Integrating Ecology and Economics in the Mathematical Modelling of Marine Ecosystems

PhD Thesis

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Marine Population Modelling

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This thesis is the result of the author's original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.

Abstract

This thesis focusses on the integration of economics and ecology in the mathematical modelling of marine ecosystems. The project addressed two aspects of the problem, 1) the relationship between fish price and landings, and 2) the formulation and solving a mean field game model for fisheries. Regarding price flexibility, we performed statistical analysis on ex-vessel fish prices, landings, and other market variables for the whole UK market, and estimated negative own-price flexibilities for numerous individual fish species, and for some broadly defined guilds of species. These provided relationships between the quantity landed and the price received, which generates a feedback between the marine ecosystem and human activity. Regarding mean field games, we developed a model which considers an economic game with a large number of players exploiting a common resource, whose payoffs depend on the collective the actions of all other players. The application of mean field game approaches to a common resource situation was novel and absent in the literature. Solving the mean field game model numerically, we were able to dynamically simulate the feedback between the marine ecosystem and fishing activity. This allowed us to investigate how the dynamics of the coupled economic-ecological system depended on ecological and economic factors, including the price flexibility identified earlier. We found interesting results relating to the impact of price flexibility and its interaction with stock growth rate, showing that higher price flexibilities resulted in increased fishing pressure for stocks with lower growth rates, but decreased fishing pressure for stocks with very high growth rates. Finally, we modelled the implementation of regulations in the mean field game, and demonstrated how these regulations affect the distribution of fishing activity in a mean field game model for a North Sea cod fishery.

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Preface/Acknowledgements

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Chapter 1

Introduction

1.1 Motivation

In this thesis we are considering the broad problem of developing models which will better allow the dynamics of integrated economic-ecological marine systems to be investigated and analysed. There are strong feedbacks between the state of the marine ecosystem and decisions made concerning exploitation and management of marine resources. Many models exist which simulate marine ecosystems, however in these models the activities of humans are usually represented as external boundary conditions. There is a need for models which allow the dynamics of integrated social-ecological systems to be analysed as a whole, which require models which extend beyond the boundaries of ecology to include economics and social science, and incorporate the feedback between the state of the ecosystem and the decisions made by humans.

Figure 1.1 demonstrates this, showing a diagram which represents the connected social-ecological system associated with a marine resource being exploited by human fishing activity. The blue box represents the marine ecosystem, consisting of interacting populations and physical drivers of the environment, for which mathematical models may draw on biology, ecology, and physical science, using tools such as ordinary differential equations. The grey boxes represent human components of the system, divided approximately into groups of decision makers who interact with each other, with the marine ecosystem, and with the wider economic and political environment. Modelling these interactions mathematically may draw from economics, finance and social science, and use tools such as game theory.

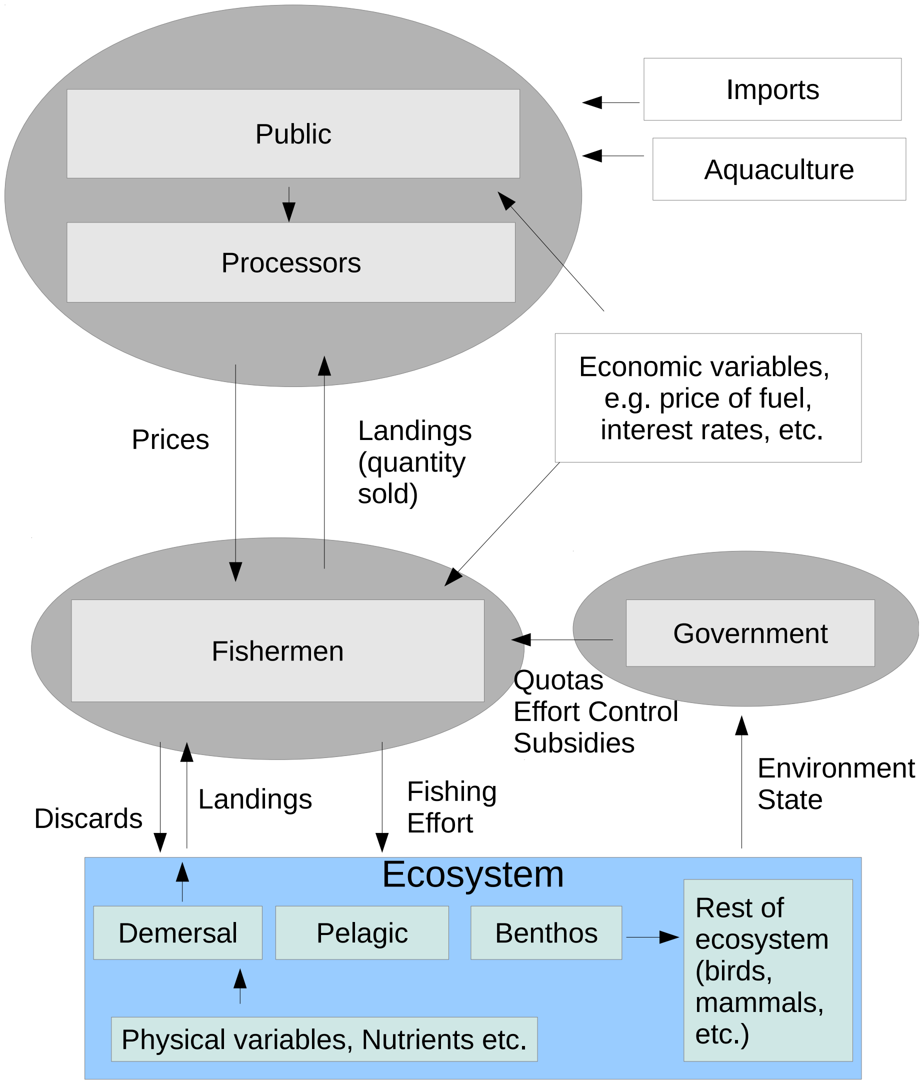


Figure 1.1: Diagram showing some of the connected components of the integrated ecological-economic system associated with a marine fishery.

In this scenario, the fishermen interact with the marine ecosystem by applying fishing effort to targeted fish species or groups, harvesting some of these populations. As well as direct removal of these fish, their activities may impact the ecosystem in other ways, e.g. by disrupting other non-targeted species, or through the effect of discarding some fish back into the marine environment. This fishing activity results in landings which will be sold, usually to processors to be sold to other consumers, but sometimes more directly to the public. The amount landed will depend on several factors; the population or stock of the targeted fish, the price that can be received, any regulations imposed by the government, and other economic factors affecting the cost of fishing. Similarly, the price fishermen will receive may depend both on demand from the public, affected by economy-wide variables such as fuel prices and interest rates as well as competition from aquaculture and imports, but also on the quantity landed by the fishermen as a whole. The regulations imposed on the fishery will likely depend on the state of the ecosystem, not just on the population of targeted fish species but also other indicators from the marine ecosystem such as populations of other species, which may be impacted by the fishery via complex food web interactions.

So there are numerous feedbacks between the different human components of the system and the marine ecosystem as a whole. Capturing these feedbacks requires us to couple mathematical models can simulate the populations within the marine ecosystem with mathematical models that can describe the activities of humans driven by rational decision making. In this thesis, we focus on the interface between economic and ecological modelling, allowing dynamic interaction between model components so that we can capture feedback between them.

In a mathematical context, the focus of the project is in developing techniques to allow the better coupling of mathematical population modelling with mathematical modelling of economic systems in a way that allows us to capture the feedback between the two. Progress made in dealing with this topic could be readily adapted to mathematical modelling in a variety of contexts - wherever there is a desire to balance environmental, economical and social benefits provided by a natural resource which can itself be modelled by mathematical population models. The project has direct relevance to the commercial fishing industry, and may be relevant to other industries benefitting from marine ecosystem services (e.g. wildlife tourism and recreational angling). In addition, by contributing to knowledge about the dynamics of marine ecosystems and the economic systems which interact with them, the project has the potential to benefit society by informing better management our marine resources.

1.2 Approach

Numerous bioeconomic models applicable to fisheries exist (e.g. see Prellezo et al. 2009 [1]), which apply a variety of different methods, aim to fulfil different functions, and are suited to different particular areas and fisheries. However, there is always a trade-off between scope and complexity, which is particularly apparent in bioeconomic models due to the different approaches in modelling ecological and economic systems, meaning different models will prioritise one over the other to different extents (some may lack a biological component entirely), depending on the particular goal. Our aim is to focus on modelling the interface between the economic and ecological components of a model, allowing us to produce a model that combines a fully interacting ecological model with a model of fisheries economics that includes dynamic feedback between the two.

There is an extremely wide variety of existing models of marine ecosystems. One example are models based on ordinary differential equations (ODEs), which are of particular interest due to their relative simplicity and because of research on creating end-to-end ODE-based ecosystem models, such as StrathE2E (Heath 2012 [2]) which simulates all trophic levels of the marine ecosystem of the North Sea at a broad scale. One of the key appeals of ODE-based models is the ability to use a relatively small number of variables but allow interactions between different parts of the ecosystem to be modelled, for example, by representing commercially important fish species as coarsely defined guilds rather than populations of each individual species. In this project, the focus is on developing an approach that could dynamically couple an ecological model like this with an economic component representing the fishing activity, so that, rather than setting the fishing activity as boundary conditions in an ODE-based ecosystem model, we can run a model which simulates fishing activity based on economic and ecological conditions and captures the interactions between the ecological and economic part of the system.

The approach we consider in this thesis is the use of mean field game theory, introduced by Lasry & Lions [3] and Huang, Caines and Malhamé [4], a particular branch of game theory where the number of players or agents in the game is high enough that it can be approximated by a continuum, and complex interactions between individual agents are replaced by a "mean field" representing the overall effect of the mass of other agents on an individual, with each infinitesimal agent contributing to and affected by the mean field. In this case each agent's individual contribution can be considered small and interchangeable enough that a given agent's payoff does not depend on the action of any other individual agent, but does depend on the action of the statistical mass of all other agents in the system.

We also consider the price of fish, in particular the relationship between quantity landed and the price received by fishermen. To investigate this relationship we performed regression analysis using data on UK ex-vessel fish prices and landings for a large number of species over a long time period, as well as data on other factors which may impact the price such as fish imports, exports and substitutes. We also performed the analysis for broadly defined guilds of species in addition to the individual species. This analysis allowed us to estimate price flexibilities for different fish species in the UK market, and provided evidence about the type of relationship between quantity and landings we could expect.

Chapters 2 and 3 contain the background information and concepts for the mathematical modelling in this thesis. In Chapter 2 we present an introduction to existing mean field games literature, with some discussion of how we will adapt it to the problem of a fishery. In Chapter 3 we discuss useful concepts from optimal control and game theory in fisheries modelling, some background information on ecosystem modelling and bioeconomic models, and some literature relating to fish prices. In Chapter 4 we discuss the elements from the background chapters that we can use to adapt mean field games to fisheries, and then we describe and formulate the mean field game model for fisheries in full. In Chapter 5 we present the results of statistical analysis of fish prices within the UK, which helps provide a basis for how to treat this subject within our mean field game model. In Chapter 6 we consider the numerical solution of the mean field game model, and present results of simulations of this model demonstrating the behaviour of the solution. In Chapter 7 we perform more numerical simulations of the mean field game model with focus on how the solution behaviour depends on different ecological and economic conditions, including how price dynamics impact the solution. Finally Chapter 8 considers the implementation of regulations in the mean field game fishery model.

Chapter 2

Background: Mean Field Games

2.1 Introduction

In this chapter we present an introduction to mean field games, a branch of game theory originally developed by Lasry and Lions [3] and independently by Huang, Caines and Malhamé [4], which concerns games with a large number of players or agents, where each individual agent is "small" compared to the total mass of all other agents. In mean field game theory, the equilibrium solution of a differential game with many players can be represented by the solution to a coupled set of partial differential equations, comprising a **Hamilton-Jacob-Bellman** (HJB) equation and a **Fokker-Planck** (F-P) equation (also called a **Kolmogorov Forward** equation).

A typical example of the system of partial differential equations arising from a mean field game of the type that we will consider is given below:

$$\frac{\partial u}{\partial t} + \frac{\sigma^2}{2}\Delta u + H(x, \nabla u) = F(x, m) \quad \text{in } \Omega \times (0, T) \tag{2.1}$$

$$\frac{\partial m}{\partial t} - \frac{\sigma^2}{2}\Delta m + \nabla \cdot (m\ H_p(x, \nabla u)) = 0 \quad \text{in } \Omega \times (0, T) \tag{2.2}$$

$$\int_\Omega m(x, t)\ \mathrm{d}x = 1,\ m \geq 0 \quad \forall t \in [0, T] \tag{2.3}$$

$$\nabla u \cdot \mathbf{n} = 0,\ \text{and } \nabla m \cdot \mathbf{n} = 0,\ \text{on } \partial\Omega \times (0, T) \tag{2.4}$$

$$u(x, T) = G(x(T), m(x, T)),\ m(x, 0) = m_0(x) \quad \text{in } \Omega. \tag{2.5}$$

The unknowns u and m are scalar functions of the state $x \in \Omega \subset \mathbb{R}^n$ and time $t \in [0, T]$ (where T is the terminal time) which solve the coupled Hamilton-Jacobi-Bellman equation (2.1) and Fokker-Planck equation (2.2). The function $H(x, p)$ is known as the Hamiltonian with $H_p(x, q)$ denoting its derivative with respect to the second variable at the point $p = q$, and $\mathbf{n}$ is the outward unit normal vector on $\partial\Omega$. We require that m is a valid probability density function, i.e. that conditions (2.3) hold. Here we assume Neumman boundary conditions boundary given by (2.4), and a terminal condition for u and an initial condition for m are given (2.5).

The aim of this chapter is to explain the interpretation of this system in terms of a game with many players, note some of the key results for this system and give an overview of some of the existing applications. In the first section we discuss some preliminary definitions and concepts from game theory and optimal control. Then we consider the mean field game equations above in more detail, explaining how they relate to a differential game with many players and some results from the literature. The following section examines some applications, particularly those that are most relevant to the application of mean field games to fisheries.

2.2 Preliminaries

2.2.1 Game Theory

Game theory studies the interaction between rational players or agents (these terms may be used interchangeably in this thesis) who each must select a strategy with the aim to optimise a payoff which depends on the strategies selected by the players. Explanations of some of these terms which will be useful for understanding mean field games are given below:

- **Rational**: Players or agents are rational if they always make the decision which maximises their expected payoff.
- **Strategy**: A player's strategy determines the action taken by the player in the game. The decision that a player can make involves selecting some strategy from a given set of possible strategies.

- **Payoff**: The payoff, usually given in terms of a utility function, is the quantity which players are trying to optimise.

A game arises when rational agents have some interaction where each agent's payoff depends on the agent's own strategy and on the strategy chosen by the other agents in the game. If the payoff depends only on the strategy of the individual player then it would be an optimisation problem rather than a game as there would be no interaction between players to consider. Likewise if the payoff of an agent depended only on what other agents did, then they could not be considered a player in the game.

In economics, the payoff is often treated as being total expected profit, but an agent which pursues profit along with other goals can be represented by including other terms in that agent's utility function.

A "solution" to a game is usually some sort of equilibrium. There are different kinds of equilibria defined in game theory, but the best known and the one we are concerned with in this thesis is the **Nash Equilibrium**. A Nash Equilibrium is reached when no player in the game can improve their own payoff by unilaterally changing their own strategy. That is, assuming that for each player the strategies of all other players are given, then that player has selected the optimum strategy.

A **differential game** is one in which the strategy to be chosen by the agents involves controlling a state variable which evolves according to a differential equation. In a differential game, the strategy of a given player is then usually given by their choice of control in a controlled differential equation. Hence differential games are closely related to problems of optimal control, where a single agent selects a control for a controlled dynamical system to maximise some payoff.

2.2.2 Optimal Control

Consider a state $x(t) \in \Omega \subset \mathbb{R}$ that evolves according to the differential equation:

$$\dot{x} = f(x, \alpha(t), t), \quad x(0) = x_0, \tag{2.6}$$

over the interval $[0, T]$.

The control $\alpha : [0,T] \to \mathbb{R}$ is selected from the set of admissible controls $A \subset L^2([0,T],\mathbb{R})$ for the control problem. Selecting a control $\alpha(t) \in A$ which is optimal in terms of a given payoff is an example of an optimal control problem. For consistency with the terminology from game theory above, we will say that the control is selected by an agent with the ability to exert some control over the state of system.

The agent selects its control with the aim to optimise some payoff. In many optimal control problems this takes the form of trying to minimise a cost given by

$$J(\alpha) = \int_0^T F(x(\tau), \alpha(\tau), \tau)\ \mathrm{d}\tau + G(x(T)), \tag{2.7}$$

from initial time $t = 0$ to a terminal time $t = T$ by selecting a control $\alpha \in A$ on the time interval $[0, T]$. In this form, $F(x, \alpha, t)$ is known as the running cost (since it is the cost accrued over time depending on the state and control) and $G(x(t))$ is the terminal cost (as it occurs at the terminal time T). The **optimal control**, denoted by $\alpha^*(t)$, is the control in A which minimises the cost $J(\alpha)$.

A key concept from optimal control is that of the value function, which is the optimal payoff that can be achieved by an agent from a given state x and time t by behaving optimally from this point until the terminal time. In this case the value function is defined as

$$u(x,t) = \min_{\alpha(t)\in A} \left(\int_t^T F(x, \alpha, \tau)\ \mathrm{d}\tau + G(x(T)) \right). \tag{2.8}$$

So the value function $u(x, t)$ is a function of the current time t and state $x(t)$ given by the minimum possible value of the payoff function that can be obtained by controlling optimally from time t and state x to the terminal time T. Note that if one can obtain the value function, one can usually then obtain the optimal control to the system.

The value function is important for mean field games because it can be shown that it solves the Hamilton-Jacobi-Bellman equation, as explained in the following section.

2.2.3 Hamilton-Jacobi-Bellman Equation

For the controlled system (2.6) and optimisation criteria (2.7) described above, with $x(t) \in \Omega \subset \mathbb{R}$ and where $\alpha(t)$ is a function $\alpha : [0,T] \to A$ for some viable set of parameters $A \subset \mathbb{R}$, we have the value function defined at time t and state x given by:

$$u(x,t) = \min_{\alpha(t)\in A} \left(\int_t^T F(x,\alpha,\tau)\,\mathrm{d}\tau + G(x(T)) \right).$$

We select some time t_m such that $t < t_m < T$, and split the integral as follows:

$$u(x,t) = \min_{\alpha(t)} \left(\int_t^{t_m} F(x,\alpha,\tau)\,\mathrm{d}\tau + \int_{t_m}^T F(x,\alpha,\tau)\,\mathrm{d}\tau + G(x(T)) \right)$$

$$u(x,t) = \min_{\alpha(t)\in A} \left(\int_t^{t_m} F(x,\alpha,\tau)\,\mathrm{d}\tau \right) + \min_{\alpha(t)\in A} \left(\int_{t_m}^T F(x,\alpha,\tau)\,\mathrm{d}\tau + G(x(T)) \right).$$

From the definition of the value function, we have that

$$u(x(t_m),t_m) = \min_{\alpha(t)\in A} \left(\int_{t_m}^T F(x,\alpha,\tau)\,\mathrm{d}\tau + G(x(T)) \right),$$

and hence

$$u(x,t) = \min_{\alpha(t)\in A} \left(\int_t^{t_m} F(x,\alpha,\tau)\,\mathrm{d}\tau \right) + u(x(t_m),t_m).$$

This means that the path of the optimal control starting from $t < t_m$ passing through $x(t_m)$, on the interval $[t_m, T]$, will be the same as the optimal control starting from t_m and $x(t_m)$. This concept is known as Bellman's principle of optimality.

Now let $t_m = t + \Delta t$, and $x(t_m) = x(t + \Delta t) = x(t) + \Delta x$. Assuming $u(x,t)$ is sufficiently smooth, we can perform a first order Taylor series expansion on $u(x(t_m),t_m)$ to obtain

$$u(x,t) = \min_{\alpha(t)\in A} \left(F(x,\alpha,t)\Delta t + u(x,t) + \frac{\partial u}{\partial x}(x(t),t)\Delta x + \frac{\partial u}{\partial t}(x,t)\Delta t \right).$$

Dividing by Δt and letting $\Delta t \to 0$, we have

$$0 = \min_{\alpha(t)\in A} \left(F(x,\alpha,t) + \frac{\partial u}{\partial x}\dot{x} \right) + \frac{\partial u}{\partial t}. \tag{2.9}$$

Note from (2.6) that $\dot{x} = f(x, \alpha, t)$, and so rearranging (2.9) we obtain the following Hamilton-Jacobi-Bellman (HJB) Equation:

$$-\frac{\partial u}{\partial t} = \min_{\alpha(t)\in A}\left(F(x,\alpha,t) + \frac{\partial u}{\partial x} f(x,\alpha(t),t)\right). \tag{2.10}$$

Hamilton-Jacobi-Bellman equations are often written in terms in terms of the **Hamiltonian** for the problem. Here, the Hamiltonian is defined as $H(x, p, \alpha(t), t) = \min_{\alpha(t)\in A}(F(x, \alpha(t), t) + pf(x, \alpha(t), t))$. With this definition of the Hamiltonian, the minimum on the right hand side of equation (2.10) can be denoted by $H(x, \frac{\partial u}{\partial x}, \alpha(t), t)$, and so the HJB equation can be written as:

$$-\frac{\partial u}{\partial t} = H\left(\frac{\partial u}{\partial x}, x, \alpha(t), t\right). \tag{2.11}$$

2.2.3.1 Higher Dimensions

The section above derives the Hamilton-Jacobi-Bellman equation for an optimal control problem in one dimension, but similar arguments can be used the HJB equation for higher dimensions. Let the state $\mathbf{x}(t) \in \Omega \subset \mathbb{R}^n$ be a vector of n dimensions that evolves according to

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \boldsymbol{\alpha}, t), \quad \mathbf{x}(0) = \mathbf{x}_0, \tag{2.12}$$

where the control $\boldsymbol{\alpha}(t)$ is given by $\boldsymbol{\alpha}[0,T] \to \mathbb{R}^n$ where $A \subset L^2([0,T],\mathbb{R}^n)$.

The Hamilton-Jacobi-Bellman equation for the problem can be derived following the same arguments as in the one dimensional problem [5], to obtain:

$$-\frac{\partial u}{\partial t} = \min_{\boldsymbol{\alpha}(t)}\left(F(\mathbf{x}, \boldsymbol{\alpha}, t) + (\nabla_{\mathbf{x}} u)^T \mathbf{f}(\mathbf{x}, \boldsymbol{\alpha}, t)\right).$$

The n-dimensional Hamiltonian is defined as follows:

$$H(\mathbf{x}, \mathbf{p}, \boldsymbol{\alpha}(t), t) = \min_{\boldsymbol{\alpha}(t)}\left(F(\mathbf{x}, \boldsymbol{\alpha}, t) + \mathbf{p}^T \mathbf{f}(\mathbf{x}, \boldsymbol{\alpha}, t)\right).$$

So the Hamilton-Jacobi-Bellman equation can be written as

$$-\frac{\partial u}{\partial t} = H(\nabla_{\mathbf{x}} u, \mathbf{x}, \boldsymbol{\alpha}, t).$$

2.2.3.2 Stochastic Hamilton-Jacobi-Bellman Equation

We now consider a stochastic control problem, where the state dynamics are given by the following stochastic differential equation (SDE):

$$\mathrm{d}\mathbf{x} = \mathbf{f}(\mathbf{x}, \boldsymbol{\alpha}, t)dt + \sigma(\mathbf{x}, \boldsymbol{\alpha}, t)dW_t. \tag{2.13}$$

Here evolution of the state is subject to some Brownian noise with intensity σ (where σ is a scalar).

The Hamiltonian for an optimal control problem as described in the section above but with state dynamics given by (2.13) is given by:

$$H(\mathbf{p}, \mathbf{x}, \boldsymbol{\alpha}, t) = \min_{\boldsymbol{\alpha}(t)} \left(F(\mathbf{x}, \boldsymbol{\alpha}, t) + \mathbf{p}^T \mathbf{f}(\mathbf{x}, \boldsymbol{\alpha}, t) + \frac{\sigma^2}{2} \mathbf{p}_{\mathbf{x}} \right),$$

where $\mathbf{p}_x = \nabla_x \mathbf{p}$. The Hamilton-Jacobi-Bellman equation for this stochastic control problem is then given by

$$-\frac{\partial u}{\partial t} = \min_{\boldsymbol{\alpha}(t) \in A} \big(F(\mathbf{x}, \boldsymbol{\alpha}, t) + (\nabla_{\mathbf{x}} u)^T \mathbf{f}(\mathbf{x}, \boldsymbol{\alpha}, t) \big) + \frac{\sigma^2}{2} (\Delta_{\mathbf{x}} u). \tag{2.14}$$

This is the type of Hamilton-Jacobi-Bellman equation that arises when considering mean field games.

2.3 Mean Field Game Equations

Here we describe the interpretation of the mean field game system as the limit of an N-player differential game as $N \to \infty$. In the setting introduced by Lasry-Lions [3] and Cardaliaguet [6], the N player game to which the mean field game system is related has $N \geq 1$ agents whose individual state X_t^i evolves according to

$$\mathrm{d}X_t^i = \alpha_t^i \mathrm{d}t + \sigma \mathrm{d}W_t. \tag{2.15}$$

and where each agent is aiming to minimise a cost functional of the form

$$\int_0^T L^i(X_t^i, \alpha_t^i) + F^i(X_t^1, ..., X_t^N)dt + G^i(X_t^1, ..., X_t^N), \tag{2.16}$$

It is shown by a verification theorem in Lasry-Lions [3] that if players are homogeneous, then as $N \to \infty$ the Nash equilibrium to the game described above corresponds to the solution to a coupled set of partial differential equations comprising a Hamilton-Jacobi-Bellman equation and a Fokker-Planck equation.

This means that the solution (here meaning the Nash Equilibrium) to a game with a large number of homogeneous players where each player has a state that evolves according to:

$$dx_t = \alpha(x,t)dt + \sigma dW_t \tag{2.17}$$

and has to maximise a payoff given by

$$\int_0^T L(x, \alpha(x,t)) + F(x, m(x,t))dt + G(x, m(x,t)), \tag{2.18}$$

can be approximated by the solution of the mean field game system

$$\frac{\partial u}{\partial t} + \frac{\sigma^2}{2}\Delta u + H(x, \nabla u) = V(x,m) \tag{2.19}$$

$$\frac{\partial m}{\partial t} - \frac{\sigma^2}{2}\Delta m + \nabla \cdot (m\ H_p(x, \nabla u)) = 0. \tag{2.20}$$

$$u(x,T) = G(x(T), m(x,T)),\ m(x,0) = m_0(x), \tag{2.21}$$

where $\nabla = \nabla_x$ and $\Delta = \Delta_x$. So instead of solving a stochastic game with a large number of equations, we instead have to solve a set of two coupled partial differential equations.

Here, $m(x,t)$ is now the distribution of agent's states over x, which must be a valid probability density function. Note that an initial condition is given for m, meaning the initial distribution of the agent's states must be known and a terminal condition is given for the value function, u, which is a terminal cost depending for an agent depending on their state and the distribution of agent's states. The boundary conditions associated with the system above will depend on the conditions imposed on the state x; in this thesis we will normally use Nuemman boundary conditions.

The existence and uniqueness of solutions to the mean field game system described under various regularity conditions and for certain forms of payoff functionals was proven in Lasry-Lions [3] and Cardaliaguet [6].

2.4 Applications of Mean Field Game Models

Mean Field Games have been used in a variety of applications including crowd dynamics, [7], economics [8] [9] [10]. The applications of emissions permit trading [11] and exhaustible resource production [12] are those which are closest to the problem considered in this thesis. The problem of emissions permit trading [11] is similar to fish stock exploitation in that it relates to the Tragedy of the Commons (see Chapter 4); there is an externality associated with pollution but an individual can gain more by polluting, and if they do not themselves pollute, they will lose out anyway if the rest of the population pollutes.

This scenario lacks an equation for some quantity that is common to all agents (as emissions are produced and expelled, rather than being harvested from a common pool), but the form of costs and SDEs used may be informative.

The example of exhaustible resource production is very similar to fish stock exploitation, particularly in the form of functions used since it concerns agents optimising profit by producing and selling some resource. To provide an example of an application of mean field games from the literature that will be informative to the problem we are considering, we will present here the modelling assumptions in detail and show how a problem of many agents with individual optimisation problems leads to the mean field game system.

2.4.1 Application of Mean Field Games in the Oil Industry

2.4.1.1 Introduction

Here we present a summary of a mean field game approach applied to the example of production of an exhaustible resource, which we will refer to as "oil". This is one of the example models presented in Guéant, Lasry and Lions' "Mean Field Games and Applications" [12], and a more thorough review of several contributions on the subject can be found in "A mean field game approach to oil production" [13]. These texts also show that the purely deterministic model is a special case that can be solved by more classical tools.

2.4.1.2 Oil Production Model

First, we assume that there are a large enough number of producers that we can treat the population as a continuum of agents. In the case of oil production, these producers can be thought of as individual oil wells or, if interested in a larger scale, perhaps oil companies [12]. Each producer has a reserve of oil, $R(t)$, with initial value R_0 at time $t = 0$. Let the distribution of reserves among all producers at time t be given by $m(t, R)$, so that the initial reserves among producers is given by $m(0, R)$. For example, if all reserves were initially equal, $m(0, R)$ would be the uniform distribution.

For any individual agent, their reserves will evolve according to the stochastic differential equation:

$$\mathrm{d}R(t) = -q(t)\mathrm{d}t + \nu R(t)\mathrm{d}W_t. \tag{2.22}$$

Here $q(t)$ is the agent's production of oil, and the stochastic term may represent changes in the estimation of remaining reserves or complications in extracting oil or finding new reserves [13]. In the deterministic case, $\nu = 0$. Note that $q(t)$, $R(t)$ and the Brownian motion are specific to each individual agent, though ν (the intensity to which reserves are affected by noise) is considered the same for all agents.

Each agent controls its production $q(t)$ subject to some cost function $C(q(t))$, which in the general oil production example is assumed to be a quadratic:

$$C(q(t)) = \alpha q + \beta \frac{q^2}{2}. \tag{2.23}$$

We assume that each individual producer is small enough that they are "price-takers"; that is, they must accept the market price of oil and are not large enough to alter the price single-handedly (however, the price of oil will depend on the collective oil production of all agents). The prices are determined according to a supply-demand equilibrium, where the supply is the total production of all oil producers which must be equal to some demand function. The demand function which is used to determine the price $p(t)$ for this example is given by:

$$D(t,p) = Ve^{\rho t} p^{-\sigma}, \tag{2.24}$$

where $Ve^{\rho t}$ represents wealth subject to constant growth rate ρ, and σ is a measure of the elasticity of demand for oil.

Given their initial reserves and the cost and price dynamics above, and requiring that $q(t) \geq 0$ and $R(t) \geq 0$, each agent now has an optimisation problem of the form:

$$\max_{q(t)} \mathbb{E} \left(\int_0^\infty (p(t)q(t) - C(t)) e^{-rt} \mathrm{d}t \right). \tag{2.25}$$

Here, r is the discount rate which accounts for the time value of money. This represents the fact that some amount of money received at some future time t is worth less than the same amount of money received today, as money received now could be invested and gain some return by time t. The discount rate r should be equal to the cost of capital for the oil producers.

So each individual producer must choose their production rate $q(t)$ to try to maximise their profit given above. Their profit will depend on their own production costs but also on the actions of the rest of the oil producers; in this example, this dependence is purely through $p(t)$, the market price which depends on the total supply.

Let the optimal production at a given time (i.e. the optimal control) be given by $q^*(t, R)$ (that is the optimal production at a time t where reserves are currently $R(t)$).

Also, recall that the distribution of oil reserves across all producers at time t is given by $m(t, R)$.

Now, we define the value function $u(t, R)$ as the maximum expected profit from behaving optimally (i.e. following $q^*(t, R)$) at time t with reserves $R(t)$, i.e.

$$u(t, R) = \max_{q(\tau)} \mathbb{E} \left(\int_t^\infty (p(\tau)q(\tau) - C(\tau))e^{-r\tau} \mathrm{d}\tau \right). \tag{2.26}$$

Then applying the standard result of mean field game theory [12] [6] gives us that the problem satisfies the following Hamilton Jacobi Bellman equation:

$$\partial_t u(t, R) + \frac{\nu^2}{2} R^2 \partial_{RR}^2 u(t, R) - ru(t, R) + \max_{q \geq 0}(p(t)q - C(q) - q\partial_R u(t, R)) = 0, \tag{2.27}$$

and the following Kolmogorov Forward equation:

$$\partial_t m(t, R) + \partial_R(-q^*(t, R)m(t, R)) = \frac{\nu^2}{2} \partial_{RR}^2 (R^2 m(t, R)). \tag{2.28}$$

A method for solving these particular HJB - Kolmogorov equations is presented in [13].

However, as well as this standard HJB - Kolmogorov from mean field games, it is also shown [12] [13] that the deterministic model (that is, where $\nu = 0$) can be solved without the HJB - Kolmogorov equations, using more standard tools (and again, a numerical method is again presented in [13]). This is particularly interesting as it could suggest an approach for a more simple model for fisheries. The mean field games PDEs are required for the stochastic case, and for including externalities or other complications in the model.

2.4.2 Numerical Methods for Mean Field Games

Solving a mean field game means solving a coupled Hamilton-Jacobi-Bellman and Fokker-Planck equation, where the HJB equation is solved backward in time and the Fokker-Planck equation is solved forward in time. This presents a significant numerical challenge and there are a range of different numerical methods that have been applied in the literature.

Existing numerical methods for means field game systems include finite difference schemes presented by Achdou et al. [14–16] (including recent application in economics [17] and to multiple populations [18]), a central scheme for mean field games [19].

The finite difference method found in Achdou [14–16] is one of the more general and frequently used methods. We may start by writing the finite difference scheme used to solve dynamic mean field systems, noting any alterations necessary to account for the common resource. Existence, uniqueness and convergence results have been shown for the finite difference method. However, although it is perhaps the most general method, it requires solving the entire discretized system of PDEs at once, and this requires an initial iterate of the solution over the entire space and time that is close to the actual solution. This may not be practical in many applications.

We also have other methods such as that found in the application to oil production [20], which relies on specific features of the problem studied and is not as general as the finite difference methods. This is also the case for various other applications such as emissions trading [11], crowd dynamics [7], or for other mean field games with specific cost functions [21]. Each method is suited towards its particular application, and though they may not be suitable for application to fish stock exploitation, they may help inform our choice of numerical method (discussed in Chapter 6).

2.5 Discussion

The sections above give an overview of mean field games, including the terminology from game theory, the derivation and interpretation of the HJB equation from optimal control, and some of the existing results of analysis of the MFG equations. We also presented some existing applications, most notably oil production, that are relevant to the application we are interested in.

We are interested in adapting the mean field games approach similar to that used in the application of oil production (described above) to wild capture fisheries or similar activities related to the marine ecosystem. Oil production shares some similarities (extraction of an exhaustible natural resource, competition in the market between many agents extracting the resource, potential externalities) so it may serve as a good basis for how similar techniques could be applied to a model of a fishery. There have also been many applications of optimal control and game theory in fisheries in the literature (reviewed in the following chapter) which can be adapted to a mean field game (as opposed to a differential game with few players). However, there are also some crucial differences that will mean we require some fundamentally different assumptions in any model compared to existing oil production models, rather than addressing only changing the optimal control part of the problem to a formulation which is suited to fisheries.

Chapter 3

Background: Fisheries Modelling

In this chapter we will consider relevant background information from existing fisheries modelling, marine ecosystem modelling and bioeconomic modelling, as well as literature on fish price dynamics. As one of our aims is to adapt existing research to a mean field games model for fisheries, we will focus on the existing literature on the application of optimal control or game theoretic models to fisheries. We will also briefly review the options for ecological models, particularly the factors that are important when making the choice of ecological component for any integrated ecological-economic model. Finally we consider the existing research on the dynamics of fish prices, particularly to identify possible methods and data that we will need in our analysis.

3.1 Optimal Control and Differential Games in Fisheries

Prior to the 1950s, most fisheries research was purely biological, with little focus on economics. The seminal paper by Gordon [22] was among the first to apply economic theory to the subject of fisheries or other common resources, and introduced the concept of a bioeconomic equilibrium to fisheries research. However, at this point the analysis was purely static, with the value of the fish stock simply given by the natural equilibrium stock level with the harvest removed. This bioeconomic equilibrium argument was used to show how an open-access fishery can lead to economic inefficiencies, as the bioeconomic equilibrium is different from that of the “socially optimum” equilibrium where total economic yields are maximised.

With the development of optimal control theory [23], the economic theory of fisheries could now be applied to fisheries in dynamic terms, with a fish stock that evolves with time [24] [25]. This led to dynamic fisheries models featuring a single stock, evolving according to a deterministic ODE, with an optimal control problem faced by a single owner, nation or manager aiming to maximise the total yield from the fishery with either finite or infinite time horizon. In the basic case of an infinite time horizon problem with linear costs, it was shown that the optimal control is given by a "bang-bang" feedback control [23, 24].

More complex ODE models may be used to represent the stock dynamics in similar optimal control problems [26] (and there is an entire field of research on population modelling to draw from when deciding the form). However, two key potential changes to the treatment of stock dynamics to note are the inclusion of multiple interacting stocks, and allowing stochasticity in the stock dynamics. Considering multiple stocks extends the dimension of the optimal control problem faced by the fishery owner or manager, and introducing stochasticity leads to a problem of stochastic control.

A natural extension is to consider not a single owner with an optimal control problem, but multiple players trying to maximise their yield while harvesting from the same fishery, leading to differential games. This extension from a single player optimal control problem to a dynamic game can be done with either a single stock or multiple stocks, and with either deterministic or stochastic dynamics for the stock [27,28]. With more than one player, the game can either be treated as cooperative (where agreements that are enforced can be put in place between the players) or noncooperative [29–31]. Applications of differential games in fisheries include examining cooperation between two or more nations [32], or investigating a noncooperative fishery that can be treated as a duopoly (or oligopoly) with two or more players [33].

3.1.1 Strategic Variable and Costs in Fisheries Optimal Control

We wish to investigate the ways that costs are treated and regulations are introduced in existing models of optimal control or games applied to fisheries.

Generally speaking, optimal control or game theoretic models of fisheries use either effort as the control variable [34–36] or harvest as the control variable [37, 38]. Hannesson, 2011 [39] discusses the difference between using effort and stock level explicitly, and notes that although stock and effort may be used, game theoretic models in fisheries most commonly use stock as the strategic variable. Both are viable, and some papers use either or some note that the other could be used instead. Using a model to examine the difference between using different strategic variables, Hannesson, 2011 [39] finds that stock as the strategic variable is a more competitive variable, and that effort as strategic variable means that things may be more dependent on initial conditions than if using stock. This finding makes sense, because if stock is the strategic variable, we can reach a lower stock quicker than if effort is limited or costly to increase quickly. Models where the effort is treated as a dynamic controlled variable, in particular governed by a differential equation [36], are closest to the type of model we will use. However information can be gained about costs and potential ways to treat regulations from a wide variety of fisheries models, whether they are static or dynamic and whether they use effort or harvest as the controlled variable.

Across many models, treating cost per unit effort as a linear function (in the absence of regulation) is common. If harvest is treated as the control variable, then instead the cost of harvest should be a function that depends on the level of the stock [37]. Often quadratic costs are used instead - depending on the formulation, these may be exactly equivalent to a linear cost per unit effort (e.g. [38]).

3.1.2 Regulations in Fisheries Optimal Control

A common and easily implemented form of regulation is a tax on catch, i.e. an extra cost paid for every unit catch [34] [33]. This can be enforced across the whole fishery or on a particular refuge area. A tax on catch is equivalent to decreasing the price received per catch. It is a regulation that is fairly simple to implement, as the catch is recorded when landed (after discards). It could also be considered analogous to purchasing quota, assuming there are no unused quota (also ignoring individually traded quota that can be traded amongst agents).

Another form of regulation is a fine for exceeding a particular quota of catch [38], which may also be subject to some proportion of enforcement. This can be interpreted as the use of regulation (through fines) to keep total catch below a certain total allowable catch (TAC). If a fine for exceeding a certain quota is implemented, then the total catch per player must be known - this is simple when the controlled variable is the total harvest over a period (e.g. [38]), but requires that an additional variable be known in dynamic models where effort or harvest rate varies with time.

In some games, tradeable quotas are used as a means of regulation [33] - this is more complex and must be considered in more depth in an agent's strategy, for example as an additional control variable (level of quota held) for each agent. A fine for exceeding quota as described above could also be combined with this approach, as otherwise it is assumed that quotas are never exceeded.

Another form of regulation is the creation of marine protected areas where fishing is restricted. This can be implemented in an optimal control model by considering the population inside protected areas and the population outside protected areas separately, with migration between the two, and allowing fishing only on the population outside protected areas [34, 35].

Different regulation methods may be implemented separately or in combination, and it may be more effective to achieve management goals through a combination of regulation methods rather than a single one, particularly if there is uncertainty [40].

3.2 Marine Ecosystem Models

Many models exist which simulate marine ecosystems. Challenges in ecosystem modelling are not the focus of this thesis, as there are already a wide range of existing ecological models which fulfil different purposes, and have very different data requirements and purposes, and there is a wide field of active research on improving ecosystem models [41]. In the course of this research, I contributed to a paper arising from a workshop on ecosystem modelling which discussed the range of different marine ecosystem models in use in the UK, how these models could be used to address certain questions, and addressed the issues surrounding using models to inform policy decisions. This paper can be viewed in Appendix B of this thesis.

One key category of ecosystem models is surplus production models, or what may be called dynamic biomass production or stock production models. A surplus production model [42] in general is one where the population state and fishing activity are represented by single variables, and the change in population state per unit time depends on the population and fishing activity. The surplus production model may use the number of individuals in the population for its variable, but it is usually biomass, the change in which is given by an ordinary differential equation. Typical surplus production models include exponential growth, the logistic growth or some generalised logistic. The Schaefer Model is among the most notable of these models, which uses the logistic population growth model as the basis of its ODE.

There are also age based models, which are detailed population models usually for a single species, which can achieve quite accurate and realistic modelling, and provide information on potential size and quality of fish from a fisheries perspective. There are also size structured (within species) which may be considered quite similar to age-based models, and size spectrum models which may group different species by size. More detailed data is usually required for age based than for biomass models, and they often require estimates of biological parameters that may be very hard to estimate in practice. This means that in many scenarios where extensive additional data collection is difficult, management parameters may be better informed by biomass models due to their lower data requirements and versatility.

Another factor in different ecosystem models is the degree to which they simulate dynamics between different populations, say via a food web, or are focused more on single or distinct populations. Any ecosystem model will account for the dynamics between different parts of the ecosystem in some way, but for some it is more likely to be in terms of parameters in an equation representing the impact on the population in question, where as others may have a more detailed and complex formulation of the interaction between different ecosystem features.

3.2.1 Bioeconomic Modelling

We consider a bioeconomic model to be one which simulates the dynamics of a natural resource, and the changes in net economic yields that result from its exploitation. We are particularly concerned that the system should include feedbacks between the economic yield and the rate of exploitation of the resource. A wide range of bioeconomic models applicable to fisheries and marine systems are available, and there have been several reviews of fisheries bioecomomic models (e.g. see Prellezo et al. 2009, 2012 [1] [43], Foley et al. 2012 [44]). There are a variety of different methods used both for the economic and ecological components, and bioeconomic models aim to fulfil different functions. In particular many models are suited to regions, fishing fleets, fish species or even specific policy or management problems. This may mean that existing bioeconomic models could be difficult to apply to scenarios outside of the one they were developed for. This can be due to significant underlying differences in the fish stock dynamics or fishing fleet composition, unique management challenges, or due to very different data requirements and availability.

Models can also be quite different in their aims and approach, such as whether they are driven by input (fishing effort) or output (yield), or whether they are simulation models (aiming to simulate the results of a particular scenario) or optimization (aiming to obtain the best result for a given function). The choices in model design are often based on the particular priorities of the fishery or region being modelled.

There is always a trade-off between scope and complexity in modelling, and this is particularly apparent in bioeconomic models due the intricacy and complexity of the systems and the very different approaches that are required in modelling ecological and economic systems. Since it would be very difficult to model both ecological and economic components with great precision, different models will prioritise the other to different extents (some may lack a biological component entirely). The choice may depend on data availability or on the particular goal in mind.

Surplus Production Models, as described above, provide one of the simplest ecological components to use in a bioeconomic models. These models relate the stock size to the yield, and so can be used for calculation of the maximum sustainable yield.

3.3 Fish Prices

The ex-vessel price (or first-hand market price) of fish is one of the key factors affecting a fishery's economic return, since the revenue of the fishery depends on this price and the quantity landed to market. When modelling economic returns from a fishery, for example to assess the impact of management decisions [45] or environmental pressures, or to compare the effectiveness of different fishing strategies [46], the assumptions made about the ex-vessel price can be crucial in determining the conclusions (e.g. Lorentzen and Hannesson, 2005 [47]).

Past investigations of fisheries economics often assumed that prices remain constant, i.e. that the fishery is small enough that its dynamics have a negligible effect on price [48]. A constant price is still assumed in many analyses, but increasingly, models assume a dynamic price that is usually dependent in some way on the quantity supplied to the market [1, 43]. Dynamic pricing can imply that, for example, earnings due to reduced catch may be compensated to some degree by an increase in the ex-vessel price. However, the relationship between price and quantity landed for a particular fishery is not always obvious, as the ex-vessel price is affected by a variety of factors and will depend on the nature of the fishery or market under consideration. Also, the heterogeneity of fish markets means that studies on fish prices in one market may not necessarily be applicable in another (Asche 2005). Hence the relationship between ex-vessel price, quantity landed, and other factors takes various forms across different bioeconomic models that utilise a dynamic (rather than constant) price [1].

The assumption in dynamic pricing models of fisheries is generally that price is the dependent variable and that an inverse price-quantity relationship applies at the levels of each species, such that the price obtained decreases as the quantities landed increase. This differs from conventional demand systems where quantity is the dependent variable. Conventional demand systems are more commonly seen in manufacturing applications, whilst inverse demand systems have been more frequent in the context of agriculture [49, 50] and fisheries [51, 52]. In the case of fisheries, the assumption of total supply being exogenous can be justified by the fact that landings of many key species are limited by the total allowable catch (TAC) or by the fish stock available.

Empirical studies in UK fish markets have also found evidence that price is determined by quantity landed rather than the other way round [51, 53]. However the assumption of total quantity supplied being exogenous is only realistic within a certain range of prices (and assuming costs remain within a certain range) such that it is commercially viable to land the species.

However, several studies have found evidence of species fungibility, i.e. market integration across substitute fish species [54,55], and across international markets, such as in the European whitefish market [56, 57]. Market integration implies that inverse price-quantity relationships for a single species are unlikely to be strong if the landings are small compared to the overall national or regional supply.

In addition to the fungibility issue, it is conceivable that positive price-quantity relationships may exist for low-value by-catch species where these are prone to being discarded at sea rather than being landed when the market price is low. Changes in the EU Common Fisheries Policy which came into force in January 2016 to limit the practice of discarding unwanted or low values components of catches may alter these relationships

Timescales are also fundamental to the relationship between landed quantities and ex-vessel prices of fish. Some markets show evidence of seasonal trend in price [58, 59] – usually because the landings themselves vary with season, but also due to seasonality in consumer preferences due to, for example holiday traditions [60]. Price will also vary at individual fish-markets or auctions depending on day to day changes in landings due to weather conditions or weekly working patterns [61], even if there is little effect on the average monthly or annual price.

The globalisation of the seafood market has a major effect on the fish price in many markets [52], and overseas trade is likely one of the major factors affecting UK ex-vessel price; most seafood consumed in the UK is imported and the UK is a net importer of fish (UK MMO Statistics [62, 63]). Both imports into the UK and exports out the UK have increased dramatically over the past 50 years. The ex-vessel price for fish landed in the UK will depend on the demand for processed fish by consumers, and this demand will depend greatly on the availability and cost of imports (even if the species imported are not the same as those landed in the UK). As well as changes in global supply (from wild capture or farmed fish), changing consumption patterns around the world [64] may affect the cost of imports and the demand for exports from the UK. Globalisation of the market also means the price may be affected more by pressures from the overall global economy and other markets, e.g. exchange rates [65].

Increases in aquaculture production worldwide since the 1980s [66] have greatly increased global supply and represent competition for wild capture products. However, they may also influence the price of wild fish via environmental factors, competition for space, and increasing demand for wild species that are used for fishmeal [67–69]. In addition to the rise in global aquaculture production, there has been an explosion of aquaculture production in the UK, particularly over the past 20 years [70, 71] which offers a competing source of seafood in the UK.

3.4 Discussion

Our aim is to produce a model that includes dynamic feedback between an ecological model and the economic system that interacts with it. We also wish to formulate a model with potential to be fairly general with potential to be adapted to fit various fish species and to be useful for answering a number of different questions, including informing management decisions, testing sensitivity to economic parameters, and changes in fish stock dynamics.

As discussed in the previous chapter, our aim is to use mean field games to produce a model that better allows the dynamics of the integrated social-ecological marine system to be investigated and analysed as a whole. One should aim to pick a model simple enough to form one module in the integrated model, but which captures enough of the important dynamics to be considered usable.

Modelling the interaction between different components within an ecosystem model often requires a somewhat wider scope and less detail in a single component or equation, either because the data requirements become more onerous or because it is harder to functionally link the different components. In this thesis we will primarily be using ODE-based dynamic biomass production / stock production models - on their own these are usually fairly simple and have less detailed and specific data requirements, which allow them to be easily extended to allow modelling of end to end ecosystems that can capture interactions between different parts of the ecosystem. This also means that ODE-based dynamic biomass models are suitable candidates to model the interaction between ecological and economic parts via simulation of fishing activity changing in response to the changing stock biomass and the interaction between price and harvest/landings.

For the economic dynamics of the fishery, we will use the existing mathematical field of mean field games (as described in Chapter 2) and apply those techniques to model fish stock exploitation. This means that our approach has quite a wide scope (as we must consider a fishery large enough and with enough strategically interacting agents to allow us to approximate by considering a continuum of agents). This has the advantage of allowing us to answer questions about feedback between different parts of the connected ecological and economic system that more granular models may not be able to capture, but it does mean that we should aim to reduce complexity in other parts of the model.

Using a game theory based model provides a lot of flexibility in terms of what questions can be addressed with the model. This is partly because game theory considers agents trying to optimise some payoff which can be defined differently for different problems, and so allows the consideration of agents who are trying to do something other than, say, maximise total profit. But it is largely because game theory involves the strategic interaction of agents who each have their own optimization problem, which means that our model is in a sense a simulation and optimization model, as the optimization of individual fishermen is handled by the mean field game model, which can be used to obtain a simulation that is based on the optimization of fishermen. Alternatively, the aim could be to optimise some other combination of variables in the model while the model itself already incorporates the individual aim to maximise profit - this is quite in line with the reality of governing bodies trying to enact management decisions to achieve certain desired outcomes while fishing vessels and firms try to achieve maximum profit.

To use mean field games to model the economic component of a fishery, we will consider existing optimal control and game theoretic models in the field, as the formulation of a mean field game model is similar to optimal control problems, so the model assumptions and some of the issues raised in optimal control will be useful for the mean field game formulation.

Chapter 4

A Mean Field Game Model for Fisheries

4.1 Introduction

Our aim is to develop a mean field game (MFG) formulation of fish stock exploitation. Game theory has been frequently applied to fisheries in the past, with a variety of aims and different methods used [30, 31, 39]. However, a mean field game approach has yet to be applied in an analysis of fish stock exploitation. This is despite the fact that two of the key properties required for the mean field assumption (a large number of agents and interchangeability of agents) are present in many fisheries.

Problems that have been tackled in the mean field games literature include production of an exhaustible resource (such as oil) [12] and emissions permit trading [11]. These applications share some properties with fish stock exploitation and we can take inspiration from them when formulating a fisheries model. However, they lack the renewable common resource present in the case of fish stock exploitation, and this difference is too fundamental to allow a straightforward application of the same techniques.

In the case of fishing, the interaction between agents occurs through their collective impact on the fish stock and landings. This situation of a common renewable resource being exploited by a large number of agents is the basis for the idea of the Tragedy of the

Commons [72] – and indeed, fishing is among the most frequently discussed examples of where the Tragedy of the Commons may occur. The Tragedy of the Commons has been examined extensively from a game theory perspective in the past, however mean field games have not been used despite the fact that the scenario features a large number of interchangeable players who are subject to the effect of the statistical mass of all other players' actions. So applying a mean field approach to the case of fish stock exploitation means extending mean field games to a typical Tragedy of the Commons scenario with a common resource.

Thus, although mean field games have been used for somewhat similar applications, we need to make some adaptations to account for the presence of a common resource, which will introduce another variable and lead to a different form of coupling in the mean field game equations, but it may also lead to some possible simplifications.

In this chapter, we consider the problem of applying mean field game to fisheries first in terms of how the presence of a common resource must be handled and the differences between this and previous similar applications described above. Then, we formulate the model in terms of a fishery informed by previous work in game theory and optimal control, as reviewed in Chapter 3.

4.2 Mean Field Games With A Common Resource

Problems such as the exploitation of an exhaustible resource [12] which was presented in detail in Chapter 2, or other related problems like emissions permit trading [11] lack the renewable common resource present in the case of fish stock exploitation. Thus, although mean field games have been used for somewhat similar applications, we need to make some adaptations to account for the presence of a common resource. This will introduce another variable and lead to a different form of coupling in the mean field game equations, but it may also lead to some possible simplifications.

Since the example of oil production has some strong similarities to fish stock exploitation, we may consider it as a useful example of an MFG model of agents aiming to maximise their profit through production (or extraction) of an exhaustible resource. To apply a similar technique to modelling a fishery, we need to identify which assumptions in this MFG model are appropriate and which parts we need to address differently. We aim to:

- Identify the elements of the oil production models that are similar to those required in fisheries.
- Identify the key differences between the oil production and fishery scenarios, and discuss how a new model must approach for these differences.

First we note the parts that are similar. For the scale that we are interested in (such as whole of the UK or UK North Sea fishing fleets), the representation of fishing vessels as a continuum of agents seems comparable with the representation of oil producers as a continuum. In the case of fisheries we would be considering individual vessels or larger fishing fleets.

The profit criterion to maximise is also very similar; the profit for an individual agent will be the quantity extracted (or landed, in the case of fisheries) multiplied by the current market price, less the cost of extraction (in this case the cost of fishing effort). For the cost function, it is usually assumed that cost is directly proportional to fishing effort or quadratic in fishing effort.

The market price for fish will similarly depend on the total amount supplied (or landed) - though the exact form of the function linking total supply and price will be different from the function in the case of oil production (see Chapter 5). The discount factor e^{-rt} can also be adapted readily to fisheries, though this requires knowing an appropriate discount rate to set. We may or may not want to consider the same infinite time horizon used in the general oil production example. If an infinite time horizon is used, then an appropriate discount factor is necessary. If we only want to consider a set time period, then we need to choose a terminal time T and set a terminal cost $K(T)$ (this terminal cost may be zero).

So there are similarities between the oil production and fishery scenarios; a large number of agents aim to maximise their profit by extracting some natural resource for some cost, where the price received depends on the total amount supplied by all agents. Adapting the cost and price function and selecting the appropriate discount rate (or time horizon and terminal cost) are relatively superficial changes.

However, there are two much more fundamental differences in applying a similar model to our fishery problem:

- The resource being extracted (wild fish) is not a constant reserve (subject to some noise) being depleted, but a renewable resource which evolves independently according to some population dynamics.
- All agents are extracting from the same pool of resources (the fish stock) and don't control their own reserve.

There are various other differences between the two contexts, however these are the two which require a fundamentally different model formulation.

