matrix
C	creep factor in Bligh's formula
c	crest width of a flood embankment
$Cov[X, Y]$	covariance of X and Y
C_V	coefficient of velocity in the calculation of discharge
C_w	weighted creep factor in Lane's formula (1935)
D	thickness of the coarse-grained layer in soil profile prone to under-piping
d	thickness of the fine grained covering layer in soil profile prone to under-piping
f_g	coefficient of grass cover condition
$F_X(x)$	cumulative distribution function of X
$f_{\mathbf{X}}(\mathbf{x})$	joint probability density function
$g(\mathbf{X})$	performance function
k	coefficient of permeability
k_s	sand-equivalent hydraulic roughness according to Nikuradse
L_h	total length of the horizontal segments of the line of creep in Lane's formula
L_v	total length of the vertical segments of the line of creep in Lane's formula
LL	liquid limit
N	total number of trials in a Monte Carlo simulation

N_f	number of trials resulting in failure in a Monte Carlo simulation
p	probability of an event to be approximated via Monte Carlo simulation
$\hat{p}$	estimator of p in Monte Carlo simulation
P	probability of an undesired event
P(B)	probability of breaching
$\mathrm{P(B)}_{\mathrm{crest}}$	probability of breaching with water level at the crest
$P(B)_{ot}$	probability of breaching with water at the reference level for overtopping conditions
$\mathrm{P}(B_{se})_{ot}$	probability of breaching by surface erosion with water at the reference level for overtopping conditions
$P(B_{tp})_{crest}$	probability of breaching by through-piping with water at the crest level
$P(B_{up})_{crest}$	probability of breaching by under-piping with water at the crest level
$P(F)$	probability of failure
P_{avg}	average annual frequency of failure by through the embankment in the UNSW method
P_{BE}	probability of Breaching in Extreme hydraulic conditions
P_E	probability of failure by piping thorough the embankment in the UNSW method
P_{ref}	probability of breaching by piping through the earthfill for the reference embankment in the subjective probability elicitation
Q	flow in a river section
$\overline{Q}$	mean annual flood
q_a	discharge over the embankment in overtopping conditions
q_c	critical discharge over the embankment for failure of the grass cover
R	risk
r	coefficient of influence reduction in the subjective probability elicitation
SOP	standard of protection
T	return period
t	duration of overtopping
$T_p(0)$	time to peak of the Instantaneous Unit Hydrograph

u	pore water pressure
$Var[X]$	variance of X
v_c	limiting velocity for failure of the grass cover
w	weighting factor in the UNSW method an in the subjective probability elicitation
$z_{\alpha/2}$	standard Normal variable that is exceeded with probability $\alpha/2$
ρ	density;
θ	fluctuation scale
β	reliability index
α	slope angle
λ	leakage factor in the analytical model for under-seepage developed by TAW
ε	tolerated error in Monte Carlo simulation
$\Phi()$	cumulative distribution function of the standard Normal distribution
ΔH_d	admissible difference in hydraulic head
γ_{sat}	saturated unit weight
σ_V	vertical total stress
ρ_{XY}	correlation coefficient of X and Y

Part I

1

INTRODUCTION

This thesis presents and discusses the development of a methodology, named the Reliability Rating System, which makes possible the rapid quantification of the expected performance of flood embankments in extreme hydraulic conditions. The proposed method makes use of a performance indicator, which is the result of a simplified and approximated reliability assessment, to rank and compare the embankments in an inventory according to their estimated capacity to withstand water levels above and immediately below the crest. The performance indicator, which represents the probability of breaching related only to this limited range of loads, is named Probability of Breaching in Extreme hydraulic conditions P_{BE}. The proposed methodology addresses significant geotechnical aspects that are not satisfactorily implemented by the approaches currently in use and represents a helpful tool when dealing with the remarkable lack of knowledge about the actual characteristics of structures, which is typical of flood defences safety assessment.

1.1 Reliability of flood embankments

The structural reliability analysis, i.e. the determination of the probability of breaching, of flood defences is a crucial element in the assessment of flood risk. A range of innovative methodologies have recently been developed in the United Kingdom and the British flood risk community is currently working towards a "whole system" model capable of integrating all phases of the flooding process: from the sources of the hazard to the damage to people and properties. The response of defences to hydraulic loading, in terms of the probability of breaching is a fundamental link along the chain of integrated flood risk modelling. However the determination of the probability of breaching of flood embankments under the full range of possible water levels is a particularly challenging task. In fact the review of the current practice and recent advances in flood embankment reliability carried out in this work has highlighted several problematic aspects.

The most important point is perhaps the incorrect consideration of the process of piping through the earthfill, which in the current methodologies is treated with a criterion for foundation erosion whose use is stretched far outside of its field of applicability. Essentially the current practice, being fully based on the classical methods of reliability analysis, requires a mathematical model for the description of the limit between failure and non-failure states. However the literature on piping through water-retaining structures shows that a credible mathematical model capable of representing this process in engineering applications is not available to date. In the absence of a large amount of data on the performance of flood embankments of different characteristics in different loading conditions, the probabilities of failure due to piping through the earthfill have to be determined by a process of subjective judgement elicitation. Expert elicitation is used in several fields of technical and scientific activity and, notably, is widely employed in the quantitative risk assessment of large embankment dams all over the world. A growing body of research is now available for expert judgement elicitation applied to geotechnical engineering (Beacher & Christian 2003, Vick 1999 and 2000, USACE 2006) and examples of its application have been published, particularly with reference to embankment dams (BC Hydro 1995, Landon-Jones *et al.* 1996, Johansen *et al.* 1997).

A part of this thesis outlines a possible approach to the determination of the probabilities of breaching by through-piping based on subjective judgement elicitation. However the implementation of the subjective probabilities approach in integrated flood risk modelling requires the estimate of the probability of breaching by through-piping for embankments with different characteristics and, most of all, for the whole range of possible water levels. This is a task of remarkable size and complexity, outreaching what is realistically achievable in an individual doctoral research programme. Instead a cross disciplinary panel of experts should be created, who could go through all the codified phases of a structured process of judgement elicitation (Beacher & Christian 2003). The author argues (Redaelli & Dyer 2008) that the mobilisation of resources for such a goal should have high priority in the plans for future research of the flood risk community.

On a shorter timescale the urgency of effectively incorporating through-piping in the safety assessment of flood embankments, and of improving other aspects of the current methodologies, needs to be addressed. For this reason the Reliability Rating System presented in this thesis has been developed.

1.2 The Reliability Rating System

1.2.1 Reliability rating

The Reliability Rating System is a tool which enables the final user to quickly quantify the expected performance, in terms of resistance to breaching during floods, of fluvial embankments. The system makes use of a performance indicator which is the result of a simplified and approximated reliability evaluation, performed only for a limited range of water levels in proximity of the embankment's crest. The result of such an evaluation has been named the Probability of Breaching in Extreme hydraulic conditions P_{BE}. Three different modes of failure are included in the methodology:

a. failure by erosion of the natural soil in the foundation (under-piping);
b. failure by piping through the manmade earthfill in the embankment's body (through-piping);
c. failure initiated by the superficial erosion due to water flowing above the embankment in overtopping conditions.

1.2.2 Reliability analysis

Two of the abovementioned failure modes, namely under-piping and surface erosion, are accredited to credible mathematical modelling. In the development of the rating system the related probabilities of failure have been calculated using the classical methods of reliability analysis. The First Order Reliability Method (FORM) and Monte Carlo simulation have been used in this research project.

1.2.3 Subjective probability

The process of through-piping is currently not amenable to credible mathematical modelling (Richards & Reddy 2007). Hence the traditional methods of reliability, which require an analytical or numerical definition of the limit between failures and safe states, cannot be employed. In the development of the presented methodology the probability of failure by piping through the earthfill has been estimated making use of subjective judgement. The proposed approach considers only a limited range of water levels and because of this the number of cases to be considered in the judgement elicitation process is reduced to a tractable size. The probabilities of breaching by through-piping for representative embankments with different characteristics have been estimated by a small panel of engineers, composed by the author and two co-workers, who followed a structured and rigorous procedure whose details are discussed in the thesis. The full elicitation process is articulated in five phases according to state-of-the art recommendations (Beacher & Christian 2003, USACE 2006), as follows:

1. Motivating phase;
2. Training phase;
3. Structuring (or deterministic) phase;
4. Assessment (or probabilistic) phase;
5. Documentation.

In most applications the subjective probability elicitation is combined with the event tree method, which is a technique for decomposing the processes leading to failure into organised chains of simpler events. Event trees, for example, are widely used in quantitative risk assessments of dams (Fell *et al.* 2000). However the scarcity of information about the actual conditions and characteristics of structures and sites, typically encountered in the safety assessment of fluvial defences, makes the application of the event tree technique problematic. In this study an alternative approach, inspired by a formulation originally developed for the analysis of embankment dams, has been adopted. The framework of the original method, based on the historical performance of an extremely large sample of these structures (Foster *et al.* 2000b), has been adapted to subjective judgement elicitation and tailored to fluvial embankments.

1.2.4 Epistemic uncertainty

A recurrent problem in the safety assessment of flood defences is the very low amount of information available about the actual characteristics and conditions of the structures. In fact, due to the remarkable extension of the flood defence network and its heterogeneity, some of the key features affecting the structures' performance are often un known to the safety assessors. This kind of lack of knowledge, also called epistemic uncertainty, is one of the main obstacles that decision makers have to face when trying to establish strategies for maintenance, repair, upgrading or replacement of the existing flood defence assets.

In the development of the Reliability Rating System the quantities required to calculate P_{BE} have been evaluated beforehand by the author, with the help of his co-workers, in a finite number of scenarios. This approach aims at covering the possible situations that can be encountered in practice when dealing with flood embankments in the UK. In this way the final users can quickly obtain an indicative measure of the expected performance, once they know the relevant characteristics of an embankment. Moreover, they can rapidly quantify the influence on the performance indicator of different assumptions on the characteristics that are uncertain.

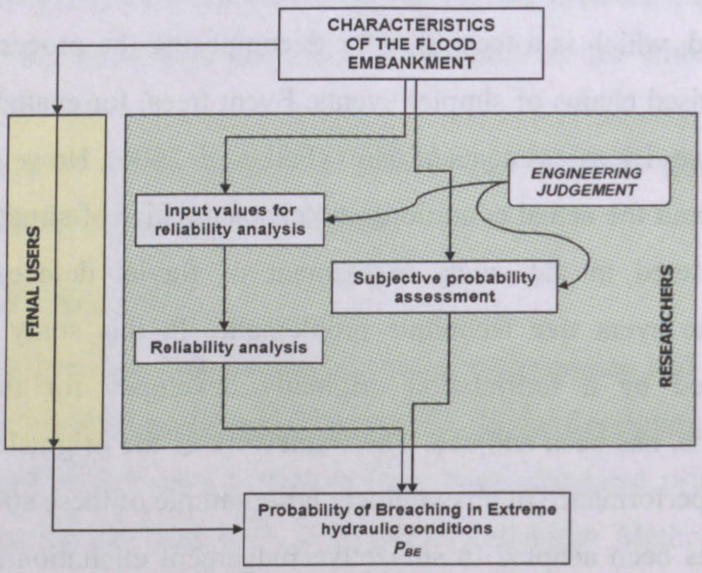

Figure 1.1. Chart of the development of the Reliability Rating System for flood embankments: the reliability calculations and the subjective probabilities elicitation are performed by the researchers in order to offer a ready-to-use tool to the final users.

The process of developing the Reliability Rating System is schematised in Figure 1.1: the author (and his co-workers for the subjective probabilities elicitation) has determined the quantities needed for the calculation of the indicative measure of performance, P_{BE}, in a finite number of representative scenarios. The final users can rapidly know the value of this performance indicator for any of the considered scenarios. A considerable amount of engineering judgement, supported by experience and informed by the available literature, has entered the development of the system. In particular a judgemental component is present in the choice of the input parameters for the reliability analysis and in the estimation of the subjective probabilities. These elements are made explicit and openly discussed throughout the thesis.

1.3 Organization of this thesis

This thesis is organised in three parts. The first, introductory part, reviews the recent developments in integrated flood risk modelling and the implementation of flood embankments reliability in that context, highlighting aspects that need modification or further development. It also summarises the literature on reliability of flood defences and the related methods and techniques developed in various countries. Finally it shows how some problematic aspects of the current approaches to flood embankments' reliability have led to the proposal of Reliability Rating System and explains the formulation of the performance indicator P_{BE}.

The second part of the thesis illustrates the reliability calculations and the subjective probability estimates performed in order to develop the Reliability Rating System. First the methodology adopted for the reliability analysis is presented and the use of judgement elicitation in geotechnical engineering is introduced. Then the three modes of failure are considered and the choices made to cover a wide range of practical scenarios are discussed. For the through-piping mode of failure the structured process of subjective probabilities elicitation undertaken by the panel of assessors is described

The third and last part of the thesis discusses advantages and limitations of the proposed methodology and illustrates how its use can help in handling epistemic uncertainty. Conclusions and recommendations for further research close the final part of the thesis.

The results of the reliability assessments for the three modes of failure, whose body is too voluminous to be fully included in the main text, are contained in the appendices.

2

RELIABILITY OF FLOOD EMBANKMENTS AND INTEGRATED FLOOD RISK ASSESSMENT IN THE UK

In the United Kingdom, and in other developed countries, the assessment of the threat posed by floods and the management of the relative defence assets is gradually shifting toward a risk based approach. Flood engineering in Britain is exploring the innovative concept of integrated flood risk management in which the performance of flood defences and their possible failure is included among other components as a crucial element for realistic modelling. Flood embankments are a very common type of defence whose behaviour under extreme hydraulic loading conditions is not easily predictable. The current approach and the recent developments in flood embankments' reliability leave some open questions that this thesis tries to address.

2.1 Flood hazard and defence management in the United Kingdom

The assessment of the threat posed by floods and the reduction of their impact on the built environment and society is a relevant issue in many countries. In the United Kingdom over 5% of the population live in the 12,200 km^2 area that is at risk of flooding by rivers and the sea (Halcrow *et al.* 2001). These people and their properties are protected by some 34,000 km of flood defences. A collective reflection, triggered by the serious floods that hit the UK in 1998 and 2000, demonstrated the importance of improved management of flood defences (Bye & Horner 1998, EA 2001, ICE 2001, Penning-Roswell *et al.* 2002). Significant resources for improving standards of flood and coastal defences have since then been released by the government (HM Treasury 2002). During the advancement of the doctoral research presented in this thesis, in the summer of 2007, other floods affected large areas of the United Kingdom. The independent technical report (Pitt 2007) prepared for the Government following these events calls for urgent measures including a better identification of areas at highest risk from flooding. The document also puts an emphasis on the need for efficient national planning and coordination. The Cabinet have recently stated (Benn 2008) their intention to invest £1.8 billion over a period of three years, directly with the authorities operating on flood management, for tasks including the construction of new or improved defences, monitoring, and the creation of flood warning systems.

The overall policy responsibility for flood risk in England and Wales is assigned to the Department for the Environment, Food and Rural Affairs (DEFRA[2]). DEFRA funds most of the Environment Agency's flood management activities and research. The Scottish Executive formulates national policy on flood prevention and warning for Scotland and provides resources to enable local authorities to address flooding risks. The Scottish Environment Protection Agency (SEPA) operates flood warning schemes, and gives advice to local authorities on flood prevention and planning issues. The Rivers Agency, which is an Executive Agency of the Department of Agriculture and Rural Development, is the body responsible for drainage and flood defence in Northern Ireland. The technical and scientific base for flood management and the related studies for Scotland and Northern Ireland are carried out by the Scotland and Northern Ireland Forum For Environmental Research (SNIFFER), which also cooperates with the Environment Agency in some joint research programmes.

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[2] Since October 2008 the new Department of Energy and Climate Change (DECC) is responsile for the climate change mitigation policy, previously with DEFRA; however at the time of writing DEFRA is still responsible for flood risk.

2.2 Integrated flood risk management

2.2.1 Traditional approach to flood defence design

For many years the conventional approach to the design of flood defences has been based on the concept of Standard of Protection. The Standard of Protection is the hydraulic load, generally expressed in term of return period (years), that the defence structure is designed to withstand. A fluvial defence for instance would be designed for a water level with a specified return period; similarly a coastal defence would be designed for a combination of storm surge and wave height with an assigned return period. The appropriate Standard of Protection for a specific defence is generally depending on the land use of the protected area. Once the corresponding design load is estimated the defence asset is constructed to structurally endure it. Some hydraulic safety margin is generally included, such as freeboard allowance, on the basis of local circumstances.

In the last decade, decision makers in charge of the flood defence network management have increasingly perceived the conventional approach as limited in providing help for large scale and long term planning. In fact the Standard of Protection concept does not offer clear guidance when choices have to be made between structural measures (like building, upgrading and maintaining defence structures) and non-structural measures (like investing in preparedness, response, improvement of legislation etc.). As more and more emphasis is put on the options appraisal process, managers have been seeking support in instruments which deal more directly with consequences, like risk based techniques.

2.2.2 Risk based approach to flood defence management

In the language of engineering analysis risk is a quantity accounting for both the probability of an undesired event occurring and the magnitude of the consequences expected if the event actually takes place. Formally the risk R is hence a function of the probability of the undesired event P and of the consequence of the event C:

$$R = f(P, C) \tag{2.1}$$

The function defining the quantitative measure of risk can vary depending on the context, for instance from one industry to another. In flood engineering a simple and convenient definition adopted for risk is the product of the probability of the undesired event times the consequence (Sayers *et al.* 2003):

$$R = P \times C \tag{2.2}$$

When more than one adverse event is possible the equation is extended to a set of events:

$$R = \sum_i (P_i \times C_i) \tag{2.3}$$

Depending on the context the risk can be expressed as an economical loss (£) or in terms of victims (number of people injured/number of lives lost).

Risk analysis can be a powerful support for rational decision making. Risk minimization, in fact, can be adopted as a criterion to make a choice between different options in complex management problems. Whatever the discipline, a risk based approach to decision making enables the comparison of different measures so that limited resources can be best targeted.

2.2.3 The challenge of integrated system modelling

Modelling flooding systems, like catchments or coastlines, with the purpose of accounting for the potential management choices that may alter the resulting risk, is a complex and difficult task. In fact loads, defence system responses, inundation and the impact of flooding on the built environment need to be integrated in a "whole system" model (Sayers *et al.* 2002).

A scheme of the component sub-models required is sketched in Figure 2.1. The frequency of the load (e.g. water level in a water course) and the response of the defence structure to loading, in terms of probability of failure, are combined to give the probability of failure of the specific defence in a defined period of time (generally one year). A defence failure can be due to overtopping (hydraulic failure) or breaching (structural failure). In both cases the result is the inundation of the locations protected by the defence. An inundation model is then needed to predict the flood level in the affected locations given by the water which propagates after being discharged from the defence. In the case of structural failure the discharge is affected by the growth of a breach.

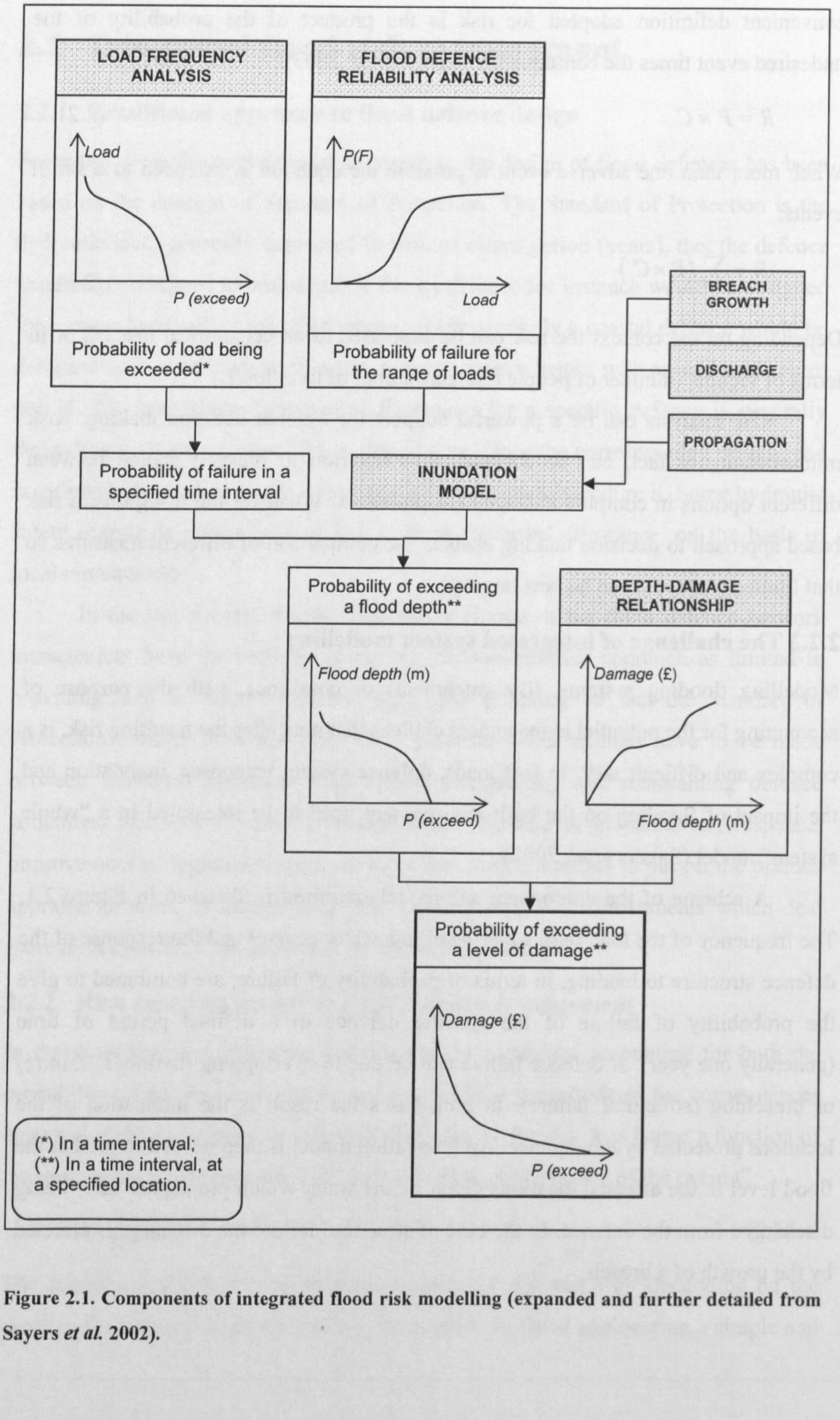

Figure 2.1. Components of integrated flood risk modelling (expanded and further detailed from Sayers *et al.* 2002).

Once the probability of exceeding a flood depth within a defined time interval at a specified location has been calculated, using the inundation model, this result can be combined with a depth-damage relationship to obtain the probability of damage within the defined time interval at the specified location. Integrating the damage over the probability gives the risk, in the form of the expected damage within a defined time interval. Summing the expected damage of all the affected locations gives the total risk.

The various scientific and technical disciplines involved in such a complex modelling task have progressed to a point which makes the attempt of developing an integrated model feasible (Sayers *et al.* 2002). Nevertheless the task is challenging and many issues still need to be investigated. These challenges include the modelling of complex and uncertain responses by man-made flood defences. Apart from the actual understanding of the physical processes involved in the structural response of flood defences a major obstacle is represented by the very limited amount of information typically available about the actual conditions of the defences in the system.

Following a widely adopted conceptual framework (DETR, 2000) the processes involved in flooding can be classified according to a "source-pathway-receptor" model (Sayers *et al.* 2002). In the case of flood risk the source of the hazard is a meteorological event while the receptor is the built environment, with population and properties. The response of the system, which includes the behaviour of the defences and the inundation process, is the pathway between source and receptor, as illustrated in Figure 2.2.

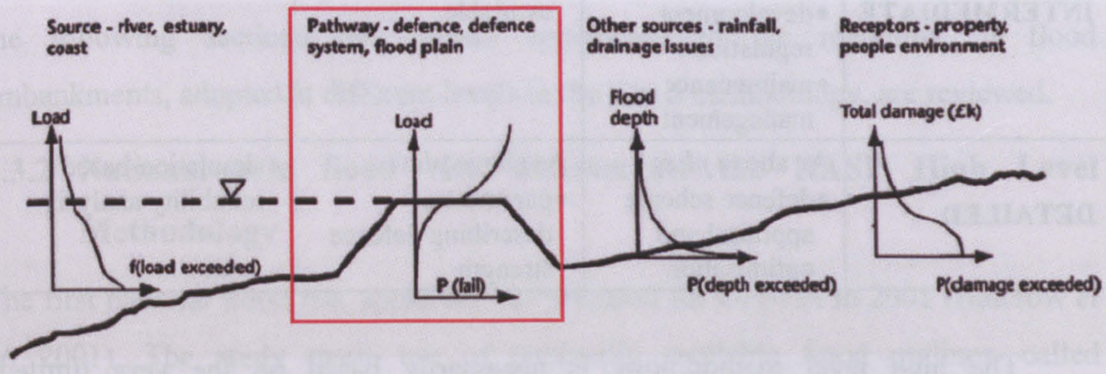

Figure 2.2. The "source-pathway-receptor" concept applied to flood risk (Sayers *et al.* 2003). This thesis is dedicated to the response to loading of flood defences, which is part of the "pathway" element.

2.2.4 Hierarchical risk assessment

Flood risk assessment is required to support decisions at very different scales, ranging from national policies to the development of individual flood defence schemes. The available information and the affordable level of detail in the modelling are quite different at the extremes of this range. In an attempt to remain consistent, while accounting for different levels of uncertainty, the risk assessment methodology which is being developed in the UK has been conceived with a tiered structure. The tiered methodology is described in Table 2.1, which lists the decisions to be informed at different levels. The data typically available and the methodologies adopted are also reported, referring exclusively to the reliability of flood defences (which is the focus of this thesis) and disregarding other aspects, like inundation or estimate of impacts.

Table 2.1. The tiered risk assessment methodology which is being developed in the UK (modified after Sayers *et al.* 2002).

LEVEL	DECISIONS	INFORMATION	METHODOLOGIES
HIGH	• national assessment of risk to economy, life, environment • expenditure prioritisation • regional planning • flood warning planning	• defence type • condition grades • standard of service	• generic probabilities of defence failure based on condition assessment
INTERMEDIATE	As above *plus*: • flood defence strategy planning • development regulation • maintenance management	As above *plus*: • crest level and other dimensions where available	• probabilities of defence failure from reliability analysis
DETAILED	As above *plus*: • defence scheme appraisal and optimisation	As above *plus*: • parameters describing defence strength	• simulation-based reliability analysis

The high level methodology is necessarily based on the very limited information available at the national level. At this scale the data regarding flood defences come from an inventory of assets. This inventory includes information on the defence type and the Condition Grade, which is the quantitative result of periodical visual inspections. The Standard of Protection is also generally known. A progressive reduction of uncertainties and more detailed modelling inform more site-specific decisions at the intermediate and detailed levels.

2.3 Reliability of embankments for flood risk modelling

2.3.1 Probability of failure of flood embankments

The process of assessing the probability of failure of an engineering system is called a reliability analysis. The reliability of flood defences is a crucial component in the integrated flood risk modelling. Among flood defence structures, earth embankments are very common and their expected performance in extreme hydraulic conditions is particularly difficult to predict. In fact the quality of the design and construction is extremely variable from case to case (Dyer 2004) and factors which are generally unknown, like the nature of the fill material and its level of compaction, or the underlying geology, have a great influence on the performance (Dyer & Gardener 1996, Morris *et al.* 2007).

The present work deals exclusively with the reliability of fluvial flood defence embankments (also simply referred to as flood embankments as opposed to coastal embankments). This is only one of the component modules of the "whole system" approach and refers to a specific type of defence structure. It will become clear as this text unfolds, that this apparently narrow field is not devoid of unsolved challenges and that its investigation is crucial for realistic modelling of flood risk. In the following sections the current approaches to the reliability of flood embankments, adopted at different levels in the tiered methodology, are reviewed.

2.3.2 National-scale flood risk assessment: the RASP High Level Methodology

The first national flood risk appraisal was prepared for DEFRA in 2001 (Halcrow *et al.* 2001). The study made use of nationally available flood outlines, called "indicative floodplain maps" but did not consider explicitly the role of flood defences. In 2002 the Environment Agency introduced the National Flood and

Coastal Defence Database (NFCDD): a digital inventory of flood defences and their condition as assessed through visual inspection. The availability of such a database allowed the first attempt to introduce the performance of flood defences in risk modelling on a national scale (Hall *et al.* 2003). The adopted approach was developed in a joint DEFRA/EA research project named RASP: Risk Assessment for flood and coastal defence system for Strategic Planning. The related modelling methodology, formulated by HR Wallingford & University of Bristol (2003), is known as RASP High Level Methodology (RASP-HLM)

In a national scale methodology the probability of failure of flood defences has to be estimated on the basis of the information contained in the NFCD database which includes location, type, condition and Standard of Protection (SOP). Information on crest level, crest width and other geometrical properties of the structure is generally not available.

Theoretically the probability of failure of a flood defence could be calculated with the methods of structural reliability (CUR/TAW 1990). These methods require an analytical or numerical expression of the limit state which separates failures and safe states for each relevant mode of failure of the structure. The parameters of this function, or numerical procedure, are random variables. The aleatory uncertainty about the input parameters can be propagated through the model of the system with approximated solution techniques. In this way the probability of failure can be calculated. In practice the application of these methods to flood defences require the knowledge of the parameters describing the defence response to hydraulic loading. These are essentially the crest level, for the hydraulic failure modes (i.e. continuous overtopping by a slow-varying water level or cyclical wave-overtopping) and the strength parameters of the defence for the structural failure modes (i.e. breaching mechanisms). The application of the structural reliability approach on a national scale, where only the information contained in the NFCDD is available, is problematic.

In the RASP High Level Methodology the reliability of flood defences is estimated using the available information on the Standard of Protection, which expresses the frequency with which overtopping is expected, and expert judgement on the structure's response to loading (Hall *et al.* 2003). Two ways of failure are considered in the analysis: overtopping (hydraulic failure) and breaching (structural failure). The difference is substantial because the impact of flooding, which in the national scale methodology is modelled by a "rapid" simplified routine, is obviously very different in the case of a moderate release of water by overtopping or an uncontrolled, conspicuous release if the structural integrity of the defence is lost by breaching.

In order to analyse the defence system each defence i is assigned a conditional probability of failure F_i given the load x – written as $P(F_i|x)$ – for the relevant range of values of the load. In reliability engineering such a conditional probability distribution is called the fragility curve (Casciati & Favarelli 1991). In the High Level Methodology the load is expressed as a multiplier of the Standard of Protection (SOP). For instance if a structure has a SOP of 50 years (i.e. it is expected to be overtopped on average once every 50 years) then $x = 0.5$ corresponds to the loading event with return period of 25 years, while $x = 2$ corresponds to an event with return period of 100 years. An example of fragility curves for overtopping is shown in Figure 2.3. The curves were constructed on the basis of expert judgement by Hall *et al.* (2003). Uncertainty about the response is expressed by introducing upper and lower bounds on the conditional probability of failure.

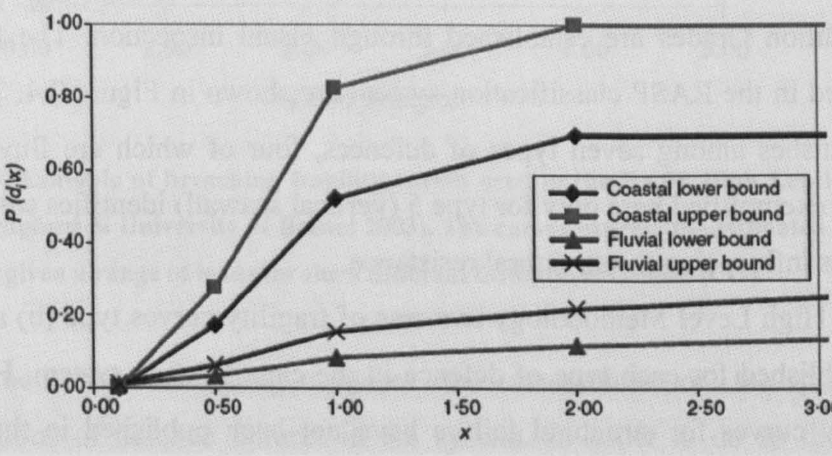

Figure 2.3. Overtopping fragility curves used for fluvial and sea defences, in national flood risk assessment with the RASP High Level Methodology (Hall *et al.* 2003). The curves represent the conditional probability of hydraulic failure given the load (ordinate axis) as a function of the load expressed as a multiplier of the Standard of Protection (abscissa axis).

Combining the response to the load, in terms of the fragility curve, with the frequency of the load $p(x)$ and integrating over all possible loading conditions the unconditional probability of failure of the i-th defence $P(F_i)$ is obtained:

$$P(F_i) = \int_0^{\infty} p(x)P(F_i|x)dx \tag{2.4}$$

As anticipated, not only the probability of overtopping but also the probability of breaching is crucial for flood risk assessment. Breaching and overtopping are not independent modes of failure: overtopping is, in fact, a common initiating event for a breach. It can be shown (Hall *et al.* 2003) that in order to calculate the probability of breaching of a defence structure three fragility curves are required giving the probability of failure conditional on the load:

a) pure overtopping, which is given in Figure 2.3;
b) breaching given overtopping occurs;
c) breaching given overtopping does not occur.

The probabilities of breaching conditional on the load, in presence or absence of overtopping, were estimated through expert judgement by Hall and his co-workers. In developing the fragility curves the influence of the Condition Grade was considered. The Condition Grade is a description of the condition of a structure in the form of an integer ranging from 1 (meaning "very good") to 5 (meaning "very poor"). Condition Grades are established through visual inspection. The types of defences listed in the RASP classification system are shown in Figure 2.4. The first level distinguishes among seven types of defences, four of which are fluvial. The second level, exemplified here only for type 5 (vertical seawall) identifies some basic characteristics influencing the structural resistance.

In the High Level Methodology two sets of fragility curves type (b) and type (c) were established for each type of defence in the classification system. However these fragility curves for structural failure have not been published in the RASP reports (HR Wallingford & University of Bristol 2003, 2004) or in the related journal paper (Hall *et al.* 2003). An example of conditional probability of breaching given the load by obtained combining suitable fragility curves of type (a), (b) and (c), is shown in Figure 2.5 for a non-specified type of structure (HR Wallingford & University of Bristol 2003).

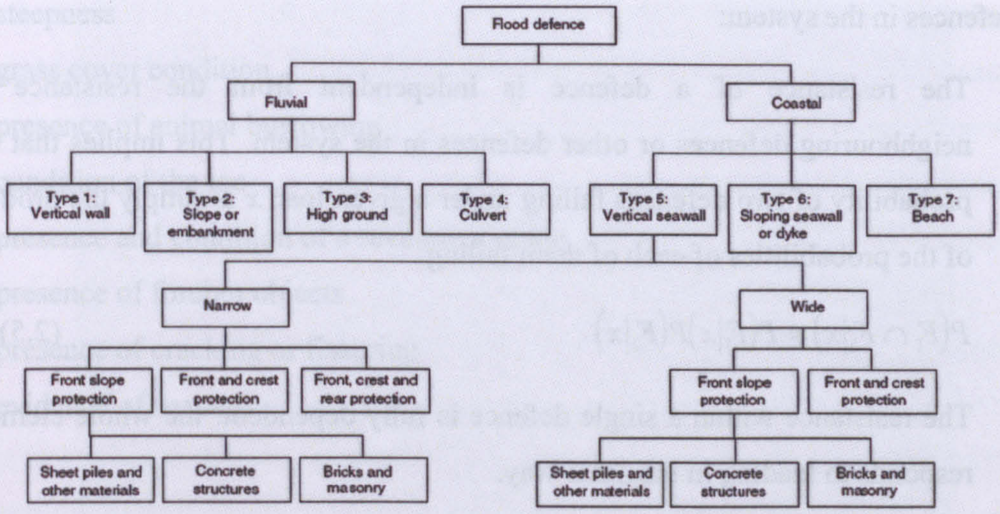

Figure 2.4. Classification of flood defences in the RASP project (Hall *et al* 2003).

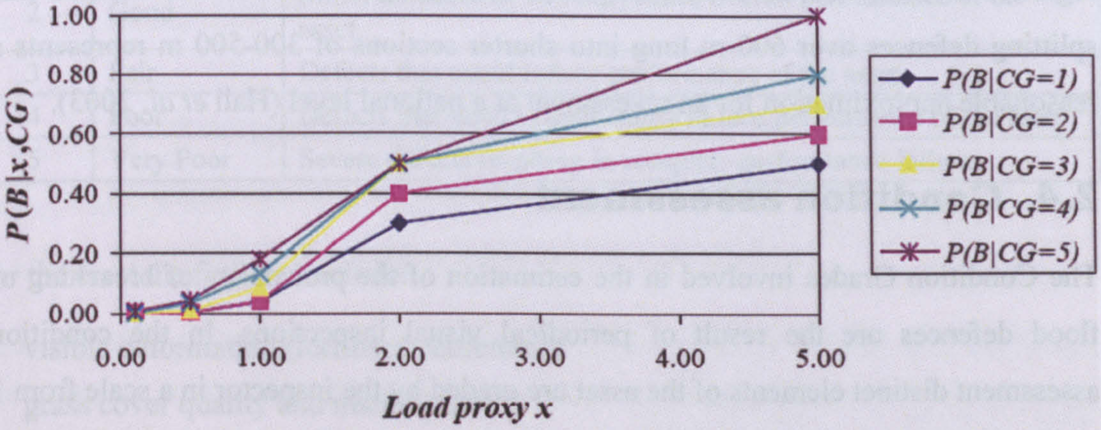

Figure 2.5. Example of breaching fragility curves used in the RASP High Level Methodology (HR Wallingford & University of Bristol 2003). The curves express the estimated probability of breaching given a range of loads for the 5 different Conditions Grades (CG).

Flood risk modelling requires the assessment of the probability of combinations of defence failures in the system. In order to do so the correlation between loads at different locations and between the responses of different defences has to be considered. In the methodology for national-scale analysis the loading (i.e. the SOP multiplier) is considered fully dependent: all defences are subject to the same load at the same time. The following hypotheses are made on the resistance of the defences in the system:

a) The resistance of a defence is independent from the resistance of neighbouring defences or other defences in the system. This implies that the probability of two defences failing under a given load x is simply the product of the probabilities of each of them failing:

$$P(F_1 \cap F_2|x) = P(F_1|x)P(F_2|x) \quad (2.5)$$

b) The resistance within a single defence is fully dependent: the whole element responds to loading in the same way.

The latter assumption is unrealistic for very long defences. In fact some parameters affecting the defence performance can show strong autocorrelation. CUR/TAW (1990) suggests that over distances grater than 500 m the relevant parameters can be regarded as independent. Considering this, in the RASP HLM it was deemed that splitting defences over 600 m long into shorter sections of 300-500 m represents a reasonable approximation for an assessment at a national level (Hall *et al.* 2003).

2.4 Condition assessment

The Condition Grades involved in the estimation of the probability of breaching of flood defences are the result of periodical visual inspections. In the condition assessment distinct elements of the asset are graded by the inspector in a scale from 1 to 5, 1 meaning *very good* condition and 5 *very poor* condition, as explained in Table 2.2. The frequency of inspections is planned every 6, 12 or 36 months depending on the assumed likelihood of failure of the defence and the impact of the potential failure. In the case of flood embankments the elements considered in the assessment are the crest and the slopes.

In order to assist the inspectors and achieve uniformity in grading a Condition Assessment Manual where images and verbal descriptions of Condition Grades can be found is provided (Environment Agency 2006). As an example, the page illustrating Condition Grade 3 for embankment slopes is shown in Figure 2.6. For a slope the features include:

- visible deformation
- steepness
- grass cover condition
- presence of animal burrowing
- condition of the toe
- presence and condition of a revetment at toe
- presence of foreign objects
- presence of cracking or fissuring
- evidence of seepage

Table 2.2. Condition Grades in the Environment Agency's Condition Assessment System (Environment Agency 2006).

Grade	Condition	General Description
1	Very Good	Cosmetic defects that will have no effect on performance
2	Good	Minor defects that will not reduce overall performance of the asset
3	Fair	Defects that could reduce performance of the asset
4	Poor	Defects that would significantly reduce performance of the asset
5	Very Poor	Severe defects resulting in complete performance failure

For the crest the features include:

- visible deformation (localised settlement)
- grass cover quality and maintenance
- erosion or rutting, evidence of overtopping
- cracking or fissuring
- presence of animal burrowing
- presence of foreign objects
- saturation or pooling of water

Importantly the grade for each element is established considering the lumped contribution of all the listed features. As only the condition grade is recorded as the outcome of the assessment, it is not possible in any subsequent phase to know which features determined the assigned grade. This poses problems when it comes to the modelling of the structure's performance.

Embankment slopes

Condition 3: Fair

General: Defects that could reduce performance of the asset

Specific description: Moderate defects – loss of vegetation or scour at toe (lower right insert). Slope too steep to prevent damage during grass cutting or undesirable type of vegetation cover (lower left insert). Shallow surface slips (Main photo) and cracking parallel to crest.

Key features: Burrowing evident in sandy fill material. Heave or erosion around 'light' foreign objects. Slope apparently too steep to be confident of integrity, or isolated bare patches, or isolated animal burrows, or damaged revetment or potential lack of integrity of toe. Some movement of revetment material but no fill wash-out. Local uplift near toe.

Figure 2.6. The page of the Condition Assessment Manual (Environment Agency 2006) regarding embankment slopes graded at Condition 3 (Fair).

2.5 The introduction of structural reliability methods

2.5.1 RASP High Level Methodology *PLUS*

During the last phase of the RASP project (2002-2004) some progress was made regarding the availability of more detailed information (for example regarding the geometry) about flood defence assets on a national level. This fact generated more interest in the application of structural reliability methods, clearly inspired by the practice in the Netherlands, where the probabilistic approach to flood defences design was already consolidated (CUR/TAW 1990). This option was also explored and developed in another DEFRA/EA project called "PAMS: Performance-based Asset Management System" (HR Wallingford 2005), which was carried out in parallel with RASP and whose second phase was still ongoing at the time of writing. The structural reliability concepts underpinning RASP and PAMS were developed in a third project on the "Performance and Reliability of Flood and Coastal Defences" (Buijs *et al.* 2007).

The adoption of a structural reliability approach in a national scale assessment led to a new methodology that was named High Level Methodology *Plus* (RASP – HLM+). In the High Level Methodology *Plus* the load is no longer expressed as a multiplier of the Standard of Protection. Instead the hydraulic loading is represented explicitly in the form, for fluvial embankments, of water level. In the structural reliability approach it is possible to deal explicitly with different mechanisms leading to breaching. In the HLM+ approach two modes of structural failure are considered for flood embankments:

a) Erosion of the grassed surface of the landward slope by water running on the embankment in condition of continuous overtopping, shown in Figure 2.7.a. This mode of failure is sometime referred to as overflowing.
b) Failure for piping thorough the earthfill, shown in Figure 2.7.b.

In order to apply the methods of structural reliability to a mode of breaching a mathematical expression of the limit state which separates failures from safe states is required. In the HLM+ analytical expressions used in the Dutch practice have been adopted as limit state functions for overflowing and piping. Similarly the probability distributions of the input random variables have been chosen in compliance with the Dutch guidelines (Vrijling & van Gelder 2000).

If a limit state function is defined and the probability distributions of its input parameters are known the probability of failure for a given load can be determined with some well known method of reliability analysis, like the First Order Reliability Method (FORM) or the Monte Carlo simulation (these are discussed in more detail later in the thesis). Repeating the calculations for several values of the load, in its relevant range of variation, leads to the construction of a fragility curve. The crucial point is, nevertheless, how well the assumed input parameters reflect the actual condition of a flood embankment. The application of structural reliability techniques to a situation where the level of information is low, like in a national risk assessment, poses some problems. In order to illustrate these issues the reliability calculation at the base of the HLM+ are explained in the following.

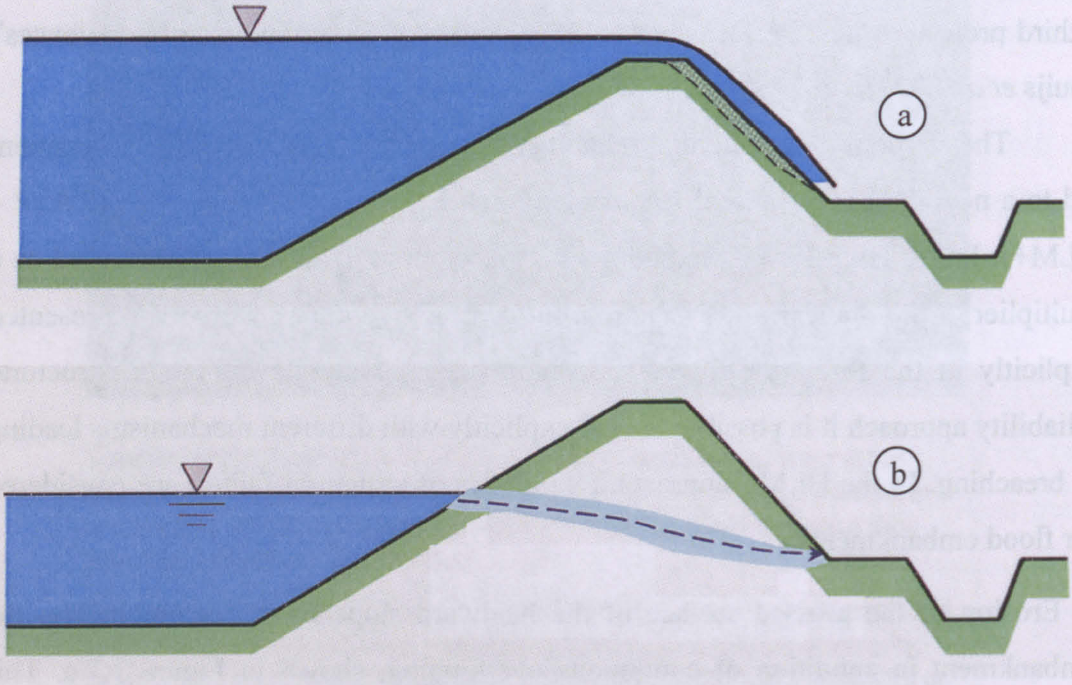

Figure 2.7. The reliability analysis of flood embankments in the RASP High Level Methodology Plus (Buijs et al. 2007) intends to include overflowing (a) and piping through the earthfill (b) as modes of failure (Pictures modified from Allsop et al. 2007).

2.5.2 Failure by overflowing

In the High Level Methodology *Plus* the limit state for overflowing is a function of the following variables (Buijs *et al*. 2007):

c_g erosion endurance of the grass

t duration of overflowing

d_w depth of grass roots

c_{RK} erosion endurance of the earthfill

L embankment width

k_s hydraulic roughness, according to Nikuradse, of the submerged grass on the landward slope

α angle of the landward slope

The assumptions regarding the probability distribution, the mean and the variability, expressed as a coefficient of variation, for these input variables are reported in Table 2.3. The mean of the coefficient of erosion endurance of the grass c_g, is linked to different Conditions Grades as illustrated in Table 2.4. A distinction is introduced between high permeability and low permeability earthfill, without further specifying the limit between the two classes. If a surface cover different from grass is present a correction factor is introduced to take into account the appropriate type of revetment.

Table 2.3. Input parameters for fluvial embankment fragility curves regarding the overflowing mode of failure, high level methodology *Plus* (HLM+) developed by Buijs *et al.* (2007).

Variable			Unit	pdf	Mean	COV
Width	Narrow	L	m	LN	7.5	0.027*
	Wide				20	0.010*
Landward slope	Narrow	$\tan\alpha$	-	N	0.5	0.05
	Wide				0.25	
Grass erosion strength		c_g	ms	LN	Depends on Condition Grade	0.30
Root depth		d_w	m	LN	0.1	0.20
Nikuradse slope roughness		k_s	m	LN	0.015	0.25
Fill erosion strength		c_{RK}	ms	LN	23,000	0.30

pdf = probability density function; COV = Coefficient of Variation.
N = Normal distribution; LN = Lognormal distribution.
(*) Derived assuming standard deviation $\sigma = 0.2$ m.

Table 2.4. Mean values of the coefficient of grass erosion strength c_g adopted in the high level methodology (HLM+) developed by Buijs *et al.* (2007) – grassed landward slope and crest.

Condition Grade	High Permeability Fill	Low Permeability Fill
1	1.0×10^6	1.5×10^6
2	8.5×10^5	1.3×10^6
3	6.0×10^5	9.0×10^5
4	4.2×10^5	5.0×10^5
5	3.3×10^5	3.3×10^5

The following observations can be made regarding the modelling assumptions:

- The required input is rather detailed for an analysis on a national-scale.

- The discretization of the embankment base width (L = 7.5 m or L = 20 m), constrained by the presence of only two classes in the national database (narrow or wide embankments), appears quite coarse.
- The assumptions on the duration of overflowing t are not stated.
- The alleged effect of the fill permeability on the grass erosion resistance is not discussed and the limit for high versus low permeability classification is not stated.
- Linking the strength of the grass to the Condition Grade can be a source of error. In fact the condition grade lumps together the contribution of several factors. An embankment can be graded as poor, for instance for the presence of intense animal burrowing, and still have a perfect grass cover. In this case the HLM+ would assign a very high probability of breaching by overflowing while the likelihood of this event is actually very low and the probability of breaching by piping would dominate instead.

2.5.3 Failure by piping

The reliability analysis of failure by piping in HLM+ is very controversial. The analysis is intended to address the mechanism of failure by piping through the embankment (Buijs *et al.* 2007), however, as will be extensively discussed in this thesis, there are no currently usable mathematical models of this physical process. A model for piping through the foundation of an impervious water-retaining structure and known as the "weighted creep" formula (Lane 1935) is used instead, overstretching this criterion far outwith its area of applicability. It has been shown in fact (Richards & Reddy 2007) that foundation erosion and piping through the earthfill are linked to physically different phenomena, namely backward erosion and internal erosion, the first being mainly related to intergranular flow in the ground, the second being related to the flow in cracks, openings and zones of concentrated leakage.

The use of Lane's formula in relation to earthfill flood defences originates from the Dutch practice where, however, it is used exclusively for assessing the safety against foundation erosion. It is very common, for specific geological reasons, to have sea-dikes lying on fine grained, recent deposits, characterised by low permeability and high resistance to erosion (TAW 1999). In most cases under this top layer some coarse grained soil, generally fine sand in the Netherlands, is encountered. This soil, much more permeable and prone to erosion, can be easily washed away, undermining the structure's stability, if the top layer cracks and originate a preferential seepage path. This situation is shown in Figure 2.7. In the Dutch practice the check against erosion of the coarse grained layer is done with Lane's formula or with the more advanced Sellmeijer's criterion (Weijers & Sellmeijer 1993).

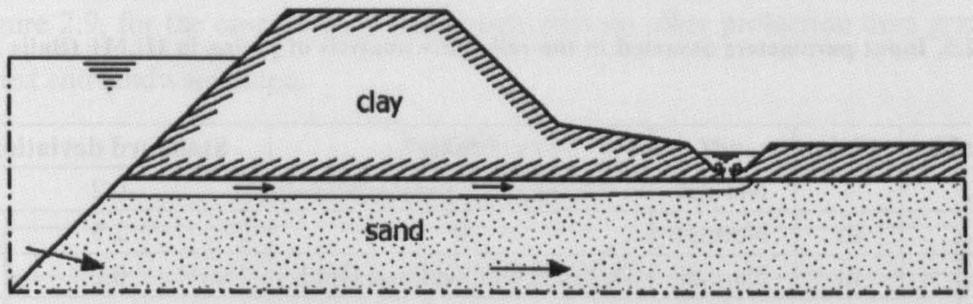

Figure 2.8. Evolution of under-piping, leading to foundation erosion, in the geological and constructive setting typical of sea-dikes in the Netherlands (TAW 1999).

According to Lane (1935) the hydraulic head difference ΔH_d that can be safely sustained by a foundation of an impervious water retaining structure is

$$\Delta H_d = \frac{L_v + \frac{1}{3} L_h}{C_w} \tag{2.6}$$

where L_v is the total length of the vertical segments of the seepage path along the foundation, L_h is the total length of the horizontal segments of the same seepage path and C_w is a coefficient, named the weighted creep factor, which depends on the type of soil in the foundation. When this criterion is employed to check the resistance to erosion of a sand layer the system composed by the compacted clay sea-dike and the clay top-layer is seen as a virtually impermeable hydraulic structure and only the sand is treated as foundation soil.

In the report on the development of fragility curves for HLM+ (Buijs *et al.* 2007) an attempt is made to justify the application of Lane's criterion to the breaching induced by piping through the earthfill. It is stated that "the probability of failure due to piping through the embankment was calculated assuming that the thickness of the impervious layer underneath the embankment is zero and the [horizontal] distance $L_{[h]}$ is equal to the width of the embankment". It is here argued that this approach gives the probability of failure along the base of an embankment directly founded on erodible soil. It cannot be claimed that the outcome of reliability calculations performed under these assumptions gives the probability of failure by piping through the earthfill. Also the choice of input parameters for the reliability analyses is problematic. This is summarised in Tables 2.5 and 2.6.

Table 2.5. Input parameters assumed in the reliability analysis of piping in HLM+ (Buijs *et al.* 2007).

Variable	Unit	pdf	Mean	Standard deviation
C_w	-	Normal	Depends on soil type	0.2
L_v	m	Normal	0	2.5
L_h	m	Normal	Depends on Condition Grade	0.2

Table 2.6. Mean values of the horizontal seepage length for reliability analysis of piping in HLM+ (Buijs et al. 2007).

Condition Grade	1 Very Good	2 Good	3 Fair	4 Poor	5 Very Poor
$\mu(L_h)$	60 m	60 m	60 m	10 m	6 m

It can be observed that:

- The soil type either in the manmade earthfill or in the natural foundation is not always known. This type of knowledge uncertainty, common to other relevant characteristics of defences, needs to be addressed.
- The choice of a normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 2.5$ m for L_v allows for negative values of the vertical seepage length.
- Linking the horizontal seepage length L_h to the condition grade deprives this variable of its physical meaning and makes it merely a parameter for inducing variation of the probability of failure with the embankment's condition.
- L_h should correspond to the base width of the embankment L; in the overflowing reliability analysis its value is either L = 7.5 m (narrow embankments) or L = 20 m (wide embankments), in the piping reliability

analysis L_h ranges from 6 m (Condition Grade 5) to 60 m (Condition Grades 1, 2 and 3).

2.5.4 RASP HLM+ fragility curves for flood embankments

For fluvial embankments the RASP HLM+ fragility curves are obtained by calculating the probability of failure by overflowing and the probability of failure by piping for a range of loads defined as the difference between the water level in the watercourse and the crest level (Buijs *et al.* 2007). The probabilities of failure were then combined assuming independency of the two failure modes. The result is shown in Figure 2.9, for the case of an embankment with no other protection than grass on the crest and landward slope.

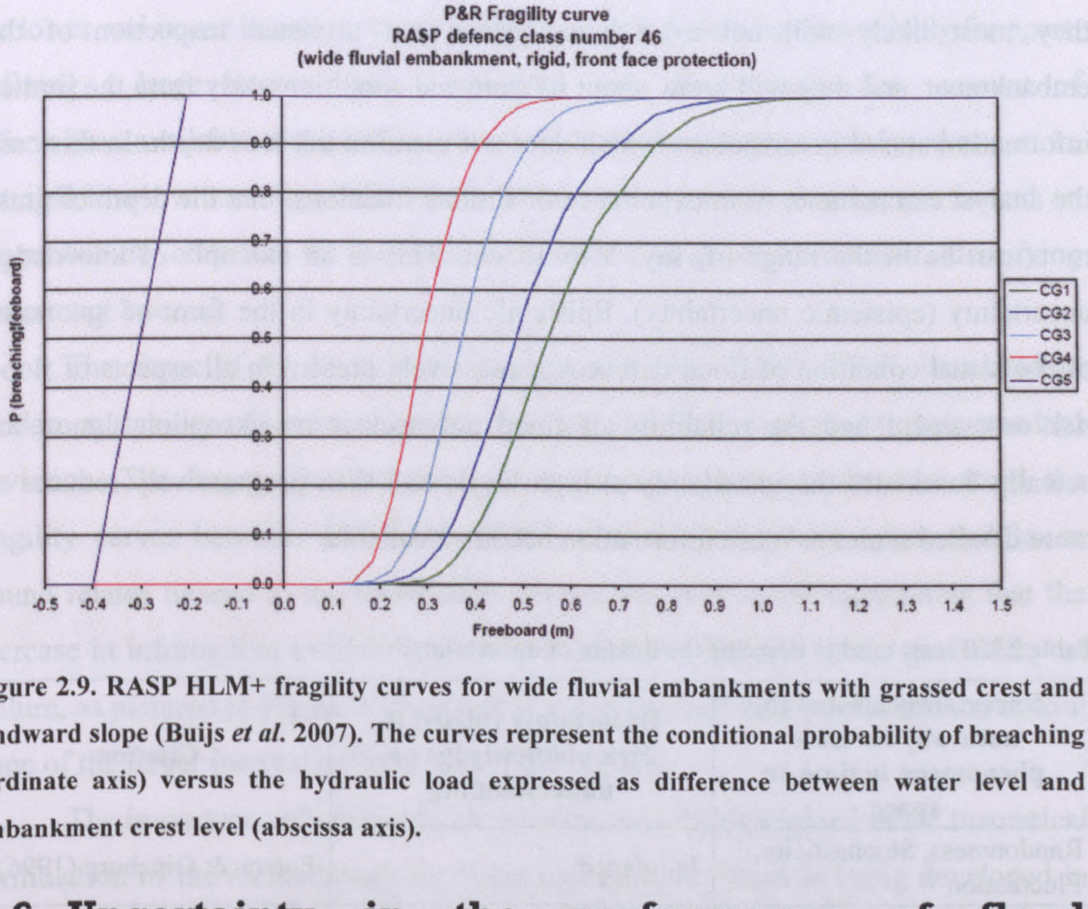

Figure 2.9. RASP HLM+ fragility curves for wide fluvial embankments with grassed crest and landward slope (Buijs *et al.* 2007). The curves represent the conditional probability of breaching (ordinate axis) versus the hydraulic load expressed as difference between water level and embankment crest level (abscissa axis).

2.6 Uncertainty in the performance of flood embankments

When considering engineering systems two kinds of uncertainties can be recognised: uncertainties related to the natural variability of phenomena in time and space and uncertainties related to lack of knowledge or understanding about the system (see for example Beacher & Christian 2003). The first type is generally called random uncertainty or aleatory uncertainty while the second is referred to as knowledge uncertainty or epistemic uncertainty. Table 2.7 summarises some terms used in the literature to reflect this duality.

The difference between aleatory variability and epistemic uncertainty can be exemplified, in the case of flood embankments, considering the depth of grass roots. If it were possible to measure the depth of grass roots exactly in several locations along the stretch of an embankment a sample of different values would be found rather than exactly the same value repeated in all measurements. This is an expression of random fluctuation (aleatory variability). However the risk analyst in most cases not only will not have access to measures of the actual depth of the roots, they most likely will not even have carried out a visual inspection of the embankment, and they will know about its state and conditions only from the limited information stored in a database which does not mention the root depth. In this case the analyst can assume, from experience of similar situations, that the depth of grass roots can be in the range of, say, 5 to 15 cm. This is an example of knowledge uncertainty (epistemic uncertainty). Epistemic uncertainty in the form of ignorance of the actual condition of flood defences is massively present in all aspects of flood risk assessment and the reliability of flood defences is no exception. Ignorance actually dominates the uncertainty at high levels and then progressively reduces at more detailed scales as more information become available.

Table 2.7. Terms used to describe the duality of uncertainty.

Uncertainty related to naturally variable phenomena in time or space	Uncertainty related to lack of knowledge or understanding	Citation
Randomness, Stochasticity, Fluctuation	Ignorance	Ferson & Ginzburg (1996)
Aleatory uncertainty	Epistemic uncertainty	Baecher & Christian (2003)
Natural variability	Knowledge uncertainty	NRC (2000)
Objective uncertainty	Subjective uncertainty	Chow *et al.* (1988)

It is widely accepted that in practically all risk assessments random variability and epistemic uncertainty coexist and most researchers recognise that they should be treated separately (Ferson & Ginzburg 1996), even if there is no universal consensus on the best way to do so (Ferson *et al.* 2004). Ferson and Ginzburg (1996) showed how propagating fluctuation with probability theory and ignorance with interval analysis is a correct and efficient approach to risk analysis. Probability theory and interval analysis can be combined in the theory of bounds on probability (Walley & Fine 1982, Tessem 1992).

These concepts were incorporated in RASP and PAMS from the very beginning as shown by Figure 2.10, which represents bounds on probability of failure at different levels of detail. Ideally the ignorance about the characteristics of flood defences and their conditions can be progressively reduced moving to more detailed levels, where more information is available. For a certain load, for example, the lower and upper bound on the probability of failure will be unavoidably far apart, defining a large interval, when modelled with the typically low amount of information available at the national level. If the same defence is analysed at a more detailed level, where more information is available, the lower and upper bound on the probability of failure for that fixed load, will be closer to one another, identifying a narrower interval.

The shape of the bounds on fragility, typically "inclined Ss", is related to random variability of the input parameters, quantified for instance by their standard deviation. The larger the fluctuation of the input parameters the less steep the fragility curves become. The width of the intervals defined by upper and lower bound relates instead to the knowledge uncertainty. It is worth mentioning that the increase in information availability results in narrower bounds on the probability of failure, as pictured in Figure 2.10, but does not imply convergence toward the central zone of the larger interval defined at the previous step.

The importance of epistemic uncertainty, clearly highlighted in the theoretical formulation of the methodology for flood risk analysis which is being developed in the UK (HR Wallingford & University of Bristol 2003, 2004), has later been overshadowed in the implementation phase (Buijs *et al.* 2007) by the attention for the random variability of input parameters. As a result the HLM+ fragility curves for the national level are always shown as best estimates, without the associated bounds, as shown in Figure 2.9. Moreover a discussion of the determination of the lower and upper bounds is not found in the literature and no exemplification is given regarding the procedure for narrowing the uncertainty interval, when moving to a more detailed level. In some circumstances the confusion between epistemic and aleatory uncertainty is substantial. Buijs *et al.* (2007), for example, suggest that fragility curves for the Intermediate Level Methodology can be obtained from a procedure identical to the High Level Methodology but with "a lower value of standard deviation to capture the uncertainty reduction in the parameters value".

