MATHEMATICAL MODELLING OF THE OPTIMAL POWER DISPATCH PROBLEM

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By

HABIB K. ALI

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

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DEDICATION

This Work is Affectionately Dedicated

To My Family and Friends

Not necessarily two separate groups of people

To the memory of those of whom, who are no longer with us, but left us the fondest and sweetest of memories

To those who still are, gracing our life with their presence and filling it with joy and making it all worthwhile

And to those with whom we have lost contact, in the hope that one day they will read these words and know that they have never been forgotten

To all of these, who, for me, are the past, the present and the future.

ABSTRACT

This thesis is concerned with the optimum operating conditions in a power system. The various aspects of the problem are modelled and solved as a number of optimization problems applying linear programming techniques. A generalized linear mathematical model has been developed for this purpose. A two-stage formulation is adopted to represent the various problems considered. In each case one power system quantity is chosen as an objective function to be optimized under a number of constraints and operating limits relating to the power system relationships and upper and lower bounds on the variables. These include constraints derived from the power flow equations and transmission network capacity. Limits are also imposed on bus voltage magnitudes and generator outputs.

With the appropriate selection of the combination of objective function and constraints, the model can be used to minimize the overall generation cost, the total system losses or the total reactive power generation. The two-stage modelling of the problem also allows optimizing two different objective functions at the same time. Two such combinations are possible. In one case the total system losses can be minimized in the first stage and the generation cost minimized in the second stage. The other combination minimizes the total system reactive power output and the active power generation cost.

Using the same model, the problem is then solved using decomposition techniques. These imply breaking up the original problem into a number of smaller problems that can be solved almost independently. The mathematical model has been developed in general terms and the associated computer program is written for a general power system. A sample system of medium size has been used to test the validity of the various aspects of the suggested model and produce numerical results.

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Thanks are also extended to friends and colleagues, members of the Power Systems Research Group, past and present, for their interest and encouragement. The occasional and casual discussions with them during the course of the research and the useful comments received from them proved very valuable and have, in various ways, helped directly and indirectly in the completion of this work. Being among them as a member of the group has been a very rewarding and fulfilling experience and I would like to take this opportunity to wish them all, continued success, progress, happiness and the best of luck.

CONTENTS

SYMBOLS AND NOTATION

A : Constraints Coefficient Matrix

AM_{ij} : An Element of the Constraints Coefficient Matrix Associated with the Bus Voltage Magnitude

AN_{ij} : An Element of the Constraints Coefficient Matrix Associated with the Bus Phase Angle

BMN : Lower Bound on a Constraint

BMX : Upper Bound on a Constraint

B_o : Bounds on Coupling Constraints

C : Coefficient of an Objective Function

CB : Bus Incremental Cost Coefficient

CE : Number of Extreme Point Solutions

CG : Generator Incremental Cost Coefficient

CGD : Cost Associated with the Net Active Power Generation Needed to meet the Total System Demand

CGT : Total Cost of Active Power Generation

CLT : Cost Associated with the Total System Active Power Transmission Losses

CM : Coefficient of an Objective Function Associated with the Bus Voltage Magnitude

CN : Coefficient of an Objective Function Associated with the Bus Phase Angle

D : Independent Coefficient Matrix in a Decomposable Linear Programming Problem

DR : Generator Deloading Rate

EP : The Value of an Extreme Point Solution

FB : Bus Hourly Generation Cost

FG : Generator Hourly Generation Cost

FY : A General Function

G + j B : System Bus Admittance Matrix

H : Number of Constraints Blocks in a Block-Diagonal Matrix Structure

K & KY : General Constants

LR : Generator Loading Rate

m & n : Number of Constraints and Variables in a General Linear Programming Problem

ME & NE : Total Number of Constraints and variables in a Reformulated Decomposable Linear Programming Problem

M_o : Number of Coupling Constraints

MT & NT : Total Number of Constraints and variables in a Decomposable Linear Programming Problem

NB, NG & NL : Total Number of System Buses, Generators and Transmission Lines

N = NB − 1

NGB : Number of Generators Connected at a Particular Bus

NR : Number of Tap-changing Transformers

NS : Number of Generating Buses (Power Stations)

P & Q : Net Bus Active and Reactive Power injections

PD & QD : Bus Active and Reactive Power Demands

PDT & QDT : Total System Active and Reactive Power Demand

PF : Overall System Power Factor

PGB & QGB : Bus Active and Reactive Power Outputs

PGBMN & PGBMX : Bus Minimum and Maximum Active Power Outputs

PGG & QGG : Generator Active and Reactive Power Outputs

PGGMN & PGGMX : Generator Minimum and Maximum Active Power Outputs

PGT & QGT : Total System Active and Reactive Power Generation

PGTMX & QGTMX : Total System Active and Reactive Output Power Capacities

PL & QL : Transmission Line Active and Reactive Power Loss

PLMX : Upper Limit On Transmission Line Active Power Loss

PLT & QLT : Total System Active and Reactive Power Transmission Losses

PT & QT : Active and Reactive Tie-line Flows

QGBMN & QGBMX : Bus Minimum and Maximum Reactive Power Outputs

QGGMN & QGGMX : Generator Minimum and Maximum Reactive Power Outputs

R : Transmission Line Equivalent Series Resistance

RB : Generating Bus Spinning Reserve

RBMN : Minimum Generating Bus Spinning Reserve

RG : Generator Spinning Reserve

RGMN : Generator Minimum Spinning Reserve

S : Complex Power

S : Apparent Power

SGGMN : Generator Minimum Apparent Power Output

SGGMX : Generator Maximum Apparent Power Output

SGT : Total System Apparent Power Generation

SL : Transmission Line Apparent Power Loss

SLT : Total System Apparent Power Transmission Loss

SPC : Overall System Cost Per Unit Of Active Power Generation

SS : Transmission Line Equivalent Shunt Capacitance

ST : Apparent Power Transfer Across a Transmission Line

STMX : Upper Limit On Apparent Power Transfer Across a Transmission Line

$$T = \frac{SS}{2}$$

t & α : Turn Ratio and Phase Shift of a Tap-changing Transformer

tMN & tMX : Lower and Upper Limits on the Turn Ratio of a Tap-changing Transformer

αMN & αMX : Lower and Upper Limits on the Phase-shift of a Tap-changing Transformer

V & θ : Bus Voltage Magnitude and Phase Angle

VMN & VMX : Minimum and Maximum Bus Voltage Magnitudes

W : Number of Sets of Constraints

X : Transmission Line Equivalent Series Reactance

$$Y = 1/\sqrt{R^2 + X^2}$$

YMN & YMX : Minimum and Maximum Values of a General Function FY

Z : Objective Function

ZD : Dual Objective Function

ZP : Primal Objective Function

$$\theta_{ij} = \theta_i - \theta_j$$

μ : Coefficient Used in Connection with Convex Combinations

π : Simplex Multiplier

Δ : Incremental Change

Subscripts

i, j, k : The i'th Bus, j'th Generator, k'th Line, etc.

ij : The Transmission Line Connecting Bus i to Bus j,

o : Initial Value

r : Reference (Bus)

MATHEMATICAL MODELLING

OF THE OPTIMAL POWER DISPATCH PROBLEM

CHAPTER 1

INTRODUCTION

1.1 POWER SYSTEM OPERATION : THE PROBLEM

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1.3 THE PRESENT RESEARCH PROJECT

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CHAPTER 1

INTRODUCTION

1.1 POWER SYSTEM OPERATION : THE PROBLEM

The operation of a modern power system is a very complicated and multifaceted problem that involves a large number of interrelated variables. One aspect of the problem is the complexity and various degrees of nonlinearity of the mathematical equations that represent the relationships among the various power system quantities. Another aspect that contributes to the complexity of the problem is the continuously expanding sizes of systems with their large number of generators, busbars and transmission lines, as well as the various methods of generation and types of power station. The problem is further compounded by the interconnection and energy exchange among different sections of the system. This applies to small areas within a large system, two or more major systems or even the power systems of two different countries. Also, because of the nature of the service they provide, power systems have to operate continuously, and this involves an enormous amount of monitoring, decision-making and control.

Obviously, the main objective when operating a power system is to satisfy the load demand on the system. But the load demand itself is a time-variant quantity and, therefore, a more accurate statement is that the objective is to satisfy the load demand on the system at any specified moment in time. The consumer demand on the system varies considerably from season to season during the year, from weekdays to weekends within a week, and from hour to hour in one day. Special days of the year such as Christmas day and other public holidays have their own loading conditions which can be quite unpredictable, in contrast with seasonal or hourly variations. Also, load variations in a power system are affected by a number of diverse factors, some of which can be very unexpected. In recent years, these can range from freak weather conditions to popular television programmes. For example, normal prevailing weather conditions can be predicted to a fairly acceptable degree of accuracy, such that the consumer demand on the system can be met satisfactorily. However, a sudden sharp drop in temperatures can present the system operator with a serious problem.

The load demand variations with time are given by the daily load curve, and although the load varies slowly and gradually, there is a considerable difference between the peak load and the trough during a 24-hour period. Obviously, this variation of load has to be taken into consideration.

The solution of the power system operation problem is started by obtaining an estimate of the expected system demand for the specified period of time. This is followed by determining the number and size of generating units to be operated during that period, since, as mentioned above, the system demand varies widely between its high and low extremes. The task of estimating the required demand and its variations at various time periods is referred to as load forecasting. The systematic procedure of deciding which generating units to be operated at the various time slots is the subject of unit commitment. Both load forecasting and unit commitment are the subjects of continuous and extensive research. They are, however, outside the scope of this thesis.

In addition to such basic requirements as safe operation and reliability of supply, the system load is to be satisfied under a large number of conditions and, sometimes conflicting, operating constraints. For example there are the physical laws that govern power system relationships such as the energy balance in the system and the mathematical relationships among bus voltages and line currents. Apart from these, there is also a number of constraints based on operating limits and engineering design specifications. Examples of these are bus voltages, which are to be kept within a specified range around their nominal values, and upper and lower limits on generator outputs.

Operating conditions imply real-time decision-making and on-line control, as opposed to other power system activities such as long term load forecasting and planning of maintenance schedules. This calls for fast, efficient and reliable algorithms and computer programs to tackle the various aspects of the problem.

1.2 RESEARCH IN THE FIELD

The importance of the problem of power system operation can be judged from the tremendous amount of literature available on this vast subject, which is a clear evidence of the attention it receives from researchers and power companies, as shown by Happ [1]. A large number of research papers has been published that cover many of the varied facets of the problem and a wide range of related topics. Also a variety of techniques have been applied to solve particular aspects of the problem, as illustrated by Lo and Brameller [2]. In a general paper, Sasson and Merrill [3] discussed the application of the various mathematical optimization techniques, such as linear, nonlinear, quadratic, integer and dynamic programming to various power system problems. A review paper of a similar nature, by Stott, Marinho and Alsac [4] discusses the application of linear programming methods in particular to the solution of various power system problems such as transmission planning, security dispatch and emergency control.

Optimization methods have been used extensively to tackle the various aspects of the power system operation problem. Generally, these involve selecting one power system quantity as an objective function to be minimized or maximized subject to a number of constraints. Methods of formulation and solution techniques can be broadly classified into two main categories. These are linear and nonlinear programming methods.

Among the early contributions to the field is the formulation, in 1968, of the optimal power flow problem by Dommel and Tinney [5]. Their method was based on the Newton's algorithm, gradient adjustment for obtaining the minimum and the use of penalty factors. Since then, there has been a continuous flow of research papers in the field.

Benthall [6] described an algorithm for solving the problem of secure economic load scheduling on a large power system. However, no numerical results were reported. The paper indicated that the computation time involved is mostly suitable for off-line calculations and that on-line application of the method calls for a more complex computing system.

The problem of minimizing transmission line losses was formulated by Peschon et al.[7]. The method was based on the suitable selection of reactive power productions and transformer tap settings. The computational procedure was based on the Newton-Raphson method for solving the power-flow equations and on the dual (Lagrangian) variables of the Kuhn and Tucker Theorem. The economic dispatch problem was formulated by El-Abiad and Jaimes [8], also using Newton's method and based on the Lagrange multipliers and Kuhn-Tucker conditions.

Shen and Laughton [9] used a similar approach to minimize the cost of real power generation with a more comprehensive set of constraints including transmission line loading limits and transformer tap settings. In another paper [10], they applied dual linear programming techniques to solve the load scheduling problem with security constraints. In reference [11], nonlinear programming was applied for the minimization of the hourly operating cost constrained by area interchanges, using penalty function method and generalized reduced gradients.

Bonaert, El-Abiad and Koivo [12] addressed the problem of scheduling a hydro system. Their computation takes into account hydro dynamics with variable heads, cascaded plants, by-pass discharges, spilling, pump storage plants and navigation requirements. Some emphasis on hydro generation was also given in a relatively recent paper by Shaw, Gendron and Bertsekas [13]. They considered the problem of optimal unit commitment and economic dispatch of a large hydrothermal power system. Their work was concerned with scheduling the startup and shut down of the thermal units and the power generation of all units such that the fuel cost is minimized over T time periods.

A number of methods were used that applied decomposition techniques to reduce the size of the problem and the associated matrices. These will be thoroughly discussed in a separate chapter in this thesis.

Assessment of Existing Methods

No doubt some excellent research work has been done in the field. For example, the paper of Dommel and Tinny [5] is one of the publications that formed important landmarks in the field. However, by studying the literature and comparing the various formulations and solution algorithms available, it has been concluded that most of the mathematical models and methods of solution used so far suffer from one or more of the following main shortcomings.

1. Because of the complexity of the mathematical equations involved and as a simplifying step in the modelling process, in some models transmission line

resistance is ignored, thus neglecting system losses. Although this is a convenient analytical tool, the resulting models do not truly represent the actual power systems which are invariably lossy. Usually, this simplifying assumption is explicitly mentioned. The problem, however, is that in some cases it is not mentioned and the following rather misleading statement is encountered in the course of the analysis without any explanation.

Total System Generation = Total System Load

2. Mathematical models used so far are inflexible. They are designed to solve one aspect of the power system operation problem, with one objective function and a specific set of constraints. Some of these models are even designed for a particular power company or with one particular power system in mind. For instance, a model that minimizes system losses cannot be readily expanded or modified such that it takes into consideration other factors or achieve different objectives in addition to or instead of those for which it was originally set up.

3. Many linear or linearized models are based on the concept of incremental changes in system variables starting from some initial conditions. One of the drawbacks frequently encountered when dealing with such models is that the incremental change notation, such as the symbol (Δ), is retained throughout the analysis

and appears in all the mathematical expressions including matrices. Although, in principle, this is not wrong it does, however, tend to hinder the clarity of the resulting model. In a real power system one is usually inclined to deal with the actual values of the variables such as V and P for voltage magnitudes and real powers, rather than their incremental values, ΔV and ΔP.

Also the assumption of the availability of the set of initial conditions is made without any suggestion as to how they might be obtained. Therefore, when trying to use such models or run a computer program based on them, the first difficulty faced will be obtaining a suitable set of initial operating conditions such as an initial generation schedule. The availability and suitability of such data is important as it provides the starting point for the solution of the problem. Some of the initial data can be assumed of course, but this can prove difficult sometimes due to the large number and different types of variables involved.

4. Most models used so far either include a load flow routine in the optimization procedure or iterate between an optimization routine and a load flow. In the first category of models, for each iteration of the optimization process, a complete load flow is performed, which in itself is an iterative procedure that involves the lengthy process of inverting the system Jacobian matrix. This has the disadvantage of considerably

increasing the number of iterations and amount of computation involved as well as the CPU time. Another disadvantage is that the computer program based on such models has to be specially written to solve the power system problem, and no use can be made of any general optimization software packages available. In the second category of models, although a general-purpose optimization routine can be employed, the computer program still has to iterate between the optimization routine and a load flow routine to check for violation of constraints and make necessary adjustments.

1.3 THE PRESENT RESEARCH PROJECT

In the present work, an attempt has been made to set up a versatile and flexible mathematical model to represent the power system for the purpose of obtaining the optimum operating conditions, with emphasis on generality of the method and simplicity of the formulation. This section gives a general and brief description of the mathematical model used in this thesis and its main characteristics and method of solution [14].

A general power system is considered under normal steady-state operating conditions. The various aspects of power system operation are represented by a number of optimization problems using linear programming formulation. For this purpose all relevant power system equations are linearized. A two-stage mathematical model has been developed to represent the power system and solve the various optimization problems considered. The first stage of the model is based on bus and line quantities such as bus injections and line flows, while the second stage is based on generator quantities such as generator hourly fuel cost. The system busbars are classified into two main categories. Buses at which generators are connected are called "Generating Buses" and the rest of the system buses are called "Nongenerating Buses". One of the generating buses is selected as the reference bus in a manner similar to that used in Load Flow studies. The individual generators connected at each generating bus are replaced by a single Equivalent Bus Generator. Generating buses correspond to power stations in the actual system. Thus, each power station in the physical system is represented by one equivalent generator in the first stage of the mathematical model. The parameters and quantities of this equivalent bus generator relevant to the various optimization problems considered, are derived in terms of the corresponding values of all the individual generators connected at the bus. Constraints and objective functions based on bus and line quantities are then set up to represent a number of different optimization problems. In each optimization problem the voltage magnitudes and phase angles of all system buses, except those of the reference bus, are used as the independent or decision variables. The optimization problem is then solved by linear programming techniques and standard computer subroutines.

The solution of the first stage of the model gives the values of all system bus voltage magnitudes and phase angles under optimum operating conditions. These can be used to calculate all other relevant quantities such as bus injections and system losses. The solution of the problem of optimum operating conditions can be terminated at the end of the first stage of the model or continued in the second stage if required. This depends on whether the main objective of the problem is based on bus or line quantities, or on generator quantities. It is to be remembered here that the first stage of the model is based on the concept of the equivalent bus generator and, therefore, does not deal with individual generator quantities.

The second stage of the model can be used to determine the output of each individual generator in the system. This can be achieved by formulating a new linear programming optimization problem based on generator quantities, with the objective of minimizing the hourly fuel cost. The active power outputs of the individual generators are used as the independent variables whose values are to be determined, with appropriate upper and lower operating limits. The constraints of the new optimization problem are based on the energy balance in the system and on the active power outputs of the generating buses. The active and reactive outputs of the equivalent bus generators can be obtained from the first stage of the model and they represent the contribution of each generating bus towards the total system demand and transmission loses. The outputs of the individual generators at a particular bus, on the other hand, represent the contribution of these generators towards the total output of that bus.

The optimization problem of the second stage of the model can then be solved using the same linear programming techniques and computer subroutines used to solve the optimization problem of the first stage.

Finally, the project considers the application of decomposition methods to the suggested mathematical model. The aim of this is to reduce the size or dimensionality of the various optimization problems addressed so that the suggested solution method can be applied to large-scale power systems. Decomposition has been based on utilizing the special structure of the power system problem and the sparsity of the associated matrices.

1.4 GENERAL LAYOUT OF THE THESIS

Apart from setting up a generalized and versatile mathematical model to represent optimum operating conditions in a power system, the present thesis also endeavours to give a general overview and a comprehensive study of the problem. In a way, as well as reporting on the actual work, the thesis also relates the author's "experience" throughout the various stages of the research project from searching for literature and reading on the subject to debugging computer programs and obtaining numerical results. This experience is reported in terms of problems that have been or are likely to be encountered in connection with the various theoretical, computational and practical aspects of the project as well as useful practical suggestions as to how these problems can be avoided.

The thesis consists of seven chapters as well as some additional material in the appendix. The present chapter has introduced the problem of power system operation, Section (1.1), and its reflection in the literature, Section (1.2), over a period of about 20 years of research in the field. Section (1.2) has also presented a general assessment of the various existing mathematical models and solution methods. Section (1.3) gave a general summary of the present research project and a brief description of the modelling framework.

The modelling process on which the present work is based, is explained in detail in Chapters 2 and 3. The salient features and important characteristics of the mathematical model are detailed in Chapter 2, while Chapter 3 shows how the problem is translated into a general linear programme.

Chapter 4 considers the inclusion of several other aspects of the power system operation problem into the suggested mathematical model. Examples of these additional aspects are tap-changing transformers and loading rates of generating units. In Chapter 5, specific optimization problems are set up, and the associated numerical results obtained are given. Chapter 6 is devoted to the application of decomposition methods to solve the same optimization problems using the mathematical model developed in the present research project. Discussion, conclusions, advantages of the suggested method and its practical application as well as suggestions for future work are the subject of Chapter 7. The appendix provides supplementary background material of a mathematical nature.

Throughout the process of writing the thesis, the reader has been kept constantly in mind. Considerable attention has been paid and a lot of effort has been made in the writing and presentation of the material. The justification for this stems from the fact that, in research circles, the proper reporting, writing and presentation of a piece of research work is as important as the work itself. Therefore, an attempt has been made to write and present a self-contained "reader-friendly" thesis although, obviously, this is not always possible in a work of this size and nature. This is particularly relevant to the mathematical background of the work.

Obviously, this thesis is not intended to be a detailed treatise on optimization and related topics from a purely mathematical point of view, as there is a large number of good textbooks that cover these areas of knowledge. However, mathematical programming in general, and linear programming in particular have direct and strong bearing on the mathematical formulation adopted in the present work. Therefore, as well as the mathematical topics discussed in the Appendix, similar material is also included in the main body of the thesis at the appropriate chapters or sections where the material is of immediate relevance and where that material is necessary for the comprehension of the rest of the section or chapter in question. All these additional topics are given here to serve as background material, supporting and complementing the rest of the thesis, and also for the general benefit of the readers, especially those who intend to follow this line of research. The same argument applies to some other specific topics such as linearization. In each case, the topic in question was given the necessary amount of emphasis to clarify all the relevant points. In the context of mathematical background of the work presented in this thesis, readers with some basic knowledge of mathematical programming, linear programming, the simplex method and other related topics have an obvious advantage in grasping the various concepts introduced in the thesis. At the other end of the spectrum, however, readers who do not possess any knowledge in these topics might need to read the Appendix as a prerequisite to reading the main text.

Having said that, however, an attempt has also been made to strike a balance between comprehensiveness of coverage and brevity of presentation. Wherever possible, any undue details have been avoided. This applies in particular to the mathematical derivations used in the thesis. Where appropriate, the general method of derivation is outlined and presented in a concise and clear manner starting from fundamental mathematical principles and relevant power system theory with the final results listed or "stacked" together at the end of the respective section or subsection. Thus, at various places in the thesis the reader will encounter "blocks" of mathematical expressions. For a first reading of the thesis, these blocks of equations can be skipped without affecting the understanding of the subject matter, although the mathematical expressions themselves are necessary for the rest of the theoretical analysis and the mathematical modelling.

Finally, the references given at the end of the thesis represent rather a short but carefully selected list from the overwhelmingly large amount of publications available on the subject. A helpful guide is also given on finding more references and further reading on the subject.

CHAPTER 2

MAIN FEATURES OF THE MATHEMATICAL MODEL

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CHAPTER 2

MAIN FEATURES OF THE MATHEMATICAL MODEL

2.1 INTRODUCTION

A general description of the mathematical model used in the present work has already been given in Section (1.3). The present chapter explains in detail the central ideas and underlying concepts that forms the basis of the modelling philosophy adopted in this thesis. The various aspects and particular features of the model are discussed in detail in separate sections. The material presented in these sections constitutes the Building Blocks of the suggested modelling structure and are treated here almost independently of each other. They will be put together in Chapter 3 to present a linear programming formulation of the mathematical model that represents power system operation. In what follows a general power system is considered, with NB busbars supplied by NG generators via a transmission network of NL lines.

2.2 THE TWO-STAGE FORMULATION

Most of the equations used in power system analysis, especially those related to the power flow equations, are based on bus quantities such as bus active and reactive power injections and bus voltage magnitudes and phase angles. In those equations no reference is made to the individual generator quantities and in most cases they are not explicitly needed in the analysis. There are some situations, however, where the analysis is directly based on individual generator quantities. An example of such situations is the calculation of the system generation cost which involves the active power output and fuel cost characteristics of individual generators. Considered on their own, these problems can be handled without major difficulties. There is a third category of problems where both bus and generator quantities are involved. When dealing with situations of this type, one problem that soon becomes obvious is the lack of explicit mathematical relationships between bus and generator quantities. An example of these situations is the minimization of the total system generation cost under constraints. At a generating bus, for instance, the total bus generation can be expressed in terms of the outputs of the individual generators connected at the bus. On the other hand, it can be expressed as an explicit function of voltage magnitudes and phase angles of all other system buses. However, there is no explicit mathematical expression that relates bus voltages and individual generator outputs. This can be illustrated by the following general mathematical expression.

$$F (X_i) = G (Y_j) \qquad (2.1)$$

$i = 1, 2, 3, \ldots , n$

$j = 1, 2, 3, \ldots , m$

In the above expression, F and G are general multivariable functions of the two sets of independent variables X_i and Y_j respectively. In a mathematical relationship of this type, it is impossible to express any of the independent variables of one side of the equation as an explicit function of the variables of the other. As mentioned above, in terms of power system quantities, such an expression can have generator variables on one side and bus variables on the other. The difficulty of the problem is increased by the nonlinearity and complexity of these mathematical relationships and the fact that they occur as sets, each consisting of a large number of equations.

Therefore, one of the considerations to be taken into account when dealing with such problems is bridging the gap between two almost separate sets of variables. Many of the algorithms published in the field of optimization tackle this problem by an iterative process between the two sets of mathematical relationships involving the two separate sets of variables. Starting from some initial solution, these algorithms generally involve calculating the functions from one set and checking the accuracy of the solution using the other set. From the result of this comparison some way is devised to improve the present solution. The iterative process is continued until the optimum solution is reached depending on a specified tolerance as a termination criterion. Basically this is the essence of the Newton-Raphson algorithm used to solve the power flow equations. The incorporation of the load flow into the optimization process has already been discussed in Section (1.2), in the course of assessment of existing power system optimization methods. As mentioned previously, this has the disadvantages of long CPU times and the need for algorithms and computer routines specially designed for the solution of power system optimization problems.

In the present thesis, the difficulty caused by the lack of explicit mathematical expressions that relate generator and bus variables is overcome by introducing the concept of the equivalent bus generator and employing the two-stage solution strategy. The first stage of the model is based on bus and line quantities with bus voltage magnitudes and phase angles as the principle independent variables whose values are to be determined. Specific values are assigned to the voltage magnitude and phase angle of the reference bus beforehand. Thus, the first stage of the model has a total of 2N independent variables. The second stage of the model is based on individual generator quantities, with the NG generator active power outputs as the independent variables. No iteration is needed between the two stages and the solution of the overall problem proceeds sequentially from Stage-I to Stage-II with obvious algorithmic and computational advantages. As mentioned earlier, depending on the main objective of the optimization problem the solution can be terminated at the end of the first stage or carried on to the second stage. However, it must be noted here that the second stage cannot be solved on its own, as some of its input parameters can only be obtained after the solution of the first stage has been completed.

2.3 THERMAL GENERATOR COST CHARACTERISTICS

The relationship between the active power output of a thermal generator and its hourly input fuel cost is represented by the generator cost function. This is a nonlinear mathematical expression of the general from of (2.2) below.

$$FG = C_0 + C_1 \times PG + C_2 \times PG^2 + C_3 \times PG^3 \qquad (2.2)$$

where the C's are constants and PG is the generator active power output.

A typical graph of such a characteristics is shown in FIG.(2.1).

The constants C associated with the quadratic and higher power terms of (2.2) are usually very small and the value of the hourly generation cost is mainly dictated by the first two terms. Therefore, (2.2) can be approximated by:

$$FG = C_0 + C_1 \times PG \tag{2.3}$$

This is a straight line equation and is also shown in FIG.(2.1), where the original nonlinear characteristic is approximated by a straight line segment between operating limits. This approximation is adequate for most practical purposes and is useful where a linear mathematical model is used to handle problems involving generation cost calculations as in the present work.

Furthermore, the fixed term C_0 in (2.2) can be ignored if FG or other quantities based on it are to be used as objective functions for optimization purposes. A fixed term in the objective function of an optimization problem does not affect the optimization process, i.e., the systematic mathematical search for the optimum solution, or its outcome, namely, the set of values of the independent variables that give the optimum solution. Thus, for the purposes of the present mathematical model, the generator fuel cost function (2.3) can be further reduced to:

$$FG = CG \times PG \tag{2.4}$$

The constant CG = C_1 is usually referred to as the Incremental Generation Cost.

Generally, the reduction of the fuel cost characteristics and the number of terms discarded or retained depends on the required accuracy of calculations and results, and also on the availability of the data, i.e., the constants C_i of the cost characteristic.

The cost function can also be approximated by a number of successive straight line segments [10]. This is shown in FIG.(2.2). The straight line segments used can be of equal or different lengths depending on the shape and degree of nonlinearity of the original function. When using this method, the accuracy of the approximation depends on the number of line segments used. Increasing the number of these line segments, thus decreasing the length of each segment, gives a better fitting between the original nonlinear characteristic and the resulting linear approximation. Piecewise linear approximation of this type is not confined to generator fuel cost characteristics. It is a general method that can be applied to any smooth continuous nonlinear function. It has the advantage of combining high accuracy and applicability to linear analysis at the same time. It, however, has the disadvantage of increasing the required amount of computation. The choice of this type of linearization depends on the required degree of accuracy and also on whether it is worth the additional computations involved [15].

2.4 PARAMETERS OF THE EQUIVALENT BUS GENERATOR

As mentioned earlier, the mathematical model used in this work consists of two stages. In the first stage of the model the generators connected at each generating bus are replaced by a single Equivalent Bus Generator. In this section the relevant parameters of this equivalent bus generator are derived in terms of the corresponding parameters of all the individual generators connected to the bus. The relationships among the various bus and generator quantities and the notation used are illustrated in FIG.(2.3).

The active and reactive power output of the equivalent bus generator and its upper and lower operating limits are given by the summation of all the corresponding individual generator quantities, as shown by Equations (2.5) to (2.10) below.

$$PGB_i = \sum_{j=1}^{NGB_i} PGG_j \tag{2.5}$$

$$QGB_i = \sum_{j=1}^{NGB_i} QGG_j \tag{2.6}$$

$$PGBMX_i = \sum_{j=1}^{NGB_i} PGGMX_j \tag{2.7}$$

$$PGBMN_i = \sum_{j=1}^{NGB_i} PGGMN_j \qquad (2.8)$$

$$QGBMX_i = \sum_{j=1}^{NGB_i} QGGMX_j \qquad (2.9)$$

$$QGBMN_i = \sum_{j=1}^{NGB_i} QGGMN_j \qquad (2.10)$$

Another relevant parameter of the equivalent bus generator is the equivalent bus incremental cost. This parameter cannot be obtained by a straightforward summation as shown above for the various active and reactive power quantities. Its derivation proceeds as follows.

The generation cost at the i'th bus is given by:

$$FB_i = \sum_{j=1}^{NGB_i} CG_j \times PGG_j \qquad (2.11)$$

From Equation (2.4), the incremental generation cost of the j'th generator is given by:

$$CG_j = \frac{FG_j}{PG_j} \qquad (2.12)$$

Thus, CG_j can be defined as:

$$CG_j = \frac{\text{Generation Cost of the j'th Generator}}{\text{Active Power Output of the Generator}} \qquad (2.13)$$

Similarly, the incremental cost at the i'th bus can be defined as:

$$CB_i = \frac{\text{Generation Cost at the Bus}}{\text{Total Active Power Output of the Bus}} \qquad (2.14)$$

From (2.5) and (2.11), this can be expressed as:

$$CB_i = \frac{\sum_{j=1}^{NGB_i} CG_j \times PGG_j}{\sum_{j=1}^{NGB_i} PGG_j} \qquad (2.15)$$

Theoretically, the cost characteristic of a generating bus cannot be represented by an analytical function as that of Equations (2.2) or (2.4). However, a graph of the characteristic can be plotted by using a suitably large number of operating points along the cost characteristics of all the individual generators. Since straight line approximation is used to represent (2.2), the resulting bus cost characteristic will, also, be a straight line. Two points will be sufficient to define the required characteristic from which CB can be evaluated. A good approximation of CB will be obtained if the selected two points are well spaced along the characteristic. The extreme points of the characteristic are chosen for this purpose and the incremental cost of the i'th bus is evaluated as in (2.16).

$$CB_i = \frac{CBMN_i + CBMX_i}{2} \qquad (2.16)$$

where $CBMN_i$ and $CBMX_i$ are the values of the incremental costs calculated as in (2.15) above, corresponding to the minimum and maximum active power outputs of all the generators connected at the i'th bus respectively.

In full, CB_i is given by (2.17) below.

$$CB_i = \frac{\dfrac{\sum_{j=1}^{NGB_i} CG_j \times PGGMN_j}{\sum_{j=1}^{NGB_i} PGGMN_j} + \dfrac{\sum_{j=1}^{NGB_i} CG_j \times PGGMX_j}{\sum_{j=1}^{NGB_i} PGGMX_j}}{2} \qquad (2.17)$$

2.5 LINEARIZATION

The equations that describe power system relationships are generally nonlinear, involving quadratic and higher power terms and trigonometric functions. The present work is based on formulating the various aspects of the problem of power system optimum operating conditions as a number of linear programming problems. Therefore, before the mathematical model of the problem can be set up, all the relevant power system equations must be linearized.

There is a number of different linearization methods. Some of these have already been described in Section (2.3), in association with thermal generator cost functions. Other methods are discussed here. In complex mathematical expressions, i.e., expressions that involve real and imaginary parts, some form of linearization and simplification of the analysis can be achieved by ignoring one of the constituent parts of the complex quantity, if its value or effect on the whole is very small compared to the other part. The justification for this depends on the given numerical data and on the nature of the physical problem. It also depends on whether a very accurate representation and rigorous analysis of the physical system is wanted or only a fast approximate solution is required. Whichever part of the complex quantity is ignored, the rest of the analysis is carried out in terms of real variables since, even when the remaining variables are imaginary, the complex operator notation, i or j, is dropped. In power system analysis, the mathematical formulation can be simplified, and an approximate solution obtained, by neglecting the resistive part of the impedance of power system components, such as transmission lines and transformers, and treating them as pure reactances. As discussed in

Chapter 1, this has the disadvantage that the resulting mathematical expressions do not give true representation of the actual power system.

Another widely used method of linearization is based on the Taylor's series expansion of the given nonlinear function [16]. Various degrees of accuracy can be obtained depending on the number of terms included. A linear expression is obtained by including only the first two terms of the series. This method involves the use of derivatives, and in multivariable functions partial derivatives have to be used.

In this thesis linearization is an important aspect that plays a major role in the modelling and solution of the problem. Therefore, it is given some emphasis and is explained in some detail in this section.

The linearization technique employed here is based on the concept of incremental change in the values of the system independent and dependent variables around a known initial point. The term incremental change in this context is used to mean a change, positive or negative, in the value of a variable which is very small in comparison with the initial value of the variable. The step by step linearization procedure is given below in general terms followed by an example [17].

1. Consider the general nonlinear function $Y = F(X)$ for which the data of an initial point is available; $Y_o = F(X_o)$.

2. Assume that there is an incremental change $\triangle X$ in the value of the independent variable and substitute each X by ($X_o + \triangle X$) in the original function Y.

3. Expand the function $Y = F(X_o + \triangle X)$.

4. Ignore any terms that involve quadratic and higher powers of $\triangle X$ and any other functions of $\triangle X$ of negligible value.

5. Substitute each $\triangle X$ back by $(X-X_o)$.

The resulting equation from step (5) is an approximate linearized version of the original nonlinear function Y, and is accurate enough as long as the incremental changes in the values of the independent variables are very small compared with their initial values.

It is to be emphasized at this point that the substitution of step (5) should be performed after the reduction of step (4).

The generality of the method cannot be explained any further and is better illustrated by a specific example. The same steps 1 to 5 above are followed.

Example:

1. Consider the function $Y = X^2$, $Y_o = X_o^2$.

2. $Y = (X_o + \triangle X)^2$

3. $Y = X_o^2 + 2X_o\triangle X + \triangle^2 X$

4. $Y = X_o^2 + 2X_o\triangle X$

5. $Y = 2X_oX - X_o^2$

The last form of the function is a straight line equation that can be rewritten as follows:

$$Y = AX + B \tag{2.18}$$

where,

$$A = 2X_o$$

and

$$B = - X_o^2$$

It is to be noted here that the resulting linear equation is expressed in terms of the original variables Y and X, not the incremental changes $\triangle Y$ and $\triangle X$ as used in the literature [18]. Also, both the constants A and B are functions of the initial point X_o.

The above is a rather simplified example for the purpose of illustration but the method can be extended to multivariable functions of any degree of complexity and nonlinearity.

Useful Results

Below are some particular linearization results of general use for the derivation of the linearized versions of the various power system equations used in the thesis.

1. For a function

$$F = AF1 + BF2 \tag{2.19}$$

the following result can be deduced:

$$F = F_o + A\triangle F1 + B\triangle F2 \tag{2.20}$$

2. Trigonometric functions, in particular, can be linearized by using the following approximations.

For a small angle $\triangle\theta$ measured in radians:

$$\sin \triangle\theta \approx \triangle\theta \tag{2.21}$$

$$\cos \triangle\theta \approx 1 \tag{2.22}$$

Applying the linearization procedure explained above, the following results can be obtained.

$$\sin \theta = (\cos \theta_o) \theta + \sin \theta_o - \theta_o \cos \theta_o \tag{2.23}$$

$$\cos \theta = - (\sin \theta_o) \theta + \cos \theta_o + \theta_o \sin \theta_o \tag{2.24}$$

Both (2.23) and (2.24) can be expressed in the general form of the straight line equation (2.18) with θ as the independent variable and:

$A = (\cos \theta_o)$, and $B = \sin \theta_o - \theta_o \cos \theta_o$, for the sine function and,

$A = -(\sin \theta_o)$, and $B = \cos \theta_o + \theta_o \sin\theta_o$, for the cosine function.

3. Many mathematical expressions in power system theory have the following general form:

$$F = XY \, (A \sin \theta + B \cos \theta) \qquad (2.25)$$

The linearized version of F is given by:

$$\begin{aligned} F = {} & Y_o \, (A \sin \theta_o + B \cos \theta_o) \, X \\ & + X_o \, (A \sin \theta_o + B \cos \theta_o) \, Y \\ & + X_o Y_o \, (A \cos \theta_o - B \sin \theta_o) \, \theta \\ & + X_o Y_o \theta_o \, (B \sin \theta_o - A \cos \theta_o) - F_o \end{aligned} \qquad (2.26)$$

This can serve as a good illustration of the linearization of a multivariable function using the linearization procedure described in this section. The nonlinear function F has three independent variables, namely, X, Y and θ. The linearized version shows the coefficients associated with each of the variables and the constant terms which are functions of the initial values of the dependent and independent variables.

2.6 THE LINEARIZED POWER SYSTEM EQUATIONS

The first stage of the model is based on linearized versions of the mathematical equations that describe the relationships among the various bus and line quantities. There is a certain number of such power system equations which represent the core of power system theory and used in power system analysis. Collectively they give a complete picture of the power system steady-state normal operating conditions and, consequently they are used in the present work to represent the various optimization problems [19-22]. These equations fall into three distinct sets.

1. The net bus active and reactive power injections. These are defined below:

$$P_i = PGB_i - PD_i \tag{2.27}$$

$$Q_i = QGB_i - QD_i \tag{2.28}$$

$i = 1, 2, 3, \ldots, NB$

2. Active and reactive power line flows. For a line connecting node i to node j, these depend on whether the power flow is from node i to node j or from node j to node i. Therefore, for each line in the system there are four such quantities, namely, P_{ij}, P_{ji}, Q_{ij}, and Q_{ji}.

3. Active and reactive power losses.

For the k'th line, connecting bus i to bus j, these are given by:

$$PL_k = P_{ij} + P_{ji} \qquad (2.29)$$

$$QL_k = Q_{ij} + Q_{ji} \qquad (2.30)$$

$$k = 1, 2, 3, \ldots, NL$$

All the relevant mathematical equations have been linearized using the procedure described in Section (2.5) and the final results are given below. Each equation is given first in its original nonlinear form, followed by the linearized version with both forms expressed as explicit functions of the independent variables V_i and θ_i.

The π-equivalent circuit is used to represent transmission lines, and the relationships among the various bus and line quantities are shown in FIG.(2.4).

<u>Net Bus Active Power Injection</u>

$$P_i = \sum_{j=1}^{N} V_i \; V_j \; (G_{ij} \cos\theta_{ij} + B_{ij} \sin \theta_{ij}) \qquad (2.31)$$

$$\begin{aligned} P_i = [& \sum_{j=1}^{N} V_{jo} \; (G_{ij} \cos \theta_{ijo} + B_{ij} \sin \theta_{ijo}) \; V_i \\ & + V_{io} \; (G_{ij} \cos \theta_{ijo} + B_{ij} \sin \theta_{ijo}) \; V_j \\ & + V_{io} \; V_{jo} \; (B_{ij} \cos \theta_{ijo} - G_{ij} \sin \theta_{ijo}) \; \theta_i \\ & + V_{io} \; V_{jo} \; (G_{ij} \sin \theta_{ijo} - B_{ij} \cos \theta_{ijo}) \; \theta_j \\ & + V_{io} \; V_{jo} \; \theta_{ijo} \; (G_{ij} \sin \theta_{ijo} \\ & \qquad - B_{ij} \cos \theta_{ijo}) \;] - P_{io} \end{aligned} \qquad (2.32)$$

Net Bus Reactive Power Injection

$$Q_i = \sum_{j=1}^{N} V_i \; V_j \; (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \qquad (2.33)$$

$$\begin{aligned} Q_i = [& \sum_{j=1}^{N} V_{jo} \; (G_{ij} \sin \theta_{ijo} - B_{ij} \cos \theta_{ijo}) \; V_i \\ & + V_{io} \; (G_{ij} \sin \theta_{ijo} - B_{ij} \cos \theta_{ijo}) \; V_j \\ & + V_{io} \; V_{jo} \; (G_{ij} \cos \theta_{ijo} + B_{ij} \sin \theta_{ijo}) \; \theta_i \\ & - V_{io} \; V_{jo} \; (G_{ij} \cos \theta_{ijo} + B_{ij} \sin \theta_{ijo}) \; \theta_j \\ & - V_{io} \; V_{jo} \; \theta_{ijo} \; (G_{ij} \cos \theta_{ijo} \\ & \qquad + B_{ij} \sin \theta_{ijo}) \;] - Q_{io} \end{aligned} \qquad (2.34)$$

Active Power Transfer Across Line (I-J)

$$\begin{aligned} P_{ij} = Y_{ij}^2 \; R_{ij} \; V_i^2 + Y_{ij}^2 \; V_i \; V_j \; (& X_{ij} \sin \theta_{ij} \\ & - R_{ij} \cos \theta_{ij}) \end{aligned} \qquad (2.35)$$

$$\begin{aligned} P_{ij} = \; & Y_{ij}^2 \; [\; 2 \; R_{ij} \; V_{io} \\ & + V_{jo} \; (X_{ij} \sin \theta_{ijo} - R_{ij} \cos \theta_{ijo}) \;] \; V_i \\ & + Y_{ij}^2 \; V_{io} \; (X_{ij} \sin \theta_{ijo} - R_{ij} \cos \theta_{ijo}) \; V_j \\ & + Y_{ij}^2 \; V_{io} \; V_{jo} \; (R_{ij} \sin \theta_{ijo} + X_{ij} \cos \theta_{ijo}) \; \theta_i \\ & - Y_{ij}^2 \; V_{io} \; V_{jo} \; (R_{ij} \sin \theta_{ijo} + X_{ij} \cos \theta_{ijo}) \; \theta_j \\ & - Y_{ij}^2 \; V_{io} \; V_{jo} \; \theta_{ijo} \; (R_{ij} \sin \theta_{ijo} \\ & \qquad + X_{ij} \cos \theta_{ijo}) - P_{ijo} \end{aligned} \qquad (2.36)$$

Active Power Transfer Across Line (J-I)

$$\begin{aligned} P_{ji} = Y_{ij}^2 \; R_{ij} \; V_j^2 - Y_{ij}^2 \; V_i \; V_j \; (& X_{ij} \sin \theta_{ij} \\ & + R_{ij} \cos \theta_{ij}) \end{aligned} \qquad (2.37)$$

$$\begin{aligned} P_{ji} = & - Y_{ij}^2 \; V_{jo} \; (X_{ij} \sin \theta_{ijo} + R_{ij} \cos \theta_{ijo}) \; V_i \\ & + Y_{ij}^2 \; [\; 2 \; R_{ij} \; V_{jo} - V_{io} \; (X_{ij} \sin \theta_{ijo} \\ & \qquad + R_{ij} \cos \theta_{ijo}) \;] \; V_j \\ & + Y_{ij}^2 \; V_{io} \; V_{jo} \; (R_{ij} \sin \theta_{ijo} - X_{ij} \cos \theta_{ijo}) \; \theta_i \\ & + Y_{ij}^2 \; V_{io} \; V_{jo} \; (X_{ij} \cos \theta_{ijo} - R_{ij} \sin \theta_{ijo}) \; \theta_j \\ & + Y_{ij}^2 \; V_{io} \; V_{jo} \; \theta_{ijo} \; (R_{ij} \sin \theta_{ijo} \\ & \qquad - X_{ij} \cos \theta_{ijo}) - P_{jio} \end{aligned} \qquad (2.38)$$

Reactive Power Transfer Across Line (I-J)

$$Q_{ij} = (Y_{ij}^2 X_{ij} - T_{ij}) V_i^2 - Y_{ij}^2 V_i V_j (R_{ij} \sin \theta_{ij} + X_{ij} \cos \theta_{ij}) \quad (2.39)$$

$$\begin{aligned} Q_{ij} = & [2 (Y_{ij}^2 X_{ij} - T_{ij}) V_{io} \\ & - Y_{ij}^2 V_{jo} (R_{ij} \sin \theta_{ijo} + X_{ij} \cos \theta_{ijo})] V_i \\ & - Y_{ij}^2 V_{io} (R_{ij} \sin \theta_{ijo} + X_{ij} \cos \theta_{ijo}) V_j \\ & + Y_{ij}^2 V_{io} V_{jo} (X_{ij} \sin \theta_{ijo} - R_{ij} \cos \theta_{ijo}) \theta_i \\ & + Y_{ij}^2 V_{io} V_{jo} (R_{ij} \cos \theta_{ijo} - X_{ij} \sin \theta_{ijo}) \theta_j \\ & + Y_{ij}^2 V_{io} V_{jo} \theta_{ijo} (R_{ij} \cos \theta_{ijo} \\ & \qquad - X_{ij} \sin \theta_{ijo}) - Q_{ijo} \end{aligned} \quad (2.40)$$

Reactive Power Transfer Across Line (J-I)

$$Q_{ji} = (Y_{ij}^2 X_{ij} - T_{ij}) V_j^2 + Y_{ij}^2 V_i V_j (R_{ij} \sin \theta_{ij} - X_{ij} \cos \theta_{ij}) \quad (2.41)$$

$$\begin{aligned} Q_{ji} = & Y_{ij}^2 V_{jo} (R_{ij} \sin \theta_{ijo} - X_{ij} \cos \theta_{ijo}) V_i \\ & + [2 (Y_{ij}^2 X_{ij} - T_{ij}) V_{jo} \\ & + Y_{ij}^2 V_{io} (R_{ij} \sin \theta_{ijo} - X_{ij} \cos \theta_{ijo})] V_j \\ & - Y_{ij}^2 V_{io} V_{jo} (X_{ij} \sin \theta_{ijo} + R_{ij} \cos \theta_{ijo}) \theta_i \\ & + Y_{ij}^2 V_{io} V_{jo} (X_{ij} \sin \theta_{ijo} + R_{ij} \cos \theta_{ijo}) \theta_j \\ & - Y_{ij}^2 V_{io} V_{jo} \theta_{ijo} (R_{ij} \cos \theta_{ijo} \\ & \qquad + X_{ij} \sin \theta_{ijo}) - Q_{jio} \end{aligned} \quad (2.42)$$

Active Power Loss In A Transmission Line

$$PL_k = Y_{ij}^2 R_{ij} V_i^2 + Y_{ij}^2 R_{ij} V_j^2 - 2 Y_{ij}^2 R_{ij} V_i V_j \cos \theta_{ij} \quad (2.43)$$

$$\begin{aligned} PL_k = & 2 Y_{ij}^2 R_{ij} (V_{io} - V_{jo} \cos \theta_{ijo}) V_i \\ & + 2 Y_{ij}^2 R_{ij} (V_{jo} - V_{io} \cos \theta_{ijo}) V_j \\ & + (2 Y_{ij}^2 R_{ij} V_{io} V_{jo} \sin \theta_{ijo}) \theta_i \\ & - (2 Y_{ij}^2 R_{ij} V_{io} V_{jo} \sin \theta_{ijo}) \theta_j \\ & - 2 Y_{ij}^2 R_{ij} V_{io} V_{jo} \theta_{ijo} \sin \theta_{ijo} - PL_{ko} \end{aligned} \quad (2.44)$$

Reactive Power Loss in a Transmission Line

$$QL_k = (Y_{ij}^2 X_{ij} - T_{ij}) V_i^2 + (Y_{ij}^2 X_{ij} - T_{ij}) V_j^2 - 2 Y_{ij}^2 X_{ij} V_i V_j \cos \theta_{ij} \quad (2.45)$$

$$
\begin{aligned}
QL_k = {} & 2\,[\,(Y_{ij}^2\ X_{ij} - T_{ij})\ V_{io} \\
& - Y_{ij}^2\ X_{ij}\ V_{jo}\cos\theta_{ijo}\,]\ V_i \\
& + 2\,[\,(Y_{ij}^2\ X_{ij} - T_{ij})\ V_{jo} \\
& - Y_{ij}^2\ X_{ij}\ V_{io}\cos\theta_{ijo}\,]\ V_j \\
& + (\,2\ Y_{ij}^2\ X_{ij}\ V_{io}\ V_{jo}\sin\theta_{ijo}\,)\,\theta_i \\
& - (\,2\ Y_{ij}^2\ X_{ij}\ V_{io}\ V_{jo}\sin\theta_{ijo}\,)\ \theta_j \\
& - 2\ Y_{ij}^2\ X_{ij}\ V_{io}\ V_{jo}\ \theta_{ijo}\sin\theta_{ijo} - QL_{ko}
\end{aligned} \tag{2.46}
$$

<u>Complex and Apparent Power Relationships</u>

In the analysis so far, the two components of the complex power have been dealt with separately. The main relationships among the three quantities, P, Q and S are summarized by (2.47) to (2.50) below.

$$S = P + j\,Q \tag{2.47}$$

$$S^2 = P^2 + Q^2 \tag{2.48}$$

$$S = \sqrt{(\,P^2 + Q^2\,)} \tag{2.49}$$

$$S = \frac{P_o}{S_o}\,P + \frac{Q_o}{S_o}\,Q \tag{2.50}$$

Equation (2.50) is the linearized version of (2.49). The derivation of (2.50) can be started by expressing Equation (2.48) in terms of the incremental values of its variables as in (2.51).

$$(\,S_o + \triangle S\,)^2 = (\,P_o + \triangle P\,)^2 + (\,Q_o + \triangle Q\,)^2 \tag{2.51}$$

Equation (2.51) is then fully expanded and terms involving quadratic incremental changes are ignored. The rest of the derivation proceeds as explained in Section (2.5).

