INVESTIGATION AND COMPARISON OF SOLUTION METHODS OF THE UNIT COMMITMENT PROBLEM FOR THERMAL UNITS

A thesis presented for the degree of

DOCTOR OF PHILOSOPHY

at

STRATHCLYDE UNIVERSITY

By

Saleh Hussein A. Al-awaji
(B.Sc., M.Sc., MIEEE, AMIEE)

Department of Electronic and
Electrical Engineering. Glasgow G1 1XW.

April 1990

ABSTRACT

This thesis deals with a general investigation and comparison of methods of solution of the unit commitment problem for thermal generating units. An intensive mathematical model for this problem has been developed and presented. The most commonly implemented methods for solving the problem of unit commitment were reviewed and discussed. Every reviewed method had been coded in FORTRAN 77 on a VAX11/785 machine. On the basis of the results obtained, a comprehensive comparison of the different methods has been carried out. For the comparison to be realistic and practical, each method was tested on three power systems of different sizes.

As a result of the comparison, the unit commitment problem for a case study system has been solved by using the most appropriate method(s) from those discussed in the thesis. A new approach for solving the problem has also been proposed and tested.

ACKNOWLEDGEMENTS

I am grateful and obliged to my supervisor Prof. K. L. Lo, the Head of Power Systems Research Group, for his continuous valuable guidance, advice, suggestions and interesting discussion during the preparation of this thesis.

My deep thanks go to the staff of Saudi Arabian Standard Organization (SASO), in particular, Dr. Khalid Y. Al-Khalaf, the director general of SASO, for offering me the opportunity for higher education and for providing the necessary financial support.

I would like to thank the advisory staff and the staff of the computer centre at Strathclyde University, and my colleagues of the Power Systems Research Group for their useful contributions.

I would also like to thank the technical staff of Saudi Consolidated Electric Company, Central region for providing the data of their Power System, which has been used as a case study in this project.

Finally, I would like to thank all members of my family for their encouragement and support, and all others who have helped, directly or indirectly, to complete this work successfully.

CONTENTS

LIST OF SYMBOLS

a_i, b_i and d_i	:	unit cost function coefficients.
Cb_i	:	cost of the banking of the unit i.
Cc_i	:	cold start up cost of the unit i.
Cf_i	:	fixed cost for starting unit i.
$C_i(p)$	:	cost of production of P MW from unit i.
Ct	:	optimal total cost of study period.
D_t	:	system demand in interval t.
D^L	:	demand of subgroup L.
Ds	:	demand of the sample system.
$F_i(p)$	:	fuel cost of unit i as a function of its output power p.
Gc_t	:	total generation cost of power in interval t.
HR	:	heat rate.
Ihr	:	incremental heat rate cost of the unit i.
L	:	number of subgroups of units in the system.
Lg	:	Lagrangian function.
M	:	number of units per group.
Mdt_i	:	minimum down time of unit i.
Mut_i	:	minimum up time of unit i.
N	:	number of units in the system.
n	:	number of committed units.
Nu_L	:	number of units in group L.

Nlc_i : no load cost of the unit i.

Nd : number of shutdown units in t interval.

Ng : number of groups in the system.

ns : number of started units in t interval.

Nsu_i : number of starting up of unit i during the commitment period.

$Nsu_{i\,max.}$: Maximum limit of starting up unit i during the commitment period.

P_D : total demand of the system.

Pg_i : generation of the unit i.

Pgs : total output power of a group of units at generation station s.

Pl : system losses.

Rdp_i : deloading rate of unit i (MW/Min.).

Rlp_i : loading rate of unit i (MW/Min.).

S : system state.

Sdc_{it} : shut down cost of unit i in interval t.

Sdc_{tt} : total shut down cost of unit(s) in interval t.

Srt : total spinning reserve of the system.

Sr_i : spinning reserve of unit i.

Suc_{it} : start up cost of unit i in interval t.

Suc_{tt} : total start up cost of units in period t.

Suc_{bt} : total start up cost of units under banking.

T : time interval.

Td : time in hours for which a unit was shut down.

Ut_i	:	up time of unit i.
U_i	:	state indicator of unit i, 1=ON and 0=OFF.
α_i	:	thermal time constant of the unit i.
σ, μ	:	Lagrangian multipliers.

CHAPTER 1

INTRODUCTION

1.1 GENERAL

The electrical power industry is considered as an attractive area of investment. Therefore, economical operation of power systems is one of the main goals of the system planners and operators. The economical operation of a power system requires interaction of major control functions shown in figure (1.1). An overall solution to this set of problems must achieve the lowest (optimal) operation and production costs, while at the same time taking the following into consideration:

- keeping continuous and reliable supply of electric energy to consumers within reasonable price.
- maintaining the desired standard of voltage levels, frequency and security requirements of the system.
- meeting the safety requirements for the personnel and equipment.
- meeting safety requirements of the environment.

In most electric utilities, the demand is cyclical in nature; see typical daily load curve in figure (1.2)

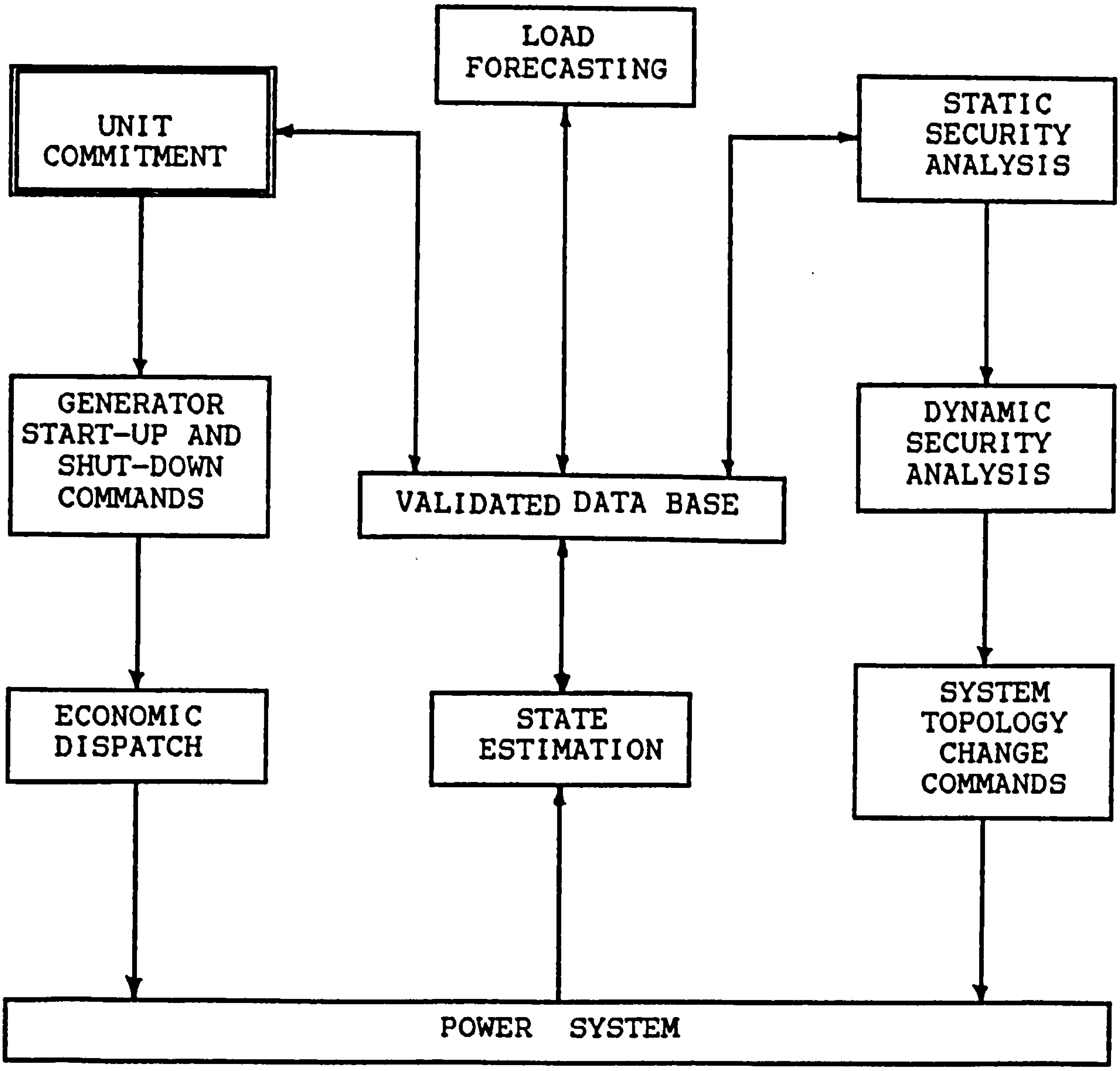


Figure (1.1): Inter-relation of UNIT COMMITMENT and other power system control activities.

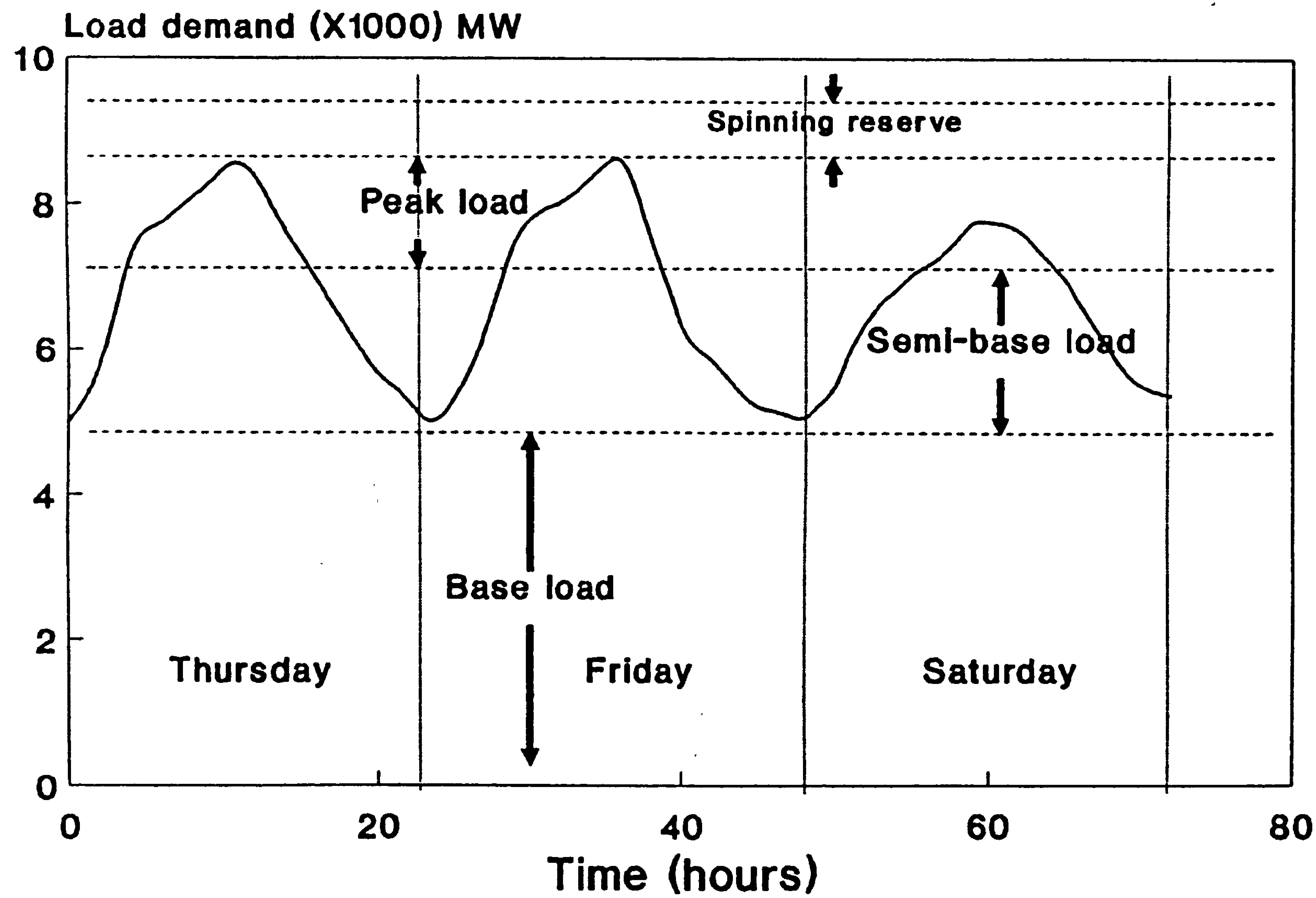
THE TIME SCALE OF THE VARIOUS
CONTROL ACTIVITIES ON POWER SYSTEM

-UNIT COMMITMENT : hours to days.

-ECONOMIC DISPATCH : minutes to hour.

-SECURITY ANALYSIS : on demand.

Figure (1.2): Typical load curve of
a power system over three days.



where the demand increases at the beginning of the day, due to consumers activities. Demand usually maintains its peak value for some time, then decreases towards the end of the day and could reach its minimum value early the next morning. The load profile may also vary from day to day, particularly from weekdays to the weekend, and also from one season to another.

In short term planning of power systems operations, the main objective of the systems operators is to meet the load demand at the lowest cost. One solution is to run enough units to meet the load at peak periods; however, at the minimum load durations these units may operate at their minimum output which is not acceptable from an economical point of view. Another possibility is to divide the units in the system into two categories; the first one represents base units that can be operated continuously to supply the base load demand, while the second includes the cycling units which can be started and shut down to cover the peak demand. The main drawback of this procedure is that it could lead to costly operation of the system.

In order to optimize the operational cost of the power system, the best method from the economical viewpoint is to involve all the available units in the system in a search operation, then try to find a combination of units which can meet the load and the system requirements at a minimum cost during any time period. Other units may be shut down.

The production cost of electrical energy can be distributed within the total capital cost, fuel cost, maintenance cost, as well as labour and administrative costs. In short term planning of power system operations, these costs can be assumed as fixed values in the economical studies of power systems, except the fuel costs. For thermal plants of electrical power generation, the fuel cost represents a significant part of the operation costs, therefore, the aim is to minimize the operation cost of the system as a function of the fuel cost. Consequently, the selection of generating units in order to meet the forecasted demand will significantly affect the production and operation costs. This selection is known as a "UNIT COMMITMENT" which can be defined as the appropriate selection of the most efficient available units in the system which would be put into service as and when the demand increases, or the less efficient units would be taken out of service, if the demand declines. At the same time, all units and system constraints must be satisfied. As a result, the lowest possible minimal (optimal) production and operating costs can be achieved. This is performed at the system control centre, at least once a day over a period of twenty four hours. It may be extended over a longer period, perhaps up to a week or ten days in advance.

1.2 UNIT COMMITMENT AND ECONOMIC DISPATCH

In short term planning and operation of power systems, unit commitment and economic dispatch are two integrated operations. However, it is useful to emphasise the essential difference between them. The unit commitment could be defined as mentioned in the previous section, while the economic dispatch is to determine the suitable allocation of generation among the operating units in order to find the optimum operating policy for the already committed units. Normally, economic dispatch is the step following the unit commitment and it can be considered as a subproblem during the solution of the unit commitment problem and is attempted as frequently as possible, typically every 5-10 minutes [15].

Unit commitment problem solution is an important element for the economical operation and short term planning of power systems because of a wide range of variation between the maximum and minimum demand throughout the day, different efficiencies of units and different start up and shut down costs of generating units. An optimal solution to the problem could lead to a remarkable saving in the system operational cost, where a small percentage reduction in cost (e.g 0.05 %) can reduce the annual operational cost of a large power system by millions of pounds [36]. Therefore, during the last three decades, there has been a considerable amount of research and development in this area.

1.3 REVIEW OF SOLUTION METHODS OF UNIT COMMITMENT

Unit commitment is a difficult problem to solve because of the numerous variables to be considered and the very large number of constraints, particularly for the large scale power systems. Furthermore, the complexity of the problem increases as a result of the following factors:

1- Different types of energy source (Coal, Oil, Gas, Hydro and Nuclear).
2- Non-linear relationship in the input-output characteristic of individual unit.
3- Non standard input-output characteristic from one unit to another and from one plant to the next.
4- Uncertainty over forecasted demand.
5- Unexpected outage of any essential elements in the power system.

One of the earlier attempts to solve the problem was proposed by Baldwin et al [16]. The optimum shut down and start up rules of the units in the system were investigated based on the priority order of the available units. Unit commitment was then formulated as an integer programming problem [17]. The method was tested on two units. A new technique was introduced by Lowyer [18], since the feasibility of using dynamic programming method to solve unit commitment had been discussed in [18]. This technique was tested on a system of 14 units. The start up cost was formulated by Kerr et al [19], for the first time as a function of the time for which the unit was shut down. Reliability cost was considered in the objective function of the problem beside the fuel and start up costs [20]. The units in the system were classified into groups according to their capacity. Mukstadt et al [21] had formulated the problem of unit commitment as a mixed integer linear programming and instead of assuming the demand process as deterministic, a probabilistic description of the demand was considered. In [22], unit commitment of a large scale power system was discussed. A method of solution based on priority order of the units had been proposed and tested on 100 units. A comprehensive study on the application of dynamic programming to the unit commitment problem has been discussed in [25]. Pang and Chen classified the units in the system into different categories and the search range was reduced to cover a small number of units. The unit commitment of 17 units was solved. Branch and bound approach has been implemented in [26] to solve the problem which was formulated as an integer programming with a probabilistic spinning reserve determination; hence a system of 16 units was tested. The problem for a multiple area pool operation with import/export constraint was discussed by Shoults and Chang [27]. Then, the method was tested on 3 interconnected areas.

It can be noted that most of the methods for solution discussed so far are only applicable to small systems, but not to realistic size of power systems, because of two reasons. The first is that dimensionality of the problem for large scale systems could go beyond the abilities of computing facilities. The second is due to the long computation time, which can be far from the practical limits of applications. Therefore, new attempts to solve the problem with a larger number of units have been carried out. For instance, Bond and Fox [28] employed a combination of dynamic programming and mixed integer-linear programming methods, and 10 units were tested. A reduction of computation time and computer memory was achieved. In [30] the unit commitment of 50 thermal units was solved by using mixed integer-linear programming algorithm with a branch and bound approach. Dynamic programming is also used by Waight et al [33] to solve the problem of 30 thermal units. Another attempt was presented in [36] to solve the unit commitment of 72 units by using a hybrid form of the discrete decision linear programming and heuristic methods.

After all, the problem of increasing dimensionality of the unit commitment problem is still the main obstacle confronting system engineers in solving the problem of the large scale systems. Therefore, decomposition methods were proposed and implemented in different attempts to overcome the dimensionality problem. The Lagrangian relaxation technique was first introduced by Mukstadt and Koeng [34] and tested on 10 units. Bertsekas et al [59] generalized and modified the Lagrangian technique in order to suit larger systems. A solution of the unit commitment of 100 units has been proposed. Lagrangian relaxation technique is improved in [35] where it is tested on a 172 units system by solving the dual problem of unit commitment. In [46,68] the Lagrangian decomposition approach is used along with successive approximation to reduce the search range in dynamic programming. Consequently, a reduction of computation requirements was achieved. It is used in [54,69,73] to solve the long term unit commitment for large scale systems, including the fuel constraints.

Benders decomposition method is another attempt to overcome the dimensionality problem of unit commitment. Turgeon [39] and Baptistella and Geromel [41] applied a technique to simplify the unit commitment problem solution by dividing the problem into two sub-problems. The first represents the unit commitment, while the second is the economic dispatch problem. A system of 10 units was tested. Benders algorithm has been modified and improved in [47,56] and tested on a system of 100 units.

It can be concluded from the previous sections that the unit commitment problem can be solved by employing one of the following techniques;

1- Heuristic methods (with priority ordering).

2- Mathematical programming (Dynamic and Mixed Integer Linear programming).

3- Decomposition methods (Lagrangian relaxation and Benders approaches).

As mentioned earlier, the problem of unit commitment is a power system element of major importance, ranking with other control activities of the system, because if the problem is solved optimally, a substantial amount of saving of the operational costs of a power system can be achieved. It is interesting, however, to note that although many valuable attempts have been made and very good research work has been devoted to solving the unit commitment problem, until now there is no method that solves the problem optimally for large power systems [60]. Therefore, the following questions arise;

- What is the best approach which can be used to solve the problem for large systems ?
- Which technique could be applied to solve the problem for a particular system ?

- What are the computational requirements of each technique ?
- Is it worthwhile to implement very complicated techniques rather than using simple and direct methods of solution ?
- What are the advantages and disadvantages of one method of solution compared with other methods ?

In order to find out answers to these questions, a comprehensive comparison of different techniques for solving the unit commitment problems becomes very important. Therefore, the fundamental aims of this thesis are summarized as follows:

1- To formulate the unit commitment problem by using an extensive mathematical model, taking into consideration all the important features and practical conditions of the thermal unit commitment.

2- To prepare and develop computer programs for solving the unit commitment problem of thermal units for every method of the following;

- Heuristic method.
- Full dynamic programming method.
- Dynamic programming sequential combinations method.
- Dynamic programming truncated combinations method.
- Mixed integer-linear programming method.
- Lagrangian relaxation method.

- Benders decomposition method.

3- To test every method on power systems of three different sizes: small, medium and large. Then, a comparison of these methods will be carried out. This comparison should be based on the following criteria:

a- Accuracy of the solution, i.e., which method can achieve optimal, or as close as possible to the optimal, solution.

b- Computation time (CPU time) for each method.

c- Storage space required by computing facilities for each method.

d- Possibilities of practical applications.

4. As an outcome of the comparison, an appropriate method(s) for solving the unit commitment problem of Saudi Consolidated Electric Company (SCECO Central) will be selected from the tested methods.

1.4 THE THESIS LAYOUT

The thesis consists of eight chapters. In the first chapter, a general introduction about power systems economical operations and control is presented. The importance of unit commitment in short term planning and operations of power systems is discussed. A brief review of solution methods of unit commitment problem is also presented. Chapter two deals with the modelling and formulation of unit commitment problem. In this chapter, power generating units' categories are explained and the characteristics of the thermal units are reviewed. The mathematical model of the objective function of unit commitment problem, as well as the necessary constraints, are developed and discussed. In chapter three, the heuristic methods of optimization are briefly outlined and their application to the unit commitment problem is demonstrated. Mathematical programming (Dynamic programming and Mixed Integer-Linear programming) are explained and used to solve the unit commitment problem in chapter four. Chapter five deals with and illustrates the decomposition techniques which can be employed to reduce the dimensionality problem in unit commitment problem of large power systems. Lagrangian relaxation and Benders decomposition approaches are presented and discussed. In chapter six, a comparison of the different methods for solving the problem of unit commitment is demonstrated. A case study system is described in chapter seven. Based on the comparison discussed in chapter six, unit commitment problem of this system is solved by implementing the most appropriate method(s). Finally, a discussion and conclusions as well as recommendations and further studies are presented in chapter eight.

CHAPTER 2

FORMULATION AND MODELLING OF THERMAL UNIT COMMITMENT PROBLEM

2.1 INTRODUCTION

Optimal solution to the unit commitment problem can strongly affect the overall operation policy of the power system. The question now is how to find the optimal solution to this problem. In answer to the question, unit commitment is similar to any optimization problem which can be optimally solved by satisfying the following general requirements:

1- The problem must be formulated and modelled in an accurate and representative form.

2- The proper technique must be implemented to solve the problem.

Therefore, this chapter is concerned with the development and formulation of the problem of unit commitment. The different classes of thermal generating units will be outlined. Unit characteristics related to the production and operation costs will be presented and discussed. The objective function of the operation cost of the system, as well as the system and unit constraints will be developed and explained.

2.2 UNIT CLASSIFICATION

A typical daily load demand for electric energy, as in figure (1.2), shows a large variation in power consumption between peak and off-peak hours during the day. This load curve changes from day to day and from working days of the week and weekends as well as from one season to another. For the purposes of economical operation of a power system and other system requirements, the generated power should approximately follow the load curve. The suitable units to supply the load can be specified by solving the unit commitment of the system. It is common, in power systems, to find a wide variation of units in terms of capacity, efficiency and age of units. Therefore, for the purpose of simplification of the solution of unit commitment, the units in the system may be classified into the following categories [1,25,43]:

i) BASE LOAD UNITS : The largest capacity and the most efficient units in addition to the units which need long starting up time will be included in this category to supply the base load of the system. Nuclear and steam units are the best example of the base-load class. These units usually run at their full-load capacity on a twenty four hour basis. The production cost of base units must be the lowest when compared to other categories.

ii) INTERMEDIATE UNITS : This class includes the units which are easy to control as well as units of low production costs. The semi-base load range of demand (see figure (1.2)) has to be met by intermediate units. Units of this class may run for twenty four hours a day with generating capacity in the range between their maximum and minimum output limits. The most appropriate units, for this class, are the hydro-powered units. However, for power utilities, where hydrogenerators are not available, easily controllable thermal units could be included in this category.

iii) PEAKING UNITS : The very fast starting units which meet the peak-load demand are usually used for peaking purpose. The high production cost units, for example, gas turbine units, are suitable for this category. Hydro-powered generators and pumped hydro-plants may also be used as peaking units. Peaking units are committed during the peak interval, which may last[a] few hours during the day and be shut down throughout the off-peak time.

2.3 UNIT CHARACTERISTICS

Electric power is generated as a result of energy produced by either steam or combustion turbines. Hydro-powered plants, where they are available, provide an excellent source of electric power. In this study, only the thermal units will be involved.

The operation costs of the thermal plants include the fuel cost, the cost of labour and maintenance. The fuel cost represents the major part of the operating costs, therefore , the selection of the fuel type directly impresses the operation policy and production cost of the system. Labour and maintenance costs are, usually, assumed as a fixed percentage of the total costs.

For the purpose of thermal unit modelling in order to deal with operation and production costs of thermal units, the unit may be described as having the following characteristics:

- Input-output characteristic.
- Incremental heat rate characteristic.
- Net heat rate characteristic.

In the following section, these characteristics will be briefly outlined.

2.3.1 INPUT-OUTPUT CHARACTERISTIC

The relationship between the input energy to the thermal unit (measured in MJ/h or MBtu/h, in SI units) and the output generated electrical power (measured in MW) is defined as the unit input-output characteristic. This relationship changes from one unit to another due to several factors such as the design considerations and unit capacity as well as the fuel type. A typical input-output characteristic of the thermal units is shown in figure (2.1). This relationship is an essential factor in the economic operation of the power systems and in unit commitment studies.

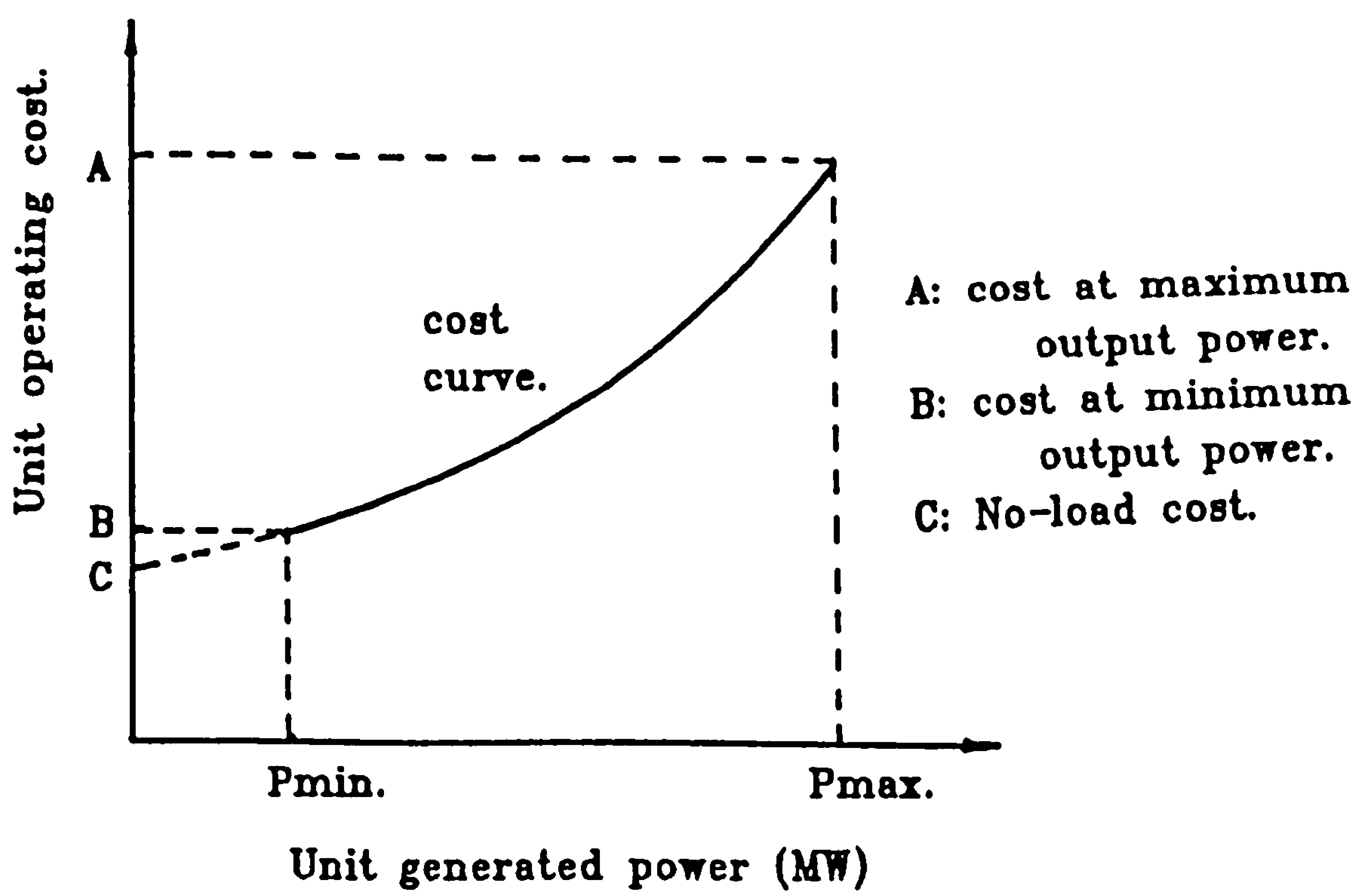


Figure (2.1); Typical input-output characteristic of thermal unit.

The cost function of the unit can be expressed from its input-output characteristic. Unit cost function may be represented, depending on the type of the unit, by one of the following mathematical forms [1,5,8,9,10] :

i) LINEAR RELATIONSHIP : In single stage units, the relationship between the input energy and the generated output power is almost linear. This relationship is illustrated in figure (2.2). The cost function may be formulated as follows;

$$C(p_i) = a_i + b_i p_i \qquad .... \qquad (2.1)$$

Where;

C = operating cost of unit i.

p = generated power from unit i.

a = no load cost of unit i.

b = incremental cost of unit i.

ii) SECOND ORDER POLYNOMIAL : In a multi-stage unit, the linear relationship does not accurately represent the input-output characteristic. Therefore, the cost function may be formulated by a second order polynomial. The cost curve is shown in figure (2.3) and the cost function can be expressed as follows:

$$C(p_i) = a_i + b_i p + d_i p^2 \qquad .... \qquad (2.2)$$

where; b_i and d_i are the polynomial coefficients in £/h

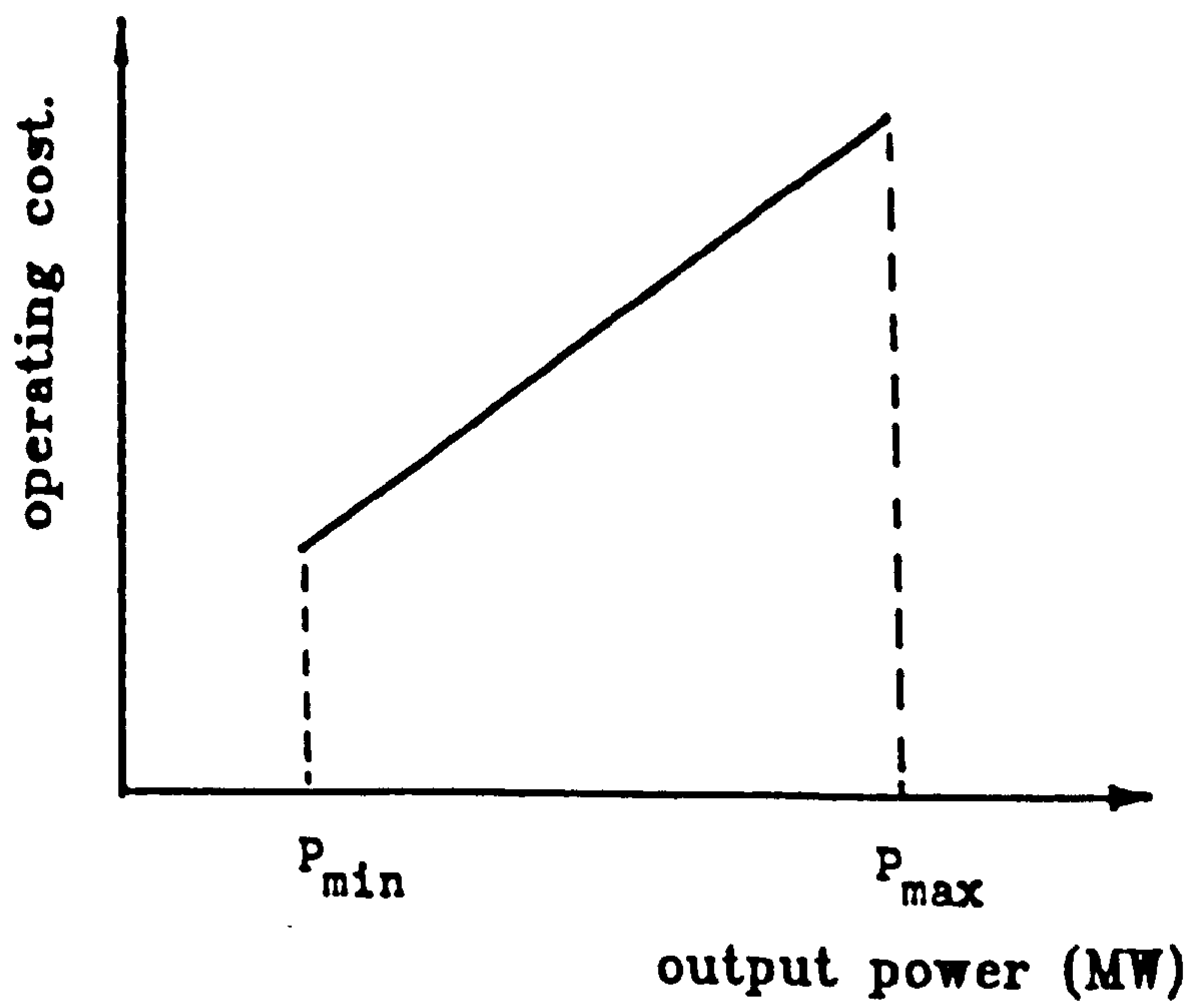


Figure (2.2) ; Linear cost function of thermal unit.

This formulation is applicable to most of the cost functions for the thermal units of power generation with reasonable accuracy.

In some of the practical applications, a linearization of non-linear input-output characteristics of the thermal units is required. Several ways of approximating the cost curve have been suggested [15] depending upon the accuracy desired. One step approximation can be used as illustrated in figure (2.5); however, if more precise and better results are needed, several linear segments (a piecewise approximation) may be implemented for linearization as shown in figure (2.4).

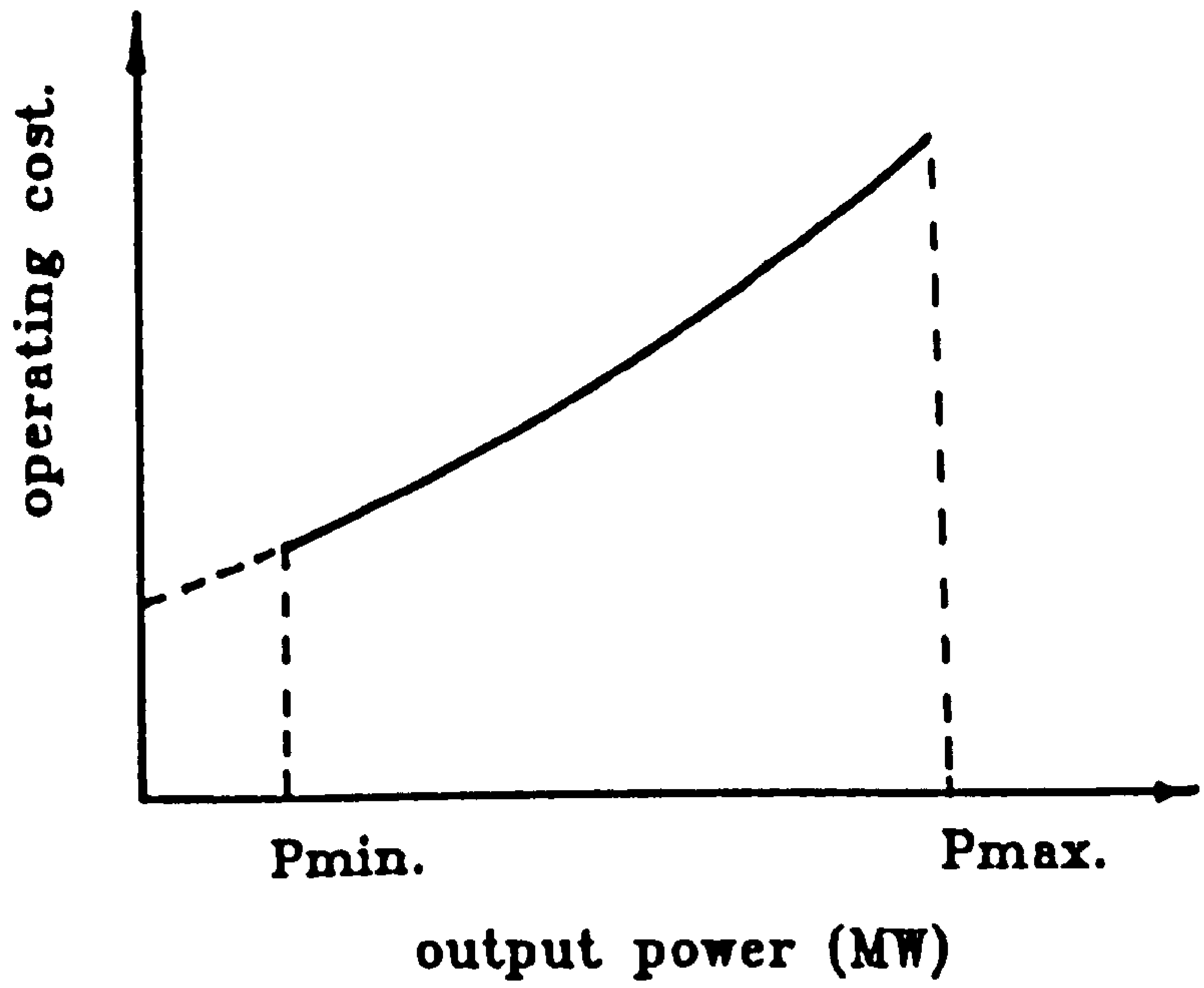


Figure (2.3); Second order operating cost curve of thermal unit.

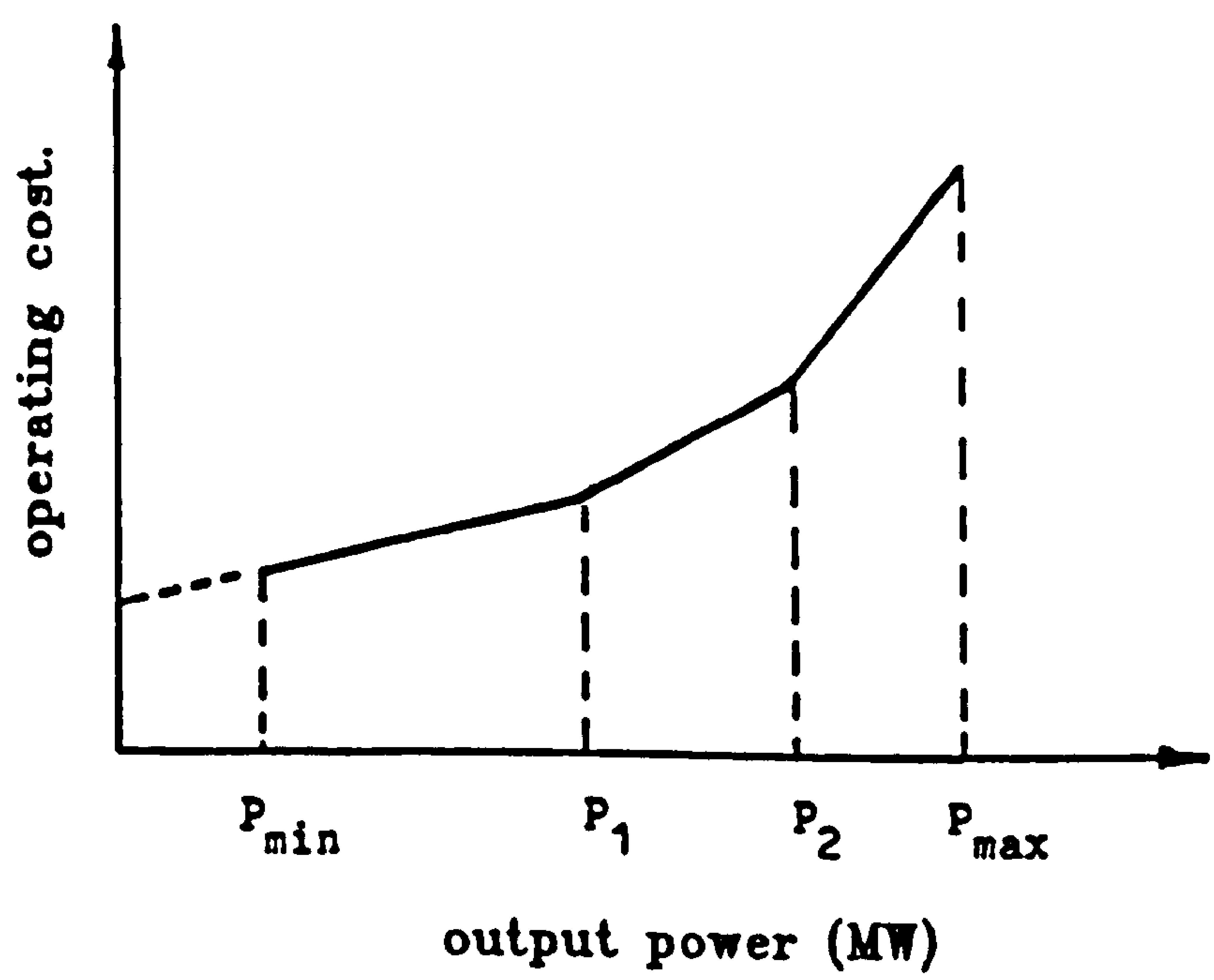


Figure (2.4) : Piecewise approximation of the cost function curve.

iii) HIGHER ORDER POLYNOMIAL : If a thermal unit characteristic cannot be represented and fitted by one of the previous methods, a suitable higher order polynomial may be used to express the unit cost function as;

$$C(p_i) = a_i + b_i p + d_i p^2 + e_i p^3 + \ldots \qquad \ldots\ldots (2.3)$$

The computation efficiency and accuracy of results can be accordingly obtained. However, the complexity of the problem formulation may be increased, and may result in difficulty in obtaining the optimal solution.

2.3.2 INCREMENTAL HEAT RATE CHARACTERISTIC

The incremental cost of generator is simply defined as the rate of change of the fuel cost as a function of the output power to the change in the output power ($\partial F_i(p_i)/\partial P_i$). Incremental heat rate characteristic (IHC) is obtained by plotting $\partial F_i(p_i)/\partial P_i$ versus P_i (measured by MJ/MWh or £/MWh). A typical plot of the IHR is shown in figure (2.5).

This characteristic is widely used in economic dispatching of the unit. The incremental heat rate plot of linear and second order input-output characteristics of units in figure (2.2) and figure (2.4) are shown in figure (2.6); a and b respectively.

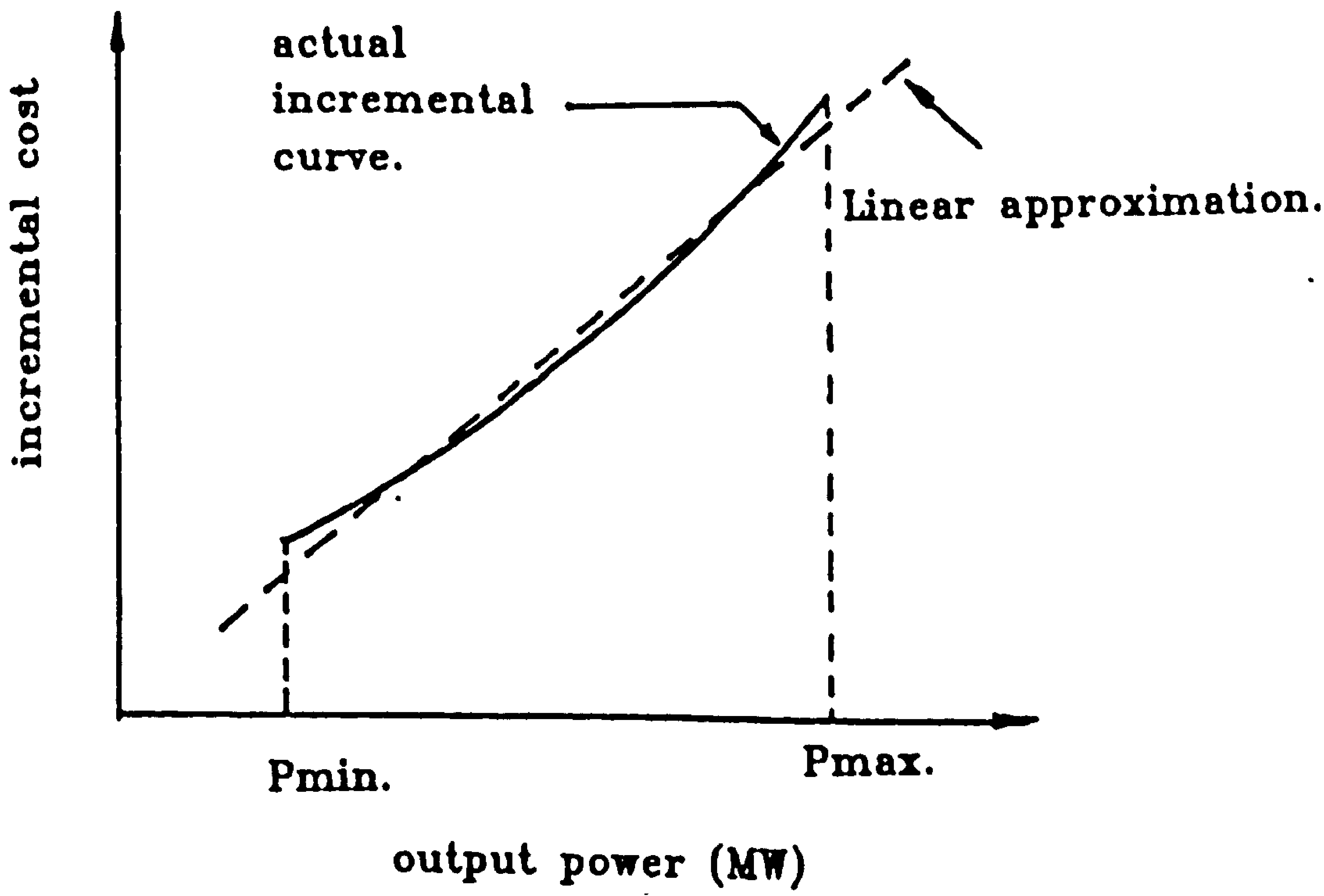
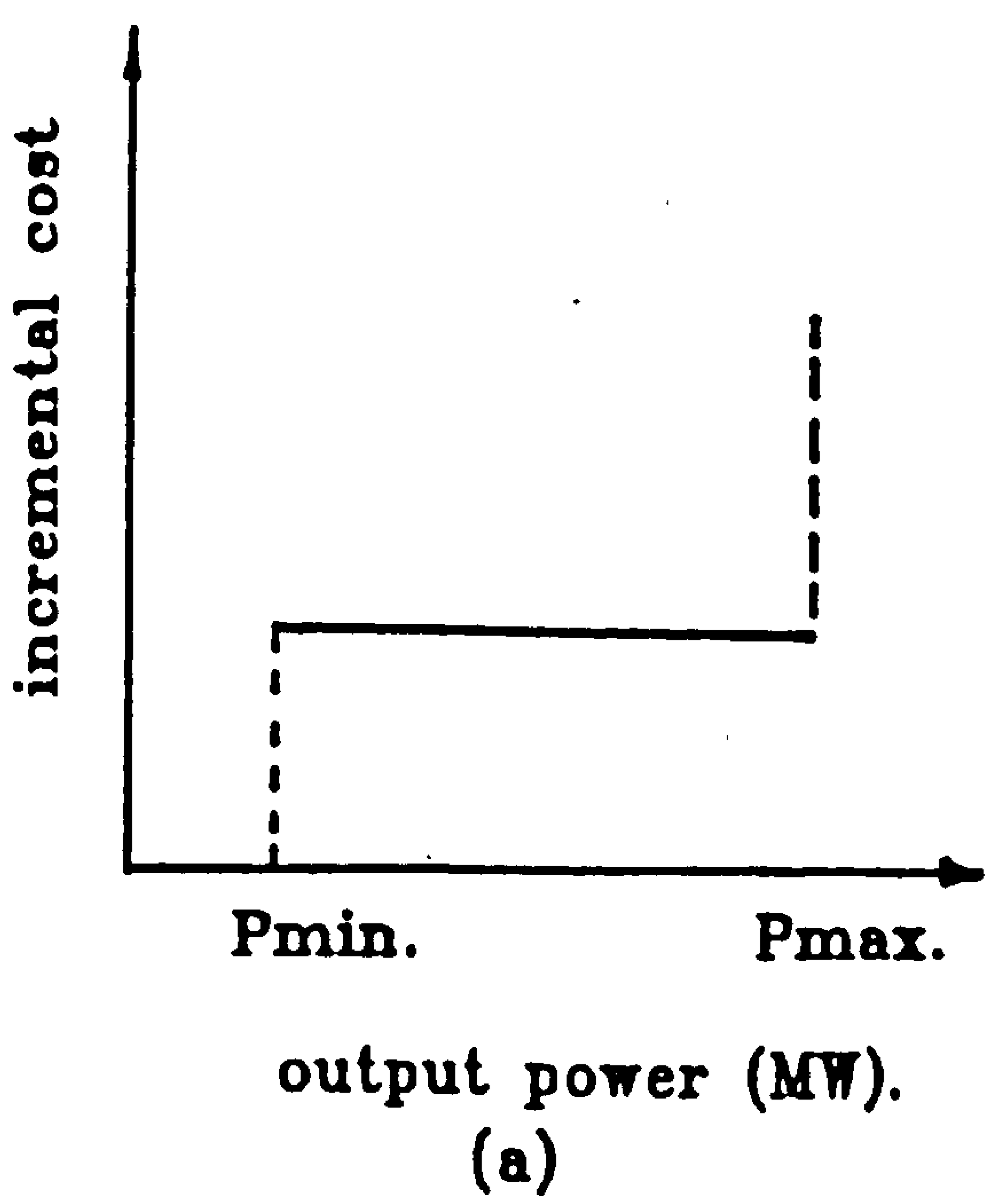


Figure (2.5) : A typical plot of incremental heat rate characteristic.