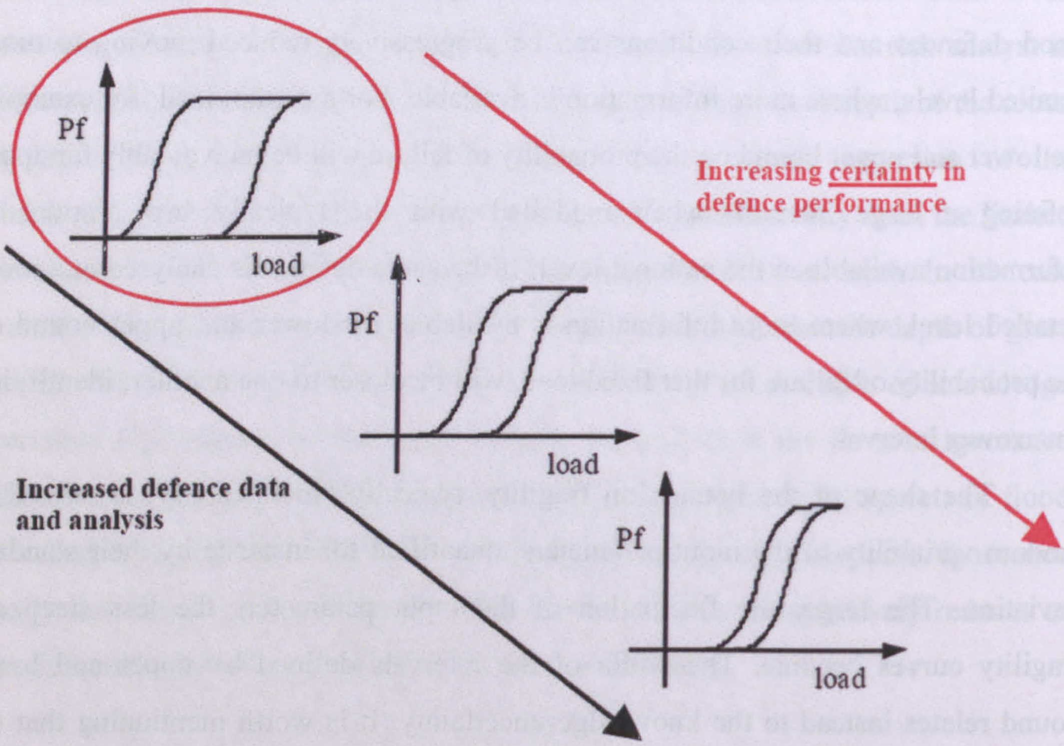

Figure 2.10. Bounds on probability of failure are introduced in the reliability analysis in order to account for epistemic uncertainty. The bounds become narrower as more knowledge about the defence characteristics and condition is acquired (Buijs *et al.* 2007).

2.7 Improved condition assessments

In a project developed within the British Flood Risk Management Research Consortium (FRMRC), Long *et al.* (2006) developed an improved approach to condition assessments. The newly proposed method grades separately the different features that can be observed on the elements of flood defences and links them to its expected performance. For instance, in the case of flood embankments, the quality of the grass cover and the presence of animal burrowing would not be lumped together like in the current approach, but would be graded separately. Keeping the features separate, it is possible to link each of them correctly to one (or more) failure mode. To quantify the influence of the observed features on different failure modes some contribution coefficients, based on expert judgement, have been established. These can be seen as weights reflecting the relative importance of each feature in making a specific failure mode more likely to occur. The new indexing system also includes a procedure to deal with uncertainty in grading. This type of uncertainty regards the possible indecision of inspectors having to grade a situation which is on the border line between two possible grades.

The improved procedure is a relevant step forward in condition assessments, introducing the important idea that features need to be graded separately and correctly linked to the different possible modes of failure. In this way the conditioning index obtained with the new methodology, which is a real number between 1 and 5, gives a quantified assessment of the structure's condition that is better than the traditional condition grade. In fact it is more closely related to the expected performance. The condition of the asset, however, is still represented by a single value. To be really beneficial to the determination of the probability of failure the relevant features should not be aggregated at all and used individually in the subsequent phase of reliability analysis.

2.8 Regional-scale methodology

Gouldby *et al.* (2008) discuss the application of a regional-scale methodology which builds on the RASP and PAMS projects. Although there are changes in the way the inundation is modelled, the part regarding the reliability of flood defences does not differ substantially from the national-scale methodology described in Section 2.5. In fact:

- The resistance of each defence to loading is considered independent from the resistance of neighbouring defences or other defences in the system.

- For the development of fragility curves, which are intended to account for piping through the embankment and overflowing, reference is made to Buijs *et al.* (2007).
- The fragility curves are related to the Condition Grade of the flood defence (Figure 2.11).
- Epistemic uncertainty is handled using upper and lower bounds on the probability of failure (Figure 2.12), but no details are given on the way these bounds have been established.

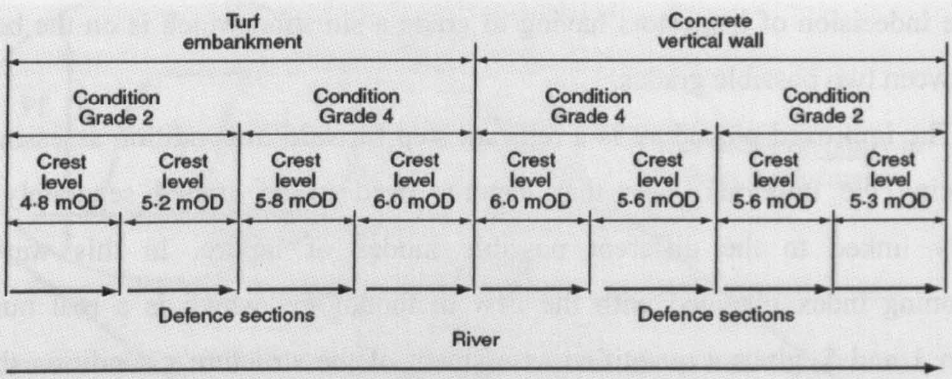

Figure 2.11. Conceptual diagram of the flood defence system discretization in the regional-scale methodology proposed by Gouldby *et al.* (2008). The reliability of a defence depends on the crest level and the condition grade.

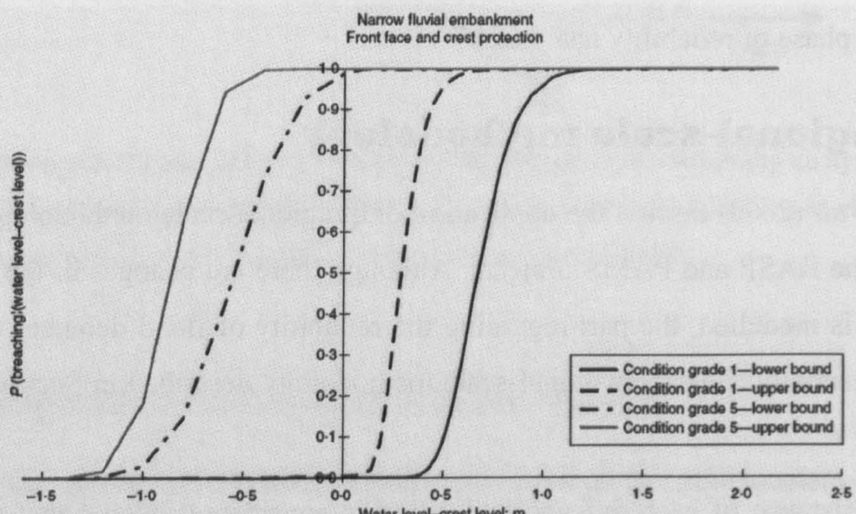

Figure 2.12. Fragility curves for flood embankments used in the regional-scale methodology proposed by Gouldby *et al.* (2008). Epistemic uncertainty is handled introducing bounds on the probability of failure.

Regarding Figure 2.12 it is worth noticing how the terms "lower" and "upper" refers to bounds on the probability of breaching (not on the loading): the lower bound fragility curve is hence at the right of the correspondent upper bound fragility curve. The two pairs of curves for Condition Grade 1 and Condition Grade 5 show that an embankment in very good condition can hardly be breached by water levels lower than the crest while an embankment in very poor conditions is extremely likely to experience breaching for water levels below the crest.

2.9 Conclusions

The reliability analysis of flood embankments currently implemented in the flood risk assessment methodologies under development in the UK is subject to some limitations. In particular:

- The process of piping through the earthfill is incorrectly modelled using an empirical equation developed for foundation erosion under impervious structures
- The practice of linking the input parameters for each failure mode to the Condition Grade is misleading because features influencing different failure modes are lumped together.
- There is lack of clarity about the way epistemic uncertainty is incorporated using bounds on the probability of failure.
- There are no published examples of how the uncertainty interval defined by upper and lower bounds can be reduced when more information is gained, moving to a more detailed level, for instance from national to regional scale.

This thesis addresses the issues listed above. The modelling of piping through the embankment is particularly problematic. As will be thoroughly discussed in this document, the physical process of piping through the earthfill is not amenable to credible mathematical modelling. This implies that the related probabilities of failure need to be determined by judgement elicitation. For an integrated flood risk assessment judgement based fragility curves for piping through the earthfill should be constructed for the relevant range of loads, also reflecting several different embankments conditions. Such task goes beyond what can be achieved in a single three-year doctoral study. For this reason most of the author's work has been devoted to the development of a simplified system for quantifying the expected performance of flood embankments. This system requires the assessment of the probability of failure by piping only in one loading condition. In this way the size and complexity of the task is brought down to a tractable level and a tool for quantifying the expected performance of flood embankments is offered to decision makers.

Hopefully, with time, the scientific and technical community involved in the research on flood defences reliability will be able to produce the effort needed for delivering credible judgement based fragility curves for the breaching due to piping through the earthfill, to be used in integrated flood risk modelling.

3

RELIABILITY OF FLOOD DEFENCES: A REVIEW

In Chapter 2 the reliability of flood defences, with particular attention to fluvial embankments, has been introduced and its fundamental role in the integrated modelling of flood risk has been discussed, reporting the innovative developments in this direction currently ongoing in the UK. In this chapter the literature on flood defence reliability which has been developed outside the UK is reviewed. A lot of emphasis is placed on the probabilistic methods developed in the Netherlands. This is because the Dutch practice, propelled by the needs of a country constantly dealing with flooding hazard, has been the first to embrace reliability techniques for design. Moreover the British research in this field is directly inspired by and linked to the methods in use in the Netherlands (Buijs *at al.* 2004). The approach to the reliability assessment of flood embankments adopted in the USA, which focuses on the geotechnical aspects, is also presented. Finally the relevant issue of combining aleatory variability and epistemic uncertainty is considered, reviewing the use of imprecise information techniques applied to flood defence reliability (Dawson & Hall 2002a, 2002b).

3.1 Probabilistic design of flood defences in the Netherlands

3.1.1 Development of reliability based design

Approximately half of the Netherlands lies below the sea level and is protected from flooding by a system of water retaining structures (Vrijling 2001). Since the Middle Ages a lot of attention has been given to the safety of dikes and other defence structures. In 1953 an extreme storm surge in the North Sea, which also severely affected England, induced overtopping and breaching of some of the Dutch sea-dikes, resulting in flooding of the South-western Delta. About 1800 people lost their lives in those tragic events and the economic costs were substantial (Dantzig & Kriens 1960). After the disaster a commission, called the Delta Committee, was created for discussing and optimising the safety level of the primary dike system. The final recommendations of the Committee focused on the definition of design hydraulic loads in relation to the land use of the protected territory. The work of the Committee also highlighted the importance of accounting for different modes of structural failure. At the time, however, the modern methods of reliability were not developed and the design requirements for structural resistance where formulated in the deterministic way.

Probabilistic models were first introduced in the late 1970s for the design of storm surge barriers (Vrijling 2001). In the 1980's the reliability approach became more widely applied, being employed for different types of water-retaining structures. In the 1990s the continuing research on reliability led to the development of an integrated computer program for calculating the probability of failure of systems of dikes (Vrouwenvelder *et al.* 1999, 2001a, 2001b). In the Netherlands a closed system of flood defences around a polder is referred to as "dike ring"; for this reason the computer program was named PC-Ring.

The use of reliability techniques allows to include in a safety assessment the chances of structural failure of the defences. The approach adopted in the Netherlands takes into account several structural modes of failure. The final aim of the reliability analysis of a dike ring is to determine the probability of a polder being flooded over an assigned time interval (generally one year). For this purpose the entire defence system has to be included in the model. Figure 3. shows the sketch of a hypothetical flood defence system with its elements. The failures of component elements are arranged in a fault tree which represents the performance of the system. The OR gates connecting the events signify that the events at the upper level -"after the gate"- occurs if any of the events at the lower level -"before the gate"- occurs. In the pictured system, for example, there is inundation if dike 1 fails or the dune fails or dike 2 fails or any other defence in the system fails. The failure of one component is sufficient to induce the failure of the system. In the reliability language this type of interaction is described as a series system. The fact that the flood defences behave as series systems translates in their structural safety being a typical weakest link problem: a single weak spot heavily influence the safety of the whole system.

Fault trees are created drawing the system failure as the top event, then adding a lower level of possible causes and so on, reasoning backward, till the initial events possibly leading to system failure are drawn at the bottom of the tree. In order to find the probability of system failure the probability of the initiating events need to be determined and then combined through logic gates, like the OR gates in Figure 3.1, until the top event is reached. In Figure 3.1, for instance, one level below the event "failure of dike 1" the possible causes of that event occurring are found. Dike 1, in fact, is thought as split in many component sections and fails if any of these sub-sections fails. In the following it will be briefly explained how the strategy of splitting the dikes in several sections accounts for the influence of the length of the structure on the probability of failure, provided the correlation between the different sections are considered. The failure probability of the elementary sections is calculated using methods of reliability theory such as Monte Carlo simulation or FORM. Admittedly in the whole process of reliability assessment a good deal of engineering judgement is required, whenever sound data on the input parameters are not available (Vrijling 2001).

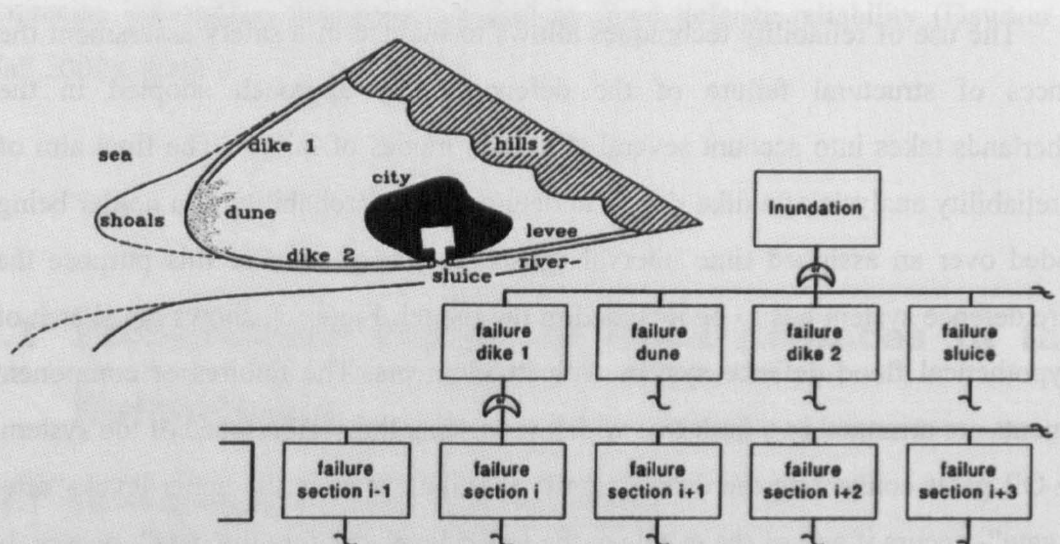

Figure 3.1. Fault tree representing for the reliability analysis of a dike ring system (Vrijling 2001).

3.1.2 The PC-Ring program package

The computer program PC-Ring (Vrouwenvelder *et al.* 1999, 2001a, 2001b) has been developed for calculating the probability of a closed system of flood defences failing, causing inundation. The program incorporates several failure modes and accounts for correlations inside and among various components. A common cross-section for Dutch sea dikes is shown in Figure 3.2. A natural fine–grained covering layer is mostly present in the local geology. Where it is not some compacted clay is put in place beneath the core of the dike and on the floodplain. The core is made of sand and a clay cover is present either on the seaward face only or all around the core. Some kind of revetment is often present on the seaward face to protect the clay from waves attack.

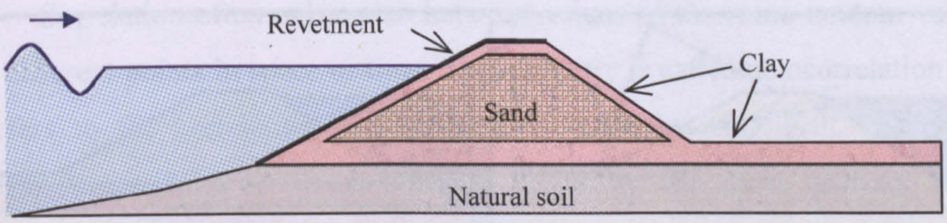

Figure 3.2. Typical section of a Dutch sea-dike.

For sea dikes four structural failure modes are considered in PC-Ring (Figure 3.3):

- erosion of the inner (landward) slope due to overflow or wave overtopping;
- damage of the outer (seaward) slope revetment and core erosion due to waves;
- rotational failure of the inner slope;
- piping through an erodible layer in the foundation, following the creation of a preferential seepage path by the rupture of the top clay cover.

For each elementary section of a dike the probability of failure can be determined, for each failure mode, with the methods of reliability analysis. In PC-Ring each mode of failure is described analytically by a limiting function which separates failure states from safe states in the space of input parameters. The input parameters are random variables described in terms of:

- probability distribution (Normal or Lognormal in most cases);

- mean;
- standard deviation (or, equivalently, coefficient of variation);

Assumptions to be used in case of lack of local information are available in the Dutch guidelines (Vrijling & van Gelder 2000). Such suggestions about reference values are available for distributions types and random variability, while mean values are more dependent on local conditions.

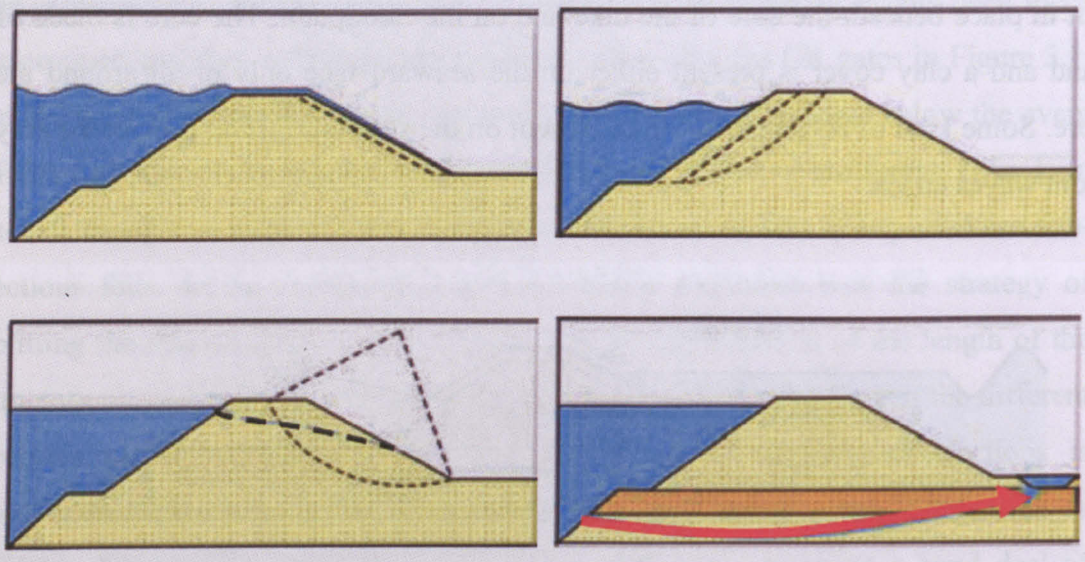

Figure 3.3. Structural failure modes in PC-Ring: inner slope erosion by overflowing/wave overtopping, erosion of the outer slope, rotational slope failure and foundation erosion (Möllmann & Vermeer 2007).

The standard technique for reliability calculations in PC-Ring is FORM (Hasofer & Lind 1974). When its use can be problematic, for instance due to convergence problems, the recommended alternative is Monte Carlo Directional Sampling. The other available options are Numerical Integration of the joint probability distribution of the limit state function, crude Monte Carlo simulation or the Second Order Reliability Method (Der Kiureghian 1987, Breitung 1984).

3.1.3 Spatial variability of parameters

Two random variables are said to be correlated if the occurrence of a determined value for one variable has an influence on the probability distribution of the other variable. The correlation coefficient of two random variables V and W, which is a measure of how well the two variables are linearly correlated, is defined as:

$$\rho_{V,W} = \frac{Cov[V,W]}{\sqrt{Var[V]Var[W]}} \tag{3.1}$$

where Cov[.,.] indicates the covariance and Var[.] indicates the variance. Positive values of $\rho_{V,W}$ indicate that if one variable assumes a value above its mean the other variable tend to be above its mean as well. Negative values of $\rho_{V,W}$ indicates inverse variation. The correlation coefficient can assume values between +1 and -1, with the extreme values indicating perfect linear correlations. If the correlation coefficient is 0 the there is no correlation between the variables. Some geotechnical parameters can be considered as correlated to one another. For instance clays with higher unit weight γ generally also have higher undrained shear strength C_u. Then when these parameters are treated like random variables γ and C_u should be positively correlated, i.e. they should have a correlation coefficient $0 < \rho_{\gamma,Cu} < 1$.

Correlation often exists also between values of the same random variable at two different points in space or time. This property is called autocorrelation because it refers to the correlation of an individual variable with itself. For resistance parameters involved in flood defences reliability the most relevant aspect is autocorrelation over space. The autocorrelation coefficient for the variable U in two points separated by a distance δ can be written as:

$$\rho = \frac{Cov[U(x),U(x+\delta)]}{\sqrt{Var[U(x)]Var[U(x+\delta)]}} \tag{3.2}$$

When several on-site measures of a parameter are available at different distances in space, the coefficient of autocorrelation can be estimated as a function of the separation distance making use of the sample autocovariance and sample variances in Equation 3.2. Experience shows that the value of the correlation coefficient tends to decrease as the separation distance increases, typically going to zero for large spacing. At small separation distances the autocorrelation is higher and by definition, it has a unit value for $\delta = 0$, i.e. a variable is perfectly correlated with itself at one point. The variation of the autocorrelation coefficient with the separation distance can be conveniently described by an autocorrelation function, which can be fitted to observed data where they're available. A common one-dimensional model is, for instance, the exponential autocorrelation function (Vanmarke 1977):

$$\rho(\delta) = \exp\left(-\frac{\delta}{d}\right) \tag{3.3}$$

where d is a constant length, called autocorrelation distance (not to be confused with the scale of fluctuation, introduced later). Other common choices for autocorrelation functions can be found in the literature (e.g. Lacasse & Nadim 1996, Beacher & Christian 2003). The scale of fluctuation θ is the distance within which a parameter shows strong autocorrelation (Phoon & Kulhawy 1996 and, for a more rigorous definition, Vanmarke 1977, 1984). The scale of fluctuation is related to the autocorrelation distance depending on the shape of the autocorrelation function. For the exponential function of Equation 3.3, for example, $\theta = 2d$. Geotechnical parameters in natural deposits exhibit a much shorter scale of fluctuation in the vertical direction than in the horizontal direction (Vanmarke 1977).

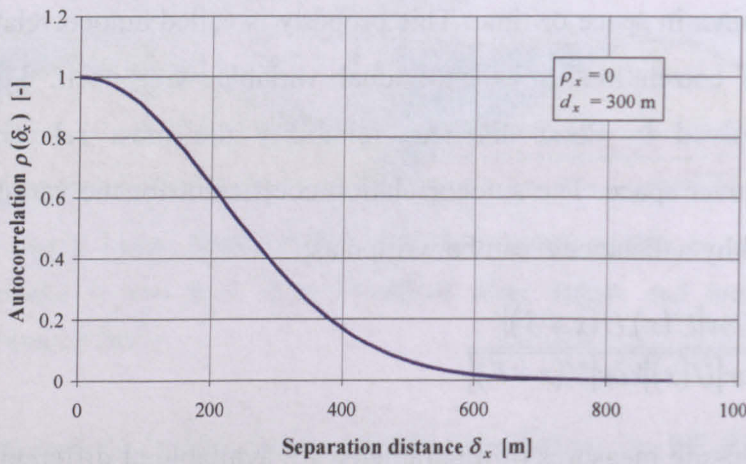

Figure 3.4. Example of one-dimensional autocorrelation function used in PC-Ring.

In PC-Ring the following function is used for describing one-dimensional autocorrelation in the direction of the dike length:

$$\rho(\delta_x) = \rho_x + (1-\rho_x)\exp\left(-\frac{\delta_x^{\ 2}}{d_x^2}\right) \tag{3.4}$$

where δ_x is the separation distance in the direction of the defence structure's length, ρ_x is the constant value that the function approaches for very large separation distances, d_x is the autocorrelation distance. An example is plotted in Figure 3.4. In slope stability calculations a two dimensional autocorrelation is considered using the following function:

$$\rho(\delta_x,\delta_y)=\exp\left(-\frac{\delta_x^{\,2}}{d_x^2}\right)\left[(1-\alpha)+\alpha\exp\left(-\frac{\delta_y^{\,2}}{d_y^2}\right)\right] \tag{3.5}$$

where δ_y is the separation distance in the vertical direction, d_y is the vertical autocorrelation distance and α is a constant, named variance ratio factor.

3.1.4 Combinatorial steps

3.1.4.1 Combinatorial steps and reliability of series systems

In the Dutch approach to the reliability of flood defences the process of combining the probability of failure of individual sections to achieve the probability of failure of the system is complex. In fact the straightforward application of a reliability method for a specified load gives the probability of failure of an individual section of a flood defence by each failure mode. Then a number of combinatory steps are required. It is, in fact, necessary to:

- combine elementary sections to obtain the probability of failure by each failure mode for an entire defence (length effect);
- combine all failure modes of a defence to obtain the total probability of failure of that defence;
- combine all defences to obtain the probability of failure of the system.

In all these steps correlation can be present. PC-Ring also accounts for the wave load being directional and all loads being autocorrelated in time. These latter aspects are not strictly connected with the core of this thesis and will be not further reviewed. The combinatorial steps listed above will instead be discussed in some more detail.

All the mentioned combinatorial steps can be seen as the determination the probability of failure of a series system for which the probability of failure of the component elements are known. The problem is, with the language of probability theory, the determination of the probability of the union of correlated events:

$$P(F) = P(F_1 \cup F_2 \cup \ldots \cup F_i \cup \ldots \cup F_n) \tag{3.6}$$

where $\cup$ is the union operator and F_i indicates the failure of the i-th component.

Depending on which specific step is considered i can indicate:

- one of n individual sections in which a long defence structure is split;
- one of n possible failure modes for a defence structure;
- one of n structures in a flood defence system.

3.1.4.2 Series system comprising two elements

For the sake of simplicity a two elements system is presented first. Such a system fails if at least one of the two elements is unable to sustain the load. The probability of system failure $P(F)$ can then be written as:

$$P(F) = P(F_1 \text{ OR } F_2) = P(F_1 \cup F_2) \tag{3.7}$$

In the special case of the two events being independent the probability of system failure can be calculated as:

$$P(F) = P(F_1) + P(F_2) - P(F_1)\, P(F_2) \tag{3.8}$$

However in most practical applications the two failures are correlated. Probability theory shows that in the case of correlated events the probability of series system failure is bounded as follows

$$\text{Max}\, [P(F_1),P(F_2)] \leq P(F) \leq P(F_1) + P(F_2) \tag{3.9}$$

These elementary bounds are generally widely spaced. Better approximations are given by Ditlevsen (1979). However when an accurate estimate is required numerical simulation or approximated analytical procedures need to be used.

In PC-Ring the combinatorial steps are solved after the reliability of each component has been determined, generally using FORM. To fix ideas the situation where two modes of failure need to be combined for a flood defence is here illustrated. From FORM analyses two reliability indices β_1 and β_2 are found. Two sets of factors of influence α_{1j} and α_{2k} are also produced as outcomes of FORM calculations. These factors express the influence of the input random variables on the probability of failure by mode 1 and mode 2 respectively. With an approximated analytical procedure (Vrijling & van Gelder 2001, Steenbergen *at al.* 2004, Vrouwenvelder 2006) an equivalent reliability index β^e for the series system can be found from the reliability indices and the influence factors of the components. Under the hypothesis of the limit function of the series system being normally distributed the probability of failure of the series system is then:

$$P(F) = 1 - \Phi(\beta^e) \tag{3.10}$$

3.1.4.2 Rolling up and out-crossing

The probability of failure of a series system comprising an arbitrary number n of elements can be calculated with a technique based on the approximation used for the two elements series system. The procedure simply begins by taking two elements and combining them with the analytical approach mentioned above. The original problem is so reduced to the combination of n-1 events. The step is repeated n-1 times till achieving the probability of system failure. The process of performing the subsequent combinations is knows as "rolling-up" (Vrijling & van Gelder 2000, Steenbergen *et al.* 2004, Vrouwenvelder 2006).

It is frequent to encounter series systems comprising n identical elements which are correlated. This is the case, for example, when the probability of failure of a long dike has to be calculated from the probabilities of failure of the component individual sections. This situations does not require the complete rolling-up procedure: an alternative, more efficient approach, named "out-crossing" can be used (Vrijling & van Gelder 2000, Steenbergen *at al.* 2004, Vrouwenvelder 2006).

3.2 Reliability assessment of dikes and levees in the USA

3.2.1 Early practice

In the USA the Corps of Engineers is in charge of the management of fluvial flood defences. The last decades have seen the focus shifting from the construction of new defences to the rehabilitation and improvement of existing ones (Wolff *et al.* 1996). Prioritisation of projects is decided according to probabilistic cost-benefit analysis supported by risk-based tools. This requires the determination of the probability of failure of the existing defence structures.

Before the 1990s the existing levees[3] that had not been designed or constructed to USACE standards used to be neglected in economic analysis. In the early 90s some documents were issued (US Army 1991, US Army 1992) to provide guidance for a judgment-based quantification of the probability of failures of existing levees as a function of Flood Water Elevation (FWE). The proposed methodology is based on the concepts of Probable Non-failure Point (PNP) and Probable Failure Pont (PFP). PNP identifies the water level which is believed to correspond to 85% chances of the levee not failing. PFP identifies the water level for which the chances of failure are judged to be 85 %. In order to define a probability of failure for all water levels a linear relationship is used to connect the two judgement-determined points. It is also common practice to assign zero probability of failure at the toe level and certainty of failure at the crest level. Hence the probability of failure is represented by a tri-linear approximation, like the one shown in Figure 3.5.

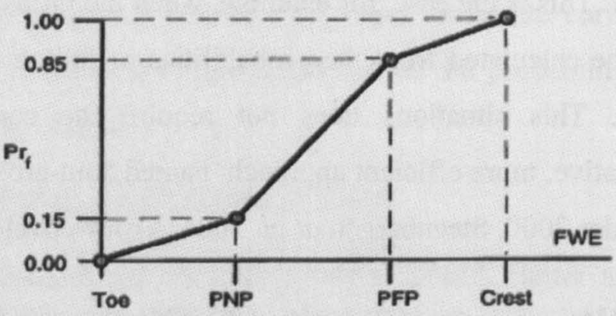

Figure 3.5. Judgement based tri-linear approximation of the probability of failure as a function of the Flood Water Elevation (FWE) employed in the USA before the introduction of reliability methods (USACE 1999).

3.2.2 The introduction of reliability methods

Since the mid 1990s Thomas Wolff and his co-workers (Wolff *at el.* 1996, Wolf 1997, USACE 1999) have carried out a research aimed at developing a more comprehensive approach to the reliability assessment of levees. As a result of these

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[3] The earthen fluvial defences, which are named flood embankments in the UK, are called levees in the USA, and in some other English speaking countries.

studies several modes of failure have been introduced and analysed with the classic limit-state reliability models. The failure modes considered are:

- excessive seepage under the levee;
- slope instability; for short and long term conditions;
- excessive seepage through the levee;
- surface erosion.

Admittedly, while under-seepage and slope failure are relatively well understood modes of failure, through-seepage and surface erosion are not satisfactorily developed in the American practice (USACE 1999).

The reliability method chosen for calculations is the First Order Second Moment (FOSM, not to be confused with FORM). This technique is based on a Taylor's series expansion of the limit state function around the mean values of the input random variables. USACE (1999) discusses the typical coefficients of variation of the geotechnical parameters required for the calculations. No reference is made to the distributional properties of these parameters (i.e. which probability distribution fits the observed values).

In the reliability analysis of levees' slopes the natural logarithm of the factor of safety *FS* is taken as limit state function. The factor of safety, in turn, is determined with one of the methods of slices.

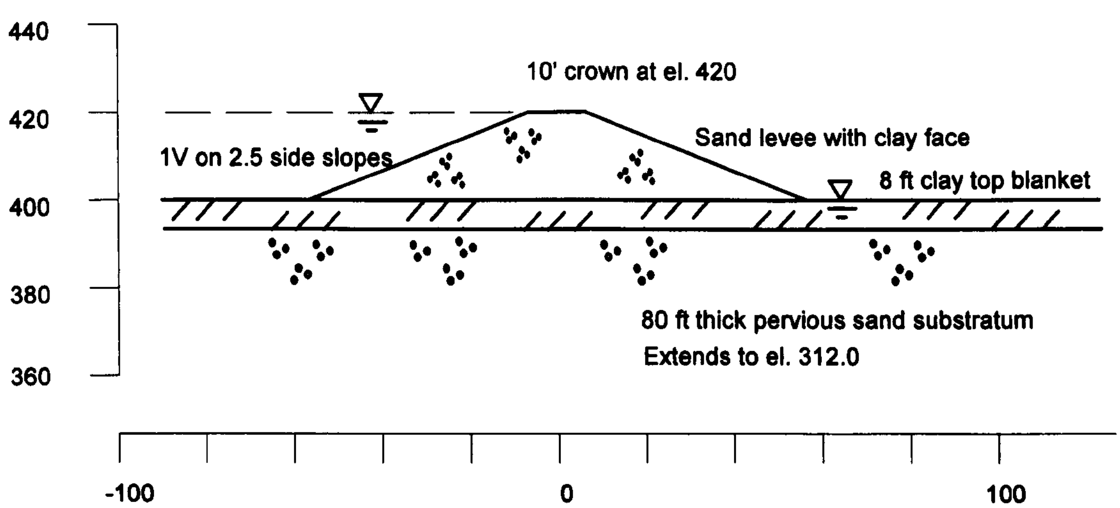

Figure 3.6. Example of levee potentially subject to failure for excessive under-seepage (Wolff *et al.* 1996).

The failure for excessive under-seepage (Figure 3.6) is modelled with a different strategy than the one used in PC-Ring. The Dutch model of foundation erosion, in fact, is composed by a check of the potential cracking of the top layer, due to uplift pore pressure at its base, plus a separate check against erosion of the coarse-grained material, which is performed with an erosion resistance criterion like Lane's (1935) or Sellmeijer's (Weijers & Sellmeijer 1993). The American model instead checks the exit gradient at the toe of the embankment, determined with a steady state seepage analysis, against a critical gradient taken as $i_c = 0.85$.

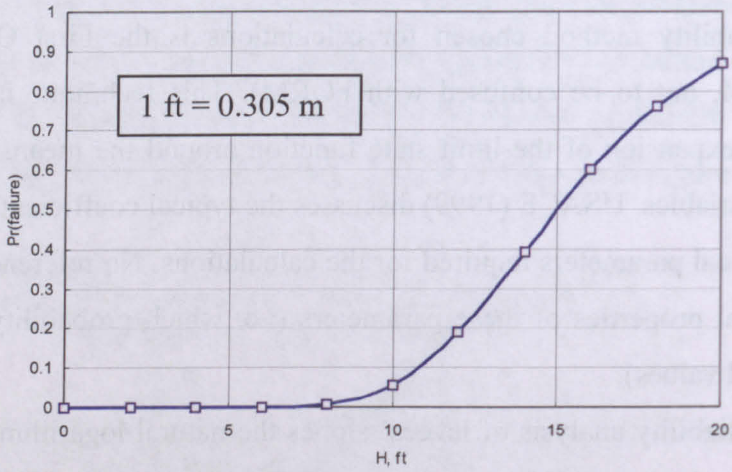

Figure 3.7. Conditional probability of failure versus flood height *H* for the excessive under-seepage mode of failure (Wolff *at al.* 1996).

The through-piping mode of failure is not mentioned by Wolff *et al.* (1996) and Wolf (1997), while USACE (1999) recognises that "there is no single widely accepted analytical technique or performance function in common use for predicting internal erosion". The document by the Corps of Engineers in fact explores two possible approaches to model these processes merely "for purposes of illustration", rather than for systematic application. One approach consists in using the erosion model of Khilar, Fogler and Gray (1985), originally aimed at predicting piping versus plugging in clay soils, and its tentative extension to coarser soils. The other approach makes use of an empirical criterion, known as the Rock Island District procedure for sand levees, which was originally developed to inform the decision of constructing embankments with or without a berm to prevent excessive through-seepage. Interestingly the application of these two approaches to the same example levee produces totally different results. In fact for a sand levee, approximately 6 m high, the Rock Island District method gives a probability of failure that is negligible for low water levels and then grows till approaching unity in when the water reaches the crest level. The Khilar equation instead gives probabilities of failure less than 10^{-6} even with water at the crest level.

Surface erosion is not considered by Wolff *et al.* (1996) and Wolf (1997). USACE (1999) discusses an illustrative example of erosion due to current velocity which assumes that the critical velocity for a grassed slope can be expressed by its mean value and coefficient of variation. The cited example is openly described as a first approximation in need of considerably more research. Waves induced erosion is not accounted for in the reliability analysis.

3.2.3 Combining conditional probabilities of failure

A curve describing the conditional probability of failure given the load can be obtained, for each failure mode, repeating the FOSM analysis for several values of the water level. Such curves are the exact equivalent of the fragility curves used in the British practice (even if such name is not explicitly used in the American approach). For example the conditional probability of failure curve for the excessive under-seepage mode of failure is pictured in Figure 3.7.

Once the conditional probability of failure curves have been calculated for all failure modes they have to be combined in order to determine the total probability of failure of the levee. This is done treating the failure modes as independent elements of a series system. Under this hypothesis of no correlation among failure modes the total probability of failure is, for each water level:

$$P(F) = 1 - \Pi\,[1 - P(F_i)] \tag{3.11}$$

where the index i identifies the different breaching mechanisms. The result of such operation is show in Figure 3.8. A curve for "judgement" is introduced to account, on the basis of the assessor opinion, for all the elements that are not captured by the formal reliability analysis, like the presence of obvious imperfections that cannot be easily included in the mathematical models (cracks, roots, animal burrows, poor maintenance, etc.). The effect of spatial correlation of parameters in not included in the approach and, for this reason, the length effect is not addressed.

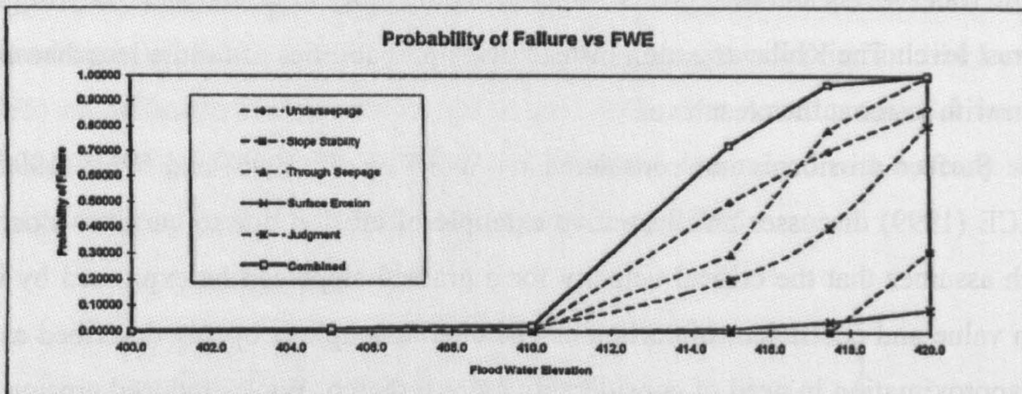

Figure 3.8. Conditional probability of failure given Flood Water Elevation obtained assuming independence of the failure modes. A judgemental curve, which allows for subjective estimate of all features not included in the four modes of failure, is introduced (USACE 1999).

3.3 The use of imprecise information

One of the salient aspects of flood defences reliability modelling is the presence, beside random variability, of a remarkable amount of epistemic uncertainty. Dawson and Hall (2002a, 2002b) have addressed this issue proposing the use of imprecise information to generate imprecise assessment of the conditional probability of defence failure. Their approach is exemplified by the reliability assessment of the rock armour of a dike, like the one shown in Figure 3.9.

A limit state equation for the armour stability is provided by Van der Meer's formula (1988) and the probability of failure conditional on the significant wave height can be calculated using FORM (Melchers 1995). The input parameters for the limit state function are:

- α the revetment's slope;
- D_{n50} the nominal rock diameter;
- S_d the damage number;
- Δ defined by $\Delta = \rho_{rock}/\rho_{water} - 1$, where ρ indicates the density;
- P the permeability factor;

N the number of waves attacking the structure;

s_m the mean wave steepness.

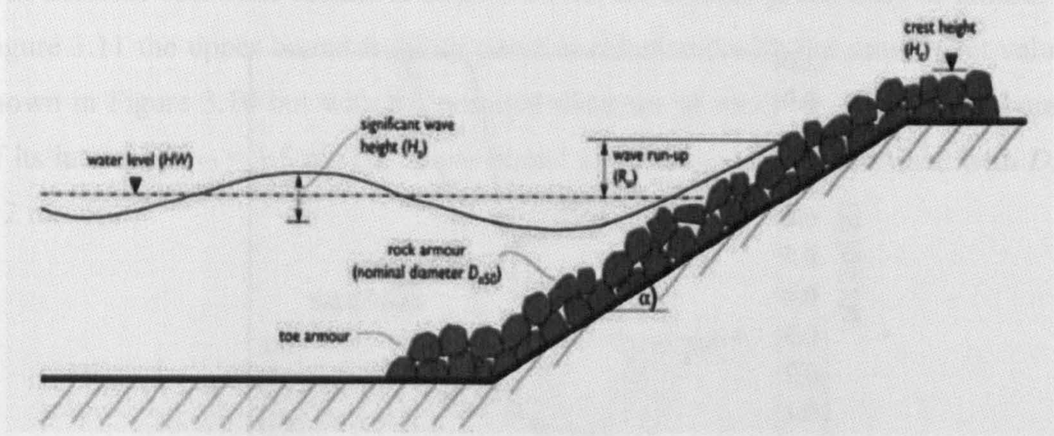

Figure 3.9. Rock armour (Dawson & Hall 2002a).

Figure 3.10 shows a fragility curve calculated assuming four of the required input parameter's for Van der Meer's formula to be normally distributed random variables and the remaining three to be deterministic constants.

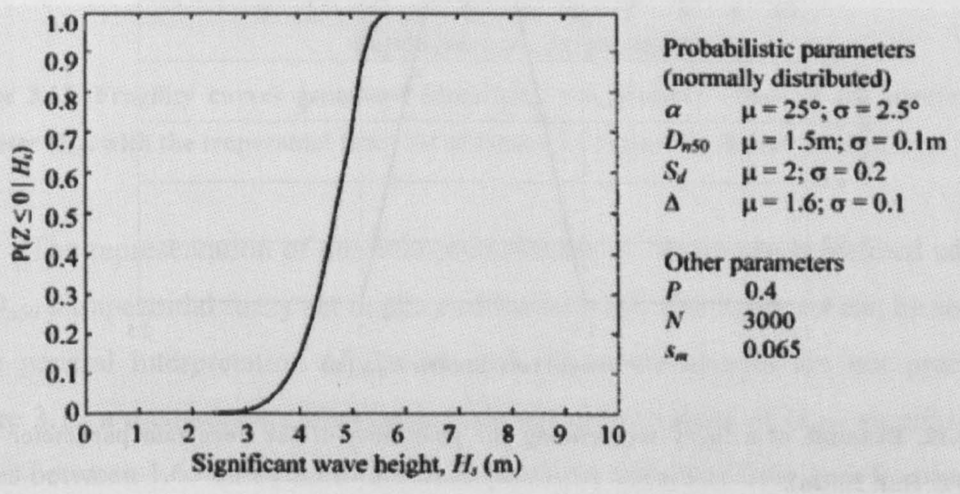

Figure 3.10. Fragility curve for rock armour revetment (Dawson & Hall 2002a).

In flood defence engineering the information on the condition of structures appears more often as vague expert judgement rather than in the form of precise measurements. The Bayesian school of probability suggests that all type of uncertain information should be mapped onto a probability distribution (Lindley 1971). However recently, as mentioned in Chapter 2, most researchers in the field of risk analysis have recognised the need of treating aleatory variation and epistemic uncertainty separately (Ferson & Ginzburg 1996).

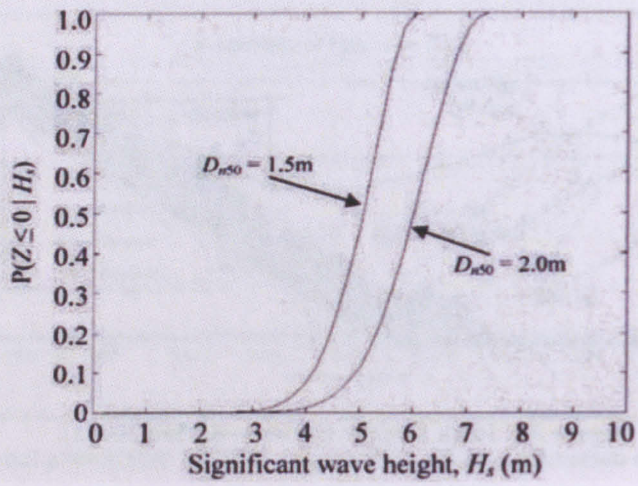

Figure 3.11. Bounds on fragility obtained identifying the possible values of the nominal rock diameter D_{n50} with an interval (Dawson & Hall 2002a).

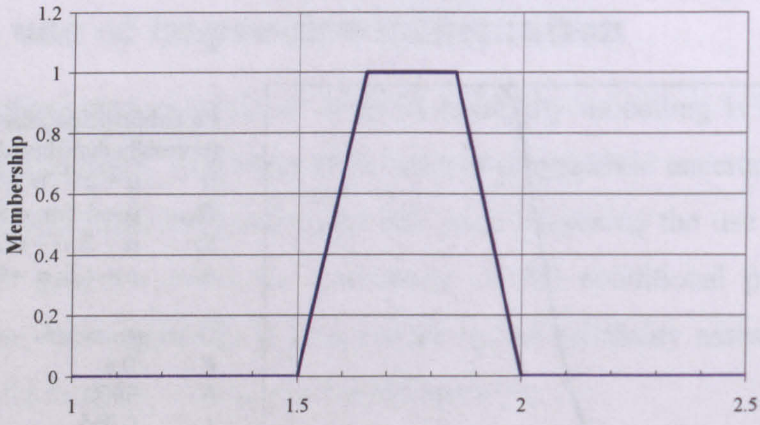

Figure 3.12. Example of a fuzzy set defining the possibility of the uncertain parameter D_{n50} (Dawson & Hall 2002a).

Dawson and Hall (2002a, 2002b) propose to incorporate the lack of precise knowledge in the reliability assessment by formally codifying the expert judgement into intervals or fuzzy sets. For example, the nominal rock armour diameter in the dike revetment may be known in term of probability distribution, like assumed for the fragility curve in Figure 3.10, for newly built structures. However, for existing structures, the precise distribution of diameters is commonly not known. Instead a small sample of measurement, or a visual inspection, can be used to estimate bounds on the rock size, for example 1.5 m $\leq D_{n50} \leq$ 2 m. This assumption on the interval of rock nominal diameters results in bounds on the conditional probability of failure. In Figure 3.11 the upper bound fragility curve is calculated with the same input values shown in Figure 3.10 but with the nominal diameter of rocks set to the lover bound of its interval D_{n50} = 1.5 m. The lower bound fragility curve in determined with D_{n50} = 2 m.

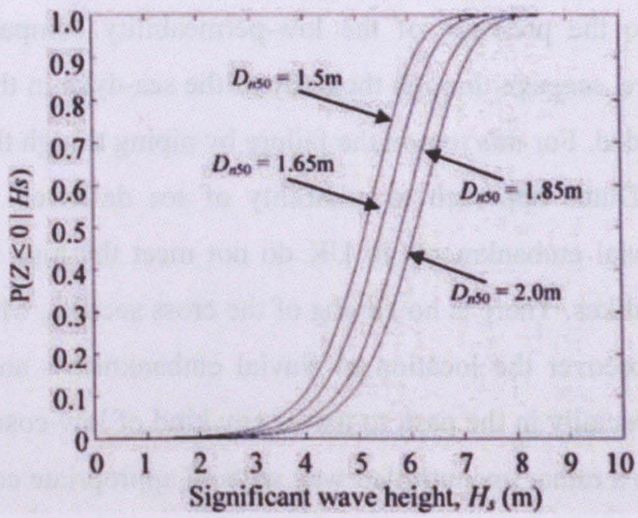

Figure 3.13. Fragility curves generated identifying the possible values of the nominal rock diameter D_{n50} with the trapezoidal fuzzy set of Figure 3.12 (Dawson & Hall 2002a).

The representation of possible probabilities of failure can be refined adopting for D_{n50} a trapezoidal fuzzy set in place of the interval. The fuzzy set can be seen as a more general interpretation of the interval where the bounds are not precise. In Figure 3.12 a membership function is assigned to the values of D_{n50}, signifying that values between 1.65 and 1.85 m are most possible; values as little as 1.5 and as great as 2 are still possible, but to a lesser extent. Four different fragility curves can be determined performing four different calculations with the deterministic values of D_{n50} corresponding to the corners of the trapezoidal fuzzy set. The middle curves in Figure 3.13 represent then the most possible fragility curves with decreasing possibility of occurrence to the outer bounds.

3.4 Conclusions

The probabilistic methods adopted in the Netherlands for the design and safety assessment of flood defence systems are a comprehensive and powerful tool. The modelling of flood defence performance accounts for several failure modes, and incorporates the length effect alongside with the presence of correlations at different levels. The Dutch practice has been the main source of inspiration for the development of the reliability techniques used for the British flood defences. Therefore it is important to recognise which aspects are different in the two cases. Importantly, due to the presence of the low-permeability compacted clay layers surrounding the core, seepage through the body of the sea-dyke in the Netherlands is almost totally impeded. For this reason the failure by piping trough the earthfill is not considered in the Dutch approach to reliability of sea defences. The design and construction of fluvial embankments in UK do not meet the high standards of the primary Dutch sea-dikes. There is no zoning of the cross section, which is made of a single material. Moreover the location of fluvial embankments and the their long length have led, especially in the past, to use of any kind of low-cost filling material, often put in place in a rather uncontrolled way without appropriate compaction (Dyer 2004). This implies that for many existing embankments, differently from the structures in the Netherlands, an excessive seepage through the earthfill, possibly resulting in breaching, is a relevant failure mode to be included in the reliability analysis.

The approach to reliability of levees developed in the USA, although less advanced than the Dutch approach in terms of probabilistic methods, is of relevance to this research for the similarity of the type of structures addressed in the study. It is important to observe that an attempt of including piping through the earthfill has been made by the American school. However this has not led to results which are sufficiently satisfactorily to define a widely accepted and applicable procedure for this specific mode of failure. As discussed later in this thesis, the difficulty in modelling through-piping is related to the role that fissures, openings and potential zones of concentrated leakage play in the process. The methodology proposed in this thesis aims at overcoming the difficulties related to this mode of failure.

Finally the use of intervals or fuzzy sets to construct families of fragility curves which incorporate imprecise information is an interesting strategy for handling the epistemic uncertainty about flood defences. It should be observed, though, that replacing a random variable with an interval, or a fuzzy set, introduces the effect of epistemic uncertainty at the expense of random variability, which for that particular variable is left out of the model. The approach proposed in this thesis intends to be a tool for handling epistemic uncertainty taking into account at the same time the aleatory variability of all the relevant parameters.

4

A RELIABILITY RATING SYSTEM FOR FLOOD EMBANKMENTS

The current approach to the reliability of flood embankments within the context of integrated flood risk assessment has some problematic aspects. The most relevant of these is the incorrect implementation of the resistance to piping through the earthfill, which is currently assessed with a criterion developed for the foundation erosion under impervious water-retaining structures (Lane 1935) that should not be used for internal erosion. To date, there are no credible mathematical models for through-piping (Richards & Reddy 2007). For this reason the probabilities of failure by this specific mechanism should be determined through the elicitation of subjective judgement. This technique is widely employed in the reliability assessment of large embankment dams (Fell *et al.* 2000). This thesis argues that the definition of judgement-based fragility curves for through-piping is a necessary step for the development of a credible methodology for the integrated modelling of flood risk. This task goes beyond what can be achieved with a single doctoral research project and will possibly require a long and committed effort by the flood risk scientific and technical community. On a shorter term, in order to support decision makers in charge of the flood defence assets management, the author has developed a simplified form of reliability quantification for flood embankments. This methodology has been named Reliability Rating System because it produces a quantitative indicator of the expected performance in extreme hydraulic conditions which is the result of a simplified and approximated reliability analysis. The system, which covers the possible scenarios that can be encountered in practice, provides a quick rating tool, helpful in dealing with the epistemic uncertainty typically associated with the performance assessment of flood embankments.

4.1 Problematic aspects of flood embankments reliability

In Chapter 2 the approach to flood embankments reliability currently adopted in the UK for the integrated modelling of flood risk has been reviewed. The methodologies in use are based on fragility curves describing the probability of failure conditional on the load. The fragility curves are obtained, in the case of fluvial defences, repeating the reliability analysis for different water levels. The calculations are performed with the methods of structural reliability, in which a limit state function discriminates between failure and non failure states and the input parameters are random variables. The methodologies currently adopted show some limitations. The main problematic aspects are related to the use of input parameters linked to the Condition Grade and to the modelling of piping. In particular:

a) Condition Grades lump together different features that affect different modes of failure; for this reason it's not possible to link correctly the actual characteristics of the embankments to the probabilities of failure by different mechanisms.
b) The reliability assessment is based only on visual features: factors like the type and origin of the soil in the earthfill, its compaction, and the geology underling the embankment, which are crucial to the performance of the structure, are neglected.
c) The necessity to include epistemic uncertainty is recognised and a strategy for doing so is proposed with the adoption of bounds of probability. However a clear definition of these bounds is missing and no illustration of the reductions of these bounds when moving to more detailed levels is given.
d) The modelling of piping through the earthfill is not satisfactory: a formula for piping under impervious structures is employed, stretching its use far outside its field of applicability. As a result through-piping is not correctly modelled and under-piping is not addressed by the methodology.

Point (a) could be improved by recording separately the characteristics of the embankment and linking them to the reliability analysis, rather than aggregating all contributions into the Condition Grade. The need for a more articulated condition assessment has already been recognised by Long *et al.* (2007). The next step would be constructing a set of fragility curves linked to the relevant characteristics rather than to Condition Grades. This would represent a more complex, but more accurate model of the structural performance of flood embankments.

Points (b) and (c) are related, in the sense that the characteristics that are not immediately recognisable in a visual inspection are responsible for a significant part of the epistemic uncertainty typically encountered. An effective and comprehensive approach to the reliability of flood embankments needs to incorporate the effect of the features that are not visually recognisable. A system which quantifies the impact of the characteristics and conditions of different structures on the reliability of flood embankments would be a valuable tool for supporting decision-making under conditions of limited knowledge.

The solution to point (d) is complex: while the inclusion of the under-piping mode of failure is straightforward and can be done using Lane's formula (1935) in its correct area of applicability, the determination of probabilities of failure by piping trough the earthfill, instead, is quite challenging.

4.2 Piping through the embankment

4.2.1 Different forms of piping

The erosion of soil in the foundation of the embankment, or under-piping, and the piping through the earthfill, or through-piping, are two distinct physical processes (Richards & Reddy 2007). Under-piping is associated with backward erosion, while through–piping is mainly related to internal erosion

Backward erosion is the removal of particles by forces due to inter-granular seeping water in a macroscopically continuous soil. The name given to this phenomenon is due to its characteristic evolution: erosion starts at an exit point, where the soil, not confined, can be more easily removed, and then evolves retrogressively. Empirical formulas for safety against backward erosion in foundations of water retaining structures have been proposed by Bligh (1910) and Lane (1935). A mathematical model of the same process has been developed by Sellmeijer and his co-workers for the specific case of sand foundations (Weijers & Sellmeijer 1993).

Internal erosion is also the removal of particles due to water seepage but, differently from backward erosion, it is related to the flow along pre existing openings like cracks in cohesive materials or voids at a soil structure interface.

Not much information is available in the literature about piping through the fill material of flood embankments. Some insight on this phenomenon can be gained from studies relating to embankment dam engineering, where investigation on piping has been undertaken for several decades. The research on earth dams, including statistical analyses of failure databases (Foster *et al.* 2000a, Richards & Reddy 2007), shows that, even if more mechanisms, including backward erosion, are contributing, internal erosion across the embankment body is responsible for most piping failures.

4.2.2 Lack of a mathematical model for internal erosion

In the geotechnical literature piping phenomena have been investigated mainly in relation to large embankment dams. A commonly adopted measure against piping through embankment dams, which is not adopted in fluvial embankments, is the construction of a granular filter downstream of the fine grained core with the aim of simultaneously acting as a drain and trapping fine particles dislodged from the core inducing the sealing of any concentrated leak (self-healing). For this reason a lot of research has gone into the behaviour of core-filter pairs (Tomlison & Vaid 2000, Reddy & Kakuturu, 2006a, 2006b), rather than in the behaviour of structures with non-zoned sections.

A theoretical formulation (Zaslavsy & Kassif 1965) and a mathematical model (Kilar *et al.* 1985) are available for piping through fine-grained soils but both refers to macroscopically homogeneous and intact materials, rather than to erosion along cracks or concentrated seepage zones.

In relation to internal erosion some authors have studied the removal of grains due to water flow in a planar opening (Louis 1969, Worman and Olafsdottir 1992, Franco and Bagtzoglou 2002). However, at the time of writing, as confirmed by Richards and Reddy (2007: 398), no mathematical model has been developed with the capacity of capturing the genesis of heterogeneities and anomalies, like cracks and loose zones of concentrated seepage, and their influence on the resistance to internal erosion.

The lack of a credible mathematical model for piping through the earthfill implies that it is impossible to define a limit state with an analytical function or a numerical procedure. For this reason no credible reliability analysis can be performed with the traditional methods of structural reliability.

4.2.3 Subjective probabilities

The difficulties in modelling through-piping are well known in the field of embankment dam engineering, where the determination of the probability of failure by piping is essential for the quantitative assessment of risk. In earth dam engineering the probability of failure by through-piping is estimated with two approaches (Fell *et al.* 2000): the historical performance approach, based on a large amount of data about failures versus successful performances, and the event tree approach, based on the decomposition of possible failure sequences in simpler events and on the use of subjective probabilities. In the case of flood embankments, differently from large embankment dams, there is clearly not enough data on the past performance to estimate the frequency of piping failure as a function of the water level.

The elicitation of subjective probabilities appears to be the only strategy to quantify the reliability of flood embankments. Subjective probabilities, in combination with the event tree technique, have been extensively employed in the reliability assessment of embankment dams by owners of large portfolios of water-retaining structures, like the U.S. Bureau of Reclamation, the British Columbia Hydro and Power Authority and others (Beacher & Christian 2003). In the event tree approach the failure is decomposed in chains of simpler events. The component events are organised in a graphical representation which starts with an initiating event and then branches repeatedly, identifying some sequences leading to failure and some others leading to a safe state. An example event tree for piping failure is pictured in Figure 4.1 (Fell *et al.* 2004). A conditional probability needs to be associated to each event, given the preceding events in the sequence, in order to determine the probability of breaching. The probability of a chain leading to failure is then calculated multiplying the probabilities of the component events along the sequence. Summing the probabilities of all failure sequences gives the probability of failure for the specified initiating event. Summing the probability of failure over all initiating events gives the total probability of failure of the system. Published examples of reliability assessment of embankment dams undertaken with the event tree approach include the Coursier Dam study by BC Hydro (1995), the study of Prospect Dam (Landon-Jones *et al.*, 1996) and the study of three dams in Norway (Johansen *et al.* 1997).

The probabilities of the component events along failure chains are assessed, in absence of a suitable mathematical model, via expert judgement elicitation. For credible results the process of subjective probability elicitation has to be structured according to a precise procedure (Vick 1999 and 2002, Beacher & Christian 2003, USACE 2006 - Appendix E). The estimate of subjective probabilities through expert judgment is used in various fields within and outside the realm of engineering (Cooke 1991) and indications are available, from studies in behavioural psychology, about how, and how well, people are able to quantify their opinions in the form of probabilities (Beacher & Christian 2003). Techniques, like the association with descriptive statements (Vick 1997, Lichtenstein & Newman 1967) or the so called "action approach to elicitation", are available for facilitating assessment. These supports will be discussed later in this thesis, when the concepts introduced here will be applied to flood embankments.

4.2.4 Need for judgment-based fragility curves for through-piping

In the flood risk methodologies which are under development in the UK the limit state function currently used for the piping through the earthfill mode of failure is taken from a criterion developed for foundation erosion and cannot correctly model the process of piping through the embankment. No other credible mathematical model of piping phenomena in water-retaining structures is available to date. For this reason the definition of judgement-based fragility curves for piping through flood embankments appears to be a necessary step.

The methods of structural reliability, recently introduced in the British practice, have been intended as a step forward from the judgement based fragility curves used in the national scale methodology originally developed by Hall *et al.* (2003). Those curves did not differentiate among different modes of failure and expressed the probability of failure conditional on a multiplier of the Standard of Protection. In order to be truly beneficial to the risk assessment the structural reliability approach needs to be accompanied by a credible estimate of the probabilities of failure by piping through the earthfill. This can only be done by judgement elicitation aimed at the construction of fragility curves for this specific failure mode only, expressing the probability of failure conditional on the water level.