The general apparent power relationships (2.47) to (2.50) apply to load, generation, bus injections, power transfer and transmission losses. The expanded forms of these equations in terms of system voltage magnitudes and phase angles can be obtained by substituting for P and Q from the appropriate equations (2.31-2.46) above.

Finally, it is to be mentioned here, in the context of linearization, that the equation of one power system quantity, namely, the hourly generation cost, has already been linearized in Section (2.3). However, the linearization principle used in that particular case is different from the procedure explained in this section. The linearization of the cost function was based on the small values of the constants associated with the variables rather than the incremental changes in the values of the variables themselves.

2.7 THE REFERENCE BUS

The two general equations (2.27) and (2.28) that define the bus injections apply to all system buses including the reference. It is to be noticed, however, that the expanded forms (2.31) and (2.33), which give the power injections in terms of system voltages, apply to all system buses except the reference. Therefore, special attention has to be paid to the reference bus when deriving the power injection equations corresponding to (2.31) and (2.33). The derivation proceeds as follows.

The total system active and reactive power generation, demand, and losses are given by Equations (2.52) to (2.57) below.

$$PGT = \sum_{i=1}^{NB} PGB_i \tag{2.52}$$

$$PDT = \sum_{i=1}^{NB} PD_i \tag{2.53}$$

$$PLT = \sum_{k=1}^{NL} PL_k \tag{2.54}$$

$$QGT = \sum_{i=1}^{NB} QGB_i \tag{2.55}$$

$$QDT = \sum_{i=1}^{NB} QD_i \tag{2.56}$$

$$QLT = \sum_{k=1}^{NL} QL_k \tag{2.57}$$

The generation and demand quantities, Equations (2.52), (2.53), (2.55) and (2.56), can alternatively be written as shown in the corresponding equations (2.58) to (2.61) below, where each total system quantity is written as the sum of two separate parts. One part corresponds to the reference bus while the other corresponds to the rest of the system buses collectively.

$$PGT = PGB_r + \sum_{i=1}^{N} PGB_i \tag{2.58}$$

$$PDT = PD_r + \sum_{i=1}^{N} PD_i \tag{2.59}$$

$$QGT = QGB_r + \sum_{i=1}^{N} QGB_i \tag{2.60}$$

$$QDT = QD_r + \sum_{i=1}^{N} QD_i \tag{2.61}$$

The energy balance equations in the system are given by:

$$PGT = PDT + PLT \tag{2.62}$$

$$QGT = QDT + QLT \tag{2.63}$$

Substituting from (2.54), (2.58) and (2.59) into (2.62) and rearranging give:

$$PGB_r - PD_r = \sum_{i=1}^{N} (PD_i - PGB_i) + \sum_{k=1}^{NL} PL_k \tag{2.64}$$

Similarly, the corresponding reactive power equation is given by:

$$QGB_r - QD_r = \sum_{i=1}^{N} (QD_i - QGB_i) + \sum_{k=1}^{NL} QL_k \tag{2.65}$$

Using the defining equations of the active and reactive power injections, (2.27) and (2.28) respectively, (2.64) and (2.65) can be written as follows:

$$P_r = \left(\sum_{k=1}^{NL} PL_k \right) - \sum_{i=1}^{N} P_i \qquad (2.66)$$

$$Q_r = \left(\sum_{k=1}^{NL} QL_k \right) - \sum_{i=1}^{N} Q_i \qquad (2.67)$$

The above two expressions give the net power injections at the reference bus in terms of the net power injections of the rest of the system buses and the total system losses. The full nonlinear and linear versions of (2.66) and (2.67) in terms of system voltage magnitudes and phase angles can be obtained by substituting from the appropriate expanded equations of Section (2.6).

An alternative method of obtaining the same results is by introducing two new quantities, PT and QT. These are the net active and reactive power injections into the whole power system and are defined as equal to the difference between the total system generation and the total system demand. In mathematical terms these are given by:

$$PT = PGT - PDT \qquad (2.68)$$

$$QT = QGT - QDT \qquad (2.69)$$

The rest of the derivation proceeds, as before, by substituting from the appropriate equations (2.58) to (2.61).

It is interesting to note that the net total power injection into the system is equal to the system losses, which is already stated mathematically in Equations (2.62) and (2.63).

Taking the reference bus into account, there is, thus, a total of 2NB power injection equations in contrast with the 2N equations used in load flow studies. In the latter case the reference bus injections are obtained after a complete run of the load flow and the calculation of all transmission losses. In the present work the effect of the reference bus and transmission losses are incorporated as constituent parts in the problem model.

2.8 THE INITIAL OPERATING POINT

As explained earlier, the relevant power system equations have been linearized using the concept of incremental changes around a given initial operating point. The data involved includes the initial values of all bus voltage magnitudes and phase angles, active and reactive power injections, active and reactive power losses and power transfers in all transmission lines. The availability of all these data is essential as a starting point for the setting up of the linear mathematical model. An approximate initial operating point can be estimated as follows.

The given total system active and reactive load is divided among all system generators in proportion to their capacities, measured by the corresponding upper limits on their outputs. Thus, the initial active and reactive outputs of the j'th generator are given by:

$$\text{Output Power of the j'th Generator} = \left(\frac{\text{Capacity of the j'th Generator}}{\text{Total System Capacity}}\right) \times \text{Total Load on the System} \tag{2.70}$$

For this purpose, the total active and reactive system capacities are defined by:

$$PGTMX = \sum_{j=1}^{NG} PGGMX_j \tag{2.71}$$

$$QGTMX = \sum_{j=1}^{NG} QGGMX_j \tag{2.72}$$

Alternatively, in terms of generating bus quantities, PGTMX and QGTMX are given by:

$$PGTMX = \sum_{j=1}^{NS} PGBMX_j \tag{2.73}$$

$$QGTMX = \sum_{j=1}^{NS} QGBMX_j \tag{2.74}$$

Using these definitions, the initial active and reactive power generations can be expressed as follows:

$$PGG_{jo} = \frac{PGGMX_j}{PGTMX} \times PDT \qquad (2.75)$$

$$QGG_{jo} = \frac{QGGMX_j}{QGTMX} \times QDT \qquad (2.76)$$

These are used to calculate the initial active and reactive power outputs of the bus equivalent generators from Equations (2.5) and (2.6). Alternatively these can be obtained using the following two equations:

$$PGB_{io} = \frac{PGBMX_i}{PGTMX} \times PDT \qquad (2.77)$$

$$QGB_{io} = \frac{QGBMX_i}{QGTMX} \times QDT \qquad (2.78)$$

In (2.77) and (2.78), the initial values of the active and reactive power outputs of the generating buses are obtained by dividing the total system demand amongst these buses in proportion to their capacities measured by their maximum generation, in a similar way to that of expression (2.70). The active and reactive bus injections can then be obtained using Equations (2.27) and (2.28) respectively.

All initial values of bus voltage magnitudes and phase angles can then be computed by performing a load flow. Finally all initial line flows and transmission losses can be calculated using the appropriate nonlinear equations from Section (2.6).

2.9 SUMMARY

This chapter has presented the main features and fundamental ideas that form the basis of the mathematical model developed in the present research project. These can be summarised as follows.

1. A two-stage formulation is adopted to represent the interrelationships among the various power system quantities. The first stage is based on bus and line quantities such as bus injections and line flows. The second stage is based on individual generator quantities such as generator outputs and generation costs.

2. The two stages of the model are linked by the concept of the equivalent bus generator which replaces the individual generators at each generating node. The respective section details the process of obtaining the relevant parameters of these lumped generators in terms of the corresponding quantities of the individual generators.

3. The concept of incremental modelling is used to linearize the various mathematical relationships involved. The linearization procedure is explained first and then the final linearized power system equations are given.

4. A method is suggested to obtain the data of an initial operating point on which the concept of incremental modelling is based.

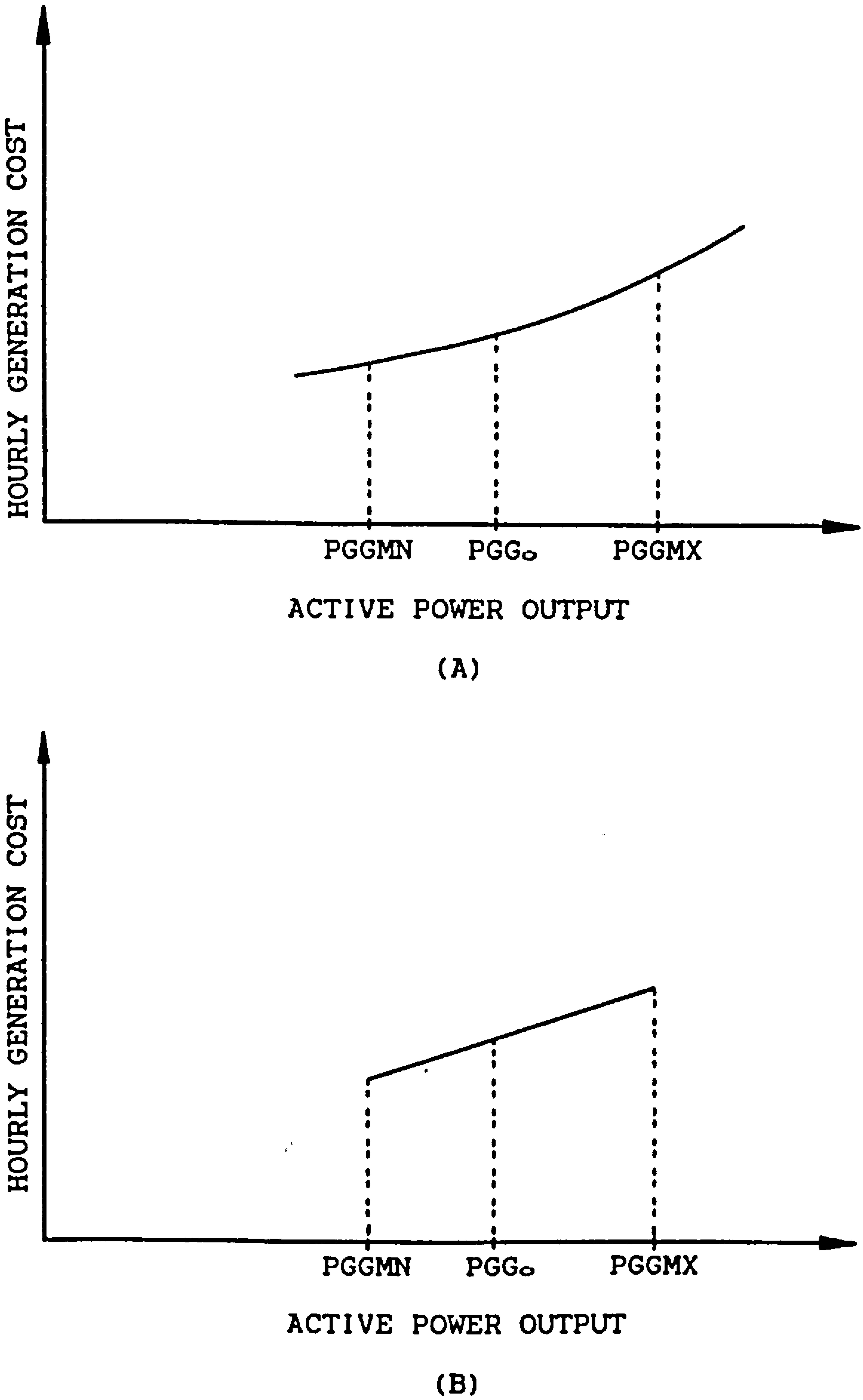

FIG.(2.1)

THERMAL GENERATOR FUEL COST CHARACTERISTICS

(A) TYPICAL NONLINEAR FORM

(B) LINEAR APPROXIMATION BETWEEN OPERATING LIMITS

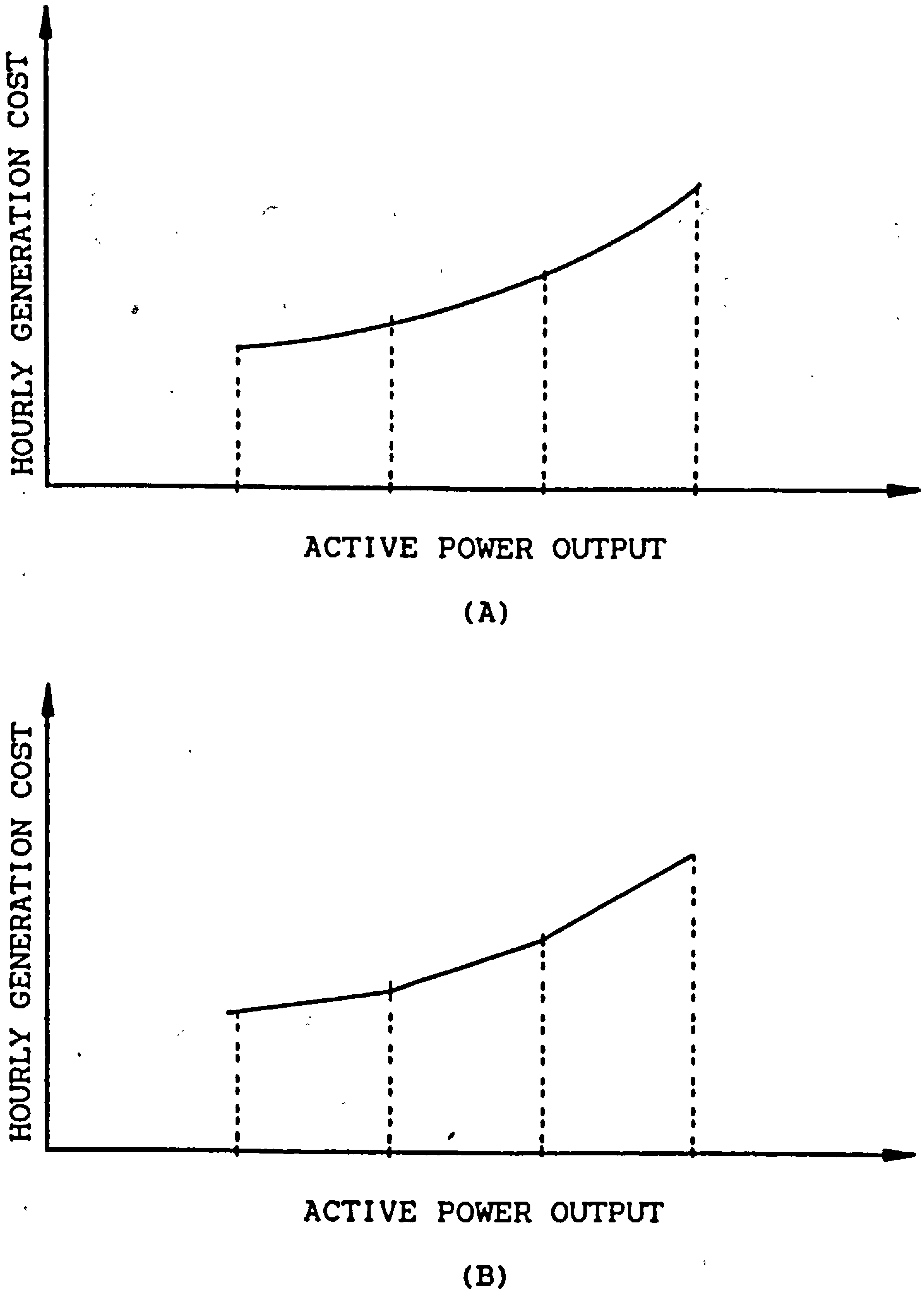

FIG.(2.2)

PIECEWISE APPROXIMATION OF A NONLINEAR COST CHARACTERISTIC USING SEVERAL STRAIGHT LINE SEGMENTS

(A) THE ORIGINAL NONLINEAR FUNCTION

(B) APPROXIMATION BY THREE STRAIGHT LINE SEGMENTS

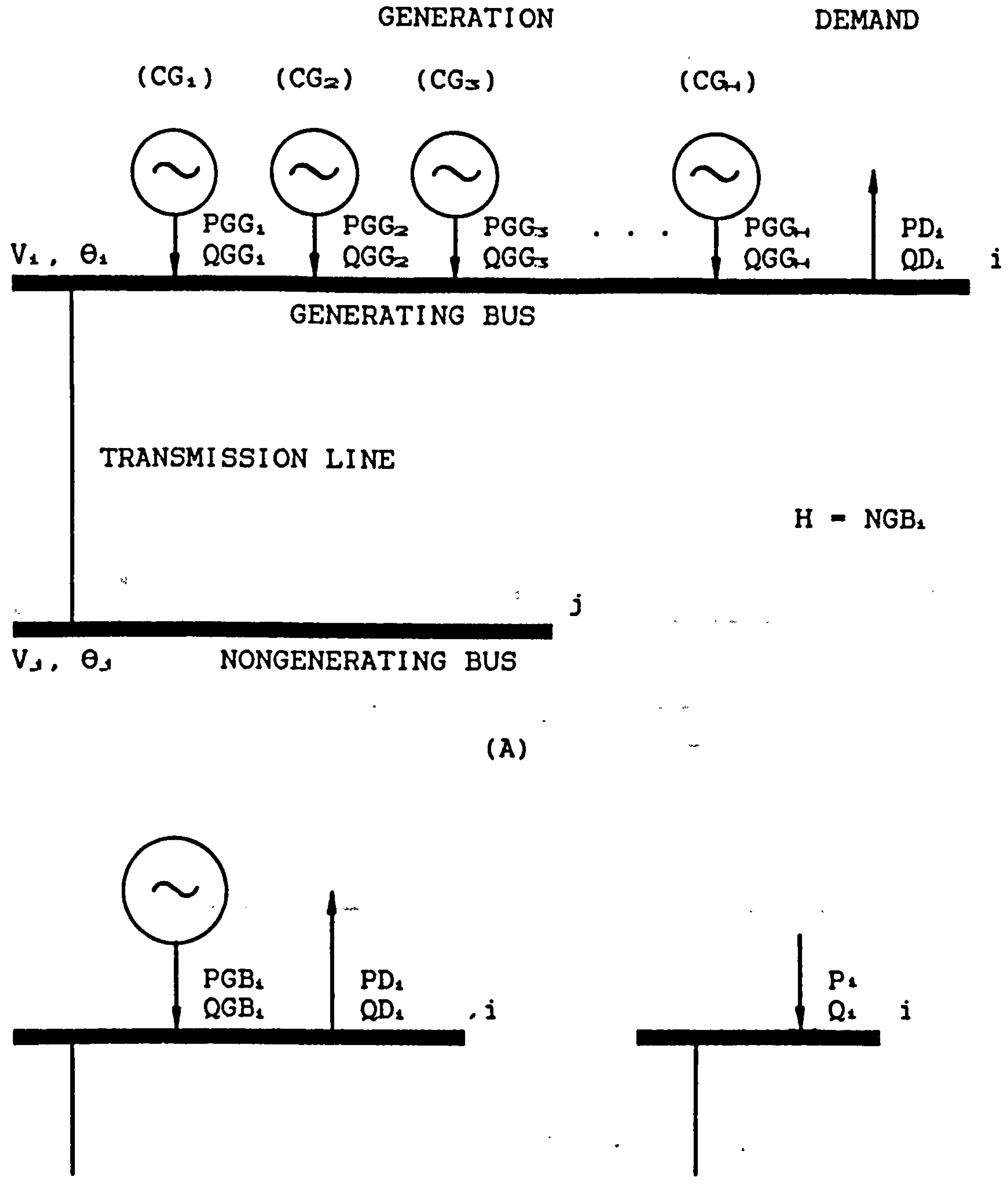

FIG.(2.3)

A SMALL SECTION OF A GENERAL POWER SYSTEM SHOWING THE NOTATION USED

(A) THE ORIGINAL DETAILED NETWORK

(B) THE GENERATING BUS LUMPED EQUIVALENT GENERATOR

(C) BUS NET POWER INJECTIONS

$\bar{V}_i = V_i \cos \theta_i + j V_i \sin \theta_i$ $\bar{Y}s = 1 / (R + j X)$

$\bar{V}_j = V_j \cos \theta_j + j V_j \sin \theta_j$ $\bar{Y}p = j T$

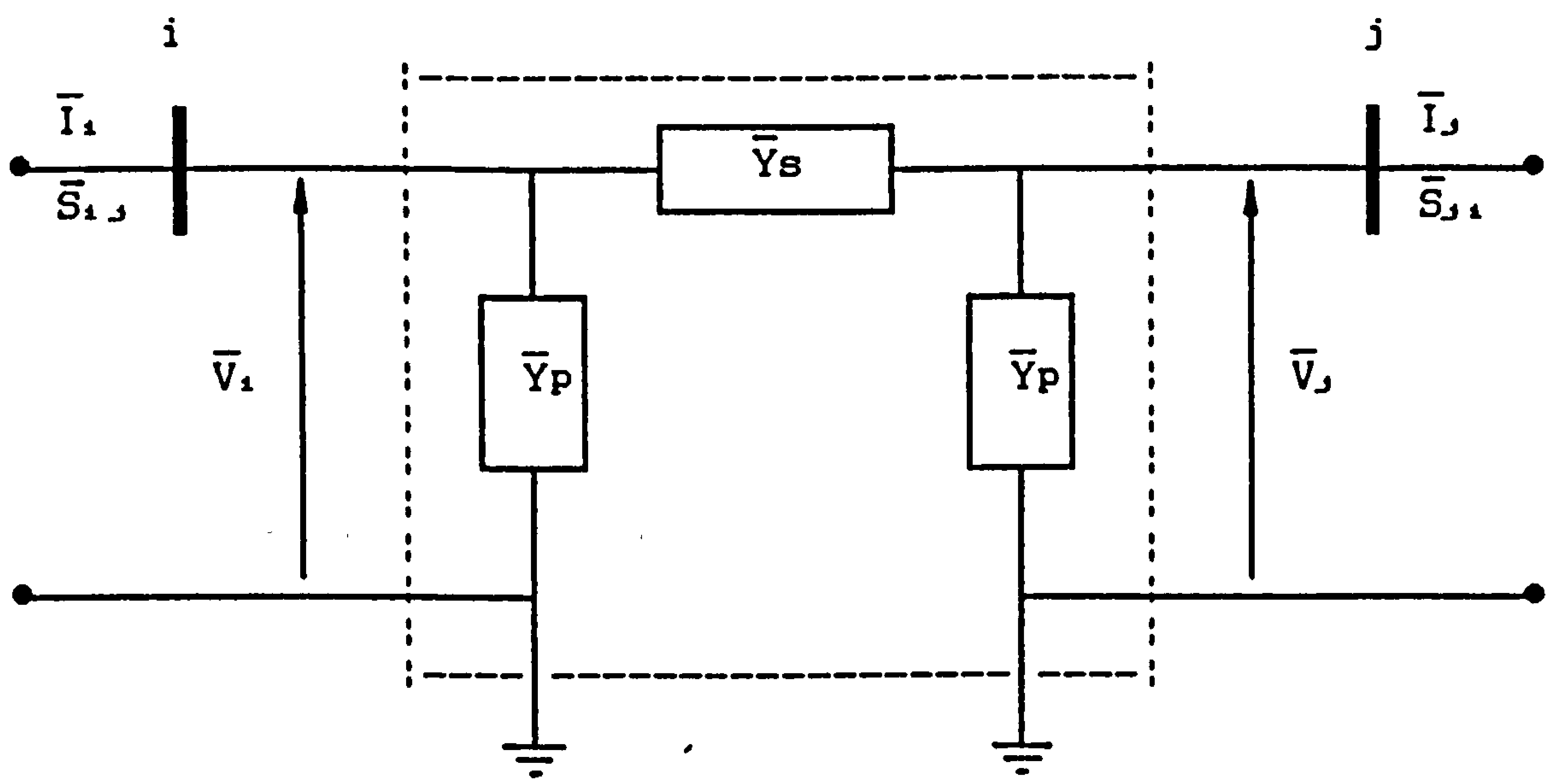

FIG.(2.4)

TRANSMISSION LINE EQUIVALENT CIRCUIT

CHAPTER 3

THE LINEAR PROGRAMMING FORMULATION

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CHAPTER 3

THE LINEAR PROGRAMMING FORMULATION

3.1 INTRODUCTION

Mathematical modelling is one of the important stages in the course of solution of a large number of practical engineering problems. It is also one of the earliest stages, as it usually follows the initial general descriptive statement of the problem. Mathematical modelling implies translating the various aspects of the physical problem into mathematical expressions. This is followed by devising an algorithm for solving the problem and then obtaining the actual numerical solution.

In many real life situations one is often faced with the need of not only solving a given practical problem, but of deciding on which solution is to be chosen from a number of alternatives available. Generally, in such cases, each alternative has its own advantages and disadvantages and it is difficult to assess all the relevant, and often conflicting factors, against one another and come up with the required solution. Choosing a solution which satisfies certain conditions may be attractive, considering those conditions alone, but might bring about severe disadvantages when taking into consideration other relevant factors. Trying to comply with the new conditions, on the other hand, might violate the first set of constraints and so on.

Such difficult decision-making is of commonplace occurrence in project planning, the operation and process control of large industrial systems, the manufacturing of goods, the allocation of resources, and in the provision and distribution of supplies and services. The operation of a large modern power system is an example of this category of complicated problems and is characterized by many of the above-mentioned features.

3.2 THE CONCEPT OF OPTIMIZATION

The field that deals with the type of problem described in the previous section is that of Operations Research [23-34]. In the general sense, operations research can be defined as the application of systematic scientific methods and techniques and quantitative tools to solve problems involving planning, decision-making and operation of systems such that optimal solutions are reached. Although problems with alternative solutions that involve difficult decision-making are not entirely new, it is only during the last 50 years or so that such problems, their impact and the need for their solutions have become so pronounced. It was at the beginning of that period that the ideas of the field of operations research started to take shape, and keen interest in related research disciplines was witnessed [23].

Operations research is a vast field which encompasses a wide range of practical problems and solution techniques. Among the various fields of operations research is that of Mathematical Programming, which is also the field that is most prominent and which received a lot of attention from researchers and, therefore, developed steadily and rapidly. Mathematical Programming is defined as a technique for determining the value of a set of decision variables that optimize a mathematical objective function and conform to a given set of mathematical constraints.

Mathematical programming, itself, consists of a large number and wide variety of methods and techniques for solving optimization problems. These include linear, nonlinear, quadratic and integer programming, binary or zero-one programming, static & dynamic programming, continuous & discrete programming, deterministic & probabilistic or stochastic programming, heuristic programming, geometric programming, separable programming and parametric programming. Each of these fields is concerned with a particular aspect of the mathematical programming problem, and the various solution techniques are suitable for different physical problems, depending on the nature of these problems and their mathematical representations, and also on the nature of the data and required results.

A relatively old category of problems which have some bearing on mathematical programming problems is that of finding the minimum or maximum of functions. These types of problem are collectively known as Extremum Point Problems. In their simplest form extremum point problems consist of a single function to be minimized or maximized in terms of one variable. This category of problem can be handled and has been successfully solved by the classical methods of calculus for a long time. However, these methods cannot handle additional constraints on the problem and the bounds on the variables which characterize optimization problems. The ability to deal with constrains is one of the main differences between the optimization methods of mathematical programming and those of classical calculus. Optimization is the technical or mathematical term equivalent to the concept of finding the "best solution", i.e., a solution that satisfies a number of conflicting conditions and imposed constraints.

One of the earliest techniques of mathematical programming is that of linear programming. This is now a well-established field and has received a wide range of practical applications [24,25]. In a linear programming problem the objective function and all the constraints are linear algebraic functions of the independent or decision variables. Linear Programming is the technique used in the present work to formulate the problem of power system optimum operating conditions.

3.3 GENERALIZED FORMULATION OF THE PROBLEM

The constrained linear optimization problem can be mathematically stated in a number of different forms. The various aspects of the original physical problem are handled differently by the various mathematical formulations and each one of these is suitable for a different solution algorithm or a different computer optimization subroutine. However, the various representations are equivalent and carry the same amount of information that describes the original physical problem. Also, transformation from one representation to another, when the need arises, is possible as explained below.

In the present work the following general representation of the linear programming problem is adopted.

$$\text{Optimize} \quad Z = C_1X_1 + C_2X_2 + C_3X_3 + \ . \ . \ . + C_nX_n \tag{3.1}$$

Subject to

$$\begin{array}{l}
BU_1 \geq A_{11}X_1 + A_{12}X_2 + A_{13}X_3 + \ . \ . \ . + A_{1n}X_n \geq BL_1 \\
BU_2 \geq A_{21}X_1 + A_{22}X_2 + A_{23}X_3 + \ . \ . \ . + A_{2n}X_n \geq BL_2 \\
BU_3 \geq A_{31}X_1 + A_{32}X_2 + A_{33}X_3 + \ . \ . \ . + A_{3n}X_n \geq BL_3 \\
\quad \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\qquad\qquad \vdots \qquad\quad \vdots \\
BU_m \geq A_{m1}X_1 + A_{m2}X_2 + A_{m3}X_3 + \ . \ . \ . + A_{mn}X_n \geq BL_m
\end{array} \tag{3.2}$$

$$\text{and} \quad XU_j \geq X_j \geq XL_j, \quad j = 1, 2, 3, \ldots, n \tag{3.3}$$

This formulation can be written in the following compact form.

$$\begin{aligned}
&\text{Optimize} && Z = \Sigma\, C_j X_j \\
&\text{Subject to} && BU_i \geq \Sigma\, A_{ij} X_j \geq BL_i \\
&\text{and} && XU_j \geq X_j \geq XL_j \\
& && i = 1, 2, 3, \ldots, m \\
& && j = 1, 2, 3, \ldots, n
\end{aligned} \qquad (3.4)$$

It can also be represented using vector and matrix notation.

The function Z to be optimized, i.e., maximized or minimized is called the **Objective Function** and the inequalities (3.2) and (3.3) are the **Constraints** to be satisfied. These consist of two distinct sets. The first set consists of the general constraints which are based on the functional relationships amongst the variables. Each constraint has a lower and upper limit given by BL and BU respectively. The second set represents the lower and upper bounds, XL and XU, on the individual variables themselves. The vectors **XL** and **XU** can be considered as subsets of the vectors **BL** and **BU** respectively. The bounds on each individual variable X_j can be treated as the (m+j)'th general constraint, with the value of the coefficient A_{ij} equal to 1 at the j'th column and zero elsewhere. The **Coefficients** A_{ij}, B_i (BL_i and BU_i) and C_j are given **Constants** and X_j are the **Unknown Variables** whose values are to be determined within the lower and upper bounds imposed. The linear programming problem is fully defined by its **A-B-C Parameters**, which is the term used in this thesis to describe the constants matrix **A**, and the vectors **B** and **C** collectively. The matrix **A** is also known as the **Constraints Coefficients Matrix** and the elements of the vector **C** as the **Coefficients of the Objective Function**.

The general-purpose formulation above should not be confused with two specific formulations used in association with the Simplex Method of solving linear programming problems, and which are known as the Standard and Canonical formulations [26].

3.4 VARIATIONS AND TRANSFORMATIONS

In the previous section a generalized form of the linear programming problem is given. In general, however, practical problems do not always occur in this particular form and the initial mathematical representation of the physical problem may need to be modified. The various possible representations and transformations are discussed below.

Maximization and Minimization

A maximization problem can be represented and solved as a minimization problem and vice versa. For example, if the objective function is to be minimized the problem can be solved by maximizing the negative of the objective function. This can be mathematically stated as :

$$\text{Minimum } (Z) = -\text{ Maximum } (-Z) \qquad (3.5)$$

and is graphically illustrated by FIG.(3.1).

Equality and Inequality Constraints

The constraints of the problem can be equalities, i.e., equations instead of inequalities or a mixture of both. Equality and inequality constraints can be transformed from one form to the other. Also transformation from one type of inequality constraint to another is possible.

An inequality of the less than or equal type ($\leq$) can be transformed into an equation by the use of a **Slack** variable. On the other hand, an inequality constraint of the greater than or equal type ($\geq$) can be transformed into an equality by the use of a negative slack variable which is also known sometimes as a **Surplus** variable. A general constraint with upper and lower bounds can be split into two separate inequalities or two separate equations. If, however, the original constraint is an equality and it is required that it is represented as an inequality for a specific purpose, then it can be written as an inequality with upper and lower bounds or two separate inequalities. The upper and lower bound in this case are equal. An inequality constraint of one type can be transformed to another type by reversing and multiplying both sides of the inequality by (-1).

If required, nonnegative variables can be taken into account in the general formulation above. This is achieved by assigning a value of zero to the lower limit of (3.3). It is to be noted here that the term "Nonnegative" is specially used in this context to include zero values of the variables, and to avoid confusion with the term "positive" which might be taken to mean only values of the variable greater then zero.

Finally, two or more inequalities can be added together, in a similar manner to the addition of equations. However, subtraction of inequalities can produce inconsistent and, therefore, unpredictable results.

It is to be noted that some of the transformations explained above might change the dimentionality of the linear programming problem concerned by increasing or decreasing the number of its decision variables or constraints or both.

3.5 STAGE-I : THE CONSTRAINTS

The constraints used in the first stage of the model are based on the various power system bus and line quantities. In this section, all these constraints are derived and set up to conform to the general linear format of (3.2). The constraints are presented here without referring to any particular optimization problem. Specific optimization problems, each with a defined objective function and sets of constraints, will be presented in Chapter 5, as well as the corresponding numerical results obtained for a small test system. The method of derivation is presented first in general terms. Then the elements of the constraints coefficient matrix A_{ij}, and the upper and lower bounds BU_i and BL_i, associated with the various power system quantities, are derived from the appropriate linearized equations of Section (2.6).

Each of the linearized power system equations of Section (2.6) can be written as a function of the general form:

$$FY_i = FY_i(V_j, \theta_j) + KY_i \tag{3.6}$$

Each equation consists of two parts. The first is a variable quantity which is a function of the independent variables V_j and θ_j. The second is a fixed constant whose value depends on the data of the initial operating point. The variable part is a function of the following form:

$$FY_i(V_j, \theta_j) = \sum_{j=1}^{N} AM_{ij} V_j + \sum_{j=1}^{N} AN_{ij} \theta_j \qquad (3.7)$$

The constant part of (3.6), in turn, consists of two parts as in (3.8) below:

$$KY_i = K_{io} - FY_{io} \qquad (3.8)$$

The general form of each constraint is an inequality of the form:

$$YMX_i \geq FY_i \geq YMN_i \qquad (3.9)$$

Substituting from (3.6) and (3.8) into (3.9) and rearranging gives:

$$YMX_i + FY_{io} - K_{io} \geq FY_i(V_j, \theta_j) \geq YMN_i + FY_{io} - K_{io} \qquad (3.10)$$

Further, the last form of the constraint can be written as:

$$BMX_i \geq FY_i(V_j, \theta_j) \geq BMN_i \qquad (3.11)$$

The upper and lower bounds of the constraint, BMX_i and BMN_i respectively, are given by ,

$$BMX_i = YMX_i + FY_{io} - K_{io} \qquad (3.12)$$

$$BMN_i = YMN_i + FY_{io} - K_{io} \qquad (3.13)$$

Below are some important points relevant to the derivation of the coefficients associated with the various system quantities.

1. The minimum and maximum bus generations are expressed by the following inequalities:

$$PGBMX_i \geq PGB_i \geq PGBMN_i \tag{3.14}$$

$$QGBMX_i \geq QGB_i \geq QGBMN_i \tag{3.15}$$

From Equations (2.27) and (2.28), the bus generations are given as in (3.16) and (3.17).

$$PGB_i = P_i + PD_i \tag{3.16}$$

$$QGB_i = Q_i + QD_i \tag{3.17}$$

Substituting from (3.14) and (3.15) into (3.16) and (3.17) respectively, and rearranging:

$$PGBMX_i - PD_i \geq P_i \geq PGBMN_i - PD_i \tag{3.18}$$

$$QGBMX_i - QD_i \geq Q_i \geq QGBMN_i - QD_i \tag{3.19}$$

Although developed and expressed in terms of generating buses, the constraints (3.18) and (3.19) are valid for nongenerating buses as well. The constraints on the power injections corresponding to nongenerating buses are a special case of (3.18) and (3.19) above, with the upper and lower limits on the bus active and reactive power generation assigned a value of zero. This reduces the two constraints to (3.20) and (3.21) below.

$$-PD_i \geq P_i \geq -PD_i \qquad (3.20)$$

$$-QD_i \geq Q_i \geq -QD_i \qquad (3.21)$$

The last two expressions are in fact just another way of writing the following two simple equalities which give the net power injections at the nongenerating buses.

$$P_i = -PD_i \qquad (3.22)$$

$$Q_i = -QD_i \qquad (3.23)$$

These can be directly obtained from (3.16) and (3.17) respectively by substituting PGB_i and QGB_i by zero.

2. A maximum limit is imposed on the active power loss in a transmission line given by:

$$PL_k \leq PLMX_k \qquad (3.24)$$

A similar upper limit is imposed on the apparent power transmitted across each line given by:

$$ST_k \leq STMX_k \qquad (3.25)$$

Each of these inequalities represent a special case of the general expression (3.11) with the lower limit implied to be zero.

3. The coefficients associated with the apparent power are derived by using Equation (2.50) and the appropriate expanded linearized active and reactive power expressions.

The coefficients AM_{ij} and AN_{ij} of (3.7), and the upper and lower bounds BMX_i and BMN_i of (3.11) associated with the various constraints have been derived as outlined above and the final expressions are listed below under the appropriate headings. It is to be noticed that they are all functions of system parameters and initial operating point data. Therefore, each will be returned as a single numerical value by the computer program in the course of the solution of the problem.