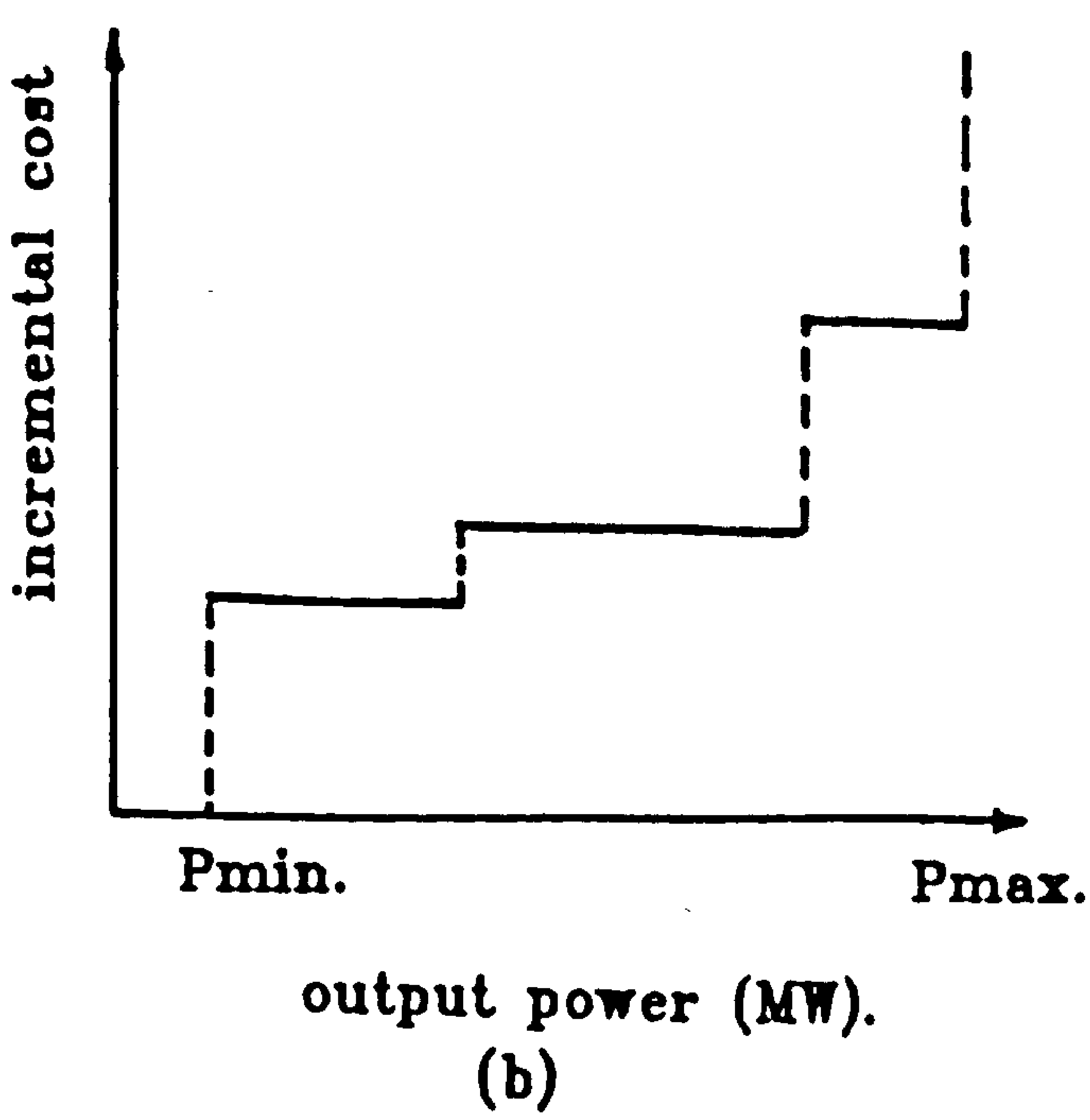


Figure (2.6) : IHR of,
(a) Linear input-output characteristic.
(b) Second order input-output characteristic.

2.3.3 NET HEAT RATE CHARACTERISTIC

Heat rate (HR) for a unit is defined as the input thermal power (Btu/h) to the unit divided by the output electrical power (MW) of the unit, i.e. (H/P). When H/P is plotted versus P, as in figure (2.7), then the net heat rate characteristic is obtained. A typical coal-fired plant heat rate is 10.5 X 10^6 Btu/MW·h [10]. This property is the reciprocal of the unit efficiency (η) in thermal units. Since 1 Btu/h is equivalent to 0.293 MW, this HR is equivalent to 3.08 MW/MW or an efficiency of 32.5%. The general relationship between HR and η is [10]:

$$\eta = \frac{3.413 \times 10^6}{HR} \qquad \text{.......... (2.4)}$$

From the net heat rate characteristic of the unit, the unit efficiency can be determined. Furthermore, the relationship between the power generated from a unit and its production cost can be specified.

2.4 TOTAL OPERATING COST FUNCTION

The objective function of the unit commitment normally consists of generation cost, unit start-up and shut-down costs, and the no-load cost. In the next sections, a brief explanation of these costs will be presented.

2.4.1 GENERATION COST

The generation cost of a thermal unit, which mainly represents the fuel cost, is assumed as a function of power generated by the unit. The total cost of the generation of on-line units in the system, at any interval of the commitment period, can be obtained by gathering the generation cost of every unit as follows;

$$Gc_t = \sum_{i=1}^{n} c_i(p_i) \quad \ldots\ldots \quad (2.5)$$

The right hand side of equation (2.5) is substituted by the proper input-output characteristics of the unit which have been expressed in equations (2.1),(2.3) and (2.4) according to the type of the unit.

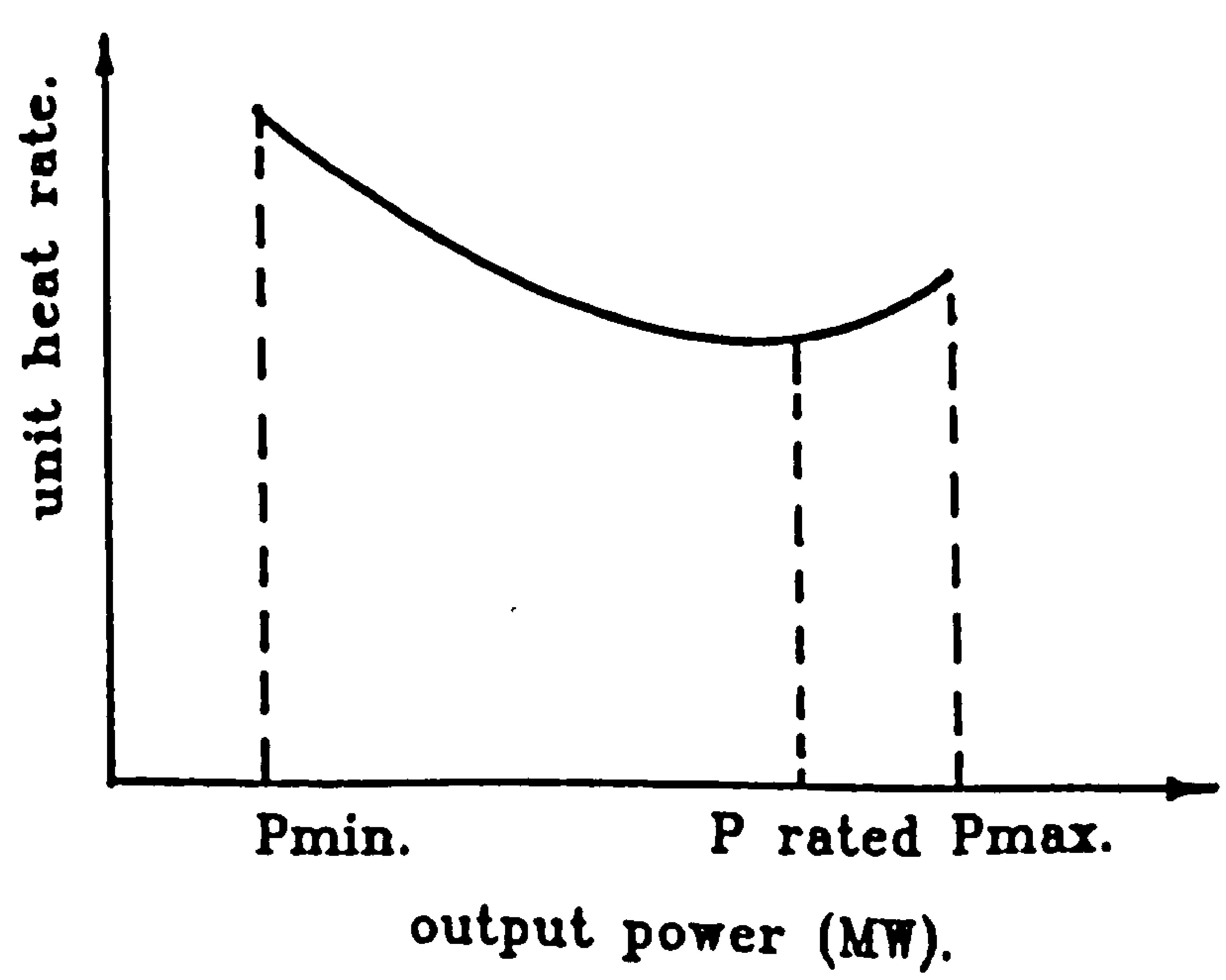


Figure (2.7): Net heat rate characteristics for thermal unit.

It could be expressed as in equations (2.1), (2.2) or (2.3), according to the type of the units and their input-output characteristics.

2.4.2 START-UP COST

The start-up cost of the thermal units can be classified into two categories;

i) **Fixed start-up cost:** (independent of shut-down time) for the units of small output capacities such as gas turbine and diesel engines.

ii) **Variable start-up cost:** for large steam units where the cost of starting is time dependent i.e. a function of time for which the unit has been shut-down [1,13,40]. It may vary from the maximum (cold start) value, if the unit has been shut down for long time (30 hours or more for a steam turbine [12]), so that its temperature becomes close to the ambient temperature, to the minimum value if the unit has been turned off recently and is still close to the normal operating temperature as shown in figure (2.8). During the shut-down period, the unit may be in either one of the following conditions. The first one is to allow the unit to cool down and return it to its normal operating temperature in time for starting up. The cooling rate of the boiler is approximately exponential with respect to shut-down time, therefore, an exponential start up cost can be assumed as shown in figure (2.8).

The cost function may be expressed as [15];

$$Suc_t = Cc_i \cdot (1-e^{-Td_i/\alpha_i}) + C_{fi} \quad \ldots\ldots (2.6)$$

where;

Suc_i : start-up cost of unit i.

C_{ci} : Cold start cost of unit i .

C_{fi} : Fixed cost of starting up a unit i.

α_i : Thermal time constant for unit i (cooling time constant for the boiler), (1/hour).

T_{di} : Time in hours for which unit i was down.

The second one requires that the boiler should have sufficient input energy in order to maintain operating temperature. This mode is called banking. The start-up cost of banking is;

$$Suc_{bt} = \sum_{i=1}^{ns} C_{bi} \cdot Td_i + C_{fi} \quad \ldots\ldots \quad (2.7)$$

Where;

C_{bi} : cost of the banking of unit i.

It can be noted from figure (2.8) that the cost of banking can be less than the cost of cooling to a certain time. This time varies from one unit to another according to the size of unit, for example, it could be in the range of 5 hours for small steam unit and 24 hours or more for large steam unit [12,13]. Therefore, banking can be used only if the unit is started before that time. Otherwise the unit under cooling is more economical.

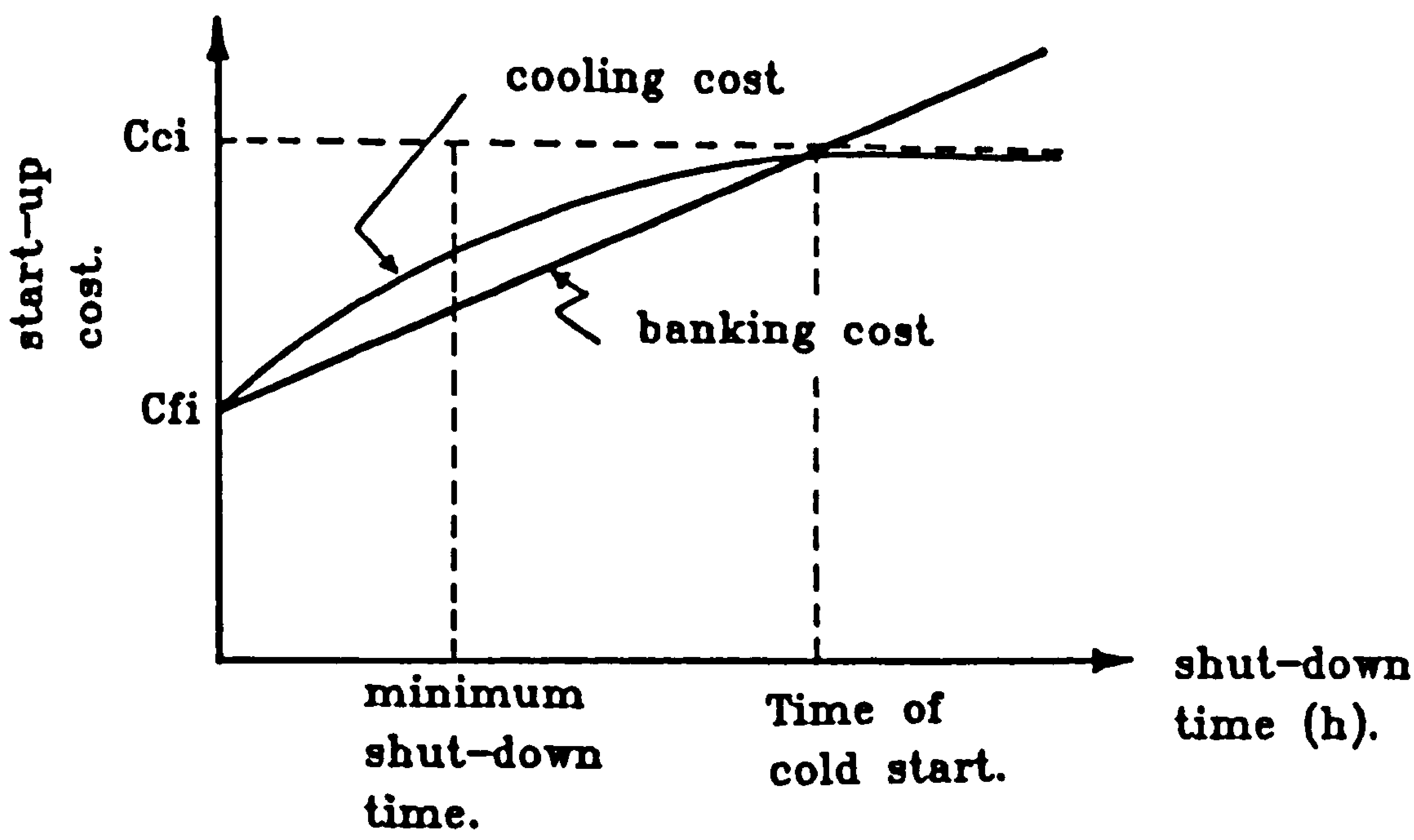


Figure (2.8): Time dependent start-up cost of thermal unit.

2.4.3 SHUT-DOWN COST

During the shut-down process, the unit is deloaded at its maximum deloading rate from its minimum output power till desynchronizing and then shutting the unit down. This procedure requires a time which may vary from few minutes to hours depending on the type and size of the unit. Therefore a certain amount of fuel is consumed throughout this operation. Consequently, the shut-down cost of the unit mainly consists of the fuel cost consumed during the shut-down time. It can be expressed mathematically as:

$$Sdc_t = \sum_{i=1}^{nd} Sdc_i \quad \ldots\ldots. \quad (2.8)$$

The total cost function of the system over the commitment period can be formulated as follows:

$$C_{total} = \sum_{t=1}^{T} \{ \sum_{i=1}^{n} Gc_{it}(u_i, p_i) + \sum_{i=1}^{ns} Suc_{it}(u_i)$$

$$+ \sum_{i=1}^{nd} Sdc_{it}(u_i) \quad \ldots\ldots. \quad (2.9)$$

This equation represents the objective function of the unit commitment problem in power systems. For the purpose of optimal operation, it can be formulated in a suitable mathematical form and minimized by the proper manipulation of some of the variables, subject to the necessary constraints.

2.5 UNIT COMMITMENT PROBLEM CONSTRAINTS

Many constraints can be imposed on the power system objective function for optimization purposes. Each individual power system may specify its own set of constraints depending on certain factors such as: network topology, reliability and security requirements, generation make-up, load curve characteristics, etc. These constraints can be classified into the following two categories:

2.5.1 SYSTEM CONSTRAINTS

System constraints are applied in the power system operating cost objective function to keep the system within the acceptable limits of stability and to meet security requirements. The most common system constraints are listed below:

i) Generation and load balance

At any time and under the steady state condition of power system, the total generation must always meet the load demand as well as the transmission losses. This relationship is known as the load balance equation which is formulated as:

$$\sum_{i=1}^{n} Pg_i = P_D + P_L \quad .... \quad (2.10)$$

ii) **Spinning reserve**

The spinning reserve (Sr) for a system can be defined as the extra amount of active power that can be obtained from committed units within a specified short interval of time (5-15 minutes) by loading them to their maximum ratings. The basic functions of the spinning reserve are :-

i) To provide a spare capacity that will cope with any errors in load prediction.

ii) To provide a spare capacity in the event of the loss of any generating unit.

The total Sr of a power system is;

$$Sr_{total} = \sum_{i=1}^{n} Sr_i \quad (MW) \quad .... \quad (2.11)$$

Where Sr_i is the spinning reserve of unit i.

Hence, the system total reserve constraint that must be satisfied is;

$$\sum_{i=1}^{n} Pg_{i\,max} - \sum_{i=1}^{n} Pg_i \geq Sr_{total} \quad ...... \quad (2.12)$$

The larger value of the system spinning reserve, the greater is the reliability of the system. However, this additional capacity increases the system operating costs. Spinning reserve can be assigned as a fixed percentage of the total demand in some electric utilities or as a fixed value in others, so that at least the outage of the largest on line unit could be made up. Spinning reserve must be spread around different generating plants of the power system to avoid any transmission system limitations and to increase the system reliability [1,11].

iii) Transmission network constraints

For each transmission line, there is an upper limit to its current carrying capacity determined by the thermal rating of the line or specified by other security requirements (e.g. bus voltage levels). This limit must not be violated under the steady state conditions, so:

$$I_{ij} \leq I_{ij}max. \qquad \ldots\ldots \qquad (2.13)$$

where I is the current in the line connecting the ith and jth buses.

iv) Fuel constraints

The fuel supply constraints, if any, vary from one utility to another depending upon the fuel type. Fuel availability can be limited because of supply problems, limited storage facilities or other reasons. Each unit in the system can use either one, two or more fuels. Fuel constraints must be considered in the long term unit commitment period because a unit can be changed from one category to another according to the fuel type. For example, a dual fuel unit can be operated with two types of fuel. When the first type is cheap but constrained, and the other is more expensive and unlimited, the unit can be used as a base-load unit only when the constrained fuel is available, while it may be operated as peak unit with the unlimited and more costly fuel.

v) Export/import constraints

In multiple area unit commitment representation, each area has its own generating units, load demand pattern and spinning reserve requirements. However, a part of the load demand of any area may be supplied by importing power from areas where it is more economical, subject to the following export/import constraints:

- Physical transmission line limitations.
- Area security considerations.
- Fuel availability.
- Regulatory restrictions.

vi) **Group or station generation limit constraints**

Due to operational limitations, fuel limitations or outages of some units, upper and lower generation limits may be imposed on the total generation levels of some stations or groups of units in a station. These constraints can be expressed as follows:

$$Pgs_{min} \leq Pgs \leq Pgs_{max} \quad .... \quad (2.14)$$

where Pgs is the available output power of group g at station s.

2.5.2 UNIT CONSTRAINTS

Unit constraints vary from one unit to another. These constraints deal with the operation of each generating unit individually. Although these constraints reduce the freedom of choice in the starting-up and shutting down of units, they must be considered in the unit commitment problem formulation. The most effective unit constraints will be outlined in the next section [1,25,39,40,43];

i) **Minimum and maximum output limits**

The unit must be operated at or above its minimum output power limit because of stability concepts, as well as thermal considerations in operating the boilers that produce steam to drive the turbines in fossil-fired and nuclear plants. It also must be operated at or below a maximum limit due to the stator thermal of the generator and design considerations, so:

$$P_{imin} \leq P_i \leq P_{imax} \quad \ldots\ldots \quad (2.15)$$

ii) **Minimum up time**

Once the unit is committed, it must be kept on line for a certain time before it can be shut down again, in order to avoid high maintenance due to cycling [71]. This time is known as the minimum up time of the unit, so it must be considered in the solution of unit commitment and can be expressed as:

$$Ut_i \geq Mnt_i \quad \ldots\ldots. \quad (2.16)$$

iii) **Minimum down time**

If the unit is shut down, a certain time must also elapse before it can be started up again. Enough time should be given for temperature equalisation within the turbine in order to maintain the stress differentials within the safe limits [71]. This time is called the minimum down time of the unit, so:

$$Dt_i \geq Mdt_i \quad .... \quad (2.17)$$

The values of the minimum up and minimum down time depend on the manufacture's specifications and vary from one unit to another. These constraints are a major influence on unit commitment.

iv) **Loading and deloading rate constraints**

In power system operations, the total generation of the committed units must always follow the load demand variations. Consequently, the output of units can be subjected to continuous fluctuation. When the unit operates in its normal operational stable range, i.e. its output power is above the minimum and below the maximum limits, and if the unit output is required to be increased or reduced, the following constraints must be satisfied:

a- Loading rate: Loading rate of the thermal generating units varies between zero and a specified maximum value. The maximum value of the loading rate also varies from one generating unit to another according to the type of unit as well as the unit condition. In the steam turbo-alternators, the loading rates are in the region of 2-5 MW/min. In contrast, it can be higher in gas turbine unit, since it could be at the rate of up to 30 MW/min.

Hydro-powered plants provide the highest loading rates [11,15]. However, they are beyond the scope of this thesis. The loading rate constraint can be expressed mathematically as:

$$0 < Rlp_i \leq Rlp_{i\,max} \quad \ldots\ldots \quad (2.18)$$

where Rlp_i is the loading rate of unit i, and $Rlp_{i\,max}$ is its maximum loading rate.

b- **Deloading rate:** If a generating unit operates at an output power above its minimum and less or equal to its maximum limits, it can be deloaded, when necessary, at a deloading rate which can be varied in a range of more than zero and less or equal to a certain maximum value, so as:

$$0 < Rdp_i \leq Rdp_{i\,max} \quad \ldots\ldots \quad (2.19)$$

where Rdp_i is the deloading rate of unit i, and $Rdp_{i\,max}$ is its maximum deloading rate.

$Rdp_{i\,max}$ is also different from one generating unit to another and it may vary in the range of 2-20 MW/min for thermal units.

v) **Frequency constraint**

Minimum up and minimum down time constraints could have small values for some types of thermal units, such as gas turbine. Consequently, a higher number of starting-up and shutting-down of these units is possible during the commitment horizon, which in turn increases the maintenance costs. Therefore, another constraint must be imposed in this case, in order to reduce the maintenance cost to the lowest possible level. This constraint is called the frequency constraint of the unit and can be expressed as:

$$Nsu_i \leq Nsu_{i\,max} \quad \ldots\ldots\ldots \quad (2.20)$$

where Nsu_i is the number of starting-up unit i and $Nsu_{i\,max}$ is the maximum limit of starting-up unit i throughout the commitment period.

The number of times a unit is started up and shut down throughout the commitment period must not exceed its frequency constraint.

vi) **Must out units**

Due to maintenance scheduling or forced outage, a number of units in the system cannot be included in the unit commitment for a certain period of time, so "must out" constraints will be imposed on these units. This results in being excluded from the unit commitment.

vii) Must run units

The large capacity units in the system cannot be easily started up due to the following factors:

- A long time is required for the starting.
- High starting cost.
- Very complicated process of starting, e.g. nuclear units.

These units can be operated over all the commitment period as "must run" units. Therefore, there is no need for them to be considered in the unit commitment.

viii) Crew constraints on plants

If a plant has a limited number of operators, crew limitation may cause a problem that prohibits starting up and/or shutting down two or more units in a plant at the same time. This is another constraint which has to be taken into account in the set of unit constraints.

2.6 CONCLUSIONS

In this chapter, a general mathematical model for the objective function of the unit commitment problem for thermal generating units has been formulated. A set of essential constraints for the system and the units has also been developed. Unit commitment objective function can be minimized by the use of an appropriate technique from the most common solution methods, which are illustrated in figure (2.9). The selection of the suitable solution method depends mainly upon the characteristics of the units, the number of units in the system and the system's load demand pattern.

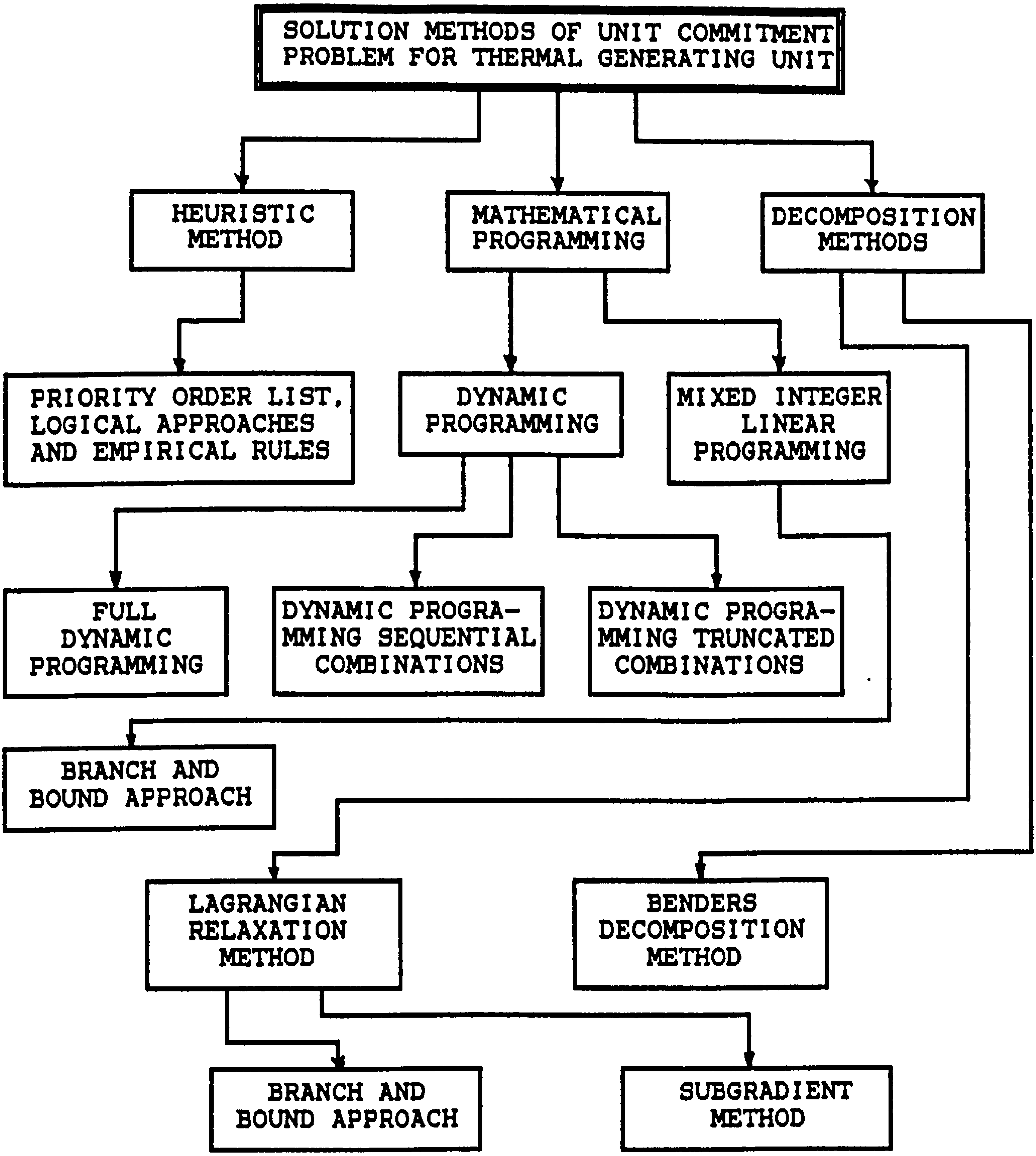


Figure (2.9): Flow chart for the commonly used methods of solution for unit commitment problem of thermal units.

CHAPTER 3

SOLVING UNIT COMMITMENT PROBLEM BY HEURISTIC METHODS

3.1 INTRODUCTION

Due to the complexity it seems very difficult to design and use exact algorithms to solve the large-scale optimization problems with a moderate computational effort. Therefore, methods have to be found that quickly produce feasible solutions which are reasonable in terms of accuracy and computational requirements [66].

Heuristic methods are widely implemented in practice with more or less satisfactory results. A heuristic procedure can be defined as a set of rules for selecting an element or taking a decision from a set, such that the outcome may or may not have desirable consequences. In practice, however, a heuristic method suggests an improvement procedure which, when applied, is expected to lead towards a superior if not optimal state. Heuristic decisions are likely to be quite intuitive and their quality is dependent upon the skill, based on experience and on the ability of observing and identifying factors that are relevant, and weighting them according to their relative importance [66,78,79].

3.2 UNIT COMMITMENT AND HEURISTIC METHODS

Heuristic method is the simplest approach as well as the earliest technique used to solve the unit commitment problem [42]. It has been developed from the manual method of solving the problem, and it mainly depends on the priority order schemes, where all the generating units of the system are listed according to their merit orders based on their average full load production cost [25]. Several attempts have been proposed to solve unit commitment by heuristic methods [1,16,19,22,27,36,63,81]. One of the earliest attempts was proposed by Baldwin et al [16]. The optimum shut down and start up rules of the generating units in the system were investigated based on the priority order. Happ et al [22] suggested and introduced a method of solution in two phases. In the first one, a feasible schedule which is close to the optimal is obtained, while in the second phase, a further reduction of the operational costs is attempted. A system of 100 units was tested. The problem of multiple area system has been discussed in [27,81]. Transmission system limitations and dynamic restrictions of generators have been taken into account. Khodaverdian et al [36] introduced a solution to the unit commitment problem by using a hybrid form of the discrete decision linear programming and the heuristic technique. Sub-optimal but fully feasible schedules were obtained and all the operational constraints have been satisfied. The proposed approach was tested on a system of 74 units over a commitment period of 48 x ½ hours intervals. Calitz et al [63] discussed the importance of the nominal solutions in heuristic unit commitment programs. The advantages of heuristic methods over other solution techniques for unit commitment problems have been briefly presented.

The basic idea of heuristic methods applications to unit commitment is to produce a feasible (sub-optimal) commitment schedule by employing the priority order list and following empirical rules and logical steps in order to satisfy the constraints of the operating units and the system with a minimum running cost. Solution procedure can be outlined as follows:

1) All the available units in the system are to be listed in ascending order according to their full load average cost (FLAC), hence a strict priority order is imposed. (FLAC) of the unit i can be calculated as:

$$FLAC_i = \frac{C_i\,(P_{i\,max})}{P_{i\,max}} \qquad \ldots\ldots \quad (3.1)$$

where C_i is the production cost of the unit i when operating at its maximum output.

2) At the beginning of the study period, the forecasted

demand and the spinning reserve for the first interval must be met by committing the most efficient units in the list.

3) At the beginning of each interval, it is necessary to check whether the forecasted demand is the same as the previous interval, whether it increases or decreases. If it is constant, the same units as in the previous interval can be kept on line. If the demand increases, then additional unit or units will be committed until the constraints are satisfied. If the demand decreases, the lowest efficient unit or units will be shut down if there is no need to commit that unit within a time less than its minimum shutdown time, otherwise it may be operated at its minimum output power.

4) If a unit is to be shut down, due to the decreasing demand, and this unit needs to be started-up after a time more than its minimum shutdown time, a comparison is made between the cost of keeping the unit on line and its startup cost when it is needed. The least cost will be selected and the necessary decision taken accordingly.

5) At each interval, after obtaining the initial minimum cost schedule of unit commitment, additional steps to improve and refine the solution may be taken, such as performing economic dispatch, in order to minimize the total operating costs. The process is terminated if no

further improvement can be achieved and the current solution is assigned as the most economical.

6) The processes in stages 2 to 5 are repeated for each interval until the commitment period is completed. The total operating cost of the system through the commitment horizon can be calculated by implementing equation (2.9).

3.3 COMPUTER PROGRAM AND TEST RESULTS

A computer program in FORTRAN 77 has been prepared and developed to solve the unit commitment by heuristic approach. The program proceeded as illustrated in the flow chart in figure (3.1).

In order to observe the effect of the size of power system on the performance and the efficiency of the program, three different systems which are described in Appendix A, were tested and their unit commitment problems solved. The units were scheduled over a twenty four hour commitment period of one hour interval. The results in this study are based on the following assumptions:

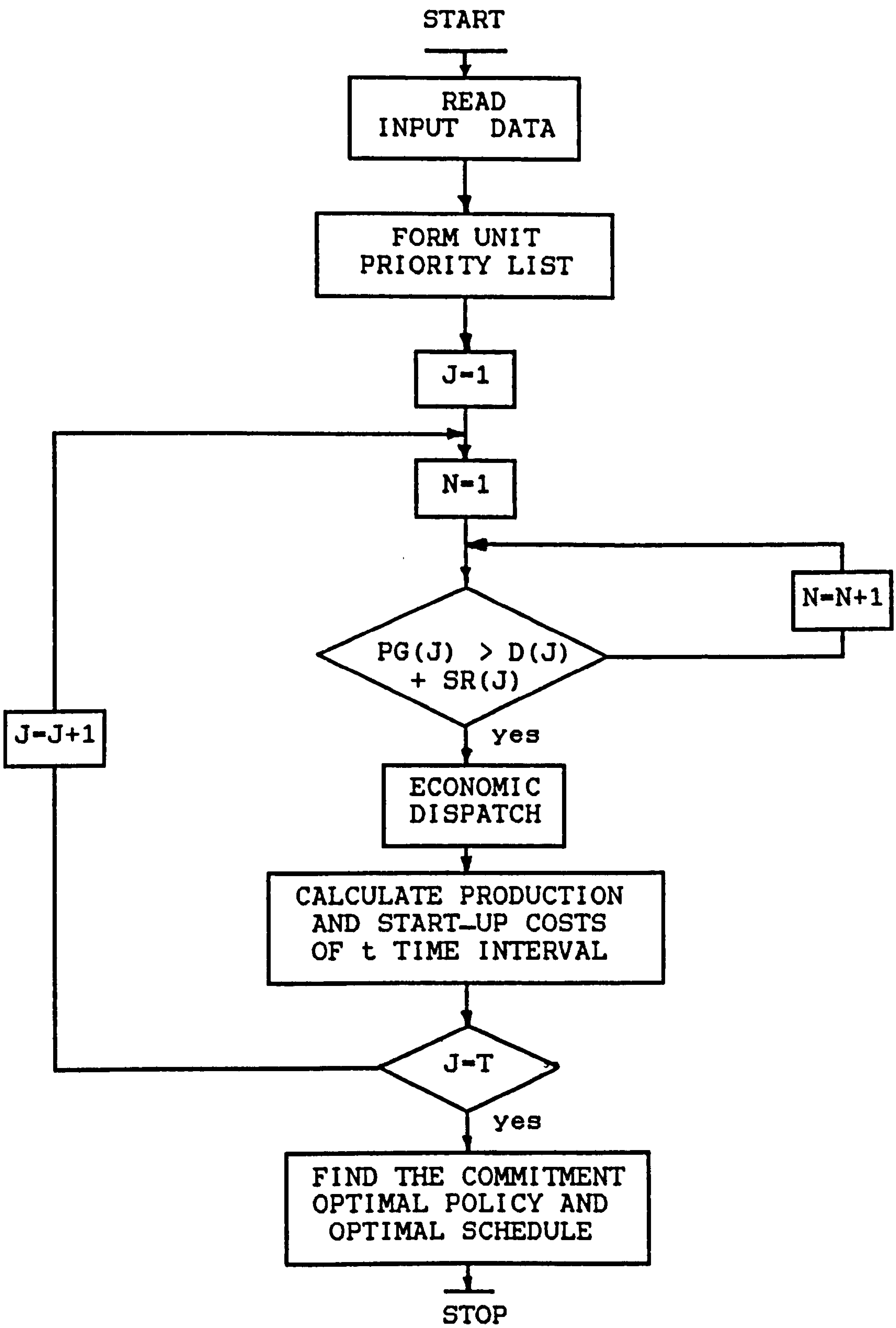


Figure (3.1): Flow chart of computer program for solving unit commitment problem by using heuristic method.

1- Step variation in the load curve, as shown in figure (3.2), i.e., the load demand is assumed to change only at the beginning of each interval, then to remain constant during the interval.

2- Deterministic behaviour of the daily load curve.

3- The shutdown cost of the unit (if it exists) is assumed to be constant.

4- Transmission losses are ignored.

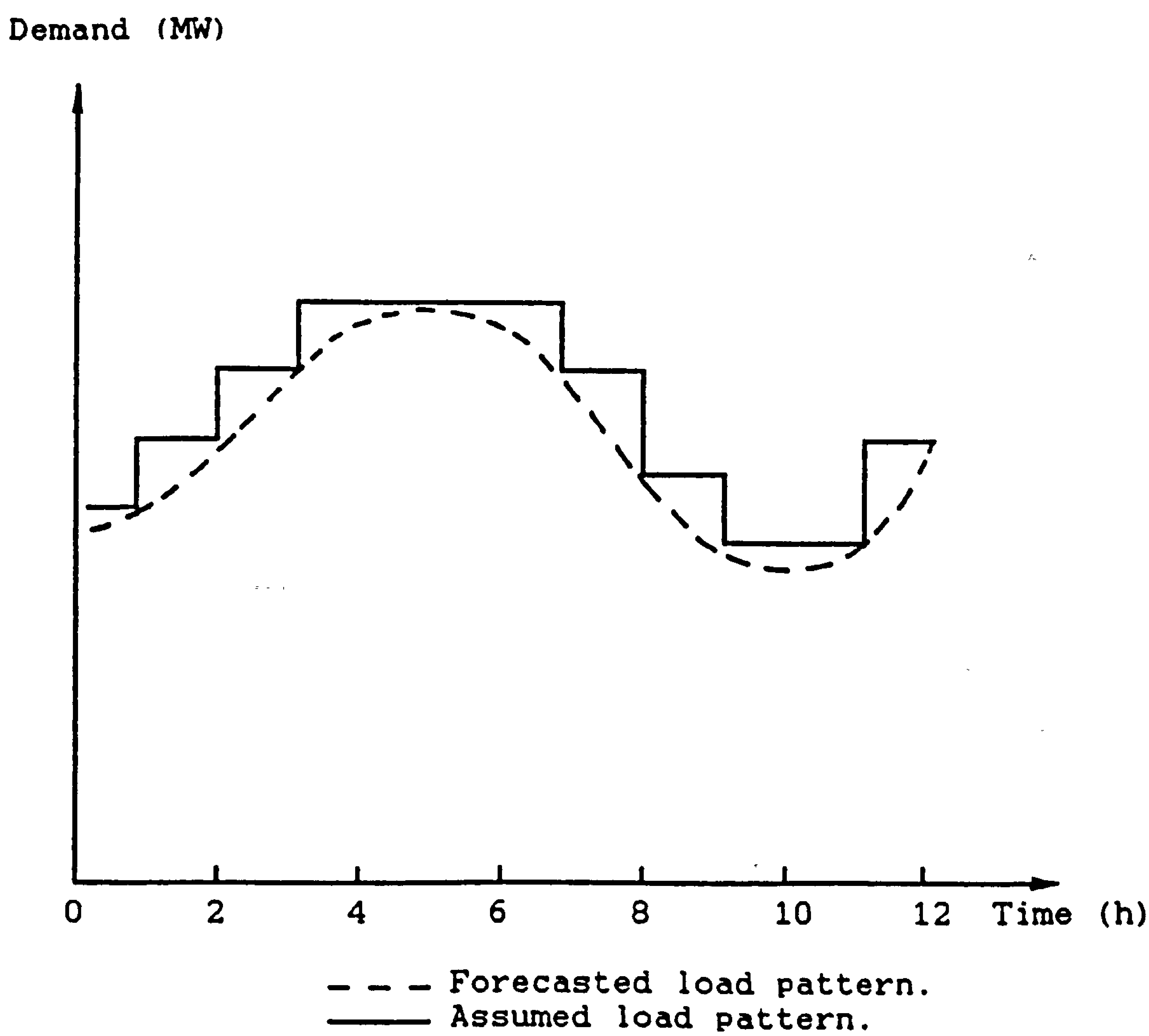


Figure (3.2): Step variation assumption of daily load demand.

In the following section, the results of the tested power systems are presented.

<u>4 UNIT SYSTEM</u>: Input data of the units and the system load demand profile are presented in tables (A-1) and (A-2) respectively. The system operational costs and other results of the commitment period are found in tables (3.1). The schedule of the units over twenty four hours is demonstrated in table (3.2).

<u>15 UNIT SYSTEM</u>: The spinning reserve of the system is assumed as a variable value depending on the demand. The input data of the generating units are found in tables (A-3) and (A-4). Data of one day load profile are listed in table(A-5). Operation costs and other output results of the system for 24 h are produced in tables (3.3) and (3.4). The schedule of units for the study period is illustrated in table (3.5).

<u>150 UNIT SYSTEM</u>: For the purpose of testing the heuristic methods on large-scale power system, the unit commitment problem is solved for a 150 unit system. Input data of the units are found in tables (A-6) and (A-7). Data of the system load demand over 24 h are shown in table (A-8). The output results are presented in tables (3.6) and (3.7)

Table (3.8) demonstrates the relationship between number of units in the system and the required computation time and computer memory storage space.

Table (3.1): Results of unit commitment for a 4 unit system.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	2	9208.36	9208.36	550.00
2	2	10648.36	19856.72	550.00
3	3	12265.36	32472.08	630.00
4	2	10828.36	43300.44	550.00
5	2	8308.36	51608.80	550.00
6	1	5573.54	57182.34	300.00
7	1	5748.14	62930.48	300.00
8	2	10108.36	73438.84	550.00
9	2	9028.36	82467.20	550.00
10	2	10468.36	92935.56	550.00
11	3	12056.56	105342.13	630.00
12	2	10918.36	116260.48	550.00
13	2	8488.36	124748.84	550.00
14	2	7948.36	132697.20	550.00
15	2	8128.36	140825.56	550.00
16	2	8668.36	149493.92	550.00
17	2	9208.36	158702.28	550.00
18	2	10108.36	168810.64	550.00
19	3	11743.36	180904.00	630.00
20	2	10648.36	191552.36	550.00
21	2	8632.36	200184.72	550.00
22	2	7588.36	207773.08	550.00
23	2	8398.36	216171.44	550.00
24	2	10108.36	226279.80	550.00

TOTAL START UP COST = £953.47

TOTAL GENERATION COST = £225326.33

TOTAL OPERATIONAL COST = £226279.80

* Total number of committed units.

Table (3.2): Unit schedule for 24 hours
(Unit status 1 = ON, 0 = OFF)
(4 unit system).

Time hours	Unit number and status 1	2	3	4
1	1	1	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	0	0
5	1	1	0	0
6	1	0	0	0
7	1	0	0	0
8	1	1	0	0
9	1	1	0	0
10	1	1	0	0
11	1	1	1	0
12	1	1	0	0
13	1	1	0	0
14	1	1	0	0
15	1	1	0	0
16	1	1	0	0
17	1	1	0	0
18	1	1	0	0
19	1	1	1	0
20	1	1	0	0
21	1	1	0	0
22	1	1	0	0
23	1	1	0	0
24	1	1	0	0

CPU TIME = 0.8400 SECOND

Table (3.3): Results of unit commitment solution by heuristic method (15 unit system).

Time (H)	No. of on-line units*	Hourly costs £	accumulated costs £	Generated power (MW)
1	5	10226.0	10226.0	2040.0
2	4	9314.1	19540.1	1840.0
3	4	8868.3	28408.4	1748.0
4	4	8598.4	37006.8	1692.0
5	4	8560.0	45566.8	1684.0
6	4	8675.4	54242.3	1708.0
7	4	9723.9	63966.1	1924.0
8	5	15393.9	79360.1	2136.0
9	5	12563.8	91923.9	2516.0
10	7	22276.7	114200.6	2856.0
11	8	19116.3	133317.0	3060.0
12	10	20746.4	154063.4	3252.0
13	10	16272.8	170336.2	3308.0
14	11	17937.9	188274.1	3404.0
15	11	16849.9	205124.0	3436.0
16	12	18532.4	223656.5	3476.0
17	12	16988.1	240644.6	3460.0
18	11	16715.7	257360.3	3400.0
19	11	16499.2	273859.5	3356.0
20	9	15829.9	289689.3	3220.0
21	9	15597.1	305286.5	3168.0
22	9	15481.7	320768.3	3144.0
23	5	12717.9	333486.2	2564.0
24	5	11822.1	334108.3	2352.0
TOTAL START UP COST = £4311.9				
TOTAL GENERATION COST = £329796.40				
TOTAL OPERATIONAL COST = £334108.30				

* Total number of committed units.

Table (3.4): Results of unit commitment solution by heuristic method (15 unit system).

Time (H)	No. of on-line units *	Available on-line generation	Load demand (MW)	Spinning reserve (MW)
1	5	2773.0	2040.0	733.0
2	4	2435.0	1840.0	595.0
3	4	2435.0	1748.0	687.0
4	4	2435.0	1692.0	743.0
5	4	2435.0	1684.0	751.0
6	4	2435.0	1708.0	727.0
7	4	2435.0	1924.0	511.0
8	5	2773.0	2136.0	637.0
9	5	2773.0	2516.0	257.0
10	7	3107.0	2856.0	251.0
11	8	3274.0	3049.0	225.0
12	10	3546.0	3252.0	294.0
13	10	3546.0	3308.0	238.0
14	11	3651.0	3404.0	247.0
15	11	3651.0	3426.0	225.0
16	12	3781.0	3476.0	305.0
17	12	3781.0	3460.0	321.0
18	11	3651.0	3400.0	251.0
19	11	3651.0	3356.0	295.0
20	9	3441.0	3216.0	225.0
21	9	3441.0	3168.0	273.0
22	9	3441.0	3144.0	297.0
23	5	2773.0	2548.0	225.0
24	5	2773.0	2352.0	421.0

* Total number of committed units.

Table (3.5): Unit schedule for 24 hours
(Unit status 1 = ON, 0 = OFF)
(15 unit system).

Time hours	Unit number and status of units 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
6	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
7	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
8	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
9	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
11	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
12	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
13	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
14	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
15	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
18	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
19	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
20	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
21	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
22	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
23	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
24	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0

CPU TIME = 1.1120 SECOND

Table (3.6): Results of unit commitment solution by heuristic method (150 unit system).

