

MODELLING OF HEATING AND AIR-CONDITIONING SYSTEM

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SYNOPSIS:

The shortage of natural resources calls for energy conservation and the changing structure of modern science and technology makes possible the efficient dynamic energy modelling. This thesis presents a general review of existing techniques in the field of building and plant system energy modelling and concentrates on the establishment of a generalised modelling tool - the control volume conservation state space approach - to cope with the developing modern technology. The theoretical basis of this approach is discussed and verified in detail and applications demonstrated to the modelling of HVAC systems and equipment. Using this approach, any component of an HVAC system can be modelled at different levels and the qualities of the model are fully supported by the theoretical background of this approach. A computer programme for heating system simulation has been developed for the purpose of model validation and the validation efforts have been involved in the international cooperated programme of IEA Annex 10 exercises. As a part of group research, the work presented in this thesis has been involved in the development of advanced energy modelling techniques and methodology which has taken place at the ABACUS unit.

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CHAPTER ONE: INTRODUCTION

1.1 The necessity of energy modelling

1.2 The development of modelling techniques

CHAPTER ONE: INTRODUCTION

1.1 The necessity for energy modelling

The demand for energy to keep up its development of science, technology and civilization of the human society is ever growing. Consideration of energy in relation to the built environment reveals that, in Europe and the United States for example, in excess of 50% of the primary energy consumption can be associated with buildings, and of this some considerable portion (more than 60% in U.K for example) is consumed to moderate spatial conditions. [Ref. 1] The reduction of energy consumption for heating of buildings may be roughly brought about in three ways (figures in brackets indicates the saving potential)

- more efficient heating system (30-50%)
- reduction of heat transmission losses by better insulation of walls, roofs and window (20-30%)
- reduction of air infiltration (10-20%)

The 1973 oil embargo has highlighted the necessity for a more rational use of energy in an attempt to reduce excessive depletion of the existing natural resources. There is a source of energy that produces no radioactive waste, very little pollution and it can do more than any of the conventional sources to help the human society deal with the energy problem it has. This source might be called energy conservation. It has been noticed since 1973 that the efforts towards energy-conscious design of building will promote increasing energy efficiency in new construction. One of the important aspects of designing "clever energy use" building is the existence of modelling systems which can be used to promote effective use of the available energy resources. Traditional design of both building and system based on the definitions of "peak loads" and simple steady state methods are getting less and less acceptance and the requirements for new techniques which allow the designer to accurately predict the

performance of building and system at early stage of design and latter of the "clever" system operation are increasing. Computer simulation of the internal behaviour of buildings and their components is a new trend recently developed for this purpose. Most of the computer simulation models are still reliant on the traditional design methods. These facts associated with the recent advancements of science and technology, the demand for the next generation simulation techniques is coming up to date.

To start from the beginning, the terminology "simulation" may be regarded as the art of representing some aspects of the real world by numbers or symbols which may be easily manipulated to facilitate their study. [Ref. 2] Over the past 35 years, the field of simulation has undergone tremendous growth in its scope and capabilities. In the early days, simulation was employed in the study of relatively simple dynamic system; large percentage of the applications originated in the aerospace researches. Today hardly an industry or a discipline does not use simulation techniques intensively. The ability to handle complex systems has advanced to the point where global socio-economic systems are being investigated with such portentous variables as population, national resources and quality of life. Parallel to the expanding of applications, the developments of simulation equipments have conscientiously tried to keep pace with the demands for greater accuracy, capacity, speed, reliability, economy and convenience of use by creating sophisticated and diverse inventory of hardware and software.

The goal of simulation may be regarded as: (1) predicting system performance under particular operation conditions; (2) testing and evaluating a system or a particular subsystem; (3) identifying those portions of the system that require further investigation. However, regardless of the particular application, it is not feasible to directly implement a complex system without a sound process of evaluation. Figure 1.1 gives a functional summation of the generic process of simulation. The functional definition of the process of simulation is probably unique in the sense of having both technical and managerial value and utility. The activities of mathematical modelling, computer implementation and model utilization (block 3, 5

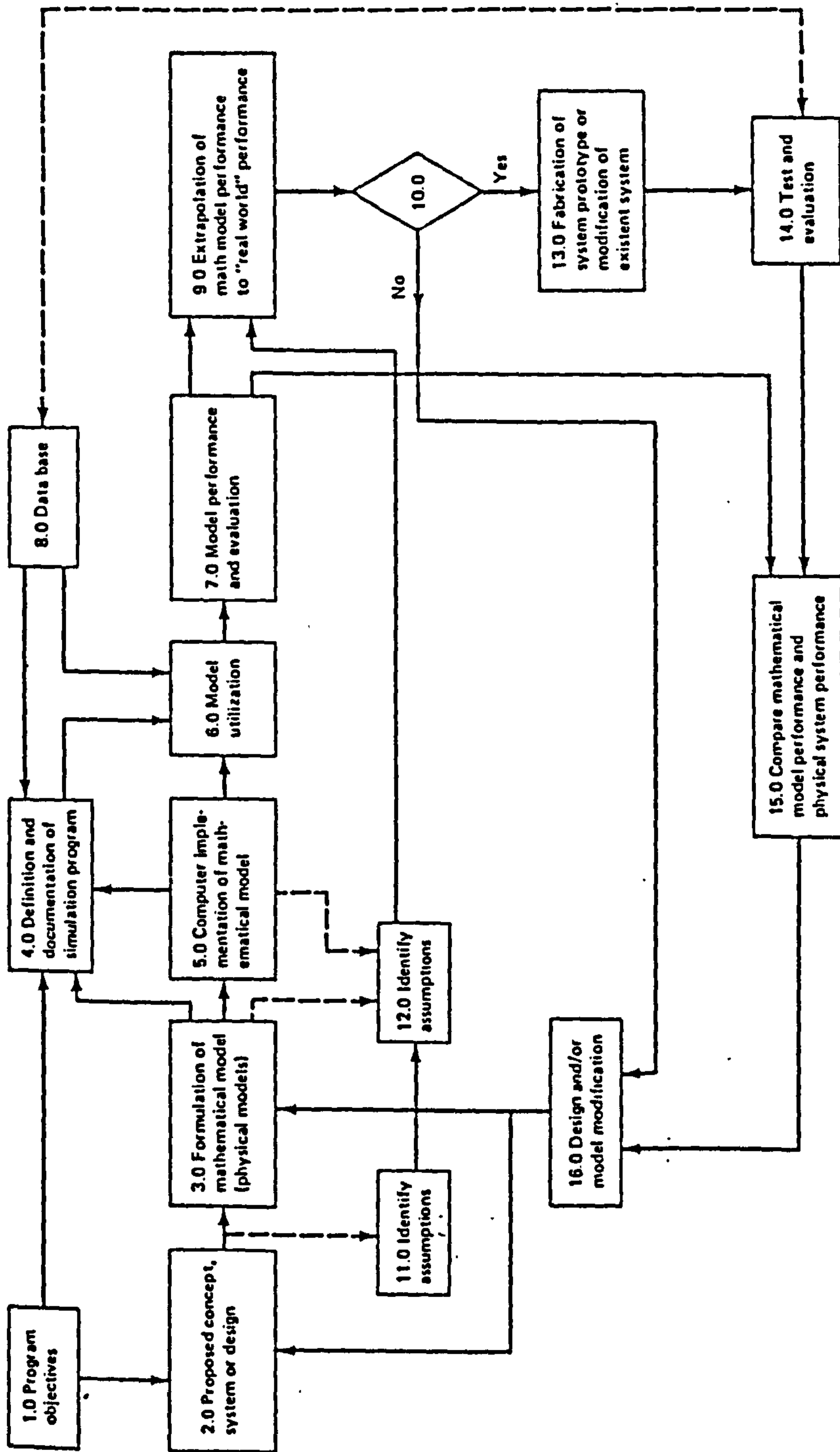


Figure 1.1 The process of simulation (Ref. 2)

and 6) may be regarded as the most important subprocesses within the overall process of simulation.

In general, simulation utility is uniquely dependent upon the validity of the mathematical model. The mathematical model, in essence, is the description of physical system in such mathematical terms as, for example, equations. However, it is extremely important that the collection of equations be formulated so that the parameters of interest are available.

Too often the mathematical models are obtained by immediately employing some existing frames that bear some alleged resemblance to the problem at hand, sufficient consideration is rarely given to the existing frames to one's particular problem. This form of insufficient mathematical modelling does not have the depth to provide information required to assess the ability of the proposed system to meet the stated program objectives. This situation can be further aggravated by those technical and managerial personnel who do not know that the information is useless, as the results, either incorrect or irrelevant conclusion are made.

However, the development of system modelling in any field has to start with the simple models and gravitate towards the detailed models. In building energy simulation, the first stage development has been experienced for ages and the new development of mathematical modelling towards the goal of energy simulation is now fully supported by the changing structures of modern science and technology, such as the development of numerical methods used to solve complex mathematical problems of modelling are largely excited by the advancement of computer technology.

1.2 The development of modelling techniques

In the pre-computer age, numerical analysis has few points of contact with mathematical analysis as practiced by the pure mathematician, such as to prove the existence of a mathematical theorem, to construct a table of numerical values of solution etc.. Even when the existence was proved, the computational demands were much too heavy to be practiced by hand. In consequence, as far as numerical solutions were concerned, great emphasis was placed on problems that permitted an explicit solution.

This situation has changed radically with the advent of electronic digital computer. Many constructive techniques of classical analysis, originally conceived mainly as tools for proving theorems, can now be put to work with a great ease to yield numerical answers. As a consequence, there has been an increasing interest in such general constructive algorithms as the detailed study of numerical analysis. At the same time, practitioners of computation are developing a keen feeling for the degree of constructiveness of a given algorithm. In addition, to the impetus given by the electronic computer, theory and practice of the numerical construction of solution of differential equations have been given an additional boost by the demands of modern technology, of which the exploration of space is only the most conspicuous example. Also, with the development of computer technology, some aspects which greatly restricted the application of numerical methods to the system modelling such as speed, memories of computer are not playing the same roles as they were in the past.

In the field of building and HVAC system modelling, mathematical problems concerned are mainly the solution of simultaneous linear or non-linear ordinary differential equations (or problems can be transformed to). Unlike other branches of numerical analysis, the study of the numerical solution of ordinary differential equation has enjoyed the advantage of being supported by a fully developed body of knowledge of theoretical concepts. These highly developed and mature results can be directly and successfully applied to the solution of complex practical problems of building and HVAC system mathematical

model at any level of accuracy required. The question left behind becomes: is the mathematical model correct or the physical phenomenon properly reflected by model? This question leads to the interests of searching for the better modelling tools for the generation of mathematical models.

A good modelling tool requires to present the ability of constantly coping with the ever developing modern science and technology, rigorous in theoretical background and flexible in use to produce either generalised or detailed models in energy simulation.

State space analysis is the terminology and approach method recently developed in control engineering. Since it has been developed from the 60s, the features of control engineering have radically changed into the group of conventional theory and modern control theory. Modern control theory employs state space analysis as a tool which enables it to be applied for the designs of more complicated systems and multi-input, multi-output systems. The vitality of this approach has excited many other fields of research.

Most of the modelling techniques existed in the field of building energy simulation can be regarded as the conventional analysis. It has been noticed recently that state space analysis can be used as a generalised modelling and simulation tool to produce formalized, documented and transferable software. [Ref. 3] The attempt to simulate the building fabric by using state space analysis has started a few years ago. The application of this approach systematically and rigorously to the HVAC system modelling still in its infancy and under blooming.

This thesis is trying to introduce the state space analysis into the field of building energy simulation and to establish, based on this approach, the generalised modelling technique and systematic procedure to cope with the special problems arisen in the model development of building and HVAC equipments and systems. Numerical methods are described in detail and employed as the basic solution medium to the non-linear system state equations. Other solution techniques are introduced briefly and they can become extremely useful and efficient

if the system mathematical problems can be simplified to the linear and time invariant system representation.

The extensive achievements and approach techniques in modern control theory based on state space analysis, such optimization problems, model simplification and automatic control to the controllable and indirect controllable parameters etc. can be introduced also to the field of building and HVAC system simulation. It can be expected that as the state of the art of state space analysis is developed, it will play a larger role in the field of building energy modelling.

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CHAPTER TWO: INTRODUCTION TO ENERGY MODELLING TECHNIQUES

2.1 Modelling of building envelope

2.1.1 Steady state method

2.1.2 Harmonic method

2.1.3 Response factor and Z transfer function methods

2.1.4 State space method

2.1.5 Finite difference method

2.1.6 Control volume heat balance method

2.2 The modelling of plant system

2.2.1 Steady state method

2.2.2 Sequential modelling

2.2.3 Transfer function modelling

2.2.4 Control volume heat balance method

2.2.5 Modelling technique of the present research

This chapter gives a general review of the development in the field of building/plant system thermal modelling. The objective is going to be introduced in two main aspects:

- (1) the modelling of building envelope; and
- (2) the modelling of plant system.

In fact, the building and plant systems modelling is a unit in which the two branches are very closely related. For example, the building envelope and its confined living space can be considered as load and boundary conditions to the plant system while the plant system in return can be considered as the disturbance to the building envelope system. This division is only because of their mathematical significances since the modelling of building envelope is mainly a heat conduction problem while the plant system is a synthesis of the heat and mass transfer and fluid flow problem. The modelling parameters for a building/plant system are usually considered to be the temperature distribution in the system for a dry thermal modelling where no mass transfer has taken place. For wet process both temperature and humidity have to be taken into account.

2.1 The modelling of building envelope

The earliest attempt to model the thermal performance of building envelope with its internal and external excitations can be traced back to the last century when J. B. Fourier developed the basic mathematics for transient temperature field in solid. This relation is given the name "Fourier heat-conduction equation". The well known differential form is given as, [Ref. 1]

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + q \quad 2.1$$

The heat flux normal to the temperature field in a solid is depicted by the relation known as Fourier's law.

$$q = -k \text{ grad}(T) \quad 2.2$$

Since then, finding solutions to the Fourier heat-conduction equation under specified initial and boundary conditions has become one of the important objectives in the fields of mathematics and thermal physics. The Fourier equation can be simplified for an isotropic, homogeneous material where the thermal conductivity k can be considered as constant, results,

$$\frac{\partial T}{\partial t} = a \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + q' \quad 2.3$$

Further simplification is made by considering the temperature distribution to be invariant with time, this gives us the Poisson equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + q' = 0 \quad 2.4$$

When the heat generation q' equals zero, it becomes the well known Laplace equation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \quad 2.5$$

Techniques involved in the modelling of thermal performance of building envelope could be considered as the practical applications of

Fourier equation and attempts to find solutions under different initial and boundary conditions. The modelling techniques can be divided into two categories:

- (1) the transient method; and
- (2) the steady state method.

The former can be considered as different methods of solving Fourier equation coupling with the specified initial and boundary conditions related to the environments of building envelopes, while the solutions of Poisson or Laplace equations with their relative boundary conditions lead to the steady state method.

2.1.1 Steady state method

For building envelope thermal problem, the steady state method is basically the solution of one dimensional Laplace equation coupling with convective boundary conditions of:

$$\left\{ \begin{array}{l} \frac{d^2T}{dx^2} = 0 \\ h_o(T_o - T) = -k \frac{dT}{dx} \quad x = 0 \\ h_i(T_i - T) = -k \frac{dT}{dx} \quad x = L \end{array} \right. \quad 2.1.1$$

where,

h_o = External surface heat transfer coefficient

h_i = Internal surface heat transfer coefficient

T_o = External air temperature

T_i = Internal air temperature

The solution of equation 2.1.1 yields the temperature distribution in the fabric as:

$$T(x) = - \frac{T_o - T_i}{L + \frac{k}{h_o} + \frac{k}{h_i}} x + T_o - \frac{T_o - T_i}{h_o \left(\frac{L}{k} + \frac{1}{h_o} + \frac{1}{h_i} \right)} \quad 2.1.2$$

and the well know heat flux equation as:

$$q = \frac{T_o - T_i}{\frac{L}{k} + \frac{1}{h_o} + \frac{1}{h_i}} \quad 2.1.3$$

It is easy to see that the temperature distribution within the fabric is linear, both temperature distribution and heat flux are independent of time, and the thermal storage effects of fabric are not considered. The thermal storage effects play important roles especially in medium and heavy structures so that this is the main disadvantage of the steady state method. Figure 2.1.2 shows conceptional differences between the load profiles calculated by steady state method and dynamical method.

The load calculated by steady state method follows the profile same as

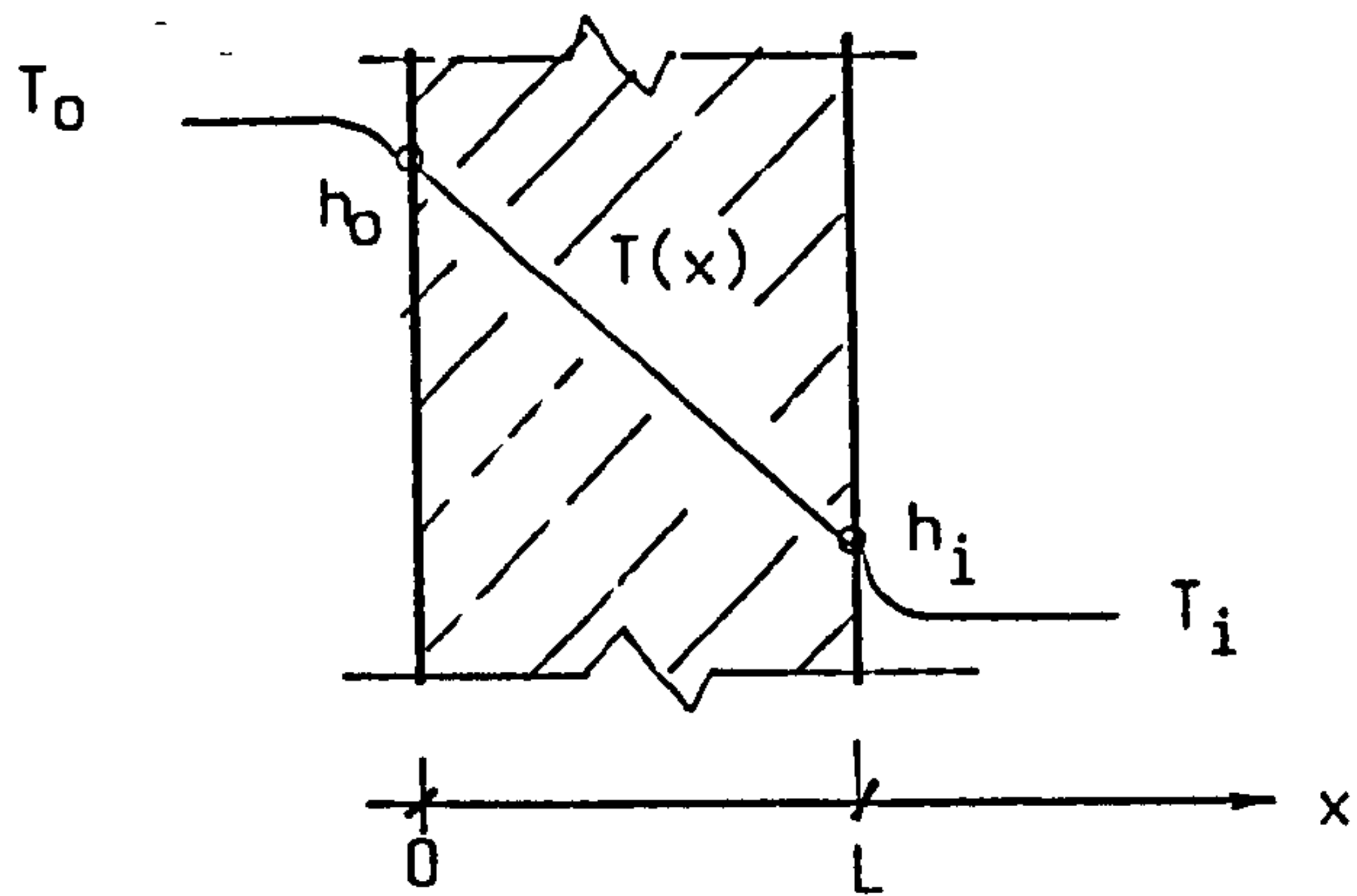


Figure 2.1.1

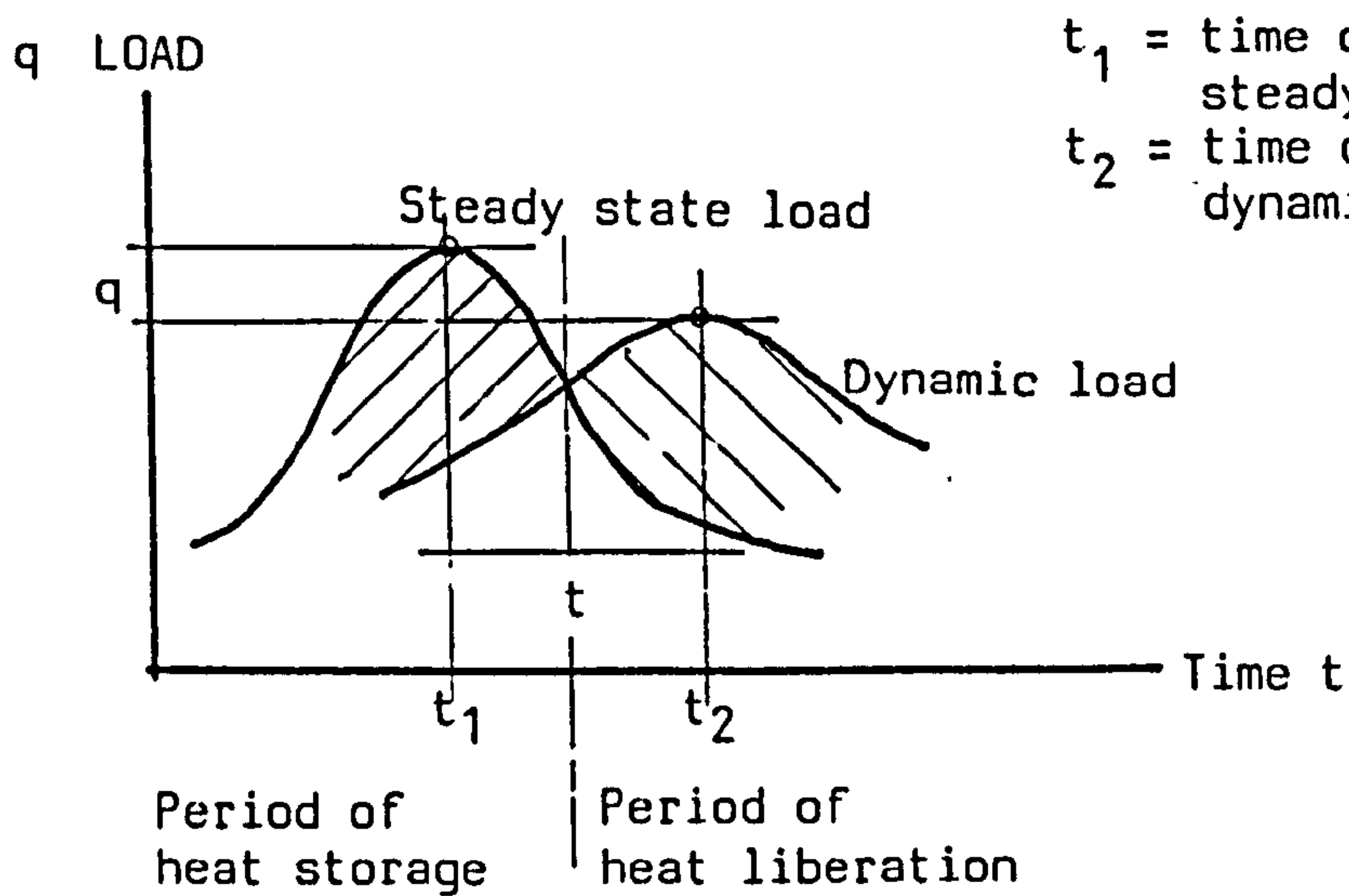


Figure 2.1.2

external excitations, while in reality, the load of a building usually behaves as the profile of dynamic load. Therefore, over-estimation of system load will occur at the storage period and under-estimation occur at the liberation period and the time of maximum load is appeared earlier and the results of simulation by this method can be seriously questioned. Efforts have been made to improve this method by introducing terms such as "sol-air temperature", "environment temperature", "decrement" and "time lag factors". The concept of "sol-air temperature" combines the effects of external excitations acting upon the external surface such as solar radiation, long wave radiative exchange between surface to surrounding and wind velocity. The effects of heat exchanges from one of the room external surfaces to room air and the long wave radiation to the remaining surfaces is combined in the term of "environment temperature". The introduction of decrement and time lag factors gives the effect of decreasing the over-estimated maximum load and catching up the time delay of maximum load by the thermal storage effect of fabric. These improvements give the steady state method possibility to cope with boundary conditions in reality and the dynamical effects. In fact, the decrement and time lag factors are concepts based on the solution of transient heat-conduction equation. Therefore, the best way to improve the modelling qualities is leading to the dynamical modelling.

2.1.2 Harmonic method

This method and the methods in the later section of Response factor and Z transfer function are considered as the analytical methods which are trials of finding analytical solutions to the Fourier heat-conduction equation under initial and boundary conditions. The difference between these three methods only lies on the different mathematical tools employed in the solution of the target problem. Harmonic method employs Fourier series to solve the target equation under periodical condition and the excitation has to be represented by superposition of harmonic waves. Response factor method uses Laplace transform as the solution tool and the superposition of triangle impulse is used as the format of excitation. Z transfer function method uses Z transformation upon the Laplace domain transfer function so that resulted its discrete time representation. The excitation of this method is the discrete sampling values of the excitation. It has been proved that these three methods will result same answer if the same input excitation is used in computation. However, since the treatments of excitation are different, they can play different roles in practical applications.

Generally, the analytical solutions can only be obtained for relatively simple bodies and temperature regions. They rapidly become complicated if the thermal properties and heat transfer coefficients are considered as functions of time and temperature. The common assumptions to the analytical methods are based on:

- (a) one dimensional heat flow;
- (b) constant thermal properties and heat transfer coefficients.

Based on these assumptions, the Fourier heat-conduction problem becomes a linear time-invariant system to which the theorem of superposition can be applied.

The fundamental of harmonic method is considering the external excitation, such as sol-air temperature, as a continuous periodical function and represented by Fourier series, as:

$$f(t) = A_0 + A_1 e^{w_1 t_j} + A_2 e^{w_2 t_j} + \dots + A_n e^{w_n t_j} + \dots \quad 2.1.4$$

where,

$f(t)$ = external excitation
 $A, \dots, A,$ = amplitude
 $w, \dots, w,$ = frequency
 t = time
 j = complex operator

The linearity and time-invariant properties of system ensures that the effect of each harmonic component of external excitation can be computed separately and then added together later. It also implies that if the input is periodic, the output is also periodic and therefore, only one term in the Fourier series is needed to obtain the basic solution. The general term in 2.1.4 can be written as:

$$f(t) = Ae^{wtj} \quad 2.1.5$$

The solution of Fourier heat-conduction equation with either the first or the third kind periodical boundary conditions can be found and the resulted heat flow through the fabric can be represented in form of frequency response, as:

$$q(t) = G(wj) Ae^{wtj} \quad 2.1.6$$

where $G(wj)$ is the frequency transfer function of the fabric representing its frequency characteristics. The steady state solution can be obtained by considering the frequency of the external excitation w equals to zero and the resulted heat flow becomes

$$q(0) = G(0)A.$$

For the external excitation in form of:

$$f(t) = A_0 + Ae^{wtj} \quad 2.1.7$$

the solution can be achieved immediately according to the theorem of linear superposition, which is:

$$q(t) = G(wj)Ae^{wtj} + A_0G(0) \quad 2.1.8$$

The complete solution of heat flow can be found as:

$$q(t) = A_0 G(0) + G(\omega_j) A_1 e^{\omega_j t} + G(\omega_j) A_2 e^{\omega_j t} + \dots + G(\omega_j) A_n e^{\omega_j t} \quad 2.1.9$$

In general, $G(\omega_j)$ is a complex quantity, it can be written in the following form:

$$G(\omega_j) = |G(\omega_j)| e^{j\phi} \quad 2.1.10$$

where $|G(\omega_j)|$ represents the magnitude which is always less than 1 in reality and $\phi = \angle G(\omega_j)$ represents the phase shift angle which is always negative. This result reveals the dynamical characteristics of building fabric in which the external excitation is subsided and delayed by the thermal storage effect.

On the development of harmonic method, an instructive work was achieved by Alford, et al. [Ref. 3] He first concentrated on the solution of the one dimensional Fourier equation based on constant coefficients, homogeneous wall with finite thickness, constant indoor air temperature and the third kind periodical boundary condition. External excitations of temperature, solar radiation and long wave radiative exchange are represented by Fourier series, each has a constant term and harmonic wave terms. The result obtained is similar to equation 2.1.9 including the steady state term and the harmonic response terms. The decrement factor and time lag phase angle are derived and proved to be independent of external excitations, only functions of building fabric properties. Later works of Mackey, et al [Ref. 47] found the treatment of composite multi-layer structure using a concept of equivalent homogeneous wall. Pipes, [Ref. 5] derived another method of transfer matrix to cope with the multi-layers problem. Nottage et al [Ref. 6] and Muncey et al [Ref. 7] removed the limitation imposed by the assumption of constant internal air temperature to increase the accuracy of modelling. The "means and swings" technique developed by Danter, [Ref. 8] which, in its simplest form, has been adopted by the Chartered Institute of Building Science (CIBS) of UK and is commonly referred as "Admittance method". The results of this method also introduced in ASHRAE Handbook of Fundamental, 1972, as the theoretical basis of TETD method (Total

Equivalent Temperature Differential) for the calculation of building heat gain.

The accuracy of this method is obvious if the external excitations can be accurately replaced by harmonic waves. However, from the basis of practical harmonic analysis, it is usually difficult to accurately replace the actual excitation $f(t)$ even if large number of harmonic waves are used. For example, the maximum number of harmonic terms can be used to approach hourly measurements of sol-air temperature over one day of 24 hours are twelve, but it still can not guarantee that the sum of harmonic waves passing through every measurement point. This approximation can be expressed as:

$$f(x) \doteq \sum_{i=1}^n A_i e^{w_i t_j}$$

or

$$f(x) = \sum_{i=1}^n A_i e^{w_i t_j} + E(t)$$

where,

$E(t)$ = an error function.

The solution only considers the harmonic waves, neglecting $E(t)$ will obviously introduce error depending on the absolute value of $E(t)$. For long term simulation, a relatively large number of harmonic waves has to be used to approach the random climatological data so as to minimize the theoretical error. This makes the practical application of this method limited. It is important to emphasize here that heat gain and cooling load are two different concepts. The result from this method only gives the heat gain of building envelopes. It has to be converted to cooling load by some further calculation. ASHRAE Handbook [Ref. 9] introduced simplified method to relate the instantaneous heat gain and cooling load through the use of weighting factors. This is done by separating the instantaneous heat gain into convective and radiant heat gain by the given proportional values; taking the convective portion of instantaneous heat gain as instantaneous cooling load while the radiant portion as reduced and averaged over a period of time by the thermal storage of the building envelope.

Other sophisticated weighting factor methods can be found elsewhere.
[Ref. 10]

2.1.3 Response factor and Z transfer function methods

Response factor and Z transfer function methods are quite similar in theory which are derived based on the concept of "linear control system". The linear time-invariant system implies that a physical system can be described by linear equation in which the coefficients are constant. According to the theory of linear control system, the definition of transfer function of a linear autonomous system is defined as "the ratio of the Laplace transform of the output to the Laplace transform of input with the restriction that all initial conditions are zero. [Ref. 11] It also states that the transfer function reflects the characteristics of the system and is independent of input and initial condition. The restriction of zero initial condition is caused by the mathematical method of Laplace transform. The building envelope with its external and internal environments can be considered as a thermal system described by a constant coefficient, linear, partial differential equation together with its boundary conditions and zero initial condition.

$$\left\{ \begin{array}{l} \frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} \\ T(x,t) = 0 \quad t=0 \\ T(x,t) = f(t) \quad x=0 \\ T(x,t) = 0 \quad x=1 \end{array} \right. \quad 2.1.11$$

where,

$$a = \frac{k}{\rho c}; \text{ thermal diffusivity, and } k, \rho, c \text{ are conductivity density, specific heat respectively.}$$

This boundary and initial value problem can be solved by taking Laplace transform to the time variable t, it results:

$$\left\{ \begin{array}{l} T(x,s) = F(s) \operatorname{ch} \sqrt{\frac{s}{a}} x - F(s) \frac{\operatorname{ch} \sqrt{\frac{s}{a}} l}{\operatorname{sh} \sqrt{\frac{s}{a}} l} \operatorname{sh} \sqrt{\frac{s}{a}} x \\ q(x,s) = -kF(s) \sqrt{\frac{s}{a}} \operatorname{sh} \sqrt{\frac{s}{a}} x + kF(s) \sqrt{\frac{s}{a}} \frac{\operatorname{ch} \sqrt{\frac{s}{a}} l}{\operatorname{sh} \sqrt{\frac{s}{a}} l} \operatorname{ch} \sqrt{\frac{s}{a}} x \end{array} \right. \quad 2.1.12$$

where,

$$s = \text{Laplace transform operator}$$

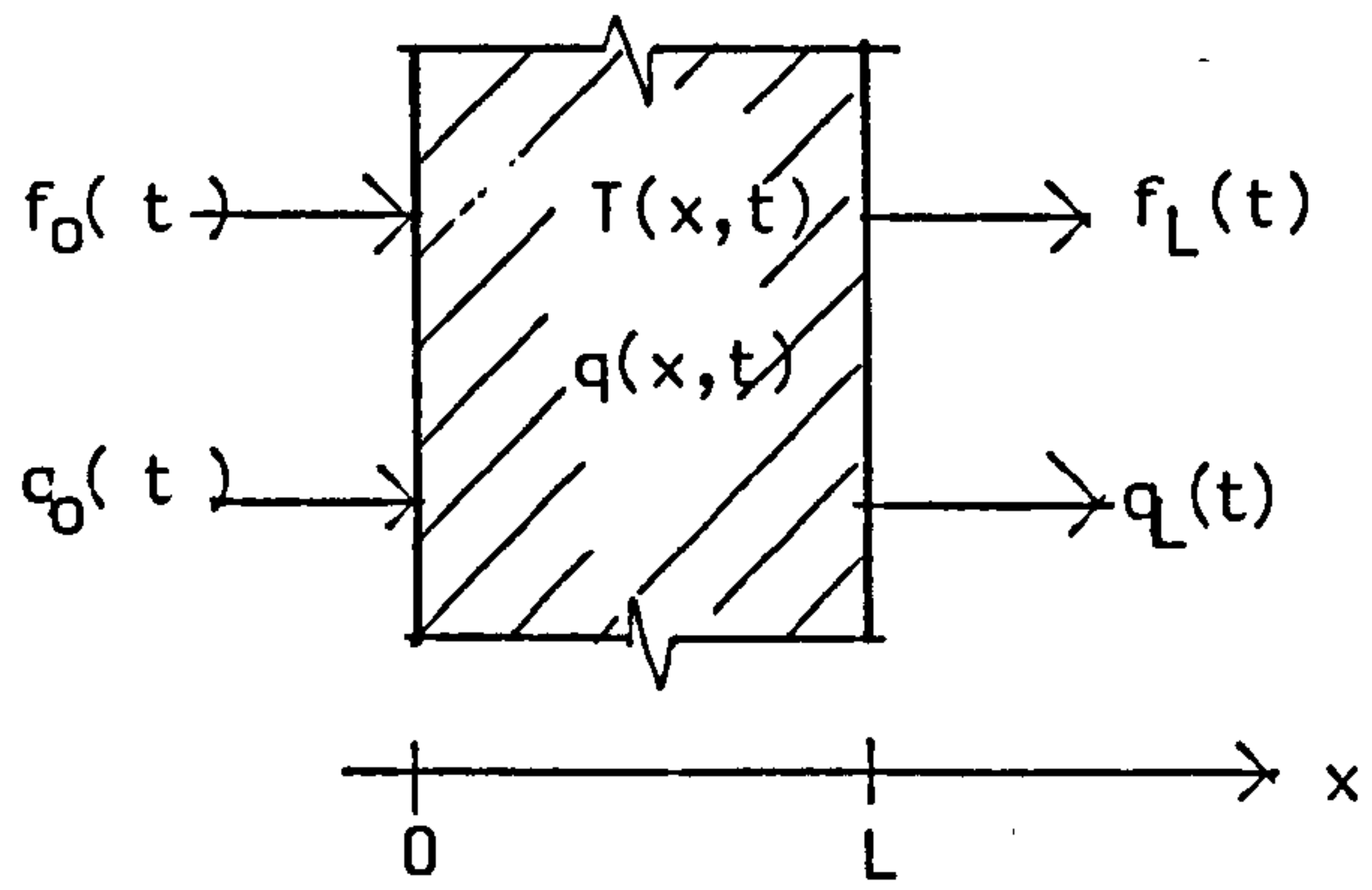


Figure 2.1.3

$F(s), T(x,s), G(x,s)$ = Laplace transform of excitation, temperature and heat flux respectively

Heat flow into internal surface can be directly obtained from equation 2.1.12 as:

$$q(1,s) = k \frac{\sqrt{s}}{a} \frac{F(s)}{\text{sh} \frac{\sqrt{s}}{a} l}$$

According to the definition, the heat flux transfer function of this thermal system is the ratio of output and input, thus:

$$G(s) = \frac{q(1,s)}{F(s)} = \frac{k \sqrt{s}}{a \text{sh} \frac{\sqrt{s}}{a} l}$$

In general, equation 2.1.12 is usually represented as the relation of excitation and response in matrix notation, which is:

$$\begin{vmatrix} T(1,s) \\ q(1,s) \end{vmatrix} = \begin{vmatrix} A(s) & B(s) \\ C(s) & D(s) \end{vmatrix} \begin{vmatrix} T(0,s) \\ q(0,s) \end{vmatrix} \quad 2.1.13$$

where

$$A(s) = D(s) = \text{ch} \frac{\sqrt{s}}{a} l$$

$$B(s) = - \frac{\text{sh} \frac{\sqrt{s}}{a} l}{k \frac{\sqrt{s}}{a}}$$

$$C(s) = -k \frac{\sqrt{s}}{a} \text{sh} \frac{\sqrt{s}}{a} l$$

The matrix in the right hand side of equation 2.1.13 is called a transfer matrix relating the input vector to the output vector, having the similar meaning as the transfer function of single variable. For multi-layer construction of homogeneous elements, the overall transfer matrix is given by:

$$\begin{vmatrix} A(s) & B(s) \\ C(s) & D(s) \end{vmatrix} = \begin{vmatrix} a_1(s) & b_1(s) \\ c_1(s) & d_1(s) \end{vmatrix} \begin{vmatrix} a_2(s) & b_2(s) \\ c_2(s) & d_2(s) \end{vmatrix} \dots \begin{vmatrix} a_n(s) & b_n(s) \\ c_n(s) & d_n(s) \end{vmatrix}$$

where,

$$\begin{vmatrix} a_i(s) & b_i(s) \\ c_i(s) & d_i(s) \end{vmatrix} = \text{Transfer matrix of homogeneous element}$$

The convective boundary condition problem in which excitations are given in air temperatures of internal and external spaces can also be treated by matrix manipulation, as

$$\begin{vmatrix} A'(s) & B'(s) \\ C'(s) & D'(s) \end{vmatrix} = \begin{vmatrix} 1 & \frac{1}{h_i} \\ 0 & 1 \end{vmatrix} \begin{vmatrix} A(s) & B(s) \\ C(s) & D(s) \end{vmatrix} \begin{vmatrix} 1 & \frac{1}{h_o} \\ 0 & 1 \end{vmatrix}$$

where, h_i = internal surface convective heat transfer coefficient
 h_o = external surface convective heat transfer coefficient

Therefore, problems concerned with multi-layer structure and convective boundary conditions can be generally represented as input and output relationship. The fundamental problem of finding the heat flux flowing into the room under external and internal excitations requires the transformation of the transfer function or transfer matrix back into their original time domain. Mitalas and Stephenson [Ref. 12,13] initiated the response factor method in 1967 by introducing a time series expression representing the excitation into the calculation process. Using the same theory, Kusada extended the calculation of response factors to multi-layer structures of various curvatures. [Ref. 14] In the time series expression of external excitation, e.g. the input of sol-air temperature, is represented by the convolution of discrete climatological values and unit triangle pulses, which is:

$$f(t) = \sum_{i=0}^{\infty} f(i\Delta) T(t-i\Delta) \quad (\text{see com.1})$$

where, $f(i\Delta)$ = discrete sol-air temperature value
 $T(t-i\Delta)$ = unit triangle pulse
 Δ = time interval

The thermal response of the building envelope under external and internal excitations also can be treated separately because of the linearity and time-invariant properties of the thermal problem. Therefore, the thermal response can always be expressed as the input/output relation as:

$$q(l,s) = G(s)F(s) \quad 2.1.14$$

where,

$q(l,s)$ = response of heat flow into the internal surface
 $G(s)$ = transfer function
 $F(s)$ = excitation

Replace excitation $F(s)$ by the time series and taking Laplace transform:

$$\begin{aligned}
 q(l,s) &= G(s) L\left\{\sum_{i=0}^{\infty} f(i\Delta)T(t-i\Delta)\right\} \\
 &= \sum_{i=0}^{\infty} f(i\Delta)G(s) L\{T(t-i\Delta)\} \\
 &= \sum_{i=0}^{\infty} f(i\Delta)G(s) \frac{e^{-i\Delta s} e^{\Delta s}}{\Delta s^2} (1 - e^{-\Delta s})^2
 \end{aligned} \tag{2.1.15}$$

(see com. 2)

and the heat flow in original time domain can be obtained by taking inverse Laplace transformation to the above equation, results

$$q(l,t) = \sum_{i=0}^{\infty} f(i\Delta) L^{-1}\left\{G(s) \frac{e^{-i\Delta s} e^{\Delta s}}{\Delta s^2} (1 - e^{-\Delta s})^2\right\} \tag{2.1.16}$$

It can be rewritten in the discrete time convolution form by taking discrete time values to both sides of equation 2.1.16 and therefore results the fundamental expression of the response factors method, which is:

$$\begin{aligned}
 q(l,t)\Big|_{t=k} &= \sum_{i=0}^{\infty} f(i\Delta) L^{-1}\left\{G(s) \frac{e^{-i\Delta s} e^{\Delta s}}{\Delta s^2} (1 - e^{-\Delta s})^2\right\}_{t=k\Delta} \\
 &= \sum_{i=0}^k f(i\Delta) Y(k-i)\Delta
 \end{aligned} \tag{2.1.17}$$

(see com. 3)

where,

$Y(k-i)\Delta$ = response factor of envelopes under certain excitation.

Here the summation is up to the limit of calculation time k rather than infinity, this is because of the zero initial condition.

It results in all values of $Y(i)$ equal to zero when $i < 0$. The response factors can be specified to four kinds in heat flow through walls, which are the response factors of heat flow at:

- a) outside surface for outside surface temperature excitation.
- b) inside surface for outside surface temperature excitation.

- c) outside surface for inside surface temperature excitation.
- d) inside surface for inside surface temperature excitation.

The response factors, $Y(i)$, can be obtained by inverse Laplace transformation of the products of system transfer function and unit triangle pulse. The results are usually achieved by Heaviside's expansion theorem of residues calculation or numerical methods rather than that direct inverse transformation by complex variable integration. For long term simulation, the heat flow expression of response factors is almost an infinite summation process. Truncation errors will possibly exist when the summation is limited to a certain extent. It has, however, been proved that the set of response factors is a convergent array. The definition of "common ratio" is introduced by Kimura [Ref. 10] as an approximation to the infinite summation, which results the relationship of the present calculation time heat flow to the flow one time step in arrear. This simplification contributes a substantial reduction in computer time for practical use.

Stephenson Mitalas later introduced the Z transform to the transient heat conduction problem leading to the method of Z transfer function. [Ref. 15] The principles underlying this approach are closely related to the response factors method. This method is also considered to be a discrete time system method in which the Z transformation plays a similar role as the Laplace transformation in a continuous time system.

In this method, the external excitation is replaced by the convolution of the discrete values and the Dirac delta function $\delta(t)$, which is:

$$f(t) = \sum_{i=0}^{\infty} f(i\Delta) \delta(t - i\Delta) \quad (\text{see com. 5})$$

The physical meaning of this process may be considered as a sampler. The system with triangle impulse input may be considered as a discrete time system with sampler and triangle pulse Hold Element. Therefore, the transfer function of this discrete time system consists of the transfer function of the continuous system, $G(s)$, and the transfer function of the hold element of triangle impulse. Referring to

equation 2.1.14 and 2.1.15, the heat flow output in Laplace domain becomes:

$$\begin{aligned}
 q(1,s) &= G(s) F(s) \\
 &= \left[G(s) \frac{e^{\Delta s} (1 - e^{-\Delta s})^2}{\Delta s^2} \right] L \left[\sum_{i=0}^{\infty} f(i\Delta) (t - i\Delta) \right] \\
 &= \left[G(s) \frac{e^{\Delta s} (1 - e^{-\Delta s})^2}{\Delta s^2} \right] \sum_{i=0}^{\infty} f(i\Delta) e^{-i\Delta s}
 \end{aligned} \tag{2.1.18}$$

Convert this expression from s domain into function of z by Z transformation, results:

$$\begin{aligned}
 \sum_{n=0}^{\infty} q(n\Delta) z^{-n} &= G(z) \sum_{n=0}^{\infty} f(n\Delta) z^{-n} \\
 \text{in which,} \\
 G(z) &= Z L^{-1} \left[G(s) \frac{e^{\Delta s} (1 - e^{-\Delta s})^2}{\Delta s^2} \right] \\
 &= Z \{ Y(t) \} = \sum_{n=0}^{\infty} Y(n\Delta) z^{-n}
 \end{aligned} \tag{2.1.19}$$

where,

$$Y(n\Delta) = \text{response factor}$$

It can be seen that the Z transfer function $G(z)$ can be obtained directly from the known transfer function of response factor method. $G(z)$ can also be represented as the ratio of two polynomials in order of z , e.g.

$$G(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots}{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots} \tag{2.1.20}$$

Heat flow at time k of $q(k)$ can be formed by the rearrangement of equation 2.1.20 with the Z transformations of input and output,

$$q(k\Delta) = \sum_{i=0}^k a_i f[(k-i)\Delta] - \sum_{i=1}^k b_i q[(k-i)\Delta] \tag{2.1.21}$$

where

$$a_i, b_i = Z \text{ transfer factors}$$

It has been proved that the convergency of the Z transfer factors series, a and b in 2.1.20, are very fast so that the summations of equation 2.1.21 can be less than k . Compare to the response factors, this method is much economical in term of computer memory space and

running time.

The actual cooling load of a building can be determined by response factors and Z transfer function methods by carrying out the zone energy balance to find out the overall response factors and Z transfer function. The Z transfer function method is the fundamental of the CLTD (Cooling Load Temperature Difference) method recommended by the recent ASHRAE Handbook. [Ref. 16]

2.1.4 State space method

State space is a concept introduced from the modern theory of control which has been developed since 1960. Different from the classical control theory, the theory of modern control is a time domain method based on the concept of state and state space rather than on the complex frequency domain. However, the concept of state is not new, since it has been in existence for a long time in fields of classical dynamics and other fields, such as in HVAC engineering, where the state of air can be represented by the state variable of pressure, density and temperature.

According to the definition, [Ref. 17] the state of a dynamic system is defined as "the minimum set of variables specified at initial time, $t = t_0$, together with the given input, determine the state at future time, $t > t_0$ ". The state variables are considered as "a minimum set of variables determining the state of the dynamic system". The selection of state variables are not unique and a vector formed by a set of state variables is called a state vector. A state space is defined as a N dimensional space with axes of state variables. Therefore, any state can be represented by a point in the state space.

The state space representation of a dynamic system is achieved by a set of first-order differential equations. To the solution of the state equations, many powerful tools such as linear algebra, vector matrix, numerical method and methods of modern control theory can be applied to analyse the dynamics of the system and optimal problems. This approach can also be applied to the solution of most nonlinear, time varying, stochastic and sampled data configurations.

J. Yi, [Ref. 18] developed a building thermal model based on this theory. In this model, the system state equation of the building envelope is derived based on the Fourier equation coupling with the dynamic boundary conditions. By taking semi-discretization to the space variable and given the coefficient as constant, the Fourier equation is converted into set of first-order linear time-invariant differential equations. Assuming the temperature distribution in the interval between two adjacent points is a quadratic function, the

state equation representing the thermal process of the building envelope can be written in matrix form, as:

$$\begin{cases} C \dot{T}(t) = A T(t) + W U(t) \\ T(t_0) = T_0 \end{cases} \quad 2.1.22$$

where,

$T = [T_1, T_2, \dots, T_n]^T$; state vector, in which T_1, T_2, \dots, T_n are state variable of temperatures in the envelope

$C = nxn$ matrix of thermal capacity of the building envelope

$A = nxn$ matrix of thermal capacity conductivity of the building envelope

$U = [u_1, u_2, \dots, u_n]^T$; vector of external disturbance

$W = nxm$ matrix, relating the disturbances to the thermal system, also may be considered as a sampler

$T_0 =$ the initial conditions

The semi-discretization is done on the spatial variable of Fourier equation by replacing the partial differential by difference expression of dividing the whole series section of wall into several layers. This treatment makes it possible to solve the problem of multi-layer structure. Each layer of the division is represented by a node representing the uniform temperature of this layer. This model can also be used to determine the thermal behaviour of a complete room. This can be achieved by building up the system model as a combination of several walls and the volume of room air in which long wave radiant exchanges between walls, infiltration etc. are also considered. In this model, the state equation is solved analytically by using the concept of "state-transition matrix" which is a typical method in modern control engineering. Since the state equation in this model is linear and time invariant, the eigenvalues of the state equation can be found to construct the "state-transition matrix".

According to system state equation 2.1.22, if C is a constant matrix and non-singular, a linear transformation can always be found so that system matrix similar to a diagonal matrix in which coefficients in the diagonal are eigenvalues of system matrix $(C^{-1}A)$. If the linear transformation of 2.1.22 can be represented as $X = PT$, and let $B = PC^{-1}W$, equation 2.1.22 becomes:

$$\begin{cases} \dot{X}(t) = \Lambda X(t) + B U(t) \\ X(t_0) = P T_0 \end{cases} \quad 2.1.23$$

where, $\Lambda = \text{diag}\{\lambda_i\}$, $(i=1, \dots, n)$; a $n \times n$ diagonal matrix in which λ_i , $(i=1, \dots, n)$ are eigenvalues of matrix $(C^{-1}\Lambda)$.

The analytical solution of the new state equation 2.1.23 is very similar to that of a first order ordinary differential equation. It has the form of:

$$X(t) = e^{\Lambda t} X(t_0) + e^{\Lambda t} \int_{t_0}^t e^{-\Lambda \tau} B U(\tau) d\tau \quad 2.1.25$$

where,

$$e^{\Lambda t} = \text{diag}\{e^{\lambda_i t}\}; \text{ the state transition matrix of the building envelope thermal system}$$

The analytical solution can also be converted to the form of time discrete expression which is suitable for the application of digital computer, which is:

$$X(\tau) = G X(\tau-1) + h_\tau \quad 2.1.26$$

where,

$$G = e^{\Lambda \Delta \tau} = \text{diag}\{e^{\lambda_i \Delta \tau}\}; \text{ and} \\ h_\tau = \Delta \tau \int_0^1 G^\xi B U(\tau-1+\xi) d\xi$$

If the final result is only considered to be the heat transmission, the expression of 2.1.26 will have a similar form as those resulted from response factor and Z transfer function methods.

The advantage of this method also can be found by treating the external excitations. Different forms of excitation can be easily handled, e.g. in solution 2.1.26, one only needs to change the term h to cope with the types of excitation. The disadvantages of this solution method are obvious. It is difficult to be applied to a multi-zone problem, since the system state equation will become very large, finding matrix C^{-1} , P , P^{-1} then becomes very complicated and time consuming. The assumption of constant coefficient, time invariant, also limits the accuracy of simulation.

However, the state space method itself has the potential of solving

time varying problems and the efficiency of the solution techniques can be improved by different grades of numerical methods, therefore, it becomes quite popular in the field of modelling and simulation. Benton. et al [Ref. 19] have emphasized the advantages of the state space method in energy modelling of buildings and recommended it as a generalized modelling technique which is best for present study and further development.

2.1.5 Finite difference method

Numerical methods are playing an increasing important role in the analysis of heat transfer problems and building energy modelling. The applications are usually applied to the problems which are insoluble by analytical methods. Among the numerical methods, the finite difference method is most popularly used in the field of building energy modelling. The essence of finite difference method is to replace the differential or partial differential equations with a set of simultaneous algebraical equations which can more easily be solved. This method has actually, been used in the field of heat transfer for quite a long time. The applications were limited in the past since the solutions of the simultaneous algebraical equations are still very time consuming by hand calculation. The advent of the digital computer provided the possibility for numerical methods to be used with speed and accuracy. Finite difference method uses the difference approximation to replace the original differential or partial differential problems. To obtain this replacement, the region covered by the original problem has to be divided into sub-regions by meshes with shapes of rectangular or polygon. Instead of rigorous discussion, the essence of finite difference formulation can be revealed by Taylor expansion. There are many different methods for obtaining the approximating difference formulation and no one method is considered superior to the others. An approximate difference formulation suited to the original problem has to pass certain tests of consistency, stability, convergency and accuracy.

In the field of building energy simulation, problems encountered are the solution of the Fourier equation and the initial, boundary problems. A number of finite difference schemes can be formulated, the most instructive are the ones given as follows of explicit and implicit formulations. [Ref. 20]

Assume the exact value of function $T(x,t)$, at $x = mh$, $t = nk$ is $T(mh,nk)$, using Taylor's theorem:

$$T[mh, (n+1)k] = T_m^{n+1} = T(mh, nk) + k \left(\frac{\partial T}{\partial t} \right)_{mh, nk} + \frac{1}{2} k^2 \left(\frac{\partial^2 T}{\partial t^2} \right)_{mh, nk} + \dots \quad 2.1.27$$

$$T[(m+1)h, nk] = T_{m+1}^n = T(mh, nk) + h \left(\frac{\partial T}{\partial x} \right)_{mh, nk} + \frac{1}{2} h^2 \left(\frac{\partial^2 T}{\partial x^2} \right)_{mh, nk} + \dots \quad 2.1.28$$

$$T[(m-1)h, nk] = T^n = T(mh, nk) - h \left(\frac{\partial T}{\partial x} \right)_{mh, nk} + \frac{1}{2} h^2 \left(\frac{\partial^2 T}{\partial x^2} \right)_{mh, nk} + \dots \quad 2.1.29$$

To simplify the discussion here, we assume a time-invariant, homogeneous to the one dimensional heat conduction problem of:

$$\frac{\partial T(x, t)}{\partial t} = a \frac{\partial^2 T(x, t)}{\partial x^2}$$

Explicit formula will be formed if variables in the above Taylor series are so arranged, as:

$$T_m^{n+1} - (1-2r)T_m^n - r[T_{m+1}^n + T_{m-1}^n] = k \left(\frac{\partial T}{\partial t} - a \frac{\partial^2 T}{\partial x^2} \right)_{mh, nk} + \frac{1}{2} k^2 \left(\frac{\partial^2 T}{\partial t^2} - \frac{1}{6r} \frac{\partial^4 T}{\partial x^4} \right)_{mh, nk} + \dots$$

where, $r = ak/h^2$

The principal part of the local truncation error is defined as,

$$\frac{1}{2} k^2 \left(\frac{\partial^2 T}{\partial t^2} - \frac{1}{6r} \frac{\partial^4 T}{\partial x^4} \right) = O(k^2 + kh^2) \quad 2.1.30$$

Ignoring the error term, the explicit approximation is:

$$T_m^{n+1} = (1-2r)T_m^n + r(T_{m+1}^n + T_{m-1}^n) \quad 2.1.31$$

Following the same process, the implicit approximation is found to be:

$$(1+r)T_m^{n+1} - \frac{1}{2}r(T_{m+1}^{n+1} + T_{m-1}^{n+1}) = (1-r)T_m^n + \frac{1}{2}r(T_{m+1}^n + T_{m-1}^n) \quad 2.1.32$$

this scheme is also called the "Crank-Nicolson formula".

The principal part of local truncation error is:

$$-\frac{1}{12} \left[k^3 \frac{\partial^3 T}{\partial t^3} + kh^2 a \frac{\partial^4 T}{\partial x^4} \right] = O(k^3 + kh^2) \quad 2.1.33$$

Compare to the above two schemes, the Crank-Nicolson formula has a higher order of truncation error than that explicit.

It has been proved that the convergent and stable region for the explicit formula is $0 < (ak/h^2) \leq \frac{1}{2}$, and for Crank-Nicolson formula is any $(ak/h^2) > 0$.

By the same process, it is possible to construct finite difference formulations for more than one space dimension, time varying and non-homogeneous problems. Other methods of constructing the difference formulae can be found in a wide range of relevant books and literature.

Different difference schemes can be used for a certain problem. For example, higher accuracy can be obtained if higher-order of central difference is retained. However, care has to be taken in choosing a finite difference formula for dealing with a special problem, since some schemes do have higher order of accuracy but cannot pass the condition of stability or can only be used in relative limited time and space steps.

2.1.6 Control volume heat balance method

The fundamentals of this method is based on the physical law of energy conservation of applying the energy conservation equation to the selected finite control volumes, building up the physical system. Figure 2.1.4 shows a control volume I in thermal contact with its surroundings, internal heat generation is also considered in volume I.

Assuming that each inter-region heat exchange can be represented by a linear function of temperature difference so that all the complexities are left to the coefficients. Any heat exchange flow path to control volume I can be written as:

$$q_{J,I}(t) = K_{J,I}(t)[T_J(t) - T_I(t)]; \quad J=1,2,\dots,n$$

where,

$$K_{J,I}(t) = \text{time dependent heat exchange coefficient between region J and I}$$

An energy balance applied on this control volume yields:

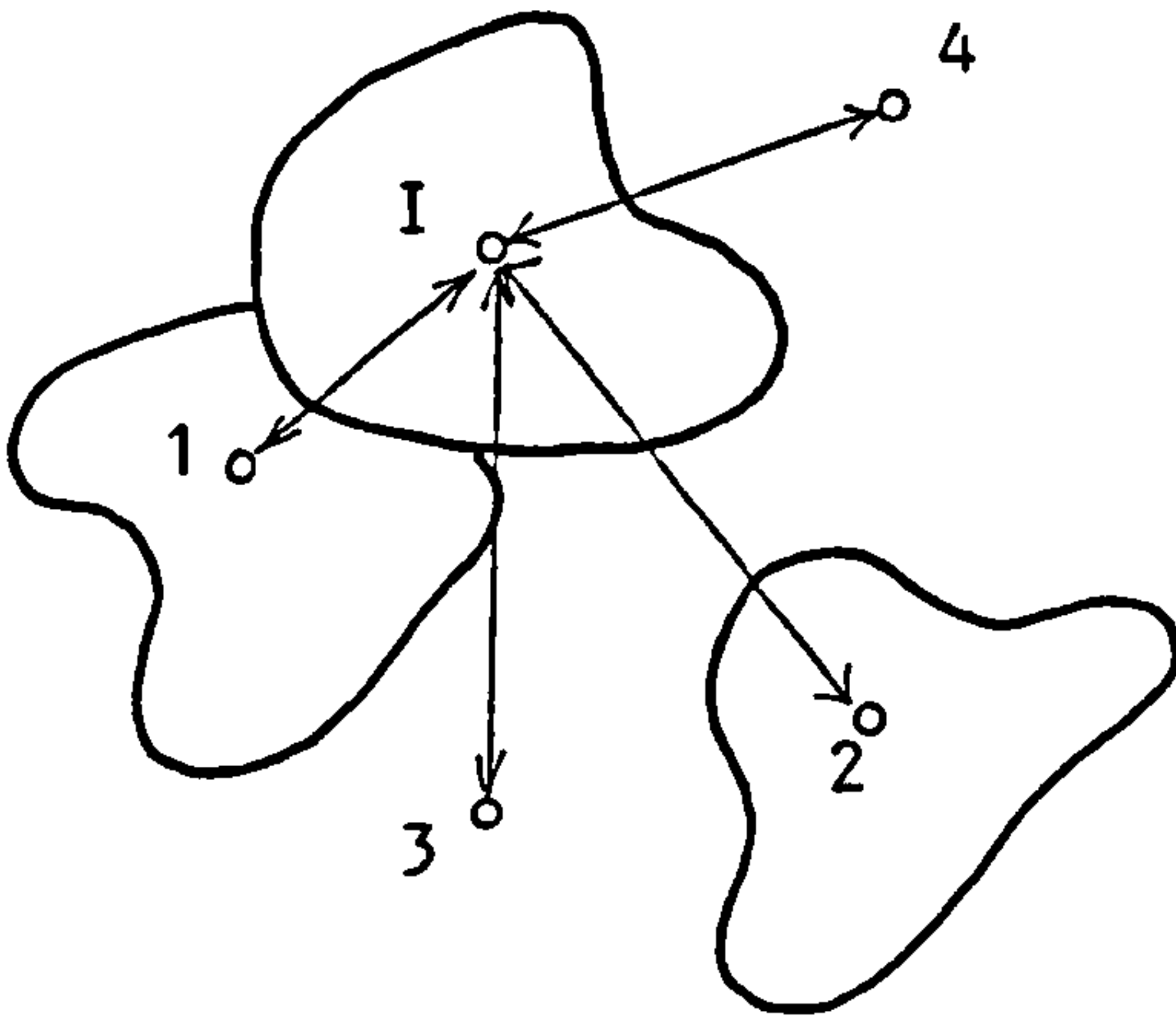
$$\left| \begin{array}{l} \text{Heat stored} \\ \text{within region} \end{array} \right| = \sum \left| \begin{array}{l} \text{heat flow into} \\ \text{the region} \end{array} \right| + \left| \begin{array}{l} \text{Heat generated} \\ \text{in the region} \end{array} \right|$$

this results in the general heat balance equation, which is:

$$\frac{\rho(\xi)c(\xi)v_r}{\delta t}[T(I,t+\delta t)-T(I,t)] = \sum_{j=1}^N K_{j,I}[T(j,\xi)-T(I,\xi)] + q_I(\xi) \quad 2.1.34$$

Evaluation of the heat flux and generation terms at the present time-row, $\xi = t$, gives the explicit formulation and evaluation at future time-row, $\xi = t + \delta t$, gives the full implicit formulation. The Crank-Nicolson scheme can be derived directly from equation 2.1.34 under the appropriate condition of equal weighting explicit and implicit formulations.

The arbitrariness of selection of the control volume makes equation 2.1.34 possible to apply to any object with irregular shape, non-homogeneous structure, time varying thermophysical properties and more



I/1 : conduction
I/2 : radiation
I/3 : convection
I/4 : fluid flow

Figure 2.1.4

than one dimension energy flow. The simultaneous presence of multiple heat transfer processes acting on the object, (such as conduction, convection, radiation and heat generation) can be treated separately by single heat flux or generation term in the general heat balance equation. Furthermore, based on the law of conservation, this method can not only be applied to the thermal system, but also has the potential to describe non-thermal systems such as mass exchange systems, mechanical systems or a mixture.

Clarke [Ref. 23] developed this method and applied to the modelling of thermal performance of building and its environments. In his model, the building under consideration is taken as a unit and divided into zones. In each zone, the general heat balance equation is applied to each pre-specified control volume represented by node. A complete set of simultaneous equations can be so derived for the building thermal system. Nodes can also be placed at window, room-air, air-conditioning plant and composite wall, so that the dynamics of a complete building can be totally described. A computer model called ESP was developed based on this theory. Within this model, each of the heat transfer and related processes are based on first principle techniques. For example: shading, insulation and view factor prediction by ray tracing techniques; air movement simulation by a modified Newton-Raphson iterative technique; window property by solar spectrum wavelength subdivision and beam tracking; solar radiation by separate processing of direct and diffuse beam using an anisotropic sky model; longwave radiation exchanges considered non-linear and evaluated separately for internal and external surface; surface convection determined on the basis of empirical data correlations; and full account of controller sensed temperature. Taking together all the nodal heat balance equations results the system mathematical model. ESP employs an implicit finite difference scheme, to achieve time-stepping energy balance as simulation proceeds, and relies on customised matrix inversion techniques to achieve fast and convergent solutions. By this algorithms, it is possible to preserve the spatial and temporal integrity of combine building/plant system by arranging that the whole system energy balance equation set (up to 2000 equations say) be solved simultaneously and repeatedly at each computational time-step; the technique has no need to simplify

flowpaths or assume superimposition and can handle non-linearities as well as complex (time-varying) boundary conditions. [Ref. 24]

This method and the methods introduced in the previous sections of state space method and finite difference method can be considered as the trials of approximate solution, in different level, for the Fourier heat conduction equation. State space method uses spatial approximation to reduce the partial differential problem into a set of ordinary differential equations. Finite difference method applies both spatial and temporal approximations to the target equation and results a set of implicit or explicit algebraic equations. Control volume heat balance method is similar to finite difference method in mathematics and further more it defines physical meanings to the spatial and temporal approximations for the practical applications. It will be proved in the later chapter that if the numerical integration is applied to solve the state equation of state space method, the representations of these three method will be identical.

2.2 The modelling of plant system

A plant system is a combination of mechanical components, the modelling concerned is not only the matter to model the performance of the single component but also the inter-relationship of the components and with relation to the building. Fewer researches had been directed to the modelling dynamically of the plant system compared with the modelling of building envelopes. Unlike the building envelopes modelling, the plant modelling synthesizes together the phenomenon of transient conduction, single and two phase fluid flow, heat and mass transfer and mechanical, electrical driving forces.

The idea was accepted for a long time in the past that the steady state method is suitable for the simulation of plant system. Even in the middle of the seventies, ASHRAE task group on "Energy Requirements for Heating and Cooling of Buildings" still recommend that steady state simulation of a plant-system is adequate for most energy calculation because of its fast dynamic response. [Ref. 23]

Recent developments no longer accept this idea and are concerned with the developments of dynamic models of plant components to form a plant system. However, even the modelling of a single component will become a full and complicated research topic if this modelling goes into much details. Such modelling can be found in the relevant field of engineering research.

Thermal modelling only concerns the basic parameters with direct relation to the thermal behaviour of plant system, such as temperature and humidity. Modelling techniques used in plant energy simulation usually are composite and can not be classified as clear as the modelling techniques of building envelope. Some models incorporate different kinds of techniques for each individual component to achieve the simplifications and required level of accuracy. The terminologies used here for the plant modelling techniques (steady-state, sequential, transfer function, control volume energy balance) are only used as classification in this thesis. It is important to note that the main point of plant modelling is to establish a fine inter-relationship between the components, the building envelope and the

living spaces. A general review will be given as follows to different models as a unity rather than to the modelling of individual component.

2.2.1 Steady state method

Most of the calculation methods presented in the HVAC designing handbooks can be considered as the steady state modelling techniques of components and systems. In such methods, the thermal capacities of components are ignored based on the considerations of their fast thermal responses compared to that of buildings. Heat and mass balances are carried out to achieve the steady-state heat and mass transfer rates of a certain component, and they are usually expressed by linear functions of temperature and humidity differences, the accuracy and reliability are largely depended on the sides of heat and mass transfer coefficients. Steady state method has been used in plant modelling for quite a long time and it produced acceptable results, especially in practical designs. The results will be acceptable for long term simulation provided that a careful consideration of the crucial coefficients has been made to obey every restricted condition of their applications.

2.2.2 Sequential modelling

Sequential modelling is the replacement of each component by some equivalent input/output relationship so that when connected to comprise a complete system, the output from one component in the calculation stream, becomes the input to the next. Individual component can be modelled by steady state method, lumped parameter models or even higher order differential equation by iterative solution.

TRNSYS is a computer simulation model based on this theory. It was developed by Solar Energy Laboratory at the University of Wisconsin, Madison as part of a joint project with Colorado State University. [Ref. 25] It consists of a number of component subroutines and executing programs that enable the user to simulate the thermal performance of a complete system. The initial version of TRNSYS was released in March 1975, and it has been continually updated. For version 10.1, a list of the library of system components and ancillary subroutines is given in table 2.1. The mathematical models for components are developed ranging from simple models of steady state (e.g. pump and fan), to complex models described by high order differential equations, such as modelling the effects of thermal stratification in a water storage tank. Iterative solution is employed inside the component model to solve the differential equations between each fluid section. The sequential technique of TRNSYS implies that the computation for a complete system is achieved individually and sequentially along the system flowing process. For an individual component it receives information from the previous components in connection and after a complete computation within it, passes the results to the downstream components.

This process will carry on through out every component in the system to complete a system simulation of one circle of time increment, which seems quite similar to the proceedings of hand calculation. This model can easily handle any number of components with interconnections as the number of components involved in a simulation doesn't make any difference to the complexity of the whole system.

| Component Type | Description |
|----------------|--|
| 1 | Flat-plate solar collector |
| 2 | On/Off differential controller with hysteresis |
| 3 | Pump or fan |
| 4 | Stratified fluid storage tank |
| 5 | Heat Exchanger |
| 6 | On/off auxiliary heater |
| 7 | Absorption air conditioner |
| 8 | Three-stage room thermostat |
| 9 | Data reader (for input of meteorological and other data) |
| 10 | Rock bed thermal storage |
| 11 | Tee piece, flow diverter, and flow mixer |
| 12 | Energy / (degree-hour) space heating |
| 13 | Relief valve |
| 14 | Cyclic time-dependent forcing function |
| 15 | Algebraic operator |
| 16 | Solar radiation data processor |
| 17 | Wall |
| 18 | Roof and attic |
| 19 | Room and basement |
| 20 | Heat pump |
| 21 | Liquid collector-storage subsystem |
| 22 | Air collector-storage subsystem |
| 23 | Domestic hot water subsystem |
| 24 | Quantity integrator |
| 25 | Printer |
| 26 | Plotter |
| 27 | Histogram plotter |
| 28 | Pipe and duct |
| 29 | Simulation summarizer |
| 30 | CPC collector |
| 31 | Cooling coil |
| 32 | Psychrometric data processor |
| 33 | Shading overhand and wingwalls |
| 34 | Window |
| 35 | Collector-storage wall |

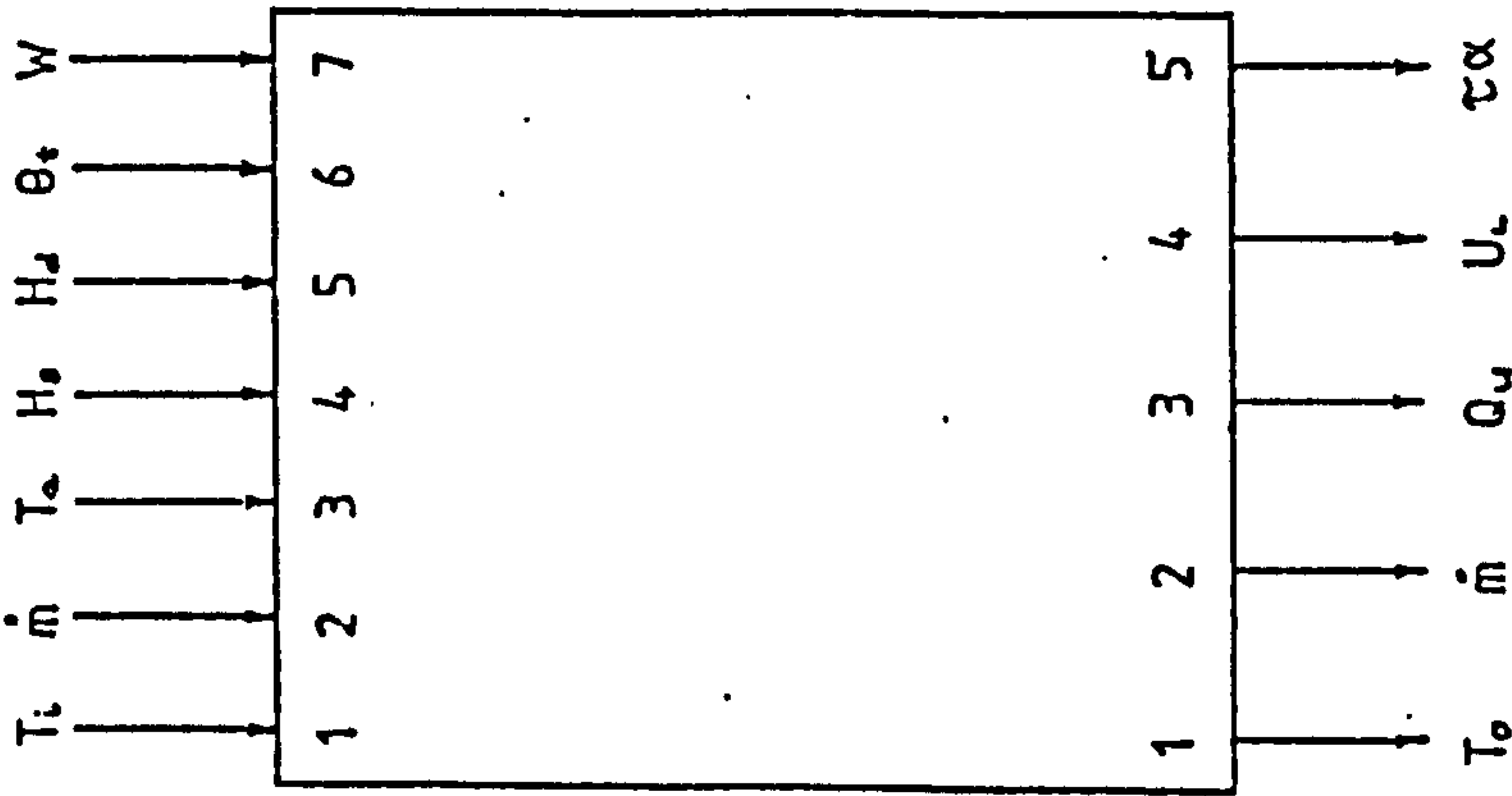
Table 2.1 Components in the library of TRNSYS solar process simulation programme

A component is represented by its pre-specified input and output parameters and static parameters. As an example, a flat plate solar collector can be represented by input/output relation as figure 2.2.1 and Table 2.2, which are based on the mathematical model of Hottel-Whiller-Bliss equation. [Ref. 26]

The complete system model is achieved by specifying the inter-relationship of the individual component. Figure 2.2.2 shows an example of a solar collector system to be modelled and figure 2.2.3 gives the signal flow diagram of a complete system out of the component flow diagrams plus a number ancillary components. [Ref. 26]

TRNSYS is a very flexible model which can analyse many different kinds of system to a reasonable degree of accuracy. This technique has distinct advantages: different modelling techniques, simple or complex, can be used for different plant components, allowing the individual component to be developed as technical knowledge improves; and the underlying modularity allows component models to be expanded and updated whilst minimising subsequent integration difficulties. Problems will arise, however, when control dynamics are to be incorporated or where component evaluation relies on information as yet uncomputed. It is also usual to experience difficulty when recirculating loops are presented and various components linking protocols are being pursued by sequential modellers. Another difficulty is the estimation of theoretical errors for a complete system, since quite different mathematical and numerical methods are used in the modelling of the components. For example, it is possible to find individually the computational error for a complex component model described by high-order differential equations and solved by iteration numerical method. It will however be difficult to find out the theoretical errors of boundary conditions which are the information sending from computations of previous component models. To overcome this, the consistency of the mathematical method for a complete system is required.

INPUTS



PARAMETERS

- 1 Mode
- 2 A
- 3 F'
- 4 Cp
- 5 α
- 6 N
- 7 εp
- 8 Ube
- 9 S
- 10 KL
- 11 n

Parameter No.

| Parameter No. | Specify | Description |
|---------------|---------|---|
| 1 | MODE | Specify A |
| 2 | A | Collector area (M ²) |
| 3 | F' | Collector efficiency factor |
| 4 | Cp | fluid thermal capacitance (kJ K ⁻¹ m ⁻²) |
| 5 | α | Collector plate absorptance |
| 6 | N | Number of glass covers |
| 7 | εp | Collector plate emittance |
| 8 | Ube | Bottom and edge loss coefficient |
| 9 | S | Collector tilt (deg.) |
| 10 | KL | Extinction coefficient x thickness of covers |
| 11 | n | Refractive index of cover |

Input No.

| Input No. | Description |
|-----------|---|
| 1 | Ti - Inlet fluid temperature |
| 2 | ṁ - collector fluid flowrate |
| 3 | Ta - Ambient temperature |
| 4 | Hb - Beam irradiation on collector surface |
| 5 | Hd - diffuse irradiation on collector surface |
| 6 | θ - angle of incidence of solar beam (deg) |
| 7 | W - Windspeed (MS ⁻¹) |

Output No.

| Output No. | Description |
|------------|--|
| 1 | To - Outlet fluid temperature |
| 2 | ṁ - collector fluid flowrate |
| 3 | Qu - rate of energy gain |
| 4 | UL - collector loss coefficient |
| 5 | τα - transmittance x absorptance product |

Figure 2.2.1 Information block diagram for a TRNSYS flat plate solar collector (for notation see Table 2.2)

Table 2.2 Data required for a TRNSYS flat plate solar collector

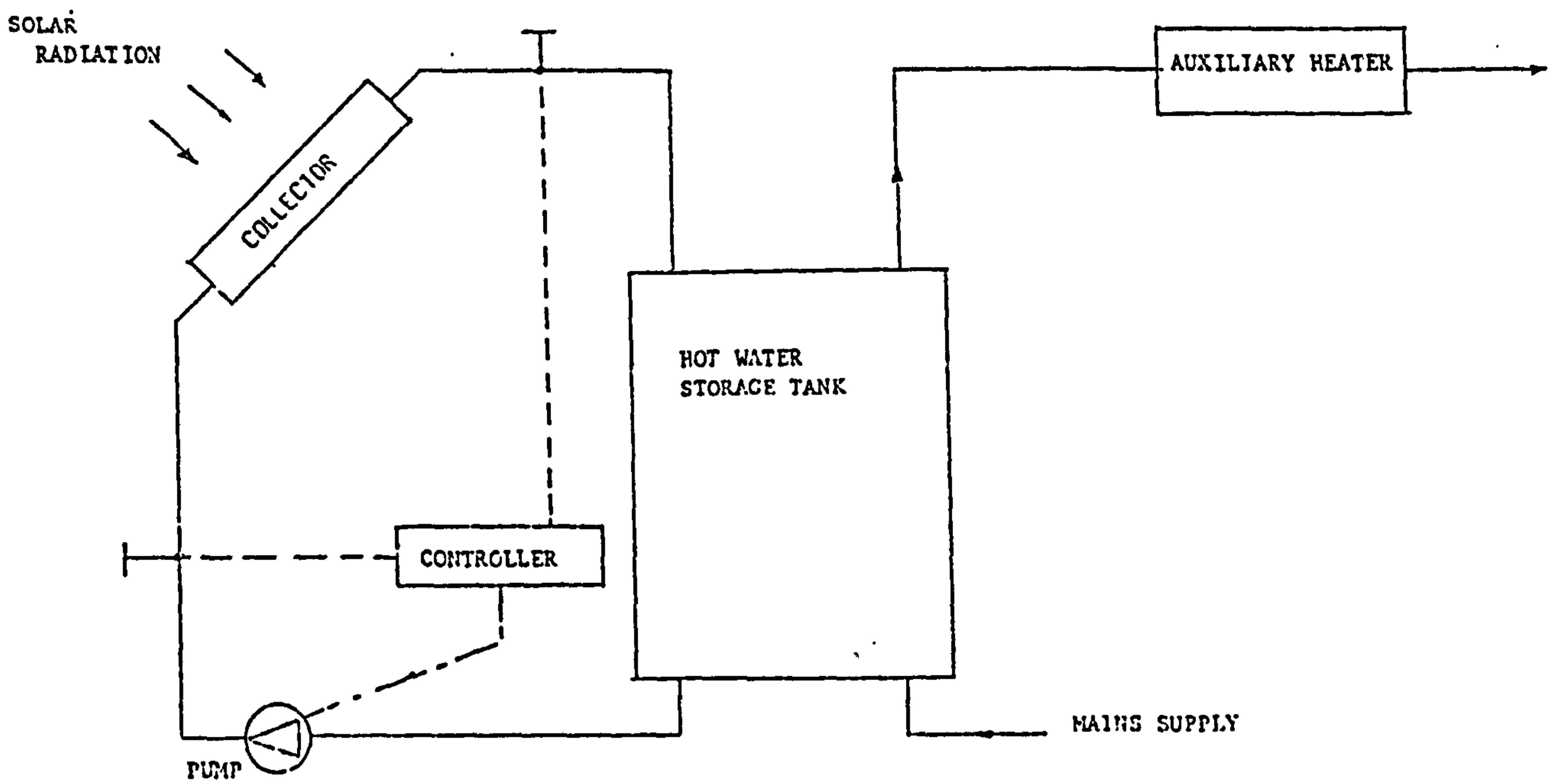


Figure 2.2.2 A solar water heating system

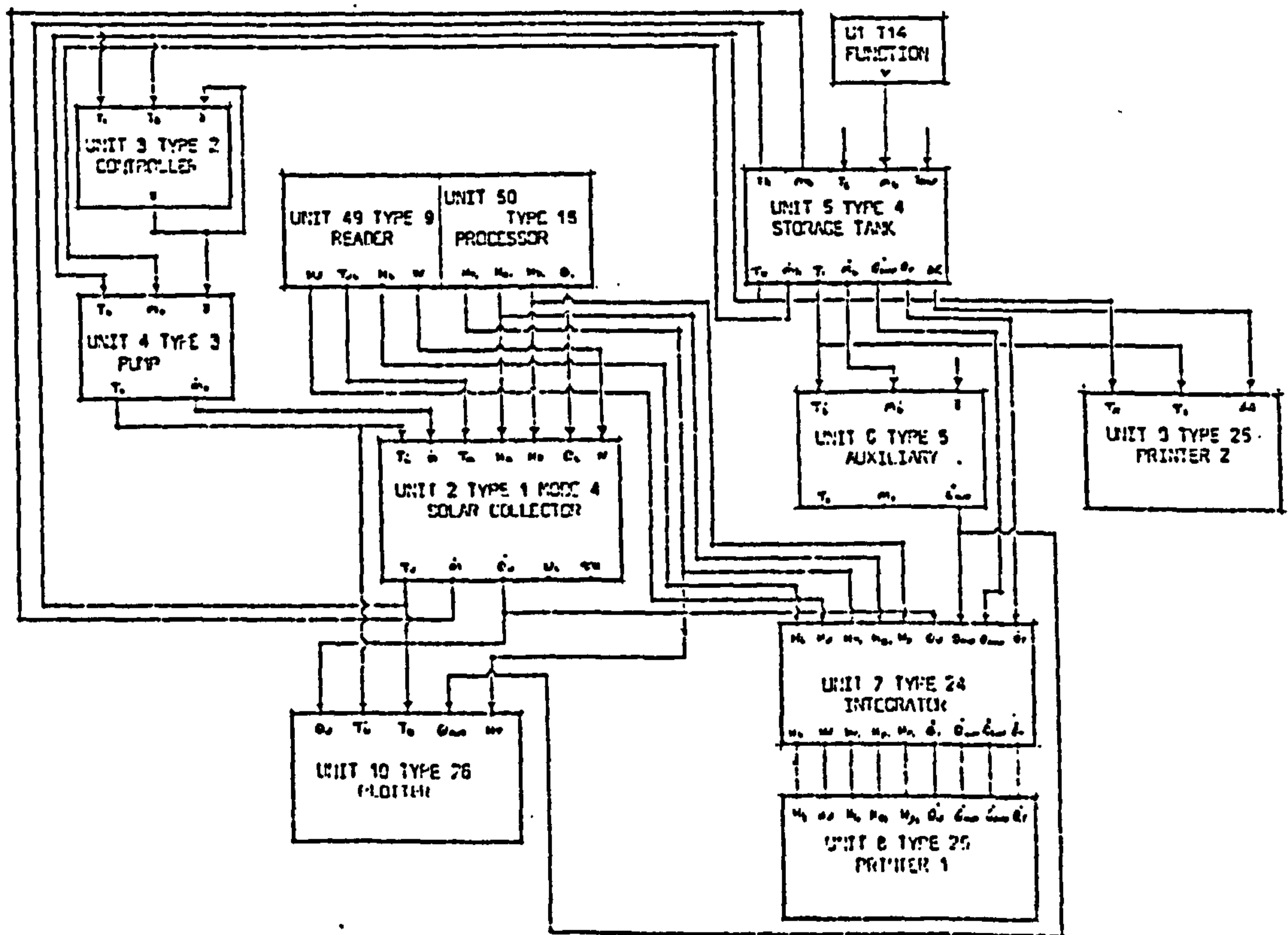


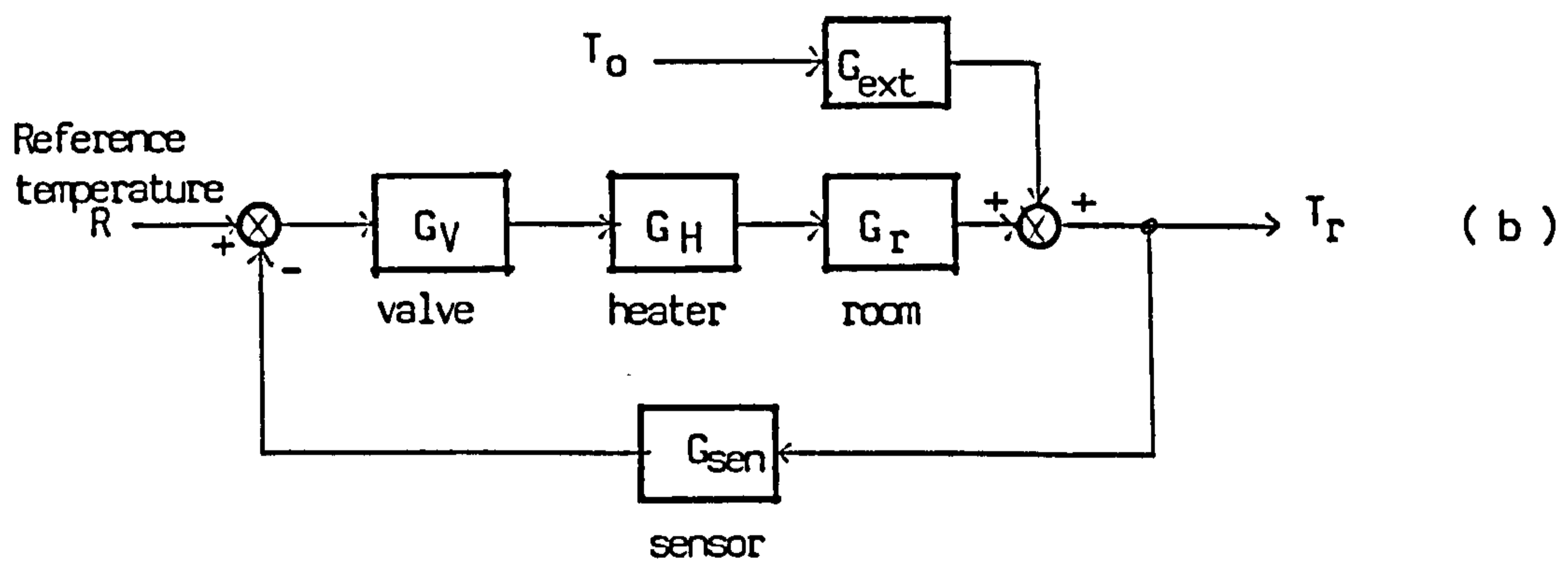
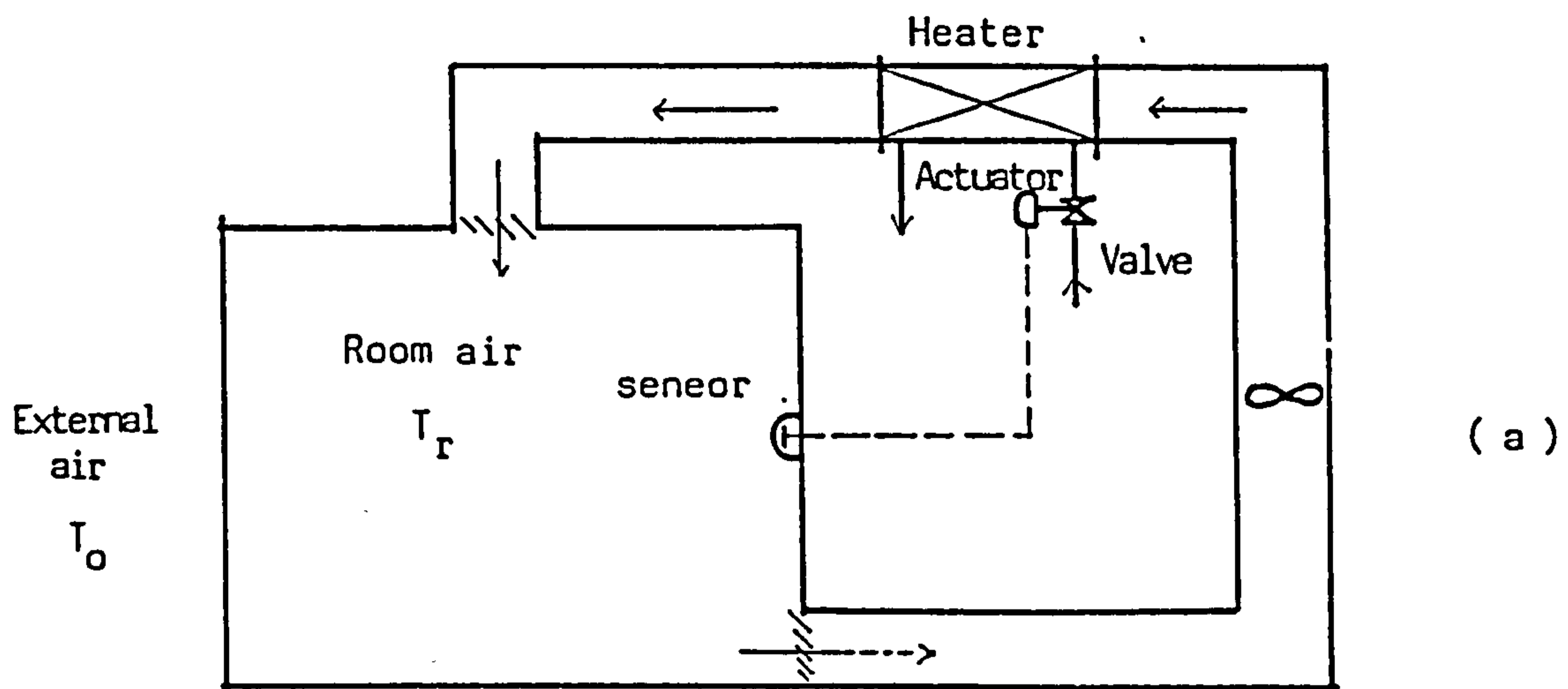
Figure 2.2.3 The block information flow diagram of the system under analysis

2.2.3 Transfer function modelling

Transfer function modelling of a plant system is a simultaneous approach. It employs the Laplace transformation to achieve the output of system response against the input excitation in frequency domain. This method originated from the automatic control theory and, in this approach, a component is usually considered as a lumped parameter, single variable system described by a linear, time-invariant differential equation. There are usually two ways to obtain the transfer function for a component, i.e. theoretically or experimentally. The former is directly derived from the component governing differential equation based on the definition of transfer function. Another way is using the concept of "black box", in which the component is firstly considered as a first-order or second-order system with unknown time constant and amplitude which will be later determined from experimental data. The transfer functions of building envelopes and living space are usually developed to cope with the simplified conditions of Laplace transformation and achieve the consistency of a complete system mathematical model. There are few steps to develop the plant model in completion if the transfer functions of components are known. Firstly, the system has to be studied and abstracted to a system signal flow block diagram in which each component in the system is represented by a functional block, and then the system response can be derived by applying the rules of cascade and parallel connections to each functional block. Figure 2.2.4 shows this process, in which

- (a) a system of room with warm air heating is studied;
- (b) the transfer functions of component are developed as well as the system signal flow block diagram; and
- (c) is the resulted expression of system transfer function.

The earliest effort of modelling plant system by component transfer functions might be referred to the works of control engineering, in which the response of a thermal system is usually taken as a typical example to illustrate the classical control theory. The main interest of their study is concentrated on analysing the quality of control loop so that the transfer functions of component are usually simplified to their simplest extent. However, this approach provides



$$T_r(s) = \frac{G_v G_H G_r}{1 + G_v G_H G_r G_{sen}} R + \frac{G_{ext}}{1 + G_v G_H G_r G_{sen}} T_o \quad (c)$$

Figure 2.2.4

a useful tool in modelling a plant system if the sophisticated transfer functions of component are developed.