Elements of the Constraints Coefficients Matrix and Upper and Lower Bounds Vectors

Elements Associated with the Net Bus Active Power Injection

$$AM_{ii} = 2\, G_{ii} \, (V_{io})^2 + \sum_{\substack{j=1 \\ j \neq i}}^{N} V_{jo} \, (G_{ij} \cos \theta_{ijo} + B_{ij} \sin \theta_{ijo}) \tag{3.26}$$

$$AM_{ij} = V_{io} \, (G_{ij} \cos \theta_{ijo} + B_{ij} \sin \theta_{ijo}) \qquad j = 1, 2, 3, \ldots, N, \quad j \neq i \tag{3.27}$$

$$AN_{ii} = \sum_{\substack{j=1 \\ j \neq i}}^{N} V_{io} \, V_{jo} \, (B_{ij} \cos \theta_{ijo} - G_{ij} \sin \theta_{ijo}) \tag{3.28}$$

$$AN_{ij} = V_{io} \, V_{jo} \, (G_{ij} \sin \theta_{ijo} - B_{ij} \cos \theta_{ijo}) \qquad j = 1, 2, 3, \ldots, N, \quad j \neq i \tag{3.29}$$

$$BMX_i = PGBMX_i + P_{io} - PD_i + \sum_{j=1}^{N} V_{io} V_{jo} \theta_{ijo} (B_{ij} \cos \theta_{ijo} - G_{ij} \sin \theta_{ijo}) \quad (3.30)$$

$$BMN_i = PGBMN_i + P_{io} - PD_i + \sum_{j=1}^{N} V_{io} V_{jo} \theta_{ijo} (B_{ij} \cos \theta_{ijo} - G_{ij} \sin \theta_{ijo}) \quad (3.31)$$

Elements Associated with the Net Bus Reactive Power Injection

$$AM_{ii} = - 2 B_{ii} (V_{io})^2 + \sum_{\substack{j=1 \\ j \neq i}}^{N} V_{jo} (G_{ij} \sin \theta_{ijo} - B_{ij} \cos \theta_{ijo}) \quad (3.32)$$

$$AM_{ij} = V_{io} (G_{ij} \sin \theta_{ijo} - B_{ij} \cos \theta_{ijo})$$

$$j = 1, 2, 3, \ldots , N \quad j \neq i \quad (3.33)$$

$$AN_{ii} = \sum_{\substack{j=1 \\ j \neq i}}^{N} V_{io} V_{jo} (G_{ij} \cos \theta_{ijo} + B_{ij} \sin \theta_{ijo}) \quad (3.34)$$

$$AN_{ij} = - V_{io} V_{jo} (G_{ij} \cos \theta_{ijo} + B_{ij} \sin \theta_{ijo})$$

$$j = 1, 2, 3, \ldots , N \quad j \neq i \quad (3.35)$$

$$BMX_i = QGBMX_i + Q_{io} - QD_i + \sum_{j=1}^{N} V_{io} V_{jo} \theta_{ijo} (G_{ij} \cos \theta_{ijo} + B_{ij} \sin \theta_{ijo}) \quad (3.36)$$

$$BMN_i = QGBMN_i + Q_{io} - QD_i + \sum_{j=1}^{N} V_{io} V_{jo} \theta_{ijo} (G_{ij} \cos \theta_{ijo} + B_{ij} \sin \theta_{ijo}) \quad (3.37)$$

Elements Associated with Active Power Loss in a Transmission Line

$$AM_{ii} = 2 Y_{ij}^2 R_{ij} (V_{io} - V_{jo} \cos \theta_{ijo}) \qquad (3.38)$$

$$AM_{ij} = 2 Y_{ij}^2 R_{ij} (V_{jo} - V_{io} \cos \theta_{ijo}) \qquad (3.39)$$

$$AN_{ii} = 2 Y_{ij}^2 R_{ij} V_{io} V_{jo} \sin \theta_{ijo} \qquad (3.40)$$

$$AN_{ij} = - 2 Y_{ij}^2 R_{ij} V_{io} V_{jo} \sin \theta_{ijo} \qquad (3.41)$$

$$[AN_{ij} = - AN_{ii}]$$

$$BMX_k = PLMX_k + PL_{ko} + 2 Y_{ij}^2 R_{ij} V_{io} V_{jo} \theta_{ijo} \sin \theta_{ijo} \qquad (3.42)$$

$$BMN_k = PL_{ko} + 2 Y_{ij}^2 R_{ij} V_{io} V_{jo} \theta_{ijo} \sin \theta_{ijo} \qquad (3.43)$$

Elements Associated with Apparent Power Transfer Across a Transmission Line (I-J)

$$AM_{ii} = \frac{1}{S_{ijo}} \{ 2 V_{io} [P_{ijo} Y_{ij}^2 R_{ij} + Q_{ijo} (Y_{ij}^2 X_{ij} - T_{ij})] + Y_{ij}^2 V_{jo} [(P_{ijo} X_{ij} - Q_{ijo} R_{ij}) \sin \theta_{ijo} - (P_{ijo} R_{ij} + Q_{ijo} X_{ij}) \cos \theta_{ijo})] \} \qquad (3.44)$$

$$AM_{ij} = \frac{Y_{ij}^2 V_{io}}{S_{ijo}} [(P_{ijo} X_{ij} - Q_{ijo} R_{ij}) \sin \theta_{ijo} - (P_{ijo} R_{ij} + Q_{ijo} X_{ij}) \cos \theta_{ijo})] \qquad (3.45)$$

$$AN_{ii} = \frac{Y_{ij}^2 V_{io} V_{jo}}{S_{ijo}} [(P_{ijo} X_{ij} - Q_{ijo} R_{ij}) \cos \theta_{ijo} + (P_{ijo} R_{ij} + Q_{ijo} X_{ij}) \sin \theta_{ijo}] \qquad (3.46)$$

$$AN_{ij} = \frac{Y_{ij}^2 V_{io} V_{jo}}{S_{ijo}} [(Q_{ijo} R_{ij} - P_{ijo} X_{ij}) \cos \theta_{ijo} - (P_{ijo} R_{ij} + Q_{ijo} X_{ij}) \sin \theta_{ijo}] \qquad (3.47)$$

$[\ AN_{ij} = -\ AN_{ii}\ ]$

$$BMX_k = STMX_k + ST_{ko} + \frac{Y_{ij}^2\ V_{io}\ V_{jo}\ \theta_{ijo}}{S_{ijo}} [\ (\ P_{ijo}\ X_{ij} - Q_{ijo}\ R_{ij}\ )\ \cos\ \theta_{ijo} + (\ P_{ijo}\ R_{ij} + Q_{ijo}\ X_{ij}\ )\ \sin\ \theta_{ijo}\ ]$$

(3.48)

$$BMN_k = ST_{ko} + \frac{Y_{ij}^2\ V_{io}\ V_{jo}\ \theta_{ijo}}{S_{ijo}} [\ (\ P_{ijo}\ X_{ij} - Q_{ijo}\ R_{ij}\ )\ \cos\ \theta_{ijo} + (\ P_{ijo}\ R_{ij} + Q_{ijo}\ X_{ij}\ )\ \sin\ \theta_{ijo}\ ]$$

(3.49)

<u>Elements Associated with Apparent Power Transfer Across a Transmission Line (J-I)</u>

$$AM_{ii} = \frac{Y_{ij}^2\ V_{jo}}{S_{ijo}} [\ (\ Q_{ijo}\ R_{ij} - P_{ijo}\ X_{ij}\ )\ \sin\ \theta_{ijo} - (\ P_{ijo}\ R_{ij} + Q_{ijo}\ X_{ij}\ )\ \cos\ \theta_{ijo}\ )\ ]$$

(3.50)

$$AM_{ij} = \frac{1}{S_{ijo}} \{\ 2\ V_{jo}\ [\ P_{ijo}\ Y_{ij}^2\ R_{ij} + Q_{ijo}\ (\ Y_{ij}^2\ X_{ij} - T_{ij}\ )\ ] + Y_{ij}^2\ V_{io}\ [\ (\ Q_{ijo}\ R_{ij} - P_{ijo}\ X_{ij}\ )\ \sin\ \theta_{ijo} + (\ P_{ijo}\ R_{ij} - Q_{ijo}\ X_{ij}\ )\ \cos\ \theta_{ijo}\ )\ ]\ \}$$

(3.51)

$$AN_{ii} = \frac{Y_{ij}^2\ V_{io}\ V_{jo}}{S_{ijo}} [\ (\ P_{ijo}\ R_{ij} - Q_{ijo}\ X_{ij}\ )\ \sin\ \theta_{ijo} - (\ P_{ijo}\ X_{ij} + Q_{ijo}\ R_{ij}\ )\ \cos\ \theta_{ijo}\ ]$$

(3.52)

$$AN_{ij} = \frac{Y_{ij}^2\ V_{io}\ V_{jo}}{S_{ijo}} [\ (\ Q_{ijo}\ X_{ij} - P_{ijo}\ R_{ij}\ )\ \sin\ \theta_{ijo} + (\ P_{ijo}\ X_{ij} + Q_{ijo}\ R_{ij}\ )\ \cos\ \theta_{ijo}\ ]$$

(3.53)

$[\ AN_{ij} = -\ AN_{ii}\ ]$

$$BMX_k = STMX_k + ST_{ko}$$

$$+ \frac{Y_{ij}^2\ V_{io}\ V_{jo}\ \theta_{ijo}}{S_{ijo}} [\ (\ Q_{ijo}\ X_{ij} - P_{ijo}\ R_{ij}\ )\ \sin\ \theta_{ijo}$$

$$+\ (\ P_{ijo}\ X_{ij} + Q_{ijo}\ R_{ij}\ )\ \cos\ \theta_{ijo} \qquad (3.54)$$

$$BMN_k = ST_{ko}$$

$$+ \frac{Y_{ij}^2\ V_{io}\ V_{jo}\ \theta_{ijo}}{S_{ijo}} [\ (\ Q_{ijo}\ X_{ij} - P_{ijo}\ R_{ij}\ )\ \sin\ \theta_{ijo}$$

$$+\ (\ P_{ijo}\ X_{ij} + Q_{ijo}\ R_{ij}\ )\ \cos\ \theta_{ijo} \qquad (3.55)$$

<u>The Independent Variables</u>

In a real power system the magnitudes of all bus voltages have to be kept within a specified operating range around their nominal values. For the i'th bus this is expressed by:

$$VMX_i \geq V_i \geq VMN_i \qquad (3.56)$$

No bounds are imposed on the individual phase angles and they are used as unconstrained variables that can assume any real values. However, for stability considerations, the difference between the phase angles across any transmission line should not exceed a certain limit. But this is taken into account when specifying the constraints on the transmission line quantities, especially the upper limits on the apparent power transfers across the lines.

3.6 STAGE-I : THE OBJECTIVE FUNCTIONS

The objective functions used in the first stage of the various optimization problems are also based on bus or line quantities and derived from the linearized power system equations of Section (2.6). Again the derivation will be presented here in general terms with the objective function presented in the general format of (3.1). The specific objective functions will be discussed in more detail in Chapter 5.

The objective function based on the general power system quantity FY_i is given by:

$$Z = \sum_{i=1}^{NF} (C_i \times FY_i) \qquad (3.57)$$

where NF is the number of functions involved.
Substituting from (3.6) and (3.7) into (3.57) gives:

$$Z = [\sum_{i=1}^{NF} C_i \sum_{j=1}^{N} (AM_{i,j} \; V_j + AN_{i,j} \; \theta_j)] + \sum_{i=1}^{NF} C_i \; KY_i \qquad (3.58)$$

This can be rewritten as in (3.59) below:

$$Z = \{ \sum_{i=1}^{NF} [\sum_{j=1}^{N} C_i \; (AM_{i,j} \; V_j + AN_{i,j} \; \theta_j)] \} + Z_o \qquad (3.59)$$

where Z_o is a constant given by,

$$Z_o = \sum_{i=1}^{NF} C_i \; KY_i \qquad (3.60)$$

As explained in Section (2.3), a constant term in the objective function of an optimization problem can be neglected without affecting the optimization process. Thus (3.59) can be reduced to (3.61) below.

$$Z = \sum_{j=1}^{N} (CM_j \; V_j + CN_j \; \theta_j) \tag{3.61}$$

The coefficients CM_j and CN_j of (3.61) are given by (3.62) and (3.63).

$$CM_j = \sum_{i=1}^{NF} C_i \; AM_{ij} \tag{3.62}$$

$$CN_j = \sum_{j=1}^{NF} C_i \; AN_{ij} \tag{3.63}$$

By selecting the appropriate objective function and a set or sets of constraints, a number of different optimization problems can be set up to represent particular aspects of the power system operating conditions. The various objective functions used will be derived in Chapter 5. The various particular optimization problems will also be presented and formulated along with numerical results obtained for a small test system. Each optimization problem is formulated as a linear programming problem. The solution of any of these problems yields the values of all bus voltage magnitudes and phase angles under optimum operating conditions. As these are the principle variables in terms of which all other bus and line quantities are expressed, once their values are known, the values of all other dependent quantities can be computed using the appropriate extended nonlinear equations from Section (2.6)

3.7 THE SECOND STAGE OF THE MODEL

From the solution of the linear programming problem of stage-I, the active and reactive power outputs of the equivalent bus generators under optimum operating conditions can be obtained. However, these equivalent bus generators and their generation costs are rather "artificial", and they have been introduced mainly as a modelling aid. They give the collective outputs at each generating bus, but not the outputs of the individual generators. To obtain the outputs of the individual generators in the system, which constitute the actual generating schedule, a new linear programming problem is formulated. The objective of this problem is to minimize the hourly fuel cost, with the active power outputs of the individual generators used as the independent variables. The new problem has NG independent variables and (NS + 1) constraints, as well as the upper and lower limits on the independent variables. One constraint is based on the active power balance in the whole system, and the other NS constraints correspond to the outputs of the generating buses as obtained from stage-I. The complete mathematical representation of this linear programming problem is given in Equations (3.64) to (3.67) below.

$$\text{Minimize } Z = \sum_{j=1}^{NG} CG_j \times PGG_j \quad (3.64)$$

Subject to

$$\sum_{j=1}^{NG} PGG_j = PGT \quad (3.65)$$

$$\sum_{h=1}^{NGB_i} PGG_h = PGB_i \quad (3.66)$$

$$PGGMX_j \geq PGG_j \geq PGGMN_j \quad (3.67)$$

$i = 1, 2, 3, \ldots , NS$

$j = 1, 2, 3, \ldots , NG$

In the above formulation the objective function represents the total instantaneous generation cost. The single constraint of (3.65) represents the active power balance equation in the system, and the NS constraints of (3.66) give the total active power output of the generating buses in terms of the active power outputs of their individual generators. The active power outputs of the generating buses represent the transition link between the two stages of the model. The upper and lower operating limits on the NG generators are given by (3.67).

It is to be noticed that the new problem is already linear and in a much simpler form than that of stage-I. Therefore, no initial conditions are required to obtain the solution of stage-II whose formulation is based on the values of the independent variables themselves rather than their incremental values. Consequently, most of the modelling, mathematical formulation and computational effort involved in the present work is directed towards the first stage of the model, especially the linearization of the relevant power system equations and the derivation of the A-B-C coefficients of the linear programming problem.

3.8 SUMMARY

This chapter has explained the formulation of a linear programming model to represent power system optimization problems. The concept of optimization is introduced first, followed by a generalized form of the linear programming model. Variations and transformations of the generalized formulation are also explained. The constraints of the first stage of the mathematical model are developed as well as a general objective function. The detailed mathematical expressions that represent the elements of the coefficients constraints matrix are then given. Finally, a complete formulation of the second stage of the model is presented.

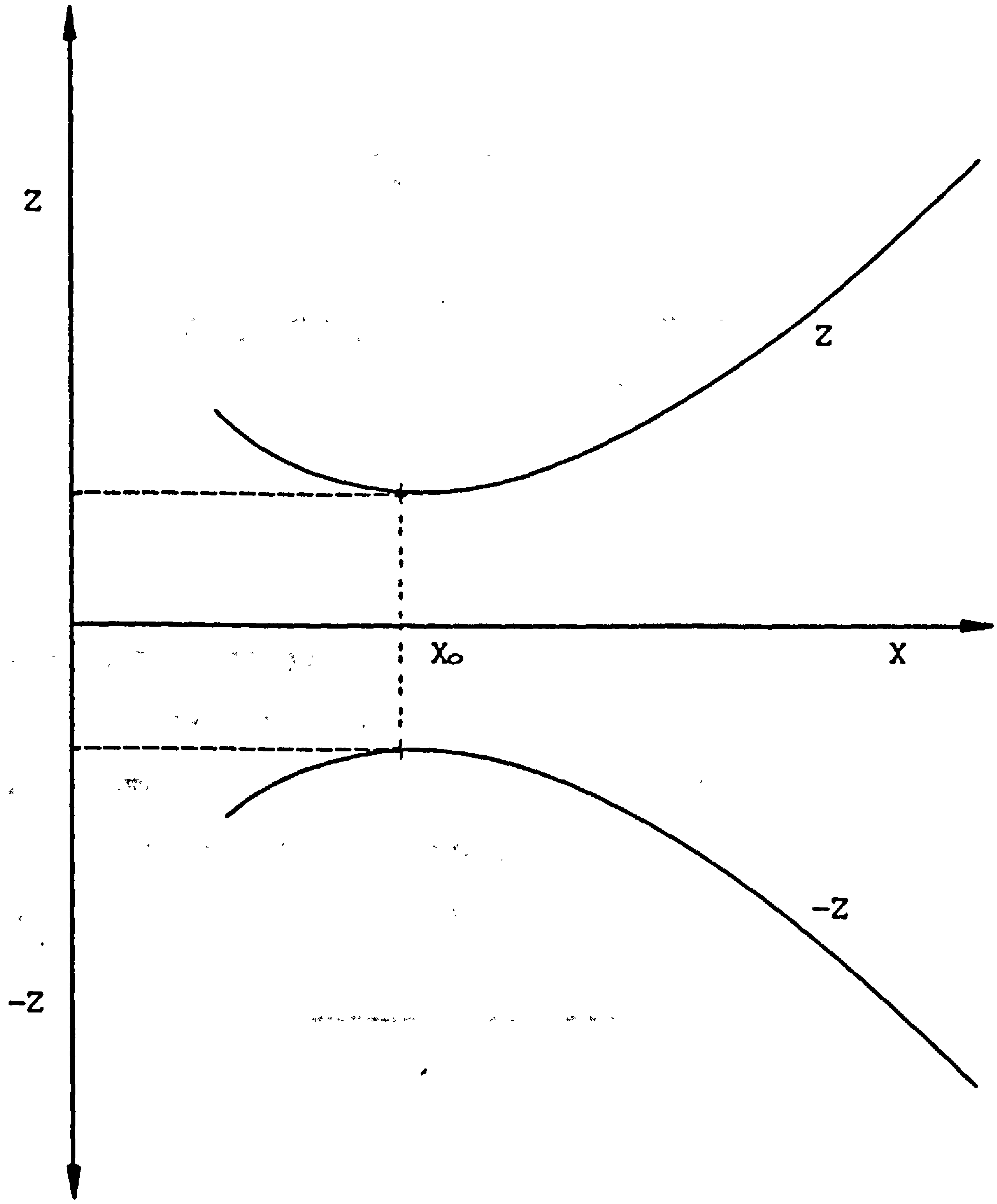

FIG. (3.1)

MINIMIZATION AND MAXIMIZATION

THE POINT (X_0) CORRESPONDS TO THE MINIMUM OF THE FUNCTION (Z) AND THE MAXIMUM OF (-Z)

CHAPTER 4

ADDITIONAL CONSIDERATIONS

4.1 INTRODUCTION

4.2 TAP-CHANGING TRANSFORMERS

4.3 ENERGY EXCHANGE AND TIE-LINE FLOWS

4.4 SPINNING RESERVE AND AREA RESERVE

4.5 LOADING AND DELOADING RATES OF GENERATORS

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CHAPTER 4

ADDITIONAL CONSIDERATIONS

4.1 INTRODUCTION

Many aspects of the power system operation problem have been introduced and discussed in the previous chapters. It has been shown that power system quantities, such as generation cost, system losses and reactive power production, can be used as objective functions, constraints or both. These by no means constitute an exclusive set of the many factors that affect power system operation. Many other factors can be taken into account in addition to, or instead of, the ones used in the mathematical model so far. The generality of the method used to set up the model developed in this thesis allows for these factors to be taken into consideration. Some of these additional considerations are discussed in the following sections. In each case, the new aspect is introduced, defined and discussed. This is followed by mathematical representation of the concept and the derivation of the necessary mathematical expressions. Where appropriate, the corresponding parameters of the equivalent bus generator are also derived. Finally, the method of incorporating the new quantity into the optimization model of the system is explained, as well as its effect on either or both of the stages of the model and their A-B-C parameters. The various additional factors considered below affect the mathematical model of the power system optimization problem and its numerical solution to various extents. Some of them involve major modifications to the mathematical formulation of the problem and all its parameters while, in the other extreme, some cases only produce minor changes in the associated numerical results.

4.2 TAP-CHANGING TRANSFORMERS

The power transformer is one of the major pieces of equipment in a power system. Its main function is to change the voltage level in various sections of the system for various purposes. Transformers are used to step up bus voltages at the generation end of the system for transmission purposes, and to step it down again at the distribution or load end for consumer use. The transformation between the various voltage levels is sometimes accompanied by a relative phase shift, depending on the connection of the primary and secondary windings of the transformers. There are certain "nominal" transformations of the voltage level for transmission and distribution purposes such as 11/132 kV or 33/11 kV. Phase shifts are usually multiples (positive or negative) of 30 degrees, resulting from star/delta connections.

Transformers can also be used for voltage control purposes. Off-nominal voltage taps can be used to improve the voltage profile and compensate for voltage drops along transmission lines. The additional off-nominal voltage changes are normally in the range of ± 10 %. Usually, this type of voltage magnitude control is performed at local buses on the high voltage side. Additional phase shifts can also be induced to control the reactive power flow in the system. Transformers that can perform both of these control functions have complex transformation ratios. The additional changes in the nominal voltage magnitude and phase angle can be administered under no-load or loaded conditions. The following discussion is concerned with on-load off-nominal tap-changing transformers. Henceforth, in this thesis, these will be variably referred to as tap-changing transformers or simply transformers, for the obvious reason of brevity.

Generally, any form of power system analysis is not considered complete unless it takes into consideration the effect of the presence of transformers in the system, or at least discusses the subject or touches on it. In many cases, however, the practice is to present the analysis, first, neglecting the effect of the transformers. Tap-changing transformers are then introduced, discussed and dealt with separately, with the method of incorporating them into the mathematical model explained. This method is followed in the present thesis. One of the reasons for this practice is that including the transformers into the problem model complicates the analysis to a considerable extent. At the early stages of a research work, the immediate aim is to set up a "working" mathematical framework based on the conceived ideas and theoretical concepts. This is then translated into a computer program which is tested on a study case or cases. Only when that stage of the work is completed, satisfactory results are obtained, and confidence is gained, that additional considerations such as tap-changing transformers are taken into account and included in the analysis, as a further stage of the research work. The above argument is particularly true in the case of the present work.

In the optimization model used so far in this thesis the effect of tap-changing transformers has not been taken into account. They were assumed to have fixed tap settings. A more comprehensive optimization model can be formulated by considering the complex transformation ratios of the tap-changing transformers as additional independent or control variables. This will cause considerable changes to the original mathematical model and the associated computer program. Unlike other additional considerations discussed in the rest of this chapter, taking the effect of these transformers into consideration alters the very equations that describe the relationships among the various power system quantities. It also adds to the dimensionality of the problem. However, the same fundamental ideas on which the original mathematical model is based and the same method of derivation can be applied to formulate the new "augmented" model.

First, the nonlinear equations of Section (2.6), that describe the relationships among the various power system quantities have to be modified to take into account the effect of the turn ratios and angular settings of the tap-changing transformers [7,9]. The resulting equations should then be linearized, applying the procedure of Section (2.5). The new linearized equations are then used to set up the objective functions and the constraints of the new model. Also, lower and upper bounds are imposed on the two new sets of independent variables constituting the complex transformation ratios of the tap-changing transformers.

Instead of each of the single power system equations of Section (2.6), there are three equations. These correspond to lines where no transformer is connected, lines where a transformer is connected at the near end, and lines where a transformer is connected at the far end. The three different situations are shown in FIG.(4.1). The descriptions "near" and "far" are in reference to node i in the figure. The designations sending and receiving ends are deliberately avoided in this context, as either end of any transmission can be described as sending or receiving with respect to the other end. The case where no transformer is used has already been fully discussed and analysed in this thesis. The two other cases are discussed in the rest of this section.

The relevant points in the analysis of the tap-changing transformer for the purposes of the present work are highlighted below.

1. The complex transformation ratio of the transformer is represented by $t\underline{/\alpha}$. Each tap-changing transformer is replaced by an ideal transformer of turn ratio t connected in series with a phase-shifter of pure angular shift α.

2. The impedance or reactance of the transformer is included with the impedance of the transmission line to which the transformer is connected. The combined impedance of the line and the transformer can still be represented by the π-equivalent circuit of FIG.(2.4). The new equivalent circuit and the values of its elements are affected also by the tap positions. However, these changes are very small and can be ignored in the analysis.

3. The system admittance matrix has to be modified to take into account the modification in the transmission network. Apart from the changes in the resulting numerical values of the parameters of the new transmission line equivalent circuit and the system admittance matrix, both the equivalent circuit and the admittance matrix are no longer symmetrical. The analysis of the new equivalent circuit and the method of obtaining its parameters and the elements of the associated admittance matrix are covered adequately in the literature [21,35,36].

4. In the modified versions of the various power system equations, the effect of the tap-changing transformers appears as a multiplier of t per unit associated with the voltage magnitude of the bus at which the transformer is connected, and as an angle α added to the phase angle of the bus.

The modified nonlinear equations corresponding to bus injections and line flows are given below. Under each category there are two equations corresponding to cases B and C of FIG.(4.1) respectively.

The Modified Power System Equations

Bus Injected Active Power

$$P_i = \sum_{j=1}^{N} t\, V_i\, V_j\, [\, G_{ij} \cos(\theta_{ij} + \alpha) + B_{ij} \sin(\theta_{ij} + \alpha)\,] \quad (4.1)$$

$$P_i = \sum_{j=1}^{N} t\, V_i\, V_j\, [\, G_{ij} \cos(\theta_{ij} - \alpha) + B_{ij} \sin(\theta_{ij} - \alpha)\,] \quad (4.2)$$

Bus Injected Reactive Power

$$Q_i = \sum_{j=1}^{N} t\, V_i\, V_j\, [\, G_{ij} \sin(\theta_{ij} + \alpha) - B_{ij} \cos(\theta_{ij} + \alpha)\,] \quad (4.3)$$

$$Q_i = \sum_{j=1}^{N} t\, V_i\, V_j\, [\, G_{ij} \sin(\theta_{ij} - \alpha) - B_{ij} \cos(\theta_{ij} - \alpha)\,] \quad (4.4)$$

Active Power Transfer Across Line (I-J)

$$P_{ij} = Y_{ij}^2\, R_{ij}\, t^2\, V_i^2 + Y_{ij}^2\, t\, V_i\, V_j\, [\, (\, X_{ij} \sin(\theta_{ij} + \alpha) - R_{ij} \cos(\theta_{ij} + \alpha)\,] \quad (4.5)$$

$$P_{ij} = Y_{ij}^2\, R_{ij}\, V_i^2 + Y_{ij}^2\, t\, V_i\, V_j\, [\, (\, X_{ij} \sin(\theta_{ij} - \alpha) - R_{ij} \cos(\theta_{ij} - \alpha)\,] \quad (4.6)$$

Active Power Transfer Across Line (J-I)

$$P_{ji} = Y_{ij}^2 \; R_{ij} \; V_j^2 - Y_{ij}^2 \; t \; V_i \; V_j \; [\; (\; X_{ij} \sin (\theta_{ij} + \alpha) + R_{ij} \cos (\theta_{ij} + \alpha) \;] \qquad (4.7)$$

$$P_{ji} = Y_{ij}^2 \; R_{ij} \; t^2 \; V_j^2 - Y_{ij}^2 \; t \; V_i \; V_j \; [\; (\; X_{ij} \sin (\theta_{ij} - \alpha) + R_{ij} \cos (\theta_{ij} - \alpha) \;] \qquad (4.8)$$

Reactive Power Transfer Across Line (I-J)

$$Q_{ij} = (Y_{ij}^2 \; X_{ij} - T_{ij}) \; t^2 \; V_i^2 - Y_{ij}^2 \; t \; V_i \; V_j \; [\; (\; R_{ij} \sin (\theta_{ij} + \alpha) + X_{ij} \cos (\theta_{ij} + \alpha) \;] \qquad (4.9)$$

$$Q_{ij} = (Y_{ij}^2 \; X_{ij} - T_{ij}) \; V_i^2 - Y_{ij}^2 \; t \; V_i \; V_j \; [\; (\; R_{ij} \sin (\theta_{ij} - \alpha) + X_{ij} \cos (\theta_{ij} - \alpha) \;] \qquad (4.10)$$

Reactive Power Transfer Across Line (J-I)

$$Q_{ji} = (Y_{ij}^2 \; X_{ij} - T_{ij}) \; V_j^2 + Y_{ij}^2 \; t \; V_i \; V_j \; [\; (\; R_{ij} \sin (\theta_{ij} + \alpha) - X_{ij} \cos (\theta_{ij} + \alpha) \;] \qquad (4.11)$$

$$Q_{ji} = (Y_{ij}^2 \; X_{ij} - T_{ij}) \; t^2 \; V_j^2 + Y_{ij}^2 \; t \; V_i \; V_j \; [\; (\; R_{ij} \sin (\theta_{ij} - \alpha) - X_{ij} \cos (\theta_{ij} - \alpha) \;] \qquad (4.12)$$

Active Power Loss in a Transmission Line

$$PL_k = Y_{ij}^2 \; R_{ij} \; t^2 \; V_i^2 + Y_{ij}^2 \; R_{ij} \; V_j^2 - 2 \; Y_{ij}^2 \; R_{ij} \; t \; V_i \; V_j \cos (\theta_{ij} + \alpha) \qquad (4.13)$$

$$PL_k = Y_{ij}^2 \; R_{ij} \; V_i^2 + Y_{ij}^2 \; R_{ij} \; t^2 \; V_j^2 - 2 \; Y_{ij}^2 \; R_{ij} \; t \; V_i \; V_j \cos (\theta_{ij} - \alpha) \qquad (4.14)$$

Reactive Power Loss in a Transmission Line

$$QL_{ik} = (Y_{ij}^2 \; X_{ij} - T_{ij}) \; t^2 \; V_i^2 + (Y_{ij}^2 \; X_{ij} - T_{ij}) \; V_j^2 - 2 \; Y_{ij}^2 \; X_{ij} \; t \; V_i \; V_j \cos (\theta_{ij} + \alpha) \qquad (4.15)$$

$$QL_{k} = (Y_{ij}^2 \; X_{ij} - T_{ij}) \; V_i^2 + (Y_{ij}^2 \; X_{ij} - T_{ij}) \; t^2 \; V_j^2 - 2 \; Y_{ij}^2 \; X_{ij} \; t \; V_i \; V_j \cos (\theta_{ij} - \alpha) \qquad (4.16)$$

Notice that the original power system equations are a special case of the modified equations and can be readily obtained by substituting the values $t=1$ and $\alpha=0$. Also transformers with only voltage magnitude or angular control can be represented by substituting $\alpha=0$ or $t=1$ respectively.

The complex transformation ratios of the transformers are "Bus Quantities", and taking them into account, therefore, only affects the general mathematical formulation of the first stage of the mathematical model. All the three parameters, namely, A, B and C of the linear programming problem of the first stage of the mathematical model, are affected when the transformers are included in the analysis. The second stage of the model need not be modified, although adding the effect of the transformers in the first stage affects the "numerical results" obtained from both stages.

The number of the general constraints in the modified model is the same as that of the original model. The number of the independent variables, however, is increased by 2NR. There is also a set of upper and lower bounds on the new variables. For the k'th transformer, these are given by:

$$tMX_k \geq t_k \geq tMN_k \tag{4.17}$$

$$\alpha MX_k \geq \alpha_k \geq \alpha MN_k \tag{4.18}$$

4.3 ENERGY EXCHANGE AND TIE-LINE FLOWS

The optimization model has been constructed so far considering a single isolated power system well-defined in terms of its boundaries. Many small power systems are nowadays connected to each other through tie-lines to form one large system for the purpose of exchange of energy, the advantageous use of resources, and to insure the continuity and reliability of supply to customers. Based on the same reasoning, the modern practice is to integrate all the power generation and transmission networks of an entire country into one "National Grid". In recent years this practice has even been extended to the connection of the national grids of more than one country. The constituent parts of a large interconnected system are usually operated autonomously, but can exchange energy depending on their load requirements.

Two systems connected through a tie-line are shown in FIG.(4.2), which also illustrates the nomenclature and notation used. The system under consideration is called the internal system and any of the other systems is called the external system. In any of the interconnected systems, the bus at which the tie-line is connected is called the boundary bus. When analysing any system the effect of all neighbouring external systems must be taken into account one way or another. This applies generally to any form of analysis such as load flow or security considerations and not only for optimization purposes as in the present work. One important aspect to be taken into account in this respect is the power flow through the tie-line. This, of course is just one aspect among several others to be considered concerning the interconnection between two power systems. Another aspect, for example, is the question of stability associated with the transmission of large amounts of electric power through very long transmission lines. This particular aspect, however, is beyond the scope of this thesis.

The exchange of energy between two interconnected systems depends to a large extent on the contractual agreements between the two individual systems, and this fact is relevant to the problem of optimum operating conditions considered in this thesis. Various situations and other relevant points are discussed briefly below.

1. In terms of the direction of the tie-line flow, there are two distinct cases depending on the capacities of the two systems involved, their loading conditions and also on the time of day or season of the year. In one case the energy exchange between system A and system B is

consistently either import or export which means that the tie-line power flow is always unidirectional. Another case is when the loading patterns of the two systems change such that system A buys energy from system B at one time or season, and sells energy to it at another time or another season.

Apart from these two cases there is the situation where the two connected systems do not normally exchange energy because the generating capacity of either system is generally sufficient to satisfy its own load requirements. However, energy exchange takes place in cases of emergency or unexpectedly high demands.

2. In any of the cases above, usually the energy transfer agreement specifies that the amount of energy transferred should fall within two, upper and lower, limits.

3. In terms of utilization of resources, there are also two different cases. In one, the importing company decides that energy should only be imported from a neighbouring system when all the internal system generating capacity has been used. In this case provision has to be made in the optimization algorithm, and the associated computer routine, to insure that this condition is satisfied. Otherwise, if unaltered, the optimization program will "see" the power available for import across the tie-line as just another generator and, therefore, depending on the associated cost, it can be

included in the resulting generating schedule. Thus, energy might be imported even when there is enough internal generating capacity to satisfy the local demand at the time.

On the other hand, the internal system may allow the import of energy even when it still has enough generation capacity available to satisfy its load requirements. This is possible when the loading conditions necessitates the operation of the less efficient or the costlier sets of the system in terms of input fuel, and when the imported energy proves cheaper than that produced by the internal system's own resources. This is also advantageous to the exporting system, since, although this energy is usually sold cheaply, it is sold in bulk. Also, it means that the external system resources are utilized, especially when its own loading conditions permits this energy transfer, since, otherwise the system will have unused generation capacity.

4. Imported power can be considered simply as generation at the boundary bus with the associated "buying cost" and therefore, for the specific period of energy transfer, it is treated like any other generator within the internal system. In a similar manner, exported power can be considered as load. However, this is rather a "special" type of load for two reasons. It is a load that has upper and lower limits as mentioned in (2) above. Also, the

energy exchange agreement between the two systems might involve imposing penalties when this load is not satisfied. Thus, this load can be looked upon as a "negative generator" with associated upper and lower operating limits and generation cost and this can then be incorporated into the optimization model. Of course in power system analysis any load can be treated as negative generation, but for optimization problems this is more than just a sign convention because of the presence of the operating limits and the associated generation cost or contract penalties.

5. If a boundary bus of a system is normally a nongenerating bus, then when energy is imported this bus must be reclassified as a generating bus for optimization purposes in the present mathematical model.

The above has introduced the various factors that affect the exchange of energy between two power systems. However, including this into the mathematical analysis and the optimization model is not as complicated as it sounds!

At the specific moment in time when the optimization program is required, the tie-line flow can be considered as a net generation or load depending on whether the power is being imported from or exported to the external system. The state of the tie-line flow can be decided or predicted from a (short-term) load forecast which is a prerequisite to the running of the optimization program. The active and reactive power injections at the boundary bus are then modified as follows:

$$P_i = PGB_i - PD_i \pm PT_i \tag{4.19}$$

$$Q_i = QGB_i - QD_i \pm QT_i \tag{4.20}$$

These additional terms in the power injection equations will affect the upper and lower limits of the corresponding constraints in the first stage of the mathematical model. Thus, the corresponding B-parameters have to be modified; but only slightly. The coefficients A_{ij} of the constraint matrix will not be affected. The effect on the C-parameters depends on the choice of the objective function. In the case of energy import from an external system, an extra generator will be added to the second stage of the model. This increases the number of independent variables and the corresponding A-parameters, as well as the number of terms of the objective function. Also, the upper and lower limits of the constraints, i.e., the B-parameters have to be modified. The export of energy has no effect on the formulation of the second stage of the mathematical model although it affects the numerical results.

4.4 SPINNING RESERVE AND AREA RESERVE

For a generator connected on line and which is already loaded, the spinning reserve or spare capacity of the generator can be defined as the amount of extra output that can be made available within a short time period to cater for load increase on the system [37]. In mathematical terms this is given by the difference between the maximum capacity of the generator and its actual output at a given time, as shown in (4.21) below:

$$RG_j = PGGMX_j - PGG_j \tag{4.21}$$

This also defines the maximum spinning reserve of the generator.

In a power system, the generators are usually loaded such that there is an adequate operating margin to take into account the possibility of additional load on the system. A minimum margin is specified for this purpose given by the following inequality:

$$RG_j \geq RGMN_j \tag{4.22}$$

Substituting from (4.21), this can be written as:

$$PGGMX_j - PGG_j \geq RGMN_j \tag{4.23}$$

This means that generators are loaded not only such that their maximum operating limits are not exceeded, but also within a specified operating margin below that. The immediate effect of this, is that it reduces the actual upper operating limits of the generators. The new modified upper limit constraint on the generator output will now be given by:

$$PGG_j \leq PGGMX_j - RGMN_j \qquad (4.24)$$

This is illustrated in FIG.(4.3)

A similar argument applies to a generating bus where several generators are connected. The spinning reserve of the equivalent bus generator is given by:

$$RB_i = PGBMX_i - PGB_i \qquad (4.25)$$

where,

$$RB_i = \sum_{j=1}^{NGB_i} RG_j \qquad (4.26)$$

The upper bound on the bus output can be obtained by the summation of the Inequalities (4.24) corresponding to all the individual generators connected at the bus. This is given by (4.27) below:

$$PGB_i \leq PGBMX_i - RBMN_i \qquad (4.27)$$

where,

$$RBMN_i = \sum_{j=1}^{NGB_i} RGMN_j \qquad (4.28)$$

The other two quantities in (4.25) and (4.27), namely PGB_i and $PGBMX_i$, have already been defined in Chapter (2) in a similar way to (4.26) and (4.28).

The inequality (4.27) can be incorporated into the first stage of the linearized model by following the same method of derivation used in Sections (3.5) and (3.6). The coefficients of the constraint matrix, i.e., the A-parameters, will not be affected by the new term in (4.27). The B-parameters have to be modified, while the effect on the C-parameters depends on the choice of the objective function. In the second stage of the model only the B-parameters are affected.

Sometimes spinning reserve specifications are imposed on a group of buses in the system constituting an "area", or on a group of areas constituting a subsystem and so on. The area spinning reserve can be defined and treated in the same way as discussed above. The relationships between the various quantities of the area and those of the individual buses are similar to the relationships between the quantities of the generating buses and those of the individual generators.

4.5 LOADING AND DELOADING RATES OF GENERATORS

When the load on the system changes, the outputs of some or all of the system generators have to be adjusted to satisfy the new loading conditions. However, a generator output cannot be changed instantly. A certain finite time TM must elapse before a specified required change in the active power output of a given generator can be effected. This time response of the generator depends on the generator design, and is measured by the loading and deloading rates of the generator. These correspond to increasing and decreasing loads respectively and are measured by units of power per unit of time [38].

The relationship between the change in the generator output and its time response is expressed as in (4.29) and (4.30) below, corresponding to increasing and decreasing outputs respectively.

$$\triangle PGG_j = LR_j \times TM \quad (4.29)$$

$$\triangle PGG_j = DR_j \times TM \quad (4.30)$$

These two equations can be rewritten as in (4.30) and (4.31) below:

$$PGG_j - PGG_{jo} = LR_j \times TM \quad (4.31)$$

$$PGG_{jo} - PGG_j = DR_j \times TM \quad (4.32)$$

It is to be noted here that both LR_j and DR_j are positive quantities.

To take into account the loading and deloading rates, the upper and lower operating limits on the generator output are modified as follows:

$$PGG_{jo} + LR_j \times TM \geq PGG_j \geq PGG_{jo} - DR_j \times TM \quad (4.33)$$

Recall that, without these modifications, these limits are given by:

$$PGGMX_j \geq PGG_j \geq PGGMN_j \quad (4.34)$$

The Inequalities (4.33) and (4.34) represent a set of "overlapping" constraints, i.e., two upper limits and two lower limits on the same quantity. This has the effect of reducing the overall operating range of the generator as illustrated by FIG.(4.3). In the figure, one set of lower and upper constraints, PL1 and PU1 overlaps another set, PL2 and PU2. The resulting operating region is defined by the least of the two upper limits and the greatest of the two lower limits. A similar situation may occur when spinning reserve and time response considerations are combined.

Loading and deloading rates are Generator Quantities. They affect the parameters of the equivalent bus generator and the corresponding constraints in both stages of the model. This effect appears as a modification in the B-parameters of the optimization problem. In both stages of the model, the A and C parameters are not affected.

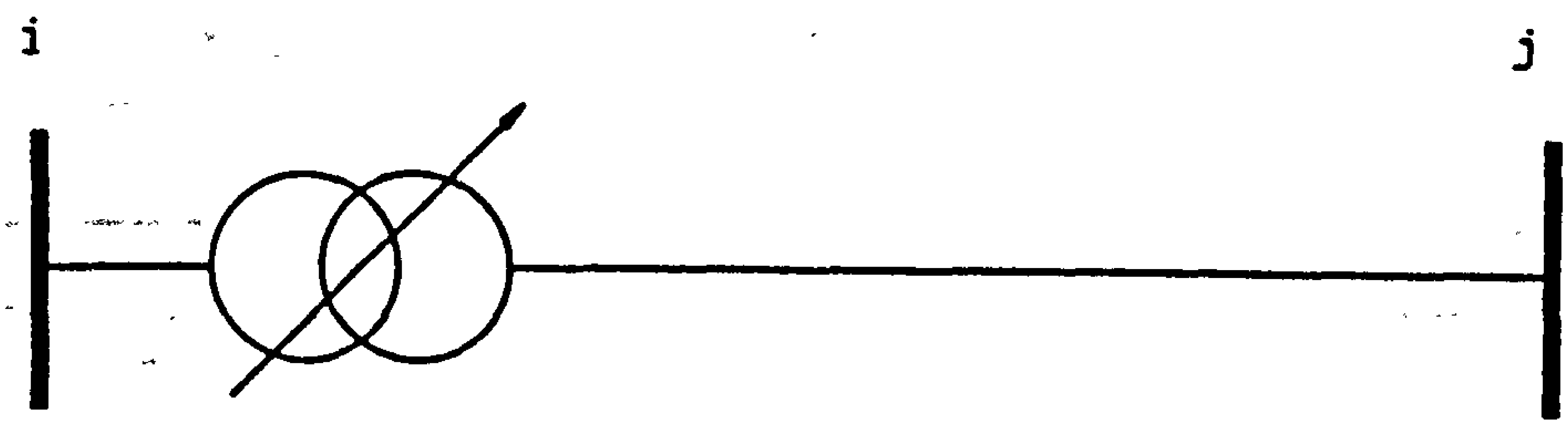
In terms of loading and deloading rates, a generating bus, where several generators are connected, is as fast as its slowest responding generator. For a certain required change in the demand $\triangle$PD, this generator will slow down the overall time response of the bus, no matter how fast the rest of the bus generators are. On the other hand, for a given period of time $\triangle$TM, the change in the bus output is given by (4.35) and (4.36) below, for increasing and decreasing outputs respectively.

$$\triangle PGB_i = \sum_{j=1}^{NGB_i} (LR_j \times \triangle TM) \qquad (4.35)$$

$$\triangle PGB_i = \sum_{j=1}^{NGB_i} (DR_j \times \triangle TM) \qquad (4.36)$$

Finally, the loading and deloading rates affect the "dynamics" of power system operation. In other words, they are only relevant when the optimization is performed in several successive short time steps. Usually the time intervals and frequency of performing the optimization procedure presented here are such that there is sufficient time for the various generators to respond to changes in the loading conditions.