Time (H)	No. of on-line units*	Hourly costs £	accumulated costs £	Generated power (MW)
1	123	29335.0	29335.0	5416.0
2	118	28016.0	57351.0	5194.0
3	117	27753.9	85104.8	5150.0
4	116	27552.7	112657.6	5121.0
5	111	26206.0	138863.6	4892.0
6	102	24244.1	163107.7	4566.0
7	109	25806.8	188914.5	4821.0
8	117	27847.6	216762.1	5155.0
9	124	29624.7	246386.8	5451.0
10	124	29638.0	276024.7	5470.0
11	125	29803.3	305828.0	5488.0
12	125	29757.5	335585.5	5479.0
13	131	31455.4	367040.8	5760.0
14	137	32992.4	400033.3	6015.0
15	137	32949.7	432983.0	6018.0
16	130	31170.8	464153.8	5724.0
17	124	29556.4	493710.2	5450.0
18	118	27987.6	521697.8	5187.0
19	125	29886.9	551584.6	5495.0
20	124	29572.8	581157.4	5454.0
21	124	29597.2	610754.6	5460.0
22	123	29294.2	640048.8	5406.0
23	126	30141.0	670189.8	5546.0
24	123	29367.6	699557.4	5424.0

TOTAL START UP COST = £459.16

TOTAL GENERATION COST = £6990557.24

TOTAL OPERATIONAL COST = £699557.40

CPU TIME = 7.06 SEC.

* Total number of committed units.

Table (3.7): Results of unit commitment solution by heuristic method (150 unit system).

Time (H)	No. of on-line units	Available on-line generation	Load demand (MW)	Spinning reserve (MW)
1	123	5805.0	5416.0	388.9
2	118	5585.0	5194.0	390.9
3	117	5541.0	5150.0	390.9
4	116	5497.0	5121.0	375.9
5	111	5277.0	4892.0	384.9
6	102	4911.0	4566.0	344.9
7	109	5189.0	4821.0	367.9
8	117	5541.0	5155.0	385.9
9	124	5849.0	5451.0	397.9
10	124	5849.0	5470.0	378.9
11	125	5893.0	5488.0	404.9
12	125	5893.0	5479.0	413.9
13	131	6157.0	5760.0	396.9
14	137	6421.0	6015.0	405.9
15	137	6421.0	6018.0	402.9
16	130	6113.0	5724.0	388.9
17	124	5849.0	5450.0	398.9
18	118	5585.0	5187.0	397.9
19	125	5893.0	5495.0	397.9
20	124	5849.0	5454.0	394.9
21	124	5849.0	5460.0	388.9
22	123	5805.0	5406.0	398.9
23	126	5937.0	5546.0	390.9
24	123	5805.0	5424.0	380.9

Table (3.8): Final results of unit commitment for three different power systems by heuristic method.

Number of units	Start up cost £	Generation cost £	Total operational costs £	CPU Time Seconds	Computer storage space K Bytes
4	953.47	225326.33	226279.80	0.84	3.2
15	4311.9	329796.40	334108.30	1.11	9.7
150	459.16	699098.24	699557.40	7.06	78.8

3.4 DISCUSSION:

Heuristic methods have the ability to overcome the dimensionality of the unit commitment problem which could arise if other mathematical techniques are applied. Therefore, heuristic methods may be used to solve the unit commitment regardless of the system size. In addition, it has the following advantages [1,36,63]:

i) Heuristic approach is simple in term of algorithm preparation and program development.

ii) All the operational constraints of the system and the units can be considered and satisfied.

iii) Feasible solutions are usually obtained.

iv) The solutions are economically reasonable.

v) The computational requirements in terms of computer memory and running time are acceptable, as can be seen in table (3.8).

The main shortcoming of the heuristic methods is that they cannot guarantee optimal solution [36]. This aspect becomes rather important especially in large scale power systems, where a small percentage reduction in the cost of power production represents a significant amount of annual financial saving. Consequently, it is worthwhile to investigate and search for other alternative techniques, even if sophisticated, which are capable of achieving more rigorous economical solutions within acceptable computational efforts.

CHAPTER 4

UNIT COMMITMENT SOLUTION BY MATHEMATICAL PROGRAMMING

4.1 INTRODUCTION

Over the last three decades mathematical programming has become a tool widely used in the problem of decision making. The development of digital computers generated a great deal of interest in mathematical programming throughout various branches of businesses.

The term "programming" in these expressions has a different meaning from that in the phrase "computer programming". Mathematical programming is a technique for determining the values of a set of decision variables which optimize a mathematical objective function, subject to a given set of constraints. It is the application of scientific methods, techniques and tools to problems involving the operation of a system in order to achieve optimal solutions. This approach consists of the following steps:

- Understanding and describing the system.
- Building a model for the real-life system.
- Using the model as a basis for predicting future situations.

Mathematical programming includes linear programming, nonlinear programming, integer programming, dynamic programming, and other variants of programming problems [3].

In the application of mathematical programming to unit commitment problem, dynamic programming and mixed integer programming are the most widely used. Therefore, these techniques will be explained in the following sections.

4.2 DYNAMIC PROGRAMMING

Dynamic programming is a mathematical optimization technique used for making a series of interrelated decisions. Usually, a multi-stage decision process is transformed into a series of single-stage decision processes. This algorithm is based on the principle of optimality formulated by R. Bellman in 1957, which states that " an optimal sequence of decisions has the property that whatever the initial state and decisions are, the remaining decisions must constitute an optimal decision sequence with regard to the state resulting from the first decision "[1,2].

Dynamic programming starts with a small part of the problem and finds the optimal solution for this smaller problem. It then gradually enlarges the problem by finding the solution to the enlarged problem, based on the previous one. This is continued until the entire problem is completely resolved.

In contrast to other mathematical programming techniques (such as linear programming), there does not exist a standard mathematical formulation of the dynamic programming problem [2]. Dynamic programming is a general strategy for optimization rather than a specific set of rules. Consequently, particular equations used must be developed to fit each problem. The nature of the decision variable identifies the type of the problem. The problem is said to be continuous if the variable can take only real value, while it is said to be discrete if the decision variable is restricted only to integer values.

Dynamic programming is essentially a recursive form of the optimization technique. The typical dynamic programming recursive function can be expressed as follows;

$$F(J,K,X) = Z[C(J,K,X) , f_{J-1}(K')] \quad ...... \quad (4.1)$$

where;

F : the cost function.

J : the stage of the problem.

K : the state of the system at stage J.

X : the decision (policy) being evaluated at stage J.

C(J,K,X) : the immediate cost associated with making decision X at stage J when the state of the system is K.

K' : the state of the system at J-1 stage resulting from decision X.

$f_{J-1}(K')$: the cost associated with the optimal sequence of the decision at stage J-1 when the state is K'.

$f_{J-1}(K')$ will be, in most cases, added to or multiplied by C(J,K,X). At each stage, the results of the recursive formulation are calculated for all the feasible values of X subject to the constraints of the problem, and the optimal decisions are retained for subsequent use.

Dynamic programming procedure proceeds as illustrated in the flow diagram in figure (4.1) and can be summarized as follows:

1) Formulation of the recursive function and all constraints.
2) Searching for the optimal value of $f_1(K)$ and $X_1(K)$ for J = 1 . The optimal value of $f_J(K)$ will be retained for use during the next stage, while the value of $X_J(K)$ will be retained but will not be

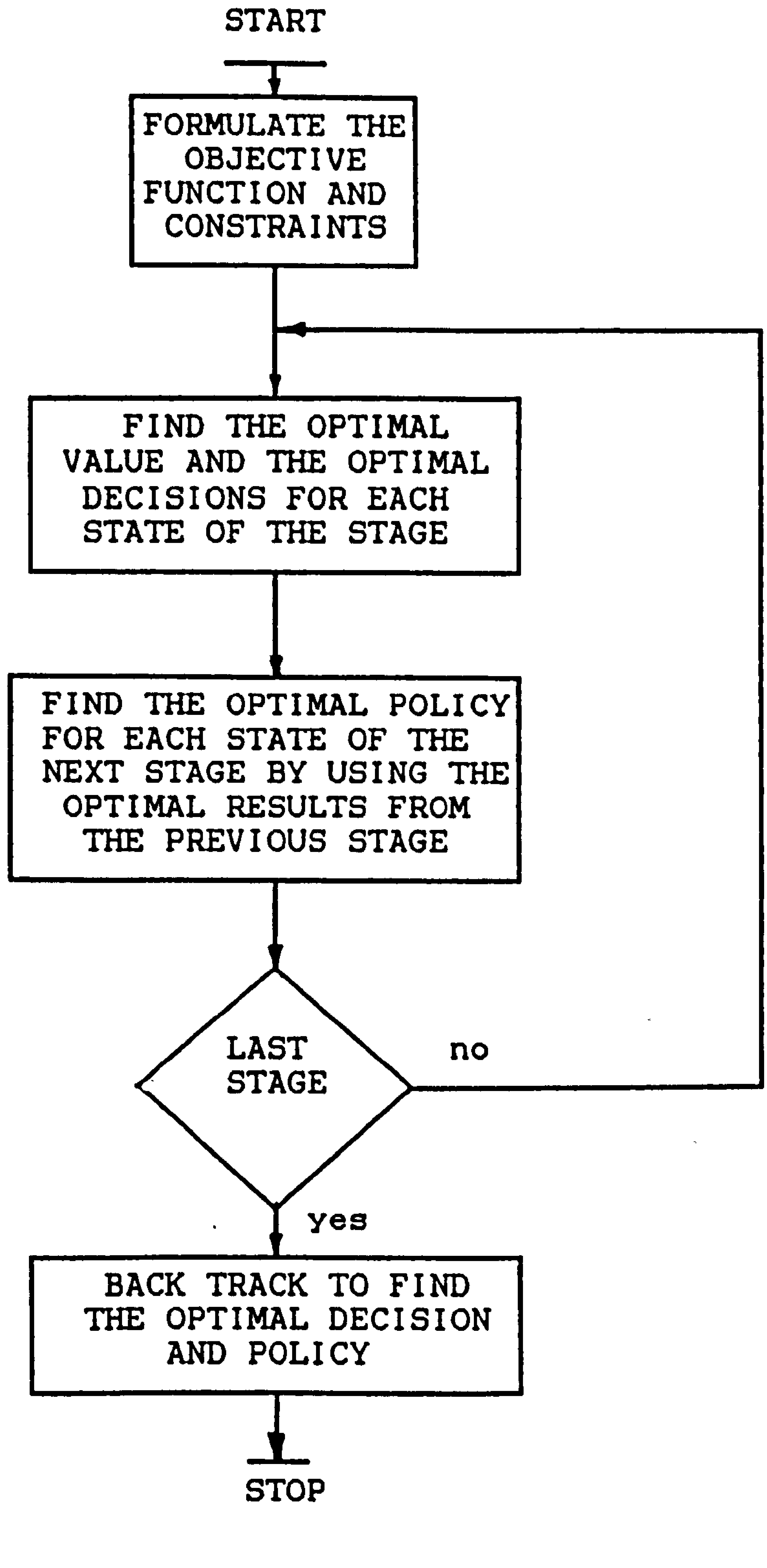


Figure (4.1):
Flow chart of the dynamic programming algorithm.

used until the final stage computations are completed.

3) When the final stage of calculation is completed, the solution procedure then moves backward, stage by stage to obtain the optimality decision policy for each state of each stage until the optimal value of $f_J(K)$ and decision $X_J(K)$ are found.

4.2.1 UNIT COMMITMENT BY DYNAMIC PROGRAMMING

In recent years dynamic programming has been recognized as an effective technique for obtaining the optimal solution to the unit commitment problem [22]. It offers considerable improvement over the priority order method. One of the earliest attempts to solve the unit commitment by using dynamic programming was introduced by Lawery [18]. The problem was solved by testing all the possible combinations of the generating units in order to find the optimal solution for a 14 unit system. Pang and Chen [25] proposed a new technique to reduce the search range of the solution by classifying the units in the system into different categories. Consequently, the method could be applied to a larger system, where a system of 17 units was tested. Dynamic programming was combined with mixed integer programming [28] in an attempt to reduce the computation time. Yamayee et al [40] suggested a hybrid form of dynamic programming and the branch and bound approach to reduce the computation requirements. Different approaches of dynamic programming which are applied to the unit commitment problem have been presented and described in [32]. Further improvements and developments of the dynamic programming methods and their application to the unit commitment problem can be found in [29,46,62,75]. The basic dynamic programming technique as applied to the unit commitment has been described in [1,25]. Before describing the procedure, the following terms are defined:

COMBINATION : A combination is any subset of a given set of units, e.g., if there are N units in the system, $(2^N - 1)$ different combinations of units can be found.

STRATEGY: A strategy denotes the transition or path from one combination at a given interval to another combination at the next interval, and the optimal strategy is the transition or the path to the optimal combination at the next interval.

In dynamic programming, the unit commitment solution can be divided into two major parts. The first covers the formation of a unit selection list, (priority list), while the other part involves a search technique which determines the optimal (i.e. least total cost) or near optimal feasible schedules for the available units of the system during a given study period [22,25].

A priority list of the units is formed, as described in chapter three, in order to reduce the number of combinations of units which are to be examined at each interval. The possible combinations of units at each interval would result in 2^N-1. Hence, for practical purposes, some form of limit is necessary. Some of these combinations can be discarded without the necessity of full consideration as they do not meet the system constraints. However, for a large system, a considerable number of feasible combinations will remain. Consequently, the size and computational requirements of the problem cover a wide space in a computer memory and could exhaust the capabilities of even the largest computers [25].

To control the size of the problem, units in the system can be categorised, as illustrated in figure (4.2) into the following groups:

i) Unavailable units.
ii) Base units.
iii) Must run units.
iv) Search range units.

Unavailable units are the units under scheduled maintenance course or forced outage. Base units are the most efficient as well as the largest capacity units. Must run units are next in priority to the base units. However, because of their complicated process in the start-up and the shut-down, these units cannot be included in the search range class. Therefore, a must-run status can be enforced on them. The search range units are to be arranged in order of priority and may be divided into three groups, usually threshold, window and excess units [46]. Each group can be specified by the demand level of each interval and may be defined as:

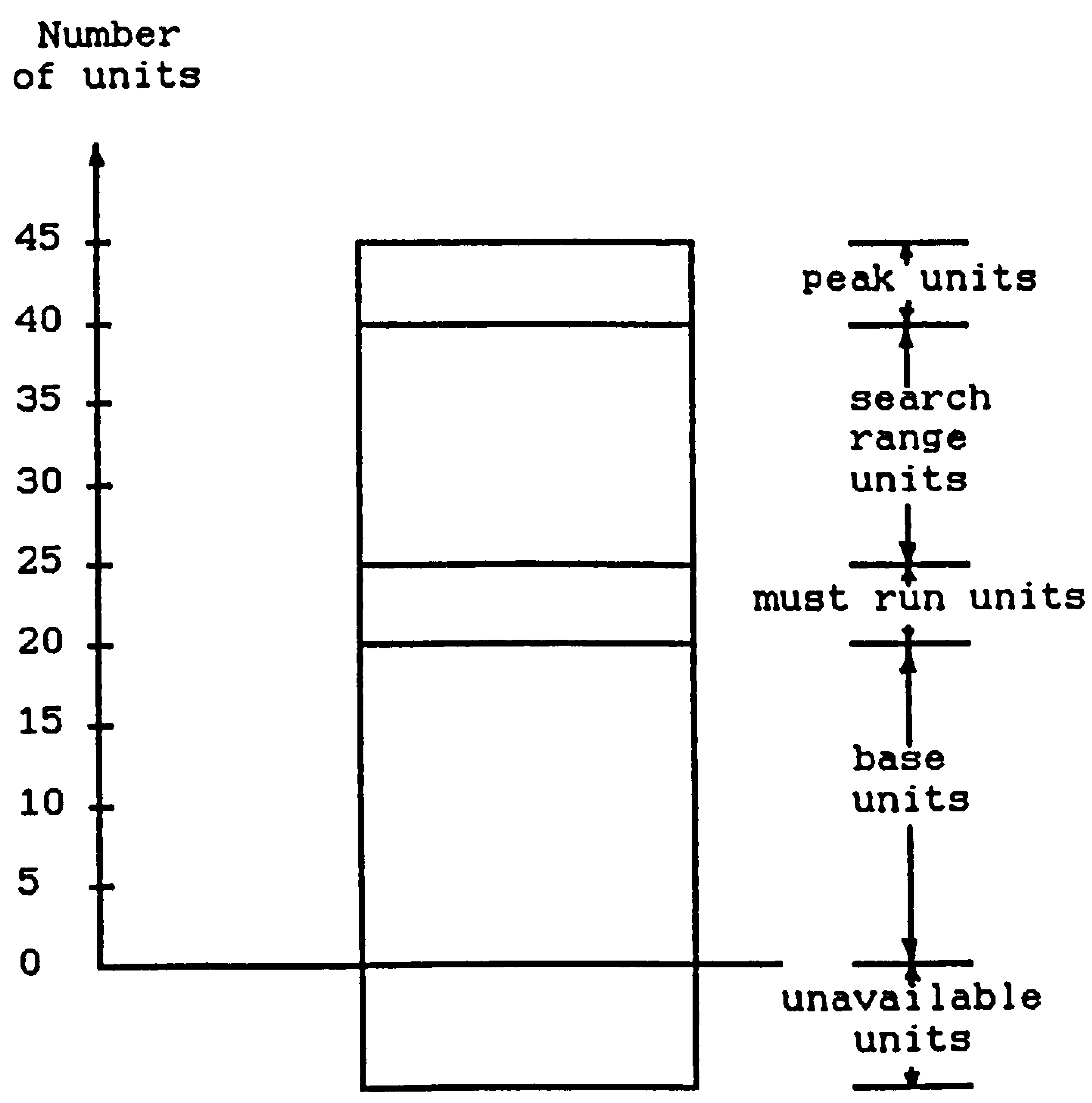


Figure (4.2): Unit categories block diagram.

THRESHOLD : In addition to the base units and the must run units, these units have been ordered as the most efficient in the priority list and also have a top priority to be committed in order to satisfy the system constraints. The threshold units could change from one interval to the next according to the rate of change in load demand.

WINDOW: Window units are the next highest priority in the search range after threshold units. These units may or may not need to be committed. They can also change from interval to interval depending on the load demand variations.

EXCESS : These are available units in the search range, but are not included in the threshold and in the window. These units are relatively inefficient and have low priority in the search range units. Therefore, there is no advantage in committing them except as standby units, for example as peaking units during off-peak demand.

An example of a power system with 45 available units is illustrated in figure (4.2). If dynamic programming is used to solve the problem of unit commitment for this system, then the total number of possible combinations is $2^{45} - 1 = 3.5 \times 10^{13}$. However, this number can be reduced by classifying the units as illustrated in figure (4.2) to $2^{15} - 1 = 32767$ combinations, since only 15 units are included in the search range by dynamic programming. This procedure can significantly reduce computational requirements. However, it is only suitable when the variations in load demand are not very large. This method could lead to suboptimality, if the rate of changes in the demand is large, due to the small search range of units.

The costs in each interval will be computed for each feasible combination.

The second part of the solution is the back-track operation. It takes place after the first part is completed. Back-track procedure starts from the final interval going back to the initial stage of the problem. At each stage, the feasible combination of units associated with the lowest cost must be selected.

In unit commitment problem, due to the nature of the decisions, the discrete dynamic programming technique is applied. This technique determines the optimal sequence of decisions in a multistage decision process. The decisions are restricted to integers and the number of sequential decisions must be specified. Constraints are considered and satisfied. The objective function of the unit commitment problem can be expressed mathematically as follows [1,25,30] :

$$Ct(T,S) = \underset{\{LL\}}{Min}\{I_{B}(M-K,N).[G_{ct}(T,S) + IC(T,N).Suc_{t}(T-1$$

$,S;LL:T,S')+IC(T,N).Sdc_t(T-1,S:LL:T,S)\}$

$$+ Ct(T-1,S)] \quad \ldots\ldots\ldots \quad (4.2)$$

where;

LL = $I_A(M,N)$, which is the commitment matrix of all possible combinations of the units in the system with the elements of 1 if the unit is ON and 0 if the unit is OFF, where N represents the available units in the system and $M = 2^N - 1$, which represents the total possible combinations of the units.

$I_B(M-K,N)$: represents the number of feasible combinations of units, where K is the number of infeasible combinations in $I_A(M,N)$ during the interval T.

$I_C(T,N)$: a vector which indicates the status of units.

Minimization of the operational cost of a power system represented by equation (4.2) can be performed by using discrete dynamic programming as follows;

1) Formulate $I_A(M,N)$ commitment matrix as a function of the number of units.

2) At the beginning of each interval, formulate the feasible combinations of the units $I_B(M-K,N)$ for the interval T from $I_A(M,N)$ as a function of the load demand and spinning reserve of the system.

3) At the first interval of the commitment period, the generation cost of each feasible combination is calculated as a function of the generated power of each unit. The feasible combination associated with the lowest cost is selected as the optimal combination. Start up and shut down costs are ignored at the first interval.

4) At the second interval and the subsequent intervals, the $I_c(T,N)$ vector is produced to monitor the status of the units and to update the down and up time of the units throughout the commitment period as follows:

I=1 to N

If $I_c(T,I)= 1$ and $I_c(T-1,I)= 1$, then unit i keeps the ON status and the up time of the unit is increased by 1.

If $I_c(T,I)= 0$ and $I_c(T-1,I)= 0$, then unit i keeps the OFF status, and its down time is increased by 1.

If $I_c(T,I)= 1$ and $I_c(T-1,I)= 0$, then unit i is started up at the interval T and its startup cost is calculated as a function of the time for which the unit was down. The up and down time of the unit are updated.

If $I_c(T,I)= 0$ and $I_c(T-1,I)= 1$, then unit i is shut down at the interval T and its shut down cost, if any, is added to the combination operating cost and the up and down time of the unit are updated.

5) At any interval, the combination which meets all the constraints with the lowest cost is assigned as the optimal combination and the transition cost to this combination is the optimal strategy.

6) The total optimal cost of the study period consists of the accumulated costs resulting from all optimal combinations and all optimal strategies specified throughout the period. In other words, it represents the summation of the least cost combination in each interval plus the lowest transition costs to that combination.

The procedure of solving the unit commitment problem by using full dynamic programming is illustrated in the flow chart diagram of figure (4.3), and it is demonstrated by the example of 3 units, as in figure (4.4).

Equation (4.2) can be optimized and solved by implementing the appropriate approach of dynamic programming from the following [18]:

i) **Full Dynamic Programming (FDP)** : In this technique all the possible combinations are tested during the search for the optimal solution. For example, if a system of 10 units is considered then the number of possible combinations is $2^{10}-1$ (i.e. 2023 combinations). This

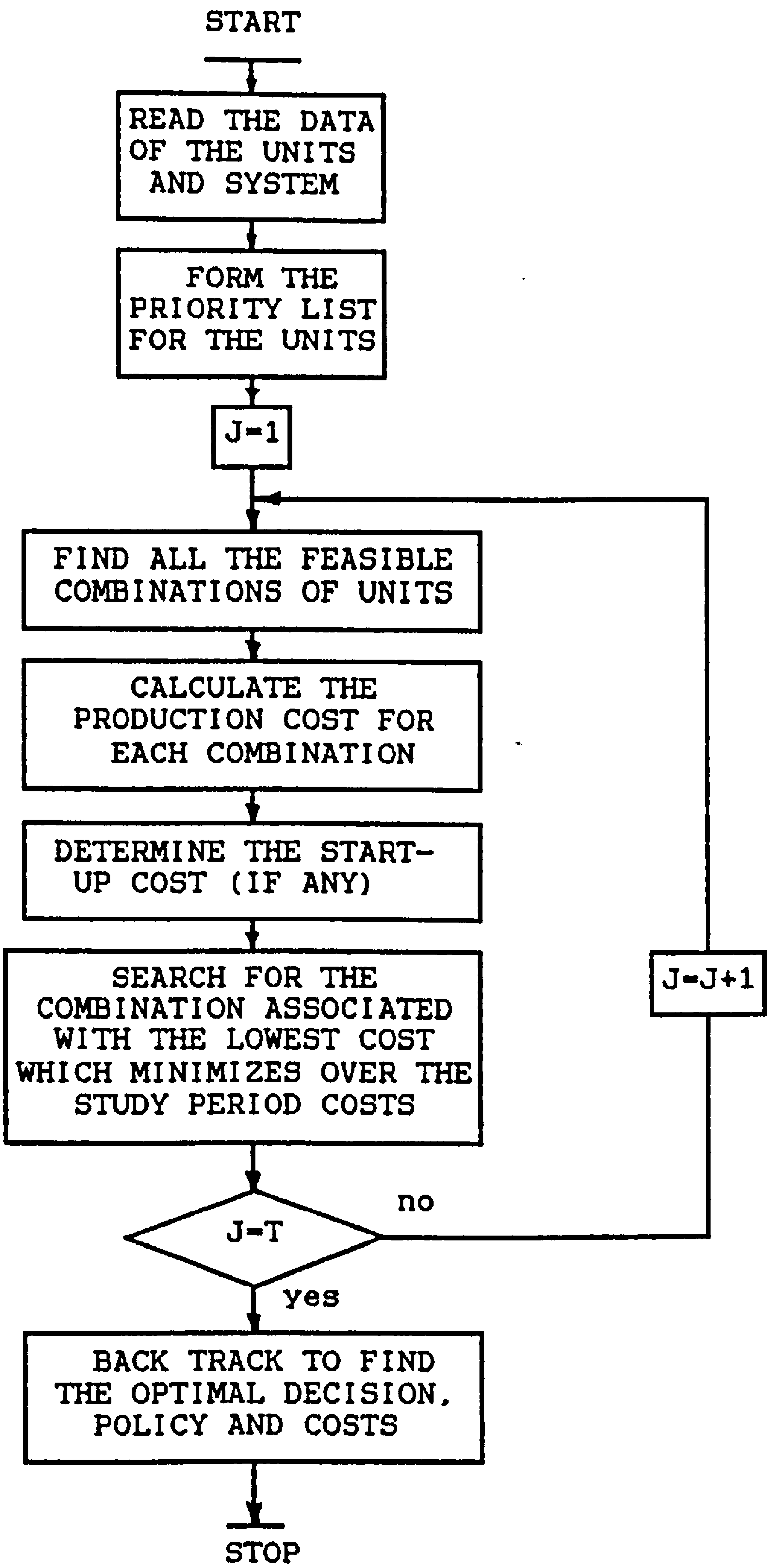


Figure (4.3): Flow chart of program for solving the unit commitment by dynamic programming.

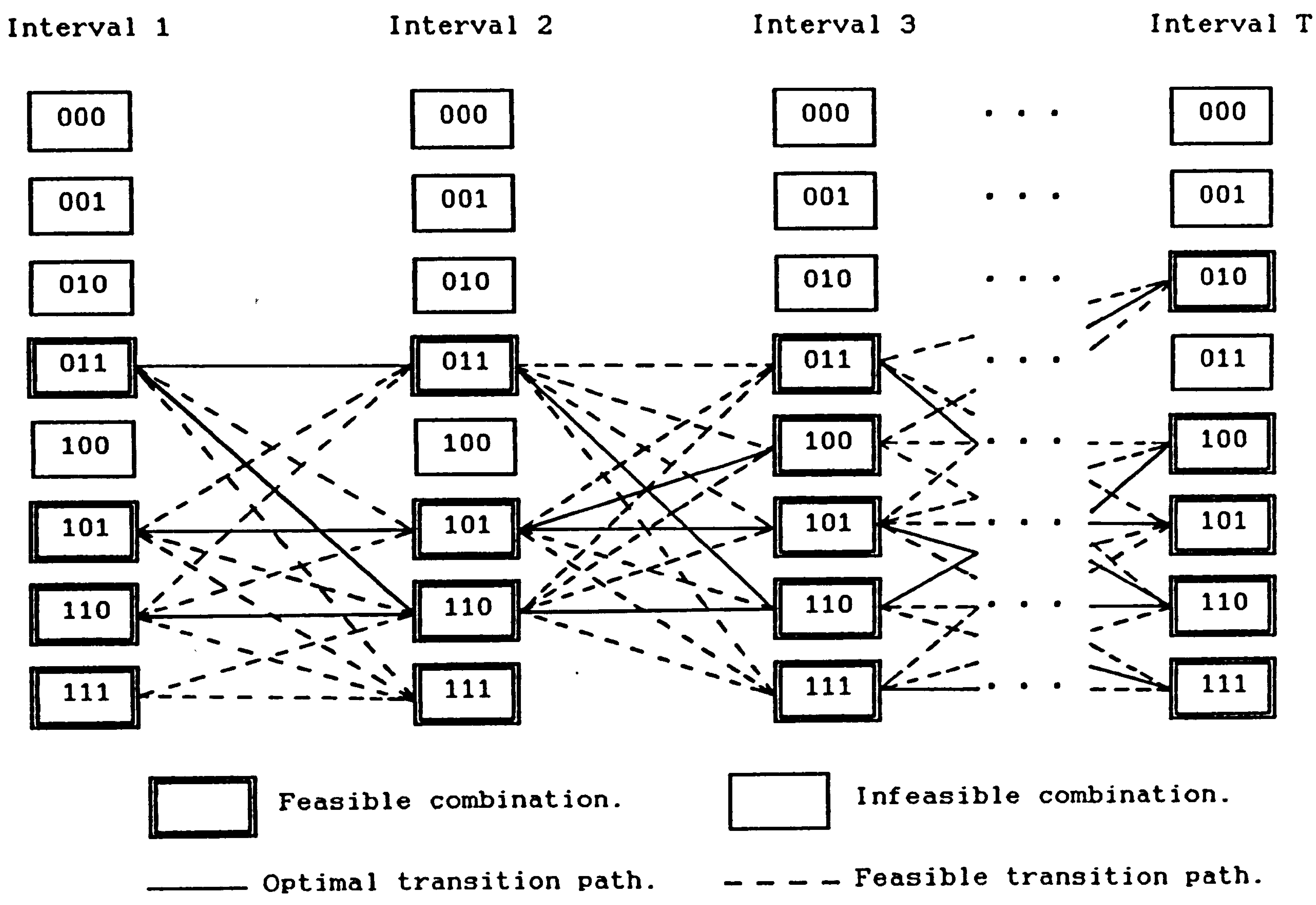


Figure (4.4): State-space diagram of the FDP for a 3 unit system.

number becomes very large if the number of the units in the system is in the range of 20 or more. Consequently, the computation requirements can increase rapidly and probably go beyond the practical limit of computing facilities. Therefore, full dynamic programming is only applicable to small power systems.

ii) Dynamic Programming Sequential Combinations (DPSC):
The DPSC is started by preparing a priority list of the units, and from the list a subset of the combination is generated by starting up each unit in the priority list in a sequence until all the operational constraints are satisfied. The infeasible combinations are discarded and a search for the optimal solution is performed on the feasible combinations. If a system of 5 units is tested, only 5 combinations can be evaluated as illustrated in table (4.1).

Table (4.1): Number of possible combinations for 5 units by the DPSC method.

Number of combinations	Unit number and status (1 = ON, 0 = OF)				
	1	2	3	4	5
1	1	0	0	0	0
2	1	1	0	0	0
3	1	1	1	0	0
4	1	1	1	1	0
5	1	1	1	1	1

iii) Dynamic Programming Truncated Combinations (DPTC): This approach is based on the priority list and on full dynamic programming. The idea is to commit the higher units in the priority list to meet the base load and to limit the search range only to the cycling units. Therefore, the combination of the search range units will be evaluated.

Further details of dynamic programming are found in appendix C.

4.3 MIXED INTEGER LINEAR PROGRAMMING (MILP)

Linear programming is applied to many real-life problems. However, it was found that it was not suitable for some cases of the decision making problems, particularly if the decision variables are to be integers rather than continuous. For example, if the decision variable is a number of employees or machines, in this case there will be no fractional portion and the results must be integers. The integer restriction appears to be a modification of linear programming which is called integer programming. The first successful method for solving the integer programming problem was suggested by Gomory (1958), and Land and Doig (1960). Since then, many other methods of solution have been introduced [3].

Integer programming is a programming operation in which all the quantities must be integer variables. In practical problems, it is more common that there are both continuous variables and integer variables. Such a model is called a mixed integer programming problem.

There are many special types of integer programming problems. However, in this study, a category of mixed integer linear programming is considered where both integer and continuous variables are allowed along with continuous objective function. Integer programming problems are more complicated to solve comparing with the corresponding continuous problems, regarding the cost and the existence of a solution [23]. Branch and bound method [26-28] is claimed to be one of the most appropriate approaches for solving this type of problems, However, it does not always guarantee optimal solution [23].

4.3.1 UNIT COMMITMENT SOLUTION BY MILP

At the earliest attempts to solve the unit commitment problem, it was formulated as an integer programming problem by Garver [17]. He tested two thermal generating units. Muckstadt and Wilson [21] proposed a theoretical application of mixed integer linear programming in order to produce an optimal schedule for thermal units. Dillon et al [26] modified the method of solution with integer programming by implementing the branch and bound approach to simplify the solution method and to reduce the computational requirements. Hamam et al [30] formulated the unit commitment problem in a mixed integer form, taking into account the linear and nonlinear fuel cost relations. The cost function was minimised, subject to the operating constraints, by a method of solution based on the branch and bound capaciated transhipment algorithm. 50 thermal generating units were examined. Further applications of mixed integer linear programming to the unit commitment can be found in [36-39,41].

Unit commitment problem has been formulated in a mixed integer linear programming form [24-26] so that:

1- Unit status ON and OFF are represented by 1-0 integer values respectively.

2- The output power of each unit and forecasted demand take integer variables.

3- The start-up, shut-down and no-load costs can also be represented by integer values.

4- The total cost is calculated as a continuous variable.

In the unit commitment problem formulation by MILP, the most important condition which must be satisfied is that the input-output characteristics of the units in the system have to be linear functions . If they are not, a linearization of the unit characteristics is required.

A piecewise linearization of unit input-output characteristic results in more accurate approximation. However, it may lead to the following disadvantages:

1- The increasing sophistication of the problem, where each segment of the input-output characteristic is to be treated as an individual unit in the system. Consequently, the constraints number becomes very large, particularly for large systems.

2- The optimal solution may not be expected, due to the linearization approximation.

The total operation cost of the power system for one interval can be represented by;

$$C_t = \sum_{i=1}^{n} A.Gc_{it}(p_{it}) + \sum_{i=1}^{Ns} B.Suc_{it}$$

$$+ \sum_{i=1}^{Nd} E.Sdc_{it} \qquad \text{.........} \qquad (4.3)$$

where;

A = 1 if unit i is ON.

= 0 otherwise.

B = 1 if unit i is to be started at the interval.

= 0 otherwise.

E = 1 if unit i is to be shut down at the interval.

= 0 otherwise.

The first term of the right hand side of equation (4.3) represents the production cost of the power by the committed units. It can be expressed as:

$$Gc(p) = \text{Min} \sum_{i=1}^{n} h.(Ihr_i .P_i) \quad .... \quad (4.4)$$

Subject to the system and units constraints.

The solution procedure of equation (4.3) is divided into two parts. The first one is to find the optimal feasible combination of the units as a function of their output power. The second part is to add the associated start-up and/or shut-down costs to the generation cost in order to obtain the total costs. Overall optimization of the objective function is performed and the feasible combination which satisfies the minimum total cost will be assigned as the optimal committed combination of the interval. In practice, start-up and shut-down costs are relatively small compared with the total operating cost, therefore the approximation for these costs will not significantly affect the solution [15]. On the other hand, this will remarkably reduce the computation requirements. It has been suggested that these costs could be averaged over an estimated operation period and then combined with the running cost of the unit.

Solution procedure is performed as shown in the flowchart in figure (4.5). For solving unit commitment problem by MILP, the branch and bound procedure [15] is implemented.

The basic idea of the branch and bound method is that the feasible region of the linear programming problem is partitioned into subsets. Upper bound of each objective function of each subset is obtained so that the integer constraints are satisfied. The subset which meets the problem requirements and gives the best solution is selected. The associated value of the objective function is assigned as the optimal solution.

In the branch and bound technique application to unit commitment problem, instead of restricting h in equation (4) to the binary states 0 or 1, a third state for the unit is allowed so that [15]:

$$0 \leq h^*_i \leq 1 \qquad \ldots\ldots\ldots \qquad (4.5)$$

The value of h* represents the unit contribution of

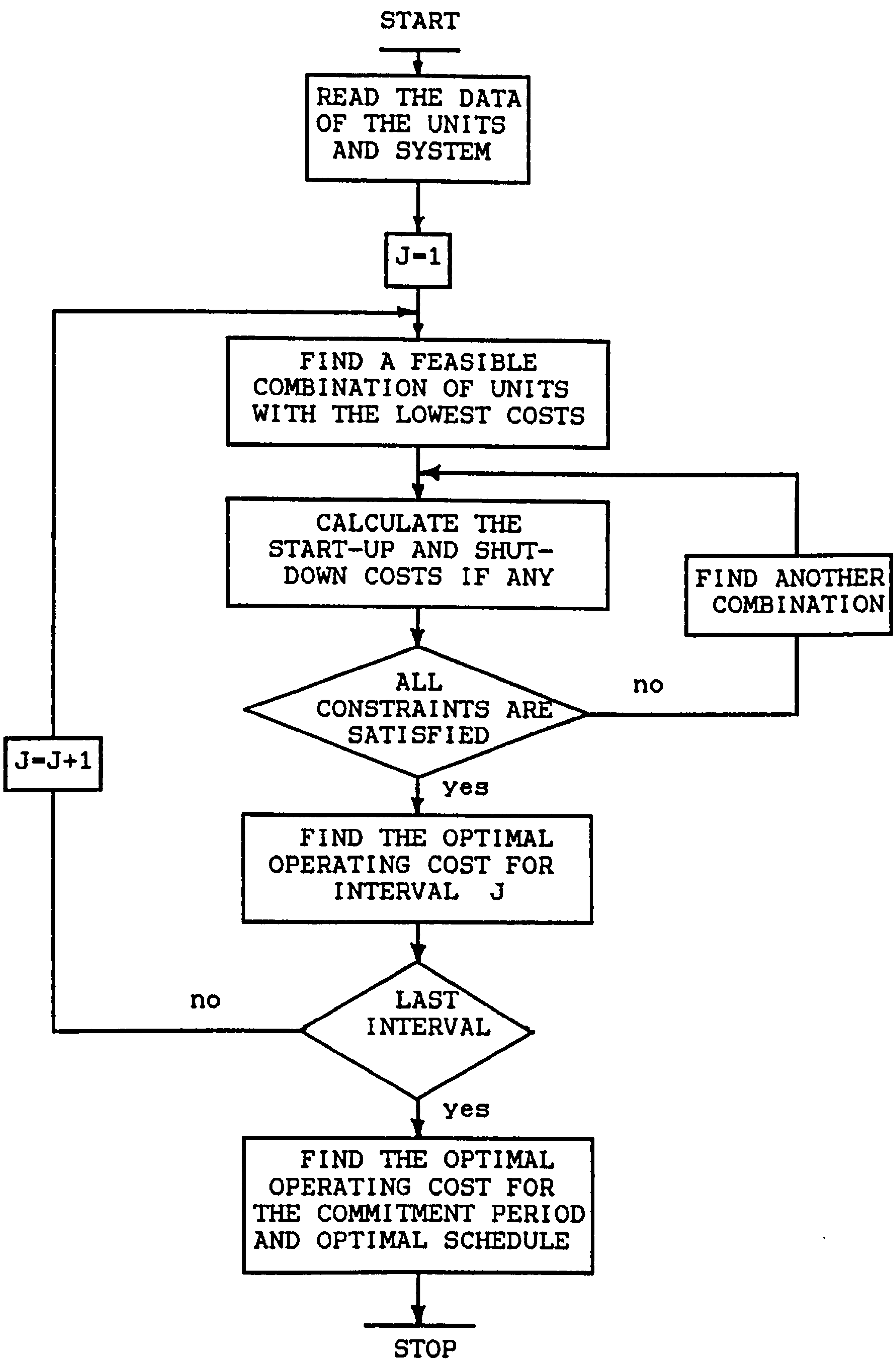


Figure (4.5): Flow chart of program for solving the unit commitment by mixed integer programming.

the power. Therefore, if $h^*=1$ then the unit operates at its full-load. If $h^*=0$, then the unit is OFF. If h^* has any value between 0 and 1, then its value multiplied by the unit full-load results in the unit output power. (E.g. for $h^*= 0.5$, the unit output will be 50 % of its full-load capacity)

During the optimization procedure, if h^*_i is assigned the value 0 or 1, then that value is considered fixed and no more manipulation is required. In order to produce a feasible solution, many variables of h^*_i are fixed while some variables are allowed to vary between 0 and 1. MILP by branch and bound method proceeds as follows [15];

1- Start with initial solution S^o with the cost C^o corresponding to a set of unfixed variables h^{*o}.

2- Near optimal solution S^1 with the cost C^1 is obtained by fixing a set of h^{*1}.

3- C^1 is specified as the least costly integer solution found so far and represents the current optimum solution C^*.

4- The procedure continues by following the tree of all possible states such that only the cheapest branch and bound at each node is followed.

5- All infeasible solutions and solutions with the costs C_i , where $C_i \geq C^*$ are eliminated.

6- At each node, if the solution S_i with the cost C_i is less than S^*, then C^* is updated to C_i. Otherwise, S_i is rejected and the corresponding node is discarded from the tree.

7- The process is repeated for the whole remaining tree starting from the node corresponding to C^* until no node with a cost less than C^* can be found.

The program is executed in the following main steps:

i)- Integer linear programming objective function is solved and an optimal solution of the running cost of a committed combination is obtained.

ii)- All units and system constraints are checked.

iii)-If all the constraints are satisfied, then the necessary decision of starting up or shutting down any units is taken. The associated cost of the decision will be added to the operating cost. The total cost is assigned as the optimum cost.

iv)- If any constraint of any unit or the system is violated, the combination is considered as infeasible. Then go to step (i).

v)- The optimum schedule of units for the interval is produced.

vi)- Steps from (i) to (iv) are repeated for all the time intervals of the study time horizon.

vii)-The total optimal operating cost of the power system is found by adding the least cost of each interval to the cost of the previous one.

viii)- The results are printed and the units status table which leads to optimal unit commitment is presented for the whole study period.

4.4 COMPUTER SIMULATION AND TEST RESULTS

For the purpose of testing the application of the mathematical programming methods, which have been discussed in this chapter, to the unit commitment problem, the systems described in appendix A have been examined by using dynamic and mixed integer programming techniques. The results are presented in the following section.

4.4.1 DYNAMIC PROGRAMMING TEST RESULTS

A computer program in FORTRAN 77 has been prepared and developed for solving the unit commitment problem by using the dynamic programming technique. In order to observe the performance of the program, the three power systems of different sizes have been examined over a study period of 24 hours. These systems are described in

Appendix A. The output results of the program are outlined in the following section.

i) <u>4 unit system</u>: Input data of the units are found in table (A-1). The data of the load demand profile of the system over 24 hours are listed in table (A-2). Full dynamic programming approach is applied to this system. The output results of the system operating costs and the schedule of the units throughout the commitment period are produced in tables (4.2) and (4.3) respectively.

ii) <u>15 unit system</u>: Input data of the generating units are presented in tables (A-3) and (A-4). Data of one day load profile are listed in table(A-5). This system is tested by three different methods of dynamic programming. In the first method, a full dynamic programming approach is used. Output results are produced in table (4.4) and table (4.5). In the second method, dynamic programming sequential combination technique is implemented. The results of unit commitment by this method are presented in tables (4.6) and (4.7). Dynamic programming truncated combination technique is employed in the third method and the results are found in tables (4.9) and (4.10).

<u>150 unit system</u>: In order to examine the application of dynamic programming to a large scale power system, a 150 unit system has been tested. However, due to the dimensionality problem, dynamic programming sequential combination technique is the only dynamic programming method

which can be used to solve the unit commitment problem for large scale power systems. Input data are found in tables (A-6), (A-7) and (A-8). Output results are printed in table (4.8).

4.4.2 MILP TEST RESULTS

Unit commitment problem of the power systems, described in the previous section, has been solved by using mixed integer linear programming. Two computer programs in FORTRAN 77 have been developed to solve the problem. The first program utilises the NAG library [84]. Routine H02BAF has been used to solve the pure integer programming problem with all integer coefficients by using Gomory's method, enhanced by including the technique of Wilson's cut. The results obtained by this program for the unit commitment problem are shown in tables (4.11), (4.12), (4.13) and (4.14). This program was found not efficient enough to solve the unit commitment problem because of the following reasons:

1- It is limited only to small and medium systems because of the dimension restriction imposed by the NAG library.

2- It is restricted to pure integer variables.

Therefore, it was necessary to develop another program with the capability to solve the unit commitment problem for any size of power system and also to accept both integer and real variables. The solution procedure in [2] has been adopted and modified in order to fit the unit commitment problem. The second program was developed and tested on the case study systems. The results of the mixed integer second program are presented in tables (4.15), (4.16) for the 4 unit system, (4.17), (4.18) for the 15 unit system and table (4.19) for the 150 unit system respectively. Table (4.20) illustrates the final results of the unit commitment problem solution for three power systems by implementing mathematical programming techniques.

4.5 DISCUSSION

The total operational costs obtained for the three systems by heuristic method, discussed in chapter three, are assumed as the base values that represent 100% of the cost of the study period. The results discussed in this chapter are compared to these base values.

It can be noted from table (4.20) that the full dynamic programming method (FDP) has achieved the best results in solving the unit commitment problem of the small size power systems. For example, a reduction of 0.264% in the total operating costs of the 4 unit system has been gained by using the FDP method compared with the heuristic methods. The computation time is almost the same for both methods, i.e. in the range of 0.5 seconds.

For the 15 unit system, the FDP has also accomplished the best solution with an 8.2% reduction of the total operational costs over the study period compared with those obtained by the heuristic methods. On the other hand, CPU time of FDP increased very rapidly from 1.11 seconds for heuristic methods to 334.6 seconds.

Although the FDP method achieves competitive results in solving the unit commitment problem for small power systems, it cannot be used to solve the problem of systems with 18 units or more, which is due to the dimensionality problem. The dynamic programming sequential combination (DPSC) technique overcomes the dimensionality problem and high CPU time of FDP; however, it does not provide a considerable improvement in reducing the operating cost for small systems. In a large system, for instance the 150 unit system tested in this study, the DPSC produced an operating cost lower by 1.1% than that of heuristic methods for the same system. The computation time was increased from 7.06 seconds for the heuristic methods to 1182.93 seconds for the DPSC.

The mixed integer linear programming technique is a proper tool for solving the problem of the unit commitment, subject to the condition that the generating units have linear input-output characteristics. If the units have nonlinear input-output characteristics, then a suitable linearization of the cost curve of the units is required. The MILP technique is applicable to any size of power system. The computational time is relatively small, regardless of the number of units as shown in table (4.20). On the other hand, despite the fact that MILP is a sophisticated technique, no remarkable improvement was achieved in the reduction of costs compared to the results obtained by heuristic methods, particularly for large-scale power systems.

A comparison of different methods for solving the problem of unit commitment that have been presented in this thesis, and of their performance as well as further discussion of the results will be carried out in chapter six.

Table (4.2): Results of unit commitment for a 4 unit system by full dynamic programming.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	3	9208.4	9208.4	630.0
2	3	10648.4	19856.7	630.0
3	3	12450.4	32307.1	630.0
4	3	10828.4	43135.4	630.0
5	2	8308.4	51443.8	550.0
6	1	5573.5	57017.3	300.0
7	1	5748.1	62765.5	300.0
8	2	10508.4	73273.8	550.0
9	2	9028.4	82302.2	550.0
10	2	10468.4	92770.6	550.0
11	3	12212.4	104982.9	610.0
12	2	10918.4	115901.3	550.0
13	2	8488.4	124389.6	550.0
14	2	7948.4	132338.0	550.0
15	2	8128.4	140466.4	550.0
16	2	8668.4	149134.7	550.0
17	2	9208.4	158343.1	550.0
18	2	10108.4	168451.4	550.0
19	3	11855.4	180306.8	610.0
20	2	10648.4	190955.2	550.0
21	2	8632.4	199587.5	550.0
22	2	7588.4	207175.9	550.0
23	2	8398.4	215574.2	550.0
24	2	10108.4	225682.6	550.0

TOTAL START UP COST = £263.16

TOTAL GENERATION COST = £225419.44

TOTAL OPERATIONAL COST = £225682.60

* Total number of committed units.

Table (4.3): Unit schedule of a 4 unit system over a 24 h period by full dynamic programming.

Time (H)	Unit number and status 1	2	3	4
1	1	1	1	0
2	1	1	1	0
3	1	1	1	0
4	1	1	1	0
5	1	1	0	0
6	1	0	0	0
7	1	0	0	0
8	1	1	0	0
9	1	1	0	0
10	1	1	0	0
11	1	1	0	1
12	1	1	0	0
13	1	1	0	0
14	1	1	0	0
15	1	1	0	0
16	1	1	0	0
17	1	1	0	0
18	1	1	0	0
19	1	1	0	1
20	1	1	0	0
21	1	1	0	0
22	1	1	0	0
23	1	1	0	0
24	1	1	0	0
CPU TIME = 0.55 SECOND				

Table (4.4): Results of unit commitment for a 15 unit system by full dynamic programming.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	11	9811.4	9811.4	2069.0
2	10	8826.3	18637.8	1911.0
3	10	8372.4	27010.1	1806.0
4	9	8156.7	35166.8	1744.0
5	7	8095.6	43262.5	1744.0
6	7	8205.2	51467.7	1744.0
7	9	9473.2	60940.9	2016.0
8	12	10540.2	71481.1	2159.0
9	9	12402.2	83883.2	2581.0
10	9	14198.7	98081.9	2913.0
11	11	15058.5	113140.4	3071.0
12	11	16007.8	129148.2	3279.0
13	11	16142.3	145290.6	3356.0
14	12	16646.1	161936.7	3409.0
15	13	16875.5	178812.2	3454.0
16	14	17145.9	195958.1	3499.0
17	10	17286.4	213244.6	3509.0
18	12	16752.2	229996.8	3409.0
19	13	16637.0	246633.8	3369.0
20	10	15702.7	262336.5	3226.0
21	10	15455.4	277791.9	3226.0
22	9	15348.6	293140.4	3146.0
23	9	12442.5	305583.0	2581.0
24	8	11427.6	317010.6	2414.0

TOTAL START UP COST - £4210.18

TOTAL GENERATION COST - £312800.42

TOTAL OPERATIONAL COST - £317010.60

* Total number of committed units.