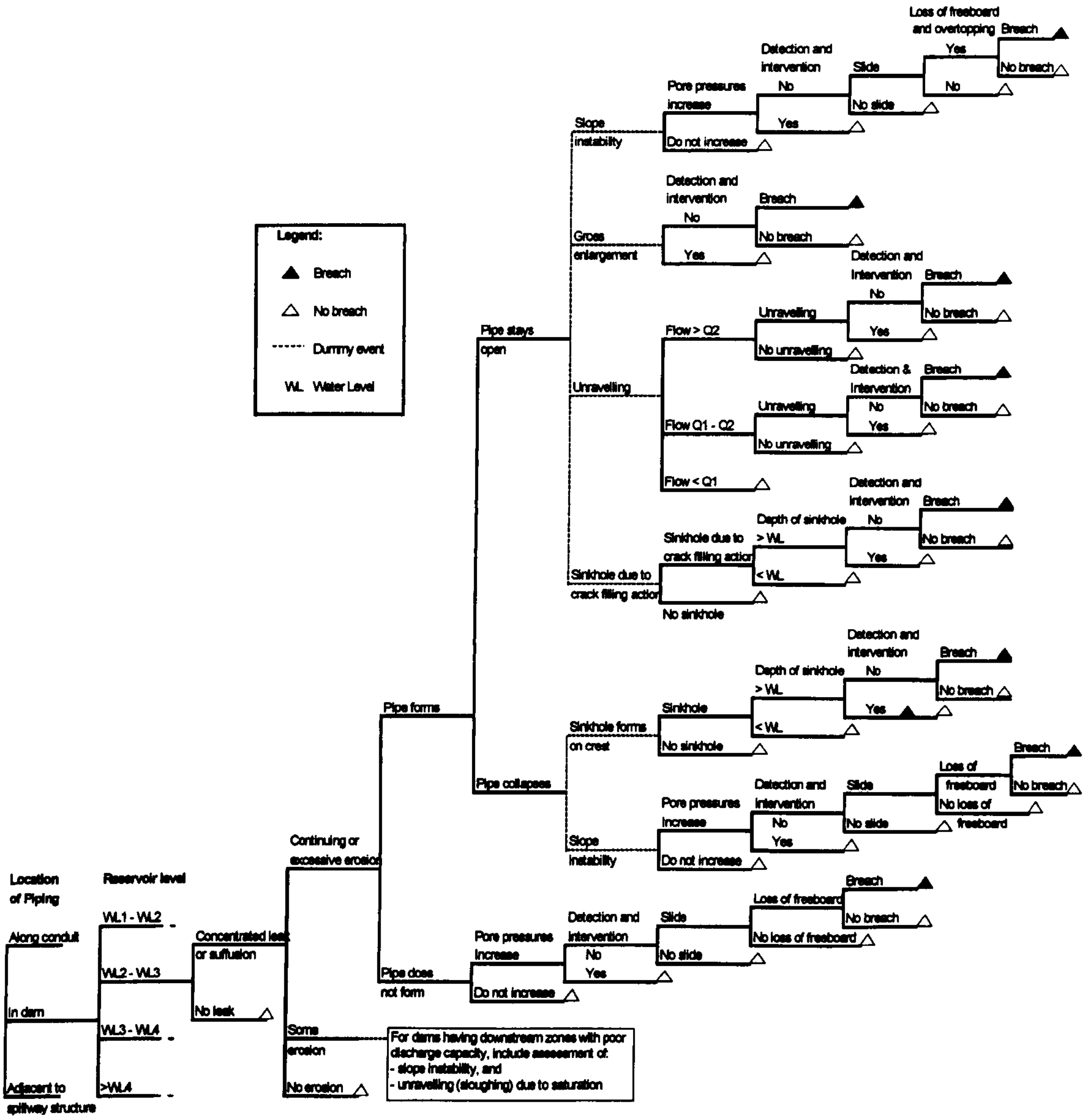

Figure 4.1. Typical event tree, referred to operating and flood conditions, for embankment dams (source: Fell *et al.* 2004). The dotted branches in the figure connect "dummy events", i.e. events that are not mutually exclusive. The use of dummy events allows for a more compact graphical representation. As an alternative different event trees could be drawn each including one of the dummy events.

4.2.5 Complexity and extension of the task

The accomplishment of the task outlined above is unfortunately neither simple nor straightforward. Different curves should be developed for combinations of the relevant characteristics which can influence the resistance to piping. For example the soil type used for the fill, its compaction, and the presence of local weaknesses should be reflected in the development of the set of fragility curves.

In fact a panel of experts should be arranged encompassing several disciplines like geotechnical engineering, river hydraulics, safety and reliability of engineering systems. It would also require the involvement of experts from both academia and industry. Moreover the composition of the panel should also recognise and optimise the mixture of different roles the individuals can play in the elicitation process (Beacher & Christian 2003). Some individuals for example are versed in facilitating or evaluating the elicitation process. Others are specialists, with a deep knowledge in very specific issues. A process of subjective probabilities elicitation should then be rigorously structured in five phases (Beacher & Christian 2003):

1. Motivating phase, developing the rapport among the experts and clarifying the aims of the process.
2. Training phase, which highlights the biases potentially affecting the assessment in order to avoid or mitigate them.
3. Structuring phase, in which the problem is analysed and decomposed to an appropriate level of detail.
4. Assessing phase, consisting in the quantification of probabilities by individuals and their subsequent discussion within the panel.
5. Documenting phase, which records, for verification and credibility, how the conclusions have been reached.

The structural failure of a flood embankment can be studied with simpler event trees than those for earth dams (Figure 4.1). In fact the homogeneous earthfill section does not require a check for erosion continuation because in absence of a filter internal erosion certainly progress. Moreover the chance of detection of initiating breaching and repair are negligible, at least as a first approximation. Considering these aspects and lumping together some detailed branches the event tree proposed in Figure 4.2 is obtained. For a fixed water level the probability of failure by through piping can be calculated if a subjective probability is assigned to each event along the branches leading to failure. The analysis should be repeated for several values of the water level in order to construct a judgement based fragility function. A number of cases need to be evaluated accounting for different types of filling material, different compaction, different local weaknesses, etc.

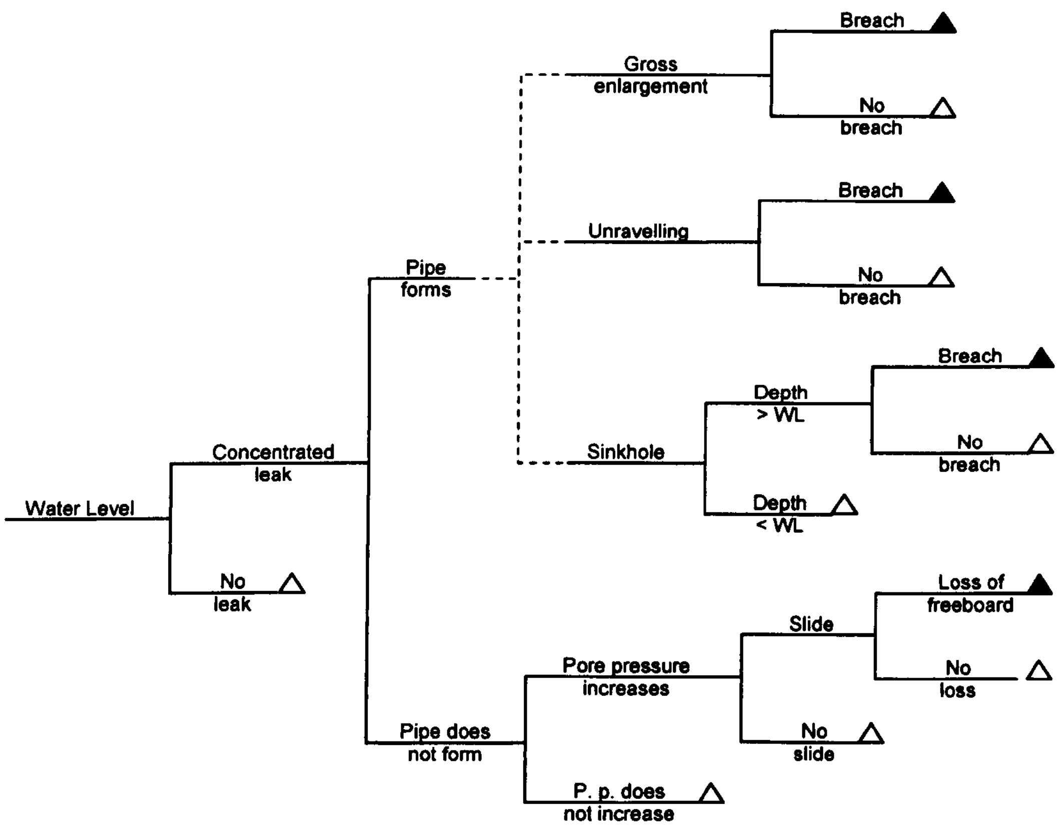

Figure 4.2. Proposed event tree for the failure by piping through the earthfill of flood defence embankments. The dotted branches connect "dummy events", i.e. events that are not mutually exclusive.

The extension of the task and the required effort in terms of number of people involved in the structured elicitation procedure, their working hours and the timescale of the whole process is far beyond what can be achieved by a single doctoral research. This thesis argues that, on a shorter timescale, it is urgent to define a simpler procedure for correctly incorporating the factors influencing the likelihood of piping through the earthfill in the reliability assessment of flood embankments.

For this reason a new methodology, called the Reliability Rating System, is presented in this thesis. This method ranks the embankments according to a performance indicator which expresses an estimate of their probability of breaching associated with only a limited range of loads, above and immediately below the crest. Taking into account only a limited number of water levels reduces the task of evaluating the subjective probabilities to a tractable level of complexity. The formulation of this approach, and its limitations, are discussed in the following.

4.3 The Reliability Rating System

4.3.1 The concept of Reliability Rating

The determination of the annual probability of structural failure of a fluvial flood defence requires the knowledge of the estimated frequency of the load $p(l)$, in this case the water level, and a probabilistic description of the structural response of the flood defence, i.e. a fragility curve giving the probability of breaching conditional on the load $P(B|l)$. If both these elements are known the total probability of failure on annual basis can be determined integrating the conditional probability of failure over all the possible loads:

$$P(B) = \int_0^{\infty} p(l)P(B|l)dl \tag{4.1}$$

The conditional probability of breaching is given by the contribution of different modes of failure. Assuming independent modes of failure for instance, like in the current British and American practice, for every value of the load l:

$$P(B|l) = 1 - \prod_i [1 - P(B_i|l)] \tag{4.2}$$

where $P(B_i|l)$ is the probability of breaching by the i-th mode of failure, conditional on the load. In the case of flood embankments piping through the earthfill is not amenable to mathematical modelling and the related fragility curve cannot be obtained with the traditional methods of reliability.

In order to provide flood defence managers with a supporting tool for decision making this doctoral research has developed a simplified and approximated system for quantifying the expected performance of flood embankments in extreme hydraulic conditions. This approach, rather than performing a complete reliability analysis which considers all possible loads, determines the probability of breaching only for a limited range of water levels in proximity of the crest. This quantity is used as an indicative measure of the earthen structure reliability that is then used to compare embankments and rank them from the most to the less prone to failure.

4.3.2 Formulation

The Reliability Rating System for flood embankments is based on a performance indicator which is related to a simplified and approximated reliability analysis. The performance indicator is called Probability of Breaching in Extreme hydraulic conditions P_{BE} and is defined by the following formula:

$$P_{BE} = AEP_{crest}\,[P(B)_{crest} + P(B)_{ot}] \qquad (4.3)$$

where AEP_{crest} is the annual exceedance probability of the embankment's crest level; $P(B)_{crest}$ is the probability of breaching with water level at the crest; $P(B)_{ot}$ is the probability of breaching in overtopping conditions. The probability of the crest level being exceeded in one year AEP_{crest} corresponds to the probability of the defence being overtopped in the same time span. In the British practice the Standard of Protection of fluvial defences is defined as the flood return period above which the defence level is exceeded (Greenyer & Pinnel 2007). This implies that the inverse of the Standard of Protection SOP can be taken as an approximation of the value of the annual exceedance probability of the crest level[4] AEP_{crest} :

$$AEP_{crest} \cong 1/SOP \qquad (4.4)$$

The probability of breaching with water at the crest level $P(B)_{crest}$ is calculated considering two failure modes, namely under-piping and through-piping, which are treated as fully independent. This leads to the equation

$$P(B)_{crest} = 1 - [1 - P(B_{up})_{crest}][1 - P(B_{tp})_{crest}] \qquad (4.5)$$

where $P(B_{up})_{crest}$ is the probability of breaching by under-piping with water at the crest level and $(B_{tp})_{crest}$ is the probability of breaching by through-piping in the same

[4] The equivalence is not perfect because flood defences are designed with some freeboard allowance, established locally. For the present purpose the approximation is considered acceptable.

loading condition. The probability of breaching in overtopping conditions is calculated considering, beside the two already introduced failure modes, the breaching by surface erosion, according to the following equation:

$$P(B)_{ot} = 1 - [1 - P(B_{up})_{crest}][1 - P(B_{tp})_{crest}]\,[1 - P(B_{se})_{ot}] \tag{4.6}$$

where $P(B_{se})_{ot}$ is the probability of breaching by surface erosion. While the probability of failure by under-piping and through-piping are referred again to water at the crest level, the probability of failure by surface erosion is calculated in correspondence of a water level h_{ot} , taken as representative of overtopping states. The reference water level is defined by the condition

$$AEP_{ot} = 0.5\, AEP_{crest} \tag{4.7}$$

where AEP_{ot} is the annual exceedance probability of the representative water level for overtopping h_{ot}.

In summary to calculate the probability of structural failure in extreme hydraulic conditions through equations (4.3), (4.5) and (4.6) it is necessary to determine:

- the annual exceedance probability of the crest level: AEP_{crest};
- the probability of failure by under-piping and through-piping with water at the crest level: $P(B_{up})_{crest}$, $P(B_{tp})_{crest}$;
- the probability of failure by grass erosion for a reference water level above the crest: $P(B_{se})_{ot}$

The formulation of the Probability of Breaching in Extreme hydraulic conditions P_{BE} on which the Reliability Rating System is based, includes, therefore, a synthetical measure of the load frequency, AEP_{crest}, and the probabilities of breaching by three modes of failure - under-piping, through-piping, surface erosion - calculated in only two loading conditions: water level at the crest for the piping phenomena and a reference level above the crest for grass erosion (Figure 4.3).

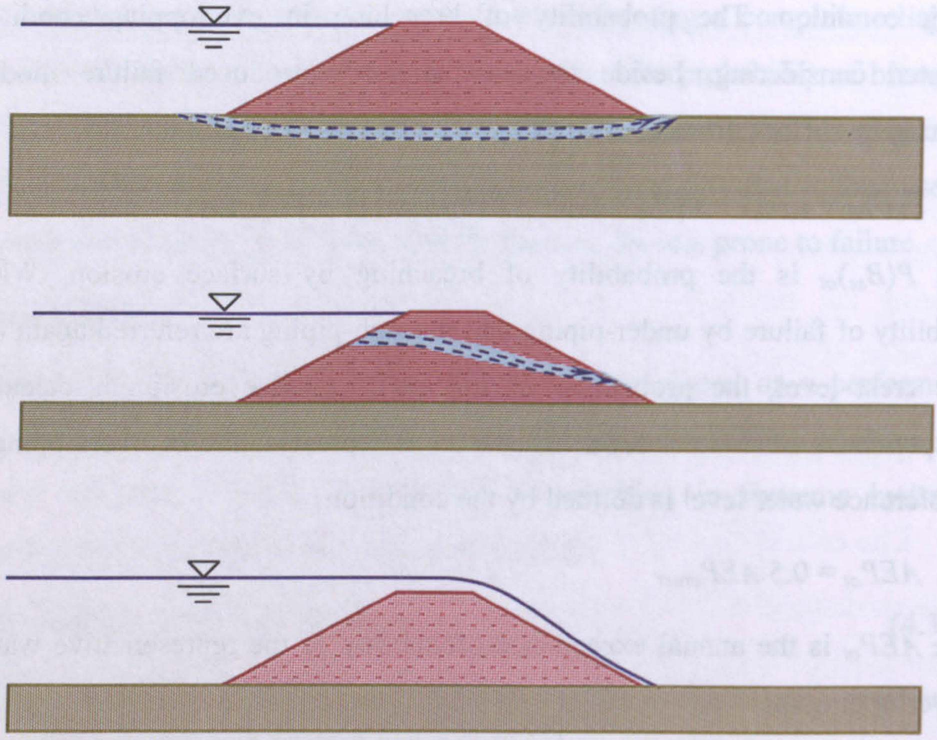

Figure 4.3. Schematic representation of the three failure modes considered in the determination of P_{BE}. The probabilities of failure by under-piping (a) and through-piping (b) are calculated with the water at the crest level. The probability of failure by surface erosion (c) is calculated for a reference water level above the crest.

4.3.3 Meaning of the Probability of Breaching in Extreme hydraulic conditions P_{BE}

The Reliability Rating System has been conceived so to require the estimate of the probability of breaching by piping through the earthfill only in one loading condition: with the water reaching the crest level. In this way the estimation of subjective probabilities of breaching by through-piping is reduced to a manageable level of complexity. In the following it is shown how the Probability of Breaching in Extreme hydraulic conditions P_{BE}, calculated with Equations (4.3), (4.5) and (4.6) is an approximation of the annual probability of failure associated only with the water levels above or in proximity of the crest.

In a complete reliability analysis the annual probability of breaching of a structure $P(B)$ is obtained integrating the conditional probability of structural failure over the load as shown in Equation 4.1. For fluvial defences, where the load is represented by the water level h, one can write:

$$P(B) = \int p(h) P(B|h) dh \tag{4.8}$$

In many practical cases the integral is approximated with a summation over intervals of water level Δh_i :

$$P(B) \cong \sum_i P(h_{low,i} < H \le h_{up,i}) P(B | h_{low,i} < H \le h_{up,i_i}) \tag{4.9}$$

where $P(h_{low,i} < H \le h_{up,i})$ is the probability of the water level being within the i-th interval, defined by its lower endpoint $h_{low,i}$ and upper endpoint $h_{up,i}$, while $P(B|h_{low,i} < H \le h_{up,i_i})$ is the conditional probability of breaching given the water level is in the same i-th interval. The conditional probability $P(B|h_{low,i} < H \le h_{up,i_i})$ can be conveniently taken as the probability of breaching for a water level h_i central to the interval. Equation (4.9) then becomes:

$$P(B) \cong \sum_i P(h_{low,i} < H \le h_{up,i}) P(B|h_i) \tag{4.10}$$

The frequency of water levels can be described by a cumulative distribution function $F_H(h)$, like the one pictured in Figure 4.4. The cumulative distribution function, or CDF, gives the annual non-exceedance probability $P(H \le h)$, i.e. the probability that the random variable water level H does not exceed each value h. The annual exceedance probability of the crest level h_{crest} is the probability of the defence experiencing overtopping in one year. As exceedance and non-exceedance are mutually exclusive and collectively exhaustive events it can be written:

$$AEP_{crest} = P(H > h_{crest}) = 1 - P(H \le h_{crest}) = 1 - F_H(h_{crest}) \tag{4.11}$$

In Figure 4.4 the annual exceedance probability of the crest level is the vertical distance between the asymptotic value of 1 and the point on the CDF identified by h_{crest}. This interval includes all the loading states resulting in overtopping.

The representative water level for overtopping level h_{ot} has been defined in Equation (4.7) as having an annual exceedance probability which is half of that of the crest level. This implies that the representative point for overtopping is located, along the vertical direction, at midway between 1 and h_{crest}. It also means that, in case of overtopping there is the same chance in one year of having water levels below the representative overtopping level ($h_{crest} < H \leq h_{ot}$) or above it ($H > h_{ot}$).

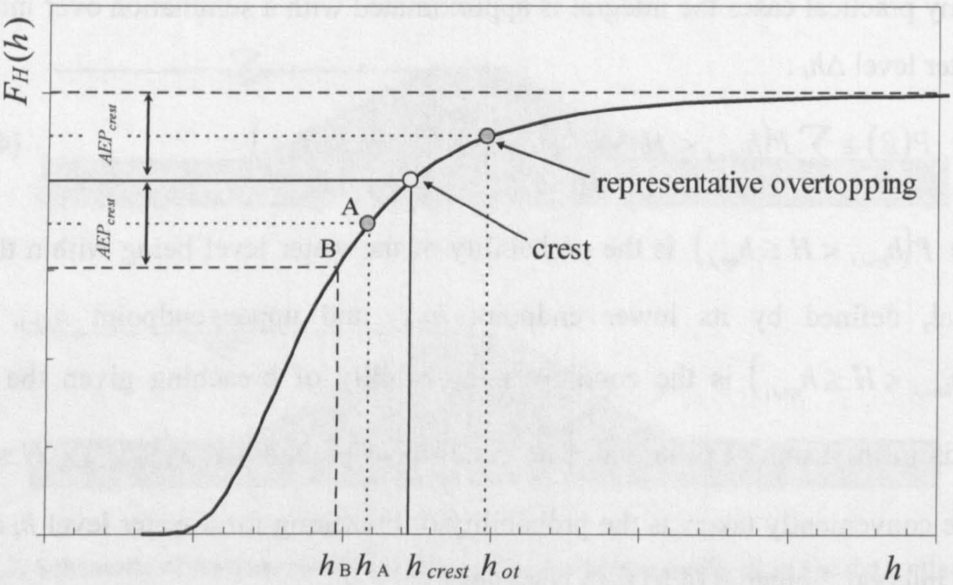

Figure 4.4. Cumulative distribution function of water level and discretization used in the derivation of P_{BE}.

Let now a second interval of the same amplitude AEP_{crest} be considered along the frequency axis. In this way an upper portion of the range of loads is subdivided in two intervals. The first interval contains all water levels above the crest, while the second interval contains some water levels immediately below the crest. The crest level h_{crest} separates the first interval from the second. The annual probability of breaching associated with these two intervals, calculated neglecting the remaining range of loads is:

$$P(B^*) = P(h_{crest} < H < +\infty)P(B|h_{ot}) + P(h_B < H \leq h_{crest})P(B|h_A) \qquad (4.12)$$

where h_B is the lower endpoint of the second interval, h_A is the point in the middle of the second probability interval and the asterisk in $P(B^*)$ reminds that it is not the total probability of breaching $P(B)$ which is being calculated.

Because of the way these intervals have been constructed the probability of the water level being in either of them equals the annual exceedance probability of the crest level AEP_{crest}. :

$$P(h_{crest} < H < +\infty) = AEP_{crest}$$

$$P(h_B < H \leq h_{crest}) = AEP_{crest}$$

The probability of structural failure can then be written as

$$P(B^*) = AEP_{crest}\,[P(B|h_{ot}) + P(B|h_A)] \qquad (4.13)$$

For water levels below the crest the embankment can fail by under-piping or through-piping; assuming the independence of these failure modes the probability of failure given h_A is

$$P(B|h_A) = 1 - [1 - P(B_{up}|h_A)][1 - P(B_{tp}|h_A)] \qquad (4.14)$$

For water levels above the crest, failure can also be due to surface erosion; assuming again independence among modes of failure the probability of failure given h_{ot} is

$$P(B|h_{ot}) = 1 - [1 - P(B_{up}|h_{ot})][1 - P(B_{tp}|h_{ot})][1 - P(B_{se}|h_{ot})] \qquad (4.15)$$

In order to reduce the load cases to be assessed, the probabilities of failure for under-piping and through-piping, instead of being evaluated twice in h_A and h_{ot}, are evaluated in h_{crest}. This will result in some over estimation of $P(B|h_A)$ and some under estimation of $P(B|h_{ot})$. Accepting this simplifying hypothesis, and renaming $P(B^*)$ as P_{BE}, equations (4.3), (4.5) and (4.6) are retrieved.

Summarising, the Probability of Breaching in Extreme conditions P_{BE} is calculated:

- Considering only water levels h with annual exceedance probability AEP_h lower than twice the probability of overtopping AEP_{crest}

  $$AEP_h < 2\,AEP_{crest} \cong 2/\mathrm{SOP} \qquad (4.16)$$

  Equivalently the water levels included in the analysis can be expressed in terms of cumulative distribution function:

  $$F_H(h) > 1 - 2\,AEP_{crest} \cong 1 - 2/\mathrm{SOP} \qquad (4.17)$$

- Including three failure modes, under-piping, through-piping and surface erosion, assumed to be independent.
- Approximating the probabilities of failure by under-piping and through-piping, assessed for the level intervals immediately above and below the crest, with the same probabilities calculated with the water being exactly at the crest level.

It is worth remembering that the assumption of failure modes independence gives an upper bound approximation on the probability of breaching for the sub-set of loads considered. The indicator P_{BE} is used as a rating for comparison among different embankments in an inventory; there is no attempt in the reliability rating system to quantify the probability of breaching of an entire defence scheme at any scale. For this reason correlation among different embankments is not considered. An individual embankment can be identified with the same procedure in use for the national and regional-scale flood risk methodologies for integrated flood risk assessment, i.e. splitting sections longer than 600 m. In addition to that, ideally, subdivisions should be introduced to reflect changes in the underlying geology.

4.3.4 Use and limitations

The formulation of P_{BE} derives from the same approach that would be used for a complete reliability performed approximating the integral in Equation (4.8) with a summation. However the proposed formulation restricts the analysis to only a portion of the load range and introduces further approximations and simplifications. As a result ranking an inventory of flood embankments according to the calculated value of P_{BE} does not strictly reflect a hypothetical ranking based on the annual probability of breaching. The reason for not using the annual probability of breaching $P(B)$ is that it has not been possible to produce a system able to cover the variety of scenarios existing in practice regarding the resistance to through-piping. However it is important to recognise that the use of P_{BE} as indicative measure of proneness to breaching, in place of $P(B)$, introduces biases in the rating. In particular it can be observed that a large part of the load range is disregarded and it cannot be granted that the contribution of this part would be negligible. For some embankments the contribution of low water levels can be more significant than for other. The use of

P_{BE} would than tend to assign to these structures a better performance-based ranking than $P(B)$.

Moreover the part of load range which is disregarded is not uniform throughout the inventory; it rather depends on the Standard of Protection. In fact P_{BE} for structures with a high Standard of Protection is calculated for a load range which is narrower than the range adopted for structures with a lower Standard of Protection. This bias is partly compensated by the fact the structures with a higher *SOP* are generally better designed and constructed. It is known that this translates in lower probabilities of failure (Beacher & Christian 2003). This aspect, which is not explicitly quantified in any part of the methodology, introduces a bias whose effect is opposite to the one mentioned above.

Finally it can be observed that disregarding the water levels far below the crest P_{BE} neglects part of the contribution to the total probability of breaching by under-piping and through-piping, while all the water level contributing to grass erosion are taken into account, even if in doing so a rough discretization is used. For this reason the reliability rating system tends to overemphasise, among the different modes of failure, the relative importance of surface erosion.

4.4 Developing the Rating System for the final user

4.4.1 General strategy

The Reliability Rating System is conceived to provide the end user with a tool for a quick quantification of reliability. To achieve this it is important to construct a methodology able to cover, even if in an approximated way, a wide range of scenarios possibly encountered in practice. This has been done with the following procedure:

- The basic characteristics influencing embankments performance in flooding conditions have been identified. Considering the level of information typically available, this operation has been constrained by the need to include only characteristics which can be known before any detailed site investigation.
- A realistic range of variation has been identified for each of these characteristics; the ranges have then been subdivided in quantitative or qualitative classes.

- The probabilities of failures involved in Equations (4.5) and (4.6) have been determined for each relevant combination of classes. This has been done making use of classical reliability methods for under-piping and surface erosion, and of quantified judgement elicitation for through piping.

Table 4.1. Basic characteristics affecting flood embankments' performance, subdivision in classes for the development of the Reliability Rating System and influence on failure modes.

<table>
<tr><th>BASIC CHARACTERISTIC</th><th colspan="60">CLASSES</th><th>FAILURE MODES</th></tr>
<tr><td colspan="62">GEOMETRY</td></tr>
<tr><td>Height (m)</td><td colspan="12">[0.5, 1.5)</td><td colspan="12">[1.5, 2.5)</td><td colspan="12">[2.5, 3.5)</td><td colspan="12">[3.5, 4.5)</td><td colspan="12">[4.5, 5.5)</td><td>TP, UP</td></tr>
<tr><td>Slope</td><td colspan="20">[2:3, +∞)</td><td colspan="20">[1:3, 2:3)</td><td colspan="20">[1:6, 1:3)</td><td>SE, TP, UP</td></tr>
<tr><td colspan="62">MATERIALS</td></tr>
<tr><td>Earthfill soil type *</td><td colspan="10">SW/SP</td><td colspan="10">ML</td><td colspan="10">SC/SM</td><td colspan="10">MH</td><td colspan="10">CL</td><td colspan="10">CH</td><td>TP</td></tr>
<tr><td>Earthfill geologic origin</td><td colspan="15">Alluvial</td><td colspan="15">Aeolian, Colluvial</td><td colspan="15">Residual, Lacustrine, Marine, Volcanic</td><td colspan="15">Glacial</td><td>TP</td></tr>
<tr><td>Compaction</td><td colspan="20">No</td><td colspan="20">Some</td><td colspan="20">Good</td><td>TP</td></tr>
<tr><td>Foundation soil</td><td colspan="60">Additional Table **</td><td>UP</td></tr>
<tr><td>Grass cover condition</td><td colspan="20">Poor</td><td colspan="20">Normal</td><td colspan="20">Good</td><td>SE</td></tr>
<tr><td colspan="62">DEFECTS / LOCAL ANOMALIES</td></tr>
<tr><td>Vegetation - roots effect</td><td colspan="20">Trees</td><td colspan="20">Bushes</td><td colspan="20">Grass</td><td>TP</td></tr>
<tr><td>Animal burrowing</td><td colspan="20">Yes, likely to completely cross the earthfill</td><td colspan="20">Yes, unlikely to completely cross the earthfill</td><td colspan="20">No</td><td>TP</td></tr>
<tr><td>Culvert through the embankment</td><td colspan="15">Many poor details</td><td colspan="15">Few poor details</td><td colspan="15">Optimal condition</td><td colspan="15">None</td><td>TP</td></tr>
<tr><td>Differential settlement</td><td colspan="20">Visible distortion and cracking</td><td colspan="20">Visible Distortion</td><td colspan="20">No visible Distortion</td><td>TP</td></tr>
<tr><td>Desiccation cracking</td><td colspan="20">Major cracks (interconnected)</td><td colspan="20">Minor cracks (disjoint)</td><td colspan="20">None</td><td>TP</td></tr>
<tr><td>Evidence of leakage</td><td colspan="15">Muddy Leakage</td><td colspan="15">Clear Leakage</td><td colspan="15">Pooling Water</td><td colspan="15">None</td><td>TP</td></tr>
<tr><td colspan="62">HYDROLOGY</td></tr>
<tr><td>Depth of the cross section *** (m)</td><td colspan="12">[1, 3)</td><td colspan="12">[3, 5)</td><td colspan="12">[5, 7)</td><td colspan="12">[7, 9)</td><td colspan="12">[9, 11)</td><td>SE</td></tr>
<tr><td>Time to peak of Instantaneous Unit Hydrograph $T_{p(0)}$ [hours]</td><td colspan="10">[0, 1.5)</td><td colspan="10">[1.5, 3)</td><td colspan="10">[3, 6)</td><td colspan="10">[6, 12)</td><td colspan="10">[12,24)</td><td colspan="10">[24,48)</td><td>SE</td></tr>
</table>

(*) Soil identification according to the Unified Soil Classification System (ASTM 1985)

Additional sub-table

(**) Foundation material: an additional table is required to account for different soil types and layered stratigraphy.

Failure modes
SE: Surface erosion
TP: Through-Piping
UP: Under-piping

(***) *Depth of the cross section*: vertical distance between the crest of the embankment and the lowest point of the river cross section.

The final users have just to locate the properties of the embankment in the appropriate class to obtain the values of the probabilities of failure required in Equations (4.5) and (4.6). Knowing the Standard of Protection they can then estimate the annual exceedance probability of the crest level AEP_{crest}. Finally the Probability of Breaching in Extreme conditions P_{BE} is determined from Equation 4.3.

The basic characteristics needed for the rating, the subdivision in classes chosen for developing the system, and the influence on failure modes, are reported in Table 4.1Table 4.. The usual notation for intervals is used, with round parentheses and square parentheses indicating exclusion or inclusion of the extremes respectively. The subdivision is meant to cover the situations possibly encountered in the UK. The basic characteristics can be grouped as geometric measures, characteristics of the manmade embankment and of the natural soil in the foundation, defect and anomalies. An additional table, which will be presented and discussed in Chapter 6, is required for the foundation soil, in order to deal with the presence of layers of different soils. A hydrologic parameter is also included aside the Standard of Protection: the time to peak of the Instant Unit Hydrograph influences the duration of overtopping and this information is needed to calculate the probability of breaching by surface erosion.

4.4.2 Reliability analyses

For two of the considered failure modes, namely foundation erosion (under-piping) and grass cover erosion, credible mathematical models are available and the probability of failure has been determined with traditional reliability analyses. In this approach the performance of the system is described by a limit state function and the input parameters are random variables. For each of the relevant classes included in Table 4. the mean, the standard deviation and the probability distribution of the input random variables have been established through literature review and judgement. The reliability calculations have been performed with the First Order Reliability Method (FORM), in the spreadsheet implementation proposed by Low and Tang (1977), and with Monte Carlo simulation (see for example Rubinstein 1981). These methods are discussed in more detail in Chapter 5, while the reliability analyses for the two modes of failure are described in Chapter 6, 7 and 8.

4.4.3 Subjective judgement elicitation

The probabilities of failure by piping through the embankment have been assessed with a subjective judgement approach. The related process of structured elicitation has been undertaken by a small panel which includes the author, Prof. M. Dyer from the Trinity College (Dublin) and Dr. S. Utili from the University of Oxford. The process has been conducted according to the recommendations by Beacher and Christian (2003), Vick (1999) and USACE (2006, Appendix E).

The development of the reliability rating system requires the evaluation of the influence of different factors on the probability of failure by piping through the earthfill. This has been done adopting, instead of the event tree technique, an innovative approach that makes use of a formulation originally developed for an historical performance method for embankment dams, known as the University of New South Wales Method (Foster *et al.* 2000b), adapting it to engineering judgement elicitation.

4.4.4 The role of engineering judgement

In estimating the reliability of flood defences a good deal of engineering judgement is involved. In the introductory chapter Figure 1.1 anticipated how different forms of judgement enter the development of the Reliability Rating System. The role of opinions quantification is very explicit in the case of the subjective probability elicitation. Nevertheless also in the reliability analyses, which are based on mathematical models, a judgemental component is introduced in the choice of the appropriate input values linked to the pertinent classes in Table 4.1. The pictured chart shows how, thanks to the effort of the researchers involved in the system development, a ready-to-use tool is available to the final users. The assumptions involving engineering judgement are made explicit and discussed throughout this thesis.

4.5 Handling epistemic uncertainty

4.5.1 Dealing with uncertain characteristics

If all the basic characteristics of the embankment listed in Table 4.1 are known the final user can directly determine P_{BE}; however, in most practical situations, some characteristics will be uncertain. In this case assumptions need to be made on the basis of partial knowledge of the local situation. When doing so it is important to quantify the uncertainty introduced in the prediction. The Reliability Rating System can be used to quantify the impact of this form of lack of knowledge on the expected performance. In fact, as the elements for calculating P_{BE} have been determined beforehand for all combinations of parameters listed in Table 4.1, the final users can quantify the variation in performance indicator for different assumptions on the uncertain parameters.

It is rather common for example to have no detailed information about the soil type used for the earthfill. In this case the assessors can make an assumption on the basis of their experience and partial knowledge of the local situation deciding, say, to calculate P_{BE} for an embankment made of high plasticity silt. With The Reliability Rating System it is possible to check what would happen in terms of performance indicator if the assumption is not true. In total absence of information on the earthfill material decision makers may want to know what are the best and the worse possible scenarios. In Table 4.1 classes in the "material" section have been organised from left to right in order of decreasing adverse influence on the embankment performance. In the case of the earthfill soil type for instance clean sands (SW/SP in the Unified Soil Classification, ASTM 1985) represent the class of soils, among those listed in the Table, which gives the lowest resistance to through-piping. At the opposite end high plasticity clay (CH) grants the highest resistance to piping. Final users, if no indications are available about fill materials, may want to know the values of P_{BE} for these two scenarios. These bounds on the performance indicator will be very wide. Depending on the context in terms of expected consequences in case of breaching the risk managers can decide that this amount of ignorance is not acceptable. If this is the case the epistemic uncertainty can be partially reduced with more investigation. With a quick and economic site visit a trained geotechnical engineer or geologist can distinguish between coarse-grained and fine-grained soils and give a first assessment of their plasticity. If, for example low plasticity fine-grained earthfill is recognised on site the bounds on P_{BE} are then narrowed to values calculated by setting the "soil type" class to low plasticity silt (ML) for the upper bound and low plasticity clay (CL) for the lower bound. Again flood defence managers will decide on the acceptability of the amount of epistemic uncertainty on the basis of consideration of potential consequences. If the uncertainty is deemed unacceptable it can be further reduced by an even more detailed investigation.

4.6 Conclusion

4.6.1 Features of the Reliability Rating System

The Reliability Rating System developed in this research project aims at providing engineers and managers in charge of the flood defence network with a tool for quickly quantifying the expected performance of flood embankments in extreme hydraulic conditions. The system makes use of the Probability of Breaching in Extreme hydraulic conditions as a quantity indicating the proneness to breaching of flood embankments. This performance indicator is obtained by an approximate calculation of the probability of breaching for a limited range of water levels, above and immediately below the embankment crest. The methodology allows for a correct association of characteristics and condition of flood embankments to the relevant failure modes. This results in a representation of the structure's performance which is more realistic than any other approach based on the Condition Grades, where different features, affecting different failure modes, are lumped together. The system accounts for several embankments' characteristics which are affecting the performance in flooding conditions, as illustrated in Table 4.1. The reliability rating system considers three failure modes:

a) surface erosion in overtopping condition
b) erosion of the natural soil in the foundation, or under-piping
c) piping through the manmade earthfill, or through-piping

For the first two failure modes the probabilities of failure are determined with the methods of reliability analysis, precisely FORM and Monte Carlo simulation. For through-piping the probability of breaching is estimated by quantification of engineering judgement. The recognition that the failure by piping through the earthfill is not amenable to credible mathematical modelling is at the basis of the development of the Reliability Rating System. This approach aims at overcoming the limitations of the current practice where the process in unsatisfactorily modelled with a criterion valid for foundation erosion.

The formulation of P_{BE} is conceived for requiring the evaluation of the probability of failure by through-piping only in one loading condition, i.e. water at the crest level. In this way the task of determining the related probabilities of failure through subjective opinion elicitation is reduced to a tractable level of complexity.
In order to generate a tool for the final users the probabilities of breaching by the various failure modes have been determined for a finite but representative number of cases aiming at covering the range of scenarios possibly encountered in practice.

4.6.2 Limitations

The formulation of P_{BE}, constrained by the need to limit the assessment of through-piping probability to one loading condition, unavoidably introduces some biases. In particular, if compared with the annual probability of failure, P_{BE} tends to overrate the reliability of flood embankments having significantly high probabilities of breaching for water levels far below the crest. It also tends, in relative terms, to underrate the reliability of structures with high Standards of Protection. Finally it tends to overestimate the influence of surface erosion with respect to other failure modes. P_{BE} is an indicative measure of performance intended for structures' ranking and comparison. The biases listed above are therefore to be interpreted in relative terms.

The definition of acceptable and unacceptable values of P_{BE} is an open question. Similarly to the annual probability of failure its acceptability should be decided in a risk framework depending on the expected consequences of a breach. However while the acceptable probability of failure can be easily calculated if the acceptable risk is defined this cannot be done with P_{BE}. In the end reference values for P_{BE} can only be decided by a collective discussion within the flood risk community, in a way which is not dissimilar to the definition of safety factors in geotechnical or structural engineering.

4.6.3 A support to judgement

For risk assessment purposes and intervention prioritization a possible approach would be pairing P_{BE} with an indicative measure of the consequences. The definition of a limited number of classes of P_{BE} and, beside it, the definition of an equal number of classes of the quantified measure of consequences, would allow for a semi-quantitative classification of embankments with low-safety/high-consequences structures being associated with the highest risk and high-safety/low-consequences structures being associated with the lowest risk. In this thesis however the definition of an indicative measure of the consequences is not developed or further discussed.

The Reliability Rating System gives, through P_{BE} a quantified measure of the expected performance of flood embankments in extreme hydraulic conditions, supporting and assisting decision makers in the evaluation and comparison of the safety of these defence structures. Despite the limitations discussed above it is a tool which offers an unprecedented capacity to incorporate geotechnical and geological aspects in the reliability evaluation of flood embankments. In particular the influence of the type of soil in the earthfill, its compaction, and the geology underlying the embankment, largely overlooked by the current practice, is included in the approach. The methodology highlights the effect of factors which are not directly recognisable in a visual inspection but that, nevertheless, heavily affect the performance of flood embankments. The Probability of Breaching in Extreme hydraulic conditions P_{BE} offers a quick way of quantifying the expected performance under different hypothesis on the actual characteristics and conditions of flood embankments. In doing so the Reliability Rating System forces the final user to critically revise the available information. Giving a measure of the effect of lack of knowledge on the performance it also offers a support for deciding if it is necessary to acquire more detailed information or the uncertainty can be accepted.

Part II

5

DETERMINATION OF THE PROBABILITIES OF BREACHING

In Chapter 4 the performance indicator P_{BE} has been proposed as a quantitative predictor of the structural performance expected from flood embankments in extreme hydraulic conditions. In order to calculate P_{BE} the probabilities of breaching related to three failure modes, in two loading conditions, must be estimated. In the development of the Reliability Rating System these probabilities have been determined by the author (and his co-workers for the judgement elicitation phase) in a finite number of representative scenarios, covering most of the cases that can be encountered in the UK when assessing the safety of existing flood embankments. In this way the final users of the methodology can quickly quantify the performance indicator and also asses the impact on it of possible uncertainties about the relevant characteristics of the structures under assessment. The probabilities of breaching by under-piping and surface erosion have been calculated with traditional methods of reliability analysis, precisely FORM and Monte Carlo simulation. The probabilities of breaching by through-piping have been estimated via elicitation of subjective judgement.

5.1 Reliability analysis

5.1.1 Fundamentals of reliability analysis

The reliability analysis of an engineering system is the calculation of its probability of performing adequately or failing to do so. In this work the focus is on the structural performance of flood embankments and failure is intended as breaching of the earthen structure. For the present scope an embankment which is overtopped but retains its structural integrity is delivering a satisfactory performance. In other contexts the focus may be on the ability of the flood defence in totally preventing the release of water outside of the watercourse. In that case an overtopped embankment would not be performing the expected function and the situation would be classified as hydraulic failure.

A number of reliability methods, initially developed in relation to structural mechanics, are currently available to solve problems in different engineering fields, including geotechnical engineering, and are receiving attention by an increasing number of researchers. A mathematical model, defining the limiting state which separates safe states from failure states, is needed in order to calculate the probability of failure of an engineering system with the reliability methods. The probabilitistic description of the parameters of this model is also required. The level of detail of this probabilitistic description varies with the chosen method: it generally includes the knowledge of the mean and the variance, but can also include the specification of the probability distribution and autocorrelation, of the parameters.

When an analytical description of the engineering system's behaviour is available, its performance, in terms of failure or success can be expressed by a limiting state function, also called performance function. More precisely the performance function $g(X_1, X_2, \cdots, X_n)$ is a random function, possibly depending on several parameters, which gives the limiting state between failures and safe states through the equation:

$$g(X_1, X_2, \cdots, X_n) = 0 \tag{5.1}$$

In particular

$g(X_1, X_2, \cdots, X_n) = 0$	identifies the limiting state;
$g(X_1, X_2, \cdots, X_n) < 0$	identifies failures;
$g(X_1, X_2, \cdots, X_n) > 0$	identifies safe states.

From the definition of performance function it follows that the probability of the system failure is given by the integration of the joint probability distribution of the model parameters $f_{X_1,\ldots,X_n}(x_1,\ldots,x_n)$ over the region, in the n-dimensional space of the parameters, where $g(X_1, X_2, \cdots, X_n)$ is negative:

$$P(F) = \int_{g(x_1,\ldots,x_n)\le 0} \cdots \int f_{X_1,\ldots,X_n}(x_1,\ldots,x_n)dx_1 \cdots dx_n \tag{5.2}$$

Another convenient measure of reliability is the reliability index β, which is defined as the ratio of the mean of the performance function μ_g to its standard deviation σ_g:

$$\beta = \frac{\mu_g}{\sigma_g} \tag{5.3}$$

In the specific case of a normally distributed performance function, the following relation between the reliability index and the probability of failure holds.

$$P(F) = 1 - \Phi(\beta) \tag{5.4}$$

where $\Phi(.)$ indicates the cumulative distribution function of the standard Normal distribution. This is a very useful expression because values of $\Phi(.)$ are widely tabulated and are available in spreadsheets and mathematical software.

The reliability analysis can be seen as a way to propagate through a model the random variability of the input parameters in order to obtain the random variability of the engineering system's performance. In almost all cases of practical relevance the solution of the mathematical problem, as set in Equation (5.2) is not straightforward and the use of approximated analytical techniques or numerical approaches is necessary. This body of techniques has become known simply as "methods of reliability analysis". The most widely used methods of reliability analysis in civil engineering are:

- the First Order Second Moment method (FOSM);
- the Point Estimate method (PEM);
- the First Order Reliability Method (FORM), or Hasofer-Lind approach;
- Monte Carlo simulation.

Each method implies different approximations and has its own advantages and limitations, along with a different computational effort, accuracy and insight in the influence of different parameters. In this doctoral research FORM and Monte Carlo simulation have been used. The illustration of the different reliability methods is beyond the scope this thesis. In the following sections only the methods used in this work are briefly introduced. A detailed explanation and discussion of reliability methods in civil engineering can be found the books by Ang and Tang (1984), Harr (1987). For an up-to-date discussion of reliability in geotechnical engineering, the interested reader is referred to the book by Baecher and Christian (2003).

5.1.2 First Order Reliability Method (FORM) or Hasofer-Lind approach

The first Order Reliability Method has been adopted in this study because it takes into account the probability distributions of the input parameters. On the contrary other reliability methods, for instance the First Order Second Moment and the Point Estimate Method, make use only of the mean and variance of the model parameters and are not able to accommodate significantly skewed random variables. As illustrated in the following chapters, many relevant parameters involved in the safety of flood embankments are believed to follow a Lognormal distribution with non negligible skewness.

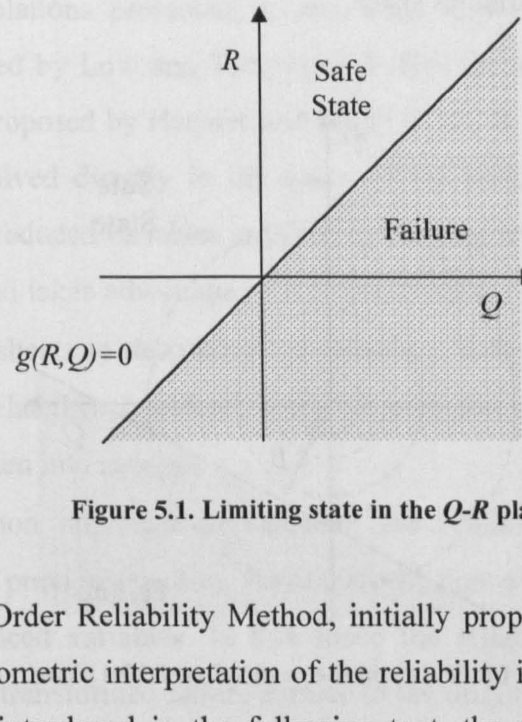

Figure 5.1. Limiting state in the *Q-R* plane.

The First Order Reliability Method, initially proposed by Hasofer and Lind (1974) gives a geometric interpretation of the reliability index. In its simplest form, which is briefly introduced in the following text, the method deals with normal independent variables. However, with suitable techniques, it can be extended to the cases of correlated variables and non-normal variables. For the sake of simplicity let the resistance R and the load Q be, in this introductory explanation, the only parameters of the model which describes the engineering system performance. Figure

5.1 shows the limiting state in the *Q-R* plane identified by the performance function $g(R,Q) = 0$.

In the Hasofer-Lind approach normalised dimensionless variables – or reduced variables – *R'* and *Q'* are introduced to replace the original variables *R* and *Q*:

$$R' = \frac{R - \mu_R}{\sigma_R}$$
$$Q' = \frac{Q - \mu_Q}{\sigma_Q} \tag{5.5}$$

where μ_R and μ_Q are the means of resistance and load respectively, σ_R and σ_Q are the corresponding standard deviations. In the space of the reduced variables the limiting state is displaced from the origin, as shown in Figure 5.2. The joint probability density function of *R'* and *Q'* is a bivariate Normal distribution centred on the origin of the axis in the reduced variables space.

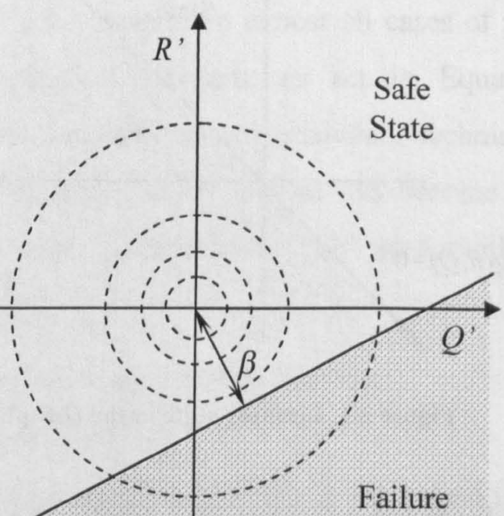

Figure 5.2. Limit state in the reduced variables plane.

It can be shown (Hasofer & Lind 1974) that the distance of the limiting state function from the origin in the reduced variables space coincides with the reliability index

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{5.6}$$

The result can be generalised to n variables. In this case the distance of a generic point from the origin is

$$d = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2} \tag{5.7}$$

The definition of the reliability index then becomes a problem of constrained optimisation. In fact the distance of a point has to be minimised providing the point satisfies the limiting state.

$$\begin{cases} \beta = d_{\min} = \min\left(\sqrt{X_1^2 + X_2^2 + \cdots + X_n^2}\right) \\ g(X_1, X_2, \cdots, X_n) = 0 \end{cases} \tag{5.8}$$

Such an optimization problem can be solved efficiently with a spreadsheet or mathematical software.

5.1.3 Low and Tang's approach

The FORM calculations presented in this work have been performed with the approach developed by Low and Tang (1997). This technique is a variation on the original method proposed by Hasofer and Lind (1974). In Low and Tang's approach the problem is solved directly in the space of the random variables, without the transformation to reduced variables required by the original method. The technique is straightforward and takes advantage of the optimisation procedures readily available in modern spreadsheets to determine the reliability index β. The procedure can be applied to uncorrelated or correlated variables and also distributions other than the Normal can be taken into account.

The common approach to calculate the reliability index β requires, as mentioned in the previous section, the transformation of the failure surface in the space of the reduced variables. In this space the reliability index is the shortest distance from the transformed failure surface to the origin. However a different point of view can be adopted considering the formulation of the Hasofer-Lind reliability index for correlated variables given in Ditelevsen (1981), also citing Veneziano (1974):

$$\beta = \min_{\mathbf{x} \in F} \sqrt{(\mathbf{x} - \boldsymbol{\mu})^T \mathbf{C}^{-1} (\mathbf{x} - \boldsymbol{\mu})} \tag{5.9}$$

where:

$\mathbf{x}$ is the vector of the random variables;

$\boldsymbol{\mu}$ is the vector of the mean values of the random variables;

$\mathbf{C}$ is the covariance matrix of the random variable;

F is the failure region, where the performance function is null or negative.

In fact Equation (5.9) indicates that the reliability index β can alternatively be calculated minimising the quadratic form directly in the space of the original variables. The quadratic form represents an ellipsoid with its centre in $[x_1, x_2, \ldots, x_n] = [\mu_1, \mu_2, \ldots, \mu_n]$ and, at failure, it must have at least one point in common with the failure region. Low and Tang's procedure leads to the determination of the ellipsoid which is tangent to the failure region in the space of the original variables, as illustrated in Figure 5.3 for the bivariate case.

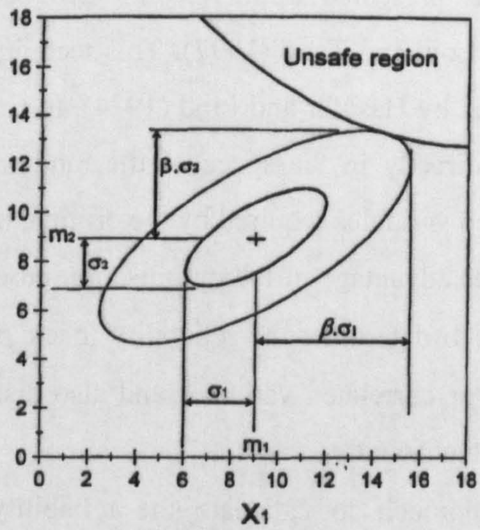

Figure 5.3. Expanding ellipse in the space of model parameters (source: Low 2007).

In Low and Tang's procedure the following steps are implemented in a commercial spreadsheet:

1. The parameters of the model are initially set to their mean values, i.e. $\mathbf{x} = \boldsymbol{\mu}$, and the functional form of the following equation is entered in a cell:

$$\beta^2 = (\mathbf{x} - \boldsymbol{\mu})^T \mathbf{C}^{-1} (\mathbf{x} - \boldsymbol{\mu})$$

2. The limiting state $g(\mathbf{x})$ is expressed, in another cell, as a function of the parameters in the vector $\mathbf{x}$.
3. The built-in optimization procedure, readily available in the spreadsheet, is invoked to find the minimum value of β^2 which satisfies the condition $g(\mathbf{x}) = 0$; this is done by changing the values of the parameters in $\mathbf{x}$.
4. Finally β is obtained from β^2 and the probability of failure follows from Equation (5.4).

Low and Tang's approach is easy to use and efficient, regardless of how low the probability of failure is. However in the development of the Reliability Rating System a conspicuous number of reliability calculations have been repeated with several different combinations of the input parameters. The drawback of Low and Tang's approach, in this context, is that, even if the single calculation is fast and straightforward, performing the optimisation for all the required combinations of input parameters is a time consuming and repetitive task, potentially dragging the analyst into gross mistakes in handling the input parameters. The situation is only marginally improved by the possibility of making part of the process automatic with the use of "macros" in Microsoft Excel™. In conclusion Monte Carlo simulations have resulted more appealing for the parametric studies required in this work. For this reason FORM has been used extensively only for one type of failure, namely under-piping in homogeneous foundation, and employed in the rest of the research project for occasional checks on the results obtained by Monte Carlo simulation.

5.1.4 Monte Carlo simulation

In the Monte Carlo approach to reliability input vectors for the performance function are created in a large number of trials, generating them in such a way that each variable follows the probability distribution required by the specific problem. The performance of the engineering system is then checked for each input vector and the probability of failure estimated as the ratio of the number of simulations resulting in failure to the total number of simulations. The method is conceptually simple, but can require a large number of randomly generated values to achieve the desired accuracy. In the present work the simulations have been performed using @Risk™ (Palisade 2005) an add-in for the program Microsoft Excel™ (Microsoft 2003)

@Risk™ generates random numbers with a desired distribution and feeds them to the spreadsheet, keeping track of the statistics of the required output over the sequence of trials.

When performing a Monte Carlo simulation it is necessary to asses how many trials are needed to achieve the desired level of accuracy in the results. In Monte Carlo analysis the probability of the studied event p is approximated by its estimator $\hat{p}$. In the case of reliability analysis[5] the event under investigation is the engineering system not meeting the desired performance and the probability of failure $p = P(F)$ is approximated by the estimator:

$$\hat{p} = N_f / N \tag{5.10}$$

where N_f is the number of trials resulting in failure and N is the total number of trials. The number of trials required to estimate the probability of an event depends on the desired accuracy of the estimate. A confidence level and an acceptable error have to be chosen by the analyst. The confidence level (1-α) is defined as the probability that the estimator differs from the actual probability p less than the accepted error ε:

$$1-\alpha = P\left(\left|\hat{p}-p\right| < \varepsilon\right) \tag{5.11}$$

Usually a confidence level of 90% or 95% is specified. It can be shown (see for example Ang & Tang 1984) that for a sufficiently large number of trials (say $N > 30$) an approximation of the number of trials needed to achieve the tolerable error ε with confidence level (1- α) is given by

$$N \geq \frac{z_{\alpha/2}^2 (1-p)}{\varepsilon^2 p} \tag{5.12}$$

where $z_{\alpha/2}$ is the standard normal variable that is exceeded with probability $\alpha/2$ and p is the probability to be found by numerical approximation; in this case the probability of failure. Equation (5.12) shows how, once the desired confidence level

[5] Monte Carlo simulation is used not only for reliability analysis but also for other applications, like the numerical solution of integrals and a variety of simulation techniques for engineering design.

is fixed, the number of required trials has to increase if a lower error needs to be achieved. Also more trials are needed for lower probabilities of failure.

In the development of the Reliability Rating System, considering the approximations incorporated in the whole methodology, an error of less than 10% with 90% confidence level has been considered acceptable. This condition has been satisfied for a probability of failure as low as 9×10^{-5}, implying 3.01×10^{6} trials. For the through-piping failure mode, which is discussed in Chapter 9, the elicitation of subjective probabilities has been used and a probability of breaching with water at the crest level as low as 9×10^{-5} has been estimated for an embankment with optimal resistance to internal erosion. For this reason it has been deemed unnecessary to require the Monte Carlo simulations to produce accurate results below this value. Some techniques are available to increase the accuracy of Monte Carlo simulation given a fixed number of trials. The Latin Hypercube sampling (McKay 1979, Iman *et al.* 1981) has been adopted throughout this work to improve accuracy.

5.1.3 Correlation

5.1.3.1 Correlations at different levels

In the reliability analysis of flood defences correlations are present at several levels. Different variables related to the same mode of failure can be correlated. Moreover some parameters display autocorrelation in space and, when a defence system is split into component defences, their structural responses are generally correlated.

5.1.3.2 Correlation between parameters

Some of the model parameters used in this work are likely to be correlated. For example, the saturated unit weight and the coefficient of permeability of fine-grained soils, used in the modelling of under-piping (Chapter 7) can be correlated. Although the reliability methods adopted in the development of the Reliability Rating System can deal with correlated variables, in this work this type of correlation has been neglected.

In fact the information available in the literature about parameter correlations is somewhat less rich and robust than the information on the probability distributions and statistical moments of the single variables. The choice of ignoring correlation among parameter has also been made with the aim of keeping the first version of the methodology as simple as possible. An improved version of the Reliability Rating System could be developed in a later phase, after gathering more material about correlations among the relevant parameters.

5.1.3.3 Autocorrelation in space

Autocorrelation in space, i.e. correlation among values of the same parameter in different points of the same material, is particularly important when structures of remarkable extension are considered. This is certainly the case for flood embankments and for linear structures and infrastructures in general.

Different approaches to modelling autocorrelation in space can be applied to structures and infrastructures of significant length. Essentially three main choices are possible:

1. Ignore the fluctuation in space and treat all materials as perfectly autocorrelated.
2. Introduce one-dimensional autocorrelation along the structure's length.
3. Model the full tri-dimensional random fluctuation in space.

Although it may seem an overly simplified approach the first option offers a first insight into the structural performance also for structures particularly extended in one dimension. Christian *et al.* (1994) for example analysed the dykes of the James Bay hydroelectric project, which have a total extension of about 50 km, without considering the effect of autocorrelation in the longitudinal direction and thus neglecting the length effect[6]. Nevertheless the reliability analysis of two different types of cross sections, and the study of the contribution of different sources of variability, resulted in a deeper understanding of the stability of the embankments on sensitive clay than what was achievable with a deterministic analysis.

A more advanced approach, mentioned as second option in the list, is to consider the autocorrelation of parameters in one dimension, along the length of the structures. This is done, for example, in the Dutch approach to flood defences reliability by means of the autocorrelation function introduced by Equation (3.4) in

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[6] Auto-correlation of parameters in the cross section was instead included in the modelling; in the stability analysis, in fact, the variation of the shear strength was averaged integrating along the failure surface.

Chapter 3. This approach allows for the incorporation of the length effect in the analysis by splitting a dike of length L in m subsection length $\Delta L = L/m$ and considering it like a series system where the component elements, i.e. the subsections, are correlated (Vrijling & Van Gelder 2000). For smaller ΔL the number of subsections increases but so does the correlation among them; these effects contrast one another in terms of their influence on the probability of failure of the whole structure. Vrouvenweldr (2006) describes an "outcrossing" procedure which also identifies the optimal subdivision in subsections.

Considering the third option, a suitable way of including in geotechnical modelling the random fluctuation of parameters in space is the generation of random fields (Vanmarke 1983, Fenton 1990) and their mapping into a finite element mesh for the solution of the mechanical problem (Griffiths & Fenton 1993, Fenton & Griffiths 1993). This powerful approach can be used for solving 2D and, with a higher computational effort, 3D problems, regarding a wide range of geotechnical issues including, for instance, seepage (Griffiths & Fenton 1993), foundation settlements (Paice *et al.* 1996) and static liquefaction of manmade slopes (Hicks & Onisiphorou 2005). The generation of random fields with the required properties can be done with the Turning Bands Method (Mantoglu & Wilson 1982) or with the Local Average Subdivision (Fenton 1990, Fenton & Vanmarke 1990). Numerous realisations are analysed via the finite elements method within a Monte Carlo framework. This approach takes into full account the effects of soil heterogeneity but requires high performance machines to deal with the remarkable computational effort required, especially for 3D simulations (Hicks *et al.* 2005).

The aim of this research project is to develop a tool for the first quantification of the expected performance of flood embankments which are included in an inventory of flood defence assets. Such a task can be satisfactorily accomplished with the simplest approach available. Therefore in the present work the fluctuation of parameters in space is neglected and all materials are treated as perfectly autocorrelated. It should be noticed that the same simplification is adopted by the methodologies currently employed in the UK at national and regional scale, and by the methods in use in the USA.

Ideally the approach to flood defences reliability should be tiered, with an increasing level of detail achievable when moving to a more local scale. The modelling strategy should change accordingly, adopting progressively more refined techniques. This should also be reflected in the way autocorrelation in space is included in the model. Therefore moving to a more detailed scale other methodologies than the Reliability Rating System should be used and option 2 in the above-mentioned list, or option 3 for a very advanced assessment of single flood defences, should be considered.