The substantial works of applying this method to HVAC system modelling can be considered to have started in the sixties, when Gartner and Harrison developed the transfer functions for the most fundamental component, the tube flow, in the system. [Ref. 27] Zermuehlen and Harrison modelled a simplified room with no thermal capacity and a dual-duct system with thermostat controlled damper for temperature control. The transfer functions of component of this are simplified to first-order. Instead of simulating the air-conditioning system, the main purpose of this work was to observe the principle of control theory as applied to heating and air-conditioning systems. [Ref. 28]

Nakarishi et al modelled a simplified room and heating system in which thermal capacity of room and heat flows through walls and windows were neglected, the main purpose was also to demonstrate the application of the optimal control theory. [Ref. 29]

Hanby developed a system model combined with thermostatic radiator, valve, together with the living space. This model used a more accurate representation of the elements in the system. The transfer functions of components were represented by a first-order exponential lag combined with a transit delay. The room thermal storage effect was neglected and represented by similar expression as the components. The main purpose of this work was to show how control theory may be applied to heating systems. The stability analysis of the system was achieved by using a modified Nyquist diagram. [Ref. 30]

Mehta et al developed a rational model for thermodynamic analysis of occupied spaces. In his model, the thermal storage effects of structure were neglected for the reason of slow thermal response, the occupied space thermal model was derived based on an energy balance on the room air side, in which a uniform mixing was assumed. The resulting model was a single parameter output with multi-input parameters each connected by its own transfer function. Figure 2.2.5 shows a block diagram of such transfer function. Transfer functions of components, such as heat exchanger, fan, ducts, sensor, controller,

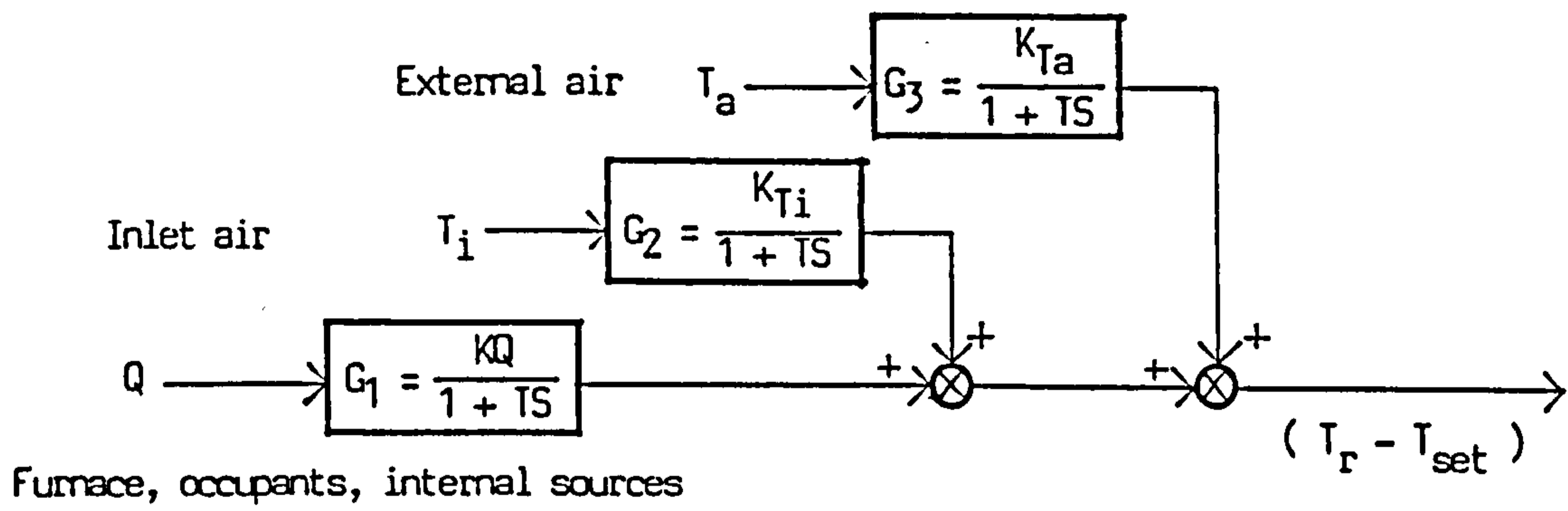


Figure 2.2.5

actuator, were developed and the modular nature of this model permitted its coupling to different types of systems to derive dynamic models for buildings. [Ref. 31]

Thompson developed a simulation model for a room with fan and coil heating system. The simulation was used to study the effect on room temperature and energy consumption and the short and long term dynamics of the heating system. [Ref. 32] Recent efforts with this method in plant modelling are leading to the purposes of energy simulation with automatic control loops and optimum operations.

This method in essence is based on the theory of classical control, by knowing the component transfer functions any system can be modelled. It also has the advantages of directly applying the methods of control theory. Difficulties will be formed when it is applied to a large system. The system block diagram then becomes much more complicated and the inverse transform of response in Laplace domain to time domain will be almost impossible without further simplifications. For this reason, the component transfer functions are usually derived as a lumped parameter, first-order systems. Limitations are also found in such Laplace domain transfer function that it is only applicable to linear, time-invariant system having single input and output parameters, it is powerless for time-varying system, nonlinear systems and multi-input, output systems. A transfer function only describes the input/output relation and no information concerning the thermal structure of system can be revealed from the description. Because of the restrictions of the classical control theory and the necessity of meeting increasingly stringent requirements on the performances of control systems, the increase in system complexity, and easy access to large-scale computer, the modern control theory of state-space analysis has been developed for the past twenty-five years. [Ref. 11]

2.2.4 Control volume heat balance method

The application of control volume heat balance method to the simultaneous plant modelling is a new development field involving the representation of plant components by discrete nodal schemes and the derivations of energy and mass flow equation-sets which represent whole system, inter-node exchanges over time and space dimensions. The plant model to emerge can then be integrated with the multi-zone building model, and the combined model can be solved simultaneously and repeatedly as it steps through its finite time increments.

As introduced previously, based on the law of energy conservation, and the arbitrary selection of control volume, the general form of heat balance equation can be equally applied to describe the instantaneous characteristics of any energy related object. In plant modelling, the general heat balance equation can be further generalised to an energy or mass balance equation. The system model can be developed on a component basis, the component can not only be modelled as lumped parameter, but also multi-parameters can be used to reach a higher level of modelling.

Based on this method, McLean developed a computer model of FLAIR for the simulation of active solar collector system. [Ref. 26] In the model, the system was broken down into component basis and a library of the component subsystem models, such as flat plate collector, heat exchanger, thermal storage units, pipes and controller, were developed to allow the user to specify any system arrangement. The general heat balance equation was modified to a general form of difference equation which allowed it to be applied to solid, fluid and multi-dimensional problems. In each component, a sub-division was made and each sub-volume was represented by a lumped parameter node, which allows some important components to be modelled to a certain level of accuracy, for example, the three dimensional heat flow was considered in a solar collector so that the sub-division was made in three directions.

A number of commonly encountered difficulties were considered and solved in this model, such as:

- the temperature dependency of fluid thermophysical properties;

- the variation of absorber plate temperature with surface position;
- the variable heat transfer coefficients depends on surface temperature, fluid flow rate, etc.
- storage medium complexities involving stratification or phase change phenomena;
- control dynamics inherent in collector tracking and energy diversion between load demand and remote storage.

The whole system model to result was N dimensional difference equation of taking together all the component sets and the system equations set was solved simultaneously and repeatedly for each time step. After the user specified the system being modelled program FLAIR searched out the relevant component models and formed the system model. The output facilities were flexible either in tabulate, interrogative and graphical.

Figure 2.2.6 shows the solar collector system being modelled. Figure 2.2.7 is the coefficient matrix of this system.

Figure 2.2.8 shows the flow diagram of programme FLAIR.

Compare with the previous methods, this simultaneous approach conducts a much more detailed analysis of inter-and-intra-connection between system and component nodes respectively. Any number of nodes may be treated as a control point allowing computer control strategies to be specified, and further more, compared to TRNSYS, this model does not rely on complex information flow diagrams and generates more data upon which design decision may be based. This approach also has the advantages of operating simultaneously with other plant systems as well as with any multi-zone building thermal model. The solution technique of a large system is critical since the dimension of matrix is too large. The conventional matrix inversion is considered time consuming and unefficient and special treatment of solution technique has been used.

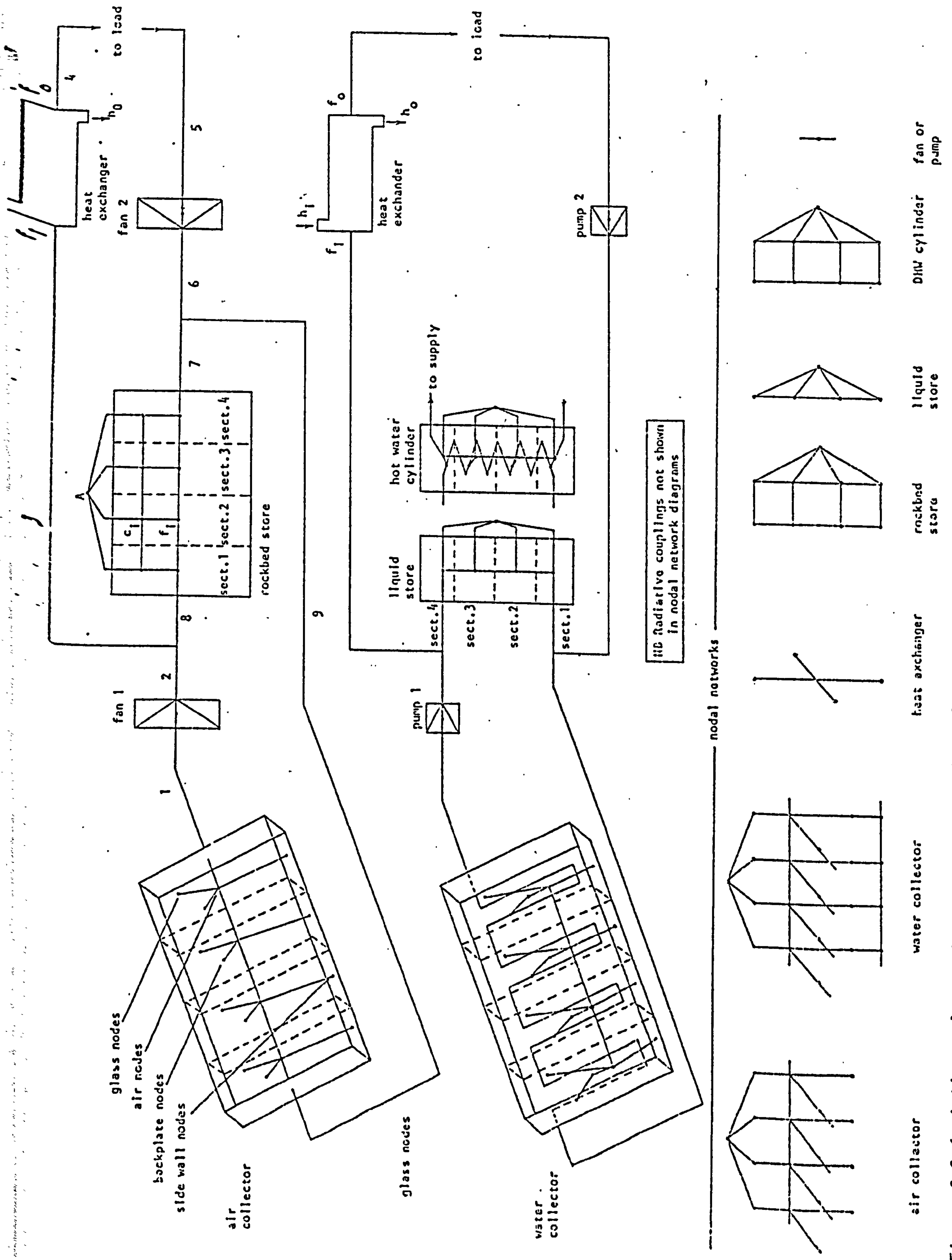


Figure 2.2.6 Active solar system components and nodal scheme

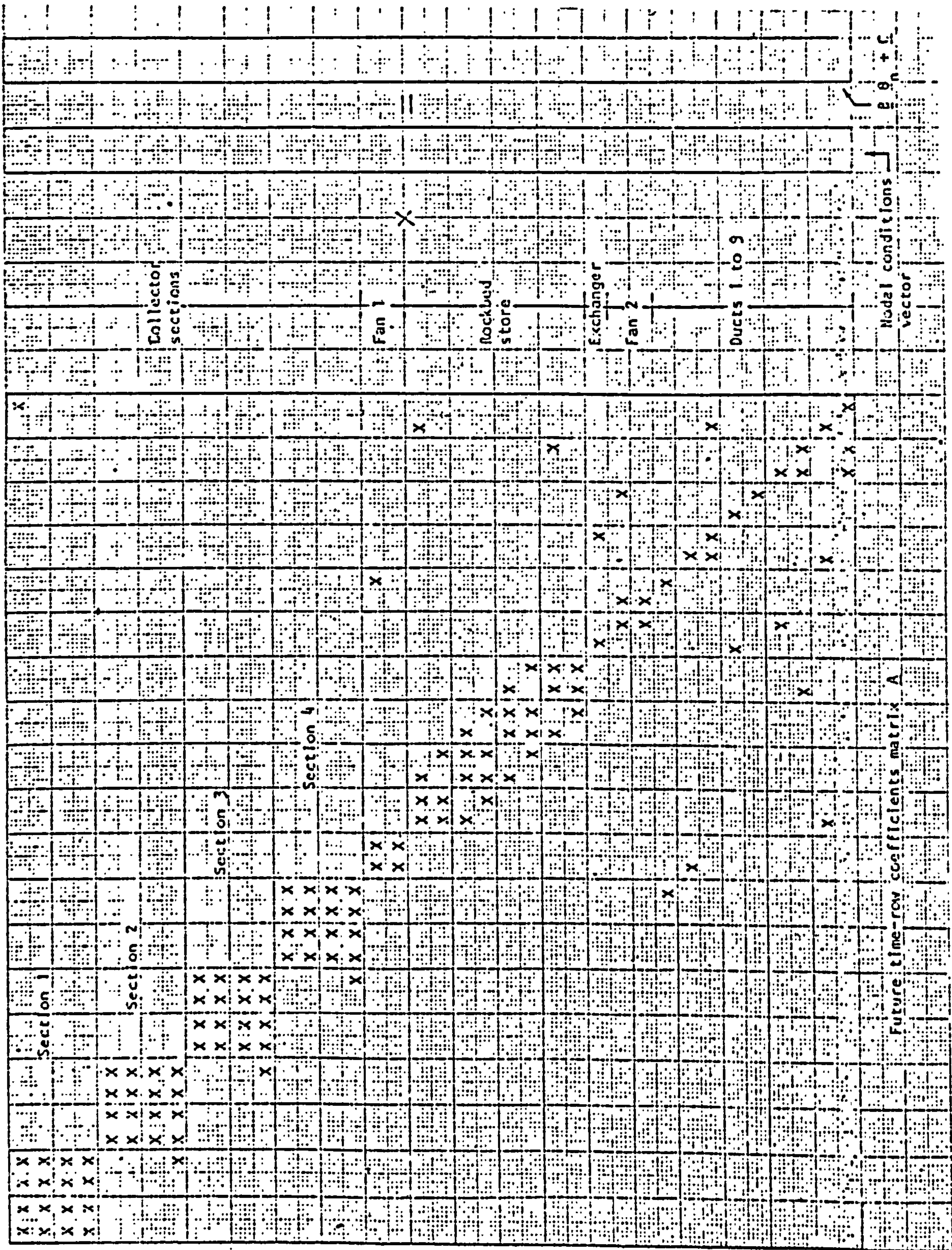


Figure 2.2.7 Active solar system energy balance matrix equation

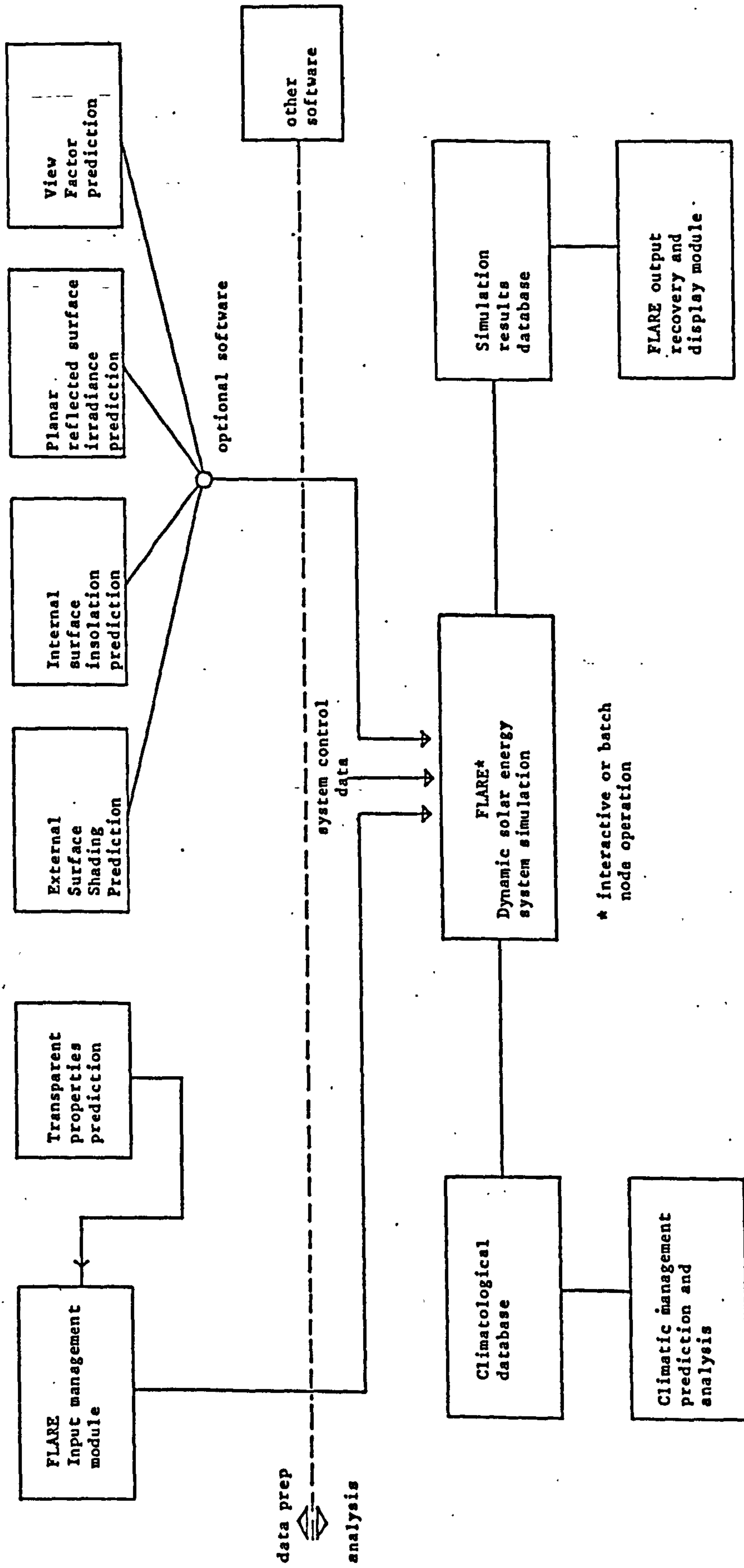


Figure 2.2.8 The FLARE system

2.2.5 Modelling technique of the present research

Based on the previous reviews and study of different modelling techniques, the method used in this research of plant modelling is found as "control volume conservation, state-space approach" which in general combines the advantages of both control volume heat balance method and state-space method.

The plant system modelling is going to be achieved by a modular approach of component basis. Each component is described by a set of state equations which are derived by the concepts of substance and energy conservations of control volume. The plant system is a combination of component models forming a complete set of state-equations of the whole system. Similar to the heat balance method and the state-space method, this technique has not only the ability to handle any complex boundary conditions, time-varying parameters, multi-dimensional problems, but also the potential of applying the methods of modern control theory leading to a higher level of modelling and development.

Plant modelling deals with most of the heat and mass transfer processes which are usually simplified to their extreme extent by other methods. Using this modelling technique, instead of handling the major aspects which can be modelled by the traditional methods, it also incorporates the considerations of:

1. The time varying heat and mass transfer coefficients, including the thermal conductivities and mass diffusivities, forced and free convective heat and mass transfer coefficients in laminar, transient and turbulent regions, radiative heat transfer coefficients. This is important in plant modelling since the plant operation requires the flow rates of working substances, water and air, constantly be changed so as to satisfy the required conditions. These changes in fluid flow directly change the convective coefficients to an extreme extent from zero, if the valve is closed and to a certain maximum value, if the valve is fully opened. Other coefficients can be influenced indirectly.

All of the time-varying coefficients are represented by relations of empirical expressions.

2. Convective and radiative heat transfer coefficients are treated separately. This is because in plant modelling, the change rates between these two coefficients are quite different, and it also provides the possibility of modelling the radiative heat exchanges of a component to its surroundings such as a radiator to the surfaces of a room.
3. Combustion process. This process is modelled by applying the laws of chemical reactions so as to find out the necessary parameters such as the composition of combustion products, supply air for combustion etc.,
4. Phase change heat and mass transfer, which play important parts in plant modelling such as in boiler, refrigeration circle and wet heat transfer in cooler etc.,
5. The control devices and function of control loops. Each control device is modelled as a single component so that the responses of control loops and their stabilities can be modelled simultaneously during the system simulation.
6. The potential for combination not only with simple building models but also with building models developed in similar technique and complexities. The simple building model means that the thermal response of the building can be constant for certain periods, or simple function of energy it receives. Combination of plant model with a full developed building model in similar modelling technique gives a consistency of mathematical concept. This can be achieved by uniquely locating the plant model into the matrix structure of a multi-zone building model connected by the cross-coupling coefficients. Simultaneous solution of the combined building and plant model gives the overall system time-series solution, defining the building and plant components status throughout the simulation period.

Generally speaking, the modelling parameters of a HVAC system are not unique, the basic parameters of moisture air are linked with each other by equations of idea gas. Parameters of temperature and humidity are defined as the modelling parameters in this research. The general thermal and mass conservation equations can be derived based on the first law of thermodynamics and the law of substance conservation. Consider an isolated element of finite volume with boundary Ω . The thermal and mass states are represented by E and W respectively. The laws of energy and mass conservation certify the relations of:

$$\left| \begin{array}{l} \text{Change rate} \\ \text{of Energy} \\ \text{in region } \Omega \end{array} \right| = \sum \left| \begin{array}{l} \text{Net energy exchange} \\ \text{in and out of} \\ \text{boundary } \Omega \end{array} \right| + \left| \begin{array}{l} \text{Net energy} \\ \text{generated in} \\ \text{region } \Omega \end{array} \right|$$

and,

$$\left| \begin{array}{l} \text{Change rate} \\ \text{of mass in} \\ \text{region } \Omega \end{array} \right| = \sum \left| \begin{array}{l} \text{Net mass exchange} \\ \text{in and out of} \\ \text{boundary } \Omega \end{array} \right| + \left| \begin{array}{l} \text{Net mass} \\ \text{generated in} \\ \text{region } \Omega \end{array} \right|$$

These two relations can be further written in form of first-order differential equations in which time is the independent variable, as:

$$\rho_I c v_I \frac{dT(I,t)}{dt} = \sum_{i=1}^N K_{i,I}(t)[T(i,t) - T(I,t)] + q(I,t) \quad 2.2.1$$

$$\rho_I v_I \frac{dW(I,t)}{dt} = \sum_{i=1}^N H_{i,I}(t)[W(i,t) - W(I,t)] + g(I,t) \quad 2.2.2$$

where,

$\frac{dT}{dt}; \frac{dW}{dt}$ = temperature and humidity change rates of control volume I

$\rho c v$ = density, specific heat, volume of control volume I

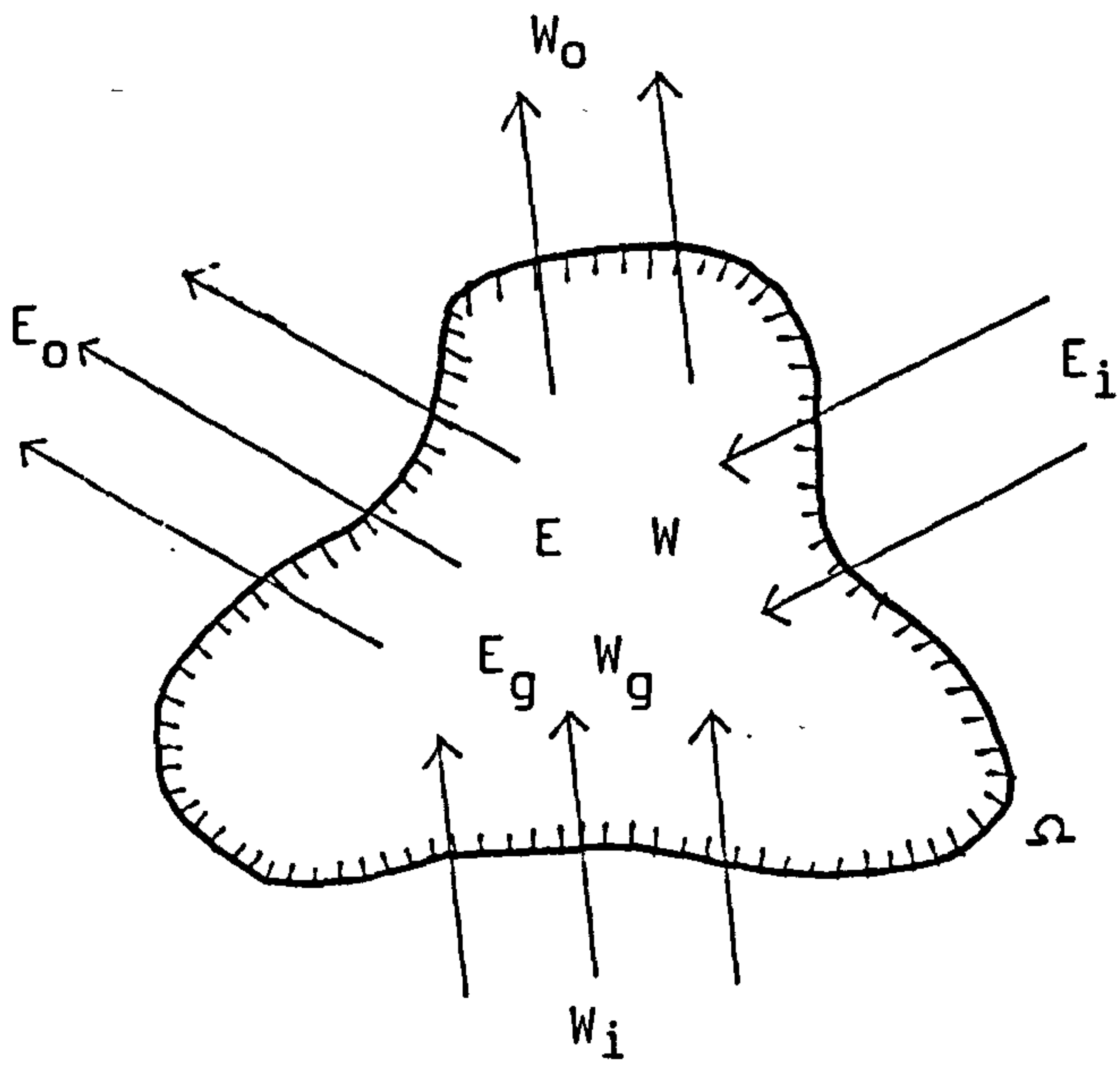
$T(I,t); W(I,t)$ = temperature, humidity of control volume I

$T(i,t); W(i,t)$ = temperature, humidity of control volume i

$K(t); H(t)$ = heat, mass transfer coefficients between control volume i and I

$q(I,t); g(i,t)$ = heat, mass generation of control volume I

In this modelling, any component in the plant system can be divided



- E_i = energy in
- E_o = energy out
- E_g = energy generated
- W_i = mass in
- W_o = mass out
- W_g = mass generated

into imaginary isolated control volumes having contact with each other and applying equation 2.2.1, 2.2.2 to each of these control volume. For dry processes, such as in a heating system, only the energy conservation differential equation is necessary, while in wet processes, such as an air-conditioning system, both energy and mass conservation differential equations are required. The application of energy and mass conservation differential equations to the control volumes of a component in plant system results in the state-equation of the component.

The mathematical basis of the state equation, the solution of state equation by implicit enumeration, the consistency, convergency and stability of the numerical method as well as the basic rules of determining the step size of spatial and temporal variables are going to be studied in the coming chapter.

Comment 1:

Unit triangle pulse is a generalised function which takes the form of

$$T(t) = \begin{cases} 0 & t \leq -\Delta \\ \frac{1}{\Delta}t + 1 & -\Delta < t \leq 0 \\ -\frac{1}{\Delta}t + 1 & 0 < t \leq \Delta \\ 0 & t \geq \Delta \end{cases}$$

Time series, $f(t) = \sum_{i=1}^{\infty} f(i\Delta) T(t-i\Delta)$, is considered as always identically convergent.

Comment 2:

Here, the Laplace transform of unit triangle function is used, as:

$$L [T(t - i\Delta)] = \frac{e^{-i\Delta s} e^{\Delta s}}{\Delta s^2} (1 - e^{-\Delta s})^2$$

Comment 3:

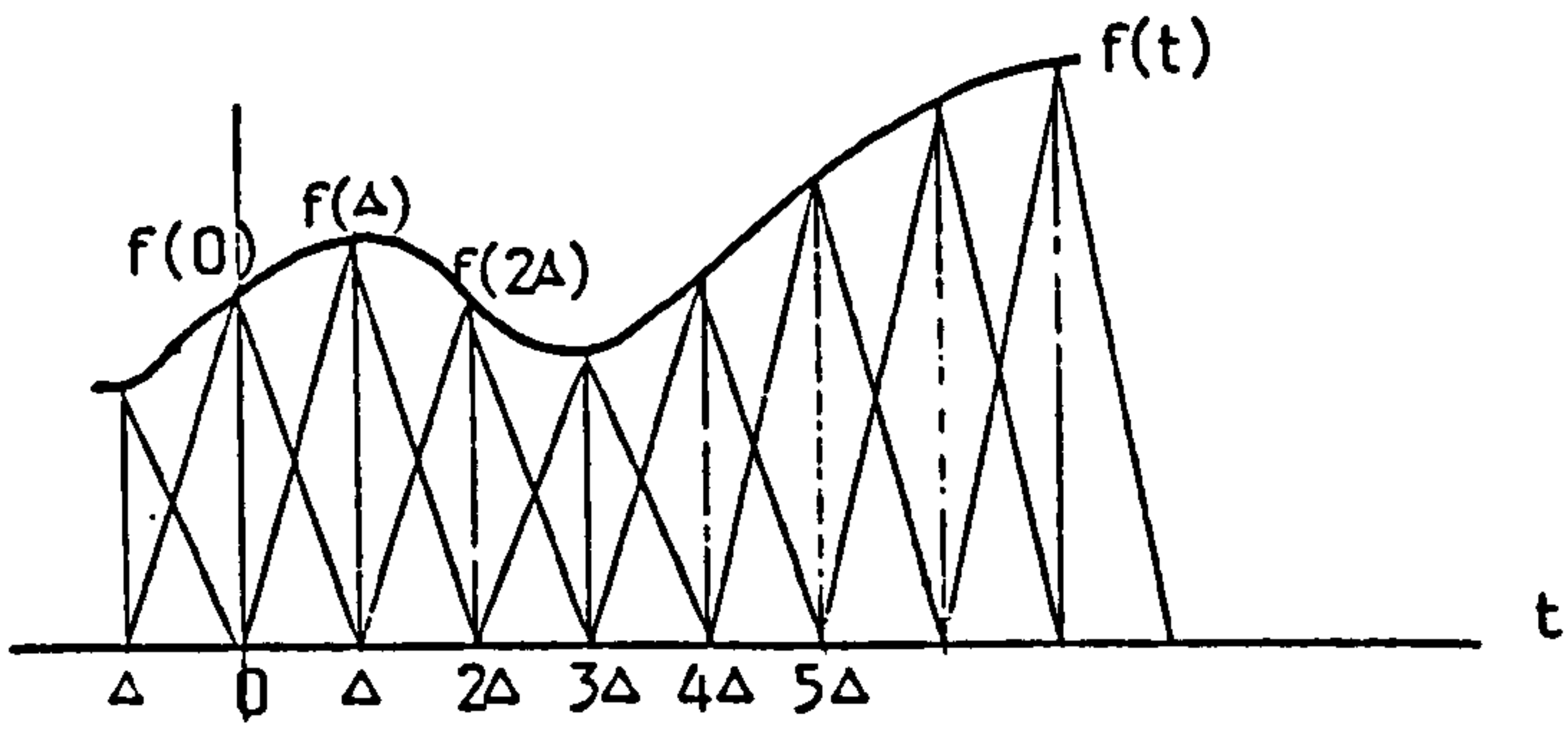
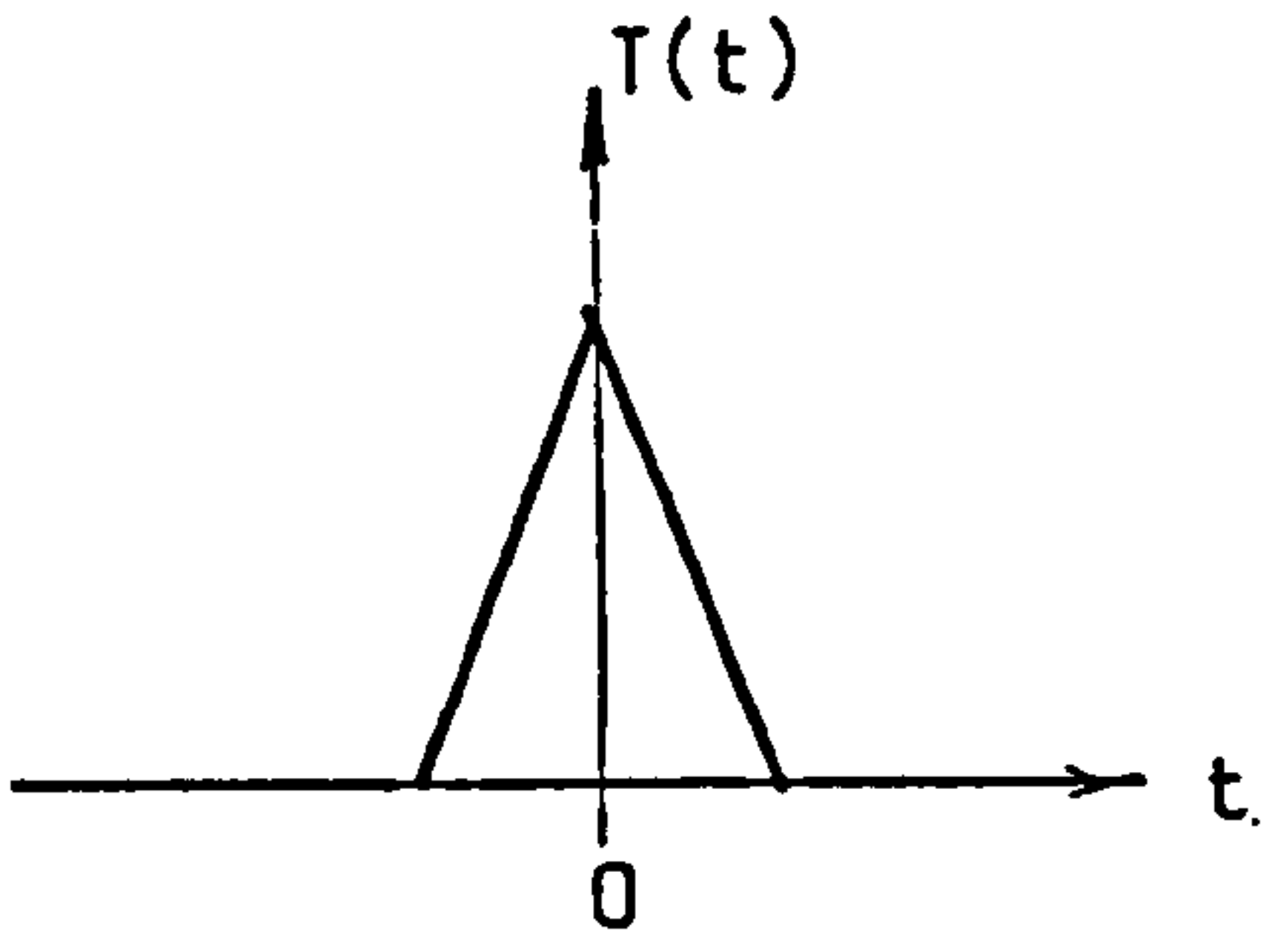
Here, we use this relation of,

$$\begin{aligned} L^{-1} \left\{ G(s) \frac{e^{-i\Delta s} e^{\Delta s}}{\Delta s^2} (1 - e^{-\Delta s})^2 \right\}_{t=k\Delta} &= L^{-1} \left\{ G(s) L [T(t-i\Delta)] \right\}_{t=k\Delta} \\ &= L^{-1} \left\{ G(s) L [T(\tau)] \right\}_{\tau=(k-i)\Delta} = Y(k-i)\Delta \quad (t-i\Delta = \tau) \end{aligned}$$

Comment 4:

By definition, the Z transform of a discrete time sequence $\{u(k), k = 0, 1, \dots\}$ is defined as:

$$Z [x(k)] = X(z) = \sum_{k=0}^{\infty} x(k) z^{-k}$$



where, z is a complex variable.

Z transform of a continuous function, $x(t)$, equals to the same transform applying to a discrete time sequence having the same sampling values as the continuous function, which

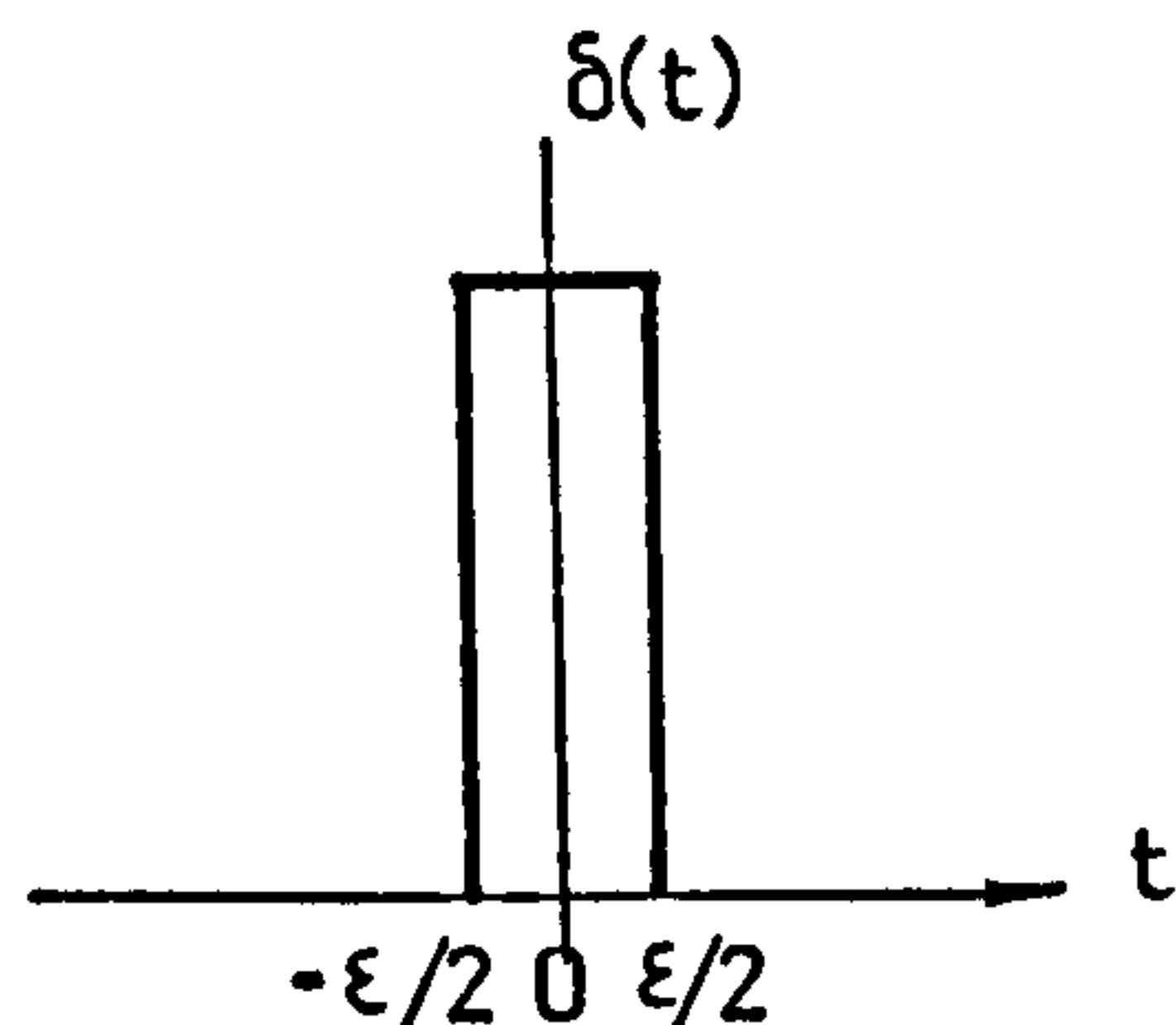
$$Z [x(t)] = X(z) = \sum_{k=0}^{\infty} x(k)z^{-k}$$

Inverse Z transform of $X(z)$ only gives the time discrete values of continuous function $x(t)$.

Comment 5:

Generalised function $\delta(t)$ is defined as a function of time characterised by:

$$\delta(t) = \begin{cases} 0 & t < -\frac{\epsilon}{2} \\ \frac{1}{\epsilon} & -\frac{\epsilon}{2} < t < \frac{\epsilon}{2} \\ 0 & t > \frac{\epsilon}{2} \end{cases}$$



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CHAPTER THREE: THEORETICAL BASIS OF CONTROL VOLUME CONSERVATION STATE
SPACE APPROACH

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CHAPTER THREE: THEORETICAL BASIS OF CONTROL VOLUME CONSERVATION STATE
SPACE APPROACH

3.1 State space representation of system

As developed in the previous chapter, the general control volume conservation equations of energy and mass are:

$$\rho_I(t)c_I(t)v_I \frac{dT_I}{dt} = \sum_{i=1}^m K_{I,i}(t)[T_i(t) - T_I(t)] + q_I(t) \quad 3.1$$

$$\rho_I(t)v_I \frac{dW_I}{dt} = \sum_{i=1}^{m1} H_{I,i}(t)[W_i(t) - W_I(t)] + g_I(t) \quad 3.2$$

These two equations are derived based on the concept of control volume and the laws of energy and substance conservations. Applying equation 3.1 and 3.2 to the pre-specified control volumes within the component of HVAC systems, the thermal state of the component can be properly modelled, the complete system state equation can be formed by the sequential combination of the components models. The differences of the component state equations are totally depended on their physical structures and thermal natures. According to the basis of derivation, the states of temperature and humidity are uniformly and homogeneously distributed through out the control volume so that a point in the control volume is chosen as a nodal temperature and nodal humidity to represent the states of the whole control volume. For thermal modelling, pressure and velocity distributions in the system are not modelled and considered as known facts. Energy conservation equation is applied to every component in the system and the mass conservation equation is only applied to those components in which mass transfer take place. As a fact, those component with no mass transfer, the humidity concentration within the component remains constant and its change rate dw/dt remains zero.

To develop the component state equation, any component in the system can be generally considered as a combination of control volumes of solid materials, static or flowing fluid, also these control volumes

can be made as lumped represented by one node or further divided into multi-dimensions represented by nodes in multi-dimensions to achieve much details and accuracy.

Instead of directly using equation 3.1 and 3.2, more specific forms of the control volume conservation equation for certain elements encountered in HVAC plant system will be derived as follows which will be easier to be applied to the component modelling.

3.1.1 The basic control volume conservation equations

The primary derivation will be done on the general elements of control volumes of homogeneous and composite solid materials, static and flowing fluids. The surface heat and mass transfer coefficients encountered in this chapter are only conceptual factors, detailed values and expressions will be discussed in the later chapters. The basic control volume conservation equations are still in their general form and can be equally applied to any component. However, variations sometimes have to be made to account for the special physical structure and thermal natures of a component. The discretization, subdivision of the component into control volumes, is another problem which has to be carefully taken into account, criteria of such discretization will depend on the expected modelling requirements and thermal physical properties of the component as well as the complete system.

I For solid material with one node

Figure 3.1 shows a control volume of solid material in an arbitrary shape, considered as homogeneous, with density ρ , specific heat c , volume v and external surface area A , external heat transfer coefficient K . In general, non-homogeneous element also can be considered, in such case, the equivalent homogeneous thermal physical properties have to be constructed by the volumetric weighting of different materials comprising the control volume.

Applying energy conservation to this control volume results:

$$\rho c v \frac{dT}{dt} = - KAT + KAT_a + q \quad 3.3$$

Equation 3.3 is a first order linear differential equation and if ρ , c , K are variables of temperature, it is considered as time varying. This one node representation is obviously an idealized case since a temperature gradient must exist in the material if heat is to be conducted in and out of the material. In general, the smaller the size of the body and higher the thermal diffusivity, the more realistic result may be expected. Such application is usually considered as a lumped-parameter analysis, reasonable estimates also

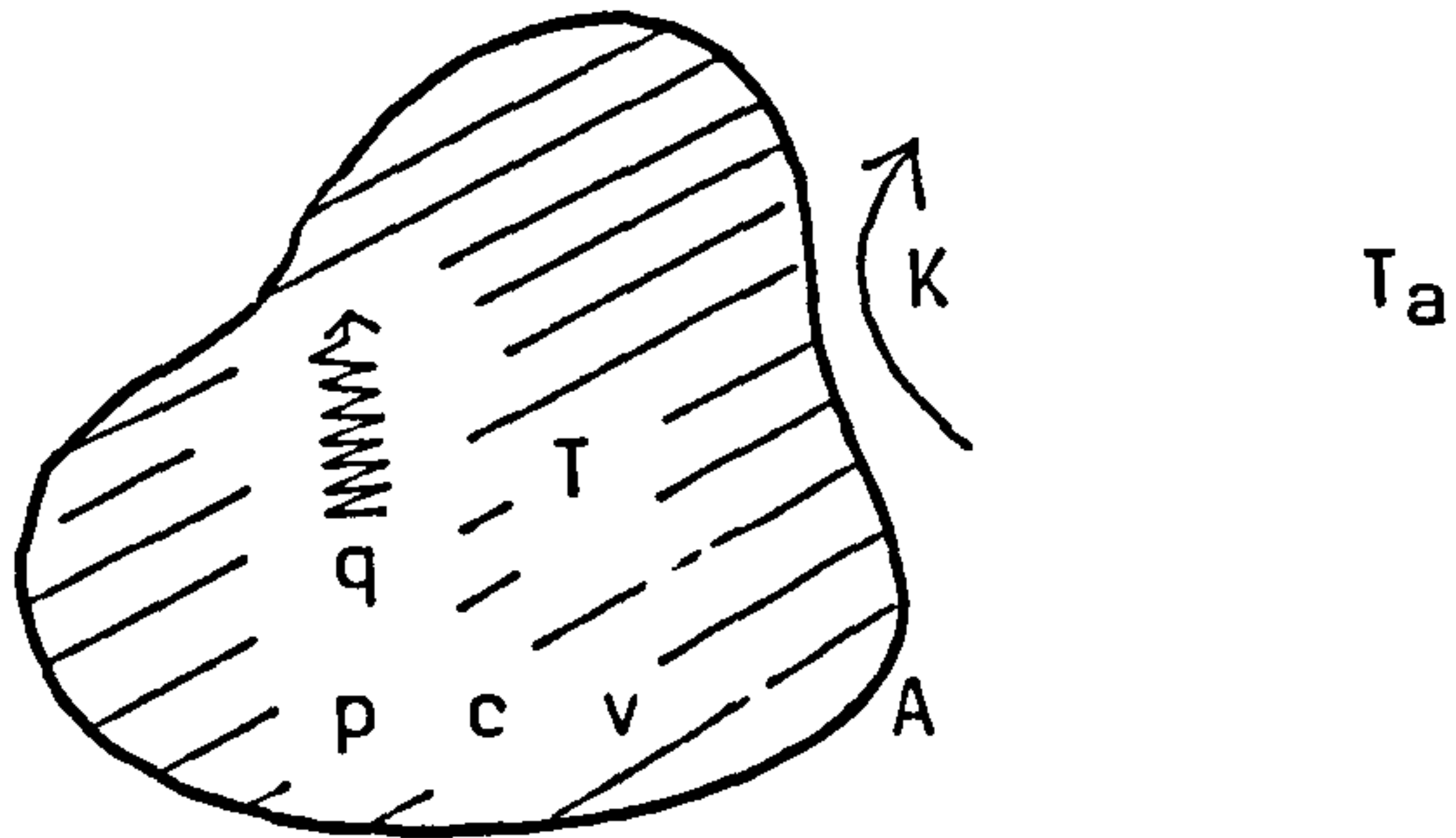


Figure 3.1 Control volume of solid

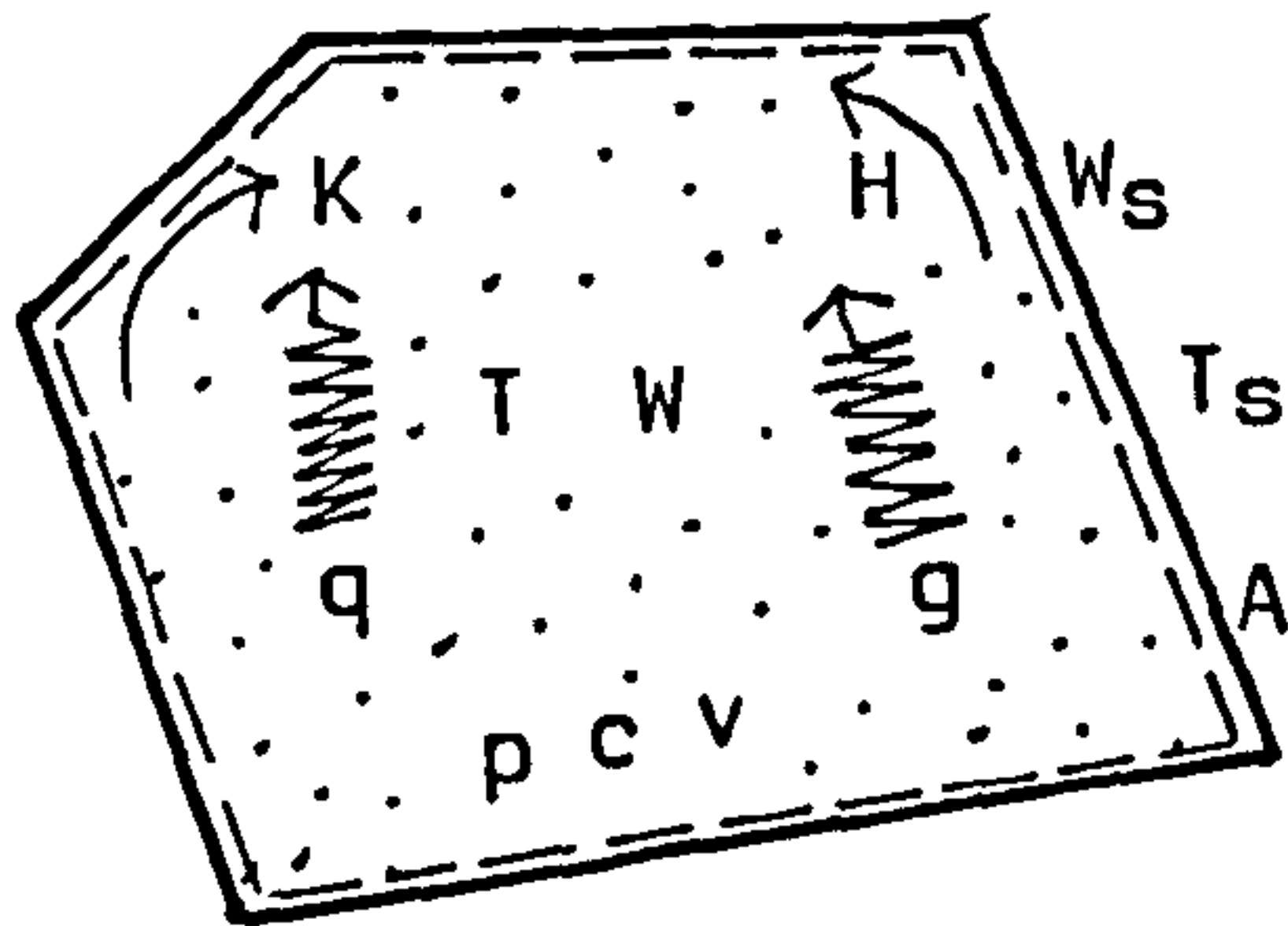


Figure 3.2 Control volume of static fluid

may be expected when the surface convective resistance is larger compare with the internal-conduction resistance of solid. Equation 3.3 usually reasonable to be applied to the metal structures of components in plant system modelling. Fairly good examples of application can be found in modelling the thermometers of mercury and thermocouples.

Equation 3.3 can also be applied to a volume of fluid bounded by surfaces confined in an enclosed space. The assumption made to this application are: the confined fluid is considered homogeneous, well mixed in the control volume, no chemical reaction, uniform pressure distribution, kinetic energy caused by thermal effect of natural convection negligible and work done by surface stress of viscosity is not considered.

As general case, humid air with temperature T and humidity W is enclosed in control volume v with enclosure A , heat and moisture generations are q and g , surface heat and mass transfer coefficients K and H , the average temperature of enclosure T_s , and W_s is considered as concentration of humidity at enclosure A corresponding to the saturation condition of T_s when it is lower than the dew point of enclosed air. The energy and mass conservation applied to this control volume result the state equations which are:

$$\rho c v \frac{dT}{dt} = - KAT + KAT_s + q \quad 3.4$$

$$\rho v \frac{dW}{dt} = - HAW + HAW_s + g \quad 3.5$$

As stated in the assumptions, the fluid movement in the control volume is ignored, however, the effects of heat and mass exchanges caused by the internal fluid disturbances can be properly described by finding good values or formulae of the surface heat and mass transfer coefficients.

II Solid material with multi-nodes

For solid material having lower thermal diffusivity and relatively large volume, a subdivision of the physical body has to be made. This is required by the accuracy of modelling since the internal heat conduction becomes dominant, and one node representation will not be accurate enough. The subdivision is confined by the criteria depending on the thermal physical properties of the body and the modelling error allowed will be discussed in the later sections. The modelling of building envelopes and thermal insulations can be considered as typical examples of this application.

Figure 3.3 shows one dimensional heat flow through a thick slab which is divided into control volumes of thin layers represented by nodes. In general, it can be considered as a multi-layer structure and each layer is a homogeneous element. The subdivision will be made to each homogeneous element to be further divided into three sections as control volume represented by nodes, one node being placed at each boundary between two different homogeneous elements and one at the central plane. This homogeneous element and three nodes representation have been proved to be good accuracy and efficient computation.

(A) For node at centre of homogeneous element

The control volume I is made as in Figure 3.4 in which x_I is the semi-thickness of the element and the thickness of control volume I, $K_I \rho_I c_I$ are conductivity, density and specific heat of this element, T_I the temperature of control volume I and T_{I-1}, T_{I+1} are temperatures of left and right adjacent control volumes. Energy conservation applied to this control volume results:

$$\rho_I c_I \Delta x_I \frac{dT_I}{dt} = \frac{k_I}{\Delta x_I} T_{I-1} - \frac{2k_I}{\Delta x_I} T_I + \frac{k_I}{\Delta x_I} T_{I+1} \quad 3.6$$

(B) For nodes at boundary between two different homogeneous elements

Figure 3.5 shows control volume I in which element 1 and 2 are homogeneous, node is placed at the boundary between two elements and the control volume I represents mixed thermal properties region.

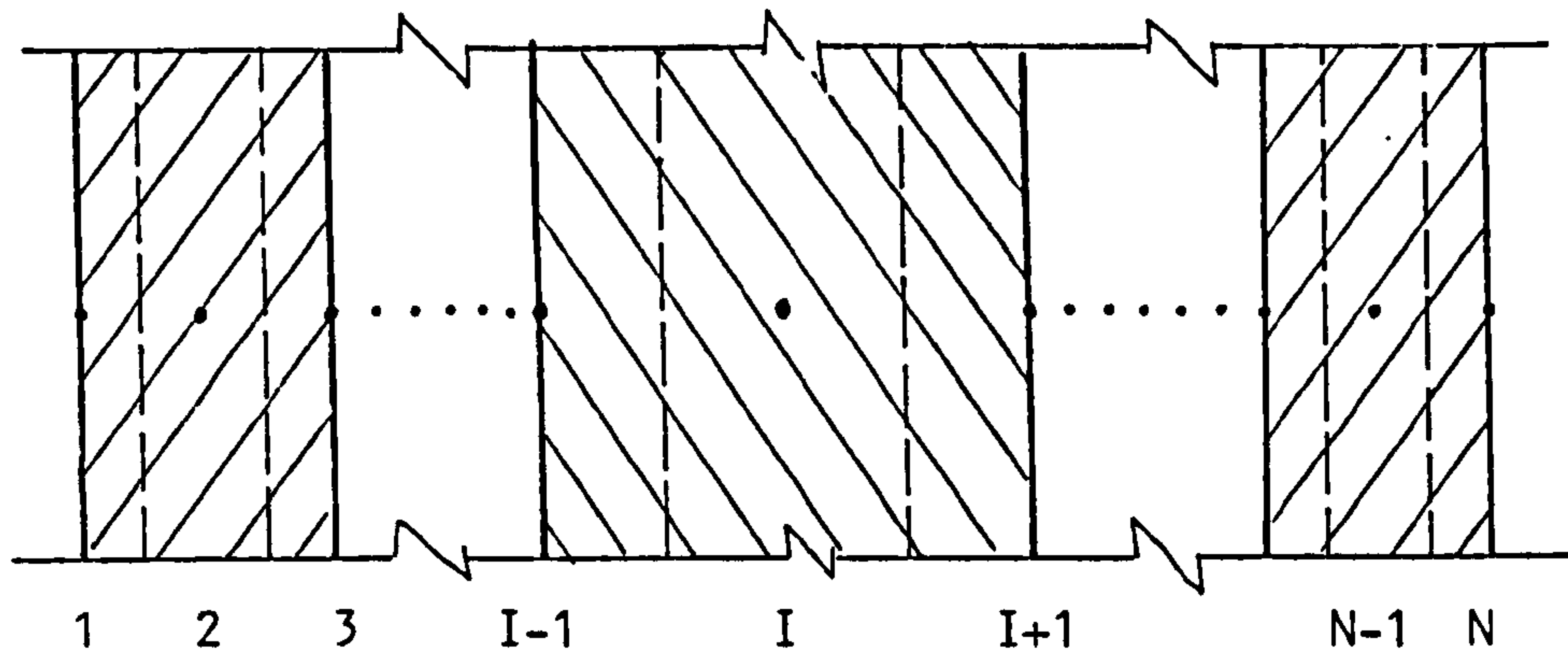


Figure 3.3 Control volume of multi-layered wall

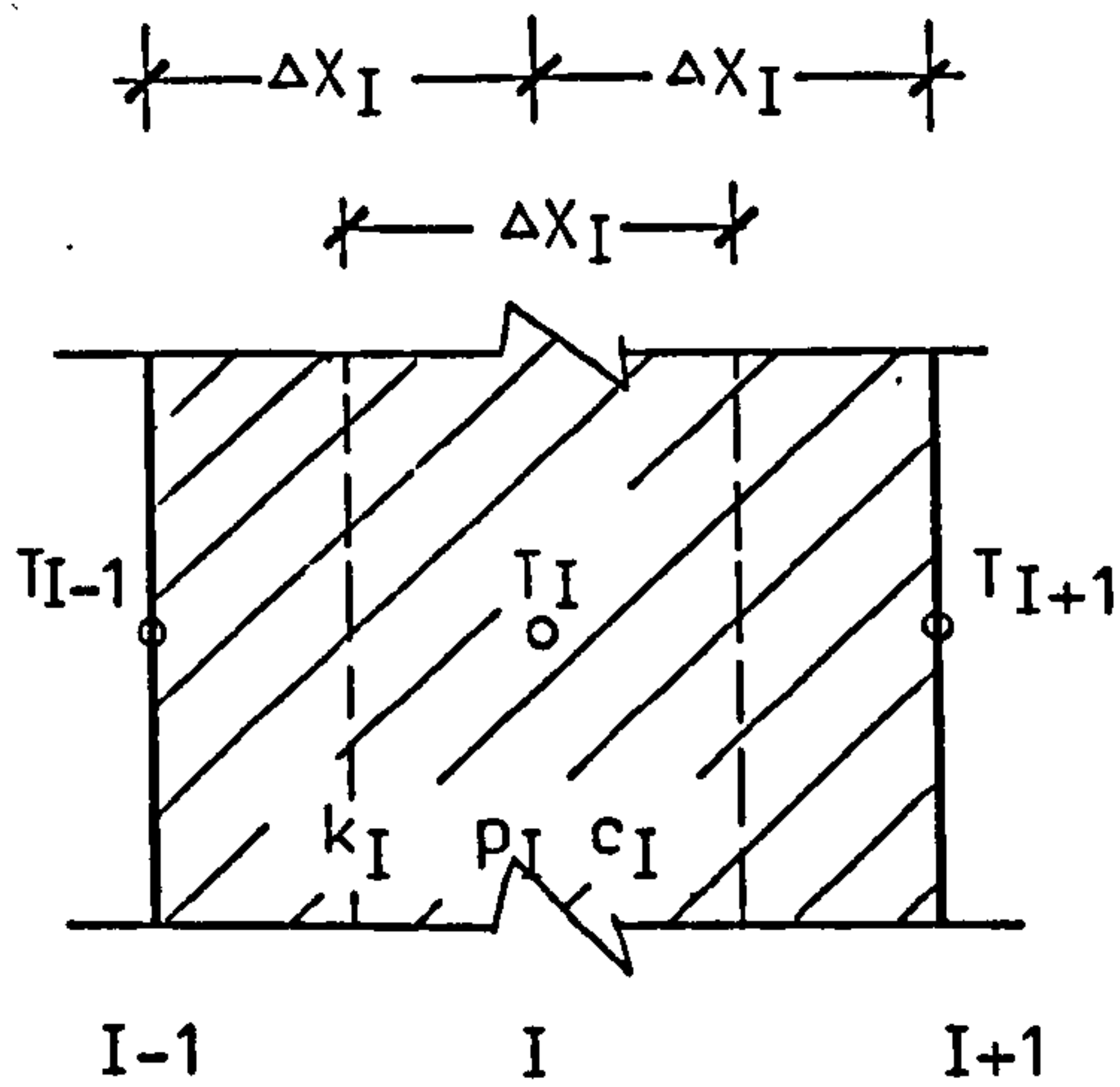


Figure 3.4 Control volume of a homogeneous element

Conductivities, specific heat, and densities of element 1 and 2 are k_1 , $c_1\rho_1$ and $k_2, c_2\rho_2$ respectively. Applying energy conservation to this control volume results:

$$\frac{(\rho_1 c_1 \Delta x_1)}{2} + \frac{(\rho_2 c_2 \Delta x_2)}{2} \frac{dT_I}{dt} = \frac{k_1}{\Delta x_1} T_{I-1} - \left(\frac{k_1}{\Delta x_1} + \frac{k_2}{\Delta x_2} \right) T_I + \frac{k_2}{\Delta x_2} T_{I+1} \quad 3.7$$

(C) For node at extreme surfaces

Figure 3.6 shows control volume I, node is situated at extreme surface contacted to its surrounds, control volume only takes half of the semi-thickness of the element, K_{in} and K_{out} are internal and external surface heat transfer coefficients, q_{in} and q_{out} are internal and external heat fluxes and T_{in} , T_{out} are internal and external air temperatures.

Energy conservation applied to internal extreme control volume results:

$$\left(\frac{\rho c \Delta x}{2} \right) \frac{dT_I}{dt} = K_{in} T_{in} - \left(K_{in} + \frac{k}{\Delta x} \right) T_I + \frac{k}{\Delta x} T_{I+1} + q_{in} \quad 3.8$$

and for external extreme control volume is:

$$\left(\frac{\rho c \Delta x}{2} \right) \frac{dT_I}{dt} = \frac{k}{\Delta x} T_{I-1} - \left(\frac{k}{\Delta x} + K_{out} \right) T_I + K_{out} T_{out} + q_{out} \quad 3.9$$

Bring together all the nodal energy conservation equations in form of equation 3.6 to 3.9, the state equation of a multi-layer structure in one dimensional heat flow can be formed as:

$$C \frac{dT}{dt} = AT + BU \quad 3.10$$

where,

$T = [T_1, T_2, \dots, T_n]^T$ is a n-dimensional vector, each variable represents the nodal temperature in the multi-layer structure and T is the state vector of it.

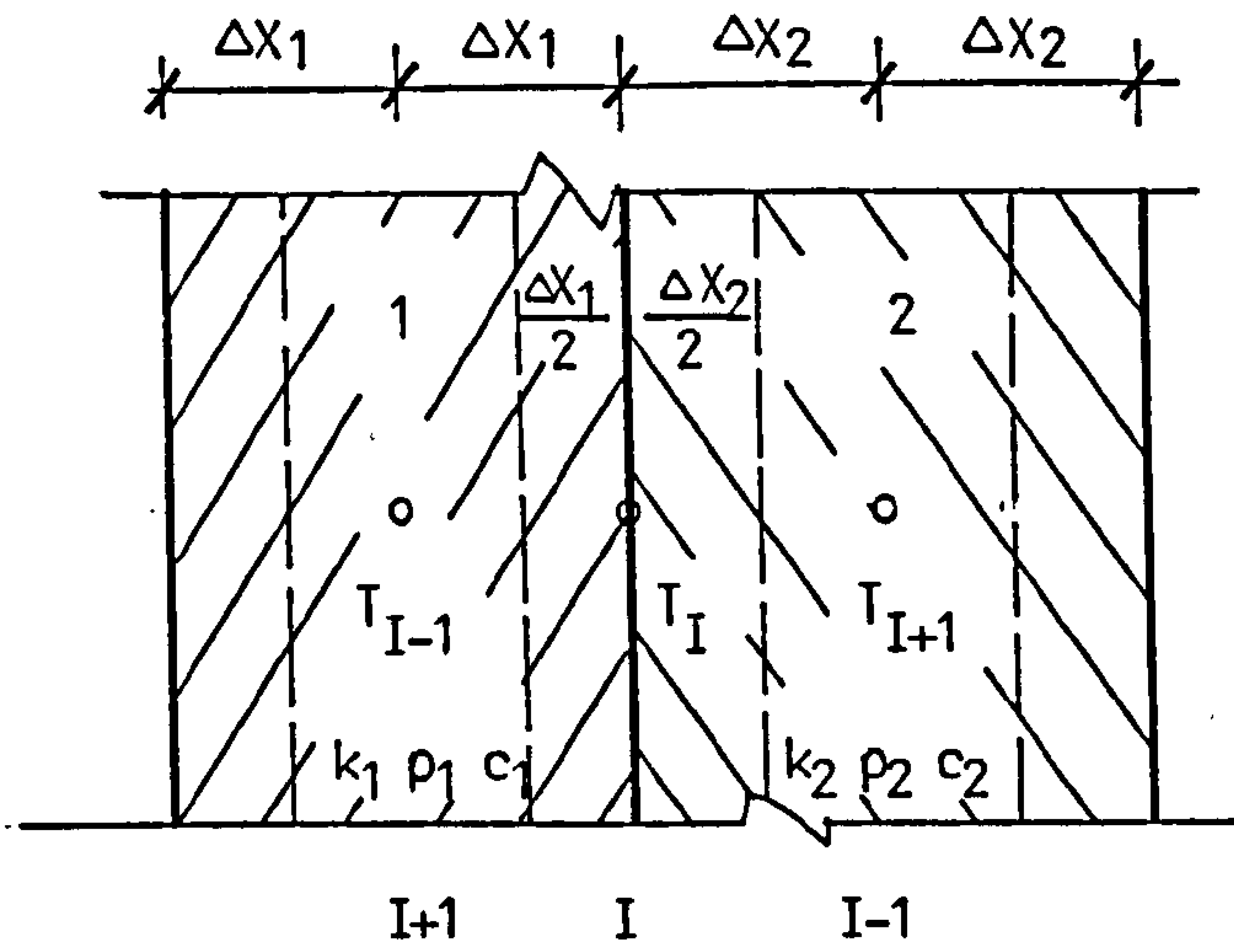


Figure 3.5 Control volume between two homogeneous elements

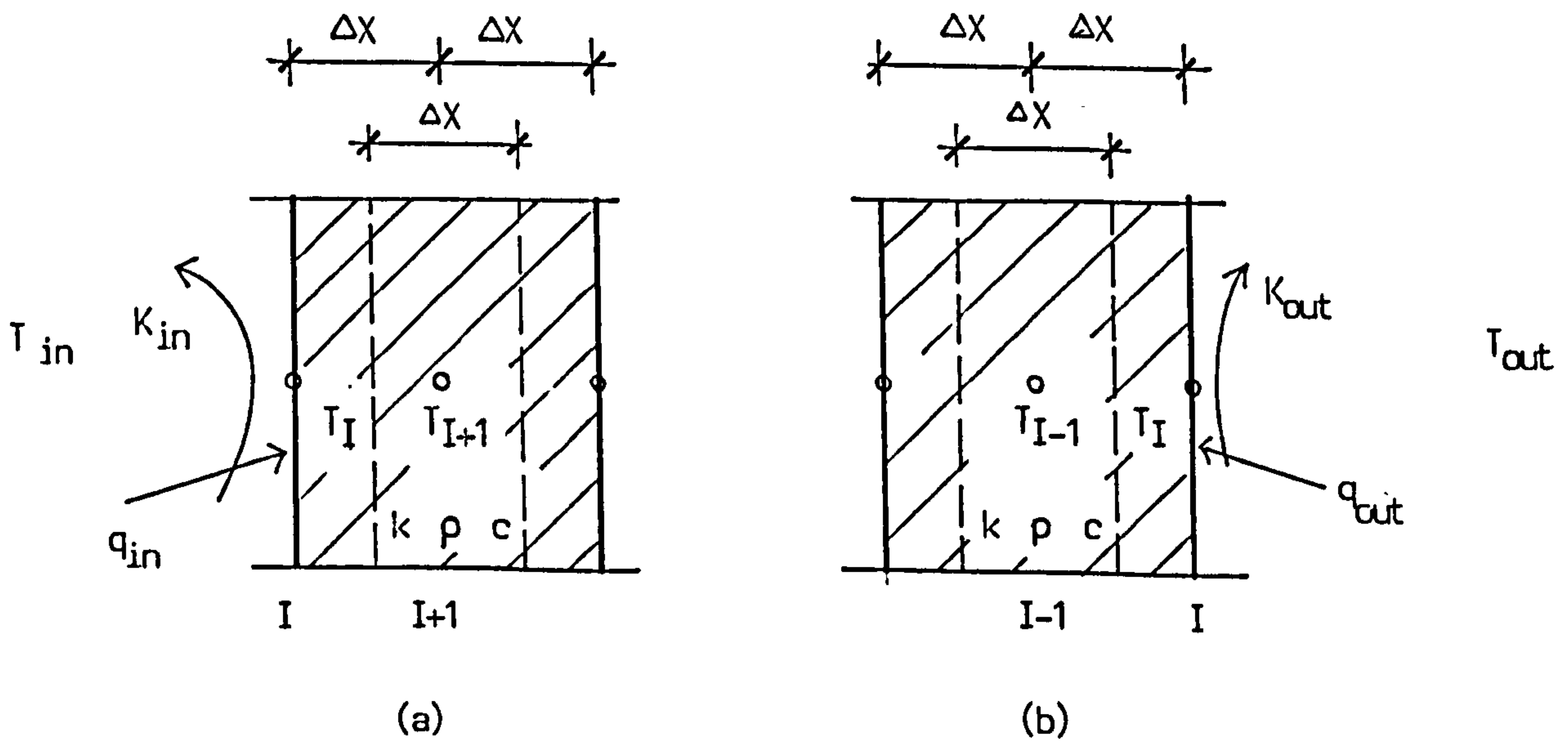


Figure 3.6 Control volume at extreme end of element

$C = \text{diag}(c_1), i = 1, 2, \dots, n.$ is a $n \times n$ diagonal matrix, each coefficient in the diagonal represents the thermal capacity of the control volume in the multi-layer structure and gives the thermal capacity of the whole structure.

$A = \text{diag}(a_{i-1}, a_i, a_{i+1}), i = 1, 2, \dots, n.$ is a $n \times n$ tri-diagonal matrix, each coefficient in the tri-diagonal is the thermal resistance between nodes in connection, and matrix gives the thermal resistance of the multi-layer structure.

$U = [T_{in}, q_{in}, T_{out}, q_{out}]^T$ gives external disturbances to the multi-layer structure.

B is a $n \times 4$ matrix which gives the connections of external disturbances to the corresponding nodes and influencing the thermal state of the multi-layer structure.

Equation 3.10 can also be applied to a thick homogeneous slab, in such case, the thermal-physical properties will not change with positions. Subdivision of the element also can be made to two or three dimensions, to cylindrical and spherical objects, the resulted state equations will be similar as equation 3.7. In two dimensional subdivision, matrix A will become penta-diagonal matrix and the coefficients are thermal resistances connecting the adjacent nodes to the central node of control volume.

Consider a homogeneous slab subdivided into control volume as Figure 3.3, state equation given same as equation 3.10, as the space steps Δx_i approach to limit this state equation will become one dimensional coordinate Fourier conduction equation with convective boundary conditions:

$$\left\{ \begin{array}{l} p c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) \\ - k \frac{\partial T}{\partial x} = K_{in} (T_{in} - T) + q_{in} \quad x=0 \\ k \frac{\partial T}{\partial x} = K_{out} (T_{out} - T) + q_{out} \quad x=L \end{array} \right.$$

Similar to the cases of two or three dimensional subdivisions, the two or three dimensional coordinate Fourier heat conduction equation with boundary conditions will be resulted.

III For node in flowing fluid

This is the most interesting case in plant modelling, since the main purpose of a plant system is to deliver the working substances, water, humid air and other fluids, from the central plant to the distribution system and components, to achieve the required design conditions. as an example of humid air flowing in a duct in which heat and mass transfer are taken place, figure 3.7 shows a control volume taken as air confined in a section of duct.

Air enters the control volume at temperature T_0 and humidity W_0 with mass flow rate at m , and leaves at temperature and humidity of control volume T and W , duct internal surface temperature at T_s and concentration at W_s . Assumptions are made to the control volume same as that in the static fluid, kinetic energy caused by forced air flow and work done by air internal viscosity are ignored, no leakage from the control volume other than the flow direction. Applying energy and mass conservation to the control volume result:

$$\rho cv \frac{dT}{dt} = \dot{m}cT_0 - (\dot{m}c + KA)T + KAT_s \quad 3.11$$

$$\rho v \frac{dW}{dt} = \dot{m}w_0 - (\dot{m} + HA)W + HAW_s \quad 3.12$$

(see com. 3.1)

Also, the control volume as Figure 3.8 with section length x can be further divided into several smaller sections, in this case, the resulted state equation of energy and mass will be a set of differential equations respectively consisted of the nodal equation in sequential connections. It can be written as same form as equation 3.10, and here the coefficients in the tri-diagonal line will be reduced to two. Longer section length x also can be made to increase

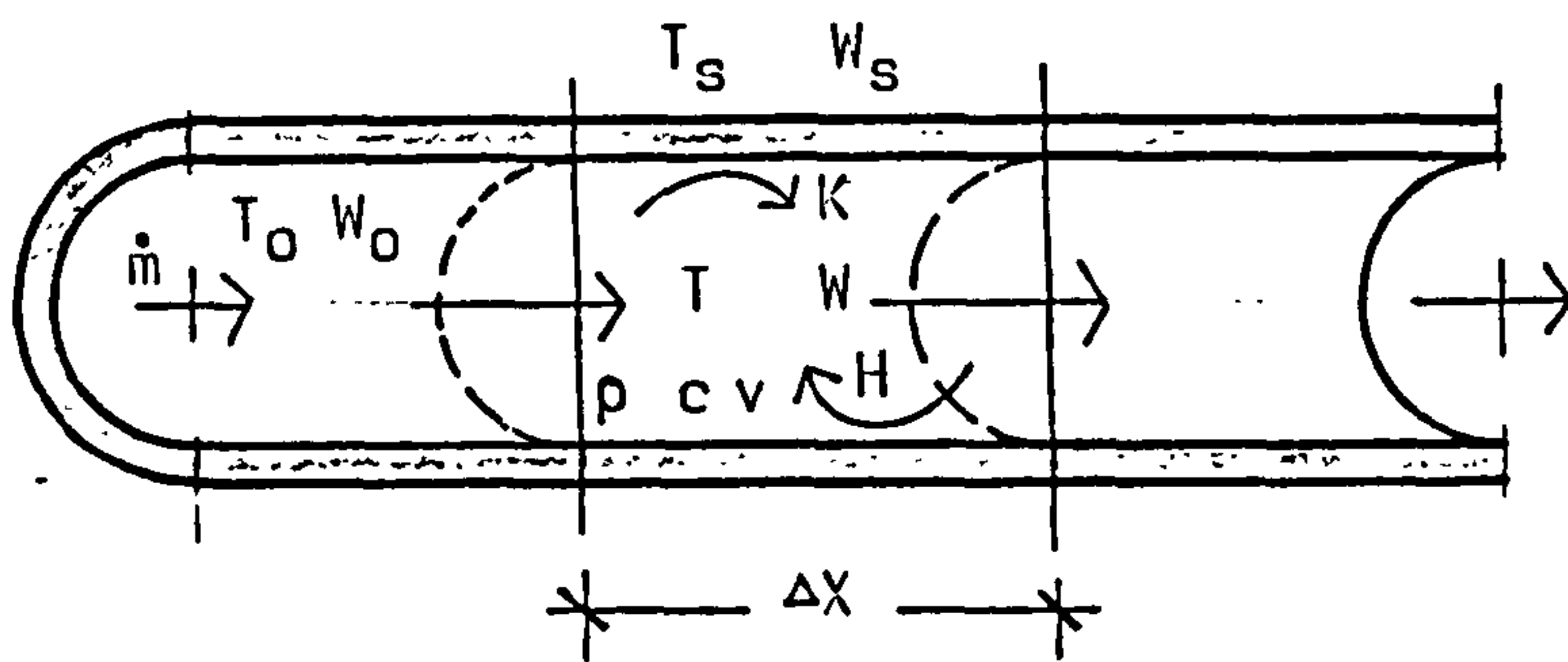


Figure 3.7 Control volume of duct flow

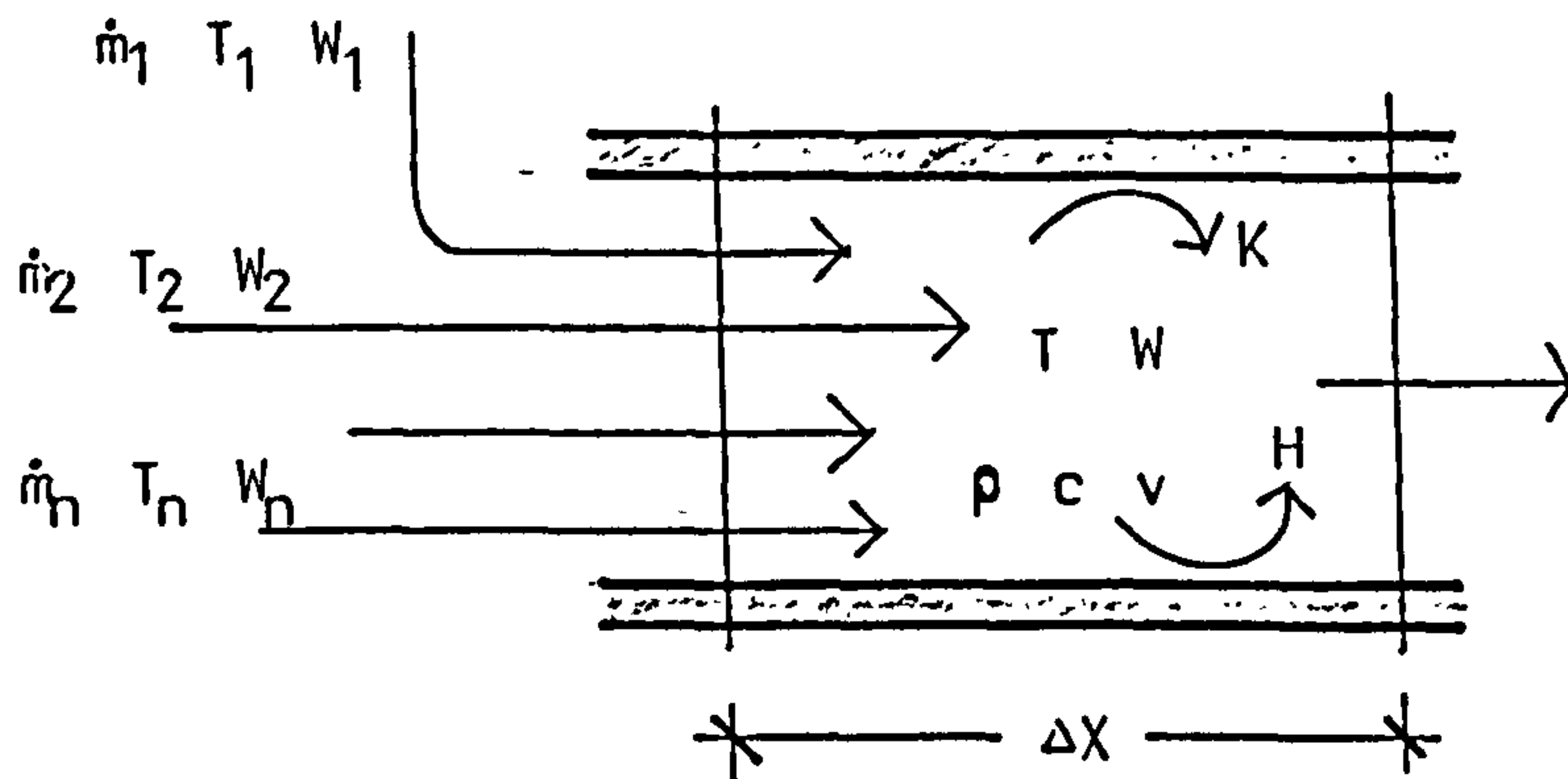


Figure 3.8 Control volume of duct with convergent flow

the size of control volume, however, it is restricted by the criteria based on error estimation.

Equation 3.11 and 3.12 can be expanded to a general case in which there are several air streams flowing into the control volume at their own flow rate m , temperature T , humidity W , the energy and mass conservation equations then become:

$$\rho_{cv} \frac{dT}{dt} = \sum_{i=1}^n \dot{m}_i c_i T_i - \left(\sum_{i=1}^n \dot{m}_i c_i + KA \right) T + KAT_s \quad 3.13$$

$$\rho v \frac{dW}{dt} = \sum_{i=1}^n \dot{m}_i W_i - \left(\sum_{i=1}^n \dot{m}_i + HA \right) W + HAW_s \quad 3.14$$

It is interesting to see that when the space-step x approaches limit, equation 3.11 and 3.12 will become:

$$\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} + q'$$

$$\frac{\partial W}{\partial t} = -u \frac{\partial W}{\partial x} + g'$$

Here, u is air velocity, and q' and g' are terms of heat and mass flux respectively. For circular duct with diameter D , it follows $q' = 4K(T_s - T)/\rho c D$ and $g' = 4H(W_s - W)/\rho D$. As space step x approaches to limit, equation 3.13 and 3.14 will become:

$$\frac{\partial T}{\partial t} = - \left(u_1 \frac{\partial T}{\partial x_1} + u_2 \frac{\partial T}{\partial x_2} + \dots + u_n \frac{\partial T}{\partial x_n} \right) + q'$$

$$\frac{\partial W}{\partial t} = - \left(u_1 \frac{\partial W}{\partial x_1} + u_2 \frac{\partial W}{\partial x_2} + \dots + u_n \frac{\partial W}{\partial x_n} \right) + \phi'$$

Here, u_1, u_2, \dots, u_n are imaginary air velocities corresponding to the cross-sectional area of control volume and x_1, x_2, \dots, x_n are imaginary space coordinates.

Now, the basic control volume conservation equations are derived and they are ready to be used as further applications of developing the component models. Undoubtedly, different physical components demand

different treatments, these basic equations have to be changed slightly to take account for special conditions. It is difficult (and not necessary) to derive the proper state equation at one time for each case of control volume in building and plant system, the basic control volume conservation equations derived above will serve the purpose of providing a convenient way to develop models of the specific components and their parametric studies.

3.1.2 Structures of components and system models

1. Component mathematical models

Utilizing the basic control volume conservation equations derived in the previous section, any component state equation of building/plant system can be possibly established. The component state equations can be divided into two independent parts: the thermal state equation and the mass state equation. For those components in which have no mass exchanges taken place, the mass state equation part can be neglected. As a fact, the mathematical expressions of thermal and mass state equation are similar, the only differences are just the symbols. Therefore, the general discussions on component mathematical models are going to be based on the energy conservation, all the results can be directly applied to the mass conservation state equations without vital difference.

A component model can be represented by one node as well as multi-nodes. In all cases, the component state equations can be generally written as a matrix differential equation:

$$C_i(t) \frac{dT_i(t)}{dt} = A_i(t) T_i(t) + B_i(t) U_i(t) \quad 3.15$$

Generally, for a m nodes component, c_i is a $m \times m$ diagonal matrix. It is clear that for components in which the thermal capacity of each control volume is non-zero, matrix c_i is non-singular. It can be proved that for those components in which the thermal capacities of some nodes are comparatively so small that can be considered as zero, matrix c_i still can be non-singular and diagonal but with lower order. [App. A1]

Matrix A_i gives the inter-relationships between nodes. In a m nodes component, A_i is a $m \times m$ sparse matrix called the basic matrix of component. In most cases, coefficients in the diagonal line are non-zero and negative and coefficients other than in the diagonal are positive. This characteristics of matrix A_i usually results the

eigenvalues of the state equation have real and negative parts. It can be further proved that in most cases, eigenvalues of component state equation are real and negative. [App. A2]

U_i is disturbance vector, each item represents a disturbance acting onto the system. If the order of U_i is r , the order of matrix B_i is $m \times r$, which represents the connections between the disturbances and the system.

Three kinds of basic conservation equation have been derived in the previous section, the static solid and air, the multi-layer solid, and the fluid flow. A component model can be represented by one kind of the basic conservation equation as well as by two or three kinds of them. A multi-layer wall, for example, represented by equation 3.10, is developed by using the basic conservation equation of multi-layer solid. For water flowing in pipe with thermal insulation, the model to result is the combination of all the three kinds of basic equations. If the thermal storage effects of pipe metal and insulation are neglected, the model will become a set of equation only consists of the fluid flow basic equation.

2. The system model

The thermal state equation of a complete system is established by the combination of all the component models with extra considerations of the inter-relationships among all these components. Some variables considered as disturbances (and included in vector U in equation 3.15) in a component model will become state variables of another component with thermal contact to it in a complete system.

Therefore, the complete system state equation can be written in a matrix form as:

$$\begin{vmatrix} C_1 & & & & \\ & C_2 & & & \\ & & - & & \\ & & & - & \\ & & & & - \\ & & & & & C_n \end{vmatrix} \begin{vmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \vdots \\ \dot{T}_n \end{vmatrix} = \begin{vmatrix} A_1 & d_{12} & \dots & d_{1n} \\ d_{21} & A_2 & & d_{2n} \\ \dots & & \dots & \\ \dots & & \dots & \\ \dots & & \dots & \\ d_{n1} & d_{n2} & \dots & A_n \end{vmatrix} \begin{vmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{vmatrix} + \begin{vmatrix} B_1 & & & \\ & B_2 & & \\ & & - & \\ & & & - \\ & & & & - \\ & & & & & B_n \end{vmatrix} \begin{vmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{vmatrix}$$

$$c(t) \frac{dT(t)}{dt} = A(t) T(t) + B(t) U(t) \quad 3.16$$

where, subscript i ($i = 1, \dots, n$) denotes component i .

$C = \text{diag}\{c_i\}$; ($i=1, \dots, n$) is a diagonal matrix of system thermal capacity. Component thermal capacity matrices c_i ($i=1, \dots, n$) are on the diagonal.

The diagonal of matrix A consists of basic matrix of all components, A_i ($i=1, \dots, n$). Other than the diagonal in matrix A , d_{ij} ($i=1, \dots, n$; $j=1, \dots, n$; $i \neq j$) are sub-matrices describing the inter-relationships among all components. d could be zero matrix, if two components have no direct thermal contact. $T = [T_1, T_2, \dots, T_n]^T$ is system state vector including all component state variables.

Disturbance variables U_i ($i=1, \dots, n$) will be changed in system equation, since those variables treated as disturbances in component model now become state variables of other component. Therefore, some of U_i ($i=1, \dots, n$) will become zero. Some as sub-matrices B_i ($i=1, \dots, n$) will be changed in system equation accordingly.

Figure 3.9 shows a simple building/plant system comprising six components, the building envelopes, room air, boiler, radiator, supply and return water pipes. Suppose the component state equations can be written in matrix form as equation 3.15, the system state equation then becomes,

$$\begin{bmatrix} C_1 & & & & & \\ & C_2 & & & & \\ & & C_3 & & & \\ & & & C_4 & & \\ & & & & C_5 & \\ & & & & & C_6 \end{bmatrix} \begin{bmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \dot{T}_3 \\ \dot{T}_4 \\ \dot{T}_5 \\ \dot{T}_6 \end{bmatrix} = \begin{bmatrix} A_1 & X & X & X & X & X \\ X & A_2 & \emptyset & \emptyset & \emptyset & \emptyset \\ X & \emptyset & A_3 & & X & \\ X & \emptyset & X & A_4 & & \\ X & \emptyset & & X & A_5 & \\ X & \emptyset & & & X & A_6 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} B_1 & & & & & \\ & B_2 & & & & \\ & & B_3 & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{bmatrix} \quad 3.17$$

or,

$$C(t) \frac{dT(t)}{dt} = A(t) T(t) + B(t) U(t)$$

where, subscripts 1,2,3,4,5,6 represent the components of room air,

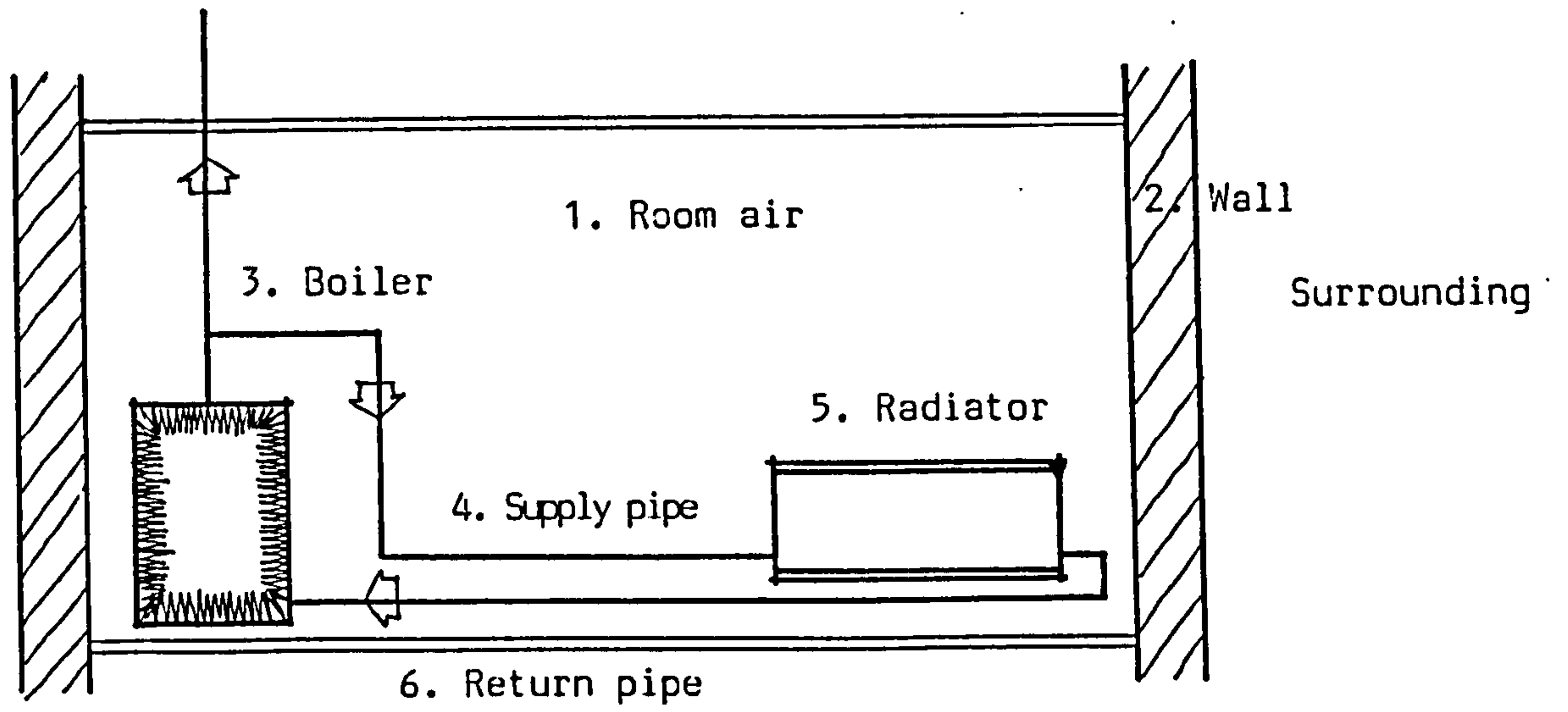


Figure 3.9 Building and plant system

wall, boiler supply pipe, radiator and return pipe respectively. X, \emptyset and $*$ are sub-matrices in system matrix A. X represents the convective heat exchange connection between two components, \emptyset represents the radiative heat exchange connection between two relevant component, $*$ represents water flow connection between two relevant components.