Table (4.5): Unit schedule of a 15 unit system over a 24 h period by full dynamic programming.

Time hours	Unit number and status of units														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	0
2	1	1	1	1	0	1	1	1	1	1	0	0	1	0	0
3	1	1	1	0	0	1	1	1	1	1	0	1	1	0	0
4	1	1	1	0	0	1	1	1	1	0	0	1	1	0	0
5	1	0	0	0	1	1	1	1	0	0	1	1	0	0	0
6	1	0	0	0	1	1	1	1	0	0	1	1	0	0	0
7	1	0	0	0	1	1	1	1	1	1	1	1	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1
9	1	1	0	0	1	1	1	1	1	1	0	1	0	0	0
10	1	1	1	0	0	1	1	1	1	1	0	1	0	0	0
11	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0
12	1	1	1	0	1	1	1	1	1	1	1	0	1	0	0
13	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
14	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
15	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0
16	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1	0	1	0	1	0	0	0
18	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
19	1	1	1	0	1	1	1	1	1	1	1	0	1	1	1
20	1	1	1	0	1	1	1	1	1	1	1	0	0	0	0
21	1	1	1	0	1	1	1	1	1	1	1	0	0	0	0
22	1	1	1	0	1	1	1	1	1	0	0	1	0	0	0
23	1	1	0	0	1	1	1	1	1	0	1	1	0	0	0
24	1	1	0	0	1	1	1	1	0	0	1	1	0	0	0

CPU TIME = 334.60 SECOND

Table (4.6): Results of unit commitment for a 15 unit system by dynamic programming sequential combination technique.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	5	10226.3	10226.3	2773.0
2	4	9110.0	19336.4	2348.0
3	4	8668.1	28004.5	2348.0
4	4	8400.5	36405.0	2348.0
5	4	8362.4	44767.4	2348.0
6	4	8476.9	53244.2	2348.0
7	4	9515.9	62760.1	2348.0
8	5	16028.8	78789.0	2773.0
9	6	16975.9	95764.8	2940.0
10	8	22281.5	118046.4	3274.0
11	9	19153.9	137200.3	3441.0
12	11	17886.7	155087.0	3651.0
13	12	17697.6	172784.5	3781.0
14	12	16718.3	189502.9	3781.0
15	13	17485.0	206987.8	3834.0
16	13	17086.4	224074.2	3834.0
17	13	17009.0	241083.2	3834.0
18	12	16699.2	257782.4	3781.0
19	12	16490.3	274272.7	3781.0
20	11	15852.7	290125.4	3651.0
21	10	15601.4	305726.8	3546.0
22	10	15489.5	321216.3	3546.0
23	6	12911.4	334127.7	2940.0
24	5	12717.9	346845.6	2773.0

TOTAL START UP COST = £2117.95

TOTAL GENERATION COST = £344727.65

TOTAL OPERATIONAL COST = £346845.60

* Total number of committed units.

Table (4.7): Unit schedule of a 15 unit system over a 24 h period by dynamic programming sequential combination technique.

Time	Unit number and status of units														
hours	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
6	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
7	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
8	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
9	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
11	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
12	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
13	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
14	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
18	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
19	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
21	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
22	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
23	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
24	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0

CPU TIME = 1.98 SECOND

Table (4.8): Results of unit commitment for a 150 unit system by dynamic programming sequential combination technique.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	119	28995.3	28995.3	5629.0
2	114	27688.0	56683.3	5409.0
3	113	27412.5	84095.8	5365.0
4	113	27306.7	111402.5	5365.0
5	107	25864.4	137266.9	5101.0
6	99	24053.3	161320.2	4797.0
7	105	25510.5	186830.7	5025.0
8	113	27516.0	214346.7	5365.0
9	120	29284.7	243631.4	5673.0
10	120	29311.8	272943.2	5673.0
11	121	29463.2	302406.4	5717.0
12	121	29417.0	331823.4	5717.0
13	127	31130.3	362953.8	5981.0
14	133	32651.7	395605.5	6245.0
15	133	32609.1	428214.6	6245.0
16	126	30830.9	459045.5	5937.0
17	120	29216.4	488262.0	5673.0
18	114	27659.3	515921.2	5409.0
19	121	29546.9	545468.1	5717.0
20	120	29243.4	574711.5	5673.0
21	120	29257.5	603969.0	5673.0
22	119	28954.2	632923.3	5629.0
23	122	29814.9	662738.2	5761.0
24	119	29041.0	691779.2	5629.0

TOTAL START UP COST = £1539.47

TOTAL GENERATION COST = £690239.77

TOTAL OPERATIONAL COST = £691779.24

CPU TIME (SECOND) = 1182.91

* Total number of committed units.

Table (4.9): Results of unit commitment for a 15 unit system by dynamic programming truncated combination technique.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	11	9851.5	9851.5	2114.0
2	9	8813.4	18664.9	2016.0
3	8	8382.6	27047.5	1886.0
4	9	8751.2	35798.7	1939.0
5	8	8094.7	43893.4	1886.0
6	8	8201.2	52094.7	1886.0
7	9	10676.3	62771.0	2016.0
8	10	15722.7	78493.6	2441.0
9	13	16981.9	95475.5	2584.0
10	12	20066.3	115541.8	3164.0
11	11	15078.9	130620.7	3111.0
12	11	22091.5	152712.2	3651.0
13	11	16272.5	168984.7	3651.0
14	12	17344.1	186328.8	3704.0
15	12	18339.6	204668.4	3781.0
16	12	17065.3	221733.7	3781.0
17	12	16988.0	238721.7	3781.0
18	12	16703.6	255425.3	3781.0
19	11	16499.3	271924.6	3651.0
20	11	15874.5	287799.1	3651.0
21	10	15439.7	303238.8	3226.0
22	12	18734.3	321973.1	3401.0
23	9	12551.5	334524.6	2888.0
24	9	11536.2	346060.9	2888.0

TOTAL START UP COST = £1679.12

TOTAL GENERATION COST = £344381.76

TOTAL OPERATIONAL COST = £346060.88

* Total number of committed units.

Table (4.10): Unit schedule of a 15 unit system over a 24 h period by dynamic programming truncated combination technique.

Time hours	Unit number and status of units														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	base units							search range units							
1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	0
2	1	1	1	1	1	1	1	0	0	0	1	1	0	0	0
3	1	1	1	1	1	1	1	0	0	0	1	0	0	0	0
4	1	1	1	1	1	1	1	0	0	0	1	0	1	0	0
5	1	1	1	1	1	1	1	0	0	0	1	0	0	0	0
6	1	1	1	1	1	1	1	0	0	0	1	0	0	0	0
7	1	1	1	1	1	1	1	0	0	0	1	1	0	0	0
8	1	1	1	1	1	1	1	0	0	1	1	1	0	0	0
9	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0
11	1	1	1	1	1	1	1	1	0	1	1	1	0	0	0
12	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
13	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
14	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
18	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
19	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
21	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0
22	1	1	1	1	1	1	1	1	1	0	1	1	0	1	0
23	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
24	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0

CPU TIME = 22.76 SECOND

Table (4.11): Results of unit commitment for a 4 unit system by mixed integer programming. (NAG library routine)

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	2	9222.0	9222.0	550
2	2	10742.0	19964.0	550
3	3	12685.0	32649.0	630
4	2	10932.0	43581.0	550
5	2	8272.0	51853.0	550
6	1	5445.0	57298.0	300
7	1	5615.0	62913.0	300
8	2	10572.0	73485.0	550
9	2	9032.0	82517.0	550
10	2	10552.0	93069.0	550
11	3	12485.0	105554.0	630
12	2	11027.0	116581.0	550
13	2	8462.0	125043.0	550
14	2	7892.0	132935.0	550
15	2	8082.0	141017.0	550
16	2	8652.0	149669.0	550
17	2	9222.0	158891.0	550
18	2	10172.0	169063.0	550
19	3	12185.0	181248.0	630
20	2	10742.0	191990.0	550
21	2	8614.0	200604.0	550
22	2	7512.0	208116.0	550
23	2	8367.0	216483.0	550
24	2	10172.0	226655.0	550

TOTAL START UP COST = £953.47

TOTAL GENERATION COST = £225701.53

TOTAL OPERATIONAL COST = £226655.00

* Total number of committed units.

Table (4.12): Unit schedule of a 4 unit system over a 24 h period by mixed integer programming. (NAG library routine).

Time hours	Unit number and status 1	2	3	4
1	1	1	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	0	0
5	1	1	0	0
6	1	0	0	0
7	1	0	0	0
8	1	1	0	0
9	1	1	0	0
10	1	1	0	0
11	1	1	1	0
12	1	1	0	0
13	1	1	0	0
14	1	1	0	0
15	1	1	0	0
16	1	1	0	0
17	1	1	0	0
18	1	1	0	0
19	1	1	1	0
20	1	1	0	0
21	1	1	0	0
22	1	1	0	0
23	1	1	0	0
24	1	1	0	0
CPU TIME = 0.53 SECOND				

Table (4.13): Results of unit commitment for a 15 unit system by mixed integer programming. (NAG library routine)

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	7	10209.2	10209.2	2346.0
2	7	9288.2	19497.4	2346.0
3	7	8864.5	28361.8	2346.0
4	7	8606.6	36968.4	2346.0
5	7	8569.7	45538.2	2346.0
6	7	8680.3	54218.4	2346.0
7	7	9675.0	63893.4	2346.0
8	7	10651.3	74544.7	2346.0
9	8	14132.2	88677.0	3016.0
10	8	14203.9	102880.9	3016.0
11	9	15415.1	118296.1	3069.0
12	11	16807.9	135104.0	3304.0
13	12	17270.4	152374.3	3409.0
14	12	17161.8	169536.2	3409.0
15	12	18982.9	188519.1	3729.0
16	12	17580.3	206099.3	3729.0
17	12	17496.1	223595.4	3729.0
18	12	17438.8	241034.2	3409.0
19	12	16909.2	257943.4	3409.0
20	11	16148.7	274092.1	3304.0
21	10	15830.3	289922.4	3199.0
22	10	15703.9	305626.3	3199.0
23	8	12859.2	318485.5	3016.0
24	8	11882.9	330368.4	3016.0

TOTAL START UP COST = £4727.49

TOTAL GENERATION COST = £325640.92

TOTAL OPERATIONAL COST = £330368.41

* Total number of committed units.

Table (4.14): Unit schedule of a 15 unit system over a 24 h period by mixed integer programming. (NAG library routine).

Time hours	Unit number and status of units 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	1	1	0	1	1	1	1	1	0	0	0	0	0	0
2	0	1	1	0	1	1	1	1	1	0	0	0	0	0	0
3	0	1	1	0	1	1	1	1	1	0	0	0	0	0	0
4	0	1	1	0	1	1	1	1	1	0	0	0	0	0	0
5	0	1	1	0	1	1	1	1	1	0	0	0	0	0	0
6	0	1	1	0	1	1	1	1	1	0	0	0	0	0	0
7	0	1	1	0	1	1	1	1	1	0	0	0	0	0	0
8	0	1	1	0	1	1	1	1	1	0	0	0	0	0	0
9	1	1	1	0	1	1	1	1	1	0	0	0	0	0	0
10	1	1	1	0	1	1	1	1	1	0	0	0	0	0	0
11	1	1	1	0	1	1	1	1	1	0	0	0	1	0	0
12	1	1	1	0	1	1	1	1	1	0	1	1	1	0	0
13	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
14	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
15	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0
16	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0
17	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0
18	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
19	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
20	1	1	1	0	1	1	1	1	1	0	1	1	1	0	0
21	1	1	1	0	1	1	1	1	1	0	0	1	1	0	0
22	1	1	1	0	1	1	1	1	1	0	0	1	1	0	0
23	1	1	1	0	1	1	1	1	1	0	0	0	0	0	0
24	1	1	1	0	1	1	1	1	1	0	0	0	0	0	0

CPU TIME = 3.99 SECOND

Table (4.15): Results of unit commitment for a 4 unit system by mixed integer programming.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	2	9208.4	9208.4	550.0
2	2	10648.4	19856.7	550.0
3	3	12615.4	32472.1	630.0
4	2	10828.4	43300.4	550.0
5	2	8308.4	51608.8	550.0
6	1	5573.5	57182.3	300.0
7	1	5748.1	62930.5	300.0
8	2	10508.4	73438.8	550.0
9	2	9028.4	82467.2	550.0
10	2	10468.4	92935.6	550.0
11	3	12406.6	105342.1	630.0
12	2	10918.4	116260.5	550.0
13	2	8488.4	124748.8	550.0
14	2	7948.4	132697.2	550.0
15	2	8128.4	140825.6	550.0
16	2	8668.4	149493.9	550.0
17	2	9208.4	158702.3	550.0
18	2	10108.4	168810.6	550.0
19	3	12093.4	180904.0	630.0
20	2	10648.4	191552.4	550.0
21	2	8632.4	200184.7	550.0
22	2	7588.4	207773.1	550.0
23	2	8398.4	216171.4	550.0
24	2	10108.4	226279.8	550.0

TOTAL START UP COST = £953.95

TOTAL GENERATION COST = £225325.87

TOTAL OPERATIONAL COST = £226279.82

* Total number of committed units.

Table (4.16): Unit schedule of a 4 unit system over a 24 h period by mixed integer programming.

Time hours	Unit number and status 1	2	3	4
1	1	1	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	0	0
5	1	1	0	0
6	1	0	0	0
7	1	0	0	0
8	1	1	0	0
9	1	1	0	0
10	1	1	0	0
11	1	1	1	0
12	1	1	0	0
13	1	1	0	0
14	1	1	0	0
15	1	1	0	0
16	1	1	0	0
17	1	1	0	0
18	1	1	0	0
19	1	1	1	0
20	1	1	0	0
21	1	1	0	0
22	1	1	0	0
23	1	1	0	0
24	1	1	0	0
CPU TIME = 0.51 SECOND				

Table (4.17): Results of unit commitment for a 15 unit system by mixed integer programming.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	10	10340.5	10340.5	2686.0
2	9	9149.7	19490.2	2016.0
3	9	8711.0	28201.2	2016.0
4	9	8444.0	36645.2	2016.0
5	9	8405.8	45051.0	2016.0
6	9	8520.3	53571.3	2016.0
7	9	9550.2	63121.6	2016.0
8	10	11951.9	75073.5	2686.0
9	10	12610.2	87683.7	2686.0
10	11	16118.0	103801.7	3356.0
11	11	15441.3	119243.0	3356.0
12	11	16356.8	135599.7	3356.0
13	11	16623.8	152223.5	3356.0
14	12	17398.5	169622.1	3409.0
15	13	19004.3	188626.4	3834.0
16	13	17605.9	206232.3	3834.0
17	13	17519.9	223752.2	3834.0
18	12	17113.8	240866.0	3409.0
19	11	16852.7	257718.7	3356.0
20	11	16204.2	273922.9	3356.0
21	11	15956.2	289879.1	3356.0
22	11	15841.8	305720.9	3356.0
23	10	12839.1	318560.0	2686.0
24	10	11828.2	330388.2	2686.0

TOTAL START UP COST = £4679.93

TOTAL GENERATION COST = £325708.31

TOTAL OPERATIONAL COST = £330388.24

* Total number of committed units.

Table (4.18): Unit schedule of a 15 unit system over a 24 h period by mixed integer programming.

Time hours	Unit number and status of units														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	0
2	1	0	0	0	1	1	1	1	1	1	1	1	0	0	0
3	1	0	0	0	1	1	1	1	1	1	1	1	0	0	0
4	1	0	0	0	1	1	1	1	1	1	1	1	0	0	0
5	1	0	0	0	1	1	1	1	1	1	1	1	0	0	0
6	1	0	0	0	1	1	1	1	1	1	1	1	0	0	0
7	1	0	0	0	1	1	1	1	1	1	1	1	0	0	0
8	1	1	0	0	1	1	1	1	1	1	1	1	0	0	0
9	1	1	0	0	1	1	1	1	1	1	1	1	0	0	0
10	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
11	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
12	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
13	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
14	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
18	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
19	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
20	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
21	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
22	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
23	1	1	0	0	1	1	1	1	1	1	1	1	0	0	0
24	1	1	0	0	1	1	1	1	1	1	1	1	0	0	0

CPU TIME = 1.07 SECOND

Table (4.19): Results of unit commitment for a 150 unit system by mixed integer programming.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	115	29604.3	29604.3	5453.0
2	110	28263.5	57867.8	5233.0
3	109	27997.0	85864.8	5189.0
4	108	27790.4	113655.2	5145.0
5	102	26448.1	140103.3	4911.0
6	94	24579.7	164683.0	4571.0
7	100	26028.2	190711.2	4835.0
8	109	28017.0	218728.2	5189.0
9	115	29744.2	248472.4	5453.0
10	116	29910.8	278383.2	5497.0
11	116	29982.8	308366.0	5497.0
12	116	29946.8	338312.8	5497.0
13	122	31614.1	369926.9	5761.0
14	128	33177.5	403104.4	6025.0
15	128	33189.5	436293.9	6025.0
16	122	31470.2	467764.1	5761.0
17	115	29740.2	497504.3	5453.0
18	109	28144.9	525649.2	5189.0
19	116	30010.8	555659.9	5497.0
20	116	29846.9	585506.8	5497.0
21	116	29870.9	615377.7	5497.0
22	114	29473.7	644851.3	5409.0
23	118	30396.0	675247.3	5585.0
24	115	29636.3	704883.6	5453.0

TOTAL START UP COST = £1592.11

TOTAL GENERATION COST = £703291.51

TOTAL OPERATIONAL COST = £704883.62

CPU TIME (SECOND) = 36.64

* Total number of committed units.

Table (4.20): Final results of unit commitment problem for three power systems by using mathematical programming techniques.

Number of units	Method of solution	Start up cost £	Generation cost £	Total operational costs £	CPU Time Sec.
4	FULL DP	263.160	225419.44	225682.60	0.55
	MILP (NAG)	953.470	225701.53	226655.00	0.53
	MILP modified	953.47	225325.87	226279.82	0.51
15	DP (FULL)	4210.18	312800.42	317010.60	334.6
	DPSC	2117.95	334727.65	346845.60	1.98
	DPTC	1679.12	334381.76	346060.88	22.76
	MILP (NAG)	4727.49	325640.92	330368.41	3.99
	MILP modified	4679.93	325708.11	330388.24	1.07
150	DPSC	1539.47	690239.77	691779.24	1183
	MILP modified	1592.11	703291.51	704883.62	36.64

CHAPTER 5

AN APPLICATION OF DECOMPOSITION METHODS TO THE UNIT COMMITMENT PROBLEM

5.1 INTRODUCTION

It can be noted from the previous chapters that the unit commitment problem could be solved either by heuristic methods or by mathematical programming techniques. Although the heuristic methods are applicable to any size of system, they do not always guarantee optimal solutions. On the other hand, mathematical programming approaches are the most rigorous methods of solution; however, these techniques are only suitable for small systems. The computational requirements of the CPU time and the computer storage space increase rapidly with the increasing number of units. They could go beyond the practical limits and exhaust even the large computer if these techniques are applied to large-scale systems. This is because of the large number of variables and constraints which are involved in the calculation. Therefore, a simplified technique is necessary to achieve the following conditions:

1- A reduction in the CPU time and computer memory space to acceptable limits required for the unit commitment.

2- An achievement of optimal solution to the unit commitment problem for large scale power systems.

At present, there is no method that solves the problem optimally for large systems [60]. Therefore, a precise solution approach based on mathematical programming methods and applicable to any size of power system has become the main goal of current unit commitment research. Decomposition methods, which show some potential in dealing with large-scale systems, have been recently applied to solve the problem of unit commitment. The principal idea of the decomposition technique is that the main problem is divided into smaller subproblems so that each subproblem can be solved independently. The overall solution to the main problem can be obtained by gathering optimal solutions to the subproblems.

In this chapter, the application of the Lagrangian relaxation method and the Benders decomposition method to solve the unit commitment problem will be described.

5.2 LAGRANGIAN RELAXATION METHOD

Lagrangian relaxation approach is used to simplify the optimisation problem by decomposing the main problem into subproblems. The principle of Lagrangian relaxation is that the constraints of the problem are incorporated into the objective function by using Lagrangian multipliers. Next, the Lagrangian dual objective function is obtained and formulated. The dual problem is then decoupled into smaller subproblems which could be solved more easily than the original objective function. The dual problem provides a lower bound on the optimal solution of the original problem as a function of Lagrangian multipliers [34,35,41,59]. Further details about Lagrangian relaxation decomposition method are outlined in Appendix D.

The application of the Lagrangian relaxation method to solving the unit commitment problem was introduced in the late 1970s. In the earliest attempts [34,38], the problem was formulated in the form of mixed integer programming, and near optimal solution to the problem was obtained. The solution procedure was simplified by joining only the constraints of the load demand and the spinning reserve requirements to the objective function using the Lagrangian multipliers. In these attempts, Lagrangian relaxation was employed to substitute the common linear programming relaxation approach in the fathoming process of the branch and bound algorithm. An improvement of computational efficiency was achieved compared with the branch and bound method. However, the determination of the upper bounds was still necessary in these methods, and it required the economic dispatch solution for the feasible schedules obtained in the branch and bound tree. The solution of the economic dispatch for the large-scale system required extensive computational time resources. Merlin [35] suggested another methodology of determining the dual optimal solution. In this algorithm, a modified subgradient method was proposed to update the Lagrangian multipliers during the search for the suboptimal feasible solution. Although these methods may not guarantee a true optimal solution, a suboptimal solution which is reasonable enough for practical applications can be obtained [81].

Recently, more advanced methods of solving the unit commitment problems by using Lagrangian relaxation technique were proposed [47,50,54,57,65,68,69,73]. However, a practical method for producing the optimal solution for the large-scale unit commitment problem has not been found yet [60].

Lagrangian relaxation method can be applied to the unit commitment problem by using one of the following approaches:

1) Decomposing the original problem into i single

generator subproblem.

2) Decomposing the study period into t single period subproblem.

The first approach has an advantage over the second one in that the cost function and the constraints, which depend on the state of the generators from one period to another, can easily be considered in the subproblem. The problem of unit commitment can then be suitably formulated in the basic model of the problem and, for the purpose of simplification, only the reserve requirement and demand constraints will be joined to the objective function. Other constraints, e.g. minimum up and minimum down time of units, can be considered in the solution without affecting the basic structure of the problem [34].

The mathematical model of the unit commitment problem represented by equation (2.9) can be formulated by assigning Lagrangian multipliers σ and μ to the constraints in equation (2.10) and (2.12) respectively, and the Lagrangian dual function of the unit commitment primal problem can be expressed as follows:

$$Lg(\sigma,\mu) = \min_{u,p} \{ Ct(u,p) + \sum_{t=1}^{T} [\sigma_t (D_t - \sum_{i=1}^{n} P_{it})$$

$$+ \mu_t (Sr_t - \sum_{i=1}^{n} Sr_{it})] \} \qquad .... \qquad (5.1)$$

subject to system and unit constraints where $D_t = P_D + P_L$.

$Lg(\sigma,\mu)$ can be expressed by reforming equation (5.1) (separable structure of cost function "Ct") as:

$$Lg(\sigma,\mu) = \sum_{i=1}^{n} Lg_i(\sigma,\mu) + \sum_{t=1}^{T}(\sigma_t D_t + \mu_t Sr_t) \quad \ldots\ldots \quad (5.2)$$

where

$$Lg_i(\sigma,\mu) = \min_{u,p} \sum_{t=1}^{T} [Gc_i(u_i, p_i) + Suc_{it}(u_i) + Sdc_{it}(u_i) - \sigma_t P_{it} - \mu_t r_i] \quad \ldots \quad (5.3)$$

subject to the system and unit constraints.

Equation (5.3) represents a single generator objective function which can be solved by dynamic programming. The Lagrangian multipliers (σ,μ) can be assumed so that their initial values produce a feasible solution to equation (5.3). This feasible solution is used as initial values for solving equation (5.2). Consequently, the optimal value of $Lg(\sigma,\mu)$ is calculated by updating Lagrangian multipliers by using a proper method. The dual problem is solved by maximizing equation (5.2) as:

$$\max Lg(\sigma,\mu) \quad \ldots\ldots\ldots \quad (5.4)$$

subject to $\sigma \geq 0$ and $\mu \geq 0$

From the duality theorem; if Ct^* is the optimal value of $Ct(u,p)$, then $Lg(\sigma,\mu) \leq Ct^*$ for all $\sigma \geq 0$, $\mu \geq 0$. If Lg^* is defined as the dual optimal value of $Lg(\sigma,\mu)$, then:

$$Lg^* = \max_{\substack{\sigma \geq 0 \\ \mu \geq 0}} Lg(\sigma,\mu) \leq Ct^* \qquad \ldots\ldots \qquad (5.5)$$

The duality gap can be expressed as $(Ct^* - Lg^*)$. The smaller the value of the duality gap, the closer the value of Ct to the optimal. The best results are achieved when the quality gap is in the range of 0.1% to 0.5% [38]. The dual objective function is usually maximized by using the subgradient method for updating Lagrangian multipliers. The variable metric method [69] is used to maximize the Lagrangian function as an alternative to the subgradient method. The Lagrangian technique is implemented in [47] to reduce the search range of the dynamic programming method.

The problem of unit commitment, in this thesis, is solved by implementing the Lagrangian relaxation method to optimize the objective function using a sequential augmented Lagrangian method. The solution can proceed as follows:

1- Initial values of unit output and Lagrangian multipliers are assumed; then equation (5.3) which represents a single generator subproblem can be solved (minimized) by dynamic programming, subject to the maximum and minimum limits of the unit output power.

2- A feasible solution to the augmented Lagrangian objective function in equation (5.2) is obtained, subject to the system constraints, by using the results of step 1 as initial values. As a result, the unit commitment decision is specified.

3- Equation (5.2) is minimized by updating the Lagrangian multipliers by using subgradient method, subject to the constraints. In this phase, the economic dispatch is performed.

4- The combination of units which satisfies the system and unit constraints with the lowest cost is assigned as the committed units for the interval and the associated cost is assigned as the optimal solution.

5- Steps 1 to 4 are repeated for each interval until the unit commitment problem of the study period is completely solved.

6- The unit commitment schedule for each interval is determined and the optimal operational policy of the system is produced.

The solution of the unit commitment problem by Lagrangian relaxation technique is illustrated in figure (5.1).

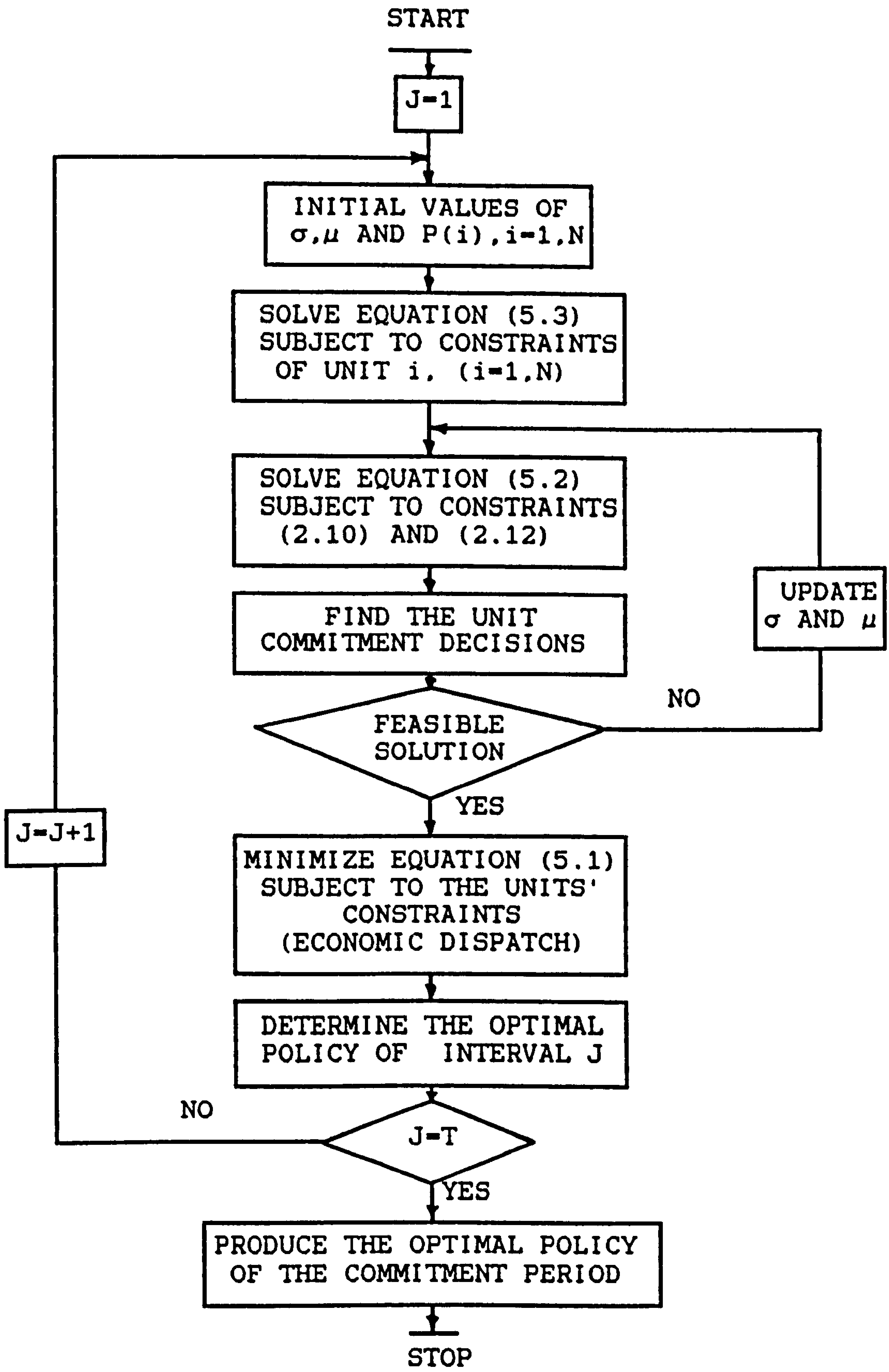


Figure (5.1): Flow chart of the program for solving the unit commitment by Lagrangian relaxation method.

5.3 BENDERS DECOMPOSITION METHOD

The basic idea of Benders decomposition technique is that the optimization problem is decomposed into two subproblems. The first subproblem must consist of pure linear programming and could be identified as the inner problem. The second one is an integer programming problem and could be identified as the outer subproblem. The feasible region of the inner subproblem is less restricted than that of the outer one. Therefore, the solution of the main problem is mainly dependent on and bounded within the solution of the outer subproblem [76]. Mathematically, Benders decomposition method can be represented in the following matrix form:

$$\text{minimize} \quad Z = [C_x]^T[x] + [C_y]^T[y] \quad \ldots\ldots \quad (5.6)$$

$$\text{subject to} \quad [A][x] + [B][y] \geq [D] \quad \ldots\ldots \quad (5.7)$$

$$[y] \geq 0$$

$$[x] \in X \quad \text{(integer)} \quad \ldots\ldots \quad (5.8)$$

where vector [x] corresponds to integer variables and vector [y] corresponds to continuous variables. $[C_x]$ and $[C_y]$ are the cost coefficient vectors in the objective function corresponding to integer and continuous variables respectively. Matrices [A] and [B] are coefficient matrices and vector [D] contains the right hand side of the constraints.

This optimization problem can be considered as a two stage decision process [56]. In the first stage, a feasible decision $[x^*]$ is assumed for vector $[x]$. A decision $[y]$ is calculated in the second stage, as the optimal solution to the following problem:

$$\text{minimize} \quad [C_y]^T [y] \qquad \ldots\ldots \qquad (5.9)$$

subject to

$$[B][y] \geq [D] - [A][x^*] \qquad \ldots\ldots \qquad (5.10)$$

$$[y] \geq 0 \qquad \ldots\ldots \qquad (5.11)$$

The second stage problem is a function of decision $[x]$ taken in the first stage. Therefore, the solution to the first stage problem can be obtained from the following equations :

$$\text{minimize} \quad [C_x]^T [x] \qquad \ldots\ldots \qquad (5.12)$$

$$\text{subject to} \quad [x] \in X \qquad \ldots\ldots \qquad (5.13)$$

Benders approach was generalized by Geoffrion [52] so that the inner subproblem needs no longer comprises a linear programming. The solution of the problem starts from the outer subproblem by finding an initial feasible solution to this subproblem. The inner subproblem is solved by using the results of the outer one. Then, the obtained results are checked to ensure that the optimality conditions of the main problem are met. If the solution violates any constraints, a new constraint is imposed on the outer subproblem. An improved solution to the inner and the main problems is tried. This procedure continues iteratively and it is terminated when no further improvement in the solution to the main problem can be obtained. Figure (5.2) illustrates the application of Benders method to unit commitment problem.

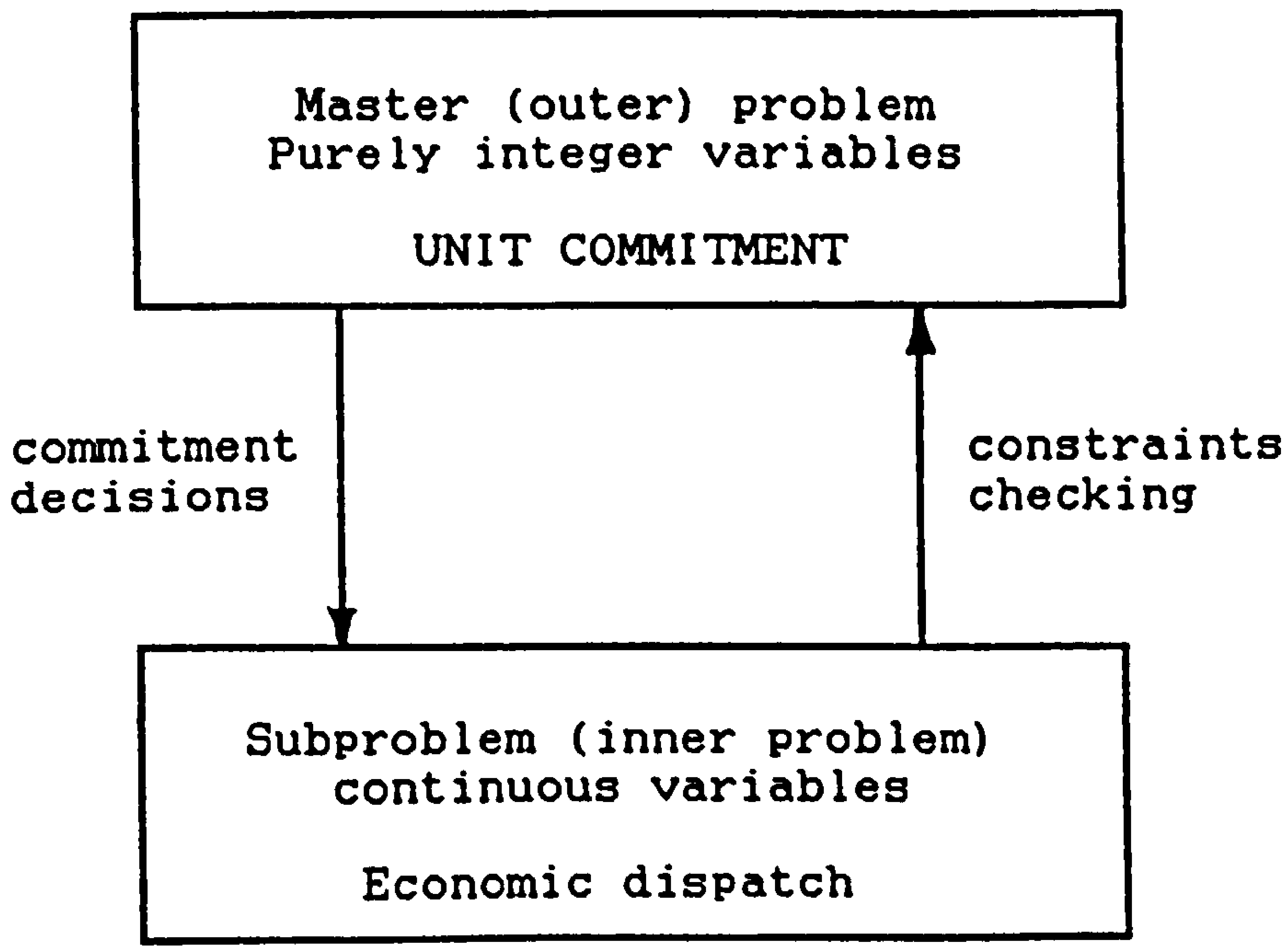


Figure (5.2): Block diagram representing application of Benders principle to unit commitment problem.

The principle of Benders decomposition [52,76] has been employed to solve the unit commitment problem. Early attempts formulated the problem by mixed integer linear programming [39,56]. It was then decomposed by the

Benders method into a master problem (outer subproblem) of integer variables, which represents the unit commitment of thermal plants, and a subproblem of continuous variables (inner subproblem), which represents the economic dispatch. For the purpose of computation simplification, the master problem can be further decomposed into smaller subproblems so that each subproblem contains a suitable number of units. In [48], the successive approximation method and a hierarchical approach were suggested to solve the unit commitment by implementing dynamic programming with Benders decomposition technique. The fuel cost function was formulated as a second order polynomial form. The start up cost is expressed as an exponential form, that is a function of the time the unit was down. The study period was divided into small intervals of variable values depending on the slope of demand. The units in the system were divided into a suitable number of subgroups, and each subgroup represented a subproblem.

A global solution to the problem was proposed in [41] by the use of generalised Benders decomposition [52]. The cost function was assumed to be non-linear. The start up cost was also non-linear and time dependent. The mathematical model of the problem represented a stochastic mixed integer non-linear form. The hierarchical structure of the problem consisted of three levels: the resolution of the unit commitment problem in the upper level (master problem), the resolution of the hydro-power problem in the middle level, and the analysis of the thermal problem (economical dispatch) at the third level.

In this study, the units in the system are divided into L subgroups. Each one has M units. The total demand and spinning reserve requirements are divided among the subgroups so that each subgroup shares a certain amount of the load. The subgroup portion from the demand and spinning reserve in interval t are assigned as D^L_t and Sr^L_t respectively. The unit commitment problem of every subgroup is solved individually; then, an overall solution to the unit commitment problem can be obtained by gathering the solutions to the subgroups. The objective function of the subgroup can be formulated as:

$$C^L(U_{i,t},P_{i,t}) = \min \sum_{t=1}^{T} [\sum_{i=1}^{M}(U_i . Gc_{it}(p_i))$$

$$+ Suc_{it}(u_i) + Sdc_{it}(u_i)] \quad ..... \quad (5.14)$$

subject to

$$\sum_{i=1}^{M} U_i . P_{i\,max} \geq D^L_t + Sr^L_t \quad ...... \quad (5.15)$$

$$\sum_{i=1}^{M} U_i . P_{i\,min} \leq D^L_t \quad ...... \quad (5.16)$$

where $D^L{}_t + Sr^L{}_t$ are the contribution of subgroup L to the total demand and the system spinning reserve which can be calculated as follows:

$$D^L{}_t + Sr^L{}_t = D_t + Sr_t - (\sum_{i=1}^{n} U_i . P_{i\,max} . - \sum_{J=1}^{m} U_{JL} . P_{JL\,max}) \quad \ldots(5.19)$$

The difference between equations (2.9) and (5.14) is the dimensionality of the main problem, i.e. the number of possible combinations of units can be reduced from 2^N-1 in equation (2.9) to the value of $L.(2^M-1)$ in equation (5.14), where M is the number of units per subgroup L. Other variable numbers and constraints remain the same. For example, if the unit commitment problem of a 20 unit system is to be solved by dynamic programming, then the number of possible combinations equals 1048575 . When the system is decomposed into two subgroups so that each subgroup has 10 units and the Benders principle is used with dynamic programming, the number of possible combinations can be reduced to $2(2^{10}-1) = 2046$.

Equation (5.14) can be solved by employing a suitable technique with one of the following approaches [48]:

5.3.1 SUCCESSIVE APPROXIMATION

The number of units in the system is divided into L subgroups with M units in each. At any interval t, a sequential solution of these subgroup problems is found, starting from subgroup 1 to subgroup L . The variables of the subgroup under investigation are allowed to change, while those of the other subgroups are kept fixed during the optimization.

The solution procedure is described as follows:

i) The values of M and L are specified according to the power system size.

ii) The contribution of each subgroup to the total demand and to the spinning reserve for an interval t is determined.

iii) Equation (5.14) is solved, then a feasible solution for each subgroup is obtained sequentially, starting from subgroup 1 to subgroup L.

iv) In order to improve the results obtained in step (iii), it is repeated in such a way that the variables of the subgroup under manipulation are allowed to change, while the variables of the other subgroups are fixed.

v) Step (iv) is continued until no further improvement to the solution is achieved. In this way the committed units and their output power are obtained for the interval t.

vi) Steps (ii) to (v) are repeated until the commitment period is covered.

The interaction of the variables throughout the solution procedure is illustrated in figure (5.3).

5.3.2 <u>HIERARCHICAL APPROACH</u>

In this approach, the number of units in the system is divided into L subgroups with M units per subgroup. Hence, each subgroup unit commitment problem is solved independently, starting from interval 1 to the last

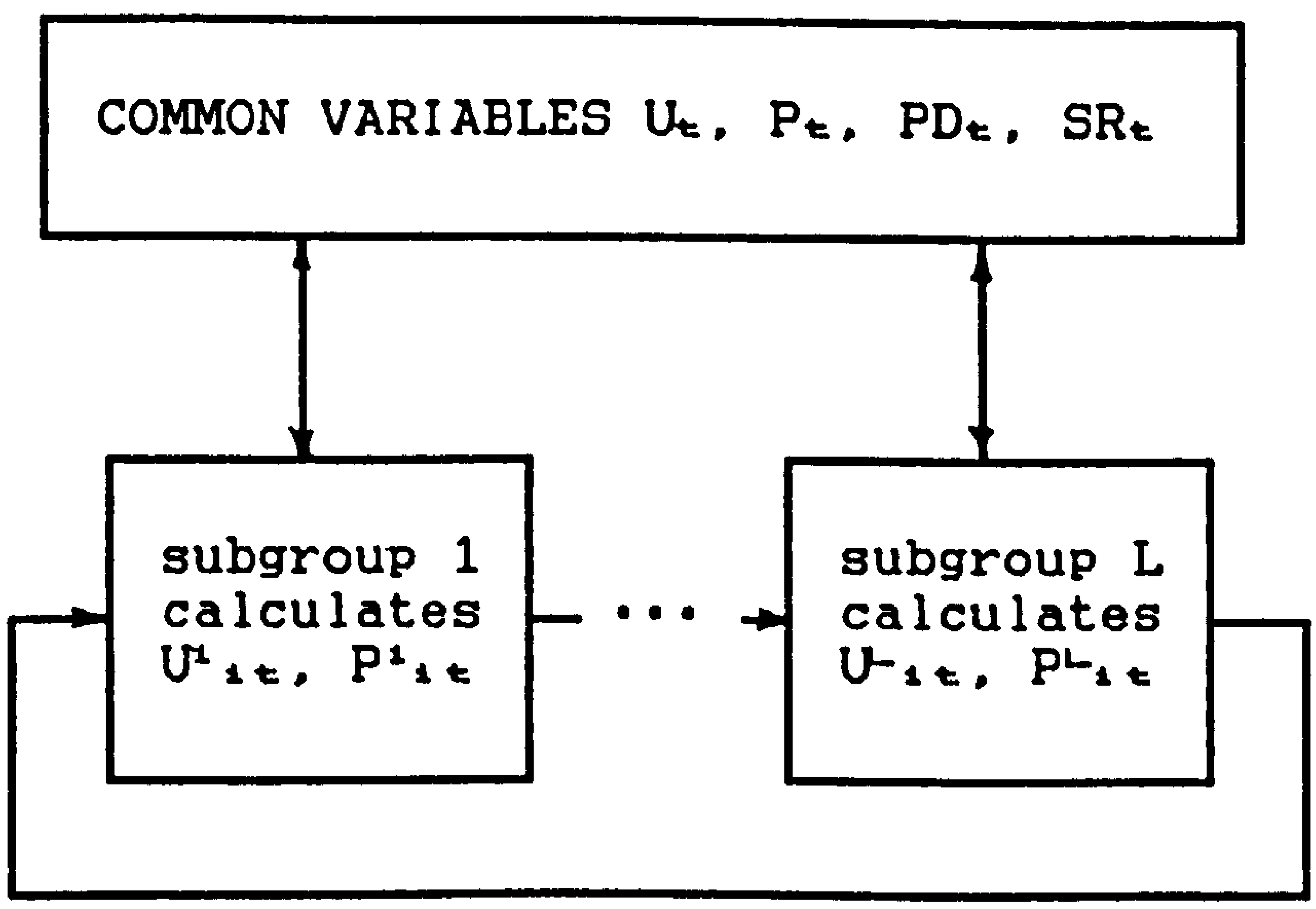


Figure (5.3): Interaction between different subgroups in successive approximation.

interval of the study period. The only linking factor between the subgroups are the demand and reserve requirements, therefore:

$$\sum_{i=1}^{L} D^{i}_{t} = D_{t} \quad \ldots\ldots\ldots \quad (5.18)$$

and

$$\sum_{i=1}^{L} Sr^{i}_{t} = Sr_{t} \quad \ldots\ldots\ldots \quad (5.19)$$

A co-ordinator, as illustrated in figure (5.4), is needed to determine the optimal value of D^{L}_{t} and Sr^{L}_{t}, in order to link the different subgroups during the optimization performance. An optimal solution for each subgroup unit commitment problem is obtained by solving equation (5.14) subject to the system and unit constraints by implementing an appropriate method.

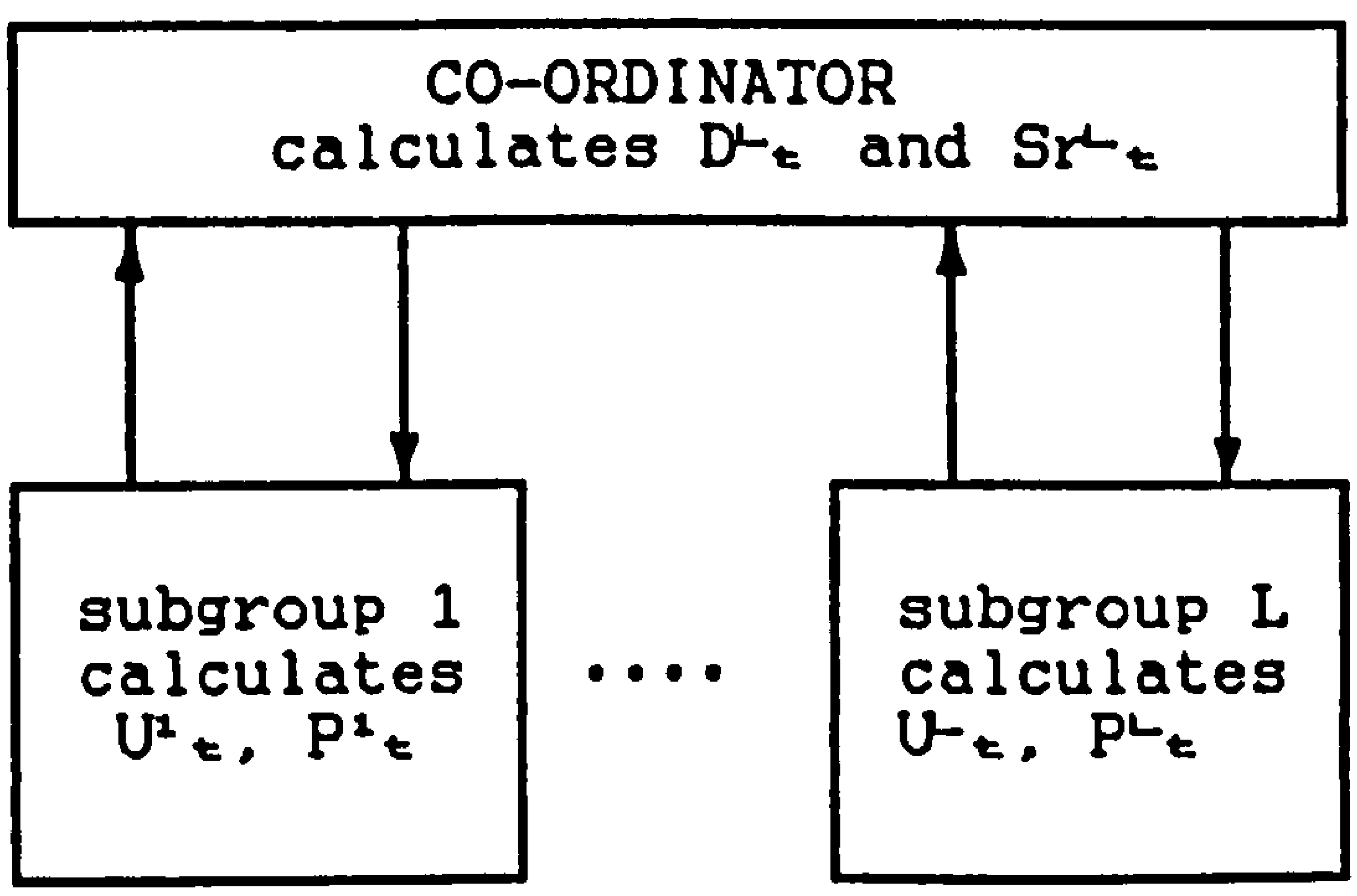


Figure (5.4): Coordination of the variables in the hierarchical approach.