5.1.3.4 Correlation between defences

Correlation can be present not only among subsections of the same long defence structure but also among different adjacent structures. The Reliability Rating System neglects this correlation. This is analogous with what is done by the current British methodologies at national and regional scale. The expedient, already mentioned in Chapter 2, of splitting longer defences (L > 500m) in more parts is maintained in the methodology proposed here. In the case of the Reliability Rating System the choice of ignoring correlation among defences is justified by the aim of the method, which is not the determination of the whole system performance but the ranking of individual defences. The author suggests that, on the contrary, for the purpose of integrated flood risk modelling an effort should be made to introduce one-dimensional autocorrelation in space at least at regional-scale.

5.1.4 Model factors

In reliability analysis the fluctuation of parameters is not the only source of uncertainty on the performance prediction. The model used in the formulation of the limiting state often incorporates some uncertainty. In fact models are always approximations of complicated physical condition. A common way to account for model error is to assign to its output a unit mean multiplier with standard deviation reflecting the estimated model uncertainty. This multiplier is generally called model factor and its standard deviation is often assessed by expert judgment.

Model factors are widely used throughout this work. Their stochastic properties are shaped on assumptions commonly made in the Dutch practice (Vrijling

& Van Gelder 2000), but some changes, discussed in the relevant chapters, have been introduced for a better adaptation to the environment of British rivers.

5.1.5 Analyses for the development of the Reliability Rating System

In the development of the new methodology it is necessary to define, for each one of the required input parameters, the probability distribution, the mean and the standard deviation. Most parameters involved in the calculations exhibit some regularity in their random variability. It is possible, for instance, to recognise the tendency to have some type of distribution and even identify typical values the coefficient of variation. Such indications are increasingly published in the literature (Beacher & Christian 2003). Examples of typical random fluctuation of the parameters involved in flood defences reliability are found in Vrijling and Van Gelder (2000). For the wider field of geotechnics standard values of the coefficient of variation for several soil properties are reported, among others, by Lumb (1974), Lee (1983) and Lacasse & Nadim (1996).

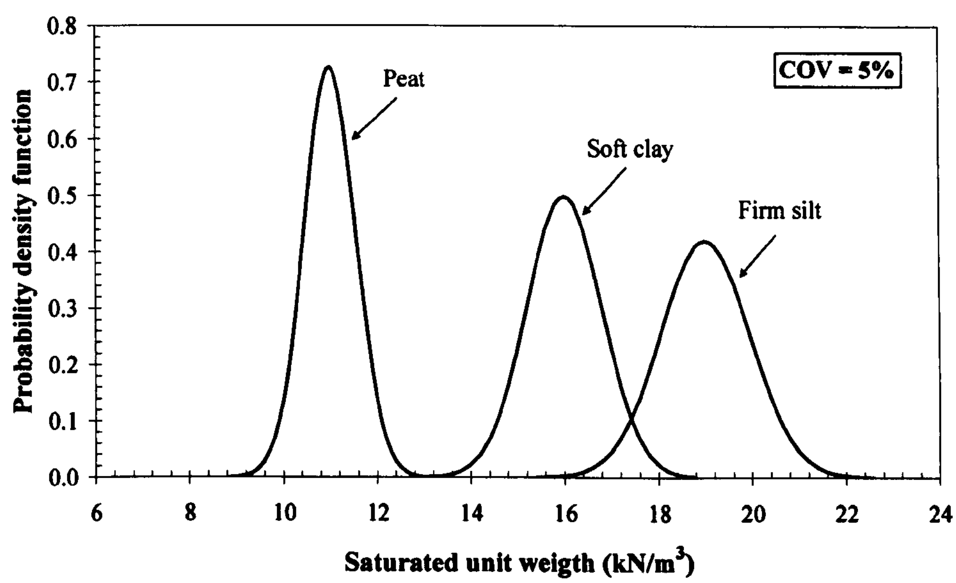

Figure 5.4. Example of different probability distributions for the saturated unit weight of fine grained soils. While the coefficient of variation can be assumed constant the mean can vary significantly depending on the type of soil.

While it is possible to make some sensible assumptions on the random fluctuation of relevant parameters, expressed by the coefficient of variation, it is much more difficult to predict their mean if no site specific information is available. This concept can be illustrated considering the example of the unit weight of fine-grained soils. This parameter is crucial, as explained in Chapter 7, to the piping-resistance of natural soil underneath flood embankments. Many authors are in consensus in indicating a coefficient of variation around 5% for the saturated unit weight, which has also been found to follow a Normal distribution (Wolff *et al.* 1996, Vrijling and Van Gelder 2000). However, with no further specification than the superficial soil being fine-grained, the mean of the saturated unit weight can assume a wide range of values. With a reasonable degree of accuracy one can identify an interval going roughly from slightly above 10 kN/m^3, in the case of soils with high organic content, to values around 20 kN/m^3 for firm clay or silt (Craig 1996). Some hypothetical examples of probability density functions are plotted in Figure 5.4 for different soil types, in order to illustrate the range of distributions that can be encountered in practice for this specific parameter. Of course a slightly different mean and coefficient of variation can be found, depending on the site, for the type of soils mentioned in the graphic. However the values chosen here can be seen as indicative of each soil type.

In the light of these observations on the variability of parameters the following procedure has been established to develop the Reliability Rating System.

1. Identify for each failure mode a suitable mathematical model describing the limit state.
2. Estimate the probability distribution and the random fluctuation of the model parameters in terms of coefficient of variation (or standard deviation) on the basis of indications available in the literature.
3. Identify the range of variation of the mean of the model parameters on the basis of literature and engineering judgement.
4. Link the mean of each parameter to a characteristic that is easily detectable on site through inspection or minimal site investigation and identify a convenient subdivision in classes for this characteristic
5. Define a reference value for the mean associated with each class.
6. Perform reliability analyses using the random variables associated to the various classes as input parameters, covering all their possible combinations.

As the usability of the methodology is of paramount importance the characteristics mentioned in point (4) should be easily recognisable on site without a detailed geotechnical investigation. The subdivision in classes is the result of a compromise between the need for an accurate prediction – which would require a large number of narrow classes – and the low level of information typically available to the final user – which is best incorporated in a small number of wide classes.

5.2 Subjective probability elicitation

5.2.1 Philosophy of judgemental probability

When the large amount of data needed for a statistical analysis is not available and reliability analysis is not supported by a credible mathematical model a third way of quantifying the probability of failure of an engineering system is required. The quantification of engineers' opinion in the form of subjective probabilities offers a possible basis to formally incorporate uncertainty in a reliability analysis (Gilbert *et al.* 1998). In most cases these opinions are based on intuition, qualitative knowledge, approximated theory, personal experience, and other sources which are not easy to represent in a rigorous mathematical framework. Nevertheless this judgement is important in analysing the probability of unsatisfactory performance of several engineering systems. Within the Bayesian school of probability, personal opinion is an acceptable basis for probability estimation as long as consistency with the corresponding mathematical theory is maintained (Ramsey 1978). In principle subjective probabilities can be mixed with probabilities obtained as relative frequencies from the statistical observation of data or with probabilities of failure from traditional reliability analyses.

5.2.2 Heuristics and biases

5.2.2.1 The quantification of subjective judgement

Quantifying judgement in the form of subjective probabilities requires the integration in a consistent framework of information of various types. The mental processes behind this estimation are studied by cognitive psychology. In the quantification of subjective probabilities two properties are particularly desirable: coherence and calibration. Subjective probabilities are said to be coherent if they are concordant with probability theory, i.e. reflect the correct mathematics. They are said to be well calibrated if they reflect the frequencies actually observed in the physical world, i.e. they have predictive value.

Research in cognitive psychology has highlighted as humans, though being able to successfully deal with uncertainty in daily life, are not particularly well equipped to achieve coherence and good calibration in subjective probability estimates. In other words probability theory shows how uncertainty ought to be processed, not how people naturally tend to handle it (Beacher & Christian 2003). In practice people use rules of thumb and simple strategies for estimating subjective probabilities. These informal methods are called heuristics. Several studies in cognitive psychology have shown that, in some circumstances, heuristics can lead to systematic errors named cognitive biases. Much of the work on heuristics and biases in human decision-making was ignited by Tversky (Edwards & Tversky 1967), Kahneman (Kaneman *et al.* 1982) and their co-workers. The use of these concepts in geotechnical engineering has been reported, among others, by Folayan *et al.* (1970), Beacher (1972) and Hynes & Vanmarke (1976). Some well known heuristics, relevant for quantification of engineering judgement, are illustrated in the following text, along with how their use can result in cognitive biases. Some of the heuristics and biases explained in the next sections apply to any subjective estimate of a quantity; like the judgemental estimate of the deterministic value of a parameter. Others specifically apply to probabilistic concepts.

5.2.2.2 Anchoring and adjustment heuristic

The anchoring and adjustment heuristic is related to people trying to estimate a typical value of some quantity as a reference point (the "anchor") and then make adjustments to it, in light of more specific information or context, to reach their final estimate. For example geotechnical engineers having to estimate the undrained shear strength of a clay often think about a typical value for clay, and then consider in which way the specific site differs from an average situation. Failure to adjust the prediction sufficiently, sticking too close to the initial value results in a bias, called anchoring bias. A way to prevent this bias is to state first the largest conceivable value, then the lowest, and only afterwards set a central value.

5.2.2.3 Representativeness heuristic and bias

Representativeness is a heuristic which reflects judgement based on the perceived recurrence of a particular condition in several circumstances considered similar to one another. If representativeness is overemphasised it can result in discarding of other relevant information and the representativeness bias occurs as a result.

A proof of this was given in an experiment by Kahneman *et al.* (1982), who provided two groups of subjects with a detailed description of the behaviour and personality of a hypothetical person and then asked those groups to estimate the probability that the person is a lawyer versus an engineer. Subjects who were told that the individual was randomly drawn from a group of 70 lawyers and 30 engineers gave the same estimate as subjects who were told that the group was formed by 30 lawyers and 70 engineers. Probability theory, in particular Bayes' Theorem, states that the two predictions should be different. However the subjects in the experiment neglected the information on the composition of the group, which was overshadowed by the representativeness bias. In making similar kind of estimates the assessors should not neglect the prior frequencies.

5.2.2.4 Conjunction fallacy

The conjunction fallacy is a cognitive bias which consists of the assumption that the combination of several events is more probable than the occurrence of each single constituent event (Taversky & Kahneman 1983).

An illustration of the concept by means of a geotechnical example is provided by Beacher and Christian (2003). Let a small dam be considered, built across a stream with similar geology at both abutments. The regional geology is flat-laying sedimentary rocks. A flowing spring of muddy water appears at the downstream toe of the dam, creating a small cone of silty sand. An engineer could have to decide if it is more likely that:

a) There is a geologic fault beneath the dam; or

b) There is a geologic fault beneath the dam allowing internal erosion of the embankment.

Many experienced engineers instinctively judge answer (b) to be correct. This cannot be true because probability theory states that the joint occurrence of two events – "there is a fault" and "internal erosion is taking place" – cannot be more likely than one of the events alone – in this case "there is a fault". While it is right to assign, on the basis of the described situation, a high probability to the combined event (b) a still higher probability should be assigned to the individual event (a).

5.2.2.5 Availability heuristic and bias

The availability heuristic operates when people estimate the frequency of an event on the basis of how easily an example can be brought to mind. Easier to imagine events and events which are recalled more vividly, causing a stronger emotional impact, are erroneously considered more likely to occur than they actually are. The opposite happens for less mentally "available" events.

Vick (1999) argues that it is important to guard against availability bias introduced by the review of failures case histories, especially well documented case histories taken as salient. They indeed represent fundamental information of remarkable importance. However, when used in probability estimation, it is useful to keep in mind that case histories can be isolated occurrences and that few geotechnical structures, even in adverse conditions, actually fail. Dealing with dam risk analysis, Vick proposes to counterbalance the availability bias due to case histories by including in the review documented cases of non-failure incidents. However, while this way of proceeding is appealing in the case of embankment dams, it appears to be hardly feasible, due to the recognised lack of information regarding flood defences.

5.2.3 Overconfidence

5.2.3.1 Misperception of extreme probabilities

The tendency to be more confident than the evidence warrants is a very pervasive bias in subjective probability assessment. Overconfidence bias appears in the tendency to discount outliers, to assign probability distributions on parameters that are too narrow about the mean, or to assign probabilities at the high or low ends of the probability scale that are more extreme than they should be. Fischhoff *et al.* (1977) investigated how often people are wrong when they are certain that they know the right answer to a question. In an experiment, for a variety of general-knowledge questions, three groups of subjects were first asked to choose the most likely answer and then to indicate their degree of certainty that the answer they had selected was, in fact, correct. Figure 5.5, shows the original results as reported by Vick (1997). The plot shows that when the actual frequency of errors was greater than 0.2 the subjective estimates were in good agreement with the actual rates. When the actual error rates fell below 0.2 the subjective rates dropped dramatically. As shown at the left side of the picture, when the actual rates were between 0.04 and 0.1 the subjects assessed that their error rates were 10^{-6}. In fact the subjects were shockingly overconfident by about five orders of magnitude. The experiment also showed that people have problems in understanding the meaning of very low probabilities. In fact the subjects showed little cognitive discrimination among extreme probabilities giving estimates ranging from 10^{-2} to 10^{-6} for substantially constant error frequencies.

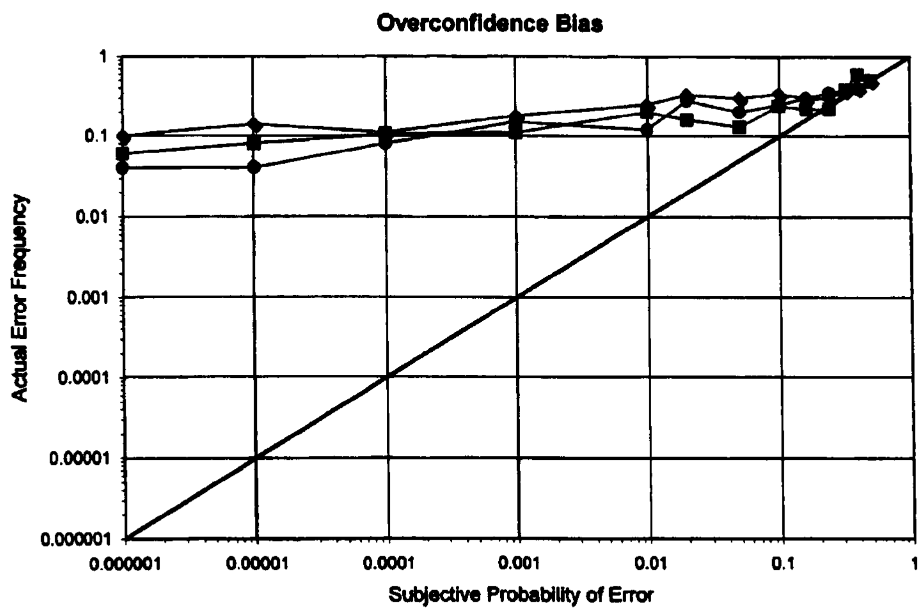

Figure 5.5. Subjective probabilities versus actual frequencies in an experiment; data from Fishhoff *et al.* (1977), plotted by Vick (1997).

5.2.3.2 An example from geotechnical engineering

The literature on expert elicitation indicates that experts tend to be good at estimating mean or median values. Moreover the average of the opinion of several experts tends to be better than individual estimates. However another important indication is that experts are generally too confident in their estimates and are inclined to underestimate the associated uncertainty. The results published by Hynes and

Vanmarcke (1976) about predictions of failure height for a test embankment on soft clay offer a good illustration of these points. Figure 5.6 shows the estimates, made by seven internationally recognised geotechnical engineers, of the amount of additional filling needed to fail the embankment named I-95 in a test set up by the Massachusetts Institute of Technology. Together with their best estimate the seven experts provided the range which they believed to correspond to a chance of failure of 50%, also called interquartile range. The large square points represent the best estimates while the vertical bars are the interquartile ranges. The average of the expert estimates proved to be a reasonably good estimate of the actual event. However the actual amount of fill to cause failure does not fall within any of the 50% confidence limits. Probability theory would predict that about half of the vertical bars (in this case 3 or 4) should enclose the actual result. The confidence intervals are clearly too narrow revealing the presence of an overconfidence bias.

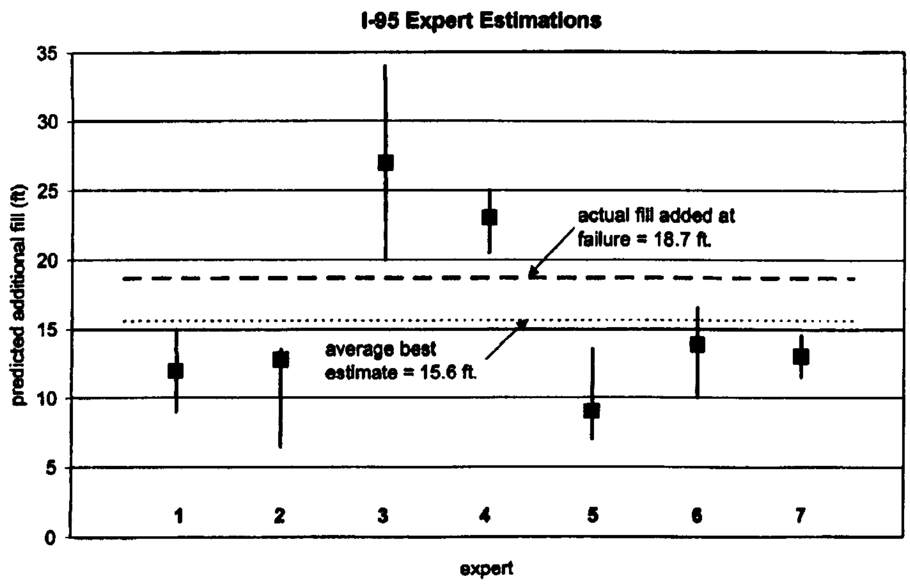

Figure 5.6. Predictions for embankment MIT I-95: best estimate and 50% confidence ranges of added height to cause failure reported by Hyens & Vanmarke (1997); plot by Christian (2004).

The seven experts where then asked to also provide the minimum and maximum value of the additional height of fill to failure. Their answers are shown in Figure 5.7. In some cases the interval does intersect the actual result. Moreover it should be noticed that the values are remarkably inconsistent with the previous prediction. In almost all cases the maximum to minimum range is very close to the interquartile range. In other words the experts are identifying an interval which should have null or at least very low probability of seeing an outcome falling outside itself as almost identical to an interval which has 50% probability of seeing an outcome outside itself. In some cases the minimum to maximum range was even smaller than the interquartile range. The problem, in this case, arises from lack of clarity in the definition of "minimum" and "maximum" which leads to very different individual interpretation of the terms. As observed by Hynes and Vanmarcke (1976) these widely used terms are essentially meaningless unless related to a precise probability.

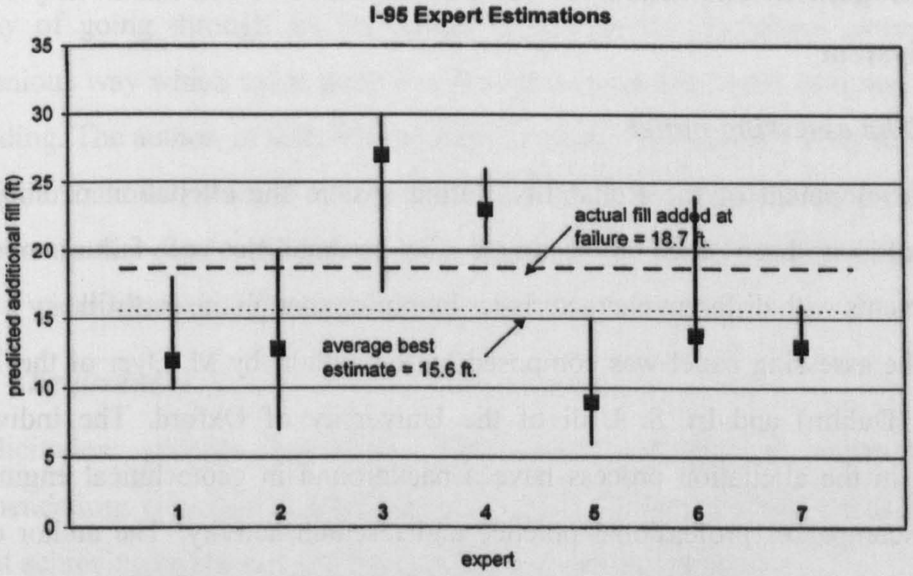

Figure 5.7. Predictions for embankment MIT I-95: maximum-minimum ranges of added height to cause failure reported by Hyens & Vanmarke (1997); plot by Christian (2004).

5.2.4. Can people learn to be well calibrated?

Several studies have tried to establish whether people's calibration in probability assessments can be improved with training. The answer seems to be positive but it appears that the results of training are expertise-specific and cannot be generalised well to new tasks (Ferrel & McGoey 1980; Keren 1994; McClelland & Bolger 1994). Not unexpectedly, people tend to estimate better calibrated probabilities when asked questions within their fields of competence. The whole question is, anyway, still controversial, with some reports (Alpert & Raiffa 1982) showing estimators to be nearly impervious to repeated training and feedback attempts.

An encouraging result comes from the realm of weather forecasting. In a series of studies Murphy and Winkler established that the forecasters working at the National Weather Service in the US are very well calibrated probability assessors (Winkler & Murphy 1968; Murphy & Winkler 1977a, 1977b). Murphy and Winkler suggest that this is due to practice, immediate feedback, quantitative scoring of performance and pay incentives for accuracy. The implication for geotechnical practice is that continuous exercises in quantifying judgement may improve the ability of handling uncertainties, whether or not a formal reliability analysis is required in a project.

5.2.5 Judgement elicitation for the development of the Reliability Rating System

5.2.5.1 The assessing panel

In the development of the Reliability Rating system the elicitation of subjective judgement has been used to estimate the probabilities of failure of flood embankments with different characteristics by piping though the earthfill.

The assessing panel was composed by the author, by M. Dyer of the Trinity College (Dublin) and by S. Utili of the University of Oxford. The individuals involved in the elicitation process have a background in geotechnical engineering which encompasses professional practice and research activity. The author of this thesis has an MSc in Civil and Environmental Engineering awarded by the Technical University of Milan. His studies focused on the protection of the built environment form natural hazards giving him a background in water engineering disciplines as well as ground engineering. Before starting this research project he had been working for a leading Italian consulting company in the fields Geotechnical, Geo-environmental and Geo-seismic Engineering. During this professional experience was involved in the design of several coastal and maritime works, including the flood defence scheme for the protection of Venice from high waters.

Mark Dyer has been collaborating with the Environment Agency (EA) for many years as a consultant on the geotechnical stability of flood embankment and is the author or co-author of many publications on the subject (Dyer & Gardener 1996, Morris *et al.* 2004, Dyer 2004, Redaelli *et al.* 2008). He has been involved in the EA project on the establishment of a Performance-based Assets Management System for Flood Defences (PAMS). At the time of writing Prof. Dyer has the role of McNamara Chair in Construction Innovation at Trinity College (Dublin).

Stefano Utili was a Post-Doctoral Research assistant at the University of Strathclyde in 2006 and 2007, working, in liaison with the Environment Agency, on the integration of geotechnical aspects in the analysis and management of flood embankments. Since 2008 he has been a Lecturer in Civil Engineering at the University of Oxford. He is currently coordinating a cross-national research project on the fissuring of flood embankments induced by seasonal desiccation.

The small size of the panel involved in this work has been dictated by the necessity of going through all the phases of a rigorous elicitation process in a parsimonious way which made good use of limited resources, both in terms of time and funding. The author, in fact, felt the need to rapidly develop this body of work in order to illustrate its potential and ignite discussion. It is recommended that further developments are conducted involving a larger panel, which ideally should also include a wider spectrum of competencies.

5.2.5.2 The protocol

The elicitation process has been structured, according to state-of-the-art recommendations (Baecher & Christian 2003, Vick 1999 and 2000, USACE 2006) aimed at achieving coherence and good calibration, in five phases:

1. Motivating phase
2. Training phase
3. Structuring phase
4. Assessing phase
5. Documenting phase

The main function of the motivating phase is to clarify the objectives of the judgmental probabilities elicitation and to explain to the assessors how the results will be used in practice. In this phase the basic concepts behind judgemental probabilities are also reviewed and the possible motivational biases in subjective judgement are highlighted. Motivational biases are those factors leading the members of the panel to provide, consciously or unconsciously, assessments which do not reflect completely or accurately their actual beliefs. Motivational biases produce a difference between the true opinion an individual holds and the opinion which is communicated to the rest of the panel. Motivational biases include under-reporting uncertainty for the desire of appearing more knowledgeable or accommodating probabilities for the desire of influencing the final decision. In the panel arranged for this specific project these issues have been discussed at the beginning of the elicitation process, attempting to achieve a common positive approach, based on constructive assertiveness and openness to feedback and debate.

The main goal of the training phase is to familiarise the panel with the psychological processes involved in the quantification engineering judgement and make them aware of related problems and limitations. In this project the relevant basics of cognitive psychology, discussed in Section 5.2, have been reviewed; heuristics and biases involved in the estimate of subjective probabilities have been thoroughly discussed within the panel, also with the help of examples and warm-up exercises.

The function of the structuring phase is to conceptualise the physical processes under investigation and articulate the problem in a way that makes the subsequent phase of probabilistic assessment more easily handled by the panel. The structuring phase has been introduced by the preparation of a report reviewing the available research on failure of water-retaining earthen structures by piping. The appropriate level of detail to describe the factors influencing through-piping has then been discussed.

In the assessing phase the subjective probabilities are estimated as a quantification of the assessors' opinion. The estimation is first conducted individually by each member of the panel. The results are then compared, differences are discussed and agreement is sought. Supporting procedures like the association of numerical values of probability with verbal descriptors, or the action approach to elicitation, which compares the problem to hypothetical gambling, are available in the literature (Beacher & Christian 2003). Generally the structuring and the assessing phases, which are deeply interrelated, make use of the event tree technique. In this work, because of the particularly low level of information available for flood embankments, an alternative formal structure, thoroughly discussed in Chapter 9, has been used.

The documenting phase, which is best conducted alongside the other phases, is essential for future checks or improvements of the produced probability estimates. A summary of the main actions undertaken during the process is reported in Appendix D.

5.3 Conclusions

In this research project the probabilities of breaching in reference loading conditions, needed for the calculation of the performance indicator P_{BE} have been determined with reliability analyses and with the elicitation of subjective probabilities for through piping.

The probabilities of breaching for under-piping and surface erosion have been calculated using Monte Carlo simulations and occasional checks have been done with the FORM in the implementation proposed by Low and Tang (1997). Due to the nature of the methodology developed in this work the simplest mathematical models capable of effectively acting as performance functions have been used in the reliability analyses and correlations have been neglected at various levels. Model factors have been used extensively.

The probabilities of breaching by through-piping have been estimated by a panel of engineers through a rigorously structured process of judgement elicitation. This codified procedure is aimed at mitigating possible biases and achieving well calibrated subjective probabilities.

6

UNDER-PIPING: HOMOGENEOUS FOUNDATION

Breaching by erosion of the foundation, also referred to as under–piping, is the first mode of structural failure considered in the Reliability Rating System. Two situations can be recognised when studying the safety of a flood embankment with regards to under-piping: if most of the seepage takes place in the top layer of natural soil on which the structure is directly founded then the foundation can be regarded as homogeneous; if, on the contrary, most of the seepage takes place in a deeper stratum then the layering affects the resistance to piping and must be included in the analysis. This chapter reports the reliability analyses performed in the case of a homogeneous foundation adopting a performance function based on the weighted creep criterion (Lane 1935). The more complex case of a layered foundation is presented in Chapter 7.

6.1 Failure by foundation erosion

In flooding conditions a difference in hydraulic head is established across a flood embankment and its foundation, resulting in seepage in the structure's body and in the natural soil beneath it. Depending on the soil type and its state the seepage both in the embankment and in its foundation can be significant. In some cases erosion phenomena induced by seeping water, and indicated with the generic name of piping, can affect the flood defence, potentially compromising its integrity. In this chapter and in Chapter 7 the safety against foundation erosion is considered. Modes of failure associated with excessive seepage through the earthfill are discussed in Chapter 9.

When a soil prone to erosion is present beneath a flood embankment the seepage in the foundation can result in the removal of soil particles. The process initiates with spring pits on the inland side and, under certain conditions, can continue, developing retrogressively under the embankment and forming pipe-like preferential seepage paths. as sketched in Figure 6.1. As more particles are removed pipes enlarge beneath the structure, potentially leading to its sinking and breaching. The whole process is indicated in the literature as regressive erosion or backward erosion (Richards & Reddy 2007). Sand, especially if fine, is the foundation soil most prone to backward erosion; for this reason the name "sand boiling" is also found in the literature to describe this process (Gabr *et al.* 1996, TAW 1999). However other soils can also be subject to under-piping and, particularly in the development of a reliability methodology for a large inventory of flood defences, it is important to account for the whole range of situations that can be found in practice.

In this Chapter embankments built on deposits that are homogeneous down to the depth where significant seepage occurs are considered. More complex situations where the layering of the soil profile has a significant influence on the seepage and on the resistance to piping are also relevant to practice and will be discussed in the next chapter.

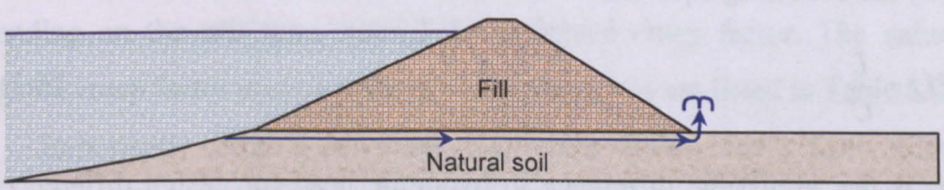

Figure 6.1. Erosion of the soil layer immediately beneath the flood embankment. The removal of natural soil in the foundation (under-piping) can compromise the structural integrity of the flood defence and lead to breaching.

6.2 Mathematical models for under-piping

6.2.1 Empirical "creep" formulae

The form of piping considered here consists of the dislodgement of soil particles in the foundation of flood embankments due to the action of inter-granular seeping water. Mathematical models are available in the literature for evaluating the safety against this process in engineering applications.

The criteria for under-piping presented in the following were originally developed for impervious water-retaining structures but their application is often extended to earth structures (TAW 1999, Vrijling & van Gelder 2000). This approximation is acceptable when the fill material is less permeable than the soil in the foundation; a condition which is generally satisfied for embankments made of well compacted fine grained soils. When the embankment is more permeable than the foundation, other, more relevant safety issues arise and under-piping ceases to be of primary concern.

The first criterion for evaluating safety against piping through the foundation was the empirical rule proposed by Bligh (1910). Considering several cases of collapses of small dams, Bligh developed a formula for evaluating the difference in hydraulic head, between upstream and downstream of the structure, that can be safely withstood by the foundation.

The formulation is based on the assumption that the resistance to under-piping is related to the length of the line which extends from the entry point to the exit point of the seeping water, running at the contact between the base of the dam and the founding soil. Such a line is named the line of creep. Bligh's criterion is expressed by the following equation:

$$\Delta H_d = \frac{L}{C} \tag{6.1}$$

where ΔH_d is the admissible difference in hydraulic head (or design difference in hydraulic head[7]), L is the length of the creep line, C is a factor depending on the type of soil, named the creep factor. The values for the creep factor suggested by Bligh for different soils are listed in Table 6.1.

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[7] Originally the symbol ΔH_c is used in Bligh's formula (and Lane's formula presented later), with the subscript "c" meaning critical. However within the current engineering terminology the adjective "critical" should be reserved for values associated with the limit state, rather than a safe state. For this reason in the present work the diction "*design* difference in hydraulic head" and the notation ΔH_d have been preferred.

Table 6.1. Values of the creep factor for different types of soil according to Bligh (1910).

Soil Type	Creep factor C (Bligh 1910)
Fine sand and silt	18
Fine micaceous sand	15
Coarse sand	12
Gravel and sand	3
Boulders, gravel and sand	4-6

It should be noted that Bligh didn't include clays in his table. This is because, recognising how granular soils are more prone to backward erosion than cohesive soils, he considered clays to be intrinsically safe.

In a later development, Lane (1935) improved Bligh's approach working on a more extensive set of case histories and arguing that the vertical parts of a creep line contribute more to the overall resistance to piping than the horizontal parts. Accordingly he proposed a modified empirical formula which is know as the "weighted creep" criterion and is expressed by the following equation

$$\Delta H_d = \frac{L_v + \frac{1}{3} L_h}{C_w} \tag{6.2}$$

where L_v is the total length of the vertical segments of the line of creep, L_h is the total length of the horizontal segments of the line of creep and C_w is again a factor depending on the soil type, named the weighted creep factor. The values of the weighted creep factor proposed for different soil types are listed in Table 6.2.

Importantly Bligh's and Lane's formulas define, for a fixed difference in hydraulic head, the minimum acceptable length of the creep line, i.e. a length that is considered safe enough for design. They do not define a limit state. In other words the two criteria incorporate an implicit safety factor whose magnitude is not clearly defined but is considered acceptable on the basis of empirical observations and experience. This has important implications for the reliability analysis. In fact reliability methods require the definition of a limit state in order to calculate the probability of failure. The two creep rules do not define a limit state but a safe state and therefore need some manipulation, as will be explained in Section 6.3.2 if they are to be used as performance functions in a reliability analysis.

Table 6.2. Values of weighted creep factors for different types of soils according to Lane (1935).

Soil Type	Weighted creep factor C_w (Lane 1935)
Very fine sand, silt	8.5
Fine sand	7
Medium sand	6
Coarse sand	5
Fine gravel	4
Medium gravel	3.5
Coarse gravel and cobbles	3
Boulders with some cobbles and gravel	2.5
Soft clay	3
Medium clay	2
Hard clay	1.8
Very hard clay	1.6

6.2.2 Sellemijer's model for sand

Sellmeijer developed a process-based mathematical model for piping in sand foundations of impermeable structures (Sellmeijer 1988, Sellmeijer & Koenders 1991, Koenders & Sellmeijer 1992). The original model refers to a structure built on a semi-infinite sand layer. In a subsequent evolution (Weijers & Sellmeijer 1993) the model was modified to match a geometry which is often encountered in practical cases, with a sand layer of finite thickness and a thin, impervious covering layer, representing a fine grained soil with very low hydraulic conductivity. Sellmeijer's model has been validated and calibrated through physical tests, displaying a good predictive ability, especially for fine grained sands. It differs from the two previously presented creep line criteria in that it defines a limit state, and it is hence suitable to be used in reliability calculations without requiring further adaptation. This criterion is currently adopted in the Dutch practice (TAW 1999, Vrijling & van Gelder 2000). Sellmeijer's model is a more advanced approach than the two empirical formulae by Bligh and Lane, but is only applicable to sand. Despite the latter being the type of soil which is most prone to erosion, for the development of the Reliability Rating System it is highly desirable to have a criterion which enables the calculation of the probability of failure for embankments also built on other soil types. For this reason Lane's criterion has been selected to be used in this work for the reliability calculations.

6.3 Reliability analyses

6.3.1 Input parameters: range of variation of the mean

6.3.1.1 Creep length

In order to develop the Reliability Rating System the probability of breaching must be calculated for a number of possible scenarios covering the range of variation of the relevant parameters that can be encountered in practice. As illustrated in the previous section Lane's criterion relates the admissible hydraulic head to the length of the creep line (giving different weights to its horizontal and vertical segments) and to the creep factor, which accounts for the susceptibility to piping of the soil in the foundation. For flood embankments the line of creep coincides with the base of the earthen structure and generally does not include vertical segments. The horizontal length of the creep line is a function of the embankment geometry and can be calculated if its crest width, height and the inclination of slopes are known.

In the Reliability Rating System different classes are defined for each relevant parameter and indicative reference values are assigned to each class. Reliability calculations are performed adopting the reference values of the mean and making reasonable assumptions on random variability according to literature information and engineering judgement. For the height of the embankment A five classes are defined, covering a range up to 5.5 m. The subdivision in classes and the correspondent reference values for the mean are listed in Table 6.3. The inclination of the slopes, which is indicated as the tangent of the slope angle α and expressed as a ratio of vertical to horizontal, is subdivided in three classes ranging from very steep slopes to slopes as mild as 1:6, as shown in Table 6.4. Symmetry of the embankments is assumed. The mean crest width $\mu(c)$ is always taken as 3 m. This measure, which makes the transit of vehicles possible, is representative of most embankments in the non-tidal sections of British rivers.

The combination of five reference heights, three reference slope inclinations and one assumed crest width leads to 15 different geometries, all producing a particular combination of difference in hydraulic head and length of the creep line. All the mentioned geometric quantities can be regarded as random variables with their own probability distribution, mean, standard deviation and auto-correlation. However here, for the sake of simplicity, rather than propagating the random fluctuation through the simple equation relating the geometric variables to the horizontal creep length L_h, the mean of L_h is approximated by the equation:

$$\mu(L_h) \cong \mu(c) + 2\frac{\mu(H)}{\mu(\tan\alpha)} \tag{6.3}$$

The probability distribution of L_h and its standard deviation are imposed according to the suggestion in the literature (Vrijling & van Gelder 2000), as detailed in section 6.3.2. The structure of random variability in space is neglected, for all parameters in the reliability calculations for the development of the Rating System, as discussed in Chapter 5.

Table 6.3. Height of the embankment: chosen class subdivision and the relative reference values of the mean used in reliability calculations.

Mean value of the height of the embankment $\mu(H)$ (m)						
Class	From	0	1.5	2.5	3.5	4.5
	To	1.5	2.5	3.5	4.5	5.5
Reference		1	2	3	4	5

Table 6.4. Considered range of variation for the embankment slopes inclination, with the chosen class subdivision and the relative reference values of the mean.

Mean value of the slopes inclination $\mu(\tan\alpha)$				
Class	From	3:2	1:3	1:6
	To	∞	3:2	1:3
Reference		3:4	1:2	1:4

In all analyses the loading condition corresponds to water level being equal to the crest level. The watertable on the landward side is taken as coincident with the ground level. Consequently a difference in hydraulic head equal to the embankment's height is applied through the foundation.

6.3.1.2 Weighted creep factor

The classification of soils as proposed by Lane (1935) and the relative creep factors are adopted in this calculation. In the reliability analysis the creep factor is a random variable with the mean equal to the value originally adopted in the deterministic analysis.

Eight types of soils have been considered in the analysis, as shown in Table 6.5. All fine grained soils have been grouped in one class with the exception of silt of low plasticity, i.e. soils labelled with MH in the Unified System (ASTM 1985), with a liquid limit *LL* less than 50. Assuming a prevalence of soft and medium clay in the alluvial deposits an indicative value of 2.5 has been chosen as the mean of the creep factor for fine-grained soils.

Table 6.5. Assumed values for the mean of the creep factor for different types of soil.

Soil Type		$\mu(C_w)$
Clay, Silt ($LL \geq 50$)		2.5
Sand	Very Fine, Silt ($LL < 50$)	8.5
	Fine	7.0
	Medium	6.0
	Coarse	5.0
Gravel	Fine	4.0
	Medium	3.5
	Coarse	3.0

It is important to notice that the resistance to erosion of silt is extremely variable and appears to be related to its plasticity. Fine-grained, cohesive soils seem to be relatively immune from piping erosion. This can be explained by the attractive inter-particle forces typical of fine grained soils. Zaslawsky and Kassiff (1965) developed a model that relates the resistance to piping of cohesive soils to their tensile strength. These observations would support a relatively high resistance to piping of silt, at least larger than the typically low resistance of purely granular soils. However the works by Bligh and Lane, not always reported correctly in recent publications, assign to silt the same high vulnerability to piping of very fine sand.

Clearly there must be a transition between the behaviour of fine sand, extremely prone to piping, and the behaviour of clay, extremely resistant to piping, which takes place in the particle-size window of silt. To the author's knowledge, though, this transition is not been clearly located. The detailed investigation of this point is beyond the scope of this research project. In the development of the reliability rating system the working hypothesis is made that, with regard to piping erosion, silts of low plasticity according to the Unified Soil Classification System (ASTM 1985), i.e. with a liquid limit less than 50, can be assimilated to very fine sands, while silts with a higher liquid limit can be treated as clays. These assumptions are in agreement with observations on the resistance to piping-erosion in embankment dams (Foster *et al.* 2000a, 2000b).

6.3.2 Performance function and random fluctuation of parameters

To perform the reliability calculations Lane's criterion needs to be put in the form of a performance function $g(x)$ which defines the limit state. Rearranging the original formula, and introducing two model factors, the following expression is obtained for the performance function (Vrijling & van Gelder 2000):

$$g(x) = m_L\left(L_v + \frac{L_h}{3}\right) - m_C C_w \Delta H \tag{6.4}$$

where m_L is a model factor applied to the weighted creep length and m_C is a model factor applied to the weighted creep coefficient. For flood embankments the creep line coincides with the base of the earthen structure and therefore L_v is null. Indications on the random variability of parameters can be found in the Dutch guidelines (Vrijling & van Gelder 2000). These recommendations, partly based on judgement and intuition, are summarised in Table 6.6. All the variables are considered to be mutually independent.

Lane's criterion does not define a limit state but a safe state and, in order to use it as a performance function, some modification is required. This is achieved through the model factor applied to the creep length m_L, which has a mean of 3, rather than the unit value usually adopted for model factors. This is intended to correct the original model in order to identify a limit state rather than a safe state. In other words it has been assessed that in a deterministic analysis the formula by Lane would give a safety factor for the difference in hydraulic head of about 3.

Table 6.6. Random variability of input parameters for the under-piping mode of failure as used in Dutch practice (Vrijling & van Gelder 2000).

Parameter	Probability distribution	Mean	Random Variability
L_h	Normal	-	$\Omega(L_h) = 0.1$
C_w	Normal	-	$\sigma(C_w) = 0.1$
Δh	Normal	-	$\sigma(\Delta h) = 0.1$
m_L	Lognormal	3	$\sigma(m_L) = 0.2$
m_C	Normal	1	$\sigma(m_C) = 0.2$
$\Omega(X)$ is the coefficient of variation of X $\sigma(X)$ is the standard deviation of X			

6.3.3 Reliability methods

For the under-piping mode of failure the reliability calculations have been performed using the First Order Reliability Method, with occasional checks using Monte Carlo simulations. Good agreement has been found between the two methods, taking into account accuracy issues for Monte Carlo simulation.

6.4 Results for homogeneous foundations

The probability of breaching induced by piping through the foundation has been calculated, in the top of bank condition, for the 15 geometries chosen as representative and for the 8 soil types identified as relevant. In each case the water level was taken to be at the crest level. The results for the 3 m high embankment are summarised in Table 6.7 and plotted in Figure 6.2. Figure 6.3 shows analogous diagrams for the 1 m high and the 5 m high embankment, the lowest and the highest respectively in the set of embankments chosen as representative.

The reliability analysis predicts in all cases extremely high probability of breaching for embankments directly founded on sand and silt of low compressibility ($LL \leq 50$). In practice, for geological reasons related to sediments deposition, further explained in the following chapter, this situation is rather uncommon and most flood embankments, especially in river sections where the bed is not particularly steep, will be founded on fine-grained cohesive alluvium. Existing embankments on coarse-grained soil prone to under-piping will most likely have been constructed with berms aimed at lengthening the creep line. The Reliability Rating System, for the sake of conciseness and simplicity, does not include features as berms among its pre-set scenarios. However in the case of embankments on soil prone to under piping the rating would correctly highlight a potential threat and force the user to check the actual situation in order to further investigate the safety of the structure.

Table 6.7. Probability of breaching by under-piping for a 3 m high embankment with water at the crest level.

Probability of failure by foundation erosion with water at crest level 3 m high embankment					
		Slope	**3:4**	**1:2**	**1:4**
Soil Type	Cohesive	Clay, Silt ($LL \geq 50$)	1.12×10^{-2}	2.82×10^{-5}	1.22×10^{-11}
	Sand	Very Fine, Silt ($LL<50$)	1.00	9.99×10^{-1}	3.66×10^{-1}
		Fine	1.00	9.82×10^{-1}	6.14×10^{-2}
		Medium	9.99×10^{-1}	8.74×10^{-1}	6.75×10^{-3}
		Coarse	9.73×10^{-1}	5.05×10^{-1}	2.18×10^{-4}
	Gravel	Fine	7.09×10^{-1}	8.70×10^{-2}	1.35×10^{-6}
		Medium	3.92×10^{-1}	1.54×10^{-2}	4.93×10^{-8}
		Coarse	1.13×10^{-1}	1.16×10^{-3}	1.03×10^{-9}

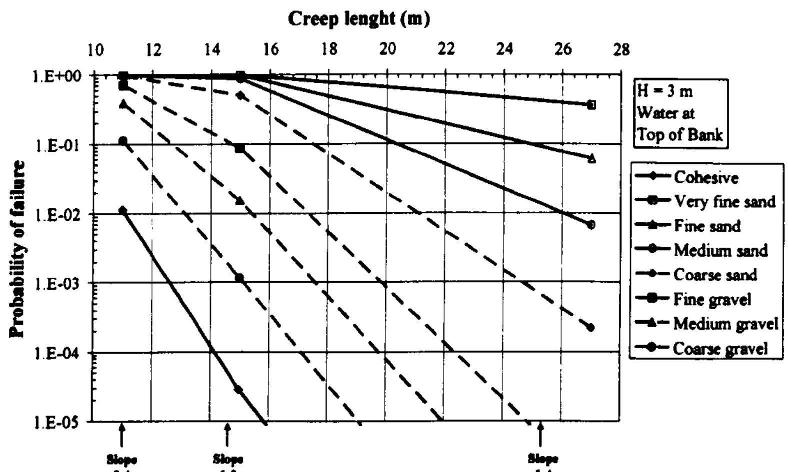


Figure 6.2. Probability of failure by piping through the foundation as a function of the creep length: 3 m high embankment with water at crest level.

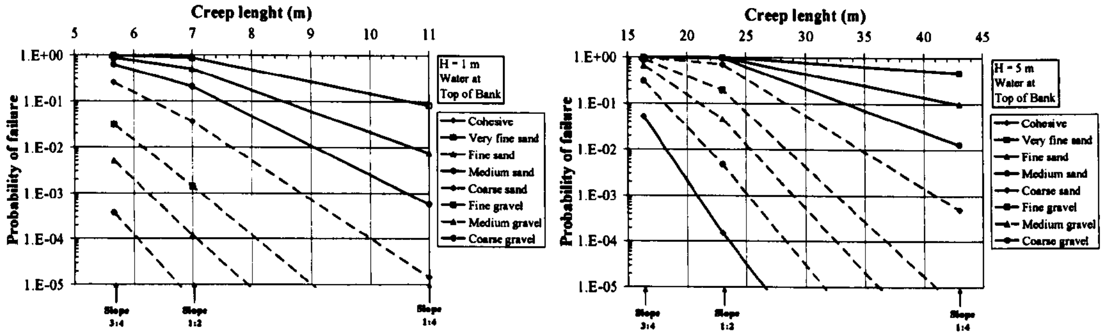


Figure 6.3. Probability of failure for piping through the foundation as a function of the creep length for 1 m and 5 m high embankments.

Higher embankments, which have a wider base and hence a longer creep line but also need to sustain a larger difference in hydraulic head across their foundation, are less safe, when the water level is equal to the crest level. This can be seen from Figure 6.2 and Figure 6.3. The full body of results regarding the probability of breaching by under-piping in a homogenous foundation is reported in Appendix A.

6.5 Conclusions

Under-piping is a mode of failure amenable to mathematical modelling and the reliability calculations needed for the development of the Reliability Rating System have been carried out with a performance function obtained from Lane's criterion (1935). A number of different scenarios have been analysed and the input parameters have been chosen in such a way as to represent the possible situations encountered in practice. Different geometries have been considered for the defence structure, resulting in different lengths of the creep line. The range of calculations also accounts for different types of soil underneath the embankment.

The adopted approach assumes that the embankment can be approximated as an impervious structure. This approximation is acceptable if the coefficient of permeability of the natural soil is significantly higher than the coefficient of permeability of the manmade fill. This condition is generally satisfied for good quality embankments built with well compacted fine-grained soils. The assumption may not hold for embankments constructed with coarse-grained soils or for poorly compacted fills. In these cases much of the seepage would occur within the defence body rather than in the foundation and the adopted approach would return an over estimated probability of failure by under-piping. However in such situations the probability of failure by through-piping would be rather high and the process of under-piping would not dominate the overall reliability.

The Reliability Rating System is conceived to be used in a first screening of the defence network. Where a high probability of breaching by under-piping related to coarse-grained founding soils is highlighted by the rating a subsequent analysis, incorporating details of the local situation and making use of Sellmeijer's criterion (Weijers & Sellmeijer 1993) is recommended.

7

UNDER-PIPING: LAYERED FOUNDATION

It is common for river embankments to be located on soil profiles which are characterised by a thin layer of fine-grained soil which covers a deeper layer of coarse-grained soil. In this setting the low permeability of the top fine-grained layer prevents significant seepage underneath the embankment. However, if the coarse-grained layer is in hydraulic communication with the watercourse, the pore pressure in it will respond rapidly to a flooding condition generating an uplifting force at the base of the covering layer. Such a force can lead to cracking of the fine grained soil, eventually creating a possible path for piping erosion. The analysis of this mode of failure requires the study of two sub-mechanisms: the cracking of the covering layer and the erosion of the coarse-grained layer once cracking has occurred.

7.1 Relevant soil profiles

The reliability calculations presented in Chapter 6 are based on a model that treats the foundation as a homogeneous soil. In reality all foundations are layered. However, due to the nature of the piping process the safety of the structure against backward erosion is dictated by the characteristics of only the shallow strata of the site. In a layered foundation fine-grained layers, characterised by low permeability and high resistance to erosion, alternate with coarse-grained layers, characterised by high permeability and possibly low resistance to erosion.

The most common geological setting in fluvial deposits in the UK is a superficial layer of fine-grained soil covering a deeper layer of coarse grained soil. In fact the coarse-grained layer is the often the result of deposition that occurred during the last glaciation, when lower sea levels induced higher hydraulic gradients and water velocities in rivers which, as a consequence, were transporting coarser sediments. The fine grained covering layer is the result of more recent deposition that has taken place in a regime of slower flows, with finer sediments transported in the same river section. However the stratigraphy of fluvial deposits can certainly be more complex than that: successive cycles of erosion and deposition can result in fluvial terraces, where older sediments lie at a higher elevation than more recent sediments. Also man-induced changes, due to excavation for quarrying or engineering works, cannot be excluded. In principle both coarse-grained and fine-grained soils can be found as the uppermost layer on which the embankment is built.

The analyses illustrated in the previous chapter, performed taking the base of the embankment as the creep line, are valid for two configurations: when the uppermost layer is fine grained and thick enough to prevent the coarse-grained deeper layer to outcrop in the river bed or when it is coarse grained. In the former case, sketched in Figure 7.1 no rapid or substantial changes are expected in the pore water pressure in the coarse-grained material during floods. In the latter case, sketched in Figure 7.2, the coarse-grained layer on which the embankment is built is highly permeable and can be prone to under-piping. The presence of fine-grained soil beneath it is irrelevant.

Figure 7.3 exemplifies the rather common situation mentioned in the opening of this section. A superficial fine-grained layer acts as an almost impervious barrier to seepage. The underlying coarse grained layer, however, outcrops in the riverbed establishing hydraulic communication with the water course. In such a setting the coarse-grained layer responds rapidly to flood water levels with an increase in pore pressure. If the resulting uplifting force at the base of the covering soil is sufficiently high a crack may form and create a preferential seepage path. The intense seepage can result in piping-erosion of the coarse-grained layer potentially leading to collapse.

In order to determine, in this geological setting, the probability of the flood embankment breaching as a result of foundation erosion the probabilities of the superficial layer cracking (called event C in the following) or not cracking (event $\overline{C}$) need to be combined with the probability of the coarse layer being eroded (event

CLE) given cracking occurs and the probability of the fine grained layer being eroded (event FLE) given no cracking occurs, according to the following formula.

$$P(B_{up}) = P(C) \times P(CLE|C) + P(\overline{C}) \times P(FLE|\overline{C}) \qquad (7.1)$$

where B_{up} is the event "failure by under piping".

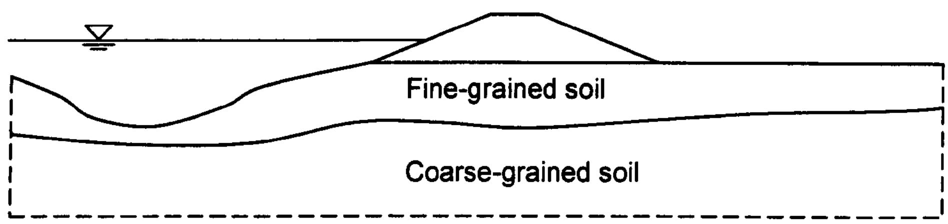


Figure 7.1. Fine grained covering layer sufficiently thick to isolate the subjacent coarse grained layer from the watercourse. The resistance to under-piping can be assessed with Lane's criterion taking the base of the embankment as the creep line. Very high resistance to under-piping is to be expected.

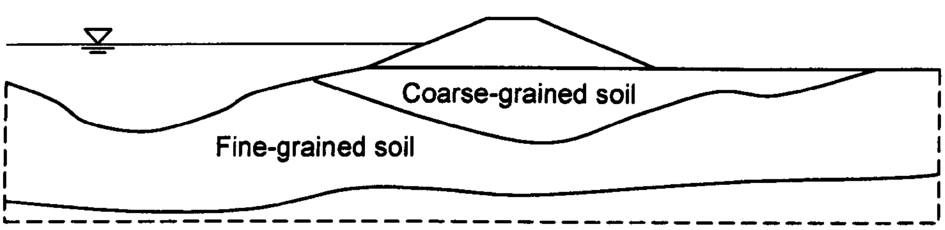


Figure 7.2. Embankment built on a coarse-grained layer. Although this is not the most common situation this setting can be found in presence of "river terraces". The resistance to under-piping can be assessed with Lane's criterion. Low resistance to under-piping is to be expected.

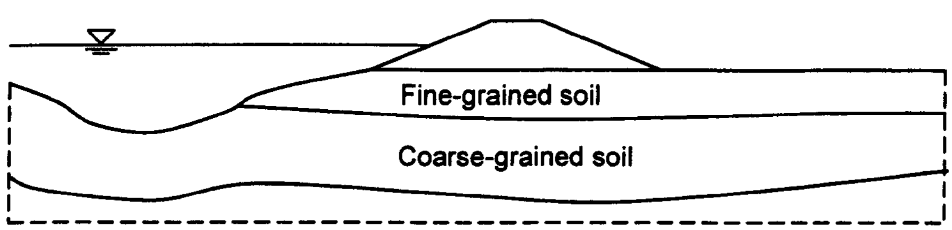


Figure 7.3. Fine-grained layer covering a coarse-grained layer in hydraulic communication with the river. In this rather common situation piping erosion of the deep layer can occur if the covering layer cracks due to the uplifting pore pressure at its base.

The probability of piping through a fine grained soil immediately under the embankment has already been calculated in the previous Chapter. The probability of piping through the underlying layer of coarse grained soil can be obtained with the same approach, only with a different creep line length because of the mutated geometry.

7.2 Probability of cracking of the covering layer

7.2.1 Mathematical model

The Dutch Technical Advisory Committee on Flood Defences (TAW 1999) suggests that cracks originate when the uplifting water pressure at the interface between coarse grained and fine grained soil overcomes the weight of the overlying ground. Assuming the covering layer is fully saturated the total vertical stress at the interface, σ_V, representing in this case the resistance of the system, is

$$\sigma_V = \gamma_{sat} \cdot d \qquad (7.2)$$

where γ_{sat} is the saturated unit weight of the fine grained covering soil and d is the layer thickness. For the uplifting induced cracking the loading on the system is represented by the pore water pressure u at the top of the coarse-grained layer. In order to check if the covering layer is subject to cracking this pore water pressure has to be calculated and compared with the total vertical stress expressed by Equation (7.2). Different models are available for studying the seepage in the two layered foundation of the embankment. An analytical model developed by TAW (1999) for stationary groundwater flow has been chosen for developing the Reliability Rating System as it is the most suitable compromise between simplicity and accuracy. The model, which assumes steady seepage and neglects the consolidation in the fine grained layer, is extensively described in TAW (2004). The approach is based on the scheme in Figure 7.4, where Zone 1 includes the portion of ground on the riverside, Zone 2 is the ground beneath the embankment and Zone 3 is indefinitely extended on the landward side. The model accounts for water flowing horizontally in the coarse-grained layer, from the river to the land, and vertically through the fine-grained layer. In particular the flow is oriented downward in Zone 1 and upward in Zone 3. The seepage in the embankment and in the covering layer in Zone 2 is considered negligible with respect to the flows in other parts of the ground and thus not included in the model. Writing the mass balance for the three zones and solving the equations with adequate boundary conditions leads to a solution for the hydraulic head h at the layers interface as a function of the horizontal coordinate x.

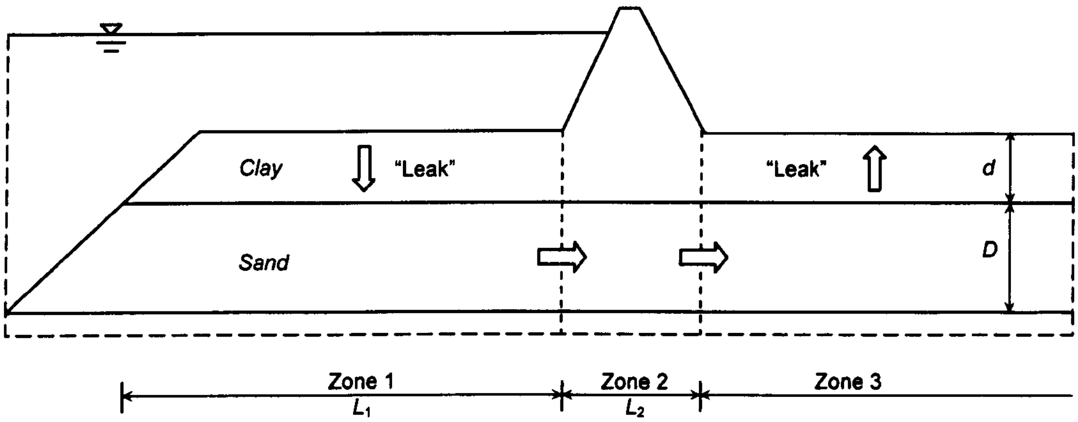


Figure 7.4. Scheme of the analytical model by TAW (1999) adopted in this work.

If the thickness of the covering layer is uniform the critical zone for cracking, where the pore pressure exceeds the total vertical stress, coincides with the toe of the embankments on the landward side, as illustrated in Figure 7.5. Therefore the analysis of this location is enough to determine the safety against cracking. According to the analytical model the hydraulic head at the interface of the layers under the toe of the embankment, which is obtained for $x = L_2$, is:

$$H_{toe} = H_3 + \Delta H \frac{\lambda_3}{L} \tag{7.3}$$

where H_3 is the hydraulic head on the landward side of the embankment, ΔH is the difference in hydraulic head across the foundation, λ_3 is the "leakage factor" for the covering soil in Zone 3 and L is a function of length and leakage factors in different zones. The leakage factors are defined as:

$$\lambda_i = \sqrt{\frac{k_{coarse} D d_i}{k_{fine}}} \qquad i = 1, 3 \tag{7.4}$$

where k_{coarse} is the coefficient of permeability of the coarse grained-soil, D is the thickness of the coarse grained layer, d_i is the thickness of the covering layer in the i-th Zone and k_{fine} is the permeability coefficient of the fine grained soil. The expression for L is given by the following equation:

$$L = \lambda_1 \tanh\left(\frac{L_1}{\lambda_1}\right) + L_2 + \lambda_3 \tag{7.5}$$

Once the hydraulic head in the critical location H_{toe} has been determined the value of the correspondent pore pressure u_{toe} immediately follows. Taking the ground level as the reference level the expression of the pore pressure is:

$$u_{toe} = H_{toe}\gamma_w + d \tag{7.6}$$

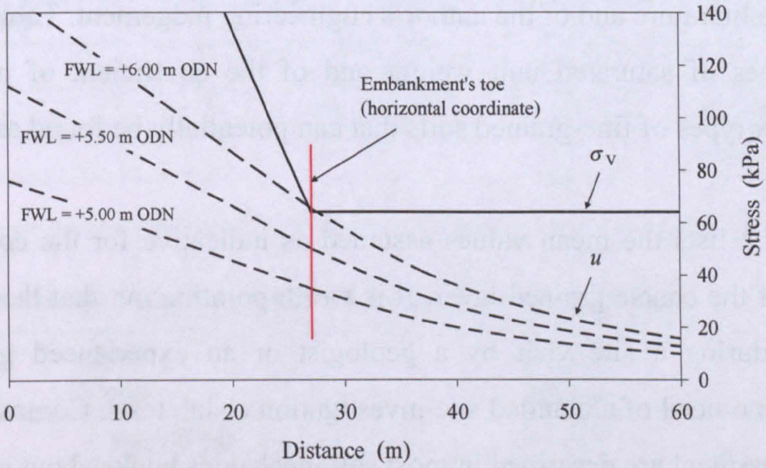


Figure 7.5. Example of pore pressure at the interface of the layers increasing with flood water level (FWL), which is measured in meters above the national reference elevation (Ordnance Datum Newlyn); the first portion of covering soil experiencing uplift and cracking is at the toe of the embankment.

7.2.2 Input parameters and simplifications

Summarising, from the previous section, it can be observed that for assessing the safety of the system against the rupture of the covering layer a number of parameters are needed. In particular in order to determine the effective total vertical stress at the interface between the two layers one needs to know:

γ_{sat} the saturated unit weight of the fine grained covering soil

d the thickness of the covering soil layer

In order to determine the pore water pressure at the critical location six additional variables are needed:

L_1 the width of the covering layer in front of the embankment

L_2 the base width of the embankment

k_{fine} the permeability coefficient of the covering soil

D the thickness of the coarse-grained layer

k_{coarse} the permeability coefficient of the coarse-grained soil

ΔH the difference in hydraulic head across the foundation

For the development of the reliability rating system reference values of the parameters have been assigned to different soil types on the basis of information available in the literature and of the author's engineering judgement. Table 7.1 shows the mean values of saturated unit weight and of the coefficient of permeability assumed for six types of fine-grained soils that can potentially be found as a covering layer.