(A)



(B)

(C)

i : NEAR END

j : FAR END

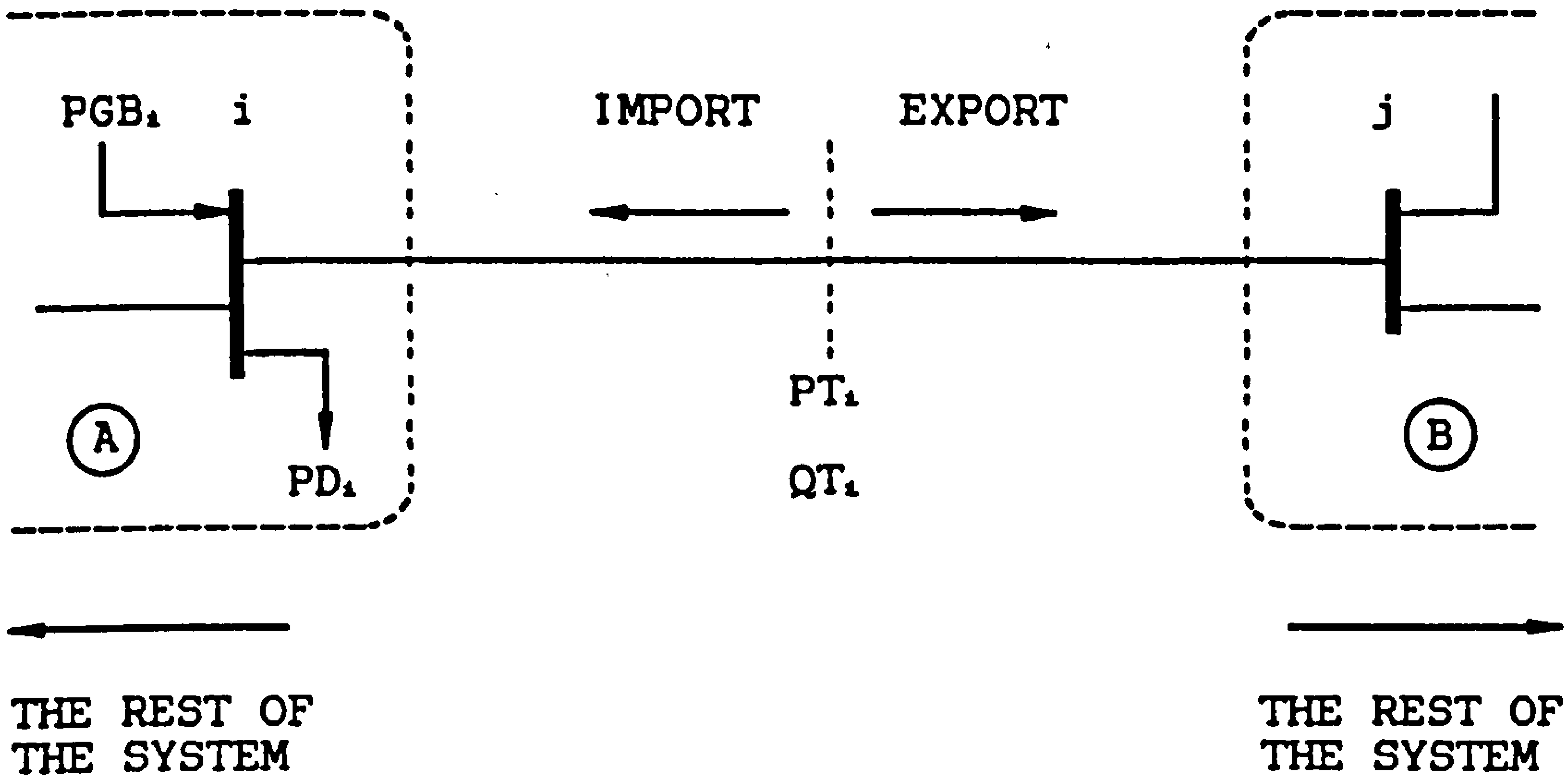
FIG.(4.1)

TAP-CHANGING TRANSFORMERS

(A) A LINE WHERE NO TRANSFORMER IS CONNECTED

(B) TRANSFORMER CONNECTED AT THE NEAR END

(C) TRANSFORMER CONNECTED AT THE FAR END



A : INTERNAL SYSTEM

B : EXTERNAL SYSTEM

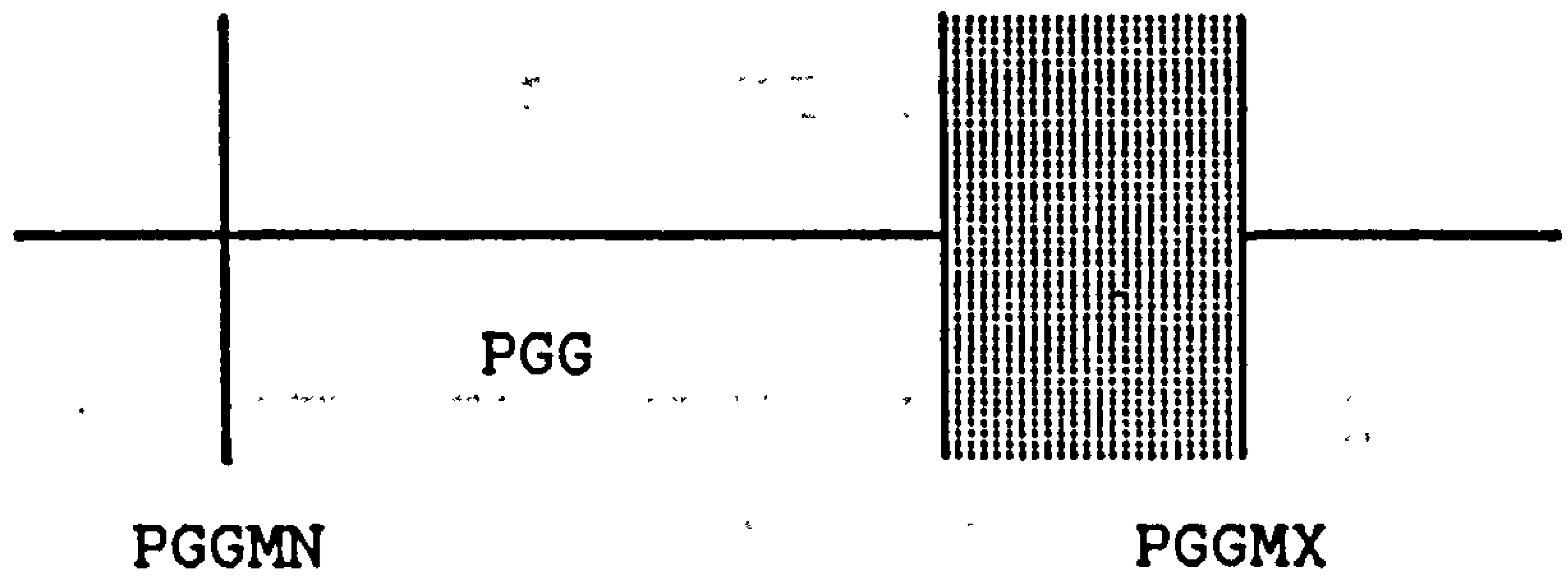
i & j : BOUNDARY BUSES

IMPORT - BUYING ENERGY - GENERATION

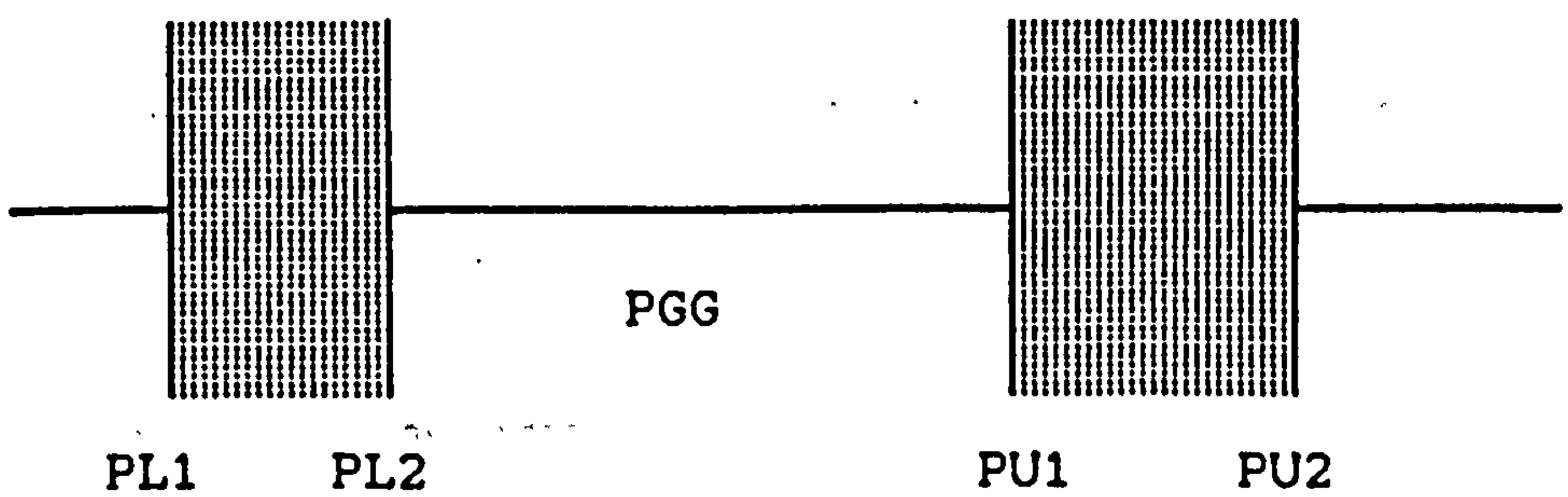
EXPORT - SELLING ENERGY - LOAD

FIG.(4.2)

INTERCONNECTION OF TWO POWER SYSTEMS THROUGH A TIE-LINE



(A)



(B)

THE UNSHADED AREAS REPRESENT THE OPERATING REGIONS

FIG.(4.3)

MODIFICATION OF GENERATOR OPERATING LIMITS

(A) SPINNING RESERVE OPERATIONAL MARGIN
(REPRESENTED BY THE SHADED AREA)

(B) OVERLAPPING CONSTRAINTS

CHAPTER 5

OPTIMIZATION PROBLEMS AND NUMERICAL RESULTS

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CHAPTER 5

OPTIMIZATION PROBLEMS AND NUMERICAL RESULTS

5.1 INTRODUCTION

The methodology of translating the operation of a general power system into a two-stage linearized mathematical model for optimization purposes has been explained in Chapters 2 and 3. The various "ingredients" of the modelling process were introduced in Chapter 2, while in Chapter 3 a generalized mathematical model has been developed, using the large number of interrelationships amongst the various generator, bus and transmission line quantities. The derivation of the constraints imposed on the various power system quantities as well as a general objective function were presented. So far, the presentation has been kept in general terms, in the sense that no particular set of constraints was chosen and no specific quantity was optimized. This is the subject of the present chapter.

A number of optimization problems will be addressed, each with a specific objective function and a selected set of constraints. Again, the formulation will be introduced, first, for a general power system of NG generators, NB nodes, including NS generating buses, and NL transmission lines. The A-B-C representation of each problem will then be set up and numerical results obtained by application to an actual sample system.

5.2 OPTIMIZATION PROBLEMS AND TEST SYSTEM

Before a detailed discussion of the optimization problems presented in the chapter, and the procedure of setting them up and obtaining the numerical solutions, this section briefly introduces these problems and gives a general description of the sample power system used in testing the various concepts introduced in the thesis.

The Optimization Problems

Three different optimization problems will be considered in this chapter and they will be discussed in detail in respective sections. In each case the problem is introduced, the objective function defined and derived, and the constraints are specified. Only the objective function is discussed with some detail in these sections. The derivation of the various constraints has already been thoroughly covered in Chapter 3. The three problems considered are listed below.

Problem 1. Minimization of the Total System Hourly Generation Cost.

Problem 2. Minimization of the Total System Transmission Losses.

Problem 3. Minimization of the Total System Reactive Power Production

These optimization problems refer to the first stage of the mathematical model. Henceforth, the respective problems will be referred to as Problems 1, 2 and 3 for short. The optimization problem of the second stage in each case is the same and is concerned with the minimization of the instantaneous generation cost. The complete formulation of this problem has already been presented in Section (3.7).

The Test System

For test purposes, a system of medium size is used which represents part of the British 132/275 kV Grid [9,10]. The system has 24 generators, 23 buses and 30 lines. The same system is used throughout, so that results from the various optimization problems can be compared, the effect of changes and variations in the mathematical model on the numerical results monitored, and the merits of the various methods assessed. A one-line diagram of the test system is shown in FIG.(5.4) and the relevant data is given in Tables (5.1) to (5.5).

5.3 THE INPUT DATA

This section introduces the various input data required to run the computer programs associated with the various optimization problems. For each optimization problem the necessary input data consists of two main parts as described below.

1. System Data

This consists of the following :

General System Data

This gives the number of power stations, generators, buses and lines in the system. It also gives the total generating capacity of the system, its nominal voltages and the Base MVA which is relevant to the transmission line parameters given in per-unit values.

Generator Data

This specifies upper and lower operating limits on generator outputs, and gives the incremental cost as well as the number of the bus at which the generator is connected. For each generator, the original data of the test system specifies the following four operating limits only:

$PGGMN_j$, $PGGMX_j$, $QGGMN_j$ and $SGGMX_j$.

To obtain the other two quantities, namely $QGGMX_j$ and $SGGMN_j$, the following two equations are used:

$$QGGMX_J = \sqrt{SGGMX_J^2 - PGGMN_J^2} \quad (5.1)$$

$$SGGMN_J = \sqrt{PGGMN_J^2 + QGGMN_J^2} \quad (5.2)$$

While the second of these equations is self-explanatory, the first needs some clarification. For a specified maximum apparent power output capacity of the generator, there are two extreme cases. One is when the generator delivers maximum active power and minimum reactive power and the other when it delivers minimum active power and maximum reactive power. This is illustrated in FIG.(5.1).

Transmission Line Data

This consists of the three parameters of the π-equivalent circuit of each transmission line, namely the equivalent series resistance, the equivalent series reactance and the equivalent shunt susceptance. The data also includes upper limits on the thermal power loss in the lines and the line apparent power transfer capacities.

2. Demand Data

This gives the loading schedule of the system, i.e., the active and reactive load on each bus in the system.

From the demand data two different loading schedules are available. These are referred to as Schedules A and B and were both used in performing the numerical tests.

5.4 SUMMARY OF THE METHOD

The general method of setting up and solving the various optimization problems of this chapter is summarized below. Generally, the various steps described also correspond to the computer program associated with each optimization problem. Detailed explanation of these steps as well as the necessary theoretical background and the associated mathematical expressions are given in Chapters 2 and 3. Reference to these is given here where appropriate.

The setting up of each optimization problem is started by obtaining the parameters of the equivalent bus generators using the corresponding parameters of the individual generators connected at the bus in question. This basically means that each power station in the system is replaced by one generator. For the test system considered, the parameters of the various generating buses thus obtained are given in Table (5.3). In a similar manner to load flow studies, one of the generating buses is chosen as the reference or slack bus of the system for which the voltage magnitude and phase angle are assigned fixed values beforehand. From the line data, the system admittance matrix is set up.

The next step is to obtain an estimated initial generating schedule, i.e., the initial set of active and reactive power outputs of all the generators in the system. From these, the active and reactive power outputs of the generating buses can be obtained. Using the demand data, the net bus injections can be determined. The initial values of all bus voltage magnitudes and phase angles can then be obtained by performing a load flow. Using the appropriate equations, all initial line flows can be computed.

From the initial operating quantities, the first stage of the incremental mathematical model of the problem is set up. This implies computing the A-B-C parameters of the corresponding linear programming problem. These are given below:

A : Elements of the constraints coefficients matrix. This is a matrix of dimension (NC X NV), where

NC is the number of constraints, and

NV is the number of independent variables.

The voltage magnitudes and phase angles of the system buses are used as the independent variables in the first stage of each optimization problem. Thus, NV = 2N and for the test system used, NV = 44. The number of constraints depends on the particular optimization problem considered.

B : Lower and upper bounds on the constraints. These form a vector of NC elements.

C : Vector of coefficients of the objective function. This vector consists of NV elements.

The resulting linear programming problem is then solved to obtain the values of the independent variables under optimum operating conditions. Any method and any computer routine available which are suitable for the solution of linear programming problems can be used to obtain the numerical solution. Alternatively, a computer program can be written for this purpose.

Using the values of the bus voltages obtained from the first stage, all other dependent power system quantities are computed. These quantities include all active and reactive power bus injections, line flows and transmission losses. The active and reactive power outputs of the generating buses can then be obtained using the corresponding injections and demands. These outputs form part of the parameters required for the formulation of the optimization problem of the second stage of the mathematical model.

As mentioned previously, the first stage of the mathematical model of the problem is based on bus and line quantities. Therefore, it does not deal with individual generator variables. These are obtained by solving the second stage of the problem. The A-B-C parameters of this problem are set up and the solution can be obtained by using the same linear programming method and computer subroutine as that used in Stage-I. As explained in Section (3.7), the linear programming problem of Stage-II of the mathematical model has NV = NG variables and NC = (NS+1) constraints. For the particular sample system used, these are given by NV = 24 and NC = 7 respectively.

Both stages of all the optimization problems discussed in this chapter can be represented by the general matrix format of Fig.(5.2). This also represents the generalized formulation of the linear programming problem of Section (3.3). The constraints coefficient matrix of the first stage of the model is shown in more detail in FIG. (5.3). As shown in the figure, this matrix consists of a number of submatrices which reflect the nature of the corresponding constraints, such as bus injections, line losses and so on. The number of rows in each submatrix block depends on the power system quantities it represents. Generally they are in sets of NB or NL constraints, depending on whether they are based on bus or line quantities.

Also, the constraints coefficients matrix, and each of its constituent blocks, consists of two distinct sets of columns. One set of columns consists of elements associated with the bus voltage magnitudes and the other consists of matrix elements associated with the bus phase angles. The number of columns in each set is equal to N, which, in the case of the test system is equal to 22.

5.5 THE TABULATED NUMERICAL RESULTS

Each of the three optimization problems has been tested on the two load schedules available from the input data. The two corresponding study cases are referred to as Case A and Case B respectively. This gives a total of six study cases. The final output results of all the problems are given in Tables (5.6) to (5.35) at the end of the chapter.

Apart from the following notes, the tables themselves are self-explanatory. For each study case the tabulated numerical results give a complete picture that defines the overall steady state of the power system under optimum operating conditions, in terms of all its dependent and independent variables and other related quantities. The output results are arranged as follows.

1. Generator Results

These are the active, reactive and apparent power outputs of each generator as well as its hourly generation cost.

2. Bus Results

These consist of three tables. The first two tables give results of all busses in the system while the third is particular to the generating buses. The first table gives the voltage magnitude and phase angle of each bus in the system. The voltage magnitudes are given in per-unit values as this is more meaningful in power system analysis and it shows the deviation of the actual voltages from their nominal values. The phase angles are given in degrees to give a clearer impression than units of radians which are used in the computer programs. The second table gives the active, reactive and apparent power injections at the buses. In a similar manner to generator results, the third table gives the total station active, reactive and apparent power outputs as well as the generation cost. It also gives the number of generators at each power station. This has already been given amongst the system data, Table (5.2), but is repeated here for completeness.

3. Transmission Line Results

These consist of three tables giving the active, reactive and apparent power flows in each line. Each table gives the corresponding power transfer across the line, in both directions, as well as the associated power loss.

4. Overall System Results

These give the total system active, reactive and apparent power generations, and total transmission losses. They also include the total hourly generation cost of the system, the overall system cost per MWhr and the hourly cost associated with the total transmission losses, as well as the cost associated with the net active power. This is the part of the total active power generation needed to meet the demand on the system. The last row in the table gives the overall system power factor.

Calculation of The Overall System Results

The total system active and reactive power generation and losses are obtained by the summation of the corresponding individual quantities from the respective tables. This also applies to the total system generation cost. The various apparent power quantities are obtained from Equation (2.49).

The cost per unit active power generation is then obtained using the following equation:

$$SPC = \frac{CGT}{PGT} \tag{5.3}$$

The total cost associated with the transmission losses is obtained form (5.4) below.

$$CLT = SPC \times PLT \tag{5.4}$$

The total cost associated with the net active power generation is obtained as in Equation (5.6) below.

$$CGD = CGT - CLT \tag{5.5}$$

The overall system power factor is obtained from (5.5)

$$PF = \frac{PGT}{SGT} \tag{5.6}$$

All the numerical results obtained will be compared and thoroughly discussed in Section (5.11).

5.6 MINIMIZATION OF GENERATION COST

Generation cost is one of the very important aspects to be considered when operating a power system. The cost incurred by the continuous running of a large power system is enormous. This has been further escalated by increasing fuel prices, especially during the last two decades. Therefore, methods have to be devised to minimize the overall generation cost of the system.

The need for the economic operation of power systems has been recognized and emphasized since the very early days of the power industry. It started by the economic division of load between two generators and the economic operation of small systems with few generators or few generating stations, followed by the economic dispatch of larger systems. As power systems grew larger and their operation and control became more demanding, optimal load flows and optimization methods came into use. More specifically, the cost of active power generation has been used in recent years as the objective function to be minimized under constraints by applying mathematical programming techniques [9,10,11,15].

In this thesis, the hourly fuel cost of the power system is minimized by formulating the problem as a linear programme and using the two-stage mathematical model. In the first stage the hourly generation cost, based on bus outputs, is used as the objective function with four sets of constraints.

The objective function is given by the sum of the hourly fuel cost at all the generating buses, i.e., power stations, in the system. This is expressed by:

$$Z = \sum_{i=1}^{NS} (CB_i \times PGB_i) \qquad (5.7)$$

Substituting from Equation (3.16) gives:

$$Z = \sum_{i=1}^{NS} CB_i \times (P_i + PD_i) \qquad (5.8)$$

By using the expanded linearized expression of P_i from Section (2.6), and ignoring any constant terms as explained in Section (3.6), Z can be expressed as:

$$Z = \sum_{i=1}^{NS} CB_i \sum_{j=1}^{N} (AM_{ij} V_j + AN_{ij} \theta_j) \qquad (5.9)$$

The coefficients AM_{ij} and AN_{ij} of (5.9) are given by the expanded expressions of (3.26) to (3.29).

The final form of the objective function Z can be written as:

$$Z = \sum_{j=1}^{N} (CM_j V_j + CN_j \theta_j) \qquad (5.10)$$

where,

$$CM_j = \sum_{i=1}^{NS} CB_i \ AM_{ij} \tag{5.11}$$

and

$$CN_j = \sum_{i=1}^{NS} CB_i \ AN_{ij} \tag{5.12}$$

In this optimization problem, four sets of constraints have been used. These are listed below:

1. Upper and lower limits on the bus active power outputs
2. Upper and lower limits on the bus reactive power outputs
3. Upper limit on the active power loss in each transmission line, and
4. Upper limit on the apparent power transfer across each transmission line.

The first two sets of constraints are based on bus quantities while the last two sets are based on line quantities. Each of the constraints sets 1 and 2 have NB constraints, while sets 3 and 4 have NL constraints each. Thus, the total number of constraints in this problem is NC = 2(NB + NL). For the test system used this is equal to 106.

This optimization problem can be viewed from a different angle. Under the given loading conditions and introducing electricity selling prices as well as input fuel cost, the problem of minimizing the generation cost can be used to maximize the profit. This will only involve minor modifications to the objective function and the corresponding computer program. If required, additional, consumer and load-related, constraints can also be incorporated in the formulation of the problem.

The numerical results of this problem are given in Tables (5.6) to (5.10) for Case A, and Tables (5.11) to (5.15) for Case B.

5.7 MINIMIZATION OF TRANSMISSION LOSSES

Apart from the generation cost required to meet the actual consumer electricity demand on the system, a certain amount of cost is also incurred by energy loss in the various components of the power system. A significant part of this energy loss is caused by the resistance of the transmission lines in the system. Although transmission losses generally constitute a small percentage of the total system output or demand, in the long run they incur a substantial cost due to the continuous operation of the power system. Therefore, as in the case of generation cost, transmission losses have received a considerable attention from researchers in the field of optimization [7,17]. The problem of transmission losses is usually tackled by formulating the optimization problem such that these losses are minimized. As in other optimization problems various numbers and types of constraint are imposed to take into account other relevant power system quantities.

In the present work, the first stage of the mathematical model can be used for this purpose, while the second stage can still be used to minimize the generation cost. This useful attribute of the two-stage formulation will be discussed further in Chapter 7.

Minimization of the total system transmission losses can be expressed by the following objective function:

$$Z = \sum_{k=1}^{NL} PL_k \tag{5.13}$$

Again, this can be expressed in terms of the independent variables, i.e., the system bus voltage magnitudes and phase angles, by using the expanded linearized expressions of PL_k from Chapter 3 and neglecting the constant terms. This gives:

$$Z = \sum_{i=1}^{NL} \sum_{j=1}^{N} (AM_{ij} V_j + AN_{ij} \theta_j) \tag{5.14}$$

The coefficients AM_{ij} and AN_{ij} in this case are those given by Equations (3.38) to (3.41).

The final form of Z can be written as follows:

$$Z = \sum_{j=1}^{N} (CM_j V_j + CN_j \theta_j) \tag{5.15}$$

with the coefficients CM_j and CN_j given by:

$$CM_j = \sum_{i=1}^{NL} AM_{ij} \qquad (5.16)$$

and,

$$CN_j = \sum_{i=1}^{NL} AN_{ij} \qquad (5.17)$$

The constraints used in this problem are similar to those used in the previous one. The corresponding computer results for the two study cases are presented in Tables (5.16) to (5.25).

5.8 MINIMIZATION OF REACTIVE POWER PRODUCTION

In a power system, the generation of reactive power does not result in a running cost once the required apparatus of the appropriate rating is installed. However, with reactive power, the question is that of control. Reactive power flows affect system currents which in turn affect transmission losses and bus voltages. For a given amount of active power generation in the system the rest of the system quantities are functions of the reactive power flow. Examples of quantities affected in this manner are apparent power in the system, current magnitudes and the overall system power factor. Since all power system variables are interrelated, the control of reactive power flow virtually affects many other system variables, either directly or indirectly. Therefore, correct decisions have to be made in this regard to insure the proper operation of the system.

Reducing or minimizing the amount of reactive power production in the system can be used as a means of reducing transmission losses and improving the voltage profile of the system. Obviously, reduction of transmission losses also reduces the associated cost. With a given amount of active power generation, reducing the total reactive power production also improves the system power factor.

Minimization of the total reactive power production in the system is expressed by the following objective function:

$$Z = \sum_{i=1}^{NB} QGB_i \tag{5.18}$$

Following the derivation steps of the two previous optimization problems, the final form of (5.18) can be written as in (5.19) below:

$$Z = \sum_{j=1}^{N} (CM_j \, V_j + CN_j \, \theta_j) \tag{5.19}$$

The coefficients CM_j and CN_j in this case are given by:

$$CM_j = \sum_{i=1}^{NB} AM_{ij} \tag{5.20}$$

and,

$$CN_j = \sum_{i=1}^{NB} AN_{ij} \tag{5.21}$$

The expanded forms of the coefficients AM_{ij} and AN_{ij}, in the two expressions above, are given by Equations (3.32) to (3.35).

In this problem, an expanded version of the mathematical model has been used in which the constraints consist of five sets. The first three sets are similar to those used in the first two problems. However, the fourth set, concerned with the apparent power transfer across transmission lines, is replaced for each line by two constraints. For the k'th line, connecting the i'th and j'th nodes, these two constraints are given by:

(a) Upper limit on S_{ij}, the apparent power transfer across the line from node i to node j, and

(b) Upper limit on S_{ji}, the apparent power transfer across the line from node j to node i.

Thus, the total number of constraints of this optimization problem is given by NC = 2NB + 3NL, which, for the present test system, is equal to 136.

It is to be recalled that the above objective function and five sets of constraints belong to the first stage of the model. Again, the second stage of the solution can be used to determine the minimum-cost generating schedule of the system, under minimum reactive power production as determined from Stage-I above.

The numerical results associated with this optimization problem are shown in Tables (5.26) to (5.35).

5.9 SUPPLEMENTARY COMMENTS AND USEFUL POINTERS

This section gives some complementary remarks relevant to the optimization problems discussed above and the associated computer programs. It also introduces and discusses an additional number of points concerning various related general, special and computational aspects under appropriate subheadings.

The Choice of the Reference Bus

The power station with the highest generating capacity has been chosen as the reference bus in all the optimization problems discussed above. This is more or less similar to the practice used in load flow studies. The reason for such a choice is to insure that there is enough generation to take the total system transmission losses into account and to satisfy the overall system energy balance equation. Otherwise, this may lead to infeasibility of the optimization problem of Stage-I of the model and erroneous results from Stage-II.

Numbering of Buses

Unlike the cases of generators and transmission lines, the serial numbering of buses in a computer program based on power system analysis is significant. Usually, in load flow studies, after the reference bus has been chosen, the buses are numbered such that the reference bus is considered as either the first or the last bus in the system; in most cases the last. Although in principal any order of numbering is acceptable, giving the reference bus any number between the two extreme cases mentioned, renders the associated computer program lengthy, difficult to handle and unnecessarily cumbersome. I have tried it!

The Reference Bus Voltage

In all the optimization problems discussed above, the voltage magnitude and phase angle of the reference bus were fixed at 1.05 p.u. and 0 degrees, respectively. The choice of the voltage magnitude is significant. While experimenting with the computer program in the early stages of the work, it has been found that at a certain minimum value, say 1.03 p.u., the optimization problem in question might become infeasible. This is due to the fact that at a certain low voltage level the reference bus fails to "drive", or "push" power into, the system. The choice of the reference phase angle, however, is completely arbitrary. The value of zero is selected here merely because it is generally customary to do so in power system analysis. For example, as they stand, the bus phase angles of Problem 1, given in Table (5.7), are almost all negative, with a minimum value of about -23°. As a test, the associated computer program was run with the value of 25° assigned to the phase angle of the reference bus. The resulting phase angles were all positive with the minimum being just a few degrees above zero, and there was no change in the rest of the numerical results.

Bounds on the Bus Voltage Magnitudes

The available data does not specify particular limits on the bus voltage magnitudes. This is mainly because constraints specifications on bus voltages are operational rather than design limits, in contrast with generator ratings and line loadings for instance. Voltage magnitudes of about ± 10 % around the nominal value are generally acceptable in power system operation. In this particular work a range of ± 5 % has been used throughout. Ranges of 4 or 6 % are equally acceptable and have also been tried, the emphasis here being on the general method rather on the specific numerical values.

The Initial Generating Schedule

Generally, the data of the initial operating point needed to set up the incremental linearized mathematical model were obtained by the method outlined in Section (2.8). Some minor variations were also used. In Problems 1 and 2, the active power schedules were obtained by roughly dividing the total system load amongst the system generators in proportion to their output capacities. Since only an estimate of the initial operating point is required in the first place, approximate round figures were used instead of the exact ones according to Equation (2.70), before entry to the computer program. From the total reactive load on the system, the reactive power output of each generator was estimated at 20 MVAR.

In Problem 3, on the other hand, the estimation process of both the initial active and reactive generation schedules was embedded in the computer program. The active part was strictly according to Equation (2.75), while the reactive part was obtained simply by sharing the total system reactive load equally among the system generators.

Limits on Reactive Power Generation

At the beginning of the project, a reduced set of the test system data was used in which no bounds were specified on the reactive power outputs of the generators. To keep the generality of the problem formulation, the following assumptions were made.

$$QGGMX_j = PGGMX_j$$

$$QGGMN_j = - QGGMX_j$$

Although these limits are not genuine, the corresponding results obtained from the problem of minimization of total system generation cost were not very different from those obtained at a later stage of the work which are reported here.

The Reactive Power Generation Schedule

The reactive power generation schedule on the generator level, in contrast with that on the bus level, has not been given much emphasis. The main reason for this is that reactive power generation affects the power system operation mostly on the bus level because of its relationship with bus voltages and line flows. Another reason is that very small, if any, fuel cost is associated with reactive generation in comparison with active power dispatch. In the present work, the bus reactive power outputs under optimum operating conditions are determined from the first stage of the problem model.

The reactive power contribution of the individual generators towards the respective bus outputs are determined simply by sharing the bus outputs among the generators in proportion to their reactive power capacity, in a similar way to that used to obtain the initial operating schedule. This is the very last step in the solution of the problem and, thus, will not affect any other quantity. Recall that the corresponding active power generation schedule is obtained by formulating the second stage of the model as another optimization problem based on generator active power outputs. The reactive power problem can be formulated in a similar way with a suitable objective function. For this purpose incremental costs similar to those of the active power problem can be used if required and if the corresponding data is available. However, as discussed above, this is not really necessary.

5.10 THE COMPUTER PROGRAMS

The computer programs associated with the various optimization problems addressed in the thesis have all been written in FORTRAN. They were run on the mainframe machines of the VAX Computer Cluster of the University of Strathclyde and the Micro-Vax Computer of the Power Systems Research Group where the project was conducted.

The basic Newton-Raphson method has been used to write the load flow computer routine needed to obtain some of the data of the initial operating point. The bulk of the computations are performed using per-unit values on a Base MVA of 100.

Routines from the NAG (Numerical Algorithms Group) Library were employed to obtain the actual numerical solutions of the linear programming problems of the two stages of the mathematical model. Two such routines designed to solve general linear programming problems were considered [39,40]. In most cases the routine E04MBF has been used. In the course of testing the suggested method and the associated computer programs, the routine H01ADF was also used to solve the linear programming problem of Stage-II of the model instead of E04MBF. Identical results were obtained.

Finally, it is possible to use these subroutines to determine the reactive power generation schedule on the generator level. As discussed above no optimization is required in this case and, therefore, no objective function is involved. The NAG optimization routines mentioned above are designed such that, when no objective function is specified for the given linear programming problem, they can determine a feasible point of the problem instead of the optimum solution. One of the input parameters of the routines is set to indicate the existence or nonexistence of the objective function.

5.11 COMPARISON AND DISCUSSION OF THE NUMERICAL RESULTS

In the present chapter three main optimization problems relating to power system operation have been introduced, formulated and solved, and the numerical results obtained have been reported. Relevant comments on the formulation and solution of these problems have also been given as well as useful remarks concerning the computational aspect of the work. In the present section all these numerical results will be analysed and discussed. They will also be compared among themselves as well as with similar results published by other contributors to the field.

Comparison with Published Results

In the early stages of the project, the numerical results obtained from Problem 1 were extensively compared with those published by Shen and Laughton [9]. The reason for this comparison is that these authors used the same sample system in their work with a similar objective function. This gives an agreeable basis for the comparison, although, of course, their formulation and method of solution are different. After accounting for system losses which were ignored in the cited work, the results obtained here were almost identical to the published ones, apart from the many advantages of the present method. These will be discussed in detail in Section (7.2).

Comparison of the Three Optimization Problems

To compare the results obtained from the three different problems two comparative tables are constructed corresponding to the two study cases. These are given in Tables (5.36-A) and (5.36-B) respectively. The two tables basically represent a summary of the total system results which are shown in six separate tables, each one at the end of the respective study case under the title of "Overall System Results".

Studying the results presented in Tables (5.36-A) and (5.36-B), the following main conclusions and comments can be made:

1. The results are consistent among themselves. The same conclusions that can be obtained from Case A can be obtained from Case B. For example, the tendency of one system quantity to decrease from Problem 1 to Problem 2 occurs in both Cases A and B. In a similar manner quantities that tend to increase from Problem 2 to Problem 3 do so in both study cases. Any of the entries of Table (5.36) can be used to confirm this.

2. Apart from being consistent among themselves as explained above, the results are consistent with the background theory. Problem 1 produces the least generation cost among the three problems, while Problem 2 shows the least transmission losses.

3. The results of problems 2 and 3 are very similar. Basically these two problems achieve the same final objective in the physical system using different mathematical objective functions. Problem 2 minimizes the total transmission losses directly by explicitly using this quantity as the objective function in the linear programming formulation. The same aim is achieved indirectly in Problem 3 by minimizing the total reactive power generation in the system. This leads to reduced current magnitudes through the transmission network and consequently reduces the associated I^2R or thermal losses. This confirms the reasoning behind the use of reactive power control as a means for reducing system losses. It also confirms the work presented by various researchers who used only one or the other of the two different objective functions of Problems 2 and 3. In the various published works, confirmation of this was not possible because of the inflexibility of the mathematical models used, which did not allow the use of a different objective function.

However, there are slight differences between the results of Problem 2 and those of Problem 3. These are mainly caused by the considerable difference in the number of constraints between the two problems. In Problem 3 there is a total of 136 constraints, i.e., 30 constraints more than those of problem 2 which has only 106. These extra constraints are based on the transmission of apparent power across the lines. Problem 3 considered the transmission of apparent power in both directions across the line, while problem 2 was concerned with the transmission of apparent power across the line in one direction only. This particular point will be discussed in more detail in Section (7.3).

4. Although the total system transmission losses and their associated cost were kept at a minimum in Problem 2, the overall generation cost is noticeably higher than that of Problem 1. This does not represent any discrepancy in the numerical results or in the mathematical formulations of the problems as discussed below.

In problem 1, the linear program is concerned with the selection of the least expensive generators, in terms of their fuel consumption, to supply the required system demand under the imposed constraints. In Problem 2, on the other hand, the optimization process is concerned with the "routing" of the generated power so that it is transmitted across the lines with the least thermal loss, i.e., the lowest resistance. This is done regardless of the generating units that produced the power in the first place. Also, the total transmission losses are supplied by the system generators collectively and it is not possible to know which generator supplies which part of the losses. Therefore, minimization of the total losses does not necessarily mean minimization of their associated cost.

This can be further clarified by considering the total energy balance equation in the system which gives the total generation as the sum of the total demand and the total transmission losses:

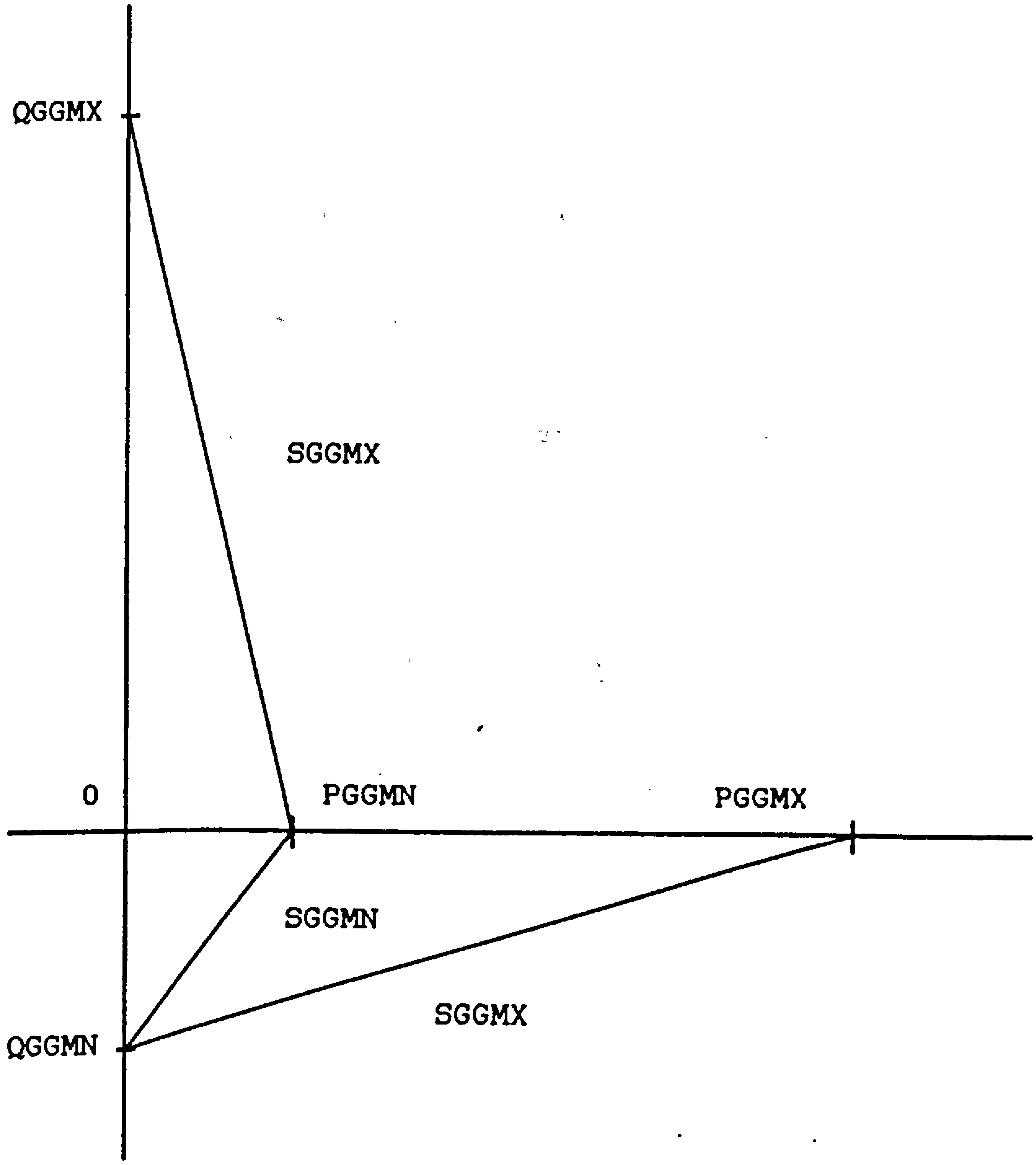
$$PGT = PDT + PLT$$

In general, PLT represents a small percentage of PGT. In the particular system considered, the ratio of PLT to PGT is less than 2 %. Problem 2 minimizes this small percentage without considering the cost associated with the production of the whole.

5. It can be concluded from the above that the best way to operate the system is under minimum generation cost as obtained from the solution of Problem 1. Obviously, this conclusion is based on numerical results of the sample system. However, it can be safely extended to power system operation in general. It is quite possible that the ratio of the total transmission losses of other systems to their total generation is not very different from those obtained here, especially since the sample system used is an actual one. Apart from that, even when the problem is concerned with the minimization of the generation cost, the constraints on transmission losses and line loading are satisfied.

6. In parallel with the minimum total system generation cost in Problem 1, the cost per unit of active power generation is also minimized although the associated transmission losses are higher than those associated with the two other problems.

7. The tables show a noticeable improvement in the overall system power factor in Problems 2 and 3, as compared to Problem 1. This is due to the reduction of the reactive power generation associated with these two problems. Again this is consistent with the underlying theory.



$$SGGMX = \sqrt{PGGMX^2 + QGGMN^2} = \sqrt{PGGMN^2 + QGGMX^2}$$

$$SGGMN = \sqrt{PGGMN^2 + QGGMN^2}$$

FIG.(5.1)

LIMITS ON APPARENT POWER GENERATION

$$Z = \begin{bmatrix} CX \end{bmatrix}$$

$$\begin{bmatrix} BU \end{bmatrix} \geq \begin{bmatrix} AX \end{bmatrix} \geq \begin{bmatrix} BL \end{bmatrix}$$

$$\begin{bmatrix} XU \end{bmatrix} \geq \begin{bmatrix} X \end{bmatrix} \geq \begin{bmatrix} XL \end{bmatrix} .$$

FIG.(5.2)

MATRIX REPRESENTATION OF A GENERAL
LINEAR PROGRAMMING PROBLEM

$$
\begin{bmatrix} CX \end{bmatrix} = \begin{bmatrix} CM & \vdots & CN \end{bmatrix} \begin{bmatrix} V \\ \cdots \\ \theta \end{bmatrix}
$$

$$
\begin{bmatrix} AX \end{bmatrix} = \begin{bmatrix} AM1 & \vdots & AN1 \\ \hline AM2 & \vdots & AN2 \\ \hline AM3 & \vdots & AN3 \\ \hline \vdots & \vdots & \vdots \\ \hline AMW & \vdots & ANW \end{bmatrix} \begin{bmatrix} V \\ \cdots \\ \theta \\ \hline V \\ \cdots \\ \theta \\ \hline V \\ \cdots \\ \theta \\ \hline \vdots \\ \hline V \\ \cdots \\ \theta \end{bmatrix}
$$

FIG.(5.3)

DETAILED REPRESENTATION OF THE OBJECTIVE FUNCTION VECTOR AND THE CONSTRAINTS COEFFICIENTS MATRIX

W = NUMBER OF CONSTRAINTS BLOCKS
(SETS OF CONSTRAINTS)

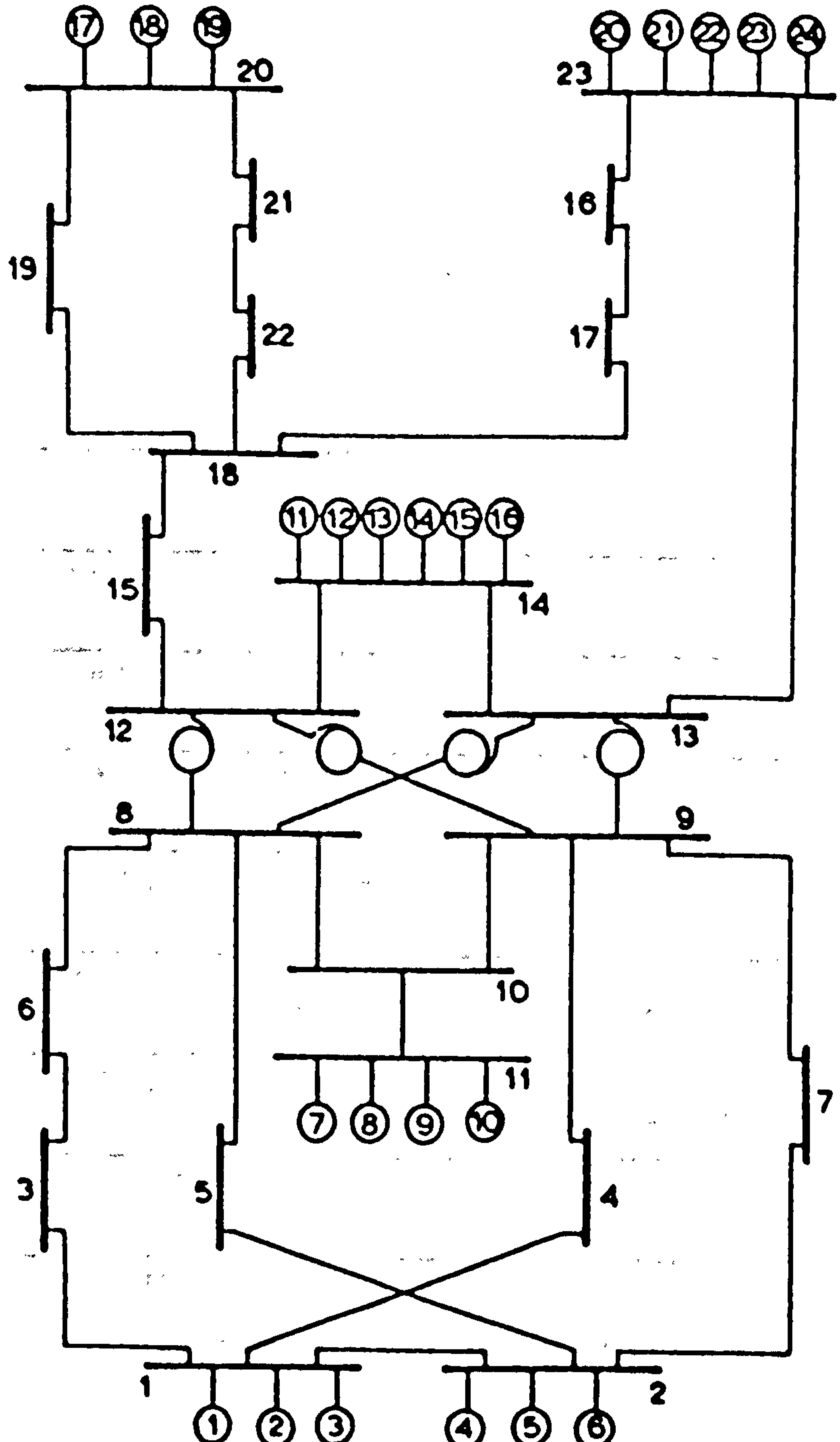


FIG.(5.4)

DIAGRAM OF THE TEST POWER SYSTEM

TABLE (5.1)

GENERAL SYSTEM DATA

NUMBER OF POWER STATIONS	6
NUMBER OF GENERATORS	24
NUMBER OF BUSES	23
NUMBER OF TRANSMISSION LINES	30
TOTAL ACTIVE POWER CAPACITY (MW)	2930
TOTAL REACTIVE POWER CAPACITY (MVAR)	3277.953
TOTAL APPARENT POWER CAPACITY (MVA)	4396.576
BASE APPARENT POWER (MVA)	100
NOMINAL VOLTAGES kV	
BUSES 1 - 11	132
BUSES 12 - 23	275

TABLE (5.2-A)

GENERATOR DATA

ACTIVE POWER OUTPUT

GENERATOR j	BUS i	$PGGMN_j$ MW	$PGGMX_j$ MW	CG_j POUNDS PER MWhr
1	1	15.000	61.000	3.220
2		15.000	61.000	3.220
3		15.000	61.000	3.220
4	2	15.000	61.000	3.220
5		30.000	61.000	2.200
6		30.000	61.000	2.200
7	11	43.000	58.000	2.160
8		43.000	59.000	2.190
9		43.000	59.000	2.170
10		43.000	59.000	2.140
11	14	83.000	83.000	0.850
12		83.000	83.000	0.850
13		83.000	83.000	0.850
14		83.000	83.000	0.850
15		83.000	83.000	0.850
16		83.000	83.000	0.850
17	20	22.000	112.000	1.710
18		135.000	334.000	1.420
19		143.000	357.000	1.210
20	23	22.000	112.000	1.670
21		22.000	112.000	1.710
22		22.000	112.000	1.670
23		135.000	334.000	1.350
24		143.000	358.000	1.150

TABLE (5.2-B)