The unit commitment problem algorithm described in chapter two is coded in FORTRAN 77. Forward full dynamic programming method and Benders decomposition technique with hierarchical approach are implemented to solve the problem. Hierarchical approach is used in this study because it is faster than successive approximation, while both approaches have the same degree of accuracy. However, feasible solution may not be easily produced. The solution procedure is illustrated as shown in figure (5.5) and can proceed as follows:

1- The number of units in the system is divided into a suitable number of subgroups with a certain number of units in each. In order to keep the number of units within acceptable limits, no more than 10 units per subgroup may be considered.

2-The load of each subgroup at any interval is determined as follows:

$$D_{Lt} = D_t \quad X \quad \frac{\text{subgroup max. capacity}}{\text{system max. capacity}} \quad \ldots. \quad (5.20)$$

3- Each subgroup is manipulated as an independent problem which can be solved with hierarchical approach as follows:

i) All the feasible combinations of units are specified for the interval t.

ii) Unit commitment is solved for every feasible combination.

iii) The start up cost, if any, is calculated.

iv) Economic dispatch is performed for each feasible combination and the generation cost is determined as a

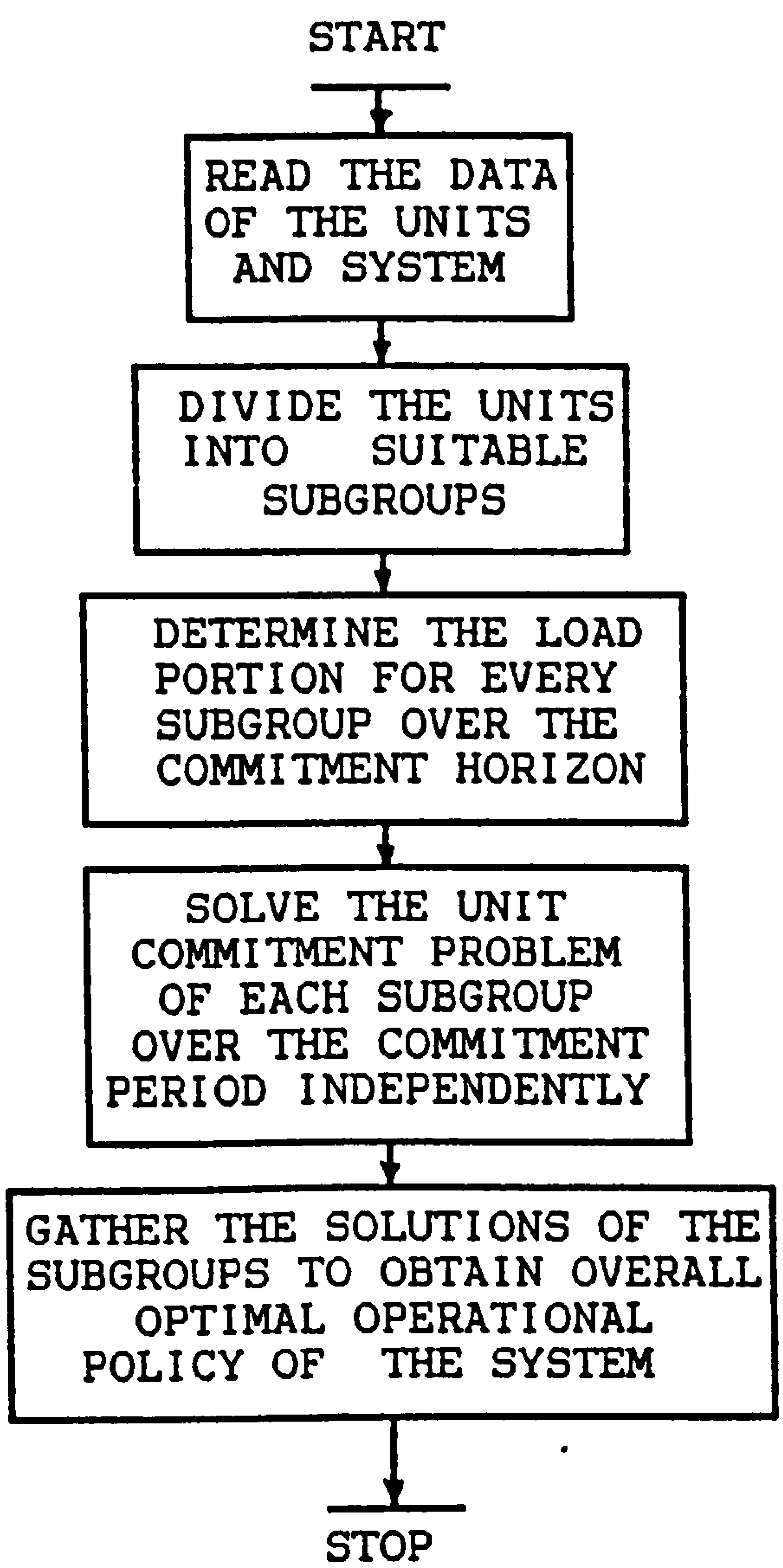


Figure (5.5): A block diagram for solving the unit commitment problem by Benders decomposition technique.

function of the generated power of every unit.

v) The feasible combination which satisfies the lowest cost is assigned as the committed combination of the interval and the associated cost is specified as the interval optimal cost of the subgroup.

vi) The total optimal cost of each subgroup is obtained by adding the optimal cost of its previous intervals to the optimal cost of the current interval.

vii) Steps (i) to (vi) are repeated until the last interval of the study period is reached.

4- Steps (i) to (vii) are repeated until all subgroups are covered.

5- The system's total optimal operation cost for the study period is found by gathering the optimal cost of every subgroup.

5.4 COMPUTER SIMULATION AND TEST RESULTS

In order to demonstrate the practical applications of the decomposition techniques of the Lagrangian relaxation and the Benders methods, computer programs for solving the unit commitment problem by employing these techniques were developed. The three different power systems described in Appendix A were tested. The results are presented in the following sections.

5.4.1 TEST RESULTS OF LAGRANGIAN METHOD

The unit commitment problem is solved by Lagrangian method using two computer programs in FORTRAN 77. The first program utilizes the NAG library by calling the NAG subroutine E04UAF. This program finds a minimum of function of several variables subject to fixed bounds on the variables and to general equality and/or inequality constraints. A sequential augmented Lagrangian method is used [84]. The results of the unit commitment problem produced by this program are reasonable. However, the NAG subroutine is not flexible enough to accommodate the necessary changes in the objective function and in the constraints of the problem. It is essential to deal with an integrated program, so that any required modification can be handled easily. Therefore, the solution procedure proposed in [82] has been adopted and the computer program has been modified and developed to suit the mathematical model of the unit commitment problem. All the necessary constraints could be imposed easily on the objective function. The results obtained were more realistic than those of the first program, and the results for this study are as follows:

i) 4 unit system: Input data of the units are listed in table (A-1), while the data of the load demand over a twenty four hour commitment horizon are found in table (A-2). Table (5.1) presents the output results obtained by Lagrangian relaxation technique, and the schedule of the units throughout the commitment period is demonstrated in table (5.2).

ii) <u>15 unit system</u>: The input data of the generating units are found in tables (A-3) and (A-4), while data of one day load demand are listed in table (A-5). The unit commitment problem of this system was solved by applying the Lagrangian relaxation method. The output results of the problem throughout twenty four hours are presented in tables (5.3) and (5.4).

5.4.2 TEST RESULTS OF BENDERS DECOMPOSITION METHOD

In the application of Benders decomposition principle to the problem of unit commitment, any suitable technique can be applied to solve the outer subproblem, which represents the unit commitment decision, and the inner subproblem of economic dispatch. Since dynamic programming seems to be the best technique to solve the problem of small size systems (up to 20 units), therefore it will be used with the Benders method in this study.

i) 15 unit system: The units in the system were divided into two subgroups, the first one containing the first seven units and the second including the remaining eight units in tables (A-3) and (A-4). Data of one day load demand are listed in table (A-5). The contribution of each subgroup to the load demand is determined from equation (5.12). Each subgroup is manipulated and solved independently, then overall solution to the main problem is obtained by adding the solutions of the subgroups. Tables (5.5) and (5.6) contain the results of the first subgroup, while the results of the second subgroup are shown in tables (5.7) and (5.8). The schedule of the units and the final results of the 15 unit system are presented in table (5.9).

ii) 150 unit system: The system with the input data outlined in tables (A-6), (A-7) and (A-8) respectively is decomposed into 15 subgroups, of 10 units each. The unit commitment of every subgroup is solved independently. Sample of the results of the subgroup unit commitment are found in tables (5.10) and (5.11). The total operational costs of the system are determined by adding the solutions of the subgroups, as seen in table (5.12).

5.5 DISCUSSION

It is possible to solve the unit commitment problem by implementing decomposition methods. In Lagrangian relaxation application to the problem, the objective function is decomposed into a single generation subproblem, then feasible solutions to the subproblems are produced. These solutions are used as initial values to obtain an overall solution of the main objective function, as a function of Lagrangian multipliers. It has been concluded that the rate of convergence and the commitment decision depend mainly on the initial values of the variables, initial values and the variation of the Lagrangian multipliers. For example, if a system consists of several units whose cost characteristics are nearly identical, a slight modification of the Lagrangian multipliers at a particular interval may turn all of these units ON or OFF. Hence, modification and updating of the multipliers should be determined properly; otherwise, the number of committed units may be greater than required, as it can be observed from tables (5.3) and (5.4). The number of units in the system also strongly affects the rate of convergence and the CPU time. It is noted that the system with 150 units could not be solved by Lagrangian relaxation method, since no convergence could be reached due to the difficulty in selecting suitable initial values and in updating

Lagrangian multipliers for this size of systems. Although some researchers claimed having solved the unit commitment for large scale power systems with the Lagrangian relaxation approach, it is clear that this method was implemented to solve the economic dispatch only, as a part of the method of unit commitment solution, as in [35,38]. It was used to solve the problem where the number of units involved in the search was reduced to a certain limit to suit the Lagrangian relaxation technique [47,54].

In the Benders method, the number of units in the system was decomposed into subgroups so that each one contained a certain number of units representing one independent problem. In this technique, the problem of the subgroup is also decomposed into an outer subproblem, representing the unit commitment, and an inner subproblem, representing economic dispatch. Each level of the problem is solved by employing the proper technique, then the overall solution to the problem is obtained by gathering the solutions to the subproblems.

It can be concluded that decomposition methods simplify unit commitment problems in terms of dimensionality. Consequently, computer space as well as the CPU time can be remarkably reduced. The total cost, however, may not always be optimal. The real advantages of decomposition methods are realized only when applied to large systems because additional efforts are required for problem formulation and computer program preparation which are not worthwhile if applied to small systems.

A comparison of the results obtained by decomposition methods discussed in this chapter and of the other methods discussed in previous chapters will be presented in chapter six.

Table (5.1): Results of unit commitment for a 4 unit system by Lagrangian relaxation method.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	2	9017.9	9017.9	550.0
2	2	17213.9	19431.7	550.0
3	3	19647.4	31729.1	630.0
4	3	10850.9	42579.9	630.0
5	3	8407.9	50987.8	630.0
6	2	5744.0	56731.8	380.0
7	1	11943.7	62375.5	300.0
8	2	9890.4	72665.9	550.0
9	2	8843.4	81509.2	550.0
10	2	10239.4	91748.6	550.0
11	3	11753.1	103851.7	630.0
12	3	16138.1	114789.8	630.0
13	3	8582.4	123372.1	630.0
14	3	8060.6	131432.8	630.0
15	2	11470.9	139403.6	550.0
16	2	8494.4	147898.0	550.0
17	2	9017.9	156915.8	550.0
18	2	9890.4	166806.2	550.0
19	3	11461.6	178617.8	630.0
20	3	10676.4	189294.2	630.0
21	3	8722.0	198016.1	630.0
22	3	7718.6	205734.7	630.0
23	2	8932.6	213967.3	550.0
24	2	9890.4	223857.7	550.0

TOTAL START UP COST = £953.47

TOTAL GENERATION COST = £222904.23

TOTAL OPERATIONAL COST = £223857.7

* Total number of committed units.

Table (5.2): Unit schedule of a 4 unit system over a 24 h period by Lagrangian relaxation method.

Time hours	Unit number and status 1	2	3	4
1	1	1	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	1	0
5	1	1	1	0
6	1	0	1	0
7	1	0	0	0
8	1	1	0	0
9	1	1	0	0
10	1	1	0	0
11	1	1	1	0
12	1	1	1	0
13	1	1	1	0
14	1	1	1	0
15	1	1	0	0
16	1	1	0	0
17	1	1	0	0
18	1	1	0	0
19	1	1	1	0
20	1	1	1	0
21	1	1	1	0
22	1	1	1	0
23	1	1	0	0
24	1	1	0	0
CPU TIME = 3.43 SECOND				

Table (5.3): Unit commitment results for a 15 unit system by Lagrangian relaxation method.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	13	10274.6	10274.6	2940.0
2	12	9234.7	19509.3	2773.0
3	12	8898.7	28407.9	2773.0
4	12	8547.3	36955.2	2773.0
5	12	8553.0	45508.2	2773.0
6	11	8491.9	54000.2	2773.0
7	12	9624.9	63625.1	2940.0
8	13	10814.9	74440.0	3107.0
9	12	12353.7	86793.7	2940.0
10	12	13914.2	100707.9	2940.0
11	12	14923.8	115631.7	2940.0
12	13	16742.6	132374.3	2940.0
13	14	16482.3	148856.6	3107.0
14	13	16776.7	165633.4	2940.0
15	14	17614.9	183248.2	3107.0
16	14	17232.9	200481.1	3107.0
17	14	17228.1	217709.3	3107.0
18	13	16819.2	234528.5	2940.0
19	14	16731.7	251260.2	3107.0
20	14	16011.9	267272.1	3107.0
21	13	15638.6	282910.7	2940.0
22	12	15361.2	298272.0	2940.0
23	11	12483.2	310755.1	2773.0
24	12	11552.5	322307.6	2940.0

TOTAL START UP COST = £4180.34

TOTAL GENERATION COST = £318127.26

TOTAL OPERATIONAL COST = £322307.60

* Total number of committed units.

Table (5.4): Unit schedule of a 15 unit system over a 24 h period by Lagrangian relaxation method.

Time hours	Unit number and status of units														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
5	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
6	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
7	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
8	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1
9	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
10	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
11	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
13	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
18	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
22	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
23	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
24	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0

CPU TIME = 417.13 SECOND

Table (5.5): Unit commitment results for a 15 unit system by Benders decomposition method. (Subgroup No. 1 with 7 units)

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	6	8016.7	8016.7	2682.0
2	6	7282.7	15299.4	2682.0
3	6	6947.5	22246.9	2682.0
4	4	6741.2	28988.1	2177.0
5	4	6875.7	35863.8	1600.0
6	4	6704.8	42568.6	1600.0
7	5	7605.0	50173.6	1767.0
8	4	8709.0	58882.7	1932.0
9	5	10027.5	68910.1	2270.0
10	5	11316.7	80226.8	2270.0
11	5	12343.0	92569.8	2773.0
12	6	12830.9	105400.6	2682.0
13	6	13174.8	118575.4	2940.0
14	6	13400.2	131975.6	2940.0
15	7	13611.4	145587.0	3107.0
16	7	13655.4	159242.4	3107.0
17	7	13596.5	172838.9	3107.0
18	6	13384.9	186223.9	2940.0
19	6	13137.4	199361.2	2682.0
20	6	12854.5	212215.7	2940.0
21	6	12471.3	224687.0	2940.0
22	6	12305.3	236992.4	2682.0
23	5	10218.6	247210.9	2270.0
24	5	9300.4	256511.4	2270.0

TOTAL START UP COST = £4485.97

TOTAL GENERATION COST = £252025.43

TOTAL OPERATIONAL COST = £256511.40

* Total number of committed units.

Table (5.6): Unit schedule of a 15 unit system over a 24 h period by Benders decomposition method. (Subgroup No. 1 with 7 units)

Time hours	Unit number and status of units 1	2	3	4	5	6	7
1	1	1	1	0	1	1	1
2	1	1	1	0	1	1	1
3	1	1	1	0	1	1	1
4	1	1	1	0	0	1	0
5	1	0	0	1	1	1	0
6	1	0	0	1	1	1	0
7	1	0	0	1	1	1	1
8	1	1	0	1	0	1	0
9	1	1	0	1	1	1	0
10	1	1	0	1	1	1	0
11	1	1	1	1	1	0	0
12	1	1	1	0	1	1	1
13	1	1	1	1	1	1	0
14	1	1	1	1	1	1	0
15	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1
18	1	1	1	1	1	1	0
19	1	1	1	0	1	1	1
20	1	1	1	1	1	1	0
21	1	1	1	1	1	1	0
22	1	1	1	0	1	1	1
23	1	1	0	1	1	1	0
24	1	1	0	1	1	1	0

CPU TIME = 5.81 SECOND

Table (5.7): Unit commitment results for
a 15 unit system by Benders decomposition method.
(Subgroup No. 2 with 8 units)

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	4	1983.3	1983.3	569.0
2	3	1789.9	3773.3	439.0
3	3	1698.7	5471.9	439.0
4	3	1643.6	7115.6	439.0
5	3	1635.8	8751.4	439.0
6	3	1659.3	10410.7	439.0
7	3	1874.1	12284.9	439.0
8	4	2100.4	14385.3	569.0
9	4	2457.0	16842.3	569.0
10	5	2820.7	19663.0	674.0
11	5	3006.5	22669.5	674.0
12	6	3251.0	25920.5	727.0
13	6	3285.0	29205.5	727.0
14	6	3389.6	32595.0	727.0
15	6	3425.3	36020.3	727.0
16	6	3471.7	39492.0	727.0
17	6	3452.9	42944.9	727.0
18	6	3385.2	46330.1	727.0
19	6	3337.0	49667.1	727.0
20	5	3178.9	52846.0	674.0
21	5	3122.2	55968.2	674.0
22	5	3096.3	59064.5	674.0
23	5	2502.8	61567.2	674.0
24	4	2291.1	63858.3	569.0

TOTAL START UP COST = £184.24

TOTAL GENERATION COST = £63674.06

TOTAL OPERATIONAL COST = £63858.30

* Total number of committed units.

Table (5.8): Unit schedule of a 15 unit system over a 24 h period by Benders decomposition method. (Subgroup No. 2 with 8 units)

Time hours	Unit number and status of units 1	2	3	4	5	6	7	8
1	1	1	1	0	1	0	0	0
2	1	1	1	0	0	0	0	0
3	1	1	1	0	0	0	0	0
4	1	1	1	0	0	0	0	0
5	1	1	1	0	0	0	0	0
6	1	1	1	0	0	0	0	0
7	1	1	1	0	0	0	0	0
8	1	1	1	0	1	0	0	0
9	1	1	1	0	1	0	0	0
10	1	1	1	1	1	0	0	0
11	1	1	1	1	1	0	0	0
12	1	1	1	1	1	1	0	0
13	1	1	1	1	1	1	0	0
14	1	1	1	1	1	1	0	0
15	1	1	1	1	1	1	0	0
16	1	1	1	1	1	1	0	0
17	1	1	1	1	1	1	0	0
18	1	1	1	1	1	1	0	0
19	1	1	1	1	1	1	0	0
20	1	1	1	1	1	0	0	0
21	1	1	1	1	1	0	0	0
22	1	1	1	1	1	0	0	0
23	1	1	1	1	1	0	0	0
24	1	1	1	0	1	0	0	0
CPU TIME = 10.034 SECOND								

Table (5.9): Final schedule of units and operational costs of a 15 unit system by Benders, Decomposition.

Time hours	Unit number and status of units — Subgroup no. 1							Subgroup no. 2							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	0	1	1	1	1	1	1	0	1	0	0	0
2	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0
3	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0
4	1	1	1	0	0	1	0	1	1	1	0	0	0	0	0
5	1	0	0	1	1	1	0	1	1	1	0	0	0	0	0
6	1	0	0	1	1	1	0	1	1	1	0	0	0	0	0
7	1	0	0	1	1	1	1	1	1	1	0	0	0	0	0
8	1	1	0	1	0	1	0	1	1	1	0	1	0	0	0
9	1	1	0	1	1	1	0	1	1	1	0	1	0	0	0
10	1	1	0	1	1	1	0	1	1	1	1	1	0	0	0
11	1	1	1	1	1	0	0	1	1	1	1	1	0	0	0
12	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
13	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0
14	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
18	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0
19	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
20	1	1	1	1	1	1	0	1	1	1	1	1	0	0	0
21	1	1	1	1	1	1	0	1	1	1	1	1	0	0	0
22	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
23	1	1	0	1	1	1	0	1	1	1	1	1	0	0	0
24	1	1	0	1	1	1	0	1	1	1	0	1	0	0	0

TOTAL START UP COST = £4670.214046287850
TOTAL GENERATION COST = £315699.4466436457
THE SYSTEM TOTAL COST = £320369.6606899336
CPU TIME (seconds) = 15.48999023437500

Table (5.10): Results of unit commitment for a 150 unit system by Benders decomposition method. (sample results for a subgroup of 10 units)

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	8	1975.3	1975.3	379.0
2	8	1912.2	3887.4	379.0
3	8	1899.8	5787.2	379.0
4	8	1891.6	7678.8	379.0
5	7	1767.2	9446.0	341.0
6	7	1674.7	11120.7	341.0
7	7	1746.8	12867.5	341.0
8	8	1908.7	14776.2	379.0
9	8	1985.3	16761.5	379.0
10	8	1990.8	18752.3	379.0
11	8	2027.5	20779.8	385.0
12	8	2015.8	22795.6	385.0
13	9	2162.2	24957.8	423.0
14	9	2257.3	27215.1	417.0
15	9	2249.0	29464.0	417.0
16	9	2156.2	31620.2	423.0
17	8	1985.0	33605.3	379.0
18	8	1910.2	35515.5	379.0
19	8	2029.5	37545.0	385.0
20	8	1993.7	39538.7	379.0
21	8	1987.9	41526.6	379.0
22	8	1972.4	43499.0	379.0
23	8	2044.2	45543.2	385.0
24	8	1991.4	47534.6	379.0

TOTAL START UP COST = £78.33

TOTAL GENERATION COST = £47456.26

TOTAL OPERATIONAL COST = £47534.60

* Total number of committed units.

Table (5.11): Unit schedule of 10 units as a subgroup from a 150 unit system over a 24 h period by Benders decomposition method.

Time hours	Unit number and status of units									
	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	0	0
2	1	1	1	1	1	1	1	1	0	0
3	1	1	1	1	1	1	1	1	0	0
4	1	1	1	1	1	1	1	1	0	0
5	1	1	1	1	1	1	1	0	0	0
6	1	1	1	1	1	1	1	0	0	0
7	1	1	1	1	1	1	1	0	0	0
8	1	1	1	1	1	1	1	1	0	0
9	1	1	1	1	1	1	1	1	0	0
10	1	1	1	1	1	1	1	1	0	0
11	1	1	1	1	1	1	1	0	1	0
12	1	1	1	1	1	1	1	0	1	0
13	1	1	1	1	1	1	1	1	1	0
14	1	1	1	1	1	1	0	1	1	1
15	1	1	1	1	1	1	0	1	1	1
16	1	1	1	1	1	1	1	1	1	0
17	1	1	1	1	1	1	1	1	0	0
18	1	1	1	1	1	1	1	1	0	0
19	1	1	1	1	1	1	1	0	1	0
20	1	1	1	1	1	1	1	1	0	0
21	1	1	1	1	1	1	1	1	0	0
22	1	1	1	1	1	1	1	1	0	0
23	1	1	1	1	1	1	1	0	1	0
24	1	1	1	1	1	1	1	1	0	0

CPU TIME = 55.66 SECOND

Table (5.12): Results of unit commitment for a 150 unit system by Benders decomposition method.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	135	29089.5	29089.5	5595.0
2	135	28095.8	57185.2	5265.0
3	135	27811.8	84997.0	5265.0
4	135	27693.6	112690.6	5265.0
5	129	25709.0	138399.6	4935.0
6	129	24451.3	162851.0	4935.0
7	129	25424.5	188275.4	4935.0
8	130	27932.8	216208.2	5265.0
9	130	29368.3	245576.5	5595.0
10	130	29308.3	274884.8	5595.0
11	130	29381.8	304266.6	5595.0
12	130	29345.0	333611.6	5595.0
13	135	31648.5	365260.2	5925.0
14	135	32921.6	398181.8	6255.0
15	135	32796.1	430977.9	6255.0
16	135	31506.5	462484.5	5925.0
17	129	29226.6	491711.0	5595.0
18	128	28066.0	519777.0	5265.0
19	129	29547.5	549324.6	5595.0
20	129	29243.0	578567.5	5595.0
21	129	29267.4	607834.9	5595.0
22	129	29049.0	636883.9	5595.0
23	129	29619.9	666503.9	5595.0
24	129	29129.4	695633.2	5595.0

TOTAL START UP COST = £1139.12

TOTAL GENERATION COST = £694494.08

TOTAL OPERATIONAL COST = £695633.20

CPU TIME (SECOND) = 828.29

* Total number of committed units.

Table (5.13): Final results of the unit commitment problem for three power systems by decomposition methods. (Lagrangian relaxation LR, and Benders decomposition BD)

Number of units	Method of solution	Start up cost	Generation cost	Total operational costs	CPU Time
4	LR	953.47	222904.23	223857.70	3.43
	BD	263.16	225419.44	225682.60	0.55
15	LR	4180.34	318127.26	322307.60	417.1
	BD	4670.21	315699.45	320369.66	15.49
150	LR**	No feasible solution was obtained			
	BD	1139.12	694494.08	695633.20	828.3

** See discussion in section 5.4.

CHAPTER 6

COMPARISON OF SOLUTION METHODS OF THE UNIT COMMITMENT PROBLEM FOR THERMAL UNITS

6.1 INTRODUCTION

In previous chapters the problem of unit commitment was formulated and solved for three different systems. In chapter three the problem of these systems was solved by implementing heuristic methods. Mathematical programmings (dynamic, integer and mixed integer linear programming) were used to solve the problem in chapter four. Chapter five presented the decomposition techniques and their application to the problem of unit commitment. Results obtained have been attached to each chapter.

A comprehensive comparison of the used methods, based on these results is the main aspect of this chapter. The aim of the comparison is to determine the difference between the solution methods in terms of the following factors;

- Quality of the solution, i.e. which method can achieve the optimal or as close as possible to the optimal solution.

- Computation time (CPU time) for each method.
- Storage space required by the computing facilities.
- Possibilities of practical application.

As an outcome of this comparison, appropriate methods for solving the unit commitment problem of Saudi Consolidated Electric Company will be selected and then presented and developed in chapter seven.

6.2 RESULTS

The final results of the different methods of solution described in chapters three, four and five for 4 unit, 15 unit and 150 unit systems are presented in tables (6.1), (6.2) and (6.3) respectively. These results have been obtained by a VAX11/785 computer. Figures (6.1) to (6.12) illustrate the start up cost, total cost, CPU time and computer memory space required for the programs to solve the unit commitment problem of the three power systems tested by different methods. Explanations of the abbreviations used in the graphs which identify the solution method are as follows:

HM : Heuristic method.
FDP : Full dynamic programming.
DPSC : Dynamic programming sequential combinations.
DPTC :Dynamic programming truncated combinations.

MILP-N : Mixed integer linear programming which calls NAG library routine.

MILP-M : Mixed integer linear programming (Modified).

LR : Lagrangian Relaxation.

BD : Benders Decomposition

Table (6.1): Final results of unit commitment for a 4 unit system by different methods.

Method of solution	Start up cost £	Generation cost £	Total operational cost £	CPU Time Seconds	Computer storage space K Bytes
HM	953.47	225326.33	226279.80	0.84	3.2
FULL DP	263.160	225419.44	225682.60	0.55	6.3
MILP (NAG)	953.470	25701.53	226655.00	0.53	4.4
MILP modified	953.47	225325.87	226279.82	0.51	15.5
LR	953.47	222904.23	223857.70	3.41	4.9
BD	263.16	225419.44	225682.60	0.55	11.4

Table (6.2): Final results of unit commitment for a 15 unit system by different methods.

Method of solution	Start up cost £	Generation cost £	Total operational cost £	CPU Time Seconds	Computer storage space K Bytes
HM	4311.90	329796.40	334108.30	1.11	9.7
FULL DP	4210.18	312800.42	317010.60	334.60	2959.9
DPSC	2117.95	334727.65	346845.60	1.98	86.6
DPTC	1679.12	334381.76	346060.88	22.76	2346.8
MILP (NAG)	4727.49	325640.92	330368.41	3.99	10.99
MILP modified	4679.93	325708.11	330388.24	1.07	41.3
LR	4180.34	318127.26	322307.60	417	59.1
BD	4670.21	315699.45	320369.66	15.49	2782.5

Table (6.2): Final results of unit commitment for a 150 unit system by different methods.

Method of solution	Start up cost £	Generation cost £	Total operational cost £	CPU Time Seconds	Computer storage space K Bytes
HM	459.16	699098.24	699557.40	7.06	78.8
DPSC	1539.47	690239.77	691779.24	1183	557.4
MILP modified	1592.11	703291.51	704883.62	36.64	2338.9
LR*	No feasible solution was obtained				
BD	1139.12	694494.08	695633.20	828.3	7770.2

* No feasible solution was obtained for this size of system (see discussion about this method in the following section)

6.3 DISCUSSION

It can be noted from figures (6.1) to (6.12) that the unit commitment problem of power systems of different sizes can be solved by one or more of the following methods:

- Heuristic method
- Full dynamic programming.
- Dynamic programming sequential combinations.
- Dynamic programming truncated combinations.
- Mixed integer linear programming.
- Lagrangian relaxation.
- Benders decomposition.

The results presented in tables (6.1), (6.2) and (6.3) are obtained when the problem of unit commitment is solved if the constraints of the system and the constraints of units, which are listed in the data in appendix A, are all satisfied. Lower costs, however, could be obtained if some of the constraints, such as minimum shut down, start up time or the spinning reserve constraints are relaxed.

In this study, the results obtained by heuristic methods for the unit commitment problem are assumed as the base values which represent 100% of the total cost, the start up cost, the CPU time and the computer memory space. Therefore, the results produced by the other methods of solution will be measured against these base values.

For the system of 4 units, which represents a small power system, it can be noted from figure (6.2) that the lowest cost over a 24 hour commitment period was obtained by Lagrangian relaxation method where a reduction of 1.07% of the total cost was achieved. The reason is that in the Lagrangian relaxation technique, the unit commitment decision is based on the economic dispatch of the units, i.e., the optimisation problem at any interval t is converted into a single generator problem. Then each generating unit is loaded at its optimal rate of output power, subject to its constraints. The combination of units which satisfied the load and spinning reserve constraints was selected as the committed combination of interval t. Although this procedure produced the best results for a small system in terms of operational cost, it is time consuming since the problem is solved in 3.43 seconds, which is relatively long compared with 0.84 seconds for the heuristic method. This is due to the large number of iterations necessary for updating the Lagrangian multipliers. Furthermore, the convergence of the solution is very sensitive to initial values of the variables and Lagrangian multipliers.

Full dynamic programming method also achieved a reduction of cost by 0.264%. The major part of this reduction was in the start up cost. In full dynamic programming technique all the possible combinations of units at any interval were tested. Hence, the lowest cost feasible combination was selected as the optimal committed combination for the interval, and then a further attempt to improve the solution was tried by performing economic dispatch. Mixed integer linear programming produced almost the same results as the heuristic method. The results obtained by Benders decomposition method are almost same to those produced by full dynamic programming. However, memory space required for this method is increased because of the further additional modification of the computer program. The space of computer memory required for any method of solution of the problem for a 4 unit system is relatively small and in the range between 3 to 16 K bytes.

For a medium power system, which for this study was represented by a 15 unit system, the problem of the unit commitment was solved over a 24 hour period by using all of the solution methods listed at the beginning of this section. Figure (6.5) to (6.8) illustrate the final results of each method. It is clear from figure (6.6) that the lowest operational cost was obtained by full dynamic programming method. 5.1% reduction of cost was gained; however, the CPU time increased rapidly from 1.1

second for heuristic method to 334 seconds for full dynamic programming, due to the large number of combinations of units tested. Memory storage space also jumped rapidly from 9.7 K bytes in heuristic to 2960 K bytes in dynamic programming because of high dimensionality of the matrix arrays of the units' status. Dynamic programming sequential and truncated combination methods did not yield any reduction in cost compared with heuristic method for the unit commitment of this system. Mixed integer linear programming method achieved a 1.1% reduction in cost in short CPU time and small computer memory space. Lagrangian relaxation technique produced better results than MILP, since a reduction of 3.53% of the cost was accomplished compared with the heuristic method. However, the CPU time was the highest. With Benders decomposition method the operational cost was reduced by 4.1% against heuristic method. The results were produced in short CPU time. However, computer memory space required for this method was large.

For a large-scale power system of 150 units, the final results are demonstrated in figure (6.9) to figure (6.12). Not all the solution methods listed previously are applicable to the large-scale system. These non-applicable methods are full dynamic programming, dynamic programming truncated combinations and Lagrangian relaxation. The first two methods are excluded because of the large dimensions of the problem; consequently, a huge computer memory space is required. If the computation resources were possible to provide, another difficulty could prevent the use of these methods due to the high CPU time required. As mentioned earlier, Lagrangian relaxation method is highly sensitive to the initial values of the variables and the Lagrangian multipliers. This phenomenon increases with the increase in the number of units and causes non-convergence in large systems.

As can be seen from figure (6.10), dynamic programming sequential combination method produced the best results in terms of operational cost. By this approach, 1.11% reduction of cost was gained compared with heuristic method. The CPU time, however, was very large (1183 seconds). Computer memory requirement was within practical limits (557 K bytes). Mixed integer linear programming method did not give any improvement, which was probably due to the linearization approximation in the input-output characteristics of the generating units. Although Benders decomposition technique offered a 0.56% reduction in cost, the CPU time of 828.3 seconds was relatively large. Computer memory space requirement was the highest for this method of solution with a value of 7770 K bytes.

6.4 CONCLUSION

It is possible to solve the problem of unit commitment for any size of power system by implementing the appropriate technique from the methods outlined in the last section. The selection of the suitable method of solution depends on a number of factors. These factors vary from one system to another and include the size of the system, the type of units, input-output characteristics of units, load demand patterns of the system, reliability and security requirements of the system, etc.

Decomposition techniques provide a significant contribution to solving the unit commitment problem of larger power systems. It should be noted, however, that the advantages and benefits of these methods can only be realized when they are applied to large systems because formulation of the problem and preparation of computer programs require additional efforts which are not worthwhile when applied to small systems.

Further conclusions and discussion as well as advantages and disadvantages of different methods are included in chapter eight.

Figure (6.1): Start up cost of a 4 unit system for a 24 hour commitment period

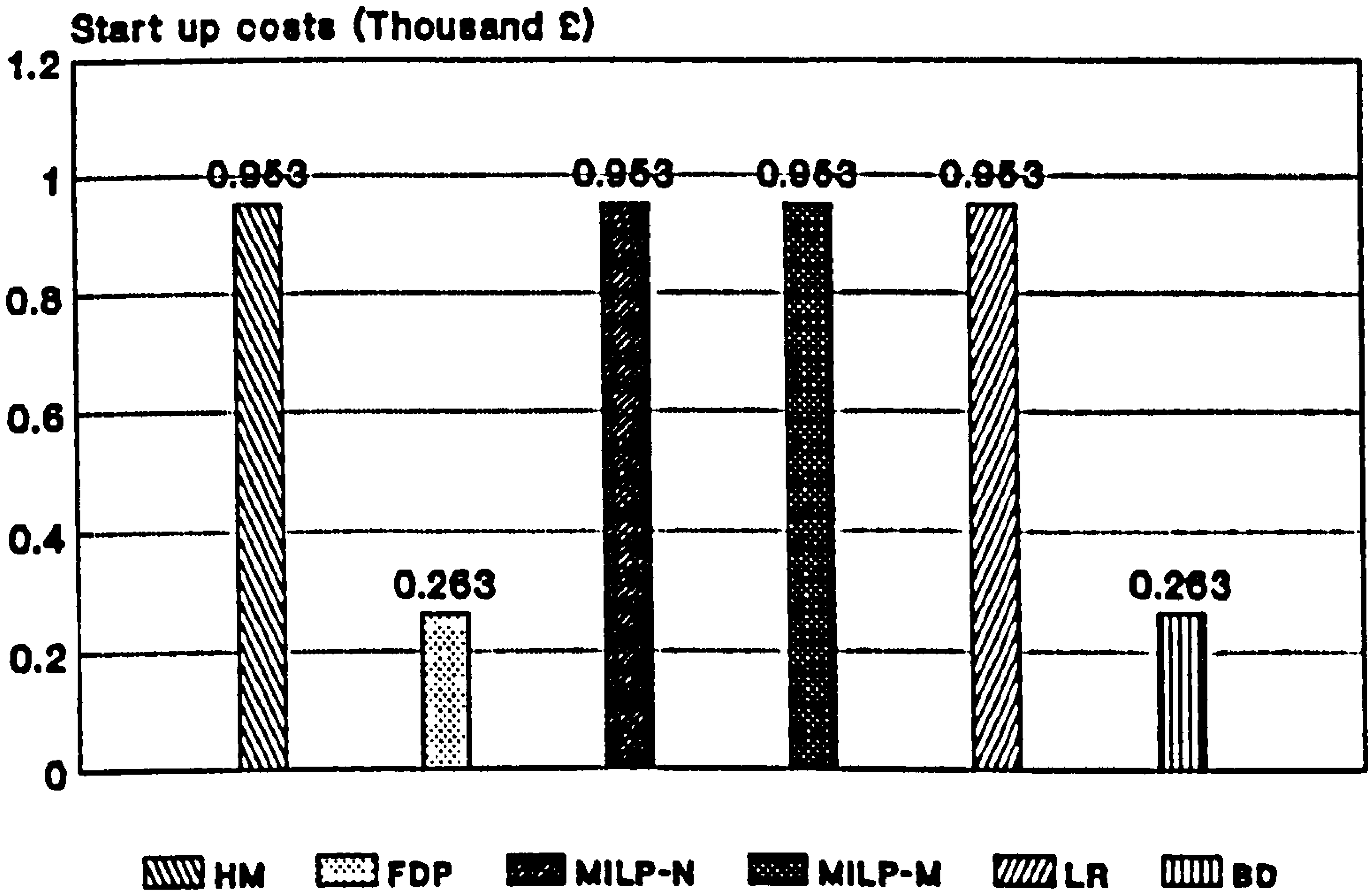


Figure (6.2): Total cost of a 4 unit system for a 24 hour commitment period.

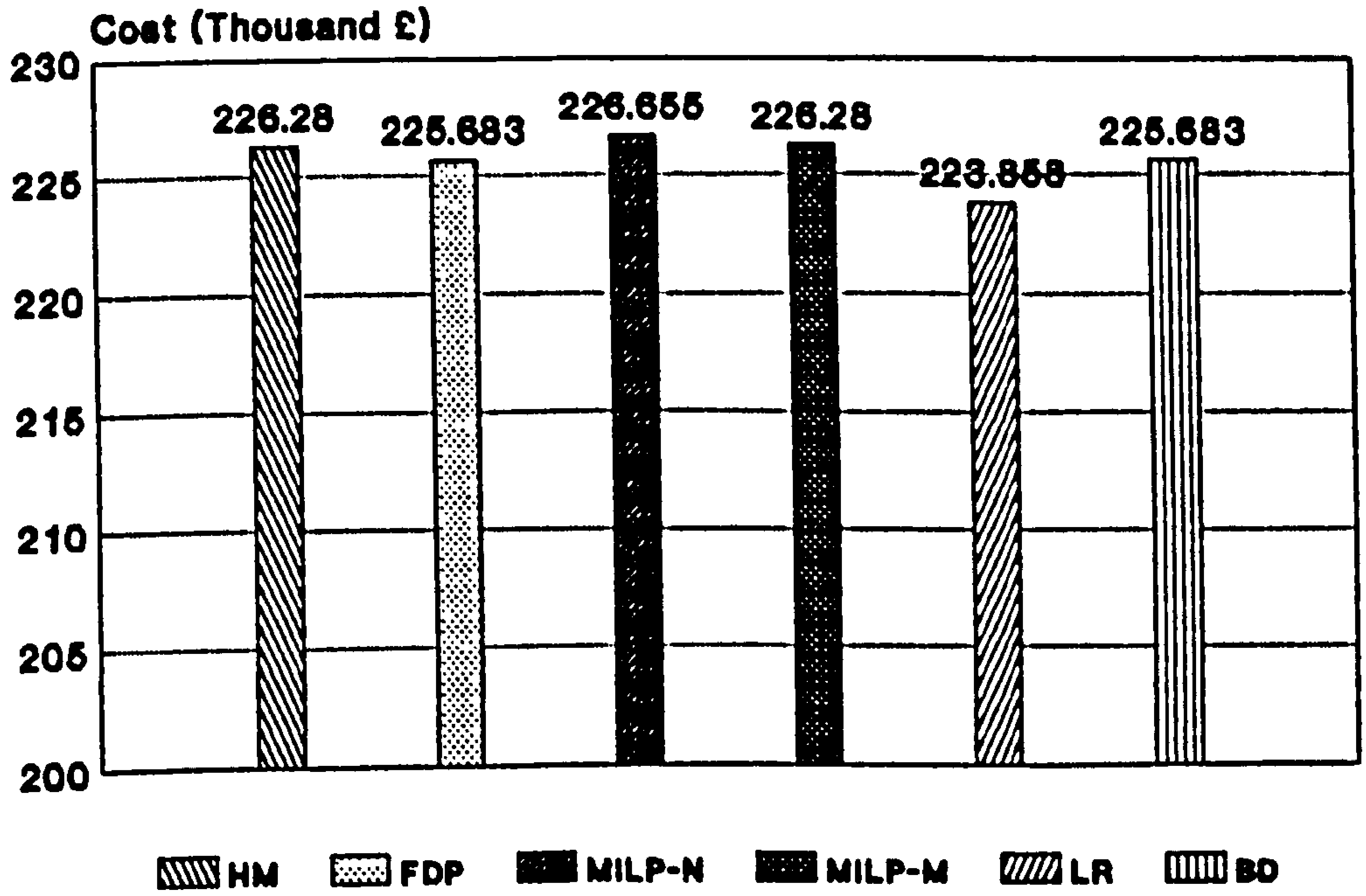


Figure (6.3): CPU time for solving the unit commiment of a 4 unit system.

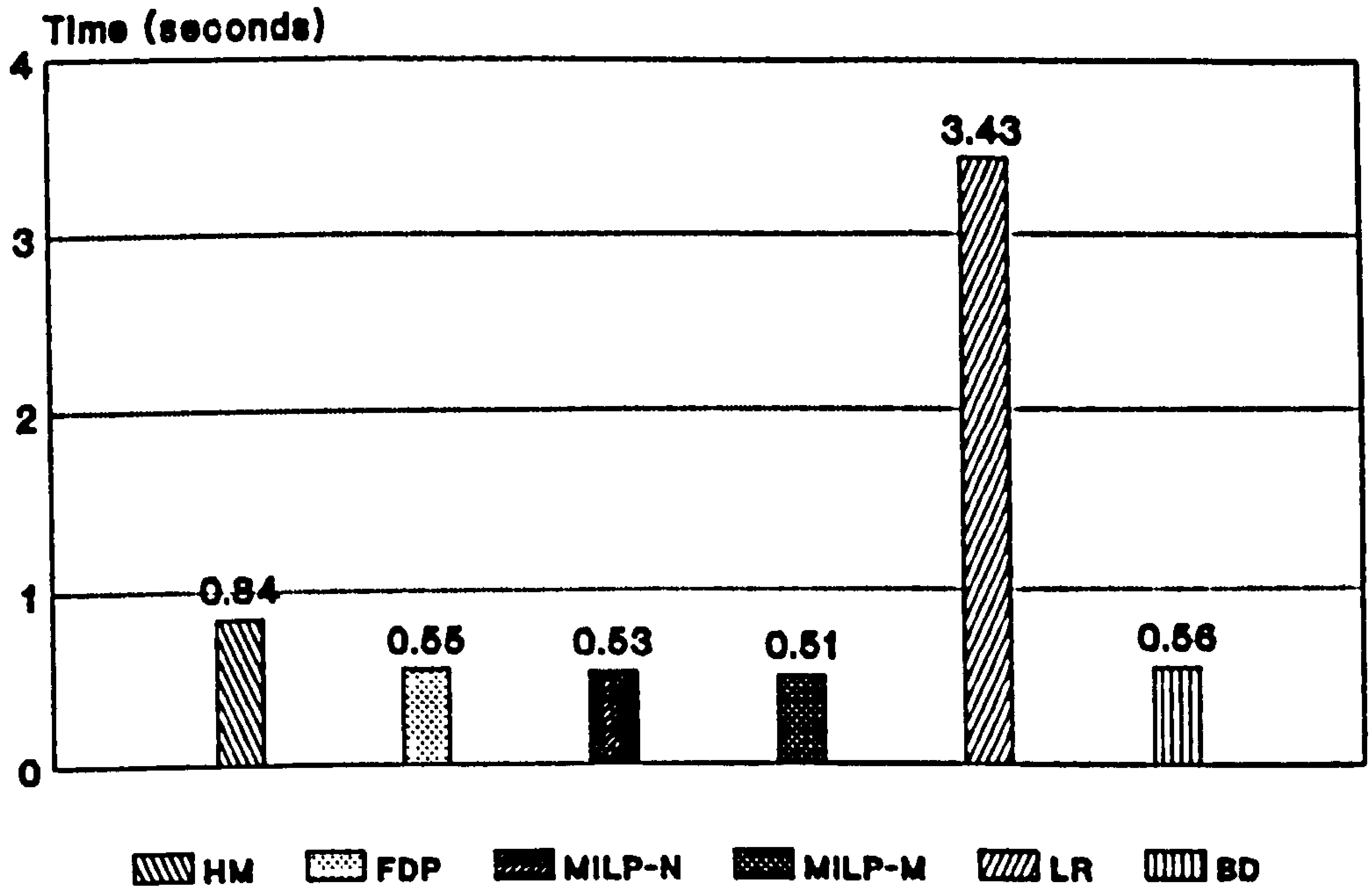


Figure (6.4): Computer memory space for the programs used for a 4 unit system.