The first of these issues is perhaps the easiest to reconcile with the general model for oil production described above. Rather than treat the resource $R(t)$ as a reserve with initial value R_0 which can only be depleted by harvesting (subject to some Brownian noise representing imperfect estimates or surprise complications / discoveries), instead let the resource be represented by $N(t)$ (to better match fisheries terminology) which, in the absence of any fishing, evolves according to some population dynamics given by, say:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = f(N, t). \tag{4.1}$$

Indeed, a similar example can be found in Halkos and Papageorgiou [37]; optimal control theory techniques are applied to a problem of a resource extractor aiming to maximise a profit function essentially the same as the one described previously through extraction of a renewable resource which evolves according to some ODE (with logistic growth being given as a particular example). This example differed from the MFG model presented above in that it considered only a single extractor with purely deterministic dynamics, and the price $p(t)$ was considered an independent function (since we are not considering the total supply from a large number of extractors). But this optimal control approach for each individual extractor could be combined with the continuum of agents and price dynamics from the MFG model.

It should be noted that while the example of oil production considered a single resource, we can extend this idea to multiple interacting resources such as different species or groups of fish. So in general our fish stock $N(t)$ is a vector of dimension n (with n being the number of distinct fish groupings considered), and the population dynamics $f(N, t)$ will include interactions between different groups. Theoretically this is not a problem, however considering more than one dimension could make a big difference in the complexity of any computations.

The other difference between fisheries and the exhaustible resource model described above is perhaps even more crucial. Each individual agent does not control its own fish stock, but takes its catch from the same stock as everyone else in the system. This changes the approach from the outset, as no longer can each agent be considered to have its own private reserve of $R(t)$ (or stock of $N(t)$) to represent its state (which is determined, whether deterministically or stochastically, through the control q). Instead, each agent could perhaps be considered to determine its extraction rate (or catch rate) through the control of "effort", with the quantity produced (or landed) now depending not just on their catch rate but on the current stock level. Additionally, an individual's profit will depend on the collective action of the other agents through their impact on the fish stock.

The example of mean field games applied to the oil industry presented by Guéant, Lasry and Lions [12] [13] provides some useful ideas for application of similar techniques to fisheries and marine population modelling. It shows an example of agents maximising profit based on extracting a natural resource at some cost and selling it in a competitive market. It also provides some numerical techniques for solving a particular set of HJB - Kolmogorov equations, and demonstrates how the deterministic case produces a simplified set of equations that can be solved without solving the MFG PDEs. As well as incorporating random noise, the stochastic case allows generalisation and the inclusion of externalities in the oil production model, but the idea of taking a simple case of a mean field game and solving it without having to solve the HJB-Kolmogorov PDEs is somewhat appealing.

Some of the ideas from the application of MFG to oil production can be taken into account. However to apply similar tools in a fisheries context requires significant change from the approach of the oil production model to deal with the two main differences we have identified, which are:

- Our resource is a dynamic, interacting population of different fish groups (each of which can be landed and sold at market).
- The fish stock is common to all fishermen in the system, rather than a reserve that each agent controls.

4.3 Mean Field Games within Fisheries

For mean field games, adding any common resource to the game is a new development, and the mean field game equations present a considerable challenge to solve numerically in themselves. For this reason, when applying mean field games to fish stock exploitation, it makes sense to begin by coupling a very simple ecological model with a mean field games system, which can be adapted or improved depending on the particular case in question. So in this thesis, we will normally focus on what is arguably the simplest usable model, the logistic growth equation, for our fisheries component, where the growth rate of a fish population N is given by the equation:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = aN\left(1 - \frac{N}{K}\right). \tag{4.2}$$

For simulations and case study (see Chapters 6, 7 and 8), we desire a simple surplus production model with logistic growth parameters based in reality. We would also want the chosen parameters to be based on a fish species for which we have some estimation about the price flexibility (see Chapter 5) or at least to be based on a fishery where price flexibility relationships of the kind examined make sense.

Another consideration is what the choice of control or strategic variable should be. From this we can take information from other game theoretic models that have been applied to fisheries before. In game theory or optimal control models of fisheries, the two most common choices of control variable are the fish stock level (i.e. how much fish stock is left behind after fishing) or the fishing effort. In this case we will use effort as the control variable rather than stock level. Effort being used as the strategic variable means it can only be changed at considerable cost and perhaps some time delay, as opposed to being able to fish at any fishing capacity. Taking effort as the strategic variable means we are being more cautious about accounting for the additional costs associated with varying fishing effort, avoiding the problems discussed in [73] of having rapid frequent transitions in effort would be difficult due to the logistics of fisheries, additional economic or social costs due to low or very variable efforts. In addition, using effort as the strategic variable for a single agent is more feasible as using the stock level after fishing assumes that all agents have precise knowledge and control over what the stock after fishing is able to be (within a degree of randomness given by the random noise in the agent's own SDE). With a large number of agents who are individually "negligible" on the whole system, it is more feasible to consider that each agent controls his own individual fishing activity, than that they try to fish until a certain stock level is reached.

4.4 Model Formulation

Let $\mathbf{N}(t) \in \mathbb{R}_+^n$ represent the common resource, in our case a fish population, which we will refer to as the **stock** at time $t \in [0, T]$, comprised of n different type of fish; thus $N_j(t)$ is the stock of fish of type j. The horizon time T is the terminal time in the agents optimal control problem, which represents the end of the planning period considered in the game.

In the absence of any fishing, the fish stock evolves according to the ODE system:

$$\frac{\mathrm{d}\mathbf{N}}{\mathrm{d}t} = \boldsymbol{f}(\mathbf{N}, t), \tag{4.3}$$

with $\mathbf{N}(0) = \mathbf{N}_0$.

Now let $\mathbf{x}(t) \in \Omega := \mathbb{R}_+^n$ be an n-dimensional state space for a continuum of interchangeable, anonymous agents, representing the agent's fishing **effort** at time t on the n-dimensional fish stock $\mathbf{N}(t)$ (so the component x_j represents effort on fish stock N_j). The fishing effort can be thought of as an "encounter rate" with the fish stock.

We make the crucial assumption that an agent's effort $\mathbf{x}(t)$ evolves according to the

SDE

$$\mathrm{d}\mathbf{x}(t) = \boldsymbol{\alpha}(t, \mathbf{x})\mathrm{d}t + \sigma \mathrm{d}\mathbf{W}_t, \tag{4.4}$$

where $\boldsymbol{\alpha}(t, \mathbf{x})$ is the agent's control (each component $\alpha_j(t)$ representing the increase or decrease in fishing effort on stock N_j), and $\mathbf{W}(t)$ is the independent Brownian noise associated with that agent. The constant σ, often referred to as the volatility, is the noise intensity (which is assumed to be constant across the population of agents). This noise represents the change in the actual "encounter rate" with the fish stock that is not controlled directly by the agent (e.g. the uncertainty in locating groups of fish, unpredictable weather, etc.); although fishing effort is normally assumed to be completely controllable, we will still refer to "effort" here even when there is a stochastic component.

Let $m(t, \mathbf{x})$ be the probability distribution of the population of agents across the state space Ω at time t (i.e. the distribution of fishing effort among the population of fishermen), and let $m(0, \mathbf{x}) = m_0(\mathbf{x})$ be an initial distribution.

Over the time period from $t = 0$ to terminal time T, each agent wants to maximise their total profit given by

$$\mathbb{E}\left(\int_0^T R(\mathbf{x}(t), \mathbf{N}(t), m(t, \mathbf{x})) - C(\mathbf{x}(t)) - S(\boldsymbol{\alpha}(t, \mathbf{x}))\ \mathrm{d}t + G(\mathbf{x}(T), m(T, \mathbf{x}), N(T))\right).$$

Here R is the agent's revenue which depends on their own effort, the actions of all other agents and the state of the fish stock. The running costs are divided into $C(\mathbf{x}(t))$, the cost per unit time associated with the current level of effort, and $S(\boldsymbol{\alpha}(t, \mathbf{x}))$ the cost associated with changing the level of effort. Finally $G(\mathbf{x}(T), m(T, \mathbf{x}), N(T))$ is the terminal cost at time T, which in general may depend on effort, overall distribution of effort and the end state of the fish stock.

In line with common assumptions in fisheries modelling, we may assume that the cost per unit effort is linear and so we have $C(\mathbf{x}(t)) = \mathbf{c} \cdot \mathbf{x}$ for some constant vector $\mathbf{c}$. For $S(\boldsymbol{\alpha}(t, \mathbf{x}))$ we will first assume a quadratic form such that $S(\boldsymbol{\alpha}(t, \mathbf{x})) = \gamma \frac{|\boldsymbol{\alpha}|^2}{2}$, for some constant γ, which represents the increased difficulty of making large changes compared to small ones (e.g. investing in more vessels or different gear). Quadratic cost of control of this form is frequently used to describe the cost of agents controlling their state in a wide variety of applications [11] [12]. Note that we have $C(\mathbf{0}) = S(\mathbf{0}) = 0$. The terminal cost G may represent consideration of future payoff beyond the horizon time by the agents, or a form of regulation on the fishery (discussed below).

The revenue for each agent can be written as

$$\sum_{j=1}^{n} Q_j(\mathbf{x}(t), \mathbf{N}(t)) N_j(t) p_j(t),$$

where $\mathbf{q}(\mathbf{x}(t), \mathbf{N}(t)) = (q_1, ..., q_n)$ is the catch rate, which is in general a function of effort and stock, and the ex-vessel price of fish is given by $\mathbf{p}(t) = (p_1, ..., p_n)$ (so each component $p_j(t)$ represents the price per unit landed of fish from stock $N_j(t)$). For simplicity we can assume that the catch rate is proportional to effort, i.e.

$$\mathbf{q}(\mathbf{x}(t)) = \boldsymbol{q}\mathbf{x}(t), \tag{4.5}$$

where the catch rates $(q_1, ..., q_n)$ are constants.

The landings of fish from stock j per unit time for each agent are given by $q_j(\mathbf{x}, t)N_j$ (we assume for now all fish caught are landed, with none being discarded). The total quantity of fish from stock j landed per unit time is hence given by:

$$L_j(t, m) = N_j(t) q_j \int_{\mathbb{R}^n} \mathbf{x}(t) m(t, \mathbf{x}) \, \mathrm{d}\mathbf{x}. \tag{4.6}$$

So the landings per unit time are given by the vector functional $\mathbf{L}(m, \mathbf{N})$.

Recall that, if there is no fishing activity, the fish stock evolves according to the ODE (4.3). When fishing effort is applied, this equation becomes

$$\frac{\mathrm{d}\mathbf{N}}{\mathrm{d}t} = \boldsymbol{f}(\mathbf{N}, t) - \mathbf{L}(m, \mathbf{N}), \tag{4.7}$$

Hence $\mathbf{N}(t)$ depends on $\mathbf{L}(m, \mathbf{N})$. A simple example is the case where $f_j(\mathbf{N}, t)$ are independent logistic growth terms, in which case we have

$$\frac{\mathrm{d}N_j}{\mathrm{d}t} = a_j N_j \left(1 - \frac{N_j}{K_j}\right) - L_j(t).$$

Finally, the ex-vessel price $\boldsymbol{p}(t)$ depends on the quantity landed i.e. for some decreasing function $\boldsymbol{D}$,

$$\mathbf{p}(t) = \mathbf{D}(\mathbf{L}(m, \mathbf{N})).$$

So if we define

$$F(\mathbf{x}(t), \mathbf{N}(t), \mathbf{L}(m, \mathbf{N})) := \sum_{j=1}^{n} \Big(q_j(\mathbf{x}(t)) N_j(t, \mathbf{L}(m, \mathbf{N})) p_j(t, \mathbf{L}(m, \mathbf{N})) \Big) - C(\mathbf{x}(t)),$$

each agent then has the optimization problem of using controls $\boldsymbol{\alpha}(t, \mathbf{x})$ aiming to maximise $J(t, \mathbf{x}, \boldsymbol{\alpha}, m, \mathbf{N})$, where

$$J(t, \mathbf{x}, \boldsymbol{\alpha}, m, \mathbf{N}) = \mathbb{E}\left(\int_0^T \underbrace{F(\mathbf{x}(t), \mathbf{N}(t), \mathbf{L}(m, \mathbf{N}))}_{\text{Cost on state and distribution}} - \underbrace{S(\boldsymbol{\alpha}(t, \mathbf{x}))}_{\text{Cost on control}} \ \mathrm{d}t + \underbrace{G(\mathbf{x}(T), m(T, \mathbf{x}), N(T))}_{\text{Terminal Cost}} \right).$$

So far we have not considered discounting in our payoff, i.e. we have considered that costs and revenue are valued equally by the agents regardless of when they occur. Multiplying the net profit by a discount factor e^{-rt}, where r is the discount rate, we have

$$J(t, \mathbf{x}, \boldsymbol{\alpha}, m, \mathbf{N}) = \mathbb{E}\left(\int_0^T \left(F(\mathbf{x}(t), \mathbf{N}(t), \mathbf{L}(m, \mathbf{N})) - S(\boldsymbol{\alpha}(t, \mathbf{x})) \right) e^{-rt} \mathrm{d}t + G(\mathbf{x}(T), m(T, \mathbf{x}), N(T)) \right).$$

This means that future profits are discounted at the rate r, which is equal to the fisherman's cost of capital. This discounting of future payoff to its present value is important for modelling time periods of more than a year or so, and particularly for very long time horizons. It is also necessary if, instead of considering a fixed horizon time T, an infinite time horizon could be considered, in which case the payoff to maximise is then:

$$J(t, \mathbf{x}, \boldsymbol{\alpha}, m, \mathbf{N}) = \mathbb{E}\left(\int_0^\infty \left(F(\mathbf{x}(t), \mathbf{N}(t), \mathbf{L}(m, \mathbf{N})) - S(\boldsymbol{\alpha}(t, \mathbf{x}))\right) e^{-rt} \mathrm{d}t\right).$$

The discount factor is required for the integral to be finite in the case with an infinite time horizon. Including a discount factor introduces an extra term in the HJB equation (see below).

4.5 Mean Field Game PDEs

The value function $u(t, \mathbf{x})$ is defined as the maximum value of J that can be obtained by behaving optimally from time t until terminal time T, that is

$$u(t, \mathbf{x}) = \max_{\boldsymbol{\alpha}(t)} J.$$

This value function then satisfies the following general Hamiltonian-Jacobi-Bellman (HJB) equation [3, 4, 21]:

$$\frac{\partial u}{\partial t} + H(\nabla u) + \frac{\sigma^2}{2}\Delta u + F(t, \mathbf{x}, \mathbf{N}, \mathbf{L}(m, \mathbf{N})) = 0$$

and the distribution $m(t, \mathbf{x})$ will evolve forward in time according to the Kolmogorov Forward (Fokker-Planck) equation:

$$\frac{\partial m}{\partial t} + \nabla \cdot (mH'(\nabla u)) - \frac{\sigma^2}{2}\Delta m = 0.$$

With terminal and initial conditions given by $u(T, \mathbf{x}) = G(m(T), \mathbf{N}(T)))$ and $m(0, \mathbf{x}) = m_0(\mathbf{x})$. Note that when a discount factor e^{-rt} is included in the payoff as described above, the HJB equation becomes [12]:

$$\frac{\partial u}{\partial t} + H(\nabla u) + \frac{\sigma^2}{2}\Delta u - ru + F(t, \mathbf{x}, \mathbf{N}, \mathbf{L}(m, \mathbf{N})) = 0$$

Here $H(\boldsymbol{p})$ is the Hamiltonian defined as $\max_{\boldsymbol{\alpha}(t)} (\boldsymbol{\alpha} \cdot \boldsymbol{p} - S(\boldsymbol{\alpha}))$. With the choice of quadratic costs $S(\boldsymbol{\alpha}(t, \mathbf{x})) = \gamma\frac{|\boldsymbol{\alpha}|^2}{2}$, we have $H(\boldsymbol{p}) = \frac{|\boldsymbol{p}|^2}{2\gamma}$ and so the HJB and Kolmogorov equations can be written as:

$$\frac{\partial u}{\partial t} + \frac{|\nabla u|^2}{2\gamma} + \frac{\sigma^2}{2}\Delta u - ru + F(t, \mathbf{x}, \mathbf{N}, \mathbf{L}(m, \mathbf{N})) = 0$$

$$\frac{\partial m}{\partial t} + \frac{1}{\gamma}\nabla \cdot (m\nabla u) - \frac{\sigma^2}{2}\Delta m = 0$$

For boundary conditions on u and m, a reasonable choice would be to select Neumann conditions on $\partial\Omega$ that impose zero flux on the density of agents at the boundary [18, 74, 75]. In general this takes the form

$$\nabla u \cdot \mathbf{n} = 0,$$

$$mH'(\nabla u) \cdot \mathbf{n} - \frac{\sigma^2}{2}\nabla m \cdot \mathbf{n} = 0,$$

where $\mathbf{n}$ is the outward unit normal vector on $\partial\Omega$. With quadratic Hamiltonian we have

$$\nabla u \cdot \mathbf{n} = 0,$$

$$\frac{1}{\gamma}m\nabla u \cdot \mathbf{n} - \frac{\sigma^2}{2}\nabla m \cdot \mathbf{n} = 0,$$

$$\implies \nabla m \cdot \mathbf{n} = 0$$

So we have homogeneous Neumann conditions for u and m on $\partial\Omega$. We also require that m goes to zero as $|\boldsymbol{x}| \to \infty$, with a corresponding condition on u. This is similar to the choice made in [7, 9, 75, 76].

These HJB and Kolmogorov PDEs for u and m and the ODE for the evolution of the fish stock give us the following coupled system:

$$\frac{\partial u}{\partial t} + \frac{|\nabla u|^2}{2\gamma} + \frac{\sigma^2}{2}\Delta u - ru + F(t, \mathbf{x}, \mathbf{N}, \mathbf{L}(m, \mathbf{N})) = 0, \tag{4.8}$$

$$\frac{\partial m}{\partial t} + \frac{1}{\gamma}\nabla \cdot (m\nabla u) - \frac{\sigma^2}{2}\Delta m = 0, \tag{4.9}$$

$$\frac{\mathrm{d}\mathbf{N}}{\mathrm{d}t} = f(\mathbf{N}, t) - \mathbf{L}(m, \mathbf{N}), \tag{4.10}$$

$$\nabla u \cdot \mathbf{n} = 0; \quad \nabla m \cdot \mathbf{n} = 0 \quad \text{on } \partial\Omega. \tag{4.11}$$

$$u(T, \mathbf{x}) = G(\mathbf{x}(T), m(T), \mathbf{N}(T)); \qquad m(0, \mathbf{x}) = m_0(\mathbf{x}); \qquad \mathbf{N}(0) = \mathbf{N}_0. \tag{4.12}$$

For the purposes of our numerical solutions in subsequent chapters, we will consider the case of a single fish stock (so variables are in one dimension), where $x \in [0, x_{max}]$, and as discussed earlier in this chapter we will consider using a simple logistic model for the resource evolution equation. So in one dimension and with logistic growth, the full system is given by

$$\frac{\partial u}{\partial t} + \frac{(u_x)^2}{2\gamma} + \frac{\sigma^2}{2}u_{xx} - ru + F(x, N, L(N, m)) = 0, \tag{4.13}$$

$$\frac{\partial m}{\partial t} + \frac{1}{\gamma}(mu_x)_x - \frac{\sigma^2}{2}m_{xx} = 0, \tag{4.14}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = aN\left(1 - \frac{N}{K}\right) - L(N, m). \tag{4.15}$$

$$u_x(t, 0) = u_x(t, x_{max}) = m_x(t, 0) = m_x(t, x_{max}) = 0. \tag{4.16}$$

$$u(T, x) = G(x(T), m(T), N(T)); \qquad m(0, x) = m_0(x); \qquad N(0) = N_0. \tag{4.17}$$

4.6 Steady state solution to MFG system

Although in this thesis we will proceed to find solutions of the dynamic problem with finite time horizon, we may also wish to consider the steady state solution of the system. In particular this may be useful if there are cases where we can solve the stationary system numerically more easily than the dynamic problem.

At the steady state solution the time derivatives of u, m and N are zero, and the system described reduces to

$$\frac{(u_x)^2}{2\gamma} + \frac{\sigma^2}{2} u_{xx} - ru + F(x, N, L(N, m)) = 0, \tag{4.18}$$

$$\frac{1}{\gamma}(mu_x)_x - \frac{\sigma^2}{2} m_{xx} = 0, \tag{4.19}$$

$$aN\left(1 - \frac{N}{K}\right) - L(N, m) = 0. \tag{4.20}$$

$$u_x(0) = u_x(x_{max}) = m_x(0) = m_x(x_{max}) = 0. \tag{4.21}$$

We also require that $m(x)$ is a valid probability density function, so

$$\int_{\mathbb{R}} m(x)\, \mathrm{d}x = 1.$$

In this steady state, we may be able to write m as a function of u. From equation 4.19, we have

$$\left(\frac{1}{\gamma} mu_x - \frac{\sigma^2}{2} m_x\right)_x = 0$$

$$\Leftrightarrow \frac{1}{\gamma} mu_x - \frac{\sigma^2}{2} m_x = A,$$

for some constant A. Noting that from the boundary conditions (4.21) we have that $m_x(0) = u_x(0) = 0$, we obtain $A = 0$. Writing $B = -\dfrac{2}{\gamma\sigma^2}$, we have

$$m_x + Bu_x\ m = 0.$$

Solving this first order linear ODE we obtain

$$(m\ e^{Bu})_x = 0 \Leftrightarrow m\ e^{Bu} = C$$

$$\Leftrightarrow m = Ce^{-Bu}$$

for some constant C. Since we require $\displaystyle\int_0^{x_{max}} m(x)\, \mathrm{d}x = 1$ for $m(x)$ to be a valid probability density distribution, we obtain

$$m = \frac{e^{-Bu}}{\displaystyle\int_0^{x_{max}} e^{-Bu}\,\mathrm{d}x}.$$

This means that the solution $m(x)$ can be obtained as a function of u, which in terms of the model parameters is given by:

$$m = \frac{e^{\frac{2}{\gamma\sigma^2}u}}{\displaystyle\int_0^{x_{max}} e^{\frac{2}{\gamma\sigma^2}u}\,\mathrm{d}x}.$$

Given $m(x)$, we can obtain the landings $L(N,m) = Nq\displaystyle\int_0^{x_{max}} xm(x)\,\mathrm{d}x$, and hence solve equation (4.20) for N, meaning that m and N can be obtained as functions of u. This leaves the single equation (4.18) for u, which, noting that m and N can be written as functions of u, and writing $F(x,N,L(N,m)) = -f(x,u)$, can now be written as

$$\frac{(u_x)^2}{2\gamma} + \frac{\sigma^2}{2}u_{xx} - ru = f(x,u) \tag{4.22}$$

Solving this single equation in u numerically would present an alternative approach for finding the steady state solution to the system instead of solving the dynamic system.

4.7 Model Formulation Discussion

In this chapter we have formulated a mean field game model for a fishery, adapting the approach of mean field games seen in the literature to the case where we have a common exploitable resource, and drawing from similar game theoretic and optimal control models in fisheries to determine the functions and parameters to include in the model. Given the above formulation, the first thing we note is that the presence of the common resource adds another key variable N and another coupled equation to the mean field game system; the resource equation, for which we are using an ODE. This introduces a new element to the general mean field game framework, but there may also be some simplification gained by the fact that the eventual impact of the distribution of agents m is always described through the landings L.

In our model formulation, we have used a discount factor e^{rt} in our cost function, to discount future payoffs with the discount rate r. Here we have assumed that the discount rate r is equal to the agent's cost of capital, which makes sense when considering the payoff purely as financial profit. However, there is some debate over how discount rates should best be applied when considering the environment and natural resources, particularly as assumptions about environmental benefits being worth less in the future are more questionable [77, 78]. Since in our formulation, the discount rate is applied to the payoff of an individual agent from the perspective of that agent trying to optimise their own net profit, then applying a discount factor as is standard for economic modelling seems appropriate. However, if considering a more complex payoff function that includes other factors than financial gain, then careful consideration of what discount factor is appropriate would be in order.

Another aspect worth discussing is precisely what the solution to the mean field game represents. We have obtained a coupled set of partial and ordinary differential equations for the value function u, the distribution m, and the stock N. The solution to this system corresponds to the Nash Equilibrium of the mean field game with infinitely many players, which serves as an approximation to a game with a large number of players. Assuming we have a method to solve the PDEs numerically, we will have obtained an approximation for the Nash equilibrium of a fishery with a large number of interchangeable agents - however, although the Nash equilibrium is the most commonly used solution concept within game theory, it is not guaranteed that a real situation will always move towards the Nash equilibrium in practice.

It is also worth noting the difference between the Nash equilibrium and the steady state solution (which can be referred to as an equilibrium) to the mean field game system. Although for a system of PDEs, an equilibrium would usually refer only to the steady state solution, in the context of mean field games, any solution of our system represents a Nash equilibrium to the mean field game, meaning that agents cannot unilaterally alter their strategy (meaning their control $\alpha(t, x)$ throughout the entire time period) and improve their payoff. So although $u(t, x)$, $m(t, x)$ and $N(t)$ are the solution to the PDES, the idea that they describe "the solution" to the game is based on our assumption that the Nash equilibrium is the relevant solution concept for the game. If however some agents behave differently than would be assumed (e.g. they are not rational), the game may not reach or remain at its Nash equilibrium.

So there is some question of the stability of the solution, not only in terms of the stability of the solution to the differential equations (i.e. is there a stable steady state solution), but the stability of the Nash equilibrium for the game itself - it is possible that if we move away from the Nash equilibrium, then the behaviour of the system would be quite different and so the functions $u(t, x)$, $m(t, x)$ and $N(t)$ would evolve differently. This is something we should keep in mind when interpreting our solutions to the mean field game in later chapters. However, the fact that the Nash equilibrium may not occur exactly in reality is true for any game theory model, but we may still draw conclusions from the solution to the game based on the Nash equilibrium even if it is an idealised solution, since in the long term it is likely that rational players would perform better than irrational ones in a large game with significant financial incentives.

For obtaining solutions to our system, firstly we considered ways to obtain the steady state solutions of this system. We will be interested in the dynamic problems (considered in chapters below), but in particular, it would be interesting to determine whether a nonzero steady state value of N exists (in the absence of any regulation) and identify the necessary conditions. In this chapter we have described how, at the steady state, m may be written as a function of u, resulting in a single nonlinear differential equation to solve for u to obtain the steady state solution. We have also presented an approach based on fixing the value of the stock N in Appendix A.1 then iteratively solving the other equations numerically to obtain the next iterate of N.

These approaches could be developed further to obtain a numerical solution for the steady state. However, since one of the key focuses of this thesis was on developing models that could capture dynamic feedback between ecosystem and economic components, and since a numerical method capable of solving the finite time horizon problem could also be used to locate steady state solutions, we will proceed with obtaining a numerical algorithm for solving the dynamic finite time horizon problem.

Given that the formulation of the model is essentially similar to a situation that has been argued to lead to the Tragedy of the Commons [72], it is possible that there may be no steady state with nonzero stock. If so, this would provide a case of the mean field approach producing the Tragedy of the Commons from a continuum of agents exploiting the common resource. If the tragedy of the commons can be represented using a mean field game, then it should allow us to examine what regulations are necessary to impose in order to allow a nonzero steady state solution to exist.

We wish to use this mean field game model model to investigate the problem of a dynamic fishery with common resource, and so in the following chapters we will analyse different scenarios of the finite time horizon problem using numerical simulations. We may also wish to find nonzero stationary solutions, or demonstrate that nonzero stationary solutions may not exist by running long time or repeated numerical simulations. We then wish to investigate ways to effectively incorporate different types of possible regulation into the model, such as in the form of taxes, effort control, total allowable catch quotas or marine protected areas. These regulations and their impact on the solution will be considered in a later chapter, using a case study of a fishery based on North Sea Cod.

Chapter 5

Fish Price Dynamics

5.1 Introduction

The ex-vessel price of fish is a key factor affecting a fishery's economic return. Many fisheries economics studies assume that this price is constant, i.e. the fishery is small and has a negligible effect on price. The main alternative to this is the assumption that negative price flexibilities exist so that price is negatively related to the quantity landed. However, the relationship between price and quantity landed for a particular fishery is not always obvious, and studies in one market may not necessarily be applicable in another. There is also the matter of other factors that may affect the price, and taking these into account when estimating price flexibilities. In this chapter we analyse annual average UK ex-vessel prices for various individual fish species and their annual UK landings, investigating whether there is evidence of non-zero price flexibilities and estimating them for the UK market. From a preliminary analysis, we see evidence of negative own-price flexibilities for several species, but find that a high proportion of variability remains unexplained considering own-price flexibility alone. We extend our analysis to include the landings of likely substitute species to identify evidence of cross-price flexibilities, and account for other factors that may affect the price by considering other market variables such as UK seafood imports and exports, UK population and disposable income, and food supply and price of meat substitutes. Doing so we find evidence of negative own-price flexibilities for more species, and are better explain the variation in price. In addition, we similarly analysed the average price and total landings of more coarsely defined guilds of fish species, and find a negative price flexibility between the landings of aggregated guilds of demersal finfish and shellfish species, and the average price of these aggregated guilds.

Here we consider annual fishery landings into the UK and the annual average UK ex-vessel price for a wide range of different species. Hence the market is relatively large and landings of substitute species can be incorporated in the analysis. We address this by investigating how the price of each individual species is affected by the landings of their substitutes (as well as their own landings), or by determining if a relationship exists between the average price and total landings of more coarsely defined guilds of fish and shellfish, rather than individual species. We divide the finfish into demersal and pelagic guilds, and the shellfish (molluscs and crustaceans) into filter/deposit feeding, and carnivore/scavenge feeding guilds. Demersal finfish are those that live mainly close to the seabed and feed on benthic (seabed-living) invertebrates and other fish. We also further separate the demersal fish into quota demersal fish (which have a Total Allowable Catch quota in the UK) and non-quota demersal fish which do not have a TAC quota in the UK. Pelagic finfish are those that live mainly up in the water column and feed on plankton. The guild-level approach is especially important in relation to efforts to integrate economic and ecological models where resource dynamics are often defined in terms of interacting guilds rather than individual species [79]. A guild-level approach to analyzing ex-vessel prices may also provide information on broader relationships that may not be revealed by a focus on individual species.

5.2 Data and Methods

5.2.1 Sources of Data

Price data were obtained from annual UK Sea Fisheries Statistics Reports currently produced by the Marine Management Organisation (MMO) [62] for a variety of demersal, pelagic and shellfish species, reflecting average first-hand price for each species over the year. Annual average first-sale prices (measured in GBP$\times$(tonne fresh weight)$^{-1}$)

of fish species landed by UK and foreign vessels into UK ports were extracted from published annual summaries for the years 1965-2014. Reporting formats and units have varied over this period. For the main commercially important species, UK annual average prices have been consistently published every year. However, prices for some other species have been reported more sporadically. In particular, the reporting format since 2000 has focussed on a more restricted range of species than in the past. Luckily, since 2000 at least, the price data for these missing species can be recovered from separately listed data on the annual quantities landed and the total sale value, either integrated over the UK or separately for the major ports. For a few of the species with significant landings there were very limited or no data on first sale values (e.g. blue ling, mullet).

Annual average first sale prices were indexed to a standard year (2000) by reference to the UK annual average Consumer Price Index (CPI) compiled by the UK Office of National Statistics. The CPI is a compound summary statistic based on the prices of a basket of key indicator commodities, designed to track the underlying price inflation of essential raw materials and products over time. Index corrected fish prices (Vs,y) were derived as:

$$V_{s,y} = v_{s,y} \left(\frac{CPI_{y=2000}}{CPI_y} \right),$$

where $v_{s,y}$ was the uncorrected annual average price of species s in year y.

Landings data were obtained from ICES FAO database, which breaks down landings by species, country and region. All landings into the UK from every region are considered, with necessary data on price and landings available for 23 demersal species, 3 pelagic species and 6 shellfish species extending back to 1965. In addition to individual species (or small groups of similar species treated the same in landings and price data), we also consider the total landings of each of four broad guilds of species (demersal finfish, pelagic finfish, shellfish carnivores and shellfish filter feeders), as well as the weighted average ex-vessel price for each of these guilds. Table 5.1 lists the groups of species along with the guild they belong to, together with an abbreviation that may be used to identify the species in plots or equations.

Data on imports and exports were also obtained from the MMO UK Sea Fisheries

Species Group	Scientific Name (Main Species)	Identifier	Guild
Atlantic cod	Gadus morhua	COD	Demersal
Haddock	Melanogrammus aeglefinus	HAD	Demersal
Whiting	Merlangius merlangus	WHI	Demersal
Saithe	Pollachius virens	SAI	Demersal
European plaice	Pleuronectes platessa	PLA	Demersal
Common sole	Solea solea	DSO	Demersal
Anglerfish	Lophidae	ANG	Demersal
Pollock	Pollachius pollachius	POL	Demersal
European hake	Merluccius merluccius	HAK	Demersal
Megrim	Lepidorhombus whiffiagonis	MEG	Demersal
Ling and blue ling	Molva molva	LIN	Demersal
Wolffish	Anarhichadidae	WOL	Demersal
Lemon sole	Microstomus kitt	LSO	Demersal
Atlantic halibut	Hippoglossus hippoglossus	HAL	Demersal
Flounder	Platichtys flesus	FLO	Demersal
Turbot	Psetta maxima	TUR	Demersal
Brill	Scophthalmus rhombus	BRI	Demersal
Witch flounder	Glyptocephalus cynoglossus	WIT	Demersal
Common dab	Limanda limanda	CDA	Demersal
Tusk	Brosme brosme	TUS	Demersal
Spurdog and dogfish	Squalus acanthis	SPU	Demersal
Gurnards	Triglidae	GUR	Demersal
Skates and rays	Rajidae	RAY	Demersal
Herring	Clupea harengus	HER	Pelagic
Mackerel	Scomber scombrus	MAC	Pelagic
European pilchard	Sardina pilchardus	PIL	Pelagic
Nephrops	Nephrops norvegicus	NLO	Shellfish Carnivore
Crab	Cancer pagurus	CRA	Shellfish Carnivore
European Lobster	Homarus gammarus	LOB	Shellfish Carnivore
Cockles	Cerastoderma edule	CKL	Shellfish Filter
Scallops	Pecten maximus	SCA	Shellfish Filter
Mussels	Mytilus edulis	MUS	Shellfish Filter

Table 5.1: The different species included in the analysis, along with their associated guild and identifier for other tables, equations or figures

statistics: global trade data for individual species groups across the whole period was not available, but average annual import and export price of finfish (demersal and pelagic combined) and shellfish (carnivore and filter feeder species combined) were used instead, as well as the annual quantity imported and exported in tonnes.

There has been an increase in aquaculture production in the UK over the study period, and although most species provided by aquaculture are not the same as those provided by UK wild fisheries, they may still represent market competition for wild capture fish and hence impact the price. Scotland accounts for around 80% of the UK's aquaculture production [80], with the industry dominated primarily by salmon production which has grown rapidly since the 1980's [70, 71]. The majority of UK marine aquaculture production occurs in Scotland, and data on annual finfish production (though not the average farm-gate price) [81] were collated as a potential driver of ex-vessel prices of wild capture fish in the UK. Shellfish aquaculture has also greatly increased in the past decades [82], however available data does not go as far back as the data available for finfish, so shellfish aquaculture production was not included in the analysis.

Since we are considering the ex-vessel price, the demand is actually from processors rather than directly from the consumers of fish, however since increased demand from consumers should lead to increased demand from processors, consumption of fish in the UK should be a good proxy for demand.

The price of other foods alternative to seafood may be another variable with an impact on demand, and could serve as an indicator of the overall price of food. The price of chicken is a good variable to use as a substitute for fish.

We may also consider UK income per capita; income or disposable income is commonly used as a proxy for demand. It may not be necessary if consumption data already gives a proxy for demand, but considering income per capita may be a way to account for the composition of fish consumed differing (even if the weight remains the same). Lastly, since we consider many variables per capita, we can also include the total UK population as a potential demand variable.

Total demand for fish will depend on the number of consumers and on how much they are willing to spend. The UK total population (obtained from the FAO) and UK gross disposable income per capita (obtained from the ONS) are included as drivers of demand. Food supply per capita of meat (from FAO data) is included as a proxy for trends in consumption of a comparable food source that may indicate a change in demand. The UK price of chicken (obtained from the ONS) is included as the price of a substitute source of protein.

5.2.2 Trends in Landings

The data show that UK demersal and pelagic landings have decreased substantially over the period studied (Figure 1(a)). This is primarily due to the decrease in landings of whitefish such as cod and haddock since the 1960s, partly due to disputes with Iceland over fishing in the North Atlantic [83, 84], and also the collapse of herring landings. The herring fishery in the North Sea was closed for a period in the 1980s, and landings eventually recovered, although not to their previous level. Mackerel landings have also decreased overall from their peak in the 1970s.

In contrast, Figure 5.1(c) shows that the landings of both shellfish carnivore species and filter feeders have increased substantially over the period. This is in particular due to increases in landings of Norway lobster (Nephrops norvegicus), crabs and scallops.

Total imports of finfish (demersal and pelagic) increased greatly over the period (Figure 1(b)), so that supply from imports now exceeds supply from landings in the UK. Shellfish imports and exports also increased greatly over the period (Figure 1(d)), although shellfish landings in the UK increased similarly. The competition from imported seafood could mean that changes in UK landings have less impact on price – even if imports are not sold on the first-hand market they increase the total supply of fish to the final consumers. In addition the increase in finfish and shellfish exports over the period could lead to increased demand for fish landed in the UK to be sold abroad.

5.2.3 Inverse Demand Model

To analyse the relationship between quantity and price, we could consider a simple inverse demand system of the form:

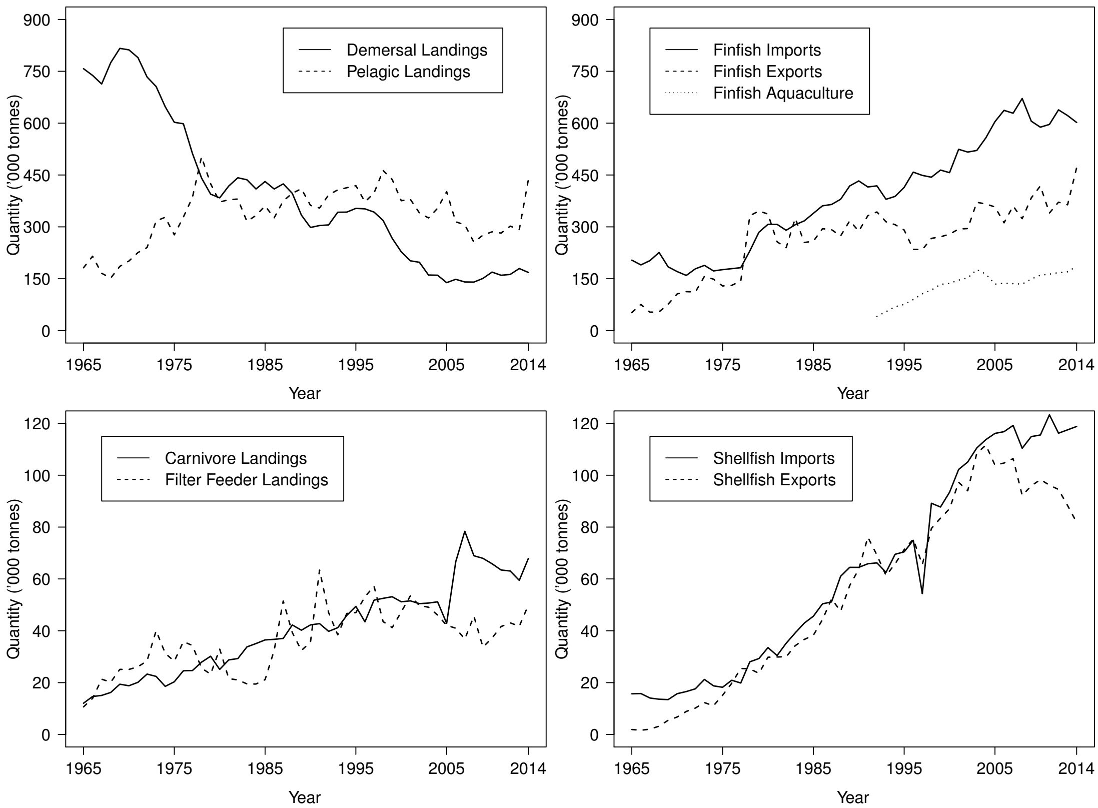


Figure 5.1: (a) UK total demersal and pelagic landings; (b) UK finfish (demersal and pelagic combined) import quantity and export quantity, and UK finfish aquaculture production (from 1990 onwards); (c) UK Benthos (shellfish) carnivore and filter feeder landings; (d) UK Shellfish (carnivore and filter feeders combined) import and export quantity.

$$\ln p_t = \alpha + \beta \ln q_t + \epsilon_t \tag{5.1}$$

where at time t, p_t is the price of the species i, q_t is the quantity of that species landed in the time period, α and β are constants, and ϵ_t is an independent and identically distributed (i.i.d.) normally distributed stochastic error term. Note that in this inverse demand system price is the dependent variable, which differs from conventional demand systems (which are more commonly seen in other economic applications) where quantity is the dependent variable; inverse demand systems are more frequently used in the context of agriculture and fisheries [51, 52]. In the case of fisheries, the assumption of total supply being exogenous can be justified by the fact that landings of many key species are limited by the total allowable catch (TAC) or other forms of quota (such as effort control), or by the fish stock available. Empirical studies in UK fish markets have also found evidence that price is determined by quantity landed rather than the other way round [51].

Estimating an inverse demand model of the form in (5.1) using time series data is problematic, as it requires that all the data are stationary; using nonstationary data can lead to spurious regression [85] and invalid conclusions. A Phillips-Perron unit root test [86] reveals that almost all of the price and landings data are nonstationary, but that the first differences of all are stationary (this result is the same both before and after taking the logarithm of the data). This indicates that almost all of the variables have a unit root (i.e. they are integrated of order 1, written as $I(1)$). Since the first difference is stationary for all, then we are able to use simple linear regression to estimate a model of the form:

$$\Delta \ln p_t = \alpha + \beta \Delta \ln q_t + \epsilon_t \tag{5.2}$$

This relates the change in price over the last time-step to the change in landings over the last time-step. Here β is the price flexibility coefficient (Note: in some contexts the definition of price flexibility may strictly refer to the percentage change in price due to a 1% change in quantity supplied - our price flexibility is approximately equal to this as long as the relative change is small).

In order to improve the amount of variability explained, we extended the analysis to include the landings of other species, which may be substitutes (or compliments). In addition, we also accounted for the effects of imports, exports and aquaculture production in the UK. Quantity and average price on imports and exports into and out of the UK for two broadly defined groups - "finfish" (demersal and pelagic species) and "shellfish" (benthos species) were obtained from the MMO UK Sea Fisheries Statistics for inclusion in the analysis [62]. Data from the Scottish government on finfish aquaculture production in Scotland (which makes up a significant proportion of total UK production) was available going back approximately 20 years (farm-gate price data for aquaculture production was not available). Phillips-Perron unit root tests comfirm that all import, export and aquaculture data has unit root (i.e, is I(1)), meaning that the first difference of the logarithm is stationary. So for each species i we can estimate a model of the form:

$$\Delta \ln p_t = \alpha + \beta^{ii} \Delta \ln q_t + \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta^{ij} \Delta \ln q_t^j + \sum_{k=1}^{m} \lambda^{ik} \Delta \ln s_t^k + \epsilon_t \tag{5.3}$$

Where $i = 1, ..., n$ are all the individual species, β^{ii} is the own-price flexibility for species i, β^{ij} is the cross-price flexibility (i.e. the effect of a change in the quantity of species j on the price of species i), and the import, export and aquaculture variables are given by s^k (λ^{ik} with $k = 1, ..., m$ gives for species i the flexibility coefficient with respect to the quantity of finfish and shellfish imports and exports, the price of finfish and shellfish imports and exports, and the quantity of finfish aquaculture production).

5.3 Results

5.3.1 Price Flexibilities: Single Species

The results of the Phillips-Perron test for unit root concluded that all species' price and landings (both before and after taking logs) are I(1), except for the prices of Gurnards and Rays, which were found to be stationary at the 5% significance level. Gurnards and

Rays were excluded from any tests of cointegration between prices for this reason. For the estimated models including only each species' own landings, significance at the 5% level was found for 7 of the 23 demersal species considered, 2 of the 3 pelagic species, 2 of the 3 shellfish filter feeders and none of the shellfish carnivores. The estimated coefficients, p-values and adjusted R^2 for each of these models are shown in Table 5.2.

The intercept terms are all close to zero and they are not significant at the 5% level in any of the models. In one of the models (Spurdogs), the price flexibility coefficient is positive, the opposite of what be expected according to theory. This could indicate a different sort of relationship between price and landings (e.g. a species caught mainly as bycatch, which is landed in larger quantities when the price is higher and discarded more often when the price is lower), or the price may be driven primarily by factors other than landings. However, in the rest of the models we have a negative price flexibility as would be predicted by economic theory. Three of the four major demersal and pelagic species by landings (cod, haddock, herring) were found to have negative price flexibilities, and in each of those cases a similar value (-0.4) was estimated.

However, it is clear from the adjusted R^2 values shown in the table that there is a large proportion of price variability left unexplained by these changes in landings.

Species	Intercept	Intercept p-value	Adj R^2 (%)	Price Flexibility	p-value
COD	-0.012	0.48	18.5	-0.390	0.0012
HAD	-0.010	0.67	17.3	-0.430	0.0018
PLA	-0.023	0.09	7.7	-0.251	0.0304
LSO	-0.009	0.44	35.6	-0.517	<0.0001
HAL	0.008	0.65	18.6	-0.170	0.0011
SPU	-0.002	0.94	13.2	0.102	0.0060
RAY	-0.020	0.18	14.3	-0.419	0.0043
HER	-0.013	0.72	31.3	-0.411	<0.0001
PIL	-0.004	0.88	17.8	-0.193	0.0015
CKL	0.022	0.69	8.4	-0.255	0.0245
MUS	-0.011	0.83	10.8	-0.236	0.0122

Table 5.2: The results of an OLS regression using the form shown in Equation (5.2) (i.e. including only the changes in the species' own landings) for the species where there was evidence that price flexibility was nonzero at the 5 % significance level. The estimated coefficients for the intercept and price flexibility are shown, along with the p-value for the intercept and the Adjusted R^2 value as a percentage

In addition, changes in the landings were only found to be significant predictors for these 11 species (and only 10 species were estimated to have negative price flexibilities) – for the rest of the species changes in landings of that species alone were not found to affect the changes in price at the 5% significance level.

Using an autoregressive distributed lag model, i.e. including lagged variables for price and landings, failed to improve the models in most cases. Lagged changes in landings improved the fit for only 3 species (saithe, cockles, nephrops), while lagged changes in price improved the fit for only 2 species (gurnards, crabs). This could indicate that the prices of these particular species depend on landings and price history in a fundamentally different way to the other species analysed, however without a convincing explanation for why that should be the case it is difficult to conclude too much. More apparent is that there is little evidence that using an autoregressive distributed lag model is an improvement for most of the species, suggesting that (after the variables are stationarised) in general past values of the dependent and explanatory variables do not have a significant effect.

5.3.2 Price Flexibilities: Multiple Species

For every species, we consider the species' own landings and the landings of species considered to be potential substitutes, as well as considering the landings of the species with the highest proportion of landings in each guild (e.g. cod and haddock for demersal), as the supply of these species is most likely to have the biggest impact on the market. Using the landings of these species as well as the finfish and shellfish import and export data, we can perform bidirectional stepwise regression to select an optimal model in terms of the Bayesian Information Criterion (BIC) [87].

Table 5.3 shows the results of the stepwise regression models for each species, displaying the number of variables included in the model (excluding the intercept), the coefficient of the species' own landings (if it was included in the model), as well as the adjusted R^2 value, which gives an indication of how well the model explains the data. The stepwise regression procedure produced a model for every species and the species' own landings were included in the optimal model for 17 species, and the coefficient was negative in all but one case (Spurdogs). Additionally, for several of the species where own landings were found to be significant alone, we can see from the adjusted R^2 value that a higher proportion of the variability in price changes is explained by including landings of other species and global trade data.

This method has the advantage of being a systematic way to narrow down which of the pool of possible variables to test for an effect on price, however despite the use of the Bayesian Information Criterion there remains the potential for overfitting or including extraneous variables which fit the data by chance for which the relationship isn't meaningful or plausible in a practical sense.

Species	Other Variables	Adj. R^2	Own Price Flexibility
Cod	3	52.8	-0.45
Haddock	2	41.1	-0.53
Plaice	3	25.7	-0.23
Common Sole	6	21.9	-0.52
Anglerfish	4	30.0	-0.30
Lemon Sole	2	43.9	-0.49
Halibut	1	25.3	-0.18
Turbot	3	26.6	-0.22
Brill	4	21.3	-0.20
Spurdog and Dogfish	3	35.2	0.11
Gurnards	7	34.7	-0.16
Skates and Rays	6	48.5	-0.42
Herring	3	55.1	-0.37
Pilchard	3	32.4	-0.18
Lobster	4	49.2	-0.36
Cockles	9	33.6	-0.24
Mussels	3	25.3	-0.29

Table 5.3: Estimated own price flexibility and adjusted R^2 for species with significant own price flexibilities. Other variables selected from import and export quantities and prices, aquaculture production and substitute and complement species using Bayesian information criterion.

In particular we are sceptical of high numbers of variables, positive cross price flexibilities (since these indicate price increases as landings increase, i.e. that the species are complimentary goods rather than substitutes), and cases where landings of a species caught on a small scale have an effect on species landed in much higher quantities. This means that while we can be fairly confident in results with a smaller number of variables with a clear economic interpretation (such as cod, which has a negative own price flexibility with dependence on finfish imports and exports), the results where the optimal model in terms of the BIC contain more variables and lack an obvious interpretation (such as Nephrops, which includes 10 variables and no own price flexibility) are less meaningful.

5.3.3 Price Flexibilities: Aggregated Guilds

As well as considering the price of individual species, we also investigate for relationships between landings of our aggregated guilds and the average price across the entire guild. For these we fit a model of the same form as above for the average price of each of the four guilds, but only using aggregated guild landings and global trade data (so no data on individual species).

The results show that negative own price flexibilities were obtained for the quota and non-quota demersal and shellfish filter feeder guilds. For demersal, finfish import price and finfish export price and quantities are also included (all with positive coefficient) while for shellfish filter feeder, shellfish export price and quantity are included (both with positive coefficient).

Guild	Variables	Coefficient	Adj.R^2 (%)
Quota Demersal Species	Quota Demersal Landings	-0.585	52.47
	Finfish Import Price	0.64	
	Finfish Export Price	0.29	
	Finfish Export Quantity	0.23	
Non-Quota Demersal Species	Non-Quota Demersal Landings	-0.21	46.79
	Finfish Export Price	0.284	
	Finfish Export Quantity	0.293	
	Shellfish Import Price	0.41	
	Shellfish Import Quantity	0.212	
Pelagic Fish	Finfish Export Price	0.366	19.69
	UK Chicken Price	1.1	
Shellfish Filter Feeders	Shellfish Filter Feeder Landings	-0.412	20.93
	Finfish Import Price	1.01	
	Finfish Export Quantity	0.388	
Shellfish Carnivores	Shellfish Export Price	0.405	24.88
	Finfish Import Quantity	0.435	
	Finfish Export Price	0.358	
	Finfish Export Quantity	0.218	

Table 5.4: The results of analysis applied to aggregated guilds of species rather than individual species . Price flexibilities significant at the 5% level were found for demersal and shellfish filter feeders, with imports and exports found to be significant for demersal guild and exports significant for shellfish.

There was no evidence of significant own price flexibility for pelagic fish or shellfish carnivores. Instead, the significant drivers of pelagic fish price were export price and the price of chicken, while shellfish carnivore price was found to depend on a combination of import and export variables.

This provides evidence that there are significant negative own price flexibilities in the UK market at the aggregated guild level for demersal species and shellfish filter feeders, and that controlling for the impact of imports and exports of finfish is the most important additional variable to consider when estimating the price of demersal or shellfish filter feeders using the own price flexibility. Quota demersal species (that is, demersal species that have a TAC quota) in particular have a higher negative own price flexibility of -0.585, suggesting that the quotas set by the UK (and hence the landings into the UK) for demersal species should be expected to have an impact on the ex-vessel price of demersal species, with reduced quotas leading to an increase in price (after accounting for the price of imports and the overall demand for UK exports of finfish).

5.4 Discussion

We have included many variables in our analysis, to try and best account for changes in demand for ex-vessel fish and how this might affect the ex-vessel price. However, inevitably there are going to be factors which could affect the price that we have not been able to include.

The fact that landings are limited by the TAC and the state of the fish stocks suggests supply from landings can be treated as exogenous, which justifies the assumption that ex-vessel price depends on landings. However the price received in first hand fish markets will also depend on the price received for processed fish, which will in turn depend on the final retail price, and there is evidence that first hand market prices follow prices received by processors [88]. The quantity and average price of imports and exports serve as indicators of the supply and demand attributable to global trade, which may impact retail and process price. We have also included several variables that might affect the overall consumer demand for fish in the UK, such as disposable income or meat consumption. However there may be shifts in demand due to other factors not included in the analysis.

As well as changes in total population, changes in demographics could affect the total UK demand for fish, or the demand for different species. Consumer habits or preferences for fish consumption could also have been impacted by factors such as marketing and health campaigns. Cultural shifts in preferences for different fish species could also affect demand – this may have been particularly applicable to mackerel and herring, which both underwent abrupt and extreme changes in both price and landings.

Another factor which could impact the price is the quality of the fish landed; we only compare the total weight landed of each species, however changes in fish populations and fishing techniques over time mean it is possible that there could be differences in the attributes of fish landed from year to year. For example smaller fish may command a lower price per unit weight, so the average weight of each individual fish landed could affect the price as well as the total weight of the species landed [89]. Other variables such as fuel price (indicative of the cost of fishing effort and possibly of imports and exports) could also have an impact. The costs associated with fishing will determine what price fishermen will accept in the first hand market.

In this research, one of our main goals in investigating the price dynamics was to learn more about the relationship between price and landings. This means that in some sense a model which best predicts ex-vessel price was not our primary goal, instead it was one that would best allow us to determine the relationship between price and landings; e.g. whether there are own or cross price flexibilities and what the best estimate of those were. The fact there are so many possible factors that could somewhat affect the price means that, even though we have not included everything, we had a lot of variables to consider when fitting our model. We could have pursued a method that restricted us to fewer variables based on a priori reasoning that certain variables would be the most important or most interesting to us - however, the fact that many variables exist that could affect the price is one of the reasons that the relationship between price and landings remained uncertain, so we wanted to try and account for as many confounding variables as possible.

Considering all of these variables meant either fitting one model including all variables - which may not provide a very good estimate of the coefficients we are most interested in, or may be susceptible to overfitting - or using some sort of model selection procedure. We used stepwise regression, utilising the BIC to penalise the inclusion of more variables so we could hopefully end up with an estimate for price flexibilities that only included the most significant confounding variables. However, when using stepwise regression as a model selection tool, the calculation of p-values on the final model may be incorrect as the distribution will be biased from the model selection procedure that led to it. There is also the possibility of fitting variables that artificially improve the fit by coincidence. For our purposes, these drawbacks were not as significant as in some cases because we were less focused on whether our identification of the most significant demand and supply variables was the most accurate and more interested in accounting for them as confounding variables, and also because our price model is fitted in large part to try to estimate the price flexibility coefficients, rather than because we wanted to use the model with all its variables to predict price as accurately as possible. However, future work on this area could involve using other model selection techniques such as elastic net regularization [90].

We naturally assume that future changes in explanatory variables have no effect on current prices. However, this may not be strictly true, as observation or forecasting from scientific models could allow participants in the market to predict changes in supply or global trade which will occur next year or later, and might influence the price today. For example if it is expected or announced that the TAC for a species will be heavily reduced next year, then there may be a corresponding increase in price in the current year in anticipation of this, even if the supply had not yet decreased.

The average import price was found to be significant for many species, particularly finfish, and for the demersal and finfish guilds, with a positive coefficient as would be expected. What was unexpected was that the total import quantity did not appear to have a significant effect on the price for almost all species or guilds, despite the huge increase in seafood imports over the time period. However, this is likely because the quantity actually imported to the UK each year is not necessarily a good indicator of the availability of substitutes on the global market; if the demand for seafood exceeds the supply from UK landings and aquaculture production then that demand will be fulfilled by imports (as long as the price of imports is not so high that there would be no demand for goods at that price). The average price of imports better represents the competition from global trade – a higher import price indicates that the availability of fish on the global market is lower, or the cost of transportation is higher.