GENERATOR DATA

REACTIVE AND APPARENT POWER OUTPUTS

GENERATOR j	$QGGMN_j$ MVAR	$QGGMX_j$ MVAR	$SGGMN_j$ MVA	$SGGMX_j$ MVA
1	-10.000	78.581	18.028	80.000
2	-10.000	78.581	18.028	80.000
3	-10.000	78.581	18.028	80.000
4	-10.000	78.581	18.028	80.000
5	-10.000	74.162	31.623	80.000
6	-10.000	74.162	31.623	80.000
7	-10.000	61.449	44.147	75.000
8	-10.000	61.449	44.147	75.000
9	-10.000	61.449	44.147	75.000
10	-10.000	61.449	44.147	75.000
11	-20.000	72.187	85.376	110.000
12	-20.000	72.187	85.376	110.000
13	-20.000	72.187	85.376	110.000
14	-20.000	72.187	85.376	110.000
15	-20.000	72.187	85.376	110.000
16	-20.000	72.187	85.376	110.000
17	-25.000	148.378	33.302	150.000
18	-65.000	376.530	149.833	400.000
19	-65.000	394.906	157.080	420.000
20	-25.000	148.378	33.302	150.000
21	-25.000	148.378	33.302	150.000
22	-25.000	148.378	33.302	150.000
23	-65.000	376.530	149.833	400.000
24	-65.000	394.906	157.080	420.000

TABLE (5.3-A)

STATION DATA

ACTIVE POWER OUTPUT

BUS i	NGB_i	$PGBMN_i$ MW	$PGBMX_i$ MW	CB_i POUNDS PER MWhr
1	3	45.000	183.000	3.220
2	3	75.000	183.000	2.472
11	4	172.000	235.000	2.165
14	6	498.000	498.000	0.850
20	3	300.000	803.000	1.354
23	5	344.000	1028.000	1.360

TABLE (5.3-B)

STATION DATA

REACTIVE AND APPARENT POWER OUTPUTS

BUS i	$QGBMN_i$ MVAR	$QGBMX_i$ MVAR	$SGBMN_i$ MVA	$SGBMX_i$ MVA
1	-30.000	235.744	54.083	298.436
2	-30.000	226.905	80.777	291.505
11	-40.000	245.797	176.590	340.060
14	-120.000	433.124	512.254	660.000
20	-155.000	919.814	337.676	1221.011
23	-205.000	1216.570	400.451	1592.742

TABLE (5.4-A)

TRANSMISSION LINE DATA

LINE PARAMETERS

LINE k	R_k PER UNIT	X_k PER UNIT	SS_k PER UNIT
1	0.0242	0.0540	0.0118
2	0.0309	0.0693	0.0151
3	0.0404	0.0888	0.0197
4	0.0325	0.0709	0.0157
5	0.0615	0.1620	0.0342
6	0.0576	0.1520	0.0320
7	0.0266	0.0700	0.0148
8	0.0229	0.0504	0.0112
9	0.0446	0.1003	0.0218
10	0.0233	0.0514	0.0456
11	0.0597	0.1315	0.0291
12	0.0597	0.1315	0.0291
13	0.0043	0.0351	0.2373
14	0.0043	0.0351	0.2373
15	0.0038	0.0307	0.2078
16	0.0035	0.0288	0.1951
17	0.0089	0.0720	0.4871
18	0.0010	0.0080	0.0543
19	0.0021	0.0167	0.1133
20	0.0016	0.0127	0.0862
21	0.0045	0.0362	0.2451
22	0.0024	0.0192	0.1298
23	0.0019	0.0156	0.1056
24	0.0014	0.0114	0.0770
25	0.0020	0.0164	0.1109
26	0.0023	0.0839	0
27	0.0023	0.0839	0
28	0.00185	0.1300	0
29	0.0023	0.0839	0
30	0.0025	0.2000	0

TABLE (5.4-B)

TRANSMISSION LINE DATA

LINE LOADING LIMITS

LINE k	BUSBARS CONNECTED i - j	$STMX_k$ MVA	$PLMX_k$ MW
1	1 - 3	100	2.42
2	1 - 4	100	3.09
3	2 - 5	100	4.04
4	8 - 5	100	3.25
5	2 - 7	100	6.15
6	3 - 6	100	5.76
7	4 - 9	100	2.66
8	9 - 7	100	2.29
9	8 - 6	100	4.46
10	11 - 10	200	9.32
11	8 - 10	100	5.97
12	9 - 10	100	5.97
13	13 - 14	620	16.53
14	14 - 12	620	16.53
15	15 - 12	620	14.61
16	18 - 15	620	13.45
17	23 - 13	620	34.21
18	16 - 17	620	3.84
19	17 - 18	620	8.07
20	19 - 18	620	6.15
21	20 - 19	620	17.30
22	22 - 18	620	9.23
23	20 - 21	620	7.30
24	21 - 22	620	5.38
25	23 - 16	620	7.69
26	12 - 8	155	0.55
27	13 - 8	155	0.55
28	12 - 9	180	0.60
29	13 - 9	155	0.55
30	1 - 2	90	0.20

TABLE (5.5)

SYSTEM DEMAND DATA

BUS i	SCHEDULE A		SCHEDULE B	
	PD_i MW	QD_i MVAr	PD_i MW	QD_i MVAr
1	64	16	56	14
2	101	25	88	22
3	0	0	0	0
4	47	12	41	10
5	51	13	45	11
6	41	10	36	9
7	48	12	42	10
8	1	0	1	0
9	150	38	133	33
10	177	44	157	39
11	130	32	115	29
12	6	0	6	0
13	-4	0	-4	0
14	480	120	425	106
15	201	50	177	44
16	132	33	117	29
17	344	86	304	76
18	104	26	92	23
19	376	94	333	83
20	-100	-25	-100	-25
21	375	94	332	83
22	-210	-52	-210	-52
23	129	32	114	28
TOTAL	2643	660	2300	572

TABLE (5.6)

MINIMIZATION OF GENERATION COST

CASE A

OPTIMUM GENERATION SCHEDULE

GENERATOR j	PGG_j MW	QGG_j MVAR	SGG_j MVA	GENERATION COST POUNDS PER HOUR
1	61.000	20.195	64.256	196.420
2	54.837	20.195	58.437	176.575
3	61.000	20.195	64.256	196.420
4	15.000	22.280	26.859	48.300
5	30.000	21.027	36.635	66.000
6	30.277	21.027	36.862	66.609
7	51.240	28.193	58.484	110.678
8	43.000	28.193	51.418	94.170
9	43.000	28.193	51.418	93.310
10	59.000	28.193	65.390	126.260
11	83.000	5.107	83.157	70.550
12	83.000	5.107	83.157	70.550
13	83.000	5.107	83.157	70.550
14	83.000	5.107	83.157	70.550
15	83.000	5.107	83.157	70.550
16	83.000	5.107	83.157	70.550
17	64.606	20.164	67.680	110.477
18	334.000	51.169	337.897	474.280
19	357.000	53.666	361.011	431.970
20	112.000	45.305	120.816	187.040
21	72.860	45.305	85.797	124.590
22	112.000	45.305	120.816	187.040
23	334.000	114.968	353.233	450.900
24	358.000	120.579	377.761	411.700

TABLE (5.7-A)

MINIMIZATION OF GENERATION COST

CASE A

OPTIMUM BUS VOLTAGES

BUS i	V_i PER UNIT	θ_i DEGREES
1	1.005	-16.612
2	0.989	-20.251
3	0.995	-17.011
4	0.968	-18.722
5	0.965	-19.326
6	0.966	-18.165
7	0.950	-20.510
8	0.973	-16.588
9	0.955	-19.162
10	0.950	-23.216
11	1.008	-22.337
12	0.954	-13.614
13	0.971	-11.466
14	0.950	-12.242
15	0.950	-12.183
16	1.002	-4.760
17	0.983	-6.633
18	0.967	-7.313
19	0.960	-7.224
20	0.996	0.987
21	0.976	-3.228
22	0.978	-3.871
23	1.050	0.000

TABLE (5.7-B)

MINIMIZATION OF GENERATION COST

CASE A

OPTIMAL BUS INJECTIONS

BUS i	P_i MW	Q_i MVAR	S_i MVA
1	112.837	44.586	121.326
2	-25.723	39.333	46.998
3	0.000	0.124	0.124
4	-47.007	-11.673	48.435
5	-50.960	-12.909	52.570
6	-41.000	-9.847	42.166
7	-47.968	-11.902	49.422
8	-0.844	0.111	0.851
9	-149.852	-37.802	154.547
10	-176.273	-44.095	181.705
11	66.240	80.773	104.460
12	-6.021	0.015	6.021
13	4.371	0.092	4.372
14	17.843	-89.358	91.122
15	-201.086	-49.981	207.205
16	-132.382	-32.718	136.365
17	-344.256	-85.716	354.766
18	-104.285	-25.716	107.409
19	-375.688	-93.920	387.250
20	855.606	150.000	868.655
21	-374.301	-93.964	385.916
22	209.589	52.081	215.963
23	859.860	339.463	924.443

TABLE (5.8)

MINIMIZATION OF GENERATION COST

CASE A

OPTIMUM STATION OUTPUTS

BUS i	NGB_i	PGB_i MW	QGB_i MVAR	SGB_i MVA	GENERATION COST POUNDS PER HOUR
1	3	176.837	60.586	186.928	569.415
2	3	75.277	64.333	99.022	180.909
11	4	196.240	112.773	226.336	424.418
14	6	498.000	30.642	498.942	423.300
20	3	755.606	125.000	765.876	1016.727
23	5	988.860	371.463	1056.328	1361.270

TABLE (5.9-A)

MINIMIZATION OF GENERATION COST

CASE A

OPTIMUM ACTIVE POWER LINE FLOWS

LINE k	P_{ij} MW	P_{ji} MW	PL_{k} MW
1	17.639	-17.537	0.101
2	63.543	-62.093	1.450
3	-4.279	4.627	0.348
4	56.762	-55.587	1.175
5	10.184	-9.869	0.315
6	17.538	-17.266	0.272
7	15.085	-14.972	0.113
8	38.489	-38.098	0.390
9	24.013	-23.734	0.279
10	66.240	-63.649	2.591
11	75.305	-71.623	3.682
12	42.306	-41.001	1.305
13	42.072	-41.865	0.208
14	59.708	-59.524	0.183
15	71.250	-71.020	0.230
16	275.218	-272.336	2.882
17	294.703	-286.763	7.940
18	424.864	-422.739	2.125
19	78.483	-78.188	0.295
20	4.168	-4.111	0.056
21	386.928	-379.856	7.072
22	299.473	-297.204	2.269
23	468.678	-464.320	4.357
24	90.019	-89.884	0.135
25	564.277	-557.246	7.031
26	56.790	-56.697	0.093
27	100.474	-100.228	0.246
28	67.734	-67.640	0.093
29	148.588	-148.035	0.554
30	31.655	-31.628	0.027

TABLE (5.9-B)

MINIMIZATION OF GENERATION COST

CASE A

OPTIMUM REACTIVE POWER LINE FLOWS

LINE k	Q_{ij} MVAR	Q_{ji} MVAR	QL_{k} MVAR
1	9.989	-10.944	-0.955
2	25.800	-24.017	1.782
3	27.750	-28.865	-1.115
4	-14.865	15.956	1.091
5	18.244	-20.631	-2.387
6	11.068	-13.428	-2.361
7	12.344	-13.415	-1.071
8	-9.031	8.729	-0.302
9	-5.002	3.582	-1.420
10	80.773	-79.429	1.344
11	-14.145	19.565	5.420
12	-15.533	15.768	0.236
13	41.579	-61.775	-20.196
14	-27.583	7.578	-20.005
15	-29.052	12.082	-16.971
16	26.714	-20.929	5.785
17	80.400	-65.977	14.423
18	178.189	-166.535	11.654
19	80.820	-89.249	-8.429
20	-60.770	53.215	-7.555
21	66.597	-33.150	33.448
22	22.264	-16.397	5.867
23	83.402	-57.891	25.511
24	-36.073	29.817	-6.256
25	256.883	-210.907	45.977
26	-21.445	24.843	3.398
27	-0.294	9.280	8.986
28	1.799	4.761	6.561
29	24.784	-4.585	20.200
30	8.797	-6.660	2.136

TABLE (5.9-C)

MINIMIZATION OF GENERATION COST

CASE A

OPTIMUM APPARENT POWER LINE FLOWS

LINE k	S_{ij} MVA	S_{ji} MVA	SL_k MVA
1	20.271	20.672	0.960
2	68.581	66.576	2.298
3	28.078	29.234	1.169
4	58.677	57.832	1.604
5	20.894	22.870	2.408
6	20.738	21.873	2.376
7	19.492	20.103	1.077
8	39.534	39.086	0.493
9	24.529	24.003	1.447
10	104.460	101.784	2.919
11	76.622	74.247	6.552
12	45.068	43.929	1.326
13	59.152	74.624	20.197
14	65.771	60.005	20.006
15	76.945	72.040	16.972
16	276.512	273.139	6.463
17	305.473	294.255	16.464
18	460.718	454.359	11.846
19	112.656	118.654	8.434
20	60.913	53.374	7.555
21	392.618	381.300	34.187
22	300.300	297.656	6.291
23	476.041	467.916	25.880
24	96.978	94.701	6.257
25	619.998	595.823	46.511
26	60.704	61.901	3.400
27	100.474	100.656	8.989
28	67.758	67.808	6.561
29	150.641	148.106	20.207
30	32.854	32.322	2.137

TABLE (5.10)

MINIMIZATION OF GENERATION COST

CASE A

OVERALL SYSTEM RESULTS

TOTAL ACTIVE POWER GENERATION (MW), PGT	2690.820
TOTAL REACTIVE POWER GENERATION (MVAR), QGT	764.797
TOTAL APPARENT POWER GENERATION (MVA), SGT	2797.396
TOTAL ACTIVE POWER TRANSMISSION LOSSES (MW), PLT	47.820
TOTAL REACTIVE POWER TRANSMISSION LOSSES (MVAR), QLT	104.797
TOTAL APPARENT POWER TRANSMISSION LOSSES (MVA), SLT	115.192
TOTAL ACTIVE POWER GENERATION COST (POUNDS PER HOUR), CGT	3976.040
SYSTEM COST PER UNIT GENERATION (POUNDS PER MWhr), SPC	1.478
COST ASSOCIATED WITH TRANSMISSION LOSSES (POUNDS PER HOUR), CLT	70.660
COST ASSOCIATED WITH NET ACTIVE POWER (POUNDS PER HOUR), CGD	3905.380
OVERALL SYSTEM POWER FACTOR, PF	0.962

TABLE (5.11)

MINIMIZATION OF GENERATION COST

CASE B

OPTIMUM GENERATION SCHEDULE

GENERATOR j	PGG_j MW	QGG_j MVAR	SGG_j MVA	GENERATION COST POUNDS PER HOUR
1	15.000	19.937	24.949	48.300
2	15.000	19.937	24.949	48.300
3	48.474	19.937	52.413	156.085
4	15.000	15.421	21.513	48.300
5	30.000	14.554	33.344	66.000
6	41.519	14.554	43.996	91.342
7	43.000	12.837	44.875	92.880
8	43.000	12.837	44.875	94.170
9	43.000	12.837	44.875	93.310
10	45.098	12.837	46.889	96.509
11	83.000	22.831	86.083	70.550
12	83.000	22.831	86.083	70.550
13	83.000	22.831	86.083	70.550
14	83.000	22.831	86.083	70.550
15	83.000	22.831	86.083	70.550
16	83.000	22.831	86.083	70.550
17	22.000	-1.898	22.082	37.620
18	276.900	-4.815	276.942	393.198
19	357.000	-5.050	357.036	431.970
20	22.000	38.616	44.444	36.740
21	22.000	38.616	44.444	37.620
22	106.856	38.616	113.620	178.450
23	334.000	97.995	348.079	450.900
24	358.000	102.777	372.461	411.700

TABLE (5.12-A)

MINIMIZATION OF GENERATION COST

CASE B

OPTIMUM BUS VOLTAGES

BUS i	V_i PER UNIT	θ_i DEGREES
1	1.017	-17.586
2	1.002	-17.713
3	1.011	-17.386
4	0.991	-17.985
5	0.988	-16.825
6	0.989	-16.800
7	0.977	-18.013
8	0.999	-14.435
9	0.983	-16.931
10	0.950	-19.329
11	0.977	-17.806
12	0.988	-11.383
13	1.008	-9.808
14	1.009	-9.931
15	0.967	-10.042
16	1.003	-3.927
17	0.984	-5.426
18	0.963	-5.686
19	0.950	-5.500
20	0.962	2.344
21	0.954	-1.627
22	0.963	-2.255
23	1.050	0.000

TABLE (5.12-B)

MINIMIZATION OF GENERATION COST

CASE B

OPTIMAL BUS INJECTIONS

BUS i	P_i MW	Q_i MVAR	S_i MVA
1	22.474	45.810	51.025
2	-1.481	22.529	22.578
3	0.012	0.192	0.192
4	-40.577	-9.605	41.698
5	-44.796	-10.624	46.039
6	-35.752	-8.849	36.831
7	-41.738	-9.887	42.893
8	-1.324	2.351	2.698
9	-132.057	-30.591	135.553
10	-154.842	-39.034	159.686
11	59.098	22.349	63.182
12	-6.648	0.471	6.665
13	-0.715	2.849	2.937
14	72.591	30.986	78.927
15	-175.939	-43.489	181.234
16	-119.016	-28.141	122.298
17	-305.061	-75.255	314.206
18	-92.534	-22.311	95.186
19	-329.969	-83.112	340.275
20	755.900	13.236	756.016
21	-328.443	-83.040	338.778
22	208.509	51.943	214.881
23	728.856	288.621	783.922

TABLE (5.13)

MINIMIZATION OF GENERATION COST

CASE B

OPTIMUM STATION OUTPUTS

BUS i	NGB_i	PGB_i MW	QGB_i MVAR	SGB_i MVA	GENERATION COST POUNDS PER HOUR
1	3	78.474	59.810	98.668	252.685
2	3	86.519	44.529	97.306	205.642
11	4	174.098	51.349	181.512	376.869
14	6	498.000	136.986	516.497	423.300
20	3	655.900	-11.764	656.005	862.788
23	5	842.856	316.621	900.364	1115.410

TABLE (5.14-A)

MINIMIZATION OF GENERATION COST

CASE B

OPTIMUM ACTIVE POWER LINE FLOWS

LINE k	P_{ij} MW	P_{ji} MW	PL_k MW
1	-1.173	1.209	0.035
2	22.433	-22.055	0.378
3	-8.178	8.365	0.187
4	54.140	-53.161	0.979
5	7.910	-7.776	0.134
6	-1.197	1.317	0.121
7	-18.522	18.709	0.187
8	34.243	-33.962	0.281
9	37.725	-37.069	0.656
10	59.098	-58.098	1.000
11	65.976	-63.315	2.662
12	34.213	-33.430	0.783
13	5.817	-5.815	0.002
14	78.405	-78.031	0.374
15	63.629	-63.240	0.390
16	241.810	-239.569	2.242
17	256.756	-251.229	5.526
18	347.132	-345.550	1.582
19	40.489	-40.159	0.331
20	10.579	-10.394	0.185
21	346.387	-340.549	5.838
22	285.925	-283.791	2.133
23	409.513	-406.068	3.445
24	77.625	-77.416	0.209
25	471.400	-466.149	5.252
26	62.285	-62.190	0.095
27	97.193	-96.976	0.217
28	72.338	-72.238	0.100
29	147.504	-146.984	0.521
30	1.214	-1.213	0.001

TABLE (5.14-B)

MINIMIZATION OF GENERATION COST

CASE B

OPTIMUM REACTIVE POWER LINE FLOWS

LINE k	Q_{ij} MVAR	Q_{ji} MVAR	QL_k MVAR
1	11.633	-12.767	-1.134
2	26.843	-27.517	-0.674
3	18.990	-20.530	-1.539
4	-9.320	9.906	0.586
5	10.765	-13.764	-2.999
6	12.959	-15.840	-2.881
7	17.912	-18.862	-0.950
8	-4.488	3.877	-0.611
9	-7.671	6.991	-0.680
10	22.349	-24.375	-2.027
11	8.277	-5.179	3.098
12	8.486	-9.480	-0.994
13	-15.129	-9.009	-24.138
14	39.995	-60.614	-20.619
15	-84.096	67.387	-16.709
16	-40.331	40.606	0.276
17	23.476	-30.391	-6.914
18	193.763	-186.466	7.297
19	111.211	-119.324	-8.113
20	-105.450	99.027	-6.423
21	2.216	22.338	24.554
22	-33.297	38.317	5.020
23	11.020	7.571	18.591
24	-90.611	85.239	-5.372
25	253.278	-221.904	31.374
26	-12.557	16.026	3.469
27	12.891	-4.961	7.930
28	6.255	0.764	7.019
29	35.478	-16.491	18.988
30	7.333	-7.226	0.107

TABLE (5.14-C)

MINIMIZATION OF GENERATION COST

CASE B

OPTIMUM APPARENT POWER LINE FLOWS

LINE k	S_{ij} MVA	S_{ji} MVA	SL_k MVA
1	11.692	12.824	1.134
2	34.983	35.264	0.773
3	20.676	22.169	1.551
4	54.937	54.076	1.141
5	13.358	15.809	3.002
6	13.014	15.895	2.884
7	25.766	26.567	0.969
8	34.536	34.183	0.672
9	38.497	37.723	0.944
10	63.182	63.004	2.260
11	66.494	63.526	4.085
12	35.250	34.748	1.266
13	16.209	10.723	24.138
14	88.017	98.808	20.622
15	105.455	92.414	16.713
16	245.151	242.986	2.259
17	257.827	253.061	8.851
18	397.549	392.651	7.466
19	118.353	125.900	8.119
20	105.979	99.571	6.426
21	346.394	341.280	25.239
22	287.857	286.366	5.455
23	409.661	406.139	18.908
24	119.315	115.147	5.376
25	535.134	516.271	31.811
26	63.538	64.222	3.470
27	98.044	97.103	7.933
28	72.608	72.242	7.020
29	151.711	147.906	18.995
30	7.433	7.327	0.107

TABLE (5.15)

MINIMIZATION OF GENERATION COST

CASE B

OVERALL SYSTEM RESULTS

TOTAL ACTIVE POWER GENERATION (MW), PGT	2335.847
TOTAL REACTIVE POWER GENERATION (MVAR), QGT	597.531
TOTAL APPARENT POWER GENERATION (MVA), SGT	2411.063
TOTAL ACTIVE POWER TRANSMISSION LOSSES (MW), PLT	35.847
TOTAL REACTIVE POWER TRANSMISSION LOSSES (MVAR), QLT	25.531
TOTAL APPARENT POWER TRANSMISSION LOSSES (MVA), SLT	44.010
TOTAL ACTIVE POWER GENERATION COST (POUNDS PER HOUR), CGT	3236.695
SYSTEM COST PER UNIT GENERATION (POUNDS PER MWhr), SPC	1.386
COST ASSOCIATED WITH TRANSMISSION LOSSES (POUNDS PER HOUR), CLT	49.672
COST ASSOCIATED WITH NET ACTIVE POWER (POUNDS PER HOUR), CGD	3187.023
OVERALL SYSTEM POWER FACTOR, PF	0.969

TABLE (5.16)

MINIMIZATION OF TRANSMISSION LOSSES

CASE A

OPTIMUM GENERATION SCHEDULE

GENERATOR j	PGG_j MW	QGG_j MVAR	SGG_j MVA	GENERATION COST POUNDS PER HOUR
1	55.421	18.038	58.283	178.456
2	15.000	18.038	23.460	48.300
3	61.000	18.038	63.611	196.420
4	33.812	12.282	35.974	108.875
5	61.000	11.591	62.092	134.200
6	61.000	11.591	62.092	134.200
7	58.000	13.546	59.561	125.280
8	59.000	13.546	60.535	129.210
9	59.000	13.546	60.535	128.030
10	59.000	13.546	60.535	126.260
11	83.000	33.759	89.603	70.550
12	83.000	33.759	89.603	70.550
13	83.000	33.759	89.603	70.550
14	83.000	33.759	89.603	70.550
15	83.000	33.759	89.603	70.550
16	83.000	33.759	89.603	70.550
17	22.000	18.143	28.516	37.620
18	264.823	46.042	268.796	376.049
19	357.000	48.289	360.251	431.970
20	112.000	23.410	114.420	187.040
21	98.100	23.410	100.854	167.751
22	112.000	23.410	114.420	187.040
23	334.000	59.407	339.242	450.900
24	358.000	62.306	363.381	411.700

TABLE (5.17-A)

MINIMIZATION OF TRANSMISSION LOSSES

CASE A

OPTIMUM BUS VOLTAGES

BUS i	V_i PER UNIT	θ_i DEGREES
1	1.050	-14.045
2	1.036	-13.851
3	1.042	-14.327
4	1.019	-15.706
5	1.018	-14.668
6	1.019	-15.125
7	1.004	-16.304
8	1.028	-13.565
9	1.009	-15.842
10	0.983	-16.956
11	1.019	-14.093
12	1.026	-12.259
13	1.037	-10.023
14	1.050	-11.101
15	1.006	-11.866
16	1.016	-5.131
17	1.005	-7.161
18	1.003	-8.283
19	0.997	-8.551
20	1.031	-1.988
21	1.012	-5.417
22	1.015	-5.675
23	1.050	0.000

TABLE (5.17-B)

MINIMIZATION OF TRANSMISSION LOSSES

CASE A

OPTIMAL BUS INJECTIONS

BUS i	P_i MW	Q_i MVAR	S_i MVA
1	67.421	38.114	77.449
2	54.812	10.464	55.802
3	0.053	0.057	0.078
4	-46.577	-11.930	48.080
5	-50.309	-12.768	51.904
6	-40.658	-10.003	41.870
7	-47.456	-11.950	48.938
8	-0.016	2.745	2.745
9	-147.898	-35.512	152.102
10	-174.713	-43.021	179.932
11	108.502	22.183	110.746
12	-5.126	0.604	5.162
13	0.380	1.913	1.950
14	17.080	82.556	84.304
15	-199.552	-49.019	205.484
16	-132.379	-32.939	136.415
17	-343.390	-85.955	353.984
18	-103.369	-25.323	106.426
19	-371.073	-93.755	382.734
20	743.823	137.473	756.421
21	-369.792	-93.788	381.500
22	208.484	52.161	214.910
23	885.100	159.944	899.435

TABLE (5.18)

MINIMIZATION OF TRANSMISSION LOSSES

CASE A

OPTIMUM STATION OUTPUTS

BUS i	NGB_i	PGB_i MW	QGB_i MVAR	SGB_i MVA	GENERATION COST POUNDS PER HOUR
1	3	131.421	54.114	142.126	423.176
2	3	155.812	35.464	159.797	377.275
11	4	235.000	54.183	241.166	508.780
14	6	498.000	202.556	537.618	423.300
20	3	643.823	112.473	653.574	845.639
23	5	1014.100	191.944	1032.105	1404.431

TABLE (5.19-A)

MINIMIZATION OF TRANSMISSION LOSSES

CASE A

OPTIMUM ACTIVE POWER LINE FLOWS

LINE k	P_{ij} MW	P_{ji} MW	PL_k MW
1	13.911	-13.851	0.059
2	55.260	-54.247	1.012
3	22.046	-21.815	0.230
4	28.748	-28.493	0.255
5	31.016	-30.416	0.600
6	13.904	-13.737	0.167
7	7.671	-7.625	0.046
8	17.107	-17.041	0.067
9	27.237	-26.921	0.316
10	108.502	-105.726	2.776
11	51.200	-49.627	1.573
12	19.658	-19.360	0.298
13	52.862	-52.671	0.191
14	69.751	-69.409	0.342
15	14.495	-14.306	0.189
16	215.690	-214.047	1.643
17	264.362	-258.713	5.649
18	463.047	-460.892	2.155
19	117.502	-117.214	0.288
20	-42.433	42.494	0.060
21	333.578	-328.640	4.938
22	245.775	-244.339	1.437
23	410.246	-407.113	3.133
24	37.320	-37.292	0.029
25	602.548	-595.425	7.122
26	28.589	-28.571	0.018
27	78.766	-78.630	0.136
28	50.000	-49.952	0.048
29	127.464	-127.087	0.377
30	-1.749	1.750	0.001

TABLE (5.19-B)

MINIMIZATION OF TRANSMISSION LOSSES

CASE A

OPTIMUM REACTIVE POWER LINE FLOWS

LINE k	Q_{ij} MVAR	Q_{ji} MVAR	QL_k MVAR
1	8.070	-9.229	-1.159
2	22.776	-22.122	0.654
3	10.180	-11.752	-1.572
4	-0.071	-1.016	-1.087
5	7.452	-9.431	-1.979
6	9.286	-12.244	-2.958
7	10.192	-11.593	-1.400
8	1.369	-2.519	-1.150
9	-3.812	2.241	-1.570
10	22.183	-20.632	1.551
11	11.145	-10.622	0.523
12	9.534	-11.766	-2.232
13	-57.355	33.078	-24.277
14	49.477	-72.264	-22.787
15	-79.984	60.056	-19.928
16	-37.136	30.965	-6.172
17	-17.360	10.022	-7.338
18	86.280	-74.581	11.698
19	-11.374	2.239	-9.135
20	-48.386	40.239	-8.147
21	59.883	-45.369	14.514
22	28.937	-30.664	-1.727
23	77.591	-62.887	14.703
24	-30.901	23.224	-7.677
25	165.787	-119.219	46.568
26	-1.763	2.416	0.653
27	11.885	-6.934	4.952
28	14.575	-11.229	3.347
29	37.361	-23.594	13.767
30	7.269	-7.167	0.101

TABLE (5.19-C)

MINIMIZATION OF TRANSMISSION LOSSES

CASE A

OPTIMUM APPARENT POWER LINE FLOWS

LINE k	S_{ij} MVA	S_{ji} MVA	SL_{k} MVA
1	16.082	16.644	1.161
2	59.769	58.585	1.205
3	24.283	24.779	1.589
4	28.748	28.512	1.116
5	31.899	31.844	2.068
6	16.720	18.402	2.963
7	12.756	13.875	1.401
8	17.162	17.226	1.152
9	27.503	27.014	1.602
10	110.746	107.721	3.180
11	52.399	50.751	1.658
12	21.848	22.655	2.252
13	78.000	62.197	24.277
14	85.517	100.198	22.790
15	81.287	61.736	19.929
16	218.864	216.275	6.386
17	264.931	258.907	9.261
18	471.016	466.887	11.895
19	118.051	117.235	9.139
20	64.357	58.523	8.147
21	338.910	331.757	15.331
22	247.473	246.255	2.246
23	417.519	411.941	15.033
24	48.453	43.932	7.677
25	624.939	607.243	47.109
26	28.643	28.673	0.654
27	79.658	78.936	4.953
28	52.081	51.199	3.347
29	132.827	129.258	13.773
30	7.476	7.378	0.101

TABLE (5.20)

MINIMIZATION OF TRANSMISSION LOSSES

CASE A

OVERALL SYSTEM RESULTS

TOTAL ACTIVE POWER GENERATION (MW), PGT	2678.156
TOTAL REACTIVE POWER GENERATION (MVAR), QGT	650.735
TOTAL APPARENT POWER GENERATION (MVA), SGT	2756.080
TOTAL ACTIVE POWER TRANSMISSION LOSSES (MW), PLT	35.156
TOTAL REACTIVE POWER TRANSMISSION LOSSES (MVAR), QLT	-9.265
TOTAL APPARENT POWER TRANSMISSION LOSSES (MVA), SLT	36.356
TOTAL ACTIVE POWER GENERATION COST (POUNDS PER HOUR), CGT	3982.600
SYSTEM COST PER UNIT GENERATION (POUNDS PER MWhr), SPC	1.487
COST ASSOCIATED WITH TRANSMISSION LOSSES (POUNDS PER HOUR), CLT	52.279
COST ASSOCIATED WITH NET ACTIVE POWER (POUNDS PER HOUR), CGD	3930.321
OVERALL SYSTEM POWER FACTOR, PF	0.972

TABLE (5.21)

MINIMIZATION OF TRANSMISSION LOSSES

CASE B

OPTIMUM GENERATION SCHEDULE

GENERATOR j	PGG_j MW	QGG_j MVAR	SGG_j MVA	GENERATION COST POUNDS PER HOUR
1	54.367	14.931	56.380	175.061
2	15.000	14.931	21.165	48.300
3	61.000	14.931	62.801	196.420
4	15.000	6.971	16.541	48.300
5	57.743	6.579	58.116	127.034
6	61.000	6.579	61.354	134.200
7	58.000	11.704	59.169	125.280
8	43.000	11.704	44.564	94.170
9	56.880	11.704	58.072	123.430
10	59.000	11.704	60.150	126.260
11	83.000	24.028	86.408	70.550
12	83.000	24.028	86.408	70.550
13	83.000	24.028	86.408	70.550
14	83.000	24.028	86.408	70.550
15	83.000	24.028	86.408	70.550
16	83.000	24.028	86.408	70.550
17	22.000	17.068	27.845	37.620
18	285.159	43.313	288.430	404.926
19	357.000	45.427	359.879	431.970
20	22.000	13.249	25.682	36.740
21	22.000	13.249	25.682	37.620
22	22.000	13.249	25.682	36.740
23	256.926	33.622	259.116	346.850
24	358.000	35.263	359.733	411.700

TABLE (5.22-A)

MINIMIZATION OF TRANSMISSION LOSSES

CASE B

OPTIMUM BUS VOLTAGES

BUS i	V_i PER UNIT	θ_i DEGREES
1	1.050	-8.357
2	1.032	-8.623
3	1.043	-8.731
4	1.023	-10.141
5	1.020	-9.497
6	1.023	-9.773
7	1.009	-10.945
8	1.031	-8.639
9	1.014	-10.595
10	0.993	-11.364
11	1.025	-8.678
12	1.033	-7.382
13	1.041	-6.015
14	1.049	-6.093
15	1.020	-7.213
16	1.027	-3.354
17	1.019	-4.536
18	1.021	-4.301
19	1.017	-4.177
20	1.050	2.481
21	1.033	-0.849
22	1.034	-1.346
23	1.050	0.000

TABLE (5.22-B)

MINIMIZATION OF TRANSMISSION LOSSES

CASE B

OPTIMAL BUS INJECTIONS

BUS i	P_i MW	Q_i MVAR	S_i MVA
1	74.367	30.794	80.490
2	45.743	-1.872	45.781
3	0.076	0.005	0.076
4	-40.682	-9.989	41.891
5	-44.469	-11.000	45.809
6	-35.823	-8.973	36.929
7	-41.631	-9.968	42.808
8	0.127	-0.218	0.252
9	-131.666	-33.236	135.796
10	-155.310	-38.931	160.115
11	101.880	17.816	103.426
12	-5.350	0.223	5.354
13	4.581	0.296	4.590
14	73.033	38.169	82.406
15	-176.385	-43.818	181.746
16	-116.952	-28.996	120.493
17	-303.907	-75.996	313.265
18	-91.373	-22.842	94.185
19	-332.450	-82.918	342.635
20	764.159	130.808	775.274
21	-331.466	-82.932	341.683
22	210.096	52.061	216.450
23	566.926	80.633	572.631

TABLE (5.23)

MINIMIZATION OF TRANSMISSION LOSSES

CASE B

OPTIMUM STATION OUTPUTS

BUS i	NGB_i	PGB_i MW	QGB_i MVAR	SGB_i MVA	GENERATION COST POUNDS PER HOUR
1	3	130.367	44.794	137.848	419.781
2	3	133.743	20.128	135.249	309.534
11	4	216.880	46.816	221.875	469.140
14	6	498.000	144.169	518.448	423.300
20	3	664.159	105.808	672.534	874.516
23	5	680.926	108.633	689.537	869.650

TABLE (5.24-A)

MINIMIZATION OF TRANSMISSION LOSSES

CASE B

OPTIMUM ACTIVE POWER LINE FLOWS

LINE k	P_{ij} MW	P_{ji} MW	PL_k MW
1	15.820	-15.758	0.062
2	55.920	-54.960	0.960
3	20.508	-20.338	0.170
4	24.318	-24.130	0.187
5	27.859	-27.398	0.461
6	15.834	-15.667	0.167
7	14.278	-14.215	0.063
8	14.283	-14.233	0.049
9	20.330	-20.156	0.174
10	101.880	-99.487	2.393
11	42.210	-41.134	1.076
12	14.878	-14.689	0.188
13	1.394	-1.372	0.022
14	74.405	-74.124	0.281
15	4.591	-4.518	0.073
16	182.095	-180.976	1.119
17	159.289	-157.241	2.048
18	278.134	-277.359	0.775
19	-26.548	26.563	0.015
20	13.271	-13.249	0.022
21	350.965	-345.721	5.243
22	288.692	-286.782	1.910
23	413.194	-410.149	3.046
24	78.682	-78.595	0.087
25	398.188	-395.086	3.102
26	27.898	-27.881	0.017
27	58.926	-58.849	0.077
28	45.394	-45.355	0.040
29	101.501	-101.257	0.244
30	2.627	-2.625	0.002

TABLE (5.24-B)

MINIMIZATION OF TRANSMISSION LOSSES

CASE B

OPTIMUM REACTIVE POWER LINE FLOWS

LINE k	Q_{ij} MVAR	Q_{ji} MVAR	QL_k MVAR
1	5.162	-6.316	-1.154
2	16.403	-15.873	0.530
3	4.268	-5.968	-1.700
4	3.790	-5.031	-1.241
5	2.922	-5.272	-2.350
6	6.321	-9.296	-2.976
7	5.884	-7.253	-1.369
8	3.493	-4.695	-1.202
9	-2.231	0.323	-1.907
10	17.816	-17.177	0.639
11	9.973	-10.582	-0.609
12	8.655	-11.172	-2.516
13	-36.242	10.493	-25.749
14	27.675	-51.096	-23.421
15	-55.278	33.987	-21.291
16	-22.563	11.460	-11.103
17	-25.553	-11.137	-36.691
18	62.944	-62.427	0.517
19	-13.569	1.903	-11.666
20	-39.727	30.953	-8.774
21	59.191	-43.191	16.000
22	34.708	-33.134	1.574
23	71.617	-58.064	13.553
24	-24.868	17.353	-7.515
25	105.417	-91.940	13.477
26	2.155	-1.539	0.616
27	13.029	-10.211	2.818
28	15.177	-12.385	2.792
29	34.647	-25.747	8.900
30	9.229	-9.062	0.167

TABLE (5.24-C)

MINIMIZATION OF TRANSMISSION LOSSES

CASE B

OPTIMUM APPARENT POWER LINE FLOWS

LINE k	S_{ij} MVA	S_{ji} MVA	SL_k MVA
1	16.641	16.976	1.155
2	58.276	57.206	1.096
3	20.948	21.196	1.709
4	24.611	24.649	1.255
5	28.012	27.901	2.395
6	17.049	18.217	2.980
7	15.443	15.958	1.371
8	14.704	14.988	1.203
9	20.452	20.158	1.915
10	103.426	100.959	2.477
11	43.372	42.474	1.236
12	17.212	18.455	2.523
13	36.269	10.583	25.749
14	79.386	90.029	23.422
15	55.468	34.286	21.291
16	183.487	181.338	11.160
17	161.326	157.635	36.748
18	285.167	284.298	0.931
19	29.814	26.631	11.666
20	41.885	33.669	8.774
21	355.921	348.409	16.837
22	290.771	288.690	2.474
23	419.355	414.238	13.891
24	82.519	80.488	7.516
25	411.905	405.642	13.829
26	27.981	27.924	0.616
27	60.349	59.728	2.819
28	47.864	47.015	2.793
29	107.252	104.479	8.903
30	9.596	9.435	0.167

TABLE (5.25)

MINIMIZATION OF TRANSMISSION LOSSES

CASE B

OVERALL SYSTEM RESULTS

TOTAL ACTIVE POWER GENERATION (MW), PGT	2324.074
TOTAL REACTIVE POWER GENERATION (MVAR), QGT	470.348
TOTAL APPARENT POWER GENERATION (MVA), SGT	2371.191
TOTAL ACTIVE POWER TRANSMISSION LOSSES (MW), PLT	24.074
TOTAL REACTIVE POWER TRANSMISSION LOSSES (MVAR), QLT	-101.652
TOTAL APPARENT POWER TRANSMISSION LOSSES (MVA), SLT	104.464
TOTAL ACTIVE POWER GENERATION COST (POUNDS PER HOUR), CGT	3365.920
SYSTEM COST PER UNIT GENERATION (POUNDS PER MWhr), SPC	1.448
COST ASSOCIATED WITH TRANSMISSION LOSSES (POUNDS PER HOUR), CLT	34.866
COST ASSOCIATED WITH NET ACTIVE POWER (POUNDS PER HOUR), CGD	3331.054
OVERALL SYSTEM POWER FACTOR, PF	0.980

TABLE (5.26)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE A

OPTIMUM GENERATION SCHEDULE

GENERATOR j	PGG_j MW	QGG_j MVAR	SGG_j MVA	GENERATION COST POUNDS PER HOUR
1	61.000	11.130	62.007	196.420
2	53.939	11.130	55.075	173.684
3	61.000	11.130	62.007	196.420
4	29.057	17.647	33.996	93.564
5	61.000	16.654	63.233	134.200
6	61.000	16.654	63.233	134.200
7	58.000	18.071	60.750	125.280
8	52.265	18.071	55.301	114.461
9	59.000	18.071	61.705	128.030
10	59.000	18.071	61.705	126.260
11	83.000	23.671	86.309	70.550
12	83.000	23.671	86.309	70.550
13	83.000	23.671	86.309	70.550
14	83.000	23.671	86.309	70.550
15	83.000	23.671	86.309	70.550
16	83.000	23.671	86.309	70.550
17	49.062	13.513	50.889	83.897
18	334.000	34.291	335.756	474.280
19	357.000	35.964	358.807	431.970
20	60.305	34.513	69.483	100.709
21	22.000	34.513	40.929	37.620
22	112.000	34.513	117.197	187.040
23	334.000	87.582	345.292	450.900
24	358.000	91.857	369.597	411.700

TABLE (5.27-A)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE A

OPTIMUM BUS VOLTAGES

BUS i	V_i PER UNIT	θ_i DEGREES
1	1.016	-10.086
2	1.016	-11.579
3	1.009	-10.749
4	0.985	-12.792
5	0.992	-12.683
6	0.987	-12.644
7	0.975	-14.360
8	0.997	-11.742
9	0.977	-13.943
10	0.960	-15.844
11	1.004	-13.430
12	0.992	-10.516
13	1.009	-8.529
14	1.009	-9.439
15	0.975	-10.073
16	1.006	-4.346
17	0.990	-5.999
18	0.977	-6.255
19	0.967	-6.205
20	0.996	1.828
21	0.979	-2.324
22	0.984	-2.926
23	1.050	0.000

TABLE (5.27-B)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE A

OPTIMAL BUS INJECTIONS

BUS i	P_i MW	Q_i MVAR	S_i MVA
1	111.939	17.389	113.282
2	50.057	25.955	56.386
3	0.247	-0.081	0.260
4	-45.364	-12.283	46.997
5	-49.988	-13.155	51.690
6	-40.083	-10.046	41.323
7	-47.069	-12.096	48.599
8	0.491	-0.530	0.723
9	-146.040	-38.675	151.074
10	-172.584	-44.217	178.158
11	98.265	40.284	106.202
12	-5.949	0.184	5.952
13	5.370	0.725	5.419
14	13.146	22.023	25.649
15	-199.576	-49.835	205.704
16	-131.322	-32.803	135.357
17	-343.056	-85.782	353.618
18	-103.086	-25.745	106.252
19	-375.271	-93.936	386.850
20	840.062	108.768	847.075
21	-374.308	-93.951	385.919
22	209.628	52.025	215.988
23	757.305	250.978	797.810

TABLE (5.28)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE A

OPTIMUM STATION OUTPUTS

BUS i	NGB_i	PGB_i MW	QGB_i MVAR	SGB_i MVA	GENERATION COST POUNDS PER HOUR
1	3	175.939	33.389	179.079	566.524
2	3	151.057	50.955	159.420	361.964
11	4	228.265	72.284	239.437	494.031
14	6	498.000	142.023	517.856	423.300
20	3	740.062	83.768	744.788	990.147
23	5	886.305	282.978	930.383	1187.969

TABLE (5.29-A)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE A

OPTIMUM ACTIVE POWER LINE FLOWS

LINE k	P_{ij} MW	P_{ji} MW	PL_k MW
1	23.734	-23.598	0.136
2	74.753	-73.016	1.736
3	28.736	-28.324	0.412
4	21.821	-21.663	0.157
5	34.769	-33.942	0.827
6	23.846	-23.505	0.341
7	27.653	-27.443	0.210
8	13.169	-13.127	0.042
9	16.707	-16.578	0.129
10	98.265	-95.613	2.652
11	54.496	-52.696	1.800
12	24.658	-24.275	0.383
13	45.650	-45.562	0.088
14	58.708	-58.488	0.220
15	17.358	-17.218	0.140
16	218.693	-216.934	1.759
17	224.208	-219.964	4.244
18	379.729	-378.040	1.689
19	34.984	-34.845	0.139
20	-2.795	2.888	0.093
21	379.152	-372.476	6.676
22	291.936	-289.822	2.114
23	460.910	-456.754	4.156
24	82.446	-82.308	0.138
25	516.909	-511.051	5.858
26	25.023	-25.007	0.016
27	67.633	-67.525	0.108
28	44.734	-44.694	0.040
29	112.050	-111.730	0.320
30	13.452	-13.448	0.004

TABLE (5.29-B)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE A

OPTIMUM REACTIVE POWER LINE FLOWS

LINE k	Q_{ij} MVAR	Q_{ji} MVAR	QL_k MVAR
1	3.325	-4.232	-0.907
2	13.988	-11.606	2.382
3	14.040	-15.122	-1.082
4	-3.177	1.967	-1.209
5	11.641	-12.855	-1.214
6	4.151	-6.439	-2.288
7	-0.678	-0.195	-0.873
8	-1.887	0.759	-1.128
9	1.751	-3.607	-1.856
10	40.284	-38.831	1.453
11	3.888	-2.711	1.177
12	0.788	-2.675	-1.888
13	-15.568	-7.864	-23.432
14	29.887	-51.828	-21.941
15	-66.392	47.437	-18.954
16	-20.655	16.557	-4.098
17	21.209	-38.528	-17.319
18	160.859	-152.758	8.101
19	66.976	-76.826	-9.850
20	-77.654	70.247	-7.407
21	46.368	-16.282	30.085
22	2.946	1.490	4.435
23	62.401	-38.577	23.824
24	-55.374	49.080	-6.294
25	229.972	-193.662	36.310
26	-6.958	7.534	0.576
27	14.467	-10.526	3.941
28	11.533	-8.711	2.821
29	40.354	-28.670	11.684
30	0.076	0.274	0.350