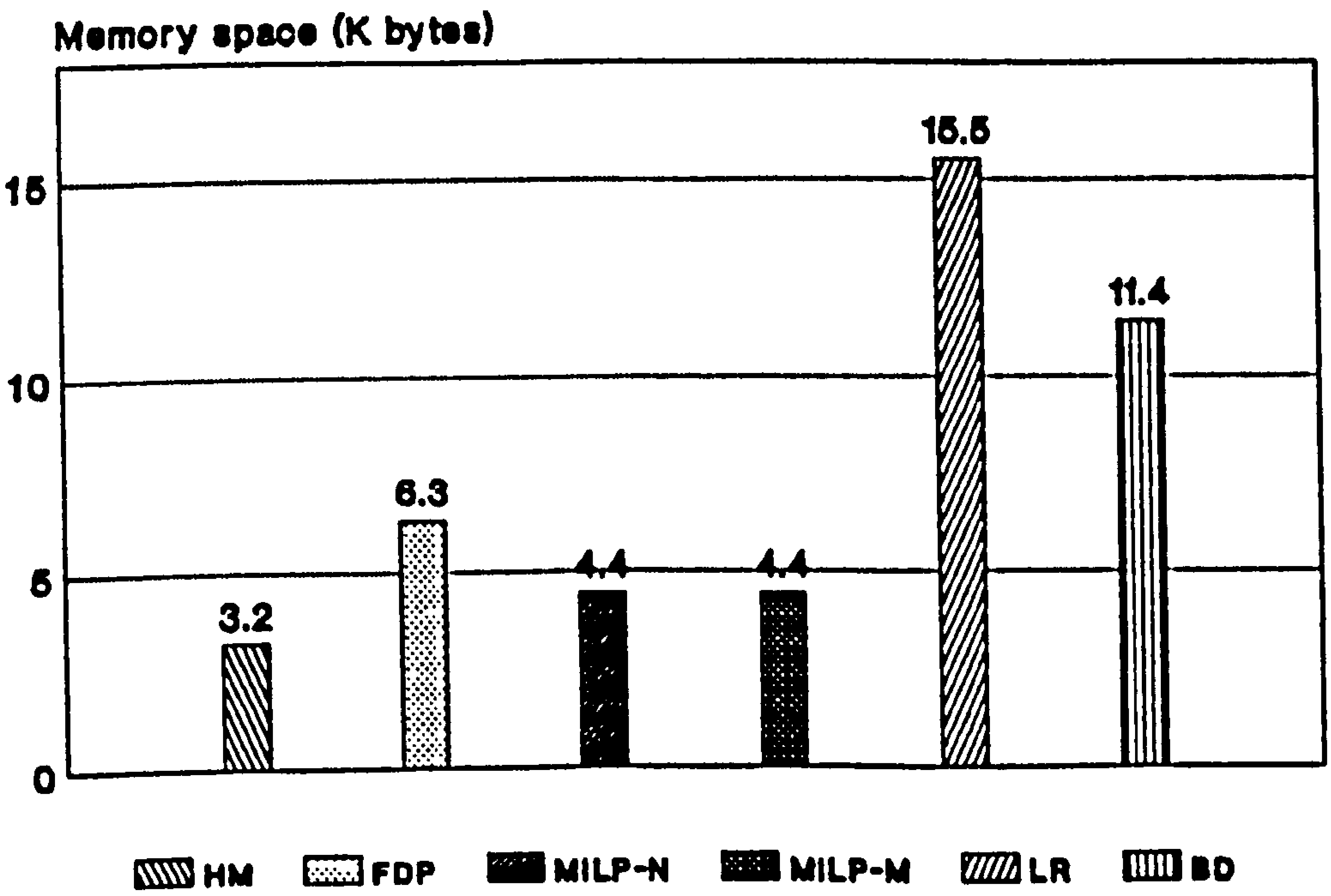


Figure (6.5): Start up cost of a 15 unit system for a 24 hour commitment period

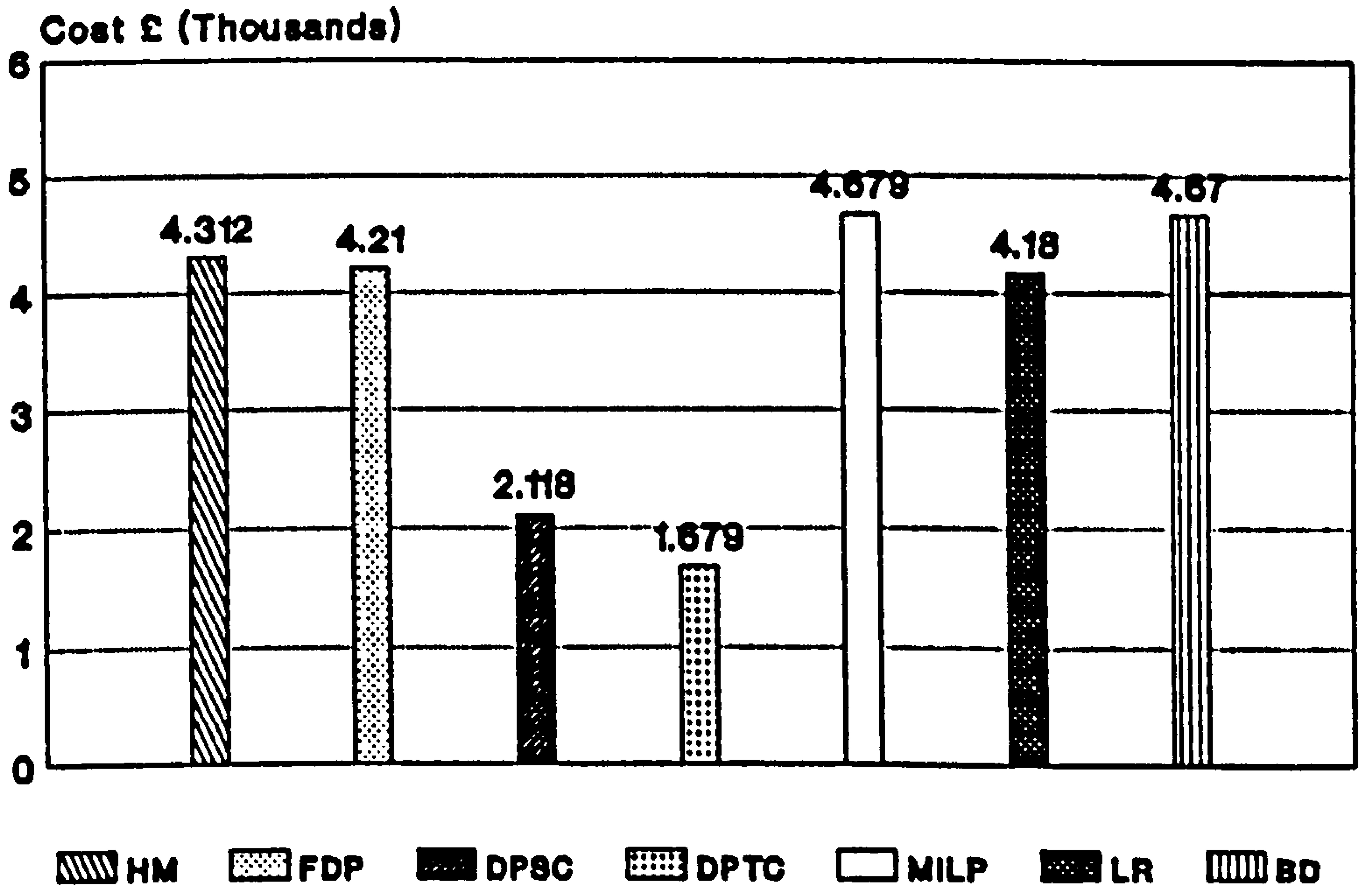


Figure (6.6): Total cost of a 15 unit system for a 24 hour commitment period.

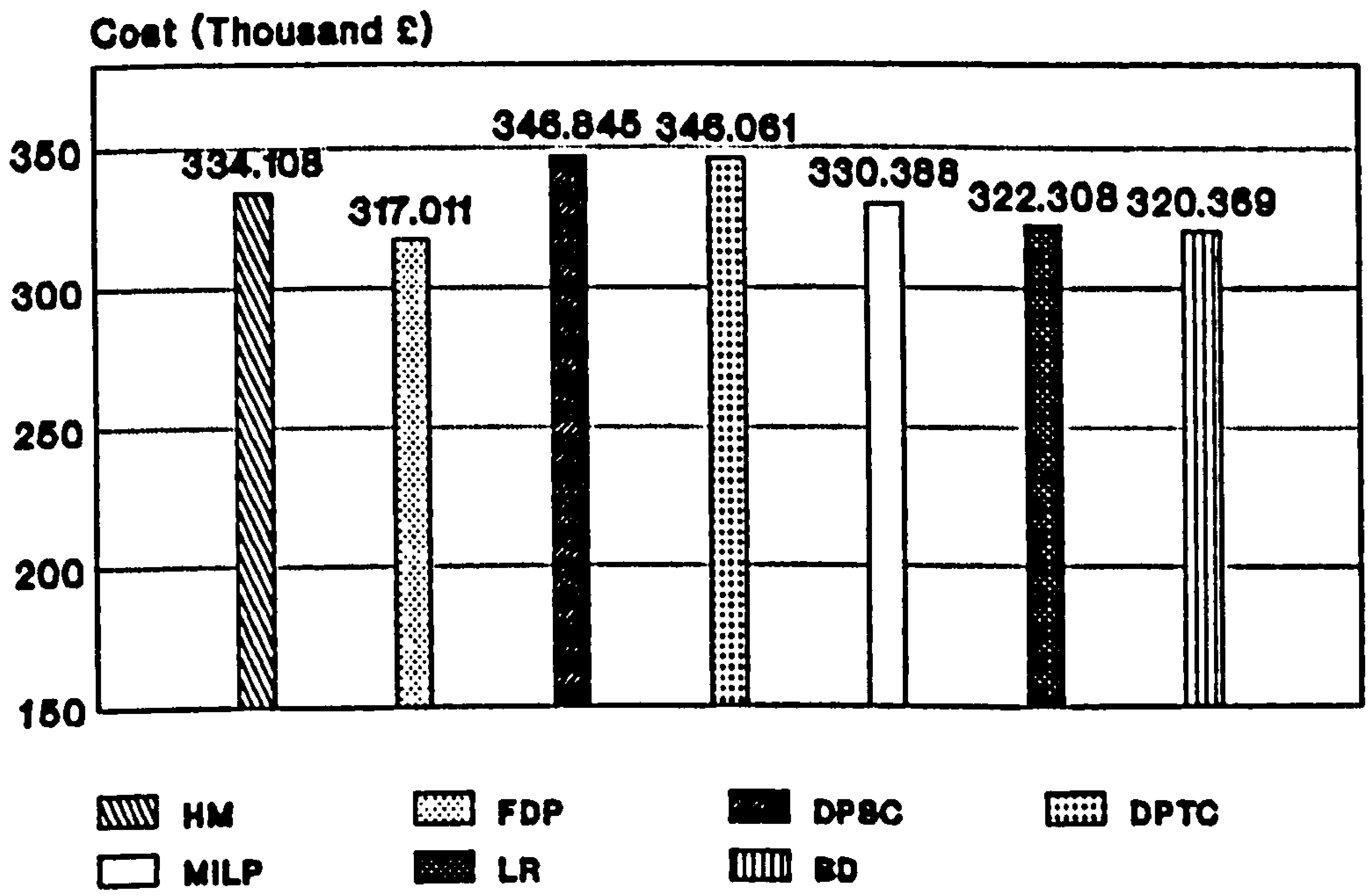


Figure (6.7): CPU time for solving the unit commitment of a 15 unit system.

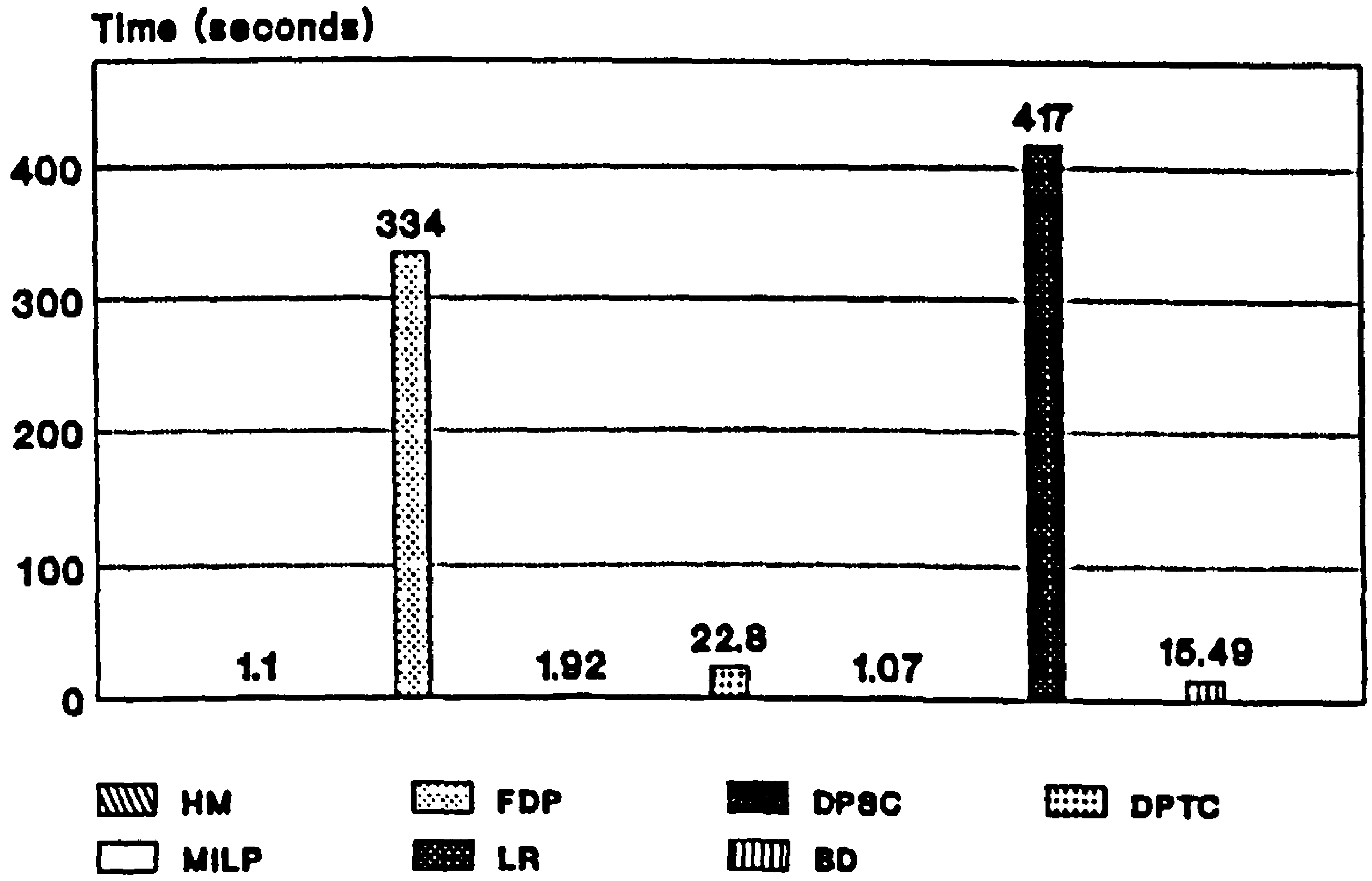


Figure (6.8): Computer memory space for the programs used for a 15 unit system.

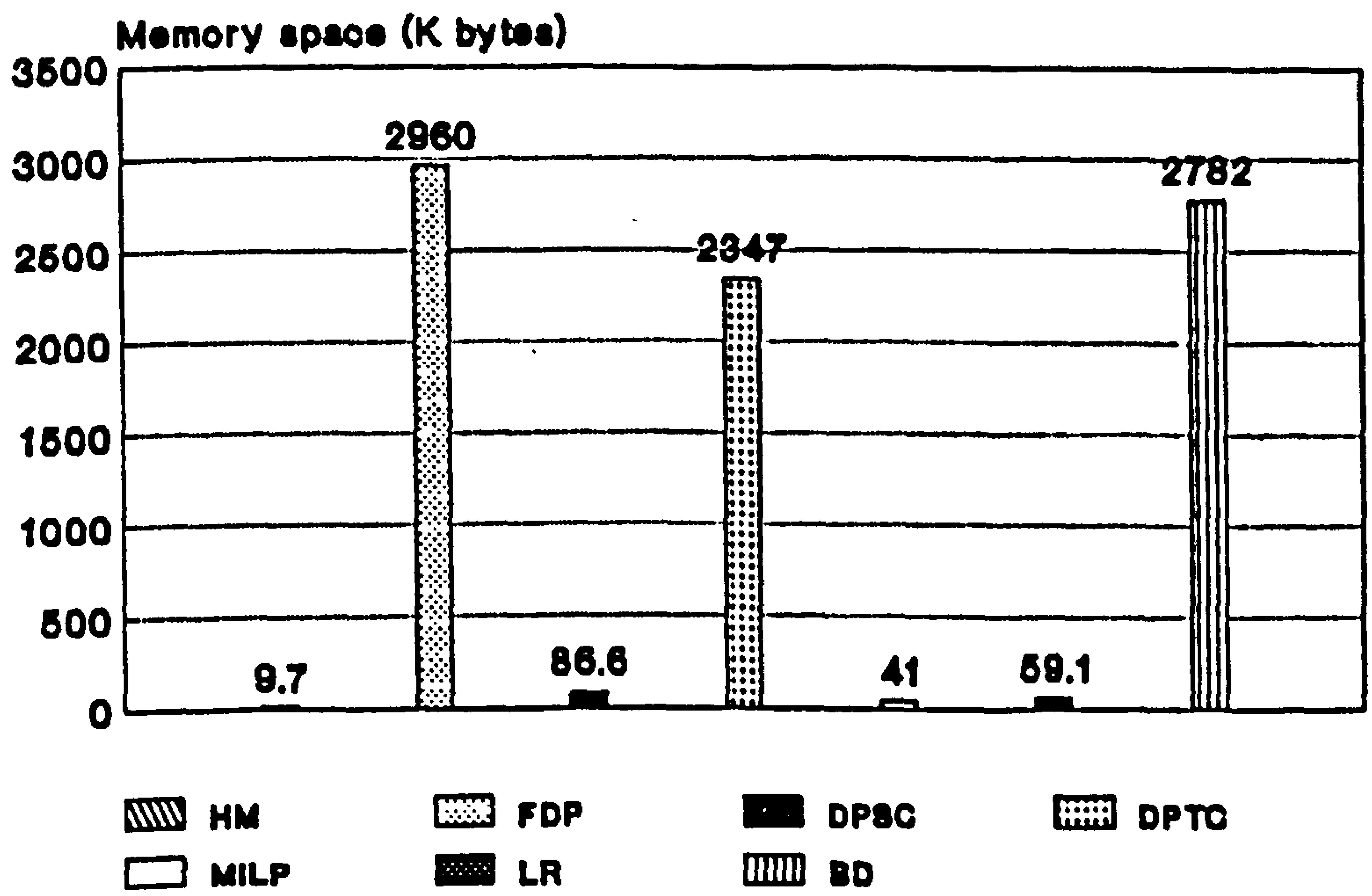


Figure (6.9):Start up cost of a 150 unit system for a 24 hour commitment period

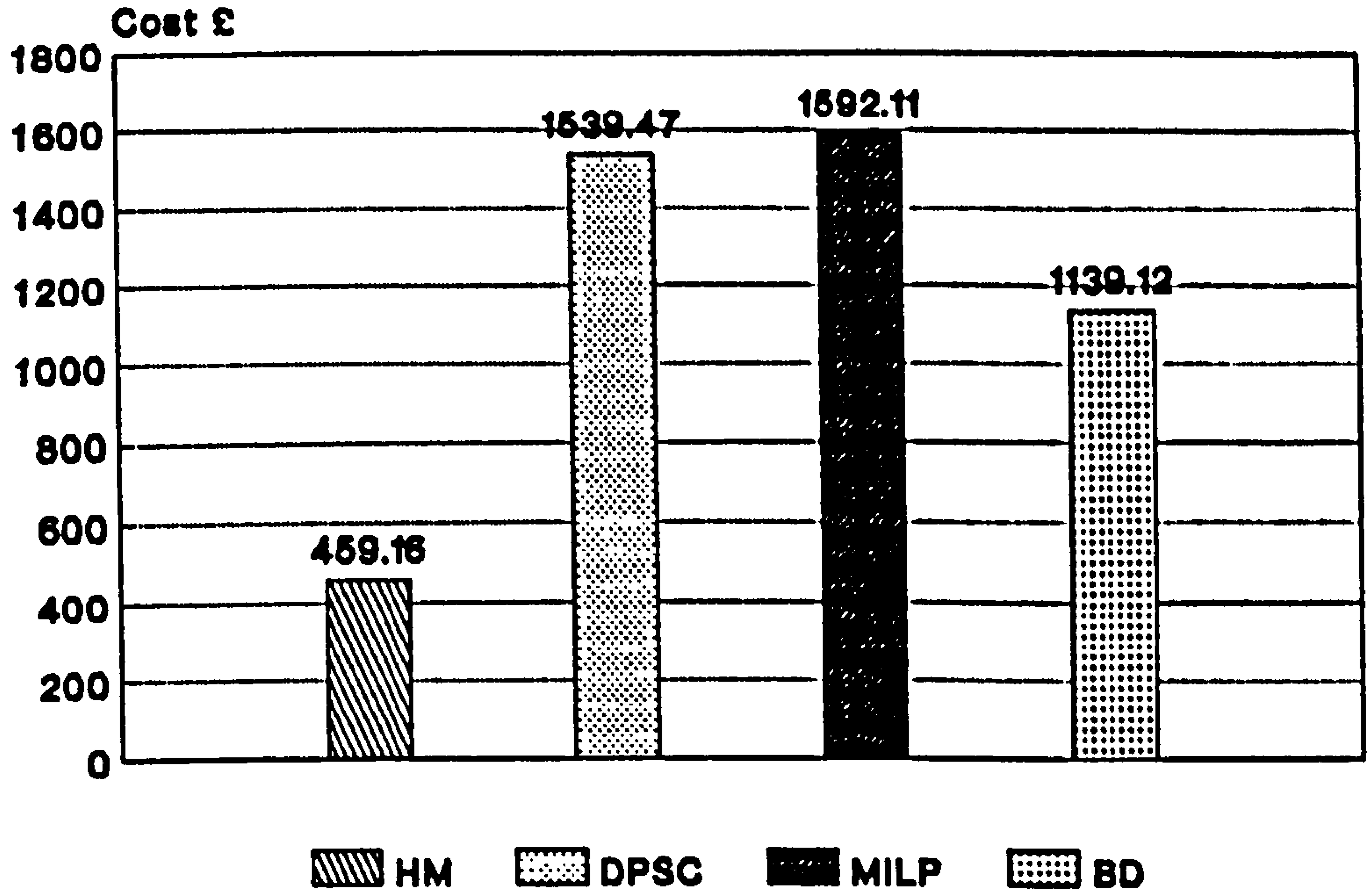


Figure (6.10): Total cost of a 150 unit system for a 24 hour commitment period

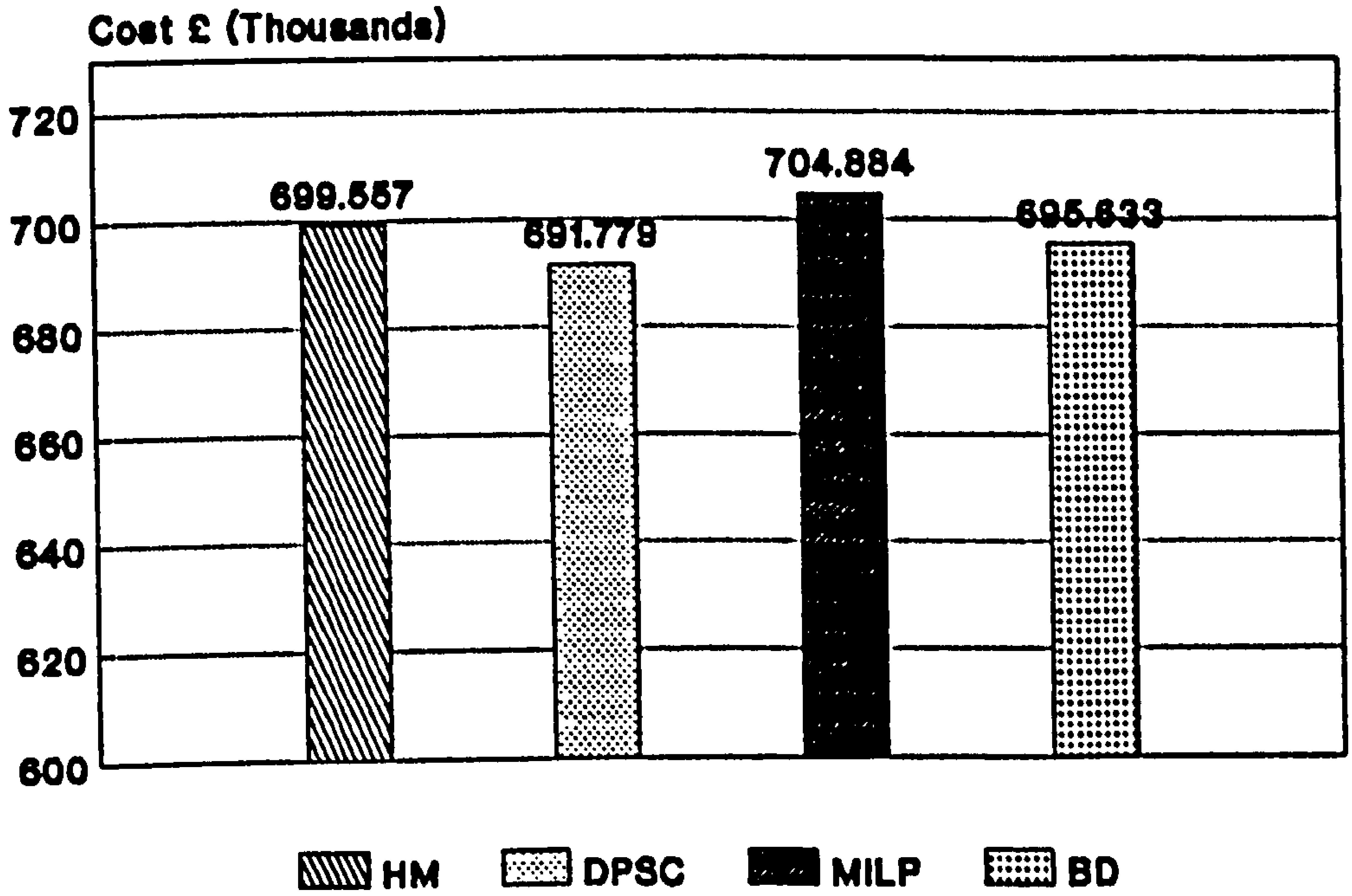


Figure (6.11): CPU time for solving the unit commitment of a 150 unit system.

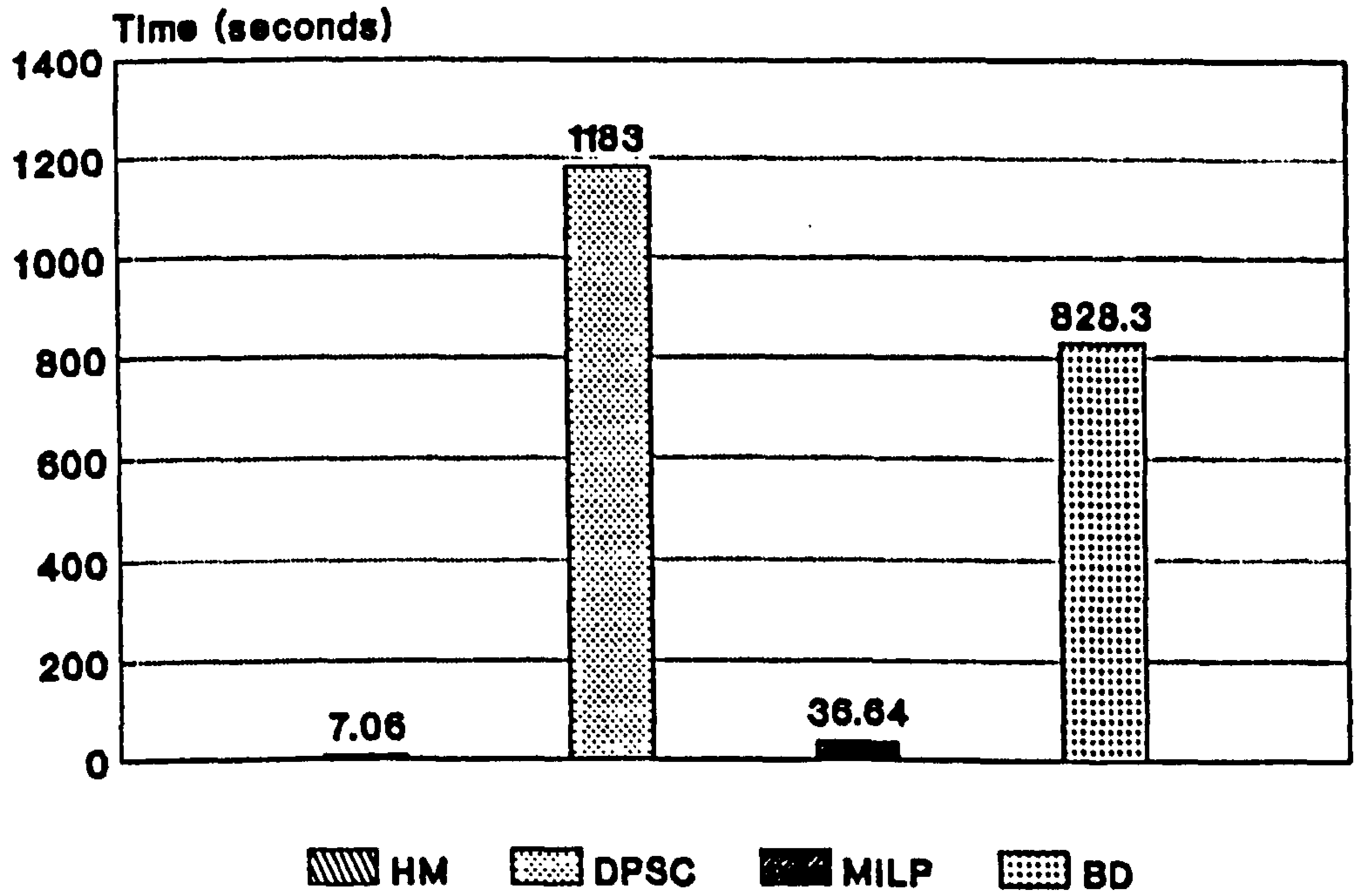
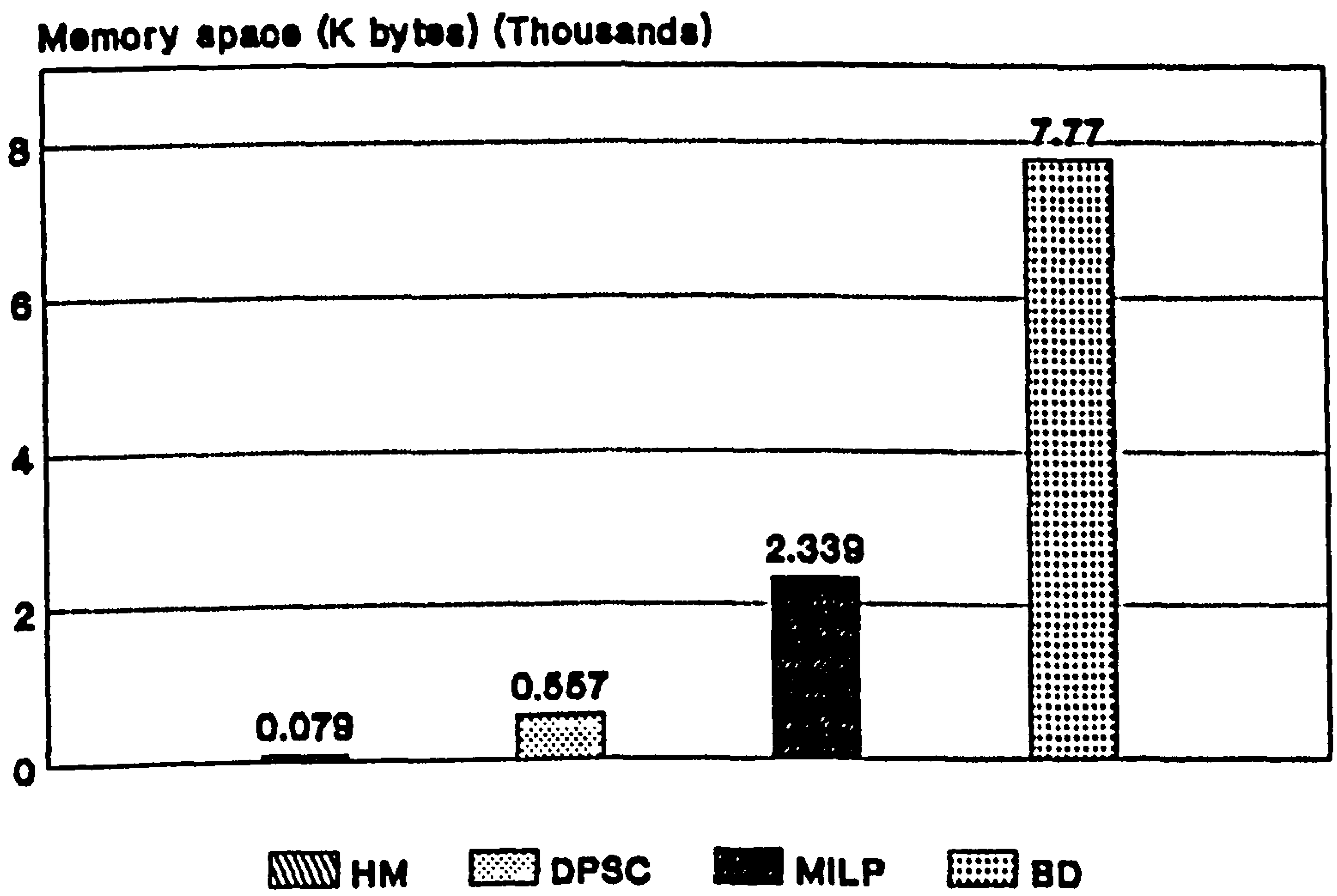


Figure (6.12): Computer memory space for the programs used for a 150 unit system



CHAPTER 7

UNIT COMMITMENT SOLUTION FOR SAUDI CONSOLIDATED ELECTRIC COMPANY, CENTRAL REGION

7.1 INTRODUCTION

This chapter is concerned with selecting an appropriate method for solving the unit commitment problem for Saudi Consolidated Electric Company from the methods discussed in previous chapters.

Saudi Consolidated Electric Company (SCECO Central) is one of the four Electric companies in the Kingdom of Saudi Arabia. SCECO central covers the midland region of the country. A brief description and information about the system, along with the necessary input data for solving the unit commitment problem of SCECO Central are given in Appendix B.

7.2 METHODS OF SOLUTION

On the basis of the comparison of solution methods of the unit commitment problem in chapter six, it can be concluded that the unit commitment problem of SCECO Central system can be solved by implementing one of five methods from those presented and compared previous chapter. These five methods are :-

1. Heuristic method.
2. Dynamic programming sequential combinations.
3. Mixed integer linear programming.
4. Lagrangian relaxation.
5. Benders decomposition.

These methods are described in chapters three to five. The quality of solution, however, varies from one method to another. Therefore, all these methods will be tested on the SCECO Central system to find out the most appropriate approach for solving its unit commitment problem.

7.3 INPUT DATA

Input data of generating units of SCECO Central, which were used for solving the unit commitment problem are listed in tables (B-6) and (B-7) according to the priority order of the plants. An hourly load demand profile of the system for one day is found in table (B-8). Table (B-9) presents the system spinning reserve requirements corresponding to the load levels.

7.4 RESULTS

Unit commitment problem of SCECO Central was solved for a 24 hours commitment period by implementing the methods of solution listed in section 7.2. The following conditions were assumed:

1. Quadratic cost function of the units.
2. No base units or must run units.
3. All the units were available for commitment subject to their constraints.
4. The start up and the shut down costs were assumed as fixed values.
5. No fuel or transmission line constraints.

A VAX11/785 machine has been used for running the programs. Results of the tested methods are presented in tables (7.1) to (7.17).

7.5 A NEW APPROACH FOR SOLVING SCECO UNIT COMMITMENT

In solving the unit commitment problem, the main cause of difficulty is the large number of units for large-scale power systems. The problem cannot be solved if all units are involved in the search for the optimal solution, since computational facilities could be exhausted. Therefore, several approaches were suggested to reduce the requirement for computational resources to the appropriate limit either by classifying the units into different categories, with only the cycling units being included in the search for the lowest cost, or by implementing one of the decomposition techniques discussed previously. Although these methods achieved a remarkable reduction of the computation requirement, further improvement of the quality of the solution is still desired.

Traditional decomposition involves the decomposing of generating units into different groups without particular reference to their input-output characteristics. A new method for solving the problem of unit commitment is proposed which takes these particular criteria into consideration. This method is different from other decomposition methods applied to unit commitment problem. The units in the system are decomposed into groups according to their input-output characteristics so that identical units form one group. Each group is represented by one sample unit. Consequently, a reduced system consisting of sample units only is generated. The basic idea of the proposed method is that the power system is represented by a reduced system (sample system) of lower number of generating units such that the unit commitment problem of the sample system is more easier to solve than that of the original system. When the solution of unit commitment of the reduced system is obtained, then solution to the problem of the original system can be accordingly determined.

The solution method can be proceeded as follows:

i) A reduced system with a lower number of units is generated. It consisted of the sample units only.

ii) The load demand for the reduced system (sample system) at any interval t is calculated as follows:

$$Ds_t = \{ (\sum_{L=1}^{Ng} \frac{Ps_{max}^{L}}{P_{max}^{L}})/Ng \} . D_t \quad ..... \quad (7.1)$$

where:

Ng : Number of groups

Ps_{max}^{L} : The maximum capacity of the sample unit of group L.

P_{max}^{L} : The total maximum capacity of units in group L.

iii) The unit commitment problem for the reduced system (the sample units of the groups and the sample of the load determined by equation (7.1)) is solved by using a suitable technique (full dynamic programming has been used in this study), subject to the constraints.

iv) Each group of units is treated according to the status of its representative unit. Therefore, if the sample unit of group L at interval t is on, then all the units in the group should be on, and operated at the same output as the sample unit, and vice versa.

v) If a sample unit of group L is started or shut down at interval t, then all units of the group are subjected to the same state.

vi) The total operational cost of the commitment period is calculated from the following equation:

$$C_{Total} = \sum_{t=1}^{T} \{ \sum_{L=1}^{Ng} GC_{sLt}(P_{sL}, U_{sL}).Nu_L$$

$$+ Suc_{sLt}(U_{sL}).Nu_L + Sdc_{sLt}(U_{sL}).Nu_L \} \quad ..... \quad (7.2)$$

where Nu_L is the number of units in group L.

It was noted in chapter six that full dynamic programming technique is the best method for solving the unit commitment problem for systems of 20 units or less. Therefore, the problem of the reduced system was solved by using this technique. Input data of units and of the reduced system and the load demand are shown in table (B-10), (B-11) and (B-12) respectively. Results of the sample system over the commitment period are found in table (7.13). Table (7.14) demonstrates the schedule of the sample units, while results of the whole system, i.e, operational cost and schedule of units over the horizon study are presented in tables (7.15) and (7.16) respectively.

Final results of the different methods tested on SCECO Central system are produced in table (7.17). Figure (7.1) and figure (7.2) demonstrate start up and shut down costs of each of the used methods respectively. Generation costs are shown in figure (7.3), while total operational costs of SCECO Central over a commitment period of 24 hours are illustrated in figure (7.4). Figures (7.5) and (7.6) show the computation resources requirements in terms of CPU time and computer memory storage space for the implemented computer programs.

Table (7.1): Results of unit commitment for SCECO system by heuristic method.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation	Spinning reserve
1	47	11239.0	11239.0	2237.5	130.47
2	46	10958.0	22197.0	2187.5	131.47
3	46	10886.7	33083.8	2187.5	146.47
4	45	10659.0	43742.8	2137.5	134.47
5	41	10082.4	53825.2	2007.5	97.63
6	39	9527.5	63352.6	1924.5	117.62
7	40	9840.2	73192.9	1966.0	96.13
8	45	10663.4	83856.3	2137.5	141.47
9	46	10955.6	94811.8	2187.5	132.47
10	47	11217.9	106029.7	2237.5	138.47
11	47	11271.8	117301.5	2237.5	122.47
12	47	11210.3	128511.8	2237.5	137.47
13	48	11476.1	139987.9	2287.5	145.47
14	49	11874.0	151861.9	2337.5	118.47
15	49	11920.0	163781.9	2337.5	104.47
16	48	11478.6	175260.5	2287.5	144.47
17	47	11228.5	186489.0	2237.5	135.47
18	44	10523.0	197012.0	2087.5	102.47
19	45	10615.7	207627.8	2137.5	145.47
20	46	11016.9	218644.7	2187.5	117.47
21	46	10716.2	229360.9	2187.5	140.47
22	44	10470.2	239831.1	2087.5	110.47
23	45	10742.3	250573.4	2137.5	114.47
24	47	11204.9	261778.3	2237.5	144.47

TOTAL START UP COST = £165.16

TOTAL SHUT DOWN COST = £129.48

TOTAL GENERATION COST = £261483.66

TOTAL OPERATIONAL COST = £261778.30

CPU TIME = 1.47 SEC.

* Total number of committed units.

Table (7.2): Unit schedule of SCECO Central system over
a 24 hours period by heuristic method.

Time (H)	Unit number and status of units (1 = ON, 0 = OFF)																																																									
	1				5					10					15					20					25					30					35					40					45					50					55				59
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0

Table (7.3): Results of unit commitment for SCECO system by dynamic programming sequential combinations.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	46	11157.3	11157.3	2187.5
2	45	10876.0	22033.3	2137.5
3	45	10804.4	32837.7	2137.5
4	44	10576.6	43414.3	2087.5
5	41	10073.1	53487.4	2007.5
6	39	9528.1	63015.5	1924.5
7	40	9840.9	72856.4	1966.0
8	44	10569.3	83425.6	2087.5
9	45	10873.5	94299.1	2137.5
10	46	11136.0	105435.1	2187.5
11	47	11283.4	116718.5	2237.5
12	47	11138.5	127856.9	2237.5
13	47	11394.3	139251.3	2237.5
14	49	11885.6	151136.9	2337.5
15	49	11919.9	163056.8	2337.5
16	47	11406.9	174463.7	2237.5
17	46	11146.7	185610.4	2187.5
18	44	10513.0	196123.3	2087.5
19	44	10521.6	206644.9	2087.5
20	46	11028.5	217673.4	2187.5
21	45	10716.2	228389.6	2137.5
22	45	10470.2	238859.9	2137.5
23	45	10742.3	249602.2	2137.5
24	46	11111.3	260713.5	2187.5

TOTAL START UP COST = £155.583

TOTAL SHUT DOWN COST = £129.48

TOTAL GENERATION COST = £260428.44

TOTAL OPERATIONAL COST = £260713.51

CPU TIME = 58 SEC.

* Total number of committed units.

Table (7.4): Unit schedule of SCECO Central system over a 24 hours period by dynamic programming sequential combination.

Time (H)	Unit number and status of units (1 = ON, 0 = OFF)																																																									
	1				5					10					15					20					25					30					35					40					45					50					55				59
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0

Table (7.5): Results of unit commitment for SCECO system by mixed integer linear programming.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	45	11437.3	11437.3	2137.5
2	44	11150.9	22588.3	2087.5
3	44	11080.4	33668.7	2087.5
4	43	10850.8	44519.5	2037.5
5	40	10279.3	54798.7	1913.0
6	38	9727.5	64526.2	1830.0
7	39	10039.9	74566.2	1871.5
8	42	10765.3	85331.5	1996.0
9	44	11156.6	96488.1	2087.5
10	45	11416.7	107904.7	2137.5
11	45	11469.6	119374.3	2137.5
12	45	11409.1	130783.4	2137.5
13	46	11680.8	142464.2	2187.5
14	47	12082.1	154546.3	2237.5
15	47	12127.0	166673.3	2237.5
16	46	11683.2	178356.5	2187.5
17	45	11427.2	189783.7	2137.5
18	42	10724.4	200508.2	1996.0
19	42	10725.5	211233.7	1996.0
20	44	11217.1	222450.7	2087.5
21	43	10905.5	233356.2	2037.5
22	42	10673.1	244029.4	1996.0
23	43	10927.0	254956.4	2037.5
24	45	11404.1	266360.5	2137.5

TOTAL START UP COST = £137.783

TOTAL SHUT DOWN UP COST= £113.983

TOTAL GENERATION COST = £266108.70

TOTAL OPERATIONAL COST = £266360.49

CPU TIME = 7.01 SEC.

* Total number of committed units.

Table (7.6): Unit schedule of SCECO Central system over a 24 hours period by mixed integer linear programming.

Time (H)	Unit number and status of units (1 = ON, 0 = OFF)																																																									
	1				5					10					15					20					25					30					35					40					45					50					55				59
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table (7.7): Results of unit commitment for SCECO system by Lagrangian relaxation method.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	51	11106.7	11106.7	2524.5
2	49	10785.0	21899.3	2441.5
3	53	11004.1	32941.2	2586.0
4	50	10613.4	43565.6	2400.0
5	51	10397.2	53991.6	2460.0
6	55	10188.8	64229.5	2637.5
7	57	10654.5	74913.2	2687.5
8	55	10903.6	85758.0	2577.5
9	51	10941.6	96721.6	2507.5
10	59	11514.5	108294.2	2717.5
11	56	11360.7	119654.8	2627.5
12	54	11302.1	130891.8	2510.0
13	59	11682.3	142612.1	2717.5
14	51	11620.8	154156.3	2507.5
15	53	11775.5	165956.7	2586.0
16	49	11107.2	177071.5	2441.5
17	54	11289.5	188398.7	2627.5
18	59	11107.3	199538.9	2717.5
19	53	10826.7	210365.6	2542.5
20	51	10897.2	221289.6	2471.5
21	59	11219.1	232568.3	2717.5
22	57	10995.7	243564.1	2687.5
23	52	10821.6	254385.7	2527.5
24	53	11242.4	265657.3	2577.5

TOTAL START UP COST = £149.93

TOTAL SHUT DOWN COST = £107.37

TOTAL GENERATION COST = £265400.0

TOTAL OPERATIONAL COST = £265657.3

CPU TIME = 1738.48 SEC.

* Total number of committed units.

Table (7.8): Unit schedule of SCECO Central system over a 24 hours period by Lagrangian relaxation method.

Time (H)	Unit number and status of units (1 - ON, 0 - OFF)																																																								
	1				5					10					15					20					25				30					35					40					45					50					55				59
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0
9	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0	0	1	1	1	1	1	0	0	1	0
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0

Table (7.9): Results of unit commitment for SCECO system by Benders decomposition method.
(RESULTS OF SUBGROUP NUMBER 1)

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	8	1913.4	1913.4	361.5
2	7	1842.8	3756.2	350.0
3	7	1820.4	5576.6	350.0
4	7	1779.1	7355.7	341.5
5	7	1706.0	9061.7	341.5
6	7	1635.0	10696.6	341.5
7	7	1678.3	12374.9	341.5
8	7	1766.1	14141.0	341.5
9	7	1842.1	15983.1	350.0
10	8	1923.5	17906.6	361.5
11	8	1919.1	19825.7	361.5
12	8	1908.4	21734.1	361.5
13	8	1961.1	23695.2	391.5
14	8	2035.6	25730.8	391.5
15	8	2033.9	27764.7	391.5
16	8	1961.5	29726.2	391.5
17	8	1917.5	31643.7	361.5
18	7	1758.4	33402.1	341.5
19	7	1763.3	35165.3	341.5
20	8	1895.0	37060.3	361.5
21	7	1815.0	38875.3	350.0
22	7	1760.8	40636.1	341.5
23	7	1851.8	42487.9	350.0
24	8	1930.5	44418.5	361.5

START UP COST OF SUBGROUP NO. 1 = £167.02

SHUT DOWN COST OF SUBGROUP NO. 1 = £137.78

GENERATION COST OF SUBGROUP NO. 1 = £44113.7

* Total number of committed units of subgroup No. 1.

Table (7.10): Unit schedule of subgroup 1
of SCECO system for a 24 h commitment period.
(Benders decomposition)

(Unit status 1=ON, 0=OFF)

Time (H)	Unit number and status of units 1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	0	0	1
2	1	1	1	1	1	1	0	1	0	0
3	1	1	1	1	1	1	0	1	0	0
4	1	1	1	1	1	1	1	0	0	0
5	1	1	1	1	1	1	1	0	0	0
6	1	1	1	1	1	1	1	0	0	0
7	1	1	1	1	1	1	1	0	0	0
8	1	1	1	1	1	1	1	0	0	0
9	1	1	1	1	1	1	0	1	0	0
10	1	1	1	1	1	1	1	0	0	1
11	1	1	1	1	1	1	1	0	0	1
12	1	1	1	1	1	1	1	0	0	1
13	1	1	1	1	1	1	1	1	0	0
14	1	1	1	1	1	0	1	1	1	0
15	1	1	1	1	1	0	1	1	1	0
16	1	1	1	1	1	1	1	1	0	0
17	1	1	1	1	1	1	1	0	0	1
18	1	1	1	1	1	1	1	0	0	0
19	1	1	1	1	1	1	1	0	0	0
20	1	1	1	1	1	1	1	0	0	1
21	1	1	1	1	1	1	0	1	0	0
22	1	1	1	1	1	1	1	0	0	0
23	1	1	1	1	1	0	0	1	1	0
24	1	1	1	1	1	1	1	0	0	1

Table (7.11): Results of unit commitment for SCECO system by Benders decomposition method.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	46	11257.4	11257.4	2146.0
2	43	10870.2	22127.5	2071.5
3	43	10741.1	32868.6	2071.5
4	43	10557.9	43426.5	2037.5
5	42	10099.1	53525.6	2022.5
6	40	9584.2	63109.8	1966.0
7	41	9909.1	73018.9	2007.5
8	43	10541.5	83560.4	2037.5
9	43	10845.6	94406.0	2071.5
10	46	11276.3	105682.4	2137.5
11	46	11302.5	116984.9	2146.0
12	46	11260.6	128245.5	2197.5
13	46	11521.1	139766.6	2223.0
14	48	11941.8	151708.4	2322.5
15	48	11946.5	163654.9	2322.5
16	46	11507.7	175162.6	2266.0
17	46	11278.6	186441.2	2146.0
18	42	10409.8	196851.0	2022.5
19	43	10454.1	207305.1	2037.5
20	47	11109.8	218414.9	2177.5
21	43	10729.7	229144.6	2071.5
22	42	10417.7	239562.3	2022.5
23	43	10848.8	250411.2	2071.5
24	46	11305.0	261716.2	2197.5

TOTAL START UP COST = £870.76

TOTAL SHUT DOWN COST = £706.41

TOTAL GENERATION COST = £260139.03

CPU TIME = 30.42 SEC.

* Total number of committed units.

Table (8.12): Unit schedule of SCECO Central system over a 24 hours period by Benders decomposition method.

