DEPARTMENT OF
NAVAL ARCHITECTURE, OCEAN &
MARINE ENGINEERING

NM963 Theory and Practice of Marine CFD

- EXAM FOR THE DECEMBER 2023 DIET-

4 December 2023 **10.00-11.30am** **TIME 1.5hrs**

Answer TWO questions

(All Questions worth equal marks)

Calculators must not be used to store text and/or formulae nor be capable of communication. Invigilators may require calculators to be reset.

Please review the University's guidance on "Good Academic Practice and the Avoidance of Plagiarism" on this website.

Question 1 (20 marks)

Consider the *steady* heat transfer across an infinite long solid slab with a finite thickness L in Figure 1(a). Assume that the material thermal conductivity (k) is a constant.

(a) Using general transport equation (Eq. (1)), determine the temperature distribution in the slab if x=0, T=T_0 and x=L, T=T_L

$$\frac{\partial(\rho\phi)}{\partial t} + div(\rho\phi\vec{V}) = div(\Gamma\ grad\ \phi) + S_{\varphi} \qquad \textbf{Eq. (1)}$$

grad ϕ is the gradient of a scaler ϕ. In 3D Cartesian space, $\nabla\phi$ or grad ϕ is given as:

$$\nabla\phi = \frac{\partial\phi}{\partial x}\boldsymbol{i} + \frac{\partial\phi}{\partial y}\boldsymbol{j} + \frac{\partial\phi}{\partial z}\boldsymbol{k}$$

$div\ (\overrightarrow{\boldsymbol{V}})$ is the divergence of a vector field. In 3D Cartesian space, $\nabla\cdot\vec{\boldsymbol{V}}$ or $div\ (\overrightarrow{\boldsymbol{V}})$ is given as:

$$\nabla\cdot\vec{\boldsymbol{V}} = div\ (\overrightarrow{\boldsymbol{V}}) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

In this problem, you can treat ϕ as temperature T, Γ=k, there is no heat sink and source in the slab.

(4 marks)

(b) Using either a Finite Difference or a Finite Volume Method to discretize the governing equation with the nodes index shown in Figure 1(b), demonstrate that the discretised equations can be represented by the following algebraic equations. N is the number of cells.

$$a_i T_i = a_{i+1}T_{i+1} + a_{i-1}T_{i-1} + b \qquad i = 2 \ldots N-1$$

with the coefficients of a and b to be

$$\begin{cases} a_{i+1} = 1 \\ a_{i-1} = 1 \\ a_i = a_{i+1} + a_{i-1} \end{cases} \qquad and \qquad b = 0$$

(5 marks)

(c) Solve the equations to part (b) for the nodal temperatures of T_2 to T_6 in the case of 6 cells that are evenly distributed across the thickness of slab. Assume T_0=700^{0}C and T_L=100^{0}C.

(5 marks)

(d) Use this case as an example to describe and explain the following CFD terms.

- **(i)** CFD grid
- **(ii)** Governing equations discretization
- **(iii)** Boundary condition
- **(iv)** Algebraic system of equations obtained by the discretization of governing equations
- **(v)** Solving equation

(6 marks)

Question 2 (20 marks)

(a) Apply the mass conservation concept to a three-dimensional control volume indicated in Figure 2. Provide the ***derivation*** for the unsteady continuity equation in the form of a Partial Differential Equation (PDE) as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \qquad \textbf{Eq. (2)}$$

Where ρ is the fluid density, t is the time, u, v, w are the velocity components in x, y and z directions. In the above PDE form, grid spacing Δx, Δy, Δz *are* reduced to *dx, dy, dz*.

(6 marks)

(b) Assume a two-dimensional incompressible steady flow, using the mesh nodes index in Figure 3 and Eq. (3), based on a second-order central differencing scheme, derive a ***discretized*** continuity equation with a Finite Volume Method.

$$\frac{\partial}{\partial t}\int_{\Omega} \rho d\Omega + \oint_{\partial\Omega} \rho \vec{V} \cdot d\vec{S} = 0 \qquad \textbf{Eq. (3)}$$

where t is the time, Ω is the control volume with boundary $\partial\Omega$, $\vec{V}$ is the fluid velocity vector with u, v, and w as its three components.

(5 marks)

(c)

(i) Using Eq. (1), Explain which term in the equation represents the physical problem is transient.

(ii) Using Eq. (1), explain what is the time discretization?

(iii) Consider a general unsteady problem using following equation,

$$\frac{d\phi}{dt} = f(\phi, t) \qquad \text{for some arbitrary function } f.$$

By using Taylor series expansions, illustrate that the explicit Euler discretization,

$$\phi^{n+1} = \phi^{n} + f(\phi^{n}, t^{n})\Delta t$$

is first-order accurate, where the superscript (n) denotes the time level n.

(9 marks)

Question 3 (20 marks)

(a) Consider a one-dimensional flow domain as shown in Figure 4. Given that the cell-size is ***unequal***, provide the Central Differencing Scheme (CDS) expression of ϕ_e in terms of nodal values of ϕ_E and ϕ_P. Assume that the coordinates of e, E, P are x_e, x_E, and x_P. What the order of this CDS is?

(4 marks)

(b) Consider the case of simulating a flow over a high-speed vehicle in a box-shaped test section of a wind-tunnel. The inlet velocity is known as 30m/s. Describe with the aid of sketches what boundary conditions and their mathematical formula are assigned at the test-section's in- and outlet, the test-section walls and the vehicle surface? Can the cost of the computation be reduced with an appropriate boundary condition?

(6 marks)

(c) Provide your answers to the following questions and give sufficient explanations.

(i) Given a specific CFL number, when the CFD mesh is refined, how will you adjust the time step in order to get a convergent solution?

(3 marks)

(ii) Explain the necessity of turbulence modelling, what are the differences between solving laminar and turbulent flow.

(3 marks)

(iii) Explain with the aid of examples what is meant by modelling error or CFD accuracy, how it arises and how its magnitudes may be evaluated.

(4 marks)

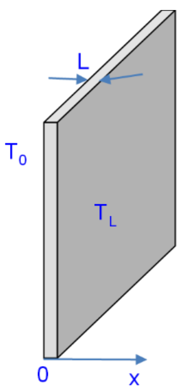

Figure 1(a)

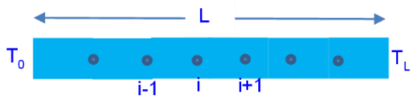

Figure 1(b)

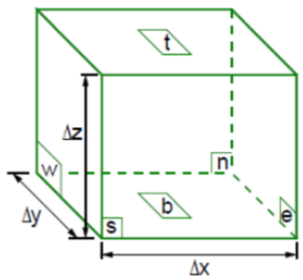

Figure 2

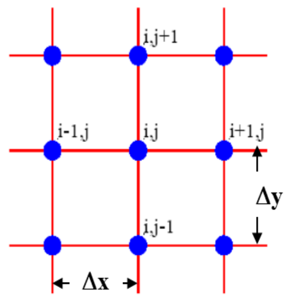

Figure 3

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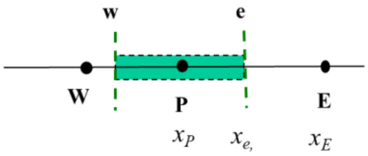

Figure 4
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