Exports were also found to have significant effect on the price of various species and guilds (usually with a positive coefficient as expected), however for exports both the average price and the quantity exported were often significant, in contrast to imports where only the price was a significant factor. This could be because an increase in quantity exported, even if there is no increase in export price, could be interpreted as an increase in demand for UK fish. Most of the relationships found fit with economic intuition – negative coefficients for the landings of the species itself and close substitutes, with positive coefficients for exports price and quantity (which indicate increased demand in the global marketplace) and positive coefficients for import price (indicating competition from global supply). For some species, the species' own landings were not found to be significant predictors but other species' landings were. This makes sense for species which have relatively low landings compared to the landings of close substitutes. For example, landings of pollock were not found to be a significant predictor for the price of pollock, but landings of cod were, with a cross-price flexibility close to the value commonly found for similar species' own price flexibility.

For certain species the models appear to indicate different relationships. Positive coefficients for landings of other species may indicate that the species are complements rather than substitutes. However, despite aiming to reduce the number of different species considered as potential substitutes it is still possible that positive cross price flexibilities are due to coincidence in the data, rather than any real economic relationship. Positive coefficients for a species' own landings may indicate that supply is not exogenous but instead depends on the price (determined by other factors influencing demand), with fishermen willing to provide more of the species when price increases. This would especially make sense for species with relatively low levels of landings, as fishermen may only choose to apply effort to catching those species (or else find it worthwhile to land them when caught as bycatch, rather than discarding) when the price is high enough.

For the aggregated guilds (demersal, pelagic, shellfish filter and shellfish carnivore), a relationship between guild price and guild landings was identified for two out of the four guilds – demersal and shellfish filter feeders. These guilds were found to have negative own price flexibilities, indicating that – after controlling for changes in the global market – decrease in total landings of these guilds could be accompanied by an increase in the average ex-vessel price, and similarly an increase in landings could lead to a decrease in average price. For the other two guilds, no own price flexibility was obtained. This could be because demand for species in these guilds is such that changes in supply have little impact on the price, however it is also possible that there have been either large shifts in demand or changes in the composition of the species landed within the guild that are not identified by considering total weight and average price.

Finding significant negative own price flexibilities for several species and for two of the aggregated guilds of fish indicates that negative own price flexibilities are to be expected when considering major UK fisheries, so we will incorporate price dynamics of this nature into the mean field game model developed in Chapter 4. In the following chapters we will perform numerical simulations using the mean field game model, and one of the questions we will investigate is how different price flexibilities affect the behaviour of the solution.

Chapter 6

Numerical Solutions of the MFG Fisheries Model

6.1 Introduction

In this chapter we will present some numerical solutions to the mean field game (MFG) model of a fishery from Chapter 4, with the aim of showing the solution of the model under the assumptions described in the model formulation chapter and investigating the effect of different parameters on the results. Here we will focus on simple functions for the individual ecological and economic components, rather than more complex models for these components which would introduce many more parameters. However, by testing the numerical solutions of the mean field game model for simple functions, we want to demonstrate how the mean field game model might be used with more complex and realistic ecological or economic models in future.

First, a sensible choice is to focus on investigating effect of parameters such as the terminal time T, the volatility σ, and the initial and terminal conditions $u(T, x)$ and $m_0(x)$, and show how sensitive the solutions are to these parameters. These initial results, shown to demonstrate the output from the model and its dependence on some of the parameters, will help give context to the results of the scenario analysis for different conditions. This is what we will consider in this chapter. Then, we wish to investigate how the solution behaves due to:

- Changes to the ecological component of the model, i.e. the resource equation. This may include changing the growth rate or carrying capacity, changing the initial condition of the stock N, changing the form of the function in the stock evolution ODE completely, or potentially adding a new ODE to represent another interacting fish population (if this is not also fished by the same agents, then it merely changes the evolution of N in the model, while if it is also fished then it increase the problem to a two dimensional mean field game). Changes of this type will be considered in Chapter 7.

- Changes to the economic conditions, such as the introduction of price flexibilities to the model, changing the cost parameters, or changing the form of the cost function. These will also be considered in Chapter 7.

- Implementation of regulations in the model, such as by additional cost terms which would need to be imposed by a regulator. In terms of how they enter into the model equations, these regulations could be similar to the second category (e.g. a tax on catch is equivalent to a change in the ex-vessel price received) or more similar to the first (e.g. an MPA which would affect the stock evolution). These will be considered in Chapter 8.

6.2 Numerical Method for MFG Fisheries Application

In one dimension, the MFG system for a fisheries model is given by:

$$\frac{\partial u}{\partial t} + \frac{(u_x)^2}{2\gamma} + \frac{\sigma^2}{2} u_{xx} - ru = -F(x, N, L(N, m)), \tag{6.1}$$

$$\frac{\partial m}{\partial t} + \frac{1}{\gamma}(m u_x)_x - \frac{\sigma^2}{2} m_{xx} = 0, \tag{6.2}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = aN\left(1 - \frac{N}{K}\right) - L(N, m). \tag{6.3}$$

$$u(T, x) = G(x(T), m(T), N(T)); \qquad m(0, x) = m_0(x); \qquad N(0) = N_0. \tag{6.4}$$

$$u_x(t, 0) = u_x(t, x_{max}) = m_x(t, 0) = m_x(t, x_{max}) = 0. \tag{6.5}$$

We wish to solve this system numerically, then use the numerical solutions to investigate the behaviour of the solution. We will use a cost function given by

$$F(x, N, L(N, m)) = qxNp - c_1 x - c_2 x^2, \tag{6.6}$$

where $p = p(L)$, with some maximum price p_{max}. We require an initial condition for $m(0, x)$ and $N(0)$, and a terminal condition for $u(T, x)$ to solve this system. Note that the Fokker-Planck (or Kolmogorov Forwards) (6.2) equation can be rewritten as:

$$\frac{\partial m}{\partial t} + \frac{1}{\gamma}(m_x u_x + m u_{xx}) - \frac{\sigma^2}{2} m_{xx} = 0.$$

We will consider an iterative method for finding a numerical solution to this coupled system of equations.

As we are solving the Fokker-Planck (or Kolmogorov Forward) equation and the stock (or resource) evolution equation forwards in time (with an initial condition for m and N) and solving the Hamilton-Jacobi-Bellman (HJB) equation backwards (with a terminal condition for u), then to solve either equation individually we require either m and N from the terminal time backwards or u from the initial time forwards. In other words, if we want to solve one equation for either u or m, we need some initial guess for either m or u, respectively.

Without more information it is hard to decide on what a reasonable initial guess for u might be, as to use the terminal condition as the initial condition would not make sense for many situations due to the definition of u - e.g. if the terminal cost $u(T, x) \equiv 0$ then setting this as the initial condition means we are setting there to be net zero payoff for an agent no matter what they do, which may mean enforcing an unnecessarily unrealistic scenario in our first guess or iterate. However, since we have an initial distribution m_0, it makes sense to consider what would happen if this same distribution was kept constant over the period. So we will solve the coupled system with an iterative approach: first we consider setting $m(t, x) = m_0(x)$ for all t, then solve the resource evolution equation to get $N(t)$ over the time period - this is what would happen to the stock assuming fishing patterns remained the same as they are at

$t = 0$.

Using this $m(t, x)$ and $N(t)$ where fishing effort is held constant, we may then solve the Hamilton-Jacobi-Bellman equation backwards in time to obtain an initial iterate of u. This gives us the starting point to iterate, solving the Kolmogorov and resource equations forward then the HJB equation backwards, until the error between iterates is small enough to indicate we have a good approximate solution to the coupled system.

We discretize the system using a central difference in space and implicit Euler method in time to solve the Fokker-Planck and Hamilton-Jacobi-Bellman equations - the Fokker-Planck equation we are marching forward in time while in the Hamilton-Jacobi-Bellman equation we must solve this backwards in time from the terminal condition.

So the iterative algorithm is as follows:

- Solve the HJB equation backwards in time, where $m(t, x) = m_0(x)$ for all t to obtain u_1.
- For u_i, obtain m_i and N_i by solving the Kolmogorov and the resource ODE forward in time given u_i.
- Obtain u_{i+1} by solving the HJB equation backwards in time given m_i and N_i.
- Repeat the last two steps until the error between iterates is below a selected tolerance.

We determine the error between the iterates using the 2-norm of the matrices for m_i, u_i and also N_i, and for our numerical solutions we will set the tolerance to require $||m_i - m_{i=1}||_2 + ||u_i - u_{i-1}||_2 + ||N_i - N_{i-1}||_2 < 10^{-5}$. So when the sum of the error between the iterates for these three solution variables is less than 10^{-5}, we will accept the iterates as our approximate numerical solution of the mean field game system of PDEs.

6.3 Numerical Simulation - Base Case Example

We will use the numerical algorithm described above to solve the mean field game system with different test parameters. First, we will solve the equations with very simple initial parameters to see what the solution using our numerical algorithm looks like, and how to interpret it. We will at first assume constant price (and investigate the effect of adding in price flexibility later), and linear costs. We will set $x_{max} = 1$, so our units of fishing effort or activity are simply the proportion of the maximum total catch rate that can be reached by an agent. We will choose $q = 1$ for a simple catchability. We will likely be more interested in scenarios where the intrinsic growth rate is lower than the fishing mortality if all of our agents are fishing close to x_{max}, so we will select the intrinsic growth rate $a = 0.75$. We will set the carrying capacity K of the stock equal to 100 units of biomass. We will set the $\sigma = \gamma = 1$, and the price $p = 1$. We set linear costs term $c_1 = 2$ (and $c_2 = 0$, i.e. only linear costs for now), and let $r = 0.05$.

Since we will later be selecting parameters guided by analysis done on an annual timescale, we will consider the units of time to be years. We will select a horizon time of $T = 10$ years. As we have set $K = 100$, we will select the initial stock value $N_0 = 50$, halfway between zero and the carrying capacity.

Parameter	Value
x_{max}	1
T	10
σ	1
r	0.05
q	1
p_0	1
c_1	2
a	0.75
K	100
γ	1
N_0	50

Table 6.1: Table of starting test parameters for the base case

We will treat these parameters as a base case for a few simulations to see what the solution looks like as various parameters are varied.

Lastly, we must set the initial condition on m and terminal cost on u. The initial distribution $m_0(x)$ must be a valid probability density function - we will select a uniform distribution for the base case. For the terminal condition for u, we do not have any particular requirements in mind to start with, so we will set the terminal cost u to be uniformly zero.

To proceed with the numerical algorithm, we need to decide on the number of grid points in our discretization in time (n_t) and space (n_x) used to perform the implicit numerical method for the HJB and Fokker-Planck equations. We will set $n_x = 100$, which we will treat as our default for every simulation unless explicitly stated otherwise. For this basic case we will start with $n_t = 500$, but we expect we are more likely to have to vary the number of timesteps as we perform more simulations.

6.3.1 Results

The results of our numerical algorithm give us an approximate solution $u(t, x)$, $m(t, x)$ and $N(t)$ to the mean field game equations and associated common resource ODE with the selected parameters. In the first instance, we want to check what this solution looks like (and see that it fits with our understanding of what we expect should occur in a model of a fishery with these parameters), and how we can interpret it.

Figure 6.1 shows a surface plot of $u(x, t)$ from our solution, and Figure 6.2 shows a surface plot of $m(x, t)$, while Appendix B.1 contains additional figures including heat maps and plots of u and m at given times. From Figure 6.1 we see we that $u(t, x)$ is decreasing with time overall, which makes sense as we have parameters where it should be profitable to fish (so the value function should be positive), heading towards a terminal condition $u \equiv 0$. A more detailed inspection of $u(t, x)$ (see Appendix B.1) shows that at each point in time, $u(t, x)$ is increasing with x, and that the steepness of the curve decreases as time increases, meaning the curves become flatter, although still increasing with x, until they reach the terminal condition $u(T, x) \equiv 0$. However, this is not easy to see from a plot of $u(t, x)$ in full, and requires comparing curves for individual values of t.

The surface plot for $m(t, x)$ in 6.2 is easier to interpret. We can see that after one

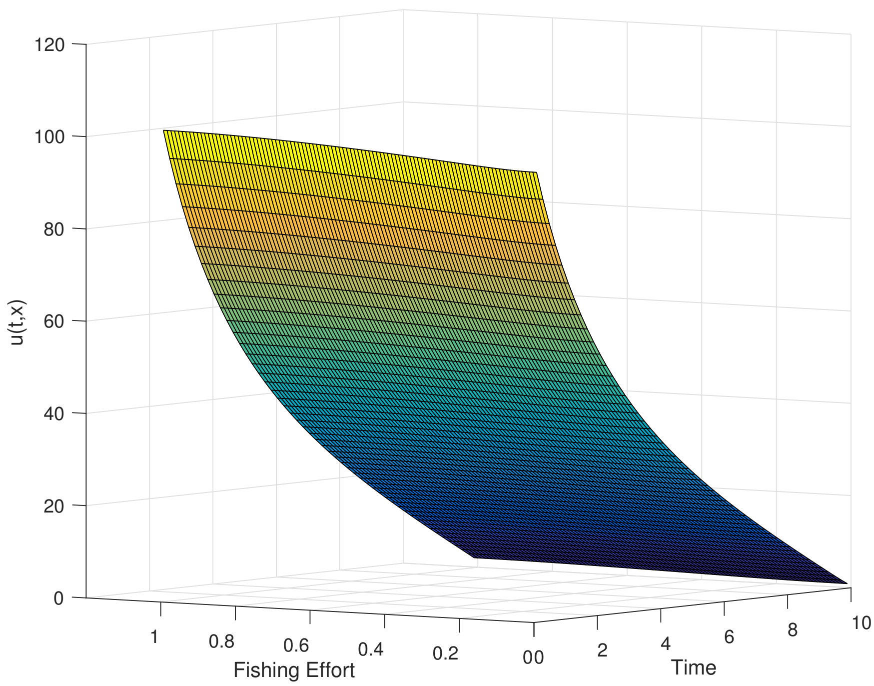


Figure 6.1: Surface plot of $u(t,x)$ from the solution to the basic case.

timestep, the distribution of agents moves from the initial uniform distribution towards a distribution with most agents fishing at higher levels, as indicated by the fact the peak is closer to $x_{max} = 1$. We can also see that after a small number of timesteps the distribution has a high proportion of agents fishing close to the maximum, before the distribution smooths out and becomes a flatter curve as time increases (this can also be seen from the heat map of $m(t,x)$ in Appendix B.1). This indicates that there is an initial spike in the overall fishing effort, before it reduces as the distribution curve flattens.

We can see the result of this distribution of fishing effort $m(x,t)$ by looking at the plot of fish stock $N(t)$ in Figure 6.3. The stock decreases sharply from its initial value $N_0 = 50$, then the decrease slows over time as the overall fishing effort starts to decrease. This behaviour makes sense given our prior knowledge of fisheries, since as $N(t)$ falls the expected catch per unit effort will decrease and it will become less profitable to fish, so the overall fishing effort decreases.

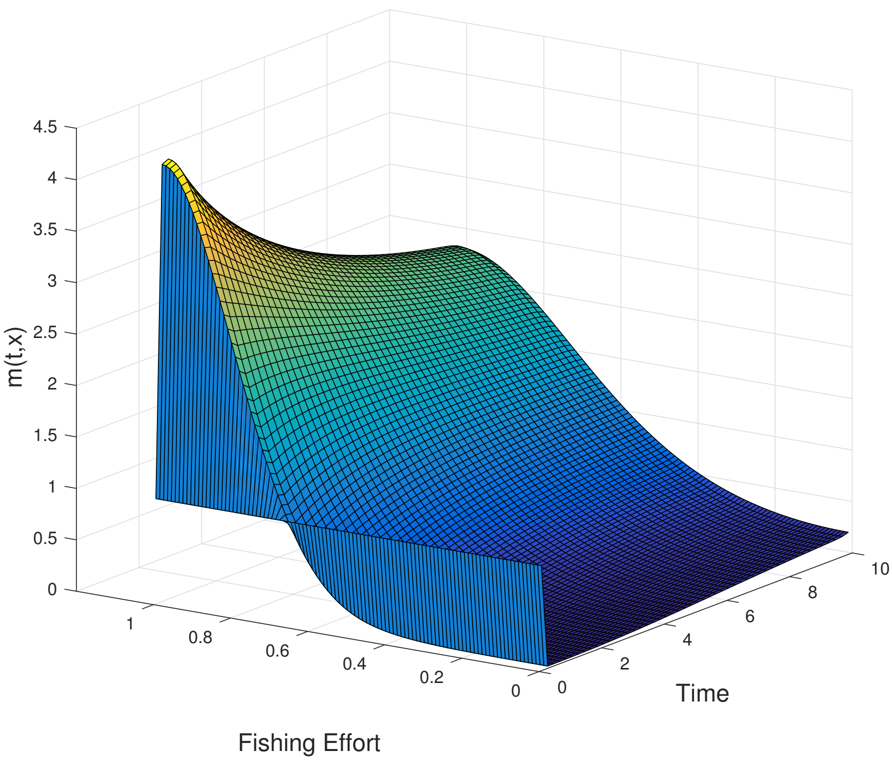


Figure 6.2: Surface plot of $m(t, x)$ from the solution to the basic case.

We can also compare the graphs of $m(t, x)$ for different values of t to see how the shape of the distribution of fishing effort evolves with time, as in Figure 6.4. The surface plot of m was already fairly clear to interpret, however in other cases it may be worthwhile to compare the curves of $m(t, x)$ at different values of time to see some of the subtle differences in them. We can see from Figure 6.4 here that it is easy to see the differences between the distribution at different values in time, even if they have approximately the same shape.

So, examining the numerical solution to this basic case, we see that our solution seems to make sense and fit with what we would expect from the model; the overall fishing effort increases from a constant initial distribution because it is very profitable to fish with a high value of $N(t)$, however as fishing effort erodes the stock, it becomes less profitable to fish so some agents no longer fish at as high intensity, resulting in a distribution that is less closely packed towards the maximum, and less fishing pressure

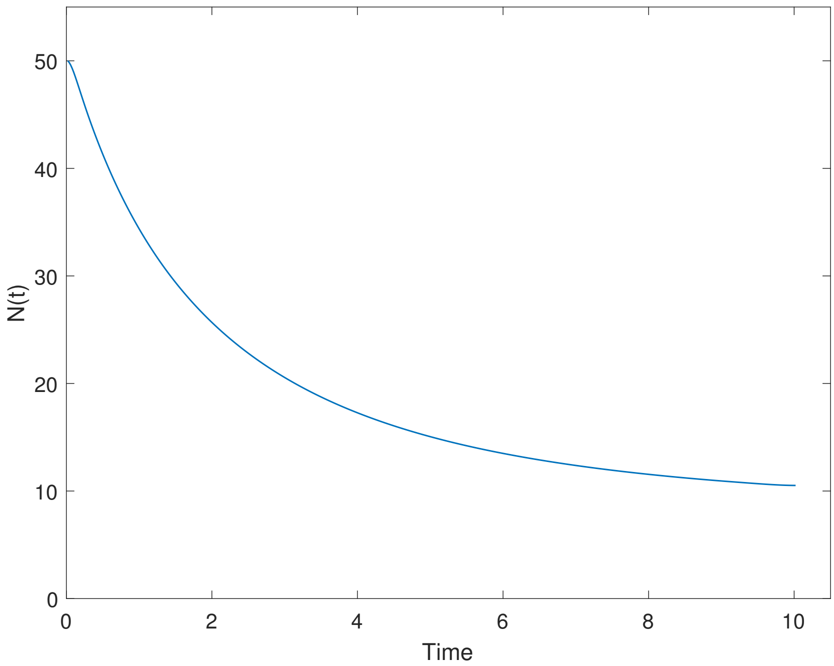


Figure 6.3: Plot of the stock $N(t)$ from the solution to the basic case.

overall.

One thing that is striking from viewing this first result is that, although the dependence of the function $u(t, x)$ on x is fairly subtle and requires closer inspection of an individual curve at a point in time to clearly see how it depends on fishing effort x, this was enough to have a significant impact on $m(t, x)$, and it was easy to see from the shape of m (either in the surface plot or in the plot showing multiple distribution curves at different points in time) what was happening. It is also easy to follow what happened to the fish stock $N(t)$ as a result of the distribution of fishing effort, and by following the way the distribution evolves over time, we can deduce the shape of the value function $u(t, x)$. Also, at the end of the time period, we can see immediately from the value $N(T)$ what the impact on the fish stock has been. This means that, if we are interested in the state of the fish stock at the end, that $N(T)$ is probably the best single piece of information to quickly compare different simulations of the mean field game model, while viewing the surface plot showing the evolution of $m(t, x)$ over the time period is the easiest way to get a full picture of the behaviours of the solution.

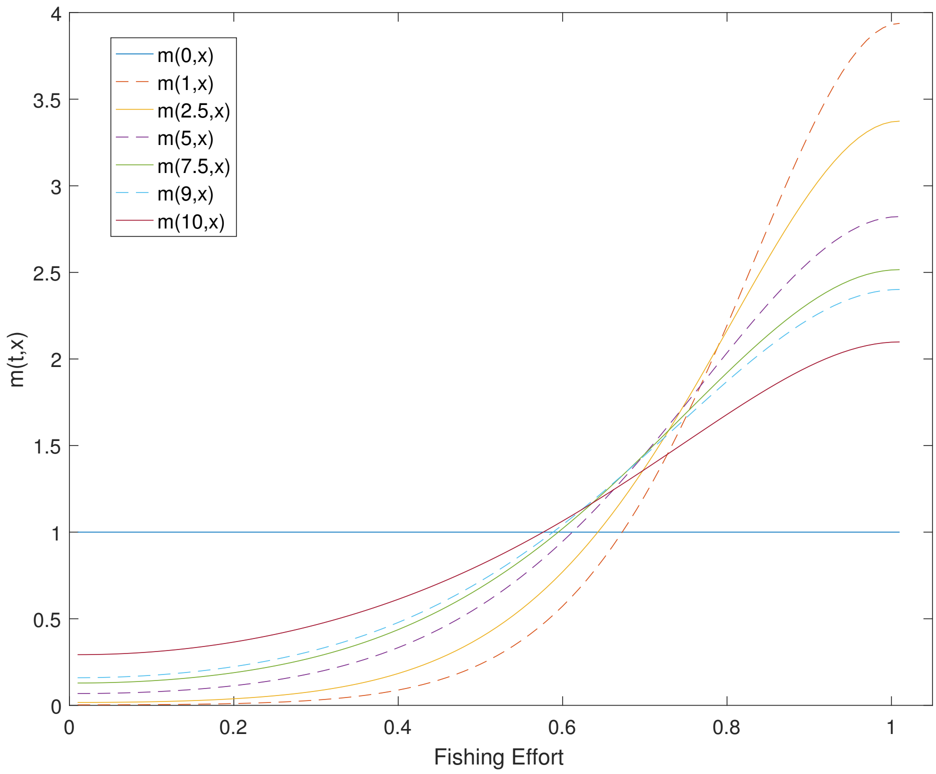


Figure 6.4: Plot showing the curve $m(t, x)$ for different values of t.

So when we present the results of future simulations, we will normally present plots of $m(t, x)$ and $N(t)$, and perhaps other information that we might want to extract from the solution such as the harvest, (which is the same as landings as we have not incorporated discarding into the model).

Now, equipped with an algorithm that can obtain a numerical solution to the dynamic mean field game, and an understanding of how to interpret the solution, we wish to investigate the sensitivity of the model and numerical method to some of the parameters or conditions. We will first consider some of the aspects that are perhaps more likely to be at the discretion of the modeller, or else ones which we want to be aware of how our conclusions have been affected by them. After we have explored some of these aspects by running a few test simulations, we will use our mean field game model to explore some different scenarios.

6.4 Terminal cost $u(T, x)$

Considering the solution to the example above, we noted that the value function $u(t, x)$ became less steep with respect to x as time increased. An obvious question is how much this was due to the choice of a constant terminal cost $u(T, x) \equiv 0$ rather than the dynamics of the rest of the system (note that simply using a different constant for the terminal cost has no effect other than to translate the entire function $u(t, x)$ by a constant). So we will consider a different form for the terminal cost.

The terminal cost is a change to the agent's payoff which is incurred at the horizon time, which can in general depend on x, $m(T, x)$ or $N(T)$. It is standard to refer to this as a terminal cost, rather than say a terminal payoff, but it does not have to be negative. Unless there is some form of penalty resulting in losses at the end of the time period, there is little reason for there to be an overall negative value function at the horizon time.

In fact, unless the specific scenario indicates that the fishery will close at the horizon time (rather than the horizon time simply being a choice of how far into the future we wish to model), then if the fishery was profitable previously, we could expect the terminal condition on u to be a positive increasing function of x, since it would presumably be profitable for an agent to be at a higher value of x after the horizon time as well. This suggests it may be a good idea to use an alternative terminal cost function instead of a terminal cost that is uniformly zero, to better represent the possibility of the fishery being a going concern at the time horizon.

So we will consider instead a linear increasing function, with $u(T, 0) = x$. This captures the increasing value functions we saw during the time period considered in the previous results, so could be a good approximation for the fact it is generally profitable to keep fishing after the time horizon. Figure 6.5 shows $m(t, x)$ with this linear terminal cost, and by inspection we see that it appears to have the same overall shape as $m(t, x)$ with a constant terminal cost (shown in Figure 6.2). A similar comparison between $u(t, x)$ with constant and linear terminal costs shows that there is very little difference

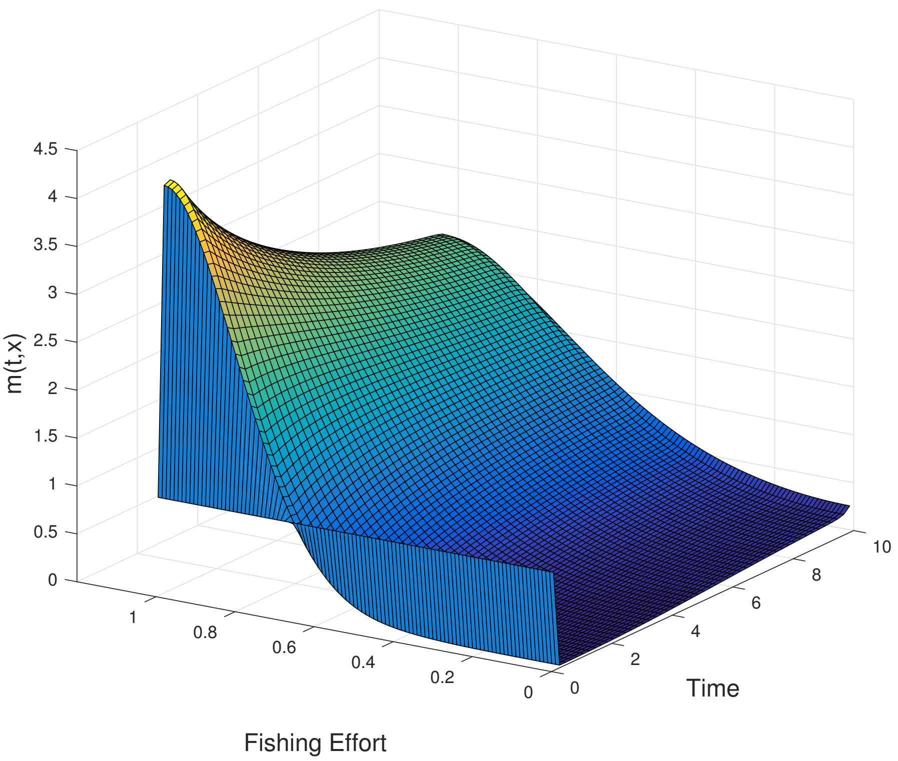


Figure 6.5: Plot of $m(t, x)$ with linear $u(T, x) = x$

between the solutions (Appendix B.2). We find that increasing the slope of the linear function used for the terminal costs results in more pronounced difference between the solutions very close to the horizon time (see Appendix B.2 for more details), but the overall shape of the solutions remains the same throughout the vast majority of the time period. So the flattening of u and the overall dynamics of the solution did not come solely from the constant terminal cost.

Since a linear terminal function for u seems slightly more realistic than a constant function (since it rewards higher fishing effort at the horizon time), we will use a default terminal cost $u(T, x) = x$ instead of $u(T, x) \equiv 0$. Depending on the problem, we could also set different forms of terminal cost functions, such as quadratic.

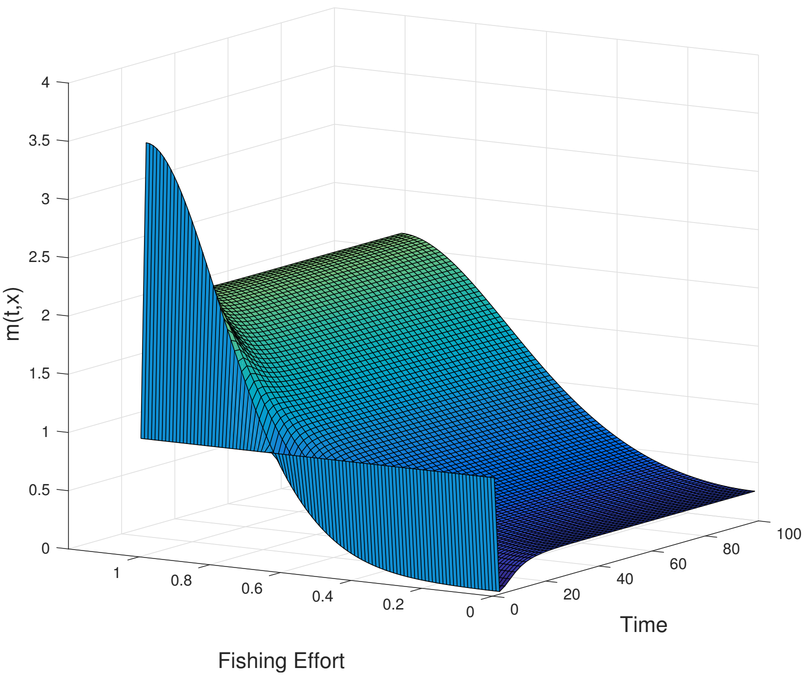
6.5 Horizon time T

As discussed in the previous section, the horizon time T is usually a modelling parameter that we set when we determine how far into the future we want our MFG model to run. We may have a hard horizon time T in some cases, e.g. for seasonal fisheries or before a fishery completely closes, but most of the time we would have some choice over the horizon time T depending on what questions we wanted to answer.

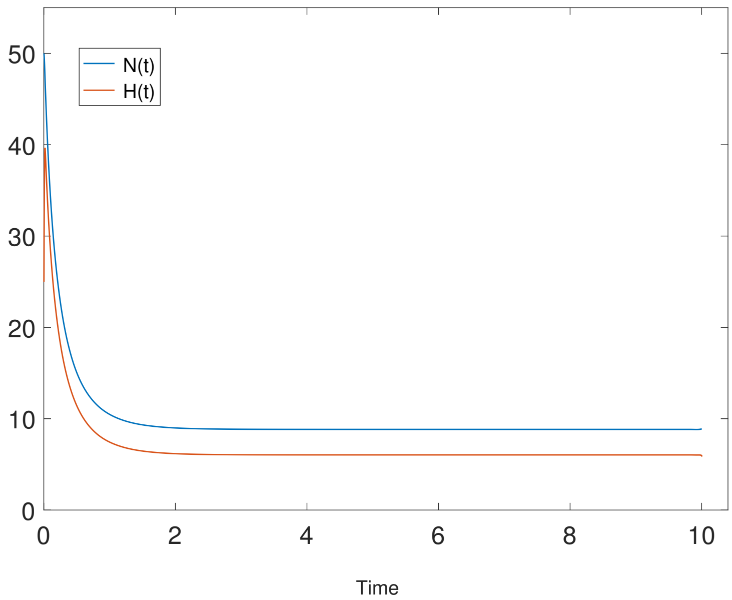
We ran the basic case simulation for $T = 1, 2, 5, 10, 15, 20, 30, 40, 50, 100$. As we might expect, particularly in this simple case, we find little discrepancy between the solutions for $m(t, x)$ and $N(t)$ during the same time periods of simulations with different horizon time. Also, we notice that for long horizon times, the solution m seems to settle into a certain distribution, while N reaches a value where it remains similar. A plot of $m(t, x)$ (Figure 6.6(a)) and a plot showing both stock $N(t)$ and harvest rate $H(t)$ (Figure 6.6(b)) for the $T = 100$ case shows this clearly. Inspecting the values in the solution we find that the distribution $m(t, x)$ remains approximately the same for the majority of the of the time period, with values $m(t, 1) \approx 2.13$ and $m(t, 0) \approx 0.25$ at the boundaries, while the values of $N(t)$ and the harvest $H(t)$ reach steady values of $N \approx 8.83$ and $H \approx 6.04$).

This is a good indicator that the model has reached steady values for m and N during the time period, but it is not completely at a steady state because even though N and m remain approximately constant in time, the value function u does not, as shown in Figure 6.7(a). However, examining $u(t, x)$ more closely in Figure 6.7(b), comparing values at successive timesteps close to the end of the time period (when the m and N solutions are remaining steady), we see that the shape of $u(t, x)$ remains very similar, although it is shifted down with each timestep.

So we can see that the solution has reached something similar to a steady state, except for the (approximately) constant decrease in $u(t, x)$ with each timestep. Returning to the original definition of the value function - the expected payoff for an agent behaving optimally until horizon time T - this makes sense, as with each successive timestep there is one timestep less to keep earning a profit. This is because the at the end of the time horizon, an agent expects to earn only their terminal payoff (in this



(a)



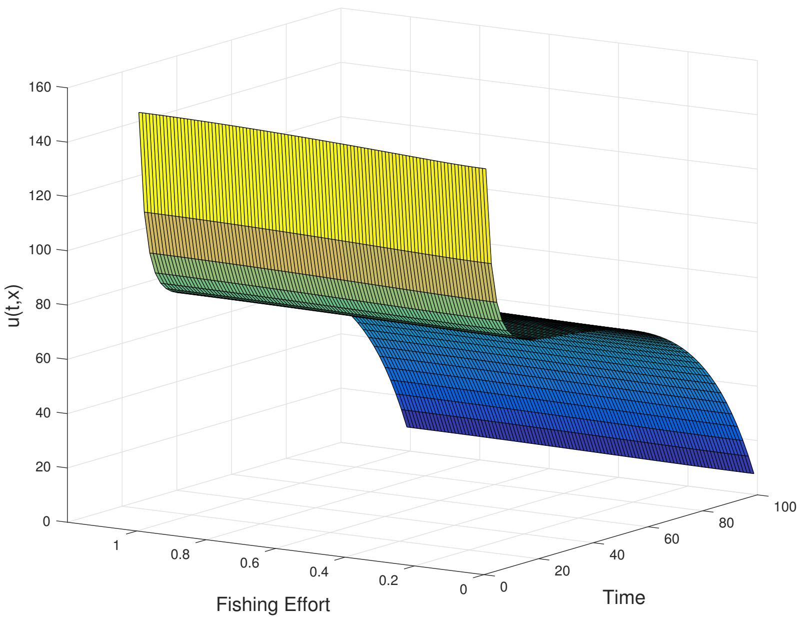
(b)

Figure 6.6: (a) Surface plot of $m(t,x)$, and (b) Plot of stock $N(t)$ and harvest $H(t)$ in the case $T = 100$

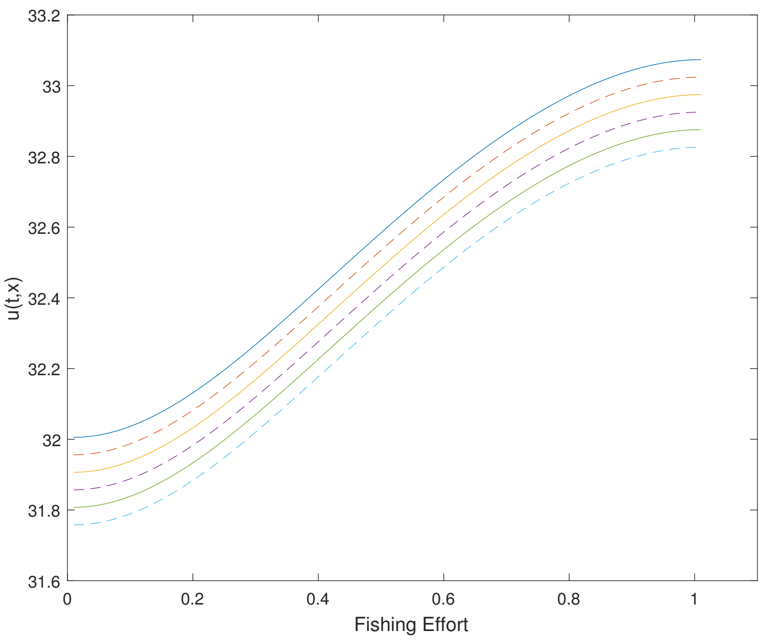
case a small linear function of x).

This suggests that in order to reach the true steady state, we should run a new simulation of the mean field game, taking as its terminal cost, say, the initial value function $u(0,x)$, and setting the initial distribution $m_0 = m(T,x)$ and $N_0 = N(T)$. Running this multiple times (so $u(0,x)$ takes the shape of our steady curve for u) we reach a point where $u(t,x)$ no longer decreases with time, because the terminal cost is the same as the value function for the beginning of the game.

Of course, in terms of seeing the dynamics of $m(t,x)$ (giving us the fishing activity of the agents) and $N(t)$ (giving us the stock), we reach something close to the steady state before $u(t,x)$ is truly constant with time. So in practice we may not always seek the true steady state, instead being more interested only in when we reach the steady state solution in $N(t)$.



(a)



(b)

Figure 6.7: (a) Surface plot of $u(t,x)$ in the case $T = 100$, and (b) plot showing the curves $u(90,x)$, $u(90{+}\Delta t,x)$, $u(90{+}2\Delta t,x)$, $u(90{+}3\Delta t,x)$, $u(90{+}4\Delta t,x)$,$u(90{+}5\Delta t,x)$ in the case $T = 100$; at each successive timestep, the curve of $u(t,x)$ is lower.

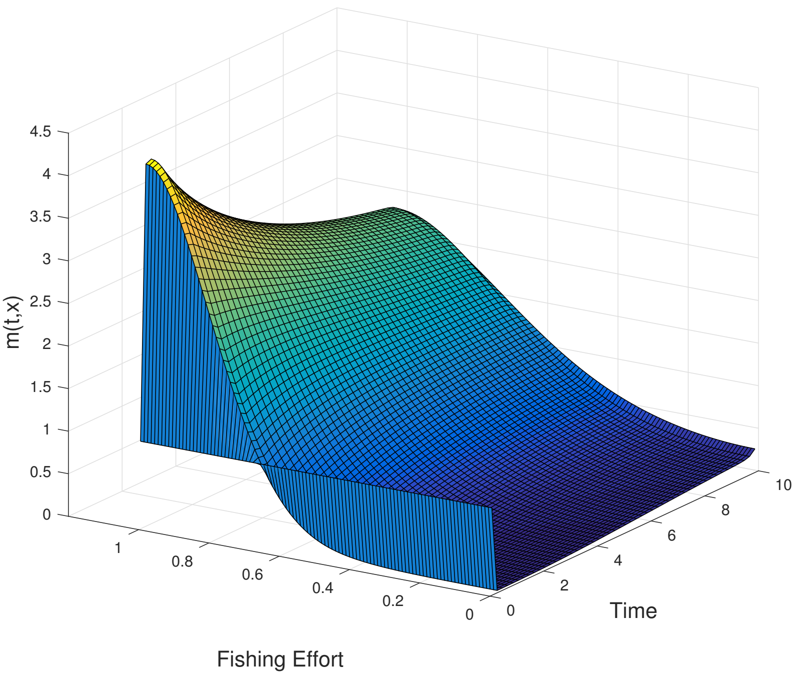
6.6 Volatility σ

Next we will consider the parameter σ. This is the intensity of the Brownian motion in our original stochastic differential equation that the MFG PDEs are based on, which we will here refer to as the volatility. Our basic case had $\sigma = 1$. Increasing σ to higher values than this confirms that the solution to our mean field game becomes more diffuse. In terms of the underlying game the PDEs are based on, this is because the increased volatility means that the distribution of agent's efforts will become more spread out as some individual agents are driven further from the theoretical optimum state by their individual Brownian motion. Figure 6.8 demonstrates this for the case $\sigma = 2$.

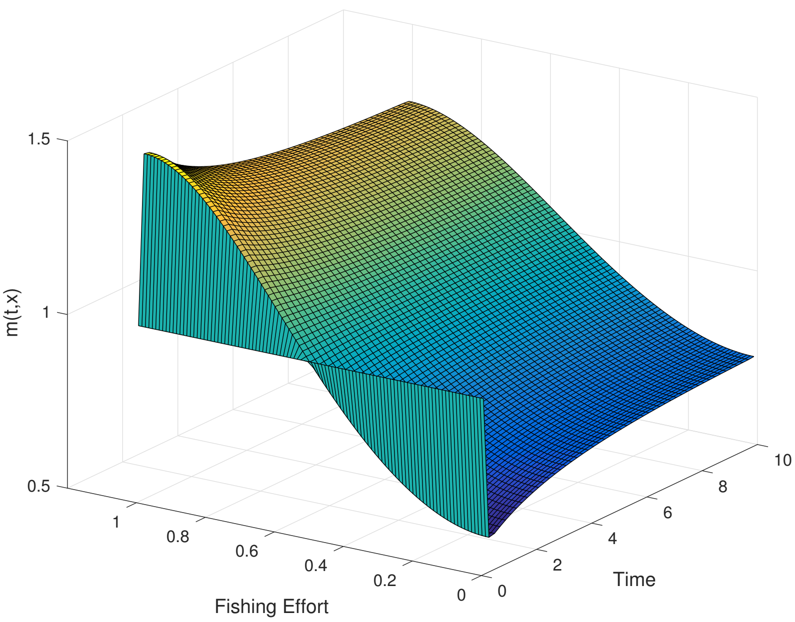
Comparing the impact on the stock with different values of σ, we confirm that the higher volatility and more diffuse effort distribution results in decreased fishing pressure and higher final stock $N(T)$, e.g. $N(T) = 27.686$ for $\sigma = 2$ compared to $N(T) = 10.518$ with $\sigma = 1$ (at least this is the case where high fishing effort is profitable - if very low fishing effort was profitable then increasing σ should increase fishing pressure).

Since increasing σ results in more diffusion in the distribution of effort, decreasing the volatility should result in sharper and less diffuse effort distributions. Testing this with different values $\sigma < 1$ confirms that this is the case, as shown in the case with $\sigma = 0.6$ in Figure 6.9.

However, decreasing the volatility from this base case to $\sigma = 0.5$, it was found we had to increase the number of timesteps n_t, and hence decrease the timestep size $\delta t = \frac{T}{n_t}$ in order to solve the mean field game system. Lowering the volatility σ appears to be one of the main reasons we may need to reduce the timestep size, so it is worthwhile to examine how σ will impact the necessary number of timesteps.

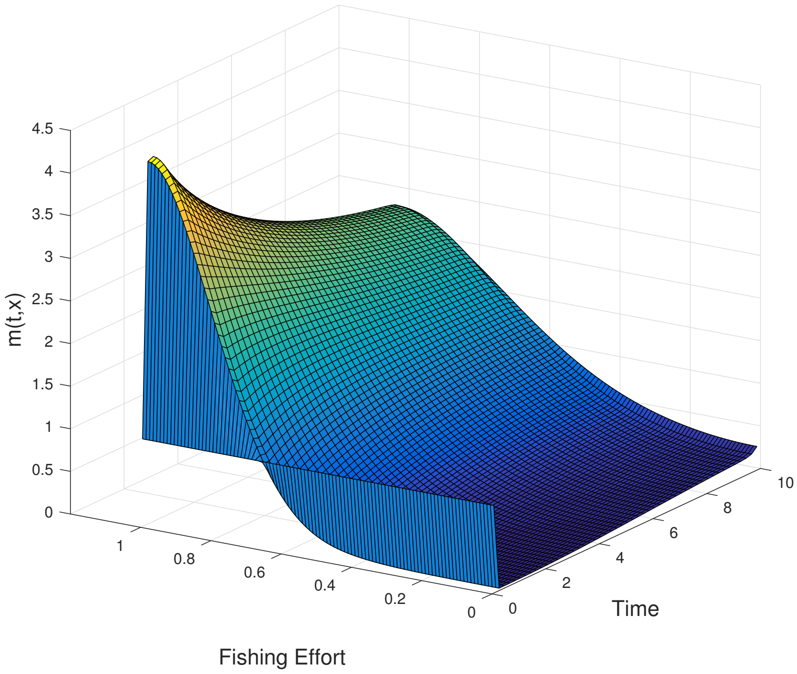


(a) $\sigma = 1$.

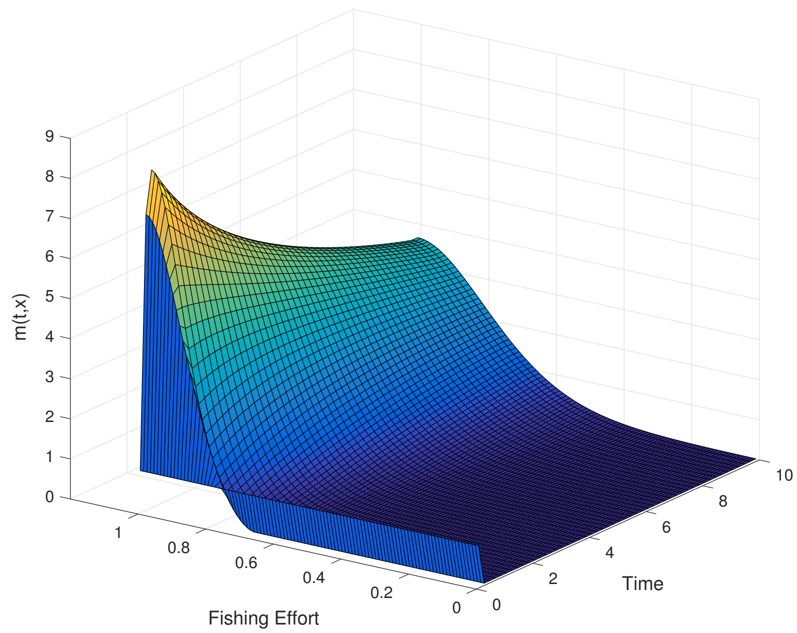


(b) $\sigma = 2$.

Figure 6.8: Surface plots of $m(t, x)$ with (a) volatility $\sigma = 1$ and (b) volatility $\sigma = 2$.



(a) $\sigma = 1$.



(b) $\sigma = 0.6$.

Figure 6.9: Surface plots of $m(t, x)$ with (a) volatility $\sigma = 1$ and (b) volatility $\sigma = 0.6$.

6.6.1 Stability and timestep size

As we reduced σ in the above example, we found that we required more and more timesteps and smaller timestep size in order for the numerical algorithm to solve. We wish to investigate how the number of timesteps required depends on the value of σ. To test this, we first set $T = 1$ so that we can solve equations requiring smaller timesteps more quickly, and then plot the number of timesteps required to obtain a solution against the value of σ. The plot showing the number of timesteps required for different values of sigma is shown in Figure 6.10.

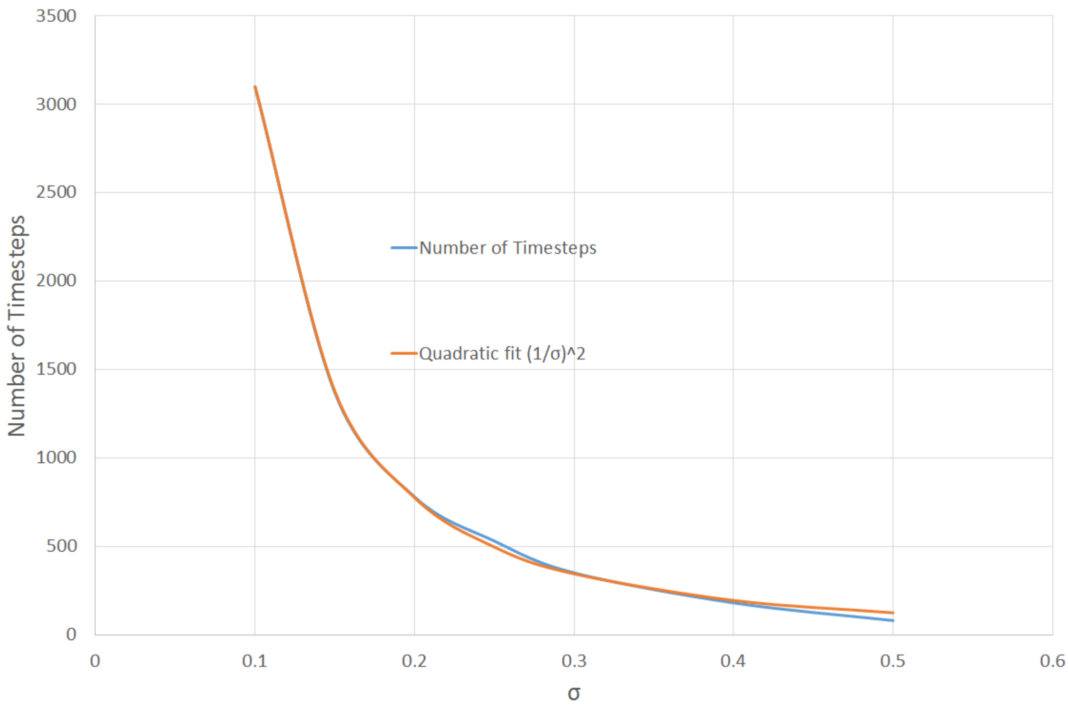


Figure 6.10: Plot showing number of timesteps required for different values of σ along with a curve of the form $y = c\left(\frac{1}{\sigma^2}\right)$.

We find the number of timesteps increasing steeply as we near $\sigma = 0.1$. Fitting a curve of the form $y = c\left(\frac{1}{\sigma^2}\right)$ against the plot of number of timesteps against σ we see that the growth in the number of timesteps appears very close to this, suggesting that the stability of our problem is of the order $\left(\frac{1}{\sigma}\right)^2$. We must keep this in mind when selecting our parameters and knowing how many timesteps might be expected to solve the equations, as a very low value of σ may result in a steep increase in the number of timesteps required.

In terms of why the timestep size depends on σ in this fashion, we find that with very low values of σ, the numerical method fails on the first stage, when attempting to solve the HJB equation using the initial guess for m and N. By inspection we can determine that this occurs due to values of u at each successive timestep (moving backwards from T) increasing rapidly until they become too large to continue to be processed using our algorithm in MATLAB.

This instability with low σ results from the fact that, for low values of the volatility, the distribution m tends towards a narrower peak, and when trying to solve the HJB equation using a distribution m that is different from this actual solution, this results in a bigger mismatch than occurs with a smoother distribution, resulting in large sharp increases in u that have the potential to blow up if the timestep is not sufficiently small.

As well as being aware of this dependence on σ when selecting our parameters, there are some other strategies that we can use to mitigate the issue. First, since the algorithm may fail because the values at individual grid points blow up, we can reduce the chance of this simply by using smaller values for parameters that will effect the value function u. Although it appears that the number of number of timesteps required is always of the order $\left(\frac{1}{\sigma}\right)^2$, we find that using high values for the parameters in the cost function (e.g. setting linear cost $c_1 = 200$, and price $p_0 = 100$) means we are more likely to run into the problem of failing to solve the HJB equation on the first pass without a high number of timesteps, while for low values (e.g. $c_1 = 0.2, p_0 = 0.1$) we can solve the system without needing to increase the number of timesteps beyond $n_t = 1000$ until we get to very low values of σ. This suggests it may be a good idea generally to scale the cost accordingly so that we may use lower values for the cost function within our actual algorithm.

Also, another parameter which affects the number of timesteps required is γ. In terms of the mean field game, the parameter γ is the quadratic cost of control, meaning it represents how difficult it is for an agent to quickly change their position. The parameters γ and σ are the two values that directly affect how an individual agent changes position in the mean field game, and in the coupled system of PDEs they are the two parameters directly included in the Fokker-Planck equation. So the value of σ in relation to γ is what will impact the timestep size required to solve the system - a higher value of γ means we can more easily deal with a lower value of σ, while lowering the value of γ means we may need to decrease timestep size more quickly.

When we do want to solve a system with low σ and low γ, rather than purely relying on decreasing the timestep size , we may consider an approach along the lines of that used by Achdou [18,91] to deal with mean field game systems with low volatility, which is to first solve for higher values of σ, then decrease the value of σ and use the previous solution as an initial guess. In our case, we use the solution for a higher value of σ as our initial guess for m and N when first solving the HJB equation. Using this approach, we are able to solve some systems with low σ using less timesteps than are required to solve it on its own - and when the number of timesteps that would be required is very high, then using this approach can be quicker even though it requires using the algorithm multiple times and gradually decreasing σ.

So when performing numerical simulations, we will be aware of how low values of σ and γ, as well as parameters that would lead to very high values in the cost function, may require a lower timestep size and hence a high number of timesteps to solve (particularly if we are using a high horizon time T). We also have the option of finding a solution with higher values of σ then gradually lowering σ to get the numerical solution when σ is low.

6.7 Initial Distribution m_0

The initial conditions are a key factor which will affect the solution of our system of equations. We have two initial conditions - the initial distribution, m_0, and the initial value of the stock or resource, N_0. How the results of the mean field game model simulation depend on the initial stock is a key question that we will consider when investigating different ecological scenarios. However, we also want to know how the choice of initial distribution affects our model.

Sometimes we may have a specific initial effort distribution for which we will want to see how it evolves, however in many cases we will be less interested in exactly what the initial distribution may be and more interested in the dynamics of the mean field game independent of the initial distribution. So in particular, we want to have an understanding of just how sensitive our model is to the initial distribution, and to have an idea of how quickly we may expect the choice of initial condition to be smoothed out by the dynamics of the MFG equations.

First, we compare the results of some simulations different with initial distributions m_0, using the parameters from the base case with $T = 1$, $\sigma = 0.5$ and $\gamma = 10$. Note that the higher value of γ, i.e. higher quadratic cost of control, means that agent's movement towards a different position should be slower, meaning the evolution of the distribution should move more slowly away from its initial conditions.

Figure 6.11 shows the resulting plots of $m(x, t)$ from two different initial distributions - a constant initial distribution (as we previously used in the base case), a Gaussian shaped initial distribution. As we can see the resulting distribution ends up looking similar well within the $T = 1$ horizon time, with the difference in the shape from the initial distributions fading away very quickly - there is also little difference between the resulting end value of the stock, with $N(T) = 35.23$ with constant initial effort and $N(T) = 35.07$ for Gaussian initial effort distribution (from an initial stock $N_0 = 50$).

Figure 6.12 shows $m(t, x)$ with an initial distribution where overall fishing effort is initially very low (with $m_0(x)$ linearly decreasing between $x = 0$ and $x = 0.2$, and $m_0(x) = 0$ for $x \geq 0.2$). This very low initial effort distribution is of particular interest as we have seen that with the base parameters used above, the distribution of effort tends to end up shifting towards the maximum effort, so we want to see how quickly the distribution would approach this shape if it initially is very low (meaning overall fishing effort is close to zero).