Time (H)	Unit number and status of units (1 = ON, 0 = OFF)																																																									
	1				5					10					15					20					25					30					35					40					45					50					55				59
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	0	1	0	1	0	0	0	1	1	0	0
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	0	1	0	1	0	0	0	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	0	1	0	1	0	0	0	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	0	1	0	0	0	0	0	1	1	0	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	0	0	0	0	1	1	1	1	0	1	0	1	1	1	1	0	0	0	0	0	0	0	0
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	0	1	0	1	0	0	0	1	1	0	0

Table (7.13): Results of unit commitment for reduced sample system from SCECO system by Proposed method.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	6	1392.8	1392.8	265.0
2	6	1352.9	2745.8	256.5
3	6	1337.4	4083.1	256.5
4	6	1318.5	5401.6	256.5
5	5	1249.2	6650.8	241.5
6	5	1198.2	7849.0	241.5
7	5	1229.3	9078.3	241.5
8	6	1320.8	10399.1	256.5
9	6	1344.4	11743.5	256.5
10	6	1400.4	13143.9	265.0
11	6	1396.9	14540.8	265.0
12	6	1389.3	15930.1	265.0
13	6	1410.5	17340.7	265.0
14	7	1509.6	18850.3	276.5
15	7	1500.9	20351.1	276.5
16	6	1422.7	21773.8	265.0
17	6	1390.3	23164.1	265.0
18	5	1309.2	24473.3	250.0
19	5	1312.7	25786.0	250.0
20	6	1364.4	27150.4	256.5
21	6	1324.1	28474.5	256.5
22	5	1316.8	29791.2	250.0
23	5	1328.4	31119.6	250.0
24	6	1375.9	32495.5	256.5

TOTAL START UP COST = £109.83

TOTAL SHUT DOWN COST = £67.20

TOTAL GENERATION COST = £32318.474

TOTAL OPERATIONAL COST = £32495.507

* Total number of committed units of the reduced (sample) system.

Table (7.14): Unit schedule of reduced sample system of SCECO for a 24 h commitment period.
(PROPOSED APPROACH)
(Unit status 1=ON, 0=OFF)

Time (H)	Unit number and status of units 1	2	3	4	5	6	7	8
1	1	1	1	0	1	1	1	0
2	1	1	1	1	1	0	1	0
3	1	1	1	1	1	0	1	0
4	1	1	1	1	1	0	1	0
5	1	1	1	1	0	0	1	0
6	1	1	1	1	0	0	1	0
7	1	1	1	1	0	0	1	0
8	1	1	1	1	1	0	1	0
9	1	1	1	1	1	0	1	0
10	1	1	1	0	1	1	1	0
11	1	1	1	0	1	1	1	0
12	1	1	1	0	1	1	1	0
13	1	1	1	0	1	1	1	0
14	1	1	1	1	1	1	0	0
15	1	1	1	1	1	1	0	0
16	1	1	1	0	1	1	1	0
17	1	1	1	0	1	1	1	0
18	1	1	1	0	0	1	1	0
19	1	1	1	0	0	1	1	0
20	1	1	1	1	1	1	0	0
21	1	1	1	1	1	1	0	0
22	1	1	1	0	0	1	1	0
23	1	1	1	0	0	1	1	0
24	1	1	1	1	1	1	0	0

Table (7.15): Results of unit commitment for SCECO system by proposed approach.

Time (H)	ON-line units*	Hourly costs £	Total costs £	On-line av. generation
1	50	11287.7	11287.7	2230.0
2	45	9737.1	21024.8	2137.5
3	45	9687.0	30711.8	2137.5
4	45	9560.7	40272.4	2137.5
5	43	9496.2	49768.7	2007.5
6	43	9167.6	58936.2	2007.5
7	43	9363.4	68299.6	2007.5
8	45	9537.5	77837.1	2137.5
9	45	9733.8	87570.8	2137.5
10	50	11263.6	98834.4	2230.0
11	50	11311.9	110146.3	2230.0
12	50	11266.6	121412.9	2230.0
13	50	11393.9	132806.8	2230.0
14	53	12418.0	145224.8	2317.5
15	53	12478.2	157703.0	2317.5
16	50	11396.9	169099.9	2230.0
17	50	11272.6	180372.5	2230.0
18	48	11126.1	191498.6	2120.0
19	48	11147.3	202645.9	2120.0
20	53	11798.6	214444.4	2317.5
21	53	11530.9	225975.4	2317.5
22	48	11101.9	237077.2	2120.0
23	48	11241.4	248318.7	2120.0
24	53	11910.9	260229.6	2317.5

TOTAL START UP COST = £389.83

TOTAL SHUT DOWN COST = £297.20

TOTAL GENERATION COST = £259542.57

TOTAL OPERATIONAL COST = £260229.6

CPU TIME = 3.95 SEC.

* Total number of committed units.

Table (8.16): Unit schedule of SCECO Central system over

a 24 hours period by the proposed approach.

Time (H)	Unit number and status of units (1 - ON, 0 - OFF)																																																										
	1				5					10					15					20					25					30					35					40					45					50					55				59
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0

Table (7.17): Final results for unit commitment of SCECO system obtained by different methods of solution over a commitment horizon of one day.

Method of solu. *	Start up cost £	Shutdown cost £	Generation cost £	Total operational costs £	CPU Time Seconds	Computer storage space K Bytes
HM	597.18	129.48	261051.64	261778.30	1.47	32.6
DPSC	155.58	129.48	260428.44	260713.50	58.02	558.7
MILP	137.76	113.98	266108.70	266360.50	7.01	1063.2
LR	149.93	107.37	265400.0	265657.30	1738	58.4
BD	870.76	706.41	260139.03	261716.20	30.42	4435.7
PM	389.83	297.20	259542.6	260229.60	3.95	2340.7

* Methods of solution.

<u>Definitions of abbreviations</u>:

HM : Heuristic method.
DPSC : Dynamic programming sequential combinations method.
MILP : Mixed integer linear programming method.
LR : Lagrangian relaxation method.
BD : Benders decomposition method.
PM : Proposed method.

7.6 DISCUSSION

It is clear from the results obtained that it is possible to solve the problem of unit commitment for SCECO Central system, which consists of 59 thermal generating units, by using one of the following methods:

- Heuristic methods.
- Dynamic programming sequential combinations.
- Mixed integer linear programming.
- Lagrangian relaxation.
- Benders decomposition.
- A new proposed approach.

The results produced by the heuristic method were assumed as the base values, as mentioned in chapter six, which denote 100% of the unit commitment problem's results. Therefore, the results obtained by the other methods were compared with the base values. Based on this assumption and the fact that unit commitment does not essentially require high speed of solution, methods of solution have been ordered according to total operational costs as shown in table (7.18).

Figure (7.4) demonstrates the operational cost of the different methods of solution for SCECO Central over a commitment horizon of 24 hours. It can be observed from figure (7.4) and table (7.18) that the best results of SCECO unit commitment were achieved by the proposed method. This method, however, is not applicable to every

Table (7.18): Recommended priority order of solution methods for unit commitment of SCECO Central system.

Order of method	Method of solution	% difference in operational costs
1	Proposed method.	-0.592 %
2	Dynamic programming sequential combinations.	-0.407 %
3	Benders decomposition.	-0.024 %
4	Heuristic method.	0.00 %
5	Lagrangian relaxation.	+1.482 %
6	Mixed integer linear programming.	+1.654 %

power system because it requires that groups of identical units must exist in the system in order to create the reduced system. The proposed method seems to be the most suitable for solving the problem of the SCECO Central system with a modest requirement of computation resources. The next best method for the SCECO Central system is the dynamic programming sequential combination technique, which solves the unit commitment problem with acceptable quality of results as well as with reasonable computational efforts. Benders decomposition method achieved a gain in the cost reduction, which makes it of the third priority. Nevertheless, the computer memory space needed for the program is the largest, as can be seen in figure (7.6). Mixed integer programming and Lagrangian relaxation methods are not recommended for solving the unit commitment of the SCECO Central system since these methods have not offered any improvement of results compared with heuristic method. The reasons for this have been mentioned in chapter six.

It is noted, however, that the accuracy of the solutions and the speed of the computer in producing the results as well as the computer memory requirement for the programs could vary from one method to another. Therefore, it is always a compromise between the quality of solution and the speed of computational facilities.

Figure (7.1): Start up cost of SCECO system for a 24 hour commitment period

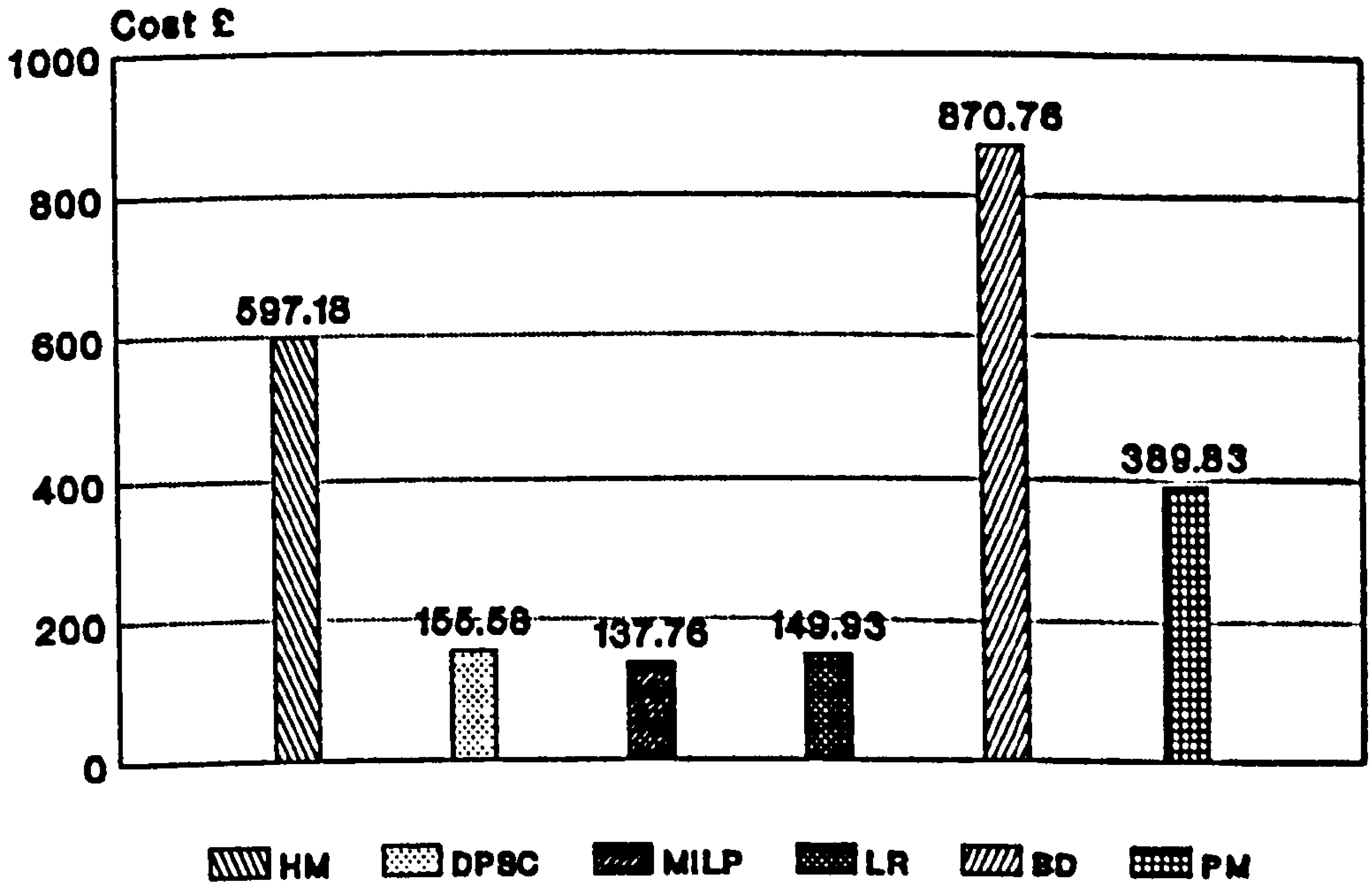


Figure (7.2): Shut down cost of SCECO system for a 24 hour commitment period.

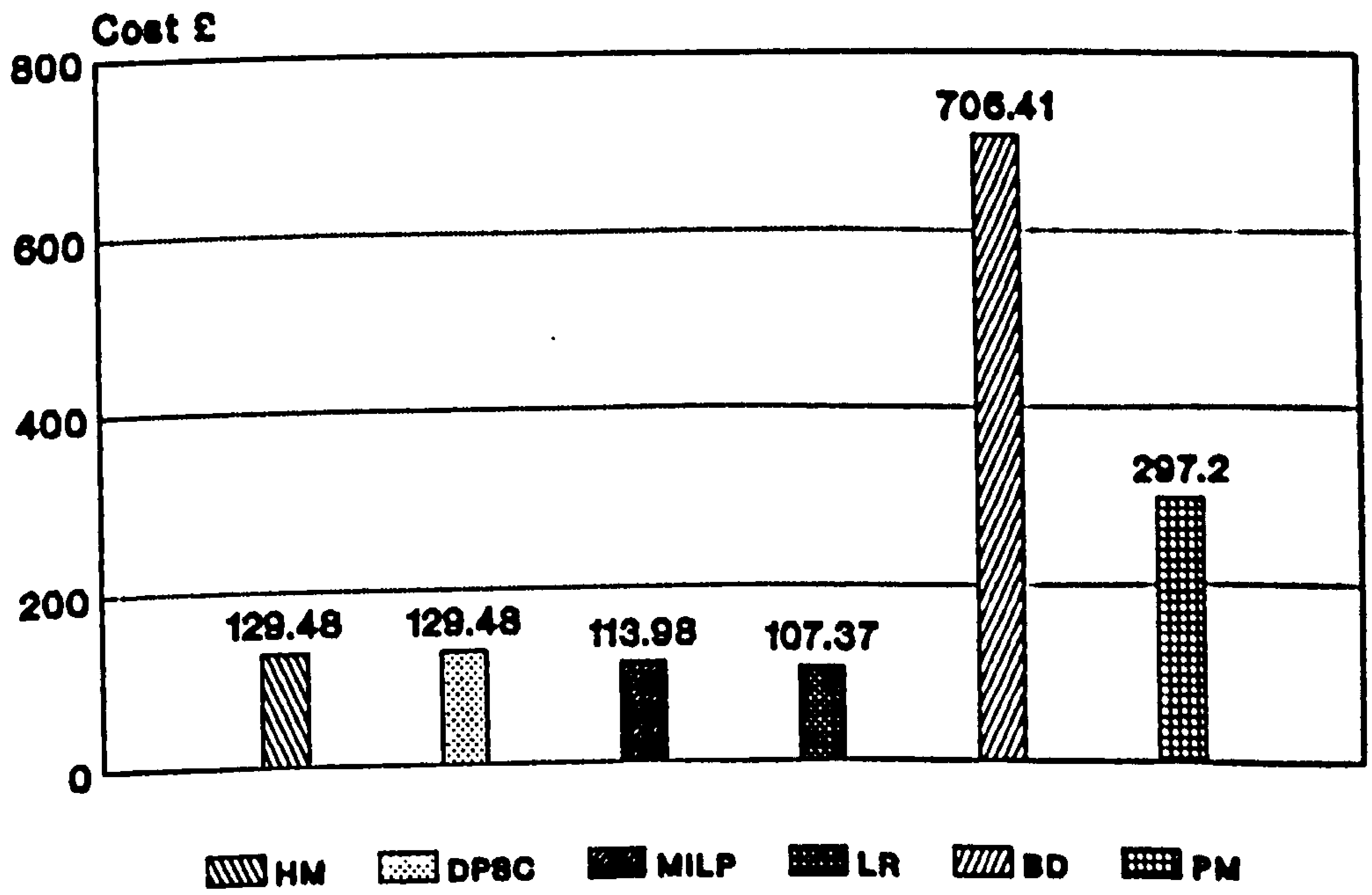


Figure (7.3): Generation cost of SCECO system for a 24 hour commitment period.

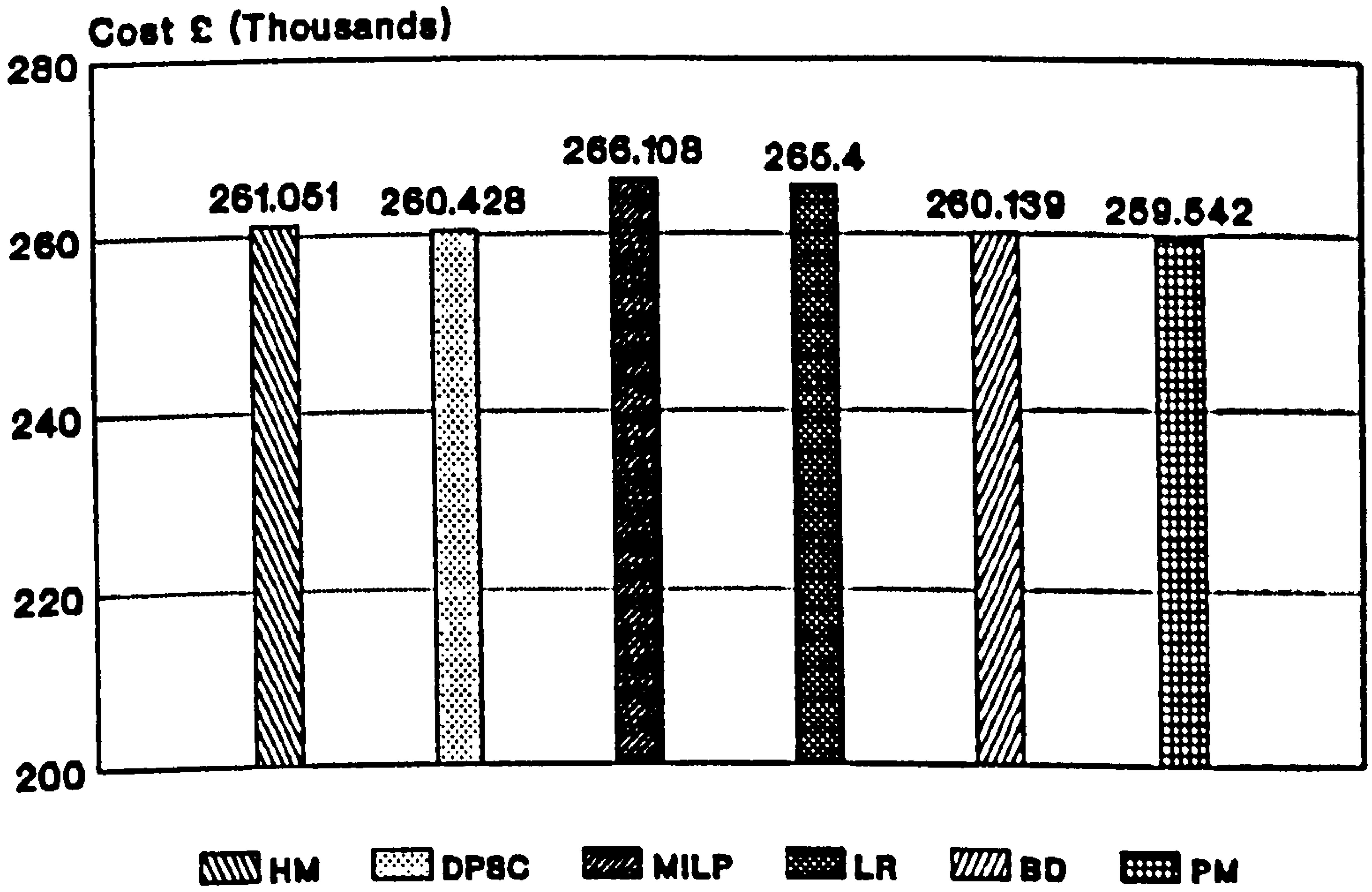


Figure (7.4): Total cost of SCECO system for a 24 hour commitment period.

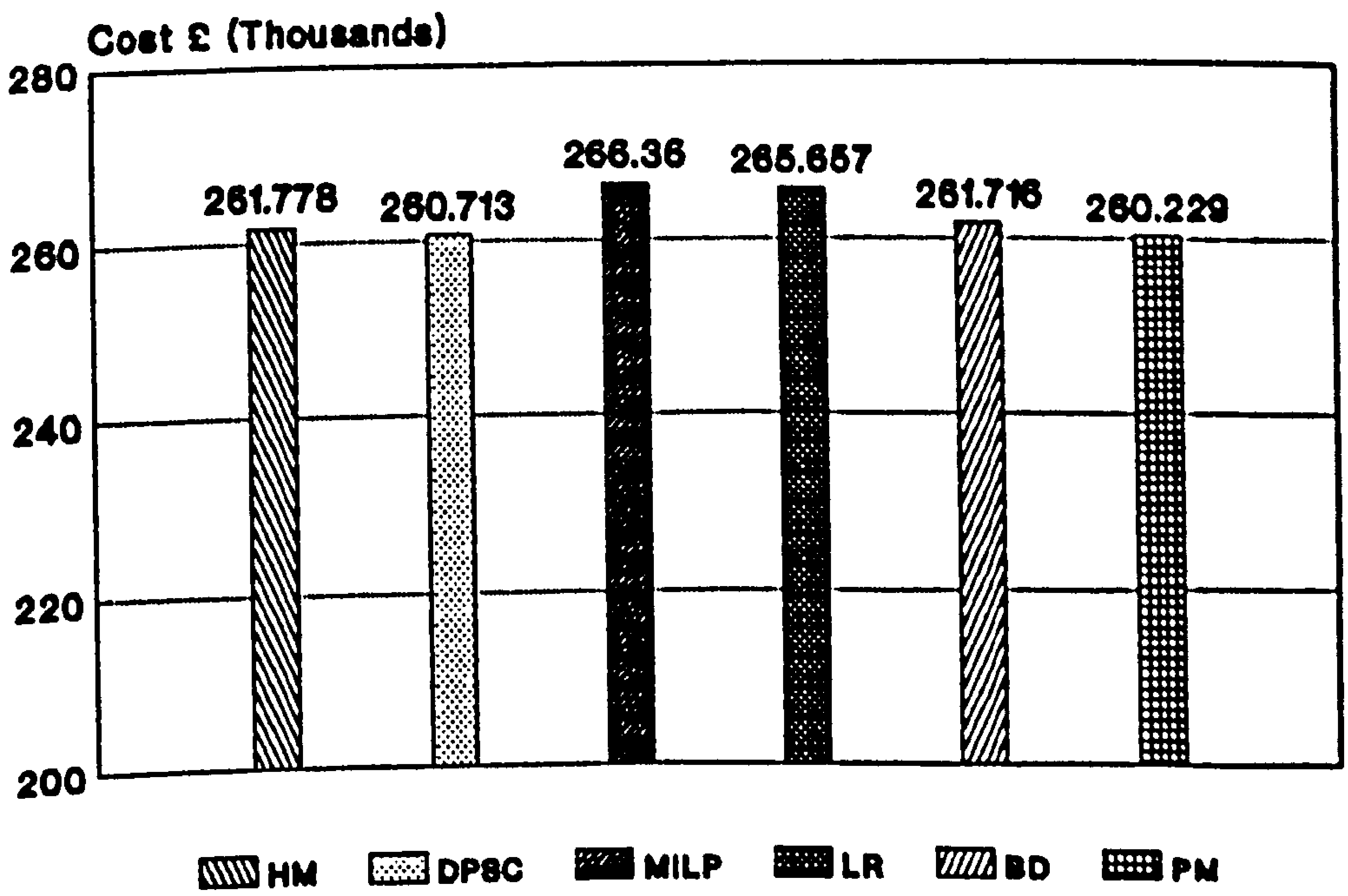


Figure (7.5): CPU time for solving the unit commitment of SCECO system.

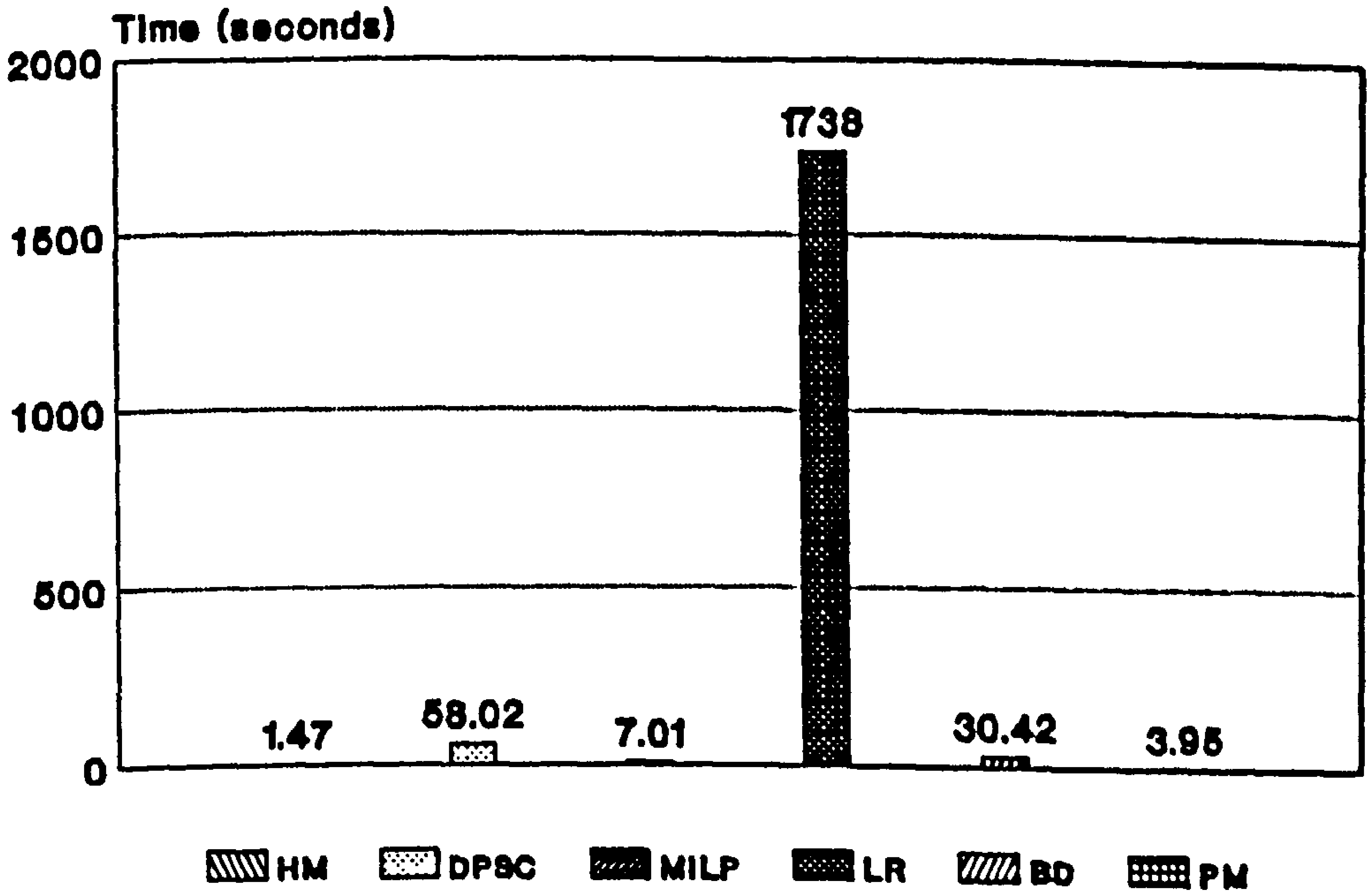
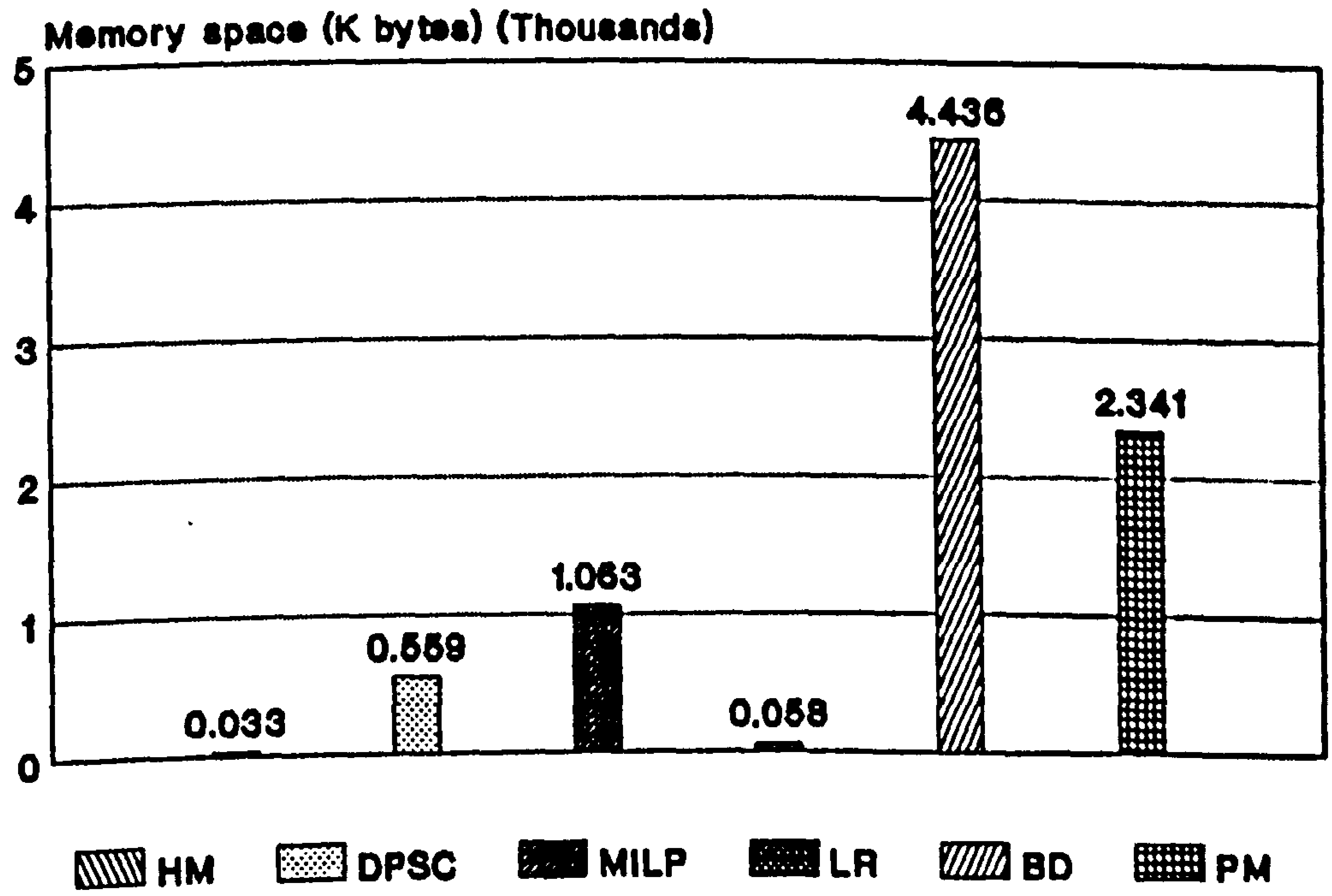


Figure (7.6): Computer memory space for the programs used for SCECO system.



CHAPTER 8

DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDIES

8.1 INTRODUCTION

Electricity cannot be economically stored and must be produced on demand. That is, the supply of electricity must equal the demand at all times. Electric utilities utilise and operate complex generation and transmission networks by implementing well coordinated planning and operations to ensure that the supply of electric power is always equal to consumer demand at acceptable economical cost to both sides.

One of the major operations of power systems is the unit commitment which can be simply defined as scheduling unit start up and shut down to meet the system load demand and spinning reserve in a way that minimizes operational costs without compromising system security and reliability requirements. At the same time, all the constraints of the system and units must be taken into consideration.

A great deal of research work was devoted to solving the problem of unit commitment during the last three decades. This thesis is concerned with an investigation and comprehensive comparison of the existing solution methods of the unit commitment problem for thermal generating units. The most commonly implemented methods of solution are presented along with a comparison of their performance. For the comparison to be practical and realistic, these methods were tested on three power systems of different size. As a result of this comparison, A new approach for solving the problem of unit commitment for Saudi Consolidated Electric Company (SCECO Central) has been developed and presented in chapter seven. So far in this thesis, there has been no presentation of advantages and disadvantages of the used methods for solving the problem of unit commitment. Therefore, this chapter fills this gap, and demonstrates general conclusions as well as recommendations for further studies.

8.2 DISCUSSION

Unit commitment problems are usually solved by using the appropriate method from those listed in chapter six. It is possible, however, to combine two or more methods in order to improve the solutions [32,36,81]. In this section, advantages and disadvantages of each method are presented.

8.2.1 Heuristic methods

Heuristic methods can be used to solve the problem of unit commitment with the following advantages:

i) It is easy to solve the problem for any size of power system.

ii) Heuristic approach is simple in terms of algorithm preparation and computer program development.

iii) All the constraints of the system and unit can be easily considered and met.

iv) Feasible solutions are usually obtained.

v) The solutions are economically reasonable.

vi) The computational requirements in terms of computer memory storage and running time are moderate and acceptable.

The drawback of heuristic methods is that they cannot always guarantee optimal solutions because the solution of the problem by using these methods is based on the priority order of units as a function of their full-load average cost. Therefore, it is possible to commit a large unit at an output rate of power close to its minimum output limit with a higher production cost, while it could be more economical to run a smaller unit at an output power near to its maximum limit, even if it is ranked lower than the other unit in the priority list.

8.2.2 Full Dynamic Programming (FDP)

Full dynamic programming is a comprehensive search technique for optimal solutions. However, its requirement for large computational resources limits its applications to small problems. Despite that, it has the following advantages:

i) It is simple in terms of algorithm preparation and computer program development.

ii) It is capable of incorporating all operational constraints of the system.

iii) It has the potential to achieve the best results of unit commitment problem for small power systems.

The main drawbacks of full dynamic programming are listed below:

i) It is limited to small and medium power systems (20 units or below), for the reasons already mentioned.

ii) It is sensitive to some constraints, such as minimum shut down and minimum running times of units. Consequently, feasible solutions may not be obtained.

iii) CPU time and computer memory space increase rapidly with the increasing of number of units in the system.

8.2.3 Dynamic Programming Sequential Combinations (DPSC)

This technique is one of the forms of the full dynamic programming technique modified so as to reduce computational requirements and to make it possible to solve the problem of larger systems. DPSC can be used to solve the unit commitment problem with the following advantages:

i) It can solve the problem regardless of the system size.

ii) Computational requirements in terms of both CPU time and computer memory space are significantly reduced compared with full dynamic programming.

iii) Feasible solutions can be easily obtained.

iv) DPSC is flexible enough to account of all operational constraints.

v) DPSC is suitable for implementation when the rate of change in load demand is high.

Although DPSC has all of these advantages, it could, nevertheless, lead to suboptimal results because of limited number of possible combinations in the search for optimal solution, since many combinations used in FDP are neglected in DPSC. Therefore, the optimal path may not be found.

8.2.4 Dynamic Programming Truncated Combinations (DPTC)

The dynamic programming sequential combination approach is applicable only when the system load is changing rapidly. If the change in the load is small then dynamic programming truncated combination is used. In this technique, a must-run status is imposed to the base units, and the search for the optimal solution is limited only to the cycling units. This method has the same advantages as DPSC. Apart from that, it is well suited to systems with small changes in load demand. However, if the number of cycling units is high, i.e. in the range of 15 units or more, the CPU time as well as computer memory space requirements become very high. Therefore, DPTC is limited to small and medium power systems.

8.2.5 Mixed Integer Linear Programming (MILP)

If the generating units have linear input-output characteristics, then it is possible to implement mixed integer linear programming method to solve the problem of unit commitment for thermal units with the following advantages:

i) It can be used to solve the problem for any number of units since the relationship between the increasing number of units and computational requirements is almost linear.

ii) Feasible solutions can be obtained easily.

iii) Computational requirements in terms of CPU time and computer memory space are modest.

iv) All the constraints can be taken into consideration and met.

The main drawback of mixed integer linear programming is that it is applicable only to a system of units with a linear relationship between input energy and output power. If this property does not exist, then a linearization of the input-output characteristic of the units is required. This, however, would lead to an approximation of the cost function which could result in suboptimality in solutions.

8.2.6 Lagrangian Relaxation method (LR)

Lagrangian relaxation method has recently been employed for solving the unit commitment problem. Over the last decade, many attempts were made to develop this technique because it has the following advantages:

i) It is capable of producing more rigorous solutions, particularly for small and medium size power systems.

ii) It has the potential to handle all the operational constraints.

iii) Its computational requirements are moderate.

On the other hand, Lagrangian relaxation method has the following disadvantages:

i) It is sophisticated in terms of computer program preparation and mathematical modelling.

ii) The convergence of solutions is highly sensitive to the initial values of Lagrangian multipliers and to the output of units, particularly if the number of units is large, or when there is no wide variation between input-output characteristics of the units. Consequently, feasible solutions may not be obtained.

8.2.7 Benders Decomposition method (BD)

Benders decomposition technique is one of the latest attempts to solve the problem of unit commitment for large-scale systems. It is implemented to overcome the dimensionality problem which arises if other techniques, for instance the dynamic programming method, are used. It is distinguished from other methods by the following advantages:

i) It can produce more rigorous solutions than heuristic methods or Lagrangian relaxation technique in solving the problem of unit commitment for medium and large-scale power systems.

ii) All constraints of the system and units can be incorporated.

iii) A remarkable reduction of computational requirements can be achieved for medium and large systems.

The shortcoming of this methods is that the dimensionality problem could arise again if the number of units is large (in the range of 200 units). Another drawback of this technique is due to its sensitivity to some of the unit constraints, for example, minimum run and down times and minimum and maximum output limits. Consequently, feasible solutions may not be acquired.

8.2.8 The new proposed method

This method was explained extensively in chapter seven. Its aim is to overcome the dimensionality problem of the unit commitment problem for large-scale systems. Competitive results were achieved by using this new approach for solving the problem of Saudi Consolidated Electric Company system. Advantages of this method are listed below:

i) A more rigorous solution can be produced.

ii) A significant reduction of computational requirements in terms of CPU time and computer memory space can be achieved.

iii) It is easy to take all operational constraints into consideration.

iv) Feasible solutions can be easily obtained.

However, the drawback of the proposed method is that it is applicable only when a system consists of groups of units which are identical in terms of input-output characteristics.

8.3 CONCLUSIONS

The main topic of this thesis is a general investigation of solution methods of the unit commitment problem for thermal units. It has been observed that in order to answer the questions posed in chapter one of this thesis regarding unit commitment, a comparison of the existing methods of solution is essentially required. It has also been noted that a comparison of results obtained by different existing methods in the literature of the subject is difficult and infeasible because of two reasons. The first reason is the lack of common bases for the comparison. The second is because most of the various methods of solution available in the literature were designed for particular power systems. Consequently, there is no general procedure that can be applied successfully so as to solve the unit commitment problem for any size of power system.

On the basis of these facts, a comprehensive comparison of the most common solution methods of the unit commitment problem has been carried out by following these steps:

1. Formulating a general mathematical model for the problem of unit commitment.

2. Writing and developing a computer program for each method of solution.

3. Testing each method on three power systems of different sizes.

4. Comparing and discussing the results obtained by the different methods for different systems.

In the first stage, an extensive mathematical model for the problem of unit commitment has been developed and presented in chapter two. The model is composed of cost function (objective function) and a set of constraints. The cost function mainly includes fuel cost as well as start up and shut down costs of units, while the set of constraints incorporates almost all the operational constraints which can be encountered in the practical operation of power systems.

In the second step, brief descriptions and mathematical formulations for the used methods were given, then computer programs in FORTRAN 77 were prepared and developed for solving the unit commitment problem for the tested systems by using the following methods:

- Heuristic methods.
- Dynamic programming methods (Full dynamic programming, dynamic programming sequential combinations and dynamic programming truncated combinations methods).
- Mixed integer-linear programming method.
- Lagrangian relaxation method.
- Benders decomposition method.
- New proposed method.

Tabulated results were produced for each method to demonstrate its application for solving the problem.

The third step of this thesis dealt with the comparison of the methods of solution discussed in step two. The comparison has been carried out and presented in chapter six along with a number of illustrative graphs. In chapter seven, this stage was completed by recommending the most appropriate methods, from those discussed so far, to solve the unit commitment problem for Saudi Consolidated Electric Company (SCECO Central).

In general conclusion, it can be said that the decision as to which method of solution is applicable and suitable to solve the unit commitment problem for a particular power system is a difficult one because it depends on several factors that must be taken into consideration. These factors are listed below:

- Number and type of units in the system.
- Input-output characteristics of the units.
- Daily load curve pattern.
- Rate of change in load demand.
- Available computing facilities.
- Desired quality of solutions in terms of accuracy and speed.
- Physical and operational constraints.
- Reliability and security requirements.

Therefore, for the purpose of selecting an appropriate solution method of the unit commitment problem for a specific system, it is worthwhile to try more than one method of solution from those which are believed to be the best, and then decide which one is the most suitable for the system in question.

Based on the results obtained in this thesis regarding the different methods of solving the problem for different sizes of power systems, the following conclusions can be drawn:

1. It is possible to solve the problem of unit commitment for any size of power system by using the most appropriate method from these discussed so far. However, it is a matter of compromise between accuracy and speed of solutions.

2. Full dynamic programming technique is the most attractive method for solving the problem of power systems with 20 units or less.

3. Dynamic programming truncated combinations technique increases dynamic programming ability to solve the problems of larger systems, for example in the range of 50 units. However, the degree of improvement depends on and is indirectly proportional to the rate of change in load demand. If the rate of change in load demand is high, then dynamic programming

sequential combinations can be used instead.

4. If the units in the system have linear input-output characteristics, then mixed integer linear programming can be used to solve the problem of small and medium size systems.

5. Until now, the unit commitment problem for large systems has been solved by implementing heuristic methods only. The reason for this is the dimensionality problem that arises if other methods are used. Attempts have been made to overcome this problem by employing decomposition techniques to large-scale systems.

6. Lagrangian relaxation method was one of the earliest attempts to tackle the unit commitment problem for large systems. Despite its disadvantages mentioned in the previous section, it can be successfully used with the branch and bound approach to solve the problems of medium and large systems in the range of up to 100 units.

7. Benders decomposition is another endeavour to overcome the dimensionality problems. The principle of Benders decomposition is used to break up the problem into subproblems that can be solved easier than the original one. Then, the overall solution can be found by gathering the solutions of the subproblems. This method can be used to solve the

unit commitment problem for medium and large systems of up to 200 units.

8. It is possible that the system's engineers and planners may design an approach to solve the unit commitment problem for a particular system, in accordance with the system's parameters and requirements, rather than use the commonly known methods, as was done in the proposed method of solution described in chapter seven for the system of Saudi Consolidated Electric Company.

Finally, it is hoped that the modelling and simulation of the most common solution methods of the unit commitment problem presented in this thesis are helpful to interested researchers. However, the art of the engineer will still be the key to successful operation of power systems because he is the source of technical skill and creativity. The mathematical models and the computer programs are only an aid in the decision making process since they take the burden of data handling and the performance of huge amounts of calculations which can provide power system operators with valuable guiding information.

8.4 RECOMMENDATIONS FOR FURTHER STUDIES

The optimal solution to the unit commitment problem for very large-scale power systems remains undetermined yet. Hard work and a lot of research is required to achieve this goal. Further advances in unit commitment research can be made in two areas. First, methods must be found that can obtain optimal, or as close as possible to the optimal solution. The second challenge is to include more aspects of the system in the modelling and problem formulation. This includes finding better ways of handling unit constraints, multi-area constraints, and fuel constraints. The following recommendations, hopefully, will lead to some improvements in this field of research:

1. As a result of this study, an integrated package of computer programs has been developed for solving the problem of unit commitment by using different methods. Therefore, it is worthwhile, as far as the author is concerned, to utilize these programs in order to build an expert system for solving the unit commitment problem.

2. Apart from the dynamic programming method, start up and shut down costs are not easy to include inside the optimization loop. Consequently, they are added to the total costs after the decision of committing units is taken. Therefore, further research is

required to incorporate them during optimization stages.

3. Decomposition techniques constitute a promising approach to solving the problem. More investigation is needed for Lagrangian relaxation to improve and modify the procedure of updating Lagrangian multipliers. In Benders decomposition method, further research is required to incorporate unit constraints properly in order to speed up the solution. These efforts could lead to a reduction of computational requirements, as a result of which the problems of larger power systems could be solved.

4. Although a great deal of research work has been devoted and many attempts have been made to improve the existing methods of solution for unit commitment, the progress has been very slow and quite limited. Therefore, it is recommended that researchers investigate and try new alternative methods of solution.

REFERENCES

(1) A. J. Wood and B. F. Wollenberg: Power Generation, Operation and control, John Wiley & Sons, 1984.

(2) A. L. Kuester and J. H. Mize: Optimisation Techniques with FORTRAN, McGraw - Hill Book Company, 1973.

(3) B. Lev and H. J. Weiss:Introduction to Mathematical Programming, Quantitative Tools for Decision Making, Edward Arnold, 1982.

(4) D. A. Wismer and R. Chattergy: Introduction to nonlinear Optimization, North Holland publishing Co.,1978.

(5) O. I. Elgerd: Electric Energy Systems theory, McGraw - Hill Book Co. 1982.

(6) K. L. LO and A. Brameller: Review of Minimization Techniques with Reference to Power System Engineering, Proceedings of the Symposium on Real-time Control of Electric Power Systems. Brown, Boverl and Company Ltd, Baden, Switzerland, 1971.

(7) J. B. Hiriart-Urruty: Fermat Days 85; Mathematics for Optimization, North Holland publishing Co., 1986.

(8) I. J. Nagrath and D. P. Kothari; Modern Power System Analysis Data, McGraw - Hill Publishing co., Newdelhi, 1982.

(9) A. M. Abbood; A Rule-Based System for the Unit Commitment of Thermal Generation, Msc dissertation, UMIST,1987.

(10) C. A. Gross; Power System Analysis, John Wiley and Sons, 1986.

(11) E. R. Laithwaite and L. L. Freris: Electric Energy: Its Generation, Transmission and Use, McGraw - Hill book Company, 1980.

(12) T. C. Elliott; Standard Handbook of Power Plant Engineering, McGraw-Hill Publishing Company, 1989.

(13) CEGB, Modern Power Station Practice: Vol. 2; Boiler, Fuel and Ash-handling. Vol. 3; Turbines and Auxiliary Equipment. Vol. 7: Operation and Efficiency, Pergamon Press, 1971.

(14) Survey of Mathematical Programming, Proceeding of 9th International Mathematical Programming Symposium, Budapest, Aug. 23-27, 1976.

(15) M. J. H. Sterling;Power System Control; IEE Control Engineering Series 6, Antony Rowe Ltd, 1986.

(16) C. J. Baldwin, K. M. Dale and R. F. Dittrich; A

study of the Economic Shutdown of Generating Unit in Daily Dispatch, IEEE Transaction on Power Apparatus and Systems, Vol. PAS,1959, pp 1272-1284.

(17) L. L. Garver; Power Generation Scheduling by Integer Programming - Development of Theory, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, Feb. 1963, pp 730-735.

(18) P. G. Lowyer; Generating Unit Commitment by Dynamic Programming, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, May 1966 pp 422-427.

(19) R. H. Keer, J. L. Scheidt, A. J. Fontana and J. K. Wiley; Unit commitment, IEEE Transaction on Power Apparatus and Systems, Vol. PAS 25, Mo. 5, May 1966 pp 417-421.

(20) K. Hara, M. Kimura and N. Honda; A method for planning economic unit commitment and maintenance of thermal power systems, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, May 1966, pp 427-436.

(21) J. A. Muckstadt and R. C. Wilson; An application of Mixed Integer Programming Duality to Scheduling Thermal Generating System, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, Dec. 1968, pp 1968-1978.

(22) H. H. Happ, R. C. Johnson and W. J. Wright; Large

Scale Hydro-thermal Unit Commitment Method and Results, IEEE Transaction on Power Apparatus and Systems,Vol. PAS-90, 1971, pp 1373-1384.

(23) J. D. Gay: Security Constraint Unit Commitment, IEEE Transaction on Power Apparatus and Systems, Vol. PAS-90, 1971, pp 1385-1391.

(24) A. K. Ayoub and A. D. Patton: Optimal Thermal Generating Unit Commitment, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, Jul./Aug. 1971, pp 1752-1756.

(25) C. K. Pang and H. C. Chen: Optimal Short-term Thermal Unit Commitment, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, Jul./Aug.1976 pp 1336-1346.

(26) T. S. Dillon, K. W. Edwin and H. D. Kochs: Integer Programming Approach to the Problem of Thermal Unit Commitment with Probabilistic Reserve Determination, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, Nov./Dec. 1978, pp 2154-2166.

(27) R. R. Shoults and S. Kang Chang: A practical Approach to Unit Commitment Economic Dispatch and Saving Allocation for Multiple Area Pool Operation with Import/Export Constraints, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, Mar./Apr. 1980, pp 625-635.

(28) S. D. Bond and B. Fox; Optimal Thermal Unit Scheduling Using Improved Dynamic Programming Algorithm, IEE Proceedings, Jan. 1986, pp 1-5.

(29) F. Eugenio Villasenca and B. Fardanesh ;Fast Thermal Generation Rescheduling, IEEE Transaction on Power Systems, Vol. PWRS-2,No.1 , Feb., 1987, pp 65-71.

(30) K. Hamam, Y. M. Hamam, K. S. Hindi and A. Brameller; Unit Commitment of Thermal Generation, IEE Proceedings, Vol. 127, Pt. C, No. 1, Jan, 1980. pp 3-8.

(31) K. W. Edwin, R. D. Machate and R. J. Taud; Comparative Evaluation of Unit Commitment Approaches, IEE International Conference on Power System Monitoring and Control, 24- 26 June 1980, London, pp 132-138.