TABLE (5.29-C)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE A

OPTIMUM APPARENT POWER LINE FLOWS

LINE k	S_{ij} MVA	S_{ji} MVA	SL_k MVA
1	23.966	23.975	0.918
2	76.050	73.933	2.948
3	31.982	32.108	1.157
4	22.051	21.753	1.220
5	36.666	36.295	1.469
6	24.204	24.371	2.313
7	27.661	27.444	0.898
8	13.303	13.149	1.128
9	16.798	16.966	1.861
10	106.202	103.197	3.024
11	54.635	52.766	2.151
12	24.670	24.422	1.926
13	48.232	46.236	23.432
14	65.878	78.147	21.942
15	68.623	50.466	18.955
16	219.666	217.564	4.460
17	225.209	223.312	17.832
18	412.395	407.737	8.275
19	75.562	84.359	9.851
20	77.704	70.306	7.407
21	381.977	372.832	30.817
22	291.951	289.826	4.913
23	465.115	458.380	24.183
24	99.316	95.830	6.296
25	565.758	546.515	36.780
26	25.973	26.118	0.576
27	69.163	68.341	3.942
28	46.197	45.535	2.821
29	119.095	115.349	11.688
30	13.453	13.451	0.350

TABLE (5.30)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE A

OVERALL SYSTEM RESULTS

TOTAL ACTIVE POWER GENERATION (MW), PGT	2679.628
TOTAL REACTIVE POWER GENERATION (MVAR), QGT	665.398
TOTAL APPARENT POWER GENERATION (MVA), SGT	2761.007
TOTAL ACTIVE POWER TRANSMISSION LOSSES (MW), PLT	36.628
TOTAL REACTIVE POWER TRANSMISSION LOSSES (MVAR), QLT	5.398
TOTAL APPARENT POWER TRANSMISSION LOSSES (MVA), SLT	37.024
TOTAL ACTIVE POWER GENERATION COST (POUNDS PER HOUR), CGT	4023.934
SYSTEM COST PER UNIT GENERATION (POUNDS PER MWhr), SPC	1.502
COST ASSOCIATED WITH TRANSMISSION LOSSES (POUNDS PER HOUR), CLT	55.003
COST ASSOCIATED WITH NET ACTIVE POWER (POUNDS PER HOUR), CGD	3968.931
OVERALL SYSTEM POWER FACTOR, PF	0.971

TABLE (5.31)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE B

OPTIMUM GENERATION SCHEDULE

GENERATOR j	PGG_j MW	QGG_j MVAR	SGG_j MVA	GENERATION COST POUNDS PER HOUR
1	60.395	15.468	62.345	194.473
2	15.000	15.468	21.547	48.300
3	61.000	15.468	62.931	196.420
4	15.000	10.613	18.375	48.300
5	47.977	10.016	49.012	105.550
6	61.000	10.016	61.817	134.200
7	54.206	11.525	55.417	117.084
8	43.000	11.525	44.518	94.170
9	43.000	11.525	44.518	93.310
10	59.000	11.525	60.115	126.260
11	83.000	30.085	88.284	70.550
12	83.000	30.085	88.284	70.550
13	83.000	30.085	88.284	70.550
14	83.000	30.085	88.284	70.550
15	83.000	30.085	88.284	70.550
16	83.000	30.085	88.284	70.550
17	22.000	0.094	22.000	37.620
18	135.000	0.239	135.000	191.700
19	352.267	0.251	352.267	426.243
20	33.651	24.032	41.352	56.198
21	22.000	24.032	32.582	37.620
22	112.000	24.032	114.549	187.040
23	334.000	60.986	339.522	450.900
24	358.000	63.962	363.669	411.700

TABLE (5.32-A)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE B

OPTIMUM BUS VOLTAGES

BUS i	V_i PER UNIT	θ_i DEGREES
1	1.050	-10.739
2	1.035	-11.494
3	1.043	-11.118
4	1.021	-12.520
5	1.019	-12.125
6	1.021	-12.186
7	1.007	-13.460
8	1.027	-11.069
9	1.011	-12.992
10	0.984	-14.358
11	1.013	-12.131
12	1.024	-9.939
13	1.039	-7.869
14	1.049	-8.392
15	1.000	-10.052
16	1.013	-4.525
17	1.000	-6.317
18	0.991	-7.279
19	0.980	-7.658
20	0.994	-1.837
21	0.986	-4.858
22	0.994	-4.922
23	1.050	0.000

TABLE (5.32-B)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE B

OPTIMAL BUS INJECTIONS

BUS i	P_i MW	Q_i MVAR	S_i MVA
1	80.395	32.405	86.680
2	35.977	8.645	37.002
3	0.069	-0.038	0.079
4	-40.333	-10.154	41.591
5	-44.485	-11.073	45.842
6	-35.632	-9.040	36.761
7	-41.531	-10.063	42.733
8	0.118	-0.185	0.219
9	-130.736	-33.234	134.894
10	-153.072	-39.210	158.015
11	84.206	17.100	85.924
12	-6.545	1.258	6.665
13	3.830	0.504	3.863
14	66.609	74.511	99.943
15	-175.916	-42.878	181.066
16	-118.078	-28.571	121.485
17	-304.599	-75.621	313.846
18	-92.526	-21.788	95.056
19	-331.973	-82.474	342.064
20	609.267	25.585	609.804
21	-330.369	-82.683	340.558
22	208.674	52.405	215.153
23	745.651	169.045	764,573

TABLE (5.33)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE B

OPTIMUM STATION OUTPUTS

BUS i	NGB_i	PGB_i MW	QGB_i MVAR	SGB_i MVA	GENERATION COST POUNDS PER HOUR
1	3	136.395	46.405	144.073	439.193
2	3	123.977	30.645	127.709	288.050
11	4	199.206	46.100	204.470	430.824
14	6	498.000	180.511	529.706	423.300
20	3	509.267	0.585	509.267	655.563
23	5	859.651	197.045	881.945	1143.458

TABLE (5.34-A)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE B

OPTIMUM ACTIVE POWER LINE FLOWS

LINE k	P_{ij} MW	P_{ji} MW	PL_k MW
1	16.476	-16.406	0.070
2	56.667	-55.663	1.004
3	17.934	-17.770	0.164
4	26.938	-26.714	0.224
5	25.292	-24.884	0.408
6	16.475	-16.288	0.187
7	15.331	-15.253	0.078
8	16.710	-16.647	0.063
9	19.506	-19.344	0.163
10	84.206	-82.510	1.696
11	49.551	-48.080	1.471
12	22.853	-22.483	0.370
13	24.162	-24.092	0.070
14	90.700	-90.211	0.490
15	-16.225	16.465	0.240
16	160.690	-159.691	0.999
17	208.009	-204.514	3.495
18	411.196	-409.399	1.797
19	104.800	-104.531	0.268
20	-60.050	60.203	0.153
21	275.392	-271.923	3.469
22	209.960	-208.887	1.073
23	333.874	-331.722	2.152
24	1.353	-1.286	0.067
25	535.137	-529.274	5.864
26	24.618	-24.605	0.014
27	71.386	-71.273	0.113
28	42.582	-42.548	0.034
29	112.796	-112.498	0.299
30	7.252	-7.249	0.003

TABLE (5.34-B)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE B

OPTIMUM REACTIVE POWER LINE FLOWS

LINE k	Q_{ij} MVAR	Q_{ji} MVAR	QL_{k} MVAR
1	6.204	-7.340	-1.136
2	18.418	-17.786	0.632
3	9.598	-11.316	-1.718
4	-1.398	0.243	-1.155
5	6.624	-9.116	-2.492
6	7.302	-10.215	-2.912
7	7.632	-8.956	-1.323
8	-0.220	-0.947	-1.167
9	-3.094	1.175	-1.920
10	17.100	-17.908	-0.808
11	10.444	-10.148	0.296
12	9.071	-11.154	-2.084
13	-47.138	21.841	-25.298
14	52.670	-74.183	-21.513
15	-88.142	68.801	-19.341
16	-56.360	45.264	-11.096
17	-21.974	-2.887	-24.861
18	121.016	-112.138	8.878
19	36.518	-45.604	-9.086
20	-78.992	71.844	-7.148
21	7.506	-3.482	4.024
22	-12.526	8.332	-4.193
23	18.079	-10.757	7.322
24	-71.926	64.930	-6.996
25	185.865	-149.588	36.278
26	-3.913	4.410	0.497
27	14.676	-10.547	4.129
28	10.553	-8.168	2.386
29	35.853	-24.962	10.891
30	7.782	-7.577	0.205

TABLE (5.34-C)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE B

OPTIMUM APPARENT POWER LINE FLOWS

LINE k	S_{ij} MVA	S_{ji} MVA	SL_k MVA
1	17.606	17.974	1.138
2	59.585	58.436	1.186
3	20.341	21.067	1.725
4	26.974	26.716	1.176
5	26.145	26.501	2.525
6	18.021	19.226	2.918
7	17.125	17.687	1.326
8	16.711	16.674	1.168
9	19.750	19.379	1.926
10	85.924	84.431	1.878
11	50.640	49.139	1.500
12	24.587	25.098	2.116
13	52.970	32.518	25.298
14	104.884	116.795	21.518
15	89.623	70.744	19.342
16	170.287	165.982	11.141
17	209.166	204.534	25.106
18	428.634	424.479	9.058
19	110.980	114.046	9.090
20	99.225	93.733	7.150
21	275.495	271.946	5.313
22	210.333	209.053	4.328
23	334.364	331.896	7.632
24	71.939	64.943	6.996
25	566.496	550.007	36.749
26	24.928	24.997	0.497
27	72.879	72.049	4.131
28	43.870	43.325	2.386
29	118.357	115.234	10.895
30	10.637	10.486	0.205

TABLE (5.35)

MINIMIZATION OF REACTIVE POWER GENERATION

CASE B

OVERALL SYSTEM RESULTS

TOTAL ACTIVE POWER GENERATION (MW), PGT	2326.496
TOTAL REACTIVE POWER GENERATION (MVAR), QGT	501.291
TOTAL APPARENT POWER GENERATION (MVA), SGT	2379.890
TOTAL ACTIVE POWER TRANSMISSION LOSSES (MW), PLT	26.496
TOTAL REACTIVE POWER TRANSMISSION LOSSES (MVAR), QLT	-70.709
TOTAL APPARENT POWER TRANSMISSION LOSSES (MVA), SLT	75.510
TOTAL ACTIVE POWER GENERATION COST (POUNDS PER HOUR), CGT	3380.388
SYSTEM COST PER UNIT GENERATION (POUNDS PER MWhr), SPC	1.453
COST ASSOCIATED WITH TRANSMISSION LOSSES (POUNDS PER HOUR), CLT	38.499
COST ASSOCIATED WITH NET ACTIVE POWER (POUNDS PER HOUR), CGD	3341.889
OVERALL SYSTEM POWER FACTOR, PF	0.978

TABLE (5.36-A)

COMPARATIVE TABLE OF NUMERICAL RESULTS

CASE A

QUANTITY	PROBLEM 1	PROBLEM 2	PROBLEM 3
PGT	2690.820	2678.156	2679.628
QGT	764.797	650.735	665.398
SGT	2797.396	2756.080	2761.007
PLT	47.820	35.156	36.628
QLT	104.797	-9.265	5.398
SLT	115.192	36.356	37.024
CGT	3976.040	3982.600	4023.934
SPC	1.478	1.487	1.502
CLT	70.667	52.279	55.003
CGD	3905.380	3930.321	3968.931
PF	0.962	0.972	0.971

TOTAL SYSTEM DEMAND	
ACTIVE POWER	2643 MW
REACTIVE POWER	660 MVAR
APPARENT POWER	2724.160 MVA

FOR DEFINITIONS OF ALL QUANTITIES IN THIS TABLE AND THEIR UNITS, SEE TABLE (5.35) ABOVE.

TABLE (5.36-B)

COMPARATIVE TABLE OF NUMERICAL RESULTS

CASE B

QUANTITY	PROBLEM 1	PROBLEM 2	PROBLEM 3
PGT	2335.847	2324.074	2326.496
QGT	597.531	470.348	501.291
SGT	2411.063	2371.191	2379.890
PLT	35.847	24.074	26.496
QLT	25.531	-101.652	-70.709
SLT	44.010	104.464	75.510
CGT	3236.695	3365.920	3380.388
SPC	1.386	1.448	1.453
CLT	49.672	34.866	38.499
CGD	3187.023	3331.054	3341.889
PF	0.969	0.980	0.978

TOTAL SYSTEM DEMAND	
ACTIVE POWER	2300 MW
REACTIVE POWER	572 MVAR
APPARENT POWER	2370.056 MVA

FOR DEFINITIONS OF ALL QUANTITIES IN THIS TABLE AND THEIR UNITS, SEE TABLE (5.35) ABOVE.

CHAPTER 6

DIMENSIONALITY AND DECOMPOSITION

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CHAPTER 6

DIMENSIONALITY AND DECOMPOSITION

6.1 INTRODUCTION

As in many other practical engineering research fields, one of the notorious problems that face the power system analyst is that of dimensionality. The general problem of dimensionality is usually superimposed on the particular complexity of the problem in hand. Dimensionality is an indication of the size of the problem in terms of the number of its unknown variables and the associated matrices involved in its mathematical representation. The problem of dimensionality can be so serious in systems analysis that it has been described as a "curse" in some publications [41]. In the present work, the various Stage-I optimization problems presented in Chapter 5 can be used to illustrate this point. For a system defined by NB buses and NL lines, the linear programming representation of each of optimization Problems 1 and 2 involved 2N variables and 2(NB+NL) constraints. These represent the dimensions of the constraints coefficients matrix. For a system as small as the one used for test purposes in the present work, the dimension of this matrix is (106 X 44). Many other matrices of similar range of dimension have to be processed and stored by the computer during the course of solution of the problem, which only worsen the situation, especially if the solution algorithm involves matrix inversion.

Initially, solution methods and computer programs are usually designed and developed irrespective of problem sizes. This means that, in principle at least, an algorithm, and the corresponding computer program, that can solve a small problem, in terms of dimensionality, can also solve a large one. However, the problem of dimensionality manifests itself in terms of the storage requirements of the computer memory and is usually accompanied by longer CPU times to perform the required computations. Because of these difficulties, solution algorithms have to be modified to take problem sizes into account.

6.2 METHODS OF DECOMPOSITION

In the literature, a number of methods and decomposition techniques have been used to reduce the sizes of optimization problems and their associated matrices. Decomposition implies breaking up the large problem into a number of smaller problems that can be handled and solved almost independently, with obvious reduction in the computer storage requirements. Decomposition methods, as applied to power system optimization problems, usually fall in one of two main categories. In the first category, sometimes referred to as P-Q decomposition, the problem is divided into two subproblems. One subproblem is known as the active power problem or P-problem, and the other as the reactive power problem or Q-problem. This category of decomposition method is based on the weak coupling between two sets of power system variables on one hand, and the strong coupling among the variables of each set on the other. One of these two sets of variables consists of the bus real power outputs and phase angles. The variables of the other set are the bus reactive powers and voltage magnitudes. In the strict sense of the word, therefore, this is some form of decoupling rather than decomposition. Basically, it is the same as the decoupling technique applied in load flow studies, based on the Newton-Raphson method, which is used to produce a reduced form of the system Jacobian matrix. One of the papers that fall into this category is that of Sjelvgren and Bubenko [42], which presented a method of solving the optimal power flow problem taking into consideration transmission line outage contingencies in addition to normal operating constraints. Another paper, that utilized the same decoupling principle, was published by Shoults and Sun [43]. Their method was based on first order gradient method and nonlinear minimization technique. The P-problem is used to minimize the hourly production cost by controlling generator real power outputs and tap settings on phase-shifting transformers.

The Q-problem is used to minimize the real power transmission losses by controlling generator terminal voltages, transformer tap settings and shunt capacitor/reactor outputs.

The second category of decomposition techniques utilizes the special structure of the power system problem. One way of achieving this is by dividing the physical system into smaller areas and solving a number of smaller optimization problems such that some overall objective is achieved. Another way is to formulate the mathematical model of the whole system and then exploit the special structure and sparsity of the resulting matrix.

In some research papers the decomposition is directed towards hierarchical or multilevel control [44]. This is based on control functions performed by separate areas of the power system which are independent of each other, but are coordinated by a central area control. In the same way the required computations are performed by separate lower level area computers coordinated by a central higher level computer.

Another method of decomposition applies tearing or diakoptics techniques [45,46,47]. These are procedures by which networks are solved piecewise. Happ and Young [46] described the logic of tearing algorithms and their embedding into the Newton-Raphson load flow. They also presented the implementation of the method for both load flow and stability programs.

Benders decomposition method is useful for mixed integer programming problems. These are linear programming problems in which some of the variables are continuous and some of them are constrained to be integers. Benders method decomposes the problem into two parts. One part deals with the integer variables while the other deals with the continuous variables. Based on this method, a relatively recent paper by Habibollahzadeh and Bubenko [48] described a decomposition technique in a model for short-term operation planning of a large-scale hydrothermal power system with high share of hydro generation. The problem was divided into a Master problem and a Subproblem. The master problem considered the integer variables of the unit commitment of thermal plants. The subproblem dealt with the continuous variables of the economic dispatch problem. Further decomposition was also achieved by dividing the master problem with respect to different plants and the subproblem with respect to hydro and thermal generation.

A relevant topic, in the context of dimensionality, is that of Sparsity. This is a property of a matrix, especially a large one, where most of the elements have zero values. Usually, in such case, the number of zero elements is much larger than the number of non-zero elements. The property itself is a reflection of the nature of the physical system which the matrix represents. Sparsity techniques are solution methods directed towards exploiting this property of matrices. They are based on the efficient storage and handling of these matrices such that only non-zero elements are used [49-52].

Another topic in this context, particularly relevant to linear programming optimization problems, is Duality. Computational experience has shown that the difficulty of a linear programming problem depends mostly on the number of constraints involved rather than on the number of variables [24]. Duality allows representing the original linear programming problem, called the Primal, by an equivalent problem, called the Dual, such that the solution of either problem can be readily obtained from the solution of the other. Generally, one of the two equivalent formulations is less demanding than the other in terms of its memory storage and computation time requirements. Thus, it can be advantageous to solve one of the problems and obtain the solution of the other indirectly. Among other properties of duality is that the number of constraints in the dual is equal to the number of variables in the primal and vice versa. Considering the number of constraints of each problem, the one with the least number of constraints is solved [34]. The concept of duality is treated in more detail in Section (A.4) of the Appendix.

6.3 SPECIAL MATRIX STRUCTURES

As mentioned above, some decomposition methods are based on exploiting the sparsity of the problem matrix. Apart from being sparse, the matrix representations of many practical systems have a special structure in the sense that the non-zero elements of the matrix tend to form a particular pattern that gives the matrix a noticeable shape. The particular shape of the sparse matrix can be very helpful in finding a suitable decomposition or solution method. Sparse matrices in which the non-zero elements are randomly scattered are not very useful in this respect, and they need more effort in devising the sparsity-orientated method of solution and the preparation of the associated computer program.

In the context of applying decomposition methods to the solution of linear programming optimization problems, the relevant matrix is that of the constraints coefficients. Among the various special-structure matrices, of particular interest is the one shown in

FIG.(6.1). This is known as the **Block-diagonal** or **Block-angular** structure. This particular shape of the constraints coefficients matrix characterizes many practical optimization problems where the decision variables of the problem appear in a number of separate sets combined by a few coupling constraints. In different publications on the subject, these coupling constraints are also referred to as the combining, common, linking or simply general constraints. It is to be mentioned that, in general, the blocks of non-zero elements, that define the particular shape or structure of the matrix, can also contain zero elements. However, these are usually much fewer than the non-zero elements and their occurrence in the block has no particular tendency. Before proceeding with this discussion, it is to be mentioned that block diagonal matrices should not be confused with diagonal matrices. The latter are square matrices where all off-diagonal elements are zero, i.e., for a matrix **A**, the value of the element $a_{ij} = 0$ for all $i \neq j$ [53]. Block diagonal matrices are not necessarily square and the reference here is to groups of elements rather than to individual elements.

Matrices that possess the special structure described above, usually result from the nature of the physical system that the mathematical problem represents. Physical systems of this nature consist of a number of "islands" that represent decentralized activities of a large organization, connected by a few distribution routes or communication links. Apart from electric power systems which are addressed here, practical examples include a number of various industries where items or goods are manufactured in a small number of factories sparsely located in a large geographical area that covers the market or the area of consumer demand. The optimization problem in these cases involves determining the best way of operating these factories or "production centres" collectively such that some global objective is accomplished. The aim can be the minimization of the total production cost, maximization of profit or selecting the least-cost routes for transporting the manufactured goods. This has to be achieved under constraints of available resources such as machinery or manpower as well as meeting the consumer demand in terms of quantities and deadlines.

The physical structure of a power system is characterized by many of the properties described above. Electricity is generated in sparsely-placed power stations and transmitted to remote load centres. The general structure of the whole system is such that it consists of a number of areas or subsystems, the mathematical representation of which has the block diagonal matrix form described above.

This property can be exploited to reduce the dimensionality of the problem by solving a number of small subproblems corresponding to the blocks of the constraints matrix, with the solutions of the individual subproblems coordinated by a combining algorithm designed to take into account the effect of the common constraints and achieve the main objective of the whole problem. This has also been found advantageous in tackling large scale problems which are, otherwise, computationally difficult to solve [28]. Another advantage of this principle is that it automatically makes use of the sparsity of the constraint matrix, resulting in a reduction in the associated computer storage requirements.

As discussed above, the block diagonal structure of the constraints coefficients matrix virtually presents a mathematical image of the physical problem which, in our case, is the electric power system. However, in many cases, the shape of the constraints coefficients matrix is not readily predictable from the power system network configuration and a considerable effort is needed to decompose the system starting, for example, from its network diagram. One reason is that the boundaries between subareas of the system are not always very clear. Also, decomposing the system network will involve taking into consideration the power or current flows or injections at the division points. The correct values of these are only obtainable after the problem has been solved. In this case, the solution of the overall problem, in turn, depends on its decomposition which brings us back to the starting point. It might be possible to overcome this difficulty by some elaborate iterative procedure but then this might cancel out the very objective of the decomposition process.

In this thesis, rather than dividing the actual physical power system into smaller areas, decomposition of the linear programming problem is achieved through the constraints coefficients matrix of the problem. The decomposition method applied here, is the **Dantzig-Wolfe Decomposition Principle**. This is a solution algorithm particularly designed to solve optimization problems whose constraints coefficients matrix is of the block diagonal structure discussed above. Often, in the literature, this particular technique is simply referred to as **The Decomposition Principle**, although decomposition is a term of a more general nature as discussed earlier. In the rest of this chapter both long and short versions of the name will be encountered. The technique is based upon the simplex method of solving linear programming optimization problems.

A number of recent publications have shown some interest in applying this decomposition method to power system optimization problems [15]. However, as the main mathematical models used suffer from the shortcomings discussed in Chapter 1, such as the complexity of formulation and iterative nature of the suggested solution method, the application of the Dantzig-Wolfe decomposition principle does not seem very beneficial. The mathematical model developed in this thesis, on the other hand, offers a simple generalized linear representation of power system optimization problems the solutions of which can be obtained by straightforward application of standard computer routines. Reducing the dimensionality of these problems by applying the above-mentioned decomposition principle can, therefore, be of great advantage. The application of the Dantzig-Wolfe decomposition principle to the linearized mathematical model developed in this project will be discussed in Section (6.5).

A general description and a summary of the Decomposition Principle will be given in Section (6.4). Both the Dantzig-Wolfe decomposition principle and the Simplex method upon which it is based are well documented in mathematical programming literature [34,54,55]. As mentioned in Chapter 1, this thesis is concerned with the application of linear programming methods to the formulation and solution of power system optimization problems rather than with linear programming as an abstract mathematical topic. However, to understand the decomposition principle and the related terminology, it is necessary to know at least the main theme of the Simplex method. Therefore, for the purpose of completeness, a general descriptive introduction to the solution of the general linear programming problem and the Simplex method is given in the Appendix. Otherwise, the explanation of the Dantzig-Wolfe decomposition principle becomes ambiguous and out of context. For the same reason some other relevant Definitions and Theorems are also given. Only the necessary minimum of this supporting mathematical background material will be presented, and only briefly. No attempt will be made to go into the details of the "mechanics" of the simplex method or such related aspects as optimality conditions. The relevant theorems will be mentioned without proofs. These can be found in the cited literature along with detailed treatment of many other related topics.

6.4 THE DANTZIG-WOLFE DECOMPOSITION PRINCIPLE

Based on the simplex method, the Dantzig-Wolfe Decomposition Principle, published in 1960, is an efficient procedure designed to solve linear programming problems whose constraints coefficients matrix is of the special structure discussed in Section (6.3) and shown in FIG.(6.1). A complete mathematical representation of this type of decomposable linear programming problem is given in (6.1) to (6.3).

Minimize

$$Z = C_1X_1 + C_2X_2 + C_3X_3 + \ . \ . \ . + C_HX_H \qquad (6.1)$$

Subject to

$$\begin{array}{lllll} A_1X_1 + & A_2X_2 + & A_3X_3 + & . \ . \ . + A_HX_H & = B_0 \\ D_1X_1 & & & & = B_1 \\ & D_2X_2 & & & = B_2 \\ & & D_3X_3 & & = B_3 \\ & & & \ddots & \vdots \\ & & & D_HX_H & = B_H \end{array} \qquad (6.2)$$

$$X_j \geq 0 \qquad (6.3)$$

In the above representation A_i and D_i are matrices, and B_i, C_i and X_i are vectors. This is in contrast with the usual representation of the linear programming problem where the symbols stand for single elements. In general, the number of elements of each vector or matrix of the various independent blocks in the decomposable problem are not necessarily equal. The number of constraints and variables of the i'th block are given by M_i and N_i respectively. Thus, the total number of constraints MT, and variables NT in the decomposable problem are given below by (6.4) and (6.5) respectively.

$$MT = M_0 + \sum_{i=1}^{H} M_i \qquad (6.4)$$

$$NT = \sum_{i=1}^{H} N_i \qquad (6.5)$$

The first M_0 constraints in (6.4) are associated with the general constraints **A** and the rest correspond to the H independent blocks D.

The important feature to be noted in the matrix structure of the above problem is that X_i, the subsets of the matrix X which represents the independent variables of the problem, do not overlap. In fact, without **A**, rightly called the complicating constraints by some authors [27], the solution of the whole problem can be obtained by solving H independent linear programming problems corresponding to the separate blocks of constraints.

The Dantzig-Wolfe decomposition principle divides the original problem into a **Master Program** based on the common or coupling constraints and a number of **Subproblems** corresponding to the independent blocks. The decomposition algorithm solves the overall problem by iterating between the master program and the subproblems and passing information forward and backward between the the two levels. To understand how the decomposition principle works let us first introduce the concept of convex combinations.

Convex Combinations :

A convex combination of a number of points $X_1, X_2, X_3, \ldots, X_n$ (written as vectors) is defined as:

$$X = \mu_1 X_1 + \mu_2 X_2 + \mu_3 X_3 + \ldots + \mu_n X_n \qquad (6.6)$$

$$\mu_1 + \mu_2 + \mu_3 + \ldots + \mu_n = 1$$

$$\mu_1, \mu_2, \mu_3, \ldots \mu_n \geq 0$$

For the two-dimensional case this can be reduced to (6.7) below:

$$X = \mu X_1 + (1 - \mu) X_2 \qquad (6.7)$$

$$1 \geq \mu \geq 0$$

Basically, (6.7) means that on a straight line segment between the points X_1 and X_2, any other point X can be represented as a convex combination of the coordinates of these two points. This is illustrated in FIG.(6.2). The coordinates of the point X(8,6) can be expressed in terms of the points $X_1(2,12)$ and $X_2(10,4)$ with $\mu = 0.25$. This can be verified by using the equation of the straight line segment between X_1 and X_2.

Before applying the Dantzig-Wolfe decomposition principle, the original decomposable linear programming problem of (6.1) to (6.3) is reformulated such that the independent variables of each subproblem are expressed in terms of the extreme points of that subproblem. The equivalent new representation of the problem is given in (6.8) to (6.11).

Minimize $$Z = \sum_{i=1}^{H} \sum_{j=1}^{CE_i} (C_i EP_{ij}) \mu_{ij} \qquad (6.8)$$

Subject to $$\sum_{i=1}^{H} \sum_{j=1}^{CE_i} (A_i EP_{ij}) \mu_{ij} = B_i \qquad (6.9)$$

$$\sum_{j=1}^{CE_i} \mu_{ij} = 1$$

$$i = 1, 2, 3, \ldots, H \qquad (6.10)$$

$$\mu_{ij} \geq 0$$

$$i = 1, 2, 3, \ldots, H$$

$$j = 1, 2, 3, \ldots, CE_i \qquad (6.11)$$

In the transformed representation above, CE_i represents the number of the extreme point solutions associated with the i'th independent block of constraints. This is defined in terms of the number of variables N_i and constraints M_i. The numerical values of these solutions are given by EP_{ij}. In the new representation, the independent variables are now given by μ instead of X. The total number of constraints and variables of the new problem are given by:

$$ME = M_o + H \qquad (6.12)$$

$$NE = \sum_{i=1}^{H} CE_i \qquad (6.13)$$

Again the number of constraints consists of two parts. The first part corresponds to the general constraints and represents the master program. The second part correspond to the H separable blocks of the original problem. Each of these represent a subproblem in the new formulation.

The new representation of (6.8) to (6.11) is completely equivalent to that of (6.1) to (6.3). However, the number of variables has been increased from NT to NE and the number of constraints reduced from MT to ME. The main advantage of this transformation is the great reduction in the number of constraints which, from the computational point of view, facilitates the solution of the problem as discussed in Section (6.2) in connection with duality. Thus, we have a new linear programming problem with its own constraints and independent variables. This can be solved by the same methods that are used to solve the original problem such as exhaustive enumeration or the simplex algorithm. The total number of extreme points corresponding to the new formulation of the problem is given by the total number of possible combinations given by (6.14) below:

$$NTE(NE,NE) = (NE!)/(ME!(NE - ME)!) \qquad (6.14)$$

Even for small problems, this number can be so large that it requires a formidable amount of computational effort to enumerate all the possible extreme point solutions and solve the problem. Direct application of the simplex method to the transformed problem will involve a large number of iterations and will be accompanied by high computer storage requirements and long computation time. It is, therefore, doubtful that by mere reformulation, using the transformation above is of any value in easing the computational burden of the original problem. However, by using this formulation and applying the Dantzig-Wolfe decomposition algorithm, the solution of the problem can be obtained quickly and efficiently.

The Dantzig-Wolfe decomposition principle is based on the notion that the solution of the overall problem consists of the convex combination of a small selection of all the possible solutions of the subproblems. The decomposition principle recognizes and selects from the solution of the subproblems only those which also satisfy the general or common constraints. The decomposition algorithm is a step-by-step procedure of this recognition and selection process.

In the Dantzig-Wolfe decomposition algorithm, rather than explicitly tabulating all the columns of the constraints coefficients matrix, the solution of the master program is started by using a very small basis of the master program. This is solved to yield the values of an initial small number of the independent variables of the problem. This starting solution can be obtained in a similar manner to obtaining the initial solution in the simplex method such as the use of artificial variables. New columns are created by a special process called

"Column Generation" and these are appended to the basis. These new columns, corresponding to new independent variables of the problem, are only generated when they are needed in the course of the solution and added to the existing basis. It is to be remarked here that in the simplex method, on the other hand, the size of the basis is fixed and only two variables are interchanged in each iteration. The decomposition algorithm continues by using the simplex multipliers corresponding to the present basis of the master program to formulate and solve the subproblems. The solution of these are then used to form a new column to enter the basis of the master problem. The new master problem is solved and the whole process is repeated. Similar to the simplex method, the decomposition algorithm has a criterion for the selection of the new columns to enter the basis and for terminating the process when the optimal solution is obtained.

Summary of the Decomposition Algorithm

The iterative procedure of the Dantzig-Wolfe decomposition principle can be summarized in the following concise steps.

1. Assume that there is an initial basic feasible solution for the master program. Let B and π be the corresponding basis matrix and vector of simplex multipliers.

2. Partition the simplex multipliers vector into two parts, $\pi = (\pi_0, \pi_1)$. The first part π_0 consists of M_0 elements corresponding to the common constraints of (6.9) and the second part π_1 has H elements corresponding to the constraints of (6.10).

3. Formulate and solve the following H linear programming problems.

$$\text{Minimize} \quad Z_i = (C_i - \pi_0 A_i) X_i \tag{6.15}$$

$$\text{Subject to} \quad D_i X_i = B_i$$

$$X_i \geq 0$$

$$i = 1, 2, 3, \ldots, H \tag{6.16}$$

4. Let the optimal solutions of the above problems be denoted by $X_i(\pi_0)$ and the corresponding value of the objective function by ZO_i. Calculate the following coefficients:

$$F_i = ZO_i - \pi_{1i} \tag{6.17}$$

and determine the minimum of these coefficients FM.

5. If $FM \geq 0$ then the process is terminated and the optimum solution to the overall problem is given by:

$$XO = \sum_j \mu_{ij} EP_{ij} \tag{6.18}$$

$$i = 1, 2, 3, \ldots, H$$

with all values of j corresponding to basic variables.

6. If FM < 0, form the column:

$$P = \begin{bmatrix} A_i X_i (\pi_o) \\ \hline UT_i \end{bmatrix} \qquad (6.19)$$

where UT_i is a vector of H elements, with 1 at the i'th position and zero elsewhere.

7. Update the master problem. Transform the column of (6.19) by multiplying it by B^{-1} and append to the current basis of the master program.
Obtain a new basis inverse and a new vector of simplex multipliers.

Go to step 2 above.

It is to be appreciated that the above is only a brief presentation of the decomposition algorithm. It mainly shows the steps that can be translated into a computer program. Detailed explanation of these steps and general analysis of the method as well as step-by-step solution of numerical examples can be found in the cited references.

6.5 APPLICATION TO THE PRESENT MODEL

This section considers the possibility of applying the Dantzig-Wolfe decomposition principle to the solution of power system optimization problems, using the two-stage generalized mathematical model developed in the present work. To do this, we need to examine the structures of the constraints coefficients matrices of the optimization problems of the two stages of the model. We are mainly concerned here with the handling and manipulation of these matrices such that they are presented in the block diagonal structure suitable for the application of the Decomposition Principle. This is achieved through an example from one of the optimization problems of Chapter 5, using the sample test system.

As discussed in Section (6.3), decomposition is applicable to certain linear programming problems whose constraints coefficients matrix posses a particular structure. The possibility of applying some form of decomposition to the solution of the power system optimization problem becomes obvious whether considering the physical system itself or its mathematical representation. In a large system, utilizing this property becomes more attractive because of the associated computational advantages such as the improvement in convergence properties and lower storage requirements. In very large systems, decomposition might become necessary as a need rather than an alternative solution technique.

Pictorial Representation of Matrix Structures

As discussed earlier, a practical power system consists of a number of segregated parts connected to each other by few transmission links. This physical property appears as a very sparse admittance matrix with the small number of non-zero elements forming a narrow "band" around the diagonal. To clarify this fact, a pictorial representation of the structure of the admittance matrix of the sample power system used in this thesis is shown in FIG.(6.3). This is done by computing the elements of the admittance matrix of the given system and then substituting each non-zero element by an X and each zero element by a space. The specific numerical values of the non-zero elements are immaterial for this purpose. For a system with NB buses the dimension of the admittance matrix is given by (NB x NB), i.e., the total number of elements in the matrix is NB^2. For the sample system which has 23 buses this number is 529. From the matrix structure of FIG.(6.3), it is obvious that the number of the non-zero elements is much smaller than the total number of elements. Also, apart from being highly sparse, the general shape of the matrix is encouraging in terms of application of the decomposition principle.

However, the matrix to be used for linear programming decomposition purposes is that of the constraints coefficients of the system. The same procedure is used to obtain a pictorial representation of this matrix. Application of decomposition to the two stages of the mathematical model will be discussed below using Problem 1 of Chapter 5 as an illustrative example.

Decomposition of Stage-I

A pictorial representation of the structure of the constraints coefficients matrix for the sample system is shown in FIG.(6.4). Due to the large size of the matrix, it is not possible to fit the figure on one page and, therefore, it is continued on the following two pages. However, the division of the matrix among the pages was not done arbitrarily. The figure was divided such that the whole pictorial effect, in terms of the particular structure of the matrix, is not spoiled. This will be made clear by the rest of the discussion below. The sequential numbers of the rows of the matrix are given to the left.

As can be seen from the figure, this matrix possesses a very distinctive structure. After carefully studying this structure, the various attributes that characterize the matrix and their interpretation in terms of the formulation of the optimization problem and the configuration of the power system are summarized in the following points. Again, it should be kept in mind that, in the following discussion, we are not concerned with the specific numerical values of the non-zero elements. What we are interested in here is the distribution of these elements within the matrix.

1. The matrix consists of 106 rows and 44 columns corresponding to the number of the constraints and independent variables of the optimization problem respectively. Obviously the matrix is very sparse. The total number of elements of this matrix is more than 4000. However, the number of the non-zero elements is about 600, which means that approximately 85 % of the elements have zero values.

2. It consists of four distinctive blocks of elements corresponding to the four sets of constraints of the problem. The first and second blocks are identical, in terms of the row-column positions of their individual elements, although, obviously, the numerical values of the corresponding elements in the two blocks are different. The same observation applies to the third and fourth blocks, which are identical to each other although they are different from the first two blocks.

3. Each of the first two blocks of elements consists of 23 rows corresponding to the system buses. In particular, the "full" row in each block corresponds to the reference bus. Similarly, the third and fourth blocks

each consists of 30 rows of elements corresponding to the transmission lines in the system.

4. In addition, each single row consists of two identical parts, each consisting of 22 columns. The elements of the first half of each row correspond to the bus voltage magnitudes while the elements of the second half correspond to the bus phase angles. Thus, the whole constraints coefficients matrix consists of eight "Sub-blocks".

5. In general, the matrix of FIG.(6.4) is a special case of FIG.(5.3) with the number of blocks W=4. Thus, this matrix can be alternatively represented as shown in FIG.(6.5). The difference in size among the various blocks is also reflected in the figure.

6. The submatrix consisting of the first 22x22 elements of each of the first four sub-blocks, is identical to the first 22x22 elements of the admittance matrix of the system shown in FIG.(6.3). This can be generalized for a system of NB buses in which case each of these sub-blocks has NxN elements, thus representing a submatrix that does not include the reference bus.

7. The non-zero elements of each of the eight individual blocks tend to form a band around its own principle diagonal.

The above has been a description of the constraints coefficients matrix of the first stage of Problem 1. The corresponding matrix of Problem 2 has exactly the same structure. The matrix of Problem 3, on the other hand, has 136 rows. The matrix structure of this problem can be predicted from the formulation of the problem and the discussion above. The matrix will be consisting of five blocks of rows. The first 106 rows have the same structure as that of problems 1 and 2. The structure of the last block of 30 rows will be similar to that of the third and fourth blocks of problems 1 and 2, as these are all based on line constraints.

It can be seen from FIG.(6.4) that the general structure of the constraints coefficients matrix is even more encouraging in terms of application of decomposition. However, the whole matrix structure does not look similar to that of FIG.(6.1) and, therefore, the Dantzig-Wolfe decomposition principle cannot be readily applied. First, the matrix has to be manipulated and the repeated blocks need to be rearranged to form one large block with a single "string" of separable diagonal blocks of non-zero elements. The first two blocks of constraints can be merged to form one large block. The same can be done with the third and fourth blocks. Moreover, the two right-to-left parts of each block can be merged to form one wide band of non-zero elements. This process of "reshuffling" the matrix is equivalent to changing the order of the constraints and variables in the original problem. The same effect can be achieved by reformulating the problem or changing the computer program if this particular shape of the matrix has been anticipated. However, this is not necessary and rearranging the matrix elements can be done after they have all been calculated and the pictorial structure of the matrix has been obtained. The structure of the resulting matrix, modified as suggested above, is shown in FIG.(6.6).

The new matrix consists of two main blocks of non-zero elements. The first block contains 46 rows corresponding to the combined bus constraints associated with the 23 system buses. Similarly, the second block contains 60 rows corresponding to line constraints. In general, the matrix now consists of two main bands of non-zero elements in addition to some full rows and a number of scattered elements at the end of the matrix, namely, rows 95 to 106. Obviously, the process can be continued such that the two main bands of non-zero elements can be combined to form one long band. However, it should be noted that each of the two individual bands form a "Staircase" rather than a block diagonal structure. Consequently, this will also be the structure of the resulting combined band. A matrix of a general staircase structure is shown in FIG.(6.7). The required block diagonal structure can be obtained by dividing this matrix into a number of horizontal bands and then selectively moving some of these bands to form part of the set of the constraints of the master program leaving the rest of the matrix as individual blocks. An example of such a process is performed along the broken lines of FIG.(6.7). The resulting rearranged matrix is shown in FIG.(6.8). In both figures, the various parts of the matrix are given the same identification, A1, D2, and so on. As can be seen from FIG.(6.8), the modified matrix is in the block diagonal structure of FIG.(6.1). Note that the resulting block of common constraints **A** is also sparse, but the distribution of its non-zero elements is such that it cannot be decomposed into independent well-defined blocks as in D.

The same process of division into bands can be applied to the actual matrix of the sample system of FIG.(6.6) such that the final form of the resulting matrix is suitable for the application of the Dantzig-Wolfe decomposition principle. It is to be emphasized that all these transformations of the matrix structure from its original to the final form are performed by the computer.

Decomposition of Stage-II

The general shape of the constraints coefficients matrix of the first stage of the mathematical model is predictable although the exact values of the non-zero elements are not. In the second stage of the model, however, both the structure of the coefficient matrix and the values of its non-zero elements are predictable. The values of the elements of this matrix are either zero or one. For the 24-generator sample system the constraints coefficients matrix of stage-II is shown in FIG.(6.9). As can be seen from the figure, the matrix is already in a block diagonal form ideal for the application of the Dantzig-Wolfe decomposition principle.

This stage has been chosen to illustrate the application of the Dantzig-Wolfe decomposition principle to the solution of power system optimization problems. A FORTRAN computer program has been written to implement the algorithm outlined in Section (6.4) above. The numerical results are based on Problem 1 of Chapter 5. After solving the first stage of the problem in the usual way, i.e., without decomposition, the second stage was solved by applying the decomposition technique. The linear programming problem of Stage-II was decomposed into six subproblems corresponding to the six power stations. The numerical results obtained were identical to those reported in Chapter 5. It is possible to decompose the problem into two or three subproblems instead of six. In this case each subproblem represents the combination of more than one station, with the number of stations divided equally or unequally among the subproblems.

6.6 STAGE-II : AN ALTERNATIVE DECOMPOSITION METHOD

It is to be reiterated here that the Dantzig-Wolfe decomposition principle is primarily designed to solve linear programming problems whose constraints coefficients matrix consists of a number of independent blocks coupled by a few combining constraints. Without these combining constraints, the linear programming subproblems corresponding to the diagonal blocks can be solved independently. This suggests an alternative method to decompose and solve the linear programming problem of Stage-II of the mathematical model. As shown in FIG.(6.9), there is only one full-row constraint in the problem. This constraint which, in fact, is the sum of the other NS constraints, can be removed from the formulation of the problem leaving NS separate equations. An objective function can then be added to each of these equations forming a small linear programming problem of NGB_i variables and one constraint. This results in NS such linear programming problems. The general mathematical formulation of the NS problems is presented by (6.20) to (6.22).

$$\text{Minimize} \quad Z = \sum_{j=1}^{NGB_i} CG_j \times PGG_j \tag{6.20}$$

Subject to

$$\sum_{j=1}^{NGB_i} PGG_j = PGB_i \tag{6.21}$$

$$PGGMX_j \geq PGG_j \geq PGGMN_j \tag{6.22}$$

$i = 1, 2, 3, \ldots, NS$

$j = 1, 2, 3, \ldots, NGB_i$

The above mathematical formulation simply states that, at each generating bus, the active power output is equal to the sum of the active power outputs of the individual generators connected at the bus, and that the generating cost at the bus is to be minimized satisfying the operating limits of the generators.

It is to be noted, however, that the presence of the single combining constraint is necessary if the original problem is to be solved as a whole, i.e., without decomposition. Attempts to remove this constraints, while experimenting with the computer program, resulted in infeasibility of the problem.

This alternative way of solving the second stage of the problem has been tested on the sample system and the results obtained were identical to those obtained by the previous two methods, namely, solving the problem without decomposition and solution by application of the Dantzig-Wolfe decomposition principle.

An interesting practical interpretation can be associated with these results. The first stage of the model can be considered as System or "Global" problem, while the decomposed generating bus problems constituting the second stage can be looked upon as Bus or "Local" problems. It is possible, thus, to solve the complete optimization problem on two different levels with their corresponding control computers. A higher level Central system computer can be used to obtain all system voltages, line flows, and bus generations. The global operation of the system can be optimized according to some criterion as illustrated by the various optimization problems of Chapter 5. Information is then sent to the various power stations about the amount of output they are expected to contribute towards the system total demand and losses. Then, it is up to the individual power stations to fulfil their commitment towards the system by operating their generators under optimum operating conditions, measured for instance by minimum input fuel cost. Communication is needed only in one direction from the main computer to the local computers, in the sense that the individual stations do not need to communicate among themselves and they can operate completely independently of each other. They only need to receive information about their power allocation from the central computer. This is illustrated in FIG.(6.10).

On the other hand, two or more stations can operate together to satisfy their collective commitment toward the system. Thus, they may form subareas of the system depending on their geographical proximity and location within the whole system as well as their individual and total loading conditions. This is shown in FIG.(6.11). In this case, apart from the central system computer and the individual station computers, there are medium-level subarea computers. The information is first passed from the central computer of the system to the various subareas. From these, information is then passed down to the individual stations in each subarea. Situations similar to those discussed above and illustrated in FIG.(6.10) and FIG.(6.11), can also arise in a number of other industries.