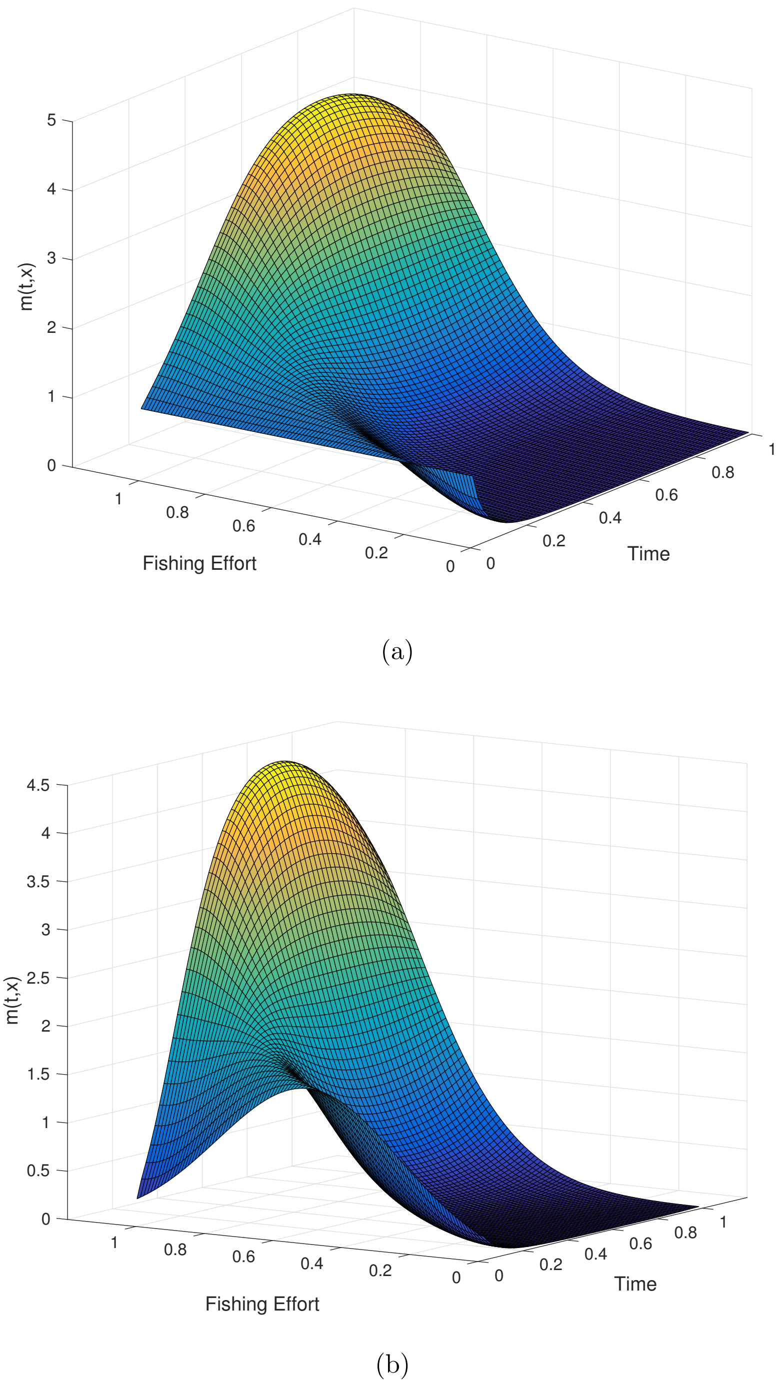


Figure 6.11: Surface plots of $m(t, x)$ with $T = 1$ and (a) constant initial distribution (b) Gaussian initial distribution.

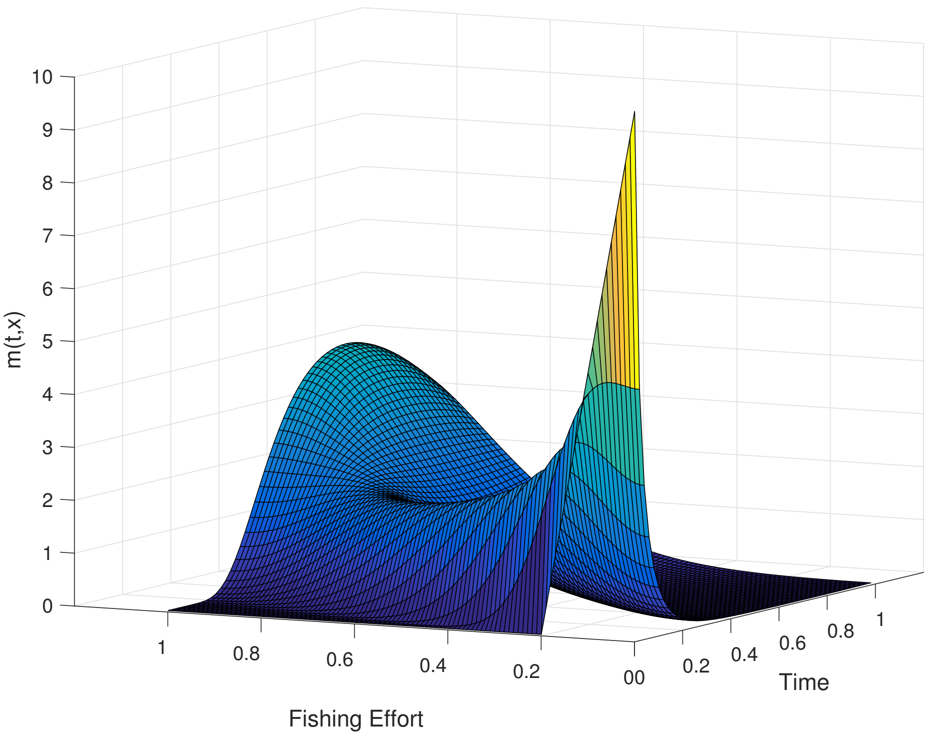


Figure 6.12: Surface plot of $m(t, x)$ starting from an initial distribution $m_0(x)$ where fishing effort is very low.

There is a more noticeable difference in the early stages of the case with very low starting distribution of effort, as we can see the evolution away from the low starting values of effort. However, it does not take very long before the distribution reaches a similar shape as in the examples above, occurring by around $t = 0.4$. The final value of the stock is $N(T) = 38.25$, so the time taken for the distribution of fishing effort to increase from its initial low value does an impact on the amount of stock that is left after one year. However, it is clear that the effect of the initial distribution in terms of the effort distribution $m(x,t)$ has mostly faded within one year, even with a very different starting initial distribution.

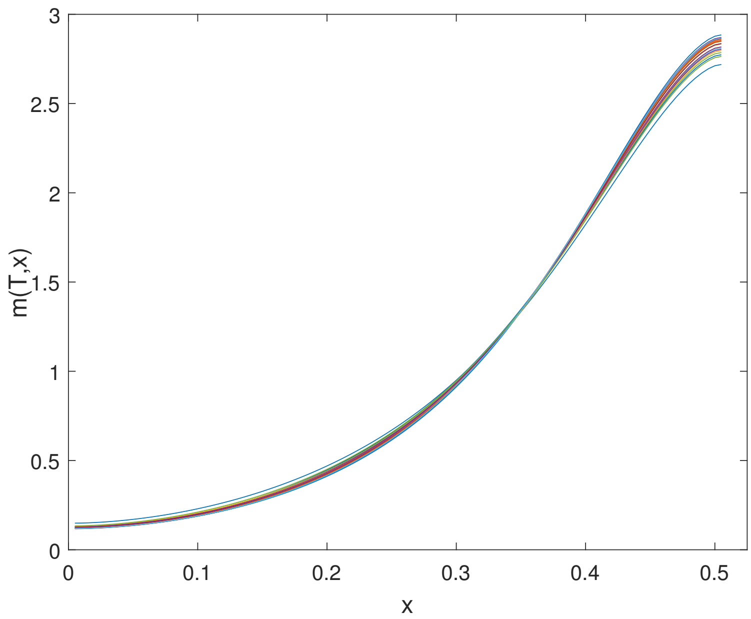


Figure 6.13: Plot showing $m(T,x)$ from 20 different simulations using randomly generated initial distributions m_0, with $T = 1$, $\gamma = 10$, $\sigma = 0.5$.

Figure 6.13 shows a plot of the resulting distributions $m(T, x)$ from 20 simulations each using initial distributions with the values for $m_0(x)$ drawn randomly from a uniform distribution at each value of x on the grid (then scaled to ensure it is a valid probability density function). The distribution at the horizon time $T = 1$ appears similar for all the different initial distributions, demonstrating that fluctuations in the initial starting distribution are smoothed out by the end of the horizon time for these parameters.

Since it appears that in general the initial distribution is smoothed out quite quickly, we will consider now only the difference between two extreme cases of initial distribution - one with very high initial effort, and one with very low initial effort. Figures 6.14 and 6.15 show the plots of $m(t, x)$ for at different values of t for the case where $m_0(x)$ is linearly decreasing between $x = 0$ and $x = 0.2$, with $m_0(x) = 0$ for $x \geq 0$ (labelled $m_{\text{low}}(x, t)$) and for the case where $m_0(x)$ is linearly increasing between $x = 0.8$ and $x = 1$, with $m_0(x) = 0$ for $x \leq 0.8$ (labelled $m_{\text{high}}(x, t)$).

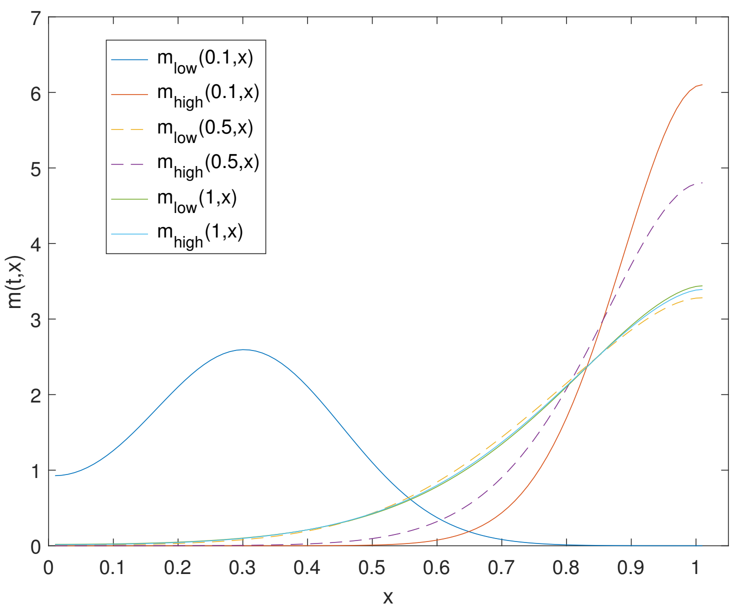
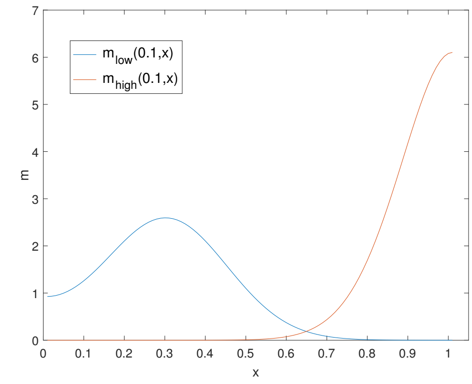
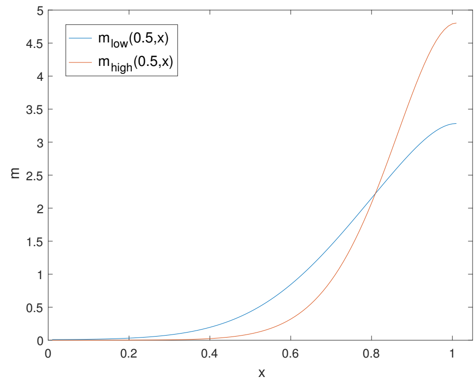


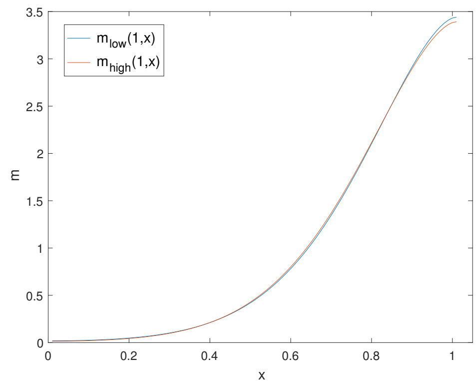
Figure 6.14: Plot comparing curves of $m_{\text{low}}(t, x)$ and $m_{\text{high}}(t, x)$ (low and high initial distributions of effort, respectively) at $t = 0.1$, $t = 0.5$, and $t = 1$, for $T = 1$, $\gamma = 10$ and $\sigma = 0.5$.



(a) $t = 0.1$



(b) $t = 0.5$



(c) $t = 1$

Figure 6.15: Plots showing $m_{\text{low}}(t, x)$ and $m_{\text{high}}(t, x)$ (low and high initial distributions of effort, respectively) for $T = 1$, $\gamma = 10$ and $\sigma = 0.5$ at (a) $t = 0.1$, (b) $t = 0.5$ and (c) $t = 1$.

From Figure 6.15(a) we can see that the distributions remain very different after a short time at $t = 0.1$, however as time progresses they become closer and from Figure 6.15(c) it is clear that by $t = 1$ the two distributions are very similar. This shows that even with the two extremes of high effort initial distribution and low effort initial distribution, the effort distribution at $t = 1$ with these parameters becomes very similar.

The simulations above had $\sigma = 0.5$ and $\gamma = 10$ - these parameters affect how quickly the distribution m evolves with time, so how long it will take for differences in the initial distribution to be smoothed out will depend on the value of σ and γ. A detailed comparison of the evolution from the initial conditions with different values of σ and γ is shown in Appendix B.3. There we see that higher values of σ and γ mean that it takes longer for simulations with two very different initial distributions to approach the same shape of distribution, while lower values of γ mean simulations will reach the same distribution more quickly - however, even for the high value of γ used and high value of σ we can see the distributions becoming similar in shape by $t = 1$.

So we have seen that differences in the initial conditions tend to be overcome by the dynamics of the mean field game problem quite quickly and do not dominate the results of the simulation beyond short periods in time. Initial conditions that are very different in shape (e.g. very low initial starting effort vs. very high initial starting effort) will have a more lasting effect on the results as it takes longer for them to reach the same distribution, but even then the effects are unlikely to be significant beyond $t = 1$, unless σ and γ are quite high. If γ and σ are low, the distribution will move away from the initial distribution quite quickly, as cost of movement for an agent and diffusion of agents is low.

We note that we have mainly focused on the effect on the distribution $m(t, x)$; of course, if the effort distribution in different solutions remains quite distinct for a longer period of time, then there will be a corresponding difference in the fishing pressure and hence on the stock value N. However, once the effects of the initial distribution have faded and the effort distributions become quite similar, the fishing pressure on the stock will be similar and N will follow a similar trajectory. So although we must make sure to take account of the impact that different initial conditions on the effort may have on the state of the stock, if we are more interested in the dynamics inherently associated with the ecological and economic parameters in the problem than how the result depends on the choice of initial effort distribution, we expect that we would not have to run the model for very long to get to the stage where the results don't depend greatly on the initial distribution.

Also, we have investigated the dependence on initial conditions with the parameters from our base case - we would not expect major qualitative differences to occur based just on changing the values of most parameters, as the same arguments regarding $m(t, x)$ moving towards optimal effort distribution for the current situation would still apply. However, if for example the cost function was quite different then we may expect to see more complicated dependence on the initial conditions.

6.8 Conclusions

We have presented an iterative algorithm for solving the coupled system of PDEs associated with the MFG model of fish stock exploitation, based on numerically solving the Hamilton-Jacobi-Bellman and Fokker-Planck equations individually and iterating until we find a numerical solution which satisfies both PDEs to a given tolerance. Testing this algorithm for a numerical solution with some basic parameters we can confirm that we can obtain a numerical solution which fits with our expectations, and see how we can interpret the output in terms of m, u and N. By testing the model with some different parameter values we have gained some understanding of how the results depend on the terminal cost, the horizon time T, the parameters σ and γ and the initial condition on the distribution m.

We found that the terminal condition on u did not appear to affect the solution significantly until it was close to the horizon time T. The initial conditions on m_0 may have more of an effect particularly if the initial distribution is quite different from the distribution that arises during the time period due to the parameters in the model. However, even quite different initial conditions in the distribution tend to become similar relatively quickly, this depends on the parameters γ and σ which determine how quickly the distribution can shift and how diffuse it is -in general, lower γ and lower σ mean that the initial distribution will have less impact on the overall solution.

We also identified situations where we may require decreasing timestep size in order to be able to solve the discrete system using our numerical method. Low values of σ, low values of γ, and high values in the cost or terminal cost function resulting in higher values of u are things that may require us to increase the number of timesteps. In particular the timestep size required appears to decrease as σ^2 decreases, meaning that very low values of σ may mean a very high number of timesteps is required. Using low values for the parameters within the cost function may help avoid issues with low σ, or we may wish to use a higher value of γ (meaning the distribution shifts more slowly). We can also solve the system for low values of σ by gradually decreasing the value from one which can be solved more easily, requiring fewer timesteps to do so when using a solution from a slightly higher value of σ as our initial guess.

Chapter 7

MFG Fisheries Model with different Ecological and Economic Conditions

In this chapter, we will investigate the impact of different ecological or economic conditions on the solution of our mean field game model of fish stock exploitation. Using the iterative algorithm described in the previous chapter to obtain numerical solutions to the MFG system formulated in Chapter 4, we will investigate how the solutions of the MFG system behave when we vary ecological or economic parameters or change the form of the functions describing the ecological or economic conditions in the model.

7.1 Ecological Conditions - Growth Rate

We will first examine how the solution of the MFG fisheries model behaves when we vary the ecological conditions. Recall that we are using a simple ODE based logistic growth model for our stock evolution:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = aN\left(1 - \frac{N}{K}\right) - L(N, m).$$

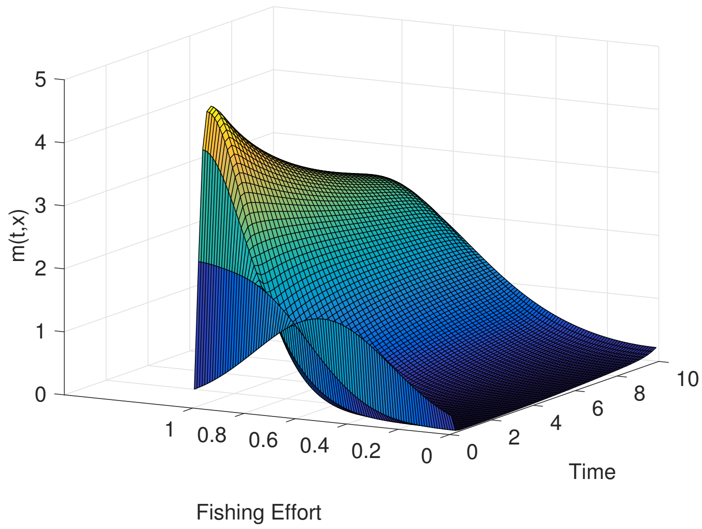
This simple growth model has two parameters, the carrying capacity K and the intrinsic growth rate which we have called a. Assuming that fishing pressure means that the fish stock is likely to remain at levels lower than the ecosystem's carrying capacity (which is likely to be the case in most scenarios that we would be interested in modelling), then the main impact of changes in K will be on the overall growth rate of the stock (since it is unlikely to approach the carrying capacity). So rather than considering changes in K, we will first consider changes in the intrinsic growth rate a, as this parameter is the one that more directly affects the overall growth rate of the stock.

For a base case to make comparisons, we will use the parameter set shown in Table 7.1, which is similar to the parameters in the previous base case but with $\sigma = 0.5$ and $\gamma = 10$. This means that the volatility σ is lower but that the cost of control (i.e. the cost of changing the level of fishing effort) is higher, so shifts in the distribution of fishing effort will be more gradual than with a lower γ. We will use $n_x = 100$ grid points in space and $n_t = 1000$ grid points in time for our dicretization by default, increasing only if required to solve the numerical system (though the high value of γ paired with a modest value of σ means we are unlikely to require a higher number of timesteps). We will set $u(T, x) = p_0 x$ as our terminal condition on u, (i.e. a linearly increasing function of x meaning that higher fishing effort at the horizon time T is more profitable, scaled to the fish price p_0) and set a Gaussian shaped initial distribution $m_0(x)$.

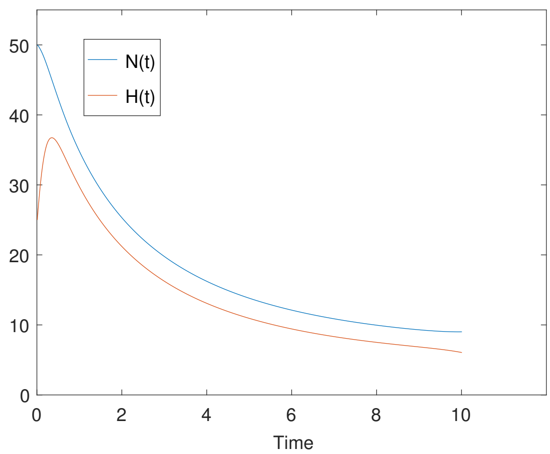
Parameter	Value
x_{max}	1
T	10
σ	0.5
r	0.05
q	1
p_0	1
c_1	2
a	0.75
K	100
γ	10
N_0	50

Table 7.1: Table of starting test parameters for the base case

Figure 7.1 shows the resulting $m(t, x)$ for the base case $a = 0.75$, as well as a plot of the stock $N(t)$ and harvest $H(t)$ over time. As in previous simulations, we see the effort distribution shift from the initial distribution towards the maximum effort, and then shift towards a distribution that remains steady for the rest of the time period. The stock $N(t)$ decreases steadily over the time period, while the harvest rate $H(t)$ (i.e. the total amount caught per unit time) first increases, then decreases along with the declining stock.



(a)



(b)

Figure 7.1: (a) Surface plot of $m(t, x)$ for growth rate $a = 0.75$, (b) Plot of $N(t)$ and harvest $H(t)$ for growth rate $a = 0.75$.

To examine what happens to the solution with different growth rates, we run the MFG model with various different growth rates from $a = 0$ to $a = 2$. The plots of results from selected values of a are shown in Figures 7.2-7.4. First, considering $a = 0$, i.e. there is no growth rate and so the fish stock is purely a consumable resource and not renewable resource, we see from Figure 7.2(b) that the stock N and the resulting harvest approaches zero as time progresses. In terms of the fishing effort distribution, there is first a shift towards the maximum effort until the resource is depleted to very low levels, after which the fishing effort more gradually decreases (shown in Figure 7.2(a) by the peak towards the left or maximum early on, followed by the shift towards low fishing effort afterwards).

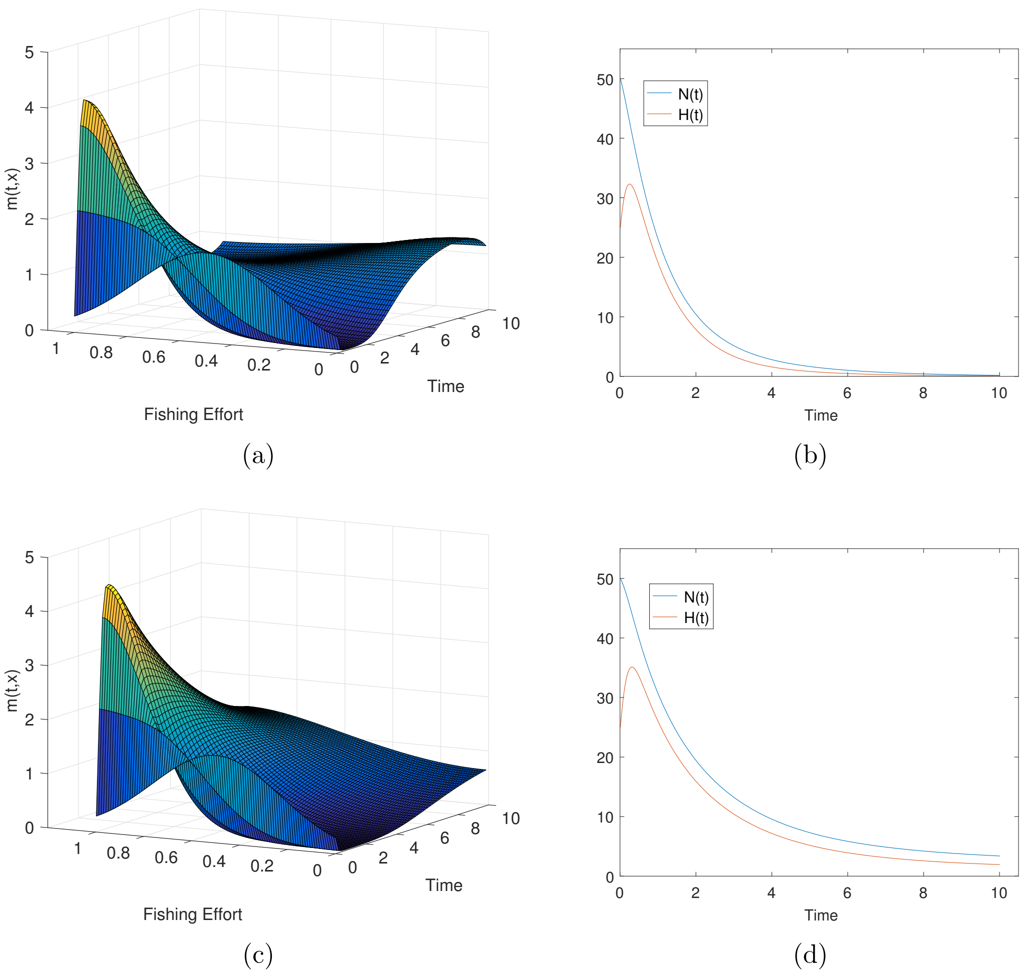


Figure 7.2: (a) Surface plot of $m(t, x)$ for $a = 0$, (b) Plot of $N(t)$ and $H(t)$ for $a = 0$, (c) Surface plot of $m(t, x)$ for $a = 0.5$, (d) Plot of $N(t)$ and $H(t)$ for $a = 0.5$.

Figure 7.2(c) and (d) show the results for $a = 0.5$, here we see that the stock is not quite brought towards zero within the horizon time but is significantly depleted, with the fishing effort distribution flattening out towards the end of the time period as it is no longer as profitable to fish at very high levels so more agents move away from the maximum.

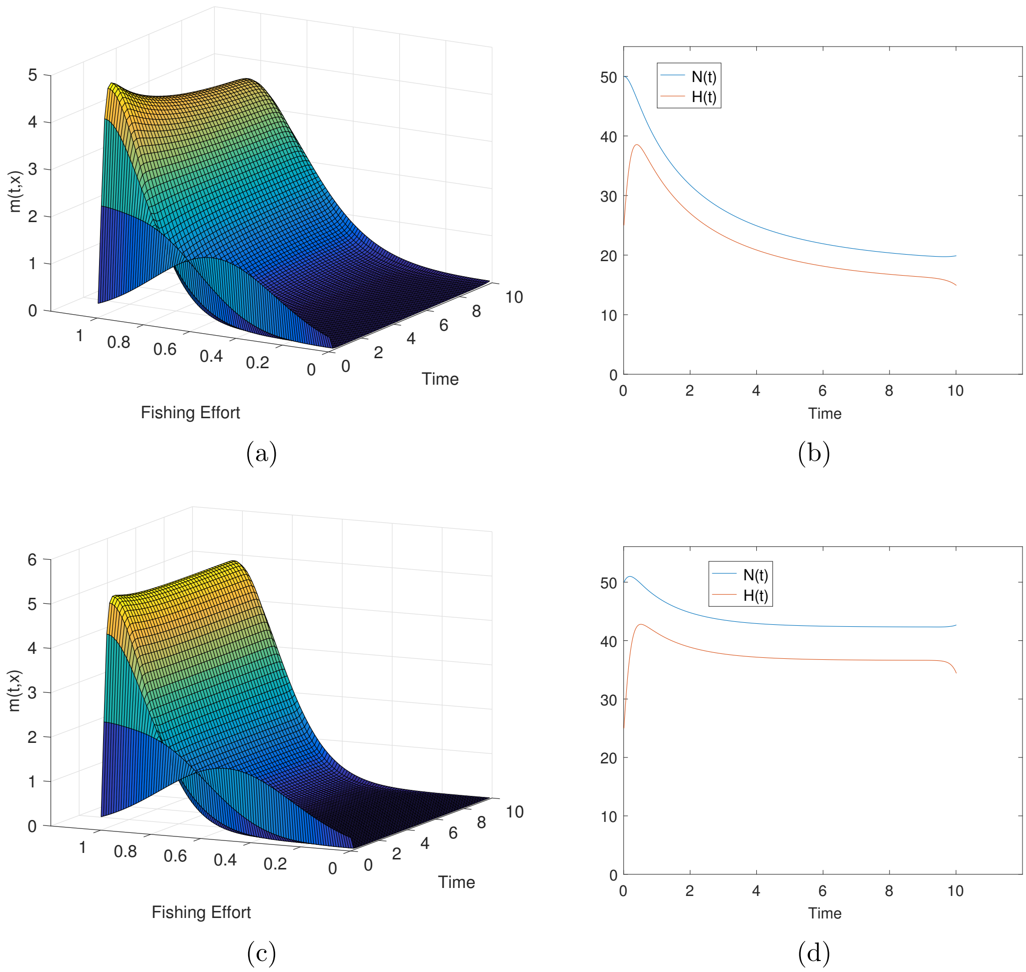


Figure 7.3: (a) Surface plot of $m(t,x)$ for $a = 1$, (b) Plot of $N(t)$ and $H(t)$ for $a = 1$, (c) Surface plot of $m(t,x)$ for $a = 1.5$, (d) Plot of $N(t)$ and $H(t)$ for $a = 1.5$.

In Figure 7.3(a) and (b), we see the resulting distribution and stock evolution for a growth rate $a = 1$, higher than our base case $a = 0.75$. Here we see that although the stock declines over the period, it is not depleted to such low levels, while the effort distribution appears to remain essentially steady throughout the time period after an initial spike in effort. Increasing a to 1.5 (as shown in Figure 7.3(c) and (d)) we see the stock initially increase from its initial value $N_0 = 50$, then remain fairly steady at a value around $N = 42$. The distribution m remains fairly steady throughout the time period.

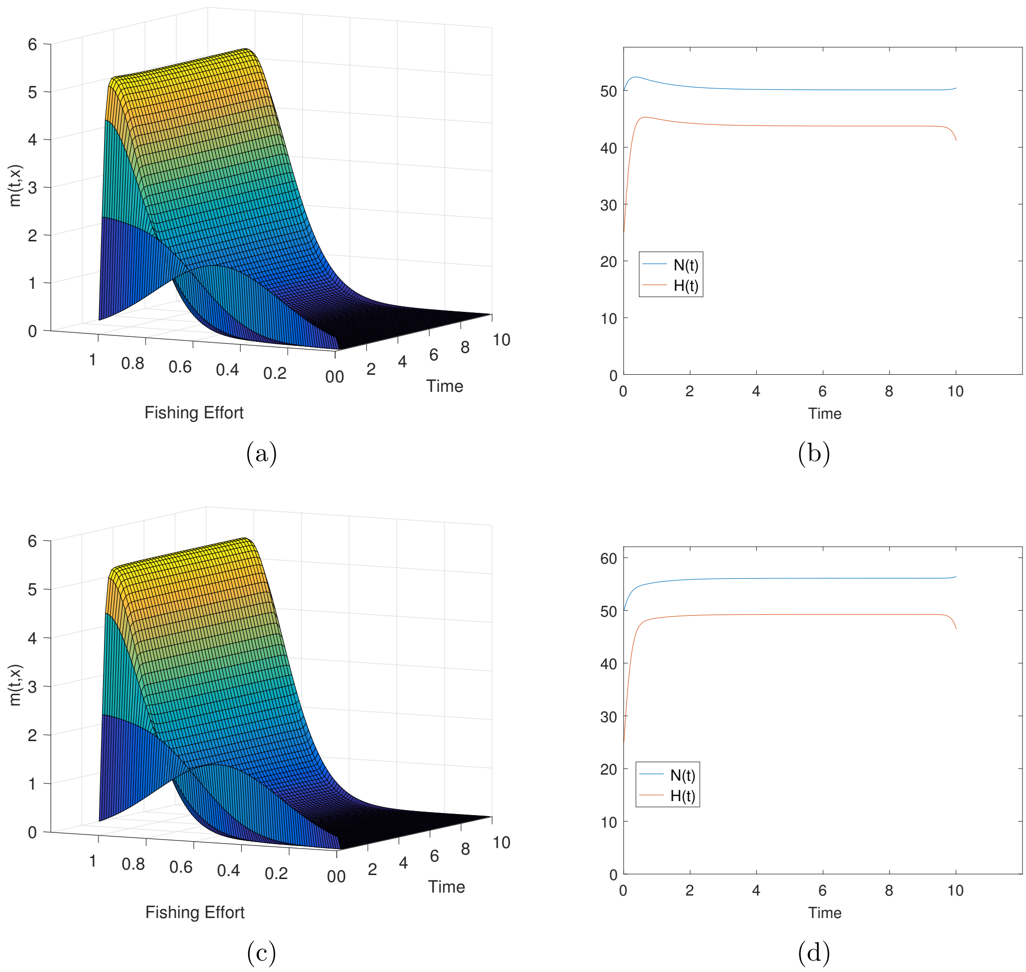


Figure 7.4: (a) Plot of $m(t, x)$ with growth rate $a = 1.75$, (b) Plot of $N(t)$ and $H(t)$ for $a = 1.75$, (c) Plot of $m(t, x)$ for $a = 2$, (d) Plot of $N(t)$ and $H(t)$ for $a = 2$.

When $a = 1.75$, we see in Figure 7.4(b) that the stock levels out around the value $N = 50$, which is half the carrying capacity of the ecosystem in the model, while the fishing effort distribution remains close to the maximum throughout the time period. Finally when $a = 2$, we see for the first time a noticeable increase in the fish stock over the time period (Figure Figure 7.4(d)). The fishing effort distribution is grouped very close to the maximum fishing level permitted in the model throughout the time period, as the high growth level allows high fishing activity while still resulting in a net increase in the fish stock.

Here we note that since we are using values of x between 0 (no fishing effort) and 1 (maximum effort permitted by an agent within the model), then the catchability parameter q gives the fishing mortality rate when fishing effort is at its maximum. So we expect that if q is higher than a then the fishing activity should be capable of driving the fish stock to very low levels, while if a is significantly higher than q then even if fishing activity is at the maximum possible within the model, the fish stock's growth rate should overtake it once the stock level is low enough. So since we have set a value $q = 1$, we know that for values of a much larger than this the stock will always increase to some level which it cannot be brought below, as the growth rate will be higher than the maximum mortality due to fishing.

Comparing different growth rates we have an idea of how the solution to the MFG Fisheries model will behave depending on how the growth rate relates to the catchability q (and maximum value of x). As expected, low growth rates will lead to the stock collapsing or reaching very low levels - we can also see how the effort distribution will evolve, with the overall effort increasing towards the maximum when it is profitable and then decreasing once the stock is depleted, thus providing evidence of the Tragedy of the Commons type dynamic in the mean field game when the growth rate of the stock is low. When the growth rate is very high we can see that it supports very high levels of fishing effort, and in cases where the growth rate is high (but not significantly higher than the maximum possible fishing mortality rate) we have seen examples of the decline in fish stock appearing to level off.

7.2 Ecological Conditions - Initial Stock

We have a good idea of how we would expect our solutions to depend on the growth rate of the fish stock. However, a key question is how the solution of the MFG depends on the initial stock value. As well as knowing just how sensitive to the initial conditions our dynamics are, this is also crucial for considering problems of stock recovery.

First, we will consider the example above where $a = 1.75$. As shown in Figure 7.4(b), in this case the fish stock ended up remaining relatively constant around the value $N = 50$, which was the initial value. We want to establish to what extent this depends on the initial stock - did the stock end up remaining close to the value 50 because it was the initial value, or was this value intrinsic to the problem parameters? To test this we ran the MFG simulation with the same parameters setting the initial stock to two extremes; $N_0 = 100$ (the carrying capacity) and $N_0 = 1$ (stock near collapse).

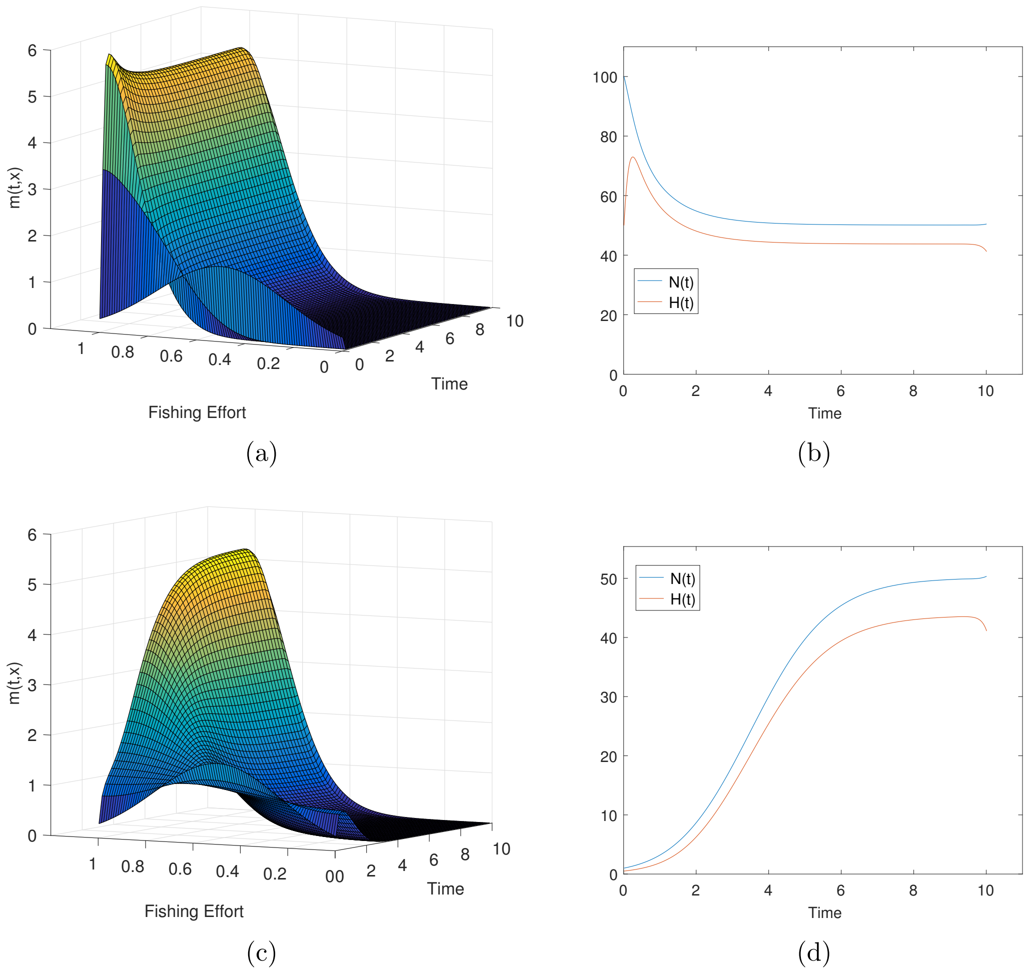


Figure 7.5: (a) Surface plot of $m(t, x)$ with growth rate $a = 1.75$ and initial stock $N_0 = 100$, (b) Plot of $N(t)$ and $H(t)$ with $a = 1.75$ and $N_0 = 100$, (c) Surface plot of $m(t, x)$ with $a = 1.75$ and $N_0 = 1$, (d) Plot of $N(t)$ and $H(t)$ with $a = 1.75$ and $N_0 = 1$.

The results shown in Figure 7.5 confirm that the fish stock does approach the value of approximately 50 even starting from very high or very low initial stock value. In Figure 7.5(a) and (b) we can see that with $N_0 = 100$, the distribution of fishing effort quickly approaches a steady shape close to the maximum effort, and the stock declines quite quickly from 100 before levelling off close to 50. When $N_0 = 1$, Figure 7.5(c) shows that the distribution of fishing effort takes longer to move towards the same peak at the maximum as seen in Figure 7.5(a), but the overall fishing effort does increase right from the start of the time period, even though the initial value of N is low. Figure 7.5(d) shows that despite the high fishing effort, the stock grows from its initial low value, with the growth slowing as it approaches $N = 50$, appearing to start levelling off as we get towards the end of the time period for this simulation.

So we can see that the stock appears to be approaching the same value regardless of the initial value of the stock. This fits with what would be expected considering that we have a very high growth rate (compared to the potential fishing activity within the model), so the stock is able to grow enough to overcome fishing pressure even when the distribution of fishing effort is close to the maximum, up to a certain value of N where the growth rate is balanced against the fishing mortality from the profitably distribution close to the maximum.

However, a more interesting scenario is one where the growth rate is not high enough that will always overcome even the maximum fishing effort. So we will return again to use the growth rate $a = 0.75$, one which is lower than our catchability q but high enough that the stock should not collapse unless there is relatively high fishing effort. In particular, we are interested in if the agents in the MFG do indeed allow the stock to recover even though it would be possible to keep fishing to overcome the stock's intrinsic growth rate.

Figure 7.6 shows surface plots of $m(t, x)$ and plots of $N(t)$ and $H(t)$ for the case with $a = 0.75$ with $N_0 = 100$ and $N_0 = 1$. From Figure 7.6(a) and (b) we see that with $N_0 = 100$, the distribution of effort moves sharply towards the maximum at first, before smoothing out into a flatter distribution (still at high fishing effort) for the rest of the time period, while the stock N decreases throughout the time period, though more rapidly at first. In Figure 7.6(c) however, we can see that at first the distribution of fishing effort shifts towards zero from its initial condition of a Gaussian distribution centred around $x = 0.5$, meaning that the overall fishing effort decreases at first due to the low value of the stock. However, it does not take long before the effort starts to increase again - a close inspection reveals that shortly after $t = 1$ is when the overall effort starts increasing again, although slowly at first, even though the stock N has only reached a value of $N = 1.32$ by that point. The distribution of fishing effort shifts towards higher fishing effort, although it does not yet reach the same high level of fishing effort as the distribution when $N_0 = 100$. From 7.6(d) we can see that the stock increases from its initial low value of $N_0 = 1$, and continues to increase throughout the time period even as the fishing effort starts to increase again. Although the increase in the stock becomes slower towards the end, we do not appear to have reached the point where the stock levels off within this time period.

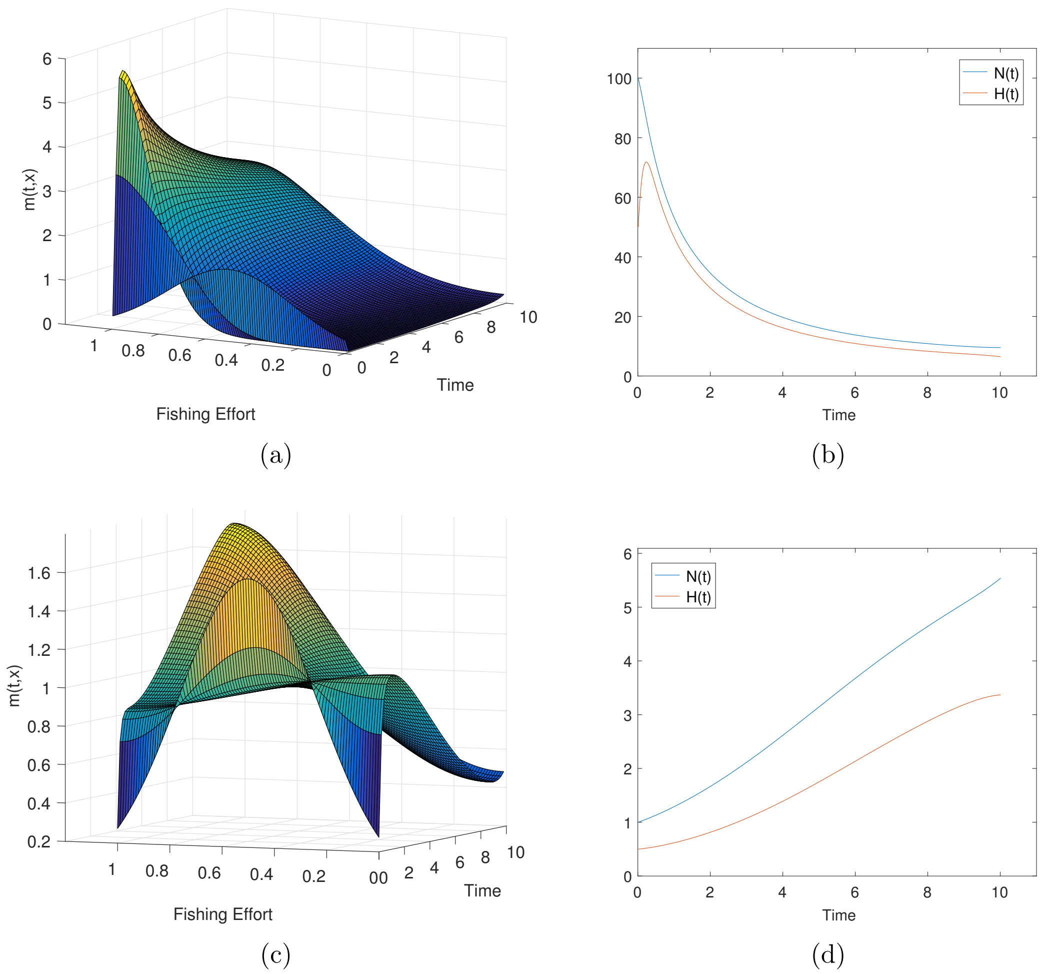


Figure 7.6: (a) Surface plot of $m(t,x)$ with growth rate $a = 0.75$ and initial stock $N_0 = 100$, (b) Plot of $N(t)$ and $H(t)$ with $a = 0.75$ and $N_0 = 100$, (c) Surface plot of $m(t,x)$ with $a = 0.75$ and $N_0 = 1$, (d) Plot of $N(t)$ and $H(t)$ with $a = 0.75$ and $N_0 = 1$.

To continue to see how the initial stock value affects the solution and to find if there's a value that serves as a steady state for the stock, we run the MFG model starting from $N_0 = 10$ (which was close to the end value of N in the case where $N_0 = 100$ where the curve appeared to be flattening).

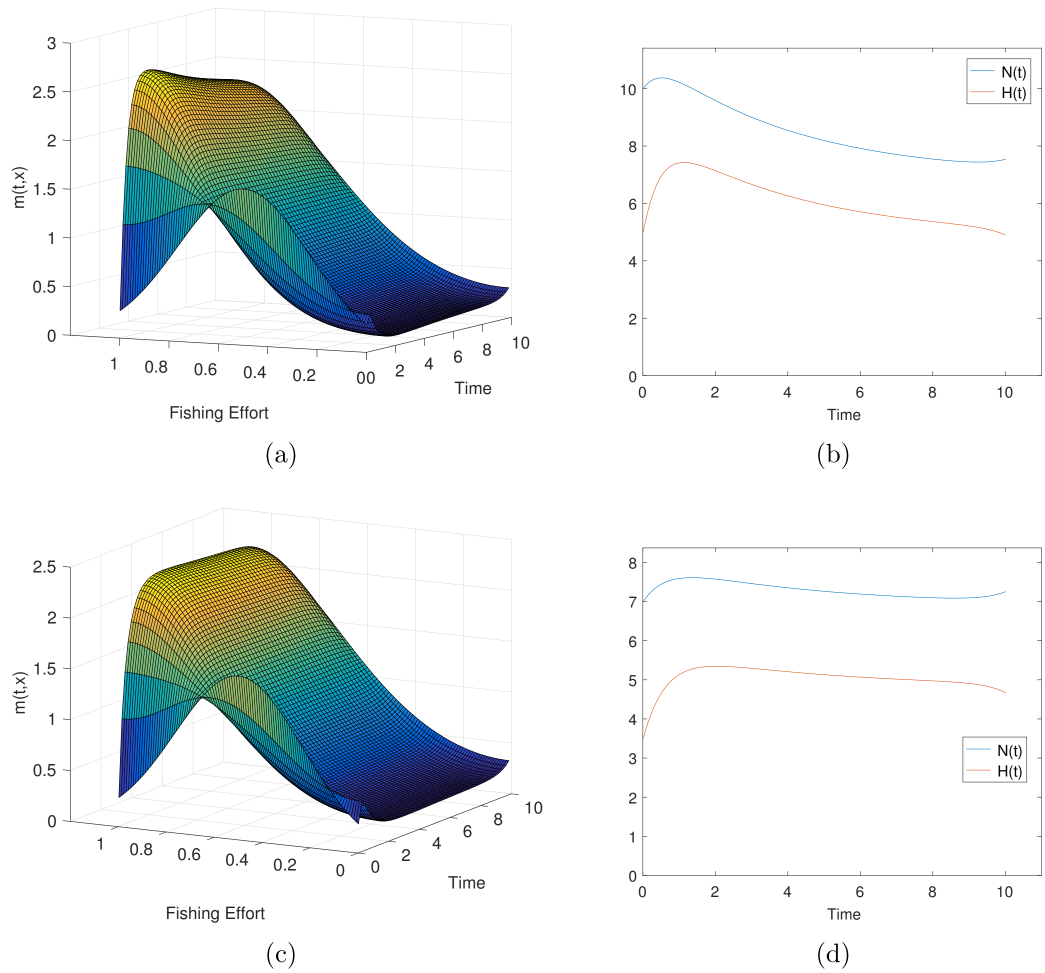


Figure 7.7: (a) Surface plot of $m(t, x)$ with growth rate $a = 0.75$ and initial stock $N_0 = 10$, (b) Plot of $N(t)$ and $H(t)$ with $a = 0.75$ and $N_0 = 10$, (c) Surface plot of $m(t, x)$ with $a = 0.75$ and $N_0 = 7$, (d) Plot of $N(t)$ and $H(t)$ with $a = 0.75$ and $N_0 = 7$.

We see from Figure 7.7(b) that the stock decreases, ending up at a value $N(T) = 7.54$. Checking an initial condition lower than this, ($N_0 = 7$ as shown in Figure 7.7(c) and (d)) we find that the stock value increases slightly over the period to $N(T) = 7.255$. Repeated runs using the previous values $m(x, T)$ and $N(T)$ as the initial distribution for the next run find that $N(t)$ remains at a value close to $N = 7.2$, and running the MFG simulation from different initial stock values over longer time horizons, we find that the stock moves towards this steady state value from any initial condition.

By testing some different initial stock values, we have seen that the initial stock definitely makes a difference to the evolution of the stock itself and to the effort distribution, and this impact is not one that is lost from the system after a short period of time (with quite different final values of the stock and effort distribution possible even after a horizon time $T = 10$). So the result of a dynamic mean field game simulation will definitely be sensitive to the initial stock value. However, in the longer run it appears that we can expect the stock to approach the same value (and the effort distribution to approach a similar curve), and we should be able to tell something about the dynamics of the system regardless of our choice of initial stock value.

In particular, we also found that it is possible for stocks to recover somewhat, even when the growth rate is lower than the catchability q. This occurs because it is not profitable to fish at very low stock values and so m shifts to a lower effort distribution, allowing the stock to recover. So, at least for these cost parameters where it is unprofitable to fish at very low stock values, the mean field game does not result in complete collapse of the stock (unless the growth rate is zero or extremely low). But this does not mean that it does not result in an example of the "Tragedy of the Commons", as the eventual level that the stock will be reduced to is unlikely to be the ecological, economic or social optimum. And of course, in our model so far we are using a simple logistic growth ODE, which allows recovery of the stock from any non-zero population. In reality reducing fish stocks to very low levels may mean that they cannot recover.

7.3 Economic Conditions - Price Flexibility

Having determined how the solution will behave for different growth rates in the ecological model and different initial states of the fish stock, we now want to determine the effect of changes to the economic conditions in the model. As well as being able to represent different fish stocks or ecosystem states through the choice of ecological model, this is the main way that we will be able to make use of the MFG Fisheries model - changes to the utility function of the agents will lead to different solution to the mean field game.

Assuming that the ecological component remains unchanged, then economic factors that will affect the utility function of the agents include the costs as a function of effort, the price received for landing fish, the cost of control (i.e. the cost of changing position in terms of fishing effort), and the discount rate (i.e. the rate of return by which future profit is discounted by to account for the cost of capital).

How the solution to the MFG depends on the price per unit landing is one of the key things we want to investigate. In our analysis in Chapter 5, we have shown that, at the UK level, ex-vessel fish price depends on the total landings of that fish (both at the individual species level and at the level of broader functional groups) for several fish species and some broad guilds of fish, particularly demersal fish. So far in our numerical simulations we have considered price to be constant, and this may be appropriate depending the scale of the fishery considered. However, particularly since we have identified negative own-price flexibilities for several species in the UK market, we wish to consider how the solution is impacted when comparing a constant price with a price which depends on landings via a negative own-price flexibility.

We know that price depends on a number of other factors such as imports and exports, and so we could have a more complicated price function with various external parameters. However, similar to the choice to use a logistic population growth function, rather than one of the multitude of other more complicated ecological models available, we will simply set a price flexibility $-\delta$ (as described in Chapter 5), which captures the dependence on landings that we are interested in for our model. We will also set within our model a very high price p_{max} which the price can't go above, to ensure we do not end up with extremely high prices (either in the numerical solution or in any of the iterations within the numerical algorithm).

This essentially assumes that other factors which may affect the price remain constant, but that they have the effect of limiting the dependence of price on landings up to a certain maximum price p_{max} (this could represent, for example, that the price may not increase to more than a certain level above the global import price). More complex dynamics are possible but since they depend greatly on species or functional group and region (see Chapter 5) we may consider this general case to examine the impact of price flexibility on the results of our simulations.

For the base case with constant price (i.e. $\delta = 0$) we will use the same base case as in our comparison of the growth rates, as shown in Figure 7.1 (with a growth rate $a = 0.75$). We will now include a price flexibility of $-\delta$, which means that the relative change in price per unit time is approximately $-\delta$ times the relative change in landings or harvest per unit time. We will only consider negative own-price flexibility here (i.e. $\delta \geq 0$), meaning price will increase if landings decrease and vice versa.

We first run the MFG model with $\delta = 0.4$, i.e. a price flexibility of -0.4, which as shown in Chapter 5 is close to the estimated price flexibility of commercially important species such as cod or herring. Figure 7.8 shows results of the simulation with $\delta = 0.4$, and from Figure 7.8(a) and (b) we see that the shape of the distribution $m(t, x)$ and the trajectory of the stock appears similar to that in Figure 7.1(a) and (b). However it was found that the overall fishing pressure was slightly higher in the case with price flexibility 0.4 as the distribution of fishing effort is grouped slightly closer to the maximum, resulting in a lower final stock value $N(T) = 7.390$ compared to $N(T) = 9.013$ in the case with constant price.

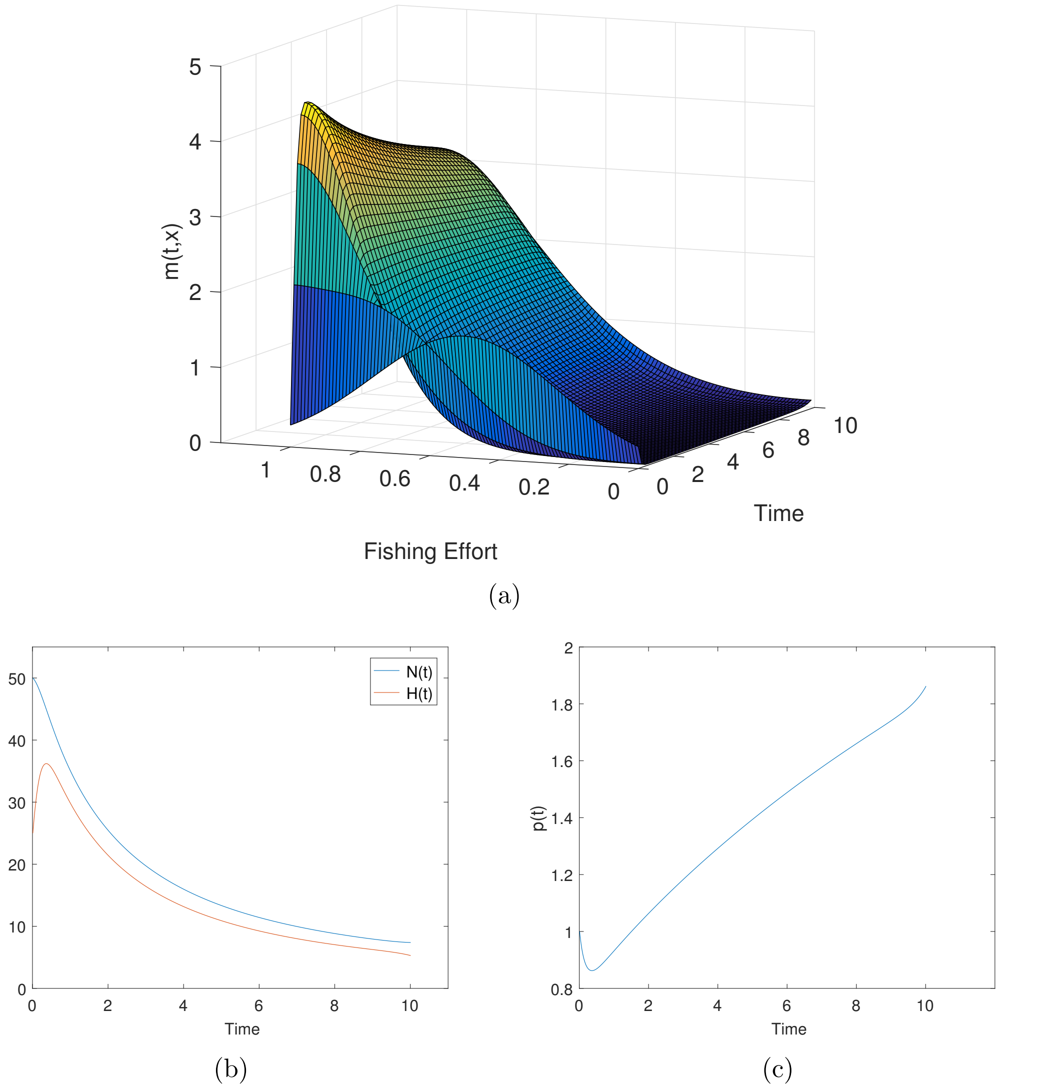


Figure 7.8: (a) Plot of $m(t,x)$ with price flexibility parameter $\delta = 0.4$, (b) Plot of stock $N(t)$ and harvest $H(t)$ with $\delta = 0.4$, (c) Plot of price $p(t)$ with $\delta = 0.4$.