(32) C. K. Pang, G. B. Sheble and F. Albuyeh; Evaluation of Dynamic Programming Based Methods and Multiple Area Representation for Thermal Unit Commitment, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, No. 3, March 1981, pp 1212-1218.

(33) J. G. Waight, F. Alboyeh and A. Bose; Scheduling of Generation and Reserve Margin Using Dynamic and Linear Programming, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, No. 5, May, 1981, pp 2226-2230.

(34) J. A. Muckstadt and S. A. Koenig; An Application of Lagrangian Relaxation to Scheduling in Power Generation Systems, Operation Research Vol. 25, No. 3, May-Jun, 1977, pp 386-402.

(35) A. Merlin and P. Sandrin; A New Method for Unit Commitment at Electricite De France, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, No. 5, May, 1983, pp 1218-1225.

(36) E. Khodaverdian, A. Brameller and E. D. Farer ; Semi-rigorous Thermal Unit Commitment for Large Scale Electric Power System, IEE Proceedings, Vol. 133, Pt. C, May, 1986, pp 157-164.

(37) A. Turgeon; Optimal unit commitment, IEEE Transaction on Automatic Control, vol. AC, No.4 Apr. 1977, pp 223-227.

(38) G. S. Lauer, N. R. Sandell, D. P. Bertsekas and T. A. Posbergh; Solution of Large Scale Optimal Unit Commitment Problem, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, Jan., 1982 , pp 79-86.

(39) A. Turgeon; Optimal Scheduling of Thermal Generating Units, IEEE Transaction on Automatic Control, Vol. AC-23, No. 6, Dec.1978, pp 1000-1005.

(40) Z. A. Yamayee, A. H. Alabiad and T. L. Morin;

Optimal and Near Optimal Unit Commitment Scheduling, Proceeding of the 6th Power System Computation Conference (PSCC-6) Darmstadt, 1978, pp 517-529.

(41) L. F. Baptistella and J. C. Geromel; Decomposition Approach to Problem of Unit Commitment Scheduling for Hydro-thermal System, IEE Proceedings, Vol. 127, Pt. D, No. 6, Nov. 1980, pp 250-258.

(42) E. Khodaverdian, A. Brameller and E. D. Farmer; Review of Unit Commitment Algorithms, 17th Universities Power Engineering Conference, UMIST, March, 1982, pp 2A3.

(43) J. Gruhl, F. Schwepp and M. Ruane; Unit Commitment Scheduling of Electric Power System, System Engineering for Power Status and Prospects, Henniker new Hampshire, 1975, pp 116-129.

(44) R. Vieva, A. Inda and J. Frausto; A Digital Computer Package for Solving Short-term Hydro-Thermal Coordination and Unit Commitment, IEEE Transaction on Power Systems, Vol. PWRS,No. 3 Aug., 1986, pp 168-174.

(45) A. Al-Sayed; Application of Decomposition Techniques to Large-scale Optimal Unit Commitment, First Symposium on Electric Power System in Fast Developing Countries, Riyadh, Saudi Arabia, March, 1987, pp 255-258.

(46) W. L. Snyder, H. David Powell and J. C. Rayburn; Dynamic Programming Approach to Unit Commitment, IEEE Transaction on Power Systems, Vol. PWRS-2, No. 2, May, 1987, pp 339-350.

(47) R. Nieva, A. Inda and I. Guillen; Lagrangian Reduction of Search Range for Large-scale Unit Commitment, IEEE Transaction on Power Systems, Vol. PWRS-2, No. 2, May, 1987, pp 465-473.

(48) P. P. J. Vanden Bosch and G. Honderd; A solution of Unit Commitment Problem VIA Decomposition and Dynamic Programming. IEEE Transaction on Power Apparatus and Systems, Vol. PAS, No. 7, July, 1985, pp 1684-1690.

(49) G. L. Viviani; Practical Optimization, IEEE Transaction on Power Apparatus and Systems, Vol. PAS, No. 7, July, 1985, PP 1691-1694.

(50) G. Cohen; Optimization by Decomposition and Coordination A unified Approach. IEEE Transaction on Automatic Control, Vol. AC 23, No. 2, Apr., 1978, pp 222-232.

(51) L. T. Anstine, J. H. Henderson, F. A. Karmer and C. H. Wetter; Application of Digital Computer Techniques to the Economic Scheduling of Marginal Generating Units for Pennsylvynia, IEEE Transaction on Power Apparatus and Systems, Vol. PAS-83, 1964,

pp 316-321.

(52) A. M. Geoffrion; Generalized Benders Decomposition, Journal of Optimization Theory and Applications; Vol. 10, No. 10, 1972, pp 237-260.

(53) S. Virmani, E. C. Arian, K. Imhof and S. Mukherjee; Implementation of Lagrangian Relaxation Based Unit Commitment Problem, IEEE Transaction on Power Systems, Vol. 4, No. 4, Oct. 1989, pp 1373-1379.

(54) F. Zhuang and F. D. Galiana; Towards More Rigorous and Practical Unit Commitment by Lagrangian Relaxation, IEEE 1987 Power Industry Computer Applications Conference, PICA 87, Montreal, Quebec, Canada, May 18-22, 1987, pp 385-392.

(55) S. Mokhtari, J. Singh and B. Wollenberg; Unit Commitment Expert System, IEEE 1987 Power Industry Computer Applications Conference, PICA 87, Montreal, Quebec, Canada, May 18-22, 1987, pp 400-405.

(56) H. Habiballazadeh and J. A. Babenkc; Application of Decomposition Techniques to Short-term Operation Planning of Hydro-thermal Power Systems. IEEE Transaction on Power Systems, Vol. PWRS-1, Feb. 1986, pp 41-46.

(57) K. Ea and M. Monti; Daily Operational Planning of the EDF Plant MIX Personal for a New Method. IEEE

Transaction on Power Systems, Vol. PWRS-1, No. 3 Aug. 1986, pp 251-256.

(58) P P J Van den Boschs ; Dynamic Optimization in Large-scale Power Systems. Proceedings of IFAC Symposium on Computer Applications in Large-scale Power System, Newdelhi, 1979, pp 97-105.

(59) D P Bertsekas, G S Lauer, N. R. Sandell and T. A. Posbergh; Optimal Short-term Scheduling of Large-scale Power System, IEEE Transaction on Automatic Control, Vol. AC.-28, No. 1, Jan 1983, pp 1-11.

(60) A. I. Cohen and V R Shekat; Optimization Based Methods for Operations Scheduling. Proceedings of the IEEE, Vol.75, No. 12, Dec 1987. pp 1574-1591.

(61) W J Hobbs, G Hermon, S Warner and G B Sheble; An Enhanced Dynamic Programming Approach for Unit Commitment, IEEE Transaction on Power Systems, Vol. 3, No.3, Aug. 1988, pp 1201-1205.

(62) P H Ashmole; Energy Managements Developments for the CEGB Power System, Proceedings of the Nineth Power Systems Computation Conference, Cascais, Portugal, 30 Aug.- 4 Sept. 1987, pp 767-771.

(63) A C Calitz, T R Fennel and S J McDiarmid; The Importance of A Nominal Solution in A Heuristic Unit Commitment Program, Proceedings of the Ninth Power

Systems Computation Conference, Cascais, Portugal, 30 Aug.- 4 Sept. 1987, pp 652-656.

(64) A M Oscar and A A Alfonso; Generation Solution In Systems With Power and Energy Costs, Proceedings of the Ninth Power Systems Computation Conference, Cascais, Portugal, 30 Aug.- 4 Sept. 1987, pp 674-678.

(65) F N Lee ; Short-Term Thermal Unit Commitment, A New method, IEEE Transaction on Power Systems, Vol. 3, No.2, May 1988, pp 421-428.

(66) B. Korte (Editor); Modern Applied Mathematics Optimization and Operation Research, Amesterdam Oxford North-Holland, 1982, pp 463-508.

(67) M. Held, P. Wolfe and H. P. Crowder; Validation of Subgradient Optimization, Mathematical Programming 6, 1974, pp 62-88, North-Holland Publishing Company.

(68) A. I. Cohen and S. H. Wan; A method for Solving the Fuel Constrained Unit Commitment Problem. IEEE Transaction on Power Systems, Vol. PWRS-2,No. 3, Aug. 1987, pp 608-614.

(69) K. Aoki, T Satoh, M Itoh, T Ichimori and K Masegi; Unit Commitment in Large-Scale Power Systems Including Fuel Constrained and Pumped Storage Hydro. IEEE Transaction on Power Systems, Vol. PWRS-2, No. 4, Nov. 1987, pp 1077-1084.

(70) R. Billinton and N A Chowdhury; Operating Reserve Assessment in Interconnected Generating Systems. IEEE Transaction on Power Systems. Vol. 3, No. 4, Nov. 1988, pp 1479-1486.

(71) M. Grubb; The Inclusion of Dynamic Factors in Statistical Power System Cost Models: Part I: Assessment of Startup and Banking Costs. IEEE Transaction on Power Systems, Vol. 4, No.2, May 1989, pp 419-425.

Part II; Loading and reserve costs pp 524-529.

(72) G. C. Contaxis, S D Kavatza and C D Vournas; An Interactive Package for Risk Evaluation and Maintenance Scheduling. IEEE Transaction on Power Systems, Vol. 4, No. 2 May 1989, pp 389-395.

(73) K. Aoki, M Itoh, T. Satoh, K Nara and M Kanezashi; Optimal Long-Term Unit Commitment in Large Scale Systems Including Fuel Constrained Thermal and Pumped-Storage Hydro. IEEE Transaction on Power Systems, Vol. 4, No. 3, Aug. 1989. pp 1065-1073.

(74) N. Chowdhury and R. Billinton; Assessment of Spinning Reserve in Interconnected Generation Systems With Export/Import Constraints. IEEE Transaction on Power Systems, Vol. 4, No. 3 August 1989, pp 1102-1109.

(75) F N Lee; A Fuel Constrained Unit Commitment Method.

IEEE Transaction on Power Systems, Vol. 4, No. 3, August 1989, pp 1208-1218.

(76) J. F. Benders; Partitioning Procedures for Solving Mixed- Variables Programming Problems. Numerische Mathematik 4, 1962, pp 238-252.

(77) S. D. Bond and B. Fox; Constraint-Directed Search Technique for Generating Unit Commitment. 19th Universities Power Engineering Conference, UPEC 19, Dundee, 10-12 April, 1984, Technical Session 15.

(78) A. Kaufman and A. H. Laborder; Integer and Mixed Programming Theory and Applications, Vol. 137 in Mathematics in Science and Engineering. Academic Press 1977.

(79) C. McMillan; Mathematical Programming: An Introduction to the Design and Application of Optimal Decision Machines. John Wiley and Sons Inc. 1970

(80) S. R. Browning, P. H. Plumptre, H. R. W. Lewis and P. G. Lucas; Development of the Generating Ordering and Loading Program, The 20th Universities Power Engineering Conference (UPEC), Huddersfield, 2-4 April, 1985, pp 61-64.

(81) S. K. Tong and S. M. Shahidehpour; Combination of Lagrangian Relaxation and Linear-programming

Approach for Feul-constrained Unit Commitment Problem. IEE Proceedings, Vol. 136, pt. c, No. 3 May 1989, pp 162-174.

(82) D. A. Pierre and M. J. Lowe; Mathematical Programming via Augmented Lagrangians. Addison Wesley Publishing Company, 1975.

(83) W. Conley; Computer Optimisation Techniques, Petrocelli Books Inc. 1980.

(84) Numerical Algorithm Group, NAG Library Manual Routine Document, Mark 12. The Numerical Algorithm Group Limited, 1987.

APPENDIX A

INPUT DATA FOR CASE STUDY SYSTEMS

In this appendix, a brief description of three power systems of different sizes is given. These systems have been tested by the methods of solution of the thermal unit commitment problem which were studied in this thesis. The tested power systems are briefly described as follows;

4 UNIT SYSTEM: The generating units have linear input-output characteristics [1]. Input data of the units and the system load demand profile are presented in tables (A-1) and (A-2) respectively.

15 UNIT SYSTEM: The system is described in [40]. The input-output characteristics of the units are represented by a second order polynomial form. The spinning reserve of the system is assumed as a variable value depending on the demand. The input data of the generating units are presented in tables (A-3) and (A-4). Data of one day load profile are listed in table(A-5).

150 UNIT SYSTEM: For the purpose of testing the solution methods on a large-scale power system, a 150 unit system has been used. Input data of the units are found in tables (A-6) and (A-7). Data of the system load demand over 24 h are shown in table (A-8).

Table (A-1): Input data of 4 units.

unit No.	output (MW) MAX.	MIN.	IHR BTU/KW)	no-load cost £	start-up cost £	min time up	down
1	300.0	75.0	8730	684.74	1100.0	5	4
2	250.0	60.0	9000	585.62	400.0	5	2
3	80.0	25.0	10440	213.00	350.0	1	1
4	60.0	20.0	11900	252.00	0.0	1	1

Table (A-2): Load profile of a 4 unit system.

Time hours	Load demand (MW)							
1-8	450.0	530.0	600.0	540.0	400.0	280.0	290.0	500.0
9-16	440.0	520.0	590.0	545.0	410.0	380.0	390.0	420.0
17-24	450.0	500.0	575.0	530.0	418.0	360.0	405.0	500.0

Table (A-3): Input data of units for a 15 unit system.

Unit number	Output (MW)		Unit cost coefficients		
	MAX.	MIN.	a_i	b_i	c_i
1	670.00	250.00	360.40	6.49	0.000827
2	670.00	250.00	360.40	6.49	0.000827
3	670.00	250.00	360.40	6.49	0.000827
4	425.00	200.00	128.23	7.69	0.000768
5	338.00	125.00	162.03	5.86	0.002269
6	167.00	35.00	89.01	5.68	0.005206
7	167.00	35.00	89.01	5.68	0.005206
8	167.00	35.00	89.01	5.68	0.005206
9	167.00	35.00	89.01	5.68	0.005206
10	105.00	30.00	68.55	5.45	0.012167
11	105.00	30.00	68.55	5.45	0.012167
12	130.00	30.00	94.26	5.03	0.011685
13	53.00	20.00	42.02	6.39	0.023020
14	45.00	15.00	43.98	7.10	0.046586
15	45.00	15.00	43.98	7.10	0.046586

Table (A-4): Data of units for a 15 unit system.

Unit number	Start cost coefficients		minimum time	
	fixed cost $	thermal time constant	up	down
1	9234.00	0.0353000	6	4
2	9234.00	0.0351000	6	4
3	9234.00	0.0352000	6	4
4	8115.00	0.0257000	5	4
5	7150.00	0.0273000	4	3
6	6178.00	0.0256420	4	2
7	6178.00	0.0256420	4	2
8	6178.00	0.0256420	4	2
9	6178.00	0.0256420	3	2
10	1037.00	0.1917740	2	1
11	1827.00	0.0214740	2	1
12	2229.00	0.0161460	3	1
13	899.00	0.0453550	1	1
14	2951.00	0.0258220	1	1
15	2951.00	0.0258220	1	1

Table (A-5): Load profile of
a 15 unit power system for 24 hours.

Time hours	Load demand (MW)					
1-6	2040.0	1840.0	1748.0	1692.0	1684.0	1708.0
7-12	1924.0	2136.0	2516.0	2856.0	3060.0	3252.0
13-18	3308.0	3404.0	3436.0	3476.0	3460.0	3400.0
19-24	3356.0	3220.0	3168.0	3144.0	2564.0	2352.0

Table (A-6): Input data of units for a 150 unit system.

Unit number	Output (MW) MAX.	MIN.	Unit cost coefficients a_i	b_i	c_i
1-43	47.00	15.67	454.72	16.97	0.076000
44-97	50.00	16.67	498.30	19.11	0.058660
98-107	38.00	12.67	433.58	20.55	0.053600
108-142	44.00	14.67	544.04	21.83	0.036630
143-150	22.00	7.33	440.42	21.42	0.114500

Table (A-7): Input data of units for a 150 unit system.

Unit number	Start cost coefficient fixed cost SR	ther. time constant	minimum time up	down
1-43	60.00	0.00	2	1
44-97	70.00	0.00	2	1
98-107	45.00	0.00	1	1
108-142	55.00	0.00	2	1
143-150	40.00	0.00	1	1

Table (A-8): Load profile of a 150 unit power system for 24 hours.

Time hours	Load demand (MW)					
1-6	5416.0	5194.0	5150.0	5121.0	4892.0	4566.0
7-12	4821.0	5155.0	5451.0	5470.0	5488.0	5479.0
13-18	5760.0	6015.0	6018.0	5724.0	5450.0	5187.0
19-24	5495.0	5454.0	5460.0	5406.0	5546.0	5424.0

APPENDIX B

SYSTEM DESCRIPTION AND INPUT DATA OF SAUDI CONSOLIDATED ELECTRIC COMPANY, SCECO CENTRAL

B.1 INTRODUCTION

Saudi Consolidated Electric Company (SCECO) is the electric utility in the Kingdom of Saudi Arabia. It is divided regionally into four companies, SCECO Central, SCECO Eastern, SCECO Western and SCECO Southern. These companies cover most of the area of the country, as can be seen from the map in figure (B-1).

SCECO central covers the midland region of the Kingdom. The system consists of five generation plants connected by 380/132 KV transmission network as illustrated in figure (B-2). Each plant contains a number of identical gas turbine units. Number of the units at each plant and the unit ratings are presented in table (B-1). The total installed capacity of the system is 2750 MW. Samples of the daily load curve of SCECO for the four seasons are shown in figure (B-3). Tables (B-2) and (B-3) provide information about demand, temperature, energy and load factors for the summer season during July, and for the winter season during January of 1989 respectively. The priority order of the plants in terms of the average full load cost per MHh can be determined from table (B-1). The power plants of SCECO Central are listed as seen in table (B-5).

Table (B-1): SCECO Power plants.

Power plant	Number of units	Manufacturer and model	Max. output of unit at 50°C	FLAC* SR/MWh
PP8	20	BBC, WY18L 06611T	50 MW	32.014
PP7	16	GE, D 225 T 3	50 MW	30.120
PP5	12	BBC, 18L-006 GK	50 MW	35.81
PP4	4	BBC, WT16L-052LL3	20 MW	43.955
PP4X	5	HITACHI, EFZB1 LA	41.5 MW	34.04
	2	HITACHI, EFZL-K	15 MW	34.06

* FLAC : Full Load Average Cost, £1 = 6.0 Saudi Riyals.

Figure (B-1): The administrative regions which relate to the four SCECOs in the Kingdom of Saudi Arabia.

- SCECO Central.
- SCECO East.
- SCECO West.
- SCECO South.

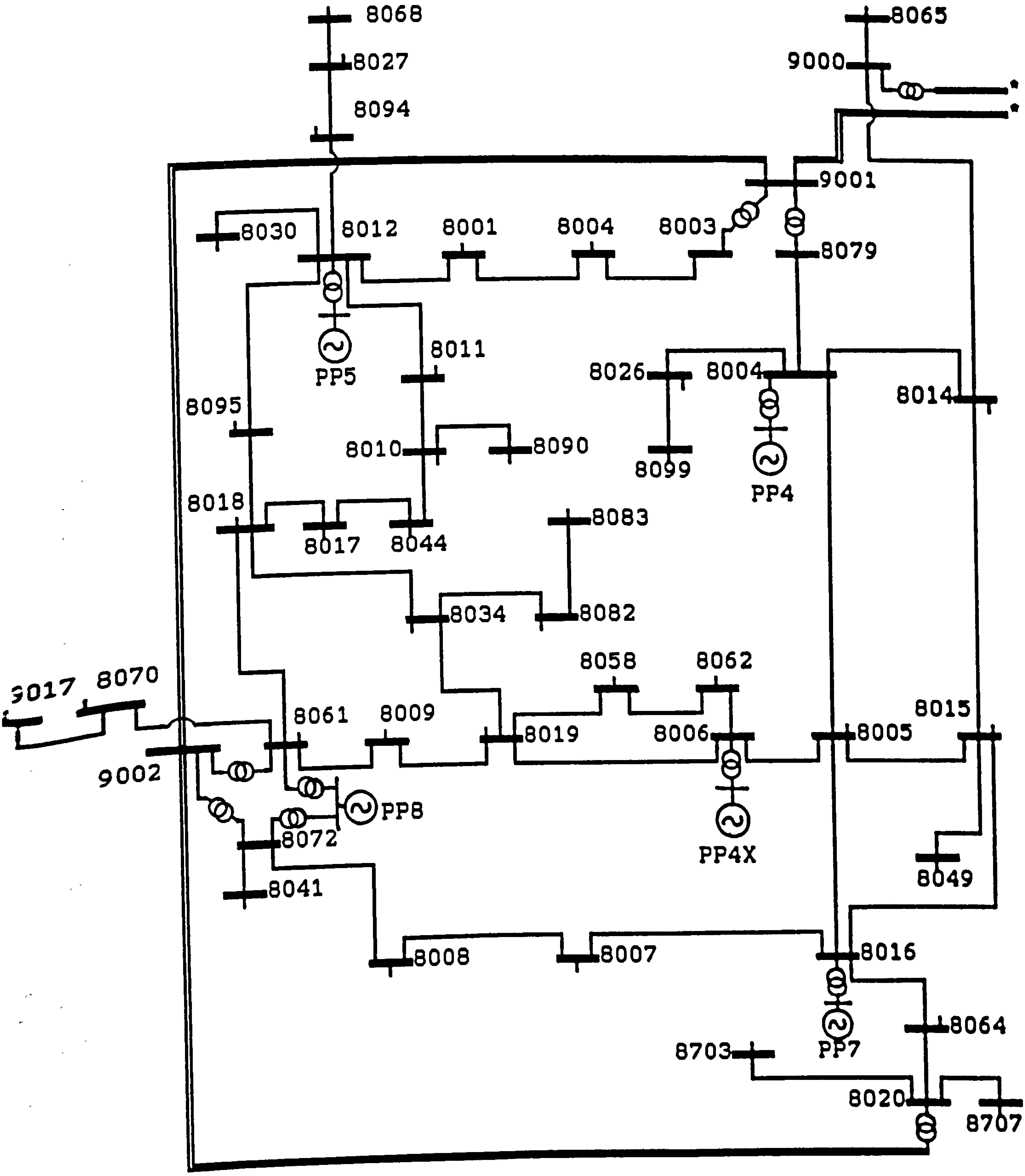


Figure (B-2): SCECO Central 380/132 KV Transmission Network.

380 KV. —— 132 KV.

* To the Eastern Region (SCECO East).

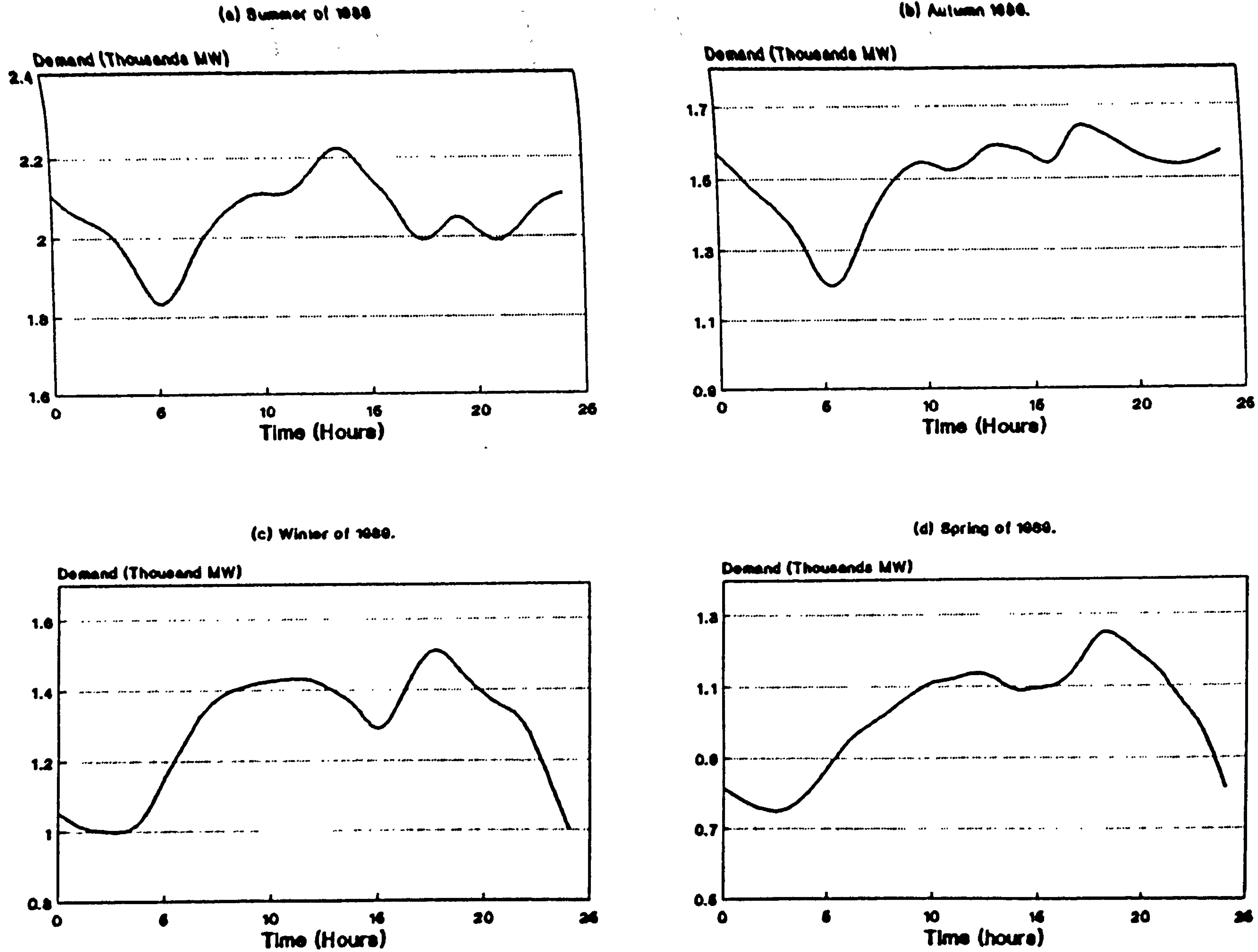


Figure (B-3): Samples of SCECO Central daily load curve for the four seasons.

Table (B-2): Heat rate and input-output characteristics of the units.

Power Plant	Unit numbers	Heat rate (HR) MBTU/MWH	Input-output charact. Saudi Riyals/h
PP8	1-16	190.93/P + 7.32 + 0.00139P	$498.30 + 19.1P + 0.0587P^2$
	17-20	190.21/P + 7.13 + 0.00137P	$495.41 + 19.0P + 0.0582P^2$
PP7	1-16	248.30/P + 4.87 + 0.038P	$454.72 + 16.9P + 0.0761P^2$
PP5	1-10	191.21/P + 7.28 + 0.0152P	$544.04 + 21.8P + 0.0366P^2$
	11-12	189.72/P + 7.11 + 0.0146P	$546.32 + 21.9P + 0.0368P^2$
PP4X	1-5	153.76/P + 7.44 + 0.0144P	$433.58 + 21.4P + 0.0536P^2$
	6-7	162.52/P + 7.71 + 0.0149P	$235.22 + 16.5P + 0.1250P^2$
PP4	1-4	166.65/P + 10.5 + 0.0003P	$440.42 + 21.4P + 0.1145P^2$

Table (B-3): System Maximum and Minimum Demand, Temperature, Energy generation and Load factor. SCECO (Central Region), January, 1989.

DAYS	DATE	PEAK		TEMP.		TIME OF PEAK LOAD		TOTAL MWHrs	L.F. %
		MAX.	MIN.	MAX.	MIN.	MAX.	MIN.		
SUN.	01	1486	915	20.0	09.0	1745	0400	28503	79.92
MON.	02	1430	880	26.0	11.0	1745	0400	27266	79.45
TUE.	03	1478	798	17.0	12.0	1745	0330	27223	76.75
WED.	04	1501	863	17.0	08.0	1800	0400	28444	78.96
THU.	05	1723	954	11.0	04.0	1800	0400	31992	77.37
FRI.	06	1796	1139	12.0	00.0	2130	0400	34631	80.34
SAT.	07	1902	1156	15.0	-0.5	1800	0330	36357	79.65
SUN.	08	1828	1202	18.0	01.0	1800	0330	35757	81.50
MON.	09	1644	1126	21.5	05.3	1800	0400	32906	83.40
TUE.	10	1560	1014	22.0	09.0	1800	0400	30734	82.09
WED.	11	1435	931	24.0	10.0	1800	0400	28573	82.96
THU.	12	1580	952	15.0	05.0	1800	0400	30325	79.97
FRI.	13	1525	1052	18.0	01.0	2200	0400	30353	82.93
SAT.	14	1641	1005	18.0	03.0	1800	0400	31574	80.17
SUN.	15	1655	1014	16.0	04.0	1800	0330	31795	80.05
MON.	16	1522	1025	23.0	08.0	1800	0400	31143	85.26
TUE.	17	1607	987	16.0	06.0	1800	0330	31341	81.26
WED.	18	1676	1074	14.0	04.0	1800	0400	33411	83.06
THU.	19	1604	1165	15.0	01.0	1800	0400	33261	86.40
FRI.	20	1512	1141	17.0	01.0	1800	0700	31441	86.64
SAT.	21	1502	1047	22.0	05.0	1800	0400	30904	85.73
SUN.	22	1371	991	24.0	10.4	1815	0400	28211	85.74
MON.	23	1462	884	17.0	07.3	1815	0400	28208	80.39
TUE.	24	1518	985	16.0	03.2	1815	0330	29636	81.35
WED.	25	1487	1019	17.0	04.6	1815	0330	29677	83.16
THU.	26	1409	1016	21.0	02.0	1815	0400	28346	83.82
FRI.	27	1271	927	25.0	08.0	1815	1700	25898	84.90
SAT.	28	1397	844	20.0	10.0	1815	0400	26921	80.29
SUN.	29	1437	887	18.0	09.0	1815	0400	27576	79.96
MON.	30	1458	929	16.6	05.8	1815	0330	28259	80.76
TUE.	31	1465	945	18.0	07.0	1815	0330	28656	81.50
MONTHLY TOTAL AND PEAK		1902	789	26.0	-0.5	1800	0330	939322	66.38
AVR. VALUES		1545	996	18.4	05.6			30301	81.80

Table (B-4): System Maximum and Minimum Demand, Temperature, Energy generation and Load factor. SCECO (Central Region), July, 1989.

DAYS	DATE	PEAK		TEMP.		TIME OF PEAK LOAD		TOTAL MWHrs	L.F. %
		MAX.	MIN.	MAX.	MIN.	MAX.	MIN.		
SAT.	01	2383	1874	43.0	28.0	1515	0600	52563	91.91
SUN.	02	2403	1872	43.0	25.0	1515	0600	52811	91.57
MON.	03	2410	1872	45.0	24.0	1400	0600	53024	91.67
TUE.	04	2464	1908	45.0	25.0	1515	0630	54065	91.42
WED.	05	2470	1960	45.0	26.0	1400	0600	54305	91.61
THU.	06	2369	1908	46.0	26.0	1515	0600	52707	92.70
FRI.	07	2229	1874	46.0	27.0	1515	0600	49885	93.25
SAT.	08	2391	1875	46.0	27.0	1500	0630	52594	91.65
SUN.	09	2372	1909	45.0	30.0	1400	0630	52018	91.38
MON.	10	2263	1831	44.5	32.0	1430	0600	50224	92.47
TUE.	11	2195	1812	43.0	31.0	1515	0600	48646	92.34
WED.	12	2230	1812	43.0	31.0	1430	0600	48672	90.94
THU.	13	2062	1587	44.0	30.0	0100	0630	44556	90.03
FRI.	14	1981	1638	43.0	28.8	0100	1830	43367	91.21
SAT.	15	1982	1675	44.0	31.0	1515	0600	44278	93.08
SUN.	16	2070	1685	45.0	31.0	1515	0530	45520	91.63
MON.	17	2160	1754	43.0	26.0	1515	0600	47665	91.95
TUE.	18	2247	1757	45.0	25.0	1515	0630	49146	91.13
WED.	19	2277	1812	44.0	26.0	1515	0600	49721	90.98
THU.	20	2246	1794	44.5	25.0	1500	0600	49512	91.85
FRI.	21	2226	1864	44.0	31.0	2400	1200	49079	91.87
SAT.	22	2424	1984	43.0	31.0	1530	0600	54111	93.01
SUN.	23	2394	1945	43.0	30.0	1530	0600	53588	93.27
MON.	24	2411	2012	44.0	32.0	1515	0600	54730	94.58
TUE.	25	2412	1948	44.0	29.0	1515	0530	54345	93.88
WED.	26	2428	1949	44.0	27.0	1515	0600	53460	91.74
THU.	27	2293	1911	45.0	24.0	1500	0600	51884	94.28
FRI.	28	2218	1849	45.0	26.0	2400	1200	49961	93.86
SAT.	29	2438	1910	45.0	25.0	1515	0600	53812	91.97
SUN.	30	2407	1943	44.0	25.0	1515	0600	53978	93.44
MON.	31	2388	1950	43.0	28.0	1530	0600	53867	93.99
MONTHLY TOTAL AND PEAK		2470	1587	46.0	24.0	1400	0630	1578094	85.87
AVR. VALUES		2298	1854	44.2	27.9			50906	92.28

Table (B-5): Priority order
of SCECO Central power plants

Plant order	Power plant	Average full load cost SR/MWh
1	PP7	30.120
2	PP8	32.140
3	PP4X	34.040
4	PP5	35.810
5	PP4	43.955

B.2 INPUT DATA:

Input data of generating units of SCECO Central, which had been used for solving the unit commitment problem, are listed in table (B-6) according to the priority order of the plants.

Table(B-6): Input data of the units.

UNIT NUMBER	OUTPUT MW		UNIT COST COEFFICIENTS		
	MAX.	MIN.	A	B	C
1-16	50.0	15.67	454.72	16.965	0.07600
17-32	50.0	15.67	498.30	19.115	0.05866
33-36	50.0	15.67	495.41	19.031	0.05682
37-41	41.5	12.67	433.58	20.549	0.05360
42-43	15.0	5.00	235.22	16.542	0.12500
44-53	50.0	14.67	544.04	21.835	0.03663
54-55	50.0	14.67	546.14	21.974	0.03676
56-59	20.0	7.33	440.42	21.420	0.11450

Table(B-7): Input data of the units.

UNIT NUMBER	START-UP COST	SHUT DOWN COST	LOADING RATE MW/MIN.	DELOADING RATE MW/MIN.	MINIMUM TIME (HOURS)	
					UP	DOWN
1-16	44.3	24.2	10.0	20.0	2	1
17-36	67.9	60.8	7.2	15.0	2	1
37-41	48.8	37.6	6.3	13.0	1	1
42-43	35.0	20.3	4.0	7.5	1	1
44-53	69.7	60.1	7.2	15.0	2	1
54-55	69.8	60.4	7.2	15.0	2	1
56-59	46.2	33.5	5.2	11.5	1	1

Table(B-8): One day load demand.

TIME HOURS	LOAD DEMAND (WM)					
1-6	2107.0	2056.0	2041.0	2003.0	1910.0	1807.0
7-12	1870.0	1996.0	2055.0	2099.0	2115.0	2100.0
13-18	2142.0	2219.0	2233.0	2143.0	2102.0	1985.0
19-24	1992.0	2070.0	2017.0	1977.0	2023.0	2093.0

Table(B-9): Spinning reserve.

SYSTEM DEMAND (MW)	SPINNING RESERVE (MW)
Below 900	160
900-1100	120
Above 1100	80

Table(B-10): Input data of the sample units for the reduced system.

No. OF UNITS PER GROUP	OUTPUT MW		UNIT COST COEFFICIENTS		
	MAX.	MIN.	A	B	C
16	50.0	15.67	454.72	16.965	0.07600
16	50.0	15.67	498.30	19.115	0.05866
4	50.0	15.67	495.41	19.031	0.05682
5	41.5	12.67	433.58	20.549	0.05360
2	15.0	5.00	235.22	16.542	0.12500
10	50.0	14.67	544.04	21.835	0.03663
2	50.0	14.67	546.14	21.974	0.03676
4	20.0	7.33	440.42	21.420	0.11450

Table(B-11): Input data of the sample units for the reduced system (continuation of table(B-10).

No. OF UNITS PER GROUP	START-UP COST	SHUT DOWN COST	LOADING RATE MW/MIN.	DELOADING RATE MW/MIN.	MINIMUM TIME (HOURS) UP	DOWN
16	44.3	24.2	10.0	20.0	2	1
16	67.9	60.8	7.2	15.0	2	1
4	48.8	37.6	6.3	13.0	1	1
5	35.0	20.3	4.0	7.5	1	1
2	69.7	60.1	7.2	15.0	2	1
10	69.8	60.4	7.2	15.0	2	1
2	69.8	60.4	7.2	15.0	2	1
4	46.2	33.5	5.2	11.5	1	1

Table(B-12): A sample one day load demand for the reduced system of SCECO Central.

TIME HOURS	LOAD DEMAND (WM)					
1-6	253.15	247.02	245.22	240.65	229.48	217.11
7-12	224.67	239.81	246.90	252.19	254.11	252.31
13-18	257.35	266.60	268.29	257.47	252.55	238.49
19-24	239.33	248.70	242.34	237.53	243.06	251.47

APPENDIX C

DYNAMIC PROGRAMMING

Dynamic programming is an optimization method which is particularly applicable to multistage decision process. Such a process contains three variables, called the **stage** variable, the **state** variable, and the **decision** variable. The process is described by mathematical equation involving these three variables. The stage variable is scalar-valued and can be defined either on an interval (continuous dynamic programming) or on a discrete set of points (discrete dynamic programming). The values of the stage variable are considered to be instants of time.

The state variable completely describes the decision process at any instant of time. It can be assumed that at every interval of time the values of the state variable are finite-dimensional vectors called **state vectors**.

A decision variable represents the independent decision inputs inherent in a multistage decision process. Its values are also assumed to be finite-dimensional vectors, called **decision vectors**.

As mentioned in chapter four, dynamic programming is an approach to solving a wide variety of problems; however, there does not exist a standard mathematical formulation of the dynamic programming problem. Therefore, particular equations used must be developed to suit each problem.

The problem of unit commitment has been formulated in chapter four so that it could be solved by dynamic programming. In this appendix, three forms of the dynamic programming technique presented in chapter four, and used for solving the unit commitment problem will be demonstrated by examples.

C.1 Full dynamic programming

Assume that unit commitment problem of a 5 unit system is to be solved by full dynamic programming. Then, total number of possible combinations of units at any interval is determined as:

$$2^N-1 = 2^5-1 = 31 \text{ combinations} \qquad (C.1)$$

If ON status of units is represented by 1 and OFF status by 0, then all possible combinations can be illustrated by the following table:

Table (C.1): All possible combinations of a 5 unit system by full dynamic programming.

Number of combination	Unit number and status (1 - ON, 0 - OFF)				
	1	2	3	4	5
1	1	0	0	0	0
2	0	1	0	0	0
3	1	1	0	0	0
4	0	0	1	0	0
5	1	0	1	0	0
6	0	1	1	0	0
7	1	1	1	0	0
8	0	0	0	1	0
9	1	0	0	1	0
10	0	1	0	1	0
11	1	1	0	1	0
12	0	0	1	1	0
13	1	0	1	1	0
14	0	1	1	1	0
15	1	1	1	1	0
16	0	0	0	0	1
17	1	0	0	0	1
18	0	1	0	0	1
19	1	1	0	0	1
20	0	0	1	0	1
21	1	0	1	0	1
22	0	1	1	0	1
23	1	1	1	0	1
24	0	0	0	1	1
25	1	0	0	1	1
26	0	1	0	1	1
27	1	1	0	1	1
28	0	0	1	1	1
29	1	0	1	1	1
30	0	1	1	1	1
31	1	1	1	1	1

The total number of all possible combinations becomes huge and may cause a serious dimensionality problem to computing facilities if the number of units is large. Therefore, dynamic programming sequential combinations and dynamic programming truncated combinations were suggested to reduce the number of possible combinations.

C.2 Dynamic Programming Sequential Combinations (DPSC)

If DPSC is applied to the system, i.e. a strict priority order is imposed on the units, then a remarkable reduction of unit combinations can be achieved. There are only five combinations to be tested for a 5 unit system at any interval, as shown in table (C.2).

Table (C.2) All possible combinations of a 5 unit system by DPSC.

Number of combination	Unit number and status (1 = ON, 0 = OFF)				
	1	2	3	4	5
1	1	0	0	0	0
2	1	1	0	0	0
3	1	1	1	0	0
4	1	1	1	1	0
5	1	1	1	1	1

C.3 Dynamic Programming Truncated Combinations (DPTC)

DPSC can be used only if the rate of change in the load demand is high. Otherwise, DPTC is better to use. This approach is based on the priority list of units and on full dynamic programming. The units are divided into two categories: base units (most efficient units) to supply base load, and cycling units (less efficient units) to supply peak load, which are included in the search for optimal solution. In the example of a 5 unit system, unit number 1 and unit number 2 are assumed as base units, and the rest are cycling units. Table (C.3) illustrates all possible combinations at any interval by DPTC for this example.

Table (C.3): All possible combinations of a 5 unit system by DPTC.

Number of combination	Unit number and status (1 = ON, 0 = OFF)				
	base units		cycling units		
	1	2	3	4	5
1	1	1	1	0	0
2	1	1	0	1	0
3	1	1	1	1	0
4	1	1	0	0	1
5	1	1	1	0	1
6	1	1	0	1	1
7	1	1	1	1	1

APPENDIX D

LAGRANGIAN RELAXATION DECOMPOSITION METHOD

Lagrangian is a mathematical approach first proposed by Lagrange in 1760 to convert the constrained optimization problem into unconstrained problem by appending the constraints to the objective function with Lagrangian multipliers. Assume the following function:

$$\min \quad F(x) \qquad \text{(D.1)}$$

subject to

$$G_i(x) = 0 \quad i=1,2,\ldots,n$$

The problem is to determine a value of x which yields to optimal minimum for F(x) and also satisfies the constraints. Obviously the feasible region is greatly reduced by the presence of constraints. If Lgarange's method is applied to this problem, then Lagrangian function is defined by [4]:

$$L(x,\sigma) = F(x) + \sum_{i=1}^{n} \sigma_i G_i(x) \qquad \text{(D.2)}$$

The new objective function in equation (D.2) is called augmented Lagrangian which is easier to solve than the original one. It can be minimized as if it were an unconstrained function, i.e. partial derivatives of L with respect to all variables are taken and equated to zero [10]:

$$\frac{\partial L(x,\sigma)}{\partial x} = 0 \quad \text{and} \quad \frac{\partial L(x,\sigma)}{\partial \sigma} = 0$$

The general method of Lagrangian multipliers for n variables and m equality constraints can be formulated as follows:

$$\min F(x_i) \quad i = 1,2,\ldots,n \qquad \text{(D.3)}$$

subject to

$$G_j(x_i) = 0 \quad j = 1,2,\ldots,m \qquad \text{(D.4)}$$

The Lagrangian augmented function is:

$$L(x_i,\sigma_j) = F(x_i) + \sum_{j=1}^{m} \sigma_j G_j(x_i) \quad :i=1,2,\ldots,n \qquad \text{(D.5)}$$

Differentiate equation (D.5) with respect to all x_i and σ_j to get n+m simultaneous equations which can be solved to determine the n variables and the m Lagrangian multipliers.

In some practical problems, as is the case in unit commitment, not all the constraints can be incorporated into the objective function. Therefore, some of these constraints can be relaxed by using Lagrangian multipliers, while the rest can be taken into consideration during the search for optimal solution. This technique is called Lagrangian relaxation method.

D.1 LAGRANGIAN DECOMPOSITION

Large-scale optimization problems are difficult to solve due to dimensionality problems. Decomposition techniques lead to the creation of smaller subproblems which can be solved independently. The final step is to combine the subproblem solutions in such a way that they solve the original optimization problem. Lagrangian decomposition is widely implemented in this field. A brief description of Lagrangian decomposition is given in the following section [4]:

If the following form of optimization is considered:

$$\min \sum_{i=1}^{n} F_i(x_i) \qquad \text{(D.6)}$$

subject to

$$G_i(x_i) \geq 0 \ , \ i=1,2,\ldots,n$$

Then forming the Lagrangian yields

$$L(x,\sigma) = \sum_{i=1}^{n} F_i(x_i) + \sum_{i=1}^{n} \sigma_i^T G_i(x_i)$$

or

$$L(x,\sigma) = \sum_{i=1}^{n} [\ F_i(x_i) + \sigma_i^T (x_i)\ ] \qquad \text{(D.7)}$$

In order to decompose the problem, we want to obtain a form for L(x,σ) such that the terms in square brackets in equation (D.7) are functions of the ith subsystem only. We begin by defining some additional slack variables s_j, where

$$s_j = x_j, \quad j = 1,2,\ldots,n \qquad (D.8)$$

This substitution can now be made in equation (D.7) if we also satisfy equation (D.8) by adding it to the Lagrangian. Thus

$$L(x,\mu,\sigma,s) = \sum_{i=1}^{n} [F_i(x_i) + \sigma_i^T G_i(x_i, s_1, s_2, \ldots, s_j, \ldots, s_n)_{j\neq i}$$

$$+ \mu_i^T (x_i - s_i) \qquad (D.9)$$

where $\mu_i = 0$ if $G_i(x_1,\ldots,x_n)$ is not a function of x_i for all $j \neq i$

The Lagrangian in (D.9) can now be decomposed in two deferent ways. The first way is to regard s_i as fixed parameters. Then the Lagrangian for each subproblem can be written as:

$$L_i(x_i, \mu_i, \sigma_i; s_i) = F_i(x_i) + \sigma_i^T G_i(x_i; s_1, s_2, \ldots, s_n)_{j\neq i}$$

$$+ \mu_i^T (x_i - s_i), \quad i = 1,2,\ldots,n \qquad (D.10)$$

The terms have been arranged in (D.10) so that the only variables not associated with the ith subsystem are fixed parameters s_i. The stationary points for each Lagrangian function $L_i(x_i, \mu_i, \sigma_i; s_i)$, $i = 1,2,\ldots,n$, can now be determined independently as a function of s_i. This task is referred to as a first-level optimization, and it is accomplished by applying the necessary conditions presented at the beginning of this appendix.

It is now necessary to determine a means of selecting the optimal values for the parameters s_i. This task is referred to as a second-level optimization, and can be accomplished by methods to be discussed briefly. This method of Lagrangian decomposition is called feasible decomposition, since the interconnection constraints given by equation (D.8) are always satisfied. Thus, even if the second-level optimization problem is terminated before complete convergence to the optimal values of s_i is obtained, the resulting solution to equation (D.10) is feasible.

Another way to decompose the Lagrangian in equation (D.9) is to regard the μ as fixed parameters. In this case we let

$$s_{ij} = x_j, \quad i,j = 1,2,\ldots,n, \quad i \neq j \qquad \text{(D.11)}$$

where s_{ij} represents the presence of x_j in the ith subsystem.

Then, substituting in (D.7), and adding (D.9) with Lagrangian multipliers σ_{ij} we have

$$L(x,\sigma,\mu,s) = \sum_{i=1}^{n} [F_i(x_i) + \sigma_i^T G_i(x_i, s_{i1}, s_{i2}, \ldots, s_{in})_{i \neq j}$$

$$+ \sum_{i=1}^{n} \mu_{ij}^T (x_j - s_{ij})]$$

where $\mu_{ij} = 0$ if $G_i(x_1, x_2, \ldots, x_n)$ is not a function of x_j. Now the subsystem Lagrangians become

$$L_i(x_i, \sigma_i, s_{ij}; \mu_{ij}) = F_i(x_i) + \sigma_i^T G^i(x_i, s_{i1}, s_{i2}, \ldots, s_n)_{i \neq j} + \sum_{j=1}^{n} \mu_{ji}^T x_i - \sum_{j=1}^{n} \mu_{ij}^T s_{ij}. \qquad (D.12)$$

In equation (D.12) the stationary points with respect to x_i, σ_i, and s_{ij} can be determined as functions of μ_{ij} by the Kuhn-Tuker theory. These values are determined in the first-level optimization, and the second-level optimization determines μ_{ij}. This method of Lagrangian decomposition is called dual feasible because it can be shown that the second-level problem arising from equation (D.12) is the dual of the one arising from (D.10).

D.2 DUALITY

The concept of duality is highly refined in linear programming and leads to several very efficient computational algorithms in this field. Every linear programming problem has associated with another linear programming problem; one of them is called the **PRIMAL** problem and the other one its **DUAL** [4]. If the following maximization problem is assumed as the primal problem:

$$\max_{x \in M} (c,x)$$

where $M = \{x | x \in E^n\}$

subject to

$$Ax \leq b$$

$$b \geq 0$$

Then the associated dual problem is:

$$\min_{z \in N} (b,z)$$

where $N = \{z | z \in E^m\}$

subject to

$$A^T z \geq c$$

$$z \geq 0$$

It can be noted that each constraint in one problem corresponds to a variable in the other problem. The constraint vector of the primal problem generates the objective function of the dual problem and vice versa. The direction of the constraint inequalities ($\leq$ in the primal problem) is reversed ($\geq$ in the dual problem).

Two problems and their optimal solution are related, and the optimal solution of one problem yields complete information about the optimal solution of the other.

A similar duality theory exists for non-linear programming problems. However, the dual non-linear programs are usually much more difficult to solve than the corresponding primal. The dual does, however, often lead to computational efficient algorithms when combined with conditions arising in the primal problem.