6.7 COMPUTATION TIME CONSIDERATIONS

One of the aspects to be considered when solving practical problems on digital computers is the time required to run the associated computer program. This is especially important when the solution method is to be considered for on-line applications where results are to be presented in real time. This section looks into this aspect of the problem.

The computation time of a computer program is affected by many factors. Two relevant factors are discussed here. On a particular machine and with a given solution algorithm, the computation time depends on the problem size. On the other hand, for a problem of a given size and using a specific solution method, the computation time depends on the type of computer used; more specifically, its processor or Central Processing Unit (CPU).

The various optimization problems addressed in this thesis are based on the real-time operation of electric power systems which involves on-line application of the associated computer programs. The two aforementioned cases, namely, problem size and processor type will be discussed in relation to these problems. To run the various FORTRAN programs associated with the project, there is a total of five computers at the author's disposal. One is the Power Systems Micro-Vax Computer known as PSMV. The other four are the machines of the VAX Computer Cluster of the Computer Centre of the University of Strathclyde, identified as VAXA, VAXB, VAXD and VAXE. The proper Model names of the five machines are shown at the end of Table (6.1).

Each of the five computers mentioned above is used on a time-sharing basis. However, it is possible, by a special computer library routine to find the CPU time allocated for a particular user, a particular program or any part or parts of a program. The name of the routine is CPUTIM and can be called by the user's program [56].

Comparison of computation times based on processor type is given in Table (6.1). The computer programs associated with the three optimization problems of Chapter 5 have been used for this purpose. Two entries are associated with each machine corresponding to the two test cases. Each numerical value represents the average of several runs.

There is obvious differences among the running times on the various machines. The PSMV, being an old version of the VAX, is the slowest and VAXE with the most advanced and powerful processor among the five machines is the fastest. The fact that VAXA and VAXB are identical is also reflected in the table where the corresponding CPU times are very close.

Using one machine, the PSMV, a comparison based on problem size is shown in Table (6.2). For each problem the table gives the number of constraints, NC, and variables, NV. The product of these, which gives the total number of elements of the constraints coefficients matrix, is also shown in the table. Again, for each problem two values corresponding to test cases A and B are shown, with each value, in turn, being the average obtained from several runs of the computer program. The PSMV is deliberately chosen to obtain the results of

Table (6.2) because, as indicated by Table (6.1), it is the slowest of the five machines which makes it possible to monitor the differences in the values of the CPU time associated with the various optimization problems. Otherwise, using a fast processor, these differences can be too small to detect, especially in optimization problems of very small size.

The connection between the running time of the computer program and the size of the corresponding linear programming problem is obvious from the table. The first three entries in the table correspond to the three main optimization problems of Chapter 5. The fourth entry represents the linear programming problem of Stage-II of the mathematical model. This problem has seven constraints. One of these is a general constraint based on the total energy balance in the system and the other six correspond to the six generating buses in the system. The 24 variables of the problem correspond to the number of generators in the system. The rest of the table consists of six small problems, each with a single constraint and few variables. Collectively, these problems represents the decomposed solution of Stage-II. They correspond to the power stations in the system with the number of variables corresponding to the number of generators of each station. The solutions of these have been obtained by the alternative decomposition method suggested in the Section (6.6). It should be noted that, because of the very small sizes of these six problems, the associated CPU times are not very accurate. Therefore, they should not be compared among themselves and should not be taken as the basis to establish firm conclusions. However, when compared with the first four larger problems they clearly reflect the relation between the size of the problem and the corresponding computation time.

In general, this also apply in a way to the results presented in Tables (6.1) and (6.2). Mainly, both tables are intended to give a general indication of the computation times associated with the various problems addressed in the thesis. It is possible to produce more detailed results in this respect and apply more rigorous analysis. However, only the more important and relevant points have been selected and discussed here.

The one main conclusion of all this is that the suggested mathematical formulation and the corresponding computer programs are suitable for on-line application in the operation of an actual power system. This point will be discussed in more detail in Chapter 7.

6.8 DISCUSSION

The problem of dimensionality in power system optimization problems and the application of decomposition techniques have been extensively dealt with in this chapter. The following relevant comments and conclusions are worth mentioning.

1. Generally, decomposable optimization problems can be solved by the usual methods, the main aim of decomposition being the reduction of the problem size and overcoming the problem of dimensionality.

2. The real benefits of decomposition can only be appreciated when it is applied to optimization problems of very large systems involving very large number of variables and constraints. The application of decomposition methods and the preparation of the associated computer programs requires special additional effort which is not worthwhile when applied to small problems.

3. The descriptions of problems as small and large are rather relative terms that depend on the computing facilities available. What used to be considered as large problems, say 20 years ago, are now only moderate, and problems which required special computing systems in the past can now be solved on available computing facilities of general access. In this respect, the problem of dimensionality in recent years is not as severe as it

used to be in the past due to the great development in computer technology.

4. Having said that, however, there is always a continuous and parallel expansion in the size of system problems and in the available computer facilities.

5. Decomposition of a large-scale problem does not necessarily reduce the total computation time. In general, the sum of the computation times needed to solve the individual small subproblems constituting the large problem, is approximately equal to the computation time needed to solve the original problem without decomposition. Again, the main advantage of the decomposition is the considerable reduction in the problem size, and the corresponding computer memory storage requirements, not the total computation time.

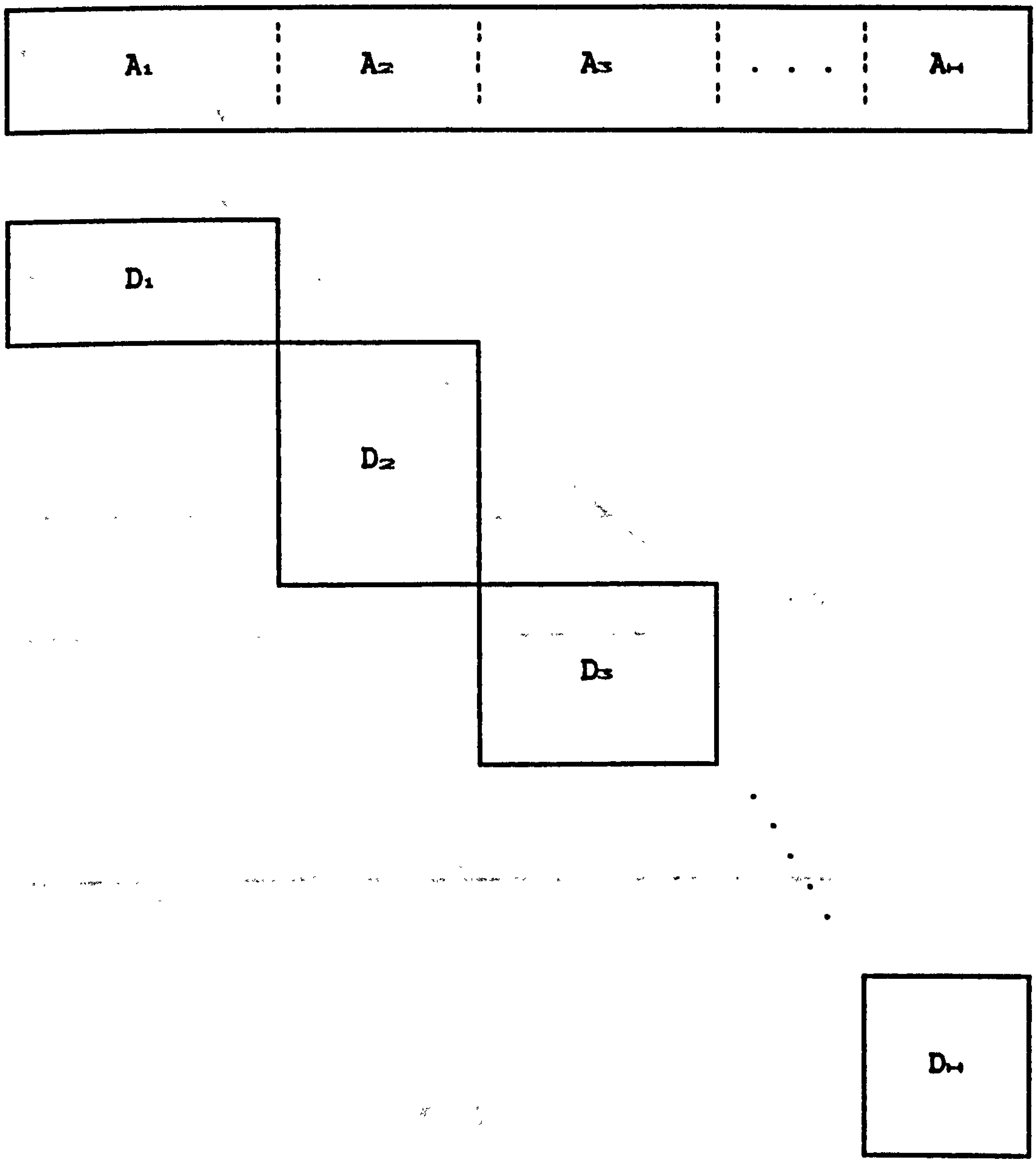


FIG.(6.1)

GENERAL STRUCTURE OF THE CONSTRAINTS COEFFICIENTS MATRIX OF A DECOMPOSABLE OPTIMIZATION PROBLEM OF THE BLOCK DIAGONAL TYPE

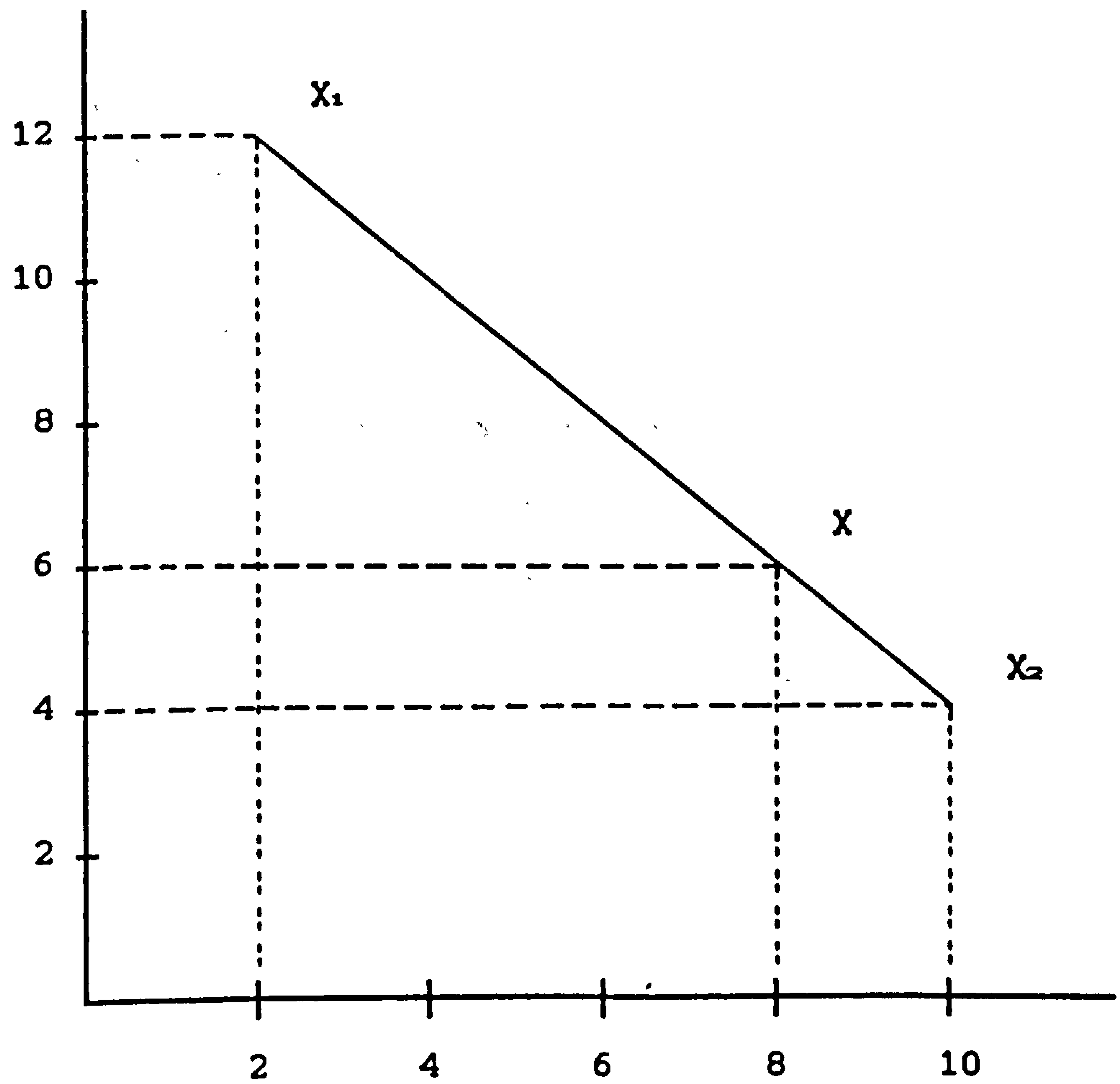


FIG. (6.2)

ILLUSTRATION OF CONVEX COMBINATIONS

1	X	X	X	X																			
2	X	X			X		X																
3	X		X			X																	
4	X			X					X														
5		X			X			X															
6			X			X		X															
7		X					X		X														
8					X	X		X		X		X	X										
9				X			X		X	X		X	X										
10								X	X	X	X												
11										X	X												
12								X	X			X		X	X								
13								X	X				X	X									X
14												X	X	X									
15												X			X			X					
16																X	X						X
17																X	X	X					
18															X		X	X	X			X	
19																		X	X	X			
20																			X	X	X		
20																				X	X	X	
22																		X			X	X	
23													X			X							X

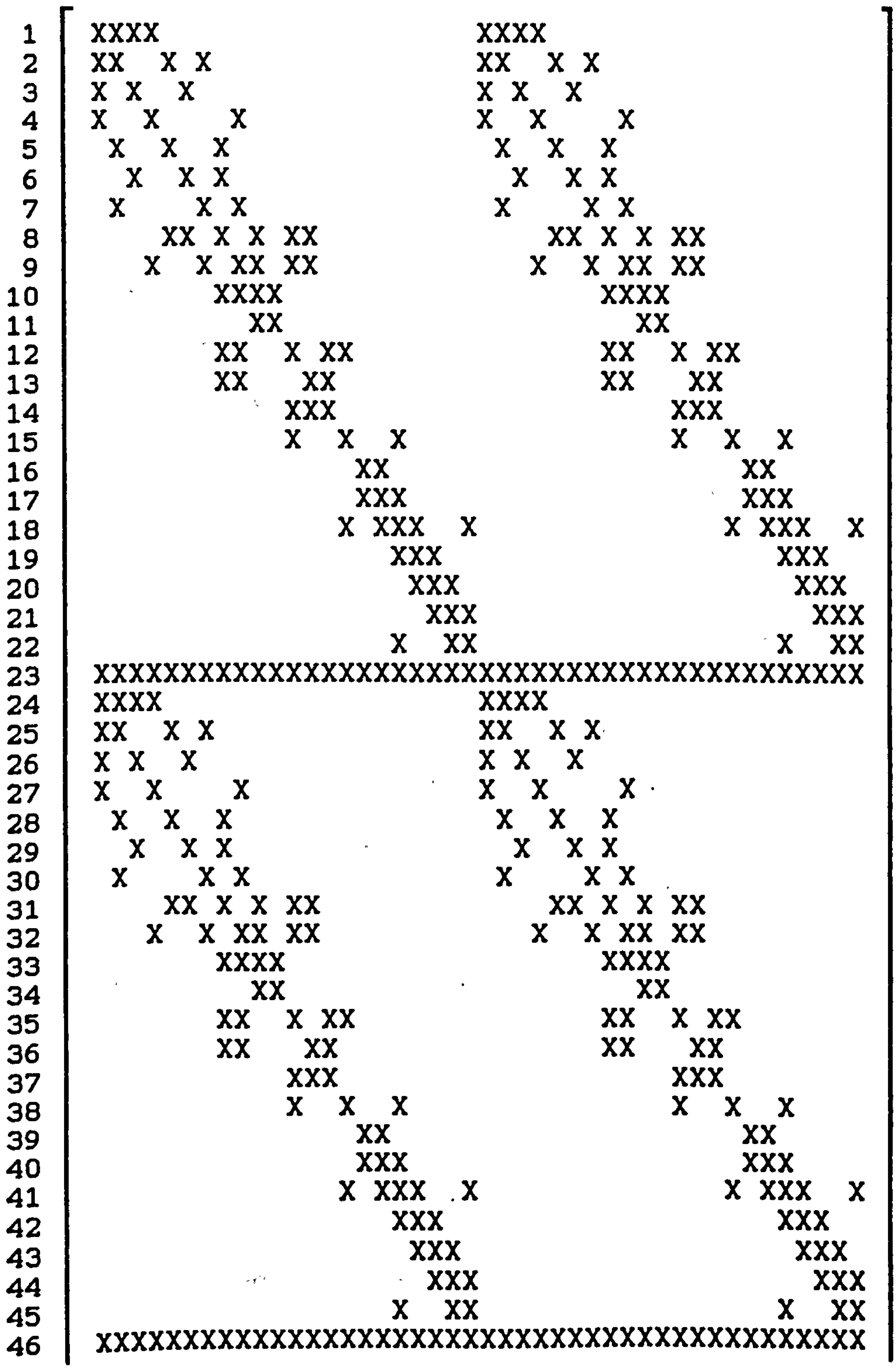
FIG.(6.3)

STRUCTURE OF THE ADMITTANCE MATRIX OF THE SAMPLE POWER SYSTEM

FIG.(6.4)

STRUCTURE OF THE CONSTRAINTS COEFFICIENTS MATRIX
OF THE SAMPLE POWER SYSTEM

The figure is continued on the
following two pages



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```
47 | X X                   X X                    |
48 | X  X                  X  X                   |
49 |  X  X                  X  X                  |
50 |     X  X                  X  X               |
51 |  X    X                Y    X                |
52 |   X  X                  X  X                 |
53 |    X    X                X    X              |
54 |       X X                   X X              |
55 |      X X                   X X               |
56 |          XX                    XX            |
57 |        X X                   X X             |
58 |         XX                    XX             |
59 |             XX                    XX         |
60 |            X X                   X X         |
61 |            X  X                  X  X        |
62 |               X  X                  X  X     |
63 |             X                     X          |
64 |                XX                    XX      |
65 |                 XX                    XX     |
66 |                  XX                    XX    |
67 |                   XX                    XX   |
68 |                  X   X                 X   X |
69 |                    XX                    XX  |
70 |                     XX                    XX |
71 |                X                     X       |
72 |        X   X                 X   X           |
73 |        X    X                X    X          |
74 |         X  X                  X  X           |
75 |         X   X                 X   X          |
76 | XX                    XX                     |
```

Continued on the next page ...

... Continued from previous page

```
 77 | X X                   X X
 78 | X  X                  X  X
 79 |  X  X                  X  X
 80 |     X  X                  X  X
 81 |  X    X                X    X
 82 |   X  X                  X  X
 83 |    X    X                X    X
 84 |       X X                   X X
 85 |      X X                   X X
 86 |          XX                    XX
 87 |        X X                   X X
 88 |         XX                    XX
 89 |             XX                    XX
 90 |            X X                   X X
 91 |            X  X                  X  X
 92 |               X  X                  X  X
 93 |             X                     X
 94 |                XX                    XX
 95 |                 XX                    XX
 96 |                  XX                    XX
 97 |                   XX                    XX
 98 |                  X   X                 X   X
 99 |                    XX                    XX
100 |                     XX                    XX
101 |                X                     X
102 |        X   X                 X   X
103 |        X    X                X    X
104 |         X  X                  X  X
105 |         X   X                 X   X
106 | XX                    XX
```

AM1	AN1
AM2	AN2
AM3	AN3
AM4	AN4

FIG.(6.5)

GENERAL STRUCTURE OF
THE CONSTRAINTS COEFFICIENTS MATRIX
OF THE SAMPLE SYSTEM

FIG.(6.6)

STRUCTURE OF THE MODIFIED CONSTRAINTS COEFFICIENTS MATRIX OF THE SAMPLE POWER SYSTEM

The figure is continued on the following two pages

```
 1  XXXXXXXX
 2  XXXXXXXX
 3  XXXX    XX  XX
 4  XXXX    XX  XX
 5  XX  XX    XX
 6  XX  XX    XX
 7  XX    XX        XX
 8  XX    XX        XX
 9    XX    XX    XX
10    XX    XX    XX
11      XX    XX  XX
12      XX    XX  XX
13    XX        XX  XX
14    XX        XX  XX
15          XXXX  XX  XX  XXXX
16          XXXX  XX  XX  XXXX
17        XX    XX  XXXX  XXXX
18        XX    XX  XXXX  XXXX
19                XXXXXXXX
20                XXXXXXXX
21                    XXXX
22                    XXXX
23                XXXX    XX  XXXX
24                XXXX    XX  XXXX
25                XXXX      XXXX
26                XXXX      XXXX
27                        XXXXXX
28                        XXXXXX
29                        XX    XX    XX
30                        XX    XX    XX
31                                XXXX
32                                XXXX
33                                XXXXXX
34                                XXXXXX
35                              XX  XXXXXX    XX
36                              XX  XXXXXX    XX
37                                    XXXXXX
38                                    XXXXXX
39                                      XXXXXX
40                                      XXXXXX
41                                        XXXXXX
42                                        XXXXXX
43                                    XX    XXXX
44                                    XX    XXXX
45  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
46  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

... Continued from previous page

```
47 | XX  XX
48 | XX  XX
49 | XX    XX
50 | XX    XX
51 |   XX    XX
52 |   XX    XX
53 |         XX    XX
54 |         XX    XX
55 |   XX          XX
56 |   XX          XX
57 |     XX    XX
58 |     XX    XX
59 |       XX          XX
60 |       XX          XX
61 |             XX  XX
62 |             XX  XX
63 |           XX  XX
64 |           XX  XX
65 |                   XXXX
66 |                   XXXX
67 |               XX  XX
68 |               XX  XX
69 |                 XXXX
70 |                 XXXX
71 |                         XXXX
72 |                         XXXX
73 |                       XX  XX
74 |                       XX  XX
75 |                       XX    XX
76 |                       XX    XX
77 |                             XX    XX
78 |                             XX    XX
79 |                         XX
80 |                         XX
81 |                               XXXX
82 |                               XXXX
83 |                                 XXXX
84 |                                 XXXX
85 |                                   XXXX
86 |                                   XXXX
87 |                                     XXXX
88 |                                     XXXX
89 |                                   XX      XX
90 |                                   XX      XX
91 |                                       XXXX
92 |                                       XXXX
93 |                                         XXXX
94 |                                         XXXX
```

... Continued on next page

... Continued from previous page

```
 95 |                               XX               |
 96 |                               XX               |
 97 |             XX    XX                           |
 98 |             XX    XX                           |
 99 |             XX      XX                         |
100 |             XX      XX                         |
101 |               XX  XX                           |
102 |               XX  XX                           |
103 |               XX    XX                         |
104 |               XX    XX                         |
105 | XXXX                                           |
106 | XXXX                                           |
    L______________________________________________|
```

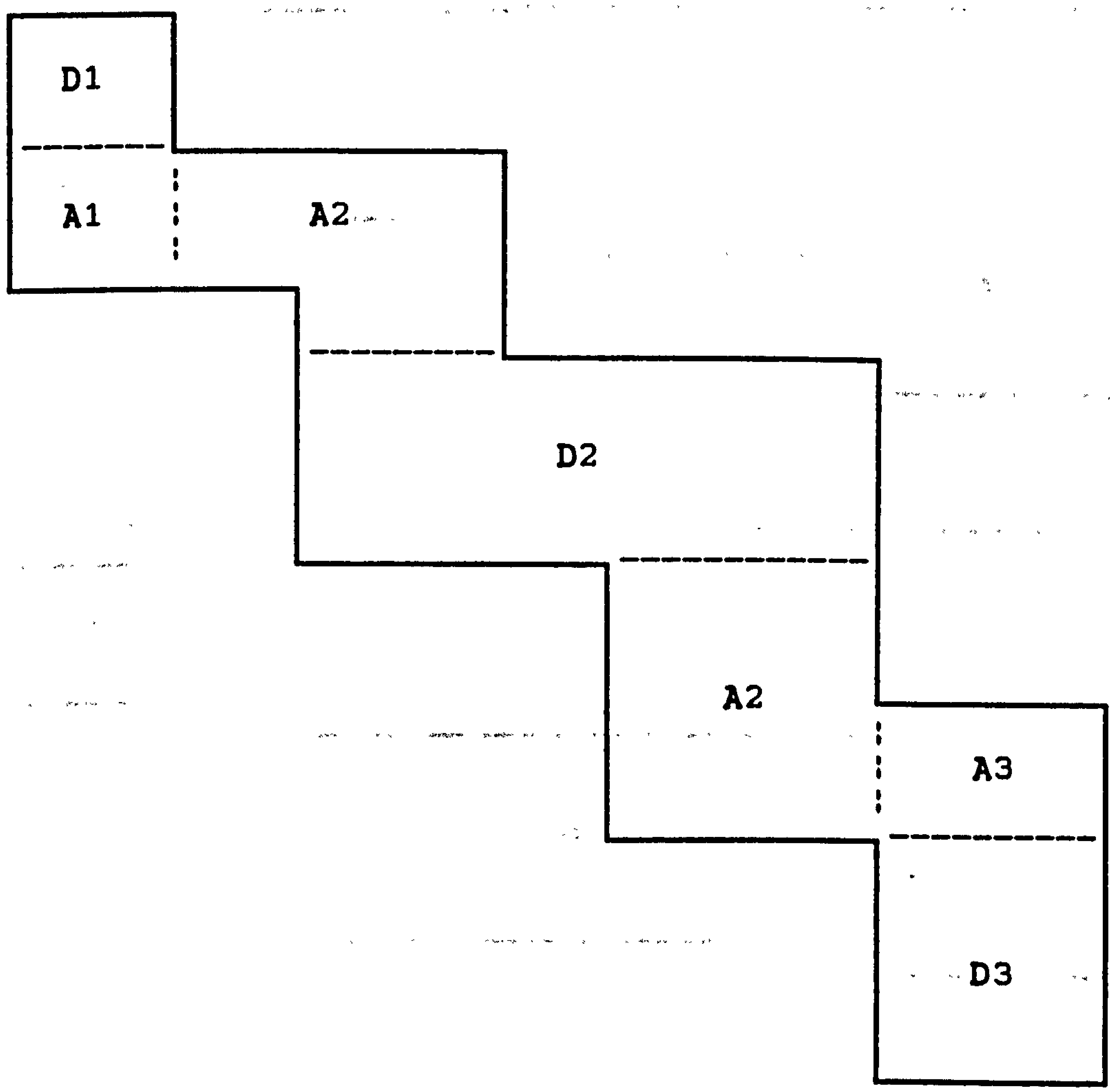

FIG.(6.7)

GENERAL STRUCTURE OF A STAIRCASE MATRIX

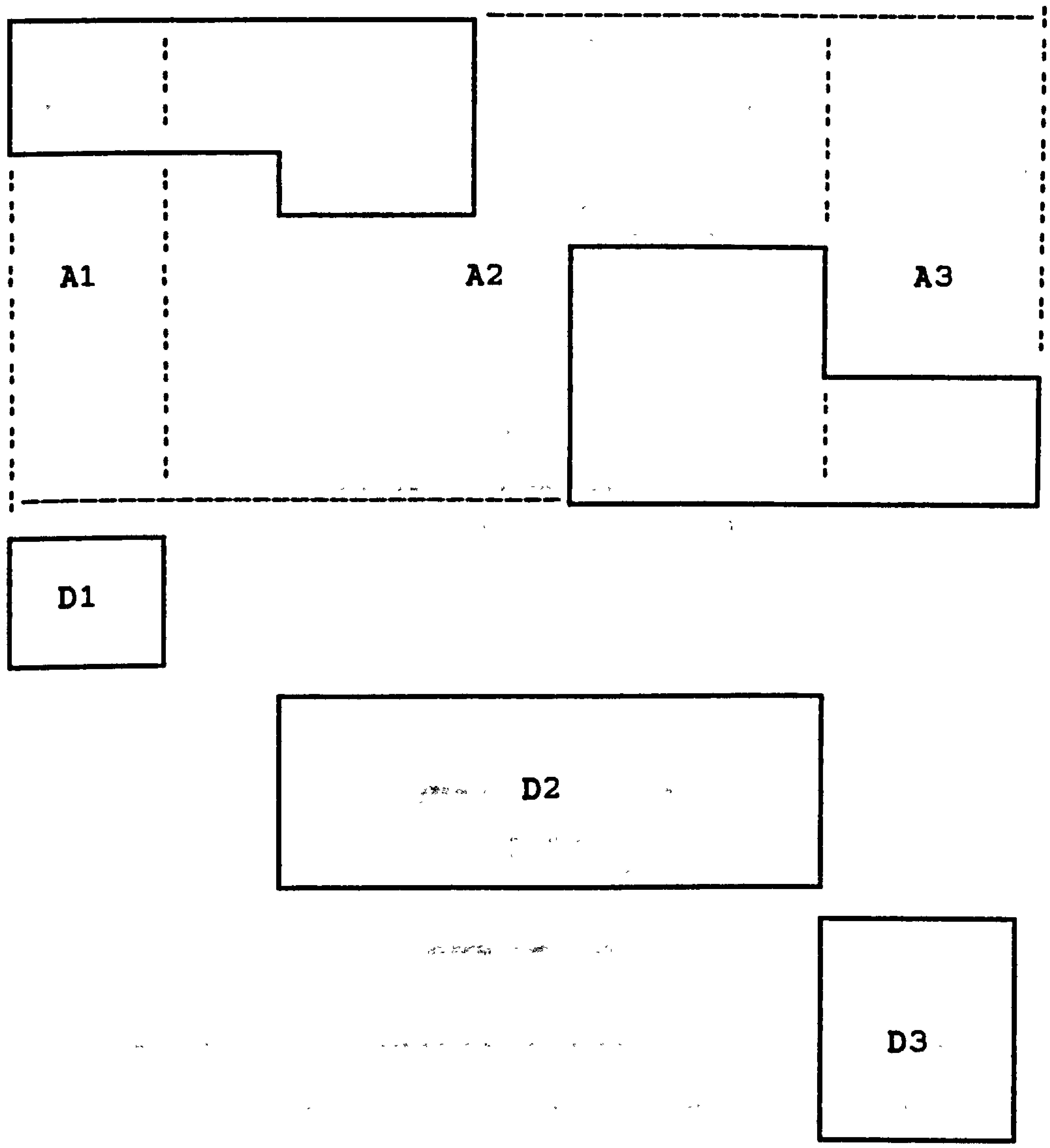

FIG.(6.8)

THE MATRIX OF FIG.(6.7), MODIFIED
TO CONFORM TO THE DECOMPOSABLE
BLOCK DIAGONAL STRUCTURE

$$
\begin{array}{c}
1\\2\\3\\4\\5\\6\\7
\end{array}
\left[\begin{array}{cccccccccccccccccccccccc}
1&1&1&&&&&&&&&&&&&&&&&&&&&\\
&&&1&1&1&&&&&&&&&&&&&&&&&&\\
&&&&&&1&1&1&1&&&&&&&&&&&&&&\\
&&&&&&&&&&1&1&1&1&1&1&&&&&&&&\\
&&&&&&&&&&&&&&&&1&1&1&&&&&\\
&&&&&&&&&&&&&&&&&&&1&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1
\end{array}\right]
$$

FIG.(6.9)

THE CONSTRAINTS COEFFICIENTS MATRIX OF THE SECOND STAGE OF THE MATHEMATICAL MODEL

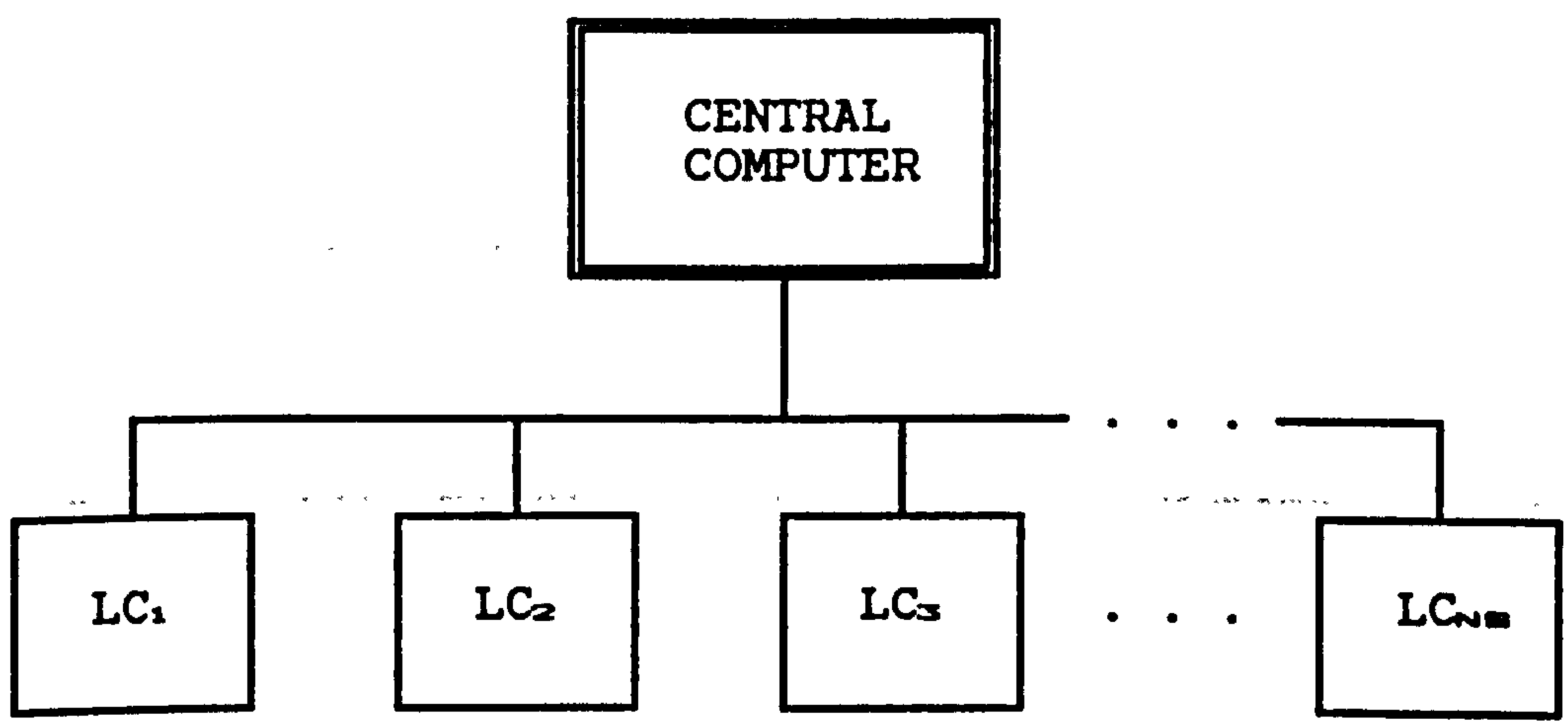

FIG.(6.10)

TWO LEVEL COMPUTING AND CONTROL

LC : LOCAL (STATION) COMPUTER

POWER SYSTEM

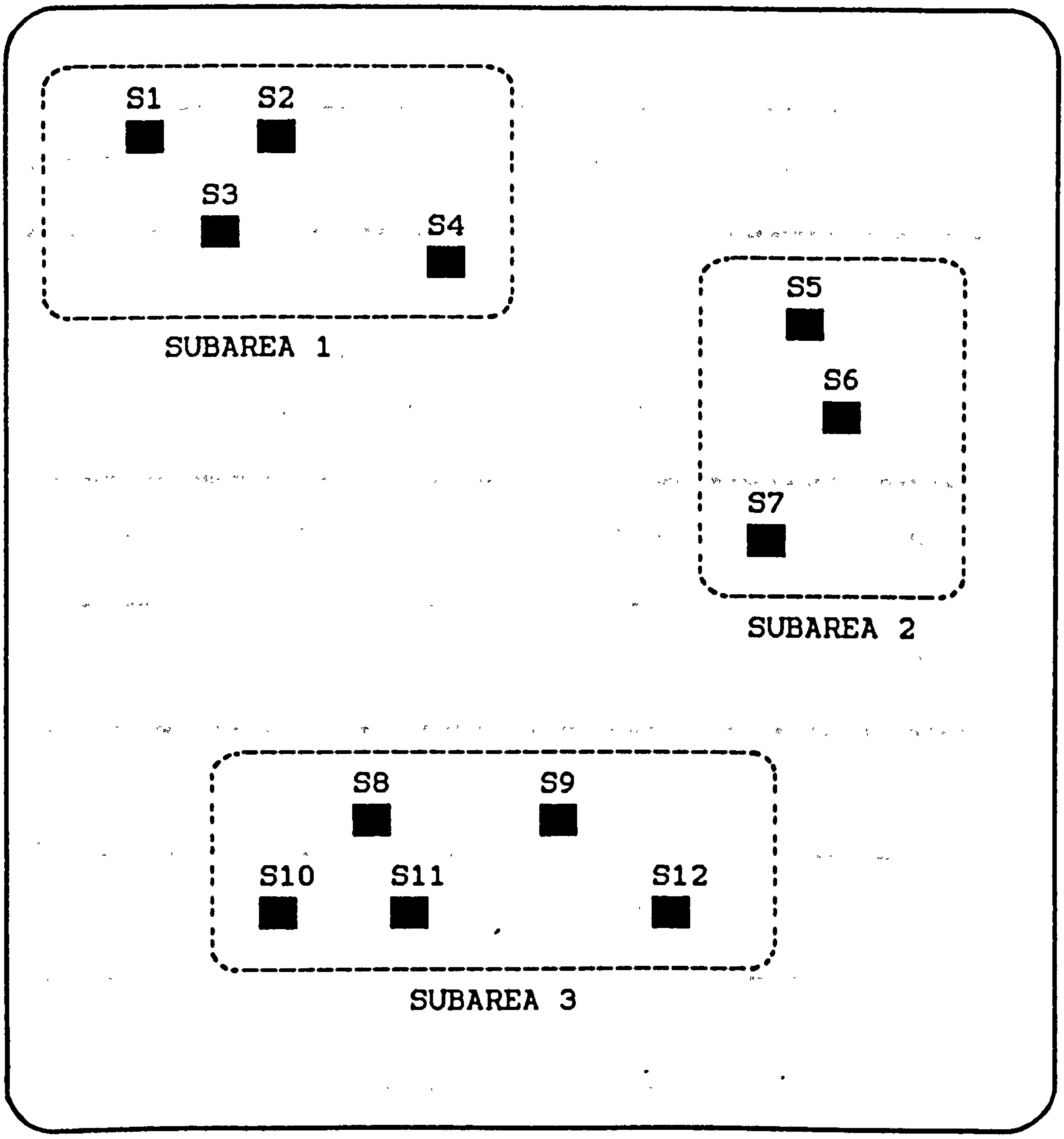

FIG.(6.11)

A SCHEMATIC REPRESENTATION OF A LARGE POWER SYSTEM DECOMPOSED INTO A NUMBER OF SUBAREAS, EACH CONSISTING OF A SMALL NUMBER OF POWER STATIONS

S : POWER STATION

TABLE (6.1)

COMPARISON OF COMPUTATION TIMES BASED ON TYPE OF PROCESSOR

ALL CPU TIME VALUES ARE IN SECONDS

PROBLEM & CASE		PSMV	VAXA	VAXB	VAXD	VAXE
1	A	33.150	7.780	7.610	19.770	5.650
	B	25.840	5.970	5.970	14.960	4.640
2	A	35.040	8.200	7.950	19.980	6.230
	B	25.820	5.890	5.780	14.730	4.450
3	A	34.940	7.480	7.350	18.960	5.860
	B	32.830	7.400	7.300	18.900	5.720

COMPUTER MODELS

VAXA	VAX 6330
VAXB	VAX 6330
VAXD	VAX 11/785
VAXE	VAX 8650
PSMV	Micro VAX II

TABLE (6.2)

COMPARISON OF COMPUTATION TIME BASED ON PROBLEM SIZE

ALL CPU TIME VALUES ARE IN SECONDS

LINEAR PROGRAMMING PROBLEM	NC	NV	NC x NV	CPU TIME	
				A	B
PROBLEM 1 STAGE - I	106	44	4664	11.500	8.500
PROBLEM 2 STAGE - I	106	44	4664	13.590	8.425
PROBLEM 3 STAGE - I	136	44	5984	15.470	15.470
PROBLEM 1 STAGE - II	7	24	168	0.9	0.9
PROBLEM 1 STAGE - II BUS 1	1	3	3	0.115	0.075
PROBLEM 1 STAGE - II BUS 2	1	3	3	0.075	0.075
PROBLEM 1 STAGE - II BUS 11	1	4	4	0.105	0.090
PROBLEM 1 STAGE - II BUS 14	1	6	6	0.090	0.095
PROBLEM 1 STAGE - II BUS 20	1	3	3	0.130	0.155
PROBLEM 1 STAGE - II BUS 23	1	5	5	0.160	0.185

CHAPTER 7

DISCUSSION AND CONCLUSIONS

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CHAPTER 7

DISCUSSION AND CONCLUSIONS

7.1 GENERAL

The present thesis has developed a generalized and versatile two-stage mathematical model to represent optimum operating conditions in electric power systems. A unified approach has been adopted to represent the various aspects of power system operation, the main aim being the development of a comprehensive mathematical model suitable for the application of linear programming methods, with emphasis on clarity, simplicity and flexibility throughout. The various concepts on which the model is based were all introduced and thoroughly explained, with the relevant mathematical expressions derived starting from basic power system theory. This was then supported by numerical results obtained by applying the suggested method to the solution of a number of optimization problems using an actual test system. The numerical results were compared and extensively analysed. This was followed by a thorough discussion of the problem of dimensionality and the introduction of decomposition methods. Application to the present project was then considered and discussed in detail.

Concise, but adequate, theoretical background has been presented to explain the various constituent parts of the model. This included supplementary material of a mathematical nature mostly related to linear programming which plays a major role in the development of the suggested method. The theoretical analysis was also supported by argument related to the practical side of power system operation. Useful "tips" gained from computer programming experience were also given in connection with the computational part of the work.

However, the presentation so far has been given in a factual manner, in the sense that the mathematical model in general has been presented without mentioning its advantages over existing methods, or pointing out how its various aspects are different from those of available publications on the subject and how the whole work fits among similar research in the field.

This will all be done in the course of the present chapter which gives an extensive discussion of the various general and particular aspects of the mathematical model, its advantages and practical application as well as suggestions for future work. Some questions that might arise in connection with the various aspects of the research project were not only anticipated but asked and answered. In answering some of these questions, the various, and sometimes conflicting, points of view and different sides of the argument are presented.

7.2 FEATURES AND ADVANTAGES OF THE MODEL

The present method of formulation and solution of the problem of optimum operating conditions in a power system has several advantages over existing methods. The various advantageous features of the suggested model are discussed below. Some of these advantages can also be considered as the contributions of the present work to this field of research.

Generality and Flexibility

The method developed here is very general in the sense that it can be applied to a power system of any size and configuration. This follows from the fact that the work is mainly based on linear mathematical programming which is now a well-established field that can be applied to a wide range of practical problems. One general mathematical model has been used to solve power system optimization problems that involve minimization of generation cost, transmission losses or reactive power generation. Even these have been presented here in the way of illustrative examples rather than an exclusive set, and the mathematical model can be applied to the solution of other power system optimization problems if required. This important feature of the model offers the possibility of comparison so that the best way of operating a power system can be chosen using a common criterion. This is in contrast with many available models which are designed to solve particular optimization problems with a limited set of constraints. Most of the published work is usually concerned with one or very few aspects of the power system optimization problem. For instance, whole works have been devoted to the problem of minimizing generation cost. Others are only concerned with the minimization of transmission losses. The associated mathematical models and computer programs are limited, inflexible, and do not offer much choice which limits their applicability. Also, comparison of results obtained from different optimization problems solved by different existing methods is very difficult because of the lack of a common basis for the comparison. This is <u>not</u> the case with the present work. The same general method can be used to optimize different objective functions with as many different constraints as required. Three main optimization problems have already been formulated and solved in Chapter 5 of the thesis with various number and types of constraints. These problems have been chosen because they are widely and frequently used in power systems research work. Detailed as they are, the numerical results produced are but examples of the possibilities this project can present. A number of additional aspects were also considered in Chapter 4 and their inclusion in the mathematical model was explained.

Apart from electric power systems, the various ideas and concepts presented in the thesis can be applied to other systems which have similar structure to that of electric power networks or can be represented by similar mathematical expressions. This will be discussed further in Section (7.5).

Simplicity and Clarity

Simplicity of the structure of the solution algorithm and clarity of presentation are two important factors in mathematical modelling of practical engineering problems which seem to have been overlooked by many researchers. It is the author's opinion and belief that when two methods of solution to a given problem are presented, and when all other factors kept unaltered, then it is preferable to choose the one with the simpler and clearer presentation. No real advantage is gained by choosing unnecessary complication.

Two of the powerful attributes of the present model are its simplicity and clarity. It solves a complicated power system optimization problem involving a large number of variables and different sets of constraints without the use of B-Coefficients or penalty factors. Also, apart from applying the concept of incremental modelling, it does not involve sensitivity factors or derivatives.

When representing any complex system problem, there is no simpler way than expressing the various relationships amongst the system variables by linear algebraic equations. This has been accomplished in the present project by the systematic and comprehensive application of the linearization procedure discussed in Section (2.5). As a result of this formulation it has been possible to use linear programming to represent and solve the various optimization problems. Linear programming, itself, is probably the simplest and most straightforward technique for presenting and solving optimization problems. All these factors add to the simplicity and clarity of the model.