It is clear that in the complete system state equation, matrix C is non-singular and diagonal since $C_i (i = 1, \dots, 6)$ are non-singular and diagonal. The component sub-matrices $A_i (i = 1, \dots, 6)$ are positioned at diagonal of system matrix A. This means that coefficients at diagonal of system matrix A are non-zero and negative, since all coefficients at the diagonal of component sub-matrices are non-zero and negative.

3. Analysis of control volume

It has been proved that when the control volumes approach to limit, the basic control volume conservation equations will become partial differential conservation equations which can usually be found in the classical problems of heat transfer and fluid mechanics. Generally speaking, such governing equations encountered in building/plant modelling can be classified into three catalogues:

- a) the first order ordinary differential equation describing the element with lumped thermal capacity such as solid, static fluid etc. It can be represented as:

$$\frac{dT}{dt} = f(t, T(t)) \quad 3.18$$

- b) the parabolic partial differential equation describing the solid heat conduction which is usually referred as Fourier heat conduction equation. It considers more of internal thermal phenomena of solid, can be written as:

$$\frac{\partial T}{\partial t} = a \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + q_p \quad 3.19$$

- c) the first order hyperbolic partial differential equation,

usually referred to as the Energy equation. In its simplest form, it is commonly used to describe fluid flow coupling and thermal contact with surrounding. It can be written as:

$$\frac{\partial T}{\partial t} = - \left(u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z} \right) + q_h \quad 3.20$$

It is clear that equation 3.18 is a time variable differential equation which has no spatial discretization problem. It can be further written as the same form as general conservation equation. It can be proved that equation 3.19 and 3.20 will be identical to the general conservation equation if "semi-discretization" is applied to the space derivatives and neglect the discretization error. For parameter study, it is enough to consider the simple one dimensional problems.

Assuming the function $T(x,t)$ is continuous and can be expanded by Taylor series, the parabolic and hyperbolic equations can be reduced to the first-order ordinary differential equations, as:

$$\frac{dT(t)}{dt} = \frac{a}{\Delta x^2} [T(t,n+1) - 2T(t,n) + T(t,n-1)] + q_p + R_p \quad 3.21$$

$$\frac{dT(t)}{dt} = - \frac{u}{\Delta x} [T(t,n) - T(t,n-1)] + q_h + R_h \quad 3.22$$

where, R_p , R_h are semi-discretization errors of one dimensional parabolic and hyperbolic equation respectively.

The principal part of R_p and R_h can be found as:

$$R_p \leq \frac{\Delta x_p^2}{12a^2} \left(\frac{\partial^2 T}{\partial t^2} \right) \quad 3.23$$

and

$$R_h \leq \frac{\Delta x_h}{2u^2} \left(\frac{\partial^2 T}{\partial t^2} \right) \quad 3.24$$

The consistency between the semi-discretizations and the original problems can be clearly seen when the spatial step Δx approach to zero. If the accuracies for modelling are pre-specified, the control volume for parabolic and hyperbolic equations can then be determined to satisfy this condition. In such case, the one dimensional control volume Δx_p and Δx_h have to be

$$\Delta x_p \leq 3.46 a R_p \left(\frac{\partial^2 T}{\partial t^2} \right)^{-\frac{1}{2}} \quad 3.25$$

and

$$\Delta x_h \leq 2 u R_h \left(\frac{\partial^2 T}{\partial t^2} \right)^{-1} \quad 3.26$$

Therefore, for heat conduction problems, smaller control volumes have to be made to those layers having lower thermal diffusivity, it follows that $\Delta x_{p2} = (a_2/a_1)\Delta x_{p1}$. To determine the control volumes of fluid flowing problems, it is not necessary to make fine control volumes to the component in which fluid velocity is higher. It should follow the relation of $\Delta x_{h2} = (u_2/u_1)\Delta x_{h1}$. [App. A3]

3.2 The solution of system state equation

The system state equation to result is the initial value problem of a set of linear, first-order, simultaneous ordinary differential equations, it can be represented as:

$$\left\{ \begin{array}{l} C(t) \frac{dT(t)}{dt} = A(t) T(t) + B(t) U(t) \\ T(t_0) = T_0 \end{array} \right. \quad 3.27$$

The purpose of modelling is to find the solution of this initial value problem. The solution methods can be analytical, numerical. Simplification can be made to constant coefficients or keep them as variables of time.

3.2.1 Analytical solution

a) The time invariant solution

Since the $n \times n$ matrix C is non-singular, there always exists its inverse matrix C^{-1} , so that:

$$dT/dt = C^{-1}A T + C^{-1}B U$$

In most cases of plant system (see com. 3.2), matrix $(C^{-1}A)$ is non-singular, a non-singular matrix P can be formed, so that:

$$P C^{-1}A P^{-1} = \text{diag}\{\lambda_i\} = \Lambda$$

where, Λ is a $n \times n$ diagonal matrix in which $\lambda_i (i = 1, \dots, n)$ are eigenvalues in the diagonal of matrix Λ .

Let $X = PT$ and $D = PC^{-1}B$, equation 3.27 becomes:

$$\begin{cases} dx/dt = \Lambda X + D U \\ X(t_0) = X_0 \end{cases} \quad 3.28$$

Equation 3.28 can be directly solved as:

$$X(t) = e^{\Lambda t} X(0) + \int_0^t e^{\Lambda(t-\tau)} D U(\tau) d\tau \quad 3.29$$

where,

$e^{\Lambda t}$ = state transition matrix,

in which,

$$e^{\Lambda t} = \begin{vmatrix} e^{\lambda_1 t} & & & & \\ & e^{\lambda_2 t} & & & \\ & & - & & \\ & & & - & \\ & & & & - \\ & & & & & e^{\lambda_n t} \end{vmatrix} = \text{diag}\{e^{\lambda_i t}\} \quad (i = 1, \dots, n)$$

Therefore, if disturbance vector $U(t)$ is known and continuous, the solution of equation 3.29 can then be readily found.

b) Time varying solution

In plant modelling, for time varying solution of equation 3.25, coefficients in matrices C, A and B become functions of state variables. For physical existence of the models, coefficients in matrix C are always bounded, non-zero, positive and diagonal, so that its inverse C^{-1} always exists. However, the similarity transformation $P C^{-1} A P^{-1} = \text{diag}\{\lambda_i(t)\}$ will not definitely exist and the matrix P will be very difficult to find. The time varying initial value problem can be represented as:

$$\begin{cases} dT(t)/dt = F(t) T(t) + G(t) U(t) \\ T(t_0) = T_0 \end{cases} \quad 3.30$$

where,

$$F(t) = C^{-1}(t) A(t); \quad G(t) = C^{-1}(t) B(t)$$

For the solution of time varying problem, the state transition matrix $\phi(t, t_0)$ is introduced, which satisfies:

$$\begin{aligned} \frac{\partial}{\partial \tau} \phi(t, \tau) &= F(t) \phi(t, \tau) \quad , \\ \phi(\tau, \tau) &= I \quad , \\ &\dots \end{aligned}$$

The solution now can be found as:

$$T(t) = T(t, t_0) T(t_0) + \int_{t_0}^t T(t, \tau) G(\tau) U(\tau) d\tau \quad 3.31$$

where $\phi(t, t_0)$ = state transition equation.

Unfortunately, there is in general no simple relation between transition matrix $\phi(t, t_0)$ and the system matrix $F(t)$, and except for very simple cases, state transition matrix $\phi(t, t_0)$ can not be easily found. Therefore, the analytical solution by equation 3.31 is used mainly in the theoretical study only. For practical application, approximation can be made to $\phi(t, t_0)$ by using series expansion,

$$\phi(t, t_0) = I + \int_{t_0}^t F(\tau) d\tau + \int_{t_0}^t F(\tau_1) \left[\int_{t_0}^{\tau_1} F(\tau_2) d\tau_2 \right] d\tau_1 + \dots$$

This series expansion will not be given in closed form.

3.2.2 Numerical solution

Numerical methods of solving a single first order ordinary differential equation are available to solve the plant state equation only vector notations are needed to replace the single functions. However, the particular problems arisen in system model requires some special treatments according to their mathematical significances, they are:

- a) The stiffness of system equation. This problem has been encountered in many fields of application of modelling. Attempts to use numerical methods are then limited. A heuristic definition of stiffness is usually defined as, for linear system,

$$dY/dt = A Y + \phi(t),$$

- (i) if the real parts of eigenvalues of A, $\text{Re}\lambda_i < 0$, and
(ii) $\max_i \{\text{Re}\lambda_i\} \gg \min_i \{\lambda_i\}$.

In plant modelling, the physical meaning of stiffness is the time constants of some component, such as a wall, are much greater than some others, such as a thermometer. The stiffness of linear system asks for the absolute stability of numerical methods.

- b) For plant modelling, the system state equation to result is usually very large and long term simulation is always required. For solving the time varying state equation, iteration methods are usually not used if the economical computing is considered. The non-iterative numerical methods have to be applied.

The problem of stiffness has been known for some times, a number of methods for solving stiffness problem have been presented by publications. The Trapezoidal method is going to be used for solving the state equation of plant system. The reasons of this method is preferred are: Firstly, it satisfies the theorem of Dahlquist which states - (a) an explicit linear multi-step method can not be A-stable, (b) the order of an A-stable implicit linear multi-step method cannot exceed two, (c) the second-order A-stable implicit linear multi-step

method with smallest error constant is the Trapezoidal rule. Secondly, in plant modelling, the selection of time-step can be varied in great extent depended on the purpose of modelling. An absolute stable method which can be applied to suit this problem is necessary. Thirdly, the Trapezoidal method applied to the plant system state equation is equivalent to Crank-Nicolson method applied to the Fourier heat conduction and fluid flow energy equations. A-stable for stiff problem is also equivalent to unconditional stable of Crank-Nicolson method. Such a numerical method has already been employed in the generalised building/plant simulation programme ESP, developed in ABACUS. Since the development of HVAC components and system modelling carried in this project is directly linked to the further development of ESP, the consistency of numerical method in developing the mathematical models is necessary.

The plant system model of equation 3.30 now can be re-called for further discussion, as:

$$\begin{cases} \frac{dT(t)}{dt} = F(t)T(t) + G(t)U(t) = f(t, T(t)) \\ T(t_0) = T_0 \end{cases} \quad 3.32$$

The unique solution for this initial value problem is always thought to be existed. In fact, the system state equation is derived based on the real plant system, physical reality ensures that both t and $T(t)$ are bounded, and so that $f(t, T(t))$ is bounded. It satisfies the Lipschitz condition which guarantees that solution exists in space D , defined by $a < t < b$, and $c < T_i < d$, ($i=1, \dots, n$), in which a , b , c and d are finite.

(1) Numerical formation of Trapezoidal method

The Trapezoidal method can be derived either by Taylor expansion or numerical integration.

For initial value problem as equation 3.32, the equivalent integral form can be found as, let $t = t_m$

$$T(t_m + \Delta t) = T(t_m) + \int_{t_m}^{t_m + \Delta t} f(t, T(t)) dt \quad 3.33$$

Using Trapezoidal method to compute the integral, results:

$$T(t_m + \Delta t) = T(t_m) + \frac{\Delta t}{2} [f(t_m, T(t_m)) + f(t_m + \Delta t, T(t_m + \Delta t))] + R_m \quad 3.34$$

where R_m = the truncation error vector

$$R_m = \int_{t_m}^{t_m + \Delta t} f(t, T(t)) dt - (\Delta t/2)[f(t_m, T(t_m)) + f(t_m + \Delta t, T(t_m + \Delta t))]$$

since $t_m < t < (t_m + \Delta t)$,

$$\begin{aligned} f(t, T(t)) &= T'(t) = T'(t_m + \tau \Delta t) & \tau > 0 \\ &= T'(t_m) + \tau [T'(t_m + \Delta t) - T'(t_m)] \\ &\quad + \frac{\Delta t}{2} \tau(\tau - 1) T'''(t + \theta \Delta t) & \theta \leq 1 \end{aligned}$$

and

$$\begin{aligned} \int_{t_m}^{t_m + \Delta t} f(t, T(t)) dt &= \int_0^1 [T'(t_m) + \tau (T'(t_m + \Delta t) - T'(t_m))] \Delta t \, d\tau \\ &\quad + \frac{\Delta t}{2} \int_0^1 \tau(\tau - 1) T'''(t_m + \theta \Delta t) \, d\tau \\ &= \frac{\Delta t}{2} [T'(t_m + \Delta t) + T'(t_m)] - \frac{\Delta t^3}{12} T'''(t_m + \xi \Delta t) \\ & \qquad \qquad \qquad 0 \leq \xi \leq 1 \end{aligned}$$

Therefore, the local truncation error for Trapezoidal method is:

$$R_m = - \frac{\Delta t^3}{12} T'''(t_m + \xi \Delta t) \quad 3.35$$

It can be seen that the local truncation error introduced by time domain numerical approximation is of order Δt^3 , so that the numerical algorithm of equation 3.34 itself is second order time accuracy if the error vector is neglected.

As a parametric study, it is useful to investigate the time invariant case. As stated in section 3.2.1, for time invariant system, the system state equation 3.30 always can be simplified to equation 3.28

by means of similarity transformation. Considering the homogeneous case, let:

$$dT(t)/dt = \Lambda T(t) \quad 3.36$$

where Λ is a $n \times n$ diagonal matrix or at least in Jordan-block diagonal. For distinctive eigenvalues, $\Lambda = \text{diag}\{\lambda_i\}$, ($i = 1, \dots, n$) in which λ_i are eigenvalues of system after similarity transformation. Substitute equation 3.36 to the error vector expression of equation 3.35, results:

$$\begin{aligned} \|R_m\| &\leq \frac{\Delta t^3}{12} \|T''(t_m + \xi \Delta t)\| = \frac{\Delta t^3}{12} \|\Lambda T''(t_m + \xi \Delta t)\| \\ &\leq \frac{\Delta t^3}{12} \|\Lambda\| \|T''(t_m + \xi \Delta t)\| \quad 0 \leq \xi \leq 1 \end{aligned}$$

Since Λ is diagonal, if the Euclidean norms are considered and the second-order derivative of state vector is existed and bounded, the local truncation error becomes:

$$\|R_m\| \leq \frac{\Delta t^3}{12} \lambda_m M \quad 3.37$$

where,

$$\begin{aligned} \lambda_m &= \sqrt{\lambda_{\max, \Lambda^* \Lambda}} ; \text{ when diagonal and all eigenvalues are real,} \\ \lambda_m &= \max_i \{\lambda_i\}. \quad M = \max_{a \leq t \leq b} \|T''(t)\| \end{aligned}$$

Therefore, system time domain discretization error can be controlled by parameter $(\Delta t^3/12)(\lambda_m)$ for linear time invariant system. Since Λ is diagonal, the absolute value of time domain discretization error for each node can be estimated by:

$$|R_{m,i}| \leq \frac{\Delta t^3}{12} |\lambda_i| M_i \quad 3.38$$

where,

$$M_i = \max_{a \leq t \leq b} |T_i''(t)|$$

For time varying system, problems of error estimation become much complicated. However, in plant modelling, matrix $F(t)$ in equation 3.32 is bounded, and to certain extent, it can always be considered as sectionally time invariant, so that the results above still can be approximately applied.

(2) Convergence and stability of numerical method

The numerical algorithm is convergent. This is because:

- (i) The initial value problem satisfies the Lipschitz condition.
- (ii) Error vector R will approach to zero when $n \rightarrow \infty$, $\Delta t = t/n, (a < t < b)$ approaches to zero. Therefore, the true solution can be approximated arbitrarily closely by making Δt smaller enough and getting greater precision.

Dahlquist has defined the definition of A-stable for numerical solution of initial value problem which is the region of absolute stability contains the whole of left-hand half-plane. He also proved that Trapezoidal method for initial value problem is A-stable.

In fact, consider the time invariant homogeneous case, substitute equation 3.36 to equation 3.34 and neglect the error vector R_m , results:

$$\begin{aligned} T(t_m + \Delta t) &= T(t_m) + \frac{\Delta t}{2} [f(t_m, T(t_m)) + f(t_m + \Delta t, T(t_m + \Delta t))] \\ &= T(t_m) + \frac{\Delta t}{2} [\Lambda T(t_m) + \Lambda T(t_m + \Delta t)] \end{aligned}$$

or

$$T(t_m + \Delta t) = \left[\left(I - \frac{\Delta t}{2} \Lambda \right)^{-1} \left(I + \frac{\Delta t}{2} \Lambda \right) \right] T(t_m)$$

Since Λ is diagonal, $\left(I - \frac{\Delta t}{2} \Lambda \right)^{-1}$ and $\left(I + \frac{\Delta t}{2} \Lambda \right)$ are diagonal and so the product of $\left(I - \frac{\Delta t}{2} \Lambda \right)^{-1} \left(I + \frac{\Delta t}{2} \Lambda \right)$ is diagonal. It can be written as:

$$(I - \frac{\Delta t}{2} \Lambda)^{-1} (I + \frac{\Delta t}{2} \Lambda) = \text{diag} \left\{ \frac{1 + \frac{\Delta t}{2} \lambda_i}{1 - \frac{\Delta t}{2} \lambda_i} \right\} \quad (i = 1, \dots, n)$$

Let, $\Lambda = \text{diag}\{\lambda_i\} = \text{diag}\{u_i + jv_i\}$ ($i = 1, \dots, n$) u_i, v_i are real so that:

$$T(t_m + \Delta t) = \text{diag} \left\{ \frac{(1 + \frac{\Delta t}{2} u_i) + j(\frac{\Delta t}{2} v_i)}{(1 - \frac{\Delta t}{2} u_i) - j(\frac{\Delta t}{2} v_i)} \right\} T(t_m) \quad (i = 1, \dots, n)$$

It is clear that for each $\lambda_i = u_i + jv_i$, the absolute values of

$$\left| \frac{(1 + \frac{\Delta t}{2} u_i) + j(\frac{\Delta t}{2} v_i)}{(1 - \frac{\Delta t}{2} u_i) - j(\frac{\Delta t}{2} v_i)} \right| = \sqrt{\frac{1 + \frac{\Delta t^2}{4} u_i^2 + \frac{\Delta t^2}{4} v_i^2 + \Delta t u_i}{1 + \frac{\Delta t^2}{4} u_i^2 + \frac{\Delta t^2}{4} v_i^2 - \Delta t u_i}} \quad 3.39$$

less than one if each $u_i, (i = 1, \dots, n)$ is negative.

In fact, under this condition, the norm of diagonal matrix

$$\text{diag} \left\{ \frac{(1 + \frac{\Delta t}{2} u_i) + j(\frac{\Delta t}{2} v_i)}{(1 - \frac{\Delta t}{2} u_i) - j(\frac{\Delta t}{2} v_i)} \right\} \quad (i = 1, \dots, n)$$

less than one, solution $T(t_m + \Delta t)$ will decay when process the computation as t_m approaches to infinite. The absolute stable region is defined by the product of Δt and u_i . As Δt always greater than zero, the stability of algorithm has no relation with time step Δt , only requires the real part of eigenvalues $u_i (i = 1, \dots, n)$ to be negative. In other words, the Trapezoidal method for initial value problem is absolute stable for all time steps.

Similar stability conditions can be obtained when Crank-Nicolson

scheme is applied to Fourier heat conduction equation and fluid flow energy equation. It has been proved that the Crank-Nicolson schemes for both Fourier heat conduction and energy equation are unconditionally stable for all time steps. For Fourier heat conduction equation, the criteria of stability has to satisfy the condition of $\tau_p = (a\Delta t/\Delta x^2) > 0$ and for fluid flow energy equation $\tau_R = (u\Delta t/\Delta x) > 0$. It is true that both time and space steps $\Delta t, \Delta x$ are always positive. The requirement of thermal diffusivity "a" and velocity "u" greater than zero ensures that in the semi-discrete forms of these two partial differential equations, the diagonal coefficients of matrix F(t) are negative and so that the eigenvalues of system will have real negative parts which then satisfy the A-stable condition in initial value problem.

It has to be emphasized that the above discussions of convergency, stability of the numerical method are based on time invariant system. For time varying system, care has to be made for choosing suitable computational time steps. Since in numerical computational, the data oscillation some times will cause some time dependent coefficients become negative, so that the absolute stable condition will be violated. The numerically negative coefficients will equally lead the unconditional stable Crank-Nicolson schemes become unstable since the criteria of stability $\tau_p > 0$ and $\tau_h > 0$ are no longer held.

(3) Structure of non-iterative numerical schemes

Now the numerical scheme for the system state equation can be established by substituting equation 3.32 to 3.34, it results:

$$\begin{aligned} & [2I - F(t+\Delta t)\Delta t] T(t+\Delta t) + G(t+\Delta t)\Delta t U(t+\Delta t) \\ & = [2I + F(t)\Delta t] T(t) + G(t)\Delta t U(t) \end{aligned} \quad 3.40$$

If one node is considered, substitute the general control volume conservation equation 3.1 to the Trapezoidal formula, the numerical scheme becomes:

$$[2\theta_I(t+\Delta t)c_I(t+\Delta t)v_I + \sum_{i=1}^m K_{I,i}(t+\Delta t)\Delta t] T_I(t+\Delta t) - \sum_{i=1}^m K_{I,i}(t+\Delta t)\Delta t T_i(t+\Delta t)$$

$$\begin{aligned}
- q_I(t+\Delta t)\Delta t = [2\rho_I(t)c(t)v - \sum_{i=1}^m K_{x,i}(t)\Delta t]T_I(t) + \\
\sum_{i=1}^m K_{x,i}(t)\Delta tT_i(t) + q_I(t)\Delta t
\end{aligned} \tag{3.41}$$

in which I denotes the position of the node.

Substitute the basic control volume conservation equation 3.4, 3.6 (as representative of 3.7, 3.8 and 3.9), and 3.11 to 3.40, the Trapezoidal temporal difference form of three basic control volume conservation equation are resulted:

a) For solid single node type

$$\begin{aligned}
[2\rho_I(t+\Delta t)c_I(t+\Delta t)v_I + K_I(t+\Delta t)A_I\Delta t]T_I(t+\Delta t) - K_I(t+\Delta t)A_I\Delta tT_s \\
(t+\Delta t) - q_I(t+\Delta t)\Delta t = [2\rho_I(t)c_I(t)v_I - K_I(t)A_I\Delta t]T_I(t) + \\
K_I(t)A_I\Delta tT_s(t) + q_I(t)\Delta t
\end{aligned} \tag{3.42}$$

b) For solid multi node type

$$\begin{aligned}
(2\rho_I(t+\Delta t)c_I(t+\Delta t)\Delta x_I + \frac{2k_I(t+\Delta t)}{\Delta x_I}\Delta t)T_I(t+\Delta t) - \frac{k_I(t+\Delta t)}{\Delta x_I}\Delta tT_{I-1} \\
(t+\Delta t) - \frac{k_I(t+\Delta t)}{\Delta x_I}\Delta tT_{I+1}(t+\Delta t) = (2\rho_I(t)c_I(t)\Delta x_I - \frac{2k_I(t)}{\Delta x_I}\Delta t)T_I(t) \\
+ \frac{k_I(t)}{\Delta x_I}\Delta tT_{I-1}(t) + \frac{k_I(t)}{\Delta x_I}\Delta tT_{I+1}(t)
\end{aligned} \tag{3.43}$$

c) For flowing fluid node type

$$\begin{aligned}
\{2\rho_I(t+\Delta t)c_I(t+\Delta t)v_I + \dot{m}_I(t+\Delta t)c_I(t+\Delta t)\Delta t + k_I(t+\Delta t)A_I\Delta t\}T_I \\
(t+\Delta t) - \dot{m}_I(t+\Delta t)c_I(t+\Delta t)\Delta tT_{I-1}(t+\Delta t) - k_I(t+\Delta t)A_I\Delta tT_s(t+\Delta t) \\
= \{2\rho_I(t)c_I(t)v_I - \dot{m}_I(t)c_I(t)\Delta t - k_I(t)A_I\Delta t\}T_I(t) - \dot{m}_I(t)c_I \\
(t)\Delta tT_{I-1}(t) - k_I(t)A_I\Delta tT_s(t)
\end{aligned} \tag{3.44}$$

Numerical scheme of equation 3.40 or 3.41 can be applied to both time invariant and time varying systems. Generally, consider equation 3.40, disturbance vector $U(t)$ and $U(t+\Delta t)$ are always known. For time

invariant system, the coefficient matrices in equation 3.40 are always held as constant, the numerical solution of future time-row state vector $T(t+\Delta t)$ can be straightforwardly obtained by time step processing. For time varying system, however, the coefficient matrices at left-hand side of equation 3.40 are dependent on the values of future time-row state variables which are unknown until the solution has been achieved. Since iterative method is not going to be used in the modelling for the reason mentioned before, the non-iterative numerical schemes have to be devised.

A non-iterative numerical scheme can be constructed by evaluating the coefficient matrices one time-step in arrears so that the future time-row matrices are established on the basis of present time-row information with the present time-row matrices utilizing immediate past information. In such case, the numerical scheme becomes:

$$\begin{aligned} & [2I - F(t)\Delta t] T(t+\Delta t) - G(t)\Delta t U(t+\Delta t) \\ & = [2I + F(t-\Delta t)\Delta t] T(t) + G(t-\Delta t)\Delta t U(t) \end{aligned} \quad 3.45$$

Other non-iterative numerical schemes can be derived by Taylor expansion. Referring to the Trapezoidal formula as equation 3.34, a local Taylor expansion about the present time value of state vector $T(t)$ yields:

$$f(t+\Delta t, T(t+\Delta t)) = f(t, T(t)) + \left(\frac{\partial f}{\partial T}\right)_t \left(\frac{dT}{dt}\right)_t \Delta t + O(\Delta t^2) \quad 3.46$$

Substitute 3.46 to 3.34, yields:

$$\begin{aligned} T(t+\Delta t) &= T(t) + \frac{\Delta t}{2} [2f(t, T(t)) + \left(\frac{\partial f}{\partial T}\right)_t \left(\frac{dT}{dt}\right)_t \Delta t] \\ &= T(t) + \Delta t f(t, T(t)) + \frac{\Delta t}{2} \left(\frac{\partial f}{\partial T}\right)_t (T(t+\Delta t) - T(t)) \end{aligned}$$

$$\text{or } \left[2I - \Delta t \left(\frac{\partial f}{\partial T}\right)_t\right] T(t+\Delta t) = \left[2I - \Delta t \left(\frac{\partial f}{\partial T}\right)_t\right] T(t) + 2 \Delta t f(t, T(t))$$

3.47

It can be seen, this equation provides a non-iterative formula for initial value problem in which the future time-row coefficient matrices can be computed at present time-row information.

Referring to the system state equation 3.32:

$$f(t, T(t)) = F(t)T(t) + G(t)U(t)$$

matrices $F(t)$ and $G(t)$ in fact are functions of state vector $T(t)$, so that the partial differential becomes:

$$\left(\frac{\partial f}{\partial T}\right)_t = F(t) I + \left(\frac{\partial F}{\partial T}\right)_t T(t) + \left(\frac{\partial G}{\partial T}\right)_t U(t) \quad 3.48$$

Substitute equation 3.48 to 3.47, the non-iterative scheme for time varying scheme becomes:

$$\begin{aligned} & \left[2I - \Delta t F(t)I - \Delta t \left(\frac{\partial F}{\partial T}\right)_t T(t) - \Delta t \left(\frac{\partial G}{\partial T}\right)_t U(t) \right] T(t+\Delta t) \quad 3.49 \\ & = \left[2I + \Delta t F(t)I - \Delta t \left(\frac{\partial F}{\partial T}\right)_t T(t) - \Delta t \left(\frac{\partial G}{\partial T}\right)_t U(t) \right] T(t) + 2\Delta t G(t) U(t) \end{aligned}$$

In practice, a first-order approximation can be made by assuming the time dependent coefficient matrices $F(t)$ and $G(t)$ are held constant between each time step internal, then equation 3.49 will be reduced to similar form as equation 3.40 with difference of the future time-row coefficient matrices are going to be computed at present time-row information.

Using a general matrix notation, equation 3.40, 3.45 and 3.49 can be represented as:

$$A(t+\Delta t)T(t+\Delta t) = B(t)T(t) + C(t+\Delta t, t) \quad 3.50$$

in the first case,

$$A(t+\Delta t) = 2I - F(t)\Delta t$$

$$B(t) = 2I + F(t-\Delta t)\Delta t$$

$$C(t+\Delta t, t) = G(t)\Delta t U(t+\Delta t) + G(t-\Delta t)\Delta t U(t)$$

in the second case of non-iterative scheme,

$$A(t+\Delta t) = 2I - \Delta t F(t) - \Delta t (\partial F / \partial T)_t T(t) - \Delta t (\partial G / \partial T)_t U(t)$$

$$B(t) = 2I + \Delta t F(t) - \Delta t (\partial F / \partial T)_t T(t) - \Delta t (\partial G / \partial T)_t U(t)$$

$$C(t+\Delta t, t) = 2\Delta t G(t) U(t)$$

Comments: (Chapter 3)

Comment 3.1

The assumption made to equation 3.11 and 3.12 include also the mass flow rate of air leaving the control volume is same as that entering. In fact, they are not exactly the same. Since mass transfer is taken place between the air stream and surfaces, the exact expressions of equation 3.11 and 3.12 will become:

$$\rho c v \frac{dT}{dt} = \dot{m} c T_o - [(\dot{m} + \delta \dot{m}) c + KA] T + KAT_s$$

$$\rho c \frac{dW}{dt} = \dot{m} W_o - [(\dot{m} + \delta \dot{m}) + HA] W + HAW_s$$

in which,

$$\delta \dot{m} = HA (W_s - W)$$

However, this amount ($\delta \dot{m}$) is rather small compared to the magnitude of mass flow rate of main air stream, it is reasonable to be neglected.

Comment 3.2

It is true for system without control feed back. For plant system with control functions, zero or positive values could be brought in diagonal matrix ($C^{-1}A$). This becomes the problems of finding the stability of control system. It can be generally achieved by finding such control parameters that make the eigenvalues of system matrix ($C^{-1}A$) all remain negative. Therefore, for any stable system, matrix ($C^{-1}A$) at least has to be non-singular.

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CHAPTER FOUR: MODELLING OF THE CENTRAL HEATING SYSTEM

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CHAPTER FOUR: MODELLING OF THE CENTRAL HEATING SYSTEM

4.1 The central heating system

The primary purpose of a heating system is to provide a specific set of thermal environmental conditions that may be required either for comfort or by an industrial process. A water central heating system is a combination of equipments in which hot water is used as medium to convey heat to or from a conditional space or process through piping connecting a boiler, water heat exchange with suitable terminal heat transfer units located at space or process. Such systems are frequently referred to as hydronic systems. In terms of flow generation, water heating systems can be divided into two types: (1) the gravity system, in which circulation of the water is due to the difference in weight between the supply and return water columns of any circuit or system; and (2) the forced system, in which a pump, usually driven by an electric motor, maintain the necessary flow.

Figure 4.1 shows a typical arrangement of domestic central heating system with hot water supply.

The nature of a heating system is a system of heat production and distribution. In such system, the modelling parameter is temperature only, no mass transfer takes place everywhere in the system.

The pressure distribution in the system is another important topic. The knowledge of pressure distribution is very important in determining the instantaneous flow condition of the system. It can be achieved by the dynamical modelling using the knowledge of fluid mechanics. As the purpose of this study is thermal modelling, pressure distribution in the system is always considered as constant or known function, so that the water flow rate throughout the system are all known factors. Clearly, this is a simplified condition imposed on this study.

The system modelling is based on the component basis. It is achieved

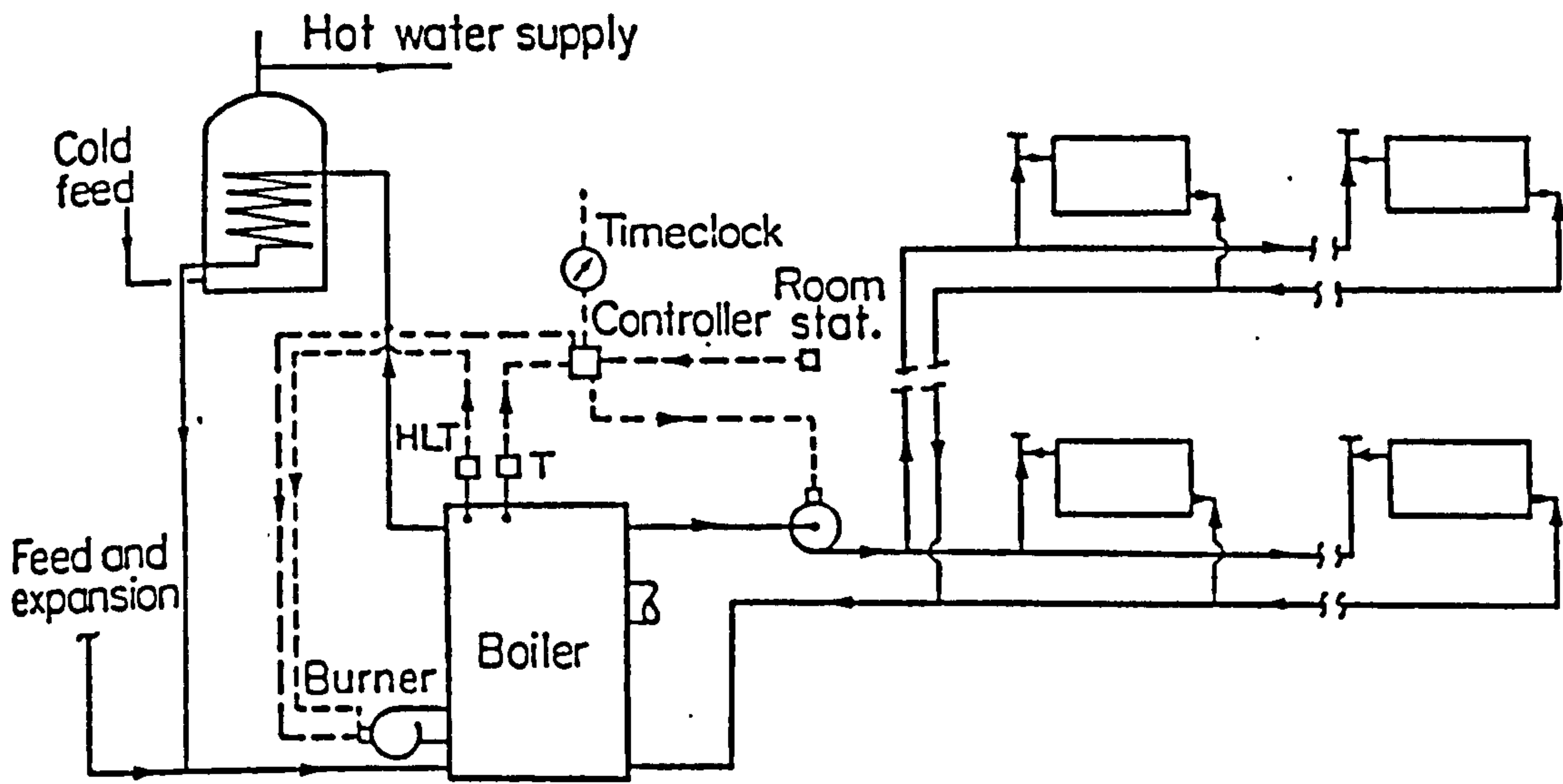


Figure 4.1 Domestic heating system with hot water supply

by breaking down the complete system into sub-systems of component and the state-space representations of sub-system will be derived individually.

In developing the component sub-system models, the time varying nature is represented by taking the thermal physical properties of fluid and heat transfer coefficients as function of time. Thermal physical properties of fluid such as density, specific heat, conductivity, dynamic viscosity, coefficient of expansion are represented by regression equations based on the data in publications. Heat transfer coefficients are represented by theoretical or empirical relations which are usually functions of local temperatures or fluid velocities. The reason of using such relations of heat transfer coefficients is that the proper functional expressions of heat transfer coefficients are not yet existed. Data given by manufactures and design books are not suitable for detailed energy modelling. Since most of the theoretical and empirical relations are derived based on standard conditions, a modification to such relations sometimes has to be made for special conditions. The accurate determination of heat transfer coefficients for special problem can be referred as the main topics in the field of heat transfer.

Although the component sub-system models can be quite different from each other depending on their structures, the technique to develop component sub-system models is unique and straightforward. A component can be divided arbitrarily into control volumes with respectation to the rules of spatial discretization, the component model can then be obtained by applying the basic conservation equations to the relevant control volumes. The complete system model can be constructed straightforwardly by taking together all the component sub-system models and specifying the inter-relationship between components. Finally the system model represented by the system equation can be cast into the non-iterative difference form and readily solved by digital computer for various purposes of simulation.

4.2 The generation of sub-system models

4.2.1 The connection sub-systems

Connections are the most fundamental components in the heating system. According to the structures, it can be divided into two kinds: bare pipe and pipe with thermal insulation. If the thermal storage effect of pipe metal is neglected, it becomes a series of flowing water element which have been studied in the previous chapter. According to its connections with other components, it can be divided as single flow and convergent flow types. Generally, types of control volume involved in this sub-system are: flowing water; lumped metal; insulations lumped or multi-layered.

Based on the derivation of the fluid flow conservation, as a general rule, in any section of pipe, water flow in the control volume water has to be in one direction and flow rate is uniform. Therefore, for convergent or divergent flow, a division has to be made at the joint.

1. Bare pipe

Figure 4.2 shows a pipe of one directional flow. It can be divided into sections along the flow direction with lengths of Δx_{I-1} , Δx_I and Δx_{I+1} . Each section consists of a control volume of water and a metal. Applying equation 3.42 to I-th control volume of pipe metal and applying equation 3.44 to control volume of water, the difference state equation of this sub-system are given as:

a) For water control volume:

$$\begin{aligned} & -a_{I,1}(t+\Delta t)T_{w,I-1}(t+\Delta t) + a_{I,2}(t+\Delta t)T_{w,I}(t+\Delta t) - a_{I,3}(t+\Delta t)T_{b,I}(t+\Delta t) \\ & = a_{I,1}(t)T_{w,I-1}(t) - a_{I,4}(t)T_{w,I}(t) + a_{I,3}(t)T_{b,I}(t) \end{aligned} \quad 4.1$$

b) For metal control volume:

$$\begin{aligned} & -b_{I,1}(t+\Delta t)T_{w,I}(t+\Delta t) + b_{I,2}(t+\Delta t) - b_{I,3}(t+\Delta t)T(t+\Delta t) \\ & = b_{I,1}(t)T_{w,I}(t) - b_{I,4}(t)T_{b,I}(t) + b_{I,3}(t)T_{\alpha}(t) \end{aligned} \quad 4.2$$

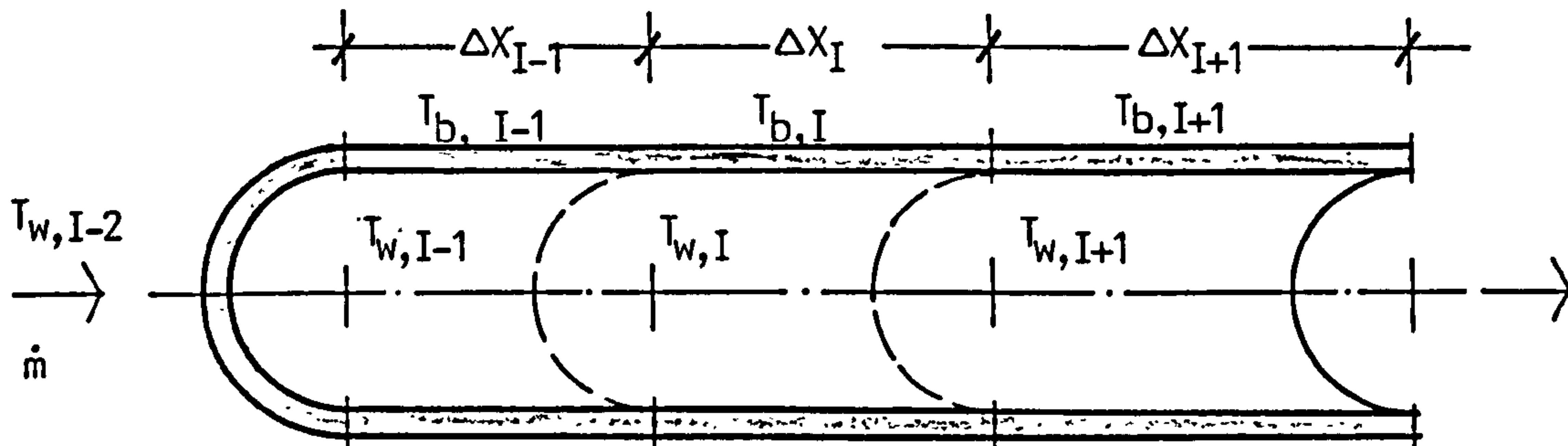


Figure 4.2 Control volume of pipe in one dimensional flow

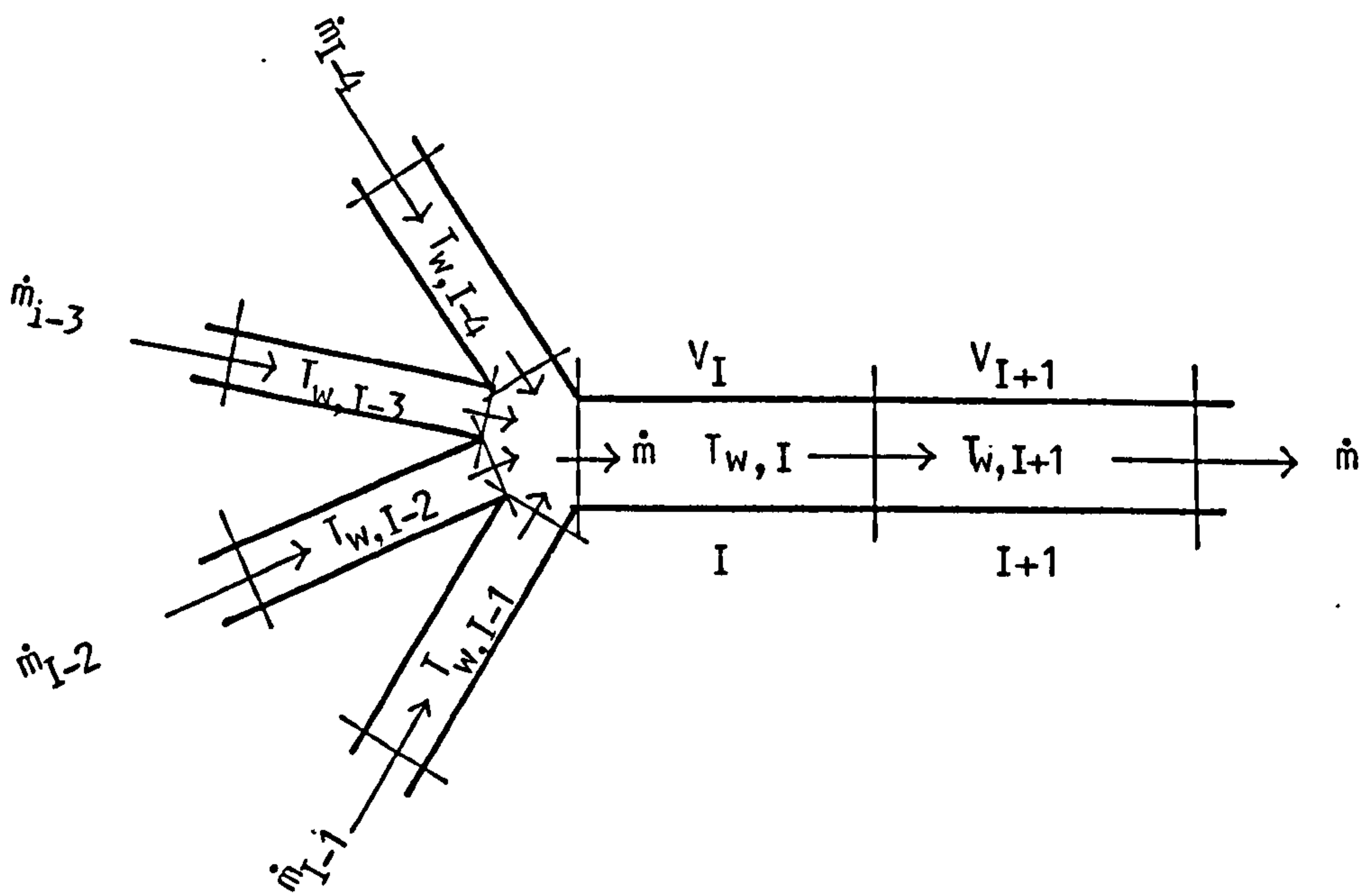


Figure 4.3 Pipe of convergent flow

where,

$T_{w,I}(\xi)$ = water temperature of control volume I, at time ξ ,
 $T_{b,I}(\xi)$ = pipe metal temperature of control I, at time ξ ,
 $a_{I,j}(\xi)$ and $b_{I,j}(\xi)$ are time dependent coefficients representing the dynamics of the relevant control volume.

Same equations can be resulted from the rest control volumes in sequential flow. Equation 4.1 and 4.2 can be written as the standard form of component state equation in matrix form, as:

$$\begin{bmatrix} a_{I,2} & -a_{I,3} \\ -b_{I,1} & b_{I,2} \end{bmatrix} \begin{bmatrix} T_{w,I} \\ T_{b,I} \end{bmatrix}^{t+\Delta t} = \begin{bmatrix} -a_{I,4} & a_{I,3} \\ b_{I,1} & -b_{I,4} \end{bmatrix} \begin{bmatrix} T_{w,I} \\ T_{b,I} \end{bmatrix}^t + \begin{bmatrix} a_{I,1} T_{w,I-1}^{t+\Delta t} + a_{I,1} T_{w,I-1}^t \\ b_{I,2} T_a^{t+\Delta t} + b_{I,3} T_a^t \end{bmatrix} \quad 4.3$$

Here, $T_{w,I-1}$ is temperature of water flowing into this sub-system, it is treated as disturbance. It will become state variable of another component when it is connected with this sub-system.

2. Pipe of convergent flow paths

In some cases, a single pipe will branch into several pipes within the system network. A common application of such arrangement is "Tee junction" in which two branches of flow come into one. In practice, such connection usually is no more than three. Figure 4.3 shows such a situation. Consider the I-th section, control volume equation of pipe metal remains the same as equation 4.2. The difference state equation of control volume water of I-th section becomes:

$$\begin{aligned} & - \left[\sum_{j=1}^4 a_{I,j}(t+\Delta t) T_{w,I-j}(t+\Delta t) \right] + a_{I,5}(t+\Delta t) T_{w,I}(t+\Delta t) - a_{I,6}(t+\Delta t) T_{b,I}(t+\Delta t) \\ & = \left[\sum_{j=1}^4 a_{I,j}(t) T_{w,I-j}(t) \right] + a_{I,7}(t) T_{w,I}(t) + a_{I,6}(t) T_{b,I}(t) \end{aligned} \quad 4.4$$

3. Pipe with thermal insulation

Figure 4.4 shows a section of pipe with thermal insulation.

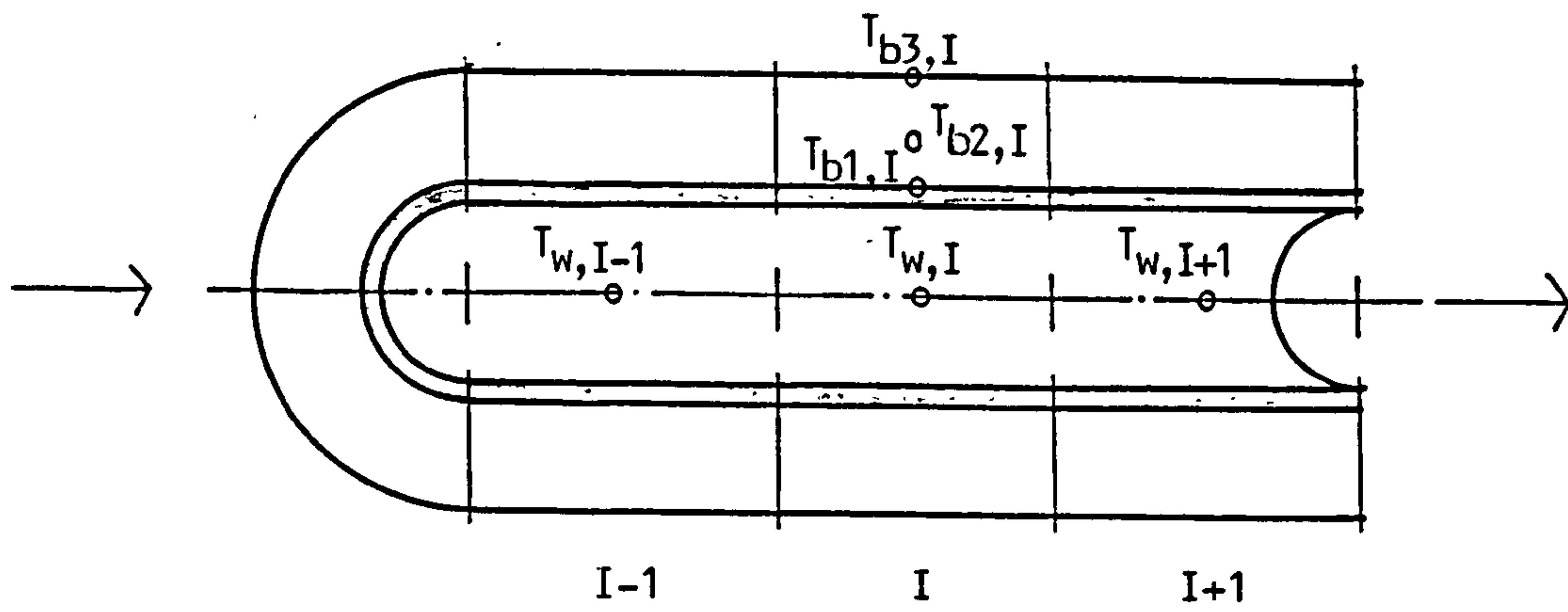


Figure 4.4 Control volume of pipe with insulation

Temperature distribution within the insulation is considered as symmetrical so that each circular layer of insulation has uniform temperature. State equation of control volume water remains the same as equation 4.1 and control volume of pipe metal has similar expression as equation 4.2 with change of disturbance T_a by the temperature of insulation in contact. Control volume of insulation can be represented by one control volume as well as by several layers of control volume. In one control volume, the expression is similar as equation 4.2. For a three control volume case, each section of insulated pipe is represented by four equations, in which the pipe metal and the first layer of insulation are considered as one mixed control volume. State equations of insulation can be obtained by applying equation 3.43 to the control volumes. Equation 3.43, however, has to be changed slightly to cope with the cylindrical coordinate of insulation layers. For the insulated pipe as shown in Figure 4.4, the sub-system state equation is given as equation 4.5.

$$\begin{array}{c}
 \left. \begin{array}{cccc|c}
 a_{I,2} & -a_{I,3} & 0 & 0 & T_{W,I} \\
 g_{I,1} & g_{I,2} & g_{I,3} & 0 & T_{b1,I} \\
 0 & -h_{I,1} & h_{I,2} & -h_{I,3} & T_{b2,I} \\
 0 & 0 & -i_{I,1} & i_{I,2} & T_{b3,I}
 \end{array} \right|^{t+\Delta t} \\
 \\
 = \left. \begin{array}{cccc|c}
 -a_{I,4} & a_{I,3} & 0 & 0 & T_{W,I} \\
 g_{I,1} & -g_{I,4} & g_{I,3} & 0 & T_{b1,I} \\
 0 & h_{I,1} & -h_{I,4} & h_{I,3} & T_{b2,I} \\
 0 & 0 & i_{I,1} & -i_{I,4} & T_{b3,I}
 \end{array} \right|^{t} + \left. \begin{array}{c}
 a_{I,1} T_{W,I-1}^{t+\Delta t} + a_{I,1} T_{W,I-1}^t \\
 0 \\
 0 \\
 i_{I,3} T_a^{t+\Delta t} + i_{I,3} T_a^t
 \end{array} \right|
 \end{array} \quad 4.5$$

4.2.2 The radiator sub-system

Radiators are the most important heat emission devices in heating system. It conveys heat energy from the working substance, water, to the living space. The types of radiator are various. According to the ways of heat emission, it can be divided to two types: (1) convective radiator, and (2) radiative radiator. In low temperature water system, the use of convective radiator is prevailed. The higher the temperature of working substance, the more heat energy will be emitted by means of radiation. The mechanical structures of radiator are also quite different from each other. Generally, it can be considered as a combination of finned or un-finned pipes. Control volume types involved in this sub-system are same as in the connections.

Since most of the radiators have finned surface, both convective and radiative heat transfer are enhanced comparing to the bare surface.

Figure 4.5 shows a parallel flow, flat plate type radiator in which the water flow rate in each vertical column is the same. The columns are tubes with vertical fins while the horizontal pipes at top and bottom are the connections of these vertical columns. According to the hydraulic nature of parallel flow, the water flow rate in each vertical column is the same.

(1) The original model

The original model is developed based on the consideration that in each control volume of flowing water, the water flow rate is constant. To develop the radiator sub-system model, a sub-division is made for the radiator as Figure 4.6. Each section in Figure 4.6 consists of a control volume of water and a control volume of radiator shell which can be either finned or unfinned surfaces. The total number of control volume for a radiator with m columns is $n = 5m$. Applying the basic control volume conservation to the relevant control volume, the component model can be obtained directly. The resulted equations of each section are similar as connections of equation 4.1 and 4.2. For

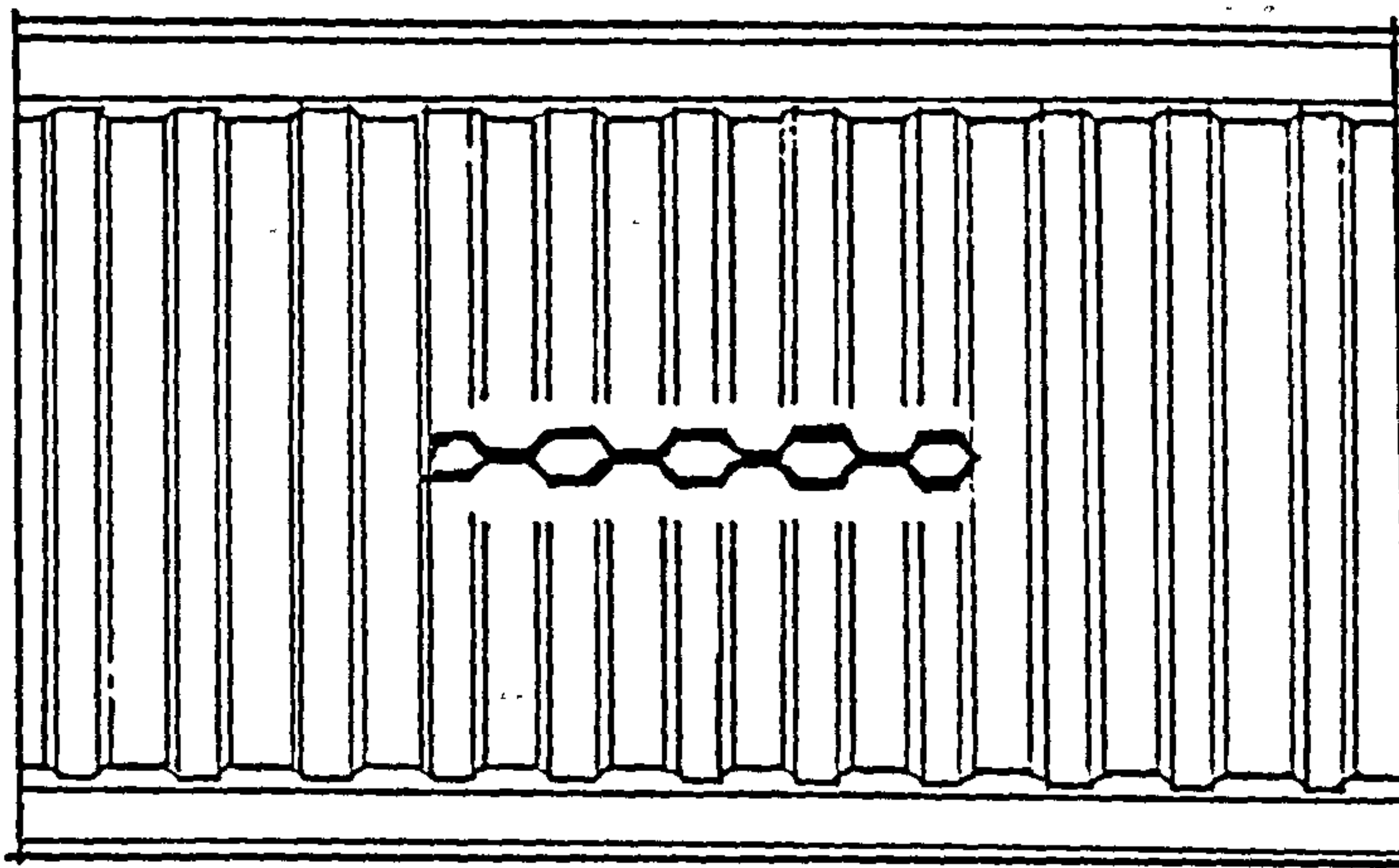


Figure 4.5 Flat plate, parallel flow radiator

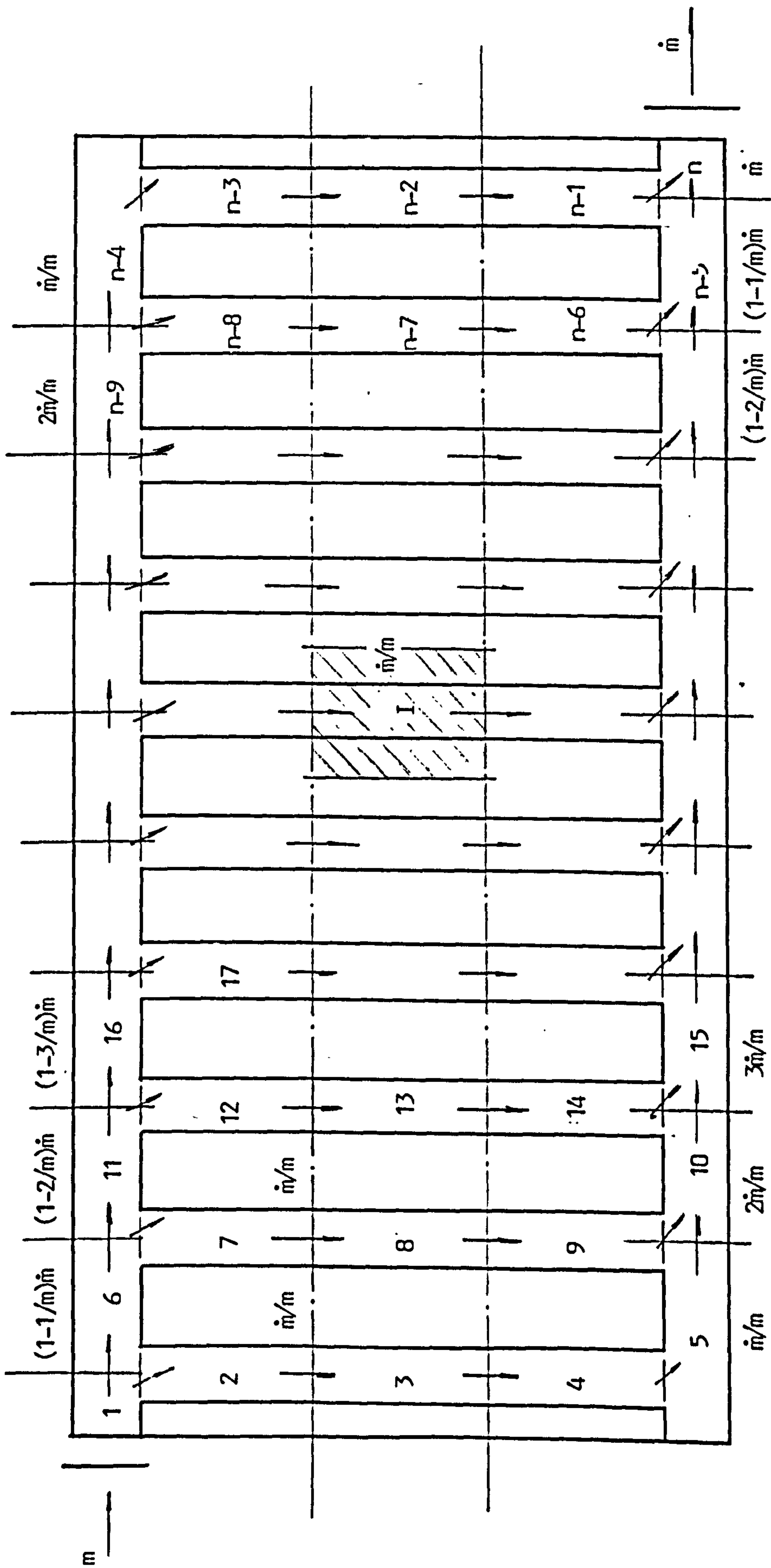


Figure 4.6 Water flow rate distribution of radiator

the radiator shell with fins, the total heat transfer coefficient is replaced by an enhanced coefficient, given by:

$$h_T^* = \frac{1}{A_{Tot}} \left(h_{rc} A_o + 2 \sqrt{h_{rc} C k A_f} t h m l \right) \left(1 + \frac{h_r}{h_{rc}} \frac{T_a - T_e}{T_b - T_a} \right) \quad 4.6$$

where,

A_o = surface area of finned tube base

C = fin parameter

k = conductivity of fin material

A_f = fin cross sectional area

l = fin length

$h_{rc} = h_c + h_r$; sum of convective and radiative heat transfer coefficients

$m = (h_T C / k A)^{\frac{1}{2}}$

A_{Tot} = total external surface area of finned pipe

T_a = room air temperature

T_s = finned tube base temperature

T_e = equivalent room surfaces mean temperature

$T_e = \left(\frac{1}{2} \sum_{i=1}^n f_{s,i} T_{R,i} \right)^{\frac{1}{4}}$

where,

f_{s-i} = view factor between radiator surface to i-th room internal surface

$T_{R,i}$ = temperature of i-th room internal surface

Total heat transfer coefficient of finned surface h_T^* can be used to describe the fin effect and radiative heat exchanges between radiator surface to different surfaces of room.

The component sub-system model can be represented as a standard form of difference equation by:

$$A_R(t+\Delta t)T_R(t+\Delta t) = B_R(t)T_R(t) + C_R(t+\Delta t, t) \quad 4.7$$

The order of matrix $A(t+\Delta t)$ and $B(t)$ are $(2n \times 2n)$. Solution of equation 4.7 will give a detailed temperature distribution in the radiator.

(2) The simplified model

The original radiators model is relatively complicated, and not suitable for large system modelling in practice. It has to be simplified.

The assumptions for the simplification are:

- a) The temperature of water entering each column of the radiator is same, so that each horizontal level in the radiator can be represented by one temperature, and the water temperature in the top and bottom sections are considered to be uniform respectively.
- b) As the velocity of water entering the top and bottom sections vary from place to place, a mean velocity is introduced. It equals to the mean value of the velocity in the inlet (or outlet) position and the velocity in the end position of each section. For a m columns radiator with inlet mass flow rate of \dot{m} , the mean velocity can be determined by:

$$U_m = \left(\frac{1+m}{2m} \right) \dot{m} / (\text{density} * \text{cross sectional area}).$$

The radiator is divided into five sections: the top and bottom horizontal sections and the middle part vertical columns are horizontally divided three sections. A schematic control volume representation of this radiator is shown in figure 4.7.

Applying the basic control volume conservation equations to the control volume of water and radiator shells, the component state equation can be obtained as:

$$A_R(t+\Delta t)T_R(t+\Delta t) = B_R(t)T_R(t) + C_R(t+\Delta t, t) \quad 4.8$$

Matrix $A_R(t+\Delta t)$ and state-vector T_R are detailed as figure 4.8.

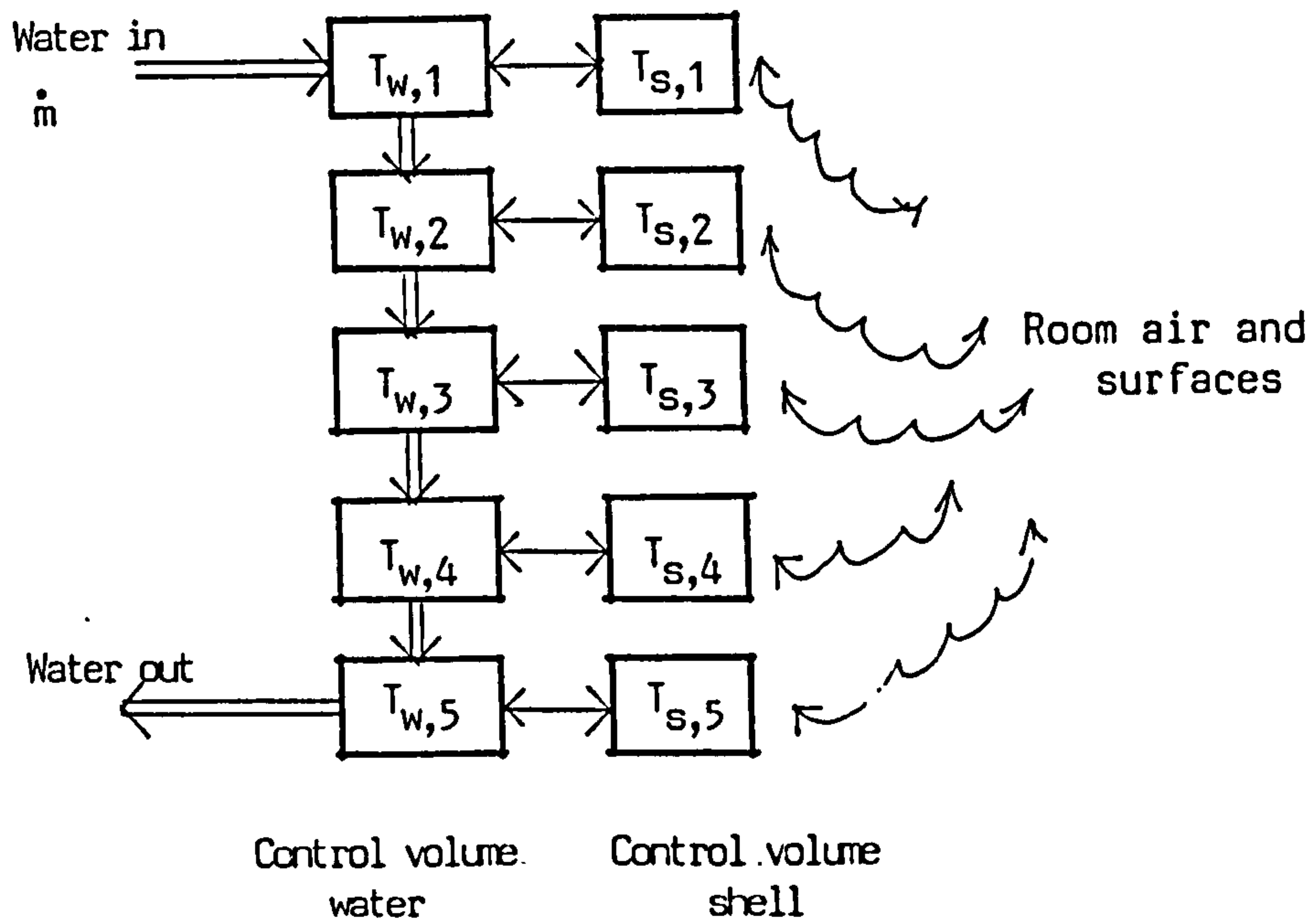


Figure 4.7 Control volume of radiator

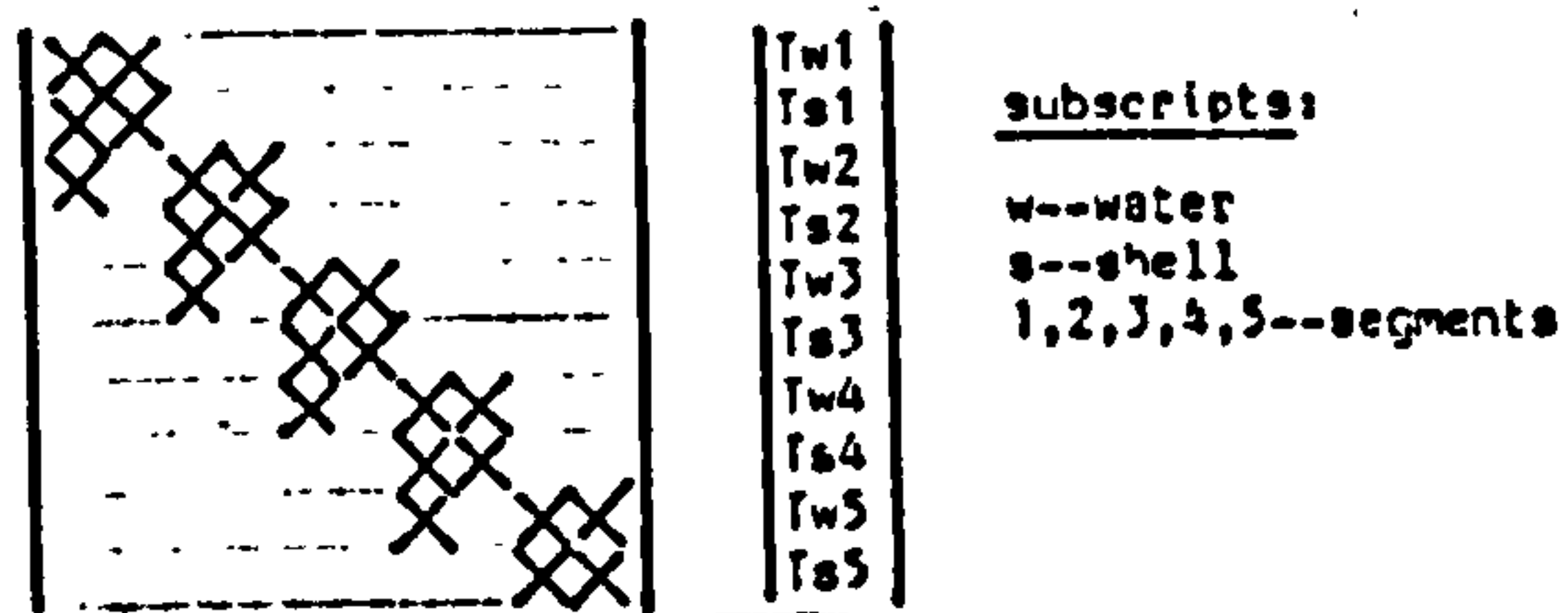


Figure 4.8 Coefficient matrix and state vector of radiator model

4.2.3 The boiler sub-systems

Boiler is the heat production unit in the plant system. It converts the fuel into heat energy and to water supplied to the system. The structures of boiler are quite different from each other, generally, a boiler usually consists of a combustion space, container of water and flow gas passage. Several different boilers are going to be discussed as follows to develop different boiler models.

1. Vertical tubless boiler

Figure 4.9 shows the structure of a steel shell, vertical type, tubless boiler. It is slightly simplified based on the structure of a real boiler. Some assumptions are introduced as follows:

- a) For domestic heating, the temperature of water leaving the boiler is always lower than its saturation temperature at atmospheric pressure, therefore, no evaporation taking place.
- b) The combustion process taking place in the furnace is considered as ideal combustion. The temperature of combustion products in the combustion chamber is uniform.
- c) The bottom of the combustion chamber is well insulated so that no heat leakage from the bottom to its surroundings.
- d) The average temperature of flue gas is considered as same as the temperature of combustion products in the combustion chamber.
- e) The heat source in the combustion chamber is considered as a radiating plane parallel to the cold surface. End effects are eliminated by assuming that the distance between the heat source and cold surface is very small compared to the extent of the surface.
- f) Forced convective subcooled boiling heat transfer takes place

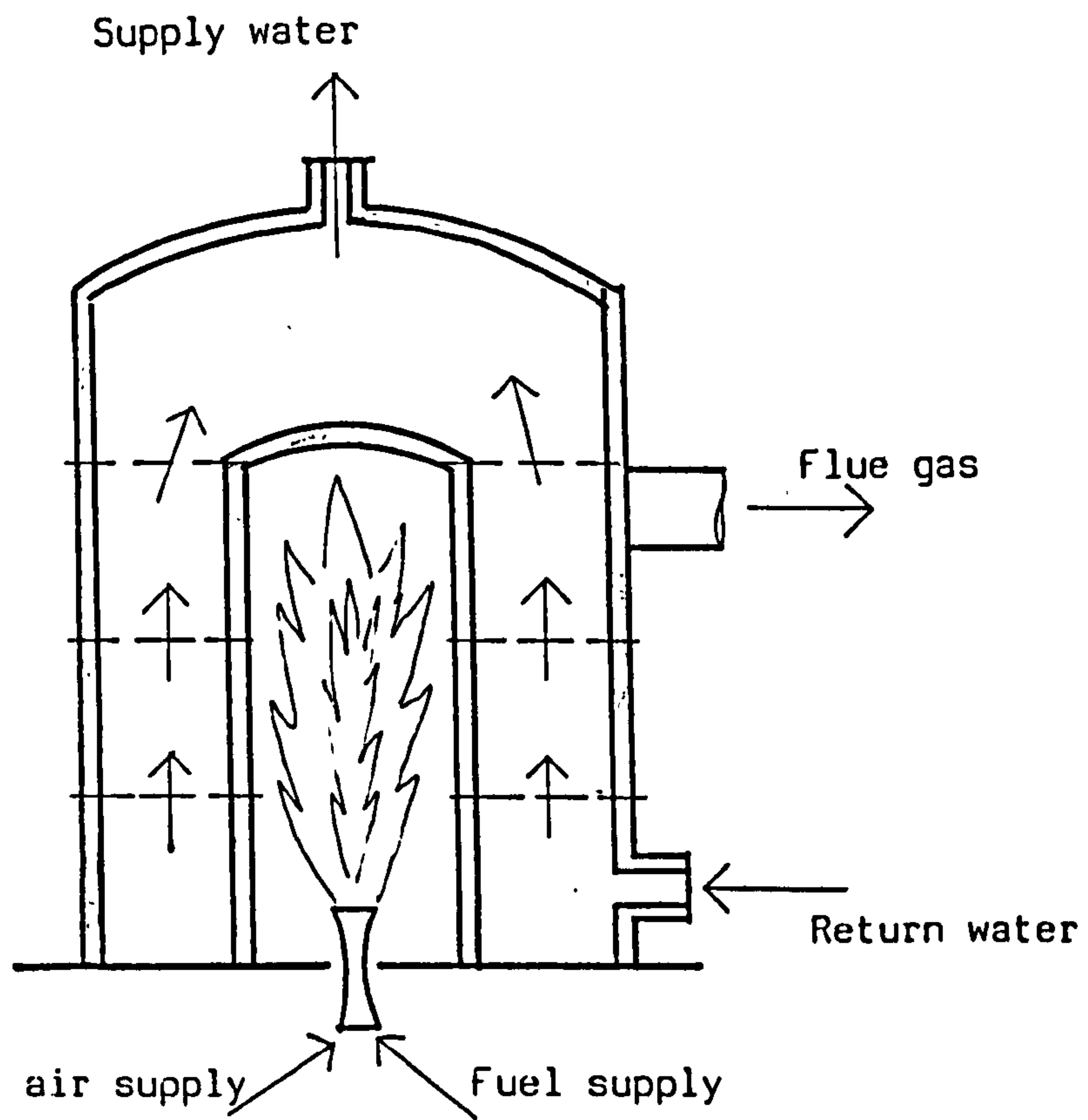


Figure 4.9 Structure of a steel shell, vertical type tubeless boiler

in the water side of the inner shell of the boiler when the inner shell temperature is higher than the water saturation temperature. It can be expressed by the relation of:

$$\left(\frac{q}{A}\right)_{TOTAL} = \left(\frac{q}{A}\right)_{\substack{\text{subcooled} \\ \text{boiling}}} + \left(\frac{q}{A}\right)_{\substack{\text{forced} \\ \text{convection}}}$$

To develop the boiler model, the spatial subdivision is made along the water flow direction. The boiler is divided to four sections, each section consists of a control volume of inner shell separating the water from the combustion product; a water volume and a outer shell volume which contacts to the room air. The combustion chamber itself is considered as a single control volume. Figure 4.10 gives the schematic diagram of control volumes and the thermal paths.

Applying the basic control volume conservation equations to the relevant control volumes, the state equation of this kind of boiler is completed in which the variables are taken as the temperatures of combustion products, inner shell, water, outer shell in each section. The fuel supply rate here is treated as disturbance. It can also be taken as variable if the extra relations of control is given to monitor the fuel supply.

The boiler state equation of difference form can be represented as the standard form of matrix notation, as:

$$A_b(t+\Delta t)T_b(t+\Delta t) = B_b(t)T_b(t) + C_b(t+\Delta t, t) \quad 4.9$$

The matrix $A_b(t+\Delta t)$ and vector $T(t+\Delta t)$ are detailed in figure 4.11.

2. Cast iron boiler

Figure 4.12 shows a domestic cast iron boiler (based on the manufacture produced of "Baxi Heating". This boiler consists of a set of finned tube-like heat exchangers made of cast iron. Combustion space is at lower part of the boiler. Heat exchangers are parallelly

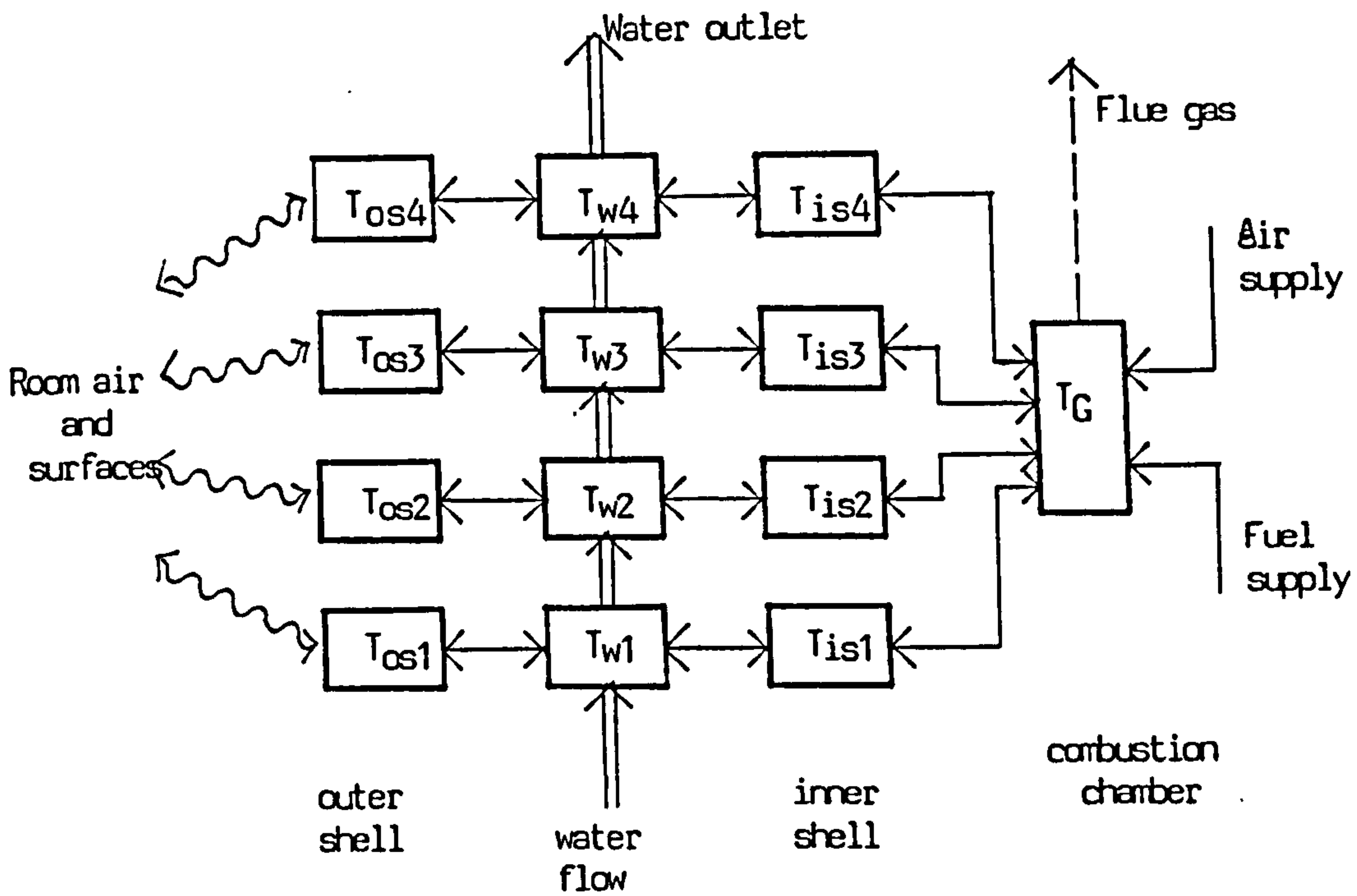


Figure 4.10 Control volume of the steel shell boiler

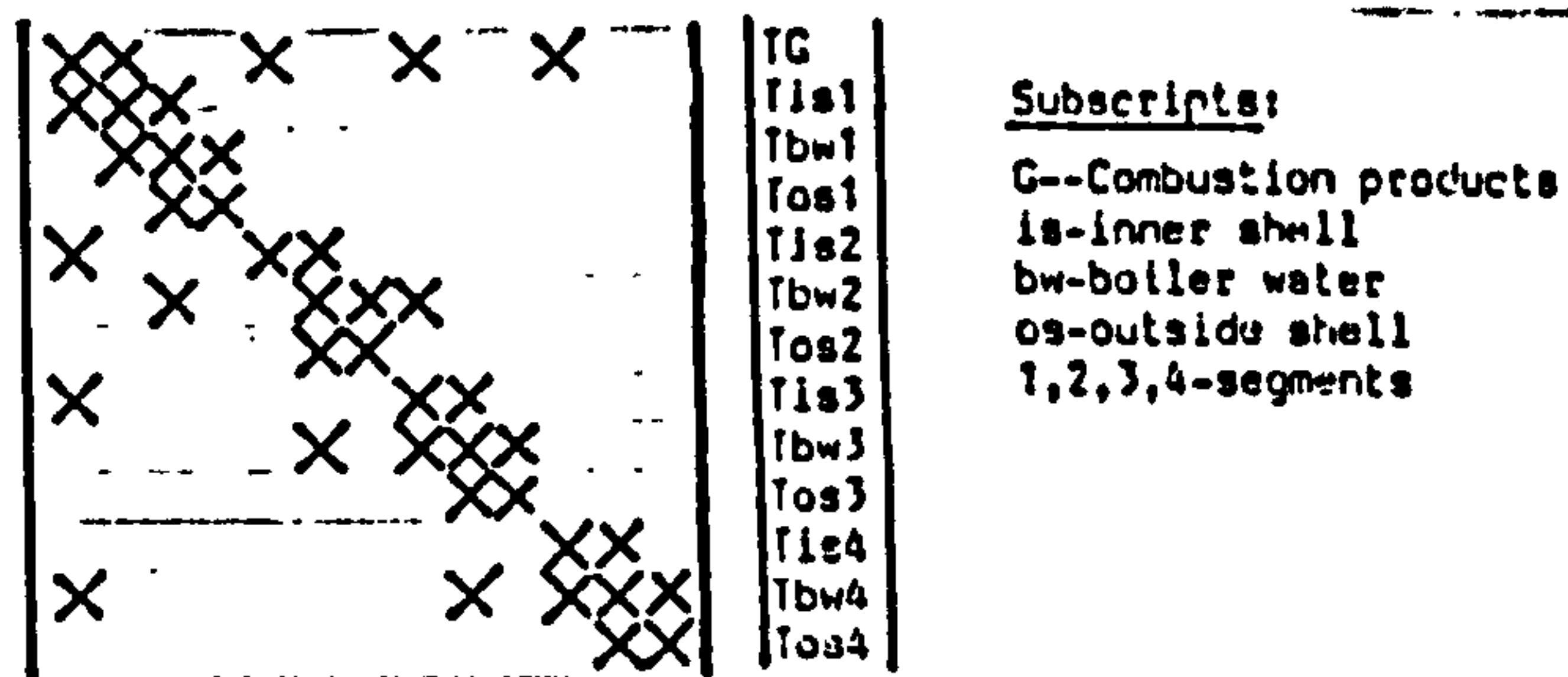


Figure 4.11 coefficient matrix and state vector of steel shell boiler model

connected. The spatial subdivision is made along the water flow direction and the control volumes are obtained as: a combustion space, four water elements, water containers and flue gas spaces. Heat exchange in combustion space is only taken place to the first two water containers. Flue gas in each section has heat exchange with the exchanger in the same section. A conceptional diagram of control volumes, water and flue gas flow and heat exchange paths is shown as Figure 4.13.

The boiler now is readily to be modelled by applying the basic control volume equations to the relevant types of control volume. The matrix equation for this boiler also has the standard form as equation 4.9. Matrix $A_b(t+\Delta t)$ and state vector $T_b(t+\Delta t)$ are detailed in Figure 4.14.

The defining of control volumes for a component is not unique. It is totally depended on the level of accuracy required for the component modelling. Different spatial subdivision can result a component model in different expressions of mathematics. However, neglect the spatial discretization error, the results of modelling should be the same. The most important thing for defining the control volumes is that the heat exchange paths have to be properly defined.

Another way to define the control volumes for this cast iron boiler can be done by dividing the heat exchanger into two parts, the lower part contacting the combustion space can be considered as a cooling surface of the combustion chamber, the upper part has not contact with combustion chamber, it can be considered as a pure convective heat exchanger. A schematic diagram of such division is shown as Figure 4.15 and Figure 4.16 gives the coefficient positions in the coefficient matrix and the state variables of the vector.