Table 7.2 lists the mean values assumed as indicative for the coefficient of permeability of the coarse grained layer. It is worth pointing out that these soils can be identified during a site visit by a geologist or an experienced geotechnical engineer, with no need of a detailed site investigation or lab tests. Common methods for a rapid assessment are described in most soil mechanics books about geotechnics. Concise guidance is given for example by Craig (1997).

The base width of the embankment L_2 is dictated by the structures' geometry. The same classes and reference values for the mean of the embankment's height and slopes inclination already used in the previous chapter for the homogeneous foundation case are adopted. The extension of the covering layer in front of the embankment L_1 depends on the local geology and on the geometry of the river bed in the cross section. Here it is assumed, for the sake of simplicity, that the riverbank is inclined like the slope of the embankment, as illustrated in Figure 7.6. In this way the top layer in front of the embankment has a cross section shaped as a right triangle where the catheti measure L_1 and d. In the analytical model of seepage it is approximated with an equivalent rectangular section of uniform thickness equal to $0.5\ d$.

One more simplification, introduced to reduce the number of possible combinations to be studied, is the assumption that the thickness of the coarse-grained layer, involved in the seepage analysis, is equal to the thickness of the covering layer. This simplification has a limited impact on the results because the sensitivity of the seepage analysis to the parameter D is quite small. With such assumptions the reliability of the system has been studied for a range of different values of the mean thickness of the covering layer d.

Table 7.1. Indicative parameters for the fine grained covering layer

Soil Type	$\mu(\gamma_{sat})$ (kN/m^3)	$\mu(k_{fine})$ (m/s)
Peat	11	1×10^{-9}
Very Soft Clay	14	1×10^{-8}
Soft Clay	16	1×10^{-8}
Firm Clay	18	1×10^{-8}
Soft Silt	17	1×10^{-7}
Firm Silt	19	1×10^{-7}

Table 7.2. Reference permeability coefficients for the coarse grained layer.

Soil Type		$\mu(k_{coarse})$ (m/s)
Sand	Very Fine	5×10^{-6}
	Fine	1×10^{-5}
	Medium	5×10^{-5}
	Coarse	1×10^{-4}
Gravel	Fine	1×10^{-3}
	Medium	1×10^{-2}
	Coarse	1×10^{-1}

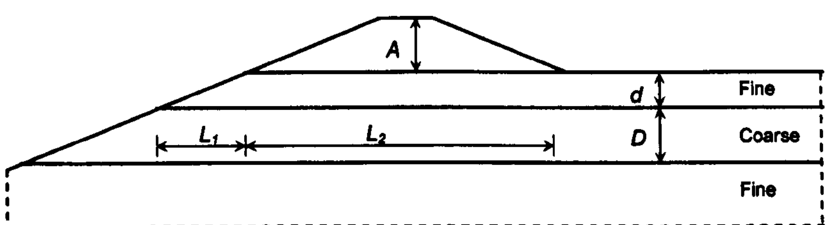


Figure 7.6. Geometry adopted in the reliability calculations.

7.2.3 Performance function and random fluctuation of parameters

The performance function adopted for the calculation of the probability of cracking of the covering layer is:

$$g(\mathbf{x}) = m_\sigma \sigma_V - m_u u_{toe} \tag{7.7}$$

where m_σ and m_u are model factors. The first model factor m_σ is introduced to represent the uncertainty in the resistance to cracking. In fact it can be observed that uplifting does not necessarily imply immediate cracking. However the author is not aware of more elaborate models in the literature. In the Netherlands, where the performance function is generally written in terms of hydraulic head, a model factor slightly larger than 1 on the critical hydraulic head for cracking is used to deal with this issue. Here a model factor with mean 1.2 has been applied directly on the total vertical stress. The second model factor m_u accounts for uncertainties in the groundwater seepage model. In Dutch practice this model factor has a unit mean value. In this work a lower mean value is proposed to account for the different hydrogeological setting of British rivers where discontinuous covering layers of limited extension offer some relief to the pore pressure built up.

Indication of the random variability of geometric variables can be found in the Dutch guidelines (Vrijling & van Gelder 2000) and has been embraced in this work. A coefficient of variation as high as 300% has been assumed for the coefficients of permeability of natural soils, in agreement with the geotechnical literature on the subject (Lumb 1974, Nielsen *et al.* 1973). The random variability of parameters used in the reliability calculations is summarised in Table 7.3. All variables are assumed to be mutually independent.

7.2.4 Monte Carlo simulation

The probability of uplift-induced cracking of the covering layer has been calculated using Mont Carlo simulation, with occasional checks carried out employing FORM. The number of trials has been chosen in such way to grant that the error would be less than 10% with a confidence level of 90% using random Monte Carlo sampling for all the calculated probabilities. This requirement has been satisfied for probabilities as little as 9×10^{-5} implying 3.01×10^{7} trials. Accuracy has then been further increased by repeating the analyses with Latin Hypercube sampling (Iman *et al.* 1980). The target value of 9×10^{-5} has been considered to be adequate in the context of the Reliability Rating System. For the through-piping mode of failure, addressed using subjective probabilities, 9×10^{-5} is the lowest probability of breaching that can be assigned to an embankment where the water level coincides with the crest level.

Table 7.3. Random variability of input parameters for modelling the cracking of the covering layer.

Parameter	Probability distribution	Mean	Random Variability
L_i	Normal	-	$\Omega(L) = 0.1$
d	Lognormal	-	$\Omega(d) = 0.3$
γ_{sat}	Normal	-	$\Omega(\gamma) = 0.05$
D	Lognormal	-	$\Omega(D) = 0.1$
k_{fine}, k_{coarse}	Lognormal	-	$\Omega(k) = 3.0$
m_σ	Lognormal	1.2	$\sigma(m_\sigma) = 0.1$
m_u	Normal	0.85	$\sigma(m_u) = 0.1$
$\Omega(X)$ is the coefficient of variation of X $\sigma(X)$ is the standard deviation of X			

7.2.5 Preliminary results and further simplifications

In order to keep the number of reliability calculations to a minimum, the sensitivity of the probability of cracking to changes in the mean of various parameters has been explored to asses the possibility of neglecting the influence of some of them. Figure 7.7 shows the probability of cracking as a function of the mean thickness of the covering layer for three different hypotheses on the mean inclination of the embankment's slopes. The probability of cracking is lower for milder slopes, which imply longer seepage paths, but the difference is in practice negligible.

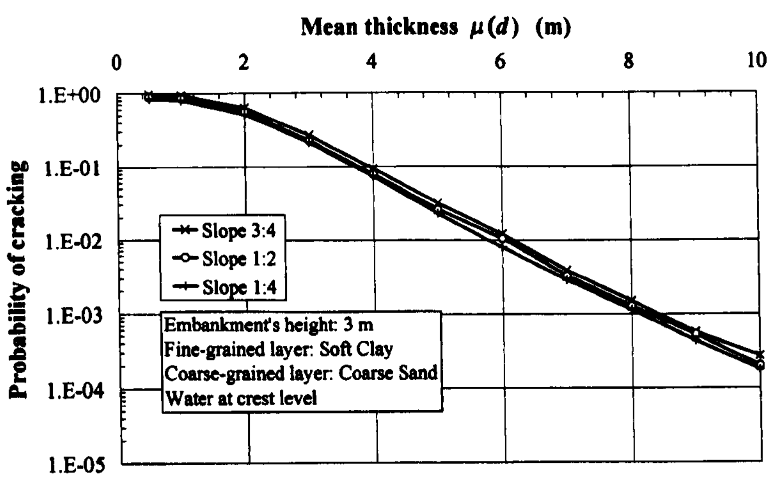


Figure 7.7. The influence on the probability of cracking of the angle of the slopes is not large.

Figure 7.8 shows the probability of cracking as a function of the mean thickness of the covering layer for three different hypotheses on the mean permeability coefficient of the coarse-grained layer. The probability of cracking is lower for lower coefficients of permeability. However the influence of this parameter is, again, irrelevant for practical purposes. For these reasons the effect of variations in the mean of the slopes inclination and in the mean of the coefficient of permeability has been ignored in the parametric studies for the development of the reliability rating methodology. The calculation of the probability of cracking has been hence performed assuming the mean of the slopes inclination is equal to 1:2 [i.e. $\mu(\tan\alpha)$ = 0.5] and adopting for the coarse grained layer a mean of the coefficient of permeability typical of coarse sand [i.e. $\mu(k_{coarse}) = 1 \times 10^{-4}$ m/s].

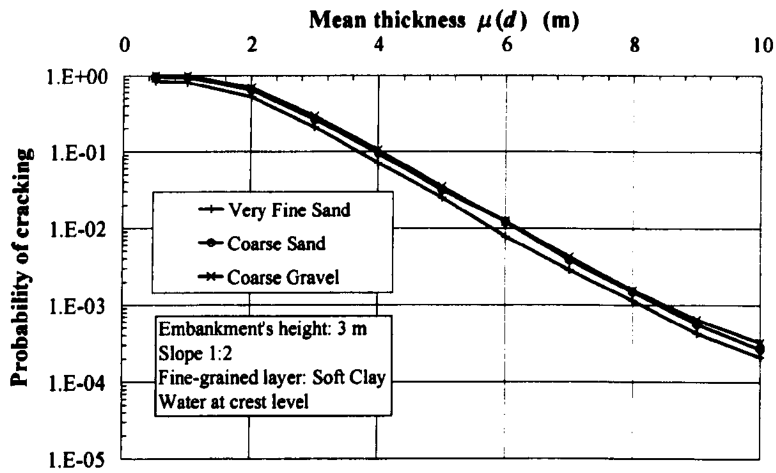


Figure 7.8. The influence on the probability of cracking of type of coarse-grained soil is not large.

7.2.6 Results

The probability of uplifting-induced cracking of the covering layer has been calculated and plotted against the mean thickness of the fine-grained layer, for the five representative heights and assuming that the water level equals the crest level. The results for the 3 m high embankment are shown in Figure 7.9 and reported in

Table 7.4.

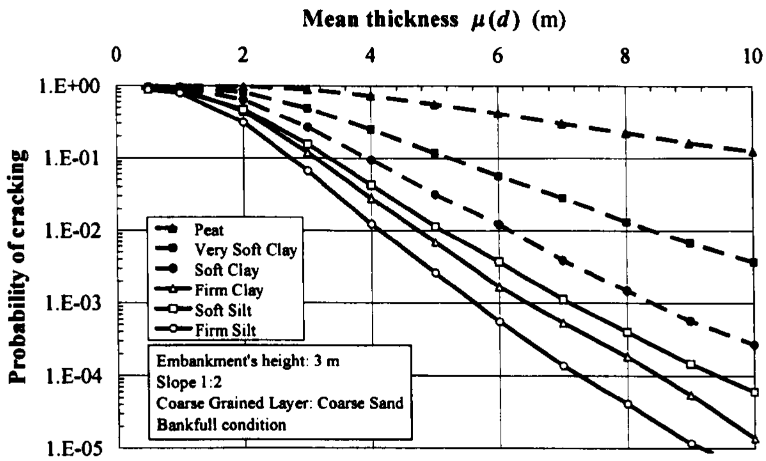


Figure 7.9. Probability of cracking of the covering layer as functions of its average thickness, 3 m high embankment in top of bank condition.

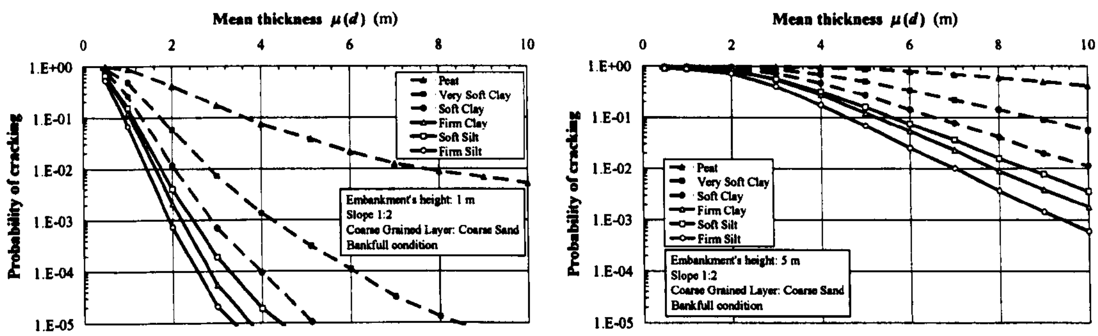


Figure 7.10. Effect of the embankment's height: in top of bank conditions the probability of cracking of the covering layer is significantly larger for higher embankments.

These results clearly indicate that knowledge of the type of soil in the covering layer is essential in order to asses the safety against cracking of a layered foundation. The comparison in Figure 7.10 between the embankment with a mean height of 1 m and that with a mean height of 5 m shows how, when the water level in the river is at the crest level, the probability of the covering layer cracking is significantly larger for higher embankments. The full body of results for the cracking sub-mechanism of failure is reported in appendix B.

Table 7.4. Probability of uplift-induced cracking of the covering layer for a 3 m high embankment with water at crest level.

Probability covering layer cracking – 3 m high embankment, top of bank						
$\mu(d)$ (m)	Fine-grained soil type					
	Peat	Very Soft Clay	Soft Clay	Firm Clay	Soft Silt	Firm Silt
0.5	9.94×10^{-1}	9.71×10^{-1}	9.65×10^{-1}	9.57×10^{-1}	8.87×10^{-1}	8.78×10^{-1}
1.0	9.93×10^{-1}	9.62×10^{-1}	9.45×10^{-1}	9.09×10^{-1}	8.43×10^{-1}	7.91×10^{-1}
2.0	9.79×10^{-1}	8.14×10^{-1}	6.40×10^{-1}	4.47×10^{-1}	4.71×10^{-1}	3.09×10^{-1}
3.0	8.96×10^{-1}	4.93×10^{-1}	2.67×10^{-1}	1.20×10^{-1}	1.57×10^{-1}	6.64×10^{-2}
4.0	7.31×10^{-1}	2.49×10^{-1}	9.45×10^{-2}	2.79×10^{-2}	4.24×10^{-2}	1.22×10^{-2}
5.0	5.62×10^{-1}	1.18×10^{-1}	3.16×10^{-2}	6.90×10^{-3}	1.13×10^{-2}	2.61×10^{-3}
6.0	4.18×10^{-1}	5.68×10^{-2}	1.22×10^{-2}	1.67×10^{-3}	3.70×10^{-3}	5.52×10^{-4}
7.0	3.01×10^{-1}	2.84×10^{-2}	3.86×10^{-3}	5.31×10^{-4}	1.12×10^{-3}	1.36×10^{-4}
8.0	2.27×10^{-1}	1.31×10^{-2}	1.48×10^{-3}	1.84×10^{-4}	3.97×10^{-4}	4.07×10^{-5}
9.0	1.64×10^{-1}	6.75×10^{-3}	5.65×10^{-4}	5.44×10^{-5}	1.45×10^{-4}	1.17×10^{-5}
10.0	1.27×10^{-1}	3.68×10^{-3}	2.68×10^{-4}	1.38×10^{-5}	6.04×10^{-5}	4.67×10^{-6}

7.3 Probability of piping given cracking

7.3.1 Mathematical model

If cracking occurs piping erosion can develop in the coarse grained layer along the contact with the covering layer. The resistance to under-piping can be studied using Lane's criterion (1935) as was the case for the uniform foundation case. The seeping water enters the erodible soil from the riverbed and exits through the crack, which is assumed to be vertical and to emerge at the toe of the embankment. The length of the line of creep in the horizontal direction is given by the length of the horizontal interface between layers in front of the embankment, L_1 in Figure 7.6, plus the width of the embankment, L_2. It is worth pointing out that, while the slope's inclination has been neglected in the analysis of uplifting-induced cracking, it is important to include it in the analysis of piping erosion because it has a significant influence on the creep length.

There are no vertical traits of the creep line within the coarse-grained soil. However an additional loss of hydraulic head must be included because of the flow in the vertical crack. Experimental tests (Sellmeijer 1981) show that a good approximation for the difference in hydraulic head across the crack is:

$$\Delta H_{crack} = 0.6d \tag{7.7}$$

Adding this further contribution to the under-piping to Equation (6.4), previously used for the uniform foundation case, the following performance function is obtained:

$$g(\mathbf{x}) = m_L \frac{L_h}{3} + 0.6d - m_C C_w \Delta H \tag{7.8}$$

Reliability analyses have been performed, with the combination of input parameters already introduced in Chapter 6, using Monte Carlo simulations, with the same level of accuracy established for the analyses in Section 7.2.4.

7.3.2 Results

The probability of piping given cracking of the covering layer has been determined for the already identified 5 representative heights, 3 inclinations of embankment slopes and 7 types of soil in the deep layer. A mean thickness of the covering layer up to 10 m has been considered. The results for the embankment with a mean height of 3 m and slopes inclined 1:2 are reported in Table 7.5 and plotted in Figure 7.11. These results clearly indicate that knowledge on the type of soil in the coarse-grained layer is essential to asses the safety against piping of a layered foundation.

Figure 7.12 compares embankments with the same slope inclination but different heights. The diagrams show how, under top of bank conditions the probability of piping through the same soil type is significantly larger for higher embankments, if the top layer cracks. Figure 7.13 compares embankments with the same height but different slope inclinations. The diagrams show how, in top of bank conditions the probability of piping through the same soil type is significantly larger for steeper embankments, if the top layer cracks. The full body of results for the probability of breaching by under-piping given uplift-induced cracking of the covering layer is reported in appendix B.

Table 7.5. Probability of piping given cracking for a 3 m high embankment, slope 1:2, water level at crest level.

Probability of piping given cracking 3 m high embankment, slope 1:2, top of bank							
	Coarse-grained soil type						
$\mu(d)$ (m)	**Sand**				**Gravel**		
	Very Fine	**Fine**	**Medium**	**Coarse**	**Fine**	**Medium**	**Coarse**
1.0	9.90×10^{-1}	8.69×10^{-1}	5.60×10^{-1}	1.61×10^{-1}	8.73×10^{-3}	7.84×10^{-4}	2.97×10^{-5}
2.0	9.35×10^{-1}	6.00×10^{-1}	2.33×10^{-1}	2.99×10^{-2}	5.68×10^{-4}	3.26×10^{-5}	6.67×10^{-7}
3.0	7.62×10^{-1}	3.02×10^{-1}	6.51×10^{-2}	3.88×10^{-3}	5.33×10^{-5}	1.51×10^{-6}	
4.0	5.15×10^{-1}	1.12×10^{-1}	1.32×10^{-2}	4.28×10^{-4}	1.67×10^{-6}		
5.0	2.75×10^{-1}	3.20×10^{-2}	2.35×10^{-3}	3.83×10^{-5}			
6.0	1.17×10^{-1}	7.65×10^{-3}	3.23×10^{-4}	3.83×10^{-6}			
7.0	4.55×10^{-2}	1.62×10^{-3}	3.83×10^{-5}				
8.0	1.41×10^{-2}	2.93×10^{-4}	3.98×10^{-6}				
9.0	3.22×10^{-3}	5.83×10^{-5}					
10.0	7.43×10^{-4}	7.41×10^{-6}					

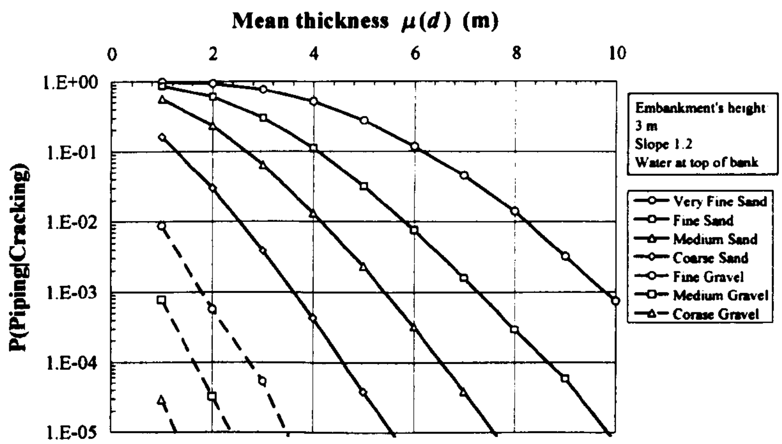
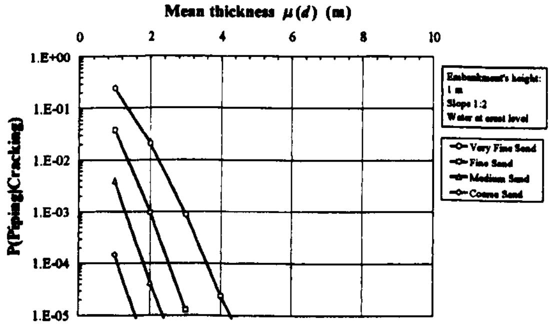
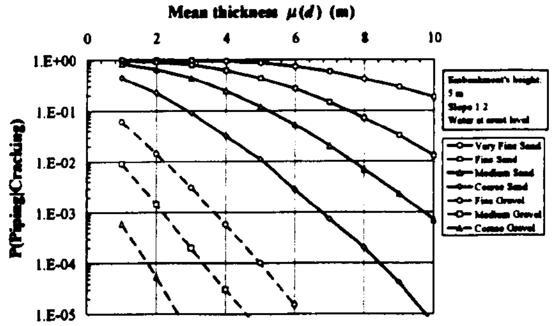


Figure 7.11. Probability of under-piping given cracking versus average thickness of the covering layer: 3m high embankment, slopes inclination 1:2.





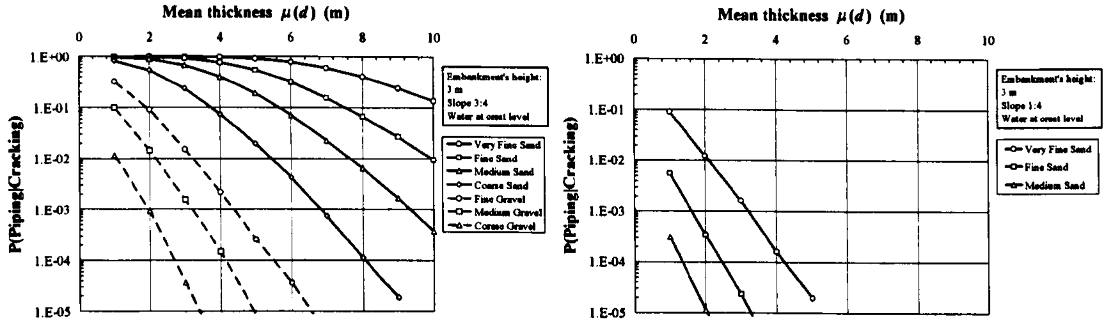


Figure 7.13. Effect of the slope angle.

7.4 Under-piping in the Reliability Rating System

The Reliability Rating System enables the final user to quantify the probability of structural failure induced by under-piping in the reference loading condition (water at the crest level) for both a homogenous and a layered foundation. However it is important to recognise in which cases the layering of the soil profile is relevant and can result in the cracking/piping behaviour. For this purpose the flow chart in Figure 7.14 synthesises the distinction between different soil profiles explained in Section 7.1.

In order to retrieve the correct probability of failure the final user must have some information on the local soil profile. Therefore an additional table has to be included in the methodology alongside the one presented in Section 4.4. The characteristics affecting the resistance to under-piping and the relative subdivision in classes chosen for the development of the Reliability Rating System are listed in Table 7.6.

Table 7.6. Characteristics affecting the resistance to under-piping.

Characteristic	Classes							
INFORMATION ALWAYS REQUIRED								
Type of soil in the covering layer	peat	very soft clay	soft clay	firm clay	soft silt	firm silt		
	very fine sand	fine sand	medium sand	coarse sand	fine gravel	medium gravel	coarse gravel	
INFORMATION REQUIRED FOR A LAYERED FOUNDATION								
Type of soil in the coarse-grained layer	very fine sand	fine sand	medium sand	coarse sand	fine gravel	medium gravel	coarse gravel	
Covering layer thickness	[0.5, 1.5)	[1.5, 2.5)	[2.5, 3.5)	[3.5, 4.5)	[4.5, 5.5)	[5.5, 6.5)	[6.5, 7.5)	[7.5, 8.5)

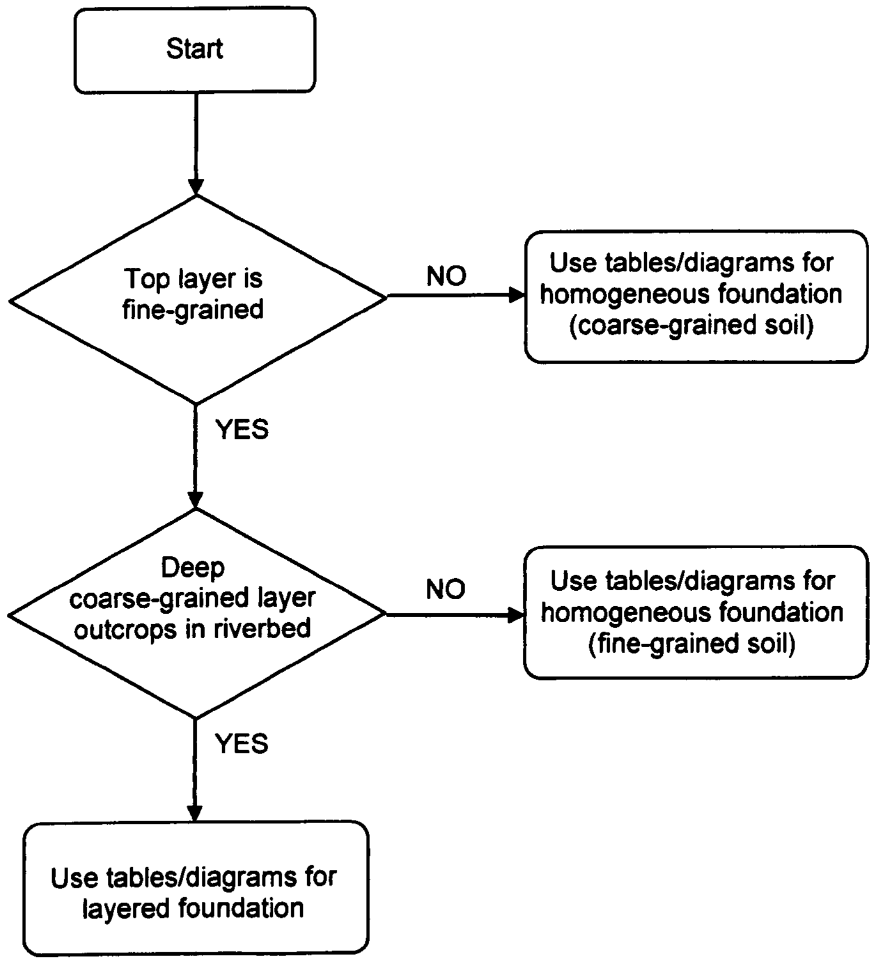


Figure 7.14. Flow chart for the determination of the probability of breaching induced by under-piping within the Reliability Rating System.

7.5 Conclusion

In order to determine the probability of under-piping in a layered foundation where a fine-grained superficial layer covers a deeper coarse-grained layer which is in hydraulic communication with the body of water two sub-mechanisms have to be considered: the cracking of the covering layer due to uplifting pore water pressure and the piping erosion of the coarse-grained layer.

The study of the uplifting induced cracking of the superficial layer requires a seepage analysis for the determination of the pore pressure at the layers interface. In the present work an analytical model, developed by TAW (1999) has been adopted for this purpose. The model assumes steady seepage and neglects the consolidation process in the fine-grained layer. The conditional probability of piping-erosion of the coarse-grained layer given cracking has occurred has been studied using Lane's criterion (1935) in analogy with what was already done for the case of a uniform foundation.

A number of reliability analyses have been performed to cover a wide range of scenarios that can be encountered in practice. In particular the end user can asses the effect of the embankment geometry, of different combinations of soil types in the covering layer and in the underlying coarse-grained layer, and of the thickness of the covering layer. The results presented show how the knowledge of the soil profile is essential to the evaluation of the safety of flood embankments against foundation erosion. In particular different types of soils in the top and deep layer and different depths of the interface between the two strata lead to totally different responses of the foundation during flood events. Most of this information is likely to be uncertain when attempting a first assessment of the reliability of the flood defence. The Reliability Rating System with its pre-calculated results, offers a tool for relating the uncertainty in the performance prediction to the uncertainty in the soil profile.

Where the Reliability Rating System highlights a significant probability of cracking and piping in a layered foundation it is recommended that a more detailed analysis is performed. In this second step numerical techniques can be adopted for solving the seepage problem, possibly in transient conditions. This allows the evaluation of the response of groundwater to a realistic flood hydrograph describing the time history of water levels in the body of water. The resistance to erosion of the coarse-grained layer should then be checked with Sellmeijer's criterion (Weijers & Sellmeijer 1993).

8

SURFACE EROSION

Surface erosion is, like under-piping, a breaching mode amenable to mathematical modelling and can be studied with the traditional methods of reliability analysis. The whole Reliability Rating System is designed to make use of only two loading conditions: one is the water level equal to the crest level; the other is a reference water level for overtopping. This approach is currently necessary because a credible quantification of the probability of breaching by through-piping for an arbitrary water level is not available. When an estimate of the probability of through piping will be available for all water levels the definition of reference loading will not be of concern. In fact the probability structural response for each failure mode will be defined for the full range of loads and a complete reliability analysis will be possible. For the time being, however, the use of a reference loading for overtopping appears to be necessary to obtain a measure of the flood embankments' performance which combines the different failure modes. In principle the reference water level for overtopping is rigorously defined as a water level with assigned return period, as explained in Chapter 4. However the exact determination of such a water level is not possible with the limited information typically available in a first assessment of the flood defence network. For this reason a simplified approach is needed and a degree of subjectivity, which influences the balance in the contribution of the different failure modes on the performance indicator, has to be introduced.

8.1 Introduction

In Chapter 4, where the formulation of the Probability of Breaching in Extreme hydraulic conditions is given, the reference water level for overtopping is defined by Equation (4.7) as the water level whose annual exceedance probability is half of the annual exceedance probability of the crest level. The adoption of such reference water level for calculating the probability of a breaching by surface erosion gives to the Probability of Breaching in Extreme hydraulic conditions P_{BE} a specific meaning: under the mentioned hypothesis, in fact, P_{BE} is an approximation of the probability of structural failure related to a defined range of water level in proximity of the crest.

However, in overtopping conditions, the determination of a water level of assigned exceedance probability is problematic. Hydrological methods are available to estimate the flow with assigned return period. Nevertheless this flow cannot easily be converted in water levels because the stage-discharge relationship, commonly used when the flow is channelled in one main direction, does not hold in presence of lateral flow over the defences which results in loss of water from the river.

In a watercourse the overtopping of a flood defence results in unloading of other defences, i.e. a lower flow in other river sections. The unloading can be very pronounced in case of breaching of a defence resulting into a conspicuous release of water. As a result, in order to establish the frequency of an overtopping water level, the water profile in an open channel has to be studied in presence of lateral loss of flow. The modelling of flood propagation is in principle feasible but requires the knowledge of data - like the topography of river sections and the hydraulic roughness of the riverbed and banks - that are unlikely to be available in the stage of preliminary assessment. For this reason a simplified procedure, involving the use of a growth curve for estimating the flow frequency, of an approximated stage-discharge relationship and some judgement-based assumptions is employed in this work. The presented approach, although adopting drastic simplifications, is able to incorporate the characteristics of the river section and of the catchment in the performance quantification.

This chapter discusses initially the mathematical model adopted for the reliability calculations in order to explain the data requirements for the probabilistic analysis. Then the simplified approach developed for establishing the loadings, in term of size and duration of the overtopping, is illustrated. Finally the details of the reliability analysis are given and the main results are explained.

8.2 Mathematical model for grass cover failure

8.2.1 Limiting velocity

When an embankment is overtopped the flow of water over the crest and the landward slope can lead to the dislodgement of the grass cover. It is here assumed that, once the bare soil is exposed, the erosion rate is sufficiently high to rapidly, and certainly, lead to breaching. Under this assumption the probability of breaching can be approximated by the probability of dislodgement of the turf.

Only the breaching initiated by grass cover failure is considered in this work. Other situations – like presence of revetments or bare soil –, although less common, are possible but have not been included in the Reliability Rating System at this stage.

CIRIA (Whitehead 1976) established three curves which identify the resistance of grass to steady water flow as a function of water velocity and flow duration. These curves, which are associated with qualitative descriptors of the grass cover condition (good/normal/poor), are reproduced in Figure 8.1. Resistance of grass to steady flow according to CIRIA (Whitehead 1976).. Despite its age this remains the most credible reference for grass erosion due to steady water flow (Young 2005).

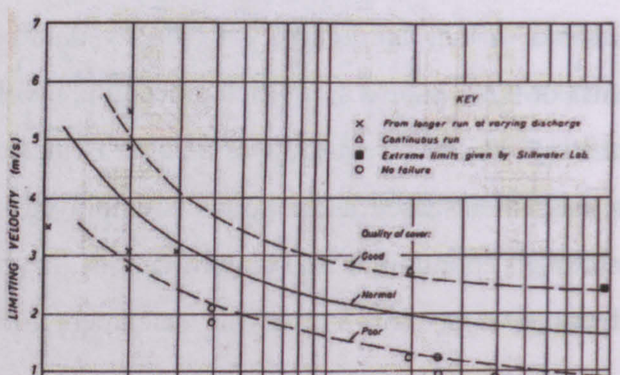


Figure 8.1. Resistance of grass to steady flow according to CIRIA (Whitehead 1976).

More recently large testing programmes were performed in the Netherlands to study the strength of dike grassland subject to wave attacks. However this body of research regards the effect of cyclical wave overtopping and is not tailored to the continuous water flow experienced by fluvial embankments. In Dutch practice, and in the computer program PC-Ring, the limiting velocity for failure of the grass cover on the landward slope is given by the equation (Vrouwenvelder *et al.* 1999, 2001a, 2001b; Vrijling & van Gelder 2000):

$$v_c = f_g \frac{3.8}{1+0.8\log_{10} t} \tag{8.1}$$

where f_g is a coefficient depending on the condition of the grass and t is the overtopping duration. Recommended values for the coefficient of grass cover condition are:

$f_g = 0.7$	for bad turf
$f_g = 1.0$	for normal turf
$f_g = 1.4$	for good turf

Equation (8.1), or a slightly more elaborate formula proposed by Vrouwenvelder *et al.* (2001)[8] are currently employed in Great Britain (Buijs *et al.* 2007) and in the rest of Europe (Allsop *et al.* 2006) to study the reliability of fluvial embankments in overtopping conditions.

In Figure 8.2 the critical velocity given by Equation (8.1) is compared it with the curves by CIRIA. The diagram shows that Equation (8.1), when applied to continuous flow, is significantly over-conservative for short overtopping durations. In this study, in order to improve the quality of predictions without radically changing the current approach, the limiting velocity has been expressed with the same functional form and the same coefficients for grass condition but recalibrating the parameters of Equation (8.1) to obtain a better fit with the CIRIA curves. The resulting equation is

$$v_c = f_g \frac{5.4}{1+1.5\log_{10} t} \tag{8.2}$$

The comparison with the curves proposed by CIRIA is shown in Figure 8.3.

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[8] The formula proposed by Vrouwenvelder *et al.* (2001) links the resistance to factors like the grass root depth and the strength of the underlying soil.

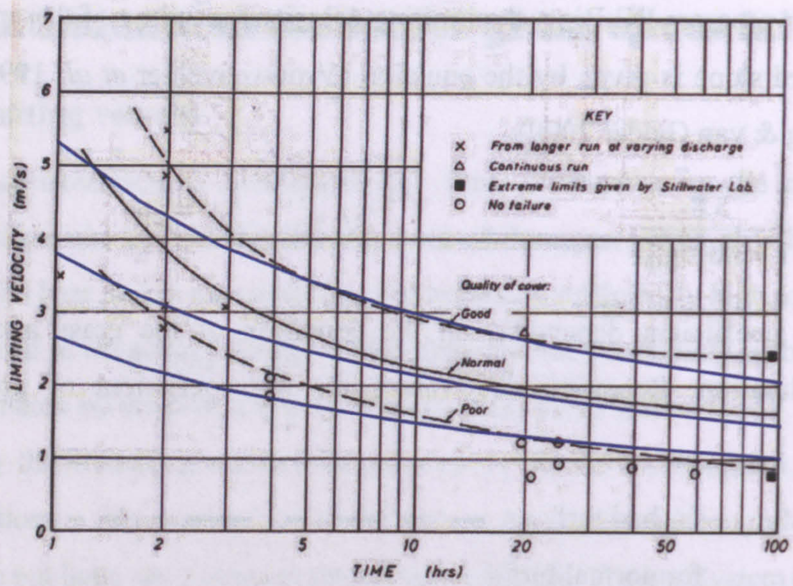

Figure 8.2. Curves for grass cover resistance to water flow used in Dutch and European practice compared with the curves proposed by CIRIA (Whitehead 1976).

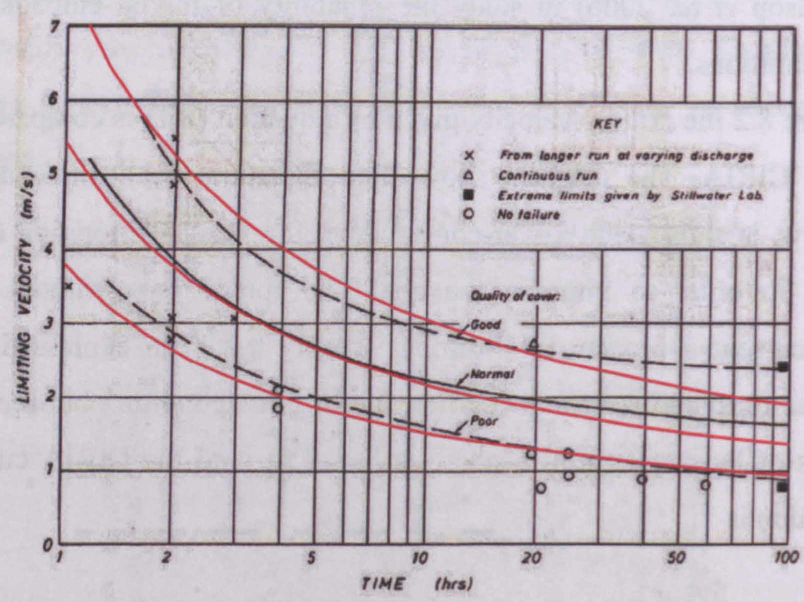

Figure 8.3. Curves for grass cover resistance to continuous water flow used in this research project compared with the curves proposed by CIRIA (Whitehead 1976).

The description of the grass conditions and the meaning of "bad", "normal" and "poor" turf are detailed in the already mentioned Technical Note from CIRIA

(Whitehead 1976) and should be used to relate on-site observations to the reliability assessment.

8.2.2 Limiting flow and actual flow

Once the critical velocity for grass resistance to erosion is known the correspondent critical flow can be determined. The critical discharge for a unit length of embankment q_c is, from uniform flow hydraulics:

$$q_c = \frac{v_c}{C^2 \tan\alpha} \tag{8.3}$$

where C is Chezy's coefficient and α is the angle of the landward slope. The Chezy's coefficient can be estimated for fully submerged grass (Ree 1949) as:

$$C = \frac{k_s^{1/6}}{25} \tag{8.4}$$

where k_s , expressed in meters, is the equivalent sand grain roughness defined by Nikuradse[9] (1933). Substituting Equation (8.4) in Equation (8.3) leads to the formula which is commonly employed to asses the critical discharge for failure of the grass cover:

$$q_c = \frac{v_c^{\frac{5}{2}} k_s^{\frac{1}{4}}}{125 \tan\alpha_i^{\frac{3}{4}}} \tag{8.5}$$

The actual discharge q_a can be found assimilating the embankment to a broad crested weir. This gives:

$$q_a = 0.385 \cdot C_V \sqrt{2g\Delta h^3} \tag{8.6}$$

where C_V is the coefficient of velocity, from the theory of discharge. In this case a value of 0.94 is appropriate for C_V.

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[9] The reader should be aware that many publications on earth flood defences reliability erroneously refer to k_s in Equation (8.5) as the roughness coefficient according to Strickler. The mistake becomes obvious if one writes the formula proposed by Strickler for C and compare it with the (8.4).

The safety of a flood embankment against surface erosion a flood is assessed comparing the actual discharge with the critical discharge for grass cover dislodgement. Summarising, for the illustrated models, the following five variables are needed:

Δh	height of water over the crest;
t	overtopping duration;
f_g	coefficient of grass cover condition;
α	angle of the landward slope of the embankment;
k_s	equivalent sand grain roughness of the submerged grass.

The first step in the development of the grass erosion failure mode for the Reliability Rating System is to define the range of hydraulic loads to be considered in the development of the methodology.

8.3 Simplified analysis of water levels

8.3.1 Reference water level for overtopping

In the formulation of the performance indicator P_{BE} the reference water level for overtopping is defined as the water level whose annual exceedance probability AEP_{ot} is half of the annual exceedance probability of the crest level AEP_{crest}. Section 8.1 has explained as the determination of such a water level is problematic. In the following text a simplified procedure that has been adopted to select the loads to be used in the development of the Reliability Rating System is described. In the adopted approach the ambition of modelling the actual flood water level is abandoned and a simpler strategy where deeper sections are treated as more likely to experience higher heights of water over the crest is embraced.

8.3.2 Difference in water levels between two events with assigned annual exceedance probability

The determination of the water height above the crest is a special case of the determination of the difference between two water levels with assigned annual exceedance probability (or in a totally equivalent way with assigned return period). Such a difference, for discharges conveyable by the river, can be estimated in two steps. First, with a growth curve, the frequency of flows in a section is determined. Then with a stage-discharge relationship each flow is linked to a water level.

The National Environment Research Council proposed (NERC 1975) growth curves based on the assumption that maximum annual floods are distributed according to a General Extreme Value distribution (GEV). The curve for flood frequency analysis is then expressed by the following Equation:

$$\frac{Q}{\overline{Q}} = u + \alpha\left(\frac{1 - e^{-\kappa y}}{\kappa}\right) \tag{8.7}$$

where Q denotes the T-year flood flow; $\overline{Q}$ is the mean annual flood; u, α and κ are parameters that can be estimated from historical data. Finally y is the standard Gumbel reduced variable, defined by:

$$y = -\ln\left(\ln\frac{T}{T-1}\right) \tag{8.8}$$

where T is the return period. Parameters calibration is available for 10 different regions in the UK. Using all national records the following values were obtained for the Great Britain growth curve: u = 0.80; α = 0.24; κ = -0.20. The national growth curve can be used for a first estimate of flow frequency when the mean annual flood for a river section is known.

In order to convert flows into water levels one needs to know the stage-discharge relationship, also called rating curve, of a river section. The stage-discharge relationship can be determined by experimental observations at gauged sections or calculated, generally assuming uniform flow, in other river sections. In order to apply this latter option the topography of the riverbed has to be known, as well as its hydraulic roughness and its slope in the direction of flow. This detailed information is seldom available at the preliminary stage of flood defences safety assessment for which the Reliability Rating System is conceived. Therefore a simpler approach has to be used for the present purpose. A common way of expressing the stage-discharge relationship is to use an interpolating equation. If the most depressed point of the cross section is taken as reference for the water level h, equations in the form

$$Q = ah^m \tag{8.9}$$

where a is a constant, are commonly used. In the following text it is shown how, knowing the growth curve and the stage-discharge relationship it is possible to determine the difference in water levels between two events of assigned annual exceedance probability.

Let the two events indicated by A and B and the difference in water levels be written as:

$$\Delta h = h_A - h_B \tag{8.10}$$

where both h_A and h_B are water levels that can be conveyed by the river without the occurrence of any overtopping occurring. From the interpolating rating curve of Equation (8.9) the higher, less frequently exceeded water level is:

$$h_A = \left(\frac{Q_A}{a}\right)^{1/m} \tag{8.11}$$

An expression for the parameter a can be found considering the lower water level h_B, for which the same rating curve is valid:

$$a = \frac{Q_B}{h_B^m} \tag{8.12}$$

Substituting Equation (8.12) into Equation (8.11) an expression for h_A is found:

$$h_A = \left(\frac{Q_A}{Q_B}h_B^m\right)^{1/m} = \left(\frac{Q_A}{Q_B}\right)^{1/m} h_B \tag{8.13}$$

The ratio of flows, Q_A/Q_B, can be determined from the growth curve. Let the second member of Equation (8.7) be written, for the sake of conciseness, as:

$$u + \alpha\left(\frac{1-e^{-ky}}{k}\right) = \xi(y) \tag{8.14}$$

The growth curve applies to both the considered water levels, leading to:

$$Q_A = \overline{Q}\xi(y_A) \tag{8.15}$$

$$Q_B = \overline{Q}\xi(y_B) \tag{8.16}$$

Substituting Equations (8.15) and (8.16) into Equation (8.13) yields:

$$h_A = \left[\frac{\xi(y_A)}{\xi(y_B)}\right]^{1/m} h_B \tag{8.17}$$

Finally substituting Equation (8.17) into Equation (8.10) leads to the following expression for the difference in water levels Δh

$$\Delta h = \left\{\left[\frac{\xi(y_A)}{\xi(y_B)}\right]^{1/m} - 1\right\} h_B \tag{8.18}$$

For natural water courses in plain levels the value of m is around 3/2 (Chow 1959); this value is adopted here as a first approximation regardless of the location of the river. Equation (8.18) expresses a linear relationship between Δh and the lower water level h_B. For example with $m = 3/2$, the annual exceedance probability of the event B $AEP_B = 0.01$, and with $AEP_A = 0.5\ AEP_B$ the described procedure gives a difference in water levels $\Delta h = 0.112\ h_B$.

8.3.3 Non-validity of the rating curve in overtopping conditions

The procedure outlined above in the case of two water levels conveyed by the river cannot be extended to overtopping conditions. In fact when part of the flow goes laterally off the river, the stage-discharge relationship no longer applies (Ervine 2008). To determine flood water levels in presence of lateral flow one has to estimate the discharge lost laterally and then use it in the equations expressing conservation of mass and momentum as it is done, for example in 1-D river modelling software such as the simplest version of ISIS (Halcrow & HR Wallingford 2001). Alternatively the energy method between cross sections can be used as in the program HEC-RAS (USACE 2002). Also in this case the lateral losses need to be quantified and the flow for the next section adjusted consequently. In a nutshell to determine the frequency of water level above embankments crest a problem of flood propagation with lateral loss of flow must be studied. Such modelling effort requires data that are seldom available in the early stages of a flood defences safety assessment. For this reason a simpler approach is needed for the development of the Reliability Rating System.

In the following section a simplified procedure is presented for the determination of the water heights above the crest to be used in the Reliability Rating

System. The presented approach introduces in the process an element of subjective judgement that will influence the relative importance of surface erosion with respect to the other failure modes.

8.3.4 Evaluation of reference water levels for overtopping

The following procedure has been adopted for the determination of the overtopping water levels to be used in the reliability calculations. In first place the height of water over the crest is calculated applying the method illustrated in Section 8.3.2 as if the correspondent flow is conveyed by the river without any lateral loss. This calculation is performed:

- taking the annual exceedance probability of the crest level as the inverse of the Standard of Protection:

  $AEP_{crest} = 1/SOP$
- imposing the annual exceedance probability of the overtopping reference condition as established by Equation (4.4)

  $AEP_{ot} = 0.5\ AEP_{crest}$
- adopting the national growth curve proposed by NERC (1975);
- assuming a stage-discharge relationship in the interpolating form given by Equation (8.9) and adopting a value of $m = 3/2$.

Subsequently only a small part of the height of water determined in such way is taken as the load for the reliability calculation. This is to account for the reduction in water level related to the loss of lateral flow in the considered section and in other sections along the river. This is a strong simplification because it neglects the actual distribution of flood water levels in space and time. Moreover the fraction of the water height calculated at the previous point which is used for the reliability calculation has been entirely determined on the basis of intuition and judgement. In the development of the reliability rating system the 10% of the height of water above the crest calculated with the abovementioned procedure is used as load in the reliability calculations for the grass cover failure. It is recommended that this hypothesis is reviewed and checked when the Reliability Rating System is applied to an increasing number of safety assessments to understand if this assumption leads to results that are in agreement with the judgement of expert assessors.

The value of 10% has been chosen, after trying other assumptions, because produces probabilities of breaching by grass cover failure which are in harmony with the rest of the Reliability Rating System. In fact the assumption has a direct influence on the relative importance of surface erosion with respect to the other failure modes; the choice made here results in probabilities of breaching by grass failure which reflect a credible relative importance of this specific mode of failure according to the author and his co-workers.

Despite the limitations discussed above the presented approach makes possible to quantify an estimated probability of breaching by surface erosion for the purpose of indicative rating without the extremely onerous need of taking into account the topography and hydraulic characteristics of the entire watercourse. With the presented approach the height of water over the crest for the reference overtopping conditions is a function of one geometric parameter of the cross section, namely the level of the crest referred to the lowest point of the cross section. The height of water above the crest obtained for different values of crest levels are reported in Table 8.1.

Table 8.1. Height of water over the crest for different crest levels measured from the lowest point of the cross section.

h_{crest}	m	2	4	6	8	10
Δh_{ot}	m	0.022	0.044	0.068	0.090	0.112

8.4 Simplified analysis of overtopping duration

The height of the water over the crest Δh which is estimated with the procedure illustrated in the previous section represents the overtopping condition at the peak of a hypothetical hydrograph. In order to estimate the duration of overtopping the shape of the hydrograph, i.e. the time history of flow, or water level, has to be known. Like in the case of flooding water levels also the duration of overtopping is heavily influenced by the local conditions and by the situation in the whole watercourse. The simplified procedure defined in this section of text has the only aim of producing loading situations that are rationally organised in a way that leads to higher overtopping durations for embankments located in catchments characterised by wider flood hydrographs, i.e. time histories where high water levels are maintained for longer. The results presented here should not be interpreted as an attempt to model local conditions.

The Flood Estimation Handbook (FEH 1999) proposes a simplified model for deriving the upper part of the hydrograph of known peak flow Q_{peak}. The first step is to estimate the width of the hydrograph at half-peak flow $W_{half\text{-}peak}$. This can be done with the formula:

$$W_{half-peak} = 2.99 \cdot T_p(0)^{0.77} \tag{8.19}$$

where $W_{half\text{-}peak}$ is expressed in hours and $T_p(0)$ is the time to peak of the Instantaneous Unit Hydrograph (IUH) defined by the Flood Study Report (NERC 1975).

The Instantaneous Unit Hydrograph is a concept used in hydrology for studying the rainfall-runoff transformation (Sherman 1932). The unit hydrograph is defined as the flow hydrograph that accommodates a volume of water corresponding to a unit depth of rainfall over a catchment. A different unit hydrograph is associated to different time periods ΔT during which the generating rain is assumed to fall uniformly. Unit hydrographs for 1-hour rainfall, 2-hours rainfall, etc., can be defined. The Instantaneous Unit Hydrograph is the unit hydrograph associated to a precipitation that releases the unit rainfall as an impulse of infinitesimal duration.

In UK practice the use of a unit rain of 10 mm is customary and a synthetical unit hydrograph of triangular shape has been proposed to develop a rainfall-runoff methodology in the Flood Study Report (NERC 1975). The FSR hydrograph is controlled by a single parameter: the time to peak T_p. The time to peak and hence the shape of the unit hydrograph proposed in the Flood Study Report can be determined for any rainfall duration from the time to peak of the Instantaneous Unit Hydrograph, indicated with $T_p(0)$. The FSR Instantaneous Unit Hydrograph is pictured in Figure 8.4. Essentially $T_p(0)$ determines the response of the catchment to a rainfall controlling how rapidly the flow reaches its maximum value and how rapidly decreases after peaking. The value of $T_p(0)$ can be estimated from different forms of analysis of past event analyses or from catchment descriptors. Tabulated values of $T_p(0)$ are available for several river sections across the UK (FEH 1999).

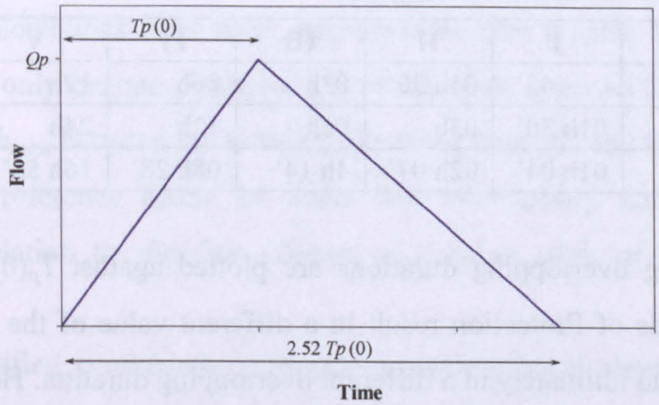

Figure 8.4. Instantaneous Unit Hydrograph proposed by the Flood Study Report (NERC 1975).

Once the width at half peak of the hydrograph is known the following formula can be applied to construct the upper part of the simplified synthetical hydrograph:

$$Q/\left(0.5 \cdot Q_{peak}\right) = 2 - 0.65\left(W / W_{half-peak}\right) - 0.35\left(W / W_{half-peak}\right)^2 \tag{8.20}$$

where W indicates the width at flow Q and Q_{peak} indicates the peak flow. It is here of interest the determination of the width of the hydrograph at the embankments crest level. This corresponds, in fact, to the duration of overtopping. Writing equation (8.21) for the top of bank condition, and noticing that Q_{peak} in this case corresponds to the reference flow in overtopping conditions Q_{ot} one gets:

$$Q_{crest}/\left(0.5 \cdot Q_{ot}\right) = 2 - 0.65\left(W_{crest} / W_{half-peak}\right) - 0.35\left(W_{crest} / W_{half-peak}\right)^2 \tag{8.21}$$

Recalling, from the previously developed analysis of flow frequency, that $Q = \overline{Q}\xi(y)$ the following equation is found:

$$2\xi\left(y_{crest}\right)/\xi\left(y_{ot}\right) = 2 - 0.65\left(W_{crest} / W_{half-peak}\right) - 0.35\left(W_{crest} / W_{half-peak}\right)^2 \tag{8.22}$$

The value of overtopping duration, which is equal to W_{crest}, can be found from Equation (8.23) once the Standard of Protection of the defence and the time to peak of the IUH are known.

In the reliability rating system six different values of $T_p(0)$, taken as representative of the values possibly encountered in UK catchments, are used, as illustrated in Table 8.2.

Table 8.2. Classes of time to peak of IUH and corresponding representative values chosen for the development of the Reliability Rating System.

Class of $T_p(0)$	I	II	III	IV	V	VI
Lower limit	45'	01h 30'	03h	06h	12h	24h
Upper limit	01h 30'	03h	06h	12h	24h	48h
Rep. value	01h 04'	02h 07'	4h 14'	08h 28'	16h 58'	33h 56'

The corresponding overtopping durations are plotted against $T_p(0)$ in Figure 8.5. Different Standards of Protection result in a different value of the first member in Equation (8.23) and ultimately in a different overtopping duration. However it can be seen that the difference between overtopping duration for Standard of Protections as small as 10 years and as large as 1000 years is negligible for the scope of this work. For this reason the overtopping duration calculated for a Standard of Protection of 100 years is used here for all the defences. These values are listed in Table 8.3.

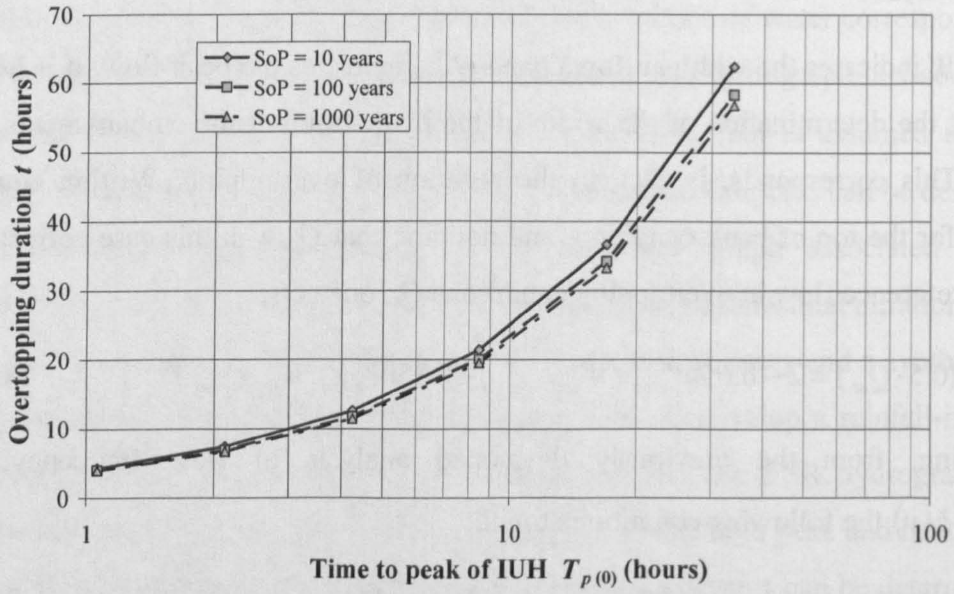

Figure 8.5. Duration of overtopping versus time to peak of the Instantaneous Unit Hydrograph for different Standards of Protection of the flood defence according to the simplified approach.

Table 8.3. Overtopping durations assumed in the Reliability Rating System for different classes of time to peak of the IUH.

Class		I	II	III	IV	V	VI
Rep. value	$T_p(0)$	01h 04'	02h 07'	04h 14'	08h 28'	16h 58'	33h 56'
Overtopping duration	t	04h 03'	06h 54'	11h 46'	20h 04'	34h 13'	58h 21'

8.5 Summary of loading conditions

The reliability calculations have been performed for five heights of water over the crest, depending only on one characteristic of the river cross section, precisely the vertical distance h_{crest} between the lowest point of the riverbed and the embankment's crest. For each reference height of water five overtopping duration have been considered in relation to the five classes of time to peak of the Instant Unit Hydrograph.

The simplified synthetical hydrograph is symmetrical about the peak (FEH 1999). In this work the upper part related to the overtopping is assumed triangular with duration $t = W_{crest}$ and peak height over the crest Δh. The available mathematical model for the grass cover resistance refers to a steady flow. For this reason the reliability calculations are performed with an equivalent height above the crest Δh_e taken as constant on the overtopping duration and identifying the same discharge as the triangular hydrograph. Because of this assumptions

$$\Delta h_e = \Delta h_{ot}/2 \tag{8.23}$$

The equivalent heights above the crest used in the reliability analyses are reported in Table 8.4.

Table 8.4. Equivalent heights above the crest used in the reliability analyses as a function of the crest level measured from the lowest point of the cross section.

h_{crest}	m	2	4	6	8	10
Δh_e	m	0.011	0.022	0.034	0.045	0.056

8.6 Reliability analysis

8.6.1 Performance function

The limiting state function for the reliability analysis of the grass cover can be written as the safety margin of discharges, i.e. the difference between the critical flow for failure of the turf q_c and the actual discharge running over the embankment q_a. Introducing a model factor for each of these quantities the following equations is obtained:

$$g(\mathbf{x}) = m_c q_c - m_a q_a \tag{8.24}$$

The critical flow and the actual discharge are calculated as shown in Section 8.2.

8.6.2 Input parameters: range of variation of the mean

The loading conditions are defined in the form of different pairs of water level and duration, regarded as deterministic values. The parameters treated as random variables are the coefficient of grass cover condition f_g, the inclination of the landward slope $\tan\alpha$, and Nikuradse's roughness of the submerged grass k_s. The mean values of f_g are taken equal to the deterministic values used in the curves presented in Section 8.2. Three values of the mean are chosen for the inclination of the landward slope, in agreement with what already done for the analysis of under-piping. These values are 3:4, 1:2 and 1:4. The mean of the equivalent sand grain roughness according to Nikuradse can be taken for submerged grass as constant $\mu(k_s) = 0.015$ m following Dutch practice (Vrijling & van Gelder 2000).

8.6.3 Random fluctuation of the input parameters

Indication on the random variability can be found in the Dutch guidelines (Vrijling & van Gelder 2000). These recommendations, partly based on judgement and intuition, are summarised in Table 8.5. All the variables are considered to be mutually independent.

8.6.4 Calculations

The reliability calculations have included 30 loading cases, deriving from the combination of 5 equivalent heights of water over the crest with 6 overtopping duration. These loads have been applied to 9 types of embankments, deriving from the combination of 3 angles of the landward slope and 3 turf conditions. In total 270 reliability calculations have been performed for the development of the section of the Reliability Rating System relating to the grass cover failure.

The reliability analyses have been carried out adopting Monte Carlo simulations with the same level of accuracy used for the under-piping mode of failure. Results have been occasionally checked with FORM, always finding good agreement between the methods.

Table 8.5. Random variability of input parameters for the grass erosion failure mode.

Parameter	Probability distribution	Mean μ	Coefficient of Variation $\Omega = \sigma/\mu$
f_g	Lognormal	-	0.20
$\tan \alpha$	Normal	-	0.05
k_s	Lognormal	0.015	0.25
m_c	Lognormal	1	0.50
m_a	Lognormal	1	0.50

8.7 Results

Figure 8.6 shows the probability of breaching by surface erosion for an embankment with a landward slope of 1:2 subject to a water level which is steadily 34 mm above the crest for the entire overtopping duration. The curves plotted in the diagram correspond to three different conditions of the grass cover: poor, normal and good. The influence of turf condition is significant and because of that the probability of breaching changes up to 2 orders of magnitude for the shorter durations and about 1 order of magnitude for the longer durations. The effect of overtopping duration is also remarkable: looking at good turf a difference of more than 2 orders of magnitude is found between the shortest and the longest duration. For bad turf this difference in probability of breaching is reduced to about one order of magnitude. The values plotted in Figure 8.6 are also listed in Table 8.6.

Figure 8.7 shows the effect of the landward slope inclination by comparing the probabilities of breaching of a slope as steep as 3:4 with those of a slope as mild as 1:4. Figure 8.8 illustrates the response of the same structure to different loadings comparing the probabilities of breaching calculated for the smallest (11 mm) and the largest (56 mm) height of water over the crest. The full body of results is reported in Appendix C.

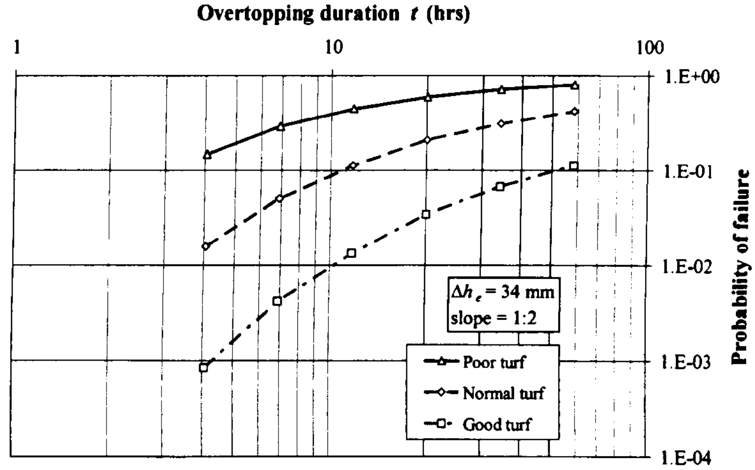

Figure 8.6. Probability of breaching vs. overtopping duration: equivalent water height above the crest Δh_e = 34 mm and landward slope inclined 1:2.