We can see in Figure 7.8(c) the evolution of the fish price $p(t)$ over the time period, with first a decline from the initial value $p_0 = 1$ as harvest increases (as shown in Figure 7.8(b)), followed by a steady increase in price as the stock and hence harvest declines.

So we have found here that introducing a negative price flexibility to the model resulted in a decrease to the fish stock over the time period compared to the case with constant price. Examining our solution suggests this occurs because, as the fish stock declines from its initial high levels, the resulting harvest also declines, and as there is a negative relationship between the harvest (or landings) and the price received, the increase in price results in it being more profitable to fish as the stock declines than in the case with no price flexibility. So slightly more agents fish at higher levels as the stock declines in the case with negative price flexibility than if the price remained independent of landings.

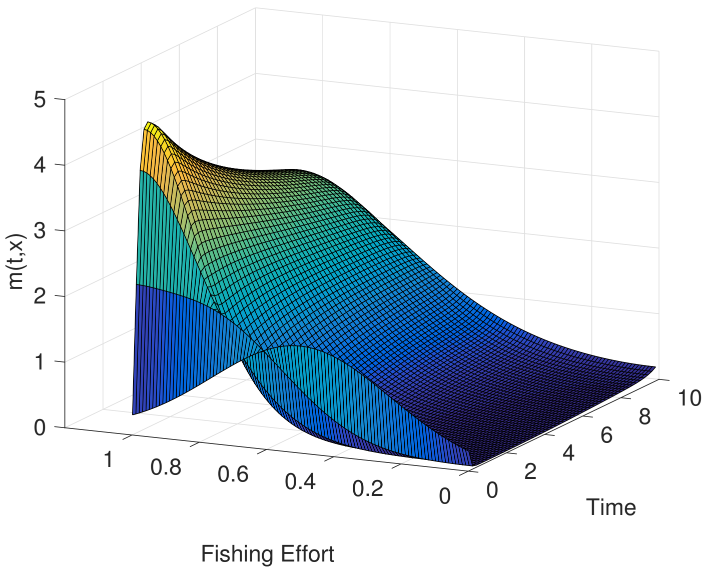
To see how different values of the price flexibility affect the solution, we run the MFG model with these parameters varying δ from 0 to 1 in increments of 0.1.

Figures 7.9 and 7.10 show plots for the cases $\delta = 0.1$ and $\delta = 1$ respectively. When $\delta = 0.1$ we get very similar results to the base case with just a slight impact due to the price flexibility, while for $\delta = 1$ we can see a noticeable shift towards higher effort in the distribution in 7.10(a), due to the very high price flexibility resulting in significantly higher prices as the harvest decreases, as shown in 7.10(c).

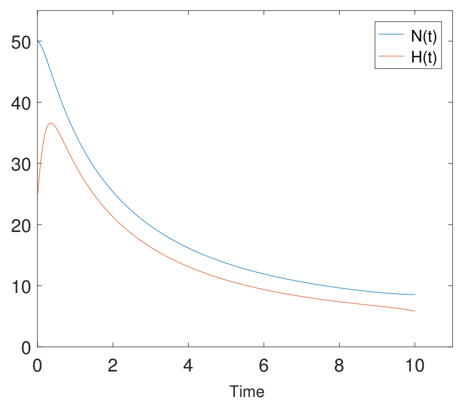
The results for different values of δ all had similar forms, so the best way to examine the impact of different price flexibilities them is to look at the final stock value $N(T)$, to see how the different distribution of fishing effort under different price flexibilities affects the state of the stock at the end of the time period.

Figure 7.11 shows the resulting final stock values $N(T)$ for different values of δ from 0 to 1, and we can see clearly that the stock level after the period of 10 years decreases as the negative own-price flexibility of the fish increases. The presence of a negative own-price flexibility means that as the harvest decreases due to the stock levels getting lower the price of the fish will increase, and the greater the negative price flexibility the greater the increase in price and hence the more profitable it becomes to keep fishing even as the stock reaches low levels, resulting in increased overall fishing effort and decreasing the stock quicker.

This makes intuitive sense, however we still find the result quite striking because, when considering the motivation for studying the dependence of fish price on landings, the scenarios of interest were often ones where landings are reduced due to quotas or other regulations to allow stock recovery. In that case, knowing that there were negative own-price flexibilities meant that the revenue lost by reducing fishing may be



(a)



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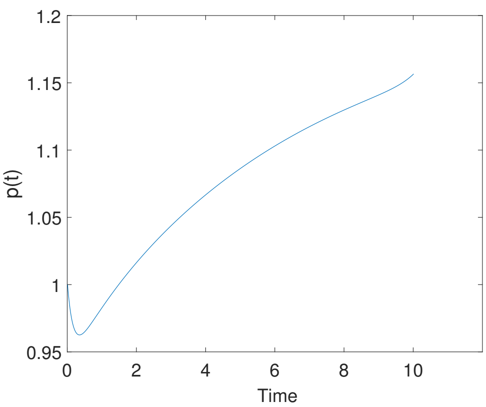
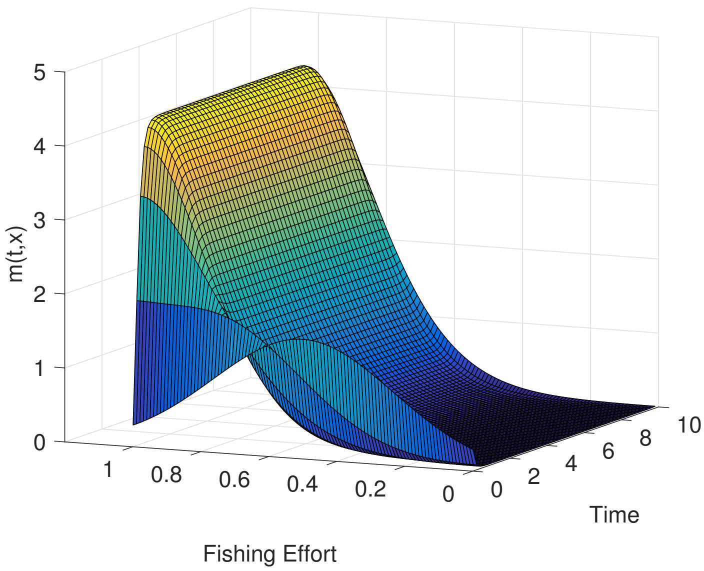


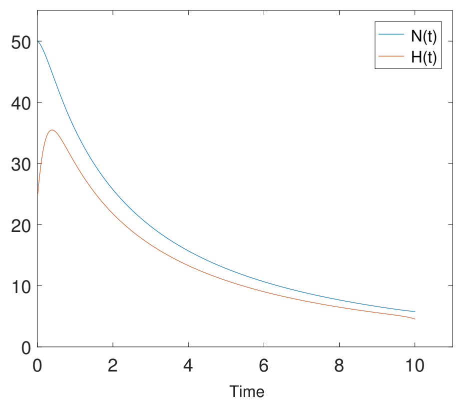
(c)

Figure 7.9: (a) Plot of $m(t, x)$ with price flexibility parameter $\delta = 0.1$, (b) Plot of stock $N(t)$ and harvest $H(t)$ with $\delta = 0.1$, (c) Plot of price $p(t)$ with $\delta = 0.1$.

partially compensated for by an increase in ex-vessel price, hence the existence of price flexibilities (as opposed to a price that would not vary with landings) could be seen as an additional argument in allowing stock to recover by reducing fishing pressure. Here though, we see clearly that (in the absence of any form of regulation), a negative price flexibility may provide additional incentive to increase fishing effort, as decreases in harvest due to declining fish stock will be partially compensated by the increase in price.



(a)



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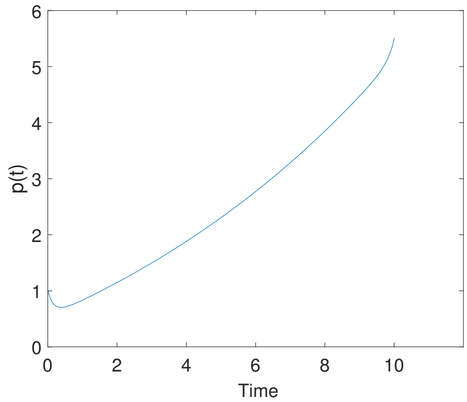


(c)

Figure 7.10: (a) Plot of $m(t, x)$ with price flexibility parameter $\delta = 1$, (b) Plot of stock $N(t)$ and harvest $H(t)$ with $\delta = 1$, (c) Plot of price $p(t)$ with $\delta = 1$.

However, we have found that increased negative price flexibility may not always result in an increase in fishing pressure and hence decrease in stock. When we considered ecological conditions with higher growth rates, we found that increasing δ in fact often caused decreasing fishing pressure and increased values of the stock by the end of the time period. Figure 7.12 shows the resulting final stock values $N(T)$ for different values of δ in the case where $a = 1.75$.

We see here that the stock remaining at the end of the time period increases as

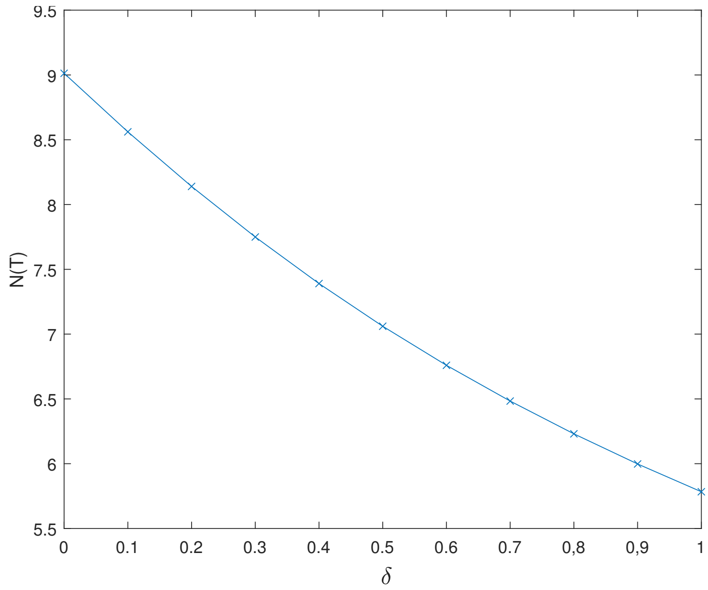


Figure 7.11: Plot of $N(T)$ against δ showing the resulting final value of the stock $N(T)$ for different price flexibilities, for growth rate $a = 0.75$

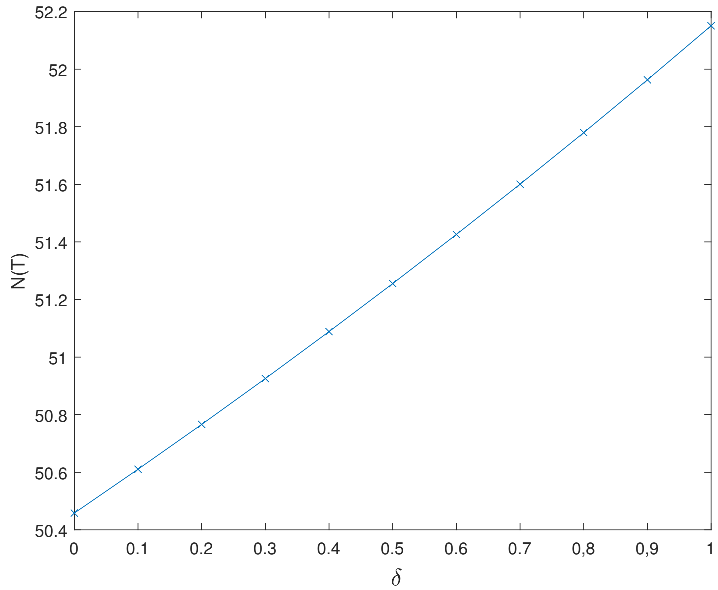


Figure 7.12: Plot of $N(T)$ against δ showing the resulting final value of the stock $N(T)$ for different price flexibilities, for growth rate $a = 1.75$

the negative own-price flexibility increases. To interpret why this is the case, recall the growth rate $a = 1.75$ is higher than our catchability q and hence higher than the maximum possible fishing effort that can be applied in the game. In this case, the agents in the game cannot lower the stock significantly (as the growth rate will become higher than the total fishing mortality), so could not drive the stock down (and hence drive the harvest down) by fishing at high levels. So instead, any increase in harvest due to increasing fishing effort towards the maximum results in a decrease in the fish price, that then means it is slightly less profitable to fish at high levels, and so the overall distribution of fishing effort will shift slightly further from the maximum.

In most situations that we would be interested in modelling, the maximum possible fishing effort we consider would likely be higher than the intrinsic growth rate of the fish stock, so the situation where the fishing pressure decreases due to price flexibility because of the very high growth rate (compared to the potential fishing mortality) is less likely to occur in practice than the previous situation where it is possible for fishing activity to drive the stock down to very low levels. However, this demonstrates the complicated feedback between the ecological and economic parts of the system, as the impact of the price flexibility on the expected distribution of fishing effort will depend on the ecological properties of the fish population. It is also another example of a "Tragedy of the Commons" type phenomenon arising from the MFG model, as the presence of a negative fish price flexibility (which could partially compensate lost revenue from decreasing fishing activity to allow stocks to maintain higher levels) actually incentivises increased fishing activity, and in fact is more likely to do so on fish stocks that have lower growth rates and are perhaps more at risk of collapse.

7.4 Economic Conditions - Changing the Cost Function

Having considered the impact of including price flexibility in the MFG model, we will now consider how the solution behaves when we alter other parameters or terms in the cost function. This is the primary way that we can investigate different scenarios using the MFG model, as for a given ecological model we can investigate how the solution will depend on different potential economic parameters, or how the solution might change depending on the inclusion of different terms in the cost function.

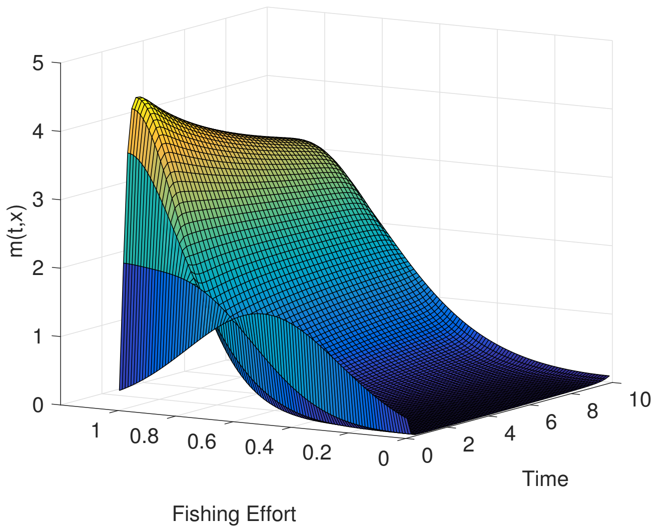
In the cases we have considered so far, it has usually been profitable to fish close to the maximum level unless the stock level became very low. This is because we have considered a cost function of the form:

$$qxNp - c_1x - c_2x^2$$

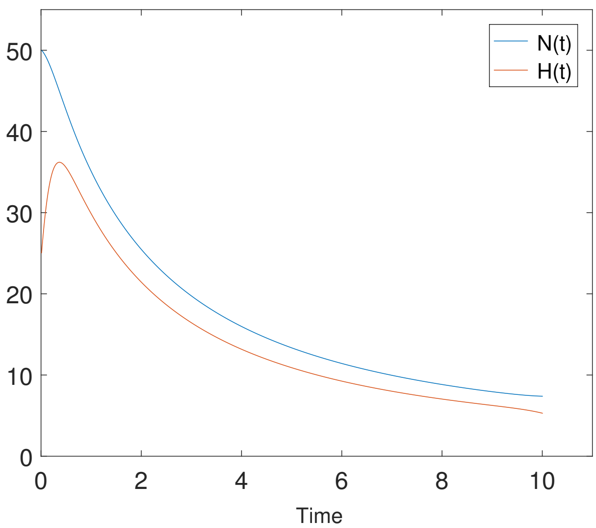
With $q = 1$, starting price $p_0 = 1$ (so $p \equiv 1$ in the cases with constant price), linear costs with $c_1 = 2$ and $c_2 = 0$, and a stock N with carrying capacity 100 and starting values such as $N_0 = 50$. With these parameters, it is clearly going to be profitable to be fishing at a high value of x for most stock levels N, as if $x = 1$ then the revenue per unit time will be with $p = 1$ will be N while the cost will be 2, so the revenue from fishing would be greater than the cost per unit effort for all but the lowest values of N. So we have mostly seen effort distributions shift towards the maximum (at least until the stock values become very low) as much as other parameters in the model allow. We will now consider situations were it is not so obviously profitable to fish at the highest level possible.

We will use start with the same base case parameters from earlier in this chapter (7.1), except since we have now tested including price flexibility in the model we will use a negative price flexibility $\delta = 0.4$ instead of a constant price. First, note that if we scale the cost parameters c_1 and c_2, the initial price p_0 and the cost of control γ then we will end up with the same solution in terms of m and N. Recalling from Chapter

6 that it is preferable to use lower values for terms in the cost function (to reduce the possibility of requiring lower timestep size when solving for u), we will decrease these three parameters by a factor of 10 from the previous base case to $c_1 = 0.2$, $p_0 = 0.1$ and $\gamma = 1$, and let this be our base case for altering other parameters in the cost function.



(a)



(b)

Figure 7.13: (a) Surface plot of $m(x, t)$ for the base case with $c_1 = 0.2$, $p_0 = 0.1$, $\gamma = 1$ and $\delta = 0.4$, (b) Plot of stock $N(t)$ and harvest $H(t)$ for this base case.

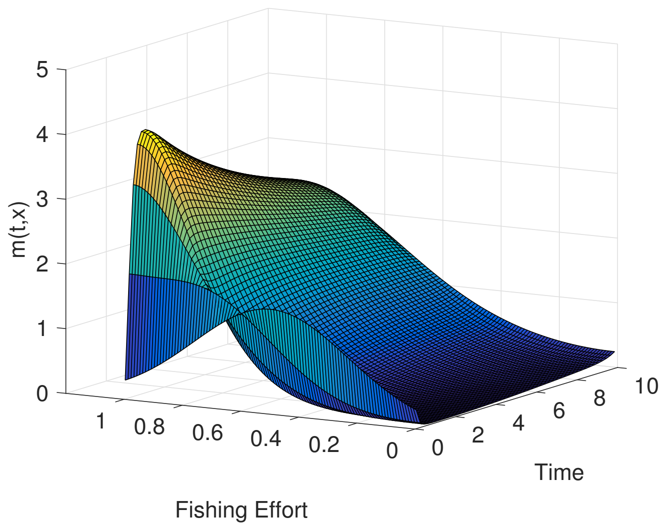
Figure 7.13 shows the resulting evolution of the effort distribution $m(x, t)$ and a plot of the stock and harvest for this base case with $\delta = 0.4$ - in fact it is identical in terms of m and N to the example with $\delta = 0.4$ shown in Figure 7.8, as we expect as we have scaled all the parameters of the agents utility function.

Now, let us first consider varying the linear cost parameter c_1. Figure 7.14 shows plots of results for $c_1 = 1$, i.e. a fivefold increase in the linear cost of effort. We see in Figure 7.14 (a) a flatter distribution of fishing effort, as it is less profitable to fish at high levels, and although the stock and harvest follow a similar trajectory as seen in Figure 7.14 (b), the reduced fishing pressure results in a higher eventual stock value of $N(T) = 9.87$ compared to $N(T) = 7.39$ for the base case.

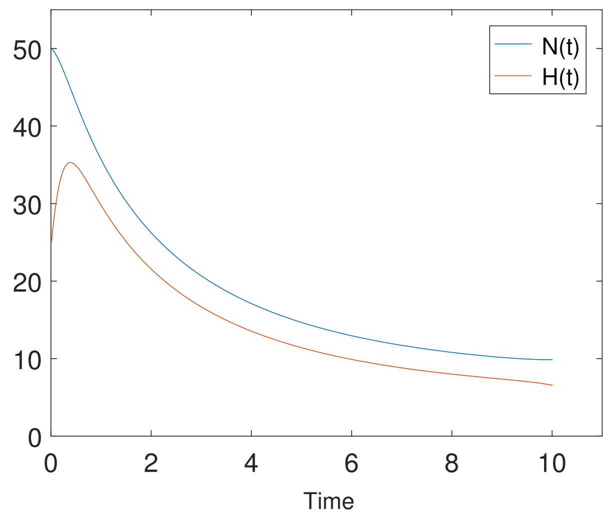
So we can see that increasing the linear cost parameter will result in reduced fishing pressure in the solution of the MFG fisheries model. Setting different values of the linear cost will change the overall fishing pressure and the resulting stock level, however, we still end up with effort distributions and stock evolution that are similar in form as long as it is still profitable to fish close to the maximum as long as N is higher than some threshold value. Let us consider what happens when costs are so high that fishing is unlikely to be profitable, by setting a very high linear cost term $c_1 = 20$. Figure 7.15(a) shows the resulting evolution of effort distribution, with fishing effort being close to zero, allowing the stock to reach a value close to the carrying capacity $K = 100$ as shown in 7.15(b).

With a very high cost term of $c_1 = 20$, it is not going to be profitable to fish even if the stock is at its carrying capacity, yet we still have some fishing effort in our mean field game model. This is because of the volatility parameter σ, which in our game formulation describes the intensity of the random noise in an individual's movement in terms of the state x. This is what results in some agents deviating from the theoretically optimum level of x. In general, the distribution of the effort x will be concentrated more tightly around optimal values of x if the volatility σ is low or the cost of control γ is low.

Figure 7.16 shows the case with $c_1 = 20$ with γ reduced to 0.1, and we see that the effort distribution is concentrated more tightly close to zero. As σ or γ approach 0 then the solution should converge to a dirac delta function located at $x = 0$, however



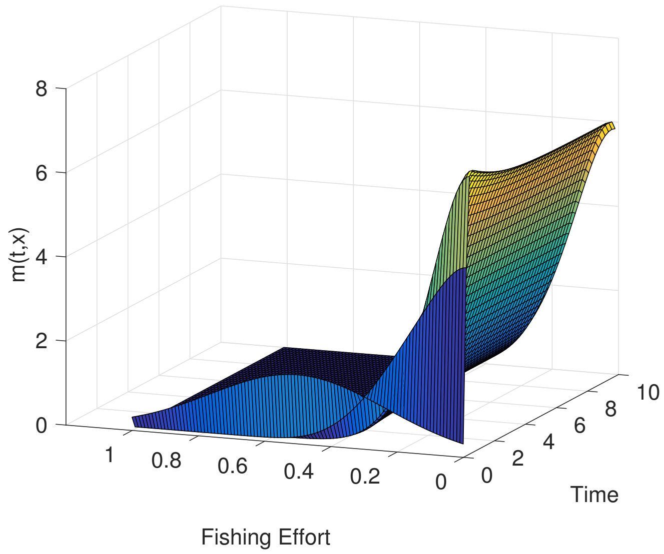
(a)



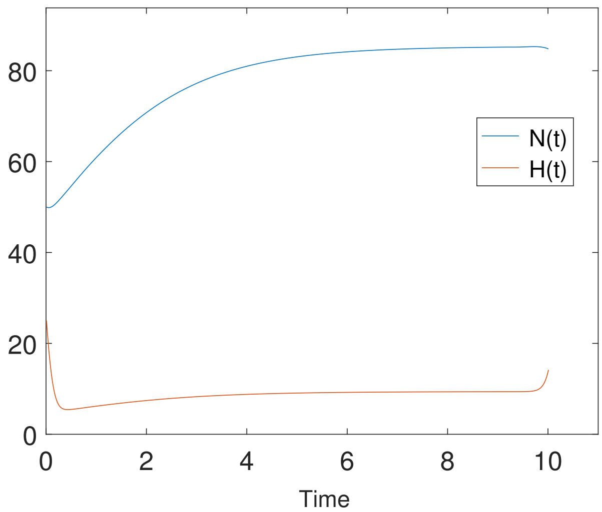
(b)

Figure 7.14: (a) Surface plot of $m(t,x)$ for the case with $c_1 = 1$, (b) Plot of stock $N(t)$ and harvest $H(t)$ with $c_1 = 1$.

approaching this solution would become more and more difficult to solve numerically. In this case, we can interpret a distribution concentrated close to zero as one where fishing activity is very low, but with some fishing activity due to individual decisions or circumstances, or due to other costs associated with reducing effort to zero such as having to get rid of gear or vessels or lay off employees.



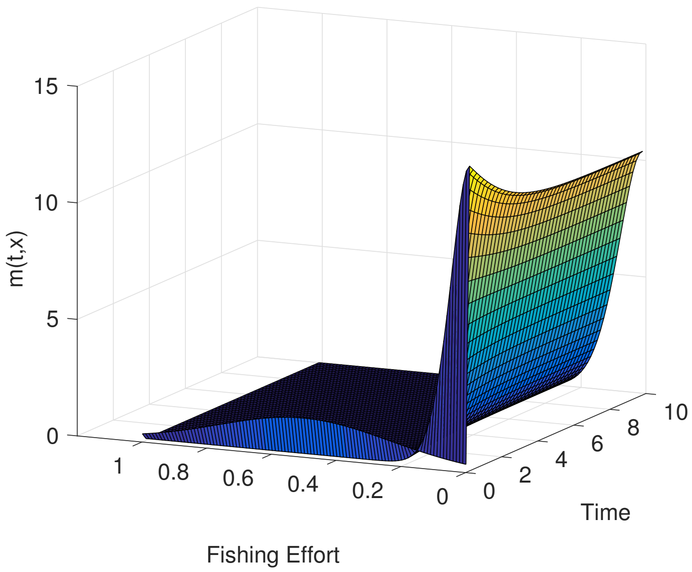
(a)



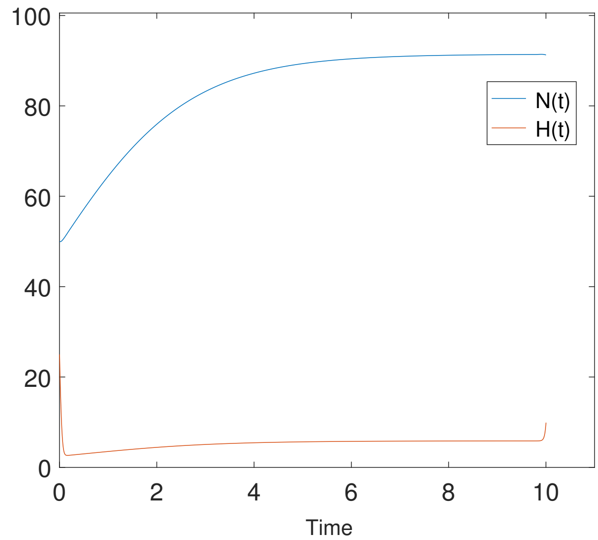
(b)

Figure 7.15: (a) Surface plot of $m(t,x)$ for the case with $c_1 = 20$, (b) Plot of stock $N(t)$ and harvest $H(t)$ with $c_1 = 20$.

We have seen varying the linear cost parameter to show a numerical decrease in the overall fishing pressure (although the form of the effort distribution remains similar), or setting the cost so high that fishing is completely unprofitable. A natural question is to investigate the cost at which things switch from fishing being generally profitable to unprofitable. For this example, Figure 7.17 shows plots of the results when we set



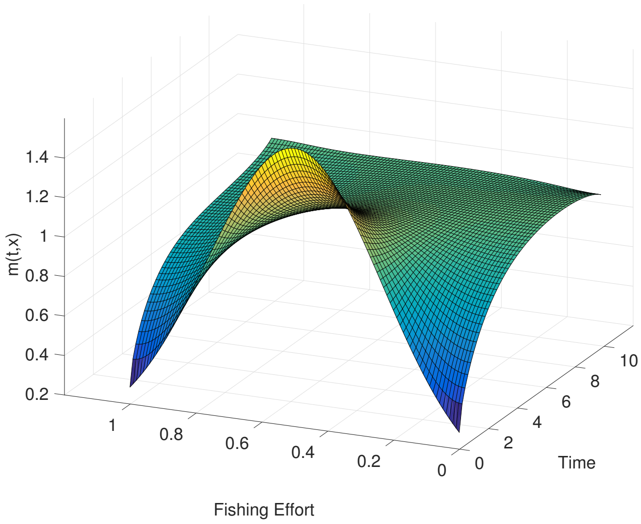
(a)



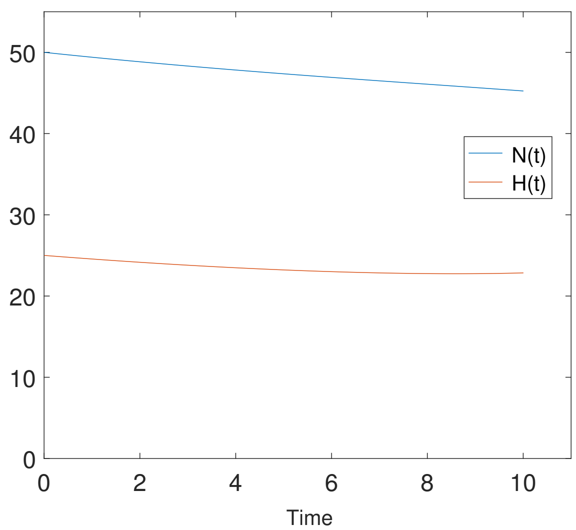
(b)

Figure 7.16: (a) Surface plot of $m(t, x)$ for the case with $c_1 = 20$ and $\gamma = 0.1$, (b) Plot of stock $N(t)$ and harvest $H(t)$ with $c_1 = 20$ and $\gamma = 0.1$

$c_1 = 5$, and we find that for this cost (which is the value at which the profit per unit effort would be zero for our initial conditions) the effort distribution does not shift towards the maximum or towards zero but instead evolves towards a flat distribution across the possible values of x.



(a)



(b)

Figure 7.17: (a) Surface plot of $m(t,x)$ for the case with $c_1 = 5$ and horizon time $T = 1$, (b) Plot of stock $N(t)$ and harvest $H(t)$ with $c_1 = 20$ and $T = 1$.

We may interpret this as indicating that when different levels of fishing effort are neutral in terms of profit, the distribution of fishing effort will be dictated more by the individual decisions or circumstances of the agents (represented by the volatility σ), rather than all agents being pulled towards a spread of optimal values.

Note that for the results shown in Figure 7.17, we have reduced the time horizon to $T = 1$ from $T = 10$. This is because when trying to solve the system numerically close to the value $c_1 = 5$ our algorithm had more difficulty and required a high number of timesteps to solve - in this case, it was not solving an individual PDE using the numerical method that caused any issue, but the iterative approach to the algorithm. Inspecting individual iterates in the algorithm reveals that the difficulty arises because the iterates switch between ones where the effort distribution is close to the maximum and ones with distributions close to the minimum, resulting in the possibility of the algorithm switching between the two types of solutions with a similar error between iterates. So we note that our iterative numerical approach may have more difficulty close to values that are switching points between distinct forms of solutions.

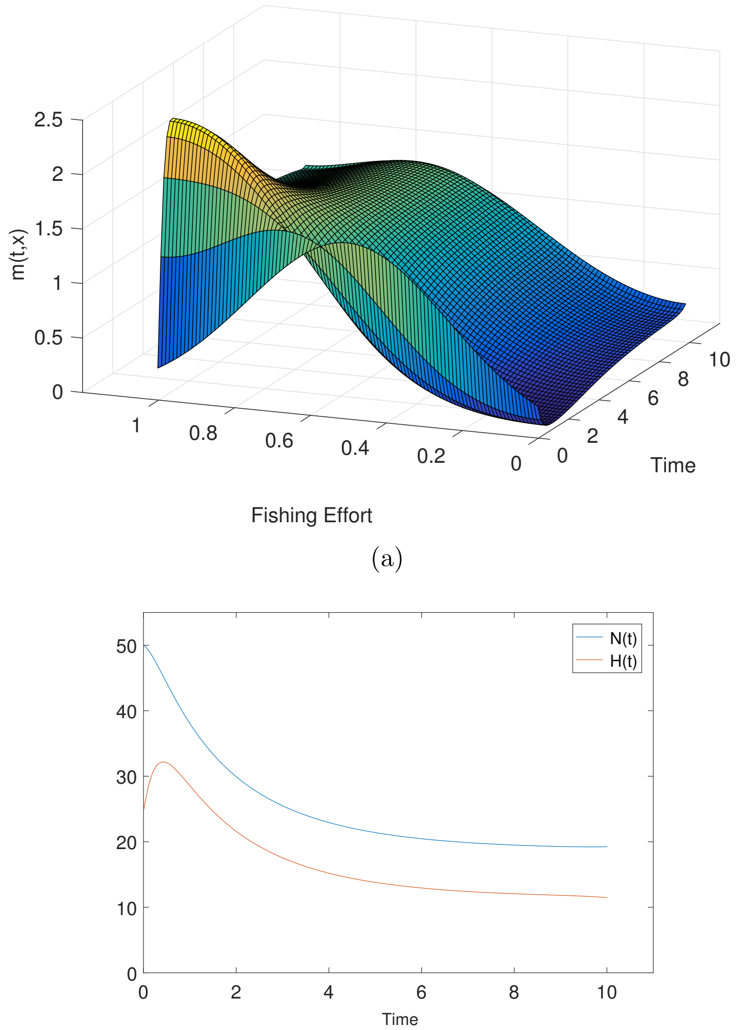
Having seen how the solution behaves for different values of c_1, we will now introduce quadratic costs by allowing a nonzero quadratic cost parameter c_2 in the cost function

$$qxNp - c_1 x - c_2 x^2.$$

Unlike purely linear costs, where for a given value of N the most profitable state would either be to fish at the maximum possible effort or to not fish at all, with quadratic costs we may expect to see more complicated distributions of effort.

We will set the quadratic cost of effort higher than the linear cost, so that we can more easily examine its effect and also to bring the overall cost of effort closer higher so that fishing at the maximum possible effort is not the obvious optimal effort for almost all values of N. Figure 7.18 shows the results of a simulation with the introduction of quadratic costs, with $c_1 = 0.2$ and $c_2 = 2$. From the plot of $m(t, x)$ in Figure 7.18(a) we see that at first the distribution shifts to a peak at the maximum value of x, shifting away from the maximum after a short time. However, unlike previous examples with linear costs where the peak at the maximum effort usually just flattened out, here we can see a distinct new peak in x (around $x = 0.7$), due to the quadratic cost term meaning it becomes less profitable above a certain value of x once the stock has decreased.

We can also see in Figure 7.18(b) that the stock and harvest have started to level out look to be close to their steady state values, which appears to occur around $N = 20$.

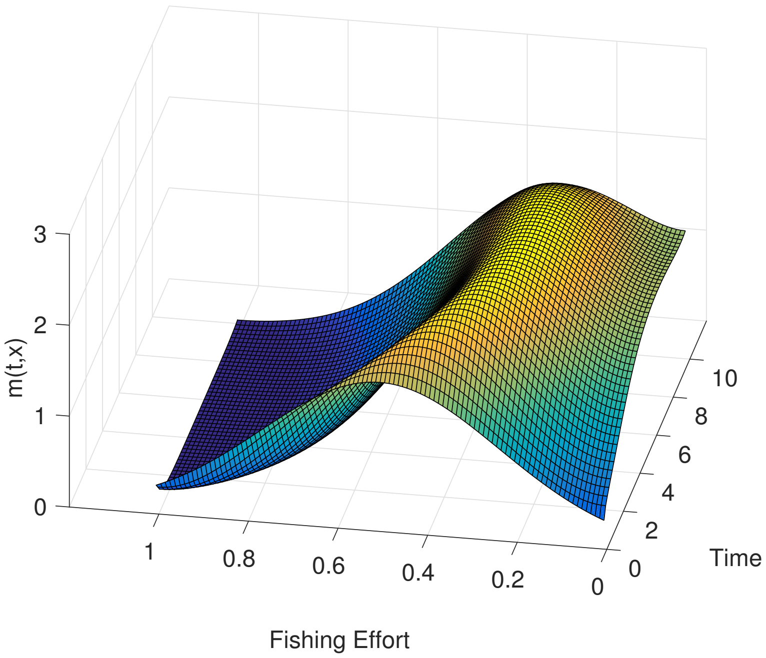


(b)

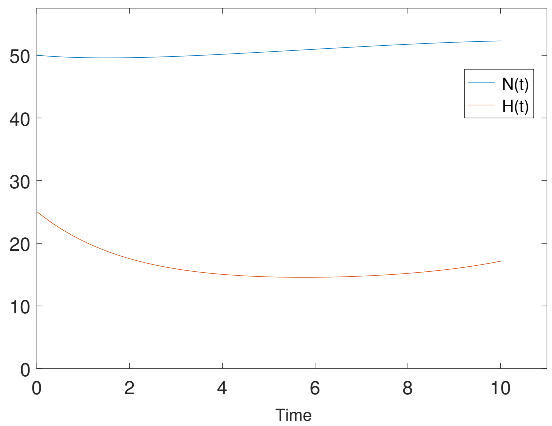
Figure 7.18: (a) Surface plot of $m(t, x)$ forthe case with $c_1 = 0.2$ and quadratic cost $c_2 = 2$, (b) Plot of stock $N(t)$ and harvest $H(t)$ with $c_1 = 20$ and $c_2 = 2$.

So with quadratic costs, when the stock is high enough that it is profitable to fix at the highest level possible the distribution will tend towards that maximum, but appears to settle around a different optimal value of x (in contrast to the case with linear costs, where the steady state distribution m would usually be some curve that peaks at the maximum possible x).

To examine this further, we set $c_2 = 10$ for a very high quadratic cost compared to the linear cost. Figure 7.19 shows the results of this simulation with $T = 1$. We use a shorter horizon time initially as selecting a high cost means we may require higher number of timesteps, so rather than running for a longer time horizon we can use shorter runs (with initial conditions coming from the end of the previous run) to approach the steady state.



(a)



(b)

Figure 7.19: (a) Surface plot of $m(t,x)$ for the case with quadratic cost $c_2 = 10$ and horizon time $T = 1$, (b) Plot of stock $N(t)$ and harvest $H(t)$ with $c_2 = 10$, $T = 1$.

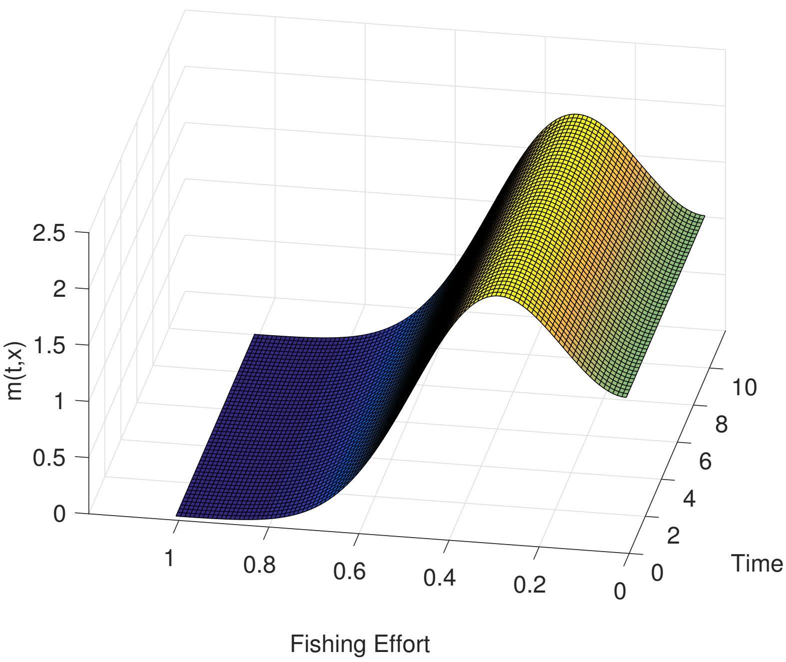
In 7.19(a), we can see the effort distribution quickly shift towards a peak around $x = 0.3$, and appear to remain in that form towards the end of the time period. The plot of stock and harvest in 7.19(b) also suggests that the solution may be approaching the steady state, as the stock appears to remain quite level.

Using the distribution $m(x, T)$ and the value for stock $N(T)$ and price $p(T)$ from the horizon time, and the value function $u(0, x)$ from the start of this simulation, we can repeat the MFG simulation until we confirm the solution has reached a steady state (determined here by seeing when $N(T)$ did not vary more than 0.01% between successive simulations). Note that, although it may require a large number of timesteps to try to solve the initial value problem over a long time horizon, once the solution is close to the steady state we can run the MFG model over longer time horizons without having to increase the number of timesteps.

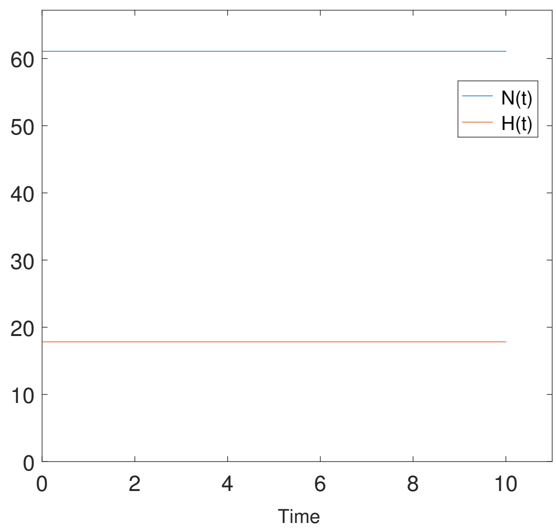
Figure 7.20 shows the resulting steady state distribution $m(t, x)$ and plot of the steady state stock $N(t)$ and harvest $H(t)$ for this example. From the plot we can see that with quadratic cost term $c_2 = 10$, we end up with an effort distribution with a peak close to $x = 0.3$, with a steady state stock of $N = 61.1$. Inspection of the value function $u(t, x)$ shows that for each t, the maximum value of $u(t, x)$ is located at $x = 0.28$; this is what results in the distribution $m(t, x)$ with having its peak at that value of x. The overall effort is fairly low, however in contrast to the case with purely linear cost of effort, we clearly have a distribution of fishing effort that is low but not grouped close to zero. So with quadratic costs, we may see distributions of fishing effort that peak around different values of x.

7.5 Conclusions

We have now considered some different scenarios in terms of the ecological and economic conditions in our MFG model of fish stock exploitation. In particular we have examined how different growth rates affect the solution of the MFG model and how it depends on the initial conditions. We found that as long as the maximum total fishing mortality is higher than the intrinsic growth rate of the fish stock then the agents will fish at high levels until the stock reaches some steady state value (which is likely to be quite low as



(a)



(b)

Figure 7.20: (a) Surface plot of $m(t, x)$ for the case with quadratic cost $c_2 = 10$ and horizon time $T = 1$, after repeated simulations to reach the steady state. (b) Plot of the stock $N(t)$ and harvest $H(t)$ with $c_2 = 10$, $T = 1$ at the steady state.

long as fishing is fairly profitable), although the agents in the game will reduce fishing pressure and allow the stock to recover to this value if it is below it.

Considering changes to the economic parameters of the model, we introduced negative own-price flexibilities (based on the analysis of price dynamics in Chapter 5). We found that the effect of increasing negative price flexibilities depended on the growth rate of the fish stock, and resulted in increased fishing pressure in the MFG model solution when the growth rate of the stock was lower than the maximum fishing mortality rate. This demonstrates further the importance of understanding and estimating price flexibilities for fish species, and in models that can capture the feedback between the economic and ecological components of the system, as understanding that price dynamics may mean some fish stocks are even more at risk of being overfished due to economic pressures could be crucial.

Lastly we have considered varying the parameters in the cost function, to examine the impact of different linear and quadratic costs of effort. Clearly, changing the parameters in the cost function can have a quantitative effect on the overall fishing effort, and new terms (such as the quadratic cost term introduced) can affect the form of the effort distribution more strongly. This will be useful for implementation of regulations in the MFG model, which is considered in the Chapter 8.

Chapter 8

Application of Regulations in MFG Fisheries Model

In this chapter, we aim to use the MFG model formulated in Chapter 4 and the numerical algorithm used in Chapter 6 and 7 to examine the impact of different methods of regulation. The MFG model in one dimension is given by:

$$\frac{\partial u}{\partial t} + \frac{(u_x)^2}{2\gamma} + \frac{\sigma^2}{2} u_{xx} - ru = -F(x, N, L(N, m)), \tag{8.1}$$

$$\frac{\partial m}{\partial t} + \frac{1}{\gamma}(m u_x)_x - \frac{\sigma^2}{2} m_{xx} = 0, \tag{8.2}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = aN\left(1 - \frac{N}{K}\right) - L(N, m). \tag{8.3}$$

$$u_x(t, 0) = u_x(t, x_{max}) = m_x(t, 0) = m_x(t, x_{max}) = 0. \tag{8.4}$$

$$u(T, x) = G(x(T), m(T), N(T)); \qquad m(0, x) = m_0(x); \qquad N(0) = N_0. \tag{8.5}$$

The cost function $F(x, N, L(N, m))$, without regulation, is given by

$$F(x, N, L(m, N)) = qxNp(L) - C_1 x - C_2 x^2. \tag{8.6}$$

Here we will consider various ways in which regulations can be incorporated into the MFG model, such as adding additional terms or equations to the model.

8.1 Regulations

Since our model is based upon a large number of homogenous interacting agents each trying to optimise their personal utility function (expected payoff), the model is well suited towards implementing regulation at the level of an individual agent's utility function. This allows us to consider ways to achieve management objectives (such as higher stock or landings of a certain quantity, while maximising economic gains as much as possible) by changing the utility function of individual fishermen. The change in the utility function implies a certain type of regulation – and these are ones that may be more acceptable or practical to agents and to regulators, such as license fees, or may imply realistic ways to implement other types of regulation that may not always be possible in practice (e.g. achieve quotas by imposing fines for anything caught over a certain amount).

We may consider regulations which, in terms of their formulation in the model, are very simple (such as altering one parameter), or regulations which are more complicated. One such criteria in which regulations may differ is whether they are considered feedback or non-feedback regulations - that is, whether they are regulations which can be implemented uniformly at all times, or whether they are regulations which respond to some state in the model and require feedback. Feedback regulations would require a higher level of information to be available to the fishery managers than non-feedback regulations

8.1.1 Tax on Catch

A tax on catch is a common regulation in fisheries modelling [34] [33], which can be fairly easily implemented in practice. A proportional tax per unit catch would be implemented in the MFG model by setting the cost function F as

$$F(x, N, L(m, N)) = qxN(p(L) - C_P) - C_1x - C_2x^2, \tag{8.7}$$

where the parameter C_P is the additional cost per unit landed to an agent due to the regulation. In cases where the price per unit is constant, a proportional tax per unit landed is equivalent to a decrease in the parameter for the price per unit. In our case, where the price also depends on the total landings, a tax per unit landed is slightly more complicated, and may be expected to have more of an impact; as overall landings increase, the marginal revenue received before tax will decrease, but the increase in tax will remain steady even for high levels of landings.

This regulation is straight forward in terms of the information required for managers, as the regulation only depends on the landings of each agent.

8.1.2 Tax on Effort

Since our control variable in the model is effort (rather than, for example, harvest), it is natural to consider regulation which focuses on this variable. An obvious first choice would be imposing additional costs based on the level of effort. A common form of regulation considered in fisheries modelling is a tax on catch – essentially decreasing the price per unit received. An equivalent measure on effort would be an additional tax on effort. As we assume a linear cost per unit effort, a proportional tax on effort this simply amounts to increasing the parameter for cost per unit effort. So the cost function F would then be

$$F(x, N, L(m, N)) = qxNp(L) - (C_1 + C_R)x - C_2x^2, \tag{8.8}$$

where the cost per unit effort now comprises an operating cost c and a proportional tax per unit effort from the regulations C_R.

This regulation requires managers to know the fishing effort of each individual agent, which is the state variable in our mean field game model. This requirement of more detailed information so may make the regulation less preferable in practice than one which requires knowing only the landings of each agent, however it is information that it would be reasonable to expect to be available, particularly if the fishery has limited access.

8.1.3 Effort Control - Fines

An alternative and more significant change to the cost function would be to add different cost terms which depend on effort in different ways, rather than just linearly. One example would be a fine for exceeding a certain level of effort, which is proportional to the amount that effort goes beyond a certain threshold level of effort. It would make sense for this fine to get larger than the regular cost per unit effort when the effort goes well above the threshold level, so a quadratic function makes sense as an approach. This means an additional term would be included in the cost function $F(x, N, L(N, m))$ of the form:

$$F(x, N, L(m, N)) = qxNp(L) - C_1x - C_2x^2 - C_M[x - x_M]^+(x - x_M) \tag{8.9}$$

where C_M is a constant and x_M is a threshold value of x above which the regulator wishes to regulate activity. This cost term is only applied if $x > x_M$, and due to the quadratic form this term would eventually dominate the revenue and cost per unit effort terms (which are linear in x) if x is large enough. This is similar to selecting the boundary value x_{max}, which is chosen as the maximum value of x. Imposing some fine on very high levels of effort which lets the profit diminish for high x is an effective way of limiting the maximum effort an agent can apply in a more smooth fashion. This helps with issues relating to the distribution of agents approaching the boundary – if there is a large additional cost term which will eventually dominate revenue (which, assuming constant stock and price, would be linear with effort), then there is something limiting agents from wishing to increase effort indefinitely.

As this is a feedback regulation, it requires managers to know the fishing effort of each agent. It may be an unreasonable assumption for managers to know the precise effort at each point in time, however the model may still serve as a good approximation for the regulation in continuous time, as it is very reasonable to expect managers to be able to know about an agent's historic fishing effort.

Other methods of effort control are possible by adding further cost terms that depend on x. Alternatively we could consider a regulation that does not allow individual agents to exceed a certain threshold of fishing effort by changing x_{max} (the maximum possible effort for an individual agent) to limit the range of x.

8.1.4 License Fees

An appealing form of regulation would be a flat license fee paid per vessel to then operate freely in the fishery, as this is a simple and easy to implement (practically) regulation for fishery managers. Our MFG model is not well suited to including one off payments unless they are incorporated into the terminal cost, but that would not serve the purpose of a license fee well as it is usually paid at the end of the period considered. A constant term in the running cost however would represent a license fee, with the assumption that an agent can pay for a license for only a proportion of the time period, whenever teir fishing activity is above a given level which requires a license.

This flat license fee would then apply for any agent with nonzero effort, or alternatively for any agent above some fairly low threshold value of effort (this concession may allow the numerics to be more stable, and may represent the potential for a small amount of unlicensed fishing to take place without being caught). So the additional term in the cost function F would be of the form:

$$F(x, N, L(m, N)) = qxNp(L) - C_1x - C_2x - C_L I_{x_L}(x), \tag{8.10}$$

where $I_{x_L}(x)$ is equal to one if $x \geq x_L$ and is zero if $x < x_L$:

$$I_{x_L}(x) = \begin{cases} 1, & \text{if } x \geq x_L \\ 0, & x < x_L \end{cases} \tag{8.11}$$

Additionally, assuming each agent has the capability to invest in more vessels, license fees may be included by selecting further threshold values of effort (which another vessel would be necessary to exceed), and then applying an additional flat licensing fee above that threshold value.

The implementation of license fees in the MFG model may not be a perfect representation of the reality, as it is included in the running costs rather than as a strict one-off payment, however it is appealing due to the comparative ease with which license fees may be implemented in practice compared to regulations that are more onerous such as TACs. This is a feedback regulation which requires managers to know the fishing effort of an agent, though in practice it is likely easier for the regulator to know only where an agent's fishing effort exceeds some threshold than to know their precise fishing effort at every time as required by effort control regulations above.

8.1.5 Marine Protected Areas

Another form of regulation is one which does not introduce any extra terms into the agents cost function, but which changes the evolution of the fish stock by including a marine protected area (MPA) which serves as a refuge to some of the fish.

We can consider regulation in the form of a marine protected area (MPA) or refuge where fishing is prohibited with the inclusion of an additional ODE describing the population of fish within the MPA, which are subject to no fishing (and perhaps to different growth rate and carrying capacity), with transference between the population of fish inside the refuge and outside.

In principle this is similar to including another species or group of fish in the model, such as a predator or competition. In the general case of including a new species, we assumed that effort could be applied to that species which introduces a new dimension to the state variable in the mean field game. However, if we assume that there is no fishing of the new fish population (in this case a population of the same species but located in a refuge) then there is no increase in dimension of the state space. If, however, it was assumed that fishermen could fish in the refuge, but would pay a different cost to do so (e.g. a fine for breaking the rules) then inclusion of an MPA would be a specific case of the two dimensional MFG.

This would involve, for the one dimensional problem above, having two fish resource evolution ODEs of the form:

$$\frac{\mathrm{d}N_U}{\mathrm{d}t} = a_U N_U \left(1 - \frac{N_U}{K_U}\right) - s_1 N_U + s_2 N_P - L(N, m). \tag{8.12}$$

$$\frac{\mathrm{d}N_U}{\mathrm{d}t} = a_P N_P \left(1 - \frac{N_P}{K_P}\right) + s_1 N_U - s_2 N_P. \tag{8.13}$$

Here, N_U is the stock outside of some protected area and N_P is the stock inside the protected area. The parameters a_U, a_P, K_U, and K_P are the growth rates and carrying capacity for the unprotected and protected areas (as there may be difference between the environment of the two areas), and s_1, s_2 are the rates of diffusion across the boundary of the area. Fishing is only possible outside the protected area, and so landings are only harvested from the stock which is outside the refuge.

By allowing fish a refuge area, the fish stock will likely be allowed to maintain a higher level and possibly be prevented from nearing extinction. However, if the MFG model otherwise suggests very high levels of fishing, it is possible that it may lead simply to very low stock levels outside a protected area and high levels inside, which may not be realistic depending on the nature of the MPA chosen.

This is a non-feedback regulation, and after setting the MPA the manager does not need to know any more information or make any active changes. However, in the model we assume that once the MPA has been identified there is strictly no fishing inside, which may not be entirely realistic. So we implicitly assume that the manager has access to information about attempts to fish inside the refuge and a means to enforce the protected area.

8.1.6 Discussion on Quotas

As well as regulation involving tax or fines on effort or catch, we may wish to consider quotas, in the form of Total Allowable Catch (TAC). The setting and allocation of quota is complicated in practice, with different decision makers at regional and local levels, and methods such as ITQs (Individual Tradeable Quotas) being used to distribute them between fishermen.

In fisheries modelling, TAC is normally considered by simply setting the landings or maximum landings and assuming that this is implemented perfectly. Models might then be used to investigate questions about the choice of appropriate TAC by varying the landings. Alternatively they more be focused on the problem of appropriate allocation of quota to different regions or different agents at the local level, or to investigate the optimal harvesting strategy to maximise economic yield for a given total landings.

For a particular fishery, a quota in the form of a Total Allowable Catch is often given. The manager of a fishery with a given TAC may allocate that quota to individual fishermen, or alternatively they may wish to simply allow fishermen to operate freely within the fishery (subject to whatever other regulations are in effect), up until the TAC approached. Since the MFG model is concerned with the utility functions of individual agents, then rather than simply assume that a manager wants to (or is even able to) force a stop to all fishing activity, we may investigate ways to achieve the same management goals by imposing an economic cost on the agents in the form of a tax related to the TAC.