3. Condensing boiler

The idea of using a pre-heat section to recover heat from flue gas is not new, it has been used for long time in large industrial boiler design. Normally, in such design, flue gas is at temperature higher than its dew point to prevent condensation as well as corrosion

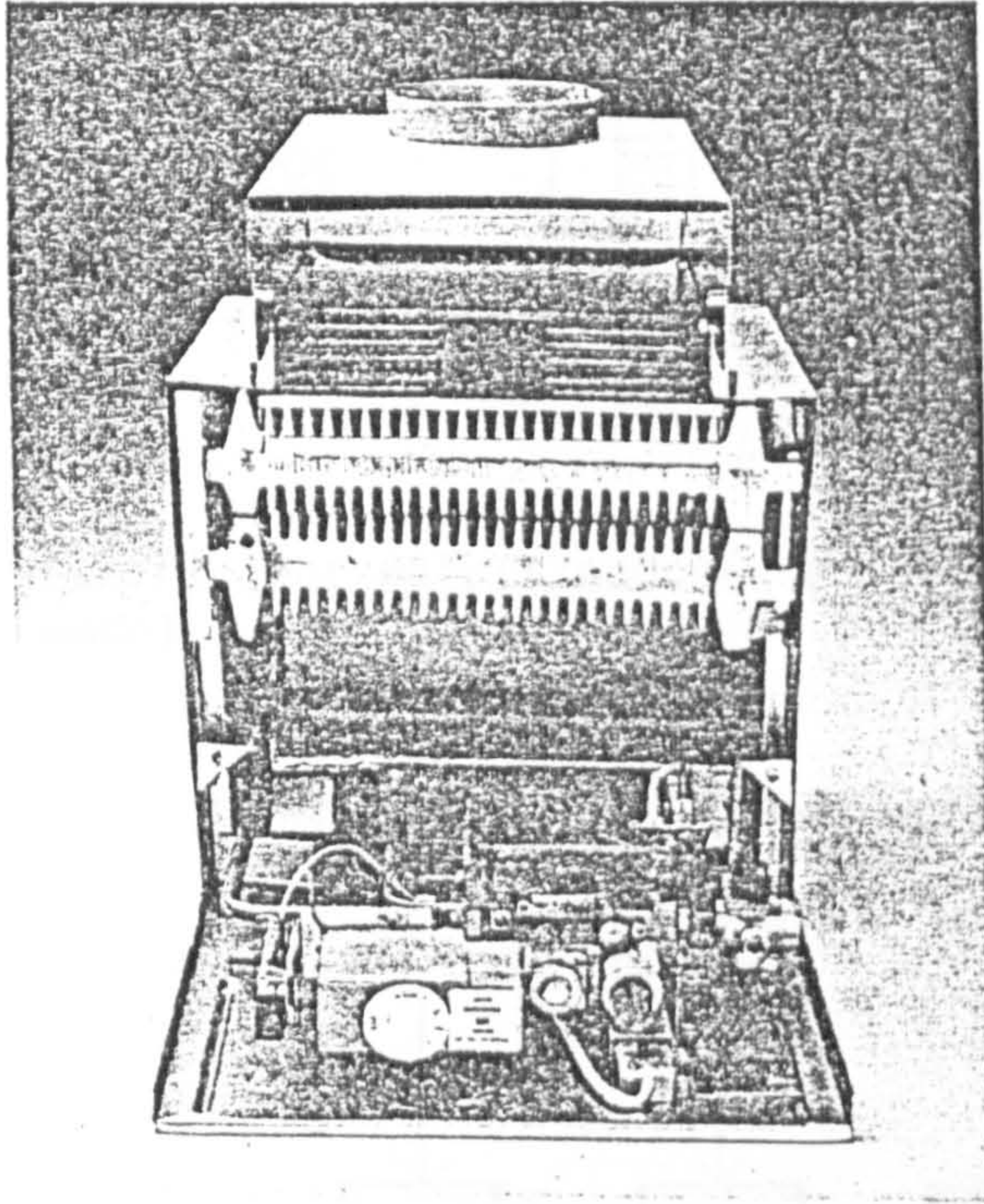


Figure 4.12 Domestic cast iron boiler (the manufacture products of "Baxi Heating")

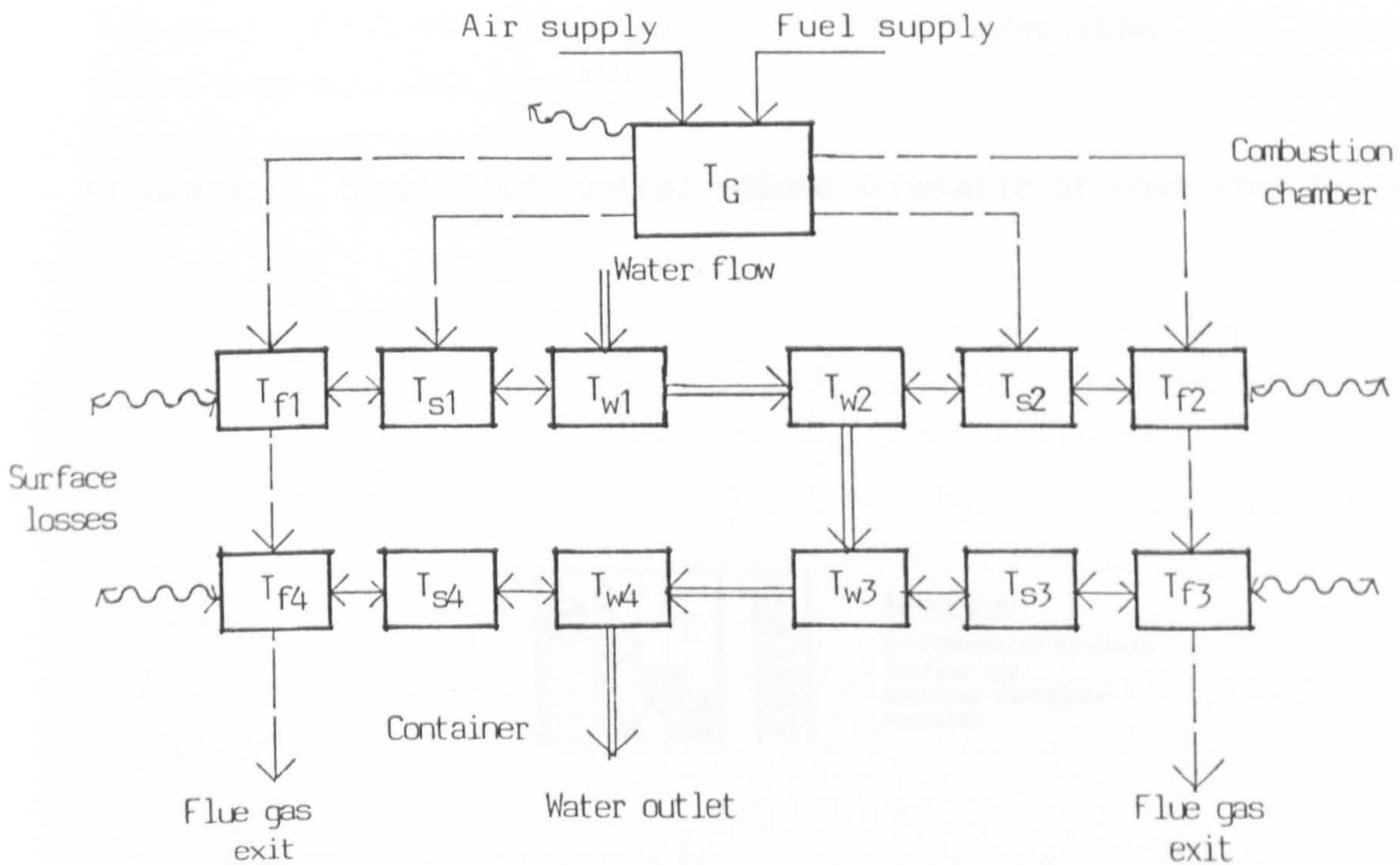


Figure 4.13 Control schematic of cast iron boiler

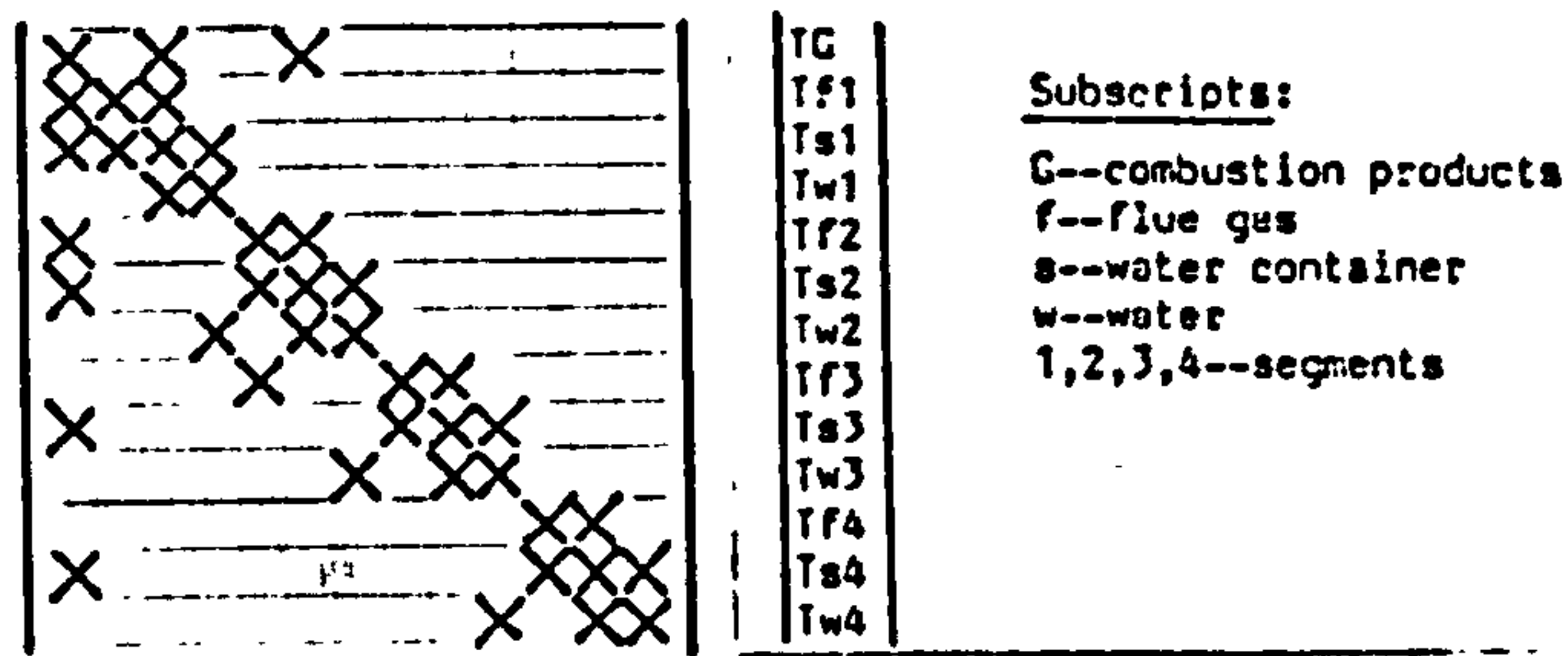


Figure 4.14 Coefficient matrix and state vector of cast iron boiler model

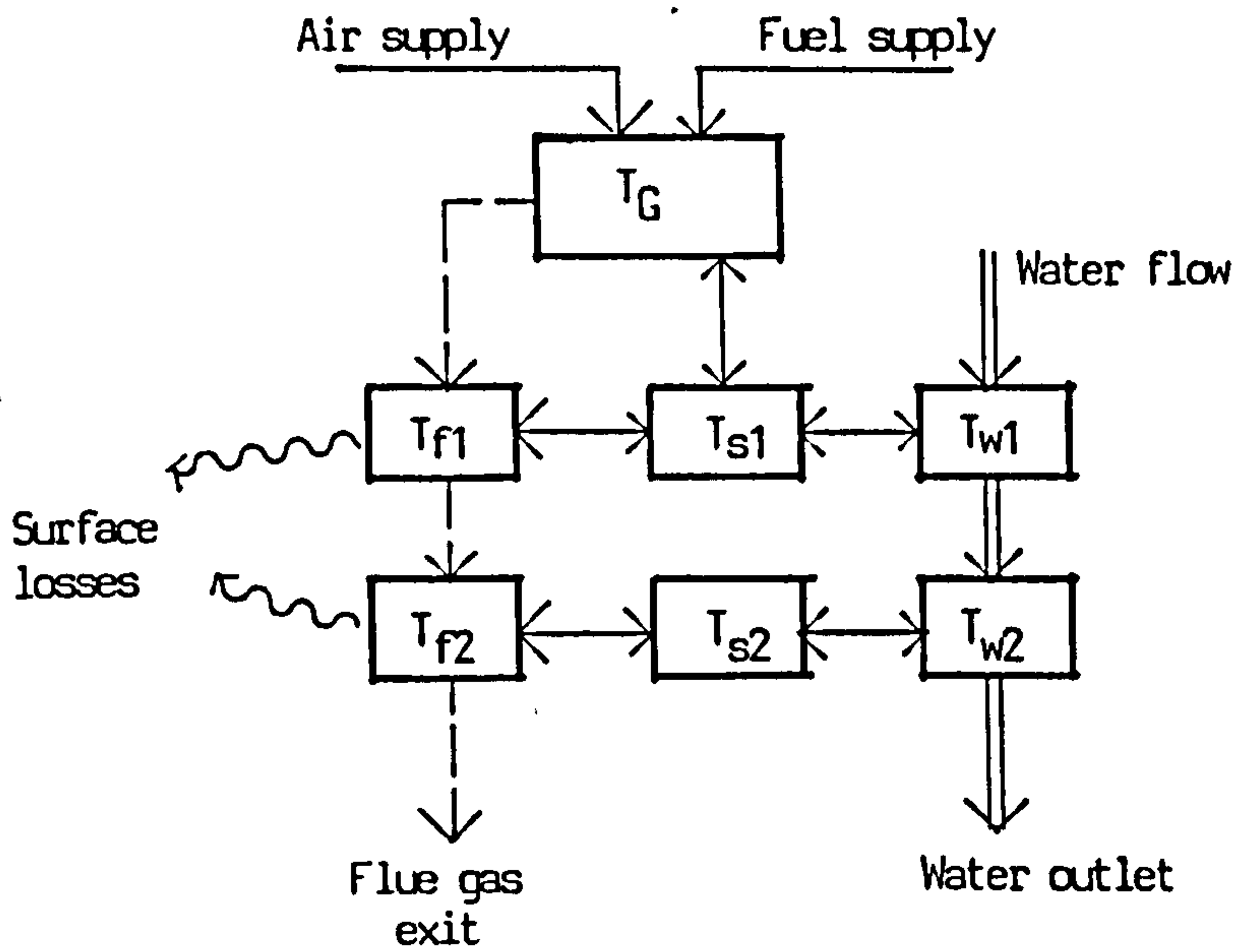


Figure 4.15 Simplified control volume schematic of cast iron boiler

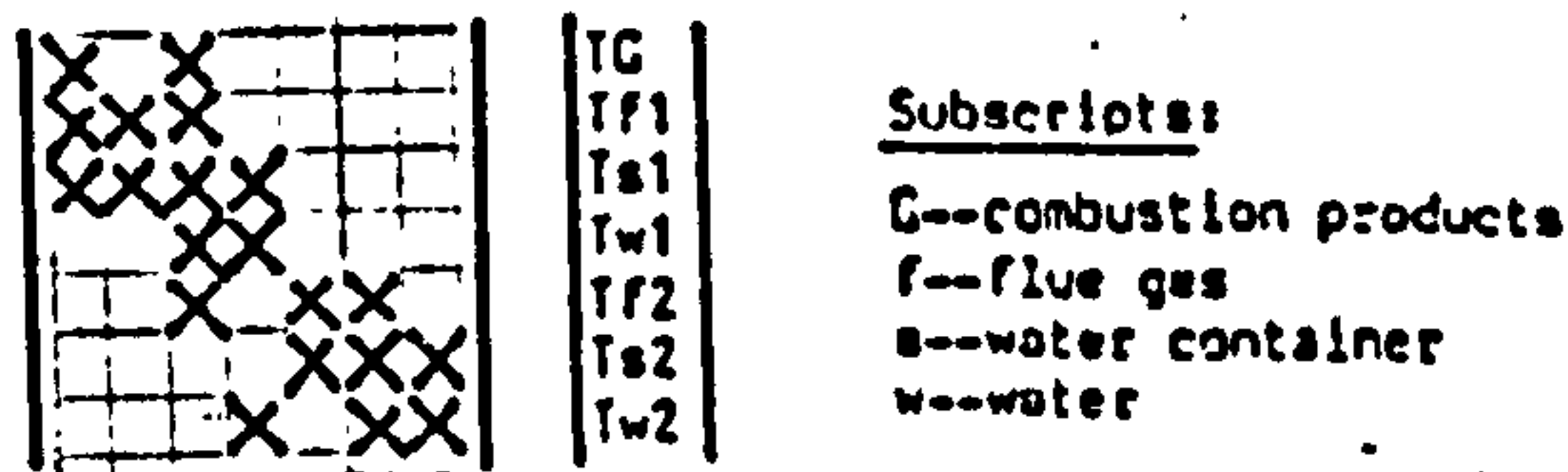


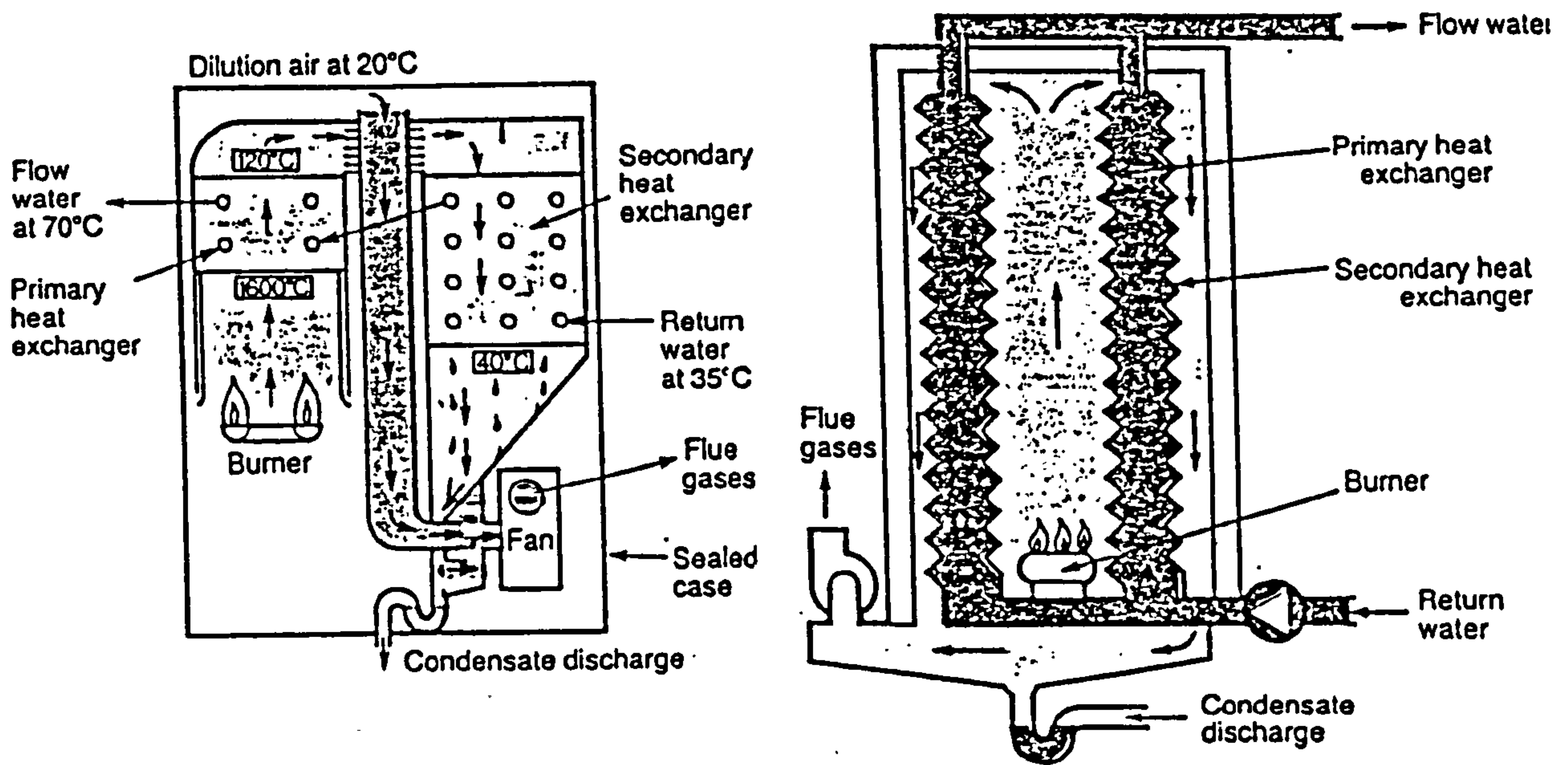
Figure 4.16 coefficient matrix and state vector of simplified cast iron boiler model

problems occurred. Recently, with the improvement in material and the importance of energy conservation, such designs start to be introduced into the field of domestic heating. Domestic condensing boiler is based on this idea which uses an extra heat exchanger in the flue gas passage to cool the flue gas temperature down below its dew point so as to extract the sensible and latent heat from the flue gas. Researches have found that by using this method, great amount of heat losses can be recovered and the total thermal efficiency of boiler can lead up to the range of 90 - 97% on gross calorific value or 100% on nett calorific values of fuel provided that the flue gas temperature is reduced to the temperature lower than the dew point of flue gas. Fundamentally, a condensing boiler can be considered as a conventional boiler plus a heat exchanger working at wet condition placed in the flue gas passage.

Figure 4.7 shows three different types of condensing boiler used in domestic heating. The structure of condensing boiler can be found as two main types: (1) an extra heat exchanger is used in the flue gas passage to recover the sensible and latent heat, such as Figure 4.17a and c. (2) One heat exchanger has the dual role of extracting both sensible and latent heat from the combustion chamber and the flue gas passage, such as in Figure 4.17b. The former type can be considered as a conventional boiler plus a heat exchanger and so the model can easily be achieved by combining the models of a conventional boiler and a heat exchanger working side by side.

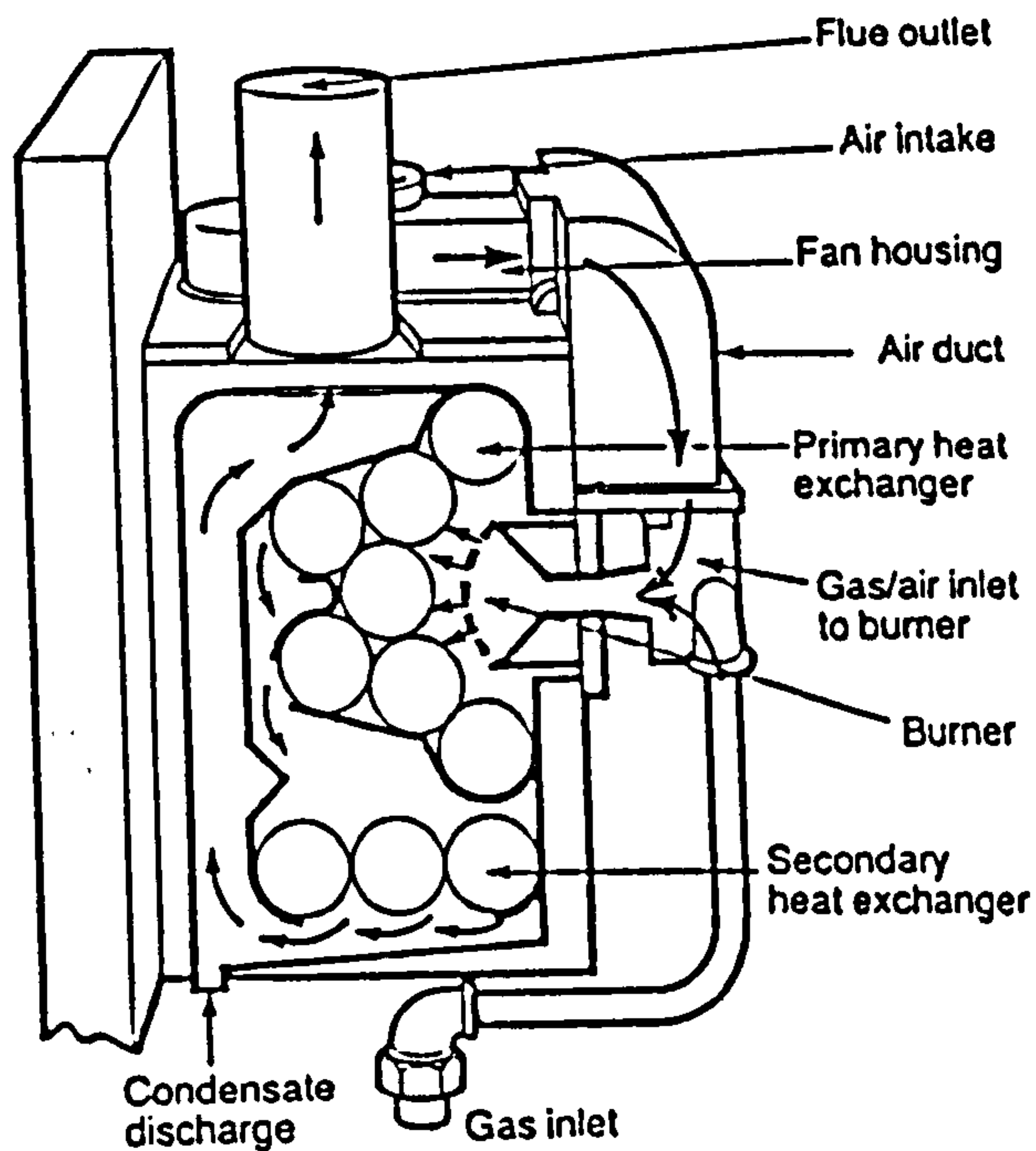
To develop the model of condensing boiler, the consideration different from the conventional boiler model is that the condensation heat transfer has to be considered on the side of secondary heat exchanger. This can be treated by using the condensation heat transfer coefficient to replace the sensible heat transfer coefficient on the heat exchanger surfaces. Such condensation heat transfer coefficients for regular surfaces can be found in various heat transfer publications.

To develop the condensing boiler model as Figure 4.17b, the spatial discretization is made along the water flow direction. Figure 4.18 shows the schematic control volume diagram of this boiler which is



(a) Schematic design of the Watson House two exchanger design option for a condensing boiler.

(b) The design of a condensing boiler of similar concept to that in figure 3 but produced by a manufacturer.



(c) A forced draught design of condensing boiler at present produced by a manufacturer. It utilises the option of a dual role arrangement in the design of such boilers for the domestic market.

Figure 4.17 Condensing boilers

(after M. White, "Condensing choice", J.CIBS, 1983')

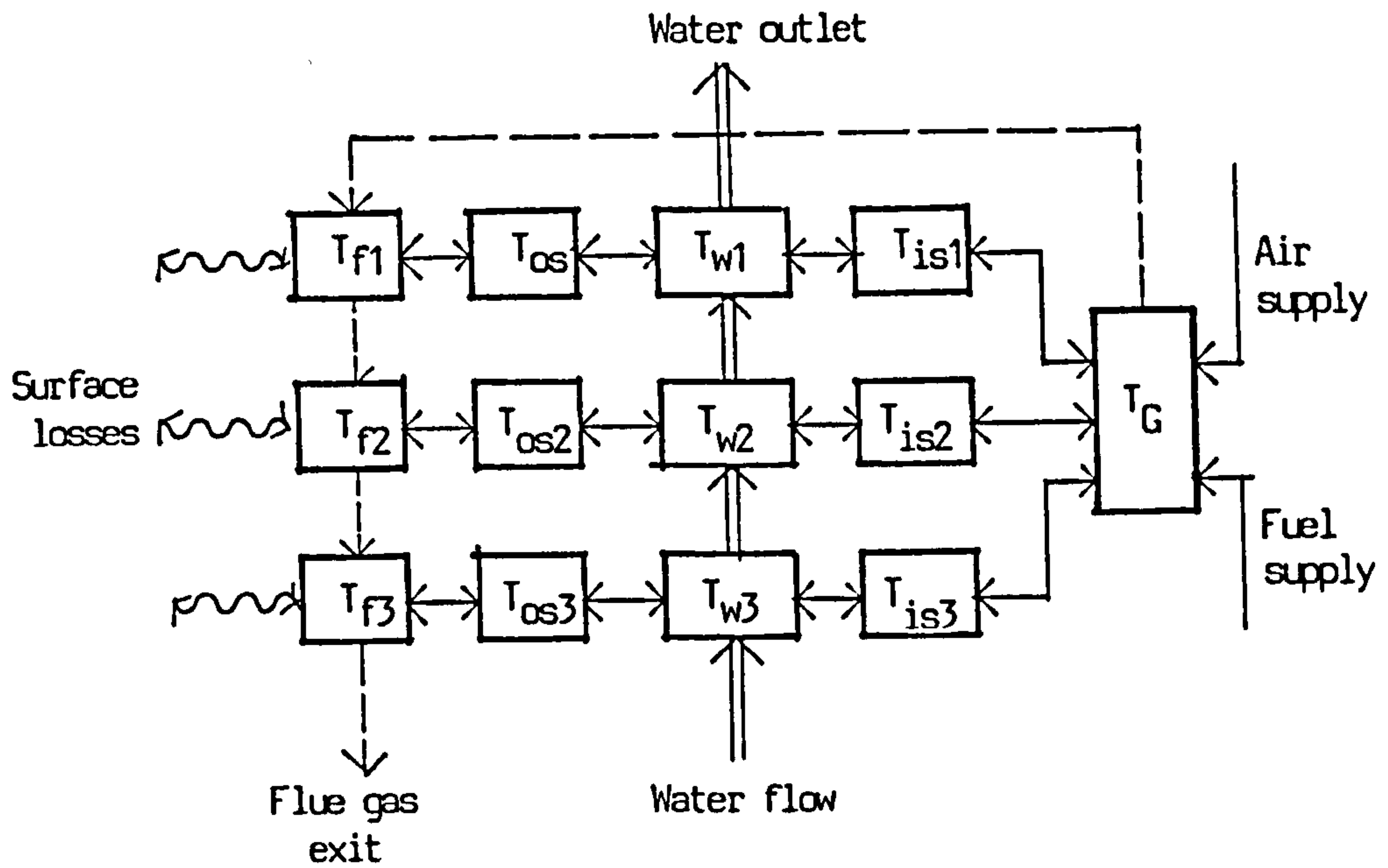


Figure 4.18 Control volume schematic of the condensing boiler

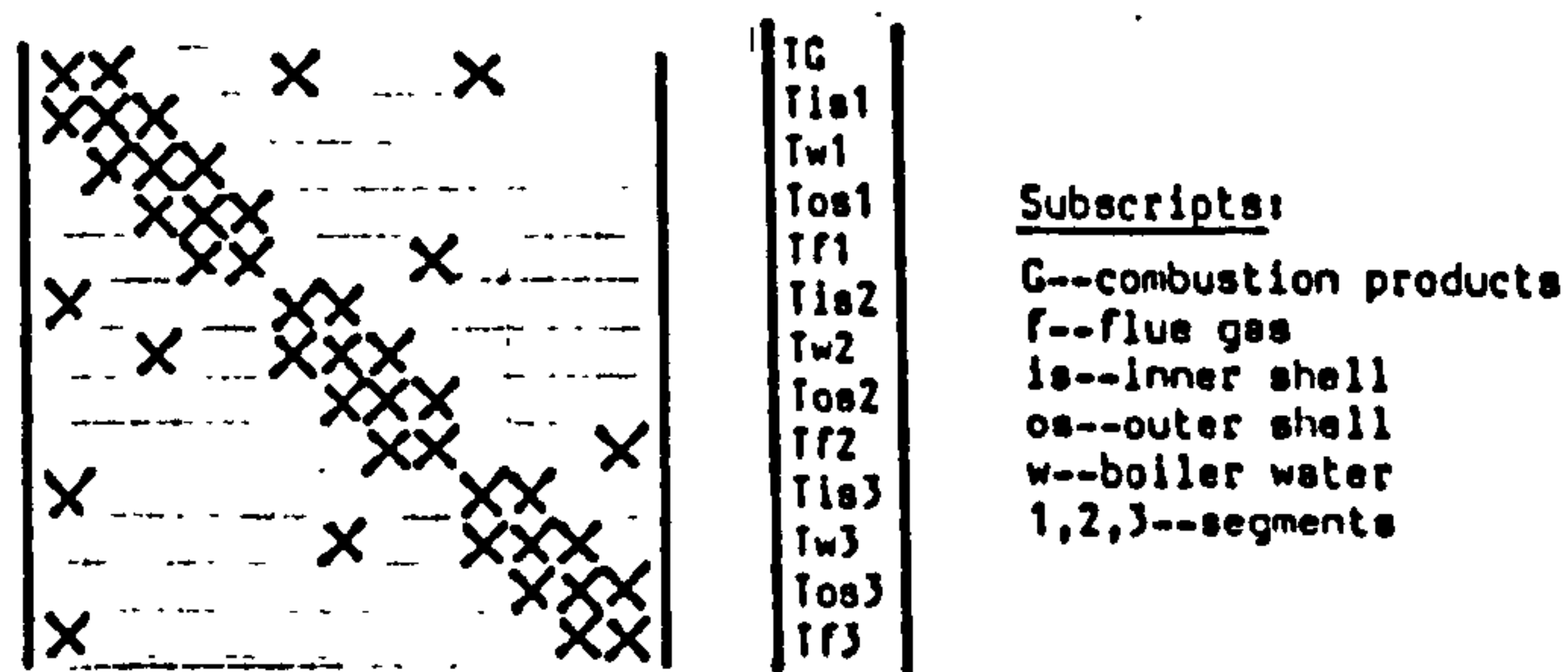


Figure 4.19 Coefficient matrix and state vector of the condensing boiler model

divided into three sections, each section consists a control volume of inner shell , water, outer shell and flue gas, the combustion chamber is taken as a unit. From Figure 4.18, the heat flow paths for each control volume can be easily defined. Applying the governing equation to each of the relevant control volume, the difference state equation for this condensing boiler can be obtained as the standard expression. Coefficients position in matrix $A(t+ t)$ and $B(t)$ and the state variables in vector T are given in Figure 4.19.

Until now, three kinds of boiler model have been derived. Because of the varieties of boiler structure, no general boiler model can be developed to satisfy all kinds of boiler. On the other hand, it is impossible and unnecessary to derive models for all different kinds of boiler. However, the technique of constructing the component models by control volume conservation is unique. Any boiler model (or other components) can be obtained by:

- (1) breaking down the boiler (or other component) into control volumes;
- (2) defining the state variable for each control volume.
- (3) applying the control volume conservation equations to the relevant control volumes and collecting them to form the boiler (or component) model.

The arrangement of state variables in state vector is arbitrary. However, it is usually conceptually better to arrange them in the sequence of main working substance (i.e. water) flow direction.

The existing boiler models also can be used to model other boilers having different structure provided that a boiler to be modelled can be broken down into control volumes to suit the schematic structure of control volumes and heat exchange paths of an existing boiler model. In such case, the new boiler will have the same expression of state equation as the existing one, only the coefficients in the coefficient matrices have to be computed differently according to the structure difference.

As an example, a typical commercial boiler as shown in Figure 4.20 can

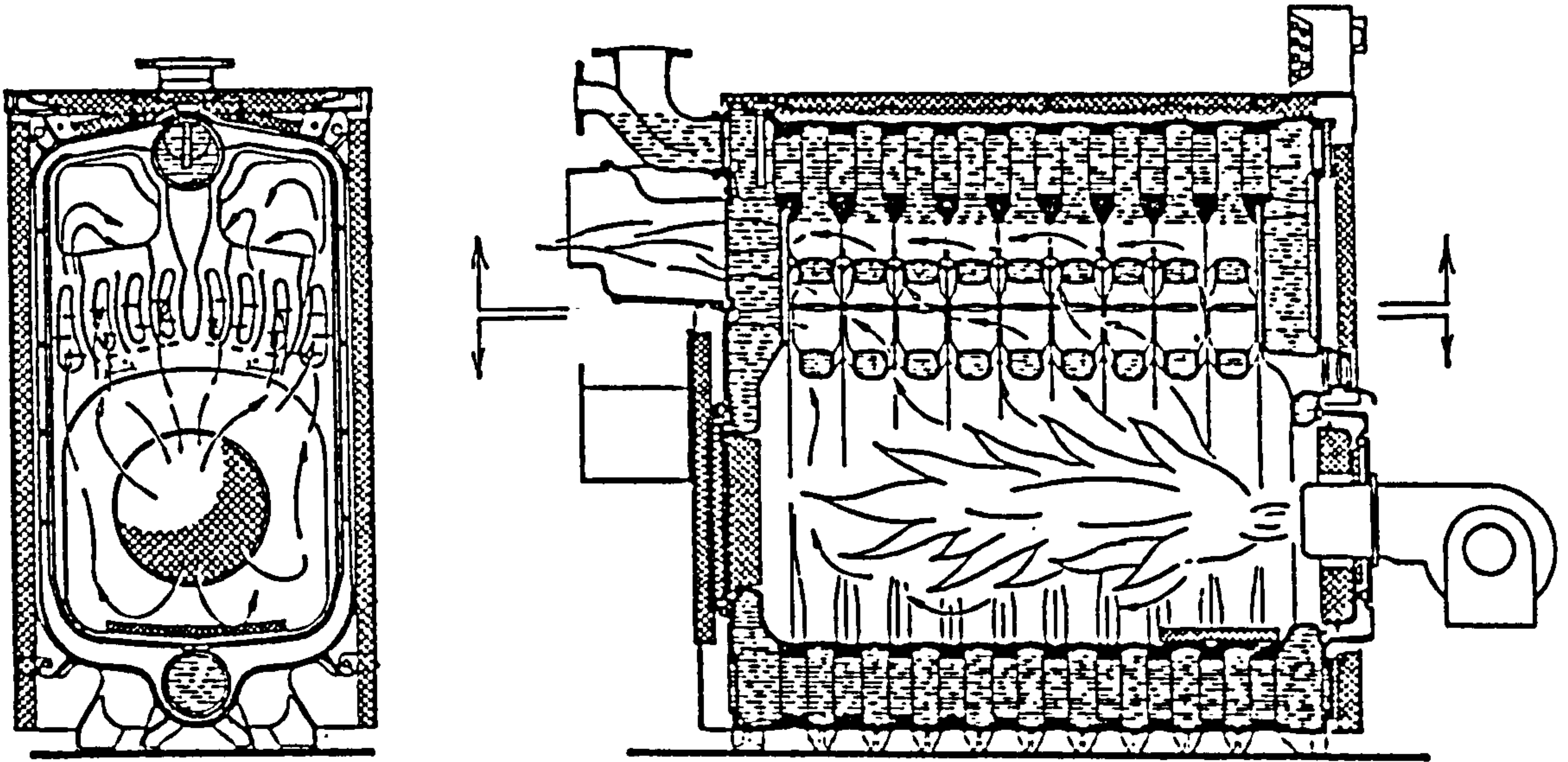


Figure 4.20 Example of a commercial cast iron boiler

be modelled by using the model as derived previously as the cast iron boiler. The control volumes can be defined by dividing this boiler into two parts, the lower part directly contacts to the combustion chamber and the upper part acts as a convective heat exchanger. By such division, its control volumes and heat exchange paths can be cast into the control volume scheme as figure 4.13, and the state equation as figure 4.15 can be directly used.

4.2.4 The pump sub-system

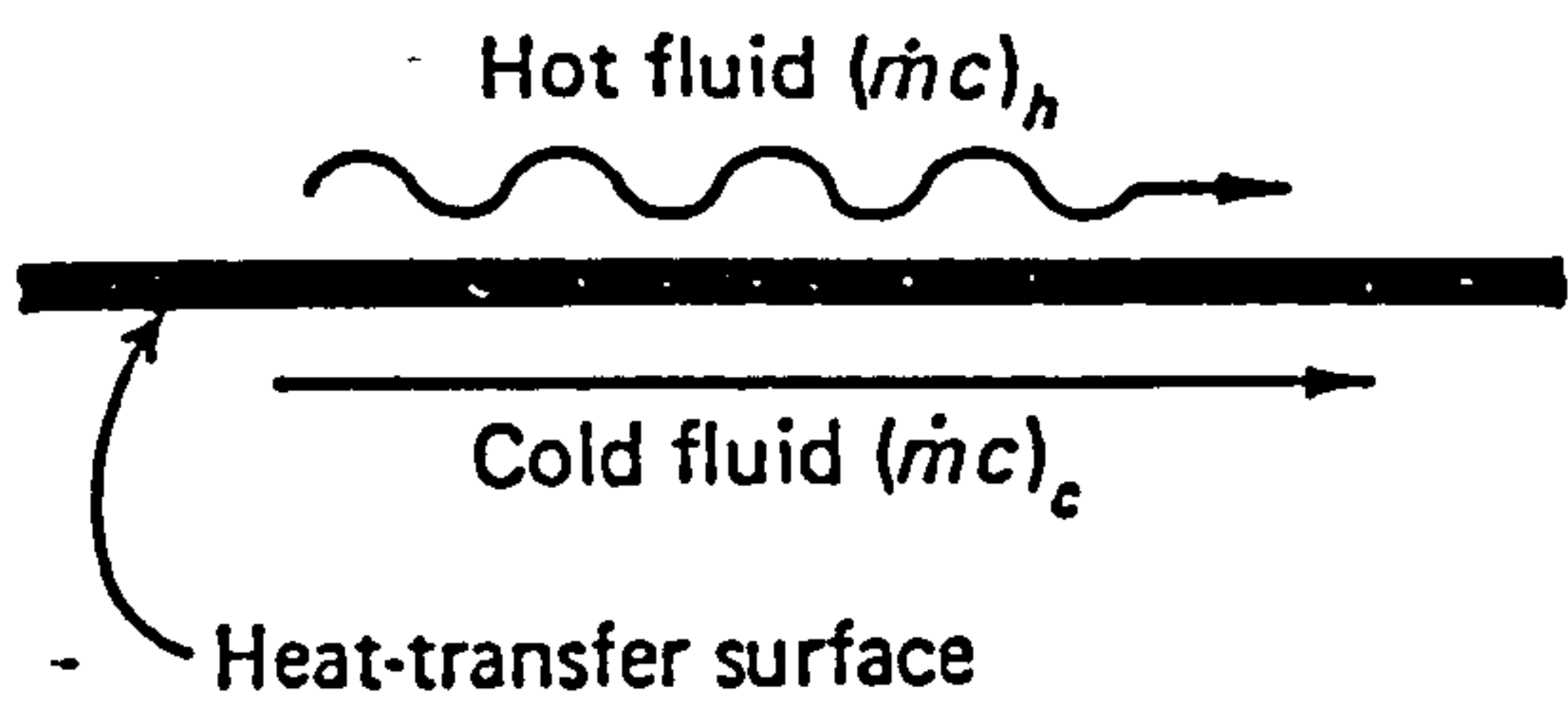
It is possible to model the thermal performance of the pump as a section of straight pipe with "equivalent length" and internal heat generation. In this case, the shell of the pump is considered as a control volume with lumped thermal capacity. Internal space of the pump is thought of full of water which can be represented by a bulk volume. Electrical power added to the pump to drive the system water circulation is partly converted into heat energy due to friction losses and absorbed by water, this amount of heat generation is relatively small in comparison to the heat generated by boiler. By knowing the water volume in the pump and the mass of the pump itself, the "equivalent length" of the pump can be easily determined. The determination of heat transfer coefficient of pump water to the shell is quite difficult. However, the main thermal resistance is existed on the side of shell external surface to surroundings which can be considered as only natural heat convection. The state equation of pump with "equivalent length" will have the same expression as a section of straight pipe with single inlet and outlet.

4.2.5 The hot water storage tank

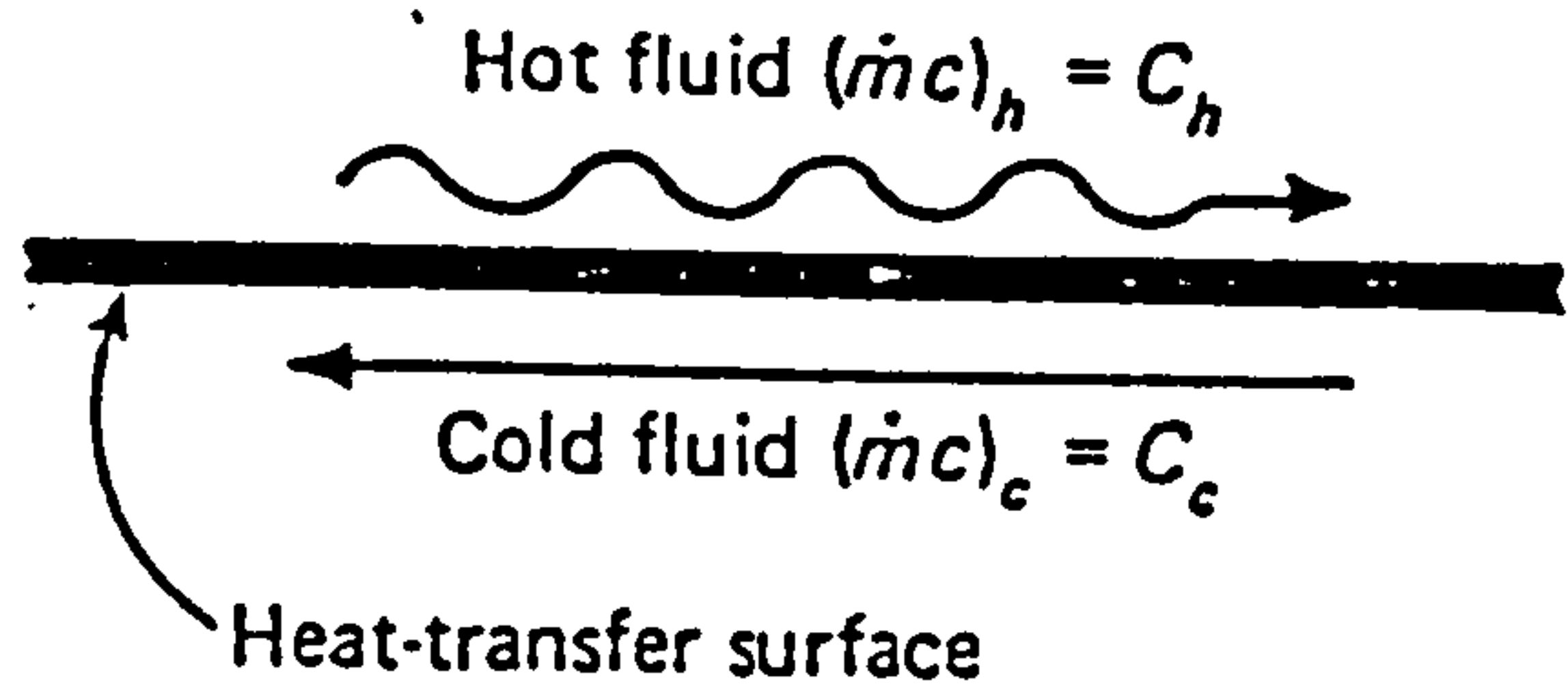
The hot water storage tank used in heating system is a heat exchanger which takes heat from the hot water provided by the heating system and produces hot water having lower temperature to serve various purposes. The structures of manufactured hot water storage tank for various usages are quite different. Consider the flowing patterns, the hot and the heated water can be mixed or unmixed; the hot water can be unmixed flowing in a tube bundle and the heated water can be mixed flowing across the tube bundle; the hot and heated water flowing directions can be parallel-flow, counter-flow, cross-flow and multipass counter-flow. Figure 4.21 shows various types of such hot water storage/exchanger in their flowing patterns.

Conventional designs of such heat exchanger are devices in which the hot and heated water are separated from each other to prevent disturbances to the system and the boiler operation. For domestic heating systems, the hot water storage tank is used to produce hot water for daily usages, so that the structures are usually much simpler than those used in industrial processes. Mixed or unmixed cross-flow types are usually used in water-air exchangers and large exchangers. For domestic purposes, simple parallel and counter-flow types are commonly used. For the consideration of thermal efficiency, counter-flow is usually recommended.

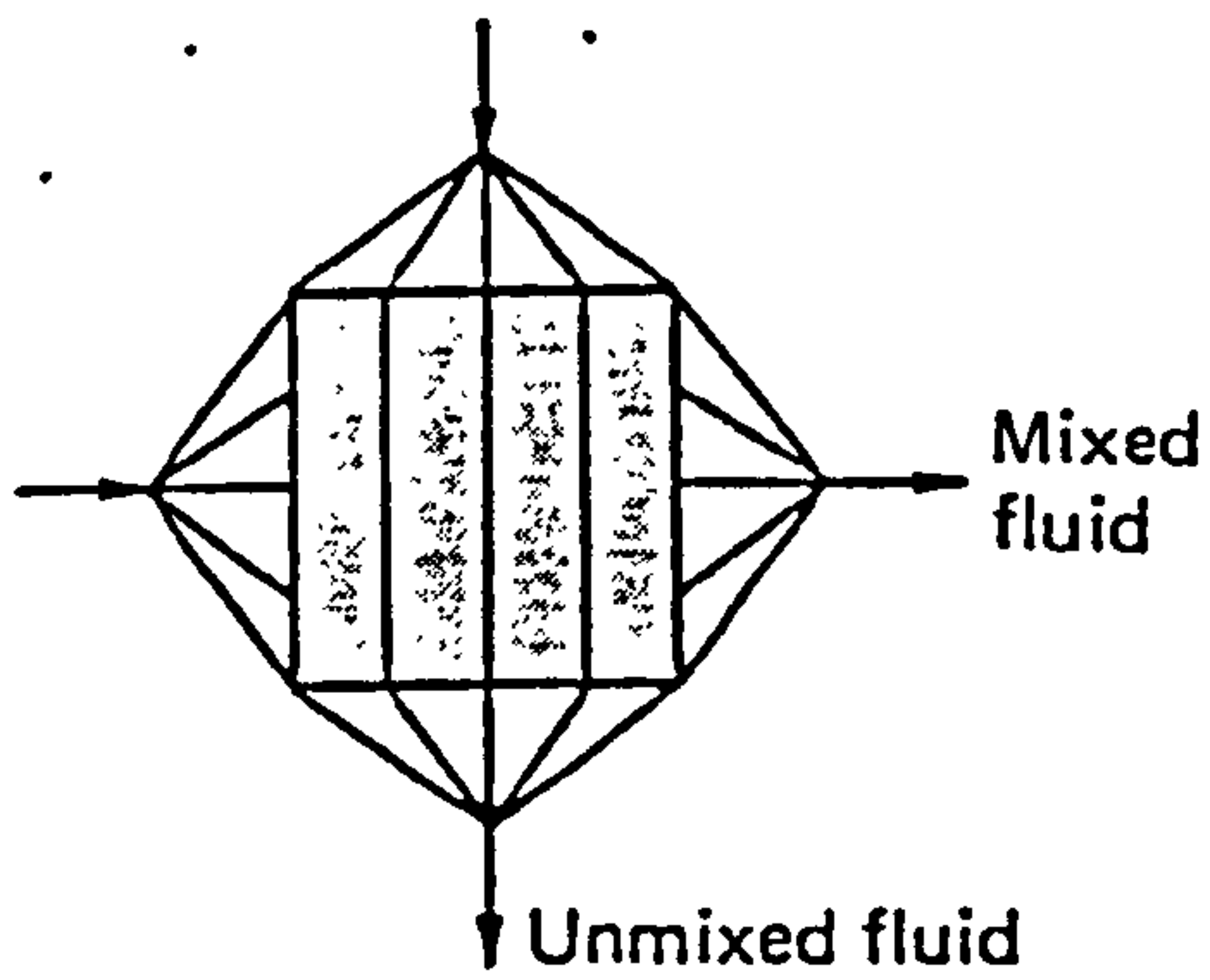
Figure 4.22 shows a vertical, counter-flow hot water storage tank in which the hot water is separated from the heated water. The separation can be a single tube, a bundle of tubes, a coil or other shaped materials. Hot water flowing in the internal passage is a part of the heating system circulation water driven by the pump. The flow rate of the heated water in the vessel is a random factor depending on the daily usage of the domestic hot water which can be considered as a function of time. Velocity of water flow in the heated water side is relatively slow comparing to the hot water and some times could be zero, so that a thermal stratification can be resulted in the heated water vessel. Therefore, an internal flow caused by the density difference in each layer has to be considered especially when the heated water is stationary.



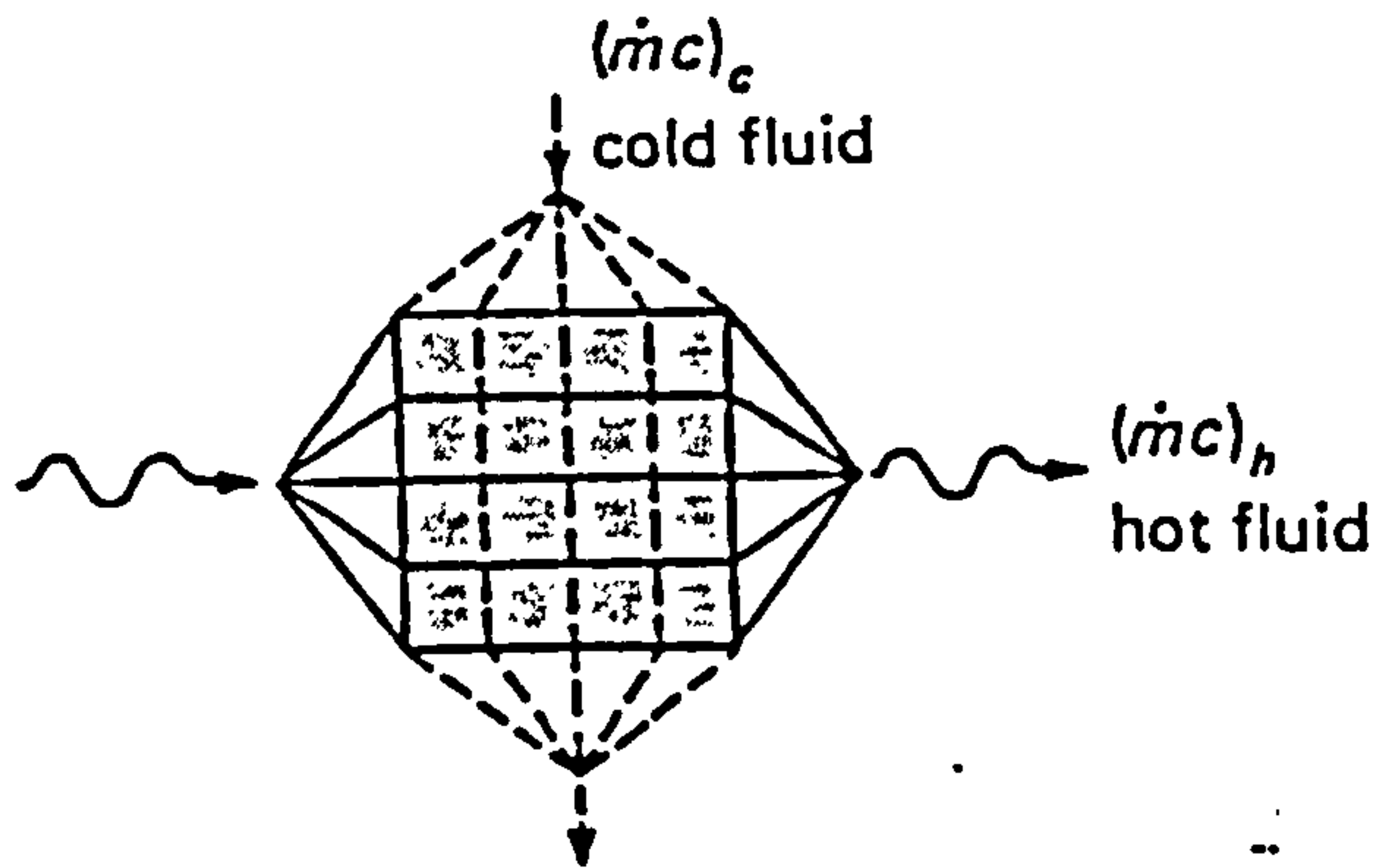
a) Parallel-flow exchanger



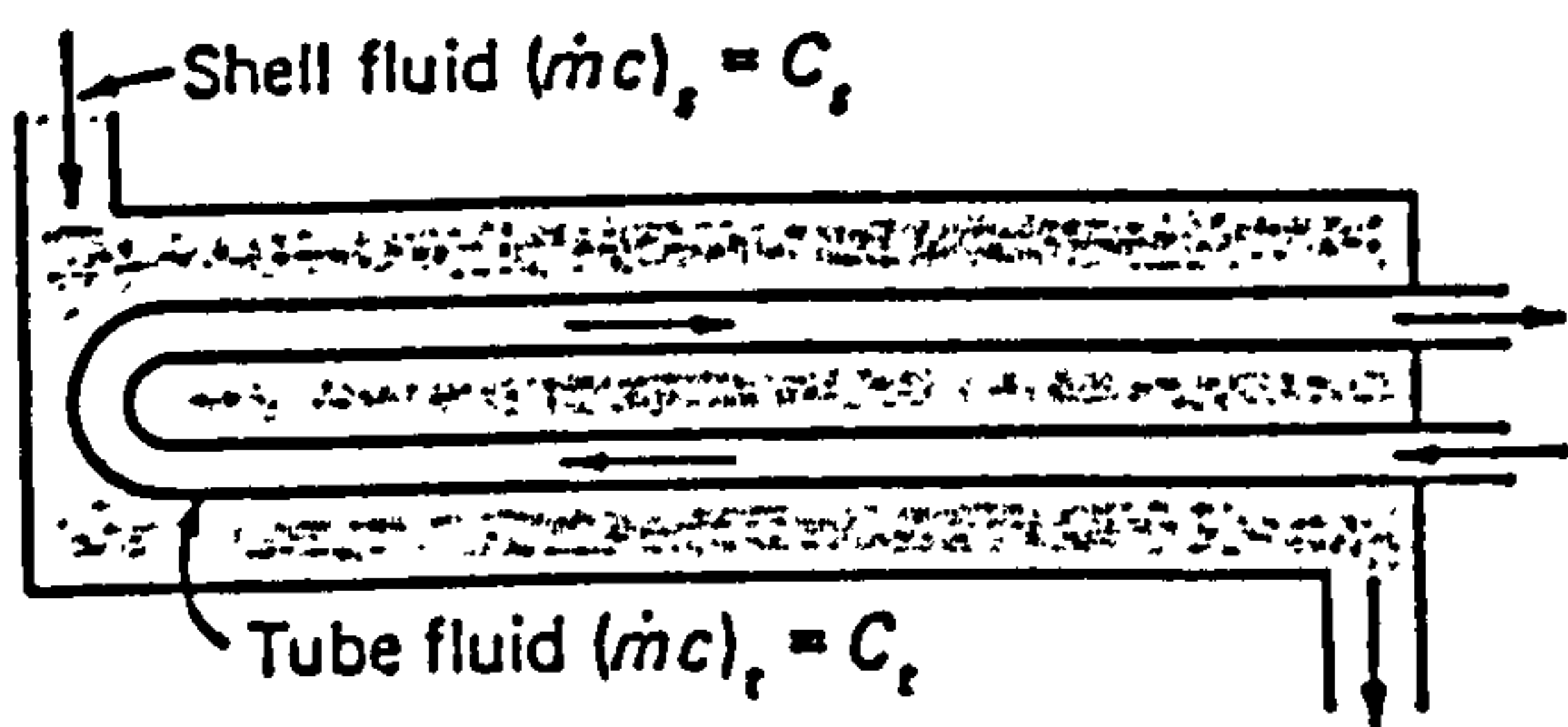
b) Counter-flow exchanger



c) Cross-flow exchanger, one fluid mixed

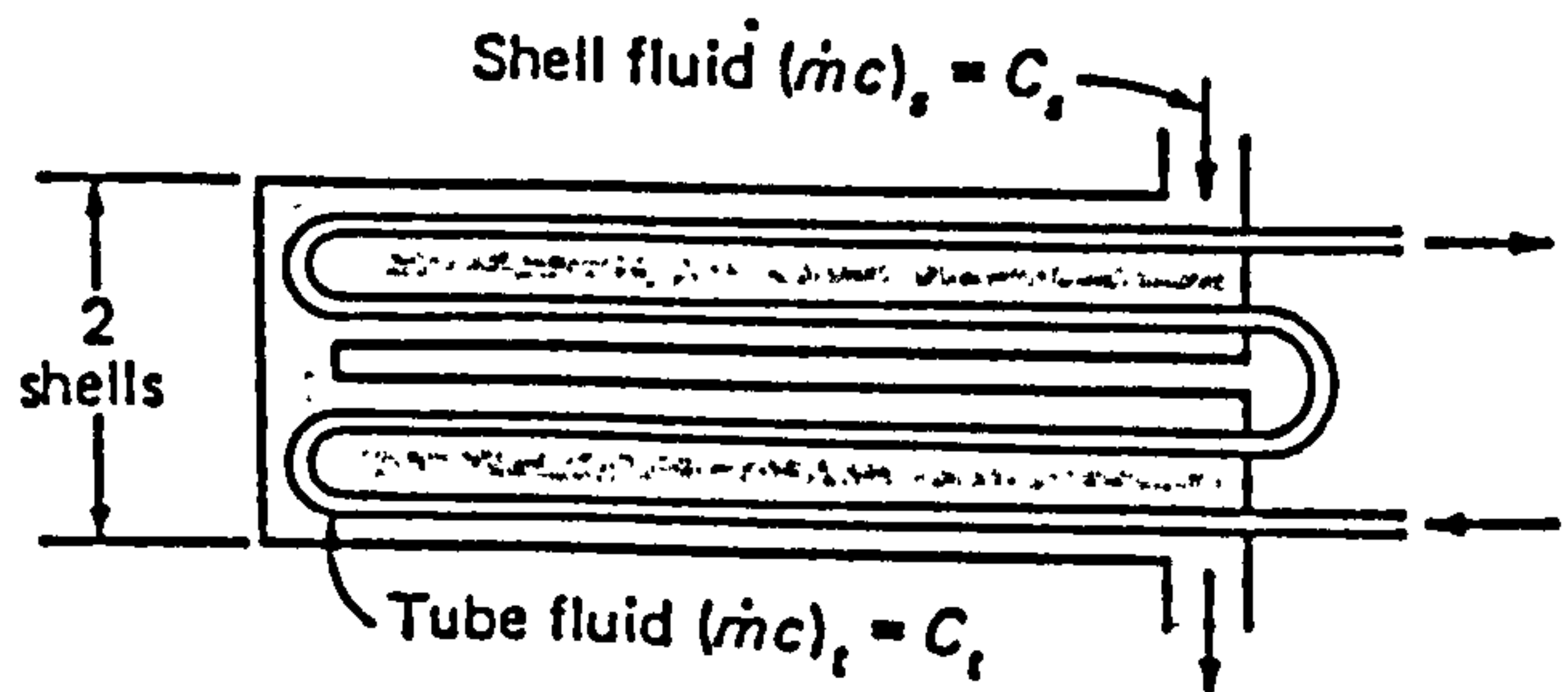


d) Cross-flow exchanger, fluids unmixed



One shell pass
2, 4, 6 etc., tube passes

e) Parallel counter-flow exchanger



Two shell passes
4, 8, 12 etc., tube passes

f) Multipass counter-flow exchanger

Figure 4.21 Types of hot water tank/exchanger

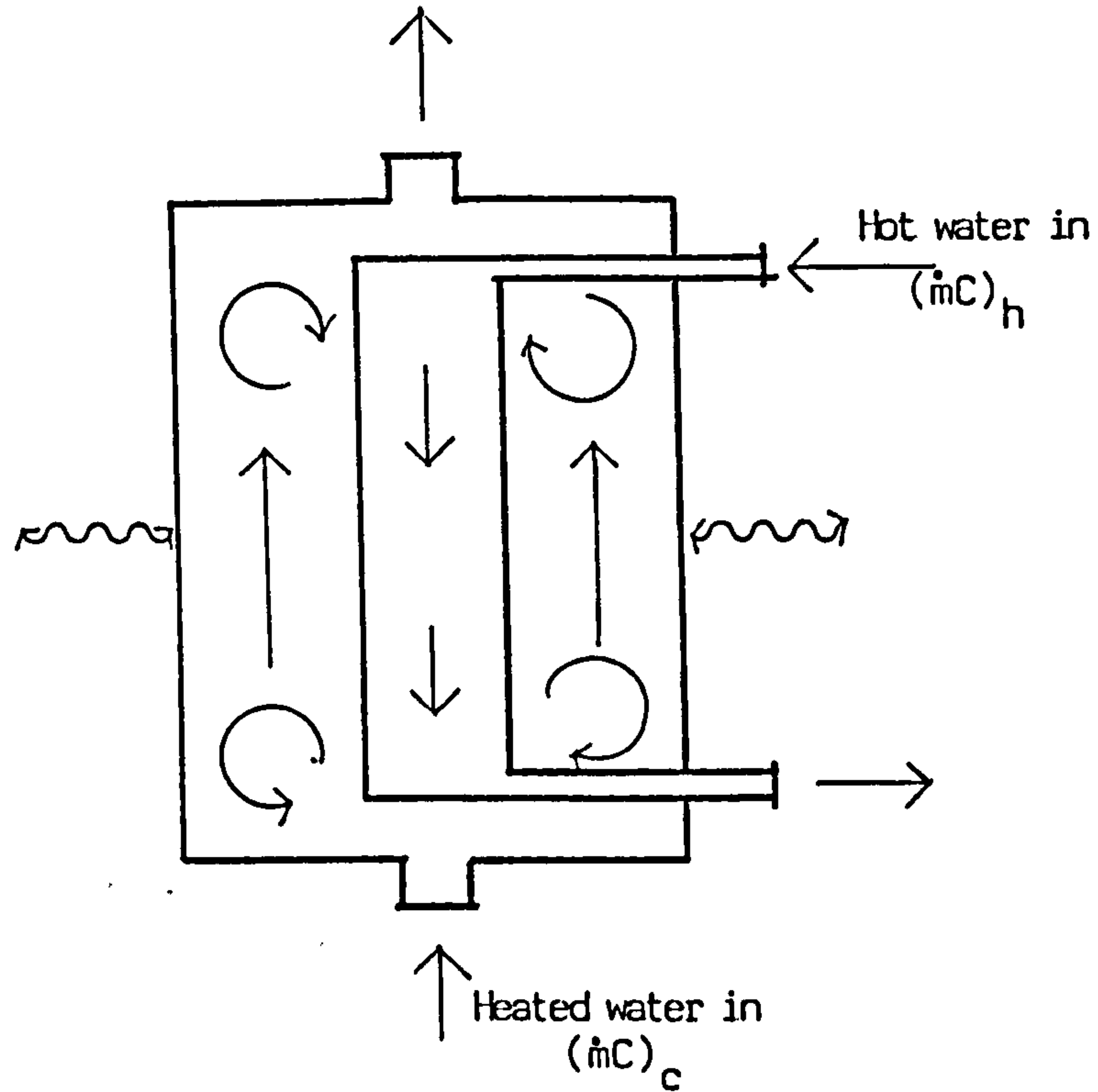


Figure 4.22 Conceptual structure of a vertical counter-flow heat exchanger

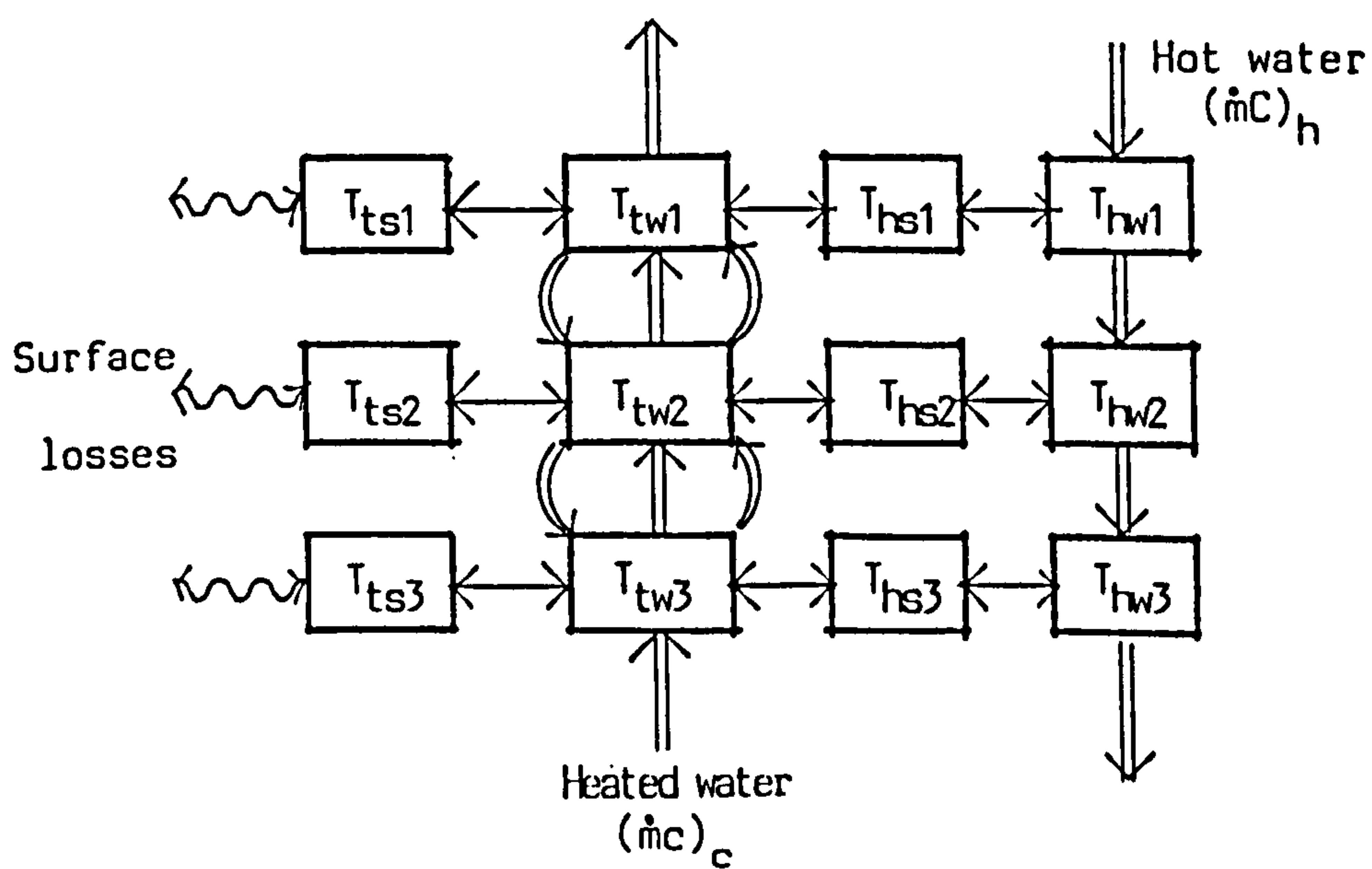


Figure 4.23 Control volume schematic of the vertical counter-flow heat exchanger

Figure 4.23 shows the control volumes and the paths of heat exchanges of this hot water storage tank. The tank is stratified to three layers according to the direction of hot water flow, the internal flow at the heated water side is considered. The assumption is made as:

- a) The internal flow only takes place between two adjacent layers;
- b) The internal flow happens when the lower layer is having higher temperature than the higher layer. In the reverse situation, no internal flow will take place.
- c) The flow rate of internal flow from the lower layer into the higher equals to that of the higher to the lower according to the substance conservation.

The state variables are defined as the temperature of each control volume given in Figure 4.21. Apply the basic conservation equations to the relevant control volumes, the component model of this hot water storage tank can be resulted. Figure 4.24 gives the positions of coefficient in the matrix and the relevant state variables in the vector.

As emphasized in the previous section, this model can also be applied to hot water storage tanks of other structure provided that the control structures of those hot water storage tanks can be so arranged to suit the pattern as this one.

As an example, Figure 4.25 shows a shell and tube heat exchanger. The flow pattern in this exchanger is cross counter-flow. To use the model as derived as Figure 4.22, the discretization has to be made horizontally along the hot water flow direction. For this model, the internal flow can be considered as always zero. This consideration is reasonable since such kind of exchanger is usually industrially used and a constant flow of the heated water is dominant.

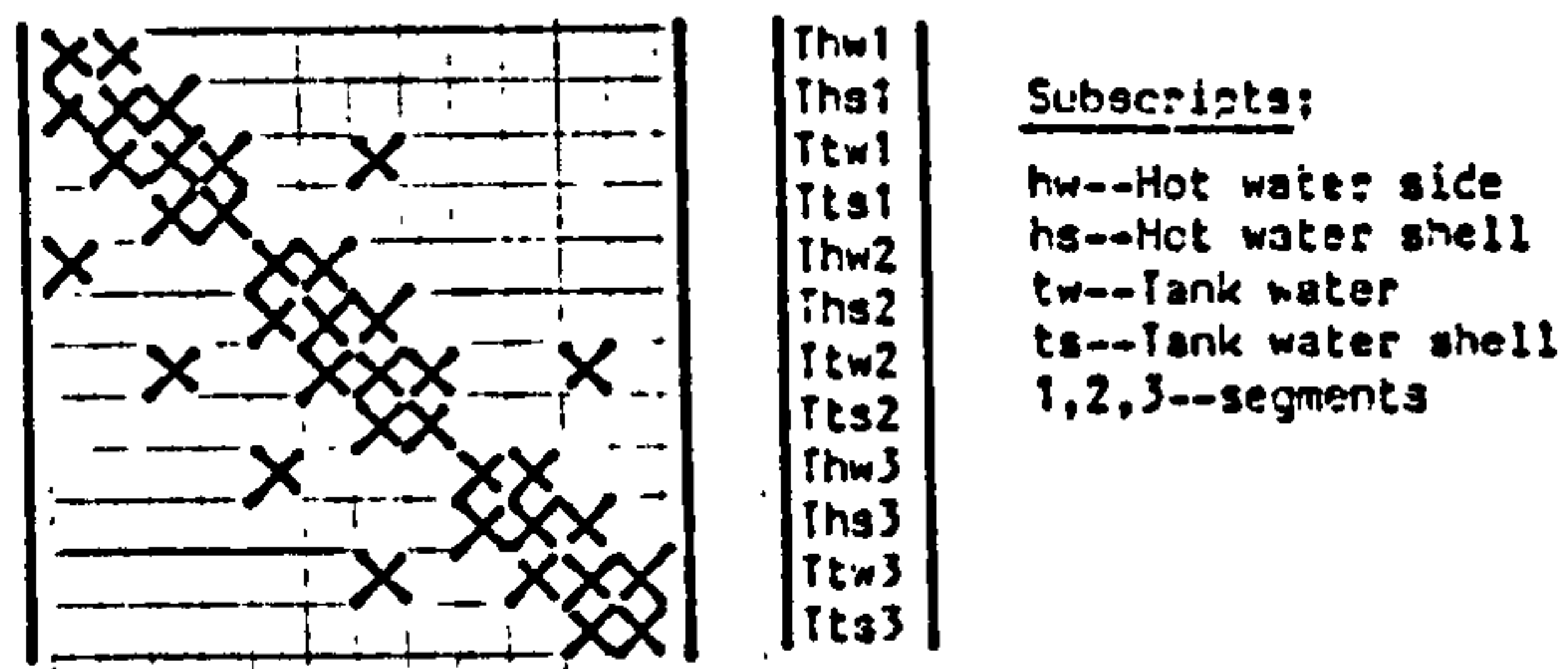


Figure 4.24 Coefficient matrix and state vector of the counter-flow heat exchanger model

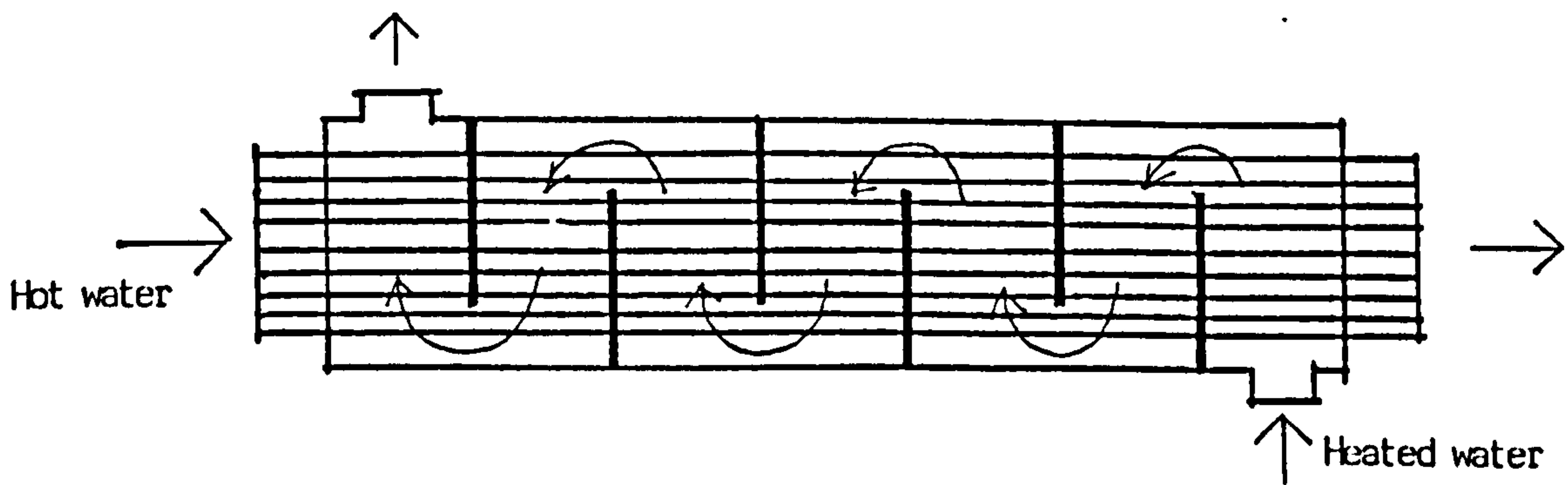


Figure 4.25 Conceptual structure of a shell and tube heat exchanger

4.2.6 The control devices

The control devices used in plant system are mainly the sensor, controller, actuator and regulator. The duty of a control loop is to translate the signal (e.g. temperature) at set position in the system into a control signal and then to regulate the controlled parameter (e.g. water flow rate). Two kinds of control loops are commonly used, the feedforward and the feedback control loops. In feedforward loop, the disturbance is measured before it arrives at the process of plant system and the controlled parameter is regulated in advance to counteract the disturbance effect. In feedback loop, the output of control loop is directly influenced by the output of process. Feedforward loop is easy to design and operate, usually no stability problem. However, the accuracy of control is very difficult to achieve. Feedback loop gives better response to irregular disturbance, high accuracy of control is possible to achieve, however, stability always is a important problem in feedback loop design. Feedback control loops are much more common for plant systems. Figure 4.26 shows a feedback control loop of controlling the boiler water outlet temperature at a required level.

Although the basic conservation equation can be applied to the control devices to obtain the mathematical models, it seems helpful to derive the models for control devices from the beginning of state equation in differential form since there are other variables other than temperature are involved.

1. Sensor

The sensors used in plant system are usually mercury thermometer, thermal couple and humidity detector. For heating system, only temperature measurement is required. The measuring head of a sensor can be considered as a lumped solid material. Applying the energy balance to the measuring head of a temperature sensor, it can be represented by:

$$(\rho c v)_s \frac{dT_s}{dt} = h A_s (T_a - T_s) \quad 4.10$$

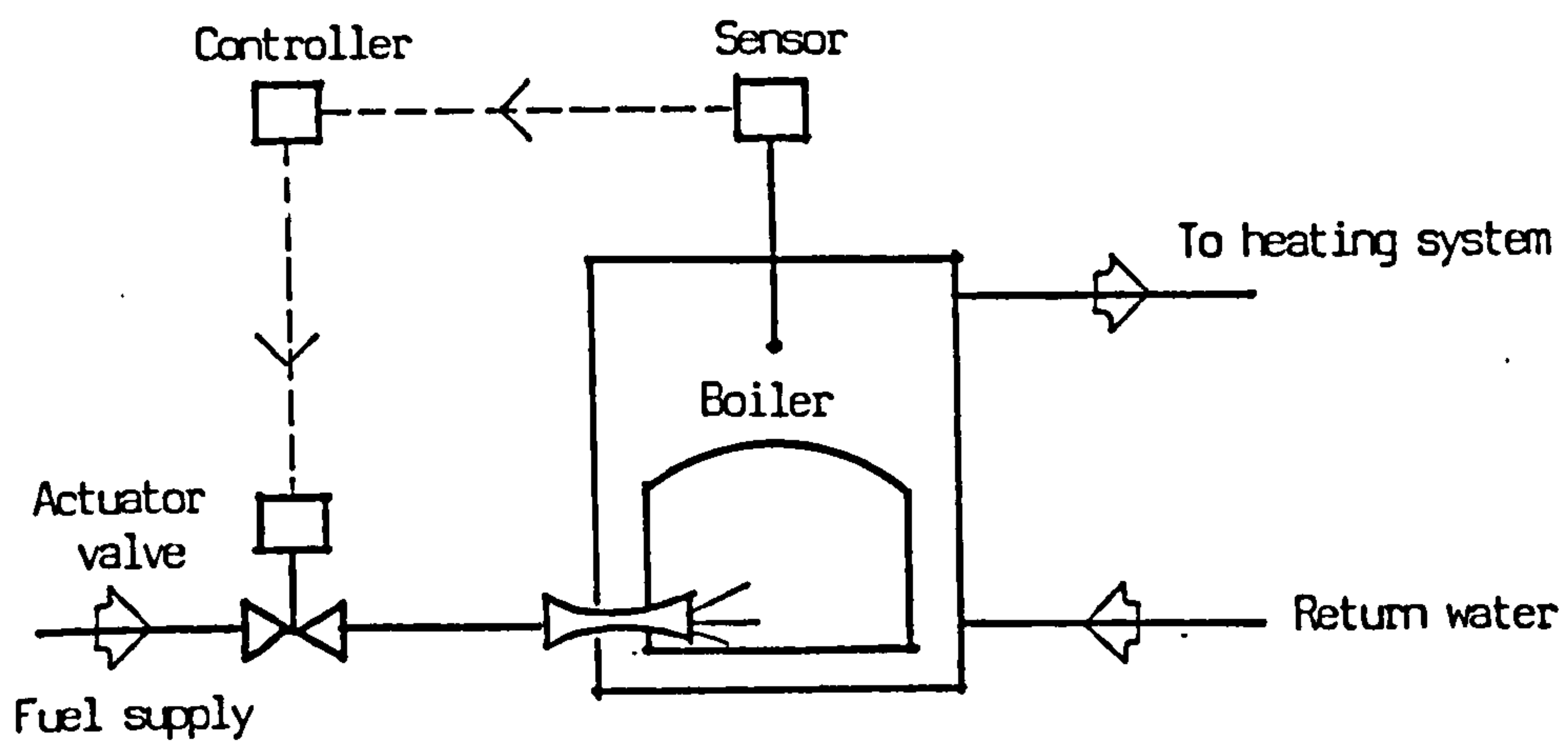


Figure 4.26 Feed back control loop of boiler temperature control

where,

T_s = sensor temperature

T_a = temperature of measured position

$(\rho cv)_s$ = density, specific heat, and volume of the sensor measuring head

h = heat transfer coefficient between sensor and the object

A_s = surface area of sensor measuring head

Parameter group

$$\tau_s = \frac{(\rho cv)_s}{h A_s}$$

is defined as the time constant of the sensor in unit of time. Time constant of a sensor is usually very small comparing to most of the thermal processes. If the time decay caused by the sensor can be neglected, equation 4.10 then becomes:

$$T_s = T_a$$

2. Controller

Controllers are usually electronic or pneumatic devices, it receives signal sent by the sensor, after the operation of the specified control function, sends an output signal to the actuator.

Six kinds of basic control function are very commonly used in industrial processes, which are position/on-off control, proportional-integral-derivative control. To simplified the discussion, assuming that the transformation of temperature signal into electronic or pneumatic signal can be taken place in the controller, the relation between the input of temperature and the static outputs of controller are given as:

a) positional/on-off control,

$$V_{st}(t) = \begin{cases} V_{min} & e(t) \geq 0 \\ V_{max} & e(t) < 0 \end{cases} \quad 4.11$$

b) proportional control,

$$V_{st}(t) = K_p e(t) \quad 4.12$$

c) integral control

$$V_{st}(t) = K_I \int_0^t e(t) dt = \frac{1}{T_I} \int_0^t e(t) dt \quad 4.13$$

d) proportional-integral control,

$$V_{st}(t) = K_p e(t) + \frac{1}{T_I} \int_0^t e(t) dt \quad 4.14$$

e) proportional-derivative control,

$$V_{st}(t) = K_p e(t) + T_D \frac{d e(t)}{dt} \quad 4.15$$

f) proportional-integral-derivative control,

$$V_{st}(t) = K_p e(t) + \frac{1}{T_I} \int_0^t e(t) dt + T_D \frac{d e(t)}{dt} \quad 4.16$$

where,

$e(t)$ = error signal, difference of sensor & set temperature.

$e(t) = T_s(t) - T_{set}$

T_{set} = set point temperature

$V_{st}(t)$ = controller static output signal

V_{min}, V_{max} = minimum and maximum set output signal

K_p = controller proportional gain

K_I = controller integral action factor =

T_I = integral action time

T_D = controller derivative action time

The dynamics of the controller can be obtained by introducing the time constant of the controller τ_c , the state equation of a controller becomes:

$$\frac{dV}{dt} = \frac{V_{st} - V}{\tau_c} \quad 4.17$$

where,

V = dynamic output signal of controller

τ_c = time constant of controller

The time constant of a electronic or pneumatic controller is usually much faster than most of the thermal processes. If the time constant of a controller can be neglected, the output signal of the controller equals to the static values.

$$V(t) = V_{st}(t)$$

3. Actuator and regulator

Actuator transforms the electrical (or pneumatic) signal received from the controller into displacement and then gives an action to the regulator. The actuator and the regulator (e.g. a valve) are physically linked to each other. Since the actuator/regulator is mainly a mechanical device, there is a time lag in the actuator/regulator in response to the change of controller output. The time constant of actuator/regulator is usually much greater than the controller. Defining the time constant of the actuator/regulator as τ_a , the dynamics of an actuator/regulator can be obtained as:

$$\frac{dD}{dt} = \frac{\alpha v - D}{\tau_a} \quad 4.18$$

where,

D = displacement of the regulator

τ_a = time constant of actuator/regulator

α = transformation coefficient of control signal to displacement

The displacement of actuator/regulator output can be represented by percentage index as $0 \sim 1$. The relationship between the controlled parameter and the displacement can be readily found from the characteristics of the regulator. Figure 4.27 shows the linear and non-linear characteristics between the water flow rate and the opening

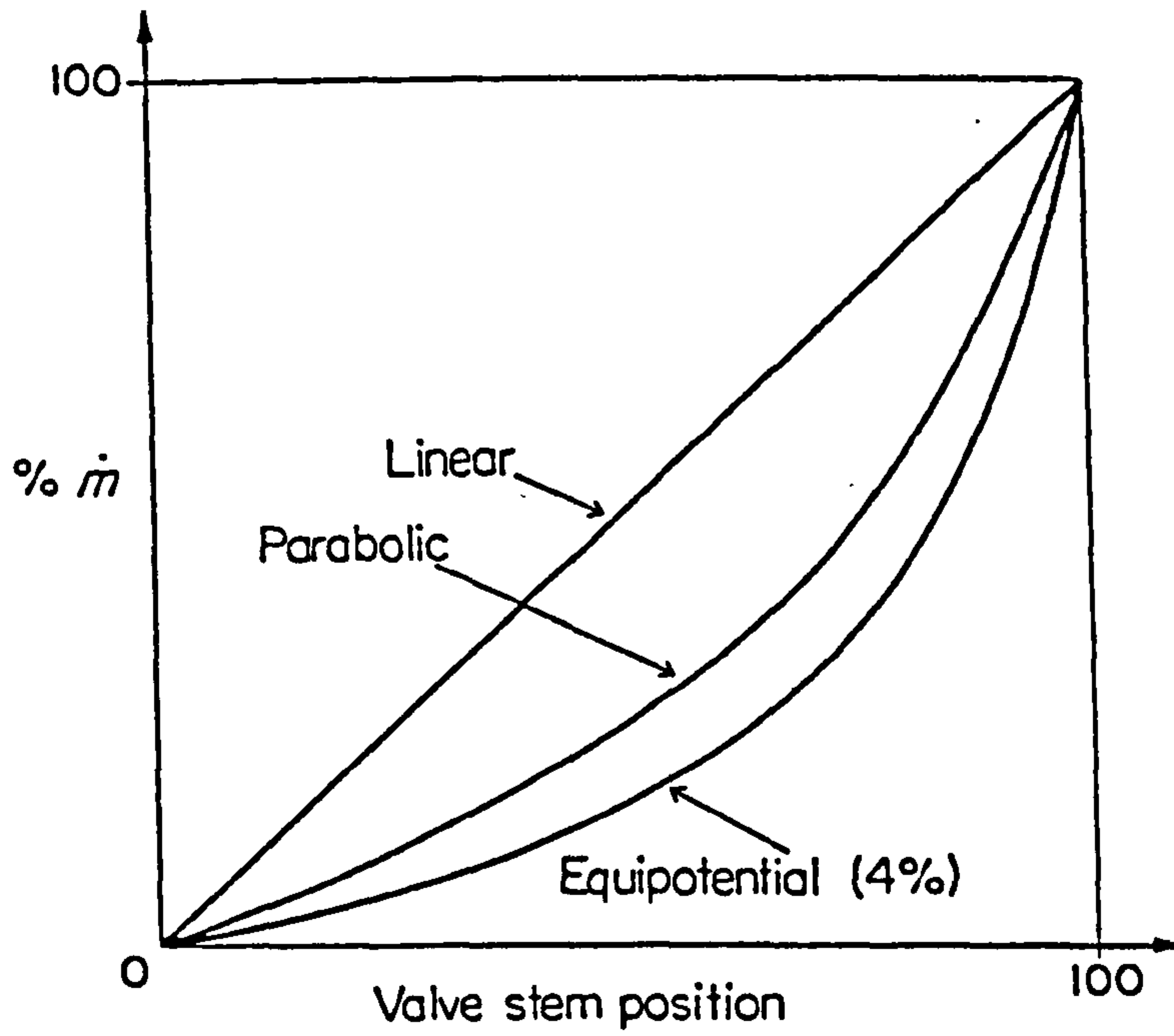


Figure 4.27 Valve characteristics

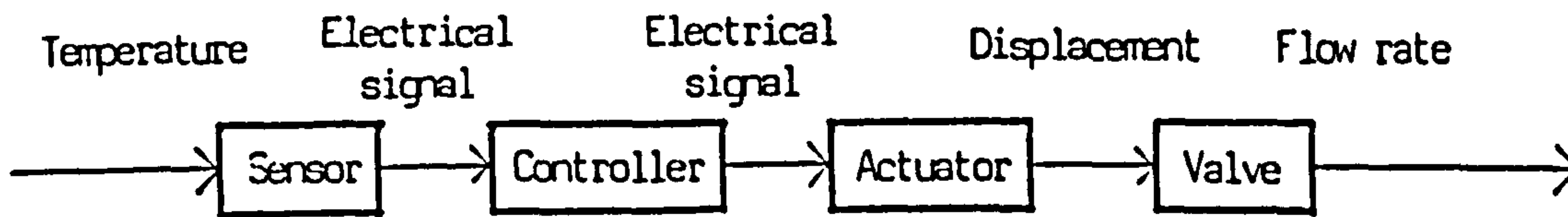


Figure 4.28 Control volume schematic of temperature-flow rate control

of the valve. In any case, the water flow rate is only a function of the valve stem position. The relationship between them can be represented by:

$$\dot{m} = f(D) \quad 4.19$$

A control volume flow diagram of a control loop is shown as Figure 4.28. This control loop can be connected to any state variable which is controllable in the complete system.

Using the definition of error signal, equation 4.10 can be rewritten as:

$$\frac{de}{dt} = \frac{(e_a - e)}{\tau_s} \quad 4.20$$

where,

$$e_a = T_a - T_{set}$$

T_{set} = set temperature of the controller

The state equation for the control loop can be obtained by collecting equation 4.20, 4.18, 4.19 and specifying the required control function as any one of equation 4.11 to 4.16. For example, for proportional control, substitute equation 4.12 into 4.17, the control loop state equation becomes:

$$\left\{ \begin{array}{l} \left| \begin{array}{c} \dot{e} \\ \dot{v} \\ \dot{D} \end{array} \right| = \left| \begin{array}{ccc} -\frac{1}{\tau_s} & 0 & 0 \\ \frac{k_p}{\tau_c} & -\frac{1}{\tau_c} & 0 \\ 0 & \frac{\alpha}{\tau_a} & -\frac{1}{\tau_a} \end{array} \right| \left| \begin{array}{c} e \\ v \\ D \end{array} \right| + \left| \begin{array}{c} \frac{1}{\tau_s} \\ 0 \\ 0 \end{array} \right| |e_a| \\ \dot{m} = f(D) \end{array} \right. \quad 4.21$$

For proportional-derivative control function, substitute equation 4.15 to 4.17, the control loop state equation becomes:

$$\left\{ \begin{array}{l} \left| \begin{array}{c} \dot{e} \\ \dot{v} \\ \dot{D} \end{array} \right| = \left| \begin{array}{ccc} -\frac{1}{\tau_s} & 0 & 0 \\ \frac{T_D}{\tau_i \tau_c} + \frac{k_p}{\tau_c} & -\frac{1}{\tau_c} & 0 \\ 0 & \frac{\alpha}{\tau_a} & -\frac{1}{\tau_a} \end{array} \right| \left| \begin{array}{c} e \\ v \\ D \end{array} \right| + \left| \begin{array}{c} \frac{1}{\tau_s} \\ \frac{T_D}{\tau_c} \\ 0 \end{array} \right| |e_a| \\ \dot{m} = f(D) \end{array} \right. \quad 4.22$$

State equation of control loop for other specified control function can be obtained by following the same process. The resulted control loop state equation can be easily written as the standard form of difference equation.

It has to be emphasized that the selection of the optimum values of gain is the most important problem in control system design. A large proportional gain can minimize the controller droop, however, it can also cause instability in the control loop. The selections of integral and derivative gains are more difficult, in practice, they are usually determined by experience in control systems, analysis and fine tuning. Analytical determination of such gains for large system requires further knowledge in control system design which is the main topic for control engineering.

4.3 Example of system representation

Until now, most of the component models encountered in heating system have been derived. The formulation of a complete heating system model has been discussed in chapter three. It can be obtained by combining all the component models together and specifying the inter-relationship between each two components. It has to emphasize again that some variables which are treated as disturbances in the single component model now become state variables of other component model in the system. The disturbances for the complete system model are those external factors acting upon the system.

4.3.1 The system

Figure 4.29 shows a domestic wet central heating system including a boiler, radiator, pump and hot water storage tank linked by pipes and subjected to a proportional controller acting on the boiler fuel supply. The components of the system are numbered along the water flow direction for the purpose of setting up the system state vector. Pipes are divided into sections according to the principle of water flow in each section is in one direction.

Components used in this system are:

1. Boiler-cast iron boiler represented by 13 control volumes as given in Figure 4.13 and having matrix form as Figure 4.14.
2. Pipe -bare pipe, represented by two control volumes as given by equation 4.1 and 4.2
3. Pump - heating similar formula expression as pipe 2
4. Pipe - as pipe 2
5. Pipe - as pipe 2
6. Radiator - flat plate parallel flow radiator, represented by 10 control volumes and having matrix form as Figure 4.5
7. Pipe - as pipe 2
8. Pipe - as pipe 2
9. Hot water tank - vertical counter-flow exchanger represented by 12 control volumes as given in Figure 4.21 and having matrix form as Figure 4.22
10. Pipe - as pipe 2
11. Pipe - pipe with two flow-in paths, represent by two control volumes and given by equation 4.4. and 4.2.
12. Control loop - includes sensor, proportional controller, actuator and valve to control the boiler fuel supply rate, represented by equation 4.22

External parameters acting as disturbances to the system are: room air temperature, room surface temperature and temperature of cold water fed into the hot water tank. water flow rate in the system is taken as constant or known function of time.

12. Control loop

Room

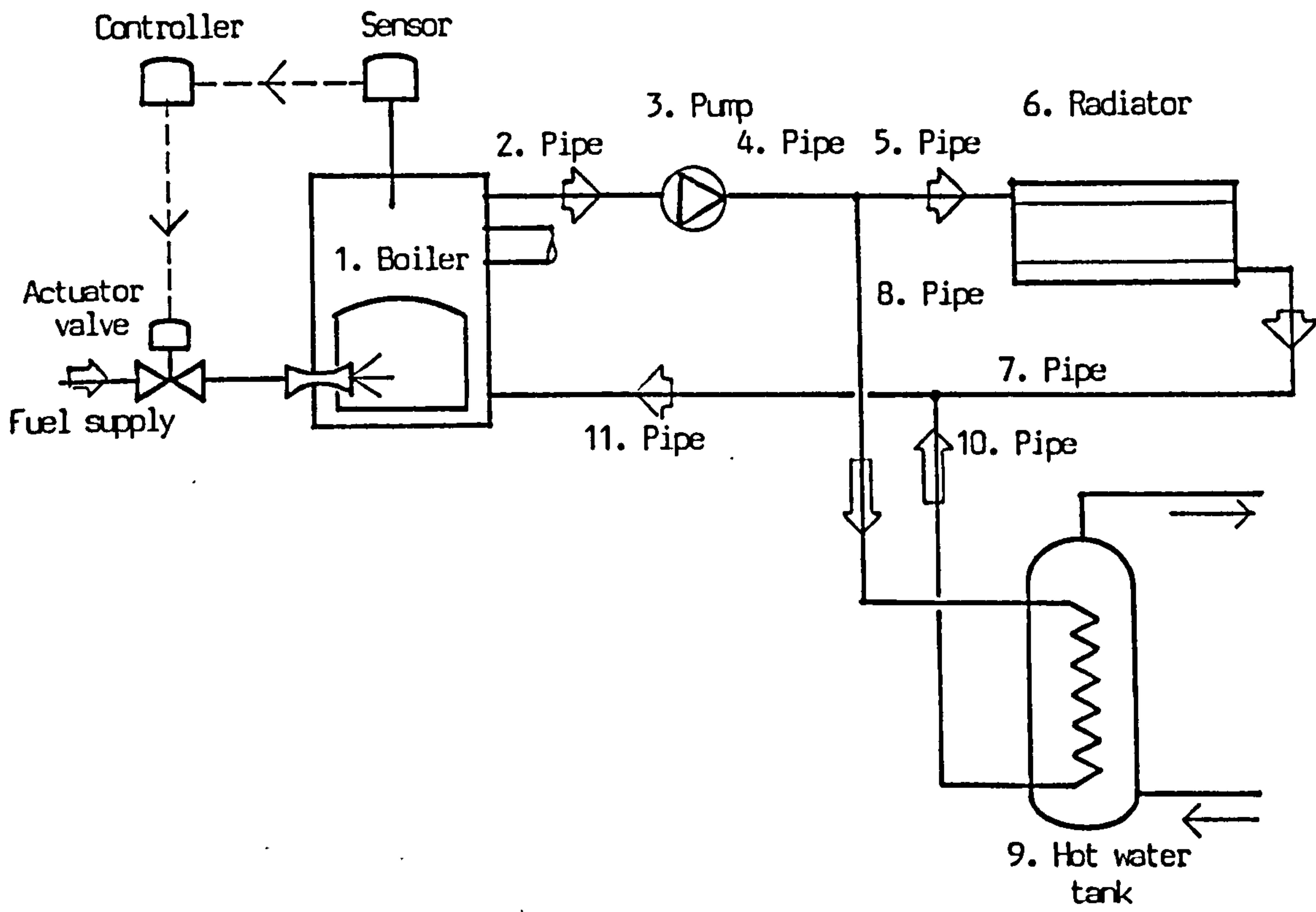


Figure 4.29 Domestic central heating system and equipments

4.3.2 The system state equation

The component state equations can be represented by the standard form of temporal difference equation. For the 12 components in the system, the component state equations are:

$$\begin{aligned} A_i(t+\Delta t)T_i(t+\Delta t) &= B_i(t)T_i(t) + C_i(t+\Delta t,t) \\ i &= 1,2,\dots,12 \end{aligned} \quad 4.23$$

For each component state equation, the positions of coefficient in both matrices $A(t+\Delta t)$ and $B(t)$ are same, vector $C(t+\Delta t,t)$ contains the external disturbances and input condition.