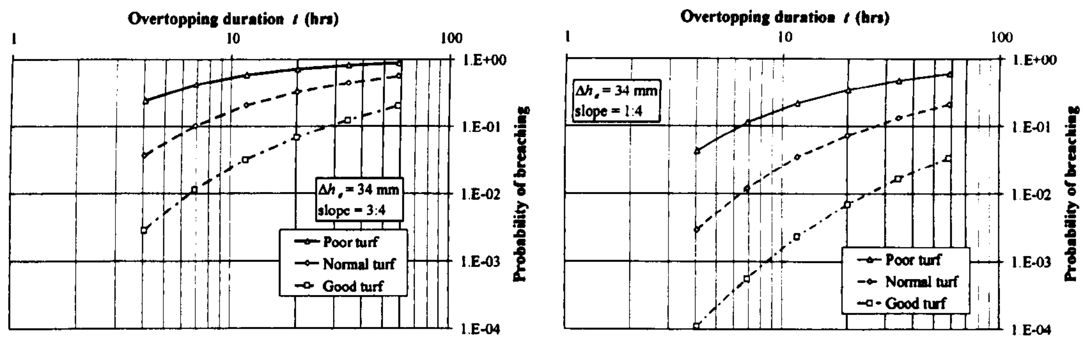

Figure 8.7. Effect of the landward slope angle: a steeper slope leads to higher probability of breaching.

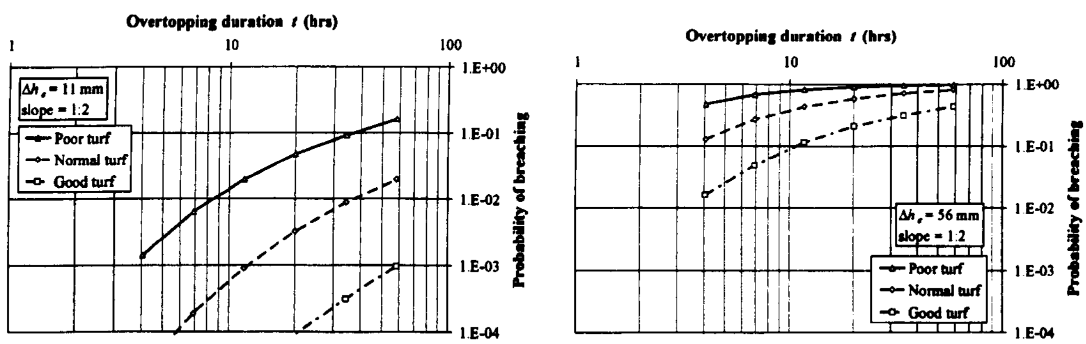

Figure 8.8. Response to loading: the lowest equivalent water height above the crest used in the Reliability Rating System, leads to probabilities of breaching much smaller than the highest adopted value.

Table 8.6. Probabilities of breaching for an equivalent water height above the crest Δh_e = 34 mm and landward slope inclined 1:2.

Probability of breaching for Δh_e = 0.034 m Landward slope inclined 1:2							
Turf	f_g	**Overtopping duration** t					
		4h 03'	**6h 54'**	**11h 46'**	**20h 04'**	**34h 13'**	**58h 21'**
Poor	0.7	1.49×10^{-1}	2.92×10^{-1}	4.47×10^{-1}	5.91×10^{-1}	7.14×10^{-1}	8.02×10^{-1}
Normal	1.0	1.56×10^{-2}	5.05×10^{-2}	1.13×10^{-1}	2.09×10^{-1}	3.09×10^{-1}	4.19×10^{-1}
Good	1.4	8.41×10^{-4}	4.17×10^{-3}	1.33×10^{-2}	3.42×10^{-2}	6.77×10^{-2}	1.12×10^{-1}

8.8 Conclusion

The probability of a breach initiated by grass cover failure have been determined in the present work using a mathematical model developed in the Netherlands (Vrouwenvelder *et al.* 1999, 2001a, 2001b; Vrijling & van Gelder 2000) with some adaptation to achieve a better fit with the situation of British rivers.

An accurate prediction of water levels with assigned annual exceedance probability is problematic in presence of overtopping. The hydraulic modelling tools to achieve this goal exist but require an amount of information which is typically not available at an early stage of flood risk assessment for which the reliability rating system is conceived. In the development of the Reliability Rating System the loadings have been analysed with an extremely simplified approach. The adopted strategy consists in evaluating the water level of desired exceedance probability with a growth curve and an approximated stage-discharge relationship, like in the absence of overtopping. Then only a portion of the height of water above the embankment's crest, calculated under the mentioned hypotheses, is taken as the load to be applied in the reliability analysis. Although not reflecting some aspects of the real behaviour of fluvial systems, where some sections will be much more loaded than others, the proposed approach supplies a rational way for incorporating some characteristics of the river section in the reliability rating.

Also the determination of overtopping durations incorporated in the system is made through a simplified strategy. Flood embankments located in catchments with a more rapid response to rainfall, i.e. with a narrower flood hydrograph, are associated with shorter overtopping durations.

Several reliability calculations have been performed with input values covering a wide range of possible situations that can be encountered in practice. This parametric study shows the influence of the grass cover condition and of the inclination of the landward slope on the resistance to surface erosion by flowing water. The Reliability Rating System provides the final user with a probability of grass cover failure once the depth of the river cross section and the time to peak of the Instantaneous Unit Hydrograph $T_p(0)$ of the catchment are known.

9

THROUGH-PIPING

The breaching of flood embankments by piping through the earthfill is currently not amenable to credible mathematical modelling. In fact, even if models for a macroscopically homogeneous and intact mass of soil can be found in the literature, in engineering applications the influence of discontinuities, defects and local anomalies - all resulting in zones of concentrated leakage - makes earth structures behave differently from such a homogeneous material. This problem is well known to geotechnical engineers involved in the risk assessment of large embankment dams. In this field the probability of breaching induced by piping through the embankment is often determined with a structured process of judgement elicitation carried out by a pool of experts.

In this research project the probability of breaching of flood embankments by through-piping has been estimated with a similar approach based on the subjective judgement of a small panel of geotechnical engineers. Due to the complexity and extension of the task, breaching probabilities have been estimated only for one loading condition: with the water level equal to the crest level. It is this current limitation that has led to the development of the performance indicator P_{BE} which gives a quantified measure of the flood defence expected performance without the need of knowing the structural response to all possible water levels. In the future, when fragility curves for through-piping will be developed it will be possible to perform an accurate reliability analysis for the full range of water levels.

9.1 Piping through water-retaining earth structures

Piping phenomena in earth structures can take different forms. The term piping itself has been used in different contexts to indicate different physical processes. For flood embankments the most relevant forms of piping are internal erosion – initiated along cracks or zones of concentrated leakage – and backward erosion – initiated at the exit point of seepage in a homogeneous and macroscopically continuous granular material. Criteria for evaluating the resistance to backward erosion can be found in the literature and have been applied, in this work, to study the piping through the foundation of flood embankments (Chapters 6 and 7).

By contrast not much information is available about piping through the fill material of which the embankment's body is made. Some understanding of this phenomenon can be gained from studies relating to embankment dam engineering, where investigation of piping processes has been undertaken for several decades. The research on earth dams, including statistical analyses of failures and accidents, shows how internal erosion across the body of the embankment is responsible for most piping failures. A mathematical model of this process, suitable of engineering application, is not available to date. Therefore, in the case of flood embankments the estimate of the probability of breaching by piping through the fill has to be based on engineering judgement.

The method used in this research project for the development of the Reliability Rating System builds on the embankment dam engineering practice, modifying it in order to suit the situation of fluvial embankments. The structure of this approach is designed to address the lack of data about the past performance of flood embankments and the typically low level of information about the actual characteristics of the structures to be assessed.

9.2 Distinction among different piping phenomena

The study of piping has been historically approached by researchers working in different disciplines and based in countries with different technical and scientific cultures, backgrounds and terminologies. The result is a rather heterogeneous body of work, in which definitions change or overlap when moving from a context to another. At least six different processes can be identified under the more generic name of piping (Richards & Reddy 2007). For the safety of flood embankments two of these processes are crucial, namely backward erosion and internal erosion. Suffosion can also play a role in some embankment dams but, as discussed later in this text, has not been considered of primary importance for flood embankments in the United Kingdom.

Backward erosion is the removal of particles by forces due to inter-granular seeping water in a macroscopically continuous soil. This form of piping has been discussed, among others, by Bligh (1910), Lane (1935), Terzaghi (1939) and Sherard *et al.* (1963). The name given to this phenomenon is due to its characteristic evolution: erosion starts at an exit point, where the soil, not confined, can be more easily removed; then evolves retrogressively across or beneath the structure. A typical example, already mentioned in this thesis, is the erosion of the founding soil under an impervious hydraulic structure. The empirical formula proposed by Lane (1935) to assess the safety against this type of failure has been used in Chapters 6 and 7.

Internal erosion is also the removal of particles due to water seepage but, differently from backward erosion, is related to the flow along pre-existing openings, like cracks in cohesive materials or voids at a soil-structure interface. Some models are available for the study of piping in homogeneous soil, for instance the well known model for piping/plugging in fine-grained soils proposed by Khilar *et al.* (1985). Nevertheless the role of discontinuities in the actual process makes all models which assume a homogeneous and macroscopically continuous soil the not suitable to describe internal erosion. Some authors have studied the erosion of a granular medium due to water flow in a planar opening (Louis 1969, Worman & Olafsdottir 1992, Franco & Bagtzoglou 2002). However there is currently no mathematical model capable of capturing the genesis of zones of concentrated leakage and the piping erosion occurring in them. In the risk analysis of earth dams the probability of breaching by internal erosion is estimated with methods based either on the historic frequency of accidents and failures or on the elicitation of subjective probabilities (Fell *et al.* 2000).

Suffosion (Kral 1975, Galarowski 1976), also known as internal instability or segregation piping, can occur in soils where the coarse part of the solid skeleton has sufficiently wide pores to allow the movement of particles belonging to its fine part under the effect of seeping water. The proneness of soils to be internally unstable can be assessed with criteria similar to those used for filter stability, thus assimilating the fine and the coarse fraction of the same soil to a core / filter pair (Kenny and Lau 1985; Åberg 1993). The advances in internal stability of soils represent a rich branch in the recent research on piping phenomena (see for instance Adel *et al.* 1998, Skempton & Brogan 1994). However only specific types of soil are subject to internal instability: as illustrated by Fell *et al.* (2004) the soils that can experience suffosion are granular gap graded soils or extremely well graded granular soils with a low, but not negligible, percentage of fines. Particle size distributions for such soils are exemplified in Figure 9.1. These soils are relatively rare and not commonly used in the construction of flood embankments in the UK. For this reason a detailed criterion for checking internal stability is not included in the Reliability Rating System.

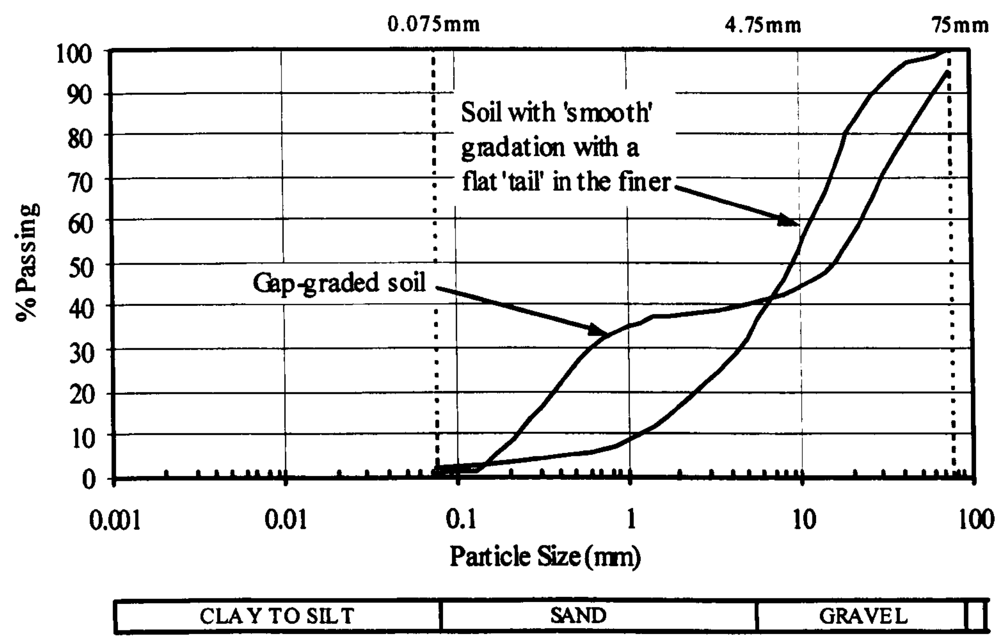

Figure 9.1. Gradation of soils subject to internal instability after Fell *et al.* (2004).

9.3 Statistics of embankment dam failures

The breaching of river embankments is generally sudden and part of the interested structure is rapidly washed away, thus making extremely difficult the reconstruction of the failure mode (Dyer 2004). This fact, and the impossibility of a constant monitoring of the extended flood defence network, explains the lack of data on breaching of fluvial embankments. Observations on failure modes are instead available for earth dams and, always remembering the differences between the two types of structure, can be used to gain more insight into the processes related to the breaching of fluvial embankments.

Foster *et al.* (2000a, 2000b) performed a detailed analysis of large embankment dam failures and accidents, gathering data from studies of the International Commission on Large Dams (ICOLD 1983, 1984, 1995), from the literature and from dam portfolio owners. Their study also investigated the influence of several factors, including the fill zoning, on the frequency of failure by different mechanisms. The subset of dams which do not have any fill zoning - i.e. without a core/filter pairing or a more refined subdivision - is of particular interest for the present purpose because of their resemblance with flood embankments. The relative frequencies of structural failure modes found by Foster *et al.* (2000a, 2000b) for homogeneous earthfill dams are shown in Figure 9.2.

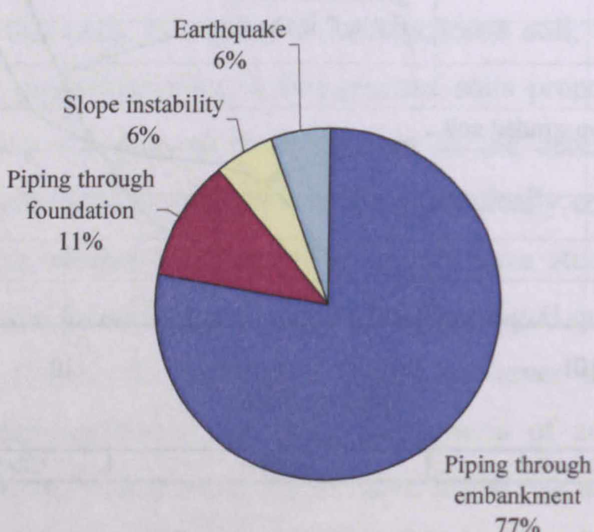

Figure 9.2. Structural failure statistics for homogeneous earthfill dams according to Foster *et al.* (2000a,b).

The frequency of all piping failures almost approaches 90% and piping through the embankment represents almost 80% of structural failures.

A key finding in the work of Foster *et al.* (2000a, 2000b) is that in piping accidents and failures the hydraulic gradient is far less important than other factors, like the presence of potential seepage paths along conduits and structures, cracks and other local defects. This implies relevance of internal erosion higher than previously assumed and a diminished role for backward erosion. Other recent works have emphasized the distinction between the flow of water through a granular medium versus the flow through cracks and structural contacts (McCook 2004). The role of internal erosion in the piping failure of earth dams is also supported by the findings from Richards and Reddy (2007), who published statistics on the failures of large and small dams, using data from Lane (1943), Jones (1981) and the National Performance of Dams Programme of Stanford University (http://npdp.stanford.edu) for a total of 267 piping failures. The failures have been divided in four categories:

a) piping failures related to the foundation;

b) failures induced by internal erosion along conduits or in the fill;

c) failures induced by backward erosion and suffosion in the fill;

d) piping failures induced by biological activity.

The results are shown in Figure 9.3.

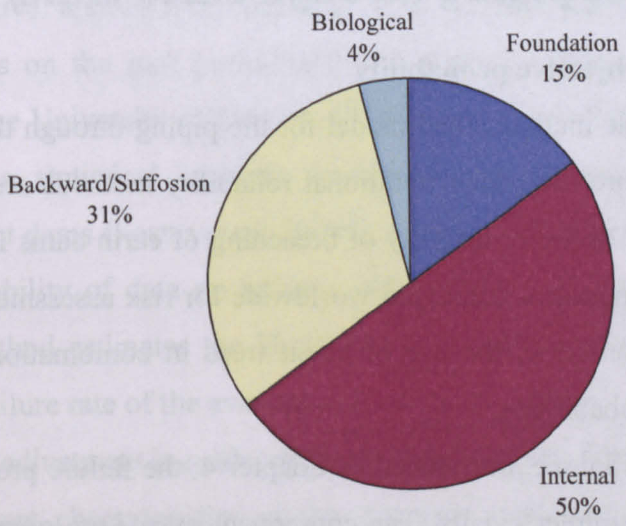

Figure 9.3. Statistics of earth dam piping failures according to Richards and Reddy (2007); elaboration based on data regarding 267 dam failures.

Internal erosion in the fill and erosion along conduits account for half of the failures, while backward erosion and suffosion are deemed responsible for almost one third of the failures through the embankment. Richards and Reddy (2007: 383) believe that if more details were available several failures for backward erosion or suffosion would be reclassified as due to internal erosion.

Although these results cannot be directly extended to flood embankments because of some differences between the two types of structures, the observed failure statistics of embankment dams can give some indication about the relevance of piping phenomena in water-retaining earthen structures. The trends emerging from statistical analysis of embankment dam failures are:

- the prevalence of piping through the embankment versus piping through the foundation;
- the prevalence of internal erosion versus backward erosion, implying a crucial role for cracks, structural contacts, and zones of high permeability and concentrated leakage.

Beside the observations on piping-erosion it is worth noticing the low frequency of breaches induced by slope failure.

9.4 Probability of piping through earth dams

9.4.1 Event trees and subjective probability

Given the lack of a credible mathematical model for the piping through the earthfill of embankments other approaches than traditional reliability methods are currently used for the determination of the probability of breaching of earth dams induced by this process. A common approach, employed worldwide for risk assessment by dam owners and authorities, consist in the use of event trees in combination with the elicitation of subjective probabilities.

In the event tree technique, introduced in Chapter 4, the failure processes are decomposed in chains of simpler events. The component events are organised in a graphical representation which starts with an initiating event and then branches repeatedly, identifying some sequences leading to failure and some others leading to a safe state. The calculation of the total probability of failure requires that a conditional probability is associated to each event along the failure chains, given the occurrence of preceding events in the sequence. These conditional probabilities are assessed, in absence of a suitable mathematical model, with a subjective probability approach by a panel of experts. In order to obtain credible results the process of subjective probability elicitation, already introduced in Chapter 5, has to be structured according to a precise procedure (Beacher & Christian 2003; Vick 1999 and 2002, USACE 2006).

Some guidance on the estimate of subjective probabilities is available, in the case of embankment dams, as qualitative or semi-quantitative aids to subjective judgment. In particular Fell *et al.* (2004), in a report summarizing a decade of research, give a number of indications and suggestions in relation to the probability elicitation of the events involved in piping failures. Despite the differences between large earth dams and flood embankments, some of this guidance can shed light into the process of piping through fluvial earth structures. This information has been used in the judgment elicitation process undertaken to develop the Reliability Rating System. In particular the guidance available for embankment dams has contributed to a better understanding of the means by which erosion is initiated and of the factors influencing the creation of high permeability zones in the embankment.

9.4.2 Historical performance approach: the UNSW method

An alternative approach to the reliability of embankment dams is the use of observations on the past performance of a large number of similar structures. In particular the University of New South Wales method (Foster *et al.* 2000b) uses the results of a statistical analysis carried out on a remarkably large sample of embankment dams (Foster *et al.* 2000a). For this kind of structures, in fact, there is a wide availability of data on failures and accidents. In its original formulation the UNSW method estimates the likelihood of failure of a specific dam adjusting the observed failure rate of the average dam of the same type.

The adjustment is made with individual weights which reflect the influence of some relevant characteristics of the structure under assessment. In particular the annual probability of piping through the embankment[10] for a dam is calculated with the following equation:

$$P_E = w\,P_{\text{avg}} \tag{9.1}$$

where P_{avg} is the annual average frequency of failure by through piping and w is the global weighting factor. The global weighting factor for piping through the embankment is obtained multiplying eight individual weighting factors.

$$w = w_{\text{f}} \times w_{\text{cgo}} \times w_{\text{cst}} \times w_{\text{cc}} \times w_{\text{con}} \times w_{\text{ft}} \times w_{\text{obs}} \times w_{\text{mon}} \tag{9.2}$$

The subscripts identify the relevant characteristics of the dam. In particular they indicate:

f	the presence and quality of a filter;
cgo	the geological origin of the core;
cst	the type of soil in the core;
cc	the quality of the compaction;
con	the presence of conduits and the condition of the fill-conduit interface;
ft	the treatment of the foundation;
obs	possible observation of seepage;
mon	the frequency and type of monitoring.

The weighting factors were determined comparing the characteristics of the dams that have experienced piping accidents with those of the whole statistical sample, using the following formula:

$$w_i = \frac{(\%\text{ failures with the particular characteristic})}{(\%\text{ dams in the sample with the particular characteristic})} \tag{9.3}$$

The procedure developed in this work for the determination of the probability of breaching of flood embankments by through piping, although based on subjective judgement rather than on statistical data, has been structured in a framework similar to the UNSW method.

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[10] The UNSW method also takes into account failure induced by piping through the foundation and by piping from the embankment to the foundation; only piping through the embankment is of interest here.

9.5 Flood embankments

9.5.1 Differences from earth dams

Before evaluating the implications for flood embankments of the trends observed on piping through embankment dams, it is convenient to examine the differences between the two types of structure. The most obvious difference is the size: while flood embankments in the United Kingdom are small structures less than 10 m high, earth dams can be large structures, several tens of meters high. However, while the difference in hydraulic head across the structure increases with the height of the structures so does the length of possible seepage paths. This implies that there is no reason for expecting substantially different hydraulic gradients along seepage paths in structures of different size. An important effect of size is that for large, heavy dams, a significant settlement is to be expected, as a result of compression in the fill and, depending on type of soil or rock, in the foundation. Large absolute settlements are generally accompanied by differential settlements and ultimately by a potential for fractures. Comparatively this issue is of less importance for flood embankments. Foster *et al.* (2000b) point out that smaller dams, being perceived as less hazardous structures, are more likely to be poorly constructed. By comparison the control on the construction of flood embankments is expected to be even lower. In fact the flood defence network in the United Kingdom comprises flood embankments which have been constructed in different times, with a range of various materials, and with a variable control on the construction process (Dyer 2004). The considerable lack of information about the construction is a pervasive obstacle to the safety assessment of existing flood embankments.

Another difference between earth dams and flood embankments is the longitudinal extension: limited by the abutments against which they are constructed in the case of dams; remarkable in the case of flood embankments. This implies that a whole class of problems related to the abutments geometry – like the uneven distribution of stress favouring hydraulic fracturing or the large differential settlements inducing cracking - is not of concern for flood embankments. Nevertheless flood embankments are still subject to problems related to differential settlement induced by irregularities in the foundation - like a change of geology between two soils with very different compressibility or local zones of high compressibility.

Most dams are designed and constructed with a zonation of the fill material where the finest soil is placed in the central core, to prevent excessive seepage, and progressively coarser materials are located on both sides, to ensure good drainage and stability. In many dams a granular filter is placed downstream of the core, with the precise function of trapping particles dislodged from the core and thus granting self-healing of possible leakages. Fluvial embankments do not have this kind of zoning and no filters are included in the design. This makes flood embankments more vulnerable to piping. It is indeed not a coincidence that failure statistics indicate for large dams with homogenous fill a much higher probability of failure for piping than the average dam (Foster *et al.* 2000a). A class of problems typical of large zoned dams is related to the difference in compressibility between the fine-grained, deformable core and the coarse, stiff material in the external zone. This clearly does not apply to flood embankments.

Finally the monitoring of flood embankments is, due to the extension of the network, generally far less frequent and accurate than the monitoring of earth dams. In addition to that, visual inspections generally happen to be undertaken in concomitance with low water levels that are quite far from the flooding conditions in which the embankment performance need to be assessed. It is more common for embankment dams to experience a fairly constant water level, not too far from the crest, for most of their operational time.

9.5.2 Probability of piping through the fill

The absence of a credible mathematical model for piping through the embankment precludes the use of the classical methods of reliability analysis for the determination of the probability of breaching. Moreover not enough data on the past performance of flood embankments are available to estimate the frequency of piping failures with an historic performance approach. In principle the probability of failure could be estimated applying the event tree technique, with the elicitation of subjective probabilities. This could be done with the event tree already presented in Chapter 4 (Figure 4.2). However in embankment dams engineering the use of event trees is supported by a reasonably detailed knowledge of the structure to be assessed. The typical level of information gathered for earth dams includes, according to Fell *et al.* (2004), details of the construction process, as-constructed drawings, particle size distribution and Atterberg's limits, placement and compaction specifications, achieved relative density of the fill, photographic documentation of the construction, seepage data, pore pressure and settlement data, Cone Penetration Tests and test pits. A comparable level of knowledge for flood embankments can realistically be achieved only for extremely detailed local studies on a single defence scheme, while is not likely to be available at a less detailed scale. For this reason an alternative approach which makes use of the formal structure of the UNSW method and adapts it to a subjective judgement elicitation has been developed in this work.

9.5.3 Proposed formulation

In order to develop the Reliability Rating System the probability of breaching induced by through piping has to be determined for a number of reference embankments characterised by different combinations of the factors which affect the susceptibility to piping. The Reliability Rating System has been conceived to make use, for this failure mode, of only one loading condition, i.e. water at the same level of the crest. In the elicitation process undertaken in this research project the probability of breaching by through-piping for this reference water level $P(B_{tp})_{crest}$ has been expressed with a formulation similar to the UNSW method:

$$P(B_{tp})_{crest} = \left(\prod w_i\right) \cdot P_{ref} \tag{9.4}$$

This approach borrows the formal structure of the UNSW method but has some important differences. In this case, in fact, the individual weights have to be estimated through engineering judgement rather than being calculated from available statistics. In absence of a large sample of data it is also impossible to define an average frequency of failure. For this reason the probability of failure of a reference embankment P_{ref} is used in Equation (9.4) in place of the likelihood of failure of the average dam P_{avg} adopted in Equation (9.1). Also the reference probability of breaching has to be estimated by subjective judgement. Moreover it should be noticed that, while P_{avg} in the UNSW method is an average frequency, i.e. it expresses a rate of failure, P_{ref} in the modified approach is a probability of failure associated with a specific loading condition and is not directly referred to a time interval. In principle the reference embankment can be chosen arbitrarily and have any combination of characteristics, provided the weights are then defined in a coherent way that reflects the quantified belief of the assessing panel about the influence of the relevant factors on the performance of the embankments.

9.6 Judgement elicitation process

9.6.1 Structured procedure

The quantification of subjective judgement has been carried out by a small panel of engineers composed by the author and two co-workers, as explained in Chapter 5. The panel has found convenient to choose as the reference an embankment with optimal characteristics for the resistance to piping. The judgement quantification procedure has been organised according to literature recommendation (Beacher & Christian 2003; Vick 1999, 2002; USACE 2006). The assessors, after the preparatory phases of motivating and training, have proceeded to the estimation of the individual weights w_i and of the probability of breaching by through-piping for the reference embankment P_{ref}. In particular in the structuring phase:

- The panel has identified the relevant characteristics influencing the probability of piping through the embankment.
- For each characteristic the level of information available in a real assessment has been discussed and an appropriate subdivision in classes has been chosen accordingly.

These operations have been guided by the literature available on the condition assessment and the performance estimation of flood embankments (Dyer & Gardener 1996, Morris *et al.* 2007, Environment Agency 2006, Long *et al.* 2006). In the subsequent assessing phase, in which the values of w_i and P_{ref} have been estimated:

- The panel has chosen a reference embankment and assigned an individual weight of 1.0 to the reference condition for each of its characteristics.
- The individual weights for each class of the different characteristics have been established as a quantitative measure of subjective judgement.
- The probability of failure for the reference embankment has been estimated.

This phase has been informed and guided by the literature available on the performance of embankment dams (Fell *et al.* 2000, Fell *et al.* 2004, Foster *et al.* 2000a,b, FEMA 2005a,b, Richards & Reddy 2007), combined with a significant dose of engineering judgement necessary to tailor the process to the case of flood embankments.

In the proposed approach the subjective judgement is applied to the estimate of ratios of probabilities (the individual weights) rather than directly to conditional probabilities. The probability of breaching by through-piping for the reference embankment also needs to be quantified. This probability is difficult to estimate, partly because it is the result of a complex process, partly because it is likely to be a very low probability, thus falling in the field affected by the overconfidence bias illustrated in Chapter 5 (Fischhoff *et al.* 1977, Vick 2002). However some guidance is available, in the case of flood embankments, on the meaning of the probability of failure of structures in terms of verbal descriptors of the performance level. In particular USACE (1999) proposes the equivalence between probabilities of breaching of levees and expected levels of performance reported in Table 9.1.

Table 9.1. Target probability of failure for different expected levels of performance (source: USACE 1999).

Performance level	Probability of failure
Hazardous	0.16
Unsatisfactory	0.07
Poor	0.023
Below average	6×10^{-3}
Above average	1×10^{-3}
Good	3×10^{-5}
High	3×10^{-7}

It is worth noticing that these reference values are disjoint from a discussion of the consequences of the structural failure and must hence be referred to some unspecified average consequence scenario. Nevertheless Table 9.1 gives some useful guidance and has become a reference criterion, not only for the safety of water-retaining earth structures, but for a wider range of geotechnical problems (Nadim

2006). For this reason the target probabilities of failure proposed by USACE (1999) have been extensively used to assist the judgement elicitation presented in this work.

9.6.2 Independent weights

During the structuring phase the panel has identified eight main characteristics which are believed to significantly affect the resistance to through-piping of flood embankments. These are:

1) The presence of animal burrows and their extension across the embankment section.
2) The observation of seepage across the embankment.
3) The presence and size of differential settlement, possibly inducing cracking.
4) The compaction of the earthfill material.
5) The presence and condition of culverts.
6) The presence of roots, representing a preferential seepage path.
7) The type of soil used in the earthfill.
8) The geological origin of the soil used in the earthfill.

The panel has identified a convenient subdivision in classes for each of the listed characteristics. The adopted classes are the result of a compromise between the accuracy of the system, the capacity of the panel to estimate the different responses, and the amount of information realistically available to the final users of the methodology.

Initially the judgement quantification has been attempted with the approach described so far and precisely using the formulation given by Equation (9.4). However the use of this formal structure, with the individual weights assumed to be independent, has proven problematic. In fact the assessors found that it was impossible for them to define weights that reflected a sufficient worsening of the performance when only one or few negative factors were present and, at the same time, satisfied the condition $P(B_{tp})_{crest} \leq 1$ for a combination of several negative factors. To overcome this obstacle a more refined formulation has been introduced.

9.6.3 Refined approach: coefficients of influence reduction

In order to reproduce a sufficiently high influence on the expected performance of a single negative characteristic, and simultaneously keep the probability of failure suitably below the unit, some form of interdependence among the individual weights has to be introduced. In the present work this has been achieved by means of coefficients of influence reduction. The idea is to make a list of the characteristics affecting the performance, ordered from the most influential to the least influential. Then each individual weight is multiplied by a coefficient of influence reduction which has a unit value for the first, most influential characteristic and progressively lower values for the following, less influential characteristics.

Adopting this formulation an individual characteristic can heavily affect the performance when it is present on its own but has a reduced influence when it is combined with other, more influential characteristics. As the reference embankment is assumed to be in optimal conditions for piping resistance all other combinations of characteristics will have a higher probability of breaching. For this reason the product of an individual weight multiplied by the coefficients of influence reduction r_i, cannot be less than 1. The mathematical formulation of this approach is expressed by the following equation:

$$P(B_{tp})_{crest} = \Pi \max[w_i r_i, 1] \times P_{ref} \tag{9.5}$$

where i is the position of the individual weight in the list arranged in decreasing order. The panel has found it convenient to express the coefficients of influence reduction with a law of the type:

$$r_i = 1/a^{i-1} \tag{9.6}$$

where a is a constant, that becomes part of the judgement elicitation process.

9.7 Results

9.7.1 Outcome of the judgement elicitation

At the end of the elicitation process the panel has agreed on the values of the individual weights, of the constant used in the determination of the coefficients of influence reduction and of the probability of breaching for the reference embankment under the designated loading conditions. Convenient coefficients of influence reduction have been found for:

$$a = 3 \tag{9.7}$$

This choice leads to:

$$r_1 = 1, r_2 = 1/3, r_3 = 1/9, \dots \tag{9.8}$$

The individual weights for the different classes of the eight relevant characteristics are reported in Table 9.2. Some remarks are required for a better understanding of the Table.

The compaction has been considered "Good" if in compliance with the recommendations of the British Code of practice for Earthworks (BS 6031:1981 – Section 9) or an equivalent modern standard. Embankments which underwent some procedure for compaction without achieving the required modern standard are placed in the class "Some compaction".

Statistics on earth dams show that a structure with a state-of-the-art conduit crossing its embankment has significantly higher chances of experiencing piping compared to structures without a conduit (Foster *et al.* 2000a). Poor details in the construction further increase the likelihood of failure. There is a degree of subjectivity about how many poor details should discriminate between the two classes "few poor details" and "many poor details". Moreover different details may not have the same importance. However, for the sake of simplicity, it has been decided to keep culverts with 1 or 2 poor details in the first class and culverts with 3 or more poor details in the second class. In the case of flood embankments poor details are to be intended both as poor construction practice and as deterioration from the intended condition. Examples of poor construction details are (see also Foster *et al.* 2000a):

- outlet located in a deep narrow trench on a non protected slope, which can be eroded;
- corrugated metal formwork used for the concrete surround compromising compaction;
- culvert geometry preventing an effective compaction around it;
- metal pipe not encased in concrete;

- pipe directly funded on soil;
- all situations which can have made an effective compaction around the culver difficult or that can have induced displacement or distortion of the culvert.

Examples of deterioration are (see also Environment Agency 2006):

- metal corrosion;
- cracks;
- settlements;
- damage at joints;
- distortion;
- mortar loss in brickwork or masonry;
- pipe-work and corrugated elements seated with visible gaps;
- vegetation penetrating trhough culvert walls.

In general conditions listed in the Condition Assessment Manual (Environment Agency 2006) under culverts with Condition Grade 4 or 5 can be regarded as significantly affecting the performance and should be included in the count.

The embankment chosen as reference, with an optimal resistance to piping, is made of glacial clay of high compressibility, compacted to modern standards and with no defects or anomalies. The individual weights for the characteristics of the reference embankment are in grey cells in Table 9.2. The probability of failure for a 500 m long embankment with such characteristics has been estimated by the panel to be in the range of

$$P_{ref} = 9.0 \times 10^{-5} \tag{9.9}$$

when the water level equals the crest level.

The results of the elicitation process have been submitted to a group of ten experts of flood risk from the academia, the civil engineering industry and the Environment Agency (Redaelli 2008b, 2008c). Although with different levels of consensus on the details no one has raised substantial objections (Bramley 2008).

Table 9.2. Individual weights estimated by the panel as quantification of subjective judgement.

CHARACTERISTIC	CONDITIONS AND WEIGHTS					
Animal Burrowing w_{burr}	Yes, likely to completely cross the earthfill **200**	Yes, unlikely to cross completely **50**	No **1**			
Observation of seepage w_{seep}	Muddy Leakage **50**	Clear Leakage **30**	Pooling Water **10**	None **1**		
Differential settlement w_{sett}	Inducing recognisable cracking **30**	Visible, no recognisable cracking **10**	Not visible **1**			
Compaction w_{com}	No Compaction **25**	Some Compaction **10**	Good Compaction **1**			
Culvert w_{cul}	Many poor details **25**	Few poor details **10**	Optimal condition **5**	None **1**		
Soil Type w_{soil}	Silt $LL < 50$ * **21**	Clean sand **15**	Clayey sand Silty sand **6**	Silt $LL \geq 50$ **5**	Clay $LL < 50$ **4**	Clay $LL \geq 50$ **1**
Vegetation roots w_{roots}	Trees **15**	Bushes **5**	Grass only **1**			
Soil geologic origin w_{geo}	Alluvial **8**	Aeolian, Colluvial **5**	Residual, Lacustrine, Marine, Volcanic **4**	Glacial **1**		

(*)LL = liquid limit; soil identification according to the Unified Soil Classification System (ASTM 1985).

9.7.2 Range of variation of the probability of breaching

In order to evaluate the width of the interval of probability of breaching covered by the proposed approach the situation in which all the relevant characteristics are set to the most unfavourable case regarding the through-piping resistance is exemplified. The conditions inducing the highest proneness to through-piping are listed at the left end of Table 9.2. The appropriate individual weights are arranged in decreasing order and paired with the correspondent coefficients of influence reduction in Table 9.3. It can be seen how the application of the influence reduction coefficients attenuates the effect of the characteristics with lower influence on the performance. In this case the first three characteristics - animal burrowing, observation of seepage and major differential settlement - contribute to the increased probability of breaching.

Table 9.3. Individual weights paired with influence reduction coefficients for the embankment with the least favourable characteristics for piping-resistance.

i	r_i	CHARACTERISTIC	CONDITION	w_i	$\max[w_i r_i, 1]$
1	1	Animal Burrowing	Yes, likely crossing the earthfill	**200**	200
2	1/3	Observation of seepage	Muddy leakage	**50**	16.7
3	1/9	Differential settlement	Inducing recognisable cracking	**30**	3.33
4	1/27	Compaction	No compaction	**25**	1
5	1/81	Culvert	Many poor details	**25**	1
6	...	Soil type	Silt LL<50	**21**	...
7	...	Vegetation roots	Trees	**15**	...
8	...	Soil geologic origin	Alluvial	**7.5**	...
				$\Pi \max[w_i r_i, 1] =$	1.10×10^4

From Equations (9.5), (9.8) and (9.9) the probability of breaching by piping through the earthfill in the reference loading conditions is:

$$P(B_{tp})_{crest} = (9.00 \times 10^{-5}) \times (1.10 \times 10^{4}) = 0.99$$

With the quantities established as the outcome of the judgement elicitation the "worse possible" embankment has an estimated 99% chance of breaching with water at the crest level. The probability of through-piping failure for this embankment is compared, in Figure 9.4, with the probability of breaching for the reference embankment. The verbal descriptors of the expected performance proposed by

USACE (1999) are also included in the picture. The reference embankment is judged to have a "good" performance also for the severe loading condition assumed here, while the "worse possible" embankment is almost certainly breaching. As the embankment with an the optimal resistance to piping has been chosen as reference, Figure 9.4 gives the full range of probabilities of breaching that the proposed approach can cover. All the combinations of characteristics included in the Reliability Rating System will fall somewhere between these two extremes.

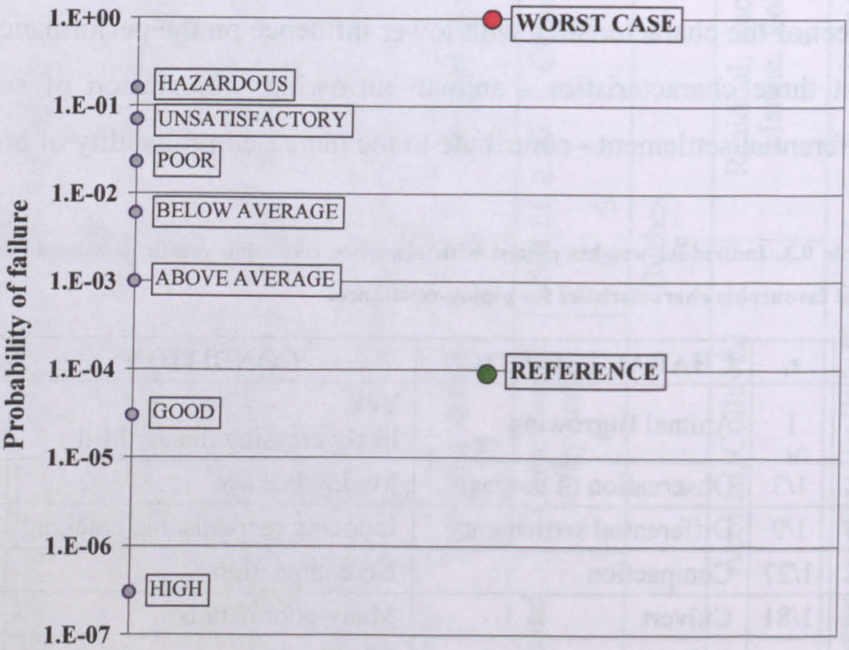

Figure 9.4. Expected performance with water level at the crest level for the reference embankment (most favourable situation for resistance to piping) and for the embankment with the least favourable characteristics.

9.8 Conclusions

The process of piping through flood embankments is currently not amenable to credible mathematical modelling and the probability of breaching by this failure mode has to be estimated with a judgment elicitation similar to those carried out in embankment dam risk assessments. In order to develop the Reliability Rating System a structured process of subjective probability elicitation has been undertaken by the author and his co-workers. The judgment elicitation has estimated the probability of breaching by piping through the earthfill for a specific loading condition corresponding to water level at the embankment's crest.

The use of the event tree technique, normally employed for embankment dams, proved problematic for the purpose of this work due to the difficulty in estimating the probability of events occurring in different hypothetical embankments with various combinations of characteristics affecting the piping-resistance. For this reason the assessing panel has adopted the formal framework of another method used for embankment dams which, in its original formulation, is based on the historical performance of a large sample of water-retaining structures. The framework has been adapted to suite the judgement elicitation process referred to flood embankments.

The subjective probabilities elicitation has been conducted according to a rigorously structured procedure, following state-of-the-art recommendations (Beacher & Christian 2003, Vick 1999 and 2002, USACE 2006). However the small size of the panel and the relatively uniform background of the participants suggest that better results could be obtained by a larger panel including a wider spectrum of competencies. The presented approach has been dictated by the need of going through all the phases of the rigorous elicitation process in a parsimonious way which made good use of limited resources.

It is recommended that in a further development of the Reliability Rating System benchmark embankments are studied with the event tree approach in order to better estimate both the probability of failure of the reference embankment and the values of the individual weights.

Part III

10

DISCUSSION

The modes of failure included in the Reliability Rating System expand the current practice of flood embankments reliability assessment which, at national or regional level, includes only breaching due to surface erosion and one type of piping. The presented methodology correctly operates a distinction between under-piping and through-piping. Seepage-induced slope failure is not addressed by a dedicated reliability analysis. However slope failure triggered by locally intense seepage is taken into account in the through-piping mode of failure. The Reliability Rating System not only enables the comparison and ranking of flood embankments, it also quantifies the effect of the lack of knowledge about the actual characteristics of the earthen flood defences and captures the reduction in uncertainty on the performance prediction when more information is gathered.

10.1 Failure modes and other geotechnical aspects

10.1.1 Failure modes

The three failure modes included in the Reliability Rating System do not cover all the possible mechanisms by which a flood embankment can fail. Under-piping, through-piping and surface erosion have been chosen as the most relevant failure modes on the basis of indications in the literature. This is in line with the methodologies currently in use at national and regional scale. In fact the High Level Methodology Plus (HR Wallingford 2005, Buijs *et al.* 2007) and the intermediate-level methodology proposed by Gouldby *et al.* (2008) include only two modes of failure for fluvial embankments: surface erosion and piping. The effort to correctly model piping has led, in this research project, to the important distinction between under-piping (related to backward erosion) and through-piping (mainly related to internal erosion) and to a total number of three modes of failure.

The reader with experience in geotechnical engineering will have noticed the absence, among the selected modes of failures, of slope instability. This absence is due to several reasons. In the first place, although the information in the literature is fragmented, this mode of flood-induced failure seems to occur less frequently than surface erosion and the two forms of piping. This assumption is certainly true for embankment dams, as proved by the statistics from Foster *et al.* (2000a) and Richards & Reddy (2007) mentioned in Chapter 9.

In the second place the correct modelling of slope instability induced by high flood water levels is rather complicated. The flood-induced failure of the landward slope is related to seepage across the embankment, which induces the rise of the phreatic surface and a change in pore pressures in the fill material (Figure 10.1).

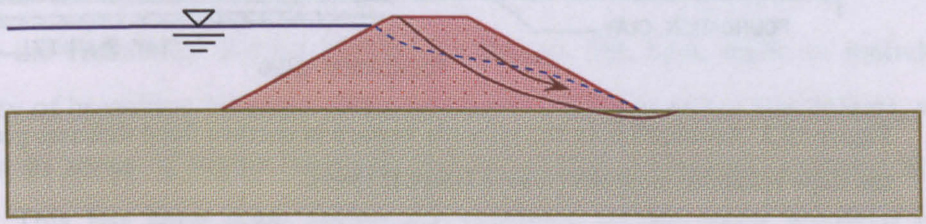

Figure 10.1. Landward slope failure triggered by rising of the phreatic surface.

The literature on the reliability of flood defences reports different methods to determine the probability of slope failure. These range from algorithms to find the probability of slope instability with a finite number of deterministic analyses (Hassan & Wolff 1999) to calculations carried out with dedicated probabilistic codes also taking into account autocorrelation in space (GeoDelft 1993). The reliability analysis of slopes can also make use of deterministic finite elements. For example Schweiger *et al.* (2001) combined finite elements with the Point Estimate Method (PEM) while Xu & Lo (2006) made use of the surface response method combined with the spreadsheet-based FORM implementation by Low & Tang (1997).

However all the mentioned approaches, while assuming fluctuation of the mechanical parameters, require the knowledge of the phreatic surface whose random variability in not included in the modelling. Even the works specifically dedicated to flood defences do not include a probabilistic description of the seepage across earth structures and instead rely on judgement to assume a deterministic phreatic surface. In American practice for example (Wolff *et al.* 1996, USACE 1999) the piezometric level within the fill material is generally taken as a straight line connecting the free surface in the watercourse to the landward toe of the embankment (Figure 10.2).

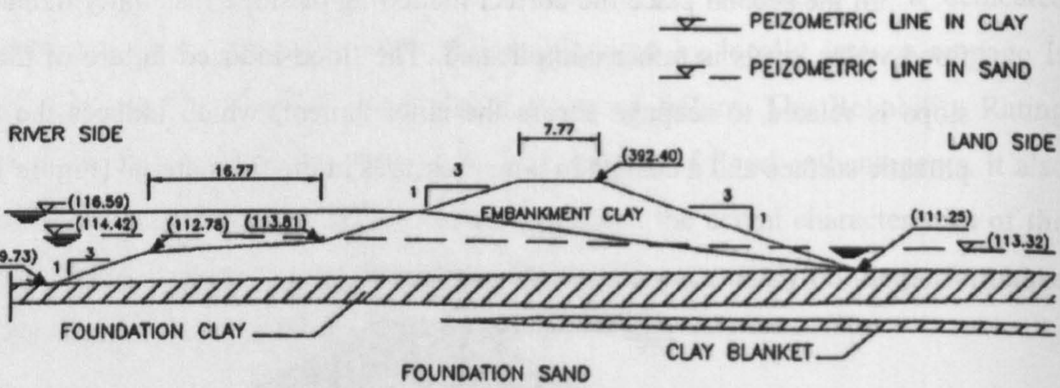

Figure 10.2. Assumption on the phreatic surface in earthen flood defences generally adopted for the slope reliability analysis (source USACE 1999).

The author argues that the hydraulic conditions within the fill material actually represent the main uncertainty in the slope reliability analysis. In fact the random fluctuation of the coefficient of permeability is much more pronounced than those of the mechanical parameters, with reported coefficients of variation, also for manmade earthworks, in the range of 200%. Moreover in the case of river embankments the changes in water level during floods are often very fast if compared with the evolution of the pore pressure within the fill. For this reason, in order to capture the real behaviour of the flood defence, a transient seepage analysis is required and some hypothesis on the shape of the flood hydrograph is needed.

In summary the slope stability analyses required to include a credible representation of flood embankments behaviour in a comprehensive reliability methodology should be:

- In terms of effective stress parameters, in order to model the influence of the changes in pore pressure, with hydro-mechanical coupling.
- Under transient hydraulic conditions, to model the response of pore pressures to the water level history described by the flood hydrograph.
- Probabilitistic; including the stochastic description of the permeability coefficient of the fill material and the foundation.
- Accounting for the influence of partial saturation of the fill, certainly on the coefficient of permeability, possibly on the mechanical behaviour.

Although these aspects of slope stability have been solved separately in the literature the author is not aware of any published work dealing simultaneously with all these issues. There is no conceptual impediment to the solution of such a problem, for example using the Random Finite Element Method (Griffiths & Fenton 1993, Fenton & Griffiths 1993). However the effort required to obtain the solution seems out of proportion for the purpose of the development of the rating methodology presented here, at least in its first version.

In the Reliability Rating System an attempt has been made to include the probability of breaching by slope slides related to local anomalies and defects, which can result in zones of higher permeability and induce a localised increase in pore pressure. This has been done asking the members of the panel involved in the judgement elicitation for the estimation of the probabilities of breaching by through-piping (Chapter 9) to include in their evaluation not only processes strictly related to fill erosion but also the failure by sliding induced by changes in pore pressure related to zones of high permeability. Therefore, in principle, these localised slope failures are implicitly taken into account in the probabilities of breaching determined for the through-piping failure mode.

There are two other situations, well known from soil mechanics literature, in which slope failure is a significant possibility: these are the construction of an embankment on soft soil and the rapid drawdown (Morgenstern 1963). The former situation, although obviously relevant in the case of the construction of new embankments, is of no relevance for already existing embankments and bears no relation to the variation of water level experienced during floods. In the latter situation the lowering of the water level, especially if rapid compared to time required by the pore pressure in the fill to re-equilibrate, can significantly reduce the stability of the embankment and trigger a failure of the riverward slope. In fact in any earth slope with relatively low permeability an unfavourable condition for stability is determined when the water level rapidly drops. In this situation the portion of slope which was previously submerged no longer benefits from the stabilising action of the water pressure while still having a high unit weight due to persistent saturation. In addition the pore pressure can take time to dissipate. A notable analysis of this process in a fluvial environment is reported by Pauls *et al.* (1999). Slope failures induced by drawdown in the decreasing branch of the flood hydrograph should, in principle, be considered in the reliability methodology. However in this first version of the Reliability Rating System this failure mode has been neglected. In fact having to prioritise the modes of failure it has been considered that happening after the peak of flood, drawdown failures appear to be less threatening in term of inundation of the built environment, than breaches occurring at or before the peak.

Considering nevertheless that repeated flooding events are possible and that the lack of monitoring of the defence network makes a prompt repair of damaged embankments unlikely, the inclusion of drawdown-induced slope failure is a desirable feature to be added to a future version of the methodology.

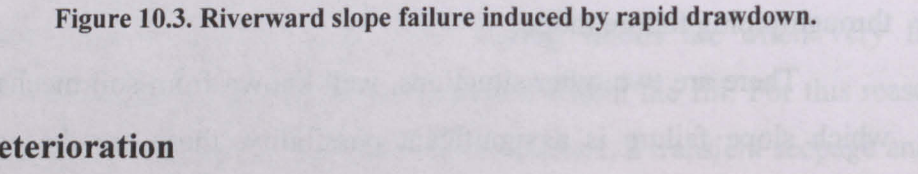

Figure 10.3. Riverward slope failure induced by rapid drawdown.

10.1.2 Deterioration

The reliability rating system is intended as a tool for quantifying the expected performance of flood embankments during a flood given their characteristics and conditions immediately before the extreme hydrologic event. Therefore this methodology does not deal with processes evolving on a longer time scale.

For instance the river morphodynamics, which can affect on the long run embankments, as much as natural banks, is not included in the Reliability Rating System. Similarly settlement, resulting in lower crest levels, is not taken into account in the proposed methodology. Although deterioration phenomena are important for the planning and management of the flood defence network in this work the priority has been attributed to an improved understanding and quantification of flood embankments performance and its relationship with random fluctuation of the relevant parameters and with the epistemic uncertainty.

10.2 Epistemic uncertainty

10.2.1 Lack of knowledge

The Reliability Rating System takes into account the characteristics of an embankment and of the natural soil in the foundation to quantify their influence on the structural performance in flooding conditions. However some of these characteristics are likely to be uncertain in the first phase of a safety assessment. The capability of handling this type of lack of knowledge (epistemic uncertainty) is one of the main features of the methodology and has influenced all its development. This goal has been achieved making a performance indicator readily available in a number of different scenarios. Moreover the Reliability Rating System permits a fast assessment of the impact of different situations on each failure mode.

The uncertain characteristics of the embankment and its foundation will be different from case to case. However some characteristics, intrinsically more difficult to assess and verify, are likely to be uncertain in most cases.

10.2.2 Epistemic uncertainties influencing under-piping

The modelling of breaching induced by foundation erosion involves parameters which are likely to be uncertain when assessing the safety of the embankment unless a detailed site investigation has been carried out. In particular the detection of a coarse-grained layer of soil in the foundation, under a covering fine-grained layer, requires some knowledge of the soil profile which is seldom available in the phase of preliminary screening of the flood defence network. The quantification of the expected performance cannot ignore:

- the existence of such a layer;
- the type of coarse-grained soil;
- the thickness of the covering layer.

The Reliability Rating System, with its pre-defined cases, offers a tool to handle the epistemic uncertainty about the three characteristics listed above.

Let an illustrative example be considered. Suppose that a flood embankment is built on fine-grained soil but, from geological maps and partial knowledge of the site the presence of a deeper layer of gravel can be inferred. This layer also outcrops at some distance from the embankment allowing for a check on the type of soil which displays particle sizes in the range of fine gravel. The depth of the interface between the covering layer and the deeper coarse-grained layer, however, remains uncertain in the zone of the embankment. The probability of breaching by under-piping with water at the crest level can be determined employing the Reliability Rating System. The final users will have to locate the characteristics which are known in the appropriate classes and to assume, on the basis of the available information an interval of variation for the unknown characteristics. In this way an upper and a lower bound on the probability of breaching by under-piping can be determined. These values can then be combined with the probabilities of breaching by other failure modes to obtain bounds on the performance indicator P_{BE}.

Table 10.1. Characteristics influencing the resistance to under-piping on an hypothetical flood embankment. The thickness of the covering layer is uncertain and can vary within an interval.

Characteristic	Value
Height (m)	about 3 m
Slopes inclination (v:h)	about 1:2
Type of fine–grained soil in covering layer	Soft Clay
Thickness of covering layer (m)	[0.8 , 2.0]
Type of coarse-grained soil in deeper layer	Fine Gravel

The probability of breaching for the embankment described in Table 10.1Table 10. can be determined with Equation (7.1)

$$\mathrm{P}(B_{up}) = \mathrm{P}(C) \times \mathrm{P}(CLE|C) + \mathrm{P}(\overline{C}) \times \mathrm{P}(FLE|\overline{C})$$

where C indicates the cracking of the superficial layer, $\overline{C}$ indicates that the superficial layer stays intact, CLE indicated that the coarse layer is eroded and FLE indicates that the fine grained layer is eroded. In this example it is supposed that the final user, on the basis of geological maps and partial information on the site, has estimated that the thickness of the covering layer could be between slightly less than 1 m and about 2 m. The Reliability Rating System provides values for all the variables in Equation (7.1).

Two calculations have to be performed with the unknown parameter assuming reference values of the two classes which enclose the interval; in this case 1 m and 2 m respectively. The probability of the covering layer cracking is available from Table 7.4, for the two reference values. The probability of the covering layer not cracking is easily found considering that "cracking" and "not cracking" are exhaustive, mutually exclusive events and therefore their probabilities add up to 1.0. The probability of the coarse layer experiencing piping-erosion given cracking has occurred is available form Table 7.5, for the two different reference values of thickness. The probability of the fine grained layer experiencing piping-erosion is found in Table 6.7 and does not depend on the thickness of the layer.

The calculations in Boxes 10.1 and 10.2 show how for the considered embankment the approximated probability of breaching, for water at the crest level, belongs to the interval:

$$2.49 \times 10^{-3} \leq P(B_{up})_{crest} \leq 2.83 \times 10^{-2}$$

or, with the set builder notation:

$$P(B_{up})_{crest} \in [2.49 \times 10^{-3}, 2.83 \times 10^{-2}]$$

Box 10.1. Calculation of the probability of breaching by under-piping for covering layer 1 m thick

Covering layer 1 m thick	
$P(C) = 9.45\times10^{-1}$	probability of the covering layer cracking
$P(\overline{C}) = 5.50\times10^{-2}$	probability of the covering layer not cracking
$P(CLE\|C) = 2.99\times10^{-2}$	probability of the coarse-grained layer being eroded given cracking occurred
$P(FLE\|\overline{C}) = 2.82\times10^{-5}$	probability of the fine-grained layer being eroded given no cracking
$P(B_{up}) = 2.83\times10^{-2}$	probability of breaching by under-piping for covering layer 1 m thick

Box 10.2. Calculation of the probability of breaching by under-piping for covering layer 2 m thick

Covering layer 2 m thick	
$P(C) = 6.40\times10^{-1}$	probability of the covering layer cracking
$P(\overline{C}) = 3.60\times10^{-1}$	probability of the covering layer not cracking
$P(CLE\|C) = 3.88\times10^{-3}$	probability of the coarse-grained layer being eroded given cracking occurred
$P(FLE\|\overline{C}) = 2.82\times10^{-5}$	probability of the fine-grained layer being eroded given no cracking
$P(B_{up}) = 2.49\times10^{-3}$	probability of breaching by under-piping for covering layer 2 m thick

It has to be observed that an error is introduced by using reference values for the input parameters rather than the exact values. For example the assumed lower bound on the top layer thickness, which is equal to 0.8 m, is converted into 1 m in the Reliability Rating System. Despite these kind of approximations, necessary to the development of the methodology, this approach give a first precious indication to the final users revealing that an uncertainty of about 1 m on the layer thickness translates into an uncertainty of about one order of magnitude on the probability of breaching by under-piping in the reference loading condition.

The results presented above are valid if the coarse-grained layer outcrops in the river bed. However if the fine-grained layer isolates the coarse-grained from the watercourse the probability of breaching would be due only to the chances of piping-erosion immediately beneath the embankment:

$$P(B_{up})_{crest} \equiv P(FLE|\overline{C}) = 2.82 \times 10^{-5}$$

This example illustrates how the soil profile has a massive influence on the performance of flood embankments in extreme hydraulic conditions. In the presented case, for the reference water level, the probability of breaching by under-piping changes by one order of magnitude for a variation of 1 m in the thickness of the covering layer. If the hydraulic communication between coarse-grained layer and river is also uncertain the variation in probability of breaching is of three orders of magnitude.

The epistemic uncertainty on one characteristic described here can be combined with other uncertainties. Remaining on this example the type of soil in the coarse-grained layer might be not known. In this case bounds on $P(B_{up})_{crest}$ can still be determined with two calculations: one with both the uncertain characteristics set to the worse situation, another with both the uncertain characteristics set to the best situation. If the final user believes that the deep layer is made of gravel but the particle size is not known with certainty the upper bound on $P(B_{up})_{crest}$, corresponding to the worse performance, would be calculated assuming a covering layer 1 m thick and fine gravel (more prone to piping) in the erodible layer. The lower bound on $P(B_{up})_{crest}$, corresponding to the best performance, would be calculated assuming a covering layer 2 m thick and coarse gravel (less prone to piping) in the erodible layer. This way of combining uncertainties is never ambiguous because it is always known, for each characteristic, which of the possible classes results in the worse or in the best performance.

The Reliability Rating System does not replace the judgement of the final users who have to decide which conditions are realistically possible on site. However the methodology translates the uncertainty on some basic characteristics into uncertainty on the expected performance in a clear and direct way which is not found in any of the other reliability methodologies in use for flood defences.

10.2.3 Epistemic uncertainties influencing through-piping

Also the resistance to through-piping depends on factors which are most likely uncertain in the case of existing embankments apart from those of extremely recent construction. In particular the procedure adopted for the compaction of the earthfill and the geologic origin of the fill material are very rarely know. The soil type is often unknown but can be determined by a trained geologist or geotechnical engineer with a rapid assessment procedure requiring a site visit but no detailed site investigation (Craig 1997).

In analogy with the previous example bounds on the probability of breaching by through-piping $P(B_{tp})_{crest}$ can be calculated assuming a worst case, with all the uncertain characteristics set to the class which most negatively affects the performance, and a best case with all uncertain characteristics set to the class which produces an optimal performance. The valued of $P(B_{tp})_{crest}$ can be found applying the procedure derived in Chapter 9. Table 10.2 shows the calculation of the upper bound on $P(B_{tp})_{crest}$ for an illustrative example in which the compaction, the soil type used for the earthfill and its geologic origin are uncertain.

Table 10.2. Individual weights and coefficients of influence reduction for the upper bound on $P(B_{tp})_{crest}$ of an embankment with typical uncertainty about compaction, type and geologic origin of the soil in the earthfill.

i	r_i	FEATURE	CONDITION	w_i	$\max(w_i r_i, 1)$
1	1	Compaction	No compaction	25	25
2	1/3	Soil type	Silt LL<50	21	7
3	1/9	Soil geologic origin	Alluvial	8	1
			$\Pi \max[w_i r_i, 1] =$		1.75×10^2

The calculation leads to an upper bound on the $P(B_{tp})_{crest}$, corresponding to the worse performance, equal to:

$$P(B_{tp})_{crest} = 9.0 \times 10^{-5} \times 1.75 \times 10^2 = 1.58 \times 10^{-2}$$

The lower bound on $P(B_{tp})_{crest}$, corresponding to the optimal performance, coincides with the probability of breaching by through-piping of the reference embankment defined in Chapter 9:

$$P(B_{tp})_{crest} = 9.0 \times 10^{-5}$$

The bounds on $P(B_{tp})_{crest}$ are plotted in Figure 10.4 along with the verbal descriptors of performance proposed by USACE (1999). It can be seen that the uncertainty on these three characteristics, which represent the typical level of epistemic uncertainty for old flood embankments, results in a significant uncertainty in $P(B_{tp})_{crest}$ of more than two orders of magnitude.