Suppose that when the total quantity of fish landed during a period exceeds some proportion θ of the TAC for that fishery, then the regulator wishes to reduce the total landings from the fishery after that point so they do not exceed the TAC. So they implement a tax on all further catch, with the aim to reduce the level of catch in the future to the appropriate level.

In the MFG model, this means we need to include a cost term which depends on the total landings from the start of the period to time t. This is an output which we would be interested in from the solution, but this form of regulation would introduce this directly into the HJB equation.

The specific choice of additional cost imposed once total landings reach a proportion θ of the TAC may be different (for example we could introduce a tax on effort), as long as it depends on when total landings exceed a certain amount. However, since this measure is specifically about reducing the future catch to an appropriate level, and because the state of the stock when the threshold proportion of the TAC is reached may vary, using a tax on catch for this seems appropriate.

Models (including differential game theoretic models) incorporating Individual Trading Quotas can be used to investigate the impact of different quotas assuming that agents can trade quotas on the market to try to optimise their yield from fishing and from selling off any excess quota.

Our MFG model is suitable for investigating optimal harvesting strategies for a given TAC, since we consider individual agents and can look at the evolution of the distribution of effort through time. However, allocating quotas (including Individual Tradeable Quotas) to individual agents is more complex, as the assumptions of MFG modelling mean that at any given time, while we know the distribution of agents' effort, we do not know the history of any individual agent. The agents are considered homogenous, which means that the only thing describing an individual agent at a given time is their state. So in order to implement individual quotas, we would require another state variable as well as effort for each agent – either in the form of held quota (particularly if we are interested in ITQs) or in the form of total landings from the start of the period to the current time. Although this is certainly possible, it would increase the dimension of the MFG, and the new state variables may need to be treated quite differently.

An alternative way to consider a TAC quota is not to consider allocating quota to individuals, but just to consider the TAC for the whole fishery. While the MFG model does not allow us to consider the total landings of an individual, we know the total landings from the whole population of agents up to a given point in time, and so we can easily determine when the total landings approach or reach some TAC value.

So instead of considering an individual's quota, we may consider purely the quota for the whole fishery – and rather than treating a set TAC as an absolute limit, we may instead consider a threshold level of total landings above which an additional tax will be applied (either to effort or to catch). This means that once the TAC threshold is reached or approached, then all fishermen will be taxed equally if they wish to fish more, rather than individuals who have landed more (and hence used up more of the quota) running out of individual held quota or being taxed more. However, while this means that we cannot consider individual quotas, a regulation which is imposed uniformly on all participants based on a total for the fishery is a reasonable and perhaps preferable option, both from a practical point of view (assuming regulators prefer more simple regulations that require less information) and because it gives another example of individual agents being impacted by the overall activity of the total population of agents, which is one of the strengths of mean field game modelling.

8.1.7 Combining Regulations

The regulations above may be implemented separately, however it is likely that management goals can be achieved more efficiently by combining one or more form of regulation, such as implementing effort control in addition to a marine protected area. The regulations may interact in more ways too - for example, we may use the same sort of fine suggested under effort control but triggered only if some proportion of TAC has already been landed.

In order to test the potential impact of different types of regulations, we could add some of the above regulations separately to see how the model solution changes from the case with no regulation, then consider adding multiple regulations and comparing how well management objectives can be obtained compared to only using one at a time. In addition to combining regulations, we may also wish to examine how regulations interact with changes to the other parameters in the model, such as the scenarios discussed earlier.

8.2 Applying Regulations Example - North Sea Cod

For an example of using the mean field game model to investigate the impact of different regulations in a fishery, we will consider a simple example based on the case of North Sea cod. Cod in the North Sea has historically undergone overfishing that required significant regulation to allow the stock to recover. However, although it was thought that the stock had recovered and hence regulation was no longer as strict, the North Sea cod is considered at risk of overfishing again and it is questionable whether the assessment that the stock had recovered sufficiently was accurate [92]. In addition, with fisheries still a key point in the Brexit negotiations, even after the UK has left the EU, there are more questions about how fishing in the North Sea, including cod, will be approached by the UK and the EU nations [93], as it remains to be determined exactly how the UK will approach its fisheries policy in relation to the EU's Common Fisheries Policy and vice versa. These factors mean that the choice of fishing policy by the UK and other nations regarding North Sea cod will be a crucial question moving forward.

A more complicated ecosystem model such as an age-based model would be best suited to providing an accurate model for North Sea cod, however in order to demonstrate the application of regulations in our MFG model with a view to consider North Sea Cod, we use the same simple logistic growth model as used previously, estimating the intrinsic growth rate and carrying capacity based on ICES assessment for cod in the North Sea.

We obtained an annual intrinsic growth rate (for spawning stock biomass of cod $N(T)$) estimate of a = 1.78, and the carrying capacity was estimated at K = 741.200 (‘000 tonnes). We assume that an individual agent in the fishery is one whose fishing effort x is in [0,1], where an individual's catch is given by qxN, with $q = 2$. The distribution of the total mass individual agents is a probability density function, so the maximum total catch rate that can be applied to the stock occurs in the case where all agents are fishing at maximum capacity $x = 1$ resulting in a mortality rate of 2. We could consider higher values of maximum fishing effort, but since this mortality rate is higher than the maximum intrinsic rate of growth (a = 1.7786) we know that it is high enough that all individuals fishing at maximum fishing effort would drive stock towards extinction; any fishing effort higher than this would only do so faster.

8.2.1 Applying Regulation - Short Term

We can solve the mean field game model for North Sea Cod (NS Cod) to get an idea of how different regulations would impact a fishery and what it looks like in the solution of our mean field game. First we will consider the NS Cod model without regulation - for our base case, we set the cost function so that Figure 8.1 and Figure 8.2 show the evolution of effort distribution, the stock biomass and harvest from a simulation running for one year using the NS Cod mean field game model with no regulation.

With this serving as our baseline, we can run the mean field game model for NS Cod using different examples of regulations described previously in this chapter.

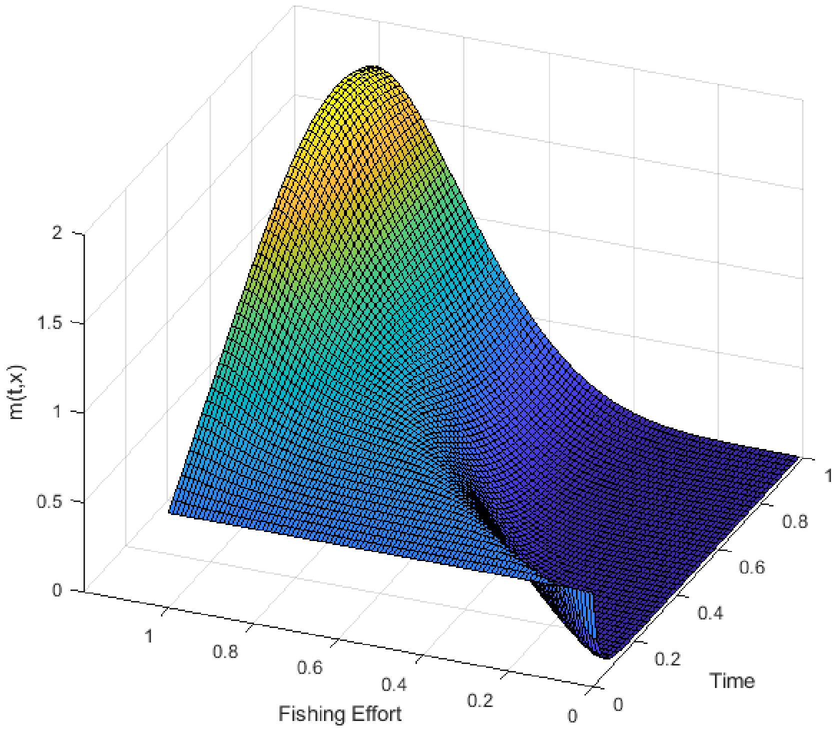
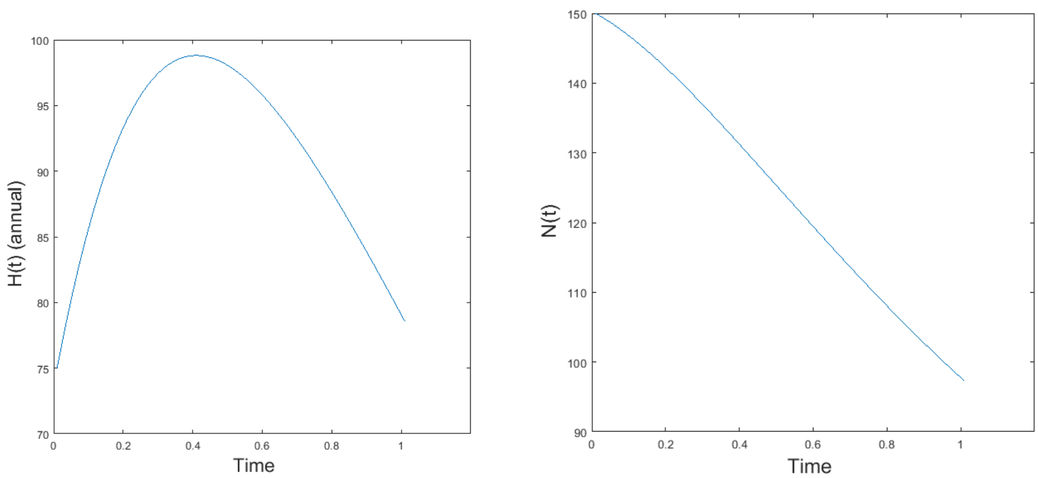


Figure 8.1: Surface plot showing the evolution of effort distribution $m(x,t)$ in the North Sea Cod model over one year without regulation.



(a) (b)

Figure 8.2: Plot of (a) the harvest $H(t)$, and (b) the stock $N(t)$, from the North Sea Cod model over one year without regulation

First we consider one of the simplest regulations to implement practically, which is also easily implemented in the mean field game model - a tax on catch. This may take the form of a constant tax that's paid when landing, or it could represent the cost per unit of quota - in order to land and sell the cod you must hold the appropriate quota, and assuming these are available for a constant price then this is equivalent to our tax on catch in the cost term of the MFG.

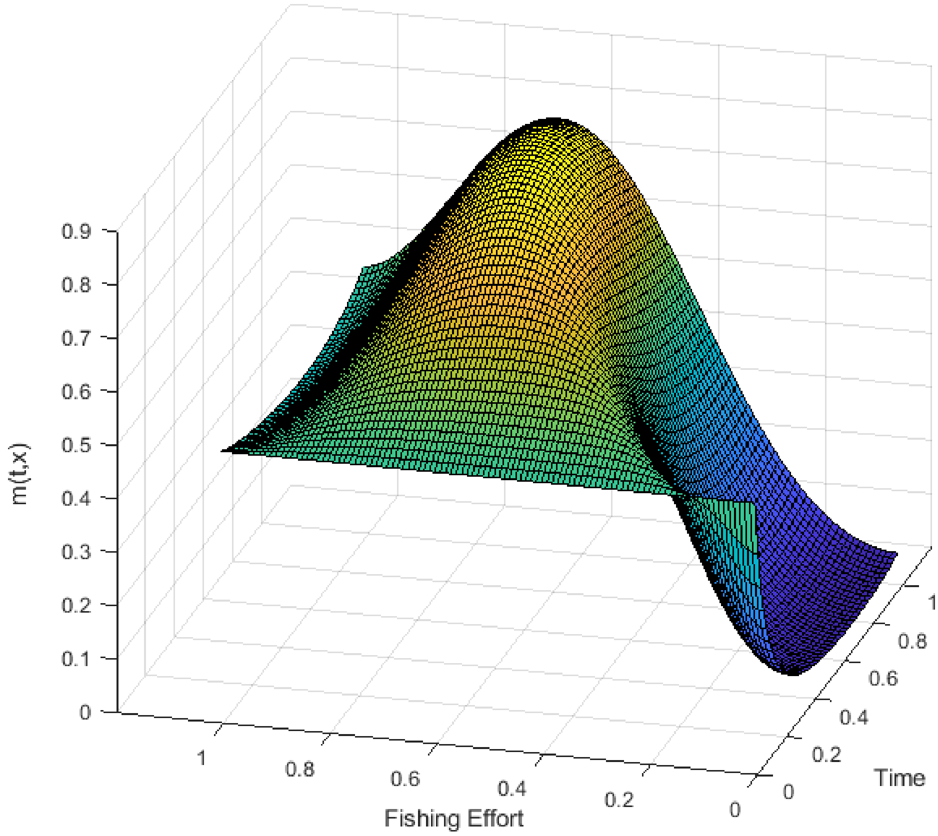
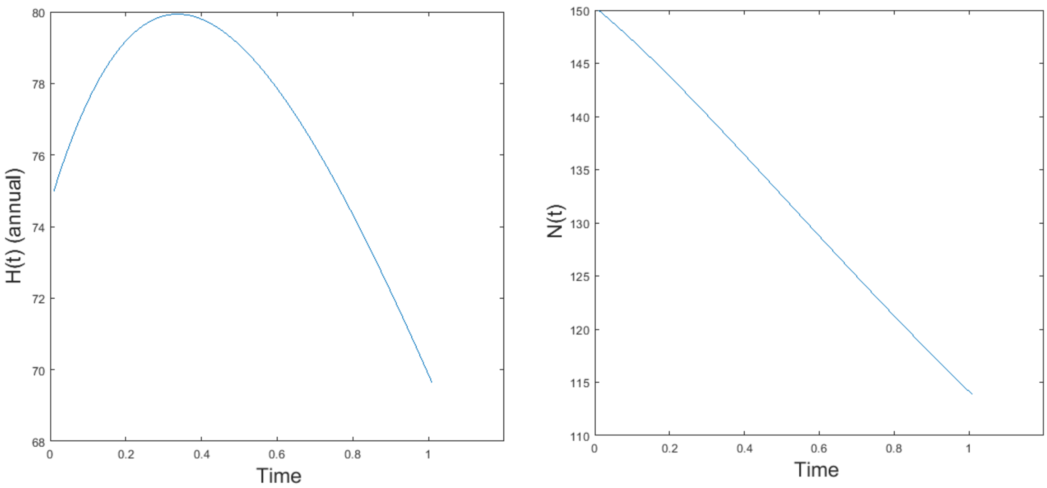


Figure 8.3: Surface plot showing the evolution of effort distribution $m(x, t)$ in the North Sea Cod model over one year under a tax on catch regulation.

Applying different levels of tax on catch, we find that for lower taxes relative to the price per landings, although the overall form of the distribution, cod and harvest looks similar, there is reduced overall fishing effort, harvest and fishing mortality on the stock. Setting a higher tax on catch we see the distribution $m(t, x)$ peak around a lower level of fishing effort that becomes the most profitable - an example of this level of regulation applied to the NS Cod MFG model for one year is shown in Figure 8.3 and 8.4.



(a) (b)

Figure 8.4: Plot of (a) the harvest $H(t)$, and (b) the stock $N(t)$, from the North Sea Cod model over one year under a tax on catch regulation.

Another form of regulation that is appealing due to its practical applicability is setting a license fee to operate in the fishery. As described in the section on regulation above, in our MFG model we are required to adapt to our continuous cost function, and set the minimum fishing effort that requires payment of a license fee to be a low level rather than zero to avoid issues with the numerical solution.

Similarly to the case of a tax on catch, we find that at low levels of a license fee regulation, there is some reduction in the overall fishing effort due to a small proportion of agents that do not fish at as high level. Setting a very high license fee however, as in the example in Figure 8.6 and 8.6, and we can see more clearly the effect on the distribution of effort that a significant regulation of this type may have.We can see the very high proportion of effort distributed among very low fishing efforts (below the level of 0.2 of maximum fishing effort per agent, where we impose a license fee), while it is profitable for a smaller proportion of agents to pay the license fee, and then fish close to the maximum level of effort.

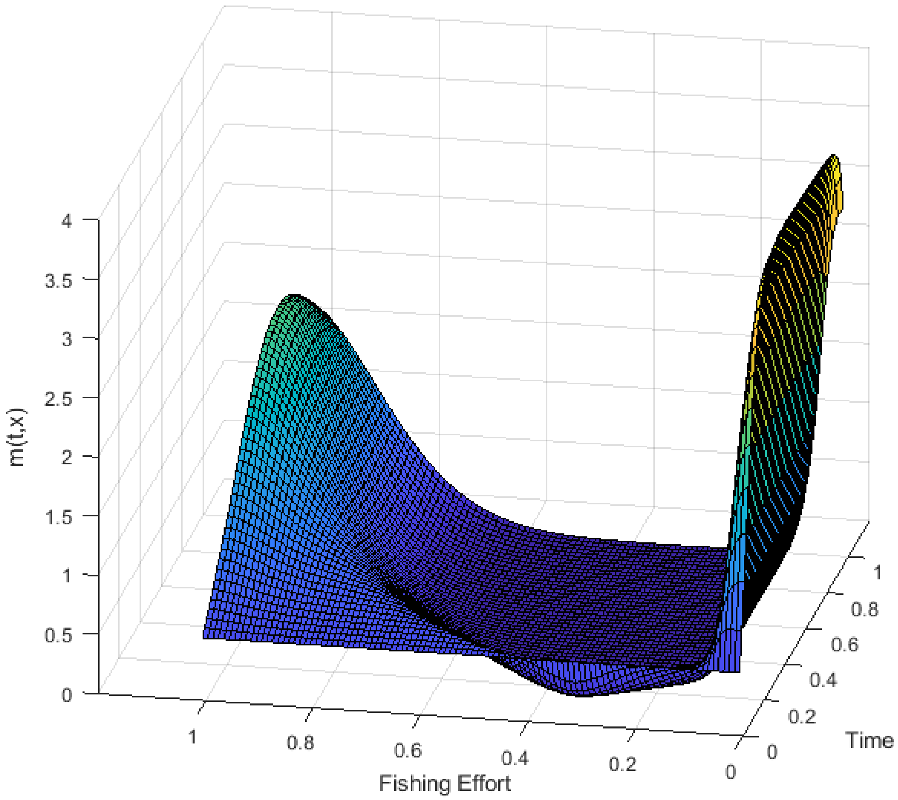
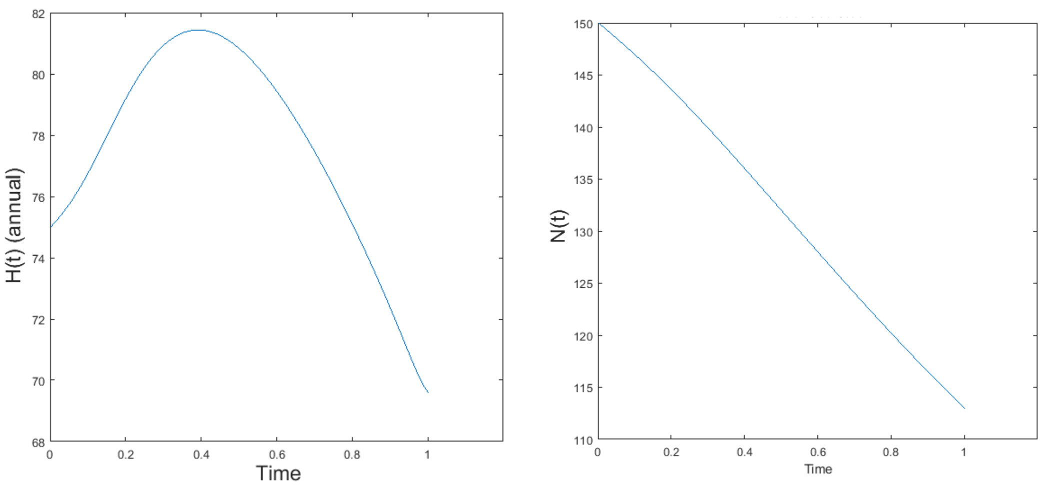


Figure 8.5: Surface plot showing the evolution of effort distribution $m(x,t)$ in the North Sea Cod model over one year under a license fee regulation.



(a) (b)

Figure 8.6: Plot of (a) the harvest $H(t)$, and (b) the stock $N(t)$ from the North Sea Cod model over one year under a a license fee regulation.

8.2.2 Steady State Without Regulation for NS Cod Mean Field Game

An approximately stationary solution to the MFG model of the NS cod fishery can be obtained by first running the model over a short period (say one year) from any starting distribution of effort and initial condition on the stock, then using the distribution and stock at the terminal time, as the initial conditions for a long run (say 50 years). We can check that the resulting solution is approximately the steady state solution by running again with the stock and distribution at terminal time as initial conditions, and the value function at time zero as the terminal condition. This results in a distribution (of effort), and stock that remain approximately constant in the long run. An example of such a steady state is shown below.

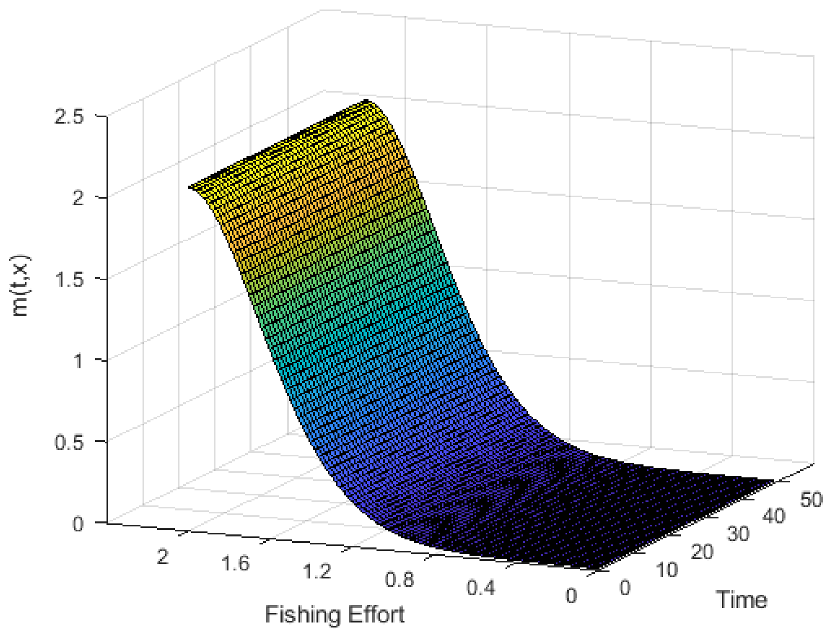


Figure 8.7: Surface plot of the steady state distribution $m(t, x)$ from the NS Cod model over 50 years without regulation.

Since the units of fishing effort are arbitrary (requiring only that catch is proportional to effort), we can for now set the cost per unit effort relative to the price per unit catch to an appropriate level. We know that all agents fishing at maximum intensity

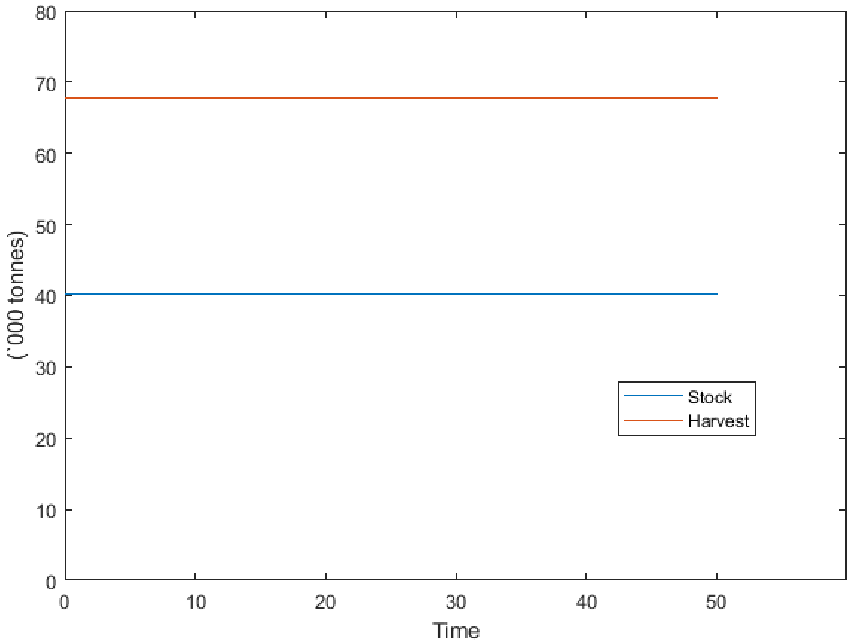


Figure 8.8: Plot showing the steady state stock $N(t)$ and harvest $H(t)$ over 50 years in the NS Cod model without regulation.

would drive the stock towards extinction. We will choose to set the costs (without regulation) to a value that result in a steady state with significant fishing effort, but not enough to drive the stock towards zero. The resulting stationary stock (around 40,000 tonnes) is somewhat below the lowest level of NS cod spawning stock biomass in the past 50 years. This seems a reasonable starting point for NS cod without regulation (but also not with the species near extinction).

8.2.3 Applying a Regulation - Long Term

Using this steady state as our starting condition, we can impose some form of regulation on the system by changing the cost term of individual agents. Here we will apply a tax on catch (or a price of buying quota) to the individual agents, set at 12.5% of the initial price (which the actual price varies slightly around). We can consider applying this regulation over shorter period of times (i.e. to see how quickly it makes an impact or what the short term result will be), and perhaps update the choice of regulation as time evolves. However for now we will run the model over 50 years, starting from the unregulated steady state, with a constant tax on catch. The results are shown below.

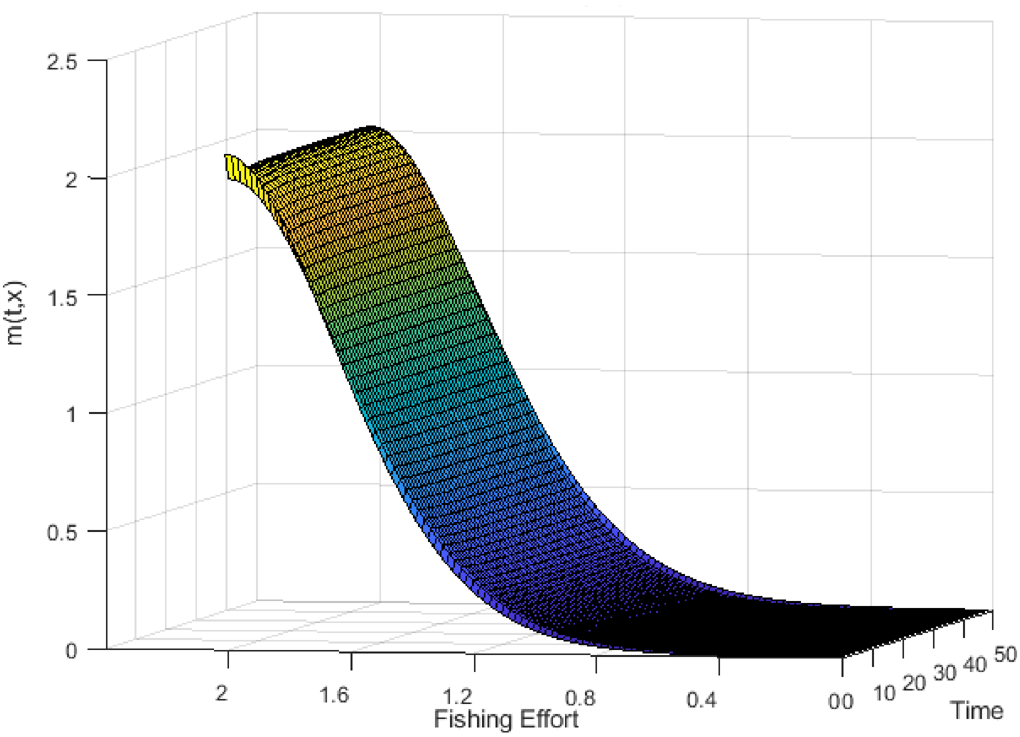


Figure 8.9: Distribution of Fishing Effort $m(t,x)$ over 50 years with tax on catch

From the distribution of effort we see a modest shift towards lower fishing intensity in the early years, which then stabilises over the rest of the time period. The stock and harvest also rise slightly (from around 40 to 45 ('000 tonnes) and 67 to 75 ('000) tonnes

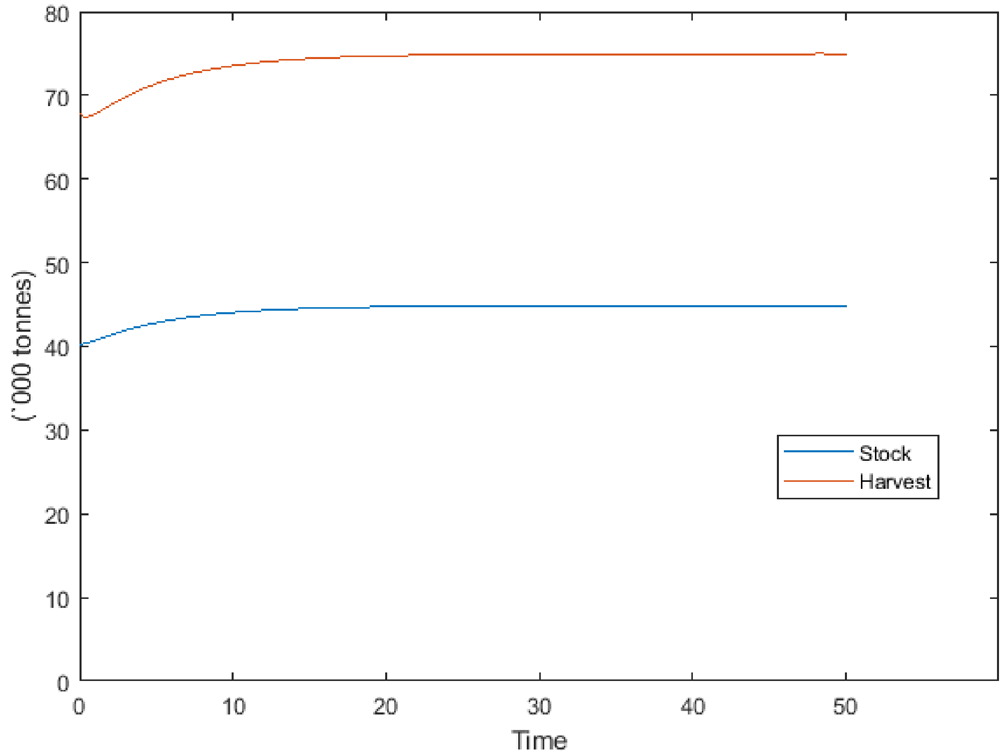


Figure 8.10: Distribution of Fishing Effort $m(t, x)$ over 50 years with tax on catch

respectively). The system remains relatively constant once those values are reached. Obviously this approach only achieves a modest increase in the stock. If the stock is below critical values then more drastic methods of regulation may be necessary in the short term.

8.2.4 Conclusion

We are able to use the MFG model to investigate both imposing short term regulations (such as those that might be desired to help a stock recover), and to try to identify optimum long term regulation schemes that would achieve a steady state that meets the criteria we want. Running the MFG model over a short period of time could be used to test regulations meant to allow recovery of a stock (such as the effort control on north sea cod in previous years). But regulations intended to achieve such stock recovery might never be intended to be maintained indefinitely or over a long period, so it is not necessarily important to model the results of such a regulation over the long term or to find out what the approximately steady state is. For example in some cases the fishery might be closed entirely, but if this is only a short term measure and the fishery will be reopened then the steady state of the closed fishery is not important, only the impact of its closure in the short term. A longer time period (say at least 10 years) should be used to investigate regulations that might be successful at keeping the fishery open sustainably, with the same regulations in place for many years with perhaps only minor changes. Regulations which would work in the long term (e.g. result in a steady state close to the maximum sustainable yield or some other desirable state) may not have much of an impact in stock recovery within a year or two of being introduced; similarly regulations which allow greater stock recover may not be optimal in the long term. So we can use the MFG model to consider these two questions separately, investigating new regulations imposed over a short term (say 1-5 years), and regulations intended to remain in place for the long term (say 10 or more years).

Chapter 9

Discussion

In this thesis, we have considered the challenging topic of the integration of economics and ecology in mathematical modelling. The eventual goal of such research would be to allow the entire social-ecological system surrounding marine resources to be modelled as a whole, including feedback between all parts of the system. Here, we have taken steps towards this primarily by developing the understanding of modelling the interface between economics and ecology, in ways that allow us to capture the dynamic feedback between the two systems and which provide tools for better understanding the complexity of the whole system.

We have contributed to the research on integration of ecological and economic mathematical modelling of marine systems in two broad categories. First we have performed some statistical analysis on ex-vessel fish prices, landings, and other market variables for the whole UK market, and established evidence for the existence of negative own price flexibilities for a number of individual species and for coarsely defined guilds of species, as well as fitting models for ex-vessel prices depending on landings and other market variables.

The other main contribution in this thesis is the development of a mean field game model that can be used to model a fishery. This introduces a new tool for modelling human activity coupled with a model of a marine ecosystem - in this thesis we have laid out the concept and formulation of the model, and presented the results of numerical solutions to numerous different versions of the mean field game model, with parameters selected to demonstrate the qualitative properties of the solution in different scenarios, and to demonstrate how the model can be used to provide some insight when trying to answer questions such as the impact of different economic or ecological conditions, or which choice of regulation or implementation of regulation is best for a fishery.

In the introduction we described an example of a coupled social-ecological system based on a wild capture fishery, and how the feedbacks between the marine ecosystem, the fishermen, regulators, and demand from the market resulted in a complex system of interacting components that would be difficult to model as a whole. While we cannot say that our work has resulted in a model which simulates all components of this system in great detail, we have gained better understanding of many of the interactions and processes in this system.

Firstly our work on fish prices provided evidence of the feedback between landings and price, as well as identifying some of the other most influential factors on the ex-vessel price such as import prices. The application of our mean field game model to a fishery demonstrated how, even modelled as driven to rationally maximise profit, fishers would react to changes in the stock, including reducing fishing effort as the stock became too low. However, it was still clear that the resulting fish stock was not the social optimum, confirming that there is still a "Tragedy of the Commons" effect when all players in the game seek to maximise their profit, demonstrating the need for regulation.

By adding regulations to the mean field game model, we saw how different types of simple regulation could result in very different distributions of fishing effort, such as for a license fee compared to a tax on catch. In this thesis we have only begun to look at application of regulations, but it is clear that the impact of different regulations will depend on economic factors and the current state of the ecosystem. So our model, which allows dynamic simulation of both the ecosystem and fishing activity in response to changing economic variables and new regulations, provides a useful tool for investigating the complex interaction of these factors.

Some of the complex interactions between different components of the system were demonstrated by the impact of price flexibility in the mean field game model, and its interaction with the growth rate of the species. Only by modelling multiple components of the system - the ecosystem model for the fish stock, the game theoretic model of fishermen, and the demand from consumers represented by the price flexibility - are we be able to understand how the interaction between growth rate and price flexibility impacts the fish stock. Further investigation of this, and other interactions between components that are somewhat removed from each other, would be a high priority for future work using this modelling approach.

Investigating the relationship between price and landings, we have also performed one of the broadest fish price analyses in the literature, being performed on a large market (the whole UK), incorporating lots of data which had to be compiled from old non-computerised sources in some cases, and covering a large number of species. Establishing negative price flexibilities for fish in the UK market contributes to the wider body of evidence demonstrating these price flexibilities in different fisheries, offers possible estimates for use in any model which concerns a large part of the UK market or one where the price flexibility in the UK market might serve as a good possible estimate, and it has demonstrated an approach for fitting a model of price or estimating price flexibilities for a different fishery or market.

Better understanding of ex-vessel price dynamics, particularly the relationship between price and landings, allows us to capture more of the interaction between the economics and ecology in a mathematical model - if we did not have any idea of the price dynamics, and so assumed price was constant or a relationship between price and landings that was incorrect, then we may miss feedback between the human economic activity and the marine ecosystem. For example, a change in fish stock drives a change in landings, which in turn has an impact on fish price, which could then change the decision making process of fishermen, and hence change the resulting impact of fishing activity on the fish stock, and so on.

As well as using a new type of model in the context of fisheries, we have extended the research on mean field games in general by considering games with a common resource. Since real world scenarios involving many players interacting with a common resource are common, the inclusion of a common resource in a mean field game could allow the techniques to be used to investigate other situations as well as fisheries. Understanding the complex interaction between natural resources and human activity is something that would be broadly applicable, and our approach of coupling a mean field game model with a resource equation is well suited to investigating those interactions.

The mean field game model of a fishery allows us to run a simulation of ecological and economic activity, with full feedback between the two, and it does so in a way that can potentially allow us to see the possible impacts of different scenarios in more detail. Although representing many different parties' fishing activity as a continuous distribution is of course an approximation, and one that is not necessarily the most natural or intuitive step when modelling fisheries, it allows us a way to incorporate the idea of a distribution of effort in a model in a way that would otherwise not be possible without a fine level of granularity (e.g. modelling the activities of a large number of fishing vessels individually).

There are many steps that could be taken to directly follow on or improve on some of the research in this thesis. We identified interesting results relating to the impact of price flexibility on the stock, and how this interacts with the stock growth rate; we found that higher price flexibilities led to increased fishing pressure for stocks with lower growth rates, but decreased fishing pressure for stocks with very high growth rates. Further detailed analysis of this, to examine for example the quantitative relationship between the growth rate, the price flexibility and, say, the stock after a certain time period or in the steady state, would be of interest. Similar analysis could also be carried out for other parameters or the interaction between them.

Further investigation of regulations is something that can also be explored using the model developed in this thesis. We examined some different regulations and gained some insights from the comparison. But simply performing similar analysis to what was carried out for price flexibility on one type of regulation, to investigate the results more closely over a range of different parameters, would help advance our understanding of how different regulations might impact a fishery, accounting for dynamic feedback between the stock and the fishing activity. Extending the analysis to look at the interaction between economic or ecological parameters and different types of regulation would then be a natural next step.

We could also take steps to improve the mean field game model. For example, we solved our mean field game equations numerically using an iterative algorithm with a fairly simple implicit method for solving the Hamilton-Jacobi-Bellman and Fokker-Planck PDEs. An obvious step would be to return to this stage and consider using the same iterative approach but with different numerical methods for the PDEs, which may result in improved performance or stability. There is also the work of Achdou [91] [18], which uses finite difference methods for the mean field games by solving a discretized version of the whole system using Newton's method. Adapting this method to solve the mean field game PDEs coupled with a common resource ODE was one of the numerical methods that we considered after clearly formulating the system. However, one of the concerns about using this approach was that the Newton method requires a guess for the whole system, including $u(x,t)$ and $m(x,t)$ at all time points. As we did not find any examples in the literature of mean field games with a similar enough formulation or any application in fisheries that had functions resembling the mean field game variables, we did not already have a strong idea of what these variables may look like in the solution, and if the method did not work with very general initial guesses then we may be left trying to find an initial way to estimate the form of u and m anyway. This meant that our iterative algorithm which required attempting to solve the Hamilton-Jacobi-Bellman equation using the initial m as input was a more suitable method, and this method proved effective enough for our initial simulations.

However, using a finite difference method similar to the work of Achdou may be a way to improve the performance or stability of numerical solutions of the mean field game model studied in this thesis - particularly as we have seen approximations of the solution and have some idea of what it should look like, which means we can more easily ensure that we start with a reasonable guess for the solution to the whole system for the first iteration of Newton's method. Although we did not go further and attempt to use the method in practice, we present in Appendix A.2 the initial steps of a modified version of the finite difference method which would work with a general mean field game with the addition of a common resource.

Another possibility for further work on mean field games in fisheries is to increase the dimension of the game - this could mean adding new species of fish, with the effort variable now having as many dimensions as the types of fish. Alternatively, the dimension of the game could be increased by adding other factors under the agent's control in addition to effort. One example discussed before was the inclusion of individual transferable quotas (ITQs), which can be bought or sold as well as used to land fish. Adding another dimension to the game would likely benefit from further work on the numerical methods to solve the mean field game with a common resource as models with more dimensions may take more time to solve.

As the bulk of the work in this thesis was on developing and obtaining numerical solutions for a mean field game model that is suited to model a fishery, we paired this mean field game only with simple ODE-based ecological models, essentially proving the concept of a mean field game with a common resource. The obvious next stage of using this mean field game model is to pair it with other ecosystem models, allowing the mean field game component to simulate fishing activity in a way that can allow feedback between the ecosystem and the activity in the fishery.

Pairing a mean field game component with a detailed ecosystem model for a specific fishery, we could then try to fit parameters for the mean field game model to historical fishing activity and economic data. In this thesis we considered North Sea cod a particularly relevant fish stock to consider due to its history and the current issues of the UK and EU's fisheries policy in the wake of Brexit. When demonstrating the example of applying a regulation to the mean field game we used a simple logistic ODE with parameters fitted to North Sea cod stock, but a good step for future work would be to pursue this further, coupling the mean field game approach with a more involved ecosystem model and using information such as UK fleet segment data to let our agents in the mean field game represent specific fleets (e.g. just the UK fleet, or the collected fleets of UK and other European nations).

As well as seeking ecological models to fit specific cases, we could also continue to investigate mean field games with different types of broad ecological model. One example which can be incorporated easily is including Allee effects [94] [95] in the stock evolution equation, meaning that growth rate decreases or becomes negative once the population decreases below some threshold value. This would allow us to investigate the results of a mean field game of fish stock exploitation where the fish stock is at greater risk of collapse if it is reduced to lower levels.

Using different ecosystem models also opens up other possibilities for different formulations of mean field games to be used. For example, using a spatial model for the marine ecosystem would allow us to include a spatial element to the mean field game. This could be done either by adding more dimensions to the game to describe the movement in space, or, considering only a one dimensional mean field game, we could assume that agents operate at a consistent level of fishing effort, but their catch depends on their location as the fish population density is location dependent. This type of approach may synergize well with existing mean field game literature on crowd dynamics, as for example an agent may be able to catch less if there are more agents fishing around the same location, meaning that agents will aim to avoid crowding.

Overall, this project has provided tools and information for the integration of economic and ecological marine models in the form of greater understanding of price dynamics and estimates of price flexibilities for the UK market, and the introduction of the mean field game approach to marine economic modelling. The demonstrations of numerical solutions to the mean field game model of a fishery have shown how such a model can be used to investigate the likely outcome of an unregulated fishery under different economic or ecological conditions, or the effects of the application of different regulations.

Investigating these solutions can help us deepen our knowledge of the interactions and feedback between economics and ecology. Being able to alter both economic and ecological variables at the same time and then simulate both the ecosystem and human activity based on agents' own optimisation behaviour allows us to investigate interactions between these variables more effectively. There is ample scope for further work, either applying price flexibility estimations or mean field game approaches alongside different ecological models, or formulating new mean field game models that take a similar approach to this one.

Appendix A

Alternative Numerical Methods

A.1 Numerical iteration method for locating stationary solution to MFG system

One approach for solving the stationary system described in Chapter 4 is to note that to solve equation (4.20) we require that

$$L(N, m) = aN\left(1 - \frac{N}{K}\right).$$

This means that at equilibrium L can be written purely as a function of N. In addition, from the form of $L(N, m)$, we obtain

$$Nq \int_{\mathbb{R}} xm(x)\,\mathrm{d}x = aN\left(1 - \frac{N}{K}\right).$$

This is satisfied either when we have $N = 0$ or when

$$\int_{\mathbb{R}} xm(x)\,\mathrm{d}x = \frac{a}{q}\left(1 - \frac{N}{K}\right). \tag{A.1}$$

This suggests the following approach to locating possible non-zero equilibria. Fix $N_* \in (0, K)$ and solve equations (4.18)–(4.19) for this given N_*; As the result we obtain the probability distribution $m_{N_*}(x)$. Now use (A.1) to find the new N_{*+1} for the given distribution. This probably cannot be done analytically, but it suggests a numerical iteration scheme: starting with some N_0, we set

$$N_{k+1} = K\left(1 - \frac{q}{a}\int_{\mathbb{R}} x m_{N_k}(x)\,\mathrm{d}x\right)$$

for $k = 0, 1, \ldots$, if it so happens that all N_k are positive.

This method may be used to determine if a nonzero equilibrium exists (that is, a solution to the stationary system where $N > 0$), or if N will eventually go to zero (i.e. if $N_{k+1} < N_k \;\forall\; N_k$).

A.2 Finite Difference Method for Mean Field Games

Here we outline the finite difference method developed by Achdou and others [14–16,91], before presenting a scheme for how this could be applied to a mean field game with a common resource. First consider a mean field game system of the form:

$$-\frac{\partial u}{\partial t} - \nu\Delta u + H(x, \nabla u) = V[m](x) \quad \text{in } \Omega \times (0, T) \tag{A.2}$$

$$\frac{\partial m}{\partial t} - \nu\Delta m - \nabla\cdot(mH_p(x, \nabla u)) = 0 \quad \text{in } \Omega \times (0, T) \tag{A.3}$$

$$\nabla u \cdot \mathbf{n} = 0, \text{ and } \; mH_p(x, \nabla u)\cdot\mathbf{n} - \frac{\sigma^2}{2}\nabla m \cdot \mathbf{n} = 0 \text{ on } \partial\Omega \times (0, T) \tag{A.4}$$

$$u(x, T) = G_T[m(T)](x), \; m(x, 0) = m_0(x) \quad \text{in } \Omega, \tag{A.5}$$

where $H(x, p)$ is the Hamiltonian and $H_p(x, q)$ is its derivative with respect to the second variable at the point $p = q$, $\mathbf{n}$ is the outward unit normal vector on $\partial\Omega$, $G_T[m(T)](x)$ is the cost payed at terminal time T, and $m_0(x)$ is the initial distribution. Here we have a finite time horizon problem with Neumann boundary conditions as in [18] (but with a single population).

To describe the discretisation, we will consider two dimensions with $\Omega = [0, 1]^2$, and define Ω_h as a uniform grid with step size h (with $h = 1/N_h$, where N_h is a positive integer). We let N_T be a positive integer with $\Delta t = T/N_T$, and define $t_n = n\Delta t$ where $n = 0, 1, ..., N_T$. The value of u and m are approximated at $(x_{i,j}, t_n$ by the grid functions $U^n_{i,j}$ and $M^n_{i,j}$ respectively.

A.2.1 Finite Difference Operators

Here we describe the elementary finite difference operators we require for the scheme. Define:

$$(D_1^+ U)_{i,j} = \frac{U_{i+1,j}}{h} \qquad \text{and} \qquad (D_2^+ U)_{i,j} = \frac{U_{i,j+1}}{h}$$

and let $[D_h U]_{i,j}$ be the collection of all four possible one-sided finite differences at $x_{i,j}$, i.e.

$$[D_h U]_{i,j} = \left((D_1^+ U)_{i,j}, (D_1^+ U)_{i-1,j}, (D_2^+ U)_{i,j}, (D_2^+ U)_{i,j-1}\right).$$

We also require the standard five-point stencil discrete Laplacian given by

$$(\Delta_h U)_{i,j} = -\frac{1}{h^2}\left(4U_{i,j} - U_{i+1,j} - U_{i-1,j} - U_{i,j+1} - U_{i,j-1}\right).$$

We define the numerical Hamiltonian $g(x, q_1, q_2, q_3, q_4)$, and approximate $H(x_{i,j}, \nabla u)$ by $g(x_{i,j}, [D_h U]_{i,j})$, i.e.

$$g(x_{i,j}, (D_1^+ U)_{i,j}, (D_1^+ U)_{i-1,j}, (D_2^+ U)_{i,j}, (D_2^+ U)_{i,j-1}).$$

It is assumed that for the numerical Hamiltonian, the following assumptions hold:

- Monotonicity: g is nonincreasing with respect to q_1 and q_3, and nondecreasing with respect to q_2 and q_3.
- Consistency: $g(x, q_1, q_1, q_2, q_2) = H(x, q)\ \forall x \in \Omega,\ \forall q = (q_1, q_2) \in \mathbb{R}$.
- Differentiability: $g \in \mathcal{C}^1$.
- Convexity: g is convex.

Note that for Hamiltonians of the form

$$H(x, p) = \mathcal{H}(x) + |p|^\beta$$

for $\beta \in (1, \infty)$, we may select a numerical Hamiltonian of the form

$$g(x, q_1, q_2, q_3, q_4) = \mathcal{H}(x) + \left((q_1^-)^2 + (q_1^+)^2 + (q_3^-)^2 + (q_4^+)^2\right)^{\frac{\beta}{2}},$$

where for $r \in \mathbb{R}$, $r^+ = \max(r, 0)$ and $r^- = \max(-r, 0)$.

Finally we let $V_h[m](x)$ be the discrete version of the operator $V[m](x)$. If V is a local operator (i.e. $V[m](x) = F(m(x))$) then the discrete operator is given by $(V[m])_{i,j} = F(M_{i,j})$. For non-local operators that include terms of the form

$$\int_\Omega x\, m(x)\, \mathrm{d}x,$$

we may approximate them via

$$h^2 \sum_{r,s} x_{r,s}\, M_{r,s}.$$

A.2.2 Finite Difference Scheme

Using the discrete operators and functions defined above, the discrete version of the HJB and Fokker-Planck equations are given as follows:

$$\frac{U_{i,j}^{n+1} - U_{i,j}^{n}}{\Delta t} + \nu(\Delta_h U^{n+1})_{i,j} - g(x_{i,j}, [D_h U^{n+1}]_{i,j}) = -(V_h[M^n])_{i,j}, \tag{A.6}$$

$$\frac{M_{i,j}^{n+1} - M_{i,j}^{n}}{\Delta t} - \nu(\Delta_h M^{n+1})_{i,j} - \mathcal{T}_{i,j}(U^n, M^{n+1}) = 0, \tag{A.7}$$

where we have introduced the notation:

$$\begin{aligned}
\mathcal{T}_{i,j}(U, M) = \frac{1}{h}(&M_{i,j} \frac{\partial}{\partial q_1} g(x_{i,j}, [D_h U]_{i,j}) - M_{i-1,j} \frac{\partial}{\partial q_1} g(x_{i,j}, [D_h U]_{i-1,j}) \\
&+ M_{i+1,j} \frac{\partial}{\partial q_2} g(x_{i,j}, [D_h U]_{i+1,j}) - M_{i,j} \frac{\partial}{\partial q_2} g(x_{i,j}, [D_h U]_{i,j}) \\
&+ M_{i,j} \frac{\partial}{\partial q_3} g(x_{i,j}, [D_h U]_{i,j}) - M_{i,j-1} \frac{\partial}{\partial q_3} g(x_{i,j}, [D_h U]_{i,j-1}) \\
&+ M_{i,j+1} \frac{\partial}{\partial q_4} g(x_{i,j}, [D_h U]_{i,j+1}) - M_{i,j} \frac{\partial}{\partial q_4} g(x_{i,j}, [D_h U]_{i,j}))
\end{aligned}$$

These discretisations use a semi-implicit Euler scheme for the HJB equation, while the discretisation of the Fokker-Planck is obtained by considering the weak formulation for the Fokker-Planck equation [15, 16, 18]. The discrete HJB equation is implicit in U and explicit in M, while the discrete Fokker-Planck equation is implicit in M, explicit in U.

Existence, uniqueness and various convergence results have been proved for this scheme by Achdou et al [14–16, 96] with different assumptions about the operator $V[m](x)$.

A.2.3 Algorithms for Solving the Finite Difference Scheme

Algorithms for solving the discretised systems such as those in A.2.1 were proposed in proposed in [16, 91]. These involve using the Newton or Newton–Raphson method to solve the nonlinear system of equations obtained from the finite difference scheme.

In A.2.1, we have the system

$$\frac{U_{i,j}^{n+1} - U_{i,j}^{n}}{\Delta t} + \nu(\Delta_h U^{n+1})_{i,j} - g(x_{i,j}, [D_h U^{n+1}]_{i,j}) = -(V_h[M^n])_{i,j}, \tag{A.8}$$

$$\frac{M_{i,j}^{n+1} - M_{i,j}^{n}}{\Delta t} - \nu(\Delta_h M^{n+1})_{i,j} - \mathcal{T}_{i,j}(U^n, M^{n+1}) = 0. \tag{A.9}$$

This nonlinear system may be written as

$$\mathcal{F}_U(\mathcal{U}, \mathcal{M}) = 0 \tag{A.10}$$

$$\mathcal{F}_M(\mathcal{U}, \mathcal{M}) = 0 \tag{A.11}$$

Where $\mathcal{F}_U$ corresponds to the discrete HJB equation (A.8), $\mathcal{F}_M$ corresponds to the discrete Fokker-Planck equation (A.9), and $\mathcal{U}$ and $\mathcal{M}$ are vectors containing all values of the grid functions U^n and M^n. This nonlinear system of equations may be very large.

The algorithm proposed in [16,91] is to use the Newton method on the whole system of nonlinear equations. Solving using the Newton method requires solving a large linear system of equations involving the Jacobian matrix

$$A = \begin{pmatrix} A_{U,U} & A_{U,M} \\ A_{M,U} & A_{M,M} \end{pmatrix}$$

where $A_{U,U}$ denotes $D_{\mathcal{U}}\mathcal{F}_U(\mathcal{U}, \mathcal{M})$.

If we let the next iterate in the Newton method be given by

$$\begin{pmatrix} \mathcal{U}_{n+1} \\ \mathcal{M}_{n+1} \end{pmatrix} = \begin{pmatrix} \mathcal{U}_n \\ \mathcal{M}_n \end{pmatrix} + \begin{pmatrix} \mathcal{U}'_n \\ \mathcal{M}'_n \end{pmatrix},$$

where

$$\begin{pmatrix} \mathcal{U}'_n \\ \mathcal{M}'_n \end{pmatrix} = A^{-1} \begin{pmatrix} \mathcal{F}_U(\mathcal{U}_n, \mathcal{M}_n) \\ \mathcal{F}_M(\mathcal{U}_n, \mathcal{M}_n) \end{pmatrix},$$

then the next iterate is obtained by solving the linear system:

$$A \begin{pmatrix} \mathcal{U}'_n \\ \mathcal{M}'_n \end{pmatrix} = \begin{pmatrix} \mathcal{F}_U(\mathcal{U}_n, \mathcal{M}_n) \\ \mathcal{F}_M(\mathcal{U}_n, \mathcal{M}_n) \end{pmatrix}.$$

It is this linear system that is solved by an iterative strategy.

In addition, it is noted that the iterative method algorithm proposed converges quickly when the volatility parameter ν in (A.8) is high [18, 91], and so a possible method for solving the system for a given value of ν is to initially solve with a high value of ν, then gradually decrease the ν to the required value, using the solution from the previous value of ν as the initial guess for the Newton Method. This is similar to continuation methods used in other applications [20].

A.2.4 Finite Difference Scheme with Common Resource

Now we consider a mean field game with a common resource. Suppose we have a system of the form:

$$-\frac{\partial u}{\partial t} - \nu\Delta u + H(x, \nabla u) = V(x, m, y) \quad \text{in } \Omega \times (0, T) \tag{A.12}$$

$$\frac{\partial m}{\partial t} - \nu \Delta m - \nabla \cdot (m H_p(x, \nabla u)) = 0 \quad \text{in } \Omega \times (0,T) \tag{A.13}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = R(y) - \int_\Omega x\, m(x)\, \mathrm{d}x \tag{A.14}$$

$$\nabla u \cdot \mathbf{n} = 0, \text{ and } \; m H_p(x, \nabla u) \cdot \mathbf{n} - \frac{\sigma^2}{2} \nabla m \cdot \mathbf{n} = 0 \text{ on } \partial\Omega \times (0,T) \tag{A.15}$$

$$u(x,T) = G_T[m(T)](x), \; m(x,0) = m_0(x), \; y(0) = y_0 \quad \text{in } \Omega. \tag{A.16}$$

This mean field game system contains the HJB and Fokker-Planck equation with Neumann boundary conditions, terminal condition for u and initial condition for m, and an equation describing the evolution common renewable resource y with initial condition $y(0) = y_0$, while the cost operator in the HJB equation is a function of x, m and y.

Let $\Omega = [0,1]^2$ with Ω_h as a uniform grid with step size h such that $h = 1/N_h$, let $\Delta t = T/N_T$, and define $t_n = n\Delta t$ where $n = 0, 1, ..., N_T$. The functions u, m and y are approximated by the grid functions $U^n_{i,j}$, $M^n_{i,j}$ and Y^n.