The complete system state equation can be represented by the same standard form as:

$$A(t+\Delta t)T(t+\Delta t) = B(t)T(t) + C(t+\Delta t,t) \quad 4.24$$

The state vector $T(t+\Delta t)$ and $T(t)$ are defined by putting together the state vectors of all the component model according to the sequence of the system numbering. With reference to the system state vector, the system matrices $A(t+\Delta t)$ and $B(t)$ can be constructed. In matrices $A(t+\Delta t)$, the diagonal is consisted of the blocks of component matrices $A_i(t+\Delta t)$, $i = 1, \dots, 12$. Coefficients at the two sides of the diagonal blocks are given as the thermal inter-relationship between each two components. The format of matrix $B(t)$ is same as $A(t+\Delta t)$. In this system, the inter-relation between each two components is the water flow. Matrix $C(t+\Delta t,t)$ contains the product of disturbance and its related coefficient matrix, it is the sum of future time and present time values. In this system model, since the water flow rate distribution in the system is taken as known factor, the order of system matrix $A(t+\Delta t)$ and $B(t)$ equal to the order of state vector $T(t+\Delta t)$ and $T(t)$, finding solution for equation 4.24 can be achieved by the time-step processing.

Figure 4.30 shows the structure of matrix $A(t+\Delta t)$ and the positions of component in the vector $T(t+\Delta t)$. Total number of state variables is 50, matrices $A(t+\Delta t)$ and $B(t)$ are of order 50x50. $C(t+\Delta t,t)$ is of

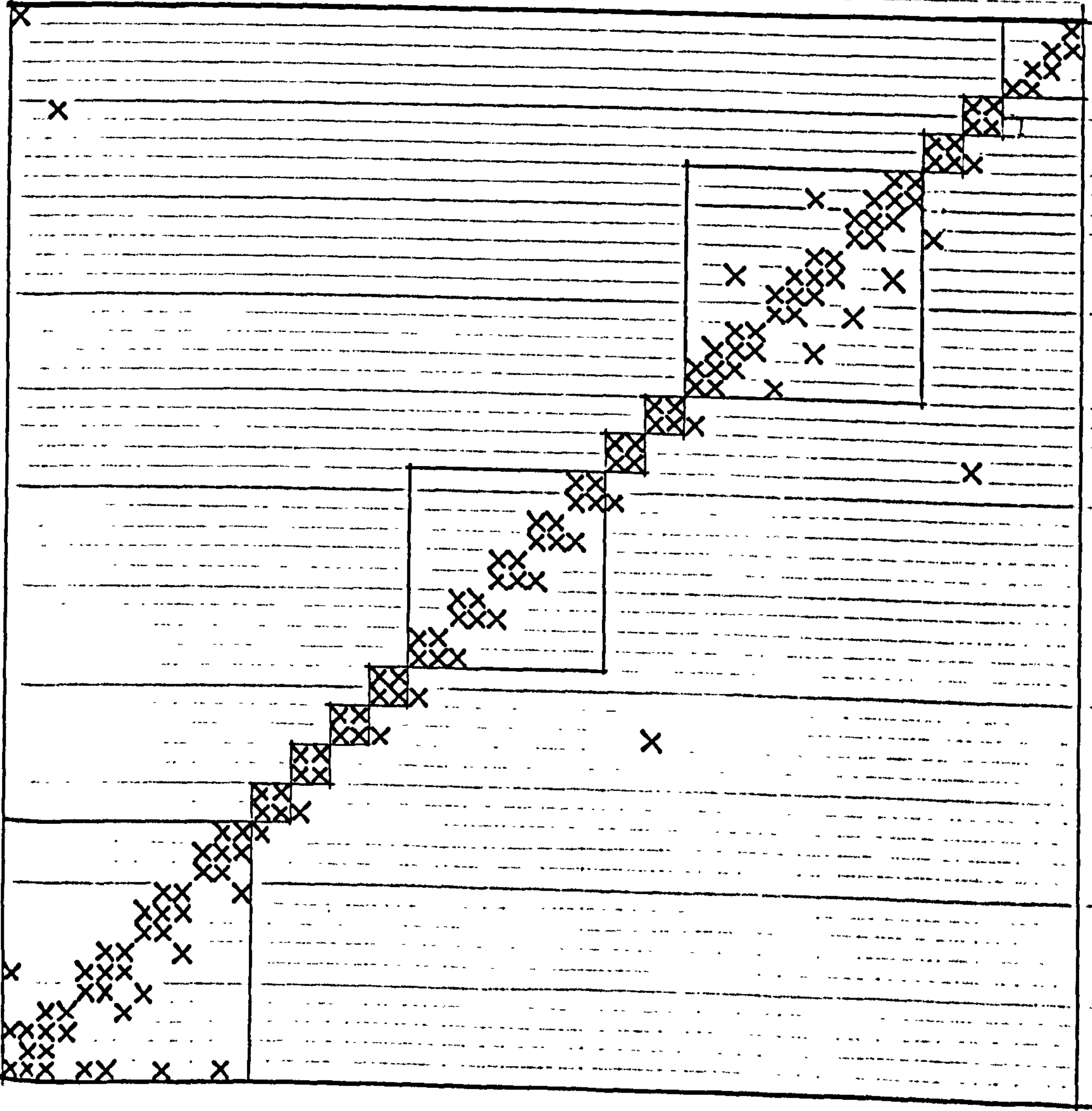
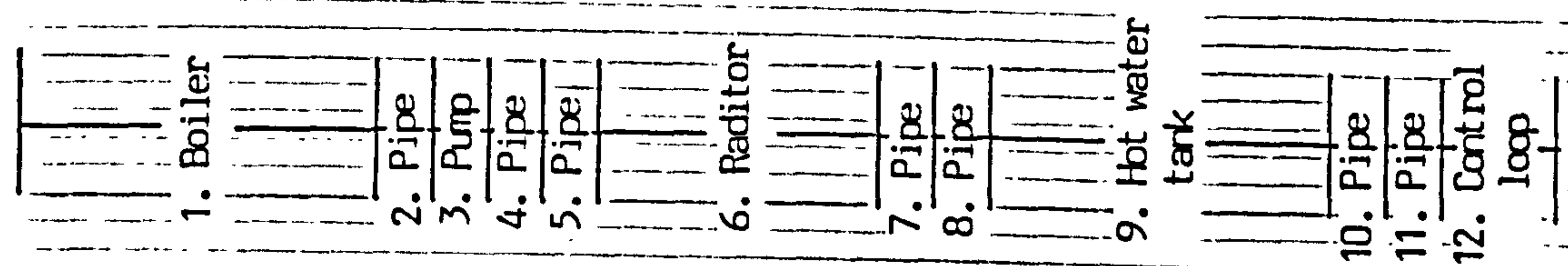
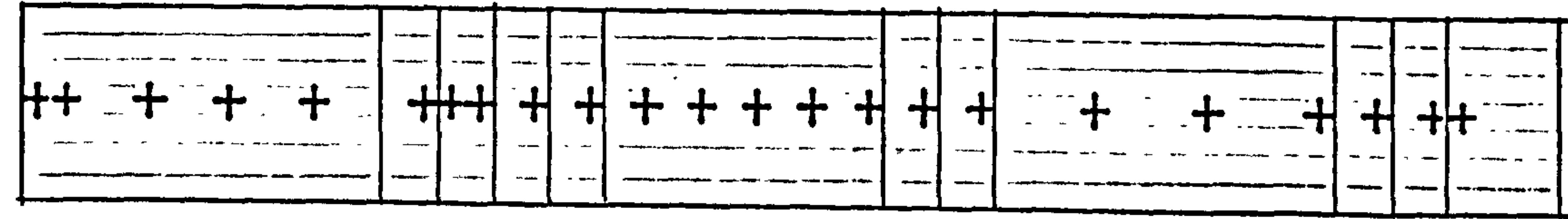


Figure 4.31 Position of disturbances in the system state equation

Figure 4.30 Coefficient matrix and the relevant state vector of the system model

order 50.

Figure 4.31 shows the positions of disturbances in vector $C(t+\Delta t, t)$.

The determination of coefficients in matrices $A(t+\Delta t)$, $B(t)$ and $C(t+\Delta t, t)$ is given in Appendix A4.3.

CHAPTER FIVE: THE AIR CONDITIONING SYSTEM

5.1 The air-conditioning system

5.2 Generation of subsystem models

5.2.1 Ducts

5.2.2 Air mixing box

5.2.3 Heater and cooler

5.2.4 Fans

5.2.5 Humidifier

5.2.6 Air washer

5.2.7 Control devices

5.2.8 Refrigeration plant

1. compressor
2. condensor and evaporator
3. expansion valve
4. the refrigeration plant

5.3 Example of system representation

5.3.1 The air handling plant

5.3.2 The complete air-conditioning system

5.1 The air-conditioning system

The function of air-conditioning system is to maintain the indoor climate - air temperature and humidity - at a required level while the external condition and the indoor heat and humidity generations are constantly changing. An air-condition system is the synthesis of heat and cold generation units and the air handling unit. Figure 5.1 shows the relations between the conditioned space and the air-conditioning plant.

As the science of air-conditioning progressed, variations in system design and new equipments have been developed rapidly. According to the types of system, it can generally be found as three categories: (1) central air-conditioning system, (2) local air-conditioning system, and (3) mixed system. The central system is usually so designed that equipments are installed outside the conditioned space and the system is capable of servicing different zones for different purposes in a building or several buildings. Local system is the compact design of central system. It is less expensive, and can be flexibly installed inside the conditioned space. In mixed system, the central plant provides primary air and the local component gives the final treatment. There are large varieties of equipment in structure which are used in air-conditioning system, however, the fundamentals and the main functions of the component are very similar.

Same as the heating system modelling, the modelling of air-conditioning system is the application of the control volume conservation state space approach. The basic air-conditioning system components are quite similar as those in heating system. The modelling parameters in air-conditioning system are temperature and humidity. For components working at wet condition, (e.g. cooler, air washer, humidifier etc.) a combined heat and mass transfer has to be considered. Pressure as well as the air flow rate distributions in the system are considered as known factors. By knowing the

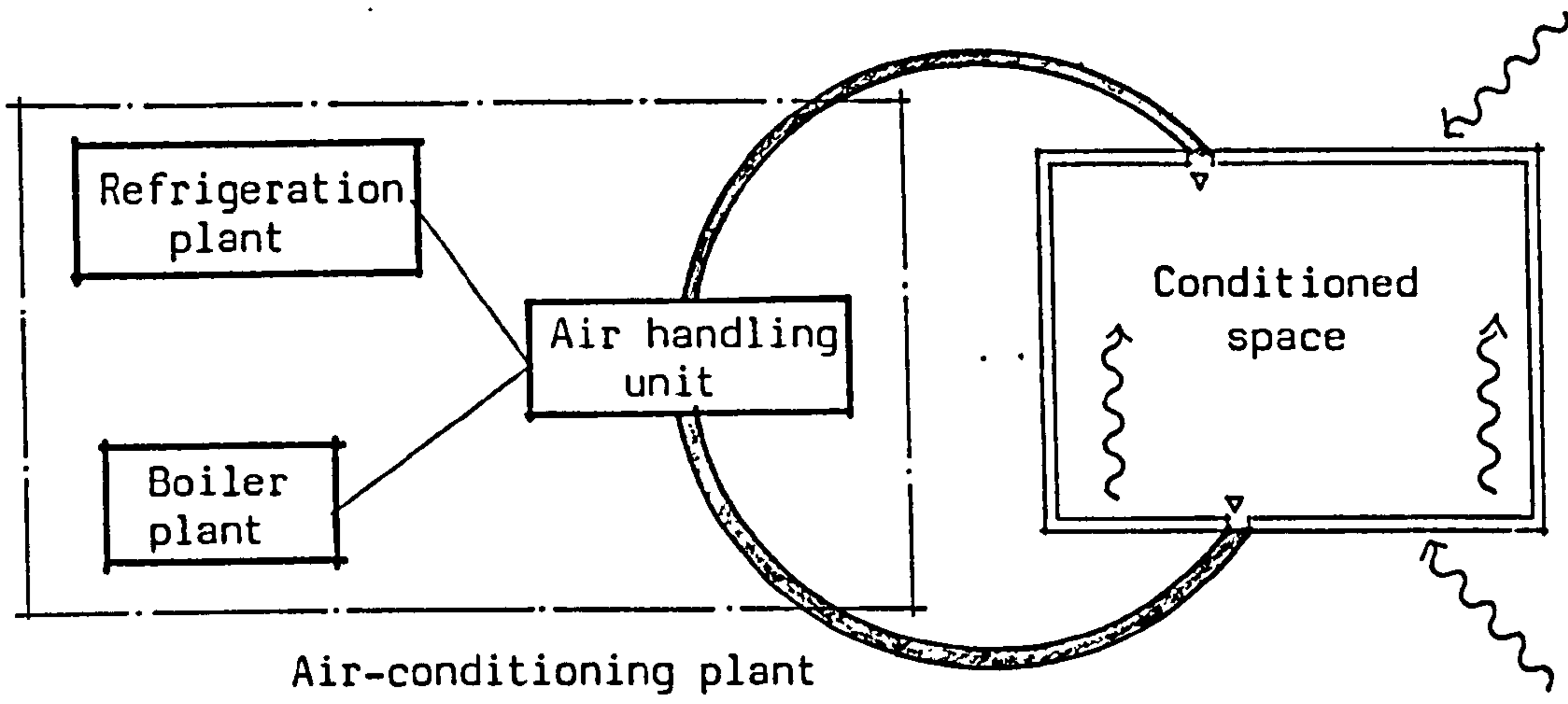


Figure 5.1 Schematic diagram of air-conditioning system

temperature and humidity, other parameters, such as enthalpy, density etc., can be readily determined according to the law of ideal gas.

This chapter is intended to demonstrate the possibility of modelling the air-conditioning components and system using the control volume conservation state space approach. Using this technique, any component can be modelled at the simplest level as an one control volume or to the higher levels as a multi-control volume models. As far as the modelling technique is concerned, there is no complexity would be imposed to develop a multi-control volume model compared to an one control volume model. The only difference is the levels of complexity in the resulted models. An one control volume model is flexible in practice. It can be applied to almost all the components having similar function in the system, the structural difference will be eliminated. A multi-control volume model will give details about the component internal conditions and can of course be much accurate. However, the more control volumes specified for a component, the more restrictions will be imposed to it, the application of the model will depend on the structure of the component. The most important thing in developing the component model is to define properly the heat and mass exchange paths to the control volumes.

Therefore, the modelling of air-conditioning components and system can be made in two different levels: (a) the one control volume representation which can be used as general models for primary simulation and practical design, and (b) the multi-control volume model for those components which detailed information is required and can be used as parameter study.

Generally, any component model and the system model is represented by two sets of state equation, the temperature and the moisture state equations. The general forms are given in chapter three as equation 3.1 and 3.2. Their finite difference forms can be represented by:

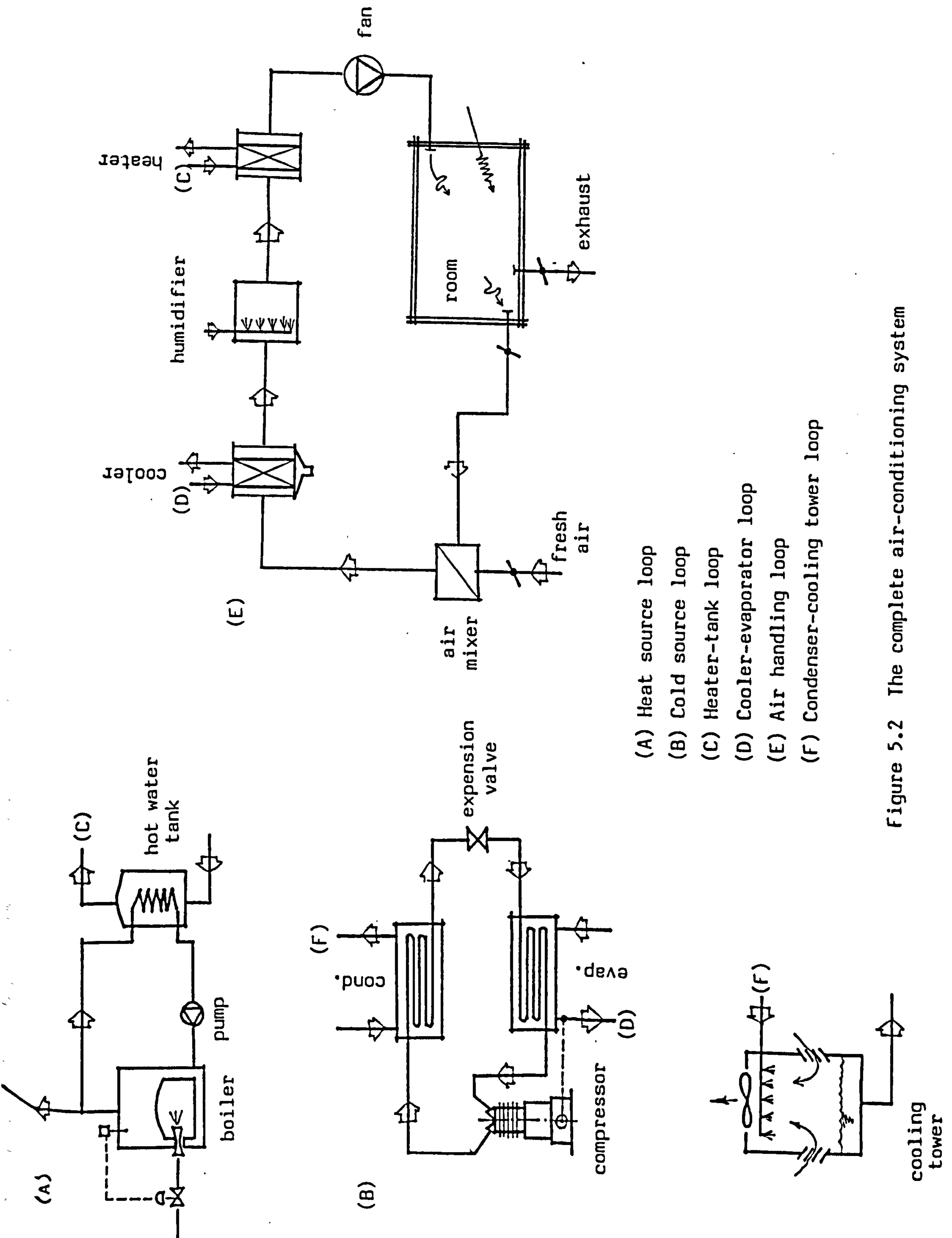
$$\begin{cases} A_T(t+\Delta t)T(t+\Delta t) = B_T(t)T(t) + C_T(t+\Delta t, t) \\ A_W(t+\Delta t)W(t+\Delta t) = B_W(t)W(t) + C_W(t+\Delta t, t) \end{cases} \quad 5.1$$

5.2 The generation of sub-system models

A complete air-conditioning system is a multi-zone system. Figure 5.2 shows conceptually an air-conditioning system which consists of six independent closed loops:

- (1) the heat source loop, which consists of boiler, part of hot water tank and the circulation water.
- (2) the cold source loop, which consists of the refrigeration plant and the circulation refrigerant.
- (3) the heater-tank loop, which consists of part of the hot water tank, heater and the circulation hot water.
- (4) cooler-evaporator loop, which consists of the evaporator, cooler and the circulation cold water.
- (5) condenser-cooling tower loop, which consists part of condenser, cooling tower and the circulation water.
- (6) the conditioned air loop, which is the main part of an air-conditioning system.

This section is mainly to develop the component and system model for the conditioned air loop.



- (A) Heat source loop
- (B) Cold source loop
- (C) Heater-tank loop
- (D) Cooler-evaporator loop
- (E) Air handling loop
- (F) Condenser-cooling tower loop

Figure 5.2 The complete air-conditioning system

5.2.1 Ducts

Modelling of ducts in air-conditioning system is very similar to pipes in heating system. The main differences are working substance in ducts are moist air. Ducts are usually made by thin steel sheets and insulated. Since the thermal capacity of moist air in duct is very small, temperature rise and drop according to the heat gain or loss through duct will become very apparent. In most of the cases, moist air flowing in ducts are dry process, no mass transfer will take place between the air and duct surface.

If the duct shell and its insulation is considered as a lumped solid material, all the derivation for connection sub-systems in the previous chapter can be directly applied to the ducts in air-conditioning system. For example, a section of air duct with thermal insulation in single flow type can be represented by equation 4.3. In this case, the moisture state equation becomes $dW_I/dt = 0$, or can be represented by difference form as:

$$W_{I-1}(t+\Delta t) - W_I(t+\Delta t) = W_{I-1}(t) + W_I(t) \quad 5.2$$

5.2.2 Air mixing box

An air mixing box is a component in which more than two air streams are met and well mixed. Such type can be found in the air handling system as tee joint of air duct in convergent flow, an 'air terminal unit' and the mixing part of an air washer.

Consider the air mixer as shown in figure 5.2. The air streams being mixed are fresh air and the air return from the room. Assume the shell and the thermal insulation of the mixer can be treated as a lumped solid material, the state equation of convergent pipe flow, equation 4.4, can be directly used for the control volume of air. Control volume of shell insulation remains same formula as equation 4.2. Since there is no mass transfer taken place, the humidity state equation remains same as in duct, equation 5.2. However, a weighted mean value has to be computed for the humidity of air stream after mixed. Since:

$$\rho V \frac{dW}{dt} = [(\dot{m}_1 w_1 + \dot{m}_2 w_2) - (\dot{m}_1 + \dot{m}_2) w] = 0 \quad 5.3$$

it leads to,

$$\begin{cases} W(t) = \left[\frac{\dot{m}_1(t)}{\dot{m}(t)} \right] w_1(t) + \left[\frac{\dot{m}_2(t)}{\dot{m}(t)} \right] w_2(t) \\ W(t+\Delta t) = \left[\frac{\dot{m}_1(t+\Delta t)}{\dot{m}(t+\Delta t)} \right] w_1(t+\Delta t) + \left[\frac{\dot{m}_2(t+\Delta t)}{\dot{m}(t+\Delta t)} \right] w_2(t+\Delta t) \end{cases} \quad 5.4$$

where,

\dot{m}_1, \dot{m}_2 = mass flow rate of air stream 1 and 2

w_1, w_2 = humidity concentration of air stream 1 and 2

w = humidity concentration of mixer

\dot{m} = total mass flow rate of air. $\dot{m} = \dot{m}_1 + \dot{m}_2$

5.2.3 Heater and cooler

Heaters used in air-conditioning system are very similar to the radiators used in heating system. Most of heaters are installed inside the air duct network, heat exchange by forced convection is dominant. The structures of cooler used in air-conditioning system are very similar as the heaters. They are usually constructed of horizontal tubes which the external surfaces are equipped with radial fins, so arranged as to facilitate the drainage of condensed moisture from the fins for coolers or to prevent the dusts accumulation on the fins for heaters. The finned tubes are usually multi-row arranged to the direction of air flow so as to achieve better heat exchange. Figure 5.3 illustrates this.

To develop the model of a heater, the radiator model derived in chapter four can be used provided the space discretization is made to the heater same as figure 4.7 for the radiator. In addition to this, a control volume of conditioned air confined in the heater which has thermal contacts with the control volumes of finned tube has to be defined. The thermal state of this control volume becomes a state variable in the state vector. External disturbances for this model are the temperature of hot water entering the heater and the temperature of conditioned air entering the control volume confined in the heater. Matrix structure and state vector for this heater model are shown in figure 5.4.

In the heater, the external surface total heat transfer coefficient has to be computed differently compared to that derived for rectangular fin. Humidity state equation of heater will be the same as for duct given by equation 5.2.

For further analysis, the control volumes of heater can also be defined in two directions. The water flow direction and the air flow direction. By constructing such control volumes, the temperature rising of air along its flow direction can be modelled.

The modelling of coolers are very similar to heaters since the structure of them are quite similar. For the cooler working at dry

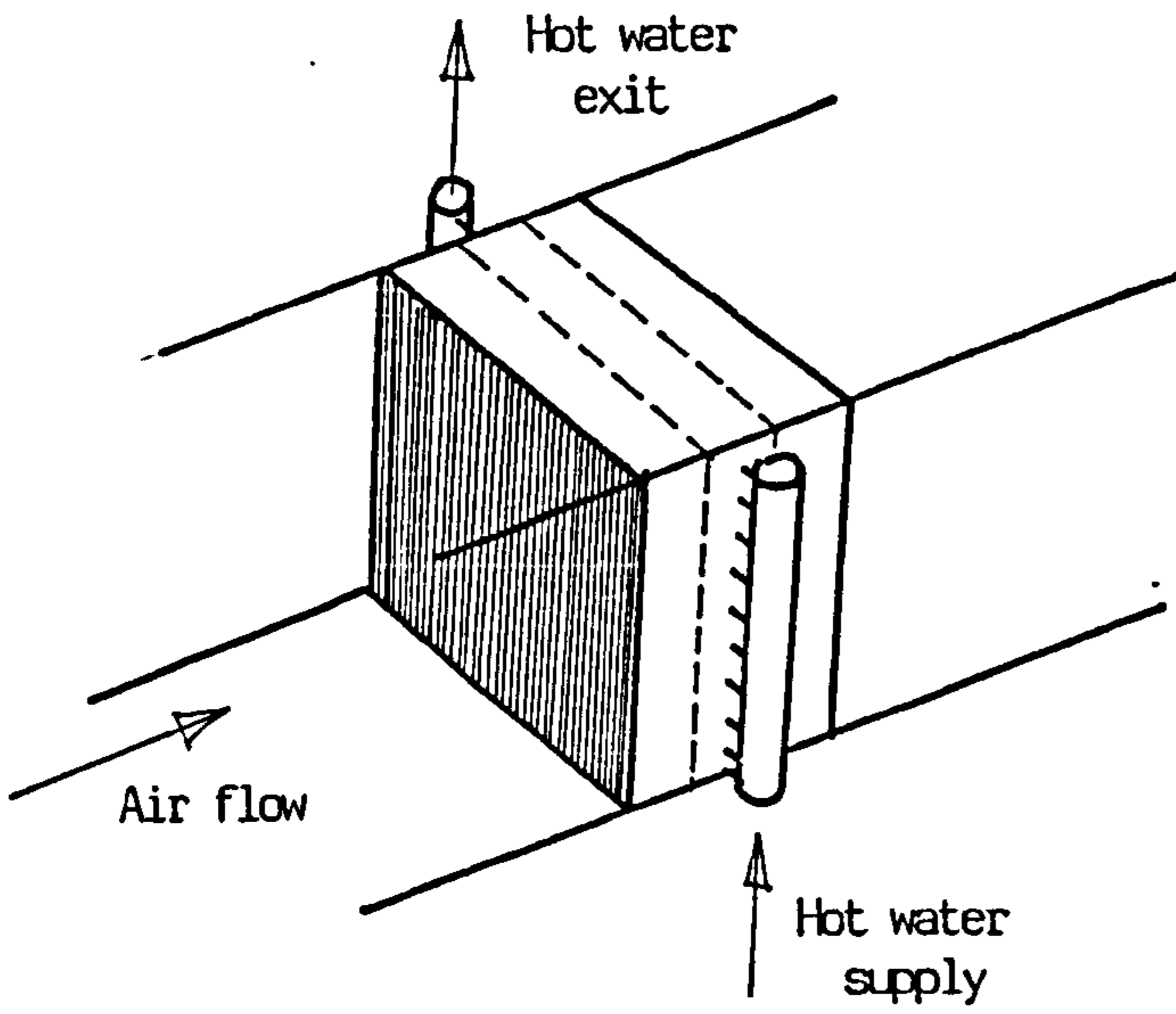


Figure 5.3 Structure of heater

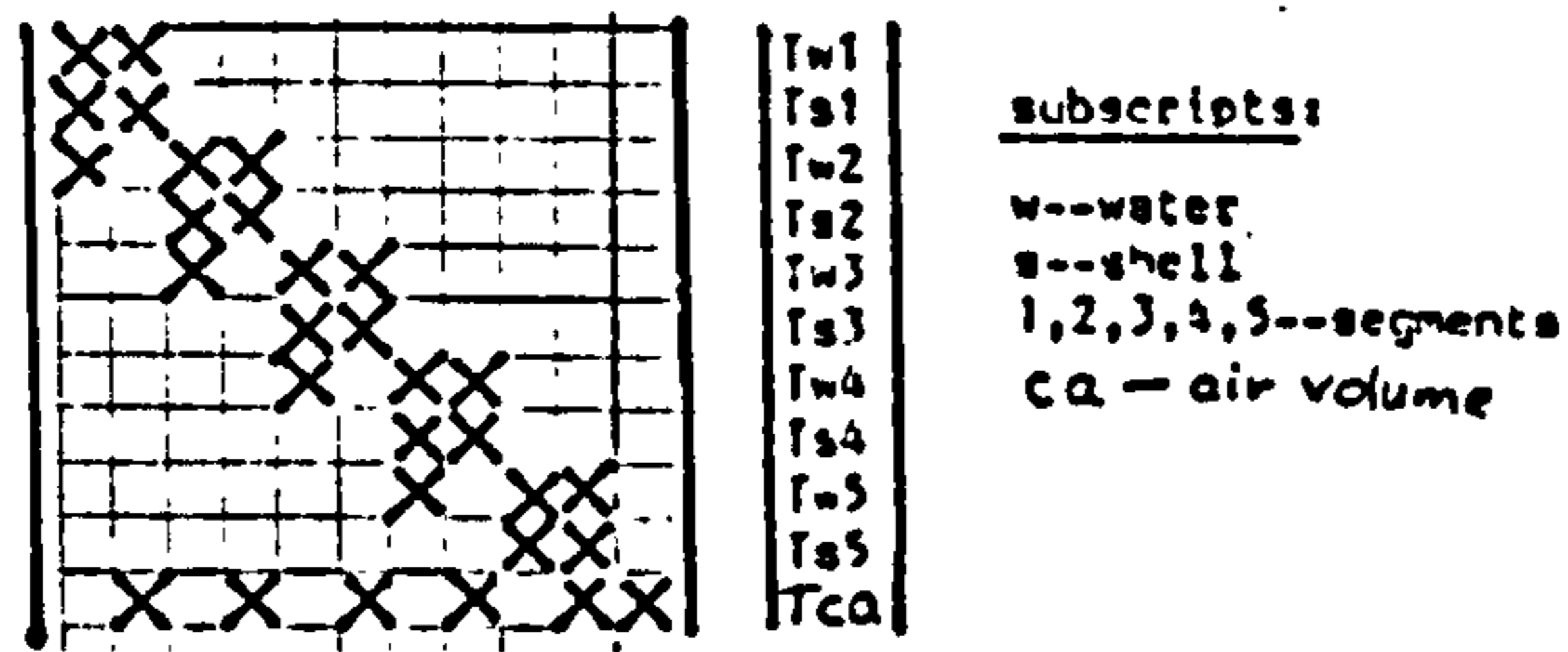


Figure 5.4 Coefficient matrix and state vector of heater model

condition, the surface temperature of cooler is always higher than the dew point temperature of conditioned air, no mass transfer will take place. The thermal and humidity state equations will exactly be the same as a heater discussed before.

For those coolers working at wet condition, condensation will be happened on the external surfaces of cooler. The thermal state equation will remain the same structure as shown in figure 5.4, but the condensation effect of the external surface of cooler has to be added to the total heat transfer coefficient.

To derive the humidity state equation, a conservation equation for the conditioned air flowing through the cooler can be written as:

$$\rho_I(t)v_I \frac{dw_I}{dt} = \dot{m}_a(t)[w_0(t)-w(t)] + \sum_{i=1}^5 h_{D,i}(t)A_i[w_{s,i}(t)-w_I(t)] \quad 5.5$$

where,

w_I = humidity concentration of control volume air

w_0 = humidity concentration of air entering the control volume

\dot{m}_a = mass flow rate of conditioned air through the cooler

$h_{D,i}$ = mass transfer coefficient between i-th surface to the control volume air

A_i = surface area of i-th section of cooler external surface

$w_{s,i}$ = humidity concentration of the i-th cooler external surface, it is the concentration corresponding to the vapor saturation condition at surface temperature $T_{s,i}$

ρ_I, v_I = density and volume of the control volume air

The difference form of equation 5.5 is given as:

$$\begin{aligned} & -a_1(t+\Delta t)W_0(t+\Delta t) + a_2(t+\Delta t)W_I(t+\Delta t) - \sum_{i=1}^5 a_{i+2}(t+\Delta t)W_{s,i}(t+\Delta t) \\ & = a_1(t)W_0(t) - a_8(t)W_I(t) + \sum_{i=1}^5 a_{i+2}(t)W_{s,i}(t) \end{aligned} \quad 5.6$$

where,

$$a_1(\xi) = \dot{m}_a(\xi)\Delta t_1$$

$$a_2(\xi) = 2\rho_I(\xi)v_I + a_1(\xi) + \sum_{i=1}^5 a_{i+2}(\xi)$$

$$a_i(\xi) = h_{D,i}(\xi)A_i\Delta t \quad (i = 3, \dots, 7)$$

$$a_8(\xi) = 2\rho_I(\xi)v_I - a_1(\xi) - \sum_{i=1}^5 a_{i+2}(\xi) \quad (i = 3, \dots, 7)$$

5.2.4 Fans

The method discussed in the previous chapter for the pump can directly be applied to the modelling of fans in air handling system. Special attention has to be paid is that the density of air is much lower than that of water, the temperature rising caused by the driving electrical power converted into heat energy becomes noticeable. The generation term in the control volume air state equation has to be properly defined.

5.2.5 Humidifiers

Types of humidifier used in air conditioning system are various. Equipment purely used as humidification usually is of the direct steam injection. The steam may be introduced from source outside the air handling system, e.g. the boiler, or a self generation steam cup mounted in the bottom of the duct. Other equipments such as the atomizing sprays and the air washers are also used for air humidification.

Here, consider the humidifier as shown in figure 5.5 is the type of direct steam injection. Assuming that the steam in the humidifier is very well distributed and mixed with the air, steam injected into the chamber at temperature T_g and flow rate \dot{m}_g , no excessive steam injected so that no condensation happened, two control volumes can be defined as the space of the chamber and the lumped shell and insulation. The thermal state equation can be obtained by taking the amount of heat bringing by the steam to the chamber as generation term in the fluid flow conservation equation, which is:

$$(\rho cv)_I \frac{dT_I}{dt} = \dot{m}_a c_p T_o - (\dot{m}_a c_p + \dot{m}_g c_{p_g} + h_A) T_I + \dot{m}_g c_{p_g} T_g + h_A T_a \quad 5.7$$

The state equation for the lumped shell insulation is similar to the duct. In spite of the coefficient differences, the thermal model for the humidifier is similar to the model of duct. External disturbances for this humidifier model are flow rate of steam injection, steam temperature and room air temperature. The humidity state equation is similar as equation 5.7 but no mass exchange between conditioned air and surface, which results:

$$(\rho v)_I \frac{dw_I}{dt} = \dot{m}_a w_o - \dot{m}_a w_I + \dot{m}_g \quad 5.8$$

or written in difference form as:

$$\begin{aligned} & -a_1(t+\Delta t)W_o(t+\Delta t) + a_2(t+\Delta t)W_I(t+\Delta t) - a_3(t+\Delta t)\dot{m}_g(t+\Delta t) \\ & = a_1(t)W_o(t) - a_4(t)W_I(t) + a_3(t)\dot{m}_g(t) \end{aligned} \quad 5.9$$

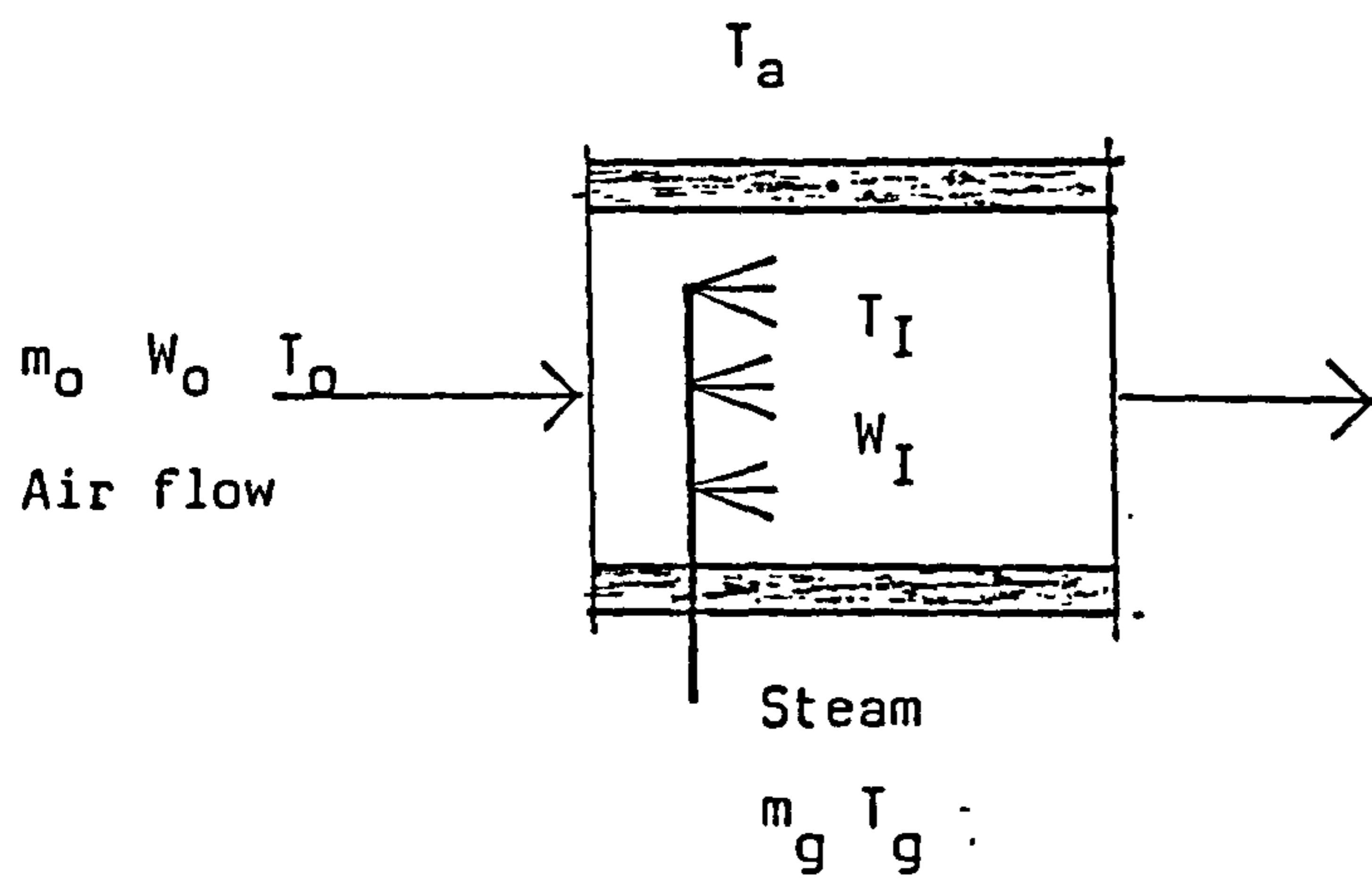


Figure 5.5 Direct steam injection humidifier

where,

$$a_1(\xi) = m(\xi)\Delta t$$

$$a_2(\xi) = 2\rho_I(\xi)V_I + a_1(\xi)$$

$$a_3(\xi) = \Delta t$$

$$a_4(\xi) = 2\rho_I(\xi)V_I - a_1(\xi)$$

5.2.6 Air washer

Air washer is very commonly used in air-conditioning system for either humidification or cooling-dehumidification processes. In air washer, water is directly contact with the conditioned air. Figure 5.6 shows an air washer with two directional water sprays. In air washer, the quantity of water circulated are very large compared with the quantity that could be evaporated to air or condensed from air. Also the rate of injection of feed water is usually greater than the air flow rate, consider the specific heat differences between water and air, temperature rising of water will be much smaller than the temperature drop of air. Air stream directly contacts the water drops atomized from the sprays, so that the heat and mass transfer will take place between the air stream and the water drops mixed with air. Since evaluating true surface areas of water drops are difficult, one approach is to work on a unit volume basis.

To develop the air washer model, the assumptions are made as follows:

- a) water drops atomized from the sprays are uniformly distributed at everywhere in the chamber.
- b) heat transfer is taken place between air and water drops at uniform temperature, mass transfer takes place between the humidity concentration of air and water drops with saturation concentration at the water temperature.
- c) a_h , a_m are the heat and mass transfer surface areas of water drops per unit volume of the chamber, respectively. They are also functions of water supply rate.
- d) heat and mass transfer coefficients between air and water drops are known, as h_D , K_D , respectively.

Control volumes of the air washer shown as figure 5.6 are defined as figure 5.7. For the control volume of shell and insulation of air washer, it is similar to that in duct. No control volume is defined for supply water since the water input condition is known and the outlet condition is not interested. For control volume of flowing air, heat exchange with water drops includes the sensible exchange and the latent heat which is caused by the loss and gain of humidity. The

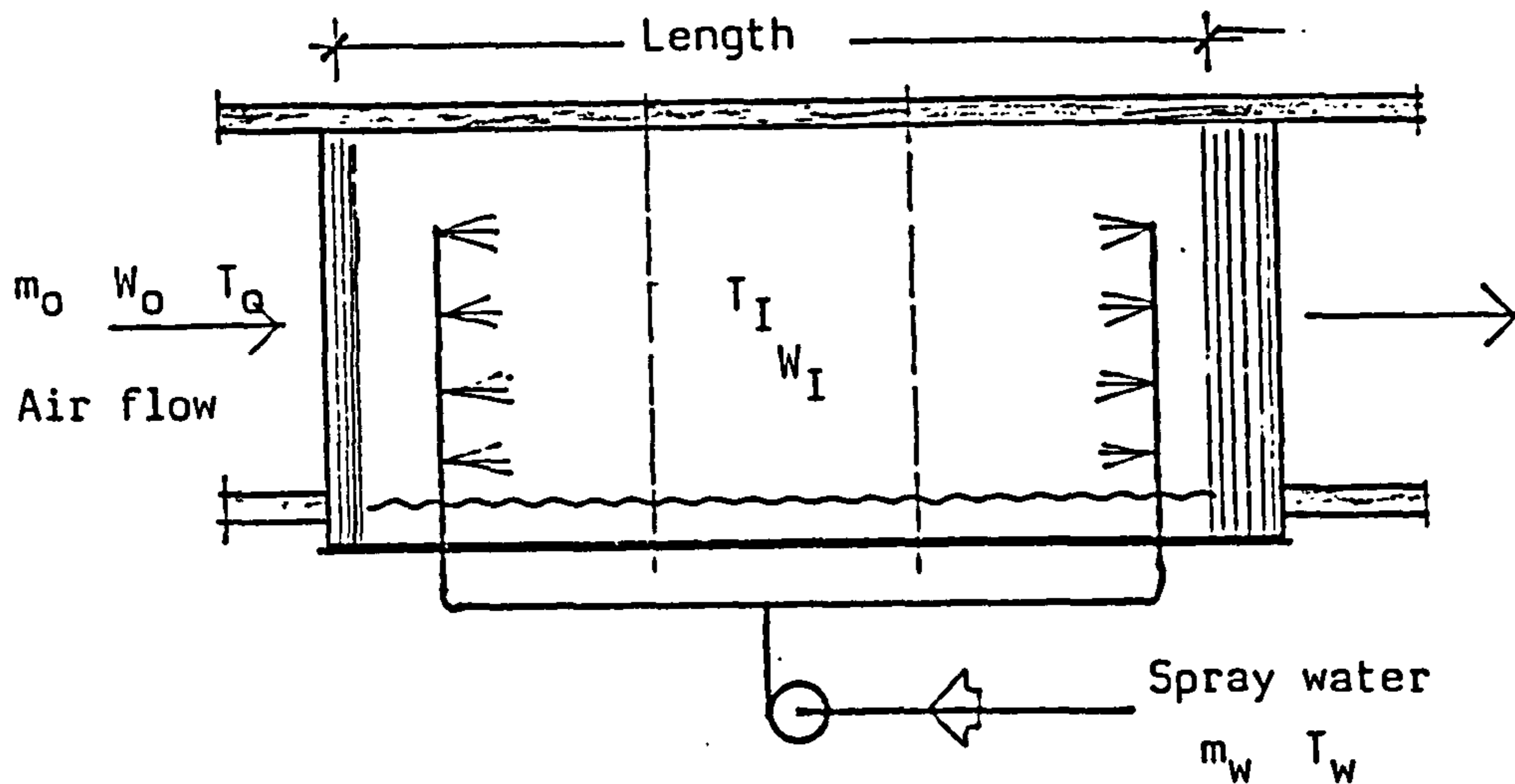


Figure 5.6 Air washer

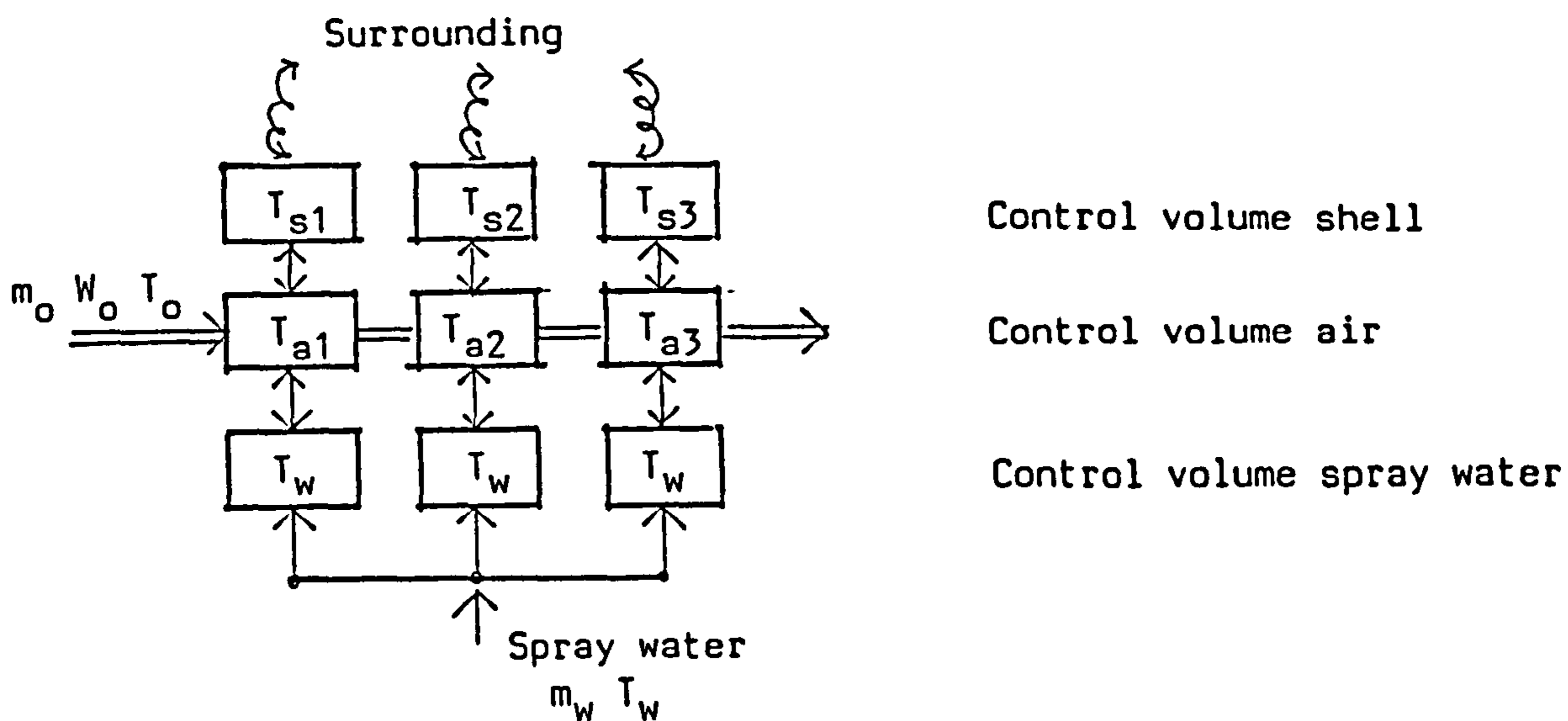


Figure 5.7 Control volume schematic of air washer

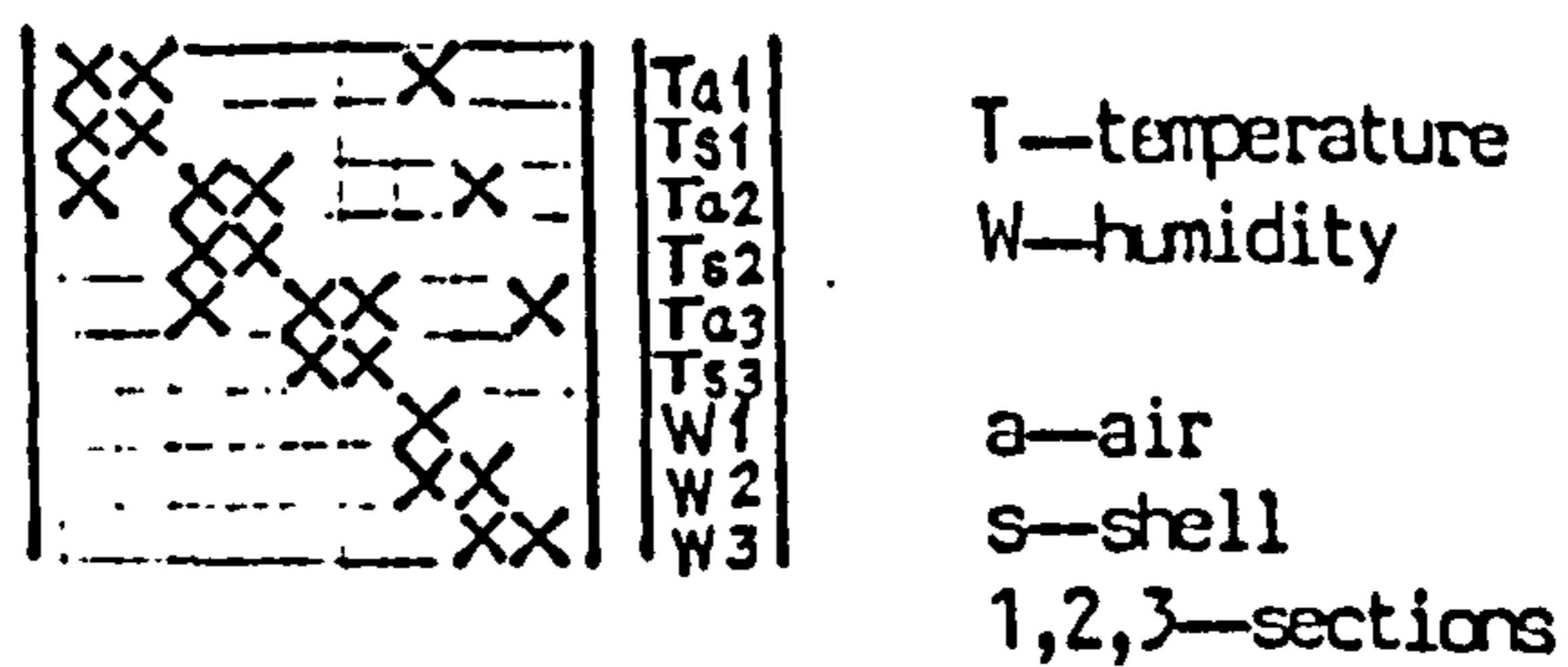


Figure 5.8 Coefficient matrix and state vector of air washer model

thermal and humidity state equations can be obtained as:

$$(\rho cv)_{a,i} \frac{dT_{a,i}}{dt} = \dot{m}_a c_p (T_{a,i-1} - T_{a,i}) + h_A (T_{s,i} - T_{a,i}) + h_D A_h (T_w - T_{a,i}) + K_D A_m (w_w - w_i) h_{fg}$$

5.10

and

$$(\rho v)_{a,i} \frac{dw_i}{dt} = \dot{m}_a (w_{i-1} - w_i) + k_D A_m (w_w - w_i)$$

5.11

where,

h_A = heat transfer coefficient between air and shell,
surface area of shell

h_D = heat transfer coefficient between air and water drops

A_h = $a_h v_{a,i}$; total heat exchange area for heat exchange
between air and water

k_D = mass transfer coefficient between air and water drops

A_m = $a_m v_{a,i}$; total mass transfer area for mass exchange
between air and water

h_{fg} = latent heat of water vaporisation or condensation

$T_{a,i}$ = temperature of air

w_i = concentration of air

w_w = concentration of water drop at saturation vapor of T_w

T_w = spray water temperature

It is clear, the thermal state equation contains variable of humidity, therefore, in this model, the thermal and humidity equations are crossed. State variables are taken as temperature and humidity of control volume air, and temperature of lumped shell and insulation. Disturbances include the inlet air conditions, temperature of supply water and temperature of environment. Matrix structure and state vector for this air washer model are shown in figure 5.8.

5.2.7 Control devices

Models derived in the previous chapter for control devices of heating system can be directly applied to the devices in air-conditioning system. The special considerations needed to pay to the regulator characteristics, such as relations between the opening of a damper and the air flow rate, opening of a three way valve and the water flow rate in each branch. In practice, control loops used in air-conditioning system are much more sophisticated than in heating system.

5.2.8 The refrigeration plant

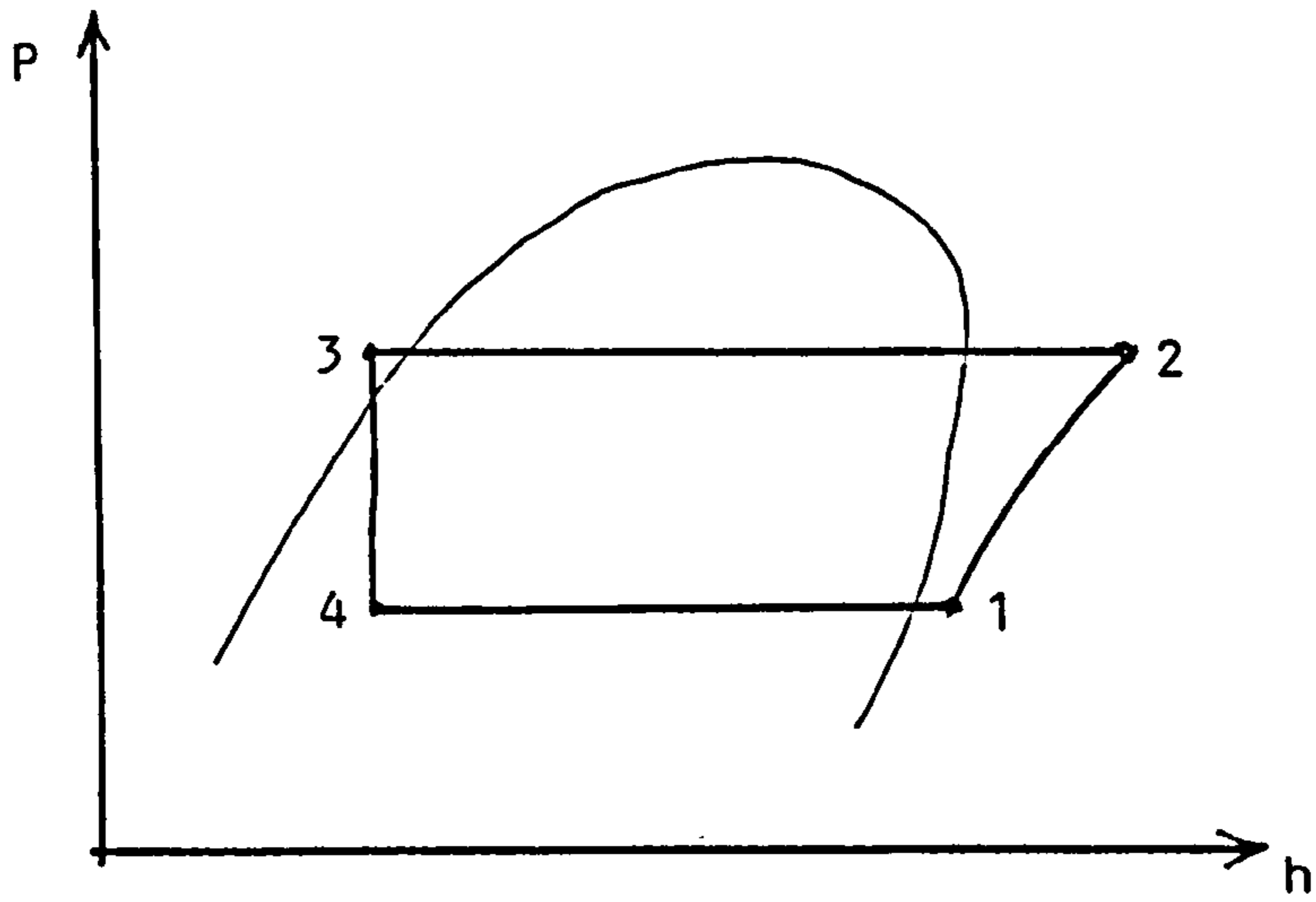
Like the heating plant, the refrigeration plant is an independent closed loop sub-system in the complete air-conditioning system. A basic refrigeration loop comprises a compressor, condenser, expansion valve and an evaporator connected parallelly to the refrigerant circuit, as shown in figure 5.2. In the refrigeration loop, the super-heated vapourised refrigerant leaving the compressor at high pressure is delivered to the condenser to be cooled down and liquidised. Liquid refrigerant leaving the condenser passes through the expansion valve depressurized to the evaporator. After absorbing heat from the evaporator, the vapourised refrigerant returns to the compressor. Figure 5.9 shows such a vapour-compression refrigeration cycle.

To develop the refrigeration plant model, the first assumption is made as: the relationship governing the gaseous refrigerant is known which can be either coorelated by the law of ideal gas or real gaseous refrigerant.

1. The compressor

The efforts of modelling the performance of compressor for different purposes have been experienced a long time. The compressor is a complicated machine, the modelling can be done both in modelling the overall performance and in modelling the detailed processes, such as heat transfer rate, fluid friction, pressure losses, valve motion, etc. The objective here is concentrated on the modelling of overall thermal performance by using the technique of control volume state space analysis.

Figure 5.10 shows the conceptual structure of a reciprocating compressor in which the piston is driven by an electrical motor and in constant motion. The control volume is defined as the gaseous refrigerant confined in the internal space of the cylinder. It can be considered as the type of flowing fluid control volume, however, the confined volume in the cylinder is changing with time according to the



- 1 - 2 Compression process
- 2 - 3 Condensing process
- 3 - 4 Expansion process
- 4 - 1 Evaporation process

Figure 5.9 Schematic diagram of a vapor-compression refrigeration cycle

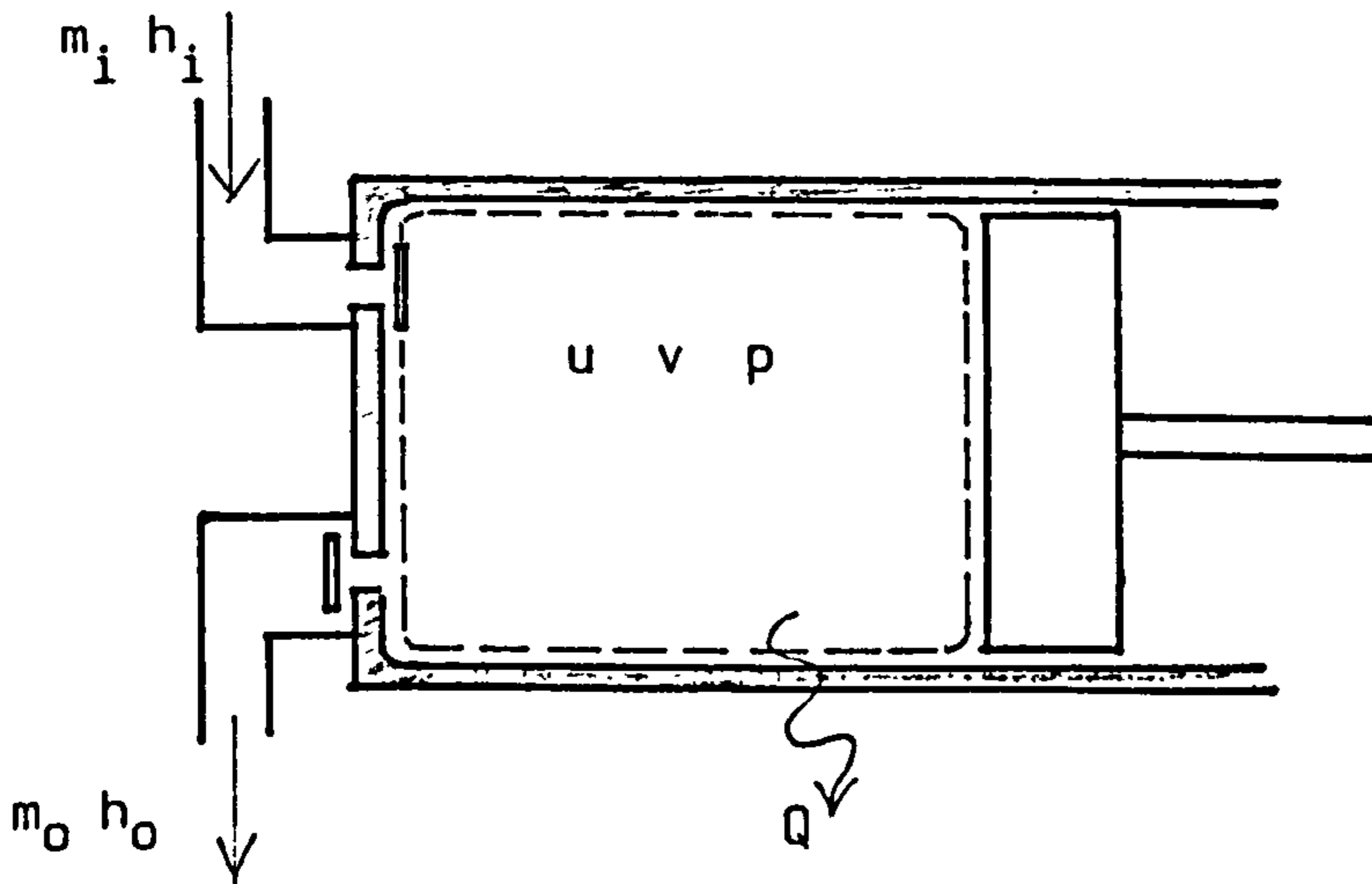


Figure 5.10 Structure and control volume of a reciprocating compressor

reciprocating movement of the piston.

The general control volume conservation can be applied to this control volume, it results:

$$\frac{d(GU)}{dt} = \dot{m}_i h_i - \dot{m}_o h + \dot{Q} - \frac{dL}{dt} \quad 5.12$$

where,

G = weight of gas in the control volume

U = internal energy of the control volume

\dot{m}_i, \dot{m}_o = inlet and outlet mass flow rate of gas, respectively

h_i, h = enthalpy of inlet gas and gas in the control volume, respectively

L = work done to the control volume

\dot{Q} = heat exchange rate of control volume gas and enclosure.

Since,

$$\frac{d(GU)}{dt} = \frac{d}{dt}[G(h-pv)] = \frac{d(Gh)}{dt} - \frac{d(Pv)}{dt} = G \frac{dh}{dt} + h \frac{dG}{dt} - p \frac{dv}{dt} - v \frac{dP}{dt}$$

and

$$\frac{dL}{dt} = p \frac{dv}{dt} \quad ; \quad \frac{dG}{dt} = \dot{m}_i - \dot{m}_o$$

equation becomes:

$$\frac{dp}{dt} = \frac{1}{v} \frac{dh}{dt} - \frac{1}{v} [\dot{m}_i (h_i - h) + \dot{Q}] \quad 5.13$$

where,

v = specific volume of gas in control volume

V = volume of the cylinder

Equation 5.13 is the governing state equation of compressor. The first term at right hand side, $(1/v)dh$, can be directly obtained if the state equation of gas, either ideal or real, is known. Volume of cylinder, V , is a function of the piston motion. Gas flow rate, \dot{m}_i , can be determined by knowing the piston swept volume and the clearance volume of the compressor. Heat exchange rate \dot{Q} is also a function of

the piston movement since the volume of gas is changing. By knowing the pressure of gas in the cylinder, temperature can be directly obtained through the gas state equation.

For example, if ideal gas is considered, according to the Maxwell relationship of thermodynamics:

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

and

$$dT = \left(\frac{\partial T}{\partial v}\right)_p dv + \left(\frac{\partial T}{\partial p}\right)_v dp$$

it follows,

$$\left(\frac{\partial h}{\partial T}\right)_p = c_p ; \quad \left(\frac{\partial T}{\partial v}\right)_p = \frac{p}{R} ; \quad \left(\frac{\partial T}{\partial p}\right)_v = \frac{v}{R} ; \quad \left(\frac{\partial v}{\partial T}\right)_p = \frac{v}{T}$$

and therefore,

$$\frac{1}{v} dh = \frac{c_p p}{R v} dv + \frac{c_p}{R} dp \tag{5.14}$$

Substitute equation 5.14 to 5.13, it follows:

$$\begin{aligned} \frac{dp}{dt} &= -\frac{\gamma p}{v} \frac{dv}{dt} + \frac{(\gamma-1)}{v} [\dot{m}_i(h_i-h) + \dot{Q}] \\ &= -\frac{\gamma p}{v} \left(\frac{dv}{dt} - v \frac{dG}{dt}\right) + \frac{(\gamma-1)}{v} [\dot{m}_i(h_i-h) + \dot{Q}] \end{aligned} \tag{5.15}$$

If no leakages from the suction and discharge valves and piston are considered, $dG = 0$, equation 5.15 becomes:

$$\frac{dp}{dt} = -\frac{\gamma p}{v} \frac{dv}{dt} + \frac{(\gamma-1)}{v} [\dot{m}_i(h_i-h) + \dot{Q}] \tag{5.16}$$

$$(\gamma = c_p/c_v)$$

The piston motion is shown in figure 5.11, the volume of cylinder gas space can be represented as a function of time, which is:

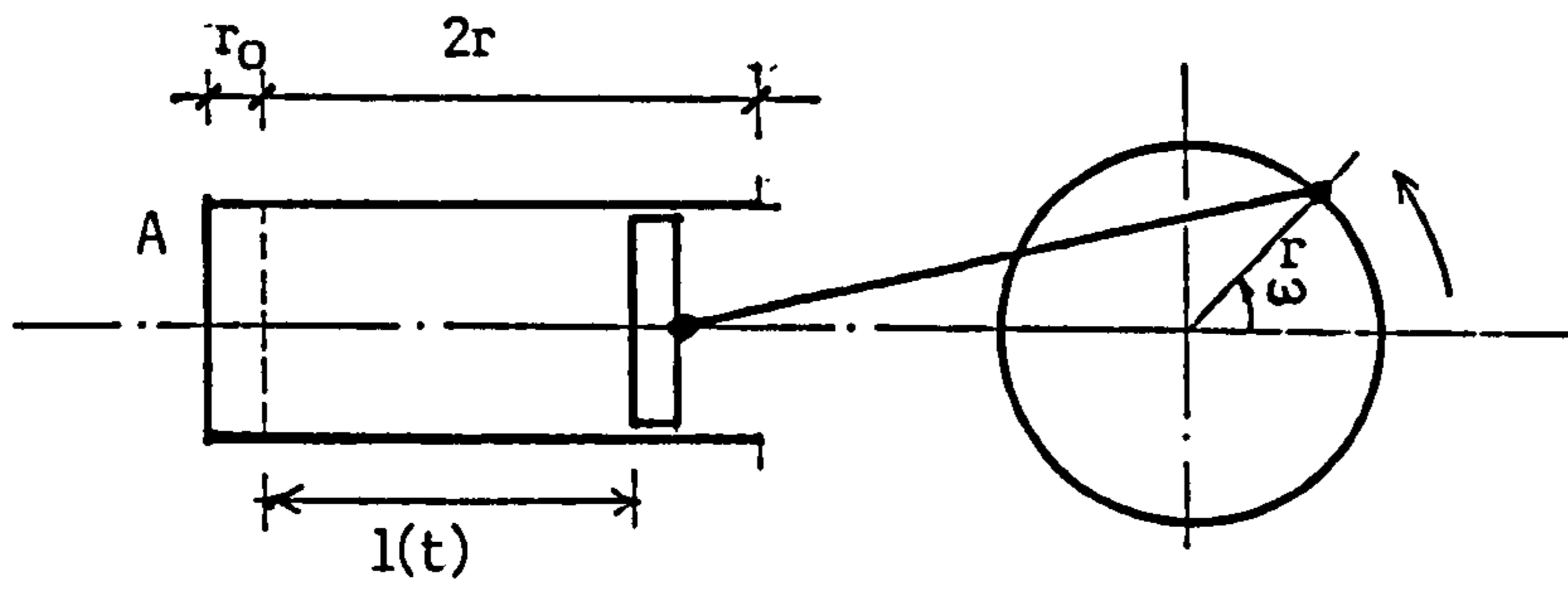


Figure 5.11 Structure of compressor piston motion

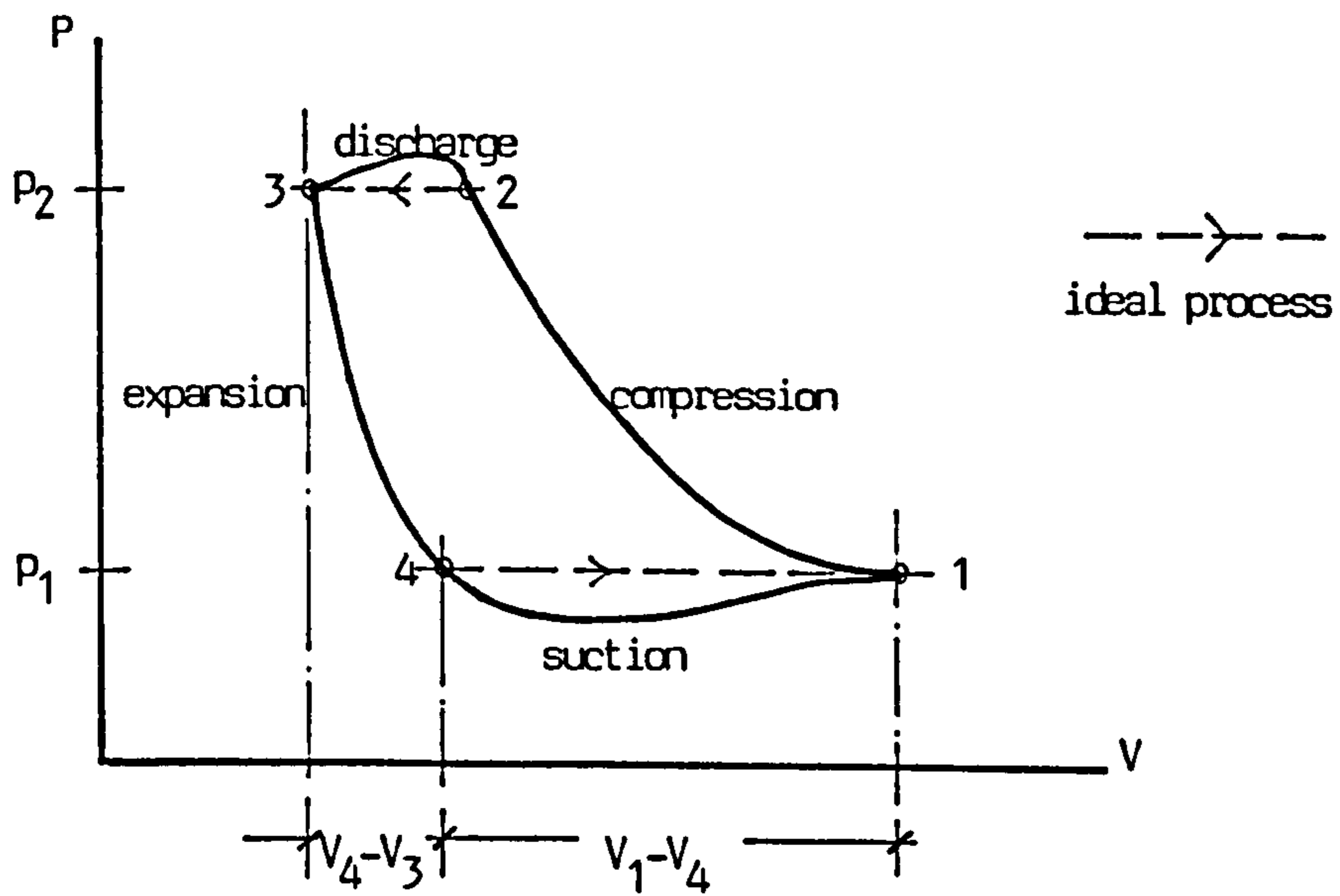


Figure 5.12 Conceptual P-V diagram of compression cycle

$$V(t) = [r(1+\cos wt) + r_0]A \quad 5.17$$

in which,

$0 < wt < \pi$: compression process

$wt = \pi$: discharge process

$\pi < wt < 2\pi$: re-expansion process

$wt = 2\pi$: suction process

Neglect the resistances of suction and discharge valves, the gas flow rate can be represented by a function of time, which is:

$$\dot{m}_i(t) = \begin{cases} 0 & \text{compression} \\ [r(1+\cos wt) + r_0] AP_i \frac{w}{\pi} \eta_v & \text{discharge} \\ 0 & \text{re-expansion} \\ [r(1+\cos wt) + r_0] AP_i \frac{w}{2\pi} \eta_v & \text{suction} \end{cases} \quad 5.18$$

Furthermore, if adiabatic processes are considered, $\dot{Q} = 0$. Substitute equation 5.17, 5.18 to 5.16, the state equation for control volume gas of a compressor is complete. In fact, this simplified example can be solved analytically. As an example, solution of equation 5.16 for compression process results:

$$p_2 = p_1 \left(\frac{2r + r_0}{r(1+\cos wt) + r_0} \right)^r = p_1 \left(\frac{v_1}{v_2} \right)^r \quad 0 < wt < \pi$$

which is a typical expression of adiabatic compression.

Analytical solution for fully developed compressor state equation 5.15 is usually difficult to be achieved. For numerical solution, time step has to be defined small enough within each process of compression and the dynamic performance can be properly modelled.

In practice, the process of vapour-compression is very fast, the movement of piston usually lies around a few hundred r.p.m. Compare to other processes in the air-conditioning system, it is quite

reasonable to model the compressor using steady state method based on the thermodynamic analysis. One approach is to use perfect gas approximation undergoing a polytropic process where the polytropical exponent is adjusted until the total work and heat agree with the real processes. The polytropical index, therefore, includes the combined effects of heat transfer, friction and deviation of gas properties from ideal gas behaviour, and thus masks the separate influences of these factors.

The real and ideal cycles of a reciprocating compressor is shown in figure 5.12. The polytropic compression and expansion process can be written as:

$$p v^n = \text{constant}$$

The total work required for the ideal cycle can be represented by:

$$w_{\text{tot}} = \frac{n}{n-1} p_1 (v_1 - v_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \quad 5.19$$

where,

n = polytropical index

Since the existence of clearance in the compressor, there is still an amount of high pressure gas remained in the cylinder after discharge. The volumetric efficiency is defined as the ratio of the actual volume of gas induced per revolution to the swept volume of compressor per revolution, which is:

$$\eta_v = \frac{v_1 - v_4}{v_1 - v_3}$$

According to the polytropical relation, it can be written as:

$$\eta_v = 1 - \left(\frac{v_3}{v_1 - v_3} \right) \left[\left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right] \quad 5.20$$

Substitute equation 5.20 to 5.19 and re-arrange it, it becomes:

$$p_2 = p_1 \left[1 + \frac{w_{\text{tot}} \left(1 - \frac{1}{n} \right)}{p_1 v_{\text{swept}} \eta_v} \right]^{\frac{n}{n-1}} \quad 5.21$$

and,

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \quad 5.22$$

Mass flow rate of vapour refrigerant is represented by:

$$\dot{m} = p w v_{\text{swept}} \eta_v \quad 5.23$$

where, w = compressor driven speed. (revolution/time)

In practice equation 5.20 usually gives higher values for volumetric efficiency since the actual losses of piston leakage, valve leakage, throttling effect and oil entrainment have not been taken into account. The accuracy of equation 5.20 may be improved by employing the effective clearance volume ratio C_{eff} , equation 5.20 then becomes:

$$\eta_v = 1 - C_{\text{eff}} \left[\left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right] \quad 5.24$$

Therefore, equation 5.21, 5.22, 5.23, 5.24 can be used as the steady state compressor model. The inputs are the state of gas sending from the evaporator and the theoretical power, outputs are the state of gas discharged by the compressor and the gas flow rate which in turns will be used by the condenser in parallel connection.

Assume the polytropical exponent n and the volumetric efficiency of compressor are known factors, equation 5.21 to 5.24 can be combined and the model of compressor can be represented by:

$$T_2 = T_1 + \left(\frac{w(n-1)}{R \dot{m} n} \right) W_{\text{tot}} \quad 5.25$$

where,

R = gas constant of vapour refrigerant

w_{tot} = theoretical power required by the compressor

According to equation 5.23, flow rate of refrigerant is proportional to the driving speed. If the driving force, electrical motor, is working at on-off condition, \dot{m} and W can be considered as constant. The term W_{tot} can be used as disturbance of known factor or state variable which can be controlled by feedback control loop. Equation 5.25 together with 5.23 are sufficient to represent the model of compressor.

2. Condenser and evaporator

Condenser and evaporator are heat exchangers, the structures can be quite different when they are used in different place. For those in which water or other liquid are used as heat exchange mediums, they are similar to the heat exchangers discussed in the previous chapter of heating system. For those use air as medium, by forced or natural convection, they are similar to the heater or cooler discussed before in this chapter. The most complicated thing in modelling the condenser and evaporator is to determine the heat transfer coefficients at the side of refrigerant.

For condenser, super-heated gaseous refrigerant is delivered into it, after the heat exchange process, it becomes liquid leaving the condenser.

The similar but reversed process is happened in the evaporator. Despite the heat transfer coefficient differences, the mathematical models of condenser and evaporator will have the same matrix structure as the models developed for heat exchangers in the previous chapter and the models of heater and cooler.

Very few theoretical relation for two phase flow heat transfer is available to be used properly as the general formulae of generating the heat transfer coefficients in condenser and evaporator. The approaches are usually accepted as the approximation of this difficulty, which are:

- a) the refrigerant flowing inside the passage of condenser or evaporator is considered as a uniform gas-liquid mixture, and therefore, a global two-phase flow heat transfer coefficient can be obtained and applied to the side of flowing refrigerant. This global two-phase flow heat transfer coefficient then contains the total heat exchange effects of liquid, gas flow, condensation and evaporation processes.
- b) the internal space of condenser or evaporator is divided into zones of pure liquid, gas and liquid-gas mixture and homogeneous heat transfer of liquid, gas and liquid-gas mixture are taken place in the individual zone, respectively. The thermal relationship between each two zones can also be specified.

Considering a water-cooled condenser having structure as a heat exchanger given in figure 4.21, for the first approach, the model matrix structure will exactly be the same as given in figure 4.23. However, the coefficients related to the side of refrigerant have to be computed according to the global two-phase heat transfer process. For the second approach, the control volumes have to be defined according to the three zones of pure gas, liquid and gas-liquid mixture. The volumes of the three zones are usually not constant, they can be functions of inlet parameters and flow rate of gaseous refrigerant, and conditions of water. It satisfies:

$$V_{total} = V_{liquid} + V_{gas} + V_{mixture}$$

Assuming the control volumes of this condenser can be defined as the heat exchanger given by figure 4.22, in which the effects of zoning are considered, the system state equation to result will have the same expression as given in figure 4.23. The coefficients in the coefficient matrix have to be computed according to the individual homogeneous heat transfer effect and the effect of time varying control volume.

The above analysis of condenser can be parallelly applied to the modelling of evaporator.

3. Expansion valve

The process taking place in the expansion valve can be considered as adiabatic expansion, no heat exchange with surroundings. Since the mass flow rate of refrigerant passing through the expansion valve is determined by the motion of compressor, the performance of the expansion valve can be considered as a pure pressure transducer.

Assuming that the refrigerant entering the expansion valve at pure liquid state and leaving at gas-liquid mixture, the pressure relationship between the inlet and outlet of the valve can be represented by:

$$p_4 = p_3 - k \frac{u_1^2 \rho_3}{2} \quad 5.26$$

where,

k = friction and local loss factor

u_1 = velocity of liquid refrigerant entering the valve

ρ_3 = density of liquid refrigerant of inlet

The loss factor k is a function of the valve openness and corresponding to the dimension and condition of the entrance.

Equation 5.26 can be re-arranged to:

$$p_4 = p_3 - k' \dot{m}^2 \quad 5.27$$

in which,

$$k' = \frac{k}{2A\rho_3}$$

where,

A = entrance cross-sectional area of valve

Instead of using equation 5.27, the relation given to the valve in section 4.2.6 of equation 4.19 can also be used to describe the pressure relationship, equation 5.27 can then be written as a single

value function of valve opening or displacement of regulator, in the general form:

$$\Delta P = (P_2 - P_1) = f(D) \quad 5.28$$

If the expansion valve is a part of a temperature control loop, it can be treated as the model of actuator/regulator as derived in section 4.2.6 of equation 4.18. Temperature of refrigerant leaving the expansion valve can be obtained by the thermodynamics of adiabatic throttling process in which the enthalpy of refrigerant is assumed to hold constant before and after throttling. According to this assumption, temperature and dryness of vapour refrigerant after throttling can be directly found from the "pressure-enthalpy" chart of refrigerant which is assumed to be known throughout the modelling.

4. The refrigeration plant

As an application, the refrigeration plant as shown in figure 5.2(B) is going to be modelled. Components comprised in the system are:

- a) reciprocating compressor, steady state model as equation 5.25, state variable is taken as the temperature of outlet gas of compressor.
- b) water-cooled condenser, matrix state equation of hot water tank is used to be this model.
- c) expansion valve, steady state model, state variable is taken as the temperature of gas after throttling represented a function of pressure after throttling, as:

$$T_4 = f(P_4) \quad 5.29$$

- d) evaporator, using water as a medium, matrix state equation of hot water tank is used to be this model.

The effects of pipes connecting the components are neglected. Pressure equations are only used in computing the coefficients. Referring to figure 5.9, pressure equation for each component is given as:

compressor:
$$P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{\frac{n}{n-1}} \quad 5.30$$

condenser : $P_3 = P_2$ 5.31

expansion valve: $P_4 = P_3 - K\dot{m}$ 5.32

evaporator: $P_1 = P_4$ 5.33

The plant system model can be represented by matrix state equation as:

$$A(t+\Delta t)T(t+\Delta t) = B(t)T(t) + C(t+\Delta t, t) \quad 5.34$$

Figure 5.13 shows the positions of coefficients in matrix $A(t+\Delta t)$ and $B(t)$ and the related state variables.

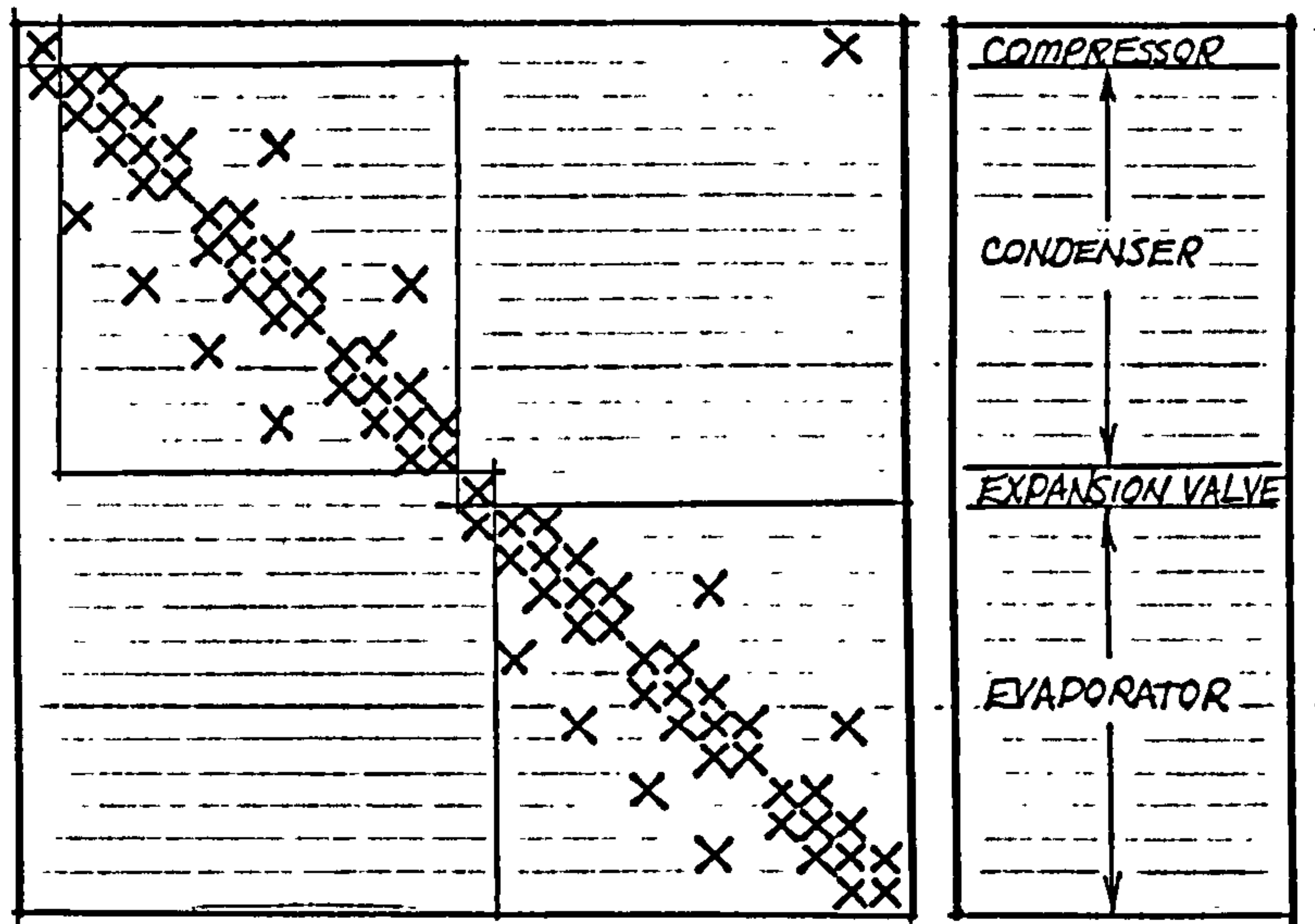


Figure 5.13 Coefficient matrix of the refrigeration plant model

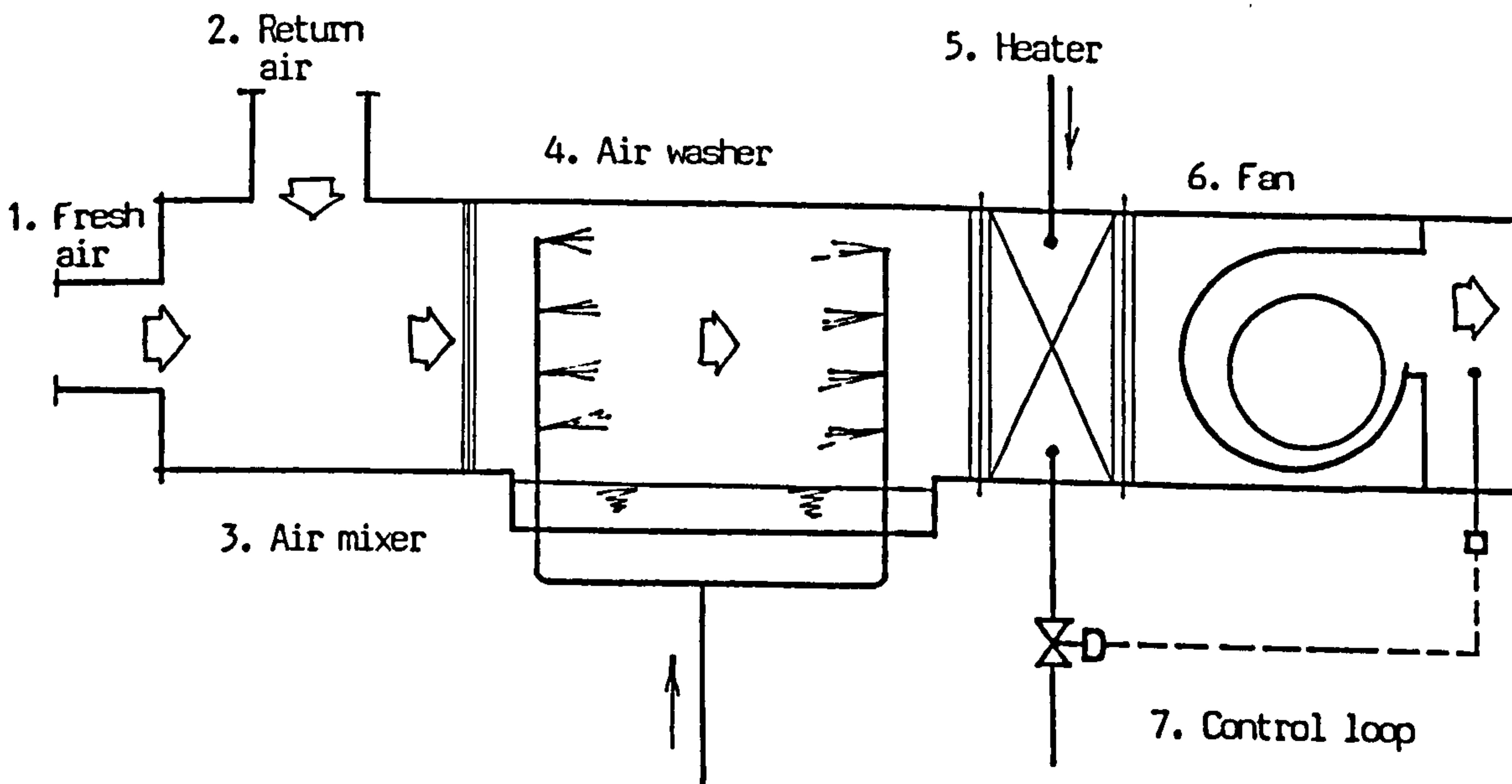


Figure 5.14 Central air handling unit

5.3 Example of system representation

The main components encountered in air-conditioning system have been discussed in this chapter. It can be found that the models of components of air-conditioning system are similar to the models derived in the previous chapter of heating system. This fact shows that other models which have not been derived can be obtained without vital difficulty by using the approach of control volume state space analysis. The examples of constructing the air-conditioning system model are going to be given as follows.

5.3.1 The air handling system

Figure 5.14 shows a central air handling unit which is quite commonly used in large system as air treatment. Fresh air intaken from the environment is mixed with return air from the conditioned space, after passing through the air washer, air reaches almost it saturation temperature closed to the spray water temperature, it is reheated again to the required supply temperature and then is supplied to the conditioned space by the fan. A temperature control loop is connected between the supply air and the flow rate of hot water of the heater to achieve accurate supply air temperature. The components are numbered along the air flow direction. Components used in this central air handling unit are:

1. Fresh air duct:

Represented by two control volumes model, thermal state equations are similar to equation 4.1 and 4.2, humidity state equation is given as equation 5.2.

2. Return air duct:

As duct 1.

3. Air mixer:

Two control volumes model, thermal state equations are similar to equation 4.4 and 4.2, humidity state equation is given by equation 5.4

4. Air washer:

Six control volumes model, consists six thermal equations and three humidity equations, model matrix structure and state variables are shown as figure 5.8.

5. Heater:

Eleven control volumes, structure of thermal state equation is shown in figure 5.4, humidity state equation is same as equation 5.2.

6. Fan:

Two control volumes model, state equations are similar to duct.