Another simplifying aspect of the suggested mathematical model is the classification of the system buses. In contrast with many published methods, especially those based on load flow solutions, the present approach divides the system buses into two main categories, namely, Generating and Nongenerating Buses with clear distinction between the two. This reduces, or indeed eliminates, any confusion that might be experienced in this respect.

Also, although the linearization of the model is based on the concept of incremental changes in the system variables, this is used only as an analytical tool. The final model is formulated in terms of the actual values of the variables rather than their incremental values, which further enhances the clarity of the model.

Completeness and Accuracy

Apart from the conceptual use of the incremental modelling, no other approximations are used in developing the mathematical model. A full representation of the power system, including transmission line resistance, is used throughout the model. This has the advantage of producing more accurate results in general, as well as taking system losses into account. Also, once the values of the independent variables are obtained by solving the linear programming problems of the two stages of the model, the rest of the system quantities and all output results are calculated by using the exact nonlinear formulas.

The Two-stage Formulation

The two stages of the mathematical model use two different sets of independent variables. In the first stage, the bus voltage magnitudes and phase angles are used as the independent variables of the optimization problem while the second stage is based on the active power outputs of the system generators. This important feature of the mathematical model has the following advantages:

1. Since the two stages of the model are solved almost independently, this presents a simple way of handling the two different sets of variables which can, otherwise, be cumbersome. This, in turn, simplifies the overall solution algorithm.

2. It eliminates the need for iteration between an optimization program and a load flow routine. This, in itself, presents several advantages. It means that the optimization program does not need to be written specifically to solve the power system problem, thus allowing the use of standard linear programming computer routines and, therefore, saving programming time and effort. Also, by avoiding the iteration between an optimization program and a load flow routine, a large reduction is effected in the required computation time. This will, otherwise, be very high due to the iterative nature of both routines, especially the load flow which involves the inversion of the system Jacobian matrix in every iteration.

3. This two-stage formulation allows the use of two different objective functions at the two levels of the solution of the overall problem. Thus, two different power system quantities can be optimized at the same time. Also, when using the same objective function in both stages, as illustrated by Problem 1 of Chapter 5, minimum generation cost was obtained on both overall system and power station levels.

Incremental Modelling and Linear Programming

It is to be admitted that some of the concepts presented in this thesis are not totally new, which might, at first, be considered as a disadvantage. However, this is not the case as discussed below. Two such concepts are linear programming and incremental modelling. The author did not invent these concepts or introduce their application for the first time. There is a certain number of well-established mathematical techniques that are widely used by a large number of contributors to the field. Yet, the various published models and suggested methods are different from one another. This comes from the way of employing these techniques and incorporating them into the various models and solution methods, and depends on the rest of the formulation of the problem in question.

The two examples mentioned above, i.e., linear programming formulation and incremental modelling, and few other existing techniques have been used in the present work to their fullest advantage and combined in such a way that their full potential is utilized. For example, linearization by incremental modelling has been applied in a systematic, consistent and very clear and simplified manner. The technique was, first, clearly explained in a number of well-defined steps. Then, it was applied to a large number of equations that represent the core of mathematical relationships amongst power system quantities relevant to the entire problem of power system steady-state operation. In many available published methods, the advantages of this useful technique have been overshadowed by the complexity of the rest of the model and ambiguity of the various other aspects of the method.

A similar argument applies to the application of linear programming techniques. Because of the nature of the mathematical formulations used so far, the advantage of the application of linear programming to power system optimization problems has been only partial. By virtue of the combination of the various advantageous features of the present mathematical model, it has been possible to apply linear programming techniques to the whole problem without recourse to other iterative procedures such that the total computational burden of the problem is alleviated. Apart from the simplicity of its formulation and clarity of its presentation, the attractiveness of linear programming comes from the fact that it lends itself easily to be translated into a computer program.

Initial Operating Conditions

Similar to the present work, many existing methods apply the concept of incremental modelling. This involves finding the solution of a problem based on some initial conditions. In this respect, the theoretical development of these mathematical models is generally acceptable. However, obtaining the required initial operating conditions might prove difficult. In contrast, the present method includes a complete procedure for obtaining the initial operating conditions from the minimum amount of information. This makes using the present method easier and more practical.

The Use Of Available Computer Routines

One of the important facts that have been kept in mind while formulating the problem in the present work is the existence, nowadays, of standard linear programming routines of a general nature that can be applied to a large number and a wide variety of practical problems. Parallel to the rapid and continuous progress in computer technology, there is now a host of "ready-made" or "canned" computer routines and software packages that perform a wide range of mathematical tasks. In recent years, solving practical problems has been reduced to the art of modelling and presenting these problems such that they can fit in these moulds. Engineers are well advised to make use of these instead of wasting valuable time in writing and testing computer programs the equivalent of which already exist and can be readily obtained and applied. Standard computer routines, based on well-established mathematical algorithms, are usually developed over long periods of time that can extend to several years. They are written by experts in computer programming and operation and extensively tested which gives them a very high standard of reliability. Those who are involved in research work that requires computer programming know only too well the amount of time and effort spent in preparing, testing and debugging a new computer program; even a very short one.

As mentioned in Section (5.10), some library routines were used in the computer programs associated with the work. However, they were included in the programs at certain clearly-defined stages such that they can be replaced by other routines without affecting the formulation of the problem and with the minimum amount of alteration to the existing computer program. This offers the option of writing a special optimization program to solve the problem if required by the user, when necessary and justifiable as discussed above, for example because of the unavailability of the suitable optimization routine. Another instance which calls for the replacement of such routines occurs sometimes when a certain library routine is withdrawn by the copyright owners and replaced by a new improved one. This happens as part of the continuous development of software libraries and computer facilities.

Application of Decomposition

In the present work decomposition is based on the constraints coefficients matrix of the system rather than on the physical system itself or its network representation. This makes possible and facilitates the application of the Dantzig-Wolfe decomposition principle, whose applicability to the power system problem is, otherwise, not very obvious. Working on the system matrix is much easier than working on the system network for the following reasons:

1. It eliminates the need to account for and deal with the problem of boundary conditions at the various division points such as bus injections and line flows. Otherwise, when dividing the system network into smaller subsystems or subareas, each of these must be treated as an internal system with the neighbouring subdivisions of the system considered as external systems that have to be accounted for in terms of their equivalent circuits and effect on the internal system. This entails a considerable additional amount of analytical and computer programming work. This aspect has previously been discussed in Section (4.3) in the context of tie-lines and system interconnection.

2. It also has the added advantage that the decomposition can be achieved by a computer algorithm. Otherwise, the system network topology would have to be split into subsystems by inspection.

3. The analysis and computation are simplified by constructing an admittance matrix for the whole system instead of several matrices corresponding to the individual areas. This in turn simplifies the rest of the analysis, including the initial load flow, and eliminates the need to select several "local" reference buses.

7.3 PARTICULAR POINTS OF DISCUSSION

This section elaborates on and discusses a number of miscellaneous specific points concerning the problem of power system operation and its presentation in this thesis.

Hard and Soft Constraints

In practical optimization problems, of which the operation of power systems considered here is an example, there are two main types of constraint in terms of their "rigidity" or "bindingness". These are "hard" and "soft" constraints. Hard constraints are usually based on equalities and must be strictly satisfied. An example of these is the total energy balance equation in the system. The total power generation in the system must be equal to the sum of the total system demand and the total transmission losses, as shown in Equations (2.62) and (2.63). Soft constraints, on the other hand, are not very binding and can be violated to a certain extent. These are usually constraints of the inequality type such as operating limits. Upper and lower limits on bus voltage magnitudes can be used as examples. Specifying operating tolerance margins on this basis might actually facilitate the solution and enhance the feasibility of some optimization problems, especially those with high number of constraints. Problems which are infeasible under the specified constraints can be rendered feasible when some of these constraints are slightly relaxed. This is permissible if the constraints involved are of the soft type described.

The numerical results have shown that the suggested method can deal with strictly binding equality constraints as well as inequality constraints which are less binding. In particular, the generating sets at one of the power stations in the test system have their operating limits given by $PGGMN_j = PGGMX_j$ as shown by the generator data of Table (5.2-A). The station involved is that of bus (14), and the reason for these operating restrictions is that this is a nuclear station where the operating requirements specify that the station should be operated at full output.

Number of Constraints

The number of constraints (or sets of constraints) of an optimization problem can be increased or decreased as required to take into account various factors that affect the problem of power system operation. Increasing the number of constraints has already been illustrated by using 136 constraints in the third optimization problem, as compared to the two other problems which have 106 constraints each. Decreasing the number of constraints can be achieved in two different ways. In one method, they are not taken into account in the formulation of the problem at all. Another approach is based on including them in the general formulation of the problem but relaxing them when running the computer program and obtaining the actual numerical solution. Relaxation of the constraints for this purpose is accomplished by assigning unrealistically high and low values to their upper and lower bounds respectively. This gives more flexibility than the first method and can be used for both increasing and decreasing the number of constraints. For instance, the problem can be formulated with a very large number of constraints, taking into account as many aspects of the power system operation problem as possible. Proper values are then assigned only to the lower and upper bounds of the constrains which are required to be included in a particular run of the computer program. For example, the problem of minimization of instantaneous input fuel cost is mainly based on active power generation. The constraints on the reactive power generation can be included in the formulation of the problem, to keep its generality, but they can be "numerically" relaxed when running the computer program.

To illustrate the above discussion, consider a generator which, by design, has the following operating limits on its reactive power output in units of MVAR:

$$75.0 \geq QGG_j \geq -15.0$$

The constraints on this generator can be relaxed by assigning them the following limits instead:

$$1000.0 \geq QGG_j \geq -1000.0$$

Under the new conditions, the generator is practically unconstrained in terms of reactive power production. Ideally or theoretically, the relaxed constraints should be given bounds of $-\infty$ and $+\infty$. On the computer, the lowest and highest negative and positive numbers that can be handled by the computer should be used. Care should be taken, however, as this might lead to numerical overflow if these values are used in further calculations.

A situation of this nature has been encountered in the debugging stage of the computer programs while specifying the bounds on the phase angles. These, as mentioned before, are used as free, i.e., unconstrained variables that can assume any positive or negative values. To reflect this fact in the computer program, extremely high absolute values of the bounds were used. It was found that the highest value that can be used without causing overflow was around 1.0E+18, although the value of 1.0E+20 is recommended by the instructions in the NAG Library routines documentation. Obviously the final actual values are those that optimize the objective function and satisfy the problem constraints and are determined by the solution of the linear programme.

Finally, the above discussion should not be confused with Constraints Relaxation as a method for solving optimization problems with relatively large number of constraints as published by Irving and Sterling [38].

Constraints on Transmission Line Quantities

The imposing of constraints on both the line losses and line power transfer is not redundant as might be thought at first. Although mathematically related to some extent, these two quantities represent two different physical properties of power system transmission lines. Constraints based on active power losses in the lines are indications of the "thermal" or current-carrying capacities of the lines as measured by their I^2R loss. Thus, they are included in the optimization problem to take account of the heating effect of the current passing through the distributed resistance of the transmission line conductors. Power transfer constraints, on the other hand, are related to the amount of maximum power that can be transferred from one end of the line to the other, which is the principle function of the line in the system in the first place. More or less, the specification of these is independent of the accompanied power loss. This power transfer is a function of the voltage magnitudes at both ends of the line and of the phase angle difference between them. This phase angle difference, in turn, is a measure of the stability of the transmission line and the power transfer process.

Effectively, the bounds on line losses and power transfer quantities might form overlapping sets of constraints, which means that some of them should be ignored as discussed in Section (4.5). However, these two quantities are represented by interrelated and complicated multivariable functions, as given by Equations (2.35) to (2.50). It is, therefore, impossible to compare the two quantities directly to decide as to which one is to be included in the optimization problem by sheer mathematical manipulation or theoretical algebraic analysis. Even when such analytical solution is possible, no generalization can be applied and the selection of the final set of constraints has to be made on a line to line bases. The overlapping of the two sets of the constraints for the individual lines can be such that the line loss constraints must be used for one line while the power transfer for another. Obviously, this will also differ from system to system.

It might be possible to obtain the solution to this problem by applying some elaborate numerical technique, as in the solution of the power flow equations. But this is lengthy, time-consuming and iterative in nature. Besides, it is one of the disadvantages the present thesis has set out to avoid in the first place. Also, even when successful, it will not solve the optimization problem or even part of it. What it achieves is merely the selection of the line constraints to be included in the <u>formulation</u> of the optimization problem concerned. This is a very small gain considering the price paid in terms of programming effort and computation time. Selection of line constraints on these basis is only worthwhile if results have already been obtained and supplied with the line data.

Therefore, the best way, in the absence of such information, is to include both constraints for each line in the system and let the optimizing routine take care of the selection process. The handling of constraints, in any case, is one of the inherent features of optimization algorithms.

A similar situation can arise in connection with the specification of upper limits on the apparent power transfer across transmission lines in both directions as in Problem 3.

<u>Lower Limits on Generator Outputs</u>

The reason for imposing upper limits on generator outputs, or line quantities for that matter, is obvious. By design, each power system component has an upper limit on the amount of power it can handle. Power being generally given by the product of current and voltage, the upper limit on power quantities is a function of the electrical endurance of the component in question in terms of its insulation properties and heat dissipation, and, consequently, cooling method. These factors, incidentally, affect, and are affected by, the actual physical size of the component.

Less obvious, however, is the reason for specifying a lower limit on the generator output. For example, one would think that a generator with a maximum capacity of 60 MW would be able to deliver any load from nought to 60 MW, and the question arises as to why a lower limit of, say 20 MW is imposed on the generator output. The issue in this case is not that of output capacity. The reasons behind the specification of lower limits on generation are related to the procedures and costs associated with the start up and shut down of generating sets. Apart from spinning reserve considerations, generating units should be operated only when it is "worthwhile" to do so from an economical point of view.

7.4 OPTIMIZATION OF COMBINED OBJECTIVE FUNCTIONS

While experimenting with the computer programs, the possibility of using a combined objective function of the following form was investigated:

$$Z = Z1 + Z2 \tag{7.1}$$

where Z1 and Z2 are two different objective functions.

In particular, an attempt was made to minimize both generation cost and transmission losses in the first stage of the model at the same time. However, the numerical results obtained had to be discarded for two reasons. Firstly, generation cost and transmission losses are two different quantities which are measured by different units. The quantity formed by direct summation of these two quantities does not have a meaningful physical interpretation. Secondly, even if the two quantities are measured by the same units, from a mathematical point of view the minimum of Z as defined by Equation (7.1) above does not necessarily correspond to the minimum of the individual functions Z1 and Z2. The same argument applies if the objective functions Z is to be maximized. The resultant function Z is a different quantity that represents neither of its two constituent parts. This argument can be easily proved by considering the case where Z1 = sin θ and Z2 = cos θ in the range 0 to 180 degrees. The maximum of Z1 is equal to 1 and occurs at θ = 90° and the maximum of Z2 is also equal to 1 but occurs at θ = 0. The combined function Z has a maximum value of 1.414 at θ = 45°. The maximum of Z can be obtained by using the usual calculus methods based on first and second derivatives.

It might be asked here, "If this concept of combined objective functions is not valid and the corresponding results are wrong, then why mention them in the first place ?" The answer to this rightly asked question is that the discussion above is included here as a cautionary tale, for the following reasons:

1. The very idea of optimizing two objective functions at the same time is attractive and it is tempting for the user to try it on the computer program.

2. The computer program works without any warning or indication of errors, giving the false impression that the combination of two objective functions in the manner described above is valid. Computer programs cannot check the validity of the theoretical and analytical concepts upon which the numerical results are based.

3. It is not possible to detect any errors in the numerical results obtained even by the user of the program because they do not look very different from those obtained in the three optimization problems of Chapter 5 which are based on valid theoretical and practical concepts. Also the results obtained seem quite acceptable and satisfactory especially since all the problem constraints are satisfied.

7.5 PRACTICAL APPLICATIONS

A load flow or power flow is probably the most frequently performed routine among power system calculations [57]. Another routine which is run on a repetitive basis is that of economic dispatch [1]. This is the routine that determines the generation schedule of the system that minimizes the operating cost while satisfying the imposed operating constraints. Generally, depending on the daily load curve, this has to be run at intervals of 15 to 30 minutes, which means that it may be performed a minimum of 48 times in a 24-hour period. A load flow routine is much inferior to the present method as its main purpose is to determine the bus voltages from a given generation schedule. Basically, it solves a set of nonlinear equations using iterative numerical techniques. Therefore, a load flow routine cannot be performed if the exact generation schedule on the bus level is not available. Also, it does not deal with the individual generator outputs and consequently any quantities dependent on them such as generation cost. Economic dispatch, on the other hand, deals mainly with the power output of system generators, with the bus voltages to be obtained from a load flow. This means that, to get the overall steady state condition of the power system, both routines must be performed iteratively. Even then, the only quantity that can be optimized is the generation cost.

An operational schedule based on the method presented here can replace the two routines mentioned above in addition to several other advantages and useful potential possibilities. The computer programs associated with the various optimization problems give all generator, bus and line quantities as well as overall system results at the optimum operating conditions. These also include Load-flow results. Thus, the practical value and the field of application of the suggested mathematical model is obvious. Apart from fulfilling the objectives of both routines, the present method is straightforward, the associated computer program is shorter and faster and the method offers more choice. From the CPU time results presented in Section (6.8), it can be seen that the suggested optimization method is suitable for on-line applications. Considering the CPU time requirements indicated by the three optimization problems addressed, and comparing these with the actual interval quoted above, there is ample time for running the three optimization programs of Chapter 5 before taking any operational decision or control action in response to the expected change in loading conditions. It is possible to devise an algorithm and a criterion such that the three programs are run, the results compared and a decision is made as to whether to operate the system under minimum generation cost, minimum transmission losses or minimum reactive power production. Thus, depending on the particular system in question and the loading conditions during the time-period of interest, the optimum operating conditions of the system can be achieved with different objective functions at different times.

A "feeling" of the sort of results that can be obtained from applying the method on an actual system has already been given in Chapter 5. The test system used does not possess any particular features and, therefore, gives a fair and typical representation of a general power system. Also, since this system includes six power stations, one of which is nuclear, it covers a considerably large geographical area which gives the presented work applicability on a practical level.

Application to Nonelectrical Systems

Many problems from other engineering disciplines can be solved by transforming the problem into an electrical one using analogies. In mechanics, for example, problems can be solved using electrical analogues where forces are replaced by voltages, velocity by current, friction by resistance and so on. More complicated problems of mechanical engineering such as vibration in springs and undesirable vibrations in mechanical equipment can also be solved in this manner [58]. This practice can be extended to a larger and more general scale such that the operation of nonelectrical systems can be represented and solved by simulation to electric power systems.

In 1971, Lo and Brameller [59] showed that the operation of a gas distribution system can be optimized using electrical network methods. Gas systems and other similar systems that can benefit from power system analysis, in general, can also benefit from the optimization method presented here. There is a number of other systems and industries whose structures bear some resemblance to that of electric power systems. As the suggested method offers many advantages to the solution of optimum operation of power systems, which is a problem of recognized complexity, then it is quite feasible that other systems can benefit in the same manner after their operation problem has been transformed into an equivalent electrical one and represented by an electric network.

7.6 CONTINUATION OF THE PROJECT AND SUGGESTIONS FOR FUTURE WORK

It goes without saying that there is always room for improvement. In a research project of this nature, this expression is particularly true. Research projects that cannot or need not be improved on or developed fall mainly into one of two categories. They are either projects that have achieved the stage of perfection, or they have reached a dead end. While the present project is far from being perfect, it does not belong to the second category. Of course, good satisfactory results are a valuable incentive for the continuation of any research project. However, incompletenesses and shortcomings can be an important motivation for improvement of the work which may, otherwise, come to a halt. In this respect, the present project possesses a great potential for development and continuation. Two main possibilities are discussed below.

Multiple-Objective Programming

The majority of optimization methods used in power systems and many other fields are based on single-objective programming. This means that the mathematical formulation of the problem specifies only one objective function to be minimized or maximized under a number of constraints. Representation of optimization problems of this type has been given in various places of this thesis and can be summarized by the following statement:

> Optimize the Objective Function Z,
>
> Subject to the Constraints C1, C2, C3 . . . CM.

Even in published methods where two different quantities were optimized, the method basically involved two separate single-objective mathematical programming problems. In the present work, it has been possible to optimize two different power system quantities using the two-stage formulation of the mathematical model. Again, this basically meant the solution of two different single-objective linear programming problems at the two different levels of the mathematical model.

The definition of Multiple-objective Programming can be summarized as follows:

> Optimize the Objective Functions:
>
> Z1, Z2, Z3, . . . ZL,
>
> Subject to the Constraints
>
> C1, C2, C3, . . . CM.

In this sense, multiple-objective programming is almost a neglected topic. Most of those who work in the field of optimization, with a reasonably adequate knowledge of its various techniques including decomposition, are not aware that the topic of multiple objective programming even exists. One of the reasons for this is the fact that in most literature on optimization it is implied that linear programming or mathematical programming, in general, means single-objective programming. In publications of this category, no hint is given about the topic of multiple-objective programming and it is not even mentioned.

An interesting future research project will, therefore, be the application of Multiple-objective mathematical programming techniques to the solution of power system optimization problems. The versatile linearized optimization model of the power system developed in the present thesis can be used as the basis. For a start, two different objective functions can be optimized simultaneously. For example both the generation cost and transmission losses can be minimized. After developing and testing the technique, it can then be extended to three and more objective functions. The final aim will be developing the technique such that any number of objective functions can be optimized under any number of constraints, which is the essence of multiple objective programming [25].

Hydro and Hydrothermal Power Systems

Basically, the suggested mathematical model can deal with any type of generation as long as the corresponding operational aspects are represented by linear or linearized mathematical expressions. Nevertheless, at the moment the work is mainly inclined towards the operation of a thermal power system. Within this inclination, however, the method can deal with the various types of thermal generation such as gas, coal and nuclear. Apart from their output capacities and dynamic responses, the main differences between these are their cost functions. However, since a linear approximation has been used to represent the various cost functions, the general formulation of the problem will not be affected by these differences. Another difference, particular to nuclear generation, is the inflexibility of the power output of the generating units. For economical and operational reasons, these have to be operated at full capacity as discussed previously but, as already mentioned, this can be easily accommodated by the suggested formulation and the associated computer routines.

Of particular interest are Hydro generation and mixed hydrothermal systems. Although the same formulation strategy presented in the thesis can be applied, hydro systems have their own set of operating conditions and constraints. These are considerably different from many of the aspects of power system operation pertaining to thermal systems discussed so far. Considerations to be taken into account when dealing with hydro systems include such factors as reservoir volume, water levels and effect on irrigation.

A possible extension of the project can, therefore, be the comprehensive formulation of the problem of optimum operating conditions of hydro and hydrothermal power systems using the modelling framework developed in the present thesis. The initial stage of the project would involve an extensive study of these systems, the various relevant factors affecting their operation and the necessary data.

7.7 POWER SYSTEM OPERATION : THE SOLUTION

In Chapter 3 of this thesis it has been mentioned that optimization is the mathematical equivalent to finding the best solution of a given problem. This implies that it is possible to have many solutions to a problem, several of which are good with some better than others, but there is only one "best" solution. So far the argument sounds consistent. However, in Chapter 5, it has been found that for the same problem, namely, power system operation, three different solutions were found each of which has been described as the "optimum"! The obvious question that follows is which one of these is the real optimum?

To answer this question it should be realized first that power system operation is a complicated problem that involves a large number of variables and even a larger number of constraints. The formulation and solution of the problem is affected by various conflicting factors some of which depend on the policy of the power company. A mathematical model, however comprehensive and sophisticated, represents a limited set of these factors. Apart from that, as the situation stands at present, optimization of power system operation inevitably implies selecting one quantity as the objective function to be maximized or minimized. Therefore, although the optimum solution is the best, at least from a mathematical point of view, the selection of the quantity to be optimized is rather a "subjective" matter that depends to some extent on the various policies of different power companies. For example, one company may decide, obviously after a lot of careful deliberation, that the best way to operate their system is under minimum running cost. For this power company, the numerical solution obtained by the corresponding optimization problem and the associated computer program is the optimum. On the other hand, the policy of another company is to reduce the transmission losses to a minimum. In this case, a different optimum applies. This further accentuates the need for multiple-objective programming as discussed in the previous section.

In conclusion, no-one can claim that he has developed or can develop the ideal, flawless and all-inclusive mathematical model to represent the operation of a power system. To make such a serious claim one has to be either a boasting novice or a genius of unlimited resources. The author is neither. The fact still stands, as stated in the opening sentence of the very first chapter of this thesis, that the operation of a large modern power system is a very complicated and multifaceted problem. However, what has been presented in this volume is but a modest attempt, hopefully in the right direction. Achieving the final goal requires a lot of hard work and continuous research on the subject. As far as the author is concerned, this is not the end but only

<u>THE BEGINNING</u>

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BIBLIOGRAPHICAL NOTE

One of the well-known facts about scientific and engineering research is that a considerable part of the time and effort allocated to any research project is spent on collecting, reading and comparing a large number of publications on the research topic in question. This also involves selecting and arranging these references in different ways for various purposes. This process continues and takes different directions throughout the research project, as does the research project itself for that matter. This has two main purposes. One is to give the researcher a general overview and an insight into the problem he intends to tackle. The other is to make him aware of previous and contemporary work in the same or related areas of research so that repetition of ideas and duplication of work are avoided, thus avoiding wastage of time and effort which can, otherwise, be exploited in developing new ideas and tackling fresh problems.

As one is expected to report on the results of his experimental or computational work in the form of a thesis or a technical paper, it is equally justifiable to report on one's findings in terms of literature available on the subject. In fact some publications, such as review papers and bibliographies, are devoted to this particular aspect of technical research.

Usually some form of reporting of this kind is included in theses as a "Literature Survey" either under that particular title or included one way or another in the introductory chapter of the thesis. Different authors have different ways of addressing the subject. Some use one section for this purpose and others devote a complete chapter for it. In the present thesis this part of the work was fulfilled by two separate sections. In Section (1.2), a concise and to-the-point literature survey was given, with discerning selection of references avoiding a lengthy listing of methods and publications as mentioned at the end of Chapter 1. This was concerned with the general problem of power system operation and its various facets. A similar survey on the particular aspect of dimensionality and decomposition methods was given in Section (6.2).

Many research projects combine two or more related or different disciplines of knowledge. For example research projects on power system stability require a considerable knowledge of control theory, and modelling of steam turbines depends heavily on thermodynamics. The present project involves two main disciplines. It is based on the application of Optimization methods to the problem of Power System operation. This interdisciplinary feature is reflected in the list of references. Most of the references from the power system side of the work are research papers while publications used here that belong to optimization and related mathematical topics are in the form of books.

The following two main points are found worth mentioning here in the context of the bibliographical side of the project.

1. When starting a research project it is very helpful to know the keywords or indexing terms under which one can find publications related to the research topic in question. In the present work the keywords or phrases given in the following short list represent a good starting point.

Algorithms
Computer Applications
Decomposition
Economic Operation
Electrical-Engineering Applications of Computers
Generators
Linear Programming
Load and Voltage Regulation
Load Dispatching
Mathematical Programming
Mathematical Techniques
Optimisation
Power System and Plant
Power Systems
Power-Systems Control
Power Transmission and Distribution
Steam Power Plants
Transmission and Distribution
Unit Commitment

Most of the entries in the above list are the recognized indexing terms of various papers published in the Proceedings of the IEE and cited in the thesis.

It is advisable, where appropriate, to look under the various spellings of certain words. In this particular work, one should be aware of the z/s spelling of such words as optimization and minimization. The relevant publications are usually classified under one or the other of the alternative spellings.

2. The usual way of finding publications in any field is to start with very few references and then try to use the list of references given by these publications to find more references, and continuing the process in a snowball fashion. In this respect, there are certain publications which are more useful than others. Some authors provide exceptionally long lists of references at the end of their written work which can be used as a very good source of references in the field. This is especially true in the case of surveys, review papers and papers of general nature. The majority of such papers are usually published by well-known and experienced contributors. This makes these publications very useful for a newcomer to a field of research who needs to have a general idea and a report of a general descriptive nature about the research topic and related practical problems of interest, rather than the specific details of a highly specialized technique used to solve a very particular

problem, or a very narrow area of the field.

In the case of the present project the following publications are very useful in this respect. These are discussed briefly below. The numbers in brackets refer to their place in the reference list of this thesis.

Happ [1], listed 112 references from 1922 to the time of publication of the paper in 1975.

Stott et al. [4] included a list of 51 references.

Sasson [3], listed a total of 101 references arranged in categories such as Optimal Load Flow and Capacitor Optimization.

Bazaraa and Jarvis [27]: This book contains a Bibliography of 490 references arranged alphabetically.

Garvin [26] contains a list of 59 reference arranged alphabetically.

Gass [24] appended a Bibliography of Linear-Programming Applications. The bibliography is divided into 12 Categories such as Industrial Applications and Economic Analysis. Some of the categories are further divided into subcategories. The entries of each category or subcategory are arranged alphabetically. This is followed by a separate alphabetical list of 105 References.

Minoux [23] included long lists of references after each of its ten chapters. In this case, one should be aware of repetition of references, a fact which also applies to the whole process of search for publications.

Zionts [34] has an alphabetical list of 316 References.

APPENDIX

MATHEMATICAL BACKGROUND

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APPENDIX

MATHEMATICAL BACKGROUND

A.1 INTRODUCTION

The optimization model developed in the present thesis is based on mathematical programming techniques. The various power system problems addressed have been formulated and solved as linear programmes. There is a large number of good textbooks that cover the general field of optimization and the various related topics such as linear programming. However, in an attempt to write a self-contained thesis, supplementary material of a mathematical nature has been selected from these fields and presented in this appendix. Only the necessary minimum of this background material is presented here. The various concepts discussed are mainly those related to the formulation and solution of the general linear programming problem with some emphasis on the simplex method. A number of related concepts are defined and some theorems are presented. The discussion is presented in a general, simplified and descriptive manner avoiding rigorous mathematical analysis. The two theorems in relation with the simplex method are stated without proof. Readers who wish to know more details about the various concepts presented are advised to consult the references cited in the thesis [23-34].

Finally, while the material in Section (A.3) is essential for the comprehension of the related concepts presented in the main body of the thesis, the historical introduction of Section (A.2) is mainly included for the general interest of the reader.

A.2 HISTORICAL BACKGROUND

From ancient times, the human race has always endeavoured to achieve the best in its varied activities. However, optimization in its present sense and related mathematical disciplines are relatively recent terms. In particular, the word "optimum" was first coined in 1710, by the German mathematician and philosopher Gottfried Wilhelm Leibniz (1646-1716).

One of the main branches of the more general field of optimization is that of mathematical programming. The use of the term mathematical programming is rather unfortunate nowadays. Newcomers to the field tend to confuse it with the modern term of computer programming. Mathematical programming and computer programming are two totally different concepts. In the early days of mathematical programming, researchers in the field were literally embarking on completely new areas of knowledge and had to invent new terminology to describe the various concepts introduced. In those days computers were not as widely used as they are now. What we now know as computer programming, which means a set of instructions to be executed by the computer, used to be known then as Computer Coding. In this sense, this is a more suitable term. Mathematical programming, on the other hand, is a mathematical technique that solves a particular category of specifically defined problems. The reason for this confusion is that nowadays scholars are introduced to computers for a wide range of applications well before acquiring any knowledge about the rather specialized field of mathematical programming.

The various fields of mathematical programming were developed almost in parallel in a relatively short period of time extending from the late 1940's to the early 1960's. The most significant contribution in the field of linear programming was by George B. Dantzig in 1947, while the foundation of nonlinear programming was laid by Kuhn and Tucker in 1951. Significant contributions to the fields of Dynamic programming and Integer programming were due to Richard E. Bellman in 1957 and Ralph Gomory in 1958.

Because of his numerous and outstanding contributions to the field, it can be safely said that George B. Dantzig is the founder of Linear programming as it is known today. It is almost impossible to find any publication on the subject that does not mention Dantzig or his famous Simplex Method. Detailed account of the method and its variations is found in many references. The development of the simplex method by Dantzig initiated an avalanche of research in the field of linear programming and mathematical programming in general.

The simplex method was developed by Dantzig in 1947. At the time he was working for the U.S. Air Force among a team called SCOOP (Scientific Computation of Optimum Programs). Dantzig's work unified the representation and solution of a large number of linear programming problems. Around the period when Dantzig published his work, linear programming and mathematical programming problems were tackled and formulated individually and given special names. Such names included the Transportation problem, Transshipment problem, Travelling Salesman problem, Diet or Dietitian problem, Warehouse problem, Caterer problem, Assignment problem, Knapsack problem and even the Marriage problem! The various problems were formulated by different contributors to the field. The transportation problem was formulated by F.L. Hitchcock in 1941 and also independently in 1947 by Tjalling C. Koopmans. In 1945, G. J. Stigler formulated the Diet problem. A. S. Cahn, Jr. formulated the warehouse problem in 1948 and W.W Jacobs formulated the Caterer problem in 1954. The first Solution of a linear programming problem on a High Speed Electronic Computer was obtained in January 1952.

A.3 THE SIMPLEX METHOD AND RELATED TOPICS

The simplex method has been devised to solve a linear programming problem of the following form:

Minimize

$$Z = C_1 X_1 + C_2 X_2 + C_3 X_3 + \dots + C_n X_n \qquad (A.1)$$

Subject to :

$$\begin{aligned}
A_{11} X_1 + A_{12} X_2 + A_{13} X_3 + \dots + A_{1n} X_n &= B_1 \\
A_{21} X_1 + A_{22} X_2 + A_{23} X_3 + \dots + A_{2n} X_n &= B_2 \\
A_{31} X_1 + A_{32} X_2 + A_{33} X_3 + \dots + A_{3n} X_n &= B_3 \\
\vdots \qquad \vdots \qquad \vdots \qquad & \quad \vdots \\
A_{m1} X_1 + A_{m2} X_2 + A_{m3} X_3 + \dots + A_{mn} X_n &= B_m
\end{aligned}$$

(A.2)

and $X_j \geq 0$ for all j (A.3)

$n > m$

A more general form of the linear programming problem has already been presented in Section (3.3). Conversion between the two forms is possible as explained previously. The particular form given above is more relevant to the Simplex method and the rest of the discussion below. This form is based on the minimization of the objective function under equality constraints and nonnegativity conditions. In the context of the simplex method, the nonnegativity conditions are sometimes called the implicit constraints as they are understood to apply even when they are not explicitly mentioned.

Ignoring the objective function and the nonnegativity conditions, the general constraints represent a set of simultaneous linear equations. However, the problem cannot be solved by the usual methods of linear algebra such as direct matrix inversion. The reason is mainly due to the fact that the number of equations m is not equal to the number of unknown variables n. Even when the number of equations matches the number of variables, there is still the problem of the nonnegativity conditions on the individual variables. Also, the solution of the set of simultaneous linear equations does not necessarily minimize the objective function.

The Simplex method is an iterative procedure to solve the linear programming problem defined above. However, before presenting the simplex method it is necessary to introduce the definitions and theorems presented below. It can be clearly seen that some of these are rather elementary , but they are mentioned here because of the dependence of other, more important, concepts on them.

Definitions

Basic Solution :

A solution of (A.2) obtained by setting n - m variables equal to zero and solving for the remaining m variables.

Basis :

The collection of m variables which are not set equal to zero in the construction of a basic solution.

Basic and Nonbasic Variables :

A basic variable is any of the variables which are in the basis of a basic solution. A nonbasic variable is any of the variables which are set equal to zero in the construction of a basic solution.

Basic Feasible Solution :

A basic solution of (A.2) which also satisfies (A.3).

Optimal Basic Solution :

A basic feasible solution which minimizes (A.1).

Simplex Multipliers

A vector of simplex multipliers π associated with a basis B is defined by the equation:

$$\pi = (\pi_1, \pi_2, \pi_3, \ldots, \pi_m) = C_B B^{-1} \qquad \text{(A.4)}$$

The basis B in (A.4) above represents the partial constraints matrix of (A.2) containing only coefficients associated with the basic variables. Similarly, C_B is a partial vector that contains coefficients of the objective function associated with the basic variables.

For historical reasons, associated with the economic interpretation of the linear programming problem, the simplex multipliers are also called Shadow Prices. This follows from the practice of using the objective function coefficients C_j to denote costs or prices. In general, this designation is not necessary, especially nowadays, as the objective function can be used to represent various other quantities. For example, in the present work, the objective function is used to represent power system transmission losses in one of the optimization problems. In a situation like this, although the mathematical definition of the simplex multipliers as given by (A.4) above is still relevant and significant, the term Shadow "Prices" is not very meaningful. The Lagrange Multipliers, mostly associated with nonlinear optimization problems, are also described by the same term of shadow prices for the same reason of economic interpretation [60].

Convex Set :

A collection of points such that if P1 and P2 are any two points in the collection, the straight line segment joining them is also in the collection.

An illustration of Convex and Nonconvex Sets is shown in FIG.(A.1)

Extreme Point or Vertex:

A point in a convex set which does not lie on a segment joining two other points of the set. This is also shown in FIG.(A.1)

Feasible Region :

The space enclosed by the constraints.

Theorems

Theorem 1 :

The collection of feasible solutions of a linear programming problem constitutes a convex set whose extreme points correspond to basic feasible solutions.

Theorem 2 :

The objective function assumes its minimum at an extreme point of the constraint set.

Solution by Exhaustive Enumeration

Since, according to Theorem 2, the optimum solution of a linear programming problem occurs at one of the extreme points of the feasible region, one obvious method of solving the problem is to evaluate the objective function at all the extreme points and select the one that gives the minimum value. For a general multivariable problem with n variables and m constraints, the number of such <u>extreme point solutions</u> is given by the number of combinations CE(n,m) defined as follows:

$$CE(n,m) = (n!)/(m!(n - m)!) \quad (A.5)$$

However, even for the smallest of practical multivariable problems, the number of extreme points CE is very large and the process of determining these points and evaluating the objective function at each one of them can be a very lengthy, time-consuming and daunting exercise.

Graphical Solution and Geometrical Interpretation

Linear programming problems of small sizes can be represented and solved graphically. In problems of two variables, the constraints are represented by straight lines with the feasible region, formed by their intersection, represented by a polygon whose vertices represent the basic feasible solutions. Various values of the objective function are represented by a number of parallel straight lines. Problems with three variables can be visualized in three-dimensional space although their graphical representation and solution can be rather cumbersome. In this case, the constraints are represented by planes whose intersections form a polyhedron that encloses the feasible region. The basic feasible solutions are given by the vertices of the polyhedron. Linear programming problems of higher dimension cannot be visualized, let alone represented or solved by graphical means. Unfortunately, problems of two or three variables are too small to have any significant practical value, and most practical linear programming problems have much higher number of variables.

The Simplex Algorithm

The simplex method is a systematic step-by-step procedure for finding the minimum of the objective function in a linear programming problem based on Theorem 2. The main advantage of the method is that it avoids the need to go through all the possible solutions of the linear programming problem.

The algorithm starts by an initial feasible basis at which the objective function is evaluated. Two of the variables of the basis are then interchanged. This means that one of the basic variables leaves the basis, thus taking the value of zero and, one of the originally nonbasic variables enters the basis by taking a non-zero value. The objective function is evaluated again and a new basis is then generated by the same process depending on the new value of the objective function. The method is designed such that the new move is towards a basis that reduces the value of the objective function. This directed minimum-seeking process of forming the new bases is continued until no further reduction of the objective function can be achieved. The method also includes a systematic procedure to obtain an initial or starting feasible solution.

This introductory summarization of the simplex method is the necessary amount required as a background to the Dantzig-Wolfe decomposition principle introduced in Section (6.4). The detailed description of the simplex algorithm is beyond the scope of this thesis.

Based on the simplex method there is a faster and more efficient solution algorithm called the Revised Simplex Method.

A.4 DUALITY IN LINEAR PROGRAMMING

An interesting and important concept in linear programming is that of duality. Associated with any linear programming problem, called the **Primal**, there is another linear programming problem called the **Dual**. There is a useful one-to-one correspondence among the various attributes of the two problems. A general primal-dual pair is represented below.

Primal

Minimize $ZP = C^T X$

Subject to $AX \geq B$

$X \geq 0$

Dual

Maximize $ZD = B^T Y$

Subject to $A^T Y \leq C$

$Y \geq 0$

In the above formulation the superscript T denotes matrix and vector transposition.

It is to be noted that any of these two problems can be called the primal and the other the dual. Also by formulating the dual of the dual problem, the original primal is obtained.

The various properties of duality are summarized in Table (A.1). Two other important relationships between the primal and dual are the following:

1. The minimum value of the primal objective function is equal to the maximum value of the dual objective function.

2. At the optimal solution the values of the simplex multipliers of the primal are equal to the values of the basic variables of the dual and vice versa.

There is also a number of theorems that relates the various aspects of the solutions of the two problems such as feasibility and optimality. The simplex and revised simplex methods of solution of the linear programming problem are modified such that the dual problem can be solved directly. The two resulting methods of solution are called The Dual Simplex Method and The Revised Dual Simplex Method. A combined Primal-Dual solution algorithm also exists. Duality in linear programming is well documented in linear programming textbooks.

TABLE (A.1)

PROPERTIES OF DUALITY IN LINEAR PROGRAMMING

PRIMAL	DUAL
Minimization	Maximization
Number of Variables	Number of Constraints
Coefficient Matrix A	Coefficient Matrix A^T
Constraints Bounds Vector B	Constraints Bounds Vector C
Objective Function Coefficients Vector C	Objective Function Coefficient Vector B
Variable $X_j \geq 0$	Constraint $AY_j \leq C_j$
Variable $X_j \leq 0$	Constraint $AY_j \geq C_j$
Variable X_j Unrestricted	Constraint $AY_j = C_j$
Constraint $AX_i \geq B_i$	Variable $Y_i \geq 0$
Constraint $AX_i \leq B_i$	Variable $Y_i \leq 0$
Constraint $AX_i = B_i$	Variable Y_i Unrestricted

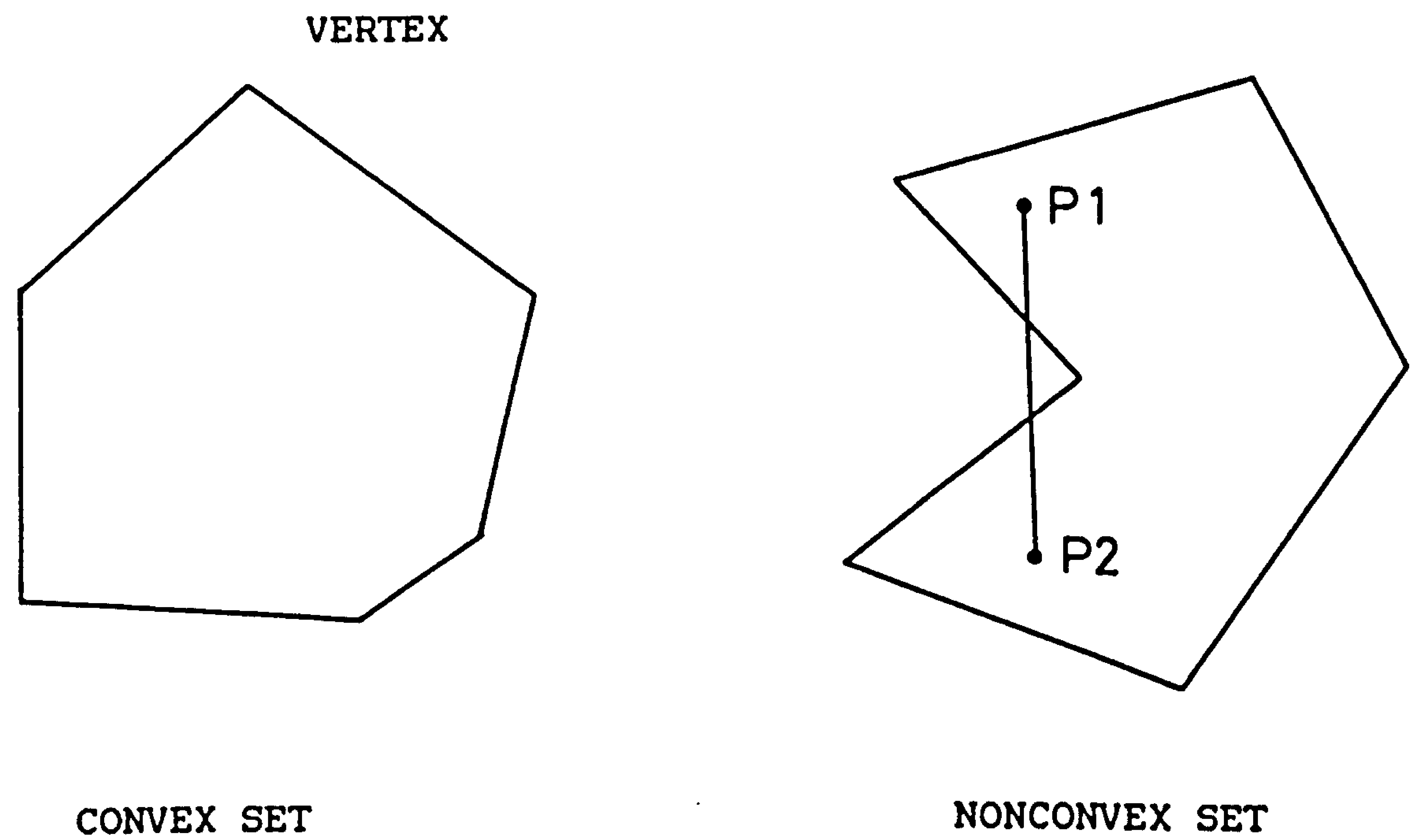

FIG. (A.1)

CONVEX AND NONCONVEX SETS

INDEXES

ALPHABETICAL INDEX OF AUTHORS

NOTE

The list of References following Chapter 7 is arranged according to the first mention of the references in the thesis. Names of all authors and co-authors are given in the following list in alphabetical order. The numbers after the entries refer to those in the above-mentioned reference list.

INDEX

NOTE

Each entry in this index is followed by a set of two numbers referring to the section and page respectively. Section numbers denoted by the letter A refer to the Appendix.

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