We define the collection of one-sided finite differences $[D_h U]_{i,j}$, the five-point discrete Laplace operator $(\Delta_h U)_{i,j}$ and the numerical Hamiltonian $g(x, q_1, q_2, q_3, q_4)$ as described in Section A.2.1.1, and set the notation:

$$\begin{aligned}
\mathcal{T}_{i,j}(U,M) = \frac{1}{h}\Big(& M_{i,j} \frac{\partial}{\partial q_1} g(x_{i,j}, [D_h U]_{i,j}) - M_{i-1,j} \frac{\partial}{\partial q_1} g(x_{i,j}, [D_h U]_{i-1,j}) \\
&+ M_{i+1,j} \frac{\partial}{\partial q_2} g(x_{i,j}, [D_h U]_{i+1,j}) - M_{i,j} \frac{\partial}{\partial q_2} g(x_{i,j}, [D_h U]_{i,j}) \\
&+ M_{i,j} \frac{\partial}{\partial q_3} g(x_{i,j}, [D_h U]_{i,j}) - M_{i,j-1} \frac{\partial}{\partial q_3} g(x_{i,j}, [D_h U]_{i,j-1}) \\
&+ M_{i,j+1} \frac{\partial}{\partial q_4} g(x_{i,j}, [D_h U]_{i,j+1}) - M_{i,j} \frac{\partial}{\partial q_4} g(x_{i,j}, [D_h U]_{i,j})\Big).
\end{aligned}$$

For the common resource evolution equation, we may approximate the integral term by

$$h^2 \sum_{r,s} x_{r,s}\, M_{r,s},$$

and use a simple finite difference scheme to obtain

$$\frac{Y^{n+1} - Y^n}{\Delta t} = R(Y^n) - h^2 \sum_{r,s} x_{r,s} \, M^n_{r,s},$$

The discretisation of the HJB and Fokker-Planck equation is the same as described in A.2.1.2, based on the finite difference methods developed in [14–16, 18, 91].

This gives us the discretised system:

$$\frac{U^{n+1}_{i,j} - U^n_{i,j}}{\Delta t} + \nu(\Delta_h U^{n+1})_{i,j} - g(x_{i,j}, [D_h U^{n+1}]_{i,j}) = -V(x_{i,j}, M^n_{i,j}, Y^n), \tag{A.17}$$

$$\frac{M^{n+1}_{i,j} - M^n_{i,j}}{\Delta t} - \nu(\Delta_h M^{n+1})_{i,j} - \mathcal{T}_{i,j}(U^n, M^{n+1}) = 0, \tag{A.18}$$

$$\frac{Y^{n+1} - Y^n}{\Delta t} = R(Y^n) - h^2 \sum_{r,s} x_{r,s} \, M^n_{r,s}, \tag{A.19}$$

The homogeneous Neumann boundary conditions for the finite difference scheme are given by setting that, at all times t_n and $i, j = 2, ...N_h$, we have:

$$U^n_{1,j} = U^n_{2,j}, \quad U^n_{N_{h-1},j} = U^n_{N_h,j}, \quad U^n_{i,1} = U^n_{i,2}, \quad U^n_{i,N_{h-1}} = U^n_{i,N_h}, \tag{A.20}$$

and that

$$U^n_{1,1} = U^n_{2,2}, \quad U^n_{N_{h1},1} = U^n_{N_{h-1},2}, \quad U^n_{1,N_h} = U^n_{2,N_{h-1}}, \quad U^n_{N_h,N_h} = U^n_{N_{h-1},N_{h-1}}. \tag{A.21}$$

The values of $M^n_{i,j}$ and Y^n are given at $n = 0$, while the values of $U^n_{i,j}$ are given at $n = N_T$.

A.2.5 Setting in One Dimension

Let $d = 1$, with $\Omega = [0, 1]$ and horizon time T. The MFG system is given by

$$-\frac{\partial u}{\partial t} - \nu \Delta u + H(x, \nabla u) = V[m, y](x) \quad \text{in } \Omega \times (0, T) \tag{A.22}$$

$$\frac{\partial m}{\partial t} - \nu \Delta m - \nabla \cdot (m H_p(x, \nabla u)) = 0 \quad \text{in } \Omega \times (0, T) \tag{A.23}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = R(y) - \int_0^1 x\, m(x)\, \mathrm{d}x \quad \text{in } (0,T) \tag{A.24}$$

$$\nabla u = 0, \text{ and } \; mH_p(x, u_{\nabla u}) - \frac{\sigma^2}{2}\nabla m = 0 \text{ on } \partial\Omega \times (0,T) \tag{A.25}$$

$$u(x,T) = G_T[m(T)](x), \; m(x,0) = m_0(x), \; y(0) = y_0 \quad \text{in } \Omega, \tag{A.26}$$

The stepsize is given by $h = \frac{1}{N_h}$, and $\Delta t = \frac{T}{N_T}$, so $t_n = n\Delta t$ and $x_i = ih$. The grid functions are defined by $u(t_n, x_i) = U_i^n$, $m(t_n, x_n) = M_i^n$ and $y(t_n) = Y^n$.

The one dimensional discrete Laplace operator is defined as:

$$(\Delta_h u)_i = \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2},$$

and we define $[D_h u]_i \in \mathbb{R}^2$ as the collection of one sided finite differences:

$$[D_h u]_i = \left(\frac{u_{i+1} - u_i}{h}, \frac{u_i - u_{i-1}}{h} \right).$$

A.2.5.1 Scheme for HJB Equation

The discrete Hamiltonian $g : \Omega \times \mathbb{R}^2 \to \mathbb{R}$ is written as $g(x,q)$, where $q = (q_1, q_2)$, and satisfies the following assumptions:

- g is continuous and $\mathcal{C}^1$ with respect to q.
- Consistency: $g(x,(p,p)) = H(x,p)$
- Monotonicity: $g(x,()q_1,q_2))$ is decreasing with respect to q_1 and increasing with respect to q_2.
- $g(x,q)$ is convex with respect to q.

In the case where $H(x,p) = |p|^\beta$, then $g(x,(q_1,q_2)) = ((q_1^-)^2 + (q_2^+)^2)^{\frac{\beta}{2}}$.

The scheme for the HJB equation is then:

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} + \nu(\Delta_h U^n)_i - g(x_i, [D_h U^n]_i) = -(V_h[M^{n+1}, Y^{n+1}])_{i,j}. \tag{A.27}$$

A.2.5.2 Scheme for Fokker-Planck Equation

To keep the structure of the MFG system, we need that g be used in the discrete version of $(H_p(x, \nabla u))$. We consider the weak formulation of the Fokker-Planck equation, which includes the term:

$$\int_\Omega \nabla \cdot (mH_p(x, \nabla u))w \, \mathrm{d}x.$$

Integrating by parts we obtain

$$\int_\Omega \nabla \cdot (mH_p(x, \nabla u))w \, \mathrm{d}x = -\int_\Omega mH_p(x, \nabla u))\nabla w \, \mathrm{d}x.$$

We approximate $\int_\Omega mH_p(x, \nabla u))\nabla w \, \mathrm{d}x$ by

$$h \sum_i M_i \frac{\partial g}{\partial q}(x_i, [D_h U]_i) \cdot [D_h W]_i$$

$$= h \sum_i M_i (g_{q_1}(x_i, [D_h U]_i), g_{q_2}(x_i, [D_h U]_i)) \cdot \left(\frac{W_{i+1} - W_i}{h}, \frac{W_i - W_{i-1}}{h} \right)$$

$$= h \sum_i \frac{M_i}{h} \left(g_{q_1}(x_i, [D_h U]_i)(W_{i+1} - W_i) + g_{q_2}(x_i, [D_h U]_i)(W_i - W_{i-1}) \right)$$

$$= h \sum_i \frac{M_i}{h} \left(W_{i+1} g_{q_1}(x_i, [D_h U]_i) - W_i g_{q_1}(x_i, [D_h U]_i) \right.$$
$$\left. + W_i g_{q_2}(x_i, [D_h U]_i) - W_{i-1} g_{q_2}(x_i, [D_h U]_i) \right)$$

$$= h \sum_i \frac{W_i}{h} \left(M_{i-1} g_{q_1}(x_{i-1}, [D_h U]_{i-1}) - M_i g_{q_1}(x_i, [D_h U]_i) \right.$$
$$\left. + M_i g_{q_2}(x_i, [D_h U]_i) - M_{i+1} g_{q_2}(x_{i+1}, [D_h U]_{i+1}) \right).$$

So if we approximate $\int_\Omega \nabla \cdot (mH_p(x, \nabla u))w \, \mathrm{d}x$ by

$$h \sum_i \mathcal{Z}_i(U_i, M_i)W_i,$$

then we have

$$\begin{aligned} h \sum_i \mathcal{Z}_i(U_i, M_i)W_i = -h \sum_i \frac{W_i}{h} \left(M_{i-1} g_{q_1}(x_{i-1}, [D_hU]_{i-1}) - M_i g_{q_1}(x_i, [D_hU]_i) \right. \\ \left. + M_i g_{q_2}(x_i, [D_hU]_i) - M_{i+1} g_{q_2}(x_{i+1}, [D_hU]_{i+1}) \right). \end{aligned}$$

Hence we approximate $\nabla \cdot (mH_p(x, \nabla u))$ with $\mathcal{Z}_i(U_i, M_i)$, where:

$$\begin{aligned} \mathcal{Z}_i(U_i, M_i) = \frac{W_i}{h} \left(M_{i-1} g_{q_1}(x_{i-1}, [D_hU]_{i-1}) - M_i g_{q_1}(x_i, [D_hU]_i) \right. \\ \left. + M_i g_{q_2}(x_i, [D_hU]_i) - M_{i+1} g_{q_2}(x_{i+1}, [D_hU]_{i+1}) \right). \end{aligned}$$

Then the scheme for the Fokker-Planck Equation is:

$$\frac{M_i^{n+1} - M_i^n}{\Delta t} - \nu(\Delta_h M^{n+1})_i + \mathcal{Z}_i(U_i^n, M_i^{n+1}) = 0. \tag{A.28}$$

The discrete HJB equation is implicit in u, and explicit in m and y, so that $(U^{n+1}, M^{n+1}, Y^{n+1}) \to U^n$, and the discrete Fokker-Planck equation is implicit im m, explicit in u with $(U^n, m^n) \to M^{n+1}$.

A.2.5.3 Scheme for Common Resource Evolution Equation

We may approximate the integral term in the resource evolution equation by

$$h \sum_s x_s \, M_s.$$

We may then choose an explicit scheme in Y and M such that $(Y^n, M^n) \to Y^{n+1}$

with

$$\frac{Y^{n+1} - Y^n}{\Delta t} = R(Y^n) - h \sum_s x_s \, M_s^n. \tag{A.29}$$

A.2.5.4 Summary of Discrete System

The full discrete system is given by the discretised system:

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} + \nu(\Delta_h U^n)_i - g(x_i, [D_h U^n]_i) = -(V_h[M^{n+1}, Y^{n+1}])_{i,j}. \tag{A.30}$$

$$\frac{M_i^{n+1} - M_i^n}{\Delta t} - \nu(\Delta_h M^{n+1})_i + \mathcal{Z}_i(U_i^n, M_i^{n+1}) = 0. \tag{A.31}$$

$$\frac{Y^{n+1} - Y^n}{\Delta t} = R(Y^n) - h \sum_s x_s \, M_s^n. \tag{A.32}$$

The homogeneous Neumann boundary conditions for the finite difference scheme are given by setting that for all n:

$$U_1^n = U_{2,j}^n, \quad U_{N_{h-1}}^n = U_{N_h}^n, \tag{A.33}$$

$$M_1^n = M_{2,j}^n, \quad M_{N_{h-1}}^n = M_{N_h}^n. \tag{A.34}$$

The values of M_i^n and Y^n are given at $n = 0$, while the values of U_i^n are given at $n = N_T$.

Appendix B

Additional Numerical Results

This chapter contains some additional results and plots that provide additional detail or supporting information to the main results presented in Chapter 6.

B.1 Numerical Simulation - Base Case

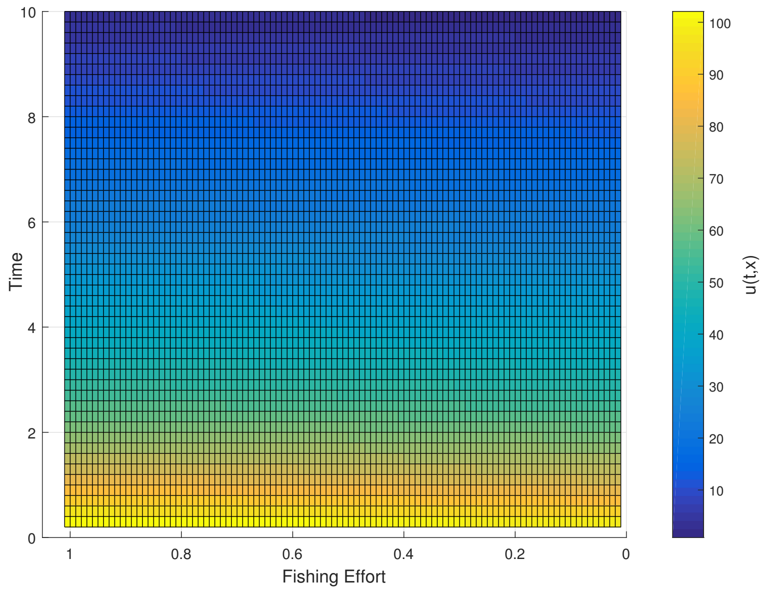


Figure B.1: Heat map plot of $u(t, x)$ from the solution to the basic case.

Here we provide some additional visualisation of the solution of the base case numerical simulation in Section 6.3. Figure B.1 shows a heat map of $u(t, x)$, corresponding to the surface plot in Figure 6.1, while Figure B.2 shows the heat map for $m(t,x)$, corresponding to the surface plot in Figure 6.2. From Figure B.2 we can see clearly how the distribution shifts from its initial uniform distribution at $t = 0$ to a distribution with high values of m close to $x = 1$ and low values of m close to $x = 0$, but then over time the values of m close to $x = 1$ decrease and there is an increase in m for lower values of x.

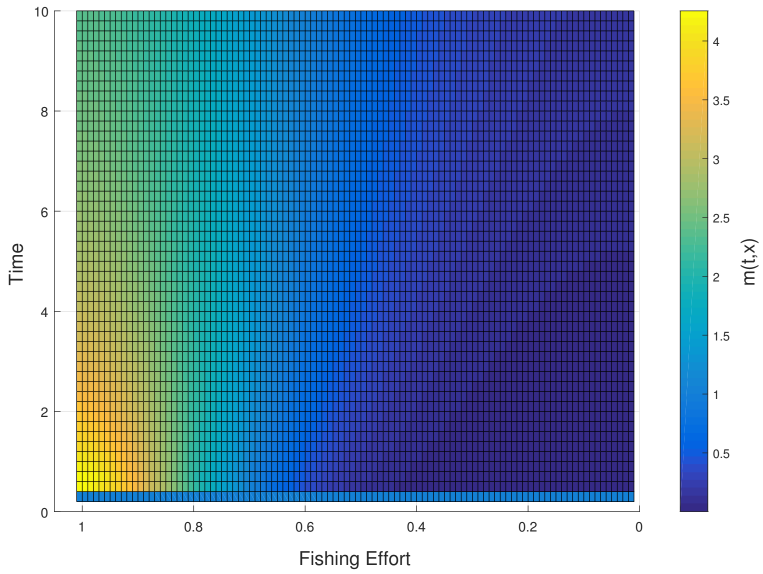
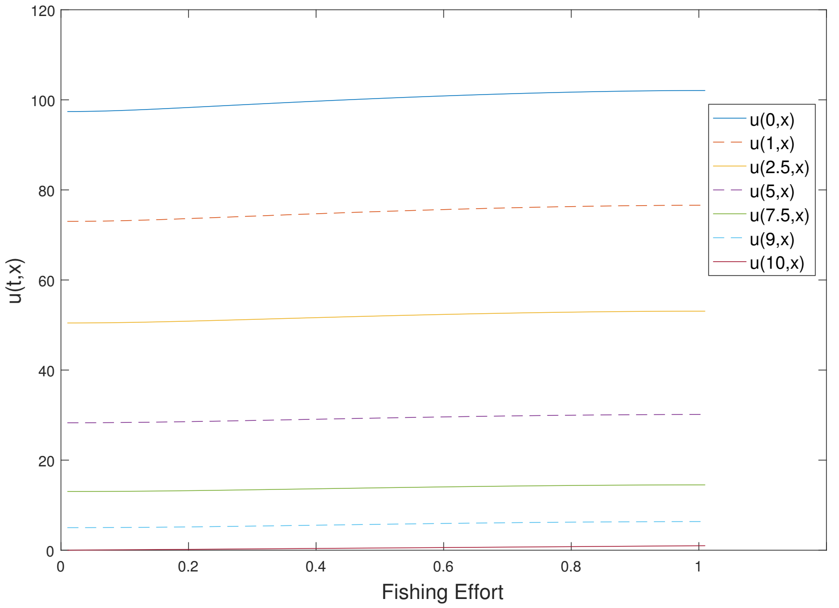
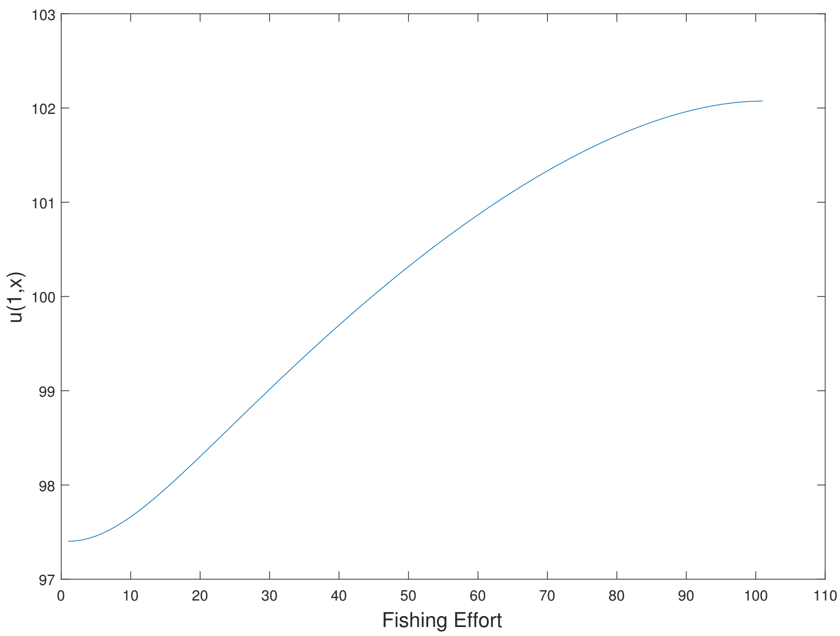


Figure B.2: Heat map plot of $m(t,x)$ from the solution to the basic case.

We can interpret what is occurring in our solution for $m(t,x)$ from the surface plot and heat map quite easily, but it is more difficult to interpret the behaviour of $u(t,x)$ from the heat map and surface plot, other than that $u(t,x)$ is decreasing with time. So we will take a closer look at $u(t,x)$ by plotting the curves of $u(t,x)$ for different values of t, allowing us to see the shape of the value function at different points in time. In Figure B.3a we can still see that the value function is decreasing with time, but we can also see that at each point in time, $u(t,x)$ is increasing with x. This is demonstrated even more clearly by looking plot of an individual curve $u(t,x)$ for a given t, as in Figure B.3b. Inspecting the curves of $u(t,x)$ for different values of time, we can determine that the steepness of the curve decreases as time increases, and the curves become flatter, although still increasing with x, until they reach the terminal condition $u(T,x) \equiv 0$.



(a) $u(t,x)$ for different values of t



(b) $u(1,x)$

Figure B.3: (a) Plot showing the curve of $u(t,x)$ for $t = 0, t = 1, t = 2.5, t = 5, t = 7.5, t = 9$ and $t = 10$, and (b) Plot showing the curve of $u(1,x)$ only, from the solution to the MFG system with base case parameters.

B.2 Terminal Cost

Figure B.4 shows $u(t,x)$ for different values of t, with the base case parameters and linear terminal cost $u(T,x) = x$ from Section 6.4. Comparing this with Figure B.3a which had $u(T,x) \equiv 0$, we see that the curves of $u(t,x)$ are almost identical other than at the horizon time, meaning that there is no major effect on the behaviour of $u(t,x)$ throughout most of the time period due to changing to a linear terminal cost.

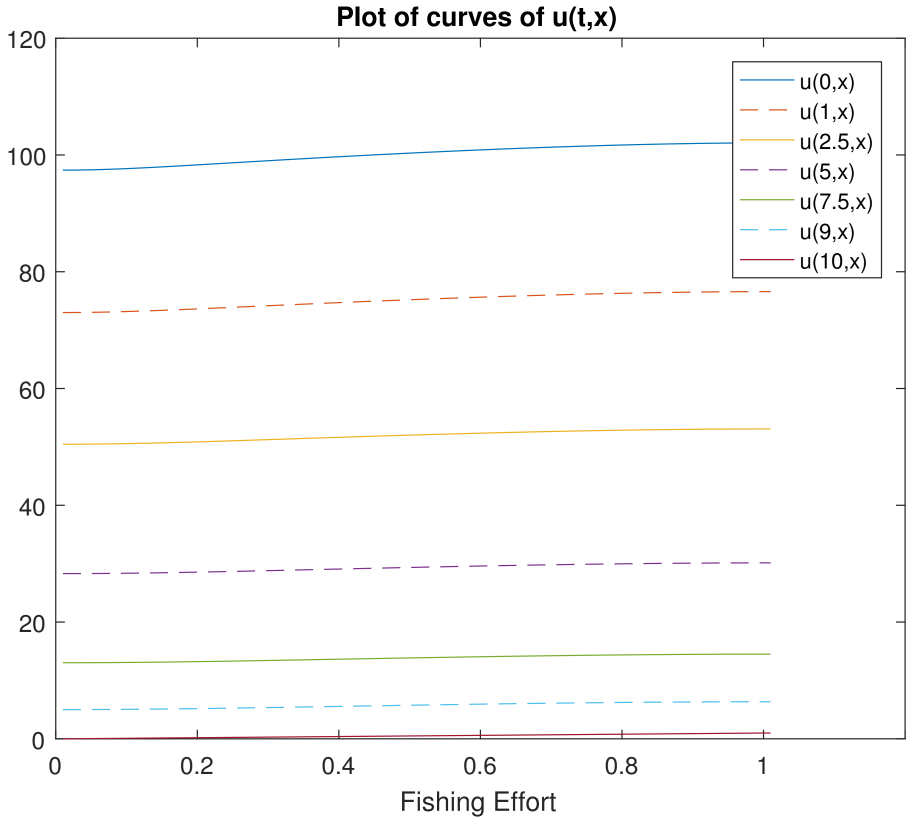


Figure B.4: Plots of $u(t,x)$ at different points in time with $u(T,x) = x$

We can also check that with this linear form for $u(T,x)$, the final stock value $N(T)$ decreases from 10.6128 to 10.5183. So there is a slight increase in fishing pressure, due to the increasing terminal function for u resulting in higher fishing rates in the timesteps immediately before the horizon time, although it did not change the overall solution by much.

To check how much the behaviour of the solution to the MFG system depends on the slope of the linear function in the terminal cost, we solved the MFG equations with parameters given in Chapter 6 and with terminal cost $u(T, x) = Cx$ for a range of values of C between 0.1 and 25, and compared them with the solution presented in Section 6.4. It was found that in all cases the evolution of m and u very similar for the vast majority of the time period, regardless of the slope of the linear terminal cost. The only notable difference occurred for steep linear functions, where the distribution differed close to the end of the time period - this can be seen in Figure B.5 for $u(T, x) = 25x$, where within the last 0.2 units of time the distribution of effort shifts back up towards the maximum again.

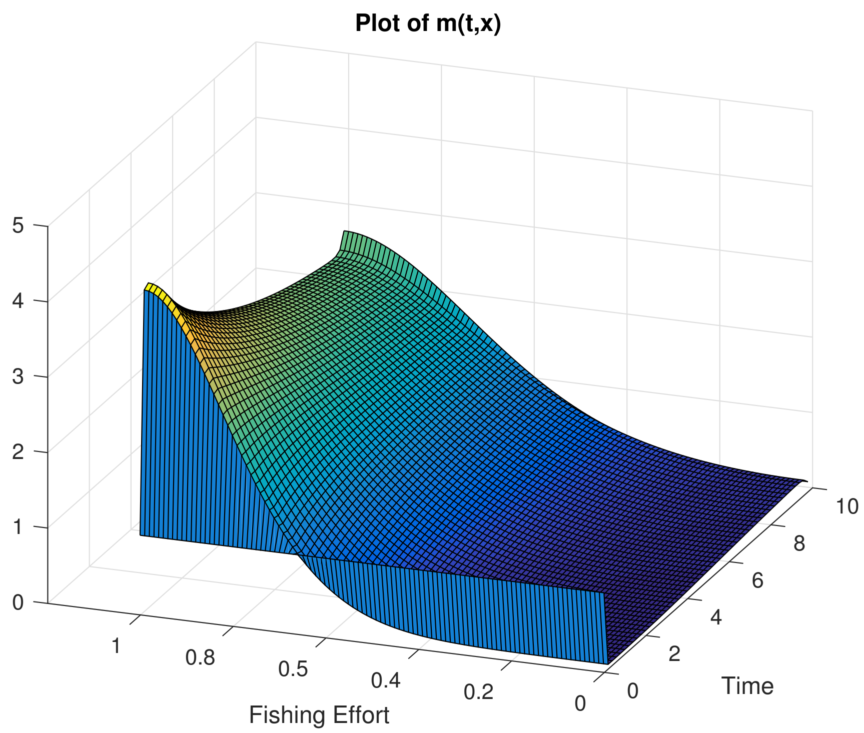


Figure B.5: Plot of $m(t, x)$ with linear $u(T, x) = 25x$

This difference for steep linear functions is due to the fact the terminal condition favours high values of x more than they were favoured by the dynamics of the game during the time period. If this discrepancy is even larger (i.e if we used even higher values of C) this shift in effort distribution would be more pronounced. Further investigation could be done to determine just how far back in time the impact of a different terminal condition extends. However, we can see that even for a terminal cost with slope 25 times higher than the value used in Chapter 6, we only see a difference in the last 0.2 units of time using a time horizon $T = 10$, suggesting that unless we choose a function with extremely high values for our terminal cost compared to the running cost, then we should not expect it to affect the dynamics of the solution throughout the vast majority of the time period.

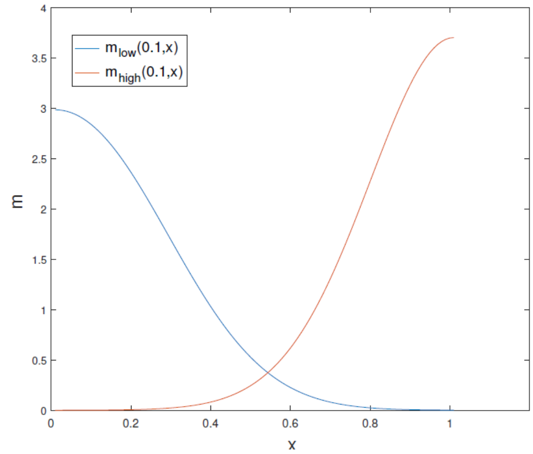
Also, we noted that solving the case with $u(T, x) = 25$ required increasing the number of timesteps n_t (solved using $n_t = 2000$ instead of 500 as previously). At lower values of n_t, the algorithm fails at the first attempt to solve the Hamilton-Jacobi-Bellman equation, using our first iteration $m(t, x) = m_0(x)$. We return to consider the number of timesteps required in a later section in Chapter 6.

B.3 Initial Distribution

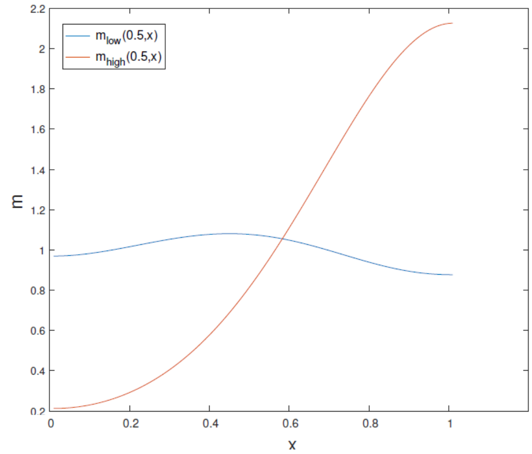
Figure B.6 shows the distribution of $m_{\text{high}}(x, t)$ and $m_{\text{low}}(x, t)$ and (using the same high effort and low effort initial distributions as in Section 6.7) at different times for the case where σ has increased to $\sigma = 1$ from $\sigma = 0.5$. In this case, we find that the effort distributions are slower to approach the same shape. Figure B.6(b) shows that the distributions are quite different even at $t = 0.5$, and in Figure B.6(c) we can see that although the distributions are quite similarly shaped by the horizon time $T = 1$, they remain quite different (compared to the curves in Figure 6.15(c) which are close to identical).

Here, increasing σ meant that it took longer for the distributions to approach the same curve and hence for the impact of the initial distribution to be smoothed out. Based on the plots in Figure B.6, we can see that it takes quite a long time for the initial low effort distribution to approach the eventual optimal distribution shape. However, the initial high effort distribution seems to approach this quite quickly as the distribution remains similar between $t = 0.5$ and $t = 1$. So it seems likely that higher values of σ mean that initial distribution matters more if it is very different from the effort distribution that would best suit the problem parameters. This occurs because, although the volatility σ has a smoothing or flattening effect on the distribution, this smoothing effect means that it takes longer for the movement towards the optimal effort distribution to take effect. If two initial distributions are already quite similar to each other, we may expect that a higher σ would mean that the difference between them would fade quicker. Similarly, if σ is very high (to the point that the eventual shape of the distribution is dominated more by σ than by the cost function) then different initial distributions may become similar more quickly as they are flattened out.

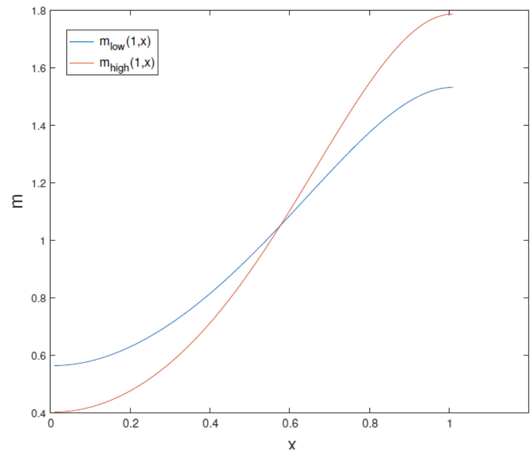
Figure B.7 shows plots of the distributions $m_{\text{high}}(x, t)$ and $m_{\text{low}}(x, t)$ starting from high effort and low effort initial conditions with $\gamma = 1$, $\sigma = 0.5$. Here we see that the lower value of γ (i.e. lower cost of control) results in the effort distributions moving more quickly towards the optimal distribution. By $t = 0.5$, the effort distribution curves are already close to identical.



(a) $t = 0.1$

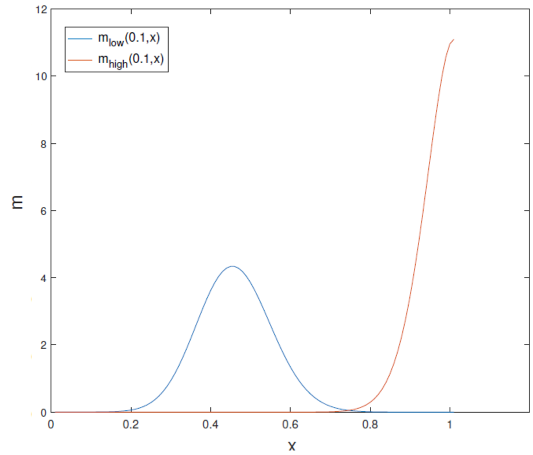


(b) $t = 0.5$

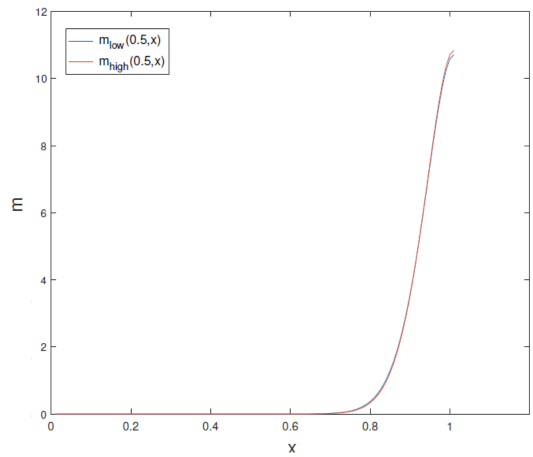


(c) $t = 1$

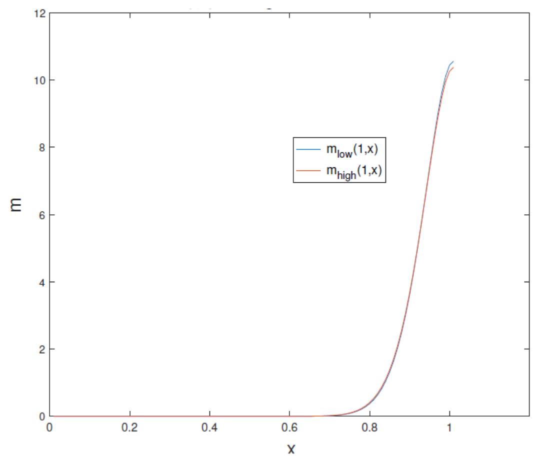
Figure B.6: Plots showing $m_{\text{low}}(t, x)$ and $m_{\text{high}}(t, x)$ (low and high initial distributions of effort, respectively) at different values of t, for $T = 1$, $\gamma = 10$ and $\sigma = 1$.



(a) $t = 0.1$



(b) $t = 0.5$



(c) $t = 1$

Figure B.7: Plots showing $m_{\text{low}}(t,x)$ and $m_{\text{high}}(t,x)$ (low and high initial distributions of effort, respectively) at different values of t, for $T = 1$, $\gamma = 1$ and $\sigma = 0.5$.

Appendix C

Papers

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Making modelling count - increasing the contribution of shelf-seas community and ecosystem models to policy development and management

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ABSTRACT

Marine legislation is becoming more complex and marine ecosystem-based management is specified in national and regional legislative frameworks. Shelf-seas community and ecosystem models (hereafter termed ecosystem models) are central to the delivery of ecosystem-based management, but there is limited uptake and use of model products by decision makers in Europe and the UK in comparison with other countries. In this study, the challenges to the uptake and use of ecosystem models in support of marine environmental management are assessed using the UK capability as an example. The UK has a broad capability in marine ecosystem modelling, with at least 14 different models that support management, but few examples exist of ecosystem modelling that underpin policy or management decisions. To improve understanding of policy and management issues that can be addressed using ecosystem models, a workshop was convened that brought together advisors, assessors, biologists, social scientists, economists, modellers, statisticians, policy makers, and funders. Some policy requirements were identified that can be addressed without further model development including: attribution of environmental change to underlying drivers, integration of models and observations to develop more efficient monitoring programmes, assessment of indicator performance for different management goals, and the costs and benefit of legislation. Multi-model ensembles are being developed in cases where many models

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exist, but model structures are very diverse making a standardised approach of combining outputs a significant challenge, and there is a need for new methodologies for describing, analysing, and visualising uncertainties. A stronger link to social and economic systems is needed to increase the range of policy-related questions that can be addressed. It is also important to improve communication between policy and modelling communities so that there is a shared understanding of the strengths and limitations of ecosystem models.

1. Introduction

Marine legislation is becoming more complex as a consequence of increasing and more diverse use of the sea [1]. Commitments to marine ecosystem-based management that influence the UK are specified in national and regional legislative frameworks including the Marine Strategy Framework Directive (MSFD) [2], Common Fisheries Policy (CFP) [3], and the Water Framework Directive (WFD) [4]. However, the funding to provide the empirical evidence base that underpins monitoring, assessment, and management in support of these policies is decreasing in relative terms, requiring increasingly cost-effective decision tools for operational management and scenario planning. The key requirements for decision-makers are to understand links between human and environmental pressures and the state of the environment, to determine suitable management measures to meet objectives, to track progress in relation to those objectives, and to assess the performance of management options based on their environmental, social and economic consequences [5–7]. Shelf-seas community and ecosystem models (hereafter termed ecosystem models) can help to meet these requirements. Specific examples of contributions could include testing the sensitivity of indicators, increasing the cost-effectiveness of monitoring programmes, and supporting practical application of theoretical concepts like maximum sustainable yield (MSY).

Ecosystem models often differ fundamentally from models of physical systems because ecosystem dynamics are rarely directly governed by physical laws alone, but result from complex biological feedbacks requiring some form of approximation. Thus, it is usually important to embrace model diversity to account for uncertainty about the most appropriate model structure [8]. Consequently, multi-model ensemble approaches similar to that used by the Intergovernmental Panel on Climate Change (IPCC) for climate projections [9] can be used to convey uncertainty that results from differences in structure; an approach that is starting to be applied to advice on the management of fisheries [8,10].

Ecosystem models could make a much greater contribution to the evidence base that underpins policy development and decision-making, because they allow a priori testing of policies and management scenarios and quantification of the risk and uncertainty. In most cases, it is impossible to assess the performance of policies and potential management measures without models. For models to fulfil a greater role in policy development and decision-making, and for the associated advice to be treated as credible, salient and legitimate, the modelling approaches used need to be more transparent, verifiable, and repeatable than they are at present.

Ecosystem models are increasingly used in support of marine environmental assessment, management, and policy development in other parts of the world including the USA and Australia (e.g. [11,12]), but are not routinely used in the UK and Europe. In this paper, the prospects for increasing the contribution of community and ecosystem models to the evidence base that underpins assessment, management and policy support is assessed. Focussing on the UK shelf-seas community and ecosystem modelling capability, the range of models available are reviewed, actions expected to increase the uptake and use of these models in environmental management are identified, and priorities for model development, application and presentation are highlighted.

2. UK ecosystem modelling capability and its impact on policy

Many different global marine ecosystem models have been developed [13] and extensive intercomparisons have been made [14], but here the focus is on regional models (e.g. shelf-wide, regional sea) as these have the most direct relevance for application to UK marine environmental policy and management including regulation. UK institutes and universities already use many classes of models that represent different components of the ecosystem (Fig. 1). These range from models of biogeochemistry and low trophic levels (e.g. [15]) to size-based approaches (e.g. [16–19]) and models of the whole food web (e.g. [20,21]). Some ecosystem models have been coupled to physical models and aim to represent the entire system from physics to fishers [22]. Models vary in structure and parameterisation since they have been developed to address different questions by researchers with different philosophies and approaches. For example, ERSEM was originally developed as an end-to-end ecosystem model to study nutrient cycling and planktonic ecosystem dynamics [15], the Population-Dynamical Matching Model (PDMM) (e.g. [23,24]) was constructed to develop theoretical understanding of food-web patterns and biodiversity [25,26], and Ecopath with Ecosim (EwE) to assess the impacts of fisheries on food webs and consequences for fisheries (e.g. [27]).

At least fourteen different marine ecosystem models are being used in the UK (Table 1 and model summaries provided at http://www.masts.ac.uk/research/marine-ecosystem-modelling/). Few of these models have directly influenced or routinely supported management and policy development, but many are likely to have influenced societal and scientific perceptions about the state of the marine environment. This has had an indirect influence on the emphasis given to ecosystem considerations in contemporary policy (e.g. [28–31]). As policy-making is normative and reflects societal values, alongside the evidence base [32], it is often difficult to ascribe direct links between models and decisions. However, there are some good examples including predicting harmful algal blooms, eutrophication, and comparisons between targets for environmental legislation as explained below.

Operational forecasting and monitoring of water quality enables timely interventions by both stakeholders and the agencies responsible for public health. The AlgaRisk monitoring tool is a prototype that provided warnings of algal blooms to support the statutory obligations of the Environment Agency [33,34]. This tool combines data from an operational physical–biological coastal model with satellite observations, and the results are available through an internet portal where users can visualise both model output and observations (http://www.neodaas.ac.uk/multiview/pa/). A demonstration AlgaRisk service was implemented in 2008 to support the European Union Bathing Waters Directive.

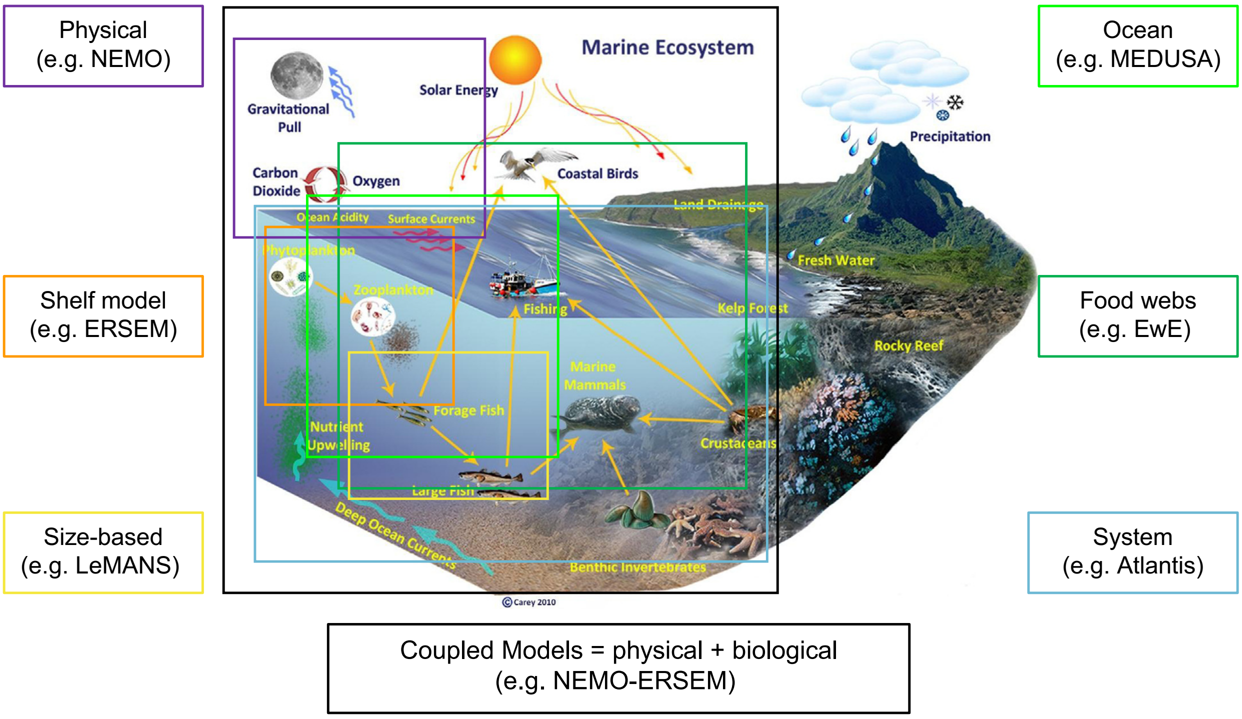

Fig. 1. Categories of ecosystem models and the parts of the ecosystem that they include.

Detection and diagnosis of eutrophication is required for a range of EU legislation (e.g. MSFD [2], WFD [4]) and by the OSPAR Convention [35]. Following the first assessment of eutrophication for OSPAR, the Netherlands and Germany identified eutrophication problem areas in their marine waters and alleged that inputs of nitrogen from the UK made a significant contribution. The OSPAR Eutrophication Committee tasked the Intersessional Correspondence Group for Eutrophication Modelling (ICG-EMO) to undertake modelling based on OSPAR riverine nutrient reduction scenarios and trans-boundary nutrient transport [36,37]. This work involved the application of seven ecosystem models by different institutes for pre-defined scenarios, using the same forcing, validation data, methods, and post-processing procedures. The resulting multi-model ensemble was used to assess uncertainty, which substantially enhanced the overall credibility of the results reported to the OSPAR Eutrophication Committee. Their subsequent influence on OSPAR decision making was far greater than would have been achieved by one national source. This modelling work was also used as supporting evidence in a case where the UK successfully defended against the European Commission in the European Court of Justice (Case C-390/07).

Advice on fisheries management is routinely supported by single-species modelling through the UK contribution to the work of ICES assessment groups. Ecosystem models are less widely used, but have been adopted to provide advice on the prospects for meeting single-species management targets simultaneously, and assessing the trade-offs between meeting targets for fisheries management and conservation. For example, three different models have been used to support advice on whether meeting MSY targets for fish in the North Sea under CFP [3] would be sufficient to meet a proposed target for the Large Fish Indicator (LFI) under Descriptor 4 of the MSFD [2,38]. It was found that, even though the rationale underlying the two targets is very different, they were indeed compatible with each other within the uncertainty of the combined model data (*Axel Rossberg, pers. comm.*).

3. Challenges for the uptake of ecosystem modelling by policy makers

3.1. Producing the right information from ecosystem models to inform policy

Policy questions are generally formulated much more broadly than scientific hypotheses [7], so there can be a mismatch between policy needs and the specific outputs produced by models. For example, the Defra Marine and Fisheries Evidence Plan [6] has the high level policy goal "*to secure healthy food supplies delivered by a more sustainable fishing industry*" that comprises of many different evidence needs including "*reducing the adverse impact of commercial fishing*". This particular evidence need is subdivided into research needs including "*developing an ecosystem approach to fisheries management through evaluating the impacts of different management scenarios*". To maximise the utility of models, high level policy goals need to be translated into evidence needs and matched against scientific questions that can be addressed using models.

Model outputs also need to be expressed in a form that is meaningful to policy makers. Knowledge of science, evidence, and policy is required to achieve this, so it is important that policy makers work closely with modellers to ensure a common understanding of, and to maximise the benefits from models. For example, policy questions are often framed in terms of socio-economic consequences, but there is often no simple way to express ecosystem model outputs in this way. Modification or development of models to allow assessment of the impact of different management measures on ecosystems in biological, social and economic value will increase the prospects for use (e.g. [39]).

3.2. Confidence in ecosystem model products

Lack of confidence in ecosystem model products may reduce their uptake by decision-makers. In contrast, managers routinely

Table 1
UK ecosystem modelling capability and impact (existing and potential). EM1-EM3 are biogeochemical formulations, EM4–EM7 are food-web formulations. EM8–EM14 are size-based formulations

Name	Description	Impact
EM1 European Regional Seas Ecosystem Model (ERSEM)	ERSEM is a lower-trophic level model designed to represent the biogeochemical cycling of carbon and nutrients (N, P, Si, O_2, Fe) as an emergent property of ecosystem interaction [15,78]. It is coupled to a number of hydrodynamic models for the north-east Atlantic. It has been validated against in situ data (e.g. [79]) and satellite ocean colour. In general predictions are reasonable for temperature, salinity, nutrients, oxygen, nutrients, but less good for chlorophyll and plankton, with predictions becoming less accurate at higher tropic levels [79]. Models capture seasonality well and can predict at spatial scales of order >50 km^2.	ERSEM has been used to assess shelf seas water quality and climate impact, ocean acidification, eutrophication, trophic amplification, and to assess potential climate impacts on harmful algal blooms, fisheries, fisheries economics and food security. For future use, the model is being developed to quantify 'blue carbon', assess nutrient budgets, and simulate changes in ecosystem function and the consequences of such changes in the context of ecosystem services.
EM2 GETM-ERSEM-BFM	This is a coupled hydrodynamic and biogeochemical model that is based on the cycling of carbon and nutrients. It represents phytoplankton, zooplankton, bacteria, macroalgae and filter feeder larvae, and has a coupled benthic system. It is available in a North Sea setup and a north-west European shelf setup that have been validated using chlorophyll, SPM, temperature, and ship-based benthic data [80].	The model has been used to investigate eutrophication and riverine nutrient transport, potential impacts of large-scale macroalgae farms, potential impact of climate change and trawling, ecosystem indicators, deep chlorophyll maximum production, *Phaeocystis* blooms, and potential impact of large-scale wind farms. In future it could be used to attribute causes of change, optimise monitoring programme, assess impacts of wind farms, tidal farms, macroalgae farms, nutrient reduction scenarios, trawling, and thermal plumes, within the context of a changing environment.
EM3 Model of Ecosystem Dynamics, carbon Utilisation, Sequestration and Acidification (MEDUSA)	MEDUSA is intermediate complexity model of lower-trophic level plankton ecosystems that is typically run within a global earth system model context to address the biogeochemical response to anthropogenic driven changes (including ocean acidification) in the oceans [81]. It has been evaluated at the global scale using observational nutrient, chlorophyll and carbon cycle fields. In general, simulations of nutrients, carbon and primary production are reasonable, though less accurate for chlorophyll. MEDUSA was selected from a UK-wide group of models to be the marine biogeochemical component of the UK Earth System Model (UKESM1) that will be used in IPCC AR6 [14].	The model is currently used at a range of resolutions (up to 1/12th-degree) to study global-scale ocean biogeochemistry and marine productivity. It is also used to make future projections of ocean biogeochemistry and acidification at the global-scale. In future, the model will provide regional predictions addressing policy issues relating to vulnerability, resilience, and adaptation to climate change. It will also be used (within UKESM1) across the suite of UK simulations submitted to IPCC AR6.
EM4 Population-Dynamical Matching Model (PDMM)	The PDMM is a simple theoretical ecosystem model that can represent typical temperate marine shelf communities, covering species of all sizes from phytoplankton to large fish. The model constructs complex and population-dynamically stable ecological model communities by mimicking the community assembly process of successive invasion. The model can reproduce size-abundance relations, distributions of species richness, species-size distributions, and key patterns in food-web [25].	The model has been used to understand mechanisms controlling size-abundance relationships, verify the theory of food-web structure, assess the Large Fish Indicator (LFI), and study biodiversity-production relationships for fish. In future, the model could be used to assess the relationship between biodiversity and ecosystem function, and the long-term implications of fisheries management strategies to reach MSY for multiple interacting stocks.
EM5 Strathclyde end-to-end ecosystem model (StrathE2E)	SrathE2E models the dynamics of nitrogen in ecosystem components including detritus, inorganic nitrogen in solution, plankton, benthos, fish, birds and mammals. Key physical, geochemical and biological processes which occur in the sea and seabed sediments are included [82]. Parameters were computationally fitted for a model of the North Sea to minimise the discrepancy between observed and modelled annual cycles and averaged abundances, production rates, and feeding fluxes [82].	StrathE2E has been used to simulate fishery yields in relation to harvesting rates, trophic cascades, sensitivity of MSY to changes in the environment, and implementation of a discard ban. In future, it could be used to assess sensitivity of fisheries to ocean acidification, disaggregate the effects of environment and fishing, compare observed fishery yields and MSY, project cumulative effects of harvesting and environmental change, and the ecological effects of the discard ban measures.
EM6 Ecopath with Ecosim (EwE)	EwE is an ecosystem modelling framework that quantifies food-web and fishery interactions. Biological components and fishing fleets can be described, and information on landings, discards and economics can be included. The 'core' of the model is determined by specifying who eats (or catches) who and how much. Models have been developed for many regions and there is a strong research community with quality standards being established. Models exist for North Sea, Celtic Sea, Western English Channel, Eastern English Channel, English channel, West Coast of Scotland, Deep West Coast of Scotland, Clyde Sea and Irish Sea, some of which have been calibrated against 20–30 years of data.	EwE has been used to evaluate the trade-offs among fishing strategies in relation to sustainable fishing and mixed fisheries, assess relative impact of fisheries and climate, investigate closed area management, evaluate impact of aggregate dredging, model dynamics of gadoid and demersal fish, and assess ecosystem based management. In future it could be used to assess the spatial impacts of fisheries and climate on the structure and function of ecosystems; quantify the performance of different management strategies; and evaluate the benefits of spatial management policies (e.g. MPA) and impacts of pressures (e.g. oil and gas) on ecosystems.
EM7 Atlantis	Atlantis is a modelling framework that contains a biophysical model that tracks nutrient flows and models consumption, production, migration, predation, recruitment, habitat dependency, and mortality. The physical environment is represented by the major geographical and bioregional	Atlantis models are being developed for the North Sea and English Channel that could be used to examine interactions between fisheries, wind farms, MPAs and climate change. The Atlantis framework has been used more extensively in other parts of the world for ecosystem based management (see

Table 1 (*continued*)

Name	Description	Impact
	features, and the biological model components are replicated at each depth. Atlantis also includes a detailed exploitation sub-model that is focused on the dynamics of fishing fleets and can address the impact of pollution, coastal development and broad-scale environmental change, in terms of economics, compliance decisions, and exploratory fishing and other complicated real world concerns such as quota trading.	[12] for a general review).
EM8 Strathclyde spatial population dynamics model (StrathSPACE)	StrathSPACE simulates the spatial and temporal dynamics of a single-species population in terms of birth, death, growth and movement of fish [83]. It has been calibrated by tuning a small number of key parameters to minimise error. In each case, the tuned model has then been compared with other independent data that were not involved in the tuning to check for compatibility.	The model has been used to address hypotheses about the mechanisms governing dynamics of copepods [84], and various fish species including cod [85] and haddock [86]. Model outputs contributed to development of the Cod Recovery Plan in the North Sea [87]. In future, it could be used for blue whiting, copepods, sand eels, and scallops
EM9 Coupled Community Size-Spectrum Model (CCSSM)	CCSSM represents the size and abundance of organisms in two coupled size-structured food chains, one based on predation and supported by primary production, and one based on energy sharing and supported by detritus [16]. Species are not represented explicitly. Predictions of size-spectrum were validated in the North Sea by comparing model predictions with empirical data on the size structure of pelagic predators and benthic detritivores [16].	Applications of this model have included the assessment of fishing impacts on community size structure and abundance in the North Sea [16], the effects of coupling pelagic and benthic food webs on responses to fishing, and prediction of the medium- and long-term effects of climate change on fish production at regional and global scales [18].
EM10 Species Size-Spectrum Model (SSSM)	SSSM is a highly simplified size-based description of the dynamics of marine species, and is unique in the fact that no assumptions about stock-recruitment relationships are made [88]. The SSSM has been shown to reproduce known classical effects at size-spectrum level [88].	After a more comprehensive validation, it could be used to inform policy makers about high-level ecosystem responses to anthropogenic pressures.
EM11 Multispecies size spectrum ecological modelling in R (MIZER)	MIZER was developed to represent the size and abundance of all organisms from zooplankton to large fish predators in a size-structured food web. An R package has been developed for application of the multi-species size spectrum model to a wide range of systems, which also contains documentation on the model equations and processes. The model provides predictions of the abundance of each species at size, and has been validated for the North Sea [17].	The model has been used to assess ecosystem responses to fishing and to determine whether meeting management targets for exploited North Sea populations will be sufficient to meet proposed Marine Strategy Framework Directive targets for biodiversity and food web functioning [17]. This modelling framework is being developed for use in management strategy evaluation and in a risk assessment framework.
EM12 Strathclyde length-structured partial ecosystem model (FishSUMS)	FishSUMS represents the population dynamics of a set of predator and prey species. For each species, the model predicts biomass by length class and includes growth, reproduction, density-dependent mortality, and losses due fishing and predation. The model produces biomass, length distributions, annual recruitment, catch, and landings. The cod-focused North Sea model has been validated against ICES stock assessment biomass, recruitment, and landings, and by comparing length distributions with IBTS survey data [89].	The model has been used to simulate cod yields and MSY in relation to harvesting rates on other species, particularly herring [89], the historical North Sea LFI and its response to changes in fishing, and changes in fish diet and biomass fluxes in the North Sea. In future, the model could be used as a length-based multispecies stock assessment tool, to make comparisons of fishery yields and MSY, to compare top-down and bottom up processes, and effects of alternative discard ban measures.
EM13 Fish community size-resolved model (FCSRM)	FCSRM represents an ecosystem including fish populations resolved by species and body size, fishing mortality, and zooplankton is included as a food source. The model predicts the types of fish communities that might coexist. This is a dynamic ecosystem size-based model [23] and a representation of the North Sea fish community has been calibrated and validated [17].	The model has been used broadly to advance ecological theory and to understand how and why ecosystems respond to fishing pressure. The model will continue to be used to address questions about the ecosystem response to different types of fishing, and competing management measures.
EM14 Length-based Multispecies Analysis by Numerical Simulation for the North Sea (LeMANS)	LeMANS is a size-structured multi-species model of a fish community with a realistic distribution of life-history attributes [19]. This approach differs from other size-based models as it maintains both the species identity and the individual population size structure. The model was validated by using fish community properties, biomass estimates from surveys North Sea, and comparisons with six assessed stocks [90]. An ensemble approach has been implemented that are screened against ICES abundance data to produce models that are consistent with data [57].	The model has been used to assess whether fishing preserves biodiversity [90]. In future it could be used in risk-based decision support including the trade-off between yield and risk of different harvest strategies in a multi-species fish community. Other potential uses include assessment of uncertainty in multi-species reference point estimates, trade-offs in fleet management, multi-species harvest control rule evaluation, and assessment of signal to noise ratios for fish community indicators.

accept results from single-species fish stock assessment models, despite uncertainties. The contrast may exist because stock assessment models are embedded in a well-established process, and there is international political acceptance of their use as the basis of advice, a good understanding of the models, and confidence in the outputs and their interpretation through quality assurance by scientific experts (e.g. ICES). In many cases, expert judgement is required to interpret the range of model outputs and these procedures can appear opaque to policy makers and lack legitimacy. Expert groups are needed that provide impartial advice on the use of ecosystem models, maintain quality standards for models, publish key validation runs, and provide clear output that can be used by decision makers (e.g. ICES Working Group on Multispecies Assessment Methods [40]). The UK Earth System Model 1 project builds on the iMARNET experience [14] to provide a common framework for marine biogeochemistry models to sit within and, as such, provides an example of how a community can be united around a common framework with common standards.