If after some investigation the type of soil used in the earthfill and its geologic origin are determined and only the compaction procedure is left to be completely unknown, the uncertainty on $P(B_{tp})_{crest}$ is reduced. Figure 10.5 shows the bounds on the probability of breaching by through-piping in the case that the fill material results to be high compressibility clay of glacial origin. The knowledge acquired about two previously uncertain characteristics has reduced the uncertainty on $P(B_{tp})_{crest}$ which is now slightly more than one order of magnitude.

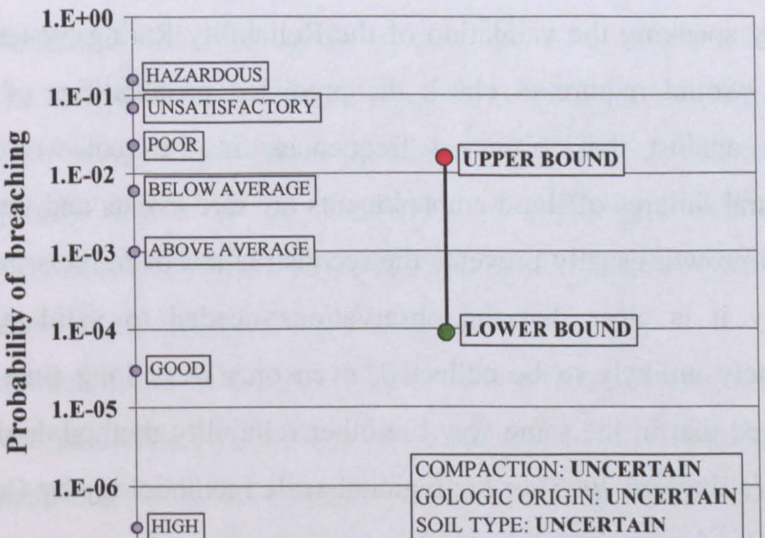

Figure 10.4. Bounds on the probability of breaching by through-piping for an embankment with typical uncertainty about compaction, type and geologic origin of the soil in the earthfill.

This example illustrates how the Reliability Rating System can capture the reduction in uncertainty associated with the collection of more information.

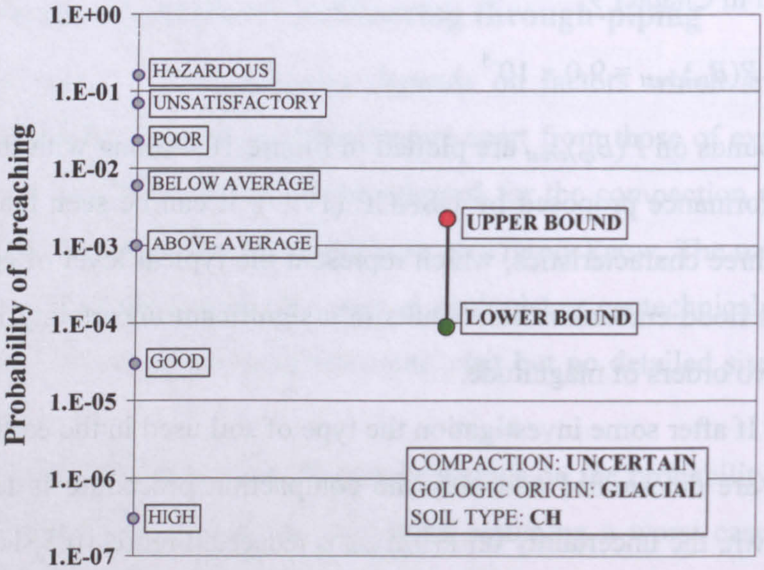

Figure 10.5. Bounds on the probability of breaching by through-piping for an embankment whose earthfill compaction is the only uncertain characteristic.

10.3 Validation

Strictly speaking the validation of the Reliability Rating System is not feasible. In fact it would require to check the predicted probabilities of different breaching modes against their observed frequencies in the real world. Considering that structural failures of flood embankments are rare events and that the rapidity of the breach growth usually prevents the reconstruction of failures that are not witnessed directly it is clear that the observations needed to validate the prediction are extremely unlikely to be collected, even on a very long time scale. It should be observed that in the same way the other reliability methodologies, like the HLM + (HR Wallingford 2005) or the regional-scale methodology by Gouldby *et al.* (2008), cannot be rigorously validated.

A partial, "loose" validation of the Reliability Rating System would be possible checking cases of breaching during flood events and seeing if the breached embankments correspond to those with highest values of the performance indicator P_{BE}, which quantifies the proneness to breaching. At the moment of writing the

Reliability Rating System has just been developed and the application to an inventory of embankments is being discussed with the Environment Agency (Redaelli 2008d, Bramley 2008). A trial application, described in the next section, has been carried out on a flood embankment on the river Irvine in Ayrshire (Scotland).

The verification of the component parts of the methodology is possible by checking the mathematical models used in the development against physical models and confronting the statistical distribution assumed for the parameters with an increasing number of real cases. Similarly the influence of different factors on the estimated subjective probabilities related to the through-piping mode of failure will be hopefully better understood in the future, possibly with the help of dedicated experimental programs. This growth of scientific and technical knowledge is constantly progressing thanks to the wider community of researchers working on the reliability of flood defences. Such an evolution should be incorporated in any future modification and improvement of the initial version of the Reliability Rating System.

10.4 Trial application

The newly developed methodology has been applied to a section 500m long of a flood embankment in the town of Galston, on the river Irvine in East Ayrshire (Scotland). The embankment, recently built, is also the object of study of a separate research project on the deterioration induced by seasonal cycles of wetting and desiccation jointly started by the University of Strathclyde and the Technical university of Milan (Castellanza *et al.* 2008). Clay from the same quarry used for the earthworks has been characterised by Zielinski (2008).

The characteristics of the embankment in Galston are summarised in Table 10.3. No investigation has been performed to determine the soil profile; however the quarry of glacial clay, which is located upstream, next to the embankment, reveals for this fine-grained deposit a thickness of several meters. Nothing suggests the existence beneath the embankment of a coarse-grained layer in communication with the watercourse. In the section examined a culvert, very well constructed to a state-of-the-art standard, is present (Figure 10.6). The grass cover although showing different levels of maintenance, is in good condition all over the embankment. No significant defects or anomalies were found inspecting the site.

Figure 10.6. State-of-the-art culvert across the flood embankment in Galston.

Table 10.3. Characteristics of the flood embankment in Galston.

Characteristic	Findings / Reliability Rating System class	
Height (m)	About 3	(2.5 , 3.5]
Slopes inclination (v:h)	About 1:2	[1:3 , 2:3)
Fill type	Clay (LL<50)	CH
Fill geologic origin:	Glacial	
Compaction	Complies with BS 6031:1981	Good
Grass cover condition:	Good	
Vegetation (roots effect)	Grass	
Animal burrowing	No	
Culvert	Optimal condition	
Differential settlement	Not visible	
Desiccation cracking	None	
Evidence of leakage	None	

For the determination of the probability of breaching by surface erosion, to be combined with the other failure modes to obtain the performance indicator P_{BE}, the time to peak of the Instantaneous Unit Hydrograph $T_p(0)$ has to be estimated. A possible, simple way of doing so is using a formula relating $T_p(0)$ to some catchment descriptors. The equation currently used is (FEH 1999):

$$T_p(0) = 4.27\, DPSBAR^{-0.35}\, PROPWET^{-0.80}\, DPLBAR^{0.54}\, (1 + URBEXT)^{-5.77} \qquad (10.1)$$

where:

- *DPSBAR* mean drainage path slope (m/km);
- *PROPWET* proportion of time when the soil moisture deficit was below 6 mm between 1961 and 1990;
- *DPLBAR* mean drainage path length (km);
- *URBEXT* extent of urban/suburban land cover.

These catchment descriptors are available for a gauged station downstream of Galston. The station of Glenfield has NRFA[11] reference 83802 and is located less than five kilometres downstream of the studied embankment. The catchment descriptors have been taken as representative of the river section in Galston without any further adjustment. The values at Glenfield are:

$DPSBAR = 64.79$ m/km

$PROPWET = 0.59$

$DPLBAR = 20.36$ km

$URBEXT = 0.0164$

These values of catchment descriptors result in a the time to peak of the Instantaneous Unit Hydrograph

$T_p(0) = 07\text{h}\ 06'$

which falls in the pre-set class of the Reliability Rating System (expressed in hours)

$T_p(0) \in (6\ ,12]$

The vertical distance from the most depressed point of the riverbed and the crest of the embankment is of about 4 m. Once the time to peak of the Instantaneous Unit Hydrograph and the depth of the cross section are found all the information needed for the determination of the Probability of Breaching in Extreme conditions P_{BE} is available. In fact the probabilities of structural failure for the reference water levels can be obtained from the tabulated results and then combined to obtain the

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[11] NRFA stands for National River Flow Archive: the national database comprising over 44,000 station-years of daily river flows in the UK.

performance indicator. In particular for the presented example the probability of breaching by under-piping $P(B_{up})_{crest}$ and through-piping $P(B_{tp})_{crest}$, both calculated with water level at the crest are:

$$P(B_{up})_{crest} = 2.82 \times 10^{-5}$$

$$P(B_{tp})_{crest} = 6.00 \times 10^{-4}$$

The probability of breaching by surface erosion $P(B_{se})_{ot}$ calculated for the representative level for overtopping conditions is:

$$P(B_{se})_{ot} = 6.84 \times 10^{-3}$$

These values result, through Equation (4.5), in a probability of breaching with water at the crest level:

$$P(B)_{crest} = 6.28 \times 10^{-4}$$

The value of the probability of breaching for the reference water level for overtopping, given by Equation (4.6), is:

$$P(B)_{ot} = 7.46 \times 10^{-3}$$

The value of the performance indicator P_{BE} follows from Equation (4.3) if the Annual Exceedance Probability of the crest level AEP_{crest}, or its approximation as the inverse of the Standard of Protection SOP, is known.

The function of the embankment in Galdston is not to directly defend the built environment from flood waters: it delimitates instead a water retention area for flood control. This implies that during flood events the embankment will have high water levels on both sides and its hydraulic boundary conditions will be different from the flood defence embankment for which the Reliability Rating System has been developed. Rather than studying the hydraulic loads actually acting on this particular structures the performance indicator P_{BE} has been calculated for a hypothetical embankment, identical to the one in Galdstone, but absolving the function of direct defence from flood water. The embankment has then been imagined into a range of different hydrologic situations in terms of Annual Exceedance Probability of the crest level AEP_{crest}. The values obtained for P_{BE} are plotted in Figure 10.7 against the Standard of Protection, here approximated by the inverse of AEP_{crest}. The effect on the performance indicator of different characteristics has been checked for illustrative purposes. The values of P_{BE} obtained assuming that no culvert is present or assuming instead the presence of a culvert with many poor details are added to the graphic to give an idea of the influence of these characteristics.

The intermediate results in terms of probability of breaching by different modes of failure sensibly indicate, for an embankment with no obvious defects or deficiencies, a prevalence of the grass cover failure as the most likely mode of failure. This is shown by the comparison of the values of $P(B_{up})_{crest}$, $P(B_{tp})_{crest}$ and $P(B)_{ot}$ calculated earlier in this Section. Consequently the total probability of breaching in reference conditions for overtopping is significantly higher than the one obtained with the water at the crest level.

The values of the performance indicator obtained for this trial application reflect the performance expected by an embankment with very good characteristics and optimal condition for the structural resistance to hydraulic loading. More case studies are certainly needed to explore the full potential and understand the limitations of the new methodology. At the time of writing the author is interacting with the Environment Agency in order to identify more locations for further testing the Reliability Rating System (Redaelli 2008d, Bramley 2008).

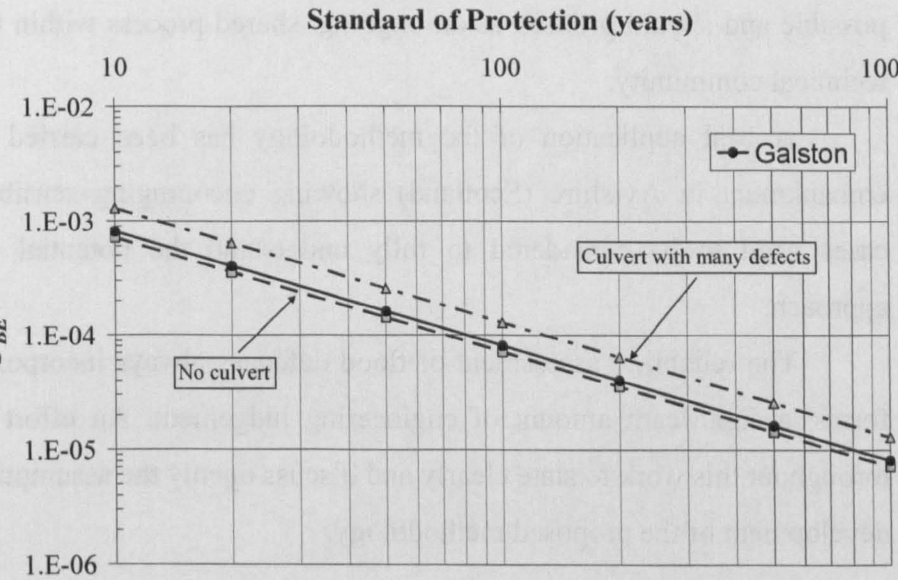

Figure 10.7. The performance indicator P_{BE} plotted against the Standard of Protection, for an embankment with the characteristics of the site in Galston, Ayrshire (Scotland), chosen for the trial application of the Reliability Rating System.

10.6 Conclusions

The Reliability Rating System addresses the breaching of flood embankments due to three different modes of structural failure: under-piping, through-piping and grass cover failure. Slope failure due to concentrated seepage is included in the through-piping mode of failure.

The proposed methodology produces a quantitative prediction of the structure's performance during floods on the basis of its relevant characteristics. Where some of these characteristics are uncertain the Reliability Rating System allows to rapidly quantify the implications of different possible scenarios, supporting the handling of epistemic uncertainty. The methodology converts the uncertainty on the characteristics of an embankment into uncertainty on its performance. The reduction in uncertainty produced by the acquisition of more information is also captured.

Strictly speaking the validation of a methodology which predicts the probability of occurrence of rare events, like the breaching of a flood embankment, would require to check the real frequencies of such events. This is not feasible in a reasonable timescale. The partial validation of the components of the methodology is possible and should proceed as an ongoing, shared process within the scientific and technical community.

A trial application of the methodology has been carried out for a flood embankment in Ayrshire (Scotland) showing encouraging sensible results. More cases need to be considered to fully understand the potential of the proposed approach.

The reliability assessment of flood defences always incorporates, in different forms, a significant amount of engineering judgement. An effort has been made, throughout this work to state clearly and discuss openly the assumptions made for the development of the proposed methodology.

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CONCLUSION

The Reliability Rating System is a new methodology for the quantification of the expected performance of flood embankments during extreme hydrologic events which allows to incorporate the effect of knowledge uncertainty. The methodology is fast, easy to use and does not require an advanced knowledge of probabilistic methods by the final user. However a final user aware of how the Reliability Rating System has been developed will be able to better understand its meaning and give the correct interpretation to the results. Clarity and transparency about assumptions have been pursued in this research project and should be maintained in any subsequent improvement or extension of the methodology.

10.1 Function of the Reliability Rating System

10.1.1 Quantitative assessment of the expected performance

The Reliability Rating System presented in this thesis is a new methodology to rank and compare flood embankments according to their proneness to breaching. It makes use of a performance indicator, called the Probability of Breaching in Extreme hydraulic conditions P_{BE} which is related to probability of structural failure of an embankment for a limited range of loads above and immediately below the crest. The methodology deals with three failure modes, i.e. breaching initiated by grass cover failure, breaching induced by piping through the manmade earthfill (through-piping) and breaching due to by piping through the natural soil on which the embankment is founded (under-piping).

Mathematical models are available for the grass cover failure and the under-piping process and the probability of breaching can be calculated with the methods of reliability analysis. In this work Monte Carlo simulation and FORM, in the spreadsheet implementation proposed by Low and Tang (1997), have been used.

This study has recognised that, as already stated by other authors (Richards & Reddy 2007, Fell *et al.* 2000), there is currently no mathematical model of through-piping which can be credibly applied to earth water-retaining structures. For this reason in the present work the probabilities of breaching by through-piping have been estimated with a structured process of subjective judgement elicitation.

The Reliability Rating System has been developed determining the quantities needed for the calculation of the performance indicator in a range of different scenarios that can be encountered in the non-tidal section of British rivers. Although the formulation of the performance indicator P_{BE} can be applied to any location and for hydraulics loads which are not exclusively fluvial, the range of reliability assessments carried out in this research project has focused on the British situation and has not included the loads that can interest estuarine and coastal embankments. An extensions of the methodology to other situations can be obtained performing additional reliability analyses and subjective probability elicitations.

10.1.2 Epistemic uncertainty

The Reliability Rating System is capable of returning the value of the performance indicator P_{BE} once the relevant characteristics of an embankment are known. If one or more of the relevant characteristics are uncertain the final users can employ the tabulated solutions presented here to study different possible scenarios. Thanks to this methodology upper and lower bounds on P_{BE} can be easily established.

10.2 Recommendations to the final users

The Reliability Rating System provides the final users with a methodology for the fast quantification of the expected performance of flood embankments in extreme hydraulic conditions and represents a tool to handle the epistemic uncertainty. The users who know the assumptions behind the development of the methodology are more likely to give a correct interpretation of the values of the performance indicator P_{BE}.

Importantly the Reliability Rating system does not replace the judgement and experience of the users. In fact the methodology, with its ability to quickly identify bounds on P_{BE}, is a support for decision making. However flood risk managers still have to assess what is an acceptable level of performance depending on the expected consequences in case of breaching. This remains a challenging task to accomplish also because the Reliability Rating System is not returning the total probability of structural failure in one year, but rather an indicative measure of performance related to the annual probability of structural failure for high water levels. Acceptable versus unacceptable values of P_{BE} will have to be defined through discussion in the flood risk community as the system is applied to more and more real cases. Nevertheless the Reliability Rating System represents an unprecedented tool for quantifying the effect of epistemic uncertainty on performance prediction and a new support for decision making able to guide choices about whether or not to invest in uncertainty reduction through targeted investigations.

10.3 Recommendations for future research

The Reliability Rating System offers a new support to the management of flood defences. However it is desirable to develop the concepts incorporated in this methodology in order to determine the probability of breaching of flood embankments for the whole range of possible loads. The achievement of this goal, which would allow the correct integrated modelling of flood risk, requires the generation of a set of judgement-based fragility curves for the through-piping mode of failure. The author believes that this goal should be a priority for the research in flood risk assessment (Redaelli & Dyer 2008).

In the meantime some aspects of the Reliability Rating System can be improved. In particular:

a. The reliability analysis of slope failure can be included as an additional failure mode. This is likely to be a challenging improvement considering the complexity of a suitable model as discussed in Section 10.1. Simultaneously a correction should be applied to the probabilities of breaching by through-piping, which currently incorporate also the failure induced by localised seepage phenomena. Rethinking this failure mode is important to avoid taking into account the same processes twice.

b. More advanced studies of the under-piping mode of failure would be a significant contribution. In particular a better understanding of the cracking mechanism of the top layer in a stratified foundation would represent a valuable improvement.

c. The probabilities of breaching by through-piping have been determined in the present work with a procedure which, although being rigorous, has unavoidably suffered from the limited resources available in terms of number of assessors in the panel and total amount of working hours. It is recommended that a more extensive elicitation process is carried out when more resources become available. This process of judgement elicitation should compare the innovative formal structure adopted here with the more traditional and time consuming event trees technique.

d. The determination of the probability of grass cover failure can be improved with an analysis which distinguishes between the crest and the landward slope. More complex models can be introduced to take into account the resistance to erosion of the soil under the grass cover.

The author believes that, in any further development or elaboration of the methodology presented in this thesis, a clear statement and an open discussion of newly introduced assumptions would be of paramount importance. In fact for its own nature the problem of the reliability assessment of extended flood defence networks still requires a remarkable amount of engineering judgement. This component must not remain hidden in implicit assumptions which are difficult to access and decode by final users and other researchers.

REFERENCES

Åberg B., 1993. Washout of grains from filtered sand and gravel materials. *Journal of Geotechnical Engineering* 119(1): 36–53.

Adel D.H., Bakker K.J., Breteler M.K., 1988. Internal stability of minestone. *Proc. international symposium on modelling soil–water–structure interaction*. Balkema, Rotterdam: 225–231.

Aitchison G.D., Wood C.C., 1965. Some interaction of compaction, permeability, and post-construction deflocculation affecting the probability of piping failure in small earth dams. *Proc. 6th Int. Conf. Soil Mech. Found. Eng.* : 442–446.

Allsop W., Kortenhaus A., Morris M., 2007. *Failure Mechanisms for Flood Defence Structures*. Technical Report T04-06-01, FLOODsite Consortium.

Alpert M., Raiffa H., 1982. A progress report on the training of probability assessors. In Kahneman D., Slovic P., Tversky A. (Eds), *Judgment under Uncertainty: Heuristics and Biases*, Cambridge University Press, Cambridge, UK.

Ang A.H.-S., Tang W.H., 1984. *Probability Concepts in Engineering Planning and Design. Volume II – Decision, Risk And Reliability*. John Wiley, New York.

ASTM, 1985. *Classification of Soils for Engineering Purposes: Annual Book of ASTM Standards*, American Society for Testing and Material 4(8): 395-408.

Baecher G.B., 1972. *Site Exploration: a Probabilistic Approach*. PhD Thesis, Massachusetts Institute of Technology, Boston.

Baecher G.B., Christian J.T., 2003. *Reliability and Statistics in Geotechnical Engineering*. John Wiley, New York..

Benn H., 2008. Flood and Coastal Erosion Risk Management (Budget Allocations and Outcome Targets). Declaration of The Secretary of State for Environment, Food and Rural Affairs, *Hansard (House of Commons Daily Debates)* 4 Feb 2008 : Column 50WS.

Bligh W.G., 1910. Dams, barrages and weirs on porous foundations, *Engineering News* 64(26): 708-710.

Bramley M., 2008. Personal communication. Visit at the University of Strathclyde, 21 Oct 2008.

Breitung K., 1984. Asymptotic approximations for multi-normal integrals. *Journal of the Engineering Mechanics Division*, ASCE 110(3): 357-366.

British Standard, 1983. *Code of Practice for Earthworks*, BS 6031:1981.

Buijs F., Simm J., Wallis M., Sayers P., 2007. *Performance and Reliability of Flood and Coastal Defences.* Joint Defra/EA Flood and Coastal Erosion Risk Management R&D Programme, Technical Report FD2318 (Available at *http://randd.defra.gov.uk/Document.aspx?Document=FD2318_5925_TRP.pdf, http://randd.defra.gov.uk/Document.aspx?Document=FD2318_5926_TRP.pdf*).

Buijs F.A., van Gelder H.A.J.M., Hall J.W., 2004. Application of reliability-based flood defence design in the UK. *Heron* 49(1): 33-50.

Burenkova V.V., 1993. Assessment of suffosion in non-cohesive and graded soils. In Brauns J, Heibaum M, Schuler U. (Eds) *Filters in geotechnical and hydraulic engineering*. Balkema, Rotterdam : 357–360.

Bye P., Horner M., 1998. *Easter 1998 floods: Report by the independent review team to the board of the Environment Agency.* Environment Agency, Bristol.

Casciati F., Faravelli L., 1991. *Fragility Analysis of Complex Structural Systems.* Research Studies Press, Taunton.

Castellanza R., Secondi M., Sentenac P., Utili S., 2008. *Long Term Deterioration of Flood Embankments*. Interim Report to the Institution of Civil Engineering, R&D Enabling Fund, Grant 0708.

Charles J.A., 1989. Deterioration of clay barriers: case histories. *Proc. Conference on Clay Barriers for Embankment Dams*, ICE, London.

Chow V.T., 1959. *Open Channel Hydraulics*. McGraw-Hill, New York.

Christian J.T., 2004. Geotechnical engineering reliability: how well do we know what we are doing? *Journal of Geotechnical and Geoenvironmental Engineering* 130(10): 985-1003.

CIRIA, 1996. *Small Embankment Reservoirs* (by Kennard M.F., Hoskins C.G. and Fletcher M.). CIRIA Report 161.

CIRIA, 2000. *Risk Management for UK Reservoirs* (by Hughes A.K., Hewlett H.W.M., Morris M., Sayers P., Moffat A.I.B., Harding A. & Tedd P.). CIRIA Report C542.

Craig R.F., 1996. *Soil Mechanics*. 6th Edition, Chapman & Hall.

CUR/TAW, 1990. *Probabilistic Design of Flood Defences*. Centre for Civil Engineering Research / Technical Advisory Committee on Flood Defences, Report 141, Gouda, The Netherlands.

Dawson R.J., Hall J.W., 2002a. Improved condition characterisation of coastal defences. In N.W.H. Allsop (Ed), *Coastlines, Structures and Breakwaters*, Proc. ICE Conf., Thomas Telford, London:123-134.

Dawson R.J., Hall J.W., 2002b. Probabilistic condition characterisation of coastal structures using imprecise information. *Proc. 28th International Conference on Coastal Engineering*, Cardiff: 2348–2359.

Der Kiureghian A., Lin H.-Z., Hwang S.-J., 1987. Second-order reliability analysis approximations. *Journal of Engineering Mechanics,* ASCE 113(8): 1208-1225.

DETR, 2000. *Guidelines for Environmental Risk Assessment and Management*. 2nd Edition, Department of the Environment, Transport and the Regions, Environment Agency and Institute of Environmental Health, the Stationery Office, London.

Ditlevsen O., 1979. Narrow reliability bounds for structural system. *Journal of Structural Mechanics* 7(4): 53–472.

Ditlevsen, O., 1981. *Uncertainty Modeling with Applications to Multidimensional Civil Engineering Systems*. McGraw-Hill, New York.

Dyer M., 2004. Construction and stability of historic flood defence embankments in england and wales. *Proc. ICE, Water Management* 157(4): 177-186.

Dyer M., Gardener R., 1996. *Geotechnical Performance of Flood Defence Embankments*. Environment Agency R&D Technical Report W35.

Edwards W., Tversky A., 1967. *Decision Making: Selected Readings*, Penguin Books.

Environment Agency, 2001. *Lessons Learned: Autumn 2000 Floods*. Environment Agency, Bristol.

Environment Agency, 2006. *Managing Flood Risk - Condition Assessment Manual*. Ref. 166_03_SD01.

Ervine A., 2008. Personal communication: e-mail to M. Redaelli, 21 Jan 2008.

FEH, 1999. *Flood Estimation Handbook* (by Faulkner D. Robson. A., Bayliss, A., Houghton-Carr H., Reed D.). Institute of Hydrology, Wallingford.

Fell R., Bowles D.S., Anderson L.R., Bell G., 2000. The status of methods for estimation of the probability of failure of dams for use in quantitative risk assessment, *Proc. 20th ICOLD Congress*, Beijing, China.

Fell R., Wan C.F., Foster M., 2004. *Methods for Assessing the Probability of Failure of Embankment Dams by Internal Erosion and Piping.* UniCiv Report No. R-428 May 2004, University of New South Wales, Sydney, Australia.

FEMA, 2005. *Technical Manual for Dam Owners: Impacts of Animals on Earthen Dams*. Federal Emergency Management Agency, US Department of Homeland Security.

FEMA, 2005. *Technical Manual for Dam Owners: Impacts of Plants on Earthen Dams.* Federal Emergency Management Agency, US Department of Homeland Security.

Fenton G.A., 1990. *Simulation and Analysis of Random Fields*. PhD thesis, Department of Civil Engineering and Operations Research, Princeton University.

Fenton G.A., Griffiths D.V., 1993. Statistics of block conductivity through a simple bounded stochastic medium. *Water Resources Research* 29(6): 1825-1830.

Fenton G.A., Vanmarcke E.H., 1990. Simulation of random fields via local average subdivision. *Journal of Engineering Mechanics* 116(8):1733–1749.

Ferrell W.R., McGoey P.J., 1980. A model of calibration for subjective probabilities. *Organizational Behavior & Human Performance* 26(1): 32-53.

Ferson S., Ginzburg L.R., 1996. Different methods are needed to propagate ignorance and variability. *Reliability Engineering and System Safety* 54(2): 133-144.

Ferson S., Joslynb C.A.,. Heltonc J.C, Oberkampfd W.L., Sentz K., 2004. Summary from the epistemic uncertainty workshop: consensus amid diversity. *Reliability Engineering & System Safety* 85(1-3), Special issues: Alternative Representations of Epistemic Uncertainty: 355-369.

Fischhoff B., Slovic P., Lichtenstein S., 1977. Knowing with certainty: the appropriateness of extreme confidence. *Journal of Experimental Psychology: Human Perception and Performance* 3(4): 552-564.

Folayan J.I., Hoeg K., Benjamin, J.R., 1970. Decision theory applied to settlement predictions. *Journal of the Soil Mechanics and Foundation Division* 96(SM4): 1127-1141.

Foster M., Fell R., Spannagle M., 2000a. The statistics of embankment dam failures and accidents. *Canadian Geotechnical Journal* 37(5): 1000-1024.

Foster M., Fell R., Spannagle M., 2000b. A method for assessing the relative likelihood of failure of embankment dams by piping. *Canadian Geotechnical Journal* 37(5): 1025-1061.

Franco G.E., Bagtzoglou A.C., 2002. An elastic contact mechanics fracture flow model. *Proc. 15th ASCE Engineering Mechanics Conference*, June 2-5, Columbia University, New York.

Gabr M.A., Wolff T.F., Brizendine A.L., Taylor H.M., 1996. Underseepage analysis of levees on two-layer and three-layer foundation. *Computers and Geotechnics* 18(2): 85-107.

Galarowski T., 1976. New observations of the present-day suffosion (piping) processes in the Bereznica catchment basin in the Bieszczady Mountains (The East Carpathians). *Studia Geomorphologica Carpatho-Balcanica* (Krakow) 10:115–122.

GeoDelft (1993). *MPROSTAB Users manual.*

Gilbert R.B., Wright S.G., Liedtke E., 1998. Uncertainty in back analysis of slopes: Kettleman Hills case history. *Journal of Geotechnical and Geoenvironmental Engineering* 124(12): 1167-1176.

Gouldby B., Sayers P., Mulet-Marti J., Hassan M.A.A.M., Benwell D., 2008. A methodology for regional-scale flood risk assessment. *Proc. ICE, Water Management* 161(3): 169–182.

Griffiths D.V., Fenton G.A., 1993. Seepage beneath water retaining structures founded on spatially random soil. *Geotechnique* 43(6): 577-587.

Halcrow, HR Wallingford, 2001. *ISIS flow user manual. Vol.1 User Guide*, HR Wallingford.

Halcrow, HR Wallingford, John Chatterton Associates, 2001. *National Appraisal of Assets at Risk from Flooding and Coastal Erosion - Final Report*. Department for

Environment, Food and Rural Affairs, Flood Management Division, London. (Available at *http://www.defra.gov.uk/environ/fcd/policy/NAAR1101.pdf*).

Hall J.W., Dawson R.J., Sayers P.B., Rosu C., Chatterton J.B., Deakin R., 2003. A methodology for national scale flood risk assessment. *Proc. ICE, Water and Maritime Engineering* 156(3): 235-247.

Harr M.E., 1962. *Groundwater and Seepage*. McGraw-Hill, London.

Harr M.E., 1987. *Reliability-Based Design in Civil Engineering.* McGraw-Hill,New York.

Hasofer A.M., Lind N.C., 1974. An exact and invariant first-order reliability format. *Journal of the Engineering Mechanics Division, ASCE* 100(EM1): 111-121.

Hassan A., Wolff T.F., 1999. Search algorithm for minimum reliability index of Earth slopes. *Journal of Geotechnical and Geoenvironmental Engineering*: 125(4): 301–308.

Hicks M.A., Onisiphorou C., 2005. Stochastic evaluation of static liquefaction in a predominantly dilative sand fill. *Géotechnique* 55(2): 123–133.

Hicks M.A., Onisiphorou C., Samy K., Spencer W.A, 2005. Implications of soil variability for geo-computations. *Proc. 13th ACME Conference*, University Of Sheffield, March 21-22.

HM Treasury, 2002. *Opportunity and Security For All: Investing in an Enterprising, Fairer Britain. New Public Spending Plans 2003–2006.* HMSO, London.

HR Wallingford, 2005. *Performance-based Asset Management System (PAMS) – Phase 1 Scoping Study*. Defra/Environment Agency Flood and Coastal Defence R&D Programme, Technical Report W5-070/TR (Available at *http://randd.defra.gov.uk/Document.aspx?Document=PAMS__9527.PDF*).

HR Wallingford, 2007. *Performance and Reliability of Flood and Coastal Defences.* Joint Defra/EA R&D Technical Report FD2318.

HR Wallingford, University of Bristol, 2003. *Risk Assessment for Flood and Coastal Defence for Strategic Planning – High Level Methodology.* DEFRA/Environment Agency Flood and Coastal Defence R&D Programme, Technical Report W5B-030/TR1.

HR Wallingford, University of Bristol, 2004. *Risk Assessment for Flood and Coastal Defence for Strategic Planning (RASP).* Defra/Environment Agency Flood and

Coastal Defence R&D Programme, Technical Report W5B-030/TR (Available at *http://randd.defra.gov.uk/Document.aspx?Document=RASP__9506.PDF*).

Hynes M.E., Vanmarke E.K., 1976. Reliability of embankment performance predictions. *Proc. ASCE Engineering Mechanics Division Specialty Conference*. Waterloo, Ontario, Univ. of Waterloo Press.

ICE, 2001. *Learning to Live with Rivers*. Institution of Civil Engineers Presidential Commission, London.

ICOLD, 1983. *Deterioration of Dams and Reservoirs*. International Commission on Large Dams, Paris.

ICOLD, 1984. *World Register of Dams – 3rd Updating*. International Commission on Large Dams, Paris.

ICOLD, 1995. *Dam Failure Statistical Analysis*. International Commission on Large Dams, Bulletin 99.

Iman RL, Davenport JM, Zeigler DK, 1980.*Latin Hypercube Sampling (Program Users's Guide)*. Technical Report SAND-79-1473 Sandia Labs., Albuquerque, NM (USA).

Johansen P.M., Vick S.G., Rikartsen C., 1997. Risk analysis for three Norwegian dams. In Broch, Lysne, FlatbÆ, Helland-Hansen (Eds), *Hydropower 97*, Balkema, Rotterdam: 431-442.

Jones J.A.A., 1981. *The Nature of Soil Piping: a Review of Research*. BGRG research monograph 3. Geo Books, Norwich

Kahneman D., Slovic P., Tversky A., 1982. *Judgment Under Uncertainty: Heuristics and Biases*, Cambridge University Press, Cambridge, UK.

Kenney T.C., Lau D., 1985. Internal stability of granular filters. *Canadian Geotechnical Journal* 22: 215–225.

Keren G., 1994. The rationality of gambling: gamblers' conceptions of probability, chance and luck. In Wright G. & Ayton P. (Eds), *Subjective Probability*, John Wiley, Chichester: 485-499.

Khilar K.C., Fogler H.S., Gray D.H., 1985. Model for piping-plugging in earthen structures. *Journal of Geotechnical Engineering* 111(7): 833-846.

Knodel P.C., 1991. *Characteristics and Problems of Dispersive Clay Soils*. Bureau of Recalmation, US Department of Interior, Denver.

Koenders M.A., Sellmeijer J.B, 1992. Mathematical model for piping, *Journal of Geotechnical Engineering* 118(6): 943-946.

Kovacs G., 1981. *Developments in Water Science — Seepage Hydraulics*. Elsevier, Amsterdam: 349–379.

Kral V., 1975. Sufoze a jeji podil na soucasnych geomorphologickych procesech v Cechach. *Acta Univ Carol Geogr* 1(2): 22–30.

Kuijper, H., Vrijling, J.K. 1998. Probabilistic approach and risk analysis. In: Pilarczyk K.W. (Ed), *Dikes and Revetments*, Balkema : 443-462.

Lacasse S., Nadim F., 1996. Uncertainties in characterising soil properties. In Shackelford C.D., Nelson P.P., Roth M.J.S. (Eds), *Uncertainty in the Geologic Environment: from Theory to Practice*, Proc. Uncertainty '96, ASCE Geotechnical Special Publication No. 58: 49-75.

Landon-Jones I., Wellington N.B., Bell G., 1995. Risk assessment of Prospect Dam. *ANCOLD Bulletin* 103: 16-31, Australian National Committee on Large Dams.

Lane E.W., 1935. Security from under-seepage: masonry dams on earth foundation. *Transaction ASCE* 100: 1235-1272.

Lassing B.L., Vrouwenvelder A., Waarts P.H., 2003. Reliability analysis of flood defence systems in the Netherlands. In Bedford T., van Gelder P.H.A.J.M. (Eds), *Safety and Reliability: Proc. ESREL 2003, Maastricht*, Balkema.

Lindley D.V., 1991. *Making Decisions*. John Wiley, London.

Long G., Mawdesley M., Simm J., 2006. *Improved Approaches to Condition Assesment*. FRMRC Research Report UR10

Louis C., 1969. *A Study of Groundwater Flow in Jointed Rock and its Influence on the Stability of Rock Masses*. Imperial College Rock Mechanics Research Report.

Low B.K., 2007. Reliability analysis of rock slopes involving correlated nonnormals *International Journal of Rock Mechanics and Mining Sciences* 44(6): 922-935.

Low B.K., Tang W.H., 1997 Efficient reliability evaluation using spreadsheet. *Journal of Engineering Mechanics* 123(7): 749-752.

Lumb P., 1974. Application of statistics in soil mechanics. In Lee I.K. (Ed) *Soil Mechanics: New Horizons*, Newnes-Butterworth, London: 44–112.

Mantoglu A., Wilson J.L., 1982. The Turning Bands Method (TBM) for simulating random fields using line generation by a spectral method. *Water Resources Research* 18(5): 1379–1394.

McClelland A.G.R., Bolger F., 1994. The calibration of subjective probabilities: theories and models 1980–94. In Wright G., Ayton P. (Eds), *Subjective Probability,* John Wiley, Chichester: 453–482.

McCook D.K., 2004. A comprehensive discussion of piping and internal erosion failure mechanisms. *Proc. annual dam safety conference*, ASDSO, Phoenix, Arizona.

McKay M.D., Beckman R.J., Conover W.J., 1979. A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. *Technometrics* 21(2): 239–245.

Melchers R.E., 1995. *Structural Reliability Analysis and Prediction.* John Wiley, Chichester.

Microsoft, 2003. *Microsoft Office Excel.*

Möllmann A.F.D., Vermeer P.A., 2007. Reliability analysis of a dike failure. *Proc. 18th European Young Geotechnical Engineers' Conference* (EYGEC 2007), Ancona, Italy.

Morris M., Dyer M., Smith P., 2007. *Management of Flood Embankments - A Good Practice Review.* Defra/Environment Agency Flood and Coastal Defence R&D Programme, Technical Report FD2411/TR1.

Murphy A.H., Winkler R.L., 1977a. Can weather forecasters formulate reliable forecasts of precipitation and temperature? *National Weather Digest* 2: 2–9.

Murphy A.H., Winkler R.L., 1977b. Credible interval temperature forecasting: some experimental results. *Monthly Weather Review* 102(11): 784–794.

Nadim F., 2006. *Risk and Reliability.* Lecture at the CISM Advanced School on Probabilistic Methods in Geotechnical Engineering, July 10 – 14, Udine (Italy).

NERC, 1975. *Flood Studies Report.* Natural Environment Research Council, London.

Nielsen D.R., Biggar J.W., Erh K.T., 1973. Spatial variability of field-measured soil-water properties. *Hilgardia* 42: 215-59.

NRC, 2000. *Risk Analysis and Uncertainty in Flood Damage Reduction Studies.* National Research Council Committee on Risk-Based Analysis for Flood Damage Reduction, National Academy Press, Washington. D.C..

Paice G.M., Griffiths D.V., Fenton G.A., 1996. Finite element modeling of settlements on spatially random soil. *Journal of Geotenical Engineering* 122(9): 777-779.

Palisade, 2005. *@RISK,* Version 4.5.

Paul C.K., 1991. *Characteristics and Problems of Dispersive Clay Soils.* USBR Report R-91-0, Denver.

Pauls G.J., Karl Sauer E., Christiansen E.A., Widger R A., 1999. A transient analysis of slope stability following drawdown after flooding of a highly plastic clay. *Canadian Geotechnical Journal* 36(6): 1151–1171.

Penning-Rowsell E.C., Chatterton J.B., Wilson T.L , 2002. *Autumn 2000 Floods in England and Wales: Assessment of National Economic and Financial Losses.* Flood Hazard Research Centre, Middlesex University.

Phoon K.K, Kulhawy F.H., 1996. On quantifying inherent soil variability. In Shackelford C.D., Nelson P.P., Roth M.J.S. (Eds), *Uncertainty in the Geologic Environment: from Theory to Practice*, Proc. Uncertainty '96, ASCE Geotechnical Special Publication No. 58: 326-340.

Pitt M., 2008. *Learning Lessons From the 2007 Floods.* Cabinet Office, London. (Available at *http://www.cabinetoffice.gov.uk/thepittreview*)

Ramsey F.P., 1978. *Foundations: Essays in Philosophy, Logic, Mathematics and Economics*. Atlantic Highlands, NJ, Humanities Press.

Redaelli M., 2008a. *Piping through the Earthfill and Reliability of Flood Embankments*. Department of Civil Engineering Internal Report, 21 May 2008, University of Strathclyde.

Redaelli M., 2008b. *Breaching of Flood Embankments by Piping through the Earthfill: Elicitation of Subjective Probabilities.* Department of Civil Engineering, University of Strathclyde, 19th Sept 2008.

Redaelli M., 2008c. *Through piping - Subjective probabilities elicitation.* E-mail to Bramley M. (EA Special Advisor), Baxter G. (EA), Mitchell C. (EA), Simm J. (HR Wallingford), Morris M. (HR Wallingford), Smith P.R. (Royal Haskoning), Hall J. (University of Newcastle), Ogunyoye F. (Royal Haskoning), Powell J. (BRE), Long G. (University of Nottingham), 19 Sept 2008.

Redaelli M., 2008d. *M. Redaelli's Doctoral Research.* E-mail to Bramley M. (EA Special Advisor), Baxter G. (EA), Mitchell C. (EA), 2 Sept 2008.

Redaelli M., Dyer M., 2008. Discussion of "A methodology for regional-scale flood risk assessment" by Gouldby *et al.*. *Proc. ICE, Water Management*, Accepted for publication, 17 Sept 2008.

Redaelli M., Utili S., Dyer M., 2008. A proposal for a reliability rating system for fluvial flood defence embankments in the United Kingdom. *Proc. 10th International Symposium on Landslides and Engineered Slopes*, June 30 – July 4,. Xi'an, China.

Reddy L.N., Kakuturu S.P., 2006a. Evaluation of the parameters influencing self-healing in earth dams. *Journal of Geotechnical and Geoenvironmental Engineering* 132(7): 879–889.

Reddy L.N., Kakuturu S.P., 2006b. Mechanistic model for self-healing of core cracks in earth dams. *Journal of Geotechnical and Geoenvironmental Engineering* 132(7): 890–901.

Richards K.S., Reddy K.R., 2007. Critical appraisal of piping phenomena in earth dams. *Bulletin of Engineering Geology and the Environment* 66(4): 381-402.

Sayers P.B., Gouldby B., Simm J., Meadowcroft I., Hall J., 2003. *Risk, Performance and Uncertainty in Flood and Coastal Defence – A Review.* Defra/Environment Agency Flood and Coastal Defence R&D Programme, Technical Report FD2302/TR1

Sayers P.B., Hall J.W., Meadowcroft I.C., 2002. Toward a risk-based flood hazard management in the UK. *Proc. ICE, Civil Engineering* 150: 36-42.

Schweiger H.F., Thurner R., Pöttler R., 2001. Reliability analysis in geotechnics with deterministic finite elements — Theoretical concepts and practical application. *International Journal of Geomechanics* 1(4): 389-413.

Sellmeijer J.B., 1988. *On the Mechanism of Piping Under Impervious Structures.* PhD Thesis, TU Delft.

Sellmeijer J.B., 1981. Piping due to flow towards ditches and holes. *Proc. Euromech* 143, Delft: 69-72

Sellmeijer J.B., Koenders M.A., 1991. Mathematical model for piping. *Applied Mathematical Modelling* 15: 646-651.

Sherard J.L., 1971. *Study of Piping Failures and Erosion Damage from Rain in Clay Dams in Oklahoma and Mississippi*. Report prepared for the Soil Conservation Service, Washington, D.C..

Sherard J.L., Decker R.S., Ryker N.L., 1972. Piping in earth dams of dispersive clay. *Proc. specialty Conference on Performance of Earth and Earth-Supported Structures*. Purdue University, Indiana, ASCE 1(1): 589–626.

Sherard J.L., Dunnigan L.P., Decker RS ,1977. Some engineering problems with dispersive clays. In Sherard J.L., Decker R.S. (Eds), *Dispersive Clays, Related Piping and Erosion in Geotechnical Projects*. ASTM Special Technical Publication 623.

Sherard J.L., Dunnigan L.P., Talbot J.R, 1984a. Basic properties of sand and gravel filters. *Journal of Geotechnical Engineering* 110(6): 684-700.

Sherard J.L., Dunnigan L.P.,Talbot J.R, 1984b. Filters for silts and calys. *Journal of Geotechnical Engineering* 110(6): 701-718.

Sherard J.L., Woodward R.J., Gizienski S.F., Clevenger W.A., 1963. *Earth and Earth-Rock Dams: Engineering Problems of Design and Construction*. John Wiley.

Sherman L.K., 1932. Stream flow from rainfall by unit-graph method, *Eng. News-Rec.* 108: 501–505.

Skempton A.W, Brogan J.M., 1994. Experiments on piping in sandy gravels. *Geotechnique* 44(3): 449–460.

Steenbergen H.M.G.M, Lassing B.L., Vrouwenvelder A.C.W.M, Waarts P.H., 2004. Reliability analysis of flood defence systems. *Heron* 49(1): 51-73.

TAW, 1999. *Technical Report on Sand Boils (Piping) – [Draft English version, August 2002]*. Technical Advisory Committee on Flood Defences, the Netherlands.

TAW, 2004. *Technisch Rapport Waterspanningen bij dijken*. Technical Advisory Committee on Flood Defences, the Netherlands.

Taylor D.W., 1948. *Fundamentals of Soil Mechanics*. John Wiley, New York.

Terzaghi K., 1922. Der Grundbruch an Stauwerken und seine Verhutung (The failure of dams by piping and its prevention). *Die Wasserkraft* 17: 445–449. Reprinted in Terzaghi K. (1960) *From theory to practice in soil mechanics*. John Wiley, New York.

Terzaghi K., 1939. Soil mechanics: a new chapter in engineering science, *Journal of the Institution of Civil Engineers* 12:106-141.

Terzaghi K., 1943. *Theoretical Soil Mechanics*. John Wiley, New York.

Tessem B., 1992. Interval probability propagation. *International Journal of Approximate Reasoning* 7: 95-120.

Tomlinson S., Vaid Y., 2000. Seepage forces and confining pressure effects on piping erosion. *Canadian Geotechnical Journal* 37(1): 1–13.

Tversky A. & Kahneman D., 1983. Extension versus intuitive reasoning: the conjunction fallacy in probability judgment. *Psychological Review* 90: 293-315.

US Army, 1991. *Benefit Determination Involving Existing Levees.* Policy Guidance Letter No. 26, Dept. of the Army, Office of the Chief Engineers, Washington, D.C..

US Army, 1992. *Stability Evaluation of Existing Levees for Benefit Determination.* Engineer Thechnical Letter No. 110-2-328, Dept. of the Army, Office of the Chief Engineers, Washington, D.C..

USACE, 2002. *HEC-RAS, River Analysis System Hydraulic Reference Manual.* U.S. Army Corps of Engineers, Hydrologic Engineering Center, Davis, CA. (Available at http://www.hec.usace.army.mil/software/hec-ras/hecras-document.html)

USACE, 1999. *Risk-Based Analysis in Geotechnical Engineering for Support of Planning Studies.* US Corps of Engineers Publication No. ETL 1110-2-556.

USACE, 2006. *Reliability Analysis and Risk Assessment for Seepage and Slope Stability Failure Modes for Embankment Dams.* US Corps of Engineers Publication No. ETL 1110-2-561.

Van der Meer J.W., 1988. *Rock Slopes and Gravel Beaches Under Random Wave Attack.* PhD Thesis, Delft University of Technology.

Vanmarke E.H., 1977. Probabilistic modelling of soil properties, *Journal of Geotechnical Engineering Division* ASCE 103 (GT11): 1227–1246.

Vanmarke E.H., 1983. *Random Fields: Analysis and Synthesis*. Cambridge, MIT Press.

Veneziano D., 1974. *Contributions to Second Moment Reliability*. Res. Rep. No. R74-33. Dept. of Civ. Engrg., Mass. Inst. of Technol., Cambridge, Mass.

Vick S.G., 1997. Dam safety risk assessment: new directions. *Water Power and Dam Construction*, 49(6).

Vick S.G., 1999. *Considerations for Estimating Structural Response Probabilities in Dam Safety Risk Analysis*. US Bureau of Reclamation Technical Report, Ref. No. 99D88313040, USBR Technical Service centre, Denver, CO, USA.

Vick S.G., 2002. *Degrees of Belief: Subjective Probability and Engineering Judgement*. ASCE, Reston, Va, USA

Vrijling J.K., 2001. Probabilistic design of water defense systems in the Netherlands. *Reliability Engineering & System Safety* 74(3): 337-344.

Vrijling J.K., van Gelder H.A.J.M., 2000. *Probabilistic Design – Lecture Notes*. TU Delft.

Vrouwenvelder A.C.W.M., 2006. Spatial effects in reliability analysis of flood protection systems. *Proc.s 2nd International Forum on Engineering Decision Making*, Lake Louise, Canada.

Vrouwenvelder A.C.W.M., 1999. *Theoretical manual of PC-Ring, Part C: calculation methods* (in Dutch), 98-CON-R1204, Delft.

Vrouwenvelder A.C.W.M., Steenbergen, H.M.G.M., Slijkhuis, K.A.H., 2001. *Theoretical manual of PC-Ring, Part A: descriptions of failure modes* (in Dutch), Nr. 98-CONR1430, Delft.

Vrouwenvelder A.C.W.M., Steenbergen, H.M.G.M., Slijkhuis, K.A.H., 2001. *Theoretical manual of PC-Ring, Part B: statistical models* (in Dutch), Nr. 98-CON-R1431, Delft.

Walley P., Fine T.L., 1982. Towards a frequentist theory of upper and lower probability. *Annals of Statistics* 10: 741-761.

Wan C.F., Fell R., 2004. investigation of rate of erosion of soils in embankment dams. *Journal of Geotechnical and Geoenvironmentl Engineering* 130(4): 373-380.

Weijers J.B.A., Sellmeijer J.B., 1993. A new model to deal with the piping mechanism. In Brauns J., Heibaum M., Schuler U., (Eds) *Filters in Geotechnical and Hydraulic Engineering*, Balkema, Rotterdam.

Whitehead E., 1976. *A Guide to the Use of Grass in Hydraulic Engineering Practice*. CIRIA Technical Note 71, London.

Whitman R.V., 2000. Organizing and evaluating uncertainty in geotechnical engineering. *Journal of Geotechnical and Geoenvironmentl Engineering* 126(7): 583-593.

Winkler R. L., Murphy A.H., 1968. 'Good' probability assessors. *Journal of Applied Meteorology* 7: 751-758.

Wolff, T. F., Demsky E.C., Schauer J., Perry, E., 1996. Reliability assessment of dike and levee embankments. In Shackelford C.D., Nelson P.P., Roth M.J.S. (Eds), *Uncertainty in the Geologic Environment: from Theory to Practice*, Proc. Uncertainty '96, ASCE Geotechnical Special Publication No. 58: 636-650.

Wolff T.F., 1997. Geotechnical reliability of levees. *Hydrology and Hydraulics Workshop on Risk-Based Analysis for Flood Damage Reduction Studies* – Paper 6, U.S. Army Corps of Engineers, Hydrologic Engineering Center.

Worman A., Olafsdottir R., 1992. Erosion in a granular medium interface. *Journal of Hydraulic Research* 30(55): 639-655.

Xu B., Low B.K., 2006. Probabilistic stability analyses of embankments based on finite-element method. *Journal of Geotechnical and Geoenvironmental Engineering* 132(11): 1444-1454.

Young M., 2005. *Wave Overtopping and Grass Cover Layer Failure on The Inner Slope of Dikes*. Unesco-IHE Thesis.

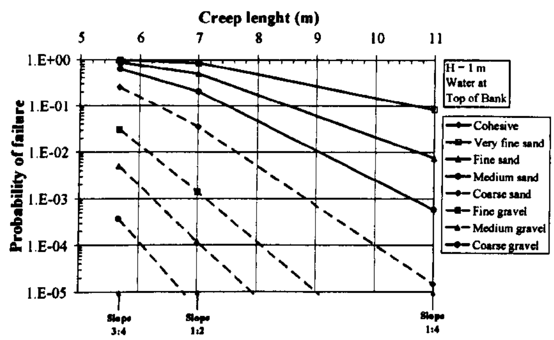
Zaslavsky D., Kassiff G., 1965. Theoretical formulation of piping mechanism in cohesive soils. *Geotechnique* 15: 305-316.

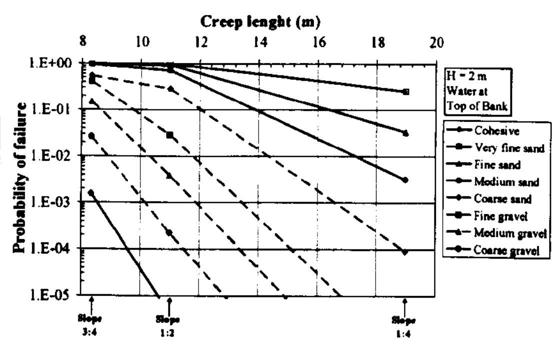
Zielinski M., Sentenac P., 2008. *Influence of the Desiccation Fine Fissuring on the Stability of Flood Embankments, Laboratory Studies.* FRMRC Final Research Report, University of Strathclyde.

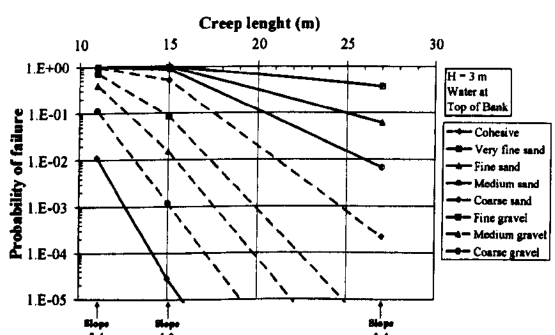
APPENDIX A

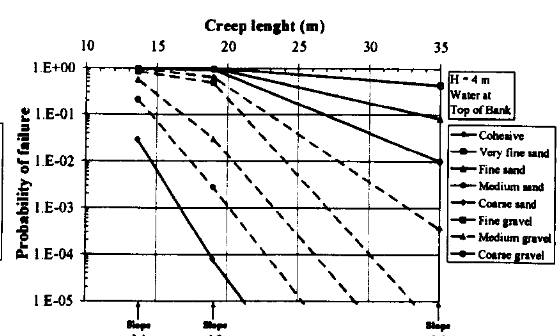
Probability of breaching by under-piping: homogeneous foundation

A.1 Diagrams









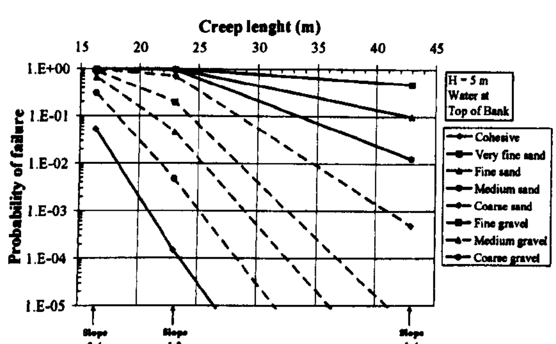


Figure A.1. Probability of breaching by under-piping for the homogeneous foundation case.

A.2 Tables

Table A.1. Probability of breaching by under-piping for the case of homogeneous foundation; embankment with a mean height of 1 m.

Probability of breaching by foundation erosion with water at crest level 1 m high embankment					
Slope			**3:4**	**1:2**	**1:4**
Soil Type	Cohesive	Clay, Silt (*LL*>50)	9.64×10^{-6}	6.55×10^{-8}	7.49×10^{-13}
	Sand	Very Fine, Silt (*LL*<50)	9.84×10^{-1}	8.52×10^{-1}	8.31×10^{-02}
		Fine	8.71×10^{-1}	4.95×10^{-1}	7.49×10^{-03}
		Medium	6.24×10^{-1}	2.06×10^{-1}	5.78×10^{-04}
		Coarse	2.53×10^{-1}	3.56×10^{-2}	1.45×10^{-5}
	Gravel	Fine	3.11×10^{-2}	1.40×10^{-3}	7.77×10^{-8}
		Medium	5.03×10^{-3}	1.18×10^{-4}	2.79×10^{-9}
		Coarse	3.74×10^{-4}	4.55×10^{-6}	5.92×10^{-11}

Table A.2. Probability of breaching by under-piping for the case of homogeneous foundation; embankment with a mean height of 2 m.

Probability of breaching by foundation erosion with water at crest level 2 m high embankment					
Slope			**3:4**	**1:2**	**1:4**
Soil Type	Cohesive	Clay, Silt (*LL*>50)	1.55×10^{-3}	4.30×10^{-6}	4.76×10^{-12}
	Sand	Very Fine, Silt (*LL*<50)	1.00	9.95×10^{-1}	2.54×10^{-1}
		Fine	9.99×10^{-1}	9.29×10^{-1}	3.38×10^{-2}
		Medium	9.86×10^{-1}	7.06×10^{-1}	3.19×10^{-3}
		Coarse	5.36×10^{-1}	2.87×10-^{1}	9.08×10^{-5}
	Gravel	Fine	4.09×10^{-1}	2.92×10^{-2}	5.15×10^{-7}
		Medium	1.52×10^{-1}	3.89×10^{-3}	1.85×10^{-8}
		Coarse	2.66×10^{-2}	2.23×10^{-4}	3.89×10^{-10}

Table A.3. Probability of breaching by under-piping for the case of homogeneous foundation; embankment with a mean height of 3 m.

Probability of breaching by foundation erosion with water at crest level 3 m high embankment					
		Slope	**3:4**	**1:2**	**1:4**
Soil Type	Cohesive	Clay, Silt (*LL*>50)	1.12×10^{-2}	2.82×10^{-5}	1.22×10^{-11}
	Sand	Very Fine, Silt (*LL*<50)	1.00	9.99×10^{-1}	3.66×10^{-1}
		Fine	1.00	9.82×10^{-1}	6.14×10^{-2}
		Medium	9.99×10^{-1}	8.74×10^{-1}	6.75×10^{-3}
		Coarse	9.73×10^{-1}	5.05×10^{-1}	2.18×10^{-4}
	Gravel	Fine	7.09×10^{-1}	8.70×10^{-2}	1.35×10^{-6}
		Medium	3.92×10^{-1}	1.54×10^{-2}	4.93×10^{-8}
		Coarse	1.13×10^{-1}	1.16×10^{-3}	1.03×10^{-9}

Table A.4. Probability of breaching by under-piping for the case of homogeneous foundation; embankment with a mean height of 4 m.

Probability of breaching by foundation erosion with water at crest level 4 m high embankment					
		Slope	**3:4**	**1:2**	**1:4**
Soil Type	Cohesive	Clay, Silt (*LL*>50)	2.96×10^{-2}	7.92×10^{-5}	2.13×10^{-11}
	Sand	Very Fine, Silt (*LL*<50)	1.00	1.00	4.33×10^{-1}
		Fine	1.00	9.92×10^{-1}	8.33×10^{-2}
		Medium	1.00	9.30×10^{-1}	1.01×10^{-2}
		Coarse	9.91×10^{-1}	6.31×10^{-1}	3.57×10^{-4}
	Gravel	Fine	8.40×10^{-1}	4.87×10^{-1}	2.35×10^{-6}
		Medium	5.64×10^{-1}	3.09×10^{-2}	8.74×10^{-8}
		Coarse	2.15×10^{-1}	2.78×10^{-3}	1.83×10^{-9}

Table A.5. Probability of breaching by under-piping for the case of homogeneous foundation; embankment with a mean height of 5 m.