7. Control loop:

Includes sensor, proportional controller, actuator and valve to control the supply water flow rate of heater, represented by equation 4.22.

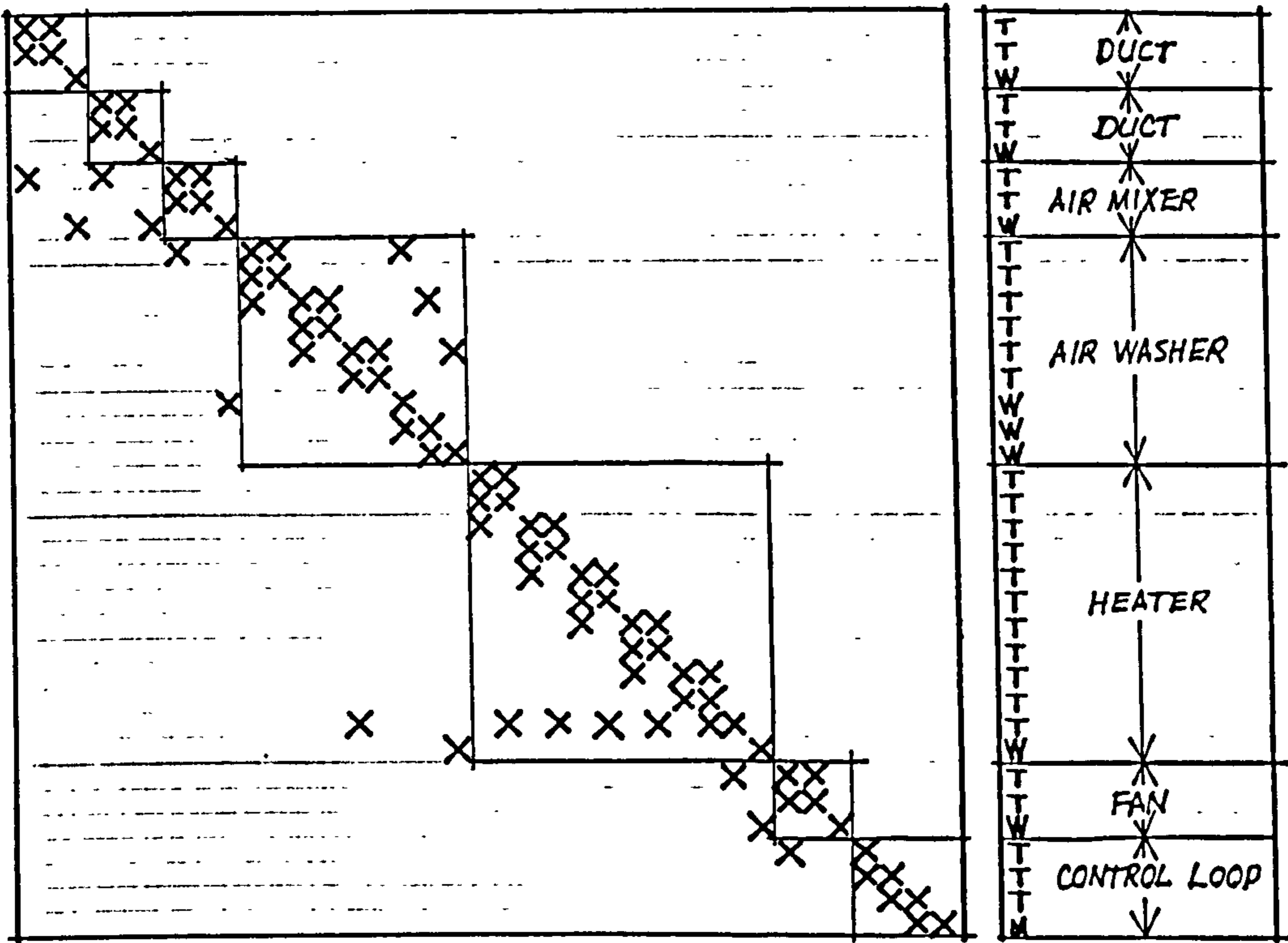


Figure 5.15 Structure of coefficient matrix and the relevant state vector of the air handling unit

The system state equation is given by the standard matrix difference state equation as:

$$A(t+\Delta t)T(t+\Delta t) = B(t)T(t) + C(t+\Delta t, t) \quad 5.35$$

Figure 5.15 shows the structure of matrix $A(t+\Delta t)$ or $B(t)$ and the corresponding state vector $T(t+\Delta t)$ or $T(t)$. In the state vector, T denotes the state variables of temperature and W denotes the state variables of humidity in each component.

5.3.2 The complete air-conditioning system

A complete air-conditioning system can be considered as a multi-zone problem which consists of a number of closed or open loop sub-systems. The same procedure of constructing the sub-system model can be used to formulate the complete air-conditioning system model. Generally speaking, the complete system model can be resulted by the collection of all the single closed or open loop sub-system models and specifying the inter-relationship among them. As demonstrate before, each single sub-system model can be represented by the state equation of:

$$A_i(t+\Delta t)T_i(t+\Delta t) = B_i(t)T_i(t) + C_i(t+\Delta t, t) \\ (i = 1, 2, \dots, n)$$

taking together all the sub-system models, the complete system model can be represented by:

$$A(t+\Delta t)T(t+\Delta t) = B(t)T(t) + C(t+\Delta t, t)$$

T , here, is the state vector of the complete system model, which is
 $T = [T_1, T_2, \dots, T_n]^T$

The complete system matrix A contains the sub-system matrices A_i on the diagonal. The coefficients representing the inter-relationship between the single sub-system models were originally contained in vector C_i now have to be moved to matrices A . These coefficients now are scattered at two sides of the diagonal block of the complete system matrices A . Variables remained in system vector C are external disturbances to the complete system model which are considered as known values or known functions of time.

The example will be given to the formulation of the complete system model for the air-conditioning system shown in figure 5.2. In figure 5.2, the air-conditioning system comprises three closed loop sub-systems: the heating plant, the refrigeration plant and the air handling plant; one open loop sub-system; the cooling tower; and three inter-connected loops: the heater-tank loop, the cooler-evaporator loop and the condenser-cooling tower loop. A central controller can

possibly be introduced in the conditioned space to control the operations of heating and refrigeration plants.

Figure 5.16 summarises these system formulation procedures. These procedures can also be extended to include any number of loops of subsystem and zones of building to present the energy dynamics of a whole building and HVAC system.

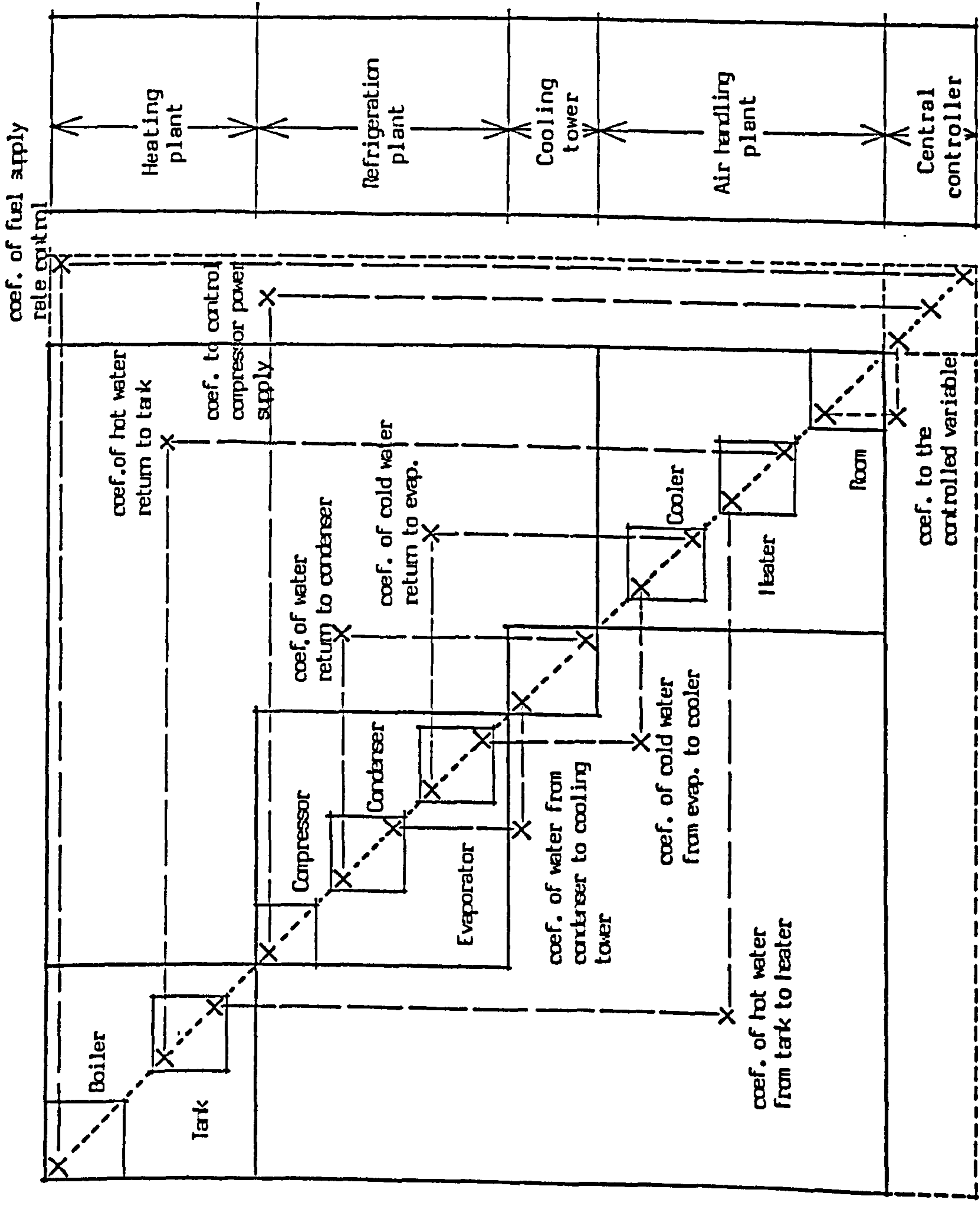


Figure 5.16 The state equation structure of the complete air-conditioning system model

CHAPTER SIX: THE COMPUTER SIMULATION MODEL

6.1 The computer program 'CENSYS'

6.2 The program and subroutines

6.2.1 The input subroutines

6.2.2 The coefficients generation subroutines

6.2.3 The main simulation subroutines

6.3 The output facilities

6.4 The development of advanced software

6.4.1 The implementation of ESP

6.4.2 The development of Data Processors

A prototype computer program CENSYS has been developed based on the theory, methodology and component models demonstrated and derived in the previous chapters. The main purpose of developing this program is for the validation of component models as well as for the complete systems. The validation itself is concentrated on the validity of the basic theory of control volume state space approach rather than to each individual component model. After this stage, the new generation software of generalised energy simulation will be developed to provide the powerful tool for theoretical analysis of HVAC systems and components and for different purposes of practical modelling.

6.1 The computer program CENSYS

CENSYS is a component based central heating system simulation program which is originally designed for the Annex 10 exercises organised by International Energy Agency. This computer program contains a library of component models of central heating system ranged from boilers, radiators, insulated and bare pipes, pump, tank and control devices. According to the component library, a variety of central heating systems can be modelled by inputting the system specifications. The resulted system model - the system matrix - will be solved by a solution subroutine in which the three different matrix solution methods can be chosen. The primary results of the system simulation are the time history of each state variable, vast amount of information can then be obtained from the known time history of state vector. A results library stores the time-dependent results for later interrogation and display. A generalised program flow chart is shown in figure 6.1.

In general, for a complete system simulation, the input of the program requires:

1. Dimensions of each component and fuel compositions.
2. The system layout specification. This is the detailing of the inter-connections of each component.
3. The external excitations which include the ambient climate and room conditions (which can be taken from the output of the building model at same time step).

Possibilities are given to choose some options such as the simulation time step, output parameters to display and control functions.

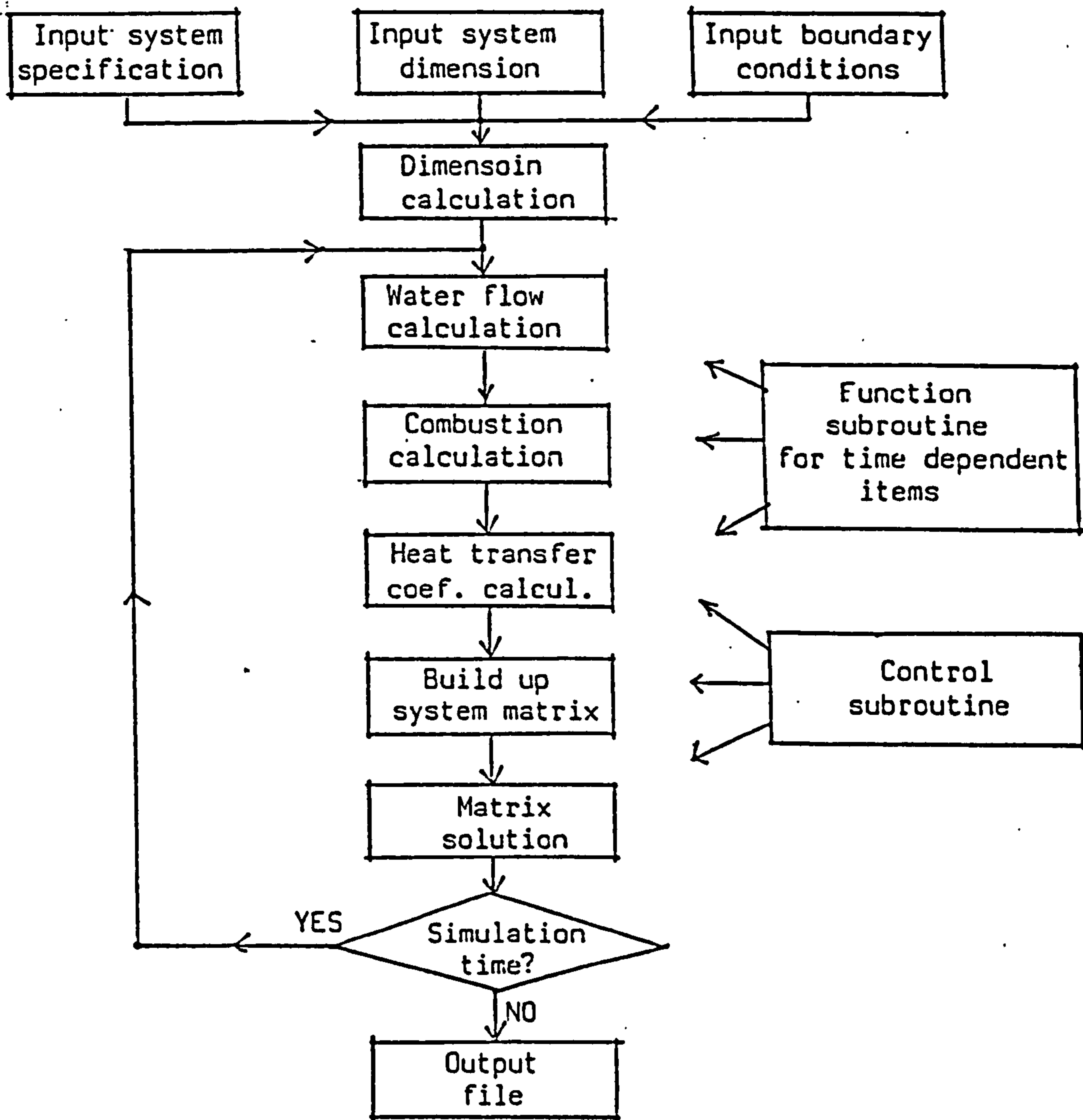


Figure 6.1 Main flow chart of programme "CENSYs"

6.2 The program and subroutines

The package program CENSYS is written in FORTRAN and run on the main fram computer of DEC-System 10. The central controlling routine CENSYS defines the flow sequences and logical connections to all the subroutines.

6.2.1 The input subroutines

Figure 6.2 shows the structure of data input management. Note, the input data of system specification are frequently used by each of the subroutine throughout the process of simulation. At the end of this stage all the input data are stored in a central data base.

Subroutines used are given as follows:

1. SYSIN - reads data from specification file, defines the types of component used in the system and specifies the interconnections of the components.
2. DIMINP - reads primary dimension from dimension data file according to the components specified by subroutine SYSIN. Fuel compositions of boiler are also inputted in this subroutine.
3. LOADIN - reads external parameters from the existing data file. The external parameters include: air and surface temperatures of each room and external air temperature. If control functions are specified to some state variables, the reference parameters are inputted in this subroutine.

Simulation options are directly inputted in the main program.

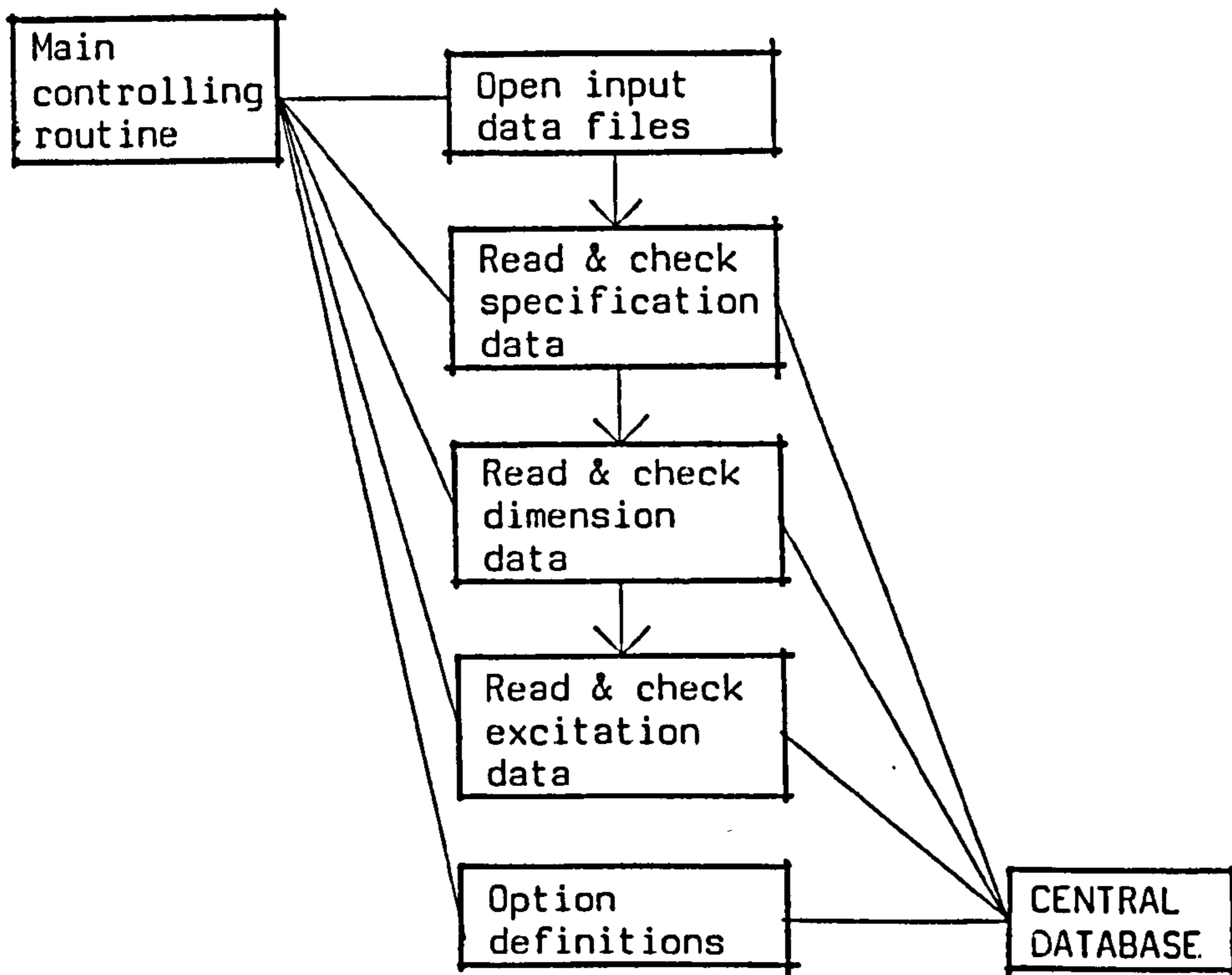


Figure 6.2 Data input management

6.2.2 The coefficients generation subroutines

This package subroutines generates the time-dependent and independent parameters. It uses the data and specification in the central primary data base to compute all the information required by energy simulation. Two kinds of subroutines involved in this package: the function subroutines provide the general form of thermal-physical properties and heat transfer coefficients; and the computing subroutines compute the required parameters by calling the function subroutines instantaneously during the computation. Figure 6.3 shows the structure of coefficients generation management.

Subroutines used in this stage are given as follows:

1. FUNCT - function subroutine including the formulae of computing radiative heat transfer coefficients, forced and natural convective heat transfer coefficients, and boiling heat transfer coefficient, etc.
2. FFLUID - function subroutine of computing thermal-physical properties of air and water.
3. FLUEGS - function subroutine of computing thermal-physical properties of gases involved in the combustion products of boiler.
4. CHDIMC - computes the detailed dimensions required by the later routines including the characteristics dimensions of each component required by the generation of time dependent heat transfer coefficients.
5. CHMFRC - computes the water flow rate distribution of system. Water flow rates of radiators are given as input data. If the water flow rates of radiators are specified as controlled parameters, this subroutine will give the predicted water flow distribution at each time step.
6. CHCPC - computes the combustion of boiler, it gives the results of compositions of combustion product, air required, thermal-physical properties of combustion product, etc.
7. CHBHTR - computes the heat transfer coefficients of boiler.
8. CHRHTR - computes the heat transfer coefficients of radiator.
9. CHCHTR - computes the heat transfer coefficients of pipes

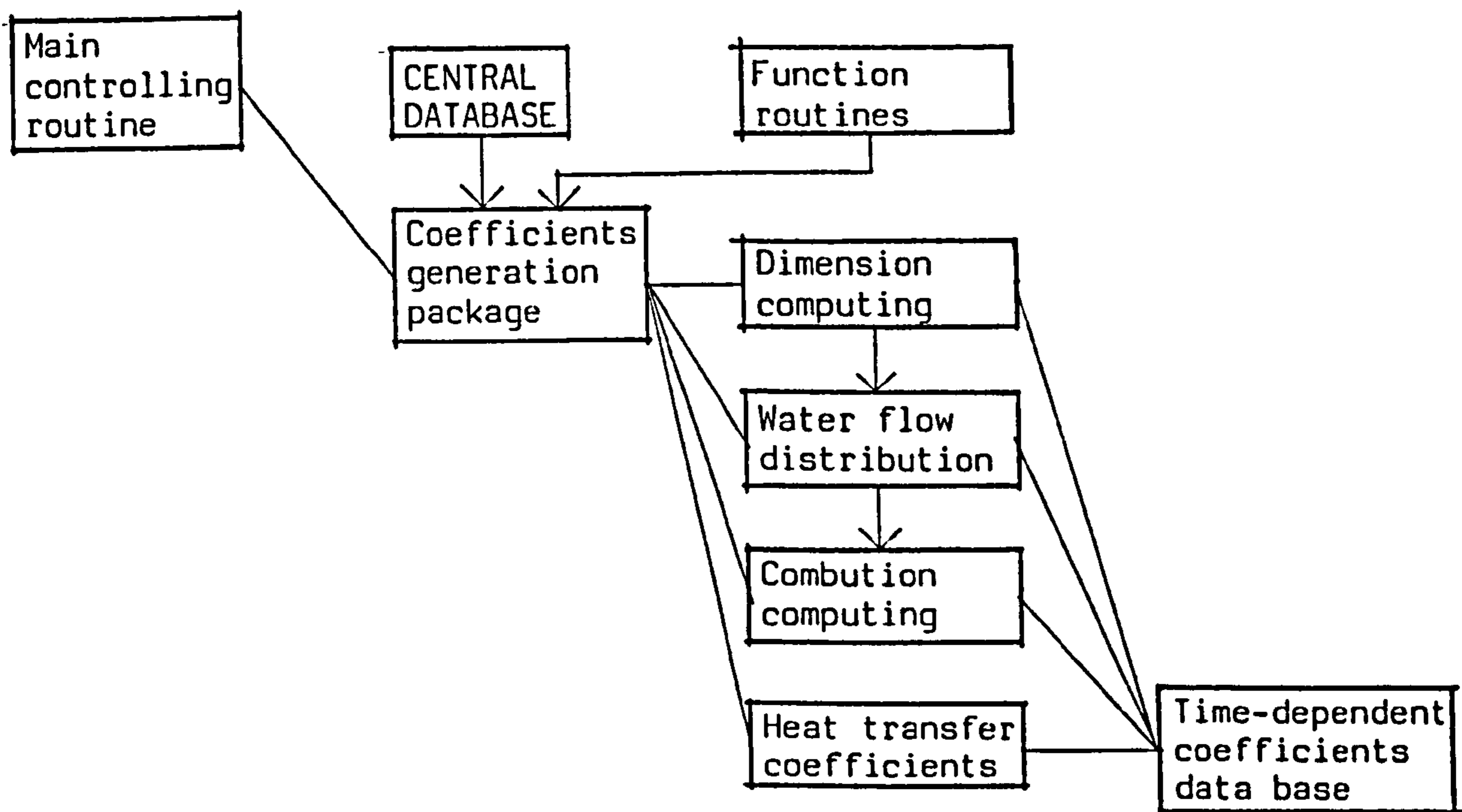


Figure 6.3 Coefficients generation management

including insulated and bare pipes and pipes with convergent flow.

At the end of this stage, all the results are stored in the time-dependent coefficients data base.

6.2.3 The main simulation subroutines

Using the system specification and time-dependent coefficients in the data bases, this package subroutines set up the system matrix equation and solve them according to the specified control function. Figure 6.4 shows the structure of the main simulation package management.

Subroutines used in this stage are given as follows:

1. CHMSTU - the main energy simulation subroutine. It computes the future and present time row coefficients in the system matrices based on the data given by the time-dependent coefficient data base and their locate them in the proper positions of system matrices according to the specification of system given by the central data base. Coefficients of matrices are then stored in one dimensional arrays.
2. CHCTRL - gives specified control functions to the controlled parameters according to the system specification. Directly controllable parameter is fuel supply rate of boiler. Indirectly controllable parameters are water flow rates of radiators.
3. CHMSOL - system matrix equation solution subroutine. Future time row state variables can be obtained at end of this subroutine.
4. CHHEAT - subroutine to convert the known future time row state variables to heat output of each component.

Up to this stage, one time step simulation is completed. Results will be transfer to a solution library defined prior to the simulation and the simulation will be directed by the main controlling routine to the next time step or to the end.

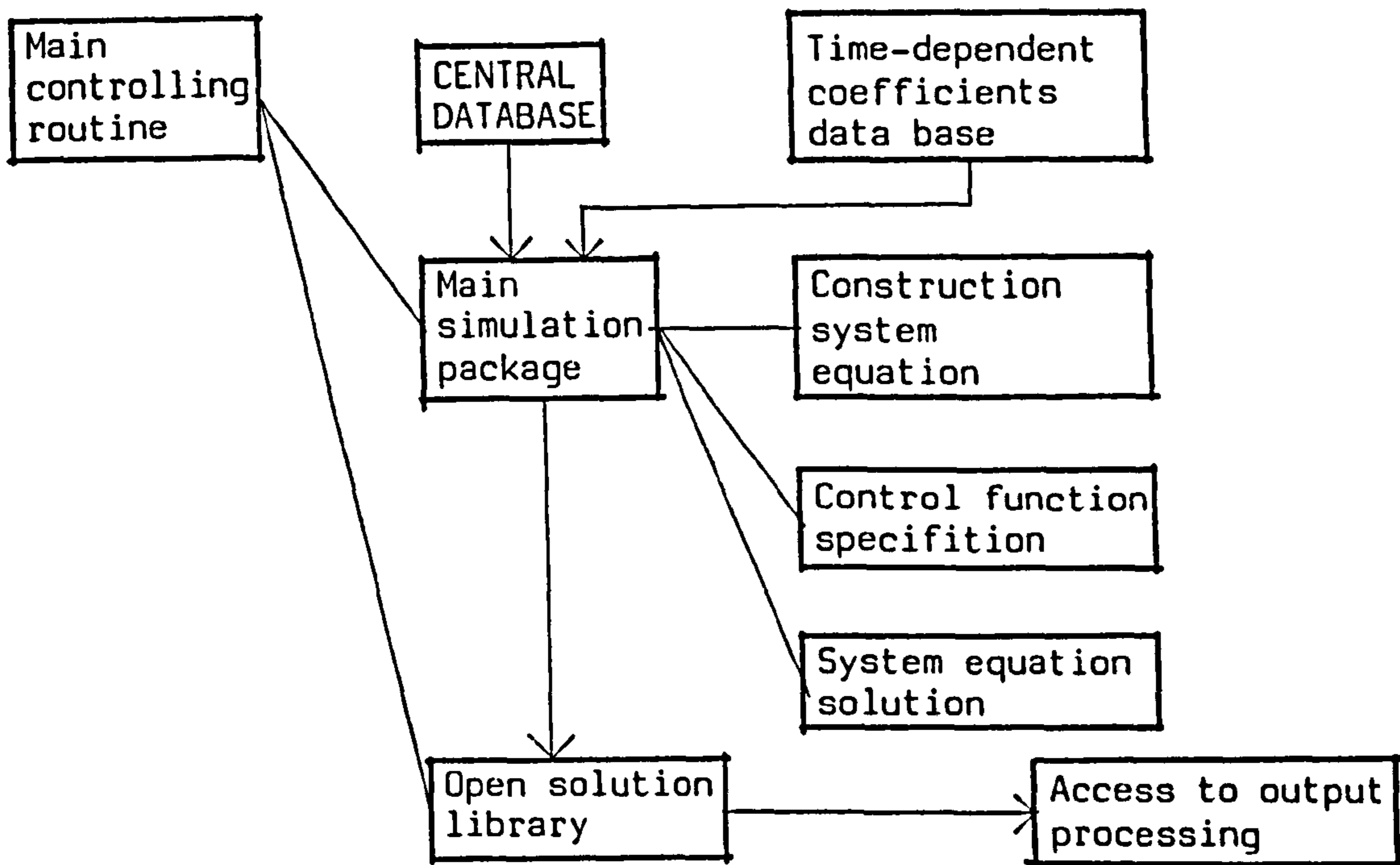


Figure 6.4 Main simulation package

6.3 The output facilities

The system simulation results can be obtained in the solution library which are given in values of state variables at each time step throughout the simulation period.

Other output facilities, such as tabular, interrogative and graphical, also can be available by using the existing software in ABACUS.

An independent program "SENOUT" is designed to process the data given in solution library into a special format required by ESPOUT - the output package of building energy simulation program ESP.

6.4 The development of advanced software

The program CENSYS is originally designed for model validation and not yet available for practical use. It is possible to expand this program to a sophisticated software by the implementation of a user-oriented interface, expanding the components library, etc. To achieve these, however, will require a vast amount of labour and time consuming. An alternative way is to use the ABACUS resources and take part in the development of advanced energy modelling software. Therefore, the work in this thesis has been directed, since the early stage, as a part of the ABACUS' development and the further developments of advanced energy simulation software are being carried out in mainly two directions, as follows:

6.4.1 The implementation of ESP

ESP is a dynamic energy simulation programme developed at ABACUS since 1977, which is originally designed for building enclosure energy simulation. [Ref. 1] In the early version of ESP programme, an ideal plant term was used to supply exact instantaneous heating and cooling loads required by the multi-zone building. However, it had left space for the further implementation of the programme. In its development, the input utilities of ESP programme has designed to allow easy insertions of new component models of plant system to update the old models. Since the modelling technique and methodology employed in this project are very much identical to those already used in ESP, the component models developed for HVAC system in this project can be directly transferred to ESP programme and to achieve the integration of HVAC system modelling together with the dynamic thermal behaviour of a multi-zone building under one unifying theoretical technique.

The recent implementations of ESP programme have achieved in the development of the central modular library and the central processing unit. [Ref. 2 & 3] Using the input utilities, the updated HVAC system components models can be stored in the central library and the central processing unit will then locate the specified component models, in their flow stream, within the multi-zone system matrix as the replacement for the previous ideal plant requirement term. Thus the multi-zone system equation can be solved by the partitioning technique to achieve the complete building and plant system performance in term of any prevailing climatic boundary condition acting upon the building and the imposed control strategy specified to the plant system.

Currently, the component models of heating system used in CENSYS have been transferred to ESP and the simulations are compared according to the IEA Annex 10 exercises of heating system simulation. A number of component models of air-conditioning system, in their simplest forms, have also been transferred into ESP programme and the results are going to be contributed to the IEA Annex 10 air-conditioning simulation of the "Collins Building" [Ref. 4]

6.4.2 The development of "Data Processors"

The development of the "next generation" generalised energy simulation software is an attractive target in the nearly future. The control volume state space approach presents the merit to be used as the foundation of the development of such software. One of the recent development at ABACUS unit includes the cooperation with LPB, Liege University of Belgium to develop the generalised energy simulation software known as the Data Processors. [Ref. 5]

A Data Processor may be regarded as a component model or a part of a component model developed according to certain theory. A central database holding the Data Processors is going to be developed. The Data Processors are constructed according to the specified format shown as figure 6.5. After the completion of this format, the component model can directly be transferred to the central database. The component models of HVAC system derived in the previous chapters can directly be inserted into this format and stored in the central database. Figure 6.6 shows the example of a radiator model constructed in the format of Data Processor. The central processing software will combine the Data Processors to construct the component matrices and then to form the complete system matrix equation. Data Processors can be regraded as software tool which is the foundation of the generalised energy modelling software. The use of Data Processors make the software flexible to use, easy to be further expanded and the component models can be easily updated. At present time, this new software is still at its developing stage. The methodology of the Data Processors development is quite similar to the implementation of ESP programme, therefore, the component models of HVAC system being structured in the ESP programme can possibly be transferred into this new frame of Data Processor.

DATA PROCESSOR SHEET

Figure 1

for further information see
data processor explanation

Author Details :

Abstract :

Processor Type : Building___, Plant___ Control___ Support___

Model Type : Algorithmic___ (supply details) Numerical___

Generic Name :

Function :

Keywords :

References :

Graphics Symbol:

Reliability : Validation references -

Input restrictions -

Sample I/O -

Photograph : (append if available)

I/O Schematic

Process Schematic

Nodal Scheme (Supply processor equation and coefficient formula)

Supplementary Pages:

Completed by:

Figure 6.5 Standard format of "DATA PROCESSOR"

SAMPLE
DATA PROCESSOR SHEET

for further information see
data processor explanation

Author Details : Various, ABACUS, University of Strathclyde, Glasgow

Abstract : Coefficient generator for a single panel radiator (top inlet/ bottom outlet)
The radiator is split into three sections: inlet header is the top section, the panel is the middle section and the bottom section is the outlet header.

Processor Type : Building___ Plant_X Control___ Support___

Model Type : Algorithmic___ (supply details) Numerical_X

Generic Name : WCH SYSTEM

Function : EXCHANGE

Keywords : RADIATOR / HEATING APPLIANCE

References : Theory available from ABACUS on request.

Graphics Symbol: 

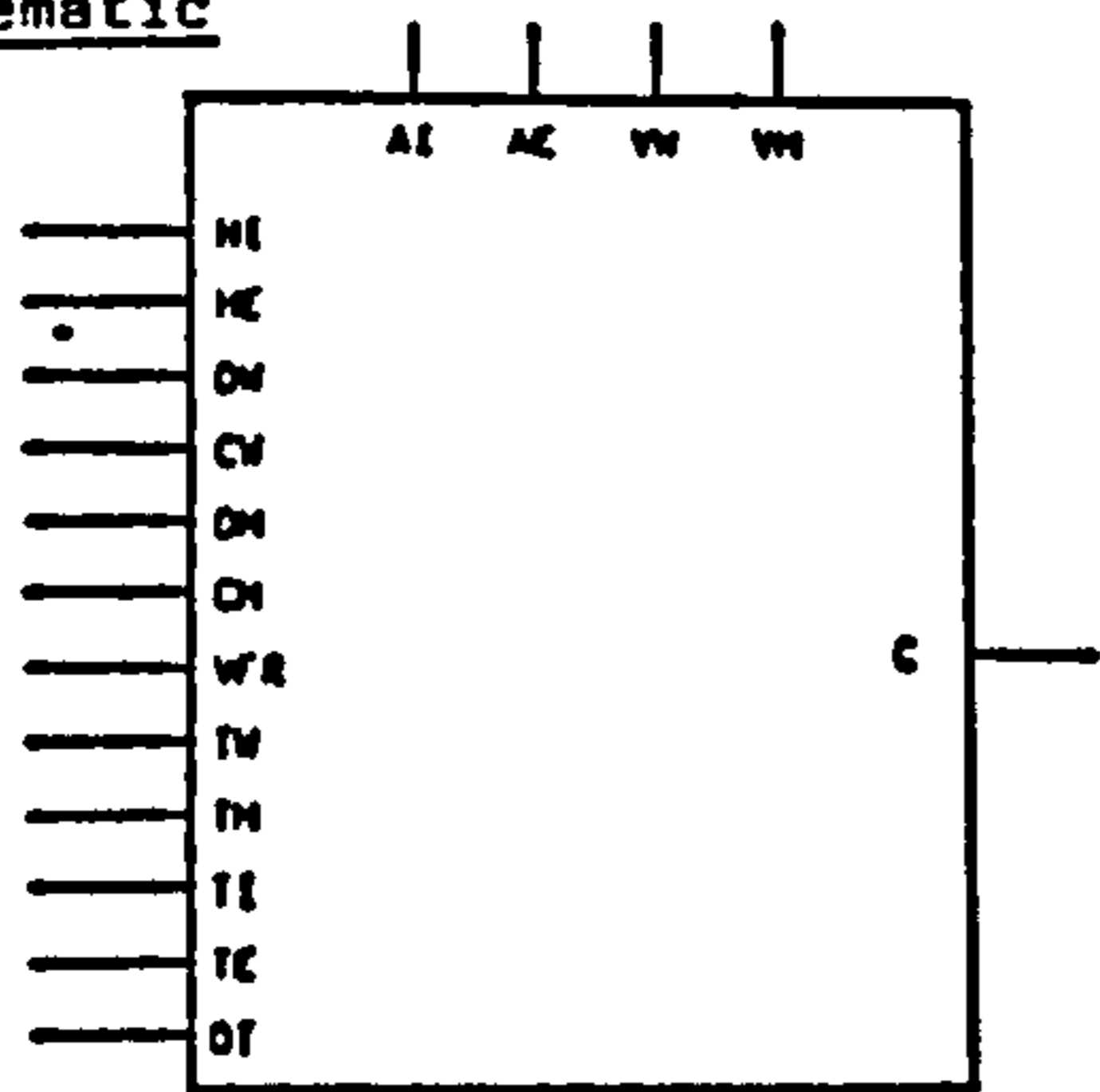
Reliability : Validation references - IEA Annex 10 reports.

Input restrictions - None

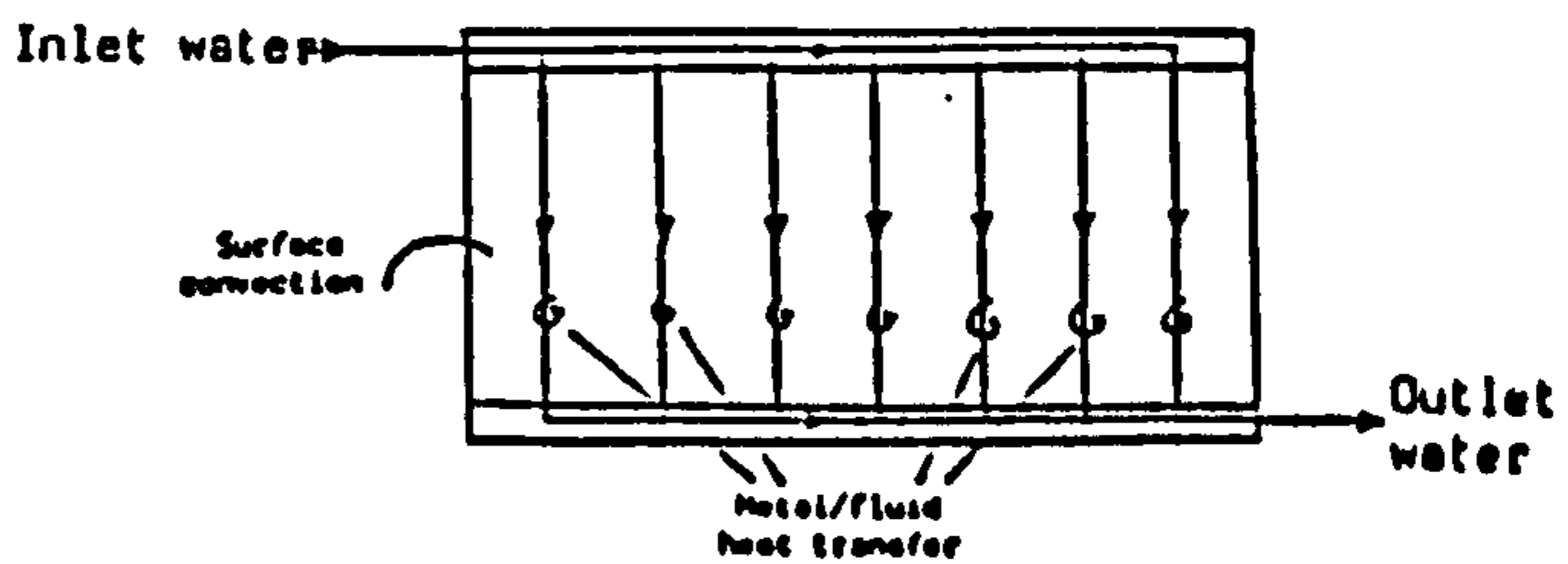
Sample I/O - Available by request from ABACUS

Photograph : (append if available)

I/O Schematic

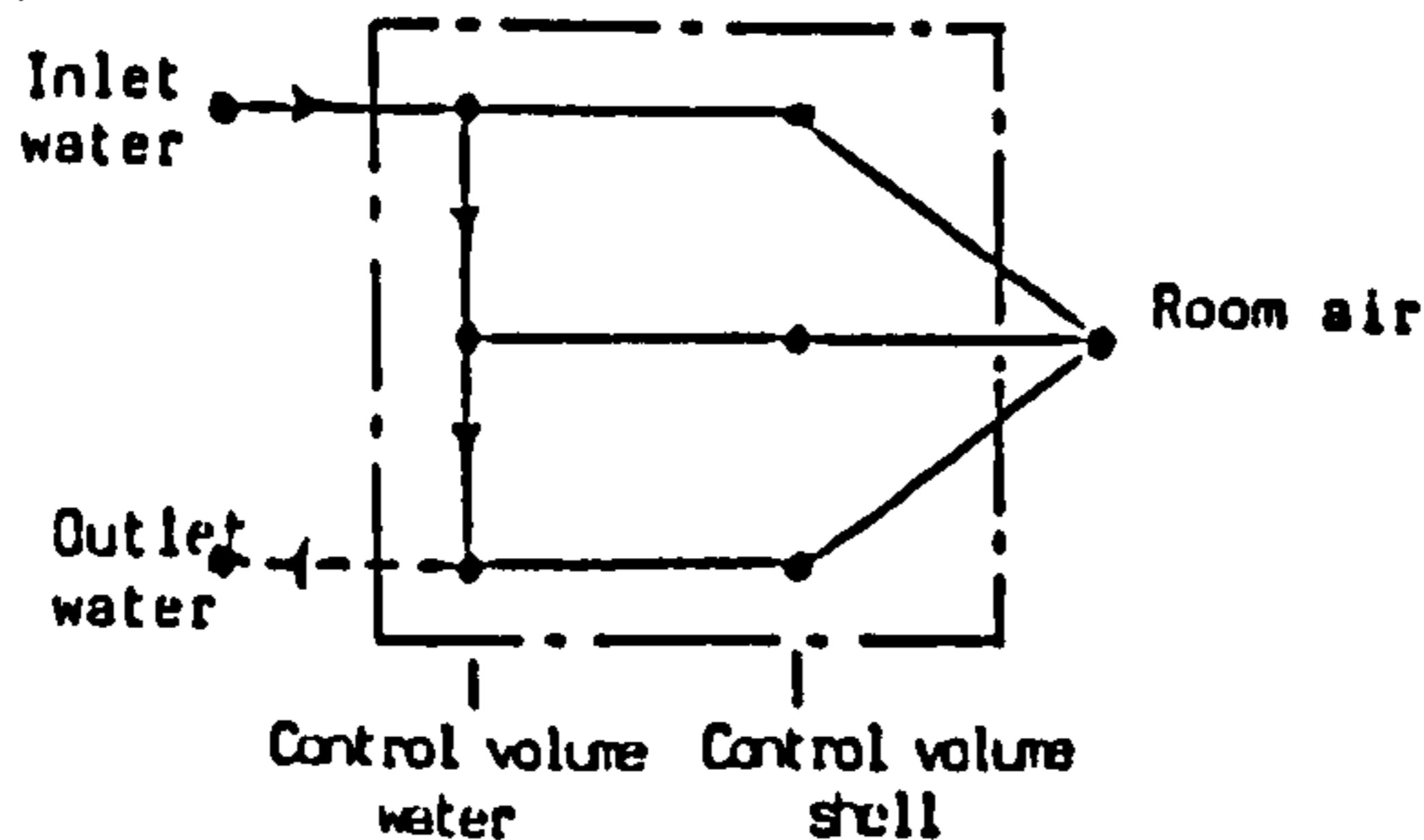


Process Schematic



See pages 1 & 2 for details

Nodal Scheme (Supply processor equation and coefficient formula) see pages 2, 3 & 4



Supplementary Pages: 4

Completed by : D.J. McLeon, ABACUS, Jan 1985

Figure 6.6 Example of radiator model in the format of "DATA PROCESSOR"

I/O Schematic Variables

All variables ending (1-6) are six element arrays where items 1 to 3 relate to the future values of the top, middle and bottom of the radiator respectively. Items 4-6 relate to the corresponding present values of the radiator.

Variables ending (1-2) are two element arrays the first term is the future value of the variable, the second item is the present value.

Variables ending (1-3) are three element arrays relating to the three sections of the radiator.

Data Sources:

AI (1-3) - Internal surface area of appropriate section

AE (1-3) - External surface area of appropriate section

VM (1-3) - Volume of water in section

VW (1-3) - Volume of metal in section

Data Inputs:

HI (1-6) - Internal heat transfer coefficients in section

HE (1-6) - External heat transfer coefficients in section

DM (1-6) - Water density in section

CW (1-6) - Water specific heat in section

DM - Density of metal

CH - Specific heat of metal

VFR(1-2) - Volume flow rate of water through radiator

IMP(1-3) - Present temperatures in radiator sections

IHP(1-3) - Present temperatures in radiator sections

IIP - Present inlet fluid temperature

ISP - Present environmental temperature

DT - Time increment

Outlet pipes:

C(1) top; fluid node; self-coupling

C(2) top; fluid node; cross-coupling to metal (top)

C(3) top; metal node; cross-coupling to fluid (top)

C(4) top; metal node, self-coupling

C(5) middle; fluid node; cross-coupling to fluid (top)

C(6) middle; fluid node; self-coupling

C(7) middle; fluid node; cross-coupling to metal (middle)

C(8) middle; fluid node; cross-coupling to fluid (middle)

C(9) middle; fluid node; self-coupling

C(10) bottom; metal node; cross-coupling to fluid (middle)

C(11) bottom; metal node; self-coupling

C(12) bottom; metal node; cross-coupling to metal (bottom)

C(13) bottom; metal node; cross-coupling to fluid (bottom)

C(14) bottom; metal node; self-coupling

C(15) top; fluid; cross-coupling to inlet fluid

C(16) top; metal; cross-coupling to surrounds

C(17) middle; metal; cross-coupling to surrounds

C(18) bottom; metal; cross-coupling to surrounds

C(19) top; fluid; present term

C(20) top; metal; present term

C(21) middle; fluid; present term

C(22) middle; metal; present term

C(23) bottom; fluid; present term

C(24) bottom; metal; present term

Processor Equations

Equ. (1) $C1 = \text{TMF}(1) + C2 + \text{TMF}(1) + C15 + \text{IIF} = C19$

Equ. (2) $C3 = \text{TMF}(1) + C4 + \text{TMF}(1) + C16 + \text{ISF} = C20$

Equ. (3) $C5 = \text{TMF}(1) + C6 + \text{TMF}(2) + C7 + \text{TMF}(2) = C21$

Equ. (4) $C8 = \text{TMF}(2) + C9 + \text{TMF}(2) + C17 + \text{ISF} = C22$

Equ. (5) $C10 = \text{TMF}(2) + C11 + \text{TMF}(3) + C12 + \text{TMF}(3) = C23$

Equ. (6) $C13 = \text{TMF}(3) + C14 + \text{TMF}(3) + C18 + \text{ISF} = C24$

where

$$C1 = \frac{2 \cdot \text{DM}(1) \cdot \text{CH}(1) \cdot \text{VM}(1)}{\text{DI}} + \text{HI}(1) \cdot \text{AI}(1) + \text{VFR}(1) \cdot \text{DM}(1) \cdot \text{CW}(1)$$

$$C2 = -\text{HI}(1) \cdot \text{AI}(1)$$

$$C3 = C2$$

$$C4 = \frac{2 \cdot \text{DM} \cdot \text{CH} \cdot \text{VM}(1)}{\text{DI}} + \text{HI}(1) \cdot \text{AI}(1) + \text{IE}(1) \cdot \text{AE}(1)$$

$$C5 = -\text{VFR}(1) \cdot \text{DM}(2) \cdot \text{CW}(2)$$

$$C6 = \frac{2 \cdot \text{DM}(2) \cdot \text{CH}(2) \cdot \text{VM}(2)}{\text{DI}} + \text{HI}(2) \cdot \text{AI}(2) + \text{VFR}(1) \cdot \text{DM}(2) \cdot \text{CW}(2)$$

$$C7 = -\text{HI}(2) \cdot \text{AI}(2)$$

$$C8 = C7$$

$$C9 = \frac{2 \cdot \text{DM} \cdot \text{CH} \cdot \text{VM}(2)}{\text{DI}} + \text{HI}(2) \cdot \text{AI}(2) + \text{IE}(2) \cdot \text{AE}(2)$$

$$C10 = -\text{VFR}(1) \cdot \text{DM}(3) \cdot \text{CW}(3)$$

$$C11 = \frac{2 \cdot \text{DM}(3) \cdot \text{CH}(3) \cdot \text{VM}(3)}{\text{DI}} + \text{HI}(3) \cdot \text{AI}(3) + \text{VFR}(1) \cdot \text{DM}(3) \cdot \text{CW}(3)$$

$$C12 = -\text{HI}(3) \cdot \text{AI}(3)$$

$$C13 = C12$$

$$C14 = \frac{2 \cdot \text{DM} \cdot \text{CH} \cdot \text{VM}(3)}{\text{DI}} + \text{HI}(3) \cdot \text{AI}(3) + \text{IE}(3) \cdot \text{AE}(3)$$

$$C15 = -\text{VFR}(1) \cdot \text{DM}(1) \cdot \text{CW}(1)$$

$$C16 = -\text{IE}(1) \cdot \text{AE}(1)$$

$$C17 = -\text{IE}(2) \cdot \text{AE}(2)$$

$$C18 = -\text{IE}(3) \cdot \text{AE}(3)$$

$$C19 = \left[\frac{2 \cdot \text{DM}(4) \cdot \text{CH}(4) \cdot \text{VM}(1)}{\text{DI}} - \text{HI}(4) \cdot \text{AI}(1) - \text{VFR}(4) \cdot \text{DM}(4) \cdot \text{CW}(4) \right] \cdot \text{TMF}(1) + \text{HI}(4) \cdot \text{AI}(1) \cdot \text{TMF}(1) + \text{VFR}(4) \cdot \text{DM}(1) \cdot \text{CW}(1) \cdot \text{IIF}$$

$$C20 = \left[\frac{2 \cdot \text{DM} \cdot \text{CH} \cdot \text{VM}(1)}{\text{DI}} - \text{HI}(4) \cdot \text{AI}(1) - \text{IE}(4) \cdot \text{AE}(1) \right] \cdot \text{TMF}(1) + \text{HI}(4) \cdot \text{AI}(1) \cdot \text{TMF}(1) + \text{IE}(4) \cdot \text{AE}(1) \cdot \text{IIF}$$

$$C21 = \left[\frac{2 \cdot \text{DM}(5) \cdot \text{CH}(5) \cdot \text{VM}(2)}{\text{DI}} - \text{HI}(5) \cdot \text{AI}(1) - \text{VFR}(5) \cdot \text{DM}(5) \cdot \text{CW}(5) \right] \cdot \text{TMF}(2) + \text{HI}(5) \cdot \text{AI}(2) \cdot \text{TMF}(2) + \text{VFR}(5) \cdot \text{DM}(5) \cdot \text{CW}(5) \cdot \text{TMF}(1)$$

$$C22 = \left[\frac{2 \cdot \text{DM} \cdot \text{CH} \cdot \text{CM}(2)}{\text{DI}} - \text{HI}(5) \cdot \text{AI}(2) - \text{IE}(5) \cdot \text{AE}(2) \right] \cdot \text{TMF}(2) + \text{HI}(5) \cdot \text{AI}(2) \cdot \text{TMF}(2) + \text{IE}(5) \cdot \text{AE}(2) \cdot \text{IIF}$$

$$C23 = \left[\frac{2 \cdot \text{DM}(6) \cdot \text{CH}(6) \cdot \text{VM}(3)}{\text{DI}} - \text{HI}(6) \cdot \text{AI}(3) - \text{VFR}(6) \cdot \text{DM}(6) \cdot \text{CW}(6) \right] \cdot \text{TMF}(3) + \text{HI}(6) \cdot \text{AI}(3) \cdot \text{TMF}(3) + \text{VFR}(6) \cdot \text{DM}(6) \cdot \text{CW}(6) \cdot \text{TMF}(2)$$

$$C24 = \left[\frac{2 \cdot \text{DM} \cdot \text{CH} \cdot \text{VM}(3)}{\text{DI}} - \text{HI}(6) \cdot \text{AI}(3) - \text{IE}(6) \cdot \text{AE}(3) \right] \cdot \text{TMF}(3) + \text{HI}(6) \cdot \text{AI}(3) \cdot \text{TMF}(3) + \text{IE}(6) \cdot \text{AE}(3) \cdot \text{IIF}$$

Figure 6.6 (Continue)

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CHAPTER SEVEN: VALIDATIONS AND APPLICATIONS

7.1 The I.E.A. organisation and exercises

7.2 Seneffe house exercise

7.3 La Chaumiere building exercise 1

7.4 La Chaumiere building exercise 2

7.5 Collins building exercise

7.6 Other efforts of application

CHAPTER SEVEN: VALIDATIONS AND APPLICATIONS

Although the modelling technique of control volume state space 'approach' based on the physical laws of energy and substance conservation is universally true, the component models derived by applying this approach will not always be correct. In other words, certain differences between the model performances and real world performances can possibly be existed. The reason for this is that in the stage of model development, it is impossible to define detailly and accurately every energy and mass exchange path of the control volume concerned, some less important facts are usually ignored and assumptions are made to the model. Therefore, validations of component model and system performance are important and necessary. From the validations, difference between the mathematical models and the real world performances can be discovered and the allowed difference caused by simplifications can then be confirmed. The results of validation is usually led to the requirement to confirm the validity of mathematical model in its presentation form of the proposed physical concept, model structure and, then, as the knowledge has gained, the modification to the model can be proceeded to achieve higher quality of model development.

Traditionally, the validation is proceeded by comparing the predictions from the mathematical model to the corresponding monitored results from actual equipments and systems. Higher accuracy and reliability can be expected from such validation provided that the measurements taken from the experiments and the actual equipments and system are well arranged and the random disturbances to the measurements can be eliminated or well treated. This can be considered as a fundamental way of model validation and seems to be attractive above all the rest. However, difficulties encountered can usually be more than the advantages obtained from such method and so that the applications are limited. Problems encountered in this area include the complexity of arrangement of experiment and field monitoring; the complexity of accurate measurement and elimination of unexpected random disturbances; the availability of instrumentation of

instantaneous data monitoring and the efficiency of organisation and data collection. This process has also been found as expensive and time consuming. Even for a single component, such as a radiator, it requires the full equipped laboratory facilities and well trained personnel and it will become much difficult, if not impossible, to perform this process to a large scale HVAC system by considering its total cost and period of measurement taking.

Instead of direct experiment and field monitoring, existing data of equipments and system operations can be used for model validation. Care has to be made since that the uncertainties involved in such data are usually not easy to be defined.

Alternative efforts seem attractive by joining the internationally agreed validation procedure. The difficulties of model validation have been found by different individual energy simulation group so that, in recent years, an international cooperation scheme has been established by the International Energy Agency to organise groups in different countries to work on the same projects. The specifications of equipments and systems, field monitored data are provided, which provides the opportunities of comparing the results with different groups as well as with the real system operations.

In this project, an extensive validation has been carried out and involved (since 1983) in the ongoing IEA Annex 10 exercises of central heating system and component performances modelling. Results have been presented in the reports circulated among the participants of IEA Annex 10 exercise. [Ref. 1, 2, 4] Part of the results have been presented at the AMSE International Conference (Athens, June, 1984). [Ref. 3]

The following sections will give the selective results from the IEA Annex 10 exercises.

7.1 The IEA Organisation and exercises

In September 1974 the United States in conjunction with other member countries of the Organisation for Economic Cooperation and Development (OECD) initiated an International energy programme concerned with energy research and development. The International Energy Agency (IEA) was subsequently established as an autonomous body of OCED to administer the programme. Twenty-one countries are currently members of the IEA, with the commission of the European Communities participating under a special arrangement.

The IEA sponsors research and development of areas related to energy, and as part of its activities set up an experts group to study the state-of-the-art in computer simulation of research of energy conservation in building and community systems. The IEA is sponsoring various exercises to predict more accurately the energy use of buildings, including comparison of existing computer programmes, building monitoring, comparison of calculation methods etc. The participants of IEA undertake cooperative activities in energy research, development and demonstration. A number of new and improved energy technologies which have potential of future development were identified for collaborative efforts.

The early stage of the IEA programme, Annex 1, consisted of a comparison of computer simulation results obtained from a number of participants when run against a hypothetical building under specified constructions and weather conditons.

Later programme, the Annex 4, was the extension of comparison of computer simulation results to include data obtained from measurements in a real commercial office building. This programme was initiated in March 1979 to comprehensively instrument an occupied commercial office building, to carry out measurements and to compare the monitored data with computer predictions carried out from six participating models of different countries. It had been found through this programme that the building models were performed reasonably well but the performances on the plant system side were poorly simulated. This led to the presently ongoing programme of Annex 10.

The programme Annex 10, has officially started since the beginning of 1983 and will continue to the end of 1986. The main objective of Annex 10 is trying to fill in the gap that remains between the definition of a building "net space heating and cooling demand" and the corresponding energy consumption of HVAC equipments and system. This programme consists of the establishment of data bank of HVAC component specifications and reference hypothesis suitable for realistic energy modelling. The suitability of the data bank is continuously assessed during the development through the simulation exercises. Figure 7.1 shows the recently confirmed brief objectives and time table for this programme. Groups from six different countries are participating as working participants and a number of groups are acting as scientific observers in this Annex 10 programme. Results from each participant are presented as report circulated among the groups and the analysis and comparison of results are made by certain groups defined as the organisers of each exercise.

The project of this thesis, as a part of ABACUS' contribution, has been participating in the "projective one" of domestic heating system simulation of Annex 10 programme since 1983. The completed exercises included in the Annex 10 programme are:

1. Seneffe house exercise - simulation of a uni-family domestic central heating system.
2. La Chaumiere house: Exercise 1 - boiler performance simulation.
3. La Chaumiere house: Exercise 2 - package of five exercises of parametric study of boiler.
4. Collins building: Exercise 1 and 2 - exercise of air-conditioning system simulation. (not yet completed)

Participants involved in this "projective one" Annex 10 programme and contribute results to the organisation are:

| Group | Organisation |
|-------|---|
| IKE | Universitat Stuttgart, West Germany |
| LPB | Universite de Liege, Belgium |
| RIT | The Royal Institute of Technology, Sweden |

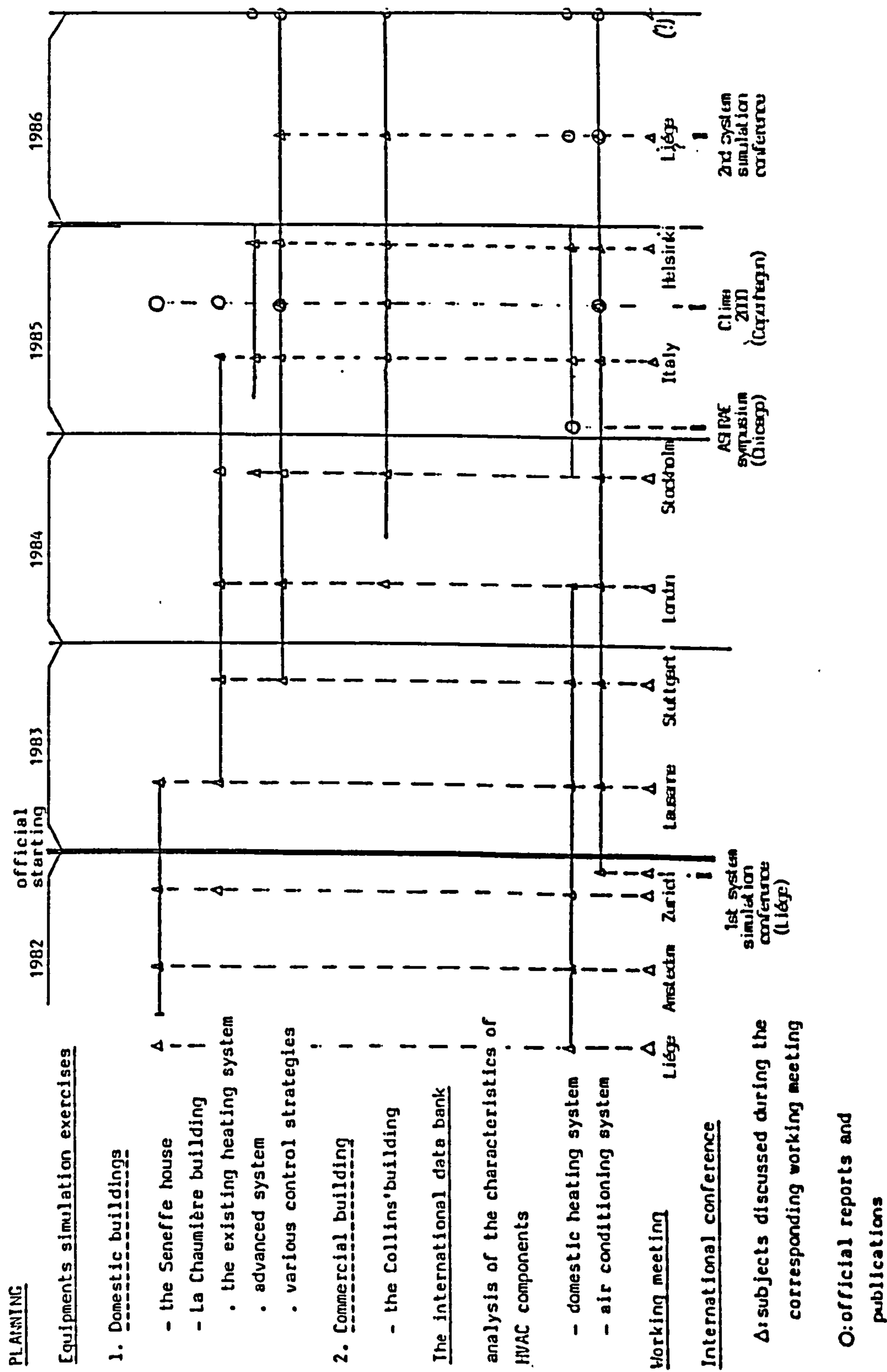


Figure 7.1 Objectives of IEA Annex 10 Programme

EMPA Swiss Fedral Laboratories of material Testing and Research,
Switzerland
PT Polytechnic of Turin, Italy
ABACUS University of Strathclyde, U.K.

7.2 Exercise of Seneffe house [Ref. 5]

The Seneffe house exercises were to simulate the performance of a water heating system in a low income house built in Seneffe, near Brussels, Belgium and this exercise was organised by LPB. Figure 7.2 and 7.3 show the two main facades of Seneffe house and the heating system layout installed in the house. The heating demand of the house as well as each room had been computed by LPB prior to the exercise by using dynamic model so that the exercise specified was purely plant simulation. Information given to the simulation exercise also included:

1. Oil-fuel boiler:

Power of boiler = 40.7 KW
Power of burner = 34.8 KW
Efficiency = 0.87
Stop losses = 837 W
Water content = 87.5 lit.
Mass = 180 kg

Assumptions were made to the outlet water temperature of boiler is constant or equals to a linear function of outside temperature.

2. Emissions of pipe, insulated or not, had to be computed as heat productions in the heated rooms or heat losses in unheated rooms.

3. Radiators:

Specifications of radiator are given in table 7.1

Assumptions made to the radiators were:

- a) each radiator is equipped with an ideal thermostatic valve to adjust its water flow rate so as to provide exact heat as demanded by the room.
- b) heat transfer coefficient of each radiator is constant.
- c) room air temperatures are constant, as given in table 7.2.

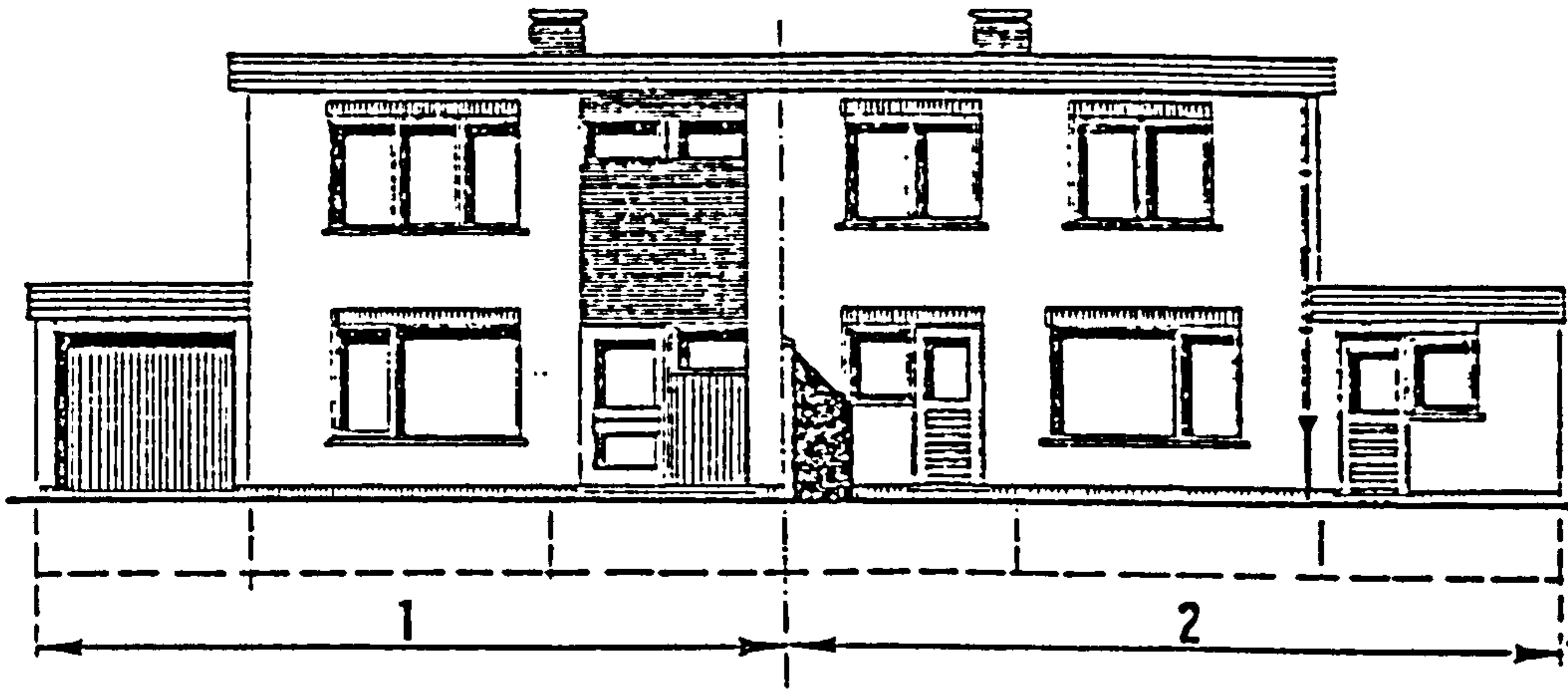


Figure 1 : the 2 main facades of the Seneffe house.

Figure 7.2 Main facade of Seneffe House (Ref. 5)

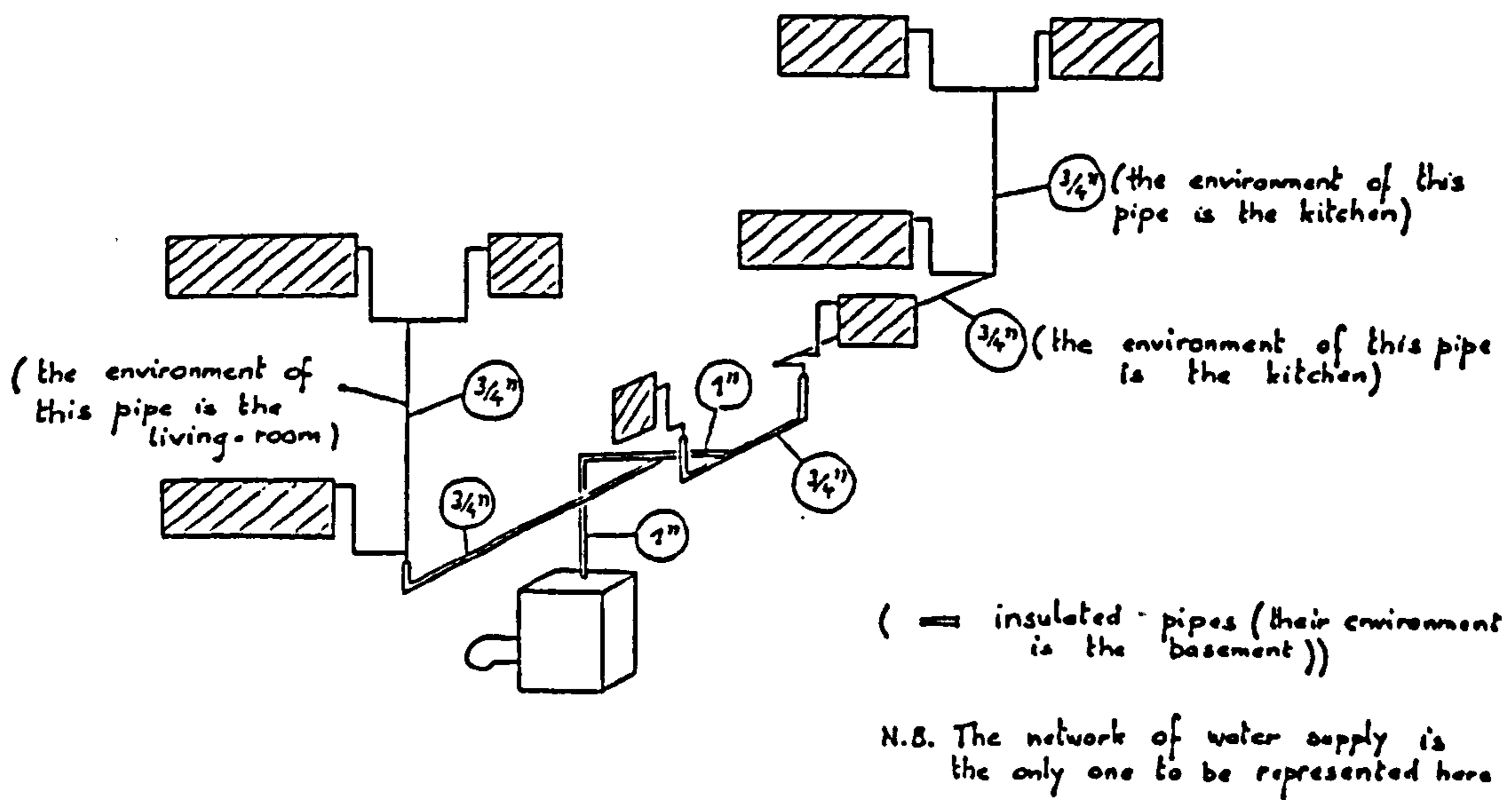


Figure 7.3 Heating system layout of Seneffe House (Ref. 5)

| Radiator location | room set temperature | length (m) | thickness (mm) | height (m) | NOMINAL POWER (W) (90-70-Δt) | SUPPLY PIPES EMISSION (W) | RETURN PIPES EMISSION (W) | Σ (W) | type | n-value | Weight (kg.) | Water content (l.) |
|-------------------------|----------------------|------------|----------------|------------|------------------------------|---------------------------|---------------------------|--------------|----------------|---------|--------------|--------------------|
| living (front) (room 1) | 20 | 2.268 | 87 | 0.4 | 3230 | 87 | 186 | <u>3503</u> | PaP | 1.273 | 63 | 5.9 |
| living (rear) (room 1) | 20 | 2.260 | 78 | 0.4 | 2643 | 87 | 155 | <u>2805</u> | PaP | 1.303 | 45 | 5.9 |
| kitchen (room 2) | 18 | 0.648 | 87 | 0.9 | 1810 | 87 | 31 | <u>1928</u> | PaP | 1.311 | 33 | 6.2 |
| Hall (room 3) | 16 | 0.648 | 70 | 0.9 | 1688 | 87 | 62 | <u>1837</u> | PaP | 1.300 | 29 | 6.2 |
| Bedroom 1 (room 7) | 16 | 2.106 | 44 | 0.5 | 2084 | 87 | 186 | <u>2357</u> | Pa | 1.252 | 30 | 6.3 |
| Bedroom 2 (room 5) | 16 | 1.458 | 78 | 0.5 | 1850 | 113 | 155 | <u>2118</u> | PP | 1.309 | 32 | 8.7 |
| Bedroom 3 (room 6) | 16 | 1.296 | 44 | 0.5 | 1283 | 87 | 724 | <u>1494</u> | P ₂ | 1.252 | 19 | 3.9 |
| Bathroom (room 8) | 22 | 1.134 | 78 | 0.7 | 1692 | 113 | 112 | <u>1917</u> | PP | 1.197 | 34 | 8.8 |

Table 7.1 Specification of radiators (Ref. 5)

| | |
|-----------------------|----------|
| Room 1 : living room | 20°C |
| Room 2 : kitchen | 18°C |
| Room 3 : hall | 16°C |
| Room 4 : W.C. | unheated |
| Room 5 : bedroom 2 | 16°C |
| Room 6 : bedroom 3 | 16°C |
| Room 7 : bedroom 1 | 16°C |
| Room 8 : bathroom | 22°C |
| Room 9 : laundry hole | unheated |
| Room 10 : garage | unheated |
| Room 11 : basement | unheated |

Table 7.2 Room conditions (Ref. 5)

4. Load:

Load of each room was dynamically computed and hourly values were given, as in table 7.3.

According to these information, each participant was asked to present the results of the 10 given days period of:

1. Water inlet and outlet temperatures of each radiator.
2. Water flow rate of each radiator.
3. Distribution losses of pipes in unheated rooms.
4. Boiler energy consumption and production efficiency.

The computer program "CENSYS" was used for the heating system simulation of Seneffe house exercise. Since "CENSYS" is a fully dynamic system simulation programme, some hypothesis and assumptions used in the above specification were modified. To simulate the Seneffe heating system, the modification to the given information were made as:

1. Heat transfer coefficients through out the system are computed dynamically at each time step and so that they are no longer held constant.
2. A control loop is used to control the fuel supply rate to the boiler so as to obtain the water outlet temperature of boiler as close to 65 C as possible.
3. Control loops are applied to each radiator to control the water flow rate so as to match heat output of radiator to the heat demand as close as possible.

Figure 7.4 to 7.7 give some results in the format required by the exercise under the given specifications and the modifications to accommodate to the dynamic model "CENSYS".

Figure 7.4 gives the inlet and outlet water temperatures of radiator number one and figure 7.5 give the same temperatures for radiator number two. Figure 7.6 shows the water flow rates of radiator number one and two respectively. Figure 7.7 shows the heating load of each room in Seneffe house and the heat provided by the radiator in each

| hour | Living front | Living rear | kitchen | Hall | Bedroom ₂ | Bedroom ₃ | Bedroom ₁ | Bathroom | t _{out} | t _{basement} |
|------|--------------|-------------|---------|------|----------------------|----------------------|----------------------|----------|------------------|-----------------------|
| 1 | 613 | 606 | 400 | 751 | 175 | 179 | 256 | 572 | 6.5 | 10.3 |
| 2 | 627 | 670 | 407 | 700 | 196 | 187 | 269 | 577 | 6.0 | 10.6 |
| 3 | 650 | 702 | 433 | 780 | 221 | 205 | 301 | 587 | 4.5 | 10.29 |
| 4 | 668 | 710 | 440 | 705 | 230 | 211 | 311 | 590 | 4.1 | 10.12 |
| 5 | 601 | 721 | 451 | 795 | 240 | 219 | 323 | 594 | 3.5 | 9.96 |
| 6 | 666 | 725 | 455 | 799 | 245 | 222 | 329 | 596 | 3.3 | 9.86 |
| 7 | 690 | 728 | 458 | 802 | 240 | 224 | 332 | 597 | 3.2 | 9.80 |
| 8 | 699 | 736 | 465 | 810 | 255 | 229 | 341 | 600 | 2.8 | 9.70 |
| 9 | 674 | 715 | 442 | 789 | 239 | 215 | 316 | 591 | 3.8 | 9.83 |
| 10 | 644 | 690 | 412 | 769 | 219 | 196 | 291 | 582 | 4.7 | 10.11 |
| 11 | 721 | 639 | 316 | 730 | 161 | 139 | 261 | 571 | 5.8 | 10.62 |
| 12 | 753 | 616 | 277 | 729 | 129 | 108 | 257 | 570 | 5.6 | 10.93 |
| 13 | 772 | 632 | 333 | 730 | 148 | 131 | 259 | 573 | 5.5 | 10.83 |
| 14 | 770 | 620 | 340 | 726 | 149 | 136 | 242 | 568 | 5.8 | 10.13 |
| 15 | 763 | 624 | 329 | 725 | 135 | 126 | 230 | 568 | 5.0 | 10.94 |
| 16 | 796 | 653 | 385 | 744 | 163 | 159 | 252 | 574 | 5.0 | 10.76 |
| 17 | 821 | 671 | 408 | 754 | 179 | 176 | 263 | 578 | 5.0 | 10.57 |
| 18 | 827 | 677 | 412 | 757 | 183 | 180 | 267 | 579 | 4.0 | 10.51 |
| 19 | 840 | 687 | 422 | 764 | 191 | 183 | 277 | 582 | 4.3 | 10.39 |
| 20 | 863 | 706 | 441 | 781 | 209 | 201 | 298 | 589 | 3.2 | 10.14 |
| 21 | 869 | 711 | 446 | 786 | 215 | 200 | 304 | 590 | 3.0 | 9.98 |
| 22 | 888 | 726 | 461 | 800 | 229 | 217 | 321 | 596 | 2.2 | 9.79 |
| 23 | 894 | 732 | 466 | 807 | 236 | 221 | 320 | 598 | 2.0 | 9.63 |
| 24 | 898 | 735 | 469 | 811 | 241 | 224 | 332 | 599 | 2.0 | 9.57 |

Table 7.3 Hourly values of heating load of Seneffe House (Ref. 5)

Result-set 1 Zone 1 Period from Day 7 Month 1 Hour 1
 (STS 4: OTSI 1) to Day 7 Month 1 Hour 24

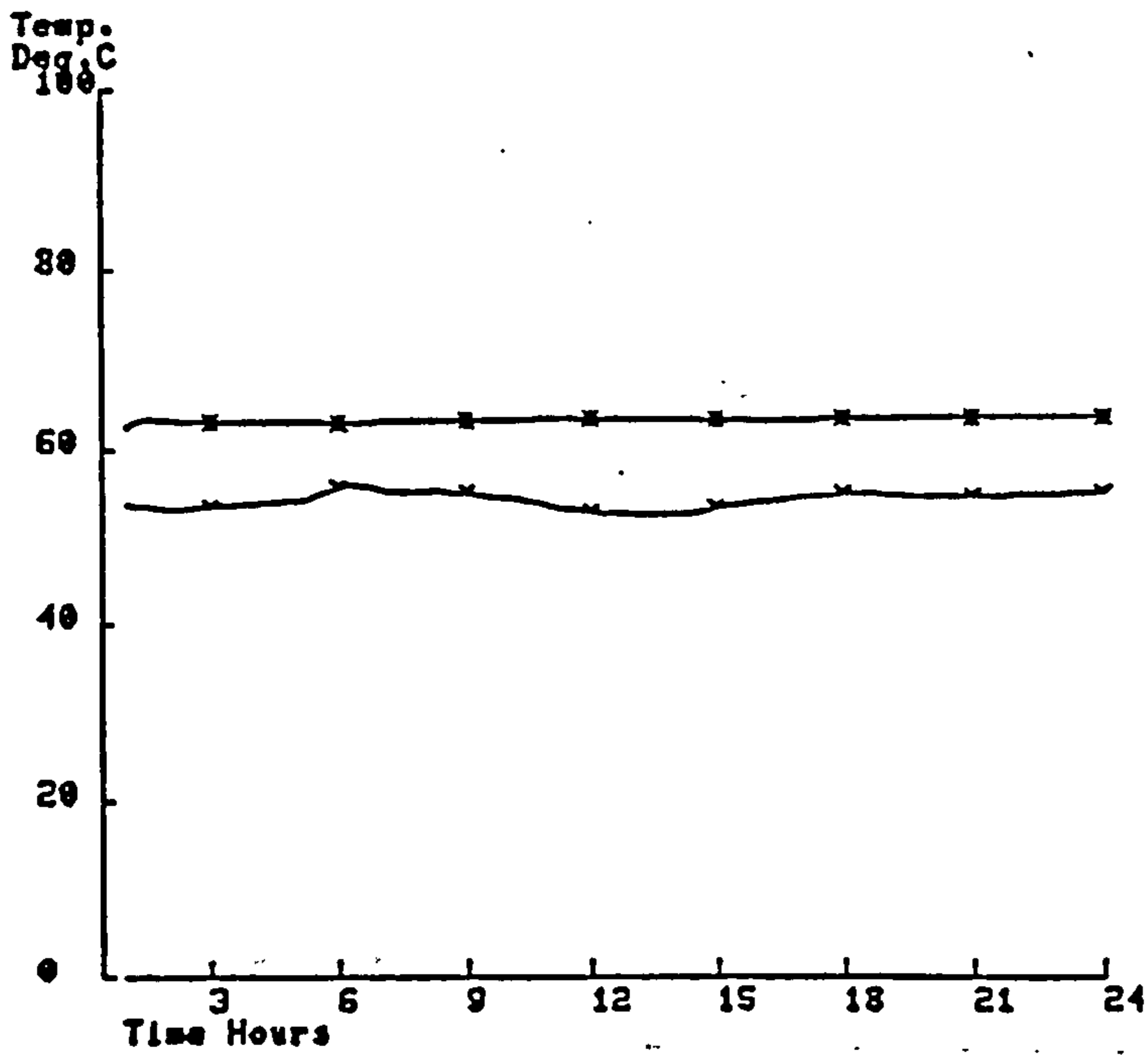


Figure 7.4 Inlet, outlet water temperature of radiator No.1

Result-set 1 Zone 1 Period from Day 7 Month 1 Hour 1
 (STS 4: OTSI 1) to Day 7 Month 1 Hour 24

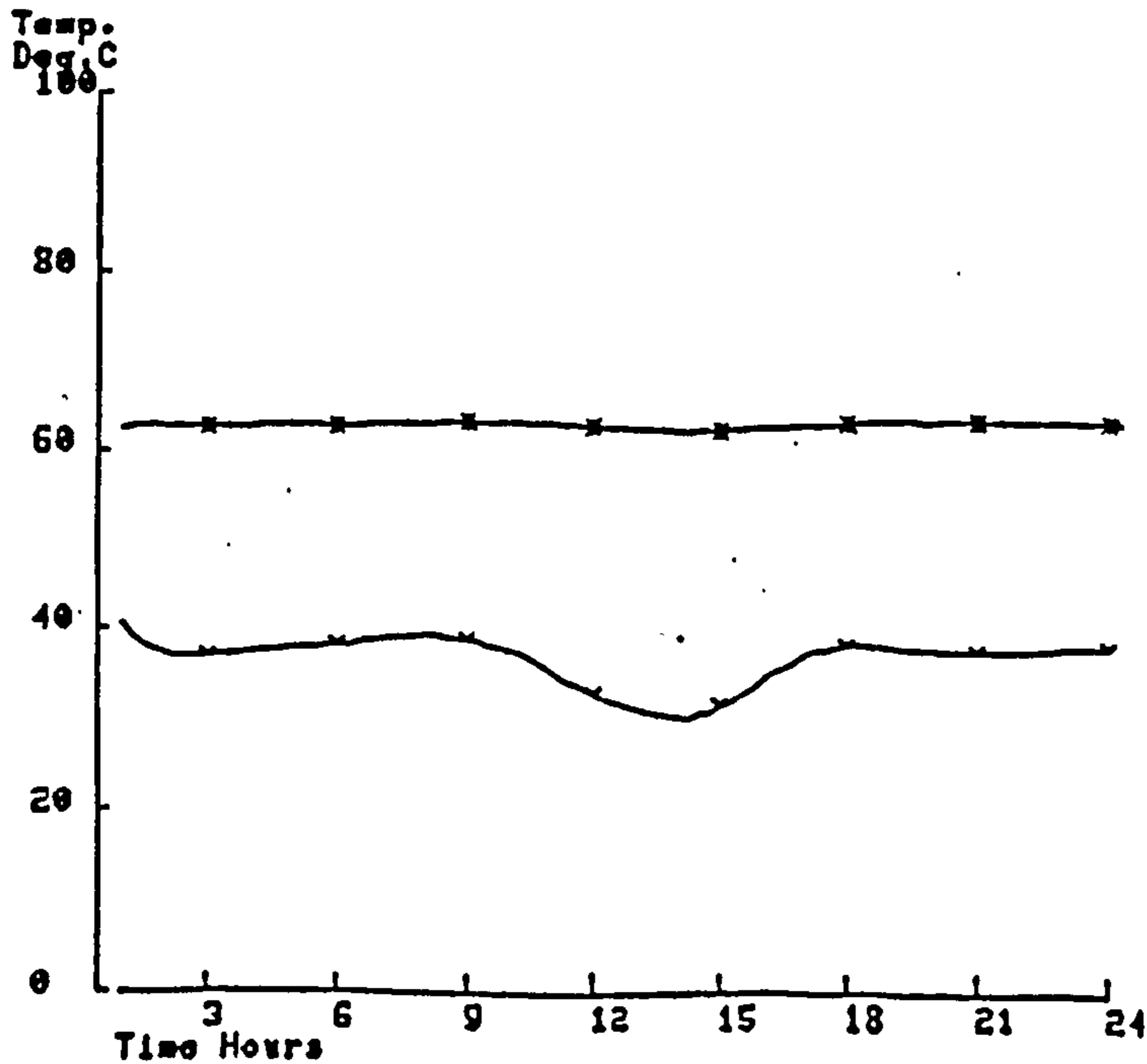


Figure 7.5 Inlet, outlet water temperature of radiator No. 2

Result-set 1 Zone 1 Period from Day 7 Month 1 Hour 1
 (STS 4: OTSI 1) to Day 7 Month 1 Hour 24

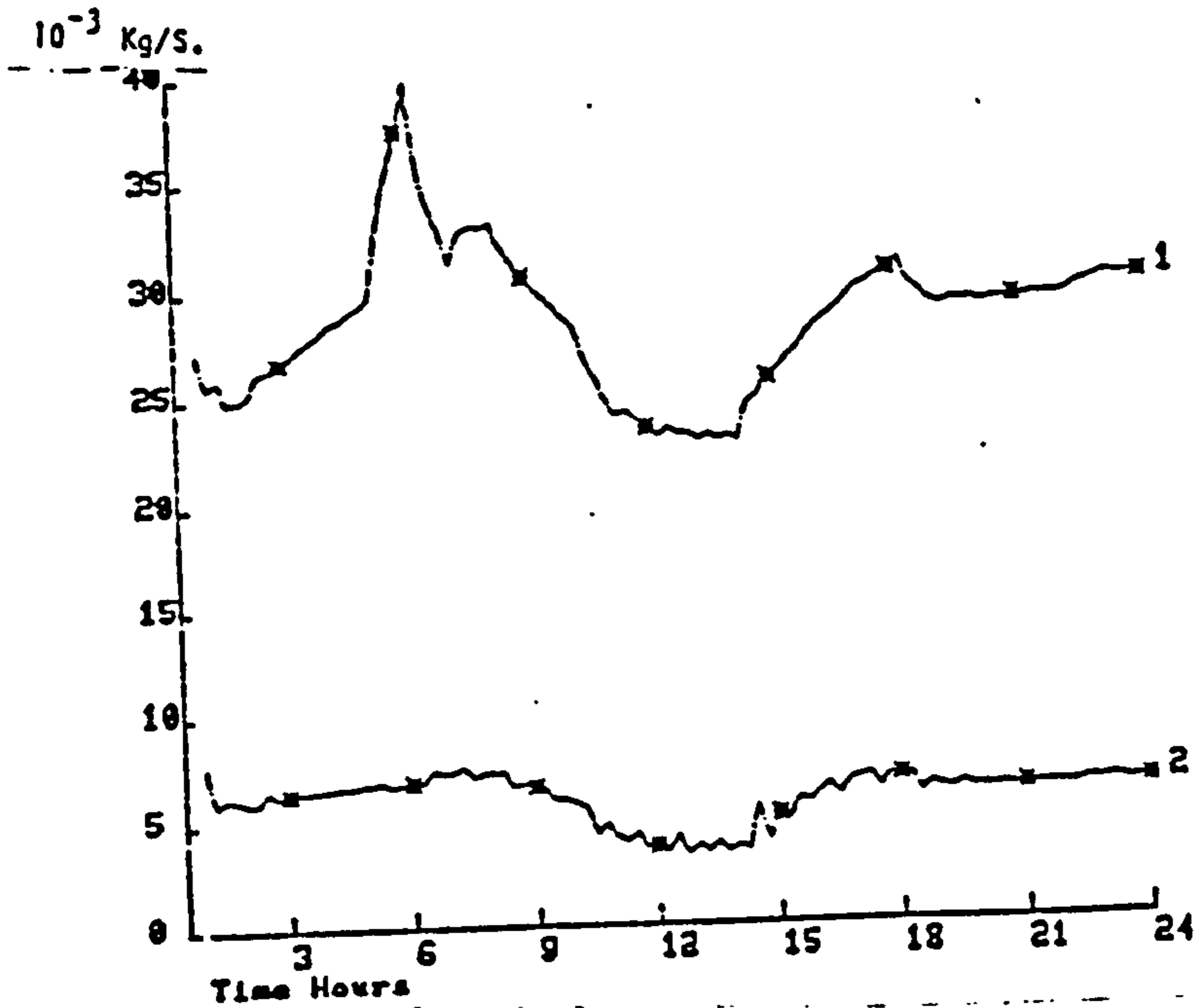


Figure 7.6 Water flow rate of radiator No.1 and No.2

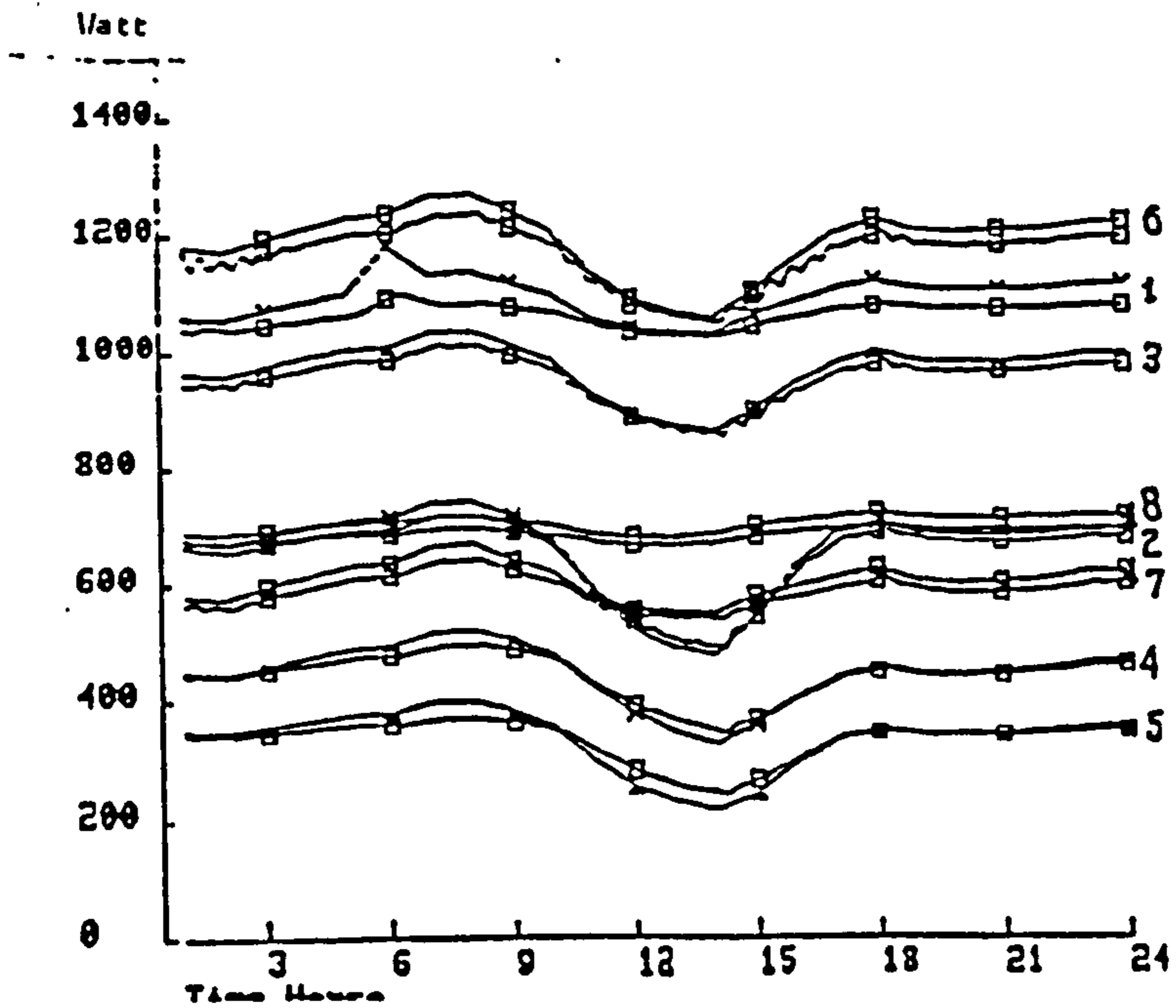


Figure 7.7 Load and heat production of radiator in each room

room.