3.3. Visibility and access to ecosystem model products

Models are often developed by the research community to answer scientific questions and are then used by modelling experts to help decision-makers [11,12,41]. For ecosystem models, this process is generally neither robust nor transparent due to the lack of visibility of existing models, difficulties accessing model products, and the absence of documentation of model metadata. This contrasts with the current initiatives on data management and data standards that provide public access to metadata catalogues and databases in order to maximise the use of existing data, and may be due to the volume and complexity of model products. However, this lack of visibility can lead to the false impression that models are not suitable for decision making.

Policy makers have often called for a "*decision support toolbox*" comprising models that can be used interactively to explore different options when negotiating and formulating policies [42,43]. Complex ecosystem models can be impractical in this context, as they generally have long runtimes, require trained operators, and produce 'big data'. It is therefore an important aim to increase transparency, and make model products available through web portals (e.g. Copernicus Prototype Marine Core Service – http://www.myocean.eu/, Marine OPEC - http://www.portaldev/marineopec.eu) and include model products in tools designed for use by evidence and decision-making communities (e.g. EMECO – http://www.emecodata.net/).

3.4. Development of ecosystem models and methods for understanding uncertainty

The quantification of uncertainty has clear importance in policy [44] yet uncertainty assessments are open to a range of interpretations that may lead to a different outcome (e.g. [45]) and communication of uncertainty can have a large impact on the decision making process (e.g. [46]). Being able to communicate uncertainties presupposes knowing what they are and this is no simple matter: ecosystem models are often extremely complex and associated with many different uncertainties. There are many different classifications of uncertainty, but a simple example relevant to marine ecosystem modelling that has been used in the context of climate change climate includes: ontological (related to underlying processes); epistemological (related to observations and model predictions); methodological (related to model structure); and axiological (related to the world view of the research) [47]. Ontological uncertainties are often accounted for through expert knowledge, and epistemological uncertainties are generally incorporated through assessment of model predictions and knowledge of the observing system.

In the context of methodological uncertainty, there are complex sets of challenges surrounding parameterisation, validation, data sets, uncertainty, visualisation, and ecosystem modelling methods that require further development. These challenges are significant, and a contrast to the physical components of earth system models that are based on well-understood physical laws and scalable processes (e.g. global predictions can be downscaled to regional seas), where the focus of development has shifted towards smaller scales, resolution, speed and numerical implementations. There is also a mismatch between the timescales associated with production of advice (weeks to months) and model development required where models do not produce the outputs needed (years to decades). Hence, there is need to anticipate how models might be used in future in order to produce advice on the timescales required.

New statistical methods are needed to analyse uncertainty in ecosystem (multi-)model ensembles that can be presented to decision-makers in order to understand the risk associated with a particular decision. The successful communication of uncertainties to decision-makers is important for transparency and robust decision-making, thus ensuring management efforts are not misplaced [48]. New visualisation methods are therefore needed to build trust and effectively communicate the outputs and associated uncertainty of ecosystem models to decision-makers and would increase the uptake of ecosystem models.

4. Increasing the use of ecosystem models in decision making

The question of how to increase the uptake and use of community and ecosystem models to support marine environmental management in the UK and Europe is addressed in this section. The conclusions are based on discussions which took place at a two day workshop that brought together 55 people from 23 organisations across the UK that included advisors, assessors, biologists, social scientists, economists, modellers, statisticians, policy makers, and funders. To understand how to increase the contribution of the models to support policy, it was important to identify policy needs and match them against models that might support these needs. The outcomes included identification of potential quick wins and gaps in existing ecosystem modelling capability in the context of biological sustainability, social benefits, and economic value.

4.1. Understanding the policy and management drivers that can be addressed using ecosystem modelling

Climate change, biodiversity, and marine evidence needs have been identified by the UK Government [5,6,49,50] and were translated into tractable modelling questions. These were categorised into the following headings: natural variability and monitoring, management measures, ecosystem goods and services, Good Environmental Status (GES) targets under MSFD [2] and pollution, and environmental change and climate adaptation (Table 2). Since it is often unclear how models have and could be used to support policy, examples of the impact of models on policy and management were identified (Table 1). A simple mapping exercise was then used to understand the potential contribution of ecosystem modelling in the policy and management arena through comparing available models against evidence needs. The utility (ranked qualitatively as "High", "Medium", or "Low") and timescale for development (1 year, 5 years, 10 years) of each type of model to deliver policy relevant goals were then used to identify:

- Gaps – new models or long-term development required.

Table 2
Policy questions derived from evidence plans [5,6,49,50] split into 5 topics and reformulated for modellers.

Policy area	Modelling questions
1. Natural variability and monitoring	A. What are the spatial and temporal scales that a particular model can address and do these match the policy requirements? B. How long would it take to quantify the uncertainty of model predictions? C. Can the model distinguish between relative performances of candidate environmental indicators? D. Can the model identify high risk areas? E. Can the model contribute to assessing the potential efficiency gains from redesigning monitoring programmes? F. Does the model have a capacity to blend models and data to get best estimate of state of system e.g. data assimilation, parameter fitting, tuning? G. Can the model be used to inform engineering the ecosystem to reach the state that you require?
2. Management measures	A. What are the expected changes in habitat extent and condition resulting from environmental change for a given network of Marine Protected Areas (MPAs)? B. How effective are given networks of MPAs in achieving their management objectives? C. How will the network of MPAs deliver objectives and outcomes in relation to environmental impacts, ecosystem structure and function? D. What are efficient programmes of measures to achieve Marine Strategy Framework Directive (MSFD) targets? E. Can the effects of changes (pressure and response) be attributed to individual and cumulative effects, and the risk (uncertainty) associated with this? F. What are the management strategies for exploitation of mixed fisheries to achieve Maximum Sustainable Yield (MSY)? G. What are the impacts of landing obligations on MSY objectives through e.g. food web interactions? H. What are the effects of changes in fisheries management on the environment, in particular through food-web effects? I. What is the risk of population decline or regional extinction of valuable, endangered or vulnerable species from CFP reform?
3. Ecosystem goods and services	A. What are the socio-economic impacts of given networks of MPAs? B. What are the costs and benefits of MSFD/Water Framework Directive (WFD)/Marine Spatial Planning (MSP) implementation? C. What are the interactions between different sectors and ecosystem services? D. What are the marginal costs/values of changes in ecosystem services? E. How are different ecosystem functions and services dynamically coupled? F. How are different ecosystem services and benefits coupled in a socio-economic system?
4. Good Environmental Status (GES) target and pollution	A. Can the model contribute to the ecosystem approach through interactions with other models? B. What is the responses of indicators to specific management measures for MSFD descriptors? C. Are there more effective MSFD indicators than those currently proposed/in use? D. What are the impacts of pollutant dispersants in the marine environment, their impacts on marine ecosystems? E. How can effectiveness of pollutant dispersants be maximised? F. What are the effects of pollution on the marine environment? G. What are the interactions between biodiversity (Descriptor 1) and other descriptors of GES Status under MSFD? H. What are the interactions between commercial fish (Descriptor 3) and other descriptors of GES under MSFD? I. What are the interactions between food web structure (Descriptor 4) and other descriptors of GES under MSFD? J. What are the interactions between sea floor integrity (Descriptor 6) and other descriptors of GES under MSFD? K. Are there alternative useful indicators that can be derived from models but not from direct observation?
5. Environmental change and climate adaptation	A. What are the impacts of regional scale climate patterns on ecosystem state (GES), and can these be valued? B. Can a change in environmental status be attributed to a combination of drivers? C. Which aspects of environmental status are sensitive to climate change? D. What are the impacts of non-native species on ecosystem state (GES)? E. What are the impacts of harmful species on human and animal health? F. How are detailed local effects of local pressures captured? G. What are the impacts of ocean acidification on ecosystem state (GES)? H. What are the impacts of changes in shelf seas biogeochemistry on ecosystem state (GES)? I. What is the impact on land/sea transition zone? J. Can the risk or impact from artificially introduced non-native species be modelled? K. What are the impacts of wind farms and other offshore structures?

- Quick wins – short development time and high utility.
- Ensembles – many models and short development times.

A matrix of future ecosystem model impact was developed for the UK (Table 3). This highlighted that there were a number of areas where we have many models that can be quickly developed to address questions (e.g. 3B – *"What are the costs and benefits of MSFD/WFD/Marine Spatial Planning (MSP) implementation?"*), some areas that few models can address (e.g. 5D – *"What are the impacts of non-native species on ecosystem state from changes in the environment or transport opportunity?"*), and some areas where it was difficult to assess if ecosystem models have any potential (e.g. 3F – *"How are different ecosystem services and benefits coupled in a socio-economic system?"*).

4.2. Identifying potential quick wins, ensembles and gaps for ecosystem modelling

The quick wins, potential ensembles and gaps were identified for each theme, with the management measures and ecosystem

Table 3
Scoring of ecosystem models (model names as in Table 1) and their ability to address policy questions (defined in Table 2). Scoring system: 0=not possible, 1= within ten years, 2=within five years, 3=within one year (darker grey indicates higher score), and diagonal hashing is not possible to assess here.

Question	EM1	EM2	EM3	EM4	EM5	EM6	EM7	EM8	EM9	EM10	EM11	EM12	EM13	EM14
1A	3	3	3	3	3	3	2	3	3	3	3	3	3	3
1B	2	2	2	2	3	3	1	3	3	3	3	3	3	3
1C	3	3	3	3	3	3	3	3	3	3	3	3	3	3
1D	3	3	3	1	3	3	2	3	1	1	1	1	1	1
1E	3	3	2		2	2	2	2	2	2	2	2	2	2
1F	3	2	2	0	3	3	3		3	2	3		2	2
1G	3	3	3	3	3	3	3	2	3	3	3	3	3	3
2A	3	3	2	1	1	3	2	1	1	1	1	1	1	1
2B	3	3	2	1	1	3	2	1	1	1	1	1	1	1
2C	3	3	2	1	1	3	2	1	1	1	1	1	1	1
2D	3	3	2	2	3	3	2	2	2	2	3	2	2	3
2E	2	2	2			3	2		3	2	3	2	2	3
2F	0	0	0	3	3	3	2	0	3	3	3	3	3	3
2G	0	0	0		3	3	2	3				3		3
2H	3	3	0	3	3	3	2	0	3	3	3	3	3	3
2I	0	0	0	0	0	2	2	0	0	0	0	2		2
3A	3	3	2	0	0	2	2	3	0	0	0	0	0	0
3B	3	3	2	3	3	3	2	3	3	3	3	3	3	3
3C	3	3	2	3	3	3	2	0	3	3	3	3	3	3
3D	3	3	3	3	3	3	2	0	3	3	3	3	3	3
3E	3	3	3	3	3	3	2	0	3	3	3	3	3	3
3F						3	2							
4A	3	3	3	2	3	3	2	2	3	2	2	2	2	2
4B	3	3	3	3	3	3	2		3	3	3	3	3	3
4C	3	3				3	2		2	3	2	2	2	2
4D	2	2	2			3	1		0	0	0	0	0	0
4E	2	2	2	0	0	0	0		0	0	0	0	0	0
4F	3	3	3	3		3	1		0	0	0	0	0	0
4G	3	3	3	3	3	3	3		1	1	1	1	1	1
4H	0	0	0	3	3	3	2		2	2	3	2	2	2
4I	3	3		3	3	3	3		2	2	2	2	2	2
4J	3	3	0			3	2		0	0	0	0	0	0
4K	3	3	3			3	3							
5A	3	3	2		3	3	2	3	3	2	2	3		2
5B	3	3	2		3	3	2	3	3	2	2	3		2
5C	3	3	2		3	3	2	3	3	2	2	3		2
5D	0	0	0	0	0	3	1	0	0	0	0	0	1	0
5E	2	2	0	0	0	1		2	0	0	0	0	0	0
5F	2	3	1	0	0	3	2	3	0	0	0	0	0	0
5G	3	3	3	0	3	3	3	0	0	0	0	0	0	0
5H	3	3	2	0	3	1	2	0	2	2	2	0	2	0
5I	2	3	1	0	0	3	2	3	0	0	0	0	0	0
5J	0	0	0	2	0	1	1	2	0	0	2	2	0	0
5K	2	3	1	0	0	3	2	3	0	0	0	0	0	0

goods and services themes combined for this purpose (Table 4). A number of policy and management issues can be addressed immediately and are brought together under the following general headings: (1) attribution of change to underlying drivers; (2) integration of models and monitoring to develop more efficient monitoring programmes; (3) assessment of indicators and the interactions between legislatory descriptors; and (4) cost-benefit of legislation (Table 4).

It was clear that multi-model ensembles could be used in some areas (Table 4), but the methods for delivering multi-model ensembles for ecosystems still need to be developed. The general methods for multi-model ensembles exist in the climate area [9], but ecosystem model structures are very diverse (e.g. food-web, size-based, nutrient cycling) making a standardised approach of combining outputs difficult. This is because it is difficult to relate the variables from different models (e.g. relating functional types to size-based groups) and this challenge increases at higher trophic levels. There are programmes underway to develop these methods (e.g. Marine Ecosystem Research Programme – http://www.marine-ecosystems.org.uk/) and includes the creation of a multi-model ensemble that builds on the ideas of Chandler [51]. The outputs are modelled using a hierarchical structure which separates individual and shared model discrepancies. This approach allows models with different outputs to inform one another through correlations and gives estimates of the true output as well as robust measurements of uncertainty. Additionally, it is possible to introduce a level to the hierarchical structure that groups models that have similar discrepancies, e.g. size-based models. Some examples of model intercomparison also exist (e.g. ocean biogeochemistry [14], nutrient transfer [36], fisheries [8]), but more work is required before multi-model ensembles can be used routinely to support policy development and management.

Potential gaps in existing ecosystem modelling capability were also identified including those relating to non-native species, disease transmission, ocean acidification, coastal zone management, marine protected areas, cumulative effects, socio-economics, and pollution and oil spills (Table 4). However, this assessment was done in the context of existing ecosystem modelling capability in the UK, and other methods exist internationally (e.g. MARXAN – http://www.uq.edu.au/marxan/ - for marine protected areas, OSCAR – http://www.sintef.no/home/SINTEF-Materials-and-Chemistry/About-us/Departments/Environmental-Monitoring-and-Modelling/OSCAR–Oil-Spill-Contigency-and-Response/ – for oil spills).

4.3. Developing the link between biological, social, and economic drivers for ecosystem management

Policy questions are often framed in terms of socio-economic value (e.g. Policy Area 3 in Table 2), but few ecosystem models express the outputs in these terms. Moreover, there are significant challenges in valuation of the marine environment and there is often a mismatch between the complexity of biological and economic models. The workshop identified a need to develop methods that use the outputs from ecosystem models to drive the valuation of ecosystem services dynamically.

Ecosystem services are the direct and indirect contributions of ecosystems to human well-being, and are made up of tangible goods (e.g. food and raw materials) and less direct and often more intangible services (e.g. the regulation of our climate and the remediation of waste) [52]. The changes in an ecosystem and how this affects value are important for policy development, with changes in ecosystem services determined from empirical data or using models. Often it is the trade-offs among the different services under alternate policies or management strategies that

Table 4
Potential for use of ecosystem model-derived products in addressing policy needs in terms of quick wins, possible multi-model ensembles (italics), and gaps that cannot currently be addressed.

Theme	Quick Wins	Gaps
Natural variability and monitoring	• *Distinguishing between the sensitivity and utility of different indicators.* • *Quantifying uncertainty.* • Integration of models with monitoring to increase efficiency. • Identifying current system state.	• Improve the ability of models to capture inter-annual variability and long term trends.
Management measures, goods and services	• *Efficient programme of measures for achieving Good Environmental Status (GES).* • Impacts of landing obligations on Maximum Sustainable Yield (MSY) through food webs interactions. • *Management strategies for achieving MSY in a mixed fishery.* • *Effects of fishery management on food webs.* • *Cost-benefit of implementation of legislation (e.g. MSFD, Water Framework Directive – WFD, Common Fisheries Policy-CFP).* • *Marginal costs / values of changes in ecosystem services.* • *Links between ecosystem function and services.*	• Assessing networks of Marine Protected Areas (MPAs) in terms of connectivity, achieving management objectives and socio-economics. • Cumulative effects. • Risk of decline of endangered species from CFP reform. • Coupling between ecosystem services and benefits in socioecological systems.
Good Environmental Status (GES) target and pollution	• *Sensitivity of indicators to management measures and identification of better indicators.* • *Effects of pollution on the marine environment.* • *Interdependencies between MSFD descriptors.*	• Impacts of pollutant dispersants. • Interdependencies between different descriptors within MSFD. • Model interoperability – modular approaches.
Environmental change and climate adaptation	• *Regional scale climate impacts and their value.* • *Attributing change in ecosystems to environmental drivers and the systems response.* • *Impacts of changes in shelf-seas biogeochemistry on ecosystem state, function and services.*	• Introductions and impacts of non-native species. • Animal and human disease. • Local effects of pressures. • Impacts of ocean acidification. • Impacts on the land-sea transition zone. • Impacts of geo-engineering. • Impacts of offshore structures.

determine the economic and social importance. The simplest way to use ecosystem models to help understand the changes in ecosystem services is to develop linkages between changes in ecosystem function and service. This has been done for Dogger Bank where indicators have been developed of changes in ecosystem services and the changes in the underlying ecological function [53].

There are a number of more complex ecosystem service frameworks, with one good example being the UK National Ecosystem Assessment Follow On (UKNEAFO [54]). UKNEAFO describes a set of strategic principles based on the adaptive management approach together with practical tools including models to inform the sustainable management of coastal and marine ecosystem services. A decision support system (DSS) was developed that adapted the Drivers–Pressures–State–Impact–Response (DPSIR) approach to assess changes in ecosystem services and their impact on human well-being, as coastal zones are increasingly affected by environmental change drivers and pressures [55]. This has highlighted key policy issues, and was adapted to include state changes and impacts specifically tailored to ecosystem services and their human welfare effects. Four main marine based scenarios which deviated from a baseline condition were explored and exposed to changes in selected environmental change (e.g. climate, socio-economic development, political, social and cultural drivers). A set of ecosystem change indicators consistent with the implementation of the MSFD were derived covering processes, intermediate and final ecosystem service delivery, in stock and flow terms [56]. The data needed for these indicators were drawn from national level observations and models. Given the uncertainty surrounding ecosystem functioning and the impact on overall biodiversity of some ecosystem changes, a number of modelling approaches were applied and tested. The UKNEAFO assessed formal models to quantify changes in ecosystem service stocks and flows, and in particular the practicality of coupling land use change, estuarine and coastal and marine models.

The incorporation of feedback between biological, social, and economic systems can be difficult in an ecosystem services framework. This is an issue because feedback loops are important for making accurate predictions of the response of systems to management measures and are inherent in the DPSIR approach. Systems dynamics is an alternative approach that is gaining support in environmental economics and is used to model complex non-linear systems including the design and analysis of policy. Current knowledge of how the 'system' functions has been used to develop a number of simple conceptual models that may not always encapsulate the entirety of the system, but include significant components (e.g. key habitats, sub-systems, human uses for fishing or renewable energy). These simple conceptual models can help to define information needs to build more information-rich system models that may be quantitative (stochastic or deterministic) or qualitative (narrative-rich models). These enable exploration of the consequences of current or proposed policy for the delivery of ecosystem services and for maintaining the integrity of the system as a whole, where different models can be employed together and the approach is not prescriptive. Promising 'wide spectrum models' that can work across the natural–social science boundary include extended Ecopath with Ecosim models [31], End-to-End models, and Atlantis [11].

4.4. Methods for analysis and visualisation of model products

The requirement for uncertainty quantification has led to a move from optimising parameter sets that fit observations [17] to finding a range of possible solutions [57,58]. However, standard methods of uncertainty analysis are difficult to conduct due to the complexity of ecosystem models and the computational power required to evaluate them (e.g. Markov Chain Monte Carlo [59]). These problems are not unique to marine ecosystem models and lessons can be learned from other disciplines including: fitting models to observations (e.g. [60]), examining structural uncertainty in decision models (e.g. [61]) and ensemble modelling (e.g. [62])

There is an abundance of scientific literature assessing the methods used to resolve the linguistic uncertainties in communicating model output [63–65], but there is little guidance about visualising the outputs and uncertainty from complex models [66]. Many of the techniques used for data visualisation ignore the presence of uncertainties or are only able to depict one source of uncertainty at a time [67,68]. More recently methods have been developed to depict multiple uncertainties within a single visualisation, although efforts have been hindered by the presence of deep uncertainties and the challenges associated with disentangling various sources of uncertainty [66].

5. Future challenges for ecosystem modelling that encompass natural, social, and economic systems

A clear limitation to the development of policy-relevant ecosystem models is the maturity of the underlying science. The link between biodiversity, ecosystem function and the flow of ecosystem services is being addressed, but is not yet well enough understood or described to fulfil the requirements for management and policy advice [69]. Concepts that are underpinned by strong evidence are regularly questioned (e.g. global warming) and others accepted before the science is fully resolved. For example, biodiversity is often "protected" because of the assumed link between biodiversity and ecosystem function and services (MSFD) although in reality this link is complex, widely debated and often recognised as context dependant [69]. There is no absolute point at which a model is sufficiently advanced to support management and policy advice, as this depends on many political and societal factors as well as the development and presentation of the science. Consequently, clear communications between scientists, modellers, statisticians, managers and policy makers is important to build understanding of the capabilities of models and the associated uncertainties.

Ecosystem functions are believed to be reliant on the organisms that inhabit the ecosystem, but predicting the functionality and how it changes with different pressures is a significant challenge. However, these uncertainties do not prevent the development of models that include biodiversity or functionality based on knowledge of the species assemblages, but this does require understanding of the limitations of scientific knowledge of the drivers of these relationships. The relative uncertainty varies depending on the ecosystem service under consideration; for example, primary production is easier to address than detoxification of xenobiotics, for which we have less specific knowledge. Progress is being made and mapping of biodiversity, habitat type and related functions and service provisions is becoming more common in terrestrial systems [70], with more information on coastal and marine systems emerging. The valuation of service in marine systems is also more problematic since the benefits of marine ecosystem services provision are less tangible than in terrestrial systems and methods of valuation (both monetary and non-monetary valuations) are more difficult to apply [71]. Hence, providing a common (comparable) currency across terrestrial and marine system can be difficult. However, the application of ecosystem models will help to focus on the most urgent issues to be addressed.

Much environmental decision-making assumes smooth cause–effect relationships, but there is increasing evidence of regime shifts at a number of different scales in both tropical and temperate marine ecosystems (e.g. [72–75]). Knowledge of 'tipping points' is empirical and conjectural, so their prediction is a huge challenge. Changes in global circulation will also affect shelf-models and represent another challenge over the next decade (e.g. [76]). Most models have to be constrained within defined spatial and temporal boundaries, and for natural systems focus on, for example, habitats, populations, or ecosystems. Social–ecological systems scales are more complex, partly because people who interact with marine systems live on the land, so operate on different scales to the natural systems they exploit. This scale mismatch presents a further challenge for modelling.

Coupled social–ecological systems suffer from 'locked-in' processes that have a profound effect on the potential options for their management. These factors can be modelled when they are properly understood but many feedback processes have not been identified as yet and can only be suspected from non-linear cause–effect behaviour, making them very difficult to model. All systems have rate limiting steps or choke points that can simplify modelling. Complex social–ecological system modelling has an added dimension, the 'on–off' behaviour of the decision-making process. This provides a challenge for 'stock and flow' models for example. Modelling the factors affecting human decisions is complex and culturally dependent, making predictions using models a significant challenge (e.g. fisher behaviour [77]).

6. Conclusions

These conclusions have been developed from this assessment of UK ecosystem modelling, but some of the challenges and solutions apply internationally. While some countries may at present be more comfortable with deploying ecosystem models to guide management and policy than others (e.g. Australia, USA), there is still a large gulf between modellers and decision-makers, and the full utility of ecosystem models has not yet been realised.

To increase the uptake and use of ecosystem models and better support marine environmental management and policy, it is important to

- Ensure that decision-makers know where and how ecosystem models can be used in the context of the limited resources for evidence generation.
- Build multidisciplinary communities of policy makers, data collectors, modellers, statisticians, and socio-economists that speak a common language and work together to develop, apply, review and compare ecosystem models.
- Define and employ rigorous quality standards to satisfy legal challenge in policy and management decisions that ensure model-derived products are available and robust.
- Put in place programmes to fill existing knowledge gaps that can only be addressed using contributions from models (e.g. linking biological sustainability, social benefits, and economic values, address the challenges of modelling dynamic systems).
- Maximise the pull-through of new modelling techniques to ensure that the latest science is being used to underpin decision-making.
- Encourage the development and use of new statistical methodologies and visualisation techniques, for inference from model ensembles and for the propagation, management and communication of uncertainty in general.

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Modelling the Effect of Fishing Levels on Commercial Fisheries Revenue using Bayesian Belief Networks

Running title: Bayesian Belief Networks for Fisheries

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Abstract

In marine ecosystem management there exist complex dependencies between fishing effort and the yield, price and revenue of important fish species in UK commerical fishing that can obfuscate the decision-making process for policymakers. In this paper we use Bayesian belief networks to investigate the effects of fishing strategies on commercial fishing revenue from the North Sea, by linking together the effects on population dynamics and landings, based on a multi-model ensemble of higher-trophic-level ecosystem models, and the effects on fish prices, based on auto-regressive time series modelling.

Keywords: Bayesian Belief Networks, Commercial Fisheries Revenue, Marine Ecosystem, Multi-model Ensemble, North Sea.

Contents

1 Introduction

In understanding the likely effects of different fishing strategies on commercial revenue, it is important to consider the indirect effects on fish prices as well as the direct consequences for fish populations and hence landings. It is also important to allow for the uncertainty in predictions in an appropriate way, distinguishing between sources of uncertainty that are specific to the fishing scenarios considered and those that are common to all cases.

Here we adopt a novel approach by taking projected landings from an ensemble (Spence et al., 2018) of ecosystem models, obtaining fish prices from an auto-regressive time series model (McPike and Heath, prep) and combining them within a Bayesian Belief Network (BBN; see §1.4). We model the aggregated commercial fisheries revenue from the North Sea areas IV a, b and c for six species of fish, chosen for their high contributions to landings in the region, under three plausible fishing scenarios: the status quo, fishing levels calculated based on the maximum sustainable yield (MSY) for each species separately, and levels based on a concept of multi-species MSY; see §1.1 for details. The total commercial fisheries revenue is the sum of the annual revenues of these six commercially-important species: plaice (*Pleuronectes platessa*), sole (*Solea solea*), mackerel (*Scomber scombrus*), cod (*Gadus Morhua*), herring (*Clupea harengus*) and haddock (*Melanogrammus aeglefinus*). The principal aim of this paper is to estimate the distribution of the difference in fisheries revenue between the three fishing effort scenarios.

We show that both yields and revenues can vary substantially between the two concepts of MSY, that the effects of the two MSY scenarios can differ greatly across the commercial species considered, and that total revenue under the multi-species MSY is around 40% greater than under status quo or single-species MSY.

The outline of the paper is as follows: in Section 1.1 the ensemble of models used to forecast commercial landings is discussed and the datasets used to inform the average annual price model are described in Section 1.3. An introduction to the idea of a BBN is given in Section 1.4. The specific BBN that we use to model commercial fisheries revenue is presented in Section 2 and the annual fish price regression model is described in Section 2.1. Finally, the results for the distributions of revenue (both at species level and in total) are shown in Section 3.

1.1 Fishing Level Scenarios

We investigate three future fishing scenarios: status quo (SQ), maximum sustainable yield (MSY), and Nash equilibrium (Nash) which is a particular concept of a multi-species MSY. Let F_i be the fishing level from 2014 for the ith species. The fishing mortality for the status quo scenario is simply the mortality from 2013, so

$$F_i^{(SQ)} = F_{i,2013}$$

where $F_{i.2013}$ is the fishing mortality of the ith species in 2013. This was taken from ICES stock-assessments ICES (2015). For each species, the fishing mortality for the maximum sustainable yield is

$$F_i^{(MSY)} = \underset{F_i}{\text{argmax}}(g_i(F.)),$$

that is, the value of F_i that maximizes $g_i(F.)$, where $F.$ is the fishing mortality for all stocks and $g_i(F.)$ is the long term yield for the ith stock. This is taken from ICES stock-assessments (ICES, 2015) and is calculated with $g_i(\cdot)$ using single-species models, which means that $g_i(F.^{(MSY)}) = g_i(F_i^{(MSY)}, F_{-i})$ for all F_{-i}, where $-i$ represents all other species except i. This is the F_i that leads to the maximum yield for the ith stock with out taking account of any of the other stocks. The Nash equilibrium (Nash, 1951), denoted by $F.^{(Nash)}$, is such that a unilateral change in any F_i would not lead to a larger yield for species i, $f_i(F.)$, i.e. for all

$$f_i(F_i^{(Nash)}, F_{-i}^{(Nash)}) \geq f_i(F_i, F_{-i}^{(Nash)}) \text{ for all } i, F_i.$$

The fishing mortality values for the Nash equilibrium were taken from Thorpe et al. (2017).

The historic values for landings used, and the landings estimated from the ensemble, are the total EU landings, not just the UK landings. As discussed later in Section 1.3, the pricing data is an average over UK ports and therefore we have a mismatch between the landings data and the pricing data. To overcome this problem we assume that the UK:EU landings ratios for the year 2013 (taken from Table 3.12 in Marine Management Organisation (2014)) will remain constant over the projected range 2014–2050 to estimate the UK average annual landings from the ensemble model output. The ratios of UK:EU landings for the six species of commercial fish for the year 2013 are given in Table 1.

Fish	**2013 EU Landings (tonnes)**	**2013 UK Landings (tonnes)**	**UK:EU Landings 2013**
Haddock	35342	32945	0.932
Cod	21941	12770	0.582
Sole	13203	858	0.065
Plaice	77988	19272	0.247
Herring	277008	58951	0.213
Mackerel	25254	1342	0.0531

Table 1: Table showing the total EU and UK landings for the year 2013 from the UK Fisheries Statistics Report 2013.

1.2 The Multi-Model Ensemble

Rather than basing our projections on a single simulation model of the marine ecosystem, we use a range of different models. We then analyze these results statistically to make inferences about the 'model consensus' and the discrepancies between different models and between the consensus and reality, in a Bayesian multi-model ensemble approach; see Spence et al. (2018) for details, and Gelman et al. (2013) for a general introduction to Bayesian statistical concepts and computation. This produces a joint posterior distribution for the annual landings for the range 2014–2050 for each of the six fish species, conditional on data from 1967-2013 as detailed below.

Each of the three scenarios were simulated with four multi-species models: Ecopath with Ecosim (EwE) (Lynam and Mackinson, 2015), mizer (Blanchard et al., 2014), FishSUMs (Speirs et al., 2010) and LeMans (Thorpe et al., 2015). Table 2 shows the different multi-species models and which species they are explicitly able to project. The model projections

Table 2: A table describing which model is able to explicitly project which species. Models that are able to project a species have a tick in the corresponding cell; those that are unable to explicitly project it do not.

Model	Cod	Plaice	Sole	Mackerel	Haddock	Herring
EcoPath with EcoSim	✓	✓	✓	✓	✓	✓
mizer	✓	✓	✓		✓	✓
FishSUMs	✓	✓			✓	✓
LeMans	✓	✓	✓	✓	✓	✓

were combined using the ensemble model of Spence et al. (2018). The parameter uncertainty for each of the multi-species models was taken from Mackinson et al. (2018) for EwE, Spence et al. (2016) for mizer, Spence et al. (2018) for FishSUMs and Thorpe et al. (2015) for LeMans. The prior distribution for the parameters for the ensemble model was elicited from one of the authors, MAS, using the methods described in Spence et al. (2018). The ensemble model was fitted to official landings from 1967 until 2013 for ICES area iv, using the No U-turn Hamiltonian Monte Carlo (Hoffman and Gelman, 2014) in the package Stan (Gelman et al., 2015).

Because this Monte Carlo sampling approach is used, the results from the ensemble analysis consist of a large number of draws from the posterior distribution of the projected landings by species. The ensemble uses common parameter values for the three fishing scenarios, which means that for each scenario the output values are correlated. Therefore, when comparing the revenues for each scenario it is necessary to use only the output from the same ensemble draw. In this way, the analysis correctly allows for components of variability that are shared between scenarios.

1.3 Datasets

The historic landing yields for the six fish species for the years 1967–2013 are available from Marine Management Organisation (2014) and corresponding reports in earlier years. The average annual price data for the six fish species was also obtained from the annual UK Sea Fisheries Statistics, with average annual prices for the years 1967–2013 indexed to 2013 using the UK annual average Consumer Price Index (CPI) as compiled by the UK Office of National Statistics (ONS); see ONS (2018b). The Commercial Landings data for the UK was taken from the ICES FAO database for each individual species and is available for 1967–2013.

To estimate the average annual fish price for the forecasted year, a regression model is fitted for each species, with covariates including the landing data and a number of related UK commercial food statistics. The additional UK food statistics in the regression model are the total annual UK import and export landings and revenue of finfish and shellfish, UK population, UK median household income, annual UK meat supply and UK chicken price. The average annual import and export price and quantity of finfish and shellfish were obtained from Marine Management Organisation (2014); the individual species information is not readily available. The UK median household income from ONS (2018a), annual UK meat supply and UK chicken price from ONS (2018c) (all adjusted for inflation using the 2013 CPI).

1.4 Background to Bayesian Belief Networks

A Bayesian Belief Network (BBN) is defined by a set of nodes and a set of directed edges that connect the nodes; for an overview see Scutari and Denis (2014). The nodes, depicted as labelled circles, represent random variables and the directed edges, depicted as arrows between two nodes, represent direct dependence among variables. Nodes that are not linked by arrows are not necessarily independent, but they are *conditionally* independent given the values of the other nodes. Note that although the arrows represent direct causal relationships between variables the reasoning process can be applied to BBNs in either direction. Within a Bayesian statistical framework, the random variables represented may be quantities that are imperfectly known through any combination of model or parameter uncertainty or inherent stochasticity. BBNs allow the user to produce a graphical representation of the dependence between variables and a method of computation of the joint probability distribution of the nodes. They also allow one to incorporate prior expert beliefs and empirical observations where available.

In addition to the graphical structure, it is necessary to specify the parameters of the model which describes the prior or conditional probability at a node. After describing the nodes and the conditional dependence between them, the aim is to make inference about nodes of interest which can be achieved by 'integrating out' any irrelevant variables.

To calculate the posterior distribution of the random variables at each node we use the statistical software *R* (R Core Team, 2015), and the package *rjags* (Plummer, 2016), to run a Gibbs sampling algorithm. Using *R* gives access to a wide range of statistical and graphical tools, and enables integration with other parts of the modelling e.g. the time series modelling of prices. An additional advantage over dedicated software such as *Netica* is that it allows nodes to be continuous random variables without having to discretise the variable's domain (i.e. without breaking up the domain into finite sections to treat it as a discrete random variable).

2 Network for Marine Revenue

The Bayesian network described in this section aims to capture the dependency between the three fishing scenarios and the commercial fisheries revenue of the six fish species. The fisheries revenue is driven by the price and total landings of the key fish species, which are dependent on the choice of fishing scenario. The Bayesian network for the commercial fisheries is shown in Fig 1. The arrows indicate the direction of dependence, e.g. Cod_Landings is a 'child' node of, and hence is conditional on, the node Scenario.

The top node in the network, Scenario, is a categorical random variable that selects one of the three sets of ensemble model outputs which represent the three fishing scenarios. The fishing scenarios directly affect the distribution of each species' projected landings. For each scenario, the ensemble model consists of 4000 Markov Chain Monte Carlo (MCMC) draws from the posterior distribution of the annual average landings at every year in the forecast range 2014-2050 given the set of fishing effort parameters that define each scenario.

As discussed in Section 1.1, the ensemble model runs for each scenario use a common set of parameters which means that we should only compare the landings and revenue between scenarios from the same draw from the ensemble model outputs. To analyze the fisheries network the first step is to sample with replacement from the integers $1, \ldots, 4000$ to choose the ensemble draw, and then for each scenario the revenue is calculated using the network of Fig 1. The landings value for each species, given the ensemble draw number and the scenario, is now deterministic.

The next level of nodes are the random variables modelling the prices of the six fish species, where each node represents a regression model that calculates the price of that species as a function of its parent nodes, the UK Import Export data and the landings distribution. The regression models for estimating the average annual fish price are discussed in Section 2.1. The final node, the fisheries revenue, is calculated by multiplying the output from the landings node (which is in tonnes per km^2) by the relevant price node output and the total area of the North Sea IV division, and aggregating across species.

2.1 Ex-Vessel Fish Price Model

To calculate the fisheries revenue from the ensemble model landings we need to estimate the average annual price (in £ per tonne) of each of the six fish species for the forecast year given the historical data. Our approach closely follows that of McPike and Heath (prep). They found that historical price data was non-stationary for each species, and hence used a regression model for the year-on-year change in price. Let $\Delta X = X_t - X_{t-1}$ be the change in X in one year, then their regression model for the change in price for one of the species is

$$\Delta \log \text{Price} = \beta_0 + \sum_{j=1}^{n_{\text{species}}} \beta_{1,j} \Delta \log X_{j,\text{Landings}} + \sum_{k=1}^{4} \beta_{2,k} \Delta \log X_{k,\text{Import/Export}} + \sum_{l=1}^{4} \beta_{4,l} \Delta X_{l,\text{UK Stat}} \tag{1}$$

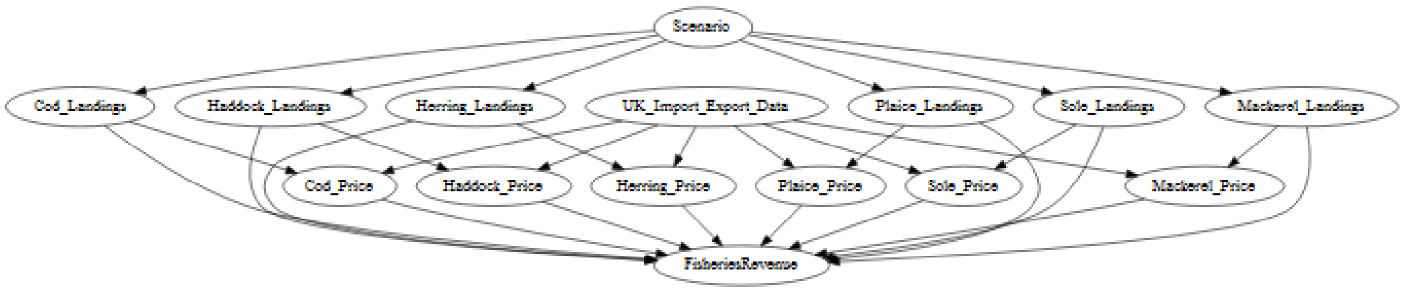

Figure 1: Figure showing the network for fisheries revenue and its dependency on fishing level.

where the landings covariates, $X_{j,\text{Landings}}$, are the average annual landings for each fish species in tonnes per km^2, the four Import Export data covariates, $X_{k,\text{Import/Export}}$, are the the UK Import and Export Total Quantity and Total Value for the aggregated finfish and shellfish imported and exported by the UK and the four UK Statistics covariates, $X_{l,\text{UK Stat}}$, are UK statistics that may be related to average annual fish prices: UK population, UK Chicken Prices, UK Meat Supply and UK Median Household Income.

For the current work, the regression models for the year-on-year change in average annual price are re-fitted with $n_{\text{species}} = 6$, using the six species discussed in Section 1. For variable selection, we using an elastic net procedure (Zou and Hastie, 2005), which identifies either a lasso model penalty function, ridge regression penalty function or a combination of both as the most appropriate model for each of the six species. The model coefficient estimates for each model are given in Appendix A.1. To use the regression models to calculate the change in average annual fish price, we need to estimate the covariate values for the forecast year. The landing values are taken from the ensemble model for the North Sea, and held constant for other regions, so that effectively a 'status quo' scenario is being used except in the North Sea. The other covariates are estimated by fitting auto-regressive integrated moving average (ARIMA) time series models; for more details see Appendix A.1.

3 Results

In this section the results from the network in Fig 1 are presented. The principal aim here is to quantify the relationship between fishing effort and landings and revenue and the plots in this section seek to illustrate this relationship. The plots show the distribution of the relative revenue of both individual species and aggregated revenue for MSY and Nash against SQ.

It is not necessarily the case that increased landings will yield increased revenue, because species price is dependent on the landings; see Table 3 for the average annual price model coefficients. The difference in the revenue of the three fishing effort scenarios can be seen in Fig 2–7. The plots show the relative revenue between MSY and Nash with SQ scenarios, per km^2 of the North Sea IV division. The combined revenue plot for the six species of commercial fish is given in Fig 8.

For a given species (or the aggregated total) the relative revenue plots are the posterior distributions for the relative revenue nodes in the BBN. The relative revenue node is calculated for the MSY and Nash scenarios by calculating the ratio of its revenue to that of the SQ scenario from same ensemble model draw. The relative revenues are plotted on a logarithmic scale.

The plots that answer the motivating question of this work are shown in Fig 2–8, which compare the relative revenue of MSY and Nash against SQ fishing scenarios. Whilst there are some exceptions for the individual fish species revenue plots, broadly the conclusion is that Nash produces a higher revenue than the SQ scenario whereas MSY returns a similar revenue. This conclusion can be seen most clearly in the aggregated revenue plot, which

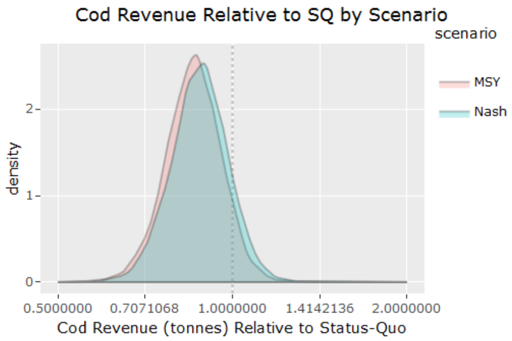

Figure 2: The relative revenue of MSY and Nash against SQ scenarios for cod landings per km^2. The x-axis has been transformed on the log-scale and the dashed vertical line at $x = 1$ represents parity in terms of revenue.

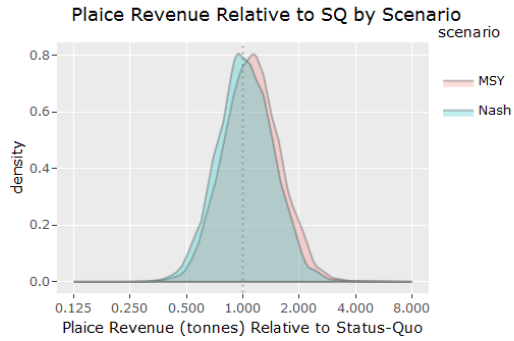

Figure 3: The relative revenue of MSY and Nash against SQ scenarios for plaice landings per km^2. The x-axis has been transformed on the log-scale and the dashed vertical line at $x = 1$ represents parity in terms of revenue.

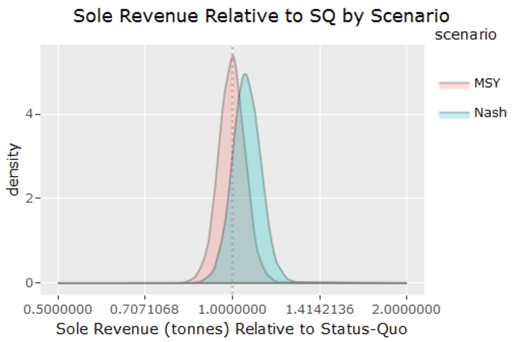

Figure 4: The relative revenue of MSY and Nash against SQ scenarios for sole landings per km^2. The x-axis has been transformed on the log-scale and the dashed vertical line at $x = 1$ represents parity in terms of revenue.

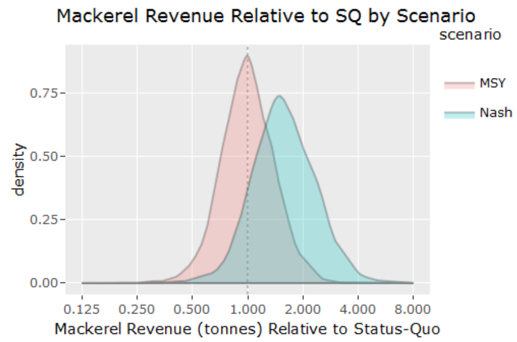

Figure 5: The relative revenue of MSY and Nash against SQ scenarios for mackerel landings per km^2. The x-axis has been transformed on the log-scale and the dashed vertical line at $x = 1$ represents parity in terms of revenue.

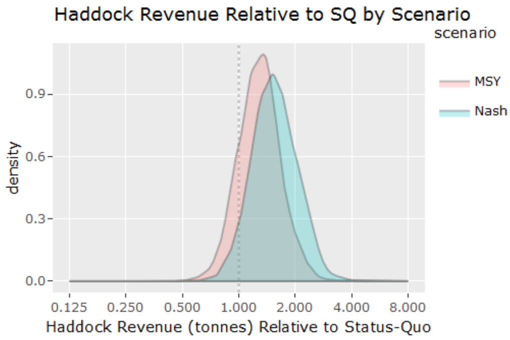

Figure 6: The relative revenue of MSY and Nash against SQ scenarios for haddock landings per km^2. The x-axis has been transformed on the log-scale and the dashed vertical line at $x = 1$ represents parity in terms of revenue.

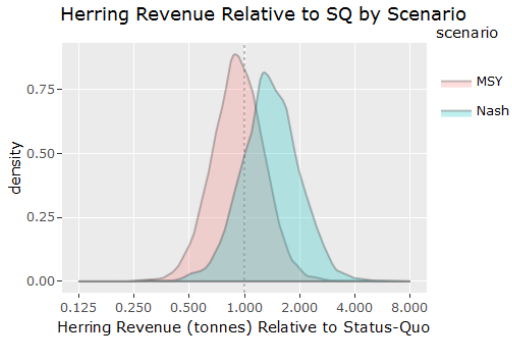

Figure 7: The relative revenue of MSY and Nash against SQ scenarios for herring landings per km^2. The x-axis has been transformed on the log-scale and the dashed vertical line at $x = 1$ represents parity in terms of revenue.

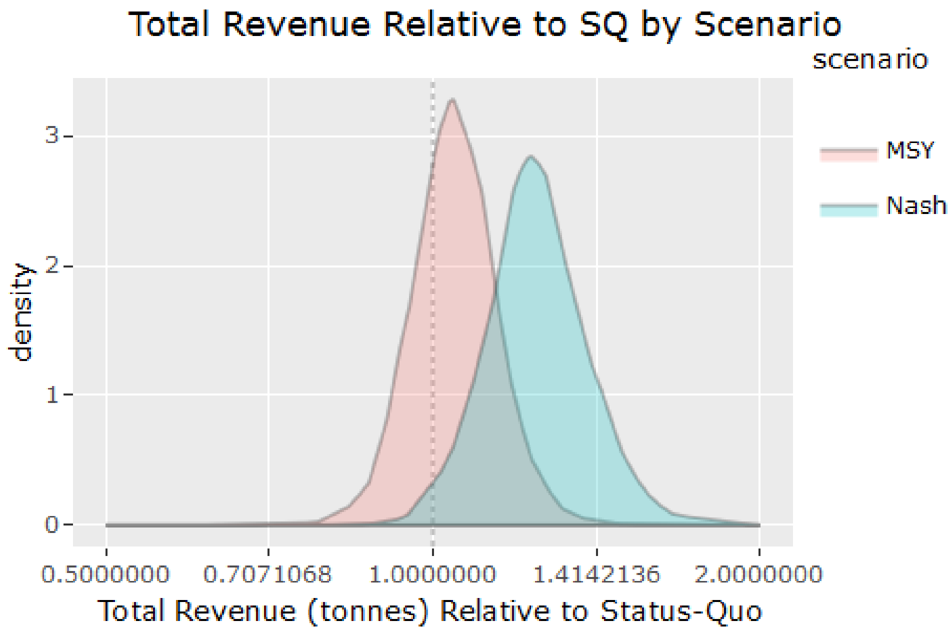

Figure 8: The relative revenue of MSY and Nash against SQ scenarios for aggregated fisheries landings per km^2. The x-axis has been transformed on the log-scale and the dashed vertical line at $x = 1$ is for reference.

displays exactly this intepretation.

4 Discussion

We have shown the feasibility of integrating ecological and social data and modelling to demonstrate both the projected financial consequences of scenarios for fisheries management and the uncertainty attached to those projections.

In contrast with existing approaches, we have synthesised the biological results from an ensemble of separate ecosystem models, rather then relying on the correctness of any single model, so that our projected yields allow for the limitations in our understanding of the relationship between management strategies and yields. In translating from these biological projections to financial consequences, we have used a statistical modelling approach to capture the responses of prices to changes in yields.

To link together these modelling elements, we use a Bayesian Belief Network, a structure which can be extended readily to incorporate other information—whether empirical data or results of modelling—as well as any other sources of uncertainty or additional policy options. As well as looking at further species and scenarios, a natural step would be to consider the effect of varying the percentage of landings that come to the UK.

The particular scenarios that we have modelled cover two key aspects of fishing management: not only the distinction between the status quo and a more sustainable approach, but also the importance of specifying exactly how we formalise the idea of sustainability. We show that the choice between the usual species-level maximum sustainable yield and a multi-species version can be crucial for the biological and economic consequences. The multi-species Nash equilibrium strategy can give an increase of revenue of around 23% compared with the status quo, whereas the corresponding figure for single species MSY is only around 4%. There is considerable uncertainy in both those figures, however, highlighting the importance of improving our understanding of both ecosystem dynamics and the relevant consumer markets.

Exit from the EU and the Common Fisheries Policy presents the UK with the opportunity to change its approach to fisheries management. Both the Fisheries Bill White Paper (Department for the Environment Food and Rural Affairs, 2018b) and the 25 Year Environment Plan (Department for the Environment Food and Rural Affairs, 2018a) reiterate aspirations to take forward an ecosystem approach for fisheries, with sustainable management to ensure healthy fish stocks and a prosperous fishing industry. The White Paper further maintains the commitment to MSY as the basis for fisheries management, while acknowledging that catch rates in mixed fisheries must take account of the interactions between harvested species. The requirement for a responsive regulation that reflects the dynamic nature of the industry is also noted (Department for the Environment Food and Rural Affairs, 2018b).

The value of fisheries is now subject to wider scrutiny, with the establishment of national natural capital accounts within which wild capture fish is currently the only marine ecosystem service valued (Office for National Statistics, 2018d).

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Data availability statement

The model ensemble outputs used are available at the repository (URL to be confirmed). All other data are available through the references cited.

Conflicts of interest

The authors declare no conflicts of interest

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A Appendix

A.1 Price Regression Models

To calculate the ex-vessel average annual UK price of the six fish species, regression models are fitted to the average annual UK price of each fish species and a group of covariates that include the historical annual UK landings data, as well as UK import and Export data and other relevant UK statistics for the period 1965–2013. A constant fisheries price is often assumed in marine revenue literature; however several studies have suggested a number of factors may affect fish price in many markets. We consider the annual fishery landings into the UK, and annual average UK ex-vessel prices which forms a sufficiently large market to determine if a relationship exists between them. In addition, fish import and export into/from the UK, UK population, UK median household income, UK meat supply and annual average chicken price (an alternative meat product) are included as regressors. With the landings for each of the six species included in the regression model, there are 18 covariates which could lead to overfitting hence a regularised regression method is used. An elastic net procedure is used because it overcomes the limitations of the lasso regression method by combining the penalisation terms of the lasso and ridge regression.

In the preliminary data analysis, the time series data for each individual species average annual price is shown to be difference-stationary and hence we use the difference in the log of price data as the dependent variable in the regression model, i.e. the change in the year-on-year average annual prices on the log scale. The full model is given in (2), note that we denote Δ as the difference operator, i.e. $\Delta x_t = \Delta x_t - \Delta x_0$,

$$\Delta \ln p_t^k = \beta_0^k + \sum_{j=1}^{6} \beta_{\text{land}}^{kj} \Delta \ln f_t^j + \sum_{j=1}^{m_s} \beta_{\text{s}}^{kj} \Delta \ln s_t^j + \sum_{j=1}^{m_d} \beta_{\text{d}}^{kj} \Delta \ln d_t^j + \epsilon_t^k, \quad k = 1, \dots, 6. \quad (2)$$

Note, p_t^k is the average annual price of species k in year t, f_t^k is the total landings of species k in year t, s_t^j is the j^{th} of m_s drivers of supply in year t and d_t^j is the j^{th} of m_d drivers of demand in year t. The coefficients to be estimated are denoted β, with the appropriate sub- and super-script, and ϵ_t^k are iid normally distributed errors. The drivers of supply are the import and export price and quantity for the UK and the drivers of demand are population, median household income, UK meat supply and chicken price. The coefficient estimates in the best-fitting models for each of the six fish species, computed using elastic net, are given in Table 3.

A.2 Time Series Model for External Factors

To estimate the annual average fish price using the regression models identified in Table 3, it is necessary to know the landings data for 2013 and the forecast year, 2014, and the other covariates (Import and Export data and UK statistics) values in 2013 and 2014. As discussed in Section 1.1 the ensemble model is used to get the projected landings data, the UK Import and Export Data and UK statistics data for 2014 is estimated using Autoregressive Integrated Moving Average (ARIMA) time series models which are given in this section.

Variable	Change in log annual average species price Cod	Sole	Plaice	Herring	Haddock	Mackerel
Intercept	0.0118	-0.00162	-0.0205	-0.00274	-0.00285	-0.00465
Cod Yield	-0.00335	0	0	0	0	0
Sole Yield	-0.000187	-0.073	0	0	0	-0.091
Plaice Yield	0.00012	0	0	0	0	0
Herring Yield	-0.000184	0	0	0	0	0
Haddock Yield	0.00024	0	0	0	0	0
Mackerel Yield	0.000115	0	0	0	0	0
Fish Import Val.	0.00102	0.0424	0	0	0	0.124
Fish Export Val.	0.000112	0	0	0	0	0
Fish Import Quant.	-0.000061	0	0	0	0	0
Fish Export Quant.	0.000308	0	0	0	0	0
Shellfish Import Val.	0.000705	0.0299	0.0331	0	0	0
Shellfish Export Val.	0.000771	0	0	0	0	0
Shellfish Import Quant.	0.000844	0	0	0	0	0
Shellfish Export Quant.	0.000606	0	0	0	0	0
Population (UK)	0.00522	0	0.192	0	0	0
Median Household Income	-0.000331	0	0.192	0	0	0
UK Meat Supply	0.00194	0	0	0	0	0.578
Chicken Price	0.00164	0	0.061	0	0	0.254

Table 3: Table showing the regression model coefficients for the year-on-year change in price for a tonne of each species.

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