Probability of breaching by foundation erosion with water at crest level 5 m high embankment					
Slope			3:4	1:2	1:4
Soil Type	Cohesive	Clay, Silt (*LL*>50)	5.25×10^{-2}	1.51×10^{-4}	3.05×10^{-11}
	Sand	Very Fine, Silt (*LL*<50)	1.00	1.00	4.76×10^{-1}
		Fine	1.00	9.96×10^{-1}	1.00×10^{-1}
		Medium	1.00	9.53×10^{-1}	1.30×10^{-2}
		Coarse	9.96×10^{-1}	7.05×10^{-1}	4.88×10^{-4}
	Gravel	Fine	9.00×10^{-1}	1.95×10^{-1}	3.36×10^{-6}
		Medium	6.72×10^{-1}	4.64×10^{-2}	1.27×10^{-7}
		Coarse	3.04×10^{-1}	4.73×10^{-3}	2.66×10^{-9}

APPENDIX B

Probability of breaching by under-piping: layered foundation

B.1 Probability of cracking of the covering layer

B.1.1 Diagrams

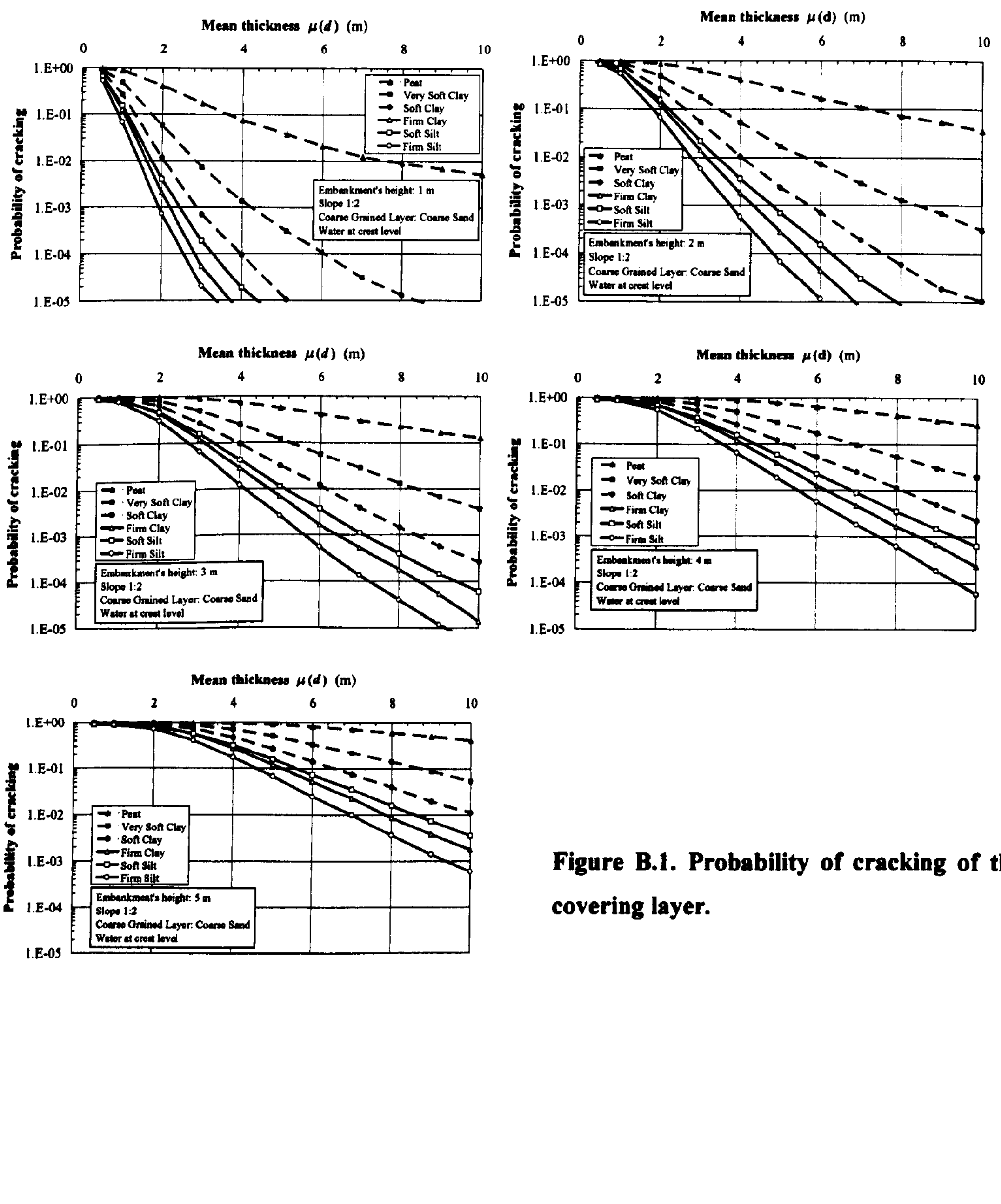


Figure B.1. Probability of cracking of the covering layer.

B.1.2 Tables

Table B.1. Probability of cracking of the covering layer; embankment's mean height of 1 m.

Probability of covering layer cracking: 1 m high embankment						
Mean thickness covering layer (m)	**Fine grained soil type**					
	Peat	**Very Soft Clay**	**Soft Clay**	**Firm Clay**	**Soft Silt**	**Firm Silt**
1	9.88×10^{-1}	9.09×10^{-1}	8.27×10^{-1}	6.89×10^{-1}	6.70×10^{-1}	5.38×10^{-1}
2	8.85×10^{-1}	4.91×10^{-1}	2.66×10^{-1}	1.24×10^{-1}	1.54×10^{-1}	6.78×10^{-2}
3	4.15×10^{-1}	5.73×10^{-2}	1.16×10^{-2}	2.11×10^{-3}	4.03×10^{-3}	7.29×10^{-4}
4	1.72×10^{-1}	7.28×10^{-3}	7.04×10^{-4}	5.46×10^{-5}	1.91×10^{-4}	2.04×10^{-5}
5	7.41×10^{-2}	1.36×10^{-3}	9.44×10^{-5}	5.34×10^{-6}	1.88×10^{-5}	2.78×10^{-6}
6	3.74×10^{-2}	3.14×10^{-4}	1.05×10^{-5}		3.31×10^{-6}	
7	2.11×10^{-2}	1.09×10^{-4}	4.47×10^{-6}			
8	1.25×10^{-2}	3.13×10^{-5}				
9	8.91×10^{-3}	1.32×10^{-5}				
10	6.78×10^{-3}	6.47×10^{-6}				

Table B.2. Probability of cracking of the covering layer; embankment's mean height of 2 m.

Probability of covering layer cracking: 2 m high embankment						
Mean thickness covering layer (m)	**Fine grained soil type**					
	Peat	**Very Soft Clay**	**Soft Clay**	**Firm Clay**	**Soft Silt**	**Firm Silt**
1	9.94×10^{-1}	9.64×10^{-1}	9.58×10^{-1}	9.48×10^{-1}	8.71×10^{-1}	8.50×10^{-1}
2	9.91×10^{-1}	9.21×10^{-1}	8.38×10^{-1}	7.11×10^{-1}	6.92×10^{-1}	5.51×10^{-1}
3	8.88×10^{-1}	4.97×10^{-1}	2.69×10^{-1}	1.26×10^{-1}	1.55×10^{-1}	6.79×10^{-2}
4	6.52×10^{-1}	1.76×10^{-1}	5.42×10^{-2}	1.41×10^{-2}	2.17×10^{-2}	5.87×10^{-3}
5	4.18×10^{-1}	5.27×10^{-2}	1.05×10^{-2}	1.82×10^{-3}	3.63×10^{-3}	5.80×10^{-4}
6	2.65×10^{-1}	1.69×10^{-2}	2.37×10^{-3}	2.87×10^{-4}	6.96×10^{-4}	6.89×10^{-5}
7	1.64x10'1	7.15×10^{-3}	7.03×10^{-4}	4.50×10^{-5}	1.53×10^{-4}	1.16×10^{-5}
8	1.10×10^{-1}	2.89×10^{-3}	1.92×10^{-4}	7.76×10^{-6}	3.04×10^{-5}	
9	7.28×10^{-2}	1.27×10^{-3}	5.90×10^{-5}	3.11×10^{-6}	8.91×10^{-6}	
10	5.31×10^{-2}	6.88×10^{-4}	1.86×10^{-5}		2.41×10^{-6}	

Table B.3. Probability of cracking of the covering layer; embankment's mean height of 3 m.

Probability of covering layer cracking: 3 m high embankment						
Mean thickness covering layer (m)	Fine grained soil type					
	Peat	Very Soft Clay	Soft Clay	Firm Clay	Soft Silt	Firm Silt
1	9.94×10^{-1}	9.71×10^{-1}	9.65×10^{-1}	9.57×10^{-1}	8.87×10^{-1}	8.78×10^{-1}
2	9.93×10^{-1}	9.62×10^{-1}	9.45×10^{-1}	9.09×10^{-1}	8.43×10^{-1}	7.91×10^{-1}
3	9.79×10^{-1}	8.14×10^{-1}	6.40×10^{-1}	4.47×10^{-1}	4.71×10^{-1}	3.09×10^{-1}
4	8.96×10^{-1}	4.93×10^{-1}	2.67×10^{-1}	1.20×10^{-1}	1.57×10^{-1}	6.64×10^{-2}
5	7.31×10^{-1}	2.49×10^{-1}	9.45×10^{-2}	2.79×10^{-2}	4.24×10^{-2}	1.22×10^{-2}
6	5.62×10^{-1}	1.18×10^{-1}	3.16×10^{-2}	6.90×10^{-3}	1.13×10^{-2}	2.61×10^{-3}
7	4.18×10^{-1}	5.68×10^{-2}	1.22×10^{-2}	1.67×10^{-3}	3.70×10^{-3}	5.52×10^{-4}
8	3.01×10^{-1}	2.84×10^{-2}	3.86×10^{-3}	5.31×10^{-4}	1.12×10^{-3}	1.36×10^{-4}
9	2.27×10^{-1}	1.31×10^{-2}	1.48×10^{-3}	1.84×10^{-4}	3.97×10^{-4}	4.07×10^{-5}
10	1.64×10^{-1}	6.75×10^{-3}	5.65×10^{-4}	5.44×10^{-5}	1.45×10^{-4}	1.17×10^{-5}

Table B.4. Probability of cracking of the covering layer; embankment's mean height of 4 m.

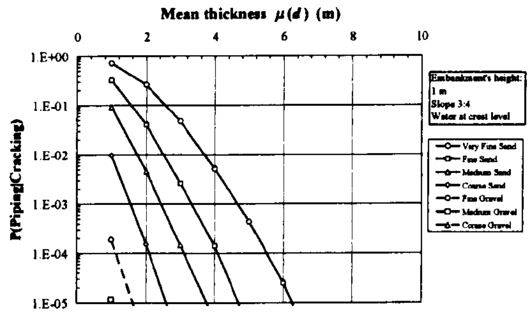
Probability of covering layer cracking: 4 m high embankment						
Mean thickness covering layer (m)	Fine grained soil type					
	Peat	Very Soft Clay	Soft Clay	Firm Clay	Soft Silt	Firm Silt
1	9.96×10^{-1}	9.72×10^{-1}	9.68×10^{-1}	9.62×10^{-1}	8.98×10^{-1}	8.88×10^{-1}
2	9.95×10^{-1}	9.67×10^{-1}	9.60×10^{-1}	9.48×10^{-1}	8.78×10^{-1}	8.58×10^{-1}
3	9.93×10^{-1}	9.20×10^{-1}	8.40×10^{-1}	7.10×10^{-1}	6.94×10^{-1}	5.60×10^{-1}
4	9.70×10^{-1}	7.41×10^{-1}	5.36×10^{-1}	3.29×10^{-1}	3.76×10^{-1}	2.16×10^{-1}
5	8.96×10^{-1}	5.00×10^{-1}	2.66×10^{-1}	1.20×10^{-1}	1.56×10^{-1}	6.54×10^{-2}
6	7.77×10^{-1}	2.99×10^{-1}	1.21×10^{-1}	3.88×10^{-2}	5.94×10^{-2}	1.86×10^{-2}
7	6.49×10^{-1}	1.70×10^{-1}	5.17×10^{-2}	1.29×10^{-2}	2.25×10^{-2}	5.67×10^{-3}
8	5.24×10^{-1}	9.58×10^{-2}	2.50×10^{-2}	4.63×10^{-3}	8.99×10^{-3}	1.84×10^{-3}
9	4.22×10^{-1}	5.35×10^{-2}	1.14×10^{-2}	1.66×10^{-3}	3.45×10^{-3}	6.12×10^{-4}
10	3.30×10^{-1}	3.02×10^{-2}	4.87×10^{-3}	6.57×10^{-4}	1.49×10^{-3}	1.85×10^{-4}

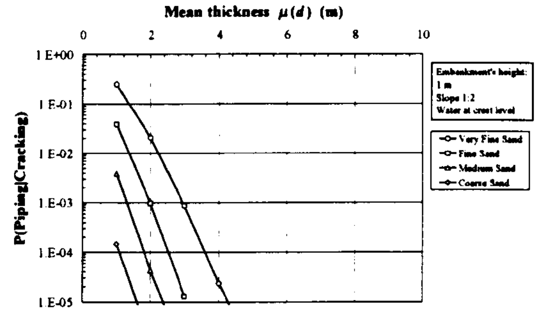
Table B.5. Probability of cracking of the covering layer; embankment's a mean height of 5 m.

Probability of covering layer cracking: 5 m high embankment						
Mean thickness covering layer (m)	Fine grained soil type					
	Peat	Very Soft Clay	Soft Clay	Firm Clay	Soft Silt	Firm Silt
1	9.95×10^{-1}	9.76×10^{-1}	9.68×10^{-1}	9.67×10^{-1}	9.02×10^{-1}	8.95×10^{-1}
2	9.95×10^{-1}	9.73×10^{-1}	9.63×10^{-1}	9.60×10^{-1}	8.89×10^{-1}	8.77×10^{-1}
3	9.93×10^{-1}	9.58×10^{-1}	9.18×10^{-1}	8.59×10^{-1}	8.04×10^{-1}	7.20×10^{-1}
4	9.86×10^{-1}	8.72×10^{-1}	7.26×10^{-1}	5.55×10^{-1}	5.66×10^{-1}	4.04×10^{-1}
5	9.54×10^{-1}	6.95×10^{-1}	4.69×10^{-1}	2.76×10^{-1}	3.16×10^{-1}	1.73×10^{-1}
6	8.97×10^{-1}	5.06×10^{-1}	2.70×10^{-1}	1.21×10^{-1}	1.58×10^{-1}	6.76×10^{-2}
7	8.03×10^{-1}	3.37×10^{-1}	1.42×10^{-1}	5.33×10^{-2}	7.35×10^{-2}	2.50×10^{-2}
8	7.04×10^{-1}	2.17×10^{-1}	7.62×10^{-2}	2.30×10^{-2}	3.61×10^{-2}	1.01×10^{-2}
9	5.96×10^{-1}	1.40×10^{-1}	4.08×10^{-2}	8.84×10^{-3}	1.58×10^{-2}	3.66×10^{-3}
10	5.02×10^{-1}	8.92×10^{-2}	1.97×10^{-2}	3.84×10^{-3}	7.53×10^{-3}	1.41×10^{-3}

B.2 Probability of piping given cracking: diagrams

B.2.1 Diagrams: 1 m high embankment





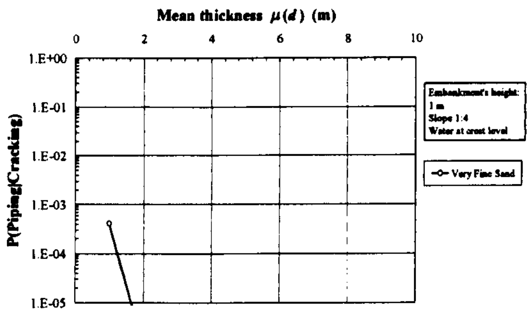
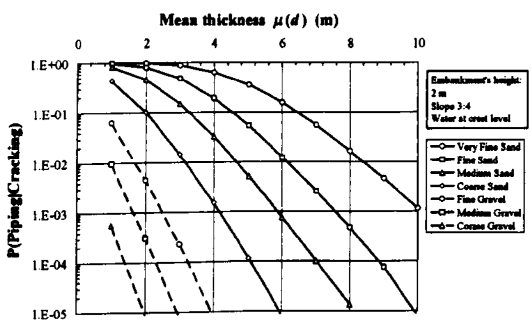
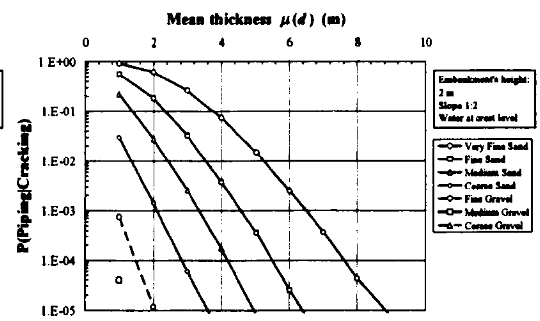


Figure B.2. Probability of piping given cracking for the embankment with mean height of 1 m.

B2.2 Diagrams: 2 m high embankment





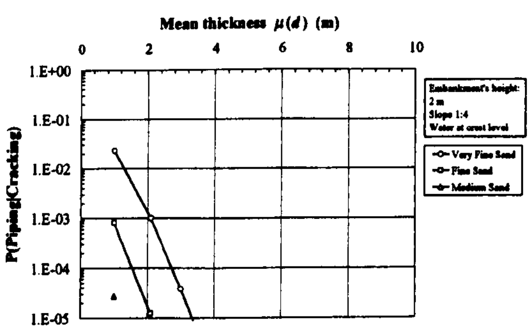
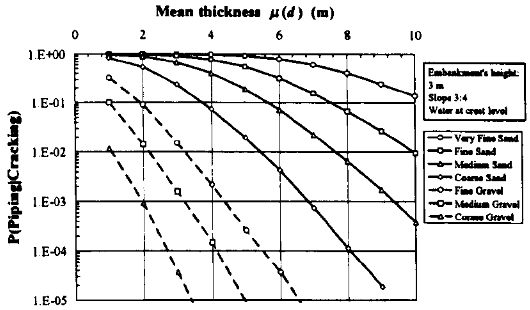
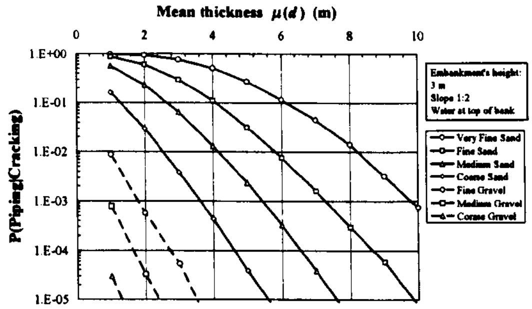


Figure B.3. Probability of piping given cracking for the embankment with mean height of 2 m.

B2.3 Diagrams: 3 m high embankment





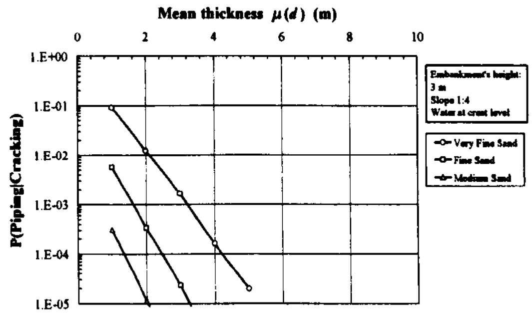
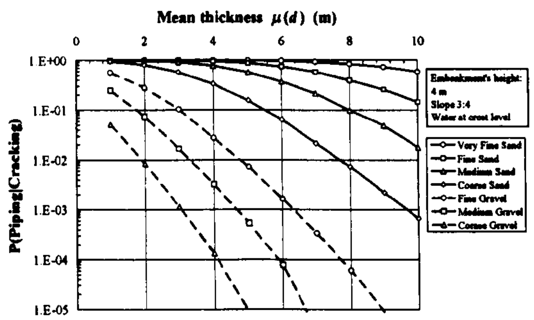
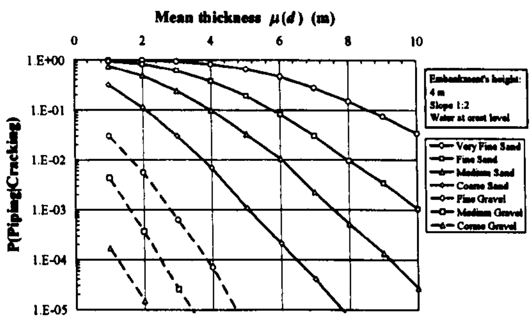


Figure B.4. Probability of piping given cracking for the embankment with mean height of 3 m.

B2.4 Diagrams: 4 m high embankment





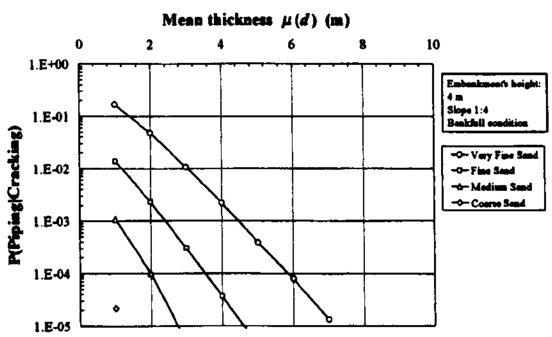
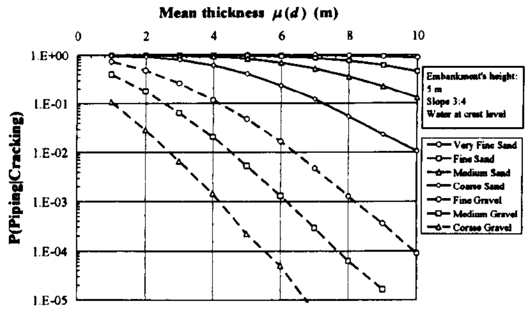
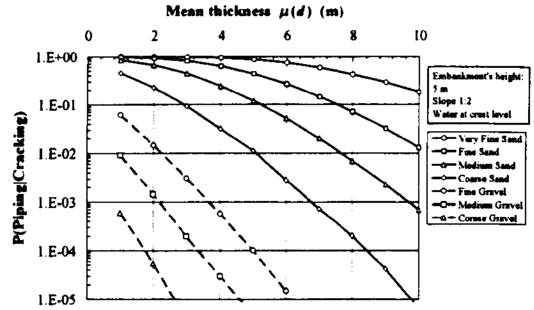


Figure B.5. Probability of piping given cracking for the embankment with mean height of 4 m.

B2.5 Diagrams: 5 m high embankment





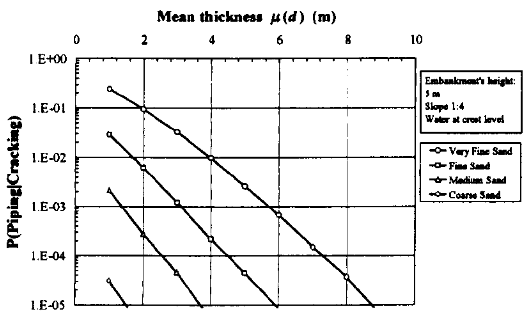


Figure B.6. Probability of piping given cracking for the embankment with mean height of 5 m.

B.3 Probability of piping given cracking: tables

B.3.1 Tables: 1 m high embankment

Table B.6. Probability of piping given cracking, inclination of slopes 3:4.

Probability of under-piping: 1 m high embankment – slope 3:4							
Mean thickness covering layer (m)	Coarse grained soil type						
	Very Fine Sand	Fine Sand	Medium Sand	Coarse Sand	Fine Gravel	Medium Gravel	Coarse Gravel
1	3.22×10^{-1}	9.22×10^{-2}	9.77×10^{-3}	1.88×10^{-4}	1.17×10^{-5}		
2	4.08×10^{-2}	4.58×10^{-3}	1.52×10^{-4}	1.67×10^{-6}			
3	2.62×10^{-3}	1.47×10^{-4}	1.67×10^{-6}				
4	1.42×10^{-4}	4.79×10^{-6}					
5	3.33×10^{-6}						
6							
7							

Table B.7. Probability of piping given cracking, inclination of slopes 1:2.

Probability of under-piping: 1 m high embankment – slope 1:2							
Mean thickness covering layer (m)	Coarse grained soil type						
	Very Fine Sand	Fine Sand	Medium Sand	Coarse Sand	Fine Gravel	Medium Gravel	Coarse Gravel
1	2.45×10^{-1}	3.85×10^{-2}	3.88×10^{-3}	1.45×10^{-4}			
2	2.06×10^{-2}	9.68×10^{-4}	4.27×10^{-5}	$1.67\times10\text{-}^{6}$			
3	8.70×10^{-4}	1.29×10^{-5}	8.71×10^{-7}				
4	2.33×10^{-5}						
5	1.17×10^{-6}						

Table B.8. Probability of piping given cracking, inclination of slopes 1:4.

Probability of under-piping: 1 m high embankment – slope 1:4							
Mean thickness covering layer (m)	Coarse grained soil type						
	Very Fine Sand	Fine Sand	Medium Sand	Coarse Sand	Fine Gravel	Medium Gravel	Coarse Gravel
1	4.20×10^{-4}	7.33×10^{-6}					
2	1.33×10^{-6}						

B.3.2 Tables: 2 m high embankment

Table B.9. Probability of piping given cracking, inclination of slopes 3:4.

Probability of under-piping: 2 m high embankment – slope 3:4							
Mean thickness covering layer (m)	**Coarse grained soil type**						
	Very Fine Sand	**Fine Sand**	**Medium Sand**	**Coarse Sand**	**Fine Gravel**	**Medium Gravel**	**Coarse Gravel**
1	9.99×10^{-1}	9.74×10^{-1}	8.31×10^{-1}	4.36×10^{-1}	6.31×10^{-2}	9.68×10^{-3}	5.67×10^{-4}
2	9.83×10^{-1}	8.04×10^{-1}	4.57×10^{-1}	1.02×10^{-1}	4.49×10^{-3}	3.09×10^{-4}	8.91×10^{-6}
3	8.76×10^{-1}	4.80×10^{-1}	1.50×10^{-1}	1.44×10^{-2}	2.28×10^{-4}	7.94×10^{-6}	
4	6.37×10^{-1}	1.92×10^{-1}	3.26×10^{-2}	1.52×10^{-3}	8.33×10^{-6}		
5	3.54×10^{-1}	5.38×10^{-2}	5.16×10^{-3}	1.15×10^{-4}			
6	1.57×10^{-1}	1.23×10^{-2}	7.55×10^{-4}	8.00×10^{-6}			
7	5.46×10^{-2}	2.57×10^{-3}	1.00×10^{-4}	6.67×10^{-7}			
8	1.66×10^{-2}	4.77×10^{-4}	1.33×10^{-5}				
9	4.64×10^{-3}	7.59×10^{-5}					
10	1.15×10^{-3}	7.94×10^{-6}					

Table B.10. Probability of piping given cracking, inclination of slopes 1:2.

Probability of under-piping: 2 m high embankment – slope 1:2							
Mean thickness covering layer (m)	**Coarse grained soil type**						
	Very Fine Sand	**Fine Sand**	**Medium Sand**	**Coarse Sand**	**Fine Gravel**	**Medium Gravel**	**Coarse Gravel**
1	9.15×10^{-1}	5.64×10^{-1}	2.22×10^{-1}	2.96×10^{-2}	7.54×10^{-4}	4.00×10^{-5}	
2	6.16×10^{-1}	1.86×10^{-1}	2.81×10^{-2}	1.42×10^{-3}	1.17×10^{-5}		
3	2.64×10^{-1}	3.28×10^{-2}	2.60×10^{-3}	6.00×10^{-5}			
4	7.43×10^{-2}	3.83×10^{-3}	1.77×10^{-4}	3.33×10^{-6}			
5	1.50×10^{-2}	3.63×10^{-4}	8.33×10^{-6}				
6	2.48×10^{-3}	2.50×10^{-5}					
7	3.67×10^{-4}	2.29×10^{-6}					
8	4.27×10^{-5}						
9	7.41×10^{-6}						
10	1.67×10^{-6}						

Table B.11. Probability of piping given cracking, inclination of slopes 1:4.

Probability of under-piping: 2 m high embankment – slope 1:4							
Mean thickness covering layer (m)	**Coarse grained soil type**						
	Very Fine Sand	**Fine Sand**	**Medium Sand**	**Coarse Sand**	**Fine Gravel**	**Medium Gravel**	**Coarse Gravel**
1	2.31×10^{-2}	8.20×10^{-4}	2.71×10^{-5}				
2	1.01×10^{-3}	1.23×10^{-5}					
3	3.82×10^{-5}						
4	7.59×10^{-7}						

B.3.3 Tables: 3 m high embankment

Table B.12. Probability of piping given cracking, inclination of slopes 3:4.

Probability of under-piping: 3 m high embankment – slope 3:4							
Mean thickness covering layer (m)	Coarse grained soil type						
	Very Fine Sand	Fine Sand	Medium Sand	Coarse Sand	Fine Gravel	Medium Gravel	Coarse Gravel
1	1.00	9.98×10^{-1}	9.81×10^{-1}	8.28×10^{-1}	3.24×10^{-1}	1.01×10^{-1}	1.18×10^{-2}
2	1.00	9.87×10^{-1}	8.92×10^{-1}	5.33×10^{-1}	9.09×10^{-2}	1.43×10^{-2}	9.17×10^{-4}
3	9.95×10^{-1}	9.25×10^{-1}	6.71×10^{-1}	2.37×10^{-1}	1.52×10^{-2}	1.58×10^{-3}	3.67×10^{-5}
4	9.77×10^{-1}	7.70×10^{-1}	4.02×10^{-1}	7.40×10^{-2}	2.20×10^{-3}	1.48×10^{-4}	1.26×10^{-6}
5	9.15×10^{-1}	5.45×10^{-1}	1.91×10^{-1}	1.95×10^{-2}	2.57×10^{-4}	8.33×10^{-6}	
6	7.86×10^{-1}	3.21×10^{-1}	7.23×10^{-2}	4.31×10^{-3}	3.67×10^{-5}		
7	6.01×10^{-1}	1.56×10^{-1}	2.26×10^{-2}	7.41×10^{-4}	3.33×10^{-6}		
8	4.09×10^{-1}	6.75×10^{-2}	6.53×10^{-2}	1.12×10^{-4}			
9	2.43×10^{-1}	2.68×10^{-2}	1.70×10^{-2}	1.87×10^{-5}			
10	1.38×10^{-1}	9.54×10^{-3}	3.78×10^{-5}				

Table B.13. Probability of piping given cracking, inclination of slopes 1:2.

Probability of under-piping: 3 m high embankment – slope 1:2							
Mean thickness covering layer (m)	Coarse grained soil type						
	Very Fine Sand	Fine Sand	Medium Sand	Coarse Sand	Fine Gravel	Medium Gravel	Coarse Gravel
1	9.90×10^{-1}	8.69×10^{-1}	5.60×10^{-1}	1.61×10^{-1}	8.73×10^{-3}	7.84×10^{-4}	2.97×10^{-5}
2	9.35×10^{-1}	6.00×10^{-1}	2.33×10^{-1}	2.99×10^{-2}	5.68×10^{-4}	3.26×10^{-5}	6.67×10^{-7}
3	7.62×10^{-1}	3.02×10^{-1}	6.51×10^{-2}	3.88×10^{-3}	5.33×10^{-5}	1.51×10^{-6}	
4	5.15×10^{-1}	1.12×10^{-1}	1.32×10^{-2}	4.28×10^{-4}	1.67×10^{-6}		
5	2.75×10^{-1}	3.20×10^{-2}	2.35×10^{-3}	3.83×10^{-5}			
6	1.17×10^{-1}	7.65×10^{-3}	3.23×10^{-4}	3.83×10^{-6}			
7	4.55×10^{-2}	1.62×10^{-3}	3.83×10^{-5}				
8	1.41×10^{-2}	2.93×10^{-4}	3.98×10^{-6}				
9	3.22×10^{-3}	5.83×10^{-5}					
10	7.43×10^{-4}	7.41×10^{-6}					

Table B.14. Probability of piping given cracking, inclination of slopes 1:4.

Probability of under-piping: 3 m high embankment – slope 1:4							
Mean thickness covering layer (m)	Coarse grained soil type						
	Very Fine Sand	Fine Sand	Medium Sand	Coarse Sand	Fine Gravel	Medium Gravel	Coarse Gravel
1	9.19×10^{-2}	5.68×10^{-3}	3.14×10^{-4}	5.33×10^{-6}			
2	1.23×10^{-2}	3.46×10^{-4}	1.13×10^{-5}				
3	1.65×10^{-3}	2.33×10^{-5}	6.67×10^{-7}				
4	1.61×10^{-4}	1.00×10^{-6}					
5	1.97×10^{-5}						

B.3.4 Tables: 4 m high embankment

Table B.15. Probability of piping given cracking, inclination of slopes 3:4.

Probability of under-piping: 4 m high embankment – slope 3:4							
Mean thickness covering layer (m)	Coarse grained soil type						
	Very Fine Sand	Fine Sand	Medium Sand	Coarse Sand	Fine Gravel	Medium Gravel	Coarse Gravel
1	1.00	1.00	9.97×10^{-1}	9.44×10^{-1}	5.69×10^{-1}	2.48×10^{-1}	5.13×10^{-2}
2	1.00	9.99×10^{-1}	9.76×10^{-1}	8.03×10^{-1}	2.79×10^{-1}	7.40×10^{-2}	8.45×10^{-3}
3	1.00	9.90×10^{-1}	9.14×10^{-1}	5.73×10^{-1}	1.04×10^{-1}	1.69×10^{-2}	1.15×10^{-3}
4	9.98×10^{-1}	9.56×10^{-1}	7.73×10^{-1}	3.33×10^{-1}	2.76×10^{-2}	3.24×10^{-3}	1.33×10^{-4}
5	9.92×10^{-1}	8.75×10^{-1}	5.74×10^{-1}	1.55×10^{-1}	7.23×10^{-3}	5.37×10^{-4}	8.33×10^{-6}
6	9.70×10^{-1}	7.46×10^{-1}	3.68×10^{-1}	6.47×10^{-2}	1.64×10^{-3}	7.77×10^{-5}	
7	9.25×10^{-1}	5.81×10^{-1}	2.14×10^{-1}	2.15×10^{-2}	3.41×10^{-4}	3.47×10^{-6}	
8	8.42×10^{-1}	3.96×10^{-1}	9.89×10^{-2}	7.20×10^{-3}	6.08×10^{-5}		
9	7.27×10^{-1}	2.57×10^{-1}	4.85×10^{-2}	2.20×10^{-3}	8.32×10^{-6}		
10	5.81×10^{-1}	1.45×10^{-1}	1.77×10^{-2}	6.65×10^{-4}			

Table B.16. Probability of piping given cracking, inclination of slopes 1:2.

Probability of under-piping: 4 m high embankment – slope 1:2							
Mean thickness covering layer (m)	Coarse grained soil type						
	Very Fine Sand	Fine Sand	Medium Sand	Coarse Sand	Fine Gravel	Medium Gravel	Coarse Gravel
1	9.97×10^{-1}	9.52×10^{-1}	7.50×10^{-1}	3.15×10^{-1}	2.98×10^{-2}	4.34×10^{-3}	1.68×10^{-4}
2	9.84×10^{-1}	8.25×10^{-1}	4.81×10^{-1}	1.13×10^{-1}	5.54×10^{-3}	3.74×10^{-4}	1.50×10^{-5}
3	9.38×10^{-1}	6.15×10^{-1}	2.43×10^{-1}	3.01×10^{-2}	6.31×10^{-4}	2.57×10^{-5}	
4	8.25×10^{-1}	3.77×10^{-1}	9.88×10^{-2}	6.90×10^{-3}	7.08×10^{-5}	2.24×10^{-6}	
5	6.56×10^{-1}	1.94×10^{-1}	3.27×10^{-2}	1.07×10^{-3}	3.33×10^{-6}		
6	4.63×10^{-1}	8.24×10^{-1}	1.06×10^{-2}	2.09×10^{-4}			
7	2.76×10^{-1}	3.06×10^{-2}	2.23×10^{-3}	3.98×10^{-5}			
8	1.51×10^{-1}	9.57×10^{-3}	5.26×10^{-4}	6.67×10^{-6}			
9	7.42×10^{-2}	3.48×10^{-3}	1.35×10^{-4}				
10	3.42×10^{-2}	1.05×10^{-3}	2.67×10^{-5}				

Table B.17. Probability of piping given cracking, inclination of slopes 1:4.

Probability of under-piping: 4 m high embankment – slope 1:4							
Mean thickness covering layer (m)	Coarse grained soil type						
	Very Fine Sand	Fine Sand	Medium Sand	Coarse Sand	Fine Gravel	Medium Gravel	Coarse Gravel
1	1.68×10^{-1}	1.39×10^{-2}	1.07×10^{-3}	2.17×10^{-5}			
2	4.83×10^{-2}	2.31×10^{-3}	9.83×10^{-5}				
3	1.07×10^{-2}	3.02×10^{-4}	4.68×10^{-5}				
4	2.23×10^{-3}	3.63×10^{-5}					
5	3.85×10^{-4}	4.07×10^{-6}					
6	7.90×10^{-5}						
7	1.33×10^{-5}						

B.3.5 Tables: 5 m high embankment

Table B.18. Probability of piping given cracking, inclination of slopes 3:4.

Probability of under-piping: 5 m high embankment – slope 3:4							
Mean thickness covering layer (m)	Coarse grained soil type						
	Very Fine Sand	Fine Sand	Medium Sand	Coarse Sand	Fine Gravel	Medium Gravel	Coarse Gravel
1	1.00	1.00	9.99×10^{-1}	9.75×10^{-1}	7.19×10^{-1}	3.97×10^{-1}	1.09×10^{-1}
2	1.00	9.99×10^{-1}	9.95×10^{-1}	9.14×10^{-1}	4.77×10^{-1}	1.79×10^{-1}	2.90×10^{-2}
3	1.00	9.99×10^{-1}	9.76×10^{-1}	7.88×10^{-1}	2.61×10^{-1}	6.41×10^{-2}	6.68×10^{-3}
4	1.00	9.92×10^{-1}	9.24×10^{-1}	6.04×10^{-1}	1.17×10^{-1}	2.08×10^{-2}	1.46×10^{-3}
5	9.99×10^{-1}	9.74×10^{-1}	8.26×10^{-1}	4.07×10^{-1}	4.83×10^{-2}	5.38×10^{-3}	2.23×10^{-4}
6	9.97×10^{-1}	9.31×10^{-1}	6.83×10^{-1}	2.33×10^{-1}	1.65×10^{-2}	1.29×10^{-3}	4.99×10^{-5}
7	9.90×10^{-1}	8.55×10^{-1}	5.14×10^{-1}	1.22×10^{-1}	4.74×10^{-3}	2.92×10^{-4}	5.75×10^{-6}
8	9.69×10^{-1}	7.43×10^{-1}	3.55×10^{-1}	5.44×10^{-2}	1.28×10^{-3}	6.23×10^{-5}	
9	9.33×10^{-1}	5.99×10^{-1}	2.25×10^{-1}	2.33×10^{-2}	3.57×10^{-4}	1.67×10^{-5}	
10	8.81×10^{-1}	4.53×10^{-1}	1.30×10^{-1}	1.05×10^{-2}	9.00×10^{-5}		

Table B.19. Probability of piping given cracking, inclination of slopes 1:2.

Probability of under-piping: 5 m high embankment – slope 1:2							
Mean thickness covering layer (m)	Coarse grained soil type						
	Very Fine Sand	Fine Sand	Medium Sand	Coarse Sand	Fine Gravel	Medium Gravel	Coarse Gravel
1	9.99×10^{-1}	9.79×10^{-1}	8.44×10^{-1}	4.45×10^{-1}	6.18×10^{-2}	9.14×10^{-3}	6.03×10^{-4}
2	9.95×10^{-1}	9.21×10^{-1}	6.57×10^{-1}	2.26×10^{-1}	1.44×10^{-2}	1.45×10^{-3}	5.46×10^{-5}
3	9.77×10^{-1}	8.02×10^{-1}	4.43×10^{-1}	9.19×10^{-2}	3.07×10^{-3}	1.95×10^{-4}	3.80×10^{-6}
4	9.42×10^{-1}	6.25×10^{-1}	2.46×10^{-1}	3.18×10^{-2}	5.78×10^{-4}	3.00×10^{-5}	
5	8.64×10^{-1}	4.34×10^{-1}	1.20×10^{-1}	1.11×10^{-2}	1.02×10^{-4}	5.01×10^{-6}	
6	7.35×10^{-1}	2.67×10^{-1}	5.21×10^{-2}	2.82×10^{-3}	1.50×10^{-5}		
7	5.81×10^{-1}	1.45×10^{-1}	2.02×10^{-2}	7.24×10^{-4}			
8	4.22×10^{-1}	7.15×10^{-2}	6.92×10^{-3}	2.02×10^{-4}			
9	2.90×10^{-1}	3.25×10^{-2}	2.31×10^{-3}	4.17×10^{-5}			
10	1.81×10^{-1}	1.30×10^{-2}	7.00×10^{-4}	6.67×10^{-6}			

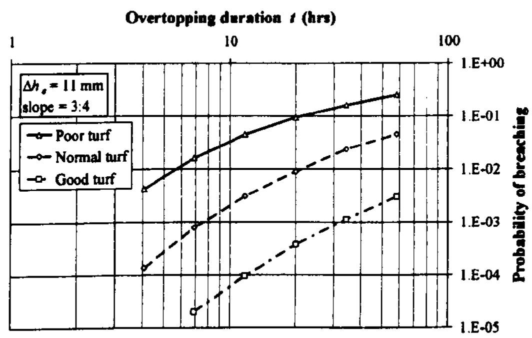
Table B.20. Probability of piping given cracking, inclination of slopes 1:4.

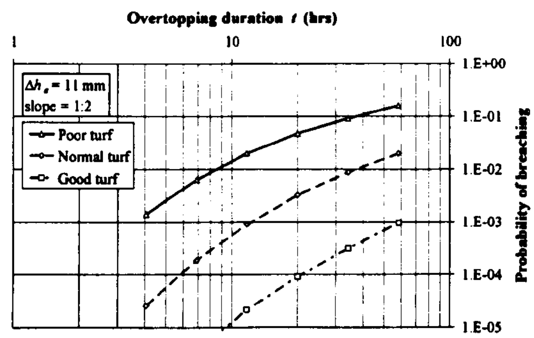
Probability of under-piping: 5 m high embankment – slope 1:4							
Mean thickness covering layer (m)	Coarse grained soil type						
	Very Fine Sand	Fine Sand	Medium Sand	Coarse Sand	Fine Gravel	Medium Gravel	Coarse Gravel
1	2.43×10^{-1}	2.88×10^{-2}	2.15×10^{-3}	3.17×10^{-5}			
2	9.41×10^{-2}	6.17×10^{-3}	2.77×10^{-4}	3.33×10^{-6}			
3	3.28×10^{-2}	1.2×210^{-3}	4.63×10^{-5}				
4	9.8010^{-3}	2.22×10^{-4}	5.50×10^{-6}				
5	2.60×10^{-3}	4.50×10^{-5}					
6	6.71×10^{-4}	8.51×10^{-6}					
7	1.51×10^{-4}	1.67×10^{-6}					
8	3.72×10^{-5}						
9	6.31×10^{-6}						

APPENDIX C

Probability of grass cover failure

C.1 Diagrams





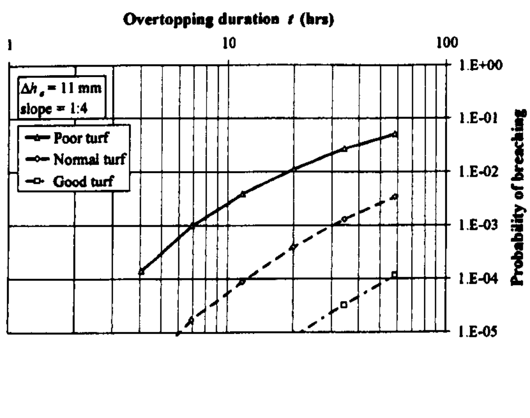
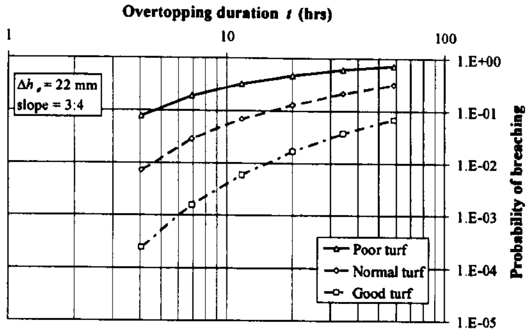
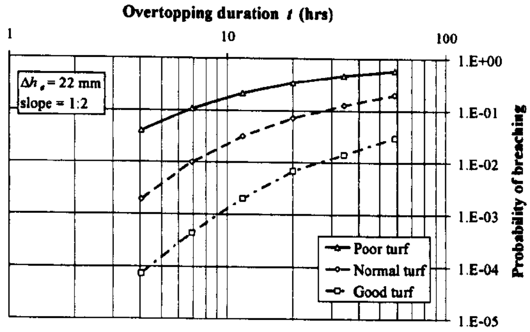


Figure C.1. Probability of grass cover failure for an equivalent height of water above the crest of 11 mm.





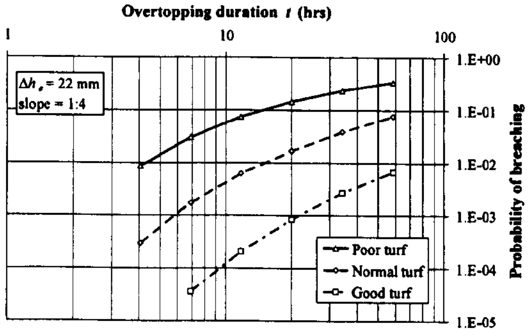
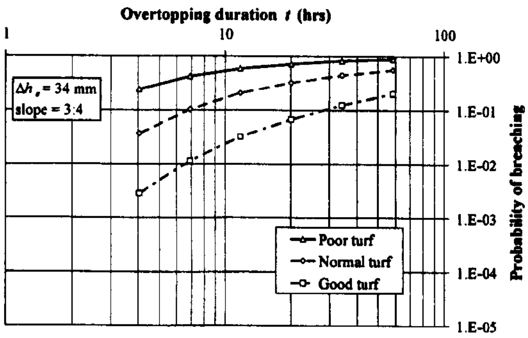
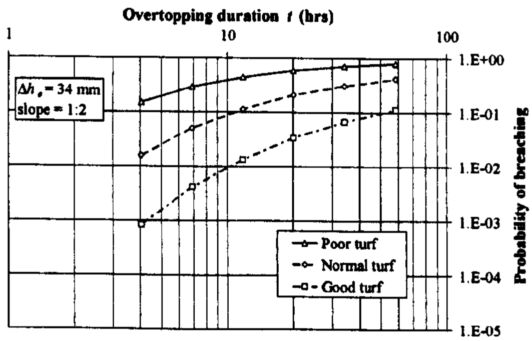


Figure C.2. Probability of grass cover failure for an equivalent height of water above the crest of 22 mm.





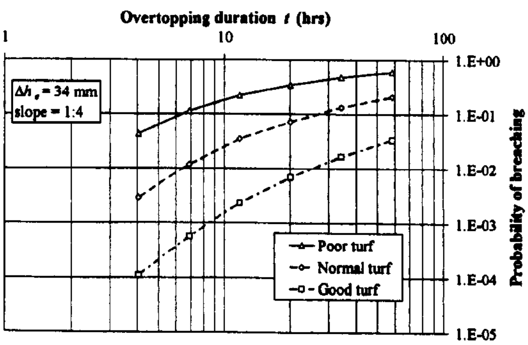
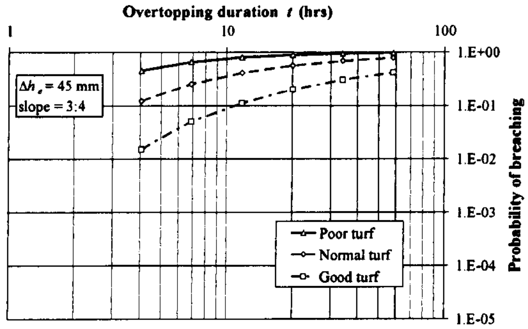
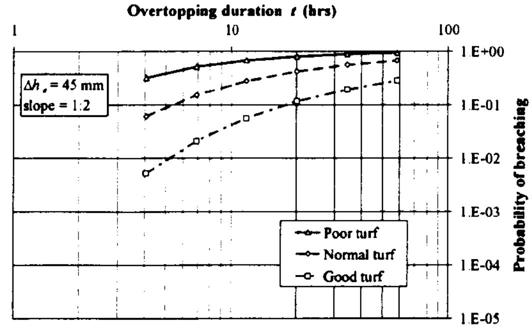


Figure C.3. Probability of grass cover failure for an equivalent height of water above the crest of 34 mm.





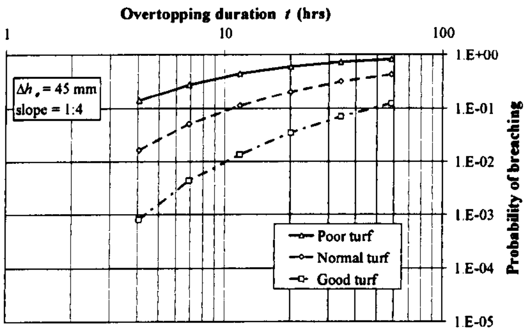
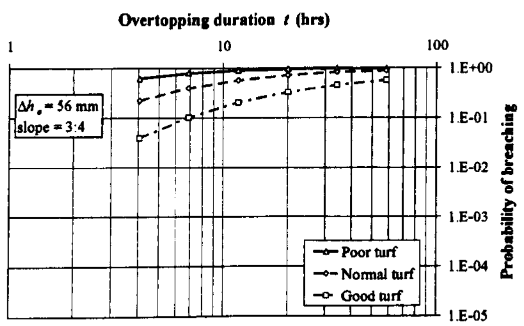
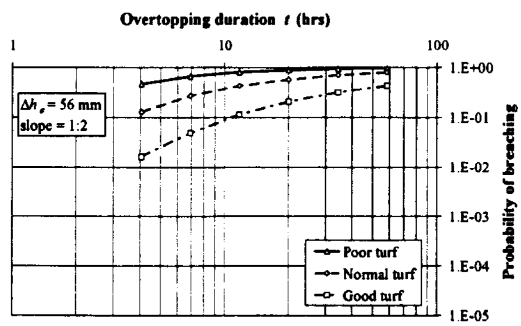


Figure C.4. Probability of grass cover failure for an equivalent height of water above the crest of 45 mm.





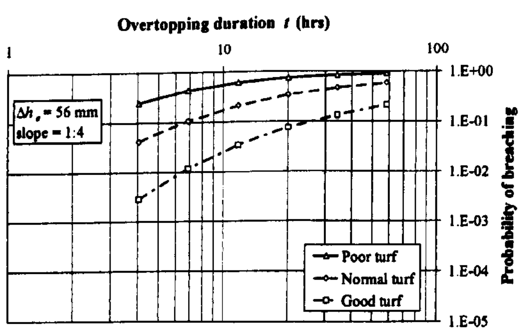


Figure C.5. Probability of grass cover failure for an equivalent height of water above the crest of 56 mm.

C.2 Tables

C2.1 Equivalent height over the crest: 11 mm

Table C.1. Probability of grass cover failure; inclination of slope 3:4.

Probability of grass cover erosion: Δh_e= 11 mm – Slope 3:4			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	4.24×10^{-3}	1.37×10^{-4}	
06h 54'	1.62×10^{-2}	7.83×10^{-4}	2.00×10^{-5}
11h 46'	4.50×10^{-2}	3.07×10^{-3}	9.52×10^{-5}
20h 04'	9.40×10^{-2}	8.98×10^{-3}	3.76×10^{-4}
34h 13'	1.62×10^{-1}	2.36×10^{-2}	1.13×10^{-3}
58h 21'	2.52×10^{-1}	4.53×10^{-2}	3.00×10^{-3}

Table C.2. Probability of grass cover failure; inclination of slope 1:2.

Probability of grass cover erosion: Δh_e= 11 mm – Slope 1:2			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	1.41×10^{-3}	2.56×10^{-5}	
06h 54'	6.39×10^{-3}	1.90×10^{-4}	3.49×10^{-6}
11h 46'	1.98×10^{-2}	8.92×10^{-4}	2.14×10^{-5}
20h 04'	4.78×10^{-2}	3.25×10^{-3}	9.07×10^{-5}
34h 13'	9.09×10^{-2}	8.72×10^{-3}	3.14×10^{-4}
58h 21'	1.60×10^{-1}	1.97×10^{-2}	9.66×10^{-4}

Table C.3. Probability of grass cover failure; inclination of slope 1:4.

Probability of grass cover erosion: Δh_e= 11 mm – Slope 1:4			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	1.41×10^{-4}	1.71×10^{-6}	
06h 54'	9.89×10^{-4}	1.72×10^{-5}	
11h 46'	3.87×10^{-3}	8.75×10^{-5}	2.76×10^{-6}
20h 04'	1.12×10^{-2}	3.90×10^{-4}	8.16×10^{-6}
34h 13'	2.70×10^{-2}	1.29×10^{-3}	3.22×10^{-5}
58h 21'	5.14×10^{-2}	3.34×10^{-3}	1.14×10^{-4}

C2.2 Equivalent height over the crest: 22 mm

Table C.4. Probability of grass cover failure; inclination of slope 3:4.

Probability of grass cover erosion: Δh_e= 22 mm – Slope 3:4			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	7.87×10^{-2}	7.08×10^{-3}	2.46×10^{-4}
06h 54'	1.90×10^{-1}	2.85×10^{-2}	1.60×10^{-3}
11h 46'	3.32×10^{-1}	6.89×10^{-2}	5.92×10^{-3}
20h 04'	4.76×10^{-1}	1.30×10^{-1}	1.69×10^{-2}
34h 13'	6.14×10^{-1}	2.14×10^{-1}	3.68×10^{-2}
58h 21'	7.26×10^{-1}	3.17×10^{-1}	6.81×10^{-2}

Table C.5. Probability of grass cover failure; inclination of slope 1:2.

Probability of grass cover erosion: Δh_e= 22 mm – Slope 1:2			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	4.03×10^{-2}	1.94×10^{-3}	7.39×10^{-5}
06h 54'	1.09×10^{-1}	1.00×10^{-2}	4.43×10^{-4}
11h 46'	2.13×10^{-1}	3.17×10^{-2}	2.05×10^{-3}
20h 04'	3.41×10^{-1}	7.14×10^{-2}	6.84×10^{-3}
34h 13'	4.74×10^{-1}	1.28×10^{-1}	1.45×10^{-2}
58h 21'	5.94×10^{-1}	2.01×10^{-1}	2.99×10^{-2}

Table C.6. Probability of grass cover failure; inclination of slope 1:4.

Probability of grass cover erosion: Δh_e= 22 mm – Slope 1:4			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	8.72×10^{-3}	2.98×10^{-4}	
06h 54'	3.02×10^{-2}	1.70×10^{-3}	3.55×10^{-5}
11h 46'	7.46×10^{-2}	6.18×10^{-3}	2.06×10^{-4}
20h 04'	1.46×10^{-1}	1.68×10^{-2}	8.42×10^{-4}
34h 13'	2.36×10^{-1}	3.86×10^{-2}	2.71×10^{-3}
58h 21'	3.41×10^{-1}	7.63×10^{-2}	6.67×10^{-3}

C2.3 Equivalent height over the crest: 34 mm

Table C.7. Probability of grass cover failure; inclination of slope 3:4.

Probability of grass cover erosion: Δh_e= 34 mm – Slope 3:4			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	2.43×10^{-1}	3.67×10^{-2}	2.85×10^{-3}
06h 54'	4.22×10^{-1}	1.02×10^{-1}	1.14×10^{-2}
11h 46'	5.94×10^{-1}	2.06×10^{-1}	3.18×10^{-2}
20h 04'	7.23×10^{-1}	3.26×10^{-1}	6.77×10^{-2}
34h 13'	8.22×10^{-1}	4.48×10^{-1}	1.24×10^{-1}
58h 21'	8.89×10^{-1}	5.62×10^{-1}	2.02×10^{-1}

Table C.8. Probability of grass cover failure; inclination of slope 1:2.

Probability of grass cover erosion: Δh_e= 34 mm – Slope 1:2			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	1.49×10^{-1}	1.56×10^{-2}	8.41×10^{-4}
06h 54'	2.92×10^{-1}	5.05×10^{-2}	4.17×10^{-3}
11h 46'	4.47×10^{-1}	1.13×10^{-1}	1.33×10^{-2}
20h 04'	5.91×10^{-1}	2.09×10^{-1}	3.42×10^{-2}
34h 13'	7.14×10^{-1}	3.09×10^{-1}	6.77×10^{-2}
58h 21'	8.02×10^{-1}	4.19×10^{-1}	1.12×10^{-1}

Table C.9. Probability of grass cover failure; inclination of slope 1:4.

Probability of grass cover erosion: Δh_e= 34 mm – Slope 1:4			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	4.28×10^{-2}	2.91×10^{-3}	1.11×10^{-4}
06h 54'	1.13×10^{-1}	1.19×10^{-2}	5.61×10^{-4}
11h 46'	2.19×10^{-1}	3.48×10^{-2}	2.32×10^{-3}
20h 04'	3.48×10^{-1}	7.25×10^{-2}	6.97×10^{-3}
34h 13'	4.81×10^{-1}	1.33×10^{-1}	1.67×10^{-2}
58h 21'	6.01×10^{-1}	2.08×10^{-1}	3.37×10^{-2}

C2.4 Equivalent height over the crest: 45 mm

Table C.10. Probability of grass cover failure; inclination of slope 3:4.

Probability of grass cover erosion: Δh_e= 45 mm – Slope 3:4			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	4.57×10^{-1}	1.21×10^{-1}	1.50×10^{-2}
06h 54'	6.64×10^{-1}	2.58×10^{-1}	5.07×10^{-2}
11h 46'	8.03×10^{-1}	4.14×10^{-1}	1.11×10^{-1}
20h 04'	8.93×10^{-1}	5.64×10^{-1}	2.00×10^{-1}
34h 13'	9.45×10^{-1}	6.91×10^{-1}	3.09×10^{-1}
58h 21'	9.73×10^{-1}	7.91×10^{-1}	4.22×10^{-1}

Table C.11. Probability of grass cover failure; inclination of slope 1:2.

Probability of grass cover erosion: Δh_e= 45 mm – Slope 1:2			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	3.24×10^{-1}	6.11×10^{-2}	5.16×10^{-3}
06h 54'	5.21×10^{-1}	1.50×10^{-1}	2.06×10^{-2}
11h 46'	6.82×10^{-1}	2.79×10^{-1}	5.56×10^{-2}
20h 04'	8.07×10^{-1}	4.26×10^{-1}	1.16×10^{-1}
34h 13'	8.87×10^{-1}	5.59×10^{-1}	1.93×10^{-1}
58h 21'	9.36×10^{-1}	6.72×10^{-1}	2.86×10^{-1}

Table C.12. Probability of grass cover failure; inclination of slope 1:4.

Probability of grass cover erosion: Δh_e= 45 mm – Slope 1:4			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	1.40×10^{-1}	1.64×10^{-2}	8.07×10^{-4}
06h 54'	2.77×10^{-1}	5.11×10^{-2}	4.37×10^{-3}
11h 46'	4.46×10^{-1}	1.12×10^{-1}	1.36×10^{-2}
20h 04'	5.96×10^{-1}	2.03×10^{-1}	3.41×10^{-2}
34h 13'	7.24×10^{-1}	3.17×10^{-1}	7.02×10^{-2}
58h 21'	8.19×10^{-1}	4.29×10^{-1}	1.22×10^{-1}

C2.5 Equivalent height over the crest: 56 mm

Table C.13. Probability of grass cover failure; inclination of slope 3:4.

Probability of grass cover erosion: Δh_e= 56 mm – Slope 3:4			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	6.15×10^{-1}	2.19×10^{-1}	3.90×10^{-2}
06h 54'	7.91×10^{-1}	3.95×10^{-1}	1.03×10^{-1}
11h 46'	8.95×10^{-1}	5.68×10^{-1}	2.03×10^{-1}
20h 04'	9.49×10^{-1}	7.11×10^{-1}	3.21×10^{-1}
34h 13'	9.75×10^{-1}	8.14×10^{-1}	4.53×10^{-1}
58h 21'	9.89×10^{-1}	8.86×10^{-1}	5.79×10^{-1}

Table C.14. Probability of grass cover failure; inclination of slope 1:2.

Probability of grass cover erosion: Δh_e= 56 mm – Slope 1:2			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	4.71×10^{-1}	1.28×10^{-1}	1.62×10^{-2}
06h 54'	6.69×10^{-1}	2.67×10^{-1}	4.87×10^{-2}
11h 46'	8.11×10^{-1}	4.32×10^{-1}	1.14×10^{-1}
20h 04'	8.95×10^{-1}	5.78×10^{-1}	2.07×10^{-1}
34h 13'	9.46×10^{-1}	7.04×10^{-1}	3.13×10^{-1}
58h 21'	9.71×10^{-1}	8.00×10^{-1}	4.34×10^{-1}

Table C.15. Probability of grass cover failure; inclination of slope 1:4.

Probability of grass cover erosion: Δh_e= 56 mm – Slope 1:4			
Overtopping duration (hours)	Grass cover quality		
	POOR	NORMAL	GOOD
04h 03'	2.44×10^{-1}	4.13×10^{-2}	2.89×10^{-3}
06h 54'	4.28×10^{-1}	1.06×10^{-1}	1.20×10^{-2}
11h 46'	6.02×10^{-1}	2.06×10^{-1}	3.40×10^{-2}
20h 04'	7.42×10^{-1}	3.34×10^{-1}	7.46×10^{-2}
34h 13'	8.35×10^{-1}	4.67×10^{-1}	1.33×10^{-1}
58h 21'	8.98×10^{-1}	5.84×10^{-1}	2.13×10^{-1}

APPENDIX D

Through-piping: chronology of the judgement elicitation process

26 March 2008	Distribution of the working note on the motivating phase.
17 April 2008	Distribution of the working note on the training phase.
22 April 2008	M. Dyer comments on overconfidence bias, need of aids to judgement, need of geotechnical examples.
24 April 2008	M. Redaelli replies to M. Dyer illustrating how the event tree technique can help to deal with the overconfidence bias and discuss an example from Whitman (2000). Additional comments on aids to judgement, verbal descriptors of likelihood, action approach to elicitation. Reflection on the importance of a sound literature review on the studied process. More information gathered on the effectiveness of assessors' training in other disciplines.
21 May 2008	Distribution of the review report "Piping through the earthfill and reliability of flood embankments" prepared by Redaelli (2008a).
26 May 2008	Distribution of the working note on the structuring phase
4 June 2008	Submission of the review report "Piping through the earthfill and reliability of flood embankments" to M. Bramley (Environment Agency Special Adviser) and C. Mitchell (Environment Agency).

16-30 June 2008	Discussion on structuring: relevance of geologic origin of the fill material; importance of animal burrows crossing the embankment's body, relatively low importance of burrows size or number. Choice of the subdivision in classes for each characteristic.
1 August 2008	Distribution of the working note on the assessing phase.
4-12 August 2008	Further discussion on the subdivision in classes chosen during the structuring phase. Number of classes reduced for some characteristics.
14 August 2008	Individual assessment by each member of the panel based on the independent weights approach.
26 August 2008	Agreement is sought and the limits of the approach based on independent weights emerge.
3 September 2008	The more advanced approach which makes use of the coefficients of influence reduction is proposed.
4 September 2008	Individual assessment by each member of the panel based on the new approach.
9 September 2008	Agreement is reached on the quantification of subjective judgement
19 September 2008	The final report on the process of judgement elicitation is sent to: G. Baxter (EA) M. Bramley (EA Adviser) J. Hall (University of Newcastle upon Tyne) G. Long (University of Nottingham) C. Mitchell (EA) M. Morris (HR Wallingford) F. Ogunyoye (Royal Haskoning)

J. Powell (BRE)

J. Simm (HR Wallingford)

P. Smith (Royal Haskoning)