Unfortunately, the Seneffe house exercises were soon abandoned by the LPB suggestion because of the lacks of monitored data and the detailed system specification, it had left too much freedom for the participants to define by themselves. These had made the results difficult to be compared. However, by observing results from different models, it can be found that the steady state model may produce reasonably good results on certain parameters which have less dynamical effect. At the same time, dynamic models may not produce better results over steady state models when simulate the systems specified by working at static conditions. Result from "CENSYS" show that the control devices of boiler and radiators are well performed. For example, results show that the performances of the proportionally controlled radiator thermostatic valves act quite similar to those devices in real world. Interesting facts can also be found, from a control view-point, that the using of larger radiator with smaller water flow rate usually better than the reverse case. In other words, the smaller the water flow rate, the easier the heat output of radiator can be controlled.

7.3 Exercise of La Chaumiere house: Exercise 1a

The specification and input data of La Chaumiere exercise were prepared by EMPA. The purpose of this exercise is made to investigate the capabilities of simulation methods for boilers as a component in the central heating system. Later the exercise is expanded to the modelling of a branch of distribution system and the combination effect of boiler and the distribution system. The exercises are based on the measurement made upon the heating system located in the building "La Chaumiere" in Lausanne, Switzerland. Table 7.4 gives the general description about the building. Figure 7.8 shows the heating plant in connection with the total heating system and table 7.5 gives the detailed information related to the heating plant. The data of weather conditions and the system load are given by hourly values on magnetic tapes. Errors appeared in the monitored data are corrected by the proposed confirmed method. The operation of boiler is controlled by the thermostat which detects the water outlet temperature of boiler and controls the water temperature between 60 °C to 70 °C. The on-off controller is considered and the loads of the plant system are given so that the water circulation in the system can be worked out. Simulation period are defined as two one-week periods on November and December respectively. The standard formats of results representation are also defined by EMPA to facilitate the latter analysis and comparisons of results.

The boiler simulation has been completed by using "CENSYS". The boiler model used in simulation is based on the structure and mathematical model presented in chapter four as figure 4.19. Table 7.6, 7.7, 7.8 are the tabulated results of simulation by the required standard formats.

An extensive study of simulation results from different participants has been carried out by EMPA and the comparisons are made between the participants and to the measurement. [Ref. 6] Part of the results extracted from the analysis are presented as follows.

Table 7.9 gives the daily performance of boiler operation as predicted by different models.

1 Location

Country Switzerland
Adress Chemin de la Chaumière 5, CH 1010 Lausanne
Coordinates: 46 deg. 32' Lat N, 6 deg. 39' Long. W.
Altitude: 636 m.
Orientation: The principal axis of the building is 13° W from the north.
Situation of the surroundings: district of small non contiguous multi-family houses.

2 Building

Four storey, six flats each. There are two 4 rooms flats at the SW and SE corners, two-3 room flats at the center of the E and W part, and two 3 room flats at both NW and NE corners.
Basement half buried, including the entrance, 5 garages, technical rooms and cellars. Central stairs and lift
Gross volume 5'268 m3
Gross floor surface 2000 m2
Heated volume 3780 m3
Heated floor surface 1'357 m2

3 Construction

Standard swiss non insulated heavy construction, with clay hollow bricks walls, wood carpentry covered with clay tiles.
Walls made of 25 cm hollow clay bricks rendered with cement mortar
Breast-walls made of 125 hollow clay bricks, rendered with cement. Interior face covered with a wooden panel.
Reinforced concrete pavement with wooden floor.
Double glazing windows on wood frames.
Ratio openings/total walls area = 2/9

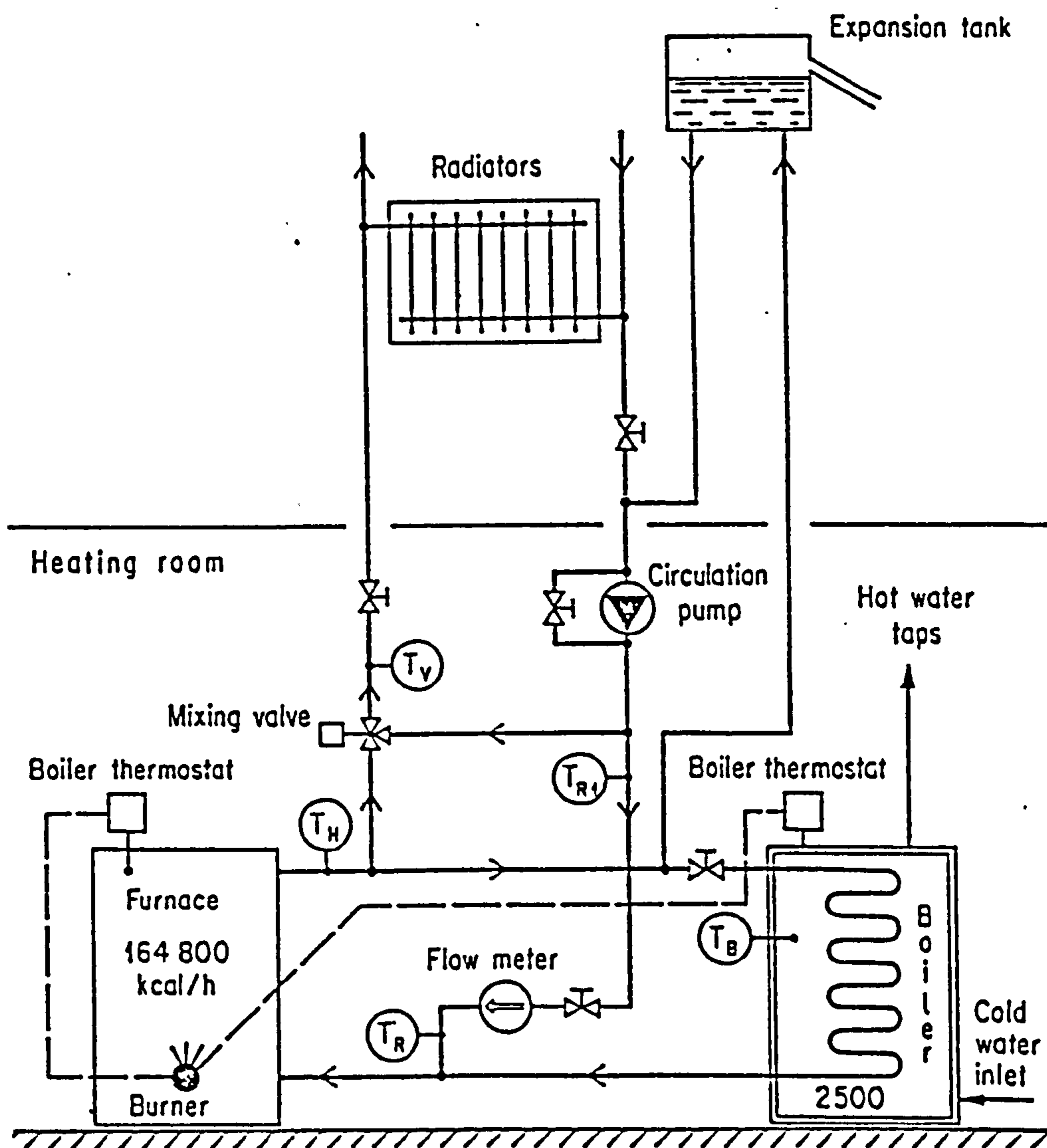
4 Heating system

Light oil plant, heating a closed water circuit for space heating by radiators and hot water heating by an exchanger in a boiler.
Yearly oil consumption 29'031 kg (335'760 kWh)
Yearly total electricity consumption 42'693 kWh
These values are averaged over 5 years

5 Heating zones

The flats are named by their principal geographic orientation and their floor number. -
Since the heating water is distributed by vertical pipes and climatic environment is varying with the orientation, the building is divided in 14 zones.

Table 7.4 Specification of "La Chaumiere House"



Heating plant of "La Chaumière" House

Figure 7.8 Heating plant of "La Chaumiere House" (Ref. 6)

Boiler

| | |
|------------------------------|---------------------------------|
| Make : | Buderus |
| Type: | Lollar P 60 W |
| Year of fabrication : | 1956/57 |
| Power : | 164'800 kcal/h = 191.7 kW |
| Width : | 1.25 m |
| Depth: | 1.20 m |
| Height : | 1.48 m |
| Mass : | 2490 kg (without water) |
| Watercontent: | 464 ℓ (in the boiler core) |
| Interior heating surface: | 20.6 m ² |
| (U·A) - value of the boiler: | 90 W/K |
| specific heat of the core : | 477 J/kg K |
| core mass: | unknown |
| mean boiler temperature: | 65°C |
| max. boiler temperature: | 70°C |
| min. boiler temperature: | 60°C |

Chimney

| | |
|-----------------|--------|
| Height : | 13 m |
| Width : | 0.5 m |
| Depth : | 0.3 m |
| pressure drop : | 900 Pa |

Burner

| | |
|---|--------------------------------|
| Make : | Shell |
| Type : | Thermo Plan Typ 2 |
| fuel flow rate : | 15.8 ℓ /h |
| fuel : | Oil extra light |
| fuel density : | 0,845 kg/ ℓ |
| Calorific value : | 42,7 MJ/kg (= 10'200 kcal/kg) |
| Minimal air flow needed : | 13.9 kg air/kg fuel |
| Ratio of the minimal air flow needed and the actual air flow : | 1.9 |

Other Data

| | |
|---|--------|
| flue gas temperature | 190°C |
| Heating room temperature | 29°C |
| CO ₂ content in the flue gas: | 8 % |
| Stand by losses (estimated) | 5.7 kW |
| Losses due to the domestic hot water supply | 0.7 kW |

Table 7.5 Information of "La Chaumiere House" heating plant (Ref. 6)

| | |
|---------------------------|----------|
| Burner on time (min.) | 669 |
| Burner on periods | 67 |
| Heat production (MJ) | 5212.168 |
| Efficiency (%) | 81.97 |
| Flue gas losses (%) | 11.36 |
| Boiler surface losses (%) | 4.36 |
| Storage effects (%) | 2.31 |
| Consumption of heat (MJ) | 6358.858 |

Table 7.6 Results of La Chaumiere House, Exercise 1a

Fuel consumption " La Chaumière " 1981

Value in Litre of oil

| | | | |
|---------|---------|---------|---------|
| 15. Dec | 152.79 | 22. Dec | 162.27 |
| 16. Dec | 143.31 | 23. Dec | 154.63 |
| 17. Dec | 148.05 | 24. Dec | 155.16 |
| 18. Dec | 154.37 | 25. Dec | 162.27 |
| 19. Dec | 178.15 | 26. Dec | 165.70 |
| 20. Dec | 176.24 | 27. Dec | 164.91 |
| 21. Dec | 164.12 | 28. Dec | 161.48 |
| Total | 1117.03 | Total | 1126.42 |

Table 7.7 Results of "La Chaumiere House", Exercise 1a

| Hour | Burner on time (min) | Fuel supply rate (lit.) |
|-------|----------------------|-------------------------|
| 0-1 | 16 | 4.215 |
| 1-2 | 11 | 2.898 |
| 2-3 | 44 | 1.054 |
| 3-4 | 16 | 4.215 |
| 4-5 | 0 | 0.0 |
| 5-6 | 24 | 6.322 |
| 6-7 | 37 | 9.747 |
| 7-8 | 37 | 9.747 |
| 8-9 | 33 | 8.693 |
| 9-10 | 37 | 9.747 |
| 10-11 | 33 | 8.693 |
| 11-12 | 31 | 8.166 |
| 12-13 | 38 | 10.01 |
| 13-14 | 31 | 8.166 |
| 14-15 | 34 | 8.957 |
| 15-16 | 36 | 9.484 |
| 16-17 | 32 | 8.430 |
| 17-18 | 31 | 8.166 |
| 18-19 | 37 | 9.747 |
| 19-20 | 30 | 7.903 |
| 20-21 | 34 | 8.957 |
| 21-22 | 34 | 8.957 |
| 22-23 | 29 | 7.640 |
| 23-24 | 24 | 6.322 |

Table 7.8 Results of "La Chaumiere House"; Exercise 1a

Table 7.10 gives some more comparison and also compare to the measurements.

Table 7.11 and 7.12 give the comparative results from a two week period, comparing different models with measurements.

Figure 7.9 shows the scattering of the results of different models.

Figure 7.10 and 7.11 provide the daily results for a two week period and linear regression lines of fuel consumption against external temperature and the system load between different models.

| | Germany | Belgium | DOE | This model |
|-------------------------------|---------|---------|--------|------------|
| Burner on time (min) | 662.85 | 650.5 | 694.59 | 669 |
| Burner on periods | 83 | 75 | --- | 67 |
| Heat output (%) | 84.70 | 84.71 | 80.57 | 81.97 |
| Flue gas losses (%) | 9.67 | 10.52 | 11.46 | 11.36 |
| Boiler surface losses (%) | 4.6 | 2.8 | --- | 4.36 |
| Load difference (%) | +0.2 | -1.5 | 0.0 * | -1.99 |
| Burner on time difference (%) | 101.82 | 100.0 * | 106.7 | 102.8 |

* value taken as standard

Table 7.9 Results of 20th Dec., compare with others

| | Measurement | Germany | Belgium | DOE | This model |
|--|-------------|---------|---------|--------|------------|
| Burner on time (min) | 780.53 | 662.3 | 650.5 | 694.59 | 669 |
| Burner on time with 15.5% correction for calculated data (min) | 780.57 | 765.0 | 751.3 | 802.2 | 772.7 |
| Difference (%) | 100 | 98.0 | 96.2 | 102.77 | 98.99 |

Table 7.10 Results of 20th Dec., compare with others and measurement

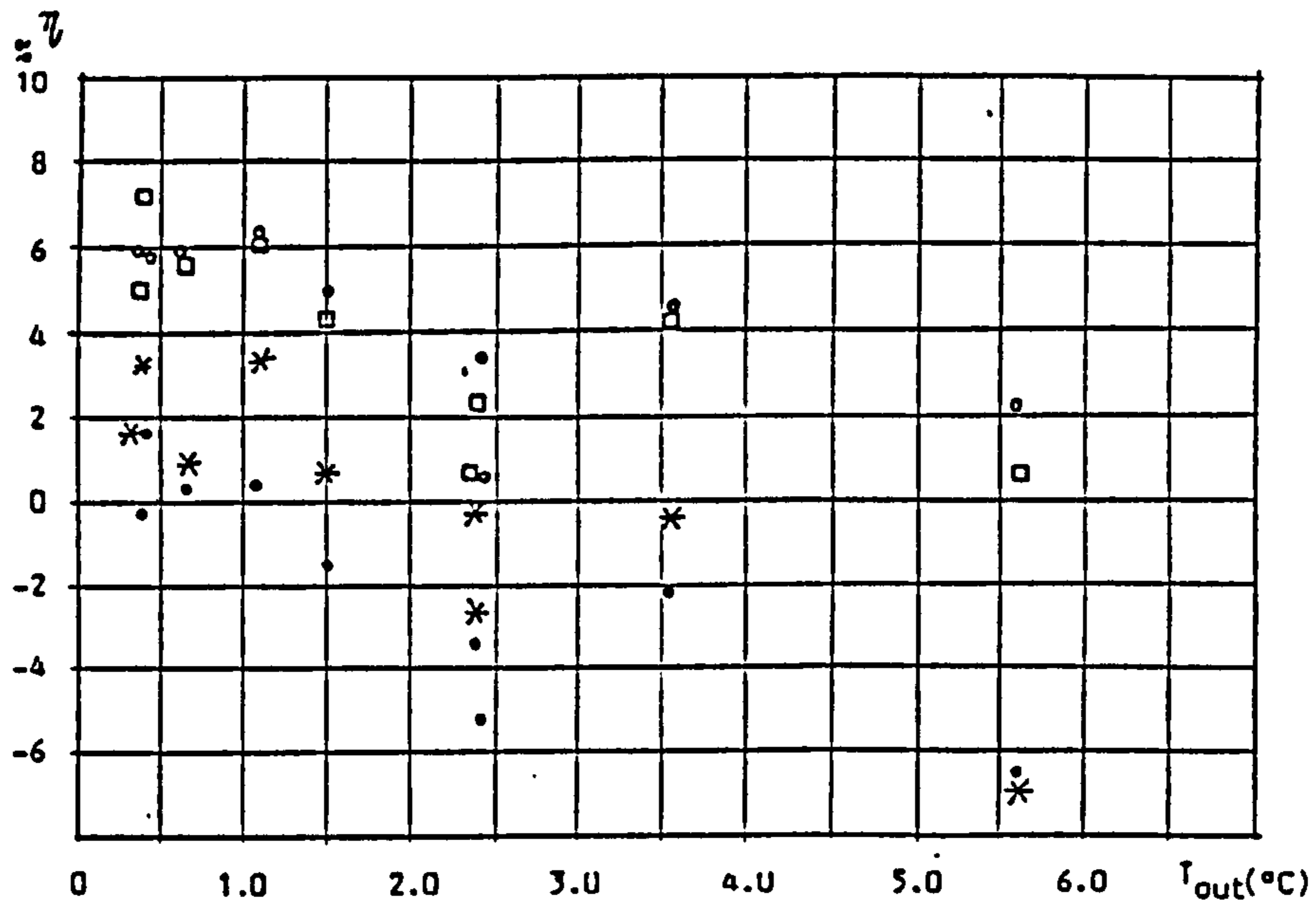
| | Germany | Belgium | DOE | This model |
|------------------------|---------|---------|---------|------------|
| Fuel consumption (lit) | 2168.95 | 2144.0 | 2293.79 | 2243.45 |
| Total efficiency (%) | 84.29 | 85.27 | 79.71 | 82.09 |
| Difference (%) | 1.2 | 0 * | 7.0 | 4.6 |

* value taken as standard

Table 7.11 Results of a 2 weeks period, compare with others

| | Measured-corrected | Germany | Belgium | DOE | This model |
|------------------------|--------------------|---------|---------|---------|------------|
| Fuel consumption (lit) | 1398.33 | 1340.02 | 1333.83 | 1421.09 | 1394.33 |
| Mean per day (lit) | 155.37 | 148.89 | 148.2 | 157.9 | 154.93 |
| Efficiency (%) | 80.68 | 84.20 | 84.59 | 79.39 | 82.11 |
| Difference (%) | 100 | 95.83 | 95.39 | 101.63 | 99.72 |

Table 7.12 Results of a 2 weeks period, compare with others and measurement



- EMPA (DOE)
- * This model
- Germany
- Belgium

$$\eta = \frac{\text{Measured corrected fuel} - \text{calculated fuel}}{\text{Measured corrected fuel}} \%$$

Figure 7.9 Comparison of relative deviation as function of external air temperature

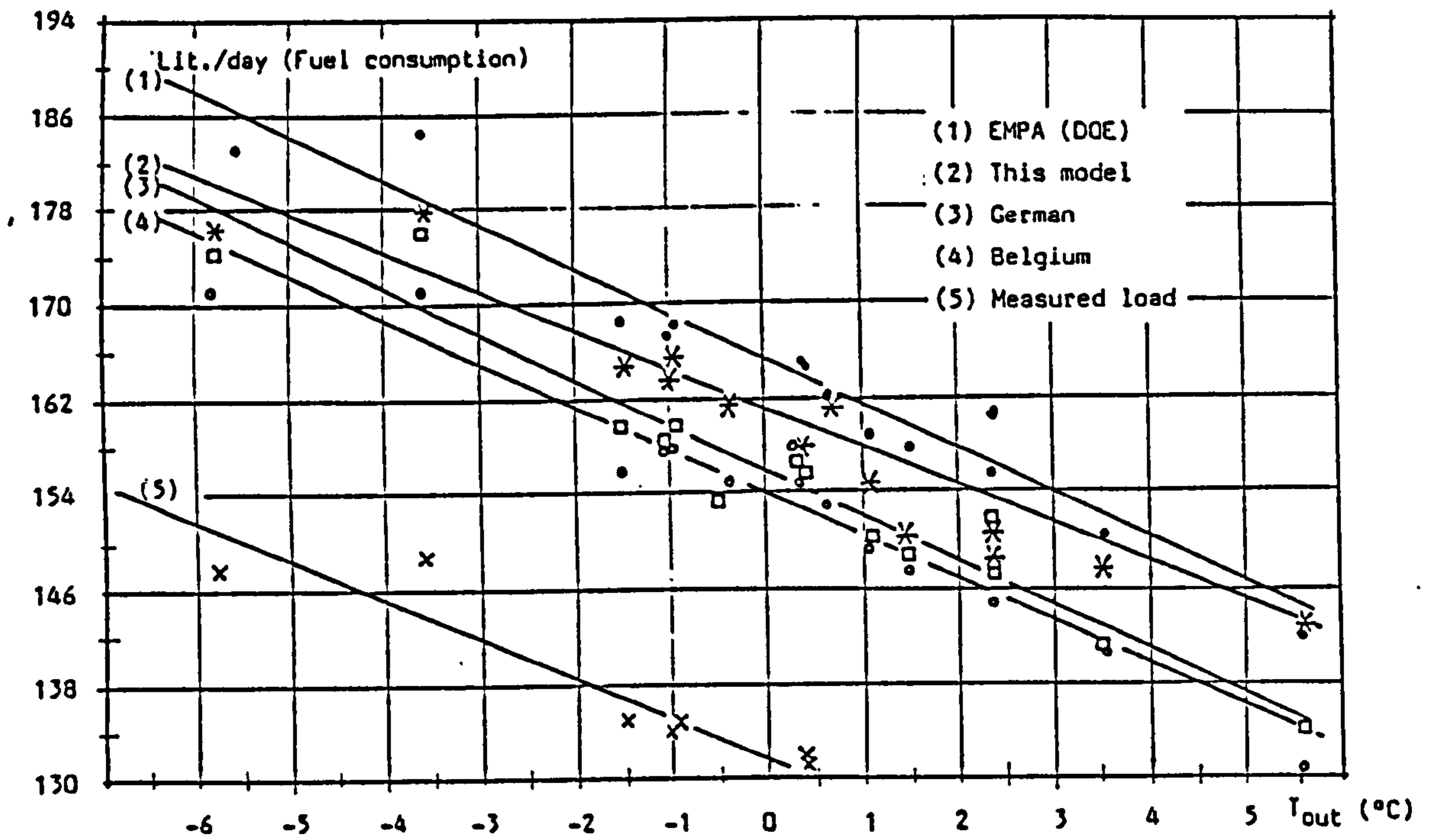


Figure 7.10 Linear regression comparison of fuel consumption as function of external air temperature

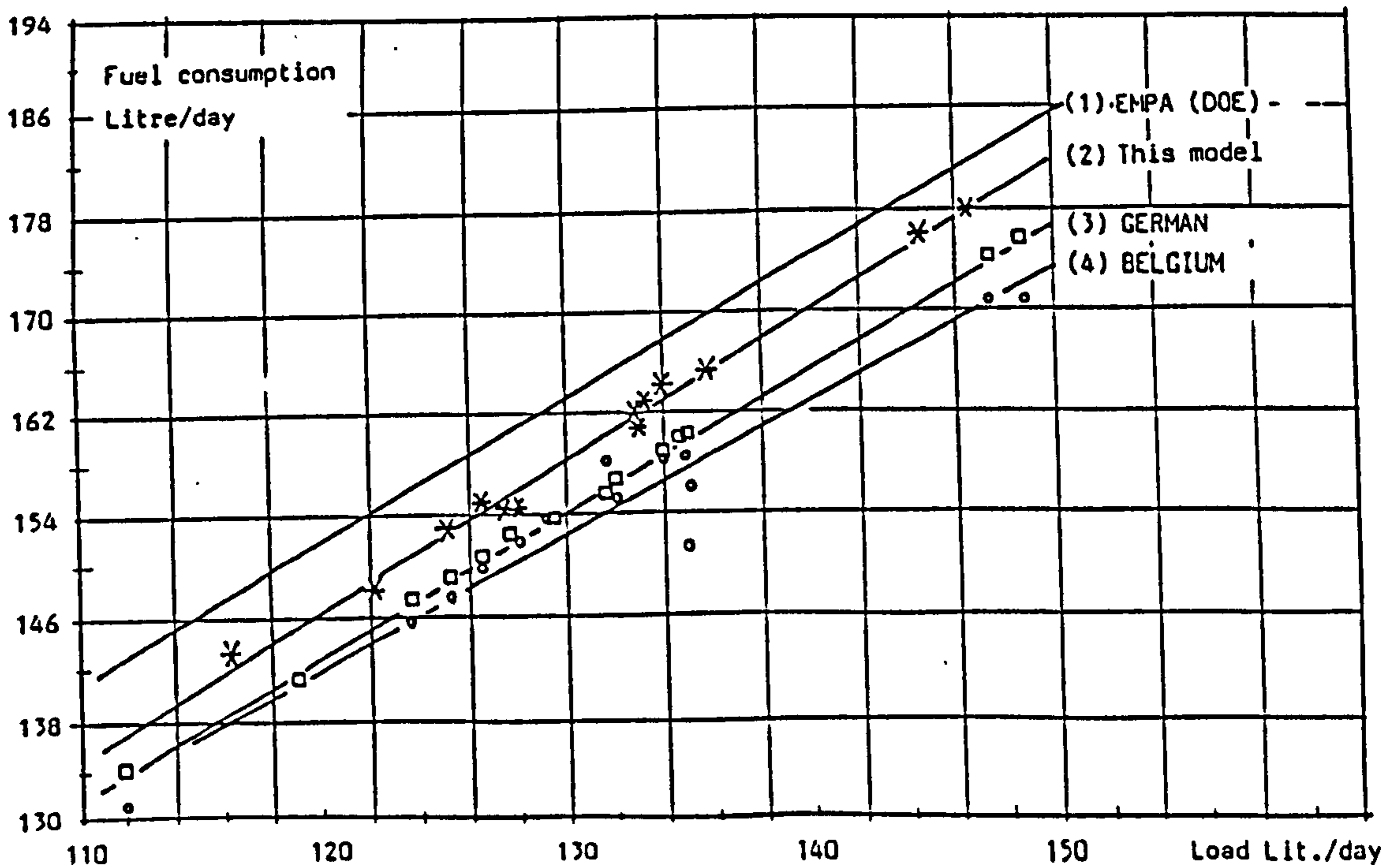


Figure 7.11 Linear regression comparison of fuel consumption as function heat production

7.4 La Chaumiere house: exercise 2

Exercise 2 of La Chaumiere house consists of six small exercises related to the simulation of boiler performance. Package of exercise 2 can be considered as a continuation of exercise 1a. After exercise 1a, interests have been found in the detailed simulation of boiler performance against models in different levels, the improvements of boiler operational efficiency are attractive to the participants. Same system as used in exercise 1a of La Chaumiere heating plant is used in exercise 2 and for the purpose of exercise 2, it is tried to find out which parameters are relevant for an accurate simulation, which parameters are relevant to the energy consumption of boiler and what are the important parameters by which the boiler operational efficiency can be apparently improved.

For all the small exercises in exercise 2, the load of the plant system are redefined in a clearer manner to avoid any mis-matching by the participants.

The six small exercises in exercise 2 are defined as

1. Exercise 2.0

Repetition of the exercise 1a, but new defined system load has to be used.

2. Exercise 2.1

The boiler remains the same as in exercise 1a and 2.0 but the burner power are adapted to the rated power of boiler. This means a larger burner is used in this boiler simulation.

New value imposed are:

| | |
|---------------------------|----------------|
| Burner power | = 240 KW |
| Fuel consumption | = 23,946 lit/h |
| Flue gas mean temperature | = 275°C |
| Plant room temperature | = 30°C |

3. Exercise 2.2

Better insulation on the boiler external surface are imposed. Suggestions give to about 5 cm to 10 cm insulation on the boiler

external surface. New values of steady state heat transfer coefficients are given, other values remain same as in exercise 1a.

4. Exercise 2.3

A new boiler which is considered as best fitting to the building, total load is used to replace the previous one. New data introduced in this exercise are:

| | |
|------------------------|----------------|
| Boiler type | = DIN 4702 |
| Rated boiler power | = 114 KW |
| Rated burner power | = 110 KW |
| Fuel consumption | = 10.975 lit/h |
| Water content | = 115 litre |
| CO in flue gas | = 12% |
| Surplus of air | = 1.3 |
| Plant room temperature | 25°C |

New values of steady state heat transfer coefficient are also given.

5. Exercise 2.4

A new boiler with higher heat capacity is introduced in this exercise. This exercise is trying to show the influence of dynamic behaviour on the energy consumption by changing the cast iron mass and water content of the boiler and the time dependency of flue gas temperature depending on the burner operation. New values introduced in this exercise are:

| | |
|----------------------|-------------|
| Boiler water content | = 335 litre |
| Cast iron mass | = 800 kg |

The rest of values remain same as in exercise 2.3.

6. Exercise 2.5

In this exercise, the boiler outlet water temperature is controlled according to the outside temperature, the controlled conditions are also changed by day and night period, which is:

$$T_b = 37.16 - 1.175T \quad \text{during the night period (0-7 o'clock)}$$
$$T_b = 61.81 - 1.368T \quad \text{during the day period (7-24 o'clock)}$$

Other values are same as defined in exercise 2.3

Standard presentation formats are defined by EMPA, including:

1. Listings and plots of hourly values of fuel consumption, boiler outlet water temperature, flue gas temperature for selected days which defined as, 19th November and 20th December.
2. Daily values are required for burner on time, energy production, flue gas losses, surface losses, etc.
3. A two week period of 18-24 of November and 16-22 of December are defined and daily values required for this period.

As part of ABACUS contribution, this package exercises has been completed by using program "CENSYS". Results have been sent to EMPA for further analysis and comparisons.

Some selective results of exercise 2 exercises presented in the standard presentation formats are given as follows. Note, that since "CENSYS" is a dynamic model, some steady state conditions imposed in the exercise specifications are not used, instead of this the time dependent parameters, such as thermal physical properties of fluids, heat transfer coefficients, are predicted at each time step of simulation.

Figure 7.12, 7.13 show the system load for the selected days of 19th November and 20th December respectively.

Figure 7.14, 7.15 show the system water supply rate for the selected days of 19th November and 20th December respectively.

Figure 7.16 to 7.20 and table 7.13 give the boiler performances on the selected day 19th November under exercise 2.4, which are:

- | | |
|-------------|--|
| Figure 7.16 | boiler water outlet temperature and boiler fuel supply |
| Figure 7.17 | boiler flue gas temperature |
| Figure 7.18 | boiler heat production |

| | |
|-------------|--|
| Figure 7.19 | boiler flue gas loss, envelope loss |
| Figure 7.20 | boiler hourly fuel supply rate |
| Table 7.13 | the detailed hourly values of boiler performance |

Figure 7.21 to 7.25 and table 7.14 give the boiler performances on the selected day of 20th December under exercise 2.4, which are:

| | |
|-------------|--|
| Figure 7.21 | boiler outlet water temperature and boiler fuel supply |
| Figure 7.22 | boiler flue gas temperature |
| Figure 7.23 | boiler heat production |
| Figure 7.24 | boiler flue gas loss, envelope loss |
| Figure 7.25 | boiler hourly fuel supply rate |
| Table 7.14 | hourly values of boiler performance |

The final report of simulation results analysis and comparisons have not yet completed by EMPA, however, a primary analysis based on the results of exercise 2.0 has been published. The following figures and table are extracted from that report. [Ref. 8]

Figure 7.26 shows the hourly fuel consumption of boiler compared among four groups for exercise 2.0 of 20th December.

Figure 7.27 shows the two week period daily energy consumptions vs. heat demand. Results compared among six groups for exercise 2.0 of 20th December.

Table 7.15 shows the fuel consumption, losses, efficiency etc. among six groups for exercise 2.0 of 20th December.

New version of analysis of exercises 2.0 to 2.5 will be released from EMPA very soon. As the programme of Annex exercises ongoing, "CENSYS" programme will still work in this project of further sophisticated system simulation. On the other hand, the newly developed generalised system simulation software at ABACUS unit - the implemented ESP programme will be equipped by the validated plant system and component models of "CENSYS", therefore, the energy simulation at generalised and advanced level can be expected in the nearly future.

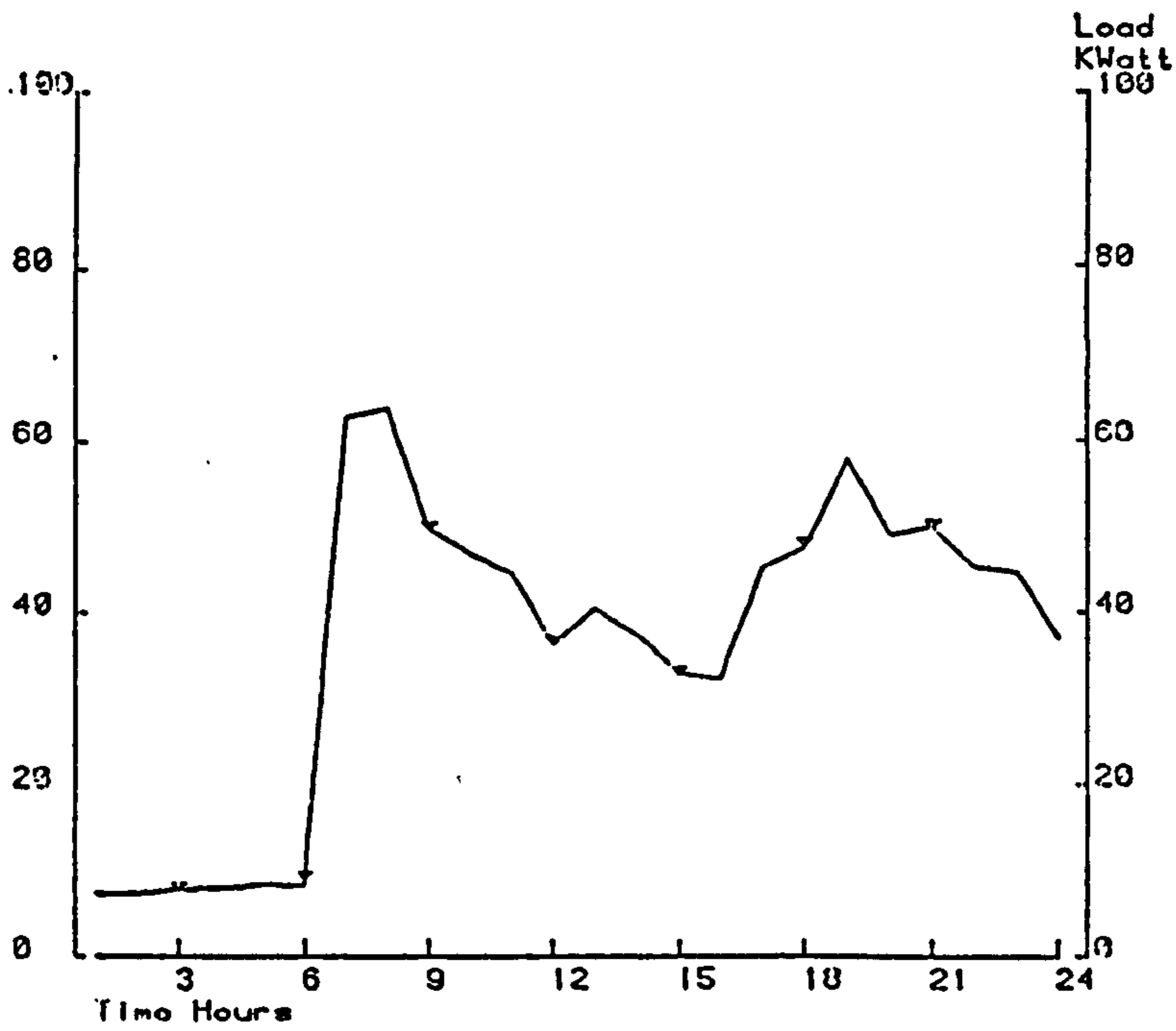


Figure 7.12 Load of system, 19th November

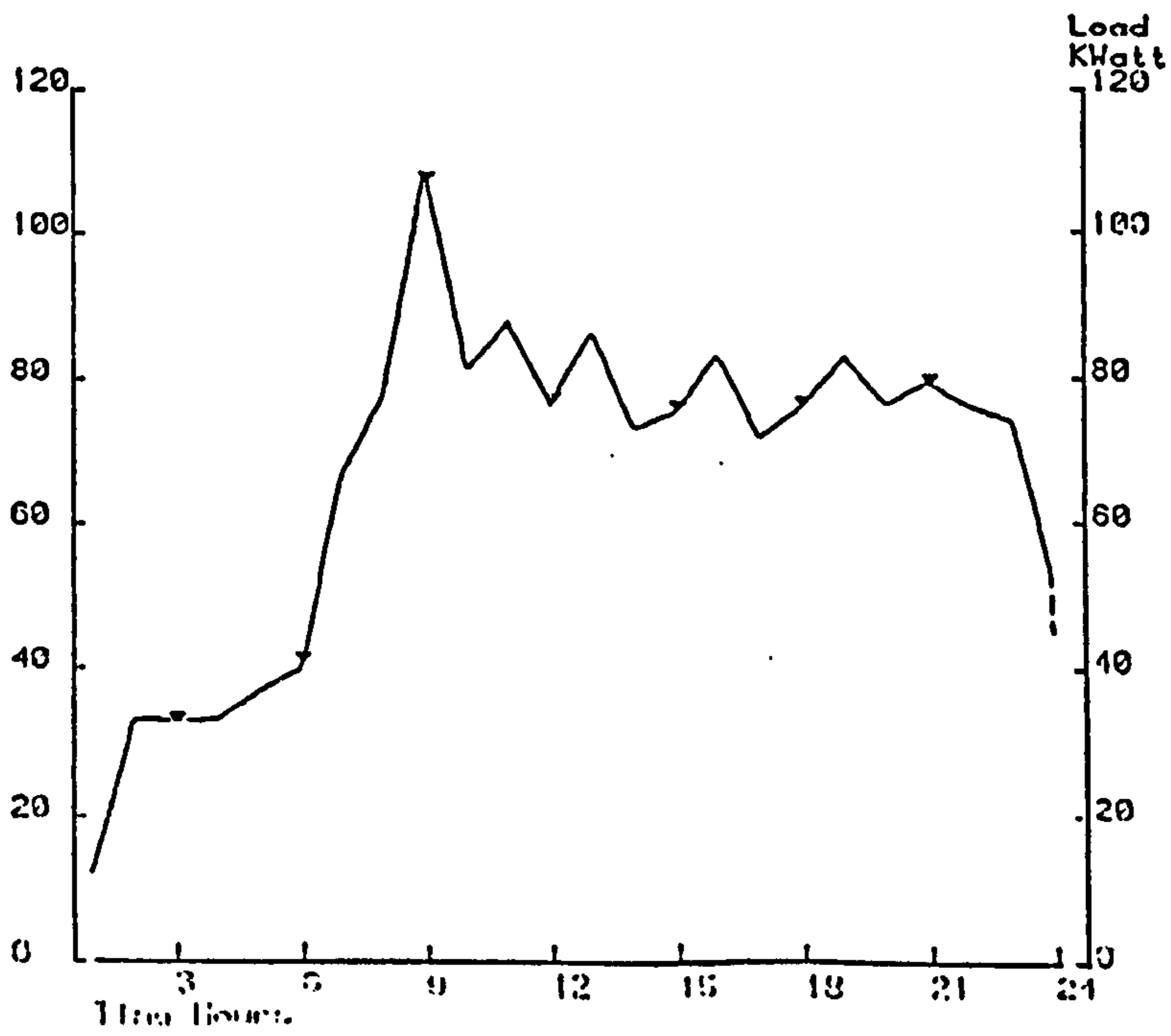


Figure 7.13 Load of system, 20th December

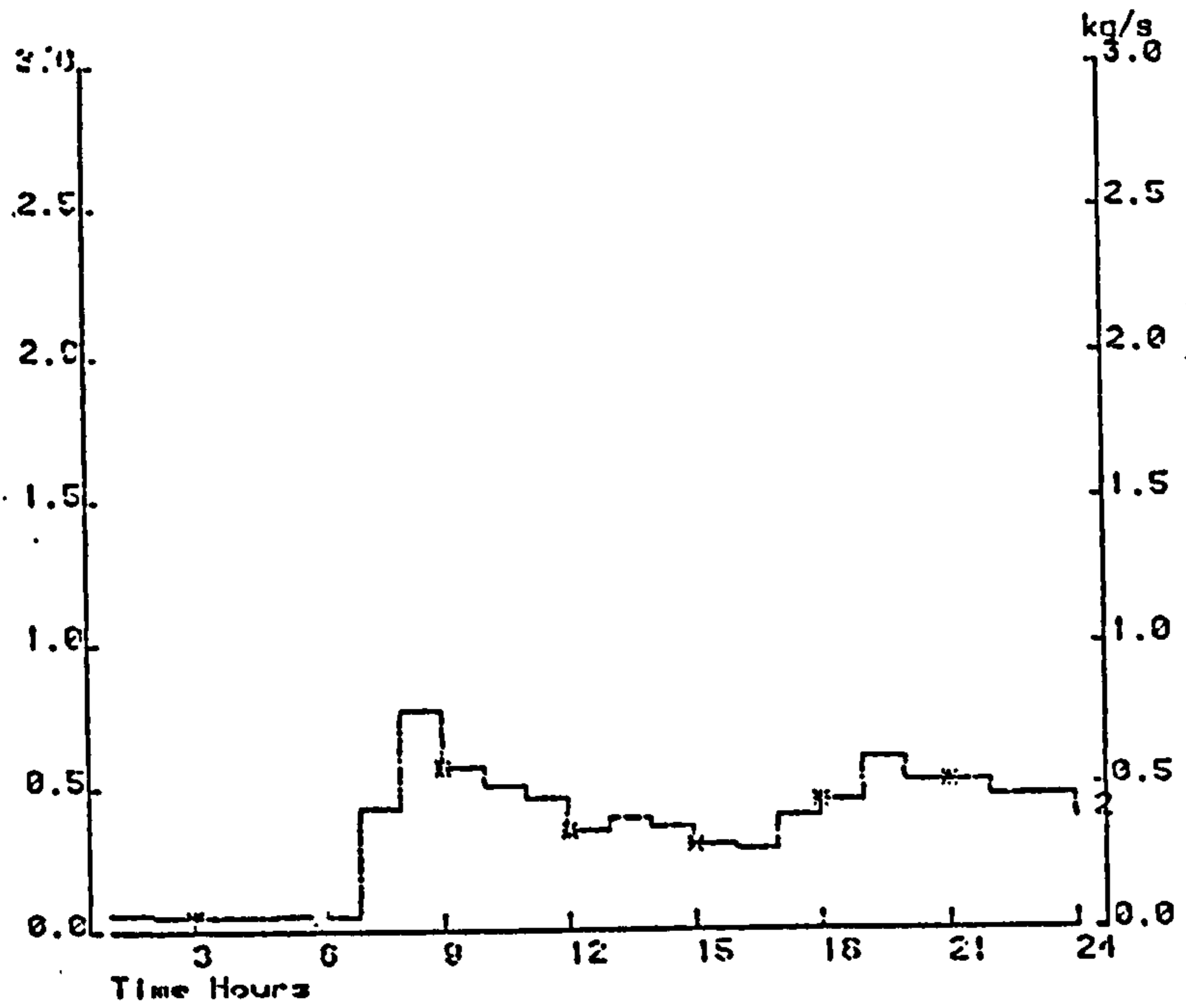


Figure 7.14 System water supply rate of 19th November

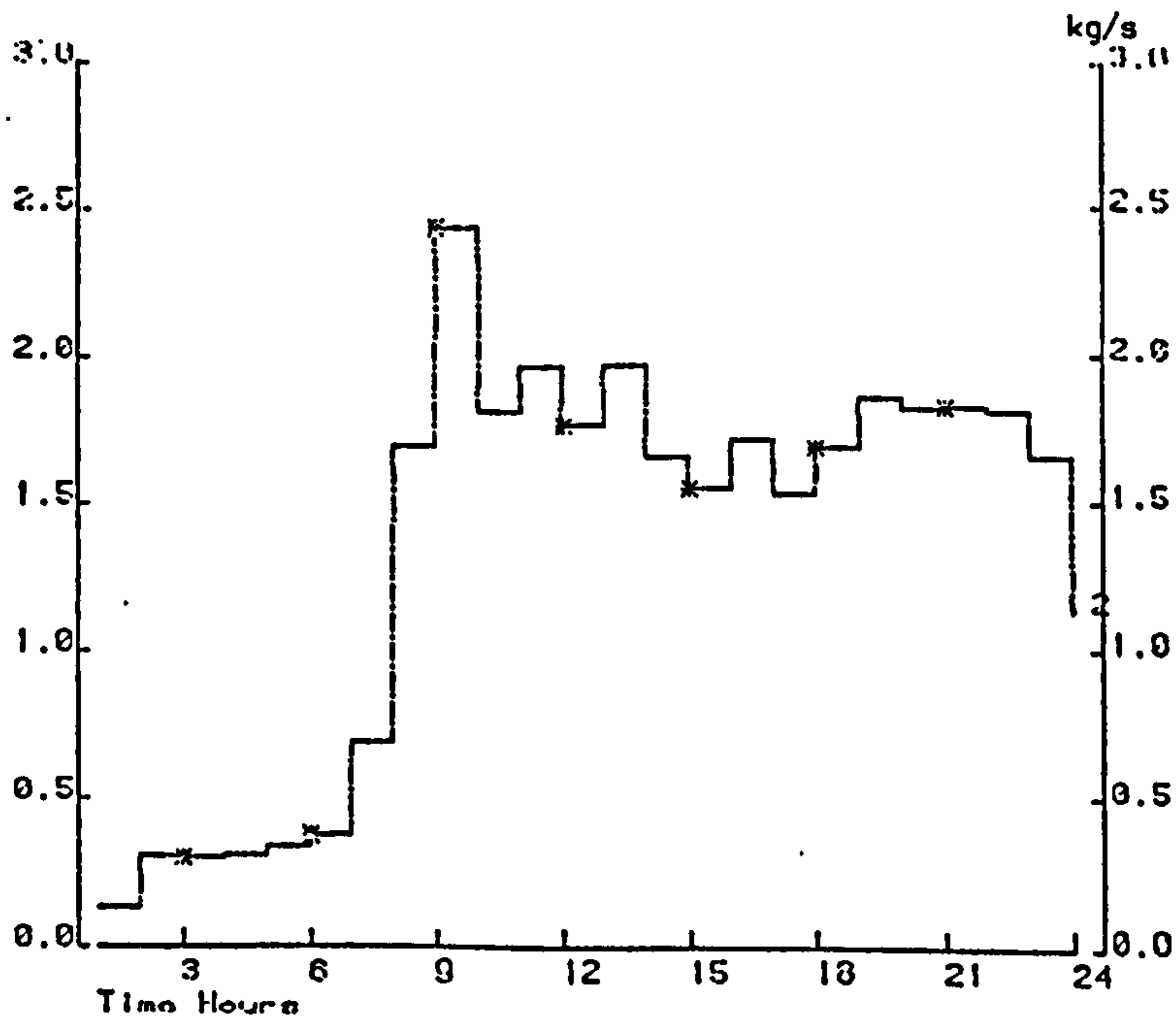


Figure 7.15 System water supply rate of 20th December

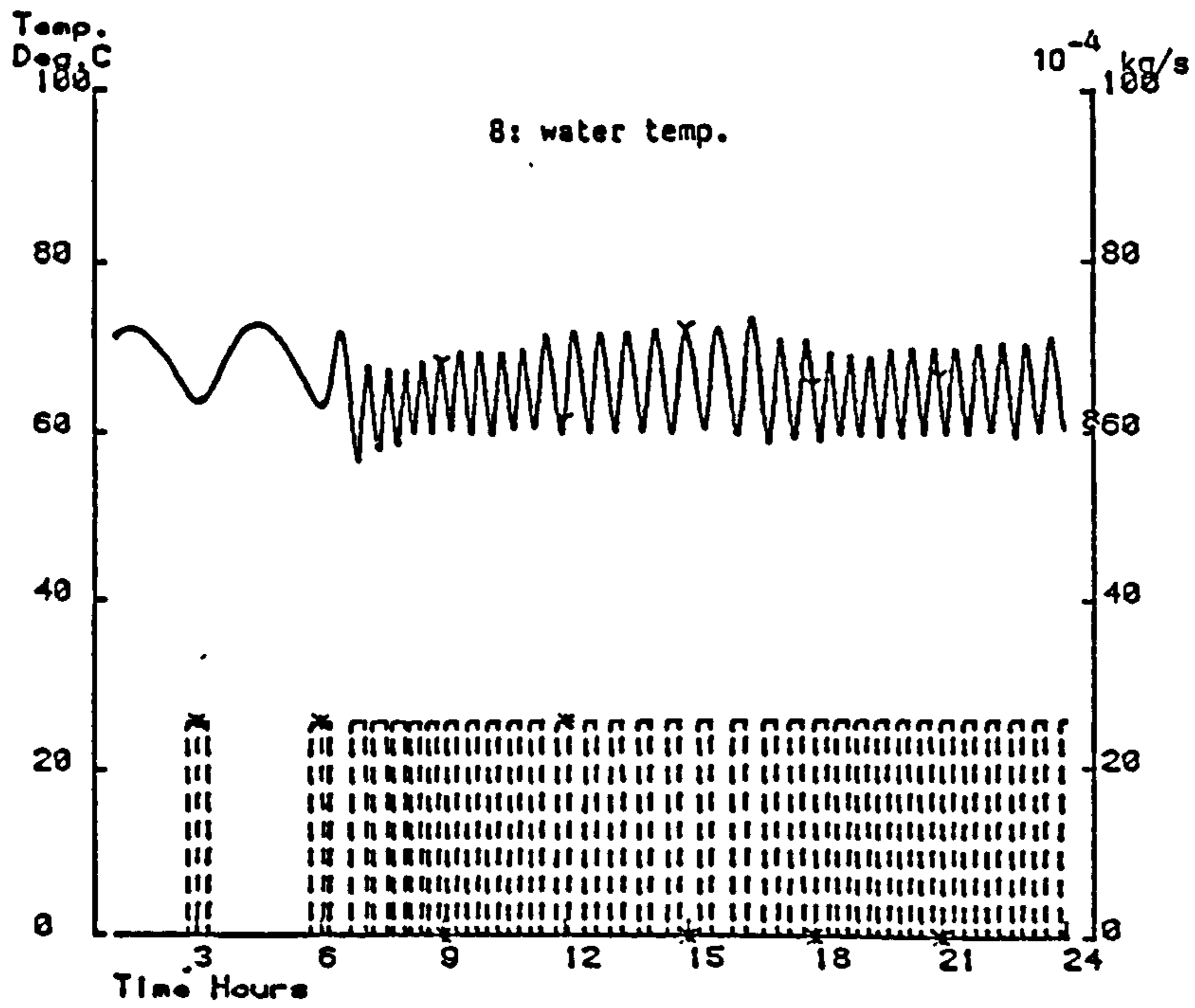


Figure 7.16 Boiler water outlet temperature and fuel supply rate of 19th November

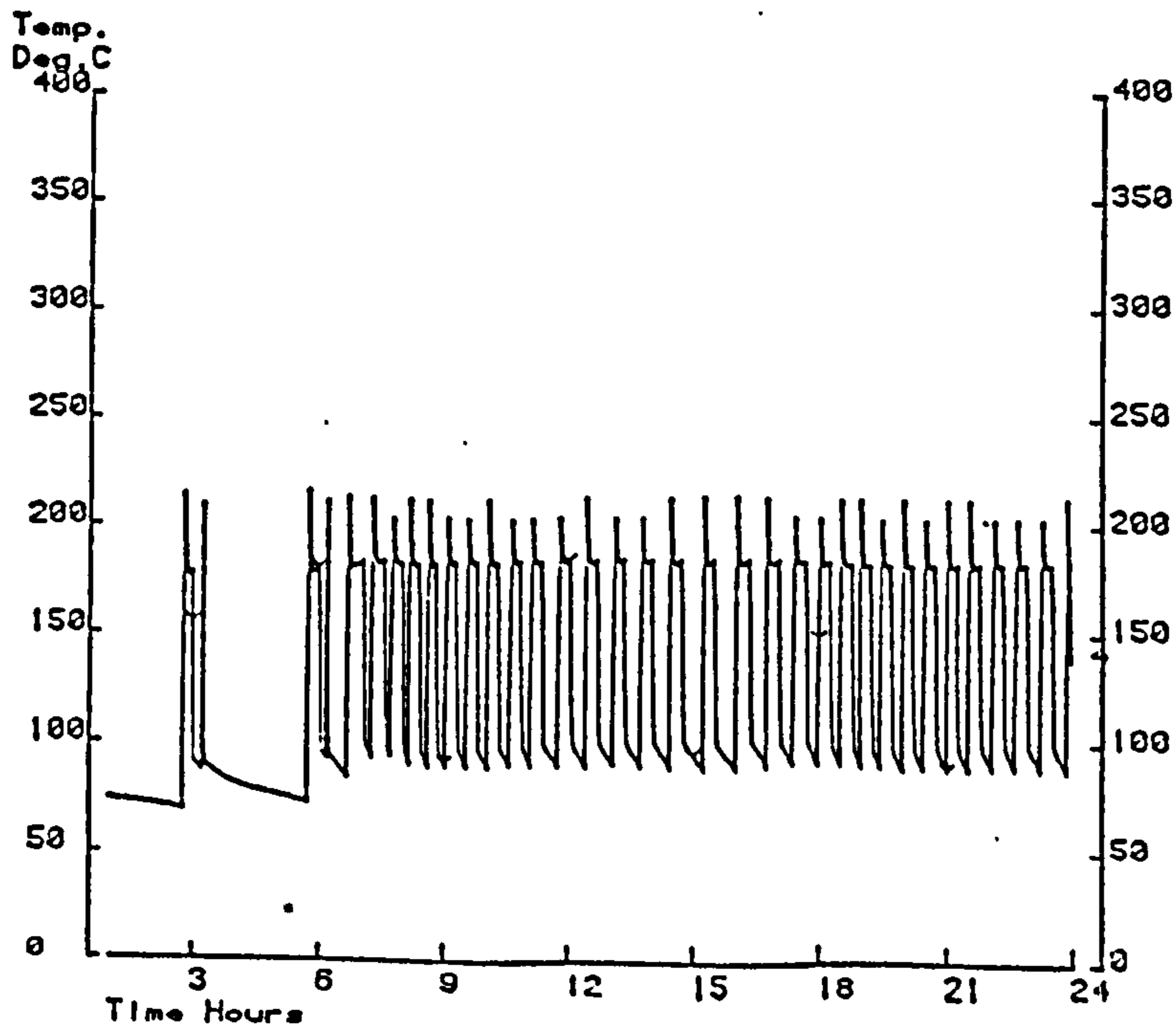


Figure 7.17 Boiler flue gas temperature of 19th November

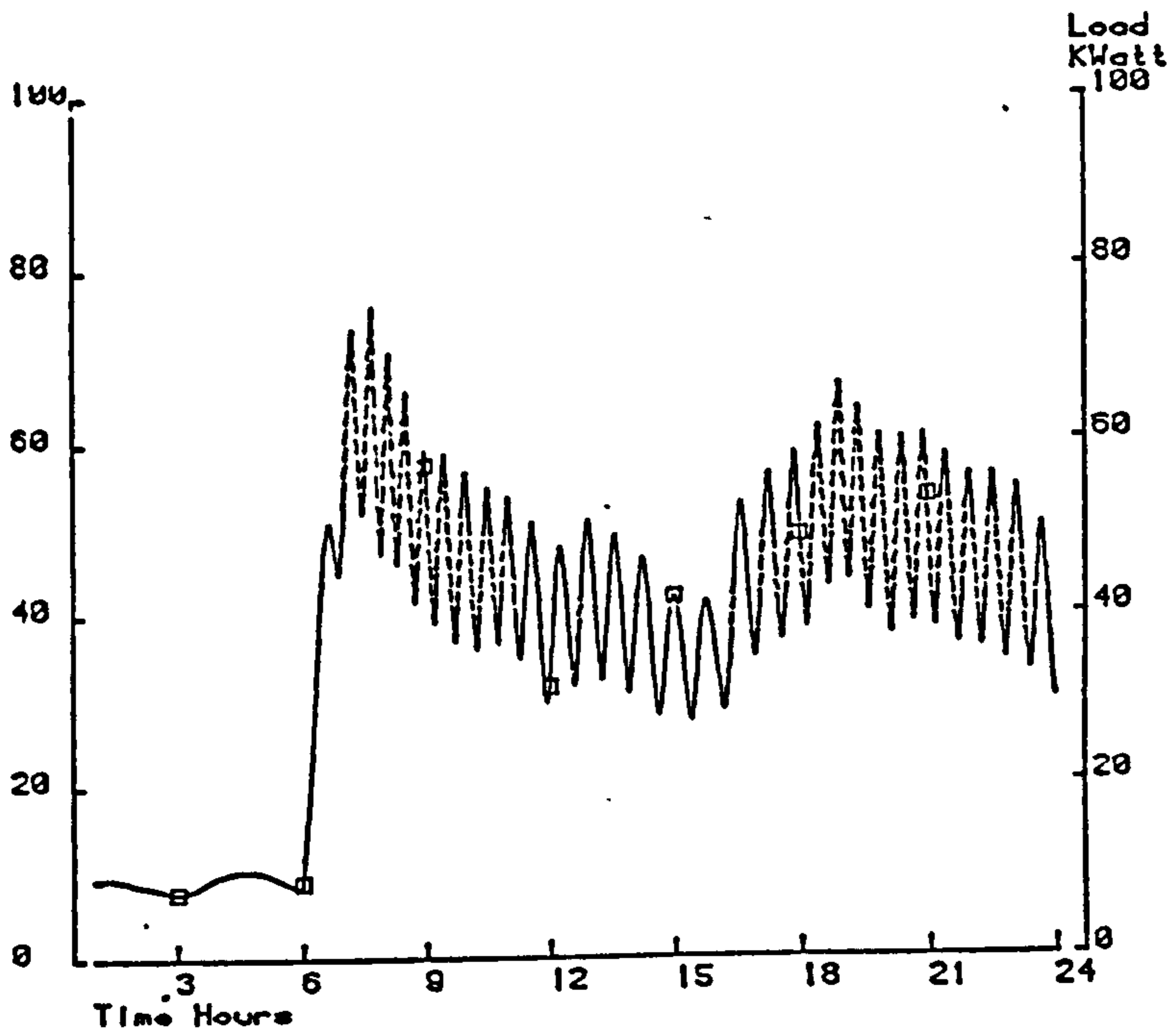


Figure 7.18 Boiler heat production of 19th November

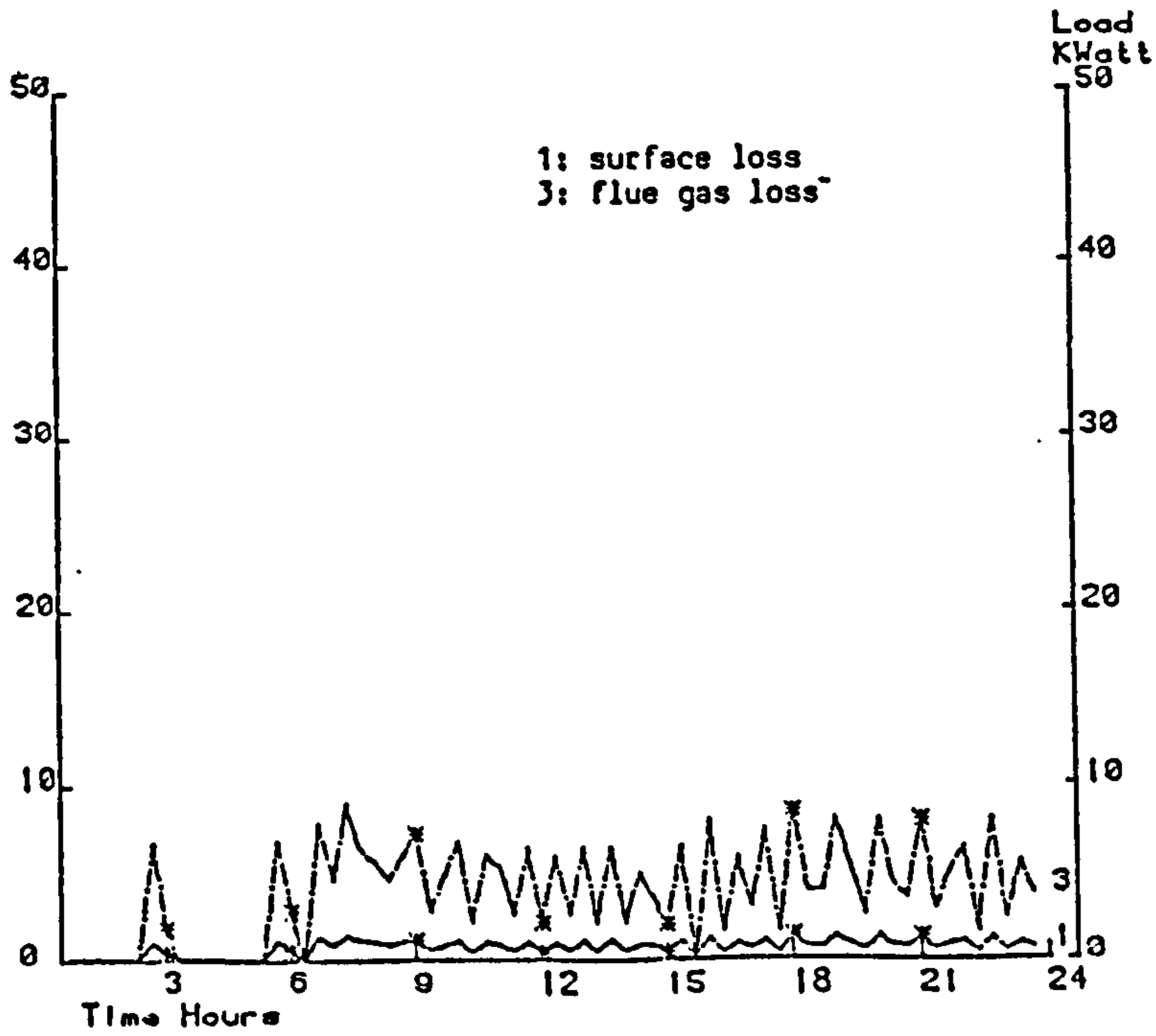


Figure 7.19 Boiler surface loss and flue gas loss of 19th November

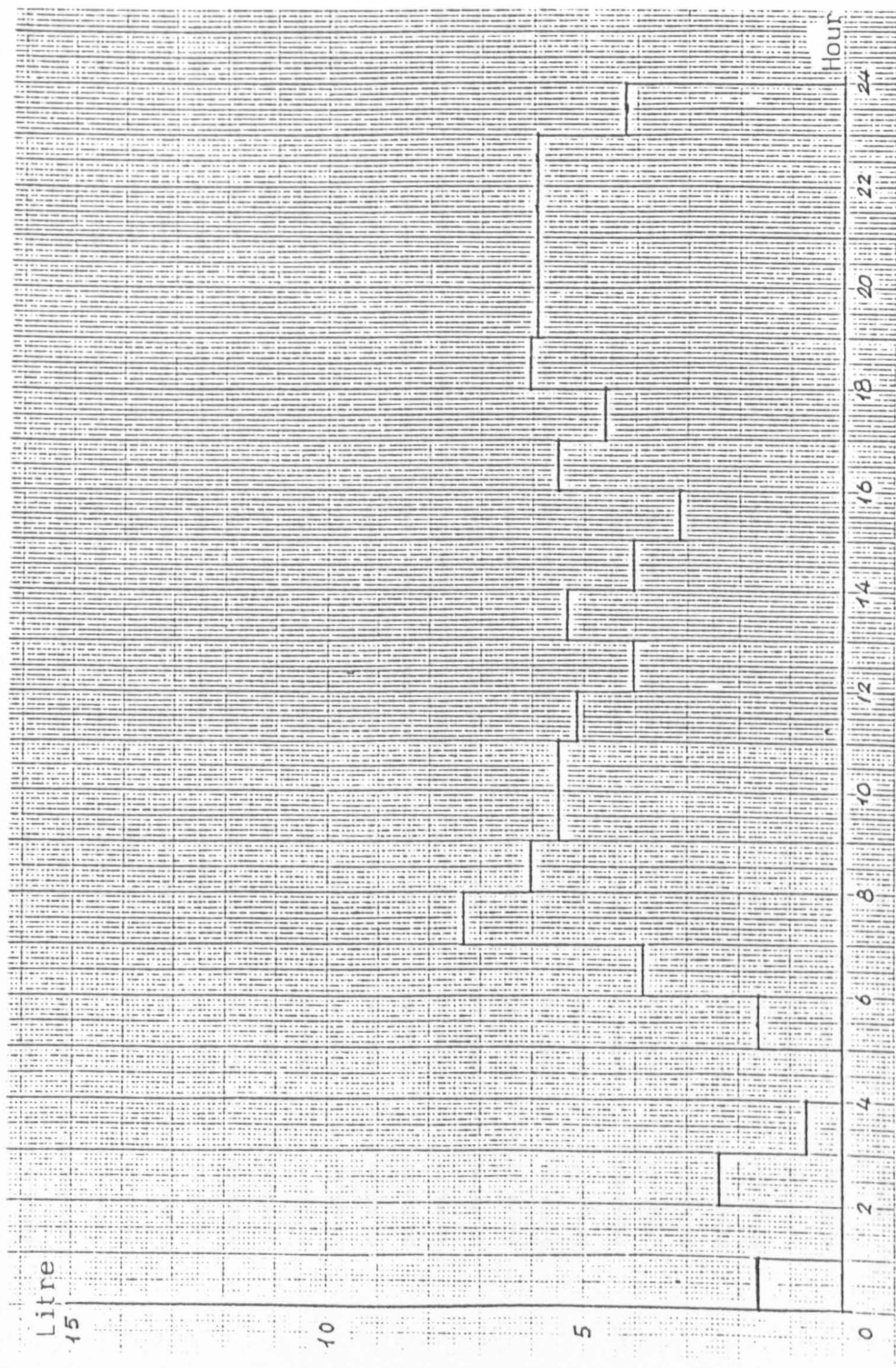


Figure 7.20 Boiler hourly fuel supply rate of 19th November (Exercise 2.4)

| UMFS (W) | QTOT (W) | QBCL (W) | QFGS (H) | TGF (K) | TBNF (K) | BMFR (G/S) | TENF (K) |
|---------------|-------------|-------------|---------------|------------|-------------|---------------|-------------|
| 16524.90 | 8289.828 | 438.8749 | 1519.339 | 94.86570 | 67.48297 | 0.6350000E-01 | 9.000000 |
| 0.0000000E+00 | 9344.571 | 174.5672 | 0.0000000E+00 | 74.53124 | 71.79635 | 0.5410000E-01 | 9.000000 |
| 23869.30 | 8364.014 | 466.3180 | 2167.951 | 96.54916 | 66.99932 | 0.5445000E-01 | 9.000000 |
| 7344.400 | 8756.553 | 330.9208 | 705.2545 | 93.39881 | 66.93153 | 0.5472000E-01 | 9.000000 |
| 0.0000000E+00 | 10301.52 | 176.7787 | 0.0000000E+00 | 79.40689 | 72.33054 | 0.5588000E-01 | 9.000000 |
| 23869.30 | 9227.744 | 465.9862 | 2182.230 | 99.11402 | 67.36127 | 0.5472000E-01 | 9.000000 |
| 38559.10 | 36579.96 | 729.1833 | 3531.715 | 124.4386 | 65.37180 | 0.4294000 | 8.000000 |
| 7344.00 | 61925.41 | 1171.738 | 6652.189 | 157.6217 | 62.74428 | 0.7651000 | 8.000000 |
| 60591.30 | 55964.50 | 1016.438 | 5498.227 | 145.2824 | 64.36040 | 0.5662000 | 8.000000 |
| 55083.00 | 48454.42 | 943.8811 | 5019.961 | 140.5576 | 64.98501 | 0.5023000 | 9.000000 |
| 55083.00 | 45507.99 | 944.2423 | 5024.592 | 140.8952 | 64.79100 | 0.4588000 | 9.000000 |
| 51410.80 | 41756.90 | 889.7017 | 4736.278 | 138.5828 | 65.53415 | 0.3475000 | 9.000000 |
| 40394.20 | 41737.62 | 753.2874 | 3644.766 | 129.3002 | 67.01972 | 0.3856000 | 9.000000 |
| 53246.90 | 39850.94 | 914.9057 | 4899.183 | 140.6876 | 65.44886 | 0.3355000 | 11.00000 |
| 40394.20 | 37817.37 | 753.4206 | 3649.021 | 129.5415 | 66.74019 | 0.2977000 | 11.00000 |
| 31213.70 | 35937.45 | 630.2451 | 2852.675 | 121.1105 | 67.60733 | 0.2822000 | 11.00000 |
| 55083.00 | 39811.70 | 937.1760 | 5061.983 | 142.4393 | 65.50335 | 0.3395000 | 11.00000 |
| 45902.50 | 48490.11 | 816.0528 | 4147.405 | 134.0837 | 65.97996 | 0.4467000 | 11.00000 |
| 60591.30 | 51912.10 | 1012.053 | 5532.586 | 146.7951 | 64.25609 | 0.5973000 | 9.000000 |
| 58755.20 | 52266.40 | 986.7226 | 5368.224 | 144.6371 | 64.48128 | 0.5195000 | 9.000000 |
| 58755.20 | 49376.01 | 986.8719 | 5366.618 | 144.7389 | 64.78063 | 0.5171000 | 9.000000 |
| 58755.20 | 47240.38 | 986.8170 | 5371.339 | 144.9603 | 64.71560 | 0.4661000 | 9.000000 |
| 58755.20 | 44629.13 | 982.7355 | 5375.150 | 144.7179 | 64.71339 | 0.4666000 | 9.000000 |
| 42230.30 | 43192.14 | 778.5420 | 3948.085 | 131.4884 | 66.19917 | 0.3861000 | 9.000000 |

ALL ENERGY ITEMS BELOW IN UNIT OF (MJ)

3635.478 . 3156.245 . 65.63485 . 332.1172 . 66.17234 . 30124.65 .

Table 7.13 Hourly boiler performance of 19th November (Exercise 2.4)

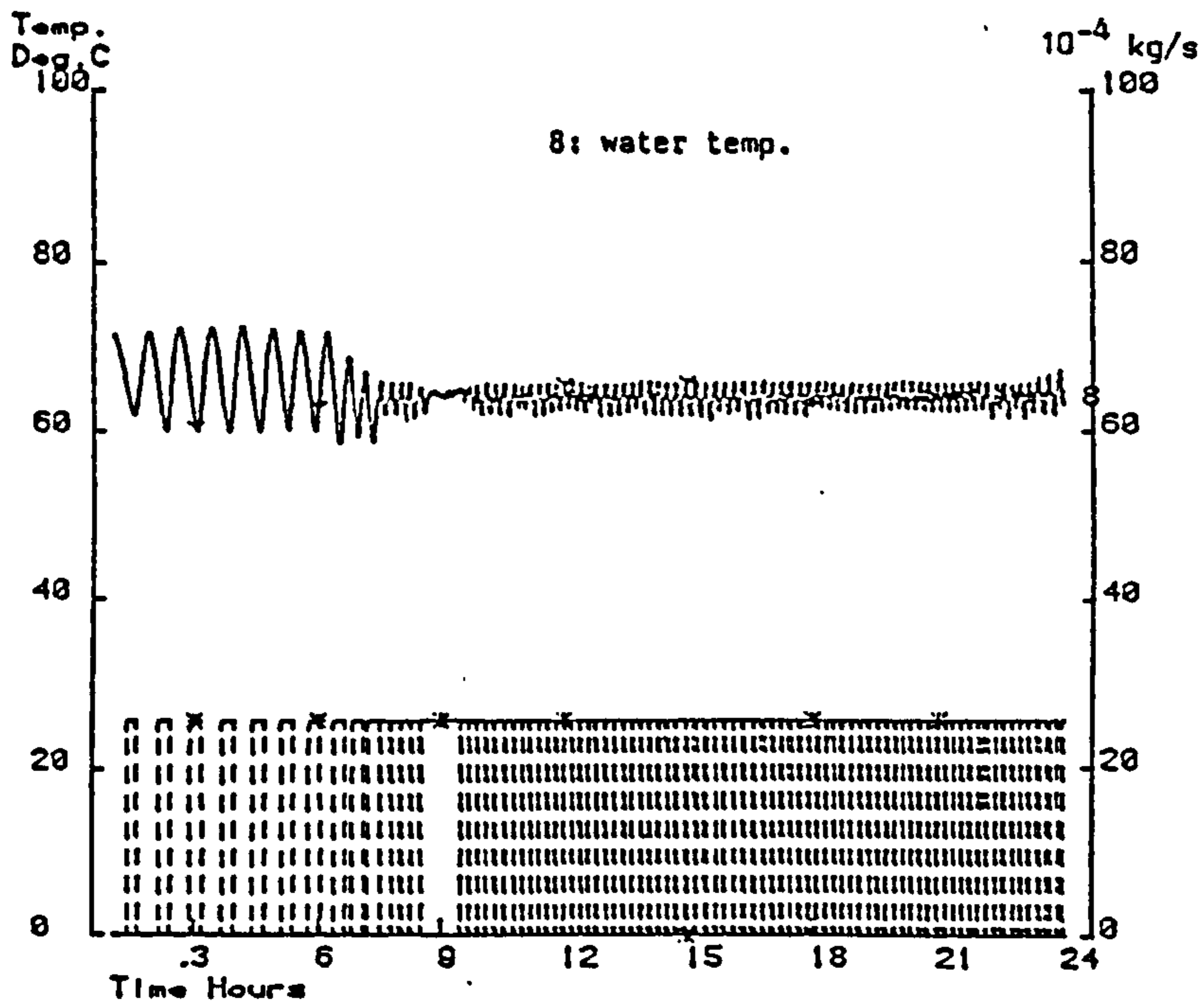


Figure 7.21 Boiler water outlet temperature and fuel supply rate of 20th December

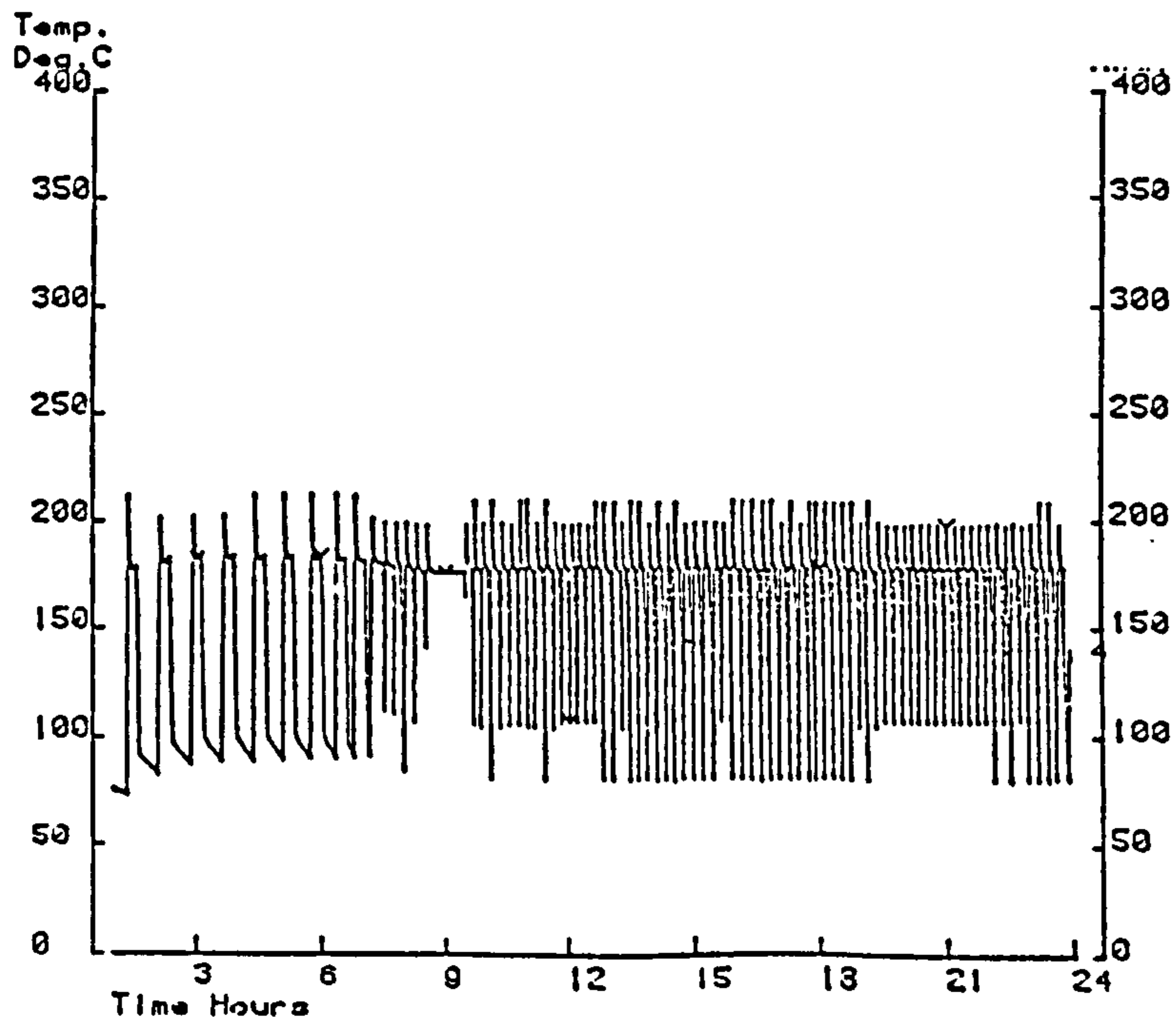


Figure 7.22 Boiler flue gas temperature of 20th December

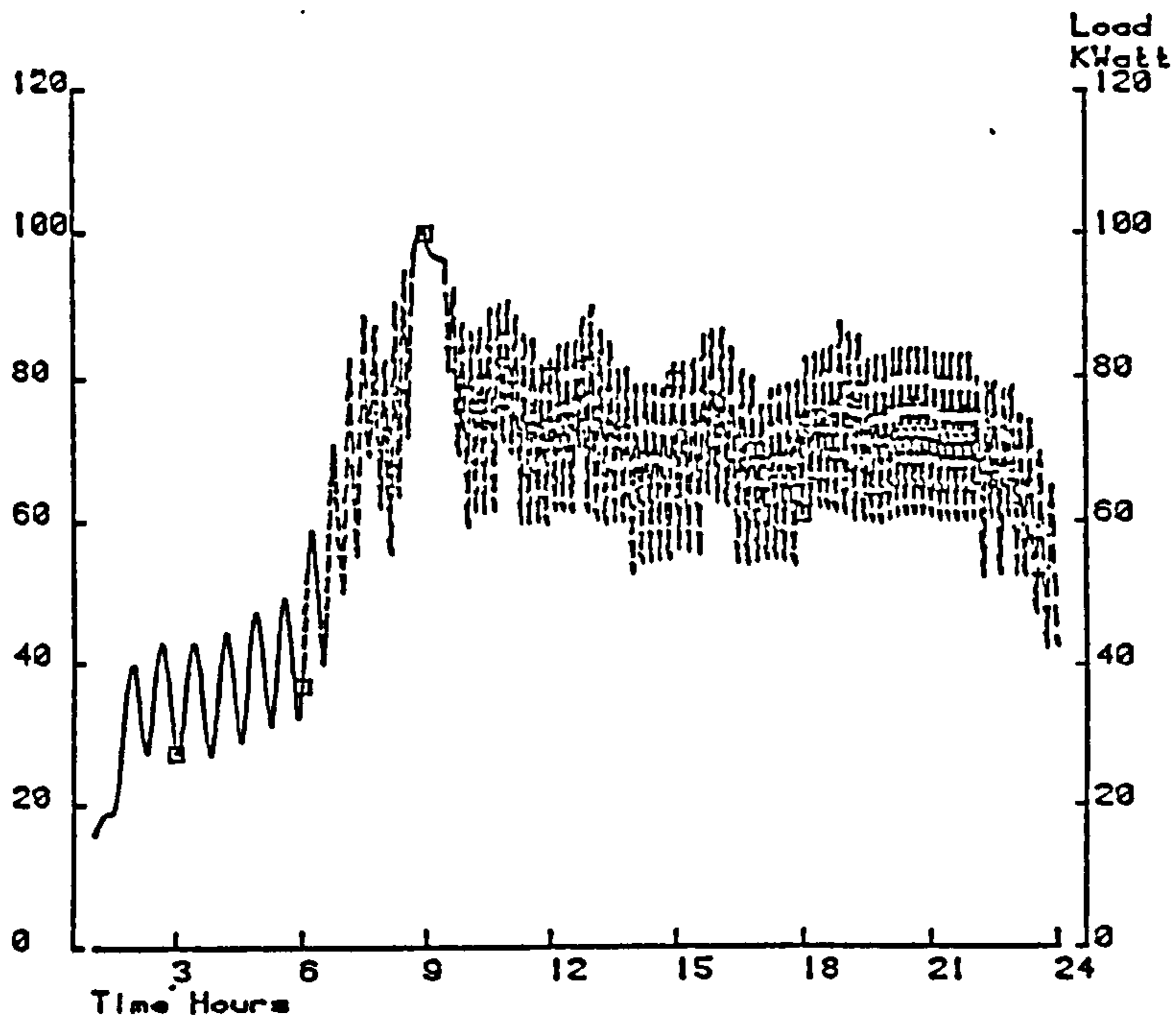


Figure 7.23 Boiler heat production of 20th December

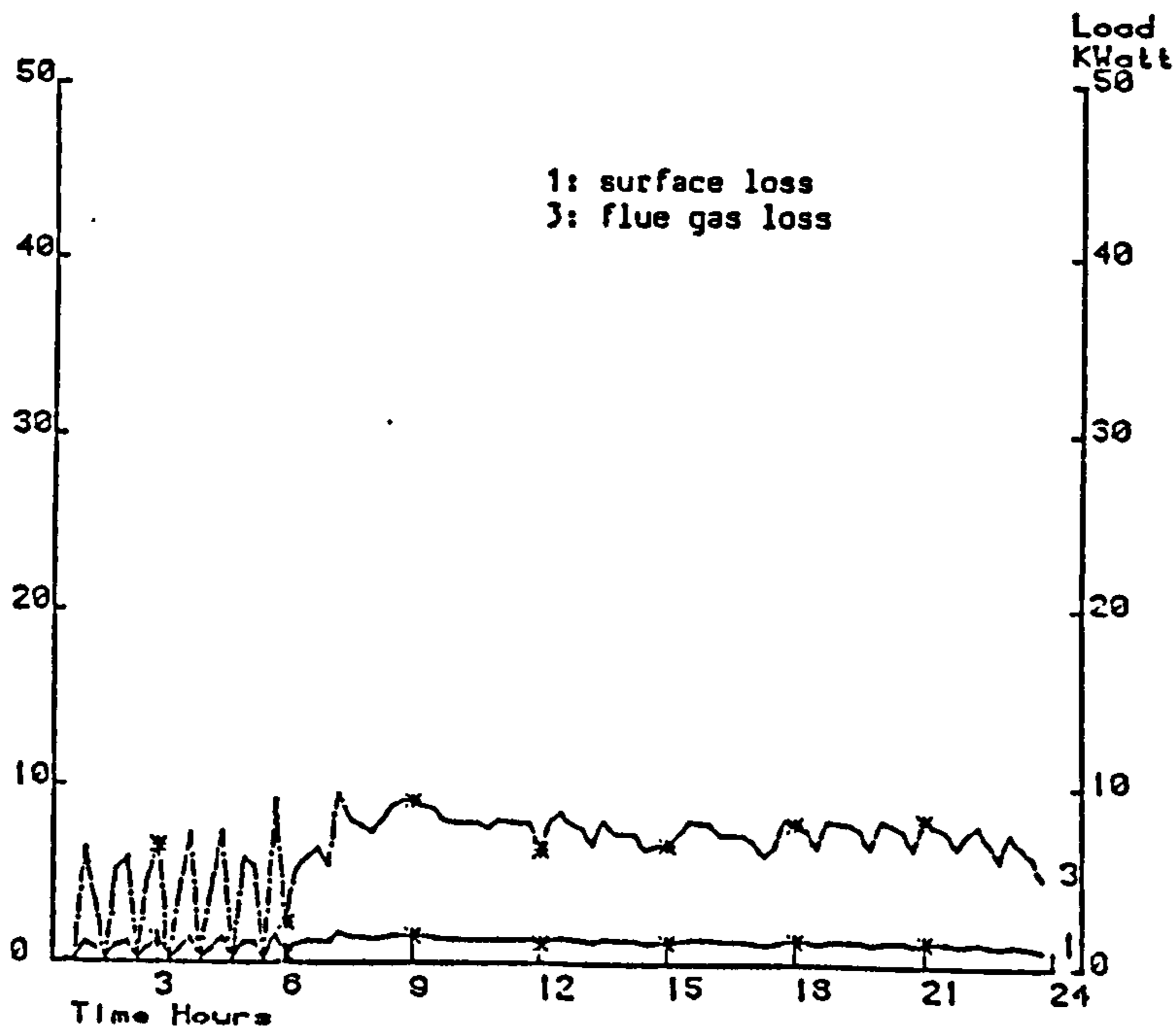


Figure 7.24 Boiler surface loss and flue gas loss (20th December)

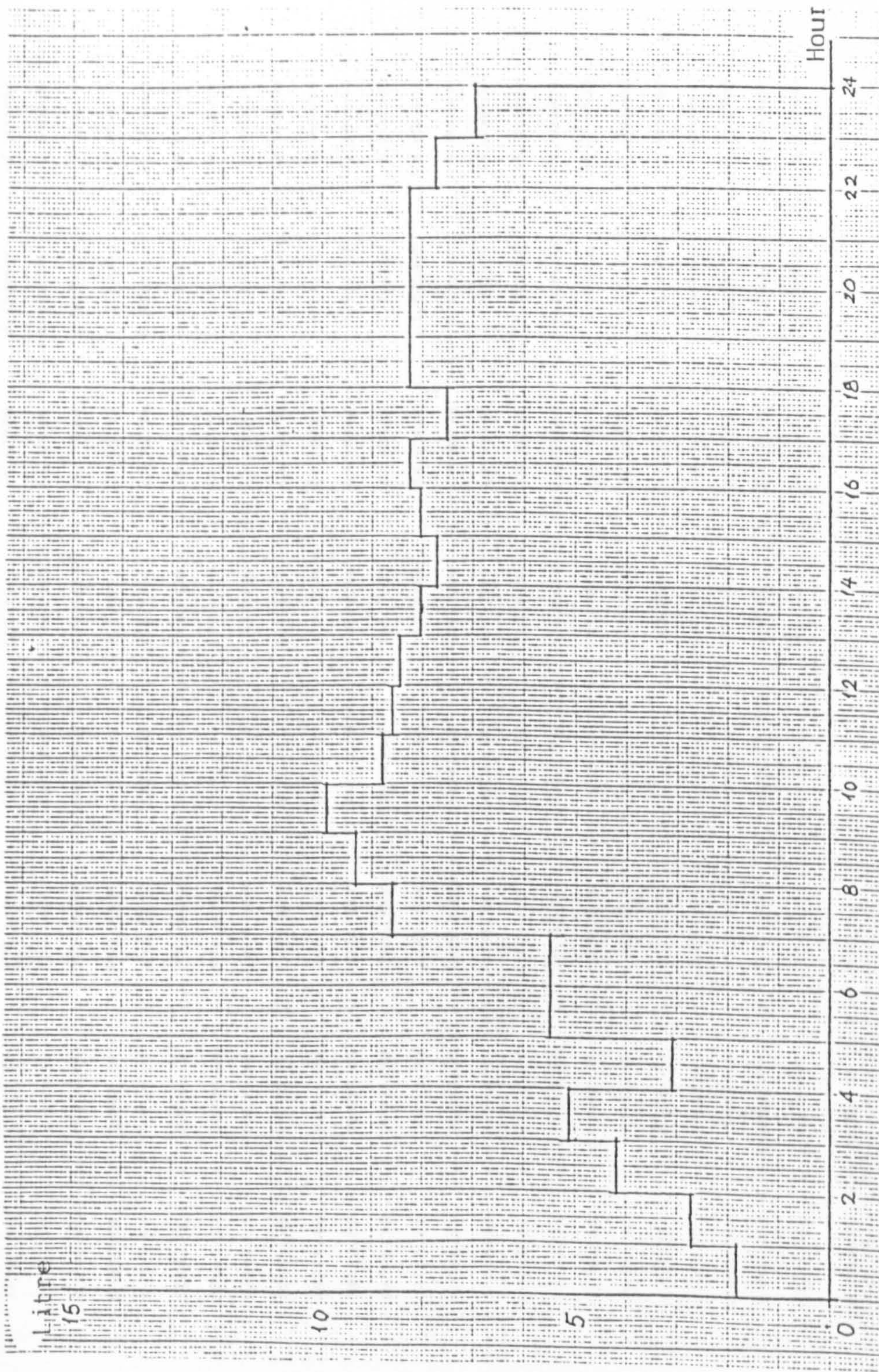


Figure 7.25 Boiler hourly fuel supply rate (20th December, Exercise 2.4)

| WFS (W) | QTOT (W) | QBCL (W) | QFOS (W) | TGF (K) | TBNF (K) | BMFR (KG/S) | TENF (K) |
|------------|-------------|-------------|-------------|------------|-------------|----------------|-------------|
| 18361.00 | 14791.33 | 474.3794 | 1824.849 | 100.1941 | 69.62928 | 0.1330000 | -7.0000000 |
| 27541.50 | 25040.48 | 564.4471 | 2474.353 | 108.0405 | 67.49661 | 0.3125000 | -7.0000000 |
| 42230.30 | 34761.54 | 772.9204 | 3897.296 | 128.6353 | 66.10764 | 0.3053000 | -7.0000000 |
| 51410.80 | 34081.31 | 685.3919 | 4608.512 | 137.5891 | 65.62448 | 0.3162000 | -7.0000000 |
| 31213.70 | 39169.63 | 635.2317 | 2847.964 | 121.5731 | 67.77254 | 0.3458000 | -8.0000000 |
| 55083.00 | 39108.19 | 937.0474 | 5055.729 | 142.1234 | 65.32035 | 0.3833000 | -8.0000000 |
| 55083.00 | 53330.33 | 946.4088 | 5041.310 | 142.2918 | 64.98671 | 0.6978000 | -8.0000000 |
| 86296.70 | 72692.66 | 1339.684 | 7770.782 | 166.5840 | 63.35541 | 1.696000 | -8.0000000 |
| 93641.10 | 84155.91 | 1425.339 | 8260.292 | 169.1240 | 63.85878 | 2.438200 | -8.0000000 |
| 99149.40 | 88541.30 | 1487.025 | 8682.503 | 173.1526 | 64.25552 | 1.812600 | -8.0000000 |
| 88132.80 | 76407.45 | 1378.649 | 7966.833 | 166.7246 | 63.92750 | 1.967100 | -7.0000000 |
| 86296.70 | 73239.70 | 1366.517 | 7937.047 | 166.9653 | 63.85284 | 1.769300 | -5.0000000 |
| 84460.60 | 73913.90 | 1349.404 | 7787.990 | 165.5109 | 64.01882 | 1.970000 | -4.0000000 |
| 80788.40 | 71889.20 | 1299.992 | 7448.349 | 161.7024 | 63.95346 | 1.656700 | -5.0000000 |
| 77116.20 | 67196.47 | 1248.785 | 7053.637 | 157.2791 | 63.92340 | 1.554000 | -4.0000000 |
| 80788.40 | 71013.65 | 1303.051 | 7511.258 | 162.9186 | 63.75828 | 1.719700 | -4.0000000 |
| 82624.50 | 69392.95 | 1306.794 | 7493.700 | 161.8676 | 63.80563 | 1.531600 | -4.0000000 |
| 75280.10 | 67048.95 | 1240.422 | 7033.988 | 157.8576 | 63.95046 | 1.685700 | -4.0000000 |
| 82624.50 | 72973.00 | 1317.638 | 7558.266 | 162.7985 | 64.08412 | 1.655100 | -5.0000000 |
| 82624.50 | 72116.86 | 1317.683 | 7549.507 | 162.5545 | 64.00122 | 1.820200 | -5.0000000 |
| 82624.50 | 72027.21 | 1326.611 | 7629.074 | 163.8211 | 64.18887 | 1.827600 | -4.0000000 |
| 82624.50 | 71398.44 | 1326.590 | 7628.144 | 163.7940 | 64.13498 | 1.812300 | -4.0000000 |
| 77116.20 | 68072.25 | 1257.237 | 7163.826 | 158.7961 | 64.00043 | 1.655000 | -4.0000000 |
| 69771.80 | 59034.07 | 1163.019 | 6517.674 | 152.2466 | 64.42340 | 1.139100 | -4.0000000 |

ALL ENERGY ITEMS BELOW IN UNIT OF (MJ)

| | | | | | | | |
|----------|----------|----------|----------|----------|----------|--|--|
| 6094.383 | 5297.030 | 99.61440 | 557.0752 | 64.76795 | 114874.7 | | |
|----------|----------|----------|----------|----------|----------|--|--|

Table 7.14 Hourly boiler performance (20th December, Exercise 2.4)

OIL CONSUMPTIONS FOR DECEMBER 20th

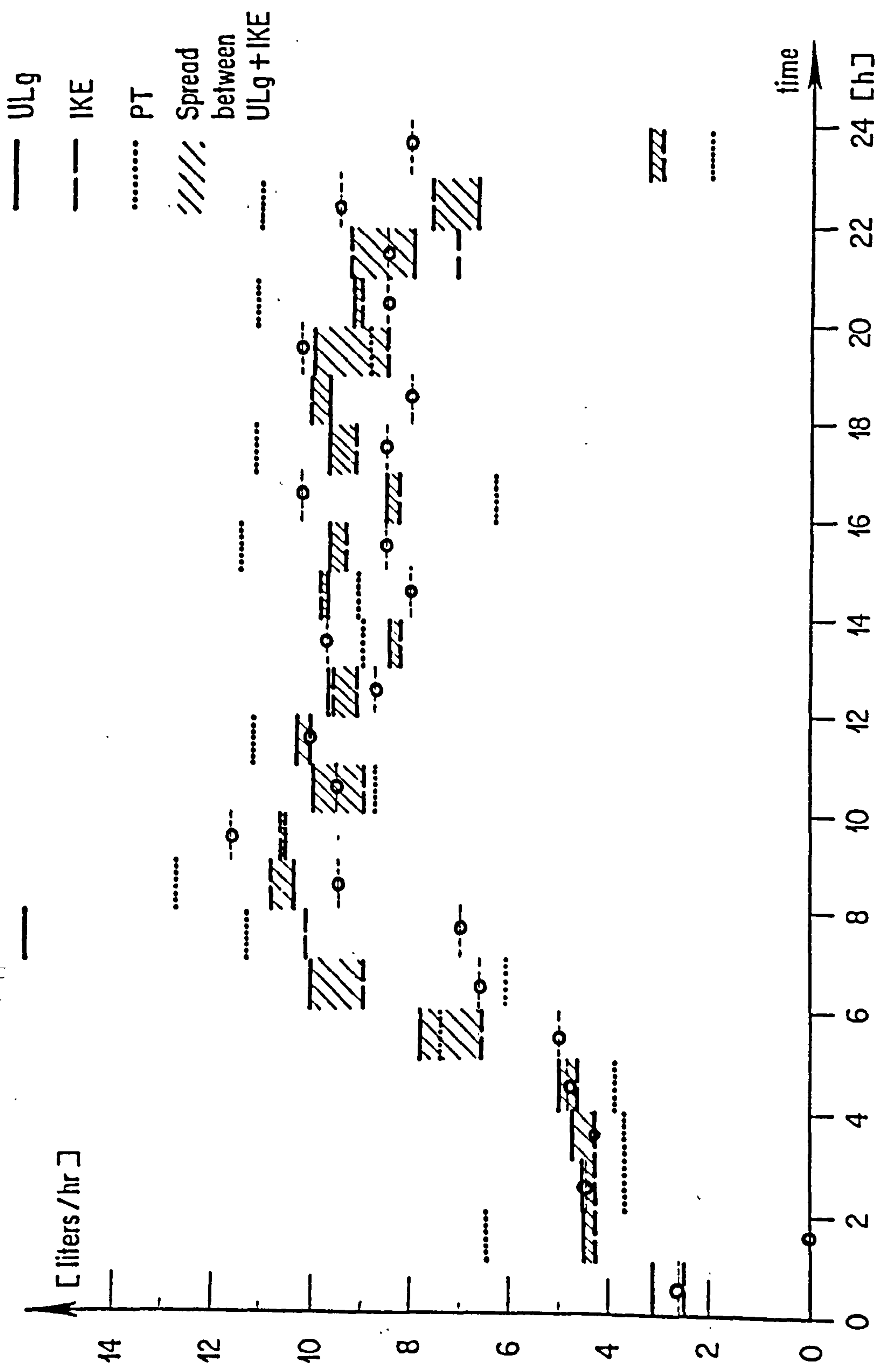


Figure 7.26 Comparison of hourly fuel consumption

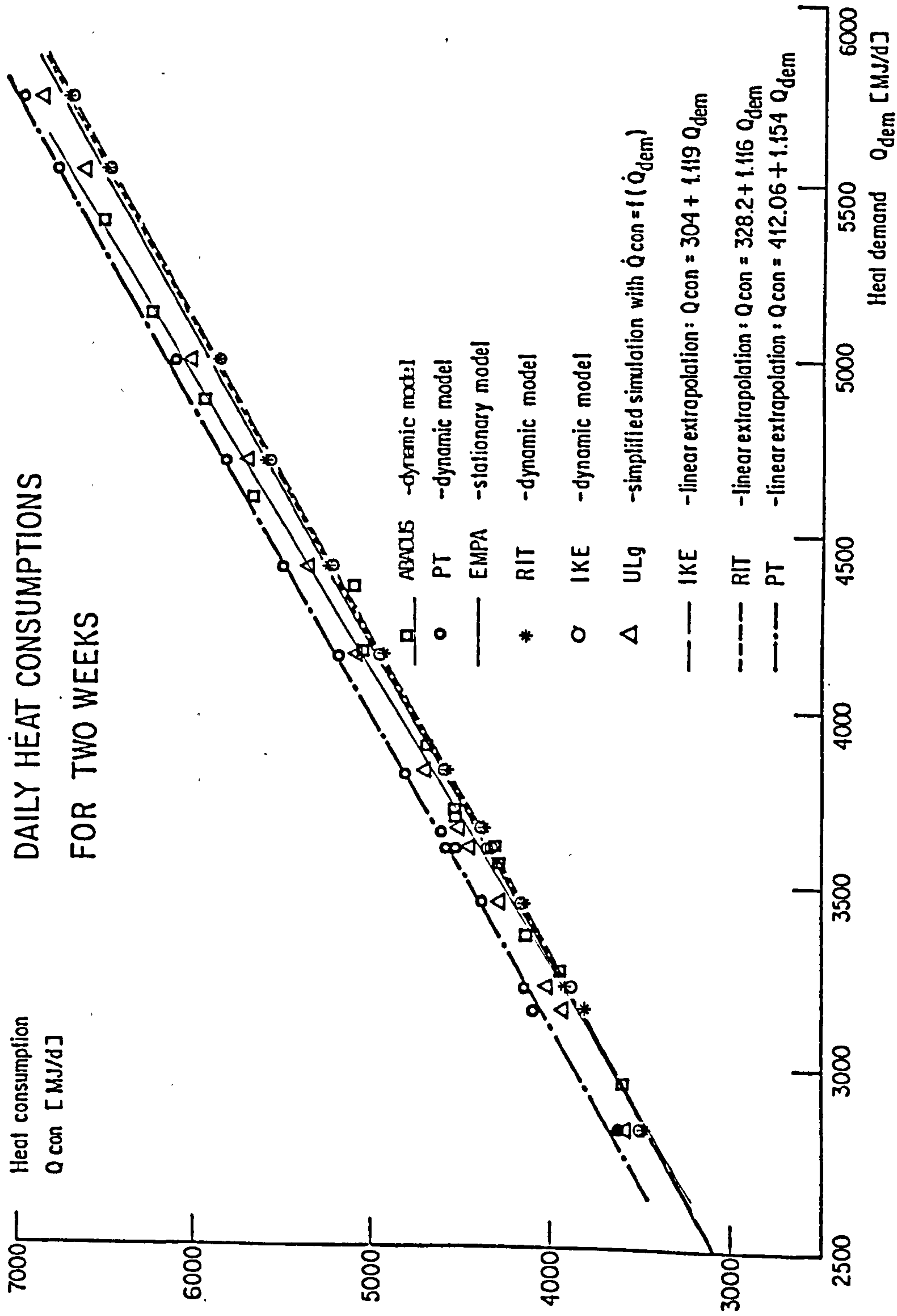


Figure 7.27 Linear regression comparison of fuel consumption as function of heat production

Comparisons of fuel consumptions and losses on December 20th

| December 20 | IKE MJ/d | EMPA MJ/d | Ulg (detailed sim) | | PT MJ/d | RIT MJ/d | Abacus MJ/d |
|--------------------|-------------|--------------|-----------------------|-----------------|------------|-------------|----------------|
| | | | MJ/d | (curve) MJ/d | | | |
| Flue gas losses | 647,1 | 761,88 | 776,22 | 10,87 | | 710 | 10,5 |
| Ventilation losses | 107,4 | 33,4 | 126,75 | 1,77 | | 81 | 1,2 |
| Total chimney | 754,5 | 795,26 | 902,97 | 12,64 | | 791 | 11,7 |
| Surface losses | 236,3 | 243,64 | 274,02 | 3,84 | | 240 | 3,5 |
| Total losses | 990,8 | 1038,9 | 1177 | 16,48 | 1322 | 1031 | 15,2 |
| Heat demand | 5730 | 5733,3 | 5966,15 | | 5733 | 5730 | 5384,2 |
| Heat consumption | 6713 | 6772,18 | 7143,14 | | 7065 | 6776,1 | 6549,0 |
| Prod. efficiency | 85,17 % | 84,66 % | 83,52 % | | 81,14 % | 84,6 % | 82,2 % |
| Burner-on-Time sec | 42400 | 42765 | 45140 | | 44619 | 42840 | 41355,0 |
| Cycles | 82 | - | 72 | | - | 89 | 69 |
| Av. min/cycle | 8,62 | - | 10,45 | | - | 8,02 | - |
| Cycle length min | 17,6 | | 20 | | | 16,2 | - |

Table 7.15 Comparison of fuel consumptions and losses on December 20th

7.5 Exercises of Collins Building

The recent Annex 10 Collins building system simulation consists of a package of six exercises on an air handling plant of the air-conditioning system installed in the Collins building in Glasgow. The given information includes the specification of the system and components; internal and external climatic conditions; air flow rate of the system and the required system operational strategy. Since these are the primary effort to simulate the performance of an air-conditioning system, the simplified models are suggested to use in the simulation. The ABACUS contribution to the Collins building simulation has been carried out by using the implemented ESP energy simulation programme as described in chapter six. The component models in the ESP component library are developed in their simplest form represented by one control volume. The air handling system model then can be formed by the ESP main simulation software according to the specification and the system state equation are solved simultaneously at each time step during the period of simulation. The primary results are given in reference [Ref. 4] of chapter six. Further results are coming soon.

7.6 Other efforts of application

1. Boiler efficiency exercise

The programme "CENSYS" was also used in the extensive studies of boiler operation at different conditions, such as water flow rate, inlet water temperature, environment conditions etc. As a result of this, the simulation results are processed and then used to generate efficiency curves to compare with the reliable results from publications and measurement.

Figure 7.28 shows the part load efficiency of the boiler, derived based on the boiler structure of figure 4.12, 4.13 of cast iron boiler. A comparison with other published results is also shown.

Figure 7.24 gives the relative part load efficiency compared with results from elsewhere. [Ref. 9, 10, 11]

2. Model simplification

It has been found in Annex 10 exercises that a complicated dynamic model can produce vast information to the objective of modelling which the simple steady state model cannot achieve. However, a simple steady state model may produce reasonable results as good as the dynamic model for certain required parameters under certain conditions. In the practical situation of modelling in which only the global parameter, such as the heat production of boiler, is interested, the complicated dynamic model sometimes needs to be simplified if the economical reason is emphasized. One way of approaching is to use component model with less control volume definition so that to reduce the order of the component state equation. However, it has been proved in the previous chapters that the fewer the control volume defined, the higher the spatial discretization error can be introduced. Alternative approach is to use the techniques of so called "system identification". The "system" here can be referred as a complete system or a component model. By using these techniques, the order of the system can be reduced, but

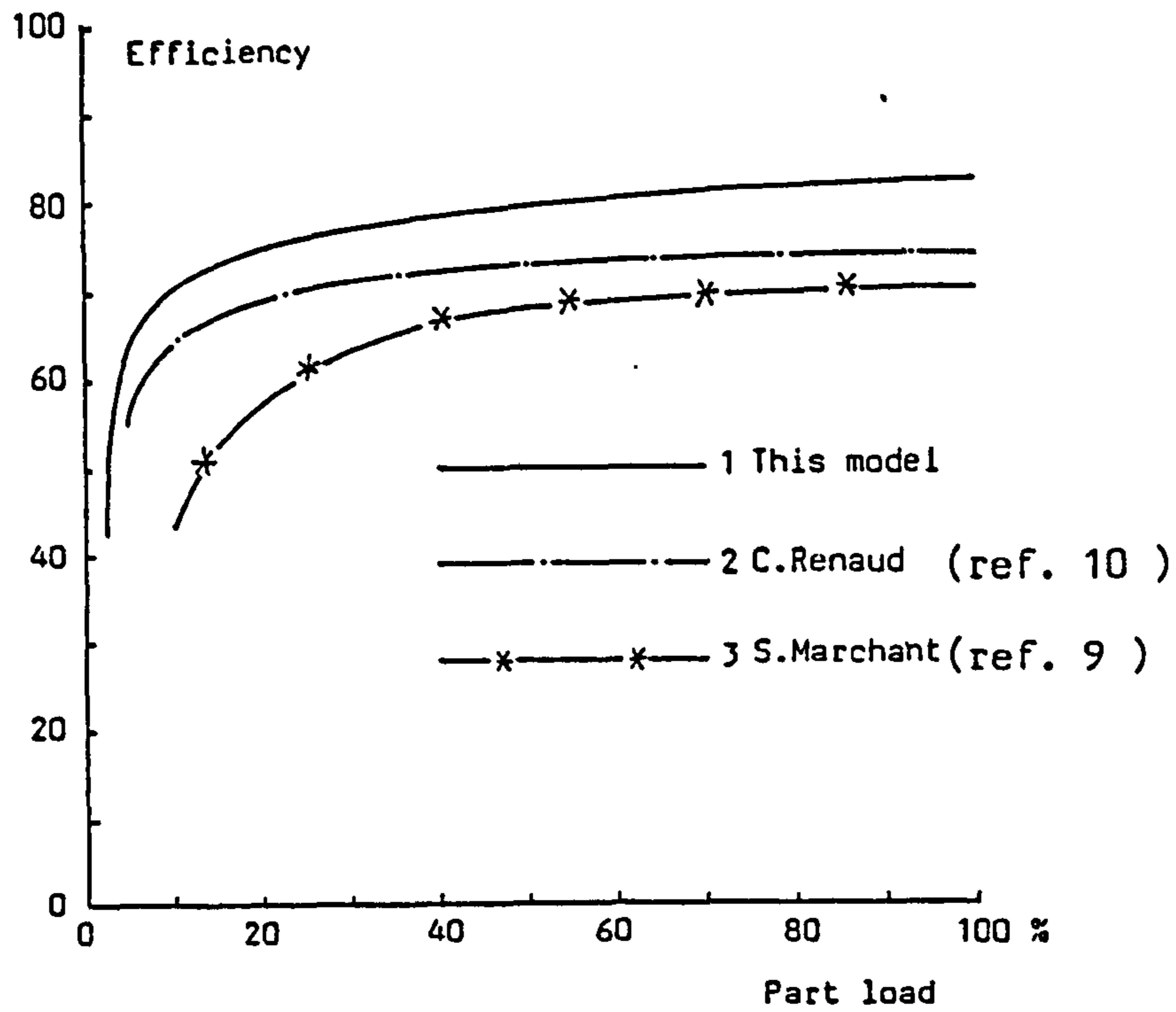


Figure 7.28 Part load efficiency compare to other results

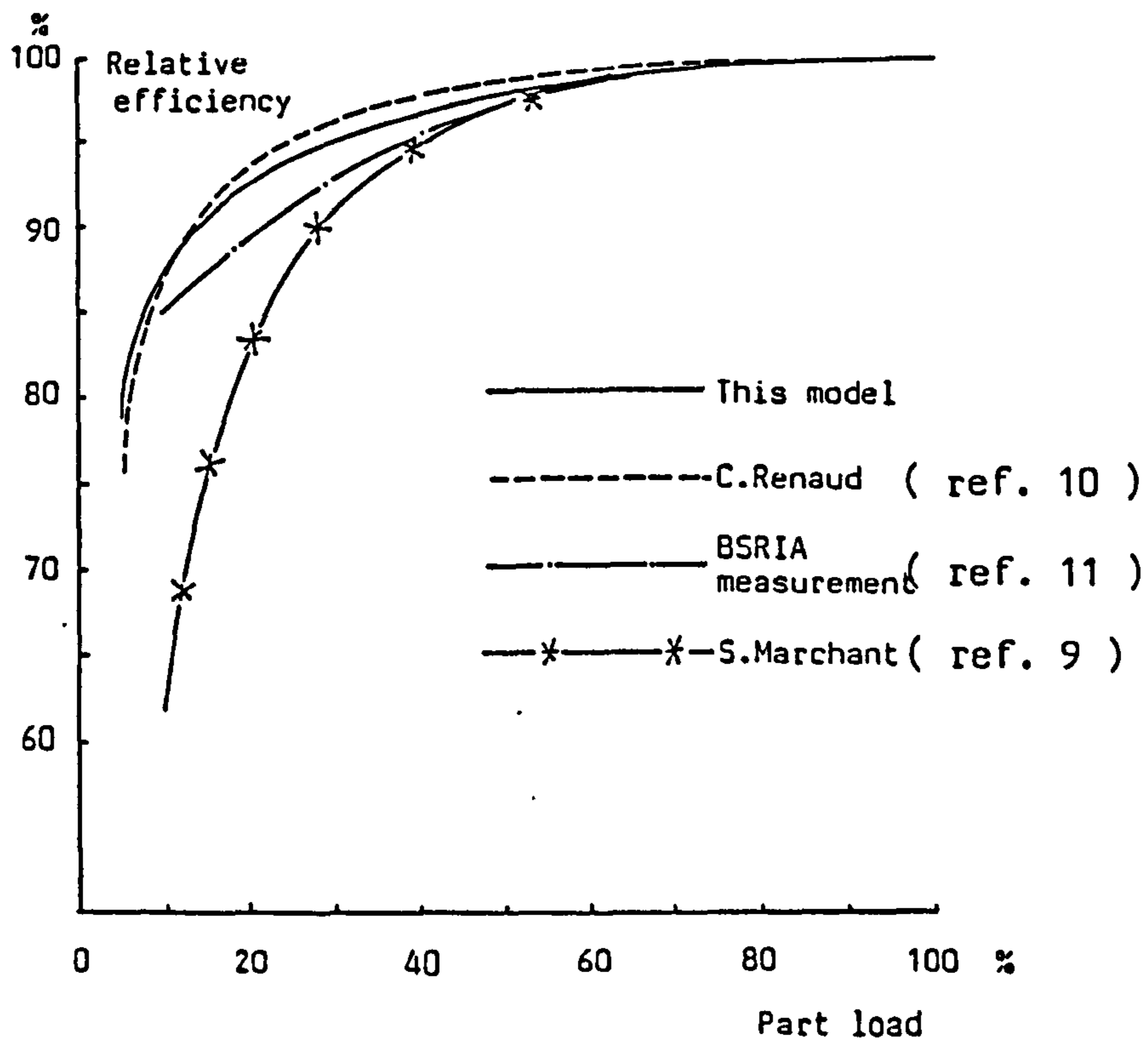


Figure 7.29 Relative part load efficiency compare to other results

the dynamics of the system performance can still be reserved to a required level. Most current identification techniques concern to reduce the problem to one parameter identification. The typical examples are the uses of Bode's diagram and parameter estimation. Both of the methods are concerned to use the results from the higher order dynamic model to determine the parameters of the lower order dynamic model of the same system.

In fact, any component model can be represented by the state equation in general form, as:

$$\dot{T}(t) = AT(t) + BU(t) \quad 7.1$$

in which $T(t)$ is a n dimensional state vector; A is a $n \times n$ dimensional matrix; $U(t)$ is a m dimensional disturbance vector and B is a $n \times m$ dimensional matrix. Matrix B can be re-arranged to a m dimensional row-vector of:

$$B = [B_1, B_2, B_3, \dots, B_m] \quad 7.2$$

in which each B_i is a n dimensional column vector.

Applying Laplace transform to equation 7.1, and re-arrange it, results:

$$\begin{aligned} T(s) &= (sI - A)^{-1} [B_1 U_1(s) + B_2 U_2(s) + \dots + B_m U_m(s)] \\ &= W_1(s)U_1(s) + W_2(s)U_2(s) + \dots + W_m(s)U_m(s) \end{aligned} \quad 7.3$$

where,

$$W_i(s) = (sI - A)^{-1} B_i \quad 7.4$$

$W_i(s)$ is the transfer function of the i -th disturbance acting upon the system. Therefore, the essence of system identification is to use a first or second order approximation to replace the $n \times n$ dimensional matrix in 7.4 and the output variable is chosen as the most interested one in the state vector $T(t)$.

Consider the boiler model developed in chapter four of equation 4.9

and figure 4.10 and 4.11. The most interested variable is taken as the water outlet temperature, the external disturbances to this model are fuel supply, room air temperature and temperature of supply water. Water flow rate can be considered as an implicit disturbance. The transfer function related between each disturbance and the interested output parameter is going to be determined. There are two kinds of state-equation involved in this model: the water flow energy state equation and the solid material state equation. It is well known that the spatial derivative term in water flow energy state equation will cause a transit delay. It has been found, from the simulation of the original boiler model, that the response of water outlet temperature is close to exponential when a step input of one disturbance is acted and the remains are kept constant. Therefore, it seems reasonable to simplified the original model to a first order system. The simplified boiler model can be represented by:

$$T_{out}(s) = W_1(s)U(s) + W_2(s)T_{in}(s) + W_3(s)T_r + W_4(s)\dot{m} \quad 7.5$$

The block diagram of this simplified boiler model is shown in figure 7.30. The coefficients in each transfer function can be determined by least square parameter estimation or curve fitting corresponding to the unit step response of each disturbance of the original model.

finally, after the determination of the coefficients of gain, time constant and transit delay of each transfer function, the simplified model of equation 7.5 in Laplace domain can be transformed back into the time domain and reserve the form as state equation. For higher accuracy, a second order can possibly be used as the simplification to the original model. It is largely dependent on the response of the original model to each of the step input disturbance.

As the conclusion to the validation efforts, the validations of component and system models of HVAC system are complicated and time consuming. It seems impossible to test accurately the validities of all the component models derived in this thesis at a short time. IEA programme of energy conservation research provides the opportunities to access the individual research into an international cooperative network. It can be expected, as the Annex exercises progressing, more

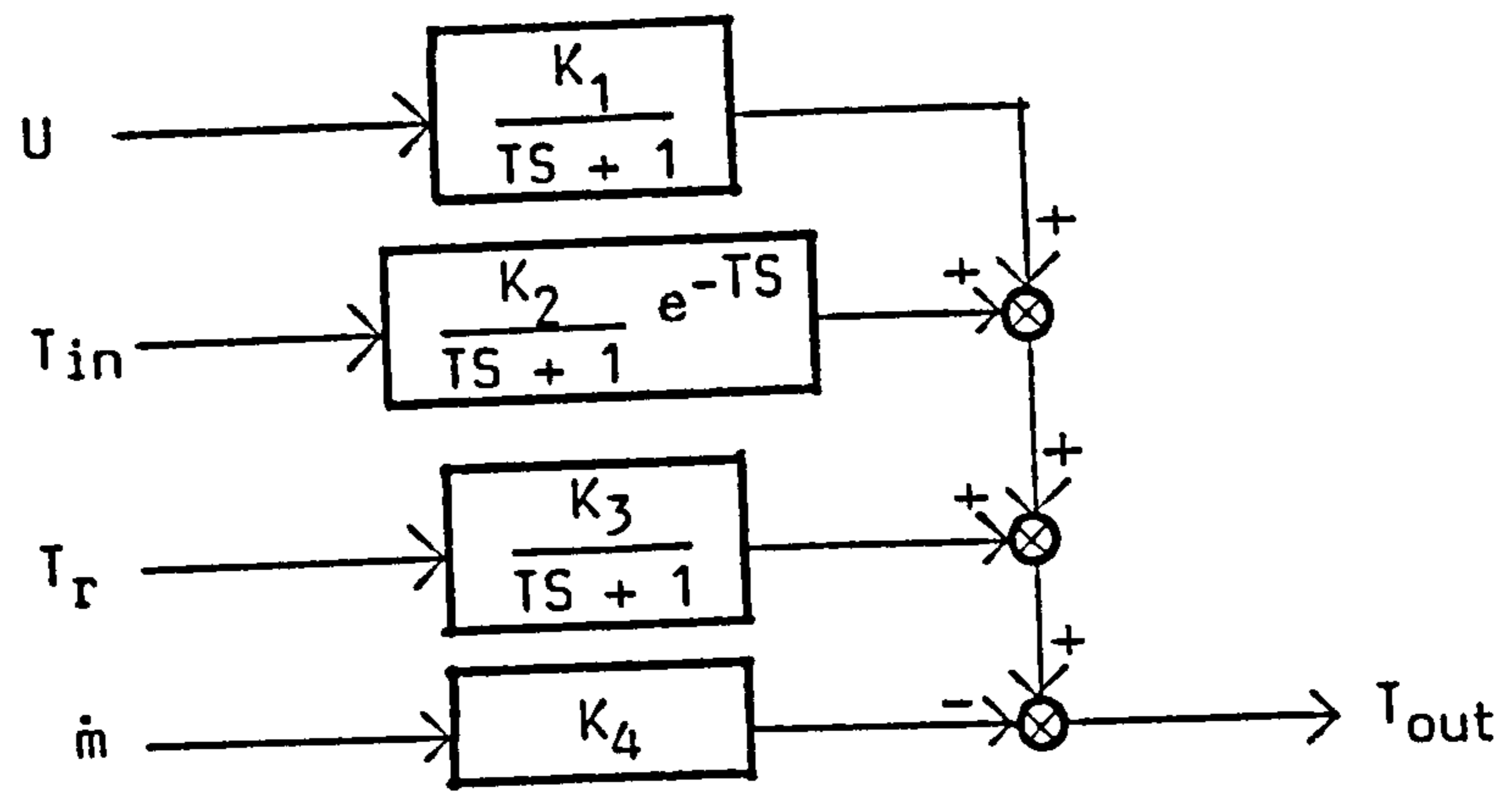


Figure 7.30 The simplified boiler model

and more components and system models can be effectively validated. On the other hand, once the modelling techniques and mathematical methods used in model development have been proved, the knowledge gained from the iterative processes of model validation can be consequently used in the stage of model development. The quality of models, as well as the target of simulations can thus be greatly improved.

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CHAPTER EIGHT: CONCLUSION AND FUTURE WORKS

CHAPTER EIGHT: CONCLUSION AND FUTURE WORKS

The goal of this thesis is to develop a generalised modelling technique for building energy and HVAC system modelling. The modelling of HVAC system and the development of HVAC component models are described in much detail as the applications of this technique. A general review to the existing techniques in the field of building and plant system modelling has been given and based on the historical experiences, a new modelling tool termed the "control volume conservation, state space approach" has been systematically developed and applied to practice. The existence and the applicability of this modelling technique to the particular problems arisen in the developments of building and HVAC equipments and system models have been thoroughly discussed and examined. Solution techniques to the resulted component and system state equations are introduced and the numerical method known as the "Trapezoidal algorithm" has been introduced as a basic tool for solving the particularly featured non-linear component and system state equations. This numerical method has also been discussed in detail so that its accuracy, convergency, stability conditions can be revealed before hand, and therefore, its applicability to the solution of particular problems encountered in the component and system models can be guaranteed. The general formulations of this approach technique to develop the building and HVAC component model have been established, and the practical applications of the formulations to the representative building elements and HVAC equipments have been demonstrated. For the case of system state equation can be simplified to linear time invariant system, the solution technique discussed can also be directly used to yield satisfactory solution and, further more, the existing methods for solving such problem are numerous and much mature. As to examine the validity of the resulted models as well as the modelling technique, a component based prototype computer programme capable of modelling the central heating system has been developed and participated in the IEA exercises for model validation. The component models derived and validated have directly been transferred to the generalised building/plant energy simulation package ESP.

As a conclusion to this project, the developed modelling tool can well cope with the goal arisen in this thesis. Experience has shown that the fundamental theory behind the "control volume conservation state space approach" is universally true and the general formulations derived based on this approach are fully supported by their theoretical background and the numerical verification. The application of this modelling technique has been proved very flexible, easy to handle and capable of the practitioner to generate component models in different level of accuracy; the theoretical error between the levels can also be detected. The only condition to generate a theoretically correct model is to define properly the energy and mass exchange paths of the control volumes within the components concerned. As a result, therefore, this thesis gives not only a number of component and system models but also a factory to produce these models.

Detailed model has higher accuracy and capable of generating more information throughout modelling. For example, a detailed boiler model can be used by the boiler manufacturer to improve its design performance and for optimal design. A detailed model also can be used as the "maternal model" to generate younger generations of models representing the same problem in different levels. However, detailed model requires more information as input parameters and its detailed structural feature will largely restrict its extensive usage. Simplified model is developed with less control volume definition so that it can easily be applied to the same type of components even with structural differences. The simple but generalised model can be found useful in energy simulations where the overall energy consumption is the main interested estimation. It also makes useful for control system designs since the control engineers usually know little about the performances of HVAC components and their main interest is the control loop quality. All the models in different level can be generated by using this presented modelling technique without vital difference.

Generally speaking, as a system modelling tool, the application of this technique and the models could be extensive. Theoretically, the method and its primary results will allow the investigation of new

design features such as control system analysis and the application to design optimal control strategies.

Further applications can be found as that of the model derived based on this modelling technique can be easily formulated, documented and made transferable, which have already been involved in the software exchange programme between ABACUS and LPB of "Data Processors Profoma".

It has to recognise that the modelling technique presented in this thesis is only the trial of searching for better techniques. The development of energy simulation techniques and their practical application is a considerably complicated task, it requires the synthesis knowledge associated with almost all fields of science researches. It is still too early to say which is the best technique or even better than others. People has realised through their experiences that the nature of contemporary research is becoming much complicated and difficult to be carried out and completed by personnel or even single constitutions. Such difficulties and complexities have been found in many countries as: ways of approach, different physical hypothesis and mathematical methods applied to the developments of models and integrated in the field of building and HVAC system simulation. As the need for sophisticated simulation tools has never been so great in history, a number of international commissions have been established in the past few years in order to strengthen the cooperation; fundings are raised for supporting researches in different countries. In Europe, the commission of the European Communities has launched several Research and Development programmes in the field of energy including the Energy modelling and Energy Conservation as the main subjects. The International Energy Agency have promoted a series of energy modelling programmes to bring together the energy modelling institutions in more than twenty countries. Recently, ABACUS has been funded by a number of international and U.K funding bodies to develop sophisticated building energy modelling software. The works which have been developed in this thesis have already linked with the existing building and plant energy simulation package programme ESP in ABACUS and results have been, as part of ABACUS researches, contributed to several cooperation

programmes.

The recent efforts to develop the next generation simulation software based on the concept of state space analysis have been carrying out in ABACUS unit. Current work is concerned with the development of additional user's interface features to the existing ESP building energy modelling system to allow model use in both as research and design context. The projective described in this thesis is part of the general systems simulation developments leading to a generalised system simulation methodology which encompasses the elements as summarised in figure 8.1, which are:

- a central database holding information on the various plant and building items capable of being modelled;
- a symbolic graphics system of user oriented interface capable of representing the components which comprise the plant configuration;
- a formally defined methodology for the interactive definition of the system being proposed for simulation;
- a methodology for establishing simulation equations, in general form, to relate the energy and mass transfer over time and between any particular component and all other components to which it can be legally connected;
- development of "intelligent" matrix processing technique complementary to those which already exist within ESP.

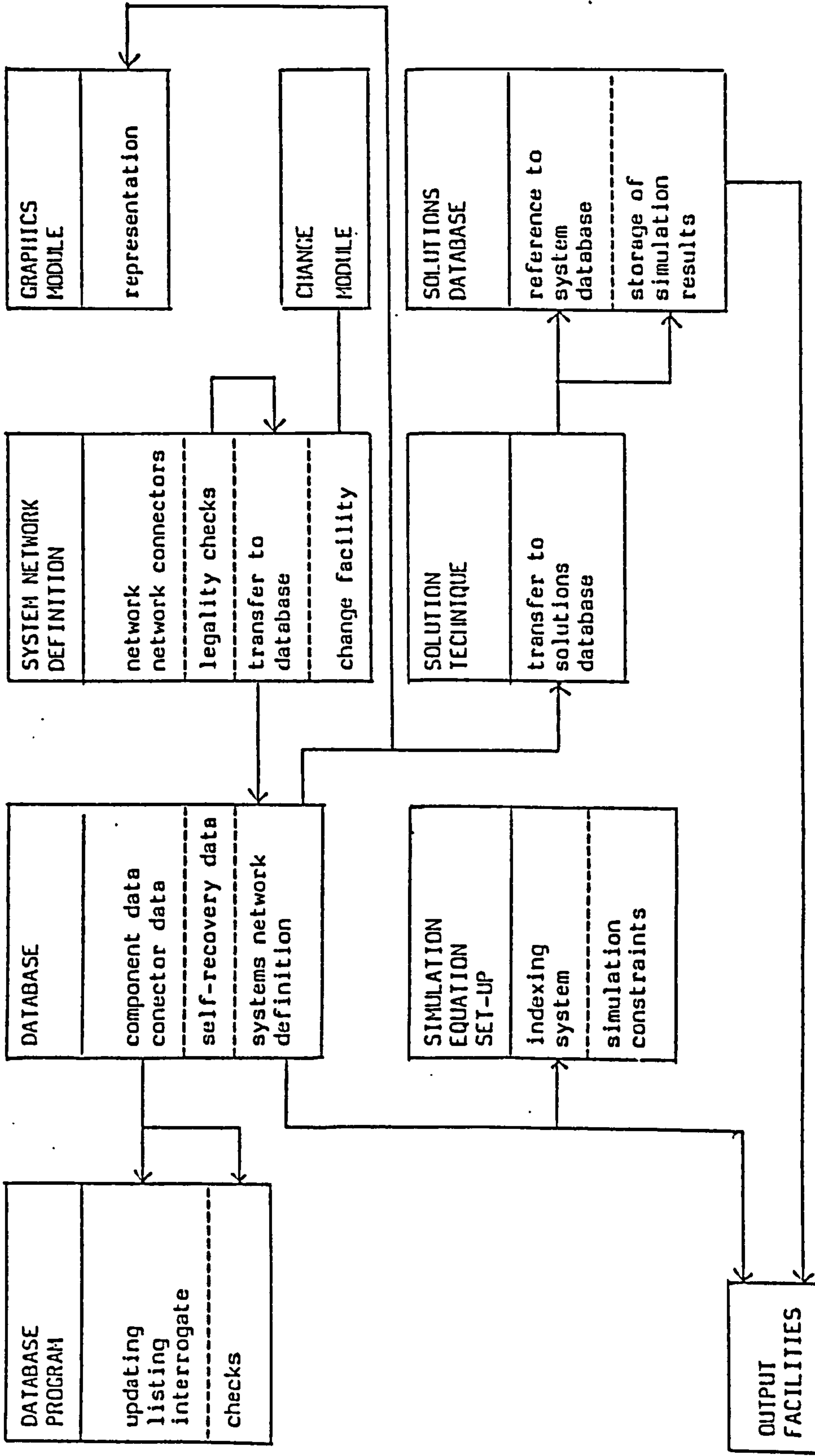


Figure 8.1 The element of building/plant simulation

APPENDICES

- A1. Proof of non-singularity of matrix C.
- A2. Proof of real-negative of component state-equation eigenvalues.
- A3. Semi-discretization errors.
- A4. Heat transfer coefficients.
- A5. Combustion of fuel.
- A6. Component models of heating system.

A1. Proof of non-singularity of matrix C

As derived in chapter three, the component state equation can be represented as:

$$C(t)[dT(t)/dt] = A(t)T(t) + B(t)U(t) \quad A1.1$$

in which

$$\begin{aligned} C(t) &= \text{diag}\{C_i(t)\}, \quad (i = 1, \dots, n); \\ A &= [a_{ij}], \quad (i = 1, \dots, n; j = 1, \dots, n); \\ B &= [b_{ij}]; \quad (i = 1, \dots, n; j = 1, \dots,); \text{ and} \\ T &= [T_1, \dots, T_n]^T; \quad U = [U_1, \dots, U_n]^T. \end{aligned}$$

In diagonal matrix C, $C_i (i = 1, \dots, n)$ are thermal capacity of each control volume in a component. In reality, $C_i (i = 1, \dots, n)$ is always positive and non-zero, so that the component thermal capacity matrix is diagonal and positive definite. However, for some control volumes in a component, c_i can be very small and so the values of (a_{ii} / c_i) become very larger. This means that the time constants of such control volume are comparatively greater than others, so that their thermal storage effects can be neglected. It is reasonable to treat the thermal capacities of such control volume as zero.

Assume that in a n nodes component model, there are l control volumes having zero thermal capacity and the rest $m = (n-l)$ are non-zero, the component state equation can be so re-arranged that the non-zero and zero thermal capacity control volumes are separated. The component state equation A1.1 becomes

$$\begin{vmatrix} C_m & 0 \\ 0 & C_l \end{vmatrix} \begin{vmatrix} \dot{T}_m \\ \dot{T}_l \end{vmatrix} = \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} \begin{vmatrix} T_m \\ T_l \end{vmatrix} + \begin{vmatrix} B_m & 0 \\ 0 & B_l \end{vmatrix} \begin{vmatrix} U_m \\ U_l \end{vmatrix} \quad A1.2$$

in which

$$C_m \neq 0, \quad C_l = 0.$$

For plant modelling problems, refer to the general control volume conservation equation:

$$\rho c v \frac{dT_I}{dt} = K_{Ii} [T_i(t) - T_I(t)] + q_I(t) \quad A1.3$$

It can be found that the diagonal of matrix A consisted of negative coefficients and the rest other than the diagonal are positive. It also satisfies:

$$\sum_{j \neq i}^{i,n} |a_{ij}| \leq |a_{ii}|, \quad (i=1, \dots, n) \quad A1.4$$

For Sub-matrix A_{11} , coefficients on the diagonal represent the thermal characteristics of each control volume. Coefficients at two sides of diagonal represent the inter-relationships of control volumes having non-zero thermal capacity.

Sub-matrices A_{12} and A_{21} represent the inter-relationship of non-zero and zero thermal capacity control volumes.

Sub-matrix A_{22} gives thermal characteristics of zero thermal capacity control volumes. Sub-matrix A_{22} is a negative and diagonal matrix since there is no thermal contact between two control volumes having zero-thermal capacity. In fact, any number of zero thermal capacity control volumes having thermal contact can always be treated as one control volume without losing modelling accuracy. Generally, the physical existence ensures that: (1) a control volume with non-zero thermal capacity only can have once contact to zero thermal capacity control volumes, (2) a zero thermal capacity control volume never can have contact to other zero thermal capacity control volumes, (3) a zero thermal capacity control volume can have more than one thermal contact to non-zero thermal capacity control volumes.

Since A_{22} is non-singular, the inversion A_{22}^{-1} exists, so that equation A1.2 can be modified as:

$$\begin{bmatrix} C_m & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{T}_m \\ \dot{T}_l \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{22}^{-1} A_{21} & I \end{bmatrix} \begin{bmatrix} T_m \\ T_l \end{bmatrix} + \begin{bmatrix} B_m & 0 \\ 0 & A_{22}^{-1} B_l \end{bmatrix} \begin{bmatrix} U_m \\ U_l \end{bmatrix}$$

State vector T_l can be represented by the linear combination of the rest state variables in T_m , so that:

$$C_m \dot{T}_m = (A_{11} - A_{12} A_{22}^{-1} A_{21}) T_m + (B_m U_m - A_{22}^{-1} B_l U_l)$$

or

$$C_m [dT_m/dt] = D_m T_m + \phi_m \quad A1.6$$

where,

$$D_m = (A_{11} - A_{12} A_{22}^{-1} A_{21}) \quad A1.7$$

D is a $m \times m$ dimensional matrix describing the thermal characteristics of the component. It can be further proved that matrix D is non-singular.

Therefore, a n control volumes component with zero thermal capacity control volumes can be represented by $m = n - l$ nodal state equations in which the component thermal capacity matrix C is non-singular.

Now consider matrix A.

Since matrix A_{22} is negative and diagonal and matrices A_{21} , A_{22} satisfy condition of equation A1.4,

let,

$$E = A_{22}^{-1} A_{21} = [e_{ij}],$$

$$-1 < e_{ij} < 0 \quad (i=1, \dots, m; j=1, \dots, l)$$

According to equation A1.4, it satisfies:

$$\sum_{j=1}^m |e_{ij}| \leq 1 \quad (i=1, \dots, l) \quad A1.8$$

Let,

$$F = A_{12} = [f_{ij}], \quad (i=1, \dots, m; j=1, \dots, l),$$

The product of F and E becomes:

$$FE = A_{12} A_{22}^{-1} A_{21} = G = [g_{ij}]$$

where,

$$g_{ij} = \sum_{k=1}^l f_{ik} e_{kj} < 0 \quad (i=1, \dots, m; j=1, \dots, m)$$

Since,

$$-1 < e_{ij} \leq 0 \quad \text{and} \quad \sum_{j=1}^m |e_{ij}| \leq 1 \quad (i=1, \dots, l)$$

It satisfies that g ($i=1, \dots, m; j=1, \dots, m$) are negative, a

$$\sum_{j=1}^m |g_{ij}| < \sum_{j=1}^l f_{ij}$$

For matrix $D_m = (A_{11} - A_{12}A_{22}^{-1}A_{21}) = (A_{11} - G)$, it can be easily found that that condition of equation A1.4 can still be satisfied. However, coefficients on the diagonal of D_m become smaller than in A_{11} .

A2. Proof of real-negative of component state-equation eigenvalues

Generally, a n-nodes component state equation can be represented as:

$$C[dT/dt] = AT + BU \quad A2.1$$

Since C is diagonal matrix and non-singular, C^{-1} exists, so that:

$$dT/dt = C^{-1} AT + C^{-1} BU = FT + GU \quad A2.2$$

The eigenvalues of matrix F will give the dynamical characteristic of this component.

According to the general control volume conservation equation, the i-th nodal equation of A2.2 is:

$$\frac{dT_i}{dt} = \sum_{j \neq i}^{1,n} \left(\frac{K_{ij}}{\rho_i C_i V_i} \right) [T_j(t) - T_i(t)] + \left(\frac{1}{\rho_i C_i V_i} \right) q_i(t) \quad A2.3$$

It can be easily found that in matrix $F = [f_{ij}]$, ($i = 1, \dots, n$; $j = 1, \dots, n$),

$$f_{ij} = \begin{cases} \left(\frac{K_{ij}}{\rho_i C_i V_i} \right) & i = 1, \dots, n; j = 1, \dots, n; j \neq i \\ -\sum_{j \neq i}^{1,n} \left(\frac{K_{ij}}{\rho_i C_i V_i} \right) & j = i \end{cases} \quad A2.4$$

In component state equation, variables of other component with connection to it are treated as disturbances, so that coefficients of matrix F satisfy:

$$f_{ii} < 0 \quad i = 1, \dots, n \quad A2.5$$

$$\sum_{j \neq i}^{1,n} f_{ij} = \sum_{j \neq i}^{1,n} |f_{ij}| \leq |f_{ii}| \quad i = 1, \dots, n \quad A2.6$$

According to Gerschgorin theorem, eigenvalues of matrix F lie in the union of the close circular discs g_i in the complex plane with centers f_{ii} and radii of:

$$\rho_i = \sum_{j \neq i}^{1,n} |f_{ij}| \quad A2.7$$

each circular disc in complex plane is defined by:

$$|z - f_{ii}| \leq \rho_i \quad \text{A2.8}$$

For each eigenvalue λ_i of F , there exists g_i in complex plane, such that $\lambda_i \in g_i$, ($i = 1, \dots, n$).

Since the coefficients f_{ii} , ($i = 1, \dots, n$) in F are non-zero, real and negative and they satisfy A2.6, the union of n Gerschörm circles of matrix F are lain at the left half of complex plane. Therefore, eigenvalues of matrix F all have negative-real part. However, according to A2.5 and A2.6, zero eigenvalues might exist.

Three kinds of basic control volume conservation equations have been derived in chapter three, any component in building/plant system can be modelled by one or the mix of two or three kinds equation.

It is useful to observe the components containing homogeneous basic equation.

1. Multi-layer wall

A multi-layer wall contains nodal equation of solid material. State equation of this component in one dimensional heat flow can be given as a standard form of:

$$C[dT/dt] = AT + BU \quad \text{A2.9}$$

The i -th nodal equation of A2.9 is:

$$C_i [dT_i/dt] = \gamma_{i-1} T_{i-1} - (\gamma_{i-1} + \gamma_i) T_i + T_{i+1} \gamma_i \quad \text{A2.10}$$

in which,

$$\lambda_{2,i} \leq \left| \frac{r_{i-1}}{c_i} \right| + \left| \frac{r_i}{c_i} \right| = \left(\frac{r_{i-1} + r_i}{c_i} \right) \quad i = 1, \dots, n-1 \quad \text{A2.15}$$

Assuming $X = (x_1, x_2, \dots, x_n)^T$ is an eigenvector corresponding to the eigenvalue $\lambda_{2,i}$, the eigenvalue problem of F_2 becomes the difference equation of:

$$\begin{cases} x_{j-1} + x_{j+1} = \left(\frac{r_{i-1} + r_i}{c_i} \right) \cos \theta x_j \\ x_0 = x_n = 0 \end{cases} \quad j = 1, \dots, n-1 \quad \text{A2.16}$$

It can be found by solving A2.16 that $\theta = i\pi/n$ ($i = 1, \dots, n-1$)

Therefore, eigenvalues of F_2 are:

$$\lambda_{2,i} = \left(\frac{r_{i-1} + r_i}{c_i} \right) \cos \frac{i\pi}{n} \quad (i = 1, \dots, n-1) \quad \text{A2.17}$$

Eigenvalues of matrix F are:

$$\lambda_i = \lambda_{1,i} + \lambda_{2,i} = - \left(\frac{r_{i-1} + r_i}{c_i} \right) \left(1 - \cos \frac{i\pi}{n} \right) \quad \text{A2.18}$$

Since, $-1 \leq \cos(i\pi/n) < 1$ ($i = 1, \dots, n$)

It is clear that eigenvalues of F are non-zero, real and negative.

2. Tube fluid flow

One dimensional tube flow is a combination of three kinds of basic conservation equation. Assume the thermal storage effects of tube and insulation can be neglected, it will become a pure flow problem described by fluid flow basic equation. This also can be considered as a simplification of a plant system in which each component is simplified as cascade flow represented by basic fluid flow equation.

A n nodes homogeneous cascade flow component can be represented by the state equation of standard form as:

$$C(dT/dt) = AT + BU \quad A2.19$$

The i -th nodal equation is:

$$\rho_i c_i v_i \frac{dT_i}{dt} = \dot{m}_i c_i T_{i-1} - (\dot{m}_i c_i + K_i A_i) T_i + K_i A_i T_a \quad A2.20$$

Let

$$F = C^{-1} A = F_1 + F_2 \quad A2.21$$

where,

$$F_1 = \text{diag} \left\{ -\frac{(\dot{m}_i c_i + K_i A_i)}{\rho_i c_i v_i} \right\} \quad (i = 1, \dots, n) \quad A2.22$$

$$F_2 = \text{diag} \left\{ \frac{\dot{m}_i c_i}{\rho_i c_i v_i}, 0, 0 \right\} \quad (i = 1, \dots, n) \quad A2.23$$

Eigenvalues of F_1 can be easily found as:

$$\lambda_{1,i} = -\frac{\dot{m}_i c_i + K_i A_i}{\rho_i c_i v_i} \quad (i = 1, \dots, n) \quad A2.24$$

Since matrix F_2 only has zero eigenvalues, eigenvalues of matrix F are:

$$\lambda_i = \lambda_{1,i} = -\frac{(\dot{m}_i c_i + K_i A_i)}{C_i} \quad (i = 1, \dots, n) \quad A2.25$$

It is clear that eigenvalues of such component state equation are non-zero, real and negative.

For a complete system, the eigenvalue problem is much complicated. Since most plant systems are closed loop, the connection of all component are complicated, no general expression can be given, special system has to be considered. However, from the discussion above, it can at least be found the eigenvalues of a closed loop complete system are real, negative and zero.

Consider a simplified plant system of closed loop cascade flow problem, the system state equation has the standard form, system matrix $F = C^{-1} A$ is given as:

$$F = \begin{vmatrix} -\frac{(\dot{m}_1 c_1 + K_1 A_1)}{\rho_1 c_1 v_1} & 0 & \dots & 0 & \frac{\dot{m}_1 c_1}{\rho_1 c_1 v_1} \\ \frac{\dot{m}_2 c_2}{\rho_2 c_2 v_2} & -\frac{(\dot{m}_2 c_2 + K_2 A_2)}{\rho_2 c_2 v_2} & & & \\ & - & - & - & \\ & & & & \frac{\dot{m}_n c_n}{\rho_n c_n v_n} & -\frac{(\dot{m}_n c_n + K_n A_n)}{\rho_n c_n v_n} \end{vmatrix} \quad \text{A2.26}$$

Eigenvalues of matrix F are

$$\lambda_i = - \left[\left(\frac{\dot{m}_i c_i}{\rho_i c_i v_i} \right) - \left(\prod_{j=1}^n \frac{\dot{m}_j c_j}{\rho_j c_j v_j} \right)^{\frac{1}{n}} \right] - \left(\frac{K_i A_i}{\rho_i c_i v_i} \right) \quad i = 1, \dots, n \quad \text{A2.27}$$

The absolute values of $\lambda_{2,i} = \left(\prod_{j=1}^n \frac{\dot{m}_j c_j}{\rho_j c_j v_j} \right)^{\frac{1}{n}}$ are close to the mean value of all $\frac{\dot{m}_i c_i}{\rho_i c_i v_i}$. However, $\lambda_{2,i}$ can be positive, negative, real and complex. System eigenvalues $\lambda_i (i = 1, \dots, n)$ can possibly have real negative part.

A3. Semi-discretization and error estimation

Assuming $T(t,x)$ is continuous function and differentiable about variable x , it can be expanded by Taylor series:

$$T(t, x + \Delta x) = T(t, x) + \Delta x \left(\frac{\partial T}{\partial x} \right) + \frac{1}{2!} \Delta x^2 \left(\frac{\partial^2 T}{\partial x^2} \right) - \frac{1}{3!} \Delta x^3 \left(\frac{\partial^3 T}{\partial x^3} \right) + \dots \quad A3.1$$

and

$$T(t, x - \Delta x) = T(t, x) - \Delta x \left(\frac{\partial T}{\partial x} \right) + \frac{1}{2!} \Delta x^2 \left(\frac{\partial^2 T}{\partial x^2} \right) - \frac{1}{3!} \Delta x^3 \left(\frac{\partial^3 T}{\partial x^3} \right) + \dots \quad A3.2$$

Consider one dimensional Fourier heat conduction equation with heat generation and constant coefficient.

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} + q_p \quad A3.3$$

the second-order spatial derivative can be represented by difference formula:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\Delta x^2} [T(t, x - \Delta x) - 2T(t, x) + T(t, x + \Delta x)] + R'_p \quad A3.4$$

where,

$$R'_p = \frac{2}{4!} \Delta x^2 \left(\frac{\partial^4 T}{\partial x^4} \right) + \frac{2}{6!} \Delta x^4 \left(\frac{\partial^6 T}{\partial x^6} \right) + \dots \quad A3.5$$

R'_p is the local truncation error of difference formula, the principal part of R'_p is:

$$R_p \leq \frac{1}{12} \Delta x^2 \left(\frac{\partial^4 T}{\partial x^4} \right) \quad A3.6$$

Equation A3.3 can be re-arranged as:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a} \frac{\partial T}{\partial t} - \frac{1}{a} q_p \quad A3.7$$

and so that,

$$\begin{aligned} \frac{\partial^4 T}{\partial x^4} &= \frac{\partial^2}{\partial x^2} \left(\frac{1}{a} \frac{\partial T}{\partial t} \right) - \frac{\partial^2}{\partial x^2} \left(\frac{1}{a} q_p \right) \\ &= \frac{1}{a^2} \frac{\partial^2 T}{\partial t^2} - \frac{1}{a} \frac{\partial^2 q_p}{\partial x^2} \end{aligned} \quad A3.8$$

Heat generation q_p usually is a function of time only, so that $\frac{\partial^2 q}{\partial x^2} = 0$.
Substitute A3.8 to A3.6, yields:

$$R_p \leq \frac{\Delta x^2}{12a^2} \left(\frac{\partial^2 T}{\partial t^2} \right) \quad A3.9$$

If the accuracy of semi-discretization is required as δ_p , so that R_p satisfies $R_p \leq \delta_p$, it follows that the spatial step Δx has to be:

$$\Delta x \leq \sqrt{12} a \delta_p \left(\frac{\partial^2 T}{\partial t^2} \right)^{-\frac{1}{2}} \quad A3.10$$

Criteria of spatial steps for two or three dimensional Fourier heat condition equation can be similarly determined by following the above derivation. However, it will become much difficult if the coefficient a is considered as spatial function.

For one-dimensional energy equation with constant coefficient and heat generation:

$$\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} + q_h \quad A3.11$$

Difference formula for the first-order derivative is:

$$-\frac{\partial T}{\partial x} = -\frac{1}{\Delta x} [T(t, x) - T(t, x - \Delta x)] + R'_h \quad A3.12$$

where,

$$R'_h = \frac{1}{2!} \Delta x \left(\frac{\partial^2 T}{\partial x^2} \right) - \frac{1}{3!} \Delta x^2 \left(\frac{\partial^3 T}{\partial x^3} \right) + \dots \quad A3.13$$

The principal part of local truncation error is:

$$R_h \leq \frac{1}{2!} \Delta x \left(\frac{\partial^2 T}{\partial x^2} \right) \quad A3.14$$

Re-arrange A3.11 to:

$$-\frac{\partial T}{\partial x} = \frac{1}{u} \frac{\partial T}{\partial t} - \frac{1}{u} q_h \quad A3.15$$

Assume q_h is only a function of time, substitute A3.15 to A3.14 yields:

$$R_h \leq \frac{\Delta x}{2u^2} \left(\frac{\partial^2 T}{\partial t^2} \right) \quad \text{A3.16}$$

For pre-specified accuracy δ_h , spatial step Δx has to satisfy the condition of:

$$\Delta x \leq 2u^2 \delta_h \left(\frac{\partial^2 T}{\partial t^2} \right)^{-1} \quad \text{A3.17}$$

A4. Determination of heat transfer coefficients

1. Radiative heat transfer coefficients

The general form of radiative heat transfer coefficient can be written as:

$$h_r = \sigma_0 F_e F_a (T_1 + T_2)(T_1^2 + T_2^2) \quad \text{A4.1}$$

where,

σ_0 = Stefan-Boltzmann constant

F_e = emissivity coefficient

F_a = geometric coefficient

T_1, T_2 = temperatures of radiative heat exchange objects

a) Uniform room surface temperature

In most cases, surface area of component installed in the room is much smaller than the room internal surface area, component can be considered as a convex object completely enclosed by a very large concave surface with uniform temperature. In such case, $F_e = e$, $F_a = 1.0$, radiative heat transfer coefficient becomes:

$$h = \sigma_0 e_s (T_s + T_R)(T_s^2 + T_R^2) \quad \text{A4.2}$$

where,

e_s = emissivity of component surface

T_s, T_R = surface temperature of component and room surface respectively.

b) Room internal surface with different temperature

In some cases, the distribution of heat emitted by the component to each room surface has to be considered. In such cases, an equivalent uniform room surface temperature is introduced as:

$$T_e = \left(\sum_{i=1}^n f_{s-i} T_{R,i}^4 \right)^{\frac{1}{4}} \quad \text{A4.3}$$

where,

f_{s-i} = view factor between component surface to i-th room surface

$T_{R,i}$ = temperature of room i-th surface

For radiator, since it is usually installed close to one side of the room surfaces, the view factor of the radiator back surface to the wall it faced becomes unit. Since the front and back surface areas of radiator are equal, according to principle of closure of radiation, the equivalent uniform room surface temperature becomes:

$$\begin{aligned} T_e^\dagger &= \left(\frac{1}{2} T_{R,n}^\dagger + \frac{1}{2} \sum_{i=1}^{n-1} f_{s-i} T_{R,i}^\dagger \right) \\ &= \frac{1}{2} \sum_{i=1}^n f_{s-i} T_{R,i}^\dagger \quad (f_{s-n} = 1) \end{aligned} \quad A4.4$$

The total radiative heat exchange between radiator and n room surfaces can be computed by:

$$Q = \sigma_0 \epsilon_s \epsilon_R (T_s^\dagger - T_e^\dagger) A_{Tot} \quad A4.5$$

Radiative heat transfer coefficient becomes:

$$h_r = \sigma_0 \epsilon_s \epsilon_R (T_s + T_e)(T_s^2 + T_e^2) \quad A4.6$$

where,

ϵ_R = emissivity of room surface

c) Radiative heat transfer coefficient of combustion products

Radiative heat exchange between the combustion products and the water container in the combustion chamber is considered as the radiative heat exchange between two parallel surface. The radiative heat transfer coefficient can be represented by:

$$h_r = \frac{\sigma_0 (T_G + T_s)(T_G^2 + T_s^2)}{\frac{1}{\epsilon_G} + \frac{1}{\epsilon_s} + 1} \quad A4.7$$

where,

T_G, T_s = temperature of combustion products and water vessel respectively

ϵ_G, ϵ_s = emissivity of combustion products and water vessel respectively

For the boiler using fuel of gas or oil, the combustion process consists of the illuminate and the non-illuminate parts which is:

$$\epsilon_G = m \epsilon_{G1} + (1 - m) \epsilon_{G2} \quad A4.8$$

where,

$\epsilon_{G1}, \epsilon_{G2}$ = emissivities of illuminate and non-illuminate combustion products respectively

m = coefficient depended on the thermal load of combustion chamber. $0 \leq m \leq 1$

For the illuminate part:

$$\epsilon_{G1} = 1 - e^{-(K_q r_\epsilon + K_{th}) \rho s} \quad A4.9$$

For the non-illuminate part:

$$\epsilon_{G2} = 1 - e^{-K_q r_\epsilon \rho s} \quad A4.10$$

where,

r_ϵ = volumetric percentage of triatomic gas

ρ = absolute pressure of combustion

s = effective thickness of radiative layer of combustion chamber

K_q = absorption coefficient of triatomic gas

K_{th} = absorption coefficient of carbon particles in combustion

Empirical relations are used to compute K_q and K_{th} , which have been compared to data given by publications.

$$K_q = \left[\frac{0.78 + 1.6 r_{H_2O}}{\sqrt{P_\epsilon s}} - 0.1 \right] \left(1 - 0.37 \frac{T_G}{1000} \right) \quad A4.11$$

$$K_{th} = 0.03 (2 - \alpha_\lambda) \left(1.6 \frac{T_G}{1000} - 0.5 \right) \frac{C}{H} \quad A4.12$$

where,

γ_{H_2O} = volumetric percentage of H₂O in combustion products

P_{Σ} = partial pressure of triatomic gas

α_L = percentage of excess air in combustion, $\alpha_L < 2$

$C_{C,H}$ = percentage composition of carbon and hydrogen in fuel

2. Convective heat transfer coefficients

Theoretical and empirical relations are used to determine the convective heat transfer coefficients in the heating system modelling.

a) Forced convective heat transfer coefficients:

$$h_w = \begin{cases} 3.66 \frac{k}{d_I} & 0 < Re \leq 2300 \\ 0.023 \frac{k}{d_I} Re_d^{0.8} Pr^{0.3} & Re > 2300 \end{cases} \quad A4.13$$

where,

k = conductivity of fluid

d_I = internal diameter of tube

Re = Raynolds number

Pr = Prandtl number of fluid

Equation can be applied to forced fluid flow in tube. For non-circular tube or other flow passages, an equivalent diameter has to be used.

b) Natural convective heat transfer coefficients

For natural convection on horizontal cylindrical surface:

$$h_w = \begin{cases} \frac{k}{d_o} \lg^{-1} \left\{ 0.01 \lg^2 (Gr Pr) + \lg [1.07 (Gr Pr)^{0.13}] \right\} & 10^4 < Gr Pr \leq 10^4 \\ 0.53 \frac{k}{d_o} (Gr Pr)^{1/4} & 10^4 < Gr Pr \leq 10^9 \\ 0.13 \frac{k}{d_o} (Gr Pr)^{1/3} & 10^9 < Gr Pr \leq 10^{12} \end{cases}$$

where,

d_o = external diameter of cylinder

Gr = Glashof number of fluid. $Gr = g\beta(T_s - T_a) d_o^3 / \nu^2$

All thermal properties are evaluated at local film temperature.

For natural convection over vertical plate

$$h_w = \begin{cases} \frac{k}{L} \lg^{-1} \left\{ 0.014 \lg^2 (Gr Pr) + \lg [1.45 (Gr Pr)^{0.1}] \right\} & 10^{-1} < Gr Pr \leq 10^4 \\ 0.59 \frac{k}{L} (Gr Pr)^{1/4} & 10^4 < Gr Pr \leq 10^9 \\ 0.13 \frac{k}{L} (Gr Pr)^{1/3} & \end{cases}$$

where,

L = characteristics dimension, i.e. height of the vertical plate

For natural convection over horizontal flat surface

$$h_w = \begin{cases} 0.54 \frac{k}{L} (Gr Pr)^{1/4} & 10^5 < Gr Pr \leq 2 \times 10^7 \\ 0.14 \frac{k}{L} (Gr Pr)^{1/3} & 2 \times 10^7 < Gr Pr \leq 3 \times 10^{10} \end{cases} \quad A4.15$$

L = characteristics dimension of horizontal surface

c) Forced convective sub-cooled boiling

The heat transfer process taking place in boiler water is, in general, considered as no boiling happened. However boiling may be happened at some points of the heated surface. When the temperature of water is below the saturation temperature, vapor will be absorbed by the water before it reaches to the surface. This is called sub-cooled boiling. Such forced convective boiling can be computed by:

$$\left(\frac{q}{A}\right)_{TOTAL} = \left(\frac{q}{A}\right)_{boiling} + \left(\frac{q}{A}\right)_{forced\ convection} \quad A4.16$$

The convective part is computed by using equation A4.13. For the sub-cooled boiling part, the Rohsenow equation can be used, as:

$$h_{ev} = \sqrt{\frac{g(\rho_l - \rho_v)}{g_c \sigma}} u_l \left(\frac{c_l}{0.013 h_{fg}^{2/3} Pr_l} \right)^3 \frac{(T_s - T_{sat})^3}{(T_s - T_w)} \quad A4.17$$

where,

c_l = specific heat of saturated liquid

h_{fg} = enthalpy of vaporization

Pr_l = Prandtl number of saturated liquid

μ_l = liquid viscosity

σ = surface tension of liquid-vapor intersurface

g = gravitational acceleration

ρ_l = density of saturated liquid

ρ_v = density of saturated vapor

T_s, T_w, T_{sat} = temperature of surface, water and saturation respectively

3. Total heat transfer coefficient

Heat transfer from external surface of component to its environment consists of the convective and the radiant parts. Because of the change rates of convective and radiant parts are different from each other, a separation has to be made. One approach is to use an equivalent total heat transfer coefficient. By this approach, the total heat transfer will correspond to the air temperature, the coefficient will handle the effects of both convective and radiant parts.

a) Ordinary surface

$$\begin{aligned} Q_{total} &= Q_{convection} + Q_{radiation} \\ &= h_c A (T_s - T_a) + \sum_{i=1}^n \sigma_0 \epsilon_s f_{s-i} (T_s^4 - T_i^4) A \\ &= h_c A (T_s - T_a) + h_r A (T_s - T_e) \quad A4.18 \\ &= \left[(h_c + h_r) + h_r \left(\frac{T_a - T_e}{T_s - T_a} \right) \right] A (T_s - T_a) \\ &= h_T (T_s - T_a) A \end{aligned}$$

in which,

h_T = total heat transfer coefficient

$$h_T = (h_c + h_r) + h_r \left(\frac{T_a - T_e}{T_s - T_a} \right) \quad A4.19$$

b) Finned surface

Heat transfer coefficients on finned surface are quite different from each other depending on the types of fin. The finned surface used in flat plate radiator is shown in Figure A4.1. It can be considered as a rectangular fin in which the end is insulated.

The temperature distribution along the length of the fin can be found by solving the following boundary problem of:

$$\begin{cases} \frac{d^2\theta}{dx^2} = \left(\frac{h_{rc}C}{kA_f}\right)\theta + \left(\frac{h_rC}{kA_f}\right)\theta_e \\ \theta(x) = \theta_1 & x = 0 \\ \dot{\theta}(x) = 0 & x = l \end{cases} \quad A4.20$$

where,

$$\theta = T - T_a; \quad \theta_e = T_a - T_e; \quad \theta_1 = T_s - T_a; \quad h_{rc} = h_r + h_c$$

The solution can be found as:

$$\theta(x) = \left(\theta_1 + \frac{h_r}{h_{rc}}\theta_e\right) \frac{\cosh m(l-x)}{\cosh ml} - \frac{h_r}{h_{rc}}\theta_e \quad A4.21$$

in which, $m^2 = \frac{h_{rc}C}{kA_f}$

The heat transfer from fin is:

$$\begin{aligned} Q_{fin} &= -kA \frac{d\theta}{dx} \Big|_{x=0} = kAm \left(\theta_1 + \frac{h_r}{h_{rc}}\theta_e\right) \tanh ml \\ &= \sqrt{h_{rc}CkA_f} \tanh ml \left(1 + \frac{h_r}{h_{rc}} \cdot \frac{T_a - T_e}{T_s - T_a}\right) (T_s - T_a) \quad A4.22 \end{aligned}$$

The total heat transfer of the finned tube as shown in figure A4.1 is the sum of the heat loss from the tube surface and that from the fins at two sides, therefore:

$$\begin{aligned} Q_{tot} &= Q_{tube} + 2Q_{fin} \\ &= \frac{1}{A_{tot}} (h_{rc}A_o + 2\sqrt{h_{rc}CkA_f} \tanh ml) \left(1 + \frac{h_r}{h_{rc}} \frac{T_a - T_e}{T_s - T_a}\right) (T_s - T_a) A_{tot} \\ &= h_T^* (T_s - T_a) A_{tot} \end{aligned}$$

where,

h_T^* = total heat transfer coefficient of finned tube

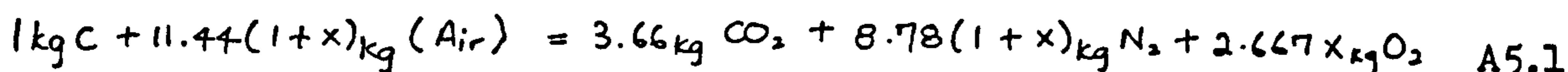
$$h_T^* = \frac{1}{A_{tot}} (h_{rc}A_o + 2\sqrt{h_{rc}CkA_f} \tanh ml) \left(1 + \frac{h_r}{h_{rc}} \frac{T_a - T_e}{T_s - T_a}\right)$$

A5. Combustion of fuel in boiler

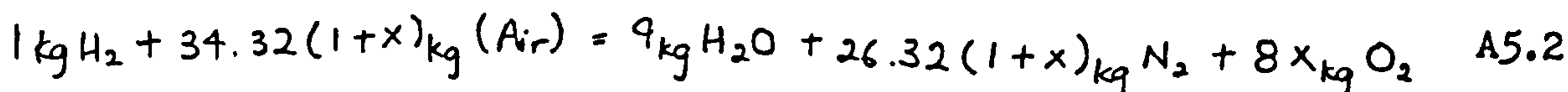
The compositions of ordinary fuel are C, S, C_aH_b , H_2 , O_2 , N_2 , H_2O etc. Combustion is a rapid oxidation process which liberates heat during the chemical reaction with oxygen. The combustion products is the sum of these reactable and unreactable elements and air supplied for the assistance of the combustion process. Air supply is usually slightly more than the exact amount needed to achieve proper combustion.

Chemical reaction formulae used for the combustable elements with x percent of excess air are:

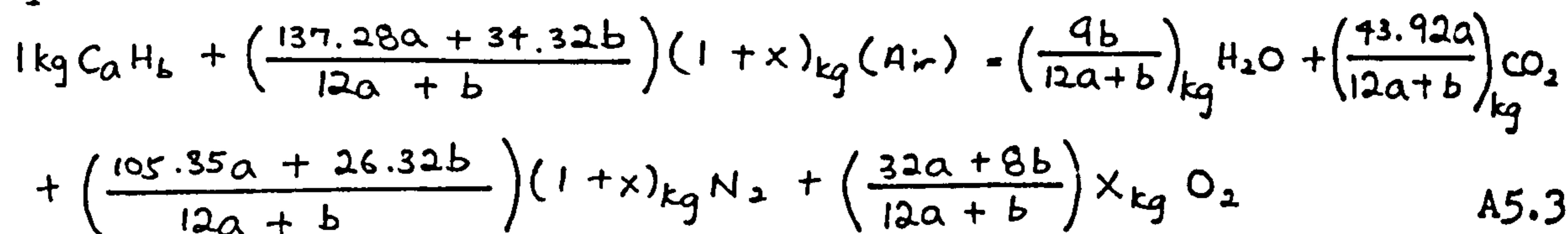
1. Carbon



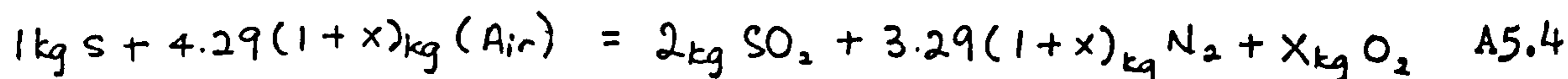
2. Hydrogen



3. Hydrocarbon



4. Sulphur



If the fuel supply rate is known as W (kg/s) as well as the composition of the fuel, the composition of combustion products can be found by weight as:

$$G_{CO_2} = 3.66W_c + \frac{43.92}{12a+b} W_{C_aH_b}$$

$$G_{SO_2} = 2W_s$$

$$G_{H_2O} = 9W_{H_2} + \frac{9b}{12a+b} W_{C_aH_b} + W_{H_2O}$$

$$G_{N_2} = \left[8.78W_c + 26.32(W_{H_2} - W_{O_2}/8) + 3.29W_s + \frac{105.35a + 26.32b}{12a + b} W_{C_aH_b} \right] (1+x) + W_{N_2}$$

$$G_{O_2} = \left[2.667W_c + (8W_{H_2} - W_{O_2}) + W_s + \frac{32a+8b}{12a+b} W_{C_aH_b} \right] X$$

Total weight of combustion products is:

$$G = G_{CO_2} + G_{SO_2} + G_{H_2O} + G_{N_2} + G_{O_2}$$

Total weight of air required for combustion is :

$$G_{air} = \left[11.44W_c + 34.32(W_{H_2} - W_{O_2}/8) + 4.29W_s + \frac{137.28a+34.32b}{12a+b} W_{C_aH_b} \right] (1+X)$$

Total kg-mol number of combustion products is:

$$m = \frac{G_{CO_2}}{44} + \frac{G_{SO_2}}{64} + \frac{G_{H_2O}}{18} + \frac{G_{N_2}}{28} + \frac{G_{O_2}}{32}$$

Total volume of combustion products:

$$V_f = \frac{mRT_G}{P}$$

where,

R = gas constant

P = pressure of combustion

The volumetric percentages of each composition in combustion products:

$$\left\{ \begin{array}{l} \gamma_{CO_2} = \frac{G_{CO_2}}{44m} \\ \gamma_{SO_2} = \frac{G_{SO_2}}{64m} \\ \gamma_{H_2O} = \frac{G_{H_2O}}{18m} \\ \gamma_{N_2} = \frac{G_{N_2}}{28m} \\ \gamma_{O_2} = \frac{G_{O_2}}{32m} \end{array} \right.$$

The volumetric percentage of triatomic gas

$$\gamma_{\Sigma} = \gamma_{CO_2} + \gamma_{SO_2} + \gamma_{H_2O}$$

A6. Component models of heating system

Component models are presented by the form of matrix temporal difference equation, as:

$$A_i(t+\Delta t)T_i(t+\Delta t) = B_i(t)T_i(t) + C_i(t+\Delta t, t)$$

The characteristics of the component model is totally depended on the state variables, matrix structure and the meanings of coefficient specified according to the specific control volumes in the component sub-system.

A selection of component models used in the system representation of figure 4.29 will be given as follows:

1. Bare pipe

| Control volume type | State variable | External disturbance | Control volume state equation |
|-----------------------|----------------|----------------------|---|
| flowing water in pipe | $T_{w,I}(\xi)$ | $T_{w,I-1}(\xi)$ | $-a_{x,1}(t+\Delta t) T_{w,x,1}(t+\Delta t) + a_{x,2}(t+\Delta t) T_{w,x,2}(t+\Delta t) - a_{x,3}(t+\Delta t) T_{b,x,1}(t+\Delta t)$ $= a_{x,1}(t) T_{w,x,1}(t) - a_{x,2}(t) T_{w,x,2}(t) + a_{x,3}(t) T_{b,x,1}(t)$ |
| pipe metal | $T_{b,I}(\xi)$ | $T_a(\xi)$ | $-b_{x,1}(t+\Delta t) T_{w,x,1}(t+\Delta t) + b_{x,2}(t+\Delta t) T_{b,x,1}(t+\Delta t) - b_{x,3}(t+\Delta t) T_a(t+\Delta t)$ $= b_{x,1}(t) T_{w,x,1}(t) - b_{x,2}(t) T_{b,x,1}(t) + b_{x,3}(t) T_a(t)$ |

| Coefficient formula | |
|--|---|
| $a_{x,1}(\xi) = \dot{m}_x(\xi) c_{w,x}(\xi) \Delta t$ | \dot{m}_x = water flow rate |
| $a_{x,2}(\xi) = 2\rho_{w,x}(\xi) c_{w,x}(\xi) v_{w,x} + a_{x,1}(\xi) + a_{x,3}(\xi)$ | $h_{w,x}$ = water side heat transfer coefficient |
| $a_{x,3}(\xi) = h_{w,x}(\xi) A_x \Delta t$ | A_x, A_o = pipe internal, external surface area |
| $a_{x,4}(\xi) = 2\rho_{w,x}(\xi) c_{w,x}(\xi) v_{w,x} - a_{x,1}(\xi) - a_{x,3}(\xi)$ | $h_{r,x}$ = total heat transfer coefficient |
| $b_{x,1}(\xi) = h_{w,x}(\xi) A_x \Delta t$ | $\rho_{w,x}, c_{w,x}$ = density, specific heat of water |
| $b_{x,2}(\xi) = 2\rho_s c_s v_{s,x} + b_{x,1}(\xi) + b_{x,3}(\xi)$ | $\rho_s c_s$ = density, specific heat of metal |
| $b_{x,3}(\xi) = h_{r,x}(\xi) A_o \Delta t$ | Δt = time increment |
| $b_{x,4}(\xi) = 2\rho_s c_s v_{s,x} - b_{x,1}(\xi) - b_{x,2}(\xi)$ | $v_{w,x}, v_{s,x}$ = volume of water and metal |

2. Pump

| Control volume type | State variable | External disturbance | Control volume state equation |
|-----------------------|----------------|--------------------------------|---|
| flowing water in pump | $T_{w,I}(\xi)$ | $q_p(\xi)$ $T_{w,I-1}(\xi)$ | $-a_{x,1}(t+\Delta t) T_{w,x,1}(t+\Delta t) + a_{x,2}(t+\Delta t) T_{w,x,2}(t+\Delta t) - a_{x,3}(t+\Delta t) T_{b,x,1}(t+\Delta t) - a_{x,4}(t+\Delta t) q_p(t+\Delta t)$ $= a_{x,1}(t) T_{w,x,1}(t) - a_{x,2}(t) T_{w,x,2}(t) + a_{x,3}(t) T_{b,x,1}(t) + a_{x,4}(t) q_p(t)$ |
| pump metal | $T_{b,I}(\xi)$ | $T_a(\xi)$ | same as pipe |

| Coefficient formula | |
|--|-------------------------------------|
| $a_{x,1}(\xi) = \dot{m}_x(\xi) c_{w,x}(\xi) \Delta t$ | $q_p(\xi)$ = heat generated by pump |
| $a_{x,2}(\xi) = 2\rho_{w,x}(\xi) c_{w,x}(\xi) v_{w,x} + a_{x,1}(\xi) + a_{x,3}(\xi)$ | |
| $a_{x,3}(\xi) = h_{w,x}(\xi) A_x \Delta t$ | |
| $a_{x,4}(\xi) = \Delta t$ | |
| $a_{x,5}(\xi) = 2\rho_{w,x}(\xi) c_{w,x}(\xi) v_{w,x} - a_{x,1}(\xi) - a_{x,3}(\xi)$ | |

3. Radiator

| Control volume type | State variable | External disturbance | Control volume state equation |
|--|-----------------------------------|--|-------------------------------|
| flowing water in radiator | $T_{w,I}(\xi)$ (I = 1, ..., 5) | $T_{w,0}(\xi)$ | same as pipe |
| radiator metal | $T_{b,I}(\xi)$ (I = 1, ..., 5) | $T_a(\xi)$ | same as pipe |
| (1) $h_{r,x}^*(\xi)$ is used to replace $h_{r,x}(\xi)$ | | (2) $A_{w,x}$ is used to replace A_o . | |

4. Boiler (cast iron 'Baxi' boiler)

| Control volume type | State variable | External disturbance | Control volume state equation |
|---|---|---|--|
| combustion products in combustion space | $T_G(\xi)$ | $T_a(\xi)$ (fuel supply) $U(\xi)$ | $-f_1(t+\Delta t)U(t+\Delta t) + f_2(t+\Delta t)T_G(t+\Delta t) - f_3(t+\Delta t)T_{b,1}(t+\Delta t) - f_4(t+\Delta t)T_{b,2}(t+\Delta t) - f_5(t+\Delta t)T_a(t+\Delta t) = f_1(t)U(t) - f_6 T_G(t) + f_3 T_{b,1}(t) + f_4(t)T_{b,2}(t) + f_5(t)T_a(t)$ |
| flue gas in passage | $T_{f,i}(\xi)$ ($i = 1, \dots, 4$) | $T_a(\xi)$ | $-c_{r,1}(t+\Delta t)T_G(t+\Delta t) + c_{r,2}(t+\Delta t)T_{f,i}(t+\Delta t) - c_{r,3}(t+\Delta t)T_{s,i}(t+\Delta t) - c_{r,4}(t+\Delta t)T_a(t+\Delta t) = c_{r,1}(t)T_G(t) - c_{r,2}(t)T_{f,i}(t) + c_{r,3}(t)T_{s,i}(t) + c_{r,4}(t)T_a(t)$ |
| Boiler water container metal | $T_{s,i}(\xi)$ ($i = 1, \dots, 4$) | | $-d_{s,1}(t+\Delta t)T_G(t+\Delta t) - d_{s,2}(t+\Delta t)T_{f,i}(t+\Delta t) + d_{s,3}(t+\Delta t)T_{s,i}(t+\Delta t) - d_{s,4}^{(w)} T_{w,i}(t+\Delta t) = d_{s,1}(t)T_G(t) + d_{s,2}(t)T_{f,i}(t) - d_{s,3}(t)T_{s,i}(t) + d_{s,4}(t)T_{w,i}(t)$ |
| boiler water | $T_{w,i}(\xi)$ ($i = 1, \dots, 4$) | $T_{w,0}(\xi)$ | $-e_{z,1}(t+\Delta t)T_{w,i}(t+\Delta t) - e_{z,2}(t+\Delta t)T_{b,i}(t+\Delta t) + e_{z,3}(t+\Delta t)T_{w,i}(t+\Delta t) = e_{z,1}(t)T_{w,i}(t) + e_{z,2}(t)T_{b,i}(t) - e_{z,3}(t)T_{w,i}(t)$ |

| Coefficient formula | |
|---|--|
| $f_1(\xi) = q_f \Delta t$ | q_f = calorific value of fuel |
| $f_2(\xi) = 2\rho_g(\xi)c_g(\xi)v_g + f_1(\xi) + f_3(\xi) + f_4(\xi) + f_5(\xi)$ | ρ_g, c_g = density, specific heat of combustion products |
| $f_3(\xi) = h_g(\xi)A_0 \Delta t$ | $v_g, v_{f,i}$ = volume of combustion chamber, and flue gas |
| $f_4(\xi) = h_g(\xi)A_0 \Delta t$ | h_g = total heat transfer coefficient of combustion product |
| $f_5(\xi) = (h_o(\xi)A_g + G_g(\xi)c_g(\xi)) \Delta t$ | A_0 = area of water container facing combustion chamber |
| $f_6(\xi) = 2\rho_g(\xi)c_g(\xi)v_g - f_1(\xi) - f_3(\xi) - f_4(\xi) - f_5(\xi)$ | G_g = weight of combustion product |
| $c_{r,1}(\xi) = G_g(\xi)c_g(\xi)(\Delta t/2)$ | $\rho_{f,i}, c_{f,i}$ = density, specific heat of flue gas |
| $c_{r,2}(\xi) = 2\rho_{f,i}(\xi)c_{f,i}(\xi)v_{f,i} + c_{r,1}(\xi) + c_{r,3}(\xi) + c_{r,4}(\xi)$ | $h_{f,i}$ = total heat transfer coefficient of flue gas |
| $c_{r,3}(\xi) = 2\rho_{f,i}(\xi)c_{f,i}(\xi)v_{f,i} + c_{r,1}(\xi) + c_{r,3}(\xi) + c_{r,4}(\xi)$ | $h_{f,0,i}$ = coefficient of flue gas enclosure loss |
| $c_{r,4}(\xi) = h_{f,0,i}(\xi)A_f \Delta t$ | A_f, A_g = enclosure area of flue gas passage and combustion chamber |
| $c_{r,5}(\xi) = h_{f,0,i}(\xi)A_f \Delta t$ | $h_{cv,i}$ = forced convection - subcooled boiling heat transfer coefficient |
| $d_{s,1}(\xi) = h_g(\xi)A_0 \Delta t$ | A_z = area of container contact to water |
| $d_{s,2}(\xi) = h_{f,i}(\xi)A_0 \Delta t$ | \dot{m} = boiler water flow rate |
| $d_{s,3}(\xi) = 2\rho_w c_w v_{w,i} + d_{s,1}(\xi) + d_{s,2}(\xi) + d_{s,4}(\xi)$ | $c_{w,i}$ = boiler water specific heat |
| $d_{s,4}(\xi) = h_{w,i}(\xi)A_z \Delta t$ | |
| $d_{z,1}(\xi) = 2\rho_w c_w v_{w,i} - d_{z,1}(\xi) - d_{z,2}(\xi) - d_{z,3}(\xi)$ | |
| $e_{z,1}(\xi) = \dot{m}(\xi)c_{w,i}(\xi)\Delta t$ | |
| $e_{z,2}(\xi) = h_{w,i}(\xi)A_z \Delta t$ | |
| $e_{z,3}(\xi) = 2\rho_{w,i}(\xi)c_{w,i}(\xi)v_{w,i} + e_{z,1}(\xi) + e_{z,2}(\xi)$ | |
| $e_{z,4}(\xi) = 2\rho_{w,i}(\xi)c_{w,i}(\xi)v_{w,i} + e_{z,1}(\xi) + e_{z,2}(\xi)$ | |

5. Hot water storage tank

| Control volume type | State variable | External disturbance | Control volume state equation |
|---------------------------|--|---------------------------------------|--|
| flowing hot water in tank | $T_{hw,z}(\xi)$ ($z=1, \dots, 3$) | $T_{hw,0}(\xi)$ (inlet hot water) | $-a_{z,1}(t+\Delta t)T_{hw,z-1}(t+\Delta t) + a_{z,2}(t+\Delta t)T_{hw,z}(t+\Delta t) - a_{z,3}(t+\Delta t)T_{hw,z}(t+\Delta t) + a_{z,1}(t)T_{hw,z-1}(t) - a_{z,4}(t+\Delta t)T_{hw,z}(t) + a_{z,3}(t)T_{hw,z}(t)$ |
| hot water vessel metal | $T_{hs,z}(\xi)$ ($z=1, \dots, 3$) | | $-b_{z,1}(t+\Delta t)T_{hw,z}(t+\Delta t) + b_{z,2}(t+\Delta t)T_{hs,z}(t) - b_{z,3}(t+\Delta t)T_{cw,z}(t+\Delta t) + b_{z,1}(t)T_{hw,z}(t) - b_{z,4}(t)T_{hs,z}(t) + b_{z,3}(t)T_{cw,z}(t)$ |
| heated water in tank | $T_{cw,z}(\xi)$ ($z=1, \dots, 3$) | $T_{cw,4}$ (inlet of heated water) | $-J_{z,1}(t+\Delta t)T_{cw,z-1}(t+\Delta t) - J_{z,2}(t+\Delta t)T_{hs,z}(t+\Delta t) + J_{z,3}T_{cw,z}(t+\Delta t) + J_{z,4}(t+\Delta t)T_{cw,z+1} - J_{z,5}(t+\Delta t)T_{cs,z}(t+\Delta t) = \dots - J_{z,6}(t)T_{cw,z}(t) + \dots$ |
| heated water shell metal | $T_{cs,z}(\xi)$ ($z=1, \dots, 3$) | $T_a(\xi)$ | $-K_{z,1}(t+\Delta t)T_{cw,z}(t+\Delta t) + k_{z,2}(t+\Delta t)T_{cs,z}(t+\Delta t) - k_{z,3}(t+\Delta t)T_a(t+\Delta t) + K_{z,1}(t)T_{cw,z} - K_{z,4}(t)T_{cs,z}(t) + K_{z,3}(t)T_a(t)$ |

| Coefficient formula | |
|---|---|
| $a_{z,1}; a_{z,2}; a_{z,3}; a_{z,4}$ same as pipe water | $h_{hw,z}; h_{hs,z}$ = water heat transfer coefficient at hot and heated water side |
| $b_{z,1}(\xi) = h_{hw,z}(\xi) A_{hw} \Delta t$ | $A_{hw,z}; A_{hs,z}$ = surface area of hot and heated vessels |
| $b_{z,2}(\xi) = 2\rho_s C_s V_{hs,z} + b_{z,1}(\xi) + b_{z,3}(\xi)$ | $V_{hw,z}; V_{hs,z}$ = volume of hot and heated water |
| $b_{z,3}(\xi) = h_{hw,z}(\xi) A_{hw} \Delta t$ | $\rho_{hw,z}; \rho_{hs,z}$ = density of hot and heated water |
| $b_{z,4}(\xi) = 2\rho_{hw,z}(\xi) C_{hw,z}(\xi) V_{hw,z} - b_{z,1}(\xi) - b_{z,3}(\xi)$ | $C_{hw,z}; C_{hs,z}$ = specific heat of hot and heated water |
| $J_{z,1}(\xi) = \dot{m}_{u,z}(\xi) C_{w,z}(\xi) \Delta t$ | $\rho_s; C_s$ = density, specific heat of metal |
| $J_{z,2}(\xi) = h_{cw,z}(\xi) A_{hw,z} \Delta t$ | $V_{hs,z}; V_{cs,z}$ = volume of hot and heated water side metal |
| $J_{z,3}(\xi) = 2\rho_{cw,z}(\xi) C_{cw,z}(\xi) V_{hw,z} + J_{z,1}(\xi) + J_{z,2}(\xi) + J_{z,4}(\xi) + J_{z,5}(\xi)$ | \dot{m}_h = mass flow rate of hot water |
| $J_{z,4}(\xi) = h_{cw,z}(\xi) A_{cw,z} \Delta t$ | \dot{m}_c = mass flow rate of heated water |
| $J_{z,5}(\xi) = (\dot{m}_c(\xi) + \dot{m}_d(\xi)) C_{cw,z}(\xi) \Delta t$ | $\dot{m}_{u,z}$ = internal flow rate of heated water with higher section |
| $J_{z,6}(\xi) = 2\rho_{cw,z}(\xi) C_{cw,z}(\xi) V_{cw,z} - J_{z,1}(\xi) - J_{z,2}(\xi) - J_{z,4}(\xi) - J_{z,5}(\xi)$ | $\dot{m}_{d,z}$ = internal flow rate of heated water with lower section |
| $K_{z,1}(\xi) = h_{hw,z}(\xi) A_{cw} \Delta t$ | |
| $K_{z,2}(\xi) = 2\rho_s C_s V_{cs,z} + K_{z,1}(\xi) + K_{z,3}(\xi)$ | |
| $K_{z,3}(\xi) = h_{cs,z}(\xi) A_{cw} \Delta t$ | |
| $K_{z,4}(\xi) = 2\rho_s C_s V_{cs,z} - K_{z,1}(\xi) - K_{z,3}(\xi)$ | |

6. Control loop (proportional control function)

| Control volume type | State variable | External disturbance | Control volume state equation |
|--------------------------------|----------------|----------------------|--|
| Sensor (sensed temp) | $e(\xi)$ | $e_a(\xi)$ | $r_1(t+\Delta t)e(t+\Delta t) - r_2(t+\Delta t)e_a(t+\Delta t) = -r_1(t)e(t) + r_2(t)e_a(t)$ |
| controller | $V(\xi)$ | | $p_1(t+\Delta t)e(t+\Delta t) - p_2(t+\Delta t)V(t+\Delta t) = -p_1(t)e(t) + p_2(t)V(t)$ |
| actuator (opening of valve) | $D(\xi)$ | | $s_1(t+\Delta t)V(t+\Delta t) - s_2(t+\Delta t)D(t+\Delta t) = -s_1(t)V(t) + s_2(t)D(t)$ |
| linear valve (fuel supply) | $u(\xi)$ | | $u(t+\Delta t) - K_v D(t+\Delta t) = u(t) - K_v D(t)$ |

| Coefficient formula | |
|---|---|
| $r_1(\xi) = 1 + \frac{\Delta t}{\tau_s}(\xi)$ $r_2(\xi) = \frac{\Delta t}{\tau_s}(\xi)$ $r_3(\xi) = 1 - \frac{\Delta t}{\tau_s}(\xi)$ $p_1(\xi) = 1 + k_p/\tau_c(\xi) \Delta t$ $p_2(\xi) = (\Delta t/\tau_c)(\xi)$ $p_3(\xi) = 1 - \frac{k_p \Delta t}{\tau_c}(\xi)$ $s_1(\xi) = 1 + \frac{\alpha \Delta t}{\tau_a}(\xi)$ $s_2(\xi) = \frac{\Delta t}{\tau_a}(\xi)$ $s_3(\xi) = 1 - \frac{\alpha \Delta t}{\tau_a}(\xi)$ | $\tau_s; \tau_c; \tau_a$ = time constant of sensor, controller, actuator k_p = proportional gain α = coefficient of ratio of control signal to displacement k_v = coefficient of ratio of fluid flow rate and valve opening |